STRUCTURAL CHANGES IN SEABORNE COAL MARKETS AND A MODELLED RESPONSE OF ECONOMIC IMPLICATIONS FOR THE SOUTH AFRICAN COAL INDUSTRY

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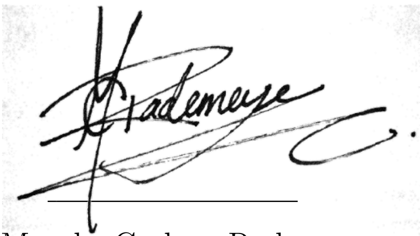
This thesis is submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in fulfillment of the requirements of the degree of Doctor of Philosophy.

Johannesburg, 2021

Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other university.



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Signed on this the 4th day of October 2021.

Abstract

Economics governs the growth and decline of all industries. The South African coal mining industry is no exception. For many years it has been believed that the profitability of coal mining in South Africa is strongly dependent on the profitability of exports. The international coal market has experienced highs and lows over the past ten years and concerns have been raised as to how South African coal production and coal supply for domestic usage would be affected over the longer term if the profitability of exports is not sustained. As such, this thesis presents the theories and applications that have been added to the field of mineral economics in order to investigate the economic implications for coal mining, and the broader coal industry in South Africa, should the market for coal exported from South Africa deteriorate. Firstly, a production function for mining operations is derived which better captures the effects of economies of scale seen in mining than other conventional cost functions. Secondly, a characterisation of the lead-time in mining project investment is created to describe the period from the time of investment until production capacity becomes operational. These two functions are then integrated into a net present value model of the cash flows over the life of a multi-product coal mining project. This mining model, GENCOMIN, is cast as a problem in static optimisation and solved computationally. The results indicate that the investment part dominates the mine owner's decision-making when seeking to maximise profit over the life of the project. Further, a general equilibrium model of the multi-product domestic South African coal market, DOTRAMOD, is developed and solved computationally as an optimisation problem. DOTRAMOD is then extended with a mining profit and an investment algorithm in accordance with the result from GENCOMIN which indicated the importance of the investment consideration. The extended DOTRAMOD problem is solved computationally, yielding the key result that a persistent decline in the demand for coal exported from South Africa would lead to the closures of

multi-product coal mines in South Africa and a subsequent reduction in coal supply for local power stations. This is expected to have implications for coal mining employment and South Africa's energy security. The work presented in this thesis is truly multi-disciplinary, and addresses topics in production economics, coal trade and marketing, and mine project investment.

Acknowledgments

I would like to thank my supervisors, Professor Richard Minnitt and Professor Rosemary Falcon, for their guidance and support.

I would like to thank Professor Bruce Watson for expanding my mathematical abilities.

A word of thanks is extended to Bart van der Steen, Ivan de Jongh, and Angel Angelov for their insight into the coal industry, as well as Brian Kennedy and Jarryd Kennedy for their insight into project investment.

Finally, I would like to thank my parents, Daniël and Heleen, for their enduring support.

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Chapter 1

Introduction

1.1 Overview of Research Papers

Five research articles were produced as part of the work presented in this thesis. Each article has been published or accepted for publication in either *Resources Policy* or *Mineral Economics*, both of which are international academic journals for work in the field of mineral resources and economics. The five articles are presented in detail in the chapters to follow. Highlights from each paper are given here.

Paper I makes a contribution to mineral economic theory by presenting a novel means for simulating production costs in models of resource extraction operations (Rademeyer *et al.*, 2018). The novel average cost function is derived from the well-established Cobb-Douglas production function and is more appropriate for operations characterised by economies of scale than conventional functional forms. The cost function is demonstrated in a simple model of a mining project which is structured and solved as a problem in optimal control[1].

Paper II presents a novel characterisation of the transformation of capital investment into production capacity to better account for the long lead-times typically

[1] This is also sometimes referred to as *dynamic optimisation*.

associated with mining projects (Rademeyer *et al.*, 2020). The derivation of this characterisation shows that the mining investment problem is actually functionally similar to a problem in signal processing. The latter is a well-established area in many engineering disciplines, suggesting that there may be methods that have already been developed for solving signal processing problems which could be applied to solving mining investment problems. The stochastic version of the lead-time characterisation is also explored.

Paper III shows how the cost model and investment model developed in *Paper I* and *Paper II* respectively are integrated into a computational optimisation model of a generic multi-product coal mine GENCOMIN (Rademeyer *et al.*, 2019). In addition, GENCOMIN describes the dynamics of the mine's coal reserves and resources over the life of the project. The application of GENCOMIN is demonstrated in a computational optimisation, and the results indicate that the investment part of the mining project problem has greater bearing on the profit-maximising miner's decision-making than the production part.

Paper IV presents a computational optimisation model of the South African coal industry, DOTRAMOD, where multi-product domestic coal trade is facilitated by a central trading hub (Rademeyer *et al.*, 2021). DOTRAMOD accounts for the three main consumer sectors served by the domestic market, i.e. the export market, the local industrial sector, and the local power generation sector. A pricing mechanism, rooted in economic theory, is also developed to allow for market prices and trade volumes to be determined by the market's supply and demand dynamics.

Paper V builds the mining investment problem into the DOTRAMOD model, as per the concept that the investment problem is a central driver in determining supply of mined commodities, as is suggested by the results from *Paper I* and *Paper III* respectively. The resulting domestic coal trade model allows for the effects of declining demand for South African coal exports to be observed (Rademeyer, 2021).

The results show that declining demand amid steady supply cause export prices to drop. The consequent reductions in profitability for multi-product mines result in the closures of some mines, and the reduction in supply of coal to the local power industry.

1.2 Novel Contributions and Applications

The work presented in this thesis made the following contributions to the theory and applications in the field of mineral economics:

A novel cost function suited to mining operations has been derived (*Paper I*).

Bulk resource mining operations, such as coal mining, are characterised by economies of scale, where average costs decrease as production volumes increase. A novel average cost function has been presented which is derived from the well-established Cobb-Douglas production function. This new average cost function has a flatter region around the turning point, i.e. flatter bottom, and as such should be more appropriate for describing costs in models of mining operations.

A novel characterisation to describe the mechanism transforming investment capital into production capacity has been presented (*Paper II*).

Mining projects typically exhibit long lead-times from the time the decision is taken to commit funds to project development, until the time at which the project becomes operational. A novel means has been presented by which to represent this inter-temporal relationship between the time at which funds are committed and the time at which production capacity becomes operational. This representation indicates that the investment problem is functionally similar to a problem in signal processing.

A computational optimisation model of a multi-product coal mine with dynamic reserves and resources has been developed (*Paper III*).

GENCOMIN is a model of a multi-product coal mine into which the cost function and capital investment characterisation developed in *Paper I* and *Paper II* respectively have been incorporated. The model also exhibits dynamics in reserves and resources over the life of the mining operation. GENCOMIN is solved as a static optimisation problem and the results obtained are similar to those that could be expected from a real-world project.

A computational optimisation model of the domestic coal market has been developed (*Paper IV*).

DOTRAMOD is a computational optimisation model of the South African coal market presented in *Paper IV*. This is the first such model of the domestic South African coal market, and it allows for investigating the effects on the domestic market of declining export demand. DOTRAMOD is equipped with a pricing mechanism which is consistent with economic theory, and also enables the exploration of the concept of a central trading hub.

DOTRAMOD is expanded upon with an algorithm for mine project development and decreasing export demand is seen to lead to mine closures and reduced domestic coal availability (*Paper V*).

An algorithm for determining mining project development and mine closures is presented and incorporated into DOTRAMOD. The optimisation problem is solved numerically, and the effects of decreasing export demand can be observed. This work delivers the key result that a persistent decrease in export demand results in mine closures and a subsequent reduction in coal supply for domestic power stations.

Chapter 2

Background and Problem Statement

2.1 The Significance of the South African Coal Industry

Coal mining plays an essential role in the South African economy for a number of important reasons, namely:

- Coal satisfies almost 70% of South Africa's primary energy needs (MCSA, 2018b).
- Over 80% of the electricity generated by South Africa's national power utility Eskom is generated by coal-fired power stations (MCSA, 2018a).
- Coal export has been one of South Africa's top earners of foreign exchange (MCSA, 2018b).
- The coal mining industry provides employment and is in general vital in that it is capable of employing a large number of South Africa's many unskilled and semi-skilled workers. The number of persons employed in South Africa's coal mining industry topped 90,000 in 2019 (MCSA, 2019), and it is further

estimated that over 170,000 jobs were created by the coal industry in the rest of the South African economy (MCSA, 2018a).

- A number of domestic industries rely on coal for energy, e.g. synthetic liquid and solid fuels, cement, brick-making, food processing, agriculture, paper and pulp, chemicals and others (MCSA, 2018a).
- The coal value chain contributes a significant amount to South Africa's gross domestic product, with sales of about 146 billion rand in 2018 (MCSA, 2018b).

According to South Africa's Department of Public Enterprises, coal will continue to play an important role in the country's industrialisation and economic growth (DPE, 2019). Sub-standard[1] coal and inadequate coal inventories are thought to pose challenges to Eskom's ability to reliably produce electricity (DPE, 2019). Furthermore, coal quality and pricing are important considerations in Eskom's coal-fired future (DPE, 2019).

However, there has been an increasing focus globally on the role of coal and other fossil fuels in ostensible climate change. Some industrialised nations have vowed to phase-out coal and there is increasing pressure on other nations globally to commit to lowering carbon emissions. A decrease in global coal demand would have implications for international coal trade, and by extension it would have implications for the demand of coal exported from South Africa. Modelling conducted for the South African Coal Roadmap (SACR, 2013a) projected that, in a low-carbon world, South African coal exports would peak in 2023 and then decline.

[1]In the case of coal, sub-standard commonly refers to coal that does not meet the specifications of the process in which it will be used. Power station plant is typically designed to burn coal of a particular composition in the most efficient way. When fed with coal containing constituents in the wrong proportions, the performance of the plant could be severely impaired. In extreme cases, the plant could be damaged.

2.2 Problem Statement

The primary objective of this research was to determine how the South African coal mining industry would evolve in response to structural or fundamental changes in seaborne coal markets. The intention was to determine how sensitive South African coal mining is to these changes, with a particular focus on the response of coal supply to adverse seaborne coal market changes, i.e. lower coal export volumes in a low export-price environment. The research set out to test the hypothesis that a persistent deterioration in global seaborne coal trade would impact negatively on the South African coal industry as it was expected that lower export volumes and lower prices would reduce coal mining profitability, and that this would result in lower coal production levels and higher domestic coal prices. It is expected that such an outcome would impact negatively on coal-fired power generation and other coal-consuming industries in South Africa.

As such, this research project set out to answer the following questions:

1. Would a decline in demand for South African export coal result in lower export prices?
2. Would the profitability of the overall South African coal mining industry decline with decreasing profitability in the export of coal?
3. Would lower coal export profitability lead to lower coal production levels owing to the closure of marginal mines and fewer new mines being commissioned?
4. Would domestic coal consumers, Eskom and local industry, be faced with reduced coal supply and/or higher coal prices?

Furthermore, the research aimed to provide an indication of South Africa's coal production profile over time as well as the evolution of coal prices faced by local consumers. An indication of future prices for securing coal supply would in theory give current coal consumers an indication of the future affordability of retaining coal-fired processes in their business operations, and whether it would be more or less viable to switch to alternative energy sources.

It was also determined that there was opportunity to contribute to the theory of mineral economics. The production cost functions typically used in commodity market models are not suitable for operations characterised by economies of scale, such as coal mining, and there was a need for a more appropriate cost function. In addition, coal mining projects are characterised by long lead-times from the time an investment decision is made until the time the mine becomes operational. There was a need to account for this lead-time in the economics of a coal mining project. The natural next step from here would be the development of the model of a coal mine into which the aforementioned cost function and investment-to-production lead-time could be incorporated.

With regard to commodity market modelling and computation, at the outset of this research it was clear that there was a need to develop a model of South Africa's domestic coal market, together with an objective pricing mechanism. That is, there was scope to create a model in which market prices are determined by the optimisation procedure and the market dynamics, and not set subjectively by the research team or based on historical price trends.

The construction of a well-specified coal industry model is of considerable value; such models provide insight into market mechanisms, enable sensitivity testing and facilitate the analysis of future industry trends. Furthermore, resource markets are particularly dynamic and market determinants often change. Models provide policy makers with a means to test the consequences of energy policy before its implementation. The volatility in international coal prices creates uncertainty. Well-formulated models assist in producing results that reduce those uncertainties which typically discourage investment in capital intensive projects such as the development of a coal mine. Furthermore, model results allow coal buyers to adjust their expectations of future coal costs and manage risks of supply-shortfalls.

Knowledge of resource industries and commodity trade has grown rapidly with the development of microeconomics and market behaviour theory. Our understanding of the mechanisms governing commodity trade has also benefited considerably from the development of computational methods owing to the expansion of computing power and information technology (IT), especially given the magnitude and complexity of a live commodity trading system. Commodity trade modelling itself has also been the subject of much interest and research in the last three decades or so owing to its application in strategic decision making by market participants in competitive industries. Optimisation has been used extensively in financial market research as well as in studies on electricity trade markets.

A large amount of work has been done on international coal trade in particular with the focus mainly on detecting strategic behaviour and market manipulation by coal suppliers as well as the buyers of coal, as discussed in Rademeyer *et al.* (2021). There remains considerable scope for the enrichment of the theory concerning the economics of mining. In contrast, prior to the work presented in this thesis, the work on the South African coal market had been limited to a handful of descriptive texts, and one or two analytical essays. The South African coal product has however been the subject of many technical academic and commercial work in coal geology, coal chemistry, coal mining, coal processing and of course coal combustion.

Chapter 3

Literature Review

Each paper presented in subsequent chapters is accompanied by its own literature review which is with respect to the topics discussed in the paper. Each paper then also has its own reference list associated with its literature review. The literature review presented in this chapter is for literature on the South African coal market. First, some general information indicating the role of coal in the South African economy is given, after which consumers of South African coal, coal products, and coal producers are discussed. This is followed by a brief overview of international thermal coal trade and South Africa's role in this market. The importance of South African coal export is then covered. The review closes with an overview of notable studies on the South African coal industry.

The literature review presented here does not go into great depth as it serves merely to provide background and context to the work covered in this thesis, as well as to indicate the state of the knowledge of the South African coal market. The associated reference list can be found at the end of this thesis, after the Concluding Remarks as given in Chapter 9.

3.1 Thermal Coal and the South African Coal Industry

South Africa has a long history of coal mining, and owing to the relative abundance of the fuel across several central coal fields, the coal industry was able to provide energy to spur the growth of key South African industries, such as the gold mining industry. To this day, coal remains one of the most widely used fuels in the country. However, the reserves of the central coal fields are diminishing, and the industry is now looking at expansion into the fields that have as yet hosted relatively little exploitation.

There are 19 coalfields in South Africa, with the largest proportion of coal sourced from only six or seven of these (Hancox and Götz, 2014). Estimates on the volumes of coal that can be economically extracted vary, with an estimate of 33.8 billion tonnes given in Jeffrey (2005a), while an estimate of 15 billion tonnes is given for the entire southern Africa in Hartnady (2010). The Witbank Coalfield is one of the most important of South Africa's coalfields and supplies over 50% of South Africa's coal for sale to both domestic and international markets (Hancox and Götz, 2014). Mining of this coalfield is known to have begun towards the end of the nineteenth century, and after more than a century of hosting mining operations, it may be described as "fairly mature in terms of its exploration and exploitation" (Hancox and Götz, 2014). Other important coalfields include the Highveld Coalfield which supplies coal to local power stations and the coal-to-liquids industry, as well as the Ermelo Coalfield which also supplies to power stations (Hancox and Götz, 2014). The Witbank and Highveld coalfields have yielded good quality, high-energy coal (Jeffrey, 2005a). Detailed information on South Africa's coalfields and their geological characteristics is provided in Hancox and Götz (2014).

The South African coal mining industry grew in conjunction with the gold mining industry, with almost all coal mines owned by gold-mining companies in the first half of the twentieth century (Hancox and Götz, 2014). Thermal coal[1] is largely consumed near where it is mined owing to the relatively high cost of transportation; the seaborne thermal coal market accounts for only about 16% of the world's total thermal coal consumption (IEA, 2018). The seaborne market grew rapidly from the 1970s as a result of the steady move away from oil for use in industry and power generation due to oil supply risks amid increasing geopolitical tensions (Haftendorn, 2012).

The South African coal industry also witnessed tremendous growth from the early 1970s resulting in production increasing five-fold to 250 million tonnes in 2006 (Eberhard, 2011). This expansion resulted from the simultaneous growth in domestic coal use for electricity generation and coal-to-liquids conversion, as well as growth in exports. Mechanisation in coal mining further drove the industry's expansion. The oil crisis of 1973/74 and consequent increases in international coal prices, opened the way for coal export to Europe, spurring growth in South African exports from less than 5 million tonnes in the early 1970s to over 70 million tonnes by the early 2000s.

The Witbank and Highveld coalfields have been supplying up to 75% of the coal produced in South Africa, but owing to over a century of exploitation, the recoverable coal in these fields is diminishing (Jeffrey, 2005a). The Waterberg Coalfield is considered to be the next great source of South African coal. It however hosts coal with high ash content together with low yields (Jeffrey, 2005b). The large scale exploitation that would be needed to make mining in this coalfield viable is currently hindered by insufficient water and inadequate transport infrastructure from the coalfield to South Africa's industrial hubs and the export market (Jeffrey, 2005b). Beneficiation of the Waterberg coal product is also expected to generate large volumes of discard which would have to be contained in an environmentally

[1]Thermal coal refers to coal used in heating processes, such as for power production or in cement making. Metallurgical coal refers to coal that is used in metallurgical processes as a source of carbon, such as in steel production.

responsible manner (Jeffrey, 2005b). The challenges posed by the Waterberg Coalfield could however encourage the development of new technologies. The feasibility of generating electricity from coal discard is investigated by North *et al.* (2015). A techno-economic study of the viability of the beneficiation of lower-energy thermal coal for export is conducted by Bergh *et al.* (2013). The beneficiation of lower-energy coal is also dicsussed in De Korte (2015).

3.2 South African Coal Consumers and Coal Products

Coal is used widely in South African industry as a source of fuel and carbon, in applications such as electricity generations, liquid fuels production, heating, and steel manufacturing. South Africa produces predominantly thermal coal (MCSA, 2018a). The largest consumers of thermal coal in South Africa are the electricity generation, steel, and liquid fuels industries (MCSA, 2018a). Of the 253 million tonnes of coal produced in South Africa in 2016, just over 72% was sold domestically (MCSA, 2018a).

National utility, Eskom, is the largest domestic coal consumer, having consumed 114 million tonnes of coal in the year ending March 2019 (Eskom, 2019). Eskom is responsible for the bulk of the electricity generated in South Africa. Coal-fired power generation represents the largest part of Eskom's fleet, representing 83% of Eskom's 44 gigawatt nominal generating capacity and supplying 90% of the electricity produced in the year ending March 2019 (Eskom, 2019). Even when accounting for other electricity suppliers, over 80% of South Africa's electricity was fueled by coal in 2016 (MCSA, 2018a).

The next largest domestic coal consumer is liquid-fuels and chemicals producer Sasol. Sasol's mining arm supplies around 40 million tonnes of coal each year for producing liquid fuels (MCSA, 2018a). Other domestic consumers use coal for the generation of heat, steam and power in their industrial processes. These include cement manufacturers, sugar refineries, breweries, brick and refractory manufacturers, as well as paper and pulp manufacturers (SACR, 2011).

Eskom's fleet typically burns coal of lower quality and with relatively high ash content, at an average of almost 30% in 2010 (SACR, 2011). The lower the coal quality, the less electricity that can be generated. Eskom's plant is however customised to use local coal (Steyn and Minnitt, 2010). The majority of Eskom's coal-fired power stations are located in the Mpumalanga province and a number of these are served coal from dedicated coal mines situated in the Witbank and Highveld coalfields (SACR, 2011). There is limited possibility for interchanging domestic coal and export coal as both product types are customised according to the preferences of different markets, and doing so would reduce the value the coal sales would otherwise achieve (Steyn and Minnitt, 2010)[2]. In addition, domestic industrial coal consumers have sizing specifications, and the screening of coal allows for the creation of additional coal products (Steyn and Minnitt, 2010). Falcon and Ham (1988) discuss the characteristics of coal in Southern Africa in detail.

South African coal typically has an ash content of 20% to 40% and must be washed to enhance its calorific value in order to be exported (SACR, 2011). There are indications that the average ash content in South African coal has been increasing, and subsequently the average calorific value has been decreasing (SACR, 2011). Coal with a lower calorific value than that which has historically been exported to European customers, has been accepted by Indian consumers (Steyn and Minnitt, 2010). Indian consumers also accept higher-sulphur content (Steyn and Minnitt, 2010). Owing to coal seam geological factors, a mining operation may cross several coal

[2]According to Steyn and Minnitt (2010), the only parameters that are comparable for domestic coal and export coal are the moisture and sulphur contents of both products. The specifications on the remaining parameters are completely different.

seams and thus mining could yield several coal products, and each of these could then be marketed to different customers with particular requirements (Steyn and Minnitt, 2010). Coal mining companies achieve efficiency and profit by mining the multiple seams they have access to (Steyn and Minnitt, 2010). This practice may result in a type of subsidisation of local coal, as suggested by Steyn and Minnitt (2010) has been the case for coal sold to Eskom.

3.3 South African Coal Producers

The South African coal mining sector is relatively concentrated. That is, a large portion of output is from a relatively small number of suppliers. In 2018, about three-quarters of South Africa's coal was produced by five major mining companies, namely Anglo American, Exxaro, Glencore, Sasol Mining, and South32 (IEA, 2018). Following the sale of Anglo American mines with total capacity of 24 million tonnes per annum to black-owned Seriti Resources in 2018 (Seriti, 2018), there are now six major South African coal producers. The remaining portion of South Africa's coal output is produced by smaller companies (IEA, 2018).

The South African coal majors produce coal for sale to domestic consumers as well as for export. As a consequence, both the domestic market and the export market provide valuable revenue streams for these coal producers. The available figures for sales or production in 2019 are given in Table 3.1[3].

Isibonelo is a mine owned by Sasol and operated by Anglo American Coal SA. Sasol's mining operations produced about 36 million tonnes in 2017, of which the majority was used by its chemicals and synthetic fuels production divisions (IEA, 2018). Some 3 million tonnes of Sasol's coal production was sold to domestic South African customers or exported (IEA, 2018).

[3]These results are taken from AA Plc (2020), Exxaro (2020a), Glencore (2020), and South 32 (2020a).

Company	Type	Sales (mt)	Production (mt)
Anglo American			
	Export	18.0	17.8
	Domestic Other	5.3	6.0
	Isibonelo	4.5	4.0
	Third Party Sales	11.0	
Exxaro			
	Export	9.1	
	Domestic	35.4	
Glencore			
	Export		13.0
	Domestic		13.9
South 32			
	Export	9.9	
	Domestic	15.0	

Table 3.1: Coal sales and production by major South African producers

Black-owned Seriti Coal purchased dedicated Eskom-serving operations Kriel, New Denmark, and New Vaal from Anglo American in 2018 (Seriti, 2018). Together these mines supply 24 million tonnes of thermal coal to Eskom each year. Seriti also owns the New Largo project which will supply 12 million tonnes per annum to Eskom's Kusile power station once fully operational (Seriti, 2020).

3.4 International Thermal Coal Trade

A considerable portion of South Africa's total coal output is exported each year, contributing a substantial portion to the country's foreign earnings. South Africa is however only one of several large suppliers competing for a share of the international coal market. Export prices are determined by the dynamics of supply from exporting countries as well as demand from international consumers. Consumer demand is in turn affected by economic activity and various government policies.

Over 1 billion tonnes of thermal coal were traded in 2017, though still representing only 18% of the total amount of thermal coal consumed globally (IEA, 2018). Countries in the Asia-Pacific region account for over 70% of thermal coal import (IEA, 2018). The largest importer is China with 212 million tonnes imported in 2017 (21% of total global import), and second largest is India at 161 million tonnes (16% of total), according to estimates by IEA (2018). The top exporters in 2017 were Indonesia (38%), Australia (20%) and Russia (15%), while South Africa took fifth place with 7% of the market, just behind Colombia with 8% (IEA, 2018).

Growing opposition to coal amidst increasing concerns regarding the contribution of fossil fuels to climate change has resulted in increasingly stringent regulations for coal-fired power generation in western nations. The Powering Past Coal Alliance was launched by the United Kingdom and Canada, with an ambition to phase out coal-fired power by 2030; more than 20 countries have already joined (IEA, 2018).

At the same time, coal-fired power generation has grown in Asia and other developing nations as an affordable, readily available source of energy to power industrial development and economic growth. As such, market trends have not firmly been signalling the end of coal. Thermal coal trade increased by 6% to over 1 billion tonnes in 2017 (IEA, 2018). Total global coal consumption is estimated to have increased by 1.1% in 2017, following two years of decline (IEA, 2018).

Thermal coal prices remained elevated through 2017 and into 2018 following a strong rise in 2016 due to tighter supply whilst demand remained firm (IEA, 2018). The gap between the higher-energy 6,000 kcal/kg product and the lower-energy 5,500 kcal/kg product reached a record 50 dollars a tonne, as measured at Newcastle in Australia (IEA, 2018), suggesting better availability of lower-energy product.

Investment in new mines and the development of greenfield projects has still not resumed after many developments came to a halt when the 2011/13 super-cycle came to an end, despite the fact that coal prices have been higher than the 2016 low (IEA, 2018). That is, prices are not strong enough to signal investment, and it is believed that increased public opposition to coal assets have resulted in investors viewing these assets as riskier (IEA, 2018). The Institute for Energy Economics and Financial Analysis has compiled data showing that major financial institutions have been divesting from coal mining and coal power projects, or have committed to limiting their exposure to coal projects (IEEFA, 2021a). Some asset managers have also done likewise (IEEFA, 2021b). In the longer term, coal's place in the global energy mix is expected to decrease as a result of increasing availability and affordability of renewable energy sources and natural gas (IEA, 2018).

3.5 The Role of South Africa in the International Thermal Coal Market

South Africa is one of several suppliers to the international coal market, and its role in the market has evolved over the last four decades as various market players have had to adjust to shifts in coal demand and supply factors. The market continues to change as key consumers of South African coal adjust their demand in response to their own economic realities and government policies. This creates uncertainty for the trajectory of demand for South African coal product, and in turn, for South Africa's role as an exporter in the future.

South Africa used to be a significant exporter to Europe with as much as 80% of South Africa's exports flowing to Europe as recently as 2004, according to the IEA (2018). As a result of the growth of Colombian and Russian supply to Europe, the share of South African shipments to Europe has declined. South Africa exported 70 million tonnes of thermal coal in 2017, representing some 28% of total production (IEA, 2018). The majority of South Africa's cargoes now flow to the Middle and Far East with India the largest buyer and taking 44% of South Africa's thermal coal exports in 2017 (IEA, 2018). Other markets for South African coal include Pakistan, Korea and Africa.

India is the world's second largest consumer of coal after China (IEA, 2018). In 2017, India consumed 825 million tonnes of thermal coal. With vast reserves of its own, India was able to satisfy the majority of its demand with its own production, which reached 673 million tonnes in 2017 (IEA, 2018). The deficit has to be satisfied however by imports. There is a focus on improving state-owned coal producer Coal India's ability to meet the country's demand and reduce the dependence on costly imports (IEA, 2018). India imported 161 million tonnes of thermal coal in 2017 (IEA, 2018). Of this, 66% was sourced from Indonesia and 24% was sourced from South Africa (IEA, 2018). South African volumes to India are not secure. These volumes dropped by 10% in 2017 from 2016 levels owing to increased competition from US suppliers (IEA, 2018). Indonesia was the world's largest coal exporter in 2017, and only about a third of its exports flowed to India (IEA, 2018).

3.6 The Role of Coal Exports in the South African Economy

The coal industry is vital to the South African economy. In addition to providing for a large part of the country's energy requirements, it also provides an important revenue stream.

The export of coal is an important source of foreign earnings for the South African economy, contributing 12% of South Africa's total foreign earnings (MCSA, 2018b). Exports generated 50.5 billion rand in sales revenue compared with 61.5 billion rand earned on domestic sales in 2016 (MCSA, 2018a). Owing to the higher prices on exported coal, this means that coal exports accounted for some 45% of total coal sales revenue in 2016, despite accounting for less than 30% of the total sales by volume (MCSA, 2018a).

Furthermore, coal mining contributes an estimated 20 billion rand annually in fixed investment (MCSA, 2019). It has been found that net investment in coal mining in South Africa seems to be correlated with international coal prices, and that "a sustained increase in export prices is soon followed by higher net investment" according to the Coal Strategy 2018 report published by the Minerals Council of South Africa (MCSA, 2018a). Furthermore, they found that coal consumption by national power producer Eskom does not appear to have a similar influence on stimulating net investment to increase production, even though Eskom tends to enter into long-term agreements with mining companies in a way that creates certainty for mining investment (MCSA, 2018a).

There are several new projects underway in South Africa that are aimed at increasing production and transport capacity towards the export market. Exxaro Resources has been progressing its greenfield Belfast project, which will produce 2.7 million tonnes of high-energy thermal coal for the export market (IEA, 2018)[4]. Resource Generation is developing the 6.5 million tonnes thermal coal Boikarabelo coal project, but has been struggling to secure adequate financing, despite already having a partnership with Asian-based trader, Noble Group (McKay, 2020). South 32 is extending the life of its Klipspruit operation for another 20 years (South 32, 2020b), which is expected to enable the export of 8 million tonnes of thermal coal (IEA, 2018). Exxaro Resources is working with national rail transport service provider, Transnet, on the construction of a rail line to transport 12 million tonnes of coal annually from Exxaro's Grootegeluk operation in the Waterberg to the Richards Bay Coal Terminal for export (IEA, 2018). There have also been plans by Transnet to expand the capacity on the dedicated rail line to Richards Bay (IEA, 2018).

3.7 Studies on the South African Coal Industry

There are a limited number of studies on the South African coal industry in the public domain that have been conducted by various research institutions and consulting firms. These are typically qualitative accounts of the industry. A very detailed description of the South African coal industry was compiled by Steyn (2009). Another comprehensive analysis is that of Eberhard (2011).

The South African Coal Roadmap (SACR, 2013a) was an industry-wide initiative that brought together representatives from coal mining, coal marketing, coal-fired power production, coal distribution and relevant government agencies to discuss the future of the South African coal industry amid growing criticism of fossil fuels and concerns about climate change. Different scenarios were considered for South

[4]The Belfast project was operational as at December 2020 (Exxaro, 2020b).

Africa's approach to handling its coal-fired fleet as well as its approach to international trends in phasing out coal, and resulting projections on the outlook for the South African coal industry were created (SACR, 2013b). The modelling showed that coal demand for power generation would increase strongly over the years to 2040 if South Africa's attitude to coal were to remain as is, while demand would remain largely flat over the same time horizon if South Africa were to impose more stringent emissions regulations or introduce more efficient coal-fired technology. In all scenarios, coal supply from the Central Basin was projected to decline until 2040, whereas supply from the Waterberg would increase in all scenarios, except in the case where South Africa introduces more stringent emissions regulations. Scenarios for coal exports were also modelled. In the cases where global coal demand either stayed the same or increased, South African coal exports were projected to peak in 2032 and then decline to 2040. In a low-carbon world, exports were projected to peak in 2023 and then decline to 2040. The accompanying technical report (SACR, 2013c) provides detailed insight into the South African coal industry.

The South African Minerals Council (formerly known as the Chamber of Mines) in 2018 published a proposal for a strategy that could be undertaken with regard to coal in South Africa (MCSA, 2018a). The report reviews the implications of stricter environmental regulations on the electricity, synthetic fuels, and steel industry, who together consume some 70% of the coal sold domestically in South Africa (MCSA, 2018a). Data on employment (both direct and indirect), production, and investment are presented to illustrate the relevance of the South African coal industry (MCSA, 2018a). Constraints on the continued viability of the industry are proposed, and four scenarios on the outlook for the South African coal industry are presented, together with proposals on how negative effects might be reduced and how positive effects could be enhanced (MCSA, 2018a).

The work of Burton and Winkler (2014) analysed the expansion of coal industry infrastructure (i.e. rail lines, coal mines and especially coal-fired power generation) in light of South Africa's climate change mitigation plans. Their analysis suggested that the country would likely exceed its self-imposed future carbon emissions limit if all planned coal-industry infrastructure is commissioned. The potential trends and outcomes for situations in which South Africa transitions away from coal-fueled energy to lower carbon energy sources were also investigated in research reports by Burton *et al.* (2018) and Steyn *et al.* (2017).

None of the aforementioned studies allow for exploring the question that this thesis sets out to answer. That is, none of them support with providing any understanding of the domestic market is tied to the export market, or how domestic coal supply would be affected by a deterioration in demand for coal exports. The coal industry is incredibly complex, comprising mines, traders, consumers, logistics service providers, and financiers. Typically, a model is constructed purposely for studying some particular attributes of an industry. Several of the studies on the South African coal industry have been particularly focused on issues concerning carbon emissions for example. The models used in these studies were developed to explore and answer these environmental questions. The South African Coal Roadmap did review scenarios for domestic and export coal demand. However, it too focused very much on the effects of environmental policy. It also treated domestic demand and export demand separately, rather than evaluating how the two markets are inter-dependent and how the one could be affected by changes in the other. Furthermore, there is not much information available on the details of the industry models themselves and how the results provided in these studies were computed. These studies do however provide useful insight into the South African coal industry which has proved useful in the research presented in this thesis.

Chapter 4

Paper I: Revisiting Operating Cost in Resource Extraction Industries

This chapter concerns the production cost function which serves as one of the two building blocks developed for the mining project model presented in Chapter 6.

This chapter has been published in the academic journal, *Resources Policy*, under the following citation:

Rademeyer, M. C., Minnitt, R. C. A., Falcon, R. M. S. (2018) Revisiting Operating Costs in Resource Extraction Industries, *Resources Policy*, **58**, 159-164.

The modelling and derivation of the proposed function, as well as the economic arguments, mathematical arguments, computational programs, and results are the work of Rademeyer. Minnitt advised on matters related to mineral economics and mining methods, and Falcon advised on matters related to coal products and coal markets.

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Abstract

A modified version of the Cobb-Douglas production function is proposed for simulating production costs in resource extraction models. The resulting average cost function is U-shaped with a wide bottom, and as such should be more representative of the economies of scale associated with bulk operations. It also possesses a minimum which is obtainable from the characteristics of the operations. The viability of the proposed cost function is demonstrated in a profit maximisation exercise constructed as a problem in optimal control, rendering results consistent with what could be seen in a real-world resource extraction operation with similar constraints.

4.1 Introduction

A resource extraction model provides the backbone for the supply-side of a resource trade model, which is often employed in the study of commodity markets. The mine operator must cover the costs of delivering minerals to the mine gate where the product is sold to an offtake agent at the prevailing market price. In modelling applications it is often desirable to have a cost function explicitly dependent on product output, such as in the coal market study of Haftendorn *et al.* (2010) and other production scheduling optimisation models where output represents a choice variable. Operating cost is of interest for other factors governing resource supply, such as the cut-off grade parameter in metals mining which determines the level of mineralisation required in order for extraction to proceed, see Zhang and Kleit (2016). Operating cost is also a determinant of the optimal price threshold for mine activation in real option value models, such as used in Zhang *et al.* (2014) as well as in Zhang *et al.* (2017).

The average cost derived from a total production cost curve is thought to be U-shaped and can be represented by a quadratic function. Though computationally convenient to work with, one is then faced with the problem of determining the turning point of the average cost function. Also, it is questionable whether a quadratic average cost renders a wide enough U-shape such that it is adequate in representing costs in bulk mining operations.

We propose a modified version of the Cobb-Douglas production cost function, in order to better approximate the economies of scale associated with resource extraction operations for applications requiring a cost function explicitly dependent on production quantity. The generalised Cobb-Douglas production function is used here due to its relative simplicity, good behaviour and the separability of input factors. A limiting term is introduced to account for diseconomies of scale that emerge as production levels approach technical capacity.

4.2 Relevant Literature

The appropriate functional form for industrial production cost curves has been under debate since Charles W. Cobb and Paul Douglas (1928) first proposed what has since become a widely used means for characterising the production behaviour of the firm. They proposed relating production Q to quantities of inputs, specifically labour L and capital K, via the equation

$$Q = AL^{\alpha}K^{\beta}, \tag{4.2.1}$$

where $A > 0$ indicates the state of technology, and $\alpha, \beta > 0$ denote the relative factor shares in the case of perfect competition. Equation (4.2.1) is a homogeneous function of degree $\alpha + \beta$; returns to scale are constant when $\alpha + \beta = 1$, increasing when $\alpha + \beta > 0$, and decreasing when $\alpha + \beta < 0$. In addition, equation (4.2.1) is additively separable and homothetic (Katzner, 1970). The constant-elasticity-of-substitution (CES) production function (Arrow *et al.*, 1961) generalises the Cobb-Douglas. The merits of using the Cobb-Douglas production function for analyses of production processes is argued by Bhanumurthy (2002).

The generalised Cobb-Douglas production function was studied by Vîlcu (2011) in a differential geometry setting, rendering interesting links, and in particular, that this function exhibits "constant returns to scale if and only if the corresponding hypersurface is developable" Vîlcu (2011).

The historical development of the production function is discussed in Mishra (2007). The relevance of technical and allocative efficiencies is highlighted, and the production function is more precisely defined as relating production inputs to the maximal technically feasible level of output.

Capacity-constrained production cost is typically considered by means of constraints imposed on an optimisation problem setting, as in Florian and Klein (1971), and Lambrecht and Vander Eecken (1977), and do not feature explicitly in the cost function itself.

The cost and production data typically available for modelling purposes are in many cases inadequate for specifying certain cost functional forms. As such, researchers have incorporated simpler cost functions in their models of commodity production and trade. Constant per-unit cost values have been used in, for instance, Leuthold *et al.* (2008), Holz *et al.* (2008), and Aune *et al.* (2004). Linear cost functions have been incorporated to account for changes in cost resulting from varying production levels, as in Yang *et al.* (2002) and Haftendorn *et al.* (2010). A quadratic marginal cost (MC) formulation has also been suggested, see Poulizac *et al.* (2012). However, for a quadratic MC one is faced with the problem of establishing the turning point of the average total cost (ATC) function. Golombek *et al.* (1995) propose a form for the marginal production cost where the cost increases exponentially as the quantity produced approaches the firm's maximum production capacity limit. This approach is adopted by Trüby and Paulus (2012).

4.3 Producer Behaviour Theory

Definitions and explanations of applicable microeconomic theory can be sourced from texts such as Bernheim and Whinston (2008), or Goolsbee *et al.* (2013).

A producer's operating cost of supply as a function of quantity produced q is a *total cost*, $TC(q)$, comprising *variable cost*, $VC(q)$, and *fixed cost*, FC. Variable costs are those costs that vary with the quantity of output produced, such as fuel, labour[1] and consumables. Fixed costs on the other hand are expenses incurred regardless of the quantities produced and include overhead items such as the cost of renting commercial space or the cost of assets such as premises or capital equipment. This distinction is important in the context of loss-making producers and determining when these producers would begin cutting back on output. In the short-run[2], loss-making suppliers would cut back volumes only when the average loss on those volumes exceeds the average variable cost, $AVC = VC/q$, associated with producing those volumes. Loss-making firms are able to increase output to spread FC over volumes, thereby reducing AFC. In the case where the firm has a higher FC, the firm might be inclined to maintain loss-making production volumes for longer. In the long-run, a firm would exit the industry if the market price P is below its long-run ATC.

A producer's *marginal cost* of production is simply the derivative of his total cost function, i.e.

$$MC = \frac{\partial TC}{\partial q}.$$

[1]Labour may be considered a variable cost in situations where it is in fact possible to adjust man-hours in response to different production output levels. In situations where labour contracts or laws stipulate relatively fixed working hours, this type of labour would instead be considered a fixed cost.

[2]The short-run is the nearer term during which a firm is unable to make adjustments to its capital of production, whereas in the long-term the firm has sufficient time to invest and adjust its means of production.

ATC is typically assumed to exhibit a U-shape.

4.4 Specifying Cost Function Form

The chosen form for operating cost function (4.4.3) is motivated by consideration of the Cobb-Douglas production function, which is a particular case of the CES or *constant elasticity of substitution* functions. The general CES function is given by (Ferris and Pang, 1995):

$$\left(\sum_{i=1}^{I} \lambda_i x_i^p\right)^{\frac{1}{p}} \qquad \lambda_i \geq 0, \quad i = 0, \ldots, I.$$

This function is assumed to be defined over $\mathbb{R}_+^I$, with continuity-defined boundary values. CES functions are concave for $p \leq 1$ and convex for $p \geq 1$. Then the Cobb-Douglas function is defined as (Ferris and Pang, 1995):

$$q = \prod_{i=1}^{I} x_i^{\phi_i}, \tag{4.4.2}$$

where q is the total quantity of product produced, x_i is the i^{th} input item, and

$$\phi_j \equiv \frac{\lambda_j}{\sum_{i=1}^{I} \lambda_i}.$$

We implement a modified version of the Cobb-Douglas by adding a term $\frac{a}{Q-q}$, where a is some positive constant, to account for cost increases owing to congestion as production levels q approach the maximum physical capacity output Q in the short-run. This function exhibits a U-shape with a longer, flatter bottom than the quadratic form and should as such be more representative of the economies of scale typical of bulk resource extraction operations.

That is, the miner is able to reduce per-unit costs by spreading overall costs over larger volumes, facing a limit to which production volumes can increase before inefficiencies begin to arise. The proposed production cost function is of the form:

$$TC = a \cdot q^b + k + \frac{\epsilon \cdot q}{Q - q}, \tag{4.4.3}$$

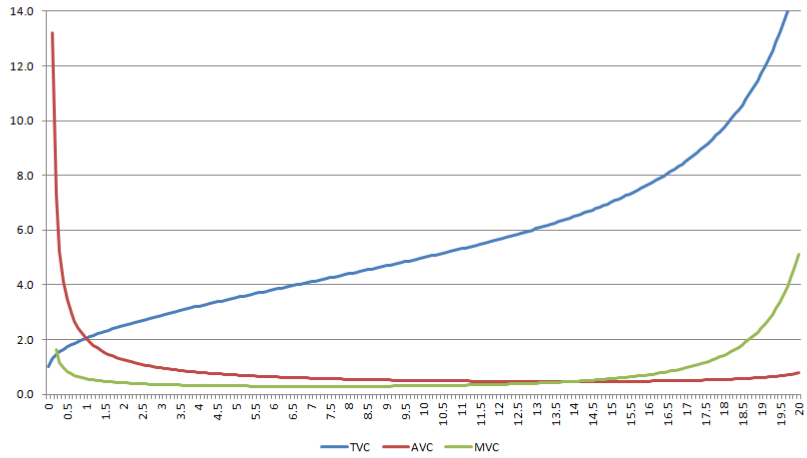

Figure 4.1: Producer Cost Functions

so that

$$AC = a \cdot q^{b-1} + \frac{k}{q} + \frac{\epsilon}{Q - q} \tag{4.4.4}$$

and

$$MC = ab \cdot q^{b-1} + \frac{\epsilon Q}{(Q - q)^2}. \tag{4.4.5}$$

These functions are plotted in Figure 4.1. Technical efficiency is assumed so that the analysis of the production function is concerned with allocative efficiency solely (Mishra, 2007).

4.4.1 Motivation for Proposed Cost Functional Form

Cost function (4.4.3) can be derived from the Cobb-Douglas production function by re-writing (4.4.2) as

$$q = \mu x_1^{\alpha_1} \dots x_n^{\alpha_n},$$

with μ some positive constant, and $x_i \in X$, where X is a vector space of dimension $n + 1$. For $\sum_i \alpha_i = 1$, returns-to-scale are constant, for $\sum_i \alpha_i > 1$, returns are increasing, and for $\sum_i \alpha_i < 1$ returns are decreasing. Now let $\frac{1}{\mu} q = \tilde{q}$, then

$$\tilde{q}^{\beta_1} \dots \tilde{q}^{\beta_n} = x_1^{\alpha_1} \dots x_n^{\alpha_n},$$

with $\sum_{i=1}^{n} \beta_i = 1$. Linearising the above equation as

$$\ln \tilde{q} = \beta_1 \ln \tilde{q} + \cdots + \beta_n \ln \tilde{q} = \alpha_1 \ln x_1 + \cdots + \alpha_n \ln x_n,$$

and letting $\ln \tilde{q} = Q$, then

$$\bar{\beta} \cdot \bar{Q} = \bar{\alpha} \cdot \bar{x} \quad \text{where } \bar{Q} = Q(1, \ldots, 1).$$

That is, the problem is then one of finding basis $\{\beta_1, \ldots, \beta_n\}$ in X such that the hyperplane $\bar{\beta} \cdot \bar{Q}$ coincides with that of $\bar{\alpha} \cdot \bar{x}$. This is achieved by choosing

$$\beta_i = \frac{\alpha_i \ln x_i}{\ln \tilde{q}},$$

so that

$$x_i = \left(\frac{1}{\mu} q \right)^{\beta_i / \alpha_i}.$$

For the desired curvature, we must have

$$0 < \ln \left(\frac{x_i}{\tilde{q}} \right) < 1.$$

That is, $\frac{\beta_i}{\alpha_i} \in (0, 1)$ if $x_i \in \left(\frac{1}{\mu} q, \frac{e}{\mu} q \right)$, where e is the natural exponent. Therefore, μ can be chosen so that the desired curvature is obtained.

Once the β_i are found, it is possible to associate with each input item a quantity factor. That is,

$$\tilde{q}^{\beta_i} = x_i^{\alpha_i}.$$

Then letting $x_i = y_i - k_i$, the cost function for input item i is given by

$$y_i = \mu^{-\beta_i / \alpha_i} q^{\beta_i / \alpha_i} + k_i. \tag{4.4.6}$$

It is required that $\beta_i / \alpha_i < 1$ and that k_i be a non-negative constant.

The Cobb-Douglas production function states production potential over a period in which adjustments can be made to capital, i.e. $Q \to \infty$. Thus, equation (4.4.6) gives the operating input cost with no capacity restrictions. In the near-term however production capacity is cost-constrained[3], which would suggest an upper limit for potential production, regardless of the addition of more input quantities. The short-run operating input cost[4] is therefore found by modifying (4.4.6) by adding a term to account for cost escalation as production quantity q nears the mine's physical production potential Q, i.e.

$$y_i = \mu^{-\beta_i/\alpha_i} q^{\beta_i/\alpha_i} + k_i + \frac{\epsilon_i \cdot q}{Q - q}. \tag{4.4.7}$$

The on-mine total cost TC formulation (4.4.3) requires a theoretic maximum mining capacity Q[5], the inclusion of which dictates the minimum value of the average total cost curve ATC. Multiplying the equation $\frac{\partial AC}{\partial q} = 0$ through by q and multiplying the equation $AC - MC = 0$ through by q^2, it is clear that the same value of q solves both equations. That is, the MC curve intersects the AC curve at the stationary point of the AC curve.

The long-run cost curve can be found by tracing the minima of the family of cost curves generated by allowing the parameters to vary with time, depending on changes in plant functionality and, in the case of resource extraction, variation in the resource deposit geology. The long-run problem is that of finding the trajectory of k_i, while the other parameters vary with deposit geology (e.g. depth, strip ratio etc.) and changes in technology.

If accounting for, say, decreasing efficiency as a result of plant aging, and by letting this situation be captured in variable $k_i(t)$, monotonically increasing with time[6], the long-run version of (4.4.4) is then found by connecting the minima of the ATC

[3]Restricted in the same sense that it is uneconomic to produce more from the same capacity.

[4]This has units in volume and not monetary values.

[5]In our model, the value of Q is given as the capacity that can be purchased using the capital spend allocated by the mine operator subject to the prevailing cost per unit capacity.

[6]It is noted that producer theory suggests that the long-run average total cost curve, $LATC$, should exhibit some U-shape as well.

functions over time.

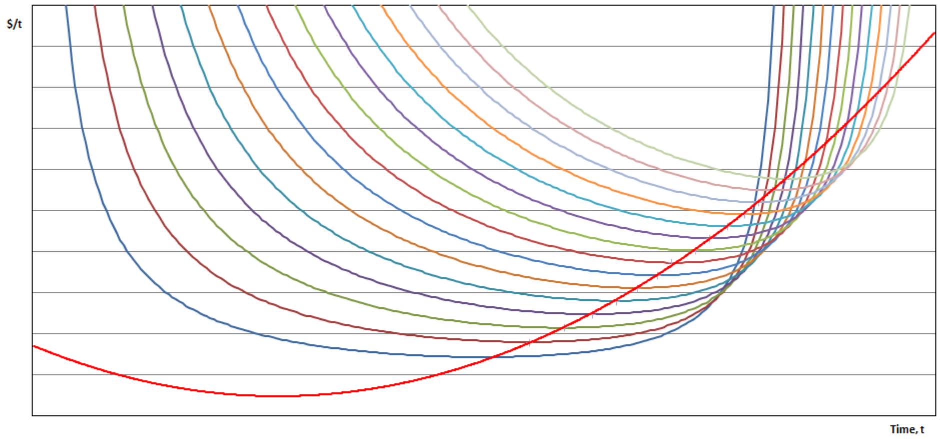

Figure 4.2: Long-run Average Cost Curve

This is illustrated in Figure 4.2. Each U-shaped curve represents a short-run average cost curve (4.4.4) at different times, with the rightmost curve being the most recent. Each U-curve is slightly higher than its predecessor due to the increase in k_i, representing higher cost associated with aging technology. The red curve connecting their minima is then the long-run average cost function which increases over time due to aging technology with no capacity expansion.

4.4.2 Concavity of Miner's Profit Function

The miner's profit function serves as the objective function in his optimisation problem. The local optimum found during the optimisation procedure is also the global optimum in the case of a concave objective function (Boyd and Vandenberghe, 2009). Concavity of the profit function in each decision variable can be seen by considering the second partial derivative of the profit function with respect to each variable. Consider a producer's profit given as a function of choice variables production quantity q and production capacity Q in the form

$$\pi(q,Q) = Pq - P^{in}\left[aq^b + d + \frac{\epsilon q}{Q-q}\right] - P^{inv}Q, \tag{4.4.8}$$

where P denotes product prices, P^{in} is factor input price, P^{inv} is the price of adding a unit of production capacity, and a, b, d, ϵ are constant parameters determined by the deposit geology.

Hence, in order to have concavity in quantity variable q, the second partial derivative must be negative, i.e.

$$\frac{\partial^2 \pi}{\partial q^2} = -P^{in}ab(b-1)q^{b-2} - 2\epsilon P^{in}\frac{Q}{(Q-q)^3},$$

is negative for $q < Q$, $a > 0$, $b \in (0,1)$, $\epsilon \geq 0$. For concavity in Q, consider

$$\frac{\partial^2 \pi}{\partial Q^2} = -2\epsilon P^{in}\left[\frac{q}{(Q-q)^3}\right],$$

which is negative on condition that $q < Q$ and $\epsilon > 0$.

4.5 Optimal Production Paths for Resource Extraction Operations

The feasibility of equation (4.4.3) representing production costs in resource extraction industries is assessed by investigating its applicability in a profit maximisation exercise, which is formulated as an optimal control problem where the optimal production path $q(t)$ is found. Refer to texts such as Kamien and Schwartz (1991) for details. With a profit function of the form given by equation (4.4.8), the present value of profit generated by an operation with a life span terminating at time T can be written as

$$V(0) = \int_0^T \pi(t)e^{-rt}dt. \tag{4.5.9}$$

Investment cost is specified by $P^{inv}(t) = \bar{P}^{inv} \cdot u^{vt}$, where $\bar{P}^{inv}$ is a positive constant, $u > 1$ and $v \in (-1, 0)$[7]. With production level $q(t)$ as state variable, and $Q(t)$ as

[7]That is, $P^{inv}(t)$ is the investment required to achieve or maintain a certain level of production capacity $Q(t)$; it is higher at the beginning of the project life and decreases exponentially over the

control variable, the state equation is

$$\frac{dq}{dt} = Q - C(t) = Q - aq^b - d - \frac{\epsilon q}{Q-q}, \tag{4.5.10}$$

and must satisfy the initial conditions $q(0) = q_0$ and $q(T) = q_T$. Further, it is required that $Q > q \ \forall t$. The current value Hamiltonian is

$$H = e^{rt}H = Pq - (P^{in} + \lambda e^{rt})\left[aq^b + d + \frac{\epsilon q}{Q-q}\right] + (\lambda e^{rt} - P^{inv})Q, \tag{4.5.11}$$

resulting in the following first order conditions

$$\frac{\partial H}{\partial q} = P - (P^{in} + e^{rt}\lambda)\left[abq^{b-1} + \frac{\epsilon Q}{(Q-q)^2}\right] \tag{4.5.12}$$

$$\frac{\partial H}{\partial \lambda} = e^{rt}\left(Q - aq^b - d - \frac{\epsilon q}{Q-q}\right) \tag{4.5.13}$$

$$\frac{\partial H}{\partial Q} = \frac{\epsilon q(P^{in} + \lambda e^{rt})}{(Q-q)^2} + (\lambda e^{rt} - P^{inv}). \tag{4.5.14}$$

Furthermore, $\frac{\partial H}{\partial Q} = 0$ implies that

$$(Q-q)^2 = \frac{\epsilon q(P^{in} + \lambda e^{rt})}{P^{inv} - \lambda e^{rt}}.$$

Keeping in mind that $Q > q$, and letting

$$\frac{P^{in} + \lambda e^{rt}}{P^{inv} - \lambda e^{rt}} = k,$$

the Q^* that maximises H is

$$Q^* = q + \sqrt{\epsilon q k}.$$

Then the state and costate equations respectively are

$$\lambda' = -\frac{\partial H}{\partial q} + r\lambda = (P^{in} + \lambda e^{rt})\left[abq^{b-1} + \frac{1}{k} + \sqrt{\frac{\epsilon}{qk}}\right] - P + r\lambda \tag{4.5.15}$$

$$q' = \frac{\partial H}{\partial \lambda} = -aq^b - d + q + \sqrt{\epsilon q}[\sqrt{k} - \sqrt{1/k}].$$

Since the arguments of the radicals in the aforestated equations cannot be negative, it follows that we must have $k > 0$, thereby implying that $\lambda < e^{-rt}P^{in}$.

The boundary value system (4.5.15) is solved numerically and the results plotted against time in figures 4.3, 4.4, 4.5, and 4.6. The profit and time values are unitless since arbitrary values are used in generating the plots.

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course of the project's life.

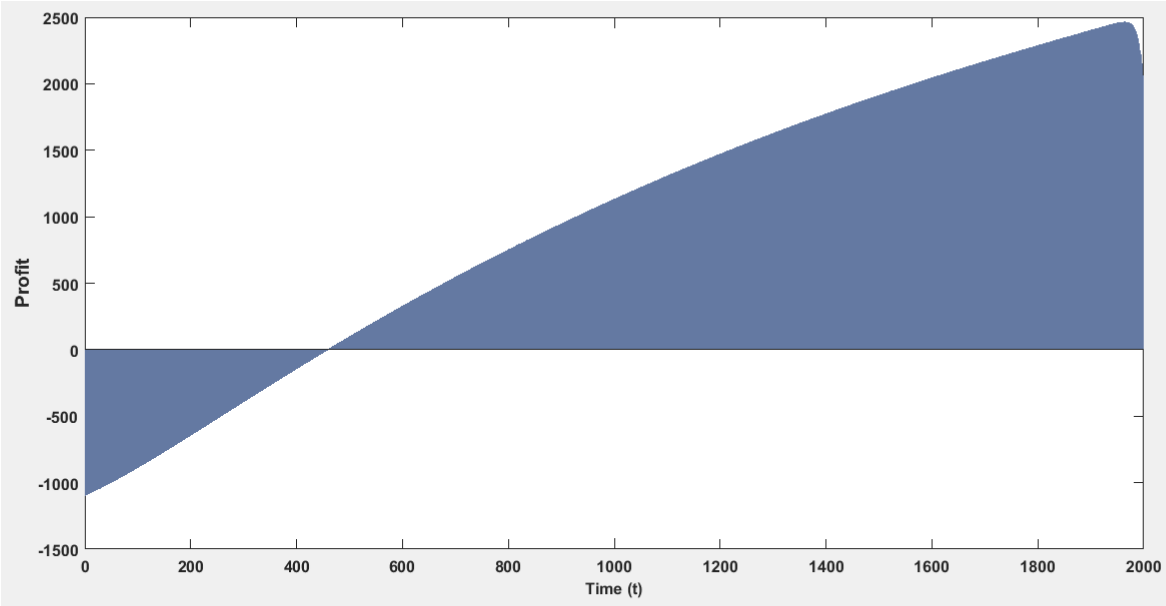

Figure 4.3: Evolution of profit over the life of the project

4.5.1 Results

As expected, profit is negative during the early stages of the project while investment in building capacity is undertaken, before turning positive and increasing for the remaining operating life, as shown in Figure 4.3.

The miner opts to build up capacity relatively quickly in the early stages of the project, thereafter raising capacity slowly for the remainder of the mine's operating life, with a sudden drop in capacity at closure. This is illustrated in Figure 4.4.

The mine operates at near-full capacity during the course of its life. The difference between capacity Q and production level q increases during this time, as shown in Figure 4.6. This could be as a result of the rise in per-tonne profit over the life of the project while investment expenditure tapers.

Setting up the problem with $Q(t)$ as control variable suggests that the capacity decision determines the production path, as opposed to the converse dependence.

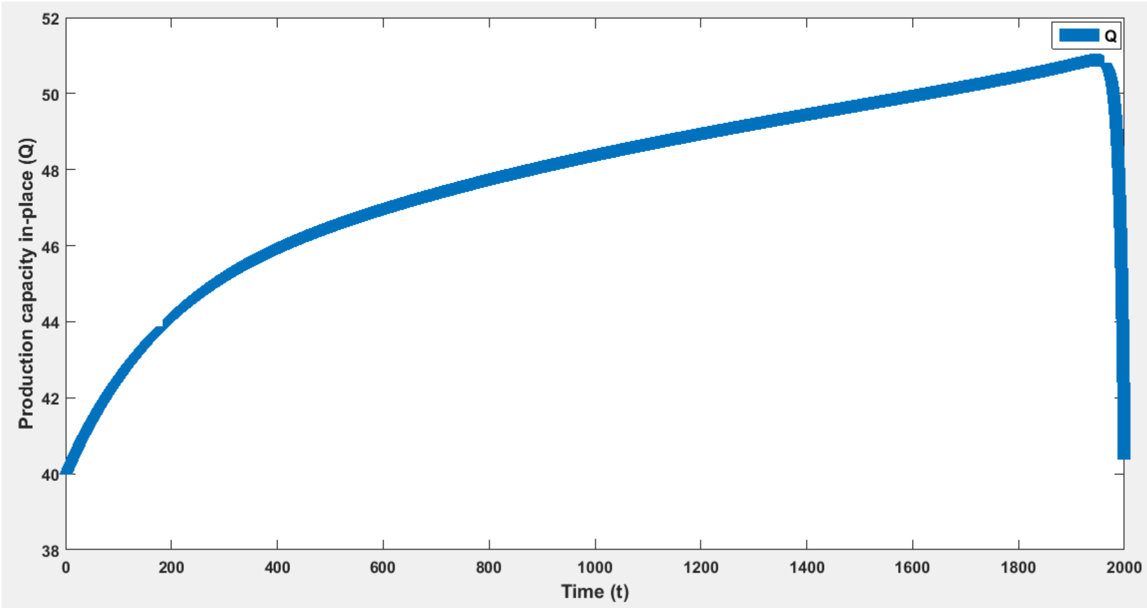

Figure 4.4: Evolution of production capacity over the life of the project

4.6 Concluding Remarks

A modified version of the Cobb-Douglas production function has been proposed for the representation of production costs in models simulating resource extraction operations. The resulting average cost function has a flatter bottom than the more commonly used quadratic variant, and as such should be more representative of the economies of scale associated with resource extraction. The proposed average cost function also possesses a minimum which is obtainable from the characteristics of the operations, as opposed to the minimum seen in the quadratic version whose location has no apparent real-world justification.

The viability of the proposed cost function is demonstrated by its application in a profit maximisation exercise constructed as a problem in optimal control. The results generated are consistent with what could be seen in a real-world resource extraction operation subject to similar conditions and constraints.

The geometry or flatness of the cost function here proposed would suggest that

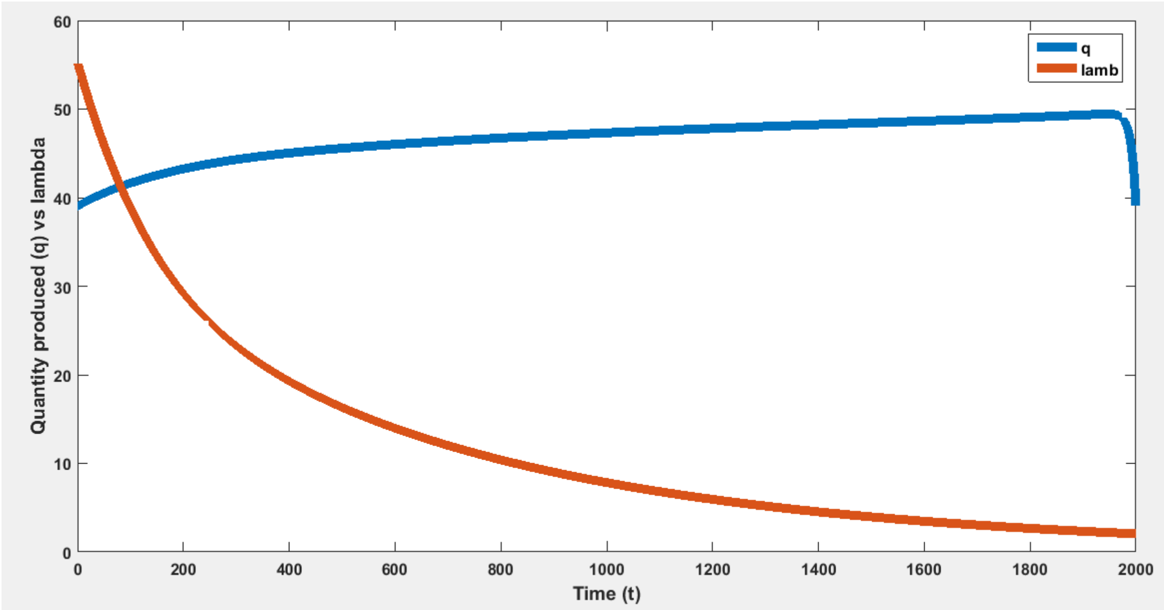

Figure 4.5: Evolution of production volume q and λ over the life of the project

the economies of scale associated with resource industries considerably reduce the effects of production-dependent variable cost on operations, as previously alluded to by Bradley (1985) who points out that the relative flatness of the average cost function renders it less influential on the miner's present value profit in comparison to substantially larger investment cost. The implication for other parameters governing resource supply, such grade selection in mining extraction, would be that these parameters are predominantly governed by capital investment cost. That is, the capital investment problem becomes of greater concern when investigating mineral economics and resource supply, while variations in production costs for fixed production capacity may be neglected.

The research presented in this paper is however limited by lack of empirical evidence to verify whether it characterises the true resource extraction cost function. Gathering the data required to test for the presence of the proposed cost function in industry would require gathering cost data for a mining operation operating at various levels of output while keeping all other factors, including production capacity, fixed. Such a task would be impractical given the difficulty controlling for external

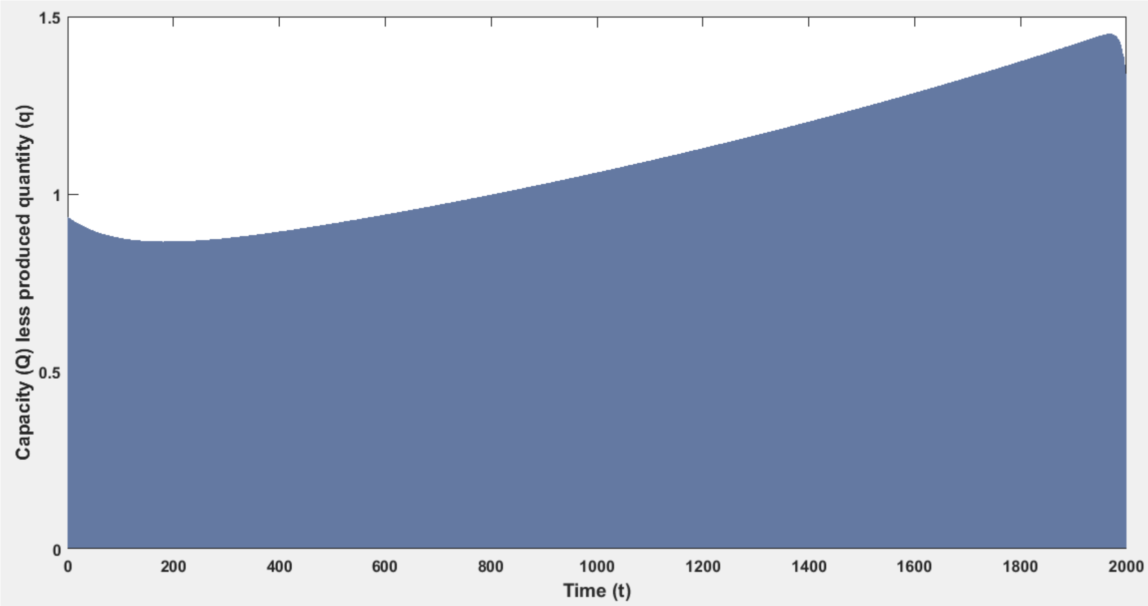

Figure 4.6: Difference between production capacity and production volumes over the life of the project

factors and the financial implications for a mine purposely operating sub-optimally.

Our future research is concerned with investigating the relative impact of production costs as compared to investment costs in governing the operations of a profit-seeking miner. This is done in a computational model and results from this effort should shed light on the implications of flat-bottomed cost curves on grade selection, as well as the classification of reserves in a mineral deposit. In addition, the nature of the resource extraction capital investment problem is investigated.

The maximum capacity has here been considered as some point value, though it may instead be represented by a region in which costs begin to rise for increasing production q. That is,

$$TC \to \infty \text{ as } Q - q \to \epsilon \text{ for } q < Q, \quad \epsilon > 0.$$

In the equation for total cost (4.4.3), the rate of increase of the limiting term $\frac{a}{Q-q}$ may be excessive regardless of how small a constant a is chosen. The rate of increase may be tempered by modifying the limiting term so that total cost (4.4.3) can instead be written as

$$TC = aq^b + k + \epsilon q \left(\frac{1}{(Q-q)^2} \right)^b .$$

Marginal cost is then

$$MC = abq^{b-1} + \epsilon \left(\frac{1}{Q-q} \right)^b \left[1 + \frac{bq}{Q-q} \right] .$$

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Chapter 5

Paper II: A Characterisation of the Mechanisms Transforming Capital Investment into Productive Capacity in Mining Projects with Long Lead-times

This chapter develops a characterisation of capital investment and is the second of the two building blocks developed for the mining project model presented in Chapter 6.

This chapter has been published in the academic journal, *Mineral Economics*, under the following citation:

Rademeyer, M. C., Minnitt, R. C. A., Falcon, R. M. S. (2020) A Characterisation of the Mechanisms Transforming Capital Investment into Productive Capacity in Mining Projects with Long Lead-times, *Mineral Economics*, **33**, 349-357.

The modelling and derivation of the proposed function, as well as the economic arguments, mathematical arguments, computational programs, and results are the work of Rademeyer. Minnitt advised on matters related to mineral economics and mining methods, and Falcon advised on matters related to coal products and coal markets.

Abstract

In the development of mining projects, there is a period in between commitment of finance and production commencement. Risks are present in various aspects of development such that production start-up could be delayed, thereby affecting the value of the project. This paper demonstrates the application of convolution in the capital investment problem for projects with long lead-times as a means of characterising the relationship between capital invested and the materialisation of capacity for production to begin. The system is functionally similar to that of a causal linear time-invariant system in signal processing. We find that the filter for capital projects, affecting the rate at which capacity becomes available for use, can be approximated by a straight-line function with positive gradient on bounded support contained in the positive time domain. The deterministic version is derived and evaluated, and uncertainty is simulated in the stochastic version, where variance results from the presence of a random process in the form of a standard Brownian motion.

5.1 Introduction

Resource extraction operations are typically capital intensive and subject to relatively long lead-times from the time a decision is first made to invest in the development of a deposit and the commencement of mineral production. This could result in capital being committed well ahead of cost realisation. Larsson and Ericsson (2014) point out in their survey of mining project investments that project costs tend to increase when a project moves from feasibility to construction. This would suggest that there are factors affecting project completion which are unknown to investors before project development actually begins. Furthermore, project value volatility was found to exceed mineral price volatility in work by Costa Lima and Suslick (2006) where a hypothetical gold mine was evaluated using the real options pricing model, indicating that it may be necessary to include other factors beyond external market- or policy-driven influences in the mining investment decision. As such, it stands to reason that a better understanding of the relationship between capital investment and physical capacity in-place would contribute towards the understanding of how lead-times affect project viability and ultimately resource supply.

In this paper a model is derived to represent the transformation of capital investment into production capacity for a project with a long lead-time. This model can be used, firstly, to aid in understanding the underlying mechanisms governing project development, and secondly, in investigations into other aspects of resource economics, such as mining operation optimisations and mineral market studies.

In what follows it is shown that this project investment problem can be considered as a problem in signal processing, by means of the convolution integral in the case of continuous-time problems, and the convolution sum for discrete-time problems. The use of convolution in capital investment theory is also found in Jorgenson (1996), and Kamien and Muller (1976). When considering linear time-invariant systems, the system is fed with an *input signal* $f(t)$ which, when convolved with the system's *impulse response* $h(t)$, produces an *output signal* $y(t)$, i.e.

$$f(t) \longrightarrow \boxed{\text{h(t)}} \longrightarrow y(t) = h(t) * f(t).$$

In situations where the input signal is known and the output signal can be measured, it is theoretically possible to calculate the system's impulse response, thereby obtaining information about the system's configuration. The linear time-invariant filter is completely described by the convolution

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau. \tag{5.1.1}$$

A system that produces a signal only in response to an input signal is termed a *causal system*, and in such a case the lower limit of integration in equation (5.1.1) is zero. The application of convolution in signal processing is discussed in texts such as Mitra and Kaiser (1993), Haykin and Van Veen (2005), and Carlson (1992).

Thus, for a mining project in particular, one might think of the capital injections as the input signal, the physical mine development process as the impulse response, and the rate at which capacity comes on-line as the output signal. It follows that since both the rate at which capital is injected and the rate at which capacity comes on-line are known or measurable, it becomes possible to gain insight into the project's characteristics and the underlying mechanisms governing the project's development. Moreover, techniques developed for solving problems in signal processing are equally valid for solving the mathematical problem of lead-time in project investment.

A similar formulation for capital investment appears as a term in the objective function of the coal mining optimisation presented in Rademeyer *et al.* (2019). There is however no motivation given for the viability of this formulation.

The research presented in this paper contributes to the knowledge on investment in mining projects by fully deriving the characterisation[1] of the relationship between capital invested and eventual production capacity that exists during the construction phase of a project. This characterisation presents a novel means for studying

[1]The terms "characterisation", "formulation" and "model" are used interchangeably throughout the paper.

investment in projects with long development lead-times, such as mine projects. To our knowledge, no earlier work discusses project development with a focus on the lead-time. Furthermore, the research presented in this paper establishes the link to signal processing, thereby making signal processing techniques available to solving variations on the lead-time problem in project investment. The derived formulation is solved and the results validate the model. In addition, we expand on the applications of the model and show how uncertainty in lead-time can be accounted for.

The research presented in this paper is useful and an improvement on previous approaches in that we look specifically at the lead-time that exists during the construction stage of the development phase. We formulate the problem such that techniques from signal processing, a well-established research area in a number of engineering disciplines, become available for solving this investment problem. The formulation could be used for improving estimates of the onset of production and the subsequent initiation of cash flow. This would support accuracy in project valuation. In addition, our approach can accommodate uncertainty by the introduction of a random variable, thereby making it viable for use in studies of project risk.

Our research is applicable to "completion risk" in particular, or the possibility that the production capacity for a mining project might not be available for utilisation as planned. This may be due to delays in construction, costs overruns requiring additional funding, and engineering flaws that require redesign (Park and Matunhire, 2011). It follows that our model relates to aspects of the design of the mine and its construction, as opposed to the mine schedule[2].

[2]Mine design versus mine plan or schedule is discussed in Savolainen (2016). The former is the design of the layout of the mine and the operational flows. The latter is the plan for the extraction of the orebody, the sequencing for the removal of individual units (Smallest, Mining Units, SMUs) by which the deposit is depleted, and the areas of operational activity. Whereas the mine design is finalised, and thus fixed, before mine construction begins, the mine plan is revised periodically and can be adjusted to support mine economics in response to changing market conditions.

We begin by discussing literature on capital investment theories, investment in mining projects, and project risk evaluation. This is followed by the derivation of the transformation of invested capital into physical production capacity, thereby showing that this problem can indeed be structured as a problem in signal processing as outlined above. We then show how, by introducing a random variable to simulate the presence of uncertainty during the development phase, one might observe how this uncertainty would affect the rate at which the physical capacity reaches completion and becomes available for utilisation. We also cover briefly how the formulation could be applied before ending with concluding remarks.

5.2 Relevant Literature

The literature discussed here is divided broadly into three areas as connected to the three main concepts addressed by the model presented in this paper. These are project investment theory, project development lead-time, and project risk. As will be seen, literature on investment theory has been concerned mainly with investor behaviour and optimal investment paths, though there has also been some interest in characterising and measuring risk associated with investments with longer lead-times. Investigations into mining project development in particular have studied project valuation, with the effects of risk restricted to those stemming from environmental and market factors. Our literature review did however not reveal any previous research concerning a focused treatment of the lead-time mechanisms relating capital outlays to physical production capacity in place.

The importance of capital investment decisions in determining resource supply is discussed by Bradley (1985) who points out that the considerable pre-production investment required in extraction industries together with the relatively flatness of the average cost function in a rather broad region around the optimal extraction rate suggests that the miner's present value profit would be more sensitive to changes in investment allocation than changes in production output. This is also shown in Rademeyer *et al.* (2018). A review by Nourali and Osanloo (2019) of models used in estimating mining capital cost reveals that production capacity is the most influencial driver of cost.

Theories of investment are generally respective to an individual firm, a single industry, or the wider economy in general. The most influential investment theories are reviewed by Eklund (2013). In particular, the neoclassical theory of investment (Jorgenson, 1963) and Tobin's Q-theory (Tobin, 1969) are discussed. Eklund (2013) also covers the accelerator principle as a particular case of the neoclassical approach with constant prices.

Jorgenson's neoclassical investment theory (Jorgenson, 1963) essentially states that a firm achieves an optimal capital stock when it maximises profit in each period over the life of its operations. Capital stock K^* is derived as a function of production output quantity Y, the price of output P and the user cost of capital c, as

$$K^* = \frac{\alpha}{c} PY,$$

with α a constant[3]. Eklund (2013) however points out that Jorgenson's neoclassical theory is really a capital theory rather than an investment theory given that the investment function falls away as a result of the theory assuming immediate adjustment to the desired level of capital stock. This may be remedied by introducing a cost to capital adjustment, as in Gould (1968).

Tobin's Q-theory introduced an adjustment-cost function, thereby restricting the rate at which capital stock can adjust so that investment is then "determined as the optimal adjusted path to the optimal capital stock" (Eklund, 2013). The result of the model proposed by Tobin (1969) suggests that the rational firm will continue investing in developing new production capital until the market value of this cap-

[3]Refer to texts on the Cobb-Douglas production function for details.

ital asset is equal to that of purchasing an already existing asset. Jorgenson and Stephenson (1967) characterised at an industry level the lag between drivers of investment and actual expenditures in US manufacturing. The implications of capital adjustment costs for the investment behaviour of the firm is discussed by Gould (1968).

The effects of time-to-build on mining project value is discussed by Zhang *et al.* (2017) in a real options valuation setting. They find that the optimal price threshold strategy is altered when accounting for time-to-build, and that time-to-build cannot be ignored when evaluating projects with high investment cost and exposure to highly volatile mineral prices. In particular, time-to-build reduces the real options value, with higher value lost as mining cost rises, and results in a higher optimal price threshold to signal mine activation.

The measurement of risk in mining projects is considered essential. Risk assessments are needed in order to account for losses that could arise during the development and operational life of a project. This is to ensure that the investment decision is made based on an accurate valuation of the project. According to Park and Matunhire (2011), mining projects are characterised by large risks, large capital investment requirements, and with payback periods that are relatively long. There are three types of risk in mining projects: technical risk, economic risk, and political risk. Abel (1983) investigates the investment behaviour of a risk-neutral competitive firm in an uncertain product price environment and when faced with convex capital adjustment costs. Teisberg (1993) uses options pricing to model the firm's investment decision when accounting for project flexibility, and shows that investment in projects with shorter lead-times is favoured amidst regulatory uncertainty and uncertainty surrounding the project's usefulness upon completion. In addition, it was found that the recovery of costs following decommissioning may provide incentive for a rational firm to continue investing and delay abandonment. Pindyck (1993) found that irreversible investment decisions concerning projects with lead-times differ when faced with technical costs uncertainty, involving physical difficulties experienced during project development, as compared to input cost uncertainty, concerning construction factor prices and government regulations. That is, investment decisions vary when confronted with costs over which the firm has influence as compared to costs that are external to the firm.

In response to the growing importance of risk assessment in project development, a couple of prominent risk measurement techniques have emerged. One such method is by Monte Carlo simulation. Heuberger (2005) demonstrates how risk can be accounted for in the widely used Net Present Value (NPV) or discounted cash flow model for mine evaluation by introducing a random variable to simulate uncertainty. Another method is the real options valuation (ROV) which argues that the decision to invest in a project can be considered as a financial option instrument. The superiority of ROV when compared to the NPV approach is discussed in Foo *et al.* (2018). ROV takes into account the ability to delay investment until such a time that the project will render the greatest value. That is, it accounts for project value arising from flexibility to benefit from uncertainty. An extensive review of real options in metals mining is done by Savolainen (2016).

Machine learning as a means for determining capital expenditure required for a mining project is discussed in Nourali and Osanloo (2019). In particular, a model based on Support Vector Regression (SVR) is developed using data on capital cost from 52 mines. Machine learning methods in general require large data sets for training the models. Gathering this data could be resource intensive and difficult in mining where financial data is typically confidential. Nourali and Osanloo (2019) also reviews other data-driven estimation methods used in developing project cost models and finds that regression-based methods are most widely used.

5.3 Deriving a Model of Capacity Investment with Lead-time

As a consequence of the size and complexity of mining operations, there is typically a lead-time of several years from the time a decision is made to invest in additional capacity and the time at which that capacity becomes available for employment. Let the actual capacity online at time t be denoted by Q_t and the additional capacity invested in at time t by I_t, and introduce a maximum lead-time parameter $L \in \mathbb{N}$. Then, assuming that parts of investment I_t become available to the miner over the years until L, effective capacity online Q_t is given by

$$Q_t = Q_{t-1} + \alpha_0 I_t + \alpha_1 I_{t-1} + \cdots + \alpha_{L-1} I_{t-L+1}, \tag{5.3.2}$$

where

$$\sum_{t=0}^{L-1} \alpha_t = 1; \qquad \alpha_i \in [0,1], \quad i = 0, \ldots, L-1.$$

Expanding equation (5.3.2) further yields an expression for the effective capacity online at time t in terms of initial capacity Q_{-1} and all preceding investments, that is,

$$Q_t = Q_{-1} + \sum_{s=0}^{L-1} \alpha_s (I_{-s} + \cdots + I_{t-s})$$

so that

$$Q_t = Q_{-1} + \sum_{s=0}^{L-1} \alpha_s \left(\sum_{\tau=0}^{t} I_{\tau-s} \right) = Q_{-1} + \sum_{s=0}^{L-1} \sum_{\tau=0}^{t} \alpha_s I_{\tau-s}. \tag{5.3.3}$$

For a new project $Q_{-1} = 0$. Assuming the limit exists, the continuous case of the problem would have capacity $Q(t)$ at time t given by

$$Q(t) = Q^* + \int_0^t \left[\int_0^L \alpha(s) I(\tau - s) ds \right] d\tau, \tag{5.3.4}$$

with $Q^* = 0$ for a new project. It is noted that the term in square parentheses is the convolution $(\alpha * I)(\tau)$, as expressed in equation (5.1.1), for $\alpha, I : [0, \infty) \to \mathbb{R}$[4].

[4]That is, functions α, I are supported on $[0, \infty)$ only, i.e. zero for negative arguments.

It is assumed that purchase costs are the same for expansion and replacement. The same assumption is made for internal costs as well, though this may not hold in reality. Plant adjustment costs should be modelled as being proportional to the adjustment rate. However, the possibility that there could be dis-economies associated with more rapid changes in capital stock (Gould, 1968) is here ignored.

Suppose investment is given by

$$I(t) = \begin{cases} Ae^{kt} & \text{for } t \geq 0, \\ 0 & \text{for } t < 0, \end{cases} \tag{5.3.5}$$

with constants $A > 0$ and $k < 0$, and suppose the lead-time response is given by

$$\alpha(t) = \begin{cases} pt + m & \text{for } 0 \leq t \leq L, \\ 0 & \text{otherwise,} \end{cases} \tag{5.3.6}$$

with constants $p, m > 0$.

There are three cases to consider:

Case I: $t < 0$

$$h_1(t) = \int_{-\infty}^{t} [0] I(t-s) ds = 0,$$

Case II: $t \geq 0$ or $t \leq L$

$$h_2(t) = \int_{-\infty}^{0} [0] I(t-s) ds + \int_{0}^{t} \alpha(s) I(t-s) ds,$$

Case III: $t > L$

$$h_3(t) = \int_{-\infty}^{0} [0] I(t-s) ds + \int_{0}^{L} \alpha(s) I(t-s) ds + \int_{L}^{t} [0] I(t-s) ds.$$

Therefore we have that

$$h(t) = \begin{cases} 0 & \text{for } t < 0, \\ \bar{A}e^{kt} - \frac{A}{k}pt - \bar{A} & \text{for } 0 \leq t \leq L, \\ \bar{B}e^{kt} & \text{for } t > L, \end{cases}$$

with constants

$$\bar{A} = \frac{A}{k}\left(\frac{p}{k} + m\right) \text{ and } \bar{B} = -\frac{A}{k}Le^{-kL}.$$

Furthermore, if $t < 0$, the production capacity in-place is given by

$$Q_1(t) = Q^* + 0 = Q^*.$$

If $0 \leq t \leq L$,

$$\begin{aligned} Q_2(t) &= Q^* + \int_0^t h_2(\tau)d\tau \\ &= Q^* + \frac{\bar{A}}{k}e^{kt} - \frac{Ap}{2k}t^2 - \bar{A}t - \frac{\bar{A}}{k}. \end{aligned}$$

If $t > L$,

$$\begin{aligned} Q_3(t) &= Q^* + \int_0^L h_2(\tau)d\tau + \int_L^t h_3(\tau)d\tau \\ &= Q^* + \frac{\bar{A}}{k}e^{kL} - \frac{Ap}{2k}L^2 - \bar{A}L - \frac{\bar{A}}{k} + \frac{\bar{B}}{k}\left(e^{kt} - e^{kL}\right). \end{aligned}$$

To ensure that $h(0) = 0$, it is required that $\bar{A} = 0$, from which it follows that $p = 1$ and $m = -\frac{1}{k}$. The equations comprising $Q(t)$ can then be reduced to

$$Q(t) = \begin{cases} Q^* & \text{for } t < 0, \\ Q^* - \frac{A}{2k}t^2 & \text{for } 0 \leq t \leq L, \\ Q^* + \frac{AL}{k^2}\left[1 - e^{k(t-L)}\right] - \frac{A}{2k}L^2 & \text{for } t < L. \end{cases} \quad (5.3.7)$$

5.3.1 Results

The result for capacity in-place $Q(t)$ as given by (5.3.7) is plotted for a particular set of parameter values and shown in Figure 5.1. The resulting curve for capacity in-place resembles a logistic curve, similar to how one might expect capacity to gradually come online for a project with a long lead-time.

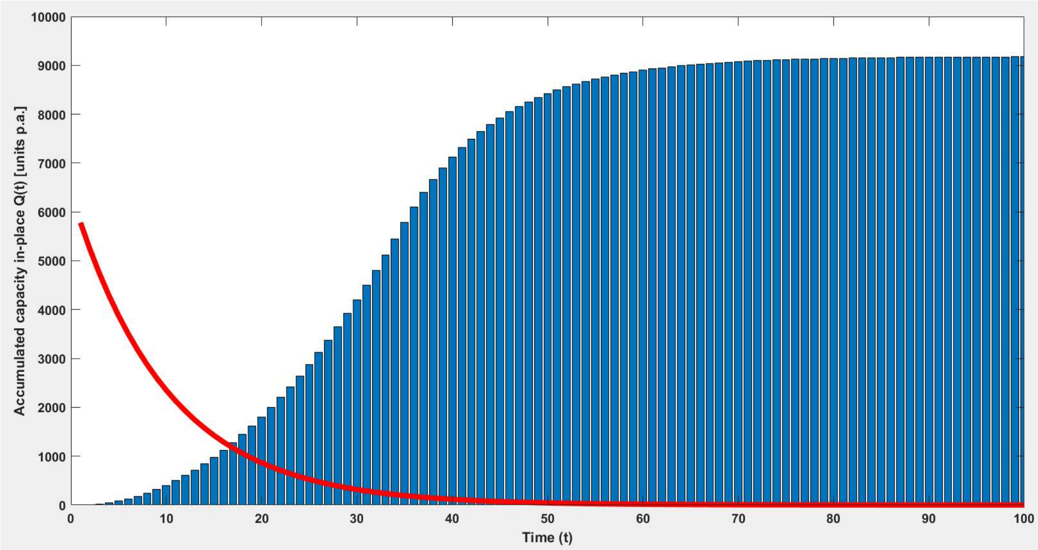

Figure 5.1: Capital accumulation (blue bars) resulting from investment (red line, rescaled) over time

5.4 Capital Investment and Accumulation with Capacity-loss due to Deterioration

As plant ages, it gradually loses functionality resulting in decreasing productivity coupled with increasing operating costs. Consequently, capacity is effectively reduced[5]. Additional capacity that comes online at time t is given by

$$dQ_t = Q_t - Q_{t-1} = \sum_{s=0}^{L-1} \alpha_s I_{t-s},$$

which, when accounting for age, would after Δt periods effectively be

$$(1 - r_A)^{\Delta t} dQ_t = (1 - r_A)^{\Delta t} \sum_{s=0}^{L-1} \alpha_s I_{t-s},$$

where r_A is the rate at which functionality is lost. Therefore, the effective capacity in-place at time t is then

$$A(Q_t) = (1 - r_A)^{t+1} Q_{-1} + (1 - r_A)^t dQ_0 + \cdots + (1 - r_A) dQ_{t-1} + dQ_t$$

[5]An alternative approach is given in Kamien and Muller (1976) where a mortality function is convolved with capacity.

$$= (1 - r_A)^{t+1} Q_{-1} + \sum_{s=0}^{L-1} \alpha_s \sum_{\tau=0}^{t} (1 - r_A)^{t-\tau} I_{\tau - s}, \tag{5.4.8}$$

where $A(Q_t) \leq Q_t$, i.e. actual capacity in-place is no more than apparent capacity invested in.

The continuous version of equation (5.4.8) is given by

$$AQ(t) = AQ^* + \int_0^t (1 - r_A)^{t-\tau} \left[\int_0^L \alpha(s) I(\tau - s) ds \right] d\tau. \tag{5.4.9}$$

Using the result given for $h(t)$ in (5.3.7) for the inner integral of (5.4.9), we find that an expression for $AQ(t)$ can be found by combining the following three cases:

Case I: If $t < 0$,

$$AQ_1(t) = AQ^*. \tag{5.4.10}$$

Case II: If $0 \leq t \leq L$,

$$AQ_2(t) = AQ^* + \frac{A}{k} \left[\frac{t}{\ln(1 - r_A)} + \frac{1 - (1 - r_A)^t}{(\ln(1 - r_A))^2} \right]. \tag{5.4.11}$$

Case III: If $t > L$,

$$AQ_3(t) = AQ_2(L) + \frac{A}{k} L e^{-kL} \left[\frac{e^{kt} - (1 - r_A)^t}{\ln(1 - r_A) - k} \right]. \tag{5.4.12}$$

5.4.1 Results

Figure 5.2 illustrates the accumulation of capital resulting from the investment profile $AQ(t)$ given by equations (5.4.10)-(5.4.12). The capital in-place is also affected by a constant rate of deterioration r_A so that the capital accumulation curve exhibits a gentler incline before reaching its maximum as compared to the curve seen in Figure 5.1. In addition, the capital in-place begins to decline after its initial peak, unlike the curve in Figure 5.1, during the time when investment tapers, thereby indicating that the tapering level of investment is inadequate to compensate for capacity loss[6]. Additional investment would thus have to be undertaken to maintain a constant capacity level.

[6]In practice, plant deterioration becomes apparent as decreasing efficiency. However, since

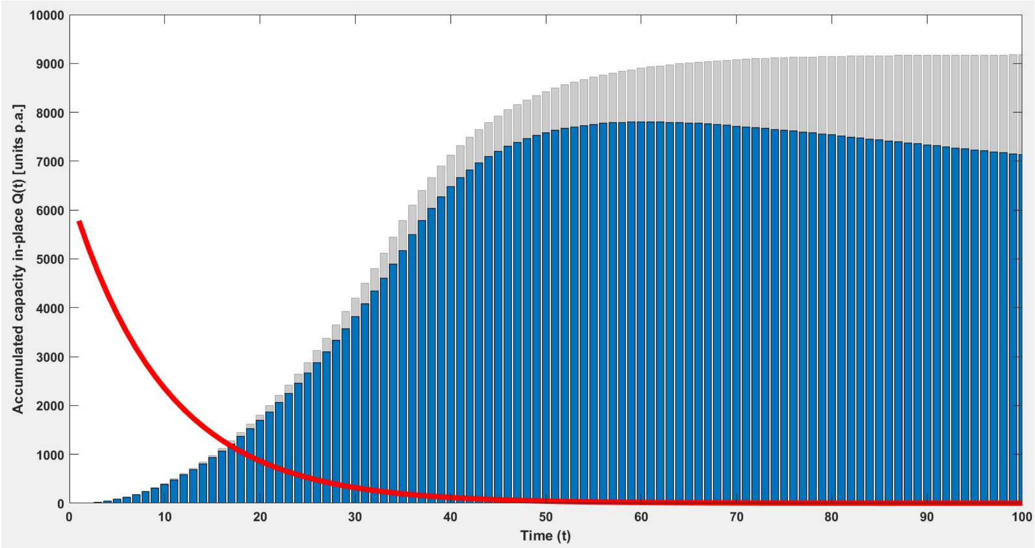

Figure 5.2: Capital accumulation (blue bars) resulting from investment (red line, rescaled) over time, and subjected to a constant rate of deterioration. Capacity unaffected by deterioration is shown in the background (grey bars) for comparison.

5.5 Capital Accumulation Subject to Uncertain Project Delivery

The problem of capital accumulation where the progress on the delivery of capital invested in is uncertain can be investigated by restating the problem with the delivery determination function $\alpha(t)$ cast as a random process. A comprehensive treatment of stochastic processes is provided in text such as Øksendal (2003). Differentiating (5.3.4) with respect to t yields the differential equation

$$\frac{dQ}{dt} = (\alpha * I)(t) = \int_0^L \alpha(s)I(t-s)ds.$$

Suppose $\alpha(t)$ consists of a deterministic part $r(t)$ and "white noise", i.e. $\alpha_t = r_t + aW_t$ where $0 \leq t \leq L$. W_t is a Brownian process satisfying the stochastic differential equation

$$dW_t = \mu_t dt + \sigma_t dB_t,$$

efficiency is the ratio of output to input, a decrease in efficiency would have to result in a decrease in output (or production) for a constant input. This would hold true for the maximum output (as designed) as well. Then, since capacity is defined as the maximum output per unit time, a decrease in efficiency per unit time results in a decrease in capacity.

where B_t is a standard Brownian motion, i.e. $B_t \sim N(0,t)$, $\mu : \mathbb{R}^n \to \mathbb{R}^n$ is the drift field and $\sigma : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is the diffusion field. Then

$$dQ_t = \left[\int_0^L r_s I_{t-s} ds\right] dt + a \left[\int_0^L W_s I_{t-s} ds\right] dt,$$

with $I_t = I(t)$ as given in equation (5.3.5).

The solution to the first term was obtained in Section 5.3 above. For the solution to the second integral, there are again three cases to consider:

Case I: $t < 0$

$$h_1(t) = \int_{-\infty}^{t} [0] I_{t-s} ds = 0$$

Case II: $0 \leq t \leq L$

$$h_2(t) = \int_{-\infty}^{0} [0] I_{t-s} ds + \int_0^t W_s I_{t-s} ds$$

Case III: $t > L$

$$h_3(t) = \int_{-\infty}^{0} [0] I_{t-s} ds + \int_0^L W_s I_{t-s} ds + \int_L^t [0] I_{t-s} ds$$

In essence, we are required to evaluate

$$\int_0^t W_s I_{t-s} ds = \int_0^t W_s A e^{k(t-s)} ds = A e^{kt} \int_0^t W_s e^{-ks} ds,$$

keeping in mind that $A > 0$ and $k < 0$. It is noted that for our choice of function $I(t)$, the integral is the Laplace transform of W_t. Let $-k = h > 0$. The presence of the positive exponential results in increasingly unstable behaviour for $t \to \infty$, suggesting that variance increases over time. That is, greater volatility in project completion is associated with longer lead-times, as expected.

Rewriting e^{hs} in its Maclaurin series representation, we have

$$\int_0^t W_s e^{hs} ds = \sum_{n=0}^{\infty} \frac{h^n}{n!} \int_0^t W_s s^n ds,$$

where the integral gives the n-th moment of W_t.

5.6 Applications

Models are useful because they give decision makers an idea about possible outcomes that may materialise. This section outlines applications where the formulations or models presented in Sections 5.3, 5.4, and 5.5 could be used.

The models would be useful to commodity analysts for simulating the lag between project investment and commodity production in market studies. This is seen in Rademeyer *et al.* (2019) where the model appears as a term representing production capacity in the objective function of the cash flow optimisation problem. In fact, it is the right-hand term of equation (5.3.3) in Section 5.3 that is used. Rademeyer *et al.* (2019) solve the cash flow optimisation and this yields results which are consistent with what might be seen for real-life mining operations.

When determining capacity requirements and accounting for delays, using the model we can determine capacity to be invested in at the time an investment decision is made. The product of capacity-invested-in, and the price of that capacity, yields the investment value at the time of decision making. The same model can be used to address both expansion investment and replacement investment.

Suppose total project capacity requirement is given by K, consisting of several distinct aspects such that total production capacity is given by

$$K = \sum_j K_j.$$

Then when taking into account lead-time from investment until operational start up, capacity to be invested in is given by

$$I = F(K),$$

where F is the function given in Section 5.3. Capacity invested in could be a sum of a series of investments in the lead-time period $[0 : T]$ so that

$$I = \sum_{t}^{T} I_t.$$

The a single investment or injection at time t would be given by I_t. Then the value of that investment would be given by

$$V_t = I_t * P_t.$$

That is, the product of the capacity invested in I_t and its price at time t as denoted by P_t.

In this way changes over time in procurement prices, due to market cycles and dynamics, can be accommodated. Economies of scale whereby procurement costs are lowered as a result of increasing project size could also be accommodated here by reformulating P_t as a function of total production capacity K. Mine size and the heterogeneity of ore material should be accounted for when determining production capacity requirements[7]. Capacity to be invested in and lead-times would vary depending on whether the investment is for replacement or expansion. Extending the model over the production time horizon would enable further investigation into the trade-off between investing in technology to expand or maintain production capacity, and the cost savings that result.

Risk analysis supports decision making by indicating what the outcomes from a decision taken might be, and how likely the outcomes are to deviate from the expected outcome or average. In practice, one might have a model with a random variable as input and one would then observe results as a range of possible outcomes. This has

[7]The determination of the capacity required to exploit the heterogenous orebody at every stage of mine-life is a very complex process. The required production capacity is non-uniform as well. To deal with investment in capacity composed of several separate parts, one could construct a model as the sum of the models as in Section 5.3 for each individual part.

been demonstrated analytically by the use of a geometric Brownian motion as input in the model in Section 5.5. Monte Carlo simulation for modelling project risk is demonstrated in Heuberger (2005).

Determining the most appropriate filter, as in equation (5.1.1) in Section 5.1, for the lead-time in mining project development could be done on a trial-and-error basis. That is, by using financial investment data from actual projects, convolving with a trial filter (as shown in Section 5.3), and then comparing the resulting production capacity with actual outcomes. Once suitable filters or "transformation" profiles are determined then the model can be used to estimate capital expenditure under uncertain scenarios.

5.7 Concluding Remarks

We have demonstrated the application of convolution in the capital investment problem for a project with a long lead-time. The system is functionally similar to that of a causal linear time-invariant system in signal processing. It was found that the filter for capital projects can be approximated by an increasing affine function on bounded support contained in the positive time domain. Such a function convolved with a decreasing exponential integral representing capital investment over time provides a feasible model for investment and the resulting capital expansion in projects with long lead-times. The deterministic version was first evaluated, with consideration given for plant deterioration due to aging. In addition, a stochastic version of the problem was considered by introducing a random process in the form of a standard Brownian motion for simulating uncertainty. The variation introduced was found to increase exponentially with time, resulting in rapidly increasing variability in situations where lead-times are large.

Our proposed approach lays a basis upon which layers of complexity seen in mine development can be built. Modifications can be introduced towards establishing a more representative model of projects with long lead-times. It is expected that the functional form for the lead-time filter will change when accounting for particular aspects of project development, such as logistics, procurement and politics. Future work should be concerned with determining viable candidates for lead-time filters. For instance, it would be useful to know what lead-time filter would render a step-wise output signal such as would more realistically represent how production capacity becomes operational, i.e. by step-wise increases in production. It would also be useful to investigate scenarios with functionality loss as determined by the rate of capacity utilisation, as well as the form of the filter in the case where investment cost is proportional to rate of investment. The implications for investment in expansion technology versus investment in replacement technology by means of varying capacity requirement in the model, present another area of future research.

We believe that the formulation presented in Section 5.5 provides insight into how uncertainty affects the mechanism at work during the time-to-build lead-time. This should support the structuring of more cost-effective risk mitigation measures. Future work should investigate how structuring financial injections or the use of appropriate financial instruments could be used to manage this risk and to compensate for delayed income resulting from a delayed start in operations.

We acknowledge that in reality, industrial production capacity is either available for use or not, and that the proposed gradual availability of production capacity is not entirely realistic. In mining for instance, a new mine would ramp up production from new capacity over time from when the capacity first becomes available for use. However, the model is still useful when the production capacity yielded by solving the problem in Section 5.3 is taken to be *level of completion* and production begins only once capacity reaches a certain level, thereby achieving the binary start-up seen in mining projects in reality.

Acknowledgments

The authors would like to thank Prof. Bruce A. Watson (The University of the Witwatersrand) for his many valuable suggestions. The authors would like to thank the reviewers for their comments and suggestions.

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Chapter 6

A Mathematical Optimisation Approach to Modelling the Economics of a Coal Mine

A model of a coal mining project is presented in this chapter. It demonstrates the application of the cost function from Chapter 4 and the investment characterisation from Chapter 5.

This chapter has been published in the academic journal, *Resources Policy*, under the following citation:

Rademeyer, M. C., Minnitt, R. C. A., Falcon, R. M. S. (2019) A Mathematical Optimisation Approach to Modelling the Economics of a Coal Mine, *Resources Policy*, **62**, 561-570.

The derivation of the proposed model, as well as the economic arguments, mathematical arguments, computational programs, and results are the work of Rademeyer. Minnitt advised on matters related to mineral economics and mining methods, and Falcon advised on matters related to coal products and coal markets.

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6.1 Abstract

A mathematical model of a multi-product coal mining project is proposed to characterise the economics governing coal production. The model accounts for mining cost variation due to varying production levels, changes in reserves available for extraction by means of a cut-off parameter, as well as lead-time delays between investment and when operations begin. The resulting constrained optimisation problem is solved computationally by maximising the net present value of future cash flows from the mining operation subject to physical limitations. We find that the capital investment problem has greater bearing on the overall project configuration decision and that changes in production capital are made throughout the life of the project while production volumes are largely unchanged.

6.2 Introduction

Coal, the organic material fueling some 40% of the world's electricity production, is extracted from the earth's crust by means of mining operations. Despite growing opposition to the use of coal amid concerns over the extent to which the carbon dioxide produced during coal combustion contributes to ostensible climate change, coal is nevertheless expected to remain an import source of relatively inexpensive fuel for baseload electricity generation.

Coal mining is capital intensive and subject to a relatively long delay from the time the decision is first made to invest in the development of a deposit and the time the first coal is produced. A better understanding of how the viability of a proposed mining project changes in response to changes in broader economic conditions would contribute towards improving the management of economic risk associated with mine development. A mine model also provides the backbone for the supply-side of a resource trade model, which is often employed in the study of commodity markets and which in turn impacts on mine valuations.

Towards this end, a computational optimisation model of a generic coal mine, GEN-COMIN, is developed in an attempt to narrow the gap between theoretical economic concepts and situations in the real-world mining industry, for application in economic or financial investigations. Due to the complexity of mining operation, models often require vast data sets. However, by exploiting relationships between variables, it is possible to reduce the number of variables in a model, thereby reducing the size of the data set required for calibration. The advantages of working with smaller data sets include reduced time spent on data mining as well as circumventing instances of erroneous, missing or unavailable data. Our model accommodates the production of multiple, distinct coal products, as well as changing resource deposit characteristics. In addition, the model provides a description of the evolution of the mine's reserves and resources over time.

Our model presents a much needed simplified description of mining operations for individuals with limited knowledge of mining who are involved in conducting economic research in coal markets, and whose research may impact on coal usage and availability. Industry analysts forecast prices based on their assessments of supply trends, which in turn affects market price expectations and can impact on investment plans. Policy makers set mineral policy and taxes based on their understanding of how these measures would affect mineral production in their jurisdictions. Policy setting can both stimulate or stunt the development of the mineral sector.

Though this research focuses on coal mining, the model developed here can be adapted to apply to the extraction of other mineral resources, especially other bulk commodities such as iron ore and copper. This would however require that reserves be determined by means of the cut-off grade parameter used for metals (refer to Section 6.4.1 for details), as opposed to the coal-specific cut-off parameter developed in this paper. In addition, the conversion between ore and energy used here for coal should instead convert ore to pure metal when analysing metals commodities.

6.2.1 Coal Mining Methods

Coal is a solid aggregate formed over millions years by sedimentation and the transformation of decomposed swamp vegetation, resulting from geological processes induced by compaction, temperature, and pressure (Speight, 2005). Though often mistakenly thought of simply as "black rock", coal is in fact a surprisingly complex material characterised by over 20 parameters (Snyman, 1998). Most often, however, only the value of carbon, moisture, volatile matter, ash, hydrogen, sulphur, oxygen, nitrogen, and specific heat content are used to to specify the quality of a coal product.

There are several mining methods applied in the coal industry today. Which method is employed will depend on the geological and geographic characteristics of the coal deposit. There are two main categories of coal mining, surface mining and underground mining.

Surface mining methods are used when the coal seams are near the surface. Often referred to as *opencast mining*, the deposit is progressively excavated as the overburden is removed to allow for the extraction of the exposed coal seam (Hartman, 1987). Open cast mining is also non-selective. That is, the entire deposit is mined out including high and low grade zones.

Underground mining methods are employed for deeper deposits. Mining rates are generally much lower than for surface mines. Underground mining is typically more selective (Hartman, 1987), meaning that less waste rock needs to be removed. Generally an access way to the seam is made during mine construction, either by sinking a shaft vertically and using a hoist, or by means of a tunnel with a conveyor system (Hartman, 1987). Underground mining is subdivided into unsupported, supported, and caving methods. Unsupported methods are used in cases where the rock structure is strong enough to support the weight of the overlying strata. In these cases, a room-and-pillar design is employed, where continuous miners excavate sections of a seam, leaving strategically placed pillars to serve as natural roof supports. For weaker ground material that is not able to support its own weight, but where it is not possible to allow the overhead material to cave in as mining progresses, the opening is immediately backfilled after the ore has been removed. Cut-and-fill stoping is an example of an unsupported method. Caving methods are used where the rock cannot support its own weight, and immediate caving after ore removal is desirable. Caving methods allow for almost the entire seam to be extracted using continuous miners with plough shearers and hydraulic supports. As mining proceeds the roof is allowed to collapse behind the advancing hydraulic supports (Hartman, 1987).

6.2.2 Mineral Economics Theory

The model presented by Hotelling (1931) is probably the most widely cited model for mineral resource economics, having gained popularity in recent years amid concerns of resource scarcity and depletion. Hotelling proposed that a producer of a non-renewable resource, when faced with constant costs over time, maximises his profit by choosing $q(t)$ such that the percentage change in rent across periods equals the discount rate. This gives rise to what is sometimes called the *flow condition*, i.e.

$$\frac{[P - MC(q(t+1))] - [P - MC(q(t))]}{[P - MC(q(t))]} = r.$$

As a consequence of maintaining this condition, the producer's output successively diminishes. Discussions on Hotelling's work are provided in Hartwick and Olewiler (1998), as well as Minnitt (2007). A qualitative review may be found in Gaugler (2015).

Preceding Hotelling and often overlooked are the essays by Gray which sought to develop the economics of resource conservation (1913) and the concept of economic rent which arises from the apparent exhaustibility of a resource (1914). Gray's contribution is assessed in Crabbé (1983).

Brazee and Cloutier (2006) developed a dynamic model based on Gray's propositions, by incorporating mineral deposit heterogeneity and differences in spatial dimensions. They found that Hotelling's flow condition does not suffice to explain a mining firm's behaviour and that an industry's extraction behaviour is not described by the linear aggregation of firms' behaviour. Campbell's (1980) dynamic model showed that mineral resource extraction is kept constant over most of a mine's life when the large pre-production investment required is taken into account. A paper by Bradley (1985) points out that Hotelling's conclusion that the optimal extraction path declines over the life of the mine is unlikely to be seen in practice since a mine operator with fixed production capital would have to forgo capital utilisation. Bradley goes on to say that the considerable pre-production investment required in extraction industries together with the relatively flatness of the average cost function in a rather broad region around the optimal extraction rate, as also investigated in Rademeyer *et al.* (2018), suggests that the miner's present value profit would be more sensitive to changes in investment allocation. That is, it is possible that the miner would rather opt to maintain a steady output rate, and that variations in output seen in practice would occur as a result of other factors.

6.2.3 Definitions and Terminology

There are several mineral codes defining reserves and resources. The classification given in the 2016 SAMREC code is stated here. Figure 6.1 provides a simple illustration of resource classification.

Resources include all quantities of coal underground, those amounts known with certainty to exist, together with those quantities that are statistically relevant, but are yet to be proved (Thielemann *et al.*, 2007). Resources are further sub-classified as *inferred*, *indicated* or *measured* resources depending on the level of geoscientific knowledge and confidence in the presence of the resources (SAMREC, 2016).

Reserves form the portion of the resources base which can be mined economically at current prices, using present-day technology (Thielemann *et al.*, 2007). Reserves are further subdivided into *proved* and *probable* reserves (SAMREC, 2016).

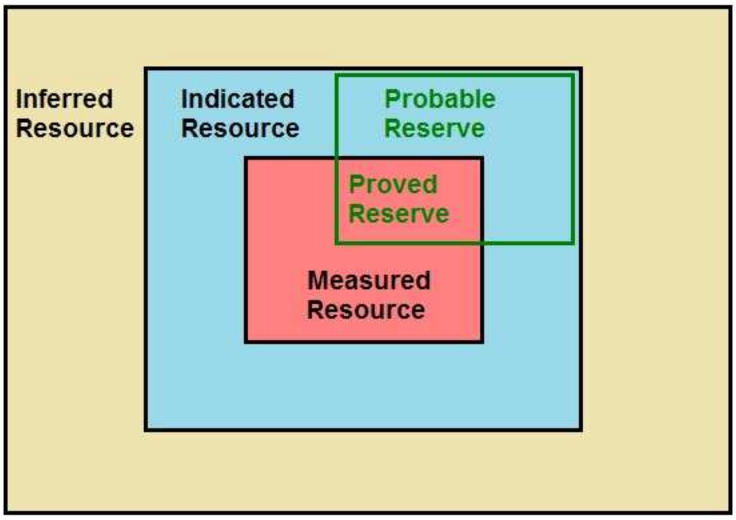

Figure 6.1: Classification of Deposit Mineral Content

6.3 Modelling the Mineral Deposit

Suppose a mine operator holds a right to mine a deposit for a period $t = 0$ to $t = T$. The deposit constitutes the total volume of material, both mineral product as well as waste product, that is to be moved during the life of the mine. We model this deposit as a rectangular prism, subdivided into block units X_{ijk} of one tonne each, as in Figure 6.2.

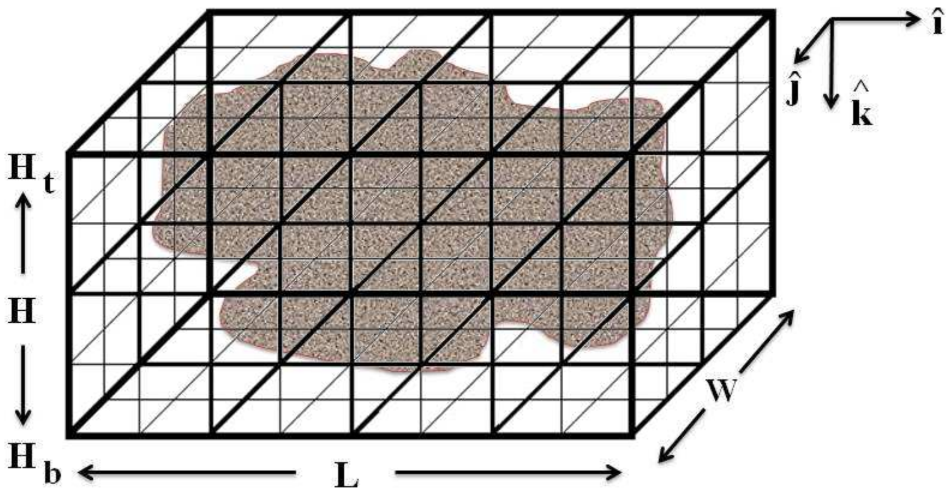

Figure 6.2: Creating a simplified model of a mineral deposit

The deposit is characterised by the following parameters[1]:

- Height[2] H, measured as the distance from the top of the deposit H_t to the bottom of the deposit H_b, in the $\hat{k}$-direction (which is oriented positively deeper underground, away from the surface). H_b is chosen such that $H \in \mathbb{N}$.
- Length L is distance measured in the $\hat{\imath}$-direction.
- Width W is distance measured in the $\hat{\jmath}$-direction.

With each X_{ijk} is associated a value for:

- Location as specified by co-ordinates $(\hat{\imath}, \hat{\jmath}, \hat{k})$.

[1]It should be noted that the deposit spatial measures are not in metric units, but rather in "block measures". That is, one length unit is the length of an edge of a one tonne cubic block of material. All measures are in terms of tonnages.

[2]In the case of coal mining, H would be shallow, and spread over a relatively large area, i.e. larger length and width.

- Quantity run-of-mine or raw coal $ROM_{ijk} \leq X_{ijk}$.

- Quantity marketable coal product $x_{ijk} \leq ROM_{ijk}$.

In industry, these fractions are typically determined by *strip ratio* and *yield*, here denoted by ρ and β respectively. Strip ratio is defined as the ratio of the volume of waste removed to the volume of ore recovered (Steffen *et. al.*, 1970). So a strip ratio of 3:1 would indicate that 3 units of waste material must be moved for each unit of raw coal moved. Hence, we define the portion of raw coal obtained from a unit of material (including both coal and waste) as

$$\frac{1}{\rho_{ijk}} = \frac{ROM_{ijk}}{X_{ijk}} \qquad \frac{1}{\rho} \in [0, 1]. \tag{6.3.1}$$

The yield is the quantity of product derived from beneficiation or washing[3] of one tonne of ROM coal and is given by

$$\beta_{ijk} = \frac{x_{ijk}}{ROM_{ijk}} \qquad \beta \in [0, 1]. \tag{6.3.2}$$

The conversion between energy-denominated coal product and tonnage-denominated coal product is captured in the variable κ_{ijk}[4].

Therefore, with each tonnage extracted is associated the following process:

$$X_{ijk} \xrightarrow{1/\rho} ROM_{ijk} \xrightarrow{\beta} x^{\kappa}_{ijk},$$

where x^{κ} is a product having energy content κ. In summary, each extracted tonne X_{ijk} is described by the following values:

- Strip ratio ρ_{ijk}

[3]Coal washing involves the separation of coal fractions based on material density and yield curves. A miner may choose to partially wash a ROM coal so that the resulting product has a lower energy value than could have been achieved had the coal been fully washed. We acknowledge that the simplified model presented here of coal product selection by washing does not completely capture the complexities involved.

[4]This conversion is necessary as thermal coal is essentially traded on energy value (in units of MJ/kg), whereas mining cost is on tonnages handled.

- Yield β_{ijk}

- Calorific value κ_{ijk}

6.4 A Simpler Model for Determining Reserves and Resources

In order to integrate the determination of reserves and resources into the computational procedure, the classification system is simplified to three classes. *Inferred and indicated resources* refer to the sum total of coal product present in a given deposit before mining begins and is denoted by X^{Roi}, so that

$$\sum_{ijk} X_{ijk} = X^{Roi}.$$

Measured resources are then that part of the total resource which has been discovered as a result of exploration[5] and given by X^{Ro}. For convenience, *resources* always means measured resources. Finally, the class for *reserves* is, as before, the portion of X^{Ro} which can be mined economically, denoted X^{Re}, and is here limited to only those reserves classified as proved reserves. This is summarised in Figure 6.3.

Resources are tied into the main optimisation model via the exploration activities which convert inferred and indicated resources into measured resources. Suppose there is some prevailing constant dollar-per-tonne exploration cost P^{EXPL}. Suppose a mining company has allocated Y million dollars for exploration, then spending \$Ym on exploration yields Y/P^{EXPL} million tonnes measured resources. Exploration is assumed to be equivalent to sustaining capital expenditure. Unlike investment spending on mining capacity, exploration spending is non-negative, i.e. a discovery cannot be undone.

[5]In a stochastic variation of the model, X^{Roi} and X^{Ro} would be a stochastic variables, whose actual values would be uncertain. In the deterministic version of the model no such uncertainty over X^{Roi} exists and X^{Ro} merely serve to constrain the quantity of coal product that can physically be removed from the deposit, using the technology available to the miner.

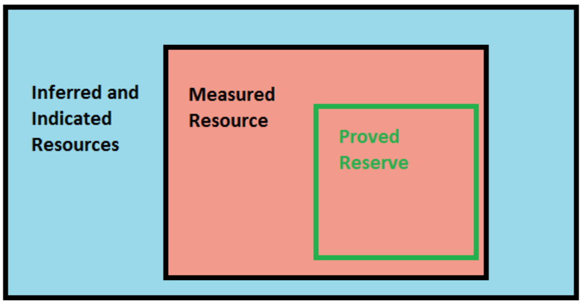

Figure 6.3: Classification of Deposit Mineral Content

6.4.1 Separating Reserves from Resources

In the precious metals industry, the boundary delineating the amount of reserve held in the resource is specified by the *cut-off grade* together with other modifying factors, i.e. environmental and social policy. Cut-off grade is defined in Thrush (1996) as "the lowest grade of mineralised material that qualifies as ore[6] in a given deposit" and is calculated as the ratio of ore-denominated operating cost $C(t)$ over grade-denominated revenue $P(t)$ at some time t, i.e.

$$g(C(t), P(t)) = \frac{C(t)}{P(t)}. \tag{6.4.3}$$

Resources for which $g \in [0, 1]$ are classified as reserves. Then the quantity of reserves is given by

$$X^{Re} = (1 - g)X^{Ro}.$$

The upper bound on g is lower than unity in practice and must include all modifying factors, i.e. policy, market etc. (see SAMREC, 2016).

[6] "The naturally occurring material from which a mineral or minerals of economic value can be extracted profitably or to satisfy social or political objectives" (Thrush, 1996)

In the coal industry, the separation of reserves from resources is instead determined by *drill-hole spacing*, a measure of uncertainty associated with coal deposit mineralisation. As part of their exploration activities, coal miners drill into the deposit to sample coal from the underground seams. Where the samples exhibit the desired mineralisation, further drilling is undertaken, thereby reducing the spacing between individual holes and increasing the certainty that extracting the entire section would yield saleable product in sufficient volumes to outweigh the cost of extraction and beneficiation. While cut-off grade is not used in coal, it is intuitively appealing. Therefore, for coal mining, we proposed that the cut-off parameter indicating a mineable section is instead given by the ratio of revenue from the section to cost of its extraction and beneficiation. That is, we propose the following cut-off grade formula for determining reserves in coal deposits

$$g_c(C(t), P(t)) = \frac{C(t) \cdot X}{P(t) \cdot x},$$

where X is the total quantity of material extracted, and x is the quantity of product sold.

6.5 The Mine Operator's Problem

The mine operator is assumed to structure his operations in such a way that the present value of profits yielded by the development of the mineral deposit over the remaining life of mine[7] is maximised (Hartwick and Olewiler, 1998). This is a problem in optimal control. Theory and examples concerning dynamic optimisation problems can be found in Turkington (2006), and Kamien and Schwartz (1991). However, the mine operator's problem can be approximated by a discrete dynamic optimisation problem, which can be solved computationally as a multi-period static optimisation problem.

[7]Life-of-mine is considered the period extending from the time the first ore is extracted t^* until the time the last ore is extracted T and the deposit has been depleted, whether economically or physically.

6.5.1 Multi-product Miner's Problem

The coal mine under consideration produces several products and faces non-uniform ρ_{ijk}, β_{ijk} and κ_{ijk}. The mine's profit is maximised over variables representing its annual production quantity q_t (a vector, with each element corresponding to the quantity in units MJ of each product produced), extraction capacity invested in I_t (a scalar in tonnage units), quantity overburden to be removed q_t^o (a scalar in tonnage units), growth in measured resources dX_t^{Ro} (a vector, with each element corresponding to a product in tonnage units), and washing capacity invested in I_t^w (a scalar in tonnage units):

$$\max \quad \Pi_m = \sum_{t=0}^{T} \left(\frac{1}{1+r_m} \right)^t (1-\xi) \left[\mathcal{F}_t(q_t, I_t, q_t^o, dX_t^{Ro}, I_t^w) \right], \tag{6.5.4}$$

where r_m is the applicable discount rate, ξ denotes the constant corporate tax rate[8] on profits and is assumed to apply to positive profit values only, and profit is operating income OI net of capital expenditure CPX as

$$\mathcal{F}_t(q_t, I_t, q_t^o, dX_t^{Ro}, I_t^w) = OI_t - CPX_t.$$

Material haulage operating costs are of the form (Rademeyer *et al.*, 2018)

$$TVC(q, Q) = a \cdot q^b + k + \frac{\epsilon \cdot q}{Q - q}, \tag{6.5.5}$$

which yields a per unit basis (not monetary) function value so that we arrive at a per unit mine cost for each tonne of material extracted – regardless of whether this tonne contains any product or not.

Effective capacities Q_t and Q_t^w, i.e. that which is actually available for utilisation, are functions of previous investments, and are of the form[9]

$$Q_t = Q_{-1} + \sum_{s=0}^{L-1} \sum_{\tau=0}^{t} \alpha_s I_{\tau-s}, \tag{6.5.6}$$

[8]The application of royalties and taxes are respective to the jurisdiction in which a mine operates and may need to be adjusted from the applications suggested here (Rudenno, 2009).

[9]This is the investment characterisation discussed in *Paper II*. *Paper III* happened to be published before *Paper II*, and so the work in *Paper II* could not formally be cited here. Instead, the mention of this function in *Paper III* is cited in *Paper II*.

where I_t is the capacity invested in at time t, α_t determines the rate at which this capacity becomes available for utilisation during development lead-time L.

Then operating income is given by revenue less haulage and washing costs as

$$OI_t = (1-\varrho)P_t \cdot q_t - (1+\theta)P_t^{IN} \cdot TVC_{mine}\left(\kappa_t \frac{\rho_t}{\beta_t} q_t + q_t^o, Q_t\right) - P_t^{WASH} \cdot TVC_{wash}\left(\frac{\kappa_t}{\beta_t} q_t, Q_t^W\right),$$

with P_t a vector holding mine-gate prices for each product in R/MJ, ϱ the constant royalty rate, P_t^{IN} a vector with per-tonne prices for each input factor required in material haulage operations, P_t^{WASH} a per-tonne washing cost, and θ an indicator function which takes the value 1 if the mine is opencast[10] and 0 if the mine is underground.

Investment expenditure is determined by the sum of a series of spending on adjustments to mining and washing capacity, together with exploration spending, and is formulated as

$$CPX_t = P_t^{INV}(\rho) \cdot I_t + P_t^{INV\_WASH}(\beta) \cdot I_t^w + P_t^{EXPL} \frac{\rho}{\beta} dX_t^{Ro}.$$

The investment cost per unit extraction capacity is given by P_t^{INV} and is dependent on ρ since the equipment used would depend on the deposit geology. Similarly, the investment cost per unit washing capacity is captured by $P_t^{INV\_WASH}$. Exploration cost in captured in P_t^{EXPL} as described in Section 6.4.

Optimisation of objective function (6.5.4) is subject to the constraints as in Table 6.1.

The rationale for each constraint is as follows:

C1 In any given period, the quantity product extracted cannot exceed the amount of reserves available.

[10]Material haulage operating cost for opencast operations is higher to account for overburden removal and reclamation works. In opencast mining, overburden is cast onto mined-out sections, enabling reclamation to proceed in conjunction with continued mining operations (Hartman, 1987).

	Constraint Equation	
C1	$\frac{1}{\kappa}X_t^{Re} - q_t \geq 0$	for each $t = 1, 2, \ldots, T$
C2	$Q_t - \kappa\frac{\rho}{\beta}q_t - q_t^o \geq 0$	for each $t = 1, 2, \ldots, T$
C3	$Q_t^W - \frac{\kappa}{\beta}q_t \geq 0$	for each $t = 1, 2, \ldots, T$
C4	$\text{TORV}_m - \sum_{t=1}^T q_t^o \geq 0$	for all t
C5	$\bar{X}^{Ro} - \sum_{t=1}^T dX_t^{Ro} \geq 0$	for all t
C6	$\sum_{\tau=1}^t \left(\frac{1}{H_o}q_\tau^o - \frac{\kappa}{H}\frac{\rho}{\beta}q_\tau\right) \geq 0$	for each $t = 1, 2, \ldots, T$
C7	$(\text{contracts})_t - q_t \geq 0$	for each $t = 1, 2, \ldots, T$
C8	$OI_t + \sum_{\tau=1}^{t-1}(OI_\tau - CPX_\tau) - CPX_t \geq 0$	for each $t = t^*, \ldots, T$
C9	$X_T^{Re} - q_T - \epsilon \leq 0, \quad \epsilon > 0$	for $t = T$.
C10	$q_t, Q_t, q_t^o, dX_t^{Ro}, Q_t^w \geq 0.$	

Table 6.1: GENCOMIN constraints

C2 In any given period, the quantity of material extracted from the deposit together with the quantity of overburden removed cannot exceed the miner's maximum haulage capacity.

C3 In any given period, the quantity of run-of-mine coal processed cannot exceed the mine's washing facility capacity.

C4 Over the life of the operation, the total quantity of overburden removed (TORV) cannot exceed the quantity of overburden present before mining begins at time $t = 0$.

C5 Exploration cannot yield more resource discoveries over the life of the operation than the amount of resources present in the deposit at time $t = 0$ as captured by $\bar{X}^{Ro}$.

C6 Sufficient overburden must be cleared in order that reserves be accessible for extraction.

C7 The miner has supply obligations, such as contracts with local utilities.

C8 Capital investment spend in any period from the time mining begins t^* is conditional on positive cash-flow availability.

C9 Mine closure occurs at economic depletion, when cash flow turns negative and where the project life ends in time T.

C10 Non-negativity applies to decision variables q_t, q_t^o, dX_t^{Ro}. Spending on capacity can be negative (i.e. divestment), but physical capacity in-place, Q_t and Q_t^w, are each non-negative.

Constraints C4 and C6 fall away if $\theta = 0$, i.e. for an underground mine. Washing[11] capacity is captured explicitly in the variable Q^W, but separation capacity (i.e. sorting the ROM coal from waste[12]) is considered as part of the extraction part of the mining process.

Overburden removal as captured by variable q_o refers to the non-coal material lying above the mineral deposit. In industry, the waste material found within the deposit, interspersed with the coal seams, is also referred to as overburden (or interburden), but in our model this waste material is accounted for by application of the parameter ρ.

Mine operators are assumed to use linearly extrapolated price projections in setting their schedules of operations. This should be done using a least squares estimate on an adequate sample of recent historical prices. However, for simplicity, the operators here are assumed to use only the two most recent data points, P_0 and P_1 say, to derive a straight-line price forecast function:

$$P_t^e = (P_1 - P_0)t + P_0.$$

Further, it is assumed that the mine is a price taker operating in a perfectly competitive environment with no opportunity for exercising market power.

[11]Includes sizing (coarse, small, fine), cleaning (cyclones, spirals), and de-watering (screens).

[12]This is also known as primary and secondary crushing.

It is assumed that the mine operator is not free to choose which sections of the deposit to mine in order to achieve the preferred strip-ratio and yield. Consequently, these parameters change exogenously over the life of the mine. Broken rock volume expands as a result of fragmentation during blasting. Thus, for simplification, our model for the deposit is assumed to represent the size of the body after blasting has taken place.

6.5.2 Hypothetical Coal Mine Data

The model's performance is demonstrated using test data for a hypothetical South African coal mine producing three products: a 25.5 MJ/kg higher-grade[13] (HG) product, a 22 MJ/kg mid-grade (MG) product, and a 14 MJ/kg low-grade (LG) product. Product grades are kept constant over time.

The mine operator's expectations for future trends are linearly extrapolated based on historic trends. Data for the two periods preceding the operational years spanning the optimisation problem are as in Table 6.2.

The mine must produce sufficient volumes to meet its contractual obligations, amounting to 100 million MJ per product (approximately 300 kilo tonnes). A royalty rate of 17% is payable on revenue, with a corporate tax rate of 15% on profit.

The discount rate is chosen to be 12.5%. This value is chosen for the purpose of demonstrating the model presented in this paper. During a valuation, an extensive amount of work is done to determine the appropriate discount factor based on capital cost considerations. This is however beyond the scope of the research presented in this paper.

[13]For coal products, the term *grade* has a different meaning in different markets. In the context of this model, *grade* is used to refer to heating value. As such, a higher-grade product in the context of this model has a higher heating value in MJ/kg than a lower-grade product.

Data Item	Period $t = -1$	Period $t = 0$
Mine-gate price on HG product (R/MJ)	0.040	0.040*(1+7.5%)
Mine-gate price on MG product (R/MJ)	0.035	0.035*(1+7.5%)
Mine-gate price on LG product (R/MJ)	0.014	0.014*(1+7.5%)
Extraction cost price (R/t)	1 100	1 100*(1+5%)
Washing cost price (R/t)	76	76.0*(1+5%)
Extraction capacity per-unit price (R/tpa)	100	100*(1+5%)
Washing capacity per-unit price (R/tpa)	50	50*(1+5%)
Exploration per-unit price (R/t)	2	10*(1+5%)

Table 6.2: Mine operation optimisation hypothetical data

The strip ratio is kept constant at 3:1 over time and throughout the deposit. The yield coefficient for each product is 0.5, kept constant over time. Overburden height is set at 100 length units (i.e. 100 tonnes of overburden must be removed to expose mineral deposit underneath.), and this parameter is here also chosen to be constant over time. Total resources present at time $t = 0$ is taken to be 4.5e7 tonnes per product.

Project development lead-time is assumed to be 5 years with mine operations starting thereafter for a duration of 20 years. The rate at which extraction capacity and washing capacity invested in become available for use is 0.5% in the year that the investment is made, 0.5% in the following year, then 12%, 37% and finally 50%, to give a total lead-time of 5 years. Divestment from a unit of production capacity is assumed to see the seller recover only 10% of the unit's value.

6.6 Results

The life-of-mine present-value profit totals R 5.5 billion, as profits from mining exceed development phase expenditure (see Figure 6.12). Return on capital invested

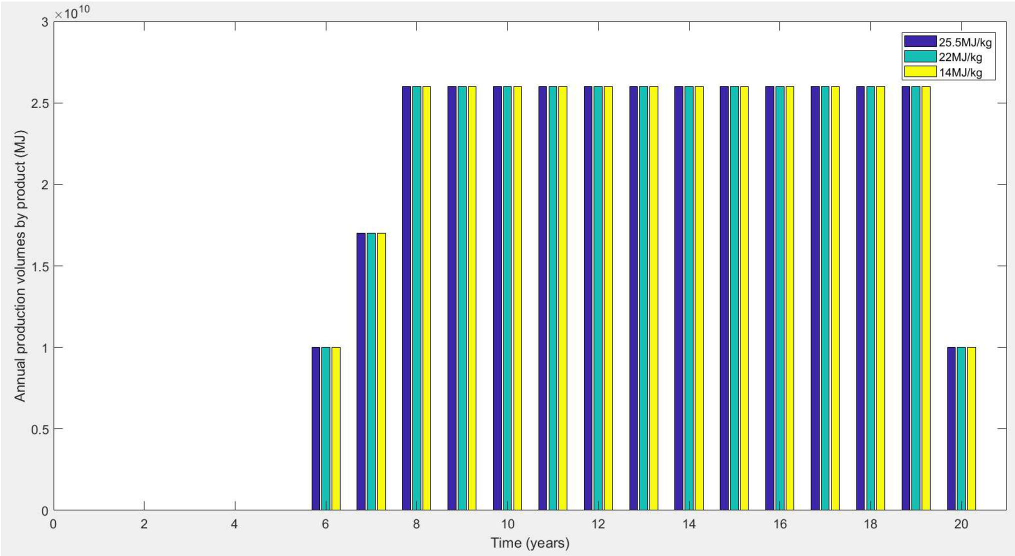

Figure 6.4: Production of three energy products is largely non-varying.

is 13.5% with the project having an internal rate of return (IRR) of 22%. The after tax profit margin is 70% on average over the life of mine operations, with operating margin at 85% on average over the life of the mine. Of course, these profit margins do not include other important expenses, such as administrative, legal, marketing, and insurance.

Production volumes ramp up to 1 mtpa HG product, 1.2 mtpa MG product, and 1.9 mtpa LG product (see Figure 6.4), remain at these levels throughout the mining years, before dropping in the final year of operations. That is, the optimisation routine suggests that deviating from a constant output rate would not render a more optimal solution. The extraction capacity utilisation as well as washing capacity utilisation are both at 84% of total on average, as shown in Figures 6.7 and 6.8 respectively. It is expected that this is owing to the cost function, and that the lowest point on the operational cost curve is at 84% of total capacity. At higher capacity utilisation, the costs rise. The optimisation routine selects capacity levels to invest in, so it is interesting that a capacity level is chose which leaves some 16% of capacity unused. However, it is expected that this is done purposely to enable

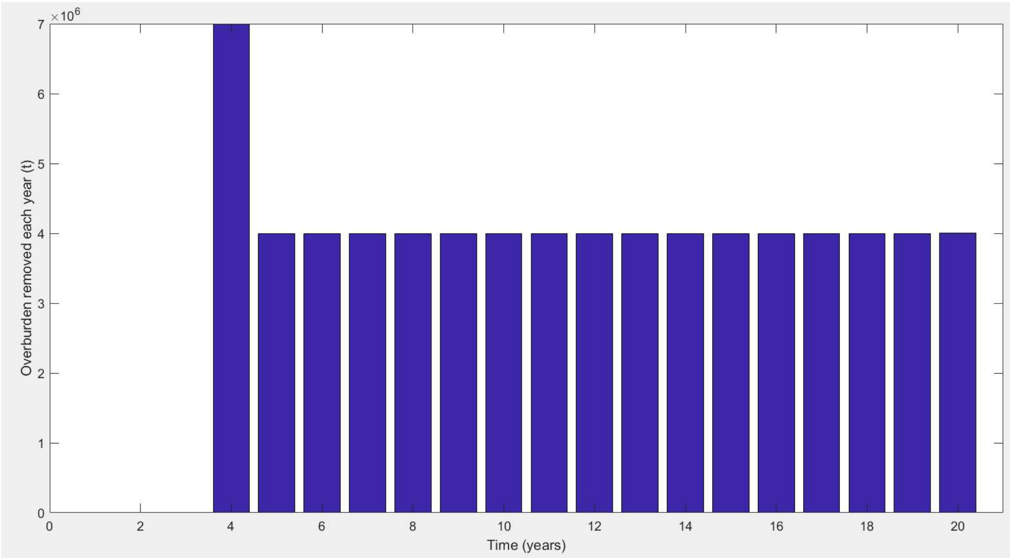

Figure 6.5: Quantity of overburden removed is higher before mining begins, but constant throughout the project's operating life.

operating at least-cost throughout the operational life of the mine. The parameters for overburden, strip ratio and yield have been kept constant in the hypothetical data used to generate these results, but it is expected that the results seen in Figure 6.5 and Figure 6.7 would show greater variation if these parameters were allowed to vary, as is typically the case in real world situations.

A large investment, some R 3.1 bn, is injected during the first year of development, dropping to R 300 million for the next four years, and tapering thereafter, as illustrated in Figure 6.11. This fits with the extraction and washing capacity profiles. That is, a large investment is required to finance the initial steep increase in production capacity. Thereafter, smaller injections are required to maintain capacity to allow for production at relatively stable levels over the operating life of the mine. Though resources available for extraction build up gradually with exploration activity seen in Figure 6.6, reserves for extraction become available only in year 6 when mining operations begin and costs are such that the reserves can be mined profitably (see Figure 6.9).

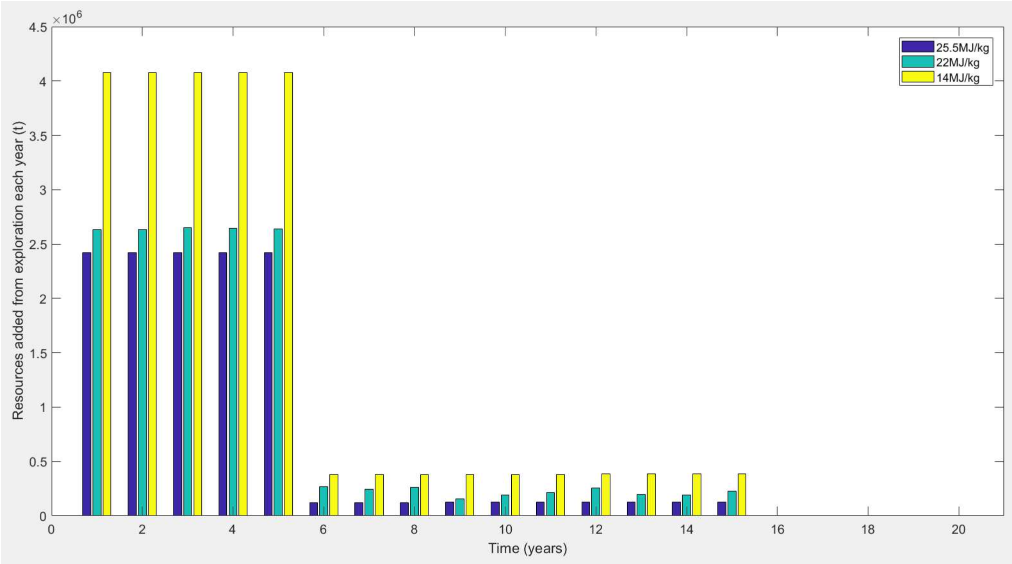

Figure 6.6: Resources added as a result of exploration are higher in the time before mining operations begin.

6.7 Concluding Remarks

Our model's results show that the profit maximising mine operator is primarily concerned with the capital investment part of his operation. That is, significant variances in physical production capacity in place is seen over time as the miner seeks to reconfigure his plant in a bid to achieve the most efficient allocation of capital spending. In contrast, there is markedly little change in coal output levels, suggesting that varying these levels would not have much effect on the location of the overall problem's optimum. Geometrically, the miner's problem is relatively flat in the product output dimension. This outcome is consistent with what was postulated by Campbell (1980) and Bradley (1985), as well as more recently in Rademeyer *et al.* (2018). Furthermore, it was found while calibrating the model that the profitability of the project as well as the return on investment are particularly sensitive to the balance between investment expenditure, strip ratio and exploration cost.

These results would suggest that mining projects are more sensitive to externalities that impact on the capital investment part of the project, such as investor

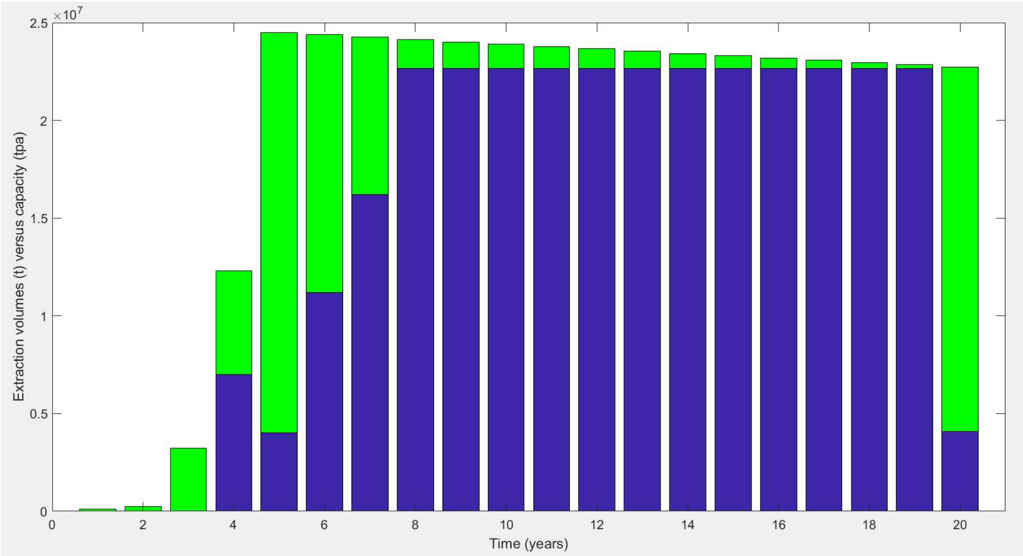

Figure 6.7: Extraction capacity utilisation is indicated by showing the split between actual volumes extracted (blue bars) together with unutilised extraction capacity (green bars), where volumes extracted include both production and overburden removal. Capacity decreases slowly over time due to aging.

sentiment and government policy, than on factors of production such as input cost. Though our model and discussion have been centered on coal mining, the generality of the model is such that it would be reasonable to expect that this flat-output feature would be characteristic for the mining of a broader range of resources.

The coal mining model presented seeks to illustrate the behavioural tendencies that emerge when incorporating certain variables in an optimisation setting. Obviously, not all of the variables present in a complex mining project have been included and as such the model results may not replicate actual mining operations in all cases. As such, the results obtained by this research are limited in that they do not account for all of the many factors involved in mining operations and project financing. The strip ratio, yield and overburden for example were chosen to be constant over the life of the project in the hypothetical data used to generate the results shown in this paper. These parameters were kept constant since the model was being presented

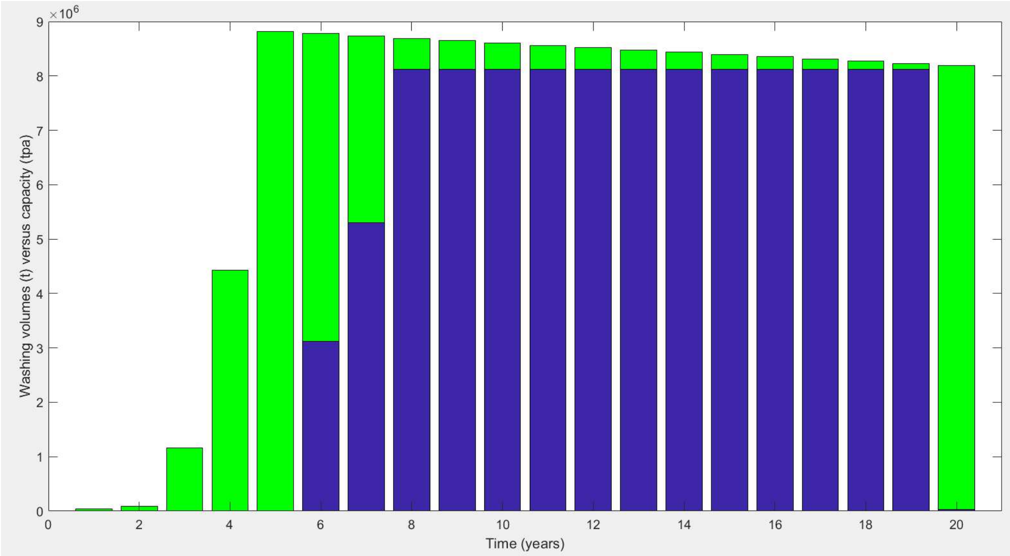

Figure 6.8: Washed ROM volumes (blue bars) are shown together with unutilised washing capacity (green bars). Capacity decreases slowly over time due to aging.

here for the first time. It was considered more important to illustrate the viability of the model. The model can then be used to investigate particular aspects of mining projects by exploring how the results change if these particular aspects are allowed to vary. It is therefore proposed that the effects of varying parameters such as strip ratio, yield and overburden should be investigated in future research.

The practical implications of using a simplified model of a mining project is that this model tells mine managers and mineral industry professionals more about the coal market as composed of a number of mines each exhibiting these general economic traits, rather than about the economics of a particular mine. Arguably, this could be achieved by more abstract models such as that of Campbell (1980), but abstract models are often discarded by industry where more tangible examples are preferred. As proposed in the introduction of this paper, the aim with the creation of our model is to bridge the gap between economic theory and the coal industry where it is applicable.

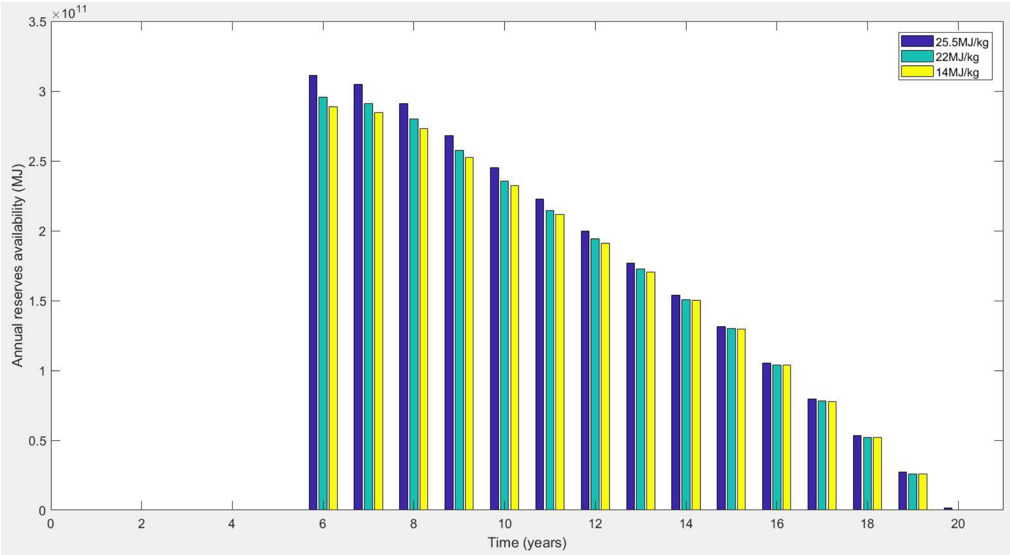

Figure 6.9: Reserves available for mining first appear only when mining begins and costs are such that discovered resources can be mined profitably.

Further, we acknowledge that coal is an incredibly complex organic material and that a coal product is specified by dozens of parameters, of which energy content is merely one. However, model results highlight general trends that assist in broadening our understanding of which factors are most influential in driving operations toward particular equilibria.

The results are based on a net present value profit maximisation. It would be interesting to compare these results with those for situations where we rather seek to maximise return on investment or the internal rate of return. Future research should investigate how modelling the exploration activity as a stochastic feature, where the rate of accumulation of mineable resources is uncertain, would change the results. It would also be interesting to adapt the model for other minerals, base metals and precious metals, and see what changes there are in the results, if any. In addition, it would be useful to investigate what profiles are rendered when setting parameters such as strip ratio and calorific value as decision variables to be determined by the optimisation procedure, while production variables are kept

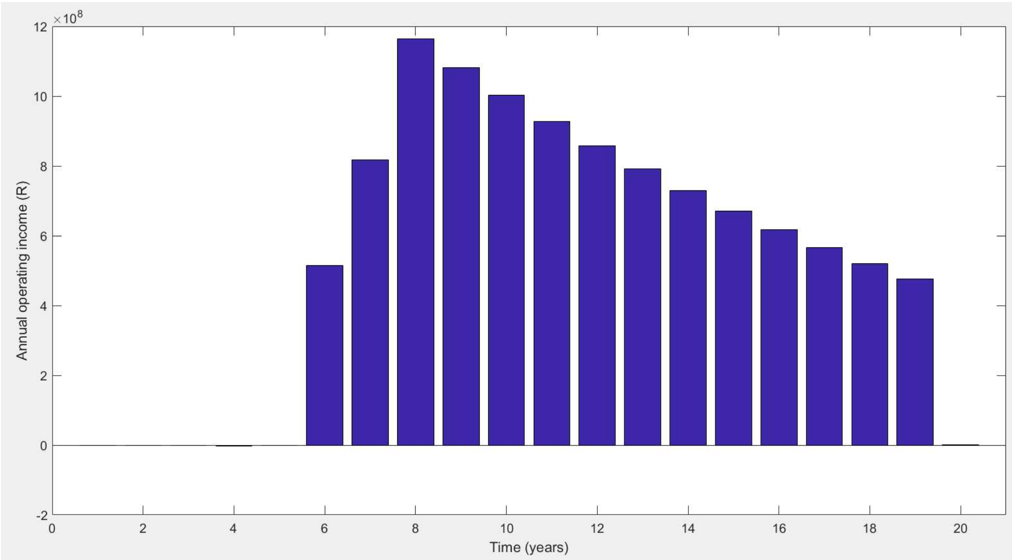

Figure 6.10: Annual operating income appears to decline over time due to operating cost pressures and discounting.

fixed.

Acknowledgements

The authors would like to thank the following individuals for their suggestions: Bart van de Steen, Ivan de Jongh, Angel Angelov, Brian Kennedy and Jarryd Kennedy.

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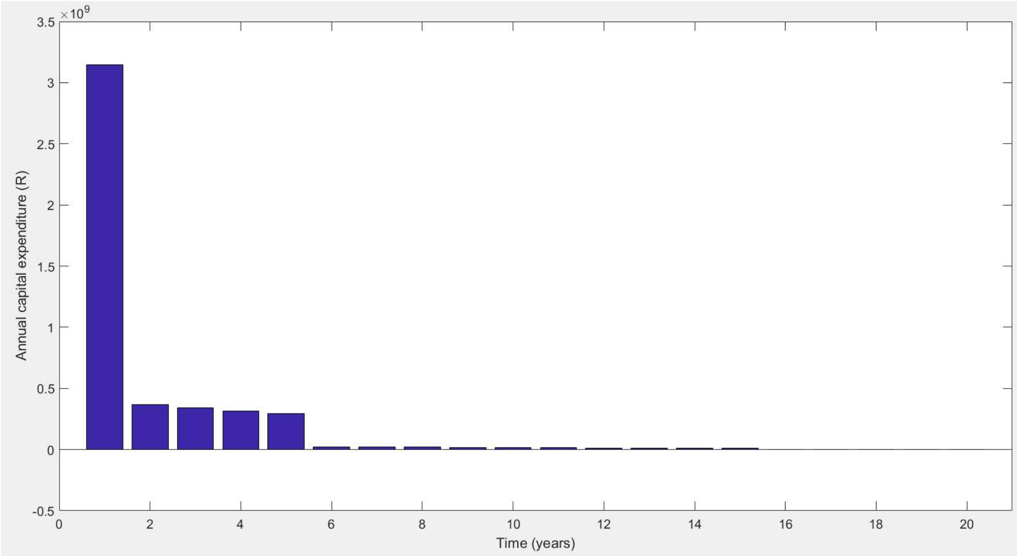

Figure 6.11: Annual capital expenditure (investment) is highest when project development begins.

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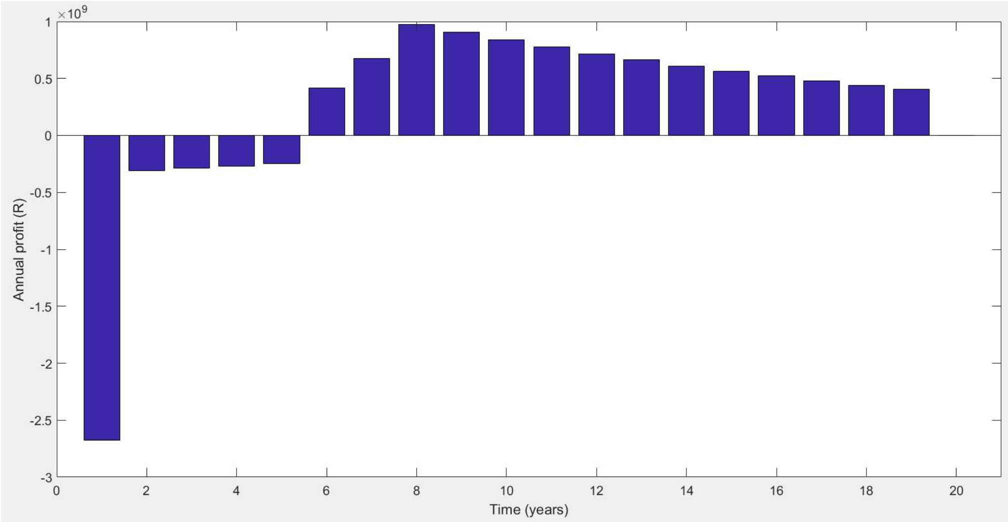

Figure 6.12: Annual profit is negative during the development phase, but turns positive after operations begin.

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Chapter 7

Multi-Product Coal Distribution and Price-discovery for the Domestic Market via Mathematical Optimisation

In this chapter is presented a model of a multi-product coal market, where coal is procured from local coal mines by a trader who then transports and sells this coal to domestic consumers, as well as the export market. This model will also be used in Chapter 8.

This chapter has been published in the academic journal, *Mineral Economics*, under the following citation:

Rademeyer, M. C., Minnitt, R. C. A., Falcon, R. M. S. (2021) Multi-product Coal Distribution and Price Discovery for the Domestic Market via Mathematical Optimisation, *Mineral Economics*, **34**, 113-126.

The derivation of the proposed model, as well as the economic arguments, mathematical arguments, computational programs, and results are the work of Rademeyer. Minnitt advised on matters related to mineral economics and mining methods, and Falcon advised on matters related to coal products and coal markets.

Abstract

This paper investigates the implications for coal supply security for the domestic South African market when faced with weaker export demand. A model is proposed of domestic coal trade via a trading hub which links coal production to consumer markets. An application of the theory for equilibrium price discovery at the mine-trader and trader-consumer interfaces is also described. It is found that the profit maximising trader seeks to sell more volumes to domestic consumers of higher-grade coal to compensate for earnings lost due to lower export volumes. This results in lower prices on higher-grade product. Supply to domestic consumers of lower-grade coal appears to be unaffected by the weaker export market, with the price on lower-grade coal constant.

7.1 Introduction

Mining is considered an important part of the South African economy, contributing 6% to gross domestic product and 35% to export earnings in 2016 (Ericsson and Löf, 2019). Almost 50% of the value of mining production in South Africa is attributable to coal mining (Ericsson and Löf, 2019). Coal sales data suggest that the value of coal mining in South Africa is strongly dependent on the revenues yielded by coal exports. Coal sales make a sizeable contribution to South Africa's earnings, contributing R 61.5 billion from domestic sales and R 50.5 billion from exports in 2016 (MCSA, 2018). These figures highlight the importance of exports in the earnings equation. Despite accounting for just under 30% of coal sales volumes in 2016[1], exports accounted for 45% of total sales value due to higher prices on exported coal tonnages. The fact that domestic coal consumers are linked to the coal export market via the inland trade market raises the question as to how the domestic coal market might be affected by changes in demand for South African export coal.

Recent industry research initiatives, such as the South African Coal Roadmap (2013) and the Chamber of Mines's Coal Strategy (2018), have looked to estimate the value of the coal industry for the South African economy and broader society. South Africa is a major supplier of coal internationally, accounting for 6% of global exports (MCSA, 2018). More importantly, however, is that coal is used in the generation of over 80% of state-owned power utility Eskom's electricity supply (MCSA, 2018). It follows that the South African economy is largely fueled by domestically produced coal.

The South African Coal Roadmap (2013) sought to bring coal industry stakeholders, i.e. producers, support services and consumers, together in an attempt to develop a strategy for successfully managing the industry into the future. This initiative was in response to changes in markets, political uncertainty, technological advance, social costs, environmental costs and the increasing awareness of climate change. Though no co-ordinated efforts have been undertaken in the years since its publication, the SA Coal Roadmap did manage to bring important characteristics of the coal industry into public knowledge[2] and does provide a reference framework for various individual research efforts in topics concerning coal mining, marketing and utilisation.

[1]Some 250 million tonnes of coal was produced in South Africa in 2016. Of the 180 million tonnes sold domestically, about 115 million tonnes was sold to state-owned power producer Eskom, 40 million tonnes was sold to liquid fuels producer Sasol, and the remainder was sold to various smaller industrial users (MCSA, 2018).

[2]The Roadmap was accompanied by a more extensive *Overview of the South African Coal Value Chain* (2011) detailing the various facets of the South African coal industry.

More recently the South African Chamber of Mines released its Coal Strategy (MCSA, 2018) in which each link in the coal value chain was quantified. Strategies were drafted for managing the coal industry subject to different market, policy and environmental scenarios.

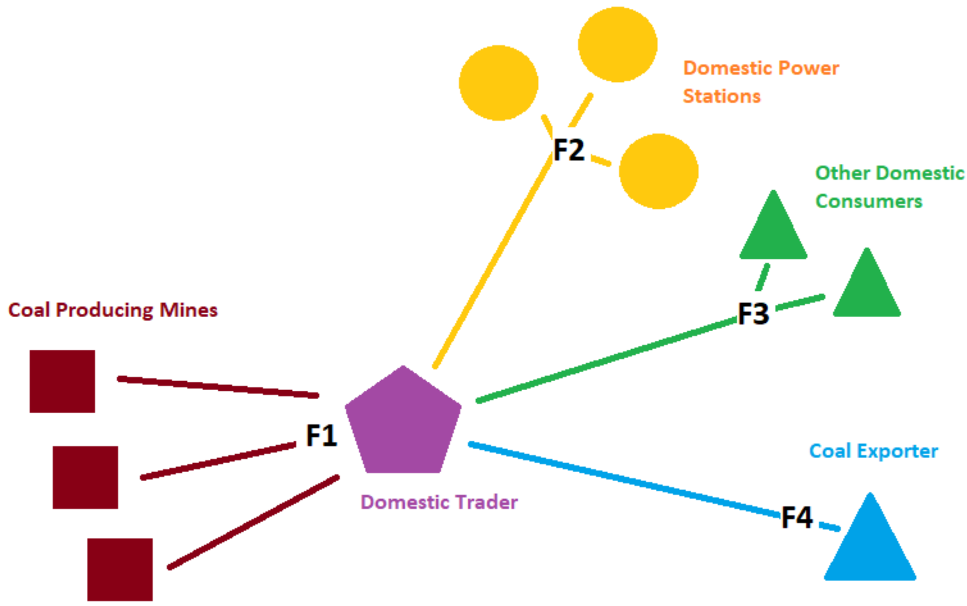

Figure 7.1: The model of domestic coal distribution proposed links coal producing mines to consumers via a domestic trader hub. Prices are established at the interface between the mines and the trader (F1), as well as between the trader and the consumers (F2, F3, F4).

Though the importance of the coal industry and coal exports in particular have been highlighted, the nature of the relationship between domestic coal availability and the demand for exports is unclear. Hence, this research paper presents a domestic trade model, DOTRAMOD, to serve as transfer mechanism between coal mines, domestic consumers and seaborne markets. The aim is to improve our understanding of the links between the supply for inland consumers and for export. DOTRAMOD is a single-agent computational spatial equilibrium model (CSEM) of domestic thermal coal trade in which a single trader is assumed to act as intermediary between mines and seaborne markets. This is illustrated in Figure 7.1.

There are generally known benefits associated with commodity trading hubs. These include the promotion of trade due to lower market supply costs that result from consolidation, as well as the promotion of standardisation and transparency. The single-trader domestic distribution model is applicable to a situation where an inland hub might be established to enable the consolidation of coal volumes from various suppliers in order to take advantage of logistics efficiencies during redistribution. This would reduce the costs for the entire coal sector. In addition, the hub would facilitate the testing and sorting of various coal products, thereby promoting the efficient allocation of coal to different coal consumers.

The establishment of a coal hub is not a novel proposition. Though no formal reference is available, the creation of a hub is an idea which has been circulating in discussions amongst South African coal industry stakeholders and contributors for some years now. The creation of such a hub has been suggested to support South Africa's emerging junior coal mining sector which is characterised by a large number of smaller volume producers.

Furthermore, given that the most efficient form of inland transport is by rail, the question of volume consolidation is a matter of concern for South Africa's state-owned rail service, Transnet. Barends (2014) investigated the hub location problem and found that a rail network having two or three hubs would provide the least cost solution and divert coal from road to rail. We believe that a market hub should naturally arise as a complement to a physical hub.

In addition to accommodating the spatial configuration of the market, DOTRAMOD makes use of a price discovery mechanism which is consistent with economic theory.

That is, the market equilibrium is found such that price reflects the interaction between demand and supply.

Optimisation problems are important tools for conducting investigations of complex problems in an objective manner. Equilibrium models, such as DOTRAMOD, allow for the investigation of a system comprising various interacting parts, so that the influence and dependence of each is well proportioned and fairly measured. The results should consequently provide a reasonable judgement on trends and outcomes for the system as a whole, as well as for the comprising parts.

Our research provides an independent study of a controversial subject, i.e. the potential value and risk associated with continued reliance on coal utilisation as a basis for South African economic and industrial activity. Continued utilisation necessitates investment in mines and coal beneficiation infrastructure. This investment question is something of a chicken-or-egg situation: Should Eskom first invest in coal-burning capacity or should miners first invest in the mines needed to secure coal supply? Should Transnet first expand the rail network or should miners first build the mines to produce the coal to be railed? Any policy introduced to impact on coal mining, trade or utilisation in South Africa may have an effect on coal revenues, coal industry jobs, South Africa's electricity supply, and ultimately the economy of Southern Africa.

The main objectives of this research paper are to:

1. Investigate how South African domestic coal supply and coal pricing would change in response to a decline in the demand for coal exported from South Africa
2. Explore the domestic coal trade hub concept as an optimisation problem
3. Present a price equilibrium mechanism, which is based on market economics theory, for use in our computational model

Our review of the current body of literature suggests that this is the first research paper in which the South African coal market is cast as an optimisation problem, with an equilibrium price mechanism, and which is solved numerically. This constitutes our original contribution to mineral economics. The results rendered challenge us to question our existing understanding of the dynamics governing the domestic coal market, as well as encourage us to improve on the model.

We also argue that the optimisation model and equilibrium price mechanism are improvements on alternative modelling approaches in that the price and volumes traded are dictated by the mathematical construction of the model itself. In that sense, the results are essentially objective as no subjective assumptions on either prices or trade volumes are made by the research team. This is especially important for this research as the research topic, i.e. the presence of coal in South Africa's energy strategy, is an extremely controversial subject, as discussed above.

It should be noted however that modelling is subject to limitations. A model is a simplification of reality since it is constructed to investigate particular aspects of reality. Computational models become increasingly difficult to solve as features are added. From our model we have excluded the mine profit consideration since the mine-life dynamics would introduce feedback in the model that, we believe, would obscure the domestic trade result. The purpose of our research is to investigate how declining export demand, via the trader's profit maximising trading strategy, affects the supply and pricing seen by domestic consumers. Allowing mines to exit or enter the market would impact on the traded supply and pricing results, thereby making it difficult to isolate the effects of the decline in export demand. We therefore defer the problem of mine profit, including the potential closures or start-ups of new mines, to future research. In addition, domestic consumer economics is omitted from this research. This is partly due to its complexity, but mostly due to the fact that the power production side invariably dominates any discussion concerning the coal industry in South Africa. The focus of this research paper is to be on the coal market and coal trade.

Though this research focuses on coal mining, the model described here can be adapted to investigate domestic supply security in other countries or for other exported commodities.

7.1.1 Commodity Market Modelling

Equilibrium modelling and mathematical optimisation techniques have been widely applied in commodity market studies, especially for investigating market structure (i.e. how many producers) and the extent to which non-competitive behaviour is present (i.e. whether suppliers are able to choose their production rate in order to influence the market price). In particular, the evolution of international seaborne coal market concentration has received considerable attention in the past two decades due to its implications for competition and the ability of players to exert market power to manipulate prices (Haftendorn, 2012).

Optimisation modelling was used to investigate the structure of the Atlantic seaborne coal market in Haftendorn (2012). This investigation was then extended to encompass all major international seaborne trade in Haftendorn *et al.* (2010) where the COALMOD-World computational model was solved as static optimisation problem with conjectural variation (i.e. each player bases their decision on what they believe their opponent will do). In this model, each mining company sought to maximise its profit, i.e. its sales revenue less cost. Trüby and Paulus (2012) replicated Haftendorn's investigation of the global market, though with a more comprehensive data set, and investigated non-competitive market behaviour by likewise employing static optimisation analysis and conjectural variation.

Moving beyond the market behaviour of private companies, the influence on seaborne trade of major coal importer China was also investigated by Paulus and Trüby (2011) using a spatial equilibrium model. This was followed by an investigation by Paulus *et al.* (2011) into strategic behaviour by major exporting regions, and the extent to which non-competitive market behaviour was displayed in export policy setting in 2008. Their model was further extended to include dedicated supply for domestic consumers in exporting nations, and they proposed that dedicated domestic supply may be diverted to export markets if the export price is at a sufficiently high level.

Equilibrium modelling has also been successfully implemented in studying behaviour in markets for other commodities. Trüby (2013) looked at pricing and trade-flows in metallurgical coal markets between 2008 and 2010. Holz *et al.* (2008) analysed the upstream and downstream sectors of the European natural gas industry using GASMOD, a model of natural gas supply.

Stochastic equilibrium modelling is often employed to account for the uncertainty associated with market outcomes and its effect on the presence of risk averse market participants. The Finnish electricity market was studied by Pineau and Murto (2003) by means of a dynamic stochastic model of oligopoly. Haurie *et al.* (1990) demonstrated that a special class of strategies emerges during stochastic equilibrium programming which the authors call *S-adapted open-loop*. Here, the solution lies between open-loop and closed-loop equilibrium solutions. Though not "sub-game perfect", it allows for the analysis of long-term supply and demand decisions under uncertainty without generating computationally challenging closed-loop strategies. Paulus (2012) applied this concept in developing a dynamic multi-stage stochastic equilibrium model to simulate investments by international mining companies under uncertainty and in the presence of Cournot-type non-competitive market behaviour.

Gabriel and Fuller (2010) developed a new Benders decomposition method for solving stochastic complementarity problems and demonstrated its efficacy by applying it to the model of electricity markets derived by Hobbs (2001). A Benders decomposition-type algorithm for large-scale stochastic multi-period mixed complementarity problems (MCPs) was used in Egging's 2013 investigation of the global natural gas market where the market power is taken into account.

Whereas the aforementioned works have mainly used optimisation modelling in investigating competitive behaviour, our research applies optimisation for investigating trends in the domestic distribution of different coal products. We also investigate the economic viability of a centralised trader hub. The approach policy makers might take to promote the creation of a commodity hub is covered by Haris and Tao (2016).

7.1.2 Characteristics of Thermal Coal Markets

Coal is transformed from decomposed vegetation over millions of years by geological processes (Speight, 2005). It is a complex material specified by over 20 parameters, including carbon, moisture, sulphur, and nitrogen (Snyman, 1998). The quality of a coal product, or its combustion properties, is determined by the relative proportions of these constituents. In the text, the term *grade* is used to refer to a coal product's calorific or heating value, so that a higher grade implies higher energy content. Thermal coal products come in a variety of grades. The energy content of a coal product, which constitutes the main component sought by a thermal coal buyer (Graham *et al.*, 1999), is by and large a homogeneous product. That is, the energy content should be priced similarly per unit for all thermal coal products in a given market; a study of coal trade should actually be based on the market for coal energy[3]. Such studies have been conducted with some success (Haftendorn,

[3] In reality there are pricing discount penalties imposed for undesirables such as moisture (consumes heat during evaporation, thereby reducing net energy content), ash (contributes to slagging and fouling when heated, and may lead to deteriorating performance and plant damage), sulphur (plant corrosion) and nitrogen (emissions contribute towards air pollution). This research paper does not consider these factors when referring to coal grade. The inclusion of these factors would however provide for a more comprehensive investigation in coal market dynamics, especially when dealing with the broader international coal market where there may be larger variation in the

2012; Haftendorn and Holz, 2010; Haftendorn *et al.*, 2010; Trüby and Paulus, 2012; Paulus *et al.*, 2011).

Thermal coal is largely consumed near where it is mined owing to the relatively high cost of transportation; about 17% of the world's total thermal coal is traded internationally, of which about 90% of this trade is seaborne (IEA, 2016). The seaborne market grew rapidly from the 1970s as a result of the steady move away from oil for use in industry and power generation due to oil supply risks amid increasing geopolitical tensions (Haftendorn, 2012; Ekawan and Duchêne, 2006; Ekawan *et al.*, 2006; Li, 2010).

Both thermal coal production and consumption benefit from economies of scale associated with mass production, bulk handling, and largely routine-based conversion and transportation processes. As a result, the industry has demonstrated a tremendous affinity for mechanisation which, though effectively lowering per-unit variable costs, has made it necessary for operators in the coal industry to invest large sums in fixed capital, thereby raising barriers for any new entrants. As a result, the thermal coal industry is characterised by a small number of large operators accompanied by a fringe of smaller players. This naturally highly-concentrated structure could enable co-ordinated operations and potentially uncompetitive market behaviour, such as collusion and cartelisation, on both the buyer and supplier sides, the potential of which has been of interest (Haftendorn and Holz, 2010; Colley, 1998; Trüby, 2013), but with no definite conclusions that the industry is not competitive. As such, the research presented in this paper assumes a competitive domestic coal market.

7.1.3 Coal Producers

Coal production comprises the extraction and beneficiation of raw coal, as well as the supply of this product to market. Theory concerning producer economics is

presence of these compounds when comparing various supplying countries and coal consumers.

given in texts such as Bernheim and Whinston (2008), as well as Goolsbee *et al.* (2013). Economic theory for the mineral extraction industries specifically is covered in Tilton (1985) and Tilton (2001).

The supply of a product is typically represented by an upward sloping curve. That is, the quantity a supplier is willing to supply to the market increases with the price level. The supplier's supply function is typically assumed to be a straight line function of the form

$$Q_S = m_S P + k_S,$$

where Q_S is the quantity of the product the supplier is willing to put up for sale at the prevailing price level P, $m_S \in [0, 1]$ is the supplier's *marginal propensity to supply*, and k_S is the lower bound on price the supplier would be willing to accept for its product. The market supply function is a linear composition of the supply functions of all the individual suppliers.

7.1.4 Thermal Coal Consumers

The consumers of thermal coal include power stations, cement producers, brick makers, chemicals manufacturers, and numerous smaller users, such as food processors and steel makers. The grades required by boiler operators vary according to coal combustion technology installed. Kiln operators typically require coal with high energy content, whereas chemical producers are able to utilise coal with lower energy content.

A buyer's demand function specifies the quantity of product that the buyer would purchase given a particular price level. It is typically represented by a downward sloping curve since, for normal goods, the quantity the buyer would be willing to purchase would decrease in response to an increase in price. The demand curve is often taken to be a straight line of the form

$$Q_D = m_D P + k_D,$$

where Q_D is the quantity purchased, P is the prevailing price level, $m_D \in [-1, 0]$ is the *marginal propensity to consume*, and k_D is the maximum price the buyer would be willing to pay for the product. A consumer's sensitivity to changes in price is captured in the *price elasticity of demand* parameter. The greater the consumer's dependence on a product, the more sensitive they are to changes in price. The converse is also true. For instance, in the case of coal, the availability of alternative energy sources, such as oil, gas and nuclear, inhibits the consumer's dependence on coal, resulting in a lower sensitivity to price changes. Topics concerning demand for mineral commodities is discussed in Tilton (1985).

7.2 Coal Trade Model Specification

There is assumed to be a single coal trader in a given country, acting as price-taker and redistributing miners' output accordingly. The trader buys coal from the various mines, and sells the coal to domestic users and exporters. We distinguish between two domestic buyers, the first being local power utilities and the second being all other users, such as cement producers, brick-makers, etc. Coal that is supplied by the mines to domestic users under long-term contracts or non market-related agreements are not included in the domestic trader's problem[4]. Contracted supply with market-linked prices are however included. The cost of transportation falls to the domestic trader; he may choose to transport coal by rail or by road. The trader may also enter into take-or-pay agreements with the rail operator(s) in order to finance rail capacity expansions. Domestic trade is conducted in local currency.

The coal miners are modelled using a supply function. The international seaborne market, as well as the domestic markets, are modelled by respective demand functions. Prices are established at each pricing interface (as given by points F1, F2, F3, and F4 in Figure 7.1) by means of the price-discovery mechanism described in

[4]These prices are established in a different manner, i.e. on the term market as opposed to the spot market. This research concerns the spot market.

Section 7.3.

In addition, an on-mine stock constraint is introduced. That is, it is assumed that production is such that a certain stock level is maintained and miners do not change production levels in response to changes in mine economics, as suggested by the results in Rademeyer et al. (2018). In the construction described in what follows, the trader takes the various products sold by the mines, consolidates these and then redistributes the products to the appropriate customer.

7.2.1 The Domestic Trader's Problem

The domestic trader l optimises his trading strategy, deciding what quantity to buy from each mine m for delivery to power stations j, other domestic buyers k, and for export i, in order to maximise profit flows over the duration of his trading business. The problem for a single trader trading in three products can be formulated mathematically as a spatial equilibrium problem as in Ferris and Pang (1995). That is, the trader maximises over objective function

$$\max \quad \pi_l = \sum_{t=1}^{T} \sum_{m=1}^{M} \left(\frac{1}{1+r_l} \right)^t [\mathcal{F}_t(x_t^{mi}, x_t^{mj}, x_t^{mk})], \tag{7.2.1}$$

in terms of decision variables $x_t^{mi}, x_t^{mj}, x_t^{mk}$[5], with profit function

$$\mathcal{F}_t(x_t^{mi}, x_t^{mj}, x_t^{mk}) = \sum_j \mathrm{CFR}_t^j x_t^{mj} + \sum_k \mathrm{CFR}_t^k x_t^{mk} + \sum_i \mathrm{FOB}_t^i \mathrm{FOREX}_t^l x_t^{mi} - COST_t^{mn}(x_t^{mn}),$$

where the first three terms represent revenue from coal sales. The CFR^j and CFR^k prices are established at the power stations and other domestic buyers respectively, and are in domestic currency values. Here, CFR is used to indicate the price at the point of sale, or transfer of ownership of the goods from the trader to customer, i.e. power stations and other domestic buyers. The export price FOB[6] is in USD and

[5]Subscripts here serve to indicate the time index; superscripts index the flow from mine m to consumer i, j, or k, i.e. these are not exponents.

[6]Incoterms used, such as CFR and FOB, are as defined by International Chamber of Commerce at https://iccwbo.org/resources-for-business/incoterms-rules/incoterms-rules-2010/ (accessed on

must be converted into the domestic currency by multiplication with the prevailing foreign currency exchange rate $FOREX$. FOB is here used to indicate the price prevailing at the port of export. Cost is composed of coal procurement costs, transportation costs, and penalties paid for transport capacity not used as stipulated by a take-or-pay agreement. Then, for $n = i, j, k$,

$$COST_t^{mn}(x_t^{mn}) = \sum_n \left[\mathrm{FCA}_t^{mn} x_t^{mn} + \tau_t^{mn} x_t^{mn} \kappa_t^{mn} + P_t^{mn} \cdot (\Psi_t^{mn} - x_t^{mn}\kappa_t^{mn}) \cdot \sigma_t^{mn}\right],$$

and

$$\sigma_t^{mn} = \begin{cases} 1 & \text{if } \Psi_t^{mn} - x_t^{mn}\kappa_t^{mn} > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{7.2.2}$$

The explanation of these terms may be expanded upon somewhat. The coal procurement cost is a product of the coal price FCA_t^{mn}, agreed between the mine and the trader, with the quantity purchased from the mine x_t^{mn}. That is, FCA is here used to indicate the price prevailing at the point of sale between the mine and the trader. Transportation cost is the product of freight rate τ_t^{mn} and coal tonnages transported $x_t^{mn}\kappa_t^{mn}$. The take-or-pay penalty is the product of the penalty fee P_t^{mn} and the amount by which the quantity transported $x_t^{mn}\kappa_t^{mn}$ is less than the contracted quantity Ψ_t^{mn}. Parameter σ_t^{mn} indicates that the penalty fee applies only when the transported quantity is less that the contracted quantity. In the calculations of transport cost and take-or-pay penalties, the energy quantities are multiplied by a factor κ to convert energy units into tonnages. That is, coal trade takes place in energy units while costs are incurred on coal tonnages.

The optimisation of objective function (7.2.1) is subject to the constraints for $n = i, j, k$ as stated in Table 7.1.

These constraints are explained as follows:

C1 Contractual obligations hold for supply to customers over the life of the trader's

21 July 2018).

	Constraint Equation	Multiplier	
C1	$\sum_m \left[\sum_{t=1}^T (x^{mn})_t - \Lambda^{mn}\right] \geq 0$	λ_t^n	
C2	$\bar{X}_t^{mn} - x_t^{mn} \geq 0$	b_t^{mn}	for each $t = 1, 2, \ldots$
C3	$D_t^n - \sum_m x_t^{mn} \geq 0$	d_t^{mn}	for each $t = 1, 2, \ldots$
C4	$x_t^{mi}, x_t^{mj}, x_t^{mk} \geq 0$	free	for each $t = 1, 2, \ldots$

Table 7.1: DOTRAMOD constraints

operations, i.e. supply to customers should be at least the contracted quantity Λ^{mn}.

C2 Tonnages taken by the trader from the mines cannot exceed the volumes the mines have in stock in any given period as given by $\bar{X}_t^{mn}$.

C3 Tonnages delivered to customers cannot exceed the customers' offtake capacity D_t^n.

C5 The non-negativity of decision variables applies.

Mine operators and traders are assumed to use linearly extrapolated projections in setting their respective schedules of operations. For simplicity, the operators and traders here are assumed to use only the two most recent data points, P_0 and P_1 say, to derive a straight-line price function:

$$P_t^e = (P_1 - P_0)t + P_0.$$

7.3 Determining Market Prices

Most often, commodity market models make use of historic price levels in setting future price levels, where changes from one period to the next are governed by a sensitivity parameter such as price elasticity. This approach ensures that price movements remain range-bound, whereas prices can in reality exhibit large fluctuations over relatively short periods. Furthermore, price elasticities may vary over time due to variable external factors, rendering the determination of these parameters from historic prices difficult. A market price-setting mechanism is here proposed to circumvent the aforementioned difficulties in computational modelling.

7.3.1 A Remark on Commodity Price Models

In what is to follow, a price setting mechanism will be presented as part of the market equilibrium model presented in this research. This may warrant a remark as to why commodity price models are not here discussed in more detail.

As indicated in Section 7.1.2, coal is an incredibly complex mineral, consisting of a variety of materials, all of which can influence its marketability. Indeed there are many forces and factors that influence price development in coal markets. These determinants may arise from the market structure, the behaviour of suppliers and buyers, as well as the product itself.

A review of proposed drivers of resource prices is conducted in Gaugler (2015). The well-known Hotelling model is covered and the validity of several proposed price drivers are investigated, with only three of these found to be supported by empirical evidence. Our paper does not seek to address any aspect of the Hotelling model. Our model is for determining the equilibrium price at which demand and supply meet, and at which transactions or trade occurs. Hotelling's model does not address demand and supply equilibrium; it does not address demand at all (Gaugler, 2015).

The purpose of the pricing mechanism introduced in this research is not to directly address commodity price models nor to address the relationship between price development and proposed price drivers. Rather, the aim is to present a mechanism which can be used in a market demand-supply equilibrium setting.

7.3.2 Market Demand

Adopting the functional form suggested in Section 7.1.4 and assuming that there is a maximum quantity which the buyer is willing to take[7], D_{max}, the buyer's inverse demand is taken to be

$$P^D(Q) = P = \begin{cases} P_{max}\left(1 - \frac{1}{D_{max}}Q\right) & \text{for } Q < D_{max} \\ 0 & \text{for } Q \geq D_{max}. \end{cases} \tag{7.3.3}$$

P_{max} is assumed to be a function of the buyer's disposable income and D_{max} is assumed to be a function of the buyer's physical offtake capacity. The market demand function is a linear composition of the demand functions of all the individual buyers. We adopt function (7.3.3) as our market demand function.

7.3.3 Market Supply

Even though the representation of supply by means of an upward sloping curve, as described in Section 7.1.3, is the generally accepted model, supply is more realistically given by a "supply stack" or what is generally referred to as the *supplier cost curve*. Graphically, this is an irregular step-wise increasing function such as is often published by commercial market analysts and is depicted in Figure 7.2[8]. A more detailed treatise of industry cost curves and profit with regard to the extraction of mineral deposits is given in Tilton (2001).

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[7]This is the quantity the buyer would take if the price level dropped to zero. In order to encourage additional offtake, the seller would have to pay the buyer, i.e. negative pricing, but this extension is here ignored.

[8]The depiction of the industry supply curve given in Figure 7.2 is generalised and does not necessarily represent any particular coal supplying industry. A supplier cost curve could be constructed for any market, whether the South African coal market or another domestic coal market, and the curve should look similar. It is of course likely that the curve may have a different shape due to larger or smaller producers being located at different places along the x-axis owing to them having different cost structures.

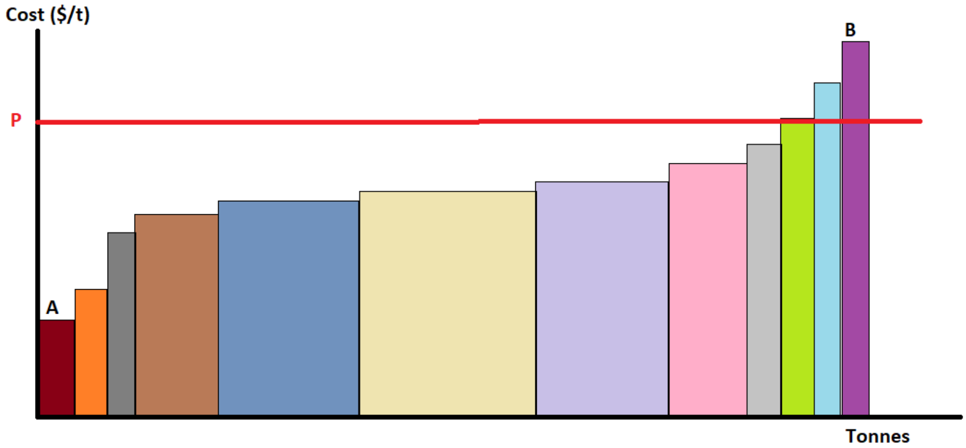

Figure 7.2: The supply curve ranks the volumes supplied by each supplier according to the average cost of bringing the tonnages to market for sale, with the least-cost suppliers on the left-most side (marked A) and the highest-cost producers on the right-most side (marked B). The profit accruing to each producer is given by the area between the red price line P and each respective cost base.

The step-like form of the supply function is as a result of the uniform cost distribution across all units supplied by a single supplier. However, the step function tends towards a smooth upward-sloping curve as the number of suppliers increases. The highest-cost suppliers are found on the right-most part of the curve. These suppliers are most likely to enter or exit the market in the short to medium term. The price at which a supplier's goods enters the stack is the *bid price*. That is, the price at which the supplier is willing to sell the goods and may be thought of as the cost[9] price plus a margin (to cover long-term investment).

For our computational model, we adopt a step-wise increasing market supply curve for two or more suppliers. The quantity supplied by each is ranked according to the cost of supplying said quantity, thereby giving rise to the "supply stack". The case of a single producer or supplier is not considered in this research; this would require

[9]There is no generally accepted definition for supply cost, though it is often taken to be the Net Direct Cash Cost or C1 cost as per the Brook Hunt definition.

that monopoly economics be accounted for.

7.3.4 Market Equilibrium

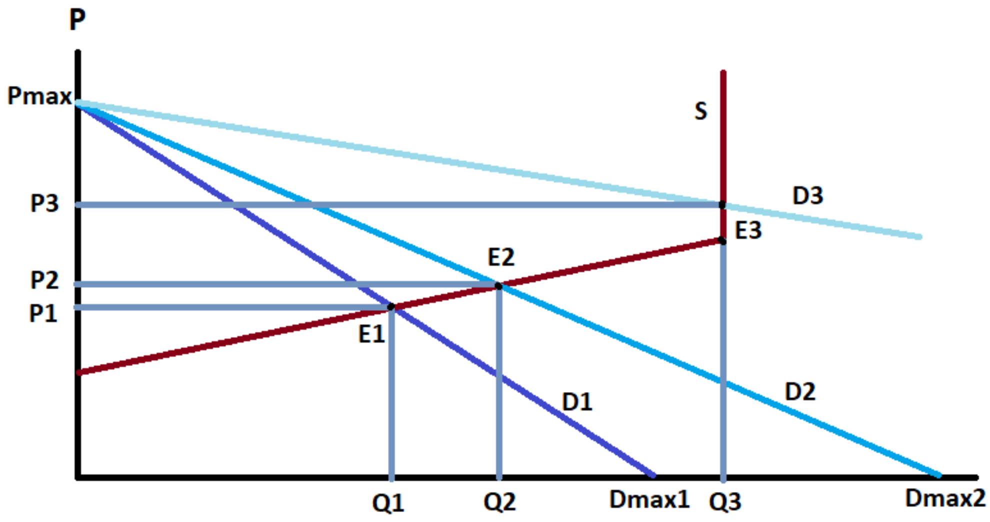

Figure 7.3: The market equilibrium occurs at the intersection of the downward sloping demand curve D and the upward sloping supply curve S. The equilibrium point E reconciles buyers and sellers to agree on quantity Q exchanged and at what price P this transaction takes place.

Market equilibrium occurs where the price is such that the quantity demanded equals the quantity supplied, i.e. where the demand and supply curves as described in Section 7.1.4 intersect. Possible intersections of demand and supply are given by points E1, E2 and E3 in Figure 8.2. That is, the equilibrium price for each year in each local trade point (i.e. each market) is established based on the intersection of the local market's prevailing (static) demand and supply curves. Theory concerning market economics is given in texts such as Bernheim and Whinston (2008), as well as Goolsbee *et al.* (2013).

The determination of price equilibrium would be similar when replacing supply curve S in Figure 8.2 with a supply curve as in Figure 7.2. The coal price for the market would be established at the level where the downward-sloping consumer demand curve D intersects the supplier stack or supply curve. The quantity traded between suppliers and buyers would likewise be at this intersection point.

Since market clearing is not assured, a means is required to deal with a situation where quantity demanded exceeds total supply. This could be addressed by having the supply curve increasing vertically at the upper supply limit, as is also suggested in Tilton (1985). Then for outward shifts in the demand curve, the price will move up the vertical line while the supply quantity remains fixed at the upper supply limit. This is illustrated in Figure 8.2 point E3 where the price P3 moves up along the vertical supply curve, though the quantity traded Q3 remains fixed. That is, for any quantity demanded that exceeds the supplied quantity, the market price is found by substituting the supply limit into the inverse demand function.

Sold and unsold volumes are determined as follows. In each market, for each supplier to that market, if the volume from a particular supplier can be supplied at a cost below the prevailing market price, then the volume is considered sold. If however the supply cost exceeds the market price, then the volume is considered unsold. For break-even volumes, i.e. when supply cost equals market price, profitable volumes are first deducted from the overall traded quantity, with the remaining portion then taken to be sold at cost.

It is noted that the miner's profitability is not considered in this investigation. Therefore the possibility of supply reduction due to mine closure is ignored. Furthermore, blending of products to create a product with any particular calorific value is also ignored. It should also be noted that as a result of the inclusion of this equilibrium market mechanism, the coal trade model as described in Section 7.2.1 is non-linear due to prices and trade volumes being implicitly defined in the market equilibrium regime.

Data Item	**Period $t = -1$**	**Period $t = 0$**
Max price for MG product (USD/MJ)	1.5	1.5*(1+0%)
Max price for HG product (R/MJ)	5.0	5.0*(1+0%)
Max price for LG product (R/MJ)	0.5	0.5*(1+0%)
Max demand for MG product (MJ)	150e11	150e11*(1+0%)
Max demand for HG product (MJ)	5e11	5e11*(1+0%)
Max demand for LG product (MJ)	17e11	17e11*(1+0%)
CV for MG product (MJ)	22.0	22.0*(1+0%)
CV for HG product (MJ)	25.5	25.5*(1+0%)
CV for LG product (MJ)	14.0	14.0*(1+0%)

Table 7.2: DOTRAMOD demand and pricing data

7.4 Demonstration of Model

7.4.1 Description of Hypothetical Test Data Used

The number of time periods over which optimisation takes place is $T = 20$. We consider a trader sourcing three products from three mines and redistributing these products to three buyers. Product sales prices are in R/MJ. It is assumed that the domestic power producers take lower grade (LG) product, exporters take mid grade (MG) product, and other domestic consumers take higher grade (HG) product. The three coal buyers are thus characterised by the properties in Table 7.2.

Therefore, in the base case, maximum tolerable prices and demand levels[10] are kept constant over time. That is, demand-side drivers are assumed constant.

Three mines are considered: a low-cost, mid-sized mine (M1), a mid-cost, large

[10]The maximum price P_{max} is the price at which demand by the buyer for the product is zero. The maximum demand D_{max} is the maximum intake in MJ where after $P = 0$, i.e. no price decrease could induce more buying.

Mine	Mid-grade	Higher-grade	Lower-grade
Mine 1	22.7	7.8	214.3
Mine 2	77.3	17.6	428.6
Mine 3	4.5	11.8	7.1

Table 7.3: DOTRAMOD mine production capacity data in million tonnes per annum (mtpa)

mine (M2), and a high-cost, small mine (M3). Each mine produces all three products. Mine costs are assumed constant over time. Table 7.3 shows the amounts in million tonnes of each product that each mine is able to supply[11] per year.

The domestic trader's transport cost is assumed to be constant at $\tau = 10$ R/t on transport routes to domestic consumers, but $\tau = 150$ R/t on the export line. The export FOB price is in USD per tonne and must be converted to a domestic currency value. The exchange rate is assumed constant at 14 ZAR per unit USD. The domestic discount rate is assumed constant at 5%.

Two scenarios are considered: a base case in which consumer demand remains constant over time and the case for declining export demand. In the latter case, it is the maximum demand for MG product which is lowered gradually over time, i.e. in each period the maximum demand is only 97% of the demand in the preceding period. This would result in a gradual flattening out of the demand curve D in the diagram depicting market equilibrium given in Figure 8.2. For non-changing supply S, it is expected that the equilibrium price P would lower over time as well.

Hypothetical data has been used to demonstrate the model since it is difficult to find real data for this purpose. One difficulty is the cash-cost-per-tonne for each

[11]The mines are theoretically able to supply these amounts. The actual sale to the trader could however be much lower, depending on market conditions.

mine, or the lowest price at which each mine is willing to sell their product. This information is very sensitive and almost never made available by mining companies. Another difficulty is the maximum price a consumer is willing to pay for a tonne of coal, as well as the maximum volume a consumer is willing to buy. These values are not explicitly available anywhere. Data indicating the exact calorific values being traded are not available to the public.

The model is evaluated via a computation program developed in MATLAB.

7.4.2 Results for Base Case

The base case serves to validate the model construction as described in Section 7.2 and Section 7.3.

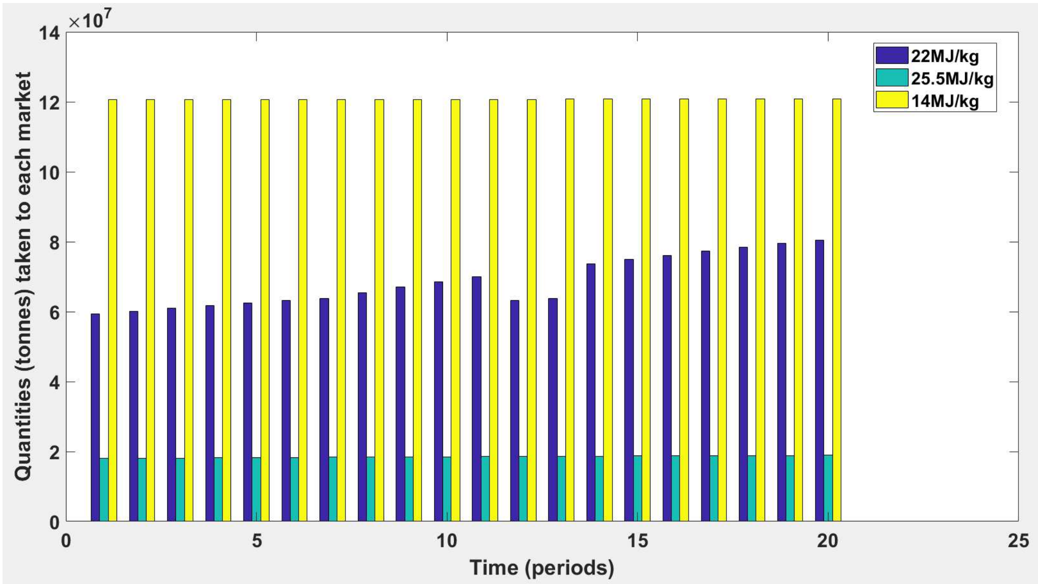

Figure 7.4: In the base case, the volumes traded with exporters (blue bars) increase firmly. There is a slow rise in HG volumes sold domestically (green bars) while the increases in volumes to domestic consumers of LG product (yellow bars) are negligible.

In the base case, increasing volumes are sold to export markets (blue bars in Figure 7.4), rising from 60 million tonnes to about 80 million tonnes over the 20-year pe-

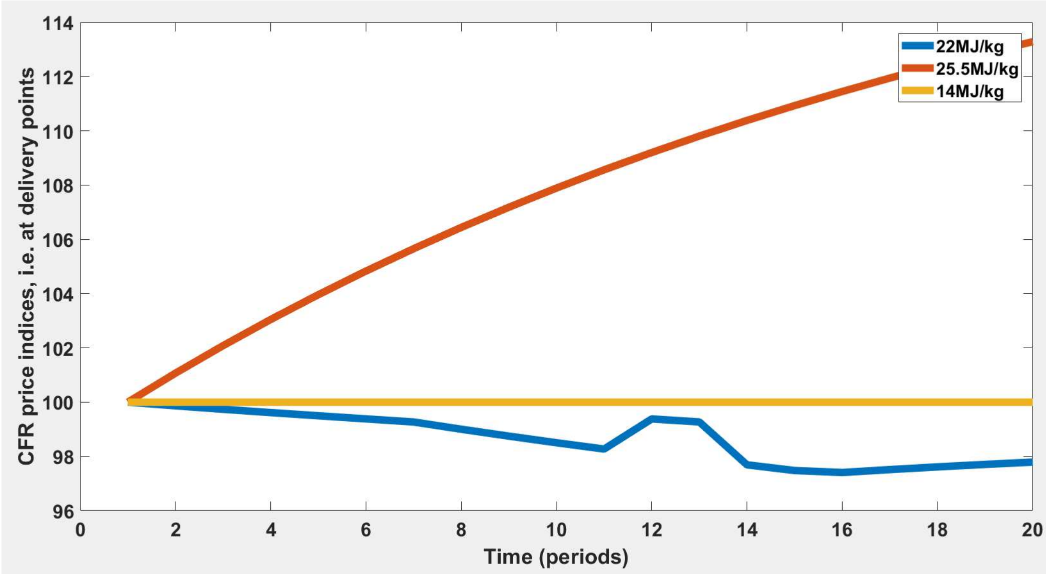

Figure 7.5: In the base case, the export price loses some value over time, whereas the price on higher grade coal increases strongly over time. The price on lower grade coal remains unchanged.

riod. Volumes sold to the domestic market for lower grade coal are essentially flat over time at 121 million tonnes (yellow bars in Figure 7.4), with sales to domestic consumers of higher grade coal increasing slowly from 18 million tonnes to 19 million tonnes (green bars in Figure 7.4).

The development of landed prices, i.e. as paid by consumers, is shown in Figure 7.5. The price on exported coal (blue line) decreases over time, though losing only 2% of its value, with a slight increase in the 12th and 11th periods corresponding to a drop in export volumes sold in those periods. The price on HG coal sold to domestic consumers (red line) increases strongly over the 20-year period, showing a 14% increase. The price on LG coal sold domestically (yellow line) remains constant over the entire time horizon.

The mid-cost, large mine (M2) is seen to be the dominant supplier of all three products (Figure 7.6). The low-cost, mid-sized mine (M1) sells mid-grade and high-grade coal, while the high-cost, small mine (M3) sells only high-grade product.

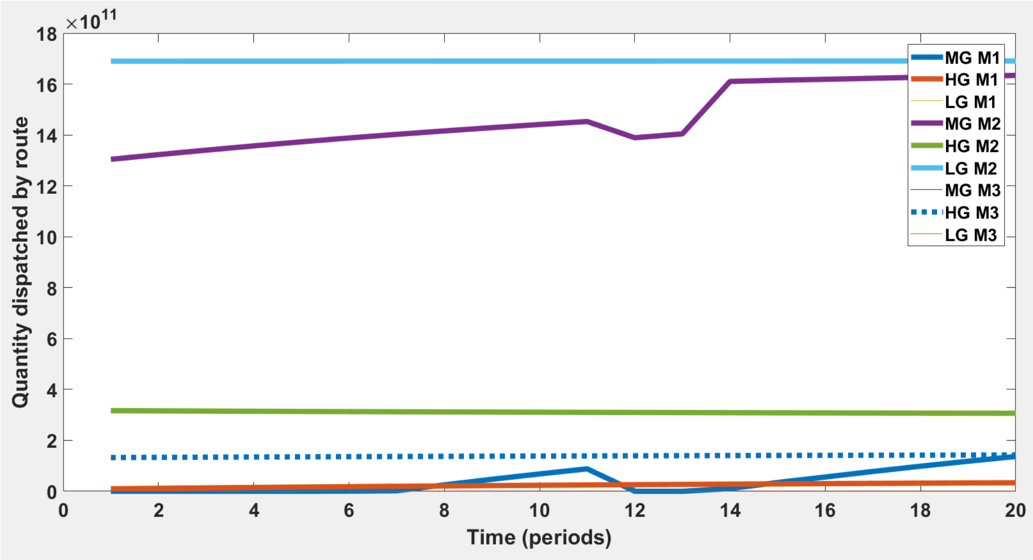

Figure 7.6: The mid-cost, large mine is the dominant supplier, supplying most of the MG, HG and LG coal.

7.4.3 Results for the Case of Declining Export Demand

The case of declining export indicates how the domestic market would, when constructed as in the base case, respond to a decline in export demand.

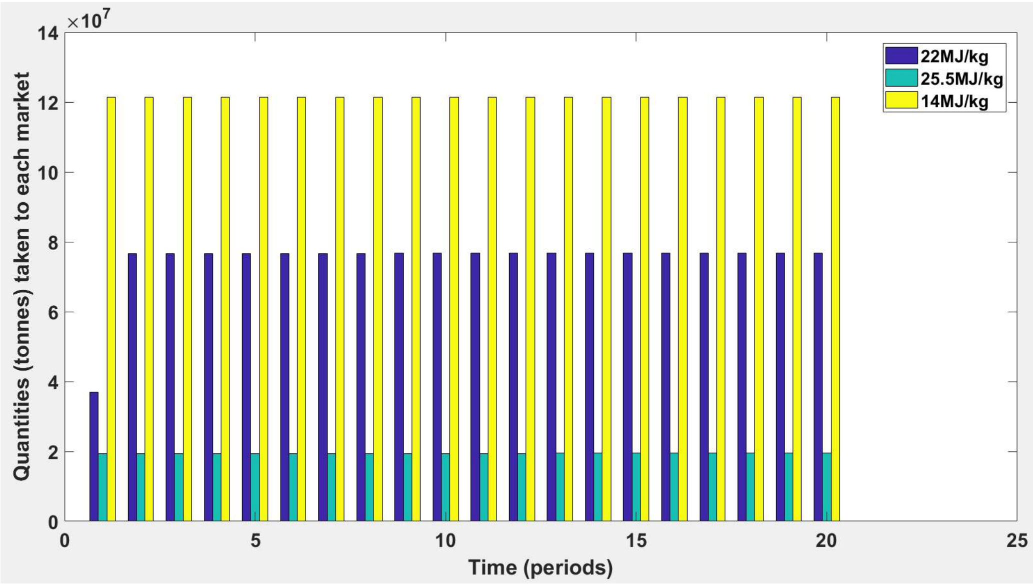

Figure 7.7: In the case of decreasing demand for coal for export, volumes traded on all three product types (bar graphs) remain largely constant over time.

In the case of decreasing demand for exported mid-grade coal, the volumes sold on

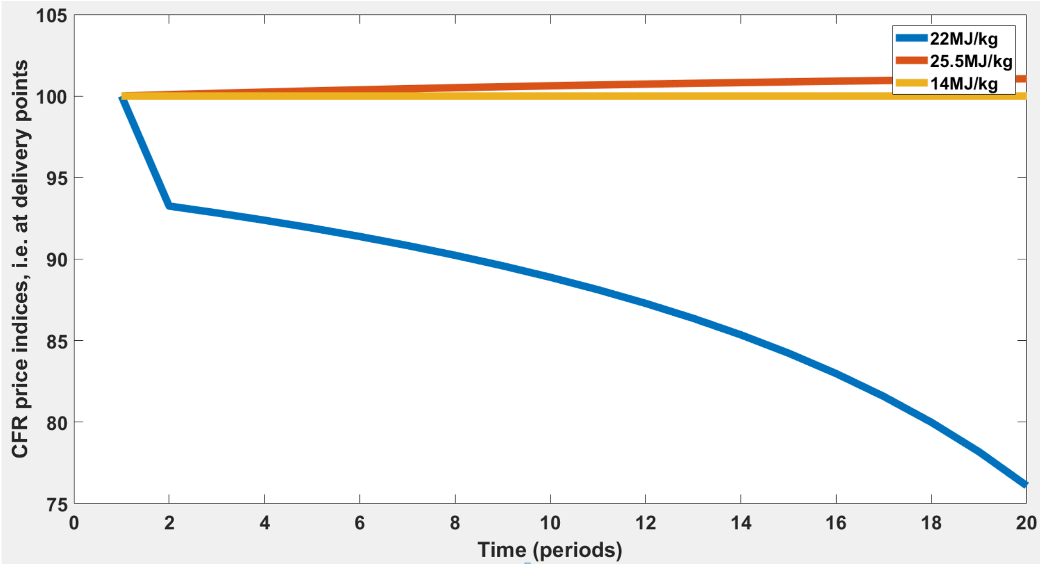

Figure 7.8: In the case of decreasing export demand, export prices drop strongly whereas prices on domestically sold HG coal increase only slowly.

all products remain constant over time, as seen in Figure 7.7. Exported sales are at 77 million tonnes, with the sale of lower-grade coal at 121 million tonnes, and higher-grade coal just under 19.5 million tonnes for the entire time horizon.

The export price drops strongly, losing up to 25% of its value over the time horizon (Figure 7.8). The high-grade coal price increases only slightly, around 1% over the time horizon, due to higher volumes being sold than seen in the base case. The price on lower-grade coal remains constant with volumes sold in line with what is seen in the base case.

The constant price on lower-grade coal may be explained by considering the optimisation solver's process. Owing to the lower price on a unit of lower-grade coal, the maximising optimisation solver, when looking to move to a higher location on the trader's profit surface, would rather move in the mid-grade and higher-grade dimensions where an increase is achieved more quickly due to the higher per-unit price on these products.

The mid-cost, large mine (M2) is the only mine to sell mid-grade and low-grade

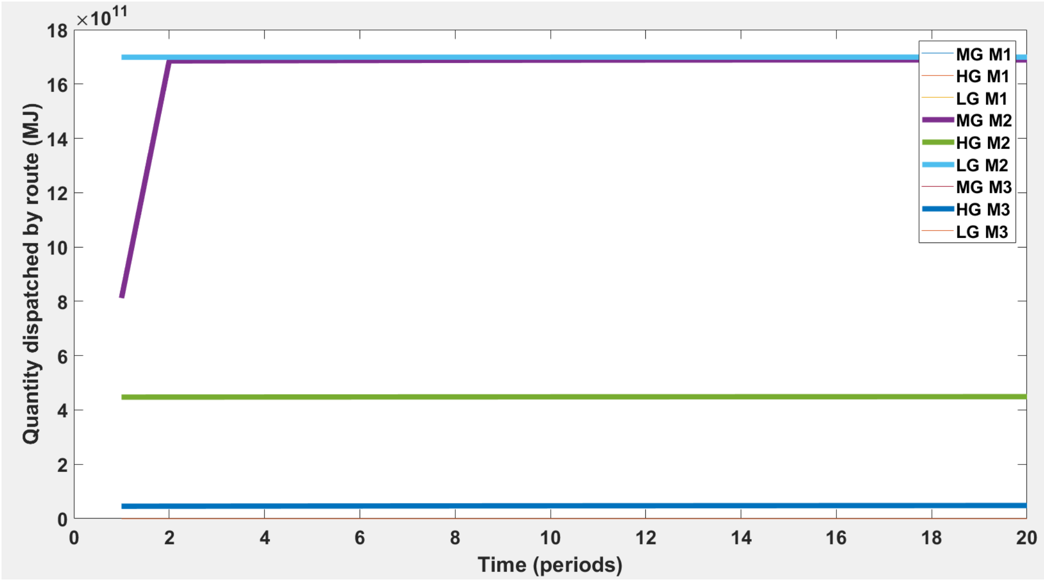

Figure 7.9: Mine 1 ceases coal sales, resulting in Mine 2 becoming the sole supplier of MG and LG product.

coal under the declining export demand scenario (Figure 7.9). The high-cost mine (M3) continues to sell high-grade coal. In all cases, it is the higher-volume suppliers that continue supplying, while mines with comparatively smaller supply cease their sales. Referring back to Table 7.3, for example, Mine 1 has lower volumes of higher-grade product compared to Mine 3.

7.5 Concluding Remarks

This research paper set out to investigate the implications for South Africa's domestic coal consumers when faced with a weakening coal export market. The single-trader domestic distribution model has been proposed and used to simulate how an inland coal trading hub might enable the consolidation of coal volumes from various suppliers for redistribution. This problem was constructed as an optimisation problem, where trade volumes and prices were determined by the interaction of demand and supply.

The results show that in the case of decreasing demand for exported coal, a single domestic profit-maximising trader would sell lower volumes of mid-grade export coal than in the base case, and these sales would be at lower prices than in the base case. The trader is further seen to sell higher volumes of higher-grade coal to inland consumers when faced with decreasing demand on the international market; these sales of higher-grade coal are at lower prices. The volumes and price on sales of lower-grade coal appear to be unaffected by the declining market for export coal.

When considering the volumes supplied by the three mines in the case for declining export, it is seen that the larger suppliers for each of the three products are preferred. This is a surprising result as the model does not inherently possess such a bias. This result might indicate that a single central hub may not be beneficial for smaller, individual suppliers. Further research is required to determine the cause of this result.

The results from this investigation are however limited by the fact that the profit problem for the miners has not been fully considered. That is, the profit accruing to the mines in the case for declining exports may result in overall profit which does not fully compensate for the capital investment required to set up a mining operation. It is expected that declining profits for miners would result in mine closures, especially for higher cost operations. This could then lead to lower coal availability for domestic consumers of both higher- and lower-grade coal. As such, future research should include the coal mining investment problem. It would be useful to know in what way investment in the domestic coal industry can be sustained or even promoted when coal exports falter.

The domestic consumer side has not been fully specified in this research paper and, as such, the economic implications of the results for the domestic consumers have yet to be determined. The application of the price-equilibrium mechanism in the model should capture the interaction between the domestic consumer and the trader, but it would be interesting to see if this dynamic changes when factors such as the consumers' profits are taken into account.

The case of more than one trade hub should also be considered as an area of future research. This would indicate whether one trade hub is adequate, or whether additional trade hubs would benefit the South African coal industry by, for instance, reducing transport costs and improving supply and pricing for consumers.

This paper has investigated the effect of only a single external factor on the South African coal industry, namely declining export demand. There are however other important socio-economic factors to address in future research, such as coal sector employment, industrial energy requirements, and environmental protection policies[12].

Though it could be argued that the South African coal market may not be representative of the global coal market (and in fact each exporting country has its own unique domestic energy market) one might from the results of this research infer that on the export market, the persistent decline in international seaborne coal demand, when faced with consistent supply availability from exporting countries, may have had a depressing effect on international coal prices. This could be interpreted as the global coal market having become more competitive since suppliers have not been able to exercise any power (by reducing supply for example) in order to lift prices[13].

[12]Environmental pressure and its effect on the coal industry is indeed a widely discussed topic today. It was however decided not to incorporate the environmental aspects into this papers as it does not seem suitable to pay special attention to one aspect affecting the coal industry, while not paying equal attention to other equally important socio-economic aspects such as coal sector employment and a country's energy requirements. In South Africa these latter two subjects are extremely complex and should be treated with care in research if the research is to have any efficacy in influencing the country's energy policy.

[13]The final paragraph of Section 7.1.2 mentions research papers that have investigated market power in seaborne coal markets.

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Chapter 8

Investigating the Outcome for South African Coal Supply to the Domestic Market when Faced with Declining Demand for Exported Coal

This chapter expands the model presented in Chapter 7 to include the project investment problem. The results from Chapter 6 highlighted the importance of the capital investment problem in the economics of a coal mining project. As a consequence of including the investment problem, the results from the expanded model presented in this chapter differ from those obtained from the original trade model in Chapter 7. Moreover, the expanded model presented here allows for addressing the primary objective of this thesis. That is, it allows for investigating what effects there would be on coal supply to domestic consumers when suppliers are faced with a decline in demand for coal exports.

This chapter has been published in the academic journal, *Mineral Economics*, under the following citation:

Rademeyer, M. C. (2021) Investigating the Outcome for South African Coal Supply to the Domestic Market when Faced with Declining Demand for Exported Coal, *Mineral Economics*.
`https://link.springer.com/article/10.1007/s13563-020-00245-0`

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Abstract

This paper investigates the implications for coal supply security for the domestic South African market when faced with weaker export demand. It expands on a previously introduced model of domestic coal trade, DOTRAMOD, by accounting for mining profit, the opening of new mines and the closing of loss-making mines. The results indicate that declining demand for exported coal would result in declining profits for multi-product mines. This would lead to the closure of some mines and in reduced coal supply to domestic power producers. These results highlight the importance of the mine investment problem in commodity market studies.

8.1 Introduction

Energy security is a key component in political discussions in many countries. The global energy mix has come under the spotlight amidst increasing concerns that carbon-based energy sources may be contributing towards irreversible and harmful climate change. Coal plays a large part in South Africa's socio-economic well-being, with some 70% of South Africa's total energy requirements, i.e. electricity and liquid fuels, satisfied by coal (MCSA, 2018). Coal exports account for 12% of South Africa's total foreign earnings (MCSA, 2018). Of the almost 460,000 people employed in the South African mining industry, 19% of these are employed in the coal sector (MCSA, 2018). A tepid investment climate and lack of development in new coal mining projects in South Africa have prompted concerns that there will not be sufficient capacity coming online to replace existing capacity when the lives of currently operating mines end (Jeffrey, 2019). Other concerns that have been raised include questions regarding the quality and price of coal sold to national power utility, Eskom. Each of Eskom's power stations was designed to utilise the coal of a particular grade or quality in its locality, though these "tied" or "captive" deposits have been diminishing (Jeffrey, 2019). Moreover, Eskom is faced with escalating coal prices as it must compete with the export market which is often seen to be an attractive alternative (Jeffrey, 2019) due to the weaker domestic currency's inflationary effect on export prices. It is uncertain how these issues would affect the broader domestic coal market.

As discussions on the role of coal in the global energy mix continue, it is worthwhile to consider what impact a global shift away from coal fuel and a subsequent deterioration in international coal trade would have on South Africa's coal market and, by extension, South Africa's economy. The DOTRAMOD model for domestic coal trade, as proposed by Rademeyer *et al.* (2021), was developed with the purpose of clarifying the relationships spanning South Africa's domestic coal industry and connecting this industry to the international coal market. However, this model does not account for the mining investment problem which drives the supply-side equation of the market equilibrium model. Mining investment is characterised by large capital investments into large projects with long lifetimes. In addition, the development of these projects is subject to long lead-times. It stands to reason that this investment problem should have a significant impact on the equilibrium of a commodity market model.

Market dynamics are driven by interaction between market participants, buyers, suppliers, cost structures and governmental policy. Markets tend to gravitate towards equilibrium states; these states may be stable or unstable. In extreme cases, instability may lead to market failure. In the case of mining, market failure could result when price volatility and the associated risk to return on investment discourages investment in new mining ventures, resulting in shortfalls in critical commodities. It may then be necessary for government to intervene and salvage affected industries through costly subsidies. Market failure due to a short-term focus by investors is argued in Cairns *et al.* (2010).

The objective of this research paper is to build the mining investment problem into the model of domestic coal trade introduced in Rademeyer *et al.* (2021) by taking account of the miner's profit consideration. That is, an algorithm is proposed in this paper that effectively allows for the miners to evaluate the future profitability of their operations and the subsequent decision on whether to continue producing coal or shut down the mine. It also allows for the development of new mining operations should investment conditions prove favourable. This research then not only extends the DOTRAMOD model, but the results highlight the importance of the production investment problem when investigating commodity market dynamics. The accompanying review of the literature indicates that many studies of commodity markets have not accounted for long-run production capacity changes, and the results presented in this paper indicate that there could be a strong difference between the results obtained when including long-run capacity change and when excluding this aspect.

As such, the contribution of this paper to the literature is the extension of DOTRAMOD with an investment algorithm. In addition, this paper contributes the key result that supply of coal for local power stations would decline when faced with a protracted deterioration in export demand, owing to mine closures due to declining profits. This research is significant in that it shows how South Africa's energy security could be threatened by a persistent decline in coal exports. It also suggests that mine closures would result and this could have implications for mining sector employment. The results obtained from this research are important at this time given that global activities to address climate change include a phase out of coal by major coal consuming countries. There is also increasing pressure on all countries to commit to phasing out fossil fuels such as coal, and the expectation is that this would result in a decline in international coal demand.

The rest of this paper is structured as follows. First DOTRAMOD is discussed, followed by a review of relevant literature. The investment algorithm is then presented. This is followed by an overview of computational scenarios and associated input data whereby the inclusion of the algorithm in DOTRAMOD is demonstrated. The results of the computation are then presented and discussed. The paper closes with concluding remarks.

8.2 Relevant Literature and Theory

8.2.1 The South African Coal Industry

The South African coal industry comprises four main markets (SACR, 2011):

1. *The market for coal supplied to Eskom, the state-owned power generation and distribution utility:* The coal supplied to this market is typically of lower energy grade and higher in ash content, and has historically been sourced from large mines under cost-plus[1] constracts. In recent years, increasing volumes have been purchased from smaller, "junior" coal mines, due to Eskom's shift towards procurement from mines owned by black South Africans.

2. *The export market for coal shipped from Richards Bay for sale to international coal consumers:* Historically, the coal exported from South Africa was higher energy, low ash coal that flowed to Europe. In recent years, these flows have reduced to negligible volumes and South Africa now exports a lower energy product to eastern markets, predominantly India.

[1]This is typically the production cost plus a margin for the mine operator.

3. *The market for coal used by smaller industrial domestic consumers:* Coal for this market is typically high energy, high quality coal.

4. *Sasol, the producers of synthetic fuels:* Sasol's operations consume large quantities of low quality coal.

Sasol's coal requirements for the production of synthetic fuels is completely satisfied by its own integrated coal mining division. As such, Sasol's operations are excluded from this investigation.

8.2.2 DOTRAMOD and Market Equilibrium

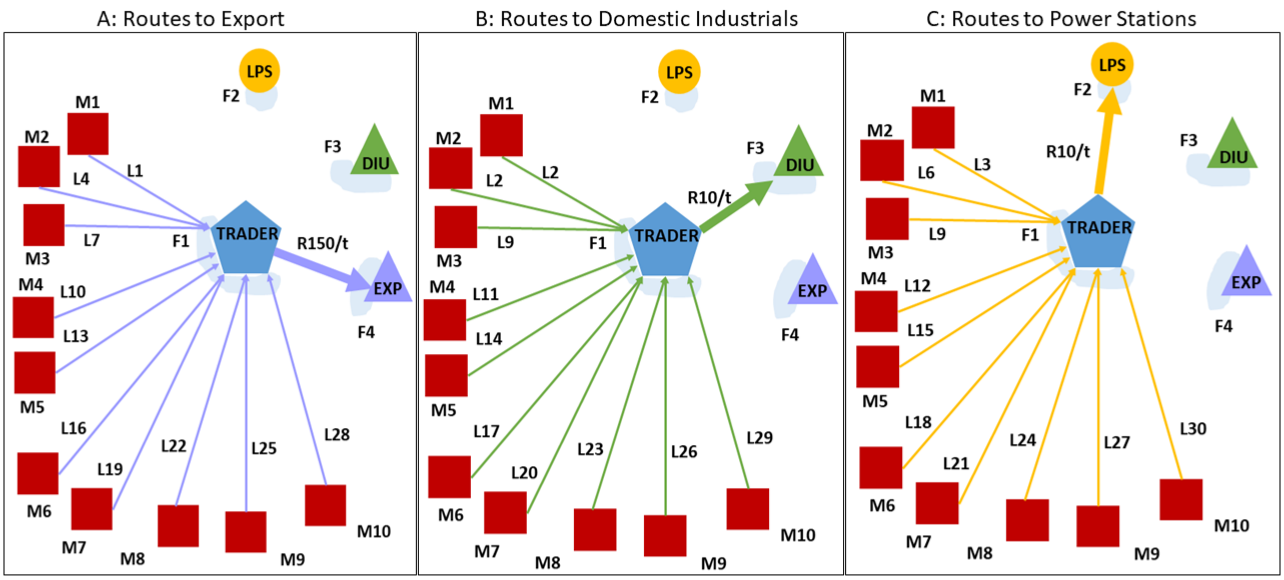

Figure 8.1: Box A shows the routes travelled by MG export coal from the mines to the trading hub and then to the export port (EXP). Box B shows routes to domestic industrial HG coal users (DIU). Box C shows routes to local power stations (LPS).

DOTRAMOD is a computational model of domestic coal trade in which trade is facilitated by a single trader intermediary (Rademeyer *et al.*, 2021). This construction is illustrated in Figure 8.1. The trader purchases coal from several mines (marked M1 to M10), consolidates the volumes and redistributes these via different transport lanes (marked L1 to L30) to three types of consumers: local power stations (LPS), other domestic industrial users (DIU), and the export market (EXP)[2]. The trader facilitates domestic trade by deciding what volumes to purchase from the mines and then sell to the three markets. The trader's decision is based on the maximisation of its profit Π over the life of its trading operation[3]:

$$\max \Pi \quad = \sum_{t=1}^{T} \left(\frac{1}{1+r}\right)_t \left[\text{REVENUE} - \text{COST}\right], \tag{8.2.1}$$

where

$$\text{REVENUE} = \sum_j P_t^{\text{LPS}} x_t^j + \sum_k P_t^{\text{DIU}} x_t^k + \sum_i P_t^{\text{EXP}} x_t^i \tag{8.2.2}$$

and

$$\text{COST} = \left(P_t^M + T_t\right) x_t + \text{TOP\_PENALTY}. \tag{8.2.3}$$

The trader covers the cost of purchasing coal and transporting the coal to each consumer. The trader maximises its profit subject to the following contraints:

1. Contractual obligations with customers must be honoured.

2. Volumes purchased from the mines cannot exceed the mines' stock levels.

3. Customers have an upper limit on volumes they can purchase.

4. Decision variables cannot be negative.

---

[2]In Rademeyer *et al.* (2021), each of these customers consumes a different coal product, i.e. lower grade (LG), higher grade (HG) and mid grade (MG) respectively. In Rademeyer *et al.* (2021), coal grade refers to the energy content such that a higher grade is associated with a higher energy value.

[3]A simplified version of DOTRAMOD is described here. The operating horizon is denoted by T. r is the discount rate. P_t^{LPS}, P_t^{DIU}, and P_t^{EXP} are the landed prices of LG, HG, and MG coal respectively, while x_t^j, x_t^k, and x_t^i are the volumes sold to power stations, domestic industries and exporters respectively. Also, $x_t = x_t^j + x_t^k + x_t^i$. Further, P_t^M is the price at which the trader purchases coal from the mines, and T_t denotes transport cost to end-users. A take-or-pay penalty, TOP_PENALTY, is paid for unused transport capacity.

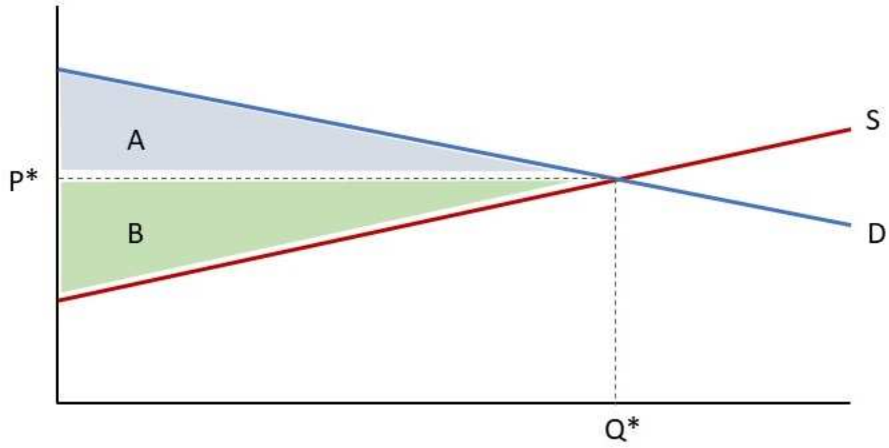

Figure 8.2: The total quantity traded in a market in equilibrium is the quantity Q^* at which the industry's supply curve S (shown in red) and its demand curve D (shown in blue) intersect. The goods are traded at the corresponding equilibrium price P^*. Region A is consumer savings and region B is supplier profits.

In DOTRAMOD, market prices are determined by a market equilibrium routine. Market equilibrium establishes the price at which trade takes place and occurs where the consumer industry's downward-sloping demand curve intersects the upward-sloping supply curve of the supplier. Prices are established at the interface between the mines and the trader (F1) as well as the interfaces between the trader and the consumers (F2, F3, F4), as illustrated in Figure 8.1. Goods supplied at a cost below the market price incur a profit margin and are thus sold. This is illustrated in Figure 8.2. Economic theory relevant to the mineral extraction industries is covered in Bernheim and Whinston (2008), Tilton (1985) and Tilton (2001). Additional details on how this theory is integrated into DOTRAMOD are provided in Rademeyer *et al.* (2021). As mentioned, DOTRAMOD does not account for possible increases or decreases in coal production capacity.

A couple of simplifying assumptions are also made. It is assumed that each buyer will purchase only a single type of coal product. That is, there is assumed to be a one-to-one relationship between coal buyer and coal product. It is further assumed that products cannot be blended. This is valid since coal product constituents do not combine linearly. That is, combining a product with an energy value of 10 MJ/kg with a 20 MJ/kg product in equal parts will not necessarily yield a 15 MJ/kg product. Finally, the coal market, comprising both local and international markets, is assumed to be competitive.

8.2.3 Investment Decision Criteria

Investment in mining projects typically requires that large capital sums are committed before the project begins generating cash flow (Savolainen, 2016). In addition, these investments are irreversible, and project outcome is subject to uncertainty and risk (Foo *et al.*, 2018). It is therefore important that good investment decisions are made which would generate value for investors. Economists have studied the investment behaviour of the firm for decades, giving rise to various theories to explain various aspects of the investment decision process. Projects are typically highly complex and there is no single, comprehensive theory or methodology that accurately captures project value when an investment decision must be taken. There does however appear to be one thing economists, financiers and investors agree on, and that is that the cash flows over the operating life of the project net of capital investment should be taken into account. That is, the net present value (NPV) should be considered

$$NPV = -C_0 + \int_0^T C(t)e^{-rt}dt, \tag{8.2.4}$$

where investment is a cash outflow $-C_0$, cash flow from project operations is given by $C(t)$, r is the discount rate, and T denotes the lifetime of the project. This is of course a highly simplified characterisation of the economics of a project. The particulars as to how this equation should be used, and how the various parameters should be interpreted and modified to be more suited to reality have been topics of discussion among economists and researchers (see Eklund, 2013). Hotelling's model for the price development of finite resources is also derived from a simple NPV, and

Gaugler (2015) extends the Hotelling model by building other explanatory factors into the simple NPV.

Discounting methods for project valuations, in particular NPV and internal rate of return (IRR) approaches, have become established as widely-used valuation methods for investment decisions (Kierulff, 2008). A review of South African mining companies by Du Toit and Pienaar (2005) revealed that 62.5% used NPV for evaluations, while 25% used IRR[4]. It is likely that many companies use both NPV and IRR methodologies as these are not mutually exclusive. Despite the popularity of these methods, they have been subject to some criticisim, primarily with regard to the choice of discount rate and their inability to handle uncertainty. The short-comings of NPV and IRR respectively are discussed in detail in Kierulff (2008), and a modified IRR is proposed as a better alternative. The factors used in the determination of the risk premium are discussed in Park and Matunhire (2011), together with a quantitative approach to deriving an appropriate risk premium. The determination of the risk-adjusted discount rate is also discussed in Jafarizadeh and Bratvold (2019).

Discounting methods are typically deterministic and as such incapable of handling uncertainty properly. A distribution of possible outcomes for NPV and IRR can be obtained by running iterative simulations where uncertain factors such as commodity prices, operating costs, and geological factors, are replaced with random variables[5]. Using a Monte Carlo simulation in mine valuation is demonstrated in Heuberger (2005), and this technique is applied for the valuation of a gold mine in Ugwuegbu (2013).

[4]Unfortunately a more recent review is not available. However, given that the mining industry is rather conservative, it is likely that these figures have not changed much.

[5]Though useful for gaining insight into what possible outcomes may materialise upon execution of a project, given uncertainty with regard to economic and technical factors, this approach may not be acceptable when conducting a proper valuation.

Real options valuation (ROV) is another approach for taking uncertainty into account that has steadily been gaining ground since the mid-1980's. Proponents of ROV claim that standard discounting methods, such as NPV and IRR, undervalue projects by failing to account for the value found in project flexibility. That is, investors have options throughout the life of a project to either begin, delay, or abandon aspects of the project, in order to maximise value or minimise loss. The review by Trigeorgis and Tsekrekos (2018) indicated that the use of real options in operations research grew between the years 2004 and 2016. The theme "Valuation Models and Other Topics" was however the smallest of the six research themes identified across all of the articles reviewed. The review concluded by highlighting challenges with real options in operations research, especially with regard to more empirical work being needed. Savolainen (2016) conducted a review of real options used in metal mining project valuation, and concluded by pointing out that the complexity in mining projects pose a challenge for analyses and valuations using real options. Real options has been applied in the valuation of mining operations by Zhang *et al.* (2015), where mean-reverting (MR) commodity prices are used[6]. An MR model was also used by Foo *et al.* (2018) and they found that the investment value of a project characterised by flexibility is higher in a volatile economic environment. The 1993 paper by Teisberg showed that, when faced with uncertainty, rational firms will invest in smaller projects with shorter lead-times, or postpone investment all together. Pindyck (1993) examined irreversible investments in projects with uncertain outcomes and found that factor cost uncertainty may have greater bearing on the value of the investment than technical uncertainty. The dynamics over the life of a project of the threshold price at which a decision is taken to invest in a mining project was investigated by Abdel Sabour (2001) using the options

[6] Geometric Brownian Motion (GBM) and MR are typically used for simulating commodity prices, however MR seems to be more appropriate for longer life projects. This is based on the assumption that in the long-term, market prices tend towards the average production cost (Savolainen, 2016).

to suspend start-up, close, or re-open. Foo *et al.* (2017) applied a binomial decision tree model in their real options approach to investigating mining investment decision-making. They looked into the effects of tax rate uncertainty and found that this was not the primary factor influencing the timing of investment.

Despite the apparent enthusiasm for ROV by the academic community, there is no evidence that ROV is gaining ground among investors when evaluating mining projects. The 1987 paper by Majd and Pindyck showed how applying the NPV rule could result in gross errors. However, a correction to their model by Milne and Whalley (2000) indicated that the simple NPV rule would at times be "an appropriate initial guide to investment".

8.2.4 Commodity Market Modelling

Mathematical optimisation is used to determine the locations of equilibria in computational models of commodity markets. This approach has been taken in several studies to investigate market behaviour under different conditions. The question of production capacity expansion or contraction is treated in different ways in these studies, if at all.

The spatial equilibrum model to investigate Cournot competition as conducted by Yang *et al.* (2002) did not address the capacity investment problem. Neither did the model of international steam coal trade described in Kolstad *et al.* (1983). Aune *et al.* (2004) modelled the short-run supply capacity problem in their model of the European energy market by allowing producers to bring additional capacity online subject to some start-up cost per unit of new capacity. Production capacity is also subject to down-time for scheduled maintenance. Capacity variation is thus linked to producer profit via the optimisation model used in their research.

In the study of pricing in the European natural gas industry by Holz *et al.* (2008), constant capacity constraints are imposed on upstream exporters and domestic producers. The paper by Haftendorn and Holz (2008) does not mention the supplier's investment problem, though it is possible for the supplier to reduce its supply. In the model of international steam coal developed by Haftendorn *et al.* (2010), it is possible for producers to invest in production and transport capacity that would then be available in the following model-year. A similar coal market study by Haftendorn (2012) did not address the supplier capacity investment problem.

In the investigation into the relationship between the Chinese electricity sector and the international coal market by Paulus and Trüby (2011), coal mine capacity is constrained. However, the possibility of mining new coal seams, when enabled to do so by suitably high price levels, is allowed. The research into the structure of the international steam coal market by Trüby and Paulus (2012) did not address the capacity investment problem. This problem was also not addressed in Paulus *et al.* (2011). In Trüby's 2013 investigation of the metallurgical coal market, supply is subject to constraints in the optimisation problem.

Paulus (2012) looked at how decisions to invest in coal supply capacity is affected by uncertainty regarding Chinese demand for coal. This entailed the development of a multi-stage stochastic spatial equilibrium model wherein investors are able to reallocate capital across time and space in order to avoid losses associated with weak demand. It was found that investors delay investment to a later time at which the likelihood of a profitable outcome is more certain. In addition, the payoff for investors in an environment characterised by uncertainty was found to be 18% lower than that seen in the deterministic case.

8.3 An Algorithm for Mining and Investment Decisions

In this paper the domestic coal trade model DOTRAMOD (Rademeyer *et al.*, 2021) is expanded upon by incorporating decisions made by miners on whether to continue operations as well as decisions by investors on whether to initiate development of new mining operations. Each mining deposit included in the model computation is assigned a mine status code. The explanation for these is as follows[7]:

0 Deposit is undeveloped

5 Deposit is in development

7 Operational mine

9 Mine has been shut down

In each period, the status code of each deposit is updated based on expectations about the future profitability of continuing with existing operations and the development of new ones. Each deposit proceeds through the following life-cycle. An undeveloped deposit, i.e. with status code 0, is evaluated for development based on the expected return on capital invested and taking into account a five-year development lead-time during which no income will be generated. Once a decision is taken to develop a deposit, the deposit's status code changes to 5. The project's status code will remain 5 for the five-year development phase after which the operation will enter production and its status code will change to 7. An operational mine with status code 7 will continually be evaluated on its expected future profitability. If it is found that the sum of the profit for the remaining operating life is negative then the mine's status code will change to 9 and the mine will be shut down. It is not

[7]The codes are not in a [0,1,2,3] sequence because it was initially thought to include "care and maintenance" as an additional stage that could be undergone between each of the four stages here mentioned. It was not however found that the inclusion of a "care and maintenance" phase would contribute meaningfully to this research, and so it was omitted.

possible to move from a higher status code level back to a lower status code level. That is, the decision to initiate mine development and the decision to shut down a mine are both irreversible decisions.

The review of the literature in Section 8.2.3 showed that discounting methods are popular among mining financiers when evaluating investment opportunities. As such, we have constructed our model's investment criteria as a simple NPV. A mine's operating profit is given by

$$\pi_m = \sum_{s=t+1}^{T} \left(\frac{1}{1+r}\right)^s \sum_{j=1}^{J} \left[FOR_s^j - C1_s^m\right] \cdot q_s^j, \qquad (8.3.5)$$

where T is the time horizon, r is the discount rate, J is the number of products produced by the mine[8], FOR_s^j is the mine-gate price on product j in period s, $C1_s^m$ is the average unit cost of production at time s, and q_s^j is the quantity sold of product j at time s. The capital investment part of an operating mine is not included since this is assumed a sunk cost, i.e. a cost incurred regardless of whether the mine is operational or not.

A decision to invest in the development of a new mine is taken when the projected return on investment from the project over its operating life exceeds the investor's required rate of return. Return on investment (ROI) is calculated as the difference between the projected income from the mine and the capital investment, as a fraction of the capital investment. That is, ROI is defined as

$$ROI_m = \frac{\text{mine income} - \text{investment required}}{\text{investment required}}. \qquad (8.3.6)$$

Mine income is taken as the profit for the period for which the mine will be operational, as per equation (8.3.5). The investment required is calculated as the product of planned mining capacity, in units of tonnes per annum tpa[9], and capacity cost,

[8]This is always set equal to the number of markets. The products that are then not produced by a particular mine are set to zero.

[9]Tonnes per annum, i.e. a unit of coal production capacity

in units of cost per unit of mining capacity $\$/tpa$. For example, the development of a one million tonne per annum operation would, at an average cost of 100 $\$/tpa$, require an investment of 100 million dollars. The mining capacity of a new project is taken to be the rate at which the deposit will be mined out in the period remaining in the computational procedure, adjusted for project development lead-time. In addition, investors are given a maximum investment level tolerance. If the required investment is significantly higher than this limit, then the new project's capacity is the ratio of this limit to the unit cost of capacity.

Capacity modification falls into one of three categories, namely short-term, medium-term and long-term. Short-run capacity expansion can be achieved by operating highly flexible plant, such as gas-fuelled electricity production which can be brought online in a couple of hours, or by adding more machines or workers to a production operation. Medium-run capacity expansion can be achieved over a couple of weeks or months, by bringing mothballed capacity back into operation. Long-run capacity expansion is achieved by developing new production capacity, and this typically takes place over a couple of years. The longer the lead-time, the greater the risk to a project being completed on time and at the budgeted cost (Rademeyer *et al.*, 2020). This paper seeks to address the question of long-run capacity availability, i.e. the large-scale capacity that is yielded by project investment.

8.4 Data and Computational Scenarios

As this is an investigation into the South African coal industry the model is calibrated using hypothetical data which is based on values seen in the industry, as in MCSA (2018). Unfortunately it is not possible to obtain operational and financial data for mining companies in the required form as this is sensitive information.

Data Item	Period $t = 0$
Max price for MG product (USD/MJ)	0.2
Max price for HG product (R/MJ)	1.5
Max price for LG product (R/MJ)	0.05
Max demand for MG product (MJ)	50e11
Max demand for HG product (MJ)	5e11
Max demand for LG product (MJ)	50e11
CV for HG product (MJ)	25.5
CV for MG product (MJ)	22.0
CV for LG product (MJ)	14.0

Table 8.1: Market demand and pricing data: Input data for economic parameters relating to products traded and customer demand at time $t = 0$.

8.4.1 Input Data used in Computation

The number of time periods over which optimisation takes place is $T = 30$ years. We consider a trader sourcing three products from seven operational mines and redistributing these products to three buyers. There are three additional unmined coal deposits which may be developed. Each mine has the potential to sell each of the three products. Product sales prices (at mine-gate) are in R/MJ. Two cases are considered, the base case and the case of decreasing export demand. The three coal buyers are characterised by the properties given in Table 8.1.

As in Rademeyer *et al.* (2021), maximum tolerable prices and demand levels[10] are initially kept constant over time. That is, all demand-side drivers are assumed constant in the base case. In the case for decreasing export demand, the maximum demand for MG export product is taken each year to be only 97% of the value seen in the preceding year.

[10]The maximum price P_{max} is the price at which demand by the buyer for the product is zero. The maximum demand D_{max} is the maximum intake in MJ where after $P = 0$, i.e. no price decrease could induce more buying.

Deposit	Status	Size	MG [MJ]	HG [MJ]	LG [MJ]	Cost [R/MJ]	Escalation
1	0	Mid	4e11	5e10	4e11	0.1	0.2%
2	7	Mid	4e11	5e10	4e11	0.1	0.2%
3	7	Mid	4e11	5e10	4e11	0.1	0.5%
4	0	Large	5e11	1e11	5e11	0.15	0.2%
5	7	Large	5e11	1e11	5e11	0.15	0.2%
6	7	Large	5e11	1e11	5e11	0.15	0.5%
7	7	Large	5e11	1e11	5e11	0.15	0.7%
8	0	Small	1e11	1e10	5e11	0.2	0.2%
9	7	Small	1e11	1e10	5e11	0.2	0.2%
10	7	Small	1e11	1e10	5e11	0.2	0.5%

Table 8.2: Mine production capacity data: Input data for deposit parameters related to mine operations.

Three mine classes are considered: a low-cost, mid-sized mine class, a mid-cost, large mine class, and a high-cost, small mine class. A diagram of the network comprising these mines and the suppliers is given in Figure 8.1. The supply of each product is assumed constant and is put up for sale if the mine is operational, i.e. status code 7. Mine costs are assumed to increase over time as per the escalations specified. The properties of the 10 coal mines considered are given in Table 8.2. The cost escalations for the operational mines in each cost class are different as well to allow for studying how projects facing stronger cost increases would perform as compared with those mines who are better able to keep cost escalations under control.

The domestic trader's transport cost is assumed to be constant at $\tau = 10$ R/t on transport routes to domestic consumers, but $\tau = 150$ R/t on the export line. The export FOB price is in USD per tonne and must be converted to a domestic currency value. The exchange rate is assumed constant at 14 ZAR per unit USD.

The domestic discount rate is assumed constant at 5%. It is assumed that investors require a rate of return of no less than 10% and the maximum investment amount is set at R 10 billion. Mine project development lead time is taken to be 5 years. The cost of new mining capacity is constant at R 100 per *tpa*.

8.4.2 Computational Scenarios

Two scenarios are considered. The evaluation period is 30 years. The first scenario considered is the base case in which demand is assumed to remain flat over the entire period under evaluation. The second scenario is the test case, or that of declining export demand over the period under evaluation.

The computational model is calibrated using the input data given in Section 8.4.1. The optimisation solver iteratively proposes quantities of each product to be dispatched by each route of the network as in Figure 8.1. It continues to do so until it has maximised the profit the trader can earn over the 30 year operation. The various results generated by the optimisation procedure are presented next.

8.5 Results

In what follows is presented the results or output from the model after the computational solver found the optimal solution for quantities of each product to be dispatched on each trade route as shown in Figure 8.1. Though there are many interesting results from a complex model, the results discussion will focus only on the key results.

8.5.1 Base Case

The base case is the control experiment. That is, it is the case in which no changes are induced on the model. The purpose of the base case is to enable comparison with the results from the test case. Prices remain constant in the results from the base case and are thus not included in the discussion. Volumes are discussed first, and then profit measures are covered. Reasons for changes in volumes are explained more fully during the discussions on profits.

Products to Market

The trader takes three products to market. The quantities of each over time is given in Figure 8.3.

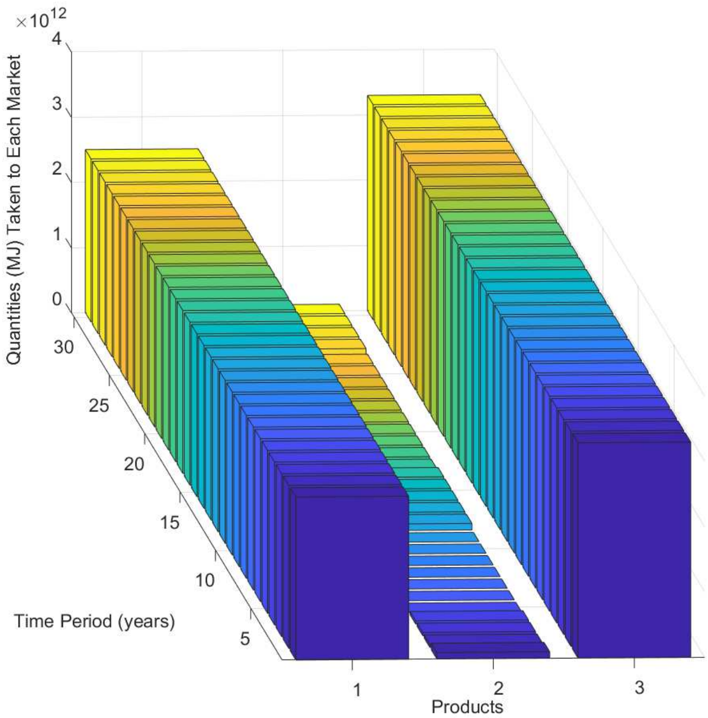

Figure 8.3: The trader delivers three products to their respective markets. Product 1 is MG coal, product 2 is HG coal and product 3 is LG coal.

Both the export product (product 1 in Figure 8.3) and the local power station (LPS) product (product 3 in Figure 8.3) are delivered to market in constant volumes over the life of the trader's operations[11]. Small quantities of higher-grade product (prod-

[11]The graphical representation is in MJ to allow for direct comparison of the three products. However, these quantities would equate to about 110 mt of export product and 240 mt of LPS product respectively.

uct 2 in Figure 8.3) are traded for most of the 30 years of the trader's operations, with no trade from years 5 to 11.

Miners' Profit from Coal Sales

The profit accruing to each mine following the sale of product to the trader is shown in Figure 8.4.

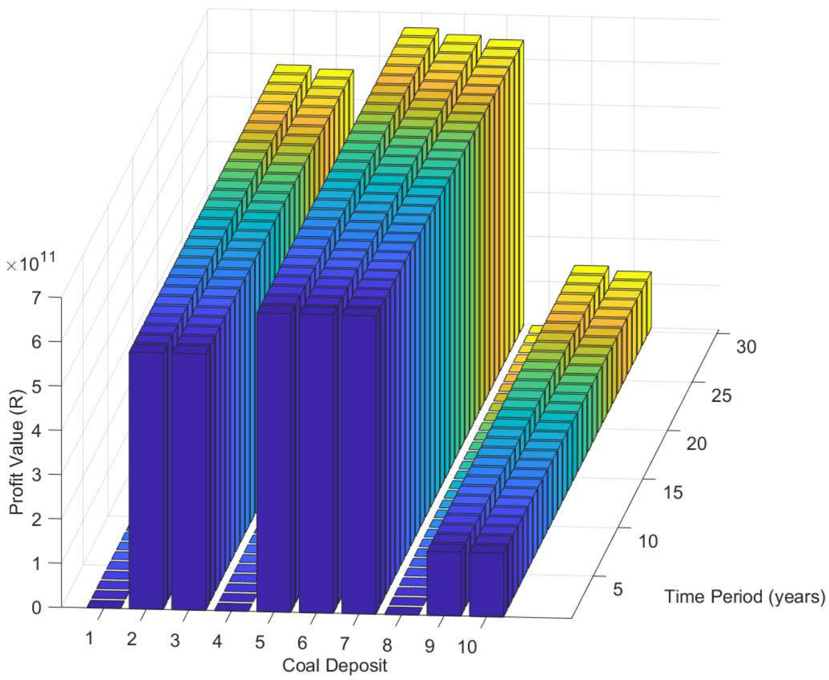

Figure 8.4: Profit accrues to each mine following product sales.

It is seen in Figure 8.4 that the mines with the slower cost escalations, i.e. mines 2, 5, and 9, experience slightly better profit growth than the other mines with stronger price escalations. The profit levels of all seven operational mines are largely stable over the model time horizon.

8.5.2 Declining Export Demand

For the test case, export demand is assumed to be declining over the 30 years of the trader's operations. Export demand in each year is assumed to be only 97% of the level seen in the preceding year. Volumes are discussed first, then prices, and finally profit measures. Reasons for changes in volumes are explained more fully during the discussions on profits.

Products to Market

The trader takes three products to market. The quantities of each over time is given in Figure 8.5.

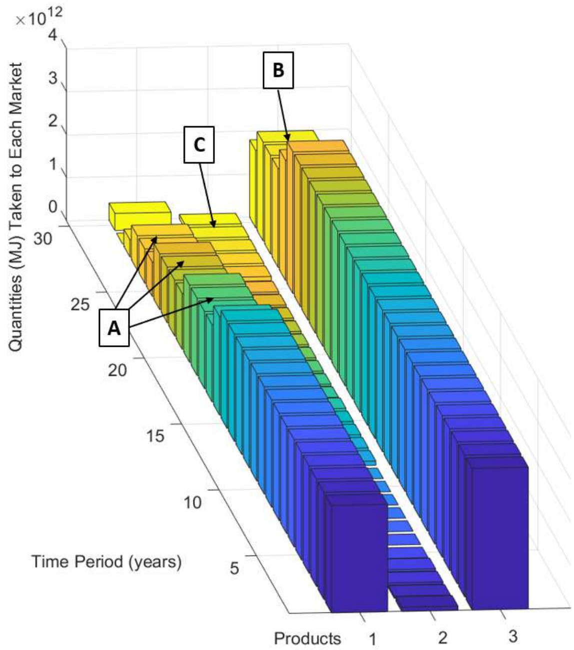

Figure 8.5: Quantities of each product are delivered to market. Labels A and B indicate declining supply levels, whereas label C indicates a slightly increasing level.

The sale of export product (product 1 in Figure 8.5) is constant over the first 15 years of the trader's operations. Thereafter sales begin to decline (at label point A). For the most part of the trader's operations small quantities of higher-grade (HG) product (product 2 in Figure 8.5) are sold, except for years 5 to 11, when no volumes are sold. A slight uptick in HG sales quantities are seen in the final five years (label point C). The local power station (LPS) product (product 3 in Figure

8.5) is delivered to market in constant volumes over the majority of the trader's operations time horizon, but declines in the final five years (label point B).

Price Development

Prices are established on the trade of products in each market. Landed prices, or CFR prices, apply at the receiving end, after transport, i.e. at trade fronts F2, F3, and F4 in the network maps in Figure 8.1. These prices are shown in Figure 8.6. The price on higher grade product (HG) is shown in the upper window and the prices for the mid grade (MG) and lower grade (LG) products are shown in the lower window.

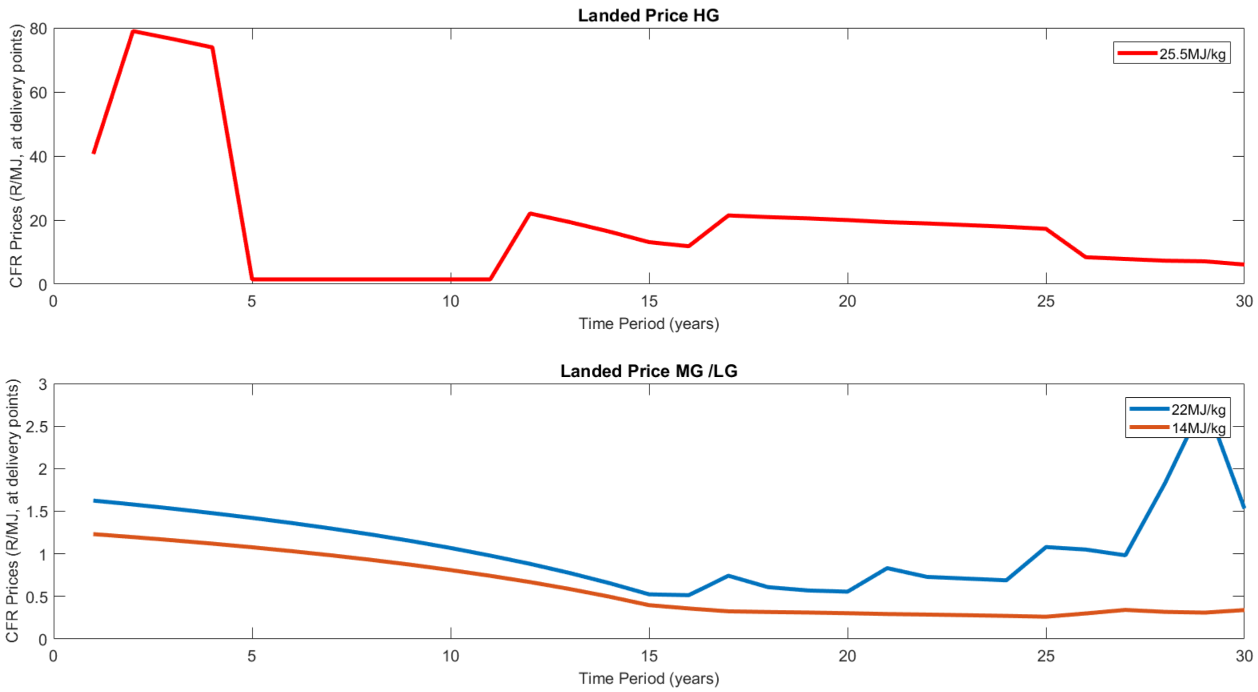

Figure 8.6: The product prices at market (customer) delivery points are shown here, with the HG price in the upper window, and MG and LG prices in the lower window.

The price on the export product (22 MJ/kg) (shown as a blue line in the lower window of Figure 8.6) declines over the first 15 years of the trader's operations due to declining demand amid consistent supply. The export price however recovers steadily over the remaining 15 years in response to supply decreases (as seen in Figure 8.5). The HG product (shown as a red line in the upper window of Figure 8.6) is elevated for most of the time horizon, indicating constrained supply. Between periods 5 to 11 the HG is price lower, suggesting higher supply as the trader attempts to recover declining export volumes. At this lower price point, trade is not viable and ceases[12]. It is interesting that the price of the LPS product (shown as a brown line in the lower window of Figure 8.6) declines as well over the first 16 periods. This would also indicate higher supply and may also be as a result of the trader trying to push more LG volumes as export revenue declines.

Miners' Profit from Coal Sales

The profit accruing to each mine following the sale of product to the trader is shown in Figure 8.7.

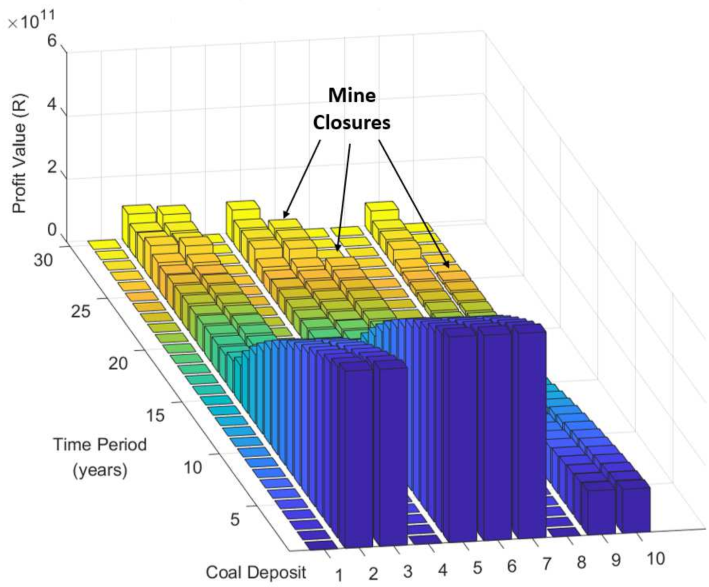

Figure 8.7: Profit accrues to each mine owing to product sales. Profits drop to zero when mines are closed, as shown.

Profits are seen to be declining at an increasing rate over the first 15 years due

[12]Supply does not equal sales. Supply is "available for sale".

Scenario	Export Demand Trend
1	100%
2	99%
3	98%
4	97%
5	96%
6	95%

Table 8.3: Scenarios for decreasing export demand

to rapidly declining export margins, but stabilise at much lower levels in the final years of the time horizon. Profits for mines 6, 7, and 10 drop to zero in the final five years prompting these mines to close. These closures and the subsequent decreases in products supplied from these mines lift prices on the export and LPS products and contribute towards improved profitability for the remaining operational mines. As expected, the higher cost mines with stronger cost escalations are the most susceptible to worsening profitability conditions.

8.5.3 Sensitivity Analysis

The results of only two scenarios have been presented here. To indicate the robustness of these results, the key results from several scenarios for export demand development over time are shown. Scenario 1 is the base case presented in Section 8.5.1 and scenario 4 is the test case presented in Section 8.5.2. A total of six scenarios are included in this analysis, with increasing strength of decline in export demand from scenario 1 through to 6.

Figure 8.8 shows how sales to the export and power stations markets respectively develop over time under the different scenarios for export demand[13]. Sales to the

[13]The heating value for each product remains constant for all periods and all scenarios. Hence the trends are the same whether shown in joules or tonnes.

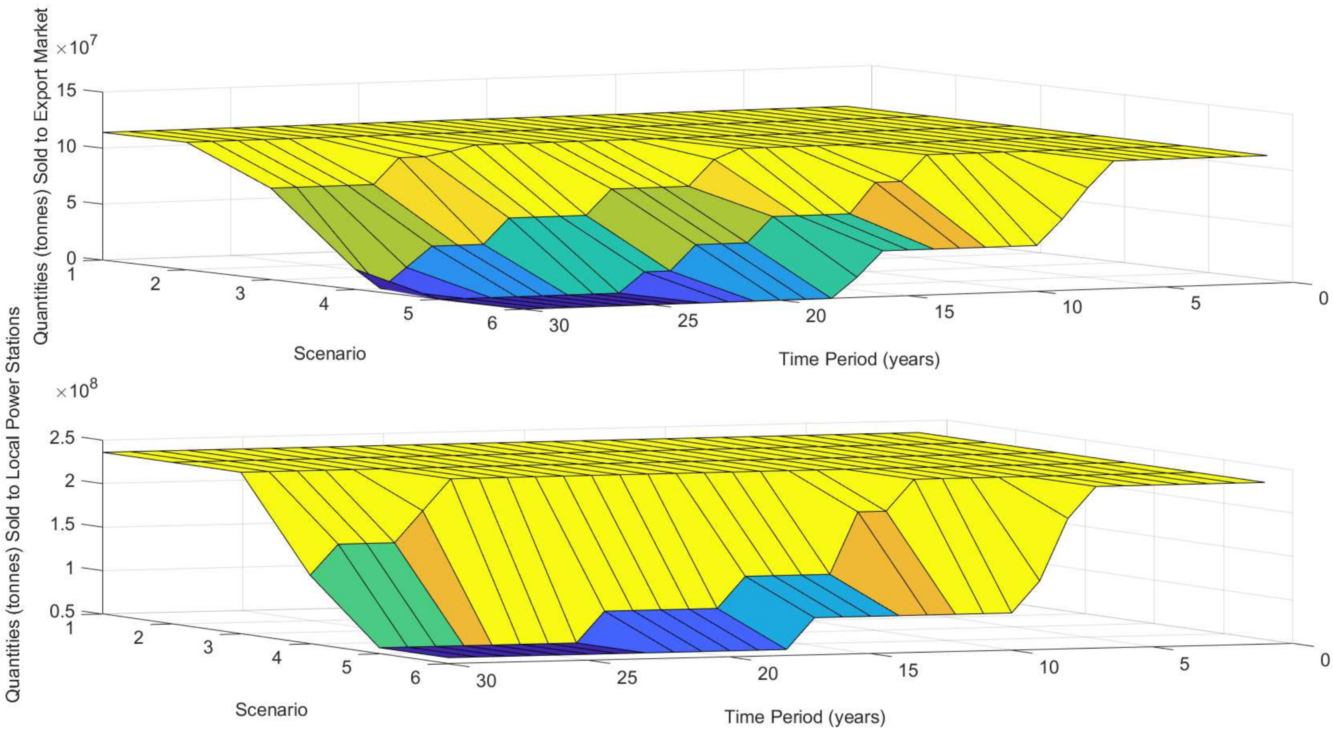

Figure 8.8: The upper frame shows the development over time for exports coal sales for different scenarios of increasingly strong export demand decline. The lower frame shows the development over time of sales to domestic power stations when faced with the same scenarios.

export market, as in the upper frame of Figure 8.8, begin to decline more strongly with an increasingly strong decline in export demand, as expected. By year 23 in scenarios 5 and 6, the export market has collapsed completely and no volumes are exported. It is interesting that precisely at this same time the market for power station coal has collapsed as well.

The profit development for mines 7 and 10 under the six scenarios is shown in Figure 8.9. These two mines, together with mine 6, are the mines that close in the test case (scenario 4). In the upper frame of Figure 8.9, it can be seen that the profit for mine 7 declines more strongly with increasingly strong decline in export demand, until the mine closes and profit is zero, as in scenarios 4, 5, and 6. The result is similar for mine 10 as seen in the lower frame of Figure 8.9. For mine 10

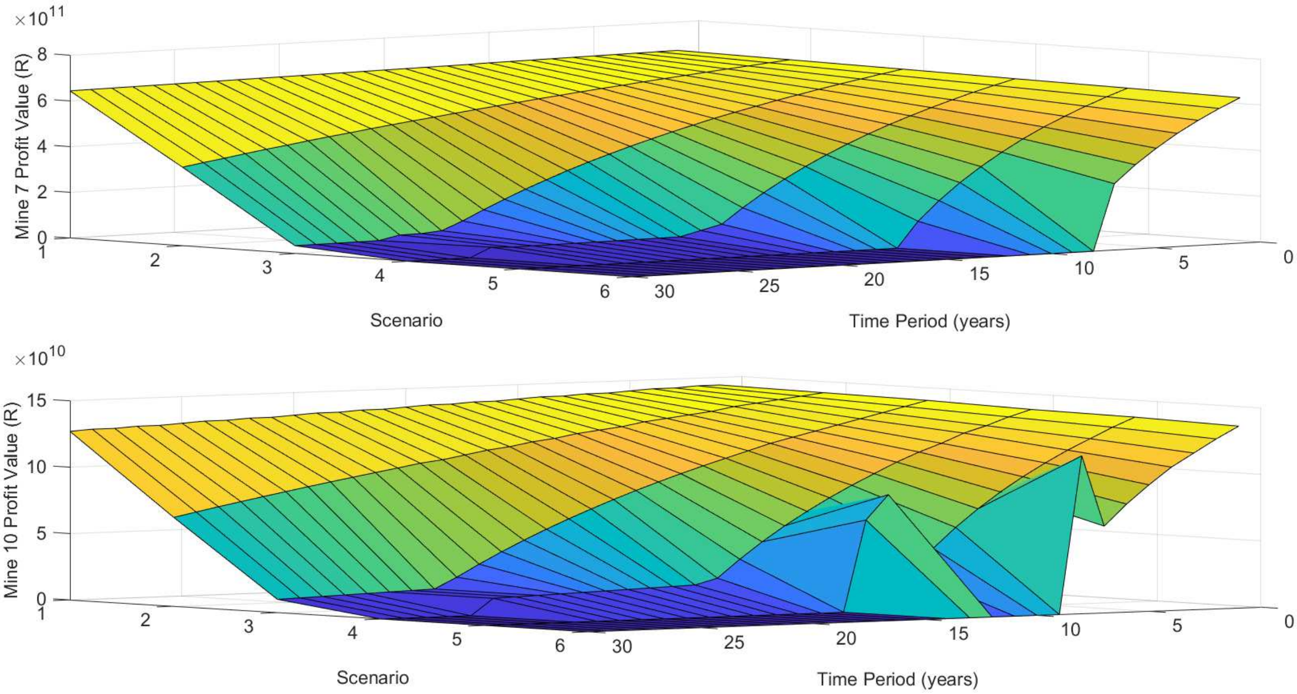

Figure 8.9: The upper frame shows the profit development for mine 7 for each of the six export decline scenarios. Similarly, the lower frame shows the result for mine 10.

however, there are brief periods of elevated profit in years 13 and 14 of scenario 5, as well as year 9 of scenario 6, where the closures of other mines enable mine 10 to sell more volumes at higher prices. The results for mine 6 are similar to those for mine 10.

Scenario four was selected for the test case as it is the case in which the effect of declining export demand is such that mine closures begin to occur and that domestic coal supply begins to be adversely affected. Below a declining export demand trend of 95%, the optimisation solver increasingly struggles to achieve feasibility.

8.6 Concluding Comments

The investigation results show that decreasing export demand leads to decreasing profits for mines which causes the eventual closure of higher-cost mines. This results in reduced trade of lower grade (LG) power station (LPS) coal which is accompanied by a gradual uptick on the LG price. The supply of higher-grade (HG) coal for domestic industrial appears to be constrained for most of the 30-year period, with the consequence being that the price on this product is elevated as well. It can therefore be concluded that declining export results in an eventual decrease in the supply of LG coal for domestic power stations, with this lower supply available at higher prices. The exact effect on HG coal is unclear. However, it appears that the smaller HG market is destabilised by dynamics between the single, common trader and the export market.

The eventual closure of loss-making mines, and the subsequent decrease in supply of exports and LPS products, lifts the prices for MG and LG products. It follows that the margins on these products stabilise so that the remaining mines are able to avoid becoming loss-making and are therefore able to continue operating. No mine development is undertaken due to unfavourable profit trends in both the base case and the case of declining export demand.

The domestic price and supply results from this research are different from those seen in Rademeyer *et al.* (2021) where supply of HG coal to local consumers increased and the price of this coal was lower than in the base case, and the price and volumes on LPS coal remained unchanged. This would suggest that the inclusion of the mining investment problem has a rather large impact on the results obtained from a commodity market study.

This paper has contributed an extension of the DOTRAMOD coal trade model with the inclusion of an investment algorithm. It has further contributed the key result that supply of coal for local power stations would decline when faced with a persistent decline in export demand. This result is significant in that it suggests that the global phasing out of coal could have implications for South Africa's energy security, as well as for mining sector employment.

There are however limitations to the research here presented and the investigation of other pertinent factors for the mining industry should be undertaken in future research. The research presented in this paper is limited in that the investment algorithm does not take other potentially important factors into account, such as tax or environmental regulations. Though of great importance to the mining industry, it was decided to keep these constant in this research and to include only the decline in export demand, so as to isolate the impact of this single parameter. The production cost function has also not been fully specified; typically per-unit mine costs vary with production level. This would have implications for project cash-flows and the overall viability of the project. The literature review on project valuation indicated the importance of accounting for various types of project uncertainty. The algorithm presented in this paper does not account for this uncertainty that investors may face and which would strongly impact on their decisions to open, close or restart a mining project. Future research should consider different market structures, i.e. competitive market, versus oligopolistic or monopolistic, as each may result in different trade and pricing outcomes. The demand side of the DOTRAMOD model is not fully specified. It would be interesting to see how the results are affected when capacity investment on the demand side is taken into account.

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Chapter 9

Concluding Remarks

The mining industry is extremely complex. It is composed of various players, such as mines, traders, customers and government agencies. These players tend to be complex themselves and have different interests. In addition, they interact with one another in complex ways. Current theories and methods in mineral economics do not adequately capture this complexity. Even when seeking to account for this complexity, computational limitations often inhibit problem solving and the derivation of meaningful results. Useful insight can however be obtained from abstract models. These models are specially designed to investigate particular topics of interest, and they can provide general insight into the behaviour of entities, or they can allow for studying exceptional outlier events. This project was truly multi-disciplinary, requiring knowledge in coal marketing, coal mining, mine economics, trade economics, mathematics, and computational optimisation techniques. While working on this thesis, it became quite clear that investigations concerning the South African coal industry require the involvement of multi-disciplinary teams, comprising individuals from academia and the various facets of the coal industry. Indeed, academia can provide valuable support to industry by delivering new methods and techniques for solving ever evolving industry problems.

The work presented in this thesis has yielded five articles that have been accepted for publication in international journals for mineral and resource economics. The criteria for the publication of academic articles stipulate that the work must make a novel contribution to the literature and broaden the knowledge in the field of mineral economics. Each of the five papers went through a peer review process during which the value of the work and its contribution to mineral economics had to be well argued and defended. Though the work presented in this thesis has been focused on coal production and coal trade, the methods and techniques developed here could be applied to other commodities. It should be possible to modify GENCOMIN to allow for investigating mine economics for other minerals, and it should also be possible to employ a modified version of DOTRAMOD for a commodity where there are links between domestic consumption and export.

For completeness, the contributions of the work presented in this thesis are restated here. *Paper I* produced a novel cost function for bulk operations characterised by economies of scale, such as is typically the case for mining operations and for coal mining in particular. This improved function is more appropriate for use in models of mining, as is demonstrated with the GENCOMIN model presented in *Paper III*. *Paper II* presented a novel characterisation of the mechanism transforming capital investment into production capacity. This is especially important for investment projects with long lead-times, such as is typically the case for mining projects. This characterisation is also demonstrated in GENCOMIN in *Paper III*. The development of an original model of a multi-product coal mine, GENCOMIN, is presented and demonstrated in *Paper III*. As mentioned, the cost function derived in *Paper I* and the investment-to-capacity mechanism derived in *Paper II* are incorporated into GENCOMIN. Results from running computational scenarios in GENCOMIN indicate that the capital consideration has more bearing on the mine project's profit outcome than the production consideration. That is, capital investment is actively adjusted over the mine project's life whereas production volumes remain unchanged. The work presented in this paper highlights the importance of the investment problem in commodity market supply and dynamics. That is, the capital investment part of the mine's profit equation becomes the most important because of the economies of scale and the resulting flatness of the average cost curve. In addition, the results from GENCOMIN in *Paper III* are consistent with the results from the more abstract models presented in *Paper I* and *Paper II* respectively. This consistency further supports the viability of these models. *Paper IV* presented a model of domestic coal trade in South Africa, DOTRAMOD, facilitated by a central trading hub. DOTRAMOD is also equipped with a pricing function that derives the market price and trade quantity from the interaction between supply and demand, and is thereby independent from subjective views or historical price levels. The purpose of the model is to allow for investigating which effects a decrease in export coal demand might have on the local market. It also explores the concept of a central trading hub. Computational results show that, in a scenario with a continuous decrease in export demand, the export price would drop and the trader would look to push more volumes onto the local market. Finally, in *Paper V*, the miner's investment problem is built into DOTRAMOD and the effects on the local market of a decrease in export demand are again investigated. This time, there are mine closures in response to lower export profitability, resulting in lower supply for local power producers. This is a key result as it suggests that a deterioration in export demand would have implications for South Africa's energy security as well as mining employment.

All of the work spanning the five papers culminate in the result obtained from the model presented in *Paper V*. This result suggests that suppliers that derive income from supplying to both the domestic and the export markets would be deterred from investing in new mines if the export market were to deteriorate, thereby undermining the export revenue stream. Furthermore, suppliers could be persuaded to exit the industry entirely if lost export sales begin to erode profitability. These results would suggest that policy makers should review the ties between coal sales for export and those for the domestic market to support the industry so that sales to the domestic market cannot be adversely affected by export market dynamics. The sale of Anglo American's coal domestic-focused coal operations to Seriti may have been beneficial in decoupling supply to the domestic market from the export market. Indeed, Seriti is now supplying to the domestic market only and as such the profitability of the supplier's coal operations are not directly exposed to export market dynamics.

A single research project could not resolve all of the questions concerning the South African coal industry. The work presented in this thesis was developed to explore the economic implications of changes in the export market. In particular, the work set out to determine how coal supply to domestic consumers could change in response to a decline in the demand for exported coal. The models presented in this thesis were developed to specifically support in exploring this question. Indeed, the mining and trade models are relatively simple and may not fully capture all of the complexities of the coal industry. Nevertheless, these models are supported by economic theory and reasoning, and the results from these models are consistent with what is seen in reality, thereby supporting the viability of these models. This research has made contributions to the field of mineral economics which could be used in continued work undertaken to understand the drivers and trajectory of the South African coal industry. It is a worthwhile challenge that other academic researchers should embark on as well.

Coal is an extremely complex mineral with an equally complicated marketing structure. The work presented in this thesis does not take into account all of the parameters distinguishing different coal products, and thus does not capture the associated marketing variations. In addition, coal mining, coal distribution, and coal marketing are very complex operations. The work presented in this thesis does not cover the very many factors that exist in these operations.

The capital investment model presented in *Paper II* could be further enhanced towards achieving a model which is more representative of the capital investment problem for mining projects. Future research should consider other variants of lead-time filters which may result when taking different aspects of project development into account, such as politics, regulations, and logistics. It would also be useful to determine what lead-time filter might be employed in order to obtain a more realistic representation of the way in which production capacity becomes available for use. This is seen in practice to occur in an almost step-wise fashion, colloquially termed "ramping up". *Paper II* only briefly addressed the uncertainty component of project investment by incorporating a random variable into the capital investment model. There may be scope to expand upon this, both by expanding the model itself as well as by incorporating other random variables. Moreover, further analysis of the stochastic configuration of the model is required.

There is much opportunity for employing GENCOMIN to investigate other aspects of mining operations. In *Paper III*, exploration is effectively deterministic, but in reality this activity is characterised by uncertainty and as such would be more accurately described by a stochastic configuration. It is expected that accounting for the uncertainty in resource accumulation may render different results. As mentioned, it is expected that GENCOMIN could be modified to allow for investigating the mining of other minerals as well. It would be interesting to know whether the main result, i.e. that the capital investment problem dominates the profit function, holds for other minerals as well. Furthermore, in reality there is scope for mine planners to determine when in the life of the mine a particular strip ratio or calorific value should be mined. It would be interesting to see what results are produced when setting these parameters as decision variables in the optimisation problem.

The value of the cost and investment functions introduced in *Paper I* and *Paper II* respectively could be further evaluated by comparing the results from *Paper III* with those obtained if other cost and investment functions were used instead. For example, the cost function could be replaced with a constant value, a linear function, or a quadratic function. Similarly, it would be interesting to see how the results from the model presented in *Paper III* would change if another approach to the capital investment function were used, or if the lead-time were reduced entirely.

This would provide an indication of just how sensitive the project value is to this development lead-time.

The model of domestic coal trade, DOTRAMOD, from *Paper IV* may have been expanded upon in *Paper V* with the incorporation of a mine profit and investment algorithm, but there still remain many extensions possible that would enable further study of the very complex and broad South African coal industry. Most notably, the domestic consumer side is yet to be fully specified, and hence it is not yet clear how domestic consumers would respond when faced with a situation of declining local coal availability. The trader in DOTRAMOD is cast as a centralised trading hub. It would be good to investigate the outcome when there are instead two or three trading hubs. This would show whether a centralised trading hub is indeed most beneficial for the South African coal industry, or whether transport cost reductions and adequate coal supply could be better served by multiple trading hubs. The mining production cost function has not been fully specified in DOTRAMOD. The fact that per-unit mine costs vary with production level would suggest that the profit results for the mines in DOTRAMOD could be different when incorporating a cost function such as proposed in *Paper I* and as in GENCOMIN in *Paper III*. In both *Paper IV* and *Paper V* the domestic coal market was assumed to be competitive.

The results from *Paper V* for the higher-grade coal product were inconclusive. This is a relatively small market in relation to the markets for power station coal and export coal. Sizing is an important factor in this market, suggesting that the market is actually further segmented by coal sizing. A separate study focusing only on the market for domestic industrial coal users may be more beneficial and render more definitive results.

The results from all five papers could be enriched by further empirical work. Adequate data, especially financial data, are not typically publicly available. Nevertheless, the evaluation of the proposed models with real-world data would be interesting areas for future research. It would be beneficial to assess the results from GENCOMIN when using data from a real mining operation for input into the model. Similarly, expanding DOTRAMOD to include more mines, as well as more particular information on the coal specifications demanded by the various domestic power stations, would yield results that could be more easily translated to particular real-world situations.

The models provided in *Paper III* and *Paper V* do not necessarily address questions in financing for mining project investment. That is, some of the principles of discount factors, weighted average cost of capital (WACC), and the capital asset pricing model (CAPM) are used in the models presented. The hypothetical values chosen for discounting are not necessarily realistic, but merely chosen for demonstration purposes. The derivation of more appropriate discounting values should consider aspects such as sovereign risk, the cost of equity and market volatility. It would be beneficial to expand the models presented in this thesis with a more comprehensive representation of project financing.

A comment is warranted on the exclusion of some features which are unique to the South African mining and coal industries. In particular, affirmative action policies such as Black Economic Empowerment (BEE) or Broad-based Black Economic Empowerment (BBBEE) are not referred to in this thesis. The reason for omitting this feature from the research is that the primary focus of the research is on the economic implications of a deterioration in the export market. The complexity of the coal industry has already been noted. Social and political issues should be treated with delicacy, especially when discussing South African mining and the coal industry. There are many critical social and political factors and these should be considered fairly to yield meaningful results. Indeed, these factors could impact on the economics of the South African coal industry, but this could be represented in the economic variables already present in the models presented in this thesis. As stated, the aims of this research are centered around economic issues, and it was therefore deemed important to maintain a firm focus on the essential economic factors only. The implications of social and political imperatives in the South African coal industry should be addressed in future research where these factors can be treated carefully and satisfactorily. Similar reasoning supported the exclusion from this research of issues around climate change and carbon emissions. These issues too would best be addressed in future research.

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