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An exploration of grade 8 learners' responses to real-world and standard mathematical problems

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CHAPTER 1: INTRODUCTION

1.1 Background to the study

There is ample evidence in the literature that learners, in the context of tests, do not pay attention to realistic considerations when constructing their responses to word problems which embed everyday context (Reusser & Stebler, 1997; Cooper & Harris, 2002; Chacko, 2004; Cooper & Dunne, 2000). In their interpretation of word problems, learners seem to follow rules and symbols without reflecting on what these rules imply in the specific context in which they are used. The general outcome is often that learners solve standard problems satisfactorily, but fail to apply their mathematical knowledge when encountering real-world problems. The aim of the research was to understand more about how learners applied their mathematical knowledge to real world events in solving mathematical problems and how learners were able to move beyond constraints associated with standard problems to apply Bernstein's (1996) concept of recognition rules as a way of making sense of learners' use of everyday knowledge in responding to the mathematical items.

1.2 Statement of purpose

The purpose of the study was to explore whether and how the use of real-life contexts influences learners' responses in solving mathematical problems.

1.3 Research questions

1) (a) How do learners respond to written real-world mathematical problems?
   (b) How do they respond to standard problems that have the same content as real world problems?

2) What do learners think about the purpose that contexts play in solving these mathematical problems?

1.4 Rationale

Recent research has documented that learners pay little attention to realistic considerations when applying their mathematical knowledge to real-life contexts. That is, during mathematical modeling of word problems, many learners solve mathematical word problems without considering the factual relationship between real-world situations (what the problem texts are about) and mathematical operations (Chacko, 2004; Yoshida, Verschaffel & De Corte, 1997; Reusser & Stebler, 1997). Therefore I want to find out why learners exclude real-world knowledge and sense making in the modeling and interpreting of solution of word problems.

It has been argued that mathematics provides tools for describing, analysing, and predicting the behaviour of systems in the real world. It is also argued that the inclusion of application of problems particularly the word problems was intended to develop learners' skill in knowing when and how to apply their mathematics effectively to various kinds of problems in everyday life (De Corte, 2000). A new classroom culture was created featuring systematic attention to the realistic mathematics modeling as an aspect of mathematical learning. Therefore I want to explore how learners use their knowledge and experience about the real world to solve mathematical problems posed in realistic contexts.

1.4.1 Location of word problems in the curriculum

Traditionally, the practice of word problem solving in school mathematics focused mainly on rote learning, decontextualised absorption of mathematical knowledge and skills. Current views on word problems argue that a major shift is that word problems are conceived primarily as a set of human sense making and problem-solving activities based on mathematical modeling and reality.

In the new South African mathematics curriculum, the Revised National Curriculum Statement (RNCS) for grades R – 9 (2002) seek to create a lifelong learner who is able to "display critical and insightful reasoning and interpretative and communicative skills when dealing with mathematical and contextualised problems" (p. 5). Word problems are important aspect in the teaching and learning of mathematics because they provide learners with an opportunity to study the interplay among and between language processes, mathematical processes, and situational reasoning (Reusser & Stebler, 1997). This suggests that learning mathematics is about learning to communicate mathematically within powerful learning environments which include acquiring fluency in words, ways of speaking, writing and presenting mathematical arguments (Setati, in press).

The new Curriculum requires incorporating everyday contexts into the mathematics. The shift in curriculum allows not only accessibility to learners' skills and knowledge in mathematics, but also their ability to use and apply their skills and knowledge. Again the shift in curriculum aims at making mathematics more meaningful and sensible to learners and to equip them with the knowledge, competencies and orientations that will enable them to succeed after leaving the school.

1.4.2 Assessment practices

Assessment is an integral component of teaching and learning in any curriculum. Clarke (1996) argues that learners' performance in mathematics must be monitored in such a way that it allows learners to demonstrate what they have learned. This suggests that learners need to be able to apply what they learned in mathematics in their everyday lives. Clarke (1996: 331) further maintains that "restriction of assessment tasks to only those requiring written responses or competency in calculations will misrepresent mathematics and leave the competencies of many students unrecognized, denying them access to further education and employment, and denying society the benefit of their particular talents". This suggests that if mathematics learners are mainly exposed to abstract mathematical concepts and procedures without linking mathematics to real

situations such learners might fail to see sense of mathematics learning in their daily lives.

Assessment in the new Curriculum is a continuous process of gathering information about the performance of learners measured against the Assessment Standards of the Learning Outcomes. The Curriculum has been reconstructed in the form of Learning Outcomes and their associated Assessment Standards in each learning area or subject and for each grade. In combination, Learning Outcomes and Assessment Standards are "cognitively dependent and supportive of each other" according to Revised National Curriculum for Grades R-9 (Department of Education, 2002: 6). That is, collectively, they show how knowledge, skills and values are acquired from grade to grade thereby enhancing continuity. To assist in the process of learner assessment, the (DoE, 2002) contextualises the Critical and Developmental Outcomes with the Learning Outcomes and Assessment Standards. This suggests Critical and Developmental Outcomes are embedded with Learning Outcomes and Assessment Standards and through that the use of both mathematical abstract concepts and contexts for learning activities, rich mathematical understanding can be enhanced.

In the above sections, I have described the importance of mathematical word problems as human sense making and problem-solving activities based on mathematical reality and how curriculum in South Africa through its Learning Outcomes and Assessment Standards allows accessibility to learners' skills and knowledge in mathematics. Assessment as a continuous process of gathering information about the performance of learners is used in achieving the Learning Outcomes.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Theoretical framework

The advent of the new Curriculum in South Africa brought changes in mathematics learning and teaching in terms of how it should be learned, what should be learned, and why should it be learned. These shifts in curriculum are in agreement with the theory of constructivism which emphasizes learning mathematics as a constructive process rather than the application of rules and formulae which develop shallow and procedural kind of knowledge (Boaler, 1997).

Learning mathematics as a constructive activity enables the learner to make his/her own discoveries and this in turn form the point of departure for the teaching-learning process in mathematics teaching. Although construction of knowledge is a personal act, it is by no means an isolated activity. When learners actively construct their own mathematical knowledge, they do so while participating in a community of practice (Lave & Wenger, 1991). Situated perspectives view learning as participation in community of practice. Thus, to learn mathematics is to become an increasingly better participant in interaction with others in a social situation. There is no contradiction between construction and sharing of knowledge since both activities influence each other. When construction is made, it is worthwhile and in some cases necessary to discuss it with others. And this discussion forms the basis for new construction (Nelissen, 2006). So to learn mathematics in an interactive, communal, and a sharing process is to become a better participant in a mathematical community using community resources according to situated perspectives.

Mathematics is often seen as a school subject concerned with abstraction and formal knowledge. According to this view, mathematical abstractions should be taught by making them more concrete. The point of departure for learning mathematics is not to memorize rules and formulae, but rather working with context. A context according to Nelissen (2006) is a “situation which appeals to children and which they can recognize in

theory. This situation might be either fictional or real, and forces children to call upon the knowledge they have gained by experience" (p. 4). Realistic contexts invite the learners to use their commonsense knowledge and experience in solving mathematical problems (Verschaffel, De Corte & Borghart, 1997). However, as previous research has shown, many learners, especially middle class learners, do not pay attention to realistic considerations when constructing their responses because they were taught that everyday contexts are merely used as vehicle for introducing mathematical tasks (Cooper & Dunne, 2000). That is, rather than functioning as realistic contexts that invite or even force learners to use their experience about real world, realistic considerations are perceived as being artificial and separate from the real world. In addition, the situation where closed or procedural problems are learned, perpetuate what Whitehead (1962, in Boaler, 1997) calls inert, inflexible knowledge and tied to the context in which it was developed. By contrast, Cooper and Harries (2002) show that many learners, when given suitable encouragement, are willing and able to engage in an extended form of realistic thinking. Cooper and Harries (2003) explored learners' responses to a real-world problem. They found that learners were able to construct their own knowledge based on experience and through appropriate interventions such as modeling and peers. My study is similar to other studies with respect to the use of real-world and standard problems, but different with regard to the aim or objective in the sense that the study aims to understand more about how learners apply their mathematical knowledge to real world events in solving mathematical problems and the impact these real-world problems have on the way these learners solve mathematical problems.

One important goal in mathematics teaching is that learners will be able to apply concepts and skills they have acquired with success. Problem-solving is considered by many to be one of the important areas of application. In problem solving the contexts can be important in the sense that they elicit knowledge which learners gained through experience and again in forming problem representation. The National Curriculum Statement (DoE, 2003) supports this when it argues that "mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives" (p. 9). Teaching mathematics should relate to learners' experiences and interests by being

learner-oriented. For these reasons, learners are allowed to express their ideas freely, to make errors because they can learn by their mistakes, and develop their own approaches.

The perception that formal mathematics has little or nothing to do with everyday contexts De Corte (2000, in Chacko, 2004) is perpetuated by public examinations which foreground the solving of standard problems rather than striking a balance between real-world problems and standard problems. Chacko (2004) defines real-world problems as those that challenge the learners to go beyond an answer and involve application of learned concepts, knowledge from real-life experiences and or commonsense knowledge. Chacko defines standard problems as those that could be answered by a rehearsal of a rule, method or formula, i.e., what Cooper and Dunne (2000) termed 'non-realistic' or 'esoteric' items.

In sum, the available research provides good support for the view that effective and worthwhile mathematics learning is a constructive, situated, and collaborative process of knowledge building and skill acquisition.

2.2 Literature review

A number of researchers have investigated the use of realistic consideration in problem solving. Most show that learners have a strong tendency to neglect commonsense and realistic consideration in solving real-world mathematical problems. Chacko's (2004) exploratory study scrutinized the solutions of real-world and standard problems by a group of primary and secondary students in Zimbabwe.

The primary school students were in Grade 7 while the secondary school students were in Form 3 or 3rd year of a four-year secondary sequence. Both groups responded to the same real-world and standard problems. The study focused on the approaches used by the primary and secondary students in solving real-world and standard problems to identify similarities and differences between (a) students at the primary and secondary levels in the solution of the types of problems and (b) high and average achieving students at the secondary level. Participants were 31 Form 3 high and average students and 35 Grade 7

students. A test of 11 items made up of 7 real-world problems and 4 standard problems was used to gather data. Here are typical examples:

Real World Problem: A bus can carry 36 children and 450 children are to be transported. How many buses are needed?
Standard Problem: Bus capacity is 60 and 600 need to be transported. How many buses are needed?

Chacko argues that recent curriculum changes encourage problem solving in real-life situations and the construction of knowledge based on experience. She further asserts that, in part, recent curricula aim to develop in students mathematical dispositions that make mathematics learning meaningful by the transfer of knowledge from school to the world outside and *vice versa*, even though Lave (1998) criticizes theories of 'learning transfer' and replace them with 'situated learning' (Lave & Wenger, 1991). Chacko draws on some mathematics educators such as Gravemeijer (1994, in Chacko, 2004) who emphasize the use of word problems as real-world problems to develop mathematical concepts and skills through informal materials. Whatever the differences in opinion of mathematics educators about the real-world problems, what is apparent from Chacko's argument is the fact that she supports learning by exploration, where concepts emerge while students attempt to solve problems.

In Zimbabwe (Chacko, 2004) most of the test items in mathematics in public examination are standard problems that can be solved using a known procedure and have a single correct answer. Teaching approaches in Zimbabwe perpetuate knowledge which Whitehead (1962, in Boaler, 1997) calls inert, inflexible and tied to the context in which it was learned. Chacko's findings correlate with educational situation that prevails in Zimbabwe because her findings suggest that, irrespective of students' educational level, they solved real-world problems as if they were standard problems.

Yoshida, Verschaffel and De Corte (1997) undertook a study which was aimed at comparing Japanese and Belgian elementary school learners' lack of activation of real-

world knowledge during understanding and solving arithmetic word problems in a school context. The world problems test used in their study was taken from a study by Verschaffel, De Corte and Lasure (1994) and administered to 91 Japanese fifth graders. They were randomly assigned to one of the two conditions. 45 fifth graders were assigned to **condition I** which involved giving learners neutral instructions, i.e., not forewarning them about problems that were difficult or impossible to solve, while **condition II** involved instructions plus warning, i.e., learners were given written instruction that the test contained difficult or impossible problems to solve and they must explain why those problems are so. The test contained a series of problematic items (real-world problems) and standard problems that was administered on two consecutive days. Here is example of a real-world problem:

Real world Problem: Grandfather gives his 4 grandchildren a box containing 18 balloons, which they share equally. How many balloons does each grandchild get?

The results of the test revealed that Japanese learners, similarly to Belgian learners, have a strong tendency to neglect commonsense knowledge and realistic considerations when solving real-world problems. This happened despite the fact that Japanese learners in condition II were given extra hints which aimed at improving their disposition towards more realistic mathematical problems since their results produced only a small non-significant effect. Yoshida *et al*., (1997) argue that learners' strong tendency to approach real-world problems in a stereotyped and non-realistic way, happened even though the mathematics curriculum in Japan highly recommends teaching that focuses on solving non-routine problems and utility of everyday experiences in mathematics lessons.

These results are in line with the small effects of different kinds of scaffolds that aimed at enhancing learners' tendency to neglect real-world knowledge and realistic considerations during mathematical modeling of word problems obtained in studies with Belgian learners (Verschaffel & De Corte, 1997) and Swiss children (Reusser & Stebler, 1997). From this study (Yoshida *et al*., (1997), it is apparent that learners disregard real-world contexts when solving real-world mathematical problems.

Another research study was conducted by Verschaffel, De Corte and Borghart (1997) which investigated pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modeling of school word problems. A test consisting of 14 word problems- half of which were real-world mathematical problems and the other half were standard problems, was administered to 332 pre-service teachers from different elementary school teacher training institutions in Flanders. For each word problem, the student-teachers were first asked to solve the problem themselves, and afterwards to evaluate four different answers given by learners. Here is an example of problems given:

Real world Problem: Steve has bought 4 planks of 2,5 metres each. How many planks of 1 metre can he get out of these planks?

The results revealed that there was a strong tendency among student-teachers to exclude real-world knowledge from their own spontaneous solutions of school word problems as well as from their appreciations of the learners' answers. The results show that not only do learners experience problems in dealing with real-world mathematical problems but (student-) teachers as well. In recent years there has been an increasing emphasis on making problem solving in school mathematics more closely related to learners' real-life experiences. However, Verschaffel *et al* (1997) argue that in order to relate mathematics to learners' real-life contexts, teachers have to be stimulated or helped to construct proper concepts, skills and beliefs that are needed for realistic modeling of problem situations.

Verschaffel and De Corte (1997) undertook a study, and in this study, they tested the hypothesis that it was possible to develop in 10- and 11- year old elementary school learners a disposition toward more realistic mathematical problem solving by immersing them into a classroom culture that focused on the importance of real-world knowledge and sense making in the modeling and interpretation of the solution of word problems. Three classes from the same school participated in the teaching experiment, one experimental class (E) and two control classes (C1 and C2).

The program consisted of five teaching/learning units of about 2½ hours each, spread over a period of about 2½ weeks. During the experiment, the learners from C1 and C2 classes continue to follow the regular mathematics curriculum. In the E class, an experimental program on realistic modeling was taught during the hours allocated for mathematics. The three classes were given the same pretest and posttest, which measured learners' disposition to realistic mathematical modeling and interpreting. Here is an example of a problem given:

Real world Problem: 300 soldiers must be transported by jeep to their training site. Each jeep can hold 8 soldiers. How many jeeps are needed?

As expected, learners' tendency to exclude real-world knowledge was again very strong during pretest. However, as hypothesized, there was a significant increase in the number of realistic reactions from pretest to posttest in the E class (from 7% to 51%), but not in the two C-classes. The positive results in the E class showed that the effect of the experimental program did not disappear after training had stopped, they even provided some evidence that it was possible to develop in learners an inclination and ability toward realistic modeling of word problems.

Cooper and Harries (2002) analysed responses of 11-12 year-old English children's responses to two realistic mathematics problems, i.e., the original lift problem and the revised lift problem, and these children were at the end of their first year of secondary schooling. The first is taken from English national tests, the second is a revised version of this item which has been rewritten to encourage a wider range of realistic responses. And through the comparison of responses to the two items it is suggested that many children, given suitable encouragement, are willing and able to enter into extended form of realistic thinking during problem solving, although the original test item renders this invisible. In my study I adopted the second item used in Cooper and Harries (2002) with the intention of encouraging a more realistic pattern of responses and in so doing to explore whether and how they do it.

In conclusion, what is apparent from most of the studies is the fact that learners seem to exclude real-life considerations when solving problems based in real-life contexts. However, what must be emphasized here is that researched items according to Cooper and Harries (2002) have the potential for the learner's response to include realistic considerations that might characterise problem-solving outside the world of the classroom. In addition, in Cooper and Harries (2002) it is shown that with encouragement a more realistic pattern of responses are likely to be revealed. In my study, I encouraged learners to consider a wider range of realistic considerations when responding to the problems posed in real-life contexts in the similar way as portrayed in the revised lift problem (Cooper & Harries, 2002). For instance, learners were presented with three competing answers on an item given by a fictitious peer. These learners were asked to see a way that the given answer could be correct by choosing "yes" or "no" and by explaining why they answered "yes" or "no". In the key literature discussed above, something is missing. That is, the influence which real-life contexts has in solving mathematical problems. In other words, the study explored the manner and extent to which real-world contexts influence learners' responses in solving mathematics items. Hence I decided on this study.

CHAPTER 3: METHODOLOGY

3.1 Introduction

In this chapter I detail the steps taken to ensure that the study was trustworthy in terms of validity and reliability. Secondly, I explain how ethical issues that arose in the study were addressed. Thirdly, I make explicit the design of the study.

3.2 Research approach

This research is informed by the constructivist approach which focuses on understanding how learners apply their mathematical knowledge in solving mathematical problems that embed real-world problems. In addition, the impact these real-world problems have on the way these learners solve mathematical problems is explored.

Creswell (1994: 2) in Leedy (1997) defines a qualitative study as “inquiry process of understanding a social or human problem, based on building a complex, holistic picture, formed with words, reporting detailed views of informants and conducted in a natural setting”. He defines quantitative study as “an inquiry into a social or human problem, based on testing a theory composed of variables, measured with numbers and analysed with statistical procedures, in order to determine whether the predictive generalizations of the theory hold true”.

As this study aims to be exploratory and interpretative in nature, i.e., seek both verbal (interview comments) and non-verbal (response categories tables) explanations and interpretations, and as such falls within both qualitative and quantitative approaches. In particular the overall design of the study was that of a case study. Leedy (1997) points out the essential features of both approaches: quantitative approach relies on deductive form of analysis (reduction of data to numbers which then is presented as results) while qualitative uses inductive form of analysis (construction of narratives from data and employ a literary style)

In this research the participants of the study were learners from one of the grade 8 classes. These learners wrote a mathematical task that consisted of three real world problems in which the mathematical modelling assumptions were not obvious and three standard problems as buffer items (see Appendix 1). With respect to each problem, learners were asked to make a choice of 'yes' or 'no' responses and to justify their choice by providing written comments.

In the real world problems, learners were confronted with the three answers given by the fictitious peers. I hypothesized that the presentation of these competing answers might cause some cognitive conflict and as such lead a learner to reflect on the features of the represented situation. Secondly, I also hypothesized that the introduction of learners' names in the mathematical task might influence the nature of their responses, possibly encouraging learners to relate more to the task by responding realistically. It had been found in earlier work that learners responding to items containing names of learners and aspects of everyday life experiences, influence nature of responses (Cooper & Dunne, 2000a). Thirdly, learners were verbally warned before writing the mathematical task that some of the questions would require their honest opinion thereby 'putting themselves in the situation' in order to answer such questions.

3.3 Validity and reliability

The mathematical task was made up of three real-world problems and three standard problems. All real-world problems have parallel standard problems and both types of problems needed knowledge of basic arithmetic operations to solve. All the real-world problems were adapted from Cooper & Harris (2002) and Reusser and Stebler (1997) while two of the standard problems were selected and adapted from Reusser and Stebler (1997) and only one was developed by the researcher.

I asked some grade 8 mathematics educators in the school to scrutinize a draft of a mathematical task and to comment on the language and the level content. In their comments, they emphasized that the language would be understood and that the content

level was easy for the grade 8 learners. They further said that even though the content was easy, the problem could be the contextualizing of mathematical problems. The mathematical task was piloted in the same school but with different grade 8 learners (not final participants) in order to ensure validity. To ensure reliability of scores, both the researcher and a colleague were jointly involved in the coding of learners' responses.

With regard to qualitative data, the interviews were used to gather the data. A major advantage of interview according to Bell (1993) is its adaptability in the sense that an interviewer can follow up ideas, probe responses and investigate motives. The process of interviewing gives rise to a type of interaction that cannot be completely neutral. In other words, interviews are subjective in nature and therefore always present the possibility of bias. In an attempt to deal with the danger of bias creeping in to interviews, I tried to reduce bias by rehearsing the interview with a colleague who was briefed beforehand about the research so that she could give feedback on the interview process. With regard to interview transcripts, and particularly statements that were used as direct quotation in the report, these were verified with interviewees first. Managing subjectivity in general, the research is accompanied by accounts of how it was actually done (Holliday, 2002).

3.4 Ethical considerations

Sieber (1992, in Opie, 2004) defines ethics as "the application of moral principles to prevent harming or wrongdoing others, to promote the good, to be respectful and to be fair" (p. 25). This definition makes it clear that the researcher must take all possible precautions to avoid any moral wrong to anyone involved in the research by showing dignity, honesty and confidentiality. As a researcher, I realized that conducting the research in the school wherein learners will be involved, the first priority was to seek validation from the participants and the authorities.

To ensure that my research was ethical, I made sure that official channels have been cleared. That is, I sought permission from the University of the Witwatersrand to conduct this study as a Masters student of the university and also approached Gauteng

Department of Education to request permission to conduct research at one of its schools. A proposal of the research was submitted to both the university and the offices of Gauteng Department of Education. The permission was granted from both institutions. I also sent letter of consent to my principal (the study was conducted at the secondary school where I taught since I had access to Grade 8 learners and it would be convenient for my research) and explained to him the purpose of the study. I explained to him that the name of the school and the names of participants will be protected. Here also the permission was granted.

The study involved one class of Grade 8 learners. At my school I am teaching only the Grades 10 and 12 and that meant that I needed to negotiate access with the Grade 8 educator and the learners. I made an appointment to meet with the educator and explained the purpose of the study and requested her participation. The educator was given a letter of consent to indicate her agreement in writing to participate in the study. The letter also assured anonymity and confidentiality. The permission was later on granted.

Learners were briefed on the study and were given two letters of consent, i.e., for the learner and parent(s)/guardian(s), which invited them to participate in the research project. The letters sought the permission of learners and their parent(s)/guardian(s). They detailed all the information about the purpose of the study. Parent(s)/guardian(s) who sought more clarity were invited to the school to discuss in detail what the research was all about. In addition to that, learners were given opportunity to agree or disagree to participate and were guaranteed anonymity. They were made aware that they were going to write a mathematical task, and that the task was not going to be used for school promotional marks but for research purposes. Learner's permission was also sought to tape- record the interviews, so that I could make an accurate record of what was said. Thereafter the learners together with their parent(s)/guardian(s) were asked to sign a consent form agreeing to their children's participation in the study. The permission was granted from parent(s)/guardian(s) and learners.

3.5 Design of study

3.5.1 Context of the study

The research was qualitative as well as quantitative in nature. That is, the research questions required learners to give verbal as well as written responses in order to explore their understanding with regard to the role that contexts play in solving mathematical problems. In addition, learners were presented with a mathematical task to answer individually and their responses were examined. Two learners were selected for the interview in order to better understand their line of reasoning with regard to their written task. The two approaches were used purposefully in the study in order to provide rich material through feedback from respondents.

In particular, the overall design of the study was a case study. Opie (2004) describes a case study as "an in-depth study of interactions of a single instance in an enclosed system" (p. 74). He also points out the essential features of a case study: data collection is systematic; the focus is on a real situation with real people; the researcher is familiar with the environment; provides a picture of an activity in a particular setting. A case study design was an appropriate research approach in this study as it allowed the researcher to explore learners' responses in solving mathematical problems.

3.5.1.1 The school and subjects

The research was conducted at one township secondary school just outside of Johannesburg. The school serves only black African learners. I chose to conduct my research at this particular school because of the familiarity with the setting. The sample was easily accessible to me. I have been teaching mathematics for 15 years in that particular school. The school has a population of 1100 learners and 31 educators of which one is the principal, two deputy principals and five head of departments (HODs). At the time of the study, the educators at the school were all qualified and of which 45% had post graduate qualification. In the mathematics department there were five educators, two of whom have a post graduate mathematics education qualification. One educator who

had post graduate mathematics education qualification was the one whom the researcher asked to participate in the research project.

The learner participants in the study were grade 8 learners who were in their first year in the secondary school. They all come from black communities and do not have English as a home language but attend an English medium school which serves learners mostly from poor socio-economic backgrounds. The school is situated in near a squatter camp (informal settlement) and is under-resourced in terms of, learning support materials, furniture, laboratories, and library resource centre. At the time of the study, there was no computer centre.

3.5.2 The sample

The study involved one class of Grade 8 learners that were of mixed ability, i.e., learners in all grade 8 classes were not grouped according to their mathematical abilities, and so were the participants. The mathematical task was given to the class which has fifty learners (13 years old and above). I then selected a sample of 6 learners whose mathematical tasks were used for analysis and of which two took part in the interviews. The selection comprised of three boys and three girls. The criteria for the selection of these learners were based on the following: they must have completed most of written tasks; provided insightful explanations of their answers; learners who showed an understanding of core mathematical practices such as justifying, making connections and communicating mathematical ideas (Rand Mathematics Study Panel, 2002). For instance, participants were able to justify their choice of ‘yes’ / ‘no’ responses through written comments and also they were able make connections by comparing their responses to those given in the task.

3.5.3 Piloting

The piloting process in this study involved both written task as well as interviews. That is, learners whose mathematical tasks were used for analysis, some were also selected for

the interviews. Mathematical task was piloted in the same school but with different grade 8 class (not participants) which was also of a mixed ability. The purpose of conducting the pilot was, firstly, that I needed to see how much time I would need to allocate in order for learners to complete the tasks and also the interview. Secondly, to establish whether the learners were able to write their responses as requested and whether they experienced any possible difficulties during the solution of the problems. Thirdly, the motivation for the piloting was to establish any unclear, ambiguous, and confusing questions during the interviews. The pilot was conducted in the second week of August 2006 after school hours. Participants took one hour to complete the mathematical task. The interview took place two days after writing the task. In addition, the lapse of two days was to allow the researcher enough time to find the "prospective interviewee" who satisfied the purpose (see analysis of data from the pilot study).

3.5.4 Administration of instruments

The researcher and the educator colleague (the one who agreed to be part of this research) were responsible for the administration of the instruments (written mathematical task and interviews). The instruments were thoroughly prepared beforehand, i.e., typed, stapled, packed; organization of suitable venues (classroom for writing and HOD's office for interview); organizing a voice digital recorder as well as calculators for learners. The preparations were done one week prior to its implementation to minimize any unprecedented disruption. The administration of instruments was carried out in September 2006. The participants were asked to write a task involving six questions and that had to be answered individually without any intervention from the educator or fellow learners. The aim of these questions was to explore whether and how the use of real-life contexts influences learners' responses in solving mathematical problems. The participants were allowed 1 hour 15 minutes to complete the written task. The interviews took place two days after writing the task.

3.5.5 Data collection

The instruments that were used for data collection as mentioned above were: a written mathematical task (see Appendix 1) as well as interview (see Appendix 3, preliminary interview questions). The interview was conducted once permission from the selected learners and their parent(s)/guardian(s) had been granted. The type of interview used was semi-structured in the sense that the research questions allowed open-ended responses and that gave rise to the type of interview that was less controlled by the interviewer and not completely predetermined (Opie, 2004).

Interviews are subjective in nature and therefore always present the possibility of bias. A preliminary interview was held to reduce bias. I rehearsed the interview with a colleague (grade 8 mathematics educator) who was briefed before about the research so that she could give feedback on the interview process. The two of us, that is, the researcher and the educator, jointly completed Problem 1 (see Appendix 2) wherein we wrote possible solutions to the problem. And subsequently an interview was conducted using preliminary interview questions (see Appendix 3). The feedback I got was that there was a high possibility that respondents would base most of their responses to what they got from using a calculator. In other words, they could stick to mathematical solutions and disregard real world solutions. In addition, participants would not be aware that, for instance, in Problem 1, the lift can still go up even if there are fewer people than the lift capacity.

The interviews took place after school in the HOD's office because it was a safe place with regard to unnecessary interruptions. My colleague also assisted during the interview process by ensuring that while the interview was underway we were not disturbed. A digital voice recorder was used to facilitate the transcription processes. Learners' permission was sought beforehand to tape- record the interviews, and permission was granted. Before the interview process started, learners were assured that their anonymity and confidentiality were protected.

The first interview was carried out with one learner from the pilot group. The second interview was with two learners from main study. The criteria for the selection of those learners were based on the fact that they showed some realistic considerations when solving the mathematical task. Moreover, they gave interesting responses which needed to be inquired more about. This process also afforded learners an opportunity to display their "insightful reasoning and interpretive and communicative skills when dealing with mathematical and contextualized problems" according to learning area statement for mathematics (DoE, 2002: 5)

I sought permission from the learners and their parent(s)/guardian(s) to record the interviews, to which they agreed. Learners were interviewed in separate days to allow the researcher enough time to go through the recorded information so as to establish whether questions intended to be asked were indeed asked or not and to check as to whether recorded interviews were audible enough for transcription processes.

Data collection was from a written mathematical task, interviews (from pilot group, research group, and grade 8 mathematics educator), written learners' perspectives as well as the educator's perspectives. For the analyses purposes, the focus was on interview transcripts and learners' written tasks.

3.5.6 Data analysis

For this study, the analysis of the data was undertaken in two phases. In the first phase, I analyzed quantitative data collected through solutions to written mathematical tasks. This analysis included learners' responses, computations and comments that were coded into the following response categories: *realistic answer* (RA), *non-realistic answer* (NA), *technical error* (TE), and *other answer* (OA) .The description of response categories will be delt with in Chapter 4. A table of response categories and frequency of categories was used to analyze learners' responses. The analysis of the frequency of categories was not enough to conclude whether and how the use of real-life contexts influences the way learners solved mathematical problems. In addition, extracts of learners' written

explanations on their solutions of given real-world problems were also analyzed in order to support and validate categorised responses. The aim in the analysis was to establish whether realistic considerations were taken into account in solving these problems or whether learners were able to move beyond constraints associated with standard problems to *recognize* the problems as contexts in which they can apply everyday reasoning (Bernstein, 1996).

In the second phase, I analyzed qualitative data collected through texts in the form of transcripts from audio recording of interviews with learners (from pilot and main study) and grade 8 educator. The aim of interviews was to get insight into learners' written comments. Hatch (2002) argues that inductive analysis begins with an examination of the particulars within data, followed by looking for patterns across individual observations, then arguing for those patterns as having the status of general explanatory statements. For this study, I used inductive reasoning in analysing extracts of learners' written and spoken comments, and that included, for instance, learners' perspectives as well as the educator's perspectives. In arguing for "particulars within data" as having the status of general explanatory statements, I used literature, theoretical orientation, personal experience and views.

CHAPTER 4: RESEARCH PROCESSES – DATA COLLECTION AND ANALYSIS

4.1 Piloting

The instrument was piloted with a different grade 8 class (not participants) which was of a mixed ability. The purpose of conducting the pilot was, firstly, that I needed to see how much time I would need to allocate for learners to complete the tasks and also the interview. Secondly, I needed to establish whether the learners were able to write their responses as requested and whether they experienced any possible difficulties during the solution of the problems. Thirdly, the motivation for the piloting was to establish any unclear, ambiguous, and confusing questions during interviews. The pilot was conducted in the second week of August 2006 after school hours. The learners made themselves available for one hour session in which the pilot was completed.

From the piloting process, two kinds of data were obtained: (a) a written mathematical task, and (b) an interview. That is, a learner whose mathematical task was used for analysis, was also selected for an interview. The piloting was administered by the researcher and the educator colleague (the one who agreed to be part of this research), and instructions were kept to a minimum. That is, learners were just given basic instructions such as motivating their choice of ‘yes’ or ‘no’ responses, but never forewarned about nature of the mathematical problems. Once the testing started, learners were not allowed to seek any intervention from the educator or fellow learners.

4.2 Main study

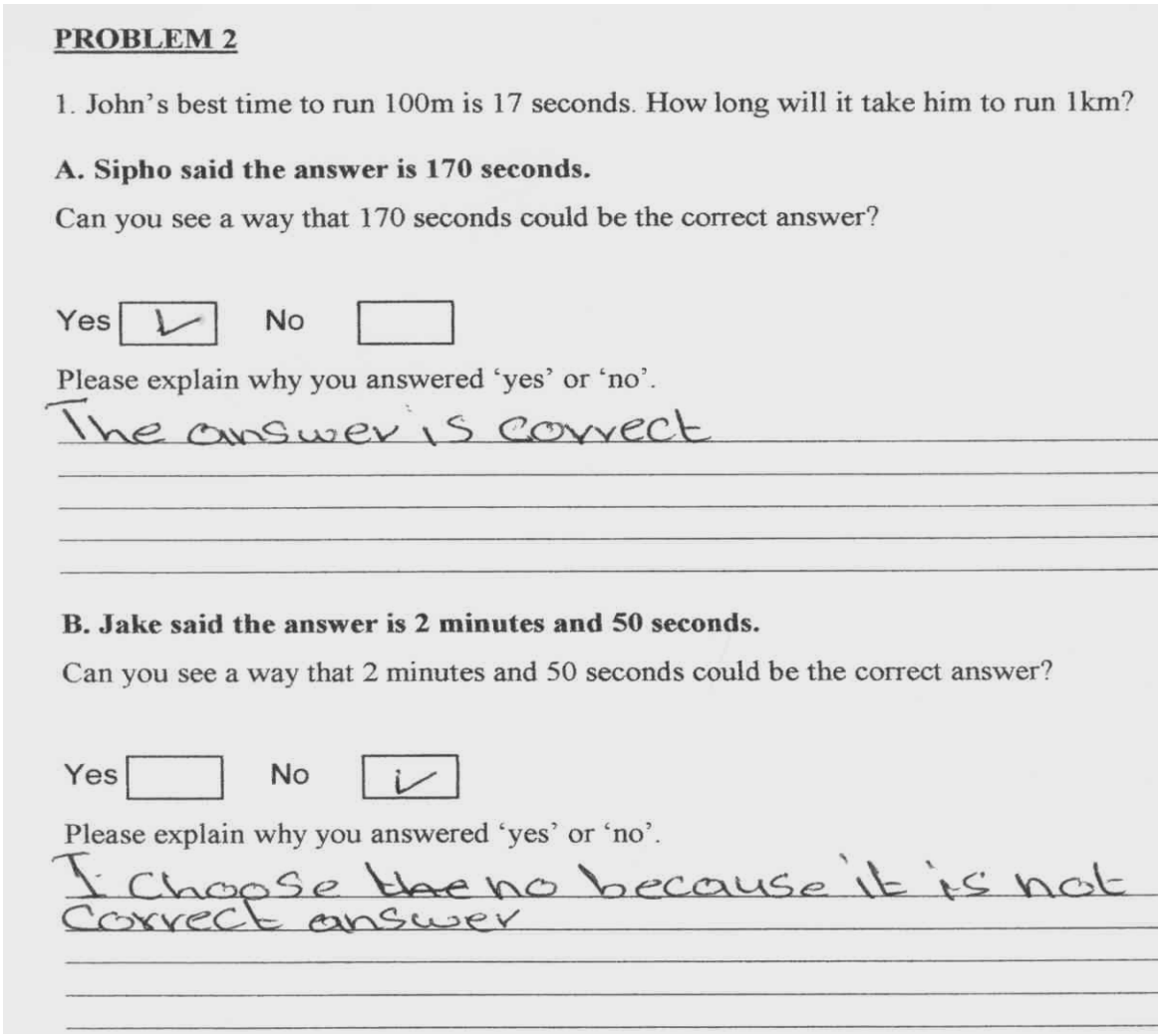
The researcher and the educator colleague were responsible for conducting written mathematical task and interviews. The study involved one class of Grade 8 learners that were also of mixed ability. The participants were asked to write a task involving six questions to be answered individually. They were allowed 1 hour 15 minutes to complete the written task. A written task was preceded by a 15 minutes introduction in which learners’ attention was drawn to the fact that the questions would require their honest

considerations. Learners were also asked to follow instructions, i.e., write their responses as requested. These were done in order to avoid problems encountered during the pilot. I went through each problem with them, asking questions on the context of the problem such as, do they know a lift? Do they know how big 1km is? At the end of each problem, I would ask them time and again whether they understood what they were supposed to do.

Learners were provided with the calculators seeing that there were no restrictions with regard to their use. Learners were given strict instructions that they were not allowed to discuss and/or help each other once the writing of the task has started. Thirty five written responses to the task were returned, 6 of which were used for analysis and two took part in the interviews. Some learners from that class abstained, saying that they were not interested in the research project.

4.2.1 The written mathematical task

Majority of participants who wrote the mathematical task 'failed' to adequately answer the questions. That is, some made their choice of 'yes' or 'no' responses but failed to adequately motivate their responses. For instance, one learner chose 'yes' and substantiate her response by just saying "The answer is correct", and the other would choose 'no' and motivate by saying "I don't agree with him" (see sample below). In this way, I struggled to find learners who were 'positively' able to motivate their responses. The way in which learners responded to questions said a lot about their classroom practice. For instance, it shows that learners are not used to motivate their responses particularly in mathematics classes. Secondly, it indicates that writing in mathematics classroom is limited to computational fluency rather than learning to argue by to putting logical ideas on paper.



PROBLEM 2

1. John's best time to run 100m is 17 seconds. How long will it take him to run 1km?

A. Sipho said the answer is 170 seconds.

Can you see a way that 170 seconds could be the correct answer?

Yes [✓] No [ ]

Please explain why you answered 'yes' or 'no'.

The answer is correct

B. Jake said the answer is 2 minutes and 50 seconds.

Can you see a way that 2 minutes and 50 seconds could be the correct answer?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

I choose the no because it is not correct answer

4.2.2 Interviews

I sought permission from the learners, i.e., L1 and L4, and their parent(s)/guardian(s) to record the interviews, to which they agreed. These learners were interviewed because some of their written responses showed that realistic situations were taken into consideration when real-world problems were solved. So in this way learners were asked to elaborate on how they solved those real-world problems.

The interview transcription process was carried out as follows: I first listened to the whole recording to get an overview of the interview and then I transcribed the entire

interview. I denoted parts of the interview with ellipse (…) to indicate a brief pause for a few seconds or less. In addition, I used square brackets to denote translation.

To analyse interview data, I specifically looked at those spoken comments wherein a learner demonstrated whether or not contexts influenced the way in which the learner solved mathematical problems. That included quoting what the learner said with reference to their written comments.

4.3 Analysis

In most of mathematical modeling of school word problems, analysis has been qualitative and/or quantitative, depending on the kinds of data obtained (Chacko, 2004; Cooper & Harris, 2002; Wyndhamn & Saljö, 1997; Cooper & Dunne, 2000; Verschaffel, De Corte & Borghart, 1997; Reusser & Stebler, 1997). That is, the presence of written comments and/or computations with respect to answers given by learners were analysed using these approaches. These written tasks are usually supplemented by reflective interviews with the aim of probing learners' understanding. In this study, as in Verschaffel, De Corte and Borghart's (1997) study, learners' responses, computations and comments were coded into four categories. According to Verschaffel, De Corte and Borghart, they called the first category *realistic answer* (RA). According to this category, these are the properties:

> A *realistic answer* is based on "effective use relevant real-world knowledge" (p. 345). In addition, this involves critical use of mathematical rules about context elicited by the problem statement. Example of RA for Problem 1(1) is 20 because lifts do not make fractional numbers of trips.
>
> The second category is called *non-realistic answer* (NA). In this category, the following are the properties: The NA results from the direct use of mathematical rule and uncritical application of the arithmetic operation without regard to reality constraints. For example, the NA for Problem 2(1) is 170 seconds or 2 minutes and 50 seconds, i.e., product of 10 times 17seconds, disregarding the fact that one gets increasingly tired while running long distances.

The third category is called *technical error* (TE). This category is characterized by the following: The TE solution structure like NA, but differs because of technical mistake or inaccuracy in the execution of the arithmetic operations. *Example*: In Problem 2(1), "John's best time to run 100m is 17 seconds. How long will it take him to run 1 km?". The TE is 1700 seconds, i.e., the product of 100m and 17 seconds.

The last category is called the *other answer* (OA): This category represents all solutions that cannot be classified into one of the other categories.

In this study, learners' responses and their elaborations of these in the interviews were analysed using the above mentioned categories. These allowed the researcher to gain insights into various dimensions of understanding which learners may have developed in their learning of mathematics. In addition, learners' responses also provided insights into understanding of whether and how the use of real-life contexts influences their responses in solving mathematical problems.

Table 1 below shows possible solutions to real-world problems. These solutions are divided into two categories.

Table 1: Real-world problems: Expected Realistic and Non-Realistic answers

Realistic Answer (RA)	**Non-Realistic Answer (NA)**
Problem 1: B: 20 times, because lift does not make fractional trips; C: 25 times, if the lift carries more than 14 people at once.	$\frac{269}{14} = 19{,}21429$
Problem 2: C: Impossible, because cannot be known of fatigue of the runner	170 seconds or 2 minus and 50 seconds, i.e., product of 10 times 17seconds
Problem 3: A or B, if month is known; C: cannot be known precisely, because month is not given.	1995 - 1978 = 17 years

Table 2 below displays solutions to the three standard problems. Because these are straightforward applications of mathematical rules, no categories were used.

Table 2: Standard Problems: Answers

Standard Problem
Problem 1: $\frac{238}{16} = 14{,}875$ *or* 15 times
Problem 2: $\frac{180}{45} =$ 4 hours
Problem 3: 60 + 90 = 150 apples

Table 3 below shows responses category of 6 learners who wrote a mathematical task. The task consisted of Problems 1(1); 1(2); 2(1); 2(2); 3(1); and 3(2).

Table 3: Response categories of participants:

(L1 = Learner 1; L2 = Learner 2; L3 = Learner 3 etc.)

PROBLEM NUMBER		**RESPONSE CATEGORY & FREQUENCY**			
		RA	**NA**	**TE**	**OA**
1(1)	**A**	0	L2 ;L3; L5; L6	0	L1; L4
	B	L1; L4	L2; L3; L5; L6	0	0
	C	L1; L4	L2; L3; L5; L6	0	0
1(2)		L1; L4	L2 ;L3; L5; L6	0	0
2(1)	**A**	0	L1; L2; L5; L6	L3	L4
	B	0	L1; L4	0	L2;L3; L5; L6
	C	0	L1;L2;L6	0	L3; L4; L5
2(2)		0	L1;L2;L3;L5;L6	0	L4
3(1)	**A**	0	L1;L2;L3;L4;L5;L6	0	0
	B	0	L1;L2;L3;L4;L5;L6	0	0
	C	0	L1;L2;L3;L4;L5;L6	0	0
3(2)		0	L1;L2;L3;L4;L5;L6	0	0

Table 4 below shows a summary analysis of responses category of participants. The task consisted of 3 real-world problems [1(1); 2(1); 3(1)] and 3 standard problems [1(2); 2(2); 3(2)].

Table 4: Summary of response categories:

PROBLEM NUMBER		**RESPONSE CATEGORY & FREQUENCY**			
		RA	**NA**	**TE**	**OA**
1(1)	**A**	0	4	0	2
	B	2	4	0	0
	C	2	4	0	0
1(2)		2	4	0	0
2(1)	**A**	0	4	1	1
	B	0	2	0	4
	C	0	3	0	3
2(2)		0	5	0	1
3(1)	**A**	0	6	0	0
	B	0	6	0	0
	C	0	6	0	0
3(2)		0	6	0	0

What do I see from the tables? (What do the tables "tell" me?)

As can be seen, response categories are divided into the following types: a *realistic answer* (RA), a *non-realistic answer* (NA), a *technical error* (TE), and the *other answer* (OA). Response categories of participants through Problem 1, 2 and 3 are divided into two tables wherein Table 3 shows each learner in a specific category. Table 4 shows a summary of response categories. In each Problem, i.e., Problem 1(1), 2(1) and 3(1), A, B, and C, represent real-world problems. Therefore, Problem 1(2), 2(2) and 3(2) as mentioned above, represent standard problems. Consequently, there are more real-world problems than standard problems.

What can be seen in Table 4 is that the majority of learners responses were categorized into two categories, and these are NA and OA. For all these learners who took part in writing the mathematical task, only two responded realistically to Problem 1(1)B, 1(1)C and 1(2). Learners responded 'correctly' in the mathematical sense to real-world problems as reflected by a high number of responses that were categorised in NA. In contrast, learners responded 'incorrectly' in the mathematical sense as observed by a high number of responses that were categorised in OA. It is also observed that two categories RA and TE have very few responses.

What do I think about what I have seen from the tables?

In the above analysis, the tables show the following:

- Categories that have high and low frequencies.
- RA is one category that has low frequencies, because L1 and L4 are the only ones whose responses were categorised into it. In other words, even though learners were given more real-world problems, that in itself did not trigger more realistic responses.
- NA is the category that has high frequencies, because most of learners' responses were categorized into it. This could suggest that the possibility of getting more responses being into NA is higher than when given realistic problems because learners are familiar with standard ways of responding to standard problems.
- In all standard problems, there is only one problem that was responded to realistically. Now the question is, why did standard problems not be responded to realistically seeing that most of standard problems were not categorized into RA?

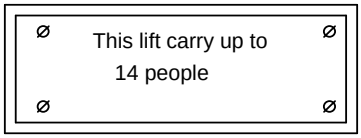
I think it is import to understand why even though learners were given realistic problems, it turned out that that did not trigger them to respond realistically. The question is why did this happen? Some of the reasons could be:

- If you give learners real-world problems and these learners are unfamiliar with this type of problems it means here that they will possibly treat such problems as standard problems because of their familiarity with standard ways of responding to standard problems.
- Learners often solve word problems by focusing on syntax of mathematics rather than the meaning of events as described (Wyndhamn & Saljö, 1997). For further discussions see "What the literature says about what I said above".
- Classroom culture that perpetuates procedural and inert forms of knowledge that cannot be used in a variety of different situations (Boaler, 1999). This is further discussed below.
- Social class differences on responding to items (Cooper & Dunne, 2000). This is further discussed below.

Evidence in relation to the fact that learners responded non-realistically to real-world problems, is seen in Learner 2 (L2)'s response below. Here L2 solves the problem by simply applying arithmetic operations without regard to reality constraints that may be taken into consideration.

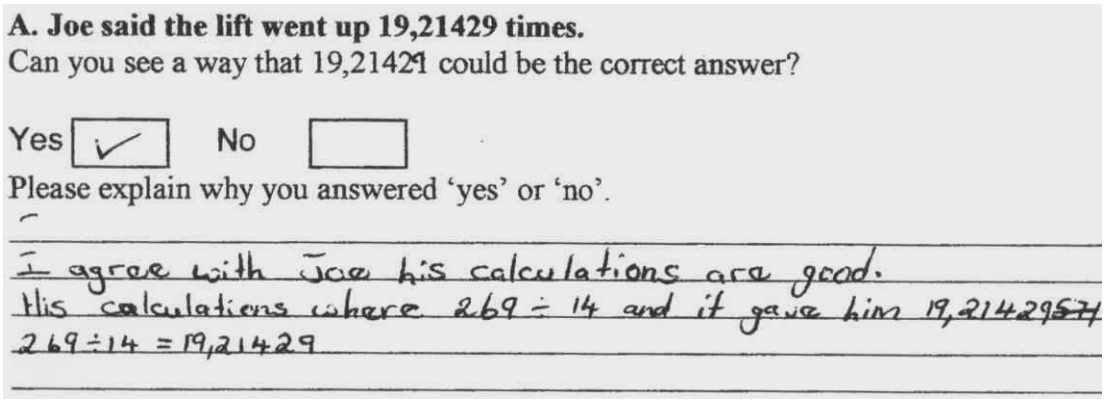
PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

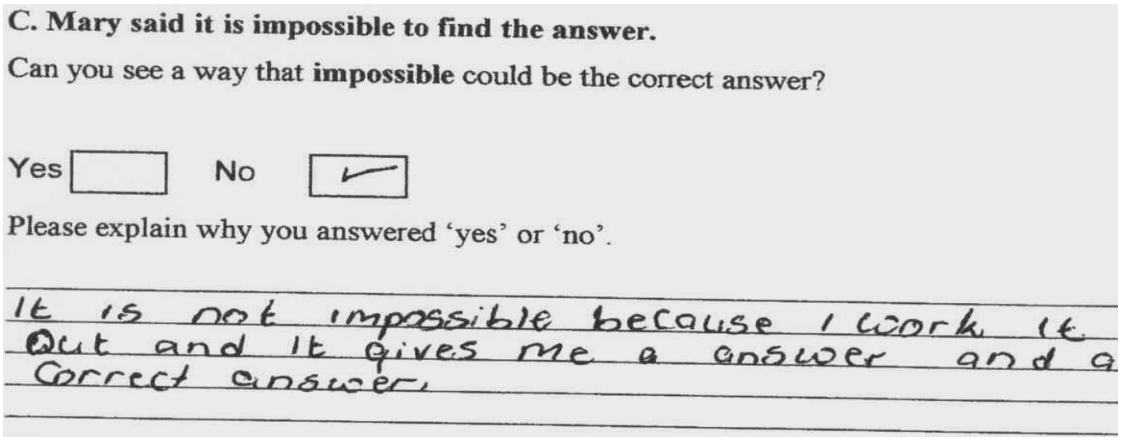


As can be seen, the learner was able to 'correctly' respond to the problem (in mathematical sense) by dividing 269 by 14. However, the learner did so without the consideration of realistic situations. That is, the learner was not conscious of realistic situations seeing that he did agree with the solution that said the lift went up fractional times, which is an acceptable solution to standard problem. This kind of response could partly be attributed to learners' background because their background is influencing the way they are responding since most of our learners come from working-class families (Cooper & Dunne, 2000). Cooper and Dunne (2000) have shown that there is a positive correlation between children's choice of 'realistic' or 'esoteric' approach and social class. Here it is important to discuss whether the lack of consideration for the realistic situations is deliberate or conscious or not deliberate or unconscious. This issue will be discussed further in the discussion section.

The majority of learners' responses demonstrated their tendency to exclude real-world knowledge when solving real-world problems and again they consider each and every mathematical problem to be having a solution, i.e., they do not agree that there could be an impossible answer as it can be seen in L6's response:

PROBLEM 2

1. John's best time to run 100m is 17 seconds. How long will it take him to run 1km?


C. Mary said it is impossible to find the answer.

Can you see a way that **impossible** could be the correct answer?

Yes ☐ No ☑

Please explain why you answered 'yes' or 'no'.

It is not impossible because I work it out and it gives me a answer and a correct answer.

Empirical support for this claim has been reported by Schoenfeld (1988, in De Corte, 2000) who found that learners have debilitating beliefs about mathematics, such as "mathematics problems have one and only one right answer, formal mathematics has little or nothing to do with real thinking and problem solving; the mathematics learned in school has little or nothing to do with real-world" (p.36). Similar perceptions were echoed by Boaler (1999) in her study wherein learners who were exposed to 'drill and practice' model could not adapt what they had learned to new situations. The question that arises is why do learners have such perceptions? This is an important issue that needs to be explored in order to provide insights into an understanding of teaching strategies.

As mentioned above that majority of learners' responses were categorized into two categories, and these are NA and OA. This could be influenced by factors such as learners not skilled at using real-world knowledge; learners' 'inability' to understand or relate to the problems; learners' 'superficial' mathematical knowledge, for instance, their inability to convert metres to kilometres and lack of understanding between whole numbers and decimal fractions. This issue will be dealt with in detail in chapter 6.

In general, there were very few inaccuracies in the execution of the arithmetic operations. As a result, only one response could be categorized into TE and this could be attributed to learners' procedural fluency (Kilpatrick, Swafford, & Findell, 2001) and the fact that they were allowed to use calculators.

What the literature says about what I have said above

The aim of putting more real-world problems in this study was to focus on ability and /or williness of learners to introduce a range of realistic consideration, to understand more about how learners apply their mathematical knowledge to real world events in solving mathematical problems and how these learners were able to relate to the meaning or "sense" (in Vygotskian terminology, Vygotsky, 1986) of the mathematical task.

The question that one may ask is why did most of learners not respond realistically to real-world problems seeing that most of the problems demanded realistic responses? Research shows that learners often solve word problems by focusing on the syntax of mathematics - the rule based relationship between mathematics symbols – rather than reflecting on the specific context as described (Reusser & Stebler, 1997; Cooper & Harris, 2002; Chacko, 2004; Cooper & Dunne, 2000). Verschaffel, De Corte & Borghart, (1997) argue that considerable experience with school arithmetic word problems develops in learners a strong tendency to exclude real-world knowledge.

The same position is confirmed by Wyndhamn and Saljö (1997) when they assert that in the paper-and-pencil world of the school, learners learn to identify word problems as mathematical exercises in which an algorithm is hidden and is suppose to be identified. Thus, learners learn that relying on common sense knowledge and making realistic consideration is harmful rather helpful in arriving at the correct answer. In contrast, according to Wyndhamn and Saljö (1997), learners should be taught that "a word problem in mathematics teaching refers to an authentic or possible situation in everyday context" (p. 265).

When learners are faced with a mathematical problem that requires them to make links to everyday contexts, they seem quite willing to accept 'unreasonable' and 'inappropriate' solutions to the problem. Because they are taught that contexts used in mathematics tasks are fantasy world in which common sense needs to be suspended for them to get the right answer (Elwood, 1998). This is confirmed in Sethole (2004) where an educator (Kevin)

consciously recruits pseudo-realistic task that foregrounds mathematical procedures over the acquisition of social aspects.

In addition, the crucial point here is that, in the sense of Bernstein's (1996) recognition rules for reading the specialized nature of contexts, most of the learners in this study were not able to 'recognize' the intended specialized problem-solving contexts. The fact that learners responded non-realistically to real-world problems, i.e., the majority of responses were categorized into NA instead of RA, showed that they have necessary mathematical skills but not skilled in using real-world knowledge in the classroom.

In this respect, it is important to stress that learning mathematics as a social construction and collaborative process of knowledge and skill acquisition with the environment can help learners, based on their involvement and experience in the modeling of authentic real-life contexts. For instance, L2 solved Problem 1A without basing his skills on involvement and experience in the modeling of authentic real-life situations and as such was unable to see that a lift cannot go up fractional times. However, meaningful learning activities based on modeling reality should also appeal to learners and create a classroom climate that induces learners' reflection on their problem-solving skills and strategies.

Learners in the study, including L2 and L6, were allowed to experience various perspectives thereby presenting them with four competing answers given by fictitious peers with the aim of showing them that there are other learners with other ideas about how to solve mathematical problems. The interaction was to stimulate active and constructive thinking when using and analysing their own solutions and the solutions of others. L2 and L6 displayed an attitude that could be seen as an expected outcome of a set of pedagogical practices which implies that correct answer rather than argumentation involved is the important point. An indication of this outcome of "routinised teaching and learning" (Wyndhamn and Saljö, 1997) is seen when learners seem quite willing to accept unreasonable solutions to problems.

CHAPTER 5: DATA ANALYSIS

5.1 Analysis of data form the pilot study

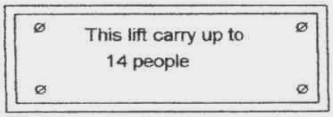
The purpose of conducting an interview was to obtain insights into the extent to which learners were capable of using mathematical and real-world knowledge in an integrated manner to solve the mathematics problems set in the task. In other words, I was trying to establish whether they were using mathematics unconsciously and as such excluded real-world knowledge or whether they consciously used mathematics and disregarded realistic situations because of being restrained by the application of word problems in the classrooms. In addition, the aim for the interview was to establish whether the task was understandable in terms of the language used, to understand the learner's written comments, and to make any adjustments to the task from the information given before the task was given to the research group.

For the piloting, I selected a learner because he was able to justify, explain, connect and communicate mathematical ideas when doing his mathematics problems (Stein, Grover & Henningsen, 1996). In addition, I also looked at completion of the mathematical task. That is, the selected learner did complete the task, and more significantly, supported the written comments that were made after a choice of yes/no responses. I selected one learner because as Opie (2004: 4) argues, "educational research involved at Masters level will be relatively small scale". He further asserts that this will in most cases not require the collection of large amounts of data. But this does not mean that it may not add 'new' knowledge to a particular field of enquiry.

In the analysis that will follow, the focus will be on how P1 (from the pilot group) responded to Problem 1. As will be seen below, a specific focus in the pilot analysis was to establish the way the learner, i.e., P1, approached the given mathematical problems that were influenced by the 'context' of the problem situation. In the discussion that we had, P1 seemed to be confused by the number 19,21429, which was the 'number of times the lift went up' in Problem 1A (see Appendix 1), by regarding it as being bigger than 269. Below are P1's responses to Problem 1:

PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes [✓] No [✓]

Please explain why you answered 'yes' or 'no'.
because if it 19,21429 it will have to go up more than the People in it

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?

Yes [✓] No [ ]

Please explain why you answered 'yes' or 'no'.
it will take 20 but 3 are going to be left. 20,3

C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

Because it will be more than 269 bec People to go up

In Problem 1(A), P1 chose ‘no’. In a followed-up interview with him, I asked him what he meant by “because if it 19,21429 it will have to go up more than the people in it”. This is what he said: (AM = researcher)

> *P1: Ke hlalosa hore e tlo tlameha ho ya ka makgetlo a mangata ho feta 269, ke hore 269 e ka kena ka makgetlo a mangata ho number ena, 19,21429, e ka kena ka makgetlo a fetang ten. [I am saying the lift will have to go more than 269 times, that is, 269 can divide many times in this number, 19,21429, it can divide into more than ten times]*

From his explanation it is clear that he regarded 19,21429 as being bigger than 269. In other words, 19,21429 was his dividend while 269 was the divisor. What is also evident is that from his explanation, “people in it”, suggests the number of people who were supposed to be transported, i.e., 269. What should be made clear here is that P1 was not confused by the fact that the lift went up fractional times, which was supposed to be the focal point here because of the ‘context’ of the problem. But surprisingly, the issue was that this number, 19,21429, besides being misconstrued as big, it was also ‘falsely’ viewed as meaning ‘the lift went up many times’. So, P1 found himself in a situation where he wasn’t sure how he was going to find out ‘the number of times the lift went up’ in order to answer the question.

P1 was quite confused by 19,21429 to such an extent that he disregarded the fact that the information on the lift capacity was given, i.e., 14. In a follow-up interview, P1 said the following in relation to the two numbers mentioned above:

> *AM: Ho 269 le 19,21429 ke efeng e kgolo, ho ya ka wena? [In your opinion, between 269 and 19,21429,which number is big?]*
>
> *P1: E kgolo? [The big one?]*
>
> *AM: Ee [Yes]*
>
> *P1: Ke … ke 19,21429 [Its …its 19,21429]*
>
> *AM: Why o nahana hore 19,21249 ke yona e kgolo? [Why do you think 19,21249 is the big number?]*

P1: Ka baka la value ya teng [Because of its value]

What can be seen from the first interview excerpt above is P1's apparent struggle to understand representation of a number. An issue arises here of: Does a number have a value?

Issue 1: Why does the learner show limited understanding of representation of a number?[1]

There were two broad scenes that were shown in the question Problem 1(A). The first was the context of the problem and the second was the mathematics of the problem. This has been mentioned in that order because the very first thing that captured my attention when I first read the question was the context. That is, the situation was portrayed wherein "269 people rush to go up the lift in the morning. The lift carries up to 14 people". A mathematically correct answer was given by a fictitious peer, Joe, who said "the lift went up 19,21429 times".

It is clear that the context in this question is foregrounded as explained above. It is clear that even though contextual conditions were amply evident, P1 "failed" to recognize that the lift cannot go up fractional times in the real-world. Now the question that arises is:

Issue 2: What is the reason for the learner not to be able to use mathematical and real-world knowledge in an integrated manner to solve the mathematical task?

In analyzing the responses of P1 in the whole mathematical task, it was surprising to see that Problem 1(B) was the only problem wherein the learner was able to use mathematical and real-world knowledge in an integrated manner. But it was interesting to see how the learner got the solution. P1 said the following when he was asked to explain how he found out that the lift went up 20 times:

[1]Issue 1 and subsequent issues will be discussed in the next chapter.

AM: *Ok. In B (Problem 1B), "Tim said that the lift went up 20 times. Can you see a way that 20 could be the correct answer?". O be o re 'yes'. [You said 'yes']*

P1: *Yes.*

AM: *O be o re "it will take 20 but 3 are going to be left. 20,3". O e thotse jwang 20,3 ena? [Then you said, "it will take 20 but 3 are going to be left. 20,3". How did you get 20,3?]*

P1: *E tlo ya hodimo ha 20, e be siya batho ba 3.[It will go up 20 times, and then 3 people will be left behind]*

AM: *Hore e ye ha 20 o e thotse jwang? [To say that it goes up 20 times, how did you get it?]*

P1: *Ke e calculatile ka … ka ho etsa… ka hore 269–14 [I calculated it, by saying 269-14]*

AM: *Ok, 269 – 14, and then?*

P1: *Be ke minus, be ke ntse ke counter ho re eya makgetlo a makae. [I then subtracted, and counted how many times it went up]*

AM: *Answer o be o ethola jwang? [How did you get the answer?]*

P1: *Answer enngwe le enngwe eo ke e tholang ke minus 14 ho fihlela ke thola hore e ya ka makgetlo a makae. Ke be ke thola hore ke 20, ho sala 3. Ke be ke ngola 20,3.[I then kept on subtracting the new answers from 14, and counted how many times it went up. The answer was 20 and 3 remained behind]*

AM: *Ok. All right. So etlo o neha answer ya 20,3 ho ya ka mokgwa oo o cauntileng ka teng?[Ok. All right. So it will give you an answer of 20,3 the way you have calculated?]*

P1: *Yes.*

In our discussion, P1 said that he calculated the answer. However, when one looks at the method he used, one can clearly see that a realistic situation was taken into consideration because he was aware that he needed to count every time the lift went up. To him, "269 – 14 = 255", represents a single trip, "255 – 14 = 241", was his second trip. He kept on

subtracting 14 from the new number (subtrahend) until three people were left after the 19^{th} trip. These 3 people were taken up the lift in the 20^{th} trip. P1 seems to confuse 19^{th} and 20^{th} trips because according to him when the lift went the 20^{th} trip, then three people were left. In fact, the lift went up the 20^{th} trip with only three in it. The interesting thing that came up in our discussion was the way P1 represented three people who were left behind, i.e., 20,3. This number, 20,3 has a different meaning to "it will take 20 but 3 are going to be left". P1 should have written $20\frac{3}{14}$ or just left it as "it will take 20 but 3 are going to be left", rather than using the representation 20,3. The issue that arises here is the understanding of numbers and how numbers are represented. That is, converting the statement to decimal form seems to be an issue. This issue is related to *Issue 1* and will be discussed in the discussion chapter. The most important thing here is that even though P1 struggled with his computational work, he was able to adapt his work to the real-world situation.

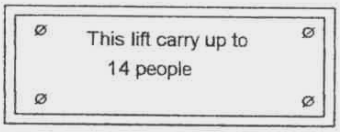
The question to be asked after realizing that Problem 1(B) was the only problem wherein P1 was able to adapt his computations to the situation was: what triggered P1 to solve this problem the way he did? Why did he not solve other problems in a similar manner as in Problem 1(B)? I think that occurred because the learner found himself in a familiar situation or maybe the learner was able to relate to the answer given by a fictitious peer called Tim. This result indicated that it would be interesting to find out how learners in the main study would respond to similar questions.

5.2 Analysis of data form the main study

As in the pilot interview, a specific focus in the analysis of data from the main study was to establish how the learners, i.e., L1 and L4, (these are the same acronyms used in the analysis of quantitative data) approached the given mathematical problems that were influenced by the 'context' of the problem situation. The link between mathematical procedures and contexts was demonstrated when both learners (L1 & L4) used similar methods to solve Problem 1(B). Below are L1's responses to Problem 1:

PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes ☐ No ☑

Please explain why you answered 'yes' or 'no'.
Because it doesn't transport many people it cannot go that many.

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?

Yes ☑ No ☐

Please explain why you answered 'yes' or 'no'.
Because it need to load 14 people over 269 people it must go 20 times

C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes ☐ No ☑

Please explain why you answered 'yes' or 'no'.

In Problem 1(A), L1 chose 'no' and gave the following comment "Because it doesn't transport many people it cannot go that many". L1 disagreed with Joe's answer because

in her opinion, 19,21429 represents a big number, i.e., in her understanding 19,21429 is the same as 1921429. In other words, 19,21429 represents the number of times the lift went up. So in her response, she chose ‘no’ because in her opinion the lift cannot take “many” trips in the sense that it only transports 269 people. In other words, L1 was aware that if she divided 269 by 14, the quotient could not be closer to “1921429”, i.e., “cannot go that many”. In an interview, she acknowledged that 19,21429 represents ‘many’ as can be seen below:

AM: *Ha o dumellane le Joe, because Joe o itse, … “Joe said that the lift went up 19,21429 times”, right? and then the question says “Can you see a way that 19,21429 could be the correct answer?”, according to your answer you said ‘no’, and then you said “because it doesn’t transport many people it cannot go that many”, right?. So ho ya ka wena, ‘many’ e bolela, 19,21429 e leng ho ya ka makgetlo a mangata?*

[You do not agree with Joe’s answer, because Joe said … “Joe said that the lift went up 19,21429 times”, right? and then questions says “Can you see a way that 19,21429 could be the correct answer?”, according to your answer you said ‘no’, and then you said “because it doesn’t transport many people it cannot go that many”, right?. So in your opinion, ‘many’ means, 19,21429 which refers to number of trips the lift took?]

L1: *Yes*

In Problem 1(B), L1 agreed that the lift went up 20 times. She supported her choice by saying “Because it need to load 14 people over 269 people it must go 20 times”. When I asked her how she got 20, she said the following:

L1: *Ke ne ke ntse ke etsa, ke entse 269 be ntse ke tlosa 14, anseng e ke e tholang be ke ntse ke tlosa 14 ho yona. [I did, I had 269 then I*

subtracted 14, the answer that I got from subtraction I kept on subtracting 14]

AM: *Ooh.*

L1: *Be ke ntse ke bala hore ke entse ha kae. [I then counted the number of times I used subtraction]*

AM: *All right, wa thola hore it went 20 times? [All right, you found that it went 20 times?]*

L1: *Yes.*

In other words, L1 used similar method to P1 (learner from pilot group). That is, she said "269 – 14 = 255", which represented a single trip, "255 – 14 = 241", was his second trip, etc. She kept on subtracting 14 from the subtrahend until she found that the lift went up 20 times. In this problem, L1 seems to be able to relate to problem to the situation in the sense that she counted every time the lift went up.

In Problem 1(C), L1 disagreed with Gita's answer because she already saw from Problem 1(B) that the lift cannot go more than 20 times. However, L1's response was limited to Problem 1(B), she did not look at other possibilities that could make the lift go 25 times. For instance, the lift could not go 25 times provided the lift was overloaded in some of the trips.

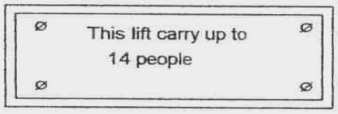
Below are L4's responses to Problem 1. In Problem 1(A), he disagreed with Joe's answer. He supported his choice by saying "I don't agree with Joe's answer because it has a lot of numbers and thats not the answer is needed in the question and thats is impossible to find the answer with that many number but it depend which question is". In an interview, when I asked him what he meant, he said the following:

L4: *Ok sir, like, ke calculatile ka calculator jwale ka ha ho thwe re e sebedise, jwale ke ile ka sebedisa calculator and ka thola hore ha e mphe dinumber tseo …[Ok sir, like, I calculated the answer using a calculator, as*

indicated, we were allowed to use them, so I calculated and discovered that it did not give me numbers…]

PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes ☐ No ☑

Please explain why you answered 'yes' or 'no'.

I don't agree with Joe's answer because it has a lots of numbers and that's not the answer is needed in the question. and that's impossible to find the answer with that many numbers but it depend which question is.

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?

Yes ☑ No ☐

Please explain why you answered 'yes' or 'no'.

I agree with Tim because I calculate, and it gives me the same answer that Tim got it.

C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes ☐ No ☒

Please explain why you answered 'yes' or 'no'.

So L4 disagreed with Joe's answer because he did not find 19,21429 as answer. He mentioned the use of a calculator in his attempt to solve the problem. Learners were allowed to use calculators. L4 was aware that to find 19,21429 a calculator had to be used

because Joe's is a decimal fraction. In his explanation L4 used the phrase "Joe's answer has a lot of numbers". This suggests that L4 in his attempt to solve the problem he found a number that has fewer digits (not lot of numbers). Because I wanted to verify his understanding of representation of numbers, I asked him the following question:

AM: *Re ntse re le hona mona e re ke o botse he. Ho ya ka answer eo ya Joe, le answer ya B le ya C. Ke number efeng e kgolo ho ya ka wena? [While still here let me ask you this question. In your opinion, if you compare answers given by Joe, Tim and Gita, which one do you consider as big?]*

L4: *E kgolo? [The big one?]*

AM: *Yes*

L4: *Ke ena ya 25 [It is this one of 25]*

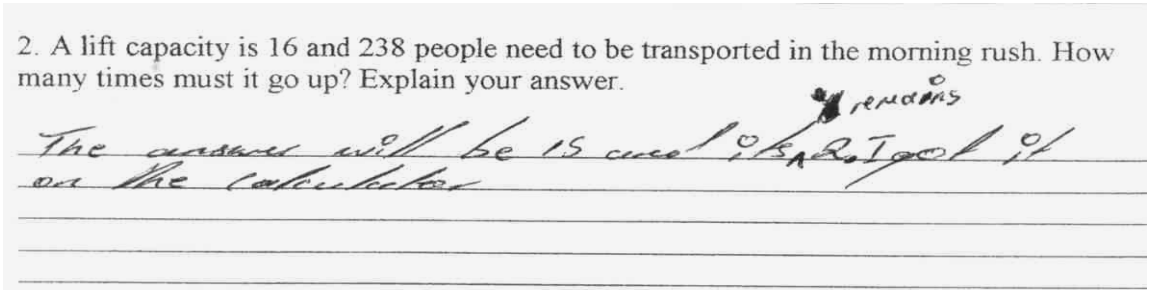
I asked L4 this question because P1 and L1 seemed not to understand that 19,21429 is not bigger than 20. However, here it was clear that L4 understood that 19,21429 was smaller than 20 and 25.

In Problem 1(B), L4 agreed that the lift went up 20 times. He supported his choice by saying "I agree with Tim because I calculate and its gives me the same answer that Tim got it". In a follow-up interview, I asked him how he got 20 and said the following:
(L4 = Learner 4)

L4: *So, ke itse 269-14=255 and then ke be ke ntse ke minus 14 ho answer enngwe le enngwe eo ke etholang until e mpha the correct answer. So, ho the correct answer ho remainile two. But two ena e ile ya nyoloha le lift ya etsa 20 times. [So, I said 269-14=255 and then kept on subtracting 14 from any new answer until I got the correct answer. So from the correct answer, two remained. But that two went up with the lift when it went up for the 20th time]*

It is also evident that the method that L4 used is similar to those used by P1 and L1. Because all of them have said "269 – 14 = 255", which represented a single trip, "255 – 14 = 241", was his second trip, etc. They all kept on subtracting 14 from the subtrahend until they found that the lift went up 20 times.

In above excerpt, L4 said that "two remained" whereas in an "Analysis of data from the pilot study", P1 said "… 3 people will be left behind". The issue that arises is: How many people actually remained behind when the lift went up for the 20th time? The acceptable answer is the one given by P1. I think L4 confused the answer he gave during the interview with the one he wrote in Problem 1(2) as can be seen below. Here L4 said "*The answer will be 15 and its remains 2. I got it on the calculator*". In other words, L4 says that the lift went 15 times and two people remained behind.


2. A lift capacity is 16 and 238 people need to be transported in the morning rush. How many times must it go up? Explain your answer.

The answer will be 15 and remains 2 I got it on the calculator

In the analysis above, L1 and L4 seem to use the same method used by P1 in linking mathematical procedures and the realistic situation. The three learners used the mathematical procedure (by subtracting 14 from each subtrahend) in conjunction with the problem situation. That is, the procedure was to be 'concretized' or drawn to the problem situation by counting the number of times the lift went up. In this problem, learners were able to show their developed skills in knowing when and how to apply their mathematics effectively to the everyday life situation. From situative perspective, these learners were able to identify with both mathematical and social requirements and expectations (Brodie, 2005). However, these learners were unable to use their mathematics effectively to solve various kinds of mathematical problems in real-world situation as will be demonstrated below.

In the mathematical task, there were more questions which demanded realistic responses rather than standard responses. So given more realistic word problems, I thought that that would trigger off realistic responses. Even though learners were given realistic world problems that in itself didn't trigger realistic responses. It is important to understand why even if learners were given realistic problems, it turned out that that in itself did not trigger them to respond realistically. The question here is why is that the case? This question will be coupled with *Issue 2* (which arose in analysis of the data from pilot study) and will be discussed in the discussion chapter.

The "failure" of learners to make proper use of real-world knowledge and sense-making when doing their mathematics activities has been seen, for instance, in L4's response to Problem 2.

AM: *Ok. Ho C, "Mary said it is **impossible** to find the answer. Can you see a way that **impossible** could be the correct answer?". O be ore "no thats not impossible, you must only use your brain or if is impossible you can use calculator". So o ne o batla horeng mona? [Ok. In Problem 2(C), "Mary said it is **impossible** to find the answer. Can you see a way that **impossible** could be the correct answer?". You said "no thats not impossible, you must only use your brain or if is impossible you can use calculator". What did you mean here?]*

L4: *Ok. Ke ne ke batla ho re ha e impossible, o ka nna wa usa your calculator or mind ho thola the answer. [Ok. I wanted to say it was possible, you can use your calculator or mind to find the answer]*

The discussion above came as a result of question posed in Problem 2 which was: "John's best time to run 100 m is 17 seconds. How long will it take him to run 1km?" A fictitious peer called Mary said it was impossible to find the answer. L4 responded by saying that it was possible to find an answer because a calculator or mind could be used to do so. The above excerpt underscores the question raised earlier in *Issue 2* that there is a strong tendency among learners to suspend real-world knowledge and realistic

considerations in solving word problems. As can be seen, L4 also thinks that all mathematical problems can be solved regardless of constraints associated with problems. In fact, by looking at this type of response, one can clearly see that L4 knew little about mathematical problems that have constraints. That is, L4 is used to solving mathematical problems that have one correct answer and if there are more than one correct answer, then they are always mathematically realistic (straightforward application of mathematics rules).

However, as will be shown below, when responding to Problem 2(A) and (B), L4 demonstrated some wiliness to engage in extended realistic thinking. L4 was responding to the question: "John's best time to run 100m is 17 seconds. How long will it take him to run 1km?". A fictitious peer named Sipho, said the answer was 170 seconds. L4 responded by saying he did not agree with Sipho's answer. When asked why he disagreed he said the following:

> *L4: Ok. Ha ke dumellane le answer ya Sipho hobane e tlamehile ebe e le answer ekgolo ho feta 170 seconds because 1km e kgolo? [Ok. I do not agree with Sipho's answer because it (answer) must be bigger than 170 seconds because 1km is quite big]*

The excerpt above suggests that L4 understood that to run 1 km could take a longer time because the distance is quite big as compared to just 100m. However, when probed, he said the runner would take around two minutes, although he was not sure. So he seemed to confuse 2 minutes and 170 seconds. He understood 170 seconds as being less than 2 minutes. In our discussion, this is what he said:

> *AM: Ha ore e be kgolo ho fata 170 seconds, e be bokae? [When you say it must be bigger than 170 seconds, how big must it be?]*
>
> *L4: E be bo ma two minutes. [Around two minutes]*

In a follow-up interview, L4 seemed not understanding that 170 seconds and 2 minutes and 50 seconds are equal. In his understanding, the two are not the same as can be seen in our discussion:

AM: *Ho ya ka wena 170 seconds e nyane ho 2 minutes 50 seconds? [In your opinion, is 170 seconds smaller than 2 minutes and 50 seconds?]*

L4: *Yes*

What can be seen in the interview excerpt above is L4's apparent struggle to appropriately verbalize his thoughts because of lack of understanding of numbers, i.e., knowledge of numbers is revealed when the learner "recognizes and uses equivalent forms of the rational numbers" as espoused in Learning Outcome 1 through Assessment Standards (RNCS, 2002: 69). The learner seems to have a problem with conversion of minutes to seconds or *vise versa.* In addition to this, the problem of understanding of numbers was also observed in our discussion with L1. She seemed to confuse 19,21429 with 20. That is, the learner views 19,21429 as representing 'a big number' because of its digits whereas 20 is regarded as being 'a small number'. The confusion came as result of L1 saying that the lift in Problem 1(A) cannot go that 'many' (19,21429). That is, the lift cannot take a lot of trips. In her understanding, 19,21429 was bigger than 20, and that was reflected in the following:

AM: *But why o re 19,21429 e kgolo ho fata 20?[But why do say 19,21429 is bigger than 20]*

L1: *Because e na le dinumber tse ngata. [Because it has a lot of numbers]*

AM: *So ha answer ena le dinumber tse ngata, then e kgolo?[So if an answer has a lot of number, then is big?]*

L1: *Yes*

The above excerpt suggests that L1 seems not to be aware that 20 can also be represented in the following ways, 20,00; 20,000 etc. This learner is not aware that we can represent

20 in some other ways and when we do that 20 will still be bigger than 19,21429. And here there seems to be a limited understanding of representation of numbers. That is, how a number is represented. The same problem of regarding 19,21429 as being big because of its digits was experienced by P1 (learner from pilot). So an understanding of numbers here is an issue. The question here is: Why do learners, even at grade 8 level still experience problems with understanding of numbers? This question will be coupled with *Issue 1* (which arose in analysis of data from the pilot study) and be discussed in the discussion chapter.

In the mathematical task, most of the learners were unable to relate their mathematics to real-world situations. However, from constructivist's perspective, learning mathematics as an activity took place as shown by the procedures and working methods thar learners have used.

5.3 Analysis of learners' perspectives

This analysis is based on learners' comments to other learners' responses to mathematical tasks. The learners' comments focused on getting their thoughts on other learners' responses. In other words, did they agree or disagree with other learners' responses, and if that was the case, why and how? But then how were the analyses organized? Learners who were selected based on their achievement in the class, i.e., LP1, LP2 and LP3, commented on the following learners' work, L1, L2 and L3 respectively. LP1, LP2 and LP3 did not comment on all responses of L1, L2 and L3. In essence, LP1 commented on L1's response to Problem 3; LP2 commented on L2's response to Problem 2; and LP3 on L3's response to Problem 1. The reason for doing that was to get different views on different questions, and from different learners. I thought that would give me a broader and better analysis rather than concentrating on only one learner's responses, for instance, LP1, LP2 and LP3 commenting on L1's responses.

In the analysis that will follow, the focus will be on how L1 responded to Problem 3. As will be seen below, in Problem 3(A), she said "I calculated it" and she further explained how the calculation was done because she used the phrase "I used subtraction/ I

subtracted", and got 17 as a solution. However, when I looked at her calculation/subtraction in Problem 3(B), I discovered that she wrote 1978 – 1995, and I wondered why she subtracted the way she did and still got 17 not -17. I realized later that what she actually meant by "calculated it", was counting from 1978 to 1995 and found 17 years. To have Problems 3(B) and 3(C) was to sensitize learners to the consideration of aspects of reality. That is, to encourage learners to respond more realistically to these items. As can be seen in Problem 3, L1 seems to act without much apparent concern for what would be realistically meaningful outside of the classroom. For instance, in Problem 3(B), L1 said the answer "cannot be 16 even if you are not calculating with a calculator". Furthermore, L1 said the answer cannot be 16 unless "she (Kathy, a fictitious peer) used a wrong sign". In other words, unless wrong method (sign) is used, there is no way that the answer could be 16. To L1, the only acceptable answer is 17 years, a mathematically "correct" answer. Below are L1's responses to Problem 3:

PROBLEM 3

1. Rob was born in 1978. Now it's 1995. How old is he?

A. Steve said the answer is 17 years.

Can you see a way that 17 years could be the correct answer?

Yes [✓] No [ ]

Please explain why you answered 'yes' or 'no'.

Because I calculated it and it gave me 17. And I used subraction/I subtracted and it still give me the same answer as Steve

B. Kathy said the answer is 16 years.

Can you see a way that 16 years could be the correct answer?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

Because 1978 - 1995 cannot be 16 even if you are not calculating with a calculater or She used the wrong sign she should've must subracted 1978 from 1995

C. Carl said the answer cannot be known precisely.

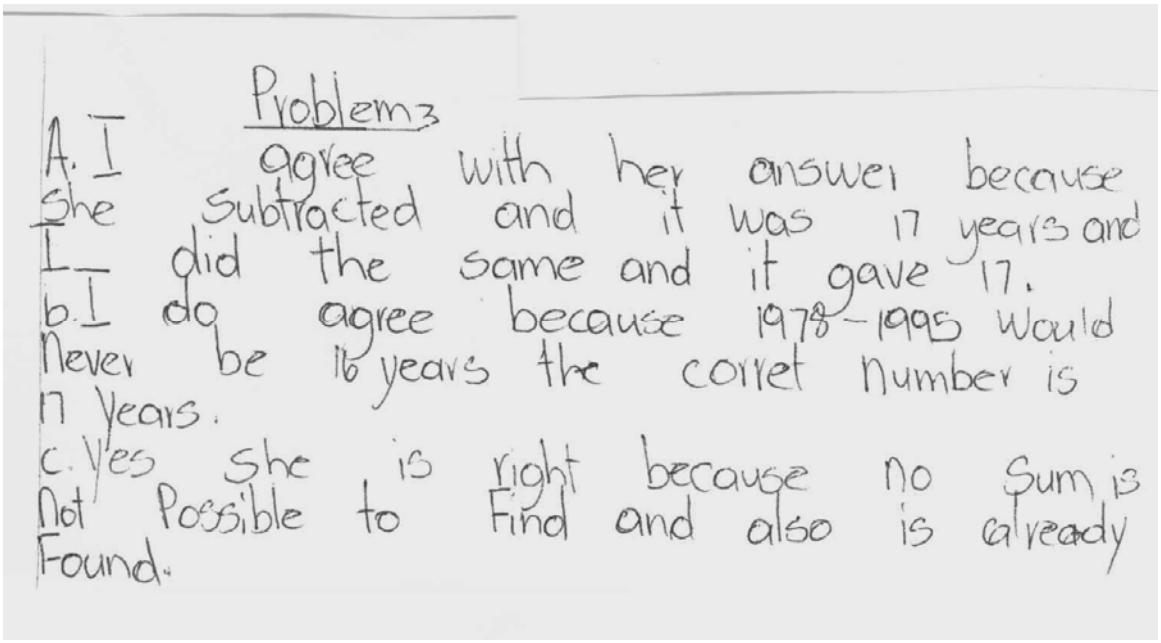
Can you see a way that the answer cannot be known precisely?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

Because weve already found the correct answer and is know

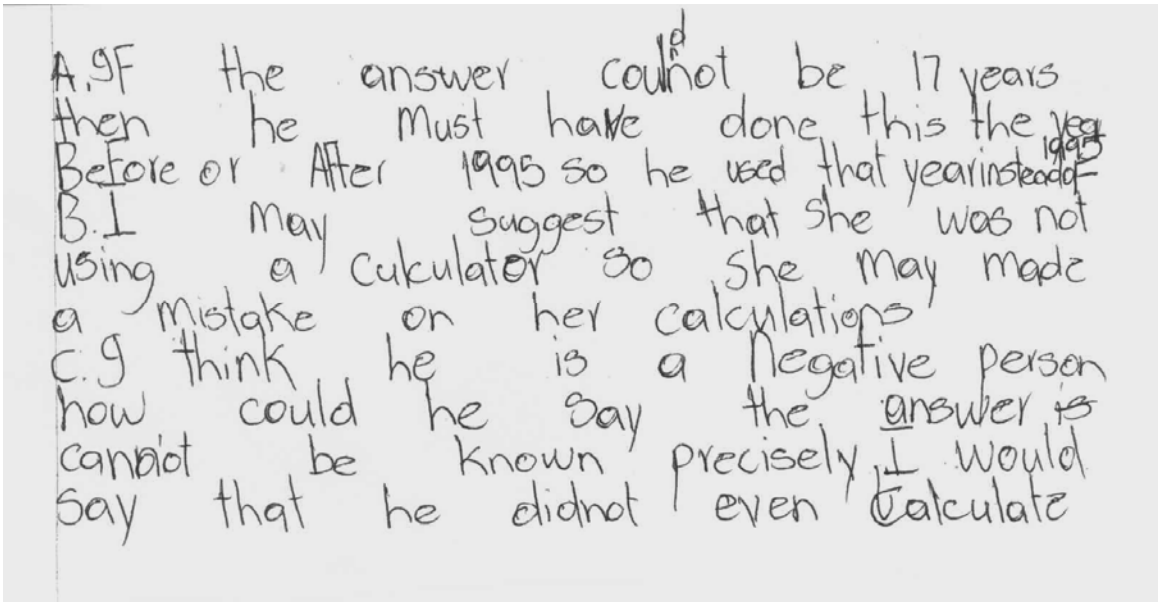
I asked LP1 to comment on L1's responses to Problem 3 as mentioned above. Below is LP1's comment:



LP1 seems to agree with all responses given by L1 as can be denoted by statements such as “I agree with her (L1)”; “I did the same”; “I do agree”; “Yes she is right”. In order words, LP1 also did his calculations in the same way, i.e., 1978 – 1995, in order to get 17 years. LP1 rejected Problem 3(B) on the grounds that “1978 – 1995 would never be 16”. In Problem 3(C), LP1 also rejected Carl’s (fictitious peer) answer by pointing out that “no sum is impossible to find”. In other words, LP1 disagreed that the “answer cannot be known precisely” and that was demonstrated by the phrase “is already found”. It is apparent from the reasons given that both learners did not make use of realistic considerations when solving the real-world items. The question here is why did they disregard constraints associated with word problems?

The analysis of both learners’ responses to Problem 3, confirms a comment by a 10 year-old from a study in De Corte (2000), when asked why he did not make use of realistic situations when solving real-world problems in the context of a school mathematics test. The learner said: “I know all this things, but I would never think to include them in a maths problem. Math isn’t about things like that. It’s about getting sums right and you don’t need to know outside things to get sums right” (p. 37).

To determine whether learners "…know about all about these things, and would never think to include them in a maths problem", I asked LP1 to comment on Problem 3 again. Because LP1 agreed with all responses given by L1, so this time I asked him how he would respond if someone disagreed, i.e., someone responded differently. The aim was to explore the extent to which LP1 would comment realistically. Below is how he commented:



In Problem 3(A), LP1 commented by saying "If the answer could not be 17 years then he must have done this the year before or after 1995 so he used that year instead of 1995". LP1 justify his agreement with Steve's (fictitious peer) answer of 17 years on the grounds that if the answer is not 17 years, then Rob must have been borne before or after 1995. In rejecting the possible correctness of Kathy's of 16 years in Problem 3(B), LP1 said that "I may suggest that she (someone) was not using a calculator she may made a mistake on her calculations". It can be seen that LP1 believed that it was through a calculator or having made a mistake that an answer of 16 could be obtained. And in Problem 3(C), LP1 argued explicitly that "I think he is a negative person how could he say the answer cannot be known precisely, I would say that he did not even calculate". In his comments, LP1 never referred to the possibility that some items might be responded to realistically. As can be seen from LP1's comment, the learner stuck to unrealistic answers even after the second attempt was made in trying to 'encourage' him to think realistically. This

supports Verchaffel and De Corte's (1997) argument that school children's tendency to exclude real world problems is strong.

In the analysis that will follow, the discussion will focus on how L2 responded to Problem 2. As will be seen below, L2 agrees with Sipho's answer in Problem 2(A), which is 170 seconds. To show that she (L2) knew how 170 seconds was calculated, she made the following calculation: 17×10 = 170. L2 was aware that 1km = 1000m and now that she already had 100m, then what she needed was 10 times the distance. In her understanding, if 100m is covered in 17 seconds, then 1km will be 17×10 = 170. Mathematically, this answer is correct and this clearly shows that L2 is able meet some of the expectations of Learning Outcome 1 as expressed through the Assessment Standards when they state that: we know this when the learner "solves problems that involve ratio and rate" (RNCS, 2002: 71).

In Problem 2(B), L2 rejected Jake's answer of 2 minutes and 50 seconds by pointing out that Jake did not calculate correctly. Furthermore L2 showed what she meant by "correct" calculations as she wrote down 17×10 = 170. There seems to be uncertainty about the gender of Jake as reflected in the pronouns "he/she". The aim of using names of learners in the mathematical task such as Joe, Tim, Gita, Jake, etc, was to influence the nature of responses, possibly encouraging more realism. As mentioned above, it has been found in earlier work that learners responding to items containing learners' names trigger out some aspects of everyday world (Cooper & Dunne, 2000a). However, as seen here, names of learners created some confusion due to unfamiliarity of such names in some communities. What follows below are L2's responses to Problem 2:

PROBLEM 2

1. John's best time to run 100m is 17 seconds. How long will it take him to run 1km?

A. Sipho said the answer is 170 seconds.

Can you see a way that 170 seconds could be the correct answer?

Yes [✓] No [ ]

Please explain why you answered 'yes' or 'no'.

I agree with Sipho when he said the answer is 170 seconds Because whe he calculate he said 17 seconds x 10 is equalls to 170 seconds
17 x 10 = 170

B. Jake said the answer is 2 minutes and 50 seconds.

Can you see a way that 2 minutes and 50 seconds could be the correct answer?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

I don't agree with Jake because he/she din't calculate correctly. He/she should have said 17 seconds x 10 which is equalls to 170 seconds. 17 x 10 = 170.
If he/she uses that method will correct answer like Sipho did.

C. Mary said it is impossible to find the answer.

Can you see a way that **impossible** could be the correct answer?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

It is not impossible to find a very easy answer like Sipho did if you could follow this method you could see that it is easy 17 seconds by 10 is equalls to 170.
which is 17 x 10 = 170

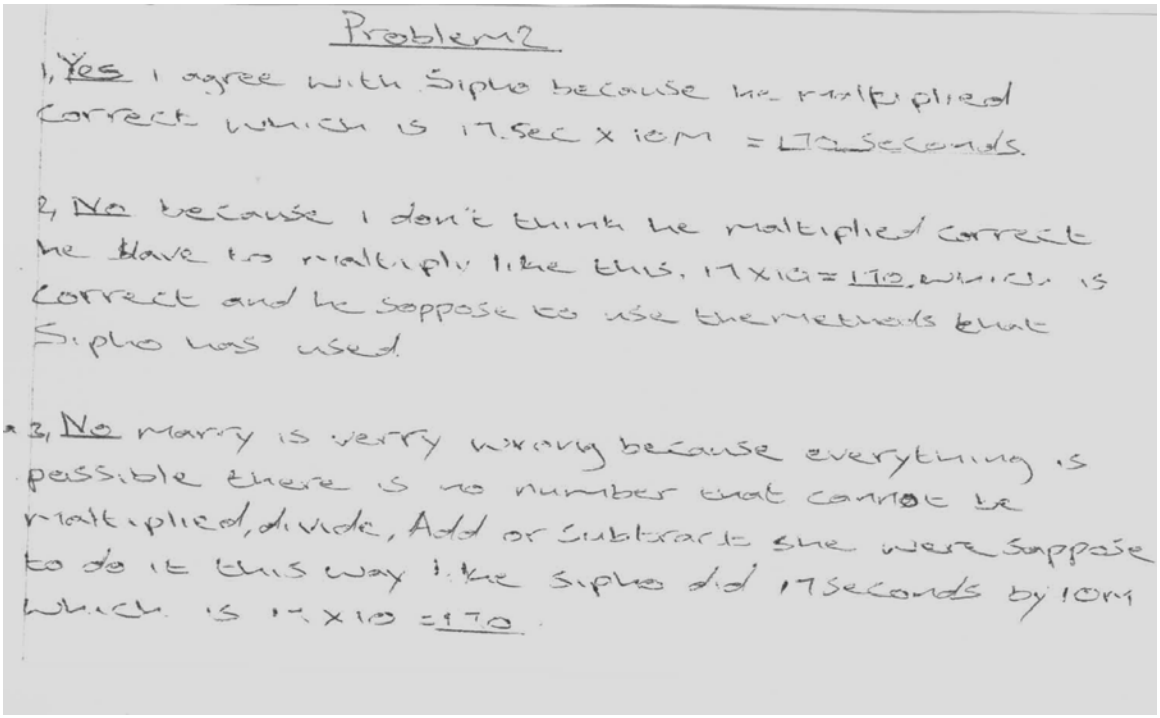
In Problem 2(B), L2 is not aware that 170 seconds is actually the same as 2 minutes and 50 seconds that is why she disagrees with Jake. The way L2 responded shows limited knowledge concerning recognition of equivalent forms of the rational numbers as

espoused in Learning Outcome 1 through Assessment Standards (RNCS, 2002: 69). In problem 2(C), L2 argues that it is not impossible to find the answer. In other words, this assumption concurs with Schoenfeld's (1998, in DeCorte, 2000) argument that learners acquired debilitating beliefs about mathematics, such as "all mathematics problems can be solved …" (p. 36).

I also asked LP2 to comment on L2's responses to Problem 2 as mentioned. Below is LP2's comment: (Retyped to improve legibility)

Problem 2

1. *Yes I agree with Sipho because he multiplied correct which is 17sec ×10m = 170 seconds*
2. *No becauseI don't think he multiplied correct he have to multiply like this 17 ×10 = 170 which is the correct and suppose to use the correct methods that Sipho has used.*
3. *No Marry is verry wrong because everything is possible there is no number that cannot be multiplied, divide, add or subtract she were suppose to do it this way like Sipho did 17 seconds by 10m which is 17×10 = 170*


Problem 2

1, Yes I agree with Sipho because he multiplied correct which is 17sec x 10m = 170 seconds.

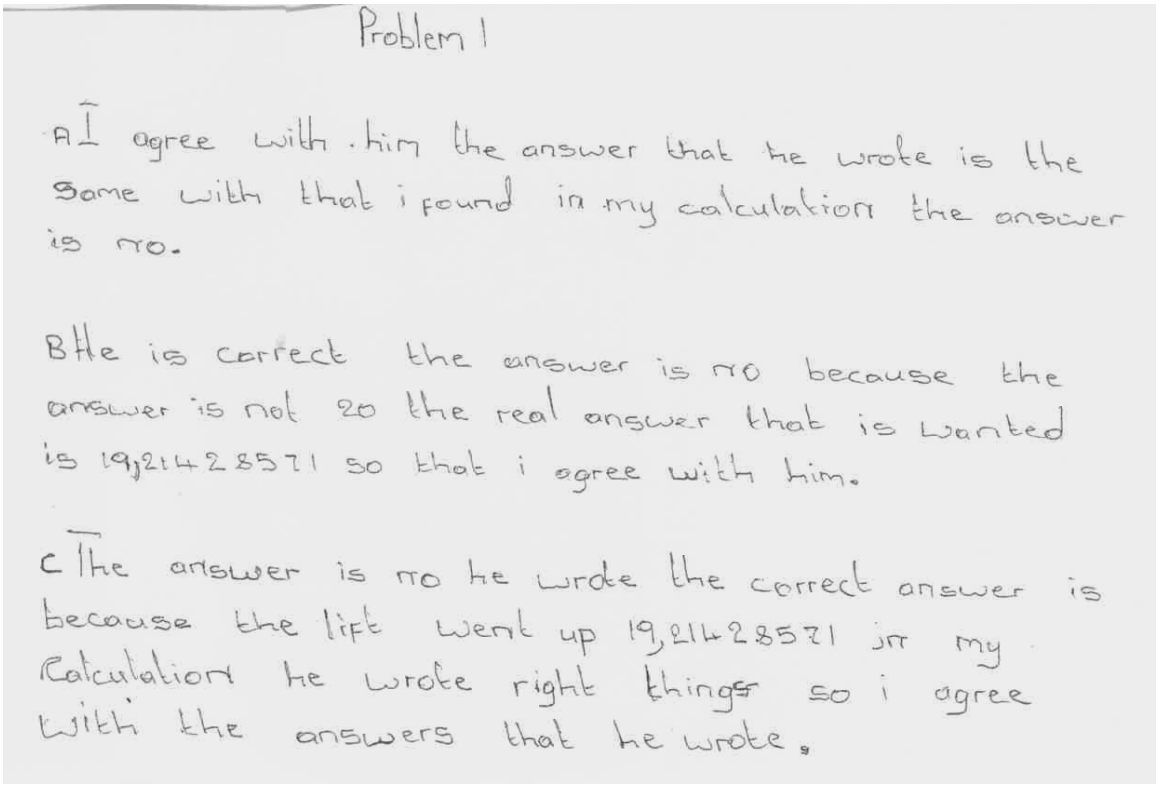
2, No because I don't think he multiplied correct he have to multiply like this. 17x10 = 170. which is correct and he soppose to use the methods that Sipho has used.

3, No Marry is verry wrong because everything is possible there is no number that cannot be multiplied, divide, Add or Subtract she were soppose to do it this way like Sipho did 17seconds by 10m which is 17x10 = 170

In Problem 2(A), LP2's comment shows that he agrees with both Sipho and L2 that the answer is 170 seconds, even though he does not show how he got the answer. In Problem 2(B), LP2's comment indicates that he, just like L2, was not aware that 170 seconds is the same as 2 minutes and 50 seconds, that is why he said that Jake did not multiply correctly. LP2 just like L2, performed formal arithmetic operation with the numbers given in the problem without seriously paying attention to the realities of the problem context. That is, they should have responded realistically because Problem 2(1) is unsolvable considering the fact that John (the runner) will not be able to run constantly at his record speed. Rather than making this kind of calculation "17×10 = 170 seconds", learners should have responded in a more realistic way.

In Problem 2(C), LP2 commented in the same way L2 did, because both believe that a mathematical problem must have an answer. That is why LP2 claims "everything is possible" while L2 says "it is not impossible" referring to the answer. Lampert (1990, in De Corte, 2000) argues that learners acquire beliefs relating to mathematics which are naïve, incorrect or both. Furthermore, according to Lampert (1990, in De Corte, 2000) these "beliefs are acquired through years of watching, listening and practicing in the mathematics classroom" (p.36). In a similar way Lampert characterizes the common view about mathematics as the one that is associated with certainty, and with being able to give quickly the correct answer.

In another analysis that will follow, the focus is on the comment made by LP3 on L3's responses:


Problem 1

A I agree with him the answer that he wrote is the same with that i found in my calculation the answer is no.

B He is correct the answer is no because the answer is not 20 the real answer that is wanted is 19,214285 71 so that i agree with him.

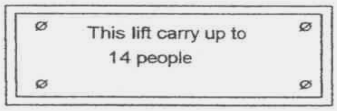
C The answer is no he wrote the correct answer is because the lift went up 19,214285 71 in my Calculation he wrote right things so i agree with the answers that he wrote.

In Problem 1(A) as can be seen above, LP3 agrees with L3 in terms of the common answer, 19,21428571. They both disagree with Joe's answer of 19,21429. In their understanding, these two numbers are not the same. This then shows limited knowledge concerning estimations and calculations by using operations appropriate to solving problems that involves rounding off as expressed in Learning Outcome 1 through Assessment Standards (RNCS, 2002: 71).

In Problem 1(B), LP3 rejects Tim's answer on the grounds that "the real answer that is wanted is 19,214298571". LP3 uses the phrase "the real answer" to denote the "actual answer given by the calculator". LP3's comments seem to agree with all of L3's responses and stuck to answer 19,214298571. The fact that three competing answers were given by Joe (19,21429), Tim (20) and Gita (25), that in itself did not cause some cognitive conflict as thought. In other words, different answers given by three different fictitious learners did not encourage LP3 to reflect on features of the represented situation. The question is why did happen? This question is related to *Issue 2* raised under "An analysis data from the pilot data" and will be discussed further in discussion chapter. Below is L3,'s response to Problem 1:

PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes ☐ No ☑
Please explain why you answered 'yes' or 'no'.

The answer is no because when I calculated it I got 19.21428571 and what I did to get this answer I took 269 and divided with 14 and it gave me the answer

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?
Yes ☐ No ☑

Please explain why you answered 'yes' or 'no'.
It mybe the calculator said 19,99 is the answer his should'nt said the lift went 20 times but because he and his calculator got the wrong answer he didint calculate well

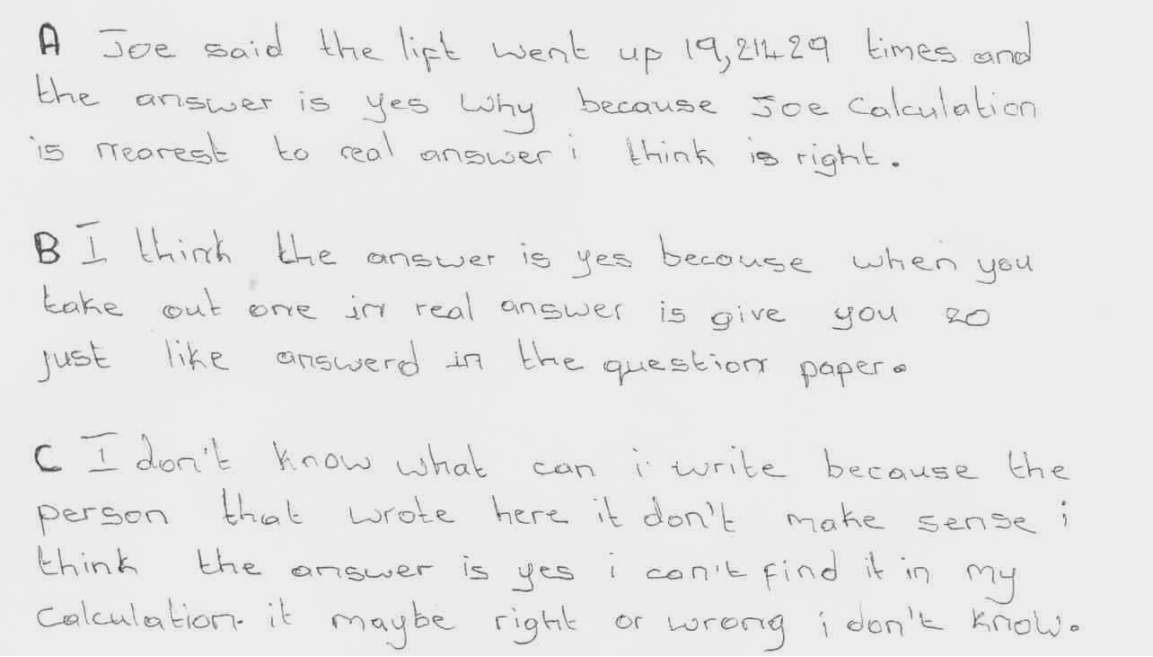
C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes ☐ No ☑
Please explain why you answered 'yes' or 'no'.

As can be seen in above, L3 clearly shows that he does not know 'rounding off', because he rejects Tim's answer on the basis that 19,21428571 is not close enough to 20. This is also supported by LP3 as seen above. Their understanding of 'rounding off' contradicts what they portrayed in Problem 1(A), because they could not see that 19,21429 was actually 'rounded off' from 19,21428571. The question is, could this be as a result of classroom practice that limits the 'rounding off' of numbers to one or two decimal digits?

This issue will be coupled with *Issue 1* raised under "An analysis of data from the pilot data" and discussed in the discussion chapter.

In Problem 1(C), L3 chose 'no' and commented by just writing "she is wrong" without explaining why. LP3 seemed to support all reasons given by L3 and stuck to the fact that the lift went up 19,21428571 times. What is of concern is that both learners 'failed' to recognize the fact that once the computational work was done, they still have to adapt their answers to the situation. LP3's comments on L3's work could merely be reflecting 'learner solidarity'. In other words, it may well be that LP3 did not wish to come across as being no-supportive. Because LP3 agreed with all responses given by L3 wherein he (L3) chose 'no', so this time around, I asked LP3 how he would comment if someone made a choice of 'yes' in all the Problems. Below is how he commented:


A Joe said the lift went up 19,21429 times and the answer is yes Why because Joe calculation is nearest to real answer i think is right.

B I think the answer is yes because when you take out one in real answer is give you 20 just like answerd in the question paper.

C I don't know what can i write because the person that wrote here it don't make sense i think the answer is yes i can't find it in my Calculation it maybe right or wrong i don't know.

As can be seen from his comments above, in Problem 1(A), he agrees because Joe's answer, 19,21429 is closer to what he got, 19,21428571. It is not known whether he used "rounding off" before concluding that "Joe calculation is nearest to real answer", or just because his was closer to that one given by Joe. LP3 used the phrase "real answer" in his comment to refer to 19,21428571 which was the 'exact answer' as reflected on the calculator. Turning to LP3's comment in Problem 1(B), he seemed to have just put "one in real answer", meaning put 1 in 19,21429 so that the answer could be 20 in agreeing

with Tim's answer. Again it was not explained where that 1 came from. In Problem 1(C), he accepted that the answer is 'yes'. However, the question is how he chose 'yes' because he could not find the answer by saying "I can't find it in my calculation".

The aim of asking LP3 to make another comment was to 'encourage' him to move beyond the assumption of a single correct answer and contemplate the possibility that there might be a range of possible legitimate answers. My assumption was that this might encourage him to think in a more realistic way. However, he constrained his comments to how he did the computational work and as such 'failed' to use commonsense knowledge base.

5.4 Analysis of educator's perspective

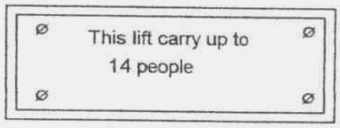
The analysis of educator's perspective is limited to responses of L5 and L6 since these were the only learners whose responses were not yet analysed. For instance, both L1 and L4's responses were analysed through interviews, and L3 and L4's responses were analysed through learners' perspectives. The analyses of responses of both L5 and L6 focused mainly on Problem 1 because of the familiarity of that particular question with the colleague educator. Problem 1 was easier for the educator to analyse because before the piloting of the interview, we (researcher and educator) were involved in the rehearsal of the interview and we discussed possible learners' responses of which were limited to Problem 1.

Below is L5's responses to Problem 1, and here the learner agrees with the answer given by Joe, and seems to have used a calculator to divide 269 by 14. After dividing, he was left with 5 numbers after the comma and he ended up by saying "will need more 2 people", meaning, in my opinion, that 2 more people will be needed for the lift to go up. This "will need more 2 people", came as a result of the answer, 19,21429, because as can be seen, 19,2 was interpreted as the lift that will go up 19 times and "will need more 2 people" for the lift to up the 20^{th} time. The same representation was observed in "Analysis of data from the pilot study" wherein P1 used the notation, 20,3 to refer to the lift that will go up the 20^{th} time and three people will be left behind. Here also converting

the statement to decimal form seems to be an issue. Below are L5's responses to Problem 1:

PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes [✓] No [✓]

Please explain why you answered 'yes' or 'no'.
I think this an ~~wrong~~ right answer, because when you take 19 five 5 numbers are left you will find the correct answer and will need more 2 people

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.
because when you multiply 20 it doesn't give me the correct answer

C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

In our discussion with the educator, she said the following with regard to Problem 1(A): (AM = researcher and ED = educator)

AM: *Ho ya ka wena, o bona response ya ngwana eo e lejwang, the way a respondileng ka teng? [In your opinion, how is L5's response, the way he responded?]*

ED: *E sharp, o itse karabo ena (pointing at 19,21429), o e thotse … o itseng (the educator tried to see where the learner has written the answer) …ok, lift eya 19 times e siya 5 numbers, ha kere? [It is fine, he (L5) said this answer (pointing at 19,21429), he got it … he said (the educator tried to see where the learner has written the answer) …ok, the lift went up 19 times and leave 5 numbers,* ***isn't it?]***

AM: *Hmm*

ED: *Ho be ho sala batho ba bang, ha kere? [Then there will be people staying behind,* ***isn't it?]***

AM: *Hmm, ho be ho sala batho ba bang [Hmm, there will be people staying behind]*

ED: *So ho ya ka yena, 19,24129 e right [In his (L5) opinion, 19,24129 is right]*

AM: *So o kgotsofetse ke the way a commentileng ka teng? {So are you happy the way he (L5) has commented?]*

ED: *Hmm, hobane o indicatile ha kere hore hona le ba bang "will need more 2 people", ha kere? [Hmm, because he indicated,* ***isn't it****, by saying that there are some people "will need more 2 people,* ***isn't it****?]*

AM: *Hmm*

ED: *So ke 19 eo ha e tlo o fa 20 [So it is 19, it will not give you 20]*

AM: *Ok, all right, because o itse "I think this an right answer, because when you 19 live 5 numbers are left you find the correct answer and will need more 2 people". Ooh, in other words, … [Ok, all right, because he said "I think this an right answer, because when you 19 live 5 numbers are left you find the correct answer and will need more 2 people". Ooh, in other words, …]*

ED: *O sebedisetse calculator, a bona hore ho na le batho ba salang. [He used a calculator, realized that there were people who would be left behind]*

The educator seems to agree with L5's response that the lift went up 19,21429 times. And this is seen in the phrase "it is fine", indicating the agreement with L5 and she (educator) further says that L5 used a calculator to find the answer. There seems to be uncertainty in the way the educator expresses her opinion as can be seen in the phrase "isn't it?" The reason for this is not known, however, this could suggest that the educator was to sure about what she was saying or she regarded the researcher as an authority and as such seeks out confirmation. If the educator is not sure about some of the responses given by learners, what about the same learners who wrote those responses like L5 and L6. Were they not experiencing the same uncertainty?

In Problem 1(B), L5 chose 'no' in his response and when I asked the educator to comment on L5's response, she said the following:

ED: *So ho B, ke hore ho yena, it is not 20 times because the 20th time hase batho ba 14 ba nyolohang le lift, ha kere? [So in Problem 1(B), to him, it is not 20 times because the 20th time, there would be less than 14 who go up the lift]*

AM: *Hmm*

ED: *So then yet a re the answer can't be 20 times, so ka mantswe a mang, ho ya ka yena, e tlamehile e be e ena le batho ba 14, so then o aware hore ha o nka 20 o e multiply ka 14 ha esa o fa 269 e se e ofa more than that, ha kere?[So then yet he says the answer can't be 20 times, so in other words, in his opinion, there must be 14 people in the lift, so then he is aware that when you multiply 20 by 14, it will not give him 269, it gives him more than that, isn't it?]*

AM: *Yes, all right.*

ED: *Ke hore leveleng ya hae, the way a answerileng ka teng, ke nahana hore e all right. [At his level, the way he answered the question, I think it's all right]*

According the educator's comment, L5 says the answer cannot be 20 because in the 20th time there will be less than 14 people in that lift. In other words, for the lift to go the 20th time, there must be 14 people in it, otherwise the lift cannot go up. In L5's opinion, the lift can only go up provided there are 14 people. This problem was highlighted by the educator earlier in our discussion (during the piloting of interview), when she said:

ED: *Ja, batla e bona e le easy and they will stick to this answer (19,21429) and ignore this one (20). Because most of them batla be ba guardwa ke calculator, but if e le hore …er, ok coming to the phrasing, for them to come to 20 they could have been told hore lift e ya tsamaya e ntse e sena batho ba 14… [Yes, they will see it as easy and they will stick to this number (19,21429) and ignore this one (20). Because most of them will be guided by the calculator, but if it happens …er, ok coming to the phrasing, for them to come to 20 they could have been told that lift will go up even if people are less than 14…]*

AM: *Oh Ja.*

ED: *So bona, etloba taba ya hore lift etlamihile ho ya hodimo provided e le hore e na le batho ba 14. So if ba le less than 14, ho bona implication ke hore, e kese tsamaye lift, hence batlo stick ho 19 times because ho bona it is impossible hore e ka tsamaya 20 times hobane ka kgetlo la bo 20 ba ya shota ka moo ka hare. [So to them, it will be an issue of the lift that must go up provided it has 14 people. So if they are less than 14, for them, the implication is that, lift cannot go, hence they will stick to 19 times because for them, it is impossible for the lift to go 20 times because in the 20th time there will not be enough people in it]*

What the educator is suggesting here is that learners after doing their calculations, they will stick to the answer 19,21429. Because for learners, 19,21429 seems an easier and understandable answer as it was provided by their calculators. So learners believed that only 14 could be travel in the lift at one time up to 19^{th} trip. Because there would be fewer people left, i.e., less than 14 in the 20^{th} trip, in their understanding, the lift could not go up. In order words, the lift could only go up provided there are 14 people in it. In this way, 20 is not acceptable answer, hence most the learners will stick to 19,21429.

Here it shows that the educator knows what her learners will produce in terms of mathematically correct answers and why they will do so. That is, she highlighted the fact that learners would fail to understand that the lift could go up even if it had less than 14 people. In other words, this indicates that learners would not pay attention to the constraints of the concrete situation involved in the problem, or be critical in their consideration of the realities of the problem context.

There is resemblance of responses between Problem 1(B) and Problem 1(C). This suggests that L5 responded the same way to the two problems. In solving Problem 1, L5 stuck to his computational work and as such 'failed' to adapt his responses to the realistic situation.

L6's responses were also analysed and the educator commented this way to Problem 1(A):

ED: *Ho A, o dividile 269 ka 14, né [In Problem 1A, she divided 269 by 14, isn't it?]*

AM: *Hmm*

ED: *So the answer is 19,21429*

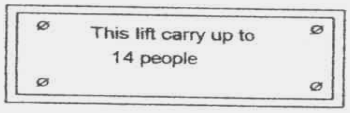
AM: *So o dumellana le answer ya Joe? [So then she agrees with Joe's answer?]*

ED: *Yes, o dividile feela ha a sebedisa any other method. [Yes, she divided, she did not use any other method]*

The educator agrees that L6 used a calculator to find an answer even though the learner did not write down the answer as 19,21429. In her response L6 just wrote "I find the answer by dividing the number of people by the capacity of the lift" to show that she agrees with Joe's answer. Below are L6 responses to Problem 1:

PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes [✓] No [ ]
Please explain why you answered 'yes' or 'no'.

because I find that answer by dividing the number of people by the capacity of the lift.

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?
Yes [ ] No [✓]

Please explain why you answered 'yes' or 'no'.

because I think is a wrong answer when I work it out it thedont give me this answer of 20 times.

C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes [ ] No [✓]
Please explain why you answered 'yes' or 'no'.

because it is a wrong answer to be it.he/she dont listen to the question

In Problem 1(B), L6 answered more or less in the same way L5 did and this is what the educator said about her (L6) response:

ED: *Ho B, …. (reads learner's answer quietly) ok, ho B o ntse a sebedisetse calculator, ka mantswe amang ke calculator, ha e mofe 20, ha a e etsa ka calculator. [In Problem 1B, … (reads learner's answer quietly) ok, In B (referring to Problem 1B) he used a calculator, in other words, she used a calculator, she did not get 20, when she used a calculator]*

AM: *O e entse jwang ka calculator? [How did she do it with a calculator?]*

ED: *Ha kere o a divida, o itse "Because I think is wrong answer when I work it out it don't give me this answer of 20", ka mantswe a mang, o ntse a e wekile out like 269 divide by 14. So then ha a e weka out ha e mofe 20 e mofa this answer (pointing at 19,21429) so yena ha a bone kgetlo lena la bo 20 le ka kenang ka wona mona. [She divided, she said, "Because I think is wrong answer when I work it out it don't give me this answer of 20", in other words, she worked it out like 269 divide by 14. So then she worked out, she did not get 20, it gave her this answer (pointing at 19,21429) so she could not see how 20 could be the answer.]*

AM: *Ka mantswe a mang, reasoning ya hae e ntse e batlile e tshwana le ya L5.[In other words, her reasoning, is more or less similar to that of L5]*

ED: *Yes.*

What the educator is suggesting here is that L6 just like L5 stuck to her answer 19,21429. When L6 divided 269 by 14 she could not get 20. L6 decided against accepting Tim's answer on the grounds that "I work it out it don't give me this answer of 20". In this way, she stuck to her known and reasonable answer of 19,21429 without showing any considerable flexibility. These two learners (L5 & L6), just like the others (L2 & L3), seem to have interpreted "correct" as referring to the act of using a calculator to generate arithmetically correct answer to 269 divided by 14 even if it isn't the best answer to Problem 1 as whole. And in that way, 'refused' to go beyond constraints associated with

standard word problems. In other words, both learners (L5 & L6), did not in any way, adjust their calculations to the given realistic situation.

It is clear that while the educator knows what her learners do mathematically, however, she cannot go beyond these solutions. That is, the educator cannot, like her learners, give adequate solutions in terms of showing how learners would use relevant context-based knowledge, especially in the construction of a mathematical model of a given problem and in the translation of the computational work in the context of the situation.

Key issues have emerged from the data analysis:

- Why do the learners show limited understanding of representation of a number?
- What is the reason for the learners not to be able to use mathematical and real-world knowledge in an integrated manner to solve the mathematical task?

The above issues are the subject of discussion in the next chapter.

CHAPTER 6: DISCUSSION OF FINDINGS

It is important to develop learners' problem-solving skills in knowing when and how to apply their mathematics effectively to various kinds of problems in everyday life. The inclusion of application problems provides a major justification for the important role of mathematics in the school curriculum. Word problems thus constitute an important setting in which learners are expected to make proper use of the problem-solving skills acquired in mathematics teaching. However, as seen from the data analysis, in their interpretations of word problems, learners seemed to follow mathematical rules without reflecting on, or analysing what those rules implied in the specific context in which they were used.

Key issues that emerged from the data analysis, i.e., learners' (P1 and L1) limited understanding of representation of a number, and reasons for learners not being able to use mathematical and real- world knowledge in an integrated manner to solve the mathematical task, are be discussed in this chapter.

6.1 Discussion issue one: learners representation of a number

The RAND Mathematics Study panel (2002) suggests a number of core mathematical practices, including representing among orders. The RAND panel asserts that the domain of representational practices includes the choice we make for expressing and depicting mathematical ideas and how we put them to use. The decimal representation of numbers, for example, is one of the historical examples of representation. For instance, it is easier to compare 4/5 and 13/16 if these numbers are represented in the same decimal form such as 0,8 and 0,8125, respectively.

As seen in "Analysis of data from the pilot study", P1 demonstrated lack of expertise in expressing mathematics and inadequate understanding of representation of a decimal number because he regarded 19,21429 as being bigger than 269. And from "Analysis of data from the main study", L1 understood 19,21429 is being same as 1921429. What is common in the two learners' representation of a number is that a decimal fraction is

regarded as a whole-number. In other words, in their understanding, the following two numbers, namely, 19,21429 and 1921429, are the same. Nasher (1987) calls such "error" or "mistake", a misconception. And she defines misconceptions as "a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic, unconnected and non-systematic errors" (p. 35). This definition is in line with constructivist view that learners' errors characterise initial phase of learning because learners' existing knowledge is inadequate and support only partial understanding. This happens because learners interpret new learning activities in terms of their prior knowledge (Smith, DiSessa, & Roschelle, 1993). Nasher further argues that misconceptions should not be treated as terrible things to be uprooted. Instead, the new knowledge should be connected to the learners' previous conceptual framework.

From a constructivist perspective, misconceptions are very important to learning and teaching. Because misconceptions form part of learner's conceptual structure that will interact with new knowledge, and influence new learning, mostly in a negative way, and because knowledge cannot be transferred ready-made, so to support learner in the construction of his/her own knowledge, misconceptions are bound to occur due to learner's existing mental structure which determines what he/she learns from experience and how it is understood. Because misconceptions differ from concepts presented in instruction, they are often taken as evidence that learners do construct their own knowledge (Hatano, 1996).

Smith *et al* (1993) argue that misconceptions usually originate in prior instruction as learners incorrectly generalize prior knowledge to grapple with new tasks. They further assert that in their efforts to understand the ordering of decimal fractions, learners apply prior knowledge of whole-numbers. Learners who apply whole-numbers knowledge to compare 3,6 and 3,471 ignore the decimal points and treat each as whole-numbers, concluding that 3,471 is greater than 3,6. This is what P1 and L1 did when they treated 19,21429 as 1921429 by applying whole-numbers knowledge on decimal fractions. From results discussed above, it is clear that prior knowledge of whole-numbers is interfering with P1 and L1's knowledge of decimal concepts. This is attributed to an

overgeneralization of the properties of whole-numbers to decimals, but mainly it means these two learners have not yet accommodated their previous schemas to include decimals. P1 and L1 view the new system of decimals as "identical" to the previous system of whole numbers, ignoring the differences between the systems.

To neutralize the interference of misconceptions, Smith *et al* (1983) argue that instruction should confront learners with the disparity between their misconceptions and expert concepts. They further assert that confrontation should begin as an external, social interaction in the classroom. Brodie (2005) explains how this social classroom interaction could take place, by asking learners to explain their thinking when they produce both correct and incorrect contribution; encourage small group work; presentations of ideas to the class; collaborative problem solving on the board; probing and pressing particular learner contributions; and encourage learners to ask questions of each other. In addition, the new curriculum expectations through the critical and developmental outcomes aims at developing learners who are able to communicate effectively with others as member of a team, critically evaluate information in order to solve problems, and participate as responsible citizens in exploring a variety of strategies to learn more effectively.

6.1.1 Learners' understanding of rounding off numbers

An issue that came up under "learners' perspectives" was limited understanding of 'rounding off' of a number as shown by L3 and LP3. In Problem 1(A) as was seen, LP3 agrees with L3 that 19,21428571 was the correct answer. They both disagree with Joe's answer of 19,21429. In their understanding, these two numbers are not the same because one is having 8 digits after the comma while the other is just having 5. They were not aware that one number is rounded off while the other is not. The rounding off numbers in RNCS (2002) is an issue that is not clearly defined in terms of number of digits after the decimal point. Because Learning Outcome 1 as expressed through Assessment Standards stipulates that "we know this when the learner: estimates and calculates by selecting and using operations appropriate to solving problems that involve- rounding off numbers to at least one decimal place" (Ibid. p. 70). As result of "rounding off numbers to at least one

decimal place", classroom practices are encouraged to limit the rounding off numbers to one, two or even three decimal digits.

In South Africa, most of test items in mathematics in the public examinations, rounding off numbers are limited to one or two decimal digits. Hence most items have the following phrases: "approximate to TWO decimal digits"; "… rounded off to one decimal digit". From the results such as those discussed above, it becomes clear that L3 and LP3's understanding of rounding off numbers could be limited to a maximum of three numbers after the comma. This limited understanding of rounding off numbers interferes with their learning because they could not understand that 19,21428571 could be rounded off to five decimal digits, i.e., 19,21429.

Wu (2001) proposes that there is convincing evidence that understanding of fractions holds the key to understanding algebra. He further asserts that the subject of fractions occupies an intermediate position in school mathematics between the arithmetic of whole numbers and algebra. In addition, one way to improve rate of fractions and decimals is naturally to improve the teaching of algebra. However, fractions according to Wu, are not properly taught because they are never defined in textbooks or professional development materials. Furthermore, Wu claims that learners are completely lost as to what a fraction is, because no clear definition is ever offered. However, with proper infusion of precise definitions and clear explanations, the teaching of fractions can eventually hope to contribute to mathematics learning.

In my opinion, these possible characteristics and causes identified in the preceding paragraph, suggest that the following aspects need more attention: educator training, curriculum planner and textbook writers. Educator training is needed because pedagogical content knowledge (Shulman, 1986) which involves knowing things like what topic learners find interesting or difficult and what the representations are that are most useful for teaching a specific content idea. Pedagogical content knowledge as a unique kind of knowledge that intertwines content with aspects of teaching and learning, seems to be an answer in improving the teaching of algebra. It is also very important that

curriculum planner allocate more time on fractions seeing that understanding of fractions holds the key to understanding algebra. And with regard to textbook writers, it is important that they are advised about the importance of definition of concepts in the teaching and learning of mathematics.

6.2 Discussion issue two: Learners' mathematical and real-world knowledge

An analysis of learners' responses to real-world problems yielded an alarmingly small number of realistic responses. This happened in spite of the fact that learners were given more realistic problems to solve. Verschaffel and De Corte (1997) argue that reasons for the learners not to be able to use mathematical and real-world knowledge in an integrated manner to solve mathematical word problems are as a result of the following classroom practice and culture: (a) the impoverished standard word problems that can always be solved through the most obvious arithmetic operation(s); and (b) the way in which word problems are used in current instructional practice, with all attention being devoted to computational proficiency at the expense of interpretive skills.

Reusser and Stebler (1997) suggest that two types of factors figure heavily in the difficulty of realistic mathematical responses in the school word problem solving, namely, textual and contextual factors. They define textual factors as those relating to stereotyped nature of most frequently used textbook problems, and contextual factors as those that are associated with practices, contexts and expectations related to classroom culture of mathematical problem solving. They further claim that only a few word problems are employed in the classrooms and textbook that invite or challenge learners to activate and use everyday knowledge and experience.

The 'failure' of learners to make proper use of real-world knowledge has been amply documented. It has been argued that current use of word problems does not foster in learners realistic considerations. Rather than providing realistic contexts that encourage learners to use commonsense knowledge and experience in the problem-solving process, school word problems are perceived as artificial, puzzle-like tasks unrelated to real world.

Learners not only know from their school mathematical experience that all word problems are undoubtedly solvable, but also that everything in the problem text is confined to relevance and no-ambiguity. And in this way, mathematical word problem statements degenerate to badly disguised equations.

Verschaffel and De Corte (1997) argue that it is useful to think of applying mathematics to solve real-world problems as a process that involves a number of phases: understanding the problem situation, constructing a mathematical model that describes the essence of those situational elements, rearranging the mathematical model in order to identify its own elements, interpreting the outcome of the computational work in terms of the practical situation, and communicate the result(s). According to this mathematical model, mathematical concepts and procedures are also considered as part of problem solver's armory which must be integrated with other kinds of knowledge and skills.

I here also want to refer to specific aspects that are relevant for improving learners' mathematical and real-world knowledge in an integrated manner when solving mathematical word problems. Skemp (1976) differentiates between relational and instrumental understanding. He suggests on one hand that instrumental understanding is "rule without reasons" (p. 20) in that learners may posses the necessary rules and ability to use them without actually understanding why and how, that rule works. This type of understanding is similar to standard way of solving mathematical word problems in that it emphasizes computational skills (mathematical knowledge) in order to improve performance in learners' work.

Relational understanding on the other hand, involves understanding both "knowing what to do and why". Knowing what to do in a specific situation and understanding why it works may enhances learners' participation in the learning activities. Relational understanding is the type of understanding that integrates learners' mathematical and real-world knowledge because it invites or encourages learners to construct their own meanings, make connections, and evaluate or make judgement of their actions. While participating in their learning, learners do so through interaction with others (social) in

familiar cultural context (Lave & Wenger, 1991), realistic and therefore also meaningful to learners.

Reusser (2000) argues that understanding and solving of mathematical word problems is a complex process that requires skilful interaction of at least three kinds of knowledge: linguistic, situational and mathematical. In many cases, the teaching and learning of mathematics is seen to be a process of learning mathematics as devoid of the language. For learners to be able to use mathematical and real-world knowledge in an integrated manner, learners must be able to display linguistic competence in mathematics. Zevenbegen (2000: 204) takes a standpoint that "within the mathematics classrooms, legitimate participation is acquired and achieved through a competence with written or spoken texts, or both". In order words, learners come to learn mathematics as a language equally in written and spoken forms in order to become effective speakers, listeners, and communicators in mathematics classrooms. Learners who are lacking in the relevant types of knowledge may need guidance of "effective pedagogical settings" (Reusser (2000: 23). This involves presenting problems in familiar contexts that induce an active thought process- thereby making learning a meaningful activity.

It is very important to maintain a balance between providing learners with success through the attainment of mathematical proficiency such as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick, Swafford, & Findell, 2001) while also providing them with activities in meaningful contexts that are relevant and purposeful.

6.3 Discussion issue two: Realistic responses to standard problems

As seen in Table 4 (summary of response categories), only one standard problem was responded to realistically. That is, most of standard problems were not categorized into RA. The question that remains is, why were standard problems not responded to realistically? And what was the reason for learners to respond realistically to that particular standard problem?

The fact that standard problems were not responded to realistically, was as a result of what Barnes *et al*. (1969, in Boaler, 1997) calls “tendency of mathematics teachers to ask closed questions with short answers which do not require any interpretation or reasoning” (p. 21). In other words, learners are not encouraged to find their own ways of solving mathematical problems and because of this, when they approach new situations they do not try to interpret what to do. Instead, they try to remember rules they learned.

The two learners (L1 & L4) who responded realistically to a standard problem, did so because the standard problem given in the mathematical task had the same content as real- world problems. Because of this, these two learners responded realistically to both real-world and standard problems in Problem 1.

In this chapter I elaborated on two key issues that emerged from the analysis of the data. In the next chapter, I conclude the research report by providing a summary of the findings of the study as well as presenting recommendations for classroom practice; teacher education and further research. Finally I reflect on the research process.

CHAPTER 7: CONCLUSION

7.1 Summary of findings

The purpose of the study was to explore whether and how the use of real-life contexts influences Grade 8 learners' responses in solving mathematical problems. A mathematical task was given to explore whether the influence exists and if so how.

7.1.1 Critical research questions

1) (a) How do learners respond to written real-world mathematical problems?
   (b) How do they respond to standard problems that have the same content as real world problems?

2) What do learners think about the purpose that contexts play in solving these mathematical problems?

a) **Learners' responses to real-world mathematical problems**

The analysis of learners' responses to real-world problems hardly matched this idea of mathematization which Freudenthal (1973, in Reusser & Stebler, 1997) describes as the structuring of reality by mathematical means. That is, most of the learners 'understood' and 'solved' mathematical word problems without considering factual relationships between real-world situations (what the problem texts are about) and mathematical operations. They therefore solved real-world problems as if they were standard problems. Analysis of real-world problems provides the evidence that the learners are not skilled in using real-world knowledge in the classroom. However, through their responses, most of them showed that they have the necessary mathematical skills as evidenced by analysis of responses to standard problems.

Learners 'failed' to respond realistically to written real-world mathematical problems even though attempts were made to encourage a more realistic perspective through:

(i) presentation of three competing answers which could encourage a learner to reflect on features of the presented situation;

(ii) introduction of learners' names might influence the nature of their responses because it had been found in Cooper and Dunne (2000a) that items containing names evoke aspects of everyday life experiences. Learners appear to work on the understanding that all mathematics problems have a solution. In addition, they seemed to have difficulty in adapting their solutions to the situation once they finished their computational work. In order words, they are not used to bringing in knowledge from the world outside in solving classroom mathematical problems;

(iii) verbally forewarning them that some of the questions would require their honest opinion thereby 'putting themselves in the situation' in order answer the questions.

b) **Learners' responses to standard problems that have the same content as real-world problems**

As indicated above, through analysis of responses, the majority of learners demonstrated that they have necessary mathematical skills in solving standard problems. This was further evidenced by learners who responded 'correctly' in the mathematical sense to real-world problems as was reflected by a high number of responses that were categorised in *non-realistic answer* (NA). The mere fact that many responses were categories in NA and not in *technical error* (TE) or the *other answer* (OA), showed that even though learners responded 'incorrectly' to real-world problems, i.e., gave answers that simply do not make sense if one takes the problem statements seriously, but they were correctly able to apply their mathematical skills in solving these problems. As Table 4 (summary of response categories) shows, learners responded correctly to standard problems (Problem 1(2), 2(2) and 3(2)) that have the same content as real world problems as evidenced by a high number of responses in NA.

c) **Learners' thoughts about the purpose that contexts play**

With reference to analyses of learners' written responses and interviews, it became evident that most learners did not think about the purpose that contexts play in solving

mathematical word problems. That is, learners' responses to real-world problems yielded an alarming small number of realistic responses or comments, indicating that they restricted their explanations to how they did their computational work. In particular, L2, L3, L5 and L6 solved all their real-world problems without consideration of realistic situations. However, L1 and L4 were able to think deeply about the implied real-world situation when solving Problem 1 in that they were able to relate the number of times the lift went up with their calculations. Even though L1 and L4 were able to take realistic considerations when solving Problem 1, nevertheless, they 'failed' to do so in Problem 2 and 3. This could suggest that they could relate their experiences to the lift in Problem 1.

Learners' limited understanding of representation of a decimal and rounding off of numbers showed that they lack the necessary mathematical skills which in turn impacts heavily on classroom practice. In addition, learners' 'failure' to respond realistically to written real-world problems provides evidence that they restricted their explanations to computational work and that they were not skilled in using real-world knowledge in the classroom. However, the majority of learners showed that they have necessary mathematical skills in solving standard problems that have the same content as real world problems.

7.2 Recommendations

7.2.1 Recommendations for classroom practice

That most of learners in this study seemed not ready to enter into the type of reasoning encouraged by given mathematical task, suggest that the current stereotyped word problems are not encouraging the breadth and depth of thinking that otherwise could be possible in our classrooms. Given that current views emphasize an application of mathematics to realistic problems, it is important to create a new classroom culture that pays attention to activities such as mathematical modeling, explaining, discussing valuable problem-solving strategies, and listening carefully to learners' explanations and justifications of their own solutions (De Corte, 2000). The new school curriculum thus strives for the creation of a classroom culture that enables learners to meet these

qualitative skills. This can be seen in the Learning Outcomes to be achieved at the end of the process thereby encouraging learner-centred and activity-based approach to education. Furthermore, the curriculum seeks to create a lifelong learner who is confident, literate, multi-skilled, and able to think logically and participate in society as a critical and active citizen (DoE, 2002).

What is also needed is a change of word problems from "puzzle-like activity with no grounding in factual real-world structure" (Reusser & Stebler, 1997: 323) to the design of more challenging, authentic ways of problem posing. Referring to research reported by Reusser & Stebler (1997), they suggest that mathematical word problems "should become exercises of realistic mathematical modeling, i.e., of both real-world based and quantitatively constrained sense-making" (p. 324). That is, mathematical problems will need permeability of boundary between learners' everyday knowledge and experience and their purely mathematical knowledge.

7.2.2 Recommendations for teacher education

Recommendation is that a variety of teaching methods is needed with regard to the teaching of word problems with an aim of stimulating active, and constructive learning activities. This could include, small-group problem solving in mixed-ability groups, whole discussion of solutions and solutions strategies, and individual assignments (De Corte, 2000). Teaching strategies should also include applying mathematical problems in real life where learners learn that they are not learning for passing examinations only, they also need to understand that mathematics uses commonsense knowledge.

The way in which word problems are used in current instructional practices, and more specifically in, public examinations, test items, class activities, and continuous assessment tasks, it is recommended that mathematical and real-world knowledge be used in an integrated manner to model and solve them properly. Rather than teaching learners to ignore the context and deal with numbers in word problems, educators need to encourage learners to focus on the argumentation involved rather than the correct answer

thereby bringing integrated mastery of knowledge and skills in their assessment. It is also suggested that the development of realistic mathematical problems and interpretation of word problems permeate the entire mathematics curriculum.

I suggest that the mindset of educators needs to change with regard to the teaching of word problems in the classrooms. That is, educators need to understand that mathematical word problems are "authentic or possible situation in an everyday context" (Wyndhamn & Saljö, 1997: 356) and not "mathematical exercises in which an algorithm is hidden and learners need to identify" (p. 366). In this way, learners will be taught proper use of real-world knowledge. In addition, introduction of new teaching and learning materials aimed at connecting mathematical problem solving and experiential world of learners are needed for realistic modeling of problem situations.

7.2.3 Recommendations for further research

- I suggest that when exploring the influence of contexts in learners' responses, a different approach might be used wherein the written task is done in small mixed-ability groups of three to four learners. The aim would be to avoid the insistence of learners on giving one single answer in spite of the fact that they realise that there could be more than one correct answer. This would also allow learners to produce different interpretations of the relationship between the everyday knowledge and mathematical knowledge and recognize both as relevant resources on which to draw.
- In all standard problems that were given, there was only one problem that was responded to realistically. Now the question is, can learners also be encouraged to respond realistically to standard problems, and if not why? I therefore propose further research that would attempt to address this question.
- Future studies should not merely assume that too little attention is paid to the relevant real-world knowledge but must also need to investigate mastery of this knowledge. One suggests that learners' problem-solving and reasoning processes be probed further through approaches such as tests, interviews, or questionnaires.

7.3 Reflections

The study enlightened me to the way learners were thinking. It showed the necessity for educators to listen to or read explanations of learners in order to understand their line of reasoning. It is through their explanations, either written or spoken, that we as educators are able to identify their mathematical reasoning as well as their impediments to such reasoning.

The problems that I encountered were learners' inability to put their thoughts on paper and as a result, most of them failed to comment or give their written responses. That happened in spite of the fact that learners were forewarned that once they have made their choices of 'yes' or 'no' responses, they needed to motivate those choices. I do not know what could have caused that problem, but I think the language could have been one of the reasons. Classroom practice which does not enforce written explanations could also be the other reason. In addition, some of the learners were unwilling to take part in the study while others did not want to be interviewed. And those who wanted to be interviewed, some refused to have interviews with them recorded.

What is interesting about the study is what came up after the selection of one grade 8 class. All other grade 8 classes that were not selected, proposed that whenever I'm through with the selected class, they would be eagerly waiting to be part of the study too. Again, learners of the selected class would come to the researcher and ask about their performance in the mathematical task. This happened even though these learners were told before writing that the task was not about performance, but that the researcher wanted to learn from them. In other words, for these learners, performance comes first.

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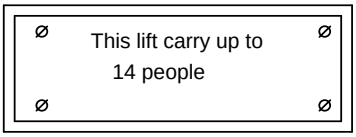
APPENDICES

APPENDIX 1: LEARNERS' MATHEMATICAL TASK

Learners will be given the following questions to answer.

PROBLEM 1

1. **Real-world problem:** This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes ☐ No ☐
Please explain why you answered 'yes' or 'no'.
_____
_____

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?
Yes ☐ No ☐

Please explain why you answered 'yes' or 'no'.
_____
_____

C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes ☐ No ☐
Please explain why you answered 'yes' or 'no'.
_____
_____

2. **Standard problem:** A lift capacity is 16 and 238 people need to be transported in the morning rush. How many times must it go up? Explain your answer.

_____
_____

PROBLEM 2

1. **Real-world problem:** John's best time to run 100m is 17 seconds. How long will it take him to run 1km?

A. Sipho said the answer is 170 seconds.

Can you see a way that 170 seconds could be the correct answer?

Yes ☐ No ☐

Please explain why you answered 'yes' or 'no'.

_____
_____

B. Jake said the answer is 2 minutes and 50 seconds.

Can you see a way that 2 minutes and 50 seconds could be the correct answer?

Yes ☐ No ☐

Please explain why you answered 'yes' or 'no'.

_____
_____

C. Mary said it is impossible to find the answer.

Can you see a way that **impossible** could be the correct answer?

Yes ☐ No ☐

Please explain why you answered 'yes' or 'no'.

_____
_____

2. **Standard problem:** A boat sails at the speed of 45km/h. How long does this boat take to sail 180km?

_____

_____

PROBLEM 3

1. **Real-world problem:** Rob was born in 1978. Now it's 1995. How old is he?

A. Steve said the answer is 17 years.

Can you see a way that 17 years could be the correct answer?

Yes ☐ No ☐

Please explain why you answered 'yes' or 'no'.

_____

_____

B. Kathy said the answer is 16 years.

Can you see a way that 16 years could be the correct answer?

Yes ☐ No ☐

Please explain why you answered 'yes' or 'no'.

_____

_____

C. Carl said the answer cannot be known precisely.

Can you see a way that the answer cannot be known precisely?

Yes ☐ No ☐

Please explain why you answered 'yes' or 'no'.

_____

_____

2. **Standard problem:** A shop keeper has two containers for apples. The first container contains 60 apples and the other 90 apples. He puts all apples into a new, bigger container. How many apples are there in that new container?

_____

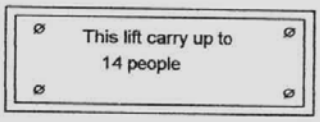
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APPENDIX 2: PRELIMINARY POSSIBLE RESPONSES

LEARNERS' MATHEMATICAL TASK

PROBLEM 1

1. This is the sign in a lift at an office block:



In the morning rush, 269 people want to go up this lift.
How many times must it go up?

- Here are the learners' different answers. Please answer the questions about them.
- You are allowed to use a calculator.

A. Joe said the lift went up 19,21429 times.
Can you see a way that 19,21429 could be the correct answer?

Yes [X] No [ ]
Please explain why you answered 'yes' or 'no'.

Possibly using a calculator to divide 269 by 14 could give the answer as 19,21429

B. Tim said the lift went up 20 times.
Can you see a way that 20 could be the correct answer?
Yes [ ] No [X]

Please explain why you answered 'yes' or 'no'.

Looking at the two numbers, there is no way that 14 can divide into 269 without any remainder.

C. Gita said the lift went up 25 times.
Can you see a way that 25 could be the correct answer?

Yes [ ] No [X]
Please explain why you answered 'yes' or 'no'.

When dividing 269 by 14 it cannot give you 25 as 14 multiplied by 25 will give you 350 which is much greater than 269.

APPENDIX 3: POSSIBLE INTERVIEW QUESTIONS (PILOT)

Note: A semi-structured interview schedule was developed after the researcher and the educator, jointly completed Problem 1 (see Appendix 2) wherein we wrote possible solutions to the problem. A specific focus in the analysis was to establish how learners would use information about the "context" of the problem situation given. As reviewed in the literature, the way a learner approaches solving a particular problem may be influenced by the context of the problem.

Here below are my initial probes that will form part of the interview protocol:

For Problem 1:

1. How did you solve **Problem 1(A)**? In what way was it the same as that given by Joe? Please explain.
2. In **Problem 1(B),** you said 'no'. How did you make that choice? Please explain.
3. In **Problem 1(C),** you also said 'no'. Please explain how you made that decision?
4. How did the answers given by Joe, Tim and Gita help you to solve the problem? Please explain.
5. Was it easy or difficult for you to solve these types of problems? Please explain.

I probed learners' responses to Problem 2 and 3 in a similar way as above. I obviously paid attention to the way the learners respond to my initial probes and steered the interview in the directions suggested by learners' responses.

APPENDIX 4: PILOT INTERVIEW: EDUCATOR

This is a rehearsal of pilot interview: researcher and the Grade 8 maths educator (AM = researcher and ME = maths educator)

AM: *For Problem 1. How did you solve Problem 1(A), which is the first one. In what way was it the same as that given by Joe? Please explain.*

ME: *Ha ke re A, ke 269 divide by 14, how many times does 14 go into 269. So then, possibly e ba obvious hore it will give you fractional number, ha kere? [As seen in Problem 1(A), it is 269 divide by 14, how many times does 14 go into 269. So then, possibly it's obvious that it will give you fractional number, isn't it?]*

AM: *Hmm (an indication of an agreement)*

ME: *So then using a calculator, possibly karabo e tlaba yona 19,21429 [So then using a calculator, possibly the answer will be 19,21429]*

AM: *If your divide?*

ME: *Ja, you divide 269 by 14, ha kere? [Yes, you divide 269 by 14, isn't it?]*

AM: *Hmm*

ME: *E tlaba how many times 14 people go into 269. So then e a tshwana le ena (pointing at 19,21429), because I'm aware hore if you divide 269 by 14 there will always be a remainder, ha kere? [It will be how many times 14 people go into 269. So then is the same as this one (pointing at 19,21429), because I'm aware that if you divide 269 by 14 there will always be a remainder, isn't it?]*

AM: *Hmm. And number 2 (referring to Problem 1(B)), you said 'no'. How did you make that choice? Please explain.*

ME: *Number 2, it can't be 20 or a whole number because 14 into 269, e ka se kene ya ofa ntho without any remainder. So then if it is 20, mathematically, the number of the decimal according to the calculator e ba nyane, ha kere? [In number 2 (referring to Problem 1(B)), it can't be 20 or a whole number because 14 into 269, can divide but leave a remainder. So then if it is 20, mathematically, the number of the decimal according to the calculator becomes small, isn't it?]*

AM: *Hmm*

ME: *So then I can't ke etsa hore ebe 1, hence, le a child a ke keng a dumela hore e be 20. But e tlisa some confusion? [So then I can't force it to become 1, hence, even the answer given by a learner cannot be 20. But it brings some confusion?]*

AM: *Jwang? [How?]*

ME: *Yes, it will go 19 times, ha kere? [Yes, it will go 19 times, isn't it?]*

AM: *Hmm*

ME: *Then ngwana o aware hore hona le batho ba salang, but when you divide, it will be 19, and 19,2 can't be 20, but truly speaking, those people, le ha ba kaba ba less than 14, they still have to go up, ha kere? [Then a learner is aware that they will be people who remain behind, but when you divide, it will be 19, and 19,2 can't be 20, but truly speaking, those people, even if those people are less than 14, they still have to go up, isn't it?]*

AM: *Hmm*

ME: So then, ka mantswe a mang, the correct answer will be 19 times with the full amount ya lift, then last trip which is the 20th time, so then e tloba le less people, that is less than 14. [So then, in other words, the correct answer will be 19 times with the full amount of lift, then the last trip which is the 20th time, so then will have less people, that is less than 14.]

AM: Ok. And ho C (referring to Problem 1(C), you also said 'no'. Explain your decision.

ME: Ho C, it's 'no' because 14 cannot go 25 times into 269. So then ke e fumane ka hore 14 multiply by 25, it gives you a number that is greater than 269. [In C (referring to Problem 1(C)), it's 'no' because 14 cannot go 25 times into 269. So then I found the solution by saying 14 multiply by 25, it gives you a number that is greater than 269]

AM: So possibly, ngwana o tlo responda in that way? [So possibly, the learner will respond in that way?]

ME: Ee, so then batho ba se bale bangata. [Yes, so then people are now many (referring to the multiplication of 14 by 25 which will give rise to more than 269 people)]

AM: Ok. Ena ha re e sheba ya number 4, in fact, how did the answers given by Joe, Tim and Gita help you to solve the problem? Please explain. [Ok. When will look at number 4 (referring to question number 4 according to Appendix 2) in fact, how did the answers given by Joe, Tim and Gita help you to solve the problem? Please explain.]

ME: Ha kere di se di ntse di o guarda ne, so then normally ha o sebedisa division, and division ke inverse ya multiplication, ke a tsaba hore if I have to divide 269 by 14, the answer I get to proof hore I am correct, I must multiply it by number that I divided with. So then what I did ke nkile the very same 14 ya batho so if I multiply it by the given numbers tsa bo Joe, Tim le Gita I'm expecting to get at least something like 269. So then it was easy. You multiply like 14 by 19 you get 19,21429, 14 by 20 o kgona ho bana hore e fana ka difference e kae, and you multiply 14 by 25 o kgona ho bona hore e feta 269. [They are guiding you, so then normally when you use division, and division is the inverse of multiplication, I know that if I divide 269 by 14, the answer I get to proof that I am correct, I must multiply it by number that I divided with. So then what I did, I took the very same 14 so if I multiply it by the numbers given by Joe, Tim and Gita, I'm expecting to get at least something like 269. So then it was easy. You multiply like 14 by 19 you get 19,21429, 14 by 20 you get a certain number and you're able to see the difference, and you multiply 14 by 25 you get number exceeding 269]

AM: So bana batla e bona e le easy? [So learners will see it as easy?]

ME: Ja, batla e bona e le easy and they will stick to this answer (19,21429) and ignore this one (20). Because most of them batla be ba gaidwa ke calculator, but if e le hore …er, ok coming to the phrasing, for them to come to 20 they could have been told hore lift e ya tsamaya e ntse e sena batho ba 14… [Yes, they will see it as easy and they will stick to this number (19,21429) and ignore this one (20). Because most of them will be guided by the calculator, but if it happens …er, ok coming to the phrasing, for them to come to 20 they could have been told that lift will go up even if people are less than 14…]

AM: *Oh Ja.*

ME: *So bona, etloba taba ya hore lift etlamihile ho ya hodimo provided e le hore e na le batho ba 14. So if ba le less than 14, ho bona implication ke hore, e kese tsamaye lift, hence batlo stick ho 19 times because ho bona it is impossible hore e ka tsamaya 20 times hobane ka kgetlo la bo 20 ba ya shota ka moo ka hare. [So to them, it will be an issue of the lift that must go up provided it has 14 people. So if they are less than 14, for them, the implication is that, lift cannot go, hence they will stick to 19 times because for them, it is impossible for the lift to go 20 times because in the 20th time there will not be enough people in it]*

AM: *Ok. Thank you.*

APPENDIX 5: PILOT TRANSCRIPT

The dialogue between the educator and the learner about the learner's mathematical task. (AM = researcher; P1 = learner from pilot)

AM: *The question says "Joe said that the lift went up 19,21429 times…"*
P1: *Yes*
AM: *"Can you see a way that 19,21429 could be the correct answer?". O be o re 'no'.[You said 'no']*
P1: *Yes*
AM: *Wa re [You said] "Because if it 19,21429 it will have to go up more than the people in it".*
P1: *Yes*
AM: *O ne o batla hore eng?[What did you mean?]*
P1: *Ke hlalosa hore e tlo tlameha ho ya ka makgetlo a mangata ho feta 269, ke hore 269 e ka kena ka makgetlo a mangata ho number ena, 19,21429, e ka kena ka makgetlo a fetang ten. [I am saying the lift will have to go more than 269 times, that is, 269 can divide many times in this number, 19,21429, it can divide into more than ten times]*
AM: *Ho 269 le 19,21429 ke efeng e kgolo, ho ya ka wena?[In your opinion, between 269 and 19,21429,which number is big?]*
P1: *E kgolo?[The big one?]*
AM: *Ee [Yes]*
P1: *Ke … ke 19,21249 [Its …its 19,21249]*
AM: *Why o nahana hore 19,21249 ke yona e kgolo? [Why do you think 19,21249 is the big number?]*
P1: *Ka baka la value ya teng [Because of its value]*
AM: *Ka ntle ho value, ke eng e etsang 19,21249 ebe kgolo? [Besides value, what make 19,21249 to be big?]*
P1: *E ne hape e bontsha hore lift e tlo ya hodimo ka makgetlo a mangata.[And again it shows that the lift will go up many times]*
AM: *Pele re ya ho B, answers tse fannweng ke Joe, Tim le Gita, na di othusitse ho solva the problem kapa ha di a othusa?[Before we get to Problem 1(B), did answers given by Joe, Tim, and Gita, help or not to solve the problem?]*
P1: *Di nthusitse. [They help me]*
AM: *Ka tsela e jwang? [How?]*
P1: *Kene ke tseba hore hona le e iwane e leng yona.Ha ke thola ena (point at Problem 1(B) ke ne ke se ke tseba hore ena (Problem 1(C) ha se yona. [I knew that out of 3 answers, one must be correct. When I got this one (pointing at Problem 1(B)), I knew that this one (pointing at Problem 1(C)) was not correct]*
AM: *Ok. In B (Problem 1B), "Tim said that the lift went up 20 times. Can you see a way that 20 could be the correct answer? O be o re 'yes'. [You said 'yes']*
P1: *Yes.*

Ellipses (…) indicate a brief pause of a few seconds or less

AM: O be o re "it will take 20 but 3 are going to be left. 20,3". O e thotse jwang 20,3 ena? [Then you said, "it will take 20 but 3 are going to be left. 20,3". How did you get 20,3?]

P1: E tlo ya hodimo ha 20, e be siya batho ba 3.[It will go up 20 times, and then 3 people will be left behind]

AM: Hore e ye ha 20 o e thotse jwang?[To say that it goes up 20 times, how did you get it?]

P1: Ke e calculatile ka …. ka ho etsa… ka hore 269 – 14 [I calculated it, by saying 269-14]

AM: Ok, 269 – 14, and then?

P1: Be ke minus, be ke ntse ke couter ho re eya makgetlo a makae. [I then subtracted, and counted how many times it went up]

AM: Answer o be o ethola jwang?[How did you get the answer?]

P1: Answer enngwe le enngwe eo ke e tholang ke minus 14 ho fihlela ke thola hore e ya ka makgetlo a makae. Ke be ke thola hore ke 20, ho sala 3. Ke be ke ngola 20,3.[I then kept on subtracting the new answers from 14, and counted how many times it went up. The answer was 20 and 3 remained behind]

AM: Ok. All right. So etlo o neha answer ya 20,3 ho ya ka mokgwa oo o cauntileng ka teng?[Ok. All right. So it will give you an answer of 20,3 the way you have calculated?]

P1: Yes.

AM: In C (Problem 1C), "Gita said the lift went up 25 times". O be ore no.[You said no]

P1: Yes.

AM: Ha o dumellene le yena. O be ore "because it will be more than 269 people to go up". O ne o batla ho re eng?[You do not agree with her. You said "because it will be more than 269 people to go up". [What did you mean?]

P1: Kene ke re e tlo hloka batho ba 269 ho re etlo tseba ho ya hodimo ha 25.[I meant, it will need 269 people to go up 25 times]

AM: O re e tlo nida batho ba 269 ho re e ye hodimo ha 25?[You say it will need 269 people to go up 25 times?]

P1: Yes

AM: O e thotse jwang ho re e tlo nida batho ba more than 269?[How did you know that it will need more than 269 people?]

P1: Hobane ke ile ka sheba ena (Problem 1B) hore e ya ha 20 e siya batho ba 3. So ena ha eno kgona ho ritjha 25. [Because I checked Problem 1B to ascertain that it went up 20 times and left 3 people behind. So this one here will not reach 25]

AM; Ooh. Ok. Ka mantswe a mang o bolela hore ha e no feta….[Ooh.Ok. In other words, you mean it will not exceed….]

P1: Ha eno feta 20. [It'll not exceed 20]

AM: Ok. Ho number 2, "A lift capacity is 16 and 238 people need to be transported in the morning rush. How many times must it go up? Explain your answer". O be ore " It will go 14,14 because will be left and it will go 14 times up". O ethotse jwang 14? [Ok. In Problem 1(1), "A lift capacity is 16 and 238 people need to be transported in the morning rush. How many times must it go up? Explain your

answer". You said "It will go 14,14 because will be left and it will go 14 times up". How did you get 14?]

P1: Ke entse jwale ka Problem 1B, ke be ke thola hore ho sala batho ba 14. [I solved it the same way I solved Problem 1B, and realised that 14 people will be left behind]

AM: So sebedisetse method o tswanang le Problem 1B?[You used similar method to Problem 1(B)?]

P1: Yes.

AM: Ok. Ho Problem 2A, "John's best time to run 100m is 17 seconds. How long will it take him to run 1km?. Sipho o ne a itse answer ya hae ke 170 seconds. Can you see a way that 170 seconds could be the correct answer?". O be o re' yes'. [Ok. In Problem 2A, "John's best time to run 100m is 17 seconds. How long will it take him to run 1km?. Sipho said the answer is 170 seconds. Can you see a way that 170 seconds could be the correct answer?". You said 'yes']

P1: Yes

AM: O be ore [You said] "because 100m is 10 times on 1km so it makes 170 seconds. I said 10×17 = 170 seconds". All right?

P1: Yes.

AM: Le mona ho Problem 2B, o thotse hore 170 seconds e ntse e le the same as 2 minutes and 50 seconds.[Even in Problem 2B, you got 170 seconds which is the same as 2 minutes and 50 seconds.]

P1: Yes, e ntse e le the same.[Yes. It is the same]

AM: Ho Problem 2C, "Mary said it is ***impossible*** *to find the answer. Can you see a way that* ***impossible*** *could be the correct answer?. O be ore 'no'… [In Problem 2C, "Mary said it is* ***impossible*** *to find the answer. Can you see a way that* ***impossible*** *could be the correct answer?. You said 'no'…]*

P1: Yes, because any sum e ka kgona ho calculatiwa. [Yes, because any sum can be calculated]

AM: Ho ya ka wena, there is no way that ho ka thweng answer e impossible?[In your opinion, there is no way that the answer can be impossible?]

P1: Yes.

AM: Always, there must be an answer?

P1: Yes sir.

AM: Ok. Number 2(2), "A boat sails at the speed of 45km/h. How long does this boat take to sail 180km?". O be ore, "It will take 45km per 3 hours". How did you get 3 hours?[Ok. Problem 2(2), "A boat sails at the speed of 45km/h. How long does this boat take to sail 180km?". You said, "It will take 45km per 3 hours". How did you get 3 hours?]

P1: Ke itse 180 ÷ 45 = 3, so 45 e iwane e fana ka 1hour. [I said 180 ÷ 45 = 3, so one 45 represent 1hour]

AM: Ok. How was the mathematical task? Difficult, easy or just moderate?

P1: E ne e ele easy. [It was easy]

AM: How about the language, na o ne o understand hore ho buwa ka eng?[How about the language, did you understand it?]

P1: Yes, kene ke understand everything.{Yes, I understood everything]

AM O ne o sena bothata?[You did not encounter any problem?]

P1: Yes
AM: Thank you.

APPENDIX 6: LETTER OF CONSENT TO THE PRINCIPAL

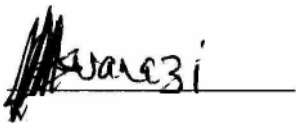
Dear Principal

Hello! My name is Abram Mkwanazi, mathematics educator at your school and student at the University of the Witwatersrand. I hereby wish to ask for a permission and approval to conduct a research with Grade 8 mathematics learners. The research is about exploring whether and how the use of real-life contexts influences learners' responses in solving mathematical problems. The information from this research will help us understand more about how learners apply their mathematical knowledge to real word events in solving mathematical problems and how learners are able to move beyond constraints associated with standard problems to recognize the problems as contexts in which they can apply everyday reasoning.

Once permission has been granted, letters of consent will be sent to learners and their parent(s)/guardian(s) to ask their participation in this research project. Learners have to decide whether they want to take part or not. If they agree to participate in this research they will be asked to write a test. A test will not be for marks but for research purposes. The date for the test will be negotiated with them. They will also be selected for an interview. The interview will focus on what they have written. With their permission, interviews will be tape-recorded so that I can ensure that I make an accurate record of what they say. In reporting on this research, I will not use their names. All records of their responses will be kept in a safe place.

I hope to keep you informed about the progress as well as findings of the research.

Thank you in advance.



M.A. Mkwanazi
Researcher

APPENDIX 7: LETTER OF CONSENT TO THE EDUCATOR

Dear Educator

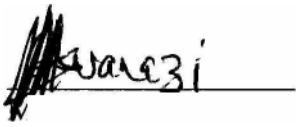
Hello! My name is Abram Mkwanazi, I'm seeking consent for your participation in a research project that is part of my Msc program at the University of the Witwatersrand. The aim of the research is about exploring whether and how the use of real-life contexts influences Grade 8 learners' responses in solving mathematical problems. The information from this research will help us understand more about how learners apply their mathematical knowledge to real word events in solving mathematical problems and how learners are able to move beyond constraints associated with standard problems to recognize the problems as contexts in which they can apply everyday reasoning.

I would like to invite you to take part in scrutinizing a draft of a mathematical task, to comment on the language and the level of content. In addition, I also like to invite you to help me in administering and scoring of the mathematical task to ensure the reliability as well as being part of rehearsal of pilot interview. With your permission, I would like to tape record our discussion so that I do not forget the important things you tell me. In reporting on this research, I will not use your names. All records of your responses will be kept in a safe place.

Your participation in this research is voluntary. If you agree to participate, you will be free to withdraw from the research at any time during the study. I do hope that you decide to participate. Your participation will be adding important ideas to this study. If you agree to participate, please complete the consent form on the next page, sign and return it to me.

Remember that you don't have to take part unless you want to. If you have any questions about this research project you should feel free to ask the researcher.

Thank you for your cooperation.



M.A. Mkwanazi
Researcher

APPENDIX 8: CONSENT FORM FOR THE EDUCATOR

UNIVERSITY OF THE WITWATERSRAND
MATHEMATICS RESEARCH PROJECT
RESEARCHER: M.A. MKWANAZI

Consent form for educator participating in a research project.

I, (educator's name) _____ , hereby agree to be a participant in the research project. I understand that the purpose of the research is to explore whether and how the use of real-life contexts influences Grade 8 learners' responses in solving mathematical problems. I also understand that, in reporting on this research project, anonymity of my name is guaranteed. All records of my responses are confidential and as such will be kept in a safe place.

I acknowledge that

- the aims, methods, anticipated benefits and consequences of the research, have been explained to me.
- I voluntarily and freely give consent to my participation in such study.
- results will be used for research purposes and may be reported in educator and academic journals.
- I am free to withdraw my consent at any time during the study.

....
Signature of educator Date

....
Signature of researcher Date

APPENDIX 9: LETTER OF CONSENT TO PARENT(S)/GUARDIAN(S)

Dear Parent(s)/Guardian(s)

Hello! My name is Abram Mkwanazi, I'm seeking consent for your child's participation in a research project that is part of my Msc program at the University of the Witwatersrand. The aim of the research is about exploring whether and how the use of real-life contexts influences Grade 8 learners' responses in solving mathematical problems. The information from this research will help us understand more about how learners apply their mathematical knowledge to real world events in solving mathematical problems and how learners are able to move beyond constraints associated with standard problems to recognize the problems as contexts in which they can apply everyday reasoning.

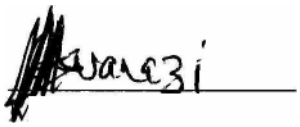
The principal has given me permission to send you this letter that asks your permission to allow your child to participate in this research project. Once you have read the letter you can decide whether you want to allow your child to participate or not.

If you allow your child to participate in this research, he/she and other learners in the class will be asked to write a test. A test will not be for marks but for research purposes. The date for the test will be negotiated with them. Your child may also be selected for an interview. The interview will focus on what he/she has written. With your permission, the interviews will be tape-recorded so that I can ensure that I make an accurate record of what he/she say. If you child is selected for an interview, he/she will not be forced to answer any questions. If he/she does not know the answer to a question that is being asked he/she will not be penalised. The research has nothing to do with his/her school report or marks for mathematics.

In reporting on this research, I will not use your names. I will remove any reference to personal information that might allow someone to guess who he/she is. All records of your responses will be kept in a safe place. If you have any questions about this research project you should feel free to ask the researcher.

If you want your child to be part of this research project, and he/she agrees, please write your names on the consent form, sign in the space provided and get your child to sign as well.

Thank you in advance.



M.A. Mkwanazi
Researcher

APPENDIX 10: LETTER OF CONSENT TO THE LEARNER

Dear Learner

I'm seeking consent for your participation in a research project that is part of my Msc program at the University of the Witwatersrand. The aim of the research is about exploring whether and how the use of real-life contexts influences Grade 8 learners' responses in solving mathematical problems. The information from this research will help us understand more about how learners apply their mathematical knowledge to real world events in solving mathematical problems and how learners are able to move beyond constraints associated with standard problems to recognize the problems as contexts in which they can apply everyday reasoning.

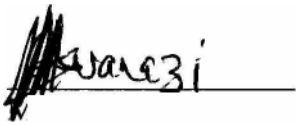
The principal has given me permission to send you this letter that asks your permission to participate in this research project. Once you have read the letter you can decide whether you want to participate or not.

If you agree to participate in this research, you and other learners in your class will be asked to write a test. A test will not be for marks but for research purposes. The date for the test will be negotiated with you. You may also be selected for an interview. The interview will focus on what you have written. With your permission and parent(s)/guardian(s)'s, the interviews will be tape-recorded so that I can ensure that I make an accurate record of what you say. If you are selected for an interview, you will not be forced to answer any questions. If you do not know the answer to a question that is being asked you will not be penalised. The research has nothing to do with your school report or marks you get for mathematics.

In reporting on this research, I will not use your names. All records of your responses will be kept in a safe place. If you have any questions about this research project you should feel free to ask the researcher. Remember that you don't have to take part unless you want to.

If you want to be part of this research project, you and your parent(s)/guardian(s) agree, please write your names on the next page, sign in the space provided and get your parent/ guardian to sign as well.

Thank you in advance.



M.A. Mkwanazi
Researcher

APPENDIX 11: CONSENT FORM FOR PARENT(S)/ GUARDIAN(S) AND LEARNER

UNIVERSITY OF THE WITWATERSRAND
MATHEMATICS RESEARCH PROJECT

RESEARCHER: M.A. MKWANAZI

Consent form for learners participating in a research project

I, (learner's name) _____ , hereby agree to be a participant in the research project. I understand that the purpose of the research is to explore whether and how the use of real-life contexts influences Grade 8 learners' responses in solving mathematical problems. I also understand that, in reporting on this research project, anonymity of my name is guaranteed. All records of my responses are confidential and as such will be kept in a safe place.

I acknowledge that

- the aims, methods, anticipated benefits and consequences of the research, have been explained to me.
- I voluntarily and freely give consent to my participation in such study.
- results will be used for research purposes and may be reported in educator and academic journals.
- I am free to withdraw my consent at any time during the study.

....
Signature of learner Date

....
Signature of parent/guardian Date

....
Signature of researcher Date

APPENDIX 12: TRANSCRIPT: LEARNER 1

An interview between the researcher and the learner about the learner's mathematical task. (AM = researcher; L1 = learner 1)

AM: *Let's look at the way o answerileng diquestion tseo o difuweng ka teng mona, [Let's look at the way you have answered those questions which were given]*

L1: *Yes*

AM: *Ermm… ho ya ka Joe, [In Joe's answer]*

L1: *Yes*

AM: *Ha o dumellane le Joe, because Joe o itse, … "Joe said that the lift went up 19,21429 times", right? and then the question says "Can you see a way that 19,21429 could be the correct answer?", according to your answer you said 'no', and then you said "because it doesn't transport many people it cannot go that many", right? So ho ya ka wena, 'many' e bolela, 19,21429 e leng ho ya ka makgetlo a mangata?*
[You do not agree with Joe's answer, because Joe said … "Joe said that the lift went up 19,21429 times", right? and then the questions says "Can you see a way that 19,21429 could be the correct answer?", according to your answer you said 'no', and then you said "because it doesn't transport many people it cannot go that many", right? So in your opinion, 'many' means, 19,21429 which refers to number of trips the lift took?]

L1: *Yes*

AM: *All right. And then ho ya ka B, Tim said the answer … "lift went up 20 times, can you see a way that 20 could be the correct answer?", and then you said 'yes', right?, because you said it "it need to load people over 269", right?, it must go 20 times, so ho ya ka wena, 19,21429 e kgolo ho feta 20? [All right. And then according to Problem B, Tim said the answer … "lift went up 20 times, can you see a way that 20 could be the correct answer?", and then you said 'yes', right?, because you said it "it need to load people over 269", right?, it must go 20 times, so in your opinion, 19,21429 is bigger than 20?]*

L1: *Yes*

AM: *That's why o reng it goes many times? That is why o sa dumellaneng le Joe? [That is why you say it goes many times? That is why you do not agree with Joe's answer?]*

L1: *Yes*

AM: *But why o re 19,21429 e kgolo ho fata 20?[But why do you say 19,21429 is bigger than 20]*

L1: *Because e na le dinumber tse ngata. [Because it has a lot of numbers]*

AM: *So ha answer ena le dinumber tse ngata, then e kgolo?[So if an answer has a lot of number, then is big?]*

L1: *Yes*

Ellipses (…) indicate a brief pause of a few seconds or less

AM: *OK. Ho B, how did you get the 20? hore o thole 20 yena o e thotse jwang? [Ok. In B (referring to Problem 1(B)) .How did you get 20 as your answer?]*

L1: *Ke ne ke ntse ke etsa, ke entse 269 be ntse ke tlosa 14, anseng e ke e tholang be ke ntse ke tlosa 14 ho yona. [I did, I had 269 then I subtracted 14, the answer that I got from subtraction I kept on subtracting 14]*

AM: *Ooh.*

L1: *Be ke ntse ke bala hore ke entse ha kae. [I then counted the number of times I used subtraction]*

AM: *All right, wa thola hore it went 20 times?. [All right, you found that it went 20 times?]*

L1: *Yes.*

AM: *All right, and then, Gita said, the lift… ha o ya ho C, C says, "Gita said the lift went up 25 times, can you see a way that 25 could be the correct answer?", and in your answer you said 'no'. [All right, and then, Gita said, the lift… when you get to C (referring to Problem 1(C)), C says, "Gita said the lift went up 25 times, can you see a way that 25 could be the correct answer?", and in your answer you said 'no']*

L1: *Yes.*

AM: *Ok. (researcher turns the page) in your answer you said "Because there are many many more people to transport more than 269 it cannot go more than 20.*

L1: *Yes.*

AM: *So, hao dumellane le Gita? [So, you do not agree with Gita?]*

L1: *Yes*

AM: *Lebaka la hao…? [The reason being…?]*

L1: *Ke hobane answer ka mane ke se ke thotse hore ke 20… and… e ke se fete 25 [Because I got 20 as an answer from the previous question… and …cannot be more than 25*

AM: *Right, er, di answer tse fanweng ke Joe, Tim le Gita, na di o thusitse ho solve problem? [Right, er, answers from Joe, Tim and Gita, did they help you to solve the problem?]*

L1: *Yes, di nthusitse, titjhere. [Yes, they helped me, teacher]*

AM: *Ka tsela e jwang? [How?]*

Learner: *E re ke re, ba ne ba re file dianswer tse 3, ha ke re? [Let me say, they gave us 3 answers, is it not so?]*

AM: *Yes, dianswer tse fanweng mona ke Joe, Tim le Gita, na di othusitse ho solva problem ee? [Yes, these answers given by Joe, Tim and Gita, did they help you to solve the problem?]*

L1: *Yes, hobane, e re kere titjhere, hape hape ne… [Yes, because, let me say, sir again…]*

AM: *Mm*

L1: *Er, ke hore dianswer tse, ke ile ka nahana pele né hore… hore ke ile ka etsa,… ke ile ka, hara dianswer tse 3 tseo ba re fileng tsona… [Er, these*

answers, I thought first… that I did,… I did, among the 3 answers given to us…]

AM: *Mm*

L1: *Ke hore… ke ile ka nna ka tlosa… eitse ha ke thola hore 25 e kgolo, ka etlohela le 19,21429 [Let me say… I kept on subtracting…when I discovered that 25 was big, I then left it together with 19,21429]*

AM: *Mm*

L1: *Ke be ke se ke ntse ke thola 20. [I then found 20 as the answer]*

AM: *Ok, in other words, ho ya ka wena, dianswer tse fanweng ke Tim, ke hore ke mastudente ana… [Ok, in other words, in your opinion, answers given by Tim, let me say, these students…]*

L1: *E re kere, titjhere, di re tlisetse haufinyana le ntho eneng e batlahala. [Let me say, sir, they brought us nearer to what was needed]*

AM: *Oh, ok, (researcher turns the page) right, and then, er, ho ya ka wena, na hona le ho tshwana kapa ho na le ho sa tshwane the way o solvileng problem ya ne e ka mane ya bo Joe, Tim le Gita (referring to Problem 1(1)), na di tshwana le number two (referring to Problem 1(2))?. [Oh, ok, (researcher turns the page) right, and then, er, in your opinion, is there any similarity or difference in the way you solved Problem 1(1) and Problem 1(2)?]*

L1: *Yes titjhere. [Yes, teacher]*

AM: *Na ke ntho e tshwanang? [Are they the similar?]*

L1: *Yes titjhere. [Yes, teacher]*

AM: *Why o re ke ntho e tshwanang, ho ya ka wena? [Why do you say they are similar?]*

L1: *Hobane ntse di sebedisa ntho e iwane, matshwao a tshwanang empa feela ba tjhentjhile feela di nomoro, yane e etsa batho ba 14, yena na eya… e entse… ke, e ya 15 times ha yane eya ntho feela,… 20 times, entse e sebedisa ntho entseng e tshwana le yane?. [Because they use the same thing, operations are the same, they just changed numbers only, in Problem 1(1), the lift takes 14 people and in Problem 1(2)… it went… 15 times while in Problem 1(1) it just,… went 20 times, this shows similarity]*

AM: *Oh, ok, er, ho ya ka wena, was it difficult or easy to solve these problems, tse kaofela tseo re qadileng ho tloha ka mane ho fihlela mona? (referring to Problem1 and 2) [Oh, ok, er, in your opinion, was it difficult or easy to solve these problems, that is, Problem 1 and Problem 2?]*

L1: *Ho ne ho le easy titjhere. [It was easy, teacher]*

AM: *Ho ne ho le easy? Why o re di ne di le easy? [Was it easy? Why do you say so?]*

L1: *E ne le easy, hape le hore titjhere … [It was easy. Again, teacher…]*

AM: *Mm*

L1: *Ke bone di le easy ho feta tsena tse ka mona hobane ha di hloke hore o etse (sound of paper) o etse … o skretjhe kelello ha ngata hona le tsena. [I found Problem 1 easy as compared to Problem 2, because you didn't need to (sound of paper) … scratch your head that much.]*

AM: *Ok, so ho ya ka wena, tse tse lateng (referring to Problem 2) di batla di le difficult… kapa di ntse di tshwana ka ofela? [Ok, so in your opinion, was Problem 2 difficult… or were they all of same?]*

L1: *E re kere hara tsena ntho enngwe…question papeng ho na le tse…tse di leng thata ha nyane ho na le tse… [Let me say, … in the task, there were those problems that were difficult and those that were…]*

AM: *Eseng thata. [Easy]*

L1: *Yes*

AM: *Yo ya ka wena di answer tse fannweng ke mastudente di ne di thusa ho ya anseng? [In your opinion, answers given by students were they helping you to get to the correct solution?]*

L1: *Yes, titjhere. [Yes, teacher]*

AM: *Ok, all right, … ehmm …ok, I don't think ho na le any other thing…, so ka mantswe a mang the overall, di question tseo le filweng ka mona, question paper ena kaofela ho ya ka wena was it easy or difficult, re sa shebe question e itseng feela, ha o esheba in … [Ok, all right, … ehmm..ok, I don't think there is any other thing…, so in other words, in general, all questions in the task, were they easy or difficult?]*

L1: *Di ne di le easy [They were easy]*

AM: *Na di ne di easy? [Were they easy?]*

L1: *Yes*

AM: *Wena o ne o sa thole mathata a itseng? [You did not encounter any problems]*

L1: *Yes Titjhere. [Yes, teacher]*

AM: *Ok, all right,… [inaudible sound]*

APPENDIX 13: TRANSCRIPT: LEARNER 2

An interview between the researcher and the learner:
(AM = researcher; L4 = learner 4)

AM: *Ho number 1 o itse, …. the question says "Joe said that the lift went up 19,21429 times. Can you see a way that 19,21429 could be the correct answer?". And you said 'no'. O ne o batla horeng mona? Because in your response you said "I don't agree with Joe's answer because it has a lots of number and that's not the answer is needed in the question and that is impossible to find the answer with that many number but it depend which question is". O ne o batla horeng?[In number 1 (referring to Problem 1(A)) you said, …. the question says "Joe said that the lift went up 19,21429 times. Can you see a way that 19,21429 could be the correct answer?". And you said 'no'. What did you mean here? Because in your response you said "I don't agree with Joe's answer because it has a lots of number and that's not the answer is needed in the question and that is impossible to find the answer with that many number but it depend which question is". What did you mean?]*

L4: *Ok sir, like, ke calculatile ka calculator jwale ka ha ho thwe re e sebedise, jwale ke ile ka sebedisa calculator and ka thola hore ha e mphe dinumber tseo …[Ok sir, like, I calculated the answer using a calculator, as indicated, we were allowed to use them, so I calculated and discovered that it did not give me numbers…]*

AM: *Ha e ofe 19,21429?[Did not give you 19,21429?]*

L4: *Yes, so ke nkile …ke itse 269 né? (he took out a calculator and showed me how he calculated) [Yes, so I did… I said 269, isn't it? (he took out a calculator and showed me how he calculated)]*

AM: *Hmm*

L4: *Minus 14, né? [Minus 14, isn't it?]*

AM: *Hmm*

L4: *Ya mpha 255, né? [It gave me 255, isn't it?]*

AM: *Hmm*

L4: *Ha ke thola ke sheba, ena (pointing at 19,21429) e na le nomoro tse ngata, so, ena ya boitwo (referring to Problem 1(B)), eo bareng re tle ho yona… [When I looked, this one (pointing at 19,21429) had a lot of numbers, so, this second one (referring to Problem 1(B)), which we are suppose to go to….]*

AM: *Re tla tla ho yona, ha re ye ho number 1(referring to Problem 1(A)). So ho number 1, ha wa thola 19.21429? [We'll come to that one later, let's go back to number 1(referring to Problem 1(A)). So in number 1, you did not get 19,21429?]*

L4: *Yes sir*

AM: *That is why oitseng 'no'? [That is why you said 'no'?]*

L4: *Yes*

Ellipses (…) indicate a brief pause of a few seconds or less

AM: *Ok. All right. But ho na le ntho eo o buileng ka yona mona ho number 1. O itse "I don't agree with Joe's answer because it has a lots of number". O ne o batla horeng ha ore "it has lots of numbers?" [Ok. All right. But there is something you said in number 1 (referring to Problem 1(A)). You said "I don't agree with Joe's answer because it has a lots of number". What did you mean by "it has lots of numbers?"]*

L4: *Ok. Eitse ha ke calculate ka thola hore answer ya ka ha e na numbers the ngata jwale ka ena ya Joe. That is why ke itseng 'no'. [Ok. When I calculated I discovered that my answer did not have a lot of numbers like Joe's. That is why I said 'no']*

AM: *Re ntse re le hona mona e re ke o botse he. Ho ya ka answer eo ya Joe, le answer ya B le ya C. Ke number efeng e kgolo ho ya ka wena? [While still here let me ask you this question. In your opinion, if you compare answers given by Joe, Tim and Gita, which one do you consider as big?]*

L4: *E kgolo? [The big one?]*

AM: *Yes*

L4: *Ke ena ya 25 [It is this one of 25]*

AM: *25 ke yona e kgolo? [Is 25 the big one?]*

L4: *Yes,*

AM: *So, ena (pointing at Joe's answer) e nyane? [So this one (pointing at Joe's answer) is small]*

L4: *Yes*

AM: *Ok. All right. Ha re ye ho B (referring to Problem 1(B)). Yona o e thotse jwang? [Ok. All right. Let's go to Problem 1(B). How did you get it?]*

L4: *B, ke e entse ka calculator [Problem 1(B), I solve it by using a calculator]*

AM: *All right. How?*

L4: *So, ke itse 269-14=255 and then ke be ke ntse ke minus 14 ho answer enngwe le enngwe eo ke etholang until e mpha the correct answer. So, ho the correct answer ho remainile two. But two ena e ile ya nyoloha le lift ya etsa 20 times. [So, I said 269-14=255 and then kept on subtracting 14 from any new answer until I got the correct answer. So from the correct answer, two remained. But that two went up with the lift when it went up for the 20th time]*

AM: *Ka mantswe a mang, for the last time, lift e nyolohile ka batho ba babedi? [In other words, for last time, the lift went up with only 2 people in it?]*

L4: *Yes*

AM: *Ok. Ena ya C (referring to Problem 1(C)), wa re ha o dumellane le Gita. How did you arrive at that? [Ok. In problem 1(C), you said you do not agree with Gita. How did you arrive at that?]*

L4: *Answer eo ke e thotseng ho B, ke ile ka e sebedisa ho C because ke thotse hore lift ha e no feta 20 times. [The answer I got in Problem 1(B), I used it to solve Problem 1 (C) because I discovered that the lift would not go beyond 20 times]*

AM: *Ok. Ho number 2 ho thwe "A lift capacity is 16 and 238 people need to be transported in the morning rush. How many times must it go up? Explain your answer?". Wena o be o re "the answer will be 15 and its remains 2. I got it on the calculator". O e entse jwang ka calculator? [Ok. In Problem 1(2) it is said "A lift*

capacity is 16 and 238 people need to be transported in the morning rush. How many times must it go up? Explain your answer?". You said "the answer will be 15 and its remains 2. I got it on the calculator". How did you get it using a calculator?]

L4: Ke e entse tjena, ke itse 238-16=222, be ke ntse ke minus 16 jwalo jwalo. [This is how I've done it, I said 238-16=222, I then kept on subtracting 16 from the new answer that I got]

AM: Ka mantswe amang o e entse seka Problem 1(B). [In other words, you did it the same way as Problem 1(B).]

L4: Yes

AM: But o itse ho ile ha sala batho ba itwo?[But you said two remained behind?]

L4: Yes, but ha eya for the last time e nkile batho bao ba itwo. [Yes, but those two people we taken along when the lift went up for the last time]

AM: Ok. Na dianswer tse fannweng ke Joe, Tim le Gita, di o thusitse ho solve problem ena? [Ok. Answers given by Joe, Tim, and Gita were they of any help for you?]

L4: Yes, hobane di nthusitse ho kgetha answer e correct. [Yes, because they help me in choosing the correct answer]

AM: Ke efe answer e correct? [Which is the correct answer?]

L4: Ke ya B. [It is Problem 1(B)]

AM: But then di o thusitse jwang? [But the how did they help you?]

L4: Hobane ho dianswer tseo ba re fileng tsona, ke thotse e tshwanang le ya Tim.[Because out of 3 given answers, I got the one similar to Tim's]

AM: Oh. Ok. And then Problem 2, question e re "John's best time to run 100m is 17 seconds. How long will it take him to run 1km?. Can you see a way that 170 seconds could be the correct answer?" Sipho said the answer is 170 seconds. Wena o be o re 'no' wa re "I don't agree with Sipho's answer that a little answer". So o ne o batla horeng? [Oh. Ok. And then Problem 2, question says "John's best time to run 100m is 17 seconds. How long will it take him to run 1km? Can you see a way that 170 seconds could be the correct answer?" Sipho said the answer is 170 seconds. You said 'no' and continued by saying "I don't agree with Sipho's answer that a little answer". So what did you mean?]

L4: Ok. Ha ke dumellane le answer ya Sipho hobane e tlamehile ebe e le answer ekgolo ho feta 170 seconds because 1km e kgolo? [Ok. I do not agree with Sipho's answer because it (answer) must be bigger than 170 seconds because 1km is quite big]

AM: Ha ore e be kgolo ho fata 170 seconds, e be bokae? [When you say it must be bigger than 170 seconds, how big must it be?]

L4: E be bo ma two minutes. [Around two minutes]

AM: O e calculatile kapa o ne o eguessa feela? [Did your calculate it or were you just guessing?]

L4: Ke ne ke sa kgone ho e etsa, so ka guessa. [I could not do it, so I guessed]

AM: Re ntse re le hona ho mona, ha re shebe B, "Jake said the answer is 2 minutes and 50 seconds". Wena o be ore "Yes I agree with Jake's story I think it will be 2 minutes and 50 seconds". Why o hanana le Sipho empa o dumellana le Jake? [While still here, let's look at Problem 2(B), "Jake said the answer is 2 minutes

and 50 seconds". You said "Yes I agree with Jake's story I think it will be 2 minutes and 50 seconds". Why did you disagree with Sipho but agree with Jake?

L4: Mona ke sebedisetse commonsense, so ha ke a etsa dicalculation. So ha ke sheba ka thola hore answer e tlamehile ebe more than two minutes, that is why ke kgethileng 2 minutes and 50 seconds. [I've used commonsense here, so I didn't do the calculations. So I realized that the answer must be more than two minutes, that is why I chose 2 minutes and 50 seconds.]

AM: Ho ya ka wena 170 seconds e nyane ho 2 minutes 50 seconds? [In your opinion, is 170 seconds smaller than 2 minutes and 50 seconds?]

L4: Yes

*AM: Ok. Ho C, "Mary said it is **impossible** to find the answer. Can you see a way that **impossible** could be the correct answer?". O be ore "No thats not impossible, you must only use your brain or if is impossible you can use calculator". So o ne o batla horeng mona? [Ok. In Problem 2(C), "Mary said it is **impossible** to find the answer. Can you see a way that **impossible** could be the correct answer?". You said "No thats not impossible, you must only use your brain or if is impossible you can use calculator". What did you mean here?]*

L4: Ok. Ke ne ke batla ho re ha e impossible, o ka nna wa usa your calculator or mind ho thola the answer. [Ok. I wanted to say it was possible, you can use your calculator or mind to find the answer]

AM: All right. Mona ho itswe "A boat sails at the speed of 45km/h. How long does this boat take to sail 180 km? Wena o be ore "2 hour it will take boat sail in 180 km". O thotse jwang 2 hours? [All right. Here it is said "A boat sails at the speed of 45km/h. How long does this boat take to sail 180 km?" You said "2 hour it will take boat sail in 180 km". How did you get 2 hours?]

L4: Ok. Ke e calculatile, ke itse 45+45=90 and 90 tse itwo di mpha 180. So 90 e le nngwe e lekana le 1 hour. [Ok. I calculated it, I said 45+45=90 and adding two 90s, they gave me 180. So one 90 equals to 1hour]

AM: 90 km represent 1 hour. Is this how you reason it?

L2: Yes

AM: Re a efeta ena. Ho Problem 3, "Rob was born in 1978. Now it's 1995. How old is he? Steve said the answer is 17 years". Wa dumella le yena, Kathy a re 16, wa hanana le yena. O e thotse jwang 17 years? [Let's pass it, In Problem 3, "Rob was born in 1978. Now it's 1995. How old is he?" Steve said the answer is 17 years. You agree with him, Kathy said 16, you disagree with her. How did you get 17 years?]

L4: Ke countile ho tloha 1978 until 1995, ya mpha 17. [I counted from 1978 until 1995, it gave me 17 years]

AM: All right. How about 16 years, na le yona o countile? [All right. How about 16 years, did you also count it?]

L4: Le yona ke e entse in the same way, empa ka thola hore ha se yona the correct answer. [I also did it in the same way, but did not get the correct answer]

AM: In C, "Carl said the answer cannot be known precisely". Wena wa re ha o dumellane le yena. Answer ya hao o itse, "No it is easy you can count from 1978 step by step to 1995 and it will give you the answer". So ho ya ka wena ha o counta ho tloha 1978 ho fihla 1995 ke wona feela method oo o sebedisetseng hore

o thole answer. [In Problem 3(C), "Carl said the answer cannot be known precisely". You disagreed with him. In your answer you said, "No it is easy you can count from 1978 step by step to 1995 and it will give you the answer". So if you count from 1978 to 1995, was this the only method you used to get an answer]

L4: Yes

AM: Thank you very much.

APPENDIX 14: TRANSCRIPT: EDUCATOR

The dialogue between the researcher and the educator about L5 and L6's mathematical task. (AM = researcher; ED = educator)

AM: *Ke tlo kopa comment ya hao ho L5, e eleng learner 5 [I'm going to ask you to comment on L5, that is, learner 5]*

ED: *Hmm (indicating acceptance)*

AM: *Ho ya ka wena, o bona response ya ngwana eo e lejwang, the way a respondileng ka teng? [In your opinion, how is L5's response, the way he responded?]*

ED: *E sharp, o itse karabo ena (pointing at 19,21429), o e thotse … o itseng (the educator tried to see where the learner has written the answer) …ok, lift eya 19 times e siya 5 numbers, ha kere? [It is fine, he (L5) said this answer (pointing at 19,21429), he got it … he said (the educator tried to see where the learner has written the answer) …ok, the lift went up 19 times and leave 5 numbers, isn't it?]*

AM: *Hmm*

ED: *Ho be ho sala batho ba bang, ha kere? [Then there will be people staying behind, isn't it?]*

AM: *Hmm, ho be ho sala batho ba bang [Hmm, there will be people staying behind]*

ED: *So ho ya ka yena, 19,24129 e right [In his (L5) opinion, 19,24129 is right]*

AM: *So o kgotsofetse ke the way a commentileng ka teng? {So are you happy the way he (L5) has commented?]*

ED: *Hmm, hobane o indicatile ha kere hore hona le ba bang "will need more 2 people", ha kere? [Hmm, because he indicated, isn't it, by saying that there are some people "will need more 2 people, isn't it?]*

AM: *Hmm*

ED: *So ke 19 eo ha e tlo o fa 20 [So it is 19, it will not give you 20]*

AM: *Ok, all right, because o itse "I think this an right answer, because when you 19 live 5 numbers are left you find the correct answer and will need more 2 people". Ooh, in other words, … [Ok, all right, because he said "I think this an right answer, because when you 19 live 5 numbers are left you find the correct answer and will need more 2 people". Ooh, in other words, …]*

ED: *O sebedisetse calculator, a bona hore ho na le batho ba salang. [He used a calculator, realized that there were people who would be left behind]*

AM: *Ho B? [In B (referring to Problem 1B), what is your comment on Problem 1(B)?]*

ED: *So ho B, ke hore ho yena, it is not 20 times because the 20^{th} time hase batho ba 14 ba nyolohang le lift, ha kere? [So in Problem 1(B), to him, it is not 20 times because the 20^{th} time, there would be less than 14 who go up the lift]*

AM: *Hmm*

ED: *So then yet a re the answer can't be 20 times, so ka mantswe a mang, ho ya ka yena, e tlamehile e be e ena le batho ba 14, so then o aware hore ha o nka 20 o e multiply ka 14 ha esa o fa 269 e se e ofa more than that, ha kere?[So then yet he says the answer can't be 20 times, so in other words, in his opinion, there must be 14 people in the lift, so then he is aware that when you multiply 20 by 14, it will not give him 269, it gives him more than that, isn't it?]*

AM: *Yes, all right.*

ED: *Ke hore leveleng ya hae, the way a answerileng ka teng, ke nahana hore e all right. [At his level, the way he answered the question, I think it's all right]*

AM: *Ok. Ho C? [Ok. In C (referring to Problem 1(C)), what is your comment on Problem 1(C)?]*

ED: *Ho C, o itse … (turns the page) o multiplyile 25 ka 14, ha kere? [In C, he said … (turns the page) he multiplied 25 by 14, isn't it?]*

AM: *Hmm*

ED: *So then 25 by 14 e ba a little bit bigger than 269, so then ka mantswe a mang, so then it can't go more than 25 times. [So then 25 by 14 it is a little bit bigger than 269, so then in other words, so then it can't go more than 25 times]*

AM: *So then o ntse a reasona se ka ya B? [So then he (L5) reasoned almost in the same way he did in problem 1(B)?]*

ED: *Yes, reasoning e ntse e batlile e tshwana, so then mara ha kere mona ha ka a hlokomela hore ka kgetlo la bo 25 even though batho ba le less than 14 e ka nna ya nyoloha. Ka mantswe a mang, e ka ya ka lekgetlo la bo 25. So ho ya ka yena, batho bana ba bang ba setseng batlamihile ba sale, ke hore … it's like ha o re bus e nka batho ba 60 ha esa tlala ha kere ha eno etsa … e tla o charga more or ha eno tsamaya, so then yena o e reasona ka jwalo hore ha e sa tlala lift ha e no kgona ho tsamaya. [Yes, his reasoning is almost the same, so then but here, he was not aware that in the 25th time even though people are less than 14, the lift can take them up. In other words, the lift can go up in the 25th time. So in his opinion, people who are staying behind, must stay behind, that is … it's like saying the bus capacity is 60, if it is not full, it will … you will pay more or it will not move.]*

AM: *As if ke bus? [As if the lift was a bus?]*

ED: *Ee, if hona le batho ba less than 14, lift e a ema, ha e ho tsamaye, so yena ha a divide mona ho sala batho ba less than 14, batho bana kaofela ba 14 ba kgona ho ya ha 19 bana ba bang baya sala and ba less than 14. So ho ya ka yena reason ya hae, e ke se nyolohe lift ena. [Yes, if there are less than 14 people in the lift, the lift will not move, so when he (L5) divides, less than 14 people stay behind. 14 are able to go up the lift 19 times and those who cannot go are less 14.]*

AM: *Ena e ne le comment ya hao ka L5, ho L6 comment ya hao ke efe? [This was your comment about L5, what is your comment about L6?]*

ED: *Ho A, o dividile 269 ka 14, né [In Problem 1A, she divided 269 by 14, isn't it?]*

AM: *Hmm*

ED: *So the answer is 19,21429*

AM: *So o dumellana le answer ya Joe? [So then she agrees with Joe's answer?]*

ED: *Yes, o dividile feela ha a sebedisa any other method. {Yes, she divided, she did not use any other method]*

AM: *Ho B? [What about problem 1B?]*

ED: *Ho B, …. (reads learner's answer quietly) ok, ho B o ntse a sebedisetse calculator, ka mantswe amang ke calculator, ha e mofe 20, ha a e etsa ka calculator. [In Problem 1B, … (reads learner's answer quietly) ok, In B she used a calculator, in other words, she used a calculator, she did not get 20, when she used a calculator]*

AM: *O e entse jwang ka calculator? [How did she do it with a calculator?]*

ED: Ha kere o a divida, o itse "Because I think is wrong answer when I work it out it don't give me this answer of 20", ka mantswe a mang, o ntse a e wekile out like 269 divide by 14. So then ha a e weka out ha e mofe 20 e mofa this answer (pointing at 19,21429) so yena ha a bone kgetlo lena la bo 20 le ka kenang ka wona mona. [She divided, she said, "Because I think is wrong answer when I work it out it don't give me this answer of 20", in other words, she worked it out like out like 269 divide by 14. So then she worked out, she did not get 20, it gave her this answer (pointing at 19,21429) so she could not see how 20 could be the answer.]

AM: Ka mantswe a mang, reasoning ya hae e ntse e batlile e tshwana le ya L5.[In other words, her reasoning, is more or less similar to that of L5]

ED: Yes.

AM: Ho C? [What about Problem 1C?]

ED: Ho C, ha a reasona ho utlwahalang, oitse karabo ke 'no', ka mantswe a mang o bona 25 e le abnormal the way a e behang ka teng. [In Problem1C, she said the answer is 'no', in other words, she saw 25 as abnormal the way she puts here]

AM: O reng? [What is she saying?]

ED: O re "Because it is wrong it is a wrong answer to be it. He/she dont listen in the question". So then ka mantswe a mang, o re enwa (referring to Gita) e abnormale ntho ena eo a engotseng, ke hore ho yena, enwa wa 20 (Tim) o ne ale betere , a le nearer to the answer. Mara enwa wa 25 (Gita) ha a mamela question e reng, implication ya hae e ne e le eno e reng bana (Joe & Tim) ba betere ba batlile ba bua ntho e tshwanang, enwa wa 25 o batla a le out. [She said "Because it is wrong it is a wrong answer to be it. He/she dont listen in the question". So then in other words, she is saying, this one (Gita) has given an abnormal answer, in her opinion, Tim was better because he gave an answer that was nearer. But Gita did not understand the question, the implication was that Joe and Tim talked almost the same language whereas Gita was quiet out.]

AM: Ha a dumellene le Gita? [She (L6) does not agree with Gita?]

ED: Ee, ho yena 25 time e out. [Yes, to her, 25 times, is out]

AM: Right. Thank you.

APPENDIX 15: LETTER OF APPROVAL FROM THE UNIVERSITY

Faculty of Humanities: Education Campus

Room 208/9, Administration Block, 27 St. Andrews Road, Parktown • Tel: +27 11 717-3021/18 • Fax: +27 11 717-3219
E-mail: senamelam@hse.wits.ac.za / moshabesham@hse.wits.ac.za

Mr MA Mkwanazi
PO Box 2606
Vereeniging
1930

STUDENT NUMBER 0316771T
Protocol 2006ECE13

15 September 2006

Dear Mr Mkwanazi

Application for Ethics Clearance Master of Science

I have pleasure in advising you that the Ethics Committee in Education of the Faculty of Humanities, acting on behalf of Senate has considered your application for ethics clearance submitted for the degree of Master of Science for your proposal entitled: **An exploration of Grade 8 learners' response to real-world and standard mathematical problems.**

The Committee has agreed that you may proceed with your research project subject to the following recommendations:

- The subject information letter is unclear – it is not clear with whom you will negotiate so that the tests can take place;
- The subject information letter to the parents and learners is confusing. You need to separate this so that parents and learners receive it and that the information is appropriate for intended group;
- Indicate whether the data will be used for conference presentations;
- Indicate where the data will be stored and indicate that it will be stored for five years;
- Include the words "anonymity" and "confidentiality" in consent forms to make meaning clearer.

You may proceed with the research once amendments have been done to the satisfaction of your supervisor.

Yours sincerely

Mathoto Senamela
Faculty Officer for Faculty Registrar
Faculty of Humanities

cc GSEC file
Supervisor: Dr W Mwakapenda
Ethics File
HDethics clearance

APPENDIX 16: LETTER OF APPROVAL FROM DEPARTMENT OF EDUCATION

UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Date:	13 July 2006
Name of Researcher:	Mkhwanazi Moleko Abram
Address of Researcher:	36 Gabro Avenue
	Waldrift
	1939
Telephone Number:	(016) 5945429
Fax Number:	(016) 5945429
Research Topic:	An exploration of Grade 8 learners' responses to real-world and standard mathematical problems
Number and type of schools:	1 Secondary School
District/s/HO	Sedibeng West

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. ***The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.***
2. ***The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.***
3. ***A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.***

Office of the Senior Manager – Strategic Policy Research & Development
Room 525, 111 Commissioner Street, Johannesburg, 2001 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0488 Fax: (011) 355-0286

4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Senior Manager (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Senior Manager: Strategic Policy Development, Management & Research Coordination with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Senior Manager concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

ALBERT CHANEE
ACTING DIVISIONAL MANAGER: OFSTED

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	Mavanazi
Date:	20/07/2006