GEOSTATISTICAL VALUATION

FOR SHORT TERM MINE PLANNING

IN SOUTH AFRICAN GOLD

MINING

MICHAEL ASHLEY FIELD

A project report submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in part fulfilment of the requirements for the Degree of Master of Science in Engineering.

Johannesburg, 1988

TABLE OF CONTENTS

CONTENTS		PAGE No:

T A B L E   O F   C O N T E N T S (Cont.)

A P P E N D I C E S

DECLARATION

I declare that this project report is my own, unaided work. The report is being submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

M A FIELD

22nd day of November 1988

ABSTRACT

The project report compares the effectiveness of two methods of kriged estimates for monthly panel valuation. The one method uses individual sample locations and sample values for kriging and the second method uses sample information regularised to a 10 metre grid for kriging.

Four data sets with different spatial characteristics were simulated to test the effectiveness of the two methods.

The method utilising individual samples proved to result in marginally better estimates compared to the method using regularised samples. The marginal improvement obtained from the individual samples is considered to be more than offset by the additional computer processing required for this method.

The actual results from the four data sets are also compared against the expected results based on geostatistical theory. Results from two of the sets conform closely with the theoretical results. However, the other two sets showed markedly different error variances to the theoretical error variances. The reason for this is that the two sets had distinct high and low grade sub-areas and the errors are proportional to the mean grade of the sub-areas.

ACKNOWLEDGEMENTS

This work was carried out while I was a member of the Geostatistics Department at Vaal Reefs Exploration and Mining Company Limited.

The computer facilities made available to me to conduct the study is greatly appreciated.

Many of the ideas in the report resulted from constructive comments made by Professor D G Krige. He and Dr. E J Magri first exposed me to the theory of geostatistics and I am indebted to them for the inspiration to pursue a career in ore evaluation.

The final document was excellently typed by Natalie Meli. Her hard work and co-operation are greatly appreciated.

Finally, I wish to thank my wife Sue for her patience, not only during the preparation of this work but also throughout my studies.

LIST OF TABLES

L I S T OF T A B L E S (Cont.)

LIST OF FIGURES

LIST OF FIGURES (Cont.)

LIST OF FIGURES (Cont.)

LIST OF FIGURES (Cont.)

1. INTRODUCTION

1.1 Problem Statement

Geostatistical methods have been used on South African Gold Mines for the past 30 years. The most common applications have been the analysis of borehole results for global estimation (Krige, 1961, Sichel, 1966) and the calculation of ore reserve estimates. (Miller 1983, Magri 1983).

To date the use of geostatistical methods for monthly valuation, monthly gold accounting and short term planning has been limited to a few isolated studies. (Krige 1962).

The routine use of geostatistics for monthly gold accounting purposes and short term decision making is becoming more important. The main reasons for obtaining more accurate estimates of stope values is that because of increasing costs and generally deteriorating grades the ability to mine selectively is or will be of paramount importance to ensure profitability.

Because the main application of geostatistics on gold mines has been for ore reserve valuation, and as a result of the large number of samples to be considered, computer systems have been developed to use regularised data as input to the kriging process (Auges User Manual, Miller 1983).

In regularisation, all sample values falling within a cell of a predefined grid are averaged. The averaged sample values are then treated as individual values located at the centre of the cell. This method has proven to be adequate in ore reserve estimation (Miller 1983) and not only simplifies the computing process but also leads to better semi-variogram models as "the averaging process removes a lot of the variability witnessed in chip sampling" (Miller et al 1987)

Although the method of regularisation has proved itself in ore reserve estimation, no research has been done on whether the method is satisfactory for estimating gold production from stoping panels on a monthly basis. This report thus compares the traditional method of using regularised data with a method whereby individual samples are used to obtain kriged estimates of gold values for stope production.

In addition, the use of geostatistics for establishing confidence limits for local estimates has recently been criticised (Philip and Watson 1986). They state "that estimation variance is meaningless in terms of local estimation" because the estimation variance depends not on local variation but on sampling density. This report thus investigates the applicability and accuracy of using kriging variance for calculating confidence limits for the two methods described previously.

1.2 Preview of Project Report

1.2.1 Chapter 2 – Methodology

The chapter gives a detailed account of the methodology used in comparing the method of kriging with individual sample data with that of kriging using regularised data.

In order to calculate the actual values of stope panels it was necessary to simulate sample values. The method of simulation used is explained. The characteristics of the four simulated deposits are presented and the basis for comparing the two methods is covered in more detail.

The choice of a sampling configuration for use in estimating stope panel values is discussed. Based on the sampling configuration the calculation of kriging weights and their use in calculating estimates of stope panel values is explained.

Finally the method of analysing the results is briefly discussed as a preview to Chapter 3.

1.2.2 Chapter 3 – Analysis of Results

This chapter compares the semi-variogram models obtained from sampling data with the theoretical semi-variogram models used in the simulation.

A comparison of the actual stope panel values with the estimated stope panel values is presented with specific attention given to the accuracy of estimates and the analysis of errors.

Finally, the analysis of the results of the simulation are compared with the expected results based on geostatistical theory.

1.2.3 Chapter 4 - Conclusion

The findings of the study are summarised in this chapter and specific recom ... ns are made.

1.3 Summary of the ... of the Study

The primary purpose of the study is to:

(a) Compare the effectiveness of using regularised sampling data with that of individual sample data in the local estimation of gold values using kriging.

(b) Compare the results obtained in the study with the expected results based on geostatistical theory.

(c) Make recommendations regarding the practical implementation of geostatistical methods for local (monthly production) valuation.

2. METHODOLOGY

2.1 General Approach

To compare the two methods, referred to as Regularised Method and Individual Method, it is necessary to have actual values of panels for comparison purposes. As stope sampling on gold mines is rarely more dense than a 5 metre by 5 metre grid, accurate measures of actual monthly stope panel values cannot be obtained from real data. For this reason sampling results were simulated on a 1 metre by 1 metre grid to facilitate the calculation of actual individual panel values.

Four data sets were simulated (see section 2.2) to analyse the effect of different semi-variogram structures on the estimation procedure. Each simulated data set covers an area of 600 metres by 150 metres which is assumed to cover sixty months of production at a ten metre face advance and five stope panels each of thirty metres face length.

Samples on a regular 5 metre by 5 metre grid were extracted from the simulated samples to represent real sampling practice on gold mines. The calculation of experimental semi-variogram models for each data set were based on samples from a 200 metre by 150 metre portion of each set.

The models were then used to estimate monthly panel values for the remaining 400 metre by 150 metre area, utilising additional sampling information as it would become available in practice. Figure 2.1.1 illustrates the above.

Based on the semi-variogram models, kriging weights were calculated for chosen data configurations for estimating the various panels. The respective data configurations used for estimating the various panels are discussed in section 2.7.

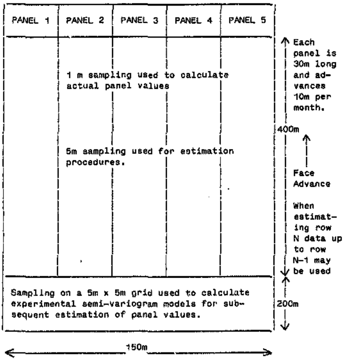


Figure 2.1.1. Illustration of layout of each data set

Two sets of estimates of panels were calculated. The first estimate of each panel represents the estimate of planned production (i.e. a forecast of gold to be mined in the following month) whilst the second estimate represents an estimate of the gold expected to reach the plant (i.e. the gold called for based on the months production and utilising the latest sampling information available).

Both sets of estimates were compared against the actual panel values to determine the effectiveness of the two methods (Regularised Method and Individual Method) in estimating the next months planned production (Planned estimates) and accounting for the current months gold production (Called estimates).

A diagrammatic sketch of the various estimates produced is shown in Figure 2.1.2.

Each simulated data set	4 Sets
↓	
Use Regularised Method and Individual Method	2 Methods
↓	
Produce estimates for: - Planned gold production from each panel. - Called gold production from each panel	2 Estimates = 16 Sets of Results for 400 Panels

Figure 2.1.2. <u>Illustration of the estimates produced</u>

The 16 sets of estimates were then compared with the actual panel values to evaluate the effectiveness of the two methods under various conditions (i.e. different semi-variogram structures and estimating Planned gold production and Called gold production).

2.2 Simulation of the Data Sets

A computer program, SIMUL (Journel and Huijbregts 1978) was used to simulate Standard Gaussian distributions with prescribed covariance models. The program uses the "turning bands" method developed by G Matheron to simulate realisations of a regionalised variable with a given covariance.

In order to analyse the effect of different semi-variogram structures on the effectiveness of the two methods of estimation, four data sets were generated.

Characteristics of the four sets are contained in Table 2.2.1. The four sets cover combinations of short and long ranges (25 metres and 100 metres) and different nugget effect to total sill ratios (0,33 and 0,5).

	Data Set 1	Data Set 2	Data Set 3	Data Set 4
Nugget Effect (Logarithmic)	0,3	0,3	0,2	0,2
Total Sill (Logarithmic)	0,6	0,6	0,6	0,6
Range (metres)	25	100	25	100
Semi-variogram Model	Spherical	Spherical	Spherical	Spherical
Mean Grade (cmg/t)	1 000	1 000	1 000	1 000
Distribution of values	Lognormal	Lognormal	Lognormal	Lognormal

Table 2.2.1. Characteristics of the Simulated Data Sets

To obtain the various model combinations, the SIMUL program was used to generate three independent simulations:

Simulation 1 = Standard Gaussian variable with a Nugget effect model.

Simulation 2 = Standard Gaussian variable with a covariance model that is Spherical with a range of 25 metres.

Simulation 3 = Standard Gaussian variable with a covariance model that is Spherical with a range of 100 metres.

Combinations of the above simulations were then used to generate the four data sets as described in Table 2.2.1.

The appropriate weighting of the simulations to maintain a Standard Gaussian distribution and to result in the required data sets is presented in Appendix A.

Using the weighting factors as per Appendix A results in weightings of the various simulations as shown in Table 2.2.2.

	SIMULATION 1	SIMULATION 2	SIMULATION 3
Data Set 1	0,7071	0,7071	-
Data Set 2	0,7071	-	0,7071
Data Set 3	0,4472	0,8944	
Data Set 4	0,4472		0,8944

Table 2.2.2. <u>Required Weighting Factors of Simulations</u>

Using the above weightings results in data sets with the required covariance models. However, the histograms of values at this stage are still Standard Gaussian histograms.

By means of a transform function the data sets were then converted to follow a Lognormal distribution with a mean of 1000 cmg/t and a logarithmic variance of 0,6.

The appropriate transform function was derived as follows:

If Z is the regionalised variable that follows a Standard Gaussian distribution and

$\mu \quad = 1000$ cmg/t (the required mean value)

$\sigma_L^2 \; = 0,6$ (the required logarithmic variance)

then a transform function that results in a variable X that follows a Lognormal distribution is required. From Lognormal distribution theory:

$Y = \ln(X) \sim N(\xi, \sigma_L^2)$

Therefore $(Y - \xi)/\sigma_L \sim N(0,1) = Z$ ....(1)

But $\exp(\xi + \sigma_L^2/2) = \mu = 1000$ (Rendu, 1978)

Therefore $\xi = \ln(1000) - \sigma_L^2/2$ ....(2)

From (1) $Y = \sigma_L Z + \xi$

Therefore $Y = \sigma Z + \ln(1000) - \sigma_L^2/2$

Therefore the required transform function to generate the Lognormally distributed regionalised variable is:

$$X = \exp\left[\sigma_L Z + \ln(1000) - \sigma_L^2/2\right]$$

or $$X = 1000 \exp\left[\sigma_L Z - \sigma_L^2/2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

After combining the simulations according to Table 2.2.2 each combined set of data *follows a Standardised Gaussian distribution* with the appropriate covariance model. Applying the transform function (3), results in the required lognormally distributed data sets each with a mean value of 1000 cmg/t, a logarithmic variance of 0,6 and the appropriate covariance model.

A total of 90 000 values (600 rows and 150 columns) per set were generated to represent values on a 1 metre by 1 metre grid.

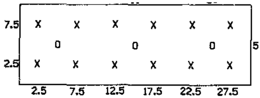
2.3 Generation of Sampling Results

Each set of 90 000 simulated values was reduced to a subset of data representing the traditional sampling practice on gold mines of sampling on a 5 metre by 5 metre grid. This *reduction resulted in* 3 600 *sample* values (120 rows and 30 columns).

2.4 Regularisation of Sampling Results

The 3 600 samples on a 5 metre by 5 metre grid mentioned in 2.3 were used as the data input for the Individual Method of kriging.

For the Regularised Method, the 3 600 samples were regularised into a 10 metre grid. The regularisation process resulted in four sample values from the 5 metre grid being averaged to form a single sample point for the 10 metre regularised data set. The co-ordinates of the 10 metre sample points were centred in the cells of the 10 metre grid as per Figure 2.4.1.



X=SAMPLE POSITIONS ON 5 METRE GRID   Ø= REGULARISED SAMPLE POSITIONS

FIGURE 2.4.1 SAMPLE POSITIONING FOR REGULARISED DATA

2.5 Calculation of Semi-Variograms

The bottom one third of the sampling subsets (as per Figure 2.1.1) were used to calculate experimental semi-variograms for each data set.

As the data sets followed a Lognormal distribution, the sample subsets were log-transformed and the variograms were calculated on the log transformed subsets of data.

In the case of regularised data, sample values were averaged into 10 metre by 10 metre cells on a regular grid. The semi-variograms for regularised data were also calculated on log transformed data. The results of the semi-variogram calculations and the fitted models are given in Chapter 3.

2.6 Calculation of Actual Panel Values

The actual panel values were calculated using the 1 metre by 1 metre simulated data points. Thus, each panel value was calculated as the arithmetic average of 300 values (30 metre face length and 10 metre face advance). Using 300 sample values to calculate the value of a 300 m^2 area provides a good estimate of the value of the area.
The reliability of this estimate was checked by using the PLAYKRIG program (See Section 2.7) and found to be adequate.

2.7 Kriging Weights of Samples for Estimation

A computer program, PLAYKRIG, developed at Centre de Geostatistique, Fontainebleau, France was used to determine suitable data configurations for estimating the various stope panels. The program allows rapid calculation of kriging weights for all samples used in kriging a specific block for a given semi-variogram model.

In addition, the program calculates the kriging variance and the expected regression slope based on geostatistical theory. The theoretical regression slope is calculated by the formula given in Appendix B. (Rivoirard 1987, Matheron 1970).

The program was thus used to determine suitable data configurations (presented as Figures 2.7.1 to 2.7.12) by considering which sample positions had significant kriging weights and by examining the kriging variance and regression slope. (Rivoirard, 1987).

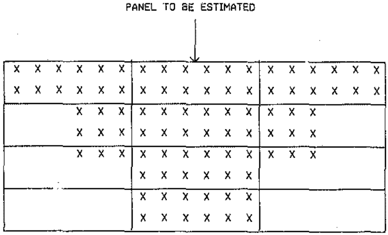


FIGURE 2.7.1 SAMPLING POSITIONS FOR KRIGING ACTUAL GOLD PRODUCTION IN COLUMNS 2,3 OR 4

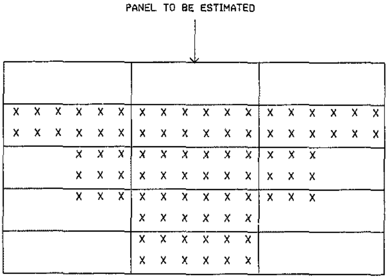


FIGURE 2.7.2 SAMPLING POSITIONS FOR KRIGING PLANNED GOLD PRODUCTION IN COLUMNS 2,3 OR 4

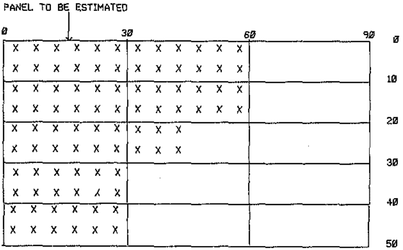


FIGURE 2.7.3 SAMPLING POSITIONS FOR KRIGING ACTUAL GOLD PRODUCED IN COLUMN 1

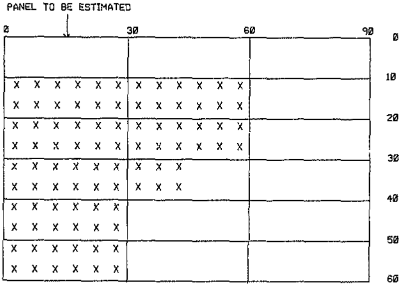


FIGURE 2.7.4 SAMPLING POSITIONS FOR KRIGING PLANNED GOLD PRODUCED IN COLUMN 1

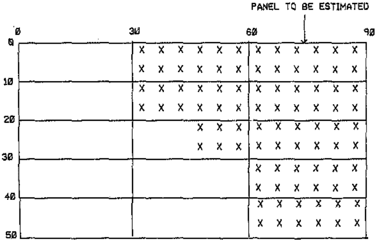


FIGURE 2.7.5 SAMPLING POSITIONS FOR KRIGING ACTUAL
GOLD PRODUCED IN COLUMN 5

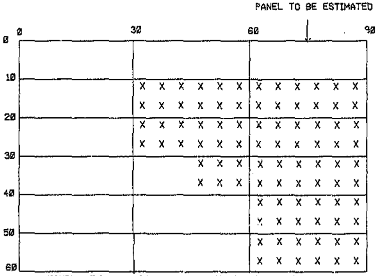


FIGURE 2.7.6 SAMPLING POSITIONS FOR KRIGING PLANNED
GOLD PRODUCTION IN COLUMN 5

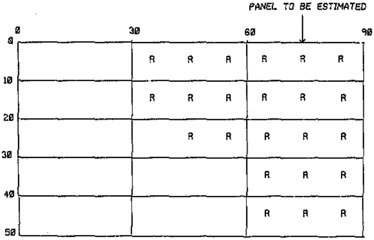


FIGURE 2.7.7 SAMPLING POSITIONS OF REGULARISED DATA FOR KRIGING ACTUAL GOLD PRODUCED IN COLUMN 5

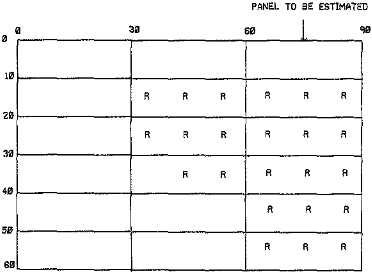


FIGURE 2.7.8 SAMPLING POSITIONS OF REGULARISED DATA FOR KRIGING PLANNED GOLD PRODUCTION IN COLUMN 5

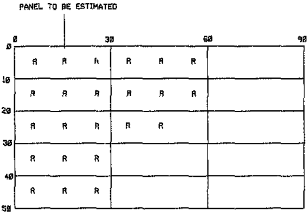


FIGURE 2.7.9 SAMPLING POSITIONS OF REGULARISED DATA FOR KRIGING ACTUAL GOLD PRODUCED IN COLUMN 1

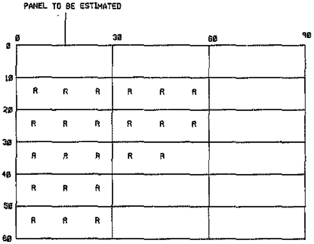


FIGURE 2.7.10 SAMPLING POSITIONS OF REGULARISED DATA FOR KRIGING PLANNED GOLD PRODUCTION IN COLUMN 1

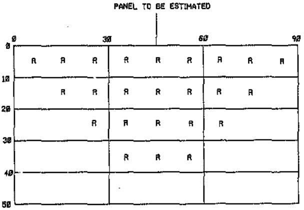


FIGURE 2.7.11 SAMPLING POSITIONS OF REGULARISED DATA FOR KRIGING ACTUAL GOLD PRODUCED IN COLUMNS 2,3 OR 4

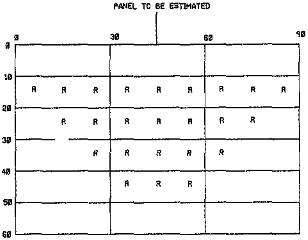


FIGURE 2.7.12 SAMPLING POSITIONS OF REGULARISED DATA FOR KRIGING PLANNED GOLD PRODUCTION IN COLUMNS 2,3 OR 4

The calculation of ancillary information such as the kriging variance and regression slope is also used in Chapter 3 to compare the accuracy of the theoretically calculated parameters with the observed results.

2.8 Analysis of Comparisons between the Methods

Chapter 3 contains a full description of the results obtained from the study and the various comparisons of the results that are carried out.

The accuracy of the Planned panel estimates and Called panel estimates are compared with the actual panel values. A full analysis of the errors is presented with specific regard to investigating whether errors and estimates are correlated and whether the five columns of panels exhibit different results.

The experimental semi-variogram models are compared against the theoretical models used in the simulation to ascertain whether the different methods require a different area to be sampled before reliable semi-variogram models can be obtained.

The observed results are also compared with the expected theoretical results based on using geostatistical theory. In geostatistical theory, knowledge of the semi-variogram model, sample positions and the block to be estimated are sufficient (without knowing the actual sample values) to calculate the expected regression effect and confidence intervals of estimates.

3. ANALYSIS OF RESULTS

3.1 Introduction

This chapter covers the detailed analysis of the results of the four simulated data sets.

The primary objectives of the analysis are to:

(a) Compare the estimates of the Individual Method and Regularised Method with the actual values to determine whether either of the two methods is superior.

(b) Calculate the expected theoretical results and compare these with the results obtained from the two methods.

(c) Examine the results of the two methods in each data set to determine the effectiveness of the two methods for *different structural relationships* (ranges, nugget effects and sill values).

An analysis of the experimental semi-variograms is discussed in section 3.2 together with the models fitted to the results. The models are then used to determine the expected results based on geostatistical theory.

The analysis of each data set is conducted separately in sections 3.4 to 3.7.

Finally, a comparison between the results for the 5 column positions is discussed in section 3.8.

3.2 Semi-variogram analysis

Experimental semi-variograms were calculated for each data set based on the sample values being log-transformed because the samples were from a Lognormal distribution.

For each data set, semi-variograms were calculated for the *individual sample values* (based on a 5m by 5m grid) as well as for the regularised sample values (based on a 10m x 10m grid).

The semi-variograms, together with the fitted models are presented in Figures 3.2.1 to 3.2.8.

The parameters of the fitted models are presented in Table 3.2.1. The theoretical models (as used in the simulations) for each data set are also included in the tables. From the table it is evident that in general the fitted models are reasonably close to the theoretical models. Previous work has shown that this is not a necessary result and that the experimental semi-variogram can be markedly different from the actual model used in the simulation (Brooker 1983).

	INDIVIDUAL SAMPLES	REGULARISED SAMPLES	THEORETICAL
DATA SET 1			
Nugget effect	0,32	0,08	0,3
Sill value	0,35	0,29	0,3
Range	27m	32m	25m
Model type	Spherical	Spherical	Spherical
DATA SET 2			
Nugget effect	0,30	0,05	0,3
Sill value	0,24	0,25	0,3
Range	60m	60m	100m
Model type	Spherical	Spherical	Spherical
DATA SET 3			
Nugget effect	0,17	0,04	0,2
Sill value	0,50	0,40	0,4
Range	25m	30m	25m
Model type	Spherical	Spherical	Spherical
DATA SET 4			
Nugget effect	0,12	0,01	0,2
Sill value	0,36	0,35	0,4
Range	60m	65m	100m
Model type	Spherical	Spherical	Spherical

Table 3.2.1. Parameters of the Logarithmic Semi-variogram Models

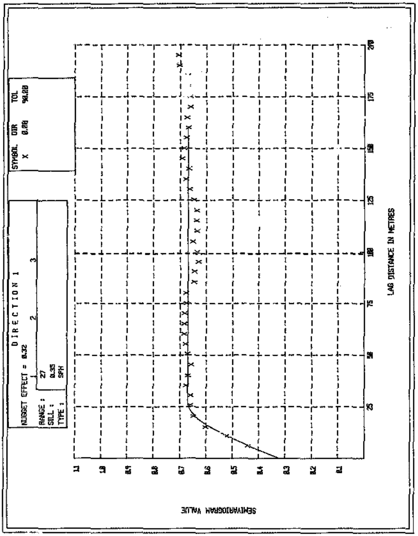


FIGURE 3.2.1 SEMI-VARIOGRAM FOR DATA SET 1
THEORETICAL MODEL IS : (R=25m CØ=.3 C1=.3)

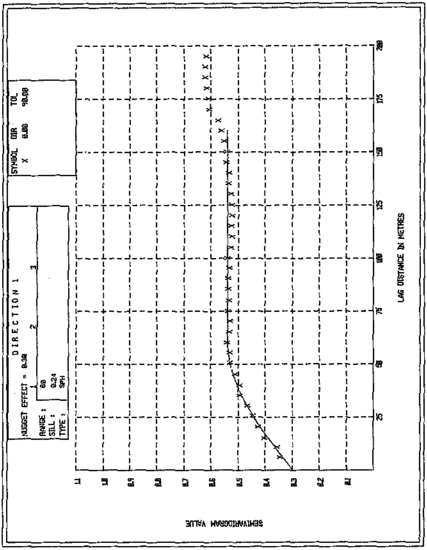


FIGURE 3.2.2 SEMI-VARIOGRAM FOR DATA SET 2
THEORETICAL MODEL IS :(R=100m C0=.3 C1=.3)

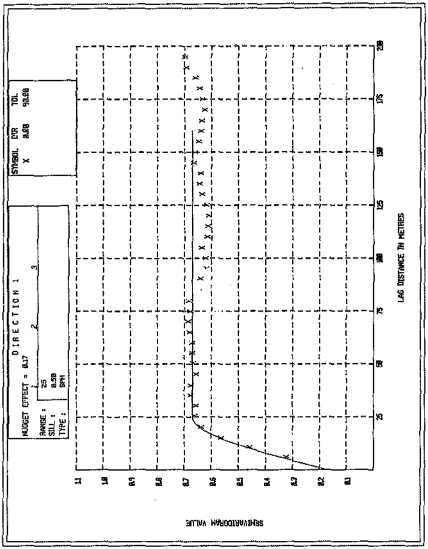


FIGURE 3.2.3 SEMI-VARIOGRAM FOR DATA SET 3
THEORETICAL MODEL IS : (R=25m CØ=.2 C1=.4)

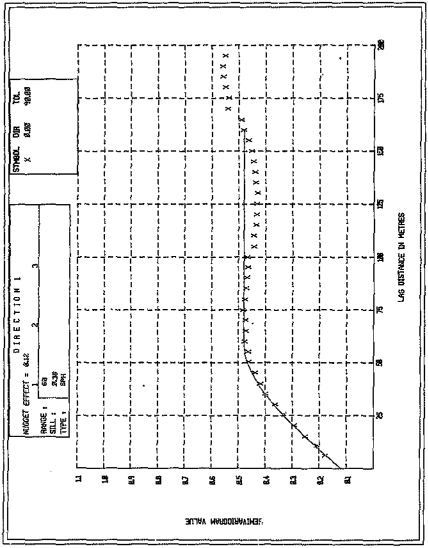


FIGURE 3.2.4 SEMI-VARIOGRAM FOR DATA SET 4
THEORETICAL MODEL IS : (R=100m C0=.2 C1=.4)

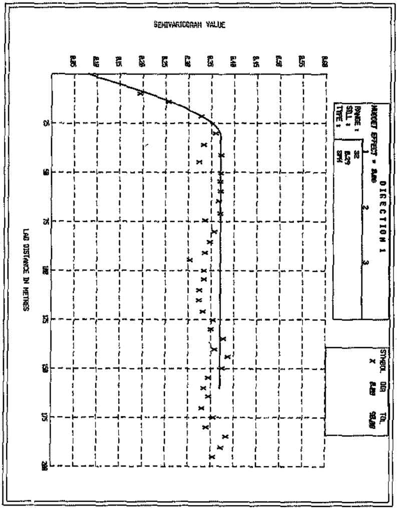


FIGURE 3.2.5 REGULARISED SEMI-VARIOGRAM MODEL FOR DATA SET I

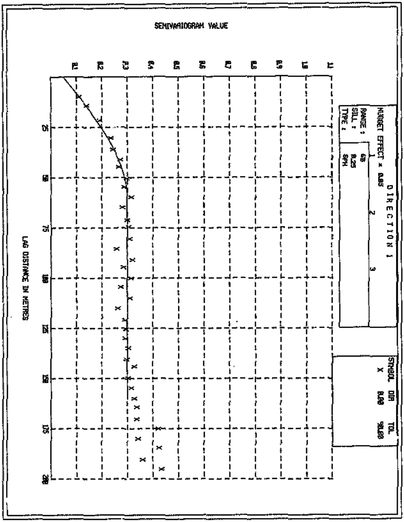


FIGURE 3.2.6 REGULARISED SEMI-VARIOGRAM FOR DATA SET 2

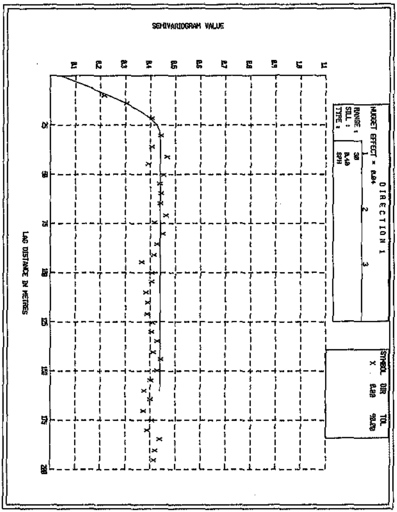


FIGURE 3.2.7 REGULARISED SEMI-VARIOGRAM FOR DATA SET 3

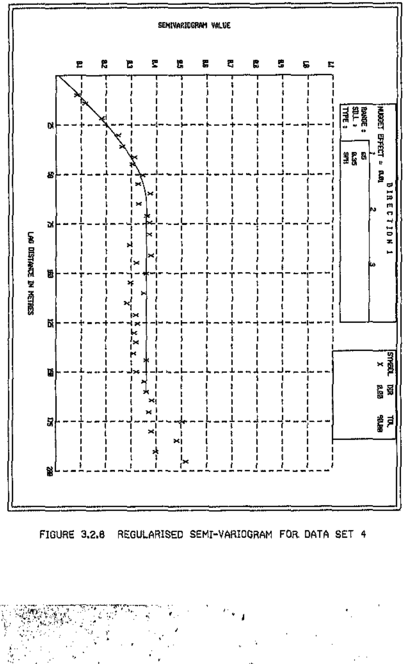


FIGURE 3.2.8 REGULARISED SEMI-VARIOGRAM FOR DATA SET 4

The following differences between the theoretical models and the fitted models were observed:

(a) The ranges of the fitted models for data set 2 and data set 4 were all approximately 60 metres compared with the theoretical range of 100 metres. However, the range has a relatively small impact on the determination of the kriging weights (Ravenscroft 1985).

(b) The ranges for the regularised data tend to increase by approximately five metres. This is as a result of regularising to a 10 metre grid.

(c) The nugget effects for the regularised data should be approximately one quarter of the nugget effects for individual sample models (Rendu 1978). This result is evident in the first three data sets but the nugget effect for data set 4 (0,01) is very low.

From Figures 3.2.1 to 3.2.8 it is also evident that the models of the individual sample semi-variograms fit the results better than the models of the regularised semi-variograms. This is primarily due to a small area (150m x 200m) being used to calculate the experimental semi-variograms. For the small area used, the number of sample pairs that are used in the calculation of the semi-

variogram values at specific distances are significantly less for the regularised samples than for the individual samples. Although the regularisation smooths the variability of the data, the results suggest that in practice a larger sampling area may be necessary to accurately construct a regularised semi-variogram model than the area required to construct a point semi-variogram accurately.

In order to work in the original units the logarithmic semi-variograms were back-transformed via the following formula:

$$\gamma_N(h) = m^2 \times e^{\sigma_L^2} \times \left[1 - e^{-\gamma_L(h)}\right]$$

Where $\gamma_N(h)$ = the semi-variogram in original units

$\gamma_L(h)$ = the log transformed semi-variogram

σ_L^2 = the logarithmic variance of the data set used to generate the semi-variogram

m = the mean value (cmg/t) of the data set.

Using the above formula the models in Table 3.2.1 were back-transformed and new models in the original units were then calculated.

The parameters of the back-transformed semi-variogram models appear in Table 3.2.2.

The back-transformed models were then used in the PLAYKRIG program to calculate the appropriate kriging weights for the various data configurations (Figures 2.7.1 to 2.7.12).

3.3 The expected theoretical results

Expected theoretical results can be calculated as the theory of geostatistics shows that quality of estimation is independent of the actual sample values used. More specifically the quality of estimation depends upon:

(a) The relative distances between the block to be estimated (the stope panel) and the positions of samples used to estimate the block.

(b) The size and geometry of the block to be estimated.

	INDIVIDUAL SAMPLES	REGULARISED SAMPLES
DATA SET 1		
Nugget effect	562 578	118 184
Sill value	440 531	357 212
Range	27m	32m
Model type	Spherical	Spherical
DATA SET 2		
Nugget effect	501 994	75 807
Sill value	306 156	327 053
Range	60m	60m
Model type	Spherical	Spherical
DATA SET 3		
Nugget effect	307 795	61 953
Sill value	653 562	500 474
Range	25m	30m
Model type	Spherical	Spherical
DATA SET 4		
Nugget effect	208 110	16 405
Sill value	493 475	482 030
Range	60m	65m
Model type	Spherical	Spherical

Table 3.2.2 **Parameters of the Back-transformed Semi-variogram Models**

(c) the quantity and spatial arrangement of the samples.

(d) the degree of continuity of the deposit which is conveyed by its semi-variogram.

The PLAYKRIG program was used with the appropriate semi-variogram models and data configurations to calculate the expected theoretical results. The results are presented in Table 3.3.1 (Individual samples) and Table 3.3.2 (Regularised samples). The results for the end panels (panel 1 and panel 5) are presented separately to the middle panels because of the different data configurations. The results in the tables clearly illustrate that there is an insignificant deterioration in the estimation of end panels compared with the estimation of the middle panels.

For all data sets, the kriging of Planned production has a significantly lower confidence level than the kriging of the Called production. This result is expected as kriging is not a good extrapolator, and the Called estimates include the additional sampling corresponding to the current months production.

	PANELS 2, 3 OR 4		PANELS 1 OR 5	
	KRIGING VARIANCE	REGRESSION SLOPE	KRIGING VARIANCE	REGRESSION SLOPE
DATA SET 1				
Called	30 483	0,963	31 707	0,955
Planned	136 542	0,762	139 437	0,746
DATA SET 2				
Called	22 933	0,975	25 276	0,955
Planned	76 505	0,871	83 262	0,843
DATA SET 3				
Called	18 486	0,985	19 109	0,981
Planned	168 265	0,814	171 284	0,801
DATA SET 4				
Called	11 793	0,991	12 575	0,986
Planned	83 564	0,913	88 544	0,901

Table 3.3.1 **Expected theoretical results of kriging panels based on individual samples**

	PANELS 2, 3 OR 4		PANELS 1 OR 5	
	KRIGING VARIANCE	REGRESSION SLOPE	KRIGING VARIANCE	REGRESSION SLOPE
<u>DATA SET 1</u>				
Called	31 754	0,948	33 052	0,939
Planned	129 969	0,738	133 616	0,716
<u>DATA SET 2</u>				
Called	18 726	0,977	20 470	0,961
Planned	81 456	0,868	87 816	0,843
<u>DATA SET 3</u>				
Called	26 756	0,967	27 695	0,961
Planned	166 624	0,763	170 569	0,744
<u>DATA SET 4</u>				
Called	9 642	0,990	10 362	0,983
Planned	89 343	0,904	95 646	0,885

Table 3.2.2 <u>Expected theoretical results of kriging panels based on regularised samples</u>

A comparison of theoretical results between the different data sets led to the following interesting results:

(a) Using kriging for calculating the Called gold shows that the *kriging* variance drops significantly for data sets 3 and 4 when the ratio between the nugget effect and sill value drops. This is consistent with previous work (Ravenscroft 1985). The results also show that the nugget effect to sill ratio has a far greater impact on the kriging variance than what the range has.

(b) Using kriging for calculating the Planned gold production shows that the regression slope increases, as can be expected, when the range increases or when the nugget effect to sill ratio decreases. However, the kriging variance increased with a reduction in the nugget effect to sill ratio (data set 1 vs data set 3 and data set 2 vs data set 4).

The validity of the expected theoretical results is tested in the subsequent sections of this chapter.

3.4 Analysis of Results of Data Set 1

3.4.1 Results using Individual Samples for Kriging

To compare the results of the estimated panel values (Planned values and Called values) with the actual panel values, only the top 400 metres of the simulated data set is used. This is because the bottom 200 metres were used to calculate the semi-variogram model. There are consequently 200 panels used in the comparison (5 adjacent panels mined for 40 months).

A colour coded diagram of the results for Planned panel values, Called panel values and actual panel values is presented in Figure 3.4.1.

A summary of the statistics of the three sets of values is given in Table 3.4.1.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	1 057 cmg/t	1 047 cmg/t	993 cmg/t
Variance	106 085	138 724	150 825
Log Variance	0,10	0,13	0,15

Table 3.4.1. Statistics of Results of Data Set 1 (Individual Samples)

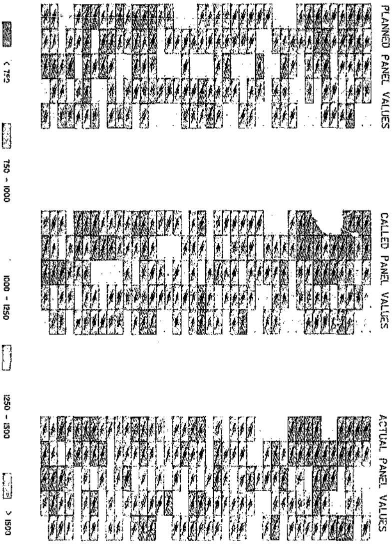


FIGURE 3.4.i: COLOUR CODED PLOTS OF THE PANEL VALUES USING INDIVIDUAL SAMPLES

From the above table it appears that both the Planned values and Called values are biased, but this is as a result of the 5 metre samples averaging 1 043 cmg/t as opposed to the 1 metre samples averaging 993 cmg/t.

The smoothing effect of kriging is also evident where the variances of Planned panel values and Called panel values are 106 085 and 138 724 respectively, compared to the actual panel variance of 150 825.

The errors in estimation (Planned panel values - actual panel values and Called panel values - actual panel values) are shown in Figure 3.4.2. The plots indicate that the errors are randomly distributed. There is no pattern in the distribution of errors nor are there clusters of small errors or clusters of large errors.

The statistics of the errors are given in Table 3.4.2. The statistics illustrate a slight improvement in the mean error for called values as opposed to planned values and there is a significant reduction in the variance of the errors of called values. This result is expected as the called values include 12 additional samples within the panel boundary being estimated.

ABSOLUTE ERRORS OF PANELS PLANNED VS. ACTUAL

ABSOLUTE ERRORS OF PANELS CALLED VS. ACTUAL

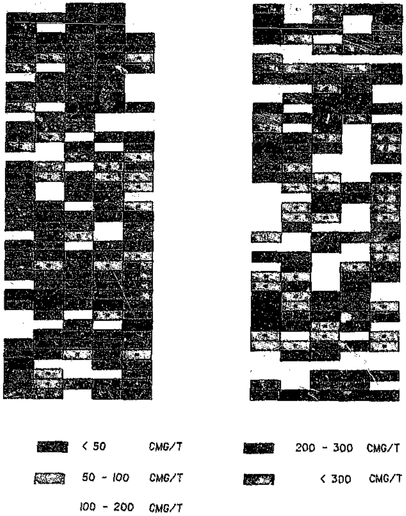


FIGURE 3.4.2 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALUES FOR DATA SET I - INDIVIDUAL SAMPLES

	MEAN ERROR	ACTUAL ERROR VARIANCE	THEORETICAL ERROR VARIANCE
Planned Values	64 cmg/t	121 560	136 542
Called Values	54 cmg/t	21 248	30 483

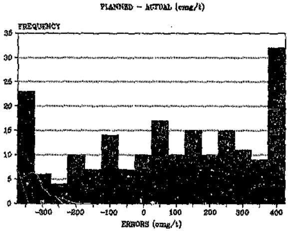
Table 3.4.2. Statistics of Errors of Data Set 1 (Individual Samples)

The actual error variances of both the Planned and Called estimates compare favourably with the theoretical error variances, although the error variance of Called values is 30% lower than the theoretical error variance.

Histograms of the errors (Figure 3.4.3) indicate that the errors of the Called estimates follow a Normal distribution. However, the errors of the Planned estimates appear to follow a Uniform distribution. This obviously affects the calculation of confidence limits, where traditionally the errors are assumed to be either Normally distributed or Lognormally distributed.

In Figure 3.4.4 the estimated panel values (Planned and Called) are plotted against the actual panel values. The scatter plot of Planned values versus the actual values, indicates the presence of the regression effect whereby the low estimates are generally underestimated and the high estimates are overestimated

The Called estimates show a very high degree of correlation with the actual values, and the regression effect is not evident.



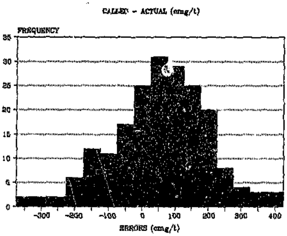
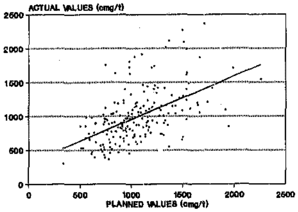


FIG 3.4.3 HISTOGRAMS OF ERRORS (SET 1, INDIVIDUAL SAMPLES)

PLANNED VS. ACTUAL PANEL VALUES



CALLED VS. ACTUAL PANEL VALUES

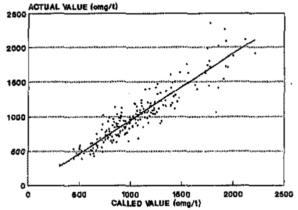


FIGURE 3.4.4 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 1 , INDIVIDUAL SAMPLES)

The statistics of the regression lines for the plots (Table 3.4.3) indicate that the actual results obtained are what could be expected based on the use of geostatistical theory for the particular data configuration and semi-variogram model.

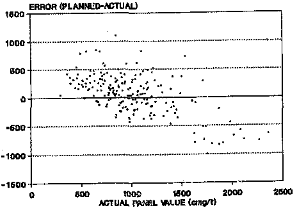
	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,638	319	,762
Called vs Actual Panel Values	0,976	-19	,963

Table 3.4.3. Statistics of Regression Lines of Set 1 - (Individual Samples)

In Figure 3.4.5, the errors of the estimates (Planned - Actual and Called - Actual) are plotted against the actual panel values.

The plot of Planned errors indicates that the panels which actually have low values are generally overestimated and that the panels with actual high values are generally underestimated The regression line fitted to the scatter plot of Planned errors versus actual values has a slope of - 0,551. The reason for the overestimation of low panel values and the underestimation of high panel values is best explained by referring to Figure 3.4.5 a. The top graph in the Figure indicates that the reasons for the negative slope of the errors versus the planned values are:

ERRORS VS. ACTUAL PANEL VALUES



ERRORS VS. ACTUAL PANEL VALUES

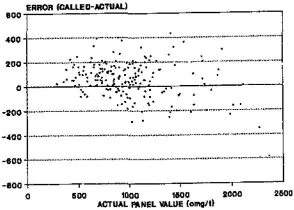
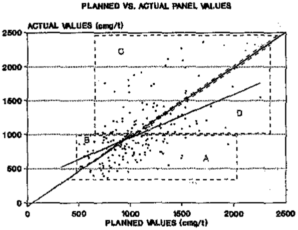


FIGURE 3.4.5 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES
(SET 1 , INDIVIDUAL SAMPLES)



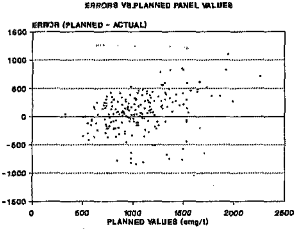


FIGURE 3.4.5A ACTUAL VALUES AND PLANNED ERRORS VS. PLANNED VALUES (SET 1 , INDIVIDUAL SAMPLES)

(a) Where the actual panel values are below 1 000 cmg/t (the mean value) the number of panels that are overestimated (Block A) far exceeds the number of panels that are underestimated (Block B).

(b) Similarly, where the actual panel values are above 1 000 cmg/t, the number of panels that are underestimated (Block C) exceeds the number of panels that are overestimated (Block D).

The bottom graph in Figure 3.4.5 a, is a scatter plot of the errors (Planned - Actual) versus the Planned values. The positive correlation between the errors and the Planned values illustrates the regression effect.

The figures illustrate that the regression effect is:

- that low estimates are generally underestimated and high estimates are generally overestimated
  and not that low actual values are underestimated and high actual values are overestimated.

3.4.2 Results using Regularised Samples for Kriging

In this section, the estimates are based on kriging data that has been regularised to a 10 metre grid. (See Figure 2.4.1).

A colour coded diagram of the results for Planned values, Called panel values and actual panel values is presented in Figure 3.4.6.

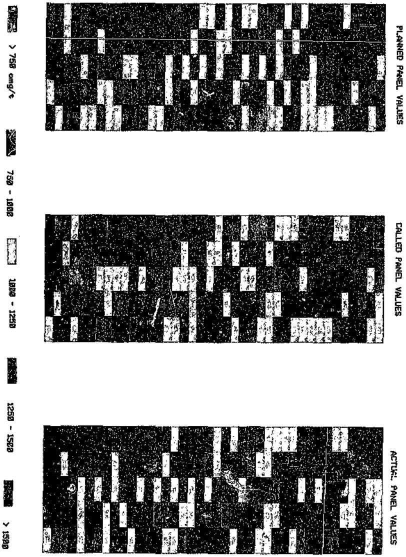


FIGURE 3.4.6 : COLOUR CODED PLOTS OF PANEL VALUES

FOR DATA SET 1 (REGULARISED SAMPLES)

Table 3.4.4 contains the summary statistics of the results of the three sets of values.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	1 051 cmg/t	1 044 cmg/t	993 cmg/t
Variance	102 837	127 549	150 825
Log Variance	0,09	0,12	0,15

Table 3.4.4. Statistics of Results of Data Set 1 (Regularised Samples)

As in the previous section, the apparent overestimation of panel values is as a result of the 5m sampling results averaging 1 043 cmg/t as opposed to the 1m sampling results averaging 993 cmg/t.

The smoothing effect of kriging is again evident where the variances of planned and called panel values are 102 837 and 127 549 respectively, versus the actual panel variance of 150 825.

The panel estimates have slightly lower variances than the corresponding estimates using individual samples. The panel estimates have thus been further smoothed using regularised data.

The absolute errors in estimation (Planned panel values - actual panel values and Called panel values - actual panel values) are shown in Figure 3.4.7.

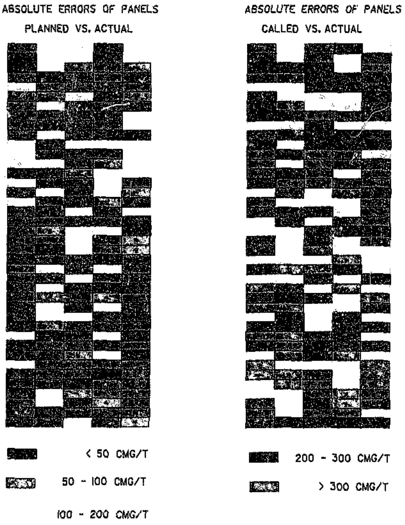


FIGURE 3.4.7 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALUES FOR DATA SET I - REGULARISED SAMPLES

The plots again indicate that the errors are randomly distributed and that there are no clusters of high or low errors.

The statistics of the errors are shown in Table 3.4.5. There is a significant reduction in the variance of the errors for Called values as would be expected.

	MEAN ERROR	ACTUAL ERROR VARIANCE	THEORETICAL ERROR VARIANCE
Planned Values	58 cmg/t	136 822	129 969
Called Values	51 cmg/t	23 713	31 754

Table 3.4.5 Statistics of Errors of Data Set 1 (Regularised Samples)

The actual error variances are similar to the corresponding theoretical error variances, although as before, the Called error variance is approximately 30% lower than the theoretical error variance.

The error variances based on regularised samples are marginally higher (for both Called and Planned errors) than the error variances based on individual samples.

Histograms of the errors (Figure 3.4.8) indicate that the errors of the Called panel estimates follow a Normal distribution.

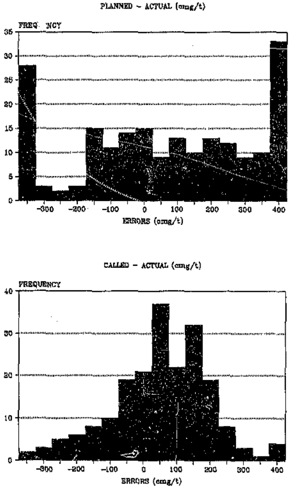


FIG 3.4.8 HISTOGRAMS OF ERRORS (SET 1, REGULARISED SAMPLES)

However, the distribution of the Planned estimates again appear to be Uniformly distributed.

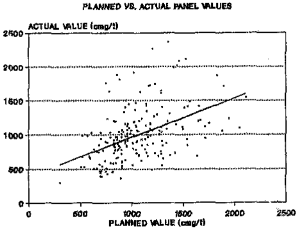
The estimated panel values (Planned and Called) are plotted against the actual values in Figure 3.4.9. The scatter plot of Planned panel values versus the actual panel values indicates the presence of the regression effect. The Called panel values show a high degree of correlation with the actual values and the regression effect is not evident.

Table 3.4.6 compares the statistics of the regression lines for the plots with the theoretical regression line slopes. The actual slopes obtained for the regression lines compare favourably with the theoretical slopes.

	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,568	396	0,738
Called vs Actual Panel Values	0,[illegible]	-49	0,948

Table 3.4.6. <u>Statistics of Regression Lines of Set 1 (Regularised Samples)</u>

The slope of the regression line for Planned estimates (0,568) is lower than the corresponding slope based on individual samples (0,638).



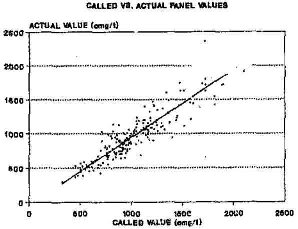


FIGURE 3.4.9 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 1 , REGULARISED SAMPLES)

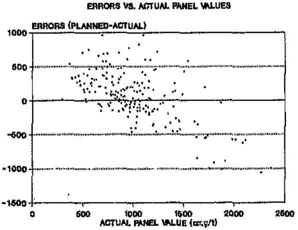
The plot of Called estimates versus the actual panel values also shows that the errors tend to get larger as the panel values increase. This proportional effect (Rendu 1978) is as a result of the variance of values being related to the mean value in Lognormal distributions. This aspect is discussed in detail in sections 3.5 and 3.7.

In Figure 3.4.10, the errors of the estimates are plotted against the actual panel values. The plot of Planned errors again shows that the panels with low value are generally overestimated and the high value panels are generally underestimated The regression line fitted to the scatter plot of planned errors versus actual values has a slope of - 0,613. This slope is steeper than the slope of - 0,551 obtained when using individual samples.

The reasons for the over and underestimation of low and high value panels respectively, are the same as discussed for the results based on individual samples.

3.4.3 Summary of Results of Data Set 1

3.4.3.1 The quality of the estimates for both Planned and Called panel values are not significantly different for individual samples and regularised samples.



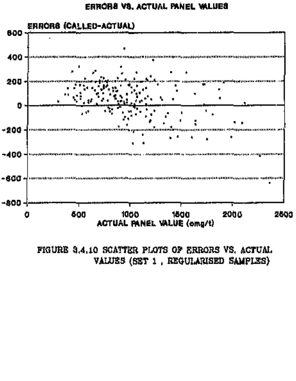


FIGURE 3.4.10 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 1 , REGULARISED SAMPLES)

3.4.3.2 The results obtained from using both individual samples and regularised samples for estimation compare favourably with the expected theoretical results.

Both methods show a slight deterioration in the regression effect for Planned values compared to the expected regression effect based on geostatistical theory.

The actual error variances of Called minus actual values are lower than the expected error variances for both the *Individual Method and Regularised* Method.

3.4.3.3 The error variances obtained from using individual samples are less than those obtained when regularised *samples are used.* The Individual Method shows a reduction of approximately 11% in error variances compared to the Regularised Method.

3.5 Analysis of Results of Data Set 2

3.5.1 Results using Individual Samples for Kriging

A colour coded diagram of the results of Planned, Called and actual panel values is presented in Figure 3.5.1.

The plot of the actual panel values clearly illustrates that the upper half of the panels have a significantly lower average value than that of the bottom half. In addition, the variability of the upper section is also lower than that of the bottom section. Lognormal distribution theory states that the variance of the distribution is directly proportional to the mean value of the distribution according to the formula:

$$V = m^2 \left(e^{\sigma_L^2} - 1\right)$$

Where V = Variance in natural units
m = Mean value
σ_L^2 = Logarithmic variance

Although there is a difference in mean values between the upper and lower sections of the simulated data set, the analysis was conducted by treating the entire set as one homogenous area.

The experimental semi-variogram calculated from the entire set compared favourably with the theoretical semi-variogram used to simulate the data set. (See Section 3.2).

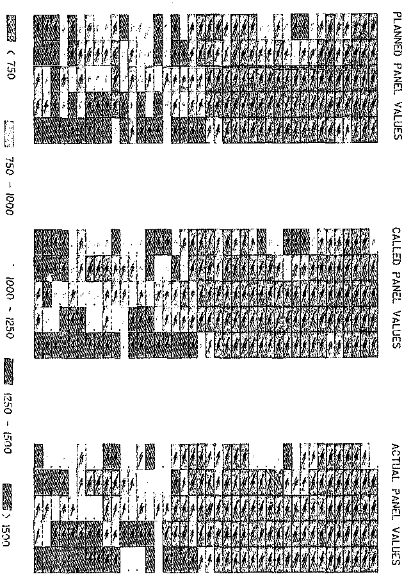


FIGURE 8.6.1 : COLOUR CODED PLOTS OF THE PANEL VALUES USING INDIVIDUAL SAMPLES

The experimental semi-variogram thus confirmed that the spatial characteristics of the simulated data corresponded to what was required from the simulation. Consequently, the entire set could be considered to be a single population of values with the desired variability and spatial characteristics.

The statistics of the Planned, Called and actual panel values are given in Table 3.5.1.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	906 cmg/t	893 cmg/t	840 cmg/t
Variance	178 860	179 341	[illegible]
Log Variance	0,24	0,26	[illegible]

Table 3.5.1. Statistics of Results of Data Set 2 (Individual Samples)

The Planned panel values and Called panel values both appear to be *biased*. This however, is as a result of the 5 metre samples averaging 888 cmg/t as opposed to the 1 metre sample results averaging 840 cmg/t.

The smoothing effect of kriging is <u>not</u> evident in the estimated panel values. The variances of the Planned and Called values are 178 860 and 179 341 respectively, compared to the actual panel variance of 164 465.

A possible reason for this is that the kriging estimates also use the samples from the region used to calculate the semi-variogram. (i.e. the bottom 200 metre section of the 600 metres simulated).

The overall actual panel variance of the entire 600 metre region is 242 082. The variability of the entire da a set is thus substantially higher than the variability of the upper 400 metres of the data set. As kriging uses samples from the entire set, the variability of the estimates are higher than the actual variability of the upper 400 metres.

The errors in estimation are shown in Figure 3.5.2. Both the Planned errors and Called errors show a distinct clustering of different magnitudes of errors. The upper half of the area has far smaller errors than the lower half of the area.

Table 3.5.2 shows the statistics of the errors for different subsets of the area.

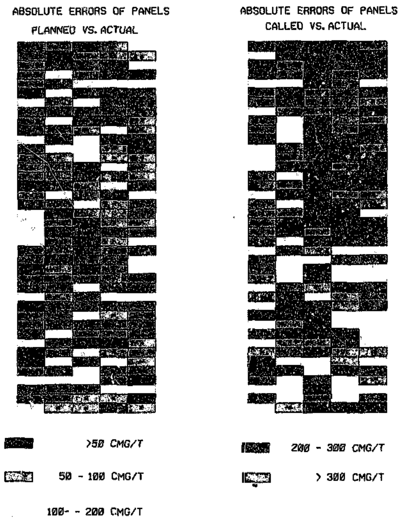


FIGURE 3.5.2 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALUES FOR DATA SET 2 - INDIVIDUAL SAMPLES

	PLANNED VALUES	CALLED VALUES
ENTIRE AREA		
Mean Error	66 cmg/t	53 cmg/t
Error Variance	46 577	11 495
Theoretical Error Variance	76 505	22 933
Mean Absolute Error	172 cmg/t	88 cmg/t
Absolute Error Variance	21 181	6 424
TOP 200 METRES		
Mean Error	43 cmg/t	37 cmg/t
Error Variance	22 969	5 809
Mean Absolute Error	117 cmg/t	61 cmg/t
Absolute Error Variance	11 061	3 470
BOTTOM 200 METRES		
Mean Error	89 cmg/t	69 cmg/t
Error Variance	69 576	16 802
Mean Absolute Error	228 cmg/t	116 cmg/t
Absolute Error Variance	25 323	7 890

Table 3.5.2. Statistics of Errors of Data Set 2 (Individual Samples)

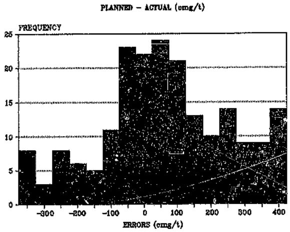
The statistics in the Table show that the mean absolute errors of the bottom 200 metre area are approximately twice the size of the mean absolute errors of the top 200 metre areas. This ratio of errors applies to both Planned and Called panel values. The absolute error variance of the bottom 200 metre area are also approximately double the value of the corresponding error variances in the upper 200 metre area.

The theoretical error variances, based on the semi-variogram model and the chosen data configurations, are approximately twice the size of the actual error variances for Planned estimates and Called estimates. The statistics in Table 3.5.2 show that the error variances of the bottom 200 metre area are approximately equal to the expected theoretical error variances. The reason for the large discrepancy between the theoretical error variances and the observed error variances for the entire area is that the errors in the upper half are very small. The errors in this section are small because the samples in this section are highly continuous and have low variability. This low variability results in more accurate estimation.

Histograms of the errors are presented in Figure 3.5.3. The errors of the Called estimates appear to be Normally distributed but the errors of Planned estimates are erratic

In Figure 3.5.4 the estimated panel values are plotted against the actual panel values. The Planned panel values show evidence of the regression effect. The Called panel values show a very close relation to the actual panel values. Although the slope of the regression line for Called values is approximately equal to 1, further evidence of the proportional effect is shown by the magnitude of errors increasing with the actual values of the panels.

Statistics of the fitted regression lines are contained in Table 3.5.3.



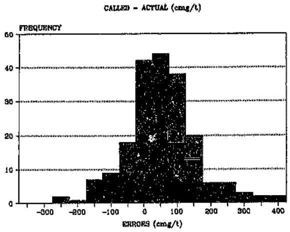
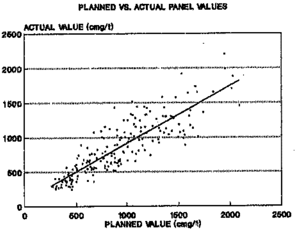


FIG 3.5.3 HISTOGRAMS OF ERRORS (SET 2, INDIVIDUAL SAMPLES)



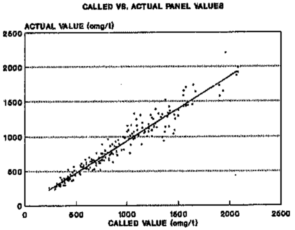


FIGURE 3.5.4 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 2 , INDIVIDUAL SAMPLES)

	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,83	88	0,871
Called vs Actual Panel Values	0,926	13	0,975

Table 3.5.3. <u>Statistics of Regression Lines of Data Set 2 (Individual Samples)</u>

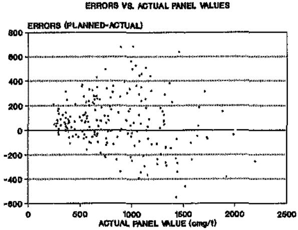
The theoretical slopes of the regression lines are approximately equal to the actual slopes obtained, where the actual slopes are within 5% of the theoretical slopes.

Figure 3.5.5 shows the errors of the estimates plotted against the actual panel values. No specific trends exist in either plot, but evidence of the proportional effect is again illustrated where the magnitude of errors tends to increase as the actual panel values increase.

3.5.2 Results using Regularised Samples for Kriging

A colour coded plot of the panel values based on Planned estimates, Called estimates and actual values is shown in Figure 3.5.6.

The statistics of the panel values are given in Table 3.5.4.



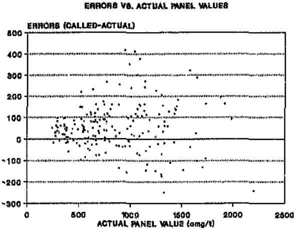
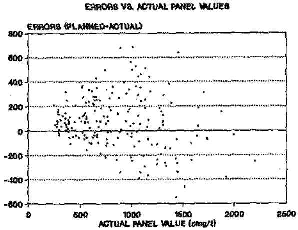


FIGURE 3.5.5 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 2 , INDIVIDUAL SAMPLES)



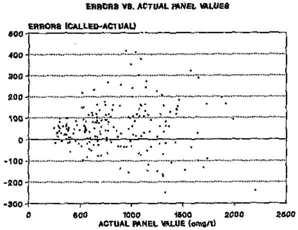


FIGURE 3.5.5 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 2 , INDIVIDUAL SAMPLES)

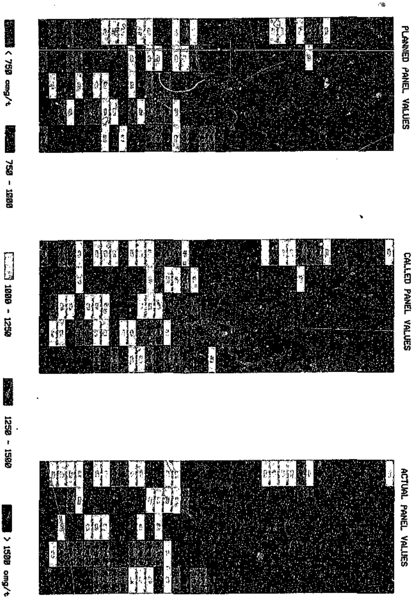


FIGURE 3.5.6 : COLOUR CODED PLOTS OF PANEL VALUES

DATA SET 2 (REGULARISED SAMPLES)

The reason for the apparent overestimation of values is because the 5 metre sampling results averaged 888 cmg/t opposed to the 1 metre samples averaging 840 cmg/t.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	903 cmg/t	891 cmg/t	840 cmg/t
Variance	183 689	185 121	164 465
Log Variance	0,24	0,26	0,25

Table 3.5.4. Statistics of Results of Data Set 2 (Regularised Samples)

The reason for the higher variances of the estimated panel values is the same as discussed in the previous section. The estimates use samples from the 200 metre section used to construct the semi-variogram model. These samples add to the overall variability of the data set where the actual panel variance is 242 082 for the entire set and only 164 465 for the 400 metre area which is compared with the panel estimates.

The panel errors for Planned values and Called values are shown in *Figure 3.5.7. The plots again show a distinct clustering of* different sized errors. The upper section has mainly small errors whereas the bottom section has errors of larger magnitude. The reason for this is again the same as that explained in section 3.5.1, where the sample results in the upper section are mainly of low value and the samples have low variability, leading to more accurate estimating.

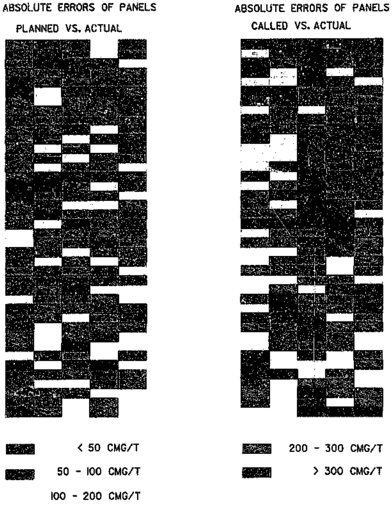


FIGURE 3.5.7 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALUES FOR DATA SET 2 - REGULARISED SAMPLES

Table 3.5.5 summarises the statistics of the errors for the different estimates and for different sub-areas of the data set.

	PLANNED VALUES	CALLED VALUES
ENTIRE AREA		
Mean Error	63 cmg/t	50 cmg/t
Error Variance	47 093	12 591
Theoretical Error Variance	87 816	20 470
Mean Absolute Error	174 cmg/t	90 cmg/t
Absolute Error Variance	20 590	7 077
TOP 200 METRES		
Mean Error	41 cmg/t	35 cmg/t
Error Variance	20 288	5 181
Mean Absolute Error	112 cmg/t	58 cmg/t
Absolute Error Variance	9 394	2 982
BOTTOM 200 METRES		
Mean Error	85 cmg/t	66 cmg/t
Error Variance	73 388	19 619
Mean Absolute Error	237 cmg/t	121 cmg/t
Absolute Error Variance	24 087	9 231

Table 3.5.5. Statistics of Errors of Data Set 2 (Regularised Samples)

The analysis of the errors for the Regularised Method shows that the results are almost identical to those of the Individual Method. The average errors for the Regularised Method show a slight improvement (3%) compared to the Individual Method.

The error variances are similar for both methods, where generally the error variances of the Individual Method are slightly lower, but the absolute error variances of the Regularised Method are slightly lower.

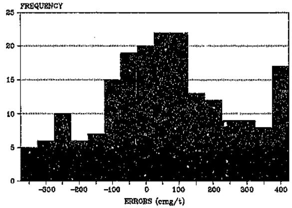
The error variances over the entire data set for both Planned and Called estimates are significantly less than the theoretical error variances. (47 093 vs 87 816 for Planned errors and 12 591 vs 20 470 for Called errors).

The reasons for this are as discussed in 3.5.1, where the area essentially consists of a high value sub-area and a low value sub-area.

Histograms, showing the distribution of errors for Planned and Called estimates based on regularised sample results are presented in Figure 3.5.8. The Called errors follow a Normal distribution but the Planned errors although not as erratic as the Planned errors for the Individual Method are not Normally distributed.

In Figure 3.5.9, the estimated panel values are plotted against the actual panel values. The Planned panel values show evidence of the regression effect where the slope of the regression line is smaller than 1,0. The Called panel values are again highly correlated with the actual values and the slope of the regression line is approximately 1. Both diagrams illustrate that the magnitude of errors increase as the actual values increase. The statistics of the regression lines are contained in Table 3.5.6.

PLANNED - ACTUAL (cmg/t)



CALLED - ACTUAL (cmg/t)

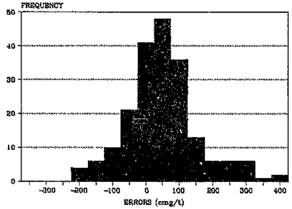
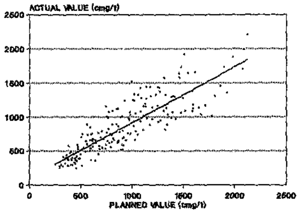


FIG 3.5.8 HISTOGRAMS OF ERRORS (SET 2, REGULARISED SAMPLES)

PLANNED VS. ACTUAL PANEL VALUES



CALLED VS. ACTUAL PANEL VALUES

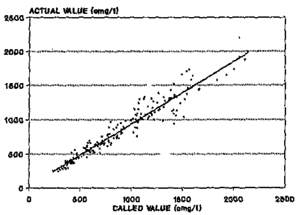


FIGURE 3.6.9 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 2 , REGULARISED SAMPLES)

The slopes of the regression lines for Planned and Called estimates are both slightly less than the corresponding slopes for the Individual Method.

The slopes are also less than the theoretical slopes by approximately 6%.

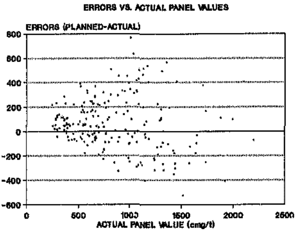
	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,819	100	0,868
Called vs Actual Panel Values	0,91	29	0,977

Table 3.5.6 Statistics of Regression Lines of Data Set 2 (Regularised Samples)

Figure 3.5.10, shows the errors of the two estimates plotted against the actual values. No specific trend in the plots is evident, but the errors in both cases increase as the actual values of the panels get larger. This proportional effect for the errors is evident for both the Individual and Regularised methods.

3.5.3 Summary of Results of Data Set 2

3.5.3.1 The quality of the estimates of Planned values and Called values is insignificantly different for the Individual Method and Regularised Method.



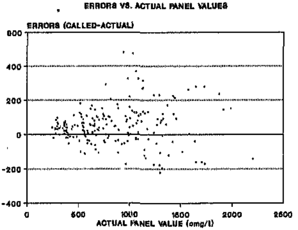


FIGURE 3.5.10 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 2 , REGULARISED SAMPLES)

3.5.3.2 The results obtained for both methods are significantly different from the theoretical results.

The theoretical error variances are approximately twice as large as the actual error variances.

Both methods show a deterioration in the regression effect compared to the theoretical regression slopes.

3.5.3.3 The simulated data set resulted in a low value sub-area (top 200 metres) and a high value sub-area (bottom 200 metres). The errors for both methods were substantially different in the two sub-areas where the errors were small in the low value area and large in the high value area.

These results illustrate the well known proportional effect of Lognormal distributions where the variance is a function of the mean value.

3.5.3.4 The large differences between the actual and theoretical results illustrate the importance of subdividing areas into statistically homogenous sub-areas. These sub-areas should then be analysed separately resulting in more representative semi-variogram models for each area. The theoretical error variances of the sub-areas should then be approximately the same as the error variances of the results because models with different sill values (variances) would be used.

3.6 Analysis of Results of Data Set 3

3.6.1 Results of using Individual Samples for Kriging

The results of Data Sets 3 and 4 are similar to the results of Data Sets 1 and 2, respectively. Consequently, the explanations of differences in results are not covered in detail in this section and section 3.7 as they are discussed in detail in sections 3.4 and 3.5.

Figure 3.6.1 contains colour coded diagrams of the Planned panel values, Called panel values and actual panel values. A summary of the statistics of the three sets of values is given in Table 3.6.1.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	1 028 cmg/t	1 018 cmg/t	988 cmg/t
Variance	137 248	194 014	206 203
Log Variance	0,13	0,19	0,20

Table 3.6.1. Statistics of Results of Data Set 3 (Individual Samples)

The reason for the apparent overestimation of panel values is that the 5 metre sampling averaged 1 016 cmg/t opposed to the 1 metre sample average of 988 cmg/t.

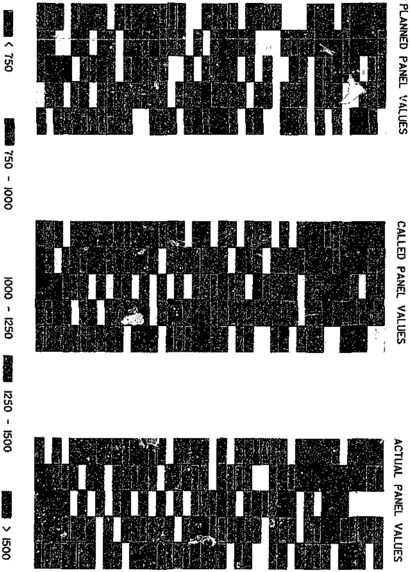


FIGURE 3.6.1: COLOUR CODED PLOTS OF PANEL VALUES
(SET 3 , INDIVIDUAL SAMPLES)

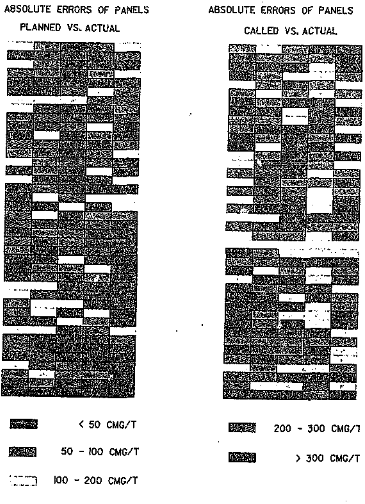


FIGURE 3.6.2 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALL
FOR DATA SET 3 - INDIVIDUAL SAMPLES

A colour coded plot of the absolute errors (Planned vs Actual and Called vs Actual) is shown in Figure 3.6.2. The magnitude of the errors are randomly distributed over the simulated area and there is no specific clustering of high or low errors.

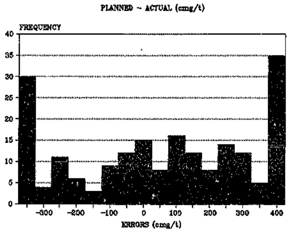
Figure 3.6.3. shows the histograms of the errors for Planned and Called estimates. The Called errors follow a Normal distribution but the Planned errors appear to be Uniformly distributed with a large variance. The statistics of the errors are given in Table 3.6.2.

The actual error variances are lower than the expected theoretical error variances. The error variance of planned estimates is 10% lower than the theoretical error variance, while the error variance of called estimates is 29% lower than the theoretical error variance.

	MEAN ERROR	ACTUAL ERROR VARIANCE	THEORETICAL ERROR VARIANCE
Planned Values	40 cmg/t	150 963	168 265
Called Values	30 cmg/t	13 150	18 486

Table 3.6.2. **Statistics of Errors of Data Set 3 (Individual Samples)**

Figure 3.6.4. shows the scatter plot of actual panel values versus the Planned and Called estimates.



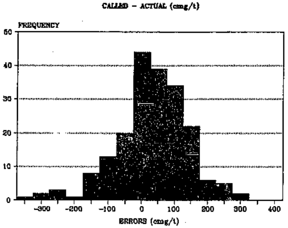
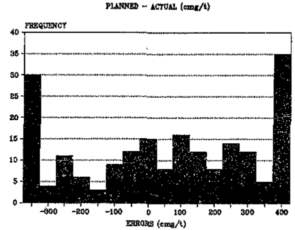


FIG 3.6.3 HISTOGRAMS OF ERRORS (SET 3, INDIVIDUAL SAMPLES)



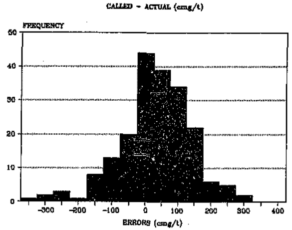


FIG 3.6.3 HISTOGRAMS OF ERRORS (SET 3, INDIVIDUAL SAMPLES)

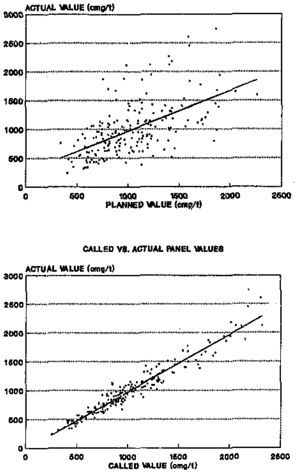


FIGURE 3.6.4 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 3 . INDIVIDUAL SAMPLES)

The scatter plots illustrate that the Planned estimates show the regression effect whereby low estimates are underestimated and high estimates are overestimated Statistics of the regression lines are given in Table 3.6.3. The Called estimates have a regression slope of 0,998 showing no evidence of the regression effect.

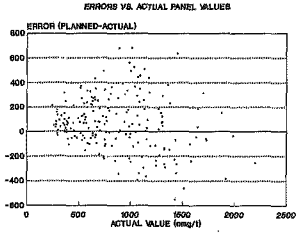
	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,701	267 cmg/t	,814
Called vs Actual Panel Values	0,998	-28 cmg/t	,985

Table 3.6.3 Statistics of Regression Lines for Data Set 3 (Individual Samples)

The slope of the regression for Called estimates is almost identical to the theoretical slope, but the slope for Planned estimates is significantly lower than the theoretical slope.

Figure 3.6.5. shows the errors of the estimates (Planned and Called) plotted against the actual panel values. As for Data Set 1, the Planned errors show that the low panel values are generally overestimated and the panels of high value are generally underestimated

The slope of the regression line (Planned errors on Actual values) is - 0,533.



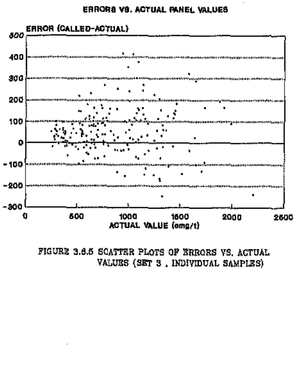


FIGURE 3.6.5 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 3 , INDIVIDUAL SAMPLES)

3.6.2 Results of using Regularised Samples for Kriging

Figure 3.6.6. shows colour coded diagrams of the Planned, Called and Actual panel values. A summary of the statistics of the three sets of values is given in Table 3.6.4.

The apparent overestimation of the Planned and Called estimates is because the 5 metre sampling results averaged 28 cmg/t more than the 1 metre sampling results.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	1 023 cmg/t	1 017 cmg/t	988 cmg/t
Variance	130 874	170 813	206 203
Log Variance	0,12	0,16	0,20

Table 3.6.4. Statistics of Results of Data Set 3 Regularised Samples)

The variances of both Planned and Called panel estimates are 37% and 17% lower than the actual panel variances, respectively.

The variances of the estimates are also lower than the corresponding variances using Individual samples (5% and 12%, respectively) showing that Regularised sampling leads to a larger smoothing effect.

A colour coded plot of the absolute errors (Planned vs Actual and Called vs Actual) is shown in Figure 3.6.7.

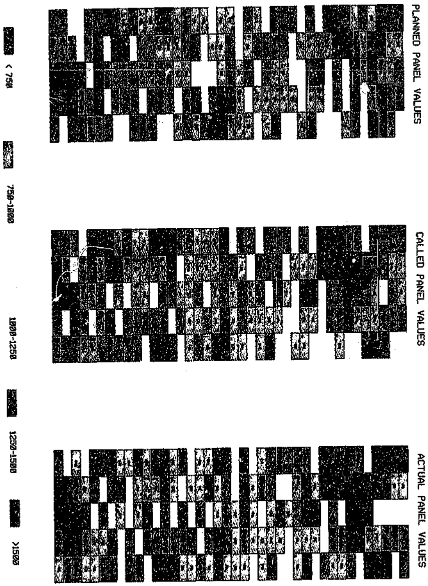


FIGURE 3.6.6 : COLOUR CODED PLOT OF PANEL VALUES FOR DATA SET 3 USING REGULARISED SAMPLES

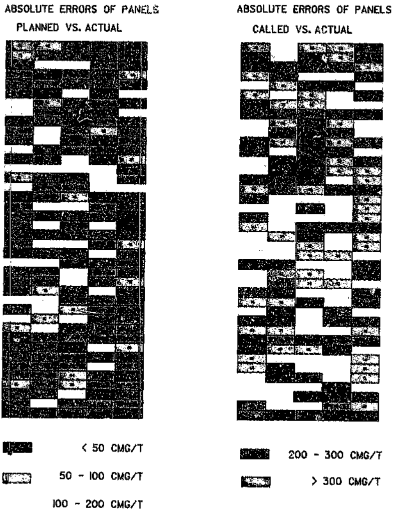


FIGURE 3.6.7 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALUES FOR DATA SET 3 - REGULARISED SAMPLES

The magnitude of the errors are randomly distributed over the simulated area.

Figure 3.6.8. shows the histograms of the errors for the Planned and Called estimates.

The Planned errors have a large variance and appear to be Uniformly distributed whereas the Called errors follow a Normal distribution.

The statistics of the errors are given in Table 3.6.5.

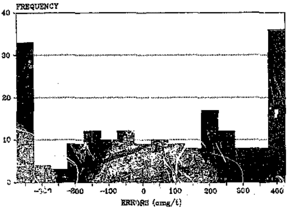
The Planned error variance is 11% higher than the theoretical error variance but the Called error variance is 37% lower than the theoretical error variance. The mean error for *Planned and Called* estimates show a slight improvement compared to the errors for Individual samples (*5 cmg/t and 1 cmg/t respectively*).

	MEAN ERROR	ACTUAL ERROR VARIANCE	THEORETICAL ERROR VARIANCE
Planned Values	35 cmg/t	185 836	166 624
Called Values	29 cmg/t	16 946	26 756

Table 3.6.5 <u>Statistics of Errors of Data Set 3 (Regularised Samples)</u>

However, the error variance for the Regularised Method are significantly higher than the error variances of the Individual Method.

PLANNED - ACTUAL (cmg/t)



CALZ20 - ACTUAL (cmg/t)

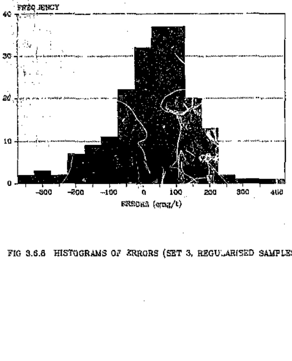


FIG 3.6.8 HISTOGRAMS OF ERRORS (SET 3, REGULARISED SAMPLES)

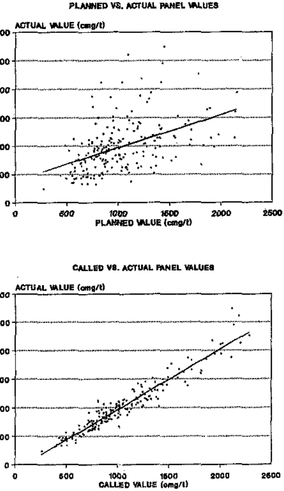


FIGURE 3.6.9 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 3 , REGULARISED SAMPLES)

The Planned error variance is 23% higher and the Called error variance 29% higher than the corresponding error variances for the Individual Method.

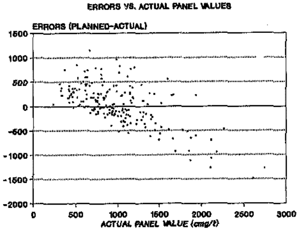
Figure 3.6.9. shows the conditional bias for the Planned estimates where the low estimates are underestimated and the high estimates are overestimated.

The Called estimates have no conditional bias and the slope of the regression line for Actual panel values on Called panel values is close to 1. The statistics of the regression lines for the scatter plots are given in Table 3.6.6.

	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,578	397 cmg/t	0,763
Called vs Actual Panel Values	1,054	-84 cmg/t	0,967

Table 3.6.6. Statistics of Regression Lines of Data Set 3 (Regularised Samples)

The slope of the regression line for Planned estimates (0,578) is significantly worse than the theoretical slope (0,763) of the regression line. This slope is also worse than the corresponding slope for the Individual Method (0,578 versus 0,701).



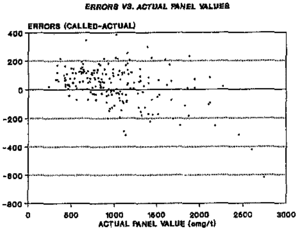


FIGURE 3.6.10 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 3 , REGULARISED SAMPLES)

Estimating ahead of the available sampling thus appears to be worse when using the Regularised Method as the conditional bias has increased and the error variance increased by 23%.

Figure 3.6.10. shows the Planned and Called errors plotted against the Actual panel values. The Planned errors show that the panels of low values are overestimated and the panels of high values are underestimated

The slope of the regression line for Planned errors on Actual panel values is - 0,633.

3.6.3 Summary of Results of Data Set 3

3.6.3.1 The quality of the estimates for the Regularised Method are slightly worse than the estimates using the Individual Method.

The Regularised Method resulted in:

(a) Larger error variances.

(b) More conditional bias of the Planned estimates.

(c) A more pronounced smoothing effect of kriging.

3.6.3.2 The results of both methods compare favourably with the theoretical results for the Called estimates, but statistics of the Planned estimates are significantly different from the theoretical results.

The error variances of Planned estimates are 11% lower than the theoretical error variance for the Individual Method and 23% higher than the theoretical error variance for the Regularised Method.

Both methods show a deterioration in the slope of the regression line for *Planned estimates. Compared to the* theoretical slopes, the Individual Method and Regularised Method show a deterioration of 14% and 24%, respectively.

3.7 Analysis of Results of Data Set 4

3.7.1 Results of using Individual Samples for Kriging

Figure 3.7.1. contains colour coded plots of the Planned panel values, Called panel values and Actual panel values. The plots clearly illustrate that the upper half of the *simulated data* set is low grade and has low variability and the bottom half is high grade and more variable. Data Set 4 is thus similar to Data Set 2 and similar results were obtained where the *"proportional effect"* of [illegible] tributions is evident.

A summary of the statistics of the three sets of panel values is given in Table 3.7.1. The results show that *the mean value* of the estimates are higher than the actual mean value. This is as a result of the 5 metre sampling results averaging 27 cmg/t more than the 1 metre sampling results.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	840 cmg/t	829 cmg/t	801 cmg/t
Variance	221 271	224 171	209 366
Log Variance	0,38	0,40	0,40

Table 3.7.1. Statistics of Results of Data Set 4 (Individual Samples)

The variances of the Planned and Called estimates are approximately the same as the variance for the *actual panel values*.

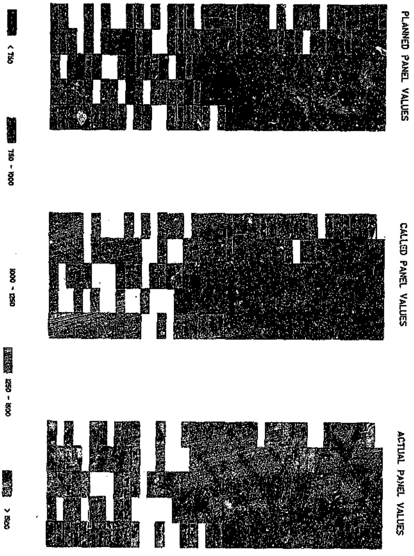


FIGURE 3.7.J: COLOUR CODED PLOTS OF PANEL VALUES
(SET 4 , INDIVIDUAL SAMPLES)

The reason for the non-presence of the smoothing effect of kriging is the same as that explained in 3.5, where the variance of the entire 600 metre simulated set is 337 445. The estimates use sampling results from the additional 200 metre area (used for calculating the semi-variogram) and because of the distinct high and low grade sub-areas the variance of the kriged estimates are higher than would be expected.

A colour coded plot of the absolute errors is given in Figure 3.7.2. The clustering of low errors in the upper position of the set and high errors in the lower portion clearly illustrates the proportional effect whereby the variance is related to the mean value. The sampling results in the upper portion have a low mean value and a relatively low variance compared to the high mean value and high variability of the lower section. The estimated values in the upper portion are thus more consistent with the actual values because the sampling results throughout the area are similar, and consequently the errors in estimation are small. Conversely, the sampling results in the lower section are highly variable and the resulting errors between estimated and actual values are thus higher.

Geostatistical theory is normally used to calculate error variances based only on the sampling positions relative to one another and relative to the block being estimated, the size and geometry of the block being estimated, and the semi-variogram model. The actual sampling results do not affect the estimation of error variances.

FIGURE 3.7.2 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALUES FOR DATA SET 4 - INDIVIDUAL SAMPLES

However, the results in this Set and Data Set 2, clearly illustrate that the errors and error variances are lower in the upper section which has a low mean value and higher in the lower section which has a high mean value. The importance of modelling areas which are statistically homogeneous is thus evident.

Data Sets 2 and 4 would produce results that are more consistent with geostatistical theory if the upper and lower sections of the *simulated areas had been modelled separately.* The model for the upper section would have a relatively low sill value and this would lead to lower theoretical error variances.

Conversely, the model for the lower section would have a relatively high sill value (because the variance of samples is *high*), and the theoretical error variance would be higher.

Statistics of the errors for various sub-areas of the simulated area are shown in Table 3.7.2.

	PLANNED VALUES	CALLED VALUES
ENTIRE AREA		
Mean Error	39 cmg/t	28 cmg/t
Error Variance	45 357	6 156
Theoretical Error Variance	83 564	11 793
Mean Absolute Error	159 cmg/t	58 cmg/t
Absolute Error Variance	21 581	3 582
TOP 200 METRES		
Mean Error	32 cmg/t	24 cmg/t
Error Variance	18 529	2 998
Mean Absolute Error	93 cmg/t	38 cmg/t
Absolute Error Variance	10 827	2 163
BOTTOM 200 METRES		
Mean Error	46 cmg/t	31 cmg/t
Error Variance	72 534	9 355
Mean Absolute Error	224 cmg/t	78 cmg/t
Absolute Error Variance	23 810	4 214

Table 3.7.2. Statistics of Errors for Data Set 4 (Individual Samples)

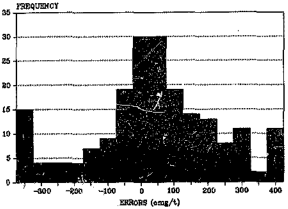
The mean absolute errors of the bottom 200 metre area are twice the size of the corresponding mean absolute errors for the top 200 metre area. The absolute error variances of the bottom sub-area are also double the value of the corresponding error variances of the upper sub-area.

The theoretical error variances, based on the semi-variogram model and chosen data configurations, are approximately twice the size of the actual error variances of the entire area for both Planned estimates and Called estimates. The primary reason for this is because the top 200 metre sub-area has very low error variances. The error variances in this sub-area are less than 50% of the theoretical error variances. This sub-area has small errors because the samples in this section are highly continuous and have low variability. The low variability of sample values corresponds with the low mean value (proportional effect of lognormal distributions) and the low variability results in more accurate estimation.

Histograms of the errors are presented in Figure 3.7.3. Both the Planned errors and Called errors are Normally distributed but the variance of Planned errors is seven times larger than the variance of Called errors.

In Figure 3.7.4. the estimated panel values are plotted against the actual panel values. The Called panel values correspond closely to the actual panel values but the errors increase in magnitude as the actual panel values increase. Further evidence of the proportional effect is thus present. The statistics of the fitted regression lines are shown in Table 3.7.3.

PLANNED - ACTUAL (cmg/t)



CALLED - ACTUAL (cmg/t)

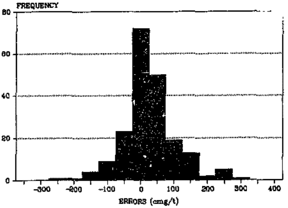
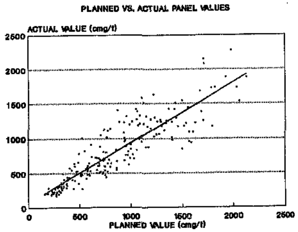


FIG 3.7.3 HISTOGRAMS OF ERRORS (SET 4. INDIVIDUAL SAMPLES)



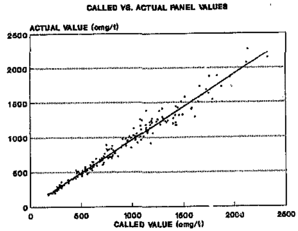


FIGURE 3.7.4 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 4 , INDIVIDUAL SAMPLES)

	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,871	70 cmg/t	0,913
Called vs Actual Panel Values	0,953	11 cmg/t	0,991

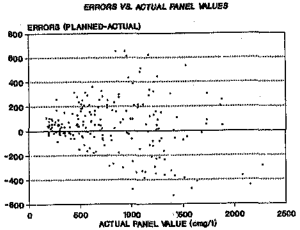
Table 3.7.3. **Statistics of Regression Lines for Data Set 4 (Individual Results)**

The theoretical slopes of the regression lines are approximately equal to the actual slopes obtained. The Planned panel values show evidence of the regression effect whereby on average the low estimates are underestimated and the high estimates are overestimated.

Figure 3.7.5. shows the errors of the estimates plotted against the actual panel v lues. No specific trends exist in either graph, but evidence of tn ,roportional effect is clearly illustrated by the increase in the magnitude of errors as the actual panel values increase.

3.7.2 Results of using Regularised Samples for Kriging

A colour coded plot of the panel values based on Planned estimates, Called estimates and actual values is shown in Figure 3.7.6. The statistics of the panel values are given in Table 3.7.4.



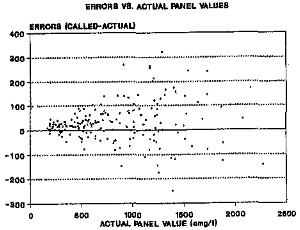


FIGURE 3.7.5 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 4 , INDIVIDUAL SAMPLES)

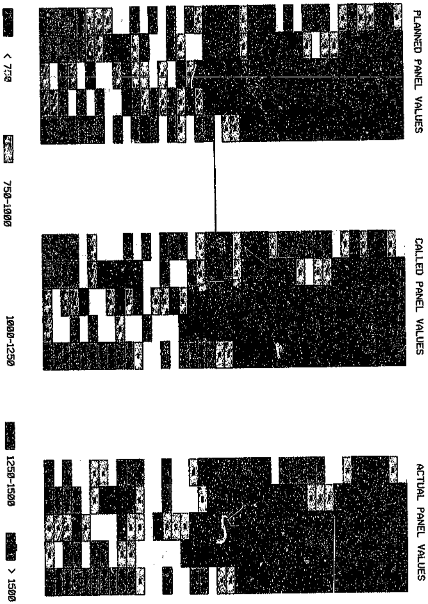


FIGURE 3.7.6 : COLOUR CODED PLOT OF PANEL VALUES FOR DATA SET 4 USING REGULARISED SAMPLES

The variances for Planned and Called panel values are approximately equal to the actual panel variance. The reason for the non-presence of the smoothing effect of kriging is the same as that explained in section 3.5.1. and section 3.7.1.

	PLANNED PANEL VALUES	CALLED PANEL VALUES	ACTUAL PANEL VALUES
Mean Value	843 cmg/t	829 cmg/t	801 cmg/t
Variance	223 315	225 900	209 366
Log Variance	0,37	0,40	0,40

Table 3.7.4. <u>Statistics of Results of Data Set 4 (Regularised Samples)</u>

The panel errors for Planned Values and Called values are shown in Figure 3.7.7. The plots again show a distinct clustering of different sized errors. The upper section has predominantly small errors which correspond to the lower mean grade and lower variability of this section. The bottom section has larger errors which correspond to the higher mean grade and higher variance of sampling results in this section.

Table 3.7.5. summarises the statistics of the errors for the different estimates within the sub-areas of the data set.

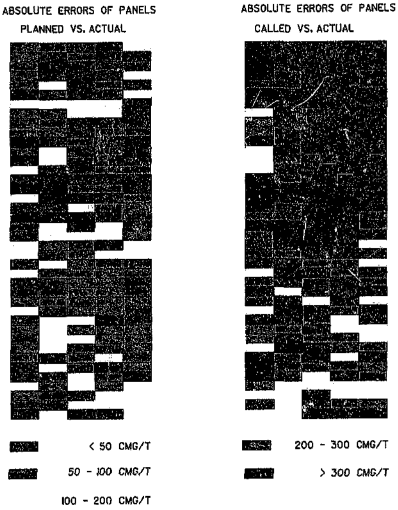


FIGURE 3.7.7 : ABSOLUTE ERRORS (cmg/t) OF PANEL VALUES FOR DATA SET 4 - REGULARISED SAMPLES

	PLANNED VALUES	CALLED VALUES
ENTIRE AREA		
Mean Error	41 cmg/t	28 cmg/t
Error Variance	42 909	6 817
Theoretical Error Variance	89 343	9 642
Mean Absolute Error	159 cmg/t	60 cmg/t
Absolute Error Variance	19 365	4 045
TOP 200 METRES		
Mean Error	27 cmg/t	22 cmg/t
Error Variance	15 450	2 545
Mean Absolute Error	89 cmg/t	36 cmg/t
Absolute Error Variance	8 186	1 715
BOTTOM 200 METRES		
Mean Error	56 cmg/t	34 cmg/t
Error Variance	70 384	11 080
Mean Absolute Error	226 cmg/t	83 cmg/t
Absolute Error Variance	20 966	5 301

Table 3.7.5. Statistics of Errors for Data Set 4 (Regularised Samples)

The analysis of the errors for the Regularised Method shows that the results are almost identical to those of the Individual Method.

The error variances of the Planned estimates using the Regularised *Method are smaller (3% to 25%) than the corresponding error variances* using the Individual Method.

The error variances over the entire data set are significantly less than the theoretical error variances for both Planned and Called estimates (42 909 vs 89 343 and 6 817 vs 9 642).

The reasons for this are as discussed in 3.7.1. where the area essentially consists of a high value sub-area and a low value sub-area.

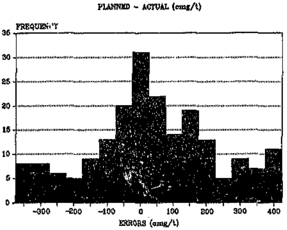
Histograms, showing the distributions of Planned and Called errors are shown in Figure 3.7.8. The errors are Normally distributed for both sets of estimates.

In Figure 3.7.9. the estimated panel values are plotted against the actual panel values. The statistics of the regression lines for the two sets of estimates are shown in Table 3.7.6. Both graphs illustrate that the magnitude of errors increase as the actual values increase. The Called panel values are highly correlated with the actual panel values but the Planned panel values show evidence of the regression effect where the slope of the regression line is 0,873.

	ACTUAL RESULTS		THEORETICAL SLOPE OF LINE
	SLOPE OF LINE	Y INTERCEPT	
Planned vs Actual Panel Values	0,873	66 cmg/t	0,904
Called vs Actual Panel Values	0,948	15 cmg/t	0,99

Table 3.7.6. Statistics of Regression Lines for Data Set 4 (Regularised Samples)

The slopes of the regression lines are approximately equal to the theoretical slopes and are almost identical to the slopes obtained using the Individual Method.



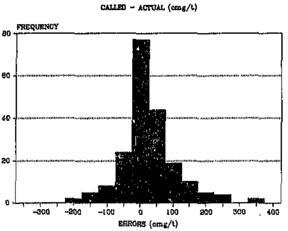
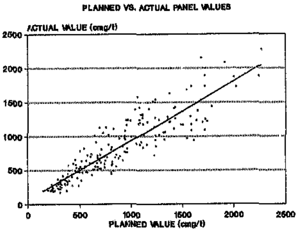


FIG 3.7.6 HISTOGRAMS OF ERRORS (SET 4, REGULARISED SAMPLES)



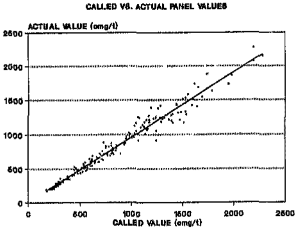


FIGURE 3.7.9 SCATTER PLOTS OF ESTIMATED VALUES VS. ACTUAL VALUES (SET 4 , REGULARISED SAMPLES)

Figure 3.7.10. shows the errors of the two estimates plotted against the actual values. These graphs again illustrate that the errors increase as the actual panel values increase.

3.7.3 Summary of Results of Data Set 4

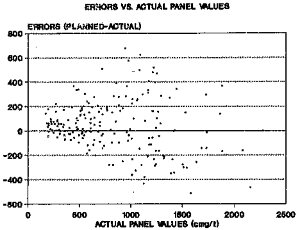
3.7.3.1 The quality of the estimates of Planned values and Called values, is insignificantly different for the Individual Method and the Regularised method.

3.7.3.2 Both methods resulted in error variances which were *significantly lower than the theoretical error variances.*

The slopes of regression lines for both methods are marginally lower than the theoretical slopes.

3.7.3.3 The errors of both methods increased as the actual panel values increased. This illustrates the proportional effect of Lognormal distributions where the variance is a function of the mean value.

3.7.3.4 The large differences between the actual error variances and theoretical variances are as a result of the simulated data set having distinct high and low grade sub-areas. The results illustrate the importance of subdividing an area into statistically homogeneous areas before applying geostatistical analysis.



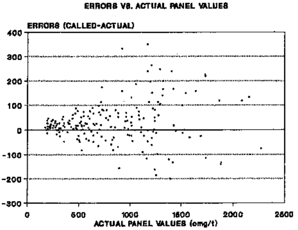


FIGURE 3.7.10 SCATTER PLOTS OF ERRORS VS. ACTUAL VALUES (SET 4 , REGULARISED SAMPLES)

3.8 Results of the 5 Column Positions

The accuracy of the estimates differ for the column positions because different data configurations are used for end panels (1 and 5) and middle panels (2, 3 and 4). A summary of the results for the Individual Method is given in Table 3.8.1.

	CALLED ESTIMATES		PLANNED ESTIMATES	
	MEAN ERROR	ERROR VAR	MEAN ERROR	ERROR VAR
DATA SET 1				
End Panels	61 cmg/t	26 084	78 cmg/t	132 826
Middle Panels	49 cmg/t	18 600	54 cmg/t	117 217
DATA SET 2				
End Panels	66 cmg/t	16 585	59 cmg/t	63 214
Middle Panels	44 cmg/t	8 027	71 cmg/t	36 743
DATA SET 3				
End Panels	29 cmg/t	16 406	50 cmg/t	159 853
Middle Panels	31 cmg/t	11 241	34 cmg/t	149 191
DATA SET 4				
End Panels	46 cmg/t	9 116	34 cmg/t	63 873
Middle panels	15 cmg/t	3 791	42 cmg/t	34 447

Table 3.8.1. Summary of Panel Errors (Individual Methods)

The results for the Regularised Method are given in Table 3.8.2

	CALLED ESTIMATES		PLANNED ESTIMATES	
	MEAN ERROR	ERROR VAR	MEAN ERROR	ERROR VAR
DATA SET 1				
End Panels	61 cmg/t	27 282	73 cmg/t	148 102
Middle Panels	45 cmg/t	21 999	49 cmg/t	132 932
DATA SET 2				
End Panels	73 cmg/t	17 274	61 cmg/t	62 575
Middle Panels	35 cmg/t	9 265	65 cmg/t	38 370
DATA SET 3				
End Panels	30 cmg/t	20 281	43 cmg/t	205 196
Middle Panels	29 cmg/t	15 228	30 cmg/t	178 233
DATA SET 4				
End Panels	49 cmg/t	9 435	32 cmg/t	55 728
Middle Panels	14 cmg/t	4 752	48 cmg/t	35 707

Table 3.8.2. Summary of Panel Errors (Regularised Method)

The results for both methods clearly illustrate that the error variances of the middle panels are significantly lower than those of the end panels.

The error variances are also lower than the expected theoretical error variances (see Table 3.3.1. and Table 3.3.2).

The error variances of the Regularised Method are also higher than those of the Individual Method in 14 of the 16 cases.

The error variances of the Regularised Method are 11,2% higher on average than those of the Individual Method.

This analysis confirms the earlier results that indicate that the error variances of the Regularised Method are generally higher than those of the Individual Method for all 4 data sets.

4. CONCLUSION

4.1 Regularised Method versus Individual Method

The results of the Individual Method were generally marginally better than those of the Regularised Method.

The error variances of the Regularised Method were higher than those of the Individual Method in seven out of the eight sets of estimates. The average increase in the error variances for the Regularised Method was 11,5%.

A summary of the slopes of the regression lines for the two methods is given in Table 4.1.1.

	PLANNED ESTIMATES		CALLED ESTIMATES	
	INDIVIDUAL METHOD	REGULARISED METHOD	INDIVIDUAL METHOD	REGULARISED METHOD
Data Set 1	,63°	,568	,976	,998
Data Set 2	,83	,819	,926	,91
Data Set 3	,701	,578	,998	1,054
Data Set 4	,871	,873	,953	,948

Table 4.1.1. **Slopes of Regression Lines for the 2 Methods**

The slopes of the regression lines for the Called estimates show virtually no differences for the two methods.

However, for the Planned estimates, the slopes of the regression lines for the Individual Method are generally better than those for the Regularised Method.

Although the Individual Method shows marginally better results than the Regularised Method, computer processing requirements also need to be considered.

The number of samples used in kriging, the computer processing time and the programming effort are all significantly reduced when regularised data is used.

In the authors opinion, the marginal improvements obtained from the Individual Method do not justify the use of this method in preference to the Regularised Method - when the additional cost of computer programming and computer processing is taken into consideration.

However, the grid size into which sampling data will be regularised needs to be carefully determined. An acceptable grid size will depend not only on the spatial characteristics of the orebody but also on the application. An application for monthly panel valuation will probably need a smaller grid size than the application for ore block valuation or global valuation.

4.2 Expected Theoretical Results

The results for data sets 1 and 3 were approximately equal to the expected theoretical results. The error variances of Called estimates were however substantially lower than the theoretical error variances for both methods. The error variances of Planned estimates are within 10% of the theoretical error variances.

The regression line slopes for sets 1 and 3 for Called estimates are almost identical to the theoretical slopes of the regression lines but both methods had lower slopes than the theoretical slopes for Planned estimates.

The error variances for data set 2 and 4 were significantly different to the theoretical error variances for both methods. The reason for these marked differences is because the simulated data sets had distinct high grade and low grade sub-areas. The proportional effect for Lognormal distributions resulted in distinct clustering of high errors and low errors corresponding to the high and low grade sub-areas.

The importance of ensuring that areas are statistically homogeneous when using geostatistical techniques for valuation is thus highlighted. This aspect is discussed in more detail in 4.3.

4.3 Confidence Intervals

As discussed in the previous section, the error variances for data set 2 and 4 showed marked differences compared to the theoretical error variances. The error variances calculated for kriged estimates are based on the semi-variogram model, the sampling configuration and the block to be estimated and not on the sample results. However, the spatial characteristics of sampling results are incorporated by their use in the calculation of the experimental semi-variogram.

The variability of sampling results is thus built into the semi-variogram model. In order to obtain meaningful and representative semi-variogram models it is imperative that, the sampling data used in the calculation of the semi-variogram and the remaining area where the model will be used, are from a statistically homogeneous area.

This condition is one of the major assumptions used in geostatistics and is stressed in all text books on the subject. However, in practice, the monitoring of sampling results, to ensure that this condition is satisfied and that semi-variogram models are representative, is a time consuming task.

The importance of ensuring statistical homogeneity is clearly illustrated in the results of data sets 2 and 4.

Both data sets had distinct high and low grade sub-areas with high and low variability respectively (the Lognormal proportional effect where variance is directly proportional to mean grade). Although the distinct sub-areas existed, the semi-variogram models obtained for both sets, were well behaved and could be modelled with par meters which were very close to the parameters used in the simulation of the data sets.

Applying the models to the entire data sets resulted in distinct sub-areas of low errors and high errors corresponding directly to the low and high variability of sampling results. The theoretical error variances are thus substantially different from the error variances for the entire area, the low grade sub-area and the high grade sub- a.

Consequently, if the theoretical error variances were used to construct confidence intervals of estimates, they would differ substantially from the actual distribution of errors. This problem can be overcome by ensuring that areas being analysed are homogeneous and the sampling results exhibit stationarity.

A further complicating factor in the calculation of confidence intervals is the actual distribution of errors. Traditionally, the distribution of errors have been assumed to be either Normally distributed of Lognormally distributed, in the case of gold. The histograms of errors given in chapter 3 appeared to be Normally distributed for Called estimates but the distributions were very errat for the Planned estimates. Probability plots of the errors are given in Appendix C.

4.4 Conditional Bias

A measure of the conditional bias in the estimates can be obtained from the slopes of the fitted regression lines.

The slopes (Table 4.1.1.) clearly illustrate that the Planned estimates are conditionally biased. The slopes ranged from 0,568 to 0,873 indicating that low value estimates are under-estimated on average and high value estimates are over-estimated.

The Called estimates had slopes which were generally very close to unity.

The slopes obtained for the Planned estimates are surprisingly low when one consideres that the blocks to be estimated were only a distance of 10 metres from the nearest sampling information. Simple kriging could be used to overcome this conditional bias if the mean value could be estimated with confidence.

4.5 Recommendations

4.5.1 This project report only dealt with the comparison of a 10 metre regularisation with using individual samples for kriging.

In addition, an excellent sampling coverage on a five metre grid was assumed.

Additional work is required to determine the effectiveness of different sized regularisations for different sampling densities. Also, the applicability of different sized regularisations for the applications of monthly valuation and ore reserve block valuation need additional research.

REFERENCES

Auges User Manual, A geostatistical evaluation system for tabular or reef deposits, Control Data Corporation of South Africa.

Brooker P.J. 1983 Semi-variogram estimation using a simulated deposit: Mining Engineering.

Journel A.G. and Huijbregts Ch.J. 1978, Mining Geostatistics, Academic Press, London.

Krige D.G. 1961, Developments in the valuation of gold mining properties from borehole results, Seventh Commonwealth Min. and Metall. Congress, Johannesburg.

Krige D.G. 1962, Economic aspects of stoping through unpayable ore, Journal of SAIMM, Johannesburg.

Magri E.J. 1983, Improved geostatistics for South African gold mining, PHD thesis in Faculty of Engineering, University of the Witwatersrand, Johannesburg.

Matheron G. 1970, The theory of regionalized variables: Centre de Geostatistique, Ecole des Mines de Paris, Fontainebleau, France.

Miller S.L. 1983, Geostatistical evaluation of a gold ore reserve system. M.Sc thesis, University of South Africa, Pretoria.

Miller S.L. et al 1987, The implementation of a production gold ore reserve system (GORS). Proceedings of the Twentieth International Symposium on the Application of Computers and Mathematics in the Mineral Industries. Volume 3: Geostatistics, Johannesburg SAIMM.

Philip G.M. and Watson D.F. 1986, Metheronian Geostatistics - Quo Vadis? Mathematical Geology Volume 18, No. 1.

Ravenscroft P.J. 1985, Kriging estimation - Effect of Nugget Effect and Variogram range. Internal report of Ore Evaluation Department, De Beers Consolidated Mines Limited.

Rendu J.M. 1978, An introduction to geostatistical methods of mineral evaluation, SAIMM.

Rivoirard J. 1987, Two key parameters when choosing the kriging neighbourhood. Mathematical Geology, Volume 19, No. 8.

Sichel H.S. 1966, The estimation of means and associated confidence limits for small samples from log-normal populations.

APPENDIX A

COMBINING SIMULATED RESULTS

In order to obtain a simulated deposit that exhibits characteristics of more than one structural model it is necessary to be able to combine independently simulated results.

Each separate simulation follows the standard Gaussian distribution and is regionally distributed according to a chosen semi-varigram model.

As an example, the combining of two simulations is illustrated below;

Simulation 1 : $Z_1 \sim N(0,1)$ Nugget Effect Model

Simulation 2 : $Z_2 \sim N(0,1)$ Spherical Model

The combining of the two simulations needs to result in a standard Gaussian distribution in order to apply a Gaussian transform function to arrive at the required distribution of values.

A linear combination of the two simulations is obtained as follows:

$a_1Z_1 + a_2Z_2$ follows a Gaussian distribution with

Mean $= a_1 0 + a_2 0 = 0$

Variance $= a_1^2 1 + a_2^2 1 = a_1^2 + a_2^2$

To follow a standard Gaussian distribution this implies:

$$a_1^2 + a_2^2 = 1 \ldots\ldots\ldots\ldots\ldots \ A_1$$

The two simulations can also be weighted in such a way so as to ensure that a certain percentage of the total sill value can be attributed to one of the models.

This weighting can be represented as:

$$a_1/a_2 = R \text{ (a desired ratio) } \ldots\ldots\ldots\ldots\ldots\ldots \ A_2$$

Therefore,

$$a_1 = a_2R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ A_3$$

Substituting A_3 in A_1 implies:

$$a_2^2R^2 + a_2^2 = 1$$

Therefore,

$$a_2 = \sqrt{1/(1 + R^2)}$$

$$a_1 = a_2R$$

The above weighting ensures that the combined simulation results follow a standard Gaussian distribution. Specified proportions of overall variance can also be ascribed to particular models.

APPENDIX B – THEORETICAL CALCULATION OF REGRESSION SLOPE

For an estimator to be conditionally unbiased then:

$$E\,[Zv/Zv^*] = Zv^*$$

where,

Zv = true grade of block

Zv^* = estimate of block

If σ^{*2} and σv^2 are the respective variances of Zv^* and Zv, then if σ^{*2} is larger than σv^2, Zv^* exaggerates the frequency of both high and low values.

The shape of the regression curve $E\,[Zv/Zv^*]$ is seldom known, but to ensure that the estimator is not too far from being conditionally unbiased, the slope of the regression line should be close to 1.

This requires that:

$$\mathrm{Cov}\,(Zv, Zv^*) = \mathrm{Var}\,(Zv^*)$$

For simple kriging this relationship is satisfied, but for ordinary kriging (OK):

$$\mathrm{Var}\,(Zv^{ok}) = \mathrm{Cov}\,(Zv, Zv^{ok}) + \mu$$

The regression slope P is given by:

$$P = \mathrm{Cov}\,(Zv, Zv^{ok})/\mathrm{Var}\,(Zv^{ok})$$

therefore,

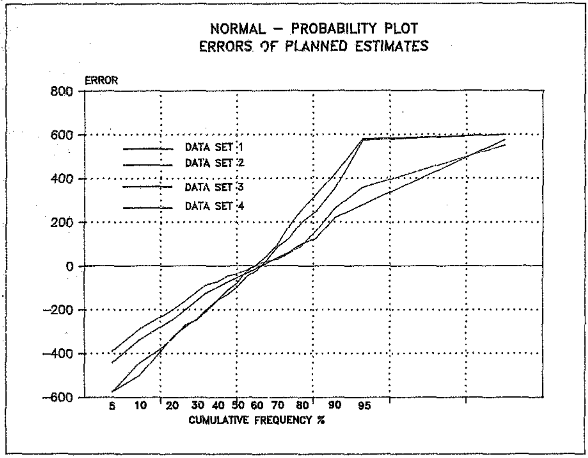
$$P = \mathrm{Cov}\,(Zv, Zv^{ok})/[\mathrm{Cov}(Zv, Zv^{ok}) + \mu]$$

Also $\mu = \zeta m\,\mu o$ where:

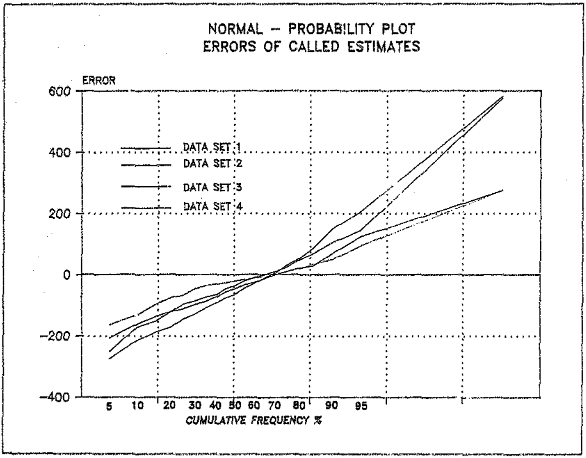
$\mu o = \mathrm{Var}\,(m^* - m)$

ζm = weight of mean in simple kriging

APPENDIX C : NORMAL PROBABILITY PLOTS


NORMAL – PROBABILITY PLOT
ERRORS OF PLANNED ESTIMATES
ERROR
800
600
400
200
0
-200
-400
-600
DATA SET 1
DATA SET 2
DATA SET 3
DATA SET 4
5
10
20
30
40
50
60
70
80
90
95
CUMULATIVE FREQUENCY %

-130-


NORMAL – PROBABILITY PLOT
ERRORS OF CALLED ESTIMATES
ERROR
600
400
200
0
-200
-400
DATA SET 1
DATA SET 2
DATA SET 3
DATA SET 4
5 10 20 30 40 50 60 70 80 90 95
CUMULATIVE FREQUENCY %