University of the Witwatersrand
School of Education

Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling

A dissertation submitted to the Wits School of Education, University of the Witwatersrand, Faculty of Humanities in fulfilment of the requirements for the degree of Master of Education (Mathematics) by Research.

By

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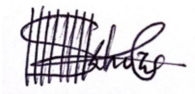
DECLARATION:

I Siyabonga Jabulane Khoza student no. 1636533 declare that the Master of Education (Mathematics) by Dissertation titled: *Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling* represents my own original research work, and that all sources directly or indirectly cited in this dissertation have been specified and acknowledged with a full reference list. I further certify that I have not submitted this dissertation before, in whole or in parts for examination at Wits School of Education or at any other higher institution for another qualification.



Mr S.J Khoza

20 July 2023
Date

The masters dissertation has been approved by the university (WSoE)

ABSTRACT:

In this study, I investigate in-service teachers’ beliefs and self-efficacy about teaching mathematical modelling. I further understood teachers’ perceptions about teaching modelling in the Grades 10 - 12 CAPS mathematics curriculum. The purpose of the study was to reveal teachers’ beliefs and SEF to support the development of teachers' modelling competency.

The study was underpinned by a structural model of professional competence for teaching mathematical modelling among in-service teachers. A structured questionnaire with a 5 Likert scale was used to collect data on the ISTs' beliefs, SEF, and prior knowledge about teaching modelling in the Grade 10 – 12 CAPS curriculum. Further semi-structured interview sessions were secured with three participants to further confirm quantitative data. Thus, a ‘Sequential explanatory research design from a mixed method research design’ was used to report the collected data.

From the questionnaire obtained results, three major themes were formulated from the research questions and used to analyse, present, and discuss the data, which were ISTs’ beliefs about mathematical modelling, ISTs’ self-efficacy about mathematical modelling, and ISTs’ prior knowledge about modelling. From the qualitative data, four themes stood out from the data during the transcription process, namely, teachers are more product-driven than process, learners should take the lead during mathematical modelling, the curriculum timeframe limits learners from exploring modelling, and the limitation of mathematics content in the curriculum. What was revealed from the data is that teachers do believe in the existence of modelling in the mathematics curriculum.

Teachers showed being constructivists in the classroom when teaching mathematics in general, including modelling. Their prior experiences with teaching modelling showed that it has contributed to their belief in teaching and learning modelling. Though teachers' beliefs and prior knowledge in this study showed to be developed and acquired respectively, to sufficiently show competencies of teaching modelling in the classroom. However, their SEF to diagnose learners' abilities during their modelling processes showed to be limited. Meaning, teachers did not show confidence in their abilities to diagnose learners' abilities when modelling, and it was not confirmed if they can identify learners' abilities when solving mathematical tasks in general. I believe in South African modelling can be taught and learnt in the classroom if it is sufficiently catered for in the curriculum and if teachers get the necessary support in teaching modelling. The value of the study is an important contribution to teachers' mathematical modelling competency.

Keywords: *Beliefs; Mathematical modelling[i]; Prior knowledge; Self-efficacy; In-service teachers.*

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"When the Time is Right, I the Lord, Will Make It Happen" (Isaiah 60:22)

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- I would like to extend my special gratitude to my fiancée Tiishetjo for supporting me throughout the research journey.
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DEDICATION:

I devote this dissertation to:

- My mom who continuously supports and take care of me from the day I was born.
- My late father who showed me love while still walking in this world.
- My fiancée Tiishetjo.
- My brother (Sibusiso) and sisters (Cindy, Rose and Thembisile).

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1.1 Introduction

This chapter discusses the research background, problem statement, aims and objectives of the study, the rationale of the study, and the significance of conducting the study. Research questions and the context where the study took place are also outlined in this Chapter.

Mathematics should be a subject that can be comprehended and valued for its significance and power in developing countries (Stillman, Blum & Biembengut, 2015). This process of education system development and change influenced research studies to be conducted in mathematics education and recommended new approaches, pedagogies, and models to be tried and used in teaching and learning mathematics and other subjects (e.g., physical sciences). One of the teaching approaches in mathematics education is approaching mathematics employing real-life situations, which in this study is referred to as mathematical modelling.

Rapid change and development of technology in our society and education system have changed the expectations of today’s societies, teachers training, and learners to be formed within the education system. Nowadays, the world expects mathematics teachers to produce learners who will be able to demonstrate abilities to come up with constructive solutions in general and mathematical problems in the events of real-world situations and use their mathematics skills and knowledge productively in their daily-life experiences (Arseven, 2015).

In this way, learners will understand the value of doing mathematics at school and will eventually enjoy doing mathematics at school instead of taking it as a boring and difficult subject that is not even applicable to their daily-life situations. Some learners are even afraid of taking mathematics at school, particularly from Grade 10 – 12 since mathematics is compulsory from Grade R until Grade 9 in the South African (SA) education system. Most learners prefer to take mathematical literacy, though it is said that mathematical literacy is more of a modelling content focus than mathematics (Conradie, 2016).

Many authors have defined modelling in different ways depending on the purpose of the study and how they wanted to use it in their studies and contexts. Greefrath, Siller, Vorhölter and Kaiser (2022) describe mathematical modelling as a process of creating mathematical models based on a real situation that will assist in explaining the situation. Such developed models will assist the modeller to make assumptions, predictions, and decisions and distributing knowledge about the given problem (Geiger, Galbraith, Niss & Holland-Twining, 2022). Mathematical modelling is considered one of the core facets of teaching and learning mathematics and provides solutions to complex real-life situations for learners, teachers, and communities (Wess, Klock, Siller & Greefrath, 2021).

Modelling and its applications are all features that show the relationship between the real world and the mathematical world. Application is the transition from the mathematical world to the real world, while modelling is the process of translating a real-life situation into mathematical equations and models (Hankeln, 2020). The integration of modelling and application in mathematics education is becoming increasingly significant in many educational curricula (Kaiser, 2020; Niss et al., 2007). The SA National Senior Certificate (NSC) (2018) report, it was indicated that most learners are lacking modelling skills. The study by Jankvist and Niss (2020) supports what was mentioned in the NSC (2018) report, they revealed that a notable number of learners have difficulty in understanding modelling tasks that are out of what they are learning daily in a mathematics classroom. Learning mathematical modelling is a cycle process that is learned over some time. Modelling entails much more than just solving mathematics word problems.

The NSC (2018) further report that in-service teachers (ISTs) do not have enough time to cover all possible problem-solving context questions to prepare learners for examination. It was then suggested by Jankvist and Niss (2020) that teachers should ensure that learners acquire all the necessary modelling skills needed for them to approach modelling questions. ISTs refer to teachers who are qualified and already teaching in the classroom. In some countries, such as Ghana, learners are discouraged from challenging their teacher's methods of approaching problem-solving questions in the classroom (Dei, 2011), while the South African Mathematics Curriculum and Assessment Policy Statement (CAPS)[1] is dominated by more procedural knowledge than conceptual knowledge of the skills and knowledge of mathematics in general and in modelling (Ndlovu, 2018). Since mathematical modelling requires learners to make assumptions, build mathematical models, mathematise, validate, evaluate, and relate the real problem in a modelling cycle (Greefrath et al., 2022; Leung et al., 2017), modelling may be

[1] CAPS is a single, comprehensive, and compact policy statement developed by South Africa's curriculum developers (DBE) for all subjects specified in the National Curriculum Statement for Grades R - 12. CAPS provides teachers with extensive guidelines on what to teach and how to assess what was taught.

difficult to be integrated with other mathematics topics (i.e., Finance, statistics, and Geometry) and subjects in the current CAPS curriculum.

Researchers urge for a transformation in teacher mathematics education, to embark on a learner-centred approach mostly in modelling to present learners with opportunities to discuss their mathematical solutions and ideas behind their workings, and to teach for conceptual understanding of the concepts (Akyeampong, Lussier, Pryor & Westbrook, 2013; Nabie, Akayuure, Ibrahim-Bariham & Sofo, 2018). Such teaching approaches depend on teachers' beliefs and self-efficacy (SEF) about the subject, in this study were teachers' beliefs and SEF about mathematical modelling. Beliefs are viewed as an individual's intellectual held understanding and assumptions about the world objects that are presumed to be true, and how it influences them to interact with reality. Their beliefs inform their SEF in certain situations, and SEF is viewed as an individual's capability to recognise someone's ability in certain situations. Furthermore, prior knowledge of a subject influences the individual's SEF.

Although my opinions are not verified and scientifically proven, looking at the SA context, according to my knowledge and experience from basic education, I would say the current mathematics education curriculum is not contextualised with mathematical problems related to learners' daily-life experiences, and when looking at the approved CAPS mathematics textbooks (such as classroom mathematics and Via Afrika)[2] the materials seemed to be borrowed from international contexts. Hence, the contexts of the mathematics textbooks used do not adequately match with SA learners' context and this might make it difficult for learners to realise the need for school mathematics in their daily-life experiences.

In addition, learning modelling is cognitively demanding (Blum, 2015), thus there should be enough time for learners to master and understand the modelling skills needed for them to approach modelling questions, which is being competent in modelling. Therefore, it is vital to note that while it is acknowledged that learners participate in modelling processes in mathematics classrooms, it is a teacher who first provides learners with the methods and procedures of mathematical modelling, which is developing learners to be competent in modelling. Hankeln, Adamek and Greefrath (2019) define modelling competence as the capability to understand the context of a given reality problem, build a mathematical model from the real-world situation by identifying relevant questions and variables, mathematise the

[2] These are some of the South African CAPS-approved textbooks used in teaching and learning mathematics at the basic education level.

model, interpret, validate, and present the obtained results back to the real-world context concerning the built mathematical model.

1.2 Background of the study

The application, teaching, and learning of modelling at universities and high school level, respectively, has become an important topic of discussion locally, in South Africa, and internationally. The discussion around modelling is because of the important rise of mathematical modelling in providing strategies, understanding, and solutions to complicated real-life situations (Wess et al., 2021) in different fields, including education, health, engineering, and natural sciences. Many researchers (see Blum & Borromeo, 2009; Kaiser, 2006; Özdemir, Üzel & Özsoy, 2017; Ramirez, 2017; Wess et al., 2021) are currently studying teachers' beliefs, SEF, prior knowledge, and perceptions about mathematics in general, applications, teaching, and learning of mathematical modelling. Commonly, many researchers would study these factors of teacher modelling professional competence within determined areas of mathematical modelling and its application. Since mathematical modelling is one of the important areas in mathematics, especially from the Grade 10 – 12 mathematics CAPS curriculum.

In mathematics education, the concept of modelling is considered one of the topics that can bring sense to learners of why they are learning mathematics at school from Grades 1 – 12, and some at a university level (Tran, Nguyen, Nguyen, Nguyen & Ta, 2020). This means modelling can show a link between school mathematics and real-life mathematics. This is because modelling mostly relates to real situations. Learning to solve complex real-life problems through modelling processes can guide one to realise the need and value of mathematics in life, and this will lead teachers and learners to understand the need of learning mathematics at school through the processes of modelling (Tran et al., 2020).

These standards of learning mathematics through modelling processes cut through geographical boundaries and educational jurisdictions. For example, CAPS's goal is to ensure that learners learn, acquire, and apply knowledge and skills in ways that are relevant to their own daily-life experiences and contexts (Department of Basic Education (DBE)[3], 2011). In this way, the curriculum plan encourages knowledge and skills in local contexts while remaining

[3] The Department of Basic Education (DBE) (2011) was created when it was separated into two forms, namely: the Department of Basic Education and the Department of Higher Education and Training. DBE focuses from Grade R to Grade 12.

mindful of global requirements because one might still be expected to explore the world. DBE (2011) urge teachers to incorporate application and real-life context in their mathematics teaching to help learners strengthen their cognitive abilities and positive attitude towards the subject. Such teaching strategies assist learners in better developing an understanding of the real world and the mathematical world, advance their mathematics competencies (including modelling competencies), and assist in the development of suitable attitudes towards the subject (Blum, 2015; Wessels, 2017).

Looking at the SA CAPS curriculum, mathematical modelling is taught at the high school level from Grades 10 – 12 by the ISTs. Although it is also taught in the lower grades (Grades 6 – 9), they are not dominating much compared to the upper grades. Modelling is also taught at the university level to the pre-service teachers (PSTs) as training for the modelling of teaching knowledge and skills on how it should be taught to the basic education learners in the classroom (Bonyah & Clark, 2022). Previous research studies based on the PSTs university training about teaching mathematical modelling have been conducted in different countries (see Blum & Ferri, 2009; Jankvist & Niss, 2020; Wess et al., 2021), and in SA (see Mudaly & Dowlath, 2016). From all these research studies, what is common in their discoveries is that pedagogical strategies used in the classroom significantly influenced learners' skills development in modelling competencies. From the studies of the authors mentioned, it was revealed that teaching mathematical modelling in the classroom is generally challenging and cognitively demanding (Blum, 2015). Looking at the ISTs, such challenges might be arising from an assumption that teachers are not well trained for teaching modelling. Further, their awareness of modelling in the curriculum might be limited, and lastly, it might be one of the factors that teachers' beliefs and SEF concerning modelling are not well developed and their attitude towards it is negative.

Looking at the ISTs competence for teaching modelling at the high school level, Grade 10 – 12, there is a knowledge gap of collective understanding on how teachers develop learners modelling knowledge and skills through their teaching approaches, and what mathematical modelling and applications to reality means in practice (Niss et al., 2007) in terms of their modelling beliefs and SEF. Studies (see Ramirez, 2017; Yang, Schwarz & Leung, 2022) conducted on teachers' beliefs about mathematical modelling revealed key findings about modelling in the classroom, but mostly focused on how in-service teachers perceive mathematical modelling in general. Teachers connect mathematical modelling with real-world situations to be modelled, others said that mathematical modelling is how a mathematical problem should be processed (Ramirez, 2017). As Tekin Dede and Bukova Güzel (2016) have disclosed, because of the teachers' uncertainty about how they should act when teaching a lesson incorporating mathematical modelling. Thus, the current study aimed to add knowledge on the in-service teachers' beliefs about teaching modelling to the learners in the classroom since most of the studies (see Mainali, 2022) were based on pre-service teachers' beliefs.

Wess et al. (2021) categorised beliefs as either being transmissive or constructivists belief. Concerning the study, transmissive belief is when the teacher gives learners step-by-step procedures for solving modelling tasks, while constructivist belief is when the teacher allows learners to be the ones that take the lead in solving modelling tasks in the classroom. White, Way, Perry, and Southwell (2005) note, belief can either be negative or positive and may contribute to teaching strategies in the classroom positively or negatively. Which may, in turn, contribute to learners' negativity, attitude, and performance. Such type of belief may continue if those learners go on to become future teachers and no intervention has been done to it. Therefore, positive teachers' beliefs in a particular topic can lead to a constructive learning environment and vice versa. It becomes a challenge for teachers to emphasise concepts that they don't believe in them, particularly for modelling, as such content requires more real-life situations and cognitive thinking.

Research studies (see Blum, 2015; Wess et al., 2021) show that teaching and learning modelling is cognitively challenging for both teachers and learners, respectively. But even more demanding for in-service mathematics teachers (ISMTs), and sometimes PSTs (Wess et al., 2021). It may be that ISMTs are not well competent in modelling themselves. Hence, it would be challenging for them to promote the development of modelling competency to the learners while teachers are not yet professionally competent in teaching mathematical modelling (Wess et al., 2021). Hence, the setting of the study was then to investigate IST's beliefs and SEF (through their prior knowledge) about the teaching of modelling in the classroom, and then understand their perceptions about modelling in the CAPS curriculum.

In the study conducted by Frejd (2012) at upper secondary schools based on teachers' conceptions about mathematical modelling, they revealed that teachers have slight experience of the concept of modelling in mathematics classes. This can support the claim that the participating teachers in their study might not have been exposed to modelling during their schooling times. Looking at the SA context and curriculum, modelling is not well emphasised and integrated across mathematics topics and other subjects. Therefore, ISMTs might not have had adequate experience in learning modelling in their secondary mathematics education, and their teacher training at higher education might not be adequate to train them to have capabilities to transmit their modelling content knowledge and skills to their learners in the classroom.

Investigating Johannesburg North and Central district schools' teachers' beliefs, SEF, prior knowledge, and understanding their perceptions about modelling in the mathematics curriculum was the main objective of the current study. Therefore, possible strategic intervention in teaching modelling may be suggested for future research to strengthen their teaching modelling competence. An explanatory study conducted by Cabassut and Ferrando (2017) among Spain and France mathematics teachers about teaching modelling confirmed that most teachers are positive about modelling, but they lack self-confidence when it comes to teaching the topic. In their study, some of the teachers acknowledged that they are experiencing some difficulties when it comes to teaching the modelling topic. Some of the factors that contributed to the challenges in teaching the topic, including not having time to prepare and teach the topic and lack of resources in their schools.

Tekin Dede and Bukova Güzel (2016) mentioned that research studies revealed that teachers sometimes do not know when and how to initiate the knowledge of modelling processes to the learners during the lesson. Tekin Dede and Bukova Güzel (2016) stated that it is regular for teachers who are beginners in the mathematics teaching profession, and teachers who did not have had adequate exposure to the modelling content knowledge from secondary education to their higher education training. Such teachers might experience challenges during the development of modelling processes for the learners (Tekin Dede and Bukova Güzel, 2016).

When reviewing relevant literature on mathematical modelling (see Chapter 2), it is discovered that many researchers have been conducting studies on this modelling topic widely, such as in Germany[4]. As acknowledged by Vorhölter, Greefrath, Borromeo Ferri, Leiß and Schukajlow (2019) and Hankeln (2020) that German universities have developed resources over a period that can be adopted by other countries to understand ISTs' competence in teaching mathematical modelling. Resources that were developed by the authors (see Wess et al., 2021) include test instruments, methods, and frameworks. A structural model (see section 2.3.2)

[4] Germany is one of the countries that has contributed comprehensive knowledge and discussions in the application and modelling of mathematics education for more than 100 years (Vorhölter et al., 2019).

explaining and correlating professional knowledge and skills for teaching modelling was formed and experimental proven within the framework by (Klock et al., 2019; Wess et al., 2021). I used some of the facets of the structural model for teaching mathematical modelling in investigating teachers' beliefs, SEF, and prior knowledge about modelling in secondary schools' mathematics education. The study included a wide range of ISMTs from Grades 10 – 12 to see their beliefs and SEF concerning modelling. The study was not focusing on the PSTs, but it was focusing on the IST's beliefs and self-efficacy about modelling.

1.3 Problem statement

Studies on secondary mathematics teachers' beliefs and self-efficacy concerning mathematical modelling in South Africa do not exist (Durandt et al., 2022). Durandt carried out an empirical study on the difference between the two teaching designs on the development of 144 first-year engineering students' modelling competency. The findings of the study were that implementing quality teaching is a very important element even at the university level, thus complementing the results of the DISUM study (for more information about the conception and the results of DISUM, see Blum, 2011; Blum & Schukajlow, 2018).

Unlike Durandt et al. (2022) study, the current study focused on the in-service secondary teachers' beliefs and self-efficacy concerning teaching and learning mathematical modelling. We chose in-service teachers because they play a big role in preparing learners for university education in modelling and applications. Given that modelling and applications is still a new topic in secondary mathematics education in South Africa, and not yet fully operationalised in the curriculum; and require a different teaching approach from the teacher-centred approach that most secondary teachers are familiar with, the current study assumed that the concept of modelling would be problematic. The study, therefore, sought to investigate in-service teachers' beliefs and self-efficacy regarding teaching modelling.

In South Africa, most high school learners graduate from school without an understanding of why they are learning mathematics at school from Grades 1 – 12 (Turner, Bennett, Granillo, Ponnuru, Roth Mcduffie, Foote & McVicar, 2022). As discovered in the study by Tran and Dougherty (2014) that learners do not see the value of mathematics in their lives and the mathematics learned might not be related to the learner's context. Hence, real problems and applications of mathematics might be compromised in the classrooms. This starts with teachers' beliefs about the topic to be learned. Meaning, learners' performance in the classroom is directly linked to the teacher's pedagogical strategies in approaching the topic and that eventually determines learners' disposition towards that topic (Mosimege & Winnaar, 2021), if a teacher is negative about a certain topic so does to the learners and vice versa.

Yang, Kaiser, König, and Blömeke (2020) postulate that to understand teachers' beliefs about mathematical modelling it is significant to also consider their beliefs about the essence of mathematics, mathematical application, and real-world and modelling. Previous studies (see Mudaly & Clark, 2016; Mutodi & Ngirande, 2014) appears to not have considered teachers' beliefs and SEF about mathematical modelling that arises from their prior knowledge in modelling. Perhaps these studies would have contributed more knowledge to the ongoing discussions about teaching and learning modelling in the classroom. It could be argued that if teachers have strong prior knowledge about modelling, then their beliefs and SEF in modelling will be sufficient to promote modelling competency in the classroom among the learners. Hence, the currently conducted study further attempted to contribute more knowledge to the ongoing discussions about teaching and learning of modelling looking at the factors of teachers' beliefs and self-efficacy in modelling.

According to Ramirez (2017), beliefs can be viewed as opinions, perceptions, and values. In this study, beliefs were investigated to see teachers' disposition towards mathematical modelling in the curriculum. I hypothesised that if teachers could believe in teaching (integrating) modelling with other topics, that will show learners the value of mathematics in their daily-life situations. Therefore, as discovered by Tran et al. (2020) that learners will exit basic education knowing how mathematics applies to their real-life situations through modelling. Furthermore, it is argued by authors (see Tran et al., 2020) that modelling is one of the significant topics that can bring sense to the learners of why they are learning mathematics at school. For learners to see the connection between the actual world and the mathematical world, they should be competent in modelling (Hankeln et al., 2019). Learners should be taught knowledge and skills of mathematical modelling so that they are competent in it, and that is the responsibility of a mathematics teacher. For one to be able to promote modelling competence to the learners he/she must be competent himself/herself in modelling processes first. Hence, the setting of the study was investigating SA, Gauteng[5] teachers’ modelling competency[6] (Beliefs, SEF, and prior knowledge) for teaching mathematical modelling using

[5] Gauteng is one of the nine provinces in South Africa. Situated on the highveld.
[6] Competency in this study was viewed as individuals' behavior and attitude in relation to what he/she has an ability to do.

a structural model of professional competence developed by Wess et al. (2021). Once the modelling level knowledge of teachers has been identified and properly developed, it might have a positive influence on learners' performance and participation in modelling classes and mathematics in general (Cevikbas, Kaiser & Schukajlow, 2022).

Tekin Dede and Bukova Güzel (2016) research studies revealed that teachers sometimes do not know when and how to initiate the knowledge of the modelling process to the learners during a lesson in a classroom. It is common to teachers who are beginners in the teaching profession, or teachers who might not have encountered often modelling content knowledge from secondary education to higher education. Teachers who did not have enough training during undergraduate studies might experience challenges during the development of modelling competencies for the learners, and sometimes for themselves (Niss & Blum, 2022). Hence, it was also significant for the study to investigate teachers' prior experiences with modelling to reveal their level of SEF. The responsibility of a teacher during the development of mathematical modelling among learners remains a crucial part of modelling lessons; then the questions arise of what teacher's beliefs about the teaching and learning of mathematical modelling can be shared with the learners for them to see the value of learning mathematics through real-world situations at school?

Looking at the ISTs beliefs and SEF for teaching modelling at Grades 10 – 12, there is a knowledge gap of collective understanding on how teachers develop learners modelling knowledge and skills through their teaching approaches, and what modelling and applications to reality means in practice (Niss et al., 2007). As Tekin Dede and Bukova Güzel (2016) have stated that teachers are not sure about how they should act when teaching a lesson utilizing mathematical modelling. Thus, the current study aimed to add knowledge on the in-service teachers’ beliefs and self-efficacy about teaching modelling to the learners in the classroom.

Wess et al. (2021) developed and empirically tested a structural model for teaching modelling that was first piloted to the PSTs. The results acquired fundamentally "represent site-specific empirical confirmations of the structures and correlations shown" (Wess et al., 2021). The authors suggested that further studies with the objective of a possible remodelling of the conceptualised structural model of teaching mathematical modelling still need to be done. The understanding of modelling competence among teachers, which is from the model for teaching modelling developed by Wess et al. (2021), only gives us knowledge and skills on how modelling competency for PSTs can be identified and enhanced. Thus, the current study needed to be conducted to add knowledge of competency for teaching modelling from the ISTs. That may lead to a comprehension of learners modelling competency and their performance in the classroom.

1.4 Purpose of the study

The purpose of the study was to investigate in-service teachers' beliefs and self-efficacy using a structural model of professional competence for teaching modelling. Furthermore, to understand teachers' perceptions about teaching modelling in the Grades 10 - 12 CAPS mathematics curriculum.

1.5 Study objectives

The study's objective was:

- To investigate in-service teachers' beliefs, self-efficacy, and prior knowledge about mathematical modelling.
- To understand teachers' perceptions about mathematical modelling content in Grade 10 – 12 CAPS curriculum.

1.6 Research questions

The following research questions served as a guide for the study:

- What are in-service teachers' beliefs about teaching and learning mathematical modelling in the CAPS curriculum?
- What are the in-service teachers' self-efficacy about teaching and learning mathematical modelling in the CAPS curriculum?
- What are in-service teachers' prior knowledge of mathematical modelling in the CAPS curriculum?
- What are in-service teachers' perceptions about the modelling content in the Grade 10 – 12 CAPS curriculum?

1.7 The rationale of the study

The proposed study aimed to investigate teachers' modelling beliefs, SEF, and prior knowledge through some of the test instruments from a structural model of professional competence for teaching modelling, in the Johannesburg North and Central district schools, Gauteng. The study further sought to understand teachers' perceptions about the teaching of modelling in the curriculum. The hidden application of modelling in mathematics by teachers and learners has made it necessary to study teachers' beliefs and SEF and to further understand their perceptions about the modelling content in the CAPS curriculum. These are factors that can contribute to modelling performance in the classroom, either positively or negatively. In addition, these factors may have a greater influence on how learners perceive mathematics learned at school with real-life mathematics experiences. The study attempted to reveal teachers' modelling competency and the knowledge they have acquired about modelling and then recommended possible interventions that can be for future research and can be considered to alleviate the faced situation.

From my firsthand experiences, firstly from beliefs, I have seen teachers with a positive belief in a certain topic giving it their best and negative belief in a certain topic given less attention. I then linked it with the CAPS curriculum that if teachers' beliefs (positive) and SEF expectations are strengthened, then the curriculum developers (from DBE) will give modelling more space across the curriculum, and training at the institutional level will further be strengthened for teachers to be prepared to teach mathematical modelling.

Wess et al. (2021) proposed that an improvement in mathematical modelling can be achieved if teachers are having a positive attitude, beliefs, motivational orientation: SEF, and well-grounded modelling content. Improving teaching and learning of modelling may mean that more teachers and learners will see the connection between school mathematics, everyday life mathematics, and reality application. As was also indicated by Tran et al. (2020) that modelling is one of the topics that can bring sense to the learners of why they are learning mathematics at school. Thus, there was a need to investigate and understand teachers' modelling competency to build positive attitudes and motivation toward modelling, particularly for the interest of the learners to see the value of doing mathematics at school.

I anticipate that the collected data from the study will reveal teachers' attitudes towards modelling, and once these are recognised, possible strategies to lighten the challenges will be recommended that will assist the Department of Basic Education curriculum designers, teachers, and learners in improving the teaching and learning of modelling.

1.8 The significance of the study

According to Niss and Blum (2020), teaching and learning mathematics with the integration of modelling (problem-solving) leads to a greater level of conceptual understanding for the learners. Thus, the findings that were obtained from this study may be of help to the CAPS mathematics curriculum developers and to encourage the whole South African, and other countries' education systems to be educational productive.

An educational productive education system should focus on the improvement of the learners' performance and concept understanding (in mathematics) while the mathematics content knowledge from the teacher is integrated with the modelling content. Mathematics teachers should be encouraged towards using pedagogical strategies that promote the development of mathematical competency in traditional and mathematical modelling competency. Additionally, the results obtained from the study will be used by mathematics teachers to reflect on their pedagogical strategies used when teaching modelling.

This pedagogy of approaching mathematics using reality situations (modelling) is one of the suitable approaches to teaching mathematics compared to the traditional way of teaching mathematical content in the classroom, where learners solve mathematical problems that are not related to their context.

1.9 Chapter summary and conclusion

From the introduction, the reader was provided with a discussion of the in-service teachers' beliefs and SEF about mathematical modelling and how it was used in the study. I further explained that mathematics teachers' prior knowledge was also investigated to help identify the level of teachers' SEF in recognising learners' abilities during modelling processes. Teachers' beliefs and SEF were investigated through a model of competence for teaching modelling. I have also pointed out that the study further understood IST's perceptions about modelling in the Grade 10 – 12 CAPS curriculum. In addition, I have also discussed the rationale and the significance of the constructs used, in mathematics education that is rapidly transforming into approaching mathematics through modelling.

1.10 Dissertation outline:

This outline shows how the dissertation is organised, as well as what each chapter's content is.

Chapter 1: Purpose and Background of the Study

The study's overview was presented in this chapter. It explained the study's context, purpose, and significance. Furthermore, it provided a clear explanation of why the study needed to be conducted, and who could benefit from it. The researcher introduced the problem, explained the study's motivation, and the research questions used to guide the study were stated here.

Chapter 2: Theoretical Framework and Review of Relevant Literature

The theoretical framework of the study and a review of related literature among teachers' mathematical modelling competency was discussed in this chapter. It included most sources with a connection to the research problem of the study that was investigated. To put it in another way, the link between existing knowledge and the research problem (knowledge gap) under investigation is addressed in this chapter.

Chapter 3: Research Design and Methodology

The research design, approach, methods, and methodology of the study were discussed in this chapter. The procedure of data collection, the type of test instruments used, and the selection criteria of the participants (teachers) were also covered in detail in this chapter. Furthermore, the chapter covered ethical considerations, reliability, validity, trustworthiness, and how the test instruments were developed and distributed.

Chapter 4: Results Presentation and Analysis

The results and data analysis chapter explained how the collected data of the study was organised and analysed. The chapter also discussed how the report's obtained information was showed, in form of graphs, tables, and data samples. The evaluation and interpretation of the findings of the study were included in the discussion.

Chapter 5: Discussion of the Results

This chapter provided the interpretation and elucidation of the obtained results. It further answered the four research questions of the study in connection to existing literature and theories. Additionally, it provided justification for the research approach used and critically evaluate the study.

Chapter 6: Conclusion, Limitations of the study, and Recommendations

This chapter drew conclusion from the study findings, provided a summary for each research question, limitations that were observed during the study process, and recommendations for future research.

The following chapter provides theoretical framework and review of relevant literature about in-service teachers' beliefs, self-efficacy, prior knowledge, and perceptions about mathematical modelling that ensured the researchability of the research problem and topic.

CHAPTER 2: THEORETICAL FRAMEWORK AND REVIEW OF RELEVANT LITERATURE

2.1 Introduction

This chapter introduced and explored the theoretical framework that guided the research study. It discusses the theoretical framework of a structural model of professional competence for teaching mathematical modelling that guided the entire study. The chapter further provided a review of relevant literature to provide what has been done locally, in South Africa, and internationally concerning my topic. The review of relevant literature was aligned with the developed four research questions (see section 1.6). The entire study was primarily based on ISTs' beliefs and SEF for teaching mathematical modelling, as mathematical modelling is cognitively demanding for both teachers and learners (Blum, 2015), the reason being, most modelling competency and competencies should be initiated during modelling processes. ISTs are the main role in the modelling processes, they should be the ones that trigger the modelling sub-competencies during modelling processes to the learners, and support learners as much as minimally as possible. Both mathematical and extra-mathematical skills and knowledge will be required during the modelling processes. Teachers are central and in control of the development and enhancement of the modelling competencies (Durandt, Blum & Lindl, 2022).

Theoretical framework

2.2. Mathematical modelling

Mathematics subject is one of the subjects that increase learners' ability to think critically and intensively. One of the objectives in mathematics education across all educational levels, including basic education and higher education, should be encouraging, and enhancing learners' mathematical modelling competency and sub-competencies. In this study, mathematical modelling is defined as a cycle process that involves translating a real-world situation into a mathematical model (Geiger et al., 2022).

Mathematics teachers’ responsibility is to show learners how school mathematics can be heuristic in real-world complex situations and different fields, including education, health, engineering, and science. Thus, learners will realise how school mathematics can be relevant to their daily-life experiences. Teachers are the foremost role in the enforcement and development of modelling in mathematics (Manouchehri, 2017). They do not only focus on its importance to the mathematics content or the learners, but they also oversee misconceptions that learners may show during the modelling processes and overcome them where possible. Hence, teachers' belief and SEF towards mathematical modelling plays a significant role in how they are supporting learners during their modelling processes in the classroom (Manouchehri, 2017).

Mathematical modelling should be taught in all grades including primary level (Blum & Ferri, 2009; Garfunkel & Montgomery, 2016). Through mathematical modelling, learners can apply mathematical concepts, independently investigate mathematical theorems, cultivate critical thinking, and advance their metacognitive and communication abilities (Wei, Zhang & Guo, 2022). This means that if modelling were to be implemented at learners' early grades or stages, they could develop modelling skills and have a greater chance of developing a productive disposition towards modelling. Learners' exposure to mathematical modelling will enhance creativity, enthusiasm, and a positive attitude towards modelling and mathematics in general. This is because young learners tend to develop knowledge and skills based on what they have learned before or based on their prior experiences (Piaget, 1971). Oberman and Sainz (2021) state that, putting modelling content in the foreground of the curriculum will encourage critical thinking and reality problem-solving skills to the learners.

CAPS's (2012) mathematics curriculum specifies some aims such as ensuring accessibility of mathematical content to the learners, also which intends to develop and enhance mathematical modelling, provide learners with an opportunity to make conclusions and justifications based on their workings, develop modelling problem-solving knowledge and cognitive skills, and prepare learners for further higher education and training, as well as the world of work across the globe. From DBE (2011) in the South African CAPS curriculum, mathematical modelling is positioned as the significance of the CAPS mathematics curriculum. One may proclaim that when applying mathematics in other subjects is comparable to when applying mathematics to real situations. Therefore, real-world problems should be and where possible be also integrated across mathematics topics. Mathematical modelling used in the classroom should be closely relevant to learners' real-life situations and not deliberated (Tran et al., 2020). Contextual problems presented to them by the teachers should reflect issues relating to education, health, economic, social, and environmental issues.

From the researcher's perspective, mathematical modelling concerning the SA context is still viewed as comprising the following components[7]: formulating a mathematical model from a real-world model, mathematical work, mathematics, Interpreting, evaluating, and exposing the results thus back to the real-world situation (Greefrath et al., 2022). Durandt (2021) states that a mathematics teacher has to continuously employ modelling processes to the learners when working through contextualised content in the mathematics classroom. The aim of mathematical modelling also contributes to the fact that mathematical context and knowledge cannot be only given to the learners to understand knowledge, skills, rules, procedures, and equations without declaring that they can link school mathematics with their everyday life experiences.

Authors (See Arseven, 2005; Lesh & Doerr, 2013), in the past years' research studies based on the use of modelling tasks in mathematics, have discovered that modelling tasks are useful tools to link school mathematics and everyday life experiences in mathematics education. The authors further argued that many countries tend to give a large space to modelling content in their mathematics curriculum, "It is emphasised that mathematical modelling is one of the main objectives of mathematics instructions in the secondary school curriculum in Germany" (Arseven, 2015). However, in the SA CAPS curriculum is not the case, it is the other way around, modelling is given a smaller place and time in the curriculum (Mudaly & Dowlath, 2016). Thus, concerning the SA curriculum it can be argued that mathematical modelling is not often integrated with other mathematics topics and subjects from basic education to higher education and mathematics teachers do not satisfactorily know its theory of origin.

2.2.1. Modelling competencies and modelling competency

From the study, I used modelling competencies as modelling processes that a modeller should embark on when solving a real problem using mathematical terms, models, and tools. Modelling competency may be viewed as modelling competencies but differs in terms of their meanings and applications, it is also defined differently from other studies (see Durandt, Blum & Lindl, 2022) depending on how it was linked to their study purpose, aims, and objectives. From previous studies (see Blomhøj & Jensen, 2007; Durandt, Blum & Lindl, 2022; Kaiser & Brand, 2015; Niss et al., 2007) modelling competency is frequently defined as an independent individual with abilities to build and use mathematical models and equations to mathematise

[7] These components are referred to as mathematical modelling sub-competencies by (Greefrath et al., 2018), and will be discussed shortly.

modelling problems, as well as to mathematise, validate and relate the given modelling task. The study by Hankeln et al. (2019) defined modelling competency as “Being competent in mathematical modelling is being able to autonomously and insightfully carry out all aspects of a mathematical modelling process in a certain context” (Hankeln et al., 2019, p. 144).

In consolidating the modelling competency definition from the author's studies, modelling competency was defined as someone who can carry out all the modelling sub-competencies in a real-world problem. For my study, modelling competency was observed as a teacher's professional competence, which associates their beliefs and self-efficacy in teaching modelling.

As mentioned, that modelling is an exercise of converting a real-world situation into a mathematical model and it is done through several steps (see Figure 2.1). For one to be competent in modelling he or she must be first competent in mathematics. Mathematical competency in this study is viewed as an individual's perceptive readiness to act on a particular kind of mathematical task or a given situation (Jensen & Jankvist, 2018). On the other hand, Niss and Højgaard (2011) explain mathematical competency as “…having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role” (Niss & Højgaard, 2011, p. 49). From the latter definition of modelling competency, once a teacher shows these kinds of characteristics in his or her teachings with deep knowledge and understanding of mathematics, he or she is in a position of being modelling competency depending on how far the teacher has gone with mathematical modelling from primary education to his or her higher education teacher training.

Thus, mathematical modelling competency follows the popular definitions of competency – in my study used and described as an individual's insightful intellectual beliefs and SEF capabilities for solving specific mathematics problems, including modelling tasks and being able to sustain all stages of mathematical modelling processes a real-life situation (Blomhøj & Jensen, 2003; Cevikbas, Kaiser & Schukajlow, 2021). These modelling processes that are applied when approaching modelling problems are often depicted as a cycle. In explanation of what is meant by carrying out "all parts of a mathematical modelling process". The model I used in this study of this modelling cycle, which is more relevant to the current literature about mathematical modelling competencies and competency is shown in Figure 2.1. These modelling sub-competencies are also acknowledged in the South African context (Stilman, 2012).

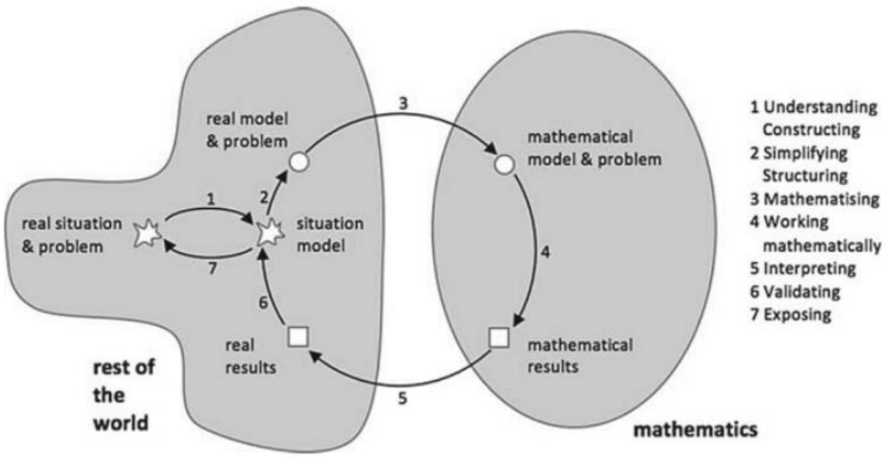

Figure 2.1 Seven-step modelling cycles (Blum & Leiss, 2007, p. 225)

The modelling cycles were analysed and presented comprehensively in (Vorhölter, Greefrath, Borromeo Ferri, Leiß & Schukajlow, 2019; Blomhøj & Jensen, 2003). This study was not focused on how learners or teachers can show or apply these modelling sub-competencies when approaching modelling problems during modelling processes. These modelling cycles (see Figure 2.1) are in turn viewed and represented as the modelling processes that one should possess when approaching modelling problems (Greefrath & Vorhölter, 2016). That is being modelled competent and it is viewed as someone who demonstrates abilities to carry out all modelling processes with insight in each modelling problem context (Hankeln et al., 2019).

Greefrath, Kaiser, Blum, and Borromeo Ferri (2013) went further by explaining how these modelling processes are applied by learners in a modelling cycle with the help of the teachers in the classroom. The modelling sub-competencies include building a mathematical model from a real-world model with an understanding of the situation context, simplifying, mathematics, interpreting, validating, and exposing the obtained results thus back to the real-world situation. Table 2.1 below provides us with explanations for each modelling sub-competencies.

Table 2.1 Modelling sub-competencies (Greefrath & Vorhölter, 2016)

Sub-competency	Description
Understanding	The students construct their own mental model for a given problem situation and thus understand the question
Simplifying	The students separate important and unimportant information about a real situation
Mathematising	The students translate suitably simplified real situations into mathematical models (e.g. term, equation, figure, diagram, function)
Working Mathematically	Students apply heuristic strategies and mathematical knowledge to solve the mathematical problem
Interpreting	The students refer the results obtained in the model to the real situation and thus achieve real results
Validating	The students check the real results in the situation model for adequacy
Exposing	The students refer the answers found in the situation model to the real situation and thus answer the question

In Germany and other countries' discussions of contextual mathematical modelling, Blum's stance (1985) is the main centre of the current study. Which considers the entire modelling processes and the connection between the mathematical world and the actual world around us, including teachers and learners. For more details about mathematical modelling discussions in German (see Greefrath, 2019; Greefrath & Vorhölter, 2016; Vorhölter, 2019), some of the discussions in the SA mathematical modelling context (see Mudaly & Dowlath, 2016). The study was not focused on the modelling sub-competencies of the learners or teachers, but it was focusing on teachers modelling competency, including beliefs, SEF, prior knowledge, and perceptions about the teaching, and learning of mathematical modelling.

In South Africa, mathematical modelling content is not mandatory in higher education (universities and colleges) teacher training courses. Only some universities (e.g., University of the Witwatersrand) modelling modules are offered regularly. If PSTs and ISTs are expected to integrate their mathematics teachings with modelling content, the innovation of their training should be based on developing and enhancing teachers' mathematical modelling competencies and competency (Greefrath & Vorhölter, 2016). Borromeo Ferri and Blum (2010) provided five distinct categories of teacher modelling competency. 1. Theory-oriented competency (includes knowledge about modelling processes, objectives, and modelling tasks), 2. Task-related competency (includes abilities to approach modelling problems and create own modelling problems), 3. Teaching competent (contains abilities of a teacher being able to plan modelling lessons and how he or she will scaffold learners to work independently during modelling processes), 4. (includes abilities to identify modelling process that learners are working at and be able to identify difficulties during those modelling processes, and provide strategic intervention for them), 5. Assessment competency (includes abilities to create appropriate modelling tasks that can be useful in assessing learners modelling competencies).

Based on the relevant literature reviewed, some of the questions remained unanswered, such as, do courses offered at higher institutions to the PSTs also show this modelling content? How teachers' modelling competency and competencies can be developed and enhanced? Because most mathematics teachers are lacking the skills of implementing modelling in their classrooms, they often assume that there are already obstacles that can prevent them from successfully implementing and integrating modelling in their teachings. Such as beliefs and attitudes need to be investigated and understood so that they are developed and enhanced positively, and where possible, also university courses (for example, LEMA) may be implemented in PSTs training programs to assist teachers in developing and enhancing their beliefs and SEF about the teaching and studying of modelling in the curriculum.

Relevant literature about teachers' modelling competency is detailed above. The following sections provide the theoretical background of a structural model for teaching modelling that was used in carrying out the study. Which included teachers' beliefs, SEF, and prior knowledge about modelling.

2.3. Mathematics teacher professional competence

The professional competence (PC) of a teacher has become important in the research discipline of mathematics education. In the classroom during a lesson, teachers make a conducive learning environment and provide opportunities for learners to explore knowledge and construct it and have a vital effect on subject-related (mathematics) and multidisciplinary accomplishment of learners' educational objectives (Malangtupthong, Nurittamont & Phayaphrom, 2022).

2.3.1. Professional competence

The PC of a teacher in any subject is understood within the notion of competence (Ibragimov, 2021). In general, being competent is described as someone who can master significant concepts in a particular field and be able to make use of the acquired knowledge appropriately and accurately. In this study, competence was defined as intellectual ability, personal beliefs about the subject, content-specific knowledge, and motivational orientations.

In general, the PC of a teacher in his or her occupation is to be understood by the learners in the classroom (Ibragimov, 2021; Weinert, 2001). PC is a concept used to elucidate the knowledge and skills of teachers required to link up their professional necessity in their teaching field (Geiger et al., 2022). Shulman (1986, 1987) developed and differentiated groups of professional knowledge as parts of professional competence, the categories include curricular knowledge, content knowledge, pedagogical content knowledge (PCK), and pedagogical knowledge (Wess et al., 2021). Moreover, among teachers, particularly in-service teachers, those with a high level of PCK use mathematical activities with a high capability for cognitive triggering and they present good scaffolding for their learners during the learning process (Kunter & Baumert, 2013; Krauss et al., 2008). Teachers’ positions, however, are allocated to specific constructs, beliefs, attitudes, and values (Kunter & Baumert, 2013). Teachers serve the community in their respective fields of education through their acquired knowledge, skills, and activities, and they have a significant influence on the learners they are teaching. Thus, teaching according to the above-mentioned features is described as a profession, and PC is considered as a combination of specific declarative knowledge, beliefs, professional values, and objectives, as well as motivational orientations: SEF and professional self-evaluation skills (Kunter & Baumart, 2013) (as cited in Wess et al., 2021).

2.3.2. A structural model of professional competence for teaching modelling

Wess et al. (2021) designed and experimentally tested a structural model (see Figure 2.2) that demonstrates the relationship between teachers' professional competency for teaching mathematical modelling. Some of the constructs in the structural model developed by Wess et al. (2021) were used as the theoretical framework that provides the path of the current research and grounds it firmly in theoretical constructs. The constructs that were adopted are teachers’ beliefs, self-efficacy, and prior knowledge about mathematical modelling. These constructs were directly linked with the 7-step modelling cycle (Blum & Leiss, 2007).

To consider the constructed structural model, the COACTIV (Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers) model was evaluated (See Kunter & Baumert, 2013). The prerequisites that teachers should possess should be identified for teachers to prepare and implement mathematical modelling methods. The understanding of the COACTIV model for the modelling world can benefit from this as well. Some of the specifications that are competence dimensions for teaching mathematical modelling are shown in Table 2.2. (Borromeo Ferri & Blum, 2010). However, the study was not focusing on the proficiency features and PC for teaching modelling. The structural model was designed to understand PSTs' modelling competency for teaching mathematical modelling during their teacher training, thus, as stated that the structural model was adopted in this study as a theoretical framework that guided the study methods and explains why the research problem under the study exists. In addition, I was not focusing on all the facets of the structural model, but I was focusing only on the aspects of teachers' professional competence, which are beliefs, motivational orientations: SEF, and prior knowledge in modelling.

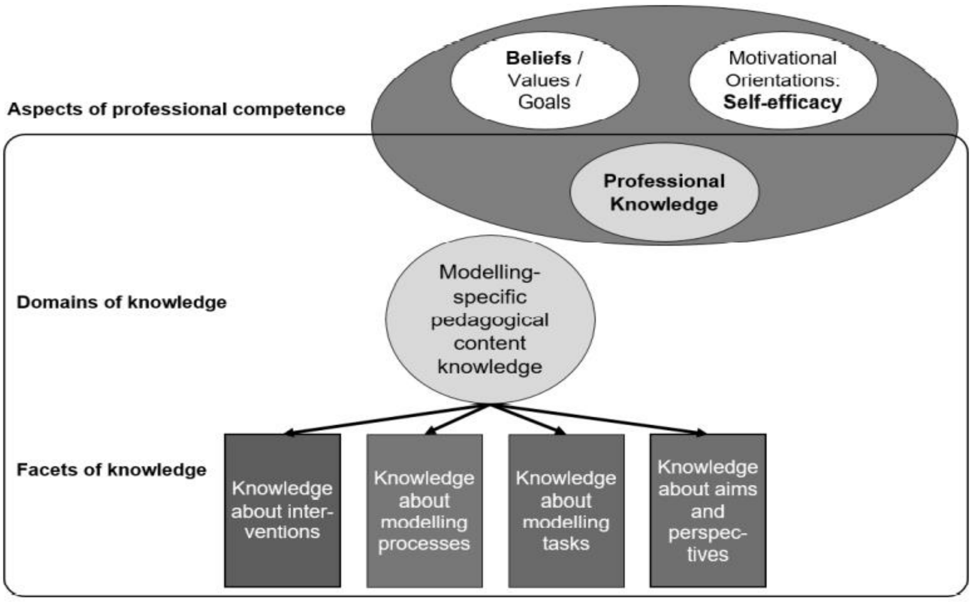

Figure 2.2 A structural model of professional competence for teaching mathematical modelling (adopted with permission from Wess et al., 2021, p. 29)

The constructs in above Figure 2.2 were designed and examined for experimental testing using a structural model for teaching mathematical modelling (Klock et al., 2019; Wess et al., 2021). The three components[8] from the structural model were constructed for the competence dimensions (see Table 2.2) for teaching mathematical modelling (Borromeo Ferri, 2018), and it further expands on the knowledge about aims, perspectives, and concepts. The study limited the constructs from the authors' study's structural model only to the aspects of professional competence (beliefs and motivational orientation: SEF). This is due to the time constraints of the current study and other factors such as heavy data analysis.

[8] Aspects of professional competence, domains of knowledge, and facet of knowledge

Table 2.2 Competence Dimensions for teaching modelling (Borromeo Ferri, 2018)

Dimension	Facets
Theoretical dimension	Modelling cycles Aims/Perspectives of mathematical modelling Criteria/Types of modelling tasks
Task-related dimension	Processing of modelling tasks Cognitive analysis of modelling tasks Development of modelling tasks
Teaching-related dimension	Planning of reality-related mathematics lessons Conduct reality-related mathematics lessons Interventions during modelling processes
Diagnostic dimension	Identifying phases in the processing process Identifying difficulties in the processing process Evaluation of modelling tasks

Theoretical, task-related, teaching-related, and diagnostic competence aspects were used in the interpretation of the COACTIV model, specifically for professional knowledge, as was explained by Borromeo Ferri (2018). A structural model of professional competency for teaching modelling was developed using the COACTIV model. As a result, several characteristics and skill set of teachers appear to be particularly pertinent in the context of mathematical modelling. Aspects of beliefs can be viewed as components of values, goals, and SEF expectations using modelling-specific factors (Wess et al., 2021). The aspect of self-evaluation, on the other hand, refers to a teacher's personality traits and it is thus unaffected by any specific concretisations. I chose beliefs and SEF because I was interested in investigating ISTs' modelling competence for teaching mathematical modelling. It is believed that learners' performance in modelling is directly linked to their teachers' knowledge, teaching methods, beliefs, and SEF expectations about modelling (Kertil & Gurel, 2016; Wess et al., 2021). Hence, I as a researcher hypothesised that learners' performance in modelling may increase after we have investigated and built teachers' beliefs and SEF in modelling using the model for teaching modelling. It should be cleared that the study did not test a hypothesis because I didn’t anticipate that I will get a sizeable number of teachers since this topic of modelling is still new in the South African education mathematics curriculum.

It has been acknowledged in the modelling literature (see Blum, 2015; Wess et al., 2021) that teaching and mastering mathematical modelling is cognitively demanding. As a result, to teach modelling, teachers must possess a wide range of knowledge and abilities, including both mathematical and non-mathematical knowledge, tasks and pedagogical strategies for teaching modelling, and appropriate attitudes and beliefs regarding modelling in mathematics education.

Without the teacher's SEF and positive views, the other objectives will be challenging to accomplish.

Review of relevant literature

2.4. In-service teachers’ beliefs about mathematical modelling.

It is recognised that for more than 20 years, German universities have researched and taught mathematical modelling. Views, beliefs, perspectives, concepts, subjective theories, worldviews, and attitudes are frequently used synonymously in their literature (Voss et al., 2013), with no clear distinction being mentioned. The word "belief" is often used interchangeably with words like "attitude," "behaviour," "opinion," "perception," "philosophy," and "values" (Leder & Forgasz, 2002; Phillip, 2007). There is no harmony in the definition of belief in the current literature (Kim, Kim, Lee, Spector & DeMeester, 2013), but there is a substantial consensus that beliefs are subjective, meaning they are not easily changed compared to attitude (Lomas, Grootenboer & Attard, 2012; Maasepp & Bobis, 2015). In this study, beliefs referred to individual personal views on a specific practice. For the interest of the study, they referred to beliefs held by the ISTs about the branch of mathematics in general and the teaching and learning of modelling in mathematics. Teachers' beliefs in their profession, particularly in their teaching subject are very important as it drives and constitutes their thoughts, opinions, and perspectives about the subject, methods of teaching, and tasks that are done by the learners in the classroom (Wess et al., 2021).

From the study by Ramirez (2017), a variety of teachers' mathematical modelling beliefs were revealed. Some teachers believed that modelling has to do with real problems, while others believed that modelling is one of the models that are used to depict mathematics abstract concepts. Three questions from the discussion of their results (see Ramirez, 2017) remained unanswered. One of the questions based on the currently conducted study was that “In which ways can the utility beliefs of a model influence the implementation of mathematical modelling?” (Ramirez, 2017, p. 978). This question was somewhat addressed in this study by revealing teachers’ beliefs and perceptions about modelling in the curriculum, which thus influences the implementation of modelling in the classroom. The author valued that teachers' beliefs are central to the development of learners modelling sub-competencies. The belief structure of mathematical models and modelling as viewed by teachers in the study by Ärlebäck (2010) suggested to be studied as partially made up of the teacher's beliefs about the actual world, the nature of mathematics, classroom mathematics, and applying and applications of mathematics. Some aspects of their suggested framework were explored using two case study interviews. It was found that two teachers did not have any well-formed beliefs about mathematical models and modelling and that the interpreted beliefs structure of the teachers contain inconsistencies. Ärlebäck (2010) further supports the notion that higher awareness about one's beliefs leads to a higher degree of integration of the beliefs in one's practice.

Ärlebäck (2010) hypothesised that the learners' views and capacities for modelling are significantly influenced by the ISTs' beliefs on discussions and engagements of the learners in the classroom throughout a lesson and in particular classroom activities. Professional knowledge, SEF, and beliefs can be seen as a crucial starting point for the education and training of mathematics instructors (Voss, Kleickmann, Kunter, & Hachfeld, 2013a; Zólyomi, 2022). Handal (2003) raised that teachers' beliefs about mathematics can be linked to what is mathematics, how teaching and studying mathematics take place in the classroom, and how the teaching and learning of mathematics should happen within the CAPS mathematics curriculum plan.

Beliefs in learning can be categorised as constructivist and transmissive beliefs. These categories of beliefs contribute to the operationalisation of mathematical modelling beliefs among teachers (Voss et al., 2013). As a result, teachers' beliefs about modelling in this study were operationalised and investigated within two scales, which are constructivist and transmissive. The constructivist-related theory was further divided into components, namely, modelling in the classroom, application of modelling, and constructivist beliefs.

2.4.1. Constructivist and transmissive beliefs

Voss et al. (2013) speak about the beliefs about learning, which are transmissive and constructivist, both contributed to the operationalisation of teachers' beliefs about mathematical modelling. The constructivist belief scale comprises three (3) theoretical sub-scales, which are referred to as components in this study. The theoretical components are constructivism, mathematical modelling in the classroom, and the application of mathematical modelling. See Table 3.2 which shows two scales of beliefs (including the three theoretical components) about mathematical modelling with the examples (from test instruments, see Appendix A) for each component.

Constructivist belief encourages learners to be competent and independent in mathematical modelling. Meaning, they should be able to discover their ways of solving modelling problems with insight and independently, thereafter discuss their notions with their friends in the classroom (Wess et al., 2021). The authors further explained that belief in a transmissive view is used to understand learning through the transmission of knowledge from teachers to learners. Means, in this belief teachers, become the leading ones during modelling activities given to the learners. In this orientation-related view, teachers provide step-by-step procedures with methods on how to approach certain modelling tasks. From this view of understanding, a one-sided process of knowledge transfer by the teacher to the learners is overcome. Hence, it is associated with negative modelling beliefs. These different types of perspectives have been entailed because constructivist perspectives are associated with positive modelling beliefs, whereas transmissive beliefs are associated with negative modelling beliefs (Kuntze & Zöttl, 2008; Schwarz, Wissmach & Kaiser, 2008). Both forms of beliefs, constructivist and transmissive, cannot be viewed as fundamentally opposite beliefs. Conversely, they are negatively associated with relatively close conceptions, as Voss et al. (2013) had empirically proven.

2.4.2. Constructivist learning theory.

Change necessitates the thorough evaluation of one's ideas and other perceptions (Patrick & Pintrich 2001). This means, for the South African education system to change how teachers and learners perceive modelling, it should start by changing teachers' beliefs and perspectives about the modelling content in the CAPS curriculum. That will enable them to develop a PC for teaching modelling. Thus, investigating SEF expectations modelling and beliefs of mathematics teachers was needed in the SA CAPS curriculum to understand the level of content understanding of teachers in modelling. Therefore, encouraging them to change their personal beliefs about modelling is very significant, and that will be achieved using the operationalised structural model of competence for teaching modelling. Kaiser (2006) makes a point that teachers modelling competency and their beliefs about mathematics must be considered as a significant factor of the low performance in mathematics in general and mathematical modelling. Teachers' beliefs towards mathematics and its application might also be a factor in learners' lack of realisation of the application and modelling in mathematics lessons. Having these determinants in mind, the study aimed to investigate teachers' beliefs about mathematical modelling through a structured questionnaire about teacher's backgrounds including their perceptions about the teaching and learning of modelling in the CAPS curriculum.

For the SA education system, it is fundamental to gain knowledge and background of teachers' beliefs about their teaching subjects, in this case, it is mathematics, including modelling. That will allow South African researchers to adopt some of the models, tools, or instruments from other countries, such as Germany, that have adequate experience in research in the field of mathematical modelling.

2.5. In-service teachers' self-efficacy about mathematical modelling.

Expectations of self-efficacy (SEF) are a practically supported component of professional competence (Kunter, 2013). In general, the idea of SEF is comprehended as an evaluation of one's abilities in certain events (Masitoh & Fitriyani, 2018) and how competent one is in a certain field. SEF is therefore suitable in my study for investigating the perspective of teachers' abilities for teaching mathematical modelling, specifically the competency of identifying learners modelling abilities during modelling processes. I used SEF in the study as a teacher's judgment about their abilities to produce the desired outcomes of learner involvement and their modelling abilities during a modelling cycle. SEF expectations are significant for teaching, learning, motivation, and beliefs to the learners in a modelling classroom.

Teachers' PC is made up of cognitive professional knowledge, beliefs, values, and motivational orientation: SEF components (Wess et al., 2021). Looking at the self-efficacy component, a teacher's efficacy is an evaluation and criticism of their capabilities to bring about desired learners' performance in their learning. Even among learners who are struggling academically or lack interest in learning mathematics (Tschannen-Moran & Woolfolk-Hoy, 2001). They have an impact on learners' performance, beliefs, and motivation and are important for teaching and learning. The latter goes hand in hand with excellent instruction, helpful instructional strategies, and subject-matter experts as teachers in the classroom (Kunter, 2013). To teach and improve student performance, beliefs, and motives, teachers' self-evaluation in modelling is important. SEF can be described and concretised about teachers' ideas about their capabilities during modelling processes and prior experiences in a certain field (Wess et al., 2021).

SEF is considered an important aspect of teacher professional competence (Kunter, 2013). Therefore, it was relevant to investigate the SEF expectations of mathematics teachers in modelling. In most cases, teachers' pedagogical strategies that they use in teaching modelling are routed from their knowledge, SEF, and beliefs about the modelling content (Wess et al., 2021). In South Africa, it will be the modelling content in the CAPS curriculum. I believe the SEF of a teacher in modelling may determine the level of competence of his or her learners in the classroom at any modelling phase. This means the content of the modelling tasks used in the classroom is determined by the dimensions of the modelling-specific pedagogical content knowledge used by the teacher in the classroom (Wess et al., 2021). Different tasks and cognitive processes determine learners' modelling process in different cycles. Analysis modelling stages are significant for different modelling phases in which the learners work. This supports the notion that teachers' SEF varies depending on the modelling phases they are at and their beliefs about modelling. With regards to modelling tasks of learners and the related systematic, it is possible to recognise unspecified modelling phases of the learners during modelling processes. Those phases can be traced by their written mathematical work and modelling abilities. Modelling phases that are specific in the modelling cycle include simplification of the built mathematical model, mathematisation, interpreting, validation, and exposing. The SEF expectations for mathematical modelling are thus operationalised between two components, which are 'mathematical modelling' and 'working mathematically'.

The study was guided by framework studies outside South Africa, which have over twenty years of research, teaching, and learning experience in mathematical modelling. The findings from this study have benefits for SA mathematics education and international, particularly in the modelling content.

2.6. In-service teachers' prior knowledge of modelling content.

It is significant to consider if the participating teachers have been exposed to mathematical modelling before, from their basic education to their university teacher training and further to the curriculum after graduating. This was considered as teachers' prior knowledge in the study. Some of the teachers might have been exposed to mathematical modelling but not been sure or aware of it from their modules and curriculum. For example, in the Grade 12 mathematics curriculum, there is no specific topic that is directly stated as 'modelling', instead they refer to it as 'optimisation'[9] under the calculus section. From Grades 7 – 11, they usually refer to them as word-problems[10]. In the study conducted by Mudaly and Dowlath (2016) in South Africa, Kwazulu-Natal, the authors concluded that the PSTs had little awareness of what mathematical modelling is, some even admitted that they are not aware of modelling from the curriculum, or they have not heard about modelling from their lectures. But when the authors analysed participants' responses, they noticed that they are using modelling methods to approach mathematics problems without being aware of their approaches. From the context of this study, teachers might solve a variety of word problems using modelling processes without being aware they are using them.

This hinders in-service mathematics teachers (ISMTs) from developing a positive attitude towards modelling to the learners since they are not aware they're modelling word problems. Furthermore, learners might lose interest in it since they won't be aware they're modelling the problems. Hence, this study also revealed teachers' awareness of modelling in the CAPS curriculum.

From the study conducted by Frejd (2012) in Sweden, 88% of the participating mathematics teachers knew mathematical modelling was used to '*simplify or describe'* things using

[9] Optimization in calculus is when you are given a practical problem and asked to find the minimum or maximum, depending on the context of the question.
[10] Word problems are application problems or traditional word problems that are usually found in learners' textbooks. They usually adapt procedural knowledge in relation to what is currently taught under a specific mathematics topic (Bliss & Libertini, 2016).

mathematical tools. From teachers' conceptions, things were referred to as events, situations, reality, objects, and connections. This showed that teachers do know what mathematical modelling is, but looking at the reviewed literature, their limitations may be arising from the little content of modelling across the curriculums. Other teachers from the authors' study argued that modelling is mostly used in other subjects such as science than used in mathematics, their notions were also discovered by (Gottfridsson, Jonasson & Lindfors, 2003). Looking at the SA CAPS curriculum, this is true because most of the mathematics content requires procedural knowledge with formulas to solve them.

The questions are not contextualised with the learner's context, and most of the given problems do not require learners to create models or formulas to solve them. From my mathematics learning experiences, in most cases, teachers were emphasising that "you do not have to know this formula by heart, it will be given to you in an exam". Those kinds of beliefs can promote procedural understanding[11] to the learners instead of conceptual understanding[12].

For the study, four components were used to measure teachers' self-reported prior experiences for teaching mathematical modelling. components used were adopted from Wess et al. (2021). Components that were used to measure teachers' previous experiences with modelling are "General role played by mathematical modelling from the ISTs training courses", "Preparedness of ISTs for teaching mathematical modelling", "Nature of modelling tasks teachers have encountered during their educational journey" and "Teachers' experiences about mathematical modelling in various contexts". The focal point is not only on basic education and ISTs context but also on extracurricular activities, such as experience in educational support (tutoring), which are recorded by the component.

2.7. In-service teachers' perceptions about mathematical modelling.

Mathematical modelling has been an important topic of discussion in mathematics education across the world. As learners' modelling competency is considered a central skill for our future mathematics education in our societies. Even though mathematical modelling is vital in mathematics education, it turned out to be having very limited research studies relating to ISTs' perceptions about the teaching and learning of mathematical modelling. From this view and

[11] Procedural knowledge (also known as how to?) is knowledge associated with how to perform a certain task or specific skill, mostly linked with procedures, methods, and facts.
[12] Conceptual knowledge (declarative knowledge) is an explicit or implicit understanding of the knowledge and principles that hegemonies a domain and of the connection between units of knowledge in a certain domain.

the relevant literature reviewed, this study aimed to understand in-service mathematics teachers' perceptions of mathematical modelling particularly in the mathematics FET Grade 10 – 12 CAPS curriculum.

Teachers' perceptions towards mathematical modelling seem to be arising on how they were exposed to modelling previously, how they define mathematical modelling, and the character of modelling in their daily-life experiences (Bonyah & Clark, 2022). Teachers’ mathematical modelling perceptions can be described as teachers’ beliefs and awareness of it regarding their content knowledge of mathematical modelling and their competency in teaching modelling (Bonyah & Clark, 2022). The authors further say most mathematics teachers in South Africa showed insufficient knowledge and pedagogical strategies in approaching modelling in their classrooms. Thus, their integration of modelling with other mathematics topics is not as good as other non-South African mathematics teachers, such as German universities. Therefore, the facet investigated may lead them to understand professional competence for teaching modelling and building a positive attitude towards their learners in the classroom.

In the study by Şandır (2010) teachers mentioned that they use modelling at the commencement of the lesson for drawing learners' attention to the lesson, motivate them towards the new concept to be learned, and allow them to visualise the created concept image. They further mentioned that they use modelling to avoid unforeseen misconceptions that may occur during a lesson. This is a good indication of the use of modelling in mathematics because it shows that teachers apply modelling possibly across all the mathematics topics to ensure that they are attracted to mastering mathematics, and they see the value of learning the subject. In this study, teachers’ perceptions about modelling were sought for getting their ideas on how and why the modelling content should be integrated across the whole mathematics curriculum. However, the study conducted by Deniz Yilmaz and Akgun (2013) revealed that teachers consider modelling to be suitable for the topics of algebra and geometry. A further research study conducted by Özdemir, Üzelm, and Özsoy (2017) concluded that teachers suggest that modelling is good when integrated with measurements, statistics, numbers, and probability. While results from this study revealed that teachers prefer to integrate it with finance and statistics. This can be viewed as contradictory between the authors (Deniz Yilmaz et al., 2013; Özdemir, Üzelm & Özsoy, 2017). But still, it indicates that modelling content can be used across most mathematics topics, and possibly to other subjects for learners to visualise the concepts and for teaching and learning reinforcement.

The study conducted by Aydın (2008) outside SA revealed that most mathematics teachers are unable to conduct their lessons in a manner of speaking that what learners learn in the classroom will apply to their daily lives experiences, and it was indicated that this obstacle arises from the way the education system is organised. The root of these obstacles was arising from the notion that the mathematics education curriculum was not given much time and the modelling concepts were too abstract for learners to comprehend (Blum, 1991). Hence, the study looked at teachers' perceptions of modelling content in the CAPS curriculum.

2.7.1. Factors affecting teachers' perceptions and implications of modelling.

Teachers' beliefs, SEF, and prior knowledge determined their perceptions and recommendations about mathematical modelling in the curriculum. Meaning some factors contributes to teachers' perceptions of mathematical modelling and its application. Therefore, in this study, beliefs, SEF, and prior knowledge are the factors that were investigated to determine teachers modelling competency and their perceptions about teaching real-life context in the classroom.

In mathematics education, there are serious discussions on what are teachers' beliefs and attitudes suitable for impressive teaching of mathematical modelling among learners. Chamberlin et al. (2022) asserted that teachers' approaches toward modelling should change for learners to develop modelling competencies. To ensure that the progress of modelling competencies is accelerated within a reasonable amount of time, teachers are expected to give indirect scaffolding of strategic help to the learners. Doerr (2007) points out that at some point teacher's role must change within their teaching style, that is to improvise[13] a lesson. This means they must have a profound and rational understanding of the distinct approaches learners may use and bring during their modelling processes. Thus, the teacher's duty in modelling class is to ensure that modelling sub-competencies among learners are enabled and enhanced through the suggested strategic intervention by (Wess et al., 2021).

Leiss (2007) developed a model for teacher intervention, the author's intervention model differentiates between three features of teacher intervention, which are primary knowledge, area, and the characteristics of an intervention with the teacher. The types of the central feature of intervention, which is an area of intervention are Organisational interventions, Affective interventions, Content-related interventions, and Strategic interventions. The latter is

[13] Improvising a lesson is to divert from what the educator initially planned his or her lesson and use a different pedagogical strategy without preparations.

considered to have high possibilities to help learners to overcome challenges during modelling processes (Stender & Kaiser, 2015). The current study did not investigate the strategic interventions among the teachers due to time limitations.

2.8. How modelling is done in other contexts.

The study conducted by Hankeln (2020) showed the comparison between German and France secondary schools on how modelling is done (taught and learned). In the study, the author was investigating how learners in Germany and France handle modelling processes. The author mentioned that they use the traditional way of teaching modelling when they teach mathematical concepts through contextualised problems. Although these teaching methods are not fixed, they're sometimes reshaped and improved, and they always influence how certain mathematics concepts should be tackled and perceived. Therefore, the German conceptualisation of problems was more relevant to the currently conducted study.

2.9. South African Grade 10 – 12 Mathematics CAPS Curriculum

Looking at the current CAPS curriculum (see DBE, 2011), it might be that modelling is not as dominant as it should be across the curriculum (so that mathematics teachers are aware of it) and integrate it across the subjects. From my personal experience, I agree that some of the universities (such as Wits University[14]) in SA do offer modelling courses to undergraduates. Though from what I have observed they are offered from the third year of schooling and not from their first year of schooling. In addition, some of the ISTs might have done mathematical modelling courses before depending on which university they enrolled to. But that does not mean their modelling competencies and competency were well developed and enhanced.

They might have been exposed to teacher-based and lecture-based methods of instruction. That might have limited them (teachers) in the exposure to creating models on their own and solving them. It is expected that mathematics teachers should see a curriculum that values development, conception, and modelling competencies (Haines & Crouch, 2001). Some of the general aims of the CAPS curriculum do show these characteristics, as one of them encourages learners to

[14] The University of the Witwatersrand is one of the top institutions in Africa. The researcher has attended his undergraduate (and postgraduate) studies at the university. Hence, he is aware of how the modelling content is structured and taught at the education school.

independently recognise and solve problems and make their solution conclusion based on their creative and critical thinking (DBE, 2011).

Bonyah and Clark (2022) argue that teachers should allow learners to construct their knowledge during the learning process and they should refrain from teaching them for examination purposes only. My concern as a researcher is that looking at the South African CAPS curriculum time frames, do we have enough time to deeply teach learners and let them create their knowledge during the learning of modelling? Does the curriculum provide knowledge and skills to mathematics teachers on how they can enhance learners' knowledge construction? According to Bonyah and Clark (2022), SA university mathematics courses need modelling-based pedagogical amendments. The author believed that modelling should be the initial phase of the introduction of an abstract concept in mathematics. Looking at the results obtained, I do agree with the authors since developing modelling competency for the learners can only occur when teachers' modelling competency is well developed. Hence, developing modelling competency for the pre-service mathematics teachers (PSMTs) can minimise workload. By the time they are qualified and teaching mathematics, including modelling, they will know how to promote a positive attitude towards modelling to the learners.

The teaching practices that are taking place in mathematics classrooms are influenced by how the ISTs were taught from basic education to higher education training. As Malambo (2021) also support this notion that learners' knowledge of mathematical concepts influences how they will teach the subject in the future. Pehkonen et al. (2013) suggested that the pedagogical strategies used in the classrooms should be based on constructivist learning theory and it is believed that this teaching approach has a greater chance of improving learners' modelling skills. However, the ongoing research studies provide ideas and recommendations on how to instruct modelling productively and overcome difficulties.

2.10. Chapter summary and conclusion

Mathematical Modelling is a fundamental aspect of mathematics in the curriculum in many countries. The literature reviewed showed that modelling (in some countries) is considered from primary education to secondary schooling, and in some countries, they also consider it at the university level to the PSTs. In SA, it is claimed that modelling is considered by the CAPS curriculum. While other studies (see Bonyah & Clark, 2022; Mudaly, 2004) revealed that teachers are unaware of their mathematical modelling competency (Beliefs and SEF) in the classroom.

The subsequent chapter 3 provides details on how the study was organised and the research designs, methods, and methodology used in realising the purpose of the research study.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

In this chapter, the research design of the study was discussed. I present the specific methods that I used in collecting, processing, analysing, and interpreting the collected data of the study to enable me to respond to the four developed research questions (see section 1.6). The specific methods used, including test instruments (questionnaire) and interviews, in the collection of data are explained in detail (see sections 3.6 & 3.7).

Firstly, I briefly presented the theoretical background comprising of research methodology and research design. Secondly, I describe the specific methods that I used to gather data. I organise the presentation of methods following the four themes that I presented in Chapter 2. The structure of presentations used to give coherence in my presentation is that each theme links directly to the research question.

3.2. Research design and approach

Research methodology and research design may be viewed as the same, but they differ in producing a research paper. Research methodology is the overall philosophy that guides the entire research paper process, research methods (Crowe & Sheppard, 2010), on the other hand, are the practical instructions or approaches that are utilised to realise the purpose of the research study, for collecting and analysing data, sampling, and procedures purposes.

3.2.1. Research approach

Table 3.1 below entails the summary of the research design and approach that was used to realize the purpose of the study and to answer the formulated research questions. It was then followed by the representation of a complete explanation of the procedure and methods used during the research study process to address the purpose of the study.

Table 3.1: Research approach and design summary (adapted from Wipulanusat et al., 2020)

Sequential explanatory research design		
Research approach	**Quantitative research**	**Qualitative research**
Research design	Descriptive design (observational study)	Explanatory design
Research purpose	To investigate in-service teachers' beliefs, self-efficacy, and prior knowledge about mathematical modelling in the CAPS curriculum using a structural model of professional competence for teaching modelling.	To understand teachers' perceptions about teaching and learning mathematical modelling in Grade 10 – 12 CAPS curriculum.
Research questions	1. What are in-service teachers' beliefs about teaching and learning mathematical modelling in the CAPS curriculum? 2. What are the in-service teachers' self-efficacy about teaching and learning mathematical modelling in the CAPS curriculum? 3. What are in-service teachers' prior knowledge of mathematical modelling in the CAPS curriculum?	4. What are in-service teachers' perceptions about the modelling content in the Grade 10 – 12 CAPS curriculum?
Objectives of the research questions	• To investigate teachers' beliefs and self-efficacy expectations about mathematical modelling.	• To understand teachers' perceptions about mathematical modelling content in Grade 10 – 12 CAPS curriculum.
Sampling procedures	A purposive sampling method was used to select the participants ($N = 40$) from the chosen schools.	3 teachers from the 40 participants were purposively selected by the researcher for the interview.
Participants	Grade 10 – 12 Mathematics teachers	
Methods of data collection	Questionnaire (55 test instruments)	Semi-structured Interview (16 questions)
Results analysis	The collected data, through a questionnaire, was analysed through statistical analysis data software (IBM SPSS).	Teachers' responses from the Interview were critically analysed by the researcher with several steps (see Table 3.3) to the questionnaire.

3.2.2. Research design

Research design reveals the fundamental methodological drive of the study and is the distinctive and study technique that is most satisfactory to respond to the predetermined research questions. Sileyew (2019). The choice of the research design is therefore influenced by the study's research problem, research questions, purpose, goals, and objectives (Sileyew, 2019).

The goal of the research design, according to Bakker (2018), is to increase the researcher's control over the research study and its reliability, validity, and trustworthiness when addressing the research topic. In deciding which research design to use, the researcher had to consider several factors, including the focus of the research study, the research problem, the context of the study, instruments used for data collection, methods of data analysis, sampling methods and time constraints. Thus, a mixed method design (sequential explanatory research design) was used to collect and analyse quantitative (a structured questionnaire) data in the first phase of the research followed by the collection and analysis of qualitative (interviews) data in the second phase that builds on the results of the initial collected quantitative results (Creswell & Creswell, 2017; Creswell, 2012).

The brief theoretical framework that was used in the study was based on a structural model of professional competence for teaching mathematical modelling developed by (Wess et al., 2021), which adopted beliefs, SEF, and prior knowledge about mathematical modelling. The model is associated with test instruments designed to provide a detailed review of PSTs' modelling competency for teaching mathematical modelling. Hence, it was adopted to provide a detailed review of ISTs' modelling competency for teaching mathematical modelling.

The focus of the study was investigating teachers' beliefs, SEF, and prior knowledge about mathematical modelling using some of the tools from the author's structural model (see Figure 2.3) of PC for teaching modelling. A descriptive design with test instruments was used in this study to measure ISTs modelling competency in teaching modelling content, and the interview questions were used to measure teachers' perceptions about teaching modelling content in the curriculum. No treatment was given to any of the participants since the teachers are qualified and already teaching the mathematics subject including modelling for quite some time. Hence, it was an observational and explanatory study. The study attempted to reveal and alleviate the knowledge gap related to teachers' modelling competence.

3.3. Context of the study

The current study was conducted in eleven (11) suburb Johannesburg secondary schools from Gauteng province, with forty (40) Grade 10 – 12 mathematics teachers. The schools are in the Johannesburg North and Central region of Gauteng Province of South Africa. All 11 schools that form part of the study were under the government of Johannesburg North and Central Districts, and the prominent languages were mixed, including English, isiZulu, Sepedi, and Sesotho[15].

3.4. Structure of the study

The study was separated into two phases. The two phases occurred concurrently. Phase one was concerned with a quantitative research design approach, which used a structured questionnaire to gather data. The second phase was concerned with the qualitative research design approach, which used interviews to collect data. In the following sections, I outlined in detail how the qualitative and quantitative designs were used. Then followed by an explanation of why mixed methods research design was an important choice for the research design of the study.

3.5. Sampling procedures and methods

Participant selection in a research study plays a significant role in ensuring a comprehensive understanding of a phenomenon being investigated and to realise the purpose of the study in connection to the topic and research problem. It is the researcher's responsibility to select specified participants who know what is being investigated (Creswell & Creswell, 2012; Sargeant, 2012). In this study, participants with knowledge and experience in teaching and learning mathematical modelling were considered and purposively selected. The following section details the selection procedures of GDE schools and participating teachers.

3.5.1. Targeted participants

Participating teachers in this study were Grade 10 – 12 mathematics teachers ($N = 40$) from suburb public secondary schools ($N = 11$), and the Gauteng Department of Education (GDE) under Johannesburg North and Central District. There were no age or gender limits in the study. However, participants that opted to complete the questionnaire were aged between 23 to 62

[15] These are four of the eleven official languages that are spoken in South Africa.

years. All the participating schools selected were government public schools. The study used a purposive sampling method to approach forty Grade 10 – 12, mathematics teachers. The selected population was mathematics teachers who have been teaching mathematics, including mathematical modelling for quite some time. Experience in teaching mathematics was an average of five (5) years in service (see Table 4.1). Some of the participants confirmed that they have seen mathematical modelling (word problems or optimisation) before from their high school education, while others started doing it at the university level. From my knowledge, mathematical modelling is sometimes called word problems (from Grades 7 – 11) and then optimisation (in Grade 12). That was explained to some of the participants who requested clarity about the modelling concept to ensure that they are familiar with how the term mathematical modelling was used in the research study.

The purposive sampling method is the researcher's intentional choice of choosing participants for a particular study due to the characteristics that the participants demonstrate and are advantageous. It is a non-probabilistic method that does not need basic theories for selecting potential participants (Etikan, Musa, and Alkassim (2016),). Other authors (Cohen, Manion, and Morrison, 2007; Kumar, 2018) further add that the primary consideration in purposive sampling is the judgment of the investigator as to whom can supply the supreme information to the goal of the study. The researcher simply decides what needs to be realised about the purpose of the study and looks out for participants who can and are willing to share their knowledge and skills about what is being investigated in the study. All participating teachers completed the same test instruments about beliefs, SEF, and prior knowledge about mathematical modelling. Out of the 40 participating teachers, only three teachers were purposively selected for an interview to confirm the obtained quantitative results.

3.5.2. Selection of the Participants' Method

The method used to select the participating teachers was also a purposive sampling method. This method was used because the researcher needed mathematics teachers that show "particular characteristics being sought after, to make sure they could engage with the research questions" (Cohen et al., 2007, p. 115). The researcher in the study tried to select mathematics teachers that demonstrated knowledge and experience in teaching mathematics, including mathematical modelling in Grades 10 – 12 within suburban secondary schools. Suitable for the study, 40 mathematics teachers were purposively selected from eleven different secondary schools, and the method used to select the participants was not too complex, but a linear process. One of the grounds for having 40 mathematics teachers was the fact that each secondary school was having approximately four Grade 10 – 12, mathematics teachers.

3.5.3. Schools' selection method

In this study, I adopted the purposive and non-probability (convenience) sampling method to select the schools that took part in the study. The schools which were used to realise the aim of the study were selected purposively on the basis that was easily accessible by the researcher. Additionally, the eleven schools were close to each other and were selected based on the advantage to move between them whenever I needed to clarify some things in person as the researcher. The short distances between the schools helped in cases where one participant cancelled an interview session because of other responsibilities, thus I could easily go to another school where the teacher was available for an interview session. The schools were under the same curriculum followed in South Africa, which is the CAPS curriculum.

3.6. Data collection (Questionnaire)

Some of the test instruments that were used in the study were in a form of closed written form, with a range of a combination including multiple-choice using a 5 Likert scale with factors. This was used to gather in-service teachers' beliefs, SEF, and modelling prior knowledge. Teachers' perceptions about teaching and learning mathematical modelling were collected through semi-structured interview questions. Three participating in-service teachers were selected for the interview sessions.

The test instruments were conducted as a single test among the Grade 10 – 12 mathematics teachers at the secondary school level. There were fifty-five (55) test instruments (questionnaires) that were used to gather in-service teachers' beliefs, SEF, and prior knowledge about mathematical modelling. As a result, the study used the following items in each factor to answer the formulated research questions about teachers' competency in mathematical modelling. 16 items were used to measure ISTs' beliefs, 15 items were used to measure ISTs self-reported prior experiences, 24 items were used to measure ISTs' SEF, and 16 interview questions were used to collect teachers' perceptions about mathematical modelling in the Grade 10 – 12 CAPS curriculum.

The questionnaire responses were collected through a Google form, downloaded, printed, and coded (see Appendix A) from version 1 to version 63 for Statistical Package for the Social

Sciences SPSS[16] software coding purposes. The duration of completing the questionnaire test instruments including instructions was approximately 20 minutes, and the duration of interview sessions varied for each participant depending on the responses they were providing. Since other participants gave responses that required follow-up questions about the research problem and topic, thus the average duration was approximately 45 minutes for each participant. Each participant in the study was provided with a book test (google form) by the researcher that had a questionnaire test instrument. Interview sessions were scheduled with the three Grade 10 – 12 mathematics teachers who contested to take part in it after the researcher had purposively selected them. It was made clear to the participating teachers that no support will be allowed during the completion of the questionnaire to ensure that the data collected demonstrate valid and authentic information about the individual ISTs modelling competency. In addition, the prohibition of support from completing the questionnaire was also to make certain that the obtained results are the true reflection of the chosen population of the study to represent the whole population, which also allows the generalisation of the study findings.

After consideration of the questions that were used in this study to investigate ISTs beliefs, SEF, and prior knowledge, and the interview questions that were used to understand teacher's perceptions about modelling, the test instruments and interviews questions were comprehensively given to participating mathematics teachers using a random sample of choosing from Johannesburg North and Central schools.

First test part of the study

The collection of data for the study began with the formation of a unique personal code for the anonymity task of the participants, for the test book (google form). The codes were created based on the participants' initial and first letters of their surnames. For example, for a teacher whose Irene Naidoo, the code would be (IN). Some of the brief questions on participants' (ISTs) general information included gender, age, sub-major[17] subject, and the number of years teaching mathematics or grade. This was followed by the test instruments (see Appendix A) for teachers’ beliefs (16 questions), SEF (24 questions), and prior experiences (15 questions) about mathematical modelling (Wess et al., 2021).

[16] SPSS (Statistical Package for the Social Sciences) is a statistical software used by different kinds of researchers for heavy statistical data analysis.
[17] Sub-major is another subject you are qualified to teach in addition to your main teaching subject.

The factors connected to the ISTs were measured using a Likert scale of five, with the expressions "strongly disagree" (ranked 1), "disagree" (ranked 2), "neither agree nor disagree" (ranked 3), "agree" (ranked 4), and "strongly agree" (ranked 5). The five-point Linkert scale was chosen to provide participants (ISTs) moderate level of expression and prevent them from being persuaded to take a position on the questions that would harm the study's attempt to collect data. Nonetheless, three participants were given the chance to express their opinions during the interview sessions in response to the last study question concerning how they felt about the teaching modelling in the CAPS curriculum for Grades 10 to 12. The Likert scale of five options to choose from also made it possible to use a verbal range to express one's self-degree of approval in a varied manner (Anjaria, 2022; Bühner, 2011). The following sub-sections provide details on how the questionnaire and interviews were conducted.

Questionnaire

The questionnaire consisted of 55 questions with the 5 Likert scale. For teachers' beliefs (16 questions), SEF (24 questions), and prior experiences of mathematical modelling (15 questions). Some of the questions were negatively worded (mostly from SEF) and were reversed scored [18]during the analysis of the data. The objective of these factors was to investigate teachers' modelling competency about learners' modelling processes.

3.6.1. In-service teachers' beliefs about teaching and learning of modelling.

This scale had 16 questions that were used to collect data from ISTs about their beliefs in teaching and learning mathematical modelling. Meaning, the participants shared their beliefs about modelling during their secondary schooling times and provided beliefs about teaching modelling in the classroom, to see if it has a certain value in the mathematics they teach. The belief scale was measured using four components. Which are modelling in the classroom, application of modelling, constructive belief, and transmissive belief.

From the test instruments (refer to Appendix A), questions 1, 5, 12, and 15 were used to test the aspect of 'modelling in the classroom beliefs'. questions 1, 3, 9, and 13 were used to test 'mathematical modelling in the classroom'. Questions 3, 8, 11, and 14 were specifically used to test 'constructivist beliefs' among the teachers. Lastly, questions 6, 7, 10, and 16 were

[18] Reverse scores occur when there is a negative formulated of statements as the opposite of positive statements and are used to show an opposing idea of what is being investigated.

specifically used to test the 'transmissive belief' among the teachers. Test instrument samples are shown in Table 3.2 below.

Table 3.2 Examples of the test instruments on beliefs about mathematical modelling (adopted with permission from Wess et al., 2021).

Components	**Test instrument example**
Beliefs about the application of mathematical modelling	"In mathematical modelling, you work on tasks that are more practical."
Beliefs about mathematical modelling in the classroom	"Learners should be given the opportunity to do mathematical modelling in the classroom."
Constructivist beliefs	"Learners learn mathematics best by discovering their own ways of solving problems."
Transmissive beliefs	"Teachers should provide detailed procedures to learners for solving application problems."

3.6.2. In-service teachers' self-efficacy about teaching and learning modelling.

Teachers modelling SEF was measured based on the notion of one's efficacy in recognising learner's performance abilities during modelling processes. 24 questions about ISTs SEF of mathematical modelling were used in this study. They required self-evaluation of one's (teacher) own efficacy and identifying the abilities of learners in the cycles of the modelling process. Teachers were not given learners mathematical modelling written work to identify their abilities, but they were generally asked about their abilities extent to identify learners' abilities from their written mathematical work.

The modelling cycles that were included in the test items included building a mathematical model from a real-world model, simplifying, mathematics, interpreting, validating, and exposing the results (Hankeln et al., 2019) since these are specific modelling sub-competencies with specific diagnostic requirements for the modelling process. It was acknowledged by the researcher that teachers might not know the mathematical modelling cycles before, but they were using other methods in approaching modelling questions. Hence, teachers during the completion of the questionnaire were allowed to ask for clarification of the modelling processes.

The test instruments from Wess et al. (2021) that were included in the SEF questionnaire required teachers to assess their effectiveness and recognise learners' skills in the phases during the modelling processes. To analyse the data, two one-dimensional components were used: 'SEF for mathematical modelling' and ‘SEF for working mathematically’ during modelling processes. Test instruments samples are shown in Table 4.10 (see sub-section 4.2.5). questions 17, 20, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, and 37 formed a scale of ‘SEF for mathematical modelling’. These are specific phases during mathematical modelling processes, they are comprised of phases such as simplifying, interpreting, and validating. Questions 21, 23, 24, 27, 28, 35, 36, and 40 formed a scale of ‘SEF for working mathematically’. These items were used to diagnose written mathematical modelling work by the learners.

3.6.3. In-service teachers’ prior knowledge of modelling content.

This component included 15 questions (refer to Appendix A) to measure teachers' prior knowledge about teaching and learning mathematical modelling. For realising teachers' prior knowledge about mathematical modelling from the questionnaire, the 15 questions were composed of four components, which are ‘General role played by mathematical modelling from the ISTs courses’ (41, 45 and 49), ‘Preparedness of ISTs for teaching mathematical modelling’ (42, 44, 46, 48 and 50), ‘Nature of modelling tasks teachers’ have encountered during their educational journey’, (47, 52 and 55) and ‘Teachers’ experiences about mathematical modelling in various contexts’ (43, 49 and 54).

3.7. Data collection (Interviews)

Interviews are still considered one of the most significant test instruments of the qualitative research method. When used appropriately by researchers in their research studies, often more details and better responses are obtained than any other method of data collection (e.g., questionnaire), since participants have room to share more knowledge and experience about the research problem. The reason for this advantage is that participants are more comfortable when talking than writing (Best & Kahn, 1998) since they get to share more of their perceptions about the research topic. It has been recommended (Kakilla, 2021) that interviews should use unstructured or semi-structured interviews so that interviewees can move freely with the discussion in any direction of their interest (both interviewer and interviewee). This method is particularly useful for investigating and understanding a topic under discussion broadly (Trochim, 2002).

3.7.1. Teachers’ perceptions about mathematical modelling content.

Teachers' perceptions and recommendations about modelling in the CAPS curriculum were collected in the form of interviews. Semi-structured interview questions were used, characterised by open-ended questions, but discussions around modelling were opened during the interview if teachers are willing to share more perceptions and recommendations beyond asked questions. As aforementioned, sixteen (16) interview questions were planned to collect data for the study, and then follow-up questions depending on the teacher's response were asked.

From the study, in addition to data collected through a questionnaire, phase 1, semi-structured interviews were conducted with three (3) teachers who contested to take part in the study. The objective of the interviews with teachers was to get more details about the teaching and learning of mathematical modelling in Grade 10 – 12 secondary mathematics education, and how it is structured from the CAPS curriculum.

During the interviews, teachers we requested to respond to the interview questions in an open-minded way to the following factors:

- Their familiarity and prior knowledge of mathematical modelling.
- Their pedagogical strategies and awareness of teaching modelling.
- Integration of mathematical modelling in Grade 10 – 12 CAPS mathematics topics.

In qualitative research design, the researcher collects and analyses the data of the study. This is because, in qualitative research design, the researcher is attempting to reveal participants understanding and perceptions about an existing natural phenomenon (Alharahsheh & Pius, 2020). In this study, the researcher used interviews as a qualitative research design to collect teachers' perceptions and recommendations about the teaching and learning of mathematical modelling in the CAPS mathematics curriculum. Semi-structured interview questions were used to collect data about teachers' perceptions, interviewee (teachers' participants) was asked to give their responses in an open-minded way of responding to ensure that the researcher gets as much information and understanding as he can about teachers' modelling competency. The open-ended interview questions (see Appendix B) used were guiding questions but also gave the participants room to respond freely. The participants and the researcher were both able to shift the discussions during the interview to their topic of interest about the asked questions.

This means the participants were able to respond broadly about mathematical modelling content with the CAPS curriculum. Furthermore, the researcher was able to ask a follow-up question after the participant has responded and there seem to be a burning topic around his or her response.

3.8. Organisation and analysis of data

Sheard (2018) indicated that the aim of analysing the findings of the research study is to lessen and consolidate the collected information and make conclusions about the population used. While the aim of interpreting the collected data is to merge the findings of the data analysis with standards, criteria, theories, and value statements to make conclusions, judgments, and recommendations for future research.

The results of the study were obtained through the predetermined research questions above, using a structured questionnaire (closed-ended) multiple choice with a 5 Likert scale and semi-structured interview questions. To see the relationship and link between teachers' beliefs, SEF, prior knowledge, and their perceptions about mathematical modelling from the responded questionnaire, the collected data were coded and analysed through a statistical analysis data software (IBM SPSS). The interview responses were critically analysed through qualitative data analysis (interview) steps (refer to Table 3.3, section 3.8.4) adopted by Kiger and Varpio (2020) concerning the results obtained from quantitative. As I was interested in investigating and understanding IST's professional competence for teaching mathematical modelling using a structural model developed by Wess et al. (2021). For this study, basic descriptive parameters were used to analyse the data collected, including the overall mean ($\bar{x}$) and standard deviation (σ). The investigation of the study sites included the Johannesburg North and Central District, Gauteng Grade 10 – 12 secondary schools. Data was gathered from GDE schools, between September - October 2022. Data analysis and discussions of the obtained results took place between November 2022 and January 2023.

Interview questions were closely related to the questionnaire. In most cases, the questionnaire limits participants since they use a 4, 5, or 7 Likert scale to choose from. Therefore, the interview responses were used to elaborate more on what teachers think about modelling the curriculum. It allowed them to provide me as a researcher with their experiences from secondary education to their teaching profession.

From the qualitative research method, the study objective was focused on teachers' perceptions and recommendations about the teaching and learning of mathematical modelling in the CAPS curriculum. The semi-structured interview questions used in this study were open-ended in nature. In this study, the relevant literature and the framework analysis are two approaches that were adopted to analyse the qualitative collected data of the study. It was significant for the data analysis process to be carefully analysed and interpreted with the key themes with teachers' responses. Framework analysis is an approach that attempts to identify similarities and differences in applied research policy (Srivastava & Thomson, 2009). In addition, it is more suitable when analysing qualitative research data that have particular questions. Hence, it was adopted from the study because there were specific questions asked to the participants for a better understanding of the research problem at hand with their knowledge and experiences.

3.8.1. Methods of answering research questions

In answering the research questions about beliefs, SEF, and prior knowledge about modelling, a structured questionnaire (closed-ended) – multiple choice questions using a 5 Likert[19] scale (strongly disagree, disagree, neither disagree nor agree, agree, and strongly agree) was used to enable the researcher to analyse the data using the 'quantitative research method'. The collected data were analysed through the SPSS software. The last research question about teachers' perceptions about the teaching and learning of modelling was answered through interview sessions, which allowed the researcher to use the 'qualitative research method'. The interview questions were informed by the quantitative data since the study was using a sequential explanatory design. Thus, the study adopted a 'mixed method research design' approach.

A mixed research method is an integration of both qualitative and quantitative research methods in one study (Cresswell, 2012). A research design of a study is a mixed research approach if it involves qualitative and quantitative research approaches in answering developed research questions and test instruments to be used in collecting data, sampling procedures, and data analysis. It is useful for gaining an understanding of the research problem, topic, and context. Quantitative study research obtains the objectivity of the participants whilst the qualitative method achieves the explanations, details, and perceptions about the research problem from the participants.

[19] A Likert scale is an ordered scale developed in 1932 by Rensis Likert from which respondents choose one best option that aligns with their perceptions. It is often used to measure respondents' attitudes, and to rate themselves on what extent they agree or disagree with a certain statement.

3.8.2. Coding participants questionnaire

During the coding of the obtained data from SPSS software, it was noticed that not all questions were answered by the participating mathematics teachers. To avoid missing values from the data that may disadvantage the process of data analysis, the following decisions were taken by the researcher.

Missing value (s): Not all 40 ISTs were able to respond to the 63 questions (according to the coding, see Appendix A) from the questionnaire. The first eight questions from the questionnaire were about participants' general information refer to Table 4.1). questions that had options including 'other' and were unanswered by the participants were coded 'other'. That was decided by the researcher based on whether the participants didn't have their options listed from the given options or the participants didn't want to reveal the information for confidentiality purposes. From questions nine (9) to sixty-three (63), which were the Likert scale options, where the participants left it blank, it was given a scale of three (3) (neither disagree nor agree).

3.8.3. Test instruments for questionnaire data analysis

In-service teachers' belief about teaching and learning of modelling.

Research question 1; teacher's beliefs about mathematical modelling were answered through four components of beliefs (modelling in the classroom, application of mathematical modelling, constructivist belief, and transmissive belief).

In-service teachers' self-efficacy about teaching and learning of modelling.

Research question 2; teachers' self-efficacy (SEF) about mathematical modelling was answered through two components of SEF (SEF for mathematical modelling and SEF for working mathematically).

In-service teachers' prior knowledge of modelling content.

Research question 3; teachers' self-reported prior knowledge and competence about mathematical modelling was answered through four components (General role played by mathematical modelling from the ISTs courses, preparedness of ISTs for teaching mathematical modelling, nature of modelling tasks teachers have encountered during their educational journey and teachers' experiences about mathematical modelling in various contexts.

Table 3.3 Example of the test instruments for measuring teachers' belief, SEF, and prior knowledge about modelling (adopted with permission from Wess et al., 2021)

Research questions	**Scale(s)**	**Component(s)**	**Example(s)**
1. What are in-service teachers' beliefs about teaching and learning of mathematical modelling in the CAPS curriculum?	Beliefs	1. Modelling in the classroom 2. Application of modelling 3. Constructivist belief 4. Transmissive belief	"Learners should be given the opportunity to do mathematical modelling in the classroom." "In mathematical modelling, you work on tasks that are more practical." "Learners learn mathematics best by discovering their own ways of solving problems." "Teachers should provide detailed procedures to learners for solving application problems."
2. What are in-service teachers' self-efficacy about teaching and learning of mathematical modelling in the CAPS curriculum?	Self-efficacy (SEF)	1. Mathematical modelling 2. Working mathematically	"… the assumptions they made when modelling." "… their handling of the mathematical symbols and operators used in modelling."
3. What are teachers' prior knowledge and competence in mathematical modelling?	Prior experiences	1. General role played by mathematical modelling from the ISTs courses. 2. Preparedness of ISTs for teaching mathematical modelling 3. Nature of modelling tasks teachers' have encountered during their educational journey.	"Mathematical modelling has already been addressed in my teacher training courses." "I have already been presented with knowledge to teach modelling." "I have solved modelling tasks myself during my teacher training."

	4. Teachers' experiences about mathematical modelling in various contexts	"Mathematical modelling played a role in my first year of teaching at school."

Research question 4 method of data analysis is discussed below (see section 3.8.4).

3.8.4. Interviews (qualitative) method of data analysis

Out of the many qualitative data analysis methods (see Charmas & Belgrave, 2012; Ritchie & Spencer, 2002), I choose to employ the thematic analysis method because it entails engaging oneself in the data to discover common patterns, concepts, or themes that arise from the phenomenon being studied and that are consistent with the research questions raised in the study. The thematic analysis method is a method used in the analysis of qualitative data that involves looking out across obtained data, identity, verifying, analysing, and reporting the repeated patterns (themes) (Kiger & Varpio, 2020). It is said to be the best and most appropriate when a researcher intends to investigate and analyse participants' thoughts, experiences, and behaviours (see Braun & Clarke, 2012).

One of the steps (see Table 3.3) in the analyses of qualitative data, a researcher needs to identify relevant themes from the transcripts to ensure it is in line with respondents' responses. In the identification of themes from the obtained qualitative data through the thematic analysis method, a researcher may opt to use an inductive approach or deductive approach (Braun & Clarke, 2012). I decided to use the inductive approach method in the identification of themes because I wanted to provide first-hand knowledge, information, and findings through observations, then finally draw general conclusions which predict what will be found in future research. The inductive method (as used in grounded theory) derived themes from the research obtained data (Varpio, Young, Uijtdehaage & Paradis, 2019). Conversely, the deductive approach method uses an already existing framework, theory or researcher focus interested in the identification of themes (Braun & Clarke, 2012; Varpio et al., 2019).

The thematic analysis method involves six (6) steps (refer to Table 3.3) during the process of data analysis; 1. A researcher familiarises himself/ herself with the data, 2. Generating codes, 3. Searching for relevant themes, 4. Review of the identified themes, 5. Naming and defining themes and 6. Reporting results. Table 3.4 shows and describes data analysis steps that were followed in analysing the semi-structured interview responses that were provided by the participating ISMTs in this study.

Table 3.4 Qualitative data analysis (interview) steps (adapted from Kiger and Varpio, 2020).

Analysis Steps	Steps/categories	Description of each step	How each step was used to analyse the data
1	A researcher familiarises himself/ herself with the data	The researcher becomes familiar with the obtained data from the participants, which involves relevant and interesting statements of the data.	I first transcribed the data and then familiarised myself with the transcribed data.
2	Generating codes	At this stage, the researcher begins to make notes and connections of likely data items of questions, interest, connections between data patterns, and another preliminary hypothesis from the researcher.	I focused on the patterns of teachers' responses and see what is it that is dominating their responses.
3	Searching for relevant themes	At this stage, the researcher examines the generated codes and looks for potential major themes from the data that may have greater potential in answering the developed research questions.	I then started to pick relevant themes for my study that may potentially answer the research questions.
4	Review of the identified themes	During this step, the researcher looks at the generated codes about the developed themes to see if they appropriately fit. This is the stage where the researcher intensively confirms if all the identity themes will sufficiently and positively contribute to the realisation of the purpose of the study and answering research questions.	I then reviewed the themes that stood out concerning the observed pattern of the teacher's responses.
5	Naming and defining of themes	During this step, the researcher names and describes each theme and provides an explanation of why it is important to the broader research questions and why should be considered in the final report.	At this stage, I was naming the themes and describing what each means and relates to the study.
6	Reporting results	This step has to do with the writing up of the report and analysis of the obtained data concerning the observed themes, the purpose of the study, other relevant studies, and theories.	During the final stage, I was reporting the results under each theme with other studies and theories.

3.9. Trustworthiness of the study

Researchers (see Kyngäs, Kääriäinen & Elo, 2020; Krefting, 1990; Schwandt, Lincoln & Guba, 2007) described the trustworthiness of research as demonstrating the true value of the research findings analysis. On the other hand, Kyngäs, Kääriäinen and Elo (2020) viewed the trustworthiness of a research study as constituting ideas such as quality, truthfulness, and authenticity of the study findings. The authors asserted that reliability and validity were phrases used to measure the trustworthiness of both qualitative and quantitative. For this study, I adopted the trustworthiness of the research discoveries for the qualitative research method that comprises credibility and dependability which was first addressed by (Lincoln, 2007) in his model. Then the validity and reliability of the study are shown with justification as well for the trustworthiness of the quantitative research method. Morse, Barrett, Mayan, Olson and Spiers (2002) argued that researchers must keep in their mind that while trustworthy of the study is useful to evaluate the standards and validity of the study, they do not themselves get to decide if their research findings will be relevant to the on-going decisions about the topic and transferable, but other scholars and researchers will decide on the transferability and use the information about their studies and experiences to come into a conclusion of relevancy.

3.9.1. Credibility

The criterion of credibility is to ensure that readers do believe that the information that is reported in this research study is trustable and valid, meaning it is credible and shows the conceptual explanation of the research. The researcher reported the results with confidence showing both the strengths and restrictions of the research study are considered credible for research in the field. I also considered that since I have experience with the topic of research from high school to university level, hence the experience and knowledge I have might affect the findings and analysis of the collected data.

The researcher also ensured that the way of asking questions, particularly during interview sessions does not influence participants to answer in a certain way that will affect the findings of the study. I as a researcher also ensured that I choose participants that show more characteristics that are relevant to the research problem and topic, and they demonstrate integrity (credibility) about the research study. Participants were also shown a sample of the research findings after data analysis for them to confirm if they are the true reflection of their experiences with the research findings. For the objective of the qualitative research method, credibility was ensured due to the descriptions of participants' experiences being immediately recognised by the researcher which showed that most participants that were interviewed showed the same perceptions about the teaching and learning of mathematical modelling (Sandelowski, 1986).

3.9.2. Dependability

Anney (2014) explains that dependability also involves participants (mathematics teachers) commenting and suggests on the interpretation of the study findings and recommendations to ensure that the results are supported by the participants about the research problem and their knowledge and experience. Thus, five participating teachers were requested to examine the research findings from the report and make recommendations to ensure that it is the true reflection of their responses, knowledge, and experiences in the mathematical modelling field.

I believe that other researchers in the discipline of mathematical modelling will follow and understand the inferences made from this study based on the data collected. Hence, this shows that the participants did not have much knowledge about the research topic, thus the research analysis was open, but considering that the researcher has knowledge about the research topic and that might affect the findings analysis of the study. With that being said, the researcher ensured that tables, graphs, figures, and quotes are included from the teacher's interview responses that explicate the data analysis process for this research report. The data of the study were analysed twice to ensure that the analysed results were consistent and valid.

3.10. Reliability of the study

According to Trochim (2006), in research, the reliability of a research study comprises consistency, repeatability, stability, and dependability of the participant's responses and the researchers' ability to record the collected data accurately. Reliability in a study is achieved if the findings of the collected data will always possibly demonstrate the same results about the study if repeatably examined. The reliability of the study is internal consistency, meaning each test instrument was considered separately. In obtaining higher internal consistency, more test items were used and considered a strong positive association between the test instruments to ensure that most of the issues related to the research topic are addressed, and interview questions were related to the questionnaire.

Babbie and Mouton (2001), suggested the following reliability measures that can be put in place for qualitative research:

- What the researcher observes during the research process.
- Posing difficult questions to the participants.
- Asking questions that do not have straight answers and confusing the participants to get more insight about the research problem.

The reliability of the study was ensured by:

- Adopting a mixed research method in the study.
- Choosing participants that are familiar with the current topic under discussion and senior teachers (including HODs[20]) that also shared their experiences about the topic and how mathematical modelling can be possibly integrated into the CAPS curriculum.
- Audio recording (interview) and transcriptions were used as one of the methods of data collection to see the response pattern between the qualitative and quantitative research methods.

3.11. Validity of the study

According to Alvis (2022), validity in research is described as the accuracy of the data measurement. Validity indicates how a specific test instrument is appropriate for a particular research problem or research topic. If the results are appropriate and accurate according to the researcher's research problem, clarifications, and assumption, then the research study is deemed valid research. If the research design, methods, and methodology that are used in the research study are accurate, then it will produce accurate and appropriate results and a valid research paper. If a research method in a study is reliable and used properly, then it is valid. In contrast, if a research method is not reliable, then it is not valid and it won't produce an appropriate and accurate research paper.

The validity of the study was ensured by using test instruments and interview questions that are accurate to measure individual (participants) characteristics, abilities, and behaviours. Test

[20] HOD (Heads of Department) are directors, guides, and supporters of the teachers within a certain subject (mathematics in the study). The person further provides development and leadership to the teachers to ensure the smoothly run of a day-to-day subject in the school.

instruments that were used ensured appropriate and immediate conclusions were drawn from participants' responses.

3.11.1. Internal validity

Internal validity explores whether the research study design, methodology, and findings analysis answer the formulated research questions without being biased (Rooney, Cooper, Jahnke, Lam, Morgan, Boyles & Lunn, 2016). For example, improper methods of choosing participants that do not show characteristics about the research topic, bias, and missing collected data can all compromise the validity of the research findings and inferences. Hence, a suitable technique for selecting participants that show characteristics of the research topic was used, data was kept in a password-protected laptop to avoid missing some of the collected data.

3.11.2. External validity

The external validity of a study according to Findley, Kikuta, and Denly (2021) refers to the potentiality of the findings of the research study to be generalised in other fields or other targeted populations. Meaning, the chosen participants of a study must represent the whole population so that it can be generalisable back to the whole population and contexts. Hence, some of the results obtained from the teachers we generalised, such as teachers' prior knowledge about modelling. It was confirmed that a good number of teachers in the profession know modelling in the curriculum.

3.12. The bias of the study

According to Rinjit (2020), bias is a systematic twisting of participants' responses by the researcher or by the way the test instruments are organised and used. Research trustworthiness and accuracy can be compromised if bias is not considered during the research process. Hence, it may result in poor reliability and validity of the study.

I minimised bias in my research study by utilising the following procedures:

- A comprehensive relevant literature review about teachers' mathematical modelling competency.
- Choosing participants (teachers) that demonstrate a representative sample of the whole population about the research problem.

- Asking questions that are not influencing the participants to respond in a certain way will advantage or disadvantage the findings of the study.
- Verified the statistical analysis (IBM SPSS) used to analyse the collected data, and the steps followed to critically analyse the interview responses.

3.13. Ethical Considerations

Regarding ethical issues, ethical clearance was requested from the Wits Human Research Ethics Committee to conduct the study, and a clearance certificate with protocol #H22/07/14 was granted. I sent the information sheets and requests to GDE (approved, see Appendix K), school principals (approved, see the sample from Appendix L & M), and participants (teachers) (see Appendix I & J) to take part in the study. Participants were also given consent forms to agree or disagree to take part in the study. Only participants who consented were allowed to get involved in the study. All minimal potential risks, what was expected from them, and the purpose of the study was explained to them in detail before taking part in the study. Participants were freed from withdrawal to participate in the study at any time.

Participating teachers were told not to share their details in any of the questionnaire test books, and their responses were stored in a password-protected laptop throughout the research process. Coding was used to differentiate teachers' questionnaire test books (e.g., NM, KN, and SN). The two letters were formed based on the teacher's first name and family name. Participants were told that though their interview responses may be audio-recorded and quoted in the research report writing, their details will not be shared in any of the publications or transcribed copies of the interview. Hence, they were quoted anonymously.

As aforementioned, the data of the study was kept in a password-protected laptop throughout the research process. Participants who chose to fill in hard copies of the questionnaire and their test books were stored in a locked drawer in an office. Research data was only shared with the student supervisor. No personal information of the participants was shared during and after research report writing (quotations or publications).

3.14. Chapter Summary and Conclusion

This chapter discussed research methods, approaches, procedures, and methodology used during the research process, and the responsibility of the researcher as a main tool for data collection and analysis. Tools that were used to collect and analyse data were described in detail. Ethical considerations during the collection and analysis of data were described, and the trustworthiness, reliability, and validity of the study issues were addressed.

The next chapter 4 presents the obtained results with the aid of diagrams, figures, and tables.

CHAPTER 4: RESULTS PRESENTATION AND ANALYSIS

4.1 Introduction

This chapter provides a report of the research results presentation and analysis that was obtained from the questionnaire (teachers' responses based on their beliefs, self-efficacy (SEF), and prior knowledge about mathematical modelling) and semi-structured interviews (teachers' perceptions about teaching and learning of mathematical modelling) with the CAPS curriculum.

The data collected and presented in this chapter which are teachers' responses from the questionnaire and interviews were aimed to answer the following research questions:

1. What are in-service teachers' beliefs about teaching and learning mathematical modelling in the CAPS curriculum?
2. What are the in-service teachers' self-efficacy about teaching and learning mathematical modelling in the CAPS curriculum?
3. What are in-service teachers' prior knowledge of mathematical modelling in the CAPS curriculum?
4. What are in-service teachers' perceptions about the modelling content in the Grade 10 – 12 CAPS curriculum?

Data collected compromises of audio-recorded (interviews) and completed questionnaires by the mathematics teachers. With some of the major themes arising from the interviews (see section 4.3).

Three major focused themes (scales) were used in the presentation, analysis, and discussion of the quantitative obtained results. From the questionnaire, the major themes are teachers' beliefs about mathematical modelling, teachers' prior knowledge about mathematical modelling, and teachers' SEF about mathematical modelling. These themes were formulated based on the predetermined research questions of the study to ensure that each theme answers each research question in line with the theoretical framework used, which is the structural model of professional competence for teaching mathematical modelling (belief, self-efficacy, and prior knowledge from Wess et al. (2021)).

After critical analysis of all teacher's questionnaires (using SPSS software) and interview responses, a few representative teachers' excerpts are provided in the sections below to show a sample of their responses. Teachers' interview responses were transcribed (see Appendix F, G & H) to restrain loss of fluency. In the excerpts, some words or terms are written in italic format for emphasis purposes.

4.2 Participated mathematics teachers in the study.

40 mathematics teachers participated in the study, with three teachers who were purposively selected to proceed to the interviews to get more insight into their perceptions of mathematical modelling in connection to the CAPS curriculum. Not all teachers responded to the given test questionnaire, this might be because the test questions were many (55 test items) or might be because teachers did not show sufficient knowledge about mathematical modelling. Hence, their responses were limited to what they know about modelling. This was analysed and will be discussed shortly.

4.3 General information about the participants

The results analysis chapter begins by providing some general information (age, gender, number of years teaching mathematics and qualification) of Grade 10 – 12 convenient participating ISMTs from both questionnaires and interviews. Table 4.1 shows participants' general information (Background). The majority of the 40 ($M = 57.5\%$) participating teachers were males. Twenty of the 40 (50%) teachers were aged range of 20 – 30 years. Surprisingly, twenty-three of 40 (57.5%) teachers reported to be currently teaching Grade 12 in secondary schools. Twenty-one of 40 (52.5%) teachers have been in the teaching field for 0 – 5 years, while eleven of 40 (27.5%) teachers were between 6 – 15 years of teaching in the mathematics field. One of the three interviewed teachers (Code: KN) with over 10 years of teaching experience was purposively selected for an interview to contribute more perceptions about mathematical modelling in the CAPS (2012 - current) curriculum. Luckily, it means the teacher also taught under (National Curriculum Statement) (NCS[21]) (2004 - 2011) curriculum, then CAPS (from academic year 2012). Two more teachers (Code: NM & Code: SN) with less than 5 years' experience of in teaching mathematics were chosen to get more insight into mathematical modelling in the CAPS curriculum recently.

Table 4.1 General information of the participants

Profiles	**Categories**	**Numbers** (N)	**Percent** (%)
Gender	Female	17	42.5%
($N = 40$)	Male	23	57.5%
	Other	0	0%
Age (in years)	20 – 30	20	50%
($N = 40$)	31 – 40	9	22.5%
	41 – 50	8	20%
	Above 50	3	7.5%
Highest qualification	Diploma	3	7.7%
($N = 40$)	B.Ed.[22]	19	48.7%
	B.Ed. (Honours)	13	33.3%
	M.Ed.[23]		
	Ph.D.[24]	2	5.1%
	Other	0	0%
		2	5.1%
Highest Grade teaching mathematics currently	Grade 10	9	22.5%
($N = 40$)	Grade 11	4	10%
	Grade 12	23	57.5%
	Other	4	10%
Number of years teaching mathematics (in years)	0 – 5	21	52.5%
($N = 40$)	6 – 15	11	27.5%
	16 – 25	7	17.5%
	Above 25	1	2.5%
Sub-major subject	Physical sciences	22	55%
($N = 40$)	Geography	4	10%
	English	1	2.5%
	Other	13	32.5%

[21] National Curriculum Statement (NCS) was a curriculum that was introduced in 2004 to 2011 that sets out the minimal standards of skills and knowledge to be achieved at each grade and sets high, achievable standards across all subjects.
[22] B.Ed. – Bachelor of Education
[23] M.Ed. – Master of Education
[24] Ph.D. – Doctor of Philosophy

4.4 Questionnaire data presentation

Concerning teachers' mathematical modelling competency, I considered three attitudinal dimensions as particularly significant (Beliefs, SEF, and Prior knowledge) (detailed in Section 3.7.2).

In this study, I decided to adopt the test instruments for beliefs, SEF, and previous experiences from a structural model (refer to Wess et al., 2021), and thus they were constructs that correspond with the determination of teachers' modelling beliefs and SEF. An additional reason for this decision of using the Wess et al. (2021) structural model was that they have already been used by the PSTs to determine their professional competence for teaching modelling in the classroom.

Research question 1: What are in-service teachers' beliefs about teaching and learning mathematical modelling in the CAPS curriculum?

Introduction

Beliefs about mathematical modelling in this study were operationalised within two scales: constructivist and transmissive beliefs. The constructivist-oriented belief scale was further divided into three components, which are 'beliefs about mathematical modelling in the classroom', 'beliefs about the application of mathematical modelling, and 'constructivist belief'. Lastly, it was a transmissive belief.

Table 4.2 Examples of the test instruments on beliefs about mathematical modelling (adopted with permission from Wess et al., 2021)

Components	**Test instrument example**
Beliefs about the application of mathematical modelling	"In mathematical modelling, you work on tasks that are more practical."
Beliefs about mathematical modelling in the classroom	"Learners should be given the opportunity to do mathematical modelling in the classroom."
Constructivist beliefs	"Learners learn mathematics best by discovering their own ways of solving problems."
Transmissive beliefs	"Teachers should provide detailed procedures to learners for solving application problems."

In response to the first research question "What are in-service teachers' beliefs about teaching and learning of mathematical modelling in the CAPS curriculum?", I looked at the four components shown in Table 4.2 of in-service teachers' beliefs about teaching and learning of mathematical modelling. Data analysis and reporting were more directed to teaching mathematical modelling than learning since the study was based on mathematics teachers (teaching) and not learners (learning).

4.4.1 Teachers' Beliefs about mathematical modelling in the Classroom

From Table 4.3 below, the N $(Valid)$ represents participated mathematics teachers $(N = 40)$. The reported results were based on the average mean and standard deviation.

Table 4.3 Teachers' Beliefs about Modelling in the Classroom

		Mathematical modelling should be part of mathematics education in the CAPS.	Learners should be given the opportunity to do mathematical modelling in the classroom.	Among other competencies, mathematical modelling competence should be taught in the classroom.	Mathematical modelling should be a specific topic in the CAPS mathematics curriculum.
N	Valid	40	40	40	40
	Missing	0	0	0	0
Mean		4.43	4.43	4.22	4.28
Median		4.50	5.00	4.00	4.00
Mode		5	5	4	4
Std. Deviation		.675	.747	.768	.751
Sum		177	177	169	171

From Table 4.3, 40 participating teachers completed the questionnaire. Out of the 40 responses, comparing the four questions being analysed for 'beliefs about modelling in the classroom', regarding the question of 'modelling should be part of mathematics education in the CAPS curriculum', only 1 teacher disagreed with the question and 20 teachers strongly agreed. From the question of whether 'learners should be allowed to do modelling in the classroom', only 1 teacher disagreed, and 22 teachers strongly agreed with the question. From the question that 'mathematical modelling competence should be taught in the classroom', only 1 teacher disagreed, and 16 teachers strongly agreed that modelling competencies should be enhanced in the classroom. From the question of whether ‘mathematical modelling should be a specific topic in the CAPS mathematics curriculum’, only 1 teacher disagreed, and 17 teachers strongly agreed that modelling should be a specific topic in the mathematics curriculum. The average mean from the four investigated questions about teachers modelling beliefs in the classroom is sitting at 4.34, showing that most of the teachers agreed and believe in mathematical modelling in the classroom. However, the qualitative method (see section 4.3) was used to confirm or further investigate why teachers did not strongly agree with the proposed question and what made them only agree with the questions. Looking at the overall Std. Deviation across the four test items is sitting at 0.736, showing that the collected data is clustered around the mean.

As stipulated that the Linkert scale of five was used to measure teachers’ modelling competency. Looking at the average median value across the four questions is sitting at 4.4, which shows that there is a strong agreement between the questions (*Mathematical modelling should be part of mathematics education in the CAPS; Learners should be allowed to do mathematical modelling in the classroom)* (see Table 4.4 and Figure 4.1) respectively. Fifteen of 40 (45%) teachers agreed that mathematical modelling should be part of the CAPS curriculum, with twenty of 40 (50%) teachers strongly agreeing on making modelling to be part of the curriculum. This indicates that teachers are aware of modelling and believe in mathematical modelling and have perceptions about mathematical modelling in the curriculum. Which should start from the CAPS curriculum and then to the classroom. I had to explain to about eight of the 40 mathematics teachers what mathematical modelling is. In avoiding being biased, I referred the participants that modelling is viewed as 'word problems' from Grades 8 to 11 and 'optimisation' from the Grade 12 content.

The mean, Std. Deviation and median values also show that participating mathematics teachers strongly agreed that learners should be allowed in the classroom to approach mathematical modelling on their own. That would also promote independence and competence in mathematical modelling. The statistics given by the teachers from these two versions were further confirmed in the following sub-section (see 4.2.3 constructivist belief and 4.3 interviews (qualitative method).

Table 4.4 Modelling should be part of mathematics education in the CAPS.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	1	2.5	2.5	2.5
	Neither agree nor disagree	1	2.5	2.5	5.0
	Agree	18	45.0	45.0	50.0
	Strongly agree	20	50.0	50.0	100.0
	Total	40	100.0	100.0	

From Table 4.3, I observed small values of Std. Deviation ($0.675 - 0.768$) with the mean. This means the data collected is clustered around the mean, and it shows that mathematics teachers' consensus about mathematical modelling is clustered and solid.

As it is evident from Figure 4.1 below, most of the participants strongly agreed that learners should be given the independency of approaching modelling tasks with little scaffolding from the teacher. Further research still needs to be done on what extent the strategic interventions (detailed by Wess et al., 2021) during learners' modelling processes will go in terms of assisting learners to complete modelling problems on their own.

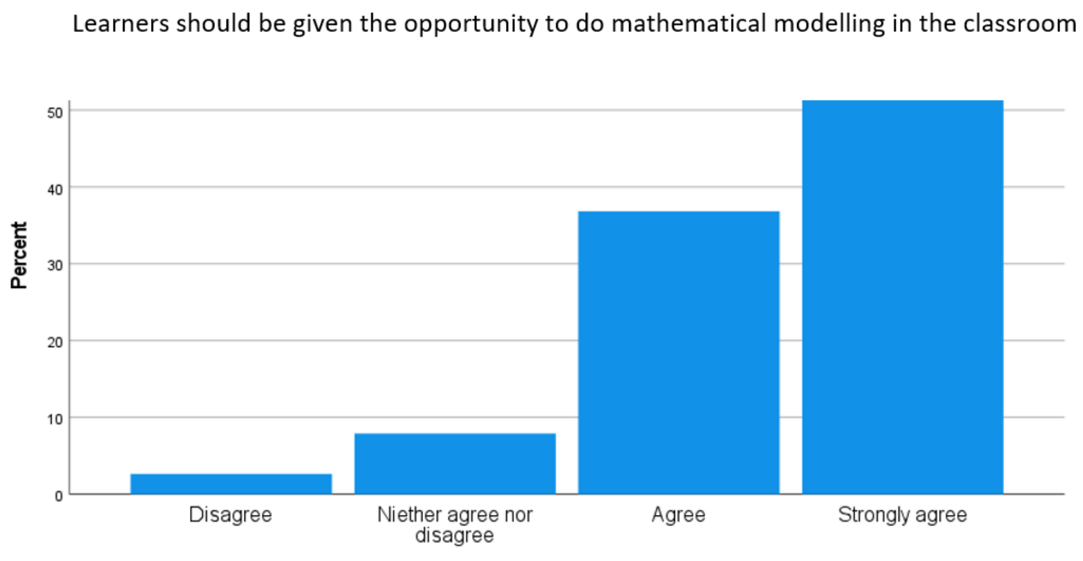

Figure 4.1 Learner’s opportunity to do modelling in the classroom.

Twenty-two of 40 (55%) teachers’ responses strongly agreed that ‘learners should be allowed to do mathematical modelling in the classroom'. This means it is not going to be challenging to engage teachers in integrating modelling content with other topics and subjects. Though the question of how teachers treat mathematical modelling in the classroom during their teachings of other topics remains unanswered.

4.4.2. Teachers' beliefs about the application of mathematical modelling

It should be noted that the question "Mathematical modelling is an unproductive game, with no concrete relation to reality." was reverse-scored due to its negative formulation. Meaning point 1 was regarded as strongly agree and point 5 was regarded as strongly disagree. Table 4.5 below displays the obtained results of teachers' beliefs about the application of mathematical modelling.

As can be seen from Table 4.5, most mathematics teachers do believe in the application of modelling in mathematics. Firstly, the mean (4.43) from whether 'mathematical modelling should be part of mathematics education in the CAPS showed that most of the mathematics teachers agreed that mathematical modelling should be part of the mathematics CAPS curriculum. The Std. Deviation (0.675) also shows a small value, depicting that the obtained data is less variable concerning the mean. Looking at the question of whether 'many aspects of mathematical modelling are used practically in reality', the mean is sitting at 4.28, showing that most of the teachers agreed that many aspects of mathematical modelling are used practically in reality. The median value also does confirm that most of the chosen data set is that teachers do believe that modelling is used practically in reality. The question has the smallest Std. Deviation (0.640), shows that most of the data values are clustered around the mean, which is between agree and strongly agree.

Table 4.5 Teachers' Beliefs about the Application of mathematical modelling

		Mathematical modelling should be part of mathematics education in the CAPS.	Mathematical modelling is an unproductive game, with no concrete relation to reality.	Many aspects of mathematical modelling are used practically in reality.	In mathematical modelling, you work on more practical tasks.
N	Valid	40	40	40	40
	Missing	0	0	0	0
Mean		4.43	2.55	4.28	4.13
Median		4.50	2.50	4.00	4.00
Mode		5	3	4	4
Std. Deviation		.675	1.239	.640	.822
Sum		177	102	171	165

Looking at the beliefs of modelling being more of a practical task, the mean (4.13) showed that most of the mathematics teachers agreed that modelling tasks are more practical to reality. The median value, which is sitting at 4 also showed that most of the data set is sitting at the scale of ‘agree’, rated 4. Looking at the overall mean of the data (3.85), it showed that most of the teachers are between agreement and uncertainty of whether modelling is an unproductive game with no concrete to reality and the overall Std. Deviation (0.877) showed less data variation. This indicates that the data set is around the mean.

Figure 4.2 below displays the mentioned results, which measured 'teachers' beliefs about modelling as being an unproductive game with no concrete relation to reality'. The results displayed in percentages show that ten of 40 (25%) teachers strongly agreed that modelling is an unproductive game with no concrete relation to reality. Further ten of 40 (25%) teachers agreed that modelling is not a productive game. This depicts that twenty of 40 (50%) teachers agreed that modelling is a productive game with concrete to reality. The results from Figure 4.2 below further show that roughly eleven of 40 (27.5%) teachers are uncertain of modelling as a productive game with concrete to reality.

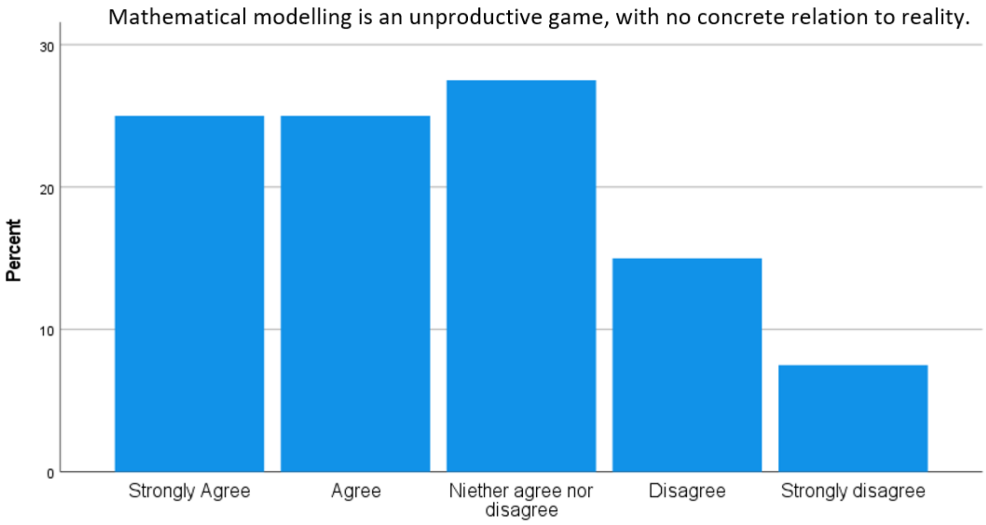

Figure 4.2 Mathematical modelling is an unproductive game, with no concrete relation to reality.

Six of 40 (15%) teachers showed disagreement with modelling being an unproductive game with no concrete reality. Further three of 40 (7.5%) teachers strongly disagreed with the used test items about modelling being concrete and practical. Looking at the overall results of the obtained data for the question 'Mathematical modelling is an unproductive game, with no concrete relation to reality.', the results showed that twenty of 40 (50%) teachers agreed that modelling is an unproductive game with no concrete to reality. Eleven of 40 (27.5%) teachers showed that they are uncertain of the question and nine of 40 teachers (22.5%) disagreed that modelling is a productive game with concrete to reality.

4.4.3. Constructivist belief

According to Wess et al. (2021), the constructivist-oriented belief view is when a teacher steps back during modelling processes and allows learners to take the lead in their modelling processes in modelling tasks. That is allowing learners to discuss and construct their knowledge and skills for approaching modelling tasks. In this situation, teachers only prompt the learners on what is expected of them, but he/ she does not provide detailed step-by-step procedure on how a modelling task should be modelled.

Table 4.6 below shows the results of teachers' constructivist beliefs about mathematical modelling in the curriculum. As evidenced from the table, looking at the sum teachers scored 701 out of 800 which shows that teachers have a positive belief about constructively teaching mathematical modelling in the classroom. Looking at the average mean (4.39), most of the teachers agreed of being positive about teaching mathematical modelling in the classroom. The overall mean of 4.39 is good as it shows that over 50% of the mathematics teachers showed that they are constructivists when it comes to teaching mathematics. The overall median (4.83) value across the three questions shows that teachers strongly agreed that learners should be allowed to do modelling on their own, to discover ways of modelling tasks in such a way that they will be able to see the link on their own. The overall Std. Deviation, which is 0.739, a small value indicates that the data is less variant.

Table 4.6 Teachers' constructivist beliefs about mathematical modelling

		Learners should be given the opportunity to do mathematical modelling in the classroom.	Teachers should allow learners to come up with their own ways of solving application problems before they show them how to solve them.	Mathematics should be taught at school in such a way that learners can see the links on their own.	Modelling helps learners to understand mathematics when they are asked to discuss their own solution thinking.
N	Valid	40	40	40	40
	Missing	0	0	0	0
Mean		4.43	4.33	4.43	4.35
Median		5.00	4.00	4.50	5.00
Mode		5	5	5	5
Std. Deviation		.747	.764	.636	.802
Sum		177	173	177	174

Table 4.7 below shows teachers' results of giving learners an opportunity to individually do mathematical modelling in the classroom. Twenty-two of 40 (55%) teachers strongly agreed of giving learners an opportunity to do mathematical modelling in the classroom. Fourteen of 40 (35%) teachers agreed that learners should be given an opportunity to do modelling in the classroom. This implies that thirty-six of 40 (90%) teachers agreed that at some stage during the lesson, they should step back and give learners the opportunity to do modelling. Only one of 40 (2.5%) teachers disagreed that learners should be given an opportunity to do modelling in the classroom, and three of 40 (7.5%) teachers showed uncertainty about whether they should give learners an opportunity to approach modelling tasks on their own before teachers teach them. From the question of whether "Learners should be given the opportunity to do mathematical modelling in the classroom.", overall, the results show that thirty-six of 40

(90%) teachers are constructivists in the modelling classrooms, and only four of 40 (10%) teachers are unconfident with the approach of allowing learners to take lead in solving modelling tasks.

Table 4.7 Learners should be given the opportunity to do mathematical modelling in the classroom.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	1	2.5	2.5	2.5
	Neither agree nor disagree	3	7.5	7.5	10.0
	Agree	14	35.0	35.0	45.0
	Strongly agree	22	55.0	55.0	100.0
	Total	40	100.0	100.0	

The question of whether "Modelling helps learners to understand mathematics when they are asked to discuss their solution thinking." was investigating teachers' awareness of whether modelling helps learners to comprehend mathematics when they are requested to justify their solution thinking. The results (see Figure 4.3) showed that thirty-four of 40 (85%) teachers agreed that modelling helps learners to understand mathematics and engage in discussions of their solutions. Only one of 40 (2.5%) teachers disagreed that no role played by modelling to the learners in terms of their solutions thinking. Furthermore, five of 40 (12.5%) teachers are not sure whether modelling helps learners to understand their mathematical working.

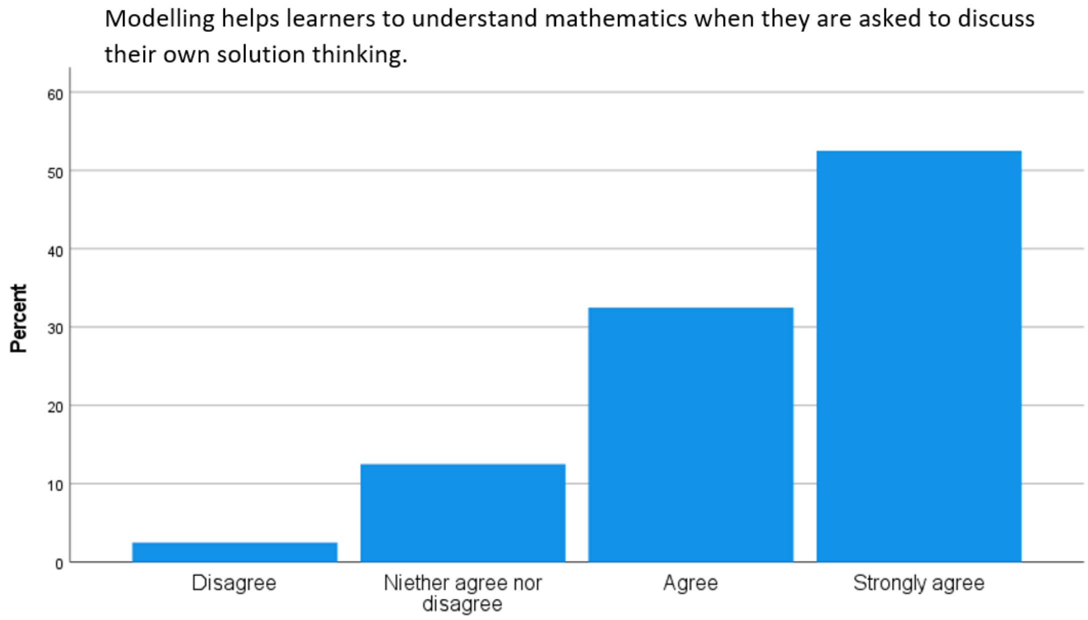

Figure 4.3 Modelling helps learners to understand mathematics when they are asked to discuss their solution thinking.

Merging the results, six of 40 (15%) teachers are not sure of whether modelling played a role in making learners understand their mathematical working, while thirty-four of 40 (85%) teachers showed strong acknowledgement of the role played by modelling to the learners in terms of making them understand their mathematical work when they are asked to discuss their written solutions.

4.4.4. Transmissive belief

It is explained that transmissive belief is one of the scales that were used in this study to operationalise teachers' beliefs about learning mathematical modelling. As described that transmissive belief is the transferring of knowledge and skills from the teacher to the learners. It is regarded as a one-way of learning in the mathematics classroom, where learners get only knowledge of the teachers without being given an opportunity to engage and construct their ways of approaching modelling tasks in the classroom. It is made clear that transmissive and constructivist beliefs cannot be regarded as fundamentally opposite (Wess et al., 2021).

Table 4.8 demonstrates the obtained results of teachers' transmissive beliefs about mathematical modelling. Based on the overall mean (3.64) calculated from the four components used to investigate ISTs transmissive beliefs, it showed that most of the teachers disagreed and are uncertain of the transmissive belief. That is because the mean of 3.64 is between agree and neither agree nor disagree. The mean (3.25) value from the question of "Learners should solve tasks in the way they were taught in class." showed that some of the teachers are uncertain of whether "Learners should solve tasks in the way they were taught in class."

Table 4.8 Teachers' transmissive belief about mathematical modelling

		Effective teachers show the right way and methods to solve an application problem.	Learners should solve tasks in the way they were taught in class.	Learners should often have the opportunity to follow their teacher's method solutions.	Teachers should provide detailed procedures to learners for solving application problems.
N	Valid	40	40	40	40
	Missing	0	0	0	0
Mean		4.00	3.25	3.73	3.58
Median		4.00	3.00	4.00	4.00
Mode		4	3	4	4
Std. Deviation		.906	1.006	.679	1.152
Sum		160	130	149	143

The overall Std. Deviation from Table 4.9 is sitting at 0.951, which is closer to one. The Std. Deviation with the mean shows some variation in the data regarding teachers' transmissive beliefs about modelling.

Table 4.9 below zooms in on the results of teachers providing detailed procedures to the learners in the classroom when approaching modelling problems. As can be seen from the table that nineteen of 40 (47.5%) teachers disagreed with the question of giving learners all procedures for solving a modelling task. Furthermore, seven of 40 (17.5%) teachers strongly disagreed with the transmissive belief question being investigated. Overall, it showed that twenty-six of 40 (65%) teachers disagreed with giving learners everything in the classroom, rather, according to the questions used, they prefer that learners are prompt and given the opportunity to explore the problem on their own. Looking at the other side of the data, six of 40 (15%) teachers agreed that learners should be given step-by-step procedures when approaching modelling problems.

Table 4.9 Teachers should provide detailed procedures to learners for solving application problems.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly agree	4	10.0	10.0	10.0
	Agree	2	5.0	5.0	15.0
	Neutral	8	20.0	20.0	35.0
	Disagree	19	47.5	47.5	82.5
	Strongly disagree	7	17.5	17.5	100.0
	Total	40	100.0	100.0	

Eight of 40 (20%) teachers are uncertain whether learners should be given step-by-step procedures when solving modelling tasks in the classroom.

4.4.5. Research question 1 results summary and conclusion

The above presented results answered research question 1 which showed that teachers do have a strong belief about mathematical modelling in the CAPS curriculum. As in-service teachers' beliefs about teaching mathematical modelling showed that over thirty-eight of 40 (95%), teachers agreed that modelling should be part of the curriculum and the opportunities to be given to the learners in the classroom to do modelling on their own. The results further showed that twenty of 40 (50%) teachers do believe that modelling is more practical to reality, and nine of 40 (22.5%) teachers do not believe that modelling is a productive game with concrete to reality.

Presented results further showed that thirty-six of 40 (90%) teachers agreed that learners should be taught in such a way that they can relate and construct their knowledge and skills during modelling processes, and only one of 40 (2.5%) teachers showed disagreement of stepping back and giving learners an opportunity to approach modelling tasks on their own. On the other hand, twenty-six of 40 (65%) teachers showed disagreement regarding giving learners step-by-step procedures for solving modelling tasks. While six of 40 (15%) teachers agreed that learners should be given step-by-step procedures during modelling processes.

Research question 2: What are the in-service teachers' self-efficacy about teaching and learning mathematical modelling in the CAPS curriculum?

Introduction

The test instruments from Wess et al. (2021) that were included in the SEF questionnaire required teachers to assess their effectiveness and recognition abilities and skills of learners in the phases during the modelling processes.

In analysing the obtained results, two one-dimensional scales were used: 'SEF for mathematical modelling' and 'SEF for working mathematically' during modelling processes. Test instrument samples are shown in Table 4.10.

Table 4.10 Examples of the test instruments for SEF about mathematical modelling (adopted with permission from Wess et al., 2021)

Components	Test instrument examples
Self-efficacy for mathematical modelling	"It is easy for me to identify the different abilities of the learners using the assumptions they made when modelling."
Self-efficacy for working mathematically	"It is easy for me to identify the different abilities of the learners using their handling of the mathematical symbols and operators used in modelling"

4.4.6. Self-efficacy for mathematical modelling

Table 4.11 shows obtained results that formed the scale of 'SEF for mathematical modelling' and were reverse scored. The results depicted that most of the teachers perceive it as challenging to use their abilities to diagnose learners' potential performance in mathematical modelling processes. Referring to the obtained scores, the highest mean (3.10) from the question "… the simplifications they have undertaken in the modelling process." shows that almost twenty of 40 (50%) teachers are not sure if they find it challenging for them to identify the distinct abilities of their learners using their judgement of the established models in the modelling process. The smallest value of the mean from Table 4.11 is 2.60, showing that most of the teachers lie between agree and are uncertain of whether they find it difficult to identify learners' abilities using their translation of mathematical results into reality. The overall mean (2.89) from Table 4.11 is more uncertain, showing that most of the teachers do not have strong confidence in their SEF capabilities to recognise learners modelling abilities during modelling processes.

Table 4.11 Teachers' Self-efficacy for mathematical modelling

		… their translation of mathematical results into reality.	... the assumptions they have made in the modelling process.	… the reasonable checks of their modelling solution.	… their writing when they separate important and non-important information about a real situation in modelling.	... the mathematical models/ methods they chose when modelling.	… the recording of the relationship between mathematical results and reality results that they produce when modelling.	… their handling of the mathematical formulae and symbols used in modelling.	… their written solution when modelling.	… their evaluation of the established models in the modelling process.
N	Valid	40	40	40	40	40	40	40	40	40
	Missing	0	0	0	0	0	0	0	0	0
Mean		2.60	2.78	2.85	2.88	2.98	2.98	2.95	2.93	3.10
Median		2.00	3.00	2.50	3.00	3.00	3.00	3.00	3.00	3.00
Mode		2	2	2	2	2	2	2	2	2[a]
Std. Deviation		.982	1.143	1.252	1.202	1.165	1.143	1.280	1.141	1.194
Sum		104	111	114	115	119	119	118	117	124

The median values depicted that 50% of the mathematics teachers do not see themselves as having the ability to identify learners' different abilities from their modelling processes, while the other 50% of the teachers do show an agreement of having the ability for diagnosing performance potentials of learners in mathematical modelling processes. The Std. Deviation across all the questions also showed a great variation, having the smallest standard deviation of 0.982 from the question "… their translation of mathematical results into reality.", and the largest standard deviation of 1.280 from the question "… their handling of the mathematical formulae and symbols used in modelling.".

Looking closer to the results for the question "… their translation of mathematical results into reality." (See Table 4.12), which refers to the translation of mathematical results into reality.

Table 4.12 Learner's translation of mathematical results into reality

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly agree	2	5.0	5.0	5.0
	Agree	22	55.0	55.0	60.0
	Neutral	8	20.0	20.0	80.0
	Disagree	6	15.0	15.0	95.0
	Strongly disagree	2	5.0	5.0	100.0
	Total	40	100.0	100.0	

The results showed that twenty-four of 40 (60%) teachers agreed that they find it difficult to identify the different abilities of the learners using their translation of mathematical results into reality, while 20% of the teachers are not sure if they can identify learners' abilities. Only 20% of the teachers showed disagreement with the difficulty of identifying learners' different abilities using their translation of mathematical results into reality. This confirmed the lowness of the average mean (2.60) from Table 4.11.

Zooming in from the question of "… their evaluation of the established models in the modelling process.", results (see Table 4.13 below) which have the largest mean, the results showed that fifteen of the 40 (37.5%) teachers showed agreement that it is difficult for them to identify the different abilities of the learners using their evaluation of the established models in the modelling process, while seventeen of the 40 (42.5%) teachers showed a positive attitude towards their SEF by showing disagreement with the question of difficulty in identifying learners abilities through their evaluation of the established models in the modelling processes.

Table 4.13 Learners' evaluation of the established models in the modelling process

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly agree	3	7.5	7.5	7.5
	Agree	12	30.0	30.0	37.5
	Neutral	8	20.0	20.0	57.5
	Disagree	12	30.0	30.0	87.5
	Strongly disagree	5	12.5	12.5	100.0
	Total	40	100.0	100.0	

Eight of the 40 (20%) teachers showed uncertainty about their SEF in identifying learners modelling abilities during their modelling processes.

4.4.7. Self-efficacy for working mathematically.

As can be seen from Table 4.14, the average mean across all three questions is 2.98. Showing that most of the teachers chose a scale of 2 (agree) and 3 (neutral). This shows that most mathematics teachers are not certain about their SEF abilities to diagnose learners' mathematical written work (results) during their modelling processes. The value of the Std. Deviation looks high, with the low being 1.141 and the high at 1.280, showing a great variation amongst teachers' responses.

Table 4.14 Teachers' SEF for working mathematically.

		… their handling of the mathematical formulae and symbols used in modelling.	… their written solution when modelling.	… the recording of their mathematical results in modelling.
N	Valid	40	40	40
	Missing	0	0	0
Mean		2.95	2.93	3.05
Median		3.00	3.00	3.00
Mode		2	2	2
Std. Deviation		1.280	1.141	1.218
Sum		118	117	122

Looking closer to the results for individual questions, starting from the question of "… the assessment of how well they improved the models that were already in use at the time of modelling." (Refer Table 4.15), eighteen of the 40 (45%) teachers showed agreement that it is difficult for them to diagnose the way learners handle mathematical formulae and symbols used in the modelling processes, while sixteen of the 40 (40%) teachers showed disagreement with the questions. In addition, they did confirm that they have abilities to diagnose learners' abilities based on how they handled their mathematical formulae and symbols. Six of the 40 (15%) teachers were uncertain of the question.

Table 4.15 Learners' handling of the mathematical formulae and symbols used in modelling.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly agree	5	12.5	12.5	12.5
	Agree	13	32.5	32.5	45.0
	Neutral	6	15.0	15.0	60.0
	Disagree	11	27.5	27.5	87.5
	Strongly disagree	5	12.5	12.5	100.0
	Total	40	100.0	100.0	

The results from the question of '… their written solution when modelling.' (Refer to Table 4.16) showed that seventeen of the 40 (42.5%) teachers showed agreement that it is difficult for them to diagnose learners' mathematical written solutions when modelling. While thirteen of the 40 (32.5%) teachers showed disagreement with the question, meaning they agreed that they have abilities to identify learners' abilities from their written solutions when modelling. Ten of the 40 (23%) teachers are not certain of the questions.

Table 4.16 Learners' written solution when modelling.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly agree	3	7.5	7.5	7.5
	Agree	14	35.0	35.0	42.5
	Neutral	10	25.0	25.0	67.5
	Disagree	9	22.5	22.5	90.0
	Strongly disagree	4	10.0	10.0	100.0
	Total	40	100.0	100.0	

Referring to Table 4.17, about the question of '… the recording of their mathematical results in modelling, sixteen of the 40 (40%) teachers agreed that it is difficult for them to diagnose learners' abilities through the recording of their mathematical results in modelling, while fifteen of the 40 (37.5%) teachers showed disagreement with the questions. This means it is easy for them to identify learners' abilities during their modelling processes through the recording of their mathematical results in modelling. Nine of the 40 (22.5%) teachers are neutral about the question; they are not sure of their abilities (SEF) to diagnose learners' capabilities through the recording of their mathematical results in modelling.

Table 4.17 Learners recording of their mathematical results in modelling.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly agree	3	7.5	7.5	7.5
	Agree	13	32.5	32.5	40.0
	Neutral	9	22.5	22.5	62.5
	Disagree	9	22.5	22.5	85.0
	Strongly disagree	6	15.0	15.0	100.0
	Total	40	100.0	100.0	

4.4.8. Research question 2 results summary and conclusion

The presented results of the respondents from the three questions demonstrated that seventeen of 40 (42.5%) teachers do agree that it is difficult for them to diagnose learners’ abilities from their mathematical work during modelling processes. Six of 40 (15%) teachers did not show confidence in the questions, meaning they are not sure of their SEF abilities to recognise learners' abilities from their written mathematical work. Fifteen of 40 (37.5%) teachers showed disagreement with the question, meaning the respondents demonstrated positiveness of their abilities to diagnose learners’ abilities from their written modelling results.

Research question 3: What are in-service teachers’ prior knowledge about the modelling content in the CAPS curriculum?

Introduction

For teachers' previous experiences with mathematical modelling, four components from the sixteen (16) test items were formed and used to control and trace their prior experiences with mathematical modelling. This was basically for teachers to apprehend their self-diagnostic of the contrasting discipline of experience in modelling, including personal knowledge about modelling and abilities to recognise learners' competencies and workings during learners' modelling processes when approaching modelling tasks.

The four components that were discussed below reflect teachers' previous experiences with modelling. The components were ‘General role played by mathematical modelling from the ISTs courses’, ‘Preparedness of ISTs for teaching mathematical modelling’, ‘Nature of modelling tasks teachers' have encountered during their educational journey', and ‘Teachers’ experiences about mathematical modelling in various contexts’. Table 4.18 below shows sample test instruments that were used for each of the four components.

Table 4.18 Examples of the test instruments for teachers' prior knowledge about mathematical modelling (adopted with permission from Wess et al., 2021).

Components	Test instrument example
The general role played by mathematical modelling from the ISTs courses	"Mathematical modelling has already been addressed in my teacher training courses."
Preparedness of ISTs for teaching mathematical modelling	"I have already been presented with knowledge to teach modelling."
Nature of modelling tasks teachers have encountered during their educational journey[25]	"I have solved modelling tasks myself during my teacher training."
Teachers' experiences with mathematical modelling in various contexts	"Mathematical modelling played a role in my first year of teaching at school."

4.4.9. General role played by mathematical modelling from the ISTs courses.

As can be seen from Table 4.19, from the question "I have previously learned mathematical modelling in my lectures", seventeen of 40 (42.5%) teachers (42.5%) with $(mean = 4.20)$ agreed that they have learned mathematical modelling during their teacher training courses. This is a good indication that most universities do offer mathematical modelling courses to pre-service teachers. But the question to what extent are they trained to teach mathematical modelling in basic education? remains unanswered. Looking at the question "Mathematical modelling has already been addressed in my teacher training courses.", Table 4.19 with a mean $(of\ 3.55)$, shows uncertainty (rated 3, neither agree nor disagree) of teachers' choice about mathematical modelling in their lectures. Concerning the question of "mathematical modelling played a role in my first year of teaching at school.", there was a contradiction of teachers' choice of selection from the questionnaire. From teachers' experiences of learning modelling from their lectures, there was an agreement that ISMTs have learned mathematical modelling in their lectures. It brings a question of how come teachers strongly agreed that they have learned mathematical modelling during their lectures, while there is an uncertainty (neither agree nor disagree) of whether 'mathematical modelling has been addressed in their teacher training courses'. Though the Std. Deviations (refer to Table 4.19) looks small, meaning the

[25] An educational journey is a continuous lifelong journey in education without a destination.

data is clustered around the mean, which confirms the disagreement between the mentioned above questions.

Table 4.19 General role played by mathematical modelling from the ISTs courses

		I have previously learned mathematical modelling in my lectures.	Mathematical modelling has already been addressed in my teacher training courses.	Mathematical modelling played a role in my first year of teaching at school.
N	Valid	40	40	40
	Missing	0	0	0
Mean		4.10	3.55	3.05
Median		4.00	4.00	3.00
Mode		5	4	3
Std. Deviation		.955	.986	.959
Range		3	4	4
Sum		164	142	122

Looking at the obtained results for the question "Mathematical modelling played a role in my first year of teaching at school". This test item measured teachers' prior knowledge of whether learned mathematical modelling has been useful to the teachers in their first year of teaching basic education. The results kept on declining, sixteen of the 40 (40%) teachers (see Appendix C) are uncertain with their responses. This means teachers are not sure of whether learning mathematical modelling from higher education has given them knowledge and skills of teaching mathematical modelling to learners in basic education. As can be seen from Figure 4.4 below, it shows that 40% of the teachers responded neither agree nor disagree. Looking at the overall mean (3.57) from Table 4.19, the results are between uncertainty and agreement.

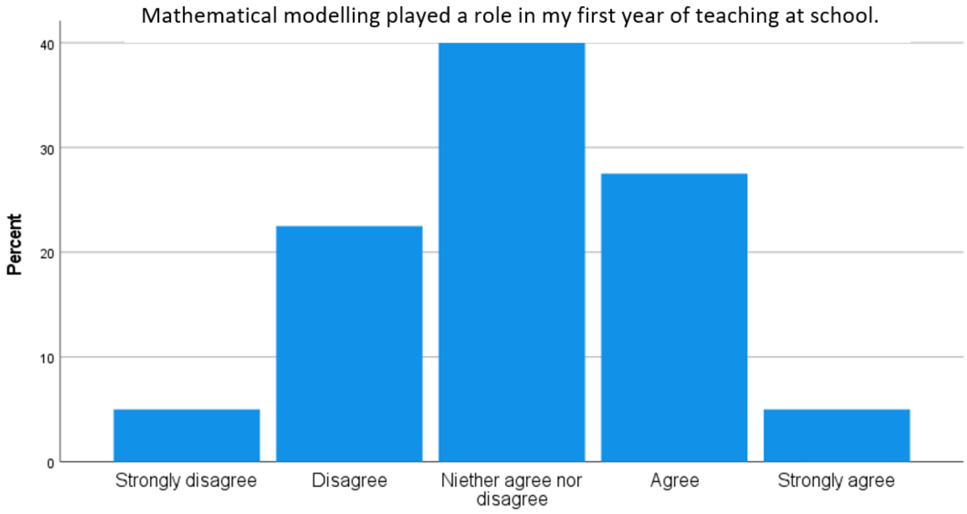

Figure 4.4 Mathematical modelling played a role in my first year of teaching at school.

This means the modelling learned from the university by the ISTs did not help them to approach modelling in the classroom with the learners. Three teachers have been interviewed on this aspect to get more insight into why the results about teachers' prior knowledge from university kept on declining. That may compromise mathematics teachers' prior knowledge about mathematical modelling in the CAPS curriculum. Hypothetically, teacher training courses at the university level do not equip teachers with the necessary knowledge and skills needed for them to teach modelling to learners from basic education. As unbiasedly acknowledged that as an investigator I am aware of the modelling courses that are offered at the University of the Witwatersrand and the University of Johannesburg (UJ)[26] (see section 2.2.1). But what I have observed is that they are offered from the upper level of study[27]

4.4.10. Preparedness of in-service teachers for teaching mathematical modelling

The selected components were used to trace teachers' previous knowledge and skills of teaching mathematical modelling from their secondary education (basic education) to their profession as qualified mathematics teachers.

Table 4.20 below displays obtained results regarding the preparedness of ISTs for teaching modelling in the classroom. The evidence showed that twenty-three of 40 (57.5%) teachers

[26] University of Johannesburg (UJ) is a public South African university located in Gauteng, Johannesburg.
[27] Generally, in South African Bachelor of Education degree takes four years (levels) to complete. The professional teacher qualification can also be obtained through a one-year course (called Postgraduate Certificate of Education (PGCE)), provided the individual has another degree equivalent to the teaching qualification (e.g., BSc).

are confident in being prepared to teach mathematical modelling to their learners. Looking at the average mean (3.33) across the questions from Table 4.20, it showed that they are all clustered around neither agree nor disagree. The result showed that the majority of Grade 10 – 12 mathematics teachers are not sure if they are competent in teaching mathematical modelling in the classroom. From the test item “I feel well prepared to teach mathematical modelling through my previous training.” Only seventeen of 40 (42.5%) teachers agreed that they feel well prepared to teach mathematical modelling through their previous experiences from their teacher training courses.

Table 4.20 Preparedness of ISTs for teaching mathematical modelling.

		I feel well prepared to teach mathematical modelling through my previous training.	I have already been presented with knowledge to teach modelling.	As part of my previous teacher training, I have been able to build solid foundations for teaching mathematical modelling.	In my education (high school to university), I have already acquired knowledge to teach modelling.	I was prepared for teaching modelling.
N	Valid	40	40	40	40	40
	Missing	0	0	0	0	0
Mean		3.25	3.53	3.30	3.25	3.30
Median		3.00	4.00	3.00	3.00	3.00
Mode		3	4	3	3	3[a]
Std. Deviation		1.080	1.062	1.018	1.104	1.159
Range		4	3	4	4	4
Sum		130	141	132	130	132

The results from Table 4.21 further depicted that thirteen of 40 (32.5) teachers are not sure if they are well prepared to teach mathematical modelling through their prior knowledge, and ten of 40 (25%) teachers show disagreement about being prepared to teach mathematical modelling in the classroom. In consolidation of the results from the tables (4.20; 4.21), twenty-three of 40 (57.5%) teachers do not seem to be well prepared to teach mathematical modelling in the classroom, only seventeen (42.5%) of the teachers showed that they are prepared to teach mathematical modelling to the learners. The Std. Deviations from Table 4.21 across all the questions are all greater than 1, which shows a bigger value of standard deviation. This means the scales (1 – 5 scales) are far away from the mean indicating a greater variation of the obtained data.

Table 4.21 I feel well prepared to teach mathematical modelling through my previous training.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly disagree	2	5.0	5.0	5.0
	Disagree	8	20.0	20.0	25.0
	Neither agree nor disagree	13	32.5	32.5	57.5
	Agree	12	30.0	30.0	87.5
	Strongly agree	5	12.5	12.5	100.0
	Total	40	100.0	100.0	

I acknowledge that most of the participating mathematics teachers in this study are newly entry teachers, with 20 teachers aged between 20 – 30 years. Twenty-one of 40 teachers with 0 – 5 years of teaching experience in the mathematics field. 23 teachers reported being currently teaching Grade 12 in their schools.

Looking at the question, "In my education (high school to university), I have already acquired knowledge to teach modelling." (see Appendix D), seventeen of 40 (42.5%) teachers agreed that their previous education from secondary school to qualified teachers equipped them with knowledge and skills of teaching mathematical modelling, while 23 (57.5%) of Grade 10 – 12 mathematics teachers disagreed (9 teachers) and unsure (14 teachers) of whether their previous education has imparted them with an understanding and expertise to teach mathematical modelling in the classroom. Overall results showed that over half (23, 57.5%) of the teachers do not feel well prepared to teach mathematical modelling in the classroom.

Interviews (see section 4.3) with the participating Grade 10 – 12 mathematics teachers were conducted to get a deeper comprehension of teachers' preparedness to teach mathematical modelling.

4.4.11. Nature of modelling tasks teachers have encountered during their educational journey.

From sub-section (4.2.2.3), teachers' educational journey I referred from teachers' secondary education to their profession as qualified teachers.

The results from Table 4.22 display results about teachers' prior experiences of solving modelling tasks during their teacher training and as qualified teachers (solving modelling tasks with learners in the classroom). From the questions indicated in Table 4.22, I was investigating if ISTs have encountered mathematical modelling tasks during their schooling time (including university training). Though the study did not engage in revealing the nature of modelling tasks they have solved before and how they were approaching those modelling tasks. That may be a recommendation for future research expanding from the current study.

Table 4.22 Nature of modelling tasks teachers have encountered during their educational journey.

		I have solved modelling tasks myself during my teacher training.	In my previous education, I also had to do mathematical modelling in the processing of tasks.	I have been working on modelling examples myself during my teacher training.
N	Valid	40	40	40
	Missing	0	0	0
Mean		3.83	3.43	3.48
Median		4.00	3.50	4.00
Mode		4	4	4
Std. Deviation		.903	1.059	1.132
Range		3	4	4
Sum		153	137	139

The three questions were used to test teachers' previous experiences of solving modelling tasks. Looking at questions (I have solved modelling tasks myself during my teacher training; I have been working on modelling examples myself during my teacher training), the results depicted that twenty-eight of 40 (70%) teachers agreed that they have solved modelling tasks themselves before during their teacher training courses. While four of 40 (10%) teachers showed disagreement in solving modelling tasks for themselves during their teacher training.

Regarding working on modelling tasks, twenty-three of 40 (57.5%) mathematics teachers did confirm that they've been working on modelling tasks themselves during their teacher training, while 9 (22.5%) of the teachers showed disagreement about working with modelling tasks themselves during their teacher training. Only eight of 40 (20%) mathematics teachers were not sure whether they have solved (worked) on modelling tasks themselves during their teacher training.

The results indicated that teachers have done mathematical modelling during their teacher training at the university level. But referring to the mean (3.43) and Std. Deviation (1.059) of the question "In my previous education, I also had to do mathematical modelling in the processing of tasks.", teachers slightly neither agreed nor disagreed about having previous experiences of solving modelling tasks. From Table 4.22 there is an observance of questions with bigger Std. Deviations ($SD = 1.059; 1.132$), show a greater variation of the values being far away from the mean.

From mathematics tasks in general, some of the tasks require one to use modelling processes when solving them. Figure 4.5 below shows that over 30% of the teachers were not sure if they have used mathematical modelling methods in solving some of the mathematics tasks. While approximately 15% of the respondents showed that they have never used modelling processes when solving mathematical tasks during their teacher training. Concurrently, approximately 15% of the mathematics teachers confirmed that they have applied mathematical modelling methods in solving tasks during their teacher training.

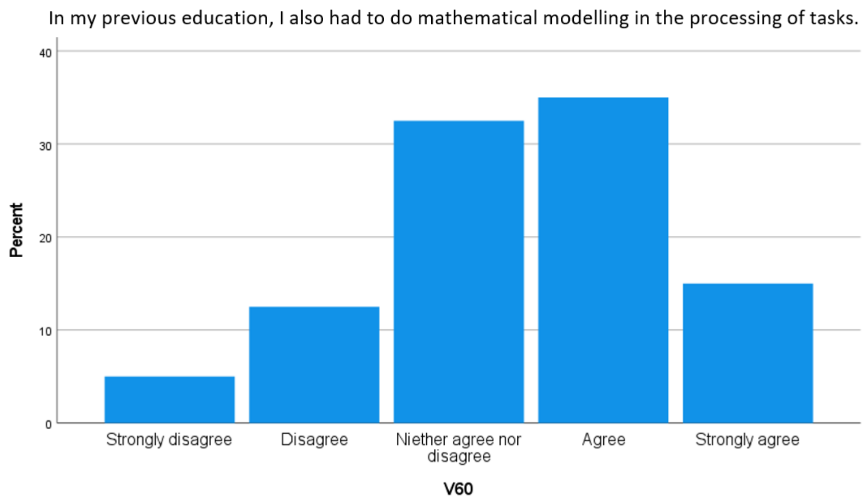

Figure 4.5 In my previous education, I also had to do mathematical modelling in the processing of tasks.

Table 4.23 below provides more information about mathematics teachers' choices of doing modelling tasks in their previous education. Looking at the cumulative percentage, 50% of the teachers are unconfident (32.5%) and disagreed (17.5%) with the question that "In my previous education, I also had to do mathematical modelling in the processing of tasks". Subsequently, 50% of the teachers agreed (35%) and strongly agreed (15%) that they applied modelling processes during the process of solving modelling tasks.

Table 4.23 In my previous education, I also had to do mathematical modelling in the processing of tasks.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly disagree	2	5.0	5.0	5.0
	Disagree	5	12.5	12.5	17.5
	Neither agree nor disagree	13	32.5	32.5	50.0
	Agree	14	35.0	35.0	85.0
	Strongly agree	6	15.0	15.0	100.0
	Total	40	100.0	100.0	

Though it was difficult (due to confidentiality purposes) to identify teachers who agreed with the questions, to interview them, and get a deeper understanding of the kinds of modelling processes they applied during solving mathematical tasks. This would be interesting to see if the modelling processes they used aligns with the ones from the modelling cycle by Blum and Leiss (2007).

4.4.12. Teachers' experiences with mathematical modelling in various contexts

The test items used to measure teachers' experiences with mathematical modelling in various contexts were not only focused on ISTs' past experiences with modelling but also on whether with their acquired knowledge are they able to transmit the modelling knowledge and skills to the learners in the classroom.

The question "I have already taught mathematical modelling with learners" was used to investigate if teachers have taught mathematical modelling before to the learners in the classroom. Table 4.24 shows that twelve of 40 (30%) teachers haven't taught modelling before to the learners. Out of the 30%, 5% of the teachers showed strong disagreement with teaching mathematical modelling to the learners. Eleven of 40 (27.5%) teachers showed uncertainty about whether they have taught mathematical modelling to the learners before. In consolidation of the data, 57.5% of the participants were not sure if they have taught modelling to the learners before, while 42.5% of the teachers showed agreement that they have taught modelling to the learners.

Table 4.24 I have already taught mathematical modelling to learners.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly disagree	2	5.0	5.0	5.0
	Disagree	10	25.0	25.0	30.0
	Neither agree nor disagree	11	27.5	27.5	57.5
	Agree	14	35.0	35.0	92.5
	Strongly agree	3	7.5	7.5	100.0
	Total	40	100.0	100.0	

Looking at the overall mean (3.13) of the data from the three questions (refer to Table 4.25 below), it showed the uncertainty of teachers of whether they have taught mathematical modelling to the learners before. The results further showed that teachers do also agree that they have gained experience in teaching mathematical modelling to the learners in the classroom.

Looking closer at the overall mean, indicated that most teachers are unconfident with teaching modelling in the classroom as the mean scale is around rate 3 'neither agree nor disagree'. A bigger Std. Deviation was observed in Table 4.25 below across all the tested questions. The questions about teachers teaching modelling to the learners showed an Std. Deviation of 1.051, indicating a greater variation of the data around the mean. looking at the 'role played by modelling to the teachers during their first year of teaching', showed an Std. Deviation of 0.959 which is considered as less variable of the data around the mean. Then teachers' experiences in mathematical modelling showed a Std. Deviation of 1.137 which portrayed a greater variance of the data around the mean. The overall Std. Deviation (0.607), which consolidates the results to be less variance around the mean. This shows different competencies and confidence among the teachers about teaching mathematical modelling in the classroom.

Table 4.25 Teachers' Experiences about mathematical modelling in various contexts

		I have already taught mathematical modelling with learners.	Mathematical modelling played a role in my first year of teaching at school.	I have already gained teaching experience in mathematical modelling.
N	Valid	40	40	40
	Missing	0	0	0
Mean		3.15	3.05	3.20
Median		3.00	3.00	3.00
Mode		4	3	4
Std. Deviation		1.051	.959	1.137
Range		4	4	4
Sum		126	122	128

Looking at the role played by modelling to the teachers during their first year of teaching basic education. Only thirteen of 40 (32.5%) teachers agreed and showed confidence that the knowledge and skills about mathematical modelling they acquired from their teacher training courses played a significant role in their first year of teaching mathematics in schools. But eleven of 40 (27.5%) teachers showed disagreement with the role played by the learned modelling in their first year of teaching. For clarity purposes, 11 mathematics teachers explicitly disagreed that mathematical modelling learned from university played a significant role in their first year of teaching mathematics at school.

Table 4.26 Mathematical modelling played a role in my first year of teaching at school.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly disagree	2	5.0	5.0	5.0
	Disagree	9	22.5	22.5	27.5
	Neither agree nor disagree	16	40.0	40.0	67.5
	Agree	11	27.5	27.5	95.0
	Strongly agree	2	5.0	5.0	100.0
	Total	40	100.0	100.0	

Furthermore, 40% (see Table 4.26 above) of the mathematics teachers showed a lack of confidence in the role played by mathematical modelling in their first year of teaching, while 45% (see Appendix C) of the teachers did confirm that they have gained knowledge and experience of teaching mathematical modelling in the classroom.

4.4.13. Research question 3 results summary and conclusion

Results presented showed that 75% of the mathematics teachers agreed that they have learned modelling from their lectures, while 17.5% of teachers were not sure of whether they have learned modelling. Further 62.5% of the teachers did confirm that modelling has been addressed in their teacher training courses and 20% of the teachers are uncertain, with 17.5% of them showing disagreement. 40% of the teachers were not sure whether modelling has played a significant role in their first year of teaching mathematics, with only 22.5% of the teachers showing agreement with the role played by modelling.

57.5% of the teachers showed disagreement with being prepared to teach mathematical modelling in the classroom, with 42.5% of the teachers showing agreement with being prepared to teach mathematical modelling to the learners in the classroom. 70% of the teachers agreed that they have been exposed to modelling tasks before during their teacher training, and 10% of them showed disagreement with modelling exposure during their teacher training courses.

57.5% of the teachers showed that they have never taught modelling content to the learners before, with some of them being unsure. 42.7% of the teachers agreed that they have taught modelling content to the learners in the classroom before.

4.5 Interviews data presentation (qualitative research method)

Research question 4: What are in-service teachers' perceptions about modelling content in the CAPS Grade 10 – 12 curriculum?

Introduction

The originally collected data comprised recorded voices. After transcribing the collected data, four utmost themes came out from the responses of the interviewed three teachers with regards to their perceptions about modelling content in the Grade 10 – 12 CAPS curriculum:

1. Teachers are more product-driven than process.
2. Learners should take the lead during mathematical modelling.
3. The curriculum timeframe limits learners from exploring modelling.
4. Limitation of content in the mathematics curriculum.

I briefly discussed each of these 4 themes in the following sections in paragraphs. All three interviewed ISTs reported teaching Grade 10 – 12 mathematics, including modelling. The researcher was the interviewer. The responses to the questions did not follow the sequence of codes (interviewees).

Table 4.27 below shows demonstrated results of the interviewed teachers to what extent are they aware of mathematical modelling in the CAPS curriculum. The results showed that the interviewed teachers are somewhat aware of modelling in the curriculum.

Table 4.27 Results linked to interview question No. 9

To what extent are you aware of the CAPS curriculum with mathematical modelling?			
1. Aware	0		
2. Somewhat aware	3		
3. Not aware	0		

The stood-out themes are briefly presented in the following sections below:

4.5.1 Teachers are more product-driven than process.

Researcher: What is your belief about learner involvement in mathematical learning tasks? Should the teacher provide step-by-step guidelines on how learners should solve modelling tasks? Or should the teacher allow learners to work on the tasks by themselves? Why or why not?

KN: …the system puts pressure, and the system is very results focused. So, teachers will get things done so those things in terms of like the problem-solving and in terms of developing those models should start at an early grade…"

NM: … you can use both I don't believe that you should use one in one of them in isolation I feel like it would be best interest of the child to use them both…"

Here, KN revealed that the education system puts pressure on teachers to complete the curriculum (syllabus). While NM supports the notion that the types of teaching should be used concurrently when teaching in the classroom. This means there's more mathematics content to be covered in a short period in an academic year, which makes it difficult for them to allow learners to engage in mathematical modelling processes. The teachers also mentioned that the system is very results-focused, which puts pressure on teachers to get good grades in the classroom. Hence, teachers will not focus on conceptual understanding, but the focus will be more on procedural knowledge. KN suggests a solution to the problem raised, the teacher mentioned that problem-solving which is modelling should start at an early grade level, which is a primary school in the South African education context. This states that such skills and knowledge should be started from lower grades where the learner's brain is still in the early development learning process. By the time they come to secondary school, it will be easy for them to develop modelling skills from what they already know or learned before.

The teacher also raised an issue that learners don't want to take up the opportunity of thinking of mathematical solutions for themselves. They mostly want to be given everything by the teachers. That ends up drifting teachers' focus to only giving them the final product of the task and not engaging learners in the process of how the modelling tasks were modelled. The teacher in the interview also mentioned that one of the reasons why teachers do not allow learners to take the lead during modelling is that they often make unrealistic or unreasonable solutions during the building of a mathematical model. Hence, teachers in most situations shave to provide learners with a starting point for them to continue to solve the modelling tasks.

4.5.2 Learners should take the lead during mathematical modelling.

Table 4.28 below shows how often participating teachers responded to the three questions (learners should take the lead, teachers should take the lead and there is not enough time) provided in the table. The question of whether 'learners should take the lead', was mentioned three times. Whether 'teachers should take the lead', was mentioned once during the interviews. The last question was checking how often teachers disclosed that 'there is not enough time' from the curriculum to let learners take the lead when solving modelling tasks, as demonstrated from the table, it was mentioned three times.

Table 4.28 Results linked to interview questions No. 3 & 4

Do you believe that learners should take the lead during modelling or teachers should take the lead during modelling processes?			
1. Learners should take the lead	3		
2. Teachers should take the lead	1		
3. There is not enough time	3		

It can be seen from the table that interviewed teachers encourages learners during modelling tasks to take the lead when approaching modelling tasks, but teachers raised that it is not always the case that learners will be given the opportunity to approach modelling tasks on their own. NM raised that letting learners take the lead in modelling "… depends on a certain situation, for example, this one I can give them step-by-step procedure but this one I can allow them to work on their own." (MN, line 7). While KN revealed that there is not enough time to allow learners to take the lead, "I think that we should allow them to come up with their ideas, however from experience that is a big task not because it cannot be done but because the system puts pressure, and the system is very results focused…"

Engaging deeply in the participating mathematics teachers' interview responses:

> **Interviewer:** do you believe that teachers should take the lead in solving modelling tasks for the learners or do you believe that learners should take the lead in solving modelling tasks?
>
> **NM:** I think learners should take the lead. Why am I saying this? because at the end of the day, I'm a teacher so I've been taught this at the university level right yeah and what steps to follow and how to teach mathematical modelling now it becomes different when it's a learner, learners should have the freedom of approaching modelling the way they understand it.
>
> **SN:** My opinion is that learners should take charge of their learning process for them to conceptually understand the ideas and theories behind mathematical tasks …

Here, from the excerpts extracted from teachers' responses during interviews, I observed that teachers NM and SN are more of a constructivist belief, showing that learners should be given the opportunity to do or approach modelling tasks the way they understand them. The teacher supported her argument by saying that giving learners freedom during the modelling cycle will allow learners to think outside the box during modelling lessons and mathematics in general. The teacher also revealed that they have done mathematical modelling at the university level during their teacher training, the teacher further added that they were taught which steps to follow during modelling and how to teach mathematical modelling as it is quite a different content when it's a learner.

> **Interviewer:** what is your belief about Leaner's involvement in mathematical learning? should the teacher provide step-by-step guidelines on how to solve modeling task or should the teacher allow learners to do the work on their own before the teacher intervene?

NM: in my personal view I feel like those two points can be utilised together and at some point, you can give them step-by-step, and some situations give them freedom of using their ways of solving mathematics.

However, the teacher also provides that giving learners an opportunity to do modelling on their own depends on each modelling type question. NM believes that during modelling processes, in some situations a teacher can take the lead on how to approach modelling in the classroom, and in some situations, learners should take the lead in approaching modelling tasks. The teacher said the decision may arise from whether the difficulties of the provided modelling task have been addressed to them before or it's a new level of modelling tasks that still need to be addressed to the learners.

4.5.3 The curriculum timeframe limits learners from exploring modelling.

Researcher: What can you say about the time given in the curriculum to teach mathematics in general and mathematical modelling?

KN: time is very limited for them to come up with that it's not an ongoing thing it's not like an algorithm that we can go whenever they feel like going to because the curriculum has a start date and end date and within that time, they need to think of those solutions which they don't want to do because of time limits.

NM: “In a mathematics class there is not enough time to complete the syllabus …as a maths teacher I sometimes need to schedule Saturday classes to push the content with the learners, because some of them are grasping the content slower than others”

It cannot be ignored that the curriculum also provides timeframes for each content to be covered in an academic year, and 100% of the content within the curriculum should be finished within the timeframe so that learners can be assessed.

KN's and NM’s responses revealed that though the curriculum claims to have given teachers enough time to cover the content, the interviewed teachers feel like the time is not enough to cover the content and let learners explore what is learned in the classroom. This is because teachers limit themselves to completing the syllabus and assessing learners. According to Bonyah and Clark (2022) at the secondary school level, learners should be taught in such a way that they will be able to construct their knowledge rather than teachers teaching them for the sake of tests and examination purposes only.

KN: "...modelling in our curriculum is not that dominating because you find that from the textbooks only 5 modelling problems out of 50 maths problems from a maths textbook are asked…"

Here, the response from the teacher demonstrated that mathematical modelling is not that visible in the curriculum, it partially appears in some topics (e.g., Finance, Statistics, and Measurements) and is not visible in some of the topics (e.g., Algebra). The teacher also mentioned that the structuring of the content and examples from the curriculum also hinders modelling in some way, during an interview under the SEF interview, the teacher mentioned that you find that there are one of fifty modelling examples in the textbooks. This limits learners from exploring more modelling content in the curriculum. This confirmed the results obtained in the study by Mudaly (2004), that mathematical modelling in SA needs modelling-based pedagogical reform for the teachers.

KN touched on how their self-efficacy for mathematical modelling was compromised, the teacher mentioned that it is difficult to evaluate their abilities of recognising learners' abilities during their modelling cycle because only a few modelling problems are asked in the textbooks. The teacher supports her claim by saying "… out of 50 mathematics problems in the curriculum only 1 or 2 of them are modelling related, the rest of the questions are just abstract mathematical problems that most are not even applicate to learners' real-life". (KN, line 9)

4.5.4 Limitation of content in the mathematics curriculum

Researcher: Regarding modelling in the CAPS curriculum Grade 10 – 12, what recommendation can you give to improve the teaching and learning of applications and mathematical modelling? Should modelling be scaled to other school levels including the primary level? Why or why not?

KN: … yeah, I feel like some of the topics don't fit into learners real-life experiences, such as algebra, it is more abstract but those skills are needed by the learners to solve mathematical problems … also it seems to be a lot to cover but in the curriculum that end up compromising the learning interactions among the learners'… so yeah I feel like the content in the mathematics curriculum should be reworked and provide more time for better learning of mathematics and modelling"

SN: I would suggest that every mathematics topic in any grade be given at least 40% of application content for them to apply maths in reality.

Here, the teachers were showing recommendations on how teaching and learning mathematical modelling can be strengthened and developed. From the excerpts, teachers suggested that the content in the mathematics curriculum should be limited in such a way that teachers will have enough time to engage learners. Additionally, learners will have time to build a model, make assumptions and mathematise the problem.

Mathematical modelling should be broadly part of the mathematics curriculum (Mudaly & Dowlath, 2016). However, teachers' responses, showed that modelling in the curriculum is neglected and only a few tasks of it are provided in the textbooks. This might be one of the factors why teachers showed low SEF and awareness of mathematical modelling in the curriculum.

4.5.5 Research question 4 Summary and conclusion

From the presented results of the interviews, four major themes stood out in the qualitative data. The themes were 'teachers are more product-driven than process', 'learners should take lead during mathematical modelling', and 'curriculum timeframe limits learners exploring modelling and limitation of content in the curriculum'. From the themes, teachers also shared their perceptions and recommendations of mathematical modelling in the CAPS curriculum. Their perceptions showed that teachers are somewhat aware of modelling, and they recommended that more modelling tasks should dominate in the curriculums and textbooks, and reality content in mathematics should be given more time to be covered.

4.6 Chapter summary and conclusion

The presented results about teachers' beliefs about modelling in the curriculum evidenced that the majority of the participating mathematics teachers do believe in the existence of modelling in the mathematics curriculum. Though they raised concerns about its little predominance in the curriculum. With their previous experiences with modelling in the curriculum, teachers demonstrated that they do have existing knowledge about modelling. Unfortunately, they raised that learned modelling knowledge did not help them during their first year of teaching. That might be the cause of the teacher's low SEF about modelling to recognising learners' abilities when modelling tasks.

The following chapter 5 provides a concise discussion of the data presented above with the theories and findings of other related studies of modelling.

CHAPTER 5: DISCUSSION OF THE RESULTS

5.1. Introduction

The objective of the discussion chapter is to explain, interpret, describe, and related the obtained results with the theories and other research that have been carried out in line with the current study. Discussion of the results further showed the significance of the results and findings demonstrated in Chapter 4 in connection with the theoretical framework (a structural model for teaching modelling) that guided the study, recent literature, and the realisation of the research problem. This chapter is important as it shows new knowledge in line with the current literature that has been reviewed from the chapter, and further contributes knowledge to the research problem pursued in the study.

The discussion chapter is arranged in terms of the four research questions which underpinned the focal point of this dissertation.

1. What are in-service teachers' beliefs about teaching and learning mathematical modelling in the CAPS curriculum?
2. What are in-service teachers' self-efficacy about teaching and learning mathematical modelling in the CAPS curriculum?
3. What are in-service teachers' prior knowledge of mathematical modelling in the CAPS curriculum?
4. What are teachers' perceptions about modelling content in the Grade 10 – 12 CAPS curriculum?

It is significant to re-visit the theoretical framework that underpinned the research and the themes that came out because of the findings before delving further into the discussion of the results. The study's goal was to investigate in-service teachers' beliefs and self-efficacy using a structural model of professional competence for teaching modelling. Furthermore, to understand teachers' perceptions about teaching modelling in the Grades 10 - 12 CAPS mathematics curriculum.

The goal of the study was realised using a structural model of professional competence for teaching mathematical modelling.

From research question 1 to research question 3 scales were used to answer each main theme, which is belief, self-efficacy, and prior knowledge. From research question 4 which is teachers' perceptions about modelling in the Grade 10 – 12 CAPS curriculum about mathematical modelling, four themes stood out during the explanation of the transcribed data from the interviews. The themes were "teachers are more product-driven than process", "learners should take the lead during mathematical modelling", "The curriculum timeframe limits learners from exploring modelling" and "limitation of content in the mathematics curriculum."

Table 5.1 Themes and Components of the Study

Research question 1	
Scales	**Components**
Research question 1: Teachers' beliefs about mathematical modelling	• Modelling in the classroom • Application of modelling • Constructivist belief • Transmissive belief
Research question 2: Teachers' prior knowledge of mathematical modelling	• The general role played by mathematical modelling from the ISTs courses. • Preparedness of ISTs for teaching mathematical modelling • Nature of modelling tasks teachers have encountered during their educational journey. • Teachers' experiences with mathematical modelling in various contexts
Research question 3: Teachers' self-efficacy about mathematical modelling	• Mathematical modelling • Working mathematically
Research question 4	**Emerged themes**
Research question 4: Teachers' perceptions about mathematical modelling in the CAPS curriculum	• Teachers are more product-driven than process. • Learners should take the lead during mathematical modelling. • Curriculum time frame limiting learners from exploring modelling. • Limitation of content in the mathematics curriculum.

5.2. RQ 1: What are in-service teachers' beliefs about teaching and learning mathematical modelling in the CAPS curriculum?

5.2.1. Introduction

Answering research question 1, from the presented results in chapter 4 it was discovered that mathematics teachers do believe in teaching and learning mathematical modelling in the CAPS curriculum. There was an interesting debate from the presented results between teachers' transmissive and constructivist beliefs in mathematics classrooms.

How learners learn mathematics in general including modelling and how they approach mathematical tasks heavily relies on teachers' beliefs about the mathematics application in the classroom (Chamberlin et al., 2022). Hence, teachers' beliefs can be viewed as a major fountainhead for teachers' pedagogy and preparation (Chamberlin et al., 2022; Kaiser & Maaß, 2006; Voss et al., 2013a). It was then necessary for this study to first investigate teachers' beliefs about teaching and learning mathematical modelling to acquire perceptions of their modelling competency.

5.2.2. Beliefs about mathematical modelling in the classroom

The presented results depicted that thirty-eight of 40 (95%) teachers showed positive belief about modelling, which also showed their willingness of making modelling to be part of the curriculum. Twenty of the 40 (50%) teachers believed that mathematical modelling is more practical than reality. This shows that our teachers do understand and acknowledge the value of modelling in the curriculum which will ensure that their learners are included during the lessons and that they see the value of mathematics in their lives. That is teaching learners what has already been experienced, other than teaching them what does not apply to their daily life experiences. Furthermore, modelling content is structured in such a way that learners link what they have learned in the classroom with their daily-life experiences. This disagreed with the hypothesis of the researcher because it was hypothesised that teachers do not show a strong belief in mathematical modelling in the classroom since it is less applied during teaching (Blomhøj, 2019). But since most of the content taught starts from the CAPS curriculum, what the curriculum caters to that area. The hypothesis was based on the literature reviewed that teachers do not integrate most of their teachings with modelling (Blomhøj, 2019; Blum, 2015) because they believe modelling is difficult and challenging to manage in the classroom during a lesson (Durandt et al., 2022; Schmidt, 2011) and they are less aware of it. According to research studies (see Blum, 2015; Passarella, 2021), it is significant for teachers to implement both modelling and problem-posing in the classroom. It can be seen from the presented results that teachers do believe in mathematical modelling in the classroom, and they are also eager if learners should be presented with a chance to approach mathematical modelling tasks independently. This support constructivist beliefs (positive) instead of transmissive belief (negative).

5.2.3. Beliefs about the application of mathematical modelling

It is observed from the presented results that teachers agreed and acknowledge that mathematical modelling is more practical than reality. This is a strong belief as it proves that teachers are conscious of the real-life context embedded in modelling content. That means teachers will know the types of modelling tasks they should present to the learners, and it gives relief that modelling tasks that will be done in the teachers' mathematics classes will be more relevant to learners' daily-life experiences, for them to perceive the link of school mathematics with their daily-life situations. As Blum (2015) put forward comprehension and learning of reality situations require a clear engagement with the appropriate application of the modelling tasks and modelling examples. Modelling tasks that comprise reality situations seen before makes it easier for the learners to make assumptions and evaluations since it will probably be situations that they have experienced before. In these kinds of situations, a sufficient shift from intra-mathematical activities cannot be anticipated. Thus, extra-mathematical activities will do as they are conceived as related to real-world situations (Niss & Højgaard, 2019).

Teachers contested that modelling is a productive game with concrete to reality. This aspect supports the notion that modelling should start from primary education so that when learners come to the secondary school level, they are aware of it, and competency will be easily developed and enhanced. These findings confirm what was discovered in the study by Wei et al. (2022) that Interventions for professional development in elementary schools can improve the quality of mathematical modelling instruction in secondary schools, which will then positively affect both lower-level education and higher education. It should be made clear that using mathematical tools to solve real-world problems is viewed as the application of mathematics. As Bahmaei (2014) acknowledges that the application into real-world situations cannot be separated from using models and modelling processes. When teachers show belief in the application of mathematical modelling it means they do believe that mathematics is indeed applicable to our daily-life experiences and that belief will work in the best interest of the learners to see the link between mathematics learned from school and their daily-life experiences. (Mazana et al., 2019; Russo et al., 2020) mentioned that learning mathematics comes with enjoyment and if only abstract concepts are often learned in the mathematics classroom, learners will be demotivated and start showing some resistance to learning mathematics with modelling. Hence, teachers will face more difficulties in promoting modelling competencies among the learners.

From the question “Many aspects of mathematical modelling are used practically in reality.” (Refer Appendix J), thirty-six of 40 (90%) teachers strongly agreed that many features of modelling are used practically. This also supports the notion that our teachers are willing to show learners the link between mathematics and reality. If our teachers strongly believe in the applicability of modelling in real life, then it will be easy to transfer this kind of belief to the learners in the classroom (Chamberlin et al., 2022).

5.2.4. Constructivist beliefs

Simamora and Saragih's (2019) proposed that constructivist theory should be the foundation of mathematics teaching pedagogies since it is likely to help learners' modelling abilities. The results from constructivist belief in this study were looking good as thirty-six of 40 (90%) teachers agreed that learners should be given independence in the classroom to do modelling tasks. During the analysis of the interviews conducted, one of the teachers showed that s/ he supports teaching modelling constructively, but the teacher further said:

> "…you need to be careful about how you do it … for example from the question you asked me under self-efficacy about making assumptions, sometimes our learners make crazy assumptions that are not even related to maths … so basically letting them to be independent needs strategic guidance from the teacher for them to make relevant assumptions" (KN, line 26).

That is to confirm that the SA teachers who partook in the study are constructivists when teaching mathematics, including modelling. As mentioned, teachers' beliefs in the classroom drive the desired learners to be constructed and developed in the mathematics classroom (Chamberlin et al., 2022). Seeing that teachers believe in teaching mathematics constructively, that is allowing learners to take the lead during solving modelling tasks. That also portrays that teachers are professionally competent in teaching the subject. As Blömeke, Kaiser, König and Jentsch (2020) described competence is the collection of capabilities and expertise, and the readiness of a learner to solve a modelling task and to evaluate their written solution. This kind of competence and willingness should start from teachers before learners can acquire them. It is good that most of the teachers are constructivists. Hence, thirty-six of 40 (90%) teachers agreed to be constructivists when teaching modelling tasks.

Biccard and Wessels (2011) raised that the existence of teaching and beliefs about mathematics can change teaching approaches of the traditional modelling approach currently used, and that may point towards positive beliefs about mathematics in general, on how it is learned and how it applies to real-life experiences. It is acknowledged (see Biccard & Wessels, 2011) that the construction of beliefs takes time, meaning if learners from the primary level are taught by teachers who are more transmissive beliefs than constructivists, those learners will have a challenge in changing to constructivist learning type, and learners who are taught by teachers who are more of constructivists will be easy for them to learn modelling constructively. Thirty-two of 40 (80%) teachers at the secondary school level (Grade 10 – 12) from the study believed that learners learn better by discovering their ways of solving mathematical tasks instead of being given step-by-step procedures. Blum and Ferri (2009) argued that for quality teaching of mathematical modelling, maintaining a permanent balance between teachers who are constructive believers and learners who can model tasks independently is very vital. Qualitative results from the interviewed teachers indirectly supported the use of strategic interventions to support learners as they are considered to have greater potential in supporting learners to overcome difficulties faced by learners during a modelling cycle (Stender & Kaiser, 2015). Teachers' constructivist beliefs were then linked with teachers modelling competence.

5.2.4.1. Teachers' modelling competence.

There were no specific questions from the questionnaire that were reserved to investigate teachers' modelling competence for teaching mathematical modelling. At this level of study, it is difficult to measure the competence of a certain aspect among teachers. This is because "competence is perceived as a complex construct that addresses key aspects of the professional debate" (Wess et al., 2021). But an attempt was made to give some knowledge about teachers' competence in teaching mathematical modelling. Revisiting teacher professional competence (PC), it was described as individual insightful abilities to act or to meet challenges in each context (Blomhøj & Højgaard, 2003). From the description of competence by Wess et al. (2021), in this study competence was viewed as teachers' knowledge and skills to constructively teach learners modelling through the modelling cycle and allow them to be competent in modelling independently. From this description of competence, an attempt to measure teacher competence for teaching mathematical modelling was made through constructivist belief. Since constructivist belief promotes constructive teaching and learning of mathematical modelling in such a way that learners will conceptually understand the processes of modelling and be able to approach and discuss modelling tasks independently.

From this study, I hypothetically linked teachers' competence with their constructivist beliefs, meaning, the more a teacher is a constructivist the more the teacher is considered as PC for teaching modelling. Hence, participating teachers seem to be competent in teaching modelling since their results showed that they are constructivists.

5.2.5. Transmissive beliefs

Beliefs are an important construct in mathematics education, and thus in the IST's mathematics education. As described by Wess et al. (2021) that transmissive and constructivist beliefs correlate negatively due to the non-identical theoretical views of teaching and learning mathematical modelling. For teachers to promote the development of modelling sub-competencies to the learners, it is advantageous to look at teachers' orientation beliefs for teaching mathematical modelling. As for “Teachers should provide detailed procedures to learners for solving application problems.”, teachers seem to be not sure if they should provide detailed procedures of a modelling task to the learners in approaching modelling tasks. This might be resulting in the IST's not knowing how to develop and enhance modelling competencies among the learners. Because it is widely acknowledged by authors (see Bonyah & Clark, 2022) that teachers, according to the structure of the South African curriculum often teach for examination purposes. Meaning they provide learners with content knowledge that will be examinable only, which is more procedural knowledge instead of conceptual knowledge. As the Ministry of Education (Wei, Zhang & Guo, 2022) in Singapore emphasises the significance of applications and modelling in the curriculum.

Looking at how teachers believe in terms of how they taught their learners, focusing on the presented results for “Learners should solve tasks in the way they were taught in class.” most of the teachers neither agreed nor disagreed. This might have resulted in teachers' experience of how learners think in the classroom during a lesson, as cited above that one of the teachers during the interviews mentioned that s/ he does not usually give learners the opportunity to bring up their ways of solving tasks because they make assumptions that are not relevant to real questions. I also put forward that for learners to be able to think of relevant assumptions and make unreasonable conclusions. That should start from primary education (Bahmaei, 2014; Wei, Zhang & Guo, 2022) so that when they get to secondary education, they have the foundation and know what is expected of them through the process of learning modelling.

The presented results showed that 65% of the participating teachers disagreed with the question of giving learners step-by-step procedures on how to solve modelling tasks in the classroom. This is good as it shows that teachers are not transmissive in teaching modelling but are constructivists. But looking at the overall mean (3.64), from sub-section 4.2.4 it showed that teachers neither agreed nor disagreed, this is worrying as teachers are not sure if they should provide a step-by-step detailed procedure to the learners or let them discover their ways of approaching modelling tasks, that is them taking the lead during modelling tasks. Overall, teachers seem to be not sure if they should teach learners conceptually or procedural, which is constructivist and transmissive respectively.

5.3. RQ 2: What are the in-service teachers' self-efficacy about teaching and learning mathematical modelling in the CAPS curriculum?

5.3.1. Introduction

In answering research question 2, the presented results showed that teachers lack SEF abilities to recognise learners' abilities during their modelling processes. The results demonstrated teachers' efficacy for teaching modelling and indicated that promoting the progression of modelling competencies of the learners and engagement during a lesson might be compromised.

Teachers' SEF in mathematical modelling was used as a main component to investigate teachers' efficacy to identify learners modelling abilities through mathematical modelling and working mathematically with learners' abilities during the modelling cycle. SEF was regarded as teachers' efficacy for their judgments based on their abilities to bring about desired results of learners' involvement and learning in the modelling classroom. They are significant for teaching and impact the performance, beliefs, and incentives of learners (Philippou & Pantziara, 2015).

5.3.2. Self-efficacy for mathematical modelling

Findings showed that ISTs gave themselves low scores based on their self-efficacy to recognise learners' abilities during modelling processes, such as building a model, simplifying, mathematics, interpreting, and validating. Only in the question of "… their translation of mathematical results into reality", ISTs gave a higher score. I believe the high level of efficacy for this question occurred because teachers do assist their learners in making assumptions about a real problem. As mentioned by one of the teachers, learners are not mathematically competent to make their assumptions with reality-posed questions. Interviewed teachers claimed that their mathematical modelling SEF increases when they are teaching more modelling content.

> "… Modelling requires a lot of time and engagement in the classroom; thus, it might not be effective in the current curriculum since time per topic is very limited …" (SN, line 5)

> "It is difficult to be a master of problem-solving while not doing it daily, as I told you before that out of 50 problems per topic you find that only 5 problem-solving are provided …" (KN, line 12)

This also proves the assumption that it's difficult for teachers to be competent in modelling while they are not teaching it daily. Moreover, the obtained results showed that ISTs have low SEF abilities for them to recognise learners' abilities during modelling phases. Hence, learners will also find it difficult to develop such modelling competencies if their teachers do not sufficiently have them. Additionally, some modelling questions from the CAPS curriculum are not related to learners' daily experiences, thus it will be difficult for them to make assumptions about something they have not experienced before. As research studies (see Lesh & Caylor, 2007; Maaß, 2006) expressed that learners made richer assumptions of modelling tasks if they met with the problem situations which were comprehensible in their reality. This is in line with what was discovered by Durandt et al. (2022), that learners who had inadequate experiences with the reality of the problem had problems in simplifying it.

Concerning teachers' prior knowledge (see section 5.3), teachers seem to have sufficient prior knowledge of mathematical modelling but insufficient competence in it because they don't engage with it in the classroom daily. It is acknowledged (Wess et al., 2021) that mathematics teachers' SEF differs according to modelling phases. Simplification, mathematisation, interpretation, and validation are stages that are particularly possible to identify in the modelling process (Blum & Lei, 2007). In particular, the modelling phases of learners can be found where the tasks can be readily followed in their written mathematical work and where the learners' modelling stages are not exclusive to the modelling process.

5.3.3. Self-efficacy for working mathematically.

Greefrath and Vorhölter (2016) put forward that learners apply heuristic[28] strategies and mathematical knowledge and skills to solve mathematical problems, under the modelling sub-competencies this is categorised as working mathematically. As mentioned, that modelling is a process that involves several steps before reaching the solution. During the modelling processes, it is possible for a modeller to experience difficulties (Galbraith et al., 2007; Schaap et al., 2011; Stillman et al., 2013). From the presented results of the study, teachers seemed to be not aware that modelling is a process that includes a 7-step modelling cycle (Blum and

[28] A heuristic approach is a temporary and logical process designed to find the answer to a specific situation at hand. (1963, p. 13; Feigenbaum & Feldman)

Leiss, 2007), as one of the emerged themes from the qualitative data analysis revealed that teachers working under the SA CAPS curriculum are ' product-driven than process'. Hence, according to the presented results, teachers focus more on the final answer than the process when learners are solving a mathematical task in general, including modelling tasks. Klock and Siller (2020) listed possible difficulties that may be faced by modellers (learners) during a modelling cycle. In this section, I only focused on difficulties that may be faced by working mathematically, which is a modelling sub-competency step during a modelling cycle. Difficulties under working mathematically include failing to use appropriate formulas/ symbols[29], struggling to use appropriate algorithms and solutions[30], failing to understand the given situation conceptually[31], failing to solve the problem due to the heavy/ complexity of mathematics and struggling to convert between units.[32]

As I examined the presented results for "… identify the different abilities of the learners using their handling of the mathematical formulae and symbols used in modelling", the results showed that eighteen of the 40 (45%) teachers do agree that it's not easy for them to identify learner's abilities based on how learners handled mathematical formulas and symbols. Furthermore, six of the 40 (5%) teachers are not sure of their working mathematical SEF abilities. This reveals that learners will be solving modelling tasks without guidance from their teachers to improve their modelling competencies. This might be resulting in teachers not having reasonable time to engage in modelling competencies. It is assumed that most of the teachers teach modelling as other mathematical tasks that do not follow the 7 steps of a modelling cycle.

As for teachers "… identify the different abilities of the learners using their written solutions when modelling.". The presented results of the study showed that teacher's selection of choices regarding this question was central, as seventeen of the 40 (42.5%) teachers agreed that it is difficult for them to identify learners' abilities from their written solutions, and further ten of 40 (25%) teachers are not sure. It cannot be argued that teachers modelling competencies are still developing as it is confirmed from the study that in-service mathematics teachers in South Africa are not sufficiently exposed to modelling tasks in such a way that they can develop them. Though Winter and Venkat (2013) argued in their study that teachers' modelling competencies

[29] Galbraith and Stillman (2006)
[30] Galbraith and Stillman (2006); Schaap et al. (2011)
[31] Stillman et al. (2013)
[32] Maaß (2005)

are still developing and show being in the early stages, it was not revealed on the kind of process or strategies they are engaging in, to prove that their modelling competencies are developing. Being competent in modelling is difficult not only for learners but for mathematics teachers as well (Jahnke & Hefendehl-Hebeker, 2019). Because according to the constructivist-oriented view (see section 5.2.3) of learning modelling, learners are encouraged to develop their models and find solutions for themselves. In this sense, teachers can only foresee what learners will do during a modelling cycle (Jahnke & Hefendehl-Hebeker, 2019), the results from this study showed disagreement with what the authors discovered in their study as teachers from my study seemed not to be confident in diagnosing and intervening spontaneously (Tropper et al. 2015) during learners modelling processes.

5.3.4. Research question 2 discussion summary

The overall discussion of research question 2, which intended to answer the in-service teachers' self-efficacy about teaching mathematical modelling in the CAPS curriculum. The results and discussions revealed that teachers have sufficient prior knowledge about mathematical modelling, but not much modelling content knowledge from the CAPS curriculum. Hence, that hinders teachers from developing capabilities of identifying learners' abilities based on their modelling phases. Teachers showed that they wish to implement mathematical modelling in their everyday lessons but do not know how to start it. This was also acknowledged by (Jahnke & Hefendehl-Hebeker, 2019).

5.4. RQ 3: What are in-service teachers' prior knowledge of mathematical modelling in the CAPS curriculum?

5.4.1. Introduction

In answering research question 3, from the presented results teachers showed that they have sufficient prior knowledge about mathematical modelling in the curriculum. Furthermore, results demonstrated that teachers feel well prepared to teach modelling in the classroom. The prior knowledge component was used to investigate teachers' prior knowledge about mathematical modelling from their early grades up until now as qualified mathematics teachers. The component from this scale focuses on the general role played by modelling from the ISTs training, on their extent to preparedness to teaching modelling, the type of modelling tasks they have seen, and their previous experiences about modelling in various contexts.

5.4.2. General role played by mathematical modelling from the ISTs courses.

There was a conflict of choices and confidence from the presented results. Teachers agreed that they have learned mathematical modelling before, but they are not sure if it has been addressed in their teacher training courses. Here, what was observed was that teachers may have done modelling during their teacher training without realising they are doing modelling. NM from the interviews said, "… actually from when I became a qualified teacher and started teaching mathematics, it came to my mind that I did modelling without realising I was modelling tasks …". From this, it can be generalised that modelling in some of the South African universities is not assigned to as a module or course on its own, but it comes from different topics randomly. As Tran et al. (2021) said teaching and learning mathematical modelling is cognitively demanding. It should be kept in mind that modelling is not about finding solutions to the problems only, but it involves several steps from the modelling cycle (see Blum & Leiss, 2007) that one should follow when solving modelling tasks. Further studies may be conducted to investigate if teachers' competency in teaching modelling is through the 7-step modelling cycle (Blum & Leiss, 2007). Overall, it means there's little contribution offered by the university modelling in the mathematics teaching profession of the ISTs during their first year of teaching.

5.4.3. Preparedness of ISTs for teaching mathematical modelling

Teachers' preparedness for teaching modelling is one of the significant requirements that can promote modelling competency among the learners in the classroom. From the presented results, it was observed that 57.5% of the teachers were not sure whether they feel prepared to teach modelling in the classroom. The results showed (depending on which university the ISTs attended) that teachers are ready for teaching modelling, but as discussed above that the curriculum does not provide enough time for them to teach modelling content. Eventually, learners will not be competent in modelling if their teachers are not competent themselves (Beswick & Fraser, 2019). Blum (2015) and Hankeln et al. (2019) would say if teachers are not prepared to teach mathematical modelling, thus they are not knowledgeable about the content, thus their modelling competency will be compromised as well.

As said, that from the consolidated results showed that 57.5% of the teachers feel not prepared to teach modelling in the classroom. It is claimed that CAPS does offer modelling content (see Mogege, 2017), but do they provide sufficient methods of teaching it to the learners? If more than 50% of the teachers lack confidence in their ability to teach modelling, then that makes it apparent that most mathematics lessons on modelling are either partially done or not done at all. Though teachers do not feel confident in teaching modelling, most of the participating teachers revealed that they have solved modelling tasks before during their teacher training. Hence, this disagreed with the study by Bonyah and Clark (2022) that the education system in Africa does not provide much of modelling tasks. Meaning, from the South African education system teachers, have done modelling before, though the results from sub-section 4.2.10 revealed that teachers are not well prepared to teach mathematical modelling and two of the three (66.67%) interviewed teachers mentioned (see the excerpt below) that they are not confident with their knowledge and skills of teaching mathematical modelling.

> … if I am not aware of modelling in the curriculum that means knowledge and skills of teaching is even worse … (KN, line 1)

> I would say I do have some knowledge and skills since I do teach word problems in the classroom, for example when teaching rates, speed, distance, and time you see? It’s a lot of wording in those questions… (NM, line 8)

5.4.4. Nature of modelling tasks teachers have encountered during their educational journey.

As acknowledged by (Blum, 2015; Wess et al., 2021) that teachers first need to be competent in modelling themselves before they can engage with learners in training them to be competent in modelling. The presented results did confirm that 70% of the teachers' have done modelling before and they have been working on modelling tasks themselves. Meaning, teachers do have experience of what modelling is, but due to the limitation of modelling content in the CAPS curriculum teachers are limited in using their prior experiences of modelling to ensure that learners are modelling competent in the classroom.

5.4.5. Teachers’ experiences with mathematical modelling in various contexts

As the results showed that 57.5% of the teachers showed disagreement and are not sure whether they have taught modelling before. This means some teachers still do not teach mathematical modelling in the classroom but looking at the Grade 10 – 12 mathematics curriculum, modelling is said to be part of the curriculum (see Mogege, 2017), and in this study, I was working only with Grade 10 – 12 teachers, thus 30% of the teachers are not teaching and integrating modelling in the classroom. 27.5% of the teachers are not sure if they are teaching modelling in the classroom or not, it might be that teachers are not aware of what modelling is, as said from the literature that from Grades 10 – 11 modelling is referred to as word-problems, and in Grade 12 is referred to as optimisation (or calculus application).

It is good that 42.5% of Grade 10 – 12 mathematics teachers are aware of mathematical modelling in their curriculum, and they did confirm that they are teaching it. From teachers' SEF results, teachers did show modelling competency (Blum & Leiss, 2007) by responding positively based on their abilities to evaluate learners' abilities based on their mathematical written work. Teachers showed that they do have experience and knowledge of teaching modelling, but it hasn't played a role in their teaching profession. What was confirmed here is that modelling is less dominant in the curriculum as teachers haven't used their modelling skills in the mathematics classroom during a lesson.

5.4.6. Research question 3 discussion summary

Briefly, to answer research question 2 about teachers' prior knowledge about teaching and learning of modelling in the CAPS curriculum. The discussions revealed that teachers are aware of modelling in the classroom, and they do have prior knowledge about mathematical modelling with some from the university during teacher training. It further confirmed that ISTs' previous knowledge about modelling was restricted during their teacher training at the university level. Hence, modelling in our education system still needs pedagogical reform (Bonyah & Clark, 2022) in terms of how teachers are trained to teach it.

5.5. RQ 4: What are teachers' perceptions about modelling content in the Grade 10 – 12 CAPS curriculum?

5.5.1. Introduction

This question addressed teachers' perceptions and recommendations about modelling content in the mathematics Grade 10 – 12 CAPS curriculums. While teachers showed similar perceptions and recommendations about mathematical modelling in the curriculum. Two of the three (66.67%) interviewed teachers showed different perspectives on whether teachers should take the lead during modelling tasks or learners should be the ones leading and working independently. The literature on this suggests that learners should be working independently when modelling tasks with little scaffolding from the teacher and the curriculum should be more of modelling content that is relevant to the learners so that learners can see the link and significance of learning mathematics at school (Bonyah & Clark, 2022; Mudaly, 2004; Mudaly & Dowlath, 2016; Tran et al., 2020).

Before discussing the stood-out themes, I firstly demonstrated teachers' awareness (see Table 5.2) about mathematical modelling in the curriculum and their knowledge and skills for teaching modelling. It seems significant to first find out to what extent are teachers aware of mathematical modelling in the curriculum. The presented results showed that teachers are aware of modelling in the curriculum. Teachers raised that they know modelling as problem-solving from Grades 8 – 11 and optimisation at the Grade 12 level.

Table 5.2 Teachers’ Awareness, knowledge, and Skills about mathematical modelling

Question (s)	**Teachers’ Scales (and Teachers’ Responses)**
How would you rate your level of awareness of mathematical modelling in the curriculum? Please say more about the scale that you have chosen.	NM (rated 7) KN (rated 5) NM: “I would say I was aware of mathematical modelling during my teacher training at university …” KN: “…because the mathematics curriculum is more of mathematical content in general than modelling itself, hence I as a teacher I’m not too exposed to modelling…”
How would you rate your knowledge and skills for teaching modelling in the curriculum? Please say more about this scale that you have	NM (rated 6) KN (rated 4) NM: “I don’t have much to say since they haven’t taught modelling often” KN: "If my level of awareness is low that means my knowledge and skills of teaching modelling will be even worse … I was never trained how to teach modelling in the classroom…"

The result of the research study uncovered that teachers do have a suitable working knowledge of modelling and they agreed that they have knowledge and skills in teaching modelling. These results disagreed with the results from the study by Mudaly and Dowlath (2016) in South Africa, Kwazulu-Natal, that revealed PSTs did not have satisfactory working knowledge and skills in mathematical modelling, though they used their modelling competencies to solve the modelling tasks that form part of the study questionnaire. It may be that PSTs are still under training, unlike the ISTs they are qualified and already teaching mathematics including modelling. Though from my study no method was meant to assess the level of teachers' knowledge and skills for teaching mathematical modelling, it was asked of them without proving it. Hence, the results cannot be generalised.

5.5.2. Emerged themes from the interviews.

The study examined the mathematical modelling competency of 40 teachers, but only three teachers were interviewed. The participating teachers were already qualified and teaching mathematics including modelling. Four major themes emerged from the interviews data analysis: 'teachers are more product-driven than process', 'learners should take lead during mathematical modelling', 'Curriculum timeframe limits learners from exploring modelling', and 'limitation of content in the curriculum'.

a) ***Teachers are more product-driven than process:*** *Teachers teach for the completion of the content in the curriculum, which focuses more on procedural knowledge than conceptual knowledge.*

From the presented results, teachers feel overwhelmed by the way the curriculum is structured which is leading them to focus on the product (results) then the process (methods). From the CAPS documents, one of the general aims of the SA CAPS curriculum (2011) is to make certain that learners acquire and use knowledge and skills in ways that give their own experiences a purpose. According to this perspective, the curriculum encourages knowledge in regional contexts while keeping in mind universal needs. To ensure that such general aims are fulfilled, the curriculum should make it possible by ensuring that there is enough time for them to teach and let learners master the concepts. But from the obtained results, teachers showed that the CAPS do not fulfil some aspects of its policy, and that ends up causing teachers to not fully engage with the mathematics concepts, rather, teaching learners for examination purposes only (Bonyah & Clark, 2022).

These results confirmed what was discovered in the study by Bonyah and Clark (2022), that mathematical modelling can be composed of some of the topics already in the mathematics curriculum taught at secondary schools. As KN did mention that topics such as finance and statistics can be taught using a modelling approach. However, the issues raised by the teachers from the implemented SA CAPS (2011) curriculum stem from the teaching methods suggested by the curriculum which mostly neglect modelling (Mudaly & Dowlath, 2016), and the nature of the final examination setting leading to certification of the learners. It was quite difficult to promote modelling competencies among the learners if the curriculum does not support the notion of it. According to the modelling expectations (Warner, Schorr & Warner, 2014), for learners to be competent in modelling they need to take the lead in their work when solving a modelling task.

b) ***Learners should take the lead during mathematical modelling:*** *Learners should find ways of approaching modelling tasks that will develop and enhance their modelling competencies.*

The insufficient modelling content in the CAPS curriculum makes it difficult for teachers to promote and develop modelling competencies among the learners. It should be acknowledged that modelling is a process that involves several steps, and modelling competencies develop over time. During interview data analysis, teachers showed belief in letting learners take the lead during solving modelling tasks, through their response teachers showed acknowledgement that the curriculum does not include much modelling content. Hence, it will be difficult for them to train learners in modelling in such a way that they can take the lead for themselves while it is not done daily (Vorhölter et al., 2019). Kaiser, Blum, Ferri and Stillman (2011) raised that there is a fundamental difference between learners in modelling class working independently with the scaffolding (strategic interventions) from the teachers in the classroom, and learners working completely alone. NM teachers confirmed what has been found by the authors that teachers play a significant role in teaching and learning modelling in the classroom. Hence, the interviewed teacher argued that these two strategies of learning modelling should occur concurrently, of whether learners should take the lead during modelling or teachers should provide a step-by-step during modelling tasks. The rationale for that might be because of the difficulty of modelling tasks that differs in terms of the provided reality situation, thus there are situations in modelling where learners can take the lead in solving them through teachers' help. But some situations will require teachers to take the lead in solving them to guide learners on how they should be approached. The findings of this study confirmed what was discovered in the study by (Geiger et al., 2022; Tan & Ang, 2016), that teachers do value the approach of giving learners the opportunity to lead their work during the modelling cycle, but they admitted that in some of the tasks it is difficult to let learners lead the work.

Giving learners an opportunity to take the lead will be good, but some challenges may be experienced by the teachers in the classroom. The study by Warner, Schorr, and Warner (2014) revealed that a teacher was having a challenge in understanding some of the learners' mathematical approaches, reasoning, explanations, and justifications. This can be possibly experienced by other teachers during solving modelling tasks, as teachers might struggle to understand learners' assumptions of a given situation. As one of the teachers in the study mentioned that learners make "crazy assumptions". In this regard, teachers need to be supported by the structure of the mathematics content in the curriculum and workshops from DBE[33] to strengthen their modelling competency.

c) ***Curriculum timeframe limits learners from exploring modelling:*** *It is significant to understand that learners require more time to understand mathematics in general and modelling, and for them to be independent and competent.*

Secondary school education does not have much modelling content in the South African educational system (Mudaly & Dowlath, 2016). Mathematics content that links learner's real-life experiences is mostly not accounted for in the curriculum. As also interviewed teachers touched on that aspect. It can be generally said that the African educational system does not include modelling as a content, though NM and SN mentioned that they did mathematical modelling as a course during his/ her training at the university. But it hasn't helped them during their first year of teaching. As said from the literature Wits and UJ do offer modelling modules, but what was observed is that they are offered from the upper levels. If the curriculum does not cater to modelling in it, it will be challenging for teachers to teach it to the learners daily. Including modelling in an everyday lesson is one of the suggested methods that can develop learners modelling competencies and enhance them (Vorhölter et al., 2019). By doing so, teachers will also acquire skills for understanding learners' written mathematical work. From teachers' SEF about teaching and learning mathematical modelling (see section 5.4), teachers are expected to be efficient in identifying learners' abilities through their written mathematical work, which seems to be possible if teachers and learners are doing modelling daily during mathematics classes.

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Teachers may, of course, attempt to do modelling often with the learners, but the curriculum is structured in such a way that every topic has a specific time limit that should be spent on it. As KN mentioned:

> "… however, from my experience that is a big task because it cannot be done because the system puts pressure on educators, and the system is very results focused. So, teachers will get things done so those things like problem-solving and in terms of developing those models should start at an early grade where children learn those skills so that when they come to high school you don't like teaching them from scratch…" (KN, lines 2 – 6)

I put forward that the curriculum in SA should be revised and spare enough time for specifically modelling content in it. That might also put some strain on teachers since it is not something they are doing daily. I suggest more workshops be conducted for teacher training purposes in modelling, and resources be made available for easy accessibility by the teachers and learners.

d) ***Limitation of content in the mathematics curriculum:*** *Limitation of content in the curriculum will give teachers more time to teach modelling deeply and allow learners to have an opportunity to explore more of the concepts and do modelling often independently.*

The time frames for each topic in the curriculum are set for specific good reasons that will ensure there's the quality of education in the South African educational system. It should be acknowledged that concepts in each topic from the curriculum are topics that learners will need in the future, and they are catered for in the curriculum. However, having topics that are related to learners' daily life experiences would be an advantage as learners will get to see the value of learning mathematics at school (Tran et al., 2020). During the interview data analysis, teachers raised that some of the contents in the curriculum are not playing much role in learners' experiences, thus it should be revised with content that will play a significant role in learners' experiences and value its existence. Though the teacher during the interview was not specific on which content or topics should be revised, he/ she mentioned and acknowledged that modelling is widely known for engaging learners in the mathematics classroom with reality examples (Bonotto & Passarella, 2018; Van den Heuvel-Panhuizen & Drijvers, 2014).

Therefore, the teacher suggested that some of the topics such as *algebra* might be difficult to teach using modelling approaches. The teacher also mentioned that modelling is viewed as problem-solving in real situations. From the teacher's statement, it can be argued that learners use algebra to solve modelling tasks after setting up a mathematical model. Bråting, Madej and Hemmi (2019) argued that algebra is considered a significant fundamental for solving general mathematical tasks, and algebra is said to be presenting problem-solving situations through algebraic expressions and equations incorporating unknowns and variables (Kieran, 2020).

It is going to be a challenge to limit the content already included in the mathematics curriculum. According to my knowledge, all topics in the curriculum are fundamentally significant to be learned at school and they prepare learners for the university. My input is that the current content in the curriculum should be revised, meaning try to integrate modelling in most of the topics and ensure it is dominant in the curriculum. In this regard, learners will be presented with more reality context tasks in mathematics and teachers will have the opportunity to use modelling teaching strategies learned during their teacher training. Thus, teachers modelling competency will be enhanced as well.

5.5.3. Teachers' recommendations about modelling in the curriculum

Provided below (see Table 5.3) are teachers' recommendations about mathematical modelling in the curriculum. From the interview data analysis, teachers recommended that mathematics textbooks used at schools should be incorporated with more modelling tasks. As was discussed in the literature and introduction (see Chapters 1 and 2) that most tasks from the currently used textbooks are mathematical tasks in general, and only one to five of the fifty tasks are modelling. Another recommendation that was appealed to by the teachers is that content learning in the mathematics classroom should be contextualised with learners' daily life experiences.

Table 5.3 Teachers' summary for recommendations about modelling in the curriculum

Categories	**Teacher's excerpts**
More modelling tasks should be incorporated into the textbooks.	"… sometimes from the textbooks you find, there's one modelling example out of fifty problems given …" (KN; SN)
More tasks in mathematics should be related to learners' real-life situations.	"Most of the problems in the textbooks and study guides are abstract and do not closely relate to learners' experiences…" (NM, line 4)
Modelling should be learned from lower grades (primary schools).	"… Developing those models should start at an early grade where children learn those skills so that when they come to high school you don't like to teach them from scratch …" (KN, line 7)

Some recommendations might be a challenge on the basis that there's a lot of diversity in South African classrooms (Taole, 2020), particularly in rural areas[34], thus, bringing textbooks that will contain learners' experiences might be a challenge as learners might exhibit different experiences in the classroom. Another significant recommendation was that modelling should start in primary education to ensure that there's a modelling content link between teachers at the primary school level and teachers at the secondary school level. This recommendation confirmed what was discovered in the study by (Tran et al., 2020). I acknowledge that modelling is regarded as a topic that can bring mathematical sense to the learners of the world around them.

5.6. Chapter summary and conclusion

The analysis of the results shows that teachers showed that they do believe in modelling and that it brings reality to the learners in the form of mathematical models. It is confirmed that their beliefs are through their modelling prior knowledge, meaning they have done or seen modelling before, and they have modelled some of the tasks from their secondary education to

[34] Rural areas (sometimes called a village) are areas (settlements) that are based outside cities and towns, and it has a low population.

qualified mathematics teachers. Furthermore, teachers showed low SEF in identifying learners' abilities during learners' modelling processes and from their written mathematical work. Though teachers showed a willingness to integrate modelling into the curriculum as they confirmed that the current CAPS mathematics curriculum insufficiently dominates modelling content in it. Additionally, they showed that they are constructivist believers as they are willing to let learners take the lead during modelling processes with the help of the teachers (Tan & Ang, 2016). It seems to be safe to generalise that such beliefs will not be applied during modelling lessons only, but also when learning mathematics in general.

The following Chapter 6 provides a conclusion about the discussion of the findings of the study, limitations that were observed during the study process, and recommendations for future research.

CHAPTER 6: CONCLUSION, LIMITATIONS OF THE STUDY & RECOMMENDATIONS

6.1. Introduction

This study investigated IST's beliefs and SEF about teaching and learning mathematical modelling through a structural model of professional competence for teaching modelling. The study was conducted at eleven public secondary schools in Johannesburg, Gauteng province in SA. The aim was to attain an understanding of the ISTs' beliefs, SEF, prior knowledge, perceptions, and recommendations about teaching modelling content with the CAPS mathematics curriculum. This was due to the lack of relevant literature on teachers' beliefs and SEF about teaching modelling in public secondary schools in the South African educational system (Mudaly & Clark, 2022).

A quantitative research design was adopted for analysing the questionnaire among the 40 participating teachers, and a qualitative research design was adopted for analysing the interviews among the 3 mathematics teachers and to answer the fourth research question as described in chapters 1 and 4.

In this chapter, I presented an overview of the central findings of the study. The next sections of the chapter provide a synopsis discussion of the contribution of the study and are linked to the wider research and debates on the teaching and learning of mathematical modelling. Then, the related limitations of the study are detailed. Finally, I conclude this chapter by discussing possible further research, recommendations of teaching modelling as well as recommendations for teacher support and training. I propose the need for more research studies in the South African ISTs to comprehend the nature of teaching mathematical modelling, focusing on the Grade 10 – 12 curriculum including primary education, and possibly modelling intervention strategies that may be adopted by mathematics teachers.

6.2. Summary of the study

6.2.1. Quantitative research findings and discussion summary (questionnaire)

Research question 1

- What are in-service teachers’ beliefs about teaching and learning mathematical modelling in the CAPS curriculum?

Measuring teachers’ beliefs was done based on the following four components, Modelling in the classroom, Application of modelling, Constructivist belief, and Transmissive belief. What was discovered from the data collected is that ISMTs do believe in mathematical modelling with the CAPS curriculum. They showed that it is significant to give learners an opportunity to do modelling in the classroom since it is more practical to reality. Additionally, teachers showed that they are constructivists as they promoted giving learners an opportunity to be the ones leading in solving modelling tasks. But they did acknowledge that often learners should be guided when solving a mathematical task. Obtained results of the study support the notion posed by (Biccard & Wessels, 2011; Chamberlin et al., 2022) that teacher and learner beliefs are key to productive modelling, and the beliefs that one possess shapes their mathematical behaviours and competence.

Research question 2

- What are the in-service teachers’ self-efficacy about teaching and learning mathematical modelling in the CAPS curriculum?

Measuring teachers' SEF was operationalised within two scales: mathematical modelling and mathematical working. Presented and discussed results revealed that teachers' SEF for recognising learners' abilities during a modelling cycle is not strong enough to be guaranteed that it will promote and develop learners modelling competencies. As explained that modelling a task contains several steps, and modelling competency needs to be acquired over some time. What came out from the results is that modelling should be included in an everyday lesson for teachers to be more exposed to it and sufficiently competent in it.

Research question 3

- What are in-service teachers’ prior knowledge about modelling content in the CAPS curriculum?

Teachers' previous knowledge about mathematical modelling was measured through the four following components, General role played by mathematical modelling from the ISTs courses, Preparedness of ISTs for teaching mathematical modelling, Nature of modelling tasks teachers have encountered during their educational journey, Teachers' experiences about mathematical modelling in various contexts. The discussions revealed that teachers have past experiences with modelling from their secondary education to their teacher training at university. They showed that they were ready to use their knowledge and skills for teaching modelling during their first years of teaching. Unfortunately, they experienced that modelling is not dominating as it should be in the curriculum. Thus, their competence in modelling was compromised.

6.2.2. Qualitative research findings and discussion summary (interviews)

Research question 4

- What are teachers' perceptions about modelling content in the Grade 10 – 12 CAPS curriculum?

There were two significant points based on teachers' perceptions and two vital points from teachers' recommendations that they emphasised. From their perceptions, firstly they showed that they are aware of mathematical modelling and what it is. Secondly, they showed that they have done modelling before from their basic education to their teacher training at universities. From their recommendations, firstly they put forward that mathematics content from the curriculum should be limited in such a way that teachers will have enough time to teach the concept conceptually and give learners enough time to explore the concept taught as well. Lastly, teachers were concerned that modelling is not dominating in the curriculum, they mentioned that only 1% of the mathematical problems provided in the textbooks are modelling problems. Hence, it will be challenging for them to be competent in modelling while they are not teaching modelling daily.

The following major themes emerged for answering research question 4: Which intended to investigate and understand ISTs perceptions and recommendations about modelling in the CAPS curriculum.

Four major themes emerged from the qualitative research (interviews):

a) ***Teachers are more product-driven than process:*** *Teachers are teaching for the completion of the content in the curriculum, which puts more focus on procedural understanding than conceptual understanding.*

b) ***Learners should take the lead during mathematical modelling:*** *Learners should find ways of approaching modelling tasks that will develop and enhance their modelling competencies.*

c) ***The curriculum timeframe limits learners from exploring modelling:*** *The significance of understanding is that learners require more time to understand modelling and for them to be independent.*

d) ***Limitation of content in the mathematics curriculum:*** *Limitation of content in the curriculum will give teachers more time to teach modelling deeply and allow learners to have an opportunity to explore more of the concepts and do modelling independently.*

From the overall report, teachers are aware of modelling content in the curriculum, but they are concerned about the way it is structured. Teachers perceived that modelling content is limited in the curriculum and should dominate more than other topics as it focuses on learners' contexts. From my experiences, my input in this is that topics from the curriculum should be given enough time to be taught and learned, eventually teaching focus will be more on conceptual understanding than procedural understanding.

6.3. Limitations of the study

Due to the time constraints of the study, and the limitation to reaching out to more participants, ISTs, the study fascinated the facet of professional competence (beliefs and SEF) to achieve the main aim of the proposed study (see Figure 2.3). Qualitative interviews were used to strengthen the quantitative data, and modelling-specific pedagogical content knowledge from Figure 2.3 was not investigated in the current study. This means that the researcher was not able to provide more information on teachers' knowledge about modelling tasks, concepts, perspectives, objectives, and knowledge about modelling processes, and strategic interventions.

6.4. Recommendations for further research

In this section, I detailed recommendations for future research that may be conducted to strengthen more teachers modelling competency and to enhance the teaching and learning of modelling in the curriculum. Recommendations were based on teaching modelling in the classroom, PSTs' development of modelling competencies, and modelling teaching methods to be used in the classroom.

6.4.1. Recommendations for teaching modelling in the classroom

This study can be considered the first part of the investigation of teachers' modelling competency. The objective of the research study was thus to investigate teachers' beliefs, self-efficacy, prior knowledge, and perceptions about their mathematical modelling competency to recommend future training and methods of teaching modelling in the classroom. Limitations that were also observed during the study urged for future research to be conducted to gain more understanding of mathematics teachers' modelling competency. I recommend research studies to be also conducted amongst the senior phase (Grade 7 – 9) and foundation phase (Grade R – 6) teachers to find out to what extent are they aware of modelling in the current curriculum, and what would be their suggestions in integrating modelling across the curriculum and make it more dominant to ensure that modelling competencies are developed and enhanced at an early grade of the learners.

6.4.2. Pre-service teachers’ development of modelling competencies

To ensure that the development of mathematical modelling competencies is developed in the classroom for the learners we need to make sure that teachers are modelling competent themselves first. I recommend future research studies be conducted on how teachers modelling courses during their training are structured. Does it train them to teach modelling in such a way that they will develop modelling competencies among learners through the 7-step of modelling cycle by Blum and Leiss (2007)? My recommendation is that the structuring of the PSTs courses at the university should include modelling that promotes the development of modelling sub-competencies (1. Building a model 2. Simplifying 3. Mathematics 4. Mathematical work 5. Interpretation 6. Validation and 7. Evaluating).

6.4.3. Modelling teaching methods to be used in the classroom.

As discovered that teaching modelling might not be necessary the same as teaching mathematics in general. Hence, for future research studies, I would suggest researchers investigate teachers' competence in teaching mathematical modelling utilising strategic interventions suggested by De Jong and Lazonder (2014). It might be of help to teachers in developing their immediate thinking skills when learners ask a question or when learners share their ideas of their written modelling solutions.

6.5. Conclusion of the study

This chapter analysed in detail the findings of teachers' responses from the questionnaire and interviews. The chapter connects the research aims, objectives, and results of the study through an analysis of teachers' responses. It gives a succinct research report about what was done in the research study and discusses the data that was collected with the research questions developed to fulfil the purpose of the study. I also drew a conclusion based on the findings from the study, relevant literature reviewed, and personal experiences about mathematical modelling and made recommendations based on the evidence obtained from the collected data.

The study aimed to investigate ISTs' beliefs, SEF, prior experiences, perceptions, and recommendations for teaching mathematical modelling using a structural model of professional competence for teaching mathematical models as a framework developed by (Wess et al., 2021). The targeted participants were Grade 10 – 12, mathematics teachers. A mixed research method approach was used to investigate teachers' modelling competency, therefore, the data collected comprised quantitative and qualitative data.

It is believed that a learner's competence in modelling, and mathematics in general is directly associated with teachers' beliefs, SEF, and content knowledge. Hence, the study attempted to identify the gaps preventing the development of mathematical modelling competence among learners by developing positive attitudes and values towards the modelling content.

In South Africa, teaching and learning through a modelling approach may profoundly add value to the enhancement of learners' beliefs about mathematics, increase motivation and positive attitude towards modelling, exposes more to complex problems and their solutions, and develop mathematical critical thinking competence (Bikić, Burgić & Kurtić, 2021).

REFERENCES

Akgün, L., Çiltaş, A., Deniz, D., Çiftçi, Z., & Işık, A. (2013). Primary school mathematics teachers' awareness on mathematical modelling. Adıyaman Üniversitesi Sosyal Bilimler Enstitüsü Dergisi, 6(12), 1-34. https://doi.org/10.14520/adyusbd.410

Akyeampong, K., Lussier, K., Pryor, J., & Westbrook, J. (2013). Improving teaching and learning of basic maths and reading in Africa: Does teacher preparation count?. *International journal of educational development*, *33*(3), 272-282.

Alharahsheh, H. H., & Pius, A. (2020). A review of key paradigms: Positivism VS interpretivism. *Global Academic Journal of Humanities and Social Sciences*, *2*(3), 39-43.

Alvis, N. (2022). Reliability and validity.

Anjaria, K. (2022). Knowledge derivation from Likert scale using Z-numbers. *Information Sciences*, *590*, 234-252.

Anney, V.N. 2014. Ensuring the quality of the findings of qualitative research: looking at Anney, V. N. (2014). Ensuring the quality of the findings of qualitative research: Looking at trustworthiness criteria. *Journal of emerging trends in educational research and policy studies*, *5*(2), 272-281.

Ärlebäck, J. (2009). Towards understanding teachers' beliefs and affects about mathematical modelling. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education (pp. 2096– 2104). Lyon, France: INRP.

Ärlebäck, J. B. (2010). TOWARDS UNDERSTANDING TEACHERS'BELIEFS AND AFFECTS ABOUT MATHEMATICAL MODELLING. *CERME 6–WORKING GROUP 11*, 2096.

Armstrong, P. K., & Bajpai, A. C. (1988). Modelling in GCSE mathematics. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *7*(3), 121-131.

Arseven, A. (2015). Mathematical Modelling Approach in Mathematics Education. *Universal Journal of Educational Research*, *3*(12), 973-980.

Bada, S. O., & Olusegun, S. (2015). Constructivism learning theory: A paradigm for teaching and learning. *Journal of Research & Method in Education*, *5*(6), 66-70.

Bahmaei, F. (2014). Mathematical modelling in primary school, advantages, and challenges. *Journal of mathematical modelling and application*, *1*(9), 3-13.

Bakker, A. (2018). What is design research in education? 1. In *Design research in education* (pp. 3-22). Routledge.

Bell, R., Warren, V., & Schmidt, R. (2022). *The application of concurrent or sequential mixed-methods research designs and their methodological implications: Investigating tacit knowledge, its use, and application in automotive development*. SAGE Publications, Limited.

Best, J.W. and Kahn, J.V. (1998) Research in Education. 8th Edition, Butler University, Emeritus, University of Illinois, Chicago.

Beswick, K., & Fraser, S. (2019). Developing mathematics teachers' 21st century competence for teaching in STEM contexts. *ZDM*, *51*(6), 955-965.

Biccard, P., & Wessels, D. C. (2011). Documenting the development of modelling competencies of grade 7 mathematics students. *Trends in teaching and learning of mathematical modelling*, 375-383.

Bikić, N., Burgić, D., & Kurtić, V. (2021). The Effects of Mathematical Modelling in Mathematics Teaching of Linear, Quadratic and Logarithmic Functions. *The Effects of Mathematical Modelling in Mathematics Teaching of Linear, Quadratic and Logarithmic Functions*, *2*(2), 129-144.

Bitsch, V. (2005). Qualitative research: A grounded theory example and evaluation criteria. *Journal of agribusiness*, *23*(345-2016-15096), 75-91.

Bliss, K., & Libertini, J. (2016). Guidelines for assessment & instruction in mathematical modeling education (GAIMME) chapter 1, USA: COPAM. *Inc. & SIAM*.

Blömeke, S., Kaiser, G., König, J., & Jentsch, A. (2020). Profiles of mathematics teachers' competence and their relation to instructional quality. *ZDM*, *52*, 329-342.

Blomhøj, M. (2019). Towards integration of modelling in secondary mathematics teaching. In *Lines of inquiry in mathematical modelling research in education* (pp. 37-52). Springer, Cham.

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching mathematics and its applications*, *22*(3), 123-139.

Blum, W. (1985). Anwendungsorientierter Mathematikunterricht in der didaktischen Diskussion. Mathematische Semesterberichte, 32(2), 195–232.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. *Trends in teaching and learning of mathematical modelling*, 15-30.

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do?. In *The proceedings of the 12th international congress on mathematical education* (pp. 73-96). Springer, Cham.

Blum, W., & Borromeo, R. (2009). Mathematical modelling: Can it be taught and learnt? Journal of Mathematical Modelling and Application, 1(1), 45-58.

Blum, W., & Leiß, D. (2005). Modellieren im Unterricht mit der "Tanken"-Aufgabe. mathematik lehren, 128, 18–21.

Blum, W., & Leiß, D. (2007). Deal with modelling problems. *Mathematical modelling: Education, engineering and economics-ICTMA*, *12*, 222.

Blum, W., & Leiss, D. (2007). How do Students and Teachers Deal with Modelling Problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), Mathematical modelling (ICTMA 12): Education, engineering, and economics: Proceedings from the twelfth International Conference on the Teaching of Mathematical Modelling and Applications (pp. 222–231). Chichester: Horwood.

Bonotto, C., & Passarella, S. (2018). MODELLING AND PROBLEM-POSING IN THE TEACHING OF MATHEMATICS: TEACHERS'PERCEPTION AND PRACTICE. *EAPRIL 2018*, 68.

Bonyah, E., & Clark, L. J. (2022). Pre-service Teachers' Perceptions of and Knowledge for Mathematical Modelling in Ghana. *Contemporary Mathematics and Science Education*, *3*(1).

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. ZDM—The International Journal on Mathematics Education, 38(2), 86–95.

Borromeo Ferri, R. (2018). Learning How to Teach Mathematical Modeling in School and Teacher Education. Springer International Publishing. https://doi.org/10.1007/978-3-319-68072-9

Borromeo Ferri, R., & Blum,W. (2010). mathematical modelling in teacher education—Experiences from a modelling seminar. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), European research in mathematics: Proceedings of the sixth congress of the european society for research in mathematics education (pp. 2046–2055). Lyon, France.

Bråting, K., Madej, L., & Hemmi, K. (2019). Development of algebraic thinking: opportunities offered by the Swedish curriculum and elementary mathematics textbooks. *Nordisk matematikkdidaktikk, NOMAD:[Nordic Studies in Mathematics Education]*, *24*(1), 27-49.

Braun V, Clarke V. 2012. Thematic analysis. In: Cooper H, editor. APA handbook of research methods in psychology. Vol. 2, research designs. Washington (DC): American Psychological Association.

Bühner, M. (2011). Einführung in die Test- und Fragebogenkonstruktion (3., aktualisierte und erweiterte Auflage). München: Pearson.

Burkhardt, H. (2006). Modelling in Mathematics Classrooms: Reflections on past developments and the future. ZDM, 38(2), 178–195.

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, 1-32.

Chamberlin, S., Payne, A. M., & Kettler, T. (2022). Mathematical modeling: a positive learning approach to facilitate student sense making in mathematics. *International Journal of Mathematical Education in Science and Technology*, *53*(4), 858-871.

Charmaz, K., & Belgrave, L. (2012). Qualitative interviewing and grounded theory analysis. *The SAGE handbook of interview research: The complexity of the craft*, *2*, 347-365.

Cohen, L., Manion, L., & Morrison, K. (2007). Research methods in education (6th ed.) London: Routledge.

Connole, H. (1993). The research enterprise. In Issues and methods in research: study guide (pp. 7–27). Underdale: Distance Education Centre, University of South Australia.

Cope, D. G. (2014, January). Methods and meanings: credibility and trustworthiness of qualitative research. In *Oncology nursing forum* (Vol. 41, No. 1).

Cresswell, J. W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research. *Lincoln: Pearson*.

Creswell, J. W. (2002). *Educational research: Planning, conducting, and evaluating quantitative* (Vol. 7). Prentice Hall Upper Saddle River, NJ.

Creswell, J. W., & Creswell, J. D. (2017). Research design: Qualitative, quantitative, and mixed methods approaches. Sage Publications.

DBE, S. (2011). Curriculum and Assessment Policy Statement: Mathematics Grades 10–12. Pretoria: Department of Basic Education.

De Jong, T., & Lazonder, A. W. (2014). The guided discovery learning principle in multimedia learning. In R. Mayer (Ed.), The Cambridge handbook of multimedia learning (2nd ed., pp. 371– 390). Cambridge: Cambridge University Press.

Dei, L. (2011). *A Comparative Study of Middle School Mathematics Education Practices in Ghana and United States Classrooms*. University of Nebraska

Deniz Yilmaz, D., & Akgun, L. (2021). Preservice Teachers' Beliefs towards Mathematics and Mathematical Modelling in Modelling Processes. *International Journal of Curriculum and Instruction*, *13*(3), 3070-3095.

Department of Basic Education. (2019). 2018 National Senior Certificate Examination Diagnostic Report. January 2019. Pretoria: Department of Basic Education.

Doerr, H. M. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In Werner Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education: The 14th ICMI study (pp. 69–78). Boston, MA: Springer.

Durandt, R. (2021). Design principles to consider when student teachers are expected to learn mathematical modelling. *Pythagoras*, *42*(1), 618.

Durandt, R., Blum, W., & Lindl, A. (2022). Fostering mathematical modelling competency of South African engineering students: which influence does the teaching design have?. *Educational Studies in Mathematics*, *109*(2), 361-381.). Fostering mathematical modelling competency of South African engineering students: which influence does the teaching design have?. *Educational Studies in Mathematics*, *109*(2), 361-381.

English, L. D., & Watters, J. J. (2005). Mathematical modelling in the early school years. *Mathematics education research journal*, *16*(3), 58-79.

Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American journal of theoretical and applied statistics*, *5*(1), 1-4.

Ferri, R. B. (2013). Mathematical Modelling in School and in Teacher Education-Conceptions and Examples. *Santiago de Chile*.

Fischer, R., & Malle, G. (1985). Mensch und Mathematik. Mannheim: Bibliographisches Institut. Franke, M., & Ruwisch, S. (2010). Didaktik des Sachrechnens in der Grundschule. Heidelberg: Spektrum.

Frejd, P. (2012). Teachers' conceptions of mathematical modelling at Swedish Upper Secondary school. *Journal of Mathematical Modelling and Application*, *1*(5), 17-40.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. ZDM, 38(2), 143–162.

Garfunkel, S., & Montgomery, M. (2016). Guidelines for assessment and instruction in mathematical modeling education. *Bedford, MA. Consortium for Mathematics and Its Applications*.

Geiger, V., Galbraith, P., Niss, M., & Holland-Twining, B. (2022). A Typology for Instructional Enablers of Mathematical Modelling. *Mathematics Education Research Group of Australasia*.

Greefrath, G. (2010). Didaktik des Sachrechnens in der Sekundarstufe. Heidelberg: Springer Spektrum.

Greefrath, G., & Vorhölter, K. (2016). *Teaching and learning mathematical modelling: Approaches and developments from German speaking countries*. Springer Nature.

Greefrath, G., Hertleif, C., & Siller, H. S. (2018). Mathematical modelling with digital tools—a quantitative study on mathematising with dynamic geometry software. *Zdm*, *50*(1), 233-244.

Greefrath, G., Kaiser, G., Blum, W., & Borromeo Ferri, R. (2013). Mathematisches Modellieren— Eine Einführung in theoretische und didaktische Hintergründe. In R. Borromeo Ferri, G. Greefrath, G. Kaiser (Eds.). Mathematisches Modellieren für Schule und Hochschule. Theoretische und didaktische Hintergründe. Wiesbaden: Springer Spektrum.

Greefrath, G., Siller, H. S., Vorhölter, K., & Kaiser, G. (2022). Mathematical modelling and discrete mathematics: opportunities for modern mathematics teaching. *ZDM–Mathematics Education*, *54*(4), 865-879.

Handal, B. (2003). Teachers' mathematical beliefs: A review. The Mathematics Educator, 13(2), 47-57.

Hankeln, C. (2020). Mathematical modeling in Germany and France: A comparison of students' modeling processes. *Educational Studies in Mathematics*, *103*(2), 209-229.

Hankeln, C., Adamek, C., & Greefrath, G. (2019). Assessing sub-competencies of mathematical modelling—Development of a new test instrument. In *Lines of inquiry in mathematical modelling research in education* (pp. 143-160). Springer, Cham.

Houghton, C., Casey, D., Shaw, D., & Murphy, K. (2013). Rigour in qualitative case-study research. *Nurse researcher*, *20*(4).

Ibragimov, A. K. (2021, May). Developing the professional competence of teachers as a pedagogical problem. In *E-Conference Globe* (pp. 333-338).

Jahnke, H. N., & Hefendehl-Hebeker, L. (2019). *Traditions in German-speaking mathematics education research* (p. 278). Springer Nature.

Jankvist, U. T., & Niss, M. (2020). Upper secondary school students' difficulties with mathematical modelling. *International Journal of mathematical education in science and technology*, *51*(4), 467-496.

Jensen, J. H., & Jankvist, U. T. (2018). Disciplinary competence descriptions for external use. *Nordic Studies in Mathematics Education, 23*(2), 3–24.

Kaiser, G. (2006). The mathematical beliefs of teachers about applications and modelling: Results of an empirical study. In J. Novotná, M. Moraová, M. Krátká, & N. Stehlíková (Eds.), Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education (Vol 3, pp. 393–400). Prague: PME.

Kaiser, G. (2007). Modelling and modelling competencies in school. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), Mathematical modelling: Education, engineering, and economics (pp. 110–119). Chichester: Horwood.

Kaiser, G., & König, J. (2019). Competence measurement in (mathematics) teacher education and beyond: Implications for policy. *Higher Education Policy*, *32*(4), 597-615.

Kaiser, G., Blum, W., Ferri, R. B., & Stillman, G. (Eds.). (2011). *Trends in teaching and learning of mathematical modelling: ICTMA14* (Vol. 1). Springer Science & Business Media.

Kakilla, C. (2021). Strengths and weaknesses of semi-structured Interviews in Qualitative research: A critical essay.

Kertil, M., & Gurel, C. (2016). Mathematical modeling: A bridge to STEM education. *International Journal of Education in mathematics, science, and Technology*, *4*(1), 44-55.

Kieran, C. (2020). Algebra teaching and learning. *Encyclopedia of mathematics education*, 36-44.

Kiger, M. E., & Varpio, L. (2020). Thematic analysis of qualitative data: AMEE Guide No. 131. *Medical teacher*, *42*(8), 846-854.

Klock, H., Wess, R., Greefrath, G., & Siller, H.-S. (2019). Aspekte professioneller Kompetenz zum Lehren mathematischer Modellierung bei (angehenden) Lehrkräften—Erfassung und Evaluation. In T. Leuders, E. Christophel, M. Hemmer, F. Korneck, & P. Labudde (Eds.), Fachdidaktische Forschung zur Lehrerbildung (pp. 135–146). Münster: Waxmann.

Krauss, S., Neubrand, M., Blum, W., Baumert, J., Brunner, M., Kunter, M., & Jordan, A. (2008). Die Untersuchung des professionellen Wissens deutscher Mathematik-Lehrerinnen und -Lehrer im Rahmen der COACTIV-Studie. Journal Für Mathematik-Didaktik, 29(3), 233–258.

Krefting, L. (1991). Rigor in qualitative research: The assessment of trustworthiness. *The American journal of occupational therapy*, *45*(3), 214-222.

Kunter, M. (2013). Motivation as an aspect of professional competence: Research findings on teacher enthusiasm. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, &M. Neubrand (Eds.), Cognitive activation in the mathematics classroom and professional competence of teachers (pp. 273–289). Boston, MA: Springer US.

Kunter, M., & Baumert, J. (2013). The COACTIV research program on teachers' professional competence: Summary and discussion. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), Cognitive activation in the mathematics classroom and professional competence of teachers (pp. 345–368). Boston, MA: Springer US.

Kuntze, S., & Zöttl, L. (2008). Überzeugungen von Lehramtsstudierenden zum Lernpotential von Aufgaben mit Modellierungsgehalt. mathematica didactica, 31(1), 46–71.

Kyngäs, H., Kääriäinen, M., & Elo, S. (2020). The trustworthiness of content analysis. In *The application of content analysis in nursing science research* (pp. 41-48). Springer, Cham.

Lazonder, A. W., & Harmsen, R. (2016). Meta-analysis of inquiry-based learning: Effects of guidance. Review of Educational Research, 86(3), 681–718.

Leder, G. C., & Forgasz, H. J. (2002). Measuring mathematical beliefs and their impact on the learning of mathematics: A new approach. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), Beliefs: A hidden variable in mathematics education? (pp. 95–114). Dordrecht: Springer.

Leiss, D., Schukajlow, S., Blum, W., Messner, R., & Pekrun, R. (2010). The role of the situation model in mathematical modelling—Task analyses, student competencies, and teacher interventions. *Journal für Mathematik-Didaktik*, *31*(1), 119-141.

Lesh, R. A., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Routledge.

Leung, F. K. S., Lai, M. C., Qiao, Z., & Chen, Y. P. (2017). Guidelines for Assessment and Instruction in Mathematical Modeling Education.

Leung, K. S. F. (2006). Mathematics education in East Asia and the West: Does culture matter?. In *Mathematics education in different cultural traditions-A comparative study of East Asia and the West* (pp. 21-46). Springer, Boston, MA.

Lincoln, Y. S. (2007). Naturalistic Inquiry. *The Blackwell Encyclopedia of Sociology*.

Lomas, G., Grootenboer, P., & Attard, C. (2012). The affective domain and mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), Research in mathematics education in Australasia: 2008-2011 (pp. 23–37). Rotterdam, The Netherlands: Sense Publishers.

Maasepp, B., & Bobis, J. (2015). Prospective Primary Teachers' Beliefs about Mathematics. *Mathematics Teacher Education and Development*, *16*(2), 89-107.

Maaß, K. (2005). Barriers and opportunities for the integration of modelling in mathematics classes: Results of an empirical study. Teaching Mathematics and Its Applications: An International Journal of the IMA, 2(3), 61–74.

Maaß, K. (2006). What are modelling competencies?. *ZDM*, *38*(2), 113-142.

Mainali, B. (2022). Investigating Pre-Service Teachers' Beliefs Towards Mathematics: A Case Study. European Journal of Science and Mathematics Education, 10(4), 412-435. https://doi.org/10.30935/scimath/12103

Malambo, P. (2021). Implicit misconceptions in prospective mathematics teachers' reasoning about trigonometric concepts. *Contemporary Mathematics and Science Education*, *2*(2), ep21011.

Malangtupthong, P., Nurittamont, W., & Phayaphrom, B. (2022). FACTORS INFLUENCING MATHEMATICAL PROBLEM-SOLVING COMPETENCY: A CASE STUDY ON HIGH SCHOOL STUDENTS. *PSAKU International Journal of Interdisciplinary Research*, *11*(2), 1-18.

Masitoh, L. F., & Fitriyani, H. (2018). Improving students' mathematics self-efficacy through problem based learning. *Malikussaleh Journal of Mathematics Learning (MJML)*, *1*(1), 26-30.

Mazana, Y. M., Suero Montero, C., & Olifage, C. R. (2019). Investigating students' attitude towards learning mathematics.

Merriam, S. B. (2002). Introduction to qualitative research. *Qualitative research in practice: Examples for discussion and analysis*, *1*(1), 1-17.

Morse, J. M., Barrett, M., Mayan, M., Olson, K., & Spiers, J. (2002). Verification strategies for establishing reliability and validity in qualitative research. *International journal of qualitative methods*, *1*(2), 13-22.

Mosimege, M., & Winnaar, L. (2021). Teachers' instructional strategies and their impact on learner performance in Grade 9 mathematics: Findings from TIMSS 2015 in South Africa. *Perspectives in Education*, *39*(2), 324-338.

Mudaly, V. (2004). Modelling of real-world problems is often the starting point for proof. Pythagoras, 60, 36-43. https://doi.org/ 10.4102/pythagoras.v0i60.125

Mudaly, V., & Dowlath, E. (2016). Pre-service teachers' use of mathematical modelling. *International Scientific Researches Journal*, *72*(8), 59-79.

Mutodi P., & Ngirande, H (2014). The influence of students' perceptions on mathematics performance: A case of a selected high school in South Africa. Mediterranean Journal of Social Sciences, 5(3), 431V445.

Nabie, M. J., Akayuure, P., Ibrahim-Bariham, U. A., & Sofo, S. (2018). Trigonometric Concepts: Pre-Service Teachers' Perceptions and Knowledge. *Journal on Mathematics Education*, *9*(1), 169-182.

Ndlovu, M. (2018). Themes in Mathematics Teacher Professional Learning Research in South Africa: A Review of the Period 2006–2015. In: Kaiser, G., Forgasz, H., Graven, M., Kuzniak, A., Simmt, E., Xu, B. (eds) Invited Lectures from the 13th International Congress on Mathematical Education. ICME-13 Monographs. Springer, Cham. https://doi.org/10.1007/978-3-319-72170-5_22

Niss, M. & Højgaard, T. (Eds.) (2011). Competencies and mathematical learning ideas and inspiration for the development of mathematics teaching and learning in Denmark. *IMFUFA tekst* no. 485. Roskilde: Roskilde University. (Published in Danish in 2002). Retrieved from: http://milne.ruc.dk/imfufatekster/pdf/485web_b.pdf

Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. Routledge.

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, *102*(1), 9-28.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In Werner Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education: The 14th ICMI study (pp. 3–32). Boston, MA: Spring.

Niss, M., Bruder, R., Planas, N., Turner, R., & Villa-Ochoa, J. A. (2016). Survey team on: Conceptualisation of the role of competencies, knowing and knowledge in mathematics education research. ZDM, 48(5), 611–632.

Oberman, R., & Sainz, G. M. (2021). Critical thinking, critical pedagogy, and climate change education. *Teaching for Social Justice and Sustainable Development across Primary Curriculum*, 69-83.

Özdemir, E., Üzel, D., & Özsoy, N. (2017). An Investigation of Teachers' Views on Applicability of Modeling in Mathematics Courses. *Journal of education and training studies*, *5*(5), 145-155.

Paola Ramirez. Teachers' beliefs about mathematical modelling: An exploratory study. CERME 10, Feb 2017, Dublin, Ireland. ffhal-01933470f

Passarella, S. (2021). Mathematics Teachers' Inclusion of Modelling and Problem Posing in Their Mathematics Lessons: An Exploratory Questionnaire. *European Journal of Science and Mathematics Education*, *9*(2), 43-58.

Patrick, H., & Pintrich, P. R. (2001). Conceptual change in teachers' intuitive conceptions of learning, motivation, and instruction: The role of motivational and epistemological beliefs. In B. Torff & R. J. Sternberg (Eds.), Understanding and teaching the intuitive mind: student and teacher learning (pp. 117–143). Mahwah, N.J: Lawrence Erlbaum.

Pehkonen, E., Näveri, L., & Laine, A. (2013). On teaching problem solving in school mathematics. *ceps Journal*, *3*(4), 9-23.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. Second handbook of research on mathematics teaching and learning: a project of the National Council of Teachers of Mathematics. Vol. 1 (pp. 257-315). Charlotte, NC: Information Age Pub.

Polit, D. F., & Beck, C. T. (2012). *Nursing research: Generating and assessing evidence for nursing practice*. Lippincott Williams & Wilkins.

Ramirez, P. (2017, February). Teachers' beliefs about mathematical modelling: An exploratory study. In *CERME 10*.

Reckase, M. D. (2000). Scaling techniques. In G. Goldstein & M. Hersen (Eds.), Handbook of psychological assessment (3rd ed., pp. 43–61). Amsterdam ; New York: Pergamon

Rinjit, K. (2020). Research methodology.

Ritchie, J., & Spencer, L. (2002). Qualitative data analysis for applied policy research. In *Analyzing qualitative data* (pp. 187-208). Routledge.

rochim, W.M.K. (2002) The Research Methods Knowledge Base. 2nd Edition, Cornell University, New York.

Rooney, A. A., Cooper, G. S., Jahnke, G. D., Lam, J., Morgan, R. L., Boyles, A. L., ... & Lunn, R. M. (2016). How credible are the study results? Evaluating and applying internal validity tools to literature-based assessments of environmental health hazards. *Environment international*, *92*, 617-629.

Russo, J., Bobis, J., Sullivan, P., Downton, A., Livy, S., McCormick, M., & Hughes, S. (2020). Exploring the relationship between teacher enjoyment of mathematics, their attitudes towards student struggle and instructional time amongst early years primary teachers. *Teaching and Teacher Education*, *88*, 102983.

Sandelowski, M. (1986). The problem of rigor in qualitative research. Advances in Nursing Science, 8(3), 27–37.

Şandır, H. (2010). Investigation of processes about modeling that mathematics teachers and preservice teachers designed and applied, Unpublished Phd Thesis, Gazi University, Ankara, Turkey.

Sargeant, J. (2012). Qualitative research part II: Participants, analysis, and quality assurance. Journal of Graduate Medical Education 4(1), 1-3.

Schaap, S., Vos, P., & Goedhart, M. (2011). Students overcoming blockages while building a mathematical model: Exploring a framework. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), Trends in teaching and learning of mathematical modelling (Vol. 1, pp. 137–146). Dordrecht: Springer Netherlands.

Schichl, H. (2004). Models and the history of modeling. In J. Kallrath (Ed.), Modeling languages in mathematical optimization (pp. 25−36). Boston: Kluwer.

Schmidt, B. (2011). Modelling in the classroom: Obstacles from the teacher's perspective. *Trends in teaching and learning of mathematical modelling*, 641-651.

Schupp, H. (1989). Applied mathematics instruction in the lower secondary level—Between traditional and new approaches. In W. Blum, et al. (Eds.), Applications and modelling in learning and teaching mathematics (pp. 37–46). Chichester: Ellis Horwood.

Schwandt, T. A., Lincoln, Y. S., & Guba, E. G. (2007). Judging interpretations: But is it rigorous? Trustworthiness and authenticity in naturalistic evaluation. *New directions for evaluation*, *2007*(114), 11-25.

Schwarz, B., Wissmach, B., & Kaiser, G. (2008). "Last curves not quite correct": Diagnostic competences of future teachers regarding modelling and graphical representations. ZDM, 40(5), 777–790.

Sheard, J. (2018). Quantitative data analysis. In *Research Methods: Information, Systems, and Contexts, Second Edition* (pp. 429-452). Elsevier.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15(2), 4–14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. Harvard Educational Review, 57(1), 1–22.

Sileyew, K. J. (2019). *Research design and methodology* (pp. 1-12). Rijeka: IntechOpen.

Simamora, R. E., & Saragih, S. (2019). Improving Students' Mathematical Problem Solving Ability and Self-Efficacy through Guided Discovery Learning in Local Culture Context. *International Electronic Journal of Mathematics Education*, *14*(1), 61-72.

Srivastava, A., & Thomson, S. B. (2009). Framework analysis: a qualitative methodology for applied policy research.

Stender, P., & Kaiser, G. (2015). Scaffolding in complex modelling situations. ZDM, 47(7), 1255– 1267.

Stillman, G. A., Blum, W., & Biembengut, M. S. (2015). *Mathematical modelling in education research and practice* (pp. 93-104). Springer.

Stillman, G. A., Brown, J. P., & Galbraith, P. (2013). Challenges in modelling challenges: Intents and purposes. In G. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), Teaching mathematical modelling: connecting to research and practice (pp. 217–227). Dordrecht: Springer Netherlands.

Stillman, G., & Galbraith, P. (2012). Mathematical modelling: Some issues and reflections. In *Mathematikunterricht im Kontext von Realität, Kultur und Lehrerprofessionalität* (pp. 97-105). Vieweg+ Teubner Verlag.

Tan, L. S., & Ang, K. C. (2016). A school-based professional development programme for teachers of mathematical modelling in Singapore. Journal of Mathematics Teacher Education, 19(5), 399–432.

Taole, M. J. (2020). Diversity and inclusion in rural South African multigrade classrooms. *International Journal of Inclusive Education*, *24*(12), 1268-1284.

Tekin Dede, A., and Bukova Güzel, E. (2016, July). How to integrate mathematical modelling into mathematics courses: A guide suggestion. Presented at 13th International Congress on Mathematical Education, Hamburg, Germany.

Törner, G. (2002). Mathematical beliefs—A search for a common ground: some theoretical considerations on structuring beliefs, some research questions, and some phenomeno-logical observations. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), Beliefs: A hidden variable in mathematics education? (pp. 73–94). Dordrecht: Springer.

Tran, D., Nguyen, D. T., Nguyen, A. T. T., Nguyen, G. N. T., & Ta, P. M. (2020). Bridging to mathematical modelling: Vietnamese students' response to different levels of authenticity in contextualized tasks. *International journal of mathematical education in science and technology*, *51*(6), 893-912.

Trochim, W. M. (2006). The qualitative debate.

Tschannen-Moran, M., & Woolfolk Hoy, A. (2001). Teacher efficacy: Capturing an elusive construct. Teaching and Teacher Education, 17(7), 783–805.

Turner, E. E., Bennett, A. B., Granillo, M., Ponnuru, N., Roth Mcduffie, A., Foote, M. Q., ... & McVicar, E. (2022). Authenticity of elementary teacher designed and implemented mathematical modeling tasks. *Mathematical Thinking and Learning*, 1-24.

Varpio L, Young M, Uijtdehaage S, Paradis E. 2019. Articulating the distinctions between theory, theoretical framework, and conceptual framework. Acad Med. Published Ahead of Print.

Vorhölter, K., Greefrath, G., Borromeo Ferri, R., Leiß, D., & Schukajlow, S. (2019). Mathematical modelling. In *Traditions in German-speaking mathematics education research* (pp. 91-114). Springer, Cham.

Voss, T., Kleickmann, T., Kunter, M., & Hachfeld, A. (2013). Mathematics Tteachers' beliefs. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand

(Eds.), Cognitive activation in the mathematics classroom and professional competence of teachers (pp. 249–271). Boston, MA: Springer US.

Warner, L. B., Schorr, R. Y., & Warner, S. J. (2014). Allowing students to take the lead in mathematical investigations. *Annual Perspectives in Mathematics Education (APME)*, 35-44.

Wei, Y., Zhang, Q., & Guo, J. (2022). Can Mathematical Modelling Be Taught and Learned in Primary Mathematics Classrooms: A Systematic Review of Empirical Studies. *Education Sciences*, *12*(12), 923.

Weinert, F. E. (2001). Concept of competence: A conceptual clarification. In D. S. Rychen & L. H. Salganik (Eds.), Defining and selecting key competencies. (pp. 45–65). Seattle: Hogrefe & Huber Publishers. Weinert, (2001).

Wess, R., Klock, H., Siller, H. S., & Greefrath, G. (2021). Measuring professional competence for the teaching of mathematical modelling: A test instrument. Springer Nature.

White, A. L., Way, J., Perry, B., & Southwell, B. (2005). Mathematical attitudes, beliefs, and achievement in primary pre-service mathematics teacher education. *Mathematics teacher education and development*, *7*(33-52).

Winter, M., & Venkat, H. (2013). Pre-service teacher learning for mathematical modelling. In *Teaching mathematical modelling: Connecting to research and practice* (pp. 395-404). Springer, Dordrecht.

Wipulanusat, W., Panuwatwanich, K., Stewart, R. A., & Sunkpho, J. (2020). Applying mixed methods sequential explanatory design to innovation management. In *The 10th International Conference on Engineering, Project, and Production Management* (pp. 485-495). Springer Singapore.

Yang, X., Kaiser, G., König, J., & Blömeke, S. (2020). Relationship between pre-service mathematics teachers' knowledge, beliefs, and instructional practices in China. *Zdm*, *52*, 281-294.

Yang, X., Schwarz, B., & Leung, I. K. (2022). Pre-service mathematics teachers' professional modeling competencies: a comparative study between Germany, Mainland China, and Hong Kong. *Educational Studies in Mathematics*, *109*(2), 409-429.

Zólyomi, A. (2022). Exploring Hungarian secondary school English teachers' beliefs about differentiated instruction. *Language Teaching Research*, 13621688221114780.

APPENDIX A Questionnaire SPSS coding sample

10/27/22, 11:44 AM Teachers' Beliefs and Self-efficacy about Mathematical Modelling

2

Teachers' Beliefs and Self-efficacy about Mathematical Modelling

Dear:
Educator.

Thank
you for participating in this questionnaire. As a principle, the information you provide in this questionnaire will be treated with confidentiality and your personal information will not be included in any publications that may result form of this study. Completion of the questionnaire (both the hard copy and online) implies consent.

Gender ① female ② male ③ other

1 Female

Age (in years)
① 20 – 30
② 31 – 40
③ 41 – 50
④ above 50

2 41 - 50

School-leaving examination grade (i.e. Grade 12)
① Grade
② A-Level
③ Other

3 Grade 12

Highest Qualification
① Diploma
② BEd
③ BEd (hons)
④ MEd
⑤ PhD
⑥ other

4 Honours Degree

https://docs.google.com/forms/d/1-EwGigE9xM5xduh_cKSflfKXacTlOy0yJ6X-HAMs-9Q/edit?pli=1#responses 374/397

10/27/22, 11:44 AM Teachers' Beliefs and Self-efficacy about Mathematical Modelling

Last grade teaching mathematics

① Grade 10
5 Grade 12 ② Grade 11
③ Grade 12

Number of years teaching mathematics (in years)

① 0 - 5
6 6 - 15 ② 6 - 15
③ 16 - 25
④ above ~~50~~ 25

① Grad 12
Current grade teaching
② Grade 11
③ Grade 10
7 Grade 12 ④ Grade 9
⑤ Grade 8

① Physical Science
Sub-major subject (e.g. Physical sciences) ② Geography
③ English
8 Information technology ④ Other

To what extent do you agree with the following statements?

		Strongly agree	Agree	Neither agree nor disagree	Disagree	Strongly disagree
		5	4	3	2	1
9	~~1.~~ Mathematical modelling should be part of mathematics education in the CAPS.	○	◉	○	○	○
10	~~2.~~ Results of mathematical modelling have benefits for our societies.	○	◉	○	○	○
11	~~3.~~ Learners should be given the opportunity to do mathematical modelling in the classroom.	○	○	◉	○	○
12	~~4.~~ Learners learn mathematics best by discovering their own ways of solving problems.	○	◉	○	○	○
13	~~5.~~ Mathematical modelling is an unproductive game, with no concrete relation to reality.	○	◉	○	○	○
14	~~6.~~ Effective teachers show the right way and methods to solve an application problem.	○	○	◉	○	○
15	~~7.~~ Learners should solve	○	◉	○	○	○

tasks in the way they were taught in class.

		5	4	3	2	1
16	~~8~~. Teachers should allow learners to come up with their own ways of solving application problems before they show them how to solve them.	○	◉	○	○	○
17	~~9~~. Among other competences, mathematical modelling competence should be taught in the classroom.	○	○	◉	○	○
18	~~10~~. Learners should often have the opportunity to follow their teacher's method solutions.	○	○	◉	○	○
19	~~11~~. Mathematics should be taught at school in such a way that learners can see the links on their own.	○	◉	○	○	○
20	~~12~~. Many aspects of mathematical modelling are used practically in reality.	○	◉	○	○	○
21	~~13~~. Mathematical modelling	○	◉	○	○	○

		5	4	3	2	1
	should be a specific topic in the CAPS mathematics curriculum.					
22	~~14.~~ Modelling helps learners to understand mathematics when they are asked to discuss their own solution thinking.	○	◉	○	○	○
23	~~15.~~ In mathematical modelling, you work on tasks that are more practical.	○	◉	○	○	○
24	~~16.~~ Teachers should provide detailed procedures to learners for solving application problems.	○	○	◉	○	○

SELF-EFFICACY

It is **easy** for me to identify the different abilities of the learners using …

		Strong agree	Agree	Neither Agree nor disagree	Disagree	Strongly disagree
		5	4	3	2	1
25	~~17.~~ … their translation of mathematical results into reality.	○	◉	○	○	○
26	~~18.~~ … their written solutions when modelling.	○	◉	○	○	○
27	~~19.~~ … the recording of the reasonable test of their solution.	○	○	◉	○	○
28	~~20.~~ … the assessment of the relationship between the mathematical result and real results that they produce.	◉	○	○	○	○
29	~~21.~~ … the recording of the solutions they get for modelling tasks.	○	◉	○	○	○
30 ~~29~~	~~22.~~ … the mathematical models/methods they chose when modelling.	○	◉	○	○	○
31	~~23.~~ … the recording of their mathematical results in the modelling process.	◉	○	○	○	○
32	~~24.~~ … the mathematical formulae and	○	○	◉	○	○

symbols they used in the modelling process.

		5	4	3	2	1
33	~~25~~. ... the assessment of how well they improved the models that were already in use at the time of modelling.	○	◉	○	○	○
34	~~26~~. ... the assumptions they made when modelling.	◉	○	○	○	○
~~34~~ 35	~~27~~. ... the learners' solutions when modelling.	○	◉	○	○	○
36	~~28~~. ... their handling of the mathematical symbols and operators used in modelling.	○	◉	○	○	○

It is **difficult** for me to identify the different abilities of the learners using ...

		Strong agree 5	Agree 4	Neither Agree nor disagree 3	Disagree 2	Strongly disagree 1
~~36~~ 37	~~29~~. ... their translation of mathematical results into reality.	○	◉	○	○	○
38	~~30~~. ... the assumptions they have made in the modelling process.	○	◉	○	○	○
~~38~~ 39	~~31~~. ... the reasonable checks of their modelling solution.	◉	○	○	○	○
~~39~~ 40	~~32~~. ... their writing when they separate important and non-important information about a real situation in modelling.	◉	○	○	○	○
41	~~33~~. ... the mathematical models/ methods they chose when modelling.	○	◉	○	○	○
~~41~~ 42	~~34~~. ... the recording of the relationship between mathematical result and reality results that they produce when modelling.	○	◉	○	○	○
43	~~35~~. ... their handling of the mathematical formulae and	○	◉	○	○	○

	5	4	3	2	1
symbols used in modelling.					
~~43~~ 44 ~~36.~~ ... their written solution when modelling.	●	○	○	○	○
~~44~~ 45 ~~37.~~ ... their evaluation of the established models in the modelling process.	●	○	○	○	○
46 ~~38.~~ ... the simplifications they have undertaken in the modelling process.	○	●	○	○	○
~~46~~ 47 ~~39.~~ ... their mathematisations of a real situation.	○	●	○	○	○
48 ~~40.~~ ... the recording of their mathematical results in modelling.	●	○	○	○	○

PRIOR KNOWLEDGE

To what extent do you agree with the following statements?

		Strongly agree 5	Agree 4	Neither agree nor disagree 3	Disagree 2	Strongly disagree 1
48 49	~~41~~. I have previously learnt mathematical modelling in my lectures.	○	○	●	○	○
49 50	~~42~~. I was prepared for teaching modelling.	○	○	○	●	○
51	~~43~~. I have already taught mathematical modelling with learners.	○	○	○	●	○
52	~~44~~. I feel well prepared to teach mathematical modelling through my previous training.	○	○	○	●	○
53	~~45~~. Mathematical modelling has already been addressed in my teacher training courses.	○	○	●	○	○
54	~~46~~. I have already been presented with knowledge to teach modelling.	○	○	○	●	○
55	~~47~~. I have solved modelling tasks myself during my teacher training.	○	○	○	●	○
56	~~48~~. As part of my previous teacher	○	○	○	●	○

	5	4	3	2	1
training, I have been able to build solid foundations for teaching mathematical modelling.					
56 57 ~~49.~~ Mathematical modelling played a role in my first year of teaching at school.	○	○	◉	○	○
59 ~~50.~~ In my education (high school to university), I have already acquired knowledge to teach modelling.	○	○	○	◉	○
~~58~~ 59 ~~51.~~ I have been attending mathematical modelling workshops with the department of education.	○	○	○	◉	○
~~59~~ 60 ~~52.~~ In my previous education, I also had to do mathematical modelling in the processing of tasks.	○	○	○	◉	○
61 ~~53.~~ If I have to design lessons for "mathematical modelling", I can draw from what I have learnt.	○	○	○	◉	○
~~61~~ 62 ~~54.~~ I have already gained teaching	○	○	○	◉	○

experience in mathematical modelling.	5	4	3	2	1
63 ~~55~~. I have been working on modelling examples myself during my teacher training.	○	○	◉	○	○

This form was created inside University of the Witwatersrand, Johannesburg.

Google Forms

APPENDIX B Interview Questions

Researcher : Mr S.J Khoza

CODE:

Research Title : Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling.

Dear Educator

Thank you for taking time to participate in the interviews. I do appreciate your consent to allow me to use the data in my research report without disclosing your identity. In this interview, I will ask you some questions about mathematics, in particular about your perception or belief about mathematical modelling in the mathematics curriculum and in the classroom.

BELIEFS:

1. Regarding the applications of mathematical modelling in real-life. Do you believe that modeling has applications in real life?
2. Do you believe that mathematical modeling is useful in mathematical education?
3. What is your belief about learner-involvement in mathematical learning tasks? Should the teacher provide step-by-step guidelines on how learners should solve modelling tasks? Or should the teacher allow learners to work on the tasks by themselves?
4. Do you believe that the teacher should take the lead in solving modelling tasks for the learners or do you believe that learners should take the lead in solving modelling tasks.

SELF-EFFICACY:

5. *How easy is it for you in your teaching, to recognise the different abilities of learners:*
   a) from their written mathematical solution solutions.
   b) about the mathematical models/methods they choose when modelling.
   c) about recording their mathematical results in the modelling process.
   d) about the mathematical formula and symbols, they use in the modelling process.
   e) about handling of mathematical symbols, they use in the modelling process.

6. *How difficult is it for you in your teaching, to recognise the different abilities of learners in terms of:*
   a) the assumptions they make in the modelling process.
   b) the reasonableness of the solutions after checking.
   d) recording of the relationship between mathematical result and reality in their modelling solutions.
   e) about handling of mathematical symbols, they use in the modelling process.

General Questions:

7. Regarding mathematics as a whole in the CAPS curriculum Grade 10 - 12, what recommendations can you offer to improve the teaching and learning of the subject? Which topics of mathematics do you recommend giving special attention to, and which ones not so much attention? Why and why not?
8. What tools or equipment do you usually use in teaching mathematics? What examples do you give when you teach different Grades? Give some examples that you remember.
9. To what extend are you aware with the CAPS curriculum Grade 10 - 12? and with mathematical modelling in the curriculum?
10. When and where did you first learn about mathematical modelling? For example, in high school when you were a learner, or in the university or both? Or did you learn about it from the CAPS curriculum after graduation?
11. How often have you encountered modelling problems from your high school experience until now as a qualified teacher?
12. As a qualified mathematics teacher, how do you implement modelling in your classroom teaching? Do you teach modelling separately or do you integrate modelling in each mathematics topic that you teach? please give examples if you can.
13. What do you like about teaching problem-solving and applications through mathematical modelling? What does not work for you when you try to teach modelling?
14. From your knowledge and experience in teaching mathematics, what do you think are the benefits of modelling? What are the possible challenges with modelling in school mathematics?
15. Regarding modelling in the CAPS curriculum Grade 10-12, what recommendation can you give to improve the teaching and learning of applications and mathematical modelling? Should modelling be scaled to other school levels including the primary school level? Why or why not?
16.

a) In a scale of 1-10 (where 1 is minimum awareness, and 10 is maximum awareness) how would you rate you level of awareness of mathematical modelling in the curriculum? Please say more about the scale that you have chosen.
b) In a scale of 1-10 (where 1 is minimum knowledge and skills, and 10 is maximum knowledge and skills for teaching modelling in mathematics) how would you rate your knowledge and skills for teaching modelling in the curriculum. Please say more about this scale that you have.

APPENDIX C Mathematical modelling played a role in my first year of teaching at school.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly disagree	2	5.0	5.0	5.0
	Disagree	9	22.5	22.5	27.5
	Neither agree nor disagree	16	40.0	40.0	67.5
	Agree	11	27.5	27.5	95.0
	Strongly agree	2	5.0	5.0	100.0
	Total	40	100.0	100.0	

APPENDIX D In my education (high school to university), I have already acquired knowledge to teach modelling.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly disagree	3	7.5	7.5	7.5
	Disagree	6	15.0	15.0	22.5
	Niether agree nor disagree	14	35.0	35.0	57.5
	Agree	12	30.0	30.0	87.5
	Strongly agree	5	12.5	12.5	100.0
	Total	40	100.0	100.0	

APPENDIX E I have already gained teaching experience in mathematical modelling.

		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	**Strongly disagree**	**2**	**5.0**	**5.0**	**5.0**
	Disagree	**11**	**27.5**	**27.5**	**32.5**
	Niether agree nor disagree	**9**	**22.5**	**22.5**	**55.0**
	Agree	**13**	**32.5**	**32.5**	**87.5**
	Strongly agree	**5**	**12.5**	**12.5**	**100.0**
	Total	**40**	**100.0**	**100.0**	

APPENDIX J Many aspects of mathematical modelling are used practically in reality.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Neither agree nor disagree	4	10.0	10.0	10.0
	Agree	21	52.5	52.5	62.5
	Strongly agree	15	37.5	37.5	100.0
	Total	40	100.0	100.0	

APPENDIX F NM Interview responses sample

NM Transcription

Interviewer: ... Do you believe that modeling has applications in real life? Should you believe that they do have application in real life what examples can I give?

NM: *Yes, I do believe let's say you have a business OK and you're trying to teach them how to get profit from whatever they sell OK. Let's say they want to sell apples they can use mathematical modeling in that aspect point.*

Interviewer: Do you believe that mathematical modeling is useful in mathematical education?

NM: *it is useful 100% because teaching Grade 10 and 11 word problems you want them to see what's happening around them and we don't call it modelling but we call it word-problems and we teach them that you are just translating the word problems into mathematical language ... how we teach it to the learners I think it's very crucial and it's telling you or it shows you the capabilities of your learners in terms of taking a word problem and translating it into reality and they do successfully translate them into mathematical language and that just tells you a lot about your learners.*

Interviewer: so, we can get learners' abilities through word problems?

NM: *yes, definitely there's a lot of them.*

Interviewer: what is your believe about Leaner involvement in mathematical learning? should the teacher provide step-by-step guidelines on how to solve modeling task or should the teacher allow learners to do the work on their own before the teacher intervene?

NM: *in my personal view I feel like those two points can be utilized together and at some point you can give them step-by-step and some situations give them freedom of using their own ways of solving mathematics and you can also give them step-by-step uh methods ... you can use both of them I don't believe that you should use one in one of them in isolation I feel like it would be best interest of the child to use them both ... because they allow them to think out-of-the-box ... because telling them to use one teaching method and you find that those methods the child does not understand but they have their own interpretation of how they can solve them because now you'll be saying that you can only be able to use this method that's what they think you can only be able to use this method and this method is the correct one so which means that you are not allowing them to feel free and use their imagination in terms of making a situation in a different way that you would perhaps you like these two things can be used in conjunction ... in other words it will depend on a certain situation this one I can give them step by step but this one I can allow them to work on their own.*

Interviewer: do you believe that teachers should take the lead in solving modelling task for the learners or do you believe that learners should take the lead in solving modelling tasks?

NM: *I think learners should take the lead. Why am I saying this? because at the end of the day I'm a teacher so I've been taught this in university level right yeah and what steps to follow and how to teach mathematical modelling now it becomes different when it's a learner, learners' should have the freedom of the way they say they understand it.*

APPENDIX G KN Interview responses sample

Interview Transcription_KN

Researcher: What is your belief about learner-involvement in mathematical learning tasks? Should the teacher provide step-by-step guidelines on how learners should solve modelling tasks? Or should the teacher allow learners to work on the tasks by themselves?

KN: I think that we should allow them to come up with their own ideas, however from my experience that is a big task not because it cannot be done but because the system puts pressure, and the system is very results focused. So teachers will get things done so those things in terms of like the problem-solving and in terms of developing those models should start at an early grade where children learn those skills so that when they come to high school you don't like teach them the way they already have ideas on how to develop solutions on their own using modelling … come up with a solution it seems like only children that go to Crawford they get those things.

Researcher: Do you believe that the teacher should take the lead in solving modelling tasks for the learners or do you believe that learners should take the lead in solving modelling tasks.

KN: Let me give an example in this one as you said no answer is right or wrong right. let's say we give the children an investigation and investigation can happen how you can derive your answer using your own methods right, but … it through a project or an investigation you can allow children to come up with modelling ideas right to solve their problems in this investigation or project. OK, the first problem on that would be that many children don't want to take up the opportunity to solve the problem that's the first thing so there will be those who will come back to you and say ma'am I tried this and so on so they eager to develop the ideas behind that and to solve a problem potion. The second problem is that the time is very limited for them to come up with that it's not an ongoing thing it's not like an algorithm that we can go whenever they feel like going to because the curriculum has a start date and end date and within that time, they need to think of those solutions which they don't want to do because of time limits.

APPENDIX H SN Interview responses sample

Interview Transcription_SN

General questions

Researcher: Regarding mathematics as a whole in the CAPS curriculum Grade 10 - 12, what recommendations can you offer to improve the teaching and learning of the subject? Which topics of mathematics do you recommend giving special attention to, and which ones not so much attention? Why and why not?

SN: … yeah I feel like some of the topics don't fit into learners real-life experiences, such as algebra, it is more abstract but those skills are needed by the learners to solve mathematical problems … also it seems to be a lot to cover but in the curriculum that end up compromising the learning interactions among the learners'… so yeah I feel like the content in the mathematics curriculum should be reworked and provide more time for better learning of mathematics and modelling…"

Researcher: When and where did you first learn about mathematical modelling? For example, in high school when you were a learner, or in the university or both? Or did you learn about it from the CAPS curriculum after graduation?

SN: I think from young age but not knowing I was doing modelling. Sometimes modelling reduces the complexity of complicated scenarios …

APPENDIX I Teachers' information sheet

TEACHERS' INFORMATION SHEET

Researcher : Mr Siyabonga Khoza

Supervisor : Dr George Ekol

Research Title : Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling.

Dear Educator

My name is Siyabonga Jabulane Khoza, I am doing Master of Education (Mathematics) by dissertation at the University of the Witwatersrand, Johannesburg. My supervisor is Dr George Ekol, a senior lecturer in the Mathematics Division at Wits School of Education. I am conducting a research study; the research topic is ***Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling.***

You are invited to participate in a research study conducted by Mr S.J Khoza. The purpose of the study is to investigate teachers' beliefs, self-efficacy (SEF), prior knowledge and perceptions about mathematical modelling.

Your participation will expect you to complete a test questionnaire of 33 questions, based on your beliefs, SEF, prior knowledge about mathematical modelling. This might take about 45 minutes to complete. There are no anticipated risks associated with this research, the risks for this study are no more than what happens in everyday life, which are minimal. There are no direct benefits for your participation in this research study. However, your participation will contribute to building modelling competency towards mathematics teachers.

Your participation is voluntary, and you may choose to withdraw your consent at any time. If you want to withdraw from the study, you can contact me (Mr S.J Khoza) at 071 518 2024 or 1636533@students.wits.ac.za You may choose not to answer any questions that you do not want to answer. You will NOT be penalized in any way should you choose to withdraw your participation from the study.

By agreeing to participate in the study, you understand and agree that your data will be shared in my research findings analysis and with my supervisor. Your personal details will not be revealed. All data will be stored on a password-protected laptop and completed test book questionnaires will be kept in a locked drawer in an office. The report will be available on the university library website. If you would like to receive a copy of this report, I will be happy to send it to you upon request. If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details provided below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the Wits University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,

Siyabonga Khoza

RESEARCHER	SUPERVISOR
Mr Siyabonga Jabulane Khoza	Dr. George Ekol
1636533@students.wits.ac.za	George.ekol@wits.ac.za
+27 (0) 71 518 2024	+27 (0) 65 829 0743

APPENDIX P Teachers' consent form

Master of Education (Mathematics) by Dissertation

TEACHERS' CONSENT FORM

Researcher : Mr Siyabonga Khoza

Supervisor : Dr George Ekol

Research Title : Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling.

	YES	**NO**
The research study was explained to me. I understand what this study is about.		
I understand that I can volunteer to take part in the study or withdraw at any time.		
I agree that my participation will remain confidential and anonymous (my personal details will not be used by the researcher in their research report)		
I agree that other researchers may use the information I provide in my interview (depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on.		

I have read this consent form and have been given the opportunity to ask questions. I will also be given a copy of this consent form for my records. I consent to my participation in the research study described from the information sheet.

____________________	________________________	___/___/2022
Participant's Signature	Participant's Printed Name	Date
____________________	SIYABONGA KHOZA ________________________	10 /07 /2022
Researchers' Signature	**Researchers' Printed Name**	**Date**

APPENDIX K GDE research approval letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	04 July 2022
Validity of Research Approval:	08 February 2022– 30 September 2022 2022/278
Name of Researcher:	Khoza S.J
Address of Researcher:	14 Juta Street
	Braamfontein
	Johannesburg
Telephone Number:	071 518 2024
Email address:	1636533@STUDENTS.wits.ac.za
Research Topic:	Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling.
Type of qualification	Master of Education (Mathematics) by Dissertation
Number and type of schools:	2 Secondary Schools
District/s/HO	Johannesburg Central and North

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

04/07/2022

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below are met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

The masters dissertation has been approved by the university (WSoE)

1. The letter would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. **Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools, or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.**
4. **The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.**
5. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s has been granted permission from the Gauteng Department of Education to conduct the research study.
6. A letter/document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs, and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
7. The Researcher will make every effort to obtain the goodwill and cooperation of all the GDE officials, principals, and chairpersons of the SGBs, teachers, and learners involved. Persons who offer their cooperation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
8. Research may only be conducted after school hours so that the normal school program is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
9. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
10. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
11. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
12. The researcher is responsible for supplying and utilising his/her research resources, such as stationery, photocopies, transport, faxes, and telephones, and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
13. The names of the GDE officials, schools, principals, parents, teachers, and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
14. On completion of the study, the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
15. The researcher may be expected to provide short presentations on the purpose, findings, and recommendations of his/her research to both GDE officials and the schools concerned.
16. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a summary of the purpose, findings, and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr. Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: ....04|07|2022....................

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX L School A permission letter

LANGLAAGTE
TECHNICAL HIGH SCHOOL

CNR. KRUGER & DU TOIT STREET
LANGLAAGTE
PRIVATE BAG X1
LANGLAAGTE 2102

TEL: 011 837 8577
011 837 8944
FAX: 011 837 8575
E-MAIL: thsll@netactive.co.za

SERVICE THROUGH KNOWLEDGE

OUR REFERENCE NO.:............................

YOUR REFERENCE NO.:............................

ENQUIRIES:

Dear Mr S.J Khoza

RE: REQUEST PERMISSION TO CONDUCT STUDY AT YOUR SCHOOL

Your letter dated 21 September 2022:

Refers your request for permission to conduct a research study titled: **Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling** with our mathematics teachers at Langlaagte Technical High School is hereby permitted.

The school will be happy if you would provide us with a copy of your Dissertation.

We wish you success in your studies.

Kind Regards.

RMokgele

Principal

GAUTENG DEPT EDUCATION
HTS LANGLAAGTE THS
TEL 837-8577
22 -09- 2022
H/V KRUGER & DU TOIT
P/SAK X1 LANGLAAGTE 2102
DISTRICT JN-D10

School stamp

Our vision is to build a disciplined learner corps with a learning culture of its own, a professional, motivated and goal orientated staff, excellent academic standards and results, effective and successful manager...

APPENDIX M School B permission letter

JOHANNESBURG SECONDARY SCHOOL

P.O. Box 42837 FORDSBURG 2033, 1 PANCRAS ROAD, HOMESTEAD PARK 2092, JOHANNESBURG TEL: 011 839 2758 E-mail: 700130062@gdeschools.gov.za

Dear Mr S.J Khoza

RE: REQUEST PERMISSION TO CONDUCT RESEARCH STUDY AT YOUR SCHOOL

Your letter dated 30 June 2022:

Refers your request for permission to conduct a research study titled: ***Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling*** with our mathematics teachers at *Johannesburg Secondary School* is hereby permitted.

The school will be happy if you would provide us with a copy of your Dissertation.

We wish you success in your studies.

Kind Regards.

Ms. M.A Mohale
JSS Principal

GAUTENG DEPARTMENT OF EDUCATION
Johannesburg Secondary School
1 PANCRAS STR, HOMESTEAD PARK, 2092
SCHOOL STAMP
12 AUG 2022
P.O. BOX 42837, FORDSBURG, 2033
TEL: 011 839 2758 FAX: 011 837 1558
mwjss@lafrica.com

APPENDIX N Turnitin receipt

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APPENDIX O Ethics clearance certificate

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Research Office

HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)
R14/49 Khoza

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: H22/07/14**
PROJECT TITLE	Investigating In-service Teachers' Beliefs and Self-efficacy about Mathematical Modelling Using a Structural Model of Professional Competence for Teaching Mathematical Modelling
INVESTIGATOR(S)	Mr S Khoza
SCHOOL/DEPARTMENT	Wits School of Education/
DATE CONSIDERED	22 July 2022
DECISION OF THE COMMITTEE	Approved Risk Level: Minimal
EXPIRY DATE	12 September 2025

DATE 29 September 2022 **CHAIRPERSON** (Professor J Watermeyer)

cc: Supervisor : Dr G Ekol

DECLARATION OF INVESTIGATOR(S)

To be completed in duplicate and **ONE COPY** returned to the Secretary at Room 10004, 10th Floor, Senate House, University. Unreported changes to the application may invalidate the clearance given by the HREC (Non-Medical)

I/We fully understand the conditions under which I am/we are authorized to carry out the abovementioned research and I/we guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to submit an amendment of the protocol to the Committee. **I agree to completion of a regular progress report. For Minimal and Low studies, this is due annually on 31 December. For Medium and High Risk studies, this is due twice annually on 30 June and 31 December.**

Signature 30 / 09 / 2022 **Date**

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

The masters dissertation has been approved by the university (WSoE)

APPENDIX Q Language editing certificate

Buzpro Evolution

Certificate of Proofreading and Editing

This document certifies that the manuscript listed below has been proofread for appropriate English language usage, grammar, punctuation, and spelling by a professional native English-speaking editor.

Author: SIYABONGA KHOZA

Dissertation Title: TEACHERS' BELIEFS AND SELF-EFFICACY ABOUT TEACHING MATHEMATICAL MODELLING

18 July 2023

Mignon Dehning

Buzpro Evolution Administration Services
Registration No. K2020692243
Marc Dehning +27 69 376 1939 | Mignon Dehning +27 71 022 3976
info@buzpro.co.za | www.buzpro.co.za

[i] The modelling term used with double l refers to the regular manner in which the term is written in the South African educational context and literature. The German, American, and European education contexts use the modelling term with a single l (Modelling). After all, the meaning of both written terms is identical.