THE SIGNIFICANCE OF POISSON'S RATIO IN THE DETERMINATION OF STRESS AND SETTLEMENT IN SOILS

H. P. RAUCH

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THIS IS TO CERTIFY THAT THIS THESIS IS A REPORT OF THE RESULTS OF MY OWN INVESTIGATIONS AND WORK CARRIED OUT WHILE ENGAGED INITIALLY IN FULL-TIME POST-GRADUATE STUDY ON A ONE-YEAR S.A.R. & H. RAILWAY ENGINEERING SCHOLARSHIP, AND SUBSEQUENTLY IN PART-TIME STUDY ON MY OWN.

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<u>H.P. RAUCH</u>.

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CONTENTS

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SYNOPSIS

This report describes investigations carried out in order to obtain a better understanding of the concept of Poisson's Ratio as related to the determination of stress and settlement in soils.

Since Poisson's Ratio is primarily a concept associated with the elastic property of materials and therefore generally only enters into soils work in connection with applying the elastic theory to soils, the investigations commence with a brief review of the basic principles of the elastic theory and the elastic properties of materials. It is found that Poisson's Ratio for elastic materials can only fall within the range zero - 0.50.

Consideration is then given to certain 2-dimensional and 3-dimensional problems of stress and strain in the theory of elasticity. It is found that the moduli of elasticity do not affect the stress distribution and hence also the strains in 2-dimensional problems. For the basic 3-dimensional problem viz. that of a vertical point load applied to the horizontal surface of a semi-infinite solid, the affect of various values of Poisson's Ratio on the resulting stress distribution is then considered. The results are presented mainly in the form of stress contour diagrams. These show that in the case of the Maximum Shear Stress (S_{max}) which is a major concern, the stress distribution is insignificantly affected by variation in the value of Poisson's Ratio below a certain comparatively shallow depth. Furthermore, on the axis of loading where the maximum stress values occur, the variation in S_{max} corresponding to the extreme limits of the range of Poisson's Ratio values applicable to elastic materials as mentioned above, is only a constant 14%.

Attention is then turned more directly towards the elastic theory and Poisson's Ratio in relation to soils, and current concepts of elasticity and Poisson's Ratio in soils work are then briefly reviewed. As a result of subsequent discussions, it becomes clear that for <u>all</u> types of saturated soils (ie. both cohesive and cohesionless), Poisson's Ratio can only have the value 0.50 for the purposes of elastic considerations.

An attempt to determine Poisson's Ratio for a soil from laboratory conducted triaxial shear tests, is then described in full. The results of 6 tests conducted show that the value of Poisson's Ratio for the silty sand test soil used (moisture content between 1-2%), lies between 0.14 and 0.24. This order of the values obtained, is found to compare favourably with the order of values obtained by overseas investigators (using methods other than that mentioned above) for various soil types at moisture contents less than saturation value.

It is finally concluded that Poisson's Ratio is only of limited significance in both the determination of stress and settlement in soils. As regards the values of Poisson's Ratio applicable to various soil types for the purposes of elastic considerations, it is concluded that Poisson's Ratio is in the first instance dependent on the moisture content of a soil; secondly, that the value of Poisson's Ratio for <u>all types</u> of <u>saturated</u> soils is 0.50; and thirdly, that in the case of partially saturated soils, the values of Poisson's Ratio for sands are less than those for clays. A suggested graphical relation between Poisson's Ratio and Degree of Saturation is then given in conclusion.

CHAPTER 1 .... /

CHAPTER 1

INTRODUCTION

The essential requirement of any foundation is that it must be able to support any form of superimposed loading in a safe and stable manner and without detriment to the latter. A soil as the most common type of foundation material encountered in practice, must comply with this requirement. A thorough understanding of the stresses and strains in soil foundations resulting from a superimposed loading is thus essential.

There are in general two approaches to foundation design in the more specific sense of bearing capacity of a soil foundation. Firstly, there is the method known as Limit Design, wherein the safe allowable bearing capacity of a soil is obtained from a determination of its ultimate bearing capacity to which a suitable factor of safety against such catastrophic shear failure is then applied, and secondly, the method of Stress Design, wherein a suitable factor of safety against possible overstress is used to obtain the allowable stress in the soil foundation.

In the case of the latter method of foundation design, some method of determining the stress induced in a soil by an applied loading is necessary. The only means by which this is possible is by the use of some theory relating force and the deformation it produces (as the fundamental alternatives for stress and strain), since it is a mechanical property of all matter that the application of load causes deformation. In some instances, however, deformation requires a certain amount of time to be effected, so that in the general case, load, deformation and time are involved. The simplest theory relating these variables is the theory of elasticity which holds for elastic materials, but this theory has the important limitation of applying only to cases wherein stress and strain are proportional and independent of time. Furthermore, as Taylor[1.1] mentions, "solutions based on the theory of elasticity are far from easy for all but the simplest of cases, and any general theory which covers even minor variations from simple, elastic conditions becomes unbelievably complex .... Thus the obtaining of a general stress:strain theory ..... especially for cases where time effects enter, must be acknowledged as an impossible goal."

It has therefore become practice to use the theory of elasticity to evaluate stress and strain in soil masses, on the assumption that soils behave as perfectly elastic, homogeneous, isotropic materials. This assumption and its implications will be discussed in more detail later.

It is in the use of the elastic theory for the purposes of the design of soil foundations by the method of Stress Design that Poisson's Ratio as a property of an elastic material becomes involved, since mathematical expressions for stress and strain derived from the theory of elasticity frequently contain terms of Poisson's Ratio.

Experiments have shown[2.1] that in the case of a prismatical bar of elastic material in simple tension, the axial elongation is a constant for a bar of perfectly elastic material and is known as Poisson's Ratio, after the name of the French mathematician who was the first to determine a value for this ratio by using the molecular theory of structure of a material (see later Section 3.1). In this report, Poisson's Ratio will be denoted by the Greek letter μ.

Hence, .../

Hence, in using expressions derived from the theory of elasticity to evaluate stress and strain in soil foundations, (the latter especially in the form of settlement), the soils engineer is faced with having to decide what value of Poisson's Ratio is applicable to the particular problem he is dealing with, and as yet there is a great deal of uncertainty in making such decisions.

The work described herein is an endeavour to obtain a better understanding of the concept of Poisson's Ratio as related to the determination of stress and settlement in soils by the use of the elastic theory.

In the first Chapters to follow, some aspects of the classical theory of elasticity and Poisson's Ratio in relation to this theory will be discussed, while later Chapters are directed towards the elastic theory and Poisson's Ratio in relation to soils. In particular, a determination of Poisson's Ratio for a soil from measurements made during a triaxial compression test, will be described and discussed.

CHAPTER 2

ELASTIC THEORY CONSIDERATIONS

Whenever dealing with problems relating to the theory of elasticity, it is important to bear in mind the assumptions on which the theory is based, as well as the limits of its applicability.

Most materials possess to some extent the property of elasticity[3.1], viz. that when deformed under the action of a system of applied forces of magnitude not exceeding a certain limit known as the elastic limit, the deformation disappears on removal of the forces and the body regains its original shape. On a broader basis, elasticity implies[4] that there is a definite relation between force and deformation which is independent of any other variable such as time, so that on imposing a load a particular value of strain (as a measure of deformation) occurs, and if this load be removed and re-imposed, the same value of strain as previously will again occur. It should be noted therefore, that elasticity implies only a definite relation between force and deformation and not necessarily any particular relation, such as for instance a linear one.* +

However, Robert Hooke[22],[5.1] was the first to investigate the elastic property of materials more closely and he succeeded in establishing by experiment that in many instances the relation between the magnitude of the forces and the deformations that they produce was approximately linear up to the elastic limit. This approximately linear relation between force and the deformation it produces, is the basis of the so-called Hooke's Law which assumes the ideal material in which the relation between force and deformation is in fact perfectly linear. Hooke's Law subsequently became the basis on which the theory of elasticity was developed, so that the term 'elasticity' used in relation to the theory of elasticity actually implies linear elasticity in accordance with Hooke's Law. Throughout this report, therefore, the term 'elasticity' is intended to be understood in this sense.

For the purposes of mathematical treatment, the theory of elasticity assumes that all bodies are perfectly elastic, i.e. the theory assumes Hooke's Law as was mentioned above. Further, it assumes that the matter of such bodies is homogeneous i.e. that at every point of the body the elastic properties are identical in identical directions; and also that the bodies are isotropic i.e. that the elastic properties are the same in all directions.

The elastic theory can thus strictly be applied, in the first instance, only to materials which have properties in accordance with these assumptions on which the theory is based, and in addition, only to that range of loading during which such properties exist. However, although no material is known

which ..../

* It is well-known[4] that a material such as rubber, for example, has a non-linear force:deformation relationship, due to the fact that as it is extended, it becomes more difficult to extend as the extension proceeds.

+ See also Ref. 1, p.208.

which satisfies all these assumptions completely, the theory has been applied to many materials with reasonable accuracy and to steel with great accuracy. In general, the accuracy with which the theory can be applied to any material will depend on how closely the abovementioned assumptions are realised.

A very important aspect of the theory of elasticity which is often overlooked, is that it only considers two variables viz. force and deformation, or alternatively, stress and strain. All other possible variables such as time and mass are considered to be constant. The magnitude of strain is thus assumed to be independent of the rate of application of stress; also, an instantaneous application of stress is considered to produce an instantaneous strain, and in all cases, during strain the body suffers no change of mass. Hence the theory is not applicable to cases where time effects or change of mass effects occur during strain. In structural materials, both instantaneous strain and constant mass during strain are invariably realised, although in the case of strain certain secondary effects(6.1),(6.2) such as creep do also occur.

With regard to Poisson's Ratio; it has already been mentioned in Chapter 1 that it is a constant in the case of perfectly elastic materials. It should, however, be pointed out in view of the above discussions that this is actually only the case with the ideal material which behaves strictly according to Hooke's Law. By implication, this also means that Poisson's Ratio for the ideal material is only constant within the elastic range of loading, i.e. while the relation between stress and strain is linear. In practice therefore, values of Poisson's Ratio are not rigidly constant(7) within the elastic range and furthermore, when the elastic limit is exceeded Poisson's Ratio becomes a variable quantity with a value which is dependent upon stress. It is known(5.1) that in the case of materials which contract under small load and expand when a state of failure is approached, Poisson's Ratio shows a considerable increase in value when the failure condition is approached.

However, it is of extreme importance to realise that when applying the theory of elasticity to any material, the value of Poisson's Ratio which is applicable, is strictly that constant value pertaining to the ideal elastic state assumed by Hooke's Law. In general, Poisson's Ratio is seldom considered other than in relation to the elastic state, especially as it will be shown in the next Chapter that the concept of Poisson's Ratio was originally derived from a consideration of the plastic state of materials. In this connection also, Taylor(1.2) mentions that: "The meaning of the elastic property known as Poisson's Ratio becomes obscure as soon as it is used in connection with non-elastic materials."

A second constant which is related to the elastic state of materials, is Young's Modulus. The linear relation between force and the deformation it produces, or alternatively, the linear relation between stress and strain, implies that the ratio of stress:strain is a constant. This constant is known as Young's Modulus (denoted by E). Most of the remarks made above in the case of Poisson's Ratio are also applicable in the case of Young's Modulus(7). For the ideal elastic material Young's Modulus therefore also has a constant value within the elastic range of loading.

These ..../

These two constants are however not the only constants of elasticity. The Bulk Modulus (K) and the Modulus of Rigidity (G) are also such constants. It can be shown (see Section 3.2), that the constants of elasticity are inter-related. Yet, from the discussion to follow in the first Section of the next Chapter, it will be seen that for the complete definition of the elastic properties of a material, the elastic theory requires the use of only <u>two</u> such constants, the Modulus of Elasticity E (i.e. Young's Modulus) and Poisson's Ratio μ being the two most commonly accepted (1.1).

CHAPTER 3

POISSON'S RATIO IN RELATION TO ELASTIC MATERIALS

3.1 Historical[5]

During the initial stages of the development of the theory of elasticity at the beginning of the 19th Century, the early investigators such as Navier, Cauchy, Poisson, Lamé, St. Venant, and others, all based their work on the so-called molecular theory viz. that the elastic property of a body can be explained in terms of some kind of attractive and repulsive forces between the ultimate particles of the body.

Navier in 1821, working from the molecular theory, was the first to establish a relation between the deformations and the molecular forces for an isotropic body. These relations were in the form of a set of three differential equations of equilibrium, which notably contained only one elastic constant. This idea, that the elastic properties of an isotropic body could be completely defined by one constant, say the modulus in tension (E), was accepted by the other early investigators already mentioned.

Poisson being very interested in the theory of elasticity based on the idea of molecular structure, obtained equations of equilibrium and boundary conditions similar to those obtained by Navier. Then applying his general equations to isotropic bodies, he found that for simple tension of a prismatical bar the axial elongation must be accompanied by a lateral contraction of magnitude μ, where $\mu = 0.25$. Thus, Poisson's value of $\mu = 0.25$, for the ideal isotropic material, was based on the uni-elastic-constant hypothesis of the molecular theory.

However, in the 1830's George Green offered a derivation of the equations of elasticity without using any hypothesis regarding the behaviour of the molecular structure of elastic bodies. His approach was a derivation of the stress:strain relations from a consideration of strain energy and showed that for an isotropic body two elastic conatants were required, instead of one as demanded by the molecular theory. His paper started a serious controversy and gave rise to two schools of thought in the development of the theory of elasticity. As a result, several physicists became interested in the determination of elastic constants and tried to settle the argument by direct tests.

Experiments by W. Wertheim and later by A.T.Kupffer to determine values of Poisson's Ratio for various materials, in both cases showed values different to Poisson's theoretical value of $\mu = 0.25$, e.g. for steel a value of $\mu = 0.294$ was found. Although these and other experimental results were in conflict with the result from the uni-constant hypothesis, and could only be justified by adopting a two-elastic-constant hypothesis for isotropic bodies, the controversy could not be settled because it was always possible to argue that the materials used had not been perfectly isotropic.

George Green's result found further support when G.G.Stokes and Franz Neumann independently verified, by different theoretical approaches remote from the molecular theory, that two elastic constants were necessary to define the elastic properties of a material completely. It was only at the start of the 20th Century that W. Voigt, one of Neumann's pupils, finally settled the controversy by determining the elastic moduli of thin prisms cut out from single crystals of a material, and in

addition ..../

addition studying the compressibility of crystals under uniform hydrostatic pressure, and then showing that the results he obtained definitely disproved those relations between the elastic constants which follow from the uni-constant theory.

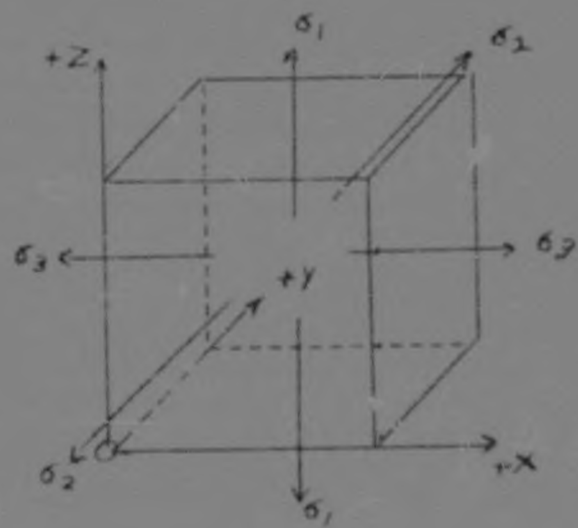
Thus the Navier-Poisson hypothesis regarding molecular forces was shown to be untenable and with it Poisson's value of $\mu = 0.25$ for the ideal isotropic material was shown to be inadmissible.

3.2 The possible Range of Values of Poisson's Ratio for Homogeneous, Isotropic, Elastic Materials

In discussing this problem it is useful to consider in the first instance the equations relating the various constants of elasticity (alternatively known as the moduli of elasticity). These equations are standard relationships in Strength of Materials and are dealt with fully in any text on the subject. Their derivations will therefore not be given again in full herein, but detailed references will be given throughout.

These relationships are obtainable by considering a cube of perfectly elastic material of unit side subjected to the action of a three-dimensional system of principal stresses as shown in Fig. 3.1 below:

Fig.3.1



It can be shown* that the sum of the strains along each of the principal axes of strain, is given by:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{1}{E}(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu) \quad \ldots\ldots\ldots 3.1$$

However, it can also shown* that the unit volume expansion or Dilation (denoted by the Greek letter Δ) of the cube as a result of the action of the applied principal stresses is approximately (neglecting quantities higher than the first order) given by:

$$\Delta = \frac{\delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

where V denotes the original volume of the cube and δV the subsequent change in volume.
Therefore, by combining 3.1 and 3.2:

$$\text{Dilation } (\Delta) = \frac{\delta V}{V} = \frac{1}{E}(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu) \quad \ldots\ldots\ 3.3$$

Based ..../

* Ref.2, p.52 and p.61-63.

Based on equation 3.3, it may be shown* that for fluid pressure conditions when $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ say, and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$ say,

$$\frac{\sigma}{\Delta} = \frac{E}{3(1-2\mu)} = K \quad \text{.......................... 3.4}$$

where K is known as the 'modulus of elasticity of volume', or simply, the 'Bulk Modulus'.
Furthermore, the condition that $\sigma_1 = -\sigma_2 = \tau$ say, and $\sigma_3 = 0$; and $\varepsilon_1 = -\varepsilon_2 = \varepsilon$ say, and $\varepsilon_3 = 0$; may be shown+ to result in a pure shear state of stress, and hence a further relationship may be obtained+:

$$\frac{\tau}{\delta} = \frac{E}{2(1+\mu)} = G \quad \text{................................ 3.5}$$

wherein τ is the pure shear stress, δ is the pure shear strain, and G is known as the 'modulus of elasticity in shear' or simply, the 'Modulus of Rigidity'.
Finally, by combining equations 3.4 and 3.5, it can readily be shown that:

$$3K(1 - 2\mu) = 2G(1 + \mu) \quad \text{.......................... 3.6}$$

With regard to the algebraic signs of the three moduli of elasticity E, G and K, it can be shown (see Appendix I) that whatever the algebraic signs of the principal stresses and strains and the pure shear stresses and strains (i.e. whether they be positive or negative), these moduli of elasticity always have a positive value.

Searle(9) referring to equation 3.6 above, points out that "if 2μ were greater than 1, or if μ were less than -1, either K or G would be negative, and hence for an isotropic solid Poisson's Ratio cannot exceed ½ and cannot be less than -1", i.e. $-1 < \mu < 0.5$. Terzaghi (8) makes no direct statement on the possible range of values of μ, but whenever discussing μ in relation to elastic behaviour, he never seems to consider any values outside the zero-0.5 range*. For instance, in dealing with(8.2) Steinbrenner's approximate solution for the computation of the settlement of loads covering a finite part of the surface of elastic layers on a rigid base, he shows as extreme limits the effects of $\mu = 0$ and $\mu = 0.5$ on the settlement influence value I_ρ, and the relevant curves for I_ρ are then only for μ values in the range zero-0.5. Taylor(1) again, makes a very clear statement regarding the possible range of values of Poisson's Ratio. He states(1.3) that: "Poisson's Ratio has values in elastic materials that are always between zero and 0.5."

There thus appears to be agreement as far as the upper limit of μ is concerned, viz. that μ cannot be greater than 0.5 in elastic materials. The physical significance of a $\mu = 0.5$ condition in elastic materials is apparent from equation 3.3. Substituting $\mu = 0.5$, gives the Dilation (Δ) equal to zero, i.e. $\delta V/V = 0$, i.e. $\delta V = 0$, which means that no volume change occurs

under ..../

*Ref.2, p.52 and p.61-63
+Ref.2, p.54-57.

*One instance was found(8.1) where Terzaghi mentions $\mu > 0.5$, but this was in connection with conditions existing when the material approached a state of failure, i.e. conditions outside the elastic range.

under the action of applied stresses. For the case where σ_1, σ_2 and σ_3 are negative (i.e. applied compressive stresses) and $\mu = 0.5$, the material is said to be incompressible. Practical materials of this type are not uncommon, rubber being probably the best example (see Section 3.3).

Regarding the physical implications of a condition $\mu > 0.5$: It can be seen from equation 3.3 that if $\mu > 0.5$, then $\delta V/V$ is negative, i.e. δV is negative, which implies that the material has suffered a volume decrease under the action of positive tensile stresses, or alternatively, the material has suffered a volume increase under the action of applied compressive stresses. It is inconceivable that either of these conditions can occur in practice under elastic conditions. All known materials show a volume decrease under compressive loading, and it will be shown (see discussion in Chapter 10) that soil over the linear range of the stress:strain curve is no exception. Further, as pointed out by Searle, $\mu > 0.5$ would render either the Bulk Modulus (K) or the Modulus of Rigidity (G) negative, and as already discussed, both K and G can only be positive for elastic materials.

Other examples such as a consideration of an isotropic elastic prism resting on a perfectly frictionless base as discussed by Terzaghi(S.3), also lead to impossible situations in the case of elastic materials. In the case mentioned, Terzaghi considers the 4 vertical sides of the isotropic, elastic prism to be confined between perfectly smooth vertical walls such that no lateral strain can occur due to the self weight of the material. From this condition he subsequently derives the relation

$$\frac{\sigma_h}{\sigma_z} = \frac{\mu}{1-\mu} = K_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3.7$$

where Ko in the theory of earth pressure is known as the "coefficient of earth pressure at rest", and is such that no lateral strains develop. If, in the above equation 3.7, $\mu > 0.5$, then Ko > 1, which means that the lateral pressure σ_h induced by the vertical self weight of the material is greater than the vertical pressure itself due to the self weight. Such a condition is unknown in the case of elastic materials. In fact, $\mu = +1$ gives Ko $= \infty$, which is clearly an impossible condition.

It therefore seems reasonable to conclude that as an upper limit for elastic, isotropic materials, Poisson's Ratio cannot exceed 0.5.

In considering next the possible lower limit of Poisson's Ratio, the significance of a $\mu < 0$ condition will briefly be investigated. Firstly, as Searle pointed out, if μ is negative, it must have a lower limit of -1, since μ less than 1 would again render either the Bulk Modulus (K) or the Modulus of Rigidity (G) negative which is an impossible condition. Further, the physical implications of a negative Poisson's Ratio are apparent from a consideration of Poisson's Ratio itself in the first instance. From its definition, Poisson's Ratio is the ratio of the lateral contraction:longitudinal extension which is observed in the case of a bar of elastic material subjected to axial tensile loading, or the reversed conditions in the case of axial compressive loading. In both cases the lateral and longitudinal strains are considered to have the same algebraic signs, so that μ is always positive. A negative Poisson's Ratio would thus mean that either the sign of the lateral contraction is negative, which implies a lateral expansion accompanying a longitudinal extension; or, the sign of the longitudinal extension is negative, which implies a longitudinal contraction accompanying a lateral contraction. Both these concepts are inconceivable for any practical elastic material.

Also ..../

Also in the case of Terzaghi's isotropic elastic prism confined between perfectly smooth vertical walls, an impossible condition arises if μ is negative. From equation 3.7 again, if μ were negative, K_o would be negative, ie. σ_h is negative, since the vertical pressure due to the self weight of the material always acts down and is therefore always positive according to the sign convention used. A negative σ_h implies that instead of the sides of the prism being acted upon by a lateral pressure due to the confining vertical walls which prevents lateral strain from occurring, the sides of the prism are acted upon by a lateral tension which prevents lateral expansion. This again is clearly an impossible condition.

Hence, it seems reasonable to conclude that Poisson's Ratio in practice cannot be negative for elastic, isotropic, materials.

However, as previously mentioned, both Terzaghi and Taylor consider $\mu = 0$ to be the lower limit of Poisson's Ratio for elastic, isotropic materials. Poisson's Ratio equal to zero means that either, a longitudinal extension is accompanied by no lateral contraction; or, a longitudinal contraction is accompanied by no lateral expansion. Such conditions are not uncommon in practice. They usually occur where lateral expansion under compressive stresses is completely prevented in some or other way. The laboratory oedometer test is a good example of this condition (see Section 6.3). In addition, Timoshenko(2.1) mentions that Poisson's Ratio for cork can be taken as equal to zero, while the value for concrete is also close to zero, viz. in the region 0.083-0.125.

Hence it seems reasonable to accept a value of zero as the lower limit of Poisson's Ratio, and it can therefore be concluded that for homogeneous, isotropic, elastic materials, Poisson's Ratio falls within the range zero-0.5.

3.3 Values of Poisson's Ratio for Practical Materials

It is of interest to note the following table of values of Poisson's Ratio μ, Modulus of Elasticity E, Modulus of Rigidity G, and Bulk Modulus K, given by Kaye and Laby(10) for various materials, which are however mainly metals. Both calculated and observed values are given for μ and G: in the case of G, calculated values were obtained by using observed values of E and μ in equation 3.5 of previously. This equation can be rewritten.

$$\mu = \frac{E}{2G} - 1 \qquad \text{.......................... 3.5(a)}$$

and hence, using observed values of E and G, calculated values of μ were obtained. The Bulk Modulus K was obtained by calculation only, by using equation 3.4 of previously. The Authors mention also that the extent of the agreement between the calculated and observed values of μ and G, gives an indication of the degree of isotropy of the material.

Table I ..../

TABLE I

MATERIAL	POISSON'S RATIO μ		YOUNG'S MODULUS E	MODULUS OF RIGIDITY G (10"dynes/cm²)		BULK MODULUS K
	OBSERVED	CALCULATED	(10"dynes/cm²)	OBSERVED	CALCULATED	(10"dynes/cm²)
Aluminium (Worked)	0.339	0.310	7.05	2.67	2.63	7.46
Copper (Worked) pure	0.337	0.356	12.3	4.55	4.55	13.1
Gold (Worked) pure	0.422	0.495	8.0	2.77	2.80	16.6
Steel (Worked) 1%C	0.287	0.287	20.9	8.12	8.12	16.4
Lead (Cast) pure	0.446	-	1.62	-	0.562	5.00
Palladium (Cast) pure	0.393	0.101	11.3	5.11	4.04	17.6
Platignum (Cast) pure	0.387	0.368	16.8	6.10	6.04	24.7
Silver (Worked) pure	0.379	0.369	7.90	2.87	2.86	10.9
Tin (Cast) pure	0.33	-	5.43	-	2.04	5.29
Bronze (Cast) ‡	0.358	0.177	8.08	3.43	2.97	9.52
Constantin (Worked) §	0.325	0.329	16.3	6.11	6.11	15.5
Manganin (Worked)+	0.329	0.329	12.4	4.65	4.65	12.1
Zinc 1% Pb	0.21	-	8.7°	3.8	-	-
Rubber (soft, vulcanised)	0.46-0.49	-	0.10-0.70	0.00016	-	-
Jena Glasses (crowns)	0.20-0.27	-	6.5 -7.8	2.6-3.2	-	4.0-5.9
Jena Glasses (flints)	0.22-0.26	-	5.0 -6.0	2.0-2.5	-	3.6-3.8

‡ 85% Cu, 7.2%Zn, 6.4%Sn; § 60% Cu, 40% Ni;
+ 84% Cu, 12% Mn, 4% Ni; ° pure Zn : E = 12.5 x 10" dynes/sq.cm.

In ..../

In addition to the values of Poisson's Ratio given in the above Table (which are mainly for metals), the following values are available for materials other than metals: As mentioned at the end of the previous Section , Timoshenko(2.1) states that Poisson's Ratio for cork can be taken as equal to zero, while the value for concrete is in the region 0.083-0.125. Further, Krynine and Judd(7) give the following values of Poisson's Ratio for various rock types: Granite 0.15-0.24, limestone 0.16-0.23, schist 0.08-0.20, marble 0.25-0.38; and, from isolated tests, the value for gneiss as 0.11, monzonite 0.17, sandstone 0.17 and tuff 0.11.

CHAPTER 4

POISSON'S RATIO IN RELATION TO ELASTIC STATES OF STRESS AND STRAIN

4.1 Introduction

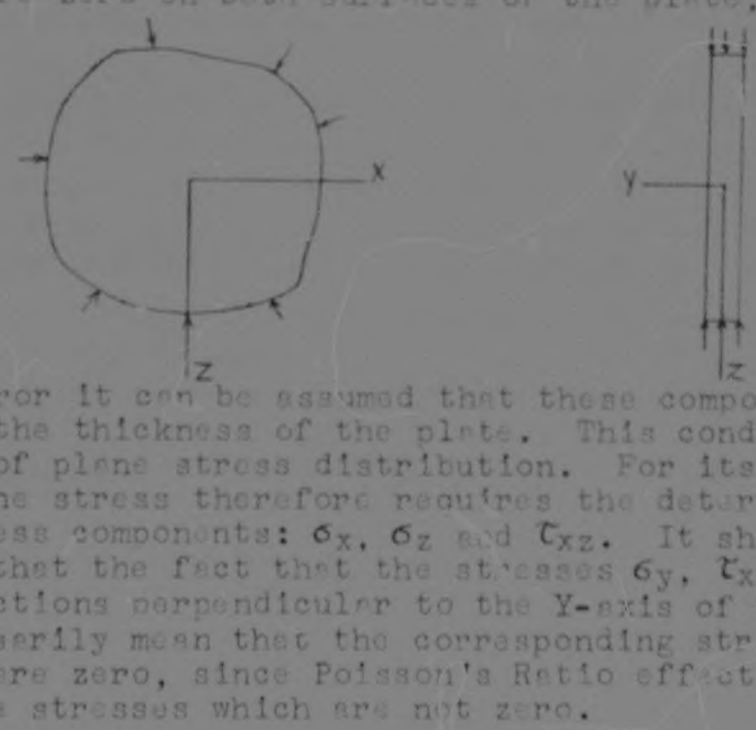
For a body in equilibrium under the action of external forces, 6 stress components are necessary(3.2) to define the state of stress at any point in the body completely. With reference to a rectangular system of coordinate axes OX, OY and OZ, these stresses are the normal stresses σ_x, σ_y, σ_z, and the shearing stresses $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$. Similarly also, 6 strain components are necessary (3.3) to define the state of strain at any point of the body completely. These are the unit strains in the three perpendicular directions ε_x, ε_y, ε_z, and the unit shear strains γ_{xy}, γ_{xz}, γ_{yz} related to the same directions.

Under two types of conditions, however, the number of stress and strain components necessary to define the states of stress and strain at a point completely reduce to only 3 in each case. These two conditions are known as a state of plane stress and a state of plane strain respectively, and in each case the stress distribution problem is reduced to a 2-dimensional problem of elasticity.

4.2 Plane Stress and Plane Strain(3.4)

If a thin plate is subjected to the action of forces applied at the boundary, parallel to the plane of the plate and distributed uniformly over its thickness (Fig.4.1), the stress components σ_y, τ_{xy}, τ_{yz} are zero on both surfaces of the plate, and without

Fig.4.1



substantial error it can be assumed that these components are zero throughout the thickness of the plate. This condition constitutes a state of plane stress distribution. For its solution, a problem of plane stress therefore requires the determination of only three stress components: σ_x, σ_z and τ_{xz}. It should be noted, however, that the fact that the stresses σ_y, τ_{xy}, τ_{yz} are zero on cross-sections perpendicular to the Y-axis of the plate, does not necessarily mean that the corresponding strains ε_y, γ_{xy}, and γ_{yz} are zero, since Poisson's Ratio effects occur as a result of the stresses which are not zero.

The same simplification of the problem as in the case of thin plates is encountered when one goes to the other extreme in which the dimension of the body in the Y-direction is very large. If a long cylindrical or prismatical body be subjected to the action of forces uniformly distributed along its length and in the plane of cross-sections perpendicular to its axis (Fig.4.2), the displacements during deformation at a considerable distance from the ends, occur only in planes perpendicular to the axis of the body, and the strain components ε_y, γ_{xy}, and γ_{yz} are zero. This condition is known as a state of plane strain.

Furthermore ..../

Furthermore, the shearing stresses τ_{xy} and τ_{yz}, which are proportional to γ_{xy} and γ_{yz} are also zero, while, since $\varepsilon_y = 0$, the normal stress σ_y (which is not equal to zero) can readily be expressed in terms of σ_x and σ_z by applying basic principles of stress and strain. A problem of plane strain is therefore reduced, as in the case of problems of plane stress, to the determination of only three stress components : σ_x, σ_z and τ_{xz}, and

Fig. 4.2

hence, in both cases, the problem of stress and strain reduces to a problem in 2-dimensions. It should also be noted in the case of problems of plane strain, that conditions are not altered in any way if a uniform extension in the direction of the axis of the body is superimposed upon the state of plane strain. Furthermore, the fact that the three strains mentioned (ε_y, γ_{xy}, γ_{yz}) are zero, constitutes an enforced condition due to the length of the body, in which the three normal stresses σ_x, σ_y, σ_z acting over cross-sections normal to the axis of the body, serve to maintain the state of plane strain.

These two conditions of plane stress and plane strain therefore constitute 2-dimensional problems in the theory of elasticity, and in all such cases the determination of stress and strain in elastic bodies is considerably simplified.

4.3 <u>The Moduli of Elasticity in relation to States of Plane Stress and Plane Strain</u>

In dealing with the problem of an elastic, isotropic solid in equilibrium under given body and surface forces, Michell(11) treats the following two 2-dimensional cases:

(a) That of a long cylinder with applied forces perpendicular to its length and the same at corresponding points along its length (a case of plane strain),

and (b) that of a thin plate with applied forces in its plane (a case of plane stress).

He determines the stress function which satisfies the equations of equilibrium and the boundary conditions in each case, and then shows that if the body forces are zero, the stresses in the solids are independent of the moduli of elasticity provided the resultant force on each boundary vanishes. He also points out that where there is a single boundary (such as in the case of a semi-infinite solid), this latter condition is always satisfied and the stresses are therefore always independent of the moduli of elasticity in this case, provided there is no body force. His final conclusions are that the stress distribution in elastic isotropic bodies for the cases considered (cases of plane stress and plane strain) are independent of the moduli of elasticity if, firstly, body forces are absent, and secondly, the boundary is simply connected. If the boundary is multi-connected, such as in the case of a perforated body, the stresses are independent of the moduli of elasticity only if the resultant force on each boundary separately vanishes.

It ..../

It thus appears that in 2 dimensional problems (cases of plane stress and plane strain) wherein the stress distribution in a semi-infinite, elastic, isotropic solid is required (the case of a single or simply-connected boundary), the stress distribution is independent of the moduli of elasticity provided the body forces (such as the self weight of the material) are disregarded.

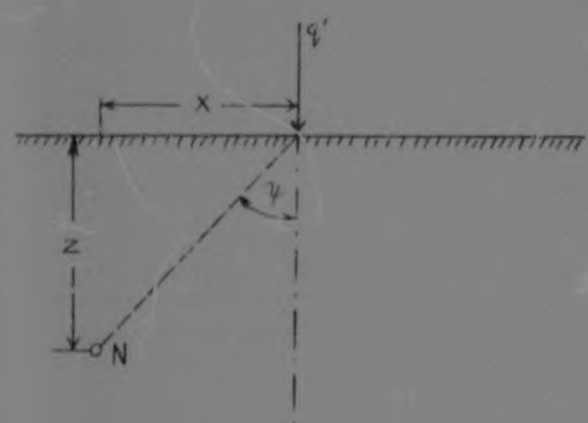
It is of interest to note(3.5) that Michell's conclusion is of great practical importance in cases where the photo-elastic method of stress analysis is used, since it enables experimental results obtained with transparent materials to be applied immediately to any other material such as steel if the external forces are the same.

4.4 Poisson's Ratio in relation to States of Stress and Strain in a Semi-Infinite Elastic Solid

With a view to the later discussion of the application of the elastic theory to soils which, for such purposes, are generally considered to be semi-infinite in extent, certain 2-dimensional and 3-dimensional problems of stress and strain in a semi-infinite elastic solid will be reviewed in this Section.

Firstly, there is the case of an infinitely extended line load of constant magnitude per unit length on the surface of a semi-infinite elastic solid (Fig.4.3). From the discussion in the previous Section, it is clear that this loading condition produces a state of plane strain in the solid, and the problem is therefore essentially a 2-dimensional one. This case was originally dealt with in a paper by Flamant(12) in which he outlines the derivation of the mathematical expressions for the stresses induced at any point in the solid due to such applied loading. These expressions have also been given in polar coordinates by Terzaghi (8.4) as follows (cf.Fig.4.3):

Fig. 4.3



$$\sigma_z = \frac{2}{\pi} \cdot \frac{q'}{z} \cos^4 \psi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4.1$$

$$\sigma_x = \frac{2}{\pi} \cdot \frac{q'}{z} \cos^2\psi \sin^2\psi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4.2$$

$$\tau_{xz} = \frac{2}{\pi} \cdot \frac{q'}{z} \cos^3\psi \sin\psi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4.3$$

where q' is the load per unit length of the straight line. It should be noted as discussed in Section 4.2, that due to the state of plane strain, only three stress components appear in this case. Furthermore, it should be noted that the equations do not contain Poisson's Ratio for the reasons discussed in Section 4.3

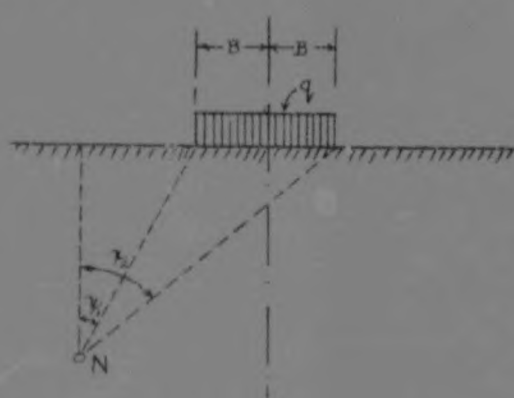
In the latter connection, it is of interest to note how Poisson's Ratio becomes eliminated from the basic expressions

of......../

of equilibrium in Flamant's original paper during his derivation of the equations 4.1, 4.2 and 4.3 given above. The actual steps in the final reduction of his mathematical expressions from which this is apparent have not been given in the paper, but are given in Appendix II of this paper.

Secondly, there is the case of a strip load of infinite extent and constant (finite) width on the surface of a semi-infinite elastic solid (Fig. 4.4). This problem was solved by Michell(13) and is actually only an extension of the problem solved by Flamant which was discussed above. It is therefore another 2-dimensional problem. The solution is obtained by integration of the basic expressions for the case of a straight line load of constant magnitude and of infinite extension on the surface of a semi-infinite solid. Terzaghi(8.4) gives these expressions for the stresses at any point in the semi-infinite solid as (cf. Fig.4.4):

Fig.4.4



$$\sigma_z = \frac{q}{\pi}\left[\sin\psi\cos\psi + \psi\right]_{\psi_1}^{\psi_2} \qquad \text{.................. 4.4}$$

$$\sigma_x = \frac{q}{\pi}\left[-\sin\psi\cos\psi + \psi\right]_{\psi_1}^{\psi_2} \qquad \text{.................. 4.5}$$

$$\tau_{xz} = \frac{q}{\pi}\left[\sin^2\psi\right]_{\psi_1}^{\psi_2} \qquad \text{.................. 4.6}$$

where q is the load per unit of area on the strip. It is clear that, for the same reasons as discussed in the case of the first 2-dimensional problem considered above, these expressions also only comprise 3 stress components which do not contain Poisson's Ratio.

Other 2-dimensional problems of stress and strain, such as those solved by Carothers(14) and Jurgenson(15), follow the same pattern as the two cases dealt with above in that for the complete determination of stress and strain, expressions for only the three stress components σ_z, σ_x and σ_{xz} are required, all of which do not contain Poisson's Ratio.

However, the case of a vertical point load applied to the horizontal surface of a semi-infinite elastic solid (Fig.4.5), which was originally solved by Boussinesq(16), is a 3-dimensional problem of stress and strain. The expressions for the stresses at any point in the solid therefore actually comprise 6 stress components in this case, but due to radial symmetry the vertical component (τ_{tz}) and the horizontal component (τ_{tr}) of shearing stress on radial planes (see Fig.4.5) are each equal to zero and only 4 stress components are therefore involved. The relevant expressions only are given by both Taylor(1.3) and Terzaghi(8.5), but their derivation has been outlined again in full by Timoshenko(3.7). The expressions are as follows:

$$\sigma_z = \frac{Q}{2\pi}\,\frac{3z^3}{(r^2+z^2)^{5/2}} = \frac{Q}{2\pi z^2}\,(3\cos^5\theta) \quad \ldots\ldots\ldots\ldots\ldots\; 4.7(a) \text{ and. } 4.7(b)$$

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3r^2z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \ldots\ldots\ldots\ldots\ldots\; 4.8(a)$$

$$= \frac{Q}{2\pi z^2}\left[3\sin^2\theta . \cos^3\theta - \frac{(1-2\mu)\cos^2\theta}{1+\cos\theta}\right] \quad \ldots\ldots\ldots\ldots\; 4.8(b)$$

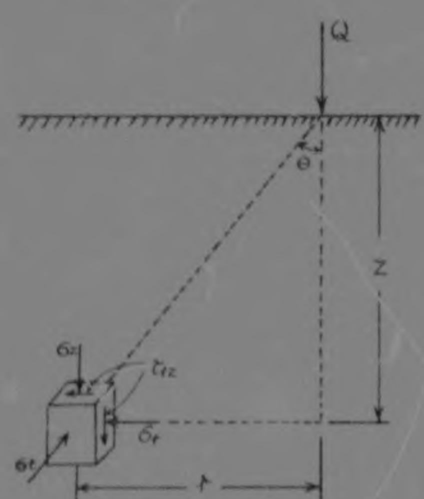
$$\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \ldots\ldots\; 4.9(a)$$

$$= -\frac{Q}{2\pi z^2}(1-2\mu)\left(\cos^3\theta - \frac{\cos^2\theta}{1+\cos\theta}\right) \quad \ldots\ldots\ldots\ldots\ldots\; 4.9(b)$$

$$\tau_{rz} = \frac{Q}{2\pi}\,\frac{3rz^2}{(r^2+z^2)^{5/2}} = \frac{Q}{2\pi z^2}\,(3\sin\theta\cos^4\theta) \quad \ldots\ldots\ldots\ldots\; 4.10(a) \text{ and } 4.10(b)$$

where Q in each case is the magnitude of the applied vertical point load. Poisson's Ratio does appear in the above expressions,

Fig.4.5



although only in the case of certain stress components viz. σ_r and σ_t. This is the case since the moduli of elasticity affect the stress distribution in 3-dimensional problems.

Extensions of the abovementioned problem in the form of a uniformly distributed loading on a finite area such as a rectangular, square or circular area, also constitute 3-dimensional problems of stress and strain, and hence, the corresponding expressions for the stress components will also contain Poisson's Ratio in certain instances. Yet, according to Terzaghi(8.6), "the computation of the stresses ......(in such cases) .... are rather involved, and the results of the computation cannot be represented by a simple set of equations. However, the problem has been solved by Love(17) and the results have been compiled in tables which make it possible to determine the stresses at any point by means of a simple process of interpolation."

Both Terzaghi(8.6) and Taylor(1.3) point out, however,

that ..../

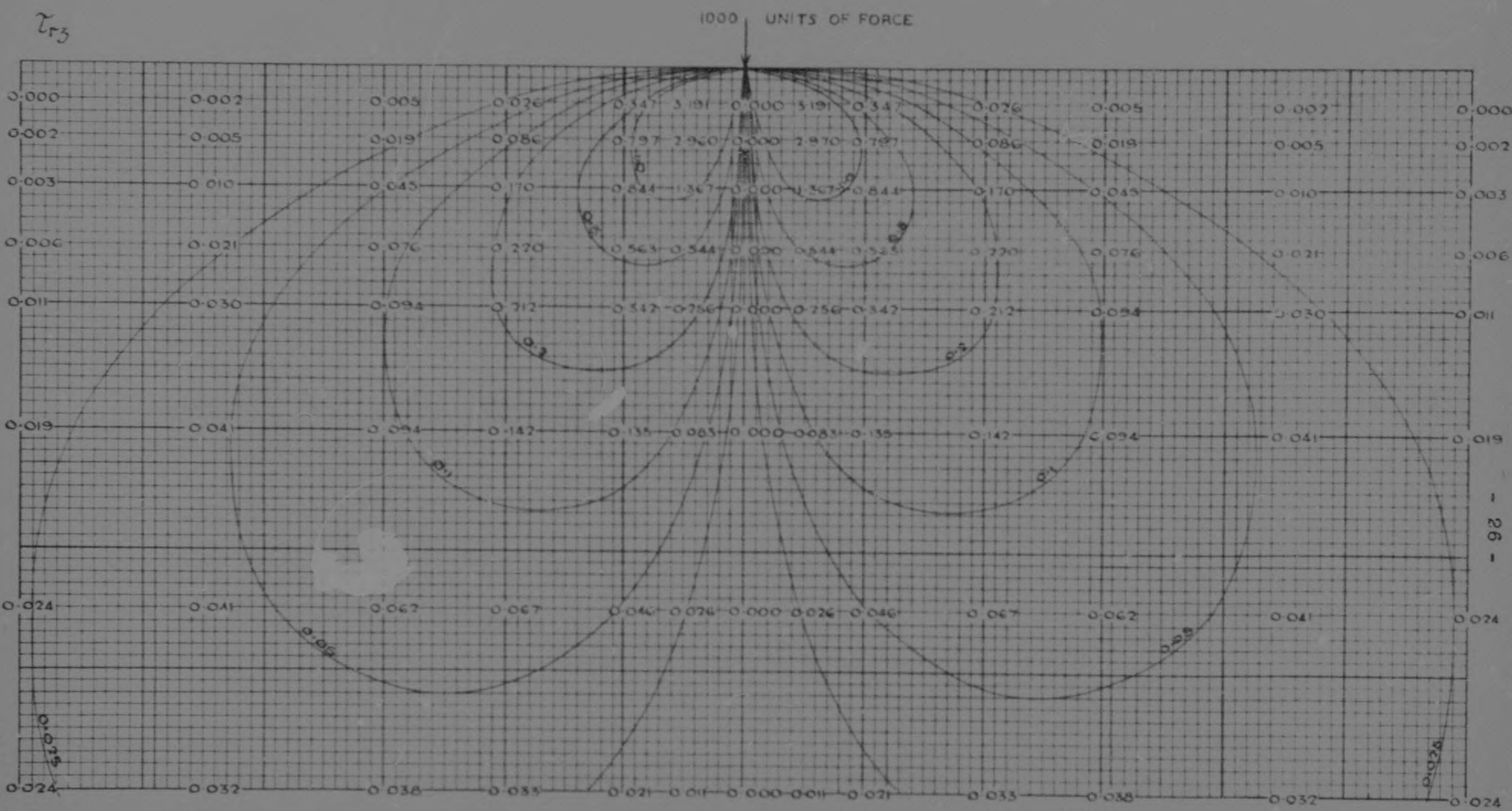


FIG 2 - CONTOURS OF VERTICAL SHEAR STRESS (τ_{rz})

FIG. 2.

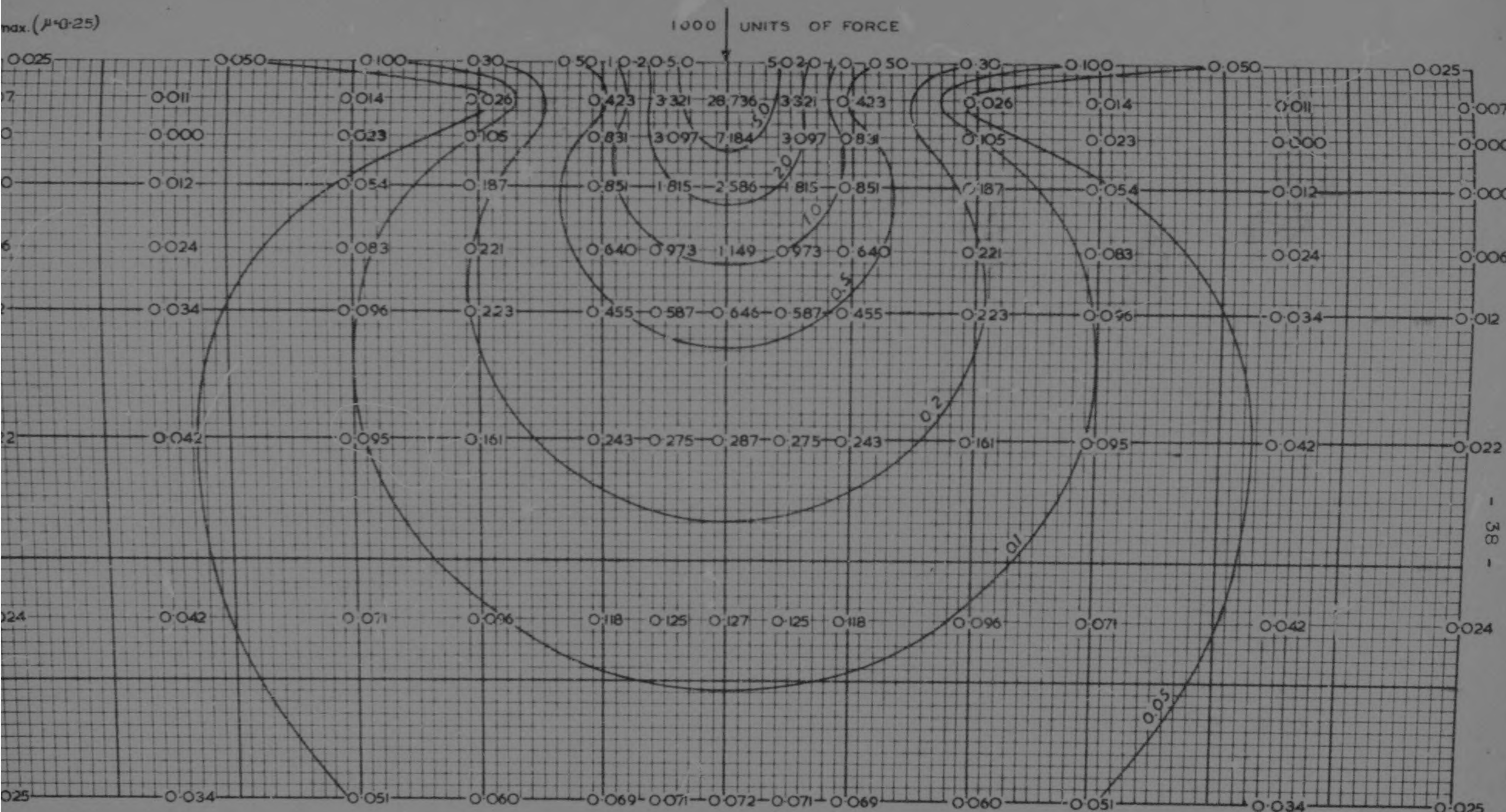
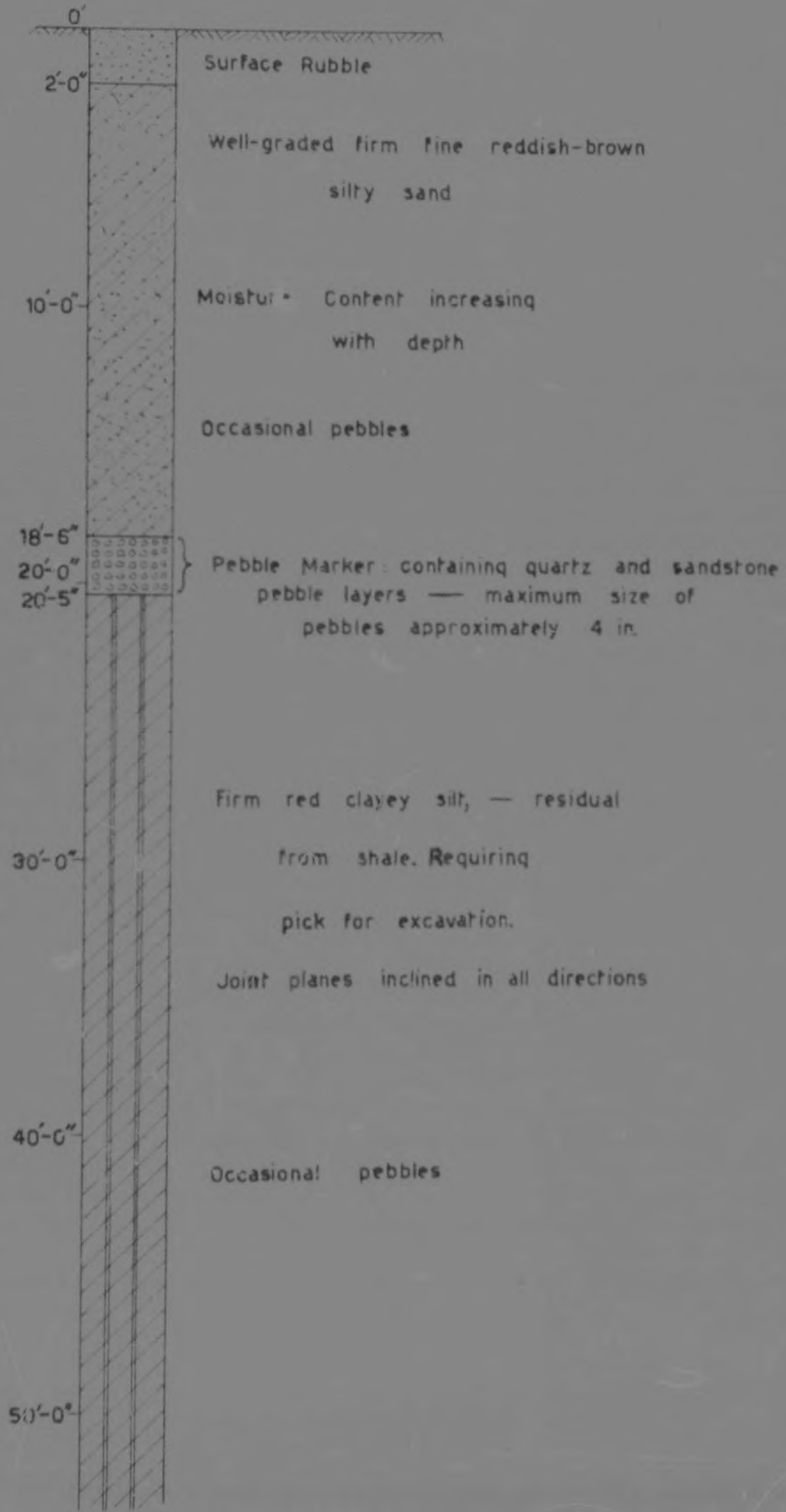


FIG. 11 — CONTOURS OF MAXIMUM SHEAR STRESS (S_{max}) FOR CONDITION ($\mu = 0{\cdot}25$)

SOIL PROFILE ON THE NORTH SIDE OF JUTA ST. NEAR BICCARD ST., BRAAMFONTEIN, J'BURG.



Horizontal Radial Stress σ_r

Equation: $\sigma_r = \frac{Q}{2\pi}\left[\frac{3r^2z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$

TABLE IV

z	r	σ_r				
		$\mu = 0$	$\mu = 0.125$	$\mu = 0.25$	$\mu = 0.375$	$\mu = 0.05$
3	0	-8.8419	-6.6314	-4.4210	-2.2105	0
	5	+2.1979	+2.9779	+3.7576	+4.5391	+5.3190
	10	+0.0191	+0.3024	+0.5873	+0.8722	+1.1555
	20	-0.1703	-0.0859	0	+0.0844	+0.1687
	30	-0.1066	-0.0668	-0.0271	+0.0127	+0.0525
	45	-0.0573	-0.0382	-0.0207	+0.0016	+0.0159
	60	-0.0350	-0.0255	-0.0143	-0.0032	+0.0064
6	0	-2.2107	-1.6584	-1.1045	-0.5523	0
	5	+0.9931	+1.3624	+1.7300	+2.0993	+2.4669
	10	+0.5570	+0.7496	+0.9438	+1.1364	+1.3289
	20	+0.0048	+0.0764	+0.1464	+0.2180	+0.2881
	30	-0.0462	-0.0111	+0.0255	+0.0605	+0.0955
	45	-0.0382	-0.0191	-0.0032	+0.0127	+0.0302
	60	-0.0271	-0.0175	-0.0064	+0.0032	+0.0127
10	0	-0.7958	-0.5968	-0.3979	-0.1989	0
	5	+0.0111	+0.1798	+0.3470	+0.5141	+0.6828
	10	+0.3772	+0.4934	+0.6112	+0.7273	+0.8435
	20	+0.1225	+0.1783	+0.2324	+0.2881	+0.3422
	30	+0.0143	+0.0446	+0.0748	+0.1050	+0.1353
	45	-0.0159	0	+0.0143	+0.0302	+0.0462
	60	-0.0159	-0.0064	+0.0032	+0.0111	+0.0207
15	0	-0.3533	-0.2642	-0.1767	-0.0891	0
	5	-0.1448	-0.0637	+0.0191	+0.1003	+0.1814
	10	+0.1082	+0.1751	+0.2419	+0.3088	+0.3756
	20	+0.1337	+0.1735	+0.2133	+0.2531	+0.2928
	30	+0.0541	+0.0780	+0.1035	+0.1273	+0.1512
	45	+0.0064	+0.0191	+0.0334	+0.0477	+0.0605
	60	-0.0048	+0.0032	+0.0127	+0.0207	+0.0286
20	0	-0.1989	-0.1496	-0.0987	-0.0493	0
	5	-0.1273	-0.0796	-0.0318	+0.0159	+0.0637
	10	+0.0016	+0.0430	+0.0859	+0.1289	+0.1703
	20	+0.0955	+0.1241	+0.1544	+0.1830	+0.2117
	30	+0.0637	+0.0828	+0.1035	+0.1225	+0.1416
	45	+0.0207	+0.0318	+0.0446	+0.0557	+0.0668
	60	+0.0032	+0.0111	+0.0175	+0.0255	+0.0334
30	0	-0.0891	-0.0668	-0.0446	-0.0223	0
	5	-0.0716	-0.0493	-0.0286	-0.0064	+0.0143
	10	-0.0366	-0.0159	+0.0032	+0.0239	+0.0446
	20	+0.0271	+0.0430	+0.0605	+0.0780	+0.0939
	30	+0.0414	+0.0541	+0.0684	+0.0812	+0.0939
	45	+0.0271	+0.0366	+0.0446	+0.0541	+0.0621
	60	+0.0143	+0.0207	+0.0255	+0.0318	+0.0382
45	0	-0.0398	-0.0302	-0.0191	-0.0095	0
	5	-0.0350	-0.0255	-0.0159	-0.0064	+0.0032
	10	-0.0286	-0.0191	-0.0095	0	+0.0095
	20	-0.0048	+0.0048	+0.0127	+0.0223	+0.0302
	30	+0.0111	+0.0191	+0.0255	+0.0334	+0.0414
	45	+0.0191	+0.0255	+0.0302	+0.0350	+0.0414
	60	+0.0143	+0.0191	+0.0223	+0.0271	+0.0318
60	0	-0.0223	-0.0159	-0.0111	-0.0064	0
	5	-0.0207	90.0143	-0.0095	-0.0048	+0.0016
	10	-0.0191	-0.0127	-0.0080	-0.0032	+0.0032
	20	-0.0095	+0.0048	+0.0016	+0.0064	+0.0111
	30	-0.0000	+0.0048	+0.0095	+0.0143	+0.0191
	45	+0.0080	+0.0111	+0.0159	+0.0207	+0.0239
	60	+0.0111	+0.0143	+0.0175	+0.0207	+0.0239

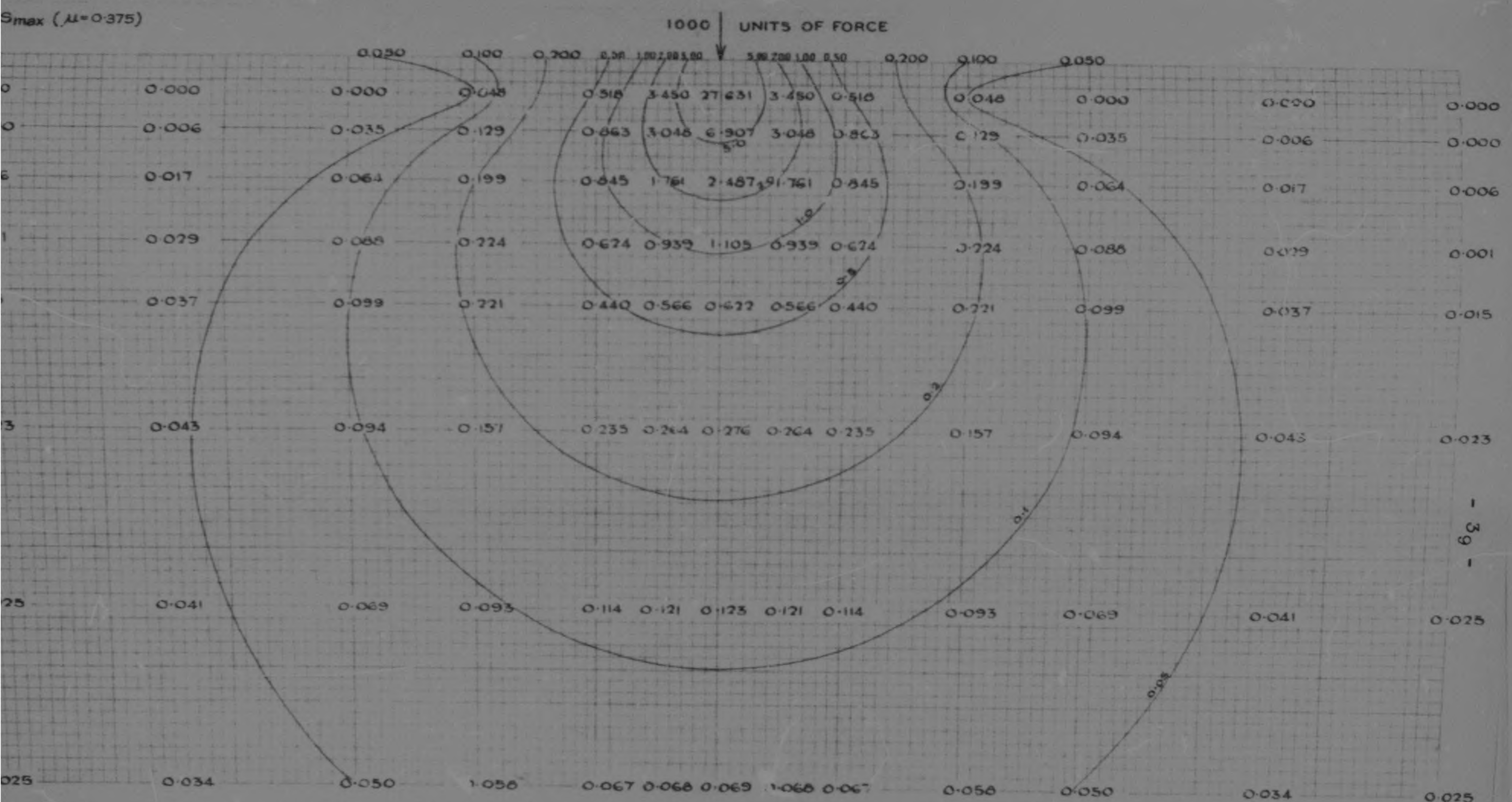


FIG 12 – CONTOURS OF MAXIMUM SHEAR STRESS (S_{max}) FOR CONDITION $\mu = 0{\cdot}375$

The results of various laboratory tests on the soil are tabulated below:

TABLE VII

Atterberg Limits	: Liquid Limit :	25.2%
	Plastic Limit :	17.0%
	Plasticity Index :	8.2%
	Linear Shrinkage :	3.3%
Grading Analysis	: Gravel :	8.5%
	Sand :	53.0%
	Silt :	30.0%
	Clay :	8.5%
Specific Gravity	: :	2.62
Void Ratio	: (Undisturbed state):	0.70
Compression Index C_c	: (" " :	0.26

The soil appears to fall into the class ML of the Casagrande Airfield Soil classification and into the class A4 of the Public Roads classification. A full grading curve and a pressure : void ratio curve determined from a normal 1-dimensional consolidometer test, are shown in Figs. 17 & 18 respectively. The moisture content of the soil at the time of sampling in the field is not known, but would appear to have been approximately 3% (see Appendix IV, Fig.53), thus indicating a Degree of Saturation of approximately 11%.

7.3 Design of the Experimental Work

The aim of the experimental work was to determine Poisson's Ratio for the selected test soil by conducting a series of Consolidated Undrained Triaxial Shear Tests (S_{cu}) with strain control on samples of the soil. The tests were to be according to British standard practice.

In view of previous discussions, this meant that the experimental work had to be designed to enable the measurement of the change in volume of a soil sample of known initial volume, due to a known change in stress within the linear range of the soil's stress:strain curve. The information thus obtained would then be used in equation 7.5 previously derived, to enable the evaluation of Poisson's Ratio for the soil.

When a soil sample is set up inside a triaxial test cell ready for test, there is a certain volume of cell fluid inside the test cell surrounding the specimen which is equal to the difference in volume between that of the soil sample and the internal volume of the test cell. Any change in volume of the soil sample must thus cause a flow of fluid either into or out of the cell, depending upon the type of volume change occurring in the sample, which is equal in volume to the change in volume of the soil sample, if the fluid is incompressible.

However, during an actual test, change in volume of the soil sample due to compressive loading is not the only cause of cell fluid entering or leaving the test cell. Entry of the loading spindle into the test cell causes a flow of fluid <u>out</u> of the test cell, while leakage from the test cell, usually where the loading spindle enters the test cell, causes a flow of fluid <u>into</u> the cell from the fluid reservoir.

If the flow of fluid either into or out of the test cell during test be accurately measured, this flow measurement will

be ..../

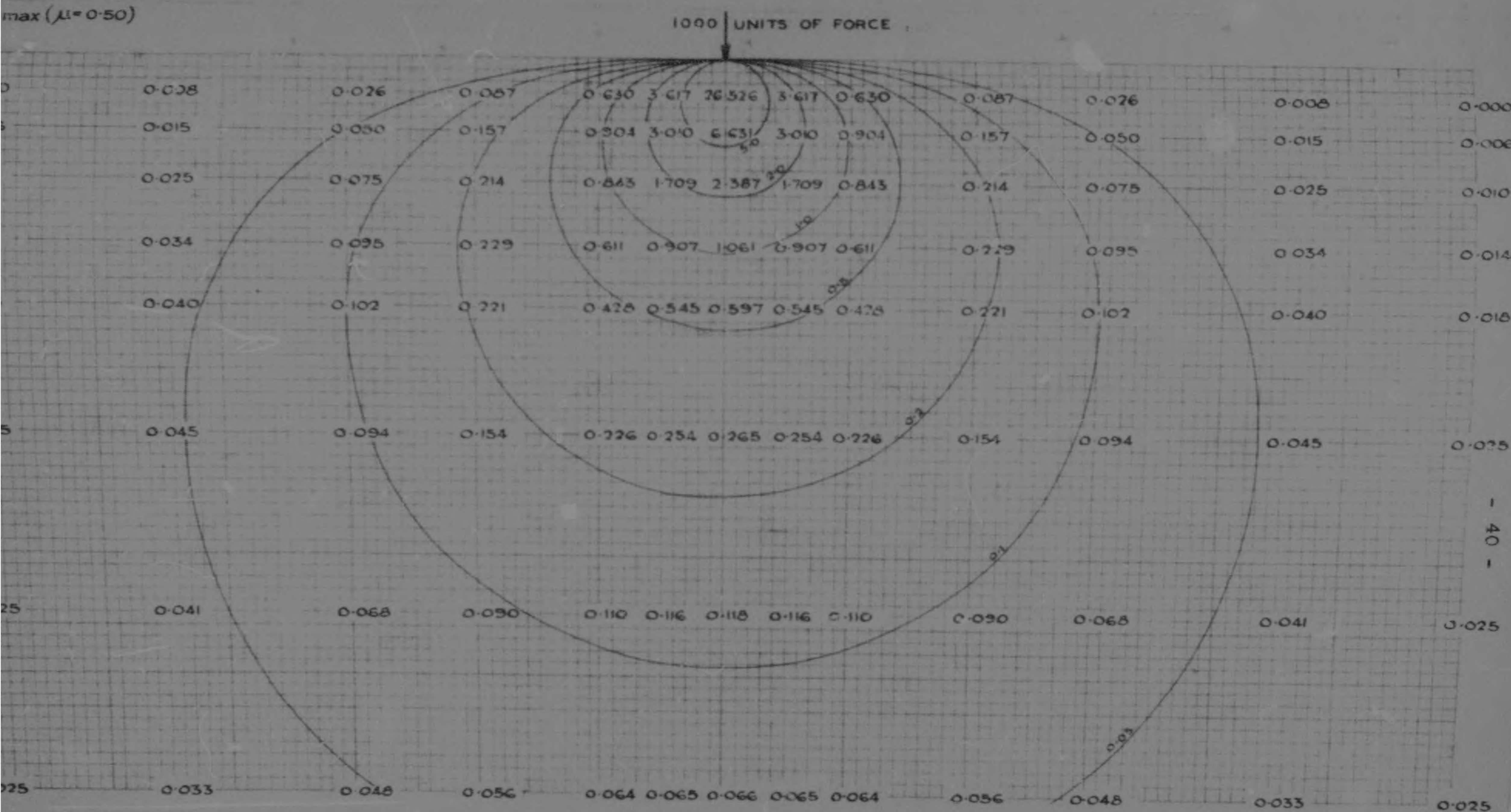


FIG 13 - CONTOURS OF MAXIMUM SHEAR STRESS (Smax) FOR CONDITION (μ=0·50)

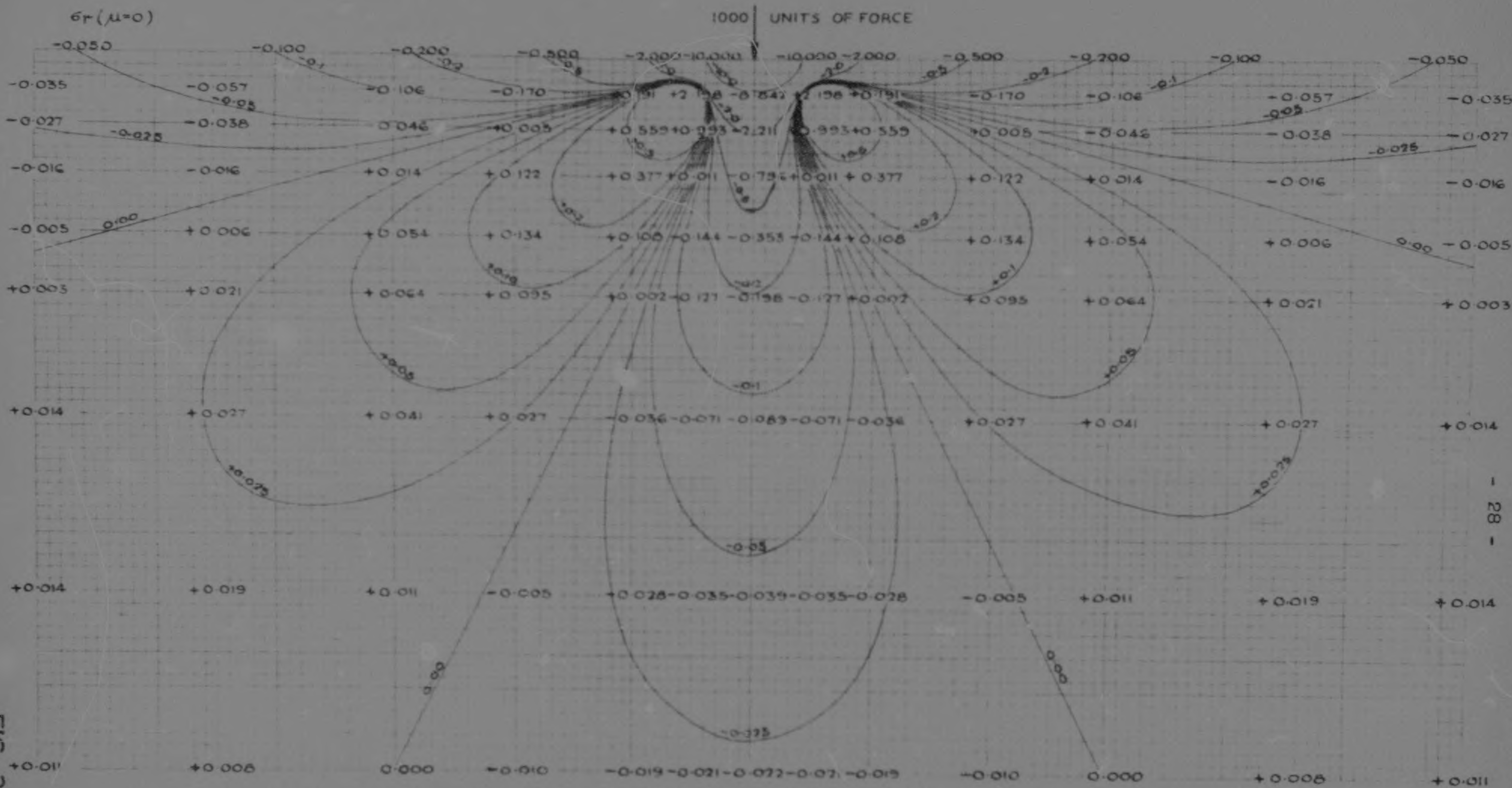
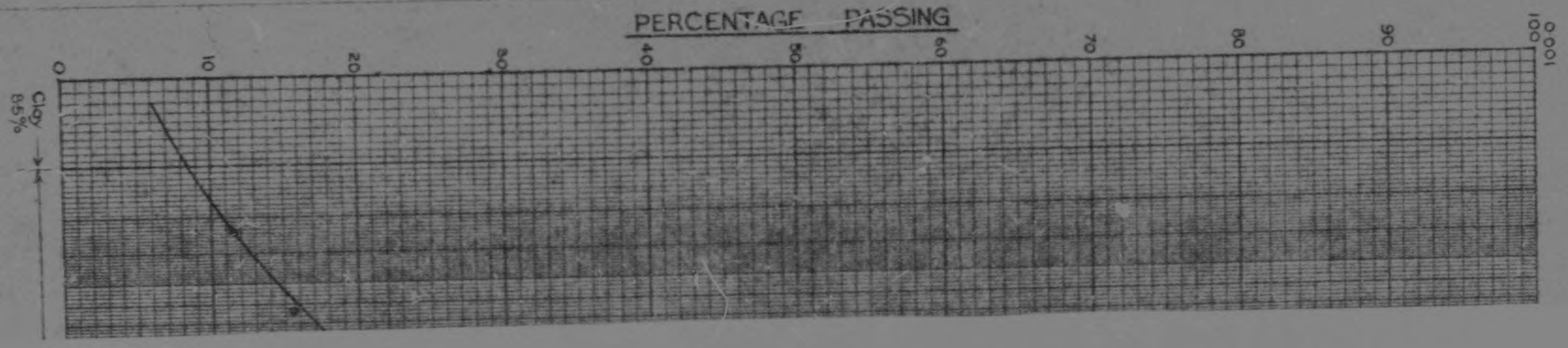


FIG 3 - CONTOURS OF HORIZONTAL RADIAL STRESS (σ_r) FOR CONDITION $\mu=0$

FIG. 3.


PERCENTAGE PASSING
0·001
100
90
80
70
60
50
40
30
20
10
0
Clay
85%

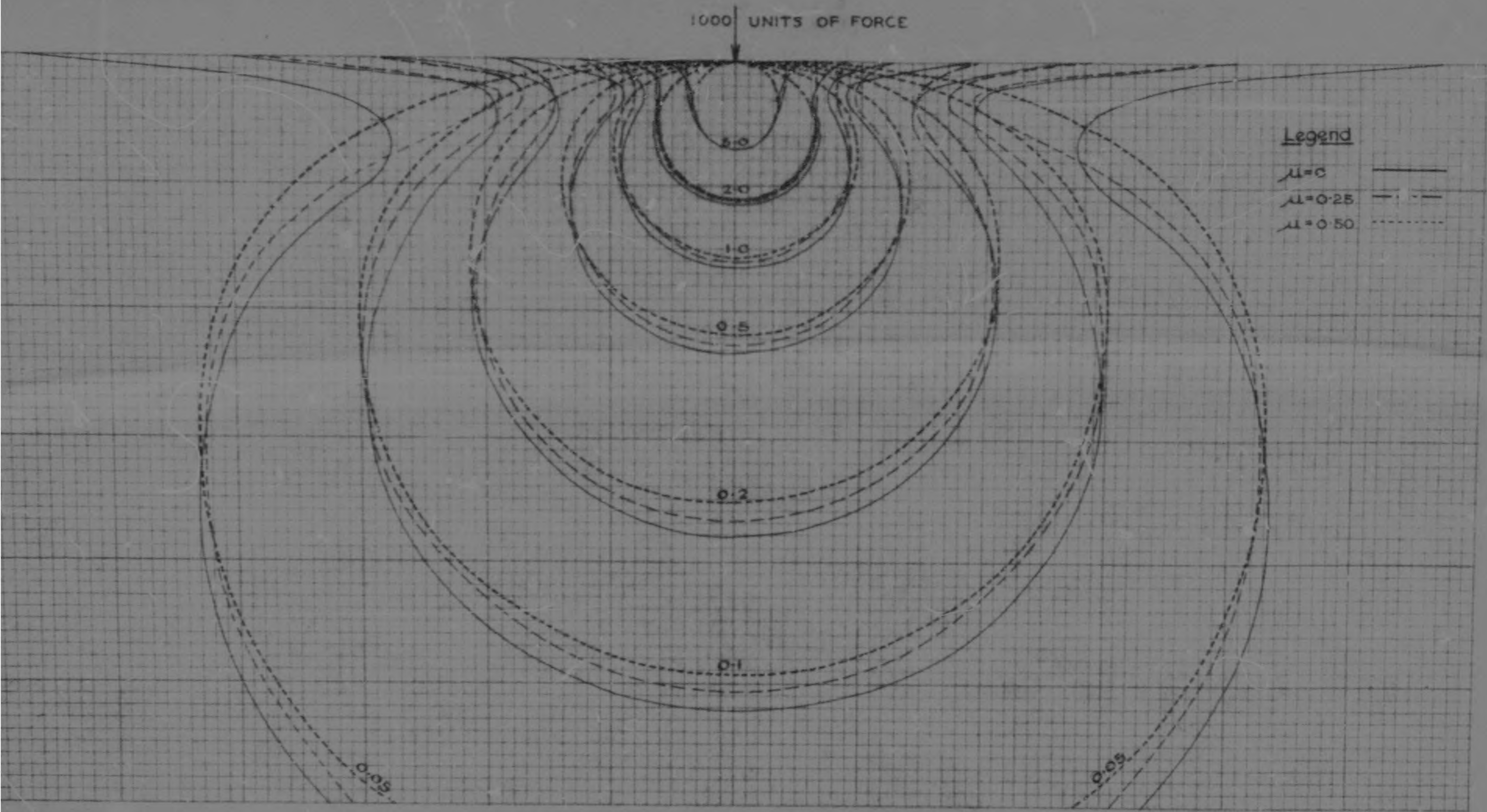


FIG 14 - COMBINED CURVES OF MAXIMUM SHEAR STRESS CONTOURS (S_{max}) FOR CONDITION $\mu = 0, 0{\cdot}25, 0{\cdot}50$.

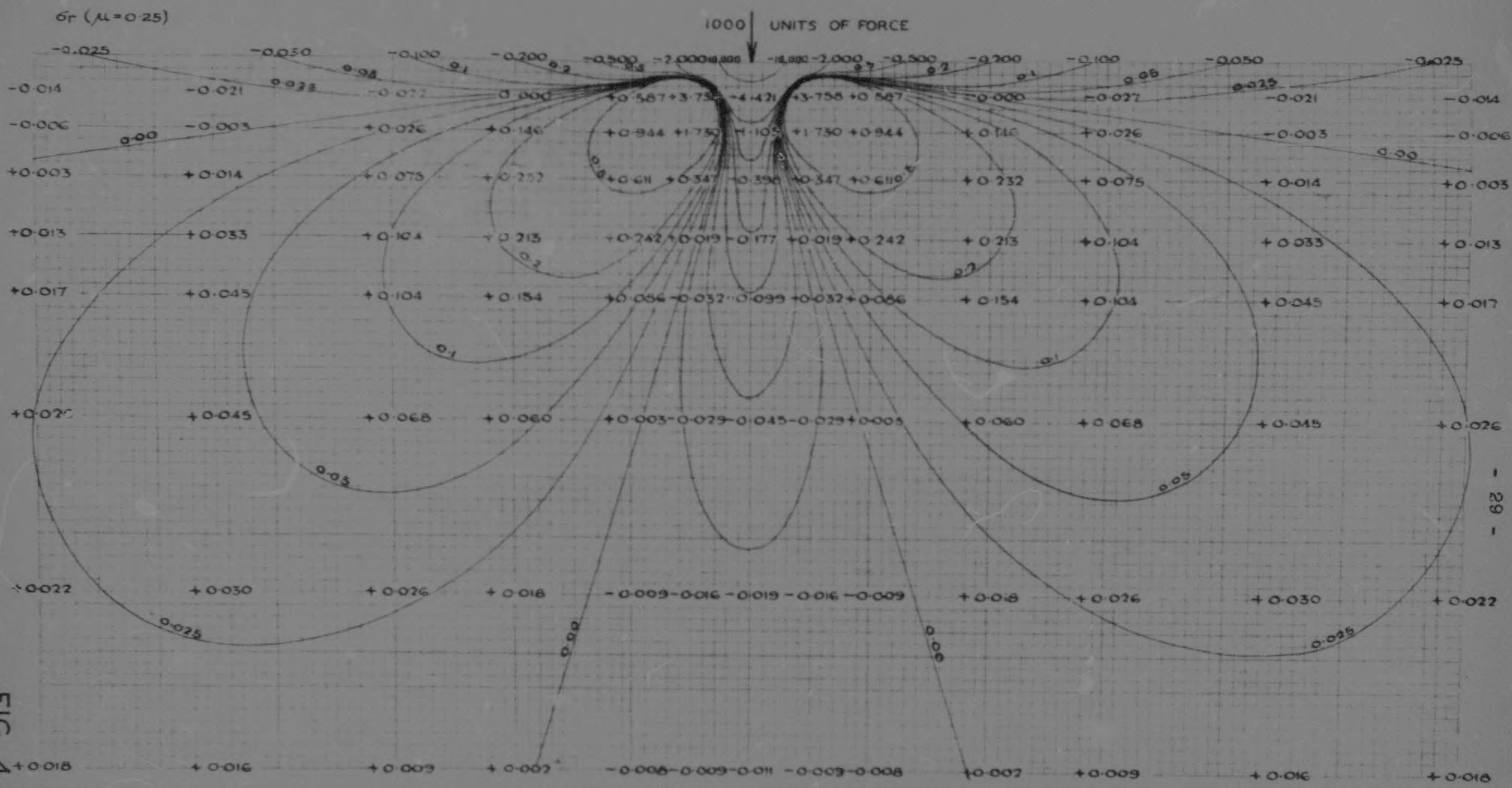


FIG 4 - CONTOURS OF HORIZONTAL RADIAL STRESS (σr) FOR CONDITION μ=0·25

FIG. 4.

be as a result of these three combined effects, and for any one of them to be known independently, both the other two effects need to be determined.

In this case, the flow of fluid as a result of volume change of the test sample only needs to be known, so that it is necessary to determine the flow of fluid during test resulting from the effect of leakage from the test cell and entry of the loading spindle into the test cell.

The latter effect mentioned can be determined by calculation from a knowledge of the cross-sectional area of the loading spindle and the distance advanced into the test cell. If leakage effects can be totally eliminated, there would be no further difficulty in determining the nett flow of fluid as a result of volume change of the sample only.

However, if leakage cannot be eliminated completely, as is more likely to be the case, a separate determination of the fluid flow into the test cell to replace leakage will be necessary.

Thus, the experimental work was designed to enable accurate measurements of the flow of fluid into or out of the test cell during test, due to the combined effects of leakage, spindle entry and sample volume change as well as independent measurements of the flow of fluid due to the effects of leakage and spindle entry only.

7.4 The Test Apparatus

This consisted of a standard model Triaxial Compression Test Machine designed for testing 1½"dia. x 3" length samples in a 3"dia. cell. The cell fluid was a mixture of glycerine and water in the ratio 30% : 70% by volume. Besides the usual pressure gauge on which the chamber pressure developed by pumping the foot pump could be read off directly, an adjustable head of mercury was connected to the bottom of a second 3"dia. cell, which was connected in series with the test cell and the fluid reservoir at the base of the machine, and was acting as an intermediate fluid reservoir. Hence, by measuring the height of the mercury column, a more accurate determination of the particular pressure selected for a test could be made.

The triaxial machine was operated by an electric motor through a variable speed gearbox, thus enabling the selection of one of several different rates of loading for any particular test. Values of applied load and deflection could be read off the respective dial deflection gauges provided for the purpose.

To enable the measurement of fluid flow into or out of the test cell during test, a circuit containing a small diameter capillary tube (approx. 1.6mm), fixed to a scale graduated in millimeters and containing a mercury slug of about 13mm. length, was connected in parallel with the main test cell fluid supply line between the intermediate fluid reservoir and the test cell. Suitable stopcocks were provided so that either of or both the capillary tube circuit and the main fluid supply circuit could be put into use as desired. Fluid flow measurements could then be effected by cutting in only the capillary tube circuit, whence any movement of fluid into or out of the test cell, would be directly indicated by the movement of the slug, accepting that the fluid is incompressible. The arrangement of the various circuits in the test set up is shown in Fig.19.

The arrangement of the test system was such that it was

found ..../

2. Vertical Shear Stress τ_{rz}: (a) and (b) as above in (1).
3. Horizontal Radial Stress σ_r: (a) Stress data for 5 values of μ in tabular form.
   (b) Stress contour diagrams for conditions: μ=0, 0.25 and 0.50.
4. Horizontal Tangential Stress σ_t: (a) as above in (3).
   (b) Stress contour diagrams for conditions: μ=0, 0.25 and 0.375.
5. Curves of Horizontal Stresses σ_r and σ_t versus Depth on the Axis of Loading, for conditions: μ=0, 0.125, 0.25, 0.375 and 0.50.
6. Maximum Shear Stress (S_{max}): (a) as above in (3).
   (b) Stress contour diagrams for conditions: μ=0, 0.25, 0.375 and 0.50.
7. Combined curves of Maximum Shear Stress contours for conditions: μ=0, 0.25 and 0.50.
8. Curves of Maximum Shear Stress (S_{max}) versus Depth on the Axis of Loading, for conditions: μ=0, 0.25 and 0.50.

The following points with regard to the various diagrams should, however, be noted. Firstly, in the case of item(4) above, stress contour diagrams for three values of Poisson's Ratio (μ=0, 0.25 and 0.375) are given in succession – but for the condition μ=0.50, σ_t = zero (see Discussion: Section 5.3) and hence, the stress contour diagram vanishes for μ = 0.50. Secondly, regarding item(5), the values of the horizontal stresses σ_r and σ_t are <u>equal</u> on the axis of loading for the <u>same</u> values of Poisson's Ratio (see Section 5.3), and hence the curves shown in Fig.9 are the same for both σ_r and σ_t versus depth on the axis of loading. Further, it needs to be mentioned that in the case of the stress contour diagrams for σ_r, σ_t and S_{max}, contour lines were frequently found to intersect the horizontal surface z = 0 of the semi-infinite solid. In these cases it was possible to calculate the exact position of the point of intersection of any stress contour line with the surface z = 0 from equations obtained by rewriting in each case the relevant stress component expression in a suitable form and at the same time applying the condition z = 0. The procedure and actual calculations are detailed in full in Appendix III Item 2.

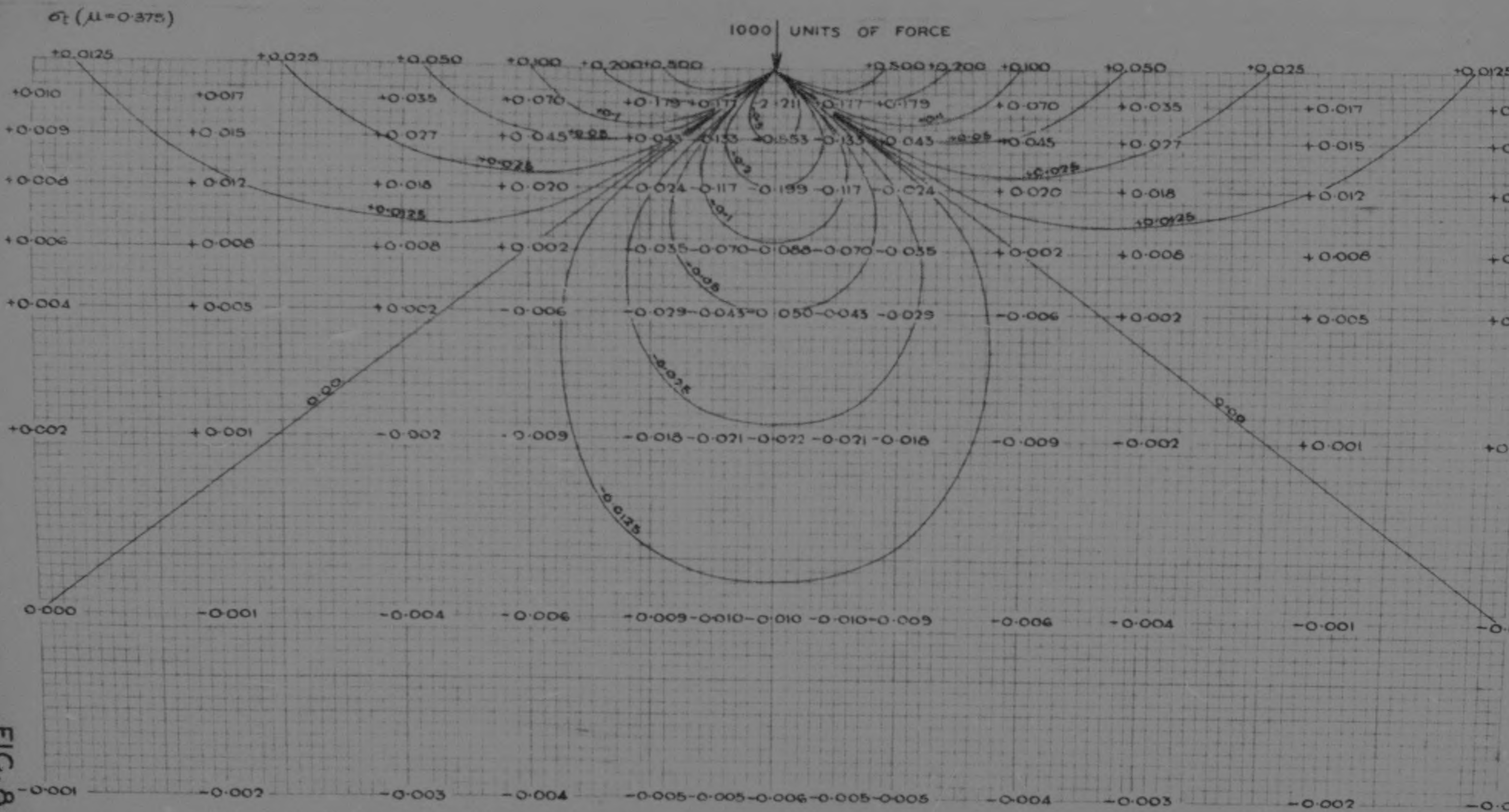


FIG 8 - CONTOURS OF HORIZONTAL TANGENTIAL STRESS (σ_t) FOR CONDITION μ=0·375

FIG. 8.

extrusion of some of the pore water from the clay (and hence with volume decrease).

With regard to these statements it should be pointed out that it is generally accepted(22) that due to the low permeability of clay soils the consolidation process does not commence immediately after the application of loading and that during this time delay the immediate settlement takes place. Furthermore, the statement that the immediate settlement takes place without change in water content in the case of a saturated soil, on a broader basis, actually implies that there is no loss of material from the soil while this settlement occurs, i.e., the soil experiences no change of mass during strain, be it a loss of mass or otherwise (see Chapter 2).

On this basis, possibly, the immediate settlement in the paper mentioned is considered to be of an elastic nature and is calculated by using an equation derived from the theory of elasticity, which is the equation 4.11 previously given and discussed in Section 4.4:

$$\rho_i = q_n . B \frac{1-\mu^2}{E} . I_\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 4.11$$

wherein, in this case, q_n denotes the nett foundation pressure, B the width of the foundation, E the Young's Modulus of the clay, μ its Poisson's Ratio, and I_ρ an influence value pertaining to settlement and with magnitude depending on the size of the foundation and the thickness of the underlying clay.

In the case of a saturated clay, no change in moisture content during the period of immediate settlement, is considered to imply no change in volume of the soil, since both the pore water and soil grain are assumed to be incompressible(8,10). Hence, for this condition $\mu = 0.5$, and the authors use this value in the above equation for calculating the immediate settlement of foundations.

No mention is made in the paper about the treatment to be afforded to clays which are partially saturated, or soils which are of a more cohesionless nature and in which drainage can take place more or less immediately after the application of loading. The question thus arises as to whether the phenomenon of immediate settlement can still be considered to occur in these cases. The answer in the case of partially saturated soils is probably closely linked up with the mechanics of the consolidation process in such soils, of which very little is known at present. If, in the case of saturated cohesionless soils, this phenomenon can still be considered to occur, a value of $\mu = 0.5$ would also be applicable for the same reasons as described above in the case of clay soils.

6.3 <u>The Laboratory Oedometer Test</u>

In the normal 1-dimensional oedometer test conducted in the laboratory, a sample of soil is subjected to a laterally-confined compression within the oedometer ring. As a result, only vertical strains can take place, all horizontal strains being of a negligible order and, in effect, can therefore be considered to be zero (22).

According to the definition of Poisson's Ratio, a case wherein no horizontal strain occurs as a result of applied loading, represents a condition of $\mu = 0$; in this case an enforced $\mu = 0$ condition.

Hence ..../

Vertical Normal Stress σ_z

Equation: $\sigma_z = \frac{Q}{2\pi} \quad \frac{3z^3}{(r^2+z^2)^{5/2}}$

TABLE II

z	r	σ_z
3	0	53.0516
	5	1.9152
	10	0.1040
	20	0.0037
	30	0.0005
	45	0.0003
	60	0.0000
6	0	13.2624
	5	3.5523
	10	0.4775
	20	0.0255
	30	0.0032
	45	0.0000
	60	0.0000
10	0	4.7747
	5	2.7343
	10	0.8435
	20	0.0859
	30	0.0143
	45	0.0016
	60	0.0000
15	0	2.1215
	5	1.6313
	10	0.8467
	20	0.1655
	30	0.0382
	45	0.0064
	60	0.0016
20	0	1.1937
	5	1.0250
	10	0.6826
	20	0.2117
	30	0.0621
	45	0.0127
	60	0.0032
30	0	0.5300
	5	0.4950
	10	0.4074
	20	0.2117
	30	0.0939
	45	0.0286
	60	0.0095
45	0	0.2355
	5	0.2292
	10	0.2085
	20	0.1496
	30	0.0939
	45	0.0414
	60	0.0191
60	0	0.1321
	5	0.1305
	10	0.1241
	20	0.1019
	30	0.0764
	45	0.0430
	60	0.0239

FIG 9

CURVES

of

HORIZONTAL STRESSES σ_r & σ_t

versus

DEPTH on AXIS of LOADING

for

VARIOUS VALUES OF μ.

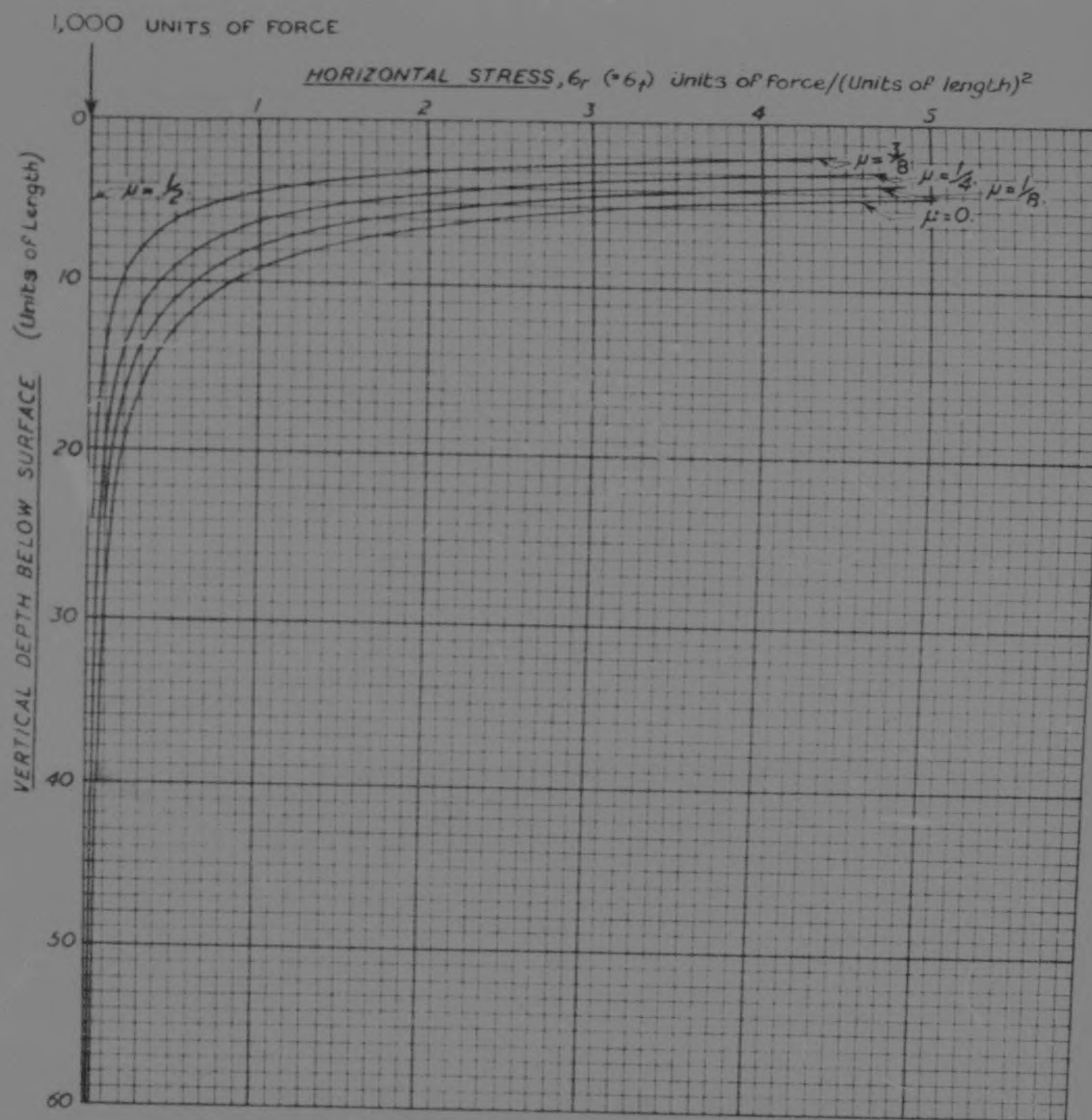


FIG. 9.

Hence, in terms of the discussion in Section 6.2, if any saturated soil be subjected to an oedometer test, it can experience no elastic settlement as a result of the applied loading, since, in the case of a saturated soil, such settlement is essentially that which takes place without change in moisture content, i.e. no loss of mass, and if soil grain and pore water are assumed to be incompressible and in addition lateral strain be prevented, no such settlement can occur. Alternatively also, if soil grain and pore water are considered to be incompressible, there can be no volume decrease in a saturated soil without the extrusion of pore water, i.e. without consolidation taking place, which therefore implies that for calculating the elastic (or immediate) settlement of any type of saturated soil, a value of $\mu = 0.5$ only is applicable. This statement thus also verifies the statement at the end of the previous Section regarding the value of μ applicable to saturated cohesionless soils.

As a result of the above discussion, the question arises as to what the nature of the initial compression(1.5) in the case of a consolidation test on a saturated soil may be. It is considered that it may possibly be due to the combined effects of very slight yield in the confining test ring (which was assumed to be zero above), and bedding down of the porous plates and other parts of the test set-up. However, for a final answer, a more detailed investigation may be warranted.

6.4 The Westergaard Equations

The Westergaard equations(1.6) are based on a condition which is not uncommon in nature, viz. that of aelotropy(8.11), in which strata of compressible material contain thin lenses of material of great lateral rigidity within them. This type of soil structure greatly accentuates the non-isotropic condition in these soils, and is also the cause of a greatly increased resistance to lateral strain.

Westergaard's equations constitute a solution based on the elastic theory, in which extreme non-isotropic conditions of this type are assumed. In the derivation, an elastic material is assumed to be laterally reinforced by numerous, closely spaced, horizontal sheets of negligible thickness but of infinite rigidity, which prevent the mass as a whole from undergoing lateral strain. In his mathematical expression (containing terms of Poisson's Ratio) for the vertical stress caused by a point load, Westergaard finds that the stresses at points directly below the load have minimum value when Poisson's Ratio has its minimum value of zero, and considers that for the case of large lateral restraint, this yields the most reliable result (see also Ref.22). Hence, his final expressions for the vertical stress caused by point loading and uniform area loading, are thus obtained by substituting $\mu = 0$.

Taylor(1.6) in comparing the Boussinesq and Westergaard expressions, mentions that within certain limits (as discussed) both point loading and uniform area loading give values of vertical stresses which are approximately equal to two-thirds of the values given by the Boussinesq equations. He considers that the stratified condition upon which the Westergaard solution is based, is certainly nearer to the conditions existing in sedimentary soils than the isotropic condition assumed by Boussinesq. Furthermore, he points out that estimates of (consolidation) settlements obtained by use of the Boussinesq equations for determination of (vertical) stresses, are in the great majority of cases larger than observed settlements, which possibly indicates that the Boussinesq equations give stress values which are too large, although admittedly, such a result could also be due to other assumptions used in settlement estimates. However, the view was also expressed that the Westergaard equations tend to

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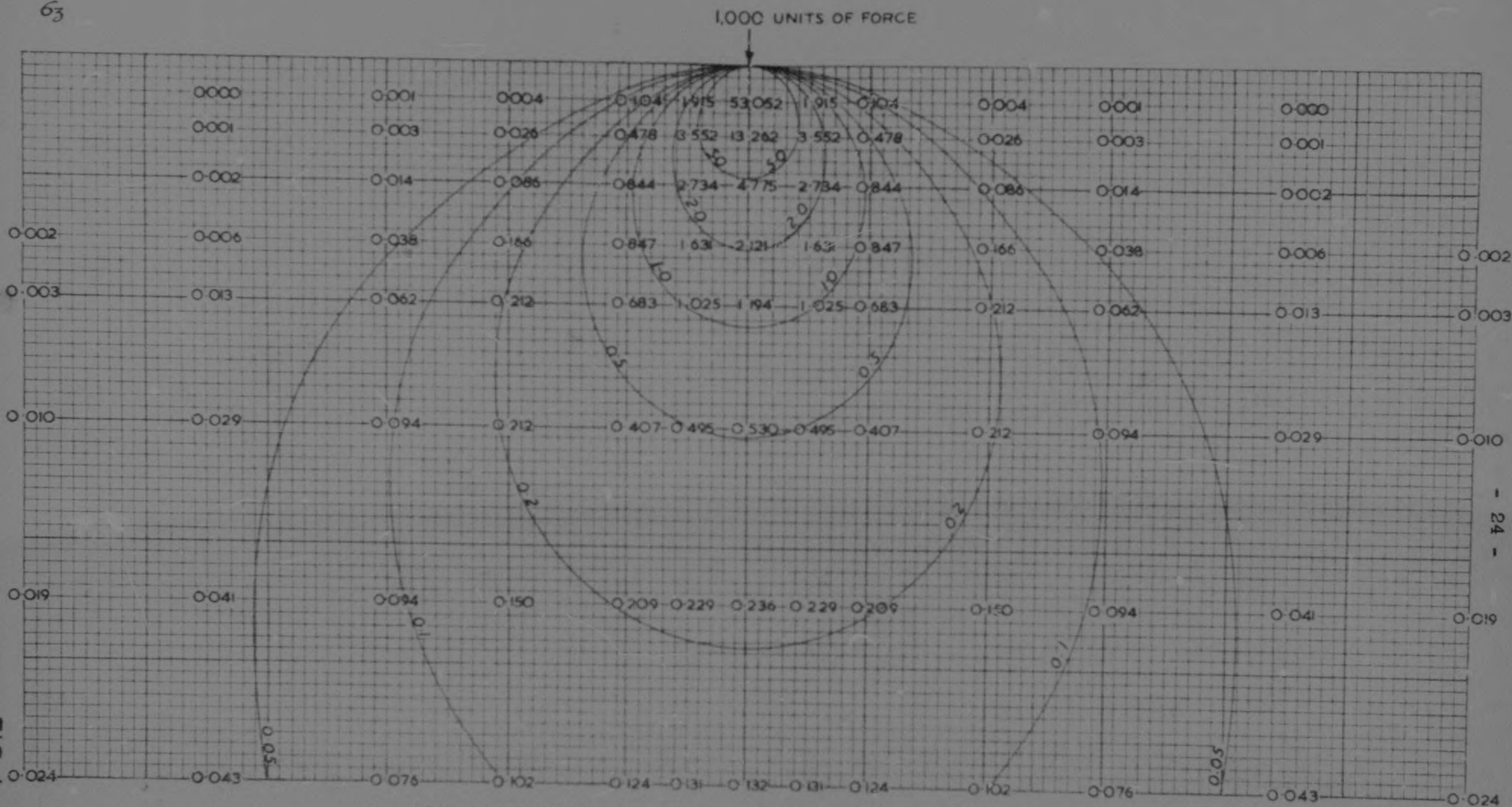


FIG 1 - CONTOURS OF VERTICAL NORMAL STRESS (σ_3)

FIG. 1.

Maximum Shear Stress S_{max}

Equation: $S_{max} = \frac{1}{2}\sqrt{(\sigma_z - \sigma_r)^2 + 4\tau_{rz}^2}$

TABLE VI

z	r	S_{max}				
		$\mu = 0$	$\mu = 0.125$	$\mu = 0.25$	$\mu = 0.375$	$\mu = 0.50$
3	0	30.[illegible]468	29.8415	28.7363	27.6311	26.5258
	5	3.1942	3.2351	3.3214	3.4502	3.6166
	10	0.3495	0.3610	0.4229	0.5176	0.6299
	20	0.0910	0.0520	0.0264	0.0483	0.0865
	30	0.0536	0.0336	[illegible].0138	0	0.0260
	45	0.0288	0.0192	0.0105	0	0.0078
	60	0.0175	0.0128	0.0071	0	0
6	0	7.7365	7.4604	7.1835	6.9073	6.6312
	5	3.2250	3.1563	3.0974	3.0481	3.0096
	10	0.7983	0.8089	0.8308	0.8627	0.9039
	20	0.0866	0.0897	0.1051	0.1291	0.1570
	30	0.0316	0.0212	0.0229	0.0350	0.0502
	45	0.0193	0.0100	0	0.0063	0.0151
	60	0.0132	0.0087	0	0	0.0063
10	0	2.7852	2.6857	2.5863	2.4868	2.3873
	5	1.9295	1.8709	1.8149	1.7612	1.7091
	10	0.8751	0.8614	0.8514	0.8454	0.8434
	20	0.1724	0.1776	0.1865	0.1990	0.2140
	30	0.0447	0.0471	0.0541	0.0636	0.0751
	45	0.0132	0.0100	0.0122	0.0173	0.0245
	60	0.0080	0	0	0.0055	0.0103
15	0	1.2374	1.1929	1.1491	1.1053	1.0607
	5	1 0416	1.0072	0.9726	[illegible].9393	0.9065
	10	0.6736	0.6560	0.6395	0.6243	0.6107
	20	0.2201	0.2196	0.2208	0.2239	0.2286
	30	0.0767	0.0787	0.0829	0.0882	0.0948
	45	0.0200	0.0212	0.0240	0.0287	0.0335
	60	0.0032	0	0.0055	0.0095	0.0135
20	0	0.6963	0.6717	0.6462	0.6215	0.5968
	5	0.6305	0.6088	0.5872	0.5658	0.5446
	10	0.4828	0.4684	0.4551	0.4402	0.4271
	20	0.2284	0.2252	0.2228	0.2214	0.2209
	30	0.0938	0.0943	0.0960	0.0985	0.1018
	45	0.0304	0 0316	0.0339	0.0367	0.0403
	60	0.0100	0.0112	0.0122	0.0150	0.0180
30	0	0.3095	0.2984	0.2873	0.2761	0.2650
	5	0.2952	0.2845	0.2747	0.2641	0.2543
	10	0.2600	0.2513	0.2432	0.2346	0.2262
	20	0.1692	0.1650	0.1607	0.1567	0.1535
	30	0.0975	0.0960	0.0947	0.0941	0.0938
	45	0.0413	0.0415	0.0421	0.0433	0.0445
	60	0.0200	0.0206	0.0218	0.0230	0.0245
45	0	0.1377	0.1328	0.1272	0.1225	0.1177
	5	0.1347	0.1301	0.1254	0.1207	0.1161
	10	0.1271	0.1226	0.1182	0.1139	0.1095
	20	0.1022	0.0987	0.0958	0.0925	0.0898
	30	0.0750	0.0728	0.0712	0.0694	0.0678
	45	0.0427	0.0421	0.0415	0.0412	0.0412
	60	0.0245	0.0244	0.0244	0.0250	0.0255
60	0	0.0772	0.0740	0.0716	0.0692	0.0660
	5	0.0763	0.0731	0.0707	0.0683	0.0652
	10	0.0744	0.0712	0.0691	0.0667	0.0636
	20	0.0648	0.0628	0.0602	0.0581	0.0561
	30	0.0543	0.0526	0.0512	0.0497	0.0482
	45	0.0361	0.0353	0.0343	0.0335	0.0331
	60	0.0255	0.0250	0.0245	0.0245	0.0245

CHAPTER 7

LABORATORY TESTS

7.1 Theoretical Considerations

The basis on which a soil can be considered to approximate the elastic state of stress, was discussed in Section 6.1. As was mentioned in that Section and as was also implied by the discussions in Chapter 2, Poisson's Ratio only has a rational meaning within the linear range of loading of a soil's stress:strain curve. In this Chapter and in those to follow, a determination of Poisson's Ratio from the observed behaviour of a soil within its linear range of loading during a triaxial compression test, will be described and discussed.

Since the elastic constants are related to volumetric strains, (see Chapter 3) it becomes possible to evaluate Poisson's Ratio for a soil by measuring the change in volume of the soil resulting from a certain measured increase in loading within the linear range. This is based on the adaption of the three dimensional stress relationship given by equation 3.3, to the conditions of a triaxial compression test.

$$\text{Dilation } (\Delta) = \frac{\delta V}{V} = \frac{1}{E}(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu) \quad \ldots\ldots\ldots \; 3.3$$

In the derivation of this equation, the principal stresses σ_1, σ_2 and σ_3 were considered to act in an outward direction on the cube of unit side, with a sign convention such that this direction is taken to be positive. Hence, the volume change δV, (a volume increase) caused by the action of this system of principal stresses, is considered to be positive, so that the Dilation (Δ) is positive for principal stresses acting in an outward direction.

However, in the case of a triaxial compression test, the direction of the principal stresses is opposite to that just considered and in addition, the lateral principal stresses are of equal magnitude. Hence, for convenience, the sign convention is altered so that compressive principal stresses are considered to be positive and hence, the Dilation (Δ) is positive for a reduction in volume. Furthermore, σ_2 in equation 3.3 is replaced by σ_3, the allround chamber pressure in the test cell, and hence, equation 3.3 becomes:

$$\text{Dilation } (\Delta) = \frac{\delta V}{V} = \frac{1}{E}(\sigma_1 + 2\sigma_3)(1 - 2\mu) \quad \ldots\ldots\ldots\ldots \; 7.1$$

However, σ_1 is inclusive of a σ_3 pressure due to the allround chamber pressure, so that if σ_d, (known as the Deviator Stress), is the nett applied pressure in the Z-direction, we have that:

$$\sigma_1 = \sigma_3 + \sigma_d \quad \ldots\ldots\ldots\ldots \; 7.2$$

and hence

$$\frac{\delta V}{V} = \frac{1}{E}(\sigma_3 + \sigma_d + 2\sigma_3)(1 - 2\mu)$$

$$\text{i.e. } \frac{\delta V}{V} = \frac{1}{E}(3\sigma_3 + \sigma_d)(1 - 2\mu) \quad \ldots\ldots\ldots\ldots \; 7.3$$

If triaxial shear tests are conducted wherein the sample of initial volume V_1 is allowed to consolidate under the action of an allround chamber pressure during the pre-shear condition, then this chamber pressure causes volumetric strains in the test sample, which at the end of the consolidation process have been completely effected, thereby resulting in a new sample volume V_2. Hence, when the actual triaxial test is commenced with the application of the Deviator Stress (σ_d), the volumetric strains then produced (which constitute the δV measured during the test), are relative to the sample volume V_2 attained at the end of consolidation, and furthermore, are caused by the action of σ_d only,

Vertical Shear Stress τ_{rz}

Equation: $\tau_{rz} = \dfrac{Q}{2\pi} \dfrac{3rz^2}{(r^2+z^2)^{5/2}}$

TABLE III

z	r	τ_{rz}
3	0	0.0
	5	3.1911
	10	0.3470
	20	0.0255
	30	0.0048
	45	0.0016
	60	0.0000
6	0	0.0
	5	2.9603
	10	0.7974
	20	0.0859
	30	0.0191
	45	0.0048
	60	0.0016
10	0	0.0
	5	1.3671
	10	0.8435
	20	0.1703
	30	0.0446
	45	0.0095
	60	0.0032
15	0	0.0
	5	0.5443
	10	0.5634
	20	0.2196
	30	0.0764
	45	0.0207
	60	0.0064
20	0	0.0
	5	0.2562
	10	0.3422
	20	0.2117
	30	0.0939
	45	0.0302
	60	0.0111
30	0	0.0
	5	0.0828
	10	0.1353
	20	0.1416
	30	0.0939
	45	0.0414
	60	0.0191
45	0	0.0
	5	0.0255
	10	0.0462
	20	0.0668
	30	0.0621
	45	0.0414
	60	0.0239
60	0	0.0
	5	0.0111
	10	0.0207
	20	0.0334
	30	0.0382
	45	0.0318
	60	0.0239

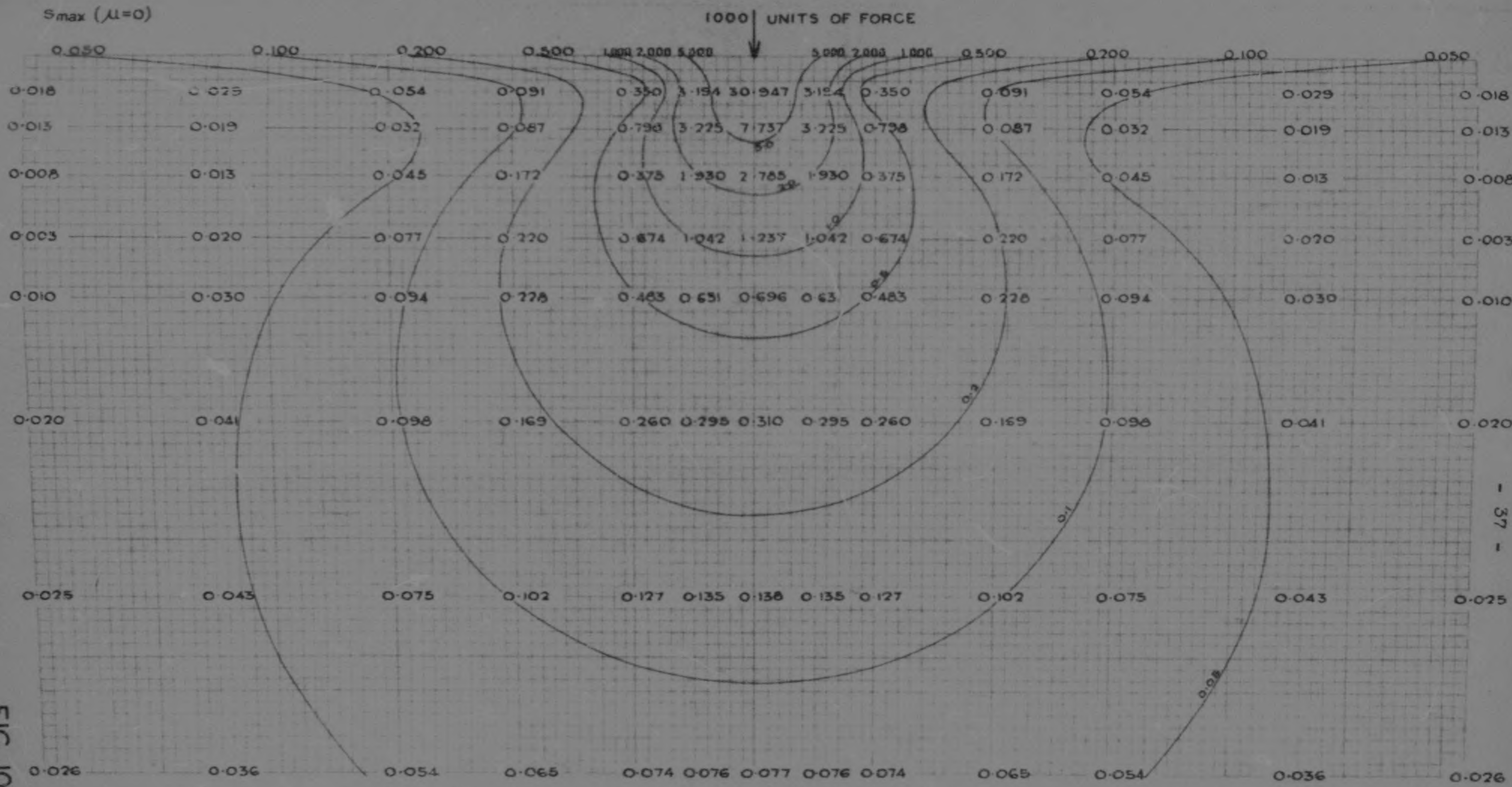


FIG 10 - CONTOURS OF MAXIMUM SHEAR STRESS (S_{max}) FOR CONDITION $\mu=0$

FIG. 10.

the chamber pressure σ_3 merely acting as a static allround pressure with no effect in so far as causing volumetric strain during test is concerned, i.e. the test conditions are similar to those of conducting a test under an allround constant air pressure, say, atmospheric pressure. In effect σ_3 thus has zero value and equation 7.3 above should be amended accordingly.

It should be pointed out, however, that the σ_3 chamber pressure controls the measured value of the volumetric strain produced by σ_d (δV as measured during the test), i.e., for different values of σ_3, the same range of σ_d results in a different measured value of δV, so that indirectly, although in the equation σ_3 has zero value, its effect is included in the calculation of Poisson's Ratio. Further, it should be borne in mind that the change in volume during the pre-shear condition resulting from the action of the σ_3 chamber pressure, is mainly as a result of consolidation effects, while the change in volume within the linear range resulting from the application of σ_d is considered to be due to elastic effects, and as such forms the valid basis of the determination of Poisson's Ratio from an equation applying to the elastic state only (cf. Chapter 2).

Hence, as it was intended to conduct Unconsolidated Undrained shear tests, σ_3, in equation 7.3, has zero value and the sample volume V, is actually the sample volume V_2 referred to in the above discussion.

Equation 7.3 thus becomes:

$$\frac{\delta V}{V} = \frac{1}{E}(\sigma_d)(1 - 2\mu) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 7.4$$

From this relationship Poisson's Ratio (μ) can be solved for by rewriting the equation as follows:-

$$(1 - 2\mu) = \frac{\delta V}{V} \cdot \frac{E}{\sigma_d}$$

$$\therefore - 2\mu = \frac{\delta V}{V} \cdot \frac{E}{\sigma_d} - 1$$

$$\therefore \mu = \tfrac{1}{2}\left[1 - \frac{\delta V}{V} \cdot \frac{E}{\sigma_d}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 7.5$$

The values of all the unknowns on the right hand side of this equation can be obtained experimentally from a triaxial compression test, and hence, the value of μ can be calculated.

Thus, the experimental work was designed to enable the measurement during a triaxial test, of the unknowns δV, E and σ_d in the right hand side of equation 7.5, V being the volume of the test sample at the end of the consolidation process during the pre-shear condition. The method by which this volume was determined, is discussed fully in Section 8.4.

7.2 The Test Soil

The soil used for the tests was a well-graded, firm, fine, reddish-brown, silty sand, taken from a depth of 13ft. from a site on the North side of Juta Street near Biccard Street, Braamfontein, Johannesburg. The full soil profile on the site was measured down to a depth of 55ft. and is shown on the next page. (Fig. 16).

As the test soil occurs above a 2ft. thick layer of pebbles which is commonly known as the "pebble marker", it is probably of a transported origin.

The ..../

that the application to practical problems of the expressions for the stress components in 3-dimensional cases, generally [illegible] requires the expression for the vertical normal stress (σ_z). Hence, the expressions for stress components containing Poisson's Ratio are seldom used. This is especially so in the case of finite area loadings, since, as discussed by both the references just mentioned, an area load is very often replaced by an equivalent point load without introducing very significant errors.

An expression for the vertical normal stress distribution below a corner of a uniform rectangular area loading on the surface of a semi-infinite elastic solid, was derived by Newmark(18) by integrating Boussinesq's expression for the vertical normal stress in such a solid due to a point load (equations 4.7 (a) and (b)). The expression is given by both Terzaghi(8.7) and Taylor (1.4), and is frequently used in soils work. As is the case with Boussinesq's equation 4.7 above, Newmark's expression therefore also does not contain Poisson's Ratio.

Expressions for the displacements in a semi-infinite elastic solid in 3-dimensional problems of stress and strain, also contain terms of Poisson's Ratio, since displacements are directly related to stress distribution. Boussinesq(16) computed the vertical, horizontal and radial displacements produced by a vertical point load applied to the horizontal surface of a semi-infinite elastic solid. These expressions have also been given by Terzaghi(8.8). Using the basic expressions of Boussinesq, Schleicher(19) obtained by integration an expression for the settlement of the corners of a uniformly loaded rectangular area of width B and length L. His expression is given by Terzaghi(8.9), who subsequently writes it in the form:

$$\Delta\rho = qB \cdot \frac{1-\mu^2}{E} \cdot I_\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4.11$$

where q is the intensity of the uniformly distributed loading, and I_ρ is a pure number known as the settlement influence value. Its value depends on the ratio L:B of the sides of the loaded area. This is another expression which is frequently used in soils work as will be seen in Section 6.2.

It is thus clear that with regard to the determination of stress and strain in a semi-infinite, elastic, isotropic solid due to applied surface forces, Poisson's Ratio only enters in 3-dimensional problems of stress and strain, since as was shown in Section 4.3, the moduli of elasticity do not affect the stress distribution and hence also the strains in 2-dimensional problems. It should be noted that in the case of a semi-infinite elastic solid wherein all loadings are applied vertically on its surface (as is generally the case in soils work), a 2-dimensional problem of stress and strain is therefore also one wherein the applied loading occurs in only 2-dimensions, and similarly in the case of 3-dimensional problems.

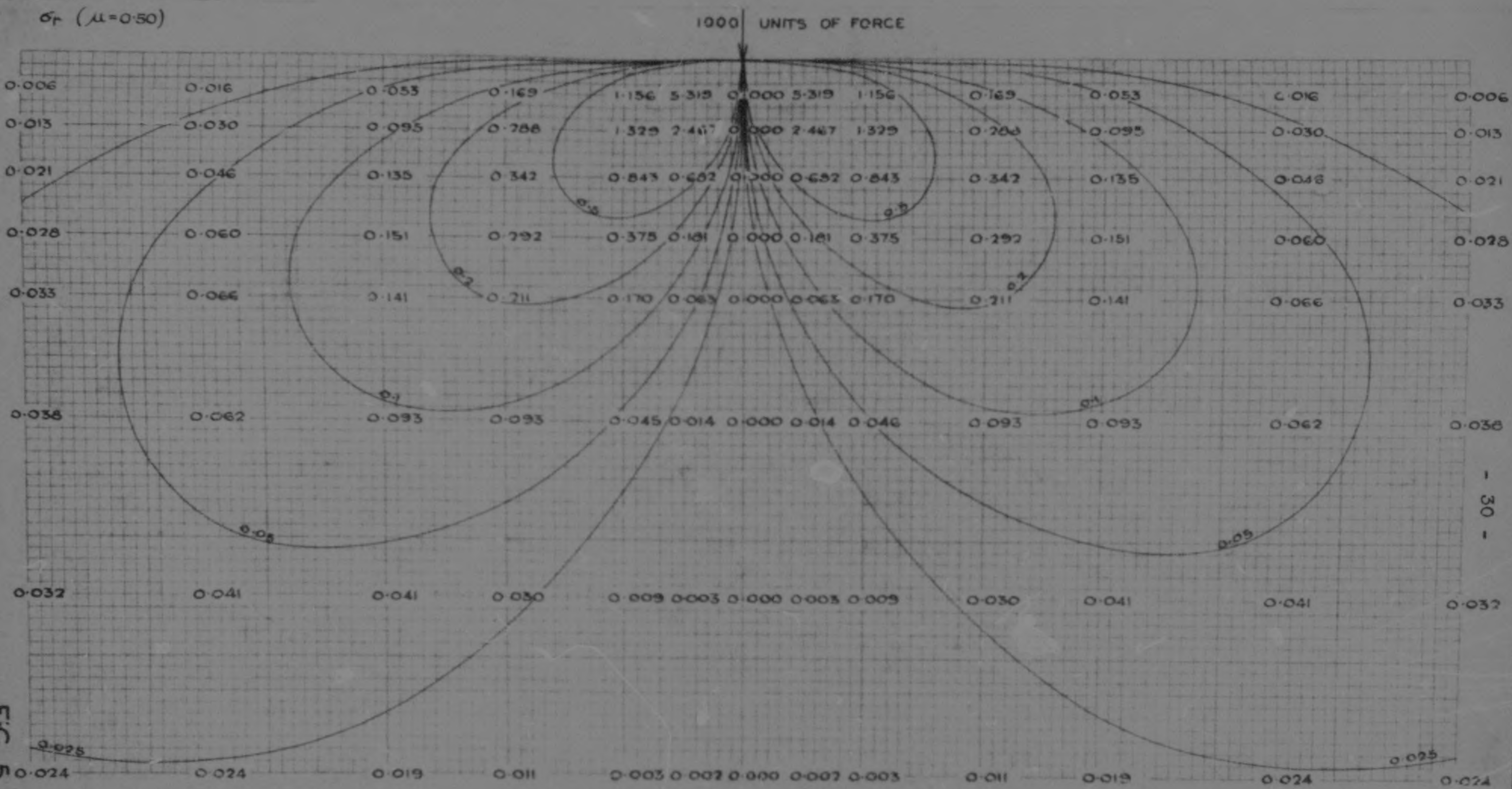
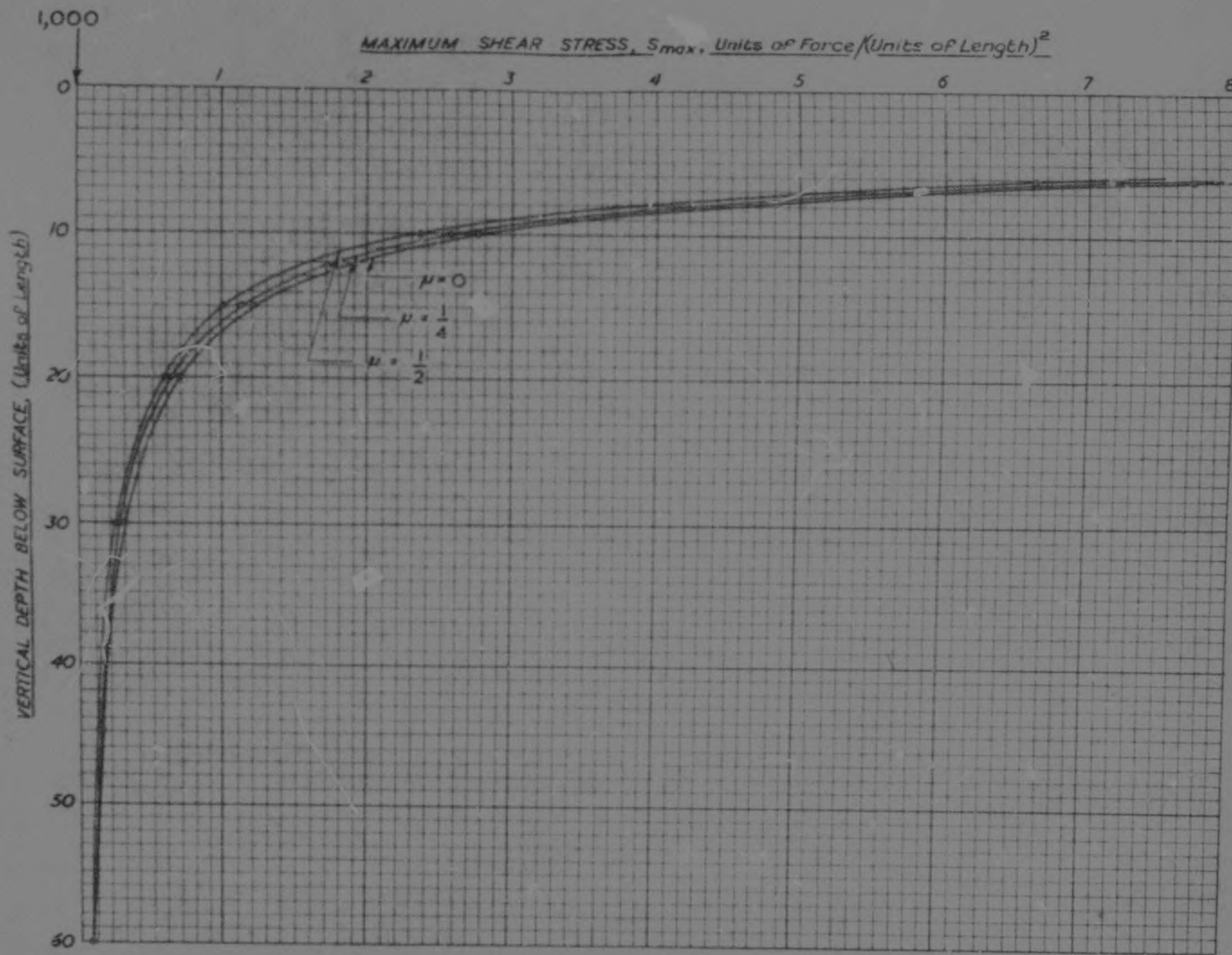


FIG 5 - CONTOURS OF HORIZONTAL RADIAL STRESS (σ_r) FOR CONDITION μ=0·50

FIG. 5.

Fig.15

CURVES
OF
MAXIMUM SHEAR STRESS,
S_{max},
VERSUS
DEPTH ON LOADING AXIS
FOR
VARIOUS VALUES OF μ.



that the application to practical problems of the expressions for the stress components in 3-dimensional cases, generally only requires the expression for the vertical normal stress (σ_z). Hence, the expressions for stress components containing Poisson's Ratio are seldom used. This is especially so in the case of finite area loadings, since, as discussed by both the references just mentioned, an area load is very often replaced by an equivalent point load without introducing very significant errors.

An expression for the vertical normal stress distribution below a corner of a uniform rectangular area loading on the surface of a semi-infinite elastic solid, was derived by Newmark(18) by integrating Boussinesq's expression for the vertical normal stress in such a solid due to a point load (equations 4.7 (a) and (b)). The expression is given by both Terzaghi(8.7) and Taylor (1.4), and is frequently [illegible]d in soils work. As is the case with Boussinesq's equation 4.[illegible]ve, Newmark's expression therefore also does not contain Poisson's Ratio.

Expressions for the <u>displacements</u> in a semi-infinite elastic solid in 3-dimensional problems of stress and strain, also contain terms of Poisson's Ratio, since displacements are directly related to stress distribution. Boussinesq[16] computed the vertical, horizontal and radial displacements produced by a vertical point load applied to the horizontal surface of a semi-infinite elastic solid. These expressions have also been given by Terzaghi[8.8]. Using the basic expressions of Boussinesq, Schleicher[19] obtained by integration an expression for the settlement of the corners of a uniformly loaded rectangular area of width B and length L. His expression is given by Terzaghi[8.9], who subsequently writes it in the form:

$$\Delta\rho = qB \cdot \frac{1-\mu^2}{E} \cdot I_\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4.11$$

where q is the intensity of the uniformly distributed loading, and I_ρ is a pure number known as the settlement influence value. Its value depends on the ratio L:B of the sides of the loaded area. This is another expression which is frequently used in soils work as will be seen in Section 6.2.

It is thus clear that with regard to the determination of stress and strain in a semi-infinite, elastic, isotropic solid due to applied surface forces, Poisson's Ratio only enters in 3-dimensional problems of stress and strain, since as was shown in Section 4.3, the moduli of elasticity do not affect the stress distribution and hence also the strains in 2-dimensional problems. It should be noted that in the case of a semi-infinite elastic solid wherein all loadings are applied vertically on its surface (as is generally the case in soils work), a 2-dimensional problem of stress and strain is therefore also one wherein the applied loading occurs in only 2-dimensions, and similarly in the case of 3-dimensional problems.

Horizontal Tangential Stress σ_t

$$\text{Equation: } \sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$$

TABLE V

z	r	σ_t				
		μ = 0	μ = 0.125	μ = 0.25	μ = 0.375	μ = 0.50
3	0	-8.8419	-6.6314	-4.4210	-2.2105	0
	5	+0.7098	+0.5323	+0.3549	+0.1774	0
	10	+0.7162	+0.5371	+0.3581	+0.1790	0
	20	+0.2817	+0.2113	+0.1408	+0.0704	0
	30	+0.1416	+0.1062	+0.0708	+0.0354	0
	45	+0.0684	+0.0513	+0.0342	+0.0171	0
	60	+0.0398	+0.0299	+0.0199	+0.0100	0
6	0	-2.2107	-1.6580	-1.1054	-0.5527	0
	5	-0.5332	-0.3999	-0.2666	-0.1333	0
	10	+0.1703	+0.1277	+0.0852	+0.0426	0
	20	+0.1782	+0.1336	+0.0891	+0.0446	0
	30	+0.1082	+0.0811	+0.0541	+0.0271	0
	45	+0.0589	+0.0442	+0.0295	+0.0147	0
	60	+0.0350	+0.0262	+0.0175	+0.0088	0
10	0	-0.7958	-0.5968	-0.3979	-0.1990	0
	5	-0.4679	-0.3509	-0.2340	-0.1170	0
	10	-0.0971	-0.0728	-0.0486	-0.0243	0
	20	+0.0780	+0.0585	+0.0390	+0.0195	0
	30	+0.0700	+0.0525	+0.0350	+0.0175	0
	45	+0.0461	+0.0346	+0.0230	+0.0115	0
	60	+0.0302	+0.0227	+0.0151	+0.0076	0
15	0	-0.3533	-0.2650	-0.1766	-0.0883	0
	5	-0.2785	-0.2089	-0.1392	-0.0696	0
	10	-0.1400	-0.1050	-0.0700	-0.0350	0
	20	+0.0064	+0.0048	+0.0032	+0.0016	0
	30	+0.0334	+0.0250	+0.0167	+0.0083	0
	45	+0.0318	+0.0239	+0.0159	+0.0080	0
	60	+0.0239	+0.0179	+0.0120	+0.0050	0
20	0	-0.1989	-0.1492	-0.0995	-0.0497	0
	5	-0.1719	-0.1289	-0.0860	-0.0430	0
	10	-0.1162	-0.0871	-0.0581	-0.0290	0
	20	-0.0239	-0.0179	-0.0120	-0.0050	0
	30	+0.0095	+0.0071	+0.0048	+0.0024	0
	45	+0.0191	+0.0143	+0.0096	+0.0048	0
	60	+0.0175	+0.0131	+0.0088	+0.0044	0
30	0	-0.0875	-0.0656	-0.0438	-0.0219	0
	5	-0.0843	-0.0632	-0.0421	-0.0211	0
	10	-0.0700	-0.0525	-0.0350	-0.0175	0
	20	-0.0350	-0.0262	-0.0175	-0.0088	0
	30	-0.0095	-0.0071	-0.0048	-0.0024	0
	45	+0.0048	+0.0036	+0.0024	+0.0012	0
	60	+0.0080	+0.0060	+0.0040	+0.0020	0
45	0	-0.0382	-0.0286	-0.0191	-0.0096	0
	5	-0.0382	-0.0286	-0.0191	-0.0096	0
	10	-0.0350	-0.0262	-0.0175	-0.0088	0
	20	-0.0255	-0.0191	-0.0128	-0.0064	0
	30	-0.0143	-0.0107	-0.0072	-0.0036	0
	45	-0.0048	-0.0036	-0.0024	-0.0012	0
	60	-0.0000	0	0	0	0
60	0	-0.0223	-0.0167	-0.0111	-0.0056	0
	5	-0.0207	-0.0155	-0.0104	-0.0052	0
	10	-0.0207	-0.0155	-0.0104	-0.0052	0
	20	-0.0175	-0.0131	-0.0088	-0.0044	0
	30	-0.0127	-0.0095	-0.0063	-0.0032	0
	45	-0.0064	-0.0048	-0.0032	-0.0016	0
	60	-0.0032	-0.0024	-0.0016	-0.0008	0

5.3 Discussion

As previously indicated, the stress components σ_z and τ_{rz} in the Boussinesq equations of stress, do not contain Poisson's Ratio and hence their stress contour diagrams are unaffected by variations in μ. However, the stress components σ_r and σ_t and the Maximum Shear Stress (S_{max}) are affected by variations in μ and therefore the following discussion is directed mainly at the effect of Poisson's Ratio on these three stresses.

From a closer consideration of the equations for the stress components σ_r and σ_t (which for convenience are given again immediately below), the following facts are apparent:

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3r^2z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \ldots\ldots\ldots\ldots 4.8(a)$$

$$\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \ldots\ldots\ldots\ldots 4.8(a)$$

Firstly, for the condition $\mu = 0.50$, σ_t disappears completely while σ_r is considerably simplified in that the second term in the bracket reduces to zero. If, in addition, either $r = 0$ or $z = 0$, then σ_r is also reduced to zero. Furthermore, on the axis of loading ($r=0$) of the vertical point load Q, the equations for σ_r and σ_t reduce to the same expressions, viz.

$$(\sigma_r)_{r=0} = -\frac{Q}{4\pi z^2}(1-2\mu) = (\sigma_t)_{r=0} \quad \ldots\ldots\ldots\ldots 5.6$$

Hence, the horizontal stresses σ_r and σ_t are equal at any depth z on the axis of loading for the <u>same</u> values of Poisson's Ratio.

These facts serve to enable a better understanding of the behaviour of the stress contour diagrams for σ_r and σ_t due to variation in the value of Poisson's Ratio. In the case of σ_r, it is observed that with increasing values of μ, the central negative pressure bulb and the negative pressure zones adjacent to the horizontal surface of the semi-infinite solid, reduce in size, while at the same time the two positive pressure bulbs increase in size. At $\mu = 0.50$, the former completely vanishes and only the two positive pressure bulbs remain with zero stress throughout on both the vertical axis of loading and on the horizontal surface $z = 0$. In the case of σ_t, it is observed that at $\mu=0$ there is only one central (negative) pressure bulb with positive pressure zones adjacent to the horizontal surface of the semi-infinite solid. With increasing values of μ, both of these reduce in size until, as previously mentioned above, at $\mu=0.50$ when σ_t attains zero value, the stress contour diagram completely vanishes.

It is thus clear that Poisson's Ratio has a considerable influence on the values of both of the horizontal stress components σ_r and σ_t. Fig.9 shows that with respect to the axis of loading the effect is in both cases most marked between the horizontal surface of the solid and a depth of 10 units of length below the surface $z = 0$. A consideration of the numerical values of $\sigma_r(=\sigma_t)$ at the various depths on the axis of loading (Table IV or Table V), shows that the stress values corresponding to the conditions $\mu = 0$, 0.125, 0.25 and 0.375

bear ..../

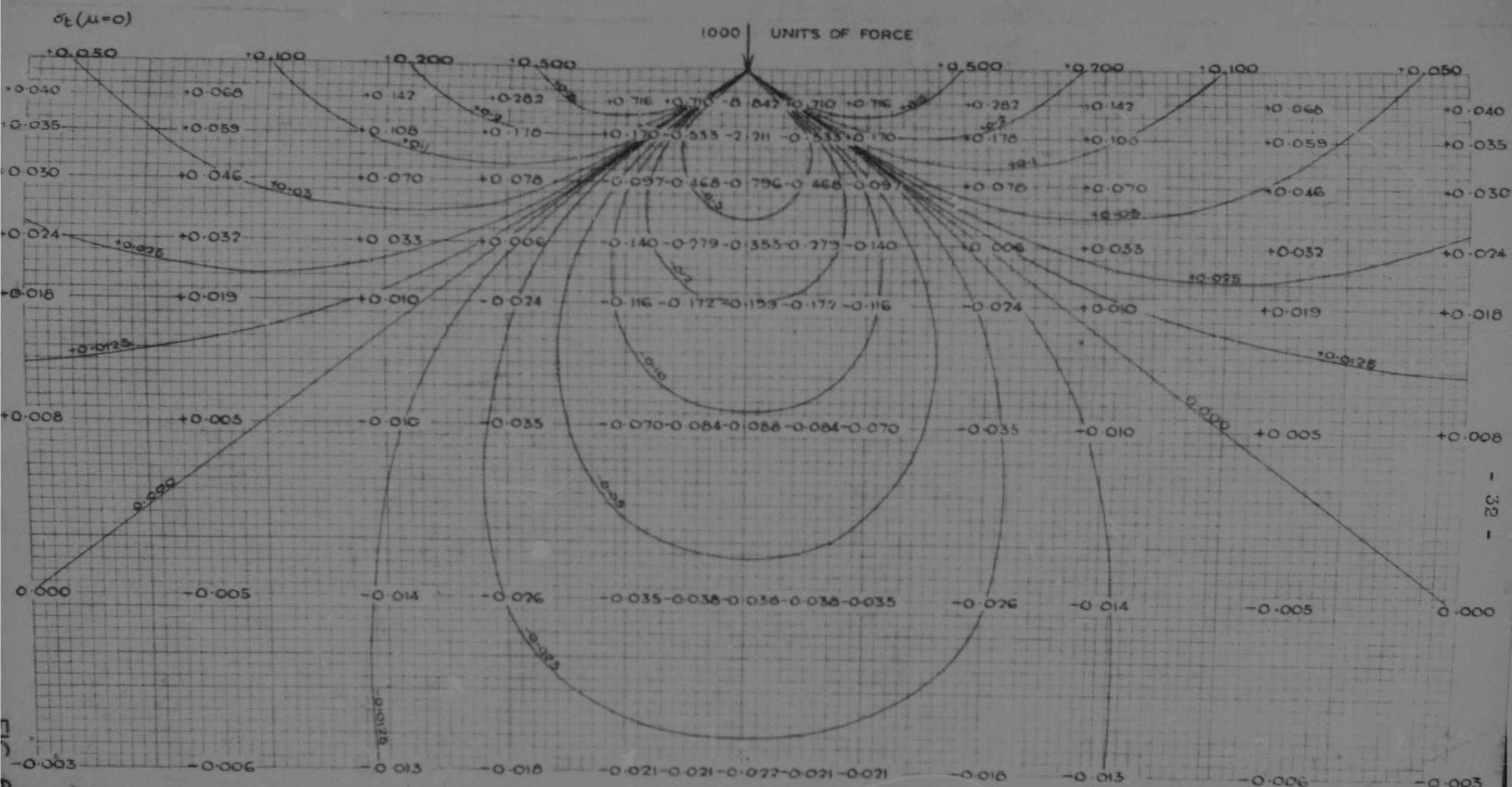


FIG 6 - CONTOURS OF HORIZONTAL TANGENTIAL STRESS (σ_t) FOR CONDITION $\mu = 0$

CHAPTER 5

THE AFFECT OF VARIOUS VALUES OF POISSON'S RATIO ON THE STRESS DISTRIBUTION IN A SEMI-INFINITE ELASTIC SOLID DUE TO AN APPLIED VERTICAL POINT LOAD

5.1 Introduction

In this Chapter the effect of various values of Poisson's Ratio within the zero - 0.5 range on the stresses calculated from Boussinesq's equations as the basic 3-dimensional case, will be outlined. The equations referred to are the equations 4.7-4.10 given in the previous Chapter.

5.2 Stress Calculations and Curves

For the purposes under consideration, the value of the vertical point load Q in Boussinesq's equations was assumed to be 1000 units of force. No specific units of either force or length were adopted, as it was intended to render the results adaptable to any system of units. The units of the stresses calculated are therefore simply : units of force/(units of length)^2, the numerical value of the stress calculated being the same for any units of force and length. That this is the case, can be seen from a consideration of the form of any of Boussinesq's equations, for example, equation 4.7(a) of previously:

$$\sigma_z = \frac{Q}{2\pi} \cdot \frac{3z^3}{(r^2+z^2)^{5/2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4.7(a)$$

Whatever the units of length may be, the numerical value of the term $3z^3/(r^2+z^2)^{5/2}$ will in each case be the same, and has dimensions : (units of length)^2, while the numerical value of the term $Q/2\pi$ is also the same whatever units of force may be used. Hence σ_z , as the product of these two terms, has a numerical value which is independent of the particular units of force and length used. The only effect is that with the later presentation of the results in the form of curves, stress contour diagrams), the scale of any diagram depends on the particular units of length used, but this is of no concern for the purposes considered herein. For this reason also, no scales are given with any of the diagrams.

For each stress component, i.e. σ_z, σ_r, σ_t and τ_{rz}, the values of the stresses have been calculated for a grid of points extending 60 units of length horizontally on either side of the axis of loading of the vertical point load, and to a depth of 60 units of length below the horizontal surface of the semi-infinite solid. Furthermore, for the grid of points just detailed, stresses were calculated for each of the stress components σ_r and σ_t (which contain terms of Poisson's Ratio) using five different values of μ, viz. μ = zero, 0.125, 0.25, 0.375 and 0.50. Stresses were in all cases evaluated by direct calculation from the relevant equations and the results have been presented, as mentioned above, mainly in the form of stress contour diagrams. The actual results of the calculations for each grid of points have been compiled in tabular form and appear in each case on the page immediately before the corresponding curves.

For the Maximum Shear Stress (S_{max}), the standard equation for 2-dimensional cases was used, although the problem being considered is essentially a 3-dimensional one. The reason for this is that the 3-dimensional case has proved to be almost insoluble (as pointed out by Kantey and Marr[20]), and the only known solution is that of Jurgenson[15] for the special case of a uniformly loaded circular area. Even in this case the solution is so complicated that Jurgenson does not give his

formulae, .../

bear a constant ratio to one another, and that the stresses for the condition $\mu = 0$ are a constant 75% of the corresponding stresses for the condition $\mu = 0.375$. Furthermore it is clear that the extreme difference in the stress values as given by the conditions $\mu = 0$ and $\mu = 0.50$ decreases rapidly with depth on the axis of loading.

In the case of the Maximum Shear Stress (S_{max}), the most serious affect of variation in μ on its stress contour diagrams appears to be only in the near vicinity of the horizontal surface $z = 0$, i.e. approximately in the region which is less than 5 units of length below the surface. This can best be seen in Fig.14, in which the stress contour lines for the three conditions $\mu = 0$, 0.25 and 0.50 are shown super-imposed. Fig.15 shows that the variation in values of S_{max} on the axis of loading as a result of variation in μ is comparatively insignificant throughout. A consideration again of the actual numerical values of S_{max} corresponding to the five values of Poisson's Ratio considered (Table VI), shows in the first instance that at any depth on the axis of loading the five stress values bear the same constant ratios to one another, and further, that in the case of the extreme conditions $\mu = 0$ and $\mu = 0.50$, the corresponding difference in the values of S_{max} is only a constant 14.3% at all depths on the axis of loading.

On the whole, therefore, it appears that S_{max} is not very critically affected by variation in Poisson's Ratio throughout the limits of its range for elastic materials. This is a very important conclusion especially as it can be seen from a study of the numerical values of the various stresses at points on the axis of loading (where the greatest stress values occur), that second only to the vertical normal stress σ_z, S_{max} yields the largest stress values and as a shear stress it may therefore be a critical factor in design. Remote from the axis of loading the values of S_{max} are quite frequently larger than the corres- [illegible]ding σ_z values. On the axis of loading the values of S_{max} for the condition $\mu = 0.50$ are 50% of the corresponding values of σ_z, while for the condition $\mu = 0$ this figure is 41.6%.

The horizontal stresses σ_r and σ_t and their extreme sensitivity to variations in the value of Poisson's Ratio are therefore possibly not as important in design as would appear at first sight, and it seems that S_{max} is a more important design factor with, in addition, the important property that its value is not seriously affected by Poisson's Ratio over the complete range of values applicable to elastic materials.

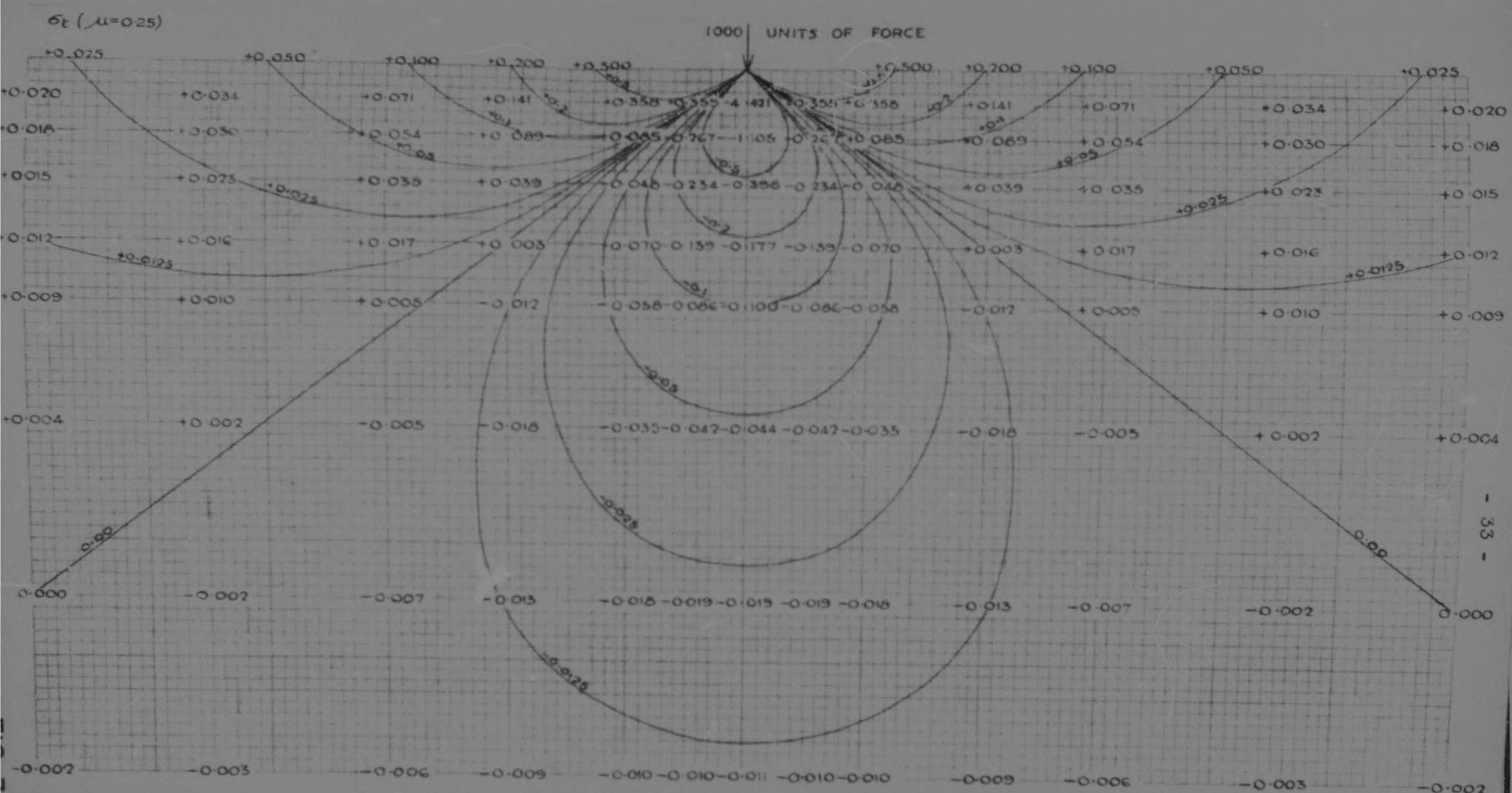


FIG 7 - CONTOURS OF HORIZONTAL TANGENTIAL STRESS (σ_t) FOR CONDITION μ = 0·25 formulae, since "they contain functions of derivatives of the solid angle subtended by the loaded circle at the point under consideration which are of interest to the mathematician only". The use of the formula for 2-dimensional cases is therefore frequently resorted to in 3-dimensional cases, especially as according to Kantey and Marr, it introduces an additional factor of safety which may be as high as 1.25.

As previously discussed (Chapter 4), 2-dimensional cases are either problems of plane stress or plane strain. In the case of plane stress (and referring now to the same orientation of coordinate axes as in Chapter 4), σ_y, τ_{xy} and τ_{yz} are zero, while in the case of plane strain, τ_{xy} and τ_{yz} are zero, but σ_y is not zero. If the normal and shearing stresses acting on the two perpendicular planes containing the OX and OZ axes are combined to give the maximum Shearing Stress, the resulting 2-dimensional formula is:

$$S_{max} = \tfrac{1}{2}\sqrt{(\sigma_z - \sigma_x)^2 + 4\tau_{xz}^2} \quad \text{.............} \quad 5.1$$

For this equation to hold, it therefore does not necessarily signify the assumption that σ_y = zero, i.e. the equation is valid even if the third principal stress is not zero since in the case of plane strain, which is a 2-dimensional problem, σ_y is not equal to zero. Hence also, if the normal and shearing stresses acting on the two perpendicular planes containing the OY and OZ axes be combined, the following 2-dimensional relationship is obtained:

$$S_{max} = \tfrac{1}{2}\sqrt{(\sigma_z - \sigma_y)^2 + 4\tau_{yz}^2} \quad \text{.............} \quad 5.2$$

In the case of the Boussinesq equations of stress, the equations 5.1 and 5.2 given above therefore become (cf. Fig. 4.5, Chapter 4):

$$S_{max} = \tfrac{1}{2}\sqrt{(\sigma_z - \sigma_r)^2 + 4\tau_{rz}^2} \quad \text{.............} \quad 5.3$$

$$\text{and} \quad S_{max} = \tfrac{1}{2}\sqrt{(\sigma_z - \sigma_t)^2 + 4\tau_{tz}^2} \quad \text{.............} \quad 5.4$$

but, since $\tau_{tz} = 0$, equation 5.4 above becomes:

$$S_{max} = \tfrac{1}{2}(\sigma_z - \sigma_t) \quad \text{.............} \quad 5.5$$

Although equation 5.3 (or basically equation 5.1) is the generally accepted expression for determining S_{max}, S_{max} was evaluated from both the equations 5.3 and 5.5 above for the purposes of comparison. Since S_{max} in both equations contains either the stress component σ_r or the stress component σ_t, both of which contain terms of Poisson's Ratio, S_{max} was also evaluated for the five conditions: μ= zero, 0.125, 0.24, 0.375 and 0.50. A tabular comparison of the results is given in Appendix III, Item 1. It was found that equation 5.3 corresponding to the generally accepted equation 5.1 for 2-dimensional cases, gave the larger values of S_{max} in all cases except a few very isolated instances.

On the following pages the results of all the abovementioned calculations and the corresponding curves are presented. These appear in the following sequence:

1. Vertical Normal Stress σ_z: (a) Stress data in tabular form.
   (b) Stress contour diagram.

2. Vertical ..../

CHAPTER 6

CURRENT CONCEPTS OF ELASTICITY AND POISSON'S RATIO IN SOILS WORK

6.1 Applicability of the Elastic Theory to Soils

The basis and validity of the application of the elastic theory to soils has been discussed in full by various Authors (see for example Ref.1, p.208-209: p.250 and p.261-266, and Ref. 8, p.3-4 and p.367), and will therefore not be discussed again herein.

It need only be mentioned again that the theory of elasticity has come into common use for estimating stresses and settlements induced in soil masses by externally applied loads, with the important provision that the induced stresses must be of a moderate order and well clear of the state of stress tending to produce failure conditions if the results are to be of any real significance. In this connection, Terzaghi mentions (Ref.8,p.367) that: "If the factor of safety of a mass of soil with respect to failure by plastic flow exceeds a value of about 3, the state of stress in the soil is likely to be more or less similar to the state of stress computed on the assumption that the soil is perfectly elastic. Hence, the state of stress in a mass of soil under the influence of moderate stresses can be estimated by means of the theory of elasticity. The importance of the error associated with the results of the computation depends chiefly on the extent to which the real stress:strain relations depart from Hooke's Law."

In practice, the concept of elasticity in soils is based on the fact that with most soils the stress:strain curves derived from tests such as triaxial compression tests and field bearing tests, show a certain range, at or near the start, over which stress and strain are proportional. Hence, a soil is said to exhibit elastic behaviour over that range of loading. In terms of the discussion in Chapter 2, the elastic theory is thus strictly only applicable to a soil mass within this so-called linear range of loading (which is usually realised by Terzaghi's proviso of a factor of safety of about 3 against shear failure), and therefore also Poisson's Ratio for the soil mass only has a rational meaning within this range of loading.

In the next Sections of this Chapter, some aspects of Poisson's Ratio in relation to currently accepted analytical and test procedures will be briefly reviewed, while in subsequent Chapters, a determination of Poisson's Ratio from the observed behaviour of a soil within its linear range of loading during a triaxial compression test, will be described and discussed.

6.2 Settlement Analysis for Saturated Clays

In discussing a development of the existing methods of calculating settlements of structures founded on clay, Skempton, Peck and MacDonald(21) commence by describing the principles on which such settlement analysis is based.

It is discussed that the nett settlement (i.e. that resulting from the increase in nett foundation pressure) at the end of construction is made up of two parts: the immediate settlement which results from deformations taking place without change in water content (and hence without volume change in a saturated clay); and the consolidation settlement arising from

extrusion ..../

SKETCH SHOWING ARRANGEMENT OF THE VARIOUS CIRCUITS IN THE TEST SET-UP.

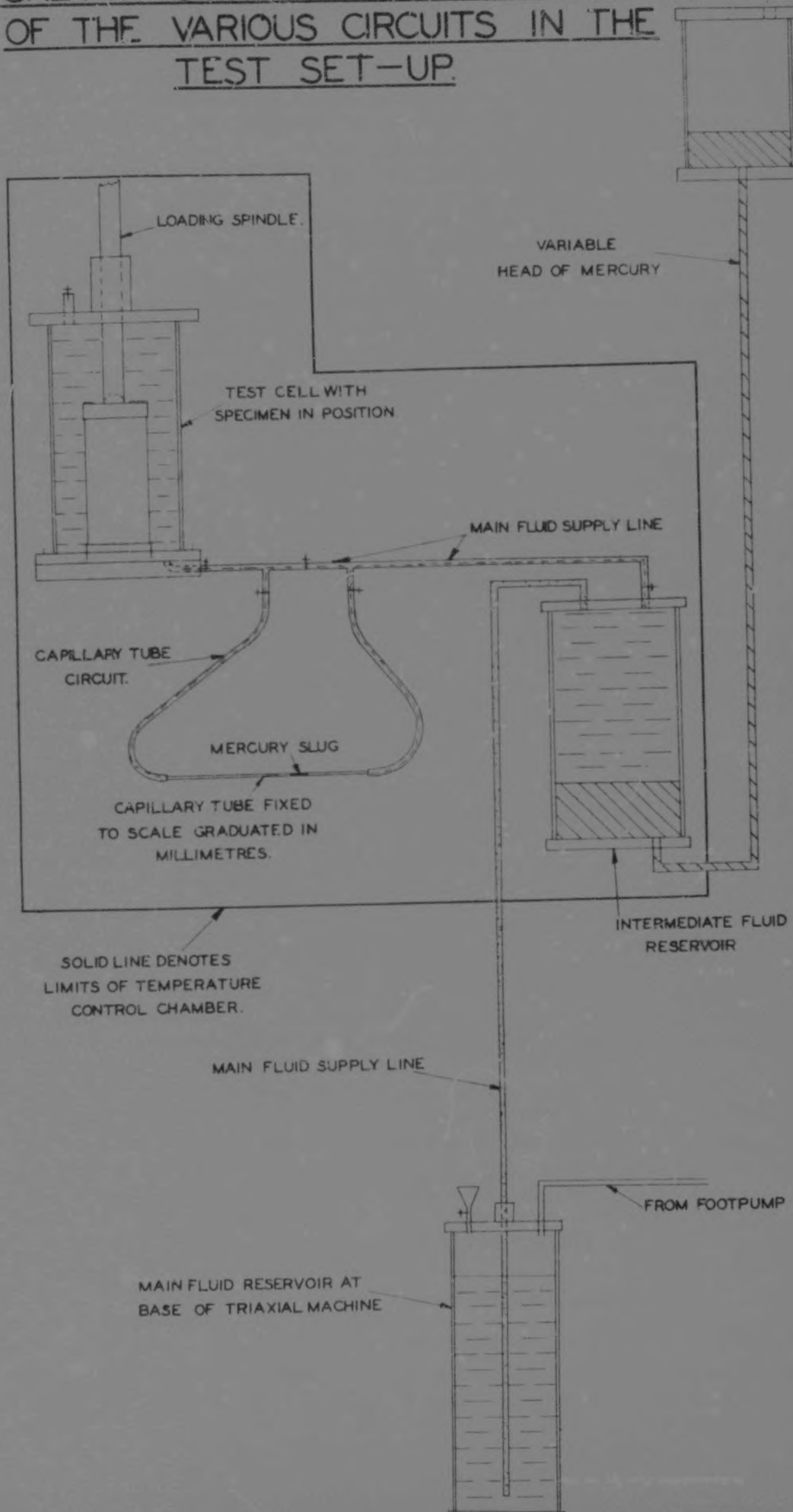


FIG. 10

found most convenient to use a tube of about 32cms. graduated length clear between end connections. In selecting a suitable tube diameter, two main factors had to be considered:

(a) The diameter would have to be small enough to enable the slug to fill the whole bore and not only part of it, otherwise it would leave a gap between its top surface and the top of the capillary tube through which fluid could flow without causing movement of the slug.

(b) The slug movement during test would have to be of a reasonable magnitude, i.e. preferably over as much of the graduated length of the tube as possible, but preferably not more than this.

A suitable balance was eventually obtained by trying out capillary tubes of different diameters in a few trial tests with the test soil.

Due consideration was given to the following matters:-

(a) Air in the test system: As the presence of air in the confining fluid system would adversely affect the flow measurements due to its compressibility it was very important to ensure that all entrapped air be expelled from the system before any test be commenced. It proved to be a very difficult matter to get the capillary tube circuit filled with fluid and its mercury slug in position with all air bubbles expelled from the circuit. However, once this was done at the commencement of the main test series, matters were so arranged by handling the stopcocks, that the circuit would remain filled with its fluid and with its slug in position over the whole series of tests.

(b) Leakage from the test cell: Leakage from the test cell was an undesirable effect for the purposes of this investigation and hence an attempt was made to eliminate it completely. However, it was found that this was not readily accomplished and hence, all that could be done was to reduce the leakage as much as possible by keeping all leakage points well greased, especially where the loading spindle enters the test cell.

(c) Temperature changes during test: Due consideration was given to the elimination of temperature changes during test as this would affect the fluid flow measurements made, and would introduce a further variable into an already complex system of test variables. For this reason the whole test bed was enclosed in a wooden box with glass windows in its access doors and a heating element and fan installed so as to enable a uniform, constant temperature to be maintained within, during test. However, it was later found that temperature conditions inside the laboratory were not very variable and that with the added protection of the wooden box against possible draughts, no significant temperature changes were liable to occur within the temperature control chamber during the comparatively short duration of a test. The heating element and fan were thus never used.

Figs. 20 to 23 inclusive are photographs showing various aspects of the test apparatus.

The test procedure would then be as follows:-

Having positioned the sample in the test cell and with the capillary tube circuit cut out, fluid is pumped from the main fluid reservoir at the base of the machine via the intermediate fluid reservoir and the main fluid supply line into the test

cell, .../

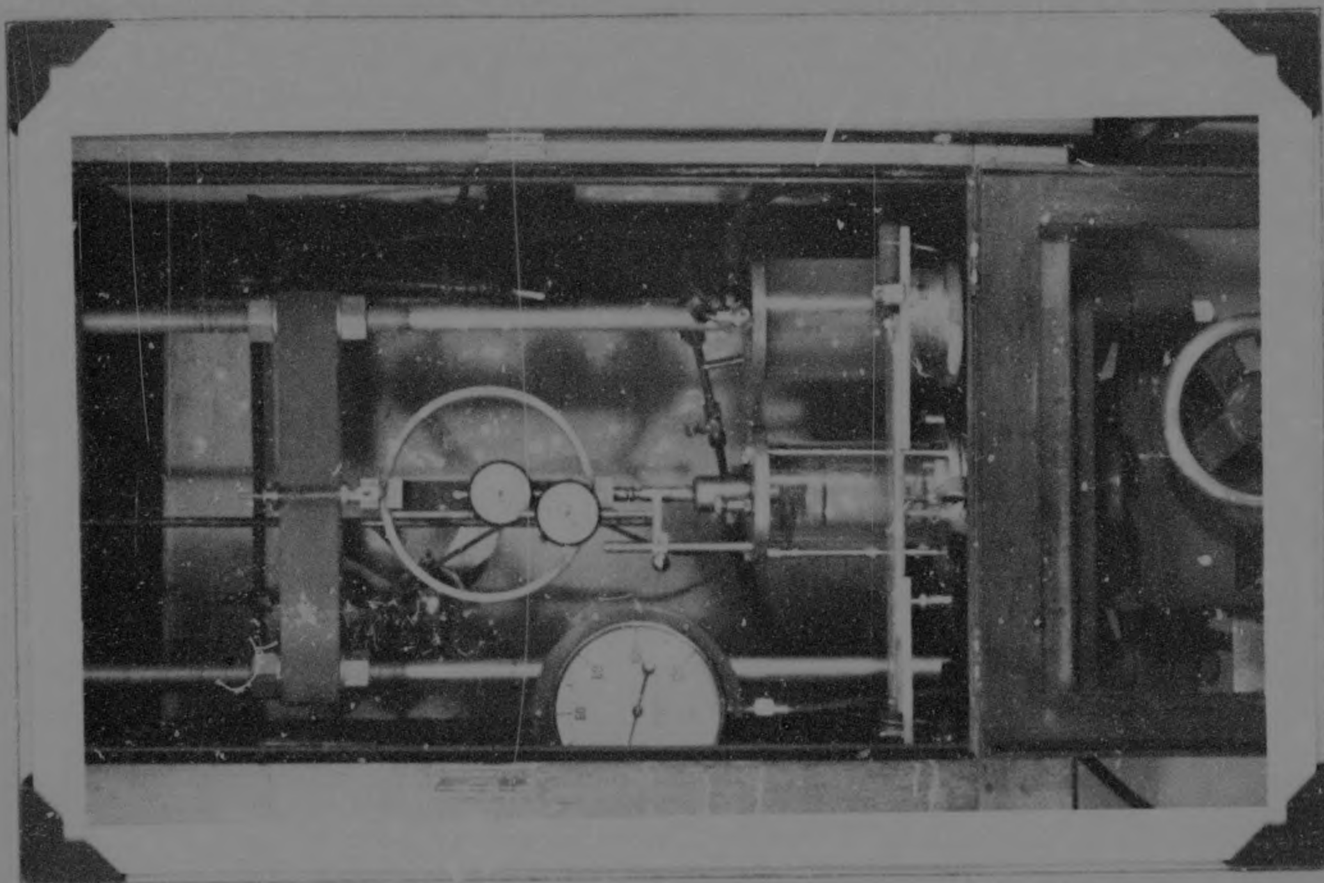


FIG. 21 — ARRANGEMENT WITHIN TEMPERATURE CONTROL CHAMBER

FIG 20 – OUTSIDE VIEW OF TEMPERATURE CONTROL CHAMBER ENCLOSING THE TEST BED

Figs. 20 & 21

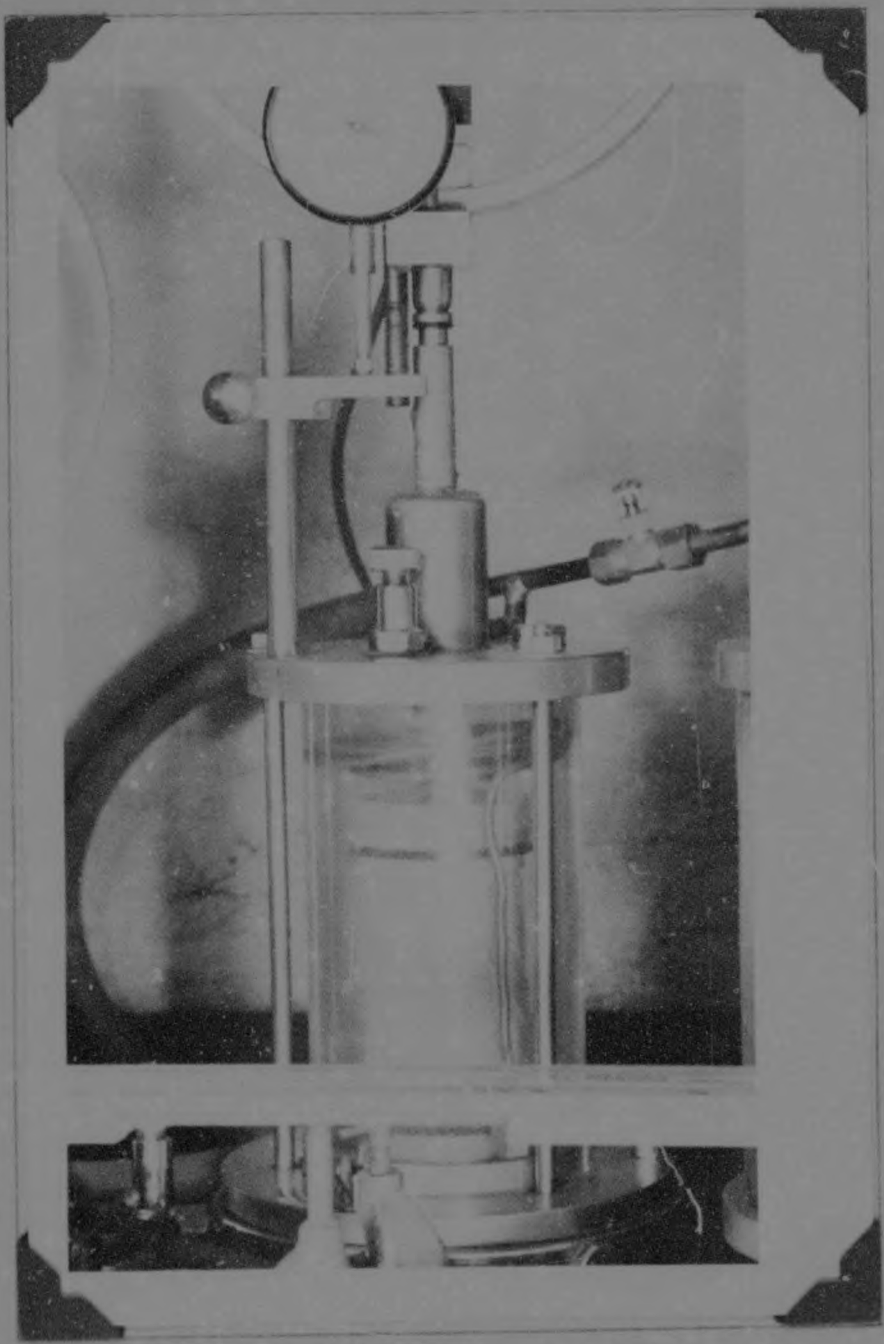


FIG.22 - TRIAXIAL TEST CELL WITH SAMPLE IN POSITION

FIG.23-CAPILLARY TUBE (WITH SLUG) FIXED TO GRADUATED SCALE.

Figs. 22 & 23

cell, as is usual procedure with triaxial tests. Having attained the preselected chamber pressure, as indicated on the pressure gauge, the variable head of mercury is suitably adjusted, if necessary, and an accurate head measurement made.

Immediately prior to testing the sample, the capillary tube circuit is opened as well, so as to produce equilibrium conditions in all circuits throughout the whole test system. Under this condition it is possible to move the mercury slug in the capillary to any desired position along the tube, by simply tilting it up or down at one end as required. Movement of the slug is made possible by the fluid moving round in the continuous closed circuit formed by the main fluid supply line in parallel with the capillary tube circuit. This knowledge serves a very useful purpose in cases where it happens that during a test the slug tends to move off the scale, as by opening the main fluid supply circuit and tilting the tube, the slug can be rapidly reset to the desired position without stopping the test to do so. After the test, each gap in the recording can easily be bridged by plotting several points on either side of a particular gap on a large scale and then, by knowing what the change in deflection during the reset interval had been, arranging, by a little trial and error, that the two sets of points be linked with a common smooth curve which thus also straddles the gap.

At the commencement of the test the main fluid supply line is closed off so that only the capillary tube circuit is cut in, and the following effects take place:

The loading spindle advances downwards into the test cell causing the displacement of cell fluid <u>back</u> into the fluid reservoirs via the capillary tube circuit, thus causing a slug movement from left to right. Superimposed on this, is a movement of fluid <u>into</u> the test cell from the fluid reservoirs via the capillary tube circuit if a decrease in volume of the sample occurs as a result of the loading, (thereby causing a slug movement from right to left), or vice-versa in the event of a volume increase of the sample. Furthermore, superimposed on these two effects is an additional fluid movement <u>into</u> the test cell via the capillary tube circuit, to replace any leakage from the test cell (which usually occurs where the loading spindle enters the cell), thus also causing a slug movement from right to left. Hence, during a test, the slug movement in the capillary tube is thus the nett movement resulting from three combined effects, and in order to determine the slug movement due to volume change of the sample only, it is necessary to separate out the slug movement which has taken place due to the other two effects.

As previously mentioned, slug movement due to entry of the loading spindle into the test cell can be calculated from a knowledge of the diameter of the capillary tube and from a determination of the volume of fluid displaced by the movement of the spindle. The latter can be found from a knowledge of the diameter of the spindle and the distance the spindle has travelled into the test cell, which corresponds to the deflection of the sample as read on the relevant dial deflection gauge.

The value of slug movement as a result of leakage from the test cell during an actual test, cannot be determined directly as during the test it is not possible to eliminate the effects of slug movement due to spindle entry and sample volume change. The only circumstances under which slug movement due to leakage <u>only</u> can be measured is under the steady conditions existing immediately before or after test. This measurement may thus not be truly representative of the actual slug movement due to leakage occurring <u>during</u> a test, as the rate of leakage may be

affected ....../

affected by the disturbed conditions during test. However, as this serves as a means of obtaining some idea of the leakage during test, these measurements were taken and in each case with the full particular chamber pressure acting and only the capillary tube circuit cut in.

To obtain some idea of the leakage under disturbed conditions similar to those existing during a test, a measurement of slug movement due to the combined effects of leakage and spindle entry only was contemplated, from which, by knowing from calculation the value of slug movement due to spindle entry only, the value of slug movement due to leakage only could be determined. As such measurements would only be possible provided no sample volume change effects were present, it was considered most suitable to carry out this determination immediately after completion of a test in the following manner:

With the capillary tube circuit closed off and the main supply circuit cut in, the spindle is raised a convenient height above its previous position on the top end plate in contact with the failed sample. With the relevant chamber pressure still acting and with only the capillary tube circuit cut in, the machine is again started and a series of slug movement : deflection readings taken. This slug movement recording is now inclusive of both spindle entry and leakage effects, but exclusive of sample volume change effects. This combined measurement would be carried out directly after the leakage measurement at the completion of the test, as discussed above.

Finally, to round off these measurements of slug movement due to leakage and spindle entry effects, a further leakage measurement is made immediately after this measurement of slug movement due to the combined effects of leakage and spindle entry.

7.5 <u>The Test Series</u>

As previously mentioned (Section 7.3), the type of triaxial compression test selected for the test series was the Consolidated Undrained Shear Test (S_{cu}) with strain control. In order to obtain some information about the possible affect of rate of strain and chamber pressure on the value of Poisson's Ratio determined for the soil, it was decided that the test series should comprise three tests at a comparatively fast speed, and three tests at a much slower speed. Each set of three tests would then be done at three different chamber pressures which were to be 10lb/sq.in., 20lb/sq.in., and 30lb/sq.in., thus six tests in all. The chamber pressure of 10lb/sq.in corresponded approximately to the overburden pressure of ≈ 13ft. depth of soil, to which the test soil had been subjected in the field.

Two convenient rates of strain, one fast and one slow, were obtained by using suitable gearbox settings. An initial determination of the resulting rates of strain was made by running the machine at each of the selected gearbox settings in turn, and taking deflection:time readings. While recording, the spindle would be entering the test cell against the resistance of the cell fluid only, which was at an arbitrarily selected chamber pressure of 30lb/sq.in. It was realised that the values determined might be dependent on the particular chamber pressure used, and might also be affected by the resistance of a sample to loading during an actual test. Hence, it was decided to refer to them as the <u>nominal rates of strain</u>. For the fast speed this was found to be 0.0125 inches per minute and for the slow speed 0.00256 inches per minute.

As all samples were to be consolidated in the triaxial cell under the respective chamber pressures selected, it was necessary to know within what length of time such consolidation would be completed. Hence, in conducting a consolidation test on an undisturbed sample of the test soil, settlement:time readings were taken in each case immediately after placing the loading increments corresponding to chamber pressures of 30lb/sq.in., 20lb/sq. inches, and 10lb/sq.in., respectively, i.e. approximately 2Ton/sq.ft., 1.5Ton/sq.ft. and 0.7Ton/sq.ft. respectively. These curves are shown on the next page. It was found that 90% consolidation would be reached within less than 24 hours in all cases and hence it was decided to consolidate all samples for about 24 hours before testing.

The following is a description of a typical test performed. A 3" length x $1\frac{1}{2}$" dia., cylindrical sample was cut from an undisturbed block sample of the soil, using the cutting cylinder provided for the purpose. To standardise the volume of all test samples cut, the same cutting cylinder was used throughout. An intermittant check was kept on the initial moisture content of test samples by using cuttings from the particular test sample being trimmed.

The sample was then placed in the triaxial machine and allowed to consolidate with drainage for about 24 hours under the particular chamber pressure selected.

When consolidation was completed, the drainage cock was closed and after noting the temperature inside the [illegible] closing wooden box, a leakage measurement was made by recording slug movement: time readings every 15 seconds for 5 minutes, according to the procedure already described in the previous Section.

The slug was then reset to a convenient position along the scale and the main fluid supply line closed off to leave only the capillary tube circuit cut in. After noting the time and again the test temperature, the test was commenced. Using deflection as a basis, accurate readings of load, slug movement, and time together with intermittant temperature readings were taken over the whole duration of the test, until definite shear failure of the sample had occurred. At the end of the test, the time as well as the temperature were recorded and a sketch of the failed sample made. On occasion, it may have been necessary to reset the slug during a test and this was then done on the lines previously described. In the initial tests, only deflection, load and slug movement were recorded, two observers being required in the case of fast tests. Continuous time recordings were initially not considered necessary, in view of the acceptance of a constant rate of strain during test, and in addition would have required a third observer which was not readily available. However, the need for continuous time recordings throughout any test (see Section 8.3) in order to determine the actual rate of strain was realised once the fact of an uneven rate of strain during test had been noticed, and it was thus necessary to call in the assistance of a third observer. Although time readings were thus not taken in some tests, it was, however, possible to utilise the information obtained from other tests in which time readings were taken to cover such cases.

Immediately after completion of the test and with the sample and loading spindle in the same position as when the machine was stopped, the slug in the capillary was reset to a convenient position and a leakage measurement made similar to the one carried out immediately before the commencement of the test. Temperature and time were appropriately recorded.

This ..../

SETTLEMENT--TIME CURVES

(FROM NORMAL 1-DIMENSIONAL CONSOLIDATION TEST.)

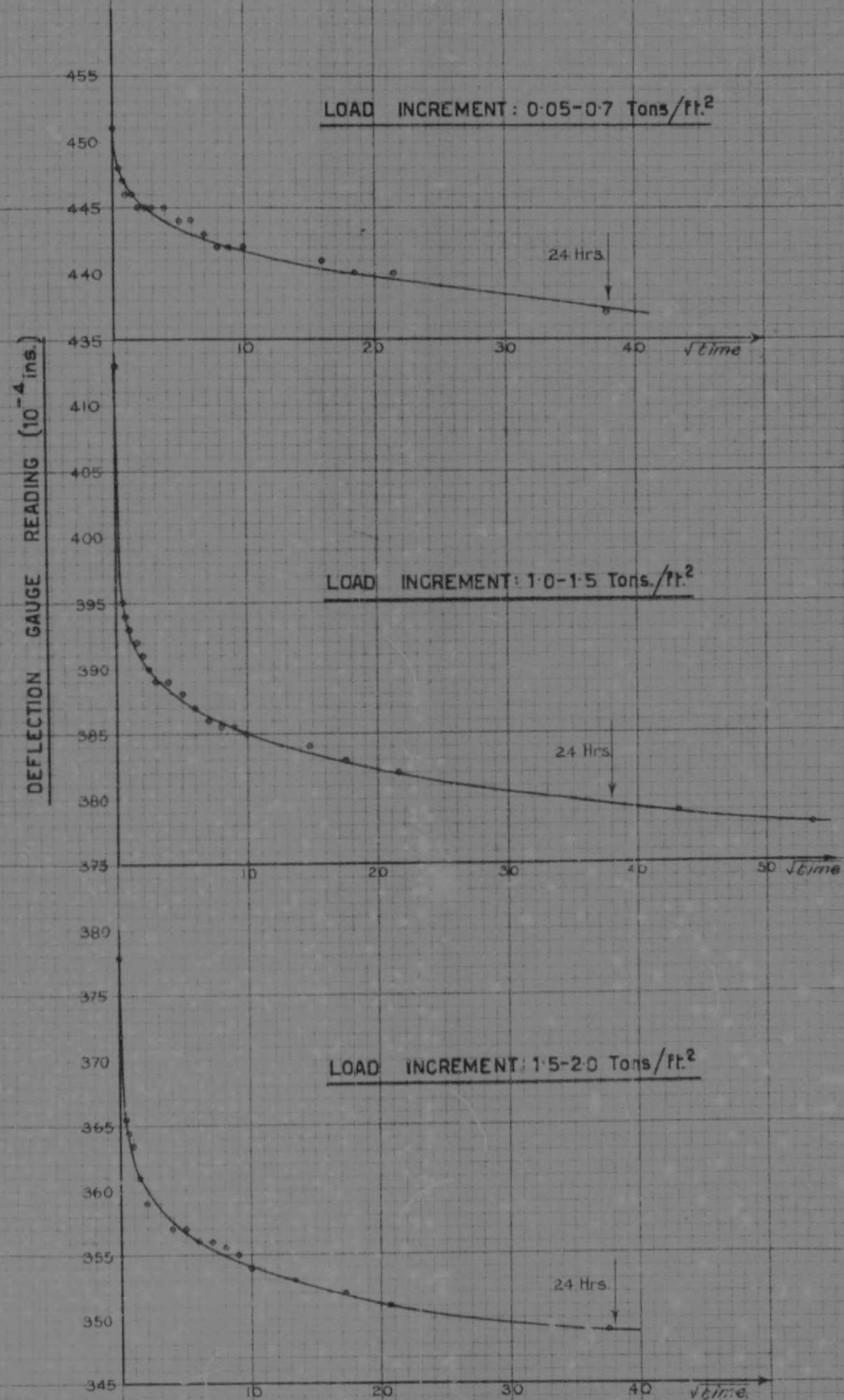


FIG. 24