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**INVESTIGATIONS INTO THE MECHANISM OF FRACTURE ONSET
AND GROWTH IN LAYERED ROCK USING PHYSICAL AND
NUMERICAL MODELLING**

Tufan DEDE

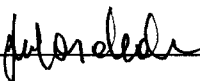
**A dissertation Submitted to the Faculty of Engineering,
University of the Witwatersrand, Johannesburg
for the Degree of Master of Science.**

Johannesburg 1996

Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

Signed this 14 day of October 1996

_____

Tufan DEDE

ABSTRACT

One of the major impediments in the field of numerical modelling in rock mechanics is limited knowledge of the mechanisms of fracture and failure of brittle rock. One important tool for improving the understanding of rock behaviour is the use of laboratory experiments under controlled conditions.

The Displacement Discontinuity Method, capable of fracture growth simulation (DIGS), has been used to model fracturing in samples under punch loading. A Finite Difference Method, capable of plastic deformations due to its explicit time marching scheme (FLAC), has also been used to model the punch tests.

By comparing numerical simulations with results from laboratory experiments of punch tests, it has been possible to define the basic failure mechanism for pillar foundation failure. Two different test set-ups were used namely, steel jacketed axisymmetric punch tests and long strip punch tests in the triaxial cell which is built for these specific tests.

The layered structure of the test specimens and in the test procedure had significant effects on the fracture pattern as well as the failure load. When the layer is near to the punch area, then both the layer and the layer conditions had a strong effect on the failure load. When the layer was frictionless, the failure stress dropped by about 20 percent. The same result occurred in both the axisymmetry and strip loading tests.

When shear fractures intersect a layer with either low or high friction it terminates. This is not the case for the tensile fractures, which can pass through the layer media. However, it is important to note that the tensile fractures which originate from near the cone area can not pass through the layers. They stop at the interface.

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I INTRODUCTION

A conceptual understanding of the nature of the fracture zone induced by tabular excavations in deep mines, established in the Witwatersrand and Orange Free State Goldfields of South Africa, has been built up over a number of years. This knowledge has been compiled from the mapping of fracture exposures on the hangingwalls of the stopes and on the sidewalls of gullies and tunnels advanced ahead of the mining face, as well as from observations of fractures intersecting boreholes drilled from tunnels below stopes, at different angles into the stope face, and vertically into the hangingwall (Napier and Hilyard, 1992).

The classification of fractures surrounding stopes is based principally on their inclinations, on the presence or absence of movement across them, and on secondary associated structures (Joughin and Jager, 1984). Three types of fractures are recognized.

1.Extension Fractures

These fractures are by far the most common, Figure 1.1. They are planar, steeply inclined, and have no displacement in the plane of the fracture when they are formed. At their time of formation, the extension fractures are almost certainly aligned with the maximum principal stress direction.

2.Shear Fractures

These fractures are far less common than the extension fractures and are inclined at angles of 60° to 75° to the horizontal (Figure 1.1). These fractures can be quite complex. In their simplest form, they are a few millimeters wide with displacements of a few millimeters. In the laboratory, fractures can be produced in extension test apparatus which resemble simple shear fractures. These fractures are formed when the minimum principal stress at failure is much higher than that when fractures resembling extension fractures are formed.

3. Flat Extension Fractures;

These fractures are inclined at 20° to 40° to the horizontal towards the direction of face advanced (Figure 1.1). They are clean planar fractures, which display no evidence of shear displacement, and are confined to the first few strata above and below the stope. They appear to occur in response to the very localized stress changes at the periphery of the stope excavation and result from the horizontal displacement rock from the fractured zone towards the mined out area.

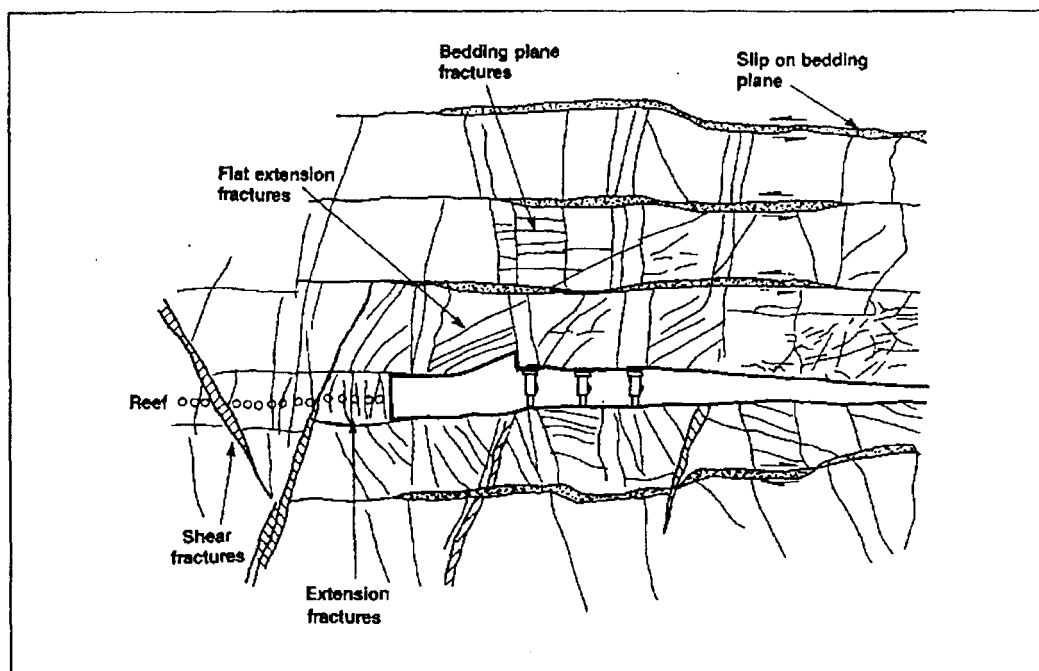


Figure 1.1 A diagram of the fracture zone around a stope face (Rockfalls and Rockburst, 1988)

It has been shown in the literature that layered strata can play a significant role in bounding the limits of fracture growth and may influence the formation of the fractures.

However in order to evaluate the numerical procedure simulating fracture formation and to gain quantitative insight into the influence of the layers on fracturing, a series of laboratory tests were carried out under controlled conditions. These tests were supplemented by number of computer runs have also been carried out by using the

numerical modelling codes, DIGS (Napier 1990) and FLAC (Itasca Consulting Group, 1992).

This dissertation is divided into three parts;

1. Literature review on stress distributions and fracture onset and growth in solid and layered media.
2. Laboratory work carried out in axisymmetry punch tests and cubical specimen strip and square punch tests. In the first set of experiments, sandstone was used and for the triaxial tests quartzite samples from gold mines were used. After completing the tests, some numerical modelling was carried out using the DIGS and FLAC two dimensional plane strain computer programs.
3. The conclusion of this work and the recommended future work are presented in the last chapter.

II LITERATURE SURVEY

The literature review covers mechanisms of stress distribution and some of the related fracture onset and growth in layered and solid media.

A number of important aspects of mechanisms of fracture onset and growth in rock remain unsolved; a.) where and how do cracks first form, b.) does their first appearance change with the characteristics of the layer and if so, how does it change? c.) if there is more than one type of crack, do the different types coincide with certain regions of the complete stress-strain curve? d.) how does the crack type affect fracture growth and its fracture pattern?

The literature review provides some fundamental understanding of these problems and serves to enable comparisons against experimental results obtained in this study.

2.1 STRESS DISTRIBUTION

When the flat surface of a solid is loaded with a hard indenter, the solid experiences a complex stress field. The detailed nature of this field depends on several factors, notably the mechanical response of the solid (linear elastic, elastic/brittle, elastic/plastic, and so on) and the geometry of the indenter (e.g. ‘blunt’, ‘sharp’ or ‘flat’).

Wagner and Schumann (1971) carried out some axisymmetric triaxial punch tests from which they were able to plot the lines of equal principal stresses and determine the magnitude of the stresses induced in the rock by an elastic stamp. (Figure 2.1). They were also able to calculate stress magnitudes due to a constant normal stress (it is so called “elastic” punch) as well as for a “rigid” punch subjected to a constant normal displacement. The results of this work showed that directly beneath the contact area of the stamp is a hemispherical region where all three principal stresses are compressive. Along the axis of the stamp the tensile stress rises

to a value of about 0.02 of the uniform pressure, p , at the contact area and at a distance of about a stamp diameter starts to decrease with increasing distance. Much larger tensile stresses, exceeding $0.3p$, occur in a shallow region near the free surface outside the contact area. These tensile stresses reach a maximum value adjacent to the edge of the stamp.

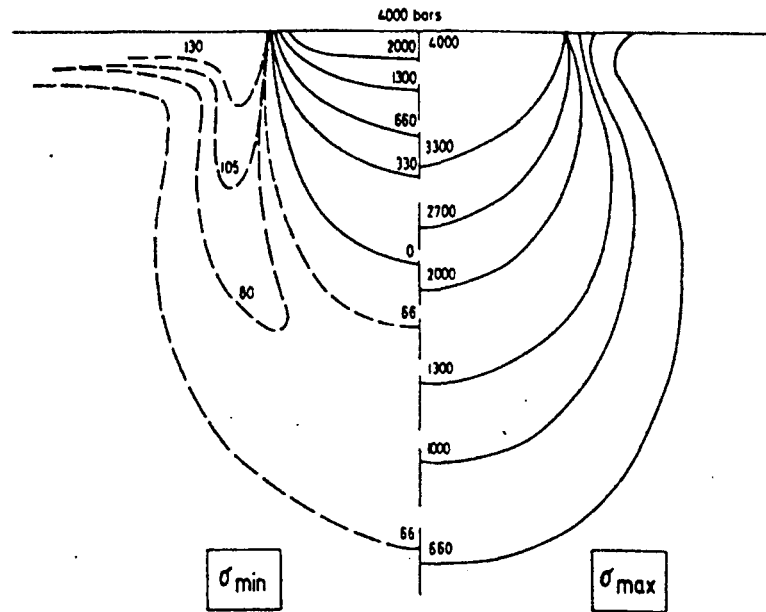


Figure 2.1 Elastic stamp-Distribution of maximum and minimum principal stresses
 ————— compression - - - - - tension (Wagner & Schumann 1971)

Comparing the stress trajectories of the elastic and the rigid stamp no significant differences were found except near the edge of the stamp where some small differences are observed which result from high stress concentrations at the edge of the rigid stamp. The general nature of the distribution of the principal stresses is very similar for the elastic and the rigid stamp, as can be seen from Figure 2.2. The major differences occur at the edge of the stamp where there are high stress concentrations which tend to infinity at the edge of the rigid stamp. Also, the maximum principal stress at the center of the rigid stamp is only half that at the center of an elastic stamp.

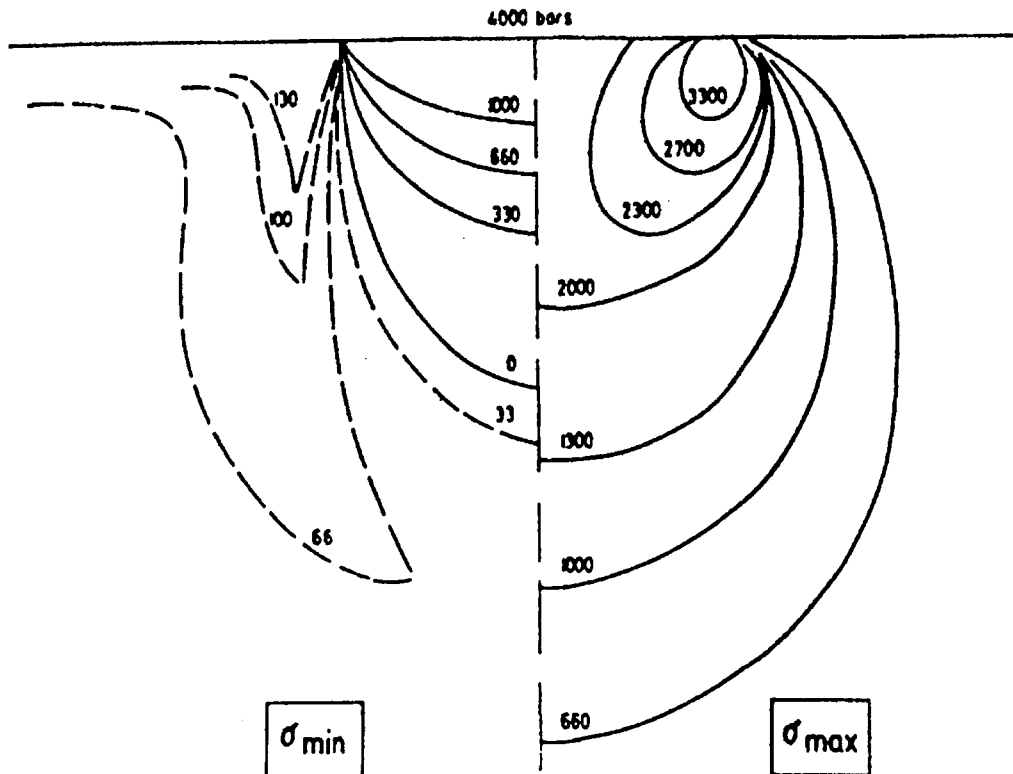


Figure 2.2 Rigid stamp-Distribution of maximum and minimum principal stresses
 ——— compression ——— tension (Wagner & Schumann, 1971)

In the layered case, however, there are some differences in the stress distributions under the punch area according to Burmister's work (1962) on a layered half plane. He considered a two layered medium based on the theory of elasticity for analysis of some foundation problems. The fundamental relations existing between the physical factors which control the magnitude and distribution of stresses and displacements in a two-layer system subjected to a radially symmetric loading is given. Out of the two layers of finite extent the surface layer is assumed to have a finite thickness which rests upon the lower layer of infinite thickness having different elastic properties. Each layer is considered to be homogeneous, isotropic and elastic for which Hooke's law is valid and has a value of Poisson's ratio equal to 0.5. The two

layers are supposed to be continuously in contact with each other but with a frictionless interface and continuity of normal stress and normal displacement. According to Burmister's findings the main effect of stratification is a greater concentration of stress beneath the loaded area.

In Lemcoe's work (1967), each layer was considered to be an elastic solid. The upper layers were considered to be of finite depth but of infinite horizontal extent and the lowest layer of infinite depth and horizontal extent. The two-layer system investigated by Lemcoe is shown schematically in Figure 2.3. On the basis of the numerical results, Lemcoe gave the following conclusions;

a.) *Effect of b/h' (b is the width of loading and h' is thickness of top layer) on stress.*
 σ_x , σ_y , and τ_{xy} increase steadily with increasing b/h' at all points when the surface loading is uniformly distributed and of constant intensity Q .

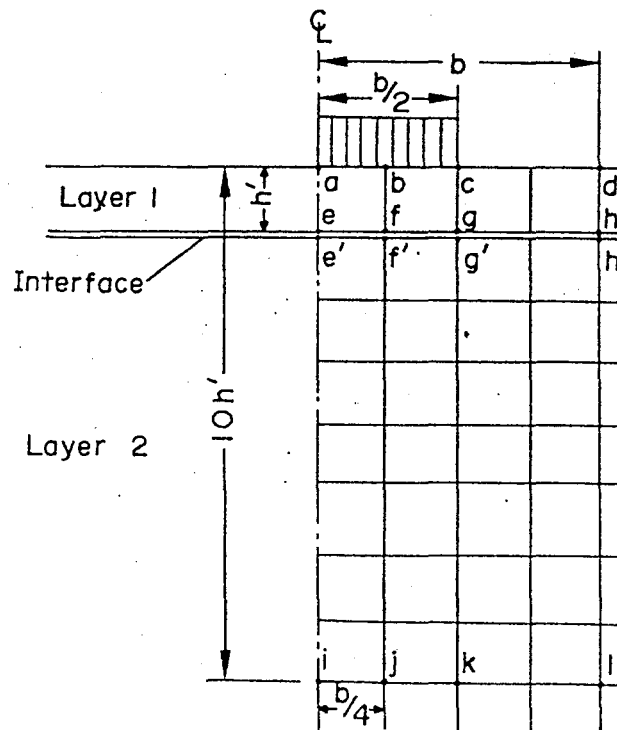


Figure 2.3 Two-layer system showing locations at which stresses were computed (Lemcoe, 1967)

b.) Effects of friction at the interface on stress. In changing the conditions at the interface from rough to smooth there is a substantial increase in σ_x (tensile stress) in the upper layer, and a considerable increase in σ_x (compressive stress) in the lower layer. There is only a very slight change in σ_y which at all points is usually a small decrease in stress, and a very little change in the value of τ_{xy} , except at the interface where, it is zero. This clearly indicates the pronounced effect friction has on the horizontal stresses in both layers, and hence the necessity to take friction into account in a rational analysis.

c.) Effects of stiffness ratio- E_2/E_1 on stress. When the friction between the layers was low, it was found by Lemcoe (1967) that increasing the stiffness ratio E_2/E_1 from 0.02 to 0.1 i.e. allowing E to decrease, nearly doubles σ_y at the interface and σ_x (compressive) is reduced by a factor of 2. Increasing the stiffness ratio results in less load being carried by flexural stresses and more by direct bearing on the supporting layer.

Iyengar and Alwar (1964) have discussed the frictionless interface case, and have shown that the stresses are affected by the different Poisson's ratios for each strata. They also observed in the case of two layer-system and for plane strain problems with $E_2/E_1=10$, that the maximum intensity of interface shear stress for $\nu_1=0.3$ and $\nu_2=0.25$. Also it is noted that this effect is especially significant when the top layer has a higher ν than the semi-infinite plane. In the case of plane stress for $E_2/E_1=1$ it was observed that the stress distribution depends on $(\nu_1-\nu_2)$ and there is a tendency for the normal stress distribution to become more and more uniform as $(\nu_1-\nu_2)$ decreases.

Moreover experimental work of Bernaix (1967) has shown wherein the distribution of stresses in a layered foundation due to uniform loading showed that the stresses can be as high as double the applied load itself, which is the same conclusion of Burmister's work (Figure 2.4)

Photoelastic studies were conducted by Maury and Habib (1970) to determine the distribution of stresses in a stratified rock continuum with different interface friction angles ($\phi=1.5^\circ$; $\phi=24^\circ$; $\phi=36^\circ$). The following observations are reported by them (Figure 2.5).

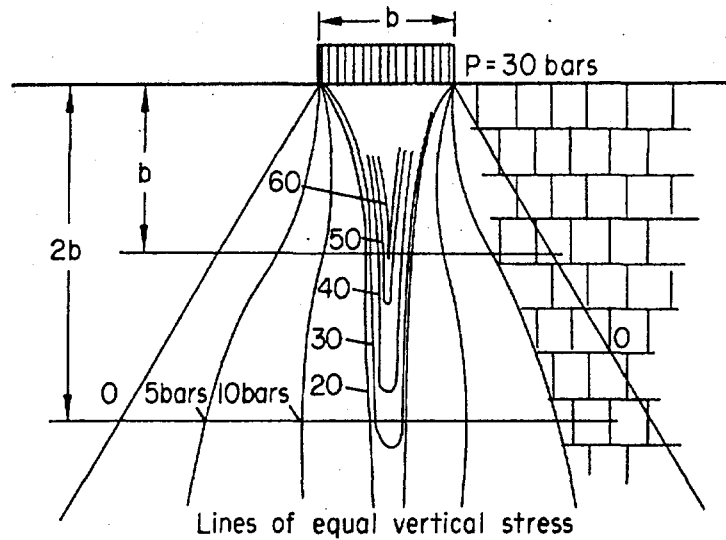


Figure 2.4 Stress in two-dimensional discontinuous system (Bernaix, 1967)

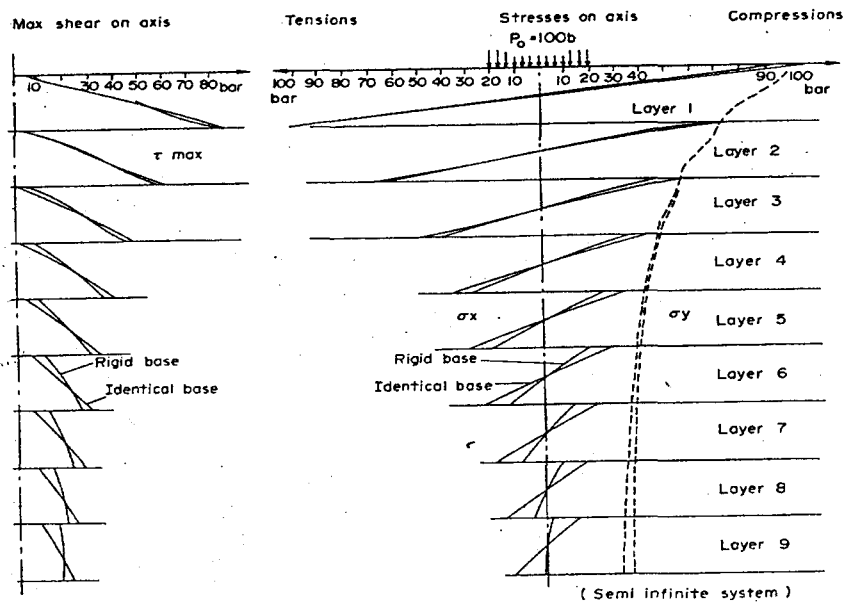


Figure 2.5 Theoretical stresses in 10-layer system (Maury & Habib, 1970)

- a.) When the friction angle is small, the order of the isochromatic values is raised and they extend quite deep and are very clear in the first six strata; this phenomenon is very much diminished, when the friction angle is 36° between the strata.
- b.) The vertical stresses are raised two times at corresponding depths compared to the continuum mass.
- c.) If the friction between the layered strata is weak, it was seen that the upper strata were observed to be under some sort of internal bending which can add to the vertical compression and which reduces slowly with the depth.
- d.) Even a friction angle of the order of one degree can modify the stress distribution significantly.

2.2 FRACTURE INITIATION

Cook, Hood and Tsai (1984) carried out the punch tests in which ground, cylindrical rock samples of 89 mm diameter and 89 mm in length were jacketed by heat-shrinking a steel tube around the rock cylinders. The dimensions of the steel tube were selected to confine the rock samples in a manner similar to the confinement provided in a semi-infinite rock mass. At low confining stress, the specimens failed in tension across a plane passing through the axis of the punch. Theoretically, compressive failure always occurs at lower punch stresses than those required for tensile failure, but the measured punch stresses at failure are significantly greater than those predicted by theory. A finite element analysis of these tests showed that the zone of compressive failure forms before that of tensile failure. Regions of tensile and compressive failure are identified in Figure 2.6 and 2.7 for mean punch pressures from 1483 to 1918 MPa at confining pressures from zero to 17.6 MPa. From Figure 2.6, it is evident that at low confining stress the region of tensile failure extends completely beneath the punch. As the confining stress is increased, tensile failure is restricted to a region adjacent to the punch corners and this region extends downwards in an approximately vertical direction as the punch load increases (Figure 2.6 and 2.7). Also shown in these figures are regions of the rock beneath the punch where the compressive strength of the rock has been reached.

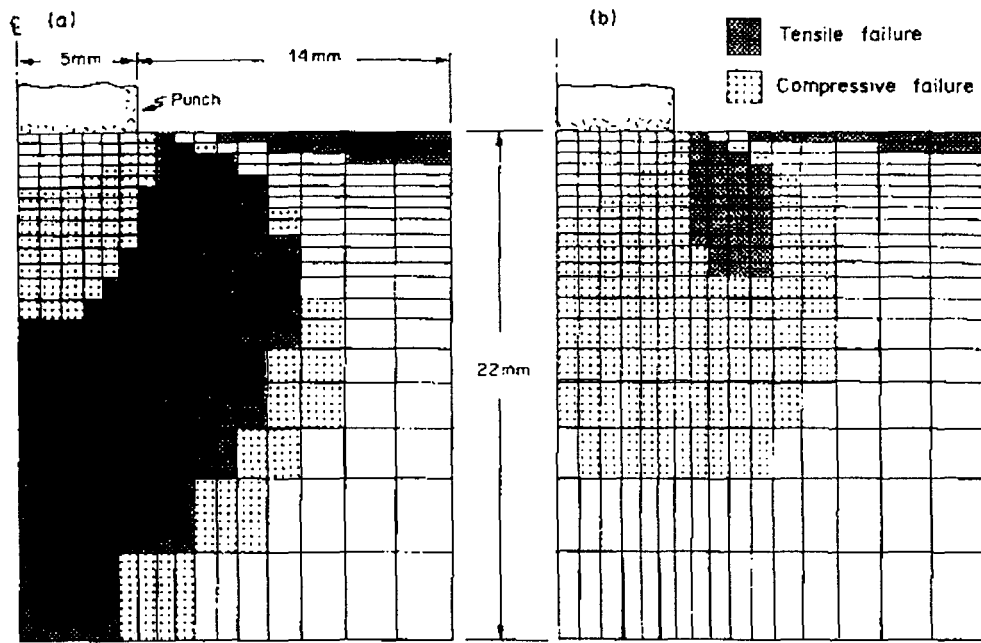


Figure 2.6 Regions of tensile (black) and compressive (shaded) failure beneath a punch with the punch pressure $p=1.48$ GPa. (a) Rock confining pressure $p_I=0$; note tensile failure extends completely beneath punch (b) Rock confining pressure $p_I=13.8$ MPa (Cook, Hood, Tsai, 1984).

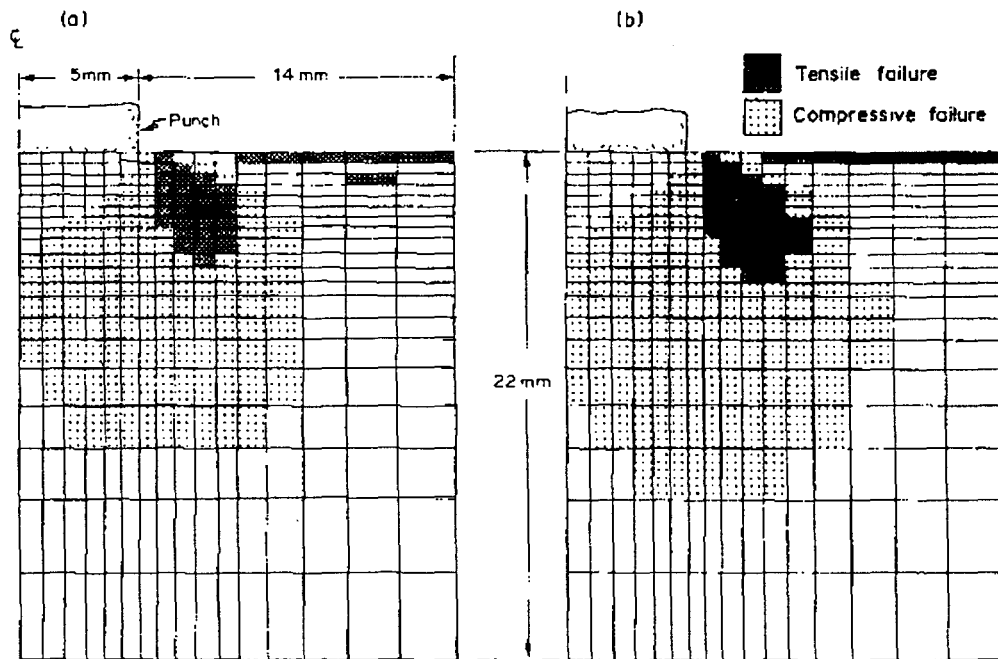


Figure 2.7 Regions of tensile (black) and compressive (shaded) failure beneath a punch with the rock confining stress. $p_I=27.6$ MPa. (a) Punch pressure $p=1.70$ GPa (b) Punch pressure $p=1.92$ GPa (Cook, Hood, TSAI, 1984).

It is interesting to note that immediately beneath the punch the rock does not fail because in this region the rock is in a state of almost hydrostatic compression. These

findings are supported by results from a plane strain, non-linear, finite element study of indentation behaviour conducted by Wang and Lehnhoff (1976).

There are two types of indentation of cracking resulting from the contacts of the indenter with the specimen surface. These contacts can be classified as blunt and sharp contacts respectively (Whittaker, Singh and Sun, 1992). The blunt contact is usually produced by blunt indentors such as spherical indentors in a brittle material under load. The main feature of this contact is that it is essentially elastic in nature until fracture takes place. The cracks which are formed are usually referred to as Hertzian cone cracks. The sharp contact, is produced by a sharp indenter. The main feature of this contact is that the contact is essentially plastic until fracture takes place (Whittaker, Singh and Sun, 1992).

Figure 2.8 shows schematically the different stages of crack formation during advanced sharp indenter loading and unloading. At stage (a), the indenter contacts and penetrates into the specimen surface. This penetration results in plastic deformed zone around the indentation site. A surface tensile zone adjacent to the contact and a substantial tensile stress field immediately below the indenter tip are consequently induced (Figure 2.8(a))

The penetration may also produce a completely interior, sub-surface penny-like crack, called a median crack, immediately below the indenter tip where the greatest sub-surface stresses are present (Figure 2.8(b)). With further increase in the applied load, the median crack extends both outwards and downwards (Figure 2.8(c)). If the applied load is further increased, then the sub-surface and propagating median crack break through the confining stress lobes and intersect the specimen surface forming a half-penny surface with median-radial cracks as shown in Figure 2.8(d).

Figures 2.8(e) to Figure 2.8(g) illustrate crack closure and the formation of lateral cracks resulting from the residual stresses upon unloading and removal of the indenter (Whittaker, Singh and Sun, 1992).

There is a possibility that if the specimen is loaded too heavily, the lateral crack may intersect the specimen surface and chips may be produced as a result.

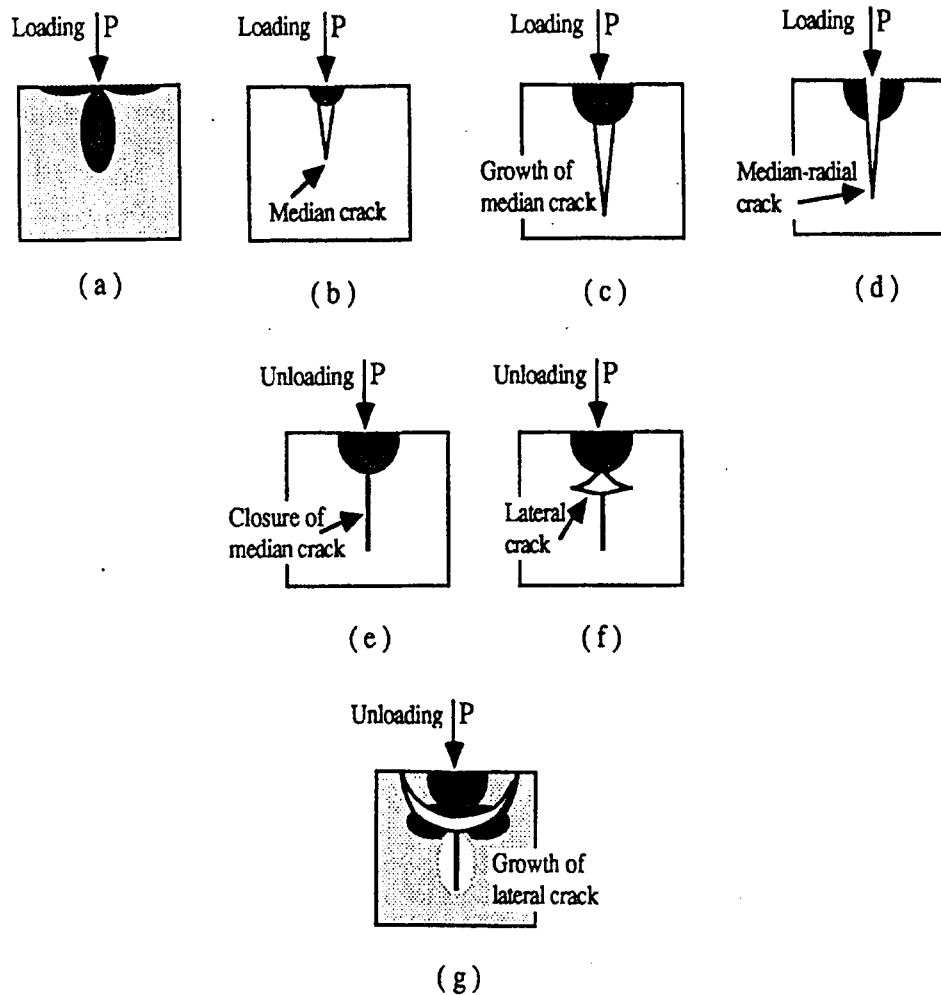


Figure 2.8 Illustration of formation of median, radial and lateral cracks during a sharp indenter loading and unloading. The heavily shaded region in each diagram indicates the fracture process zone in which the stress is tensile. The slightly shaded regions in (a) and (g) represent compressive stress fields (Whittaker, Singh and Sun, 1992).

It has been shown that the full development of the median-radial cracks results from the formation of the sub-surface penny-like crack and the surface radial cracks and their interactions upon loading. Figure 2.9 illustrates a model proposed by Lankford (1981) for fracture initiation below a sharp indenter (Whittaker, Singh and Sun, 1992).

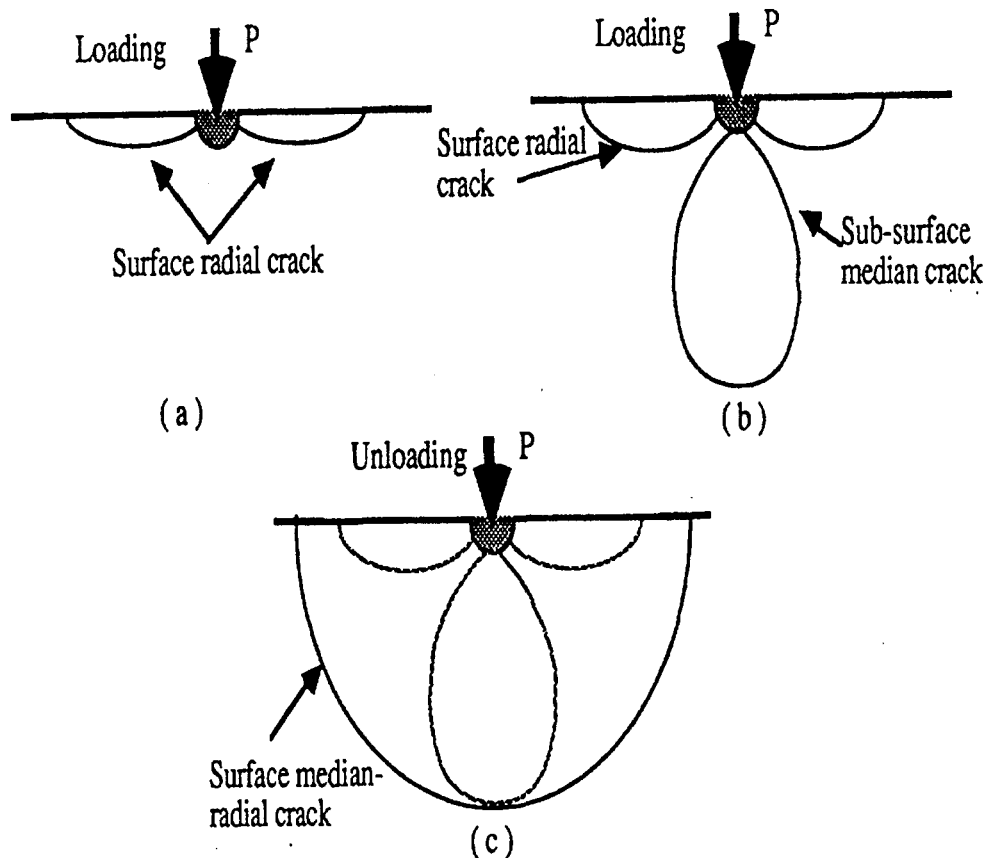


Figure 2.9 Model proposed by Lankford (1981) for fracture initiation beneath a sharp indenter. This model shows that surface radial crack initiation (a) precedes that of the sub-surface median crack.

Bieniawski's studies (1967) on glass revealed that fracture initiation occurs near the crack tip but also at some distance away. This indicates that the crack tip itself plays a minor role in the fracture process. The primary factors responsible for fracture initiation are the relative movement of the crack faces. Irregularities of the crack surface result in an uneven stress distribution along the crack length and tensile fracture initiates at the tensile stress zones which occur at points where the crack surface is relatively free to move.

In order to study fracture initiation in rock, tests were conducted by Bieniawski on cylindrical rock specimens under uniaxial and triaxial loading conditions. Fracture initiation was detected from the plot of volumetric strain ($\epsilon_1 + \epsilon_2 + \epsilon_3$), thus adding the axial strain ϵ_1 to twice the lateral strain $\epsilon_2 = \epsilon_3$) versus principal stress difference $\sigma_1 - \sigma_3$. In Figure 2.10 a typical relationship is given between volumetric strain and

axial stress for quartzite specimens, having the same diameter as the loading platens, tested in uniaxial compression in an ordinary loading machine. Fracture initiation is manifested by departure of the stress-volumetric strain curve from linearity as point A in Figure 2.10.

It can be seen from Figure 2.10 that a number of distinct regions may be observed. Region I characterizes closing of cracks, Region II, giving a linear relationship, represents the elastic deformation of rock, Region III characterizes stable fracture propagation while Region IV represents unstable fracture propagation. It should be noted from this figure that fracture initiation is also manifested by the departure from linearity of the stress-lateral strain curve but this does not apply to the stress-axial strain curve.

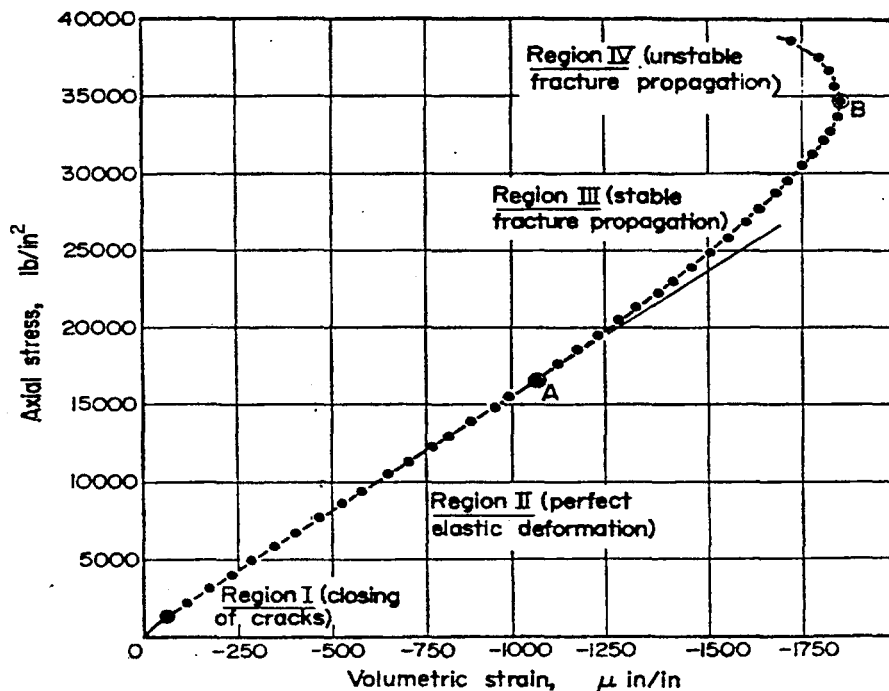


Figure 2.10 Relationship between axial stress and volumetric strain for quartzite in uniaxial compression tested in a conventional loading machine (Bieniawski, 1967).

There is, further, considerable evidence described in the literature that fracture in rock takes place in the direction of the major principal stress. Hallbauer et.al. (1973)

sectioned rock specimens that had been loaded to preset limits. These sections revealed a predominance of axial fractures (the direction of the major principal stress), and also demonstrated that microcracking takes place well below the ultimate strength of the rock. This corresponds to the result of Peng (1976) who monitored surface strains on a rock specimen. He found that fractures occurred first in areas of extension strain.

Stacey (1981) expected that the initiation of fractures would be indicated by a small noticeable increase in the lateral strains, i.e. as the extension fractures formed in planes normal to the direction of action of the confining pressure (the minor principal stress direction) the specimen would expand more rapidly in the radial direction. The expected change in lateral strain was observed as can be seen from Figure 2.11. As can be expected, the magnitude of the change was generally less for greater values of confining pressure. It was found that, for each set of rock specimens, the change point occurred at approximately the same magnitude of lateral extension strain. Stacey (1981) proposed that fracture of brittle rock will initiate when the total extension strain in the rock exceeds a critical value which is characteristic of that rock type.

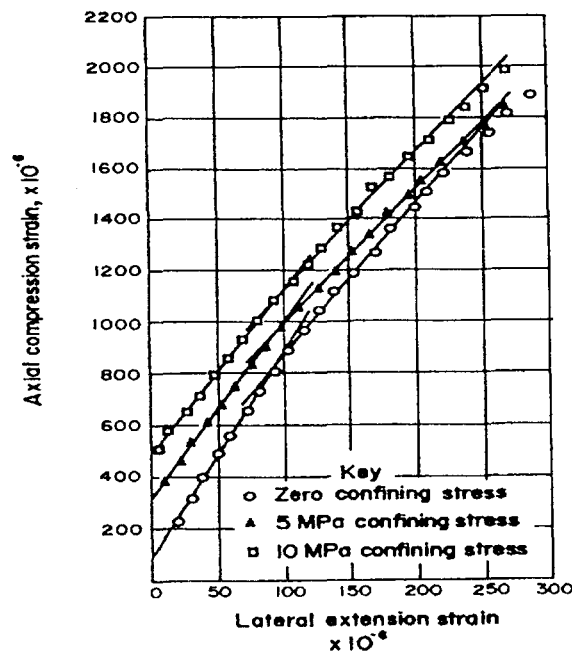


Figure 2.11. Results from rock specimen testing (sandstone) (Stacey, 1981)

The levels of strain shown in Table 2.1 are very low and in a uniaxial compressive test correspond to a stress level in the region of 30% of the UCS. This corresponds well with observations made by several other researchers. Brace & Byerlee (1968) found that dilation associated with fracture began at a stress of one third to two thirds of the ultimate fracture strength.

Rock type	Core Size	Specimen length/ diameter ratio	Critical extension strain
Quartzite A	BX	2	0.000120
Quartzite B	BX	2	0.000109
Quartzite C	AX	2	0.000081
Quartzite D	BX	2	0.000107
Quartzite E	BX	2	0.000130
Lava A	BX	2	0.000152
Lava B	BX	2	0.000138
Diabase	BX	2	0.000175
Norite	NX	2.5	0.000173
Conglomerate	BX	2	0.000086
Conglomerate	BX	2	0.000073
Conglomerate	BX	2	0.000083
Sandstone	BX	2	0.000090
Shale A	BX	2	0.000160
Shale B	BX	2	0.000150
Shale C	AX	2	0.000095

Table 2.1 Critical Values of Extension Strain (Brace & Byerlee, 1968)

Results presented by Rummel (1971) show that primary ‘tensile’ crack initiation occurs at about one third of the ultimate strength. For concrete cubes, Jones (1952) found that ‘tensile’ fractures initiated at only 25-30% of the ultimate strength.

2.3 CRACK PROPAGATION

The failure behaviour of the Witwatersrand quartzite was studied in stamp load tests by Wagner and Schumann (1971). They expressed this in terms of the maximum and minimum principal stresses. Based on triaxial compression tests, the following criterion was established ;

$$\sigma_{\max} \leq 207 \pm 0.6\sigma_{\min} \quad (\text{MPa})$$

where $\sigma_{\max}$ is the maximum principal stress and $\sigma_{\min}$ is the minimum principal stress. In this criterion compression has been taken as positive so that the negative sign in the equation is valid for the tensile region. Wagner and Schumann also assumed the tensile strength of the quartzite to be one-tenth of the uniaxial compressive strength (Figure 2.12).

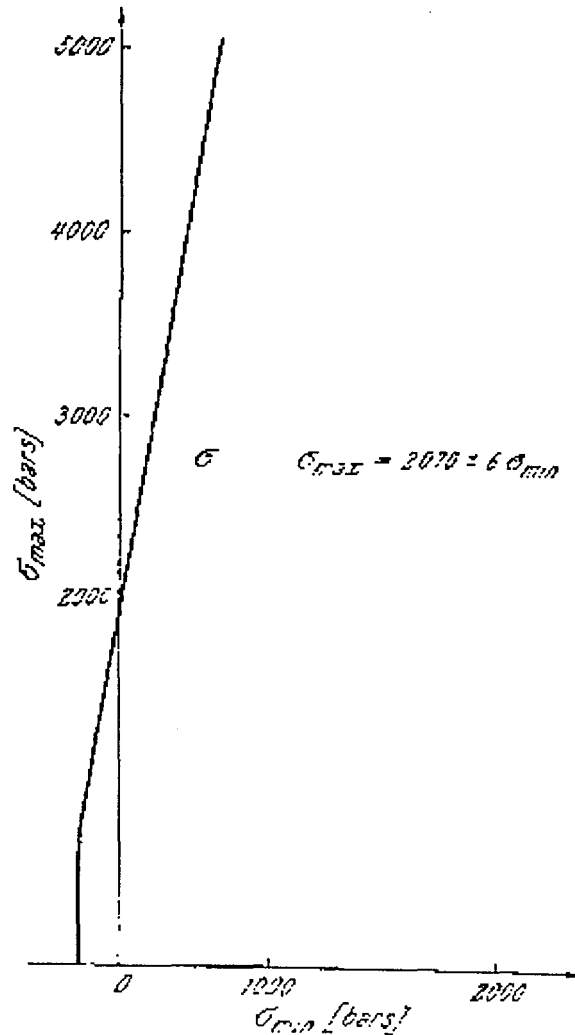


Figure 2.12 Failure criterion of Witwatersrand quartzite (Wagner & Schumann, 1971)

Four different values of stamp load were used and the average stress at the stamp were increased in equal steps from 200 MPa to 800 MPa. The resultant fracture zones are shown in Figure 2.13. It is obvious that the failure of rock starts with a tensile ring crack next to the edge of the stamp. This initial tensile crack is already well developed at a stamp load equal to the uniaxial compressive strength of the

rock. With increased stress the region of tensile failure grows but at the same time the rock starts to fail along the contour of a hemispherical region beneath the stamp, while the rock inside the hemispherical region remains solid because of the high confinement. The failure of the rock along the hemispherical region is partly compressive and partly tensile.

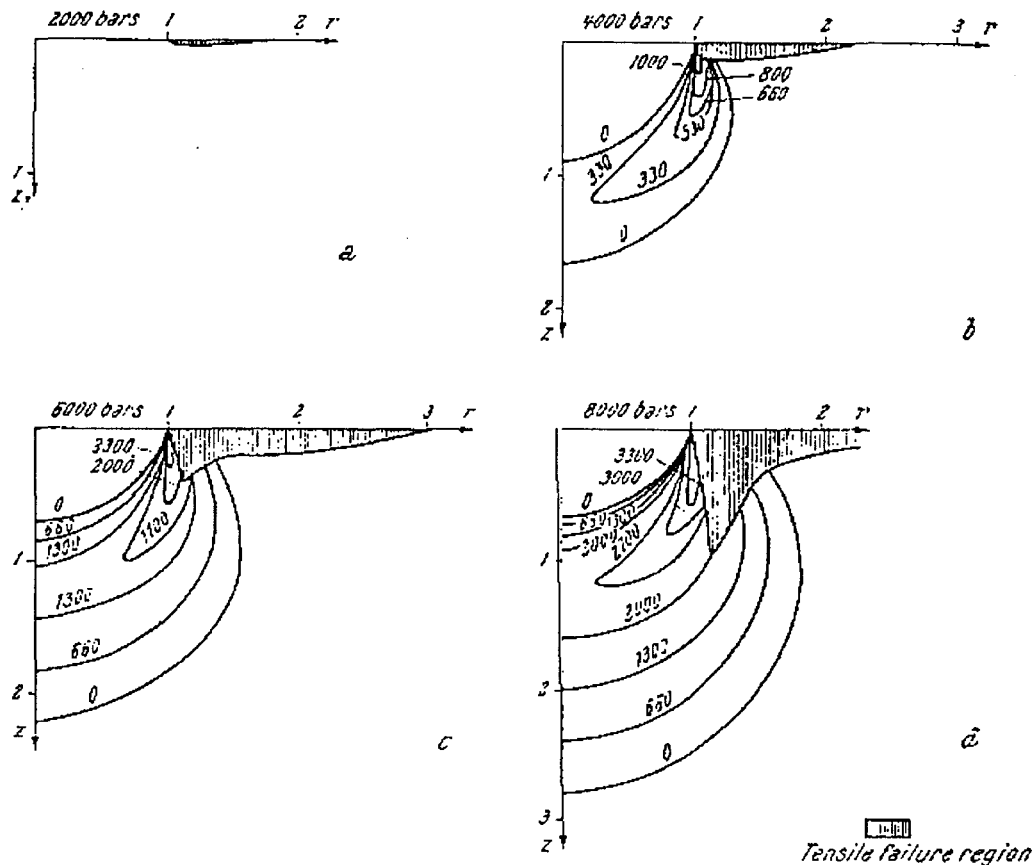


Figure 2.13 Elastic stamp- Fracture zones of four different contact stresses (Wagner & Schumann, 1971)

Failure around the hemispherical region is developed completely a uniform stress of 400 MPa over the contact area. The highest intensity of the fracture can be expected near the edge of the stamp where the excess stress reaches values of more than 100 MPa.

With further increase in stress, up to a value of three to four times the uniaxial compressive strength of the rock, the general fracture pattern remains almost the same although the intensity of fracture, as indicated by the excess stresses and extension of the fracture zone increased. Of particular interest is the fact that the

depth of the tensile failed region near the edge of the stamp has grown from about a tenth of the radius of the stamp at 400 MPa to nearly the radius of the punch at 800 MPa over the contact area. At stamp stresses of 800 MPa, excess stresses of more than 300 MPa have been calculated near the edge of the stamp. Though the excess stresses decrease with increasing distance from the contact area they still exceed 200 MPa at the axis of the stamp about a radius underneath the contact area.

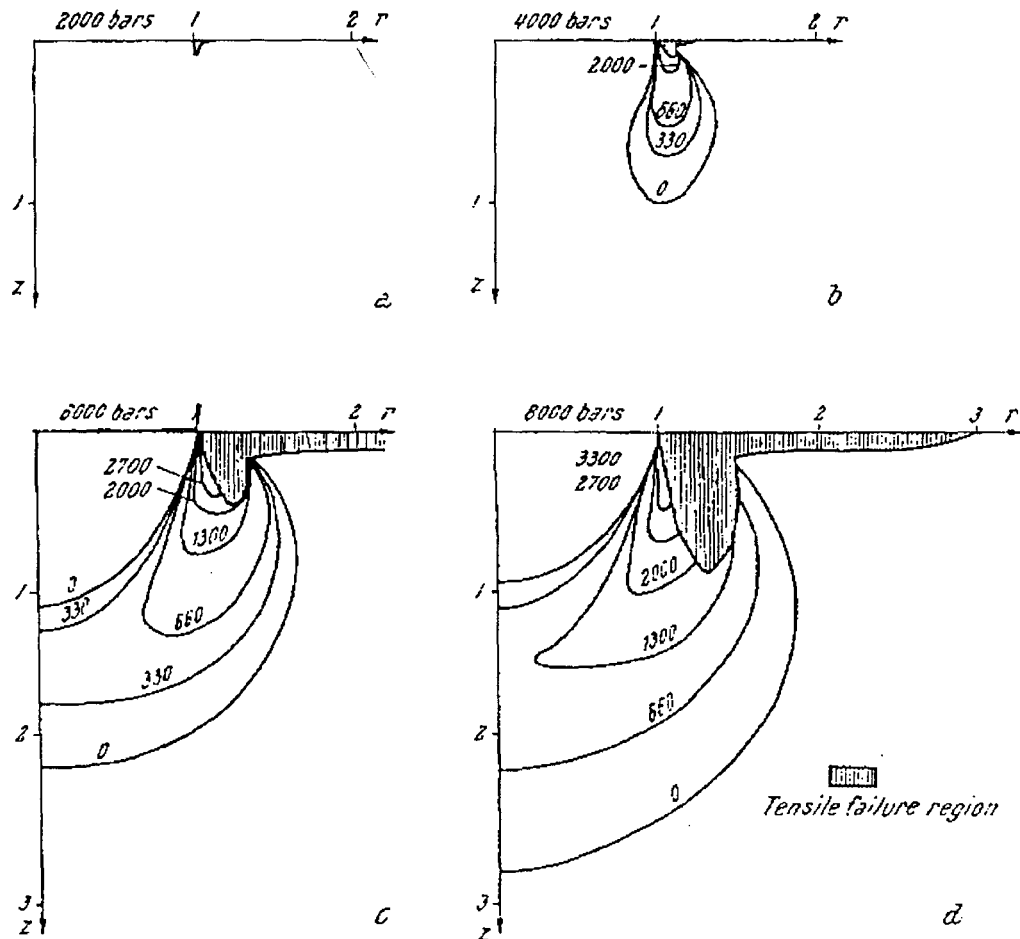


Figure 2.14 Rigid stamp- Fracture zones of four different contact stresses (Wagner & Schumann, 1971)

Wagner and Schumann analyzed the fracture zone underneath a flat-ended rigid stamp using the procedure already described for the elastic stamp (Figure 2.14). From the results obtained, it can be assumed that the loading conditions only influence the early stages of fracture and are unimportant for the final failure of rock under stamp loads.

The model proposed by Wagner and Schumann (1971) for the mechanism of the failure is shown in Figure 2.15. Part 1 of this diagram shows the initial loading stage at which the rock behaves elastically. Once the tensile ring crack has increased sufficiently, the surface moves slightly upwards and becomes distressed and stage 2 can best be described as the elastic rebound of the surface. With further increases in stamp load the rock fails under compression and the volumetric expansion becomes effective in the direction of the minimum principal stress. As a consequence of this lateral expansion, the surface moves upwards and the sign of the surface strain changes from tension to compression, stage 3. Finally, during stage 4, the volumetric expansion increases rapidly and the rock specimen fails outside the contact area.

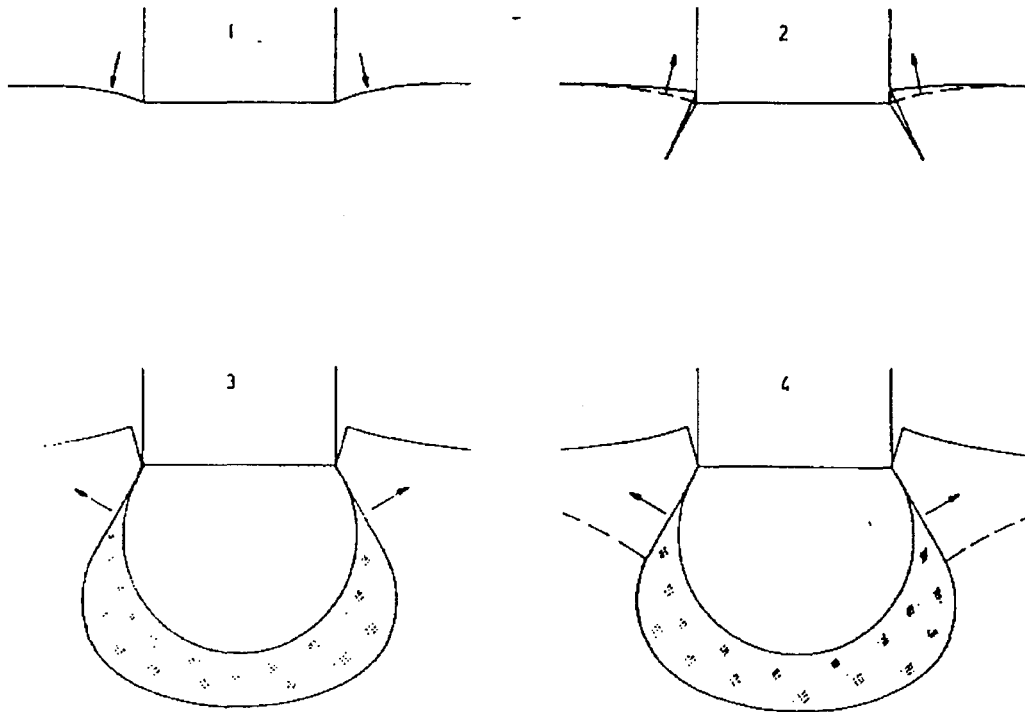


Figure 2.15 Schematic representation of the failure mechanism of rock under stamp loads (Wagner and Schumann, 1971)

The material propensity for fracture depends on the presence of discontinuities which can occur as cracks, open fissures, zones filled with moist clay or marl, very hard concretions or pebbles in a weaker matrix and or large crystals or minerals with a fissile structure such as mica. If the fracture fronts encounters such discontinuities, it may cut through them, or it may stop. In order to understand the behaviour of

cracks under these circumstances; the mechanism of propagation in front of the fracture tip: the (Gramberg, 1989. See Figure 2.16).

The pattern of the microstress field around the fracture edge travels at the same speed as the fracture edge itself. The concentrated induced tension and compression is similar to the impression of a transverse wave; the stress wave (Hall, 1953, Corten and Park, 1963). The material particles move perpendicular to the direction of the fracture propagation, and as has already been pointed out, in front of the fracture edge there must be a zone of extra transverse strain, the "scouting lane" (SL). The particles on the left hand side move to the left, whereas, those on the right side move to the right. These transverse movements are transferred from one particle or atom to the next one by cohesive forces, such as shearing. See Figure 2.17-a. This transfer of movement provides the condition for the fracture propagation. Corten and Park (1963) have developed an equation to describe this process;

$$\delta E_e = \delta \gamma + \delta W + \delta E_k \dots\dots\dots (2.1)$$

E_e stands for the elastic energy; δE_e is a function of the lateral deformation, i.e. of the transfer of the movements by shear stress. γ stands for the surface energy; W is the energy of plasticity and δE_k is the kinetic energy.

In a massive, intact anisotropic dense material, the fracture propagation is undisturbed and fast because $\delta W = 0$, $\delta \gamma$ is constant and as a result, δE_k may reach high values. See Figure 2.16-a. If the fracture front finds a discontinuity in its way, such as an open fissure or a layer of foliaceous, fissile, or laminated material, then the propagation stops. In that case the term E_e becomes zero because of the lack of contact at the interface and the poor ability to transfer shear forces. See Figure 2.17-a. This can happen in rock with a schistose structure.

However, if the fissures, or the materials with a fissile structure, are compressed at a high angle to those structures, then the axial cleavage fracture can cut cleanly

through these discontinuities. Because of the good mechanical contact, the shear forces will be transferred and δE_c can develop. See Figure 2.17-c.

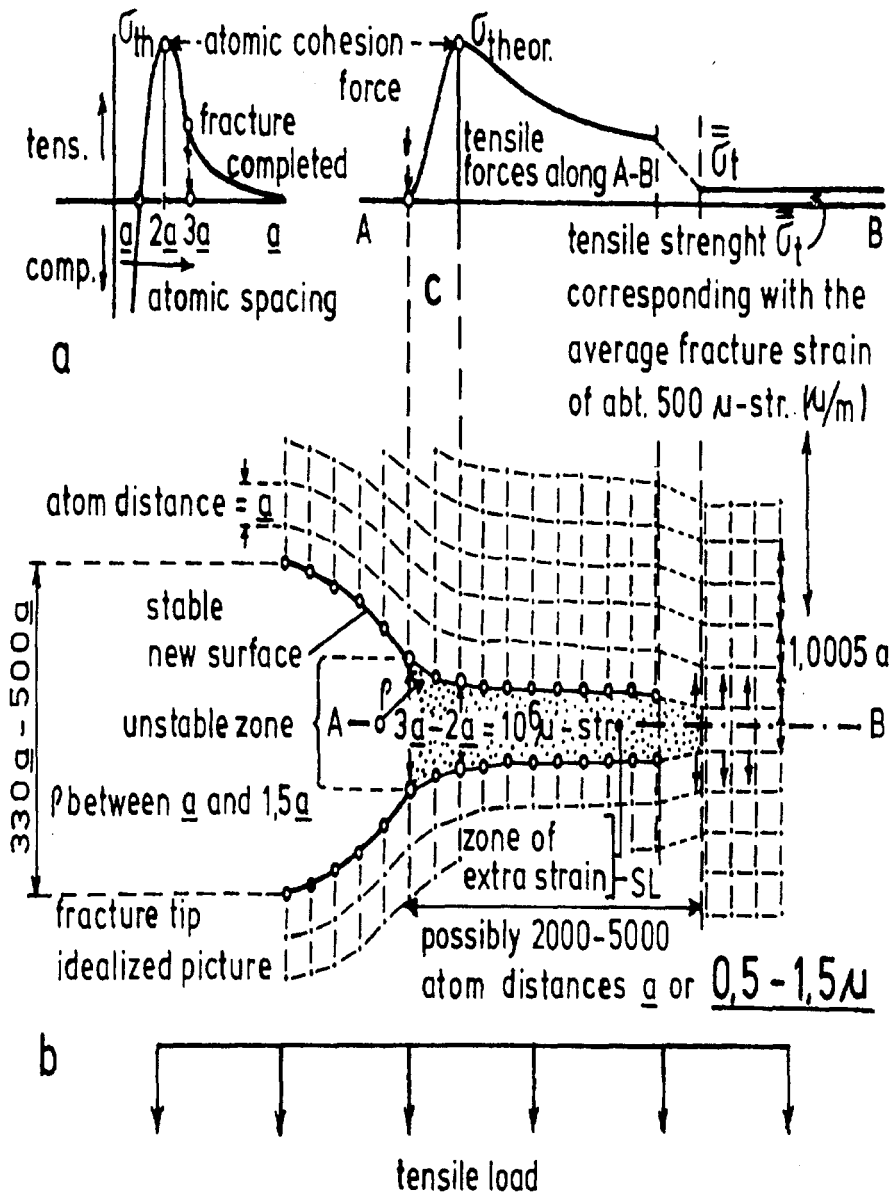


Figure 2.16 The fracture edge and tip as a regular array of points, representing atoms; direct tension
a: the interatomic forces as a function of the spacing a b: schematic representation of the arrangement of the atoms around the fracture edge and tip; maximum forces at $2a$; fracture complete at $3a$; stable new surface has generated; unstable zone $3a-2a$ wide; radius of atomic sharpness (Gramberg, 1989)

If a fracture front meets a plastic zone, such as moist argillaceous material (clay), the propagation stops because the term δW is so large that the elastic energy δE_c dies out. If on the other hand, the contacts are perfect and the material has no plastic characteristics, the fracture propagation may continue by inertia. In this case, δE_c is

very large, and $\delta\gamma$ has reached its limit and $\delta W=0$. However, δE_k is sufficiently elevated to make the fracture extend by inertia in zones where the conditions were previously unfavorable.

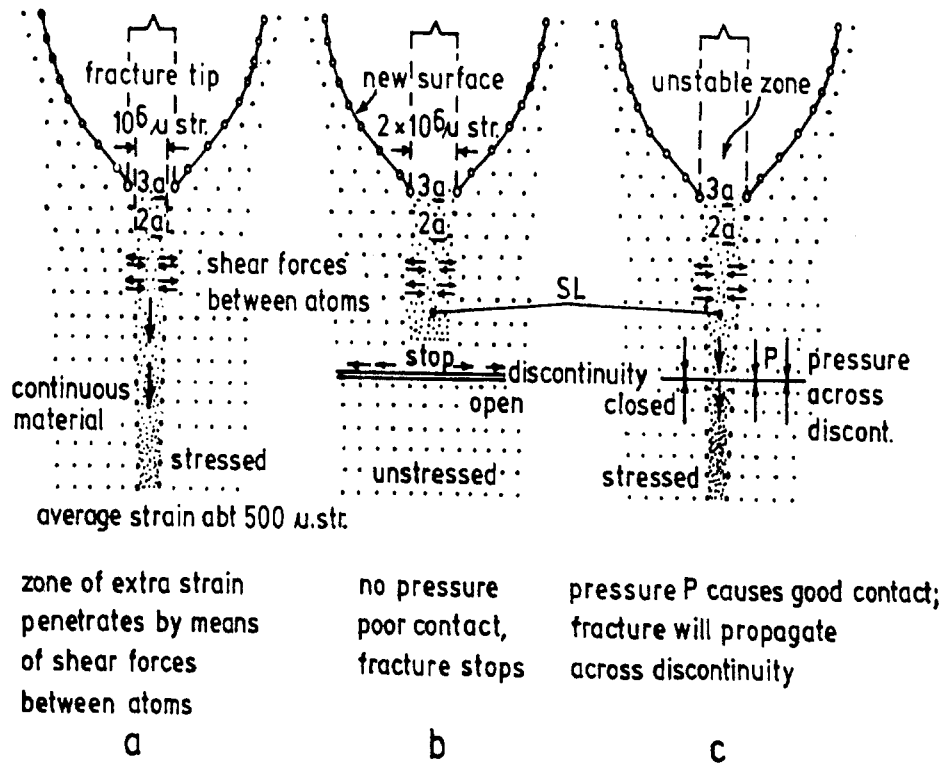


Figure 2.17 A fracture meets a discontinuity in order to explain the characteristics of the conductivity for fracture propagation referring to the picture of the fracture tip, represented by a regular array of points, standing for atoms, (Corten and Park, 1963)

Sometimes the propagating fracture meets hard pebbles or concretions embedded in a weaker matrix. Occasionally the pebbles are cut cleanly, at other sites the fracture will pass around the pebbles.

In the first case, the amount of δE_k in the fracture front of the matrix must become rather large; therefore δW must equal 0. A large δE_k is required because the term $\delta\gamma$ for hard pebbles is much larger than that of the matrix. In order to bridge the difference, the value of δE_e has to be high in order to elevate δE_k . Another point is that the cement as well as the mechanical contact between the matrix and the pebble

should be excellent. This means in many cases that the pressure, i.e. the depth, must be sufficiently large or, that a pressure zone is present (Gramberg, 1989).

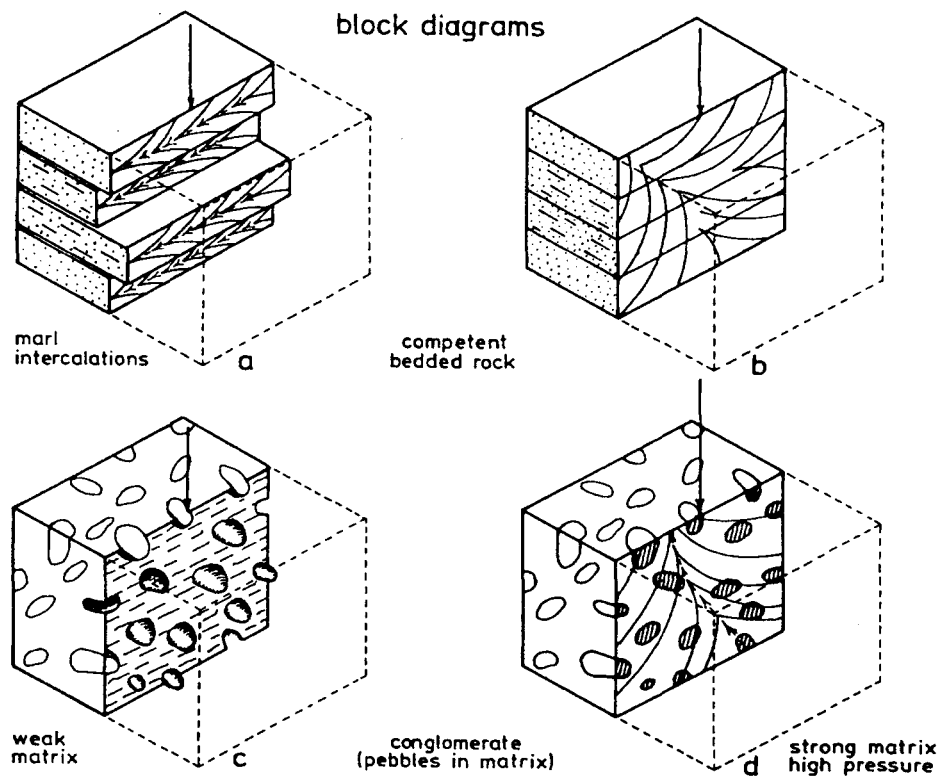


Figure 2.18 Examples of axial cleavage fractures in the cases of bad contact and good contact a: a series of brittle rock layers with marl intercalations; the marl does not transmit shear forces; every cleavage fracture has to begin in each layer separately; the mode of propagation leaves the herringbone or chevron lineation on the separate fracture planes; the total of these fractures shows an uneven composite step-like or scalariform zone b: a comparable series of brittle rock layers without weak intercalations; the cleavage fracture plane shows one single planar plane with large lineations c: a conglomerate, consisting of hard pebbles in a weak matrix, shows a cleavage fracture plane with protruding hard pebbles d: a conglomerate with hard pebbles under high pressure will conduct the cleavage fracture, cutting cleanly through the matrix as well as through the pebbles (Iyengar, Alwar, 1964)

If the depth is not great enough, or the pressure zone is not developed, then the pebbles or concretions will protrude from the cleavage fracture surface. See Figure 2.18. Occasionally, such features may be useful to estimate the depth or the existence of a pressure zone.

2.4 CRACK PATTERN

A series of tests was carried out by Nolen, Hoeksema and Gordon, 1987 in which double cantilever beam test specimens with 76 x 48 mm faces, 25 mm deep central

slots, and thicknesses between 16 and 25 mm were used for the experiments. A load frame was designed to permit the specimen to be split by a wedge driven into its end while on the stage of a reflecting light microscope. A hydraulic loading system consisting of a hand-operated screw press and a steel bellows working through a level was used to advanced the wedge.

The crack pattern at two early stages of the development of the fracture in a specimen of dolomite marble is shown in Figure 2.19. In the first stage, shown by solid lines, the applied load has reached 73% of that required to break the specimen; in the second, shown by dashed lines, it has reached 85%. Once a crack nucleates at the roof of the notch, it branches and spreads into the pattern shown, with no indication as to which of many possible paths through the crack network will eventually become the rupture surface. Additional cracks were observed to have opened within the region already containing cracks when the specimen finally failed. These are shown by the dotted lines in Figure 2.19; areas in which spalling took place are shaded. Not all the open cracks in the pre-failure network are connected to one another. Discrete crack segments form ahead of the connected network and are later incorporated into it as the level of force on the specimen continues to increase (Nolen, Hoeksema and Gordon, 1987).

The crack pattern development near the starting notch in another specimen, is illustrated in Figure 2.20. Factors that can be used to decide whether or not a particular crack was already present in the rock include the sharpness of the edges of the crack (pre-existing cracks are rounded during polishing) and the presence of unsorted, angular debris, which is formed in the cracking process and is different in appearance from polishing debris.

Drawings showing the granular development of cracks during the tests on limestone using a sharp and a truncated wedge are presented in Figure 2.21 and 2.22 respectively.

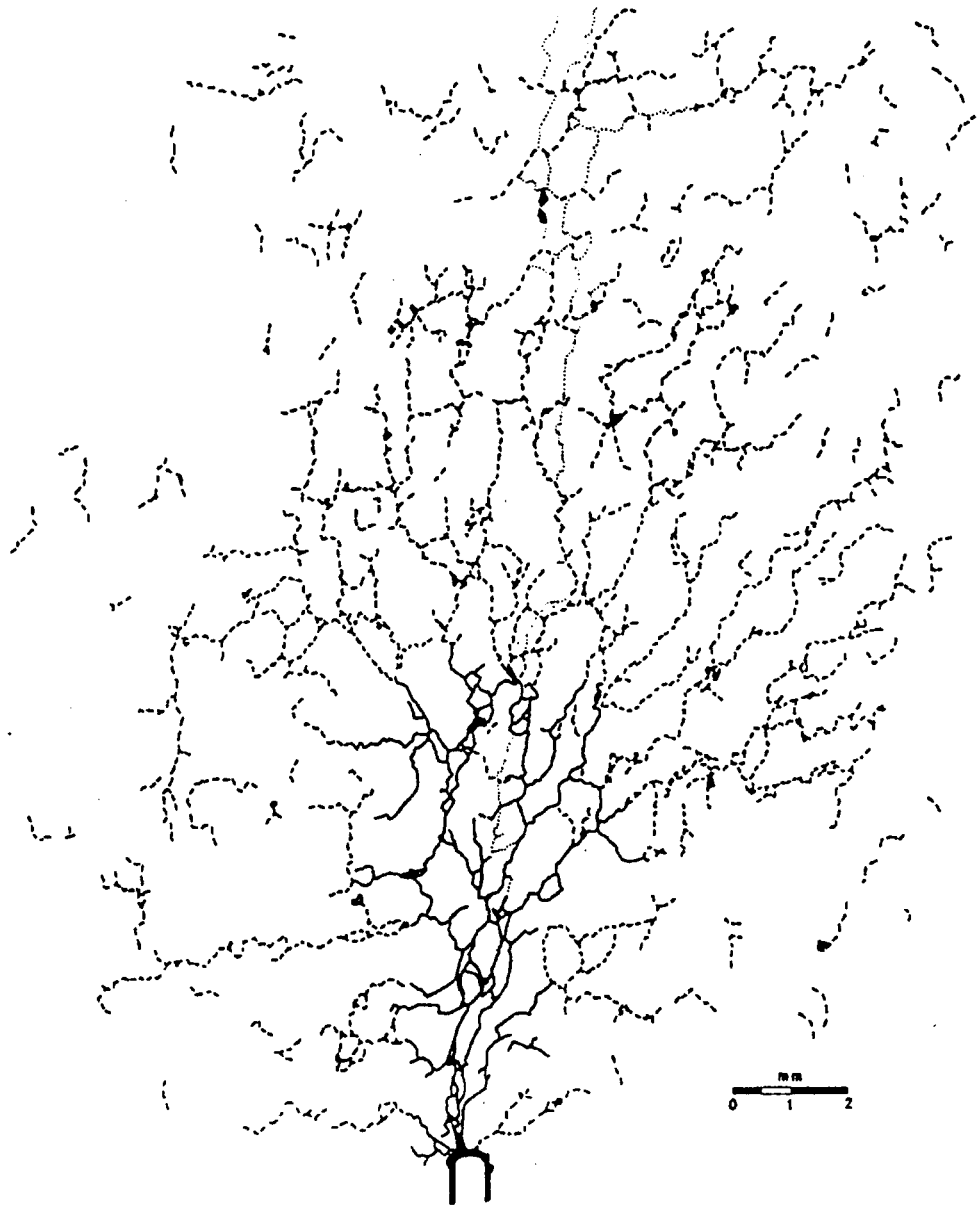


Figure 2.19 Two successive stages in the development of the crack pattern during the fracture of dolomite marble specimen. The solid lines show the pattern developed at 73% of the maximum applied force, the dashed lines, that at 85% and the dotted lines, cracks found after the specimen rupture (Nolen & Gorden, 1987).

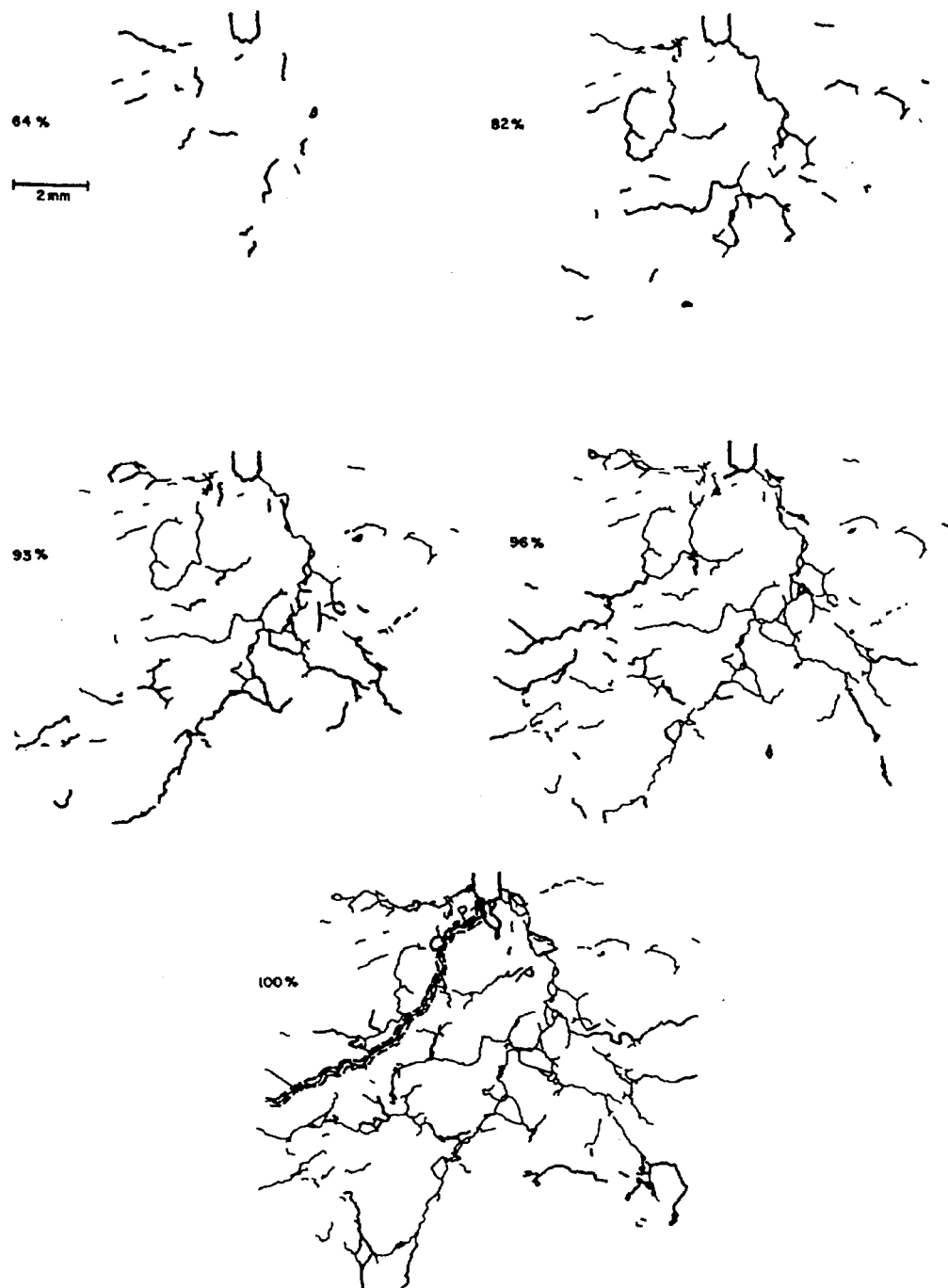


Figure 2.20 Successive in the development of the crack pattern during the fracture of double cantilever beam test specimen. Cracks formed in earlier loading shown with light lines (Nolen & Gorden, 1987).

Figure 2.21 shows a test in which the first crack initiates soon after the load is applied. The slight chamber of the right side of the sharp wedge is less than 1 mm long, the remainder of the edge being fully symmetrical. When the load is increased, the first crack propagates in steps and new cracks are initiated at the edge directed radially downwards, some of them at a wider angle to the load axis. Further loading gradually opens the dominating crack by wedging, but other radially-directed cracks may also propagate, Figure 2. 21-h.

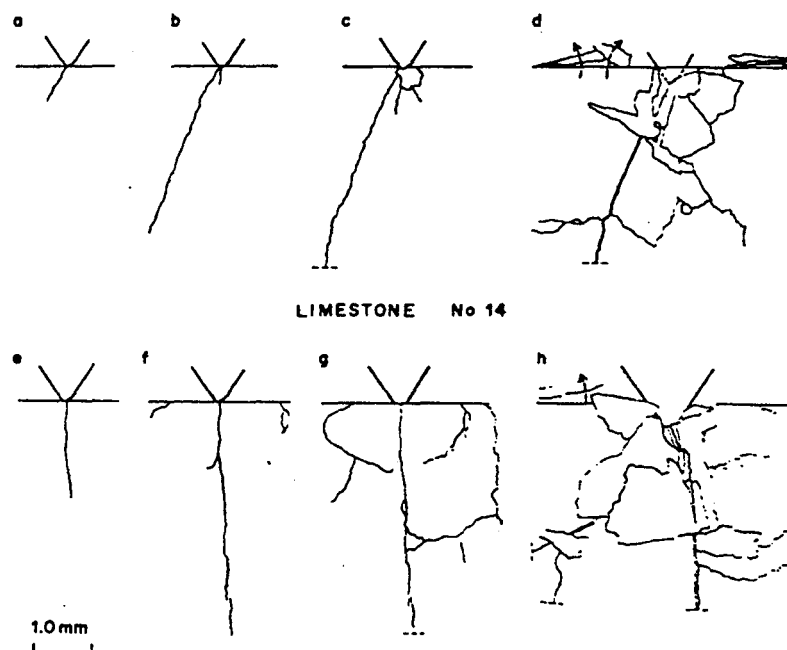


Figure 2.21 Crack pattern of sharp wedge indentation in limestone. Arrows indicate movement of fragments at chipping events (Lindqvist, Lai and Alm, 1984).

An increasing load ultimately leads to the failure of the specimen, due either to propagation of the dominating crack through the specimen or to excessive scaling of the front face (Lindqvist, Lai and Alm, 1984).

The general sequence of cracking events caused by the truncated wedge in limestone is shown in Figure 2.22. A marked difference in the character of the cracks in limestone and in the marble can be noted. While limestone exhibits straight and fairly continuous cracks, those in marble are irregular and discontinuous. This is

certainly due to different material properties such as grain size, grain bonding and the mineral cleavage planes.

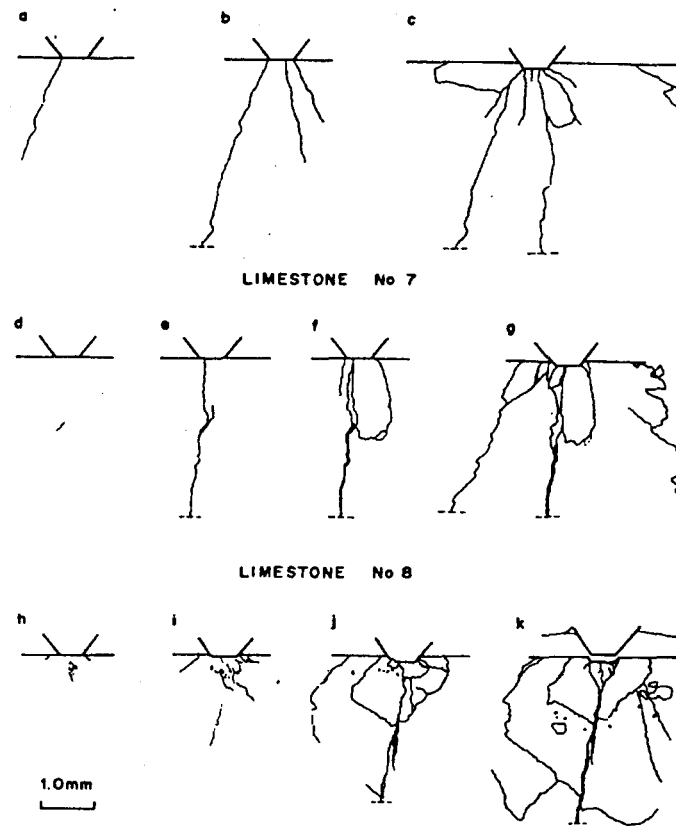


Figure 2.22 Crack pattern of truncated wedge indentation in limestone (Lindqvist, Lai and Alm, 1984).

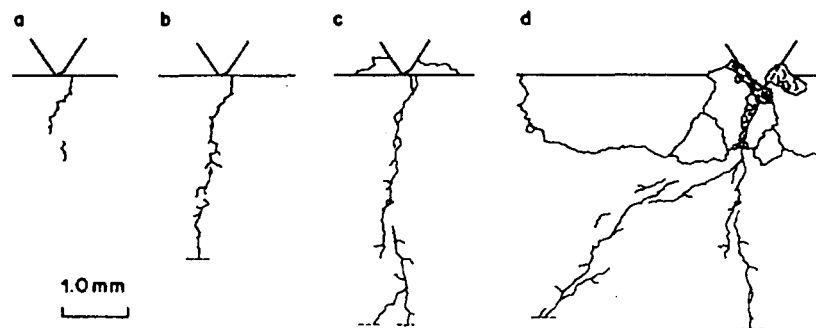


Figure 2.23 Crack patterns of sharp wedge indentation in marble (Lindqvist, Lai and Alm, 1984).

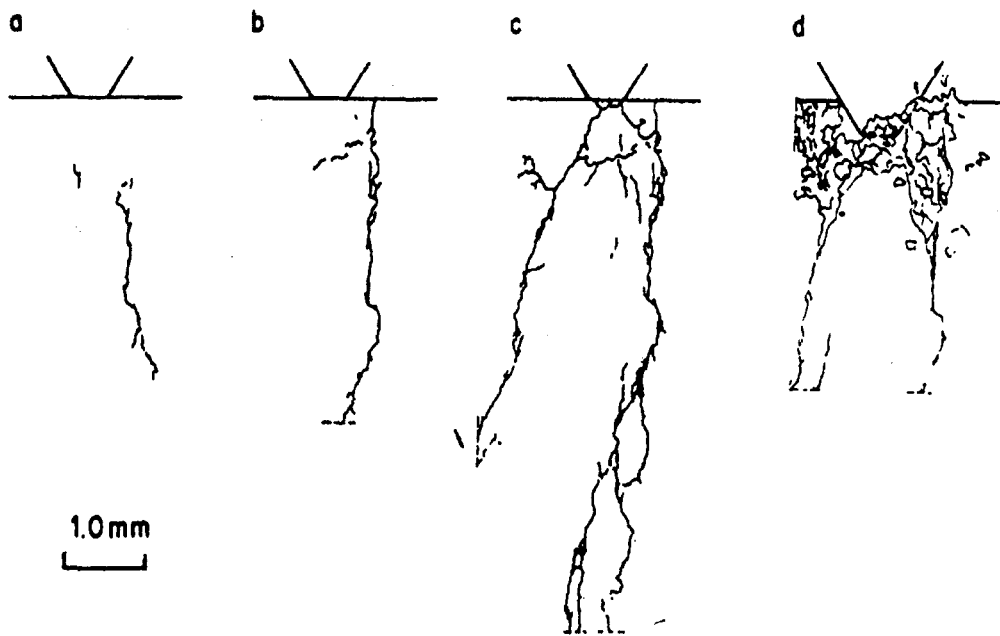


Figure 2.24 Crack pattern of truncated wedge indentation in marble (Lindqvist, Lai and Alm, 1984).

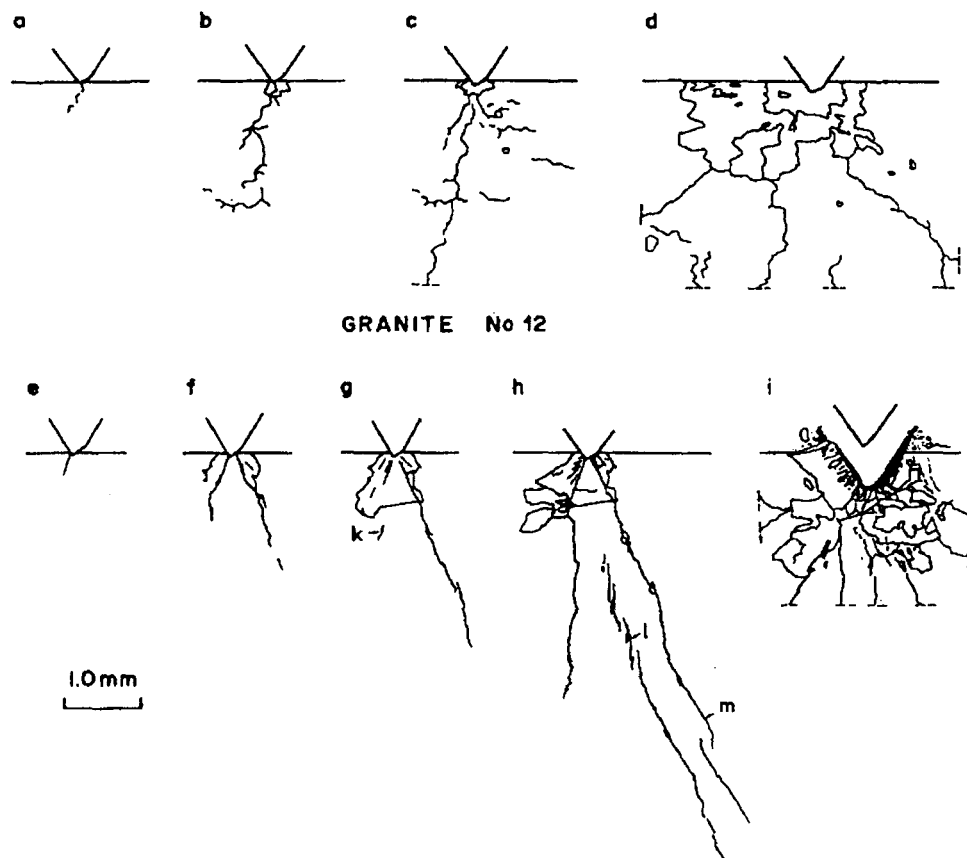


Figure 2.25 Crack pattern of sharp wedge indentation in granite (Lindqvist, Lai and Alm, 1984).

The crack pattern in granite differs somewhat from the crack patterns in limestone and marble. At an early stage of indentation, a few cracks were formed and seemed to become dominant, Figures 2.23, 2.24 and 2.25 (Lindqvist, Lai and Alm, 1984).

The following conclusions can be drawn from the literature review.

- In layered media with Young's Modulus increasing downwards, the stresses normal to the layers are of the same order of magnitude as those obtained in the case of homogeneous media where the strata have an elastic modulus the same as that of the first layer. However where the strata have a modulus which decreases in a downward direction, the stresses normal to the layers are not as low as those in rigid homogeneous strata (Varshney, 1975).
- Soft strata below the top layer reduce tension in the vertical direction as compared to homogeneous strata (Varshney, 1975).
- Stamp load bearing tests show that as the strength of rock under compression increases with confinement it is very unlikely that the rock will fail beneath the stamp where all three principal stresses are compressive (Wagner and Schumann, 1971).
- Considering the relatively small tensile strength of rock it can be concluded that the failure will start with a tensile ring crack close to the edge of the stamp, where the tensile stresses reach their maximum value (Wagner and Schumann, 1971).
- Increased stress in the region of tensile failure grows but at the same time the rock starts to fail along the contour of the hemispherical region beneath the stamp, while the rock inside the hemispherical region remains solid because of high confinement (Wagner and Schumann, 1971).

- It can be assumed that the rock will fail along the contour of the hemispherical region where one of the three principal stresses is tensile (Wagner and Schumann, 1971).
- Wagner and Schumann proposed that the loading conditions only influence the early stage of fracture and are unimportant for the final failure of rock under stamp loads. The flaw in this argument is that both “elastic” and “rigid” analyses assumed no failure of the material and hence any path dependency is lost (Napier, 1996).
- At low confining pressure, the mode of rock fracture changes and a vertical tensile crack, parallel to the axis of the punch, split the specimen before the formation of rock chips at the surface (Cook, Hood and Tsai, 1984).

III LABORATORY EXPERIMENTS AND NUMERICAL MODELLING

3.1. STRESS DISTRIBUTION UNDER A PUNCH LOAD

The purpose of selected examples of elastic modelling has been to assist in the interpretation of the stress distribution and stress changes under a punch loading in a solid and layered media. The stress distribution in the rock adjacent to the punch was calculated by making the following assumptions:

- (i) The rock behaves as a linear elastic, isotropic material
- (ii) The punch applies a uniformly distributed load to the surface of the rock, which is taken to be a semi-infinite mass.

It has been shown that some hard rocks exhibit ideal elastic behavior up to a critical stress level (Wagner and Schumann, 1971), and therefore the first of these assumptions is valid prior to major failure in the specimen. It can be shown that the stresses occurring in an elastic material underneath a punch are dependent on the assumptions made about the nature of the punch. If the punch is assumed to be rigid then, in the regions underneath the corners of the punch, the stresses tend to infinity. On the other hand, finite stress values in the material are calculated if the stress along the contact area between the punch and the elastic body is taken to be uniformly applied. Away from the contact area the stress fields induced by these two loading conditions become virtually identical (Wagner and Schumann, 1971). These stress distributions will be different if the punch shape is changed in particular if a strip punch is considered.

A cross-section view of a strip punch applied to the surface of an elastic body is given in Figure 3.1. This shows a distributed normal load P applied to the surface of a body punch with a contact length of $2b$. A number of authors have developed

the solution for the components of stress within the body when the normal force, P , in as Figure 3.1, is applied to the surface, (Love, 1923). Under uniform vertical loading, the stress distribution in the sample can be calculated by (Jurgenson, 1934)

$$\sigma_z = \frac{P}{\pi} [\sin \alpha \cos(\alpha + 2\rho)] \dots\dots\dots (3.1)$$

$$\sigma_r = \frac{P}{\pi} [\alpha - \sin(\alpha + 2\rho)] \dots\dots\dots (3.2)$$

$$\sigma_r = 2 \frac{P}{\pi} \nu \alpha \dots\dots\dots (3.3)$$

$$\tau_{rz} = \frac{P}{\pi} \sin \alpha \sin(\alpha + 2\rho) \dots\dots\dots (3.4)$$

$$\sigma_1 = \frac{P}{\pi} [\alpha + \sin \alpha] \dots\dots\dots (3.5)$$

$$\sigma_3 = \frac{P}{\pi} [\alpha - \sin \alpha] \dots\dots\dots (3.6)$$

$$\tau_{\max} = \frac{P}{\pi} \sin \alpha \dots\dots\dots (3.7)$$

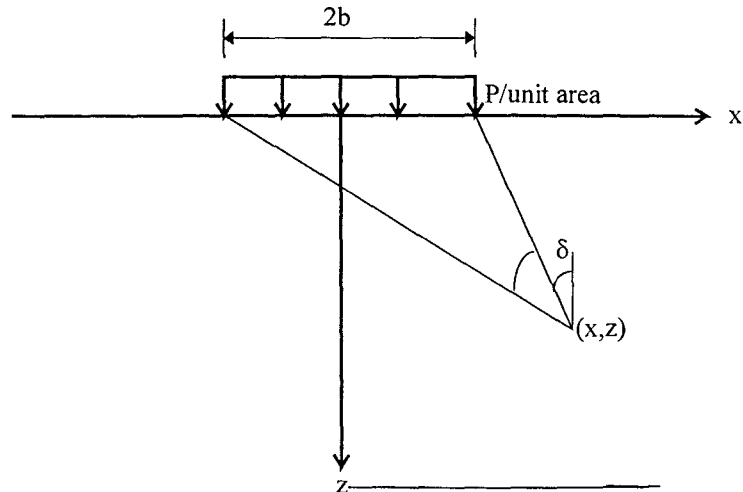


Figure 3.1 Dimensions of the semi-infinite mass, for solid case (Jurgenson, 1934)

The normal stress is taken as 400 MPa and the resultant distributions are given in Figure 3.2 and 3.3. To compare the mathematical solution with FLAC (Itasca, 1994), a plane strain elastic model is prepared in which the boundary conditions and the grid orientation are given in Figure 3.4. The FLAC results are shown in Figure 3.5 and 3.6. The general behaviour of the stress in the material are the same, for example the tensile region is present in both models just next to the punch at the free surface. Just beneath the punch area the magnitude of the stresses are the same in both case as well, for example, at a depth of $2b$ which is the width of the punch, it is around 100 MPa.

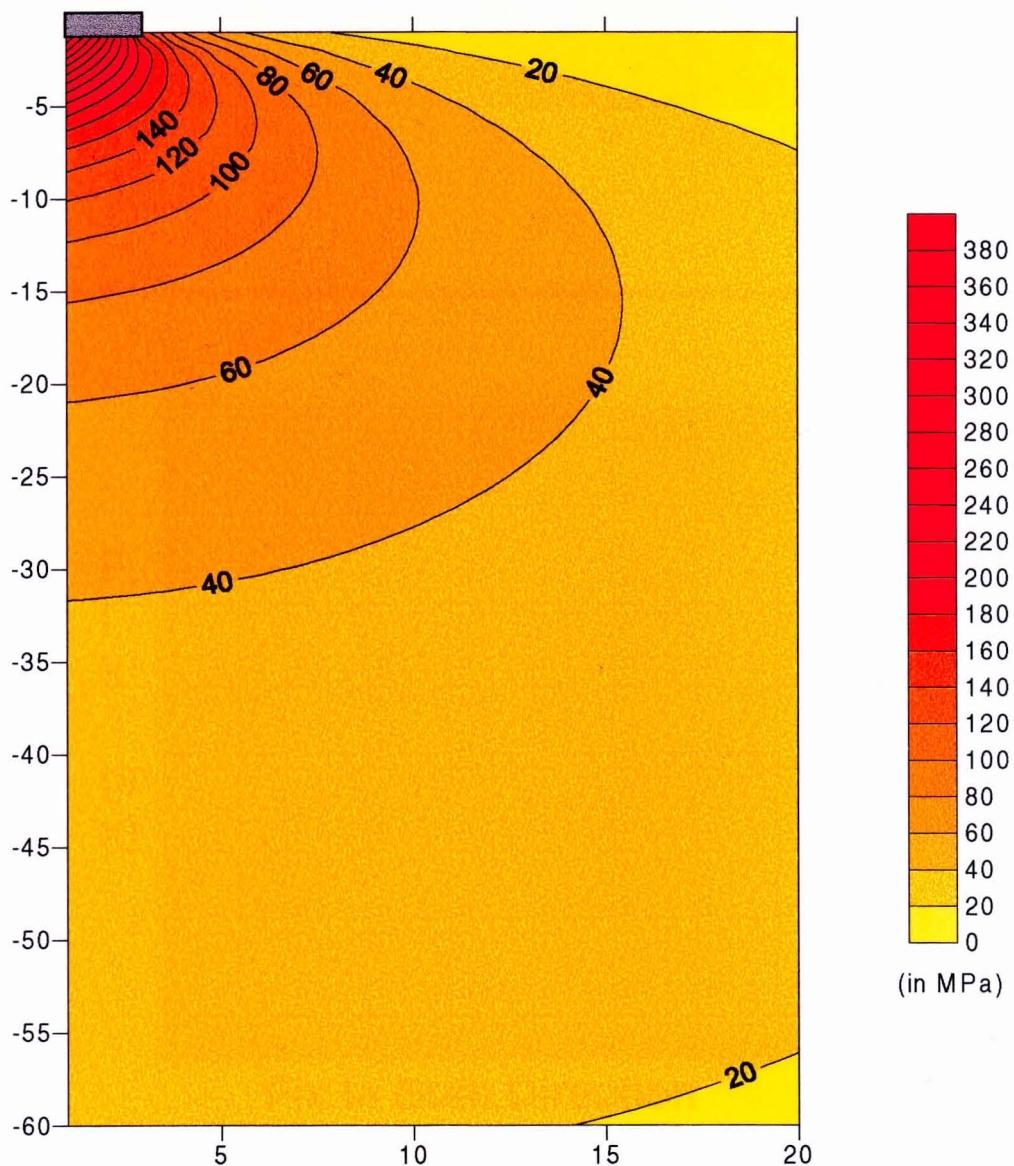


Figure 3.2 Maximum principal stress distribution under 400 MPa punch stress from mathematical solution

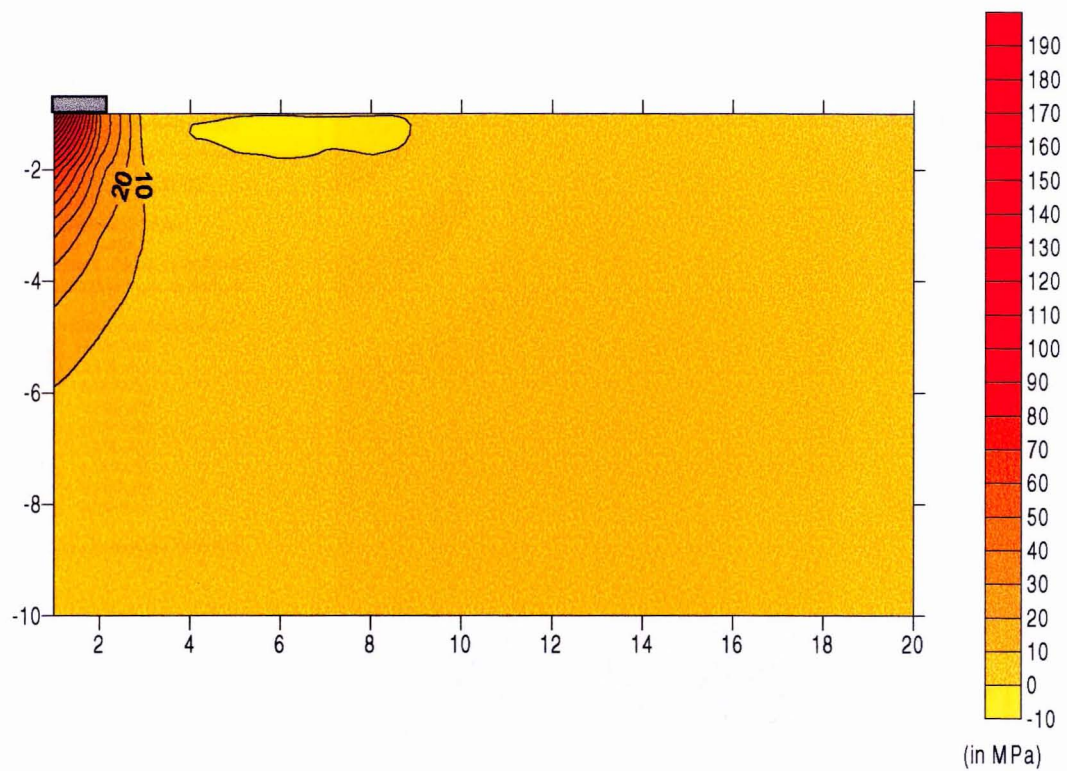


Figure 3.3 Minimum principal stress distribution under 400 MPa punch stress from mathematical solution

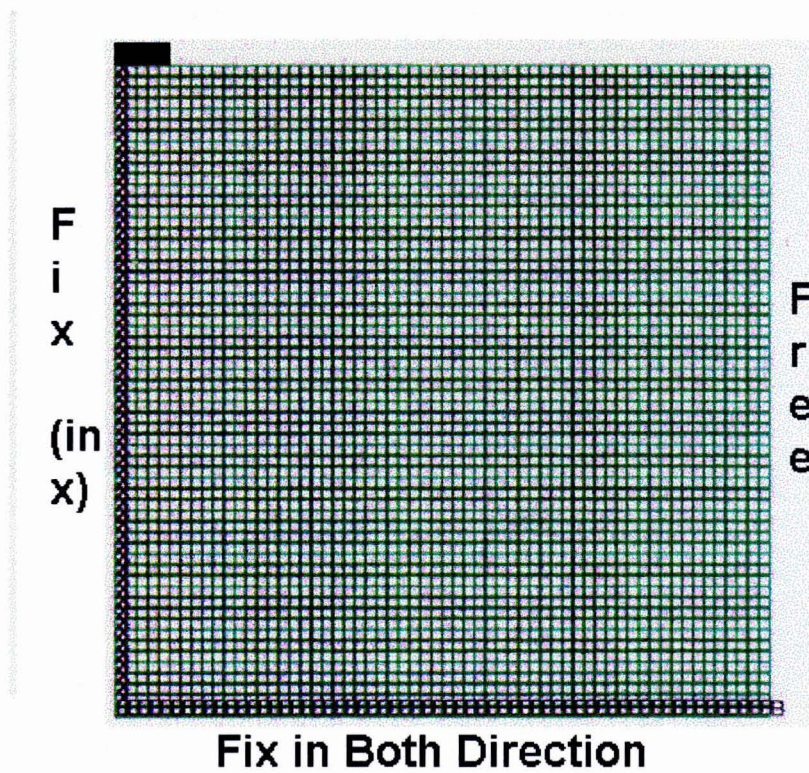


Figure 3.4 Grid orientation and boundary conditions for FLAC model

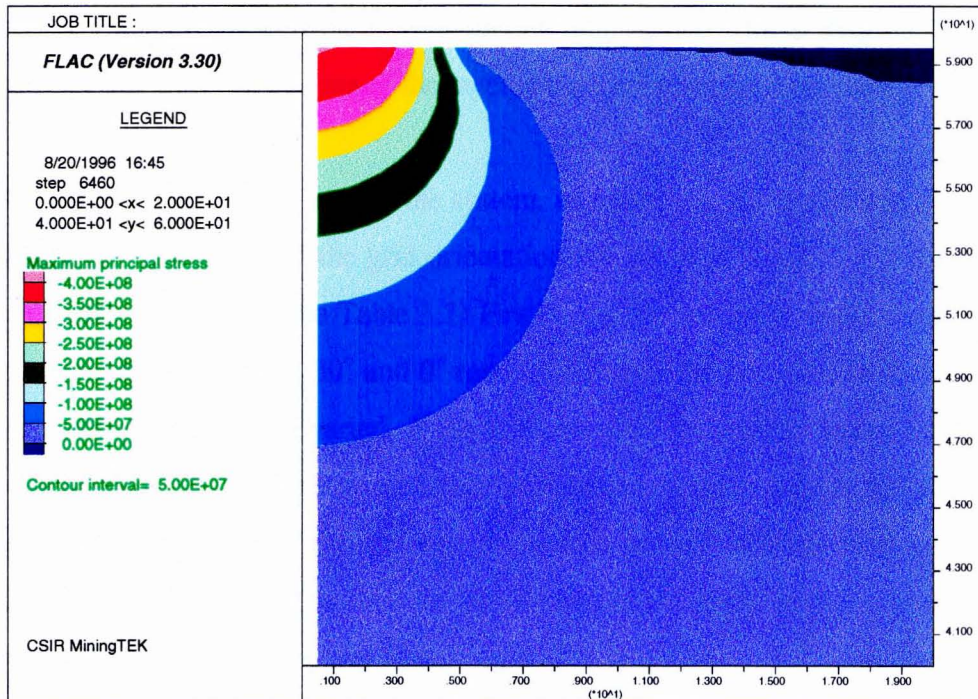


Figure 3.5 Maximum principal stress distribution under 400 MPa punch stress from elastic FLAC model

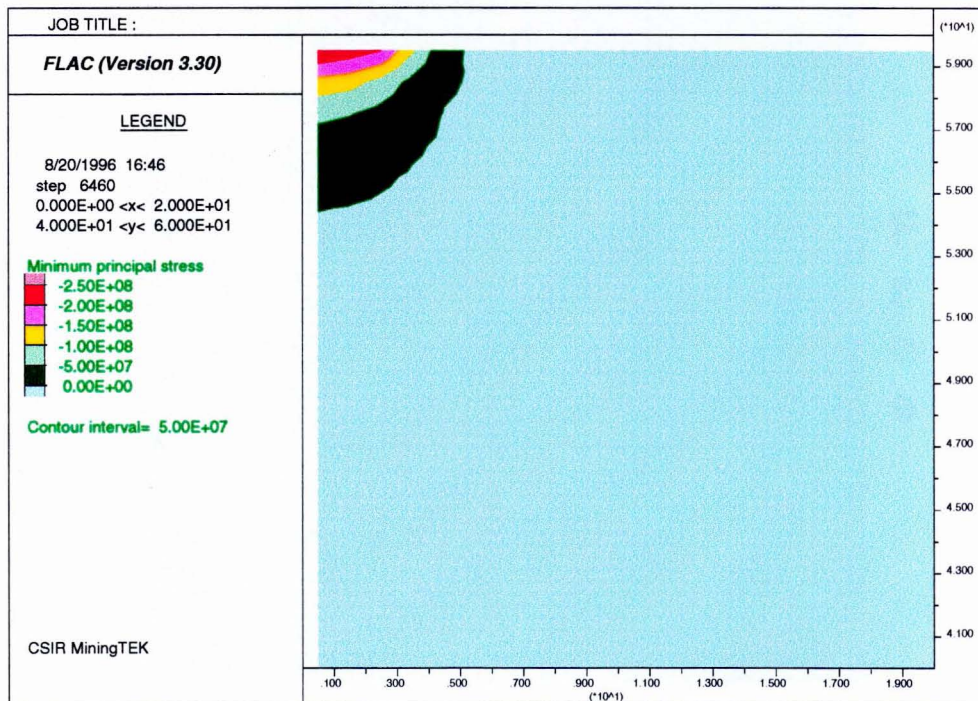


Figure 3.6 Minimum principal stress distribution under 400 MPa punch stress from elastic FLAC model

If a layer is introduced into the system parallel to the free surface, then the principal stresses in the rock are completely different. In this case the layer properties play an important role in determining the stress distribution in the sample. Numerical solutions of the layered case are given in Appendix A.

To see the effect of the layer in the system, an elastic FLAC model was prepared in which the boundary and the grid orientation are given in Figure 3.7. Various layer properties were used (see Table 3.1) Firstly, the friction angle and cohesion of the layer are kept constant, 30° and 0° respectively, but the normal and shear stiffness for the layer have been varied. If the stiffnesses of the layer is increased, then the system converges to the stresses as at the solid case (Figure 3.8,3.9,3.10). At lower stiffness, for example if the stiffness is 10 times greater then the modules which is the case-I, the principal stresses are diminished at the layer, in other words, below the layer there will not be any fracturing.

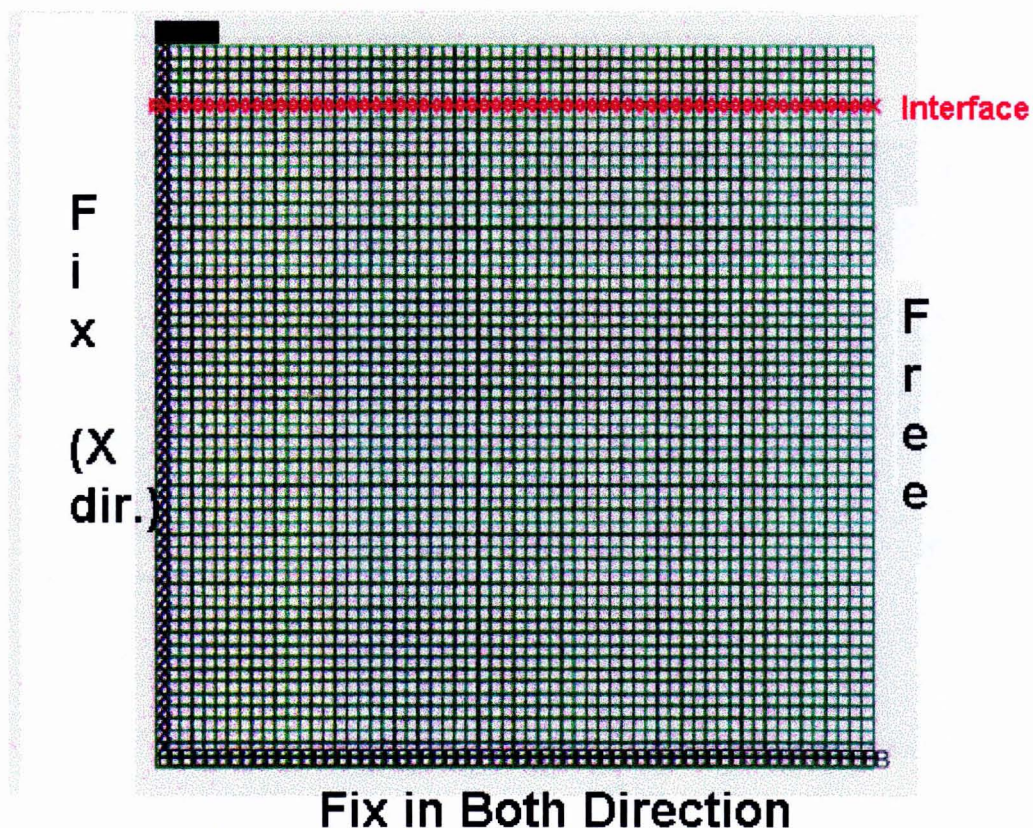


Figure 3.7 Grid orientation and boundary conditions for FLAC model for layered case

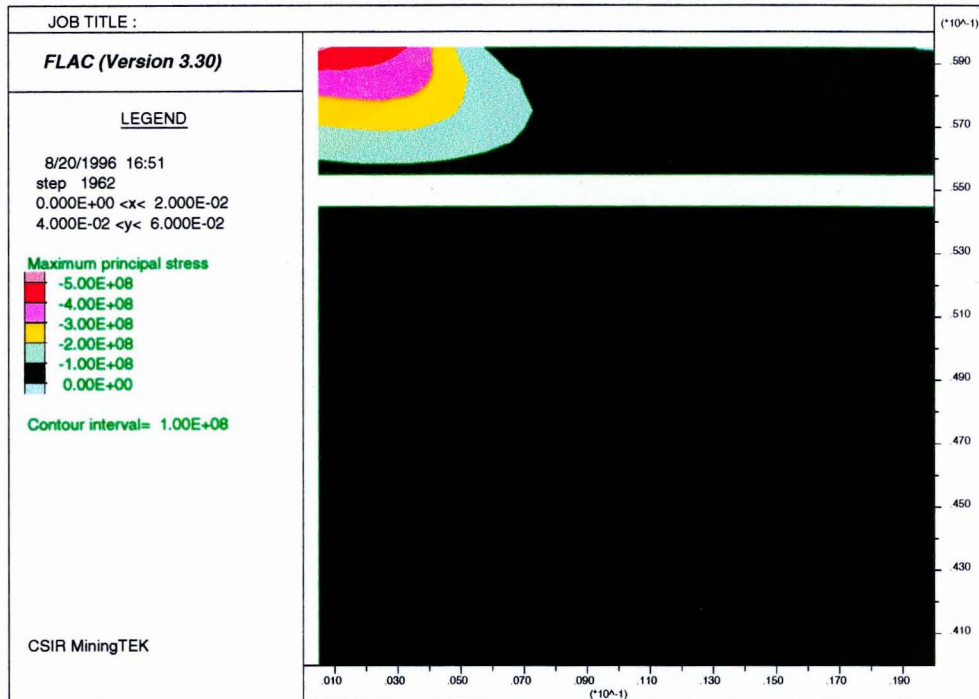


Figure 3.8 Maximum principal stress distribution when layer normal and shear stiffness are taken as 19.10^{10} Pa and 11.10^{10} Pa respectively with zero cohesion and 30° friction angle (case-I)

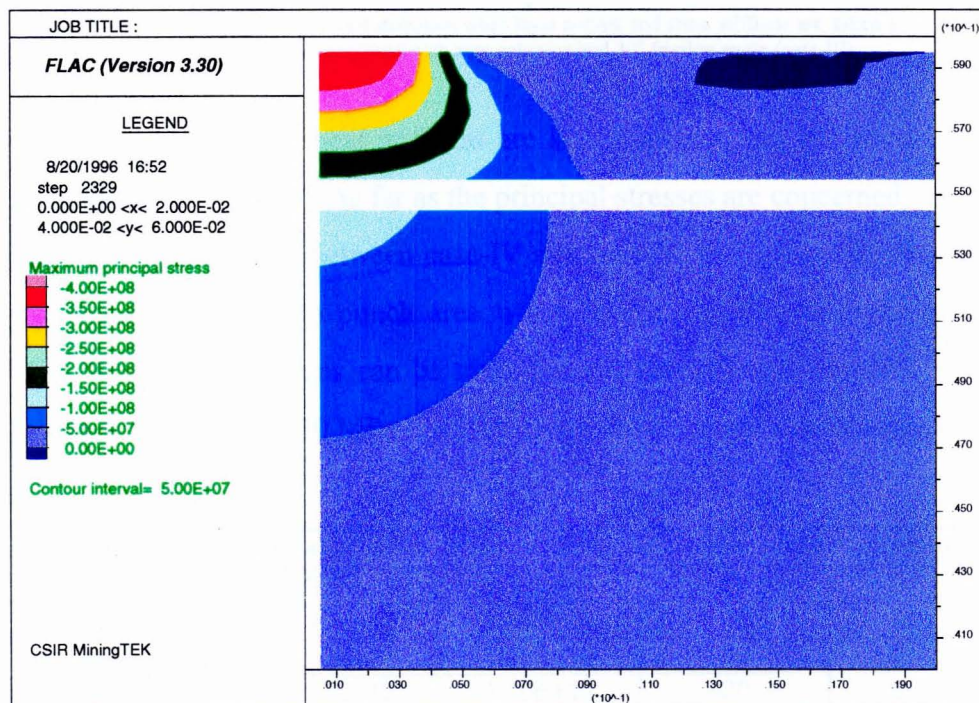


Figure 3.9 Maximum principal stress distribution when layer normal and shear stiffness are taken as 19.10^{11} Pa and 11.10^{11} Pa respectively with zero cohesion and 30° friction angle (case-II)

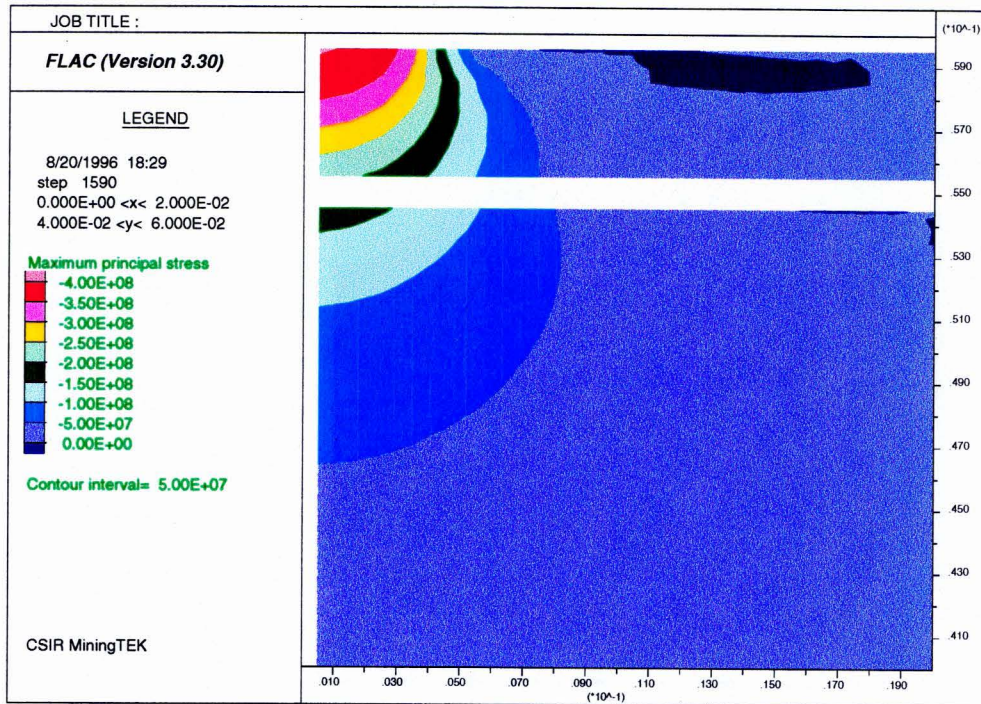


Figure 3.10 Maximum principal stress distribution when layer normal and shear stiffness are taken as 19.10^{12} Pa and 11.10^{12} Pa respectively with zero cohesion and 30° friction angle (case-III)

Secondly, normal and shear stiffness are kept constant, but the friction angle is changed from 20° to 40° . As far as the principal stresses are concerned, there is not a significant difference between case-IV and case-V; the only difference is in the tensile zone just next to punch area which is not significant (Figures 3.11 and 3.12). Similar conclusions can be made for the case-VI and case-VII. Changing the cohesion from 0 to 10 MPa does not affect the principal stresses significantly.

	Normal stiffnes	Shear stiffness	Friction angle	Cohesion
case-I	1.90E+11	1.10E+11	30	0
case-II	1.90E+12	1.10E+12	30	0
case-III	1.90E+13	1.10E+13	30	0
case-IV	1.90E+12	1.10E+12	20	0
case-V	1.90E+12	1.10E+12	40	0
case-VI	1.90E+12	1.10E+12	30	5.00E+06
case-VII	1.90E+12	1.10E+12	30	1.00E+07

Table 3.1. Layer properties used for FLAC modelling.

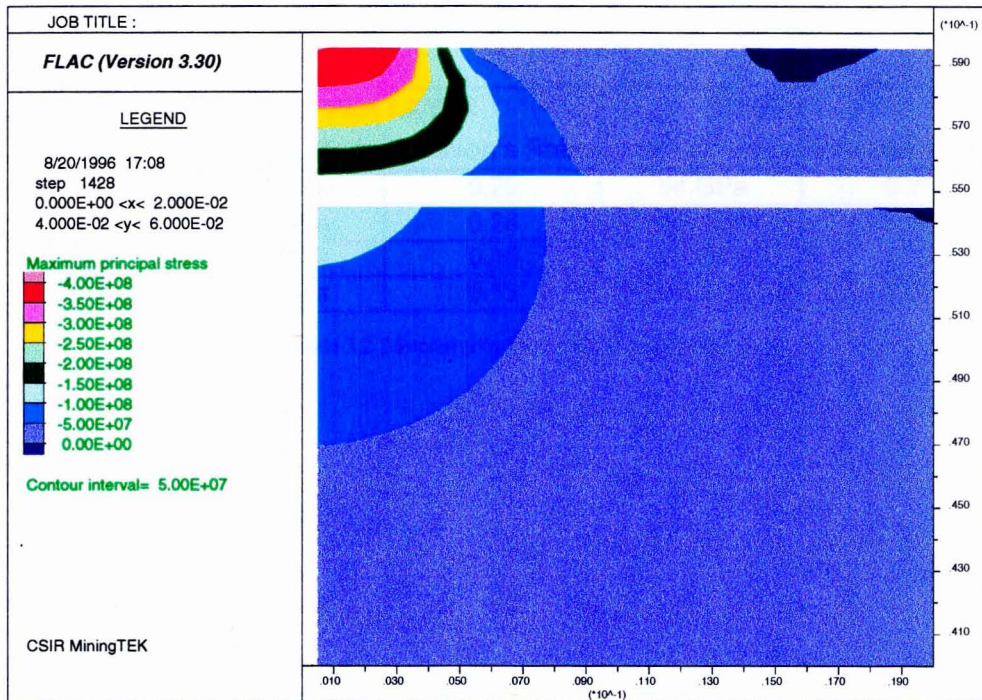


Figure 3.11 Maximum principal stress distribution when layer normal and shear stiffness are taken as $19 \cdot 10^{11}$ Pa and $11 \cdot 10^{11}$ Pa respectively with zero cohesion and 20° friction angle (case -IV)

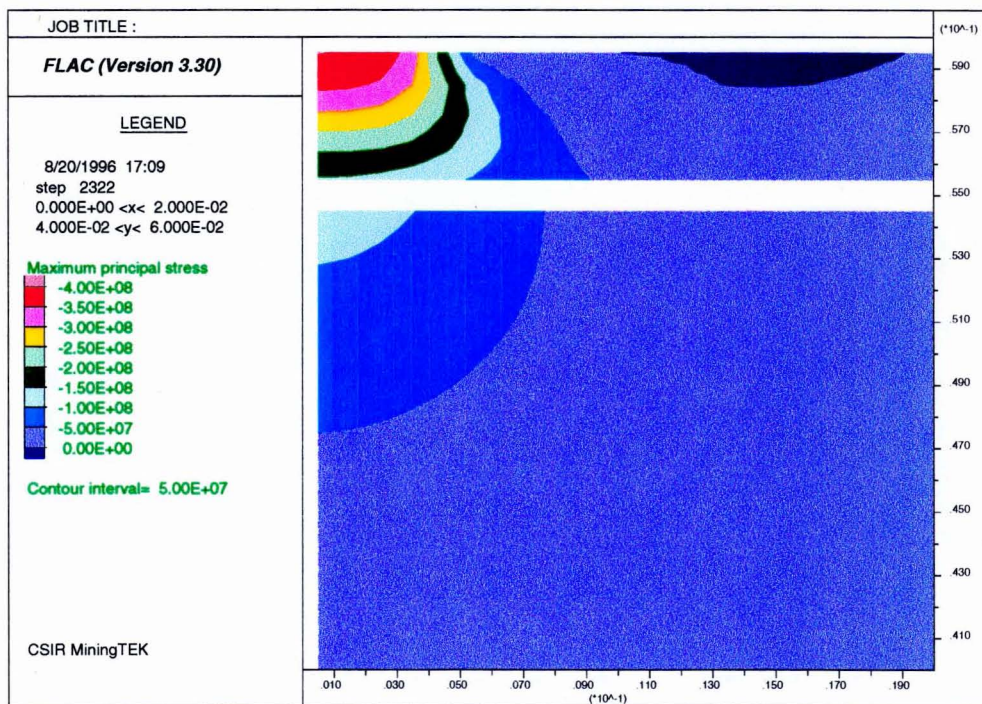


Figure 3.12 Maximum principal stress distribution when layer normal and shear stiffness are taken as $19 \cdot 10^{11}$ Pa and $11 \cdot 10^{11}$ Pa respectively with zero cohesion and 40° friction angle (case-V)

A second series of runs has been carried out to see the effect of material properties on the stress distribution when 2 layers are used in the system (Table 3.2)

	UPPER LAYER		LOWER LAYER	
	Young's Mod.	Poisson's Ratio	Young's Mod.	Poisson's Ratio
op-I	27 GPa	0.26	54 GPa	0.26
op-II	27 GPa	0.26	13.5 GPa	0.26
op-III	27 GPa	0.26	27 GPa	0.13
op-IV	27 GPa	0.13	27 GPa	0.26

Table 3.2. Material properties used for FLAC modelling

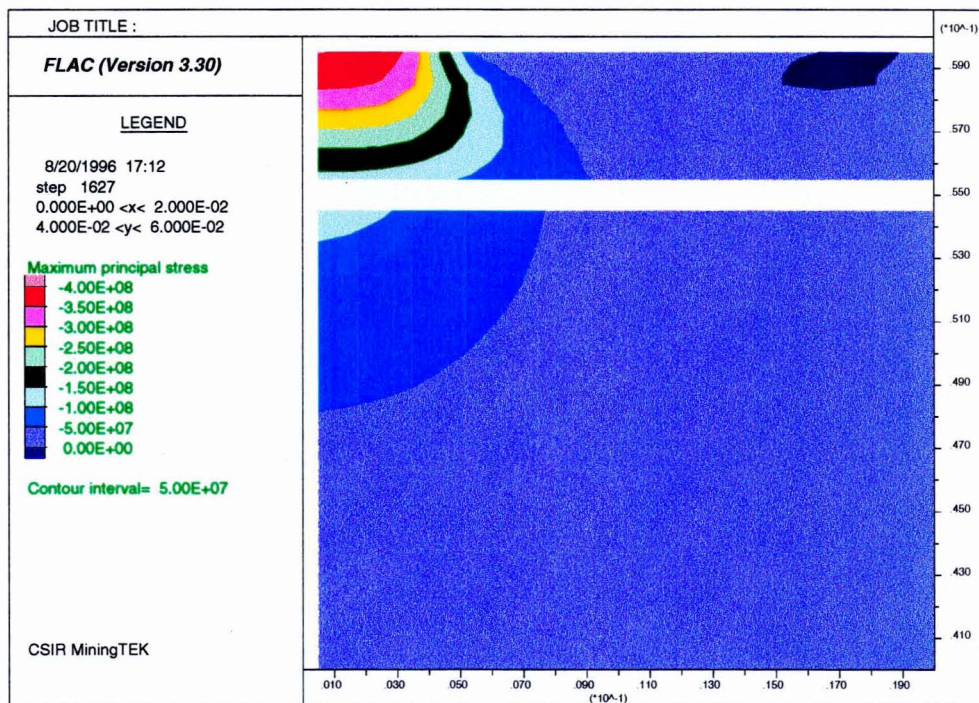


Figure 3.13 Maximum principal stress distribution when the upper layer has a bigger Young's Modulus compared to lower layer, but they have the same Poisson's Ratio (op-II)

The layer properties have been taken as the case-I from the previous runs (Table 3.1). In the first case (op-II in Table 3.2) Poisson's ratios are kept constant for both layers but the lower layer has a Young's Modulus of 13.5 GPa, which is smaller than that of the upper layer ($E_1/E_2=2$) (Figure 3.13). In this case there is a stress concentration in the upper layer compared to the other cases which have $E_1/E_2=0.5$ and $E_1/E_2=1$ (Figures 3.14 and 3.15). The most important result from these runs is that if the layer with the high Young's Modulus is the upper layer, then the failure stress is greater than the case in which the layer with low Young's Modulus is the top layer. Changing the Poisson's ratio does not affect the stress distribution in the system under the confining conditions given above.(Figure 3.15).

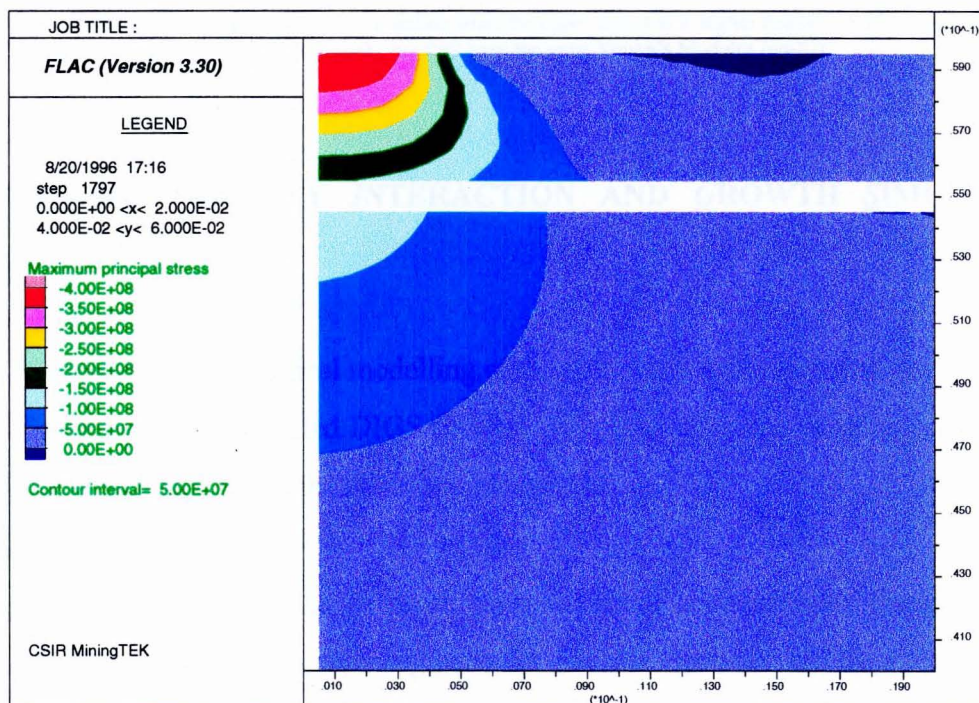


Figure 3.14 Maximum principal stress distribution when the lower layer has a bigger Young's Modulus compared to upper layer ($E_1/E_2=0.5$), but they have the same Poisson's Ratio (op-I)

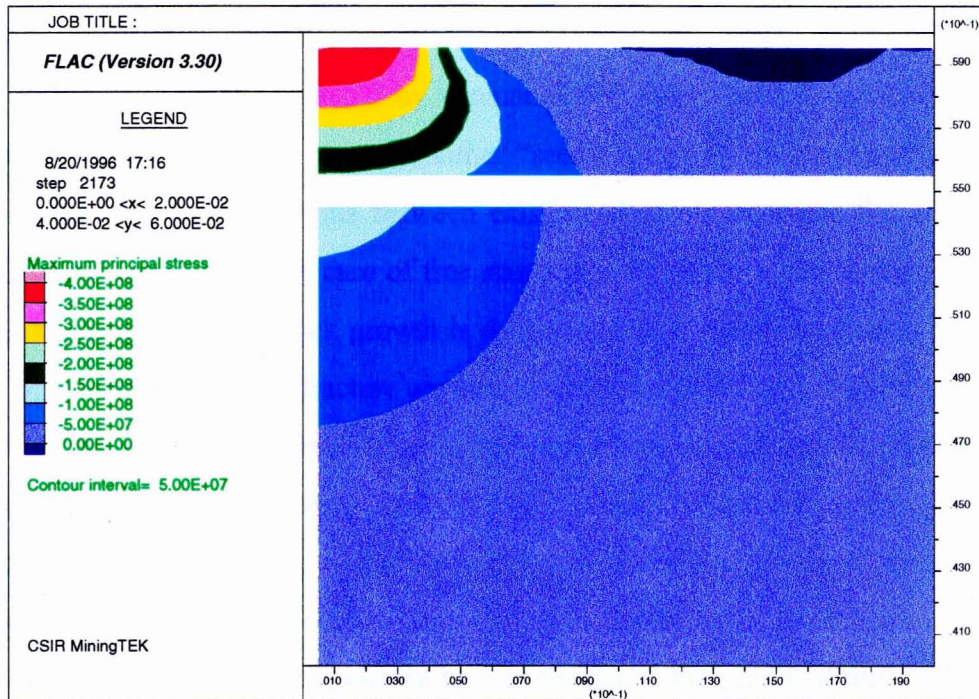


Figure 3.15 Maximum principal stress distribution when the upper layer has a higher Poisson's Ratio compared to lower layer ($\nu_1/\nu_2=2$), but they have the same Young's Modulus (op-III)

3.2 DISCONTINUITY INTERACTION AND GROWTH SIMULATION (DIGS)

To carry out the numerical modelling of the laboratory tests, extensive use was made of a computer code called DIGS (Napier, 1990), some of the working principles of which are described in the following paragraphs.

Fractures are represented in plane stress or strain geometries by linear segments jointed end to end. On each segment the crack is characterized by a displacement discontinuity density, which is assumed to vary linearly along each element as explained in the Appendix B. Stress or displacement boundary conditions are matched at two collocation points within each linear variation element. This

eliminates the inherent over-estimation of the displacement discontinuity magnitudes which arises when constant strength elements are used.

Fracture growth is controlled by evaluating the stress field at a set of potential growth sites. The sites comprise either “seed” points at arbitrary positions in the medium or at the junctions between existing elements or at the tips of growing fracture segments. In the case of free standing seed points, a characteristic measure of the propensity for crack growth is determined from the stress field at that point. For extension or tensile fracture growth, this measure is deemed to be the magnitude of the most tensile principal stress component. If the orientation of this component is at an angle α with respect to a specified global co-ordinate system, then it is assumed that the fracture will grow in either the direction $\alpha_1 = \alpha + \pi/2$ or $\alpha_2 = \alpha - \pi/2$. To make the choice specific, a base angle β is associated with each seed point and the angle α_i , $i=1,2$ that is closest to β is chosen as the growth direction. To avoid bias in the selection of growth directions from several seed points, β can be set to a random angle for each point and two complementary seeds with angle β and $\beta + \pi$ can be specified at each point (Napier and Hilyard, 1992).

In the case of a growing fracture, the evaluation of the growth direction and growth propensity is more complicated. The base angle, β , is assumed to be the angle of the last element in the segment (Figure 3.16). The optimum growth angle is found by a direct search of the stress field at a fixed radius ρ (see Appendix B) from the crack tip. In the case of extension fracture growth, the optimum angle θ^* is the position P at which $\tau_{\theta\theta}$ is a maximum (assuming tensile stresses are positive). Although it is computationally more arduous, the direct search method has been found to be more robust in avoiding oscillations in the crack growth path which can occur if the growth direction procedure is based on local estimates of the crack tip stress intensity factor.

Shear failure localisation is an inherently more complex process than extension fracture growth. At present, this is simulated using the same seed and crack growth structure described for extension fracturing but with a different rule for determining the angle and propensity for fracture initiation and growth. Specifically, the shear growth direction is determined by finding the angle at which the local strain energy release rate is a maximum (Napier et. al. 1995). This is equivalent to finding the maximum value of

$$G = (\tau_{r\theta}^2 - \mu \tau_{\theta\theta}^2) \dots \dots \dots (3.9)$$

where

r = direction of proposed element

θ = direction normal to proposed element

μ = effective friction coefficient

$\tau_{r\theta}$ = shear stress on element

$\tau_{\theta\theta}$ = normal stress across an element.

In specifying a fracture growth problem, shear fracture seeds must be distinguished from tension fracture seeds. Each seed point must refer to a constitutive definition in which the failure properties of the material are specified in terms of the Mohr-Coulomb parameters σ_c , m and T_c defined above. Once the crack element is emplaced at the seed point, a specified “post failure” friction angle and cohesion is applied to the crack faces. Some judgment has to be exercised by the modeller about the selection of the shear or tension fracture growth modes and in certain cases simulations must be restricted to only one mode (Napier, 1990).

3.3 AXISYMMETRY PUNCH TESTS

The material conductivity for fracture depends on the presence of discontinuities. These can occur as cracks, open fissures, zones filled with moist clay or marl, very

hard concretions or pebbles in weaker matrix or large crystals of minerals with a fissile structure such as mica. In the deep level gold mining environment, the fracture zone around a stope face exhibits a remarkably systematic pattern of behaviour once the main factors influencing it have been recognized and taken into consideration. Most of the fractures develop in direct response to the mining of the stope face. Thus, as the face is advanced, the fracture zone migrates forward ahead of the face. It appears that in a number of cases the new fractures form within a very short time after advancing the face and that very little fracturing takes place more than 3 hours after the face has been advanced (Joughin and Jager, 1984).

If the fracture front finds such discontinuities in its way, it may cut through them, or it may stop. In order to understand these phenomena, a number of laboratory tests were carried out. The laboratory tests consisted of a series of "punch" tests conducted on cylindrical specimens built from layers of norite, sandstone and quartzite

Five rock layers, each of 54 mm diameter and 10 or 20 mm in height, were stacked in a steel tube and tested in compression. Various layered configurations and confinement levels were used. The effect of friction at layer interfaces on fracturing was also investigated.

3.3.1. Description of the tests

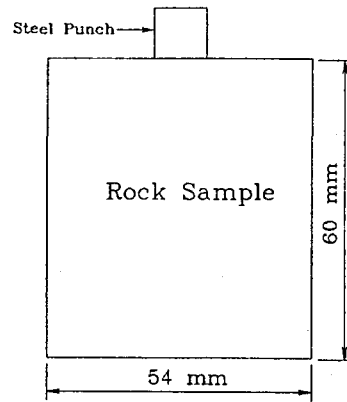
The tests were conducted on 54 mm diameter cylindrical specimens, prepared from solid blocks or layers of sandstone, norite and quartzite, Table 3.3. For the tests with confinement, the specimens were placed in a 4 mm thick steel tube and the level of confinement was monitored by strain gauges placed on the tube. The specimens were loaded to failure and sections were taken to observe the degree and pattern of fracturing that occurred in them.

	NORITE	SANDSTONE	QUARTZITE
UCS (MPa)	210	90	200
ELASTIC MOD (GPa)	112	21	70
POISSON'S RATIO	0.31	0.27	0.20
FRICTION ANGLE	53°	43°	50°

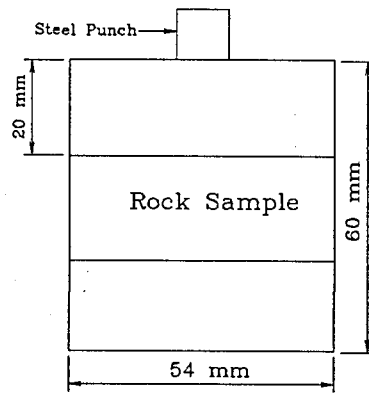
Table 3.3 Physical properties of the rock determined from laboratory tests

In Figure 3.17, the dimensions of rock samples and the punch used in the experiments can be seen. Various combinations were tested with changing interface conditions, Table 3.4.

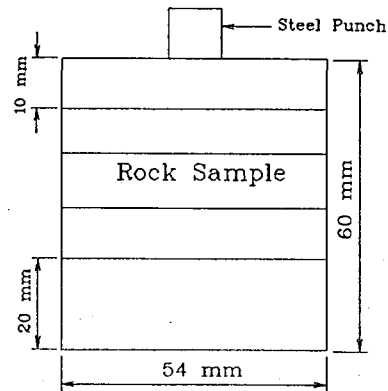
A better understanding of the fracture processes can be obtained if their origin and sequential genesis can be simulated numerically. Although the first set of experiments is carried out using an axisymmetric loading arrangement, the post-failure behaviour of the samples is fully three dimensional. Some insights can nevertheless be gained by comparing the results observed in the experiments to two dimensional plane strain simulations of fracturing by DIGS in test configurations corresponding geometrically to a diametral section through the samples (Dede, Ozbay and Napier, 1995).



a. Solid case



b. 3 layers case



c. 5 layers case

Figure 3.17 Dimensions of the specimens used in the axisymmetry tests

	TEST 1 NO FRIC.	TEST 2 FRICTION	TEST 3 NO FRIC.	TEST 4 FRICTION	TEST 5 NO FRIC.	TEST 6 FRICTION	TEST 7 NO FRIC.	TEST 8 FRICTION
1st LAYER	SST*	SST	NORITE	NORITE	SST	SST	NORITE	NORITE
2ND LAYER	SST	SST	NORITE	NORITE	NORITE	NORITE	SST	SST
3rd LAYER	SST	SST	NORITE	NORITE	SST	SST	NORITE	NORITE
4th LAYER	SST	SST	NORITE	NORITE	NORITE	NORITE	SST	SST
5th LAYER	SST	SST	NORITE	NORITE	SST	SST	NORITE	NORITE

* SST denotes sandstone

Table 3.4 Tests carried out using the with and without friction reducer

3.3.2 Unconfined Solid Tests

3.3.2.a. Physical Modelling

In this series, solid cylindrical sandstone specimens were subjected to "punch" loading using a 10 mm diameter steel piece. The mode of failure observed in the 54 mm diameter and 60 mm long solid sandstone specimen is diagrammatically shown in Figure 3.18.

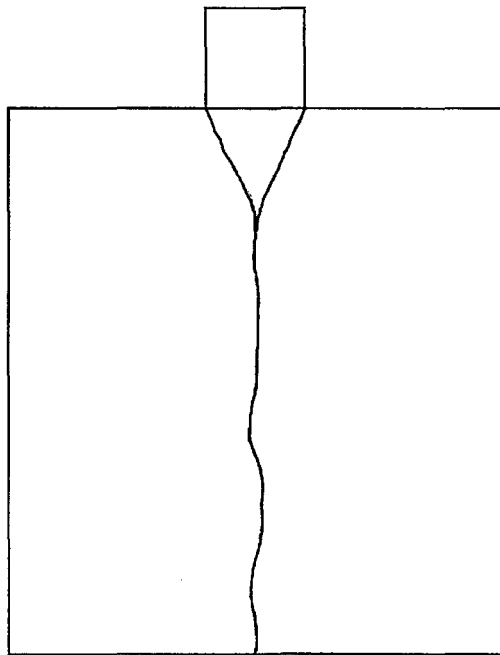


Figure 3.18 Fracturing in unconfined solid sandstone sample

The mode of failure corresponds to the formation of an inverted cone immediately under the punch and a radial tensile crack along the axis of the specimen. The base diameter and height of the cone were each about 10 mm, which is equal to the punch diameter. The cone appears to have wedged into the specimen, axially splitting it in tension into two pieces. The white powdered material observed on the side of the conesurface implies failure by shear fracturing around the cone. The failure took place suddenly when the vertical stress reached 663 MPa (see Figure 3.19).

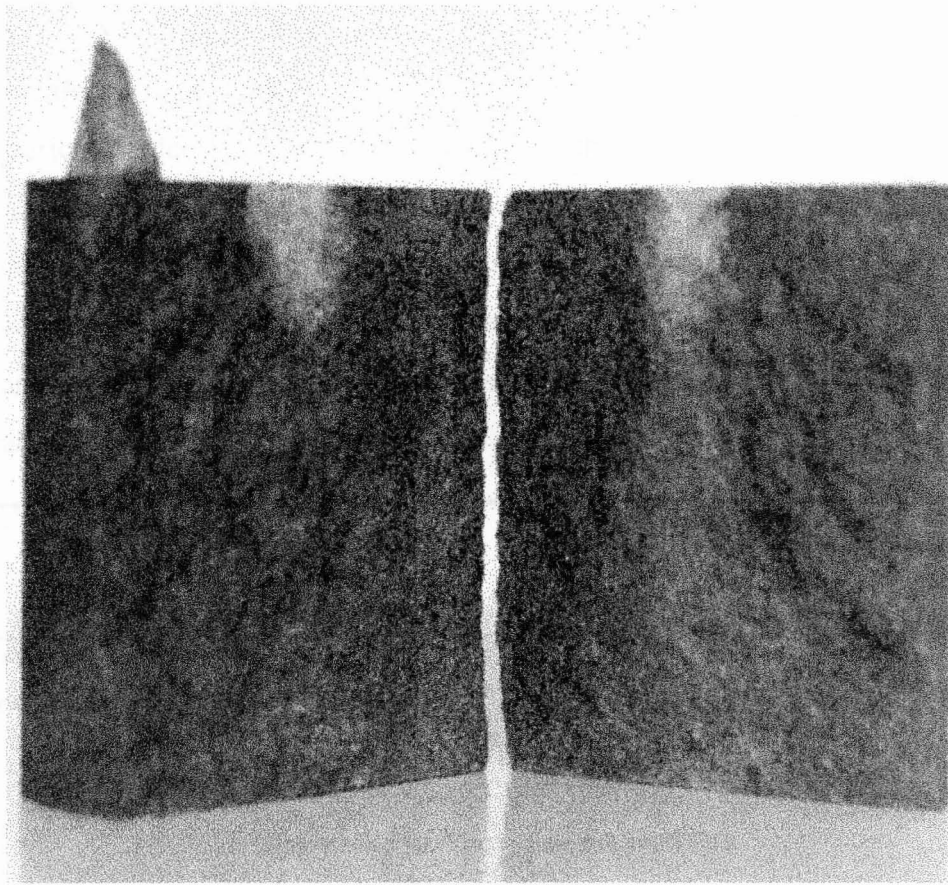


Figure 3.19 Picture of unconfined solid sandstone sample

3.3.2.b. Numerical Modelling

The punch loading of the unconfined sandstone specimen is considered first. Two shear growth fractures were allowed to initiate below the edges of the region on the surface of sample corresponding to the area covered by the punch. This area is loaded uniformly by the stress value corresponding to the maximum observed stress in the physical experiment, 662 MPa. To obtain the tensile fracture starting from the bottom of the punch and ending at the bottom of the sample a tensile seed point is introduced into the system. The material properties used in the numerical simulations are summarized in Table 3.5. The shear fractures are seen to converge to form a wedge shaped structure below the punch as shown in Figure 3.20. If a seed point is introduced subsequently to the formation of the shear

fractures in the region below the punch, it is found to grow below the wedge structure. The final fracture pattern is qualitatively similar to the observed pattern in Figure 3.18 but shows a marked difference in the wedge angle and extent. The numerical model also ignores the possibility that the tensile fracture below the wedge may form prior to the complete localization of the wedge. The shape of the simulated wedge may differ from the observed shape because of the distinct differences that can arise between an axisymmetric and a plane strain loading geometry as well as inadequacy of the shear growth rule. Figure 3.21. shows the strain-stress curve of the test from DIGS run.

	Pre-failure cohesion (MPa)	Post-failure cohesion (MPa)	Tension cut-off (MPa)	Unmobilized friction angle (degrees)	Mobilized friction angle (degrees)
Tension and shear fractures	13	0	10	25	20
Intact material: Young's modulus= 21 GPa Poisson's ratio = 0.27					

Table 3.5 Material properties used in computer simulations

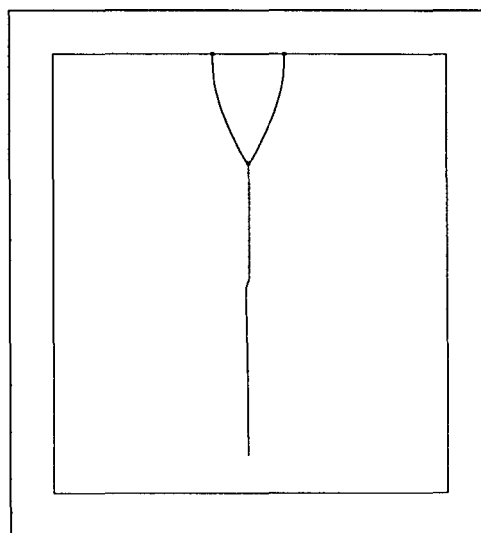


Figure 3.20 The results of the unconfined axisymmetrical sandstone punch tests from DIGS results.

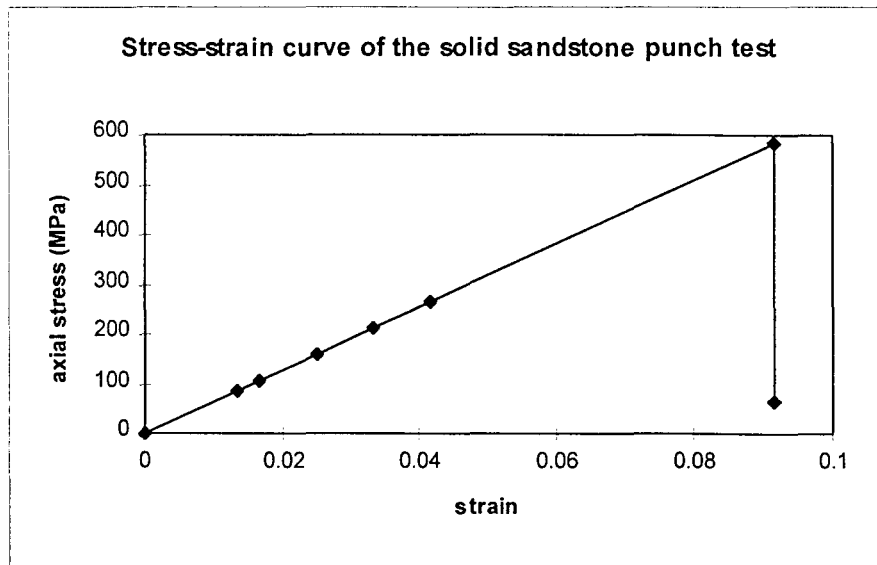


Figure 3.21 Stress-strain curve of the solid sandstone punch test from DIGS

The geometry and boundary conditions used for the FLAC simulations are shown in Figure 3.22. The geometry represents the section of the right half of a test

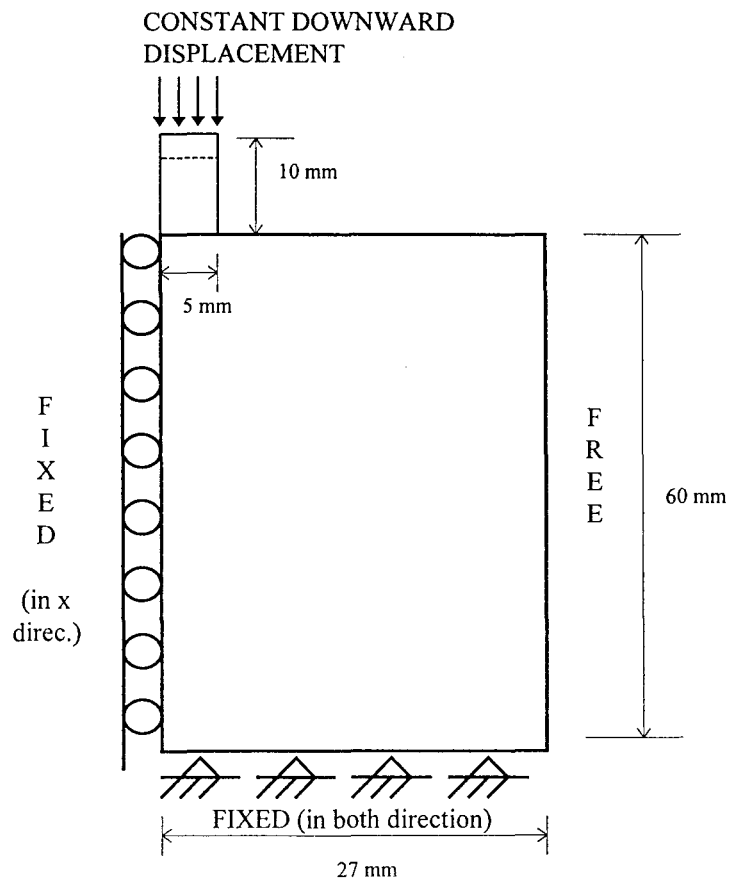


Figure 3.22 Geometry and boundary conditions used for FLAC modelling

sample. To simulate the test loading conditions, the right side was not subjected to direct loading and the load was applied through the punch area with a constant downward displacement. An interface was placed between the punch and the sample to simulate the laboratory conditions. Two different models were used, namely Elastic and the Mohr-Coulomb Plasticity Models. Examples of the data files are given in Appendix C.

The model was run until the stress under the punch reached the failure stress determined from the physical modelling. The stress-strain curve obtained from the FLAC model is shown in Figure 3.23.

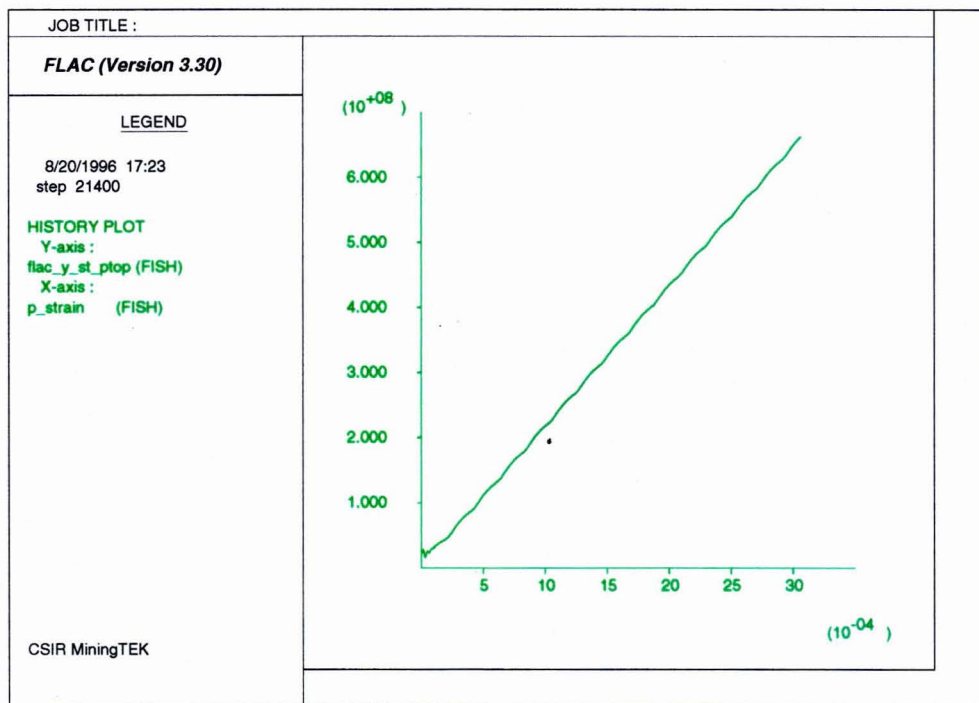


Figure 3.23 Stress-strain curve of the unconfined sandstone by an elastic FLAC model

The Mohr-Coulomb Criterion only the major and minor principal stresses are active in the simulation of fracture growth while the strength/stress ratio gives a

similar trend of failure compare to the results obtained from both DIGS and physical modelling (Figure 3.24).

The second step was the modelling of the same problem by using the Mohr-Coulomb Plasticity model. The boundary conditions were exactly the same as the previous elastic model. The material properties used for this model is shown in Table 3.6

<i>SANDSTONE</i>	
Bulk Modulus (GPa)	19
Shear Modulus (GPa)	11
Density (kg/m ³)	2500
Dilation (°)	22.8
Friction (°)	30
Tension (MPa)	2

Table 3.6 Material properties used for the FLAC model

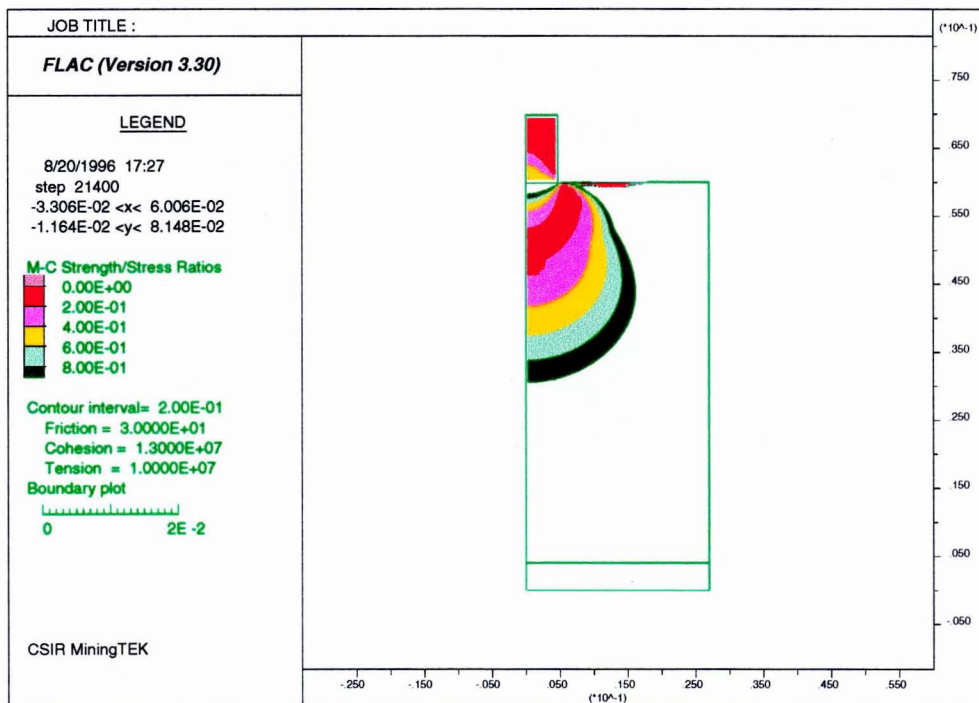


Figure 3.24 Mohr-Coulomb strength/stress ratios of the unconfined sandstone using an elastic FLAC model

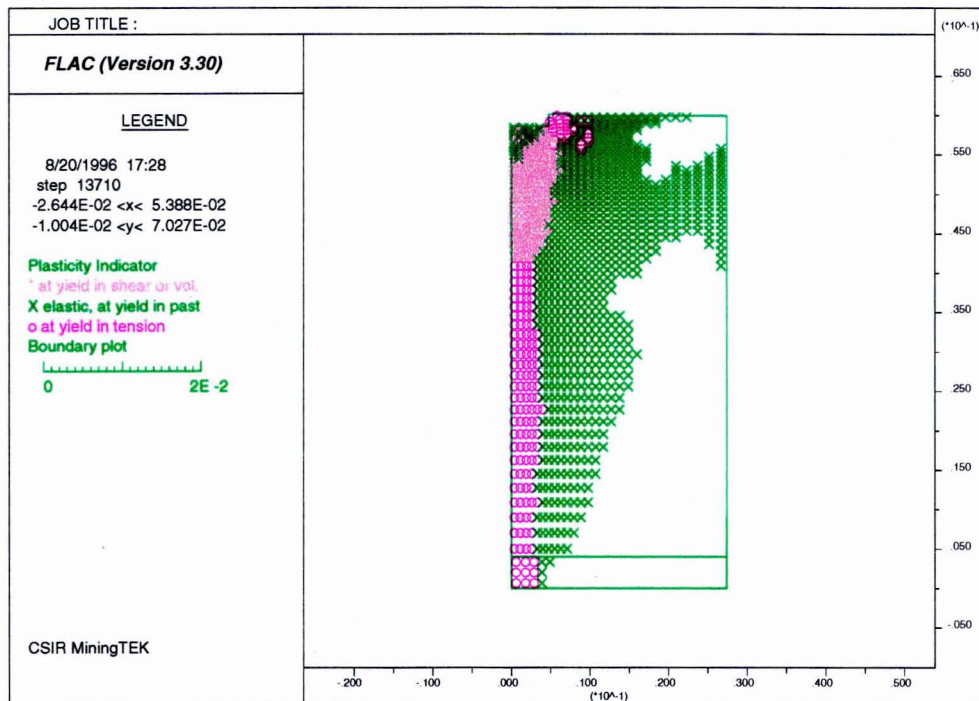


Figure 3.25 Mohr-Coulomb Plasticity plot of an unconfined sandstone using a FLAC model

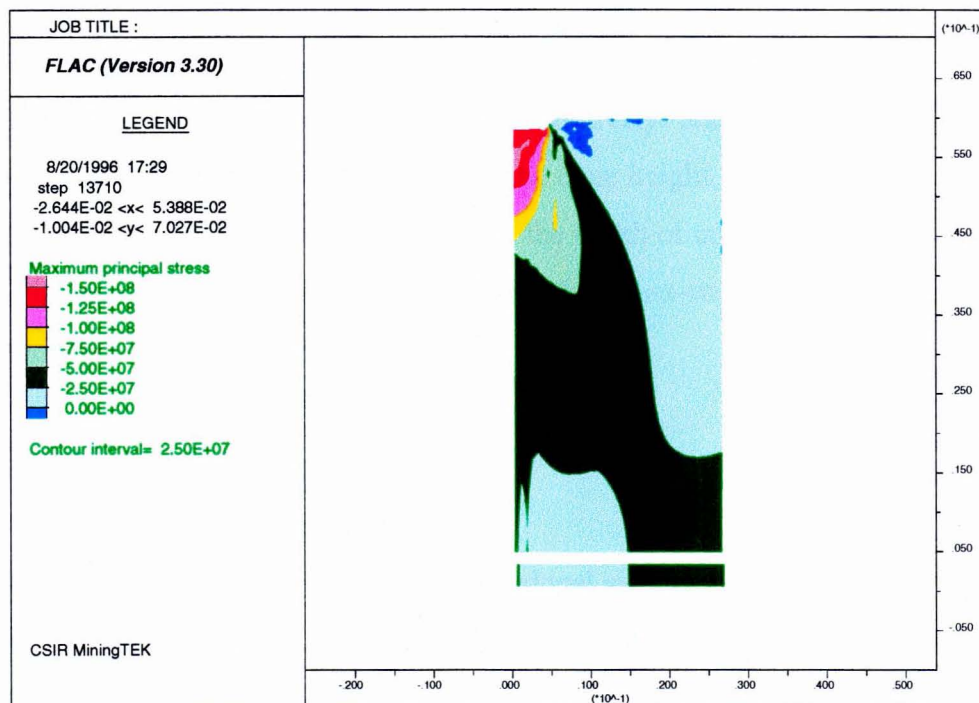


Figure 3.26 Maximum principal stress distribution using a Mohr-Coulomb Plasticity model on an unconfined sandstone sample

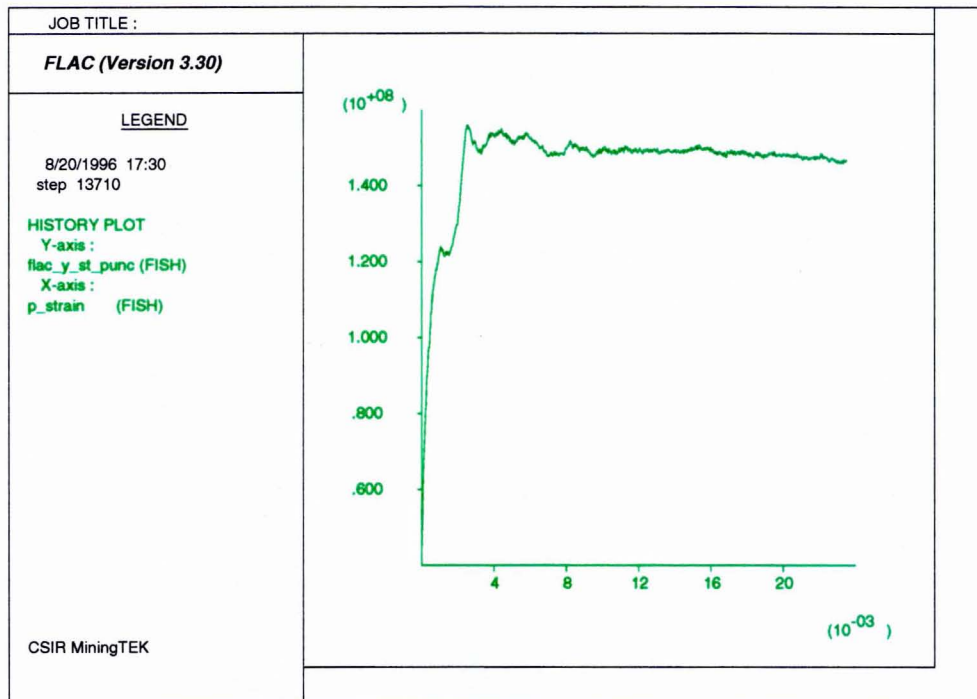


Figure 3.27 Stress-strain curve of an unconfined sandstone sample using a Mohr-Coulomb Plasticity model

The Mohr-Coulomb model gives a qualitatively similar trend to that of the physical model, except with respect to the cone height, there is a large difference which in the physical model is only half the width of cone in the numerical model (Figure 3.25). The stress distributions and the stress-strain curve of the model are shown in Figure 3.26 and 3.27.

3.3.3 Unconfined Layered Tests

3.3.3.a. Physical Modelling

When a stack of three 20 mm long cylindrical specimens were loaded by the steel punch, the splitting was observed to stop at the first interface. The punch stress at failure was, however, less than half the value for solid specimen, being 318 MPa. Two cones were formed as shown diagrammatically in Figure 3.28. A photograph of the failed specimen is shown in Figure 3.29.

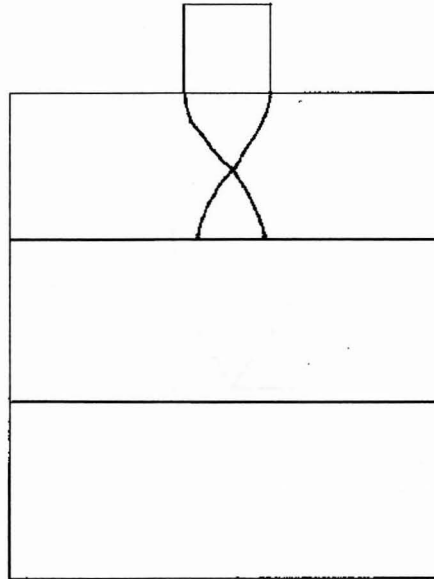


Figure 3.28 Fracturing in unconfined layered sandstone sample

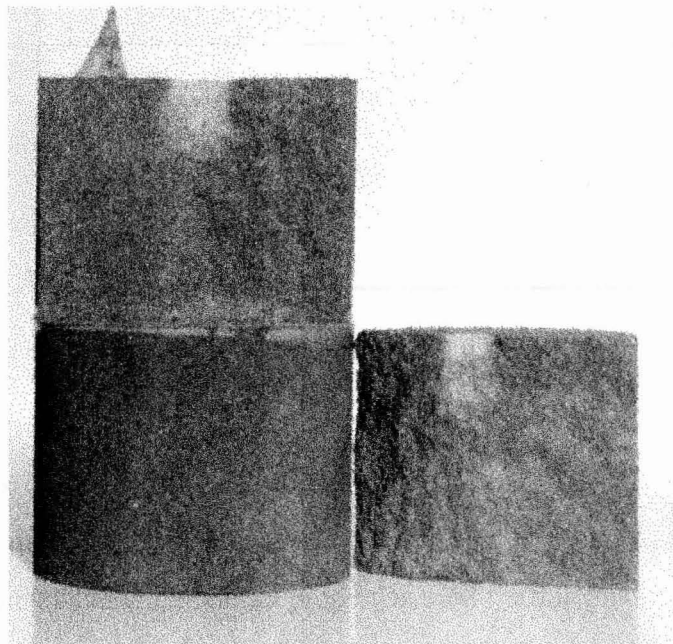


Figure 3.29 Picture of unconfined layered sandstone sample

Further unconfined tests were carried out to observe the effect of thickness and the composition of layers on fracturing under punch loading. The layer thickness was reduced to 10 mm, the same as the punch diameter, and the specimen was constructed from five layers. The maximum stress attained by the specimens in these cases was less than the case with 20 mm layers, being around 229 MPa. The cone

formation was complete and no fracturing was observed in second layer (Figure 3.30) (Further photographs are shown in Appendix D).

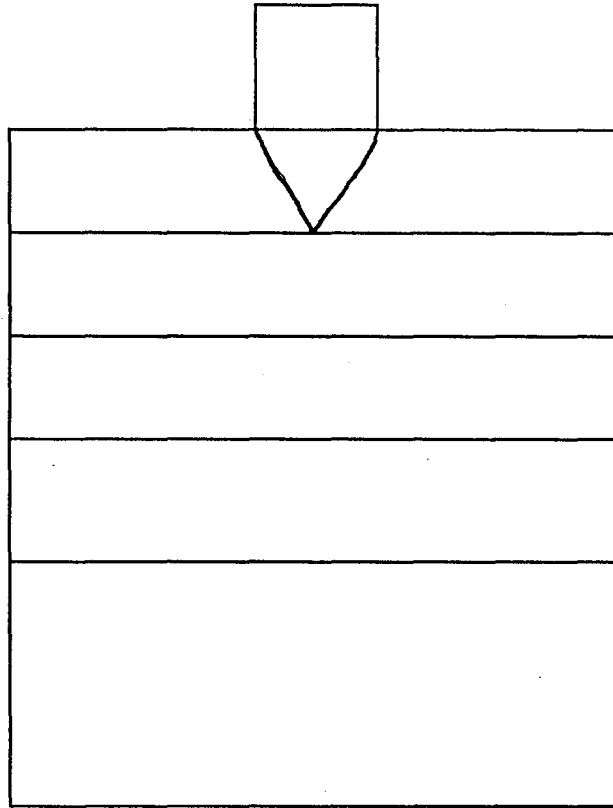


Figure 3.30 Fracturing in unconfined layered sandstone sample

3.3.3.b. Numerical Modelling

Numerical modelling of the unconfined layered test was carried out using DIGS plane strain computer code. Material properties used for this model were the same as these in the previous unconfined solid DIGS model (Table 3.5), but four layers were introduced into the system. Each layer had a friction angle of 30° and zero cohesion. Two shear seed points were placed next to the edges of the punch to obtain the cone. The result of the modeling is shown in Figure 3.31. The general pattern is similar to the one obtained in the physical modelling, in that there is no fracturing below the first layer. The main difference can be seen in the cone size. Fractures are stopped at the first layer before full formation of the cone. This is due to the incompatibility

between the plane strain assumption of the DIGS code and the actual axisymmetric loading arrangement.

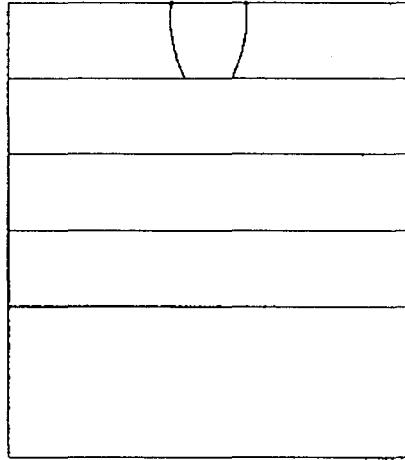


Figure 3.31 The results of the unconfined layered sandstone punch tests from DIGS result.

3.3.4 Confined Solid Tests

3.3.4.a. Physical Modelling

In this series, the specimens are enclosed in 4 mm thick steel tubes. The confining stress build up during testing is monitored by means of strain gauges placed on the outer surface of the steel tubes. The formation of surface parallel extension cracks, which initiate around the edge of the punch (known as Hertzian cracks), and the tensile fractures around the cone are the most significant differences in this series compared to the unconfined tests. Figure 3.32 is a diagrammatic illustration of fracturing developed in the solid confined specimen, and shows the repetitive formation of the fractures parallel to the free surface but a relatively poorly defined cone. It is also important to note that at the middle of the sample, fractures are almost horizontal and their spacing is wider than the spacing between fractures near to the punch. Figure 3.33 shows a photograph of the result of the laboratory test. In this test, the failure stress was found to be 2.1 GPa, which is the average punch stress under a confinement of 130 MPa from the steel jacket. The formation of axial tensile fracturing appears to be suppressed under the influence of the confining

stresses exerted by the steel tube, although an intense crushed region (shown by the dotted area in Figure 3.32) at the cone tip can be seen.

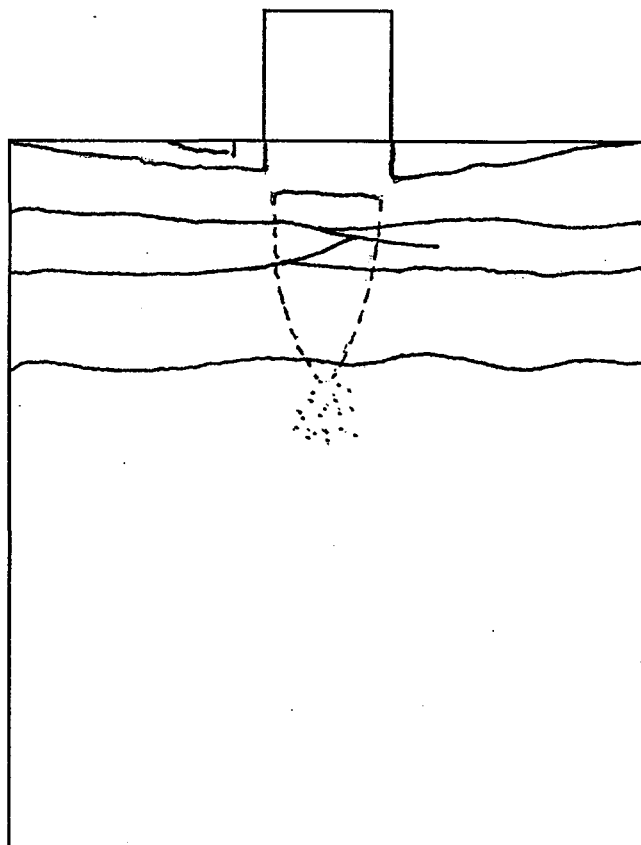


Figure 3.32 Fracturing in confined solid sandstone

3.3.4.b. Numerical Modelling

The confined sandstone experiment was modelled using the DIGS code. In the same way as the previous model, two shear growth fractures were allowed to initiate at the edges of the region below the punch area. However, unlike the unconfined case, some tensile seeds were placed around the cone appeared to form tensile fractures (Figure 3.34). Both tensile and shear fractures were allowed to grow at the same time. As before the fracture pattern was influenced by the exact placement of the seed points. According to where they were placed either next to the cone area or far away from it, the whole fracture pattern could be changed.

However, the final fracture pattern was again qualitatively similar to the observed pattern in the physical modelling, (Figures 3.33 and 3.35) although some major differences, especially in the vicinity of the cone area were observed.

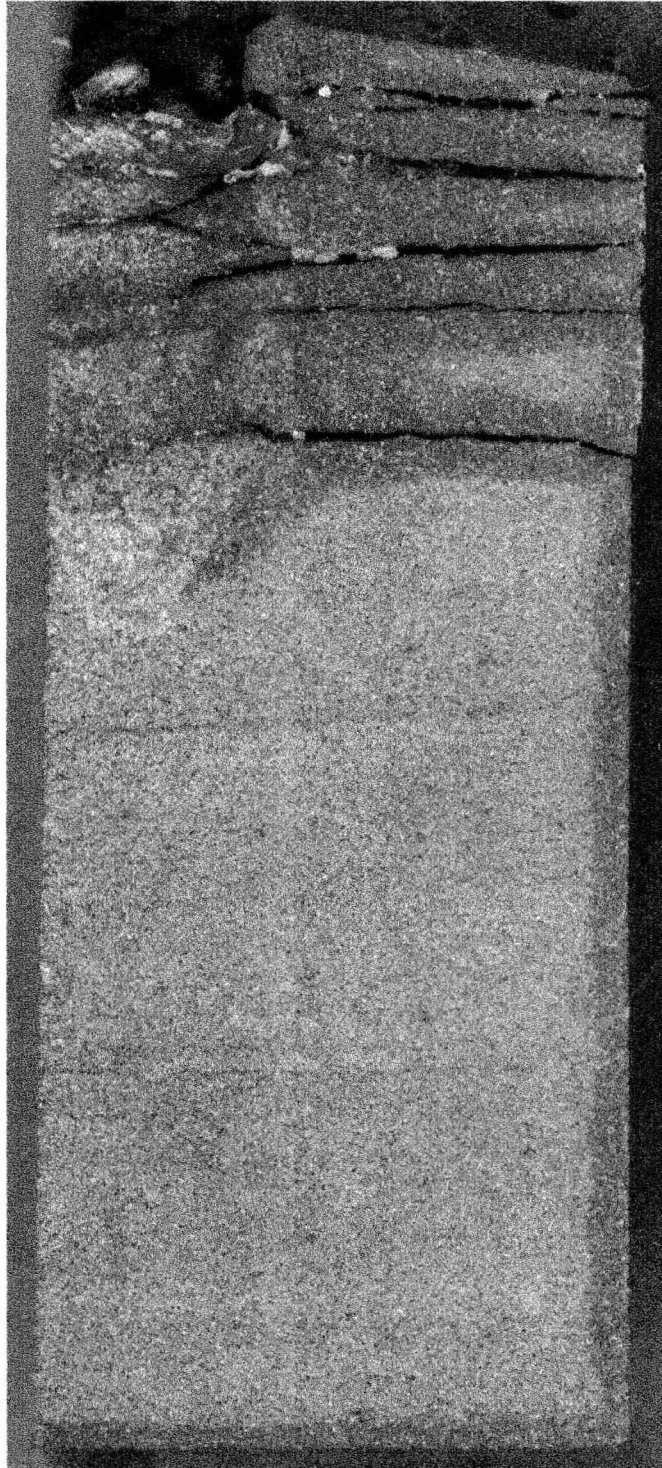


Figure 3.33 Right half of the confined solid sandstone sample picture

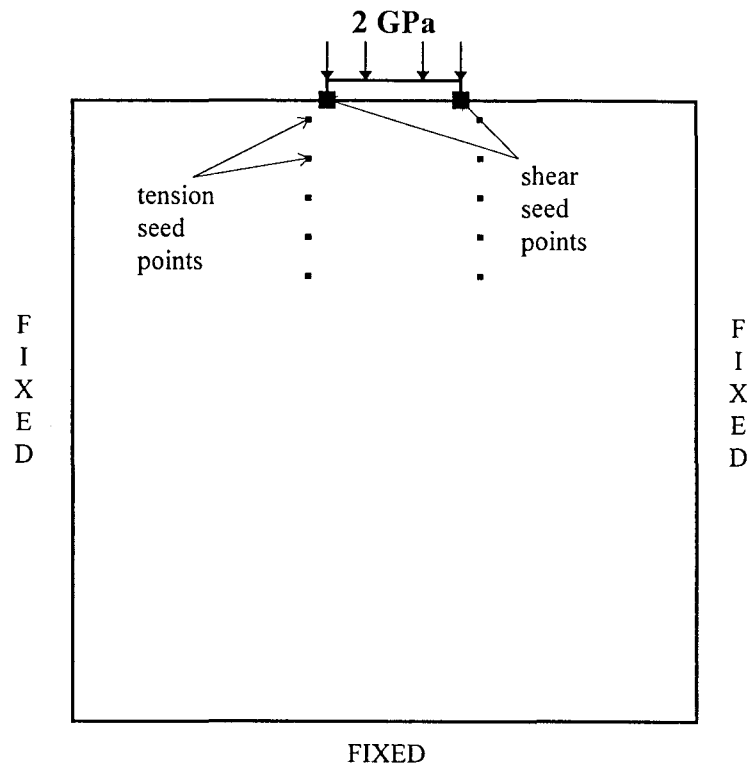


Figure 3.34 Seed points locations and boundary conditions for the DIGS model

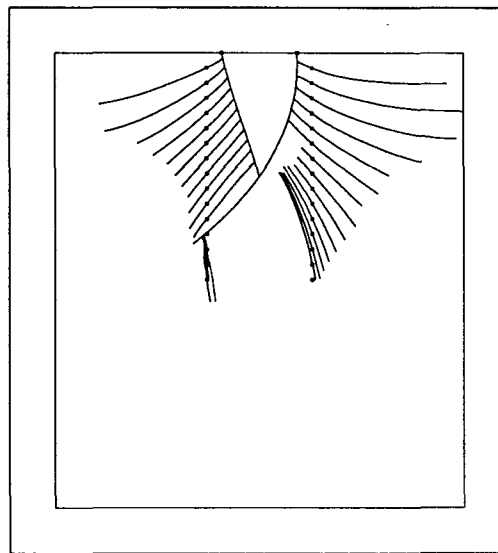


Figure 3.35 The results of the confined axisymmetrical sandstone punch tests from the DIGS

The fractures in the physical experiments propagated horizontally, but in the modeling the dip of the fractures was steep. These differences can again be

ascribed to the inappropriateness of the plane strain model with respect to the DIGS code. Further modelling of these tests was carried out using FLAC with an axisymmetric geometry. The boundary conditions are shown in Figure 3.36. The left hand side of the model is fixed due to the symmetry property of FLAC and the right side is fixed to simulate the test condition of rigid lateral confinement with zero friction.

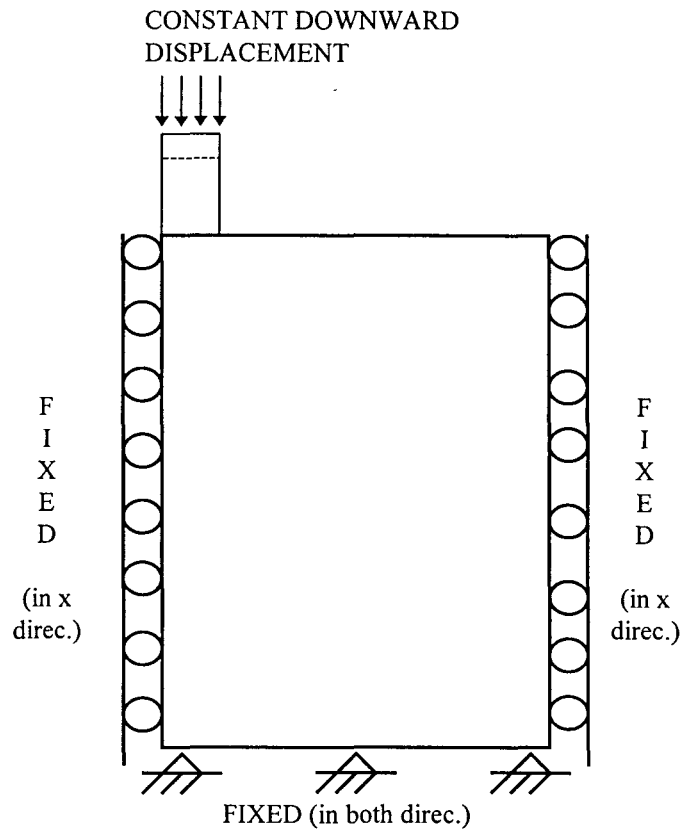


Figure 3.36 Geometry and boundary conditions used for the FLAC modelling

Modelling using the Mohr-Coulomb failure criterion, indicates the failure zone is confined to the beneath the punch and in the upper portion of the sample (Figure 3.37). This result was found from an elastic model and applying the physical properties of the material, namely friction, cohesion and tension to FLAC code.

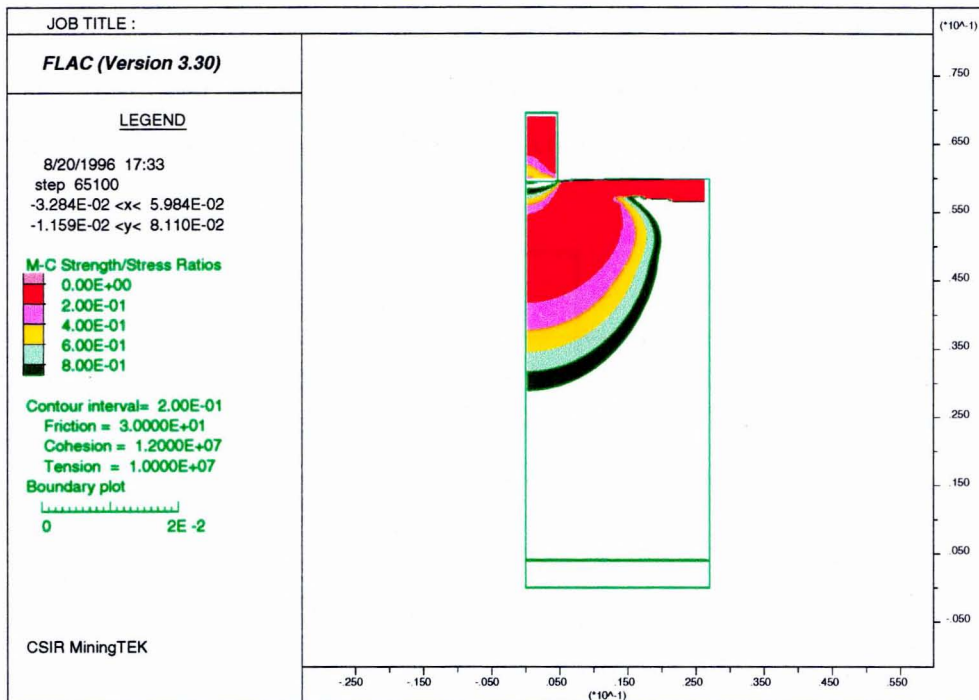


Figure 3.37 Mohr-Coulomb strength/stress ratio of the confined sandstone from an elastic FLAC model

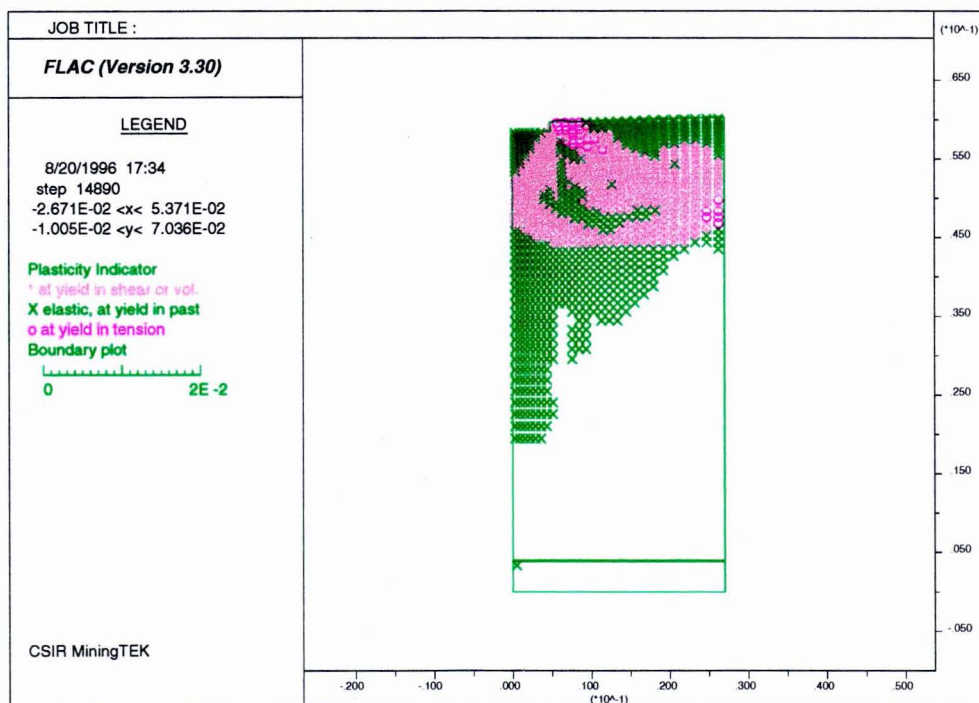
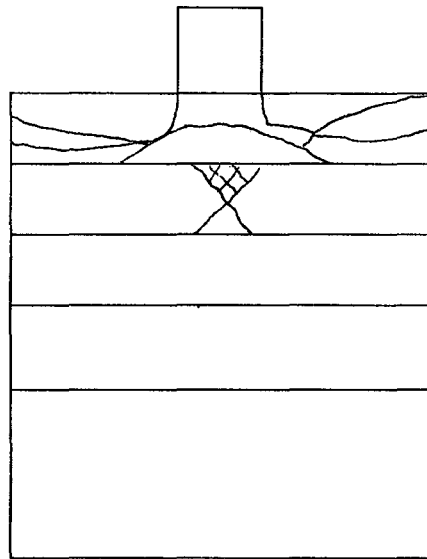
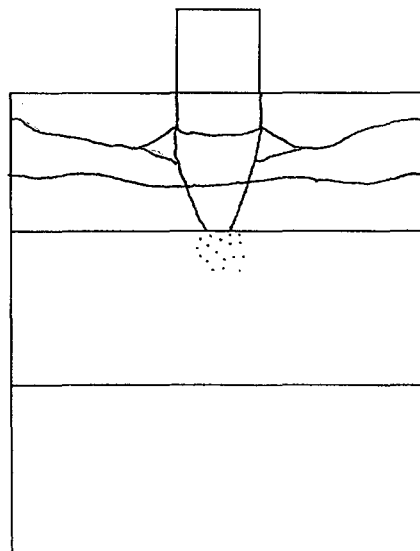


Figure 3.38 Plasticity plot of the confined sandstone sample by Mohr-Coulomb FLAC model

The second step was to model the problem by using the Mohr-Coulomb Plasticity Model. From these runs, the plasticity plot shows similar trends to the physical modelling (Figure 3.38). There is the usual cone which is surrounded by tensile and shear fractures which are restricted to the upper portion of the model, which is exactly the same behaviour as observed in the physical modelling.



(a)



(b)

Figure 3.39 Fracture pattern of a confined sandstone sample, layer thickness of a.) 10 mm b.) 20 mm

3.3.5 Confined Layered Tests

3.3.5.a. Physical Modelling

Two different layer thickness were used in the confined layered tests, namely 10 mm and 20 mm. In both tests, the surface parallel tensile cracks did not seem to propagate across the interface in the confined in layered tests case as seen in Figure 3.39.

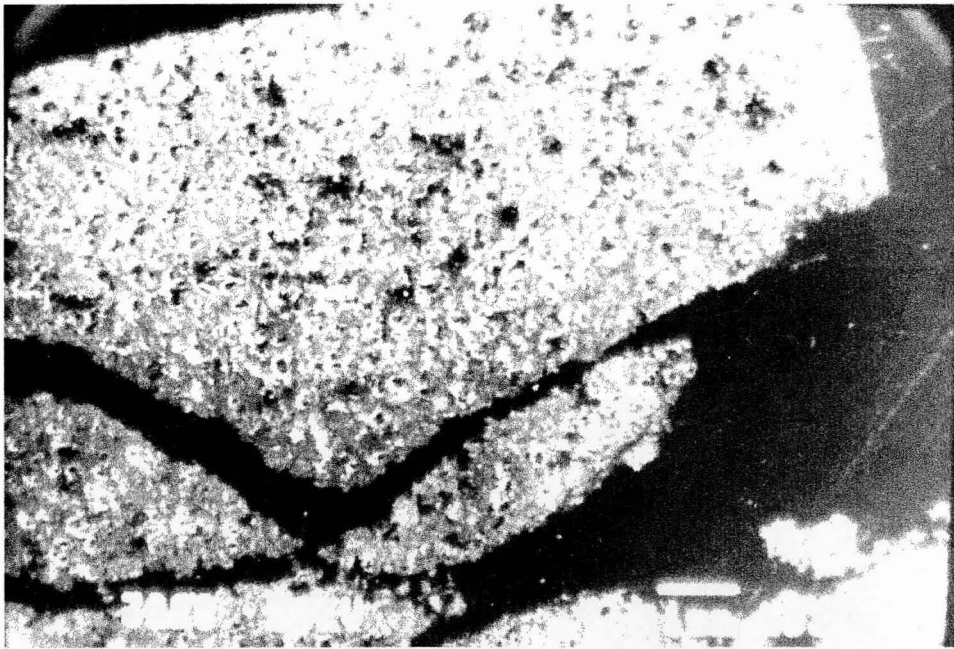


Figure 3.40 Picture of confined sandstone sample just next to cylindrical punch under electron microscope

Figure 3.40 shows the fracture pattern of the sample just next to punch in the confined tests.

To observe the effect of reducing friction at the interfaces, some of the confined tests were repeated with and without using a friction reducer such as stearic acid, between the layers. The mapped fracture pattern developed and a photograph of a sample made up of 20 mm thick sandstone layers with stearic acid between them, can be seen in Figures 3.41 and 3.42 respectively. The failure stress was 1.8 GPa under a confinement of 180 MPa . This value is 20% less than the case with no layers.

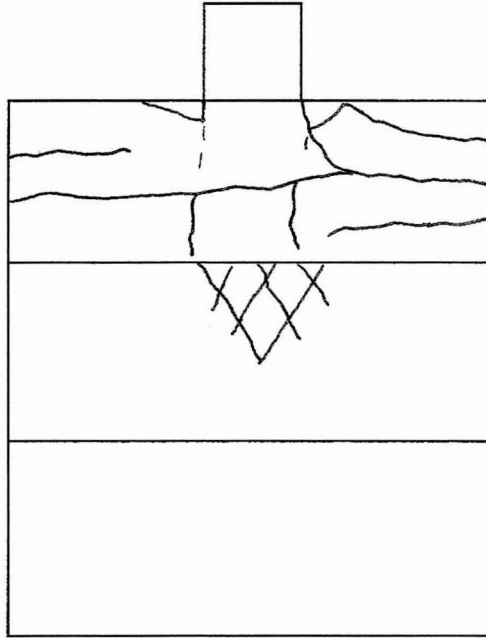


Figure 3.41 Fracturing in confined 20 mm thick layered sandstone with friction reducer (stearic acid)

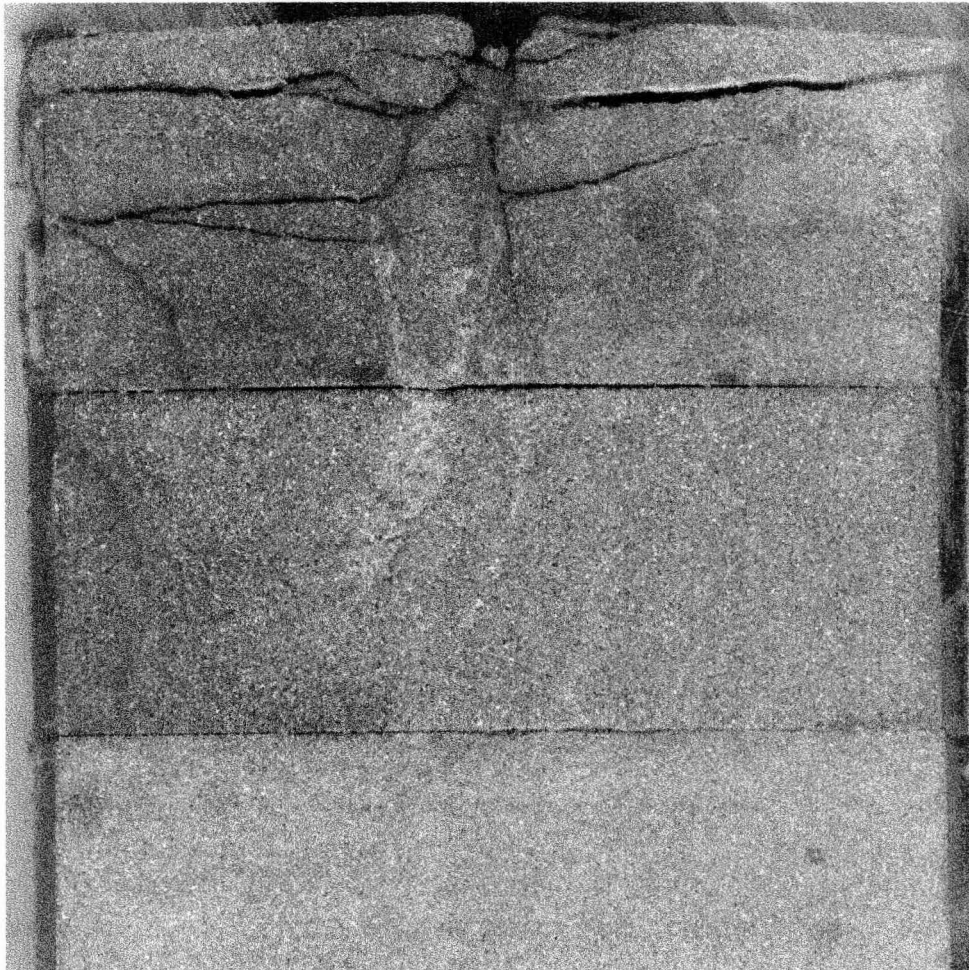


Figure 3.42 Picture of confined 20 mm thick layered sandstone with friction reducer (stearic acid)

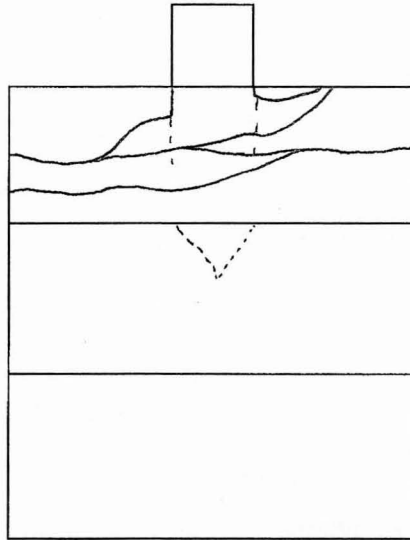


Figure 3.43 Fracturing in confined 20 mm thick layered sandstone without friction reducer

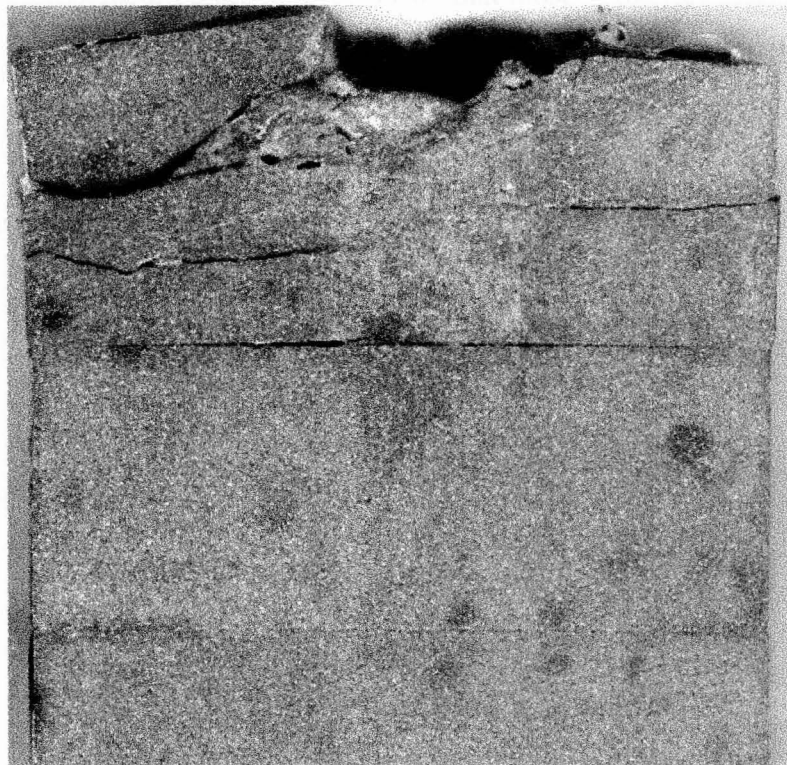


Figure 3.44 Picture of confined 20 mm thick layered sandstone without friction reducer

Figures 3.43 and 3.44 show the fracturing when a friction reducer is used on the layers. The surface parallel cracks are restricted to the first layer. However, the

significant difference is the formation of a well defined damage zone at the upper portion of the second layer where the failure stress under a confinement of 110 MPa was found to be 2 GPa which is greater than that when the stearic acid was used but less than the stress acting on the solid sample. It can thus be concluded that the strongest test set-up is the solid case. However, if an interface is introduced to system, the failure stress drops by about 20 percent.

3.3.5.b. Numerical Modelling

The final axisymmetric test simulations were carried out using DIGS. Two horizontal interfaces were introduced into the sample as shown in Figure 3.45 (a) which can be compared to the experimental results shown in Figure 3.45(b). Each interface is modelled as a sliding crack with zero cohesion and a friction angle of 30 degrees is assumed. Figure 3.45(b) shows that most of the fracturing is confined to the upper layer but the detailed damage pattern is different from the observed fracturing shown in Figure 3.45(a). These are also valid for the final model in which the thickness of the layer was decreased from 20 to 10 mm (Figure 3.46(a) and Figure 3.46(b)).

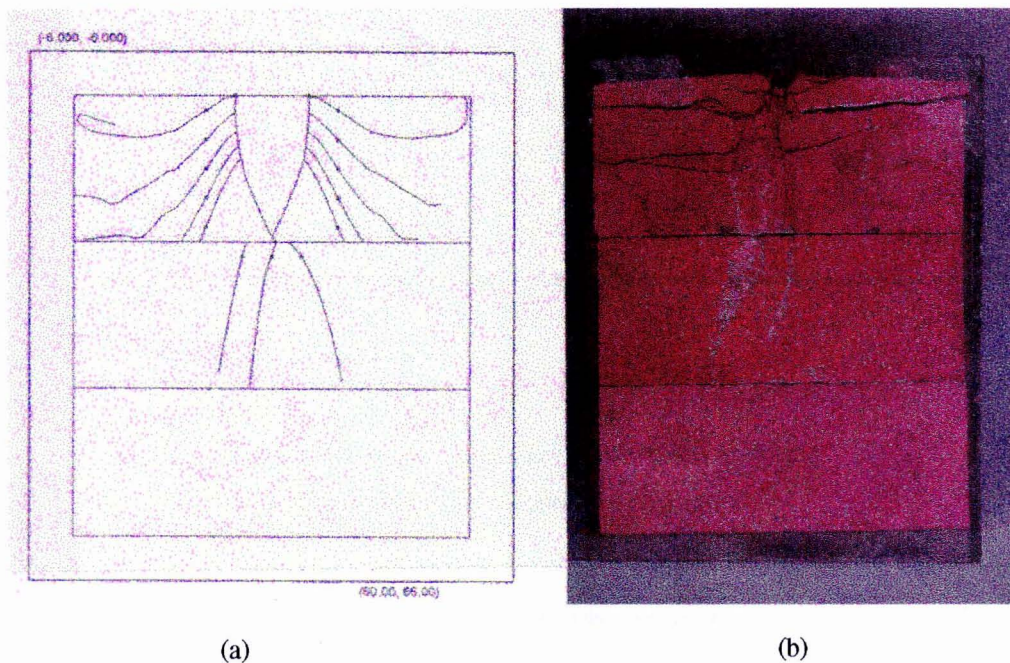
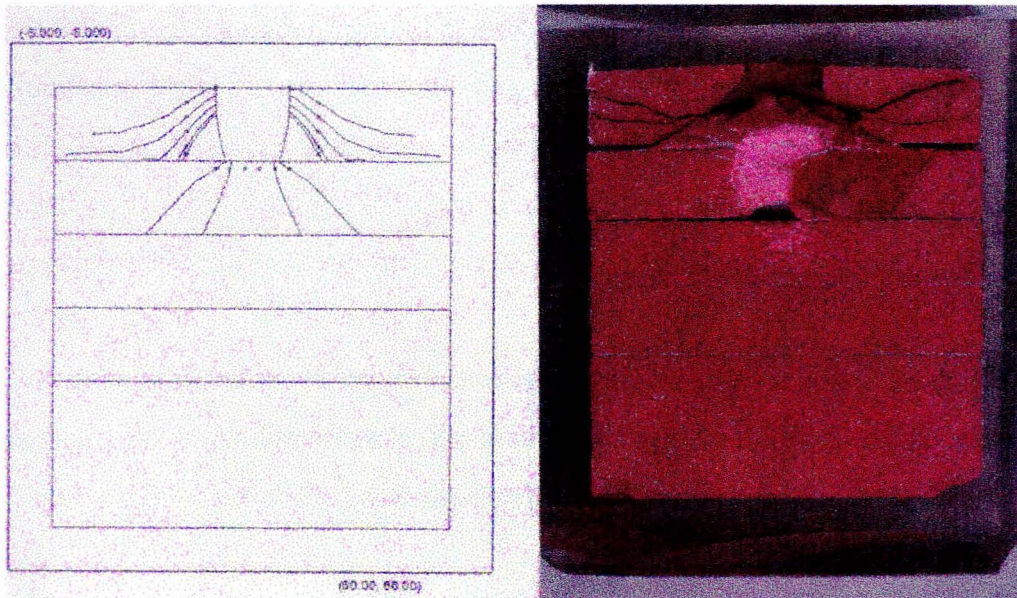


Figure 3.45 Confined sandstone sample (a) from the DIGS model (b) from the laboratory simulations



(a)

(b)

Figure 3.46 Confined sandstone sample (a) from the DIGS model (b) from the laboratory simulations

	TEST 1 [*]	TEST 2 [†]	TEST 3 ^{††}	TEST 4 [‡]
Without Friction reducer (MPa)	$\sigma_1=1337$ $\sigma_3=64$	$\sigma_1=1745$ $\sigma_3=53$	$\sigma_1=879$ $\sigma_3=27$	$\sigma_1=1210$ $\sigma_3=11$
With Friction Reducer (MPa)	$\sigma_1=942$ $\sigma_3=53$	$\sigma_1=2508$ $\sigma_3=22$	$\sigma_1=751$ $\sigma_3=22$	$\sigma_1=955$ $\sigma_3=16$

^{*}TEST 1; Sandstone layers

[†]TEST 2; Norite layers

^{††}TEST 3; Alternating Layers (Upper most layer is sandstone)

[‡]TEST 4; Alternating Layers (Upper most layer is norite)

Table 3.7 Failure stresses of the laboratory tests

3.3.6 Summary and calculations of axisymmetry tests

Summary of the tests carried out using axisymmetry geometry is given in Table 3.7. The table shows that when sandstone layers were used under confinement level of 53 MPa with friction reducer on the layers, then the failure stress was found to be 942 MPa (Test 1 in Table 3.7).

When the height of the layers is decreased, the fracture load is also decreased. There is thus a linear relation between the failure load and the thickness of the layer (see Figure 3.47) for which the linear regression best fit is;

$$y=8.64x+144 \quad R^2=1$$

where;

y= failure stress in MPa

x= thickness of layer in mm

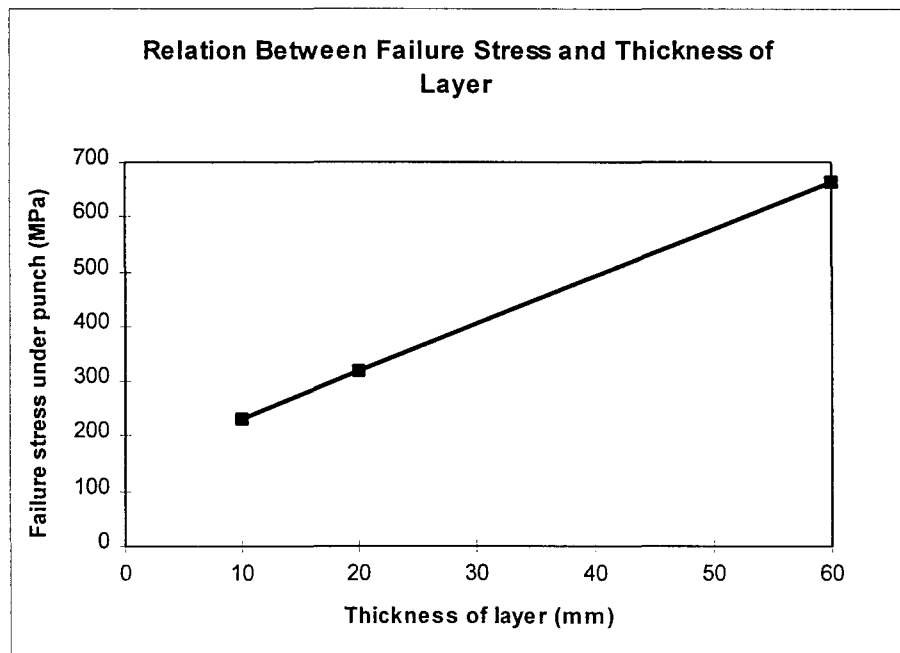


Figure 3.47 Relation between failure stress and thickness of upper most layer for sandstone

The degree of friction along the interface has a significant effect on the fracture pattern and failure load. Under reduced friction conditions, more vertical extension

fractures appear on deeper layers; for example the third layer would have more fractures than the second layer. From the failure load point of view, when friction reducer is applied the failure load decreases by about 20 percent (Table 3.7).

3.4 TRIAXIAL STRIP PUNCH TESTS

3.4.1 Description of the tests

In order to carry out 3-D tests on prismatic rock samples, a triaxial load cell, shown in Figure 3.48, was constructed. The cell can accommodate 80 or 100 mm cubic samples under confining pressure levels of up to 10 MPa. Models representing strip and square pillars were prepared from quartzite, the physical properties of which are shown in Table 3.8.

The laboratory tests carried out can be divided into four groups;

1. Long Strip Punch Tests
2. Incremental Loading Tests
3. Layered Strip punch Tests
4. Square Punch Tests

	QUARTZITE
UCS	200 MPa
UNIAXIAL TENSILE STRENGTH	27 MPa
INTERNAL FRICTION ANGLE	51°
COHESION	36 MPa
POISSON'S RATIO	0.17
YOUNG MODULUS	68 GPa

Table 3.8 Physical properties of quartzite used in the tests determined from Laboratory tests

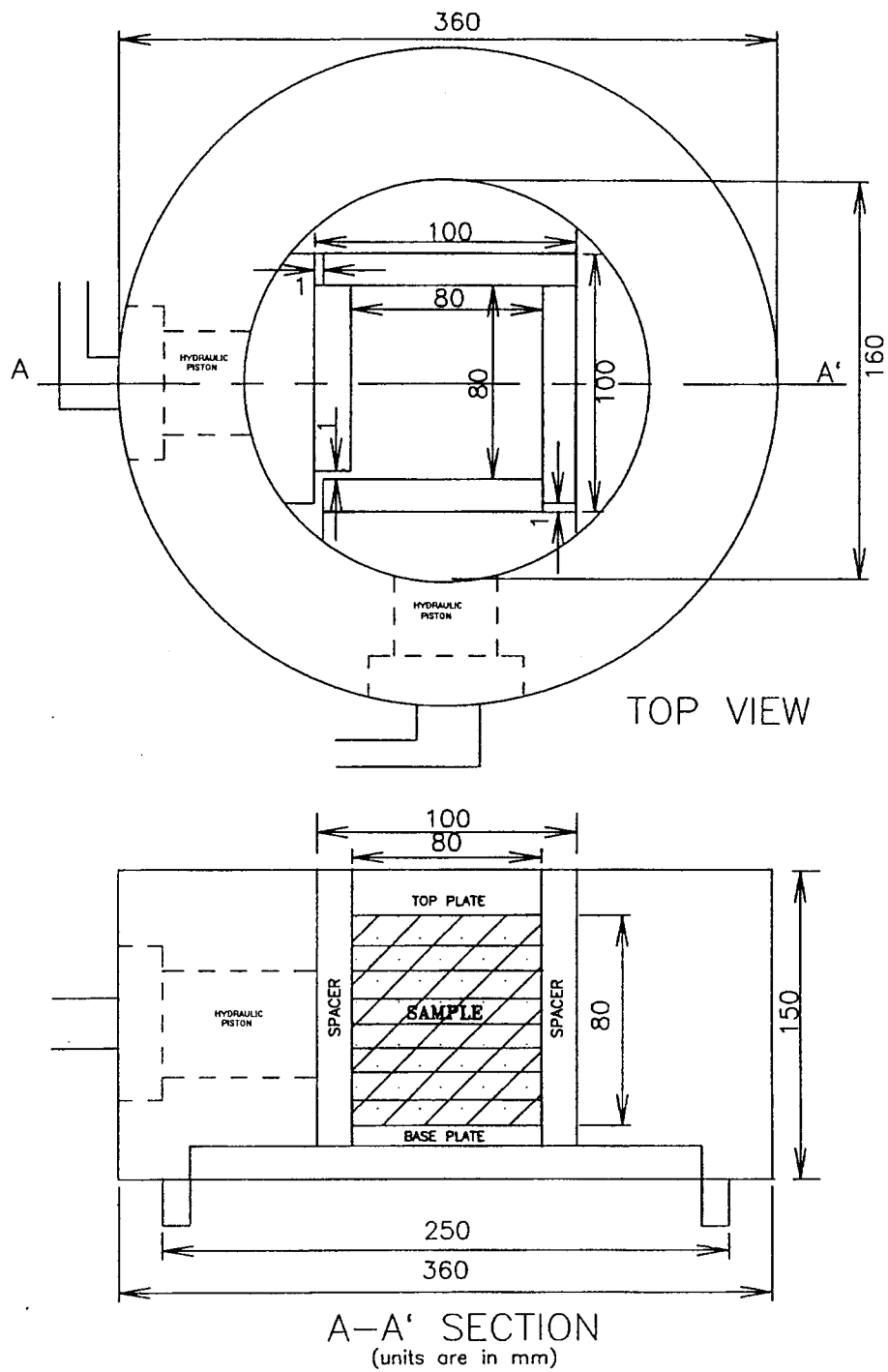


Figure 3.48 The triaxial frame (a) top view (b) A-A' section

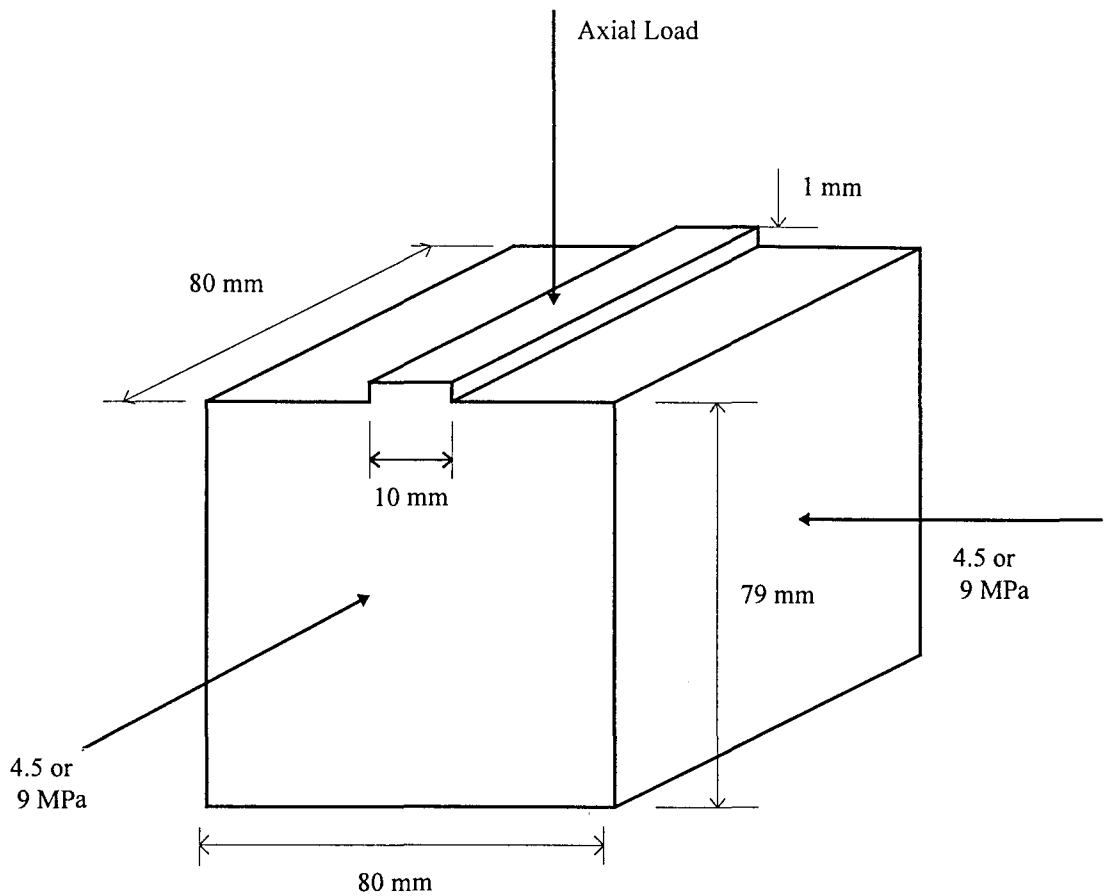


Figure 3.49 Dimensions of the strip punch tests specimen

Cubical specimens were prepared in such a way that a 1 mm high and 10 mm wide strip were left on the specimen as shown in Figure 3.49. The loading direction was through this punch (Figure 3.49) and stearic acid was applied as a friction reducer to all sides of the specimen which were in contact with the cell. Two different levels of confinement were applied: 4.5 MPa and 9 MPa.

To determine the relation between axial load applied through punch the area and the confinement applied from sides to satisfy the plane strain conditions, a number of FLAC runs were carried out. The obtained relation is given in Appendix E.

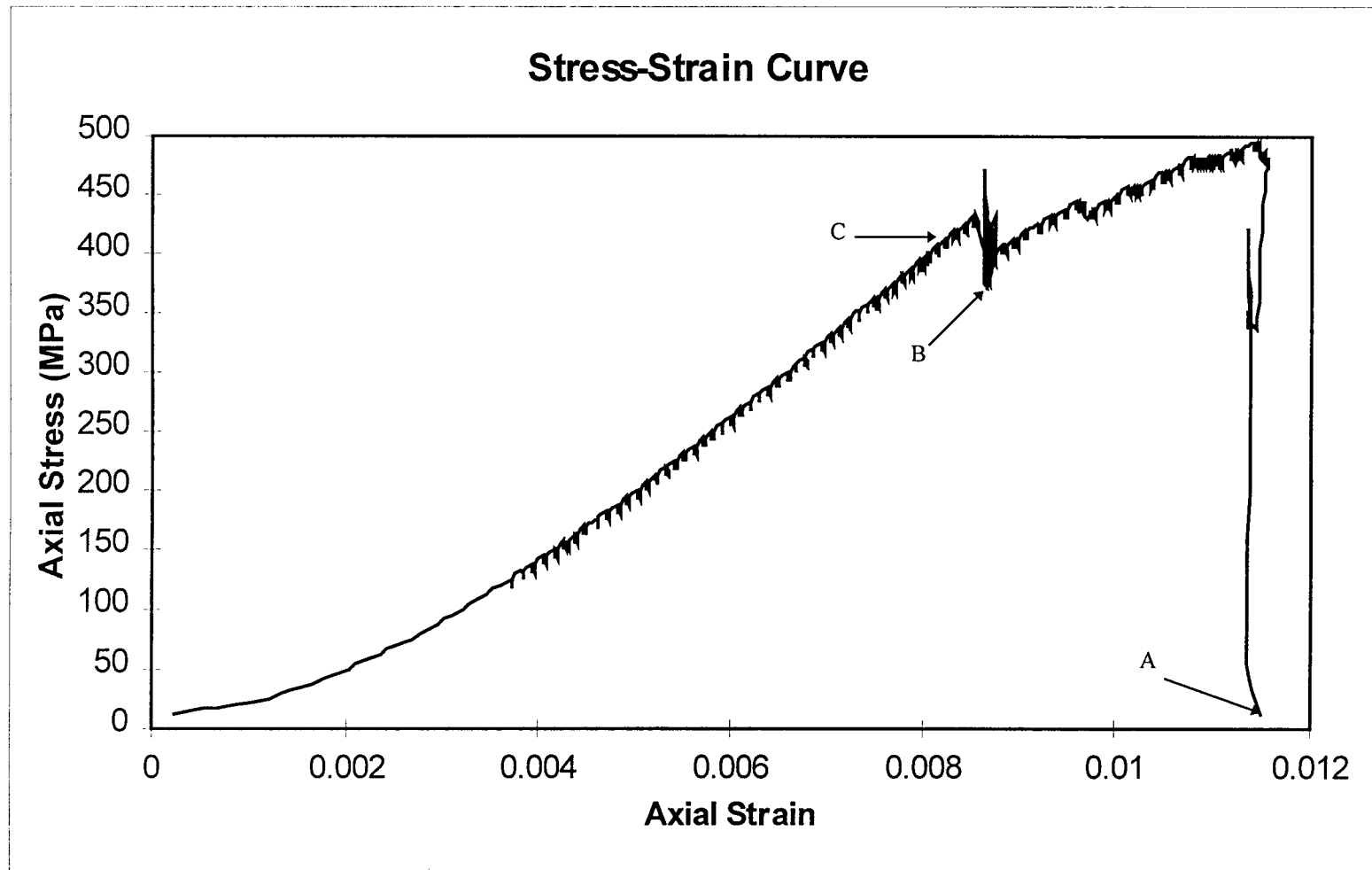


Figure 3.50 Stress-strain curve of the long strip punch test

3.4.2. Long Strip Punch Tests

The stress-strain curve of the test has two peak points where stress drops occur, as seen in Figure 3.50. The fracture pattern obtained from the 4.5 MPa confinement case is shown in Figure 3.51 for which the failure stress active on the punch was recorded to be about 490 MPa. The cone width is the same as the punch width of 10 mm. Unlike the axisymmetric tests carried out previously, the cone height is about 20 mm, which is twice the base width of the cone and the punch width. The same test was repeated at a confinement level of 9 MPa, but there was not a significant difference between the 4.5 and 9 MPa confinement cases. The main difference was that, at the higher confinement, there were more tensile fractures around the cone (Figure 3.52).

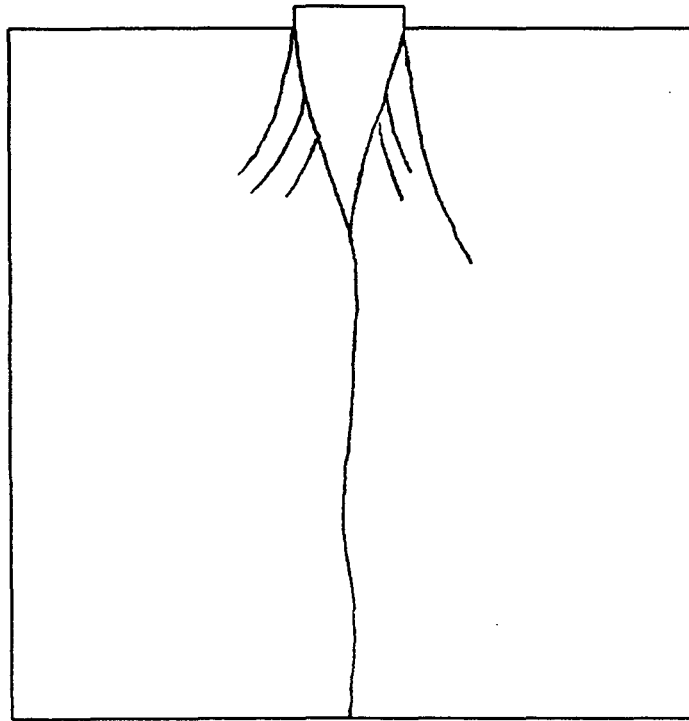


Figure 3.51 Main fracture pattern of strip punch test (at point C of Figure 3.50) under confinement level of 4.5 MPa.

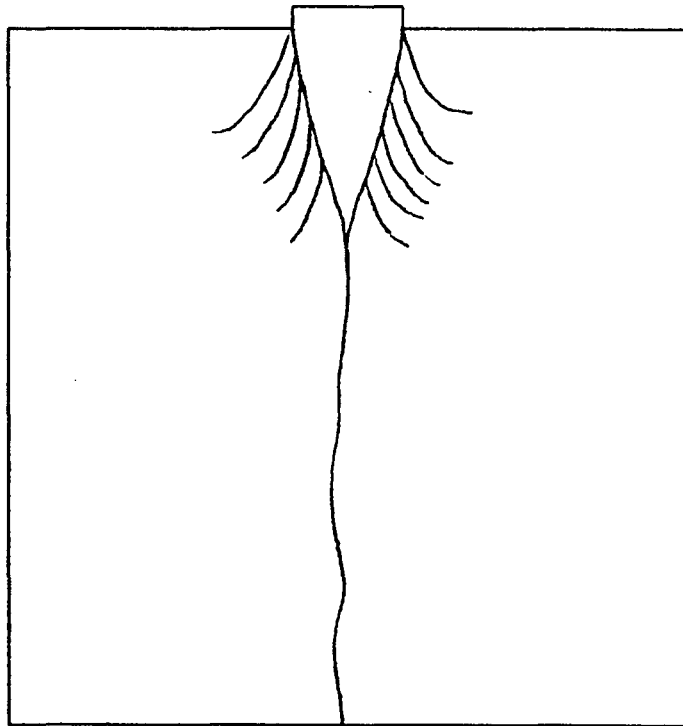


Figure 3.52 Main fracture pattern of strip punch test (at point C of Figure 3.50) under confinement level of 9 MPa

3.4.3 Incremental Loading

3.4.3.a. Physical Modelling

An understanding of sequence of the growth of fractures is crucial, when modelling of the tests with numerical codes is planned. To obtain the whole sequential growth of fracture, punch tests were conducted at different loading. In the first case, the complete fracture pattern of which was developed is shown in Figure 3.52

The main features of the fracture pattern can be summarised as follows: a cone just beneath the punch, with its width and height being 10 and 20 mm respectively. However, there is a tensile fracture which splits the sample into two pieces starting from the bottom of the cone and ending at the bottom of the sample. Apart from this main tensile fracture, there are number of additional tensile fractures around the cone (Figure 3.53.c).

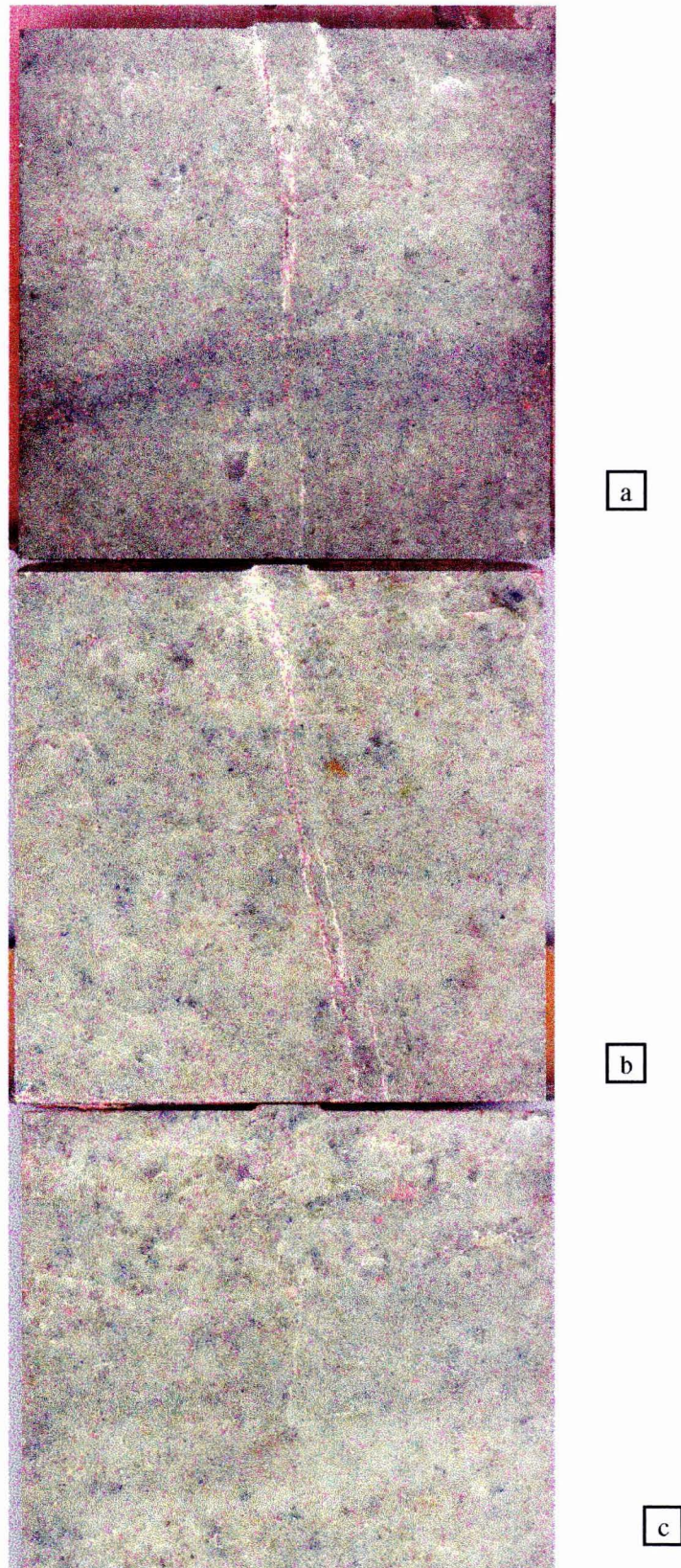


Figure 3.53 Pictures of whole sequence of strip punch test under confinement level of 9 MPa

In the second case, the test was stopped at the bottom of the first peak, which is the point B in Figure 3.50. At that point, the final stress was recorded as 520 MPa and the strain was 95% of the total strain (Figure 3.54). Although there was not a complete cone beneath the punch, the main tensile fracture could already be observed. Many tensile fractures were also observed around the cone area and it could be concluded that before the full formation of the cone, the main tensile fracture appears, as well as the tensile fractures around the cone area. (Figure 3.55 and Figure 3.53.b)

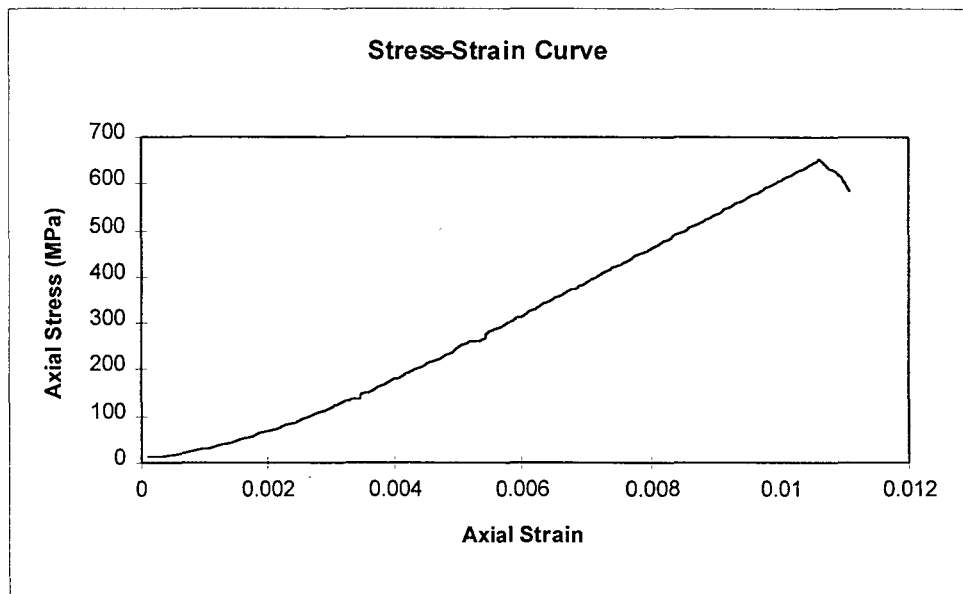


Figure 3.54 Stress-strain curve of quartzite sample under confinement level of 9 MPa at point B from Figure 3.50

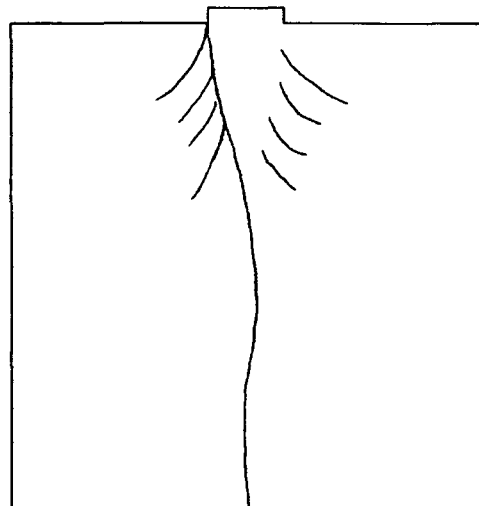


Figure 3.55 Fracture pattern of quartzite sample under confinement of 9 MPa at point B from Figure 3.50

In the third test, loading of the sample was stopped just before the first peak (at point C in Figure 3.50) and the fracture pattern observed is shown in Figure 3.56. The stress on the strip punch was 560 MPa and the strain was 90 % of the total strain when the test was stopped (Figure 3.57). It is interesting to note from Figures 3.56 and 3.53.c that the vertical tensile crack at the centre starts forming before the formation of the cone is completed.

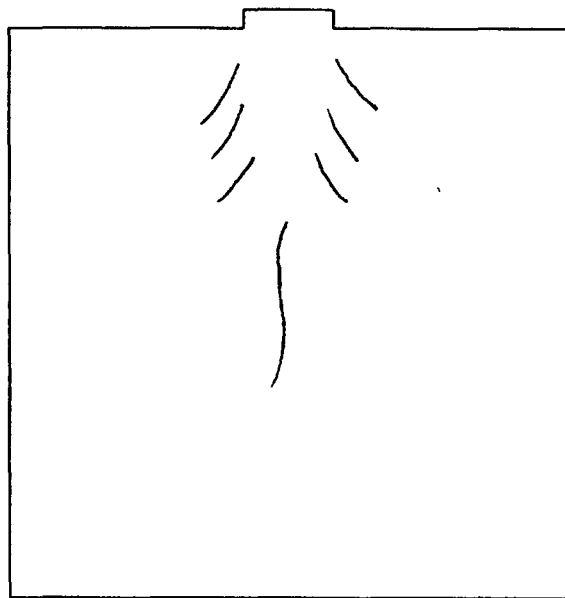


Figure 3.56 Fracture pattern of quartzite sample under confinement level of 9 MPa at point C from Figure 3.50

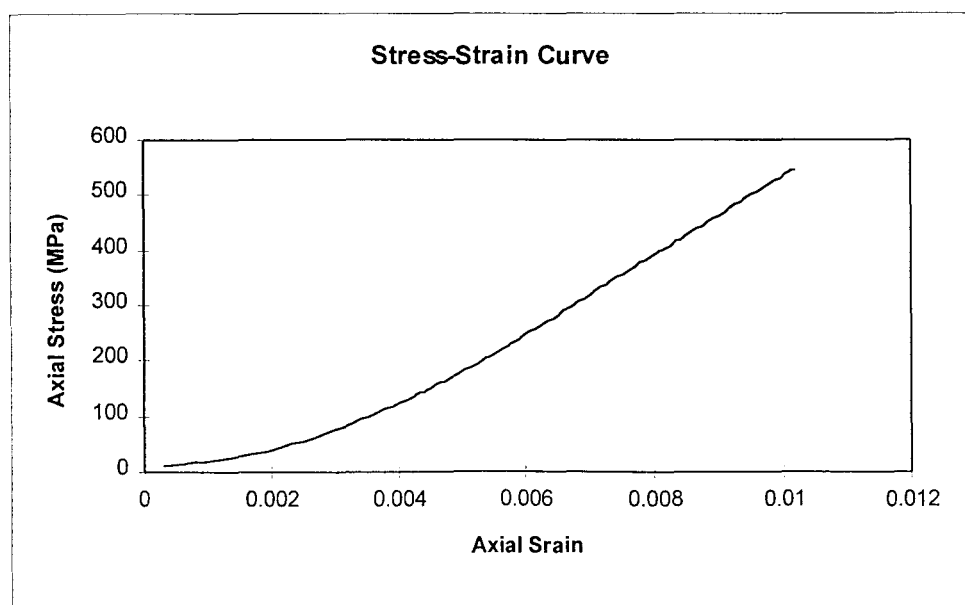


Figure 3.57 Stress-strain curve of quartzite sample under confinement level of 9 MPa at point C from Figure 3.50

3.4.3.b. Numerical Modelling

In the this model, the triaxial strip punch test was modelled using DIGS and FLAC. For the DIGS model, as in the physical modelling, lateral stress boundary conditions of 10 MPa were applied to the sides of the sample (Figure 3.58).

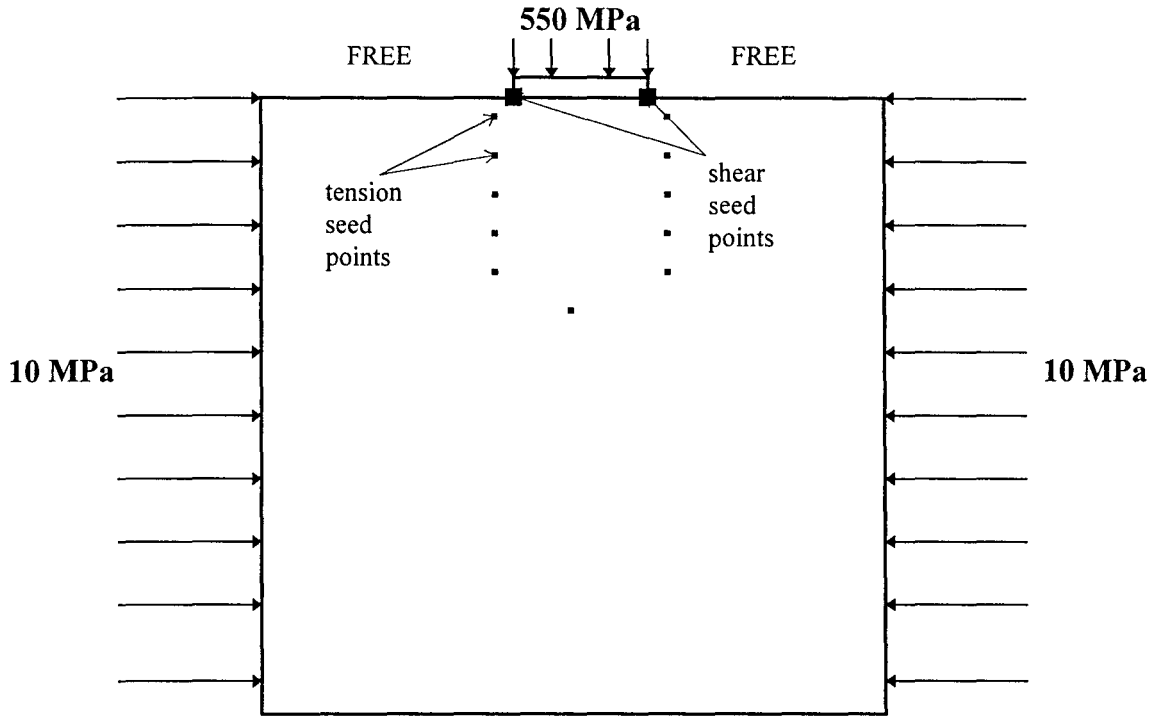


Figure 3.58 Seed points locations and boundary conditions for the DIGS model

Crack and Material Properties	M (shear type of fracture)	N (tensile type of fracture)
Unmobilized cohesion (MPa)	35.1	45
Unmobilized friction angle (°)	35	51
Mobilized dilation angle (°)	0	0
Mobilized cohesion (MPa)	0	0
Mobilized friction angle (°)	32	35
Reverse dilation angle (°)	0	0
Tension cutoff (MPa)	30	10
Young modulus (GPa)	68	68
Poisson's ratio	0.17	0.17

Table 3.9 Crack and material properties assumed for the DIGS model

A stress of 550 MPa, which was determined from the physical modelling of the failure stress, was specified through out the punch area. Two shear and as well as some tensile seed points around the cone zone were introduced in to the model to enable the cone and the surrounding tensile fractures to form (Figure 3.58). The material properties used for the model are given in Table 3.9.

From the two shear seed points, the usual cone started to grow together with the main tensile fracture at the middle of the sample. While these fractures were growing, the tensile fractures are initiated around the cone area. This result indicates that there is some consistency between the physical modelling and the DIGS model (Figure 3.59). Figure 3.60 shows the stress-strain curve of the test from DIGS run.

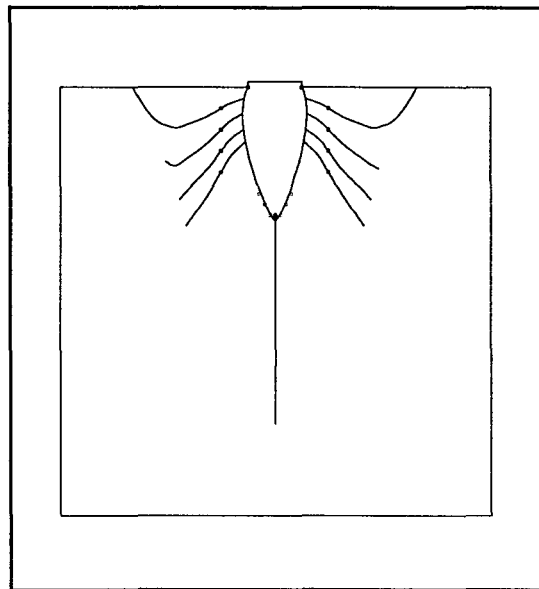


Figure 3.59 Fracture pattern of the triaxial strip punch test modelled by DIGS

Modelling of the triaxial strip punch test with FLAC gives more or less the similar results compare to those obtained from both the physical and DIGS models. Three different models were used namely the Elastic, Mohr-Coulomb Plasticity, and Strain Softening Models. The boundary condition for this modelling are shown in Figure 3.61. Only the right side of the actual sample is taken due to the symmetry

properties of FLAC. Unlike the DIGS model, a constant downward displacement is imposed through the punch area. The properties of the material (quartzite) and the FLAC model constants used for the model are given in Table 3.10.

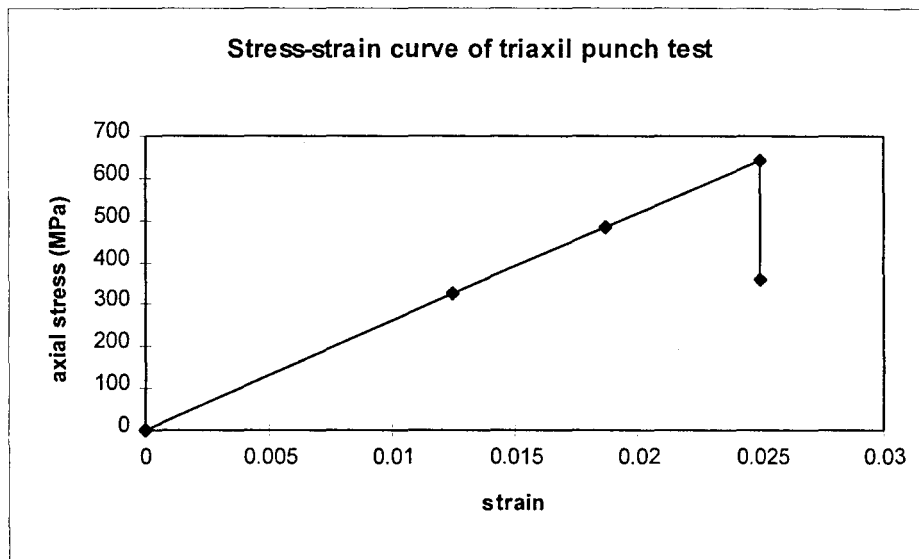


Figure 3.60 Stress-strain curve of the strip punch test from DIGS run

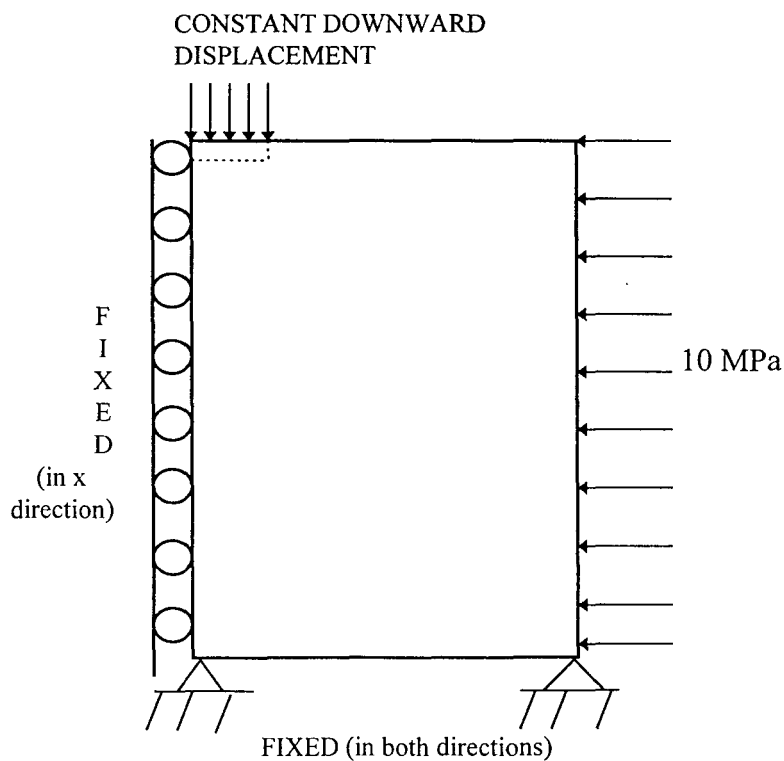


Figure 3.61 Geometry and boundary conditions used for the FLAC modelling

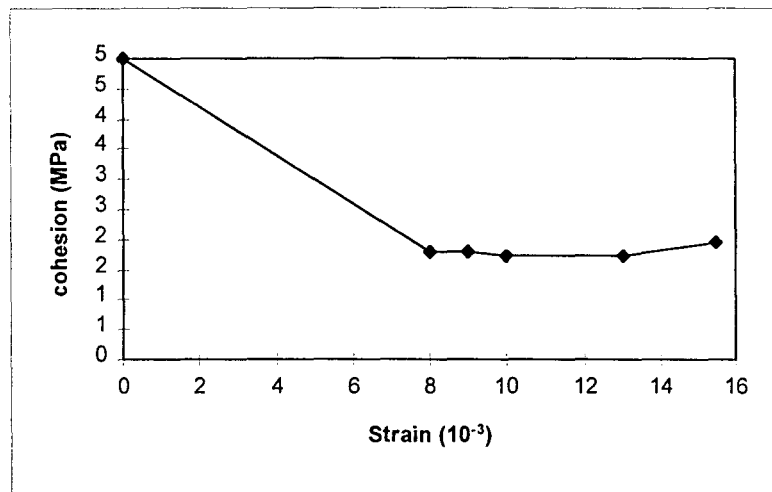
	Elastic Model	Mohr-Coulomb Model	Strain-Softening Model
Density (kg/m³)	2700	2700	2700
Bulk Modulus (GPa)	31.5	31.5	31.5
Shear Modulus (GPa)	31	31	31
Tension (MPa)	-	15	15
Cohesion (MPa)	-	20	Figure 3.62 (a)
Friction angle (°)	-	51	Figure 3.62 (b)
Dilation angle (°)	-	30	Figure 3.62 (c)

Table 3.10 Material properties used for the FLAC models

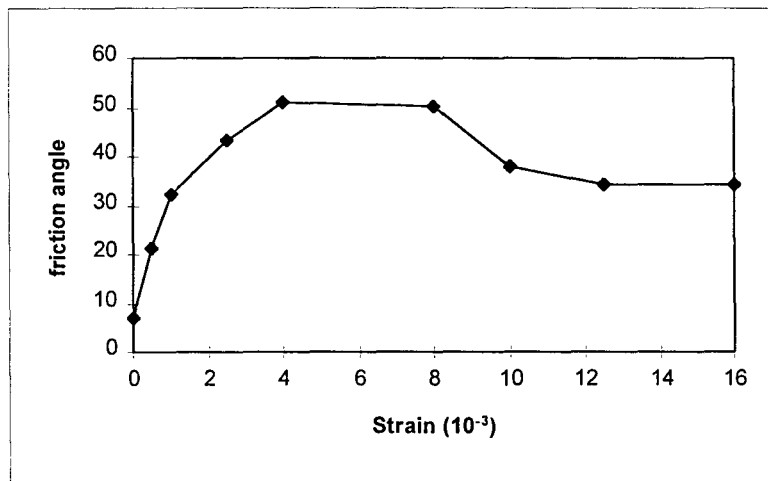
The Mohr-Coulomb Failure Criteria from the elastic run shows a similar result to the physical modelling (Figure 3.63). From the results of the elastic run it is possible to determine the failure region and elastic stress distribution in the sample using the Mohr-Coulomb failure criterion, and the strength/stress ratio which indicates where the factor of safety is less than 1.

The Mohr-Coulomb Plasticity model gives more realistic results than the elastic FLAC model if the Hertzian and tensile cracks are included. The boundary loading conditions are the same as for the elastic model (Figure 3.61). The plasticity indicator shows a similar trend to the physical model (Figure 3.64) for which the stress-strain curve monitored and is shown in Figure 3.65.

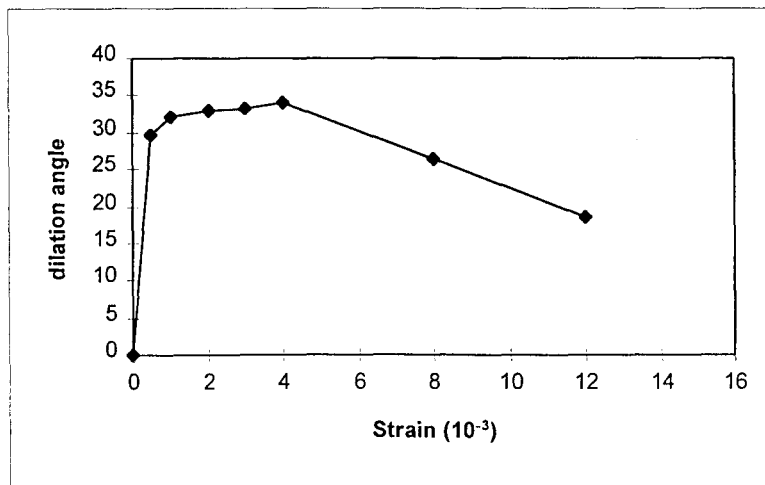
The second plasticity model used was the strain softening model with the boundary conditions the same as for the previous models (Figure 3.61). The effect of changing of material properties with increasing strain is specified by the tables which are available in the FLAC models shown in Figure 3.62.



(a) Cohesion-strain graph for the SS FLAC Model



(b) Friction angle-strain graph for the SS FLAC Model



(c) Dilation angle-strain graph for the SS FLAC Model

Figure 3.62 Strain Softening (SS) parameters for the FLAC model

The modelling shows major differences between the Strain Softening Model and the Mohr-Coulomb Model. For example, there is the usual cone beneath the punch for both models, but in the Mohr-Coulomb case, the height of the cone the width which is not valid for the Strain Softening Modelling. There are some tensile failure zones next to the punch surface in the Mohr-Coulomb Model, but it is not possible to see these cracks in the Strain Softening Model. From the physical model, one of the most important features is the tensile crack which starts from the bottom of the cone and ends at the bottom of the sample. This crack can be seen in both models.

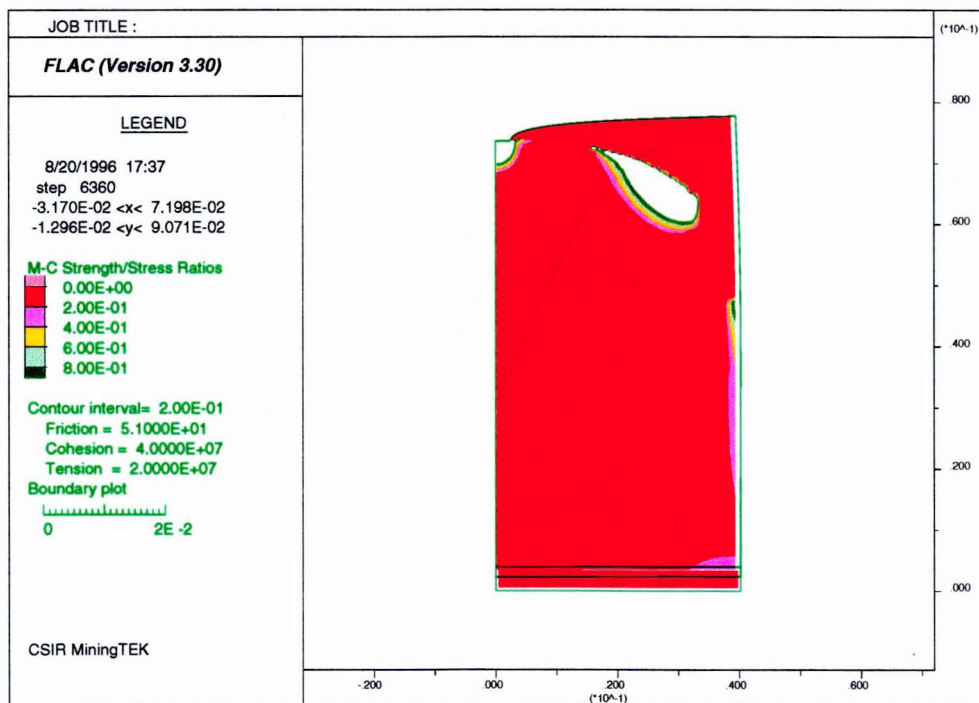


Figure 3.63 Mohr-Coulomb strength/stress ratio of the triaxial strip punch test by an elastic FLAC model

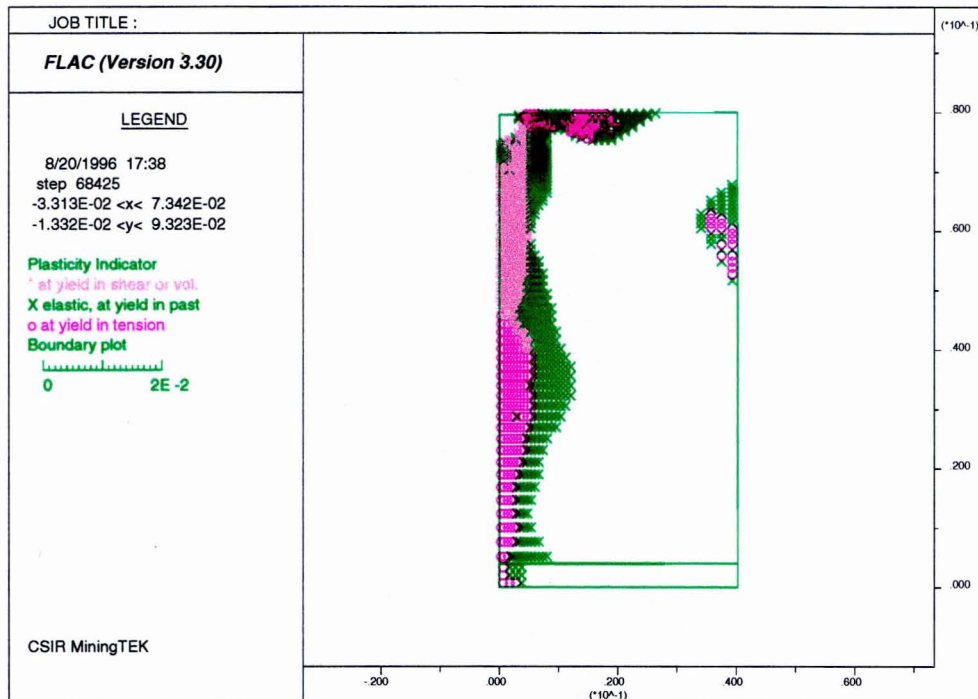


Figure 3.64 Mohr-Coulomb Plasticity plot of the triaxial strip punch by FLAC

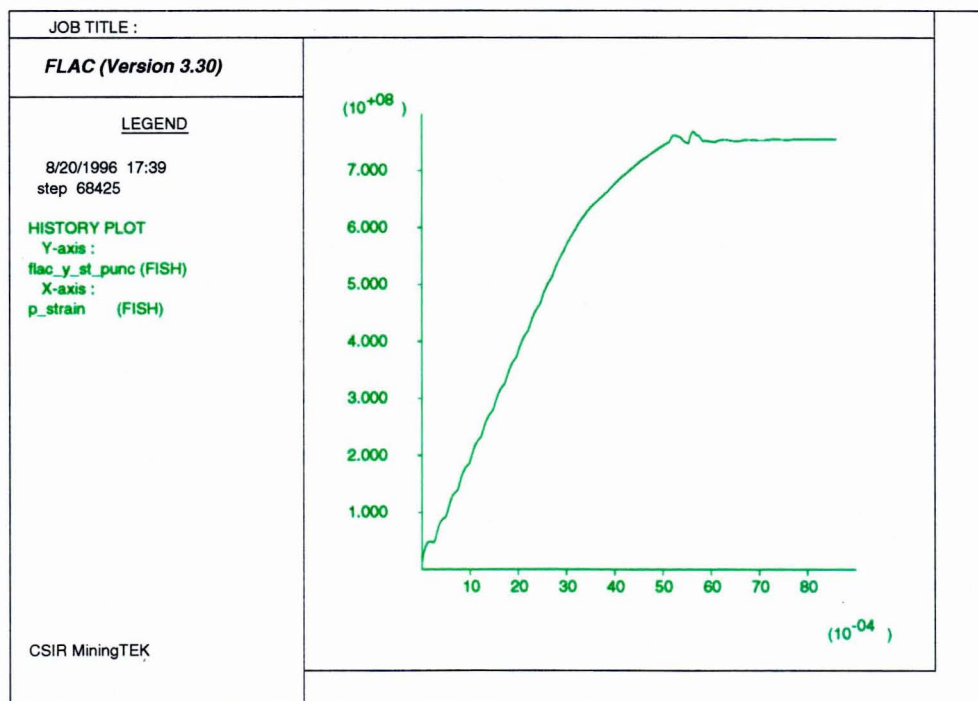


Figure 3.65 Stress-strain curve of the triaxial strip punch, Mohr-Coulomb FLAC model

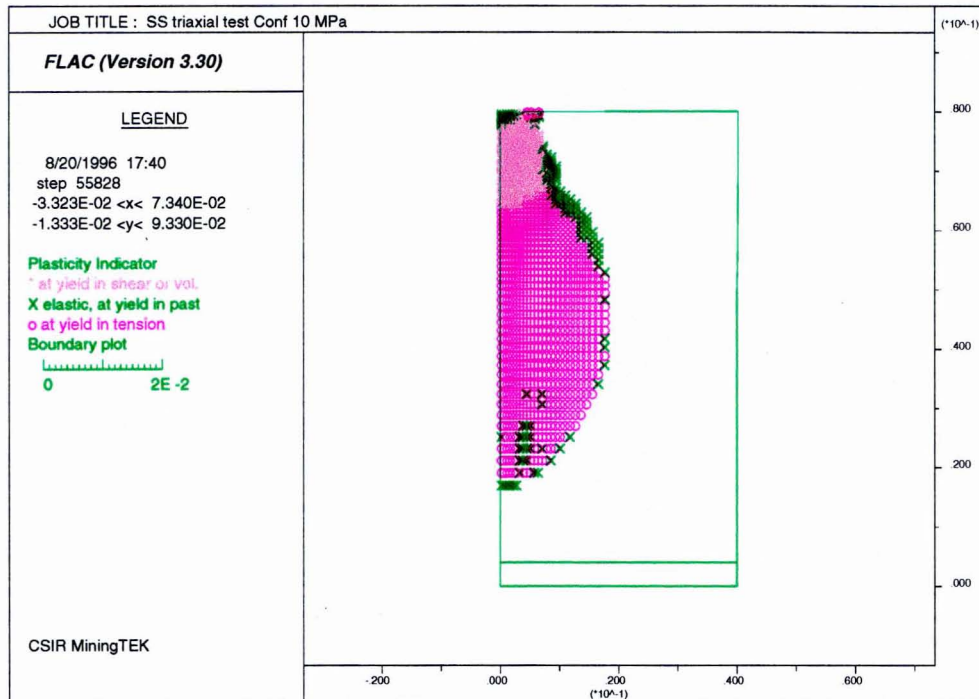


Figure 3.66 Strain softening model of the triaxial strip punch model

The area under tensile plastic strain is not the same in the SS and Mohr-Coulomb models because for the SS-Model that zone is wider than the physical models and the Mohr-Coulomb Models (Figure 3.64 and Figure 3.66).

3.4.4 Layered Strip Punch Tests

3.4.4.a. Physical Modelling

This series of tests can be divided into three groups according to the layer conditions;

1. 15 mm layered test without friction reducer in between the layers
2. 15 mm layered test with friction reducer (stearic acid) in between the layers
3. 40 mm natural discontinuity layer model test

In the first tests, the samples were prepared as in the previously described except that there was a 15 mm layer perpendicular to the punch loading direction (Figure 3.67).

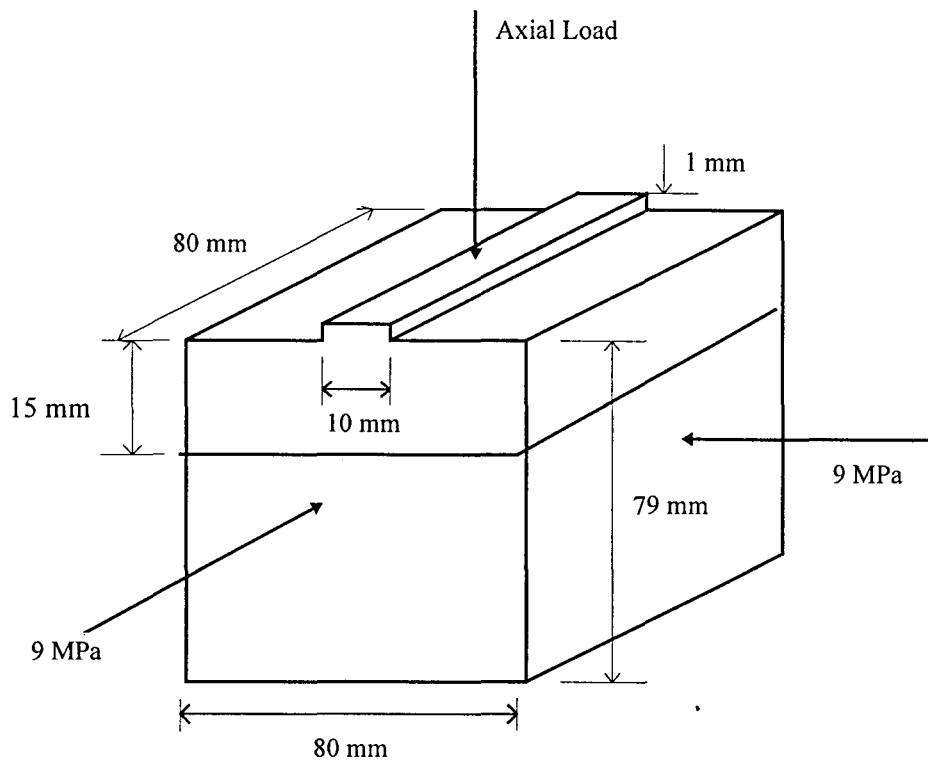


Figure 3.67 Dimensions of the quartzite sample in layered tests

In the first case no stearic acid was placed between the layers and the loading conditions were kept the same as in the previous cases. The fracture pattern resulting from this test is shown in Figure 3.68. The general characteristics were that there was a truncated cone just beneath the punch. For the solid strip punch tests the cone height was about 20 mm but in this case (Figure 3.68), the cone is truncated by the 15 mm layer preventing the formation of a complete cone. Similarly, there are tensile fractures around the cone and the main tensile fracture is seen to initiate from the interface and to grow to the bottom of the sample. The failure stress was 550 MPa, which is less than the solid case (Figure 3.69).

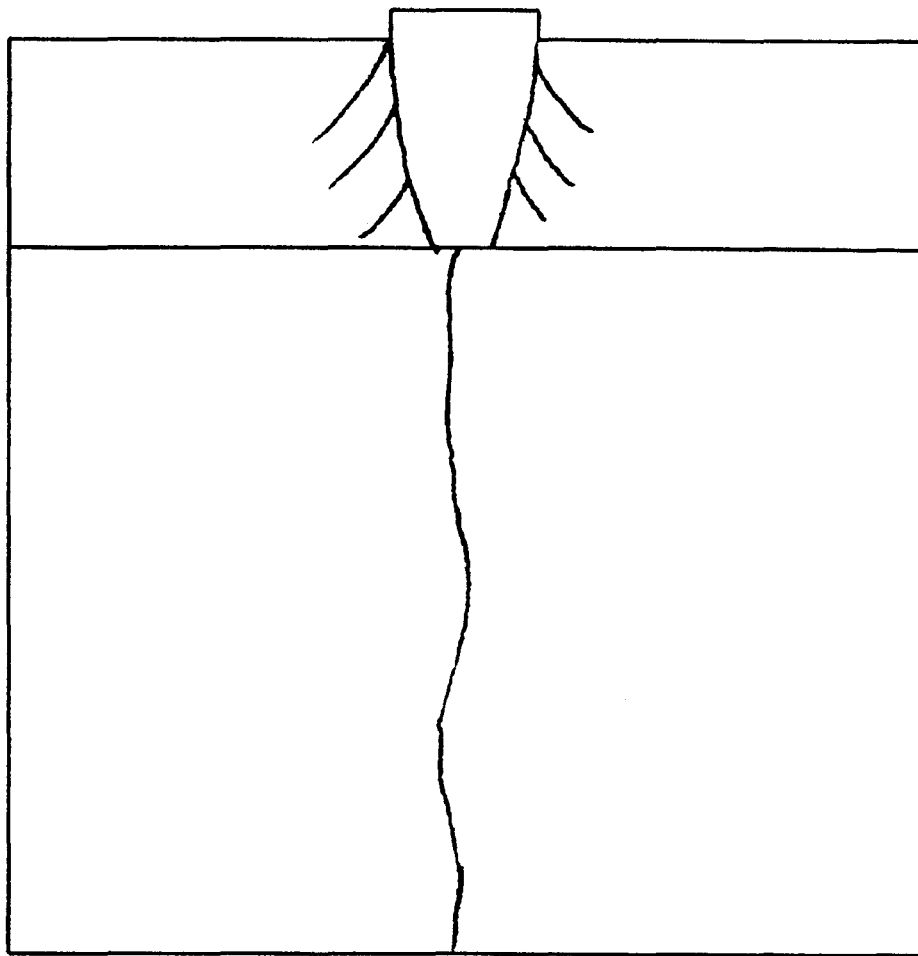
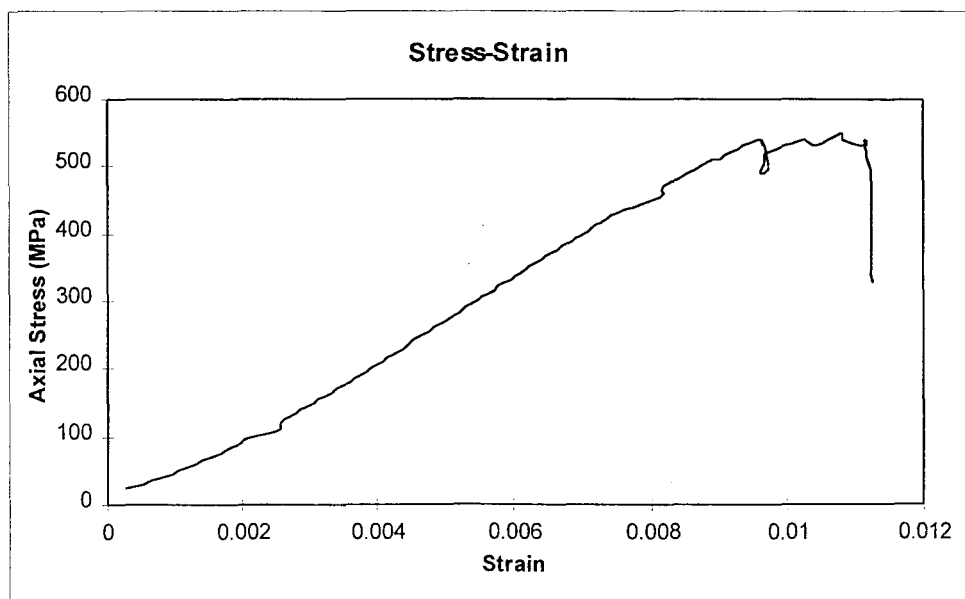


Figure 3.68 Fracture pattern of 15 mm layered quartzite without stearic acid



3.69 Stress-strain curve for a 15 mm layered quartzite sample without a friction reducer

The second test was similar, with the same geometry and the same boundary conditions, but with stearic acid placed between the layers as a friction reducer. The fracture pattern obtained is shown in Figure 3.70. The failure stress was found to be 425 MPa which is less than the failure stress in the previous test (Figure 3.71).

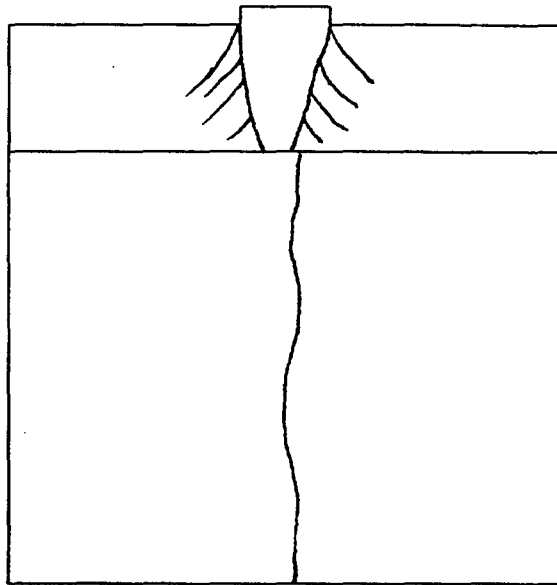
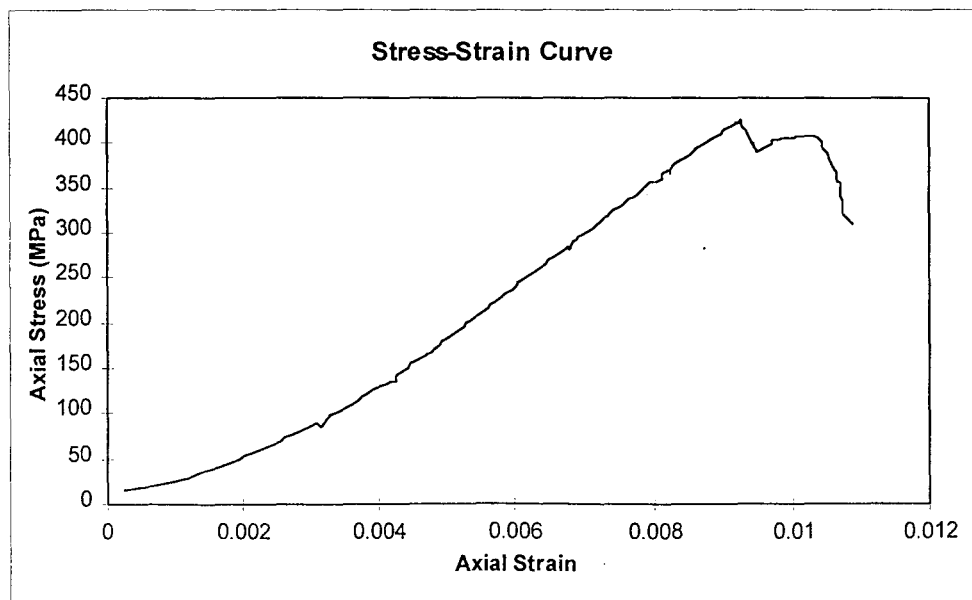


Figure 3.70 Fracture pattern of 15 mm layered quartzite with stearic acid case



3.71 Stress-strain curve of 15 mm layered quartzite sample with friction reducer

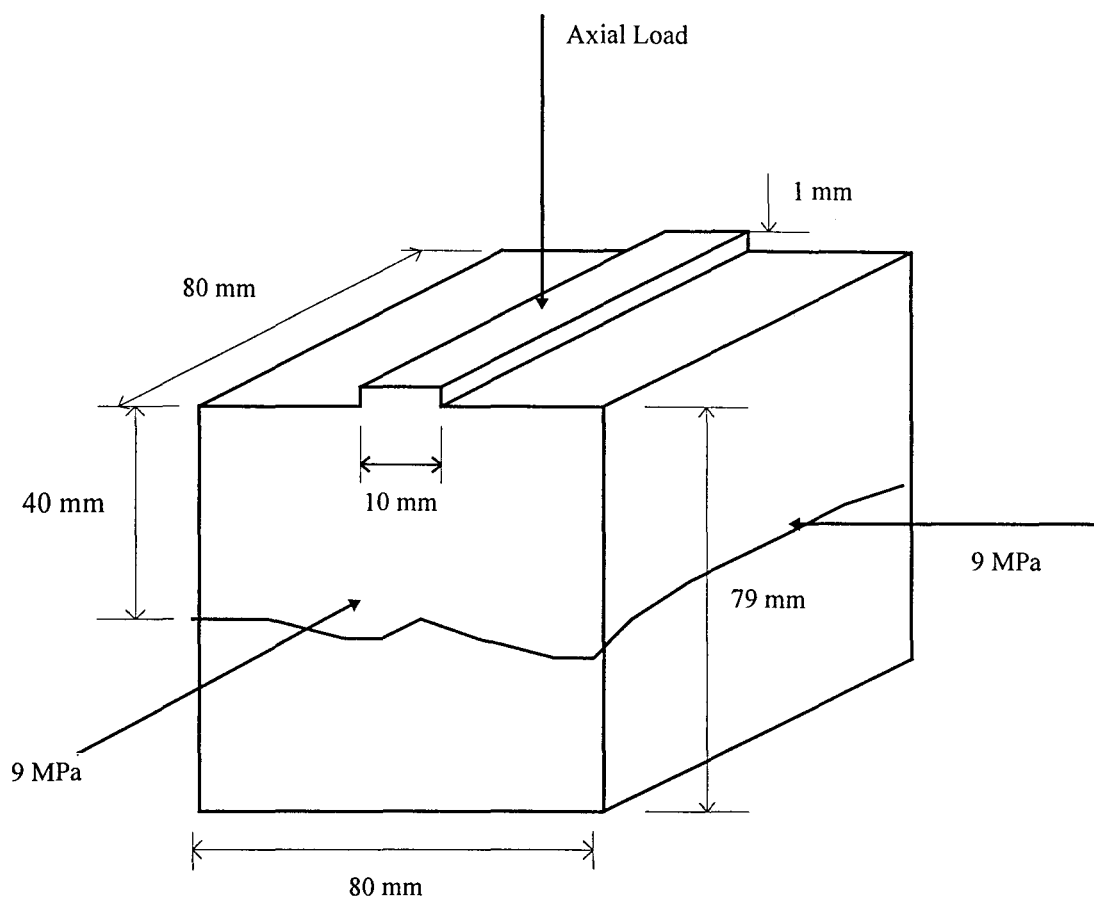


Figure 3.72 Dimensions of the quartzite sample with a natural discontinuity punch test

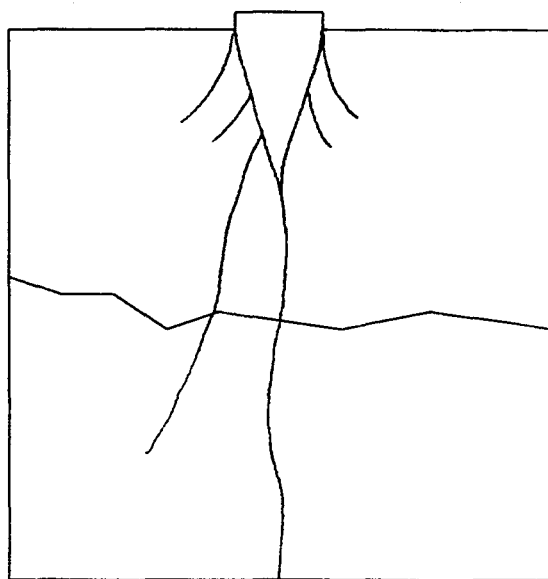
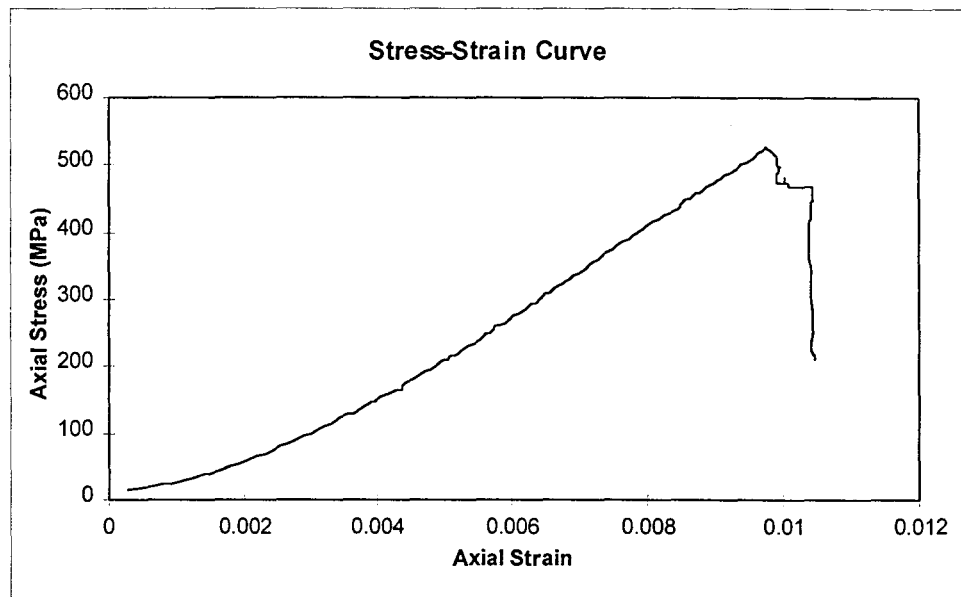


Figure 3.73 Fracture pattern of layered quartzite with natural discontinuity case

The last example is of a natural discontinuity instead of an artificial layer (Figure 3.72). The resulting pattern was that of a full cone, mainly because the layer thickness was greater than 20 mm (Figure 3.73). The failure stress was found to be 550 MPa which is the same as found in the 15 mm layered sample with no stearic acid (Figure 3.74).



3.74 Stress-strain curve of layered quartzite sample containing a natural discontinuity

3.4.4.b. Numerical Modelling

The final model consisted of a layered quartzite strip punch test for which DIGS was used. Unlike the previous solid models, for this model a crack was defined to simulate a layer which is 15 mm below the bottom of the punch. The boundary conditions were the same as the previous solid models and the properties of the cracks used for the run are given in the Table 3.11.

As in the solid punch test, two shear seed points were placed at the corners of the punch to simulate the cone formation, and several tension seeds were placed around the cone area to initiate tensile fractures. To simulate the physical model, one tension seed point was placed at the layer interface to allow tensile fracturing, which splits the sample into two from which fractures grow. The resultant fracture pattern from the DIGS analysis shows the same trend and shape as in the physical

modelling in which the cone was truncated (Figure 3.75). Below the layer the principal stresses generate a tensile region which causes the main tensile crack to grow from the layer, which was not observed in the solid case where the tensile crack started from the middle of the sample.

Crack and Material Properties	shear type of fracture	tensile type of fracture	fracture to simulate the layer
Unmobilized cohesion (MPa)	35.1	40	0
Unmobilized friction angle (°)	35	51	30
Mobilized dilation angle (°)	0	0	0
Mobilized cohesion (MPa)	0	0	0
Mobilized friction angle (°)	32	35	30
Reverse dilation angle (°)	0	0	0
Tension cutoff (MPa)	30	10	0
Young modulus (GPa)	68	68	68
Poisson's ratio	0.17	0.17	0.17

Table 3.11 Crack and material properties used for the DIGS model

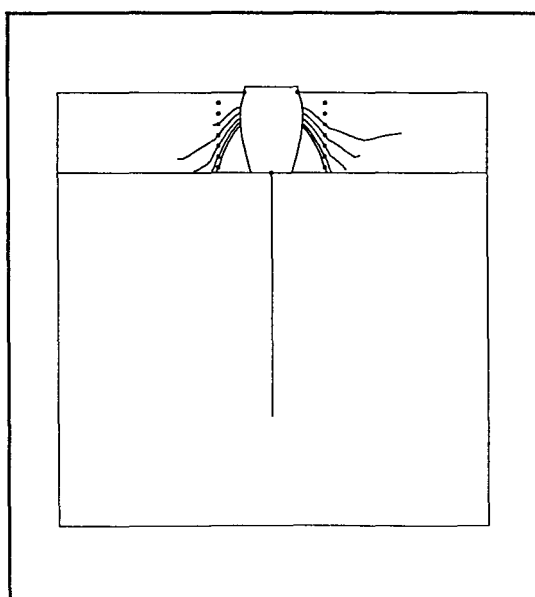


Figure 3.75 Fracture pattern of the layered quartzite model by DIGS

The stress-strain curve of the layered punch test from DIGS run is shown in Figure 3.76.

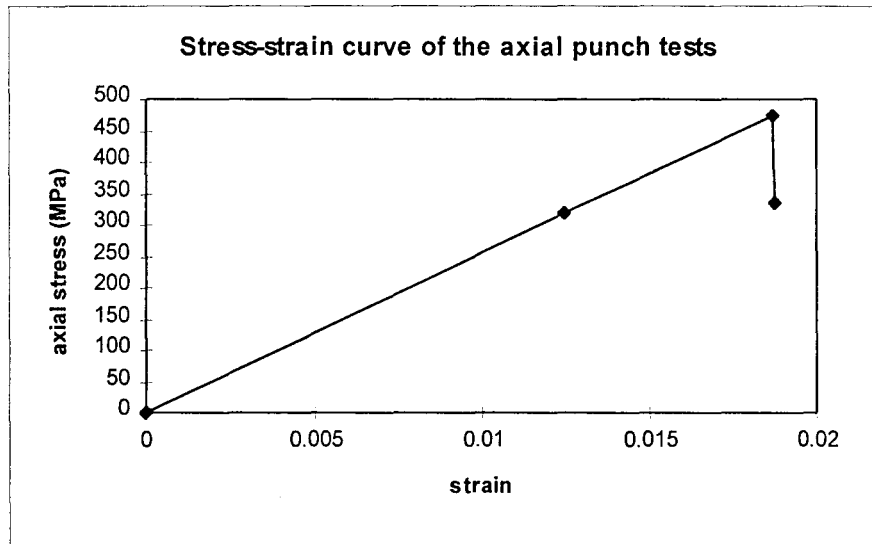


Figure 3.76 Stress-strain curve of the strip punch test from DIGS run

These tests showed some consistency between both the numerical models and the physical model experiments, provided the appropriate loading conditions of plane strain or axisymmetry were used. The general behaviour of the sample or crack growth under a punch depends mainly on the boundary conditions, the physical properties of the sample, the punch type and its shape. However, the biggest disadvantage of numerical modelling is to estimate the numerical properties of the sample that determine the fracture onset and propagation, or in other words the calibration of the solution. For example, the final fracture pattern using DIGS was known from the laboratory experiments before the numerical simulation was carried out. Two parameters (internal friction angle and tension cutoff) had to be manipulated to obtain the same pattern as in the laboratory simulations. Thus from the DIGS point of view, it is important to know the which parameter changes which characteristics of the fracture. Table 3.12 gives the original parameters as well as the changed parameters.

Crack and Material Properties	ORIGINAL PARAMETERS		MODIFIED PARAMETERS	
	shear type of fracture	tensile type of fracture	shear type of fracture	tensile type of fracture
Unmobilized cohesion (MPa)	35.4	40	35.1	40
Unmobilized friction angle (°)	51	51	35	51
Mobilized dilation angle (°)	0	0	0	0
Mobilized cohesion (MPa)	0	0	0	0
Mobilized friction angle (°)	10	35	32	35
Reverse dilation angle (°)	0	0	0	0
Tension cutoff (MPa)	20	10	30	10
Young modulus (GPa)	68	68	68	68
Poisson's ratio	0.17	0.17	0.17	0.17

Table 3.12 Parameters used for the DIGS modelling, left hand side for original parameters, right hand side for the changed parameters

Modelling with these two parameters, the final shape may be different but the trend and general fracture behaviour are the same. In fact for each specific case, only the cone shape differs. Table 3.12 shows that the wedge shape is controlled by the internal friction angle. The appropriate angle giving a satisfactory cone shape was found to be 35°.

The most favorable orientation for the growth of a shear band, according to the Coulomb failure criterion in terms of the difference between the shear stress and the frictional resistance, is equal to $45-\phi/2$, with respect to the direction of the maximum principal stress (the Coulomb angle), and where ϕ is the internal angle of friction. This relation is still very useful, provided the appropriate value for the internal friction angle can be obtained. However, the effective friction angle is not constant, but is directly associated with accumulated damage. Thus the final

orientation of the macro shear fracture will strongly depend on the amount of damage and the associated internal friction angle at the time of fracture initiation. The application of a confining stress will probably result in a reduced internal friction angle which could explain the effect of the confining stress on the shear fracture orientation. Thus it should be possible to establish a correlation between damage and the internal effective friction angles for different values of confining stress.

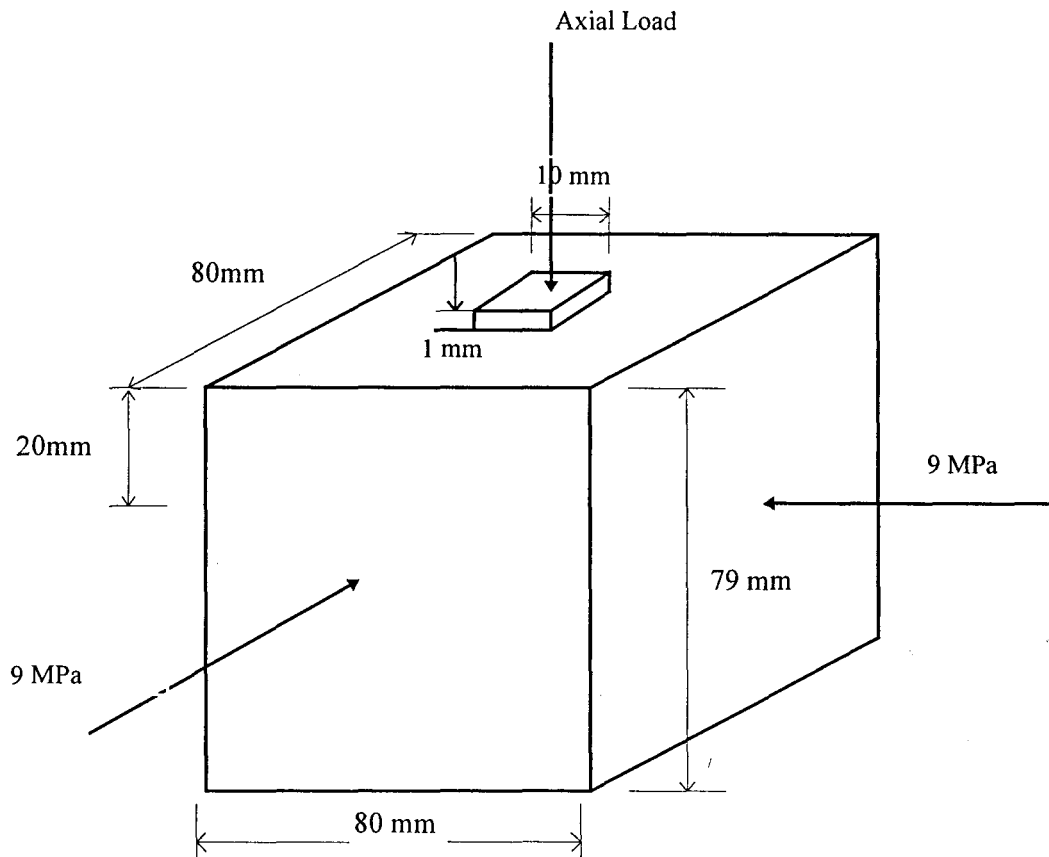


Figure 3.77 Dimensions of the square punch tests specimen

3.4.5 Square Punch Tests

For these tests, samples were prepared to form a square pillar of 1 mm in height and 10 mm in width (Figure 3.77). The boundary and loading conditions are the same as the previous tests. The fracture pattern obtained from the tests was different from the strip punch test results, especially with respect to the shape and size of the cone (Figure 3.78). The height of the cone in this figure is 2-2.5 times

greater than the punch width which is not the case in the strip punch tests, however, tensile fractures form around the cone in a similar manner to the strip punch tests. It is important to note that this fracturing occurs on all sides of the pillar, with the main difference being the absence of tensile fracture beneath the cone. All the fractures are localised around the punch area. The failure stress was measured as 1100 MPa which is much greater than observed in the strip punch cases (Figure 3.79).

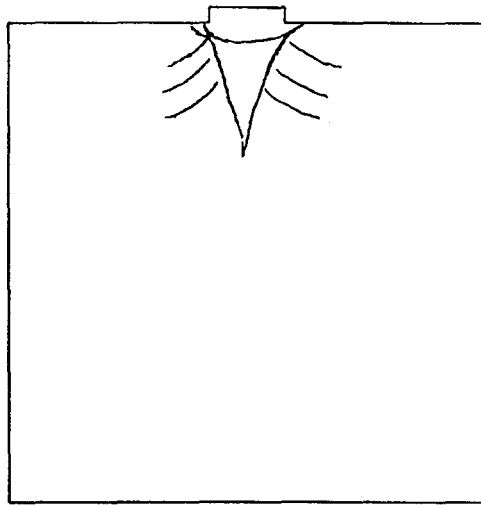


Figure 3.78 Section of the sample from the middle of the square punch

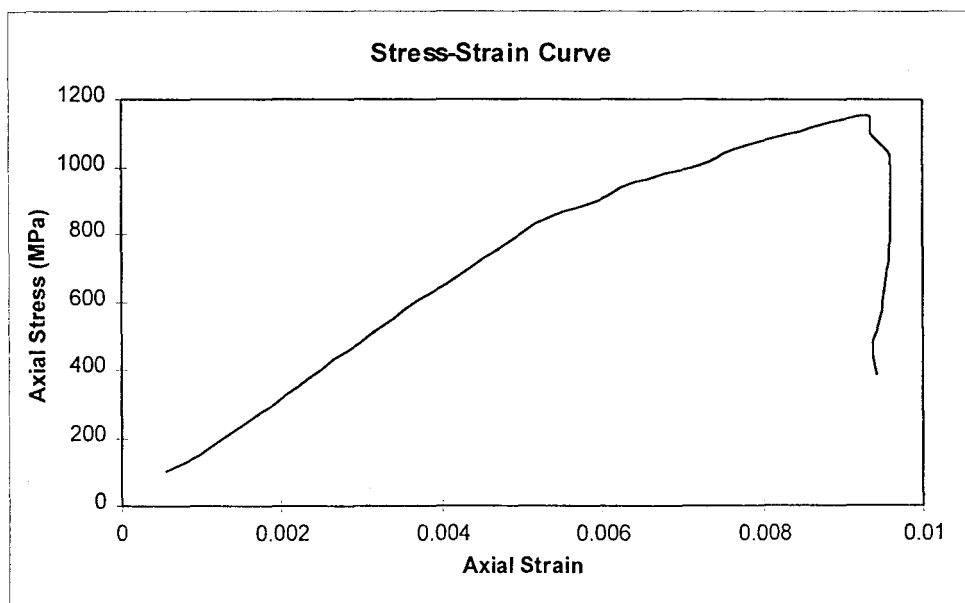


Figure 3.79 Stress-strain curve of square punch tests

From all tests carried out it can be concluded that the presence of a layer in the system has a significant effect on the fracture pattern as well as on the failure load. If the layer is near the punch area, then the presence of the layer and the layer conditions affect the failure modes strongly. Moreover when the layer is frictionless, as in the stearic acid experiments, the failure stress drops by about 20 percent in both the axisymmetric and plane strain tests.

When the shear fracture intersects a layer which is either frictional or frictionless, it stops. This is not the case for tensile or extension fractures, which can pass through the layers. However, the tensile fractures which originate from near the cone area do not pass through the layers.

The punch tests can be analyzed into 4 different phases according to the laboratory tests and numerical modelling described above.

1st phase (Elastic phase, 0-150 MPa);

During the first phase no fracturing occurs, the sample is under elastic loading and the average punch stress is less than the uniaxial compressive strength of the sample, which is 200 MPa (Figure 3.80).



Figure 3.80 Schematic representation of the punch test (1st phase)

2nd phase (Initiation of shear fractures, 150-250 MPa);

Because of the two different stress zones under the punch area (Figure 3.81), shear fractures start to grow from the corners of the punch where high stress accumulates due to the sharp corners (Figure 3.82). The average punch stress at this stage is more or less equal to the uniaxial compressive strength of the rock.

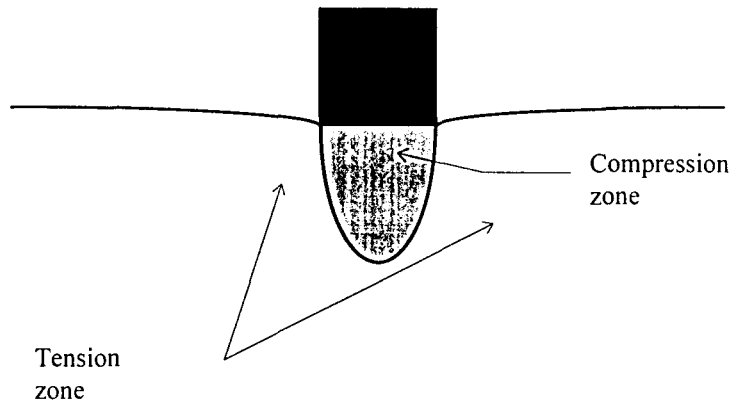


Figure 3.81 Schematic representation stresses under the punch (2nd phase)

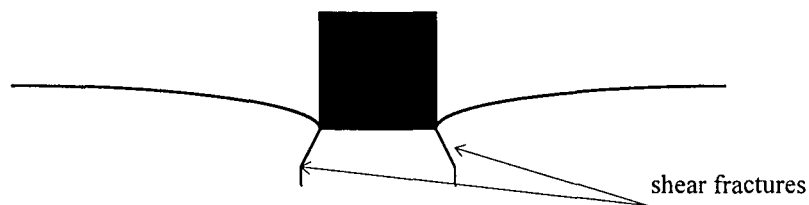


Figure 3.82 Schematic representation of the punch tests (2nd phase)

3rd phase (Initiation of tensile fractures, 250-450 MPa);

As the load is increased, the shear fractures continue to propagate to form a cone. However, as the cone starts to form, but before it is completed, a tensile fracture appears below the cone region and starts to grow. At this stage the volumetric

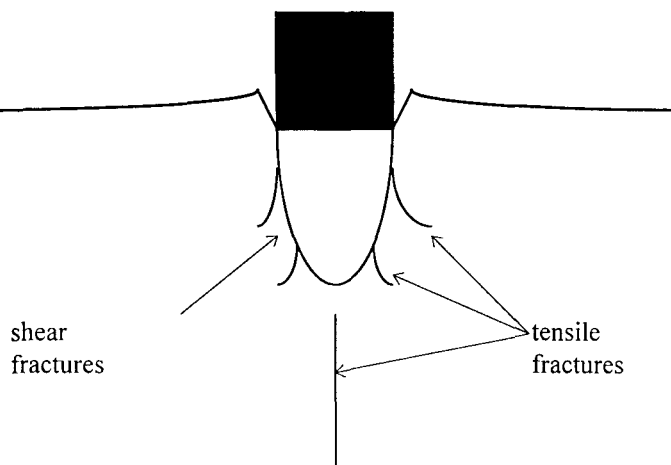


Figure 3.83 Schematic representation of the punch tests (3rd phase)

expansion increases in the direction of the minimum principal stress. This results in tensile fracturing around the wedge (Figure 3.83). During this phase the average punch stress is equal to at least twice the uniaxial compressive strength of the sample. It is also important to note that if the confinement is high enough then a tensile fracture is unlikely to form at the bottom of the wedge and during the experiment described above, the confinement was insufficient to suppress the formation of the tensile fracture below the wedge.

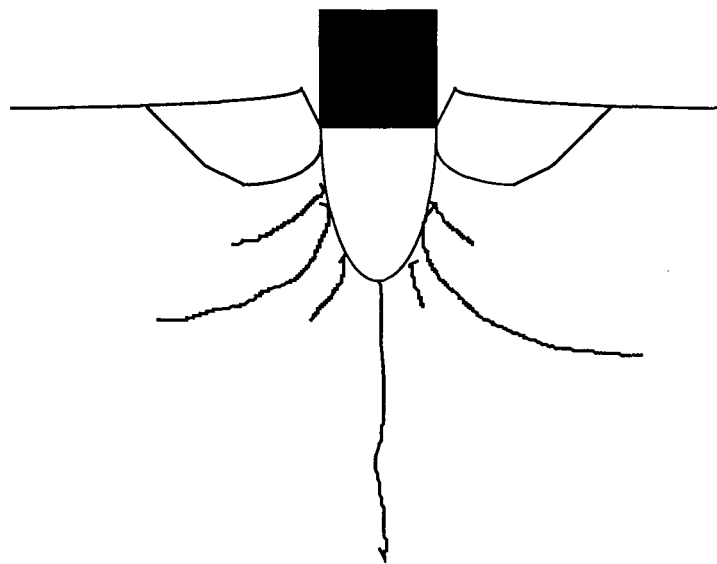


Figure 3.84 Schematic representation of the punch tests (4th)

4th phase; (growing of the fractures and the failure, >450 MPa);

In this final phase of the test, all the fractures grow until failure occurs. If the confinement is low, then the failure is sudden although the failure load depends on the confinement that has been applied (Figure 3.84).

IV. CONCLUSIONS AND RECOMMENDATIONS

Modelling of the failure processes in rock is an essential adjunct to the analysis of observations of underground rock fracturing. The identification of mechanisms of deformation represents a basic step in the understanding of rock mass behavior in different geological settings and at different working depths. Control of the rock mass deformation near deep level stopes and the avoidance of damaging incidents of violent rock failure requires a fundamental understanding of rock failure mechanisms.

Pre-existing discontinuities in rock, such as parting planes or weak joints, have been shown to play an important role in the formation of extension fracture patterns near openings.

Laboratory scale physical modelling experiments have been carried out to identify the nature of fracture initiation from discontinuity interfaces and to calibrate and verify the DIGS code. A special test rig has been constructed to allow triaxial tests to be carried out on rectangular shaped specimens. The tests in which a rectangular strip punch was used to load a biaxially confined specimen have shown the formation of the wedge-shaped shear failure zones initiating near the edges of the punch. The growth of these shear fractures can be approximately simulated using the shear growth model implemented in DIGS.

Important conclusions from the laboratory experiments carried out are as follows.

- In both confined and unconfined tests, an inverted cone forms under the punch. In the unconfined test, the cone wedges into the sample, axially splitting it into two pieces. This tensile splitting is suppressed in the confined tests and the failure stress is increased.

- In the confined tests, surface parallel cracks initiate along the punch edge and continue to grow deeper into the specimen.
- Compressive failure always occurs at lower punch stresses than those required for tensile failure.
- At low confining pressures, the mode of rock fracture changes from a shear mode of failure in high stress to a vertical tensile cracking, parallel to the axis of the punch, which splits the specimen before the formation of rock chips at the surface.
- Rock deforms elastically until the applied punch load exceeds 50% of the maximum load that the rock can sustain. At loads greater than 50% of the maximum, a crack is initiated around the perimeter of the punch and this crack propagates in the well-known conical Hertzian manner. When the crack begins to propagate, the location of its front is reasonably well defined. However, with continued application of the load, an intense zone of microcracks surrounds and conceals the crack tip. Fractures initiating within this microcracked region propagate to the surface and form rock chips. The effects of the punch size and the confining stress applied, orthogonal to the punch, results in a decrease in punch strength with increasing punch size. There is however, an increase in strength with increasing confining stress. In contrast, at low confining stress, specimens split by cleavage through the axis of the punch.
- The layer interfaces tend to halt the progression of surface parallel fractures and the shear fractures initiated from the surface of the sample.

Conclusions concerning the numerical simulations of the punch behaviour can be summarized as follows.

- Numerical simulations of the experimental observations using plane strain geometrical configurations have shown broad similarities between the experimental test observations. However, the simulation of the detailed orientation of shear bands and extension fractures was not clearly reproduced. Nevertheless the tests showed that it would be easier to model rectangular physical geometries such as the 3-D tests carried out by using the biaxial cell or, more challenging, to develop the capability of modelling three dimensional crack growth procedures.
- Accurately scaled models of mining geometries are difficult to prepare and test in the laboratory. However, the models should not be isolated from reality. The most common mine layout configuration includes the stope face and pillars and therefore the model geometry selected was biased towards pillar stability and the effect of advancing the stope.

The agreement between the observed zones of rock failure and those predicted by the numerical models is good in view of the simple nature of the model. The calculated region of tensile stress corresponds well with the observed microcracked zone. Rock damaged beneath the punch, that can be interpreted as compressive failure, is also observed.

The numerical modelling of the physical tests using DIGS gave similar results to those observed in the physical modelling. For example for the long strip punch tests, the same boundary conditions as in the physical tests (9 MPa confinement and 550 MPa axial stress) were used. Overall resemblance of the numerical modelling results to that of physical tests was achieved by introducing two shear and several tensile seed points around the potential cone zone. However, some of the material properties had to be modified to obtain the observed fracture pattern. The cone under the punch could be initiated by inserting two shear points as well as the main tensile fractures at the middle of the sample. While these fractures were growing, additional tensile fractures were initiated adjacent to the wedge, with the uppermost fractures intersecting the free surface of the sample. These

results indicate that explicit modelling of fracture growth is feasible but also shows that the modelling procedure is somewhat contrived in having to emplace appropriate seed growth sites at specific positions and following defined growth modes. More robust growth models should be developed that will not require the specification of possible fracture modes to simulate fracturing in mine stopes. As far as the future direction of this work is concerned, it is important to understand fracture processes in multiple layered materials. All the experiments and the numerical modelling in this work were carried out using 2-D plane strain models, however, in reality all fractures are in 3-D. Therefore these experiments need to be repeated and improved in 3-D as well.

APPENDIX A

Numerical Solution of a Semi-Infinite Solid Under Punch Load in Plane Strain

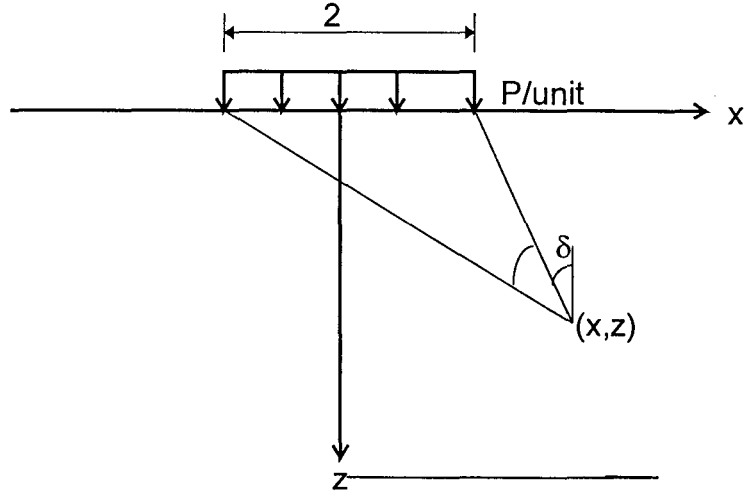


Figure A.1 Dimensions of the semi-infinite mass, for solid case (Jurgenson)

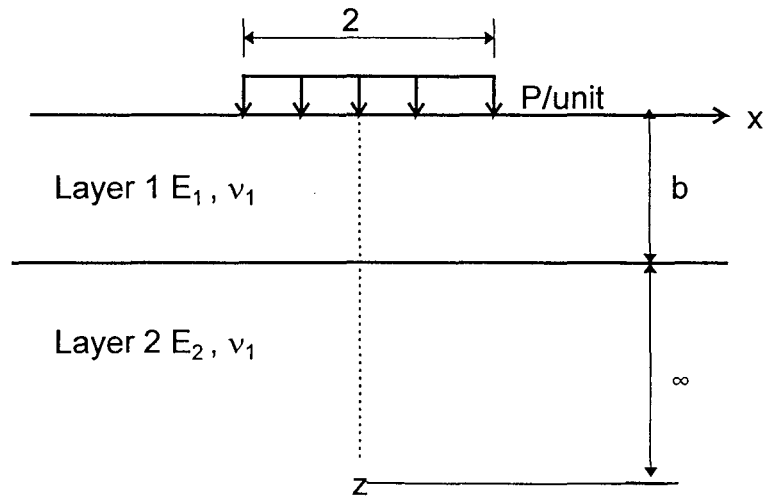


Figure A.2 Dimensions of the semi-infinite mass, for layered case (Jurgenson)

Changing of the vertical stress to applied stress ratios are shown in the Table A.1 and Figures A.3-A7. The Dimensions of the semi-infinite mass is given in Figures A.1 and A.2.

x/b	z/b	solid sigz/p	two layers sigz/p (E1/E2)=2	two layers sigz/p (E1/E2)=10	two layers sigz/p (E1/E2)=100
0	0.5	0.9594	0.85	0.69	0.57
0.5	0.5	0.9087	0.79	0.65	0.53
1	0.5	0.4969	0.39	0.33	0.25
1.5	0.5	0.0892	0.07	0.06	0.03
2	0.5	0.0194	0.02	0.04	0.02
2.5	0.5	0.0068	0.01	0.03	0.02
3	0.5	0.0026	0	0.02	0.01

x/b	z/b	solid sigz/p	two layers sigz/p (E1/E2)=2	two layers sigz/p (E1/E2)=10	two layers sigz/p (E1/E2)=100
0	1	0.8183	0.57	0.29	0.08
0.5	1	0.7352	0.49	0.26	0.07
1	1	0.4797	0.31	0.19	0.06
1.5	1	0.2488	0.14	0.12	0.05
2	1	0.0776	0.06	0.08	0.04
2.5	1	0.0357	0.03	0.07	0.03
3	1	0.0171	0.01	0.04	0.03

x/b	z/b	solid sigz/p	two layers sigz/p (E1/E2)=2	two layers sigz/p (E1/E2)=10	two layers sigz/p (E1/E2)=100
0	1.5	0.6678	0.41	0.24	0.06
0.5	1.5	0.6078	0.35	0.21	0.06
1	1.5	0.448	0.25	0.16	0.05
1.5	1.5	0.2704	0.15	0.12	0.04
2	1.5	0.1458	0.06	0.08	0.04
2.5	1.5	0.0771	0.05	0.07	0.03
3	1.5	0.0427	0.02	0.04	0.03

x/b	z/b	solid sigz/p	two layers sigz/p (E1/E2)=2	two layers sigz/p (E1/E2)=10	two layers sigz/p (E1/E2)=100
0	2	0.5508	0.3	0.18	0.06
0.5	2	0.5107	0.27	0.17	0.05
1	2	0.4095	0.2	0.14	0.05
1.5	2	0.2876	0.14	0.11	0.04
2	2	0.1847	0.09	0.09	0.04
2.5	2	0.1139	0.05	0.07	0.03
3	2	0.0705	0.03	0.05	0.03

x/b	z/b	solid sigz/p	two layers sigz/p (E1/E2)=2	two layers sigz/p (E1/E2)=10	two layers sigz/p (E1/E2)=100
0	2.5	0.4617	0.24	0.14	0.05
0.5	2.5	0.4372	0.22	0.14	0.04
1	2.5	0.3701	0.17	0.13	0.04
1.5	2.5	0.2851	0.13	0.11	0.04
2	2.5	0.2045	0.09	0.09	0.04
2.5	2.5	0.1409	0.06	0.07	0.03
3	2.5	0.0952	0.04	0.05	0.03

Table A.1 Vertical stress to applied stress ratios with changing layer conditions

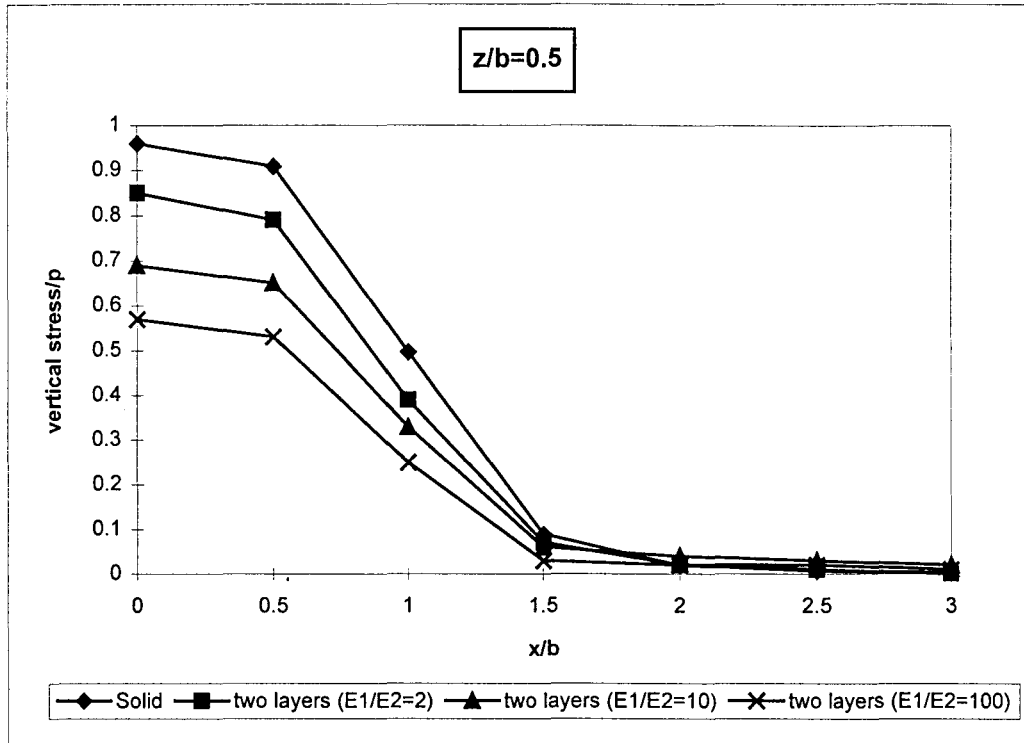


Figure A.3 Variations of the applied stress to vertical stress ratios with changing young modulus of the layers

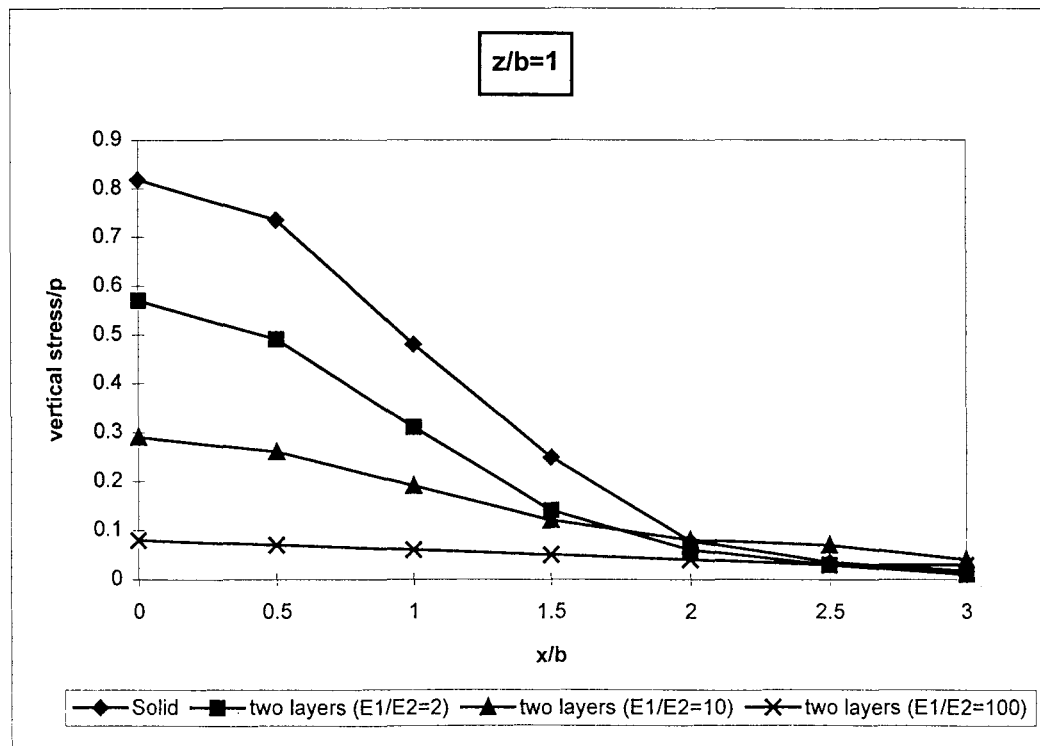


Figure A.4 Variations of the applied stress to vertical stress ratios with changing young modulus of the layers

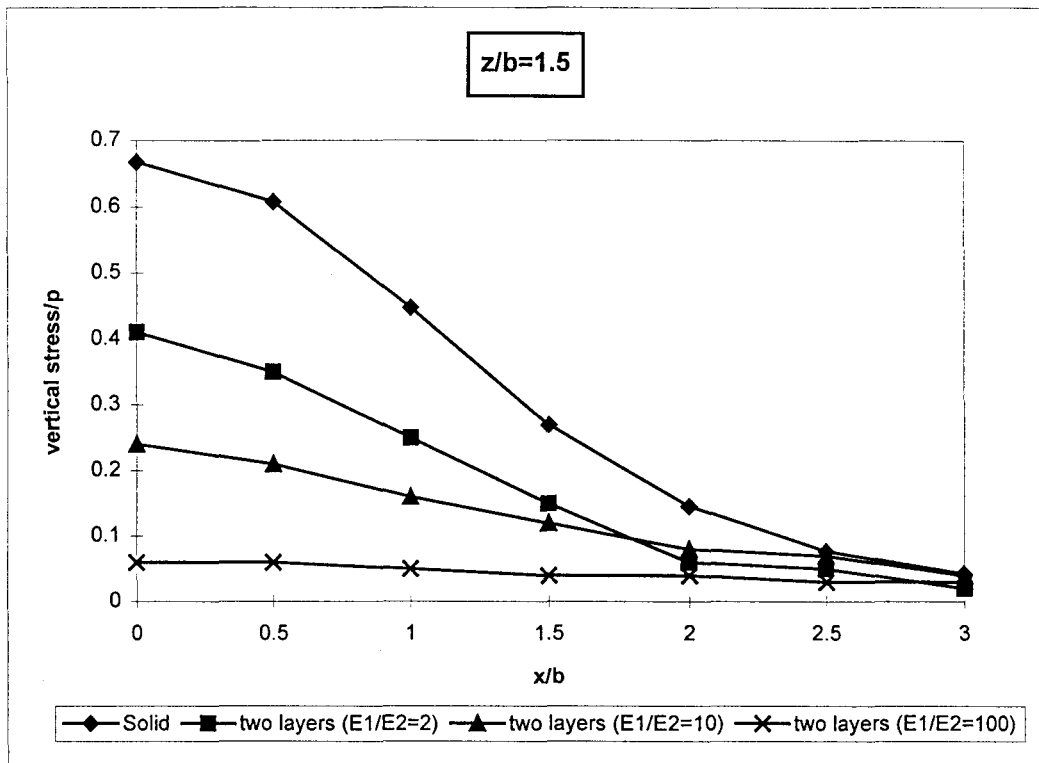


Figure A.5 Variations of the applied stress to vertical stress ratios with changing young modulus of the layers

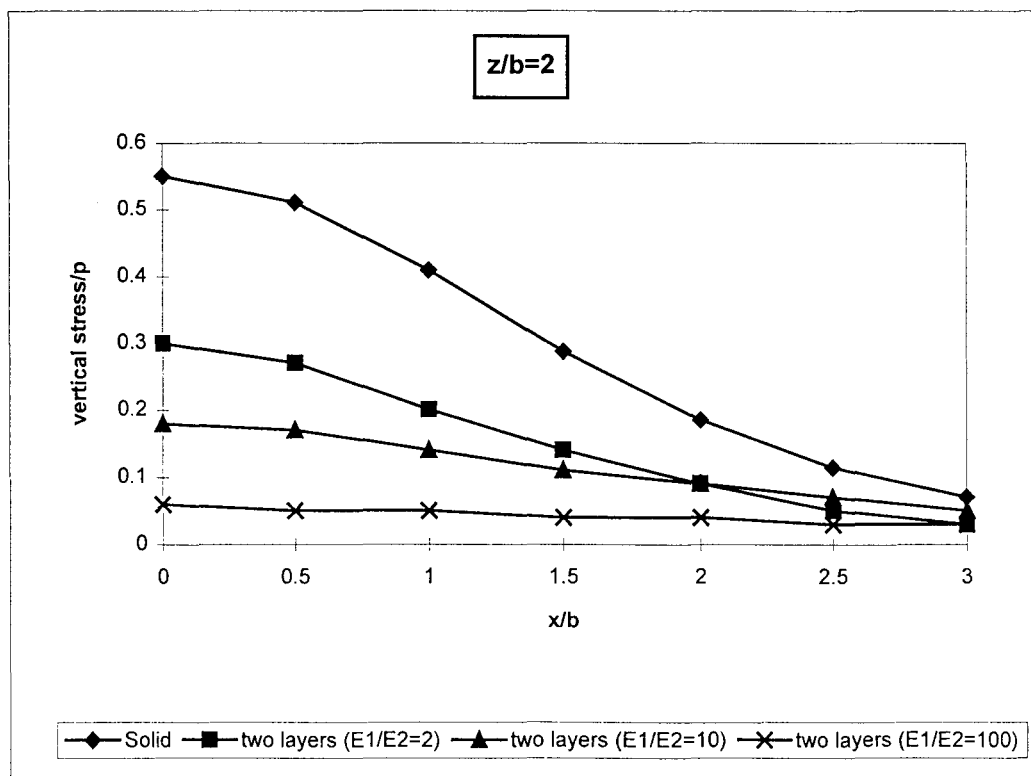


Figure A.6 Variations of the applied stress to vertical stress ratios with changing young modulus of the layers

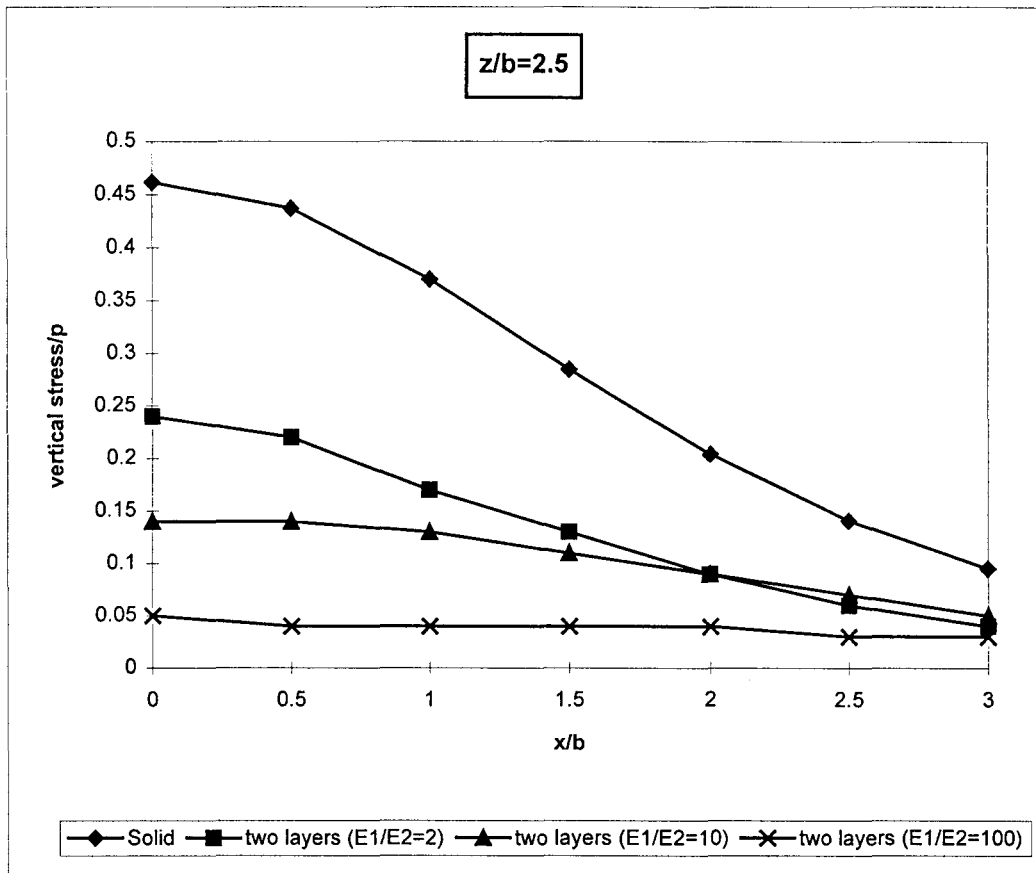


Figure A.7 Variations of the applied stress to vertical stress ratios with changing young modulus of the layers

APPENDIX B

Displacement Discontinuities Method;

The influence of distributions of dislocations or so called ‘displacement discontinuities’ throughout the region of interest is computed using integral relationships of the form

$$u_k(P) = - \int_{\delta B} G(Q_{ij}, P_k) n_j(Q) D_i(Q) dS(Q) \dots\dots\dots (B1)$$

$$\tau_{kl}(P) = - \int_{\delta B} \tau(Q_{ij}, P_{kl}) n_j(Q) D_i(Q) dS(Q) \dots\dots\dots (B2)$$

where

$u_k(P)$ = k^{th} component of the displacement vector at P

$\tau_{kl}(P)$ = kl^{th} component of the stress tensor at point P

$n_j(Q)$ = j^{th} component of an assigned normal to the surface δB at point Q

$D_i(Q)$ = i^{th} component of the displacement vector jump across surface δB at point Q.

Specifically, the displacement discontinuity is defined to be

$$D_i(Q) = u_i^-(Q) - u_i^+(Q) \dots\dots\dots (B3)$$

where $u_i^+(Q)$ is the i^{th} displacement vector component of the material in the region towards which the normal vector $n_j(Q)$ is directed and $u_i^-(Q)$ is the i^{th} displacement vector component on the opposite side of the discontinuity. In an isotropic elastic medium, the influence tensor $G(Q_{ij}, P_k)$ and $\tau(Q_{ij}, P_{kl})$ can be

expressed in terms of the derivatives of the harmonic functions $\log r^2$ and $1/r$, for two and three dimensional problems respectively, where $r=|P-Q|$.

In the present case, it is assumed that plane strain conditions obtain and that each fracture can be represented as a set of connected straight line elements. The shear and normal components of the displacement discontinuity vector are assumed to vary linearly over each element according to

$$D_i = \alpha_i + \beta_i \eta \quad \dots\dots\dots (B4)$$

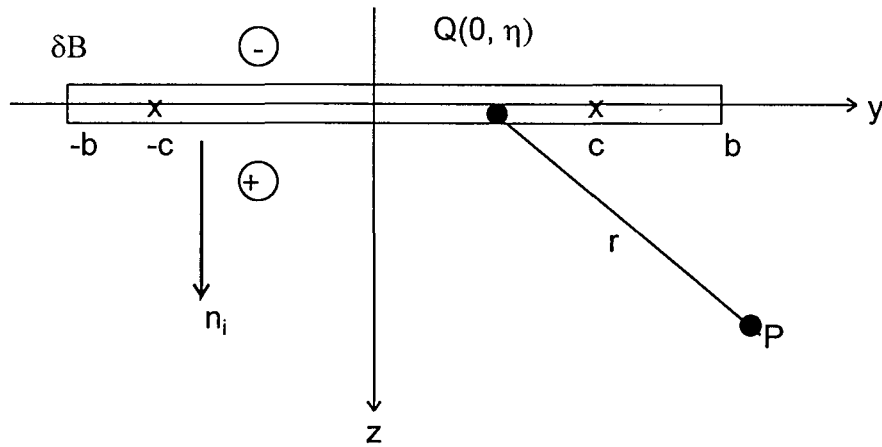


Figure B.1 Local co-ordinate system used to define the influence of a given element at a general point P.

where α_i and β_i are constants for component i of the displacement discontinuity vector and η is local variable representing the position of point Q within the element (see Figure B.1). Erecting a local co-ordinate system at the center of an element of length $2b$, as shown in Figure B.1, the incremental components of displacements and stress at point P can be shown to be given by the following expression when the material is isotropic with shear modulus G and Poisson's ratio ν .

$$u_y(y, z) = \frac{1}{8\pi(1-\nu)} [2(1-\nu)K_{y,z} + zK_{y,zz} - (1-2\nu)K_{z,y} - zK_{z,yz}] \dots\dots\dots (B5)$$

$$u_z(y, z) = \frac{1}{8\pi(1-\nu)} [(1-2\nu)K_{y,y} - zK_{y,yz} + 2(1-\nu)K_{z,z} - zK_{z,zz}] \dots\dots\dots(B6)$$

$$\tau_{yy}(y, z) = \frac{G}{4\pi(1-\nu)} [2K_{y,yz} + zK_{y,yzz} + K_{z,zz} + K_{z,zzz} + zK_{z,zzz}] \dots\dots\dots(B7)$$

$$\tau_{zz}(y, z) = \frac{G}{4\pi(1-\nu)} [-zK_{y,yzz} + K_{z,zz} - zK_{z,zzz}] \dots\dots\dots(B8)$$

$$\tau_{yz}(y, z) = \frac{G}{4\pi(1-\nu)} [K_{y,zz} + zK_{y,zzz} - zK_{z,yzz}] \dots\dots\dots(B9)$$

where

$$K_y = \alpha_y E + \beta_y F \dots\dots\dots(B10)$$

$$K_z = \alpha_z E + \beta_z F \dots\dots\dots(B11)$$

$$\begin{aligned} E(y, z) &= - \int_{-b}^b \log[(y - \eta)^2 + z^2] d\eta \\ &= \sum_{i=-1,1} \varepsilon \left\{ B \log R^2 - 2B + 2z \tan^{-1} \left(\frac{B}{z} \right) \right\} \dots\dots\dots(B12) \end{aligned}$$

$$\begin{aligned} F(y, z) &= - \int_{-b}^b \eta \log[(y - \eta)^2 + z^2] d\eta \\ &= \sum_{i=-1,+1} \varepsilon \left\{ \frac{1}{2} (2yB - R^2) \log R^2 - 2yB + \frac{1}{2} B^2 + 2yz \tan^{-1} \left(\frac{B}{z} \right) \right\} \dots(B13) \end{aligned}$$

and

$$B = y - \varepsilon b; \varepsilon = -1 \text{ or } +1; R^2 = B^2 + z^2$$

In equations (B5)-(B9), subscripts following a comma denote derivatives with respect to the local directions y and z at the field point P. For example $K_{y,yz} \equiv \delta^2 / \delta y \delta z [K_y]$. The left handed co-ordinate system in Figure B.1 and the definition (B4) implies that when slip is right lateral, the component D_y is positive. The collocation distance, c , shown in Figure B.1 is defined as the point at

which stress or displacement boundary conditions are matched in each element. The collocation factor, c_f , is defined by

$$c_f = \frac{c}{b} \dots\dots\dots(B14)$$

where b is the half-length of the element. The radius, ρ , from the crack tip at which the stress field is evaluated to determine the crack growth path is given by

$$\rho = b(1 - c_f) \dots\dots\dots(B15)$$

Following Crawford and Curran (1982) and using additional arguments relating to the opening displacement of a single element (Napier, 1990), c_f is chosen to be $1/\sqrt{2}$ for linear variation elements. At present there is no physical basis for the choice of the radius ρ and it can only be justified by a posteriori compatibility of observed and simulated fracture patterns. It may also be noted that numeric experiments indicate that at distance ρ from the crack tip the stress field generated by the linear variation elements described above is very similar to that generated by a single crack having constant shear and normal tractions applied to the crack faces (Malan, Napier and Watson, 1994).

APPENDIX C

Example Data Files from DIGS and FLAC Modelling

1) This DIGS data file has been prepared for modelling of unconfined axisymmetry punch sandstone test.

```
PR 21000.0 0.27 0.000 0.000 0.0 0.0 0.0 0.0 0.0
CW 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
CM 0.0 13.0 25.0 0.0 0.0 20.0 0.0 10.0 0.0 0.0
C-L 0.0 0.0 500.0 0.0
CZCST 0.000 0.000NT 0.000 0.000
C$CSD 0.000 0.000ND 0.000 0.000
CTCST 0.000 0.000NT 0.000 0.000
CRCST 0.000 0.000N+ 0.000 0.000
CLCST 0.000 0.000N- 0.000 0.000
CBCS- 0.000 0.000N- 0.000 0.000
CPCST 0.000 0.000NT -662.000 -662.000
FO/
FNUNSAND2.SAV
FCUNSAND2.CFG
#ISTA0 100 60 150 0.1 0.0 0.4
STTOPL 22 0.00 0.00 22.00 0.00 0.0 0.0 T T T
SPPNCH 10 22.00 0.00 32.00 0.00 0.0 0.0 T T T
STTOPR 22 32.00 0.00 54.00 0.00 0.0 0.0 T T T
STRSID 60 54.00 0.00 54.00 60.00 0.0 0.0 T T T
STLSID 60 0.00 0.00 0.00 60.00 0.0 0.0 T T T
SBBOT 54 0.00 60.00 54.00 60.00 0.0 0.0 T T T
RX
RG 20 20 17.50 0.50 0.00 1.00 1.00 30.0 5.0
#PSTA1 10 60 50 0.1 0.0 0.4
?MSCL1 30 22.00 0.00 0.0 0.0 90.00 1.000 S T T
?MSCR1 30 32.00 0.00 0.0 0.0 90.00 1.000 S T T
?MMID1 48 27.00 15.00 0.0 0.0 90.00 1.000 T T T
RX
RG 27 30 0.00 0.00 0.00 2.00 2.00 30.0 5.0
/
```

2) This DIGS data file has been prepared for modelling of confined axisymmetry punch sandstone test.

```
PR 21000.0 0.27 0.000 0.000 0.0 0.0 0.0 0.0 0.0
CW 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
CM 0.0 13.0 30.0 0.0 0.0 25.0 0.0 15.0 0.0 0.0
```


C-L	0.0	0.0	500.0	0.0						
CZCST	0.000	0.000	NT	0.000	0.000					
C\$CSD	0.000	0.000	ND	0.000	0.000					
CTCST	0.000	0.000	NT	0.000	0.000					
CRCST	0.000	0.000	N+	0.000	0.000					
CLCST	0.000	0.000	N-	0.000	0.000					
CBCS-	0.000	0.000	N-	0.000	0.000					
CPCST	0.000	0.000	NT	-2100.000	-2100.000					
FO/										
FNAX2.SAV										
FCAX2.CFG										
#ISTA0	100	60	150	0.1	0.0	0.4				
STTOPL	22	0.00	0.00	22.00	0.00	0.0	0.0	T	TT	
SPPNCH	10	22.00	0.00	32.00	0.00	0.0	0.0	T	TT	
STTOPR	22	32.00	0.00	54.00	0.00	0.0	0.0	T	TT	
SRRSID	60	54.00	0.00	54.00	60.00	0.0	0.0	T	TT	
SLLSID	60	0.00	0.00	0.00	60.00	0.0	0.0	T	TT	
SBBOT	54	0.00	60.00	54.00	60.00	0.0	0.0	T	TT	
RX										
RG	20	20	17.50	0.50	0.00	1.00	1.00	30.0	10.0	
#ISTA1	10	60	50	0.1	0.0	0.4				
?MSCL1	30	22.00	0.00	0.0	0.0	90.00	1.000	S	TT	
?MSCR1	30	32.00	0.00	0.0	0.0	90.00	1.000	S	TT	
RX										
#ISTA2	10	400	50	0.1	0.0	0.4				
?MTL01	25	20.00	2.00	0.0	0.0	0.00	1.000	T	TT	
?MTL02	25	20.00	2.00	0.0	0.0	180.00	1.000	T	TT	
?MTL03	25	20.00	4.00	0.0	0.0	0.00	1.000	T	TT	
?MTL04	25	20.00	4.00	0.0	0.0	180.00	1.000	T	TT	
?MTL05	25	20.00	6.00	0.0	0.0	0.00	1.000	T	TT	
?MTL06	25	20.00	6.00	0.0	0.0	180.00	1.000	T	TT	
?MTL07	25	20.00	8.00	0.0	0.0	0.00	1.000	T	TT	
?MTL08	25	20.00	8.00	0.0	0.0	180.00	1.000	T	TT	
?MTL09	25	20.00	10.00	0.0	0.0	0.00	1.000	T	TT	
?MTL10	25	20.00	10.00	0.0	0.0	180.00	1.000	T	TT	
?MTL11	25	20.00	12.00	0.0	0.0	0.00	1.000	T	TT	
?MTL12	25	20.00	12.00	0.0	0.0	180.00	1.000	T	TT	
?MTL13	25	20.00	14.00	0.0	0.0	0.00	1.000	T	TT	
?MTL14	25	20.00	14.00	0.0	0.0	180.00	1.000	T	TT	
?MTL15	25	20.00	16.00	0.0	0.0	0.00	1.000	T	TT	
?MTL16	25	20.00	16.00	0.0	0.0	180.00	1.000	T	TT	
?MTL17	25	20.00	18.00	0.0	0.0	0.00	1.000	T	TT	
?MTL18	25	20.00	18.00	0.0	0.0	180.00	1.000	T	TT	
?MTL19	25	20.00	20.00	0.0	0.0	0.00	1.000	T	TT	
?MTL20	25	20.00	20.00	0.0	0.0	180.00	1.000	T	TT	
?MTL21	25	20.00	22.00	0.0	0.0	0.00	1.000	T	TT	

?MTL22	25	20.00	22.00	0.0	0.0	180.00	1.000	T	TT
?MTL23	25	20.00	24.00	0.0	0.0	0.00	1.000	T	TT
?MTL24	25	20.00	24.00	0.0	0.0	180.00	1.000	T	TT
?MTL25	25	20.00	26.00	0.0	0.0	0.00	1.000	T	TT
?MTL26	25	20.00	26.00	0.0	0.0	180.00	1.000	T	TT
?MTL27	25	20.00	28.00	0.0	0.0	0.00	1.000	T	TT
?MTL28	25	20.00	28.00	0.0	0.0	180.00	1.000	T	TT
?MTL29	25	20.00	30.00	0.0	0.0	0.00	1.000	T	TT
?MTL30	25	20.00	30.00	0.0	0.0	180.00	1.000	T	TT
?MTR01	25	34.00	2.00	0.0	0.0	0.00	1.000	T	TT
?MTR02	25	34.00	2.00	0.0	0.0	180.00	1.000	T	TT
?MTR03	25	34.00	4.00	0.0	0.0	0.00	1.000	T	TT
?MTR04	25	34.00	4.00	0.0	0.0	180.00	1.000	T	TT
?MTR05	25	34.00	6.00	0.0	0.0	0.00	1.000	T	TT
?MTR06	25	34.00	6.00	0.0	0.0	180.00	1.000	T	TT
?MTR07	25	34.00	8.00	0.0	0.0	0.00	1.000	T	TT
?MTR08	25	34.00	8.00	0.0	0.0	180.00	1.000	T	TT
?MTR09	25	34.00	10.00	0.0	0.0	0.00	1.000	T	TT
?MTR10	25	34.00	10.00	0.0	0.0	180.00	1.000	T	TT
?MTR11	25	34.00	12.00	0.0	0.0	0.00	1.000	T	TT
?MTR12	25	34.00	12.00	0.0	0.0	180.00	1.000	T	TT
?MTR13	25	34.00	14.00	0.0	0.0	0.00	1.000	T	TT
?MTR14	25	34.00	14.00	0.0	0.0	180.00	1.000	T	TT
?MTR15	25	34.00	16.00	0.0	0.0	0.00	1.000	T	TT
?MTR16	25	34.00	16.00	0.0	0.0	180.00	1.000	T	TT
?MTR17	25	34.00	18.00	0.0	0.0	0.00	1.000	T	TT
?MTR18	25	34.00	18.00	0.0	0.0	180.00	1.000	T	TT
?MTR19	25	34.00	20.00	0.0	0.0	0.00	1.000	T	TT
?MTR20	25	34.00	20.00	0.0	0.0	180.00	1.000	T	TT
?MTR21	25	34.00	22.00	0.0	0.0	0.00	1.000	T	TT
?MTR22	25	34.00	22.00	0.0	0.0	180.00	1.000	T	TT
?MTR23	25	34.00	24.00	0.0	0.0	0.00	1.000	T	TT
?MTR24	25	34.00	24.00	0.0	0.0	180.00	1.000	T	TT
?MTR25	25	34.00	26.00	0.0	0.0	0.00	1.000	T	TT
?MTR26	25	34.00	26.00	0.0	0.0	180.00	1.000	T	TT
?MTR27	25	34.00	28.00	0.0	0.0	0.00	1.000	T	TT
?MTR28	25	34.00	28.00	0.0	0.0	180.00	1.000	T	TT
?MTR29	25	34.00	30.00	0.0	0.0	0.00	1.000	T	TT
?MTR30	25	34.00	30.00	0.0	0.0	180.00	1.000	T	TT

RX

RG 27 30 0.00 0.00 0.00 2.00 2.00 30.0 10.0

/

3) This DIGS data file has been prepared for modelling of triaxial strip punch tests of quartzite.

```

PR 68000.0  0.17  0.000  0.000   0.0   0.0   0.0   0.0   0.0
CW  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
CM  0.0 35.1 35.0  0.0  0.0 32.0  0.0 30.0  0.0  0.0  0.0
CN  0.0 40.0 51.0  0.0  0.0 35.0  0.0 10.0  0.0  0.0  0.0
CZCST  0.000  0.000NT  0.000  0.000
C$CSD  0.000  0.000ND  0.000  0.000
CTCST  0.000  0.000NT  0.000  0.000
CRCST  0.000  0.000NT -10.000 -10.000
CLCST  0.000  0.000NT -10.000 -10.000
CBCS-  0.000  0.000N-  0.000  0.000
CPCST  0.000  0.000NT -550.000 -550.000
FO/
FNTRIAX22.SAV
FCTRIAX22.CFG
#ISTA0 100 60  150  0.1  0.0  0.4
STTOPL 22  0.00  0.00 35.00  0.00  0.0  0.0  T  TT
SPPNCH 10 35.00 -1.00 45.00 -1.00  0.0  0.0  T  TT
STTOPR 22 45.00  0.00 80.00  0.00  0.0  0.0  T  TT
SRRSID 60 80.00  0.00 80.00 80.00  0.0  0.0  T  TT
SLLSID 60  0.00  0.00  0.00 80.00  0.0  0.0  T  TT
SBBOT  54  0.00 80.00 80.00 80.00  0.0  0.0  T  TT
STPNL  3 35.00  0.00 35.00 -1.00  0.0  0.0  T  TT
STPNCR 3 45.00 -1.00 45.00  0.00  0.0  0.0  T  TT
RX
RG 30 40 25.00  0.00  0.00  1.00  1.00 30.0 30.0
#PSTA1 10 400  50  0.1  0.0  0.4
?MSCL1 40 35.00  0.00  0.0  0.0 90.00 1.000  S  TT
?MSCR1 40 45.00  0.00  0.0  0.0 90.00 1.000  S  TT
?NMID1 45 40.00 24.00  0.0  0.0 90.00 1.000  T  TT
?NMID2 45 40.00 24.00  0.0  0.0 90.00 1.000  T  TT
?NTL03 25 30.00  4.00  0.0  0.0  0.00 1.000  T  TT
?NTL04 25 30.00  4.00  0.0  0.0 180.00 1.000  T  TT
?NTL07 25 30.00  8.00  0.0  0.0  0.00 1.000  T  TT
?NTL08 25 30.00  8.00  0.0  0.0 180.00 1.000  T  TT
?NTL11 25 30.00 12.00  0.0  0.0  0.00 1.000  T  TT
?NTL12 25 30.00 12.00  0.0  0.0 180.00 1.000  T  TT
?NTL15 25 30.00 16.00  0.0  0.0  0.00 1.000  T  TT
?NTL16 25 30.00 16.00  0.0  0.0 180.00 1.000  T  TT
?NTR03 25 50.00  4.00  0.0  0.0  0.00 1.000  T  TT
?NTR04 25 50.00  4.00  0.0  0.0 180.00 1.000  T  TT
?NTR07 25 50.00  8.00  0.0  0.0  0.00 1.000  T  TT
?NTR08 25 50.00  8.00  0.0  0.0 180.00 1.000  T  TT

```

```

?NTR11 25 50.00 12.00 0.0 0.0 0.00 1.000 T TT
?NTR12 25 50.00 12.00 0.0 0.0 180.00 1.000 T TT
?NTR15 25 50.00 16.00 0.0 0.0 0.00 1.000 T TT
?NTR16 25 50.00 16.00 0.0 0.0 180.00 1.000 T TT
RX
RG 20 20 30.00 0.00 0.00 1.00 1.00 30.0 30.0
#ISTA2 10 400 50 0.1 0.0 0.4
?NTL19 25 37.00 20.00 0.0 0.0 0.00 1.000 T TT
?NTL20 25 37.00 20.00 0.0 0.0 180.00 1.000 T TT
?NTL21 25 38.00 22.00 0.0 0.0 0.00 1.000 T TT
?NTL22 25 38.00 22.00 0.0 0.0 180.00 1.000 T TT
?NTL23 25 39.00 24.00 0.0 0.0 0.00 1.000 T TT
?NTL24 25 39.00 24.00 0.0 0.0 180.00 1.000 T TT
?NTR19 25 43.00 20.00 0.0 0.0 0.00 1.000 T TT
?NTR20 25 43.00 20.00 0.0 0.0 180.00 1.000 T TT
?NTR21 25 42.00 22.00 0.0 0.0 0.00 1.000 T TT
?NTR22 25 42.00 22.00 0.0 0.0 180.00 1.000 T TT
?NTR23 25 41.00 24.00 0.0 0.0 0.00 1.000 T TT
?NTR24 25 41.00 24.00 0.0 0.0 180.00 1.000 T TT
RX
RG 40 40 20.00 0.00 0.00 1.00 1.00 51.0 30.0
/

```

4) This DIGS data file has been prepared for modelling of triaxial strip punch tests of layered quartzite.

```

PR 68000.0 0.17 0.000 0.000 0.0 0.0 0.0 0.0 0.0
CW 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
CM 0.0 35.1 35.0 0.0 0.0 32.0 0.0 30.0 0.0 0.0 0.0
CN 0.0 40.0 51.0 0.0 0.0 35.0 0.0 10.0 0.0 0.0 0.0
CI 0.0 0.0 30.0 0.0 0.0 30.0 0.0 0.0 0.0 0.0 0.0
CZCST 0.000 0.000NT 0.000 0.000
C$CSD 0.000 0.000ND 0.000 0.000
CTCST 0.000 0.000NT 0.000 0.000
CRCST 0.000 0.000NT -10.000 -10.000
CLCST 0.000 0.000NT -10.000 -10.000
CBCS- 0.000 0.000N- 0.000 0.000
CPCST 0.000 0.000NT -425.000 -425.000
FO/
FNTRIAX25.SAV
FCTRIAX25.CFG
#ISTA0 100 60 150 0.1 0.0 0.4
STTOPL 35 0.00 0.00 35.00 0.00 0.0 0.0 T TT
SPPNCH 10 35.00 -1.00 45.00 -1.00 0.0 0.0 T TT
STTOPR 35 45.00 0.00 80.00 0.00 0.0 0.0 T TT

```

SRRSID	80	80.00	0.00	80.00	80.00	0.0	0.0	T	TT
SLLSID	80	0.00	0.00	0.00	80.00	0.0	0.0	T	TT
SBBOT	80	0.00	80.00	80.00	80.00	0.0	0.0	T	TT
STPNL	1	35.00	0.00	35.00	-1.00	0.0	0.0	T	TT
STPNCR	1	45.00	-1.00	45.00	0.00	0.0	0.0	T	TT
SICRAC	80	0.00	15.00	80.00	15.00	0.0	0.0	T	GG
RX									
RG	30	40	25.00	0.00	0.00	1.00	1.00	30.0	30.0
#PSTA1	10	400	50	0.1	0.0	0.4			
?MSCL1	40	35.00	0.00	0.0	0.0	90.00	1.000	S	TT
?MSCR1	40	45.00	0.00	0.0	0.0	90.00	1.000	S	TT
?MMID1	45	40.00	15.00	0.0	0.0	90.00	1.000	T	TT
?NTL01	25	30.00	2.00	0.0	0.0	0.00	1.000	T	TT
?NTL02	25	30.00	2.00	0.0	0.0	180.00	1.000	T	TT
?NTL03	25	30.00	4.00	0.0	0.0	0.00	1.000	T	TT
?NTL04	25	30.00	4.00	0.0	0.0	180.00	1.000	T	TT
?NTL05	25	30.00	6.00	0.0	0.0	0.00	1.000	T	TT
?NTL06	25	30.00	6.00	0.0	0.0	180.00	1.000	T	TT
?NTL07	25	30.00	8.00	0.0	0.0	0.00	1.000	T	TT
?NTL08	25	30.00	8.00	0.0	0.0	180.00	1.000	T	TT
?NTL09	25	30.00	10.00	0.0	0.0	0.00	1.000	T	TT
?NTL10	25	30.00	10.00	0.0	0.0	180.00	1.000	T	TT
?NTL11	25	30.00	12.00	0.0	0.0	0.00	1.000	T	TT
?NTL12	25	30.00	12.00	0.0	0.0	180.00	1.000	T	TT
?NTL13	25	30.00	14.00	0.0	0.0	0.00	1.000	T	TT
?NTL14	25	30.00	14.00	0.0	0.0	180.00	1.000	T	TT
?NTR01	25	50.00	2.00	0.0	0.0	0.00	1.000	T	TT
?NTR02	25	50.00	2.00	0.0	0.0	180.00	1.000	T	TT
?NTR03	25	50.00	4.00	0.0	0.0	0.00	1.000	T	TT
?NTR04	25	50.00	4.00	0.0	0.0	180.00	1.000	T	TT
?NTR05	25	50.00	6.00	0.0	0.0	0.00	1.000	T	TT
?NTR06	25	50.00	6.00	0.0	0.0	180.00	1.000	T	TT
?NTR07	25	50.00	8.00	0.0	0.0	0.00	1.000	T	TT
?NTR08	25	50.00	8.00	0.0	0.0	180.00	1.000	T	TT
?NTR09	25	50.00	10.00	0.0	0.0	0.00	1.000	T	TT
?NTR10	25	50.00	10.00	0.0	0.0	180.00	1.000	T	TT
?NTR11	25	50.00	12.00	0.0	0.0	0.00	1.000	T	TT
?NTR12	25	50.00	12.00	0.0	0.0	180.00	1.000	T	TT
?NTR13	25	50.00	14.00	0.0	0.0	0.00	1.000	T	TT
?NTR14	25	50.00	14.00	0.0	0.0	180.00	1.000	T	TT
RX									
RG	40	40	20.00	0.00	0.00	1.00	1.00	51.0	30.0
/									

5) This Mohr-Coulomb Plasticity FLAC model has been prepared for modelling of triaxial strip punch tests of quartzite

```
new  
grid 40 80
```

```
model ss
```

```
model null i=1,40 j=4 ;bottom plate
```

```
gen (0,0) (0,4e-3) (40e-3,4e-3) (40e-3,0) i=1,41 j=1,4  
gen (0,4e-3)(0,80e-3) (40e-3,80e-3) (40e-3,4e-3) rat 1.0365 0.97 i=1,41 j=5,81  
int 1 aside from 1,4 to 41,4 beside from 1,5 to 41,5  
int 1 kn=1e12 ks=1e12 fr=30
```

```
set large  
prop d=2700 sh=31E9 bu=31.5E9 ctable=1 ftable=2 dtable=3 ten=15E6 j=5,80  
table 1 0,50e6 8,18e6 9,18e6 10,17.2e6 13,17.2e6 15.5,19.6e6  
table 2 0,6.8 0.5,21.2 1,32.4 2.5,43.2, 4,51.2 8,50.4 10,38, 12.5,34.4 16,34.4  
table 3 0,0 0.5,29.6 1,32 2,32.8 3,33.2 4,34 8,26.4 12,18.4
```

```
prop d=2700 sh=31E9 bu=31.5E9 ctable=1 ftable=2 dtable=3 ten=15E6 j=1,3
```

```
fix x i=1  
fix y j=1  
fix y i=1,8 j=81  
apply press=10e6 i=41 j=1,4  
apply press=10e6 i=41 j=5,81  
ini sxx=-10e6  
ini szz=-10e6  
ini yvel=-1e-8 i=1,10 j=81
```

```
def flac_y_st_punch  
  total_area=0  
  total_force=0  
  ar=0  
  loop i (1,9)  
    ar = ((x(i+1,81) - x(i,81)) + (x(i+1,80) - x(i,80))) / 2  
    total_area = total_area + ar  
    force = syy(i,80) * ar  
    total_force = total_force + force  
  end_loop  
  flac_y_st_punch = (-1) * total_force / total_area  
end
```

```
DEF p_strain
  p_strain=-1*(ydisp(1,80)-ydisp(1,1))/(y(1,80)-y(1,1))
end
hist nstep=10
hist unbal
hist flac_y_st_punch
hist p_strain
step 10000
save ctriax.001
```

APPENDIX D

Pictures of the Tested Samples in The Laboratory Experiments

Some of the laboratory experiment results are shown in this sections (Figures D1-D8).

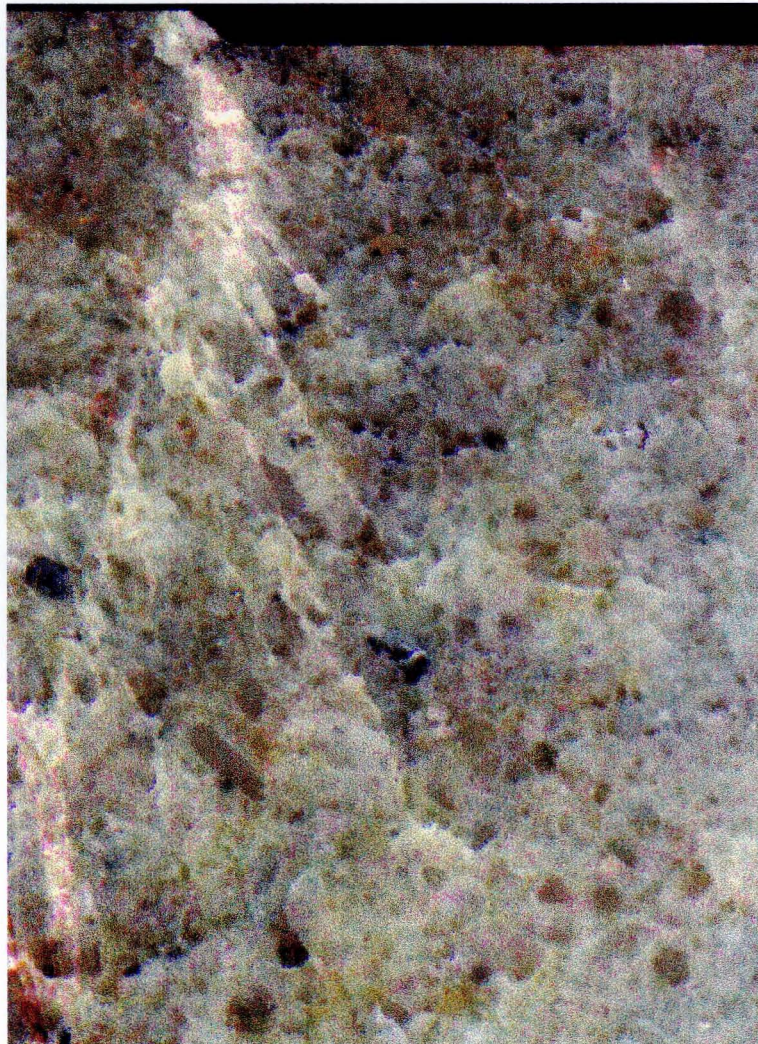


Figure D.1 Zoomed picture of the triaxial strip punch tests (final pattern)



Figure D.2 Zoomed picture of the triaxial strip punch tests (at the bottom of the first peak)

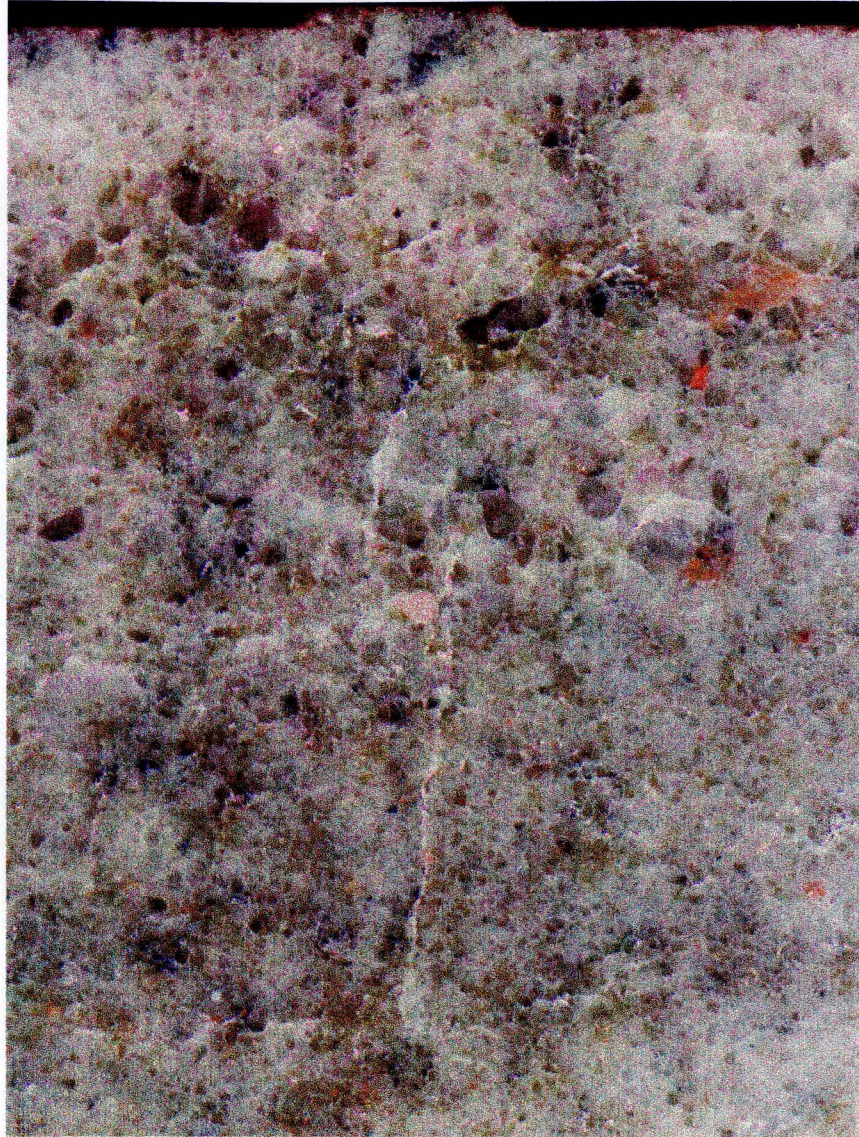


Figure D.3 Zoomed picture of the triaxial strip punch tests (before the failure)

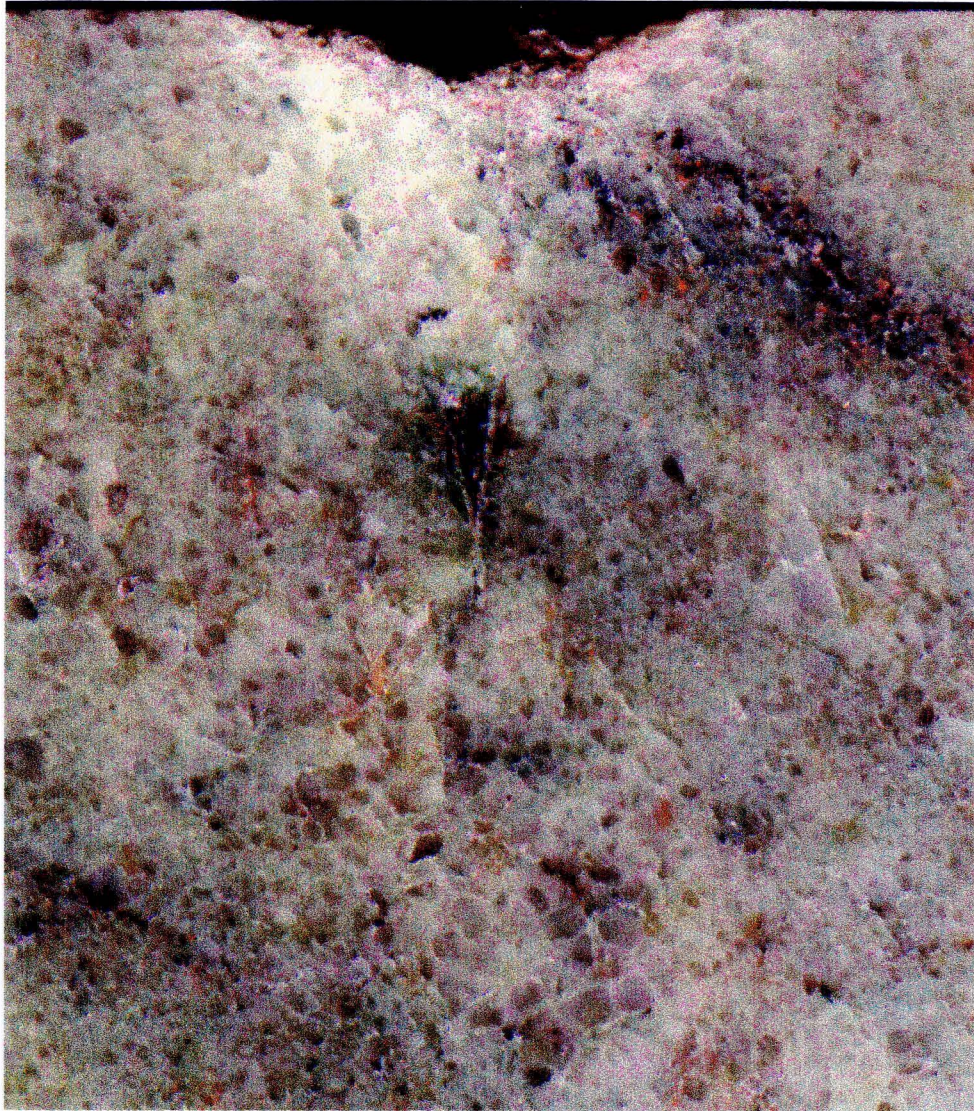


Figure D.4 Zoomed picture of the square punch test (final pattern)

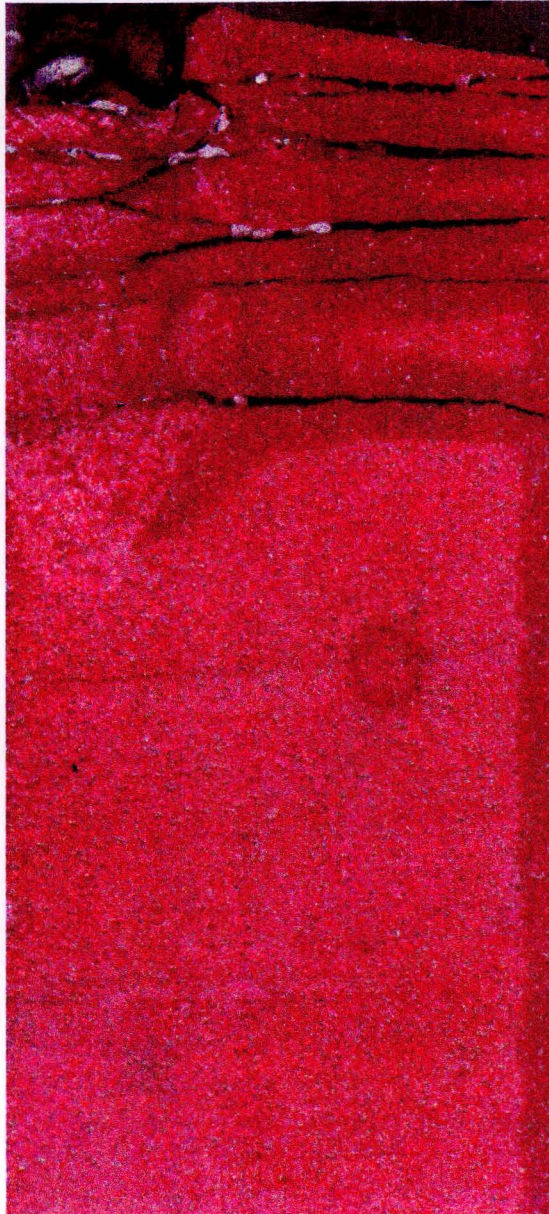


Figure D.5 Confined sandstone sample from steel punch test (only left side of the sample)

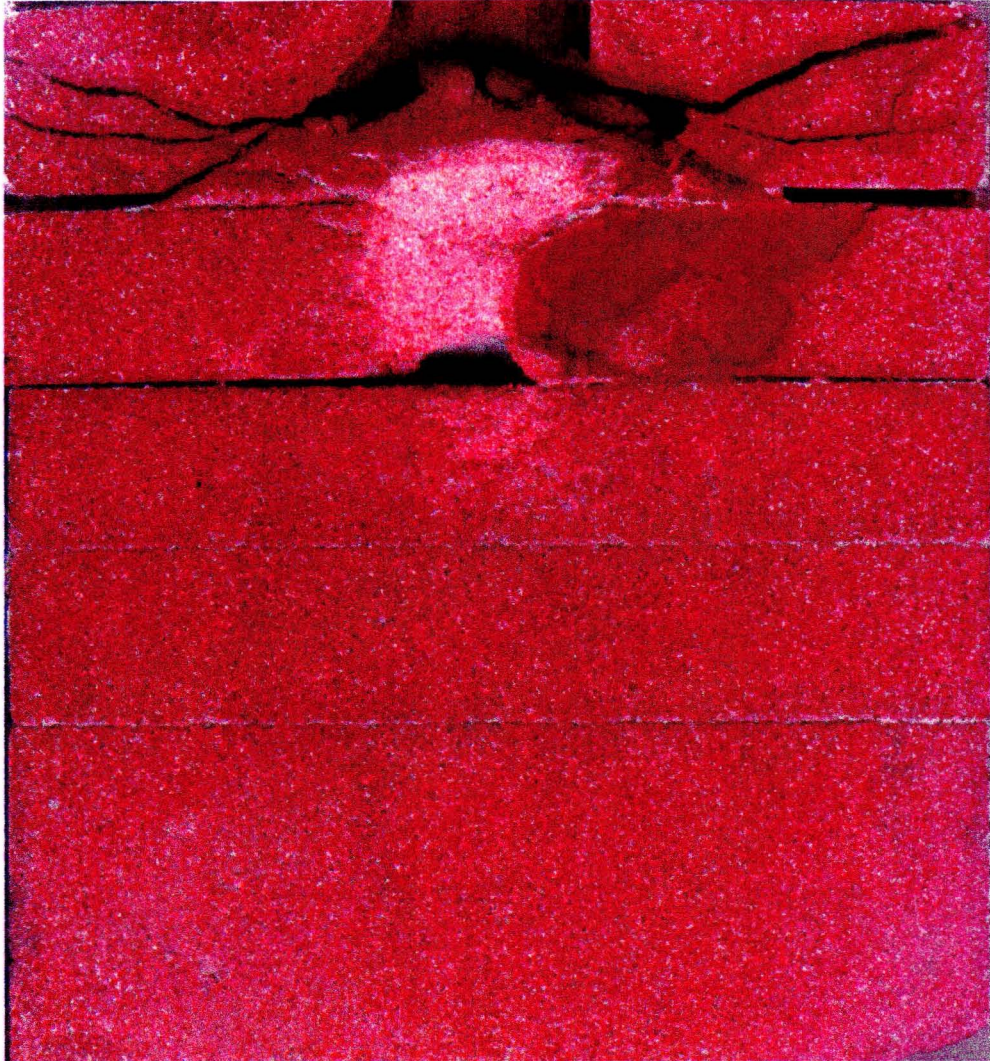


Figure D.6 10 mm layered sandstone sample from steel punch test

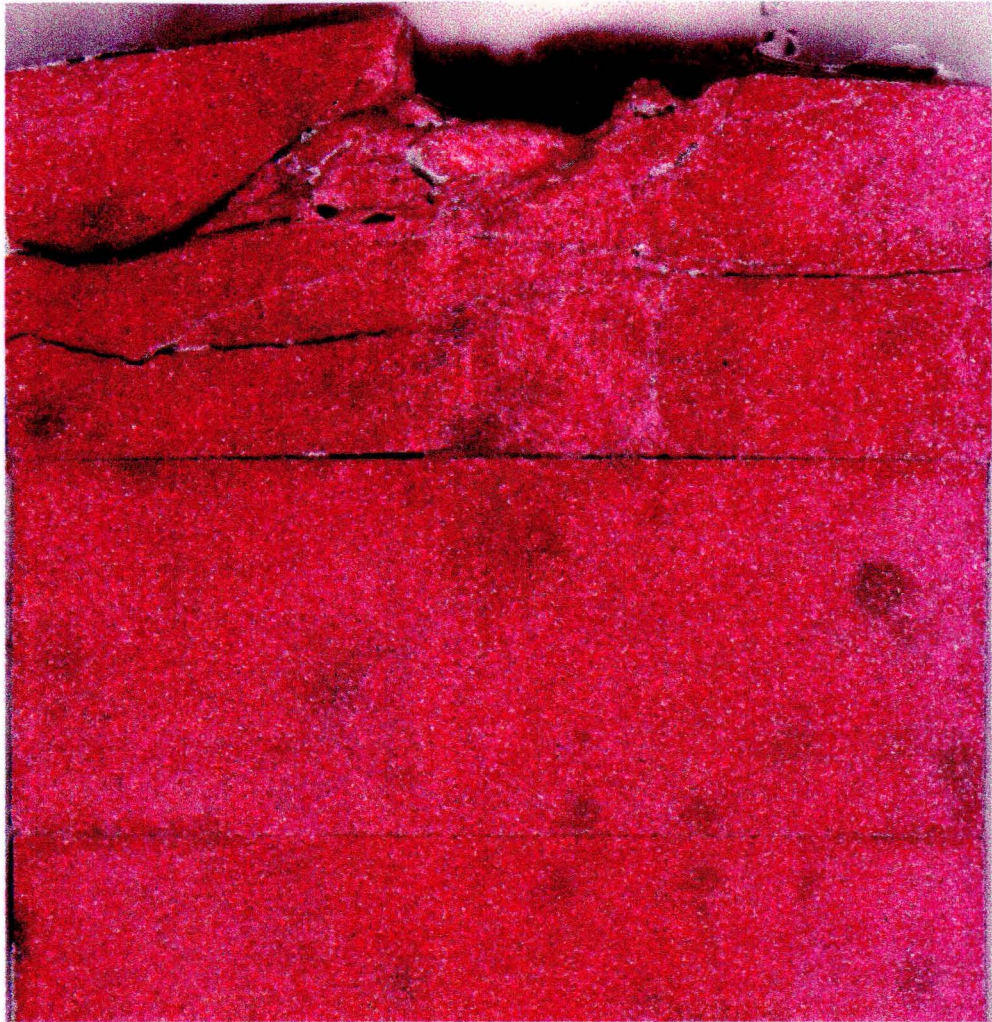


Figure D.7 20 mm layered sandstone sample from steel punch test (without friction reducer between the layers)



Figure D.8 20 mm layered sandstone sample from steel punch test (with friction reducer between the layers)

APPENDIX E

Determination of the Loading Conditions for the Triaxial Strip Punch Tests

A series of FLAC modelling tests have been carried out to determine the relation between axial stress and the confinement applied to satisfy the plane-strain loading conditions. From the FLAC runs, it has been found that the ratio of applied confinement to axial stress is 0.034 (Table E.1)

Axial Force	Axial Stress (σ_y)	Confinement (σ_x)
kN	MPa	MPa
10	12.5	0.422375
20	25	0.84475
30	37.5	1.267125
40	50	1.6895
50	62.5	2.111875
60	75	2.53425
70	87.5	2.956625
80	100	3.379
90	112.5	3.801375
100	125	4.22375
110	137.5	4.646125
120	150	5.0685
130	162.5	5.490875
140	175	5.91325
150	187.5	6.335625
160	200	6.758
170	212.5	7.180375
180	225	7.60275
190	237.5	8.025125
200	250	8.4475
210	262.5	8.869875
220	275	9.29225
230	287.5	9.714625
240	300	10.137
250	312.5	10.559375
260	325	10.98175
270	337.5	11.404125

Table E.1 Loading combination between axial stress and the confinement

During the experiments these conditions were only satisfied up to a confinement of 10 MPa. At higher stresses the confinement can not be increased because of the limitations of the experimental test cell.