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Some aspects of radial flow between parallel disks

Higgins, Brian Gavin

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APPENDIX B

REVIEW OF MODERN ASYMPTOTIC EXPANSIONS

Order Notation

Two types of order symbols will be used

$$f(\varepsilon) = O[g(\varepsilon)] \text{ as } \varepsilon \to 0$$

if there exists a positive number A independent of ε and $\varepsilon_0 > 0$ such that

$$|f(\varepsilon)| \leq A\,|g(\varepsilon)| \quad \text{for all } |\varepsilon| \leq \varepsilon_0 \tag{B-1}$$

This condition can be replaced by

$$\lim_{\varepsilon \to 0} \left|\frac{f(\varepsilon)}{g(\varepsilon)}\right| < \infty \tag{B-2}$$

$f(\varepsilon) = o[g(\varepsilon)]$ as $\varepsilon \to 0$ if for every positive number δ, independent of ε there exists an ε_0 such that

$$|f(\varepsilon)| \leq \delta\,|g(\varepsilon)| \quad \text{for all } |\varepsilon| \leq \varepsilon_0 \tag{B-3}$$

This condition can also be replaced by

$$\lim_{\varepsilon \to 0} \left|\frac{f(\varepsilon)}{g(\varepsilon)}\right| = 0 \tag{B-4}$$

Asymptotic Series

In general given a series

$$\sum_{m=0}^{\infty} \frac{a_m}{x^m} \text{ where } a_m \text{ is independent of } x$$

we say that the series is asymptotic and write

$$y \sim \sum_{m=0}^{\infty} \frac{a_m}{x^m} \text{ as } |x| \to \infty \tag{B-5}$$

if and only if

$$y = \sum_{m=0}^{N} \frac{a_m}{x^m} + o[|x|^{-N}] \quad \text{as} \quad x \to \infty \tag{B-6}$$

This condition may be written as

$$y = \sum_{m=0}^{N-1} \frac{a_m}{x^m} + O[|x|^{-N}] \quad \text{as} \quad x \to \infty \tag{B-7}$$

Asymptotic Sequences

A sequence is called asymptotic if

$$\delta_n(\varepsilon) = o[\delta_{n-1}(\varepsilon)] \quad \text{as} \quad \varepsilon \to 0 \tag{B-8}$$

where $\delta_n(\varepsilon)$ is a sequence of functions.

Asymptotic Expansions

An asymptotic expansion is given by

$$y \sim \sum_{m=0}^{\infty} a_m \delta_m(\varepsilon) \quad \text{as} \quad \varepsilon \to 0 \tag{B-9}$$

if and only if

$$y = \sum_{m=0}^{N-1} a_m \delta_m(\varepsilon) + O[\delta_n(\varepsilon)] \quad \text{as} \quad \varepsilon \to 0 \tag{B-10}$$

where $\delta_m(\varepsilon)$ is an asymptotic sequence and the a_m are independent of ε. Clearly an asymptotic series is a special case of an asymptotic expansion.

Non-uniform Expansions

In a parameter perturbation the quantities to be expanded can be functions of one or more variables besides the perturbation parameter. If we develop the asymptotic expansion of a function $f(X,\varepsilon)$ where X is in general a vector (but can be scalar) independent of ε in terms of the asymptotic sequence $\delta_m(\varepsilon)$ we have

$$f(X,\varepsilon) \sim \sum_{m=0}^{\infty} a_m(X)\delta_m(\varepsilon) \quad \text{as} \quad \varepsilon \to 0 \tag{B-11}$$

where the coefficients $a_m(X)$ are functions of X only. This expansion is said to be uniformly valid if

if and only if

$$y = \sum_{m=0}^{N} \frac{a_m}{x^m} + o[|x|^{-N}] \quad \text{as} \quad x \to \infty \tag{B-6}$$

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where the coefficients $a_m(X)$ are functions of X only. This expansion is said to be uniformly valid if

$$f(X,\varepsilon) = \sum_{m=0}^{N-1} a_m(X)\delta_m(\varepsilon) + R_N(X,\varepsilon) \qquad \text{(B-12)}$$

where $R_N(X,\varepsilon) = O[\delta_N(\varepsilon)]$ uniformly for all X of interest. Otherwise the expansion is said to be non-uniform or singular. For the uniform condition to hold $a_m(X)\delta_m(\varepsilon)$ must be small compared to the preceding term $a_{m-1}(X)\delta_{m-1}(\varepsilon)$ for each m. Since $\delta_m(\varepsilon) = o[\delta_{m-1}(\varepsilon)]$ as $\varepsilon \to 0$ one requires that $a_m(X)$ be no more singular than $a_{m-1}(X)$ for all values of X of interest if the expansion is to be uniform.

Limit Processes

In perturbation problems one defines a limit process as follows: let $f(X,\varepsilon)$ be a function of position and some parameter ε where X is an abbreviation for the co-ordinates ξ, η.

In this limit process denoted by $\lim_1$, X is held fixed while $\varepsilon \to 0$. The resulting limit function is then denoted by

$$F_1(X) = \lim_1 f(X,\varepsilon) = \lim_{\varepsilon\to 0} f(X,\varepsilon), \quad X \text{ fixed} \qquad \text{(B-13)}$$

Now by the repeated application of this limit process with a suitably chosen sequence $\delta_m(\varepsilon)$ one obtains the following expansion

$$F \sim \sum_{m=0}^{\infty} a_m(X)\delta_m(\varepsilon) \qquad \text{(B-14)}$$

This is identical to the Poincaré type expansion for some small parameter.

Now if this expansion is singular one can then define another limit process which will generate a new expansion valid in the region of non-uniformity. In the second limit process the point X will not be fixed but will move along a curve $\xi = \text{const}$ in such a way that the ratio

$$\bar{\eta} = \frac{\eta}{g(\varepsilon)} \quad \text{is fixed.}$$

The second limit process can be defined as follows: let $F(X,\varepsilon)$, as before, be a physical quantity depending on position and ε. The limiting function approached in the second limit process will be denoted by $F_2(\xi,\bar{\eta})$ or $\lim_2 F$ and defined by

$$F_2(\xi,\bar{\eta}) = \lim_2 F = \lim_{\varepsilon\to 0} F(\xi,\bar{\eta},\varepsilon) \quad \xi,\bar{\eta} \text{ fixed} \qquad \text{(B-15)}$$

Note that the first limit of F depends on F only while the second depends also on the choice of co-ordinates (see Kaplun (1967) for an exposition on optimal co-ordinate systems). A repeated application of the second limit process gives an expansion of the form

$$F \sim \sum_{m=0}^{\infty} \delta'_m(\varepsilon) B_m(\xi, \bar{\eta}) \qquad \text{(B-16)}$$

Usually in most problems it is necessary only to define two-limit processes, but this need not be the case (see van Dyke (1964)).

An expansion cannot be expected to satisfy conditions that are imposed in the region where the expansion is singular. Hence insufficient boundary conditions are generally available for the expansion. The missing conditions are obtained by matching the expansions. The possibility of matching the expansions rests on the existence of an overlap domain where any two expansions are valid (see Kaplun (1967)).

According to van Dyke (1964) it is only possible to match a parameter perturbation expansion that is singular in the co-ordinates or a co-ordinate expansion that is non-uniform in other co-ordinates. One cannot match two different parameter perturbation expansions, i.e. for large and small values of the Reynolds number. This also applies to co-ordinate type expansions.

To match the expansions one can either use Kaplun's (1967) method or van Dyke's (1964) method. They are essentially both the same, but van Dyke's method is much simpler to use.

To match two expansions the matching principle according to van Dyke is as follows:

> the m-term inner expansion of *the N-term outer expansion*
> = the n-term outer expansion of *the M-term inner expansion*. (B-17)

APPENDIX C

MATCHING PROCEDURES

a) Asymptotic Matching of the Boundary Layer Expansion with the Upstream Expansion

1-Term upstream expansion $\hat{\phi}_{\hat{z}} \sim \hat{\phi}_{o\hat{z}}(\hat{r},\hat{z})$

Rewritten in terms of boundary layer variables and expanded for large Re $= \hat{\phi}_{o\hat{z}}(\hat{r},1) + \ldots$

1-Term boundary layer expansion $\hat{\phi}_{\hat{z}} \sim \Phi_{ox}(\hat{r},x)$

Rewritten in terms of upstream variables and expanded for large Re $= \Phi_{ox}(\hat{r},\infty)$

$$\therefore \quad \Phi_{ox}(\hat{r},\infty) = \hat{\phi}_{o\hat{z}}(\hat{r},1)$$

Similarly, to determine the boundary condition (3.22)

2-Term upstream expansion $\hat{\phi} \sim \hat{\phi}_o(\hat{r},z) + \frac{1}{Re^{\frac{1}{2}}}\hat{\phi}_1(\hat{r},\hat{z})$

Rewritten in terms of boundary layer variables and expanded for large Re $= \hat{\phi}_o(\hat{r},1) + Re^{\frac{1}{2}}\hat{\phi}_1(\hat{r},1)$

1-Term boundary layer expansion $\hat{\phi} \sim \frac{1}{Re^{\frac{1}{2}}}\Phi_o(\hat{r},x)$

Rewritten in terms of upstream variables and expanded for large Re $= \frac{1}{Re^{\frac{1}{2}}}[\Phi_o(\hat{r},\infty) + \ldots]$

Now matching terms of $O(Re^{\frac{1}{2}})$

$$\hat{\phi}_1(\hat{r},1) = -\lim_{x \to \infty}[x\Phi_{ox} - \Phi_o]$$

where the l.h.s. is just the displacement thickness.

b) Asymptotic Matching of the 1-Term Upstream Expansion with a 1-term Downstream Expansion

1-Term downstream expansion $\phi \sim \frac{1}{Re^{\frac{1}{2}}}\phi_o(r,z)$

Rewritten in terms of upstream variables and expanding for large Re $= \frac{1}{Re^{\frac{1}{2}}}\phi_o(\frac{1}{\sqrt{Re}},z)$

1-term upstream expansion $\hat{\phi} \sim \hat{\phi}_o(\hat{r},\hat{z})$

Rewritten in terms of downstream variables $= \phi_o(\infty,\hat{z})$

and expanding for large Re $= \frac{1}{Re^{\frac{1}{2}}}\hat{z}$

$$\therefore \quad \phi_o\left(\frac{1}{\sqrt{Re}},\hat{z}\right) = \hat{z}$$

APPENDIX D

NUMERICAL SOLUTION OF THE BOUNDARY LAYER EQUATION WITH AN UNKNOWN PRESSURE DISTRIBUTION (THE FIRST TERM OF THE DOWNSTREAM EXPANSION

a) Finite Difference Equations

For computational purposes equation (3.28) is rewritten as

$$u^*\frac{\partial u^*}{\partial r^*} + v^*\frac{\partial u^*}{\partial z^*} = -\frac{\partial p^*}{\partial r^*} + \frac{\partial^2 u^*}{\partial z^{*2}} \qquad \text{(D-1)}$$

$$\frac{\partial}{\partial r^*}(r^*u^*) + r^*\frac{\partial v^*}{\partial z^*} = 0 \qquad \text{(D-2)}$$

$$1 = r^*\int_0^1 u^* dz^* \qquad \text{(D-3)}$$

The boundary conditions become

$$\frac{\partial u^*}{\partial z^*}(r^*,0) = v^*(r^*,0) = 0 \qquad \text{(D-4)}$$

$$u^*(r^*,1) = v^*(r^*,1) = p^*(1) = 0 \qquad \text{(D-5)}$$

$$u^*(1/\sqrt{Re},z^*) = 1 \qquad \text{(D-6)}$$

where the donwstream variables are defined as

$$u^* = \frac{u\sqrt{Re}}{U}, \quad v^* = \frac{Rev}{U}, \quad z^* = \frac{z}{h}$$
$$r^* = \frac{r}{h}\sqrt{Re}, \quad p^* = (p - p_o)\frac{Re}{\rho U^2}, \quad U = \frac{Q}{4\pi h^2} \qquad \text{(D-7)}$$

The method employed was based on a technique developed by Bodoia and Osterle (1961) (see also Hornbeck (1973) for further details). Figure D.1 explains the notation.

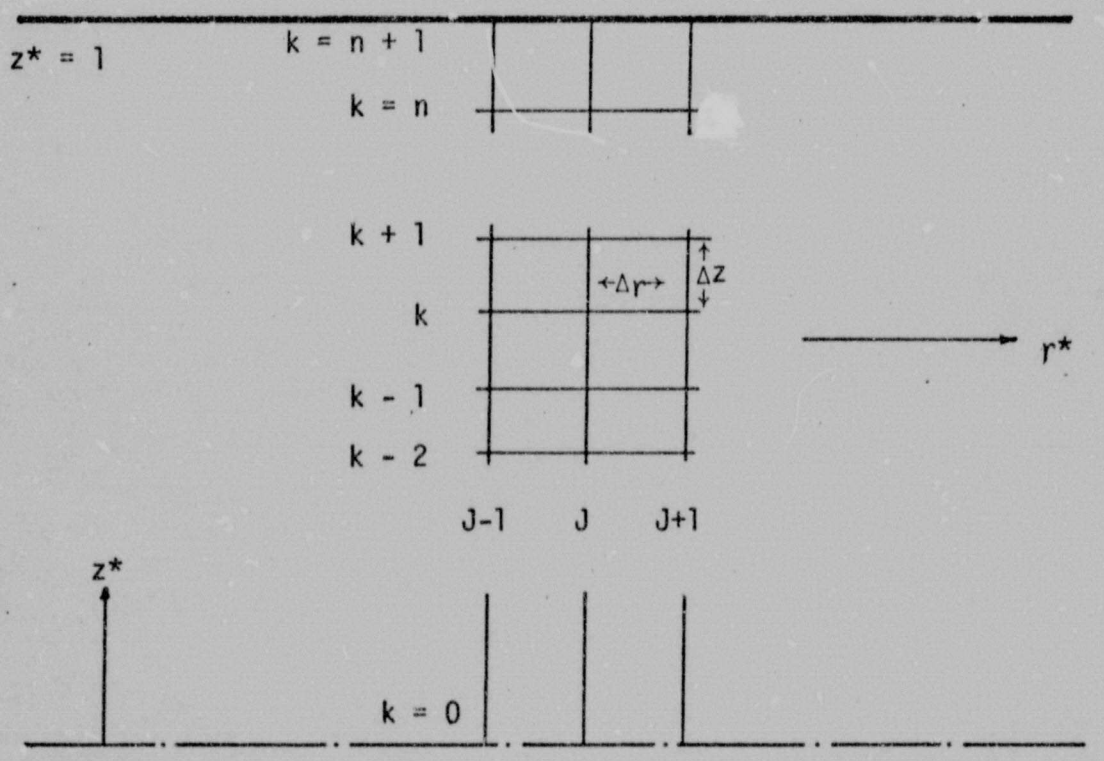

Fig. D.1 - Notation for finite difference equations

The finite difference analog for the momentum equation is

$$u(J,K)\left[\frac{u(J+1,K) - u(J,K)}{\Delta r}\right] + u(J,K)\left[\frac{u(J+1,K+1) - v(J+1,K-1)}{2\Delta z}\right]$$

$$+ \frac{p(J+1) - p(J)}{\Delta r}\left[\frac{u(J+1,K+1) - 2u(J+1,K) + u(J+1,K-1)}{\Delta z^2}\right] = 0 \qquad \text{(D-8)}$$

The continuity equation (D-2) is defined in a rather peculiar manner

$$\frac{r_{J+1}\left[\frac{u(J+1,K+1) + u(J+1,K)}{2}\right] - r_J\left[\frac{u(J,K+1) + u(J,K)}{2}\right]}{\Delta r}$$

$$+ r_{J+1}\left[\frac{v(J+1,K+1) - v(J+1,K)}{\Delta z}\right] = 0 \qquad \text{(D-9)}$$

The reason for the above finite difference analog for the continuity equation can be seen if one sums (D-9) over all K, equation (D-9) reduces to

$$u(J+1,0) + 2\sum_{k=1}^{N} u(J+1,K) = \frac{r_J}{r_{J+1}}[u(J,0) + 2\sum_{k=1}^{N} u(J,K)] \qquad (D\text{-}10)$$

which is just the finite difference analog of the trapezoidal rule for (D-3). The procedure enables the number of unknowns to be reduced from $2n + 2$ to $n + 2$ which results in a considerable saving in computing time. For convenience we rewrite (D-8) and (D-5) as

$$[-\frac{v(J,K)}{2\Delta z} - \frac{1}{\Delta z^2}]u(J+1,K-1) + [\frac{u(J,K)}{r} + \frac{2}{\Delta z^2}]u(J+1,K)$$
$$+[\frac{v(J,K)}{2\Delta z} - \frac{1}{\Delta z^2}]u(J+1,K+1) + \frac{1}{\Delta r}p(J+1) =$$
$$= \frac{u^2(J,K) + p(J)}{\Delta r} \qquad (D\text{-}11)$$

Equations (D-10) and (D-11) represent a set of $n+1$ linear algebraic equations in $n+1$ unknowns. This set can be rewritten in matrix notation as

$$Ax(J+1) = x(J) \qquad (D\text{-}12)$$

where A is the coefficient matrix evaluated at position J and $x(J)$ is the known values of $u(J,K)$ and $p(J)$ at position J.

Although A is a sparse matrix it does not have a tridiagonal character and the efficient Gaussian elimination scheme cannot therefore be used. Instead we have inverted the matrix using a Gauss-Jordan technique. A disadvantage of this method is the computing time required which is proportional to N^3 where N is the number of linear equations. To obtain accurate results Bodoia and Osterle (1961) in their studies on the parallel plate channel were forced to make N as large as 50 which results in a considerable amount of computing time. A method of circumventing this problem is to make use of a variable grid. This ensures that the grid spacing near the wall is sufficiently small to give accurate results while near the centre of the channel where the velocity gradients are not so steep, the grid is much coarser.

When forward or backward schemes of order Δr or Δz are used, a variable grid technique presents no problem since such differences only involve two points and these can be contained in a region of any given mesh size. However when central differences are used as in the momentum equation difficulties occur in evaluating the differences at a point of mesh size change.

This can easily be seen if one studies Figure D.2

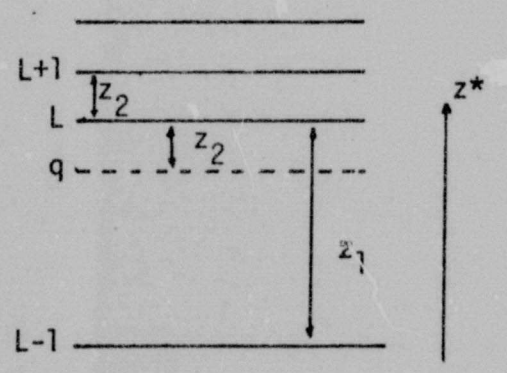

Fig. D.2 - Notation for change in step size

A central difference scheme for a velocity at L involves knowing the velocity at L + 1 as well as q. The velocity at q can be evaluated by passing a parabola through the values of u(J+1,L+1) u(J+1,L) and u(J+1,L-1) and the interpolated value is then given by

$$u(J+1,q) = \frac{2\gamma^2}{1+\gamma} u(J+1,L-1) + 2(1-\gamma)u(J+1,L) + \frac{\gamma-1}{\gamma+1} u(J+1,L+1) \qquad (D\text{-}13)$$

where $\gamma = \frac{\Delta z_2}{\Delta z_1}$.

The derivatives at L become

$$\left.\frac{\partial u^*}{\partial z^*}\right|_L = \frac{u(J+1,L+1) - u(J+1,q)}{2\Delta z_2} \qquad (D\text{-}14)$$

$$\left.\frac{\partial^2 u^*}{\partial z^{*2}}\right|_L = \frac{u(J+1,L+1) - 2u(J+1,L) + u(J+1,q)}{(\Delta z_2)^2} \qquad (D\text{-}15)$$

where $u(J)1,q)$ is given by (D-13).

The finite difference analog of the integral continuity equation is also slightly modified and becomes

$$r_{J+1}[\{u(J+1,0) + 2\sum_{k=1}^{L-1} u(J+1,K) + u(J+1,L)\}\frac{\Delta z_1}{2} +$$

$$+ \{u(J+1,L) + 2\sum_{k=L+1}^{N} u(J+1,K)\}\frac{\Delta z_2}{2}] =$$

$$= r_J[\{u(J,0) + 2\sum_{K=1}^{L-1} u(J,K) + u(J,L)\}\frac{\Delta z_1}{2} +$$

$$+ \{u(J,L) + 2\sum_{K=L+1}^{N} u(J,K)\frac{\Delta z_2}{2}] \qquad \text{(D-16)}$$

If a grid size change in the radial is required, i.e. Δr_1 to Δr_2 no problems are encountered since all the finite differences for this direction are either backward or forward.

b) Inherent Errors in the Computational Procedure

In determining the flow rate in the channel a transverse integration of the radial velocity component must be made. This automatically introduces a finite error into the calculations which can be compensated by redefining the initial velocity profile as

$$u(0,K) = \frac{1}{(1 - \frac{\Delta z}{4})^2} \qquad \text{(D-17)}$$

A detailed description of the analysis leading to the above result is given by Hornbeck (1973) and will therefore not be discussed.

c) Stability

Hornbeck (1973) and Bodoia and Osterle (1961) have done a stability analysis on the finite difference schemes given in section A. The difference equations were found to be stable for all positive values of the velocity components and the following conditions were found to be applicable when negative velocity components are encountered

$$\frac{\Delta r}{|u|\Delta z^2} \geq \frac{1}{2} \qquad \text{(D-18)}$$

and

$$v \geq \sqrt{\frac{2|u|}{\Delta r}} \qquad \text{(D-19)}$$

This is rather fortunate since it is now possible to integrate through a region of reverse flow.

d) Computer Program

Figure D.3 is a listing of the program while Figure D.4 is a simplified flow diagram. A brief description of the program is given below.

The coefficient matrix A which is only a function of the Jth step is inverted using a Gauss-Jordan elimination routine. In order to reduce round-off errors, maximization of pivotal elements is employed. Once the inversion is complete the vector x(J+1) may be determined explicitly since (D-12) can be rewritten as

$$x(J+1) = A^{-1}x(J) \qquad \text{(D-20)}$$

To evaluate the v(K,J+1) components we make use of (D-11) which is also an explicit formula for the v(K,J+1) components as all the u(K,J+1) are now known. Once the v(K,J+1) components have been evaluated it is then possible to compute the new coefficient matrix for the J+1 step, and so on.

e) Accuracy

The differences between this problem and the parallel plate channel are:

i) the form of the continuity equation, and

ii) the initial boundary condition.

These differences constitute a minor part of the program and it was possible therefore to run the program for the parallel plate channel as well. The results could then be checked against the results of Bodoia and Osterle (1961) which have been conveniently tabulated in Hornbeck's (1973) monograph.

To obtain sufficient accuracy the following requirements had to be met:

i) The program had to be written in the double precision mode.

ii) The initial Δx step had to be of the order of 1×10^{-5}. After a number of steps the Δx could be increased slowly to attain a final value of 3×10^{-3}.

iii) The variable grid spacing was very critical and after a considerable amount of numerical experimentation the following grid was adopted:

$$\begin{array}{lll} \text{For} \quad 1 \leq z < 0{,}3 & \Delta z_2 = 0{,}030435 & \\ \text{and} \quad 0{,}3 \leq z \leq 0 & \Delta z_1 = 0{,}1 & \end{array} \qquad \text{(D-21)}$$

Using this grid the difference between Bodoia and Osterle's (1961) results were

$$\left|\frac{u_C - u_{BC}}{u_{BC}}\right| \times 100 \leq 0{,}5 \qquad \text{(D-22)}$$

where

u_C = centre line velocity

u_{BC} = Bodoia and Osterle's centre line velocity

Figure D.6 is a summary of the numerical experimentation with the variable grid. The accuracy can be further improved by using a finer grid but this increases quite substantially the computing time required.(Note: the computing time required, as pointed out earlier, is proportional to N^3 where N is the number of grid points.)

```
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION ALPHI(30),BETA(30),OMEGA(30),PHI(30),CC(30,30)
      DIMENSION W(30),AA(30),PI(30),C(30,30),V(30)
      TUM=0.
      AL=5
      RP=DSQRT(79.0D0)
      B1=1./RP
      DA=.000
      N=26
      B1=N+1
      B2=N+2
      B3=N+3
      DO 150 L=1,N2
      DO 100 LL=1,N3
  100 C(L,LL)=0.
      DO 20 II=2,NM
      I=I,-1
      N2=5
      NM=II/NU
      JP=4
      IP1=IP-1
      IP2=IP+1
      DELA=0.00001
      IF(II.GT.15)DELA=0.00002
      IF(II.GT.30)DELA=0.0002
      IF(II.GT.60)DELA=0.001
      IF(II.GT.75)DELA=0.003
      DEL1=0.1
      DEL1=DELV
      DEL2=0.0500495
      THETA=PAL2/DELI
      IF(II.GT.2)GO TO 60
      DO 10 K=1,N
      W(K)=1./11.-(DEL2/2.)**2)**.5
      V(K)=0.
      PI(K)=0.
   60 CONTINUE
      W(1)=0.
      V(1)=0.
      V(2)=0.
      TUM=TUM+DELA
      B2=B1+TUM
      B3=B2-DELA
      B4=B2/B3
      DO 11 K=2,N
      C(1,1)=DELI/2.**2
      IF(K.LT.IP)C(1,K)=(DELI)**2
      IF(K.EQ.IP)C(1,K)=(DELI+DELI)/2.**2
      IF(K.GT.IP)C(1,K)=DEL2**2
      C(2,K)=0.
   11 C(3,K)=0.
      SUM=0.
      AI=I
      DO 60 J=2,N2
      IF(J.GT.IP) DELY=DEL2
      IF(J.EQ.IP) SUMM=0.
      IF(J.LE.N2) GO TO 105
      OMEGA=-2./DELY**2
      K=J-1
      ALPHI(K)=-V(K)/12.*DELY)-1./DELY**2
      BETA(K)=V(K)/(2*DELA)+2./DELY**2
      OMEGA(K)=V(K)/12.*DELY)-1./DELY**2
      PHI(K)=U(K)**2*P(I))/DELA
      IF(J-P)62,60,62
   60 C(J,K)=BETA(K)
      C(J,K+1)=OMEGA
      C(J,N)=1./DELA
      C(J,N2)=0.
      C(J,N3)=PHI(K)
      GO TO 67
   62 C(J,K-1)=ALPHI(K)
      C(J,N2)=0.
      IF(J.EQ.IP2) C(J,K-1)=0.
      IF(J.EQ.IP2) C(J,N2)=ALPHI(K)
      C(J,K)=BETA(K)
      IF(J.EQ.N1) GO TO 69
      C(J,K+1)=OMEGA(K)
   65 C(J,N)=1./DELA
      C(J,N3)=PHI(K)
      GO TO 67
  100 C(J,IP1)=2.*THETA**2/11.+THETA2
      C(J,IP)=2.*11.-THETA2
      C(J,IP2)=(THETA-1.)/(THETA+1.)
      C(J,N3)=0.
      C(J,N2)=-2.
      GO TO 60
   67 CONTINUE
      IF(J.GT.N1 GO TO 60
      IF(J.EQ.IP) GO TO 190
      SUM=SUM+U(J)
      GO TO 60
  105 SUM=(SUM*2.+U(1)+U(J))*DELY/2.
   60 CONTINUE
      SUM=SUM+(1.+SUM+U(IP))*DELY/2.
      SUM=SUM*B1
  600 FORMAT(2,2(2E5.8,E14.7))
      C(1,N3)=SUM
      WRITE(6,600)SUM1,SUM
      CALL DIMACC(,N2)
      DO 13 J=1,N2
      SM=0.
      DO 12 K=1,N2
      SM=SM+C(J,K)*C(K,N3)
   12 C(J,K)=0.
      IF(J.GT.N1 GO TO 77
      SM=SUM
      IF(SM.LT.0.)SM=-
      GO TO 13
   77 IF(J.EQ.N1) PI(1)=SM
   13 CONTINUE
      DO 70 J=1,N
      SUM(K)=0.
      V(1)=0.
      DELY=DEL1
      IF(J.GT.IP) DELY=DEL2
      C(1)=W(J+1)+W(J)-B3/B2*U(J+1)*DELJ)
      V(J+1)=(J-DELY/2.*DELA)*PI(J)
      WRITE(6,201) B2,PI(J)
  201 FORMAT(2X,D14.7,2X,D14.7)
  204 FORMAT(/,7(2X,D14.7))
      WRITE(6,205)(U(J),J=1,N)
   71 DO 66 K=1,N
   66 W(K)=U(K)
   70 CONTINUE
   80 CONTINUE
      STOP
      END


      SUBROUTINE DIMAC(CC,J3)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION CC(30,30),CC1(30)
      DO 129 I=1,J5
  129 J1(I)=J3+I
      DO 144 I=1,J3
      CC(I,I)=1.
      J1=I
      DO 139 K=1,J5
      IF(DABS(CC(I,K)).GE.0.0001 GO TO 130
  130 J1=K
      CC1(K)=CC(I,K)
  139 CONTINUE
  127 IF(I.EQ.J1) GO TO 136
  128 K=J1(K)
      J1(K)=J1(I)
      J1(I)=K
      DO 137 K=1,J5
      SM=CC1(K)
      CC1(K)=CC(I,K)
      CC(I,K)=SM
  137 CC1(K,J1)=50
  150 CC(I,J1)=1.
      DO 135 J1=1,J5
  145 CC(I,J1)=CC(I,J1)/C0
      DO 142 J1=1,J5
      IF(I.EQ.J1)GO TO 142
  129 CC(J1,I)=CC(J1,I)
      IF(I.EQ.9.) GO TO 142
  130 CC(J1,I)=0.
      DO 141 K=1,J5
  141 CC(J1,K)=CC(J1,K)-CC(I,K)*A3
  142 CONTINUE
  144 CONTINUE
      DO 143 I=1,J5
      IF(J1(I)-J3).I) GO TO 143
  151 J1=I
  152 J1=J1(I)
      IF(J1(I)-J3).I) GO TO 153
  136 IF(J1.GT.J1) GO TO 152
  153 J1(I)=J1(I)-J3
      DO 163 K=1,J5
      CC(I,K)=CC(J1,K)
  163 CC(J1,K)=C0
      J1=J3+I
  143 CONTINUE
      RETURN
      END
```

Fig. D.3 - Listing of computer program

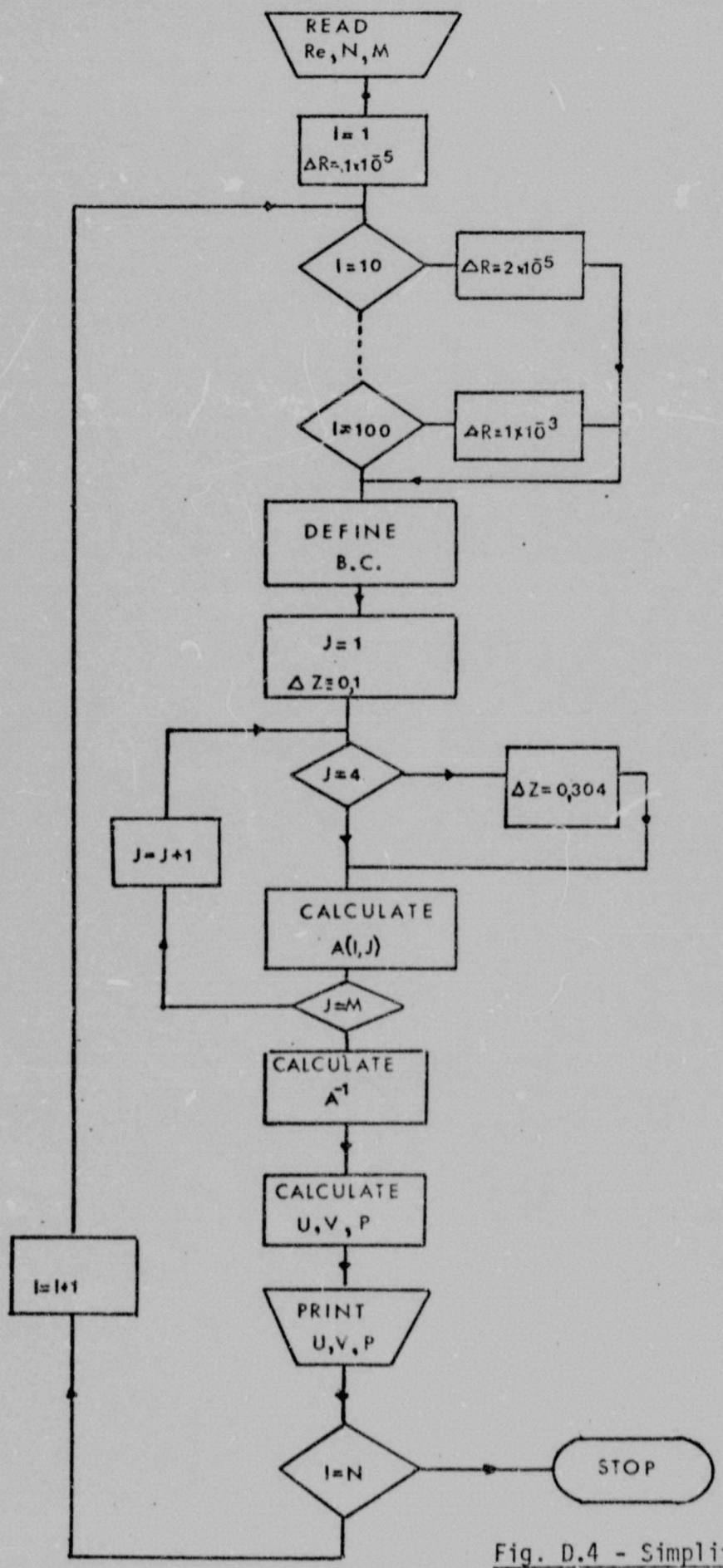

Fig. D.4 - Simplified flow diagram

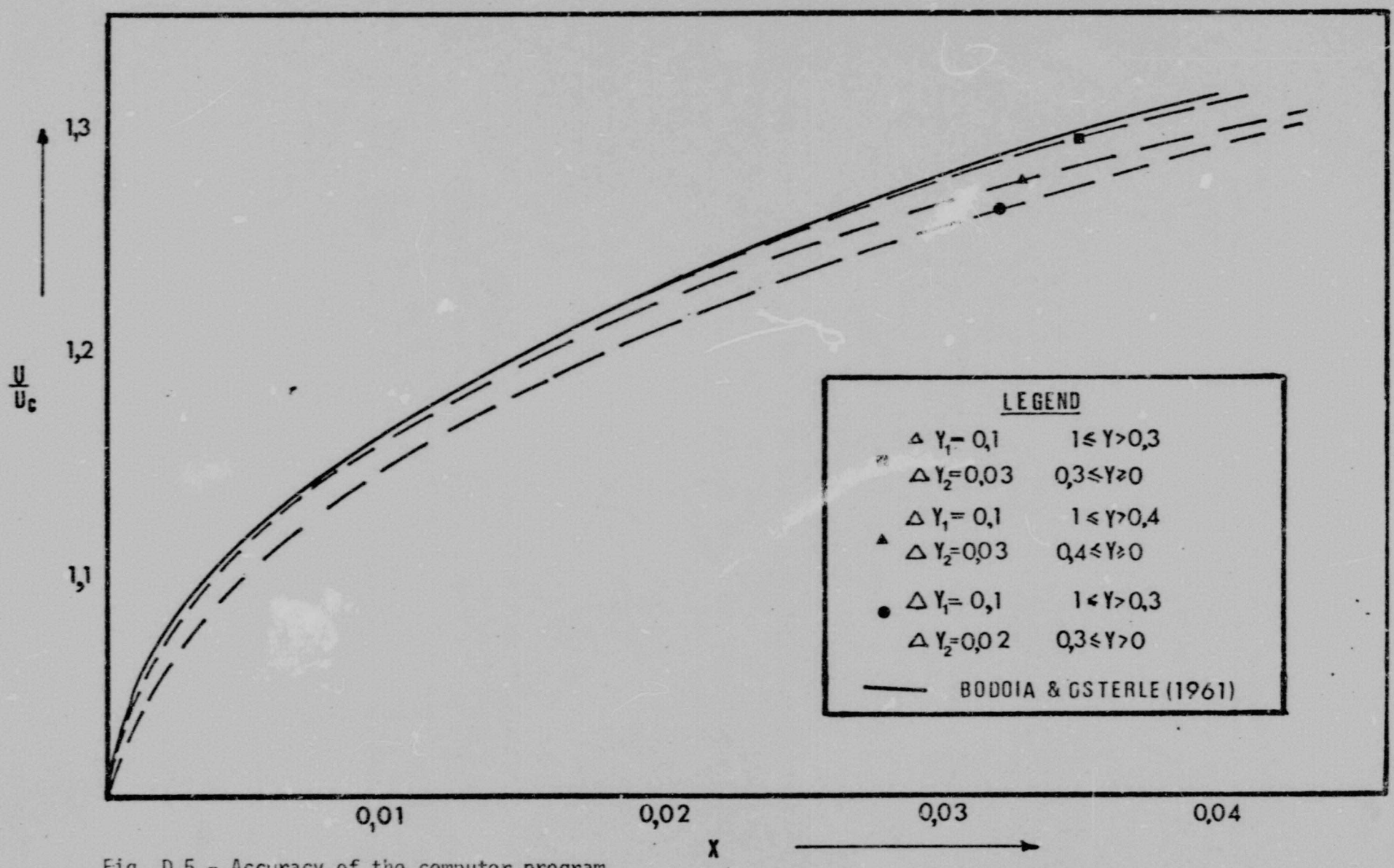

Fig. D.5 - Accuracy of the computer program

Author Higgins Brian Gavin
Name of thesis Some Aspects Of Radial Flow Between Parallel Disks. 1975

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