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MSc. Research Report



A Demonstration of Perceived Practical Benefits of Assessing Product Reliability from Degradation Data

By

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May 27, 2019

DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

_____ _____

(Signature of candidate) Date

To my late beloved mother, Mrs. Ellinah Moyo who was my role model, biggest cheerleader and best friend.

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Abstract

The traditional failure time analysis method has been proven to be less effective in assessing reliability of highly reliable products. It is a prerequisite that failures are recorded or observed when one considers the traditional censored failure time analysis approach. As for degradation data, with few or no failures, it is possible to obtain useful and significant reliability information, on condition that there is a strong correlation between the underlying degradation process and the product's failure. This research study seeks to demonstrate the advantages of using degradation data via the data on GaAs (Gallium Arsenide) lasers adopted from Meeker and Escobar (1998). In particular, to illustrate that with degradation data, meaningful conclusions can be reached much earlier. It does this by proposing the general degradation path model and infers the time to failure distribution from the registered degradation data. The lognormal distribution is deemed to be the appropriate failure time model from experience with the lasers and this information is taken into account in a simulation procedure. The laser is regarded to have failed the first time a 10% increase in current is required to maintain the constant light output. Lasers whose degradation paths have not reached the failure threshold are extrapolated to failure (generating pseudo-failure times) or to 5000 hours, the desired years of operation for the laser. Parameter estimation for the best ranked lifetime distribution is done via the median rank regression method. Important pieces of reliability information are derived from the inferred failure time distribution. Shorter and longer simulated test data were found to be comparable with respect to most of the reliability metrics and their confidence bounds. This demonstrates the advantages of using degradation data when assessing product reliability.

Keywords: Degradation data; Traditional failure time analysis; Product reliability.

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Acronyms

BX percent life estimated time when the probability of failure reaches a specified percentage point

cdf cumulative distribution function

CI Confidence Interval

DA Degradation Analysis

DESV Weighted decision variable value

D_f Critical level at which failure of a product is assumed to take place

FR Failure Rate

$F_T(t)$ Distribution Function of the Failure Time

FTA Failure Time Analysis

GaAs Gallium Arsenide

KDE Kernel Density Estimation

K-S test Kolmogorov-Smirnov test

MLE Maximum Likelihood Estimation

MRL Mean Remaining Life

MRR Median Rank Regression

MTTF Mean Time to Failure

pdf Probability Density Function

SE Standard Error

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Chapter 1

Introduction

1.1 Background

Manufacturing companies are constantly in search of new ideas to improve the performance of their products. The main variable of interest when assessing product performance is product failure time (also known as lifetime) and the main objective is to infer the failure time distribution of products under inspection. Today's products are however generally too well-made, with little or no failures observed in life tests of practical length or operational phases, even under accelerated conditions. The resulting failure time data will likely be highly censored, providing little information about product reliability which significantly limits the accuracy and precision of the conclusions.

An alternative approach (see for example Freitas, Toledo, Colosimo and Pires, 2010) and numerous references therein is to make use of degradation data. The idea is that failure mechanisms of most manufactured products are often linked to an underlying degradation process. For example, light intensity of a light emitting diode (LED) drops with usage. Industrial standards define failure as occurring when either the observable physical degradation (in tyre wear for example) or performance degradation reaches either a random or a specified failure threshold denoted here by $\mathcal{D}_f$. The registered degradation signals reflecting the health status of the product are then used to infer the failure time distribution from which important pieces of information on reliability which include the mean time to failure (MTTF) or some specified percentiles can be derived (Ye and Xie, 2014).

Over the past 40 years, it has been possible for numerous high reliability products to have degradation data available. For example, sensors and smart chips are now inserted in products to record vital in-

formation on degradation. Warranty databases are used to store this kind of information. Specialised commercial software has also been developed to analyse degradation data (Meeker, 2009). As a result, this has necessitated the need amongst industrial statisticians and engineers to continuously research and develop statistical methods to analyse degradation data (Meeker, 2009). A lot of attention is being given to degradation data because of several advantages associated with it, which authors such as dos Santos and Colosimo (2015), Ferreira, Freitas and Colosimo (2012), Lu and Meeker (1993), Meeker, Doganaksoy and Hahn (2001) and Shi, Meeker and Escobar (2009) highlight in their work.

1.2 Characteristics of Degradation Data

The following characteristics are synonymous with degradation data:

1. The time sequence in which degradation data are recorded is indicated by a subscript, t.
2. A measurable product parameter (characteristic) drifts monotonically (upwards or downwards) with time towards a specified failure level. When it reaches the failure level, the product is said to have failed.
3. The drift, measured in terms of the product parameter is linear (or can be transformed to become linear) over time with a slope (rate of degradation) that depends on the random characteristics of the product being measured and the stress under which it is operating.

Thus, the value of degradation analysis lies in how closely related the level of degradation is to product failure. If there is no strong correlation between the level of degradation and product failure, little is gained by using degradation data instead of traditional censored failure time data. Additionally, degradation paths must be well behaved with little measurement error in order for pseudo-failure times to be reasonably extrapolated. According to Chiao and Hamada (2001), Crowder and Lane (2014), Lu, Meeker and Escobar (1996), Meeker and Escobar (1998) and Nelson (1990), the degradation model for the product under consideration must also be correctly specified for degradation analysis results to hold.

1.3 Aim and Objectives of the Study

Aim

The main aim of this investigation is to demonstrate some of the perceived practical advantages associated with using degradation data when assessing the reliability of a product. In particular, to demonstrate an important practical advantage that conclusions are reached earlier with degradation analysis.

Objectives

1. To infer the product's failure time distribution based on the full simulated degradation data set.
2. To derive important pieces of reliability information from the inferred failure time distribution and quantify their uncertainty.
3. Repeat (2) for a shorter test (inspection) and compare the results of this analysis to those from the full simulated data set in order to check if similar conclusions can be reached earlier.
4. Apply the methods to a real data set and draw conclusions.

Chapter 2

Theoretical Background

The idea of degradation-threshold failures connects degradation and product failure nicely (Ye and Xie, 2014). This motivates the use of degradation data to assess product reliability. In particular, the failure time distribution and its associated parameters can be inferred from the analysis of the degradation mechanism and the data.

2.1 Degradation Data Models

Degradation models can be classified into two broad categories, namely stochastic process models and general path models. A discussion of the two degradation model classes is given next, together with a comparison and choice of the degradation model to be adopted in this study.

2.1.1 Stochastic Process Models

Degradation processes are governed by some kind of random mechanism that is represented by a stochastic process, say $\{X(t); t \in T\}$ for convenience where t is the time parameter and the index set T is the set of all possible time points (Ye and Xie, 2014). Examples of stochastic process models for degradation include the well-studied Wiener process (and its variants), Gamma process and the inverse Gaussian process.

2.1 Degradation Data Models

2.1.1.1 The Wiener Process Model

The motivation for the Wiener process (or Brownian motion) $\{B(t); t \in \mathbb{R}^+\}$ (where $\mathbb{R}^+$ is a set of non-negative real numbers including 0) as a degradation model is that the degradation increment $B(t+h) - B(t)$ as $h \to 0$ may be viewed as the sum of a large number of small random external effects (additive superposition) (Ye and Xie, 2014). Since the external effects (usually shocks) are often independent, it follows that the increments $B(t_n) - B(t_{n-1}), \cdots, B(t_4) - B(t_3), B(t_2) - B(t_1)$ over disjoint time intervals $[t_1, t_2], [t_3, t_4], \cdots, [t_{n-1}, t_n]$ for arbitrary $n \in \mathbb{Z}^*$ (where $\mathbb{Z}^*$ is a set of positive integers excluding 0) are independent and normally distributed with zero mean (Ye and Xie, 2014). In many applications, physical degradation is assumed to be a continuous process. The Wiener process is often assumed as the basic model for degradation since its sample paths are continuous functions.

The Wiener process model has certain limitations associated with it. For example, it is not recommended when considering cumulative damage processes such as fatigue growth (Shahraki, Yadav and Liao, 2017). Fatigue growth requires a degradation process model that is time heterogeneous since the rate of crack growth varies during the course of crack propagation. The Wiener process is inappropriate in this case since it is a time homogeneous process (Zhang, 2015). More importantly, the Wiener process cannot model a monotone degradation process since it is not non-decreasing in its argument.

2.1.1.2 The Wiener Process Model with Drift

In general, product degradation has a non-zero mean. To ensure non-zero mean degradation, the Wiener process is improved by including a mean (drift measure) $\nu > 0$. This results in a Wiener process with drift $\{W(t); t \in \mathbb{R}^+\}$ represented as

$$W(t) = \nu\Lambda(t) + \sigma B(\Lambda(t))$$

where $\Lambda(\cdot)$ is the monotone increasing function, σ is the volatility parameter, $B(\cdot)$ is the Wiener process and consequently $W(t) \sim N\Big(\nu\Lambda(t), \sigma^2\Lambda(t)\Big)$. The Wiener process with a linear drift is only taken into account when the mean of the degradation path is increasing in a linear fashion (Shahraki et al., 2017). Applications of the Wiener process with drift as a degradation model follows from its mathematically tractable first passage time distributions. In particular (Folks and Chhikara, 1989), the first passage time of a Wiener process with drift to a fixed threshold is distributed as an inverse

2.1 Degradation Data Models

Gaussian with density function

$$f(t,\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi t^3}}\exp\left[-\frac{\lambda}{2\mu^2}\frac{(t-\mu)^2}{t}\right], t > 0$$

where it is assumed that $\mu \in \mathbb{R}^+$ and $\lambda \in \mathbb{R}^+$.

2.1.1.3 The Wiener Maximum Process Model

The Wiener process with drift applies when the only requirement on the degradation process is to have continuous sample paths. If a monotone degradation process is to be assumed, then the maximum process

$$W^+(u) = \left\{\sup_{0\le u\le t} W(u), u \ge 0\right\}$$

would be preferred as it is non-decreasing in its argument (Hove and Beichelt, 2016). Its first passage times are also distributed as inverse Gaussian.

2.1.1.4 Gamma Process

The Gamma process is a model that works very well especially when temporal variability and unit to unit variability are a large part of the degradation process. It is suitable to use when the degradation process is either increasing or decreasing monotonically over time, for example crack growth. Based on Shahraki et al. (2017), the Gamma process $\{Y(t); t \ge 0\}$ with a shape function $\eta(t) > 0$ and scale parameter $\mu^* > 0$ is a continuous stochastic process with the following properties:

- $Y(0) = 0$ almost surely
- $Y(t+u) - Y(u)$ and $Y(s+v) - Y(v)$ are independent for $\infty > t+u > u \ge s+v > v \ge 0$
- $Y(t+u) - Y(u) \sim \text{Gamma}\Big(\eta(t+u) - \eta(u), \mu^*\Big)$ where $\eta(t)$ is a monotone increasing function with $\eta(0) = 0$

As a consequence of the sample paths of the Gamma process being monotone, it is an appropriate model for gradual damage accumulating over time.

2.1.1.5 Inverse Gaussian Process

A simple inverse Gaussian process, $\{U(t); t \geq 0\}$ has the following properties (Shahraki et al., 2017):

- $U(0) = 0$
- $U(t_2) - U(t_1)$ is independent of $U(t_4) - U(t_3)$ for $t_4 > t_3 \geq t_2 > t_1 \geq 0$
- $U(t) - U(s) \sim IG\left(\mu\Big(\Lambda(t) - \Lambda(s)\Big), \lambda\Big(\Lambda(t) - \Lambda(s)\Big)^2\right)$ for $t > s \geq 0$, where $\Lambda(t)$ is nonnegative and a monotone increasing function of time

It is usually used as an alternative when other stochastic processes fail to fit the monotone degradation data under consideration (Shahraki et al., 2017; Ye and Xie, 2014).

2.1.2 The Degradation Path Model

The degradation path models assume a path model and infer the time to failure distribution from the registered degradation data. Consider n randomly selected test products from a production process or a population of test products. For each i^{th} $(i = 1, \cdots, n)$ sample product, assume degradation measurements are registered at prespecified inspection times t_{ij} $\Big(i = 1, \cdots, n; j = 1, \cdots, m_i\Big)$ the j^{th} time that measurements from product i are taken where m_i is the number of inspection times for the i^{th} product. The inspection times t_{ij} could be real time, operating time or some appropriate measure of use such as distance or the number of cycles depending on the test. They do not necessarily need to have the same value for all products, neither are they required to be equidistant.

Denote by y_{ij} the observed sample degradation measurement taken on the i^{th} product at inspection time t_{ij}. Accordingly, the path of degradation measurements is the pairs $\Big(t_{i1}, y_{i1}\Big)$, $\Big(t_{i2}, y_{i2}\Big)$,$\cdots$,$\Big(t_{im_i}, y_{im_i}\Big)$ for $i = 1, \cdots, n$. The observed sample response y_{ij} is the product's actual degradation value at time t_{ij} plus a random residual deviation term. Thus, the general degradation model (Ferreira et al., 2012) is

$$y_{ij} = \mathcal{D}\Big(t_{ij}; \boldsymbol{\alpha}, \boldsymbol{\beta}_i\Big) + \varepsilon_{ij}; i = 1, \cdots, n; j = 1, \cdots, m_i \tag{2.1}$$

where

- $\mathcal{D}\Big(t_{ij}; \boldsymbol{\alpha}, \boldsymbol{\beta}_i\Big)$ is the i^{th} product's actual degradation value at time t_{ij}

2.1 Degradation Data Models

- $\boldsymbol{\alpha} = \Big(\alpha_1, \cdots, \alpha_p\Big)^T$ is a $p \times 1$ vector of fixed effects which describes characteristics of the population of products. These are considered to be common across all products
- $\boldsymbol{\beta}_i = \Big(\beta_{i1}, \cdots, \beta_{ik}\Big)^T$ is the $k \times 1$ vector of the i^{th} product random effects. They represent the characteristics which are specific to the product
- ε_{ij} is the i^{th} product random error term at time t_{ij}. They are assumed to be independent and identically distributed with zero mean and variance σ_ε^2

Thus, the degradation path model requires the path to be specified together with fixed and random effects. An empirical analysis of the degradation process under consideration may help establish the deterministic form of $\mathcal{D}\Big(t_{ij}; \boldsymbol{\alpha}, \boldsymbol{\beta}_i\Big)$. Otherwise, and when known from past experience for example, the physical-chemical phenomenon associated with the degradation process should be used. Values of the random vectors $\boldsymbol{\beta}_i = \Big(\beta_{i1}, \cdots, \beta_{ik}\Big)^T$ would be of interest in applications where for instance the future degradation of a particular product is to be predicted on the basis of a few registered degradation measurements. Since the interest here is in making inferences about the population of products (or a production process), it is the probability distribution of the random effects and its associated parameters that are of interest. Accordingly, the random vectors $\boldsymbol{\beta}_i = \Big(\beta_{i1}, \cdots, \beta_{ik}\Big)^T$ are assumed to be independent and identically distributed as the multivariate distribution function $G_\beta(b; \boldsymbol{\theta})$ (where b, can be any specific value of β_i), which may depend on an unknown (fixed) parameter vector $\boldsymbol{\theta} = \Big(\theta_1, \cdots, \theta_q\Big)^T$ that must be estimated from the registered degradation data. The random vectors $\boldsymbol{\beta}_i$ are further assumed to be independent of the residual deviations ε_{ij}. The underlying model parameters are therefore $\boldsymbol{\alpha}$, $\boldsymbol{\theta}$ and σ_ε^2 (Ferreira et al., 2012).

2.1.3 Degradation Model Choice

Theoretical justification of stochastic process models is based on the observation that degradation stems from enduring stress. The product has a degradation threshold and fails when it experiences a certain amount of cumulative damage. They are a natural choice when the physics behind a degradation process can be captured and has clear interpretations. Often, stochastic process models are complex and therefore not easy to understand and use in practice. In addition, the resulting distribution for the first passage time to a failure threshold may not be expressed in a closed form.

The data driven degradation path model is adopted in this research report because it is applicable in situations where physical explanations of the degradation process are not obvious. Some of the advantages of degradation path models over stochastic process models (Ye and Xie, 2014) include:

- Flexibility in incorporating random effects. This is a consequence of the fact that degradation path models are essentially mixed effects regression models.
- They are more robust in that deviations of registered degradation data from the true degradation path may be accounted for by adjusting the error term distribution for the model to apply to specific degradation data.
- Resulting distributions for the first passage time to a failure threshold often have closed form expressions. Otherwise, numerical methods are applied.

In the case where the randomness of degradation is intrinsically due to random environmental factors such as temperature, usage pattern etc., it can at least be reduced by incorporating such environmental factors as time varying covariates in the degradation process. Otherwise, randomness in environmental factors will manifest itself in the registered degradation data (Ye and Xie, 2014).

2.2 Distribution Function of the Failure Time, $F_T(t)$

The product's failure time determines its reliability and therefore, inferences in the product's failure time distribution are required. By definition, product failure in a degradation analysis occurs at time T when the actual degradation $\mathcal{D}\Big(t_{ij}, \boldsymbol{\alpha}, \boldsymbol{\beta}_i\Big)$ reaches a failure threshold $\mathcal{D}_f$. The failure time T is unique since degradation changes with time along a definite direction (monotone). Consequently, specifying the model for $\mathcal{D}\Big(t_{ij}, \boldsymbol{\alpha}, \boldsymbol{\beta}_i\Big)$ and the failure threshold $\mathcal{D}_f$ defines the product's failure time distribution. In particular, and without loss of generality, consider the case when $\mathcal{D}\Big(t_{ij}, \boldsymbol{\alpha}, \boldsymbol{\beta}_i\Big) = \beta_i t_{ij}$, giving the simple linear random effects degradation model

$$y_{ij} = \beta_i t_{ij} + \varepsilon_{ij} \tag{2.2}$$

where the common initial amount of degradation $\boldsymbol{\alpha} = (0, \cdots, 0)$ for all products. Assume that the product monotonically degrades in time and that $\mathcal{D}(t)$ is an increasing function since degradation is irreversible. For the linear random effects model in Equation (2.2), failure occurs when $\mathcal{D}_f = \beta T$ and the distribution function of the failure time is

$$\begin{aligned} F_T(t) &= P(T \leq t) \\ &= P\left(\frac{\mathcal{D}_f}{\beta} \leq t\right) \\ &= P\left(\beta \geq \frac{\mathcal{D}_f}{t}\right) \\ &= 1 - G_\beta\left(\frac{\mathcal{D}_f}{t}\right), t > 0 \end{aligned} \tag{2.3}$$

It follows from Equation (2.3) that for a fixed failure threshold $\mathcal{D}_f$, the unknown distribution function of the random effects β, $G_\beta(\cdot)$ determines the failure time distribution of the product. Denote the $100u^{th}$ percentile of $F_T(t)$ by t_u. The percentile or quantile function t_u is the inverse of $F_T(t)$ and it is defined as the period or time at which a specified proportion u of the products fails (Hong, Meeker and Escobar; 2008). According to Dakhn, Ebrahem and Eidous (2017), the required percentile is determined by solving

$$u = F_T(t_u) = 1 - G_\beta\left(\frac{\mathcal{D}_f}{t_u}\right)$$

for t_u, yielding

$$t_u = \frac{\mathcal{D}_f}{G_\beta^{-1}(1-u)}$$

As is the case with the distribution of T, the determination of the $100u^{th}$ percentile also relies on the distribution function of the random effects β. It is possible to express $F_T(t)$ in a closed form in instances where path models are simple. However, path models are often not simple and the problem is particularly complicated when the number of random parameters exceeds one. Where possible, $F_T(t)$ can be obtained using integral transformations. Otherwise, $F_T(t)$ may be obtained numerically using Monte Carlo simulations (Meeker and Escobar, 1998).

2.3 Commonly Used Lifetime Distributions

The term lifetime distributions refers to a collection of statistical probability distributions that better describe life data and have wide applications in reliability and life data analysis. The extent to which

the assumed lifetime distribution fits the times-to-failure data depends on the behaviour of its hazard function (also known as instantaneous failure rate). Recall that in terms of the unreliability function, the probability of a product failing in some interval, say $[t_1, t_2]$ is given by

$$\int_{t_1}^{t_2} f(t)dt = \int_{-\infty}^{t_2} f(t)dt - \int_{-\infty}^{t_1} f(t)dt = F(t_2) - F(t_1)$$

The failure rate (FR) is the probability of a failure occurring per unit time in the interval given that no failure occurred prior to the beginning of the interval. That is, the failure rate is given by

$$\text{Failure Rate} = \frac{F(t_2) - F(t_1)}{(t_2 - t_1)\bar{F}(t_1)}$$

Redefining the interval as $[t, t + \delta t]$, the expression for the failure rate becomes $\frac{F(t+\delta t)-F(t)}{\delta t \bar{F}(t)}$ and it is a function of time. The limit as the interval approaches zero is the hazard function (hence instantaneous failure rate). It is expressed as

$$\begin{aligned} \lambda(t) &= \lim_{\delta t \to 0} \frac{P(t < T \leq t + \delta t)}{\delta t \bar{F}_T(t)} \\ &= \frac{1}{\bar{F}_T(t)} \lim_{\delta t \to 0} \frac{F(t + \delta t) - F(t)}{\delta t} \\ &= \frac{f(t)}{\bar{F}_T(t)} \end{aligned} \tag{2.4}$$

where T is a continuous random variable and $\bar{F}_T(t_1) = 1 - F_T(t_1)$ is the reliability function. It allows for the determination of the number of failures among survivors occurring per unit time. Essentially, the instantaneous failure rate characterises typical failure patterns of products over time. A plot of the failure rate against a continuous time scale for an entire product population yields the so called "bathtub curve" (see Beichelt and Tittmann (2012) and numerous references therein). The shape of the bathtub curve is generally characterised by three phases as follows:

- Decreasing failure rate: Also called early "infant mortality" failure, this phase is characterised by a high failure rate which decreases as the survival time of products increases. Failures in this phase are largely due to poor quality control; errors in design, installations or defects in manufacturing or material etc. They can be reduced by adopting best design approaches, better quality control and proof tests before releasing the product population into the market.

- Constant failure rate: This is the phase when products are described to have normal or useful life. It is characterised by a low and approximately constant failure rate. Failures are considered to be random and occuring due to random faults and overloads with no apparent pattern. Often, product warrant periods fall in this phase.
- Increasing failure rate: Significant increase in failure rate is observed during this phase. This is when products wear-out (age) thereby rapidly increasing the failure rate.

The most commonly used lifetime distributions in reliability and life data analysis include the exponential, Weibull, lognormal and Gamma distributions. A brief discussion about them is given next.

2.3.1 Exponential Distribution

The exponential distribution is widely used due to its simplicity and constant failure rate property. The two-parameter exponential probability density function (pdf)

$$f(t) = \frac{1}{\eta}\text{exp}\left[-\left(\frac{t-\gamma}{\eta}\right)\right]$$

has a constant hazard function given by

$$\lambda(t) = \frac{1}{\eta}$$

where $t \geq 0$, $t > \gamma$, γ is the location parameter (also termed guarantee time in the sense that failure is deemed to occur only when $t > \gamma$) and η is the scale parameter (known as the characteristic product life). When $\gamma = 0$, the two-parameter exponential distribution reduces to the well-known one-parameter exponential distribution expressed as

$$f(t) = \frac{1}{\eta}\text{exp}\left[-\frac{t}{\eta}\right]$$

The one-parameter exponential form is the one which is commonly used in analysing product reliability data due to its simpler form. The constant hazard function implies that the probability of a product failing is independent of its age and this scenario is not practical for most products (Meeker and Escobar, 1998). For example, it is not suitable to model the life of electronic and mechanical products such as lasers, filament devices and bearings since they are prone to wear-out, fatigue and

corrosion. Meeker and Escobar (1998) state that there are special circumstances under which the exponential distribution is useful, for instance the times between system failures and arrivals in a queue.

2.3.2 Weibull Distribution

The Weibull distribution is widely used in modelling lifetimes of products, mainly based on theoretical (i.e., extreme value theory) and practical (i.e., provides a good fit to the lifetimes of many products) considerations (Doganaksoy, Hahn and Meeker, 2010). The more general three-parameter Weibull distribution has the cumulative distribution function (cdf)

$$F_T(t) = 1 - \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right], \eta > 0, \beta > 0 \tag{2.5}$$

where $t > \gamma$, γ is the location parameter. With regard to product reliability, γ is termed the guarantee time. The scale parameter η is the characteristic product life and β is the shape parameter. Its ability to describe failure distributions with many varied shapes is highlighted as one of its major strengths (Meeker and Escobar, 1998).

Its hazard function is given by

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \tag{2.6}$$

which reduces to

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

with corresponding distribution function

$$\begin{aligned} F_T(t) &= P(T \leq t) \\ &= 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], t > 0 \end{aligned} \tag{2.7}$$

for the commonly used two-parameter Weibull distribution when the location parameter (fault free

life) $\gamma = 0$. The flexibility of the Weibull distribution as a model in life data analysis stems from the behaviour of its hazard function depending on its parameter values. That is:

- If $\beta = 1$, the two-parameter Weibull distribution takes the form of the exponential distribution (constant failure rate)
- If $\beta < 1$, there is a decreasing failure rate. In real life there are very few situations which can be related to this type of failure rate
- If $\beta > 1$, there is an increasing failure rate (aging or wearing out of a product)

2.3.3 Lognormal Distribution

The lognormal distribution has two parameters μ' and σ'. Its pdf is given by (Reliasoft Corporation, 2015):

$$f(t;\mu',\sigma') = \frac{1}{t\sigma'\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t'-\mu'}{\sigma'}\right)^2}$$

where $\mu' \in (-\infty, \infty)$ is the mean of the natural logarithms of the failure times, $\sigma' > 0$ is the standard deviation of the natural logarithms of the failure times and t' is the natural logarithm of the failure time of a product. Its hazard function is expressed as

$$\lambda(t) = \frac{\frac{1}{t\sigma'\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t'-\mu'}{\sigma'}\right)^2}}{\int_{t'}^{\infty} \frac{1}{\sigma'\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu'}{\sigma'}\right)^2} dx}$$

To solve the hazard function, the use of standard normal tables is required.

The lognormal distribution has an unusual hazard function behaviour which monotonically increases to a peak and then eventually decreases to zero. However, many electronic products such as GaAs lasers have a hazard function which behaves in such a manner. This explains why the lognormal distribution is usually chosen to model the lifetimes of electronic products. Fatigue crack growth is also usually modelled using the lognormal distribution (Meeker and Escobar, 1998).

2.3.4 Gamma Distribution

The Gamma distribution is a very flexible distribution which provides a good fit to certain failure-time data (Meeker and Escobar, 1998; Reliasoft Corporation, 2015). Its pdf is given by

$$f(t) = \frac{e^{kz-e^z}}{t\Gamma(k)}$$

where $t > 0$, $z = \ln(t) - \mu^*$, $-\infty < \mu^* < \infty$ and $k > 0$. e^{μ^*} and k are the scale and shape parameters respectively. In reliability applications, the shape parameter k represents the number of shocks a product experiences until failure.

Its hazard function is expressed as

$$\lambda(t) = \frac{e^{kz-e^z}}{t\Gamma(k)(1-\Gamma_I(k;e^z))}$$

where $\Gamma_I(k;e^z)$ is an incomplete Gamma function defined by

$$\Gamma_I(k;e^z) = \frac{\int_0^z x^{k-1}e^{-x}dx}{\Gamma(k)}, z > 0$$

The hazard function can either be decreasing (when $k < 1$) or increasing (when $k > 1$). If $k = 1$, the Gamma distribution reduces to an exponential distribution.

2.4 Parameter Estimation Methods for the Degradation Path Model

The degradation path model parameters can be estimated by common parametric methods which include the maximum likelihood estimation (MLE) and the median rank regression (MRR) method amongst others. Overall, the MLE method is regarded to be the most robust parameter estimation technique. The MRR method is based on the popular ordinary least squares method, which is quite easy to understand, interpret and use. In reliability applications, especially with regard to degradation data of highly reliable products, small data samples are usually available. The MLE procedure works very well for large samples. It is asymptotically unbiased or efficient, meaning that the precision of the parameter estimates greatly improves as the sample size gets larger (Reliasoft Corporation, 2015). The MRR method tends to handle small sample sizes much better than the MLE method in terms of statistical accuracy.

2.4.1 Maximum Likelihood Estimation

As mentioned previously, maximum likelihood estimators are usually preferred due to their optimum properties in large samples. Discussed next, is the theory behind the MLE method in relation to degradation data.

Let $\mathbf{Y}_i = \Big(Y_{i1}, Y_{i2}, \cdots, Y_{im}\Big)^T$ represent the $m_i \times 1$ random vector for the degradation measures of the i^{th} product and $y_i = \Big(y_{i1}, y_{i2}, \cdots, y_{im}\Big)^T$ be the actual registered degradation data. The complete set of degradation measurements is represented by an $N \times 1$ random vector $\mathbf{Y} = \Big(Y_1, Y_2, \cdots, Y_n\Big)^T$ where $N = \sum_{i=1}^{n} m_i$. The complete set of the registered degradation data values is thus $y = \Big(y_1, y_2, \cdots, y_n\Big)^T$. Also let $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \cdots, \boldsymbol{\beta}_n)^T$ be the $nk \times 1$ vector that combines the n $\boldsymbol{\beta}_i$ vectors. Denote the probability density function of $\mathbf{Y}_i$ by $f\Big(y; \alpha, \beta_i, \theta, {\sigma^2}_{\varepsilon}\Big)$ and that of $\boldsymbol{\beta}_i$ by $g\Big(\boldsymbol{\beta}_i; \boldsymbol{\theta}\Big)$. Then the respective probability density functions of the complete set of actual degradation measures $\mathbf{Y} = \Big(Y_1, Y_2, \cdots, Y_n\Big)^T$ and $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \cdots, \boldsymbol{\beta}_n)^T$ are (Ferreira et al., 2012);

$$f\Big(\mathbf{y}; \boldsymbol{\alpha}, \boldsymbol{\beta}_i, \boldsymbol{\theta}, \sigma_{\varepsilon}^2\Big) = \prod_{i=1}^{n} f\Big(y_i; \alpha, \beta_i, \theta, \sigma_{\varepsilon}^2\Big) \tag{2.8}$$

and

$$g\Big(\boldsymbol{\beta}; \boldsymbol{\theta}\Big) = \prod_{i=1}^{n} g\Big(\boldsymbol{\beta}_i; \boldsymbol{\theta}\Big) \tag{2.9}$$

For the underlying model parameters $\boldsymbol{\alpha}$, $\boldsymbol{\theta}$ and σ_{ε}^2 of the degradation path model in Equation (2.2), the maximum likelihood gives parameter estimates that maximize the probability (likelihood) of getting the observed degradation data. Thus, the likelihood function is the joint probability density of the data. For the complete set of actual degradation measures $y = \Big(y_1, \cdots, y_n\Big)^T$, it is expressed as

$$\begin{aligned}
L\Big(\boldsymbol{\alpha}, \boldsymbol{\beta}, \sigma_{\varepsilon}^2 | y\Big) &= f\Big(y | \boldsymbol{\alpha}, \boldsymbol{\theta}, \sigma_{\varepsilon}^2\Big) \\
&= \int_{\Xi\boldsymbol{\beta}} f\Big(y, \boldsymbol{\beta} | \boldsymbol{\alpha}, \boldsymbol{\theta}, \sigma_{\varepsilon}^2\Big) d\boldsymbol{\beta} \\
&= \int_{\Xi\beta_1} \cdots \int_{\Xi\beta_n} f\Big(y | \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \sigma_{\varepsilon}^2\Big) f\Big(\boldsymbol{\beta} | \boldsymbol{\theta}\Big) d\beta_1 \cdots \beta_n \\
&= \int_{\Xi\beta_1} \cdots \int_{\Xi\beta_n} \left[\prod_{i=1}^{n} f\Big(y_i | \alpha, \beta_i, \theta, \sigma_{\varepsilon}^2\Big)\right] \left[\prod_{i=1}^{n} f\Big(\beta_i | \theta\Big)\right] d\beta_1 \cdots \beta_n \\
&= \prod_{i=1}^{n} \left\{ \int_{\Xi\beta_i} f\Big(y_i | \alpha, \beta_i, \theta, \sigma_{\varepsilon}^2\Big) f\Big(\beta_i | \theta\Big) d\beta_i \right\}
\end{aligned} \tag{2.10}$$

where $\Xi\beta_i$ is a real argument for the random effect parameter. Maximizing the likelihood function in Equation (2.10) with respect to $\boldsymbol{\alpha}$, $\boldsymbol{\theta}$ and σ_ε^2 is very complex unless $\mathcal{D}\Big(t_{ij}, \boldsymbol{\alpha}, \boldsymbol{\beta}_i\Big)$ is a linear function. When a simple linear degradation path model is considered, ML estimates of model parameters can be easily obtained. Otherwise, numerical approximations to the likelihood function would be required (Ferreira et al., 2012).

2.4.2 Median Rank Regression

The rank regression method is a better alternative to MLE when a small sample is involved, as it produces parameter estimation results with much less bias. The median rank regression method is essentially an ordinary least squares approach which uses median rank plotting positions (Genschel and Meeker, 2010). MRR basically linearizes the cdf of the failure time distribution, that is putting it in a linear form expressed in Equation (2.2). To see how MRR linearizes (Reliasoft Corporation, 2015) the cdf, consider the two-parameter Weibull distribution

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], t > 0$$

Rearranging and taking natural logarithms gives

$$\ln\Big[1 - F(t)\Big] = \ln\left[e^{-\left(\frac{t}{\eta}\right)^{\beta}}\right] = -\left(\frac{t}{\eta}\right)^{\beta}$$

Taking the natural logarithms for the second time yields

$$\ln\left[-\ln\Big(1 - F(t)\Big)\right] = \beta\left[\ln\left(\frac{t}{\eta}\right)\right]$$

which after rearranging becomes

$$\ln\left[\ln\left(\frac{1}{1 - F(t)}\right)\right] = \beta\ln t - \beta\ln(\eta) \tag{2.11}$$

Equation (2.11) is now in the general form

2.4 Parameter Estimation Methods for the Degradation Path Model

$$y_i = bx_i + a$$

such that

$$y_i = \ln\left[\ln\left(\frac{1}{1 - F(t_i)}\right)\right]$$

and

$$x_i = \ln(t_i).$$

The simple linear random effects degradation model can then be rewritten as

$$y_i = bx_i + a$$

where a and b are regression coefficients. Similarly, this linearisation of the cdf can be done for any assumed lifetime distribution.

The MRR method basically works by first ordering the failure and censoring times from smallest to largest and assigning ranks denoted by i ($i = 1, 2, \cdots, n$). The respective estimates of probability of failure $\hat{F}(t_i)$'s for products are determined by the use of median ranks. The Bernard's approximation method is one of the most common type of median ranks; its estimator is defined by

$$\hat{F}(t_i) = \frac{i - 0.3}{n + 0.4}$$

with n being the sample size and the failure order number from the ranking is denoted by i. When using statistical software such as Weibull ++, MRR is either implemented in the vertical or horizontal direction (RRY or RRX). Rank regression based on X produces more accurate results than RRY when a small sample size is involved. The time to failure X is generally associated with much more variability than the median ranks, hence this study shall consider analysis based on RRX. RRY is suitable for free-form data such as warranty data (HBM United Kindom Limited , 2017).

2.5 Estimation of the Target Reliability Metrics

The primary aim of this research report is to estimate the probability of failure at 5000 hours (desired lifetime of the motivational GaAs laser data set to be used in the methodology section) and other important reliability metrics such as mean time to failure, reliable life, BX% life, failure rate and some specified percentiles. The reliability metrics to be considered in this study are defined as follows (HBM United Kindom Limited , 2017):

- Probability of failure - probability of failure at a specified time
- Mean Time to Failure - average time a population of products is expected to operate before failure
- Reliable Life (Warranty time) - estimated time for a specified target reliability
- BX% Life - estimated time when the probability of failure reaches a specified percentage point, for example, if the expectation is that 30% of the products will fail in 6 years time then B30% is equal to 6 years
- Failure Rate - the expected number of product failures per unit time

2.5.1 Confidence Bounds for $F_T(t)$

Confidence intervals (CIs) are used to quantify the uncertainty of estimates of reliability metrics including lifetime quantiles (Hong et al., 2008). In reliability applications, the uncertainty usually arises from degradation measurement errors. Genschel and Meeker (2010) highlight the importance of quantifying uncertainty to do with product reliability, particularly in instances where safety issues could be of great concern and where small to moderate samples are available. Lu and Meeker (1993) and Lu, Park and Yang (1997) state that there are many methods of constructing confidence bounds or intervals and these include the normal approximation, the bootstrap and the likelihood ratio (LR) methods. Since degradation data is associated with a few observations or limited data, it is highly recommended to employ the bootstrap method or the LR pointwise confidence intervals instead of those based on the normal approximation methods which effectively deal with large sample sizes. The bootstrap confidence intervals perform on par with LR based CIs and both are actually better than normal approximation methods based CIs in terms of accuracy.

Only two types of CIs, namely the LR and bootstrap based CIs are discussed in greater detail in this research report.

2.5 Estimation of the Target Reliability Metrics

2.5.1.1 Bootstrap based Confidence Intervals from degradation data

The following steps describe how the uncertainty associated with $\hat{F}_T(t)$ is quantified (Crowder and Lane, 2014):

1. Generate bootstrap samples in such a way that they mimic the degradation data which consists of n sample paths. Sampling is done with replacement.

2. For all the n degradation sample paths, fit the degradation path model shown in Equation (2.2) and then estimate the models parameters for each path using methods such as MRR, MLE and so on.

3. If the underlying distribution of the random effect parameters from the degradation path model is not known, determine it using the estimates from the n sample paths.

4. Determine the failure time distribution $F_T(t)$ of bootstrap samples by substituting their respective estimated parameter values in Equation (2.3) on condition that $F_T(t)$ has a closed form expression. Otherwise, use the parametric distribution from step 3 to generate a large number N of random degradation paths.

5. Repeat steps 1-4 many times, for example 1000 times, in order to obtain a distribution of values for $F_T(t)$. The central $100(1-\alpha)\%$ of this distribution represents the pointwise approximate $100(1-\alpha)\%$ confidence interval for $F_T(t)$, which in other words is the uncertainty interval for $F_T(t)$.

2.5.1.2 Likelihood Ratio based Confidence Intervals

The $100(1-\alpha)\%$ LR based confidence interval for $F_T(t)$ of the percentile t_u is based on the inversion of the likelihood ratio statistic Λ. The LR CI is only applicable when there is a closed form expression for $F_T(t)$. The LR based intervals make use of the following hypothesis

$H_0 : F_T(t) = u$ vs. $H_1 : F_T(t) \neq u$ where u is the u^{th} quantile of $F_T(t)$.

Based on Lu et al. (1997), the decision rule for the hypothesis test is that the null hypothesis H_0 is rejected if the LR statistic Λ is greater than $\chi^2_{1,\alpha}$ (upper α quantile of the chi-square distribution with 1

degree of freedom). The inversion of the test of $H_0 : F_T(t) = u$ is based on the asymptotic normality of

$$V(t) = \ln\Bigg(- \ln(F_T(t))\Bigg) \tag{2.12}$$

It should be noted that the quantity of $V(t)$ is unrestricted in that it can either be greater than 1 or less than 0. The confidence interval for the estimate of the failure distribution function must strictly lie between 0 and 1. Transformation is done such that

$$F_T(t) = \exp\Bigg(- \exp(V(t))\Bigg) \tag{2.13}$$

so as to get the correct CI restriction. The $100(1-\alpha)\%$ CI for $V(t)$ based on $\hat{V}(t)$ is constructed as follows

$\left[\hat{V}(t) - z \times \text{SE}\left(\hat{V}(t)\right), \hat{V}(t) + z \times \text{SE}\left(\hat{V}(t)\right)\right]$ and this CI is equivalent to

$$\left[\exp\{-e^{\hat{V}(t)+z\times \text{SE}(\hat{V}(t))}\}, \exp\{-e^{\hat{V}(t)-z\times \text{SE}(\hat{V}(t))}\}\right]$$

Substituting $\hat{V}(t) = \ln\Bigg(- \ln(\hat{F}_T(t))\Bigg)$ into the above CI, yields

$$\left[\hat{V}(t) - z \times \text{SE}\left(\hat{V}(t)\right), \hat{V}(t) + z \times \text{SE}\left(\hat{V}(t)\right)\right] = \left[\left(\hat{F}_T(t)\right)^{e^B}, \left(\hat{F}_T(t)\right)^{e^{-B}}\right] \tag{2.14}$$

where $B = z \times \text{SE}\left(\hat{V}(t)\right)$ and SE is the standard error of $\hat{V}(t)$. To determine B, the variance of $\hat{V}(t)$ has to be calculated; the z value (the two-sided critical z-value is usually denoted as $z_{1-\alpha/2}$, where α is the level of statistical significance) and it is easily found in the statistical tables.

2.6 Mean Squared Error

An estimator's accuracy and precision can be evaluated by determining its bias and standard error respectively. Besides the use of confidence bounds, determining the MSE associated with the estimates

2.6 Mean Squared Error

of reliability metrics is another effective way of quantifying uncertainty. The mean squared error of an estimator $\hat{t}_u$ is the most commonly used measure to evaluate its accuracy. It is denoted by

$$MSE = \text{E}[(\hat{t}_u - t_u)^2] = [\text{SE}(\hat{t}_u)]^2 + [\text{Bias}(\hat{t}_u)]^2 = \text{Var}(\hat{t}_u) + [\text{Bias}(\hat{t}_u)]^2$$

where $\text{Bias}(\hat{t}_u) = \text{E}(\hat{t}_u - t_u)$, $\text{Var}(\hat{t}_u)$ is the variance of the estimate $\hat{t}_u$ and SE is the standard error (Genschel and Meeker, 2010). The formulae that are used to compute sample bias and sample variance are as follows

$$\text{Bias} = \frac{\sum_n^{j=1} \left(t_u - \hat{t}_u\right)}{n}$$

$$\text{Variance}(\hat{t}_u) = \frac{\sum_{j=1}^{n} \left[\hat{t}_u - \text{mean}(\hat{t}_u)\right]^2}{n-1} = \left[\text{SE}(\hat{t}_u)\right]^2$$

where $\hat{t}_u$ denotes the specified percentile or some other reliability metrics and n represents the number of simulated degradation data sets.

Chapter 3

Literature Review

Over the past 40 years, it has been possible for numerous high reliability products to have degradation data available (Meeker, 2009). At the same time, many researchers and reliability engineers have also developed a huge interest in degradation analysis (DA). Hence, numerous valuable DA studies (as shall be seen in the following literature review) have been conducted to explore the multiple benefits associated with degradation data analysis, particularly over traditional failure time analysis in assessing product reliability. Included in these various investigations or studies are conditions which need to be met for DA to optimally perform as desired, the different forms of degradation processes, the degradation modelling approaches and parameter estimation methods. The review of literature will cover important aspects of DA under the following headings: why degradation analysis, forms of degradation, modelling approaches and estimation methods.

3.1 Why degradation data analysis?

Over recent years, DA in relation to product reliability has become quite popular in the manufacturing and industrial sector. This is mostly due to the fact that today's products are too well-made (as previously mentioned in section 1.1) and they take ages to fail (Lu et al., 1997). This makes failure-time data less useful in tests meant for assessing the reliability of highly reliable products. Degradation is deemed to be more attractive because it utilises all the test information including scenarios where no failures occur. This is demonstrated by authors such as Hamada (2005) and Meeker et al. (2001) that use a laser data example adapted from Meeker and Escobar (1998). Shahraki et al. (2017) note that it is imperative to have a thorough understanding of degradation analysis, since the knowledge of a product's degradation behaviour can significantly help prevent potential failures in order to effectively

avoid subsequent losses. Again, if DA is implemented correctly, a valid degradation model can be used for forecasting and decision making, for example, the estimation of the failure time distribution, forecasting of warranty costs and remaining useful life prediction during field use and condition-based maintenance (Ye and Xie, 2014).

There are many other practical advantages associated with using degradation data. For example, meaningful and accurate product reliability information can be obtained from degradation data much earlier compared to when using traditional censored failure time data. The data collection process of failure time data for products that are highly reliable can be very expensive and not practical because they tend to take a long time to fail (Ferreira et al., 2012). In such a case, DA is a much better solution than traditional FTA. This benefit is of immense value (for highly reliable products), especially when there are severe time constraints for product reliability testing and when other quick logistic decisions need to be made (Shahraki et al., 2017). With few or no failures, it is also possible to extract substantial information on product reliability from the use of DA (Chiao and Hamada, 2001; Crowder and Lane, 2014; Ferreira et al., 2012; Freitas et al., 2010; Lu and Meeker, 1993; Meeker et al., 1998; Nelson, 1990; Oliveira and Colosimo, 2004; Park and Padgett, 2006; Shi et al., 2009). Statistical techniques such as DA are a very important factor for companies, in that they guarantee a competitive position in the consumer market and also reduce the overall product development time (Oliveira and Colosimo, 2004; Shahraki et al., 2017).

Statistically designed DA experiments can be used to enhance the reliability and achieve robust reliability for highly reliable products (Chiao and Hamada, 2001). With DA, it is possible to truncate paths whereas with failure time data, complete sample paths are required to carry out an analysis which results in obtaining meaningful information on product reliability (Crowder and Lane, 2014; Meeker et al., 2001). Degradation data yields more accurate life estimates than accelerated life tests, with few or no failures (Ferreira et al., 2012). Another strength of DA pertains to its ability to estimate quantiles of failure probabilities beyond the range of available data (Lu et al., 1996).

Degradation data has certain limitations to it. Admittedly, for results from DA to be statistically valid (Chiao and Hamada, 2001; Meeker et al., 2001), there must be no significant measurement errors. Hence there is need to ensure a high level of accuracy when recording a product's degradation measures over time. The actual degradation of a product at time t_{ij} is observed with error ε_{ij}. The degradation measurements must have minimal measurement errors, otherwise benefits of DA can be hugely compromised (Chiao and Hamada, 2001; Ferreira et al., 2012; Hamada, 2005; Lu et al., 1996; Meeker and Escobar, 1998; Meeker et al., 2001). However, even with inherent measurement errors

(Lu et al., 1996), DA performs better than FTA as long as there are enough inspections to cater for the measurement error variance. In addition, situations could arise where data collection of degradation measurements for some products is expensive. For instance, (Lu et al., 1996; Meeker, 2009), the scientific study of measurements (metrology) is very costly and a lot of capital resources are needed to record the measurements. In such cases, it is vital to strike an optimal balance between the benefits of taking degradation measurements and their cost effectiveness (Lu and Meeker, 1993). In the main however, the data collection process is not expensive since smart chips are generally inserted for the purpose of recording useful degradation information over time (Lu et al., 1996; Meeker, 2009).

Furthermore, for one to be able to reap maximum benefits from degradation analysis, quite a number of strict conditions have to be met, especially the presence of a strong correlation between the underlying degradation process and the product's failure. Determining the correct variables which are closely related to a product's failure time is important but also a very difficult engineering challenge of DA (Meeker and Escobar, 1998). Results obtained from degradation analysis based on an inappropriate model are of no significance. It is therefore of paramount importance to specify an appropriate model which adequately describes the degradation process of a product. The threshold level of degradation also has to be prespecified (Chiao and Hamada, 2001; Ferreira et al., 2012; Lu et al., 1996; Meeker and Escobar, 1998). In cases whereby accelerated conditions have been used, it is necessary that one applies physical models that relate the degradation rate to stress when extrapolating and estimating $F_T(t)$ at a design stress (Ferreira et al., 2012; Meeker et al., 1998; Park and Padgett, 2006).

3.2 Forms of Degradation

There are in general two forms of degradation, namely physical degradation which depends on direct observation and a degradation process based on the performance output of a component or subsystem. Meeker et al. (1998) and Meeker and Escobar (1998) explain that modelling degradation based on direct observation such as tyre tread wear and crack growth is much easier and less complicated compared to modelling a performance degradation process, such as a change in output voltage or decrease in light intensity of LEDs, for example. Due to its simplicity, the physical degradation form tends to provide more credible and precise reliability estimates and a firmer basis for extrapolation. Performance degradation is complex in the sense that there is always a possibility that the performance of a product may be affected by more than one underlying degradation process (Meeker and Escobar, 1998). In such a case, the National Institute of Standards and Technology (2003) recommends that a

standard failure time analysis be carried out if possible, in order to check if the parameter estimates obtained from standard FTA are comparable with those obtained from the predictions of DA, just as a "reality" check.

It is also crucial that the correct form of degradation is taken into account in any DA experiment. For example, not much is gained from DA when one is incorrectly recording degradation measures from a performance variable instead of the actual physical degradation. Various studies done (see for example Meeker and Escobar (1998)) explore both forms of degradation with different kinds of data, such as the fatigue crack size, distance to failure for car shock absorbers, battery life data, GaAs laser data examples and so on. The experiments by Meeker et al. (2001) and Hamada (2005) also consider performance degradation using the GaAs laser data. Other studies apply DA using the physical form of degradation by making use of the train wheel degradation data (discussed further in sections 3.3 and 3.4), crack growth data (Ferreira et al., 2012; Freitas et al., 2010; Lawless and Crowder, 2004; Meeker et al., 1998). Pan and Crispin (2011) investigate the degradation process of light emitting diodes. Oliveira and Colosimo (2004) focus on a degradation process related to an automobile's tyre and the response of interest is the tyre tread reduction.

3.3 Modelling Approaches

Degradation models can be classified into two broad categories, namely stochastic process models and general path models. Ye and Xie (2014) extensively discuss and elaborate on the theories, strengths and weaknesses associated with both the stochastic process models and general path models. General path models are usually preferred because of their simplicity in modelling continuous processes for degradation data. There are several degradation studies which assume the general degradation path model (Ebrahem et al., 2009; Ferreira et al., 2012; Freitas et al., 2010; Lu and Meeker, 1993; Meeker et al., 2001; Oliveira and Colosimo, 2004; Pan and Crispin, 2011). They have the modelling flexibility of incorporating both fixed and random effects into the degradation path function (Pan and Crispin, 2011; Shahraki et al., 2017; Ye and Xie, 2014).

Ferreira et al. (2012) and Freitas et al. (2010) assume a simple linear degradation path model in modelling train wheel degradation data. Common distributions used to model the product's lifetime such as the Weibull and lognormal are considered in modelling the train wheel data. Oliveira and Colosimo (2004) use the simple linear model as well, in a degradation process related to an automobile's tyre.

3.3 Modelling Approaches

Lu et al. (1997) consider a linear degradation model in modelling the degradation process of a semiconductor. Work is also done which includes covariates into the general path model, to ensure that external factors such as environmental conditions are taken into account in modelling the degradation behaviour of a product (Lu et al., 1997; Lu and Meeker, 1993; Meeker et al., 1998; Pan and Crispin, 2011).

Pan and Crispin (2011)'s approach in modelling and analysing LED degradation data is to some extent different from that of Chiao and Hamada (2001). Pan and Crispin (2011) propose a hierarchical modelling technique (that is, the degradation model is determined at two levels) based on a general path model suggested by Meeker et al. (1998). The first model involves a simple linear function whereas for the second one, the path is modeled by a nonlinear function. Both Chiao and Hamada (2001) and Pan and Crispin (2011) assume that the degradation path is nonlinear and take into account the unit to unit variability. Freitas et al. (2010) and Shahraki et al. (2017) highlight that for many degradation processes in practice, the models are nonlinear in nature with more than one random effect. Using a simple linear random effects model is not ideal in such cases, as it does not have the ability to adequately describe the actual degradation path. A nonlinear degradation model is the perfect candidate for such processes.

Several efforts have also been made to apply stochastic process models namely the Wiener process models, Gamma and inverse Gaussian processes in many degradation studies. The common argument presented is that degradation is usually a complex phenomenon, that is better described and captured adequately by stochastic process models instead of the general degradation path models. Peng et al. (2014) use the GaAs laser data and apply the inverse Gaussian process models. Doksum and Hoyland (1992) consider a variable stress accelerated life test in which the degradation characteristic is described by the Wiener process model with drift and the failure time distribution is inverse Gaussian. It is demonstrated that the Wiener process with drift is a very flexible model which captures very well an accumulated decay process. Lim and Yum (2011) also take into account the Wiener process in modelling an accelerated degradation test, with the goal of designing an optimal test plan using a constant stress loading method. Whitmore (1995) discusses the estimation of degradation by a Wiener diffusion process in the presence of measurement errors. The Wiener process presents a difficult challenge in practice, in that it requires several assumptions, such as a linear degradation rate and a single source of variation (Pan and Crispin, 2011).

Additionally, Tseng, Balakrishnan and Tsai (2009) work on an accelerated degradation test with a path described by the Gamma process. Lawless and Crowder (2004) also present the Gamma pro-

cess, focusing on its covariates and random effects with regard to crack growth degradation data. Advantages of using the Gamma process are highlighted, for example the random effects' ability to incorporate the heterogeneity across products, its monotonicity and independent increments property. Park and Padgett (2006) develop stochastic process models which take into account several accelerating variables. This study points out that in most cases the exact degradation value at the first passage time is not observable. As a remedy to this issue, Park and Padgett (2006) suggest the interpolation of the first passage times or the adjustment of the failure threshold. A stochastic process model is beneficial in how it takes into account the correlation of several performance measures for a product, but the manner in which it handles the nonlinear degradation functions and random effects is very complex, whereas the general degradation path model is easy to use and straightforward (Pan and Crispin, 2011).

3.4 Estimation Methods

In estimating degradation model parameters, one can consider parametric or non-parametric methods. Parametric approaches include the MLE and MRR methods, whereas the non-parametric approaches include methods such as the kernel density estimation (KDE). Studies such as of Chiao and Hamada (2001), Dakhn et al. (2017), Doksum and Hoyland (1992), Freitas et al. (2010), Lu and Meeker (1993), Lu et al. (1996), Pan and Crispin (2011) and Whitmore (1995) make use of MLE. Chiao and Hamada (2001) propose a two stage maximum likelihood method just as Lu et al. (1996) did to analyse the degradation path of a fluorescent lamp's luminosity.

Freitas et al. (2010) also apply the two stage MLE method in analysing train wheel degradation data. Before performing the two stage method, the one stage MLE approach is implemented on the data. This entails forming the appropriate likelihood function and then obtaining estimates from it. The random effects degradation model parameter estimates of the sample degradation paths are obtained using the least squares estimation method in the first stage. Lu and Meeker (1993) propose a two stage method to estimate a mixed effect path model parameters in a case study involving fatigue growth data. It is shown that both approaches for MLE have drawbacks in degradation analysis even though there are certain advantages to them. For the one stage method, it is shown that the likelihood function may not have a closed form, thereby making it necessary to use numerical approximation methods, whereas the second stage depends on asymptotic normality properties and it is generally

computationally intensive (Chiao and Hamada, 2001; Freitas et al., 2010).

Due to the fact that MLE is a parametric procedure, this implies that it can only be employed if the stochastic component (random effect parameter β_i) of the degradation model has valid assumptions on it, otherwise non-parametric methods such as KDE are a better option (Dakhn et al., 2017; Ebrahem et al., 2009). KDE is one of the best non-parametric methods to consider when there is lack of prior information pertaining to the underlying distribution of the random effects in the field of degradation data (Ebrahem et al., 2009). Ebrahem et al. (2009) use the GaAs laser example to compare the performance of KDE with that of the MLE and ordinary least squares methods in estimating degradation model parameters.

On the other hand, some authors are of the opinion that the Bayesian approach offers a practical advantage over MLE in that it allows for subjective information to be included in the degradation analysis (Hamada (2005) and Peng, Li, Yang, Huang and Zuo, 2014). This is very beneficial when observations are very few and Freitas et al. (2010) allude to these advantages too in a study whereby the Bayesian method is compared to four classical methods in the analysis of train wheel degradation data. The Bayesian method is found to handle nonlinear models with more than one random parameter and mixed parameter models much better than classical approaches.

The Bayesian approach also provides a natural approach to degradation data in how it combines prior information (about the unknown model parameters) and information contained in the actual data about the same parameters forming a pdf called the posterior (Freitas et al., 2010; Hamada, 2005; Peng et al., 2014). Nelson (1990) notes that there is a certain level of difficulty in the specification of the prior distribution for the Bayesian method. Efron (2001)'s work also criticises the Bayesian method by pointing out that it is a statistical method that does not interpret data fairly (with regard to issues such as unbiasedness, confidence intervals and significance levels) due to its subjective nature.

Other popular statistical methods for degradation data analysis which fall under the category of "classic" inference methods with MLE, include the approximate, analytical and numerical methods. These classic methods are extensively discussed by Freitas et al. (2010) and Oliveira and Colosimo (2004). Freitas et al. (2010) proposes modelling and analysing degradation data using the approximate, analytical and numerical methods on train wheel degradation data. Point estimates (such as MTTF, median distance, specified percentiles, reliability at certain chosen distances) and confidence intervals are obtained using all the three methods. In the end, it is proven that both the approximation

and numerical estimates derived from degradation measurements are more precise than those of traditional failure time data.

Oliveira and Colosimo (2004) carry out a similar study to that of Freitas et al. (2010) with the exception that the degradation process in this case is pertaining to an automobile's tyre. The approximation and analytical methods were found to produce similar results and it was also established that the numerical method has the best results since it has narrower confidence intervals, implying better accuracy and precision. It was possible to apply all three methods to the tyre wear degradation data, since the degradation path is simple in that it has one random effect. If the degradation path had been nonlinear, only the numerical method would have been appropriate in this case. Freitas et al. (2010) deliberately leave out the analytical method since the view is that for simple models, the approximation and analytical methods tend to produce similar results. The results from these methods are also compared to those of traditional FTA and the conclusion was that CIs obtained from the two classic methods are narrower and have less bias, implying better precision. The numerical method was found to be most affected by the misspecification of the random parameter distribution (Freitas et al., 2010).

Chapter 4

Methodology

4.1 Motivating Data Example

A GaAs laser (for telecommunications systems) example from Meeker and Escobar (1998) is used in this research study to demonstrate the practical advantages of degradation data analysis through a simulation procedure. This type of laser has a life time of 200 000 hours (slightly over 20 years) when operated at the normal use temperature of 20°C. It has a built-in feedback circuit which ensures that constant light output is maintained. As the laser ages, it needs more electric current to maintain the required constant light output. The laser is regarded to have failed the first time a 10% increase in current is required to maintain the constant light output. The engineers estimated from experience that when tested at an accelerated temperature of 80°C to obtain failure data quickly, the time to failure would be accelerated by a factor of approximately 40. Accordingly, a lifetime of 200 000/40=5000 hours was desired. A sample of 15 lasers are tested at 80°C and degradation measurements are taken every 250 hours within a time frame of 4000 hours. The real data set simply has two variables under consideration (*percent increase in operating current* and *time (in hours)*) for a sample of 15 GaAs lasers. There are no missing values in the data.

4.2 Simulation Procedure

A simulation study motivated by the GaAs laser data example is conducted to demonstrate the advantage of assessing product reliability from degradation data. Degradation paths for the laser data are linear with an initial amount of degradation equal to zero. Accordingly, the linear random effects

4.2 Simulation Procedure

model in Equation (2.2) is assumed with $\mathcal{D}_f$ set at 10% increase in operating current. Additionally, Meeker et al. (2001) deemed the lognormal distribution to be the appropriate failure time model from experience with the lasers. Hence, the simulation is generated according to the following model:

$$y_{ij} = \beta_i t_{ij} + \varepsilon_{ij}$$

$$\varepsilon_{ij} \sim N(0; \sigma_\varepsilon^2)$$

$$\beta_i \sim \text{Lognormal}(\mu'_\beta; \sigma'_\beta)$$

for $i = 1, \cdots, 15$ ($n = 15$), $j = 1, \cdots, 16$ and $t_{i1} = 250, t_{i2} = 500, \cdots, t_{i16} = 4000$ with a censoring time of 5000 hours, which is equivalent to the desired lifetime for the lasers at a temperature of 80°C. Two simulated data sets considered are namely:

- Simulated GaAs laser degradation paths at 4000 hours extrapolated to failure or 5000 hours as is the case with the actual data set
- Simulated GaAs laser degradation paths at 2500 hours extrapolated to failure or 5000 hours

The shorter test is essentially a subset of the longer test and the 2500-hour data is selected to ensure that standard or traditional failure time analysis does not apply since no failures are recorded at 2500 hours. The simulated degradation data for the 15 GaAs lasers are then used to assess their reliability as follows:

1. If there are failures for the 4000-hour degradation data, a failure time analysis is carried out.
2. For the i^{th} laser, the model in Equation (2.1) is used to determine the least squares estimate of β_i based on the simulated degradation data $\Big(t_{i1}, y_{i1}\Big), \Big(t_{i2}, y_{i2}\Big), \cdots, \Big(t_{im_i}, y_{im_i}\Big)$.
3. For lasers that have not failed by 4000 hours, pseudo-failure times $\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_n$ are established by solving the Equation $\mathcal{D}\Big(t_i, \hat{\beta}_i\Big) = \mathcal{D}_f$.
4. Perform degradation analysis to a combination of observed failure times, pseudo-failure times and suspensions and compare results to failure time data analysis.
5. Repeat for the 2500-hour data set and compare results to the 4000-hour data set.
6. Carry out a FTA on a data set where all GaAs lasers are allowed to fail and compare the results to the 4000-hour degradation data.

4.2 Simulation Procedure

If the results of the analysis of the 4000-hour and 2500-hour test data are comparable, then the perceived advantage of degradation data analysis that conclusions are arrived at earlier would have been demonstrated. In particular, the goal is to estimate the 5000-hour failure probability from 2500-hour and 4000-hour degradation data. Other reliability metrics which will be derived include mean time to failure, reliable life, BX% life and failure rate. The uncertainty of these reliability metrics will be quantified using confidence bounds, bias and standard error.

An application to the real data set similar to the approach adopted by Meeker et al. (2001) will also be carried out. Meeker et al. (2001) admit that the MLE method does not seem to provide a good fit to the GaAs laser data and this is supported by their lognormal probability plot. This research study seeks to employ both the median rank regression method and MLE in order to find out if the results are comparable.

Chapter 5

Data Analysis and Results

5.1 Introduction

Data analysis and results for the simulated data set and the real GaAs laser data are presented in this chapter. Specifically, standard failure time analysis applied to the simulated 4000-hour degradation data is presented in Appendix C. These data show three failures (products P5, P8 and P12) and 12 unfailed units. Results from the failure time analysis are compared to the 4000-hour degradation data analysis which entails generating pseudo-failure times for the surviving products by extrapolating their degradation paths to 5000 hours and including them in the analysis. An extension of the analysis is achieved by running the simulations beyond 5000 hours to allow all products to "fail". Traditional failure time analysis based on all products failing is then compared to the 4000-hour degradation data analysis. Benefits of assessing product reliability from degradation data are demonstrated by analysing the degradation data available after only 2500 hours and comparing the results to those from the analysis of the 4000-hour degradation data. It is important to note that standard failure time analysis fails for the 2500-hour degradation data since there are no product failures. The statistical performance of two parameter estimation methods (that is, MRR and MLE) is also compared in analysing the simulated GaAs laser degradation data.

5.2 Simulation of the GaAs Laser Data

Simulation is implemented in R, an open source software by RStudio Team (2015) (see Appendix A for the R simulation code) according to the model in Equation (2.2). The selection of the indices for the simulation is as follows:

1. The sample size is $n = 15$, corresponding to 15 GaAs lasers as in the real data set.
2. The critical degradation threshold $\mathcal{D}_f$ is set to 10, corresponding to the failure definition of 10% increase in operating current for the lasers.
3. $\beta_i \sim$ lognormal$(-6.21334129; 0.20405874)$. These parameter values are obtained by fitting the lognormal distribution to the $n = 15$ rates of degradation (slopes) for the lasers (shown in `Appendix E`) derived from the regression analysis of the real degradation data in `Appendix F`.
4. $\varepsilon_{ij} \sim N(0; 1.05)$. The error variance is selected through a visual inspection aimed at approximating the linearity assumption while ensuring that the degradation data are monotone.
5. $t_{i1} = 250, t_{i2} = 500, \cdots, t_{i16} = 4000$ are the fixed times at which degradation measures are simulated.

As a result, a 16 by 15 matrix representing the simulated laser degradation data in `Appendix C` is generated and Figure 5.1 gives a plot of these data (see R code which generated the plot in `Appendix B`). This particular data set also has 3 observed product failures (similar to the real data set - see `Appendix F`) using the 10% current increase failure definition. The three failure times are at 3319 hours, 3764 hours, 3870 hours and 12 unfailed units at 4000 hours (see Table 5.1). The degradation paths of the 15 GaAs lasers are approximately linear. Therefore, there is no need for transforming the data.

5.3 Simulated 4000-hour Data Analysis

5.3.1 Failure Time Analysis

The 15 GaAs lasers are ID coded as P1, P2, P3, P4,$\cdots$, P15 as can be seen in Table 5.1 which indicates that at 4000 hours, three products have failed, namely P5, P8 and P12 and the other 12 lasers have not failed. The observed failure times are 3319 hours, 3764 hours and 3870 hours for P8, P5 and P12 respectively. It is possible to implement FTA in this particular instance due to the presence of observed failure times.

5.3 Simulated 4000-hour Data Analysis

Table 5.1: *Failure times for the simulated 4000-hour data*

GaAs Laser ID	State (F or S)	Time to F or S (hr)
P8	F	3319
P5	F	3764
P12	F	3870
P1	S	4000
⋮	⋮	⋮
P15	S	4000

The column labelled State shows either a failure time denoted by F or a suspension time denoted by S. Figure 5.1 gives a plot of the 4000-hour simulated degradation data.

5.3.1.1 Graphical Methods

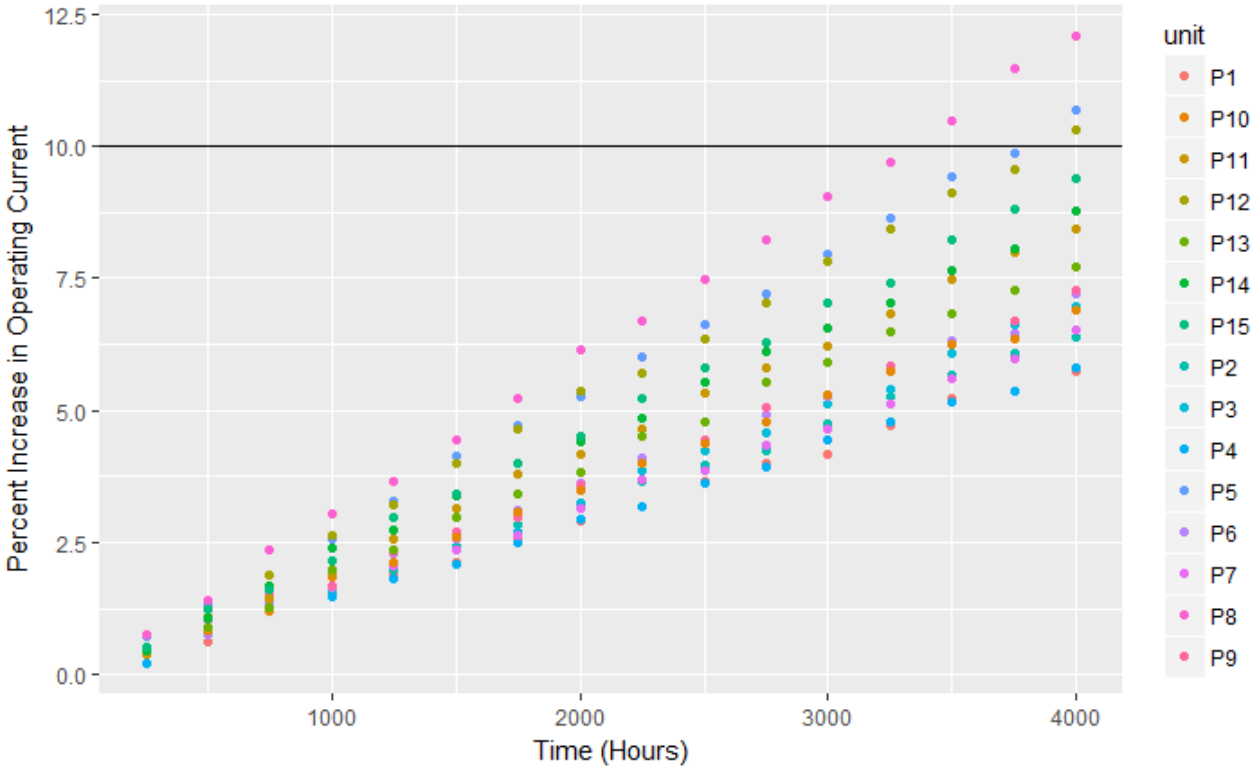


Figure 5.1: *Percent Increase In Operating Current vs. Time (Simulated 4000 hour degradation data)*

5.3 Simulated 4000-hour Data Analysis

These data were simulated using the R code in `Appendix A` and are presented in `Appendix C`.

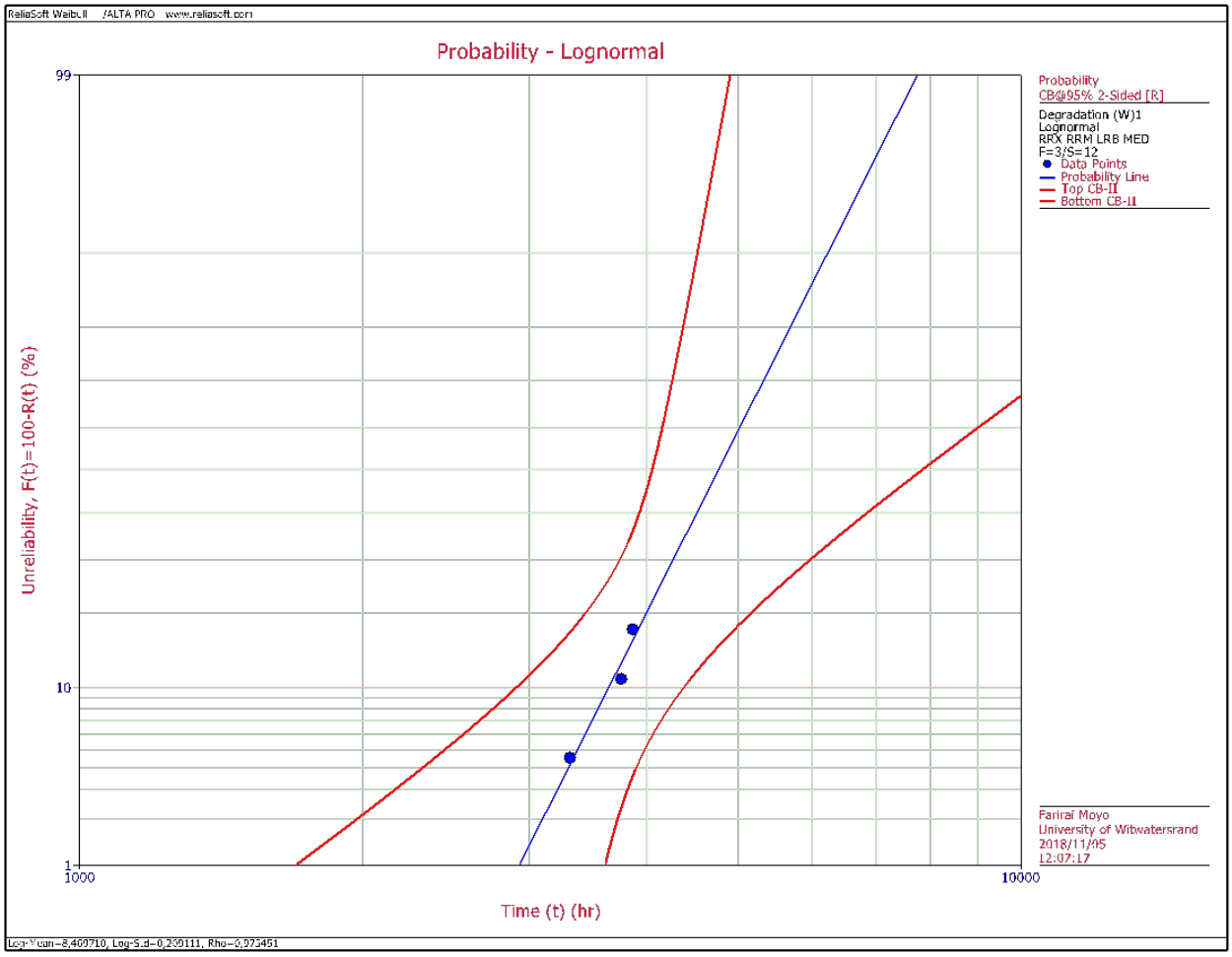


Figure 5.2: *Lognormal Probability Plot (Simulated 4000-hour FTA data)*

The Weibull ++ software is used for the analysis and the lognormal distribution was found to best fit the simulated 4000-hour degradation data. Presented in Figure 5.2 is the lognormal probability plot for these failure time data. The straight line represents the least squares estimate of $F(t)$ based on the assumed lognormal distribution taking into account the 3 observed failures and the 12 suspensions. The least squares line in Figure 5.2 seems to fit the plotted points well regardless of there being a high proportion of suspensions. Approximate two sided 95% likelihood ratio confidence bounds are included in the plot.

5.3 Simulated 4000-hour Data Analysis

5.3.1.2 Numerical Methods

Goodness of fits tests are performed to find out which specific distribution(s) provide the best fit for the simulated 4000-hour data set. Three common distributions for modelling lifetimes are considered, namely the one-parameter exponential, two-parameter Weibull and lognormal. A brief description of the three goodness fits of tests considered in this study is as follows:

- The Kolmogorov-Smirnov (K-S) test is based on the maximum difference between an empirical cumulative distribution function $F(t)$ and a fitted cdf $\hat{F}(t)$ in order to assess fit (Massey , 1951). The values labelled the K-S test are p-values.
- The normalized correlation coefficient test measures the extent to which plotted values fit on a straight line.
- The likelihood ratio test calculates the value of the log-likelihood function based on specific parameters given on a distribution.

The following results are obtained:

Table 5.2: *Goodness of fit tests for 4000-hour data (FTA)*

Criteria	1P Exponential	Lognormal	2P Weibull
Kolmogorov-Smirnov test	9.99E-11	9.99E-11	2.98E-07
Normalized correlation coefficient (ρ) test	4.61	1.27	1.14
Likelihood ratio test	-32.75	-27.99	-28.05
DESV	250	150	210
Ranking	3	1	2

With respect to the K-S test, the 1P exponential and lognormal distributions have the same ranking since they have the same values and the 2P Weibull is ranked last based on that criteria because it has the highest value. With regard to the normalized correlation coefficient test, the 2P Weibull has the lowest value and is ranked the best in that respect. As for the likelihood ratio test, the lognormal distribution is the best since it has the highest value. Ultimately, the final ranking is determined by summing the weighted values of the three criteria, giving the overall weighted decision variable value denoted by DESV (HBM United Kindom Limited , 2017). For these data, the lognormal is ranked the best based on it having the smallest DESV value. The obtained results are consistent with those of

Meeker et al. (2001) who argue its suitability based on past experience with the GaAs lasers. Accordingly, the lognormal distribution is chosen to perform data analysis for the 4000-hour failure time data.

Table 5.3 shows the lognormal distribution parameter estimates for the 4000-hour failure-time data.

Table 5.3: *Lognormal parameter estimates for the 4000-hour failure-time data*

4000-hour failure-time data		
Lognormal	Log-Mean (hr)	8.47
	Log (σ)	0.21
	ρ	0.98
	Fail/Susp	3/12

where σ is the standard deviation and ρ is the correlation coefficient.

5.3.2 4000-hour Degradation Data Analysis

It is clear from the plot in Figure 5.1 that the degradation path belonging to one of the GaAs lasers is about to reach the critical threshold level (the horizontal line at 10% which represents failure). In order to extract more useful reliability information pertaining to this particular laser and other unfailed lasers, the process of extrapolation is considered up to failure or to 5000 hours. Meeker et al. (2001) state that extrapolation must be carried out for a reasonable time as products stop functioning at some point. Failure times well beyond are not statistically useful. The laser degradation paths may have a huge probability of not being reasonably linear if the degree of extrapolation is too much.

After extrapolation is implemented to failure or 5000 hours, three more lasers (P15, P14 and P11) fail. The resulting pseudo-failure times for the respective lasers are 4308 hours, 4564 hours and 4749 hours and are displayed in Table 5.4 showing 3 observed failures, 3 pseudo-failures and 9 suspensions.

5.3 Simulated 4000-hour Data Analysis

Table 5.4: *Failure times for the 4000-hour degradation data*

GaAs Laser ID	State F or S	Time to F or S (hr)
P8	F	3319
P5	F	3764
P12	F	3870
P15	F	4308
P14	F	4564
P11	F	4749
P1	S	5000
P2	S	5000
P3	S	5000
P4	S	5000
P6	S	5000
P7	S	5000
P9	S	5000
P10	S	5000
P13	S	5000

Combining these pseudo-failure times with the observed failure times in the analysis constitutes degradation data analysis. As with failure time analysis, the lognormal distribution was found to be the best fitting distribution for the 4000-hour degradation data. The lognormal probability plot in Figure 5.3 for the 4000-hour degradation data is intended to assess fit and it provides a good fit to the data.

5.3 Simulated 4000-hour Data Analysis

5.3.2.1 Graphical Methods

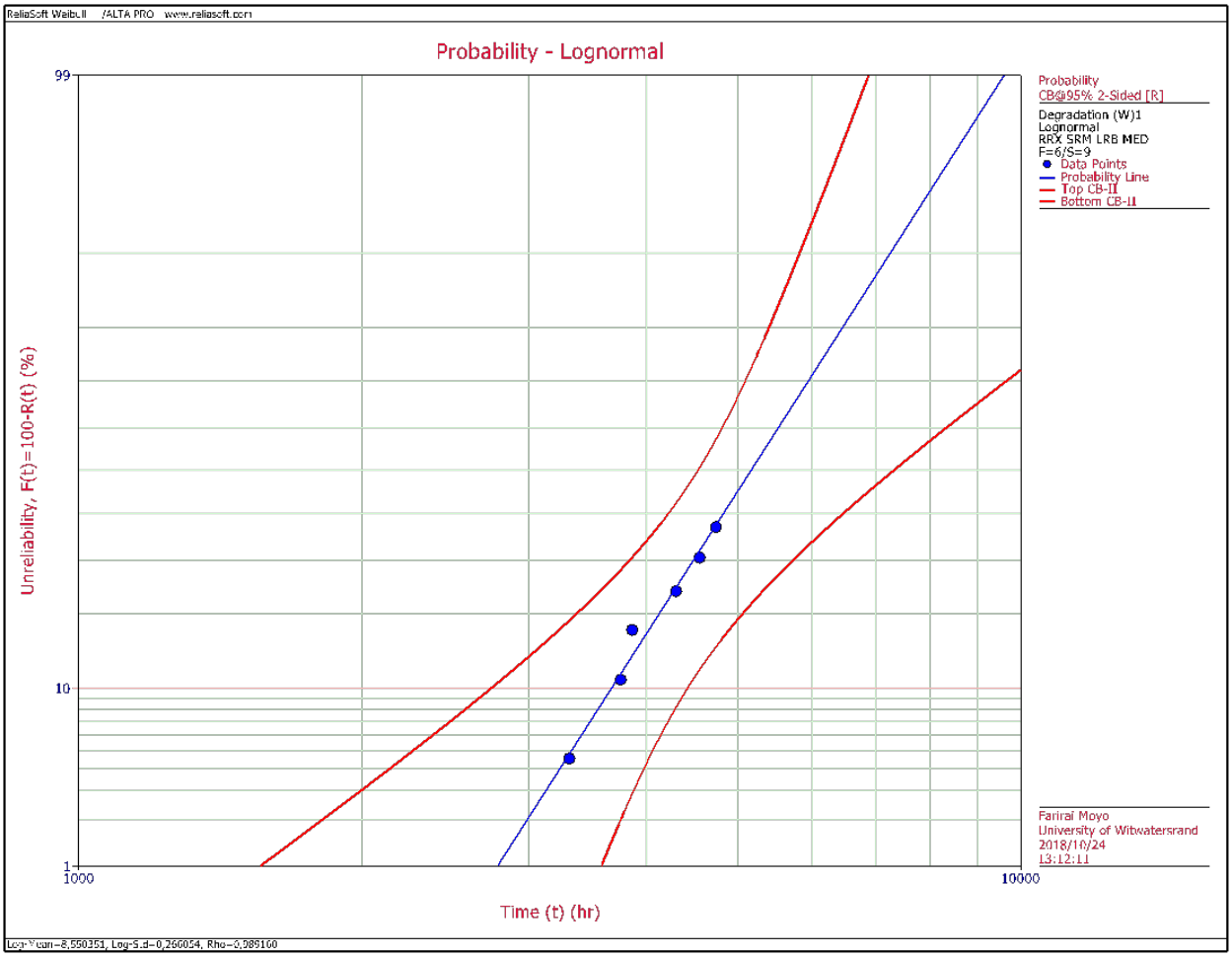


Figure 5.3: *Lognormal Probability Plot (Simulated 4000-hour degradation data)*

5.3.2.2 Numerical Methods

In the same vein with the numerical methods under failure time analysis, the lognormal distribution is ranked the best based on the smallest DESV value (see Table 5.5) for the 4000-hour degradation data. The best ranked distribution is thus chosen to perform data analysis.

Table 5.5: *Goodness of fit tests for the 4000-hour degradation data*

Criteria	1P Exponential	Lognormal	2P Weibull
Kolmogorov-Smirnov test	1.35	9.99E-11	9.99E-11
Normalized correlation coefficient (ρ) test	8.45	1.24	1.82
Likelihood ratio test	-62.21	-55.90	-56.62
DESV	300	120	200
Ranking	3	1	2

Parameter estimation for the lognormal distribution is presented in Table 5.6.

Table 5.6: *Lognormal distribution parameters (4000-hour degradation data)*

	Parameter Estimates
Log-Mean (hr)	8.55
Log (σ)	0.27
ρ	0.99
Fail/Susp	6/9

where σ is the standard deviation and ρ is the correlation coefficient.

5.3.3 Comparison of 4000-hour Failure Time and Degradation Data Analyses

The comparison of the 4000-hour FTA and 4000-hour DA for the specified reliability metrics is based on the 95% two-sided CI lengths as indicated in Table 5.7.

5.3 Simulated 4000-hour Data Analysis

Table 5.7: *Results based on the lognormal distribution (4000-hour failure-time data vs. 4000-hour degradation data)*

	Analysis		Point estimate	95% C.I	Length
Prob. of Failure	DA	Q(t=5000)	0.45	(0.19 , 0.67)	0.48
	FTA	Q(t=5000)	0.59	(0.18 , 0.99)	0.81
Mean time to failure	DA	MTTF1	5355	(4690 , 7834)	3144
(hr)	FTA	MTTF2	4874	(4099 , 8306)	4207
Reliable Life(hr)	DA	t(R=0.85)	3923	(3088 , 4746)	1658
	FTA	t(R=0.85)	3839	(3219 , 4772)	1553
BX% Life(hr)	DA	B10% Life	3675	(2739 , 4426)	1687
	FTA	B10% Life	3647	(2908 , 4380)	1472
Failure Rate	DA	FR(t=5000)	0.00432	(0.000141 , 0.001139)	0.000998
(no. of failures/hr)	FTA	FR(t=5000)	0.000907	(0.000131 , 0.007793)	0.007662

The results in Table 5.7 are based on the MRR method at this stage. Overall, the CIs derived from the 4000-hour DA are narrower than those of the 4000-hour FTA with the exception of two instances, reliable life at t(R=0.85) and BX% life at 10%. Narrower CI lengths imply better statistical precision and accuracy. Results based on FTA are depending on only three observed failures, whereas DA incorporates extra information in form of pseudo-failure times, thereby leading to narrower CIs. The advantage of degradation analysis over FTA in terms of incorporating extra information in form of pseudo-failure times is also reflected in the parameter bounds for the model estimates in Table 5.8.

Table 5.8: *Parameter bounds based on the lognormal distribution (4000-hour DA vs. 4000-hour FTA)*

Analysis	Parameter	Point estimate	95% C.I	Length
DA	Log-Mean (hr)	8.55	(8.42 , 8.90)	0.48
	Log-Standard Deviation	0.27	(0.15 , 0.59)	0.44
FTA	Log-Mean (hr)	8.47	(8.31 , 8.97)	0.66
	Log-Standard Deviation	0.21	(0.07 , 0.60)	0.53

That is, CI lengths for the former are slightly narrower than for the latter.

5.4 Failure Time Analysis when all lasers are run to failure

A further analysis where all lasers are run to failure is also carried out and the results are compared to the 4000-hour degradation data analysis. The rationale behind running all lasers to failure is to establish the trade-off between testing time and the level of statistical accuracy. Running all the lasers to failure is expensive in terms of both testing time and cost in terms of losing the lasers themselves. Its advantage is in the form of more failure time information. On the other hand, DA has less observed failure times but is cheaper in terms of testing time and resources since lasers do not necessarily need to be run to failure. Assuming the results are comparable, then an advantage of DA would have been demonstrated. The failure time of the lasers after running them all to failure are given in Table 5.9.

Table 5.9: *Failure times (when all lasers are run to failure)*

GaAs Laser ID	State F or S	FTA Time to F or S (hr)
P8	F	3319
P5	F	3764
P12	F	3870
P15	F	4308
P14	F	4564
P11	F	4749
P13	F	5113
P9	F	5576
P6	F	5630
P10	F	5735
P3	F	5843
P2	F	6239
P7	F	6280
P1	F	6857
P4	F	6881

Continuing with the lognormal distribution as the assumed model for the lasers (Meeker et al., 2001), the lognormal probability plot is given in Figure 5.4.

5.4 Failure Time Analysis when all lasers are run to failure

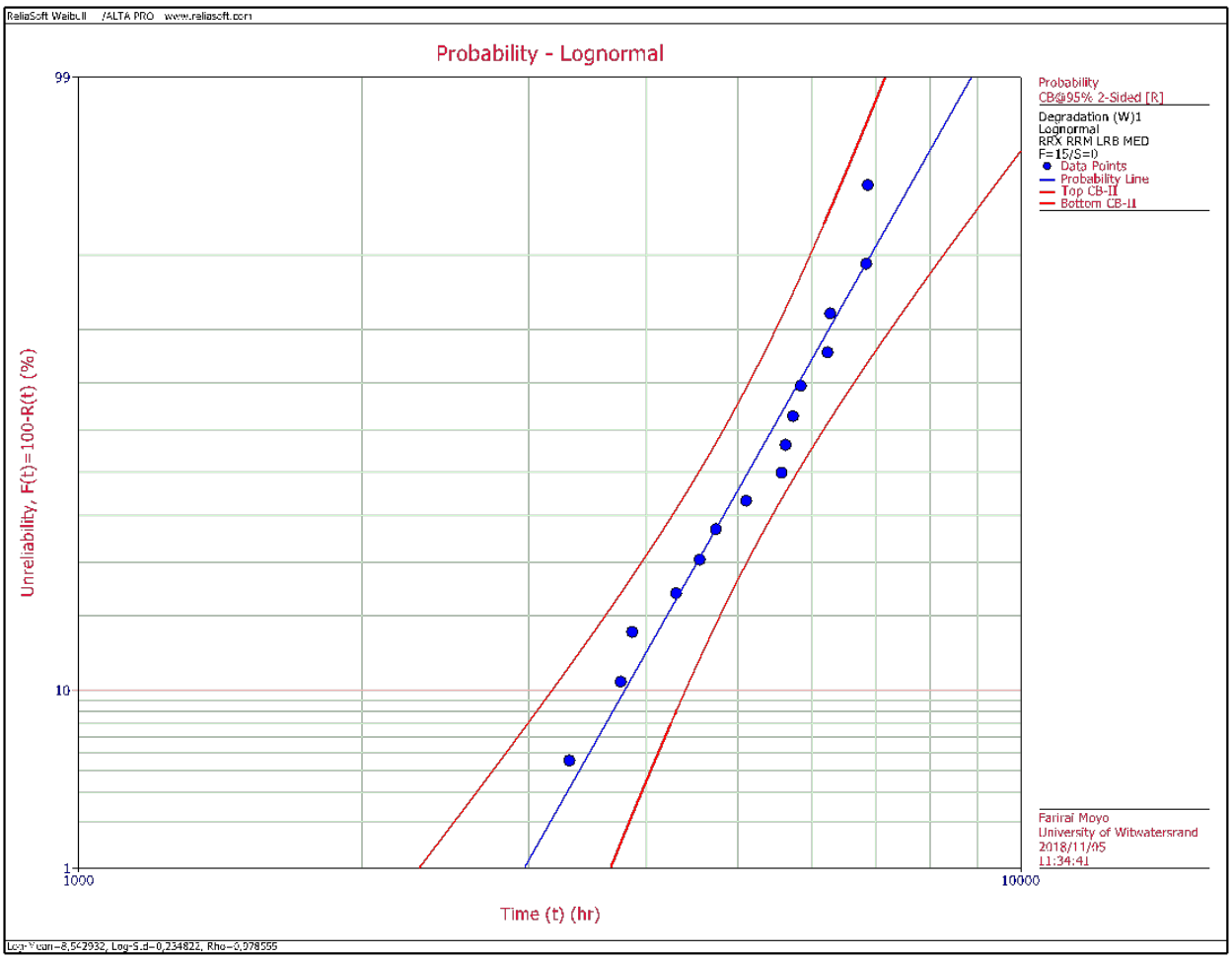


Figure 5.4: *Lognormal Probability Plot (when all lasers are run to failure)*

The lognormal probability plot provides a fairly good fit to the data, based on the least squares straight line. Admittedly, there are some points that affect the fit. This is because, although the expected lifetime for these lasers is 5000 hours, a couple of them have failure times close to 7000 hours when they are run to failure. Parameter estimates for the assumed lognormal distribution are presented in Table 5.10.

5.4 Failure Time Analysis when all lasers are run to failure

Table 5.10: *Lognormal distribution parameters (when all lasers are run to failure)*

	Parameter Estimates
Log-Mean (hr)	8.54
Log (σ)	0.23
ρ	0.98
Fail/Susp	15/0

5.4.1 Comparison with 4000-hour Degradation Data Analysis

A comparison of the 4000-hour DA and FTA when all lasers are run to failure is given in Table 5.11.

Table 5.11: *A comparison between 4000-hour degradation data analysis and a FTA when all lasers are run to failure (results based on the lognormal distribution and the MRR method)*

	Test		Point estimate	95% C.I	Length
Prob. of Failure	FTA	Q(t=5000)	0.46	(0.26 , 0.66)	0.40
	DA	Q(t=5000)	0.45	(0.19 , 0.67)	0.48
Mean time to failure	FTA	MTTF-FTA	5274	(4681 , 5961)	1280
(hr)	DA	MTTF-DA	5355	(4690 , 7834)	3144
Reliable Life(hr)	FTA	t(R=0.85)	4022	(3421 , 4613)	1192
	DA	t(R=0.85)	3923	(3088 , 4746)	1658
BX% Life(hr)	FTA	B10% Life	3797	(3179 , 4405)	1226
	DA	B10% Life	3675	(2739 , 4426)	1687
Failure Rate	FTA	FR(t=5000)	0.000621	(0.000367 , 0.001101)	0.000734
(no. of failures/hr)	DA	FR(t=5000)	0.00541	(0.000153 , 0.001264)	0.001111

All the confidence intervals derived from the 4000-hour degradation data are a bit wider than those from the traditional failure time analysis in which all GaAs lasers are run to failure, as presented in Table 5.12. However, the point estimates and the CI lengths of the two approaches are comparable with respect to all the reliability metrics. These results are consistent with what Genschel and Meeker (2010) and Meeker et al. (2001) established with regard to this particular comparison. Traditional FTA generally has narrower CIs because its results are based on a greater number of failures compared to

those of DA. When there are few or no failures observed, this widens the CIs. This is also reflected in the parameter bounds depending on the analysis as indicated in Table 5.12.

Table 5.12: *Parameter bounds based on the lognormal distribution (4000-hour DA vs. FTA (when all lasers are run to failure))*

Analysis	Parameter	Point estimate	95% C.I	Length
DA	Log-Mean (hr)	8.55	(8.42 , 8.90)	0.48
	Log-Standard Deviation	0.27	(0.15 , 0.59)	0.44
FTA	Log-Mean (hr)	8.54	(8.42 , 8.66)	0.24
	Log-Standard Deviation	0.23	(0.16 , 0.33)	0.17

However, it cannot be ignored that the DA option is much advantageous in that plenty of testing time is saved and a whole lot of unnecessary expenses are avoided in assessing product reliability since not all products are required to have failed.

5.5 Simulated 2500-hour Degradation Data Analysis

Standard failure time analysis is not applicable to the 2500-hour degradation data due to the fact that there are no observed failures in that particular test duration.This leaves degradation data analysis as the only applicable approach, since it has the ability to also assess product reliability when there are no observed failures. It does this by relying on the extra information contained in the observed degradation measures about the product's failure time. Valid results can only be obtained provided there is a strong correlation between the underlying degradation process and the product's failure. Other prerequisites for the degradation path model have been discussed in previous chapters.

5.5.1 Graphical Methods

For the 2500-hour degradation data, none of the 15 lasers have failed (reached the failure threshold level) at 2500 hours. The corresponding 15 simple linear regression lines representing the degradation paths for the lasers are shown in Figure 5.5. These lines appear to satisfactorily fit the plotted points in a linear fashion for all the lasers.

5.5 Simulated 2500-hour Degradation Data Analysis

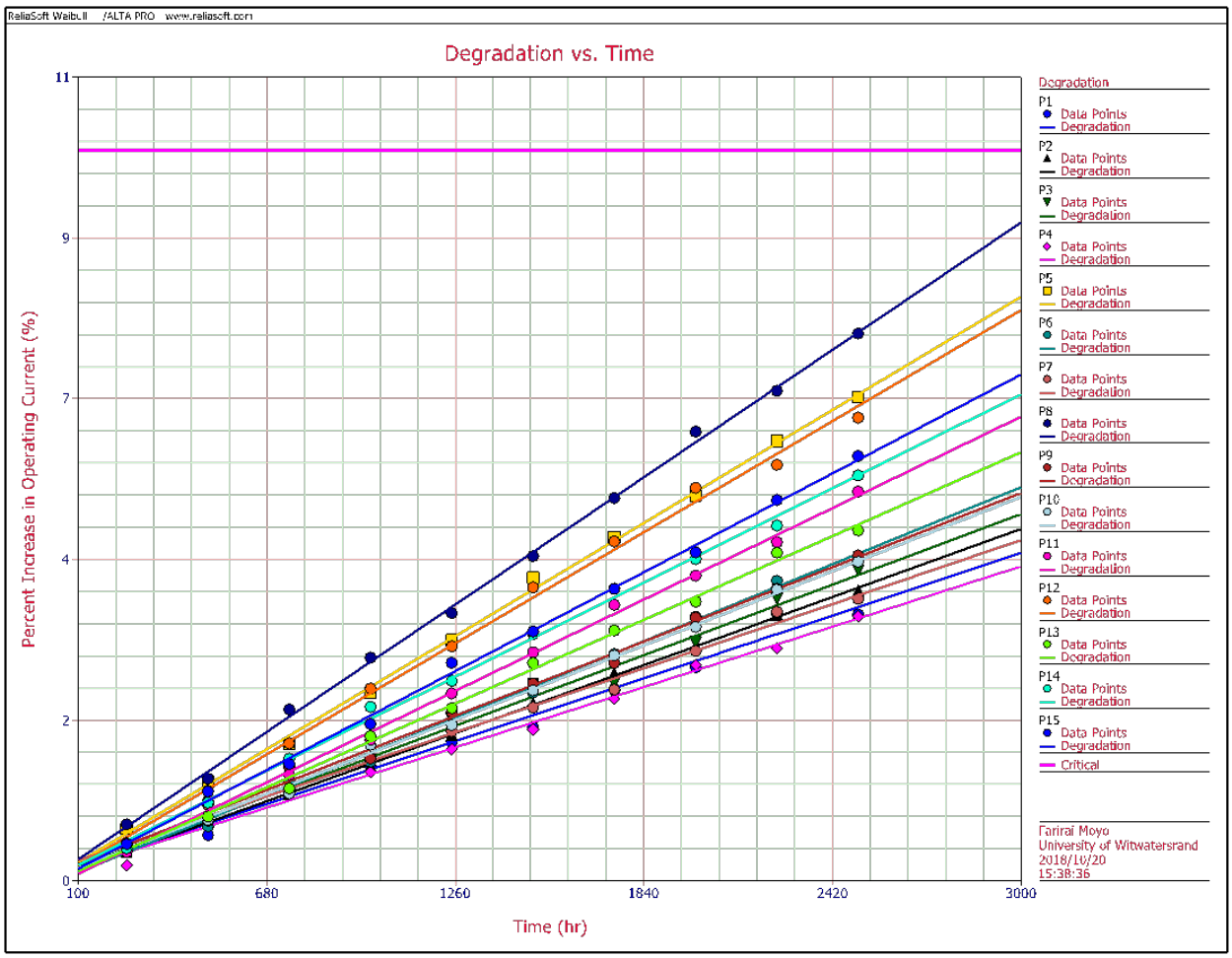


Figure 5.5: *Degradation vs. Time Plot (Simulated 2500-hour degradation data)*

A total of six pseudo-failure times are generated when the 2500-hour degradation data are extrapolated to failure or to 5000 hours. These data are shown in Table 5.13.

5.5 Simulated 2500-hour Degradation Data Analysis

Table 5.13: *Failure times for the 4000-hour and 2500-hour degradation tests*

		4000-hour test	2500-hour test
GaAs Laser ID	State F or S	Time to F or S (hr)	Time to F or S (hr)
P8	F	3319	3330
P5	F	3764	3756
P12	F	3870	3858
P15	F	4308	4360
P14	F	4564	4506
P11	F	4749	4785
P1	S	5000	5000
P2	S	5000	5000
P3	S	5000	5000
P4	S	5000	5000
P6	S	5000	5000
P7	S	5000	5000
P9	S	5000	5000
P10	S	5000	5000
P13	S	5000	5000

The lognormal probability plot for the simulated 2500-hour degradation data is given in Figure 5.6.

5.5 Simulated 2500-hour Degradation Data Analysis

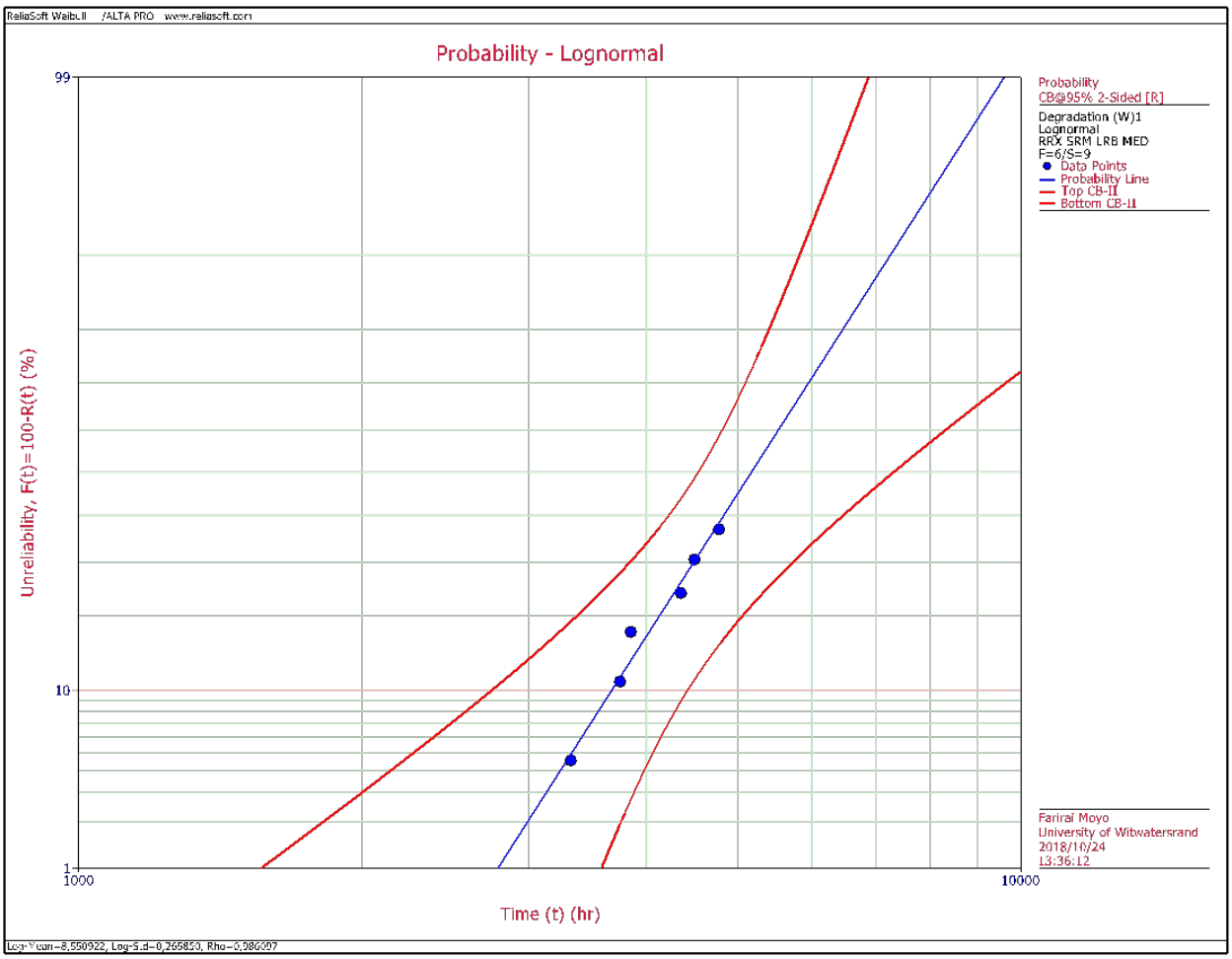


Figure 5.6: *Lognormal Probability Plot (Simulated 2500-hour degradation data)*

It follows from Figure 5.6 that the lognormal distribution provides a good fit to the data based on the least squares straight line.

5.5.2 Numerical Methods

From Table 5.14, the lognormal distribution is the best ranked among all the distributions under consideration based on the smallest DESV value.

Table 5.14: *Goodness of fit tests for 2500-hour degradation data*

Criteria	1P Exponential	Lognormal	2P Weibull
Kolmogorov-Smirnov test	1.39	9.99E-11	9.99E-11
Normalized correlation coefficient (ρ)	8.50	1.44	2.03
Likelihood ratio test	-62.22	-55.89	-56.60
DESV	300	100	200
Ranking	3	1	2

The lognormal parameter estimates for the 2500-hour degradation data are presented in Table 5.15.

Table 5.15: *Lognormal parameter estimates for the 2500-hour degradation data*

2500-hour data (simulated)		
Lognormal	Log-Mean (hr)	8.55
	Log (σ)	0.27
	ρ	0.99
	Fail/Susp	6/9

where σ is the standard deviation and ρ is the correlation coefficient.

5.5.3 Comparison of the 2500-hour and 4000-hour Degradation Data Analyses

To demonstrate the practical advantage of using DA to assess product reliability, results of the 2500-hour and 4000-hour degradation data analyses are compared. In addition, an extension of the study by Meeker et al. (2001) is considered by incorporating both MLE and MRR methods in analysing the degradation data. The MRR and MLE methods are applied on both the 2500-hour and 4000-hour degradation data with the objective of comparing the estimated reliability metrics. If the results of the 2500-hour and 4000-hour degradation data analyses are comparable, then the implication is that with degradation analysis, conclusions are reached earlier. This will in turn allow an earlier release of highly reliable products.

In addition to quantifying the uncertainty associated with the estimates of the reliability metrics, standard errors and bias are also reported for precision and accuracy respectively. To compute the

bias, the true value for t_u and β must be known. The true value for the former is based on data obtained by running all lasers to failure as this gives the actual failure times, whilst that of β is from the simulation design. The estimates $\hat{t}_u$ of a specified percentile or some other reliability metrics are based on the results from the simulated degradation data sets in `Appendix G` and `Appendix H`. Recall that the main objective is to estimate the failure probability at 5000 hours. Hence for comparison purposes, bias and standard error values are established for probability of failure at 5000 hours and a few other metrics namely MTTF and failure rate at 5000 hours. Table 5.16 reports the confidence bounds and bias based on the two estimation methods.

5.5 Simulated 2500-hour Degradation Data Analysis

TEST 1 represents the 4000-hour DA and **TEST 2** represents the 2500-hour DA.

Table 5.16: *MRR and MLE results based on the lognormal distribution (4000-hour DA vs. 2500-hour DA)*

			MRR			
	Test		Point estimate	95% C.I	Length	|Bias|
Prob. of Failure	1	Q(t=5000)	0.450	(0.190 , 0.666)	0.476	0.090
	2	Q(t=5000)	0.449	(0.190 , 0.666)	0.476	0.088
Mean time to failure	1	MTTF1	5355	(4690 , 7834)	3144	38
(hr)	2	MTTF2	5358	(4692 , 7827)	3135	286
Failure Rate	1	FR(t=5000)	0.000541	(0.000153 , 0.001264)	0.001111	0.000478
(no. of failures/hr)	2	FR(t=5000)	0.000541	(0.000153 , 0.001265)	0.001112	0.000486
			MLE			
Prob. of Failure	1	Q(t=5000)	0.4148	(0.1922 , 0.6631)	0.4709	0.0810
	2	Q(t=5000)	0.4146	(0.1920 , 0.6627)	0.4707	0.0759
Mean time to failure	1	MTTF1	5479	(4697 , 7769)	3072	131
(hr)	2	MTTF2	5479	(4699 , 7762)	3064	17
Failure Rate	1	FR(t=5000)	0.000505	(0.000156 , 0.001251)	0.001095	0.000517
(no. of failures/hr)	2	FR(t=5000)	0.000506	(0.000156 , 0.001252)	0.001096	0.000508

From Table 5.16, CI lengths for the considered reliability metrics under the two estimation methods are largely comparable. In addition, the absolute bias values for the same metrics are very low and comparable for the two estimation methods with the exception of MTTF values. This is however due to the fact that the product under consideration has a very high lifetime.

Results obtained from the 2500-hour and 4000-hour degradation tests with respect to the parameter bounds are comparable to a very large extent as shown in Table 5.17.

5.5 Simulated 2500-hour Degradation Data Analysis

Table 5.17: *Parameter bounds based on the lognormal distribution (4000-hour DA vs. 2500-hour DA)*

Estimation Method	Test	Parameter	Point estimate	95% C.I	Length
MRR	1	Log-Mean (hr)	8.55	(8.42 , 8.90)	0.48
		Log-Standard Deviation	0.27	(0.15 , 0.59)	0.44
	2	Log-Mean (hr)	8.55	(8.42 , 8.90)	0.48
		Log-Standard Deviation	0.27	(0.15 , 0.59)	0.44
MLE	1	Log-Mean (hr)	8.57	(8.42 , 8.89)	0.47
		Log-Standard Deviation	0.26	(0.15 , 0.58)	0.43
	2	Log-Mean (hr)	8.57	(8.42 , 8.89)	0.47
		Log-Standard Deviation	0.26	(0.15 , 0.58)	0.43

This also provides motivation for one to opt for the 2500-hour degradation data test in assessing product reliability.

Since the results from the two degradation tests are comparable, this then confirms the advantage of degradation data with regard to the crucial aspect of time. In other words, degradation data available after the 2500-hour test allows for an earlier release of a reliable product or swifter redesign of an unreliable one. Although, the 2500-hour test data consists of just pseudo-failure times and does not have observed GaAs laser failures, it is possible to extract meaningful and important product reliability information from it. This strongly proves both the effectiveness and efficiency of DA.

The estimates of the MRR method almost have the same level of accuracy with those of MLE (see Table 5.16), based on the absolute bias values obtained (pertaining to estimates for probability of failure at 5000 hours, MTTF and failure rate at 5000 hours). However, estimates based on MLE, are more precise (due to the lower standard error values with respect to estimates for probability of failure at 5000 hours and MTTF) than those belonging to the MRR method (see `Appendix G` and `Appendix H`). The MLE method is generally considered to be a more powerful and robust method than several methods including MRR. In product reliability applications though, the MRR method is usually preferred mostly because it is based on the well known least squares method which is easy to grasp, use and interpret. In most cases, it also provides a good fit to degradation data.

However, a study which compares MRR and MLE based on the Weibull distribution by Genschel and Meeker (2010), to some extent suggests otherwise. This particular study argues that emphasis should not only be given to bias properties of an estimator but to variability properties as well. Results obtained in this research study are in agreement with this line of thought, since the MLE technique was found to offer better precision. The view is that the amount of variance in the MRR method dominates the bias such that when the overall accuracy of MLE is evaluated against that of MRR, the MLE is a better method in practice. It further goes on to mention that the MRR method has a habit of passing through data points even when it should not, consequently producing misleading results. The other argument presented in Genschel and Meeker (2010)'s study in providing a strong basis for using MLE over MRR is that the MLE method is based on a very solid theory which provides multiple ways of CI computation (Wald method CIs, likelihood based CIs and parametric bootstrap CIs).

Other additional plots are produced, the reliability vs. time plots (for the 4000-hour and 2500-hour DA) shown in `Appendix I` and they indicate that as time goes by, product reliability decreases as expected. One can also use these plots to identify the specific reliability values of the six failed lasers at the plotted time points. However, the plots only show reliability information within the range of 0 to 5000 hours since the degradation experiment was stopped at 5000 hours. The failure rate vs. time plots are also determined (see `Appendix J`) including two-sided 95% confidence bounds. The lognormal failure rate reaches a peak very early in the lifetime of the GaAs lasers and then decreases to zero (typical behaviour of the lognormal distribution failure rate). Mean remaining life (MRL) plots are obtained as well (see `Appendix K`). The MRL function and the failure rate function are essentially a complement of each other.

5.6 Application to Real Data

In this section, the analysis is applied to the laser degradation data adopted from Meeker and Escobar (1998), Table C.17, p.642. Work by Meeker and Escobar (1998) is only based on the MLE method. This research study seeks to take a different approach by applying two parameter estimation techniques (MLE and MRR) on the laser data and then comparing their statistical performance. Failure time analysis data is not considered, as it has been demonstrated to be inferior to DA on the basis of the simulation study.

Graphical methods based on the analysis of real data are not included in this section since they are similar to those under the simulation study. Rather, only numerical results are presented. The failure data from the degradation analysis (observed failures, pseudo-failures and suspensions) for both the 4000-hour and 2500-hour data are displayed in Table 5.18.

Table 5.18: *Failure times derived from real data*

		4000-hour DA	2500-hour DA
GaAs Laser ID	State F or S	Time to F or S (hr)	Time to F or S (hr)
P10	F	3308	3241
P6	F	3613	3767
P1	F	3707	3667
P2	F	4173	4078
P13	F	4780	4830
P12	F	4949	4617
P9	F/S	5000 (S)	4904 (F)
P4	S	5000	5000
P5	S	5000	5000
P7	S	5000	5000
P8	S	5000	5000
P9	S	5000	5000
P11	S	5000	5000
P14	S	5000	5000
P15	S	5000	5000

5.6.1 Distribution selection and ranking results based on real data

Based on the failure time data in Table 5.18, goodness of fit tests are first carried out using the MRR method. Four lifetime distributions are considered, namely 1P exponential, lognormal, 2P Weibull and Gamma.

Table 5.19: *Goodness of fit tests for real data using the MRR Method*

	Criteria	1P Exponential	Lognormal	2P Weibull	Gamma
4000-hour DA	Kolmogorov-Smirnov test	1.10	2.34E-08	9.99E-11	1.04E-07
	Normalized ρ test	7.59	3.22	4.75	2.48
	Likelihood ratio test	-62.22	-56.24	-56.65	-56.66
	DESV	400	170	170	260
	Ranking	4	1	1	3
2500-hour DA	Kolmogorov-Smirnov test	9.45	9.99E-11	9.99E-11	1.84E-07
	Normalized ρ test	9.61	2.48	3.40	2.55
	Likelihood ratio test	-71.45	-63.90	-64.11	-64.19
	DESV	350	100	170	230
	Ranking	4	1	2	3

As was the case under the simulation study, the best fitting distribution for both the 4000-hour and 2500-hour degradation data is the lognormal although there is a tie with the 2P Weibull for the former.

The same four distributions considered in Table 5.19 are also considered for the MLE method as indicated in Table 5.20.

5.6 Application to Real Data

Table 5.20: *Goodness of fit tests for real data using the MLE Method*

	Criteria	1P Exponential	Lognormal	2P Weibull	Gamma
4000-hour DA	Kolmogorov-Smirnov test	5.95	3.72E-08	6.48E-08	2.08E-08
	Normalized ρ test	9.66	3.57	3.56	3.64
	Likelihood ratio test	-62.15	-56.20	-56.53	-56.28
	DESV	400	150	280	170
	Ranking	4	1	3	2
2500-hour DA	Kolmogorov-Smirnov test	22.17	9.99E-11	9.99E-11	4.98E-08
	Normalized ρ test	11.53	3.25	2.91	3.23
	Likelihood ratio test	-71.38	-63.80	-64.08	-63.86
	DESV	360	120	200	200
	Ranking	4	1	3	2

Similarly, the lognormal distribution is best ranked. These results are consistent with those established by Meeker and Escobar (1998) and Meeker et al. (2001) with regard to the chosen distribution for data analysis. The lognormal parameters obtained from the MRR and MLE methods are displayed in Table 5.21.

Table 5.21: *Lognormal parameters for real data*

Estimation Method		4000-hour DA	2500-hour DA
MRR	Log-Mean (hr)	8.58	8.55
	Log (σ)	0.31	0.29
	Fail/Susp	6/9	7/8
MLE	Log-Mean (hr)	8.58	8.54
	Log (σ)	0.28	0.25
	Fail/Susp	6/9	7/8

where σ is the standard deviation.

The comparison of the 4000-hour and 2500-hour DA for the specified reliability metrics is based on the 95% two-sided CI lengths using the MRR method first and thereafter, the MLE method. Table 5.22 contains results obtained from MRR for both the 4000-hour (denoted **Test 1**) and the 2500-hour

5.6 Application to Real Data

(denoted **Test 2**). Overall, point estimates and CI lengths for the reliability metrics under consideration are largely comparable. Therefore, using degradation data from the 2500-hour DA is more advantageous in terms of one being able to cut back on testing time and financial resources in assessing product reliability.

Table 5.22: *MRR results based on the lognormal distribution (real data)*

	Test		Point estimate	95% C.I	Length
Prob. of Failure	1	Q(t=5000)	0.42	(0.19 , 0.66)	0.47
	2	Q(t=5000)	0.45	(0.23 , 0.71)	0.48
Mean time to failure	1	MTTF1	5602	(4719 , 8232)	3513
(hr)	2	MTTF2	5393	(4576 , 7040)	2464
Reliable Life(hr)	1	t(R=0.85)	3880	(2998 , 4755)	1757
	2	t(R=0.85)	3840	(3068 , 4565)	1497
BX% Life (hr)	1	B10% Life	3597	(2636 , 4409)	1773
	2	B10% Life	3578.886	(2754 , 4293)	1539
Failure Rate	1	FR(t=5000)	0.000432	(0.000141 , 0.001139)	0.000998
(no. of failures/hr)	2	FR(t=5000)	0.000503	(0.00200 , 0.001394)	0.001194

The comparison of the 4000-hour and 2500-hour DA for the specified reliability metrics is based on the 95% two-sided CI lengths using the MLE method as indicated in Table 5.23.

5.6 Application to Real Data

Table 5.23: *MLE results based on the lognormal distribution (real data)*

	Test		Point estimate	95% C.I	Length
Prob. of Failure	1	Q(t=5000)	0.41	(0.19 , 0.65)	0.46
	2	Q(t=5000)	0.47	(0.24 , 0.71)	0.47
Mean time to failure	1	MTTF1	5568	(4713 , 8118)	3405
(hr)	2	MTTF2	5263	(4571 , 6906)	2335
Reliable Life (hr)	1	t(R=0.85)	3981	(3013 , 4743)	1730
	2	t(R=0.85)	3932	(3097 , 4546)	1449
BX% Life (hr)	1	B10% Life	3713	(2653 , 4401)	1748
	2	B10% Life	3697	(2786 , 4279)	1493
Failure Rate	1	FR(t=5000)	0.000460	(0.000143 , 0.001128)	0.000985
(no. of failures/hr)	2	FR(t=5000)	0.000597	(0.000206 , 0.001369)	0.001163

The results based on the MLE method are comparable to those of MRR with respect to all the reliability metrics considered.

Overall, the analysis of both the simulated and real GaAs laser data demonstrates the practical benefits of using DA (especially saving testing time and prevention of product loss).

Chapter 6

Summary, Conclusions and Remarks

Failures have to be recorded or observed for traditional censored failure time analysis to provide useful reliability information. As for degradation data, with few or no failures, it is possible to obtain useful and significant reliability information, on condition that there is a strong correlation between the underlying degradation process and the product's failure. In order for one to realise maximum possible benefits from DA, valid models which describe the physical/chemical phenomenon of the underlying degradation process have to be applied in degradation data analysis.

This research study adopts the general degradation path model to analyse the GaAs laser degradation data. A linear random effects model is assumed. The lognormal distribution is assumed to be the appropriate failure time model from experience with the lasers. Simulation is implemented in R to generate data which exhibits similar characteristics to the real data set. In order to demonstrate the advantages of degradation data, especially that of reaching meaningful conclusions earlier, simulated data is used to create two data sets (the 2500-hour and 4000-hour degradation data). For the longer test, the lasers whose degradation paths have not reached the failure threshold $\mathcal{D}_f$ are extrapolated from 4000 hours to failure or 5000 hours. As for the shorter test, the extrapolation begins from 2500 hours to failure or 5000 hours. At 2500 hours, there are no observed failures, due to this, standard failure time analysis does not apply to the shorter test. This (lack of observed failures for highly reliable products) makes the shorter test design ideal for demonstrating the advantages of using degradation data.

The four most common distributions for modelling lifetimes are considered and also on the basis of the distribution selection and ranking results obtained. Goodness of fits tests are conducted to assess fit (for the shorter and longer test datasets). The best ranked failure time distribution is chosen for

analysis in both the longer and shorter tests. The median rank regression and maximum likelihood estimation methods are employed in estimating parameters of the fitted distributions. Due to its excellent asymptotic properties and high level of robustness, the MLE method is usually preferred in practice. On the other hand, the MRR technique is usually favoured in reliability applications because it is based on the least squares method which is fairly easy to comprehend and use. In most cases, it also provides a very good fit to degradation data. It has to be noted that other authors such as Genschel and Meeker (2010) are of the view that the MRR method has a tendency of producing misleading results in terms of providing a good fit to data (as previously stated in section 5.5). In this research study, the MRR procedure achieved an accuracy level which is comparable to that of MLE. However, MLE fared much better on precision than MRR due to the lower standard error values realised for its reliability metrics estimates.

Degradation paths of the lasers are found to be approximately linear and hence, the least squares approximation method is used to perform the extrapolation process which results in the generation of pseudo-failure times in both the longer and shorter tests. Some specified percentiles of the inferred failure time distribution and important reliability metrics such as probability of failure, reliability, mean time to failure, reliable life, BX% life and failure rate are derived. The uncertainty associated with these metrics is quantified using two sided 95% likelihood ratio confidence bounds, bias and standard error values. Results obtained from the 4000-hour and 2500-hour degradation data tests using the proposed statistical methods and reliability metrics are found to be comparable to a larger extent. Hence, the shorter test is a better alternative to the longer test, implying that with degradation data, substantive conclusions are reached much earlier. It is not necessary to have observed failures for one to be able to objectively and effectively assess product reliability. Failure time analysis results (when all lasers are run to failure) are also compared to those of the 4000-hour degradation data. It is concluded that DA is a better choice than FTA. Analysis performed on simulated data is also applied to the real dataset.

It is important that non-parametric methods are also considered in analysing degradation data. This study employed parametric procedures which are appropriate for use when the underlying distribution of the random effects is known. Most existing literature pertaining to degradation data employs parametric methods such as MLE and regression. There is still need for non-parametric methods such as the kernel density estimation method (in relation to degradation data) to be further explored. A few authors such as Ebrahem et al. (2009) and Dakhn et al. (2017) are able to demonstrate the advantages of using non-parametric methods in the area of degradation data. However, more research can still be done with regard to applying the non-parametric approach to DA.

Appendices

Appendix A

R Code for Simulation

```
> GaAsLaser <- read.table("http://www.public.iastate.edu/~wqmeeker/anonymous/Stat533_data/
Splida_text_data/GaAsLaser.txt", header=T)
> View(GaAsLaser)

> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==101) # R code for determining the slopes for
    15 lasers
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==102) # Slopes established in dat23 for
    simulation
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==103)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==104)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==105)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==106)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==107)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==108)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==109)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==110)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==111)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==112)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==113)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==114)
> lm(Current~Hours -1, data=GaAsLaser, subset = Unit==115)

dat23<-c(0.002698,0.002396,0.001779,0.001672,0.001840,0.002768,0.001628,0.001559,
0.001974,0.003023,0.001898,0.002021,0.002091,0.001718,0.001634)
fit_par23<-fitdistr(dat23,"lognormal")
fit_par23
t<-seq(250, 4000, by = 250)
y<-matrix(c(rep(0,240)),nrow=16,ncol=15)
beta92<-rlnorm(15,meanlog = -6.21334129, sdlog =  0.20405874 )
```

```
set.seed(2307)
par(mfrow=c(1,4))
for(j in 1:15)
{err<-rnorm(length(t), mean = 0, sd = 0.105)
for(i in 1:16)
{
y[i,j]<- beta92[j]*t[i]+err[i]
plot(t, y[,j], type = "b", main = "Simulated degradation data, n=15", xlab = "Time (Hours)",
ylab = "Percentage increase in operating current")
abline(h=10, col='red')
}
}
y
beta92

#plot(t, y, type = "b", main = "Simulated degradation data", xlab = "Time (Hours)",
ylab = "Percentage increase in operating current")

#abline(h=10, col='red')
```

Appendix B

Percent Increase in Operating Current vs. time Plot (R code)

```
> library(readxl)
> MySimData <- read_excel("C:/Users/Farirai/Desktop/WITS/DEGRADATION DATA ANALYSIS/MySimData.xlsx")
> View(MySimData)
ggplot(MySimData, aes(x=Hours, y=Current, col=unit)) + geom_point() + geom_hline(yintercept = 10)+
    xlab("Time (Hours)") + ylab("Percent Increase in Operating Current")
```

Appendix C

Simulated Data set

```
>beta92
[1] 0.001463091 0.001600194 0.001715645 0.001449674 0.002651708 0.001772731 0.001576745 0.003023429
[9] 0.001791070 0.001739588 0.002084772 0.002588885 0.001953490 0.002182549 0.002332988
>y
        [P1]      [P2]      [P3]      [P4]       [P5]
[1,]  0.5031778 0.4063067 0.4893000 0.2129914  0.7118515
[2,]  0.6253099 0.8143402 0.8291647 0.8759981  1.3509607
[3,]  1.2645975 1.1874497 1.2930581 1.2210960  1.8741758
[4,]  1.5696605 1.5663282 1.6583207 1.4859330  2.5742261
[5,]  1.9050335 1.9982239 2.1015634 1.8007892  3.2966858
[6,]  2.1059546 2.4273716 2.6906388 2.0720952  4.1473525
[7,]  2.6114110 2.8423726 2.7057525 2.4924075  4.7026444
[8,]  2.9239198 3.2078552 3.2647548 2.9520573  5.2755185
[9,]  3.6613469 3.6443428 3.8604704 3.1793492  6.0203075
[10,] 3.6470081 3.9733249 4.2524679 3.6188137  6.6223663
[11,] 3.9912746 4.2480853 4.5895552 3.9313512  7.2126762
[12,] 4.1632669 4.7541281 5.1191335 4.4558099  7.9464456
[13,] 4.7035302 5.2620665 5.4121355 4.7932797  8.6248072
[14,] 5.2236493 5.6691164 6.0725997 5.1588302  9.4111145
[15,] 5.3586848 6.0821879 6.6386602 5.3760887  9.8582072
[16,] 5.7379460 6.3862663 6.9705897 5.7968211 10.6880051

        [P6]      [P7]       [P8]     [P9]       [P10]
[1,]  0.4930188 0.4145042 0.7725358 0.5101476  0.5133291
[2,]  0.7464775 1.0413780 1.3985248 0.8222317  0.8360910
[3,]  1.3495959 1.1999883 2.3481812 1.5493131  1.1961489
[4,]  1.6329968 1.6828650 3.0556277 1.6746922  1.8629928
[5,]  2.2948240 2.0479664 3.6646257 2.3135972  2.1323093
[6,]  2.5799512 2.3726444 4.4455999 2.7012501  2.6104574
```

```
[7,]  3.1043035 2.6189540 5.2417414 2.9836202  3.0857725
[8,]  3.6161574 3.1463091 6.1483712 3.6040676  3.4757662
[9,]  4.1060917 3.6827787 6.7078013 3.9994974  3.9864567
[10,] 4.4284792 3.8618346 7.4940603 4.4541627  4.3674634
[11,] 4.9116861 4.3530273 8.2372908 5.0471280  4.7756709
[12,] 5.2620003 4.6411377 9.0518438 5.3099424  5.2819294
[13,] 5.7460281 5.1412329 9.7116063 5.8383770  5.7250771
[14,] 6.3254396 5.6075966 10.4969402 6.2356776 6.2489085
[15,] 6.4548814 5.9690763 11.4597928 6.6796804 6.3631788
[16,] 7.2250738 6.5347698 12.0829419 7.2872448 6.9005264

        [P11]      [P12]     [P13]     [P14]     [P15]
[1,]  0.3934595  0.4983867 0.5164553 0.4442241 0.5036568
[2,]  1.0893656  1.2322294 0.8791314 1.0691620 1.2226644
[3,]  1.4516329  1.8819584 1.2670888 1.6724077 1.5975886
[4,]  1.9305761  2.6309786 1.9829291 2.3814043 2.1467501
[5,]  2.5664213  3.2104236 2.3643805 2.7388444 2.9845127
[6,]  3.1295352  4.0142909 2.9806114 3.3774728 3.4121184
[7,]  3.7780850  4.6451724 3.4281627 3.9990712 3.9968861
[8,]  4.1769019  5.3796282 3.8215751 4.4046598 4.4968903
[9,]  4.6366626  5.6962463 4.4946562 4.8636950 5.2133331
[10,] 5.3298154  6.3403113 4.7993816 5.5506231 5.8178673
[11,] 5.7984534  7.0205601 5.5440927 6.1082448 6.2777698
[12,] 6.2063340  7.8184831 5.9123696 6.5636467 7.0220026
[13,] 6.8375508  8.4348111 6.4757597 7.0349198 7.4147552
[14,] 7.4660775  9.1293356 6.8406529 7.6475975 8.2248089
[15,] 7.9875592  9.5702257 7.2904339 8.0654086 8.8148864
[16,] 8.4270009 10.2990201 7.7227045 8.7717541 9.3891722
```

Appendix D

Rates of degradation for 15 lasers (simulated data)

Table D.1: *Rates of degradation for 15 lasers (simulated data)*

GaAs Laser ID	4000-hour degradation data	2500-hour degradation data
P1	0.001463	0.001509
P2	0.001600	0.001606
P3	0.001716	0.001679
P4	0.001450	0.001443
P5	0.002652	0.002662
P6	0.001773	0.001781
P7	0.001577	0.001585
P8	0.003023	0.003003
P9	0.001791	0.001782
P10	0.001740	0.001751
P11	0.002085	0.002090
P12	0.002589	0.002592
P13	0.001953	0.001940
P14	0.002183	0.002220
P15	0.002333	0.002294

Appendix E

Rates of degradation for 15 lasers (real data)

Table E.1: *Rates of degradation for 15 lasers (simulated data)*

GaAs Laser ID	4000-hour degradation data	2500-hour degradation data
P1	0.002698	0.002727
P2	0.002396	0.002452
P3	0.001779	0.001904
P4	0.001672	0.001857
P5	0.001840	0.001750
P6	0.002768	0.002654
P7	0.001628	0.001554
P8	0.001559	0.001554
P9	0.001974	0.002039
P10	0.003023	0.003086
P11	0.001898	0.001969
P12	0.002021	0.002166
P13	0.002091	0.002071
P14	0.001718	0.001621
P15	0.001634	0.001634

Appendix F

Real data set

Figure F.1: *Real data set*

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Table C.17. Percent Increase in Operating Current for GaAs Lasers Tested at 80°C

Time (hours)	Unit Number														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
250	.47	.71	.71	.36	.27	.36	.36	.46	.51	.41	.44	.39	.30	.44	.51
500	.93	1.22	1.17	.62	.61	1.39	.92	1.07	.93	1.49	1.00	.80	.74	.70	.83
750	2.11	1.90	1.73	1.36	1.11	1.95	1.21	1.42	1.57	2.38	1.57	1.35	1.52	1.05	1.29
1000	2.72	2.30	1.99	1.95	1.77	2.86	1.46	1.77	1.96	3.00	1.96	1.74	1.85	1.35	1.52
1250	3.51	2.87	2.53	2.30	2.06	3.46	1.93	2.11	2.59	3.84	2.51	2.98	2.39	1.80	1.91
1500	4.34	3.75	2.97	2.95	2.58	3.81	2.39	2.40	3.29	4.50	2.84	3.59	2.95	2.55	2.27
1750	4.91	4.42	3.30	3.39	2.99	4.53	2.68	2.78	3.61	5.25	3.47	4.03	3.51	2.83	2.78
2000	5.48	4.99	3.94	3.79	3.38	5.35	2.94	3.02	4.11	6.26	4.01	4.44	3.92	3.39	3.42
2250	5.99	5.51	4.16	4.11	4.05	5.92	3.42	3.29	4.60	7.05	4.51	4.79	5.03	3.72	3.78
2500	6.72	6.07	4.45	4.50	4.63	6.71	4.09	3.75	4.91	7.80	4.80	5.22	5.47	4.09	4.11
2750	7.13	6.64	4.89	4.72	5.24	7.70	4.58	4.16	5.34	8.32	5.20	5.48	5.84	4.83	4.38
3000	8.00	7.16	5.27	4.98	5.62	8.61	4.84	4.76	5.84	8.93	5.66	5.96	6.50	5.41	4.63
3250	8.92	7.78	5.69	5.28	6.04	9.15	5.11	5.16	6.40	9.55	6.20	6.23	6.94	5.76	5.38
3500	9.49	8.42	6.02	5.61	6.32	9.95	5.57	5.46	6.84	10.45	6.54	6.99	7.39	6.14	5.84
3750	9.87	8.91	6.45	5.95	7.10	10.49	6.11	5.81	7.20	11.28	6.96	7.37	7.85	6.51	6.16
4000	10.94	9.28	6.88	6.14	7.59	11.01	7.17	6.24	7.88	12.21	7.42	7.88	8.09	6.88	6.62

All percent increase values at time 0 are equal to 0.

Appendix G

MRR results: Absolute bias and SE values based on 10 simulated data sets

Table G.1: *MRR results: Determination of absolute bias and SE using 10 simulated data sets*

Test	Data set no.	Prob. of failure estimate (at 5000hrs)	MTTF estimate	FR estimate (at 5000hrs)
4000 hrs.	1	0.638634	4838.68835	0.001898
	2	0.607030	4877.326529	0.001681
	3	0.152934	8102.221297	0.000141
	4	0.776182	4476.921549	0.001639
	5	0.647013	4718.568624	0.000981
	6	0.580885	4896.917134	0.000896
	7	0.481822	5286.382313	0.000528
	8	0.407893	5411.295341	0.000576
	9	0.411827	5416.866985	0.000563
	10	0.763595	4333.052901	0.002087
		SE = 0.188487 $\|\text{Bias}\| = 0.0904225$	SE = 1071.96254 $\|\text{Bias}\| = 37.958$	SE = 0.000675 $\|\text{Bias}\| = 0.000478$
2500 hrs.	1	0.642937	4831.696860	0.001900
	2	0.614599	4867.997296	0.001737
	3	0.119743	11018.042154	0.000083
	4	0.776932	4481.844673	0.001667
	5	0.649580	4712.668579	0.000992
	6	0.578272	4903.827176	0.000884
	7	0.466596	5363.848166	0.000496
	8	0.411863	5403.809217	0.000575
	9	0.410055	5412.136796	0.000571
	10	0.771294	4600.712512	0.002169
		SE = 0.198439 $\|\text{Bias}\| = 0.0878281$	SE = 1946.947341 $\|\text{Bias}\| = 285.876$	SE = 0.000709 $\|\text{Bias}\| = 0.0004864$

Appendix H

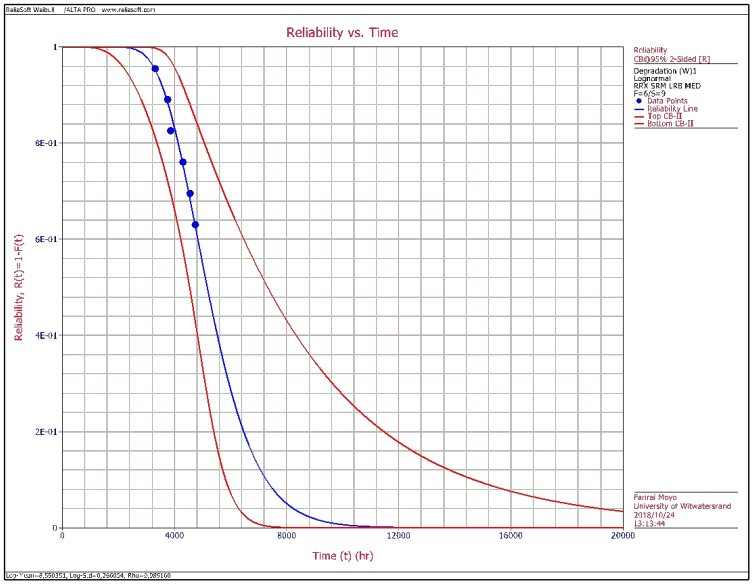
MLE results: Absolute bias and SE values based on 10 simulated data sets

Table H.1: *MLE results: Determination of absolute bias and SE using 10 simulated data sets*

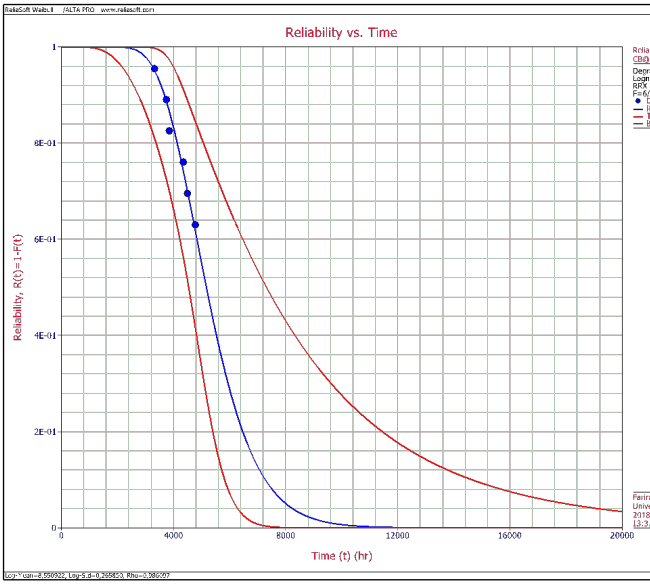
Test	Data set no.	Prob. of failure estimate (at 5000hrs)	MTTF estimate	FR estimate (at 5000hrs)
4000 hrs.	1	0.689656	4792.234871	0.002398
	2	0.656502	4825.250705	0.002122
	3	0.191587	6189.74678	0.000310
	4	0.711331	4596.940189	0.001316
	5	0.66565	4687.574509	0.001115
	6	0.599400	4854.040104	0.001030
	7	0.353325	6134.308505	0.000314
	8	0.452431	5194.755942	0.000796
	9	0.407926	5374.420871	0.000616
	10	0.64544	4777.482878	0.001366
		SE = 0.175448 $\|\text{Bias}\| = 0.0809658$	SE = 585.121041 $\|\text{Bias}\| = 131.106$	SE = 0.000699 $\|\text{Bias}\| = 0.0005173$
2500 hrs.	1	0.691941	4786.731082	0.002375
	2	0.662033	4820.101647	0.002174
	3	0.133819	7341.415894	0.000164
	4	0.711418	4601.045733	0.001335
	5	0.666369	4686.062955	0.001118
	6	0.597881	4857.285194	0.001021
	7	0.352487	6133.152887	0.000315
	8	0.453935	5196.303356	0.000785
	9	0.407216	5366.293600	0.000628
	10	0.645915	4777.637865	0.001376
		SE = 0.189294 $\|\text{Bias}\| = 0.0759424$	SE = 861.903528 $\|\text{Bias}\| = 17.179$	SE = 0.000724 $\|\text{Bias}\| = 0.0005081$

Appendix I

Reliability vs. Time Plots (Lognormal)



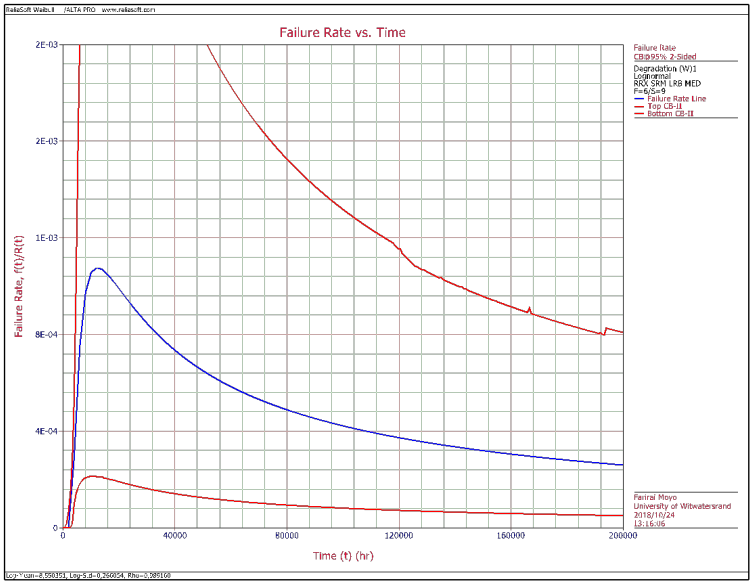
(a) 4000-hour DA



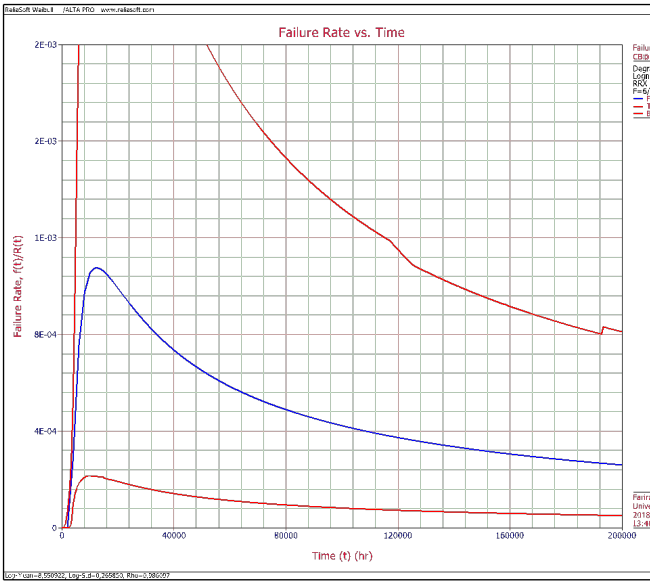
(b) 2500-hour DA

Appendix J

Failure Rate vs. Time (Lognormal)



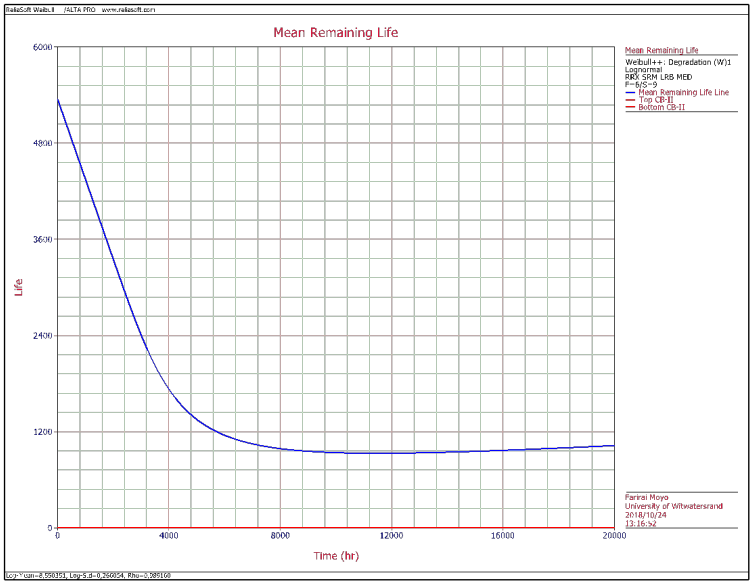
(a) *4000-hour DA*



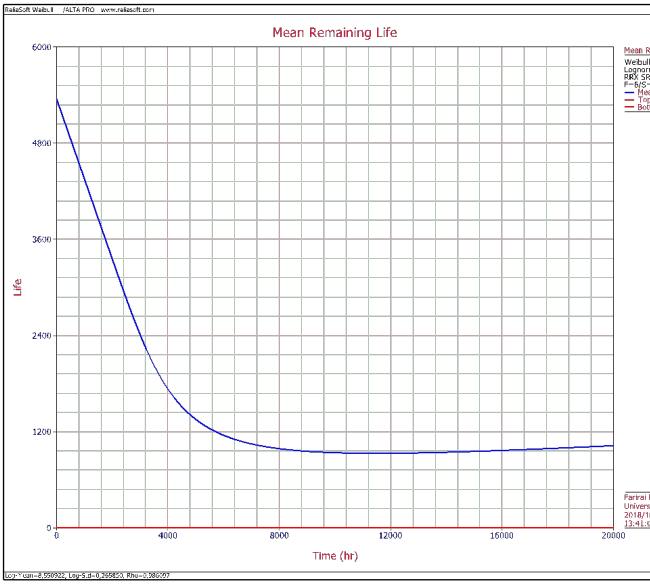
(b) *2500-hour DA*

Appendix K

Mean Remaining Life (Lognormal)



(a) 4000-hour DA



(b) 2500-hour DA

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