

Connecting Metacognition and Mathematical Proficiency: A Case Study of South African Matriculants

Charmaine Price

Student No: 579332

Protocol No: 2014ECE034M

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Signed on 10 September 2016 in Austin, Texas, USA

DECLARATION

I declare that this Dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

CS Price

(Signature of candidate)

10th day of September 2016 in Austin, TX USA

ABSTRACT

This dissertation investigated the possible connection between the two constructs of metacognition and the strands of mathematical proficiency. Using a small convenience sample of Grade 12 learners at a public all-girls high school in Johannesburg, South Africa, the teacher/researcher examined if training in metacognitive knowledge influenced the learners' conceptual understanding, procedural fluency, strategic competence and heuristic strategy use. Two instruments (pre-test and post-test) developed by the researcher and completed by the students were on-line assessments of metacognition utilising written self-reports of strategy use that occurred simultaneously with mathematical problem solving. Qualitative analyses were applied to the data and learners demonstrated declarative, procedural and conditional metacognitive knowledge in their explicit heuristic and content-specific strategies. The learners also demonstrated strategies which could be inferred from their working. The study showed evidence of mixed transferability of metacognitive training to mathematical proficiency. Questions for further research are provided.

In memory of my mother and best friend

Connie Wakefield

1924 - 2014

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