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Adaptive Market Efficiency: Evidence from the South African Stock Market

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Adaptive Market Efficiency: Evidence from the South African Stock Market

Abstract

This study examines the adaptive market hypothesis in the Johannesburg Stock Exchange market by testing for stock return predictability using daily data from January 1996 to December 2023 across six different indices. The study applies three linear and three nonlinear tests to check if the Johannesburg Stock Exchange market is efficient, moving towards efficiency, switching to efficiency/inefficiency, adaptive and inefficient. The overall findings of this study provides compelling evidence that stock returns within the Johannesburg Stock Exchange market across the six indices are inefficient, as indicated by significant evidence of inefficiency, non-randomness, the presence of serial correlation and nonlinear effects in the six tests performed. These results challenge the notion of the weak-form of market efficiency and suggest that past prices of stock returns are related to future prices, indicating the presence of predictability and exploitable opportunities within the JSE market.

Declaration

I, Ishmael Masangu declare that this research paper is my own work and that I have correctly acknowledged the work of others. It is submitted to fulfil the requirements for the degree of Master of Commerce in Finance at the University of the Witwatersrand, Johannesburg. I declare that this research paper has not been submitted for any other degree or examination in this or any other institution.

Ishmael Masangu

March 2024

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Definition of abbreviations, terms and symbols

Abbreviation/term/symbol	Full name
ADF	Augmented Dickey-Fuller
ALSI	All share index
AMH	Adaptive Market Hypothesis
AR	Autoregressive
BDS	Brock, Dechert and Scheinkman
COVID-19	Corona Virus Disease 2019
DFA	Detrended fluctuation analysis
EMH	Efficient Market Hypothesis
ER	Expected return
fin	Financial Index
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
gold	Gold Index
iid	Independent and identically distributed
ind	Industrials Index
JSE	Johannesburg Stock Exchange
min	Mining Index
TAR	Threshold autoregressive
Top40	JSE Top40
USA	United States of America
VR	Variance ratio

1. Introduction

The topic of market efficiency has been the subject of in-depth debate and discussion in the finance literature. Empirical testing have been carried out to ascertain whether or not markets are efficient since the groundbreaking work of Fama (1970) outlining the Efficient Market Hypothesis (EMH). The paradigm has changed to offer alternative, testable theories of market efficiency that can be shown to be true or incorrect. This study investigates the ability to predict stock returns over different periods and assesses whether one such hypothesis, the Adaptive Market Hypothesis (AMH) is a suitable framework for explaining the predictability on the Johannesburg Stock Exchange (JSE).

This research is motivated by the fact that, according to the weak-form EMH, stock returns follow a random walk and are unpredictable given past prices because all information is completely reflected in stock prices. that is currently available (Fama, 1970). Many authors (see Chapter 2) have found, however, that stock returns can be predictive and that the EMH is not always valid, raising doubts about the EMH's dependability. Furthermore, the majority of previous studies on market efficiency employed a conventional technique to assess the weak-form efficiency over a specified period of time, which has two major problems:

- Firstly, according to Campbell, Lo, and MacKinlay (1997), this method ignores the concept of relative effectiveness. Relative efficiency allows for the comparison of market efficiency across different markets, enabling the assessment of whether a market operates efficiently throughout a given period. It refrains from viewing market efficiency as a binary condition but rather evaluates its degree of efficacy relative to other markets.
- Secondly, the traditional approach presupposes a static level of market efficiency over a predefined duration. However, this assumption is unrealistic given the dynamic nature of financial markets. As noted by Lim and Brooks (2011), various factors continually influence market dynamics, leading to fluctuations in efficiency levels over time

Lo (2004) proposed that market efficiency and inefficiencies can concur in a way that is cognitively consistent. According to the AMH, market efficiency changes through time as opposed to adhering to the traditional notion of all-or-nothing efficiency. The number and makeup of market players as well as trading tactics are determined by natural selection, which

guarantees the survival of the fittest. Market participants employ heuristics to inform their investment choices as they adapt to a perpetually evolving environment. Lo (2005) used a rolling first-order autocorrelation test to show that the return predictability can consequently occasionally change due to shifting market conditions on the S&P Composite Index's monthly performance over the period 1871 to 2003. As a result, there is no certainty that market efficiency will eventually converge, and the degree of efficiency will rely on the players in the market and its circumstances at any given time. There have been several studies published recently that provide compelling evidence of the AMH in stock markets (see for example Ghazani & Araghi, 2014; Kim, Shamsuddin, & Lim, 2011; Lim, Luo, & Kim, 2013; Manahov & Hudson, 2014; Urquhart & Hudson, 2013; Urquhart & McGroarty, 2014). This study aims to extend to the body of literature of the AMH in South Africa and motivated by the discovery that, contrary to the weak-form (i.e., stock prices follow a random walk) of EMH stock returns may be predicted using historical price and return data. By applying linear and nonlinear methods, this study will assess the predictability of stock returns in an emerging economy like the JSE to further understand whether stock returns within the JSE are efficient (i.e., whether they follow a random walk) or are inefficient. This study is motivated by the studies of Van Heerden *et al.*, (2013), Phiri (2015); Seetharam (2016) and Yousuf and Makina (2022) when they looked at market efficiency from the evidence of the EMH and the AMH on the JSE, respectively. Similar to Van Heerden *et al*., (2013) and Phiri (2015), this study will look at six generalised stock price indices under the JSE using daily data collected between the 01 January 1996 and 31 December 2023. Using daily data provides consistency, reduces noise, and is practical to work with. Limiting the study to six indices enhances manageability, reduces risk, and allows for comparisons across the six indices

1.2 Motivation of the Study

This study is motivated by the discovery that, contrary to the weak-form of the EMH, stock returns may be predicted using historical price and return data. To evaluate the weak-form of the EMH empirically, the study uses both linear and nonlinear approaches.

This study will examine six stock price indices within the JSE, including the All Share Index (*ALSI*), the JSE Top 40 Companies Index (t*op40*), the Industrials Index (*ind*), the Financial Index *(fin)*, the Mining Index (*min*), and the Gold Index (*gold)*. Daily data collected from January 31,

2019, to December 31, 2021, will be utilised. The study will adopt the methodology employed by Ndubuisi and Okere's (2021) on stock return predictability and the AMH in emerging markets, as elaborated in section 3.2.

1.3 Knowledge Gap

In South Africa, there has been a few studies conducted to investigate the AMH (See Van Heerden, Rodrigues, Lambert, Tjaart and Phiri 2013; Phiri, 2015; Seetharam, 2016 and Yousuf & Makina, 2022). This study seeks to add to the body of literature of the AMH in South Africa by empirically analysing the AMH in contrast to the EMH.

1.4 Testable Hypothesis

Within the context of the South African JSE market, is the share price behaviour efficient?

Primary Hypothesis (further hypothesis are explained in section 4).

Null Hypothesis (H_0): The stock prices within the South African JSE market are inefficient. In other words, it implies that the JSE market stock prices do not fully reflect all available information and future prices can be anticipated using past prices. i.e., the movement of share prices is not random.

Alternative Hypothesis (H_1): The stock prices within the South African JSE market are efficient. This means that stock prices on the JSE accurately and quickly represent all pertinent information that market players have access to. This further suggests that future prices cannot be predicted using past prices i.e., the movement of share prices is random.

In order to test for market efficiency in the JSE market, there are going to be five categories of market efficiency i.e., efficient, switching to efficiency/inefficiency, moving towards efficiency, adaptive and finally, inefficient. If returns are uncorrelated and independent across the sample, a market is said to be efficient. In the event that returns were dependant, and this dependence has been eroding over time, the market is shifting in favour of efficiency. A market has transitioned to inefficiency (efficiency) if returns were independent (dependent) but then became dependent (independent); however, this could also indicate an early-stage adaptive market. A market is considered adaptive if returns have gone through at least three distinct stages of dependency

(dependent, independent, and dependent). Finally, a market is inefficient if returns are not independent throughout the sample.

1.5 Limitations

One of the limitations in this study is that there are other widely used tests to examine the AMH which are not utilised in this study. However, this study uses different tests that perform similar functions to the ones not used, as explained in sections 2.2 and 3.2.

1.6 Chapter outline

The study's remaining sections are as follows. Section 2 provides a review of the existing empirical research on market efficiency while section 3 shows the data and methodology applied in the paper. Section 4 shows the empirical results, discusses and summaries the empirical results. Section 5 shows the conclusion of the paper.

2. Literature review

The aim of this chapter is to examine the current body of literature concerning market efficiency. Section 2.1 provides an overview of market efficiency, exploring its historical evolution, as well as its theoretical and practical implications. Section 2.1.1 starts by defining the EMH while section 2.1.2 dwells on the definition and the theories of the AMH. In Section 2.2, the focus shifts to an exploration of methodologies commonly utilised in examining the AMH but not employed in the present study.

2.1 Review of existing literature

2.1.1 Defining the EMH

Fama (1965) defined the EMH as a framework where competition among numerous knowledgeable participants results in a system in which, at any given time, the actual prices of individual securities already incorporate the impact of information based on both past events and projected future events. To put it simply, in an efficient market, a security's current price is a good indicator of its inherent value at any given moment. The term "efficient" denotes a market in which prices regularly follow a random walk pattern (further explained below) and takes into account all available information (Fama, 1965).

According to Mischin (2010), the EMH informs us that we cannot anticipate obtaining an exceptionally high return that surpasses the equilibrium return when purchasing a security. The EMH claims that historical prices cannot be utilised to predict future prices economically runs counter to the fundamental principles of technical analysis. For proponents of this concept, looking for cheap stocks or attempting to forecast market movements using technical or fundamental analysis is futile.

According to the EMH, participants are always playing a game of chance rather than skill when the buy and sell stocks. One will never be able to purchase a stock at a discount if markets are efficient and up-to-date, which implies that prices always reflect all available information. This idea states that a market is considered efficient if it operates in a way that makes transactions relatively inexpensive for both buyers and sellers and ensures that all parties are promptly

informed of any new developments. Such fresh information will be reflected in market prices (Mishkin, 2010).

The EMH states that there are numerous participants in an efficient market who have access to all pertinent information influencing stock prices. These participants compete freely and equally for stocks, which results in a full reflection of the value of stocks in their current prices due to the competition and the full information available to the participants . Although individual market players may not always act rationally or possess all the information, EMH contends that their combined decisions balance one another and produce a rational result (optimists who purchase stock and raise the price are offset by pessimists who sell their stock, maintaining the price in equilibrium) (Fama, 1970).

Stock prices act in a random manner when new information is developed, acted upon, and reflected in prices. Additionally, the EMH backs up the random walk hypothesis by asserting that the prices of any tradable instrument are fundamentally unpredictable. According to the EMH, efficiency can be divided into three types of market efficiency as proposed by Fama (1965) i.e., weak-form, semi-strong, and strong-form of efficiency. Each degree of efficiency reflects a different set of information because different types of information can be made available in a market that results in these different levels. How well market prices reflect the quality of information determines how efficient the market is (Chitenderu, 2013). The three forms of market efficiency are explained below.

Weak-form of efficiency

This type of efficiency asserts that stock prices incorporate all accessible information, primarily historical prices, leading to stock returns being entirely unpredictable based on past prices alone. When a market displays weak-form efficiency, the current prices of stocks fully reflect all of their historical prices. As a result, technical analysis loses its ability to regularly outperform the market because it depends on past price patterns to forecast future price movements. In such a scenario, information pertaining to recent stock price fluctuations would offer no advantage in stock selection (Jain & Jain, 2013).

Semi-strong form of efficiency

According to the semi-strong form of market efficiency, share prices react swiftly to releases of information. A strategy based on technical or fundamental analysis won't generate returns higher than the market average. On the other hand, people with access to private data could be able to get better returns (Seetharam, 2016).

Strong-form of efficiency

Finally, share prices accurately reflect both public and private information when there is strong-form efficiency. As a result, no long-term, sustainable, superior profits after costs can be obtained (Seetharam, 2016).

This study is going to analyse the AMH in contrast to one form of the EMH which is the weak form. The AMH is discussed below.

2.1.2 Definition and the theories of the AMH

Contrary to the weak form of the EMH, Fama and French (1988) proposed that asset prices do not exhibit a random walk behaviour, and fluctuations in prices can be predicted. Opong, Mulholland, Fox, and Farahmand (1999) performed regressions on the FTSE and found that, in contrast to perfectly random series, some cycles or patterns recur more frequently, indicating that the stock index returns series exhibit non-random properties. The debate surrounding the EMH was subsequently advanced by experts in behavioural finance, with Shiller (2003) asserting that markets do not consistently function efficiently. He identified two anomalies: first, investors tended to both underreact and overreact to anomalies; and second, these anomalies tended to diminish over time or as research methodologies improved. Shiller argued that these findings do not support the notion of fully rational markets, as there is no inherent psychological principle dictating that individuals consistently overreact or underreact. Furthermore, Lo (2005) extended the discussion on behavioural biases by introducing the AMH, which asserts that market efficiency and inefficiency can coexist with predictability. This hypothesis draws on the evolutionary principles (e.g. competition, mutation, reproduction and natural selection) from behaviour economics (Lo, 2004). Market prices, according to Lo (2005), are representations of both rational and irrational ideas. The EMH has been contested by behavioural economics and finance, which offer two main grounds of contention: 1) Markets are not always logical and there

are opportunities for profit, due to inefficiencies, and 2) Investors are influenced by irrational factors such as fear, loss aversion, overconfidence, overreaction and greed, impacting their decision-making processes.

Lo (2004) states that the efficiency of markets and the performance of investment goods, companies, and sectors are determined by the effects of evolutionary forces on financial institutions and market players. AMH has some unexpectedly tangible ramifications for financial operations, despite its generally qualitative and abstract nature.Lo (2004) looked into market efficiency in addition to behavioural alternatives using evolutionary concepts such as competition, adaptability, and natural selection. He examined monthly S&P Composite index returns spanning from 1871 to 2003, analysing them over ten-year rolling windows. According to his findings, market efficiency varies cyclically throughout time.

Lim and Brooks (2006) carried out research on the changing efficiency of developing and developed stock markets. They examined daily returns of the MSCI indices for 27 developing and 23 developed nations between December 31, 1989, and December 31, 2005. Utilising the portmanteau bicorrelation test statistics introduced by Hinich (1996), they discovered that the extent of market efficiency showed cyclical fluctuations over time within a rolling sample framework. These findings were consistent with the principles outlined in the AMH. Lim and Brooks (2006) looked at both developing and developed markets while this study looks at the JSE which is a developing market using daily returns data. Lim and Brooks (2006) analysis of daily returns from MSCI indices of various countries provides insights into the dynamics of market efficiency over time, which can inform this study's phenomena in the JSE market context.

Lim (2007) conducted a study focusing on 11 emerging markets (Argentina, Brazil, Chile, Indonesia, Malaysia, Mexico, Philippines, South Korea, Taiwan, and Thailand) as well as two developed markets (Japan and the United States of America) over the time frame of January 1992 through December 2005. Employing a rolling sample portmanteau bicorrelation test statistics approach with a rolling window of 50 observations, the author discovered that the efficiency levels of each market undergo evolutionary changes over time, aligning with the principles of the AMH. Similar to the findings of Lim and Brooks (2006), Lim (2007) analysed daily returns across various indices for both developing and developed markets. These insights

into the dynamics of market efficiency over time can offer valuable implications for understanding phenomena within the context of the JSE market.

Todeo, Ulici, and Silaghi (2009), within the framework of the AMH, examined the return predictability utilising the stock market indices daily returns from six Asian countries. (HangSeng, BSSE, Kuala Lumpur, Strait Times, and Nikkei 225) over the period of 1997 to 2008. They employed a moving average strategy in their analysis. Their findings indicated that as profit opportunities vary over time, the degree of the market efficiency also fluctuates. These results support the principles of the AMH. Similar to their study, Todeo, Ulici, and Silaghi (2009) investigated the stock market indices daily returns from multiple Asian countries, shedding light on the dynamics of market efficiency over time. Their research emphasizes the necessity of using robust statistical methods to evaluate market efficiency, thus aligning with the focus of this study on market efficiency and the AMH within the context of the JSE market.

Ito and Sugiyama (2009) examined market inefficiency by computing the time-varying autocorrelation of stock returns. They analysed the S&P 500 index monthly returns from January 1955 to February 2006, utilising a rolling window approach to calculate first-order autocorrelations. This methodology allowed them to assess how the autocorrelation of stock returns varied over time, providing insights into market inefficiencies. According to their research, market efficiency varies with time, with the late 1980s being the most inefficient and the year 2000 being the most efficient. While their research focuses on the S&P 500 index and utilises a rolling window methodology to measure inefficiency, their insights into the temporal variation of market efficiency offer valuable context for understanding similar dynamics in other markets, including the JSE market.

Kim, Shamsuddin, and Lim (2011) investigated the stock returns predictability using daily data from the DJIA spanning from 1900 to 2009. They employed an automated VR test and an automated portmanteau test, utilising a rolling window approach. Their analysis revealed substantial evidence of predictability which varies over time and is influenced by market dynamics. They found that stock market crashes were not significantly correlated with return predictability, whereas economic and political crises exhibited a high degree of predictability with a moderate level of uncertainty. Furthermore, they observed that stock returns were highly predictable during periods of economic and political crises. The findings of Kim, Shamsuddin

and Lim (2011) provides empirical support for the notion that market efficiency is not a static phenomenon but rather evolves over time, aligning with the core tenets of the AMH. The findings of Kim, Shamsuddin and Lim (2011) will contribute to a deeper understanding of how adaptive behaviour shapes market outcomes and sheds light on the mechanisms underlying stock return predictability within the framework of the AMH in this study.

Alvarez-Ramirez, Rodriguez and Espinosa-Paredes (2012) provide an entropy-based measure of market efficiency. The Detrended Fluctuation Analysis (DFA), applied over a rolling window, is utilised to derive the fractional scaling exponent using the DJIA daily returns from 1929 to March 2012. Their findings were that market efficiency could change over time, similar to the AMH. The study by Alvarez-Ramirez *et al.* (2012) and its implications for market efficiency can enrich this study's analysis on the AMH in the South African context. Their findings underscore the importance of adopting innovative methods to assess market dynamics and highlight the need to account for temporal variations in market efficiency when evaluating the validity of the AMH. This study also applies various methods to assess market dynamics.

Smith (2012) employed the rolling window VR approach to investigate how European emergent stock markets are evolving and how efficient they are. The study analysed daily returns from 15 developing and three developed nations in Europe, spanning from February 2000 to December 2009. The research revealed that, on one hand, the least efficient markets were identified as Ukraine, Malta, and Estonia, while on the other hand, Turkey, the UK, Hungary, and Poland emerged as the most efficient markets. Each of the 18 markets demonstrates evidence of the return's compatible with AMH time-varying character. In the context of this research, Smith (2012) findings serve as a compelling empirical demonstration of adaptive behaviour within financial markets. The observed variations in market efficiency underscore the dynamic nature of market performance, indicative of participants' abilities to adapt their strategies in response to changing conditions over time. By integrating Smith (2012) findings into this study, the aim is to deepen the understanding of how adaptive behaviour influences market efficiency and dynamics within the framework of the AMH, thereby enriching the theoretical underpinnings of this research.

Three major US stock indices were examined for return predictability using a rolling estimation technique by Lim, Luo, and Kim (2013). They utilised the wild bootstrapped automated VR test

and automated portmanteau Box-Pierce test in their analysis. The study revealed evidence of time-varying return predictability and strong return autocorrelations, largely attributable to significant exogenous events. These results align with the principles of the AMH. Lim, Luo and Kim (2013) highlight the influence of large exogenous events on return autocorrelations, suggesting that external factors play a significant role in shaping market dynamics. In the context of this research, Lim, Luo, and Kim (2013) findings serve to strengthen the theoretical framework of the AMH. By demonstrating the influence of large exogenous events on return autocorrelations, their study provides empirical support for the notion that market efficiency is contingent upon the adaptive responses of market participants to external stimuli. Furthermore, their analysis enriches the understanding of how adaptive behaviour manifests in market dynamics, particularly in the context of return predictability.

Popovic, Mugosa, and Durovic (2013) conducted an empirical examination of the AMH within the context of the Montenegro equity market. The authors utilised the rolling window method to analyse the Montenegrin stock market's daily and weekly returns spanning from 3 January 2004 to 31 December 2011. They discovered that market efficiency changes with time, in line with the AMH. Popovic, Mugosa and Durovic (2013) found evidence that efficiency of the market changes with time in the Montenegro equity market, consistent with the principles of the AMH. This observation resonates with the overarching theme of this study, which aims to explore how market efficiency evolves over time in response to changing market conditions and participant behaviour. By corroborating their findings with this study's analysis of the JSE market, the dynamics of market efficiency can be better understood with the help of this study.

Urquhart and Hudson (2013) conducted an examination of the Japanese, UK and US stock markets. They analysed the daily returns of TOPIX, FT 30 and DJIA indices from their inception until December 31, 2009. The data was divided into five-year sub-samples, and both linear (autocorrelation, running, and variance ratio tests) and non-linear methods (McLeod Li, Engle LM, and BDS tests) were employed. Their findings provided substantial evidence for the AMH's viability as an explanation for stock return behaviour, surpassing that of the EMH.

Zhou and Lee (2013) investigated real estate investment trusts data using the automatic portmanteau test and automated VR test. According to their findings, market efficiency fluctuates over time and is impacted by the prevailing conditions of the market. This study

applies similar tests as with Urquhart and Hudson (2013) which will help in understanding of the dynamics of market efficiency.

Hull and McGroarty (2014) investigated 22 emerging markets utilising the Hurst-Mandelbrot-Wallis rescaled range test as a metric for market efficiency. Their analysis yielded significant evidence supporting the AMH. Ghazani and Araghi (2014) examined the Tehran Stock Exchange's daily data spanning from 1999 to 2013. Their findings indicated that the AMH offers a suitable evolutionary viewpoint on efficient markets. Similar to Hull and McGroarty (2014), this study looks at an emerging market using daily returns to understand the dynamics of market efficiency.

Rodriguez, Aguilar-Cornejo, Femat, and Alvarez-Ramirez (2014) employed DFA to investigate the AMH concerning the DJIA index. They analysed data spanning weekly, monthly, quarterly, and yearly time scales from 1929 to 2014. Their findings indicated that inter-day and intraday returns exhibited stronger serial connections compared to overnight returns. Furthermore, they revealed that the DJIA's efficiency does not remain constant with time, aligning with the principles of the AMH. The authors find that the DJIA index's efficiency varies with time, with inter-day and intraday returns exhibiting stronger serial connections compared to overnight returns. This observation underscores how market efficiency fluctuates with time, consistent with the principles of the AMH. By corroborating their findings with this study's findings of the JSE, this study can contribute to the understanding of how market efficiency evolves over time and its implications for market participants.

Noda (2016) used a time-varying autoregressive (AR) model to evaluate the AMH in Japan. With reference to Ito, Noda, and Wada's (2014, 2016) studies, the monthly returns of Japan's TOPIX and TSE2 indices for the years from October 1961 to December 2015 were examined using the time-varying model technique. Their findings were that both markets' efficiency varies over time. When compared to the TSE2 index, the TOPIX index is more effective. The performance of the TOPIX market, a more accurate indicator, is evolving while the TSE2 index remains static. As a result, the findings back up AMH. The reference to Ito, Noda, and Wada's (2014, 2016), Noda (2016) allows for a comparative analysis of efficiency dynamics across different indices and markets. By examining how efficiency varies between the TOPIX and TSE2 indices within the Japanese market, Noda (2016) offers insights that can be applied to

other financial markets, including the JSE market. By drawing parallels between Noda's (2016) observations and the behaviour of the JSE market, this study can enhance the applicability and generalisability of this study's findings.

In a study focused on the Ghana stock market, Gyampfi (2018) investigated the GSEFSII and the GSEALSH stock indices return predictability. The author utilised the generalised spectral test, the wild-bootstrapped automated variance ratio (VR) test, and the automatic portmanteau Box–Pierce test to analyse data from a return series spanning from 2011 to 2015. The results indicated that the GSEALSH index demonstrated greater predictability compared to the GSEFSII index across all experiments. Furthermore, Gyampfi (2018) concluded that these findings aligned with the principles of the AMH. As Gyampfi (2018) concludes that his findings align line with the AMH, this suggests that market efficiency in the Ghana stock market, as reflected by the predictability of stock indices, evolves over time in response to changing market conditions and participant behaviour. By drawing parallels between Gyampfi's (2018) observations and the behaviour of the South African stock market, you can enhance the applicability and generalisability of your research findings within the framework of the AMH.

Ertas and Ozkan (2018) investigated the AMH concerning market efficiency in both the Turkish and US equity markets. They utilised Lo's (2004) five-year rolling window approach, analysing the BIST-100 and S&P 500 indices monthly returns from 1 February 1988 to 1 February 2018. Their research revealed that the efficiency levels of these markets evolve over time. Moreover, they found that the AMH provides better insights into the behaviour of these markets compared to the EMH.

Urquhart and McGroary (2016) examined stock return predictability over time in four of the world's largest stock markets, namely the S&P500, FTSE100, NIKKEI225 and EURO STOXX 50 by evaluating the AMH using daily data from January 1990 until May 2014. Furthermore, they examine whether certain market conditions can be linked to the different degrees of stock return predictability as market conditions can have significant effects on the psychology of market players and how they interpret data, which in turn affects how they make decisions.

In order to evaluate the predictability of stock returns, they apply three bootstrapped versions of the Variance Ratio (VR) to evaluate nominal stock returns and also whiten the returns via an AR-Generalised Autoregressive Conditional Heteroskedasticity (GARCH) process to examine

nonlinear predictability after accounting for conditional heteroscedasticity through the Brock, Dechert and Scheinkman (BDS) test. According to their findings, there are periods of significant predictability and periods of no predictability for each return series, indicating that the predictability of returns in every market varies over time (Urquhart & McGroary, 2016). This is demonstrated by each variant of the VR test as well as the BDS test for nonlinear predictability.

The results suggest that predictability of linear and nonlinear stock returns varies over time and that market efficiency is not an all-or-nothing condition. Additionally, the authors note that several markets exhibit strong predictability at various points in time, which suggests that each market develops in a unique way throughout time and that markets predictability is not highly associated. The authors also discovered that specific market conditions exhibit statistically significant correlations with predictability in certain markets. However, each market demonstrates unique interactions with these various market conditions. This shows that markets interact and respond to changing market conditions differently throughout time (Urquhart & McGroary, 2016).

In contrast to the weak-form, stock returns have been demonstrated to have predictive potential from historical price and return data, which is the motivation of the study by Urquhart and McGroary (2016).

The AMH has also been investigated in South Africa. Mwamba (2011) used a parametric (the weak form of the EMH which is equivalent to the random walk) and non-parametric models to examine the predictability of stock market behaviour in South Africa. After ten thousand replications, it was discovered that non-parametric stock price modelling outperformed parametric modelling in terms of low predicted mean square error, low volatility, higher mean, and a narrower confidence interval. Mwamba (2011) empirically proved the outperformance of the non-parametric model over the random walk model for South African equity market.

Van Heerden, Rodrigues, Lambert, Tjaart and Phiri (2013), evaluate the EMH in the JSE between the years January 2001 and July 2013 deviated from the typical use of a linear approach. Through the application of a threshold autoregressive (TAR) model and related symmetric unit root tests, they found their results reconcile two opposing assertions made in earlier investigations by coming to the conclusion that a number of stock prices listed on the JSE stock

market may not develop as pure unit root processes, but rather as a highly persistent time series, and suggesting they can be considered weak-form efficient.

Phiri (2015) conducted further investigation into the weak form of the EMH for five generalised stock indices in the JSE. The study utilised weekly data collected from January 31, 2000, to December 16, 2014. Phiri (2015) employed the Augmented Dickey-Fuller (ADF) tests, the three-regime unit root tests proposed by Bec, Salem, and Carrasco (2004), as well as the two-regime TAR unit root tests described in Enders and Granger (1998). Similar to the findings of Van Heerden *et al.* (2013), Phiri (2015) concluded that although some stock prices on the JSE may not follow pure unit root processes, the time series exhibit persistence to a degree that qualifies them as weak-form efficient.

Seetharam (2016) investigated the AMH with the objective of providing a framework for analysing South African equities data, which included 44 shares and six indices spanning from 1997 to 2014. The study aimed to verify the dynamic (cyclical) nature of market efficiency. Through the application of this framework, several stylized facts emerged. Firstly, it was observed that data frequency has an impact on the effectiveness of market analysis. Using only a single frequency of data might lead to contradictory results. Secondly, data can be binned into smaller sub-samples to create patterns that indicate whether the equities market is efficient. Thirdly, one can make the results more reliable by doing a range of tests. However, this is a somewhat debatable because one may either do numerous tests, each of which would be superior to the previous ones or discuss the theoretical benefits of every examination before deciding which is most suited. Fourthly, in markets with highly concentrated sectors such as the JSE, there might be a tendency to assume that the entire JSE exhibits unpredictability. However, efficiency can be enhanced through industry analysis. Unlike other sectors, the resources sector may play a leading role in this regard. Lastly, when analysing data with less-than-ideal sample sizes, the use of neural networks as approximators is advantageous. The outcomes for the whole sample period and non-overlapping sub-samples revealed that the temporal accuracy of this model varied. One can draw the conclusion that the nature of market efficiency in South Africa can be viewed as time dependent, in line with the implication of the AMH consistent with Lim and Brooks (2011), when combined with the results of the trading strategies.

Yousuf and Makina (2022) examined the JSE to see if the AMH and bounded rationality theories are applicable to assess how behavioural risk factors affect the effectiveness of the stock market as behavioural theories show a gap in the theory of market efficiency. The gap highlighted by the authors was that some impacts on financial markets cannot be adequately explained by current fundamental economic models. One such example is the influence of political factors. To account for this, the authors looked at behavioural risk factors such as the business confidence index, consumer confidence index and in predicting future returns. The authors further used a quantile regression to show the applicability of the AMH and found that past market returns are highly predictive of future return, deviating from the random walk. Higher quantiles saw an increase in the lagged return, which varied depending on the state of the market (such as, pre-financial crisis, financial crisis and post-financial crisis). So, as market conditions change, so does the predictability of returns consistent with second issue by Lim and Brooks (2011).

Convergence and divergence among the various studies on AMH

In line with Lo (2004) AMH's, which holds that market efficiency changes over time in response to evolutionary processes like competition, adaptability, and natural selection, the studies above collectively highlight the dynamic and cyclical character of market efficiency. The above studies consistently show that efficiency is a dynamic state that varies according on participant behaviour, market conditions, and outside events. This temporal unpredictability is highlighted by studies of Lim and Brooks (2006), Lim (2007), and Ito and Sugiyama (2009), which demonstrate that market efficiency cycles between times of high and poor predictability.

Methodologically, most studies adopt rolling window approaches to assess time-varying efficiency, allowing them to capture trends and patterns over time. For example, Lo (2004) used ten-year rolling windows to analyse S&P Composite index returns, while Lim and Brooks (2006) applied a rolling sample framework to MSCI indices. Similarly, studies like Ertas and Ozkan (2018) and Ito and Sugiyama (2009) used rolling estimation techniques to explore return predictability and autocorrelations. These approaches consistently reveal that efficiency evolves cyclically and is influenced by market dynamics and external factors.

Another area of convergence is the predictability of stock returns. Numerous studies above, such as those by Alvarez-Ramirez et al. (2012) and Kim, Shamsuddin, and Lim (2011), demonstrate that stock returns are predictable during particular times, frequently associated with financial,

political, or economic crises. This result supports the AMH's claim that participants' adaptive reactions to shifting circumstances are what determine market efficiency. Additionally, the study by Gyampfi (2018) and Hull and McGroarty (2014) applies this knowledge to emerging markets, showing comparable cyclical trends in return predictability and market efficiency.

2.2 Literature on methodology comparisons

In this section, we will explore the common methods and tests utilised in examining the AMH. It should be noted that this study uses other tests explained in chapter 3 that perform similar functions with the below-mentioned tests that are specifically tailored to assess aspects of market efficiency and the AMH in financial time series data.

DFA test

DFA is a robust statistical method utilised to scrutinise the temporal correlations inherent in time series data. Originally introduced by Peng, Buldyrev, Havlin, Simons, Stanley, and Goldberger (1994), DFA has garnered significant attention due to its efficacy in discerning the presence of long-range correlations and fractal-like patterns within complex systems (Peng *et al.*, 1994).

At its core, DFA operates through a systematic process that involves the following key steps: data preparation, trend removal, division and measurement, fitting and exponent calculation, and interpretation (Hardstone, Poil, Schiavone, Jansen, Nikulin, Mansvelder, & Linkenkaer-Hansen 2012). Initially, the time series data undergoes meticulous preparation to ensure the elimination of any discernible trends. This step is pivotal as it enables DFA to focus exclusively on the inherent fluctuations within the data, disregarding any extraneous influences. Subsequently, the detrended data is partitioned into non-overlapping segments of uniform size, and the root mean square fluctuations of these segments are meticulously computed.

DFA's strength lies in its ability to discern the scaling properties of the fluctuations across various time scales. By plotting the root mean square fluctuations against the corresponding window sizes on a logarithmic scale, DFA facilitates the estimation of a scaling exponent, denoted as α. This exponent characterises the correlation properties of the time series: an α value of 0.5 suggests uncorrelated white noise, while α values greater than 0.5 signify correlated

behaviour or long-range dependence (Kantelhardt, Koscielny-Bunde, Rego, Havlin, & Bunde, 2001).

However as noted by Peng *et al.* (1994), one of the primary drawbacks of DFA lies in its sensitivity to trend and non-stationarity. This can lead to misleading results, particularly when analysing data with inherent trends or non-stationary characteristics. In addition, Alvarez-Ramirez *et al.* (2012) highlight the complexity and computational intensity associated with DFA. The method's reliance on calculating root mean square fluctuation over varying time scales can be computationally demanding, necessitating substantial computational resources and time for analysis, particularly for large datasets or high-resolution analyses.

It is also crucial to consider parameter selection when employing DFA. As emphasised by Kantelhardt *et al.* (2001), choosing appropriate parameters such as the range of time scales and degree of polynomial detrending can be challenging and may influence the interpretation of results. Optimal parameter selection often requires prior knowledge of the underlying system or extensive trial and error. Furthermore, the assumption of monofractality inherent in DFA may limit its applicability to systems exhibiting multifractal or other complex scaling behaviours. This observation is echoed by Hu *et al.* (2001), who emphasize the need for caution when interpreting DFA results in systems where nonlinear dynamics play a significant role.

For the above-mentioned reasons, the DFA was not used in this study.

Automated portmanteau Box-Pierce test

The automated portmanteau Box-Pierce test stands as a fundamental statistical tool widely employed in econometrics and time series analysis, particularly in the assessment of model adequacy through the examination of residual autocorrelation. Initially introduced by Box and Pierce (1970), the automated portmanteau Box-Pierce test has evolved to incorporate automated lag selection methods, enhancing its applicability and efficiency in empirical studies.

In scholarly literature, the automated portmanteau Box-Pierce test serves as a cornerstone for validating time series models, ensuring their appropriateness for capturing the underlying dynamics of economic and financial data. Following the estimation of a time series model, such as an autoregressive integrated moving average model, researchers often rely on this test to

evaluate the presence of autocorrelation in the model residuals, as it could lead to biased parameter estimates and erroneous inference.

The methodology of the automated portmanteau Box-Pierce test involves several essential steps, beginning with the estimation of the time series model using established techniques such as maximum likelihood estimation. Subsequently, the residuals are computed by subtracting the fitted values from the observed data. Unlike the traditional Box-Pierce test, where the lag parameter is manually determined, the automated version employs criteria such as the Akaike information criterion or the Bayesian information criterion to automatically select the lag length, ensuring robustness and reliability in the test results (Lütkepohl, 2005).

The test statistic, denoted as Q, is then calculated based on the sum of squared autocorrelations of the residuals at the selected lags. This test statistic is compared to critical values from the chi-square distribution, with degrees of freedom determined by the number of selected lags. If the test statistic exceeds the critical value, it indicates significant autocorrelation in the residuals, prompting further investigation into potential model misspecification. Conversely, if the test statistic falls below the critical value, it suggests that the null hypothesis of no autocorrelation in the residuals cannot be rejected, implying a satisfactory fit of the model to the data.

Shumway and Stoffer (2017) however noted that one of the limitations of the automated portmanteau Box-Pierce test is its sensitivity to departures from the assumed model structure. In addition, Bloomfield (1973) highlights the challenge of selecting an appropriate lag parameter for the Box-Pierce test. The choice of lag can significantly impact the test's power and performance, and there is often ambiguity in determining the optimal lag length, particularly in practice. Furthermore, the automated portmanteau Box-Pierce test assumes that the residuals are independent and identically distributed (*iid*), which may not hold true for all datasets. As discussed by Brockwell and Davis (2016), violations of the independence assumption can lead to inaccurate test results and may require alternative diagnostic procedures

Due to the above-mentioned reasons, the DFA was not used in this study.

Wild bootstrapped automatic VR test

The Wild Bootstrapped Automatic Variance Ratio (VR) test is a statistical method used in time series analysis, particularly in assessing the efficiency of financial markets and identifying

deviations from the random walk hypothesis. Developed by Lo and MacKinlay (1988), this test extends the conventional VR test by incorporating wild bootstrapping techniques to address potential heteroscedasticity and autocorrelation in the data.

In the context of market efficiency, the EMH states that asset prices represent all available information and thus follow a random walk process (Fama, 1970). The VR test, initially proposed by Lo and MacKinlay (1988), evaluates the random walk hypothesis by examining the behaviour of variance ratios derived from successive overlapping subintervals of a time series.

The methodology of the Wild Bootstrapped Automatic VR Test involves several key steps, including VR calculation, automatic lag selection, wild bootstrapping, test statistic calculation, and critical value determination.

By comparing the test statistic to the critical value, conclusions can be drawn regarding the efficiency of the market. Rejection of the null hypothesis suggests evidence of market inefficiency, while failure to reject the null hypothesis implies that the market may be efficient.

While the Wild Bootstrapped Automatic VR Test offers a robust approach to assessing market efficiency and detecting deviations from the random walk, it is essential to consider its limitations. As highlighted by Lo and MacKinlay (1988), one potential drawback is the sensitivity of the test to the chosen parameters, such as the number of overlapping subintervals and the lag selection criteria. Improper selection of these parameters can lead to biased results and affect the test's ability to accurately identify market inefficiencies.

In addition, the Wild Bootstrapped Automatic VR Test relies on the assumption of stationarity in the underlying time series data. However, as noted by Mandelbrot (1963), financial market data often exhibit non-stationary behaviour, characterised by changing volatility and trends over time. Failure to account for non-stationarity can undermine the validity of the test results and lead to erroneous conclusions regarding market efficiency. Furthermore, the use of wild bootstrapping techniques in the Wild Bootstrapped Automatic VR Test introduces additional computational complexity and resource requirements. As cautioned by Efron (1979), wild bootstrapping involves resampling from a distribution with infinite variance, which may lead to increased computational burden and longer processing times, particularly for large datasets or high-frequency trading data.

The Wild Bootstrapped Automatic VR Test was not used in this study due to the above mentioned.

ADF test

The Augmented Dickey-Fuller (ADF) test is a fundamental statistical method used in econometrics and time series analysis to determine whether unit roots are present in a particular time series dataset. Introduced by Dickey and Fuller (1979), the ADF test has become a fundamental tool for testing stationarity and analysing the behaviour of economic and financial variables.

Stationarity, a time series' ability to maintain consistent statistical properties across time, such as its mean and variance. On the other hand, unit roots indicate non-stationarity, suggesting that the series exhibits a stochastic trend and lacks a stable mean-reverting behaviour (Fusthane & Kapingura, 2017).

The methodology of the ADF test involves several key steps such as:

1. Model Specification: The ADF test begins by specifying an autoregressive model of order p, denoted as AR(p), where *p* represents the lag length. Additionally, deterministic components such as a trend and intercept may be included in the model to capture any systematic patterns in the data.

2. Hypothesis Testing: The time series' presence of a unit root, which denotes non-stationarity, is the null hypothesis of the ADF test. The time series may be stationary, which is the alternate theory.

3. Estimation: The ADF test statistic is computed by estimating the coefficients of the autoregressive model using ordinary least squares regression. The test statistic is then standardised to facilitate hypothesis testing.

4. Critical Values: Critical values for the ADF test statistic are obtained from theoretical distributions or simulation methods, such as Monte Carlo simulations, under the null hypothesis of a unit root. These critical values are used to determine the test statistic's significance and, consequently, the existence of unit roots in the data.

5. Decision Rule: An assessment of the time series' stationarity is accomplished by comparing the results of the Augmented Dickey-Fuller (ADF) test statistic to the critical values. The time series is considered stationary if the test statistic is greater than the critical values, which rejects the null hypothesis of a unit root. Conversely, if the test statistic is less than the critical values, the data will still have a unit root and be non-stationarity, and the null hypothesis cannot be ruled out.

Despite its widespread application, the ADF test is not without limitations. As noted by Cheung and Lai (1995), one disadvantage of the ADF test is its sensitivity to the specification of lag length in the regression model. The choice of lag length can significantly impact the test results, leading to potential biases and inaccuracies in assessing the stationarity of the time series.

In addition, the ADF test assumes linearity in the relationship between the lagged differences of the dependent variable and the dependent variable itself. This assumption may not hold true for all-time series data, particularly in cases where non-linear dynamics are present. As highlighted by Enders (2004), violations of the linearity assumption can lead to unreliable test results and incorrect conclusions regarding stationarity. Furthermore, the ADF test may lack power in detecting unit roots when the sample size is small or the time series data exhibit low signal-to-noise ratio. According to Schwert (1989), this limitation can undermine the test's ability to accurately identify non-stationarity in certain datasets, potentially leading to false conclusions about the underlying dynamics of the time series.

Due to the above-mentioned reasons, the ADF test was not used in this study.

Threshold autoregressive model

The TAR model is a statistical method used in econometrics and time series analysis to model nonlinear relationships in time series data. Introduced by Tong in 1978, the TAR model allows for regime shifts or threshold effects, where the relationship between variables changes depending on the value of an observed threshold variable (Tong, 1978).

The TAR model is particularly useful for capturing asymmetric responses or abrupt changes in the behaviour of economic and financial variables, which may occur due to changes in policy regimes, economic shocks, or other structural breaks.

While TAR models offer a flexible framework for capturing nonlinear dynamics in time series data, they are not without limitations. As highlighted by Terasvirta (1994), one potential disadvantage of TAR models is the requirement to specify the threshold variable and the threshold value(s) a priori. This selection process can be subjective and may introduce model uncertainty, particularly when the underlying threshold mechanism is unknown."

In addition, the estimation of TAR models can be computationally intensive, especially for datasets with large sample sizes or complex nonlinearities. As noted by Tong (1990), fitting TAR models often involves iterative procedures to optimize the model parameters, which can be time-consuming and require significant computational resources.

Furthermore, TAR models assume a piecewise linear relationship between the predictor variables and the response variable within each regime. However, as cautioned by Terasvirta (1994), this assumption may not always hold true in practice, leading to misspecification and potential biases in model estimation.

Due to the above-mentioned reasons, the TAR model was not used in this study.

Hurst exponent

The Hurst Exponent test, originating from the work of Hurst (1950), is a statistical method employed to analyse the time series data's persistence and long-term memory. The Hurst Exponent is useful for comprehending trends, volatility, and risk as well as the underlying dynamics of financial and economic processes.

In finance and economics, the Hurst Exponent is utilised to investigate the presence of trends and patterns in time series data, which can have significant implications for asset pricing, risk management, and forecasting. The Hurst Exponent measures the degree of self-similarity or fractal-like behaviour in the data, helping researchers identify persistent trends and distinguish them from random fluctuations.

The methodology of the Hurst Exponent test involves several key steps:

1. Data Preparation: Ensure the time series data is stationary and free from any trends or seasonality. Detrending may be necessary to remove any systematic patterns present in the data.

2. Partitioning: Divide the time series data into overlapping or non-overlapping segments of different lengths, known as windows or intervals.
3. Range Calculation: Compute the range of the data within each window, which represents the difference between the maximum and minimum values.
4. Rescaled Range Calculation: Calculate the rescaled range for each window, which is the ratio of the range to the standard deviation of the data within the window.
5. Hurst Exponent Estimation: Estimate the Hurst Exponent using the rescaled range analysis. The Hurst Exponent is typically computed as the slope of the log-log plot of the rescaled range against the window size.
6. Interpretation: Interpret the value of the Hurst Exponent to determine the persistence or anti-persistence of the time series data. A Hurst Exponent greater than 0.5 indicates persistent behaviour, while a value less than 0.5 suggests anti-persistence.

While the Hurst exponent test is a widely used method for quantifying long-range dependence in time series data, it is however without its limitations, as cautioned by Beran (1994), one potential drawback of the Hurst exponent test is its sensitivity to data properties and sample size. Small sample sizes can lead to increased variability and reduced reliability of Hurst exponent estimates, particularly in the presence of outliers or non-stationarities.

In addition, the interpretation of Hurst exponent values can be non-trivial. While a Hurst exponent greater than 0.5 typically indicates positive long-range dependence, values close to 0.5 may reflect either short-range dependence or a lack of dependence altogether. As discussed by Mandelbrot and Wallis (1969), interpreting Hurst exponent estimates requires careful consideration of the underlying data-generating process and potential model assumptions.

Furthermore, the Hurst exponent test assumes stationarity in the underlying time series data. However, as noted by Lo, Mamaysky and Wang (2000), financial market data often exhibit non-stationary behaviour, characterised by changing volatility and trends over time. Failure to account for non-stationarity can lead to biased Hurst exponent estimates and erroneous conclusions regarding long-range dependence.

For the above-mentioned reasons, the Hurst exponent was not used in this study.

Rolling autocorrelation test

The Rolling Autocorrelation test is a statistical method utilised to assess the presence and significance of autocorrelation in time series data over consecutive rolling windows. This test provides insights into the temporal dependence structure of the data, allowing researchers to identify patterns, trends, and potential predictive relationships.

In finance and economics, understanding autocorrelation is crucial for various applications, including forecasting, risk management, and portfolio optimization. Autocorrelation refers to the degree to which a variable is correlated with its own lagged values over time. Detecting autocorrelation helps analysts determine whether historical patterns are likely to persist or whether there are opportunities for exploiting predictive relationships.

The Rolling Autocorrelation Test offers several advantages in finance and economics, including the ability to detect changing autocorrelation patterns over time and the flexibility to adapt to evolving market conditions. By analysing autocorrelation on a rolling basis, researchers can capture dynamic relationships and adjust their models accordingly to improve forecasting accuracy and decision-making processes.

While the rolling autocorrelation test offers a dynamic approach to assessing temporal dependencies in time series data, it is important to consider its limitations. As noted by Jones and Faddy (2003), one potential drawback of rolling window analyses is the sensitivity to window size selection. Smaller windows may capture short-term fluctuations but could miss longer-term patterns, while larger windows may obscure short-term dynamics. The choice of window size can significantly impact the test results and may require careful consideration.

In addition, the rolling autocorrelation test assumes stationarity within each rolling window, which may not hold true in practice, particularly for financial and economic time series data. As highlighted by Diebold and Mariano (1995), violations of stationarity assumptions can lead to biased estimates of autocorrelation and invalidate the test results. It is essential to assess the stationarity of the data before applying the rolling autocorrelation test and to consider techniques for handling non-stationarity, such as detrending or differencing."

Furthermore, the rolling autocorrelation test may be susceptible to boundary effects, particularly at the beginning and end of the time series. As discussed by Brockwell and Davis (2002), observations at the edges of the rolling windows may have fewer neighbouring data points, leading to increased uncertainty and potentially biased autocorrelation estimates. Researchers should be mindful of boundary effects when interpreting the results of the rolling autocorrelation test and consider techniques for mitigating their impact, such as overlap between rolling windows or padding the data series.

The rolling autocorrelation test was not further used in this study due to the above reasons.

The next section presents the data and methodology applied in the paper. Section 4 presents the empirical results, discusses and summaries the empirical results. Section 5 presents the conclusion of the paper.

3. Data and Methodology

3.1 Data and Sampling

The dataset utilised in this study will be sourced from the EquityRT database and will comprise the daily closing prices of the *ALSI, top40, ind, fin, min,* and g*old* indices. The dataset covers a daily sample period ranging from January 1, 1996, to December 31, 2023.

Each index's daily return is determined by:

$$Daily\ return = \ ln(P_t) - \ ln(P_{t-1}) \qquad (1)$$

where $ln(P_t)$ is the index's natural logarithm at time *t*.

Using daily data provides consistency, reduces noise, and is practical to work with. Limiting the study to six indices enhances manageability, reduces risk, and allows for comparisons across the six indices. Additionally, it enables a deeper analysis of each index and provides a diversified representation of the JSE market as the study aims to add to the body of literature in South Africa. Overall, this approach balances data availability, analytical rigor, and practicality, leading to more robust and insightful research outcomes.

3.2 Methodology

This study is motivated by Ndubuisi and Okere (2021) where they state that according to weak-form market efficiency, analysing historical prices is useless when attempting to anticipate future prices, which results in stock prices to move in a random walk. This study examines stock returns using six independence tests to determine whether price behaviour is random. The first three tests examine linear reliance in returns, whereas the latter three examine nonlinear dependence in returns.

Based on the degree of return independence across time, this study suggests that there are five different types of behaviour for stock market returns.. These categories are efficient, moving towards efficiency, switching between efficiency and inefficiency, adaptive, and inefficient.

1. Efficient: If returns exhibit no correlation or dependence across the sample period, the market is considered efficient.
2. Moving towards efficiency: The market is said to be heading toward efficiency if returns demonstrate dependence, but this dependence is diminishing with time..
3. Switching between efficiency and inefficiency: When returns have changed from being independent (dependent) to dependent (independent), the market may have transitioned to either efficiency or inefficiency. However, this could also indicate the early stages of an adaptive market.
4. Adaptive: The market is categorised as adaptive if returns show at least three different phases of dependence, such as dependent, independent, and dependent again..
5. Inefficient: If returns demonstrate a lack of independence throughout the sample period, the market is considered inefficient.

This categorisation framework provides a comprehensive understanding of the evolving nature of market efficiency over time (Ndubuisi & Okere, 2021).

3.2.1 Linear Tests

3.2.2.1 Autocorrelation test

To determine if the observed return can be explained by a series of independent and *iid* random variables, the autocorrelation test is used. This test is valuable for assessing whether consecutive serial correlation values deviate significantly from zero.

In the study, the joint hypothesis that all autocorrelation coefficients ρ_k are collectively equal to zero is investigated using the portmanteau Q statistic, which was proposed by Ljung and Box (1978).. The test statistic, denoted as *Q*, is calculated as follows:

$$LB = n(n+2)\sum_{k=1}^{m}\left(\frac{\rho_k^2}{n-k}\right) \qquad (2)$$

where *m* indicates the number of autocorrelation coefficients under consideration or the lag length, and *n* indicates the number of observations in the sample. The data follows a chi-square ($\chi 2$) distribution.

3.2.2.2 Runs test

An examination of the randomness of a sequence of stock returns is done using the non-parametric Runs test. Despite being commonly considered a linear test, the Runs test can identify nonlinearity in a returns series (Ndubuisi & Okere, 2021). Consequently, the outcomes may exhibit slight discrepancies from those obtained through linear autocorrelation tests. If an uninterrupted sequence of data is truly random, the number of runs in the series should match the expected number of runs in the Runs test. A run refers to a sequence of consecutively identical symbols, which in this context represent positive or negative returns, and are either followed by another identical sign or preceded by a different symbol. *N* stands for the number of negative runs, and *P* for the number of positive runs. The formula to calculate the expected number of runs (*ER*) is:

$$ER = \left(\frac{X(X-1) - \sum_{i=1}^{3} c_i^2}{X}\right) \qquad (3)$$

where c_i represents the number of changes in returns for each category of sign (where *i = 1, 2,*

3), and *X* denotes the total number of runs. For sufficiently large *X* the expected number of runs (ER) in Equation (3) approximately follows a normal distribution. Hence, the study employs the standard *Z* statistic to assess the null hypothesis. The null hypothesis of the independence of the series is rejected if the computed *z*-value is greater than the critical values. If not, we draw the conclusion that the returns are independent.

Moreover, an excessive or insufficient sample's number of runs may indicate dependence among returns. Therefore, the distribution of run lengths can be examined to ascertain the independence of returns. When the actual number of runs exceeds (falls below) the projected number of runs, a positive (negative) *z*-value is obtained.

3.2.2.3 VR test

The VR test, which, according to Hoque, Kim, and Pyun (2007), has emerged as the most popular econometric method for assessing the random walk hypothesis and has been a crucial instrument for figuring out if stock returns are serially uncorrelated since Lo and MacKinlay's (1988) seminal research. The statistical property that, if a stock price follows a random walk, the variance of the return over the next k periods will be k times that of the return over the previous period serves as the foundation for the VR test. This indicates that the 10-day returns' variance is 10 times higher than the daily returns' variance. A test for this hypothesis is provided by Lo and MacKinlay (1988) using the single VR, denoted as VR *(k).* This test compares the variance of returns over different time horizons to assess whether the observed variances align with the expectations under the random walk assumption. For every time *t = 1, 2, 3*,..., let rt represent the asset return. When rt has a holding period of *k*, the VR for *rt* is:

$$VR(2) = 1 + \rho(1) \qquad (4)$$

where $\rho_{(j)}$ denotes the order *j* autocorrelation of *rt*. Put differently, the VR is equal to one plus the weighted sum of the autocorrelation coefficients for the positive and decreasing weighted asset returns. Given that returns are serially uncorrelated when $\rho_{(j)}$ = 0, the VR tests the null hypothesis that the VR equals 1 for all *k*s. Conversely, values of VR *(k)* less than 1 represent mean reversion or negative serial correlations, whereas values greater than 1 indicate positive serial correlations..

3.2.3 Nonlinear Tests

The previous tests primarily focused on identifying linear relationships in stock market returns, which might not capture nonlinear dependencies. Since it reveals possible dependencies even when linear tests show independence, nonlinear dependence in stock returns has garnered attention recently (Alagidede, 2011; Caraiani, 2012; Hiremath & Kamaiah, 2010; Lim & Hooy, 2012).

To ensure a comprehensive understanding of the series and mitigate the risk of reaching incorrect conclusions, Instead of depending just on one statistical test, this study uses a battery of nonlinear tests. The data is first pre-whitening modelled to eliminate linear structure. Once the standardized residuals are uncorrelated, as indicated by the Ljung-Box Q-statistic up to 20 lags, an AR(p) model is fitted to the data to establish the ideal model length. Subsequently, the residuals of the pre-whitened model undergo evaluation through a battery of nonlinear tests, including the ones proposed by Brock, Dechert, LeBaron, and Schieinkman (1996), as well as the Engle (1982) and McLeod and Li (1983) tests.

3.2.3.1 McLeod Li test

One portmanteau test for nonlinearity is the McLeod and Li (1983) test. They propose using the following statistic to examine a time series for nonlinear effects:

$$Q_{(m)} = \frac{n(n+2)}{n-k} \sum_{k-1}^{m} r_a^2(k) \qquad (5)$$

where the autocorrelations of the squared residuals are represented by r_a^2. If the series et is independently and identically distributed, the asymptotic distribution of Q(m) is $\chi 2$ with m degrees of freedom. The null hypothesis is the independence of returns; if it is rejected, it indicates that there are GARCH nonlinear effects/ Autoregressive Conditional Heteroskedasticity (ARCH) disruptions in the data..

3.2.3.2 Engle LM test

To detect ARCH disruptions, Engle (1982) introduced the Engle LM test. The residuals of the AR(p) model are analysed for heteroskedasticity. An extra test regression is used to produce the Engle LM statistic. This regression includes:

$$e_t^2 = a_0 + \sum_{i=1}^{p} a_i e_{t-1}^2 + v_t \qquad (6)$$

where e is the pre-whitened AR(p) model residual. The NR^2 is Engle's LM test statistic, calculated as the number of observations times the R^2 from the test regression, whereas the F-statistic is an omitted variable test for the joint significance of all lagged squared residuals. NR^2 for this regression is asymptotically $\chi 2$ (p) when the null hypothesis of a linear-generating mechanism for e_t is assumed. If the null hypothesis is not accepted, the data show that ARCH/GARCH effects exist.

3.2.3.3 BDS test

The BDS test stands as a widely utilised method, noted in the work of Chen and Yeh (2002), to investigate serial dependence or nonlinear patterns in time series analysis. Initially proposed by Brock *et al.* (1987), the version commonly employed stems from the refinement introduced by Brock *et al.* (1996). While the BDS test provides valuable insights, it does not specify the optimal data producing technique for modelling the data, necessitating consideration of alternative tests.

In the BDS test, the null hypothesis posits that the data producing processes are independent and *iid*, while the alternative hypothesis suggests a model mis-specification. Rejecting the null hypothesis implies evidence of non-independence, challenging the notion of market efficiency as the test evaluates nonlinear predictability within the sample.

The correlation integral, a key component of the BDS test, quantifies the probability that any two points in phase space are separated by a specified distance 'e'. As the distance 'e' increases, the probability scales according to the fractional dimension of the phase space, providing insights into the underlying dynamics of the time series data. The calculation of correlation integrals follows a specific method, which involves:

$$C_m(n,\varepsilon) = \frac{2}{(n-m)(n-m+1)} \sum_{s=1}^{n-m} \sum_{i=s+1}^{n-m+1} I_m\,(x_s\,, x_t\,, \varepsilon) \quad (7)$$

where the sample size is n, the embedding dimension is m, and the correlation integral's maximum difference between pairs of observations is ε.

4. Empirical results and discussions

This section discusses the empirical results of both linear and nonlinear tests carried out in the present paper. Table 1 reports the descriptive statistics for the *ALSI, the top40, the ind, the fin,* the *min,* and *gold* indices returns. The mean returns are positive for all indices. The standard deviations of all indices returns are all approximately 0.01 (rounded up to two decimal places) and show evidence of low volatility. The variance of all indices are all 0.00 (rounded up to two decimal places) showing evidence of low risk. The skewness is negative for all indices implying that returns are flatter to the left compared to the normal distribution, except for the min index. Moreover, it indicates that the negative returns have greater magnitude than the positive. The kurtosis for all indicates is above 3, which means the distribution has sharper, more peaked central tendency than a normal distribution.

Table 1 : Descriptive statistics

	Minimum	Median	Maximum	Mean	Standard deviation	Variance	Skewness	Kurtosis
ALSI	-0.1269	0.0007	0.0742	0.0004	0.0122	0.0001	-0.4906	6.4207
fin	-0.1331	0.0004	0.0798	0.0003	0.0136	0.0002	-0.5984	8.3518
ind	-0.1280	0.0008	0.0799	0.0004	0.0118	0.0001	-0.4716	6.1672
gold	-0.1566	0.0004	0.1205	0.0003	0.0182	0.0003	-0.1090	4.6239
min	-0.1589	0.0003	0.1346	0.0003	0.0195	0.0004	0.0167	4.0238
top40	-0.1429	0.0007	0.0845	0.0004	0.0133	0.0002	-0.4014	6.2341

4.1 Linear tests

4.1.1 Autocorrelation test

The present study employs Ljung-Box test to check whether all autocorrelations are simultaneously equal to zero. To check for autocorrelation, we apply the below hypothesis.

- Null Hypothesis (H_0): There is no autocorrelation in the time series data.
- Alternative Hypothesis (H_1): There is autocorrelation in the time series data.

If the p-value is below 0.05, we reject the null hypothesis, suggesting evidence of autocorrelation.

If the p-value is above 0.05, we fail to reject the null hypothesis, indicating no significant evidence of autocorrelation.

The Ljung-Box test results for the *ALSI, fin, gold, min* and *top40* indices documented in Table 2 show that p-values are all less than 0.05 indicating strong evidence to reject the null hypothesis of no autocorrelation. This suggests that the data exhibits significant dependence at various lags. The consistently low p-values across all lags imply a high level of statistical significance, revealing a pronounced time-dependent structure in the data. This dependence pattern signifies that each observation in the dataset is influenced by its past observations, highlighting a that the *ALSI, fin, gold, min* and *top40* indices are inefficient. The overall results suggests that past values of the *ALSI, fin, gold, min* and *top40* indices are related to future values and the indices do not follow a random walk.

The results of the *ind* index show that the lags ranging from 1 to 32 and at lag 36, the p-values are below 0.05, suggesting a presence of dependence in the *ind* index. However, a notable shift occurs at lag 33 to 35, where p-values are above 0.05 suggesting no pronounced evidence of dependence. Overall, the data suggests that the *ind* index is adaptive as the returns went through three different stages of dependence.

Table 2 : Autocorrelation test results

Lag	*ALSI*		*fin*		*ind*		*gold*		*min*		*Top40*	
	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value
1	13.08	0.00***	41.94	0.00***	11.97	0.00***	22.86	0.00***	15.52	0.00***	6.50	0.01***
2	13.40	0.00***	42.00	0.00***	12.18	0.00***	22.96	0.00***	16.06	0.00***	6.50	0.04***
3	15.15	0.00***	42.01	0.00***	12.64	0.01***	26.08	0.00***	17.09	0.00***	10.74	0.01***
4	24.07	0.00***	46.37	0.00***	17.25	0.00***	30.84	0.00***	18.70	0.00***	22.24	0.00***
5	27.78	0.00***	46.58	0.00***	18.79	0.00***	34.08	0.00***	22.40	0.00***	28.68	0.00***
6	32.43	0.00***	51.78	0.00***	19.51	0.00***	38.86	0.00***	25.01	0.00***	32.36	0.00***
7	43.84	0.00***	52.76	0.00***	23.72	0.00***	45.87	0.00***	27.64	0.00***	43.03	0.00***
8	44.15	0.00***	52.81	0.00***	24.70	0.00***	48.59	0.00***	30.50	0.00***	43.58	0.00***
9	44.24	0.00***	52.83	0.00***	24.75	0.00***	50.69	0.00***	30.50	0.00***	43.59	0.00***
10	44.52	0.00***	57.53	0.00***	25.09	0.01***	51.05	0.00***	30.51	0.00***	44.19	0.00***

Lag	*ALSI*		*fin*		*ind*		*gold*		*min*		*Top40*	
	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value
11	44.88	0.00***	58.86	0.00***	25.11	0.01***	52.39	0.00***	32.04	0.00***	44.65	0.00***
12	45.20	0.00***	65.79	0.00***	25.95	0.01***	52.56	0.00***	32.87	0.00***	44.70	0.00***
13	45.87	0.00***	65.81	0.00***	27.03	0.01***	53.16	0.00***	33.02	0.00***	45.52	0.00***
14	50.76	0.00***	66.10	0.00***	32.09	0.00***	54.84	0.00***	34.79	0.00***	49.43	0.00***
15	54.64	0.00***	69.65	0.00***	33.47	0.00***	55.77	0.00***	37.10	0.00***	51.42	0.00***
16	55.15	0.00***	70.22	0.00***	33.48	0.01***	56.08	0.00***	38.69	0.00***	51.82	0.00***
17	59.28	0.00***	71.54	0.00***	34.77	0.01***	56.84	0.00***	39.38	0.00***	55.43	0.00***
18	65.60	0.00***	77.93	0.00***	36.14	0.01***	61.91	0.00***	43.63	0.00***	62.24	0.00***
19	66.28	0.00***	78.12	0.00***	36.53	0.01***	62.53	0.00***	44.01	0.00***	62.49	0.00***
20	67.71	0.00***	79.62	0.00***	37.49	0.01***	62.53	0.00***	44.01	0.00***	63.89	0.00***
21	67.97	0.00***	79.89	0.00***	38.58	0.01***	62.79	0.00***	44.50	0.00***	63.91	0.00***
22	68.73	0.00***	80.93	0.00***	39.07	0.01***	63.14	0.00***	44.74	0.00***	64.35	0.00***
23	71.40	0.00***	81.26	0.00***	39.09	0.02**	69.57	0.00***	48.81	0.00***	67.11	0.00***
24	72.02	0.00***	83.45	0.00***	39.53	0.02**	69.57	0.00***	48.98	0.00***	67.81	0.00***
25	72.54	0.00***	89.39	0.00***	39.80	0.03**	70.36	0.00***	49.00	0.00***	68.13	0.00***
26	73.75	0.00***	89.72	0.00***	40.21	0.04**	72.78	0.00***	50.85	0.00***	68.71	0.00***
27	74.10	0.00***	90.79	0.00***	40.66	0.04**	72.85	0.00***	51.24	0.00***	69.38	0.00***
28	75.01	0.00***	90.94	0.00***	42.24	0.04**	73.35	0.00***	54.87	0.00***	69.95	0.00***
29	75.01	0.00***	92.54	0.00***	43.21	0.04**	73.44	0.00***	54.87	0.00***	69.95	0.00***
30	76.86	0.00***	94.50	0.00***	44.88	0.04**	74.28	0.00***	54.90	0.00***	71.54	0.00***
31	77.89	0.00***	95.56	0.00***	45.04	0.05**	75.53	0.00***	56.02	0.00***	72.47	0.00***
32	78.03	0.00***	105.60	0.00***	45.23	0.06*	75.58	0.00***	56.03	0.01***	73.12	0.00***
33	78.29	0.00***	108.69	0.00***	45.50	0.07*	78.25	0.00***	56.77	0.01***	73.66	0.00***
34	81.93	0.00***	110.20	0.00***	45.61	0.09*	79.52	0.00***	57.67	0.01***	76.92	0.00***
35	85.87	0.00***	123.98	0.00***	50.20	0.05**	79.54	0.00***	57.68	0.01***	81.41	0.00***
36	91.86	0.00***	128.61	0.00***	64.15	0.00***	83.46	0.00***	60.26	0.01***	88.45	0.00***

Note: * denotes a 10% level of statistical significance, ** denotes a 5% level of statistical significance and *** denotes a 1% level of statistical significance.

4.1.2 Runs test

The present study employs the runs test to analyse the randomness of a series of stock return rates.

To check for randomness, we apply the below hypothesis.

- Null Hypothesis (H_0): There is randomness in the time series data.
- Alternative Hypothesis (H_1): There is no randomness in the time series data.

If the p-value is below 0.05, we reject the null hypothesis, suggesting no evidence of randomness in the time series data.

If the p-value is above 0.05, we fail to reject the null hypothesis, indicating significant evidence of randomness in the time series data.

The Runs test results of the *ALSI, fin, ind, gold* and *min* indices as documented in Table 3 show that the p-values are below 0.05 indicating no significant evidence of randomness in the time series data. The data suggests that the *ALSI, fin, ind, gold* and *min* indices are inefficient.

The results of the *top40* index show that the p-value is above 0.05 suggesting significant evidence of randomness in the time series data suggesting that the *top40* index is efficient.

Table 3: Runs test results

Runs Test

Test Statistic	p-value	Runs	Index
-2.8720	0.0041	3,373	ALSI
-4.7247	0.000002	3,316	FINDI
-2.0335	0.0420	3,411	INDI
-3.0634	0.0022	3,312	GOLD
-3.3665	0.0008	3,369	Mining
-1.4550	0.1457	3,456	Top 40

4.1.3 VR test

The present study employs the VR test to analyse whether the stock returns follow a random walk.

To check for random walk, we apply the below hypothesis.

- Null Hypothesis (H_0): The stock returns follow a random walk, and there is no serial correlation in the returns ($VR = 1$).
- Alternative Hypothesis (H_1): The stock returns do not follow a random walk, there is evidence of serial correlation in the returns ($VR \neq 1$).

If the $VR \neq 1$, we reject the null hypothesis, suggesting that the stock returns do not follow a random walk.

If the $VR = 1$, we fail to reject the null hypothesis, suggesting that the stock returns follow a random walk.

The results of the VR test documented in Table 4 show that all indices are not all equal to 1 (not rounded up to the nearest whole number). The results of all indices indicate that there is serial correlation in the stock returns suggesting that the stock returns do not follow a random walk.

Table 4: VR test results

Variance Ratio Test

Test Statistic	Sum	Index
3.0057	1.0888	ALSI
3.9090	1.1308	FINDI
2.5007	1.0732	INDI
3.2755	1.1034	GOLD
3.0932	1.0924	Mining
2.1778	1.0595	Top 40

4.2 Nonlinear tests

4.2.1 McLeod Li test

This study employs the McLeod Li test to check for nonlinear effects in the time series data.

To check for nonlinear effects, we apply the below hypothesis.

- Null Hypothesis (H_0): There is no evidence of ARCH effects in the time series data; the volatility is constant over time.

- Alternative Hypothesis (H_1): There is evidence of ARCH effects in the time series data; the volatility varies over time.

If the p-value is below 0.05, we reject the null hypothesis, suggesting evidence of ARCH effects in the time series data.

If the p-value is above 0.05, we fail to reject the null hypothesis, indicating no significant evidence of ARCH effects in the time series data.

The results of the McLeod Li test documented in Figure 1 to Figure 6 show that all the p-values for all indices at different lags are below 0.05 suggesting that the time series data contains nonlinear effects in the form of ARCH effects in the returns. The results suggests that all indices were inefficient.

Figure 1: McLeod Li test results on the ALSI

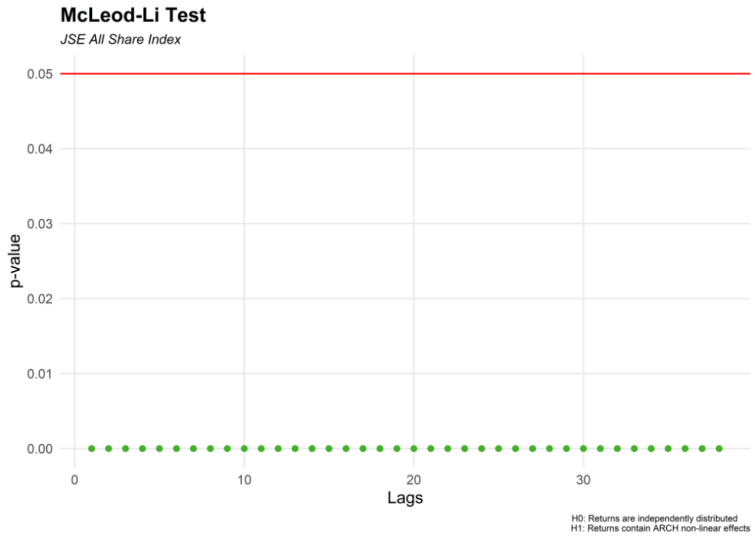


Figure 2: McLeod Li test results on the fin index

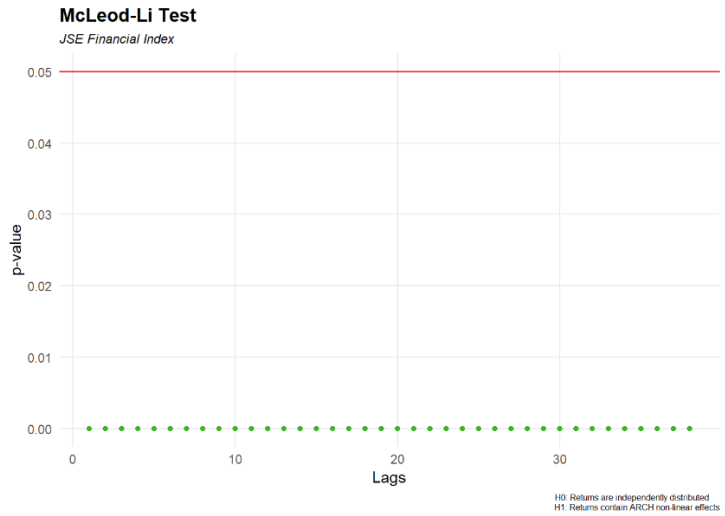


Figure 3: McLeod Li test results on the ind index

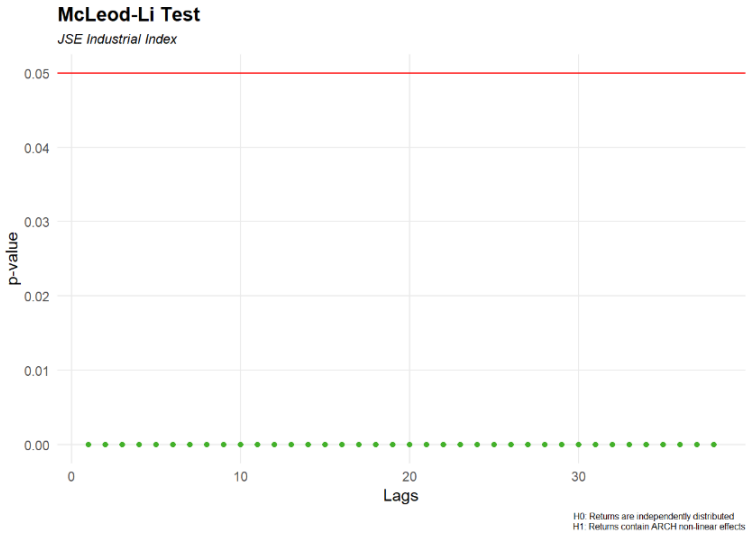


Figure 4: McLeod Li test results on the gold index

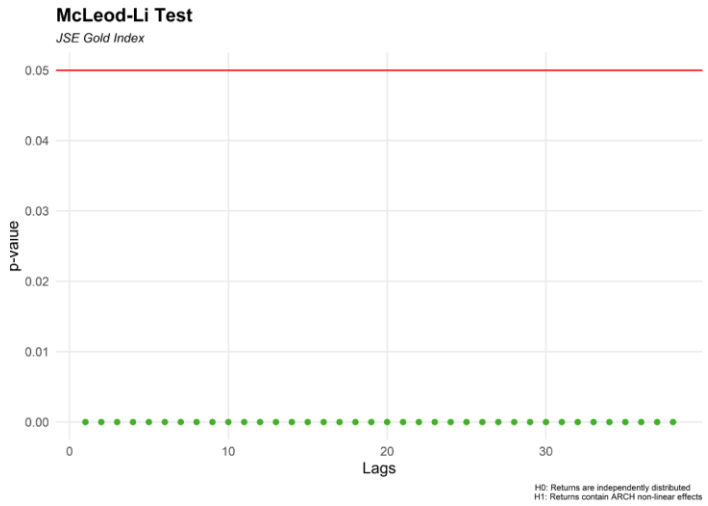


Figure 5: McLeod Li test results on the min index

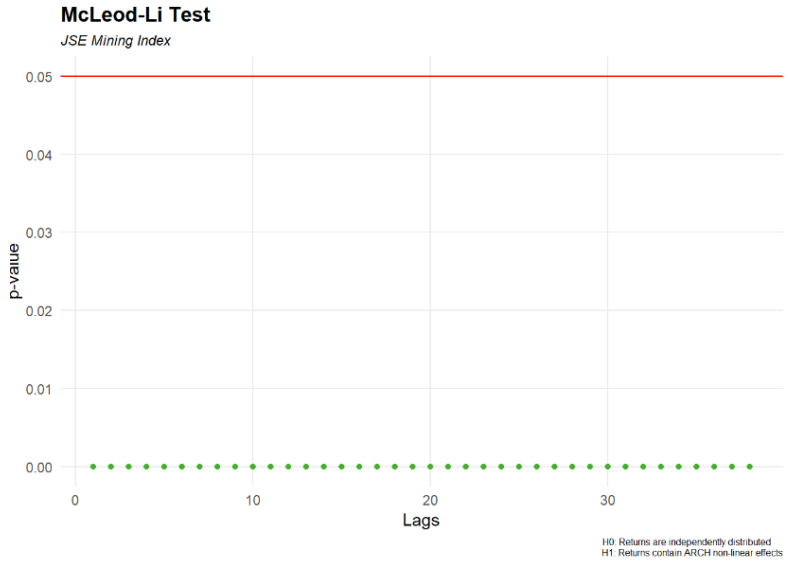
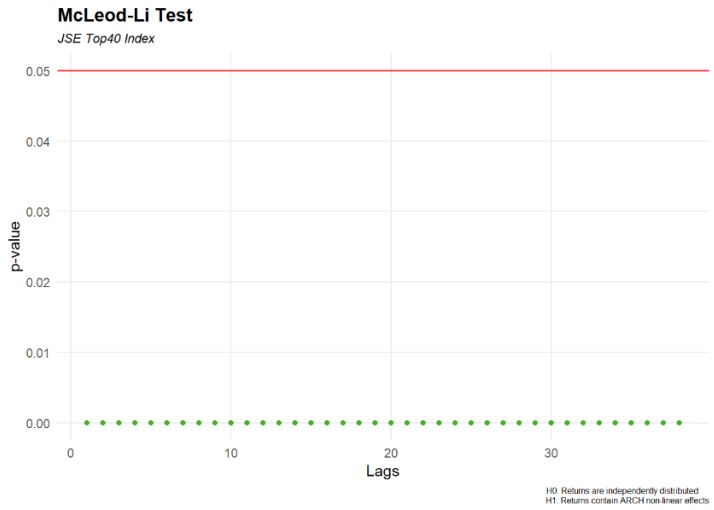


Figure 6: McLeod Li test results on the Top40 index



The McLeod test results indicate that all indices were efficient. Similar to the first three tests.

4.2.2 Engle LM test

This study employs the Engle LM test to analyse ARCH disturbances in the time series data.

To check for ARCH disturbances, we apply the below hypothesis.

- Null Hypothesis (H_0): The returns are independently and iid.
- Alternative Hypothesis (H_1): The returns contain ARCH non-linear effects.

If the p-value is below 0.05, we reject the null hypothesis, suggesting that the returns contain ARCH non-linear effects.

If the p-value is above 0.05, we fail to reject the null hypothesis, that the returns are independently and iid.

The Engle LM test results show documented in Table 5 that all the p-values for all indices are below 0.05, indicating significant evidence of ARCH effects in the time series data suggesting significant time-varying volatility. The results suggests that all indices were inefficient.

Table 5: Engle LM test results

Engle-LM Test

Test Statistic	p-value	Index
1,163.6350	0	ALSI
1,494.2020	0	FINDI
903.0620	0	INDI
1,224.2390	0	GOLD
1,115.1420	0	METALS
1,102.4510	0	Top 40

The Engle LM test results for show that all indices were inefficient.

4.2.3 BDS test

This study employs the BDS test to analyse serial dependence or nonlinear structures in the time series data.

To check for serial dependence or nonlinear structures, we apply the below hypothesis.

- Null Hypothesis (H_0): The data generating processes are independent and iid, implying no serial dependence or nonlinear structure.
- Alternative Hypothesis (H_1): The data generating processes are not independent and iid, suggesting the presence of serial dependence or nonlinear structure.

If the p-value is below 0.05, we reject the null hypothesis, suggesting the presence of serial dependence or nonlinear structure.

If the p-value is above 0.05, we fail to reject the null hypothesis, implying no serial dependence or nonlinear structure.

The results of the BDS test documented in Table 6 to Table 12 show that all the p-values for all indices at different embedding dimensions (*m*) and distances (*eps)* are below 0.05 indicating that all indices exhibits significant departure from independence or iid structure. The BDS test provides evidence of serial dependence or nonlinear structure in all indices suggesting possible predictability of future returns using the history of returns (i.e., the indices do not follow a random walk).

Table 6: BDS test results on the ALSI

BDS Test

JSE All Share Index

Epsilon	Test Statistic	p-value
eps[1] m=2	15.2108	0
eps[1] m=3	20.8496	0
eps[2] m=2	16.9185	0
eps[2] m=3	22.8546	0
eps[3] m=2	18.2551	0
eps[3] m=3	24.5489	0
eps[4] m=2	18.9971	0
eps[4] m=3	25.3384	0

Table 7: BDS test results on the fin index

BDS Test

JSE Financial Index

Epsilon	Test Statistic	p-value
eps[1] m=2	17.5112	0
eps[1] m=3	23.1786	0
eps[2] m=2	19.4400	0
eps[2] m=3	25.1427	0
eps[3] m=2	21.6761	0
eps[3] m=3	27.1690	0
eps[4] m=2	23.5033	0
eps[4] m=3	28.8373	0

Table 8: BDS test results on the ind index

BDS Test

JSE Industrial Index

Epsilon	Test Statistic	p-value
eps[1] m=2	14.3761	0
eps[1] m=3	19.9117	0
eps[2] m=2	16.1080	0
eps[2] m=3	21.5460	0
eps[3] m=2	17.0626	0
eps[3] m=3	22.1844	0
eps[4] m=2	17.6393	0
eps[4] m=3	22.4399	0

Table 9: BDS test results on the gold index

BDS Test

JSE Gold Index

Epsilon	Test Statistic	p-value
eps[1] m=2	12.5126	0
eps[1] m=3	17.1000	0
eps[2] m=2	13.3574	0
eps[2] m=3	17.8727	0
eps[3] m=2	14.7948	0
eps[3] m=3	19.6730	0
eps[4] m=2	15.7646	0
eps[4] m=3	21.3382	0

Table 10: BDS test results on the min index

BDS Test

JSE Mining Index

Epsilon	Test Statistic	p-value
eps[1] m=2	13.2407	0
eps[1] m=3	18.1145	0
eps[2] m=2	13.8481	0
eps[2] m=3	18.5188	0
eps[3] m=2	15.0877	0
eps[3] m=3	19.9664	0
eps[4] m=2	15.9520	0
eps[4] m=3	21.1479	0

Table 11: BDS test results on the Top40 index

BDS Test

JSE Top 40 Index

Epsilon	Test Statistic	p-value
eps[1] m=2	14.9271	0
eps[1] m=3	20.3783	0
eps[2] m=2	16.7468	0
eps[2] m=3	22.5220	0
eps[3] m=2	18.3190	0
eps[3] m=3	24.4503	0
eps[4] m=2	18.9825	0
eps[4] m=3	25.3443	0

4.3 Discussions

In this section, we discuss the overall results of six tests performed across the *ALSI*, *fin*, *ind*, *gold*, *min* and *top40* indices.

The Autocorrelation test results shown in Table 2 reveals evidence of autocorrelation in all indices (except for the *ind* index which was adaptive), indicating a significant dependence on past observations and inefficiency. Additionally, the Runs test showed evidence of inefficiency in the *ALSI, fin, ind, gold,* and *min* indices, contrasting with the *top40* index, which suggested significant evidence of randomness in the time series data as shown in Table 3. Furthermore, the VR test documented in Table 4 confirms that there is serial correlation in the stock returns of all indices suggesting that the stock returns do not follow a random walk.

The presence of nonlinear effects in the form ARCH effects in the returns, as indicated by the McLeod Li test results shown in Figure 1 to 6, suggests that all indices were inefficient during the tested period. Moreover, the Engle LM test results documented in Table 5 underscores evidence of ARCH effects in the time series data suggesting significant time-varying volatility suggesting that all indices were inefficient. Finally, The BDS test documented in Table 6 to 12 provides evidence of serial dependence or nonlinear structure in all indices suggesting possible predictability of future returns using the history of returns (i.e., the indices do not follow a random walk).

The aim of the study is to investigate the predictability of stock returns in an emerging economy like the JSE by applying both linear and nonlinear methods.. The literature review supports the notion that stock prices do not always follow a random walk as suggested by Lo (2005) and other authors mentioned in section 2.1. The overall findings of this study provides compelling evidence that stock returns within the JSE market across the six indices are inefficient, as indicated by significant evidence of inefficiency, non-randomness, the presence of serial correlation and nonlinear effects in the six tests performed. These results challenge the notion of the weak-form of market efficiency and suggest that past prices of stock returns are related to future prices.

According to Lo (2004) a respected figure in financial economics, the AMH suggests that the degree of market efficiency can vary over time and across different market conditions. This aligns with the findings of this study, which provide significant evidence of inefficiency and non-randomness in stock returns within the JSE market, challenging the traditional notion of market efficiency.

Furthermore, Shiller (2015) has noted that financial markets are not always efficient; they are subject to fluctuations and behavioural biases that can lead to departures from efficiency. This study's identification of serial correlation and nonlinear effects in stock returns supports Shiller (2015) findings, suggesting that past prices of stock returns are indeed related to future prices, contrary to the assumptions of the weak-form of market efficiency.

In his seminal work, Fama (1970) acknowledges that while markets may be efficient over the long term, there are periods of inefficiency that can be exploited by investors with the appropriate information and analytical tools. The findings of this study provide empirical support for this view, demonstrating that stock returns in the JSE market exhibit significant inefficiencies that may present opportunities for investors to generate abnormal returns through adaptive strategies.

According to Shleifer and Vishny (1997) stock market inefficiencies can arise due to investor sentiment and behavioural biases, leading to departures from market efficiency. This perspective aligns with the findings of this study, which indicate significant evidence of inefficiency, non-randomness, and the presence of serial correlation and nonlinear effects in stock returns within the JSE market.

Thaler (2015) emphasises that human biases and irrational behaviours can lead to market inefficiencies, creating opportunities for savvy investors to exploit mispricing's. The study findings of inefficiencies and non-randomness in the JSE market corroborate with the findings of Thaler (2015), indicating that investor sentiment and behavioural biases may play a significant role in shaping stock price dynamics.

Furthermore, Malkiel (2003) argues that efficient markets are more of an approximation than an absolute truth, as market anomalies and investor behaviour can result in deviations from efficiency. This study's identification of systematic deviations from randomness and the existence of predictable patterns in stock returns supports Malkiel (2003) assertion, suggesting that adaptive strategies may exploit these anomalies for profit.

5. Conclusion

This study examines the AMH in the JSE market by testing for stock return predictability using daily data from January 1996 to December 2023 across six different indices. The study applies three linear and three nonlinear tests to check if the JSE market is efficient, moving towards efficiency, switching to efficiency/inefficiency, adaptive and inefficient.

The findings presented in this research offer significant insights into the dynamics of the JSE market. The study's comprehensive results offer convincing proof that stock returns within the JSE, across six indices, exhibit inefficiency. This is evident from significant indications of inefficiency, non-randomness, the presence of serial correlation, and nonlinear effects observed in the six conducted tests. These results suggest that past prices of stock returns are indeed related to future prices, indicating the presence of predictability and exploitable opportunities within the JSE market.

However, it is crucial to acknowledge the limitations of this study. While the findings provide compelling evidence of market inefficiency within the JSE, further research is warranted to explore the nuances of adaptive behaviour and market dynamics in different contexts. Additionally, the evolving nature of financial markets necessitates ongoing inquiry and refinement of theoretical frameworks to capture the complexities of real-world phenomena.

In summary, this study contributes to the ongoing discourse surrounding market efficiency and adaptive behaviour within financial markets, particularly within the context of the JSE. By challenging conventional wisdom and providing empirical evidence of market inefficiency, this research paves the way for a deeper understanding of the intricacies of financial markets and offers valuable insights for both academic research and practical investment decision-making.

References

Adaramola, A. O., & Obisesan, O. G. (2021). Adaptive market hypothesis: evidence from Nigerian stock exchange. *The Journal of Developing Areas*, *55*(2) 1548-2278.

Alagidede, P. (2011). Return behaviour in Africa's emerging equity markets. *The Quarterly Review of Economics and Finance*, *51*(2), 133-140.

Alvarez-Ramirez, J., Rodriguez, E., & Espinosa-Paredes, G. (2012). Is the US stock market becoming weakly efficient over time? Evidence from 80-year-long data. *Physica A: Statistical Mechanics and its Applications*, *391*(22), 5643-5647.

Bec, F., Ben Salem, M., & Carrasco, M. (2004). Tests for unit-root versus threshold specification with an application to the purchasing power parity relationship. *Journal of Business & Economic Statistics*, 22(4), 382-395.

Beran, J. (2017). *Statistics for long-memory processes*. Routledge.

Bloomfield, P. (1973). An exponential model for the spectrum of a scalar time series. *Biometrika*, *60*(2), 217-226.

Box, G. E., & Pierce, D. A. (1970). Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. *Journal of the American statistical Association*, *65*(332), 1509-1526.

Brock, W. A., Dechert, W. D., & Scheinkman, J. A. (1987). *A test for independence based on the correlation dimension." Social Systems Research Institute, University of Wisconsin-Madison*. Working Paper 8702.

Brockwell, P. J., & Davis, R. A. (Eds.). (2002). *Introduction to time series and forecasting*. New York, NY: Springer New York.

Broock, W. A., Scheinkman, J. A., Dechert, W. D., & LeBaron, B. (1996). A test for independence based on the correlation dimension. *Econometric reviews*, *15*(3), 197-235.

Campbell, J. Y., Lo, A. W., MacKinlay, A. C., & Whitelaw, R. F. (1998). The econometrics of financial markets. *Macroeconomic Dynamics*, *2*(4), 559-562.

Caraiani, P. (2012). Nonlinear dynamics in CEE stock markets indices. *Economics Letters*, *114*(3), 329-331.

Charles, A., & Darné, O. (2005). Outliers and GARCH models in financial data. *Economics Letters*, *86*(3), 347-352.

Chen, S. H., & Yeh, C. H. (2002). On the emergent properties of artificial stock markets: the efficient market hypothesis and the rational expectations hypothesis. *Journal of Economic Behavior & Organization*, *49*(2), 217-239.

Cheung, Y. W., & Lai, K. S. (1995). Lag order and critical values of the augmented Dickey–Fuller test. *Journal of Business & Economic Statistics*, *13*(3), 277-280.

Chitenderu, T. T. (2013). *Testing Random Walk Hypothesis in the Stock Market Prices: Evidence from South Africa's Stock Exchange (2000-2011)* (Doctoral dissertation, University of Fort Hare).

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association*, *74*(366a), 427-431.

Diebold, F. X., & Mariano, R. S. (2002). Comparing predictive accuracy. *Journal of Business & economic statistics*, 20(1), 134-144.

Efron, B. (1979). Bootstrap methods: another look at the jackknife. *The Annals of Statistics*, 7(1), 1-26.

Enders, W. (2008). *Applied econometric time series*. John Wiley & Sons.

Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, *50*(4), 987-1007.

Ertaş, F. C., & Özkan, O. (2018). Piyasa Etkinliği Açısından Adaptif Piyasa Hipotezi'nin Test Edilmesi: Türkiye ve ABD Hisse Senedi Piyasaları Örneği. *Finans Politik ve Ekonomik Yorumlar*, (642), 223-240.

Fama, E. F. (1965). The behavior of stock-market prices. *The journal of Business*, *38*(1), 34-105.

Fama, E. F., & French, K. R. (1988). Permanent and temporary components of stock prices. *Journal of political Economy*, *96*(2), 246-273.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The journal of Finance*, *25*(2), 383-417.

Fusthane, O., & Kapingura, F. M. (2017). Weak form market efficiency of the johannesburg stock exchange: pre, during and post the 2008 global financial crisis. *Journal of Economics and Behavioral Studies*, *9*(5 (J)), 29-42.

Ghazani, M. M., & Araghi, M. K. (2014). Evaluation of the adaptive market hypothesis as an evolutionary perspective on market efficiency: Evidence from the Tehran stock exchange. *Research in International Business and Finance*, *1*(32), 50-59.

Ghazani, M. M., & Jafari, M. A. (2021). Cryptocurrencies, gold, and WTI crude oil market efficiency: a dynamic analysis based on the adaptive market hypothesis. *Financial Innovation*, *7*(1), 1-26.

Grossman, S. J., & Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *The American economic review*, *70*(3), 393-408.

Hardstone, R., Poil, S. S., Schiavone, G., Jansen, R., Nikulin, V. V., Mansvelder, H. D., & Linkenkaer-Hansen, K. (2012). Detrended fluctuation analysis: a scale-free view on neuronal oscillations. *Frontiers in physiology*, *3*, 450.

Hiremath, G. S., & Kamaiah, B. (2010). Non-linear dependence in stock returns: Evidences from India. *Journal of Quantitative Economics*, *8*(1).

Hoque, H. A., Kim, J. H., & Pyun, C. S. (2007). A comparison of variance ratio tests of random walk: A case of Asian emerging stock markets. *International Review of Economics & Finance*, *16*(4), 488-502.

Hu, K., Ivanov, P. C., Chen, Z., Carpena, P., & Stanley, H. E. (2001). Effect of trends on detrended fluctuation analysis. *Physical Review E*, 64(1), 011114.

Hull, J.C., 2012. Options, futures and other derivatives. Harlow, Essex: Pearson Education Limited.

Hull, M., & McGroarty, F. (2014). Do emerging markets become more efficient as they develop? Long memory persistence in equity indices. *Emerging Markets Review*, *18*, 45-61.

Hurst, H. E. (1951). Long-term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers*, 116, 770-799.

Ito, M., & Sugiyama, S. (2009). Measuring the degree of time varying market inefficiency. *Economics Letters*, *103*(1), 62-64.

Ito, M., Noda, A., & Wada, T. (2014). International stock market efficiency: a non-Bayesian time-varying model approach. *Applied Economics*, *46*(23), 2744-2754.

Ito, M., Noda, A., & Wada, T. (2016). The evolution of stock market efficiency in the US: a non-Bayesian time-varying model approach. *Applied Economics*, *48*(7), 621-635.

Jain, K., & Jain, P. (2013). Empirical study of the weak form of EMH on Indian stock market. *International Journal of Management and Social Sciences Research*, *2*(11), 52-59.

Jones, R. H., & Faddy, M. J. (2003). *A random walk test for serial correlation in regression. Biometrika*, 90(3), 777-790.

Kantelhardt, J. W., Koscielny-Bunde, E., Rego, H. H., Havlin, S., & Bunde, A. (2001). Detecting long-range correlations with detrended fluctuation analysis. *Physica A: Statistical Mechanics and its Applications*, *295*(3-4), 441-454.

Kim, J. H., Shamsuddin, A., & Lim, K. P. (2011). Stock return predictability and the adaptive markets hypothesis: Evidence from century-long US data. *Journal of Empirical Finance*, *18*(5), 868-879.

Lim, K. P. (2007). Ranking market efficiency for stock markets: A nonlinear perspective. *Physica A: Statistical Mechanics and its Applications*, *376*, 445-454.

Lim, K. P., & Brooks, R. (2011). The evolution of stock market efficiency over time: A survey of the empirical literature. *Journal of Economic Surveys*, *25*(1), 69-108.

Lim, K. P., & Brooks, R. (2011). The evolution of stock market efficiency over time: A survey of the empirical literature. *Journal of Economic Surveys*, *25*(1), 69-108.

LIM, K. P., & HOOY, C. W. (2013). Non-Linear Predictability in G7 Stock Index Returns. *The Manchester School*, *81*(4), 620-637.

Lim, K. P., Luo, W., & Kim, J. H. (2013). Are US stock index returns predictable? Evidence from automatic autocorrelation-based tests. *Applied Economics*, *45*(8), 953-962.

Ljung, G. M., & Box, G. E. (1978). On a measure of lack of fit in time series models. *Biometrika*, *65*(2), 297-303.

Lo, A. W. (2004). The adaptive markets hypothesis. *The Journal of Portfolio Management*, *30*(5), 15-29.

Lo, A. W. (2005). Reconciling efficient markets with behavioral finance: the adaptive markets hypothesis. *Journal of investment consulting*, *7*(2), 21-44.

Lo, A. W., & MacKinlay, A. C. (1988). Stock market prices do not follow random walks: Evidence from a simple specification test. *The review of financial studies*, *1*(1), 41-66.

Lo, A. W., Mamaysky, H., & Wang, J. (2000). Foundations of technical analysis: Computational algorithms, statistical inference, and empirical implementation. *The journal of finance*, *55*(4), 1705-1765.

Luo, W., Bai, Z., Zheng, S., & Hui, Y. (2020). A modified BDS test. *Statistics & Probability Letters*, *164*, 108794.

Lütkepohl, H. (2005). *New introduction to multiple time series analysis*. Springer Science & Business Media.

Manahov, V., & Hudson, R. (2014). A note on the relationship between market efficiency and adaptability–New evidence from artificial stock markets. *Expert Systems with Applications*, *41*(16), 7436-7454.

Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4), 394-419.

Mandelbrot, B. B., & Wallis, J. R. (1969). Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence. *Water resources research*, *5*(5), 967-988.

McLeod, A. I., & Li, W. K. (1983). Diagnostic checking ARMA time series models using squared-residual autocorrelations. *Journal of time series analysis*, *4*(4), 269-273.

Malkiel, B. G. (2003). The efficient market hypothesis and its critics. *Journal of economic perspectives*, *17*(1), 59-82.

Mwamba, J. M. (2011). Predictability of Stock Price behaviour in South Africa: a non-parametric approach. *African Finance Journal*, *13*(Part 1), 14.

N'dri, K. L. (2015). Variance ratio tests of the random walk in the BRVM. *Applied Economics and Finance*, *2*(2), 118-125.

Noda, A. (2016). A test of the adaptive market hypothesis using a time-varying AR model in Japan. *Finance Research Letters*, *17*, 66-71.

Numapau Gyamfi, E. (2018). Adaptive market hypothesis: evidence from the ghanaian stock market. *Journal of African Business*, *19*(2), 195-209.

Opong, K. K., Mulholland, G., Fox, A. F., & Farahmand, K. (1999). The behaviour of some UK equity indices: An application of Hurst and BDS tests. *Journal of empirical finance*, *6*(3), 267-282.

Peng, C. K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., & Goldberger, A. L. (1994). Mosaic organization of DNA nucleotides. *Physical review e*, *49*(2), 1685.

Phiri, A. (2015). Efficient Market Hypothesis in South Africa. *Managing Global Transitions*, *13*(4): 369-387

Popović, S., Mugoša, A., & Đurović, A. (2013). Adaptive markets hypothesis: Empirical evidence from Montenegro equity market. *Economic research-Ekonomska istraživanja*, *26*(3), 31-46.

Rodriguez, E., Aguilar-Cornejo, M., Femat, R., & Alvarez-Ramirez, J. (2014). US stock market efficiency over weekly, monthly, quarterly and yearly time scales. *Physica A: Statistical Mechanics and Its Applications*, *413*, 554-564.

Schwert, G. W. (2002). Tests for unit roots: A Monte Carlo investigation. *Journal of business & economic statistics*, *20*(1), 5-17.

Seetharam, Y. (2016). *The dynamics of market efficiency: testing the adaptive market hypothesis in South Africa* (Doctoral dissertation, University of the Witwatersrand, Faculty of Commerce, Law and Management, School of Economic & Business Sciences).

Shiller, R. J. (2003). From efficient markets theory to behavioral finance. *Journal of economic perspectives*, *17*(1), 83-104.

Shiller, R. J. (2015). Irrational exuberance: Revised and expanded third edition. In *Irrational exuberance*. Princeton university press.

Shleifer, A., & Vishny, R. W. (1997). The limits of arbitrage. *The Journal of finance*, *52*(1), 35-55.

Shumway, R. H., Stoffer, D. S., & Stoffer, D. S. (2000). *Time series analysis and its applications* (Vol. 3). New York: springer.

Smith, G. (2012). The changing and relative efficiency of European emerging stock markets. *The European Journal of Finance*, *18*(8), 689-708.

Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association*, 89(425), 208-218.

Thaler, R. H. (2015). *Misbehaving: The making of behavioral economics*. WW Norton & Company.

Todea, A., Ulici, M. and Silaghi, S. 2009. Adaptive Market Hypothesis: Evidence from AsiaPacific Financial Markets. *The Review of Finance and Banking, 1(1)*, 7-13.

Tong, H. (1978). On a threshold model. *Pattern recognition and signal processing*, 575-586.

Urquhart, A., & Hudson, R. (2013). Efficient or adaptive markets? Evidence from major stock markets using very long run historic data. *International Review of Financial Analysis*, *28*(June), 130-142.

Urquhart, A., & McGroarty, F. (2014). Calendar effects, market conditions and the Adaptive Market Hypothesis: Evidence from long-run US data. *International Review of Financial Analysis*, *35*(October), 154-166.

Urquhart, A., & McGroarty, F. (2016). Are stock markets really efficient? Evidence of the adaptive market hypothesis. *International Review of Financial Analysis*, *47*(October), 39-49.

Van Heerden, D., Rodrigues, J., Hockly, D., Lambert, B., Taljard, T., & Phiri, A. (2013). Efficient Market Hypothesis in South Africa: Evidence from a threshold autoregressive (TAR) model.

Vasileiou, E. (2021). Behavioral finance and market efficiency in the time of the COVID-19 pandemic: does fear drive the market?. *International Review of Applied Economics*, *35*(2), 224-241.

Yousuf, Z., & Makina, D. (2022). The behavioural finance paradigm and the adaptive market hypothesis: Evidence from the JSE. *International Journal of Finance & Banking Studies (2147-4486)*, *11*(2), 34-48.

Zhou, J., & Lee, J. M. (2013). Adaptive market hypothesis: evidence from the REIT market. *Applied Financial Economics*, *23*(21), 1649-1662.