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The ways in which three grade 10 mathematics teachers use formative assessment to select and mediate learner teacher support material

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Masters in Education: Curriculum

Research Report by Coursework

Topic: The ways in which three grade 10 mathematics teachers use formative assessment to select and mediate learner teacher support material

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Acronyms and Symbols

CK	Conceptual Knowledge
CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
DT	Diagnostic Test
EK	Everyday Knowledge
FET	Further Education and Training
KZN	KwaZulu Natal
LTSM	Learner Teacher Support Material
NEEDU	National Education Evaluation Development Unit
NGOs	Non-governmental organisations
NSC	National Senior certificate
PK	Procedural Knowledge
SAS	Side- Angle- Side
SAA	Side- Angle- Angle
SSS	Side-Side-Side
Std	Standard
T	Teacher
IEA	International Association for the Evaluation of Educational Achievement
TIMMS	Trends in International Mathematics and Science Studies
Trig	Trigonometry
ZPD	Zone of Proximal Development
Δ	Triangle
$\equiv$	Congruence

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Declaration

I declare that this research report is my own unaided work. I hereby submit it for the degree of Masters in Education: Curriculum at the University of Witwatersrand, Johannesburg, South Africa. It has not been submitted before for any degree or examination at any university.

Signature of candidate

On this _____day of _____(month)_____(year)

Abstract

The study investigated the ways in which Three Grade 10 teachers in KwaZulu Natal South Africa use LTSM to help mediate learners' knowledge gaps in mathematics. The specific focus of the investigation is on trigonometry, as a subdomain of mathematics, with which learners experience difficulties in understanding. The study used LTSM from a research project that involved a pre- and post - diagnostic test and Learner Teacher Support Materials (LTSM) as a resource and intervention method designed to mediate knowledge gaps. The conceptual framework used Elmore's (2008) instructional core as a basis for analysis. Thus, over and above the analysis of the diagnostic tests and LTSM, a significant component of the study analysed teacher instructional practices in the classroom in using the LTSM to mediate learners' knowledge gaps.

The research strategy is qualitative, making use of descriptive statistics, using case study methodology to observe three teachers. The findings showed that teachers do not use diagnostic assessment as a tool to assess learning gaps, and their mediation was not informed by the diagnostic assessment. The pre-diagnostic tests showed significant knowledge gaps, and the post-test showed significant improvement in certain types of knowledge for two teachers, where only one teacher showed significant improvement in two of three constructs considered fundamental to trigonometry.

The ways in which the three teachers used the LTSM to address learners knowledge gaps can be explained in their instructional practices, involving their pedagogical moves, and scaffolding processes in the classroom. The three teachers used the LTSM differently, and used it only in one of two lessons observed and they made choices when, how and for what purposes they used the LTSM.

The study recommends further research including a longer term comparative study. These ideas are to be integrated into policy frameworks. It also recommends research into practices that encourage learner involvement in the mediating of knowledge gaps, the type of teacher training and support required to bridge these gaps, as well as the tools, LTSM and other resources required to close them. I argue for these resources to be used to provide teachers with diagnostic assessment tools and resources in key topics of the curriculum, which learners generally find difficult and for which they possess a lack of pre-requisite i.e. foundational knowledge. Given teachers knowledge of the way in which learners learn was limited, I therefore argue that

teacher professional development ought not to only focus on the content of educative materials for addressing learning gaps, but that it requires a focus on teachers' theoretical understanding of the way in which learners learn, and the interactive relationship between their pedagogical moves and scaffolded processes in the classroom. The outcome of this argument underscores the fact that teachers require teacher training so as to integrate the theories of psychology of learning with classroom pedagogy, such that they are enabled to use their teaching practices to assist learners to overcome knowledge gaps and develop higher cognitive abilities.

CHAPTER ONE:

Introduction

This chapter aimed to give insight into the topic and problem that was investigated in the research. It framed the study and describes the context, outlines the purpose and scope of the study**.**

1.1 Research topic

This research investigates the ways in which three Grade 10 teachers in classrooms in Quintile 4 township schools in KwaZulu-Natal, South Africa, use Learner Teacher Support Material (LTSM) to assist learners in overcoming knowledge gaps.

1.2 Research problem

Research has shown that learners from low socio-economic backgrounds lack the adequate foundations to access education effectively. As a result, these learners tend to start formal schooling with knowledge gaps. Morrow (2007, p. 19) has distinguished between two types of access:

1) "Formal access", namely the number of students who access education as part of their right to schooling; and

2) "Epistemological access", which he describes as access to forms of knowledge that are valued by the modern world, specifically schools

For the purposes of this study focus is placed on "epistemological access", or access to school knowledge, according to the rationale that a lack of school knowledge contributes to gaps in knowledge for disadvantaged learners. Curriculum content and policy is central to school knowledge.

There have been considerable changes in the education system in post-Apartheid South Africa which involved major shifts in curriculum content and policy. These shifts in the main involved the introduction of Outcomes Based Education, Curriculum 2005 and Curriculum Assessment Policy Statement (CAPS) during 2012. OBE was denounced in 2010 on the basis of a Report by the Review Committee that recommended fundamental changes to the curriculum.

The shift was to another curricula initiative; CAPS (DBE, 2012). The policy emphasises implementation and teacher needs through the centrality of knowledge of subject disciplines

and outlines what content ought to be taught and assessed on a grade-by-grade and subject-by-subject basis. The main difference between CAPS and OBE is that the policy integrates high knowledge and credibility, quality and efficiency and compares with other countries, whereas the implementation of the OBE curriculum left much to be desired when considering content and quality delivery. he effect of the changes in curriculum are that a cohort of learners within the system that have not received the appropriate level of content and that is not competent or have sufficient foundational knowledge, ie they have knowledge gaps. At school level, South Africa has almost achieved universal, formal access. However, very low epistemic access remains a significant challenge. This is evident from the learner performance in international comparative studies, such as the Trends in International Mathematics and Science studies (IEA:TIMMS, 2011), where South Africa came in last. In reviewing the 2004 Senior Certificate results, Taylor (2004;p.3) revealed that 80% of South African schools are highly ineffective, producing only 15% of Higher Grade passes in mathematics whereas 7% of top performing schools produced 66% Higher grade passes in the Senior certificate examinations. The vast majority of these underperforming schools are disadvantaged township and rural schools that are poor (Spaull,2013). These underperforming schools fail to produce 30 mathematics passes in the Senior Certificate Examinations (Simkins, 2005). These research studies gives a broad sense of what could be considered underperforming schools; i.e. a school that fails to produce sufficient Higher grade passes in Mathematics or at least 30 mathematics passes. For my, study I have defined disadvantaged and underperforming schools to be quintile 4 schools, who fail to achieve above 40% in mathematics. This definition provides scope to do research in a school that is not completely dysfunctional, but has potential to benefit from the study as they are also not producing enough higher grade passes.

The IEA:TIMMS (2011) indicate that learner knowledge gaps include a lack of the basic knowledge required to access new knowledge and to make connections, as well as a lack of understanding of the theory and principles informing subject content. Furthermore, knowledge gaps consist of misconceptions from incorrect learning and gaps in subject content that was never learnt. These gaps are cumulative, and present severe handicaps to effective grasp of the curriculum by the time learners reach Grade 10 (IEA: TIMMS 2011,Taylor et al., 2010; Department of Basic Education, 2012). Recent research suggests that this problem is exacerbated by the fact that many teachers having their own knowledge gaps, due to their own inadequate training (DBE, 2012; Fleisch et al., 2011). The context of curriculum changes and

problems indicate a compelling case for CAPS to recognise learner knowledge gaps by providing appropriate content to deal with the gaps.

In response, governmental and non-governmental organisations (NGOs) have developed learner-teacher support material (LTSM), such as learner workbooks, trusting that these will support teachers in their attempts to close learners' knowledge gaps. Research by Cohen et al. (2002) has shown that learning materials are not adequate on their own, but that they are made more effective when used efficiently by teachers. Thus, if they are not mediated or facilitated well, they do not meet their intended purpose. There is a rise in demand for LTSM to provide content for teaching, but there is very little research in South Africa describing the way in which teachers use and mediate LTSM. This study contributes to reducing this lacuna.

1.3 Background to the research

Learner Teacher Support Material (LTSM) was developed by Zenex Foundation to assist teachers in addressing identified knowledge gaps. In order to do this, a Diagnostic Test (DT) was also designed to identify learner knowledge gaps, such as in the subdomain of trigonometry in the mathematics curriculum, an area which many learners struggle to understand. The DT is a 35 minute test, which identifies learners' knowledge gaps, conceptual level, and level of understanding and reasoning. The DT used in this study is specifically focused on trigonometry content readiness, and covers all the knowledge and skills required to determine readiness for trigonometry. The DT results were analysed by the researcher, and teachers received individual learner profiles that showed the results of their learners. Teachers received orientation on the materials. The LTSM contains teacher notes, and it was intended for the teacher to use to prepare and select some exercises and activities. One exercise had varying levels of difficulty that the teachers could give to a learner depending the learners' results on the diagnostic test. The exercises were intended for use with learners in group work, pair work, and individual exercises, as well as for matching with learners' knowledge gaps, as identified in the diagnostic tests. There are also exercises that aim to build learner's conceptual skills and focus on problem solving, reading, and critical thinking in the LTSM. The research assumed that the test and material were of the correct standard, so as to help identify and mediate the learners' knowledge gaps. Once the profiles from the test were provided, the teachers were expected to select the appropriate activities from the LTSM by means of which to address their learners' knowledge gaps. The research investigated the ways in which three teachers mediated the material in the classroom.

1.4 Research aim

The study investigated the way in which teachers mediate Learner Teacher Support Material (supplementary to regular teaching resources such as textbooks and lesson plans) in Grade 10 classrooms, to assist learners who have knowledge gaps. The gaps were assessed through a diagnostic tool, before and after teachers' use of the LTSM.

1.5 Research rationale

In Pedagogy Content Knowledge, Shulman (1986) has argued that "dealing with [a] learner's thinking is an important part of teachers' professional knowledge" (p. 9). Teachers' professional knowledge requires them to understand the complexity of the content learners are obliged to learn, as well as the diverse backgrounds they bring with them to the learning environment.

The research interest of this study is focused on whether teachers' proper use of LTSM proves to be an effective way to overcome learners' knowledge gaps. There are opposing views about the use of LTSM among researchers. As one view states:

> *resources are not self-enacting, and differences in their effects depend on differences in their use [and] […] teachers' knowledge, skills and strategic actions can be seen as a resource together with learners' knowledge, experiences, norms and actions* (Cohen, Raudenbush, & Ball, 2002, p. 80-81).

This means that LTSM is effective when they are used in classroom instruction to combine the experience, knowledge, and actions of learners and teachers.

Fleisch, Taylor, Herholdt, & Sapire (2011, p. 488) have investigated whether LTSM, particularly custom-designed workbooks, improve primary mathematics achievement, and are more cost effective than conventional textbooks. They undertook a combination of randomised and control trials of Grade 6 classes in 44 primary schools serving low income communities in the Gauteng Province. Their study showed that learners improved to the same extent, whether they used conventional textbooks, or specific workbooks (Fleisch et.al., 2011), and this is important because of the comparison of cost effectiveness between workbooks and textbooks as a way of improving performance, but the study did not investigate the use of LTSM in the classroom to mediate knowledge gaps. The current study aims to investigate the way in which teachers use the LTSM in the classroom to effectively mediate learners' knowledge gaps. From

this consideration, the study intends to contribute towards a better understanding of the way in which teachers' use of LTSM assists with improving learner performance.

1.6 Research question

In what ways do three Grade 10 mathematics teachers use learner teacher support material to mediate learners' knowledge gaps as determined by diagnostic assessment?

1.6.1 Sub-questions

- How do the three teachers understand diagnostic assessment, specifically in relation to identifying their learners' knowledge gaps?
- How do the three teachers understand the relationship between the learning and teaching material and the learners' knowledge gaps?
- In what ways do the three teachers use the material to address learners' knowledge gaps?
- What forms of classroom practice enable or constrain the three teachers' mediation of the learner teacher support material?

1.7 Conclusion

I provided an overview of the topic to be investigated, situated the study, and gave the background and context of the study. The overview informed the research questions which I posed and aim to address.

Structure of the report

- **CHAPTER 1: Introduction chapter**

The chapter discussed the problem of inadequate foundational knowledge and epistemic access for learners from disadvantaged backgrounds. The research aims to investigate an intervention of LTSM, comprising diagnostic assessment and Learner Teacher Support Material as a way of addressing the problem of inadequate foundations and epistemic access. The chapter frames the research questions that will be addressed in this study.

- **CHAPTER 2: Literature Review**

The research questions guided the formulation of four specific themes for the literature review, namely: 1) formative assessment, specifically diagnostic assessment; 2) The nature of mathematical knowledge and in particular trigonometry knowledge as a subdomain of mathematics; 3) the nature of LTSM; and 4) mediation in three specific areas, namely: i) mediation of LTSM, ii) cognitive mediation; and iii) mediation in the classroom by means of the teachers instructional practice. The themes informing the core of the conceptual framework are drawn together and based on the 'instructional core', "made up of the level of content that learners are taught, the skill and knowledge that teachers bring to [the] teaching of that content and the level of learners' active learning (City et al., 2009, p. 24). This framework was originally discussed in Elmore (2008), and further developed by City, Elmore, Fiarman, & Teitel (2009).

- **CHAPTER 3: Methodology**

Chapter 3 addresses the research methodology, data collection methods and the methods of data analysis utilised. I use a qualitative research strategy involving a case study of three teachers' use of LTSM and mediation of knowledge gaps, together with quantitative data and frequency analysis of the three teachers' use of LTSM, and their mediation of mathematical knowledge and instructional practices. The ethical considerations and dilemmas experienced in the research process are also described.

- **CHAPTER 4: Data Presentation and Analysis**

Chapter 4 provides an analysis of the results of the pre-and post- diagnostic test data, based on five trigonometry constructs that are pre-requisite knowledge for trig. I also analyse the test based on the different types of mathematical knowledge, namely conceptual and procedural knowledge. This chapter also contains an analysis of three teachers' use of LTSM, and their instructional practices.

- **CHAPTER 5: Discussion and Analysis of Findings**

Chapter 5 analyses the findings in relation to the research questions, and the conceptual framework devised for this study, which was based on the instructional core. Explanation is provided of the evidence from the pre- and post-diagnostic tests, as well as of teacher instructional practices, which used the LTSM to mediate learners' knowledge gaps. The teachers' instructional practices also showed their understanding of the relationship between their use of the LTSM and the results of the diagnostic test. I also describe the forms of classroom practice that both enabled and constrained the mediation of LTSM.

- **CHAPTER 6: Conclusion, Implications of the study and Central claim**

In Chapter 6, I provide a summary of the central claims, the implications for further research as it pertains to the use of LTSM, and the general instructional practice used to mediate learners' knowledge gaps in Grade 10 Mathematics classrooms in South Africa.

CHAPTER TWO:
Literature Review

This literature review aims to explore the theory and key concepts that frame and inform the focus of the research study. It elucidates the conceptual framing, presenting both the object of study and the conceptual framework on which the investigation pertaining to it is based. The study reviews literature formulated around five concepts: firstly on formative assessment, in order to better understand diagnostic assessment, which is part of the LTSM; and secondly, on the nature and types of knowledge in mathematics, focusing specifically on trigonometry as a subdomain of mathematics and the topic of investigation. I reviewed mediation of mathematical content in the classroom according to the following concepts:

1) learner-teacher support material (LTSM), focusing on its nature and use as an additional resource in the classroom;
2) learning, so as to understand cognitive learning and development, focusing on knowledge transfer and acquisition; and
3) pedagogy, focusing on teachers' classroom instructional practices.

2.1 Preface

There is very little extant research to be found on the use of LTSM, particularly regarding the way in which teachers' mediation affords or constrains its success in supporting learners to close their knowledge gaps. The empirical object of the study is the use of LTSM (material that was designed to close knowledge gaps), based on a claim which is twofold:

- the design of the teaching and learning processes using LTSM is focused on addressing knowledge gaps (the knowledge learners need to acquire in order to access the curriculum at the appropriate level of cognitive demand); and
- teachers' mediation of LTSM as an important resource, which can at turns enable or restrict the successful use of LTSM.

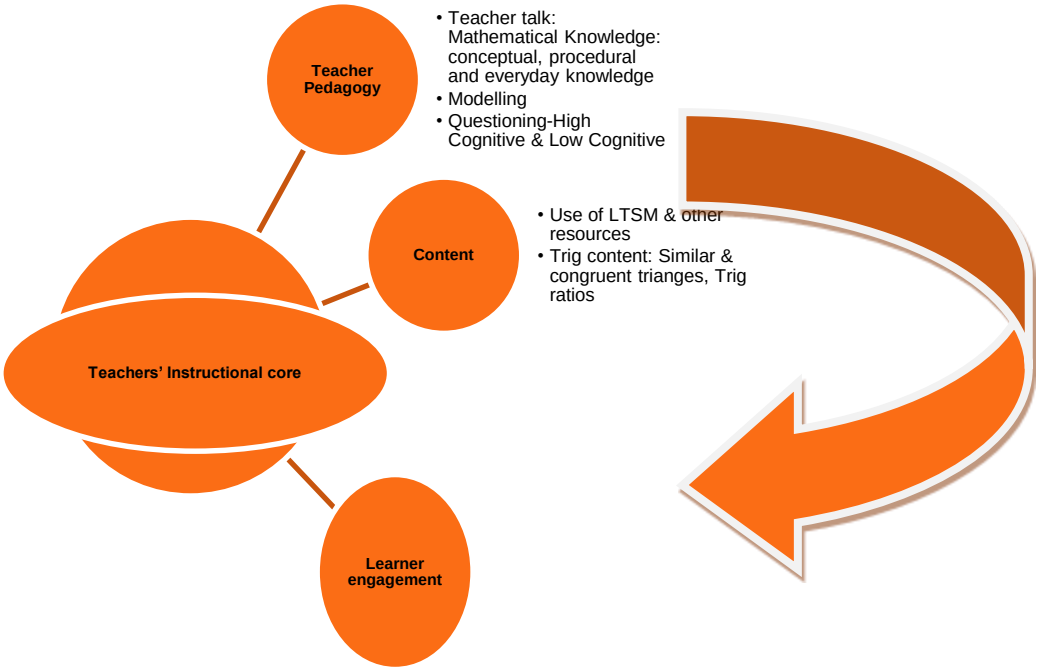
Cohen et al. (2002) define the instructional core of classroom teaching as "the relationship between the teacher, student and the content; not the qualities of any one of them by themselves" (p. 85). Simply put, the instructional core is an interdependent relationship

between the level of work (content) that learners are taught, the skill and knowledge that teachers bring to teaching this content, as well as the level at which learners take responsibility for their learning. It is this idea of the instructional core that informed my study, which was originally discussed in Elmore (2008), and further developed by City, Elmore, Fiarman, & Teitel (2009), who have advocated that:

> *[…]The core is made up of the level of content that learners are taught, the skill and knowledge that teachers bring to [the] teaching of that content and the level of learners' active learning* (City et al., 2009, p. 24).

City et al., (2009) have illustrated the way in which a skilled teacher's interaction with LTSM took the form of revision, building on concepts, and prior knowledge, by connecting the task to previous work. Among the four teachers in their study, one of the teachers used a process of questioning of learners to determine what they learnt, and this formed the basis for identifying learning gaps and learner understanding. The teacher reminded learners of a similar example, where she explained the method and approach that could be used to deal with the concept. This reinforced a previous concept, and allowed learners to draw on what they already knew. The teacher also provided the opportunity for group work, and circulated amongst the learners while they worked in order to help those who were struggling. This kind of interaction forms the basis of the type of mediation that will be investigated in the study. Elmore's instructional core theory triangulates mediation of content: 1) using the LTSM as a resource and mediation of content through cognitive transfer and acquisition; 2) through an external social mediation process by the teacher in the process of knowledge transfer; and 3) ascertaining learner engagement with content, and in the process of knowledge acquisition. The current study will not focus on teacher's knowledge, but on mediation of the content. I cannot ignore the (additional) level of content that emerges through the interaction with learners. Nevertheless, the main focus will be on the teachers' use of the material. A visual representation of the components within the instructional core are depicted in figure 1. The illustration integrates Elmore's (2008) instructional core with what will be seen in the three teachers' instructional core .

Figure 1: Model of three teachers' instructional core: A model of analysis against Elmore's instructional core



In the diagram above, the left center circle with three elements presented in the joining sub-circles depicts the "instructional core" (Elmore, 2008). The first element presented in the top circle is teacher pedagogy, which represents the skills teachers' bring into the classroom. The second (middle) sub-circle shows the content or level of work that learners are taught. This includes the type of knowledge, as well as the quality of teaching the different types of knowledge. The third (bottom) circle shows learners' engagement. The model attempts to align the three teachers' instructional core with Elmore's conception of the instructional core. The bullet points outside each sub-circle shows how teachers in this study enacted their own model of instructional practices.

I observed that teachers' instructional core showed pedagogical interactions of teacher talk, modelling and questioning. The model shows that the teachers' instructional core is informed by the interaction between teacher pedagogy, the content (partly from the LTSM and generally on trig content) and the engagement of the learners. The teachers' instructional core fell short of alignment with Elmore's notion of the instructional core, specifically in relation to learner engagement.

In the next section, I begin with a discussion of formative assessment, along with the use of diagnostic assessment, to show its importance in the approach of the design of the LTSM, and because diagnostic tests were used to identify learners knowledge gaps in trigonometry measures to indicate their readiness to learn trigonometry. I then move to discuss the nature of knowledge, to gain an understanding of the types of mathematical knowledge in the subject, and its subdomain, trigonometry. Following the discussion on the nature of knowledge is a discussion on teacher mediation of LTSM and learning theories, as well as instructional practices, and all three aspects inform the way teachers use the LTSM in the classroom and in their general teaching practices.

2.2 Formative assessment

Formative assessment plays a central role in the way(s) in which teachers use LTSM to mediate knowledge gaps, given that it evaluates whether learning has taken place, and identifies areas that require further improvement. Following Reiser's (2001b) conception that mediating encompasses evaluation of learning, and Reigeluth's (2013) view that mediation includes identifying weaknesses and looking at ways of improvement. I draw a relationship that the elements of formative assessment are an important process in the use of LTSM to mediate learner knowledge gaps. Research by Black and William (1998; 2009) suggests that formative assessments are teaching practices where teachers elicit evidence through the evaluation of learner achievement. Teachers reflect on teaching and learning, and make decisions about teaching and mediation. This reflection provides an analysis of learner achievement, helping to diagnose areas of weakness requiring further improvement. The main aim of classroom testing and assessment is to evaluate student learning by obtaining valid, trustworthy and helpful information related to student achievement (Miller, Linn, & Gronlund, 2005, p. 134).

In line with learning theories, assessment theories were also developed, strengthening and building upon specific ideas. The objectivist view of testing has shaped beliefs about the nature of evidence and principles of fairness. Shepard (1999) has drawn attention to a project where teachers and researchers were surprised to discover that despite efforts to find alternatives to standardised objective testing, teachers and researchers did not have the same understanding of how standardised testing should be implemented in classrooms. More compelling was that teachers held beliefs more consistent with behavioural principles of scientific measurement. They believed that assessment needs to be an official event, separate from instruction, which must be uniform and objective. Thus, when attempts are made to change the form and purpose

of assessment of learning, it is important to acknowledge the power of such beliefs (Shepard, 1999, p. 6).

To reconceptualise classroom assessment practices, the principles of a social constructivist conceptual framework needs to be revisited. The cognitive revolution reintroduced the concept of the mind in order to help us understand that "learning is an active process of mental construction and sense making" (Shepard, 1999, p. 6). Shepard (1999) proposed that a broader range of assessment tools is needed to capture the important learning goals and processes, and to connect assessment to ongoing instruction. This means expanding the scope of assessment data to include "observations, collections of student work and student's self-evaluations, etc., thus requiring teachers to engage in systematic analysis of evidence" (Shepard, 1999, p. 8).

The zone of proximal development tells us that the way we approach diagnostic tests requires a focus on actual development or learning that has been achieved, as well as on potential learning and mental development. This coincides with the aim of the formative test in my study, which is to assess learners' readiness for potential learning of new content. The test is formative, as it is used to determine learners' knowledge and to help improve learning, rather than to make decisions about promotion or competency.

Classroom assessments and tests also possess the potential to make expected learning outcomes explicit, and to show what types of performance are valued (Miller et al., 2005, p. 163). This can only be achieved if proper planning and preparation of assessments takes place, which require an understanding of the different forms and tools of assessment.

In view of Shepard's claim, the range of assessment tools need to be expanded (Shepard, 1999, p. 7). I now move to discuss the tools relevant to my study, namely diagnostic assessment, pre-tests and post-tests, as formative assessment tools.

Miller et al. (2005, p. 135) have defined and characterised different types of classroom tests and assessments by their location and timing within the instructional process, including pre-tests, 'during instruction' tests and end-of-instruction tests, based on the work of Airasian & Madalus (1975, pp. 53–64). Pre-tests occur before instruction, and can be used either to assess whether learners have the prerequisite knowledge needed to begin the instruction process, or to access the extent to which learners have already achieved the objectives set out in order to assist with "learner placement or modification of instruction" (Miller et al., 2005, p. 135). Pre-tests used to determine learners' readiness are usually limited in scope, and are characterised

by a relatively low level of difficulty; while those used for student placement encompass a broad sample of objectives, and consequently, display a wide range of difficulty (Airasian & Madalus, 1975, p. 53–64).

During instruction, testing can either be formative or diagnostic (Airasian & Madalus, 1975, pp. 53–64). *Formative testing*, which is based on a predefined part of instruction, is carried out periodically, with a limited sample of learning tasks, in order to improve and monitor learning progress. *Diagnostic testing* deals with common student errors and misconceptions, and is characterised by a low level of difficulty and a limited sample of specific errors on a need-only basis, in order to remedy student error (Miller et al., 2005, p. 136). Miller et al. (2005) have asserted that because these types of tests encompass a limited sample, selection of test items and performance tasks needs to be done carefully, in order to ensure that high-priority objectives are adequately represented in the assessment. The results of tests undertaken during instruction can assist in identifying poorly performing learners, and can offer them alternative methods of study, while overall poor achievement on these type of tests can be used to review and remediate group performance (Miller et al., 2005, p. 136). This shows the importance of the different forms and tools used in assessment that support the knowledge acquisition process. Therefore, it shows that teacher mediation is closely linked with assessment, and that it is important to identify and prioritise the objectives of assessment in classroom practices.

In South Africa, assessment in the post Apartheid era has incrementally shifted from only summative evaluation and high stakes Grade 12 examinations to include standardised testing through participation in international studies such as Trends in International Mathematics and Science Study (TIMMS, 1995; 1999) under the auspices of the International Association for the Evaluation of Educational Achievement (IEA) which assesses learners in Grade 7, 8 and 12. Recently the shift also involved the introduction of internal accountability tests such as the National Systemic Evaluation (2006; 2008) at Grade 3, 6 and 9 and the most recent Annual National assessments (ANA, 2011; 2012).

In this study I expected to see teachers in the classroom adapt their classroom practices from externally driven approaches to an approach that requires identifying learning needs, errors, and misconceptions through diagnostic assessment. This was to be followed by feedback and appropriate remediation and then assessing what was learnt. The diagnostic approach placed "conflicting demands" (Vandeyar ,2005, p.463) on the teachers' existing assessment practices which focussed on implementing and meeting the policy assessment requirements (p.465).The

conflicting demands posed difficulties for teachers as assessment is very closely embedded to their pedagogy (Shepard,1995). Empirical Studies by (Shepard, 1995;Steinberg,2008; Vandeyar,1995) also showed that teachers understanding, beliefs, perceptions are in close relationship to their assessment practices. This study particularly focussed on the way teachers use assessment in their pedagogy to mediate learning.

In the next section, I discuss the nature and types of knowledge. This is done in order to understand what knowledge is, and what different types of mathematical knowledge inform the subject, as well as how it is constructed. Conceptual and procedural knowledge are relevant to the focus of this study. In particular, the diagnostic tests in my study were designed to assess the two types (conceptual and procedural) knowledge.

2.3 The nature and types of knowledge

In this section, I discuss knowledge following on from Vygotsky's (1978) theory that knowledge is psycho-social process. I also explain Vygotsky's (1962) theory on concept development, which distinguishes between scientific and spontaneous concepts (everyday knowledge). I note mathematics to be a form of specialised knowledge, and then discuss trigonometry, as a subdomain of mathematics, as the key topic of investigation in this study. Vygotsky (1978) holds that knowledge is firstly, psychological, and secondly, socially constructed. Psychological processes are those processes that take place in an individual's mind or brain and include thinking, processes of understanding, and remembering, or recalling. Socially-constructed processes are those processes that involve the external environment, and within which an individual interacts, and are populated by the people with whom they interact in the environment. The development of such processes inside and outside the mind and interactions with people can help with making sense of, and understanding the world better. The process of interaction with the environment and within oneself to make sense and to understand is a process of constructing knowledge. Once this knowledge is constructed, it enables further development from knowledge that is general and concrete, to abstract thinking. This form of abstract thinking is a higher form of knowing. Vygotsky (1978) calls this further development into higher forms of knowing cognitive mediation. Higher forms of knowing help people to develop advanced, complex thinking, and enables critical thinking and problem solving abilities.

The interaction between people towards the development of knowledge involves a process of communication, specifically between adults and/or those with knowledge to impart, and

children, or in the case of this study, learners in the school context. Cognitive mediation refers to those processes mediated by someone who is more knowledgeable. It involves guided assistance to move from functional to deeper understanding, from the concrete to the abstract, or from everyday common sense knowledge to specialised scientific knowledge. The movement towards deeper understanding leads us to the space between one's actual development, and potential development through guided assistance, where the knowledge can be meaningfully acquired. Vygotsky (1978) called the space the zone of proximal development, which I will discuss later in this chapter.

In the world today, the knowledge accepted as most socially valid is school knowledge, and it is specialised and formal. The focus of my study is on Mathematics as a subject, which is a form of formal school knowledge, embedded in the curriculum.

2.3.1 Types of knowledge

Conceptual and procedural knowledge develop mathematical knowledge. Although conceptual and procedural knowledge cannot always be separated, it is important to make a distinction between the two in order to understand mathematical knowledge development (Canobi, 2009; Rittle-Johnson et al., 2001). Conceptual knowledge refers to general ideas or thoughts that have "bonds that stand in relationship to one another, which help to add and shift to another feature of abstraction" (Vygotsky, 1962, p. 49). This means that concept development happens when abstract and advanced complex thinking is developed.

Procedural knowledge refers to the sequenced steps and actions used to solve a mathematical problem (Canobi, 2009; Rittle-Johnson et al., 2001). Star (2005) describes conceptual knowledge to be a web of connected concepts that form relationships with one another, where measures of conceptual knowledge are more varied (Star, 2005); whereas procedural knowledge does not only indicate what is known, but also one way that procedures (algorithms) can be known (p. 408).

The diagnostic test in this study was designed to establish the nature of learners' conceptual and procedural knowledge. It was intended to inform the analysis of teachers' use of the LTSM, and the types of knowledge they used in their classroom instruction to mediate learners' knowledge gaps.

Instructional practices are influenced by teachers' values and beliefs about what knowledge is valid. In contemporary society, school knowledge is more valued, as a universally legitimated

form of knowledge. Epistemic access for learners from low socio-economic status (SES) backgrounds require school knowledge. Makonye (2009) explored the impact of Curriculum 2005 on equitable achievement in mathematics in South Africa. The study suggests that the curriculum policy did not reduce inequity in mathematics achievement for a number of reasons. These reasons are language, varying levels of teacher qualifications and teacher understanding of what constitutes good mathematics (pp.93-96). Thus, in order to gain epistemic access for such learners there are a number of interventions proposed. For example, Delpit (1988) posit that "children from disadvantaged groups need to be explicitly taught the culture of knowledge and power and argued that this would remain covert if it was not openly mediated to them" (p.88). Lubienski (2000) documented that "lower SES learners preferred more external direction and sometimes approached problems in a way that caused them to miss the mathematical points"(p.89). These researchers provide compelling reasons and reveal the importance of providing epistemic access through school knowledge for learners from low SES backgrounds.

It is generally argued that specialised knowledge is scientific and universally accepted, powerful, and clearly explained. Contemporary society tends to regard everyday knowledge as weak, and highly contested, where it fails to be recognised as an equally valuable form of knowledge. Vygotsky (1962) draws a clear distinction between the development of higher level scientific concepts and everyday or spontaneous concepts. Vygotsky (1962) shows that developing scientific concepts runs ahead of development. Scientific concepts can 'raise the level of understanding' of "spontaneous concepts" (p. 109). Once the child is aware and can take charge in one kind of concept, all of the previous concepts are reconstructed accordingly. According to Vygotsky (1962), "scientific concepts supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use. Scientific concepts grow down through spontaneous concepts and spontaneous concepts grow upward through scientific concepts" (p. 109). Therefore, everyday knowledge, according to Vygotsky, deals with basic, common sense experiences and practices, and is focussed on our experiences in the world (context), whereas specialised knowledge refers to formal language and specialised concepts.

In the classrooms, everyday knowledge such as the use of fruit cut up in halves and quarters to explain fractions instead of mathematical denotions of a fraction (Kazima & Adler, 2006), and the way it is used in the classroom can affect cognitive development of the learner, since access to higher forms of thinking and knowledge construction requires a process of

formalisation, notwithstanding the current discourse restoring value to formally undervalued types of knowledge. Vygotsky (1962) saw the

> *uniqueness of generalisation of thought in a way that it created a pyramid of concepts that permits one to pass mentally from one particular property of an object to another by means of a general concept. These concepts have bonds that stand in relationship to one another, which help to add and shift to another feature of abstraction. Once this relationship is developed, it enables structures that bring parts and wholes together, which enable higher forms of thinking (mental functioning)* (p .49).

Vygotsky's (1962) theory of concepts draws a relationship between concepts and generality as the former expressing itself in and through the latter. He proposes a dialectical approach to child development, informed by concept development, as well as the development of higher mental functions. An example is a teacher undertaking a concept mapping exercise in order to develop relationships between concepts.

In the current study, the LTSM focused on specialised Maths knowledge, in particular trigonometry, where the mediation by the teacher might bring in everyday explanations. However, it is argued here that these everyday explanations can either help with giving learners access, or constrain access to specialised knowledge, if the teacher does not formalise the explanations effectively. The focus of the investigation in my study on trigonometry requires me to provide a clear understanding of the nature of trigonometry. In the next section I discuss this aspect.

2.3.2 The nature of trigonometry

This section aims to provide insight into the nature of trigonometry in the context of South Africa, and its expression in the curriculum.

Trigonometry in mathematics is the study of relationships between angles and sides of a triangle. Trigonometry can be traced back to ancient Egyptian, Babolylonian and Greek civilisations (Boyer 1991, p. 162). The term trigonometry was derived from Greek "trigonometria" (Wikipedia, p. 1), where '*trigo*' means triangle, and '*metria*' derived from metrics, meaning measure. It is used in solving any problem that involves an angle, for example, calculating the height of a tree, or the distance between someone kicking a soccer ball and the ball reaching the soccer poles. Trigonometry is a subdomain of mathematics in a

way similar to that of algebra and geometry (Pournara, 2001). Trigonometry knowledge as a subdomain of mathematics is developed through both conceptual and procedural knowledge. As discussed on page 23, given that trigonometry is a subdomain of mathematics, the study similarly asserts the need to delineate between conceptual and procedural knowledge so as to understand how this form of mathematical knowledge is constructed and measured. Therefore, there is a relevant distinction in the definition used by Star (2005), between: 1) conceptual knowledge as a web of connected concepts that form relationships with one another, where measures of conceptual knowledge are more varied; and 2) procedural knowledge not only indicating what is known, but also one of the ways in which procedures (algorithms) can be known (p. 408).

The South African curriculum focuses on two subdomains of trigonometry, namely: functions and ratios (DBE, 2011). Grade 10 trigonometry is part of the FET phase of the Mathematics curriculum, where assessment policy (CAPS) is one of ten content focus areas. Trigonometry supports one of the main aims of the curriculum, which is to "to develop learners problem solving and cognitive skills, in particular spatial skills and properties of shapes and objects to identify, pose and solve problems creatively and critically" (DBE, 2011, p. 11-12) .

This study is focused on Trigonometry as a subdomain of mathematical knowledge because it proves to be a critically important topic in the curriculum, but also a difficult topic (NSC diagnostic report 2012; Pournara, 2010). The study seeks to understand how teachers prepare learners for trigonometry content knowledge.

In the section above, the concept of knowledge has been explored, including how it is constructed, three different types and forms i.e., specialised, scientific and everyday or spontaneous. The argument for mathematical knowledge as specialised knowledge was also established, and that trigonometry knowledge is a subdomain of mathematical knowledge. This provides the context of knowledge that informed the diagnostic test measures used in the design of the test. In the next section, I discuss the nature and use of LTSM as a resource. This is followed by a discussion regarding the ways in which LTSM is used to mediate knowledge in other studies in the literature.

2.4 The Nature and use of Learner-teacher support material

In reviewing the literature, a wide range of terminology is used to describe the resources (physical and human) and use(s) of learner-teacher support material. Teaching and learning material refer to the various physical and human resources used to assist learners with knowledge acquisition. Cohen, Raudenbush & Ball (2002, pp. 80-81) have observed that most literature refers to familiar resources, such as curriculum material and facilities.

These resources are assumed to influence learner performance. Cohen et al. (2002) developed a conceptual definition of a resource. They argue that the most common view implies that "resources carry capacity", and that there is no direct relationship between resources and learner performance and that "[d]ifferences in the effects of resources depend on differences in their use" (2002, p. 80).

In order for LTSM to work successfully, it needs to be employed by both teachers and learners, and this creates the relationship between human resources (teachers and learners) and physical resources (material and technology), where the one is dependent on the other to achieve its outcomes. The LTSM provides a vehicle for the delivery of teaching and learning content and strategies. The LTSM is mediated by the teacher. According to Clark (2005):

> *Learner-teacher support materials are courses of study, syllabuses, curriculum bulletins or guides, handbooks, research reports, curriculum newsletters, publishers' manuals, audio-visual aids, and other material, which serve a teacher education function* (Clark, 2005, p. 173).

LTSM is considered support material when it does not replace teachers as the primary means of instruction. Reiser (2001) has asserted that according to the definition by Clark (2005), every physical means of instructional delivery, from the textbook to the computer, would be classified as learner-teacher support material (Reiser, 2001a, p. 54). Thus, the teacher becomes the vehicle for delivering teaching and learning strategies through mediating the LTSM.

The LTSM for this research study was developed by researchers and teachers as part of a project funded by the Zenex Foundation. The LTSM consists of a Diagnostic Test (DT), exercises and teacher notes, based on the CAPS curriculum content for teachers. The material is aligned with the DT, developed through the same project, and funded by the Zenex Foundation. The material provides teachers with various entry points on the topic

(Trigonometry) to accommodate learners who require basic knowledge to access the grade curriculum. Teachers' notes are also provided. The notes explain the knowledge gap identified in terms of each subtopic. Furthermore, the notes give teachers guidance on how to prepare learners, and how to explain each subtopic, along with indicating the way in which learners will respond when they understand the concepts, and what different learning styles and levels are needed.

2.5 Teacher mediation of learner teacher support material

Teacher mediation comprises various interactions in the classroom, which are concerned with the content covered by LTSM. As defined in section 2.1, the instructional core comprises three elements:

1) the level of content;
2) the skill and knowledge that teachers bring to teaching the content; and
3) the level at which learners' take responsibility for learning.

Knowledge acquisition cannot happen outside of this interaction, where teacher mediation is a necessary feature of the instructional core. Therefore, if teachers do not directly mediate the content from the LTSM, this could affect the effectiveness of the LTSM, as well as the quality of the lesson. Bernhardt (1987) argues that text should be seen as a participant in knowledge production and not as authoritative or as a repository of truth, but rather, as a raw material for building or constructing understanding. Bernhardt has asserted that "text mediation is an interactive process, where readers use their background experience and prior knowledge to construct text[ual] meaning from text[ual] material" (Bernhardt, 1987, p. 32). That is, discussion is a way of mediating text as an interactive process, where learners can be a 'participant'.

In a research study of three science teachers, Alvermann (1989) has shown the importance of discussion between teachers and learners in text mediation, as it provides opportunities for supplementing and modifying text material. Thus, discussion provides opportunities for learners to question, and for teachers to skilfully focus learners' attention. During this process, teachers can use concrete examples, build a few concepts, develop learners' understanding of those concepts, and then introduce new ones. Alvermann (1989) forwards that text which is mediated by a teacher who is generating only simple questions, or by reading from the teachers' manual, gives rise to rote learning. This type of learning can constrain learning from a social

constructivist perspective which advocates for learning to build critical thinking and engagement in the classroom. This is where the teacher mediation also becomes authoritative, where learners respond directly and in a limited way to the text.

Teaching and learning is a professional activity undertaken by teachers, which deals with applying appropriate teaching methods, with the view to enabling learners' acquisition of knowledge. The different combination of methods, as Alvermann (1989) described in his study, could include discussion with learners, which will inform teacher's adaptation of the LTSM, or in contrast, use the LTSM as is, or whether the teacher limits learner responses. Reiser (2001b) conceptualises the field of instructional design as "encompassing the analysis of learning and performance problems, and also the design, development, and evaluation of the learning and teaching process" (p. 57). It also includes the process of describing "techniques for identifying weaknesses" (Reigeluth, 2013, pp. 8–9), how to deal with these weaknesses, and ways for improving upon them. Therefore, the way in which teachers use LTSM to mediate learners' knowledge gaps can be made more or less aligned to the different methods and techniques outlined by Alvermann (1989) and Reigeluth (2013, pp. 8–9), and this provides the basis for what will be investigated in the research study. More specifically, the study will examine Alvermann's techniques of identifying learner weaknesses, text mediation, the use of discussion to build concepts, as well as questioning techniques.

The first section of this chapter reviewed formative assessment, so as to understand the use of diagnostic tests. It then reviewed the nature and types of knowledge in order to understand the types of knowledge that inform the current study. The section above then reviewed the nature, use and mediation of the LTSM, as a resource teachers are required to use to help address learners' knowledge gaps.

The next section turns to mediation in the classroom as a cognitive process of transfer and acquisition, focusing on learning theories.

2.6 Learning theory

Two major learning theories aid an understanding of their implications for teaching when using the LTSM. Theoretically, behaviourism and constructivism are seen in stark opposition (Deubel, 2003, p. 163), however, in this study, it is expected that both theories can be used to analyse teacher mediation of LTSM.

2.6.1 Behaviourist Approach to learning and development

This section on behaviourism discusses theories of learning based on studies conducted with animals and human behaviour. More specifically, it looks at theories that have been developed through observing animal behaviour in controlled environments, and their learned responses. The human aspect of behaviourism was investigated by Bandura (1969) and he used the principles of behaviourism to expand this knowledge through observing human behaviour in classroom context. The study of human behaviour is called social learning theory. Below three principles with implications for this study are discussed: conditioning, modelling and observational learning.

Behaviourism is a theory of learning based on experimental observation of animals, in order to predict and control their behaviour. In terms of learning, behaviourism focuses on the relationship between observable behaviours and events that occur in the environment of learning, or between stimulus and response. To behaviourists, learning is a form of behavioural change. Behaviourists place emphasis on the association between different types of conditioning; that is, learning behaviour in a controlled environment through creating a stimulus and response, and thus these researchers conducted experiments to provide an objective means of investigating different associations.

Pavlov (1927) initiated the views of learning in terms of stimulus-response linkages and transfer. Pavlov's discovery has become known as classical conditioning. It proposed that a response is established by association with an environmental stimulus. The theory holds that learning occurs in the stimulus-response (S-R) association. This association was made explicit by Skinner (1950), when he elaborated further that behaviour is learned in response to the consequences generated by our past behaviour. For example, in the classroom, if the teacher praises a learner for providing a correct response, then the learner will continue to respond to questions in an attempt to gain more praise. According to Skinner (1950), "learning is a process in the behaviour of an individual. The behaviour must be observable and it must appear in the situations we study" (p. 193). The relationship between behaviour and the environment can

only be determined under conditions that can be observed and controlled. Skinner (1968) argues that we only know that learning has occurred when the behaviour in a particular area of learning has changed, or when the desired behaviour is repeated. Behaviourists exclude the internal processes of the mind in analysing learning. Therefore, in their research, change in a given behaviour only 'counts' when it can be observed and controlled.

Skinner (1950) further distinguished between classical (respondent) conditioning and operant (instrumental) behaviour. 'Classical behaviour' (p. 196) refers to voluntary responses that happen through reflexes by an individual's body, where the behaviour is not learnt, whilst 'operant behaviour' refers to responses that are produced from interactions with the environment. Classical conditioning happens upon association of a stimulus that does not ordinarily elicit a particular response. There are two important phenomena evident in experiments on classical behaviour.

The first of these is extinction. In a scenario where a conditioned stimulus is always presented alone, we would find that the response would decrease in size, such that it would disappear altogether. This means that extinction takes place when one is obliged to eradicate a negative behaviour or need to redirect the behaviour. For example, if a teacher sees that a learner is constantly distracted from the lesson, the teacher may keep quiet until the learner pays attention, thus interrupting the learner's distracting behaviour.

The second phenomenon is generalisation, which happens when a new pattern of behaviour is acquired. For example, in learning triangles and not being able to initially distinguish between a right angle and a square, visual diagrams labelling these respective categories may assist the learner in being able to make the relevant distinction. This is done through the teacher modelling, a stimulus such as visual diagrams labelled 'right angle' and 'square', and explaining the properties, the learner observes and they are able to describe the distinguishing properties of a right angle and a square, having made the relevant association. The child is therefore able to discriminate between appropriate and inappropriate responses as they observe the teacher modelling (Rachlin, 1970, p. 66-69). The principle of operant (instrumental) behaviour states that "one effect of a successful behaviour is to increase the probability that it will occur again in similar circumstances" (Rachlin, 1970, p. 73).

Behaviourism evolved from investigating animal behaviour, to focusing on human learning and social theory. Behaviourism, however, propounds that when learning something, the lesson must be reinforced through external reward; whereas social learning theory propounds that one

can learn by simply observing others experiences. Bandura (1969) investigated behaviourism using a social learning theory approach, and studied observational learning with children as observers and adults as models. He and his colleagues demonstrated that the reinforcement of a models behaviour correlated with the observer's judgement of whether or not the behaviour was appropriate to imitate. The results of such studies formed the empirical basis for Bandura's (1977) social learning theory. Bandura (1969) placed special emphasis on the roles played by vicarious, symbolic and self-regulatory processes in theories of learning. These 'vicarious processes' (p. 2) involve learning by observing, and making decisions about whether the teacher's behaviour is appropriate for copying or imitating in the classroom.

Bandura (1969) explained the way in which the interplay between people and the environment can influence or cause change in behaviour, using principles of behaviourism in observation and modelling. Bandura's (1969) specific focus was on observing modelled behaviour of a teacher. The behaviour change happens through processes and patterns of acquisition, through copying and imitating teacher behaviour.

Some key principles of behaviourism

The key principles that behaviourist theorists draw upon for successful acquisition of behaviour patterns involve observational learning and modelling, and response sequences, which involve reward and punishment. In this study, focus is placed on observational learning and its processes, otherwise known as 'learning through modelling behaviour'.

Observational learning

The human capacity to learn by observation allows us to acquire large 'units' of behaviour, without having to develop patterns gradually, by means of trial and error. Thus, learning takes place within a controlled environment, and through observation of other's behaviour. Observational learning is underpinned by six processes, namely: attentional process, retention processes, motor reproduction, modelling behaviour, reinforcement and feedback (Bandura, 1969; 1977, 1986; 1989). These ideas describe some of the different ways in which human learning can be observed and reinforced, however not all are relevant for this study. This study focuses on modelling behaviour as one of the unique principles of behaviourism that can be observed in a classroom. Models are observed with a view that the behaviour may be repeated. Therefore, there is an interrelationship between the observation, learning, and modelling of human behaviour.

Learning through modelling behaviour

Even in cases where it is possible to establish new response patterns through other means, the process of acquisition can be shortened by providing appropriate models. Miller & Dollard (1941), as cited by Bandura (1969), have argued that the modelling process involves imitative learning, which requires observers to be motivated to act. Social learning theory assumes that modelling influences learning. Cognitive skills are mediated through reinforcing effects by the teachers' modelled behaviour. Accordingly, reinforcement helps to mediate a learner's ability to understand what the teacher is showing them, and helps the learner to be able to learn from the example(s), by copying or reproducing the processes in learning new behaviours (Bandura,1969).

The interrelationship between observational learning and modelling will influence the way in which teachers mediated knowledge through the content in the LTSM, and how learners acquired that knowledge. The next section explores some ideas of the implications in the classroom context of my research study.

Critique of behaviourism

The behaviourist approach to learning is criticised on three fundamental grounds: firstly, it claims that learning takes place through behavioural changes, and that the imitation of modelled behaviour does not take into account the consciousness and mental processes of the mind as it is involved in learning; second, it claims that learning happens as an observable form of behaviour, taking cues from stimuli in the environment; and thirdly, it is not able to explain the development of new behavioural patterns, both in animals as well as in humans (Vygotsky,1962; Papert ,1993).

As a result, constructivist theory is developed not only to critique, but also to provide counter-theoretical foundations as an alternative to behaviourism.

2.6.2 Constructivism

Constructivism is based on psycho-social observation, and an investigation of the way in which people develop meaning and acquire knowledge. Constructivists believe that people construct their own understanding, and acquire knowledge of concepts and explanations through their experiences and ideas, by interacting with the social world. Constructivism holds that knowledge is constructed both socially and mentally. This explains the psycho-social

construction of knowledge. This differs fundamentally from behaviourist approaches, which assert that learning happens through a change in behaviour, as learners model behaviour. Piaget and Vygotsky are discussed below as key contributors to constructivist theory.

The Piagetian approach to learning and development

The most important concept in Piaget's theory of cognitive growth is the "schema", which refers to the "internalised mental representation of a particular action" (Athey & Rubadeau, 1970, p. 2), i.e. the way in which representation occurs in the mind. This means that the schema are functions in the mind, supportive of higher level thinking. The development of mental capacity happens through "progressive internalisation" (p. 2), and the schema is hierarchically organised. The way teachers select and organise the LTSM during the lesson can respectively create affordances, or constraints, to the development of internal mental structures. Earlier schema are further developed, are more complex, and are enhanced by interaction with other schema. This creates a new level of "sensitively balanced equilibrium" (p. 2) of internal mental structures. In the Piagetian methods, teachers should create opportunities for learners to discover structures in which to learn trigonometric concepts, rather than "transmitting structures which may be assimilated only at a verbal level" (p. 7). Piaget uses the concept of "adaptation" (p. 2) to explain change and growth in mental development. Furth has further noted that "assimilation focuses on the most important aspects of knowing, namely sameness, common and generalisable in a given situation" (1969, p. 17). In other words, assimilation takes place when trying to fit new information into a pre-existing scheme, whilst accommodation involves modifying pre-existing schemas to accommodate new information. Adaptation is defined by a balance between assimilation and accommodation. The two principles of adaptation are:

- assimilation of the environment to general schemes of knowledge; and
- accommodation of the schemes to specific knowledge.

This means that during assimilation, one identifies common and familiar features of an object, and adopts a general frame of action to deal with the object using old schema. Where the object is too different to use old schema, the existing schema must be modified. Furth goes on to add that "the child makes progress and learns to function better in so far as he/she constantly accommodates an identical scheme to particular features and most importantly the child's intelligence makes abstractions" (1970, p. 25). Therefore, accommodation helps to develop the child's intelligence, and enables the child to construct abstract or conceptual knowledge. The

process of exploration, identification, constructing, measuring and differentiating, is a process of assimilation and accommodation.

Accommodation and assimilation form the basis for developing higher forms of learning. Knowledge is developed through the external environment, whereby people construct meaning. This process helps the child to form a scheme, and to strike a balance between him/herself and the world around them. Equilibrium is achieved when a balance between assimilation and accommodation is achieved, in order to reduce the tension created in the process of incorporating new knowledge into existing schema. Piaget's concept of equilibrium is that the child seeks a balance between what is seen and experienced, i.e. his or her perceptions. Thus, equilibrium is achieved when the concepts that explain the world do not conflict with the child's mental schema. If there is a sense of conflict between the child's experiences and the way in which they perceive, then equilibrium remains elusive. Equilibrium thus refers to the constant shifts in the schema between what we know, and new forms of thinking that we incorporate into our ambit through interaction with the environment.

The different stages of child development play an important role in the process of learning, and are able to explain the levels of complexity that children can cope with in acquiring knowledge at a particular stage of development. Piaget (1977) asserted that learning happens through an active construction of meaning, where a given learner is not merely copying modelled behaviour from the teacher. Piaget's central theory is premised on the notion that knowledge comes from our thinking and reasoning with objects, and that we structure and restructure our knowledge on a continuous basis.

Piaget identified four sequential stages of development children undergo as: i) the sensory motor stage, describing a child's formation from birth to two years; ii) the pre-operational stage between aged two to seven; iii) the concrete operational stage from age seven to 12; and iv) the formal operational stage between 12 and fifteen. This study focuses on the formal operational stage, as it is concerned with concept of development, and developing abstract knowledge and higher forms of thinking.

The formal operational stage occurs between the ages of 12 to 15, and is generally also expressed as 'maturation', adolescence, or 'the higher mental development stage'. According to Piaget, formal operations mean the most superior level of "mental organisation" (Athey & Rubadeau, 1970, p. 2). At this stage, a young learner can formulate patterns of reasoning into abstract ideas. The learner's understanding of time and space develops, and he/she can

generalise from individual cases, to more cohesive concepts. The learner is able to solve problems and to build schema from sorted facts. The LTSM in the study provides exercises that require learners to use their problem-solving skills, and to develop concept maps. The learners can draw on concepts that are available to them without referring to concrete materials or images. The learners' ability to draw on concepts without referring to concrete materials or images will indicate that they have developed qualities of thinking that are mathematical and logical, and which have free flowing thought processes.

The way teachers use the LTSM, and select and sequence activities, will either help or constrain the development of concepts and the creation of equilibrium. The evidence of changes, adaptation and conceptual development will potentially be gauged in the results of the post-test in my study, which will reveal whether or not learners have made progress in their readiness and ability to understand and reason in trigonometry from the pre-test to the post-test.

Critique of Piagetian theory

The critique of Piaget's theory is that it tends to be reductive, and fails to deal in sufficient complexity with the development of higher mental functioning. Higher mental functioning is the ability to acquire higher levels of knowing, which presupposes the presence of foundational knowledge, and enables us to put such knowledge to use. The LTSM in this study are aimed at addressing gaps in learners' knowledge, with a view to developing higher mental functioning. This means developing learning foundations that will enable the balance required between assimilation and accommodation, so as to develop the conceptual knowledge or equilibrium required to access curriculum knowledge in trigonometry. Therefore, this study turns to Vygotsky's (1978) theory, which provides some explanation of the determinants and ideas on how to develop higher mental functioning.

Vygotsky's Approach to learning and development

Piaget's (1977) theory suggested that development precedes learning. In contrast, Vygotsky (1978) held that knowledge is firstly psychological, and secondly, socially constructed. Psychological processes are those that take place in the individual's mind; processes of understanding and remembering or recalling. Socially-constructed processes are those that involve the external environment within which an individual interacts, including those with whom they interact. The development of such processes, inside and outside the mind,

including social interaction, can help us make sense of and understand the world better. When these processes interact to develop better understanding and higher forms of knowing, Vygotsky (1978) calls this 'cognitive mediation'. Higher forms of knowing involve developing general and concrete (explicit) knowledge into abstract knowledge and advanced complex thinking.

The interaction between people to develop knowledge also involves a process of communication, especially that between adults and learners in the school context. Therefore, cognitive mediation is undertaken by someone who is more knowledgeable. The movement towards deeper understanding leads to the space between actual development and potential development, through guided assistance, where knowledge can be meaningfully acquired. Vygotsky (1978) called this the 'zone of proximal development'.

The zone of proximal development

The zone of proximal development (ZPD) is the distance between what one can do on one's own, and what can be done with others more knowledgeable than oneself. The more collaboration between the child and teacher (or knowledgeable other) that takes place, the more the child can form relationships between concepts, link them, and create new and creative ways of thinking. This process of concept formulation and developing new ways of thinking introduces the psychosocial aspect of pedagogic practice. In his explanation of the ZPD, Vygotsky (1978a) described the successful independent use of internal psychological tools by the child as "actual development" (p. 278), and the successful external use of tools based on adult guidance or mediation as "potential development" (Daniels, 2001, p. 278).

Mediation in the ZPD does not only happen due to engagement with the LTSM, but also because the teacher is concerned with the development of conceptual knowledge in the student. The focus on developing conceptual knowledge is very important, because, for Vygotsky (1978), the purpose of mediated action in the ZPD was developing conceptual knowledge. Mediation in the ZPD has implications for classroom discourse, as per the discussion that follows.

In the classroom, the teacher will allow learners to experiment, but will provide guidance. Giving "appropriate scaffolding" support to achieve the task initially set beyond the child's current level of development, helps the child to achieve his goal of becoming competent. "Scaffolding" (p. 318) is a way of providing specialised instructional support to help learners

fill learning gaps, to motivate learners to learn, and to achieve their learning goals. In applying the 'scaffolding method' (p. 318), teacher and students collaborate in learning and practicing key skills, namely: problem solving, demonstration, and questioning. Teachers can furthermore use concrete examples, build a few concepts, and develop learners' understanding of those concepts, and then introduce new ones. Over time, the teacher's involvement assists the learner in becoming "self-regulated" (Daniels, 2001, p. 27), where self-regulation refers to a form of learning that is guided by a learner's own thoughts and behaviours.

Teachers will rely on questioning that develops higher-order thinking, direct instruction and evaluation, in order to develop students' understanding of mathematics. The types of questions will differ, depending on the teacher's beliefs about learning, and their normal classroom practice. Access to epistemic knowledge and development of conceptual knowledge is possible if teachers use explanations which draw on specialised mathematical concepts and terminology in the LTSM.

Teachers' pedagogical practices are informed by the use of psychological tools, such as signs, symbols and languages. This process introduces the concept of semiotic mediation.

Semiotic mediation

"Semiotic mediation [refers to] the signs, symbols and language" (Daniels, 2001, p. 13-22) used to communicate in the process of knowledge acquisition and development of higher-order thinking. Any society has a system of signs, and we regulate each other through the use of these in language. Language is a powerful tool for communication. Language can be used to socialise in an informal context, and it can be used formally in professional spaces, and during classroom instruction, between learners and teachers. It can also be used to develop advanced thinking processes. Vygotsky (1962) viewed language as an important part of cognitive development (Daniels, 2001). Vygotsky (1962) posited the way in which thought and word are brought together in order to make meaning as a form of "communicative interaction" (p. 137).

In classroom instruction, teachers will use language and speech to ask questions, to explain concepts and to give instructions to learners. Learners will use speech to respond to questions or to ask questions if they do not understand. Our minds are shaped by signs to help us to separate, integrate and coordinate our thoughts. We have developed signs and symbols to represent information and patterns in society. Signs may serve as a tool for concept development or mediation of higher order thinking (mental functioning). Signs develop on the

basis of psychological development “after a series of qualitative transformations” (Vygotsky, 1978, p. 45-46 cited by Daniels, 2001, p. 182). Vygotsky’s genetic law of cultural development explains that higher mental functioning happens twice: firstly, between people, and then within oneself (hence a form of self-regulation); and secondly, with the help of guided assistance.

It must be noted that in South Africa, direct instruction is a dominant discourse, where there are hierarchical power relations in classrooms to be noted. Teachers’ pedagogical approach will affect the way in which they use the LTSM. Teachers could just hand the LTSM to learners, and instruct them to read, and do the exercises on their own, or they could use the LTSM to explain concepts as part of the lesson, and give learners exercises for homework. The instruction in these disadvantaged contexts that can be expected encompasses “teachers dominating talk time” (Hardman, 2010, p. 96). In these classrooms, learners are required to “negotiate time to talk” (p. 96). Many learners are unwilling, and even unable, to do this in school (Hardman, 2010). It is possible that learners’ lack of participation or confidence might constrain mediation and ultimately their access to trigonometry content knowledge.

Critique of the Vygotskian theory of conceptual development and semiotic mediation

One of the most compelling criticisms of Vygotsky is that it is impractical, as it requires the circumstances afforded by individual education, and consequently, it may not be possible to implement in large classrooms (Wertsch, 1985; Phillips, 1995). It is also argued that the child and the adult teacher may not connect at the same level of understanding, and the child may not be clear on the teachers’ explanations (Wertsch, 1985).

The current study will observe the way in which the teacher mediates the material in order to address gaps in learning, and to assist learners in the mastery of concepts. The study will undertake to explain the teachers’ pedagogical movements and strategies used to assist learners.

Implication for instruction and LTSM in this study

Teachers may not necessarily be conscious of the underlying theories of their teaching practice, or of the theories of learning. Teacher beliefs and views about learning are important for the use of LTSM. This study on the use of LTSM in classroom teaching will analyse their beliefs about teaching to understand whether or not behaviourism and constructivism is espoused theory in the teachers’ methods of instruction. A behaviourist approach will espouse modelling and specific types of questioning, such as closed and recall questioning. A constructivist approach will involve group work, learner engagement, and open questioning, which provides

learners with opportunities to learn. In addition, the LTSM was designed according to a constructivist approach, and it encourages the teacher to match tasks to the level of the learner, or sometimes one level higher, in order for maximum accommodation to occur (Athey & Rubadeau, 1970, p. 9). In the study, the LTSM allows teachers to use the results of the formative test to match tasks and exercises to the level of the learner.

Having discussed learning theories as a process of learnt behaviour in behaviourist theory and psychological and social construction of knowledge in constructivism, the following section addresses the social or external process of mediation. The way in which access to specialised knowledge is constrained or enabled depends on the instructional discourse in the classroom. Therefore, the concept of pedagogy is important in framing the discussion on knowledge transfer and acquisition, and more specifically, in framing teacher's instructional practice in mediating learners' knowledge gaps in the classroom.

2.7 Instructional practices

Boaler (2001) describes instructional practices as those methods and strategies that teachers use to shape learners engagement with mathematics content. Boaler (2001) has referred to the teacher's instructional practices to assist learners' in the process of "sense-making" (p. 6) of mathematics content as "teacher moves" (p. 6). This emphasises that the process of teaching and learning is a cognitive (psychological process of the mind) and sociological (external interactive) process. This study uses the term pedagogical moves to describe teachers' instructional practices. Furthermore, Vygotsky's (1978) term of scaffolding is used here to describe teachers' instructional practices as discussed on page 35. "Scaffolding" (p. 318) is a way of providing specialised instructional support, where the teacher leads, demonstrates and carefully explains concepts and procedures to help guide learners based on the teachers' understanding of learners' knowledge and learning needs.

Vygotsky (1978) describes scaffolding as the process of "Scaffolding" (p. 318) is a way of providing specialised instructional support to help learners fill learning gaps, to motivate learners to learn, and to achieve their learning goals.

Pedagogical discourse will influence the type of knowledge transmitted, as well as the way in which the teacher will use the LTSM to mediate learners' knowledge gaps. This will either create affordances or constraints for the acquisition of knowledge. Initially, I understood pedagogy to be the process of teaching and learning i.e. the methodologies that teachers employ

in the classroom to transfer knowledge. I was also of the view that pedagogy referred to the interaction with learners on curriculum content, and concerned the way learners are organised in classrooms for learning to take place. Vygotsky (1997) and Bernstein (1999) offer the following views:

The definitions show that instruction is a process of knowledge construction, and as such, is a political, social and cultural process. It also implies that assessment of learning is embedded in the pedagogy and scaffolding, where this refers to criteria and evaluation.

The importance for my study is that teachers' pedagogical and scaffolding approach will affect the way in which they use the LTSM.

2.8 Conclusion

Evidence (DBE, 2012; IEA TIMMS, 2011; Taylor et al., 2010; Schollar, 2011) shows that the curriculum context and changes in CAPS contributed to a need to recognise learner knowledge gaps. These knowledge gaps are explained by poor understanding and lack of foundational competencies, as well as learner backgrounds. The starting point for the conceptual framework is to understand formative assessment, and the way in which it informs diagnostic assessment. Formative assessment provides opportunities for evaluating whether or not the intended learning outcomes have been achieved. It informs planning for teacher mediation, and is integral to identifying knowledge gaps. The study employs a diagnostic assessment tool as a way of identifying learners' knowledge gaps and areas for improvement as part of the LTSM. The ZPD requires that potential mental development be considered in this diagnostic assessment. Key to the social constructivist framework is the attention required to change teacher beliefs about learning and assessment. The nature and type of mathematical knowledge comprises specialised mathematical knowledge, everyday knowledge, as well as scientific and spontaneous concepts. Trigonometry is a subdomain of mathematical knowledge, and will be the topic investigated. LTSM is the vehicle for teacher mediation strategies. Learner Teacher Support Material specifically aimed at addressing knowledge gaps presents one of the ways in which to support teacher mediation strategies. Learning theories are antithetical, and they specifically assist with understanding learning and development from a behavioural, social and cognitive perspective. The theories make explicit teacher mediation of knowledge, through the use of Learner Teacher Support Material.

Finally, the teacher's classroom practices focused on the pedagogical movements that the teachers will make when using the LTSM. Teacher mediation is important, as it ultimately helps to "construct [the] meaning and understanding of text" (Bernhardt, 1987, p. 32). In this study, the teacher will use the text in the LTSM and mediate learners' understanding of the text.

Conceptual framework

The empirical object of study is the use of LTSM (material designed to close knowledge gaps) based on a claim, which is two-fold, namely that: 1) the design of teaching and learning processes using LTSM is focused on addressing knowledge gaps (the knowledge learners need to acquire in order to access the curriculum at the appropriate level of cognitive demand); and 2) teachers' use of LTSM is an important resource, where their normal instructional practices can enable and/or restrict the success of the use of LTSM and assisting learners with knowledge gaps. There are three elements that Elmore (2008) refers to in the instructional core, first the level of content that learners are taught, second the teachers skill and knowledge that they bring to teaching the content and thirdly the level of learners active learning of the content. Elmore (2008) asserts that professional development only improves learning "if it influences what teachers do in the classroom and the effects lies in the level of content. Furthermore, instructional leadership is only effective when it influences the level of work in the classroom and the instructional core" (p.1). I premise my study on Elmore's instructional core theory, as it provides a good basis for investigating my claims, and it draws together: 1) using the LTSM as a resource and means for the mediation of content; 2) through an external social mediation process by the teacher in the process of teaching; and 3) ascertaining learner engagement.

The next section on methodology describes the methods used to collect data and details the way in which the data was analysed for the current study. I will also discuss my ethical considerations, as the researcher.

CHAPTER THREE : Methodology

3.1 Methodological Framework

The area of teachers' use of Learner Teacher Support Material (LTSM) to mediate learners' knowledge gaps in Grade 10 mathematics, specifically using diagnostic assessment, received very little attention from researchers. Thus, as a research strategy, a mixed methods approach was chosen, focusing mainly on a qualitative case study of three teachers, and this approach also uses a quantitative statistical descriptive analysis framework for test data. A frequency analysis framework of qualitative classroom data was also used. The framework is appropriate, because it focuses on documenting the teachers' classroom practices in mediating knowledge gaps in a real-world setting. To avoid generalisations, the framework focused on inquiry and inductive analysis, drawing themes and describing patterns and differences in the three teachers. Purposive but representative sampling was used (Schumacher, 2006), with criteria. The data collection methods included diagnostic assessment (pre-testing and post-testing) semi-structured interviews (pre-lesson and post-lesson) and two lesson observations of each of the three teachers.

3.2 Research strategy

A mixed design was used, involving mainly qualitative analysis through case studies and some quantitative techniques to analyse diagnostic test data. Denzin & Lincoln (2000) have posited the suggestion that qualitative and quantitative research methods are based on different assumptions of the world, research purpose, methods and process, prototypes, researcher roles and places different importance of context. Johnson & Onwuegbuzie (2004) have argued that:

> *mixed methods research offers great promise for practicing researchers who would like to see methodologists describe and develop techniques that are closer to what researchers actually use in practice. Mixed methods research as the third research paradigm can also help bridge the schism between quantitative and qualitative research* (Johnson & Onwuegbuzie, 2004, p. 14-15).

The research is predominantly a case study, describing the ways in which three Grade 10 Mathematics teachers in Quintile 4 township schools use LTSM to address learners' knowledge gaps. According to Yin (1981):

> *a case study is an empirical inquiry that investigates a contemporary phenomenon in depth and within its real life context, especially when the boundaries between the phenomenon and context are not clearly evident* (p. 18).

A qualitative approach was chosen to describe the chosen case study. According to McMillan & Schumacher (2006, p.23), "qualitative designs emphasise gathering data on naturally occurring phenomena." Patton (2005) has posited that:

> *[..]Qualitative research analyses data from direct fieldwork observations, in-depth, open-ended interviews, and written documents. Qualitative researchers engage in naturalistic inquiry, studying real-world settings inductively to generate rich narrative descriptions and construct case studies. Inductive analysis across cases yields patterns and themes, the fruit of qualitative research* (p. 1).

To analyse the diagnostic test data, quantitative analysis was used in the form of a t-test, to determine how significant the results of the pre- and post-tests were, and to ensure reliability of the results.

> *Whilst both techniques can be used in one study to analyse data, "quantitative and qualitative research states that many writers find it helpful to distinguish between quantitative and qualitative research*" (Bryman, 2008, p. 21, 22).

Layder (1993) provides the following distinction, which serves as an operative distinction for this study:

> *the status of the distinction is ambiguous because it is almost simultaneously regarded by some writers as a fundamental contrast and by others as no longer useful or even simply as false. There is little evidence to suggest that the use of the distinction is abating and even considerable evidence of its continued, even growing currency. On the face of it, there would seem to be little to the quantitative/qualitative distinction, other than the fact that quantitative researchers employ measurement and qualitative researchers do not* (p. 110).

The table below describes the fundamental differences between quantitative and qualitative research strategies as developed by Bryman (2008).

Table 1: Differences between Quantitative and Qualitative research strategies

	Quantitative	Qualitative
Principal orientation to the role of theory in relation to research	Deductive, testing theory	Inductive; generation of theory
Epistemological orientation	Natural science model, in particular, positivism	Interpretivism
Ontological orientation	Objectivism	Interpretivism

(Bryman, 2008, p. 22)

The study used open-ended semi-structured interviews and observations, through video recording, studying real classroom situations. Furthermore, it used LTSM from a project of the Zenex Foundation, which designed LTSM that included diagnostic tests and classroom activities, as well as exercises specifically designed to address knowledge gaps of learners in Grade 10 Mathematics.

3.3 Sampling

A purposeful sample of three Grade 10 Mathematics teachers were selected. McMillan & Schumacher (2006, p. 138) have explained that in purposeful, or purposive sampling, the researcher selects particular elements from the population that will be representative or informative about the topic of interest. The following set of selection criteria were used for this study.

3.4 School selection

Three Quintile 4[2] township schools with basic functionality were selected, which offered Grade 10 mathematics from one of the KZN Districts. The criteria were that the schools achieved at least a mean pass of 40% in mathematics in the 2012/13 National Senior Certificate (NSC), because a minimum pass is 30 percent. A criterion of 40% suggests that learners have knowledge gaps, and are likely to benefit from the teacher mediation of the LTSM. Since one

[2] Quintiles are categories of schools classified based on funding norms using the poverty levels of the community around the school.

school had two teachers who gave consent, I decided to work with only two schools, from which the three teachers participating in the study were drawn. Quintile 4 schools lack some basic resources, and very rarely receive outside intervention or support. The selected schools closely met the criteria. The schools did not benefit from a similar intervention before and during the period of the study. The table below shows the school information for schools A and B:

Table 2: School Profiles

Details	School A	School B
Total Number of learners	1500	405
2012 Performance in Maths	40%	39%
Number of Grade 10 Maths teachers	2	1

3.4. 1 Telephone contact and Meeting with Principal

Since the study took place in KZN, I contacted the principal of each of the schools, explaining my research and requesting a meeting to discuss undertaking research in their respective schools. I also offered to send information by email or fax, and after doing so, had meetings with the principals between 15 and 20 March 2014.

3.4.2 Teacher Profiles

Together with the principal(s), I selected three teachers from school A and B. In school A, two Grade 10 Mathematics teachers participated in the study based on the principal's guidance, as well as the teachers' willingness to participate. Only one teacher in school B participated, as he was the only Grade 10 mathematics teacher, and he provided consent and indicated his willingness to participate. The table below outlines information on the three teachers' experience, qualifications and classroom sizes.

Table 3: **Teacher Profiles**

Schools	**School A**		**School B**
Details	**TEACHER 1**	**TEACHER 2**	**TEACHER 3**
Teaching Experience	3 Years	7years	26 years
Other Grades taught in 2014	8,9,12	9,10,11	10,11,12
Qualifications	BTech & PGCE	BA(HDE) & ACE(Maths GET)	BA(HDE) Honours Education Management and Leadership Honours Maths (in progress)
Number of learners in class	70	78	48
Classes selected for research Diagnostic tests & videotaping	10 C	10B	10A

3.4.3 Planning Meeting with teachers

The first step took place from 20 to 31 March, 2014. The meeting took place after teachers had returned their consent forms to participate in the research to the principal. This step involved planning and discussion with teachers on the research process. The planning meeting involved what the research entails, and the protocols that as a researcher, I needed to follow with them as teachers, as well as with learners and parents. I then collected information about the number of learners that the teachers taught. We agreed on dates for the various data collection activities. The activities included an introduction to the three teacher's classes, and involved giving learners assent and consent forms to be signed and returned. We also scheduled dates for administering the diagnostic tests. This step also involved coming to agreement on the topics that the teachers would teach during the two lessons to be videotaped, and scheduling dates for videotaping of the lessons.

I expected to see similarities and differences in the three teachers' use of the LTSM, understanding of learners' knowledge gaps, and the way in which they mediated the knowledge gaps. I sought to understand the way in which teachers mediate Learner Teacher Support Material, towards assisting learners with knowledge gaps that were assessed through a diagnostic tool, before and after teachers' use of the LTSM. Based on my research questions, I wanted to see how teachers understand diagnostic assessment in relation to identifying their learners' knowledge gaps. I also wanted to see how the three teachers understand the

relationship between the LTSM and their learners' knowledge gaps, and understand what forms of classroom practices enables or constrain the teachers mediation of the LTSM.

3.4.5 Learners

Those learners undertaking Mathematics in the Grade 10 classes taught by the selected teachers participated in the study. This is due to the fact that they were the class that wrote the diagnostic assessment and with whom the teacher used the LTSM to mediate knowledge gaps. Learners' ages in Grade 10 usually range between 15 and 17 years.

Assent from Learners and Parental Consent

Each learner participant received copies of the assent and consent forms via their teachers to take home for signing. Teacher 3 only returned forms for 25 learners out of 48, while Teacher 1 returned 67 out of 70 learners forms, and Teacher 2 returned the greatest proportion of 74 out of 78 learners' forms. Not all learners returned the forms, and I excluded those who did not give consent in the study from videotaping and testing. The teacher agreed that we could move learners who gave assent and consent from their parents to the front of the class, whilst the rest of the learners sat in the back of the class, and were not videotaped. A total of 157 learners out of the 196 learners returned signed assent and consent forms from the three teachers' classes. The learners wrote the pre- and post-diagnostic test. In the pre-test, it was necessary to ascertain what knowledge gaps learners had, and in which test measures they were experiencing gaps: 2) to understand if the teachers used the LTSM to assist with mediating their knowledge gaps; and 3) provide a deeper understanding of the similarities and differences in the three teachers use of the LTSM and in their normal teaching.

3.5 Data Collection

Data was collected according to the following steps:

Step 1: Pre-diagnostic test

The teachers administered the same test (pre & post diagnostic) (Appendix 4) with Grade 10 learners at each school. The learners wrote the pre-diagnostic test at the end of the term, at least three weeks prior to the lessons that were observed. The time was planned to make it possible to mark the tests and completing the learner profiles. This meant a one month lag time (holiday time) between the pre-diagnostic test and the first lesson observed for both schools.

Overview of the test measures

Learners were tested, and data recorded in two content areas: trigonometry (trig) readiness and function readiness, but for purposes of this study, I will only focus on analysing trig readiness.

Table 1 provides the test measures of the knowledge and skills required for trig readiness. The table was developed jointly by the Zenex Foundation 2011 Research Project. The required skills and knowledge are confirmed in the literature by Pournara (2001); De Villiers & Jugmohan (2012); Canobi, (2009); Rittle-Johnson et al., (2001); and Star (2005). Refer to Table 1.

Table 4: Test measures, marks and trig knowledge and skills

	Required knowledge & skills	Type of knowledge	Question number	Max marks Total 25	Rationale	Relationship to question on test
trigonometry	Measuring and constructing angles	Procedural	1.1 and 1.2	4	Provides physical knowledge of angle sizes, and a means to check solutions physically.	The test required learners to construct and measure angles of 37 degrees.
	Properties of angles between parallel lines	Procedural	2.1	2	Comes into trigonometric problems such as those involving angle of elevation, depression.	The test required learners to identify which angles between parallel lines add up to 180 degrees.
	Similarity of triangles	Conceptual	2.2	3	That which is Fundamental to understanding trig ratios being the same for a particular angle in a right- angled triangle.	The test required learners to identify which triangles were similar to one another, among others that were not.

	Required knowledge & skills	Type of knowledge	Question number	Max marks Total 25	Rationale	Relationship to question on test
	Pythagoras' Theorem	Conceptual	3	3	Fundamental to computing the ratios and relative lengths of the sides, specifically in 90 degree angles.	The test required learners to explain, using diagrams, two features of Pythagoras theorem.
	Fraction computations	Procedural and conceptual (the explaining)	4.1 & 4.2	5	Occur in using special angles and later in identities.	The test required learners to identify appropriate use and procedure of simplifying and to explain their answer.
	Surds	Identify (Procedural) Explain (Conceptual)	5	4	Occur in special angles.	The question required learners to identify surds among other examples that were not surds and explain their answer.
	Ratio and proportion	Procedural	7.1 & 7.2	4	Fundamental to conceptual understanding of trigonometry.	The test question required learners to determine unknown proportion of numbers based on a given ratio.

Sources: Zenex Foundation (2011); Pournara (2001); De Villiers & Jugmohan (2012)

Step 2: Pre-test feedback

I appointed an external Mathematics teacher to mark both pre- and post-diagnostic tests and provided a memo (Appendix 5) to the teacher for marking. I then captured the marks for each question on an excel spreadsheet. The researchers from the Zenex project divided questions into four knowledge areas, namely: trig readiness and function readiness (although tested, the research did not focus on this area), procedural and conceptual knowledge. I recorded the final marks into three categories of knowledge Trig readiness, conceptual and procedural. See Table 1 above outlining the test measures, marks, and trig knowledge and skills required. I used the data to develop graphic profiles for each learner who wrote both the pre-and post-diagnostic tests and class group profiles showing class averages.

I provided teachers with feedback on the test results at the start of the new term. This meant a one-month lag time (holiday time) between the pre-diagnostic test and the first lesson observed for both schools. I gave feedback to teachers during the first week of term (teacher three during the second term and teacher one and two in the third term).

I used the test feedback session to show the teachers the materials, and to give them a pack of the material. I gave the three teachers these profiles to gain an understanding of their learners' trig readiness in relation to conceptual and procedural knowledge. My view was that they would use the LTSM to provide feedback to learners on the test and mediate gaps in learners' knowledge. Teacher 1 and 3 asked me to provide feedback to learners on the test. The request was a dilemma, as I (the researcher) was considered a resource to giving feedback on assessment. I decided to give the learners feedback with their results and discussed with them that the test will not be part of their marks. I explained that this was done to see how the teacher could assist them better where they had problems. I also presented a motivational talk to them, concerning how they might improve their learning. Teacher 2 provided the feedback himself to the learners.

Step 3: Interviews

Overview of Interviews

McMillan & Schumacher (2006) described structured interviews as data instruments that collect data face-to-face or telephonically as follows:

> *In a structured interview, the researcher asks the same questions of numerous individuals in a specific manner, offering each individual the same set of possible responses* (McMillan & Schumacher, 2006, p. 319).

I used semi-structured interviews in order to probe certain answers and issues of importance to the participants. Longhurst (2003) has defined semi-structured interviews as:

> *a verbal interchange where the interviewer attempts to elicit information from the interviewee by asking questions. Although the interviewer prepares a list of predetermined questions, semi-structured interviews unfold in a conversational manner offering the interviewer the opportunity to explore issues they feel are important* (Longhurst, 2003, p. 117).

The semi-structured interviews consisted of open-ended questions. According to Mouton & Babbie (2001):

> *open-ended questions provide no structure for the answer, they should be tightly focused to elicit the kind of information the researcher wants to get. These type of questions require accurate and time-consuming transcription, their use should be limited to initial research where the number of respondents is small. The object is to refine the research direction and determine more precise questions that can be structured another way* (Mouton & Babbie, 2001, p. 233).

I used a combination of a semi-structured interview and asked open-ended questions. The semi-structured nature of the interview worked differently with the three teachers. In the case of Teacher 1 and Teacher 3, the interview happened in a conversational way, but with the teachers volunteering a lot of their own information. I was obliged to try and ensure that I focus, so as to obtain responses on all the questions that I asked. In the case of Teacher 2, it was less

conversational, where he gave one sentence answers, and I was obliged to probe deeper to elicit more information.

I piloted the interview instruments with one teacher who taught Grade 10 during 2013. The instruments were refined after the pilot, and the actual pre-lesson interview with Teacher 3. For example, I removed questions that referred to teacher instructional practices, as I realised that those can be observed in the video lesson. The interview with Teacher 3 took almost two hours, and I realised that some of the information would be observed during the lesson. I therefore refined the pre-lesson instrument again, before I used it with Teacher 1 and 2. All three teachers requested me to give them the interview questions in preparation for the interviews (both pre-lesson and post-lesson). I suspected that this was because they understood that they were going to be recorded and wanted to prepare. Having sent the teachers the instruments to prepare, it appears that it did not affect their responses, given that they came across as both open and genuine in their responses. I prepared detailed transcriptions of the pre- and post-lesson interviews after they took place.

Pre-lesson interview

I completed the pre-lesson interview with the three teachers after the test feedback and before the first lesson was observed. The purpose of the pre-lesson interview was to gather some of the data outlined in the teacher profiles, but also to gather teacher perspectives about their learners' learning needs and knowledge levels.

There are four areas of data that the pre-lesson interviews helped to elicit given the questions that were asked:

- Teachers' understanding of their learners' knowledge levels and learning needs, specifically in trigonometry;
- Teachers' perspectives about their own instructional strategies;
- Teachers' perspectives on assessment feedback to learners and the types of assessment given to learners;
- Teachers' responses on the use of Learner Teacher support Material (LTSM).

Step 4: Video lessons

The purpose of the videotaped lesson observations was to record the way in which teachers use the LTSM to mediate learning gaps in their learners. It was thought that they would use the

results of the pre-diagnostic test to understand learners' gaps and learning needs. We videotaped two lessons for each teacher.

Video lesson 1

The first lesson was videotaped at least a month after the pre-test, however for both schools, the month's lag time was during the holidays. The first lesson was on similar and congruent triangles because the topic is pre-requisite knowledge for accessing trigonometry content knowledge.

Video lesson 2

This was the second lesson and it focused on trigonometry ratios. The purpose of the trig lesson was to align curriculum content, and to determine whether learners coped better with the grade-specific content once the learning gaps were filled.

Step 5: Post-test

The post-test was the same test as the pre-test, but I did not make the teachers aware of this. The teachers administered the post diagnostic tests during the week after the second videotaped lessons for each class. The post- test for school A was administered on 14 August 2014, while school B conducted the test on 27 May 2014.

Step 6: Post-lesson interview

I conducted the post lesson interview one week after the post-test for each of the three teachers. The purpose of the post-lesson interview was to gauge teachers' reflection and experiences during the videotaped lessons and their perspectives about the workings and limitations of the LTSM. The post-lesson interview reflected on:

- Teachers' overall reflection of lessons;
- Teachers' reflection on use of diagnostic assessment;
- Teachers' reflection on use of LTSM and effects on learners' knowledge;
- Teachers' learning and development from the research process.

The table below provides an outline of the data collected by teacher:

Details	Data Collected and Date	Teacher 1	Teacher 2	Teacher 3
Ethics approval	7 March 2014			
Consent Forms	Informed and Signed Forms Parents, Teachers, Learners & Principals	March to April 2014		
Diagnostic tests	Pre-test	24 April 2014	24 April 2014	14 March 2014
Diagnostic Results	Test Feedback and Profiles	27 May 2014	27 May 2014	24 March 2014
Interviews	Pre-Lesson Interview	1 August 2014	1 August 2014	9 April 2014
Video/Observation	Video Lesson One	1 August 2014	1 August 2014	20 May 2014
Video/Observation	Video Lesson Two	13 August 2014	13 August 2014	23 May 2014
Interviews	Post-Lesson Interview	15 August 2014	20 August 2014	20 August 2014
Diagnostic test	Post-Test	14 August 2014		27 ay 2014

3.6 Data coding and analysis

3.6.1 Diagnostic Tests

In analysing the test data, I began by using raw averages, and soon realised that the analysis was flawed without a statistical analysis to back up my analysis of the data. A statistician was brought in to assist with the statistical analysis. In my second analysis, there were questions showing anomalies in the test results that were difficult to explain. The lack of explanation necessitated a revision of data, during which it was discovered that there had been some duplication in the formula for conceptual knowledge and trig readiness. The data was thoroughly cleaned and rechecked, followed up by an overview by Zenex Foundation researchers to check the data for accuracy, and to redo the statistical analysis using a t-test to determine statistical significance in test scores.

The interviews and video lessons from the transcripts were coded using open coding on Atlas.ti.

Coding happens soon after the collection of initial data, and the data is broken down into parts that are named. I coded the lessons from the transcripts using ‘open coding’ on Atlas.ti. Strauss

and Corbin (1990) assert that open coding refers to breaking down, examining, comparing, conceptualising and categorising data yield concepts and later grouping these into categories.

3.6.2 Interviews

The pre and post-lesson interviews were audio-taped to ensure all the data was captured in detail, and with accuracy, to avoid researcher bias. I took brief notes during the interviews to show respect and attentiveness to the teacher's responses rather than focussing on taking detailed notes. The interview data was then transcribed and the data coded on Atlas. I coded both the pre- and post-lesson interviews in the same unit in Atlas.ti, and the chosen codes were influenced by the themes in the study. Thus, in both interviews, I coded teachers' beliefs about learner assessment. The code involved their views about diagnostic assessment, other assessments they used, how they assess their own learners, and their reflections about diagnostic assessment in this study. This was a dominant code and created a theme. Another dominant code I used was 'mediation', but this was changed to 'instructional practices', as it encompassed the teachers' beliefs about their addressing learning gaps and their pedagogical practice and beliefs about learning. In my analysis, I discussed teacher beliefs about their knowledge gaps separately from instructional practices, so it ought to have been coded that way, where I was required to refer back to the code list and the transcript when the portraits on this aspect were written. Whilst teacher beliefs of LTSM was a code, but not a dominant one, it formed a theme when the portraits were initially developed.

I also developed the teacher portraits in Appendices 1, 2 and 3. The portraits helped to draw key themes in the analysis sections that supported the findings on the classroom observation data from the video lessons. The portraits also helped to compare the similarities and differences in the three teachers under the analysis.

3.6.3 Video lessons

Two lessons by each of the three teachers were videotaped when the teachers were teaching trigonometry, according to the CAPS curriculum schedule for Grade 10 Mathematics, and a transcriber was hired to transcribe the video lessons. This proved to be a challenge, as when I checked the transcriptions against the recording, I was required to supplement large sections to record teacher's pedagogical moves, especially when they modelled examples on the board, including many of the mathematical concepts. Atlas.ti was then used to code the data and create themes called families, and a frequency count was conducted of the ways teachers used

different types of knowledge, the LTSM and the classroom pedagogy and also categorised these moves based on themes from the literature. The data was analysed comparing the three teachers, using frequency and detailed descriptive analysis.

The constructs and theoretical underpinning from the literature in the conceptual framework guided the selection of codes, and where certain codes described the same information, I refined the codes into families. Several revisions were required, each time I went back to check codes. The codes informed the description of the data presented. I coded the pedagogical moves and scaffolding as instructional practices of the teachers during the two lesson observations. 'Pedagogical moves' refer to socio-cultural interactions between learners and teachers, which include evaluation or assessment of learning (Vygotsky 1997 as cited by Daniels 2001, p. 5; Boaler, 2001).

'Instructional practices' in this study refers to the pedagogical moves and scaffolding process of the three teachers in the classroom. Knowledge construction forms part of pedagogical practices (Vygotsky 1997 as cited by Daniels 2001, p. 5; Boaler, 2001).

'Mediation' was used as a family code for instructional and pedagogic practices. For writing purposes, 'instructional practices' was found to be the most appropriate, where the separation between mediation and instructional practices will help to keep the concepts distinct. The codes that formed part of mediation were teacher talk, procedural knowledge, Modelling, Different types of questioning: open-ended, recall, closed and questions assessing learning and understanding, the last of which was coded as 'reasoning questions'. The dominant codes were 'procedural knowledge', 'modelling recall', and 'closed questioning'. These are analysed in detail in the data analysis section[3]

I identified instructional practices of the teachers as a way of understanding and gaining insight into the ways in which teachers use the LTSM to mediate learning gaps identified in the diagnostic assessment. I also wanted to understand what practices enable or constrain the teachers' mediation of the LTSM.

The table outlines the codes I used, with an explanation of each code, based on the three teachers' pedagogical moves and scaffolded processes during instruction.

[3] See Chapter 4 p. 89-106

Table 5: Codes explaining teacher instructional practices

Coding (Instructional practices)	Explanation of Code	Example
Conceptual knowledge: Teacher-led	The teacher explained or defined a mathematical concept or principle.	Teacher 2 explained the principles of proving that triangles are congruent to learners: "You only have to mention three aspects between the two triangles that are equal. The most important thing of the three aspects is that if you mention sides, you must mention that the sides are corresponding; if you mention angles, you must mention angles that are corresponding" [sic] (Teacher 2, Video lesson 1, Congruent triangles, 1 August 2014)
Conceptual knowledge: Learner reasoning	The teacher asked learners to provide reasons to explain their answer.	In the trig lesson, Teacher 1 asked the learners: "How do you see that this is My… Sin? This is (ah) my cos… how do you know to name them?" [sic]. The question requires learners to explain trig ratios in relation to the sides of the angles that make up the trig ratio. (Teacher 1, Video lesson on trigonometry, 13 August 2014)
Procedures & techniques	The teacher explained the sequenced steps and actions involved in approaching a mathematical problem.	Teacher 2 explained the steps and procedures for solving a right-angled triangle during the trig lesson as follows: "So in solving this right-angled triangle, we'll then say Sin30°= the side that is ab over the side that is ac. Right. We don't know what the length of side AB is, but we know the length of side ac. So what we do, we then say (writes on board) sin 30°=AB/5. This now has become an equation, and we call this: trigonometric equations. We call this (writes on board): "trig equation". And when we solve trig equations, we solve trig equations like we'd solve any other equation. Okay. Right, the unknown here is ab. What does it tell us? It tells us that we should then take the unknown and make it the subject of enquiry or the subject of the formula. To get rid of this 5, it means we must

		multiply by 5 on both sides and then eventually we will have 5Sin30°=ab. Now our ab is the subject of enquiry. Then we can use our knowledge of finding the value of this trig ratio in our calculators" [sic]. (Teacher 2, Video lesson on trigonometry, and 13 August 2014)
Everyday knowledge	The teacher embedded a concept in everyday context or used everyday illustrations and or examples to embed a concept.	Teacher 3 set the context of the mathematical concept through a practical illustration of three girls of similar height. He explains to the class that if you measure the height of one of the girls, there will be no need to measure the height of the other two girls, as their height would be similar. He used the everyday illustration as a yardstick to introduce the topic of similar triangles. (Teacher 3, Video lesson on congruent triangles, 20 May 2014)
Open questioning	Teacher asked a question that required learners to draw on prior trig knowledge to solve trig problems and provide supporting logic and evidence of their knowledge.	Teacher 1 asked the following questions: "How will I know which one is adjacent and which one is the opposite… yes, Sisi? Which one is the one that is next to the angle? Which will be the… the adjacent side? *Angithi uyabona*? And which one is the opposite?" [sic]. The questions require learners to use prior knowledge on triangles, specifically the principles of approaching unknown angles and in asking how, the teacher requires evidence of the knowledge. (Teacher 1, video lesson on trigonometry, 13 August 2014)
Closed questioning	Teacher asked a question that had only one correct answer.	Teacher 1 asked learners: "So we are saying that side X, Y is congruent to side A, B, because they are both how many centimetres?" [sic]. (Teacher 1, Video lesson on congruent triangles, 1 August 2014)
Recall questioning	Teacher asked a question that required learners to remember or retrieve mathematical knowledge without logical context or understanding.	During the trig lesson, teacher MA asked the learners: "remember when we introduced sin? what was the ratio?" [sic]. (Teacher 3, video lesson on trigonometry, 23 May 2014)
Assessment of learning & understanding	Teacher asked questions or gave learners tasks to elicit learners understanding or evaluate learning.	Teacher 2 checked learners understanding of their homework by asking: "Did anyone have any problem when doing the homework yesterday?"

		One learner responds: Yes. Teacher 2 follows up to assess: "Right. Where was the problem? Which sum did give you a problem? Number?" (Teacher 2, Video lesson on trigonometry, and 13 August 2014)
Modelled exercises, examples	Used for examples, exercises that the teachers used to explain or calculate mathematics on the board.	Teacher 2 modelled the trig equation answer that he gave learners for homework. SinΘ/3=0.13 SinΘ=0.39 Θ=Sin(0.39) Θ=23.0°. (Teacher 2, Video lesson 2, trigonometry, and 13 August 2014)

The table above shows the codes and explanations of the three teachers' instructional practices. It also highlights selected examples that the teachers used in their instructional practices.

3.7 Limitations

This study used interviews and classroom observations and the case study was informed by the data collected. According to Yin (1981):

> *Case studies rely on multiple sources of evidence, with data requiring convergence in a triangulation fashion, and, as a result, benefits from the prior development of theoretical propositions to guide the data collection and analysis* (Yin, 1981, p. 18).

Classroom sizes posed a constraint to teachers' use of the LTSM for groupwork, or for scaffolding with individual learners to provide them with opportunities to develop within the Zone of Proximal Development. This was evident in teachers using the LTSM for whole class teaching, and in their limited use of questioning to engage learners and to develop their high cognitive abilities.

The study was short, and expedited so as to fit in with the teachers' curriculum schedule. It is possible that more lessons could have been recorded to understand teachers' normal teaching better, and compare this with their observed lessons. Due to scheduling impediments that including aligning ethics committee decisions with teaching schedule delays, and advanced planning taking place prior to the advent of the study in November of the previous year, there are a number of ways in which the time spent with these teachers was regrettably curtailed. A further consideration in hindsight is that time taken to review the first lesson's videos with the

teachers, may have augmented the outcomes of the research since teachers may have used the LTSM and tried to improve their teaching in the second lesson. It may also have been useful to teachers to see videos of lesson 2 and get their perspectives on their own teaching. Another limitation was that the study was not a controlled trial, comparing teachers using the LTSM with teachers who do not use the LTSM.

Notwithstanding these limitations, the study will make a contribution to our understanding of how teachers mediate material based on the mathematics curriculum, although it does not directly add to broader mathematics knowledge per se. I have passed mathematics up to grade 12 had exposure to the LTSM during workshops on the Zenex Project. In addition, I was able to obtain expert assistance in my employer's organisation, where I needed a specific expert in mathematical knowledge.

3.8 Ethical Considerations

Respect plays a key role in protecting the rights and welfare of research participants, in particular, human beings. According to McMillan & Schumacher (2006), "research ethics are focused on what is morally proper and improper when engaged with participants or when accessing data" (p. 117). I showed respect to the research participants involved, for their time and efforts in contributing to my study. I engaged respectfully in a professional manner and provided participants with details of what the research entailed, and negotiated carefully and openly in respect of all the requirements and expectations from them for the study. I also advised participants that participation was voluntary, and advised them of their right to withdraw from the study at any point if they wished to do so.

I obtained signed permission from the school principal and teachers and gave participants formal letters and consent forms that explained the study and obtained relevant signatures from the principal and teachers. Permission from the KwaZulu-Natal Department of Education (KZNDOE) was paramount, given that the Department holds authority over the schools in question. The application was done through the KZNDOE ethics permission form, where assent was gained from 157 learners, and consent from their parents for them to write the diagnostic tests, to video their children during the classroom interactions, and to use the results of the study for reporting or public dissemination. Given that the research was proposed and conducted under the auspices of Wits University as an academic institution, I obtained ethical clearance from the Wits ethical clearance committee through the protocol number: 2014ECE002M. The principle of anonymity is applied in the final research report, and

pseudonyms will be used. Lastly, I used numerical coding to distinguish between characters and participants in my report.

3.9 Ethical Dilemmas

Unexpected tension was introduced between adopting a pure observer status and creating a comfortable relationship with the teacher and their learners in the case of both Teacher 1 and Teacher 3. The two teachers requested me to give feedback to learners on the results of the diagnostic tests, citing the potential novelty as a form of motivation for their learners. We agreed to schedule specific dates for the feedback to learners. I started by explaining the results, using a profile of their class averages. I also assured learners that the marks do not count in their school results and that if they did not do well, it only means that they don't know the topic and constructs yet, and that they should not feel demotivated, as the intention was that their teacher would support them to learn the work when she teaches, and would use materials to help them, based on their results. I then handed each learner their profiles and asked them if they had questions, and whether they understood the results. In addition, I undertook a motivational presentation on how they might develop resilience, some tips on how they could plan, get support and feedback from their work. Teacher 2 decided to give the feedback to his learners himself, and the researcher did not observe this process, but discussed it with the teacher in the post-lesson interview.

Another dilemma was encountered with learners that did not return their consent and assent forms, where as a result, I could not include them in the videotape, although they wanted to be part of the video. I was required to explain to them again the importance of giving consent and assent for ethical reasons. We had to re-organise the class before videotaping to have learners that gave consent and assent sit in the front of the class, and the ones who did not, to sit at the back of the class. In particular, I noticed that perhaps as a result of this, Teacher 3 focused mainly on the learners in the front. At some point during lesson one, when the teacher walked around to check learners' work, I took the opportunity – in a private way – to include the learners at the back of the class in his teaching, since I noticed when learners at the back raised their hands he did not engage them. Becoming involved in providing feedback was uncomfortable for me in my assumed position as an impartial researcher, but the teacher responded positively to my comments and rectified the matter in the rest of the lesson by engaging the learners at the back of the class.

3.10 Trustworthiness

The key strategy for trustworthiness involves the researcher's ability to show credibility in conducting research studies. Shenton (2004) refers to a number of ways in which credibility can be achieved. In this study I have managed to achieve two important ways to do this and I discuss only the two:

Firstly, I adopted a mixed methods approach mainly focussing on qualitative methodology and quantitative testing methods. In addition, I used observations, and interviews and these are well established methods in the field of research (Shenton, 2004. p.65)

Secondly, Shenton (2004) asserts the importance of achieving "triangulation through using different methods especially observation and interviews" (p.65). The two methods form the major data collection strategies for most qualitative research. This study combined video lesson observation, pre and post-lesson interviews and pre-and post- testing.

3.11 Conclusion

This chapter describes the research framework, design, data collection instruments and procedures for analysing the data gathered in this study. I have also discussed the ethical considerations and dilemmas that I faced in the process of data collection, along with the limitations and trustworthiness of the study. The analysis of the data follows in Chapter 4, and the discussion of the findings is in Chapter 5.

CHAPTER FOUR: Data Presentation and Analysis

Introduction

The aim of this chapter is to present and analyse pre- and post-diagnostic test data in order to determine whether learners were ready to learn new trigonometry content presented to them. It further analyses how teachers mediated knowledge gaps identified in the pre-diagnostic assessment. The analysis focuses on how teachers used LTSM to mediate these knowledge gaps. I made a comparison of the teachers' use of different types of knowledge namely conceptual knowledge, procedural knowledge and everyday knowledge (CK, PK and EK) in their teaching. I also analyse the teachers' instructional practices and their methods and strategies, as a basis for understanding how they mediated learning.

Boaler (2001) describes the instructional practices that teachers use to assist learners in the process of "sense-making" of mathematics content, as "teacher moves" (p. 6). In this study, I referred to "teacher moves" as "pedagogical moves" to describe how the three teachers mediated mathematical knowledge in the classroom. Furthermore, the analysis considered the way in which teachers recruited formative (diagnostic) assessment to inform the interaction between learners, teachers and the content in general, and specifically in relation to the use of LTSM in the classroom. The analysis was informed by the teachers' portraits created from the field research, contained in Appendices 1, 2 and 3.

I present the data analysis in seven sections, as outlined below. In the seventh category, I reflected on teacher learning and development from the research process, as an additional component to include their reflections of this study.

Pre- and post-test analysis of knowledge gaps. In this category, I provide a statistical analysis of the findings on pre- and post-diagnostic test data, as a basis for identifying whether learners had knowledge gaps, and whether they were ready to learn trigonometry. I conducted a statistical t-test analysis of the pre- and post-diagnostic test in the two knowledge types: conceptual and procedural knowledge. Conceptual knowledge refers to general ideas or thoughts that have "bonds that stand in relationship to one another which help to add and shift to another feature of abstraction" (Vygotsky, 1962, p. 49). Procedural knowledge refers to sequenced steps and actions used to solve a mathematical problem (Canobi, 2009; Rittle-Johnson et al., 2001). The distinction between conceptual and procedural knowledge was built into the test design. In doing the analysis, I wanted to determine whether there were changes

in readiness levels of learners from pre- to post-test, and to identify which type of knowledge contributes to overall trigonometric readiness.

The teachers' use of the LTSM. This category starts with an analysis of the frequency of use of the LTSM, and provides a descriptive analysis of the way in which teachers use the LTSM in mediating learners' knowledge gaps. I wanted to determine whether the use of the instructional practices selected by the teachers to mediate the LTSM helped to address learners' knowledge gaps, and contributed to bettering the trigonometry readiness of learners in the three teachers' classes.

The teachers' use of different types of mathematical knowledge in the classroom. This section provides an analysis of the frequency of the three teachers' use of conceptual and procedural knowledge. It also gives a description of the way the teachers employ mathematical knowledge in the classroom.

The teachers' use of instructional practices in mediating learners' acquisition of mathematical knowledge in trigonometry. The section analysis teachers pedagogical moves and scaffolding processes in the classroom

Relationship with interviews. This is an analysis of the interviews conducted in order to understand the relationship between the diagnostic assessment, instructional practices and the teachers' perspectives.

Teacher learning and development. This is an analysis of the teachers' views on what they learned, based on the interviews.

In the following section, the paper provides a broad overview of the diagnostic tests, the aim of which is to give insight(s) into the structure of the test and the test measures used for identifying knowledge gaps and determining learners' readiness to learn trigonometry. A presentation of the pre- and post-test results on trigonometry readiness follows the overview.

The data showed results for all measures that are an important form of pre-requisite knowledge in being able to learn trigonometry, after which an analysis of the data is provided, focusing firstly on each teacher's class. Secondly, comparison is drawn between the three classes, including some overall comments on the analysis of both the pre-test and post-test. Thirdly, although analysing the data on all measures, the discussion is deepened on areas such as similar triangles, Pythagoras theorem, and ratios, which are fundamental to trigonometry readiness, in

order to determine test measures contributed to the changes in the results from pre- to post-test. That is, I will analyse and compare the test data for each teacher, according to the measures considered as fundamental pre-requisite knowledge for acquiring new trigonometry knowledge. Lastly, I analyse the test data as it pertains to the different types of mathematical knowledge. In this case, my rationale is to determine which types of knowledge (conceptual or procedural) contributed the most to the learners' trig readiness or not. Thus, I will analyse and compare these results for each teacher so as to understand and explain the differences.

4.1 Diagnostic tests

In this section, I situate the diagnostic testing within the relevant theory, and I explain the overall structure of the test. The diagnostic assessment comprised a pre-lesson test and a post-lesson test to determine trig readiness. Farrel (2004) defines prior knowledge as "existing beliefs, attitudes and understandings from past experience". Prior knowledge is linked to readiness, or a learners' entry point to a particular concept or skill ("Helping all learners' Readiness", 2015). The test focuses on five mathematical concepts (henceforth measures) that are important prior knowledge for trig readiness, namely: triangles, right angles, surds, fractions and ratios. The measures were categorised into two types of mathematical knowledge: conceptual and procedural knowledge, as a part of the design of the test questions.

The analysis draws on Airasian & Madalus's (1975) understanding of the use of pre-tests to determine learners' readiness as highlighted in the literature review.

What follows is one example of the test questions, based on the procedural knowledge. In the diagnostic test, the learners' procedural knowledge was tested to determine their level of familiarity with the procedures for adding and cancelling ratios, and how to apply them correctly to solve equations. It also tested whether they could answer questions correctly, using the correct procedure.[4] Question 4.1 of the diagnostic assessment is an example of how the diagnostic test measured procedural knowledge:

[4] Refer to Appendix 4 Diagnostic assessment Grade 10.

Question 4-Example of a procedural question

4.1 Circle the example(s) showing an appropriate use of the shortcut of 'cancelling' to simplify a fraction.

A	B	C	D	E	F
$\frac{50}{70} = \frac{5}{7}$	$\frac{501}{702} = \frac{51}{72}$	$\frac{42}{72} = \frac{4}{7}$	$\frac{520}{705} = \frac{52}{75}$	$\frac{175}{100} = \frac{7}{10}$	$\frac{100}{7000} = \frac{1}{70}$

Following Question 4.1, the diagnostic assessment measured learner reasoning by asking:

How did you decide which one(s) to circle?

This question was designed to test the learners' conceptual knowledge by asking them to provide reasons for their answers to trig readiness questions. It thereby tested learners' procedural and reasoning abilities.

I gave the teachers an overview of the diagnostic test, detailing two types of mathematical knowledge: conceptual and procedural. In the section that follows, I will discuss the five test measures and, in particular, the three areas fundamental to trig readiness, namely: similar triangles, Pythagoras and ratios.

4.1.1 Pre- and post- diagnostic test data on trig readiness

According to research conducted by the Zenex Foundation (2011), Pournara (2001), and De Villiers & Jugmohan (2012), the most fundamental areas for trig readiness are prior knowledge in similar triangles, Pythagoras theorem and ratios. Firstly, similar triangles are fundamental to understanding trig ratios, because they draw on the relationships of a particular angle in a right-angled triangle to the sides opposite or adjacent to the angle. "Congruent Triangles are figures with three sides, made up of three straight lines, and the number of sides it has that are of equal length may classify it. Right angles are where one of the angles are 90 degrees" ("Helping all learners' Readiness",2015). The test measured the learners' understanding of the fact that the sum of the angles of a triangle adds up to 180˚ (degrees).

Secondly, Pythagoras' theorem is fundamental to computing the ratios and relative lengths of the sides, specifically in 90-degree angles. The theorem of Pythagoras states that "the square of the hypotenuse side (the side opposite the right angle) is equal to the sum of the squares of the other two sides" (Smith, 2012, p. 243), for example $a^2 + b^2 = c^2$. Pythagoras forms the crux of trigonometric work as it is fundamental to understanding that trigonometry ratios are

the same for a particular angle in a right angle triangle.[5] Thirdly, a surd is an irrational number written in root form, i.e. it is a square root that cannot be reduced to a whole number" (Smith, 2012, p. 243), for example $\sqrt{2}$. The square root of two is in its simplest form. We could solve it with a calculator, but the result will have endless decimals i.e. 1.414213562…

The test required learners to add, multiply and divide fractions. A fraction represents a part of the whole or any number of equal parts. The test measured whether learners understood that fractions added when the denominators (representing the number of parts the whole is divided into) of the two fractions are the same. Fractions were also tested to determine whether learners knew the principle of multiplication and division. This is important knowledge for understanding trig ratios.

Ratio and proportion are fundamental to a conceptual understanding of trigonometry. A ratio is the relationship between two numbers of a kind expressed as 5 to 3 or 5:3 or $\frac{5}{3}$. The test also measured simplification and adding of ratios.

Having provided a broad understanding of the test, its structure and measures and explained their relationship to trigonometry, the discussion moves to an examination of the pre- and post-test results regarding the trig readiness of the three teachers' classes.

Table 6 below compares the learner results from the diagnostic tests of the three teachers' classes. The table represents a sample t-test statistical analysis, showing the paired differences from pre- to post-test; it shows the mean scores and standard deviation in relation to trig readiness, and the concepts measured in the test for each question. The t-test was undertaken so as to determine the statistical significance of score differences from pre-to-post-test. For consistency, if learners only wrote one of the tests due to absenteeism or due to not giving consent, these tests were discarded. The analysis showed the pre-diagnostic results that identified the knowledge gaps of each teachers' class, as well as whether or not the learners in the three classrooms improved from pre-diagnostic test to post-diagnostic test. The table also shows whether or not the improvements were statistically significant. Where the Sig value is equal to or below 0.005, the table shows a 95% confidence level. However, where there is some average improvement, that is, where the Sig value shows above 0.005, we cannot be as much as 95% certain that it is in fact a variation in the data that is responsible for the improvement.

[5] Refer to Table 41 in Chapter 3, p. 47.

Table 6: Statistical analysis of three Grade 10 classes' pre- and post-diagnostic assessment for trig readiness

Teacher 1						Teacher 2	Teacher 3								
Number of learners:67						Number of learners: 72					Number of learners: 18				
Test Results of Trig readiness Measures	Pre-test	Post-test	Paired Differences	Std Deviation	Significance (2tailed)	Pre-test	Post-test	Paired Differences	Std Deviation	Significance (2tailed)	Pre-test	Post-test	Paired Differences	Std Deviation	Significance (2tailed)
Triangles: Draw triangle	0,10	0,09	-0,015	.615	.843	0,13	0,25	0,125	.749	.161	0,00	0,11	0,111	.471	.331
Angle: Measure angle	0,18	0,42	0,239	.872	.028	0,10	0,06	-0,042	.458	.442	0,00	0,00			
Angle: Identify 180-degree angle	0,90	0,96	0,060	.919	.597	0,81	0,78	-0,028	1.186	.843	0,44	0,61	0,167	1.098	.528
Triangle: Identify similar triangles	1,06	1,69	0,627	1.347	.000	0,97	1,08	0,111	1.449	.517	0,72	1,06	0,333	1.029	.187
Triangle: Pythagoras theorem	0,46	0,85	0,388	.778	.000	0,28	0,35	0,069	.678	.388	0,11	0,33	0,222	.647	.163
Fractions: Multiplication and Division	0,84	1,37	0,537	.927	.000	0,82	1,24	0,417	1.071	.002	0,50	0,83	0,333	.686	.055
Fractions: Adding	0,48	0,42	-0,060	.694	.484	0,17	0,89	0,722	1.078	.000	0,44	0,89	0,444	1.294	.163
Surds	0,97	1,19	0,224	.918	.050	0,92	0,96	0,042	.941	.708	0,89	1,17	0,278	.958	.236
Ratios: Adding Ratio's	0,69	1,01	0,328	1.133	.021	0,42	0,50	0,083	1.135	.535	0,56	0,56	'0,000	1.138	1.000
Ratios: Simplification	1,13	1,31	0,179	1.242	.242	1,03	0,86	-0,167	1.245	.260	0,67	1,11	0,444	1.423	.203
Trig Readiness	6,81	9,31	2,507	3,539	.000	5,63	7,13	1,500	3,580	.001	4,33	6,67	2,333	2,521	.001

In the absence of a benchmark, in this study, a trigonometry readiness level of a score of 70% to mastery of prior foundational knowledge was assumed.The total score for trig readiness was taken out of 25, with a readiness score set at 17,5 (two-thirds).

4.1.2 Pre-test on Trig readiness

The overall pre-test mean scores for the five measures (triangles, right angles, surds, fractions and ratios) of trig readiness shows mean scores of 6.81 for Teacher 1's class, 5.63 for Teacher 2's class and 4.33 for Teacher 3's class. This shows that learners understood less than a third (8.33) of the requisite knowledge to learn trig. This indicates that they are not ready to learn trig. Specific scores outlined below show that learners are not trig ready.

In the pre-test, Teacher 1's class obtained mean scores of 1.06 in identifying similar triangles out of three scores; 0,46 for Pythagoras out of three scores; 0,69 for adding and 1.13 for simplification of ratios out of four scores. The scores show gaps in areas that are fundamental to the readiness to learn trigonometry.

Teacher 2's class also shows similar trends to Teacher 1's class. Learners' lack of readiness to learn trig is evident in the mean scores of Teacher 2's class of 0,97, in identifying similar triangles out of 3 scores; 0,28 for Pythagoras out of three scores; and 0,42 for adding and 1.03 for simplification of ratios out of four scores.

In keeping with the trends shown by Teacher 1's and Teacher 2's classes, Teacher 3's class shows lower mean scores than both teachers 1 and 2 in the areas fundamental to trig readiness. The specific data is as follows: 0,72 in identifying similar triangles out of three; 0,11 for Pythagoras out of three; 0,56 for adding and 0.67 for simplification of ratios out of four.

Overall, all three classes showed knowledge gaps in all five test measures, and more specifically, in similar triangles, Pythagoras and ratios. Teacher 1's class showed a better starting base in the fundamental knowledge of skills required for trig readiness, and her class is followed by Teacher 2's class with Teacher 3's class showing the lowest starting base. All three teachers' classes have low mean scores of less than a third in the pre-test. The low scores suggested knowledge gaps and a low starting base for developing trig knowledge.

In the next section, I analyse the results of the post-test of trig readiness. By doing this, I want to determine whether learners in the three classes became ready to learn trigonometry by examining whether the scores improved in the post-test, as well as the significance of the

changes in scores. In addition, I want to determine, in cases where there was significant improvement, whether the improvement can be explained in the specific measures of trig that were tested, or in the different knowledge types. I will also consider the differences in the results of the three classes.

4.1.3 Post-test on trig readiness

Table 1 shows that all three teachers' classes improved their mean scores from pre-test to the post-test in trig readiness. The post-test shows statistically significant improved mean scores in overall trig readiness of the five measures. The specific scores are (9.31) for Teacher 1's class, (7.13) for Teacher 2's class and (6.67) for Teacher 3's class. These scores showed that learners were still not ready to learn trigonometry; Teacher 1's class showed just over a third mean score; and Teacher 2 and 3's classes still show mean scores of less than a third in overall trigonometry readiness.

The class with the highest improved score for trig concept readiness was Teacher 1's class, showing a paired difference of 2,507 and statistical significance in the t-test.[6] The statistical significant mean improvements in identifying similar triangles (0.627), Pythagoras theorem (0,388) and fractions (0,537) influenced the score. The significant improvement[7] in identifying similar triangles and the Pythagoras theorem showed improvement in two of the three areas fundamental to trig readiness.[8]

Teacher 2's class also showed statistically significant improvement in overall trig readiness showing a paired difference of 1,500. Teachers 2's classes' improvement is the lowest improvement of the three classes, but is statistically significant.[9] The class showed significant improvement in both measures of fractions; multiplication and division (0,417) and adding (0,722). None of these significant improvements were in the areas that were fundamental to

[6] In this case, the interval from pre- to post-test ranged from 1,644 to 3.371, indicating the 95% confidence level.

[7] Teacher 1's class shows average improvement in measures such as angles measuring (0,239) and identifying 180-degree angles (0,060), surds (0,224) and ratios, adding (0,328) and simplification (0,179). These average improvements were not statistically significant.

[8] Furthermore, the overall trig readiness results for Teacher 1's class of 3,539 show that there is a disbursement of results away from the mean that may result in high variation in scores that affect the overall mean results.[8] The standard deviation for identifying similar triangles (1,347), adding ratios (1,133) and simplifying ratios (1,242) contributed to an overall standard deviation of 3,539 and this variation influenced the overall mean score.[8]

[9] Teacher 2's class also improved on measures of triangles; drawing (0,125), identifying similar triangles (0,111) and theorem of Pythagoras (0,069), surds (0,042) and adding ratios (0,083). However, these were average improvements but one cannot be 95% confident that this average improvement was not due to variation within the data.

trig.[10] Teacher 3's class showed statistically significant improved results[11] in overall trig readiness. The class did not show significant improvement in the areas that were fundamental to trig.

Overall, all three classes showed statistically significant improvement in overall trig readiness of the five constructs. Teacher 1's class showed the highest improvement, followed by Teacher 2's class, and the least improvement being shown by Teacher 3's class. Teacher 1's class was the only teacher that showed significant improvement in identifying similar triangles and Pythagoras theorem, two of the three areas that are fundamental to trigonometry readiness. The other two classes did not show significant improvement in any of the three main areas that are fundamental to learning trigonometry. All three teachers' classes, however, are still not ready to learn trigonometry based on the post-test results.

4.1.4 Overall Comment on the pre- and post-test

The pre-test results on trig readiness show that all three teachers' classes had significant knowledge gaps in the main areas fundamental to understanding trig, namely: triangles, Pythagoras theorem and ratios, as evidenced in the low mean scores in Table 1 for trig readiness.

The post-test shows that all three classes showed statistically significant improvement in overall trig readiness. The post-test also shows that Teacher 1's class had a readiness score of just over a third of the requisite knowledge, while Teacher 2's class and Teacher 3's class still showed a mean score of less than a third for trig readiness. The low scores in trigonometry readines show that learners were still not ready to learn trigonometry despite the two video recorded lessons showing that the teachers taught them the topic. A closer look at the test measures shows that only Teacher 1's class improved significantly in two of the measures

[10] The mean score of the class was influenced by the standard deviations in identifying similar triangles (1,449), ratios simplification (1,245) identifying angles (1,186) ratios (adding) and both measures of fractions: adding (1,078) and multiplication and division (1,071). Teacher 2's class had more or less the same standard deviation as Teacher 1's class, an indication of some outliers of high and very low scores, which influenced the overall mean for trig readiness.

[11] Teacher 3 showed some average improvements in trig readiness with a paired difference of 2,333. This class showed average improvements in all measures, excluding angle measurement and adding ratio. None of the improvements were statistically significant, and it is not possible to be 95% confident that this average improvement was not due to variation within the data. The standard deviation of 2,521 shows that although there were some outliers, these are within three standard deviations, and this did not influence the mean scores.

(identifying similar triangles and Pythagoras theorem), which are fundamental to trig readiness. The other two teachers classes did not show statistically significant improvements in any of the three measures. These two teachers' classes (of teachers 2 and 3) shows an average improvement in the two areas - congruent triangles and Pythagoras, and showed no improvement in ratios. However, one cannot be 95% confident that variation in the data did not cause the average improvement.This means that there may have been outliers, where some learners achieved very high results, while others achieved very low results.

Teacher 1's class showed interesting patterns from pre-to post-test. First, her learners' score started off better in the pre-test. Secondly, the class showed significant improvement and higher scores for overall trig readiness in the post-test. Thirdly, it is only her class that showed significant improvement in two (identifying similar triangles and Pythagoras theorem) of the three test measures fundamental to trig readiness.

Teacher 2's class patterns show a middle path to Teacher 1's class, with the former's learners starting off second best to Teacher 1's class in the pre-test. Teacher 2's class also showed statistically significant improvement in overall trig readiness, with the second highest scores, and average improvement (showing variation in data, not 95% confidence level) in similar triangles and Pythagoras.

Teacher 3's class has a pattern of the lowest scores on all accounts, compared to patterns highlighted for Teachers 1 and 2's classes. The classes patterns are specific to the starting scores in the pre-test, with the least score in overall trig readiness. However, the pattern shows a similar trend to Teacher 2's class average improvement (showing variation in data, not 95% confidence level) in similar triangles and Pythagoras.

The section to follow explores the above differences in the three teachers' class results and the different patterns, by looking at the pre- and post- diagnostic test results in relation to conceptual and procedural knowledge. In particular, it will be examined whether there are specific explanations for the results of the two knowledge types that could explain the pattern described above. The pattern includes an explanation of Teacher 1's class significantly improved results in similar triangles and Pythagoras and the three teachers'classes significant improvements in overall trig readiness. I also want to see how the types of mathematical knowledge help explain knowledge gaps, as well as learners' trig readiness.

4.1.5 Pre-and post-diagnostic test of Grade 10 learners' conceptual and procedural knowledge

Table 7 below compares the learner pre- and post-diagnostic assessment results for the two types of mathematical knowledge: procedural knowledge and conceptual knowledge.These knowledge types were built in as part of the diagnostic test design. They represent the mean scores paired differences, standard deviation and statistical significance for both the pre- and post-test.

Table 7: Statistical analysis of three teachers' Grade 10 classes pre- and post-diagnostic assessment for Procedural Knowledge (PK) and conceptual knowledge (CK)

Teacher 1						**Teacher 2**					**Teacher 3**				
Number of learners:67						Number of learners: 72					Number of learners: 18				
Types of knowledge	Pre-test	Post-test	Paired Diff-erences	Std Devi-ation	Signifi-cance (2-tailed)	Pre-test	Post-test	Paired Diff-erences	Std Devi-ation	Signifi-cance (2-tailed)	Pre-test	Post-test	Paired Diff-erences	Std Devi-ation	Significance (2tailed)
Procedural knowledge	5,00	6,37	1,373	2.870	.000	4,08	5,24	1,153	2.958	.001	3,28	5,11	1,833	2.479	.006
Conceptual Knowledge	1,81	2,97	1,164	1.274	.000	1,54	1,86	0,319	1.555	.086	1,06	1,56	0,500	1.150	.083

The full score for procedural knowledge was 16, and for conceptual knowledge was 9.

The pre-test results of the three classes indicated a low starting base for procedural knowledge of (5,00) for Teacher 1's class, (4,08) for Teacher 2's class and (3,28) for Teacher 3's class. The results were less than a third (5,33) of the requisite procedural knowledge for trig readiness. The low result in procedural knowledge means that learners did not possess adequate knowledge of the steps and actions required to solve a mathematical algorithm in the fundamental measures of trig readiness. The results for conceptual knowledge also showed a low starting base of less than a third (3) for the three teachers' classes. Table 3 shows that the mean scores for conceptual knowledge for all three classes, were much lower than their procedural knowledge. Overall, the pattern showed that Teacher 1's class starts with better scores in procedural and conceptual knowledge, followed by Teacher 2's class, with middle scores, while Teacher 3's class earned the lowest scores in the two knowledge types.

The post-test results show statistically significant improvement in procedural knowledge and conceptual knowledge for Teacher 1's class. The significance is evident in the Significance value (two-tailed) of (.000) for the two knowledge types for Teacher 1's class. Teacher 2's class shows statistically significant improvement in procedural knowledge only.[12] Teacher 3's class did not show statistically significant improvement in either procedural or conceptual knowledge.[13]

Overall, it appears from the data that Teacher 1 and 2's classes showed significant improvement in procedural knowledge, and the type of knowledge contributing mostly to their overall trig readiness scores. The significant improvement in fractions in Teacher 2's class results may have contributed to the significant improvement in procedural knowledge, and even though fractions is not classified as fundamental to trig readiness, it is an important form of prior knowledge.

The data also showed that Teacher 1's learners' improvement in the two areas fundamental to trig (similar triangles and Pythagoras) contributed to the significant improvements in the learners' conceptual and procedural knowledge. The overall result is the three classes showed improved mean scores for procedural knowledge of (6.37) for Teacher 1's class, (5.24) for Teacher 2's class and (5.11) for Teacher 3's class. Teacher 1's class score is just above a third for procedural knowledge, and Teacher 2 and 3's classes are less than a third. The scores showed that learners still don't possess the pre-requisite procedural knowledge for trig readiness.

[12] Teacher 2 showed average improvement in conceptual knowledge, but there cannot be 95% confidence that this was not due to variation in the data.
[13] Teacher 3 showed average improvement in both knowledge types, but there cannot be 95% confidence that this was not due to variation in the data.

All three teachers' classes have less than a third of the pre-requisite conceptual knowledge to learn trig. Overall, the three teacher's classes do not have the pre-requisite conceptual and procedural knowledge and are not trig ready, even after the two videotape lessons.

Teacher 1's class seems to be doing better than Teacher 2 and 3's classes, in terms of the fundamental measures to trig readiness and the two knowledge types: conceptual and procedural knowledge. Teacher 2's class shows middle attainment in these areas, and Teacher 3's class lags behind both Teacher 1 and 2's classes in trigonometry readiness and conceptual and procedural knowledge.

4.1.6 Summary

I have shown through the analysis of the results of the pre-diagnostic test that learners have knowledge gaps and are not ready to learn trigonometry. The analysis showed gaps in all five measures of trig readiness, and in specific areas fundamental to trig readiness, such as similar triangles, Pythagoras and ratios. I show the differences in the results among the three classes in relation to their knowledge gaps and trig readiness.

In the post-test, I have shown that the three teachers' classes are still not ready to learn trigonometry. The test results of the three classes also show significant improvement in overall trig readiness. In the specific test measures that are fundamental to trig, only Teacher 1's class results show statistically significant improvement in similar triangles and the theorem of Pythagoras. Teacher 2's class results show significant improvement in fractions, which is important prior knowledge, but not fundamental to trig readiness. Teacher 3's class results did not show significant improvement in the three areas that are fundamental to trig.

In relation to the pre-test results on the types of mathematical knowledge, this study shows that all three classes do not have adequate conceptual and procedural knowledge to learn trigonometry. The post-test shows that the results of Teacher 1 and 2's classes show significant improvement in procedural knowledge, and that this type of knowledge contributed the most to overall trig readiness. Teacher 3's class did not show significant improvement. The study also shows that only Teacher 1's class improved significantly in both conceptual and procedural knowledge. Teacher 2 and 3's classes did not improve significantly in conceptual knowledge. Furthermore, the test measures that contributed to significant improvement seemed to be similar triangles and Pythagoras, in the case of Teacher 1's class as well as in the case of Teacher 2's class, fractions contributed significantly to the improvement in procedural knowledge.

It can be seen from the results that procedural knowledge contributed mostly to the gains in trig concept readiness. The contribution evident in the higher mean scores in the post-test for all three classes, as well as the statistically significant improvements in two of the three classes for procedural knowledge. It is possible that there are improvements in areas that are more focused on procedural knowledge than those requiring conceptual knowledge, given that it is easier to build procedural knowledge. It is not always possible to separate conceptual knowledge from procedural knowledge, as the two stand in relation to each other. To explain my claim, conceptual knowledge is a web of connected concepts that form relationships with each other, where measures of conceptual knowledge are more varied (Star, 2005). Procedural knowledge does not only indicate what is known, but is also one way that procedures (algorithms) can be known (p. 408). It is also evident in the lower mean scores of conceptual knowledge for all three teachers' classes in the post-test, and the fact that only Teacher 1's class showed statistically significant improvements in this type of knowledge.

I have also shown that Teacher 1's class had a better start than Teacher 2 and 3's classes. There is a pattern of Teacher 1's class doing better, Teacher 2's class being in the middle of attainment, and Teacher 3's class showing the lowest attainment levels among the three teachers.

In the next section, I wanted to see if the improvements had anything to do with the use of the LTSM. I also wanted to see if the differences between the results of the three teachers had to do with the way they used the LTSM, and whether the examples they used contributed to pre-requisite and new mathematical knowledge. The analysis of the LTSM is important, because the way teachers use the LTSM to mediate learner's knowledge gaps were the main component of my research questions.

4.2 The use of the LTSM

This section examines data from the videotaped classroom observations to see what teachers used in the LTSM, how frequent they used it, the different ways and purposes for which they used it. The previous section on testing showed differences in the results between Teacher 1's class and the other two teachers' classes, and I want to explain these differences by looking at the teachers' use of LTSM. I also give insight into the three teachers' perspectives of the LTSM from the interviews.

The teachers received brief orientation on the LTSM and the approach it took. The design of the LTSM embedded a constructivist approach and provided the three teachers with notes and

exercises. The purpose of the LTSM was to give teachers a resource to use together with the results of the diagnostic tests to help fill learners knowledge gaps. Due to this, the use of the LTSM included both group work and individual work. The intended purpose of the LTSM was also for teachers to choose exercises based on different levels of difficulty. The difficulty levels of the exercises were designed to match with the learners' level of ability. If the learner mastered the easy exercises, the teacher would let the learner move to more difficult exercises, requiring continuous scaffolding from easy levels to difficult ones.

One should notice the difference in the use of the LTSM by the three teachers. Overall, the three teachers used the LTSM in only one of the two lessons. Teacher 1 and Teacher 2 used it to fill gaps, by teaching the lesson on similar triangles and congruency, as this constituted prior knowledge to trig. Teacher 3 used the LTSM to introduce new mathematical content on trigonometry by teaching the history and background of the concept of trigonometry itself, and this gave context to the rest of the lesson.

All three teachers taught the same topics, namely: congruent triangles, similar triangles in Lesson One and Trig ratios in Lesson Two. The topics were discussed and agreed upon upfront between the three teachers and the researcher, for purposes of observing and comparing the three teachers' use of the LTSM and their instructional practices in mediating learners' knowledge gaps. The teachers could make choices about the exercises they use in the lesson, and determine how and when they used it if at all they wanted to use it to teach the topic. Research showed congruent triangles and similar triangles to be fundamental pre-requisite knowledge for understanding conceptual and procedural knowledge in trig (Zenex Foundation, 2011; Pournara, 2001; De Villiers & Jugmohan, 2012).

I move to describe each teacher's use of the LTSM by firstly looking at how frequent they used it in the two lessons, the examples they used from the LTSM, and whether they used it for the intended purposes. Secondly, I will compare the three teachers' use of the LTSM.

The table below shows the frequency of the three teachers' use of the LTSM and the number of exercises that they used.

Table 8: Frequency of teachers' use of LTSM

	Teacher 1		**Teacher 2**		**Teacher 3**	
	Frequency of LTSM use					
	Lesson 1 Congruent triangles	**Lesson2 Trig ratios**	**Lesson 1 Congruent triangles**	**Lesson2 Trig ratios**	**Lesson 1 Congruent triangles**	**Lesson2 Trig ratios**
Use of LTSM						
LTSM Use	1	0	1	0	0	1
Exercise modelled by teachers	3	1	3	7	1	6

Table 8 above shows that all three teachers only used the LTSM in one lesson, and not in both lessons. The exercises that were modelled by teachers in the table came from both the class textbook and from the LTSM. Teacher 1 and Teacher 2 used the LTSM in lesson one.

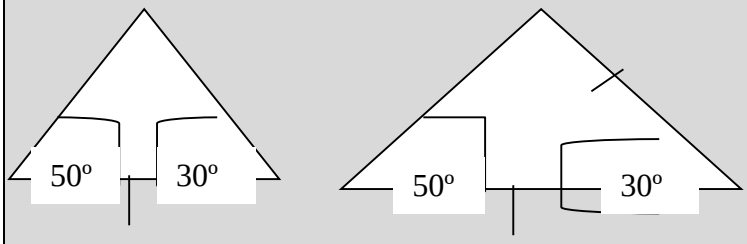
In response to the questions as to whether she used the LTSM, Teacher 1 indicated:

> *Yes, we did. I used especially the one where they had to measure the sides of a triangle; although I didn't have enough time to do all of that in class, but I have given it to them to do, just to practice at home. And one where they had to draw a triangle using the size of the angles, and constructing the triangles – I also gave them that one.* [sic]
> (T: 1 post-lesson Interview, 15 August 2014)

It is important to note that, during the interview, Teacher 1 explained that she had misplaced the LTSM (during a period of hospitalisation), and thus did not use it at the time of recording lesson two on trigonometry. However, the teacher used the LTSM in a class exercise and gave learners worksheets to finish the exercises at home during lesson 1. This was confirmed in the lesson observations on congruence and similar triangles i.e. in lesson 1.

During Lesson 1, Teacher 1 modelled three examples of congruent triangles and provided learners with worksheets from the LTSM to identify similar triangles. I observed that she had read the teacher notes from the LTSM for understanding mathematical concepts. The teacher displayed her reading of the notes from the LTSM by giving examples of congruent and similar triangles. Below is an illustration of the example given by Teacher 1:

[Teacher 1 draws two angles on the board]



Teacher 1: "*You all remember that from Grade 9*?"

Learners: "*Yes*."

Teacher 1: **"***When you are looking at those two triangles, are they congruent? Who said they are congruent by a show hands? None of you say they are congruent? None of you say they are congruent? So they are not congruent? Why do we say they are not congruent? Uh-uh, I cannot even hear a word that you are uttering from your mouths? Is your hand up? Yes, I am listening Bongani*." [sic]

[Learner responds]

Teacher 1: *"Bongi says they are not congruent because… they have the same angles... the angles are the same. We have a 30 degree angle and a 30 degree angle. But what about the size? Although they are equal, they are not in proportion. So those two triangles are not congruent. Do you get that?"* [sic]

(Teacher 1, video lesson on congruent triangles, 1 August 2014)

The teacher used the example above to build conceptual knowledge, specifically to build learners' understanding of the properties of congruent triangles. She did this by explaining to learners how to prove that a triangle is not congruent. The example was used for the intended purpose. The teacher did not make any adaptation or revision to the LTSM. The example dealt

with the knowledge gaps identified in the diagnostic test, specifically to identify similar and congruent triangles through knowing their properties.

She also gave the characteristics of similar and congruent triangles through definitions and explaining properties as discussed in the teacher notes. This is an example of one of her statements:

> "*So I gave you… I think once* in a lifetime *I gave you an example of the difference between your similar triangles and your congruent triangles*." [sic]
>
> (Teacher 1, Video lesson on congruent triangles, 1 August 2014)

She went on to explain an everyday example about twins: Teacher 1 embeds the mathematical concept of congruence in an everyday example of identical twins, and she explains it as follows:

> "*When we talk about congruency we are talking about triangles. They are twins, but they are not the same. They will have the same genes, but they are not the same. When we are talking about identical twins, their eyebrows, their* [eye]*lashes, everything about them is the same. They are identical. They are exactly the same.* [These two triangle's] *angles are the same; their sides are the same*." [sic]
>
> (Teacher 1, Video lesson on congruent triangles, 1 August 2014)

The analogy of twins was not covered in the LTSM showing that Teacher 1 adapted the LTSM. In this example, Teacher 1 used this in an abstract explanation of twins to show that congruence is defined as identical measures between two people or objects. Teacher 1's explanation, of congruent triangles being identical, is correct.

The teacher's use of an everyday example helped to build conceptual knowledge. This was given that she used the idea of twins (being an everyday concept with which learners could identify and of which they may have knowledge) to explain the concept of congruent triangles being identical. The teacher's correct explanation of congruent triangles and her use of the everyday example to build conceptual knowledge was the intended purpose of the LTSM. The use does deal with the knowledge gaps in conceptual knowledge as shown in the pre-diagnostic test results.

During Lesson 1 (on similar and congruent triangles), Teacher 1 gave learners handouts to do classwork on their own after explaining the concepts of congruent and similar triangles. Given

the time constraints of the lesson, she told learners to complete the exercises from the LTSM as their homework.

> "*Listen Grade 10, I have a hand-out for you. It is based on congruency, okay? I have labelled the triangles triangle 1, 2, 3 up to triangle eight*." [sic]
>
> (Teacher 1, Video lesson on congruent triangles, 1 August 2014)

The example shows that Teacher 1's class gave individual opportunities to do the exercises in class and at home. The researcher did not follow upon the homework due to time constraints, however this also did not form part of the study. Unfortunately, the study did not procure data on how the teacher scaffold's this work once learners have completed the class and homework tasks. It is therefore not possible to conclude whether the individual work helped to fill the learners' knowledge gaps identified in the pre-diagnostic test.

Teacher 2 used the LTSM during the lesson on similar and congruent triangles, and he used it together with the classroom mathematics textbook Pike et al. (2011). I questioned Teacher 2 on his reasons for not using the LTSM for the trig lesson. He responded in a letter as follows:

> *About the resources you suggested, I decided to design my own worksheet because of the large numbers of learners in the class. I felt it would take a long time to use your material. The worksheet used was based on the topic that has been covered in the trigonometry.* [sic]
> (Teacher 2, via email, 10 November, 2014)

Teacher 2 modelled another example on the board, asking different types of questions discussed on pages 110-119 (closed questions and questions that assessed learner understanding). The example he modelled was not from the LTSM, but from the class textbook.

Teacher 2 gave learners one handout from the LTSM for classwork. He used the LTSM to build conceptual knowledge by eliciting learners' reasoning and asking them questions. For example, he scaffolded the example from the LTSM as follows. He started by giving learners an instruction:

"*Right, this is what I want us to do. I want us to look at the worksheet I have given to you. I want us to look at the figures, study these figures and then determine which pairs of triangles are the same. And you should have reasons as to why you are saying the pairs of triangles are the same*." [sic]

The teacher asked the following questions from the LTSM.

"*Right. Can I then get one learner to raise her hand or his hand and tell me which triangles they found to be the same? Yes, sisi*?" [sic]

Learner responds: "*Number five and number six (meaning triangle number five and six)*."

Teacher checks answer by asking; "*Number five and six?"* Learner changes her mind: "*Seven*."

Teacher checks: "*Five and Seven*?"

Learner responds: "*Yes*."

Teacher: "*Right. Why do you say these triangles are the same*?" [sic]

The question recruits the learner's reasoning or understanding of congruent triangles. The learner responds correctly, although the response was inaudible, the teacher repeats the learner's response: "*Okay. You mean between the two triangles, triangle 5 and triangle 7, you found that the hypotenuse side of the two triangles is the same? Is the same length? And you also found the two other sides are of the same length? Okay*." [sic]

(Teacher 2, Video lesson on congruent triangles, 1 August 2014)

In giving the instruction, the teacher clearly sets the context for learners' reasoning when they set out to compare the triangles. He gives learners the opportunity to identify the triangles that are similar and then elicits a response from one learner: the teacher asked the learner to provide reasons for her answer. This is a conceptual question. In order for the learner to give the reason, the learner had to understand the concept of similar triangles and its properties, which is also conceptual knowledge. The teacher also follows up with questions to help learners think about the properties in order to explain the answer.

In this lesson, the Teacher 2 did not only use exercises from the LTSM, but he also used exercises from the textbook. The quote below shows that the teacher gave homework exercises for learners to do.

Homework exercise from the textbook (Classroom Mathematics): "*Right, for your homework, in your books, please do Exercise 9.3, Exercise 2. Do all of the exercises*." [sic]

Teacher 2 uses the example from the LTSM and did not adapt the actual example. The use of the LTSM, together with the classroom textbook, showed that Teacher 2 made choices about when to use and for what purpose he used the LTSM. For example, Teacher 2 used the textbook to model examples and used the LTSM to build conceptual knowledge, specifically by eliciting learners understanding of congruent triangles. This was the intended purpose of the LTSM, i.e. to fill the gaps identified in conceptual knowledge, and to use the LTSM with existing resources.

Teacher 3 did not use the LTSM for the lesson on congruent triangles (Lesson One). In a post-lesson interview, Teacher 3 explained that he left the LTSM at home. Teacher 3's response to the use of the LTSM was:

> *Remember in your lesson [referring to the LTSM] the practical example you did of a tree and calculate the height, now driving that concept because when it comes in the exam they use those things – not to say calculate normally, but calculate the side of the house, height of a cliff and so forth*. [sic]
>
> (T3: post-lesson interview, 20 August, 2014)

In the quote above, the teacher refers to the use of trigonometry in real life situations. The teacher used the above example of the tree in the lesson to embed trigonometry into a context. He did this to introduce the topic, and provided learners with the history of trigonometry.

It can be noted that Teacher 3 used examples from his memory to explain the lesson. Teacher 3 worked through at least three examples from the LTSM in the class comparing triangles and proving congruence of triangles with learners in the class, by asking learners questions. The lesson is mediated mainly through teacher talk and question, with examples from his memory of planning from the LTSM as follows:

Examples from Teacher 3's memory:

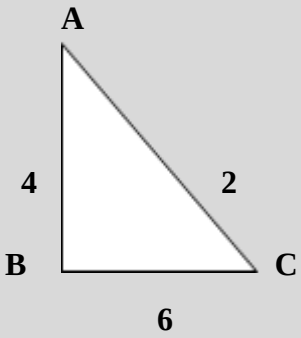
[Teacher 3 drew two similar triangles on the board ΔABC and ΔDEF]



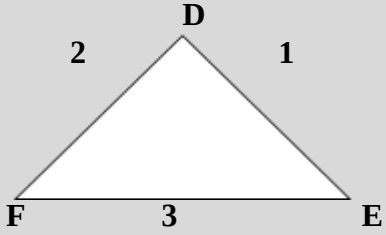
The teacher explained the concept of similar triangles as follows:

"*Now if you look at these triangles, they are a little bit the same. Now when they look alike or seem to be the same we say they are: Teacher 3 (writes on the board) similar triangles* Δ. *"Now to be similar it means they have, we check the ratio of the triangles; their ratio must be equal to be similar, ok. The shape, the angles inside, the angles might not be exactly the same, but that ratio is very important because that carries the whole thing*." [sic]

Teacher 3: "*If for instance…*"



Teacher 3: "*This one is four centimetres and this one is six centimetres and this one is two centimetres, so if I come to this…*"



Teacher 3: *"I find that this one is three centimetres, this one is two centimetres and this one is one centimetres, ok. Now we try to identify now the sides which look the same, sides have got*

the names there, we have got from that one, ab, ac, df, de, which side do you think is more or less the same as that one? Pointing by Comparing Δ ABC and Δ DEF sides or the side which is in here that is more or less the same as that one?" [sic]

Learner: "*ac and df*."

Teacher 3 Writes on board: **'**AC is equal to DF?'

Teacher 3: "*Yes. Hey, is it?*" [sic]

Learner: "*Yes*."

Teacher 3: "*I know why he is saying that, it's because I used this. Yes, that is why he is saying DF and AC, that is correct, I accept it. But it is my mistake as I look at it. That can be my mistake. mmmm. what I was supposed to be, grrrr! Teacher 3 pointed to side DE. Okay, let me change this, if I have one there (side DE), I have two here, let me put three here (side DF) and take this as four. But to be more accurate, I just take it from mine. What I wanted to explain with you is that – if we want to see whether the two triangles are similar, so according to the ratio and here it does not work, it does not work, it does not work easily. No.*" [sic]

[Teacher 3 erases from the board]

Teacher 3: "*We have this one as three, two, one. (Δ DEF: Side DE 2, DF1, EF 3) Let us try that, if we look at that one. DE is equal to AC, what is AC?*" [sic]

Learner: "*Four*."

Teacher 3: "*What is DE*?"

Learner: "*Two*."

Teacher 3: "*The ratio is AC over DE, isn't it?*"

Learner: "*Yes.*"

Teacher Writes on board: 'AC/DE = 4/2'

Teacher 3: *"...and we are getting?"* [sic]

= 2.

Teacher3: "*Two into four is two*." [sic]

Learners: "*Yes*."

Teacher 3: "*My example is not quite correct. Ok. Let us leave those examples out*." [sic]

(Teacher 3, Video lesson 1 on similar and congruent triangles, 20 May, 2014)

The discussion that follows is aimed at analysing the difference between what the teacher did in comparison to what is indicated in the LTSM.

The two triangles drawn on the board do not have the appearance of similar triangles, as one is drawn with an angle that is close to 90 degrees, and the other is not. Hence, the drawings would confuse a learner. The LTSM gave worksheets with triangles in various sizes and sides with different proportions that the teacher could have copied and handed to learners to engage them in measuring the angles.

The teachers' explanation of the concept is not clear, and simple everyday language is used. For example: of the same kind instead of saying they are similar, "they are a little bit the same, now when they look alike or seem to be the same we say they are similar. And it is important, because that carries the whole thing" [sic]. (Teacher 3, Video lesson 1 on similar and Congruent triangles. "They are a little bit the same" is not clear or indeed mathematically correct language.

The LTSM stated that congruent triangles (or any other shape) have the same shape (i.e. the same sized angles), but not the same size. Their corresponding (matching in position) sides are in proportion (have the same ratio). "The shape, the angles inside them might not be exactly the same" is in fact incorrect, because the angles will indeed be the same.

The LTSM show that if you do not know the sizes of the internal angles of two triangles, you can show that the triangles are similar by measuring the lengths of the three sides of each. If you then compare the ratio of the matching three sides, and these are the same, you can say that triangle ABC is similar to triangle DEF and this will enable you to prove that the triangles are similar.

Congruency is a different concept and shows that two shapes are identical. The LTSM encouraged teachers to use the worksheets with different triangles, which have different length sides, but the same sized angles, to get learners to measure the lengths of the three sides of each. Learners could then use their calculators to work out the ratios of the matching sides. This would focus on an approach that engages learners, using a social constructivist approach, rather than a 'talk and chalk' method.

The teacher used the textbook to give learners exercises on congruent triangles, and similar triangles, which was part of the content of the lesson. He also gave class exercises from the class textbook, as evidenced by his instruction to learners as follows:

"*Let us take our books and see what we can do from the books. Open page 113, starting on page 112. From page 112. If we look at our number one there…*" [sic].

(Teacher 3, Video lesson on trigonometry, 23 May, 2014)

I observed during lesson two that Teacher 3 used a reading for purpose exercise from the LTSM, which was a skills development exercise, and was not meant for the teacher to teach the whole class in this way alongside this example. The exercise was for learners to read individually, and intended for learners to gain background knowledge of trigonometry and to introduce right angle triangles. Teacher 3 drew a tree and depicted a right angle triangle on the tree between the trunk and the ground. He modeled a trigonometric algorithm on the basis of the embedded context to show how one can solve the right angle. Teacher 3 used the exercise from the LTSM to introduce and give context and to explain the concept as follows:

"*Right, what we do then, we will be using the technique from trigonometry. This technique was first used in Greece by the Greeks, and it was also used by the Asians. So many people out there, who are well covered as far as mathematics is concerned, so those people, they help us a lot, in saying for instance if we are working with right-angled triangles, now suppose you have a tree here, let me start if from there*" [sic].

(Teacher 3, Video lesson on trigonometry, 23 May 2014).

What follows is a screenshot of the video clip of the example of what this looked like on the board:



Teacher 3 did not adapt the example from the LTSM that he used in the class. The teacher's use of the LTSM to teach the whole class new trigonometry content, did not correlate with the

intended purpose of the LTSM. This may have been the teacher's purpose in the lesson, but the LTSM was a skills exercise, aimed at reading with purpose, so as to give context to the concept of trigonometry. However, Teacher 3 used the LTSM to build an understanding of right-angled triangles and trig ratios, which is new Grade 10 knowledge, although the tests showed that the learners were not ready to learn trigonometry.

4.2.1 Summary

The examples discussed above show that the teachers used the LTSM differently. Teacher 1 mainly taught conceptual knowledge when she used the LTSM, and Teacher 3 used everyday knowledge from the LTSM. Teacher 2 used the examples to build the learners' understanding i.e. their conceptual knowledge. Teacher 1 and 2 used the LTSM, for the intended purposes to some extent, given that they also did not vary the exercises between easy and difficult. They made choices about when and for what purposes they used the LTSM. Teacher 3 did not use the LTSM for the intended purposes. The three teachers used the same examples from the LTSM to teach the whole class, and did not use the examples to differentiate levels of difficulty based on the learners' knowledge gaps that needed to be addressed. Teacher 3 also did not use the example in group work and individual work.

The teachers indicated their enlightenment with the results of the diagnostic test, and they appreciated that it made them aware that their learners had knowledge gaps. The three teachers said that they wanted to use the LTSM, which includes the test, in future. The three teachers did use the LTSM to help with filling knowledge gaps by teaching the mathematical content on congruent and similar triangles fundamental to understanding trigonometry and by teaching new knowledge. However, this was done only to a limited extent. It could be the case that the limited induction on the LTSM and by showing teachers the results of the diagnostic assessment, the teachers may have been aware of the types of knowledge (conceptual and procedural), and that therefore, they focused on it in their teaching.

The teachers have other pressures, such as time constraints, and the challenge of large class size, as Teacher 2 mentioned in the interviews. The question of whether teachers wanted to work this way depends on a number of classroom factors such as class size and time constraints. The question also depends on the teachers' understanding of their learners' knowledge. The teachers' knowledge of how learners learn and how to create opportunities through mediation for their learners to make good use of such learning opportunities, may require development if they are to do the work well.

In addition, at least two of the teachers also used their class textbooks to teach the two lessons.

The limited and varied use of the LTSM, and the choices made about when and for what purposes they used the LTSM, does not clearly explain whether the three teachers' use of the LTSM contributed to the significant gains of Teacher 1's class, and the achievement paths of Teacher 2 and 3's classes. For this reason, I proceed to analyse their teaching practices, so as to make sense of the improvements in overall trig readiness, and the specific improvements in similar triangles and Pythagoras.

4.3 Teacher use of the different types of knowledge in the classroom

The analysis of the three teachers' instructional practices will assist in explaining the significant improvement of conceptual and procedural knowledge of Teacher 1's class and procedural knowledge for Teacher 2's class in the post-test. I start the analysis by firstly looking at the frequency of each type of mathematical knowledge. I also give a description of the ways in which the teachers used conceptual, procedural and everyday knowledge. Secondly, I describe the teachers modelling and questioning practices, which I observed in the two lessons.

4.3. 1 Frequency of teachers' uses of different types of knowledge

The three knowledge types are conceptual, procedural and everyday knowledge, and these were followed in the design of the diagnostic test and the LTSM, and can also be observed in the teacher's use during the lesson. I used conceptual knowledge (CK) in two different ways. Conceptual knowledge (CK) referred firstly to instances when the teachers' explains concepts in their instructions through teacher talk, and secondly, when they ask learners questions and elicited explanations for their answers. Procedural knowledge (PK) was used in instances when teacher explained the sequence steps and actions involved in approaching a mathematical problem. Everyday knowledge (EK) refers to instances when the teacher embedded a concept in everyday context or used everyday illustrations. Here, I begin by quantifying procedural (PK) and everyday knowledge (EK) and comparing the three teachers' practices. Table 5 is a quantitative comparison of the similarities and differences between the three teachers' instructional practices when working with conceptual, procedural and everyday knowledge. The table shows the number of times each teacher used the different types of knowledge (CK, PK and EK) during the two lessons. It is worth noting that this does not speak to the quality of

the engagement with the different knowledge forms, but only to the frequency with which the teachers draw on the different types of knowledge. It is also important to note that some content areas taught by the teachers are more amenable to some types of knowledge, as shown in the chapter on methodology. Conceptual knowledge includes such knowledge as the ability to explain similar triangles, Pythagoras, surds and fractions. Measuring and constructing triangles, knowing properties of angles between parallel lines, fraction computations and the ability to identify surds, is by contrast, procedural knowledge.

Table 9: Frequency of three teachers' recruitment of conceptual knowledge, procedural knowledge and everyday knowledge

	Teacher 1			**Teacher 2**			**Teacher 3**		
Frequency of the use of the different types of knowledge									
	Lesson 1	**Lesson2**	**Total**	**Lesson 1**	**Lesson2**	**Total**	**Lesson 1**	**Lesson2**	**Total**
Conceptual knowledge	8	9	17	9	3	12	4	3	7
Procedural knowledge	8	5	13	9	19	28	7	18	25
Everyday Knowledge	1	0	1	0	0	0	1	1	2

From Table 9 above, it is evident that Teacher 1 worked with conceptual knowledge 17 times and used this type of knowledge in four instances, which was more than she used procedural knowledge. While Teacher 2 used procedural knowledge more frequently (28 times), he worked with this knowledge in 16 instances more than he used conceptual knowledge. Teacher 3 worked more times with procedural knowledge (25 times) and worked with conceptual knowledge only seven times. Teacher 3 used everyday knowledge twice, Teacher 1 used it once, and Teacher 2 did not use it all as a vehicle for building conceptual knowledge.

Overall, the three teachers show differing frequencies in their use of mathematical knowledge. The frequency of the use of conceptual and procedural knowledge shows an interesting pattern. For example, Teacher 1 used four instances more conceptual knowledge than procedural knowledge, whereas Teacher 2 uses 16 instances more procedural knowledge than conceptual knowledge; and Teacher 3 used 18 instances more procedural knowledge than conceptual knowledge. The pattern can therefore be described that two teachers used more procedural knowledge, and only one teacher used more conceptual knowledge.

In the next section, I analyse the manner in which teachers recruit conceptual knowledge, procedural, and everyday knowledge in the classroom.

4.4 Conceptual knowledge: Teacher eliciting Learner Reasoning

Learner reasoning abilities exemplify higher cognitive functions in mathematical knowledge. The section analyses teachers' recruitment of learner reasoning. The three teachers recruited this type of reasoning when they asked learners to provide reasons, or to explain their answers to questions on trig content. In the post- test, Teacher 1's class showed statistically significant improvement in conceptual knowledge, as shown in Table 7, page 76, while Teacher 2 and 3's classes did not show statistically significant improvement.

An example of a teacher recruiting conceptual knowledge was observed when Teacher 1 asked learners a question on the theory of Pythagoras that required reasoning:

> "*And then when we did the theory of i-Pythagoras… where did we use it? Why did we use it?"* [sic]
> (Teacher 1, Video lesson on trigonometry, 13 August 2014).

The question is eliciting pre-requisite knowledge for trig, specifically on right angle triangles, and attempts to determine whether learners can reason how the knowledge is used in trigonometry.

Teacher 2 recruits reasoning in the classroom during the first lesson on congruent triangles. He starts by giving learners an instruction:

> "*Right, this is what I want us to do. I want us to look at the worksheet I have given to you. I want us to look at the figures, study these figures and then determine which pairs of triangles are the same. And you should have reasons as to why you are saying the pairs of triangles are the same*." [sic]
>
> (Teacher 2, Video lesson 1 on congruent triangles, 1 August 2014)

In giving the instruction, the teacher clearly sets the context for learners' reasoning when they compare the triangles. He gives learners the opportunity to identify the triangles that are similar and then elicits a response from one learner:

"*Right. Can I then get one learner to raise her hand or his hand and tell me which triangles they found to be the same? Yes, sisi?"* [sic].

Learner responds: "*Number five and number six [meaning triangles numbered five and six.*" [sic]

Teacher checks answer by asking; "*Number five and six?"*

Learner changes her mind: "*Seven*."

Teacher checks: "*Five and Seven?"*

Learner responds: "*Yes."*

Teacher: "*Right. Why do you say these triangles are the same?"* [sic]

The question elicits learners' reasoning or understanding of congruent triangles.

The learner responds correctly, although the response was inaudible, the teacher repeats the learners' response: "*Okay. You mean between the two triangles, triangle 5 and triangle 7, you found that the hypotenuse side of the two triangles is the same? Is the same length? And you also found the two other sides are of the same length? Okay*." [sic]

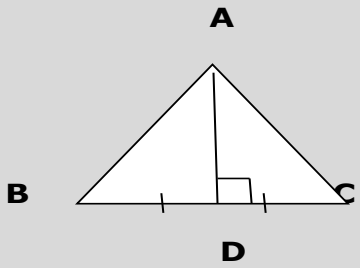
(Teacher 2, Video lesson on congruent triangles, 1 August 2014)

The response shows that the learner understands the conditions for congruency, specifically the sides of a triangle. Teacher 2 recruited reasoning eight times during the lesson on congruence.

Example 1: Teacher 3's teaching of conceptual knowledge

This shows the way in which Teacher 3 built conceptual knowledge in his normal instructional practice, specifically when he did not use the LTSM.

Congruent Δ



Teacher 3: "*This side also has an angle which is 90 degrees. Right?"* [sic]

[Learners' chorus]: "*Yes.*"

Teacher 3: "*Now here we… I would like to change on top there, we are looking at the congruent triangles, if I fold this paper here, if it was a paper, if I fold it it must lie on this exactly. Do you see that?*" [sic]

[Learners' chorus]: "*Yes*."

Teacher 3: "*Yes. This is the triangle, this is another triangle. If I fold them it must flap on this B exactly. So that means that they are of the same kind, they are equal in other words. But we don't say they are equal, we say they are?*" [sic]

[Learners' chorus]: "*Congruent.*"

Teacher 3: "*Ok, so there must be two triangles when we are checking for congruency. Let's look at D, there is a triangle a, b, d and triangle a, d, c. Do you see that?*" [sic]

[Learners' chorus]: "*Yes.*"

Teacher 3: "*So…*"

In Δ ABD and Δ ADC

Teacher3: "*So… we compare the two triangles now. Okay. In other words I have cut here and I have cut there (illustrates two angles left and right side), do you see that?*" [sic]

[Learners' chorus]: "*Yes.*"

Teacher 3: "*The triangle A, B, D and A, D, C. Alright*."

[Learners' chorus]: "*Yes*."

Teacher 3: "*Alright, now what can you say about line BD? As compared to this triangle, side bd. Yes, Nthombi?*" [sic]

[Nthombi response]: "*Yes.*"

Writes on board:

BD=DC=

Teacher 3: "*Your reason for that – (bisected by D) Shows Bisection on AD. Then it leaves two sides equal. You get me there? Alright.What other lines are equal as you look at them?"* [sic]

[Learner's chorus]: "*AD*."

Teacher 3: "*Hands up. Remember there is nothing we can say about AD*."
Laughs: "*I didn't say put your hands down*."
"*What can we say?"* [sic]

Teacher 3: "*It is AD. This triangle here (refers to Δ ABD) has got this side AD. In this triangle this side (refers to Δ ADC) has got AD, what is AD?*" [sic]

[Learner response]: "*AD =AD*"

Teacher 3: "*AD is the common side. Once you have found these two these other two fall into suit, and therefore: ΔABD≡Δ ADC (congruent to triangle ADC). Are we happy with that?"* [sic]

[Learner's chorus]: "*Yes*."

Teacher 3: "*Is it clear? Not yet? Not yet*." [sic]

[Learner's chorus]: "*Yes*."

Teacher 3: "*Yes. I don't have much exercises. Before I give you more exercises, there is something else I must give you here when we talk about congruent triangles*." [sic]

[Teacher 3 writes on board…]

'Conditions for congruency'

Teacher 3: "*Now if we talk about the conditions for congruence of triangles, They have to be congruent under certain conditions, like for instance – there is one. condition that is said to be side, side, side, or SSS, or let me start by side, side, side*." [sic]

Teacher 3: "*This means that all sides are equal in two triangles, do you get that?*" [sic]

[Learner's chorus]: "*Yes*."

Teacher3: "*For instance this one here, like this one. We proved that this side BD = AD equals this one. And this side is common. Therefore these two triangles are congruent, the condition of congruency is side, side, side (incorrect). Do you get that? This particular case is SAS, not SSS.*" [sic]

[Learner's chorus]: "*Yes*."

Teacher3: "*Yes. We have used sides. Another condition is…?*" [sic]

[Learner responds]: "*Side, angle, side."*

Teacher3: "*Yes, 2. (Side, angle, side) – SAS. What is another one?"*

[Learner responds]: "*Angle, side, angle (ASA)*."

Teacher3: "*Side, angle, angle is not an option. We can write it as angle angle side (AAS) or angle side angle (ASA). Okay?"* [sic]

[Learner's chorus]: "*Yes*."

Teacher3: "*Right. Therefore, what is the last one? Right angle, hypotenuse, side (RHS)*." [sic]

[Learner responds]: "*Right-angled triangle*."

(Teacher 3, video lesson on congruent triangles, 20 May 2014)

The above example shows Teacher 3's approach to be conceptually flawed, as he did not explain what congruency means, or confirm whether his class understands the nature of a hypotenuse. The teacher overlooked to tell learners that we name the sides according to the opposite angles, e.g. it is the side opposite to angle A, B is the side opposite to angle B, etc. "*Of the same kind*" is incorrect terminology. Triangle ADC superimposes exactly over triangle ADC, which means that they are exactly the same size and the same shape. This means that the triangles are congruent.

Chorus answers do not accurately inform the teacher about pupils understanding. Teacher 3's teaching that AB will be equal to AC is incorrect, as two sides of one triangle can measure the same as two sides of another triangle, but the third side can still be different in this case. It is *only* because the angle that is in between these equal sides is equal that the triangles will be congruent. The teacher's explanation about the condition of congruency as being side-side-side (SSS) is incorrect. This particular case is side-angle-side (SAS). Teacher 3 does not mention the fact that the side must be in a matching position for this condition - the condition is two angles with a matching side - so side-angle-angle (SAA) is correct if the side matches.

In the section above, the teachers' use of conceptual knowledge has been examined, and it showed instances when the teachers explained to learners, and when teachers involved learners, to elicit their reasoning. So far, teachers used conceptual knowledge in their pedagogy of teacher "talk" style instruction. Therefore, every time the teacher's used conceptual knowledge, there is demonstration of the pedagogical move Teacher "talk" style of instruction. Teacher 3's attempts at teaching conceptual knowledge were generally confusing and incorrect, and may have confused learners more than they assisted them in filling their knowledge gaps.

Below, I will describe the teachers' use of procedural knowledge.

Table 9 shows that Teacher 2 recruited procedural knowledge 28 times, while Teacher 3 used procedural knowledge 25 times, and Teacher 1 used it 13 times. Table 7 shows that Teacher 1 and 2's learner results showed statistical significant improvements in learners' procedural knowledge, while Teacher 3's learners did not.

Procedural knowledge refers to a teacher explaining the steps and actions that must be undertaken to solve a mathematical algorithm. The example in the box reflects Teacher 2's use of procedural knowledge in the classroom.

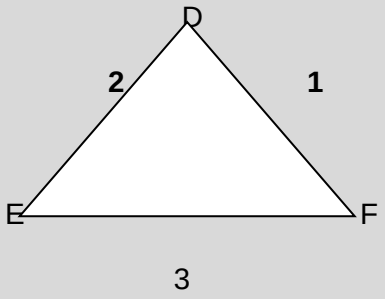
Procedural knowledge refers to sequenced steps and actions used to solve a mathematical problem (Canobi, 2009; Rittle-Johnson et al., 2001), which can also be modeled by the teacher.

"*Right, let me demonstrate how we can do this and then I would like you to calculate, if I do the angle that is* α*, you'll have to do* Θ*. Let me do* Θ*. Right, I want to find the size of angle* Θ*. Okay. And in this right- angled triangle that I have drawn, I have the three pieces of information that I need. I have my 90° angle, I have my length of the hypotenuse side, I have my length of the side that is BC. So again, how do I go about it? I use the information that is given to me. Right, to find angle* Θ*, I will check which two sides have been given the lengths of, then in relation to the position of this angle, I will then determine which trig ratio I would use. In relation to this angle, which is the unknown angle (* α*), I am given the side that is adjacent to this angle and the side that is the hypotenuse of this triangle. Okay. Let us quickly check. So this is, remember when you calculate the size of angles, we need to use the inverse function in our calculators. So you start by pressing shift if you've got a Casio™, your Sharp™, you press 2nd function and then press cos, the brackets will open up. Then you press that sign for your fraction. You put four in the numerator, you move the cursor to the denominator. You press your five then you move your cursor to the right, and then you press your bracket equals*." [sic]

In the example above, Teacher 2 explained the steps to determine the unknown angle of a right angle triangle in trigonometry. The teacher also gave a technical illustration of how to do the actual calculation using scientific calculators. The use of the calculator builds technical skills in procedures for calculating and solving algorithms, but does not build mathematical procedural knowledge.

Example 2: Teacher 3's Teaching of procedural knowledge

Teacher 3: "*Ok. Now, let us look at – like here, if I do a simple example now. Simple examples*." [sic]



Teacher 3: "*I find that this one is 3 cms, this one is 2 cms and this one is 1 cm, okay.Now we try to identify now the sides which look the same, sides have got the names there, we have got from that one, ab, ac, df, de, which side do you think is more or less the same as that one? Pointing by Comparing* Δ *ABC and* Δ *DEF sides*." [sic]

Teacher3: "*Or the side which is in here that is more or less the same as that one? AC and DF."*

[Writes on board: '*AC is equal to DF?*']

[Learners' chorus]: "*Yes*."

Teacher3: "*Hey. Is it?"*

[Learner responds]: "*Yes*."

Teacher3: "*I know why he is saying that, it's because I used this. Yes, that is why he is saying df and ac, that is correct, I accept it. But it is my mistake as I look at it. That can be my mistake. mmmm. What I was supposed to be… grrrr!, Teacher 3 pointed to side DE*."

Teacher3: "Ok let me change this, If I have one there (side DE), I have two here, let me put three here (side DF) and take this as four. But to be more accurate, I just take it from mine. What I wanted to explain with you is that – if we want to see whether the two triangles are similar, so according to the ratio and here it does not work, it does not work, it does not work easily. No."

[Erases from the board]

Teacher 3: "*We have this one as 3, 2, 1. (*Δ *DEF: Side DE 2, DF1, EF 3)*"
"*Let us try that, if we look at that one. DE is equal to AC, what is AC?*"

[Learners' chorus]: "Four."

Teacher 3: "*What is DE? Two*."

"The ratio is AC over DE. Teacher 3 writes on the board… *AC/DE*

isn't it?"

[Learners' chorus]: "Yes."

[Writes on board]

Teacher 3: "*Which equals…?"* [sic]

Teacher 3 writes on the board $\frac{AC}{DE} = \frac{4}{2}$

Teacher 3: "*…and we are getting…?"* [sic] = 2

Teacher 3: *"2 into 4 is 2"*

[Learners' chorus]: "*Yes*."

(Teacher 3, video lesson, 20 May 2014)

Teacher 3's explanation of procedural knowledge is confusing. For example, he says, "*if I have one there (side DE), I have two here, let me put three here (side DF) and take this as four. But to be more accurate, I just take it from mine. What I wanted to explain with you is that – if we want to see whether the two triangles are similar, so according to the ratio.*" [sic] Such an explanation can be extremely confusing to the learners. Teacher 3's attempt to teach that similar triangles have their matching sides in the same ratio should, rather, have started with an explanation that he is demonstrating how to calculate the sides of similar triangles. Following this introductory sentence he could then have started by revising ratio, and given a few examples of ratio. Then teacher 3 could go on to explain about proportion, when two ratios are equal e.g. $\frac{4}{8} = \frac{3}{6}$

In the section that follows I describe the teachers' use of everyday knowledge in the classroom.

"Everyday knowledge is based on context" (Bernstein, 1999, p. 158) and is generally contested and not universally accepted as mathematical or scientific knowledge. However, everyday knowledge has the potential of creating "epistemological access" Morrow 2007, p. 19) if used to build and make the transition to mathematical constructs. Teacher 1 and teacher 3 attempted to use everyday knowledge to build and make the transition to mathematical constructs in their limited use of this type of knowledge. Table 5 shows that Teacher 3 used everyday knowledge twice during the two lessons and Teacher 1 used it once. Teacher 2 did not use everyday

knowledge in both lessons. The teachers' use of everyday knowledge is very little compared with their use of mathematical knowledge. Research by Kazima & Adler (2006) showed that overall in South African classrooms, teachers use much more everyday knowledge than mathematical knowledge. The finding of the teachers' use of everyday knowledge in this study is in contrast to the evidence on the use of everyday knowledge in other studies.

The two teachers used everyday knowledge differently to explain congruence. The difference in explanation is that Teacher 1 used an abstract explanation of twins to show that congruence concerns identical measures between two people or objects. Teacher 1 embedded the mathematical concept of congruence in an everyday example of identical twins, and she explains it as follows:

> "*When we talk about congruency we are talking about triangles. They are twins, but they are not the same. They will have the same genes, but they are not the same. When we are talking about Identical twins, their eyebrows, their* [eye] *lashes, everything about them is the same. They are identical. They are exactly the same. "Their angles are the same; their sides are the same*." [sic]
>
> (Teacher 1, video lesson on congruent triangles, 1 August 2014)

Teacher 3 performed a practical illustration to provide a visual demonstration of similarity between the three girls' height. He explains to the class that if you measure the height of one of the girls, there will be no need to measure the height of the other two girls, as their heights would be similar. He used the illustration as a yardstick to introduce the topic of similar triangles.

Teacher 3 explained the concept of similar triangles as follows:

> "*Now, if you look at these triangles, they are a little bit the same, now when they look alike or seem to be the same we say they are :…* [writes on board] *similar triangles Δ, now to be similar it means they have, we check the ratio of the triangles, their ratio must be equal to be similar, okay, the shape, the angles inside, the angles might not be exactly the same but that ratio is very important, because that carries the whole thing.*" [sic]
>
> (Teacher 3, Video lesson: congruent triangles, 20 May, 2014)

Teacher 1's explanation asserting congruent triangles as identical, was correct. Teacher 3's explanation of similar triangles was correct where he noted that that they have a relationship, and the ratios of the sides are the same. However, he incorrectly suggested that the angles are not the same. The angles determine the measure of the proportion of the sides and any similar triangle's sides are in proportion. These everyday examples embed mathematical concepts. The

practical demonstration provides a real world context of everyday knowledge using the analogy of twins. Teacher 1's example shows good use of everyday knowledge, and she provided the correct definition of congruent triangles. Teacher 3's definition is not correct, and is somewhat confusing.

Overall, I showed that the three teachers used the different types of knowledge in differing frequencies. I have also shown that the teachers focused on different types of knowledge for different purposes, and that each teacher displayed a particular pattern in the post-test lesson, their use of LTSM and in the frequency of the different types of knowledge. For example, Teacher 1 used conceptual knowledge 8times to explain the concept of congruency using its properties and characteristics, while Teacher 2 used conceptual knowledge 12 times to define trig ratios and 9 times to draw on prior knowledge of similar triangles. Teacher 2 used procedural knowledge to demonstrate to learners how to follow the steps to determine the size of an unknown angle of a right-angled triangle in trigonometry. He also built learners' technical skills through demonstrating the procedures of using a calculator in order to solve the algorithm. Teacher 1 used everyday knowledge to explain congruence, and provided an abstract explanation to illustrate that twins are identical, while Teacher 3 used everyday knolwedge in a visual demonstration to teach similar triangles by using two girls to illustrate similarities in their height.

There is evidence of correlation between the frequency of use of types of knowledge with the improvement in the results of Teacher 1 in specific constructs and in the two types of knowledge; procedural and conceptual, and for Teacher 2 in procedural knowledge. Teacher 3's use of specific types of knowledge was poor, and it shows up in his class test results, which did not show significant improvement.

Summary of types of mathematical knowledge

When comparing Table 3 (diagnostic assessment results) and Table 5 (the frequency of use of different types of knowledge), a pattern emerges. The increase in the use of the type of knowledge correlates with the improved change in mean scores of the post-diagnostic test with respect to that specific knowledge type. Teacher 2 used procedural knowledge the most number of times (28), and the results of the diagnostic post-test shows significant improvement for Teacher 2's class from the pre- to post-test in procedural knowledge. Teacher 1 also showed statistically significant improvement in procedural knowledge. This improvement can be

attributed to the relationship between conceptual and procedural knowledge[14]. Teacher 3 showed a higher use of procedural knowledge than Teacher 1, but although the test results showed that there was an average improvement in procedural knowledge in his class results of the post-test, it is not significant in statistical terms.[15]

Only Teacher 1 used conceptual knowledge extensively, and her class showed statistically significant improvement in conceptual knowledge. I infer that the teachers' use of the different types of knowledge could have an influence on the gains in the post-diagnostic results, but this depends on the type of knowledge gaps that the learners have. The influence on the gains in scores also depends on the quality of the teachers' use of the different types of knowledge, investigated in this section.

The data suggests that in the instance of teacher 2, the learners' procedural knowledge developed in spite of having low levels of conceptual knowledge. Perhaps the particular areas of improvement were at a low level of cognitive demand for procedural knowledge for trig readiness, and were not too heavily dependent on conceptual knowledge. In addition, it is possible that the high frequency of use of procedural knowledge might explain the significant improvement in the post-test results in procedural knowledge for Teacher 1 and Teacher 2's classes. This means that the more procedural knowledge these two teachers used the better the learners results were in the post test.

Overall, the evidence from this study shows a relationship between the improved results on the post-test and the frequency of the teachers' use of conceptual and procedural knowledge. The relationship is evidenced in the high frequency of use of procedural knowledge by Teacher 2 and Teacher 3.

The section above examined the types of knowledge that the teachers recruited in their facilitation of learning processes. It explained the relationship between the three teachers' frequency of use and recruitment of the different types of knowledge in relation to the patterns that emerged among the three teachers. The patterns were that of the three teachers showing significant improvement in overall trig readiness in the five measures of trigonometry, where

[14] See Chapter 2, page 24

[15] Teacher 3's average improvement in procedural knowledge is not beyond the 95% confidence level beyond which one is certain that the improvement was not caused by variation in data. This may be explained by the lower starting base of knowledge of Teacher 3's class in the pre-test, or by the quality of teaching.

only Teacher 1's learners achieved significant improvement in two of the measures that are fundamental to trigonometry readiness. Teacher two's learners did not show significant improvement in the areas fundamental to trigonometry readiness, but showed improvement in fractions which is important, but not fundamental.

In the section below, I describe in detail the three teachers' pedagogical moves used to mediate learning. The teachers' pedagogy in mediating learning is of interest because it is also a core part of the current research question, which concerns whether there is a relationship between the teachers' instructional practices and the changes in performance in the diagnostic assessment that could explain the patterns shown by Teacher 1, 2 and 3, respectively.

4.5 Teacher's instructional practices in the classroom

This section will focus on the three teachers' instructional practices in the classroom, where I focus on two specific practices that I observed during the two lessons. The first refers to the teachers' modelling and the second to the teachers' questioning practices while using the LTSM to engage and assist learners with overcoming their knowledge gaps. Boaler (2001) refers to teachers instructional practices in the classroom as pedagogical moves' while Daniels (2001, p. 278) refers to mediation as a series of "scaffolded processes". I use 'pedagogical moves' to refer to the act of teaching and the moves that teachers make, while I use 'scaffolded process' to refer to when the teacher works with the learners to develop their understanding through leading and guiding them, using carefully selected steps and questions, based on the teachers understanding of how learners learn. I also want to infer whether the pre-diagnostic assessment informed the teachers' mediation of mathematical knowledge.

4.5.1 Modelling

Behaviourist theory of Learning assumes that modelling influences learning, and that observers acquire symbolic representations of modelled activities (Bandura, 1969, p. 8-9). The three teachers modelled examples of congruence and trigonometry content on the board. The modelled examples included examples of ratios and determined sides and unknown angles. Procedures for solving or calculating the solutions of the trigonometry examples were also modelled.

Teacher 2 modelled the most number of exercises (10) during the two lessons. Analysis is only provided using the example from Teacher 2, so as to avoid repeating examples already used

from Teacher 1 and Teacher 3, as they modeled few examples in their teaching. It must be noted that the modelled exercises from the LTSM during Lesson One. The examples in the trig lesson came from the *Classroom Mathematics* textbook.

Having discussed the teachers' use of LTSM, where it showed how he used the LTSM to model examples on the board, below I highlight an example of Teacher 2's modelling of a trigonometric equation that he gave learners for homework. Teacher 2 used modelling in the classroom pedagogy through homework exercises by checking the learners' answers first, which is then scaffolded through the example, using the question and answer technique. The teacher involved learners by eliciting their answers in each step of the calculations in the following way:

e) 2 x tanΘ = 1.

Teacher 2: "*Right, what size of an angle did you get for that sum*?" [sic]

Learner responds: "*26.6."*

26.6°?

Learner responds: "*Yes*."

Teacher 2: "*Right. This Θ here was 26.6°, but remember yesterday we said when we solved these equations; we need to do the following steps. We need to start by isolating the trig ratio. So in this case you must get rid of the co-efficient of this, or the two. And how do we get rid of the two? We divide both sides by two?*" [sic]

Yes

Teacher 2 : "*So we'll end up with a tanΘ = 1/2 and then our Θ will equal the tan of ½, which then will give us a Tan as, what did you say was the value of Θ?*" [sic]

Learner: "*6.6*."

Teacher 2: "*Right 26.6, because we were told to write the angle correct to one decimal place."*

The complete calculation written on the board is as follows:

e) 2 x tanΘ = 1

tanΘ = 1/2

Θ= tan(1/2)

Θ= 26.6°

In the example above, Teacher 2 used a combination of modelling and questioning as pedagogical moves used, as one attempt to scaffold trigonometry knowledge. The teacher used symbolic representations from trigonometry such as Θ, x, and = to create his representations. The example (e) above shows the complete symbolic representation of the answer as a way of modelling. He then recruited procedural knowledge by telling learners that in order to solve the equation, they need to start with the step of isolating the trig ratios. He carried on to explain how the isolation of the ratio should be done. The example also shows scaffolded processes used by Teacher 2.

Teacher 1 modelled, with the following example on the board:

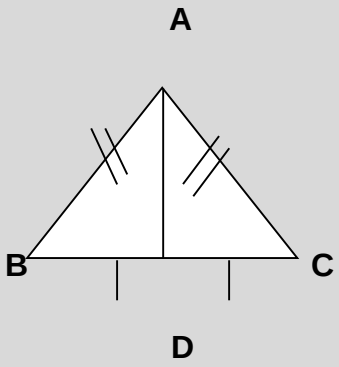
Teacher 1: "*Okay, let's do another example, but you guys will do this one*." [sic]

[writes on board**]**

Teacher 1: "*We must prove that triangle ABD and triangle ACD are congruent, angithi?* [*Not so*]*."* [sic]

Example written: Prove that

Δ ABD ≡Δ ACD Understand?



Teacher 1: "*In geometry we said in order for us to understand correctly what do we use? The table Is angithi?* [*Not so*] *yes? The statement and the reason column. So there is my table… statement…*" [sic]

[Teacher 1 writes on board]

Solution

Statement	**Reason**

Teacher 1: "*Okay, let's look at those two triangles. In order for us to see which triangle we are dealing with, I told you to use colour – you must always use colour – angithi* [not so?], *so you would be able to distinguish correctly which triangles we are talking about. We are talking about triangle A, B, D, angithi? The red one. The blue one will be... A, C, D. it is A, B, D (or) and A, C, D. So we must prove that those two triangles are congruent. So we must look between those two triangles if they have equal angles or they have equal sides? angithi? Understand?"* [sic]

Learner's Response: "*Yes."*

Teacher 1: "*When we are talking about congruency we consider the sides and we also consider ini (What)? (What)…* […]*angles. Okay. The first one. The first statement*." [sic]

Learner Response: "*Yes*."

Teacher 1: "*You can talk when you are writing. You must tell everybody why you are stating that statement. She must speak up. Don't just write, you must tell us what you are writing?"* [sic]

Teacher 1: "*Where is our white chalk? That one is not even clear? Hayibo!* [No] *I don't know!*

Statement	**Reason**
Ab=AC	Given
Ab=AC	Given

Siyavumelana?[understand?] And the second one…we said when we are proving we need three statements which goes with three reasons. And then at the end we… we conclude. Understand?" [sic]

Learner's Response: "Yes"

Teacher 1: "*So it must be three statements, three reasons and a conclusion – you get marks for those three statements and reasons, and you also get a mark for the conclusion because they are showing me that you know why you say that these two triangles are congruent and state the reason for the congruency. Okay*?" [sic]

Learner's Response: *"Yes"*

Teacher 1*:* "*The second triangle… False? No….please continue…"*

Statement	**Reason**
AB=AC	Given
BD=DC	Given

Learners respond: "*Why do you say that? We are given…* [Laughter]

You also want to do the last one too?" [sic] [Laughter]

Learner: "*I want them to repeat it. Okay. And the last one will be…"* [sic]

[Learners respond, chorus,]

Teacher writes learner response:

Statement	**Reason**
AB=AC	Given
BD=DC	Given
AD=AD	Common side

Teacher 1: "*Why do we say that? It is the common side. This side is common to both ABD and ACD.*" [sic]

Learner Response: "*Yes!"*

Teacher 1: "*So in conclusion, yes, we will say therefore, the triangle…*

Learners' chorus: *"ABD is congruent to ACD."*

Teacher 1: *And the reason is…"* [sic]

Learners in chorus: "*Side-side-side. Yes…*

Teacher 1: "*So do you understand that?"*

Teacher writes:

Δ ABD ≡Δ ACD

(Teacher 1, Video lesson on similar & congruent triangles, 1 August 2014)

In the example above, teacher 1 correctly models how to prove congruence to the learners. She also models the steps carefully using a scaffolding approach that helps learners to provide reasons for their answers.

Teacher 3's modelling is shown in example 2 on procedural knowledge on pages 98-99. The examples show that teacher 3 modelled incorrectly and were confusing.

The next section will consider how teachers recruit questioning as a pedagogical move to assist learners with knowledge gaps in their attempts to acquire mathematical (trig) content knowledge.

4.5.2 Teacher questioning practices

Questioning is one of the 'pedagogical moves' the three teachers used to garner learner involvement in the lesson. In questioning, teachers evaluate and check learner work, and understanding (Black and Williams, 1998; 2009). Here, comparative analysis is made of the three teachers' questions, dividing them into two cognitive levels: high cognitive and low cognitive questions. Cotton (1988) defines higher cognitive questions as those questions that require "students to mentally manipulate information" so as to respond or "support an answer with logically reasoned evidence" (p. 4). In Piaget's theory, the process of "accommodation" [16] is theorised to help to develop learners' understanding. This understanding enables them to conceive concept or abstract knowledge, in other words, higher cognitive functioning" (Furth, 1970, p. 25). Vygotsky (1978) refers to the ZPD. Higher cognitive development elucidates the distinction between 'actual' and 'potential' development[17]. "Lower Cognitive questions" have been referred to in the literature as 'fact', 'closed', 'direct', 'recall' and 'knowledge' questions (Cotton, 1988, p. 3).

Below, analysis is also made of the frequency of use of questions, and description provided of the pedagogical moves and scaffolding of four types of questioning, namely: 1) open-ended; 2) closed; 3) recall questions; and 4) questions that assess learning and understanding. Open-ended questions are those that require learners to draw on prior trig knowledge to solve trig problems, and which provide supporting logic and evidence of their knowledge. Closed questions have only one correct answer. Recall questions require learners to remember or retrieve mathematical knowledge with no logic or understanding. Questions that assessed learning and understanding were those used when teachers gave learners tasks to elicit learners' understanding, or when they asked questions that evaluated learning.

Numerous studies (Bloom, 1956; Brown & Wragg, 1993; Dillon,1988; 1994; Morgan & Saxton, 1991; Cotton, 1988) have provided guidelines on cognitive levels of questioning, and classroom practices that teachers could employ. Specifically in this study, teachers could draw on the guidelines of cognitive questioning to develop their questioning techniques. For

[16] See Chapter 2 page 35
[17] See Chapter 2 page 38

example, there are several conditions for effective learning that teachers could take into account. According to Bloom (1956), teachers ought to consider appropriately pitched questions that have been carefully and explicitly developed to achieve the learning outcome. Research by Dillon (1998), Morgan and Saxton (1991) and cotton (1998) has shown that: 1) teachers require knowledge of when, why and how to ask different types of questions; 2) when teachers use questioning pedagogy, it is important for them to develop an appreciation for and understand the thinking and learning process that learners undergo; 3) teachers' practices should take account of how to exploit a learner's response to drive learning through high-quality questioning techniques.

Table 10 : Comparison of the frequency of three teachers' questioning as a pedagogical move

	Teacher 1			**Teacher 2**			**Teacher 3**			
Frequency of different types of questioning										
	Lesson 1	**Lesson 2**	**Subtotal**	**Lesson 1**	**Lesson 2**	**Subtotal**	**Lesson 1**	**Lesson 2**	**Subtotal**	**Total**
LTSM Use	Yes	No		Yes	No		No	Yes		
Open questioning	2	15	17	4	8	12	3	8	11	40
Closed questioning	6	9	15	10	13	23	4	9	13	51
Recall Questions	4	3	7	1	2	3	0	2	2	12
Assessment of learning and understanding	2	4	6	0	11	11	0	6	6	23
Total Questioning	14	31	45	15	34	49	7	25	32	126

Table 10 shows that all three teachers mostly asked closed questions, and that Teacher 2 used questioning techniques the most (49) during the two lessons. Teacher 2 also used the most number of closed-ended questions (23) among the three teachers. Teacher 1 used closed questions 15 times, and Teacher 3 used this type of questioning 13 times. The second highest frequency is the use of open-ended questions, with Teacher 1 using this form the most (17 times), Teacher 2 12 times, and Teacher 3 using it 11 times. Teacher 2 and Teacher 3 used

recall questions the least, while Teacher 1 used this technique the most (7 times). Teacher 2 made more use (11 times) of questioning for assessment of learning and understanding than did the other two teachers, whose frequency of use was the same (6 times). Teachers 2 and 3 used this form of questioning in the second lesson on trigonometry.

Overall, the analysis shows that the three teachers used questioning in different ways. By using more open-ended questions and questioning for learning and understanding, teachers' approaches and techniques align to a social constructivist approach, while questioning techniques using closed and recall questioning, combined with modelling, aligns with behaviourism. Teacher 1 and 2 show alignment to constructivism and Teacher 3 aligned with behaviourism when taking into account the frequency of the different types of questioning used.

In the sections above, I analysed the type and frequency of four types of questions that the teachers used in the two lessons, and the scaffold process initiated by them in relation to the question. In the section below, I turn to analyse the level of questions that teachers recruited in terms of higher cognitive questioning and lower cognitive questions.

Higher order cognitive questioning

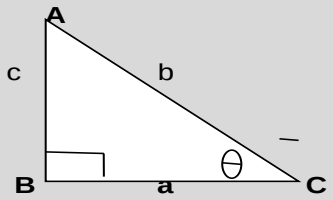
Teachers recruited higher cognitive questioning when using open-ended questions, and when recruiting questions that assessed and evaluated learning or understanding.

a) Open-ended questioning

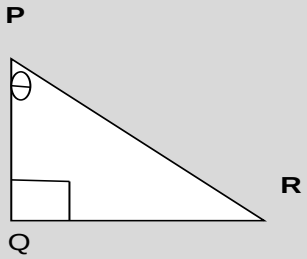
Open-ended questioning is considered a higher cognitive function, as it has been defined by Cotton (1988) in the literature. I analyse these in relation to the scaffolded process in which Teacher 1 used open-ended questioning. I analyse how Teacher 1's interactions work together with and alongside questioning and assessing their learners' on trig ratios.

Teacher 1's example of questioning to assess learner understanding:

Triangle 2a



Triangle 3a



In trigonometry, Teacher 1 asks learners an open-ended question:

Teacher 1: "*What did we say about the theory of Pythagoras?"*

(Teacher 1, Video lesson transcripts on trig,13 August 2014)

In Teacher 1's example above, the question requires an understanding of learners about how to approach the concept and an understanding of the properties of the Pythagoras theorem. The question drew on learners' prior trig knowledge. Teacher 1 did not wait for learners' responses and explained that the theory of Pythagoras meant that the class was dealing with one unknown side. She explained the answer to help learners identify which angles are relevant to trig determining trig ratios. The teacher did not reinforce that Pythagoras is only pertinent to right angle triangles. The teacher did not engage in dialogue with learners, and did not elicit responses from learners, and learners did not get to use such opportunities for learning.

Example of Teacher 1's use of open-ended questioning:

Teacher 1 asked an open-ended question: "*So how would I know which one is my sinΘ yami* [mine] *what do I look at?"* [sic] referring learners to the homework she gave them to identify trig ratios and scaffolds the question as follows:

Teacher 1: "*Let's look at triangle one. I want to find the sin of Θ.* [inaudible] *Yes, sishukuthi* [*we mean*] *which side and which side…* ?"

Learner: [inaudible]

Teacher 1: [writes on board]

'It is the…

sinΘ = <u>opposite</u>
hypotenuse'

Learner: [inaudible]

Teacher 1 says and writes: "It is the opposite side over the…

Class: "*Hypotenuse side…"* [teacher together with learners]

Teacher 1: "*Angithi* [understand], *Right?"* [sic]

Teacher 1: "Yes, I can also see: Which one is the opposite side if we look at the small letters? c. And which one is the hypotenuse side? [Points to triangle]. It's b, angithi, right."

$$\sin \Theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{c}{b}$$

Teacher 1: *"Yes? Let's go to the cos of Θ – Oo lo Θ . (The one that we are looking at). This will be the cos of Θ?"* [sic]

Class: Yes

Learner: "*When I am finding the ratio of cos Θ.*" [sic]

Teacher 1: "*Yah, ma, it will be the...* [writes on board] *Its b angiti (understand) right*."

$$\cos = \frac{\text{adjacent}}{\text{Hypotenuse}}$$

Teacher 1: "*adjacent side over the hypotenuse side, angithi (right)?*" [sic]

Class: "*Yes*."

Teacher 1: "*And also look at the small letters... in terms of the small letters. Thulani [keep quiet]. It is? A over… B… angithi (right)?"*

[teacher writes on board]

$$\cos \Theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{b}$$

Learners: "*Yes*."

Teacher 1: "[…] *tan of Θ. Noxolo: it is the… opposite side over the… adjacent side. Hanimjalu* [clap hands] *for Noxolo, hanim* [clap]." [sic]

$$\tan \Theta = \frac{\text{opposite}}{\text{adjacent}}$$

[laughter and clapping]

Teacher 1: "*Uphendule enkulu* [you answered the big one], s*iphendula ngomkhulu* [we answered with the big one], *bengimele… bengimele* [they waited]. *Bengi shaya ngomqondo* [they beat your brain]. *Siyabonga* [Thank you]. *Thank you! Okay, let's go back. Which side is the opposite side in terms of the small letters?"* [sic]

Learners chorus… "*C over A*"

[teacher writes on board]

$$\tan \Theta = \frac{\text{opposite}}{\text{Adjacent}} = \frac{c}{a}$$

Teacher 1: "*Angithi. Okay. So, why I am teaching you Ndaba [to name] your triangles is so that you know you using the small letters because in an exam uyazi nibhala i (U know, write the) external exam, you guys do know you are writing an external exam. Sometimes they will ask you to measure an angle and not make it clear when you must use small letters or big letter, and there will be a question that include the small letter 'b'. Which side are they talking about? What will come in handy for you when answering the questions you will see that some of my questions has the small letter 'b' or the small letter 'a' the first thing that you must do is you must take your pen and name them yourself so you know which side they are talking about, this is why I was teaching you how to name your sides using the small letters.*" [sic]

I observed the scaffolding above, and it showed that Teacher 1 asks questions, and she mostly provides the answers herself. Although she uses high cognitive open-ended questioning, she does not give learners the opportunity to respond to the questions. The pedagogical move of posing questions determines whether learners know the relationship between different sides of an unknown angle, and how to solve each ratio in trig. The question draws on prior mathematical knowledge and current trig knowledge for its answer. Learner number one provided the correct answer as follows:

$$\text{Sin } \Theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{a}{b}$$

The teacher recruited learner responses by asking them to answer questions or work out solutions on the board on all the trig ratios, namely: sin Θ as above, cos Θ and tan Θ from different learners. The learner responses help the teacher assess their understanding of the ratios. All three learners responded correctly, showing that they understood the relationship between unknown angles and solving the different trig ratios. Teacher 1 does not engage the class fully, and does not explore opportunities for learners, who may not know the correct answer. The questioning practice is void of scaffolding both for learners who may not know the correct answers, as well as those who were not given opportunities to provide answers.

Teacher 2 assessed learning and understanding when he started the lesson on trigonometry with homework revision. The way Teacher 2 assessed learners' understanding of trig homework was by asking questions when he revised homework. Teacher 2 also checked the learners' understanding of the work when they did class exercises, sometimes asking learners directly if they understood or whether they had problems.

During the lesson on trig for example, Teacher 2 asked learners questions as quoted below. The teacher elicited the final answer from learners and assessed whether they understood the first step that he used in simplifying the equation. He asked learners whether they understood what

he did, and upon some of them answering no, he asked which of the learners could help him to explain the step. Teacher 2 went through the calculation step by step, until the complete answer was displayed on the board. Below I also illustrate how he explained and modelled the correct answers to the learners.

Teacher 2: "*Where was the problem? Which sum did give you a problem? Number?*"

Learner: "*I.*"

Teacher 2: "*Right, number I. Did you all get number I*?"

Learners Chorus: "*Yes*."

Teacher 2: "*Right. Number I was 4 x tanΘ – 30° = 1. Right, what answer did you get for number I? Yes?"*

Learner: "*44.0*."

Teacher 2: "*44.0. Right. So this is tanΘ - 30° = 1/4. Can you all understand why I am writing 1/4 this side*?"

Class: "*No.*"

Teacher 2: "*No. Who doesn't understand why I am writing 1/4 this side? Ja, you don't understand. Who can help him understand? Why are we writing this as 1/4? Because we divided on both sides by four to get rid of this 4?"*

Class: "*Yes*."

Teacher 2: "*Right. Now the next step is to say Θ-30°=tan1/4 and Θ-30°= … what is this? Mtshongo? 14.0. And then after this to find the value of Θ we then say this is 14.0+30° and your Θ then equals to?"*

Learner: "*44*."

Teacher 2: "*44.0 degrees. Right.*"

Complete calculation on board for i)

4 tan(Θ-30°)=1
tan(Θ-30°)=1/4
(Θ-30°)= tan 1/4
(Θ-30°)=14.0°
Θ=14.0°+30°
Θ =44.0°
(Teacher 2, Video lesson on trigonometry, 13 August 2014)

In assessing learning, the teacher was able to engage which questions learners did not understand and could not do. He also engaged those learners who could do the sum, to explain to others who did not understand. Teacher 2 also provided learners with the correct answer, which helped learners who did not have the correct answer to the algorithm. In this example, the teacher scaffolded new trig knowledge in solving trig ratios using equations. The questioning helped develop higher cognitive questions for assessing learning and understanding and open-ended questions as part of the scaffolding. Teacher 2 assessed learning and understanding better than both Teacher 1 and Teacher 3.

In the next section, I look at low cognitive level questions, specifically 'recall' and 'closed questioning'.

Lower cognitive questions

In my research, I found only two types of lower cognitive questions that teachers recruited. Lower cognitive questioning comprised of closed and recall questions. Table 3 shows that lower cognitive questions are used more frequently by the three teachers. There are two types of lower cognitive questions, namely: recall questions and closed questions.

Recall questioning

Table 10 on page 111 shows that Teacher 1 drew on recall questions more than the other two teachers. The example below indicates an example where the teacher scaffolded recall, lower cognitive questions.

Example of Teacher 1's used of recall questions:

Teacher 1: *"What do you remember about congruence in Grade Nine? I am sure you were taught. Those who were taught by me I taught you about congruence in Grade Nine…What does it mean? What do we mean by the word congruent? Mmm? Grade 10? Ngiyabuza phela* [I am asking] *Zandile, Yes?"* [sic]

Learner: *"It means that the triangles are exactly the same."*

Teacher 1: *"So if we are talking about congruents we are talking about ama* [these]*... triangles, angithi (understand). Right now, we will be dealing with amatriangles. You can also have ama quadrilaterals that are congruent, but for this lesson we will be dealing with ama triangles. So I gave you… I think once in a lifetime I gave you an example of the difference between your similar triangles and your congruent triangles. What did I associate the similar triangles with? What did I say you can associate the similar triangles with? Do you remember that lesson? Yes... Kwathiwani* [what did they say]? *They are?* [sic]

Learner: *"They are sides and angles."*

Teacher 1: *"Yes... but I related them with something practical in real life. Do you remember? Yes?"* [sic]

Learner: *"The corresponding sides are in proportion."*

Teacher 1: *"Yes, but I referred to something real, in real life ...about our twins do you remember? What did I say about your similar triangles? They are like which twins? Are they identical? No, they are twins, but they are not the same."* [sic]

(Teacher 1, Video lesson on congruent triangles, 1 August 2014)

The example above shows that the teacher required learners to recall information that they had learnt previously. Specifically, the questions seek to determine whether learners remembered the concept of congruent triangles, and whether they knew that it was related to the subject of triangles. Furthermore, the recall question elicited whether learners could recall the properties of congruent triangles. The recall does not require any logic or evidence of understanding or problem-solving and are, therefore, lower order questions. Teacher 1 used recall questions the most (7 times).

Closed questions

Table 10 on page 111 shows that Teacher 2 used closed questions 23 times during the two lessons. Although the question below is a closed question, it also can serve as a recall if the teacher taught ratios previously.

Closed question from Teacher 2: *"What trig ratio do you think we should use to find the length of ab?"* [sic]

Learner: *"sin of angle c."*

Teacher 2: *"sin of angle c? And in this case sin of angle c is 30 degrees. So to find the length of this side (points to side AB on diagram), we need to use sin, trig ratio of sin, angiti* [understand]*, not so?"* [sic]

Class: *"Yes."*

Teacher 2: *"Why the trig ratio of sin? Because we are required to find the side that is opposite this angle, and we are given the side that is the hypotenuse, angiti. So in solving this right-angled triangle, we'll then say sin30°= the side that is ab over the side that is ac. Right. We don't know what the length of side ab is, but we know the length of side AC. So what we do, we then say (Writes on board) sin 30°=AB/5. This now has become an equation, and we call this: trigonometric equations. We call this a?*

[Teacher 2 writes on the board]

trig equation

[**Teacher 2:** *"And when we solve trig equations, we solve trig equations like we'd solve any other equation. Okay. Right, the unknown here is AB. What does it tell us? It tells us that we should then take the unknown and make it the subject of enquiry or the subject of the formula. Right, if we make ab the subject of enquiry in this equation, what do we do? How would you make ab the subject of enquiry?"* [sic]

Learner: [Inaudible]

Teacher 2: *"Is it true? Is it the right way of doing it? Remember five is in the denominator. To get rid of this 5, it means we must multiply by five on both sides and then eventually we will have 5sin30°=ab. Now our ab is the subject of enquiry. Then we can use our knowledge of finding the value of this trig ratio in our calculators. Remember we said that when we want to find the sizes of angles in our calculators, if you don't have a Casio™, if you have a Sharp™ calculator then your calculator should always be in degrees mode. Right. Can you then find the value of 5sin30°?"* [sic]

Learner: "2.5."

Teacher 2: *"Right. 2.5. We rounded it off to one decimal unit. It's 2.5?"* [sic]

(Teacher 2, Video lesson on trigonometry, 13 August 2014)

Teacher 2 asked the closed question that had one correct answer given by one of the learners. He scaffolds the question further, by asking why sin is the answer, but responds to his question and does not give learners the opportunity to provide the answer. He then moved to explain to learners the procedures to solve the equation. The question was a closed one, and the teacher' scaffolding process limited learner engagement.

4.4 Overall summary of three teachers' instructional practices

The three teachers' instructional practices involved 'teacher talk-type' instruction, modelling and questioning. Teacher talk was used to build conceptual knowledge through using everyday examples and explanations. Teacher 1 used teacher talk to explain pre-requisite knowledge on similar and congruent triangles. In the new mathematical trigonometry knowledge, only Teacher 3 used the LTSM to scaffold knowledge on right-angled triangles using everyday knowledge. The teachers used the pedagogical move of modelling examples of pre-requisite and new mathematical knowledge. On the pre-requisite knowledge Teacher 1 and 2 used the LTSM to scaffold knowledge in similar and congruent triangles. However, Teacher 3 did not use the LTSM for its intended purposes in that lesson. The other two teachers did not use the

LTSM to scaffold new trigonometry knowledge in their teaching of trig ratios, but they used the classroom textbook to model and scaffold learning. The teachers modelled answers through scaffolding in a procedural manner, by writing complete calculations and answers on the board.

The three teachers used different questioning techniques. Teachers questioning techniques fell short of providing opportunities for learning, and limited learner engagement, as well as opportunities to scaffold deeper learning. The difference is shown in Teacher 1 using mostly open questioning that is considered high cognitive level questioning. However, she uses it in a limited way, as she mostly answers the questions herself, and does not give the learners opportunities to respond. While Teacher 1's use of open-ended questioning limited learner engagement, the high level of use of open-ended questioning could have contributed to the statistically significant improvement in conceptual knowledge as this type of questioning develops higher cognition.

Teacher 2 used mostly closed questions and attempts to do scaffolding, but also limited learner engagement by answering questions he, and by not providing opportunities for learners to respond to the question. Teacher 2 also assessed learning and understanding the most. The scaffolding gave Teacher 2 the opportunity to help learners with homework exercises where they did not understand. Teacher 2 gave learners opportunities to point out the new trig content on ratios that they did not understand in the homework exercises.

Teacher 3 used closed questions more than he used open-ended questioning, but used both types of questions the least, when compared to Teacher 1 and Teacher 2. Teacher 2's use of questioning to assess learning shows a progressive approach towards mediated learning.

Overall, the three teachers pedagogy does give some explanation for their respective class patterns of achievement in the post-diagnostic test. For example, all three teachers assisted with filling knowledge gaps in their focus on congruent and similar triangles. Teacher 1 showed an attempt to challenge learners by using high cognitive open-ended questioning, and Teacher 2 attempted to promote high cognitive development in assessing learner understanding and learner engagement. Teacher 2 also used modelling to develop procedural knowledge. Teacher 3's pedagogy shows weakness. He showed poor quality teaching of conceptual and procedural knowledge, and used low cognitive-type questioning, showing similar limitations to Teacher 1 in assessing learner understanding and the engagement of learners.

Teacher 2's use of scaffolding helped learners' understanding on how to solve an equation in trig ratios. The scaffolding approach provides opportunities for learners to engage with the content taught by the teacher in the class. Even in cases where learners did not give the correct answers, teachers pursue other learners who can provide the correct responses, or they provide the correct answers. Learner-initiated questions were strikingly absent in the three teacher's classes.

The poor quality of questioning strategies and techniques could account for the average improvement (not statistically significant) in mean scores for conceptual knowledge for Teacher 2 and 3.

Explanation has been provided to explain teacher classroom practices in relation to the different types of mathematical knowledge, and in relation to pedagogical moves above. The effects of the types of mathematical knowledge and the pedagogical moves on the results of the post diagnostic tests, have been demonstrated and discussed, and the correlations between the diagnostic test results, teachers' use of types of knowledge and the pedagogical moves in the classroom.

In the next section, I will make the link between the diagnostic assessment and the different types of mathematical knowledge with the interviews. I do this by considering the contributions to the success of Teacher 1's class, followed by a middle path of Teacher 2's class and the lowest achievement seen in Teacher 3's class.

4.5 The relationship between interviews, diagnostic assessment and teachers' use of mathematical content knowledge.

4.5.1 Diagnostic assessment

I analysed the pre- and post-test results of the learners' of the three teachers' classes' trig concept readiness on page 68. The analysis concerned pre-requisite trig knowledge and skills areas such as angles, similar triangles, Pythagoras, fractions and ratios. I also analysed learners' readiness in relation to their conceptual and procedural knowledge. The results of the pre-test showed that in the three areas that are fundamental to understanding trigonometry i.e. similar triangles, Pythagoras and ratios, learners know less than a third of the requisite knowledge. The pre-test also showed low levels of conceptual and procedural knowledge. In sum, the tests showed that the learners demonstrated knowledge gaps, and that learners are not ready to learn

trigonometry. This is also evident in Teacher 2's response to a question on learners' knowledge levels:

> *I think the class has got a mixed ability of learners; most of them are not ready to do trig, only a few of them. Like the concept, we should have learnt in the previous classes, like Eight and Nine – I usually have to repeat those concepts. Like some of them are not clear about Pythagoras' theorem and things like that*. [sic]
> (Teacher 2 pre-lesson interview, 1 August, 2014).

Teacher 2's view that most learners' were not ready to learn trigonometry was consistent with the pre- diagnostic assessment results. Teacher 2 was of the view that learners did not have the pre-requisite knowledge of similar and congruent triangles, Pythagoras, and ratios, which is fundamental to understanding trigonometry. This view coincided with the results shown in the low scores of less than a third percentage mean scores obtained for the areas mentioned, which are fundamental to understanding trig in the pre-diagnostic test.

I analysed the teachers' recruitment of the different types of mathematical knowledge from page 92-105. In this section, the evidence shows a relationship between the improved results on the post-test, and the frequency of the teachers' use of conceptual and procedural knowledge. The teachers built mathematical knowledge through conceptual knowledge, everyday knowledge, and procedural knowledge.

The improved results also showed that procedural knowledge contributed mostly to the improvements in trig readiness. The high frequency of use of procedural knowledge by Teacher 2 and his classes significant improvent on the test, as well as Teacher 1's class's significant improvement, could explain the contribution to the improved trig readiness. Teacher 3's class did not improve significantly in the post-test, because the quality of his teaching was poor.The teachers' perspectives showed that the pre-test results of the diagnostic assessment informed their teaching as shown below.

I asked Teacher 2 whether the diagnostic tests informed his teaching, and his response was:

> *In fact, it was just a bit of the result from the test that I used*. [sic]
> (Teacher 2 post-lesson interview, 20 August, 2014)

When asked how the diagnostic assessment informed her teaching, Teacher 1 explained:

> *The diagnostic test did have some influence, especially in the trig, they knew nothing about trig, and they had never done it before. So I had to be very careful that I was introducing the lesson just to let them understand about the trig ratios. But, before I even introduced the lessons I asked the learners to go home and just read in their textbook on trigonometry about the ratios, to have an understanding before I even started teaching the lesson. So it did inform me that the learners didn't know anything about diagnostic test*[s] *– I mean the diagnostic test did help me to know the learners knew nothing about the trig. And also on congruency, some of them didn't understand. I assumed when I looked at the diagnostic test they had done congruency in Grade Nine, because that is when it was done, yet they did not do it. Or, I wouldn't say they did not do it but maybe they forgot, or they didn't teach them because they don't know*. [sic]
>
> (post-lesson interview, 15 August 2014)

Since the results of the diagnostic assessment, the three teachers' teaching focused on developing conceptual and procedural knowledge, showed that learners were not ready to learn trig and showed that learners had gaps in learning conceptual and procedural knowledge.

The link between the types of knowledge and teacher instructional practices has been shown with the results of the diagnostic test in previous sections. The link showed that in the case of teacher 1 that used more conceptual knowledge, her class improved significantly in this type of knowledge in the post- test and similarly, teacher 2 used more procedural knowledge and his class improved significantly in this knowledge in the post- test. In the section above, I have further corroborated the results with the teachers' perspectives during the interviews.

Below I present and discuss teachers' learning and professional development, drawing a relationship with the theory (Ball & Cohen, 1996; Drake et al., 2014) that argues that LTSM could be used as a mode of professional development.

4.8 Teacher learning and development

The section describes teachers' perspectives on what they learned from the research process, specifically from the LTSM and from the diagnostic assessment. I present this aspect to show that teachers only used the LTSM in part, and what they used had minimal effect on teaching and learning in the classroom. In addition to the small effects in the classroom, the teachers indicated that the LTSM also contributed somewhat to their learning, i.e. their professional development. The teachers' perspectives on what they learned is included in the quotations below.

When I asked what was different in the lesson Teacher 2 said:

> *There were more responses from the kids. Usually, when we go to class because we are rushing to cover the syllabus because we just go on and go on tell the …Learners. Yes. We just go on and go on. But with these two lessons I had to involve the kids and make them talk and respond.* [sic]
>
> (Teacher 2, post-lesson interview, 20 August 2014)

Teacher 2 expressed what he learnt as follows:

> *What I can say is that I learnt that, hmm … how can I put this…? There were some useful hints in the booklet. What I also learnt from the booklet was that the booklet, in fact, if I follow some of the lessons in the booklet, will help the learners better understand, I mean self-discovery things for on their own.* [sic]
>
> (Teacher 2, post-lesson interview, 20 August 2014)

Teacher 1 felt she learnt that she should make more effort to understand whether learners have the required knowledge and level of understanding. Her words were:

> *What I have learned is that I should not assume that the learners know, I should always try harder or have some mechanism to see how much they understand or how little they understand so I know exactly which areas to do more on, so especially the diagnostic test, I like the idea of that, to give them a test before you even teach the lesson, just to try and find their level of understanding and how far they are.* [sic]
>
> (T: 1 post-lesson interview, 15 August 2014)

Teacher 3 expressed what he had learnt in the process of teaching trigonometry using the LTSM as follows:

> *I learnt a lot, as I've just mentioned these kinds of things working with the learners that our trigonometry is not a trig just in space, but it is too practical, you can measure, say if you are faced with a problem, and you can solve it practically using your trigonometry.*[sic]
>
> (Teacher 3 post-lesson interview, 20 August 2014)

In summary, the teachers said that they learnt the importance of using diagnostic assessment to inform their teaching. All three teachers said that prior to the research, they did not use diagnostic assessment to inform their teaching. Two of the teachers indicated that they usually taught a topic or concept assuming learners were ready to learn the topic, or that they knew nothing, which shows they do not have a diagnostic approach. They said that they learnt that based on the diagnostic assessment, that their assumptions were not quite accurate, and that they needed to change by using diagnostic assessment in order to understand their learners' knowledge levels. They also learnt that the LTSM required them to involve learners in their classroom instruction. Teacher 2 highlighted that there are constraints, given the large class sizes and the time allocated to complete the curriculum. Teacher 3 said that he learnt the importance of using everyday examples to make mathematics more practical. Teacher 1 indicated that she learnt from the LTSM that providing practical examples of measuring triangles, for example, can help learners remember principles such as the angles of a triangle add up to 180˚ (degrees).

The literature (Ball & Cohen, 1996; Drake et al., 2014) shows that LTSM has the potential for use as professional development for teachers. The theorist premised the argument on the interactive and dynamic relationship between teachers and textbooks. While I agree that the use of LTSM towards professional development of teachers is possible, I also argue that this can only be successful if the training does not only focus on the LTSM content. The evidence showed that Teacher 1 and 2 made attempts at high cognitive development in their questioning techniques, and that Teacher 3 showed weaknesses by focusing on low cognitive questions. In addition, there were overall limitations, with low levels of learner engagement among the three teachers. The evidence showed that teachers do not have an informed understanding of their learner's knowledge gaps, and that they do not use diagnostic assessment in their normal

teaching. The research made them aware of the need for diagnostic assessment. In addition, teachers showed limitations in their pedagogical moves, of questioning to develop their learners high cognitive ability. This required an understanding by teachers of their learners' knowledge as well as an understanding of how to create more learning opportunities and how learners learn better. Therefore, the teachers will require training with a focus on teachers' theoretical understanding of how learners learn and the interactive relationship between their pedagogical moves and scaffolded processes in the classroom.

4.6 Conclusion

The pre-diagnostic test showed that Grade 10 learners in the three teachers' classes have knowledge gaps in similar and congruent triangles, Pythagoras theorem, and ratios, which are fundamental pre-requisite knowledge for understanding trigonometry. In particular, the low scores in the content areas of pre-requisite knowledge showed that learners were not ready to learn trigonometry. The post-test scores showed that learners overall readiness to learn trig improved significantly in the five areas of trig readiness. However, the scores of less than a third of a percent in mean scores in the core areas of trig readiness showed that learners are still not ready to learn trigonometry. The post-test also showed that procedural knowledge contributed mostly to the improvements in trig readiness. The observations showed that teachers used the LTSM to teach conceptual, everyday and procedural mathematical knowledge. They used the LTSM in their pedagogical moves of modelling and questioning. The analysis confirms Remilliard's (2005) findings that shows a relationship between the materials, the ideas they represent (pedagogy and mathematical content) and the teaching that it is supposed to support. It could be argued that since I gave the teachers the profiles of the learner's results that showed learner weaknesses in conceptual and procedural knowledge, this made teachers aware that learners need assistance and were weak in the two knowledge types and therefore they focussed their teaching on building learners conceptual and procedural knowledge. This focus on the two knowledge types shows that when there is a relationship between diagnostic assessment and teachers pedagogical practice, the practice assists with improving learners' weak knowledge areas. The point made is evident in the significant improvement of teacher 1's class in conceptual knowledge as well as in procedural knowledge and teacher 2's class significant improvement in procedural knowledge in the post test.

The test showed the different knowledge types. In some instances, teachers also did not use the LTSM for its intended purposes; as they used the same examples with the whole class and the

LTSM tried to differentiate the use based on learning gaps. As Sherin and Drake (2009) have pointed out, teachers need to consider curriculum materials with learners as their audience, and make decisions about how to use and adapt suggested activities.

In analysing the different types of mathematical knowledge, the data on the frequency of use of the different types of knowledge showed that Teacher 2 mostly used procedural knowledge, and his class improved significantly as a result of this type of knowledge. Procedural knowledge contributed the most to the significantly improved trig readiness scores. The data shows a strong relationship between the frequency in use of procedural knowledge, with the significantly improved scores in procedural knowledge in the diagnostic test, specifically in the case of Teachers 2's class and Teacher 1's class. The three teachers also used conceptual knowledge to build mathematical knowledge to a lesser extent than procedural knowledge, and used everyday knowledge minimally. Only Teacher 1's class showed a strong positive correlation of statistically significant improvement between conceptual and procedural knowledge. Teacher 2's class showed significant improvement in procedural knowledge. However, Teacher 3's conceptual and procedural knowledge showed a weak relationship. Rittleston-Johnston and Siegler (2001, p. 109) showed a positive correlation between learners' understanding of mathematical concepts and their ability to execute procedures. Whether conceptual knowledge precedes procedural knowledge depends on the topic. For example, it is possible that if learners only work with procedural knowledge by following algorithms, they could achieve average results even though they may not perceive the relation between Pythagoras (conceptual knowledge) and ratios.

The teachers' pedagogical moves and scaffolding processes showed teachers' mathematical explanations, modelling and high and low cognitive-type questioning. Research (Bloom, 1956; Elmore, 2008; Vygotsky,1978) shows that using high amounts of cognitive questioning and assessing learning and understanding through carefully scaffolded processes, provides better opportunities to learn. Although Teacher 1 used open-ended questioning, it was limited as the scaffolding, and fell short of engaging learners and of providing them with opportunities to learn. Teacher 2 provided learners with opportunities to learn when he used scaffolding in assessing their learning and understanding. The study shows that the teachers' instructional practices fall short of Elmore's instructional core, as this pertains to learner engagement.

There is a relationship between what teachers said about the learners' readiness and the results shown in the diagnostic assessment; that the learners are not ready to learn trigonometry.

Teachers' perspectives on what they learnt from the research process and the LTSM are consistent with contemporary theories. The theories argue that LTSM could be used for professional development based on an interactive and dynamic relationship (Drake et al., 2014, p. 154). The analysis begins to inform the findings; which I will discuss in the next chapter.

CHAPTER FIVE: Analysis and Discussion of Findings

5.1 Introduction

The aim of this chapter is to answer the main research and sub-questions that informed the study. I structured the questions around Elmore's (2008) instructional core theory. I made an attempt to develop a model, described above, of what I thought best analysed the teachers' scaffolded processes and pedagogical moves. These processes and pedagogical moves inform the way in which the teachers helped the learners (or not) to fill their knowledge gaps in trigonometry, identified by the diagnostic assessment and by using the LTSM.

5.2 Discussion

I begin this discussion by examining the teachers' beliefs and ideas about the nature of their learners' knowledge gaps and how they teach trigonometry content. Looking at the teachers' beliefs and ideas of their learners' knowledge levels and the way they teach, the aim has been to understand diagnostic assessment in relation to identifying learners' knowledge gaps (sub-question 1). This has been followed by a description of the ways teachers used the LTSM to mediate learners' knowledge gaps, which addresses my main research question. Insight is also sought on how, through demonstrating their instructional practices, they address their learners' knowledge gaps identified in the diagnostic assessment. More specifically, the study exposes the types of knowedge they recruit in their use of the LTSM and their general mediation of learners' knowledge gaps (main question, sub-questions 2 and 3). Lastly, it examines the teachers' perspectives on their experiences and challenges or constraints with the use of the LTSM to help mediate their learners' knowledge gaps (sub-question 4).

The presentation and analysis of the pre-diagnostic assessment indicated that learners had knowledge gaps and were not ready to learn trigonometry, as well as that they had a lower starting base in conceptual knowledge than in procedural knowledge. The interviews gave evidence that teachers do not use diagnostic assessment to inform their teaching in the classroom.

It is significant to note that even after teaching the two lessons observed and at least 3 other lessons that were not observed within one week, the learners are still not yet ready to learn trigonometry.

The results of the post-diagnostic test showed that Teacher 1's class improved significantly in conceptual and procedural knowledge, and more specifically, in two of the five measures, namely similar triangles and Pythagoras, which are fundamental pre-requisite knowledge for learning trigonometry. Teacher 2's class improved significantly in procedural knowledge and Teacher 3's class did not improve significantly in any of the types of knowledge. The improvements can be explained by the frequency with which teachers used conceptual and procedural knowledge. The frequent use of a particular type of knowledge correlated with the improved results in the two knowledge types. For example, Teacher 1 used conceptual knowledge the most, and her class improved significantly in this type of knowledge. Similarly, Teacher 2 used procedural knowledge the most, and his class improved significantly in that type of knowledge. The mean scores show higher improvement in procedural knowledge than in conceptual knowledge for Teacher 2's class. Teacher 3's class did not improve significantly in the post-test, although he had a high frequency of use of procedural knowledge. The low rate of improvement in his class the post test could reasonably be explained by the poor quality of his teaching of these lessons.

Teachers' use of the LTSM was limited, varied and not sufficient to explain the significant improvements in the post-diagnostic test.

The instructional practices that the teachers used to teach the content of the lessons were: 'teacher talk' instruction, modelling procedures and using different types of questioning: namely, high cognitive questioning (such as open-ended questions and questions which assess a learner's understanding) and lower cognitive questions (such as closed and recall questions).

The way teachers use LTSM to mediate learning, as well as for professional development, has been a topic of more recent interest (Ball & Cohen, 1996; Drake et al., 2014), and it has been investigated by a number of researchers (Cohen et al., 2002; Fleisch, Taylor, Herholdt, & Sapire, 2011, p. 488; Alvermann, 1989).

The teachers used their skills and the LTSM, as well classroom textbooks, as trig content knowledge.

The teachers taught mathematical content knowledge through three types of knowledge, namely: conceptual, procedural, and everyday knowledge, using 'teacher talk-type' instruction (Hardman, 2010, p. 96). The content involved the teaching of pre-requisite and new Grade 10 trig knowledge as observed in the two lessons. The teachers' pedagogy showed them

scaffolding learning through modelled examples and different questioning techniques. The teachers' instructional practices show limitations in the use of questioning. The limited use of questioning in carefully scaffolded processes, provides some explanation, in my view, for the small improvements in conceptual knowledge.

The teachers' engagement of learners through questioning was limited, In that it did not allow for high cognitive development, or for learners to take responsibility for their learning. For example, learners did not ask questions and teachers' did not give learners opportunities to answer high cognitive questions.

My analysis draws on sections of the literature review,[18] specifically Vygotsky's (1962; 1978) idea of developing higher mental functioning through mediation in the ZPD. The analysis also describes teachers' pedagogical practices of instruction on: conceptual, everyday knowledge, modelling of procedural knowledge (which required a lens on behaviourism)[19] and higher cognitive questioning (showing alignment with social constructivism).

I propose that an understanding of the kind of pedagogical moves that teachers make use of as they work with scaffolded processes, building mathematical knowledge and filling learners' knowledge gaps, is important. What is important is to understand how they use diagnostic assessment to inform their teaching practices and how they use the LTSM to mediate the knowledge gaps of learners.

[18] See Chapter 2 page 38.
[19] See Chapter 2 pages 31-41.

5.3 Analysis and discussion of Research Questions

Table 11 below outlines the research questions that informed the study.

Table 11: Main research and sub-questions

Main Research Question
In what ways do three Grade 10 mathematics teachers use Learner Teacher Support Material to mediate learners' knowledge gaps?
Sub- Questions
1. How do the teachers understand formative assessment (diagnostic assessment), specifically in relation to identifying their learners' knowledge gaps in trigonometry?
2. How do the teachers understand the relationship between the learning and teaching material and learner knowledge gaps?
3. In what ways do the teachers use the material to address learners' knowledge gaps?
4. What forms of classroom practice enable or constrain the teachers' use of the learner teacher support material (LTSM)?

The teachers' beliefs and ideas about their learners' knowledge gaps and how the diagnostic assessment informed the way they teach congruence (prior knowledge of trigonometry) and trigonometry.

To understand the three teachers' perspectives and ideas about their learners' knowledge gaps, I examine the ways in which the teachers' views can be seen in the types of mathematical knowledge that they recruit in relation to the diagnostic assessment. I also look at this in relation to the mathematical knowledge that they recruit that is specific to trigonometry. I further examine the relationship between the teachers' beliefs, the diagnostic assessment results and the pedagogical moves they select as they work to mediate the learners' knowledge gaps, informed by the diagnostic assessment results of the learners and the use of the LTSM.

The teachers' belief that their learners (sub-question 1) were not ready to learn trigonometry was consistent with the pre-diagnostic test results that provided evidence of their low levels of trig readiness, particularly in conceptual and procedural knowledge. Both Teacher 2 and 3 indicated the low levels of readiness of their learners in the interviews. Teacher 1 first assumed that the learners were ready and later realised that the learners were not ready when I gave her

feedback on the results and profiles of the pre-diagnostic assessment of her class. The overall pre-test for the five test measures (triangles, right angles, surds, fractions and ratios) of trig readiness shows mean scores of 6.81 for Teacher 1's class, 5.63 for Teacher 2's class and 4.33 for Teacher 3's class out of 25. The data shows that the learners understood less than a third (8.33) of the requisite knowledge required to learn trig, from which it is inferred that they were not ready. The teachers' responses in the interviews also revealed that they do not use diagnostic assessment in their normal teaching. It is evident that all three teachers' learners showed low trig readiness and low levels of both conceptual and procedural knowledge.

Overall, the way the three teachers used the LTSM was found in their instructional practices of teacher talk, modelling and questioning. The three teachers used the LTSM differently. The LTSM was used in only one of the two lessons and mainly when teaching the whole class. Teacher 1 and Teacher 2 used it to fill in gaps by teaching the lesson on congruency, as this was requisite knowledge prior to learning trig (main question). Teacher 3 used the LTSM to give context to new mathematical content (trig ratios) (main question).

The teachers' choices and decisions as to when and how to use the LTSM enabled the use of the LTSM to mediate knowledge gaps through modelling as a pedagogical move to instilling procedural knowledge. The teachers indicated that the use of the LTSM was constrained by class sizes and time pressures (sub-question 4). Teacher 1 corroborated these challenges in her interview.

The ways in which the teachers selected and used LTSM to mediate learners' knowledge gaps was observed when they recruited exercises from the LTSM. They recruited the exercises using the pedagogical move of modelling during a lesson with the whole class (main research question). This is consistent with my view that teachers make decisions about when and how the different types of mathematical knowledge are recruited. For example, the teachers (2 and 3) modelled examples using procedural knowledge. Similarly, there is consistency to be found in the teachers' decision(s) about which pedagogical moves to use in the process of using the LTSM to address learners' knowledge gaps. Teacher 1 and Teacher 2 used exercises from the LTSM once in lesson one and Teacher 3 recruited exercises once in lesson two. The teachers used the LTSM during the pedagogical move of modelling in the scaffolding process (sub-question 3). This shows that the three teachers used the LTSM to a limited extent, and that they used it differently either in their pedagogical moves or in the types of knowledge that they recruited. Teacher2 used modelling and procedural knowledge more and teacher 1 used teacher

talk style instruction in conceptual knowledge, whilst teacher 3 used everyday knowledge more than the other teachers. Modelling is a pedagogical move, and thus, the teachers used the LTSM in their instructional practices of modelling.

The teachers' beliefs about the nature of mathematics and the way in which the subject is taught can be seen in the manner in which they recruit conceptual knowledge (teacher talk and the way in which they develop low cognitive ability into high cognitive ability.[20] By selecting certain pedagogical moves during instructional practice, the teachers recruit the three different ways of building mathematical knowledge in varying frequencies. Understanding the frequency depends on knowing what the teachers' recruit, and their intention for its use. That is, the ways the LTSM is recruited, will influence the purpose according to the teachers' beliefs about what the intention of the move is, and this will increase the frequency or the sequence. In the section that follows, I will show how these claims about the teachers' instruction unfolded in my study.

The types of knowledge and pedagogical moves are in line with the model developed from the instructional core above, and the descriptive analysis answers the main research question and sub-questions 2 and 3.

There are two additional aspects that constrain the mediation of the LTSM. These are barriers to classroom practice. The first is classroom sizes, and the second is a teacher's knowledge of how learners learn. The South African education system has set norms and standards for class sizes. However, all three teachers have classroom sizes that are above the norm. Teacher 2 has a class size of 78 learners; Teacher 1 had a class size of 74 learners, while Teacher 3 had a class size of 48 learners (noting that Teacher 3 performed worst than the other two teachers with larger classes). Although class sizes posed constraints in teaching in the case of Teacher 1 and Teacher 2, Teacher 3's low quality of teaching had a negative effect on learners' progress in the post-test. In fact, the teachers' 1 and 2 could not use pedagogical moves and scaffolded processes that would engage all learners in the two lessons. These constraints included difficulty in arranging group work, giving all learners opportunities to ask questions, and the inability to effectively collaborate with all learners.

[20] See Chapter 2, page 23

5.3.1 Alignment of types of mathematical knowledge with Pedagogical Moves

The table below shows the way in which the types of mathematical knowledge are aligned with the pedagogical moves in the study.

Table 12: Alignment of mathematical knowledge with pedagogical moves

Types or modes of mathematical knowledge	Pedagogical move
1.1 Conceptual knowledge: 1.2 Everyday knowledge	1. 'Teacher talk' style instruction
2. Procedures	2. Modelling
3. High cognitive: open-ended; assess understanding Low cognitive: closed and recall	3. Questioning

In the table above, I align mathematical content of everyday knowledge and conceptual knowledge with the pedagogical move 'Teacher talk' style instruction. This is because in the teachers' pedagogical moves, whenever they use conceptual and everyday knowledge, there is demonstration of such in 'teacher talk' style instruction. I aligned 'procedures' with the pedagogical move modelling, because this is when, in their instructional practices, the teachers modelled trigonometry examples and exercises for solving the mathematical algorithms.

High cognitive and low cognitive questions are aligned with the pedagogical move of questioning, because this was when teachers questioned learners so as to develop their conceptual knowledge. It was also when the teachers asked questions so as to check whether the learners' could recall what was already taught, as well as assess their learners' understanding. Only Teacher 2 assessed learners understanding, while Teacher 3 and Teacher 2 and 3 used a number of recall questions.

The evidence from the observations in this study show alignment with teachers' beliefs about their practices, as shown in the interviews and the relationship with the diagnostic assessment on page 121. Teacher 2 spoke about the instruction method as the 'telling method' used to help learners develop trig concepts, stating that they use the 'telling method' in their teaching, and did not engage learners given the challenges of time constraints and large classes. The telling method refers to what I called "teacher talk" type instruction. Teacher 3 and Teacher 1 both

spoke about everyday knowledge as practical knowledge that learners needed to develop their understanding of congruence and trigonometry. Teacher 1 also spoke about showing the learners examples, referring to modelling. None of the teachers, however, spoke – in their own terms or otherwise - about procedures of high or low cognitive questioning as modes of building conceptual knowledge.

The three teachers worked with conceptual knowledge, which is aligned with 'teacher talk' style instruction in Table 11, in Lesson 1 (which focused on pre-trig knowledge) after the feedback on the diagnostic assessment. Teacher 1 worked with conceptual knowledge 17 times, and used this type of knowledge in four more instances than she used procedural knowledge. While Teacher 2 used procedural knowledge more frequently (28 times), he worked with this knowledge in 16 more instances than he used conceptual knowledge. Teacher 3 worked more times with procedural knowledge (25 times) and worked with conceptual knowledge only seven times. The three teachers recruited everyday knowledge the least, where Teacher 3 recruited it once in each of the two lessons, and Teacher 2 did not recruit it at all in both lessons. Vygotsky (1978) refers to a dialectical relationship between everyday concepts and mathematical concepts, where everyday concepts create the potential for developing mathematical concepts and mathematical concepts create the necessary structures for strengthening everyday concepts. The teachers worked with conceptual and procedural knowledge in their instructional practice, and they also worked with everyday knowledge. Teacher 1 and 3 embedded mathematical knowledge in different ways, for example: through embedding concepts within a context, and providing illustrations within a familiar context. In lesson one, the teachers work with prior trigonometry concepts, such as congruence, and build on the prior knowledge in lesson two, focusing on right angle triangles and trigonometry ratios. As the teacher work with trigonometry knowledge, they work with it in relation to pre-trig knowledge, and recruit everyday knowledge at the same time as they work with pedagogical moves based on 'teacher talk' style instruction. The teachers also recruit procedural knowledge to build mathematical knowledge, and work with modelling as the pedagogical move to solve algorithms. The teachers modelled examples in their instructional practice. They did this through the modelling of exercises and through recruiting procedural knowledge. Social learning theory propounds that modelling influences learning, and that knowledge is acquired through symbolic representations of modelled activities (Bandura, 1969, p. 8). The three teachers recruited modelling more (14 times) in the second lesson compared to the number of

times they recruited it in Lesson 1 (5 times). Teacher 2 modelled the most exercises, where Teacher 3 and Teacher 1 modelled the least exercises.

The teachers used different types of questioning in their instructional practice, as shown in Table 10 page 111. They recruited different types of questioning for different purposes. All three teachers mostly asked closed questions. Teacher 2 used questioning techniques the most (49 times) during the two lessons. Teacher 2 also used the most number of closed-ended questions (23) among the three teachers. Teacher 1 used closed questions 15 times, and Teacher 3 used this type of questioning 13 times. The second highest frequency of a particular type of question is the use of open-ended questions, with Teacher 1 using this form the most (17 times), and Teacher 2 (12 times) and Teacher 3 (11 times). Teacher 2 and Teacher 1 used recall questions the least, while Teacher 1 used it the most (7 times). Teacher 2 made more use (11 times) of questioning for assessment of learning and understanding than the other two teachers, whose frequency of use was the same (6 times). Teachers 2 and 3 used this form of questioning in the second lesson on trigonometry.

Teachers may believe that lower cognitive questions slowly build the foundations. Lower cognitive questions are appropriate to a scaffolded process and as the trigonometry concepts become more difficult, however, higher cognition should be built as part of scaffolding. When teachers build high cognition in open-ended questioning and in reasoning, teachers' understanding and beliefs show signs that teaching trigonometry is a psycho-social process, in line with a social constructivist approach. The teachers' instructional practices still need depth in the way they recruit questioning as a pedagogical move. My argument stems from the fact that learner engagement has to be deepened, as teachers do not scaffold learners' thinking to develop high cognitive ability within the Zone of Proximal development (Vygotsky, 1978) in their questioning practices. The limited learner engagement in teachers' classroom practice is considered a constraint in the mediation of learning gaps and the use of the LTSM (sub-question 4). The limited engagement with learners does not give teachers an understanding of learners' mathematical thinking. For example, when a learner answers a question incorrectly, the teacher does not scaffold or collaborate with the learner through the learners' level of understanding. Instead, the teacher moves on to ask another learner to provide the correct answer. Teachers also do not allow learners to question and have peer or group discussions as social constructivist theory advances.

Modelling aligns with behaviourists approaches. For this reason, teachers' use of the LTSM in modelling suggests that they believe learning gaps should be mediated through modelling exercises from the LTSM (sub-question 2). Teachers' pedagogical moves (teacher talk instruction, modelling of procedures and recall questioning) suggest that their beliefs about teaching congruence (pre-trig knowledge) and trigonometry are informed by behaviourists approaches to learning and teaching. In particular, Teacher 2 and 3's approach is aligned to behaviourism. Although the teachers' recruitment of everyday knowledge and their development of high cognitive ability through open-ended questioning and questions that assess learners' understanding is minimal, to some extent this reveals their belief that teaching trigonometry is aligned with conceptions of learning held by social constructivism. Teacher 1's learners' significant improvement in developing conceptual knowledge, her high frequency of recruiting conceptual knowledge and attempts to ask high level cognitive questions, although she did not give learners the chance to respond, seems to align somewhat with constructivist theory.

The behaviourists approach is in conflict with the intended constructivist approach of the LTSM. I would argue that the teachers' classroom practices, informed by a behaviourist approach, can be identified as one of the constraints in their mediation of the LTSM (sub-question 4).

Teacher knowledge of how learners learn, as per social constructivist theory and other learning and development theories, is the second aspect that constrains these teachers' mediation of learners' knowledge gaps using the LTSM. Generally, teachers' understanding of how learners learn could inform their choices and decisions about the pedagogical moves and scaffolding processes when mediating learners' knowledge gaps. The choices of pedagogical moves and scaffolded process could also inform the use of the LTSM, or when mediating mathematical content in general.

5.4 Conclusion

In summary, my findings show that learners in the three teachers' classes had knowledge gaps. Two teachers' beliefs and understanding of their learners' knowledge gaps tested correct in the pre-diagnostic test, and only one teacher made incorrect assumptions. However, it is important to note that all three teachers did not use diagnostic assessment in their normal teaching (sub-question 1).

The main finding is the way in which teachers used the LTSM to mediate learners' knowledge gaps. The three teachers used the LTSM in limited and different ways. The way the teachers used the LTSM to mediate knowledge gaps was to model exercises by recruiting procedural knowledge. Teachers mainly used the LTSM for whole class teaching.

The ways the teachers used the LTSM to address learners' knowledge gaps can be explained in their instructional practices, involving their pedagogical moves and scaffolding processes in the classroom (main research question and sub-question 2). Their instructional practices also explain the way they understand the relationship between the LTSM and their learners' knowledge gaps (sub-question 3). The teachers' instructional practices involved teacher talk, modelling and questioning, when using the LTSM and in their general classroom practices. Teacher talk recruited everyday knowledge and conceptual knowledge. Modelling recruited procedural knowledge and questioning recruited high and lower cognition. The frequency at which conceptual knowledge was recruited correlated with the significant improvement in the post-diagnostic test results in the case of Teacher 1's class. The same correlation between the frequency of procedural knowledge and the significant improvement of Teacher 2's class in this types of knowledge.

Teacher 1's alignment with social constructivist theory showed only marginal success in using the LTSM, and through her normal teaching practices in mediating learners' knowledge gaps. This is evident in her class, which showed significant improvement in conceptual and procedural knowledge in two of the five test measures that constitute fundamental pre-requisite knowledge to learn trigonometry.

The classroom practices that constrained the use of the LTSM (sub-question 4) were:

1. The teachers used the LTSM in whole class teaching when the LTSM intention was for group and individual work;
2. Time constraints and large classes;
3. The teachers' questioning pedagogy limited learners' engagement;
4. Behaviourist approaches used by teacher 2 and teacher 3.

CHAPTER SIX:

Conclusion and Recommendations for future research

There are three separate processes that require ongoing investigation and research in the South African context if the complexities and challenges of addressing knowledge gaps are to be understood. My study specifically investigated the ways in which three Grade 10 mathematics teachers used LTSM to mediate learners' knowledge gaps in trigonometry. It is important to bear in mind that this is a small-scale case study over a very short period of time, and presents limited findings. The intention is not to present definitive conclusions and recommendations, or to generalise for all South African classrooms. In order to generalise, a randomised control trial involving a larger scale study and sample size over a longer research cycle and timeframe would be required. The study is rather used as a means to gain some insights into whether and how the three Mathematics teachers in less than average-performing schools, use LTSM in their teaching to mediate learners' knowledge gaps. It is also about how teachers understand the relationship between the diagnostic assessments and how they incorporate the mediation of learners' knowledge gaps into their teaching.

The challenges of understanding the complexities and challenges of how to address knowledge gaps are:

1) Expanding this study to a large scale randomised control trial that will incorporate how teachers involve learners in mediating their knowledge gaps in the context of South African classrooms in order to understand whether these findings can be generalised.

2) A comparative framework and analysis of the types of classroom-based training and support is available to help teachers fill learning gaps effectively.

3) What tools and LTSM and resources exist for mediating learning gaps and how can they be embedded in the curriculum.

6.1 Main Findings and Central Implication for practice

Firstly, the findings indicate that the ways the three Grade 10 Mathematics teachers select and use LTSM to mediate learners' knowledge gaps are informed by their instructional practices. More specifically, it is informed by teachers' recruitment of mathematical knowledge and the pedagogical moves they make. In this study, teachers' pedagogical moves were teacher talk, modelling and questioning and they recruited conceptual, procedural and everyday knowledge.

Secondly, although teachers had a broad understanding that their learners had knowledge gaps, and their views about this are consistent with the findings of the diagnostic assessment, their views were not informed by diagnostic assessment, as this is not part of their normal practice.

Thirdly, there is a strong correlation between the improvement of the post-diagnostic assessment, and the frequency with which the teachers' recruited conceptual and procedural knowledge.

Finally, the teachers' instructional practices fell short of Elmore's instructional core theory, and of the goals espoused by social constructivism. In particular, Teacher 2 and Teacher 3's mediation of LTSM are predominantly informed by modelling and recall questions that align with behaviourism. This conflicts with the intended purpose of the LTSM, which used a constructivist approach in the design. The approach means that learning is a social process, involving groupwork, individual learning, active participation and learner engagement, to develop high cognitive ability and address learning needs. One teacher used a constructivist approach in the use of everyday knowledge, and high cognitive questioning in her pedagogical move of questioning.

Based on the findings of this study, the central implication of this study for professional practice is by providing an insight into the way teachers use diagnostic assessment to select and use LTSM, along with the finding that mediating learning gaps should involve a carefully planned, scaffolded, and learner-engaged process. The process requires teachers to collect information about learners' mathematical knowledge using diagnostic assessment. The teachers should then use the information to inform their instructional practices, pedagogical moves and selection of LTSM. This means that teachers instructional practices would require engagement of learners to participate in their learning by applying more social constructivist approaches, and by helping learners to become involved in the lesson. For example, learners could work with their peers in groups, ask questions in the class, and be assisted to do problem-

solving and develop critical thinking skills. This could help to address their learning gaps, as well as to improve the use of higher cognitive abilities in the process.

6.2 Limitations of the study

The main limitation of the study, as mentioned in the methodology and findings, is that this is a small case study. The findings can therefore not be used to draw generalisations. However, based on the evidence of this study, using diagnostic assessment to inform mediating learners' knowledge gaps through using LTSM is not a simple process. It is part of a carefully-planned process of diagnostic testing and analysis, selection of LTSM and scaffolding, involving the collaboration and engagement with learners.

The design of the study also had limitations. For example, it may have helped to observe more lessons in order to understand teachers' normal teaching practices better. It may have also been useful to analyse the video lesson with the teachers, and for them to have reflected on how they use the LTSM, in order to see if it would help them in their teaching.

6.3 Implications of the research (project and development intervention)

The study identifies areas for further investigation that expand the existing scope to a large scale study in the SA classroom context. It also recommends research into what exists that encourages learner involvement in mediating knowledge gaps; the type of teacher training and support needed, and the tools – LTSM and otherwise – required to address learning gaps. These resources should be used to provide teachers with diagnostic assessment tools and resources in key topics of the curriculum that learners generally find difficult, and in which they lack pre-requisite knowledge i.e. foundational knowledge. The Zenex's LTSM is one attempt to address the need. Given that teachers' knowledge of how learners learn was shown to be limited, training should not only focus on the content of educative materials for addressing learning gaps, but requires a focus on teachers' theoretical understanding of how learners learn, and the interactive relationship between teachers' instructional practices i.e, their pedagogical moves and scaffolded processes in the classroom. The aforementioned will require teacher training to integrate the theories of psychology of learning with classroom pedagogy so that teachers can use their teaching practices to assist learners in addressing knowledge gaps and developing higher cognitive abilities.

6.4 Reflection of growth

At the outset of the study, I was interested in finding a solution to the problem of poor performance in mathematics at the FET level. My personal belief was that the problem must have a solution. As I spoke to a number of teachers in schools that I work with through my employer, teachers elaborated on a vast array of problems associated with poor performance. These teachers attributed the poor performance to learners' socio-economic backgrounds and a lack of mathematical aptitude on the part of learners. Some teachers pointed out that learners come to high school without having basic knowledge in mathematics. I agree with them as this was shown in the pre-diagnostic test results. I also believe that the curriculum and current teaching do not address learners' knowledge gaps. One of the central claims of this study is that teacher's instructional practices should be informed by diagnostic assessment in order for them to understand learners' thinking and learning needs. The teachers' understanding of their learners' thinking is also important for their teaching approaches, more specifically the instructional practice and scaffolded process used to fill learners' knowledge gaps.

Over the course of the study, it became apparent that the issue is not about finding the solution to the complex problem of learners' knowledge gaps, but in understanding the problem better. I also realised that the problem is beyond the use of LTSM, designed for purposes of learning gaps. I realised that the problem is multi-layered and complex and does not have an expedient solution. I do however believe that the research has made a contribution to understanding the problem of assisting learners with knowledge gaps with greater nuance. Amongst its findings, the study has shown the relationship between teachers' understanding of diagnostic assessment, the LTSM, and their learners' knowledge gaps.

Based on the discussion above, I realise that the problem of learners' knowledge gaps and poor performance requires broader intensive investigation and research in the South African classroom context. Teachers, curriculum advisors and developers and teacher training institutions ought to understand the level of expertise and knowledge required by the teacher for diagnostic assessment to be linked to teaching practices. This understanding will help to address learners' knowledge gaps successfully, and ultimately, to improve performance at the FET level. The challenge of gaining this collective understanding, implementing appropriate training programmes, and employing the depth of thinking in the relationship between diagnostic assessment and teaching practice in South Africa remains pressing.

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APPENDICES 1, 2, 3: Teacher Portraits

Introduction:

Appendices 1, 2 and 3 present an account of the three teachers' perspectives, reflections and instructional practice. The teachers perspectives are described as pertains to their understanding of how the diagnostic assessment informs their teaching of congruence and trigonometry. The aim is also to present teachers' understanding of their learners' knowledge levels, and how they use the LTSM to mediate learning gaps based on the information from the diagnostic assessment. I describe the story of each teacher by presenting a portrait of each of the three teachers to encapsulate their perspectives and reflections. The portrait is a descriptive account of teachers' responses and actions as they participated in the research study. The data is fully analysed in Chapter 4. In this section, I provide a portrait of each teacher according to the components of my study. I start with the data collected from the pre-lesson and post-lesson interviews, showing how the three teachers' classes did in the pre-and post diagnostic test, in the specific knowledge constructs and the types of knowledge. Furthermore, I portray what I observed in the three teachers' use of the LTSM, their classroom practice, focusing on the use of the different types of knowledge and pedagogical moves.

APPENDIX 1: Teacher 1

The year 2014 was Teacher 1's third year of teaching in a public school in Umlazi Township, Durban. Teacher 1 has a Bachelor of Technology (Engineering) and Postgraduate Certificate in Education, and is, therefore, a qualified teacher in Mathematics. Teacher 1 taught 67 learners in one Mathematics class.

During the study, Teacher 1 showed better success in the pre- and post-diagnostic assessment and, therefore, her case was of significant interest to this study.

Diagnostic assessment: Pre-lesson interview on 1 August, 2014

a) Teacher 1's beliefs about her learners' readiness to learn trig.

In the interview, Teacher 1 expressed the belief that her learners were ready to learn trig, and that she just needed to do revision with them. Her response to whether her learners were ready to learn trigonometry was:

> *I think it (learners' knowledge) will be adequate because they have done similar triangles, they have done the theorem of Pythagoras, what they have not done because it only starts in grade 10, and it will be the trig ratios, so the trig ratios will be something that will be new to them.* [sic]

The teacher's view indicates that she does not work diagnostically to understand learners' knowledge levels and learning needs. It seems that she assumes that when work was covered previously, that learners would have adequate pre-knowledge to learn new trig knowledge.

Teacher1's view gave an interesting perspective that appeared to change when considering her reflection during the post-lesson interview, specifically when she talks about her reflection on the diagnostic assessment.

In order to assess learners' knowledge levels and understand their learning needs she stated:

> *What I normally do before, I will give them maybe one sum on the board and just ask them to do it themselves. So that is how I originally test if they understand. And if I see that a number of them cannot do the second example I then go to the third one, just to explain it further and find where,*

> *because the learners are not the same; some of them are sharp, they will understand it the first time round you explain it to them, but with the others we need to be patient. Sometimes you will not be even able to explain it with those three examples; you would need to have a one on one with them.* [sic]

The comment indicates that Teacher 1 understood the need for formative assessment and expressed the belief that, by repeating examples, some learners who do not understand will eventually grasp the content. She acknowledges that some learners may require individual attention beyond this, and when I explored with her how she accommodates this need, she advised me that learners would come to her before class starts in the morning and during breaks. It must be noted that the individual attention given by Teacher 1 depends on the learner initiative to approach the teacher if they do not understand. In addition, there is no structured intervention or process in place to ensure that all learners are given such opportunities.

b) Teacher 1's perspectives on assessment

Teacher 1 explained that the way she gave feedback to learners was that she would say to learners:

> *'This is where you went wrong and this is where you can improve.'*

She employed the following assessment practices as explained in the pre-lesson interview:

> *...because at the end of every chapter I usually give them a class test. The final exam will be the departmental exam, but we do have our class tests we give them, we do have the assignments we give them, we do have investigations that we give them.* [sic]

The feedback practice described by Teacher 1 sounds like a standard response. It does not indicate that the teacher applies a thoughtful, carefully-selected and well-crafted process to ensuring more engagement of the learner in the feedback process. Similarly, it appears that the assessment practices she describes are also standard, and not diagnostically informed.

Diagnostic assessment: Post-lesson Interview on 15 August 2014

When Teacher 1 was asked whether the research made a difference to her teaching, she highlighted that:

> *I would say yes in a way, because especially when you gave me the diagnostic test results, you know, we don't have time to do those with our learners, you just start a section and you will sometimes assume they will have an understanding of the previous grades, not knowing that there are so many gaps. You will just assume that they are in grade 10 and should know the angles of a triangle add up to 180, yet some of them don't. So it did change my way of thinking when I am preparing the lessons.* [sic]

The diagnostic assessment influenced Teacher 1's planning as follows:

> *The test, it showed me I couldn't just come to class and talk about trigonometry, which was something they had never seen before. They [learners] had to go home and read prior to coming to class for the lesson.* [sic]

The above shows that Teacher 1 did not use diagnostic assessment in her normal teaching, and that her teaching assumes learners have learnt and understood the pre-requisite trig content from Grade 9. According to Teacher 1, the research created awareness in her thinking and planning of lessons. This awareness may be a step towards changing Teacher 1's practice towards working more diagnostically.

When asked how the diagnostic assessment informed her teaching, she explained:

> *The diagnostic test did have some influence, especially in the trig, they knew nothing about trig, and they had never done it before, so I had to be very careful that I was introducing the lesson just to let them understand about the trig ratios. But before I even introduced the lessons I asked the learners to go home and just read on their text book on trigonometry about the ratios, to have an understanding before I even started teaching the lesson. So it did inform me that the learners didn't know anything about diagnostic test – I mean the diagnostic test did help me to know the learners knew nothing about the trig. And also on congruency, some of*

> *them didn't understand. I assumed when I looked at the diagnostic test they had done congruency in grade 9 because that is when it is done, yet they did not do it. Or I wouldn't say they did not do it but maybe they forgot, or they didn't teach them because they don't know.* [sic]

The diagnostic assessment results raised Teacher 1's awareness about its importance of identifying what learners did not understand. It seems that she places weight on her intervention of requesting learners to read up on an upcoming topic in preparation for her lessons. Learners pre-reading on a topic does not deal with their knowledge gaps and requires the teacher to plan the support that she gives learners, and how she engages them in and outside the classroom.

Teacher 1's class results on pre-and post diagnostic test

The overall pre-test mean scores for the five constructs (triangles, right angles, surds, fractions and ratios) of trig readiness show mean scores of 6.81 for Teacher 1's class. Teacher 1's class obtained mean scores of 1.06 in identifying similar triangles out of three scores; 0.46 for Pythagoras out of three scores; 0.69 for adding and 1.13 for simplification of ratios out of four scores in the pre-test. The scores show gaps in areas that are fundamental for readiness to learn trigonometry. Teacher 1's class shows a better starting base in the fundamental knowledge of skills required for trig readiness.

Teacher 1's class showed the highest improved score for trig concept readiness of the five constructs (triangles, right angles, surds, fractions and ratios) in the post-test showing a paired difference of 2.507 and statistical significance in the t-test.[21] The specific scores are 9.31 for Teacher 1's class. However, these scores show that learners are still not ready to learn trigonometry. Teacher 1 shows just over a third percent in mean score.

The statistical significance means improvements in identifying similar triangles (0.627), Pythagoras theorem (0.388) and Fractions (0.537) influenced the score. The significant improvement[22] in identifying similar triangles and the Pythagoras theorem shows improvement in two of the three areas fundamental to trig readiness.[23] Teacher 1 was the only teacher that

[21] In this case, the interval from pre- to post-test ranged from 1.644 to 3.371 indicating the 95% confidence level.

[22] Teacher 1's class shows average improvement in measures such as angles measuring (0.239) and identifying 180-degree angles (0.060), surds (0.224) and Ratios, adding (0.328) and Simplification (0.179). These average improvements were not statistically significant.

[23] Furthermore, the overall trig readiness results for Teacher 1's class of 3,539 show that there is a disbursement of results away from the mean that may result in high variation in scores that affect the overall mean results. The

showed significant improvement in identifying similar triangles and Pythagoras theorem, two of the three areas that are fundamental to trigonometry readiness. Teacher 1 shows interesting patterns from pre- to post-test. First, her learners' scores started off better in the pre-test. Secondly, she showed significant improvement and higher scores for overall trig readiness in the post-test. Thirdly, hers is the only class that showed significant improvement in two (identifying similar triangles and Pythagoras theorem) of the three test measures fundamental to trig readiness. Fourth, Teacher 1's class improved significantly in both conceptual and procedural knowledge.

Teacher 1's use of LTSM

In response to the interview questions about whether she used the LTSM, Teacher 1 indicated:

> *Yes, we did. I used especially the one where they had to measure the sides of a triangle; although I didn't have enough time to do all of that in class, I have given it to them to do, just to practice at home. And one where they had to draw a triangle using the size of the angles, and constructing the triangles – I also gave them that one.* [sic]

It is important to note that during the interview, Teacher 1 explained that she had misplaced the LTSM (due to the interruption of her hospitalisation), and thus did not use it at the time of recording Lesson 2 on trig.

The observations confirmed the quote in the video lesson on congruence and similar triangles that the teacher used the LTSM in a class exercise, and provided them with worksheets to finish the exercises at home. The way learners responded to the lesson was different according to Teacher 1:

> *It was the way they responded because sometimes if you don't tell them to go and read up and try to introduce a new concept, they will just look at you, because they don't even know what you are talking about. Now, because they went home and read through, even though they didn't*

standard deviation for identifying similar triangles (1.347), adding ratios (1.133) and simplifying ratios (1.242) contributed to an overall standard deviation of 3.539, and this variation influenced the overall mean score.

> *understand 100%, at least they did have some background or prior knowledge of what I was supposed to be teaching.* [sic]

The comment shows that Teacher 1 believed that a way of helping learners to gain prior knowledge is by giving them background reading. She also believed that her giving learners' background reading helped learners to respond better to the lesson. Teacher 1 reported that she used the following textbooks and LTSM:

> *For the trigonometry, for the activities I will use the 'Classroom', but for me to explain the mathematics to the Grade 10s, sometimes you find the 'Classroom' doesn't explain the concept better, so I will go to another book, I like this one: The Maths Handbook.* [sic]

In addition, she indicated that she used:

> *Platinum, and also there is this book called Everything Maths* (Interview 1 August, 2014).

The above shows that Teacher 1 used a variety of LTSM, as resources. She also explained the problems with the LTSM as follows:

> *I did use the material, but I had to add some of the stuff in my understanding e.g. there was an activity where they had to cut, I had to take that one out because we didn't have enough time, and there are so many of them in a class so I couldn't use that one. So I had to arrange it, just try and pick up the ones that I was able to use in class. I would say there were some problems, as I mentioned the one about the cutting, because it was not part of the current curriculum for the grade 10s I wouldn't do it with my grade 10s, I would do it with my Grade Nines.*

The comment showed that the teacher had to make choices about how she used and adapted the LTSM given the large classes. Teacher 1 only used the LTSM in Lesson 1 on congruent triangles. During Lesson 1, Teacher 1 modelled three examples of congruent triangles, and provided learners with worksheets from the LTSM to identify similar triangles. I observed that she had read the teacher notes from the LTSM for understanding mathematical concepts. The teacher displayed her reading of the notes from the LTSM by giving examples of congruent and similar triangles. The teacher used an example from the LTSM to build conceptual

knowledge, specifically to build learners' understanding of the properties of congruent triangles. The teacher used the example as from the LTSM, and did not make any adaptation or revision to the LTSM. The example dealt with the knowledge gaps identified in the diagnostic test, specifically to identify congruent triangles through knowing their properties. Teacher 1 embedded the mathematical concept of congruence in an everyday example of identical twins. The analogy of twins was not covered in the LTSM, showing that Teacher 1 adapted the LTSM to make the explanation easier to understand. Teacher 1 used this example in an abstract explanation of twins to show that congruence is about identical measures between two people or objects. The teacher's use of an everyday example to help build conceptual knowledge is part of the intended purpose of the LTSM. The teachers' use of the LTSM does deal with the knowledge gaps in conceptual knowledge as shown in the pre-diagnostic test results.

Teacher 1's instructional practices

When I asked Teacher 1 about instructional practices, such as modelled examples on the board, memory techniques, pair and group work she responded as follows:

> *Okay, when I am introducing a section or a new concept what I usually do, I give them an example on the board. I write an example for them on the board, I do it with them on the board, and I explain it to them – that is how you are supposed to apply your knowledge based on that example. And then I usually do three examples and then the second example I will let them, I will write it on the board again and I will let them do the solutions on their own, just to check if they understood what I explained in the previous example – so that is the way.* [sic]

The evidence on the observations confirmed that Teacher 1 did explain examples, mainly through teacher talk and modelling, where she also gave learners exercises in class and for homework.

> *So I will ask them to understand, we have to do three trig ratios and I will show them a way of memorising them. I usually use the one I was taught in school because it always worked, the 'so-ca-to-ha', so that is how they will memorise the trig ratios.* [sic]

I did not observe the memory technique that Teacher 1 described in the two lessons, however, it is a common technique as Teacher 3 also referred to it in his interview:

> *The pair work I usually use it during the class work but not all the time. The only time where it works for us is when we are giving them an investigation, because sometimes before we even start the section we will give them an investigation based on the section that we will start maybe the following week, we will give them an investigation just to go and read or find out more about the section which we are about to start.* [sic]

Teacher 1 did not use pair work in class as she used whole class teaching. It is unclear how pair work happens when learners are asked to "read or find out more about the section". Pair work would require a clearer explanation of what the task is, and what is expected from the joint work. Reading could be an individual task, unless pairs are assigned specific roles on what is to be read, and for what purposes it should be read. The teacher would also need to plan how she will know that learners have read and understood the work.

Teacher 1's instructional practices (pedagogical moves and scaffolded processes)

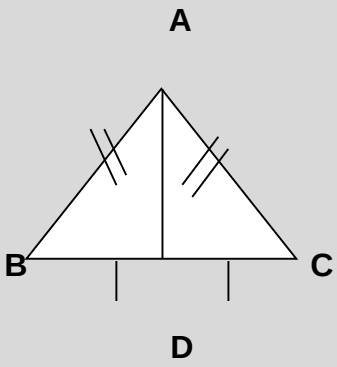
Teacher 1 worked with conceptual knowledge 17 times, and used this type of knowledge four times more than she used procedural knowledge. She only worked once with everyday knowledge.

In her pedagogical moves, Teacher 1 used high level cognitive questioning – specifically open-ended questioning – the most (17 times), and closed questions 15 times. Teacher 1 used questions that assess learning the least (6 times).

The table below gives a portrait of the coded categories and provides a description or example of Teacher 1's pedagogical moves in line with each of the codes.

Coding (content knowledge and instructional practices)	**Pedagogical move**
Conceptual knowledge	In the second lesson on trig, Teacher 1 offered the following concept explanation: *So, trigonometry is also based on these two concepts: similar triangles and the theorem of Pythagoras. So if you understand 'ama*[these]*similar' triangles and if you understand theory of Pythagoras, then you won't have any problem to apply any trig ratios once we start introducing 'amatrig' ratios used in trigonometry.* [sic] (Video lesson on trigonometry, 13 August 2014). The teacher used 'teacher talk' when she explained the concept.
Conceptual knowledge:	Teacher 1 checked reasoning in the lesson on congruency by asking learners to prove how two angles are congruent on the board, and to provide reasons for why these are congruent [Refer to triangle XYZ and triangle ABC in the transcript of the lesson on congruence]. The question she asked required learners to provide a reason as follows: *Why do we say they are congruent?* (Video lesson on congruent triangles, 1 August 2014).
Procedure & techniques	The quote provides an example of her explaining a procedure for identifying trigonometry ratios: *So in order for you to identify your trig ratio's you need to look at the position of Uheni, for example ama position ama*[these]*triangle you look at your theta, Uheni.* [sic] (Video lesson on trigonometry, 13 August 2014).
Everyday knowledge	In the lesson on congruence, the teacher embedded the following:

	When we talk about congruency we are talking about triangles. They are twins, but they are not the same. They will have the same genes, but they are not the same. When we are talking about identical twins, their eyebrows, their [eye]*lashes... everything about them is the same. They are identical. They are exactly the same. Their angles are the same; their sides are the same.* [sic] (Video lesson on congruent triangles, 1 August 2014). The teacher uses twins as an everyday example that learners can identify with to explain similar triangles.
Open questioning	Open-ended question during congruence lesson: *What did we say amasimilar triangles were? Angiti (Not so) we see amatriangles ukuthi are similar? So how did we conclude that ukuthi two triangles were similar?* [sic] (Video lesson on congruent triangles, 1 August 2014). The question required learners to draw on their concept knowledge of similar triangles and provide supporting logic and evidence.
Closed questioning	*For us to prove that triangles are congruent we need to prove how many conditions? And we have also found the unknown sides using the theory of Pythagoras, am I right? The hypotenuse side… does it ever change?* [sic] (Video lesson on congruent triangles, 1 August 2014)
Recall questioning	Two recall questions that teacher LO asked during the introduction of the concept of congruence are listed below: *What do you remember about congruence in Grade Nine? And then when we did the theory of iPythagoras… where did we use it?* [sic] (Video lesson on congruent triangles, 1 August 2014).
Assess learning & understanding	*Their angles must also be… equal, angithi* [understand]*?* [sic] and during the lesson on trig she asked: *I have angle ubani(which one) of 90 degrees and I have angle ubani of theta. Understand?* [sic]

	(Video lesson on congruent triangles, 1 August 2014).
Modelled exercises, examples	Teacher 1: "*Okay, let's do another example, but you guys will do this one*." [sic] [writes on board**]** Teacher 1: "*We must prove that triangle ABD and triangle ACD are congruent, angithi?* [*Not so*]*."* [sic] Example written: Prove that Δ ABD ≡Δ ACD Understand?  Teacher 1: "*In geometry we said in order for us to understand correctly what do we use? The table Is angithi?* [*Not so*] *yes? The statement and the reason column. So there is my table… statement…*" [sic] [Teacher 1 writes on board] Solution: She request individual learners to complete the solution and checks their answers with the class.

<u>Teacher 1's overall reflection of video lessons</u>

Teacher 1 felt more relaxed in the second lesson, and felt that both she and the learners had enjoyed it. She said:

> *Yes, it was the second time around, I was more relaxed; I was used to it. I actually enjoyed it.* [sic]

For Teacher 1 in the lesson on congruence the practical exercise worked best, as she describes:

> *The one I also enjoyed and I think the learners enjoyed and I will say it worked best was the measuring of the angles, because as I said before,*

> *some of the learners you will assume they know that the angles of a triangle add up to 180, but some of them you will ask and they will tell you 90 degrees, but when they were measuring the angles, once they drew it, according to the size of the angles using the protractor and they measured and they saw when they added all the angles it added to 180; and I think for me it was a big thing and I don't think they will ever forget that because they actually did it and measured; it was not like I was telling them. It is a different story when you are telling them and when they are actually doing it and seeing that they do add up to 180.* [sic]

Summary of Teacher 1

Teacher 1's view in the pre-lesson interview, that her learners were ready to learn trigonometry, shifted. She explained in the post-lesson interview that she realised that one could not assume that learners know the basics, even if they were taught these previously. The pre-diagnostic assessment showed that her learners were not ready to learn trigonometry. Teacher 1's class showed significant improvement in the post-diagnostic test in two constructs, similar triangles and Pythagoras that are fundamental pre-requisite knowledge to trig. Her class also showed significant improvement in conceptual and procedural knowledge

Teacher 1's use of the LTSM was minimal and not sufficient to explain her learners' success in achievement in the post-diagnostic test. The achievement of her class is attributable to the learners' significant improvement in two constructs (similar triangles and Pythagoras) that are fundamental pre-requisite knowledge to trig and in conceptual knowledge and procedural knowledge. Teacher 1 recruited concept knowledge the most when compared with the other two teachers. Teacher 1's pedagogical moves shows high frequency in use of open-ended questioning, developing higher cognitive ability in the classroom, and her pedagogical moves aligned with social constructivist approaches. There is a correlation between the frequency of Teacher 1's use of conceptual knowledge, with the significant improvement of her class in conceptual knowledge in the post-diagnostic test.

APPENDIX 2: Teacher 2

Teacher 2 has seven years teaching experience and has a Bachelor of Arts with a Higher Diploma in Education & Advanced Certificate in Education (Maths GET). Teacher 2 has 78 learners in his Mathematics class.

Diagnostic assessment: Pre-lesson Interview, 1 August, 2014

a) Teacher 2's beliefs about his learners readiness to learn trig.

In response to the question on learners' knowledge levels, Teacher 2's response was as follows:

> *I think the class has got a mixed ability of learners: most of them are not ready, actually, to do trig, only a few of them. Like the concept we should have learnt in the previous classes, like Eight and Nine – I usually have to repeat those concepts. Like some of them are not clear about Pythagoras' theorem and things like that.* [sic]

The strategies that Teacher 2 used to help learners with learning gaps usually involve revisitng that which they have already learnt in the previous term or the previous grade. He said:

> *And then I try to make them understand the concepts that will help them understand the topic that you are dealing with at the time.* [sic]

The diagnostic test confirmed Teacher 2's understanding of his learners in that the class had mixed abilities, and they were not ready to learn trig. This shows that Teacher 2 is aware of his learners' knowledge gaps, and he believe that he should repeat teaching concepts from Grades Eight and Nine.

b) **Teacher 2's perspectives on assessment**

Teacher 2 emphasised that the large classes were a challenge when it came to assessment feedback. Thus, on an individual basis his way of giving a learner feedback when they make mistakes has been:

> *I tell the learner politely, no, what you did was incorrect, and you have to come to me and check where you went wrong.* [sic]

Teacher 2's challenge of large classes shows how it constraints his assessment and feedback practices. He relies on learners approaching him for help on where they went wrong. According to Teacher 2, he used the following means and examples to give assessment feedback:

> *Like, if I give them homework, if I give them tests, then what I will do is we will go through the test, and usually I will do that and come in before and prepare and then do it in the classes. Then they will go through the test, and then I will go through the homework so that I can save time.*

I observed what Teacher 2 said in the quote above in his lesson on trigonometry. He used a carefully selected scaffolding process in class, doing s revision of homework. The process did help develop learners' cognitive abilities, as he carefully assessed their learning and understanding in his use of questioning in his pedagogical moves. In addition, Teacher 2 carefully explained that:

> *If I am able to, I will make copies like the exam papers, the common papers and then after they have written the paper and after I had marked the papers I will come with a memo, and usually they do it again in groups. It is not only the two Grade 10 classes that I have, but I also teach two Grades 11. Then, it would be no use if I give them a test now and then I gave them feedback after two weeks.* [sic]

Teacher 2 draws from a range of resources as part of his assessment practices. He advises that:

> *In most cases, I use previous exam papers; I just pick questions from the previous exam papers set by the department and the study guides ('Table Maths' and other) and I just give questions, which are separate. And I also pick up some from the set books – the 'Table Maths' or whatever.* [sic]

Teacher 2 seems to have developed a set of assessment practices that works for his teaching, given the constraints of time and large classes. He used a carefully selected scaffolding process in class and prepares learners on exam-type assessments and uses memos to facilitate group

discussions. I did not observe the practice of the group work in the two lessons, but the scaffolding was confirmed in the observation of the lesson.

Diagnostic assessment: Post-lesson Interview, 20 August, 2014

I asked Teacher 2 whether the diagnostic tests informed his teaching, and his response was: "*in fact, it was just a bit of the results from the test that I used*." Teacher 2 reported that the way he checked learners' understanding or assesses learners' knowledge to be the following:

> *And you know, in the classes, we do mostly demonstrate on the board, then we give them what to do, and you see some of them are not doing it right.* [sic]

Teacher 2's response to the diagnostic test was vague, and there is no evidence that he uses diagnostic assessment in his normal teaching. He (Teacher 2) did assess learners' understanding in the lesson on trigonometry that I observed. The observation confirmed that he also modelled examples on the board. Teacher 2 was concerned about the class sizes and offered his ideas of what he could do to manage his marking:

> *Yes, another thing that I was thinking of is, if I can use the groups that I already have, and use the groups to help me mark the other learners work, but I don't know if this is appropriate or not, or if it is just class work.* [sic]

Teacher 2 sought guidance from me as the researcher about whether he could use his learners to mark other learners' work. It was uncomfortable for me to give guidance and maintain my independence as a researcher. My response was to give examples of what I saw other teachers do that worked in their classroom environments and advised that ultimately he has also to check learners' marks if he does use learners to do each other's marking.

Teacher 2's class results on pre-and post diagnostic test

The overall pre-test mean scores for the five constructs (triangles, right-angles, surds, fractions and ratios) of trig readiness shows mean scores of 5.63 for Teacher 2's class. Teacher 2's class also show similar trends to Teacher 1's class. Learners' lack of readiness to learn trig is evident in the mean scores of Teacher 2's class, which averaged 0.97 in identifying similar triangles out of three scores, and 0,28 for Pythagoras out of 3 scores. The learners' lack of readiness also

shows when looking at the mean scores of 0.42 for adding and 1.03 for simplification of ratios out of four scores.

The scores show gaps in areas that are fundamental for readiness to learn trigonometry. Teacher 2's class are slightly less ready than Teacher 1's class, who showed a better starting base in the fundamental knowledge of skills required for trig readiness. The post-test shows statistically significant improved mean scores in overall trig readiness of the five constructs (triangles, right angles, surds, fractions and ratios). The specific scores are 7.13 for Teacher 2. However, these scores show that learners are still not ready to learn trigonometry; Teacher 2 shows mean scores of less than a third in overall trig readiness.

Teacher 2's class also showed statistically significant improvement in overall trig readiness, showing a paired difference of 1,500. The improvement is the lowest improvement of the three teachers, but it is statistically significant.[24] The class showed significant improvement in both measures of fractions; multiplication and division (0,417) and adding (0,722). None of these significant improvements were in the areas that were fundamental to trig.[25]

Teacher 2 lags behind Teacher 1 from pre-to post-test. He shows a middle attainment path between the three teachers with his learners' scores showing a slightly lower start off than Teacher 1's class, but slightly better than Teacher 3's class in the pre-test. His class showed significant improvement and second highest scores for overall trig readiness in the post-test. His class did not improve significantly in the pre-requisite constructs that are fundamental to trig. Teacher 2's class improved significantly in procedural knowledge.

Teacher 2's use of LTSM

Teacher 2 found the LTSM useful. However, he emphasised challenges of the class sizes and time constraints:

[24] Teacher 2's class also improved on measures of triangles; drawing (0.125), identifying similar triangles (0.111) and theorem of Pythagoras (0.069), surds (0.042) and adding ratios (0.083).These were average improvements, but it is not possible to be 95% confident that this average improvement was not due to variation within the data.

[25] The mean score of the class was influenced by the standard deviations in identifying similar triangles (1.449), ratios simplification (1.245) identifying angles (1.186) ratios (adding) and both measures of fractions: adding (1.078), multiplication and division (1.071). Teacher 2's class had more or less the same standard deviation as Teacher 1's class, an indication of some outliers of high and very low scores, which influenced the overall mean for trig readiness.

> *No, it is definitely the document you gave us was quite useful but I suppose it could only work nicely if we had …* (smaller numbers and more time?). [sic]

Teacher 2 raised his challenges of using the material and gave the following example:

> *Like the lesson you showed me in the booklet, I could not let them cut the triangles because it was too time consuming, otherwise they would not have done the lesson" So I would give them the triangle and let them draw and work from there.* [sic]

Teacher 2's concerns about large classes and time constraints were also raised by Teacher 1 and, in particular, this dominated Teacher 2's views in the post-lesson interview. The problem does show up in the teacher's approach to assessment and in the use of the LTSM as well as in his whole class teaching observed in the video lesson. On the question of Teacher 2's use of textbooks and LTSM he responded:

> *I use different books, books and study guides; I even got some books from the internet.* [sic]

The researcher listed the following as confirmation of the LTSM that Teacher 2 used: Classroom Mathematics, Answer series, some internet, all different…Study guides. Teacher 2 responded:

> *So whenever I do a topic I will take out the books, check out, look at which is best and then make a lesson and pick up exercises from there.* [sic]

The above shows that Teacher 2 uses a variety of LTSM as resources in his teaching. The use of a variety of LTSM does require careful planning and decisions about what LTSM he uses at what points in the lesson and to what end. Teacher 2 used the LTSM during the lesson on congruent triangles, and he used it together with the classroom mathematics textbook. I questioned Teacher 2 on his reasons for not using the LTSM for the trig lesson. He responded in a letter as follows:

> *About the resources you suggested, I decided to design my own worksheet because of the large numbers of learners in the class. I felt it would take a long time to use your*

> *material. The worksheet used was based on the topic that has been covered in the trigonometry*. [sic] (Emailed letter, 10 November, 2014).

The above response indicates that Teacher 2 did engage with the LTSM. He provided two clear reasons for not using the LTSM during the second lesson on trig. Teacher 2 modelled an example on the board, and he asked different types of questions (closed questions and questions that assessed learners' understanding). The example he modelled was not from the LTSM, but was from the class textbook. This choice of using the textbook showed that the teacher made choices about when to use the LTSM in the lesson. Teacher 2 gave learners handouts from the LTSM for classwork. Teacher 2 used the LTSM to build conceptual knowledge by eliciting learners' reasoning and asking questions from them. Teacher 2 used the textbook to model examples and used the LTSM to build conceptual knowledge, specifically by eliciting learners understanding of congruent triangles.

Teacher 2's use of the LTSM was limited to lesson one on congruent triangles, and the use does not sufficiently explain his significant improvement in procedural knowledge. It also does not explain his middle attainment path.

Teacher 2's Instructional practices

a) Teacher 2's perspectives about instructional strategies

Teacher 2 said that he grouped learners and explained as such:

> *I group them in accordance with their ability. I take the learners that are in higher level, and I take the learners that are in the lower group, the lower level. And then the one who is the stronger one becomes a group leader and then if the group leader has a problem they come to me with a problem, I help them out or come with his group and that way we can quickly go back and teach the concept.* [sic]

The observations showed that Teacher 2 used whole class teaching, and there was no evidence of the group work described above during the observations. Another way that Teacher 2 assists learners is, as he explained:

> *I revise it with them; basically I end up re-teaching some of the concepts. Similar triangles, I think it was the first time that they heard of similar triangles in grade 10 them last term.*[sic]

In the pre-lesson interview, Teacher 2 mentioned that:

> *Usually it is like we are using all the time the telling method because of the number of learners. Yes, we are lecturing to the kids.* (Pre-lesson interview, 1 August 2014).

During the lesson on trigonometry, there was evidence of revision of homework in class. The observations from the video lesson also showed that the "telling method" [teacher talk] was dominant in Teacher 2's class. Teacher 2 used procedural knowledge more frequently (28 times), he worked with this knowledge 16 times more than he used conceptual knowledge. Teacher 2 did not use everyday knowledge as a vehicle for building conceptual knowledge.

Teacher 2 used questioning techniques the most (49) during the two lessons. Teacher 2 also used the most number of closed-ended questions (23) among the three teachers. Teacher 2 and used recall questions the least. Teacher 2 made more use (11 times) of questioning for assessment of learning and understanding than the other two teachers, whose frequency of use was the same (6 times).

Teacher 2's Instructional practices (pedagogical moves and scaffolded processes)

The Table 2 below gives a portrait of the coded categories and provides a description or example of teacher 2's pedagogical moves in line with each of the codes.

Coding (Content Knowledge and Instructional Practices)	Example
Conceptual knowledge: teacher-led	Teacher 2 provided the following content definition and explanation of the concept in the lesson on congruence: *When triangles are congruent, we say the triangles are the same in every respect.* [Writes on board] *By this, we mean their corresponding sides and angles are equal. So for us to say that two triangles are equal, we mean that two triangles should be similar or the same in every respect.* [sic] (Video lesson on congruent triangles, 1 August, 2014).

	In the second lesson on trig, Teacher 2 offered the following explanation: *Remember the trig ratios we said, describe the relationship between the angle and the ratio of the sides. Okay.* [sic] (Video Lesson 2, trigonometry, 13 August, 2014).
Conceptual knowledge: learner reasoning questions	Reasoning question from Video Lesson 1 congruent triangles, on 1 August 2014, Teacher 2 asked learners to provide reasons as to why they thought that two triangles from the handouts were congruent as follows: *Right, can you then tell me why you decided that number two and number eight are the same?* [sic]
Procedures & techniques	*So when given a problem such as this one, we need to look at the position of the angle, look at the side that is given to us and then look at the side of which we are required to find the length. And then try and determine which, between the three trig ratios and their reciprocals, we should use.* [sic] (Video Lesson 2, trigonometry, 13 August).
Everyday knowledge	Teacher 2 did not embed concepts in the everyday context or used everyday illustrations or examples during the two lessons observed.
Open questioning	Open-ended question Trig lesson: *What comes into your mind when we say our topic today will be solving right-angled triangles? What do you think we will be solving?* [sic] (Video lesson 2, trigonometry, 13 August, 2014).
Closed questioning	*What trig ratio do you think we should use to find the length of ab*? (Teacher 2, Video lesson 2, Trigonometry, 13 August 2014).
Recall questioning	Teacher 2 asked the question: *Right remember in our last lesson we looked at how we calculate the sizes of angles when we are given the ratio. Can you please calculate the size of this angle for me?* [sic] (Teacher 2, Video lesson 2, Trigonometry, 13 August 2014). The question required learners to retrieve knowledge in determining

	angles using trig ratios.
Assess learning & understanding	The way the teacher 2 assessed learners' understanding of trig homework was by asking questions when he revised homework. Teacher 2 also checked learners understanding of the work when they did class exercises, or sometimes he asked learners' directly if they understand or whether they had problems. During the lesson on trig for example teacher 2 asked: *Where was the problem? Which sum did give you a problem? Number? Can you all understand why I am writing 1/4 this side? Who doesn't understand why I am writing 1/4 this side?* [sic] (Video lesson 2, Trigonometry, 13 August 2014).
Modelled exercises, examples.	The teacher modelled homework exercises by checking learners' answers first and then scaffolded through the example using question and answer, and involving learners by eliciting their answers in each step of the calculations and writing on the board in this way: **e) 2 x tanΘ = 1.** Teacher 2: "*Right, what size of an angle did you get for that sum?"* [sic] Learner responds: "*26.6."* Teacher 2 asks "26.6°?" Learner responds: "*Yes."* Teacher 2: "*Right. This Θ here was 26.6° but remember yesterday we said when we solved these equations; we need to do the following steps. We need to start by isolating the trig ratio. So in this case you must get rid of the co-efficient of this or the two. And how do we get rid of the two, we divide by two both sides?"* [sic] Learners respond: "Yes". Teacher 2: "*So we'll end up with a tanΘ = 1/2a and then our Θ will equal the tan of 1/2 which then will give us a tan as, what did you say was the value of Θ?"* [sic] Learner: "*6.6."*

	Teacher 2: "*Right, 26.6, because we were told to write the angle correct to one decimal place."* [sic] The complete calculation written on the board is as follows: **e) 2 x tanΘ = 1** **tanΘ = 1/2** **Θ= tan(1/2)** **Θ= 26.6°**

Teacher 2's overall reflection of video lessons, Post-lesson Interview, 20 August, 2014

Teacher 2 indicated that the lesson he had covered the lesson previously in another topic. Thus his response was:

> *I think the last lesson went well because, in fact, some of the work we had done when we did geometry.* [sic]

When I asked what was different about this lesson his view was:

> *There were more responses from the kids. Usually, when we go to class because we are rushing to cover the syllabus because we just go on and go on tell the… learners. Yes. We just go on and go on. But with these two lessons I had to involve the kids and make them talk and respond.* [sic]

Summary on Teacher 2

Teacher 2's view in the pre-lesson interview that his learners were not all ready to learn trigonometry correlated with the findings of the diagnostic assessment.

The pre-diagnostic assessment showed that Teacher 2's learners were not ready to learn trigonometry. Teacher 2's class did not show significant improvement in the post-diagnostic test in any of the constructs that are fundamental pre-requisite knowledge to trig (similar triangles, Pythagoras and ratios). His class showed significant improvement in procedural knowledge.

Teacher 2's use of the LTSM was minimal, and not sufficient to explain the learners significant achievement in procedural knowledge in the post-diagnostic test. Teacher 2 recruited procedural knowledge the most compared with the other two teachers. Teacher 2's pedagogical moves shows high frequency in use of questioning to assess learner understanding developing higher cognitive ability in the classroom. He also used modelling and high frequency of closed questions. His pedagogical moves aligned with behaviourism. There is a correlation between the frequency of Teacher 2's use of procedural knowledge with the significant improvement of his class in this type of knowledge in the post-diagnostic test.

APPENDIX 3: Teacher 3

Teacher 3 has been teaching for the last 26 years. He holds a Bachelor of Arts with a Higher Diploma in Education, an Honours degree in Education Management and Leadership and is currently studying towards an Honours degree in Mathematics. In the lessons observed in this study, Teacher 3 taught 48 learners Mathematics.

Diagnostic assessment: Pre-lesson Interview, 9 April, 2014

a) Teacher 3's beliefs about his learner's readiness to learn trig.

Teacher 1 told me more about his learners' background knowledge stating that:

> *Our Grade 10 this year is I don't know the right term to call them, but they are learners from other high schools around us, who have failed their Grade 10 in those high schools. They are repeating here, they come all of them, almost 90% of them. I happened to read a report of one who was repeating, and there was not a single subject passed. You see? And I wonder what we are going to do with them, because we are not a special school. I would understand if it was a specialised school, remedial work.* [sic]

Teacher 3 checked learners' understanding and their learning needs as follows:

> *At the moment I use to take them having no background on it. Only that I tell myself: 'they don't know'. Then that is my approach, then I introduce the concepts to those who know nothing about it.* [sic]

Although he indicates that he assumes that learners have no background, this is an incorrect assumption, as the results show that his learners did have knowledge gaps, rather than a total lack of knowledge as they did not score zero or null in the test.

Teacher 3's perspective on assessment

Teacher 3's perspective, on formative assessment, was directed at assessing understanding after teaching and feedback as follows:

> *Sometimes I do have what I call informal tests. I get those from the department, but those I formulate myself respective of the number, irrespective how many are they. But if I see the need maybe four or five questions to just to check how far they've grasped the concepts. The tests are very reliable and because from them I can detect whether my lesson was with it or not.* [sic]

He explained further:

> *And what I think is also important to guard against is that you must not assume that they have understood. So ask this short questions than these short questions are going to throw light whether they've understood or not.* [sic]

The quote that follows is Teacher 3's response to the issue of feedback to learners. He referred to the importance of immediate feedback:

> *After all this, this is coming to an end of a term, so the department prepares a controlled test. But before they write the control test, we do some kind of revision of all we have taught them. In this revision, they are getting short exercises, and there is immediate feedback. So it must mean something to them because if you just let them write an exercise and there is no feedback it becomes a problem to them.* [sic]

Teacher 3 described how he checks whether learners have grasped the concepts i.e. assessing learners understanding, based on the content he taught them in class. He also gives learners short exercises and revision as a way to provide them with immediate feedback. While assessing for understanding is formative assessment, it is evaluative, and does not serve the same ends as a diagnostic assessment. I observed the teacher's belief about assessing for understanding during his teaching of Lesson 2. I did not observe the revision that Teacher 3 refers to during the two lessons.

Diagnostic assessment: Post-lesson interview 20 August, 2014

Teacher 3's reflections on the diagnostic assessment are encapsulated in his own words below:

> *A diagnostic test, it helps a lot, because from it you discover the background of each and every learner as far as the lesson or subject is concerned. Now I think that can guide you a lot, as how much of work you can push into that. So your results are this, it means they have a good background, so it means even your pace then should be faster, it determines your pace. If you find they are lacking it is of no use for you to push in more work to those learners because they don't understand it. So go back a little bit and introduce a little bit, go back and introduce a little bit, until you are satisfied they are coping.* [sic]

His advice to other teachers was:

> *To me it doesn't help much if you just introduce new things all the time, yet you are sure there is a gap in this. So you first fill the gap, or try and if you have done it yourself there is a little bit of confidence and hope that maybe they have understood this, so maybe if I bring this they will understand.* [sic]

Teacher 3 indicates his belief that the diagnostic assessment was helpful, and his understanding is that it could assist in guiding how much work a teacher does. He also stated that it could help teachers with pacing, and described how scaffolding can happen to help support learners and assist them to fill the gaps. This is what the diagnostic test aims to achieve, together with the LTSM.

Teacher 3's class results on pre-and post diagnostic test

The overall pre-test mean scores for the five constructs (triangles, right angles, surds, fractions and ratios) of trig readiness shows mean scores of 4.33 for Teacher 3's class. The data shows that learners understood less than a third (8.33) of the requisite knowledge to learn trig. Teacher 3's class shows lower mean scores than both Teachers 1 and Teacher 2 in the areas that are fundamental for trig readiness. The specific data is as follows : 0,72 in identifying similar

triangles; 0,11 for Pythagoras; 0,56 for adding and 0.67 for simplification of ratios. Teacher 3 showed the lowest starting base in comparison to the other two teachers.

Teacher 3's class showed statistically significant improved results[26] in overall trig readiness. The class did not show significant improvement in the areas that were fundamental to trig. Teacher 3's class also showed the least improvement, with a pattern of the lowest scores on all accounts compared to patterns highlighted for the other teachers included in the study. The patterns are specific to the starting scores in the pre-test, with the least score in overall trig readiness. However, the pattern shows a similar trend to Teacher 2's average improvement (showing variation in data, not 95% confidence level) in similar triangles and Pythagoras. Teacher 3's class did not show statistically significant improvement in either procedural or conceptual knowledge.[27]

Teacher 3's use of LTSM

Teacher 3 only used the LTSM in Lesson 2 and did not use the LTSM for the lesson on congruent triangles (Lesson 1). In a post-lesson interview, Teacher 3 explained that he left the LTSM at home. Teacher 3's response to the use of the LTSM was:

> *Remember in your lesson* [referring to the LTSM which the researcher provided for intervention to help address learners' knowledge gaps] *the practical example you did of a tree and calculate the height, now driving that concept because when it comes in the exam they use those things – not to say calculate normally, but calculate the side of the house, height of a cliff and so forth.* [sic]

The comment shows the type of exercise to have been chosen by the teacher because he believed it was an exam-type question exercise. When I asked Teacher 3 about the LTSM and textbook that he used, he indicated as follows:

[26] Teacher 3 showed some average improvements in trig readiness with a paired difference of 2.333. This class showed average improvements in all measures, excluding angle measurement and adding ratio. None of the improvements were statistically significant, and one cannot be 95% confident that this average improvement was not due to variation within the data. The standard deviation of 2.521 shows that although there were some outliers it is within three standard deviations, and it did not influence the mean scores.

[27] Teacher 3 showed average improvement in both knowledge types, but I cannot be 95% confident that this was not because of variation in data.

> *'Siyavula' for ten, eleven and twelve. 'Platinum' is a new book, which other, yes there are those three.* [sic]

As the researcher, I confirmed with Teacher 3 that 'Mind Action Series', 'The Platinum' and 'Siyavula' are the only textbooks he uses, and he responded:

> *I also encourage them to use any book written Grade 10 if they can put their hands on any book of a concept that we are busy with at the time.*

The above shows that Teacher 3 uses a variety of resources to teach, which appears similar to other teachers' perspective about the textbooks and resources they use. In response to whether the LTSM helped or whether there were any problems, Teacher 3 explained to me how the practical example in the LTSM will help learners:

> *Remember in your lesson the practical example you did of a tree and calculate the height, now driving that concept because when it comes in the exam they use those things – not to say calculate normally, but calculate the site of the house, height of a cliff and so forth.* [sic]

He provided a positive view of what the LTSM could do to assist teaching and learning and said:

> *Ja, what I can say in closing is those lessons, maybe if one can have them time and again that would be very good, even to develop me as an educator, the learners as well, there is a lot I can say about them, because all in a nutshell, they are good. One using them can take the subject just to the highest level.* [sic]

I observed during lesson two that Teacher 3 used a reading for purpose exercise from the LTSM, which was a skills development exercise. The LTSM exercises were intended for learners to gain background knowledge of trigonometry and to introduce right-angled triangles. In the trig lesson, Teacher 3 used an everyday example from the LTSM to introduce, give context and to explain the concept. Teacher 3 drew a tree and depicted a right-angled triangle on the tree between the trunk and the ground. He modelled a trigonometric algorithm on the basis of the embedded context to show how one can solve the right angle.

Teacher 3's use of the LTSM was limited, and he did not use it for the intended purposes. His use of the LTSM and the choices made about when and for what purposes he used the LTSM does not clearly explain the weak achievement paths of Teacher 3.

Teacher 3's instructional practices

a) Teacher 3's perspectives about instructional strategies

This part of the interview provided in-depth information about Teacher 3's instructional strategies and his beliefs about how learners learn. I provide detailed extracts of quotes to exemplify some of his assumptions and perspectives on instruction:

> *Because kids are good at copying what the adult is doing. What I've done, they'll do exactly. Do what is known as remedial work so I can come to them closer and some of them you know you'll find that they don't get* [it] *correct, not because they don't know the work, but because they are shy, or they don't want to come out in the classroom. But as you get closer to them, then bit by bit they become relaxed. So, now if you let them know that you can wake them in the evening they will know it is y/r to the sine (meaning that it is the ratio sin* **Θ)**. *That is the end. So you drill that in them. Now this concept of Pythagoras, they start from Grade Eight, 9, then for you are to reinforce from that or by Pythagoras it is like this. So using Pythagoras you can also calculate the angle inside… you know, eh… repetition. I'm afraid to say this, but this changing of the curriculum, at one stage they criticised that repeating its confusing learners but that's how they mostly learn. That's how they mostly have been led because if you are repeating they get to understand it better as you repeat it. So to me this old concept of rote learning was better because if you repeat it the more you understand it.* [sic] (Pre-lesson interview, 9 April 2014)

The above shows that the teacher's beliefs about teaching aligns with behaviourism. The alignment to behaviourism was observed and analysed against Teacher 3's pedagogy in the classroom.

Teacher 3's instructional practices (pedagogical moves and scaffolded processes)

The table below gives a portrait of the coded categories and provides a description or example of Teacher 3's pedagogical moves in line with each of the codes.

Coding (Content Knowledge and Instructional Practices)	**Example**
Conceptual Knowledge:	In the trig lesson teacher 3 gave context and explained the concept of trigonometry as follows: *Right what we do then, we will be using the technique from trigonometry. This technique was first used by in Greece by the Greeks and it was also used by the Asians, so many people out there who are well covered as far as mathematics is concerned, so those people, they help us a lot, in saying for instance if we are working with right-angled triangles, now suppose you have a tree here, let me start if from there.* [sic] (Teacher 3, Video Lesson on trigonometry, 23 May, 2014).
Conceptual Knowledge:	Teacher 3 asked a learner after the learner provided a correct answer of two triangles that were similar. "*Your reason for that?*" [sic] (Teacher 3, Video Lesson on congruent triangles, 20 May 2014).

Procedures & techniques	Teacher 3 explained a technical procedure for using the calculator to calculate the answer to a trigonometry calculation as follows: *Then we say in your calculator, one calculator is written second function on one and one is written shift. Press that button shift, have you pressed? Then you go to button sin down there.* *Then you go to this fraction, (points to calculation sin Θ=3/5) three over five. Write the fraction three over five. So this is going to look like: Writes continuation of calculation:* *Sin Θ=3/5. When it is sin it will look like sin, then open bracket 4 over 5, in your calculator it must look like this: Θ=sin (4/5).* (Teacher 3, Video lesson on trigonometry, 23 May, 2014)
Everyday Knowledge	Teacher (3) introduced the topic of congruence by recruiting everyday knowledge in the form of a real world practical example. He used a practical illustration of three girls of similar height to explain to the class that if you measure the height of one of the girls, there will be no need to measure the height of the other two girls as their height would be similar. He used the illustration as a yardstick to introduce the topic.
Open questioning	Teacher 3 wrote a trig problem on the board for learners to solve. **Given: A=38,6º** **B=141,4º** ***(A)*** **cos 2A –sin (½ B).** He asked the following open questions "*How do you read A?*"

	Teacher 3: "*What do you think we shall be doing there? What do you think?*" The questions and problem require learners to consider the logic of solving a trig problem with some information provided. It also requires evidence of trig ratios, and how to calculate the answer using the calculator.
Closed questioning	One instance of closed questioning was during the trig lesson, when teacher MA explained the ratio sin Θ to learners thus: Teacher 3: "*What was the ratio?"* Learners chorused: "*Opposite over hypotenuse."* Teacher repeated: *"Opposite over hypotenuse."* Teacher 3: *"So if we were using…*" [writes on board]: **sin Θ =** "*to find an angle, we will be using that basic that says, opposite divided by? Hypotenuse. The opposite is three. The questions above had only one correct answer, and required low levels of cognition, mainly recall*."
Recall questioning	[Teacher 3 writes on the board] **:sin 30º** Teacher 3***: "****Do you remember we did that?."* [sic] Learners: "*Yes*." Teacher 3: "*At the moment, because this line is adjacent to this angle, what function will we use there? When it is adjacent to this angle (points to diagram at 60º angle). Remember adjacent over hypotenuse, what is that? The question requires learners to recall trig ratios*." [sic] Learners chorus: "…*cos*."

Assess learning & understanding	Teacher 3 walked around to check learners' work when they did a class exercise, and stopped by one learner and to say: Teacher 3: "*Ok this one has got it. Let me see if this is correct. That is correct. This one is correct."* [sic] The teacher then put a tick in learner's book.
Modelled exercises, examples.	Teacher 3 drew two triangles on the board and explained the example in the following way: *Now we try to identify now the sides which look the same, sides have got the names there, we have got from that one, ab, ac, df, de, which side do you think is more or less the same as that one? Pointing by Comparing* Δ *ABC and* Δ *DEF sides. Or the side that is in here that is more or less the same as that one?* [sic] The following exchange then took place: Learner: "AC and DF." Teacher 3 wrote on the board and asked: "AC is equal to DF?" [sic]

b) <u>Teacher 3's overall reflection of video lessons, post-lesson interview, 20 August, 2014</u>

While he was happy with the second lesson, Teacher 3 expressed that he had a problem with the video lesson on congruent triangles. He said:

> *Ja, not the second one on trigonometry was okay, it was very good, it felt comfortable but the problem was with the one on congruent triangles, what happens was all the examples which were in that book you gave me, that manual, so I left the book at home, so I couldn't pick up the*

> *relevant examples. So I just thought of examples, picked them up in my mind and only to find that when we worked together now they don't work. That is where the problem was, but the lesson as such, was okay.* [sic]

Summary of Teacher 3

Teacher 3's views about the learners' knowledge levels in the pre-lesson and post-lesson interview correlate with the pre-diagnostic test that shows a low starting base when it comes to trig readiness. The pre-diagnostic assessment showed that Teacher 3's learners were not ready to learn trigonometry. Teacher 3's class did not show significant improvement in the post diagnostic test in any of the constructs that are fundamental pre-requisite knowledge to trig (similar triangles, Pythagoras and ratios) or in any of the types of mathematical knowledge.

Teacher 3's use of the LTSM was not for the intended purpose and not sufficient, and this could explain his weak levels of achievement in comparison with the other two teachers. Teacher 3 recruited procedural knowledge 18 times more than he recruited conceptual knowledge. Teacher 3's pedagogical moves shows high frequency in use of open and closed questioning in the classroom. He also used modelling. His pedagogical moves aligned with behaviourism.

APPENDIX 4: Diagnostic Assessment

Academic Support

Research Project

MATHEMATICS DIAGNOSTIC TESTS

GRADE 10 - TEST 1

Name:_____ School: _____

INSTRUCTIONS

No calculators are allowed.

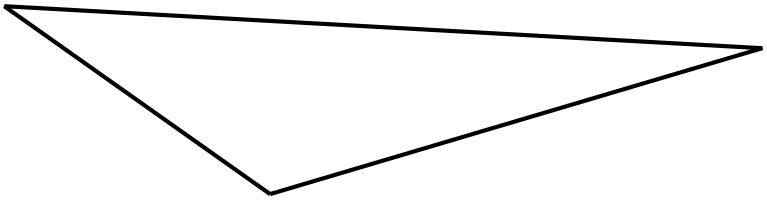
You will need pencil, eraser, ruler and protractor

1. In all multiple choice questions, circle the letter corresponding to the correct answer(s).
2. Space is provided for explanations in some questions.
3. You have **35 minutes** to complete this test.
4. Work steadily through the questions. Do not spend too long on questions that you are not sure of. Rather work to the end and come back.

Question 1

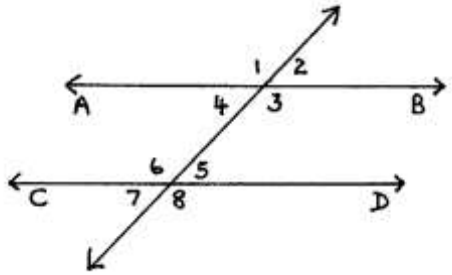
1.1. Draw an angle of 37°. Let the sides of the angle be 10 cm and 8 cm. Complete the triangle and measure the length of the third side.

1.2. Measure the angles in the triangle below. Write your answers on the diagram.



Question 2

2.1 In the figure shown, no lines are perpendicular and lines AB and CD are parallel.



The angles whose measures add up to 180° are

A ∠ 1 and ∠ 3

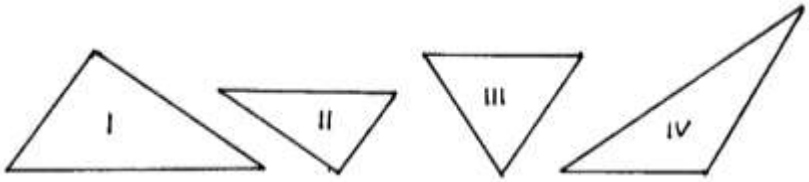
B ∠ 4 and ∠ 6

C ∠ 2 and ∠ 5

D ∠ 2 and ∠ 7

E ∠ 1 and ∠ 8

2.2 Which two triangles are similar?



A I & II B I & IV C II & III D II & IV E III & IV

How did you decide which one(s) to circle?

Question 3

Explain, with the help of diagrams, two important things that Pythagoras' Theorem tells us.

Question 4

4.1 Circle the example(s) showing an appropriate use of the shortcut of "cancelling" to simplify a fraction.

A	B	C	D	E	F
$\frac{50}{70} = \frac{5}{7}$	$\frac{501}{702} = \frac{51}{72}$	$\frac{42}{72} = \frac{4}{7}$	$\frac{520}{705} = \frac{52}{75}$	$\frac{175}{170} = \frac{7}{10}$	$\frac{100}{7000} = \frac{1}{70}$

How did you decide which one(s) to circle?

4.2 $\frac{3}{4} + \frac{8}{3} + \frac{11}{8} =$

A $\frac{22}{15}$ B $\frac{43}{24}$ C $\frac{91}{24}$ D $\frac{115}{24}$

Question 5

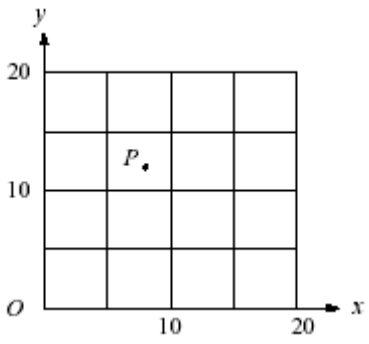
Which of these numbers are surds?

A $\sqrt{7}$ B $\sqrt{9}$ C π D $2\sqrt{3}$

Explain the reasoning for your choices.

Question 6

6.1. Which of the following are most likely to be the co-ordinates of point P?



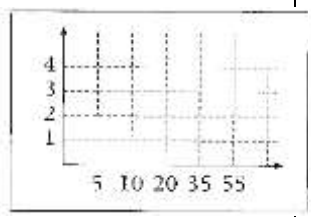
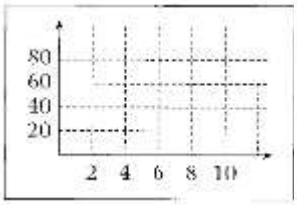
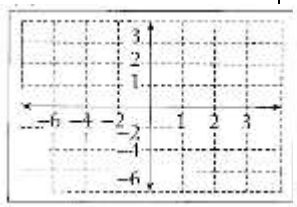
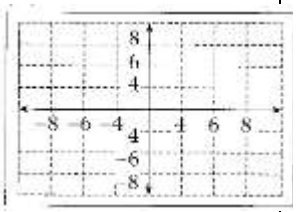
A (8 ; 12)

B (8 ; 8)

C (12 ;8)

D (12 ; 12)

6.2 Circle the label of each coordinate graph that has been drawn correctly.

A	B	C	D
			

Explain why you chose, or did not choose each of the coordinate graphs.

Question 7

7.1 The ratio of girls to boys in the choir is 3:4. If there are 12 girls in the choir, how many boys are there?

A 9 B 15 C 16 D 24

7.2 If the ratio 7 to 13 is the same as the ratio x to 52, what is the value of x?

A 7 B 13 C 28 D 364

Question 8

The table represents a relation between x and y.

x	1	2	4	7
y	1	?	7	13

What is the missing number in the table?

A 2 B 3 C 4 D 5

Question 9

9.1 Draw an accurate set of Cartesian axes in the space provided below. Both the x and y values should go from -4 to +4.

9.2 Use these axes to draw the graph of $y = x$. Show your working clearly.

Question 10

10.1 Three learners Alf, Busi and Cara each solved the equation p + 23 = 140. Whose working, notation and answer is correct?

Alf	Busi	Cara
p + 23 = 140	p + 23 = 140	p + 23 = 140
p + 23 = 140 – 23 = 117	p + 23 – 23 = 140 +23	p + 23 – 23 = 140 -23
p = 117	p = 163	p = 117

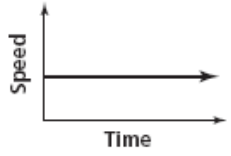
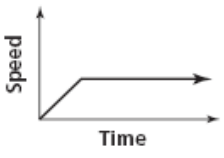
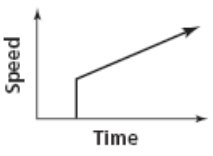
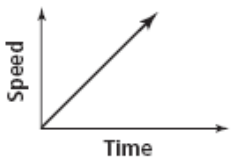
Explain the reasoning for your choice and why the others may or may not make sense mathematically.

10.2. $P = LW$. If $P = 12$ and $L = 3$, then W is equal to

A 9 B 3 C 4 D 36

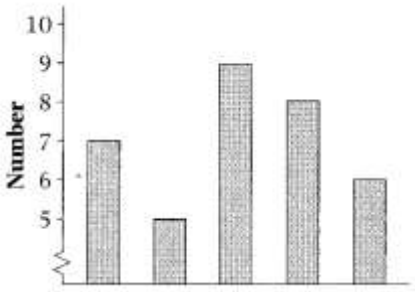
Question 11

11.1 A car speeds away from a robot and then drives at the same speed for a while. Which of these graphs shows how the speed of the car changes?

A	B	C	D
			

11.2 Look at the graph below,

Which of these could be the title for the graph?



A Number of learners who walked to school in the last 5 days

B Number of learners in 10 clubs

C Either of these could be the title

Explain your thinking

APPENDIX 5: Marking Memo

Grade 10 Term 2 2013 Readiness - Maths diagnostic test

The following content is set for Term 2, 2013.

Trigonometry Part 1 (1-3 from Term 1)

1. Know definitions of the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$ using right-angled triangles for the domain $0° \leq \theta \leq 90°$.
2. Take note that there are special names for the reciprocals of the trigonometric ratios (these three reciprocals should be examined in grade 10 only.)
3. Derive values of the trigonometric ratios for the special cases (without using a calculator), $\theta \in \{0°;30°;45°;60°;90°\}$.
4. Solve two-dimensional problems involving right-angled triangles.
5. Solve simple trigonometric equations for angles between 0^0 and 90^0.
6. Problems in two dimensions.

~~~~~~

1. Extend the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for $0° \leq \theta \leq 360°$.
2. Use diagrams to determine the numerical values of ratios for angles from $0°$ to $360°$.
3. Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [0°;360°]$.
4. Study the effect of a and q on the graphs defined by: $y = a\sin\theta + q$; $y = a\cos\theta + q$; and $y = a\tan\theta + q$ for $\theta \in [0°;360°]$.
5. Sketch graphs, find the equations of given graphs and interpret graphs.

Note: Sketching of the graphs must be based on the observation of number 4.
~~~~~~

Functions

1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations.

   Note that the graph defined by $y = x$ should be known from Grade Nine.

2. Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x; b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable).

3. Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x, b > 0, b \neq 1$

4. Sketch graphs, find the equations of given graphs and interpret graphs.

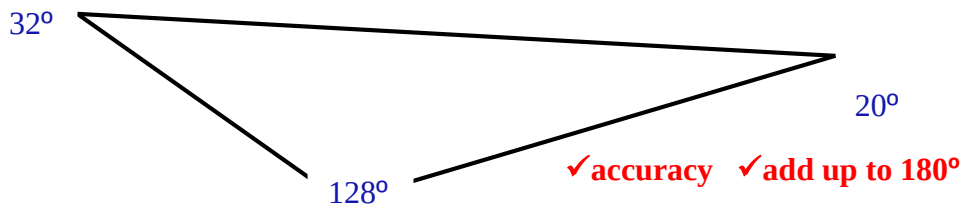
   Note: Sketching of the graphs must be based on the observation of number 3.

Question 1

1.1. Draw an angle of 37°. Let the sides of the angle be 10 cm and 8 cm. Complete the triangle and measure the length of the third side.

The third side should measure 6 *cm* ✓✓

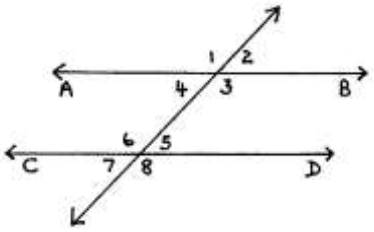
1.2. Measure the angles in the triangle below. Write your answers on the diagram.



✓**accuracy** ✓**add up to 180º**

Question 2

2.1 In the figure shown, no lines are perpendicular and lines AB and CD are parallel.



The angles whose measures add up to 180° are

A ∠ 1 and ∠ 3

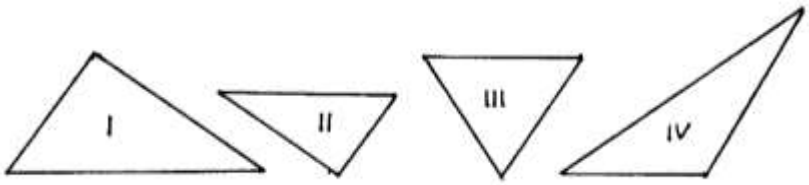
B ∠ 4 and ∠ 6 ✓✓

C ∠ 2 and ∠ 5

D ∠ 2 and ∠ 7

E ∠ 1 and ∠ 8

2.2 Which two triangles are similar?



A I & II✓✓ B I & IV C II & III D II & IV E III & IV

How did you decide which one(s) to circle?

A mathematical reason would include mention of SHAPE or EQUAL ANGLES ✓

Question 3

Explain, with the help of diagrams, two important things that Pythagoras' Theorem tells us.

Diagrams should show right-angled triangles ✓

1. If a triangle is right-angled then $a^2+b^2=c^2$ where *c* is the hypotenuse, and *a* and *b* the other two sides. ✓

2. If in a triangle, $a^2+b^2=c^2$, then C is a right angle ✓

Showing how a short side can be calculated is a variation of (1) and gets no extra credit.

Question 4

4.1 Circle the example(s) showing an appropriate use of the shortcut of "cancelling" to simplify a fraction.

	B	C	D	E	F
$\frac{50}{70}=\frac{5}{7}$	$\frac{501}{702}=\frac{51}{72}$	$\frac{42}{72}=\frac{4}{7}$	$\frac{520}{705}=\frac{52}{75}$	$\frac{175}{150}=\frac{7}{10}$	$\frac{100}{7000}=\frac{1}{70}$

A and F✓✓ penalize in next part if an extra was included

How did you decide which one(s) to circle?

Correct mathematical reason should include mention of dividing both numerator and denominator by the same number, in these cases by 10 and 100, to obtain equivalent fractions. ✓

4.2 $\frac{3}{4}+\frac{8}{3}+\frac{11}{8}=$

A $\frac{22}{15}$ B $\frac{43}{24}$ C $\frac{91}{24}$ **D** $\frac{115}{24}$ ✓✓

Question 5

Which of these numbers are surds?

A $\sqrt{7}$ ✓1/2 B $\sqrt{9}$ C π **D** $2\sqrt{3}$ ✓1/2

Explain the reasoning for your choices.

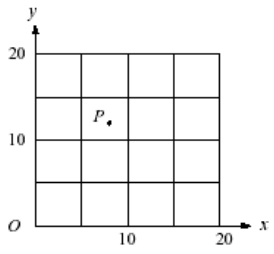
A surd is an irrational number that is the square root of a number that is not a perfect square. ✓

$\sqrt{9}$ is 3 so is a rational number and not a surd✓

π is an irrational number but not a surd. ✓

Question 6

6.1. Which of the following are most likely to be the co-ordinates of point P?



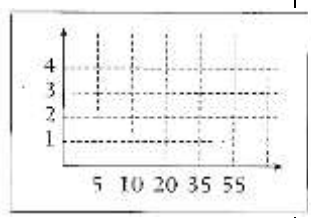
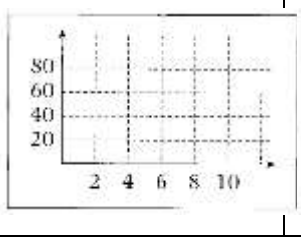
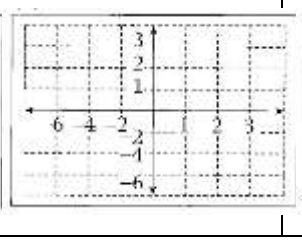
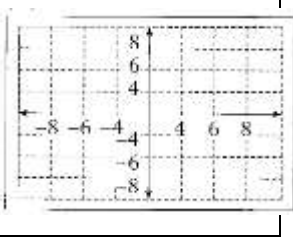
A **(8 ; 12)** ✓✓

B (8 ; 8)

C (12 ;8)

D (12 ; 12)

6.2 Circle the label of each coordinate graph that has been drawn correctly.

1.1.1 A	**B**	C	D
			

Explain why you chose, or did not choose each of the coordinate graphs.

A has inconsistent scale on X axis✓

B is correct✓ – the scale on each axis is consistent even though they are different✓

C has different scales for positive and negative values✓

D From the origin to the first marked unit is 2 and not 4 ✓

Question 7

7.1 The ratio of girls to boys in the choir is 3:4. If there are 12 girls in the choir, how many boys are there?

A 9 B 15 **C** **16**✓ D 24

7.2 If the ratio 7 to 13 is the same as the ratio x to 52, what is the value of x?

A 7 B 13 **C** **28**✓ D 364

Question 8

The table represents a relation between x and y.

x	1	2	4	7
y	1	?	7	13

What is the missing number in the table?

A 2 **B** **3**✓✓ C 4 D 5

Question 9

9.1 Draw an accurate set of Cartesian axes in the space provided below. Both the x and y values should go from -4 to +4.

Check for :

Correct and evenly spaced intervals ✓

Same scale for positive and negative values on both axes ✓

Axes labeled

Axes show at least from -4 to +4 } ✓

10.2 Use these axes to draw the graph of $y = x$. Show your working clearly.

Any method is fine. ✓I would expect a table, or point by point plotting at a basic level. Possible dual intercept method but his runs into problems in this case as only 1 point is generated.

Accuracy of graph ✓✓

Question 10

10.1 Three learners Alf, Busi and Cara each solved the equation p + 23 = 140. Whose working, notation and answer is correct?

Alf	Busi	**Cara**✓
p + 23 = 140	p + 23 = 140	**p + 23 = 140**
p + 23 = 140 – 23 = 117	p + 23 – 23 = 140 +23	**p + 23 – 23 = 140 -23**
p = 117	p = 163	**p = 117**

Explain the reasoning for your choice and why the others may or may not make sense mathematically.

Alf: Has the correct answer but his second line does not make mathematical sense ✓

Busi: Has added 23 to the LHS and subtracted 23 from the RHS so no longer has an equivalent equation✓

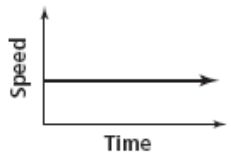
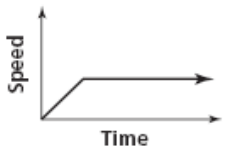
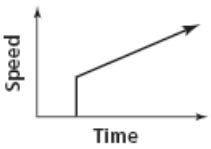
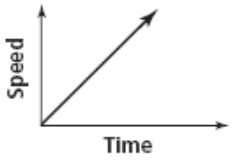
Cara: Has correctly kept equivalent equations and obtained the correct answer✓

10.2. $P = LW$. If $P = 12$ and $L = 3$, then W is equal to

A 9 B 3 **C** **4✓✓** D 36

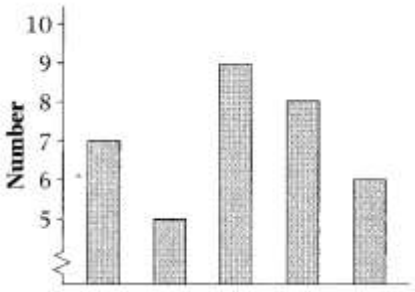
Question 11

11.1 A car speeds away from a robot and then drives at the same speed for a while. Which of these graphs shows how the speed of the car changes?

A	**B✓✓**	C	D
			

11.2 Look at the graph below,

Which of these could be the title for the graph?



A Number of learners who walked to school in the last 5 days ✓

B Number of learners in 10 clubs

C Either of these could be the title

Explain your thinking

In a bar graph the bars represent the categories. The categories in A are the 5 days, and in B the 10 clubs. The only correct option is A. ✓

APPENDIX 6: Interview Instruments - Protocol Number: 2014ECE002M

Appendix 6.1: Pre – Lesson Interview

Subject: _____ Grade_____

Researcher: _____ Date _____

Start time_____ End time_____

1. How many grade 10 mathematics classes do you teach?

2. How many learners are in each class?

3. Which class would you like me to video for lesson observation?

4. Have you taught grade 10 mathematics prior to this year?

    - 4.1 How many years have you taught mathematics?
    - 4.2 What other grades have you taught mathematics besides grade 10?

5. Tell me about your overall assessment of the learners' ability and readiness to understand the concepts in trigonometry in the class/es to be videotaped.

6. Please describe which topics or concepts have you been teaching over the past two weeks? Did any of the topics prepare learners for the concept of similar triangles and measuring triangles? Probe: Is it possible for you to show me the tasks that you did in class or homework tasks which you gave learners to do in preparation of the lesson?

7. Do you think that the learners' prior maths knowledge is adequate to learn the concept of similar triangles and measuring triangles as well as trig ratios to be covered in the lesson that will be videotaped ?
    - [Probe: How do you know this? What shows this?]

8. Please explain what would assist learners to become ready to learn this concept.

9. Please describe in detail your plans for teaching the concept of similar triangles and measuring triangles?

10. Do you give learners' quizzes and games? If yes, please give examples

11. Do you usually provide learners with examples and model answers to tasks? Please explain.

12. Do you provide incentives for learners who do well? Please explain

13. What do you think one could say to learners when they make mistakes (in class or when they do their homework)?

14. Which is concepts in trigonometry you think need to be memorised and why? Please describe any resources such as props, visual aids, charts and diagrams which you use to teach trig.

15. Do you use group or pair work in class? How often?

16. Can you show me three tasks that you gave to learners in the last two weeks – an easy, midlevel and a difficult task?

17. How many teaching periods do you use to teach the full trigonometry section of the curriculum ?

18. Do you develop your own assessments for learners to write on each concept on a regular basis, or do learners write the assessments given by the Department? How do you give feedback on the assessment results to the learners? Do you use the results of assessments to inform your teaching or lesson plans?

19. What learner teacher support material (LTSM) do you normally use to teach Trigonometry? [Probe: textbooks, workbooks, activity sheets.]

Thank you for your time. If I have any additional questions or need clarification, how and when is it best to contact you?

Appendix 6.2: Post – Lesson Interview

Subject: _____ Grade _____

Researcher: _____ Date _____

Start time _____ End time _____

1. How do you feel about the lessons that was videotaped?

2. Did the research process change anything? Is there anything you would change in the lesson given the changes you mentioned from the research process

3. Did you use any of the exercises as provided with the LTSM

to plan your lesson?

4. How did you use it? Did you use different levels of exercises with the different learners or did you use the same exercises with the whole class.

5. Did you plan this lesson or was it organised in that material? Please explain.

6. (a) What parts of the lesson do you think worked best?

    - [Probe for details and reasons.]

    (b) Were you disappointed with anything in the lesson? Please explain.

    (c) Was this different from your usual approach? Did the LTSM assist with implementing the curriculum or were there constraints? Please explain?

7. Have you taught this lesson before?
   If yes: Please explain how this lesson was different?

8. Is there anything to do with the learners level of ability or was it perhaps informed by the diagnostic results

9. How did learners respond to the lesson? [Probe: What was different in the way they responded?]

10. (a) Did you pick up any gaps in the learners' background knowledge which made trig especially difficult for them?
    (b) *If yes*, please explain what you noticed and how you picked this up.

11. Do you think the content and tasks from the lesson material had an effect on learners' understanding of the concepts? Please explain.

12. (a) Did you think that any of the trigonometry concepts in the materials presented problems for the learners? Please explain.
    (b) What do you suggest could address this challenge?

13. Did you learn anything from using the material? Please explain.

14. What do you suggest maths teachers should do to make up for the gaps in learners maths foundations? Do you think the LTSM addresses the gaps completely or only partially or not at all

15. Looking at the diagnostic tests, do the exercises sufficiently address the gaps ? Have you used the tests and profiles to plan your lessons? Please explain.

16. What other comments would you like to make?

Thank you very much for your time and participation.