Chapter 6

Teaching Sequencing in Grade 2

Whoever teaches learns in the act of teaching,
and whoever learns teaches in the act of learning.
-Paulo Freire (Pedagogy of Freedom)

Introduction

The following two chapters focus on the discussion and analysis of the pre- and post-intervention workshop lessons, which dealt with the teaching of sequencing, and focusing on changes that the teachers subsequently made to their pedagogy. I refer to these lessons as lesson set one (LS1) (one lesson per teacher before the workshops) and lesson set two (LS 2) (one lesson per teacher after the workshops). This chapter describes the general trends, relating to the teaching of growth sequences, which were observed in the 12 lessons that form part of this study. Where necessary, I will refer to individual teachers but, to avoid repetition, general characteristics across all the lessons will be discussed.

I will use percentages to indicate the prevalence of certain pedagogical moves made by the teachers. I will use tables (as illustrations) and tallies to describe observations across the two lesson sets. The tallies will be in the form of percentages and will be reported in co-ordinate pair form, with the first co-ordinate representing the percentage of instances of a concept in LS1, and the second co-ordinate the percentage tallied in LS2 after the three intervention workshops. Where clarification is necessary I will include the totals that gave rise to the percentages. These percentages and totals were calculated from the coding that was described earlier in Chapter 3 and quantify the individual occurrence of the concept under discussion.

I will argue that sequenced items that resemble counting activity, limit the pedagogical possibilities to develop relational thinking in the FP class when dealing with CA2. It will be demonstrated that activities that resemble counting are successfully completed by the learners without referring to the relational attributes embedded in growth sequences. I will further argue that when the teachers moved away from sequences that resemble counting the mathematical activity in class became more sophisticated with the teachers being more mathematically active by explaining, reasoning and writing mathematics on the board. The pedagogical shift that teachers made when working with sequences that do not resemble counting, supports more sophisticated mathematics in the classroom. I will show that the convergence of relational features of sequencing, across the two lesson sets, led to the formation of a pedagogy that foregrounds surface and systemic features embedded in sequences and uses these features to extend sequences.

I will start by describing the general trends regarding the teaching of sequencing by considering the lesson phase time and pedagogical attributes when dealing with sequencing in the FP class. The key attributes that I will focus on are the tension between counting and sequences that are formed by employing key mathematical attributes, procedural and relational instruction practices and how these lead teachers to instruct learners primarily through oral lessons with very little written on the board during LS1. I will show how a relational approach to the teaching of sequences led to teachers making certain structural aspects visible by clearly indicating these on the board and referring to the growth regularity when working with sequencing during LS2. This is followed by a focus on the pedagogical landscape in which FP sequencing operated in this study. I revisit the key relational features of sequencing; repeat and growth. I will discuss the various pedagogical resources that teachers rely on when working relationally in the domain of sequencing, and whether they are working with repeat or with growth in sequences. This leads to the meta-level details of the number sequences that teachers used in class, and revisits the key features of sequencing tasks in the FP class.

General Trends and Comments Regarding the Teaching of Sequencing in Grade 2

Lesson Phase Time

The lessons were generally 95 minutes in duration and consisted of three distinct phases. The first stage can be termed the *teaching phase* (about 45 minutes), where the teacher and learners interacted and actively engaged with repeating sequences and growth sequence problems. The teaching phase was characterised by the teacher presenting sequenced items and asking learners various questions related to sequence. The second phase, the *activity phase*, was time allocated to working individually on textbook problems or activity sheets and lasted about thirty minutes, with the teachers intervening only to provide clarification when required. During the activity phase, learners performed activities in class which were chosen by the teacher and based on the topic and content of the lesson. The final phase of the lessons was the *feedback phase* when the teachers used the last 15 to 20 minutes to mark the work that learners had done during the activity stage. The times that are assigned to each phase are based on the actual times that the teachers spent in each of the 12 lessons that were observed and are reported to present an average time across the lessons. Thus each lesson phase had variability of a few minutes either way. During the feedback phase the teachers usually asked learners to station themselves in front of the class in order to write their answers on the board and, in so doing, share them with the whole class.

Pedagogical Attributes

Each lesson was characterised by the pedagogical choices that the teacher made to deal with sequencing, the pedagogical resources that were engaged to deal with the nature of sequenced items, and the approaches to the teaching of sequencing. Figure 6.1 contains a visual depiction of both lesson sets with regards to the general pedagogical trends (as shown in coding matrix in Chapter 3, p.80) that were observed across the 12 lessons. In the first two lesson phases,

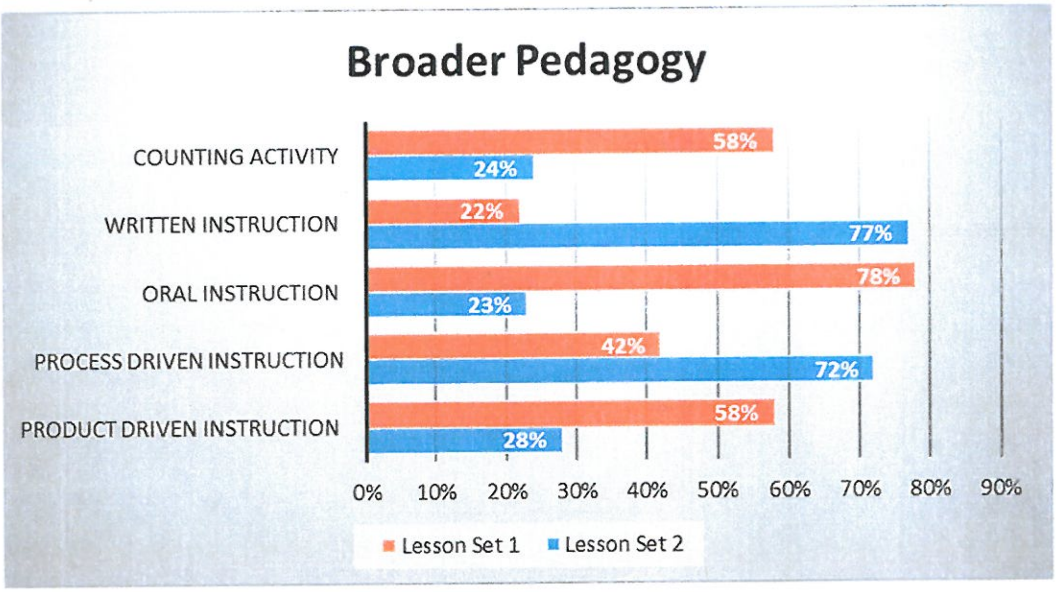


Figure 6.1: General pedagogical trends in the FP sequencing class.

there were 134 tasks covered in LS1. Across the six lessons in LS1, this came to an average of 22.3 problems per period that were worked through with the learners. Thus, the average time spent on one problem across LS1 was 4.26 minutes. There were 109 tasks completed in LS2, with an average of 18.17 problems per lesson and roughly 5.23 minutes spent on each problem. The difference in time spent per problem across the two lesson sets was due to the more complex nature of the problem types that were dealt with in LS2, and the teachers' seeking to involve learners more in classroom activity. The teachers increasingly made use of learner contributions in LS2, and appeared to move away from merely seeking correct answers, to working with both correct and incorrect answers in building understanding. I will discuss this in detail in the sections that follow by considering the key features that were generally observable across all the lessons. These features are: (i) using counting as a sequencing activity, (ii) oral as opposed to written instruction, and (iii) answer-driven teaching activity as opposed to process-driven instruction.

Counting activity and sequencing

Both lesson sets made use of counting activities (58% (78/134); 24% (26/109)). The counting activities were used mainly as teaching activities in LS1 and as warm-up activities in LS2. In LS2, skip counting activities were used to revise previously acquired number knowledge, to prime the learners for the content to follow and to involve everyone in the class in the subsequent extension of counting. In LS1, teachers frequently used counting as an example of sequencing in which learners had to count backwards or forwards in a particular unit; for example, 'count in twos between 2 and 100'. The act of counting in the selected unit was then termed a rule; thus, in the above example, counting in twos was then renamed as 'a rule of add two'.

These counting activities were carried out without writing on the board to reinforce the structural implications of sequencing. In LS1, classroom activity centred around learners only counting forwards and/or backwards and were used as examples of growth sequences as an extension of counting activity. In LS1, teachers used both skip counting (50/78), as well as counting-on activities (28/78). The activities that were coded as skip counting were activities that counted in twos, threes, fives etc. and resembled counting in multiples of the particular number starting at the lowest multiple of the counting number. Examples of such counting activities would be 2; 4; 6; 8; ... or 4; 8; 12; 16; ... Those exercises coded as counting-on activities were taken to be those counting activities that started at a higher starting point than the lowest multiple of the counting number. An activity such as 'count in threes starting at 21' is an example of an activity that was coded as counting-on. The importance of this detail and its implications for structural exploration will be discussed later in the chapter.

In LS2 not all problems started as counting and there was a drop to 24% in the use of counting as a component of the sequencing activity. Here counting was used as a warm-up activity at the beginning of the lesson or a particular problem. Counting activity tended to rely less on multiples of the growth number during LS2. The importance of this will be explored shortly in this chapter. This move away from using counting multiples shifted the classroom discussion to an exploration of the relational qualities embedded in sequenced items.

<u>Oral and written instruction</u>

During LS1, 78% (105/134) of the activities were dealt with through instructional methods that excluded writing, other than the visual display of the actual problem, and were thus only explained orally. The type of activity that was coded 'instr-oral[51]' was, for example, an activity where the teacher instructed learners to 'count in two's starting at 10'. The learners would chorus the sequenced numbers, and when the teacher felt that they had counted enough, she would stop them. She would then continue by referring orally to the numbers in the sequence, asking for the rule or the 'pattern' (add two's), and possibly then asking what came after a particular number. For the duration of this type of activity, nothing was written on the board.

LS2 was characterised by a significant increase in time spent writing mathematics on the board rather than merely 'talking mathematics'. Here 23% (25/109) of activities, were dealt with at an exclusively oral level. This implies that in 77% (84/109) of the activities teachers used explanation by way of writing down, using symbolic notation of the activity of sequencing (see Figure 6.2 for details) and discussing the mathematical structures by working on the board, whether dealing with new content or marking and giving feedback after learners had completed

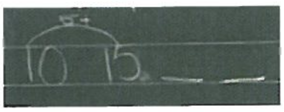
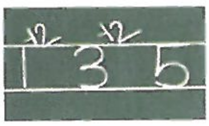
Illustration	Description
 	Teacher 4 started using hoops to indicate that there is a symbolic jump of two and five between the numbers that are sequenced in the two sets.
	Teacher 6 used superscripts to indicate the symbolic growth of two from one number to the next.

Figure 6.2: An illustration of the symbolic representation of the growth in sequenced items.

[51] See the inductive coding matrix in Chapter 3, p.80 for details.

activities in class. Activity that was coded as written instruction included an oral component, but also interaction with the mathematical sequences that were written on the board and made visually accessible, so that the embedded relations in each of the sequences could be explored. Once the learners had visual reinforcement of a particular sequence, they engaged more with the relational characteristics of counting, rather than chorusing a set of sequenced numbers to the satisfaction of the teacher.

Process-driven and product-driven instruction

There was a discernible shift towards engaging learners in mathematical processes that underscore growth sequencing from LS1 to LS2. Classroom activities that focused mainly on getting the correct answer (product-driven) would often end abruptly once the correct answer had been obtained. The process that led to the specific answer was not taken into consideration, as learners were not required to reflect on or describe how they had arrived at the answer, regardless of whether the answer was correct or not. These classroom activities were coded as product-driven, as there was no discussion of the process that was followed or any further reflection on the activity once it was finalised. An example of product-driven classroom activity (with no written reinforcement on the board) from LS1 is illustrated in the extract from the transcript below, where the class was instructed by Teacher 3 to 'count in odd numbers' starting at 17 (32 minutes into the lesson):

(1)	Teacher 3:	Count in odd numbers for me starting from the number seventeen.
(2)	Class:	(*Chorusing*) Seventeen, nineteen, twenty-one, twenty-three, twenty-five ... (up to number one hundred and one)
(3)	Teacher 3:	Alright, stop. Are we counting in ones, twos, threes, fours? If we are counting in odd numbers ...
(4)	Learner 1:	Threes.
(5)	Teacher 3:	No, try again.
(6)	Learner 2:	Ones.
(7)	Teacher 3:	Try again.
(8)	Learner 3:	Sevens.
(9)	Teacher 3:	(*Frowning disapprovingly*) Sevens? Try again.
(10)	Learner 4:	Twos (*but the teacher ignores the learner*).

(11) Teacher 3: Learner 1, what did you say?

(12) Learner 1: Threes.

(13) Teacher 3: (*To the class*) Is Learner 1 correct? (*No response*) Put your finger for me on one. Now count for me up to three.

(14) Class: (*Some say*) One, three.
(*Others say*) One, two, three.

(15) Teacher 3: Let's look at my number chart. (*Teacher refers to a number chart displayed in the front of the class*). Look up here! Is one an odd number?

(16) Class: (*Chorusing*) Yes.

(17) Teacher 3: One is an odd number...One, two (*points to three*)... One, two (*points to five*)... One, two (*points to seven*). What are we counting in?

(18) Learner 5: Sevens.

(19) Teacher 3: (*Louder*) Sevens? We are counting in odd numbers. Are we counting in twos, threes, or ones?

(20) Learner 4: Twos.

(21) Teacher 3: (*To the class*) We are counting in twos! Remember last week I told you that you can start at any number and be counting in two's. You don't have to start at two, four, six, eight, ten, and then you think you're counting in twos. You can start at any other number. So, yes, we are counting in odd numbers, but we are also counting in twos when we are counting in odd numbers. Do you understand that?

The teacher allowed for a few guesses (turns 1 to 10) before she returned to Learner 1 who had initially proposed that the counting was in threes (turn 4). She then called the learners' attention to her number chart and started by pointing at 1, and then counted the jumps by voicing 'one-two' and pointing to the next number, waiting for the learners to say the number (turn 17). She did not engage with the concept of the odd nature of the numbers 17; 19; 21; ... and how they relate to the numbers 1; 3; 5; ... to which she had pointed on the chart. Secondly, she used the auditory resource of rhythm and the visual reinforcement of pointing when pacing 'one-two', 'one-two', 'one-two' while pointing to 1; 3 and 5 on the chart, but she did not link the 'one-two' jumps to the odd nature of the numbers. The learners continued their guessing and when another learner suggested two, it was acknowledged as correct without returning to the original question.

A second transcript from LS1 shows a more obvious instance of the product-driven or answer-driven approach that was taken by Teacher 4. This is also another example of oral instruction without visual reinforcement. In this transcription, the teacher played 'guess my rule' where she proposed a set of sequenced numbers and the learners were expected to identify the rule by expressing it as counting in some number:

(1)	Teacher 4:	Now what you're going to do now is listen to me... If I count, you have to tell me what am I counting in. (*The teacher starts to count.*) One, three, five, seven, ... Who can tell me what pattern is that? One, three, five, seven ... What pattern is that? (*Waits for three seconds*) A pattern in twos or threes or fours or fives? Which one is it, Learner 1?
(2)	Learner 1:	In ones.
(3)	Teacher 4:	In ones, threes or sevens, Learner 1?
(4)	Learner 1:	Yes (*confirming her answer*).
(5)	Teacher 4:	Try to think Learner 1. You are not correct; try it again. (*Points to another learner*) Yes?
(6)	Learner 2:	In threes.
(7)	Teacher 4:	Uh? She says in threes (*to the class*). Is she correct?
(8)	Class:	(*Chorusing*) No.
(9)	Teacher 4:	Yes! Learner 2, she is correct... (*moving immediately to the next sequence*) and when I say four, eight, twelve, what am I counting in? Or what am I counting on? Yes? (*points to a learner*)
(10)	Learner 3:	In fours.
(11)	Teacher 4:	In fours yes. Now are you - will you be able to count for me in threes up to fifty?

Teacher 4 turned her attention from one learner to the next, rejecting wrong answers without pausing to reflect with the group on why the answer was not correct. In turn 9 the teacher incorrectly accepted the answer of three as correct, and immediately moved to the next set of sequenced items. This was also the case in turn 11 when the answer given by Learner 3 was acknowledged as correct, with the teacher turning to the next activity without reflecting on the learner's answer. In LS1, 58% of the activities were coded as product-driven approaches to teaching the notion of sequencing.

The shift in focus in LS2 towards a more process-driven experience enhanced the relational aspects of sequenced numbers. In LS2, the teachers were more inclined to lead the classroom discussion towards a relational approach to the notion of sequencing. Teachers worked with both correct and incorrect answers as objects of learning through which the process of obtaining an answer was made visible and discussed in class. Activities that were coded as process-driven included a relational approach to sequencing as a key component of mathematical activity. Through reflection on the structural aspects and by withholding the rule that was used to get to the next number, the sequencing activity involved more sophisticated ways of looking for and expressing regularity. Process-driven instructional practice increased from 42% (56/134) in LS1 to 72% (78/109) in LS2. A contributing factor for this shift was the type of sequence that was presented to the learners as a growth pattern. This will be discussed fully in later sections that describe the characteristics of sequenced numbers. The two transcripts that follow are examples of a process-driven approach to growth sequences taken by Teacher 3 and Teacher 1.

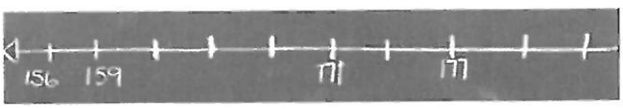
From LS2: (28 minutes into the lesson)

(1) Teacher 3: The last one (*writes on the board*).

156; 159; ___ ; ___ ; ___ ; 171; ___ ; 177; ___ ; ___ .

Okay, I can see the steam coming out of your ears. What is going on with that number line there?

(*Drawn on the board is the following number line*)



(2) Class: (*Chorusing*) Threes.

(3) Teacher 3: Why did you say threes? (*Waits*) Who says fours? (*No response*) Who says twos? (*A few hands go up hesitantly.*) Who says fives? Who says

tens? Lots of you did not put up your hands. Who says threes? (*Approximately fifteen learners raise their hands*). You're all correct.

All eyes here; I want to show you something. (*Pointing to the number 156*) One hundred and fifty-six, (*starts counting and pointing with left hand*) one hundred and fifty-seven (*one finger up on right hand*), one hundred and fifty-eight (*second finger up on right hand*), one hundred and fifty-nine (*third finger up on right hand. Turns to the class with her hand raised*). How many fingers do I have here?

(4)	Class:	(*Chorusing*) Three.
(5)	Teacher 3:	Three fingers so it means - Learner 1, look here. We starting at one hundred and fifty-six, so you don't say that number (*signalling not to start counting at 156*). The next number is one hundred and fifty-seven (*shows one finger*) ... one hundred and fifty-nine (*third finger*). So I am counting in threes (*holding up the three fingers*). All right. Now count three on. You can use your number chart. Put your finger on one hundred and fifty-nine, now count three on.
(6)	Class:	(*Chorusing*) One hundred and sixty, one hundred and sixty-one, one hundred and sixty-two.
(7)	Teacher 3:	(*Writes 162 on board*). Learner 2, am I counting backwards or am I counting forwards?
(8)	Learner 2:	Forwards.
(9)	Teacher 3:	(*To learner 2*) Is your finger on one hundred and sixty-two?
(10)	Learner 2:	Yes, ma'am.
(11)	Teacher 3:	(*To the class*) Let's count three on.
(12)	Class:	(*Chorusing*) One hundred and sixty-three, one hundred and sixty-four, one hundred and sixty-five. (*With each number that the learners call out the teacher raises one finger.*)

(13) Teacher 3: (*Teacher writes 165 on board*) Okay one hundred and sixty-five; is your finger on the number? Let's count on. You don't count one hundred and sixty-five; you have to count on from one hundred and sixty-five. So let's count, one hundred and sixty-six ...

(14) Class: (*Chorusing*) One hundred and sixty-six, one hundred and sixty-seven, one hundred and sixty-eight.

(15) Teacher 3: (*Teacher writes 168 on board*). Okay, count three on.

(16) Class: (*Chorusing*) One hundred and sixty-nine, one hundred and seventy, one hundred and seventy-one.

(17) Teacher 3: Is one hundred and seventy-one on the board? Yes, so are we still counting correctly? Yes. Okay so count on to the next number. One hundred and seventy-two ...

(18) Class: (*Chorusing*) One hundred and seventy-two, one hundred and seventy-three, one hundred and seventy-four.

(19) Teacher 3: (*Leading the counting, the teacher writes 174*) Okay count on...

(20) Class: (*Chorusing*) One hundred and seventy-five, one hundred and seventy-six, one hundred and seventy-seven.

(21) Teacher 3: Is it on the board? Yes. Right then, count three on...

(22) Class: (*Chorusing*) One hundred and seventy-eight, one hundred and seventy-nine, one hundred and eighty.

(23) Teacher 3: (*Writes 180 on the board*). Now it's very easy! One hundred and eighty plus three? (*Not pointing the fingers again*).

(24) Class: (*Chorusing*) One hundred and eighty-three.

(25) Teacher 3: (*Repeats one hundred and eighty-three and writes it on the board*) Now one hundred and eighty-three plus three?

(26) Class: (*Chorusing*) One hundred and eighty-six.

(27) Teacher 3: (*Writes 186 on board*) Now one hundred and eighty-six plus three?

(28) Class: (*Chorusing*) One hundred and eighty-nine.

(29) Teacher 3: Good job. One hundred and eighty-nine plus three?

(30) Class: (*Chorusing*) One hundred and ninety-two.

(31) Teacher 3: Now one hundred and ninety-two plus three?

(32) Class: (*Chorusing*) One hundred and ninety-five.

In the transcript above, the Teacher 3 made use of the semiotic resources of signalling with each count by raising her finger as learners counted each of the successive numbers to get to the next number on the number line. She explained that the process of one jump on the number line was equivalent to *counting on* in threes on the number chart. She did not indicate the jump of three on the number line, and only referred to it in oral instruction and by use of signals to count to the next number on the number line. The number line and signalling of three with the fingers constituted pedagogical resources to represent growth by three for each consecutive number item on the number line. At turn 24, the teacher reverted to a process of counting on to complete the rest of the number line. This activity will be revisited in Chapter 7 in the discussion of the use of structure and the number line where the teacher extended the idea of growth using the relational aspects embedded in the sequence.

From LS1: (three and a half minutes into the lesson)

(1) Teacher 1: Now I want to see if you can do this one (*writes*): 3; 5; 7; ___ ; ___ . Let's read the pattern.

(2) Class: (*Chorusing*) Three, five, seven, mmm, mmm.

(3) Teacher 1: Okay, what is the rule for this pattern?

(4) Learner 1: It's counting in ones.

(5) Class: (*Disapprovingly*) Hau!

(6) Teacher 1: Counting in ones? (*Frowning at class and suggesting that they do not have to comment when they think someone has made a mistake*). She says that we are counting in ones. (*Turns to the board*) Okay, let's see. (*Writes a superscript plus one at the top of three*). Three plus one (*pointing to the 3 then the +1*) is that equal to five?

(7) Class: (*Chorusing*) No.

(8) Learner 2: (*Teacher not acknowledging the answer from the class*) It's 4, ma'am.

(9) Teacher 1: Okay, so this is wrong (*pointing to the +1 and then removes the plus 1 from the top of the first term*). Let's see (*pointing to Learner 3*).

(10) Learner 3: We are counting in odd numbers.

(11) Teacher 1: (*Repeating*) We are counting in odd numbers; (*looking at the numbers on the board*) That is true, but now what is the rule?

(*Pointing to learner 3*)

(12) Learner 3: It's plus two, ma'am. We are counting in twos.

(13) Teacher 1: Let's check if Learner 3 is right. (*Goes to the board and writes a superscript plus two at the top of term 1.*) Three plus two (pointing to the three and then the +2). What is the answer?

(14) Class: (*Chorusing*) Five.

(15) Teacher 1: Okay, five (*writes a superscript +2 at the top of the 5*) plus two...

(*The class continues to complete the sequence up to 11 in the same manner*)

Turns 6 and 13 show that Teacher 1 worked with the answers that the learners provided, regardless of whether the answers were correct or not. In doing so, she tied the rule to the given numbers in the sequenced set and provided a check for learners to see if they were correctly identifying the surface structure that determines the regularity that exists between the numbers. She used pointing and signalling as semiotic resources to explicate the structural aspects of the sequenced set of numbers and to demonstrate that these features apply to all the terms in the sequence (turns 11 to 15).

<u>In Summary</u>

In LS1, the teachers initially chose to work with sequencing by selecting oral activities that mirrored counting activity by requiring that learners count in '*a*' starting at '*b*' and finishing at '*c*'. This counting was usually followed up by asking learners to state the 'counting number' in which they have just counted as a 'rule', without any exploration of the answers that the learners gave, regardless of whether these were correct or not. In LS 1, the teachers would acknowledge correct answers as correct, and dismiss incorrect answers immediately without any further discussion. Thus, the instruction in LS1 was mainly oral and product-driven, without paying attention to the processes that were followed to obtain an answer, and with very little explanation written on the board. Learners were thus mainly required to count correctly and provide the right answers to the questions that the teachers were asking. In LS2, the pedagogical approaches to sequencing shifted to include more written examples of sequences, making processes explicit and reinforcing ideas that were discussed in class. Lessons were enhanced by process-driven instruction which gave the relational features between sequenced items more visibility and meaning, and provided learners with the opportunity to explain their reasoning. In LS2 counting was used as a warm-up activity at the beginning of a lesson, or between problems to prime learners for what was to come. The teachers would suggest a counting sequence, and from there start working on sequences that use the same growth element as the counting sequence, but work relationally with the growth element.

Mathematical Landscape of Foundation Phase Sequencing

I will now focus on the general attributes of sequencing in the six FP classes in my study, across both lesson sets. The figures that I will use to indicate the frequency of attributes will be based on the total number of activities that were *not coded as counting activity*. I will discuss repeating and growth sequences, focusing on the meta-level detail in these two categories, and focus on the three types of sequences that were used by the six teachers in my study.

Repeated and Growth Sequences

Table 6.1 summarises the mathematical characteristics of the activities employed by the teachers when teaching sequencing with numbers in the FP classroom. In both lesson sets, repeating sequences (2% {3/134}; 6% {7/109}) and growth sequences (40% {53/134}; 70% {76/109}) were discussed in class after counting activity (58% {78/134}; 24% {26/109}) was completed.

	Lesson Set 1				Lesson Set 2			
Number of activities	134	56	53	51	109	83	76	74
Counting Activities	78/134				26/109			
Repeating Sequences	3/134	3/56	(56 = 134 – 78)		7/109	7/83	(83 = 109 – 26)	
Growth Sequences	53/134	53/56			76/109	76/83		
Different Growth		(53 = 56 – 3)	2/53			(76 = 83 – 7)	2/76	
Constant Growth			51/53				74/76	
Positive Growth			(51 = 53 – 2)	36/51			(74 = 76 – 2)	57/74
Negative Growth				15/51				17/74
Consecutiveness of given terms			Three	34/51			Three	21/74
			Two	9/51			Two	15/74
			None	8/51			None	38/74

Table 6.1: The attributes assigned to the various activities used by the six teachers when dealing with sequencing at Grade 2 level.

The number of activities focusing on growth sequencing was 53 for LS1 and 76 for LS2. Only three repeating sequences were dealt with by the teachers during LS1, while seven repeating sequence activities were carried out in LS2; this despite the fact that the Grade 2 FPRWs and the ANA exemplar and test items contain a significant number of repeating sequences.

Of the 53 growth sequences in LS1, 51 activities possessed constant growth. For the 76 activities in LS2, 74 activities possessed constant growth. Activities with constant positive growth (71% {36/51}; 77% {57/74}) or constant negative growth (29%{15/51}; 23% {17/74}) were the primary types of growth sequence used in both lesson sets. There were also activities included that did not use constant additive growth, but rather made use of a constant multiplicative growth of doubling to progress from one term in the sequence to the next. In both LS1 and LS2 two activities used multiplicative growth to form a sequence.

When counting resulted in a discussion of patterns, the focus tended to be on the rule used when counting. This rule represented the surface structure in the sequencing activity. In these types of activities, the teacher would ask the learners to tell her what the rule was for counting in twos, and the answer would be 'plus two' or 'take away two' depending on whether they had counted forwards or backwards. Number also evolved into counting activities, where counting would be initiated by the teacher and identified by the learners as a rule that is applied recursively in the sequenced numbers under consideration. Here the teacher worked with a sequenced set of numbers, identifying the relational aspects between the sequenced set of numbers, and then concluding the activity by asking learners to use the rule and count in the specific number (for example, twos).

Constant growth sequences made up 96% (51/53) of the sequences in LS1 and 97% (74/76) in LS2. Of these constant growth sequences, those where the first three consecutive terms are given (66% {34/51}; 28% {21/74}) were used, but the prevalence decreased by 38 percentage points in LS2. This downward trend was indicative of the teachers' increasing use of activities that do not offer cues, evident in counting activity, but rely more on the learners being able to investigate the embedded surface structures in the sequenced set of numbers. In LS2, the teachers made more use of two consecutive terms that are given (18% {9/51}; 20% {15/74}) where consecutive terms will usually require learners to access the surface features in a sequence and use this to extend or complete the sequence. The use of activities in which no consecutive terms (16% {8/51}; 52% {38/74}) are given increased by 36 percentage points between LS1 and LS2. This shows a significant move towards working with the relational aspects of the set of sequenced numbers, as learners could no longer rely on cues from counting activity to access surface features in the sequences. Examples of these three variants of sequences are discussed in the section that follows. Figure 6.3 shows a graphic summary of all these attributes discussed above and how they featured in each of the lesson sets[52].

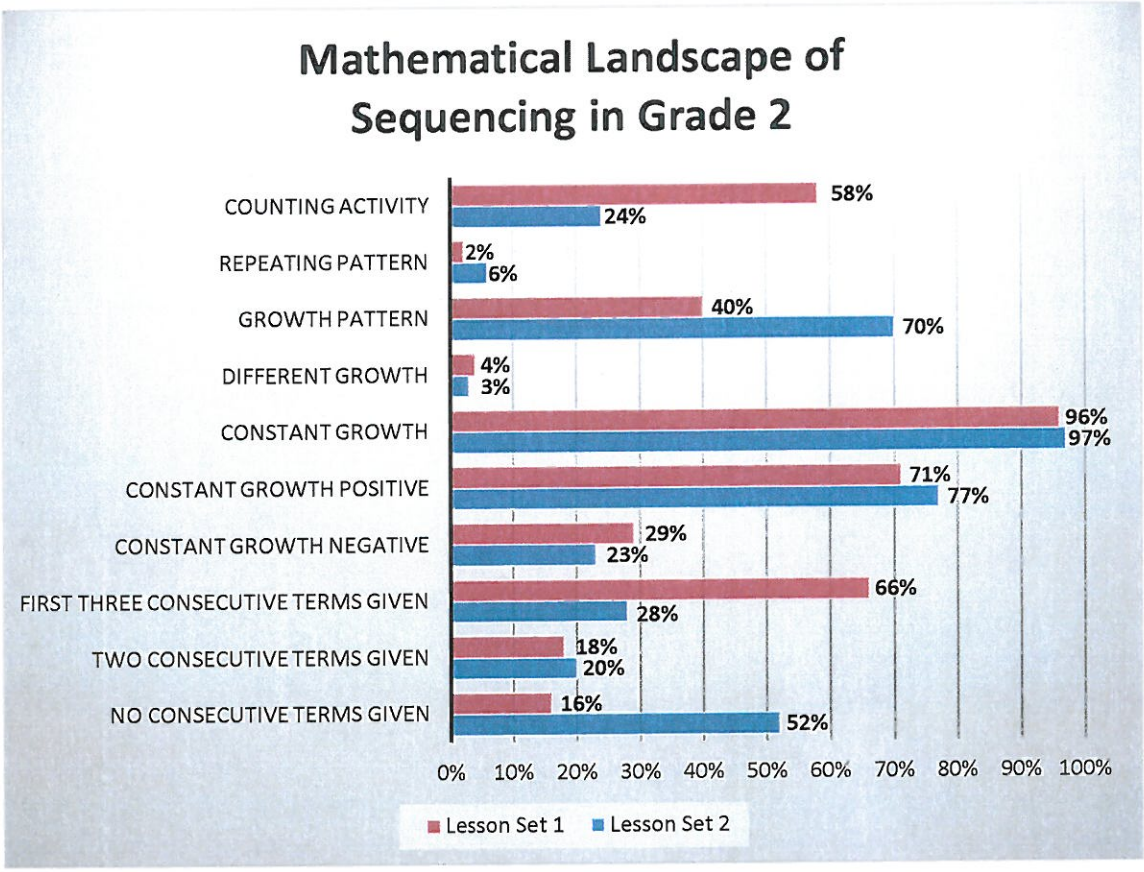


Figure 6.3: General attributes of sequencing in Grade 2 classes in this study.

Sequence Construction: Three Variants

Each type of sequence construction engaged different cognitive skills when working with patterns. The sequences 2; 4; 6; … and 1; 3; 5; … are examples of sequenced sets with three consecutive terms given. Learners were required to determine the next few numbers in each sequence. Both of these sequences are accessible through their close resemblance to counting either in twos or even/odd numbers (as labelled by the teachers). Teachers in the study would refer to these two sequences as counting in even or odd numbers and did not always link this to the fact that both sequences function within the recursive rule of adding two. Of the four

52 These attributes are a combination of coding matrices illustrated in Chapter 3, Tables 3.4(a)/(b) and Table 3.5.

teachers (Teachers 1, 3, 4, and 6) who used the notion of counting in odd or even numbers, only Teachers 1 and 3 referred to both the number types and the recursive rule of adding two when working with these sequences[53].

The sequences that made use of the given consecutiveness of two terms resembled the sequence 3; ___ ; ___ ; 9; 11; ___ . Learners would use the two consecutive terms to establish the growth quantity and then test if their hypothesis was true by examining whether the growth number fitted the missing items between 3 and 9. This was done procedurally (testing the fit) or by observation of the structural aspects of the sequence, through noting that three jumps separated the two terms 3 and 9, and that these terms differ by six which resulted in three groups of two. The transcript below demonstrates one such discussion in class during LS1:

Teacher 1: Now the next one (*writes*) 12, 9 , ___ , ___ , ___ . Do it on your white boards. (*Teacher waits for the first learner to finish and then instructs the class to put down their pens.*) Okay Learner 1 is going to tell us what he has done. Don't copy now. Put down your pens.

Learner 1: The rule is minus three.

Teacher 1: (*To the class*) Okay he says that it's minus three or take away three. Okay let's see. (*Turns to the learner who is at the board. Learner writes*: …, 6, 3, 0). Is he correct?

Class: (*Chorusing*) Yes.

Teacher 1: Okay let's see….(*writes a minus 3 at the top of 12*) What is 12 take away 3?

Class: (*Chorusing*) Nine.

(*Teacher continues to do the same until reaching 0, with the learners doing the calculations correctly.*)

Teacher 1 did not require any explanation from the learner as to how he determined the rule in the sequence. She merely wrote his rule at the top of the numbers and continued to ask the class for the answers.

[53] One such a class discussion was referred to in this chapter, pp. 194-196 and 200-201.

The sequenced items where no consecutive terms were given resembled the number sequence 4; __ ; __ ; __ ; __ ; __ ; 24 [54]. This sequence can be approached procedurally by a hit-or-miss approach, most probably based on the fact that both 4 and 24 are multiples of four, so the growth of four is a reasonable hypothesis for the growth number. The alternative approach is to work with the structure as in the previous example. These three types of classifications will be elaborated on in the section of this chapter that describes the meta-level detail of the sequencing approaches that the teachers used.

<u>In Summary</u>

Across the two lesson sets, the general attributes of sequencing at the FP level consisted of two key activities, namely counting and growth sequencing. Counting was used in both lesson sets in different ways. The teachers worked mainly with growth sequences with a constant growth that was positive. This produced sequences where different degrees of consecutiveness were used in each of the activities, from three consecutive terms being given to those sequences where no consecutive terms were given.

The most popular types of growth sequence in LS1 were sequences where three consecutive terms were given, and this changed later in LS2 to sequences where, most frequently, no consecutive terms were given being. This shift in sequence construction led to teachers and learners working on a more relational level with sequencing, which I will focus on in Chapter 7.

In the following sections, I will discuss the use of the number chart as a pedagogical resource, and then continue to look at further meta-level attributes of tasks that the teachers used to engage with sequencing.

The Number Chart as a Pedagogical Resource

In both sets of lessons, teachers used different visual depictions of sequences. A popular tool used to support these depictions was the number chart with which learners would perform a variety of procedures (counting, sequencing, checking) in order to understand a given set of sequenced numbers or to form their own sets of sequenced numbers. The number chart can be

[54] This type of sequences will be discussed in Chapter 7 pp. 247 – 250 where I will discuss the use of structure.

a powerful pedagogical tool if used skilfully. It possesses structural aspects that can be closely linked to items of growth in a sequenced set of numbers.

The sequences illustrated in Figure 6.4 are examples of using the number chart to reinforce the regularities when working with multiples; multiples of five have either a 5 or a 0 as its unit digit, and the multiples of four cycles through the unit digit values in a sequence of 4; 8; 2; 6 and 0. The chart possesses spatial structures which can be explored when learning about numbers; vertical growth ($\pm$ 10), horizontal growth ($\pm$ 1), diagonally top left to bottom right ($\pm$ 11) and top right to bottom left ($\pm$ 9). The whole number concept can be enhanced through working structurally with the 100 chart.

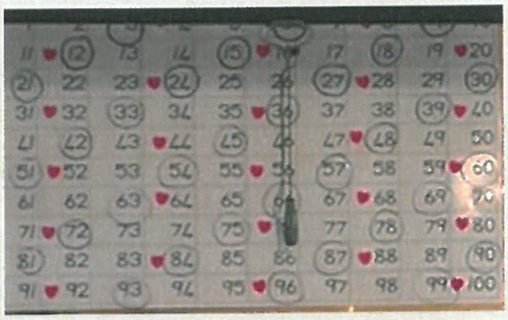
Visual depiction of sequence 4; 8; 12; 16; ...									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Visual depiction of sequence 5; 10; 15; 20; ...									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 6.4: Using the 100 chart to represent patterns formed by sequences.

The use of the number chart can, however, also obscure a growth sequences, making it appear as a repeating sequence, because the sequence is studied as an array of numbers on a chart (Liljedahl, 2004). The sequenced numbers 4; 8; 12; 16; 20... and 5; 10; 15; 20; 25... appear as vertical rows on the number chart (see Figure 6.4). This approach was encouraged by Teacher 5 in both LS1 and LS 2, and presented as a pattern that is formed when counting in twos, threes, fours, fives and tens. Learners who used their number charts tended merely to continue along the vertical lines on the chart, calling out the numbers without considering the relational aspects of the sequence of numbers. After the first three to four lines were coloured in, learners proceeded along the array, identifying the position of the numbers that fitted in the array. The teacher would then wrap up the discussion by pointing out that the 'pattern for twos is a straight line' that crosses vertically over the number chart.

Teacher 3



Here the number chart is used as a reference to remind learners of the arrays that were formed when counting in threes (circles) and fours (red hearts). Teacher 3 used the chart to remind learners where the even and odd numbers are located as arrays on the chart for easy reference to check answers if they are uncertain. She did not use the chart to teach sequencing.

Teacher 5

Teacher 5 used her number chart as a teaching resource to demonstrate the visual array that is formed by counting in threes. She did this by covering the numbers that form part of the array, and it soon revealed how the sequence should continue the number chart by simply repeating the array. Teacher 5 used the number chart exclusively to teach 'pattern' as an array on the chart.

Figure 6.5: Two depictions of arrays on the number charts which blurs the lines between repeating sequences and growth sequences.

Figure 6.5 shows two depictions of number sequences using the number chart. Two of the participating teachers, Teachers 3 and 5, used the number chart differently. Teacher 3 used the number chart as a recording tool to which she referred when learners showed difficulty in remembering basic number facts (like odd and even numbers), and to assist learners in counting activities where they used skip counting in class. Teacher 5 used the number chart exclusively to teach growth sequences and presented the sequence that was formed by counting in threes as an array that lies diagonally across the charts (as shown in Figure 6.5). The array was presented as the 'pattern' in each of the activities that she discussed in LS1. Teacher 5 did not explicate the relational features that lie embedded between the numbers that were used to form a visual array. Chapter 7 revisits the use of arrays to represent sequenced items and considers how Teacher 5 changed her strategy to draw attention to the structural aspects between sequenced numbers.

Teacher 5's use of the number chart blurred the line between growth and repeat, especially where the array of numbers lay in a connected linear way as the learners merely called out the numbers that were situated underneath one another (when counting in fives or twos). By simply following the linear array that became evident after the first three rows on the charts had been coloured in, the covering of the charts was completed by the teacher with the learners colouring in the arrays on their individual charts. The relational features were thus not discussed at all in the lesson, and there was no reference made to any growth rules that formed the arrays.

A Meta-Level Analysis of Growth Sequencing Tasks

The collective attributes that each of the activities possess at a meta-level, shape the mathematical structure that can be elicited from the sequenced items and used to develop algebraic reasoning ability. Through careful analysis of the activities that teachers conducted in class during both lesson sets, it was possible to determine the common attributes across these activities. Ten of the most common attributes were identified in LS1, and 11 common attributes were noted in LS2.

In LS1 the teachers worked most often with questions that have the following attributes: (i) positive growth number sequences where (ii) the growth was used recursively to move from one sequenced item to the next, (iii) creating, identifying or extending sequences by (iv) applying a given rule or regularity which (v) started at a multiple of the growth number, which was often the (vi) smallest multiple of the growth number, (vii) working on a random interval (viii) most likely located between 1 and 100 but (ix) but sometimes starting at zero, (x) using a growth unit size of either two, five or ten. Table 6.2 on the next page, shows examples of questions that incorporate the widely-used attributes that were identified in the classroom work on growth sequencing in both lesson sets.

		Positive growth number	Recursive use of constant growth	Creating, identifying or extending pattern	Applying a given rule	The rule is withheld	The rule is to be identified	Starting at a multiple of the growth number	Starting at a non-multiple of the growth number	Starting at smallest number	Working on a random interval > 20	Interval located between 0 and 100	Unit growth of 2	Unit growth of 3	Unit growth of 4	Unit growth of 5	Unit growth of 10
1.1	Count in twos from 0 up to 100	x	x		x			x		x		x	x				
1.2	Tell me what the rule is that I am using: 25;30;35;40.	x	x	x		x	x	x			x	x				x	
1.3	Write down the next five terms: 2; 4; 6; …	x	x	x		x		x		x		x	x				
1.4	Fill in the missing numbers: 30;___ ; 50; 60; ___ ; ___ ; ___ .	x	x	x		x		x			x	x					x
2.1	Count in twos from 11 up to 41	x	x		x				x		x	x	x				
2.2	Will 45 be part of the sequence formed by 7; 10; 13; … ?	x	x	x		x			x			x		x			
2.3	Given 6; 11; 16; … and 3; ___ ; ___ ; ___ ;23, how are these two number patterns the same? Describe each pattern in your own words.	x	x	x		x	x		x	x		x				x	
2.4	Find the rule that is used in this sequence and then fill in the missing numbers: 2; ___ ; ___ ;14; ___ ; ___ .	x	x	x		x	x	x	x	x		x			x		

Table 6.2: Examples of questions that incorporate the widely-used attributes that were identified in the classroom work on growth sequences in both lesson sets.

In LS2, there was a change in the types of problems that were posed. Shifts were mainly in the areas of choice of sequenced numbers and the growth element. Sequenced numbers were chosen more carefully, and the growth element was chosen so as to enhance the relational aspects between numbers. The problems that were used in LS2 also (i) used positive growth number sequences where (ii) the growth was identified through using a recursive approach to move from one item to the next. (iii) Creating, identifying and extending patterns were made more likely by (iv) searching for the growth regularity that (v) was withheld and not given as part of the question, (vi) and by specifying that items start at a non-multiple of the growth number and at (vii) the smallest non-multiple of the growth number, (viii) working on a less random interval located (ix) between 1 and 100 at the lower end and (x) not including zero, (xi) using growth unit sizes of two, three and four.

The percentages recorded in Table 6.3 were calculated based on the total number of growth sequencing activities used in each lesson set: 51 in LS1 and 74 in LS2. The numbers in brackets

	Attribute	Lesson set 1 (51 tasks)	Lesson set 2 (74 tasks)	Trend
1.	Positive growth number	71% (36)	77% (57)	6
2.	Recursive use of constant growth	69% (35)	91% (67)	22
3.	Creating, identifying or extending pattern	47% (24)	70% (52)	23
4.	Applying a given rule	55% (28)	43% (32)	-12
4.	The rule is withheld	41% (21)	49% (36)	8
6.	The rule is to be identified	26% (13)	46% (34)	20
7.	Starting at a multiple of the growth number	43% (22)	18% (13)	-25
8.	Starting at a non-multiple of the growth number	20% (10)	37% (27)	17
9.	Starting at smallest number	28% (14)	39% (29)	11
10.	Working on a random interval (>20)	47% (24)	28% (21)	-19
11.	Interval located between 1 and 100	41% (21)	93% (69)	52
12.	Unit growth of 2	30% (15)	32% (24)	2
13.	Unit growth of 3	10% (5)	18% (13)	8
14.	Unit growth of 4	10% (5)	16% (12)	6
15.	Unit growth of 5	24% (12)	14% (10)	-10
16.	Unit growth of 10	18% (9)	11% (8)	-7

Table 6.3: The most frequently occurring attributes in classroom practice when working with growth sequences at FP.

indicate the number of activities that were assigned to an attribute. The breakdown reveals significant shifts in some key aspects that informed the selection of items used by the teachers. The first shift was in the category of creating, identifying and extending sequences. In LS1, the 'guess my rule' game was used by five of the teachers in the study (excluding Teacher 5), where they would propose a number sequence and learners would be required to identify the growth rule that had been used by the teacher. These activities ended the moment the correct rule was identified, and there were no extension activities linked to these tasks. In LS2 the teachers worked with these sequences by identifying the growth rule and extending the sequences to items that lie beyond the perceptual. There was a 23 percentage point increase (from 47% in set 1 to 70% in set 2) in the use of surface structure to extend sequences, and this will be the focus of Chapter 7.

A second significant shift occurred in the use of the growth rule (the surface structure) of the set of sequenced items. In LS1, there was a higher incidence of the growth rule being given to the learners (55%; 43%). Instructions that were used in class either directed learners to write down the sequenced items when, for example, counting in twos, starting at 2 and ending at 30 or simply to count in twos from 0 to 100. From LS1 to LS2, questions that required a direct application of some given rule were replaced with questions that asked learners to identify the rule (26%; 46%), and to use the rule to complete and extend sequences. This indicates a 20 percentage point increase in the requirement that learners identify the rule and use it in further application. This change was accompanied by a significant shift in the choice of numbers that made up a sequenced set. The shift was away from using numbers that were relationally connected to the growth number (43%; 18%). In LS2, there was a significant shift away from sequenced numbers starting at a multiple of the growth number (drop by 25 percentage points), to include numbers that were not multiples (20%; 37%) of the growth number (up by 17 percentage points). In doing this, teachers included numbers in the sequences that were between 0 and 20 (28%; 39%), starting at times at the smallest number that is not a multiple of the growth number (up by 11 percentage points).

A third observable shift in the common attributes of sequenced numbers between the two lesson sets was the choice of interval that made up the sequenced items, combined with the choice of unit growth number. In Table 6.3, we see that the teachers tended to choose items in the range from 1 to 100 (up by 52 percentage points) rather than numbers that fell in the range of 1 to 200. In LS1 24% (12/51) of items chosen either breached the 100 limit, by starting in the interval 1 to 100 and ending in the numbers larger than 100 (13% of items in LS1), or were completely located on the interval 100 to 200 (11% of items in LS1). In LS2, a mere 1% (1/74) of items breached the 100 limit. This represents a significant decrease in the magnitude of numbers that were chosen to be included in growth sequences. By choosing lower number ranges in LS2, the teachers could focus on structural aspects by working with sequenced numbers and growth numbers that were easily accessible to learners from their counting practice, even though the sequences did not resemble counting. In the next section I will discuss this in more detail.

From LS1 to LS2, number intervals also appeared to be chosen less randomly (down by 18 percentage points) with a larger number of sequenced items falling in the range of numbers between 1 and 20. The most popular unit growth numbers in LS1 were growths of two, five and ten. This could be since learners could already fluently count in twos, fives and tens. Number sequences in LS2 showed a decline in the use of five and ten as growth numbers (drop by 9 and 6 percentage points respectively) in favour of a growth in the use of two, three and four as growth numbers in sequenced items.

In the next section I focus on this change in number range as it allowed the teachers to focus on the relational aspects of number sequences.

Selection of Starting Intervals for Sequenced Numbers

As previously observed, in LS1 the teachers used random selections for intervals starting at numbers bigger than 20. This was supplanted in LS2 by a focus on numbers that lie between 1 and 100, where starting numbers were chosen as the lowest number (up by 11 percentage points). This focus proved interesting, and can be attributed to a variety of choices converging at this point. Working (i) relationally with number sequences which (ii) contain number items that are not multiples of the growth number, whilst (iii) avoiding using consecutiveness as well as (iv) not revealing the regularity that is applied, compounded the cognitive challenge and informed the decision to keep the sequenced numbers as low as possible. This is particularly true if one considers that the growth units that were most frequently chosen were two, three and four in LS2, as opposed to the use of growth numbers two, five and ten in the first.

The shift away from using multiples of the growth numbers to form sequences (even if counting in twos, fives or tens) is naturally more challenging, as learners cannot rely on fluency and recognition of counting schemes. When accessing numbers in a higher number range, say within the range 41 to 101, learners would often struggle to get 'out of the starting blocks' in the warm-up activity of counting. In these cases, the teachers would then refer the learners to their number chart, and what should have been the building of a sequence, became a counting activity with the use of their number charts as an indicator of what came next. In order to avoid unnecessary distraction from seeking and using structure, at this level of exploration it makes sense not to raise the cognitive bar artificially[55] by using larger numbers and non-multiples in sequences. It would, therefore, appear that the focus which enhances the structural aspects of the sequence will be easily accessible if the number sequences are as rudimentary, yet relationally focused, as possible.

	Lesson set 1	Lesson set 2	Change	Lesson set 1	Lesson set 2	Change
	Range 0 to 20			Range 0 to 40		
Teacher 1	55%	55%	0%	60%	67%	7%
Teacher 2	14%	72%	58%	26%	83%	57%
Teacher 3	62%	78%	16%	74%	94%	20%
Teacher 4	29%	100%	71%	50%	100%	50%
Teacher 5	82%	61%	-21%	91%	67%	-24%
Teacher 6	73%	89%	16%	80%	100%	20%

Table 6.4: Percentage of growth sequenced number items which started within the ranges 0 to 20 and 0 to 40.

Table 6.4 provides the percentages, per teacher, for the ranges 0 to 20 and 0 to 40 of starting numbers used in activities. For the range 0 to 20, there is an average increase of 23 percentage points of sequenced items starting within this range, with increases extending from 16 to 71 percentage points for four of the six teachers. For the range 0 to 40, the average increase is 22 percentage points of sequenced number items starting in this range, with the individual teacher choice of this range increasing from 7 to 57 percentage points. Teacher 5 is the only teacher who included a focus on higher numbers in both lesson sets, on both intervals. Teacher 5 used the number chart exclusively to indicate sequences as arrays on the chart. Her approach in LS2

[55] Teacher 2 referred to 'artificially making it more difficult' when discussing the choice of number range with the researcher after the completion of her lesson. She indicated that she expected the learners to find it difficult to work relationally with sequences that have big numbers, and without using their number charts, if they are not yet comfortable with the different growth numbers. She did not allow the learners to use their number charts as most of the numbers that belong to a certain growth rule had already been coloured in on these charts.

differed significantly, as she worked with the relational aspects of number on the number charts. Details with regards to her approach will be discussed in the next chapter. Teacher 1 remained consistent in her choice of items falling into the range of 0 to 20, but showed a 7 percentage point growth over the range of 0 to 40.

Not all the teachers in the study worked with a higher frequency of larger numbers. Figure 6.6 shows the variety of number ranges that the teachers used in each lesson set. The table

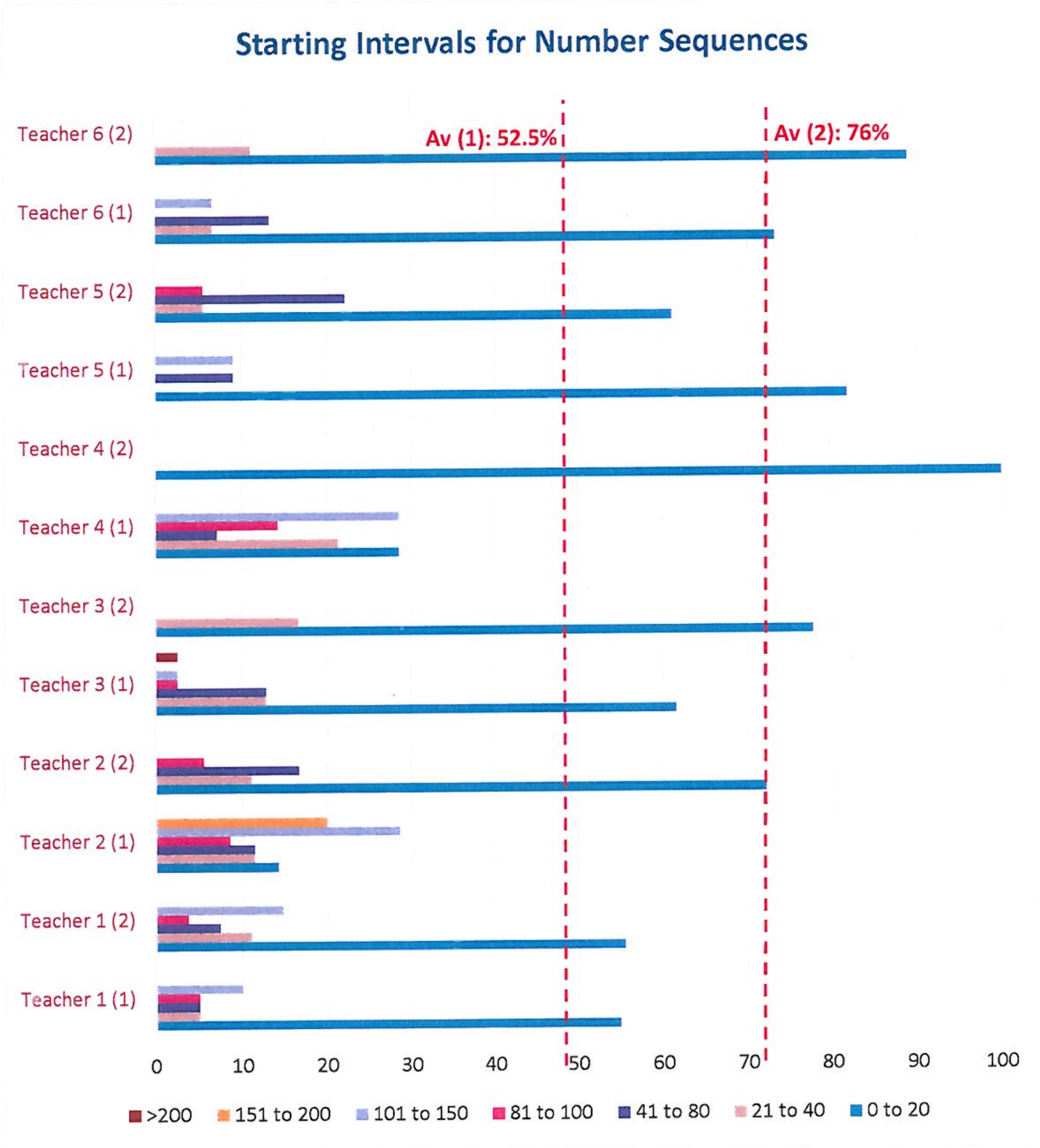


Figure 6.6: Percentage of growth sequence problems falling into a particular starting range in each of the lesson sets.

illustrates percentages which were calculated by only using the number of items working with growth sequences in a lesson, excluding all counting activity. The figures in parenthesis indicate the particular lesson set. The average of number sequence items that started on the interval zero to 20 for LS1 (52.5%) and the second (76%) are indicated in Figure 6.6 by the dotted red lines. Initially LS1 featured many items falling into this interval, primarily because of the nature of the items and the high incidence of growth sequences that were conflated with counting activity.

There was a marked preference for using smaller numbers when working with the relational aspects of number sequences that do not rely on counting activity cues, as can be observed in Table 6.4 and Figure 6.7. Teacher 4 used both number ranges zero to 20 and 101 to 150 with equal frequency in LS1. The reason for this can be attributed to her choice of textbook questions that the learners had to work on in class, as the particular exercises only focused on larger number ranges. Teachers 2 and 3 were also the only teachers who worked on numbers above the range of zero to 150. The reasons for this were different. Teacher 2 chose more sequenced number items in the range from 101 to 200 in LS1 with 49% of problems in this range, and only 26% of the problems falling in the starting range of zero to 40. This high incidence of bigger numbers was mainly due to the fact that she randomly chose intervals to count in and increased the number ranges when learners showed fluency in lower counting ranges, believing that this will raise the cognitive demand of the activities. Teacher 3 was working with learners in groups and asked learners to offer their own examples of number sequences using number cards. One learner formed a sequence of counting in thousands by forming the number sequence 1000; 2000; 3000 ...up to 9000 and another learner formed a sequence of counting in 100s by forming 100; 200; 300... up to 1000.

In Summary

The number ranges and the growth number that were assigned to each of the sequence problems, had a great effect on the ability of the learners to grasp the relational features embedded in growth sequences. When teachers kept the number range high, learners would often have to use their number chart to access the numbers that formed a sequence, and this impeded their ability to work relationally with growth sequences. By keeping the number range low, together with using numbers that are not multiples of the growth number and using growth numbers other that two, five and ten, the teachers were able to achieve the intended relational focus. Learners could check their number facts quickly when they made a mistake at the start of a growth sequence that uses low number ranges, and could also realise that different sequences could be formed by the same growth number.

In both LS1 and LS2 it was observed that by withholding the growth number and using activities where learners needed to access the surface structure, teachers created the opportunity for learners to work with growth and apply it recursively to complete sequences relationally. By increasingly using sequences that do not resemble counting, the necessity was created to 'notice' growth in sequences, and apply it more frequently to complete sequences that were given. In the next chapter, where I focus on the use of structure in the teaching of sequencing at FP, we will again revisit the findings in this chapter with regards to these changes that have been discussed in this section.

Working With or Across the Grain

The pedagogical choices made by the teachers in LS1, when working with counting activities and growth sequences, did not differ greatly from one another. In LS1 the questions that extend a sequence were closely related to counting activities, and learners did not access the structural aspects of the sequenced set of numbers in order to extend the sequence beyond the perceptual. They merely recognised the sequence from counting and wrote down the numbers that were missing or filled in the numbers that were required to extend the sequence. The dearth of sequenced numbers that focus on structural extension, and work across the grain (this notion was discussed in full in Chapter 2) is also problematic when trying to untangle the lines between counting activity, recall and activities in growth sequence that call for extrapolation to another term or set of terms. When these terms are usually the next three, or the two in between, it is easy for learners merely to recall the answer from counting activities, especially when working with sequenced numbers that are multiples of the growth number, and more so when such sequenced items start at the lower multiples. This access by recognition is also applicable to sequenced sets of numbers where the rule is withheld. In LS1, only 4% (5/134) of the growth sequence activities required generalisation across the grain[56]. One of the teachers worked with four-legged animal figures, relating the total number of animal legs to the total number of animals that were presented to the learners. Her questions were designed to show the

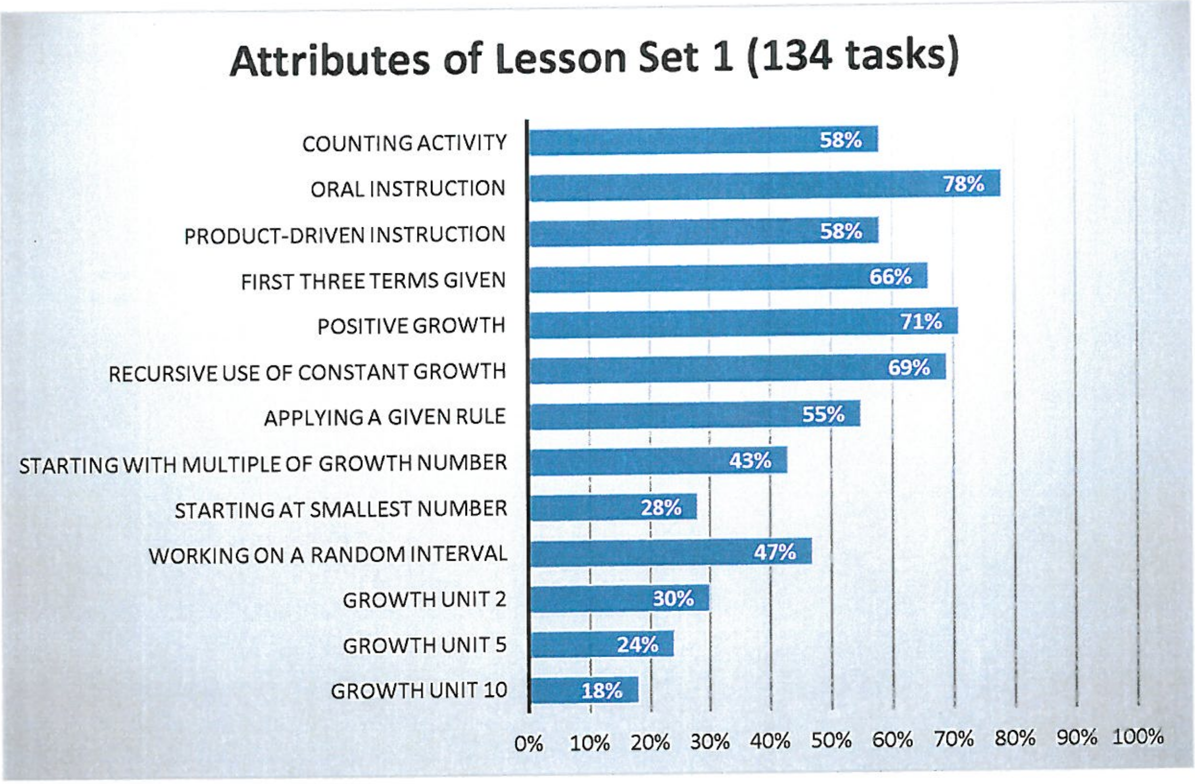
[56] This is described in the coding matrix Chapter 3 on p.77, codes T2 and T3.

relationship between input and output in the sequenced items. I would refer to these plastic animals that were used as number objects and classify them as unitised objects (similar to counters and sticks) and grouped objects (similar to toy cars with four wheels). When working with grouped objects, the notion of functional thinking was also used in recording the data, as every number object was assigned a position in a sequence based on the learners holding the toy animal, and these positions served as unit growths (learner number in the sequence) related to collective group growths (growth of 4). If the teacher referred to the second learner in the row, the two units of learners would each represent four single unit growths that collectively would make up the number 8. The teacher extended this unit growth to include 11 learners making up a sequenced set of 11 numbers which grows by 4 each time to give a total of 44 when learner (or position) 11 is reached.

The number of questions working across the grain in LS2 improved to 13% (14/109) (a growth of nine percentage points) and included work on a number line and representations in tables. This does not represent a significant shift away from a recursive view of sequencing towards a co-variant (functional) relationship. The activities that were used to work across the grain will be discussed in detail in the following chapter.

Combining Key Attributes Per Lesson Set

Through the combination of the common attributes for each lesson set, details regarding the problem types, pedagogy and the mathematical features of the sequenced number sets, which were used in class, can be derived. Figure 6.7 (on the following page), which follows, shows these features in a dual bar graph illustration, which has been compiled by combining the most common features discussed thus far in this chapter. From these illustrations, it is clear to see that counting activities were replaced with growth sequence activities, oral instruction with written instruction and a product-driven or answer-driven teaching approach was replaced by



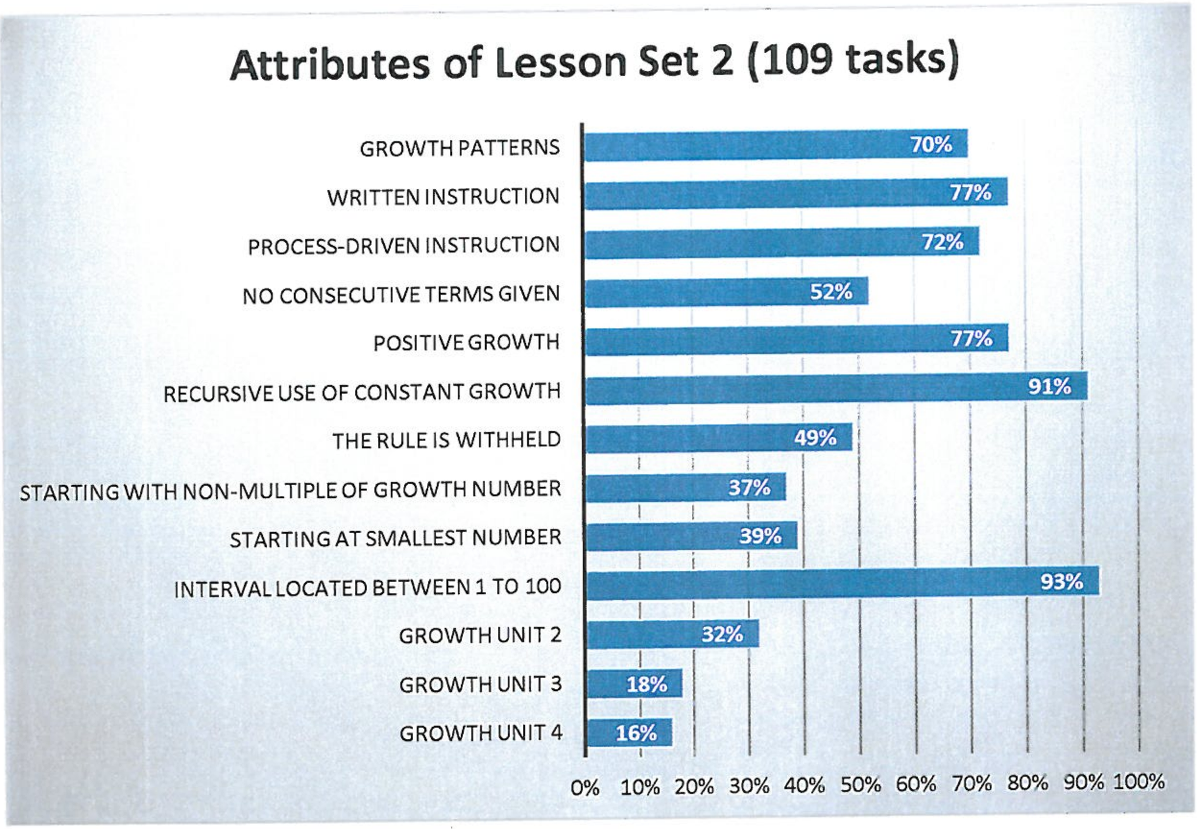


Figure 6.7: Combination of the most common attributes used in growth sequences for both lesson sets.

an approach that emphasised the process-enhancing features of mathematical activity. Both lesson sets worked mainly recursively with positive growth sequences, but activities based on revealing the first three terms in a sequence were replaced by sequences where no consecutive terms were given or sequences where two consecutive terms were revealed later in the sequence. A key change in the approach to number sequences was evident in the replacement of activities that required the application of a rule that was given to the learners with activities where these rules were withheld and learners were required to establish the regularity themselves through accessing surface structural features embedded in the number sequence. Starting at the smallest multiple of the growth number resulted in sequences that resembled counting activities. The replacement of these types of sequences with number structures, which do not use multiples of the growth number, shifted the mathematical activity away from activities resembling counting towards a more relational approach to growth sequences (even if the new number items started at the lowest non-multiples of the growth number).

Summary

This chapter gave a general overview of the items that the participating teachers used when teaching growth sequences, and how they dealt with growth sequences in the classroom. This was illustrated by examining the frequency with which certain choices were made and activities were used in the growth sequence class. There are a limited number of activities provided in the growth sequence section of workbooks and materials which build on the relational aspects of a sequenced set of numbers[57]. In fact, they are mostly related to counting in both form and choice of sequenced numbers.

In LS1 it was evident that learners could count both forwards and backwards using various growth numbers and over a variety of different ranges. These counting activities usually, as per curriculum requirement, counted using multiples of the growth number, and started at the lowest multiple of the growth number. The analysis has shown that learners often did not understand, or even use, relational features when dealing with sequences that resemble counting. This became evident when after chanting a counting sequence, the teacher asked questions that probed the relational features of sequencing. The learners often guessed the growth rule, and then change their answers if the teacher did not immediately acknowledge the

[57] See Chapter 4, pp. 122 – 126 for the details.

answer as correct. The teachers also employed a mainly oral mode of instruction, which resulted in a product driven approach to sequencing, where correct answers were acknowledged and incorrect ones dismissed. There were no mathematical discussions regarding the relational features of growth sequences in LS1. Using these types of sequences therefore limited the possibility to develop a structural approach to sequencing.

In LS2, The first significant shift was the manner in which the teachers worked with the growth in a set of sequenced numbers. By selecting to withhold the growth number in sequencing activities, combined with a movement away from multiples of the growth element, teachers were able to draw attention to the relational features of sequences. A second significant change in pedagogy was the choice of growth number and the range over which the sequenced terms were chosen. By keeping the number range as low as possible, learners could easily self-correct their minor mistakes when working on sequences, and could also work more effectively with sequences that possess growth regularity. The third important shift was the use of consecutiveness of terms to elicit a variety of different approaches to completing sequences. Sequences varied on three different levels of consecutiveness of terms. The first of these sequence types worked with three consecutive terms being given and here teachers used these terms to discuss the surface features of the sequence, and used these to complete or extend the sequence. For those sequences where two consecutive items were given, the use of the growth element in the sequences varied from identifying growth and applying it to check for fit, to combining the systemic and surface features of the given sequence to complete the sequences. The third variant made use of non-consecutive terms. Here the teachers used both systemic and surface features to build understanding of the use of growth.

The consequence of these shifts was that teachers moved away from a pedagogy that favoured oral instruction, and was predominantly product-focused and answer-driven, to one that foregrounded relational attributes of growth sequences and processes that work with the structure embedded in growth sequences. These pedagogical changes resulted from the convergence of the number of key modifications that teachers made to their teaching and choice of attributes in sequences during LS2.

Working with the structural features in sequences is only possible when teachers work with sequences where consecutiveness is limited to two or none of the terms in the sequence. Mason, Stephens and Watson (2009) maintain that thinking structurally is characterised by the use of structure when dealing with sequences, not just merely recognising surface structural features.

The teachers in the study could only start developing the approach that uses structural features in the sequences, when they chose sequences with limited consecutive terms given, and used sequences that do not resemble counting. Surface structure and systemic structure worked together to enhance methods that developed generalised forms of sequences later on.

Chapter 7 will explore the shift towards the notion of structural extension and the fostering of algebraic reasoning skills in the classroom. This discussion will compare LS 1 and LS2 lesson attributes to identify changes in the pedagogical approaches, particularly focusing on structural extension. Using the features identified in this chapter, I propose a typology for both growth and repeating sequences and use this typology to discuss the use of structure in the FP class in CA 2. I continue to discuss moves towards working structurally with sequences in the context of the classification schemes that are proposed, and how this classification can be used to distinguish items that foster a structural approach to sequences from those that do not.