

**CHANGES MADE TO THE FIRST PHD SUBMISSION ENTITLED AN
INVERSE PROBLEM FOR AN INHOMOGENEOUS STRING WITH AN
INTERVAL OF ZERO DENSITY AND A CONCENTRATED MASS AT THE END
POINT.**

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The purpose of this document is to list all the changes that have been made on the first submission of the PhD thesis, some of which are mathematical and others are non-mathematical such as sentence reconstruction just to improve readability of the thesis.

- (1) Changed the Abstract and Introduction.
- (2) Page 4 up to page 6, we improved the English, no mathematical changes.
- (3) Page 8, improved on the English and no mathematical changes.
- (4) Page 9, we capitalised Theorem 3.1 on line 11 from the bottom and improved the English.
- (5) Page 10, capitalised "I" in Theorem 3.3 (the Identity theorem).
- (6) Page 12, used the dots format to show a range of indices just before and after equation (4.1), equation (4.2) and equation (4.3). Globally we adopted the correct dots format to show a range of the index. Changed $\frac{\cos \lambda a}{\lambda^2}$ in the last term of the integration to $\frac{\cos \lambda t}{\lambda^2}$ on Theorem 4.1.
- (7) Page 13, a slightly improvement of the English, no mathematical changes.
- (8) Page 14, combined Corollary 4.3 which was in the first PhD submission with Lemma 4.2.
- (9) Page 15, indexed the zeroes λ_k of the function E_1 with the set H on line 11 from the bottom. The same indexing set was used for the zeroes of the function E_2 and E_3 .
- (10) Page 15, changed maxima and minima to maximum and minimum on line 4 from the top.
- (11) Page 15, changed order $o(1)$ to $O(1)$ in the first statement of the proof of Theorem 4.3.
- (12) Page 15, 6 lines from the bottom changed Theorem 4.1 to Lemma 4.2.
- (13) Page 17, did not change the factor $\frac{L_1^2 - L_2^2}{\lambda_k^2}$ to $-\frac{L_1^2 - L_2^2}{\lambda_k^2}$ on line 3 from the top, disagree with the referee on the suggestion.
- (14) Page 17, removed the unnecessary 2 on both sides of the equation (4.8).
- (15) Page 19, reformulated the statement of Lemma 4.5.
- (16) Page 20, 4 lines from the top, changed the $kh_k = O(1)$ from $kh_k = O(\frac{1}{k})$ and changed Lemma 4.6 to Proposition 4.4 on the same line.
- (17) Page 21, changed the English for better reading, no mathematical changes.
- (18) Page 25, added $O(\frac{1}{k^4})$ on line 5, 7 and 9 from the top.
- (19) Page 27, moved the i from Q_5 to Q_4 on the asymptotic expansion λ_k on line 12 from the top.
- (20) Page 28, second line from the bottom, attached K_1 in the denominator.

- (21) Page 30, 9 lines from the top, changed K_1^{10} to K_1^2 in the denominator for terms in the square brackets.
- (22) page 33, 9 lines from the bottom, i changed $\chi(\lambda_k)$ to $\psi(\lambda_k)$.
- (23) Page 39, added brackets inside the integral of equation (4.15) and equation (4.16).
- (24) Page 40, improved the English but nothing mathematical was changed.
- (25) Page 41, changed the definition of M_1 and M_2 in equation (5.20) and (5.21), and adjusted equation (5.19) to align with the new definition of M_1 and M_2 . All instances of M_1 and M_2 from this page onwards in the thesis were replaced with M_1^{-1} and M_2^{-1} as a result of the new definition.
- (26) Page 42, 11 lines from the bottom changed $W_2^2(0, a)$ to $W_2^2(0, l_1)$, $L_2(0, a)$ to $L_2(0, l_1)$ and $L_\infty(0, a)$ to $L_\infty(0, l_1)$ in the same area.
- (27) Page 44, an explanation of why equation (5.31) is called a characteristic equation is provided from line 11 from the top.
- (28) Page 46, removed the double instance of belong on the first line.
- (29) Page 47, re-worded the first statement of the proof of Proposition 5.5.
- (30) Page 48, improved sentence construction, and no mathematical changes.
- (31) Page 48, used the sequence $\{\lambda_k : k \in \mathbb{Z} \cup \{0^+\}\}$ in Theorem 5.6 for the spectrum from this page onwards.
- (32) Page 50, deleted the constants $K_1, i = i, 2, \dots, 8$ and merely referenced them after the function $\varphi(\lambda)$ in Theorem 5.7 on line 8 from the bottom.
- (33) Page 51, the second line of Theorem 5.8 replaced asymptote with asymptotics.
- (34) Page 51, moved i from Q_5 to Q_4 in the asymptotics of Theorem 5.9.
- (35) Page 52, deleted the constants constants $K_1, i = i, 2, \dots, 11$ and merely referenced them after the function $\varphi(\lambda)$ on line 18 from the bottom.
- (36) Page 52, replaced K_1 with K_2 on the third line from the bottom.
- (37) Page 53, replaced K_1 with K_2 on equation (5.36).
- (38) Page 53, changed definition of M_3 and M_4 on the second and fourth line from the bottom.
- (39) Page 54, changed the sign from $+$ to $-$ in front of the coefficient $P_1^2(P_4 + P_1P_2)$ located on the 11'th line from the top.
- (40) Page 55, replace M_3 with M_4 in the definition of the domain $D(A)$ on line 16 from the bottom.
- (41) Page 56, 16 lines from the top, attached i to the term $P_1 \frac{\cos \lambda a}{\lambda}$.
- (42) Page 56, the suggestion that the estimate on line 22 from the top should be replaced by $e^{|\lambda|a}$ is not implemented since the estimate is bigger than the estimate in the thesis.
- (43) Page 56, changed Definition 6.2 to start as *A function*
- (44) Page 57, changed the statement of Proposition 6.3 to now say ".....zeroes of a function S of sine type $\leq \sigma$ belonging to $L_p(-\infty, \infty)$".
- (45) Page 57, inserted i to the middle term on line 8 from the bottom.
- (46) Page 58, fixed the definition of the constants P_j on line 14 from the bottom.
- (47) Page 58, inserted the formula $\chi^0(\lambda) = \frac{\Psi(\lambda)}{K_1}$ on the second line from the top and therefore deleted equations (6.12) up to (6.16) from the first submission.
- (48) Page 58, added tilde on top of all the constants $Q_i, i = 1, 2, \dots, 5$ on the last equation.
- (49) Page 59, added tilde on top of the Q_2 and Q_4 located on the fourth line from the top.

- (50) Page 60, improved the English, added the superscript 0 to λ_k to get λ_k^0 on the 5th line from the bottom.
- (51) Page 61, added the complex conjugate on all λ in the denominator in line 3 from the bottom and improved the English.
- (52) Page 62, added the complex conjugate on λ_k^0 located on line 8 from the bottom.
- (53) Page 63, replaced $\lambda^{(0)}$ with λ^0 on line 4 from the top.
- (54) Page 63, added superscript 0 to get $\text{Im}\lambda_k^0$ on line 12 from the top.
- (55) Page 63, improved the English and added the superscript 3 to get $\frac{\psi(\frac{2\pi k}{a})}{(\frac{2\pi k}{a})^3}$ on line 10 from the bottom.
- (56) Page 63, added the term $\frac{\psi(\frac{2\pi}{a}(k+\frac{1}{4}))}{(\frac{2\pi}{a}(k+\frac{1}{4}))^3}$ on line 7 from the bottom.
- (57) Page 63, deleted k to get $\chi(\frac{2\pi}{a}(k+\frac{1}{4}))$ from $\chi(\frac{2\pi k}{a}(k+\frac{1}{4}))$.
- (58) Page 64, added $-i$ just after the $=$ on line 4 from the top.
- (59) Page 64, added $-i$ just after the $=$ on line 5 from the top.
- (60) Page 64, changed the $-$ to $+$ on line 10 from the top.
- (61) Page 64, deleted the term K_5 to get $\frac{K_2 K_7 + K_1 K_9}{K_2}$ on line 11 from the top.
- (62) Page 64, deleted the calculation just after Definition 6.7 since they were not used in the thesis.
- (63) Page 65, improved the English in the beginning of the proof of Lemma 6.8.
- (64) Page 65, changed K_4 to K_5 to get $\frac{K_2 K_3 + K_1 K_5}{K_1^2}$ on the last line.
- (65) Page 66, 8 lines from the top, changed k to $-k$ in λ_k .
- (66) Page 66, rewrote the entire proof of Corollary 6.11.
- (67) Page 70, deleted the extra -1 in the denominator on line 3 and 4 from the top.
- (68) Page 70, replaced $\frac{P}{Q}$ with $\frac{Q}{P}$ on the second line from the bottom, and also on line 5 from the top.
- (69) Page 70, replaced $\bar{\zeta}$ with ζ on line 15 from the top.
- (70) Page 70, delete typo R after the expression $M_3 - M_4 > 0$ on the last line of the page.
- (71) Page 71, changed Theorem 1 of [7] to Theorem 1 of [8] on line 5 from the top.
- (72) Page 71, changed the superscript $-$ to $+$ to get b_k^+ on line 14 from the top.
- (73) Page 72, deleted the extra $+$ before the term $+iK_9...$ on line 10 from the top.
- (74) Page 72, deleted the extra i which was on the numerator of the first term on line 19 from the top.
- (75) Page 72, improved the English.
- (76) Page 73, changed the function $u(z)$ to $y(z)$ on the second line from the top.
- (77) Page 73, changed the formula for L on line 8 from the top.
- (78) Page 73 changed the formulary for $\tilde{\varphi}$ on line 13 from the top.
- (79) Page 74, corrected the definition of the function $\tilde{\varphi}$ on line 6 from the top and corrected the definition of $\tilde{g}$ on line 8 from the top.
- (80) Page 74, improved the English.
- (81) Page 74, changed 2λ to λ^2 in the term $-iM_3....$ on line 6 from the bottom.
- (82) Page 74, changed 2λ to λ^3 on them $+M_3...$ on line 6 from the bottom.
- (83) Page 74, changed $2\lambda^4$ to $K_2\lambda^4$ in the term $+iM_4....$ on line 5 from the bottom.
- (84) Page 75, added the term $\cos \lambda a$ on line 12 from the bottom.
- (85) Page 75, improved the English.
- (86) Page 76, changed the variable of integration on equation (6.28).
- (87) Page 76, changed the definition of A on line 11 from the bottom.

- (88) Page 77, changed the definition of $g(\lambda)$ on line 15 from the top.
- (89) Page 77, changed the formula on equation (6.32) and on the line above equation above (6.32).
- (90) Page 78, changed formula on line 9 from the top.
- (91) Page 78, improved the English.
- (92) Page 79, changed Corollary 6.12 to Corollary 6.16 on line 10 from the bottom.
- (93) Page 79, improved the English.
- (94) Page 79, changed inequality on line 6 and line 8 from the bottom.
- (95) Page 80, improved the English.
- (96) Page 82, inserted $d\lambda$ on line 18 and line 20 from the top.
- (97) Page 86, corrected the asymptotics on line 8 and 9 from the top.
- (98) Page 87, corrected the domain $D(A)$ in Corollary 6.23 which was previously called Theorem 6.23.
- (99) Page 87, changed the proof of Corollary 6.23.
- (100) Page 88, added the superscripts -1 and the formula for M_1 and M_2 on line 15 and 16 from the top.
- (101) Page 88, deleted M_1 on line 22 from the top.
- (102) Page 88, replaced M_3 with M_4 on the definition of domain $D(A)$ on line 6 from the bottom.
- (103) Page 89, improved the English.
- (104) Page 89, corrected the formula for $x(s)$ on line 7 from the bottom.
- (105) Page 90, changed the superscript from -1 to -2 on line 10 from the top.
- (106) Page 91, added motivation for equation (6.12) from line 3 to line 6 from the top.
- (107) Page 91 to 92, improved the conclusion.
- (108) Page 93 to 94, change the format of the bibliography.