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Deep-lying hole states in nuclei

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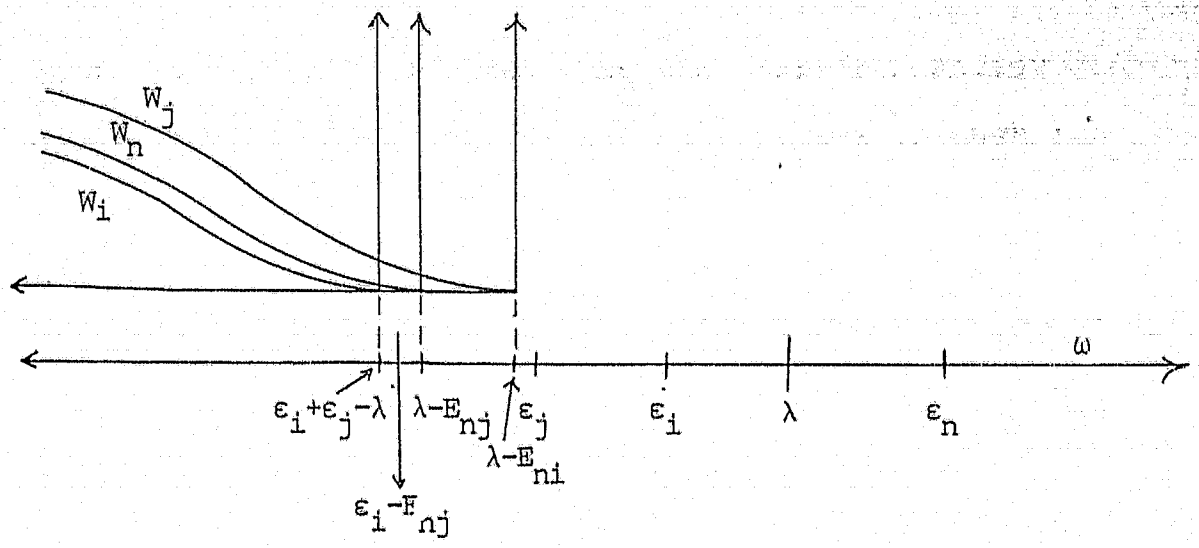


Fig.2.5: The contributions of W_n, W_i, and W_j to the second term of $\Sigma_1^d(\omega)$ are indicated schematically for an arbitrary choice of ε_n, ε_i, and ε_j.

From Fig.2.3 and Fig.2.5, it is evident that when $\omega>\lambda$ there is an imaginary contribution to $\Sigma_1^d(\omega)$ only from the first term of Eq.(2.45), the second term at most providing a real background. Therefore, the sign of $Im\Sigma_1^d(\omega)$ for $\omega>\lambda$ is uniquely determined by the sign of the imaginary part occurring in the denominator of the first term: $Im\Sigma_1^d(\omega)<0$.

Similarly, when $\omega<\lambda$, the imaginary contribution to $\Sigma_1^d(\omega)$ is due only to the second term of Eq.(2.45), since the contribution from the first term in this vicinity is purely real. Thus the sign of $Im\Sigma_1^d(\omega)$ for $\omega<\lambda$ is uniquely determined by the imaginary part occurring in the denominator of the second term: $Im\Sigma_1^d(\omega)>0$.

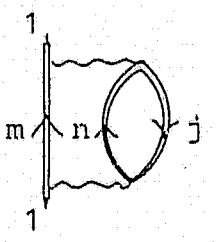
These results, viz. $Im\Sigma_1^d(\omega)\gtrless 0$ for $\omega\lessgtr\lambda$, accord exactly with

Eq.(2.14), thus confirming that the correct behavior occurs for the result Eq.(2.45) as ω passes through λ.

Let us examine the widths that are effective in the vicinity of the resonances of $\Sigma_1^d(\omega)$. The first term of $\Sigma_1^d(\omega)$, given by Eq.(2.45) resonates at a point $\omega=\varepsilon_m+E_{nj}>\lambda$, indicated in Fig.2.3. Similarly, the second term of $\Sigma_1^d(\omega)$ resonates at the point $\omega=\varepsilon_i-E_{ni}<\lambda$, indicated in Fig.2.5. Both of these figures (2.3 and 2.5) demonstrate what the effect of evaluating the W's at off-shell arguments is, viz. that the widths to be sampled in the vicinity of the resonance are very much reduced over what they would have been, had the W's been evaluated on-shell. This is seen, for example, most dramatically in Fig.2.5, for W_i. If W_i had been evaluated on-shell, it would have been non-zero for all $\omega<\varepsilon_i$, and would thus have reached a much larger value at the point of resonance. Thus, the feature of the off-shell energy dependence of W is clearly an important one.

We next demonstrate that the argument associated with each function W is such as to satisfy energy conservation requirements.

Consider first, the Goldstone diagram which represents the first term of $\Sigma_1^d(\omega)$ in Eq.(2.45):



We assume that, associated with each propagator is a width W, measured at the associated off-shell energy $\tilde{\varepsilon}$. i.e. we have $W_m(\tilde{\varepsilon}_m)$, $W_n(\tilde{\varepsilon}_n)$, and $W_j(\tilde{\varepsilon}_j)$. $\tilde{\varepsilon}_m$, $\tilde{\varepsilon}_n$, and $\tilde{\varepsilon}_j$ are determined by assuming that each individually satisfies the energy conservation requirement: the particle or hole energy should be off-shell by an appropriate amount, such that its excitation energy is degenerate with the excitation energy of the system. Hence,

$\tilde{\varepsilon}_m$ satisfies $\tilde{\varepsilon}_m+\varepsilon_n-\varepsilon_j=\omega$

or $\tilde{\varepsilon}_m=\omega-\varepsilon_n+\varepsilon_j$

$\tilde{\varepsilon}_n$ satisfies $\varepsilon_m+\tilde{\varepsilon}_n-\varepsilon_j=\omega$

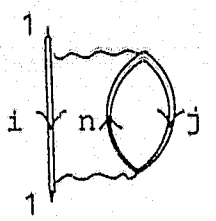
or $\tilde{\varepsilon}_n=\omega-\varepsilon_m+\varepsilon_j$

$\tilde{\varepsilon}_j$ satisfies $\varepsilon_m+\varepsilon_n-\tilde{\varepsilon}_j=\omega$

or $\tilde{\varepsilon}_j=\varepsilon_m+\varepsilon_n-\omega$

As can be seen by comparing the three above equations with the imaginary part in the first term of the self-energy $\Sigma_1^d(\omega)$ in Eq.(2.45), the energy conservation rule is indeed satisfied.

Next, consider again the Goldstone diagram specific to the second term in $\Sigma_1^d(\omega)$ in Eq.(2.45):



As before, each propagator has associated with it, a width W taken at its off-shell energy $\tilde{\varepsilon}$: $W_i(\tilde{\varepsilon}_i)$, $W_j(\tilde{\varepsilon}_j)$ and $W_n(\tilde{\varepsilon}_n)$. $\tilde{\varepsilon}_i$, $\tilde{\varepsilon}_j$, and $\tilde{\varepsilon}_n$ are determined by the energy conservation condition at the energy $-\omega$.

$\tilde{\varepsilon}_i$ satisfies $-\tilde{\varepsilon}_i+\varepsilon_n-\varepsilon_j=-\omega$

or $\tilde{\varepsilon}_i=\varepsilon_n-\varepsilon_j+\omega$

$\tilde{\varepsilon}_j$ satisfies $-\varepsilon_i+\varepsilon_n-\tilde{\varepsilon}_j=-\omega$

or $\tilde{\varepsilon}_j=\varepsilon_n-\varepsilon_i+\omega$

and $\tilde{\varepsilon}_n$ satisfies $-\varepsilon_i+\tilde{\varepsilon}_n-\varepsilon_j=-\omega$

thus confirming the structure observed in the second term of Eq.(2.45). Hence the structure of the imaginary parts of $\Sigma_1^d(\omega)$ may be understood in terms of energy conservation.

2.6 THE DRESSED PARTICLE-HOLE PROPAGATOR.

The definition of the single dressed bubble was given in Sec.2.1 as

$$iF(4321,t) = \langle\Psi_o|Tc_3^+(t)c_4(t)c_1^+(0)c_2(0)|\Psi_o\rangle$$

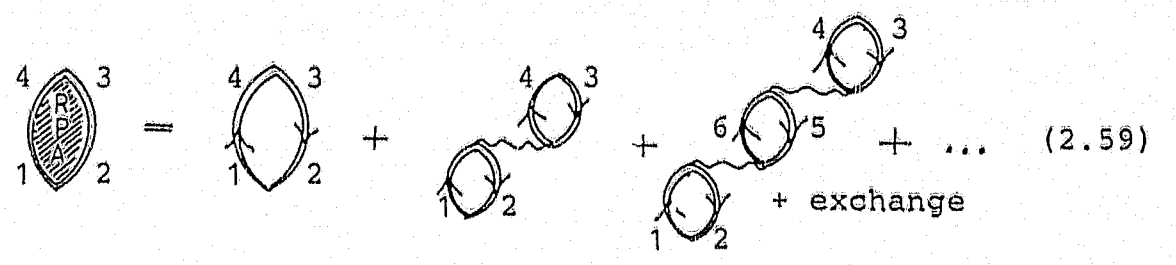
In the same way that the single particle Green's function was dealt with, the Fourier transform of this particle-hole propagator may be expressed as

$$F(4321,\omega) = \sum_\lambda \frac{(c_2^+c_1)_{\lambda o}(c_3^+c_4)_{o\lambda}}{\omega + \omega_\lambda - i\eta} + \sum_\lambda \frac{(c_2^+c_1)_{o\lambda}(c_3^+c_4)_{\lambda o}}{\omega - \omega_\lambda + i\eta}$$

$\eta\to 0$ (2.58)

a form which explicitly displays the acausal nature of its poles. Here $(c_2^+c_1)_{\lambda o}=\langle\Psi_\lambda|c_2^+c_1|\Psi_o\rangle$, where $|\Psi_\lambda\rangle$ is an eigenstate of the full Hamiltonian, with excitation energy ω_λ.

Symbolically, in Fourier space, the selective diagrammatic expansion of $F(4321,\omega)$ that constitutes the RPA, is given by:



(2.59)

Note that this differs from the usual RPA due to the fact that our particle and hole lines are dressed.

Using the diagram rules to translate this series, yields the associated expression for F:

$$-iF(4321,\omega) = -i\P(43,\omega)\{\delta_{14}\delta_{23} + (-i)(42|\tilde{V}|31)(-i)\P(12,\omega) + \sum_{56}(-i)(45|\tilde{V}|36)(-i)\P(65,\omega)(-i)(62|\tilde{V}|51) + \ldots\} \tag{2.60}$$

The effect of exchange is again accounted for in the interaction matrix element of V. (see Eq.(2.39) for example) We construct, using this expression, the quantity

$$\sum_{56}(-i)(54|\tilde{V}|63)(-i)F(6521,\omega) = \sum_{56}(-i)(54|\tilde{V}|63)(-i)\P(65,\omega)\,\delta_{16}\delta_{25} + \sum_{56}(-i)(54|\tilde{V}|63)(-i)\P(65,\omega)(-i)(62|\tilde{V}|51)(-i)\P(12,\omega)+\ldots$$
$$= (-i)(42|\tilde{V}|31)(-i)\P(12,\omega) + \sum_{56}(-i)(54|\tilde{V}|63)(-i)\P(56,\omega)(62|\tilde{V}|51)(-i)\P(12,\omega)+\ldots$$

= the second and following terms occurring in parentheses in Eq.(2.60).

Hence, in the Random Phase Approximation, built up of dressed bubbles, the particle-hole propagator still satisfies an integral equation:

$$F(4321,\omega) = \P(43,\omega)\{\delta_{14}\delta_{23} - \sum_{56}(54|\tilde{V}|63)F(6521,\omega)\} \tag{2.61}$$

Two distinct sorts of equations arise from this. Suppose label 4 represents a particle state m', and 3 a hole state i. Then, on inserting Eq.(2.37) for $\P(mi,\omega)$, the expression

$$F(mi21,\omega) = \{E_{mi} - i[W_i(\varepsilon_m - \omega) + W_m(\varepsilon_i + \omega)] - \omega\}^{-1} \times \{\delta_{1m}\delta_{2i} - \sum_{56} (5m|\tilde{V}|6i)F(6521,\omega)\} \tag{2.62}$$

results. If label 4 represents a hole state i, and 3 a particle state m, however, a different set of equations result.

$$F(im21,\omega) = \{E_{mi} - i[W_m(\varepsilon_i - \omega) + W_i(\varepsilon_m + \omega)] + \omega\}^{-1} \times \{\delta_{1m}\delta_{2i} - \sum_{56} (5i|\tilde{V}|6m)F(6521,\omega)\} \tag{2.63}$$

As yet, labels 1 and 2 are unspecified: they may represent a (particle,hole) or (hole,particle) combination. The sum over states 5 and 6 refer to all possible states of (5,6) being both (particle,hole) and (hole,particle) too. Thus, introducing the abbreviation

$$\Gamma_{mi}(\omega) = W_m(\varepsilon_i + \omega) + W_i(\varepsilon_m - \omega) \tag{2.64}$$

and writing out the sums over 5 and 6 explicitly in terms of their particle or hole character, equations (2.62) and (2.63) become, respectively:

$$\sum_{nj}[(E_{mi} - i\Gamma_{mi}(\omega) - \omega)\delta_{mn}\delta_{ij} + (jm|\tilde{V}|ni)]F(nj21,\omega) + \sum_{nj}(nm|\tilde{V}|ji)F(jn21,\omega) = \delta_{1m}\delta_{2i}$$

$$\sum_{nj}[(E_{mi} - i\Gamma_{mi}(-\omega) + \omega)\delta_{mn}\delta_{ij} + (ni|\tilde{V}|jm)]F(jn21,\omega) + \sum_{nj}(ji|\tilde{V}|nm)F(nj21,\omega) = \delta_{1i}\delta_{2m} \tag{2.65}$$

These results can be re-expressed as a matrix equation:

$$\begin{bmatrix} A-i\Gamma(\omega)-\omega I & B \\ B^* & A^*-i\Gamma(-\omega)+\omega I \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{2.66}$$

where the submatrix B has elements $B_{mi,nj} = (mn|\tilde{V}|ij)$, and is thus symmetric. Consequently $B^* = B$. The submatrix A consists of elements $A_{mi,nj} = E_{mi}\delta_{mn}\delta_{ij} + (jm|\tilde{V}|ni)$, and is thus Hermitian, consequently satisfying $A^\dagger = A$. The dimension of A and B is determined by the number of particle-hole pairs N under consideration. Both matrices are then N×N. The unit matrix appearing on the right hand side is also N×N, and $\Gamma(\omega)$ is the N×N diagonal matrix with elements $\Gamma_{mi}(\omega)$. The four submatrices F_{11}, F_{12}, F_{21}, and F_{22} are then such that the labels representing each row refer to the character of the (4,3) pair, and those representing each column are indicative of the character of the (2,1) pair:

$$F = \begin{array}{c|cc} (4,3)\backslash(2,1) & \text{hp} & \text{ph} \\ \hline \text{ph} & F_{11} & F_{12} \\ \text{hp} & F_{21} & F_{22} \end{array} \tag{2.67}$$

That is to say, F_{11}, F_{12}, F_{21} and F_{22} are all N×N matrices having characteristic matrix elements of the form:

$$(F_{11}) = F(mijn,\omega) \qquad (F_{12}) = F(minj,\omega)$$

$$(F_{21}) = F(imjn,\omega) \qquad (F_{22}) = F(imnj,\omega)$$

where each element satisfies one of the equations in the set (2.65).

It is possible to solve for F in Eq.(2.66) by inverting the matrix on the left hand side of this equation. We perform this inversion indirectly, by examining the associated homogeneous equation:

$$\begin{bmatrix} A-i\Gamma(\omega) & B \\ B^* & A^*-i\Gamma(-\omega) \end{bmatrix} \begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix} = \omega_\lambda(\omega) \begin{bmatrix} X_\lambda(\omega) \\ -Y_\lambda(\omega) \end{bmatrix} \qquad (2.68)$$

Here $X_\lambda(\omega)$ and $Y_\lambda(\omega)$ are both column vectors with N components. Note that this equation is <u>not</u> a true eigenvalue equation, since the vector $\begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix}$ is not returned on the right hand side. However, an eigenvalue equation of the standard form results on rearranging this equation such that the operator on the left becomes $\begin{bmatrix} A-i\Gamma(\omega) & B \\ -B^* & -A^*+i\Gamma(-\omega) \end{bmatrix}$, and we thus refer to $\begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix}$ as an eigenvector.

Since the matrix operator in Eq.(2.68) is not Hermitian, it becomes necessary to consider the adjoint equation also, defined by:

$$\begin{bmatrix} A+i\Gamma(\omega) & B \\ B^* & A^*+i\Gamma(-\omega) \end{bmatrix} \begin{bmatrix} R_\lambda(\omega) \\ S_\lambda(\omega) \end{bmatrix} = \omega_\lambda^*(\omega) \begin{bmatrix} R_\lambda(\omega) \\ -S_\lambda(\omega) \end{bmatrix} \tag{2.69}$$

where $R_\lambda(\omega)$ and $S_\lambda(\omega)$ are again column vectors with N components, and, as before, $\begin{bmatrix} R_\lambda(\omega) \\ S_\lambda(\omega) \end{bmatrix}$ is the eigenvector of a slighty different equation.

Two important properties can be derived from equations (2.68) and (2.69):

1) Orthogonality.

Consider first Eq.(2.68), multiplied from the left by $[R^+_{\lambda'}(\lambda) \quad S^+_{\lambda'}(\omega)]$:

$$[R^+_{\lambda'}(\omega) \; S^+_{\lambda'}(\omega)] \begin{bmatrix} A-i\Gamma(\omega) & B \\ B^* & A^*-i\Gamma(-\omega) \end{bmatrix} \begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix} = \omega_\lambda(\omega) [R^+_{\lambda'}(\omega) \; S^+_{\lambda'}(\omega)] \begin{bmatrix} X_\lambda(\omega) \\ -Y_\lambda(\omega) \end{bmatrix} \tag{2.70}$$

Then, taking the adjoint of Eq.(2.69), bearing in mind the symmetry properties of A and B, and multiplying from the right by $\begin{bmatrix} X_{\lambda'}(\omega) \\ Y_{\lambda'}(\omega) \end{bmatrix}$ yields

$$[R^+_\lambda(\omega) \; S^+_\lambda(\omega)] \begin{bmatrix} A-i\Gamma(\omega) & B \\ B^* & A^*-i\Gamma(-\omega) \end{bmatrix} \begin{bmatrix} X_{\lambda'}(\omega) \\ Y_{\lambda'}(\omega) \end{bmatrix} = \omega_\lambda(\omega) [R^+_\lambda(\omega) \; -S^+_\lambda(\omega)] \begin{bmatrix} X_{\lambda'}(\omega) \\ Y_{\lambda'}(\omega) \end{bmatrix} \tag{2.71}$$

By exchanging λ and λ' in the last equation, and subtracting this from Eq.(2.70), we arrive at the orthogonality

relation,

$$\begin{bmatrix} R^{+}_{\lambda},(\omega) & S^{+}_{\lambda},(\omega) \end{bmatrix} \begin{bmatrix} X_{\lambda}(\omega) \\ -Y_{\lambda}(\omega) \end{bmatrix} = \begin{bmatrix} R^{+}_{\lambda},(\omega) & -S^{+}_{\lambda},(\omega) \end{bmatrix} \begin{bmatrix} X_{\lambda}(\omega) \\ Y_{\lambda}(\omega) \end{bmatrix}$$

$$= \eta_{\lambda}\delta_{\lambda\lambda'} \qquad (2.72)$$

where η_{λ} is a normalisation constant, still to be set.

11) <u>The conjugate nature of the solution.</u>

The complex conjugate of the adjoint equation, Eq.(2.69) is

$$\left[A^{*}-i\Gamma(\omega)\right] R^{*}_{\lambda}(\omega) + B^{*}S^{*}_{\lambda}(\omega) = \omega_{\lambda}(\omega) R^{*}_{\lambda}(\omega)$$

and $$BR^{*}_{\lambda}(\omega) + \left[A-i\Gamma(-\omega)\right] S^{*}_{\lambda}(\omega) = -\omega_{\lambda}(\omega) S^{*}_{\lambda}(\omega),$$

which may be rearranged in the form:

$$\begin{bmatrix} A-i\Gamma(-\omega) & B \\ B^{*} & A^{*}-i\Gamma(\omega) \end{bmatrix} \begin{bmatrix} S^{*}_{\lambda}(\omega) \\ R^{*}_{\lambda}(\omega) \end{bmatrix} = -\omega_{\lambda}(\omega) \begin{bmatrix} S^{*}_{\lambda}(\omega) \\ -R^{*}_{\lambda}(\omega) \end{bmatrix} \qquad (2.73)$$

This is almost exactly the original homogeneous equation, Eq.(2.68). In fact, by letting $\omega \to -\omega$, it is clear that the result

$$\begin{bmatrix} A-i\Gamma(\omega) & B \\ B^{*} & A^{*}-i\Gamma(-\omega) \end{bmatrix} \begin{bmatrix} S^{*}_{\lambda}(-\omega) \\ R^{*}_{\lambda}(-\omega) \end{bmatrix} = -\omega_{\lambda}(-\omega) \begin{bmatrix} S^{*}_{\lambda}(-\omega) \\ -R^{*}_{\lambda}(-\omega) \end{bmatrix} \qquad (2.74)$$

retrieves the original homogeneous equation, with solution

the eigenvector $\begin{bmatrix} S^*_\lambda(-\omega) \\ R^*_\lambda(-\omega) \end{bmatrix}$, and corresponding eigenvalue $-\omega_\lambda(-\omega)$.

Furthermore, it is possible to find the eigenvector orthogonal to this, by taking the complex conjugate of the original homogeneous equation, Eq.(2.68), and letting $\omega \to -\omega$. This process yields that the vector $\begin{bmatrix} Y^*_\lambda(-\omega) \\ X^*_\lambda(-\omega) \end{bmatrix}$ with eigenvalue $-\omega^*_\lambda(-\omega)$ satisfies the adjoint equation, Eq.(2.69), and the orthogonality relation Eq.(2.72), becomes

$$\begin{bmatrix} Y^{*+}_{\lambda'}(-\omega) & X^{*+}_{\lambda'}(-\omega) \end{bmatrix} \begin{bmatrix} S^*_\lambda(-\omega) \\ -R^*_\lambda(-\omega) \end{bmatrix} = \begin{bmatrix} Y^{*+}_{\lambda'}(-\omega) & -X^{*+}_{\lambda'}(-\omega) \end{bmatrix} \begin{bmatrix} S^*_\lambda(-\omega) \\ R^*_\lambda(-\omega) \end{bmatrix}$$

$$= \eta_\lambda \delta_{\lambda\lambda'}, \qquad (2.75)$$

In summary, if we have a solution to the homogeneous equation, Eq.(2.69) as $\begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix}$ with eigenvalue $\omega_\lambda(\omega)$, then $\begin{bmatrix} R_\lambda(\omega) \\ S_\lambda(\omega) \end{bmatrix}$ is a solution to the adjoint equation, and is therefore orthogonal to it. Furthermore, $\begin{bmatrix} S^*_\lambda(-\omega) \\ R^*_\lambda(-\omega) \end{bmatrix}$ is also a solution to the original homogeneous equation, with associated eigenvalue $-\omega_\lambda(-\omega)$, and has orthogonal to it, $\begin{bmatrix} Y^*_\lambda(-\omega) \\ X^*_\lambda(-\omega) \end{bmatrix}$, the solution to the adjoint equation.

The normalisation constant η_λ, as defined by Eq.(2.72) may be directly obtained by continuity with the case of $\Gamma_{mi} \to 0$, thus ensuring that the matrix operator in the equation for F (Eq.(2.66)) is Hermitian, as discussed, for example, by Thouless [Th 61]. i.e. the set of vectors associated with the eigenvalue $\omega_\lambda(\omega)$ are normalised such that $\eta_\lambda = 1$, and those associated with the eigenvalue

$-\omega_\lambda(-\omega)$ are normalised such that $\eta_\lambda=-1$.

Having established these properties, we move on to construct F in terms of its eigenvectors.

$$\text{let} \quad F = \sum_\lambda \begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix} C_\lambda(\omega) \tag{2.76}$$

where $C_\lambda(\omega)$ is an unknown matrix, and the vector $\begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix}$ represents generally, all the possible eigenvectors satisfying the homogeneous equation, Eq.(2.68), i.e. $\begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix}$ represents each eigenvector of the form $\begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix}$ as well as $\begin{bmatrix} S^*(-\omega) \\ R^*_\lambda(-\omega) \end{bmatrix}$. Let us call the general eigenvalue corresponding to $\begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix}$, Ω_λ. Clearly, when $\begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix}$ is taken to represent $\begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix}$, Ω_λ will represent the corresponding eigenvalue $\omega_\lambda(\omega)$, and similarly, when $\begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix}$ represents $\begin{bmatrix} S^*_\lambda(-\omega) \\ R^*_\lambda(-\omega) \end{bmatrix}$, correspondingly, $\Omega_\lambda=-\omega_\lambda(-\omega)$.

Upon inserting this ansatz for F into the defining equation, the expression

$$\sum_\lambda (\Omega_\lambda-\omega) \begin{bmatrix} P_\lambda \\ \Omega_\lambda \end{bmatrix} C_\lambda(\omega) = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{2.77}$$

results. Multiplying this from the left by the associated orthogonal vector $[R^+_\lambda,(\omega) \quad S^+_\lambda,(\omega)]$ or $[Y^*{}^+_\lambda,(-\omega) \quad X^*{}^+_\lambda,(-\omega)]$, generally represented by $[U_\lambda, V_\lambda,]$, results in an expression for $C_\lambda(\omega)$:

$-\omega_\lambda(-\omega)$ are normalised such that $\eta_\lambda=-1$.

Having established these properties, we move on to construct F in terms of its eigenvectors.

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Upon inserting this ansatz for F into the defining equation, the expression

$$\sum_\lambda (\Omega_\lambda-\omega)\begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix} C_\lambda(\omega) = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{2.77}$$

results. Multiplying this from the left by the associated orthogonal vector $[R^+_{\lambda'}(\omega) \quad S^+_{\lambda'}(\omega)]$ or $[Y^*{}^+_{\lambda'}(-\omega) \quad X^*{}^+_{\lambda'}(-\omega)]$, generally represented by $[U_{\lambda'}, V_{\lambda'}]$, results in an expression for $C_\lambda(\omega)$:

$$C_\lambda(\omega) = \frac{[U_\lambda^+ \quad V_\lambda^+]}{\Omega_\lambda - \omega}\,\eta_\lambda \tag{2.78}$$

Hence, F may be written in terms of its eigenvectors and eigenvalues as

$$\begin{aligned} F = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} &= \sum_\lambda \frac{\eta_\lambda}{\Omega_\lambda - \omega} \begin{bmatrix} P_\lambda \\ Q_\lambda \end{bmatrix} [U_\lambda \quad V_\lambda] \\ &= \sum_\lambda{}' (\omega_\lambda(\omega) - \omega)^{-1} \begin{bmatrix} X_\lambda(\omega) \\ Y_\lambda(\omega) \end{bmatrix} [R_\lambda^+(\omega) \quad S_\lambda^+(\omega)] \\ &+ \sum_\lambda{}' (\omega_\lambda(-\omega) + \omega)^{-1} \begin{bmatrix} S_\lambda^*(-\omega) \\ R_\lambda^*(-\omega) \end{bmatrix} [Y_\lambda^{*+}(-\omega) \quad X_\lambda^{*+}(-\omega)] \end{aligned} \tag{2.79}$$

where $\sum'$ means that a restriction is placed on the summation to avoid double counting. We compare this with the structure of the general result for F, Eq.(2.58). In doing this, we see that the set of eigenvalues $\omega_\lambda(\omega)$ must have a positive real part, and lie below the real axis. Then it follows that the pole structure of this construction Eq.(2.79) is in accord with the general result. This can be seen pictorially in Fig.2.6, where, for completeness sake, the eigenvalues corresponding to the adjoint problem are also included.

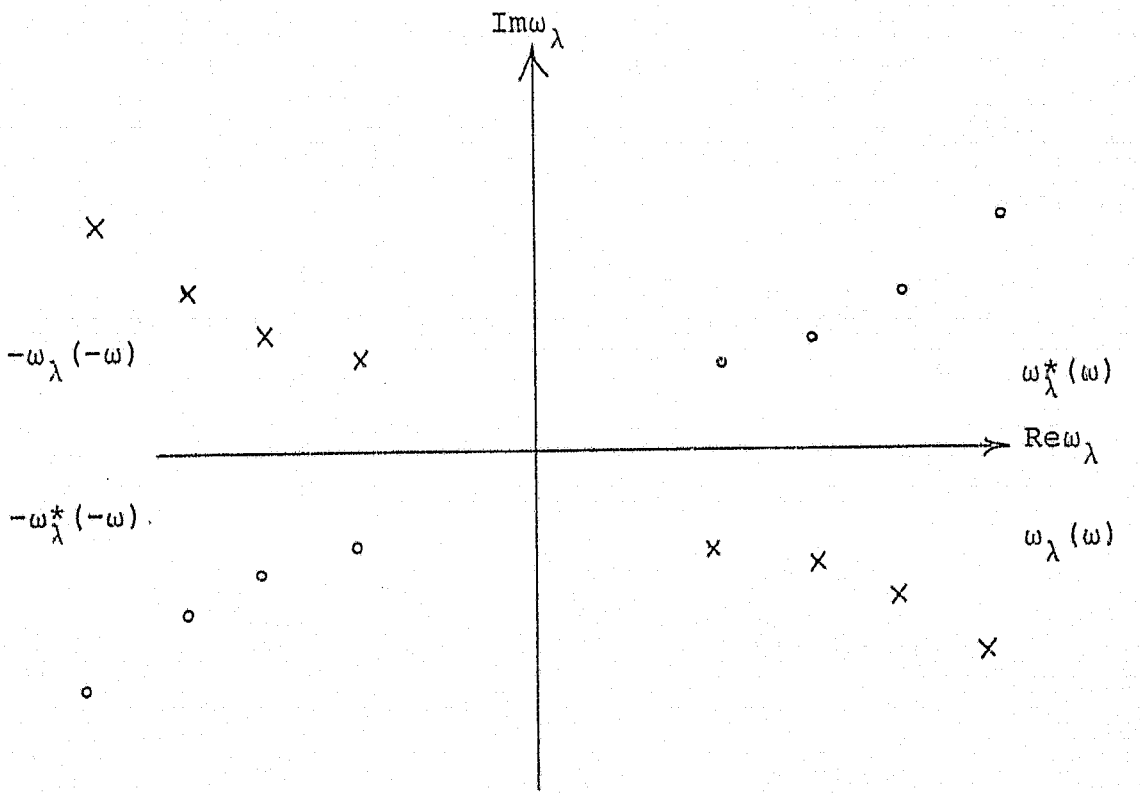


Fig.2.6 The structure of the eigenvalues of the homogeneous equation, Eq.(2.68) denoted by the symbol "x" and the eigenvalues corresponding to the adjoint equation, denoted by the symbol "o", is displayed. Note that if $\Gamma(\omega) \neq \Gamma(-\omega)$, then $\omega_\lambda(\omega) \neq \omega_\lambda(-\omega)$.

We see from this that the restriction on summation is that all sums must run over λ such that $\mathrm{Re}\,\omega_\lambda(\omega) > 0$, to avoid double counting. From the matrix equation of F, Eq.(2.79), the identification of each submatrix may be made:

$$F_{11} = \sum_\lambda{}' \left(\omega_\lambda(\omega) - \omega\right)^{-1} X_\lambda(\omega) R_\lambda^+(\omega) + \sum_\lambda{}' \left(\omega_\lambda(-\omega) + \omega\right)^{-1} S_\lambda^*(-\omega) Y_\lambda^{*+}(-\omega)$$

$$F_{12}=\sum_{\lambda}{}' (\omega_\lambda(\omega)-\omega)^{-1} X_\lambda(\omega) S_\lambda^+(\omega) + \sum_{\lambda}{}' (\omega_\lambda(-\omega)+\omega)^{-1} S_\lambda^*(-\omega) X_\lambda^{*+}(-\omega)$$

$$F_{21}=\sum_{\lambda}{}' (\omega_\lambda(\omega)-\omega)^{-1} Y_\lambda(\omega) R_\lambda^+(\omega) + \sum_{\lambda}{}' (\omega_\lambda(-\omega)+\omega)^{-1} R_\lambda^*(-\omega) Y_\lambda^{*+}(-\omega)$$

$$F_{22}=\sum_{\lambda}{}' (\omega_\lambda(\omega)-\omega)^{-1} Y_\lambda(\omega) S_\lambda^+(\omega) + \sum_{\lambda}{}' (\omega_\lambda(-\omega)+\omega)^{-1} R_\lambda^*(-\omega) X_\lambda^{*+}(-\omega) \quad (2.80)$$

and, using these, the identification of each matrix element occurring in the above submatrices, can be made. This yields

$$F(mijn,\omega)=\sum_{\lambda}{}' (\omega_\lambda(\omega)-\omega)^{-1} x_{mi}^\lambda(\omega) r_{nj}^{\lambda *}(\omega) + \sum_{\lambda}{}' (\omega_\lambda(-\omega)+\omega)^{-1} s_{mi}^{\lambda *}(-\omega) v_{nj}^\lambda(-\omega)$$

$$F(minj,\omega)=\sum_{\lambda}{}' (\omega_\lambda(\omega)-\omega)^{-1} x_{mi}^\lambda(\omega) s_{nj}^{\lambda *}(\omega) + \sum_{\lambda}{}' (\omega_\lambda(-\omega)+\omega)^{-1} s_{mi}^{\lambda *}(-\omega) x_{nj}^\lambda(-\omega)$$

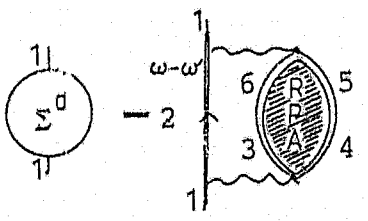
$$F(imjn,\omega)=\sum_{\lambda}{}' (\omega_\lambda(\omega)-\omega)^{-1} v_{mi}^\lambda(\omega) r_{nj}^{\lambda *}(\omega) + \sum_{\lambda}{}' (\omega_\lambda(-\omega)+\omega)^{-1} r_{mi}^{\lambda *}(-\omega) y_{nj}^\lambda(-\omega)$$

$$F(imnj,\omega)=\sum_{\lambda}{}' (\omega_\lambda(\omega)-\omega)^{-1} v_{mi}^\lambda(\omega) s_{nj}^{\lambda *}(\omega) + \sum_{\lambda}{}' (\omega_\lambda(-\omega)+\omega)^{-1} r_{mi}^{\lambda *}(-\omega) x_{nj}^\lambda(-\omega) \quad (2.81)$$

Here $x_{mi}^\lambda(\omega)$ is the $(mi)^{th}$ element of the column vector, $X_\lambda(\omega)$, associated with the eigenvalue $\omega_\lambda(\omega)$, and similarly for $y_{mi}^\lambda(\omega)$, $r_{mi}^\lambda(\omega)$, and $s_{mi}^\lambda(\omega)$.

2.7 SELF-ENERGY CALCULATION USING THE DRESSED PARTICLE-HOLE PROPAGATOR.

The approximation to $\Sigma_1^d(\omega)$ employing the renormalised single particle Green's functions in the calculation of the particle-hole propagator is represented schematically in Fourier space as in Eq.(2.27):



(2.82)

As before, exchange effects are included in this diagram via the interaction matrix elements. Using the rules for translating the diagram, we obtain

$$-i\Sigma_1^d(\omega) = \sum_{23456} (-i)\,(32|\bar{v}|41)\,(-i)\,(51|\bar{v}|62) \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\; iG_2^b(\omega-\omega')\,(-i)F(6543,\omega') \tag{2.83}$$

This may be written more suggestively as:

$$i\Sigma_1^d(\omega) = \sum_{2} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\; G_2^b(\omega-\omega') \sum_{3456} (51|\bar{v}|62)\,F(6543,\omega')\,(32|\bar{v}|41) \tag{2.84}$$

Each label 3,4,5 and 6 ranges over all possible particle and hole states. By designating each of these states specifically as to its character, the expression becomes:

$$i\Sigma_1^d(\omega)=\Sigma\int_{2-\infty}^{\infty}\frac{d\omega'}{2\pi}\,G_2^b(\omega-\omega')\sum_{minj}\{(i1|\tilde{V}|m2)F(mijn,\omega')(n2|\tilde{V}|j1)$$
$$+(i1|\tilde{V}|m2)F(minj,\omega')(j2|\tilde{V}|n1)+(m1|\tilde{V}|i2)F(imjn,\omega')(n2|\tilde{V}|j1)$$
$$+(m1|\tilde{V}|i2)F(imnj,\omega')(j2|\tilde{V}|n1)\} \qquad (2.85)$$

Now, Eq.(2.81) gives the explicit forms for the matrix elements of F, which may be inserted, to yield

$$i\Sigma_1^d(\omega)=\Sigma\int_{2-\infty}^{\infty}\frac{d\omega'}{2\pi}G_2^b(\omega-\omega')\sum_{Re\omega_\lambda>0}\Big\{\frac{1}{\omega_\lambda(\omega')-\omega'}\sum_{minj}[(i1|\tilde{V}|m2)x_{mi}^\lambda(\omega')r_{nj}^{\lambda*}(\omega')(n2|\tilde{V}|j1)$$
$$+(i1|\tilde{V}|m2)x_{mi}^\lambda(\omega')s_{nj}^{\lambda*}(\omega')(j2|\tilde{V}|n1)+(m1|\tilde{V}|i2)y_{mi}^\lambda(\omega')r_{nj}^{\lambda*}(\omega')(n2|\tilde{V}|j1)$$
$$+(m1|\tilde{V}|i2)y_{mi}^\lambda(\omega')s_{nj}^{\lambda*}(\omega')(j2|\tilde{V}|n1)]$$
$$\frac{1}{\omega_\lambda(-\omega')+\omega'}\sum_{minj}[(i1|\tilde{V}|m2)s_{mi}^{\lambda*}(-\omega')y_{nj}^\lambda(-\omega')(n2|\tilde{V}|j1)$$
$$+(i1|\tilde{V}|m2)s_{mi}^\lambda(-\omega')x_{nj}^\lambda(-\omega')(j2|\tilde{V}|n1)+(m1|\tilde{V}|i2)y_{nj}^\lambda(-\omega')r_{mi}^{\lambda*}(-\omega')(n2|\tilde{V}|j1)$$
$$+(m1|\tilde{V}|i2)r_{mi}^{\lambda*}(-\omega')x_{nj}^\lambda(-\omega')(j2|\tilde{V}|n1)]\Big\} \qquad (2.86)$$

This expression reduces by noting that the coefficients of $(\omega_\lambda(\omega')-\omega')^{-1}$ and of $(\omega_\lambda(-\omega')+\omega')^{-1}$ factor, to give the simplified expression

$$-i\Sigma_1^d(\omega)=\Sigma\int_{2-\infty}^{\infty}\frac{d\omega'}{2\pi}G_2^b(\omega-\omega')\sum_{Re\omega_\lambda>0}\left\{\frac{A_{12,\lambda}(\omega')}{\omega_\lambda(\omega')-\omega'}+\frac{B_{12,\lambda}(-\omega')}{\omega_\lambda(-\omega')+\omega'}\right\}$$

where
$$A_{12,\lambda}(\omega')=[\sum_{mi}x_{mi}^\lambda(\omega')(i1|\tilde{V}|m2)+\sum_{mi}y_{mi}^\lambda(\omega')(m1|\tilde{V}|i2)]$$
$$\times[\sum_{nj}r_{nj}^{\lambda*}(\omega')(n2|\tilde{V}|j1)+\sum_{nj}s_{nj}^{\lambda*}(\omega')(j2|\tilde{V}|n1)]$$

and
$$B_{12,\lambda}(-\omega')=[\sum_{mi}x_{mi}^\lambda(-\omega')(i2|\tilde{V}|m1)+\sum_{mi}y_{mi}^\lambda(-\omega')(m2|\tilde{V}|i1)]$$
$$\times[\sum_{nj}r_{nj}^{\lambda*}(-\omega')(n1|\tilde{V}|j2)+\sum_{nj}s_{nj}^{\lambda*}(-\omega')(j1|\tilde{V}|m2)] \qquad (2.87)$$

In order to evaluate the integral over ω', write $\omega_\lambda(\omega)$ in terms of its real and imaginary parts, as

$\omega_\lambda(\omega)=\alpha(\omega)-i\beta(\omega)$, where $\alpha_\lambda(\omega),\beta_\lambda(\omega)>0$. The expression for $G_2^b(\omega-\omega')$ is given by Eq.(2.11). We then evaluate $\Sigma_1^d(\omega)$ in two stages.

i) Suppose label 2 refers to a particle state, p.

Then

$$G_p^b(\omega-\omega') \simeq -\sum_s \frac{|(c_p^+)_{so}|^2}{\omega'-\omega+\lambda+\epsilon_s-i\eta} \tag{2.88}$$

It is evident from Eq.(2.87) for $\Sigma_1^d(\omega)$, that the only non-zero contribution to the integral will arise from multiplying $G_p^b(\omega-\omega')$ by the first term given in parentheses, since poles of this structure are at $\omega'=\omega_\lambda(\omega')$. i.e. at $\omega'=\alpha(\omega')-i\beta(\omega')$, which fall in the opposite imaginary half plane of ω' as the pole arising from $G_p^b(\omega-\omega')$. Hence, we are required to evaluate $\Sigma_1^d(\omega)$, given by

$$i\Sigma_1^d(\omega) = \sum_{p,\lambda,s}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\,\frac{|(c_p^+)_{so}|^2}{\omega'-\omega+\lambda+\epsilon_s-i\eta}\;\;\frac{A_{1p,\lambda}(\omega)}{\omega'-\omega_\lambda(\omega')} \tag{2.89}$$

On performing the integral over ω', and approximating the sum over s by the appropriate integral, the expression resulting is:

$$i\Sigma_1^d(\omega) \simeq \frac{i}{D_s}\sum_{p,\lambda}\int_o^\infty d\epsilon_s\,\overline{|(c_p^+)_{so}|^2}\;\frac{A_{1p,\lambda}(\omega-\lambda-\epsilon_s)}{\omega-\lambda-\epsilon_s-\alpha_\lambda(\omega-\lambda-\epsilon_s)+i\beta_\lambda(\omega-\lambda-\epsilon_s)} \tag{2.90}$$

Since the functional dependence of $A_{1p,\lambda}$, α_λ and β_λ on ϵ_s is unknown, as well as the dependence of $W_p(\epsilon_s+\lambda)$ occurring

in $\overline{|(c_p^+)_{so}|^2}$ on ε_s, we employ the same technique in evaluating this as was used in the evaluation of Eq.(2.42). i.e. we evaluate the functions $A_{1p,\lambda}$, α_λ and β_λ at the value of ε_s at which $\overline{|(c_p^+)_{so}|^2}$ peaks, i.e. at $\varepsilon_s=\varepsilon_p-\lambda$. This results in the functions $A_{1p,\lambda}(\omega-\varepsilon_p)$, $\alpha_\lambda(\omega-\varepsilon_p)$ and $\beta_\lambda(\omega-\varepsilon_p)$, all of which have no dependence on ε_s. We now evaluate $W_p(\varepsilon_s+\lambda)$, from Eq.(2.29), at $\varepsilon_s=\omega-\lambda-\alpha_\lambda(\omega-\varepsilon_p)$ ignoring β_λ as being small, and using α_λ taken at its new value, independent of ε_s. This results in the function $W_p(\omega-\alpha_\lambda(\omega-\varepsilon_p))\equiv W_p(\omega-\alpha_\lambda)$ appearing in the expression for $\overline{|(c_p^+)_{so}|^2}$. With all unknown functional dependences on ε_s now removed by this procedure, the integral over ε_s may be performed. (As before, the limits of integration are extended to $\pm\infty$). The resulting expression is

$$\Sigma_1^d(\omega) \simeq \sum_{p,\lambda} \frac{A_{1p,\lambda}(\omega-\varepsilon_p)}{\omega-\varepsilon_p-\omega_\lambda(\omega-\varepsilon_p)+iW_p(\omega-\alpha_\lambda)} \qquad (2.91)$$

ii) Suppose label 2 refers to a hole state, h.

This is dealt with along the same lines as for the particle state. We see that

$$G_h^b(\omega-\omega') \simeq -\sum_{s'} \frac{|(c_h)_{s'o}|^2}{\omega'-\omega+\lambda-\varepsilon_{s'}+i\eta} \qquad (2.92)$$

and the non-zero contribution to $\Sigma_h^d(\omega)$ is given by folding this with the second term in parentheses in Eq.(2.87).

i.e. we write

$$i\Sigma_1^d(\omega) = -\sum_{\substack{h,s'\\ \lambda}} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{|(c_h)_{s'o}|^2}{\omega'-\omega+\lambda-\varepsilon_{s'}+i\eta} \frac{B_{1h,\lambda}(-\omega')}{\omega_\lambda(-\omega')+\omega'} \tag{2.93}$$

Employing the same procedure as for the particle case, we perform the integral over ω', and replace the sum over s' by the appropriate integral, resulting in the expression

$$\Sigma_1^d(\omega) \simeq \sum_{h,\lambda} \frac{1}{D_{s'}} \int_o^\infty d\varepsilon_{s'} \overline{|(c_h)_{s'o}|^2} \frac{B_{1h,\lambda}(-\omega+\lambda-\varepsilon_{s'})}{\omega-\lambda+\varepsilon_{s'}+\alpha_\lambda(\lambda-\omega-\varepsilon_{s'})-i\beta_\lambda(\lambda-\omega-\varepsilon_{s'})} \tag{2.94}$$

Employing the same technique as for the evaluation of Eq.(2.42), we find that to eliminate the functional dependence of $B_{1h,\lambda}$, α_λ, and β_λ on $\varepsilon_{s'}$, these functions must be evaluated at the peak of $\overline{|(c_h)_{s'o}|^2}$. i.e., such that $\varepsilon_{s'}=\lambda-\omega-\alpha_\lambda(\varepsilon_h-\omega)$. (We neglect the contribution from the imaginary part β_λ, as before.). This results in $W_h(\lambda-\varepsilon_{s'})$ being replaced by $W_h(\omega+\alpha_\lambda(\varepsilon_h-\omega)) \equiv W_h(\omega+\alpha'_\lambda)$. As before, the limits of integration are extended to $-\infty$, and the integral over $\varepsilon_{s'}$ performed, resulting in

$$\Sigma_1^d(\omega) \simeq \sum_{h,\lambda} \frac{B_{1h,\lambda}(\varepsilon_h-\omega)}{\omega-\varepsilon_h+\omega_\lambda(\varepsilon_h-\omega)-iW(\omega+\alpha'_\lambda)} \tag{2.95}$$

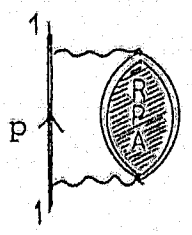
Finally, the results, equations (2.91) and (2.95) may be combined into a single general expression for $\Sigma_1^d(\omega)$:

$$\Sigma_1^d(\omega) \simeq \sum_{\lambda}{}' \sum_{2} \left\{ \frac{(1-n_2)A_{12,\lambda}(\omega-\varepsilon_2)}{\omega-\varepsilon_2-\omega_\lambda(\omega-\varepsilon_2)+iW_2(\omega-\alpha_\lambda)} \right.$$

$$\left. \frac{n_2 B_{12,\lambda}(\varepsilon_2-\omega)}{\omega-\varepsilon_2+\omega_\lambda(\varepsilon_2-\omega)-iW_2(\omega+\alpha'_\lambda)} \right\} \qquad (2.96)$$

From this result, it is evident that the damping of the single particle or hole is once again controlled by off-shell energy arguments of the imaginary parts, the width W_2 associated with the particle or hole, and β_λ, the width arising from the interacting dressed bubbles. The energy arguments occurring in these functions can be seen to accord with the requirements of energy conservation, as described in Sec.2.5. We demonstrate how this operates here.

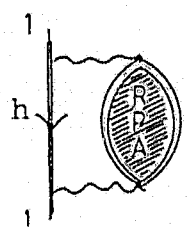
The specific (Goldstone) diagram represented by the first term of $\Sigma_1^d(\omega)$ is:



Associated with the propagator p, is the width W_p, which is expected to be off-shell by an amount $\tilde{\varepsilon}_p$, such that $\tilde{\varepsilon}_p+\alpha_\lambda=\omega$, at the point of resonance. i.e. $\tilde{\varepsilon}_p=\omega-\alpha_\lambda$. The imaginary part contributing to $\Sigma_1^d(\omega)$ from the particle-hole propagator is β_λ, which should be evaluated at the (real) energy $\tilde{\omega}_\lambda$, such that $\varepsilon_p+\tilde{\omega}_\lambda=\omega$. i.e. $\tilde{\omega}_\lambda=\omega-\varepsilon_p$. This confirms the

results obtained in the first term of Eq.(2.96).

The specific (Goldstone) diagram represented by the second term in parentheses is



Associated with h, is the width W_h, expected to be off-shell by an amount $\tilde{\varepsilon}_h$, such that $-\tilde{\varepsilon}_h+\alpha'_h=-\omega$ at the point of resonance. i.e. $\varepsilon_h=\omega+\tilde{\alpha}_\lambda$. The imaginary part contributing to $\Sigma_1^d(\omega)$ from the particle-hole propagator, β_λ, should be evaluated at $\tilde{\omega}_\lambda$ such that $-\varepsilon_h+\tilde{\omega}_\lambda=-\omega$, or $\tilde{\omega}_\lambda=\varepsilon_h-\omega$ This is confirmed by the second term of Eq.(2.96).

Furthermore, our expression for $\Sigma_1^d(\omega)$ must satisfy the requirement (see Eq.(2.14)) that its imaginary part changes sign as ω passes through λ. Examining each term in the expression for $\Sigma_1^d(\omega)$ individually, the following is evident:

From the first term, $W_2(\omega-\alpha_\lambda)$ is non-zero only if $\omega-\alpha_\lambda>\lambda$, i.e. for values of $\omega>\alpha_\lambda+\lambda>\lambda$, since α_λ is positive. However, we know from Fig.2.6 that the set of values $\omega_\lambda(\omega)$ has an imaginary part only for $\omega>0$. i.e. $\beta_\lambda(\omega)>0$ for $\omega>0$ since $\beta_\lambda(\omega)=-\mathrm{Im}\omega_\lambda(\omega)$. Hence $\omega_\lambda(\omega-\varepsilon_2)$ is non-zero only for $\omega>\varepsilon_2$ which is larger than λ, since ε_2 represents a particle. Hence, from the first term, there can only be a contribution to the imaginary part of $\Sigma_1^d(\omega)$ for ω <u>at least</u> larger than λ.

From the second term, $W_2(\omega+\alpha'_\lambda)$ is non-zero only if $\omega+\alpha'_\lambda<\lambda$ i.e. ω is at least less than λ by a positive amount α'_λ. Similarly $\omega_\lambda(\varepsilon_2-\omega)$ can only contribute if $\varepsilon_2-\omega>0$. i.e. for $\omega<\varepsilon_2$, which is less than λ since ε_2 here represents a hole. Hence, from the second term, there can only be a contribution to the imaginary part of $\Sigma_1^d(\omega)$ for ω <u>at least</u> less than λ.

Thus, since for $\omega>\lambda$, the imaginary part is non-zero only in the first term in $\Sigma_1^d(\omega)$, the sign of $\mathrm{Im}\Sigma_1^d(\omega)$ is uniquely determined from the sign of the imaginary part occurring in this denominator: $\mathrm{Im}\Sigma_1^d(\omega)<0$.

Similarly, for $\omega<\lambda$, the imaginary part is non-zero only in the second term on $\Sigma_1^d(\omega)$, the sign of $\mathrm{Im}\Sigma_1^d(\omega)$ is uniquely determined from the sign of the imaginary part occurring in this denominator: $\mathrm{Im}\Sigma_1^d(\omega)>0$.

These results, viz. $\mathrm{Im}\Sigma_1^d(\omega)\gtrless 0$ for $\omega\lessgtr\lambda$, guarantees that $\Sigma_1^d(\omega)$ changes sign correctly as ω passes through λ.

2.8 SYMMETRY CONSIDERATIONS

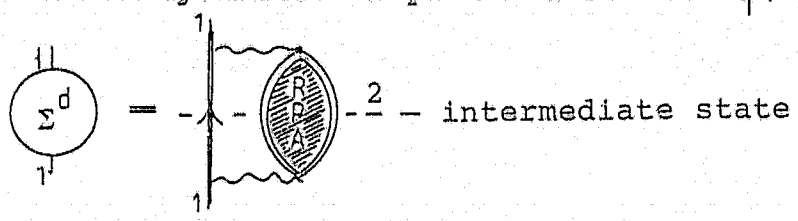
The formalism developed thus far is as yet uncommitted to any nuclear system. In order to study the self-energy in a particular nuclear system, it is necessary to consider the symmetries under which the associated nuclear Hamiltonian is invariant. The symmetries of particular importance in our applications will be those of conservation of angular momentum and isospin.

a) Angular Momentum Coupling

In a spherical nucleus, all single particle or hole states carry good angular momentum. We must therefore be explicit in including the angular momentum involved in describing the label "1" in the construction of the dressed particle hole propagator.

In order to facilitate the inclusion of angular momentum, we alter our notation slightly. Thus far, the labels 1,2, etc., refer to all quantum numbers of the states involved: $1\equiv\{n_1 l_1 j_1 m_1\}$, $2\equiv\{n_2, l_2, j_2, m_2\}$, etc. In the following, however, we deprive the symbols 1 and 2, say, of their magnetic quantum numbers, which will now be written out explicitly, thus: $\{1m_1\}$ and $\{2m_2\}$. The labels 1 and 2 now refer to only the quantum numbers $\{n_1 l_1 j_1\}$ and $\{n_2 l_2 j_2\}$. Thus for example, $F(4321,\omega)$ in this notation means $F(4m_4 3m_3 2m_2 1m_1,\omega)$.

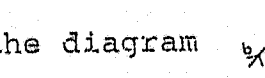
Now consider the diagrammatic representation of $\Sigma_1^d(\omega)$:



Since angular momentum must be conserved, it follows that the total angular momentum of the 'intermediate' state must be j_1, the same as that of label 1. This requires us to consider the internal line, designated by 2, and thus carrying angular momentum j_2, and the dressed RPA bubble independently. It follows that, if the dressed particle-hole propagator carries angular momentum J, then $\vec{J} + \vec{j}_2 = \vec{j}_1$. We therefore introduce the dressed particle-hole propagator of angular momentum J by

$$F^J(\overrightarrow{4321},\omega) = \sum_{\substack{m_1 m_2 \\ m_3 m_4}} (j_4 m_4 j_3 -m_3 | JM)(-)^{j_3-m_3} \times F(4m_4 3m_3 2m_2 1m_1,\omega) \times (j_1 m_1 j_2 -m_2 | JM)(-)^{j_2-m_2} \qquad (2.97)$$

The fact that the particle-hole propagator is actually independent of M is stressed by the notation F^J introduced. This will be seen from the equation that F^J satisfies. The arrow notation introduced in F^J above, indicates the order of coupling of angular momenta, which in turn prescribes the order of the angular momentum

labels appearing in the Clebsch-Gordan coefficients[1], ie that 1 is coupled to 2, and 4 to 3, both to a total angular momentum of J, in the manner prescribed. We apply this notation generally to the coupling of all angular momenta. For example, the matrix element given by the diagram  is simply $(j_5 m_5 j_4 m_4 | V | j_6 m_6 j_3 m_3)$. This may be coupled to a total angular momentum in one of several ways. For example, 5 may be coupled to 4, and 6 to 3, to a total angular momentum J with projection M, or alternatively, 6 to 5 and 4 to 3 may be coupled to (J,M). The former scheme is given by

$$(\underbrace{54}_{J}|V|\underbrace{63}_{J}) = \sum_{m's} (j_5 m_5 j_4 m_4 | JM)\,(j_6 m_6 j_3 m_3 | JM)$$

$$\times\ (j_5 m_5 j_4 m_4 | V | j_6 m_6 j_3 m_3) \qquad (2.98)$$

while the latter is

$$(\overbrace{54}^{J}|V|\underbrace{63}_{J}) = \sum_{m's} (j_6 m_6 j_5 -m_5 | JM)\,(-)^{j_5 - m_5}\,(j_4 m_4 j_3 -m_3 | JM)\,(-)^{j_3 - m_3}$$

$$\times\ (j_5 m_5 j_4 m_4 | V | j_6 m_6 j_3 m_3) \qquad (2.99)$$

Since each state in this matrix element is coupled to a conjugate state, the appropriate phase, see for example [Fe 71], has been included, and the sign of the magnetic

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[1]The Clebsch-Gordan coefficients are defined according to the conventions of A.R Edmonds [Ed 60]

quantum number on which the arrow ends suffers a sign change.

Reversing the order of coupling in any way just leads to the introduction of a phase change, ie

$$(\underset{J}{\underset{\rightarrow}{54}}|V|\underset{J}{\underset{\rightarrow}{63}}) = (-)^{j_5+j_4-J}(\underset{J}{\underset{\leftarrow}{54}}|V|\underset{J}{\underset{\rightarrow}{63}})$$

$$= (-)^{j_6+j_3-J}(\underset{J}{\underset{\rightarrow}{54}}|V|\underset{J}{\underset{\leftarrow}{63}})$$

$$= (-)^{j_3+j_4+j_5+j_6+2J}(\underset{J}{\underset{\leftarrow}{54}}|V|\underset{J}{\underset{\leftarrow}{63}}) \qquad (2.100)$$

and

$$(\underset{J}{\overset{J}{\overset{\rightarrow}{54|V|63}}}) = (-)^{j_5+j_6-J}(\underset{J}{\overset{J}{\overset{\rightarrow}{54|V|63}}})$$

$$= (-)^{j_4+j_3-J}(\underset{J}{\overset{J}{\overset{\leftarrow}{54|V|63}}})$$

$$= (-)^{j_3+j_4+j_5+j_6-2J}(\underset{J}{\overset{J}{\overset{\leftarrow}{54|V|63}}}) \qquad (2.101)$$

The relation between the two coupling schemes is given by

$$(-)^{j_4+j_5+J}(\underset{J}{\underset{\rightarrow}{54}}|V|\underset{J}{\underset{\rightarrow}{63}}) = \sum_{\lambda}(2\lambda+1)\begin{Bmatrix} j_5 & j_4 & J \\ j_3 & j_6 & \lambda \end{Bmatrix}(\underset{\lambda}{\overset{\lambda}{\overset{\rightarrow}{54|V|63}}}) \qquad (2.102)$$

and conversely

$$(\underset{J}{\overset{J}{\overset{\rightarrow}{54|V|63}}}) = \sum_{\lambda}(2\lambda+1)\begin{Bmatrix} j_5 & j_4 & \lambda \\ j_3 & j_6 & J \end{Bmatrix}(-)^{j_4+j_5+\lambda}(\underset{\lambda}{\underset{\rightarrow}{54}}|V|\underset{\lambda}{\underset{\rightarrow}{63}}) \qquad (2.103)$$

where the brackets $\left\{ \right\}$ denote 6-j symbols [Ed 60].

The matrix element $(j_5 m_5 m_4 | V | j_6 m_6 j_3 m_3)$ may also be written in terms of the coupled matrix elements by inverting either of equations (2.98) or (2.99):

$$(5m_5 4m_4|V|6m_6 3m_3) = \sum_{JM} (j_5 m_5 j_4 m_4|JM)(j_6 m_6 j_3 m_3|JM) \times (\underset{\Rightarrow J}{54}|V|\underset{\Rightarrow J}{63})$$

$$= \sum_{JM} (j_6 m_6 j_5 -m_5|JM)(-)^{j_5 - m_5}(j_4 m_4 j_3 -m_3|JM)$$

$$(-)^{j_3 - m_3}(\underbrace{j_5 \overbrace{j_4}^{J}}_{J}|V|\underbrace{j_6 j_3}_{J}) \qquad (2.104)$$

Inclusion of the exchange diagram in V results in a modification of the relation connecting the two coupling schemes. One finds

$$(\underbrace{54|\overbrace{V}^{J}|63}_{J}) = \sum_{\lambda}(2\lambda+1)\begin{Bmatrix} j_5 j_4 \lambda \\ j_3 j_6 J \end{Bmatrix}\{(-)^{j_4+j_5+\lambda}(\underset{\lambda}{54}|V|\underset{\lambda}{63}) - (\underset{\lambda}{45}|V|\underset{\lambda}{63})\} \qquad (2.105)$$

in place of Eq.(2.103).

After this detour, we return to the construction of $F^J(4321,\omega)$. We know that $F(4m_4 3m_3 2m_2 1m_1,\omega)$ satisfies an integral equation given by Eq.(2.61). In terms of our revised notation, this equation becomes:

$$F(4m_4 3m_3 2m_2 1m_1, \omega) = \P(43,\omega)\{\delta_{14}\delta_{23}\delta_{m_2 m_3}\delta_{m_1 m_4} - \sum_{\substack{j_5 j_6 \\ m_5 m_6}} (j_5 m_5 j_4 m_4 | V | j_6 m_6 j_3 m_3) F(6m_6 5m_5 2m_2 1m_1, \omega)\} \tag{2.106}$$

We have used the fact that $\P(4m_4 3m_3, \omega)$ does not carry any m labels, since the energies occurring in Eq.(2.37) depend only on $n\ell j$. An analogous equation for $F^J(\underset{\rightarrow}{43}\underset{\leftarrow}{21}, \omega)$ can be obtained from Eq.(2.106) by operating on both sides with the expression

$$\sum_{\substack{m_1 m_2 \\ m_3 m_4}} (j_4 m_4 j_3 {-m_3} | JM)(-)^{j_3 - m_3}(-)^{j_2 - m_2}(j_1 m_1 j_2 {-m_2} | JM)$$

Then the left hand side becomes $F^J(\underset{\rightarrow}{43}\underset{\leftarrow}{21}, \omega)$ by construction, while the first term on the right hand side reduces to $\P(43,\omega)\delta_{14}\delta_{23}$ due to the orthogonality relation [Ed 60]

$$\sum_{m_1 m_2} (j_1 m_1 j_2 m_2 | JM)(j_1 m_1 j_2 m_2 | J'M') = \delta_{JJ'}\delta_{MM'}$$

between Clebsch-Gordan coefficients. The second term on the right hand side is

$$-\sum_{\substack{m_2 m_3 m_4 \\ m_5 m_6 j_5 j_6}} (j_4 m_4 j_3 {-m_3} | JM)(-)^{j_3 - m_3}(-)^{j_2 - m_2}(j_1 m_1 j_2 {-m_2} | JM) \times (j_5 m_5 j_4 m_4 | \tilde{V} | j_6 m_6 j_3 m_3) F(6m_6 5m_5 2m_2 1m_1, \omega) \tag{2.107}$$

$$F(4m_4 3m_3 2m_2 1m_1, \omega) = \P(43,\omega)\{\delta_{14}\delta_{23}\delta_{m_2 m_3}\delta_{m_1 m_4} - \sum_{\substack{j_5 j_6 \\ m_5 m_6}} (j_5 m_5 j_4 m_4 | V | j_6 m_6 j_3 m_3) F(6m_6 5m_5 2m_2 1m_1, \omega)\} \quad (2.106)$$

We have used the fact that $\P(4m_4 3m_3, \omega)$ does not carry any m labels, since the energies occurring in Eq. (2.37) depend only on $n\ell j$. An analogous equation for $F^J(\underset{\rightharpoonup}{4}\underset{\leftharpoondown}{3}\underset{\rightharpoonup}{2}\underset{\leftharpoondown}{1}, \omega)$ can be obtained from Eq. (2.106) by operating on both sides with the expression

$$\sum_{\substack{m_1 m_2 \\ m_3 m_4}} (j_4 m_4 j_3 -m_3 | JM)(-)^{j_3 - m_3}(-)^{j_2 - m_2}(j_1 m_1 j_2 -m_2 | JM)$$

Then the left hand side becomes $F^J(\underset{\rightharpoonup}{4}\underset{\leftharpoondown}{3}\underset{\rightharpoonup}{2}\underset{\leftharpoondown}{1}, \omega)$ by construction, while the first term on the right hand side reduces to $\P(43,\omega)\delta_{14}\delta_{23}$ due to the orthogonality relation [Ed 60]

$$\sum_{m_1 m_2} (j_1 m_1 j_2 m_2 | JM)(j_1 m_1 j_2 m_2 | J'M') = \delta_{JJ'}\delta_{MM'}$$

between Clebsch-Gordan coefficients. The second term on the right hand side is

$$- \sum_{\substack{m_2 m_3 m_4 \\ m_5 m_6 j_5 j_6}} (j_4 m_4 j_3 -m_3 | JM)(-)^{j_3 - m_3}(-)^{j_2 - m_2}(j_1 m_1 j_2 -m_2 | JM)$$

$$\times (j_5 m_5 j_4 m_4 | \tilde{V} | j_6 m_6 j_3 m_3) F(6m_6 5m_5 2m_2 1m_1, \omega) \quad (2.107)$$

By inserting the first relation in Eq.(2.104) for the matrix elements of both the direct and exchange terms, the following expression results:

$$- \sum_{\substack{m_2 m_3 m_4 m_5 m_6 \\ j_5 j_6 J' M'}} (j_4 m_4 j_3 -m_3 | JM)\,(j_1 m_1 j_2 -m_2 | JM)\,(-)^{j_2 - m_2}\,(-)^{j_3 - m_3}$$
$$\times\ (j_5 m_5 j_4 m_4 | J'M')\,(j_6 m_6 j_3 m_3 | J'M')\,F(6m_6 5m_5 2m_2 1m_1, \omega)$$
$$\times\ \left\{ (\underset{J'}{54} | V | \underset{J'}{63}) - (-)^{j_4 + j_5 - J'} (\underset{J'}{45} | V | \underset{J'}{63}) \right\} \qquad (2.108)$$

The summation over three Clebsch-Gordan coefficients may be carried out (see for example, [Fe 71]) to yield

$$\sum_{m_3 m_4 M'} (j_4 m_4 j_3 -m_3 | JM)\,(j_6 m_6 j_3 m_3 | J'M')\,(j_5 m_5 j_4 m_4 | J'M')\,(-)^{j_3 - m_3}$$
$$= (-)^{j_5 - m_5} (j_6 m_6 j_5 -m_5 | JM)\,(-)^{j_4 + j_5 - J'}\,(2J'+1) \begin{Bmatrix} j_5 & j_4 & J' \\ j_3 & j_6 & J \end{Bmatrix} \qquad (2.109)$$

and this gives

$$- \sum_{\substack{m_2 m_5 m_6 \\ j_5 j_6 J'}} (j_6 m_6 j_5 -m_5 | JM)\,(-)^{j_5 - m_5}\,(j_1 m_1 j_2 -m_2 | JM)\,(-)^{j_2 - m_2}$$
$$\times\ F(6m_6 5m_5 2m_2 1m_1, \omega)\,(2J'+1) \begin{Bmatrix} j_5 & j_4 & J' \\ j_3 & j_6 & J \end{Bmatrix}$$
$$\times\ \left\{ (-)^{j_4 + j_5 + J'} (\underset{J'}{54} | V | \underset{J'}{63}) - (\underset{J'}{45} | V | \underset{J'}{63}) \right\} \qquad (2.110)$$

in place of Eq.(2.108). Using the definition of F^J, and Eq.(2.105), this in turn reduces to

$$- \sum_{j_5 j_6} (54|\widetilde{V}|63)^{J}_{J} F^{J}(6521,\omega) \qquad (2.111)$$

Finally then, $F^J(\underset{\sim}{4}\underset{\sim}{3}\underset{\sim}{2}\underset{\sim}{1},\omega)$ is found to satisfy the integral equation:

$$F^J(\underset{\sim}{4}\underset{\sim}{3}\underset{\sim}{2}\underset{\sim}{1},\omega) = \P\{\quad\delta_{14}\delta_{23} - \sum_{j_5 j_6} (54|\widetilde{V}|63)^{J}_{J} F^J(\underset{\sim}{6}\underset{\sim}{5}\underset{\sim}{2}\underset{\sim}{1},\omega)\} \qquad (2.112)$$

which confirms that F^J is indeed independent of M. The simplicity of this answer reflects the power and beauty of the coupling notation of using arrows, since the result Eq. (2.112) is seen to be obtainable directly from the original integral equation for $F(4m_4 3m_3 2m_2 1m_1,\omega)$, by simply choosing the coupling scheme on the left hand side, and then maintaining it throughout the equation.

With this in hand, we now turn to the question of expressing $\Sigma_1^d(\omega)$ in terms of F^J. The general expression for the self-energy, in terms of the revised notation, is

$$i\Sigma_1(\omega) = \sum_{\substack{j_2 \to j_6 \\ m_2 \to m_6}} (3m_3 2m_2|\widetilde{V}|4m_4 1m_1)(5m_5 1m_1|\widetilde{V}|6m_6 2m_2)$$

$$\times \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\, G_2^b(\omega-\omega') F(6m_6 5m_5 4m_4 3m_3,\omega') \qquad (2.113)$$

It is convenient to write

$(3m_3 2m_2|\widetilde{V}|4m_4 1m_1) = (1m_1 4m_4|\widetilde{V}|3m_3 2m_2)^*$, and to express

both matrix elements of V in terms of the coupled matrix elements given by the second expression of Eq.(2.104), including the exchange part. Then Eq.(2.113) becomes

$$i\Sigma_1(\omega) = \sum_{\substack{j_2 \to j_6 m_2 \\ JM}} (j_1 m_1 j_2 -m_2 | JM)^2 (j_4 \overrightarrow{j_1 | \tilde{V} | j_3 j_2})^* (j_5 \overrightarrow{j_1 | \tilde{V} | j_6 j_2}) \times \int \frac{d\omega}{2\pi} F^J(6543,\omega') G_2^b(\omega-\omega') \qquad (2.114)$$

on employing the definition of F^J. Using the standard relation [Ed 60] for interchanging angular momenta,

$$\sum_{m_2 M} (j_1 m_1 j_2 -m_2 | JM)^2 = \left(\frac{2J+1}{2j_1+1}\right) \sum_{m_2 M} (j_2 m_2 JM | j_1 m_1)^2 = \left(\frac{2J+1}{2j_1+1}\right) \qquad (2.115)$$

the general expression for $\Sigma_1^d(\omega)$ becomes

$$i\Sigma_1^d(\omega) = \sum_{j_2 \to j_6} (j_4 \overrightarrow{j_1 | \tilde{V} | j_3 j_2})^* (j_5 \overrightarrow{j_1 | \tilde{V} | j_6 j_2}) \left(\frac{2J+1}{2j_1+1}\right) \times \int \frac{d\omega'}{2\pi} F^J(6543,\omega') G_2^b(\omega-\omega') \qquad (2.116)$$

This result indicates that the contribution to $\Sigma_1^d(\omega)$ from polarisation bubbles carrying a prescribed angular momentum are simply additive. Furthermore, one notes once again that the answer given also follows immediately applying the arrow coupling tec.nique to Eq.(2.113), provided one includes the factor $(2J+1)/(2j_1+1)$. Using the arrow coupling method, the general expression for $\Sigma_1^d(\omega)$ that was obtained in Sec.2.7 as Eq.(2.96) may now be

written in terms of amplitudes $A^J_{12,\lambda}$ and $B^J_{12,\lambda}$ that carry good J:

$$\Sigma_1^d(\omega) = \Sigma_J\left(\frac{2J+1}{2j_1+1}\right)\Sigma'_\lambda \Sigma_2 \left\{ \frac{(1-n_2)A^J_{12,\lambda}(\omega-\varepsilon_2)}{\omega-\varepsilon_2-\omega_\lambda(\omega-\varepsilon_2)+iW_2(\omega-\alpha_\lambda)} + \frac{n_2 B^J_{12,\lambda}(\varepsilon_2-\omega)}{\omega-\varepsilon_2+\omega_\lambda(\varepsilon_2-\omega)-iW_2(\omega+\alpha'_\lambda)} \right\} \qquad (2.117)$$

where

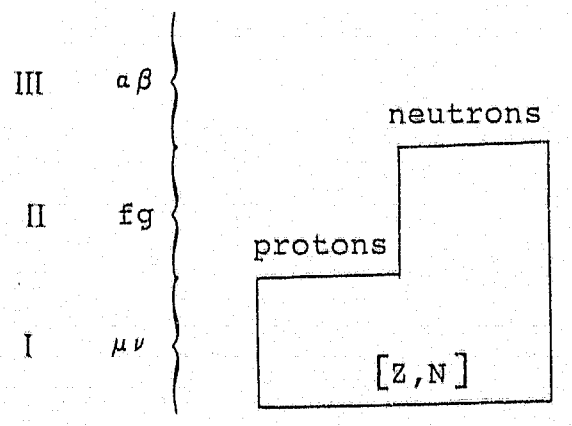
$$A^J_{12,\lambda}(\omega) = \left[\sum_{mi} x^\lambda_{mi}(\omega)\,(i1|\overbrace{\tilde{v}}^{J}|m2) + \sum_{mi} y^\lambda_{mi}(\omega)\,(m1|\overbrace{\tilde{v}}^{J}|i2)\right] \times \left[\sum_{nj} r^{\lambda *}_{nj}(\omega)\,(n2|\overbrace{\tilde{v}}^{J}|j1) + \sum_{nj} s^{\lambda *}_{nj}(\omega)\,(j2|\overbrace{\tilde{v}}^{J}|n1)\right]$$

and

$$B^J_{12,\lambda}(-\omega) = \left[\sum_{mi} x^\lambda_{mi}(-\omega)\,(i2|\overbrace{\tilde{v}}^{J}|m1) + \sum_{mi} y^\lambda_{mi}(-\omega)\,(m2|\overbrace{\tilde{v}}^{J}|i1)\right] \times \left[\sum_{nj} r^{\lambda *}_{nj}(-\omega)\,(n1|\overbrace{\tilde{v}}^{J}|j2) + \sum_{nj} s^{\lambda *}_{nj}(-\omega)\,(j1|\overbrace{\tilde{v}}^{J}|m2)\right]$$

b) The Role of Isospin

One knows now that isospin is conserved to a high degree in all nuclear systems, especially in heavy nuclei [Bo 75]. In the shell model, this feature is recognised by constructing wave functions that contain good total isospin. For nuclei with equal numbers of protons and neutrons, this is trivially done by assigning each single particle state a good isospin quantum number. However, when N>Z, a complication arises that may be seen as follows. Suppose we represent such a nucleus pictorially as in the diagram below where the proton and neutron states have been displaced laterally with respect to each other.



This system, which we denote by |C>, forms the target nucleus in subsequent discussions. In its ground state, |C> carries isospin quantum numbers $[T_o, T_o]$, where $2T_o = N-Z$ is the neutron excess. One notes that three classes of single particle states appear in |C>:

I : states occupied by both neutrons and protons

II : states occupied by neutrons only

III: states lacking both neutrons and protons.

We identify a single particle state in each of these classes by the labels μν, fg, and αβ respectively, while numerical labels, like 1,2 can lie in any of these three regions. States in region I are referred to as <u>core states</u>, while the states in region II are referred to as <u>valence states</u>.

The analog state of such a system is formed by the operation of removing neutrons from a particular state, and creating protons in exactly the same state, a process that is accomplished by the isospin lowering operator

$$T^{(-)} = \sum_1 p_1^+ n_1 \qquad (2.118)$$

Here, the operator n_1 destroys a neutron in state 1 and p_1^+ creates a proton in this same state. Applying this operator to the core, we note that only neutrons from states f,g can be destroyed, since this is the only available region to create protons in, such that the destroyed neutron and created proton are in the same state, and we have, therefore that

$$T^{(-)}|C> = \sum_f p_f^+ n_f |T_o T_o> = \sqrt{2T_o}|T_o\ T_o-1> = \sqrt{2T_o}|A> \qquad (2.119)$$

where $|A>$ is the analog state of $|C>$.

The amplitude $\sqrt{2T_o}$ follows from the properties of $T^{(-)}$, its Hermitian conjugate $T^{(+)}$, and the T_z operator, which

satisfy the same relations as the corresponding angular momentum operators (see for example [Ed 59]). Hence the analog state to the core is:

$$|A\rangle = \frac{1}{\sqrt{2T_o}} T^{(-)} |C\rangle \qquad (2.120)$$

In a target nucleus $|C\rangle$, which contains different numbers of protons and neutrons, the removal of a proton always leads to states of good isospin. However, the removal of a neutron may or may not lead to states of good isospin depending on which region (II or III) the neutron was removed from. The nucleus $[Z-1,N]$ with a single proton removed, carries isospin $[T_o + \frac{1}{2}, T_o + \frac{1}{2}]$, and is the <u>parent</u> of the nucleus $[Z,N-1]$, formed by removing a neutron from $|C\rangle$. The analog nucleus $[Z,N-1]$ carries isospin $[T_o - \frac{1}{2}, T_o - \frac{1}{2}]$. We designate the parent state as $p_\mu|C\rangle$. From this, we may build its analog state by applying the $T^{(-)}$ operator of Eq.(2.118). Thus:

$$\psi_A^{[Z,N-1]} = \frac{T^{(-)}}{\sqrt{2T_o+1}} p_\mu|C\rangle = \{-n_\mu|c\rangle + p_\mu T^{(-)}|C\rangle\}(2T_o+1)^{-\frac{1}{2}} \qquad (2.121)$$

where the factor $(2T_o + 1)^{\frac{1}{2}}$ ensures that this state is normalised to unity. In terms of the analog state of the target, $|A\rangle$, this may be rewritten as

$$\psi_A^{[Z,N-1]} = \{-n_\mu|C\rangle + \sqrt{2T_o}\, p_\mu|A\rangle\}(2T_o+1)^{-\frac{1}{2}} \qquad (2.122)$$

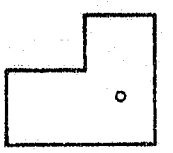
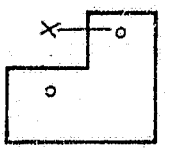
The orthogonal state to this gives the antianalog state,

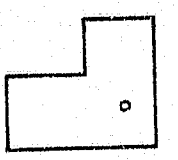
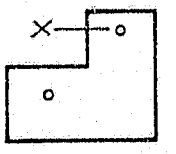
$$\psi_{AA}^{[Z,N-1]} = \{\sqrt{2T_o}\, n_\mu|C\rangle + p_\mu|A\rangle\}(2T_o+1)^{-\frac{1}{2}} \qquad (2.123)$$

which may be written in terms of the core $|C>$ only, as

$$\Psi_{AA}[Z,N-1] = \{\sqrt{2T_o}\, n_\mu |C> + \frac{P_\mu T^{(-)}}{\sqrt{2T_o}}|C>\}(2T_o+1)^{-\frac{1}{2}} \qquad (2.124)$$

Pictorially, the situations represented by equations (2.122) and (2.123) are:

Analog state in nucleus $[Z,N-1]$ = $\left\{ - \right.$  $+\sqrt{2T_o}$  $\left. \right\}(2T_o+1)^{-\frac{1}{2}}$

Antianalog state in nucleus $[Z,N-1]$ = $\left\{ \sqrt{2T_o} \right.$  $+$  $\left. \right\}(2T_o+1)^{-\frac{1}{2}}$

Energetically, the relationship between the target, parent, and analog is summarised in Fig.2.7.

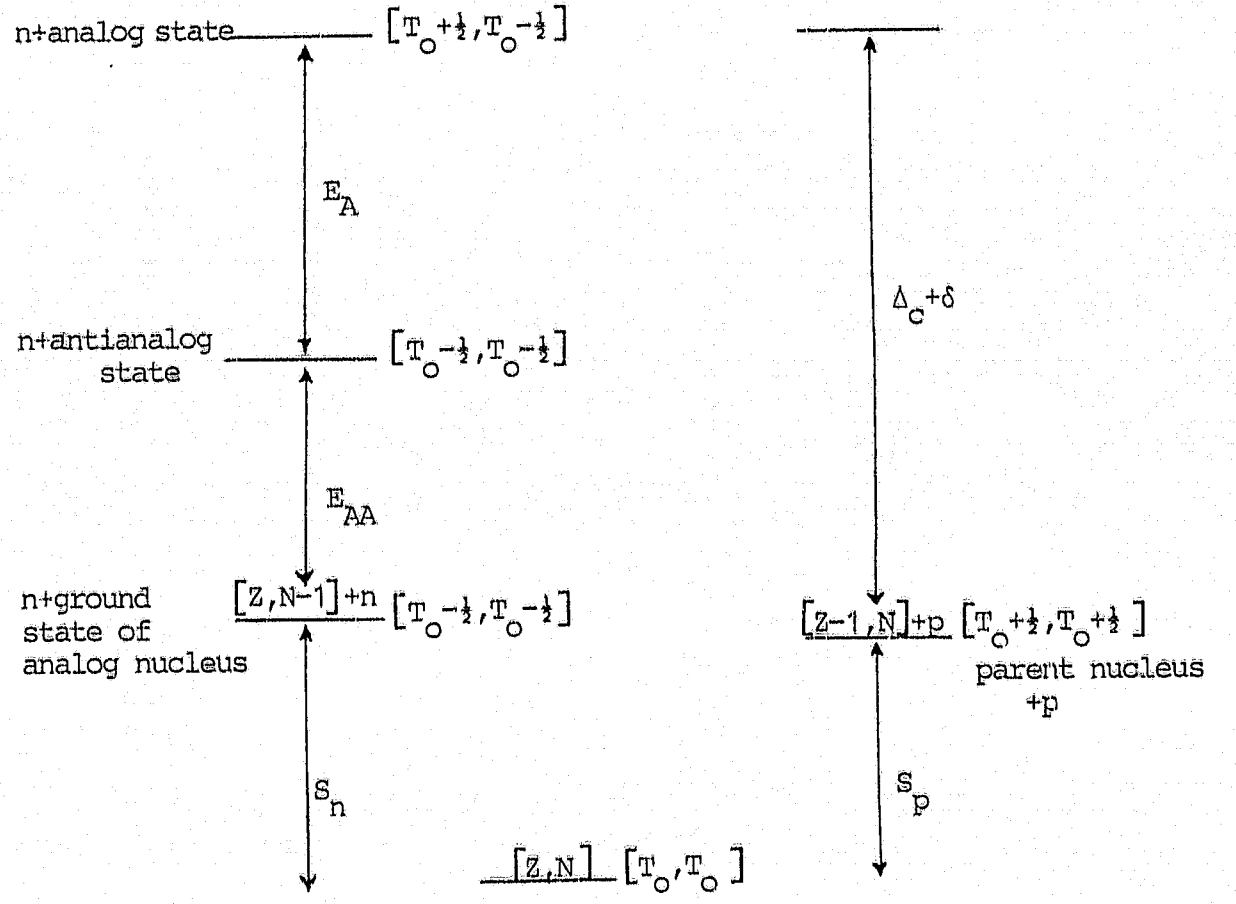


Fig. 2.7: see caption on following page.

Fig.2.7: Schematic energy level diagram showing the relationship between the states of the target, parent, and analog nuclei. S_n and S_p are the neutron and proton separation energies of the target. E_A and E_{AA} are the excitation energies of the analog and antianalog states, measured relative to the ground state of the analog nucleus. The Coulomb energy shift Δ_c can be calculated from an empirical formula $\Delta_c = 1430(Z/A^{1/3})-992$ keV [Ge 80]. δ is the neutron-proton mass difference.

From the expressions for the analog and antianalog states, which both carry good isospin, it is clear that the removal of a single neutron from a deep-lying state within the region designated by μ,ν, does not alone give rise to a state of good isospin. The additional component required to build the good isospin state has a 1 particle - 2 hole character. We therefore refer to $\Psi_A^{[Z,N-1]}$ and $\Psi_{AA}^{[Z,N-1]}$ as isospin quasihole states [Kl 82].

From the equations (2.121) and (2.124) relating the analog and antianalog states to the target, we may define the operators ϕ_μ^+ and $\tilde{\phi}_\mu^+$, which create the analog and antianalog states depicted, as

$$\phi_\mu^+ = \{-n_\mu + p_\mu T^{(-)}\}\,(2T_o+1)^{-\frac{1}{2}}$$

$$\text{and } \tilde{\phi}_\mu^+ = \{\sqrt{2T_o}\,n_\mu + \frac{p_\mu T^{(-)}}{\sqrt{2T_o}}\}\,(2T_o+1)^{-\frac{1}{2}} \qquad (2.125)$$

Eliminating $T^{(-)}$ from these, we obtain the operator relation

$$n_\mu = \{\sqrt{2T_o}\Phi^+_\mu - \phi^+_\mu\}\,(2T_o+1)^{-\frac{1}{2}} \qquad (2.126)$$

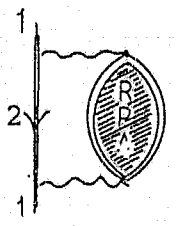
ie if we destroy a neutron in state μ, we do not have a state of good isospin; this destruction operator, however, may be written as a linear combination of analog and antianalog creation operators, which themselves carry good isospin. Hence, a neutron destruction operator can be written selectively as

$$n_1 = \begin{cases} \{\sqrt{2T_o}\Phi^+_1 - \phi^+_1\}\,(2T_o+1)^{-\frac{1}{2}} & \text{if 1 is in Region I} \\ b^+_1(n) & \text{if 1 is in Region II} \\ a_1(n) & \text{if 1 is in Region III} \end{cases} \qquad (2.127)$$

depending on the region in which it acts. Here $b^+_1(n)$ creates a neutron hole in state 1, which is in the occupied region I, and $a_1(n)$ destroys a neutron in state 1 above the occupied regions. This above description of the destruction operator $b^+_1(n)$ in Region II is as yet incomplete, in that it does not recognise the fact that the valence shell may not be completely filled. This problem will be dealt with later in terms of the Pairing Model.

The fact that a neutron in a state μ does not carry good isospin reflects itself in the construction of the

associated $\Sigma_1^d(\omega)$. Let us re-examine the self-energy in this situation. If the label 2 in the diagram below refers to a hole state in Region I, then Eq.(2.125) shows that we must consider the propagation of an analog or an antianalog excitation that isospin conservation requires.



However, if 2 refers to a hole state in Region II, isospin is automatically conserved. The former thus represents the propagation of a quasihole, the latter an ordinary hole. The Green's function puts these features in evidence. Specialising c_2^+ and c_2 in Eq.(2.1) to refer specifically to the neutron creation and destruction operators n_2^+ and n_2, one has

$$iG_2^b(t) = \langle C|Tn_2(t)n_2^+(0)|C\rangle$$

$$= \frac{2T_o}{2T_o+1}\langle C|T\tilde{\phi}_\mu(t)\tilde{\phi}_\mu^+(0)|C\rangle + \frac{1}{2T_o+1}\langle C|\phi_\mu(t)\phi_\mu^+(0)|C\rangle$$

2 in Region I (2.128a)

$$= \langle C|Tb_f^+(t)b_f(0)|C\rangle, \quad 2 \text{ in Region II} \quad (2.128b)$$

There is no contribution from Region III. In Region II, the neutron state 2 propagates according to Eq.(2.128b). in Region I according to <u>that component</u> of G_2^b, ie $\langle C|T\tilde{\phi}_\mu(t)\tilde{\phi}_\mu^+(0)|C\rangle$ or $\langle C|T\phi_\mu(t)\phi_\mu^+(0)|C\rangle$, depending on whether the antianalog or analog excitation is present

in state 2. Since the polarisation bubble in the diagrams that we consider have $\Delta T = 0$, the states 1 and 2 must refer to the same isospin. For the $T_o - \frac{1}{2}$ channel therefore, state 2 must be an antianalog state if it falls in Region I; under these circumstances, the self-energy $\Sigma_1^d(\omega)$ becomes

$$\Sigma_1^d(\omega) \rightarrow \frac{2T_o}{2T_o+1}\Sigma_\mu^d(\omega)$$

where
$$\Sigma_\mu^d(\omega) = \sum_J \left(\frac{2J+1}{2j_1+1}\right)\sum_{\substack{\text{Re}\,\omega_\lambda^2<0}}\left\{\frac{(1-n_2)A^J_{12,\lambda}(\omega-\varepsilon_{aa})}{\omega-\varepsilon_{aa}-\omega_\lambda(\omega-\varepsilon_{aa})+iW_2(\omega-\alpha_\lambda)}\right.$$
$$\left. + \frac{n_2 B^J_{12,\lambda}(\varepsilon_{aa}-\omega)}{\omega-\varepsilon_{aa}+\omega_\lambda(\varepsilon_{aa}-\omega)-iW_2(\omega+\alpha'_\lambda)}\right\} \qquad (2.129)$$

In this expression, $A^J_{12,\lambda}(\omega)$ and $B^J_{12,\lambda}(-\omega)$ are as defined by Eq.(2.117) and $\lambda-\varepsilon_{aa} = E_{AA}$, where E_{AA} is the excitation energy of the antianalog state in the nucleus $[Z,N-1]$, as depicted in Fig.2.7. The label 2 now includes the isospin quantum number also.

If, however, state 2 lies in Region II, where isospin is automatically conserved, we may take over the result for $\Sigma_1^d(\omega)$ from Eq.(2.117) with <u>no</u> modification, other than that all labels now include the associated isospin quantum numbers.

CHAPTER 3.

DEEP-LYING HOLE STATES IN THE TIN ISOTOPES.

Hole states in the isotopes of Tin have been extensively studied via neutron pickup reactions on targets ranging from ^{112}Sn to ^{120}Sn by several authors, [Va 74], [Va 77], the most recent data being due to Gerlic at al. [Ge 80] relating to the isotopes ^{112,116,120}Sn. Furthermore, since the structure of the Tin isotopes is well understood both theoretically and experimentally [Ki 60], they provide us with an ideal system for testing the theoretical ideas developed so far. However, a technical complication arises in this connection. Thus far, the basis states that we have employed in the construction of the necessary Green's function were of a pure single particle character, or at most complicated by considerations of isospin. However, in the Tin isotopes, the neutron valence shell is incomplete, and, as a consequence, the intrashell scattering of neutrons leads to strong deviations from a single particle spectrum for these isotopes that one would associate with independent neutron motion. This problem was studied some time ago by Kisslinger and Sorensen [Ki 60], who showed quite conclusively that the residual interactions between the valence shell neutrons play an essential role in

establishing the low-lying spectra of the Tin isotopes. They show further that the main features of these spectra can be accounted for in considerable detail by extracting the two main features of the two-body interaction, viz. that of pairing, and that of a weaker long range component, and implementing these features in terms of an effective "pairing plus P_2 interaction". The ground state of such a model system has the characteristics of a superconducting (BCS) ground state, while the excitations consist of elementary modes, represented by independent quasiparticle motion (see [Bs 77] for example for a concise overview of the superconducting properties of nuclei).

The pairing model has been enormously successful in accounting for features like the energy gap in the spectra of the isotopes with even neutron numbers, (and the lack of a gap for those with odd neutron numbers), as well as the occurrence of a low-lying collective 2^+ level. Consequently we must give consideration to the role of pairing in the application of our formalism to the discussion of hole states in these isotopes.

3.1 THE PAIRING MODEL IN BRIEF.

The approximation of a "pairing" interaction is introduced in nuclear physics to mimic the main feature of a short range interaction between nucleons in a nucleus. The effect of this is to lead to large matrix elements between states in which pairs of nucleons are coupled to zero total angular momentum. The essential effect of the pairing interaction is to introduce an energy gap [Ki 60] in the spectrum of elementary excitations described by the quasiparticle creation and destruction operators, A^+_α and A_α : (we follow here the notation of [Fe 71])

$$A^+_\alpha = u_a c^+_\alpha - v_a S_\alpha c_{-\alpha}$$
$$A_\alpha = u_a c_\alpha - v_a S_\alpha c^+_{-\alpha} \qquad (3.1)$$

The state labels $\pm\alpha$ are an abbreviation for $\{a, \pm m_\alpha\}$, where "a" summarizes the non-magnetic quantum numbers of the single particle states, and $S_\alpha = (-)^{j_\alpha - m_\alpha}$ is a phase factor. The operators A^+_α and A_α create and destroy quasiparticles of excitation energy

$$E_a = \sqrt{(\varepsilon_a - \lambda)^2 + \Delta_a^2} \qquad (3.2)$$

relative to the quasiparticle ground state $|0\rangle$, that acts as a vacuum state for the A_α:

$$A_\alpha|0\rangle = 0 \tag{3.3}$$

The parameters Δ_a and λ in Eq.(3.2) are the gap parameter and Lagrange multiplier (chemical potential), the former being a consequence of the pairing interaction, the latter being required to ensure conservation of particle number on the average in the presence of the transformation, Eq.(3.1). Finally u_a and v_a are given by

$$u_a^2 = \tfrac{1}{2}\left[1 + \frac{\varepsilon_a - \lambda}{E_a}\right]$$

$$v_a^2 = \tfrac{1}{2}\left[1 - \frac{\varepsilon_a - \lambda}{E_a}\right] \tag{3.4}$$

and represent the probability for the single particle state a being unoccupied or occupied respectively, in the quasiparticle ground state. They are shown in Fig. 3.1

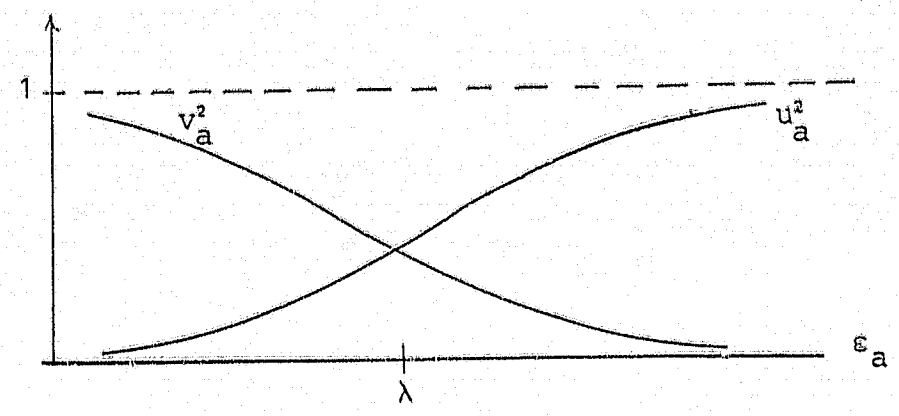
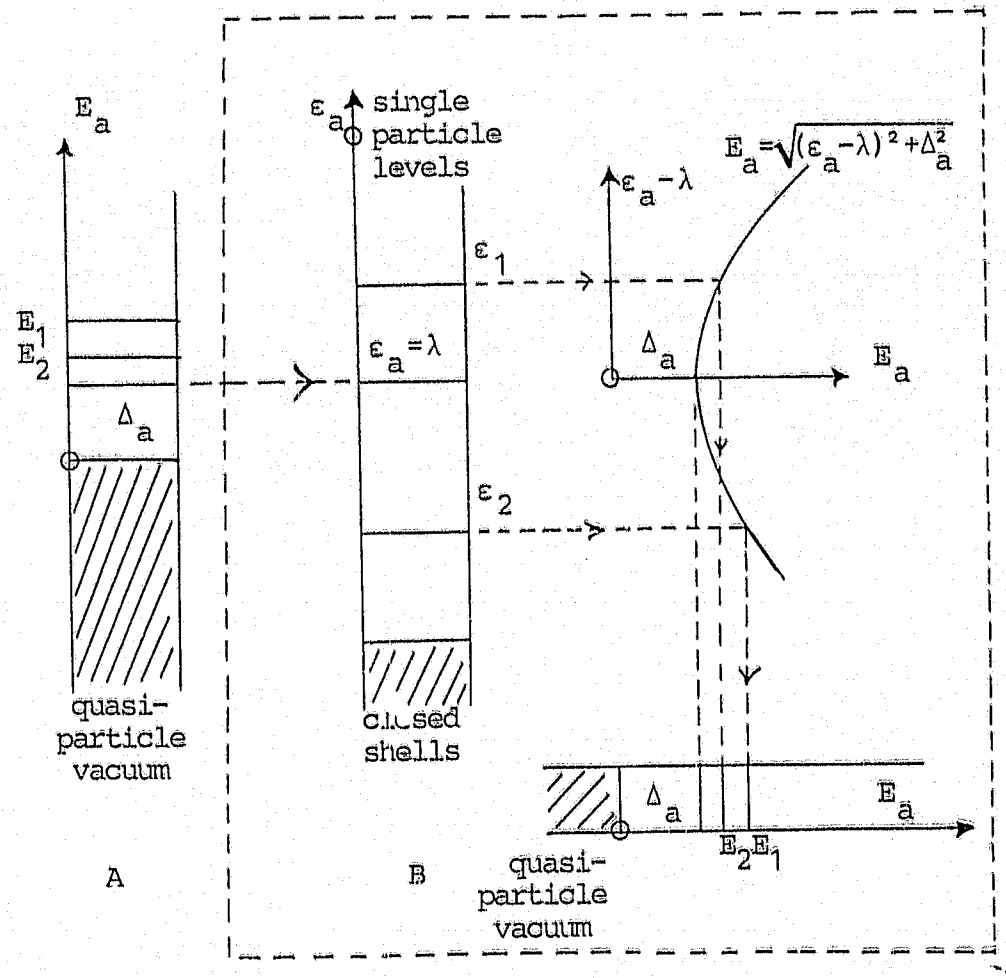


Fig. 3.1 Occupation probabilities u_a^2 and v_a^2

As the notation used suggests, the energies E_a and Δ_a, and the amplitudes u_a and v_a are independent of the magnetic quantum number m_α, in a spherical nucleus.

A concise re-derivation of these results will be found in Appendix C. Of more immediate importance to us are the relationships between the quasiparticle energies E_a, and the excitation energies of these quasiparticle energies relative to the chemical potential λ of the nucleus in which they occur. These relationships are displayed in Fig. 3.2 [Ču 63].

Fig. 3.2 Relationship between the single-particle energies ε_a, and the quasiparticle energies E_a.



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