A Comparison of Black-Scholes versus Weibull Distribution Option Pricing Models in South Africa

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ABSTRACT

The purpose of this study is to compare the accuracy of two options pricing models, namely the Black-Scholes (1973) and Savickas's Simple option pricing model (2002), in the pricing of options. The models were compared using data on options traded on JSE Securities (Ltd). This paper looks at the differences in option pricing models and the apparent shortcomings of the Black-Scholes model. The JSE data was divided into Puts and Calls, and longer dated versus shorter dated derivatives.

A chi-squared test was used in testing the results, and it was found that the Savickas option pricing model yielded results that were a better fit with the JSE data than the Black-Scholes. This is true for both longer and shorter dated options and to a lesser extent for the Puts.

The conclusion is that the Savickas option pricing model and its assumptions that share price returns follow a Weibull distribution, is more accurate in pricing options than the Black Scholes model. The Black Scholes assumption that share price returns follow a log-normal distribution seems to be unrealistic.

DECLARATION

I, Anthea Gardner, declare that this research report is my own work except as indicated in the references and acknowledgements. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration in the University of the Witswatersrand, Johannesburg. It has not been submitted before for any degree or examination in this or any other university.

Anthea Nicolette Gardner

Signed at Morgan Stanley, London

On the 20th day of October 2008

DEDICATION

This research report is dedicated to my parents, Errol and Meredith, for their commitment to ensuring that I understood the value of knowledge and hard work.

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A special thank you goes to Professor Mthuli Ncube for his patience and persistence in getting me to undertake and complete this report.

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1. INTRODUCTION

1.1. Purpose of Study

The purpose of the study is to compare the accuracy of two option pricing models, namely the Black-Scholes (1973) and the Savickas (2002) option pricing model, in the pricing of options. The Black-Scholes is based on the lognormal distribution and Savickas's model assumes returns follow a Weibull distribution. In a way, this study is a test of whether returns follow a Weibull or lognormal distribution. The models are tested on the JSE Securities Ltd options data.

1.2. Context of Study

Market makers use a variety of pricing methods for pricing options. The prices that the market makers advertise or show their clients are based mostly on the Black-Scholes formula, combined with factors that take into account their view of future market movements and the loose relationship between implied volatility and the volatility skews[1]. There have been sources of literature that contest the popularity of the Black-Scholes model. In their paper De La Peña, Ibragimov, & Jordan, (2004) make the claim that the binomial option model is currently the most widely used model in Wall Street. Whilst others such as Lim, Martin, & Martin (2003) open their study with comments claiming that the Black-Scholes (1973) model is the most commonly used model in practice.

Schönbucher (1999) emphasises his proposition that Black-Scholes will show deviations from the market, not because of pricing errors, but because of the inaccuracies of the assumptions of the Black- Scholes model.

[1] Volatility skews are the term used for way in which implied volatilities vary across different strike prices for a fixed expiration.

It is with these comments in mind that this study will make a comparison of the Black-Scholes Option Pricing model which is based on the lognormal distribution and a model developed by Robert Savickas in 2002, based on the Weibull distribution.

The Savickas study produced prices that were within or close to the bid offer spread of the S&P500 options. For options with a term of one year or more, Savickas's model is significantly more accurate than Black-Scholes.

Such a study has not been done on the South African market and therefore this study fills a gap in the literature.

1.3. Problem Statement

1.3.1. Main Problem

Black-Scholes, although widely used, is generally accepted as being inaccurate in pricing options under certain conditions. It tends to misprice options that are deep in and deep out-the-money as well as longer dated options. This has necessitated alternative models such as the Savickas model, based on the Weibull distribution. While Savickas has been tested in the US, it has not been tested in South Africa. This study will test the accuracy of the Savickas model versus Black-Scholes in pricing options listed on the JSE.

1.3.2. Sub Problem

A secondary issue that this paper researches is whether or not the Weibull distribution based option pricing model prices shorter dated options more accurately than Black-Scholes.

1.3.3. *Hypotheses*

The testable hypotheses are:

H_0 : Application of the Weibull Distribution option pricing model to the South African market for contracts produces no more accurate results than than the Black-Scholes model.

H_1 : The Weibull Distribution based option pricing formula produces more accurate option prices than the Black- Scholes option pricing model.

H_0: The Black Scholes model and Savickas model show no bias in the pricing of options with different maturities.

H_1: The Savickas option pricing model produces more accurate prices for options with a term longer than one year.

1.4. Significance of Study

The Weibull distribution based option pricing model has been tested on the US market and has been proven to price longer term options; over one year; more accurately than the Black-Scholes model. This study looks at the applicability of the Weibull distribution option pricing model by Savickas (2002) to the South African market. This study will provide guidance to all options traders as well as the South African Futures Exchange. The South African Futures Exchange currently uses Black-Scholes for extraction of the implied volatility and to calculate daily mark-to-market[2] of the open option positions.

The Savickas study produced prices that were within or close to the bid offer spread of the S&P500 options. For options with a term of one year or more, Savickas's model is significantly more accurate than Black-Scholes

[2] Mark-to-Market is the value assigned to an open options contract that has not expired yet. It is related to the market value of the contract, based on the price of the underlying and parameters assumed by the provider of the Mark-to-Market, in this case, SAFEX.

This study will test the application of Savickas's findings on the South African market.

1.5. Delimitations and limitations

1.5.1. Delimitations

This paper looks at equity index options traded on the South African Futures Exchange.

1.5.2. Limitations

Due to availability of information, only exchange traded options and not over-the-counter deals will be analysed.

1.6. Definitions of terms

Alsi: The Futures index which comprises of the forty-one top stocks by market capitalisation in the South African market.

FTSE: The FTSE group is an independent company that originated from a joint venture between the Financial Times and the London Stock Exchange, and is responsible for providing stock market indices and associated data.

Kurtosis: Is the height or sharpness of the midsection of the bell curve.

OTC: Over-The-Counter. This refers to options that are traded away from the exchange, to facilitate non-standard parameters, with counterparties happy to be governed by an ISDA (International Swaps and Derivatives Association)

SAFEX: The South African Futures Exchange.

Skewness: Is the measure of asymmetry of the bell curve.

1.7. Assumptions

When calculating the implied volatility, Safex assumes the spot price as it does not have this as one of the inputs into the trading system. This research will do a comparison of the price paid for the option using that day's closing price, and should not significantly effect the results.

2. LITERATURE REVIEW

2.1. Introduction

The following literature review introduces option pricing theory and some of the models used to price options and in particular the Black-Scholes and Weibull Distribution based option pricing formulas. It also points to the research and various opinions on option pricing theory.

2.2. Pricing Models

The Black-Scholes model has received much attention as an inaccurate model in the pricing of options. “New quants on Wall Street often are amazed at the naïveté of Black-Scholes, and immediately try to do better by adding things like jumps, stochastic volatility, correlations, and transaction costs. But traders are limited humans and data are sparse, so this extra complexity doesn’t necessarily improve things. A usable model has to provide both input and a way of speaking that comes naturally.” (Derman 2000:61)

Natenburg (1994) stated that “traditional” option pricing models were imperfect. If Natenburg is to be believed, the question then is how do traders go about pricing options if not by means of theory based valuations models? Natenberg emphasises that the traditional models do not always work perfectly, but historically, they have proven to be the best available methodology, and that it is apparent that traders who ignore these issues of theoretical models, do so at their own peril.

What makes a model better could be anything from:

1. ease-of-use

2. a model that gives more accurate theoretical pricing or

3. one that mimics the marketplace more closely.

Hewett and Igolnikov (2000) do a comparison of the Black-Scholes and Merton model, the binomial trees model of Cox, Ross and Rubenstein and the Monte Carlo Simulation, over a variety of different types of options (European, American, barriers, digitals, etc) and find that the main issues they are faced with are:

- Failure of a model to accurately represent the market
- Failure to implement the model correctly and effectively
- Inaccurate historical data or inappropriate use of data

Cox and Ross (1976) attempt to improve on the Black-Scholes formula by considering alternate forms of the stochastic process governing asset prices. They do this by introducing several jump and diffusion processes, which had not been used before.

Ahn and Thompson (1988) test some of the models based on jump-diffusion processes. They tested their theories on bond prices, and discovered that even in some cases “where the traditional expectations theory about the term structure is consistent with general equilibrium under jump diffusion processes, the traditional theory is not consistent under jump-diffusion processes” (Ahn & Thompson; 1988:155). They show that the bond prices are higher under jump risks than normal.

Heston (1993) puts forward two option pricing models. One formula is based on the log-negative-binomial distribution and as with Cox, Ross and Rubenstein’s log-binomial formula, this formula does not depend on jump probability. Heston (1993) presents a second formula based on the log-gamma distribution. Unlike Black-Scholes, which is independent of the mean of the stock returns, this formula does depend on the mean of the stock return.

Ahn and Cho (2002) later developed a closed-form solution for pricing European Calls in situations when the volatility of the underlying’s returns are governed by a diffusion process. This model differs from Black-

Scholes in that it requires volatility to be adjusted. Here the volatility movement is correlated to the stock price movement.

In a recent paper, Haug and Taleb (2007:1), assert that option hedging, pricing and trading is neither philosophy nor mathematics. They claim that it is a skill which traders learn from other traders (or by traders copying other traders) and tricks developed under evolution pressures, in a bottom-up manner.

This statement has not diminished the enthusiastic search by mathematicians to find improved options formulas.

2.3. Options

Two types of basic options exist, there are call options and put options. The call option gives the holder the right to buy an underlying asset in the future, before or on a specified date, at a price specified in the present. The put option gives the holder the right to sell the underlying asset in the future at a price specified in the present. Options can be written on physical assets such as commodities (for example: gold, oil, copper, etc.), individual equities (such as Microsoft, Gazprom, etc) and on index futures (such as the S&P, FTSE and Alsi). An index future is a highly regulated notional asset based on a specified underlying asset (example: a basket of shares). Many factors are at play in the process of pricing options. In the South African market, these factors include liquidity of the underlying asset, historic volatility of the underlying asset, expected future volatility of the same underlying asset, risk management factors of the price maker and one that is more difficult to measure, the relationship between the market- maker[3] and her client.

If the buyer of the option wishes to exercise his right, the option writer is under obligation to buy or sell the underlying asset to the holder of the

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[3] Market-makers are traders of financial instruments who provide liquidity to the market by making both buy and sell prices, and who make a profit on the bid offer spread.

option. The price of the right is called the option premium or option price. The option writer is the person who makes the price of the option and is under obligation to buy or sell if the holder of the right decides to exercise this right. The option premium is considered compensation for the risk the writer undertakes.

Although options are totally separate entities from the underlying assets, (Cohen, 2005), there are a number of factors that influence the price of the option, not least of them the price of the underlying, and the likelihood of exercise. It follows then that the more volatile the stock price, the higher the price of the option (Kolb, 2000) as there is a price to pay for uncertainty.

Although there are a number of other option pricing methodologies, such as the Binomial model (Cox-Ross-Rubenstein model) and the Whaley (Quadratic) model; most option prices are calculated using the Black-Scholes (1973) formula. The Black-Scholes is a methodology based on the assumption of riskless arbitrage between the underlying assets and options on these assets' prices (Cretien, 2006). The Black-Scholes formula calculates the price of an option using the inputs of: strike price of the option, price of the underlying, risk free rate of interest, time to expiry of the option and the volatility of the stock. Mandelbrot (2004) feels that Black-Scholes, the original options pricing formula, is now widely accepted to be imprecise at best, and misleading at worst.

The volatility is the only 'intangible' input into the formula (Strydom, 1995), and thus requires careful attention. Kolb (2000) states that if the Black-Scholes option pricing model correctly measures the factors that affect option prices, then the price of the option, which would be calculated using the Black-Scholes model, should correspond to the price observed in the market. He goes on to suggest that if this is not the case, then the model is inadequate or prices in the market are irrational. Savickas (2002) proposes a new option pricing formula based on the Weibull Distribution of share returns. His results yield option prices that are consistent with options in the S&P 500 which are traded on the Chicago Board Options Exchange (2002) and therefore prove that this new model is superior to the Black-Scholes model.

The Efficient Market Hypothesis states that in a liquid market all information regarding shares are reflected in the share price. Schönbucher (1999) raises the points that firstly it is still a hotly debated discussion as to whether the Efficient Market Hypothesis is always fully valid and secondly that there is compelling evidence that exchange-traded options prices do contain idiosyncratic information that cannot be backed out from the price information of the underlying security alone.

2.4. Pricing

The JSE regulation stipulates that listed options are traded in lots of fifty contracts with each index point being the equivalent of ZAR10. Traders are also likely to round prices up or down according to their view of the direction the market will take, rather than by the standard mathematical rules.

Mandelbrot (2004) claims that almost everyday ‘order imbalances’ occur on the New York Stock Exchange. His research showed that on January 8th 2004 the Reuters News Service reported a typical day with eight order imbalances, which were caused by either news of the Food and Drug Administration approving a medicine, an unexpected takeover offer or a windfall legal victory. In the cases of ‘order imbalances’, some exchanges license ‘specialist’ broker-dealers to step into the breach and trade when others will not. These order imbalances affect the liquidity and price of the share, which the options trader looks at when pricing the option.

2.5. Volatility

Kalyvas and Dritsakis (2003) define three types of volatility; namely: Historic or Backward-looking Volatility, Implied Volatility and Forward-looking Volatility. This report focusses on only two types of volatility, historic and implied, as they are the most widely used types of volatility in the market, and are relevant to this study.

In the pricing of options, the relevant information is information about volatility, which is not observable in the share price. Schönbucher (1999) emphasises that studies have shown that predictions of future share volatility are more accurate when based on implied volatility than when based on historic volatility.

Volatility is measured by the standard deviation of the lognormal returns.

Chriss (1997) suggests that volatility and riskiness are equal and that the more volatile the stock price movement, the more risky the stock and pricing of the option. If one is able to compute the volatility of the stock then we are able to calculate the riskiness of the stock.

Ncube (1996:1) points to the fact that much research has been done to show that the assumption that the Black-Scholes model makes that market volatility is constant, is in fact, not the case, and that “volatility is stochastic and the log returns is leptokurtic, skewed and therefore non-normal.”

2.5.1. Historic Volatility

Historic volatility is the volatility of a share calculated using historic price data, calculating the price relatives, logarithmic price relatives and the mean and standard deviation of the logarithmic price relatives (Kolb, 2000).

2.5.2. Implied Volatility

Implied volatility is the “instantaneous variance of the percentage changes in futures prices implicit in the price of options” on the underlying contract (Sutcliffe 2006:200). Ncube (1996:72) points to the fact that “implied volatility has been suggested as an estimator of market volatility, especially that implied by at-the-money options.”

Implied Volatility is calculated by inputting five of the six variables needed for Black-Scholes, of which one is the observed market price of the call option, to determine the standard deviation. “The estimated standard deviation is an implied volatility because it is the value implied by the other five variables in the model.” (Kolb, 2000:400)

For the purposes of this study it is important to note Lehnert’s (2003) observation that volatilities implied from equity options invariably show patterns that are consistently not lognormal in nature.

2.6. Volatility Smiles and Skews

The terms volatility smiles and volatility skews are references to the way in which implied volatilities vary with strike price for a fixed expiration.

When implied volatility increases as strike price decreases for out-of-the-money options it is called a negative skew. This alludes to the conclusion that out-of-the-money puts are more expensive than Black-Scholes predicts. Conversely, as strikes increase, option values drop due to the decrease in volatility, leading us to believe that in-the-money calls are less expensive than Black-Scholes predicts. In most markets, returns appear more leptokurtic than is assumed by a lognormal distribution. Market leptokurtosis would make way out-of-the-money or way in-the-money options more expensive than would be assumed by the Black-Scholes formulation. The supply and demand for options is another explanation of why markets exhibit skewness. An interesting observation by Chriss (1997) is that after the stock market crash of October 1987, volatility smiles for most equity markets, have become more pronounced. Contrary to this view, Rubenstein (1994) points out that there is a distinct possibility that the stock market crash of October 1987 changed the way market participants viewed index options. Out-of-the-money puts became much more highly valued, eventually leading to the situation where low striking price options had significantly higher implied volatilities than higher striking options. The market’s pricing of options since the crash seems to indicate an increasing ‘crash-o-phobia’. Rubenstein is supported by Foresi and Wu

(2005), where they test implied volatilities in several markets in 12 different countries and find that the negatively skewed volatility smirk is evident in every single one of these markets.

Kalyvas and Dritsakis (2003) point out that if pricing models were accurate, then options with different strike prices would have the same implied volatility. One cannot ignore the phenomenon of different implied volatilities for different strike prices. This is known as the ‘volatility smile’. Furthermore, we cannot rule out the fact that the implied volatilities from the calls and puts may be different. They assert that there is no reason to believe that the implied volatility of the put option should be higher than that of the call simply because the put is a natural hedging instrument and investors use it as an instrument of insurance.

In their study of options smiles, Ederington and Guan (2002) found that part of the smile may be due to erroneous distributional assumptions in the Black-Scholes model and that a substantial part must reflect other forces. Although traders’ portfolios are generally delta-neutral and gamma or vega neutral upon implementation, this changes through the life of the option, and frequent rebalancing can deteriorate profits. However, Ederington and Guan point out that the smile persists despite substantial pre-transaction cost trading profits.

2.7. Standard Deviation and Normal Distribution

Standard Deviation is a term used to describe, in graphic terms, how fast or slowly the normal distribution curve spreads out (Natenberg 1994). For an options trader, this information is important because it tells the trader how often a price movement can fall within a specific region from the average price movement of the stock. A positive or negative one standard deviation captures approximately 68.3% of all occurrences, negative or positive two standard deviations consider approximately 95.4% of all occurrences and negative or positive three standard deviations capture approximately 99.7% of all occurrences (Natenberg 1994:57).

According to Natenberg (1994), one logical approach to option valuation is to assign a probability to an infinite number of possible price outcomes for an underlying contract; if we multiply each possible price outcome by its associated probability we can use the results to calculate an option's theoretical value. Natenberg (1994) also points out that, the resultant infinite number of possibilities is impossible to work with and not a practical solution. Studies of the normal distribution graph have been so in-depth, "that formulas have been developed which facilitate the computation of both the probabilities associated with every point along a normal distribution curve, as well as the area under the various portions of the curve. If we assume that prices of an underlying instrument are normally distributed, these formulas represent a unique set of tools with which we can solve for an option's theoretical value" Natenburg (1994:145). This, he goes on to point out, is the reason why Black-Scholes assumed the normal distribution curve when developing their option valuation model, as a more accurate solution would be cumbersome and impractical.

Bernstein (1998) questions how closely changes in stock prices resemble a normal distribution curve. Despite his questioning, his conclusion is that there is overwhelming evidence to support the theory stock price returns are normally distributed. Bernstein (1998:170) goes on to show how Francis Galton (1822-1911) turned the "notion of probability from a static concept based on randomness and the Law of Large Numbers into a dynamic process in which the successors to the outliers are predestined to join the crowd at the center"; in other words, the normal distribution curve is an unavoidable outcome and the "driving force is always toward the average, toward the restoration of normality" (Bernstein; 1998:170).

2.8. Stochastic Volatility Option Pricing

Chriss (1997) focuses on the theory that volatility is stochastic. This theory points to the need for a model based on stochastic volatility. Chriss likens models based on stochastic modelling to models based on Geometric Brownian Motion with regard to the fact that as the stock price changes randomly, the volatility changes randomly. While Ncube (1996) believes that the theory of geometric Brownian motion movements is restrictive on the analysis of stock returns.

Mandelbrot (2004) agrees that price changes do not follow natural science movements and states that Brownian motion does not apply to a share's price movement. Mandelbrot points out that Bachelier's theory, that price movements follow a Brownian Motion, requires three assumptions to be true.

Firstly that each price is independent from the last and not affected by any historical price movement (Mandelbrot, 2004).

The second assumption needed for Bachelier's theory on Brownian Motion and price movement is statistical stationarity of the price movements. Mandelbrot (2004:87) explains that whatever the process is for generating price changes, this process remains the same over time. He uses the analogy of coin tosses to explain his theory. If you assume coin tosses decide prices, then coin does not get switched or weighted in the middle of the game; all that changes is the number of outcomes as the coin is tossed; not the coin itself.

The third assumption is that prices follow the normal distribution curve.

Mandelbrot (2004) mistrusts these assumptions and asserts that some trading strategies are misguided, and that options are mis-priced. He believes that whenever the bell curve is used in financial calculations, errors will occur.

2.9. Conclusion of Literature Review

2.9.1. Proposition 1:

As is apparent from the literature review, Black-Scholes, although widely used, is based on the contested assumption that share price movements are normally distributed. Ncube (1996:1) states that "the Black and

Scholes model misprices options that are deep in the money and deep out of the money and those that are far from expiry”. Savickas has recognised these shortcomings in the Black-Scholes model and has developed a simple option pricing formula based on the Weibull distribution. He tested the simple option pricing formula on the CBOE and found it to be statistically closer to market prices than the Black-Scholes model. This paper will examine the applicability of Savickas’s simple option pricing formula to the South African market.

2.9.2. Proposition 2:

Savickas also found that his formula priced longer dated options more accurately than the Black-Scholes model. This report compares his findings to the South African market and tests if Black-Scholes is a more appropriate model for options with expiries longer than one year.

3. RESEARCH METHODOLOGY

In this section we present the Black-Scholes and Savickas models for pricing options. We present the methodology for comparison.

3.1. The Models

3.1.1. Black-Scholes (1973)

The Black-Scholes model was developed by Fischer Black and Myron Scholes in 1973 under the assumptions that asset prices adjust to prevent arbitrage, that stock prices change continuously and that stock returns follow a log-normal distribution. The following is the Black-Scholes formula for calculating the price of a call option:

(1) $$C = S_t N(d_1) - Xe^{-r(T-t)} N(d_2)$$

where:

$$d_1 = \frac{\log(s/x) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

and:

C = current call option

S_t = current stock price

N(*d*) = The probability that a random draw from a standard normal distribution will be less than *d*. This equals the area under the normal curve up to *d*.

X = strike price

r = risk free rate (the annualised continuously compounded rate on a safe asset with the same maturity as the expiration of the option, which is to be distinguished from r_f, the discrete period interest rate)

T = time to maturity of option in years

σ = standard deviation of the annualised continuously compounded rate of return of the stock

The Black- Scholes model was developed for calls only, but European puts can be calculated by applying the put-call parity theory.

Put-call parity equation:

(2) $$P_t = c_t - S_t + Xe^{-r(T-t)}$$

Substituting the Black-Scholes call formula into the put-call parity equation gives us the formula for pricing a put option:

(3) $$P_t = S_t[N(d_1)-1] + Xe^{-r(T-t)}N(d_2)_t - S_t + Xe^{-r(T-t)}[1-N(d_2)]$$

In order to calculate the volatility or standard deviation we need to derive the returns and value returns, then,

(4) PR_t (Price relative for the day) = P_t / P_{t-1}

The formulas for the mean of the logarithmic return is:

(5) $$\overline{PR} = \frac{1}{T}\sum_{T=1}^{T}\ln PR_t$$

Then the historic volatility is given by:

(6) $$\sigma = \frac{1}{T-1}\sum_{t=1}^{T}(\ln PR_t - \overline{PR})^2$$

The standard deviation is δ

3.1.2. Weibull Distribution

The Savickas (2002) Option Pricing Formula uses a Weibull distribution, characterising the behaviour of stock returns. The Weibull Distribution was developed in 1937 by a Swedish engineer, Waloddi Weibull (1887 - 1979), and first introduced to the USA in 1951

The Weibull Distribution has been documented as working well with Life Data due to its flexibility, and is also widely used in reliability engineering and failure analysis. It has the flexibility to mimic the behaviour of other statistical distributions such as normal and exponential distributions as shown in the formula for the failure rate *h* (or hazard rate):

(7)
$$h(x;k,\lambda) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}$$

And explained as such: If the failure rate decreases over time, then *k* < 1. If the failure rate is constant over time, then *k* = 1. If the failure rate increases over time, *k* > 1. When *k* = 3, then the Weibull distribution appears similar to the normal distribution. When *k* = 1, then the Weibull distribution reduces to the exponential distribution.

The formula for the probability density function of the general Weibull distribution is:

(8)
$$f(x) = \frac{\gamma}{\alpha}\left(\frac{x-\rho}{\alpha}\right)^{(y-1)} \exp(-((x-\rho)/\alpha)^{y})$$

where $x \geq \rho; \gamma, \alpha > 0$

The case where $\rho = 0$ and $\alpha = 1$ is called the **standard** Weibull distribution.

And the case where ρ = 0 is called the **2-parameter** Weibull distribution as per the formula below:

(9)
$$f(x) = \gamma x^{\gamma-1} \exp(-(x^{\gamma})) \qquad x \geq 0; \gamma > 0$$

Savickas (2002:210) uses the Weibull distribution because it "always has a lower third central moment than that of the lognormal distribution." In their study of parametric pricing of higher order moments in S&P500 options, Lim et al (2003) conclude that there are significant gains to be made from pricing higher order moments. The extent of and investigation into pricing of the higher order moments, however, is not within the scope of this research.

3.1.3. Simple Option Pricing Formula (Savickas:2002)

Schönbucher (1999) posits that market participants do not assume that Black-Scholes applies to the actual market, but rather that market makers use Black-Scholes to make their price quotations more independent from the movements of the price of the underlying security for which there is already an efficient market. Lim et al (2003) claim that part of the Black-Scholes model's popularity is its analytical tractability as the price is simply the mean of a truncated lognormal distribution. They point out that there is strong empirical evidence that proves that the two main assumptions of the Black-Scholes model, that is to say that the distribution of the underlying stock returns is normal and that volatility is constant; are not valid.

With the Black-Scholes' shortcomings in mind, Savickas (2002) developed what he called a Simple Option-Pricing Formula, which was based on the Weibull distribution, and which he believes is comparable to the Black-Scholes model in its simplicity and ease of implementation. He then tested the model on options traded on the S&P 500 and showed that pricing biases in the Black-Scholes model were significantly reduced for long-term options (options with more than one-year expiries).

As previously discussed, it has been proven that the third central moment of the price distribution is significantly negative. To this end, there are two possible solutions: One would be to adjust the lognormal distribution by some correction skewness term, and the second would be to use a distribution with a natural negative skewness.

Savickas, in his 2002 research, however, chose to employ the second option, and used a naturally negatively skewed distribution to arrive at a simpler algebraic form and fewer parameters than the model preferred by Jarrow and Rudd (Savickas, 2002).

Assuming that the underlying price is distributed Weibull at maturity, that the current price is S_0, and the dividend yield is *q*, the value C of a call option with the strike price *k*, time to maturity τ, and risk free rate *r*, is determined as follows:

(10) $$C = S_0 e^{-q\tau}.\left[1 - F_{2(1+1/\beta)}(2\omega)\right] - Ke^{-r\tau}.e^{-\omega}$$

Where $F_d(a)$ denotes the cumulative distribution function at point a of a χ^2 – distributed random variable with the degrees of freedom equal to *d*

(11) $$\omega = \alpha.K^{\beta} e^{-\beta.(r-q).\tau},$$

$\beta = 1/(p-1)$, and $\alpha = \left[\frac{\Gamma(1+1/\beta)}{S_0}\right]^{\beta}$

3.1.4. Characteristics of the Savickas model (2002)

Corrado and Su (1997) tested the Jarrow-Rudd model and found that the third central moment of the price distribution is significantly negative. This negative skewness results in Black-Scholes underpricing in-the-money options and overpricing out-of-the-money options, and therefore, Savickas uses the Weibull distribution which is naturally negatively skewed.

Savickas (2002) demonstrates that the Weibull skewness is negative, which makes it more appropriate than the lognormal distribution for valuing options on negatively skewed underlying assets. By using the Weibull distribution, Savickas does not make the claim that actual prices follow a Weibull distribution, but rather that the Weibull distribution mimics the first three moments of the actual prices better than the lognormal distribution.

Despite there being a number of models that make allowance for the negative skewness, the Savickas model also claims ease of use and simple algebraic form as part of its practical appeal.

3.2. Methodology

Using Excel Solver, the optimal implied volatility for Black-Scholes , and the Beta for Savickas's model were extracted. These volatities and betas were input into the formula and prices for each of the formulas was extracted.

3.2.1. Validity and Reliability

Many statisticians and modern economists such as Steven Levitt (2005) agree that regression analysis can express correlation, but does not prove causality. This study uses a chi-squared test to evaluate the posited hypotheses. The chi-square test is used to test if a sample of data came from a population with a specific distribution.

3.2.2. Chi Squared Test

The Chi-squared test (also known as the goodness-of-fit test) measures the likelihood of the differences between the expected and the actual (deviations) occurring by chance.

The test is not suitable in situations where the expected frequency is less than five, and in those cases where each expected frequency is between five and ten. Where there are only two classes (one degree of freedom), Yates' correction should be applied.

(12) $$X^2 = \sum_j \frac{(n_j - \mu_j)^2}{\mu_j}$$

Greater differences $(n_j - \mu_j)^2$ produce greater X^2 values, for fixed n.

4. THE DATA

A comparison was made, using data supplied by SAFEX, between the Black-Scholes and simple option pricing models and the results validated using the chi-squared test. A comparison was made between the prices on SAFEX and the prices extracted from Black-Scholes and from the Weibull distribution based option pricing formula.

4.1. Basic Characteristics of the Data

We use data on share Index futures which are traded on Safex. While the JSE is responsible for the operation of the FTSE/JSE Africa Index Series, it is the responsibility of FTSE to calculate and disseminate the levels of the indices in this series. FTSE also maintains records of the market capitalisation of all the constituents and reserve companies.

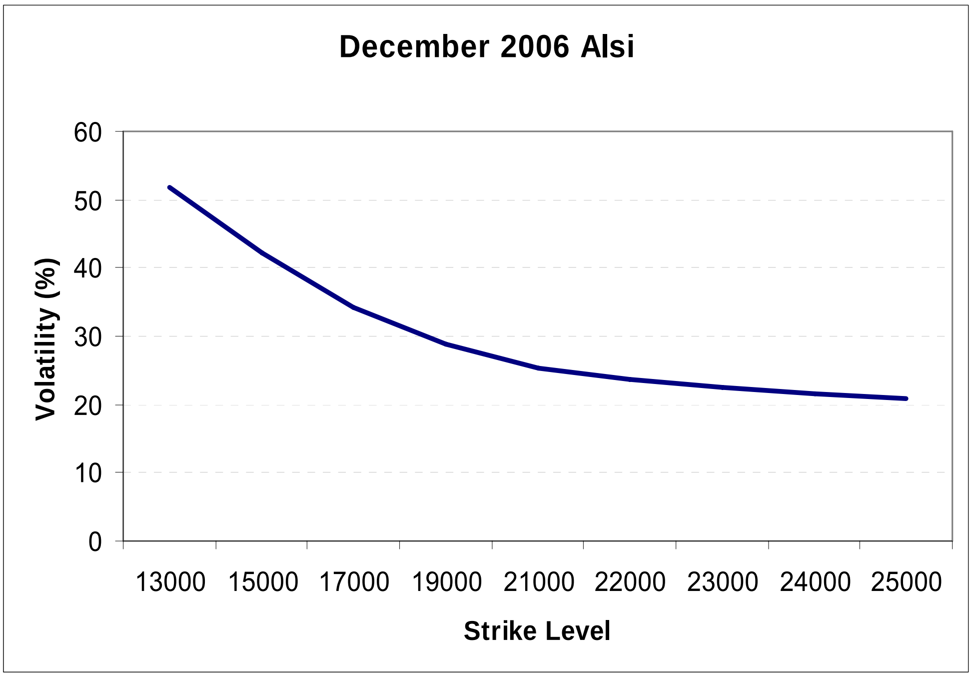
The FTSE/JSE Top40 index consists of the largest forty companies ranked by full market value; that is, before the application of any investibility weightings are applied. The value of each contract in Rand is the index level multiplied by 10. The strike prices for options on share index futures is in steps of 50 index points. However, the price of the option is listed in Rand cents.

Futures and option expiry takes place on the third Thursday of March, June, September and December; and in the case of this not being a working day, then the previous business day becomes the expiry day. Futures expiry prices are calculated by taking the average of 100 iterations of the index level taken at 1 minute intervals starting at 12noon and finishing at 13h40 (the first calculation being at 12h01) of the expiry day.

Listed options traditionally have fixed expiry dates and strike prices, whereas with an over-the-counter (OTC) option, the buyer and seller of the option will agree on expiry dates and strike levels that they want. With the introduction of Can-Do options, Safex has given the market more flexibility to list any strike and expiry that the counterparties would like.

The graph below is a plot of the December 2006 Alsi contract as at 5th October 2006, when the December 2006 Alsi future was trading at 21,000. It is an apparent skew and not a smile (Source: SAFEX: 2006)

Figure 1: Implied Volatility across different strikes for the Alsi



4.2. Population and Sample

4.2.1. Population

Option prices obtained from SAFEX from October 2007 to May 2008. This includes all exchange traded options.

4.2.2. Sample

For the purposes of this study we will look at the Alsi future which is currently the most liquidly traded index futures on SAFEX as shown in the tables on the following page:

Table 1: Volumes of Index Futures Traded on Safex in September 2006

Futures	Deals	Contracts	Value in ZAR	% Deals
ALSI	30,273	1,848,554	369,146,922,479	98%
CTOP	13	71,328	7,393,517,280	0%
DTOP	235	267,198	11,128,905,443	1%
FINI	5	300	22,643,400	0%
FNDI	419	81,961	16,455,256,173	1%
GLDX	8	2,833	84,791,670	0%
INDI	42	2,877	427,769,250	0%
RESI	11	1,237	491,641,340	0%
Totals	31,006	2,276,288	405,151,447,035	100%

Legend:

ALSI: The FTSE/JSE Top40 index is a capitalisation weighted index of the 40 largest companies by market capitalisation included in the FTSE/JSE All Share index.

CTOP: The FTSE/JSE Capped Top 40 Index is a capitalisation weighted index. The index has the same members as the FTSE/JSE Africa TOP 40 Index, but with no one member comprising more than a 10% weighting.

DTOP: The FTSE/JSE Shareholder Weighted Top 40 Index is a capitalisation weighted index. The members of the index are the same as the FTSE/JSE Top 40 Index, with the member weightings adjusted for non South African shareholdings, in addition to strategic and cross-shareholding adjustments FTSE/JSE Shareholder Weighted Top 40 Index Future

FINI: The FTSE/JSE Africa Financial 15 index is a market capitalisation weighted index of the 15 largest financial companies by free-float market capitalisation.

FINDI: The FTSE/JSE Africa Financial and Industrial 30 index is a market capitalisation weighted index of the 30 largest financial and industrial companies by free-float market capitalisation.

GLDX: The FTSE/JSE Africa Gold Mining Index is a market capitalisation weighted index. Companies included in this index are engaged in gold mining and are also included in the FTSE/JSE All Shares Index.

INDI: The FTSE/JSE Africa Industrial 25 Index is a market capitalisation weighted index. The top 25 members of the basic industrial or general industrial economic groups by market capitalization are included in this index.

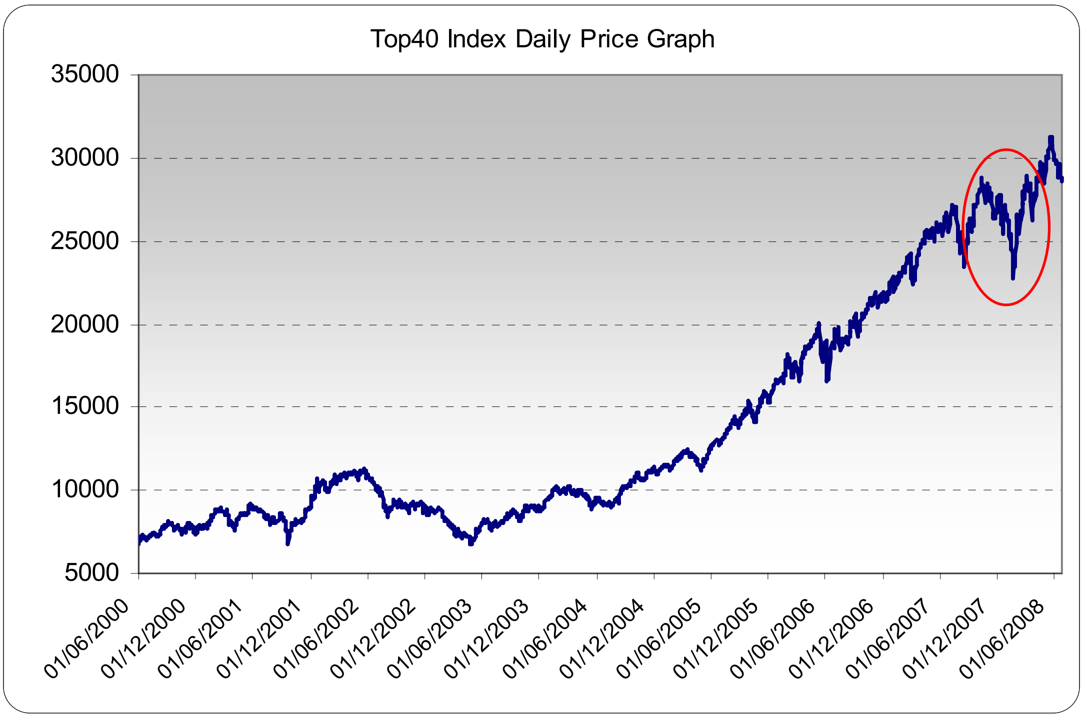
RESI: The FTSE/JSE Resource 20 Index is a capitalisation weighted index. Companies included in this index are the 20 largest companies by pre-free float market capitalisation included in the resources economic group.

Table 2: Volumes of Index Options Traded on Safex in September 2006

Options	*Deals*	*Contracts*	*Value in ZAR*	*%*
ALSI	1,153	920,702	8,931,156,172	84%
CTOP	0	0	0	0%
DTOP	72	410,438	576,261,679	5%
FNDI	138	24,839	200,378,740	10%
RESI	4	132	1,572,648	0%
Totals	1,367	1,356,111	9,709,369,238	100%

Figure two shows the daily price graph of the Top40 index for the period June 2000 to June 2008. The circled area is the period of the eight months of data. From the figure, it is apparent that the period was a volatile one.

Figure 2: Top40 index daily price graph



Source: Bloomberg

4.3. Data collection

All data was supplied by Safex and subsequently all options were exchange traded. The option prices given by Safex were compared to the prices calculated by the models (Black-Scholes and Savickas's Simple Option Pricing model). Data was categorised into options that had between 75 and 100 days to expiry, and options that had between 275 and 400 days to expiry. All options that were in-the-money were eliminated.

Data Problem: Safex does not collect the spot price used in pricing the option and this introduces the problem of nonsynchronicity. In the Savickas paper, he used the underlying closing price, which coincided with the closing of the spot market. For this report, we have applied the same methodology.

In contrast to Savickas's paper, we are not using simply listed options, we are using actual traded options, and therefore we do not eliminate options where two different prices are quoted for the same option.

Infrequency of trading: Due to the fact that we were using only traded options, the problem of infrequent trading could potentially have been a problem. However, the number of traded options on the Alsi is sufficient to overcome this issue. For the 75days to 100days to expiry category, we had 132 data points and for the category of options with 275 to 400 days to expiry we had 207 data points.

5. RESULTS AND DISCUSSION

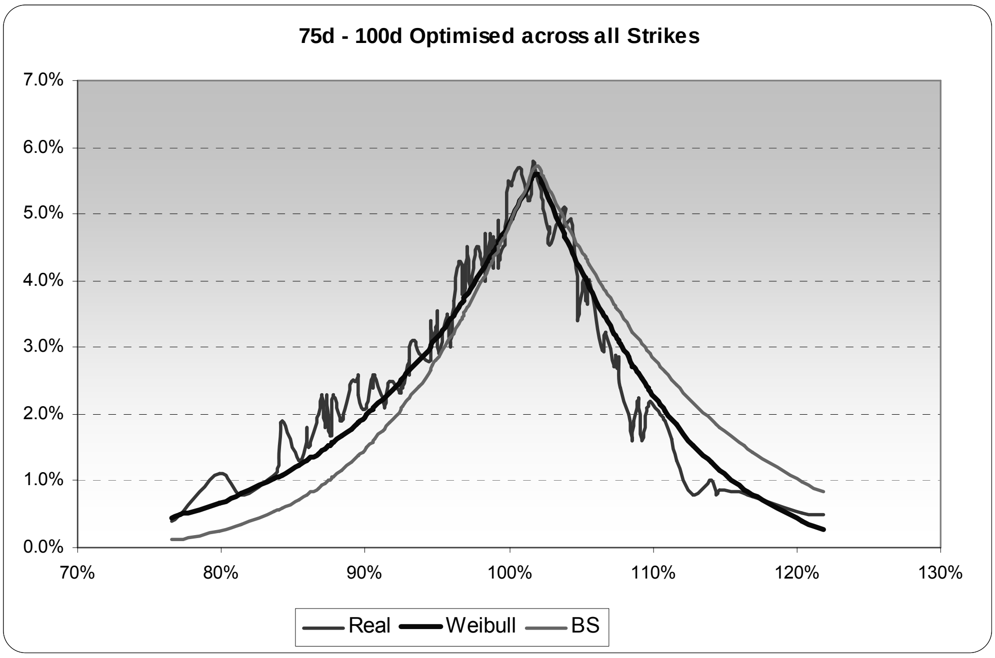
5.1. Introduction

The results are presented below in both graphic and tabular form. The graphic representations give a good idea of what to expect in the statistical outputs. The data was divided into Puts and Calls and into expiration categories to analyse whether or not the two formulas priced longer-dated options differently, and whether out-the-money-options were priced differently.

5.2. Graphic Representations

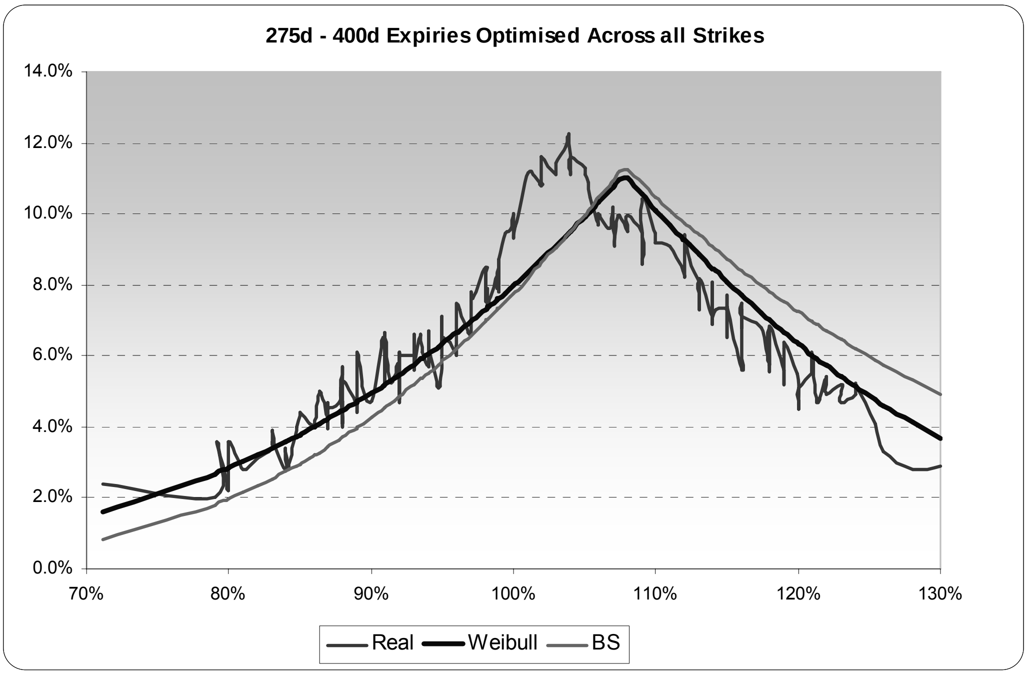
Figure three is a graphic representation of the 75day – 100day expiry category. From the graph, it would appear that Black-Scholes prices the Calls too high, and prices the Puts too low, while Savickas's formula is a lot closer to the SAFEX data.

Figure 3: 75day – 100day expiry group, optimised across all strikes



The below graph (figure four) also appears to be showing that, although to a lesser extent than the 75day – 100day category, the longer dated options, 275day - 400day, the Savickas formula is a closer fit to the SAFEX data.

Figure 4: 275day – 400day expiry category, optimised across all strikes



5.3. Chi-Square Results

Below is a table of the results of the chi-squared test, which was run across data that was divided into Puts and Calls in each maturity category.

Table 3: Chi-Squared Results

Calls		
	Chi square value	**Level of Significance**
Longer-dated Savickas	0.8578	1.85E-27
Longer-dated Black-Scholes	0.7886	1.70E-20
Shorter-dated Savickas	0.8310	1.12E-25
Shorter-dated Black-Scholes	0.7291	3.71E-17

Puts		
	Chi square value	**Level of Significance**
Longer-dated Savickas	0.9476	7.65E-56
Longer-dated Black-Scholes	0.9484	3.44E-56
Shorter-dated Savickas	0.9718	4.85E-50
Shorter-dated Black-Scholes	0.9688	2.09E-48

5.4. Discussion

The graphic representations of the comparison between the SAFEX, Savickas and Black-Scholes prices, are a preview of the results that could be expected by the chi-square test. For ease of explanation, it is worth noting early on in the discussion of results that the p-value showed good levels of significance across all eight sets of data (that is: Savicka's formula versus Black-Scholes; Calls versus Puts; Longer-dated versus shorter-dated). The confidence level chosen for the test was 0.05, and all the p-values were much lower than 0.05.

Calls:

Savickas's formula was significantly better correlated to the SAFEX data than Black-Scholes for both the longer and shorter-dated maturities.

Puts:

Here, the correlation is not as apparent as the Savickas formula was close to the Black-Scholes results for both longer and shorter-dated options.

We can thus accept the Hypothesis that the Weibull based option pricing formula produces more accurate results than the Black-Scholes option pricing model.

6. CONCLUSION AND RECOMMENDATIONS

6.1. Introduction

From the results, we can conclude that Savickas's simple option pricing formula has a better fit with the real world data than the Black-Scholes model for Calls, but we cannot say that conclusively about the Puts. The levels of significance (p-value) prove that the simple option pricing formula supports the hypothesis, but not our sub-problem of pricing longer-dated options more accurately.

The Implications:

1. From this research, it is apparent that Savickas's option pricing formula prices options closer to the SAFEX data. However, one of the three requirements that were listed earlier was ease-of-use, and this research has not conclusively proven that the Savickas model is easier to use than Black-Scholes.

2. Although, without apparent skew, the Savickas formula produces prices that are better correlated to the market prices, it has not been proven that this holds true under all circumstances.

7. FURTHER RESEARCH

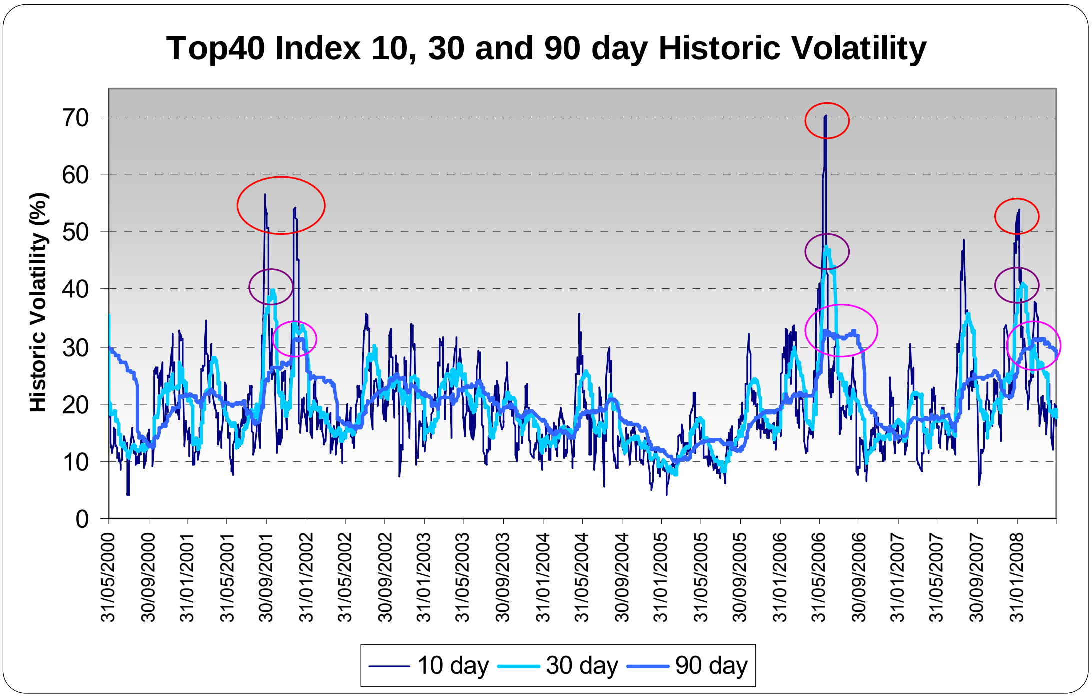
For further research we suggest the following:

1. It is prudent to note that this data was over a period of eight months, and could have been affected by market events. In this period, we experienced what has been named the “credit crunch”, which saw the market fall by 18% from 1st October 2007 to 23rd January 2008 (as indicated by the red circle in the graph on the following page – figure five) and then climb 31.44% to the end of May 2008. It is also worth noting that in August 2007, the Top40 index underwent a 14% correction (Bloomberg). Intuitively, this implies that skew would already have been high in that period, and that another correction in the market would lead to skew increasing faster than under stable market conditions.

It was in this period that 10-day historic volatility spiked to above 50% - an event only seen 3 times in the previous eight years and 90-day historic volatility only previously rose above 30% twice in the same sample.

The below graphic, taken from Bloomberg, shows the Historic 10, 30, and 90 day volatility for the Top40 index for the period 31st May 2000 to 31st May 2008.

Figure 6: Top40 Historic Volatility



Source: Bloomberg

The higher volatility environment could be influencing the results, and it is recommended that further research be done around the influence of market events and the effect on pricing models.

2. This research paper distinguishes between puts and calls. With reference to Ncube's (1996) comments that Black-Scholes misprices deep out-the-money options, it is recommended that further research be carried out to do comparisons between Savickas's formula and Black-Scholes across various degrees of moneyness.

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CONSISTENCY MATRIX

Research problem: This paper researches the possibility that a model designed by Robert Savickas (2002), based on the Weibull distribution, is a more accurate model for the South African options market.					
Sub-problem	Hypotheses/Propositions/Research questions	Source of Theory	Source of data	Type of data	Analysis
The secondary issue the paper researches is whether or not the Weibull distribution based option pricing model prices shorter dated options more accurately than Black- Scholes	H_0 : Application of the Weibull Distribution option pricing model to the South African market for contracts with a term longer than one year, produces no more accurate option prices than the Black-Scholes option pricing model.	Savickas, R. (2002): *A Simple Option-Pricing Formula*, The Financial Review 37, 207-226.	SAFEX	Exchange traded options	Chi-square test