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An investigation of the constitution of the legitimate text and opportunities to learn number pattern in Grade 11

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An Investigation of the Constitution of the Legitimate Text and Opportunities to Learn Number Pattern in Grade 11

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A research report submitted to the Faculty of Science, in partial fulfilment of the requirements for the degree of Master of Science University of the Witwatersrand, Johannesburg.

Johannesburg

10 October 2011

DECLARATION

I declare that this research report is my own work. It is submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.



Signature

Date: 10 October 2011

ABSTRACT

This study was concerned with the constitution of the 'legitimate text' - a key construct in Basil Bernstein's (2000) theory of the pedagogic device. The question the study sought to understand was: what is constituted as the legitimate text across the mathematics education literature, the official curriculum document, in the official assessment texts, and in the textbook used in the classroom observed for the topic of number pattern. These sources were compared with what was constituted as the legitimate text in a sequence of five lessons based on number patterns in Grade 11 in an inner city school.

This was a qualitative case study, the methodology of which was framed by Bernstein's theory which explains the sociological nature of knowledge, the implicitness and explicitness of the communication for the acquisition of the legitimate text and hence opportunities to learn. One teacher was observed while teaching number pattern to a G11 class in an inner-city high school in Johannesburg in South Africa. A sequence of five lessons was videotaped and transcribed. The documents were analysed. One broad evaluative event with numerous sub-events called input objects were used to chunk the data into more manageable units of analysis. A framework emanating from the literature and from the analysis of the curriculum was used to present and categorise the legitimate text from the documents and the classroom. Kieran's (2007) model of school algebra was used to do the analysis as well as Dowling's (1998) model of domains of practice.

The results of the study showed that the documents did not align with each other in terms of what they constituted as the legitimate text. It was found that the teacher aligned with the curriculum document. The results revealed that the teacher preferred working with numeric contexts. The consequence of this misalignment was that the documents created an additional work load for the teacher to understand and interpret them (documents).

Dedication

In memory of my beloved mother, Gciniwe Luxomo

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ABBREVIATIONS

AS	Assessment Standards
CAPS	Curriculum Assessment Policy Statements
CIE	Computer Intensive Environment
DoE	Department of Education
GET	General Education and Training
GTG	Generational Transformational Global/Meta
G10	Grade 10
G11	Grade 11
G12	Grade 12
FET	Further Education and Training
ID	Instructional Discourse
LO	Learning Outcome
NCS	National Curriculum Statements
NCSM	National Curriculum Statements for Mathematics
NSC	National Senior Certificate
ORF	Official Recontextualisation Field
PD	Pedagogic Device
PRF	Pedagogic Recontextualisation Field
RD	Regulative Discourse

Chapter 1 Introduction

1.1 Introduction

In South Africa, mathematics education has received a lot of attention and a lot of studies have explored or investigated the problems related to poor performance that persists from year to year. In view of this, the South African context is also of interest to researchers/teachers for a number of reasons. To mention two; the first is its history as a country emerging from apartheid and secondly the many changes that have taken place and continue to take place post-apartheid SA in terms of curriculum reform. The curriculum changes have been put in place as a means towards solving the problem of poor performance. It is within this context that I saw a need to pay attention to what Bernstein (2000) calls the *legitimate text*.

This study is about the constitution of knowledge in mathematics, specifically in the curricular materials and in the classroom. In this chapter, I begin with a discussion of the problem statement, the critical questions and the rationale for my focus. I show that the study is located theoretically within the domain of sociology of education, and in particular, Basil Bernstein's theory of the pedagogic device which contains his notion of the *legitimate text*, is the key theme of my study. The data collected (and analysed) in this study was drawn from four sources: the official National Curriculum Statements (NCS) for FET mathematics, the official national assessments (these are the external examinations at Grade 12 level) and the textbook that was used in the classroom in which empirical data was collected. The empirical evidence is drawn from a sequence of five lessons from an inner-city school based on number patterns in Grade 11 (G11). In order to locate the study within the field, I drew on the research literature relating to number patterns. The structure of the thesis will be outlined at the end of this chapter.

1.2 Background

International and national research has shown that learners' opportunities to learn have been compromised in many ways (Reeves & Muller, 2005; Venkat, 2010). Within the context of South Africa mathematics results at Grade 12 (G12) speak for themselves and performance in the TIMSS-99 (Trends in International Mathematics and Science Study),which is an international comparative test, also testify to this calamity (Howie & Plomp, 2005). The question this study seeks to explore is what opportunities to are made available for acquiring the 'legitimate texts'

related to number patterns are presented by the teacher in the process of teaching number patterns to G11.

The grounds for conducting this study are based on the work done by Shulman (1986) about two decades back and his strong argument about the role and importance of the teacher in the process of learning, particularly the quality of learning. The concern here is the teaching process and hence, the learning opportunities made available to learners. The quality of the teaching process and the interaction between learners and teacher are viewed as critical to learning and so, this study focuses on the teaching rather than the learning based on the reasons mentioned above. Research has shown that in the classroom the object of learning is often referred to without paying attention to the grounds that serve as foundations for the object (Davis2010). This lack of specific and deliberate attention to the mathematical object of learning is problematic because learners' opportunities to produce the legitimate text in oral and written forms are compromised.

The theoretical framework for the study is drawn from Basil Bernstein's (2000) theory of the sociology of knowledge and pedagogy – his theory of the 'pedagogic device'. The pedagogic device is a mechanism for describing the constitution of knowledge in pedagogic contexts. The pedagogic device is composed of a system of rules; the rules are about the social distribution, recontextualisation and evaluation of knowledge. Bernstein (2000) argues that the whole purpose of the pedagogic device is 'condensed in evaluation'. What this means is that successful pedagogy lies in the individuals that are being evaluated being able to produce the 'legitimate text' – what counts as valid knowledge in the curriculum/classroom. In this formulation, the reason why a lot of learners, specifically in South Africa, are not successful in school is because they are unable to produce the legitimate text, and hence it was necessary for me to problematise the notion of the 'legitimate text'. Legitimate text is anything that attracts evaluation and it does not necessarily have to be written, it can be an action or verbal. It is the social evaluation of what counts as correct/acceptable to do, say or write. The unit of analysis that was used to chunk the data from the transcript is called the *evaluative event* (Davis, Adler, Parker & Long, 2003) and it has its theoretical rationale resting in Bernstein's theory of the pedagogic device, particularly the significance he places on the evaluation (that evaluation is the whole purpose of the device).

Therefore, this study seeks to find out and compare what is constituted as the 'legitimate text' (Bernstein, 2000) in some of the key curricular materials and the classroom and what

opportunities for learners discerning number patterns are made available. According to Bernstein (2000), legitimate text refers to what the context has legitimised and learners are supposed to recognise and realise, when realising, that means they (learners) can produce the legitimate text. The underlying assumption is that when teachers have a sound knowledge of (a) the content (b) the curriculum and c) assessment, then the teacher is in a better position to elaborate and pay more attention to the objects of learning and its foundations and hence give learners access to recognition and realisation rules of the legitimate text.

Literature on number patterns has been reviewed and mathematics education research views this topic as the 'heart and soul of mathematics'(Zazkis & Liljedahl, 2002). Driscoll (1999) views this topic is a proper bridge for early grades into algebra and the process of generalisation should be made a 'habit of mind'. Mathematics education researchers (Kieran, 2007; Usiskin, 2004; Watson, 2009) have shown that the process of generalisation, which is the key action in activities based on number pattern, underpins the whole essence of doing algebra, specifically, and mathematics in general. Consequently, mathematics education literature on number pattern provided a vantage point from which I could read and analyse the documents and the teachers practice.

1.3 Problem statement

The 2009 Senior Certificate Examinations report (DoE, 2009c) shows that mathematical attainment throughout the country (South Africa) was poor, especially for learners coming from disadvantaged communities. The problem is not sluggish teachers because performance in this exam dropped generally for most schools. Most state schools after the 2009 exam were declared to be underperforming because they received a percentage pass rate of less than 60% (DoE, 2009c). The problem is not a lack of resources because what is mentioned here includes the so called well-resourced schools with highly qualified and experienced teachers in mathematics education. The problem generally is with the quality of education and around the notion of quality of education there are many variables. I have decided to focus on mathematics instruction and the attention given to mathematical objects and their elaboration. Seeing that the mathematical objects and their thorough explication is vital for the acquisition of the legitimate text I therefore have explored and analysed the constitution of the legitimate text in the curricular

documents first and secondly in the classroom and what opportunities for learners discerning number patterns were made available.

The objectives of the study were to find out from the official national curriculum, the official national assessment texts and the mathematics textbook used in the classroom studied, what is constituted as the legitimate text for the topic of number patterns. This information served as a basis for comparing what was constituted as the legitimate text for the topic of number patterns in a sequence of five lessons in the classroom. Below are the critical questions that framed the study and the research question that summarises the study.

1.4 Research Question

This study can be summarised as: An investigation of the constitution of the legitimate text and opportunities to learn number pattern in Grade 11.

1.4.1 Critical questions

1. What is constituted as the legitimate text for the topic of number patterns across:
    a) The key 'official' curriculum (NCS-National Curriculum Statement)
    b) The National Assessments (Matric papers 2008 and 2009)?
    c) Within the mathematics textbook that was used?
2. What is constituted as the legitimate text by a teacher within a sequence of lessons focused on number patterns in Grade 11?
3. What is the relationship between what is constituted in the classroom and what is constituted in the official curriculum and assessment texts as well as in the mathematics textbook that was used in the classroom?
4. What opportunities for learners acquiring number pattern are made available?

1.5 Rationale for focus and critical questions

The focus of this study is to understand the teaching process a teacher in South Africa embarks on to teach number patterns to a G11 class. The usefulness of this focus is backed up by national and international evidence that has shown teaching to be of critical importance to what is opened up mathematically for learners (Adler2008; Ball, Bass, & Hill, 2004).

Why have number patterns been chosen? Number pattern is a topic that stays in the curriculum from Grade 1 up to G12. The process of generalisation is one of its key mathematical practices. This process underpins the whole essence of learning algebra in particular and mathematics in general. Why is the focus on the G12 exams? The reason for this is that the content from Grade 10 (G10) and G11 is assessed in the official national G12 examinations and that is why it seemed good to look at G12 official national assessments, the G12 official exams have a particular significance and a high status because they provide the high school exit certificate which is a gate-keeper to institutions of higher learning. These are 'high stakes' examinations as learner performance in these exams determines whether and what future studies the learner would be allowed to pursue. Furthermore, the education department also uses learner performance from these assessments to rank schools. It goes without saying therefore that the assessment at national level is a projection of the legitimate texts/forms of knowledge learners are expected to have acquired. Why G11? There was opportunity to engage with a G11 teacher, this is elaborated further in the sampling section in the methodology chapter. Why the curriculum and across the FET (Further Education and Training – that is Grade 10, 11 and 12) when only looking at G11? Curriculum policy is seen as a key factor in showing the direction for teaching and learning and Parker (2006) says that school curriculum documents show symbolic images of what the state regard as valuable knowledge and forms of transmission of that knowledge for schooling. Mathematics is a progressive discipline and the current curriculum is designed and segmented according to phases, and so it made sense for me to look across the FET phase to answer the question of what is the legitimate text.

1.6 Outline of thesis by chapter

In this chapter (chapter 1) I have introduced the study by giving some background information, the research questions and their rationale. I have located the study in the field by stating the literature and the theory that informs the study and now I give the outline of the thesis.

Chapter 2 contains the literature review. In this chapter I give a report on the survey I have made on studies based on number patterns. I present the key ideas related to number pattern in mathematics education and the content/processes described, and why number pattern is viewed as important.

In Chapter 3 I discuss the theory that underpins the study which is Bernstein's (2000) theory of the pedagogic device. From the pedagogic device I am interested in the notion of evaluation which is central and the whole purpose of the device. For anyone to succeed in the evaluation they have to be able to produce the legitimate text. The notion of the legitimate text is a technical term within Bernstein's theory and does not necessarily refer to written text, but anything which attracts evaluation. It can be verbal, an act or written. The legitimate text is the social evaluation of what counts as legitimate to do, say or write within a particular community of practice.

In Chapter 4, I discuss the general methodology of the study and discussed in detail how the data for classroom observations were collected, organised and analysed.

In Chapter 5 a detailed documentary analysis for the curriculum document across the FET phase for the topic of number pattern was done. The Grade 12 national official assessments for the years 2008 and 2009 were analyzed. The last document that was analysed is the textbook that was used in the classroom which is the most dominant textbook used in South African classrooms and that is Classroom Mathematics Grade 11.

Chapter 6 is a discussion of the classroom data and the legitimate text for the teacher emerges from the actions, explanations and questions the teacher employed during the course of the five lessons. This legitimate text was then contrasted with what was constituted as the legitimate text in the documents using Dowling's (1998) model of domains of practice, Kieran's (2007) GTG model and the literature review/curriculum framework. In Chapter 7, the final chapter, I discuss the findings, the implications these findings have for policy and practice as well as the limitations and recommendations.

1.7 Conclusion

This chapter has located the study in its theoretical and empirical fields. I have also outlined the research questions that frame the study and rationale. At the end of this chapter I have given the structure of the thesis. I now move on to discuss the literature on number patterns.

Chapter 2 Literature Review

2.1 Introduction

I begin this chapter with an introduction of how 'pattern' is described within mathematics I then move on to look at the literature, where two things are focused on namely: the nature of school algebra and studies conducted by mathematics education researchers on the topic.

In mathematics, a numerical or special arrangement is described as 'patterned' if it displays 'regularity' of some sort. From just playing with number mathematicians made many discoveries in mathematics. In fact, mathematics, in general, has been described as the study of patterns, observing patterns in algebra is a natural tendency for some people and is one approach that is used to address difficulties in mathematics because expressing generality is seen as an indication of understanding (Watson, 2009). Patterns are regularities or similarities that describe sets of numbers. It is not only Mathematicians who look for patterns in their work but scientists and engineers perform this function of pattern looking to explain and describe phenomena. Number pattern provides a context that exhibits structure and regularity and hence provides rich opportunities for studying mathematics and solving problems (Watson, 2009). Number pattern is believed to be a context that enhances the beginning of mathematical thinking (Kieran, 2007) which is described as beginning with recognition of similarities among objects, and from here mathematical thinking proceeds to making generalisations and abstraction. These processes described by Kieran (2007) are stages that a learner needs to go through when involved in activities based on this topic. The investigation of pattern helps learners understand the concept of constant growth as they analyse sequences like 1, 3, 5, 7, ... Learners can contrast this type of change with other relationships such as 1, 2, 4, 8, ...; or 1, 3, 6, 10, 15, ... The study of patterns provide learners with opportunities to observe, and from this they can explain the difference and describe the underlying structure (Watson, 2009). Since mathematics is described as a language through which we express generality – the ability to generalise and abstract from particular cases – therefore, understanding and mastering the processes involved in patterns and the use of mathematical representations to describe patterns is important for general understanding of the discipline itself (Kaput, 1989).

In the school curriculum, discussed in Chapter 5, number patterns were examined as part of algebra in Learning Outcome 1. Literature has reported on what school algebra entails and will be discussed briefly so as to build an understanding of school algebra from the perspective of the literature.

2.2 What is school algebra?

From engaging with literature that defines school algebra two key aspects emerged: that it is a language of expressing generality, and an activity

2.2.1 Language of expressing generality

For Usiskin (1988), algebra is a language used for generalising in mathematics. For Watson (2009) algebra is the way generalisations about number, quantities, relations and functions are expressed and therefore good understanding of these is positively correlated to success in algebra. Vermeulen (2007) when discussing the notion of school algebra says that it (the symbol system) is used to represent deep yet simple structures.

One of the ways in which Usiskin (2004) describes the concept of school algebra is that algebra is 'generalised arithmetic'. To some extent this is true because there is a tendency amongst people not to believe that learners are doing algebra up until some letters and symbols are seen in what is written. Empirical research reviewed by Kieran (2007) shows that learners have difficulties with algebra because they are still caught up with using the methods of arithmetic, which are to calculate without looking at relations which is the focus of algebra (Kieran, 2007). However, I agree with Watson (2009) when she says to think of algebra as generalized arithmetic is to think of algebra as growing directly from arithmetic which is misleading and overly simplified.

What comes through from all the researchers mentioned is that algebra is a symbolic system, a language used for expressing relations, written or verbal between quantities. To emphasise this point more, Kaput (1989) says that algebra is a sign system from which we express reasoning about relations. Hence, algebra engages learners with the grammar of the sign system. Kaput says learners are to learn algebra as a language and have to learn to use it to communicate meaningful statements. This is true because without an understanding of the rules of the

language of algebra, legitimate communication is not possible. For example, letters are used in different ways in different contexts and learners need to know and be able to distinguish between letter as an unknown in an equation: $2n+1=4$, as unit of measurement: $36ns$ (ns denotes nanoseconds), as variable in the formula: $T_n = 4n+1$ with n representing natural numbers and hence the use of 'n' as convention to denote natural numbers.

Findings from research study conducted by Lee and Wheeler (1987, cited in Kieran, 2007) show that a few learners use or appreciate the role of algebraic notation as a tool/language for expressing the general term of geometric and numerical patterns. In other words, few learners appreciate the use of letter/symbol to represent variables and are able to generate the algebraic representation of the pattern. It goes without saying therefore that the evaluation criteria around the notation and the specific meaning denoted by the symbols needs to be explicit so that learners may have access to recognition and realisation rules. On the notion of conventions, Watson (2009) argues that learners need to understand basic operations and became confident with the notational rules to understand algebraic symbolisation. Watson argues for the learning of precise use of notation and I concur, because in the light of what Kaput (1989) has mentioned, without mastering the language of expression which is made up of the symbols, conventions and the specialised vocabulary, learners may not be able to communicate ideas in mathematics

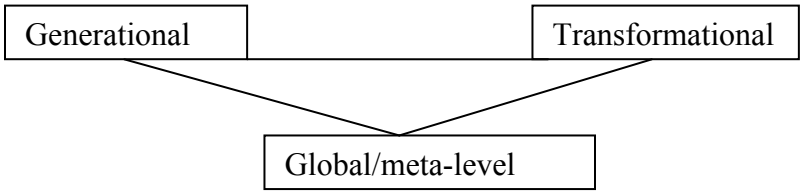
So, from this section (2.2.1 language of expressing generality), what is coming out of school algebra literature is the notion of mathematical conventions as a language used for expression within algebra. Therefore the legitimate text within this is being able to read and use the symbol system used in algebra for expression. Another perspective on school algebra dictates that school algebra is an activity; this is what I discuss in the next section.

2.2.2 School algebra as an 'activity'

From interviews conducted with mathematics teachers, mathematicians and mathematics education researchers, Lee (1997) asked the question: "What is algebra?" Seven themes emerged, namely: a school subject; generalised arithmetic; a tool; a language; a culture; a way of thinking and an activity. One of the participants, a mathematics education researcher interviewed by Lee responded and said *action* is a central theme of school algebra. Consequently, Kieran (2007) developed a model that synthesises the activities of school algebra

into three types after reviewing literature based on school algebra namely: generational; transformational and global/meta-level, known as the GTG model. It is important to note the way in which algebra has been defined here; that it is an activity and activities/actions that teachers or learners engage in when doing algebra are described below instead of content. The content is simply used to perform these actions, and this is not specified. I discuss each below and give an example from the topic of number patterns. For the global meta-level category I have chosen an example from the grade 12 National Assessment.

Figure 1: Kieran's model of school algebra



(Sourced from Kieran, 2007, p.713)

Generational activities involve the forming of the objects of algebra – that is the equations and expressions from geometric patterns or numerical sequences. For example when a number pattern is given 4; 8; 12; ... the process of getting to the algebraic expression $T_n = 4n$ is a generational one. The process of reading a diagram or word problem to generate the numerical pattern is also a generational activity and in relation to this type of questioning, Warren (2000) argues that the level of difficulty is higher because of the amount of processing involved.

Transformational activities (Kieran writes that this is 'sometimes referred to by some as the rule based activity') involve collecting like terms, factorising, expanding, substituting, simplifying. These activities involve continuing the pattern, finding the value of 'n' from the equation $T_n = an^2 + bn + c$, where a, b and c are known, and remembering that 'n' is a positive integer. Then substituting values of 'n' into the equation to test if the general term is correct is another transformational activity of school algebra within this topic of number pattern.

Thirdly Global/meta-level activities of school algebra are said to include 'problem solving, modelling, and working with generalisable patterns, justifying and proving, making predictions and conjectures, studying change in functional situations, looking for relationships or structure'

(Kieran, 2007, p. 714). An example of this is where learners are required to prove whether the statement is true or false and justify their answer about whether a pattern is arithmetic or geometric. Another example of this is the following problem from the DoE (Department of Education) examinations in figure 2 below:

Figure 2: Question from the DoE Examinations

> 5.1 Kopano wants to buy soccer boots costing R800, but he only has R290, 00. Kopano's uncle Stephen challenges him to do well in his homework for a reward. Uncle Stephen agrees to reward him with 50c on the first day he does well in his homework, R1 on the second day, R2 on the third day, and so on for 10 days.
>
> 5.1.1 Determine the total amount uncle Stephen gives Kopano for 10 days of homework well done. (5)
>
> 5.1.2 Is it worth Kopano's time to accept his uncle's challenge? Substantiate your answer. (2)
>
> (from the DoE/NCS Preparatory Examination 2008 Question 5 of Paper 1 page 4)

A problem like this one, while the global/meta-level activity is the overarching activity, requires the other activities. Problems of this kind, therefore, provide opportunities for learners to experience all three types of activities, however, the activity that is fore-grounded is the global meta-level activity because learners are required to justify and prove whether the challenge is worth Kopano's time. For example, learners have to generate the sequence of numbers: $\frac{1}{2}$;1; 2; ... from the story – this is a generational activity. To find S_{10} there will be substitution of known values into the formula: $S_n = \frac{a(r^n - 1)}{(r-1)}; r \neq 1$ – the act of substituting is a transformational activity. All these activities will be drawn on to justify and prove that the uncle's proposal is worth Kopano's time. However this is a simple problem where the number of terms are few and does not necessarily require the use of a formula, the sum of terms can be found without the formula.

The literature reviewed by Kieran (2007) shows that number pattern is located more within the first and last activities of school algebra that is generational and global/meta-level. Kieran (2007)

says from the GTG model that one activity will dominate while other activities will be drawn upon. For example, problem solving types of problems will have the global/meta-level as the overarching activity like the above problem from the national assessment, the transformational activity and the generational activity will be drawn on to achieve the aim which is to think at a meta-level. Therefore, these activities are inseparable from each other but the overarching activity within each mathematical text among the three in the GTG can usually be determined. In the case of this study, the topic concerned is number pattern and these categories in the GTG will be used to determine the kind of activity used.

What emerges from this section is that school algebra is an activity and within the activities three classifications can be made depending on which activity among the three is dominant. The three types of activity provide a way of analysing the teaching of number pattern across the sequence, and pointed to a focus on algebraic notation and language as well. This notion of school algebra being an activity is used to segment the transcript into smaller units of analysis within the broader unit of analysis called the evaluative event in Chapter 4. From the two sections focused on literature review on school algebra, two themes emerged and these are: (1) activity and (2) the notion of mathematical conventions.

What follows is a report of literature discussing recent studies conducted (over the past five years) on this topic of number pattern. The question that this report seeks to answer is what these researchers perceive/privilege as legitimate forms of knowledge. And to what extent is this recognisable in the way they report on this particular topic and what they recommend as useful ways of engaging with the topic in learning and teaching of mathematics and specifically algebra.

2.3 Literature on number pattern

Most of the studies, reported below, focused on young learners in this topic, but very little literature was found on older learners. Within the studies conducted with young learners, the focus has been on the ways pattern can be used to support generalising activity. Some of the studies recommend specific teaching actions that are helpful in helping learners generalise in a particular way. The use of visual patterns and how these foster the generation of an algebraic expression has been explored also as well as the use of technology. With older learners, the

literature has focused on how learners appropriated the concept of geometric sequence and series and didactic tools for teaching the concept of limit to first year university students.

2.3.1 Ways of generalising

Research by Driscoll (1999) shows that learner's early encounters with number patterns entails describing patterns in recursive terms before the closed explicit way which entails describing a term in relation to its position. Lannin, Barker and Townsend (2006) say "explicit rules use index-to-term reasoning that relates the independent variable to the dependant variable(s), allowing for the immediate calculation of any output value". (p. 300). Warren's (2000) findings also support this finding. In addition, Warren (2000) refers to recursive rules as *single variational thinking* (p.283) (that is finding relationships within a sequence of numbers) and to the closed, global or explicit way as *functional thinking* (that is describing pattern in terms of their positional relationship) (p.283). Driscoll (1999) continues in this line of thinking to say that it is difficult to move from the recursive representation to the 'closed global form'. For example, when learners are given the sequence of numbers: 2; 4; 8; 16; 32 ... and asked to find or describe the general term, usually they describe it in terms of what is done to the first term to get to the next term i.e. $T_2 = 2 \times T_1$, $T_3 = 2 \times T_2$, which works out to be $T_n = 2T_{n-1}$ which is the general recursive term. While this kind of a representation of the pattern is important and useful, it is restrictive/ limiting in that it is not useful for finding term number 1000 because one would need to know term number 999. Evidence here suggests that moving from the recursive to the explicit, closed global form: $T_n = 2^n$, which is the general term, is difficult for learners. Watson (2009) claims that the move from the term-to-term (recursive) formula to the functional formula (closed, explicit, global form)involves/demands a major shift in perception more than working through it as a notational problem, because it shows that learners see the pattern more than using symbols to express it. Radford (2001) in his study of the transition from the particular to the general, Radford argued that such a process takes time. This term-to-term generalisation is called naïve induction by Radford (2003) and of course the work he reported on here was done with young learners.

So, from the way mathematics education researchers have reported on the ways of generalising, what comes through is that the global/functional way is being privileged because it is the general

term that expresses the overall generality efficiently. However, their findings also suggest that starting with the recursive way of talking about number pattern and representing number pattern in class is probably useful, because it is where learners first see and describe patterns anyway. Then learners are (more) ready to move on to the global/functional way of representing a pattern in mathematical terms. This will open up opportunities for learners to compare and contrast the two ways of generalising a pattern and note different things that are highlighted by different representations. I now move on to discuss some of the aspects of generalising as described by Driscoll (1999) as habits of mind.

2.3.2 Habits of mind

Driscoll (1999), when referring to the topic of number patterns, says that number patterns are used to introduce algebra and generalisation as a thinking process that not only applies in number patterns, but throughout algebra and is cognitively demanding. He says that a high level of algebraic thinking is shown when learners make convincing arguments. He recommends two processes of generalising can be used to advance learner's algebraic thinking; these are *globalising* and *extending*. Both of these processes are part and parcel of work done in number pattern.

Globalising is the cognitive skill of being able to craft convincing arguments that the rules generated will work for all relevant cases. The types of activities include working from an empirically proven generalisation to an explicit generalisation or making the move from the particular to the general and being able to argue that it will always work in all situations. For example, when the sequence of numbers is given: 1; 4; 9; 16; ... and learners are asked to find the n^{th} term they describe the pattern in empirical terms and they usually give the response: $1 \times 1 = 1$; $2 \times 2 = 4$; $3 \times 3 = 9$; $4 \times 4 = 16$; ...; $10 \times 10 = 100$; which is empirical evidence which is supposed to lead them to the explicit/global form: $T_n = n^2$. The explicit global form requires that learners should have the proper conception of what a variable is within this context, which is a letter symbol, 'n', representing a range of natural numbers. This is structural thinking because learners need to first understand the letter symbol as it is used here.

Extending is presented as another way that a mathematical thinker can generalise. Driscoll (1999) says extending is "following the lines of further inquiry suggested by a particular

mathematical result" (p. 97). Extending, as I understand it, is exploring by asking the question: '*What if*' in relation to the pattern itself or the generated rule. The purpose is to explore what happens when one or two things change. In other words, you are extending your thinking and taking it beyond the current situation and think about what would happen in a different situation. For example, observing that the row number for constant differences is the same as the highest power of the expression shows this notion of extending ones thinking, thus ones thinking has been extended by exploring the 'what if' question.

Driscoll (1999) says that these (extending and globalising) processes should form part of regular instruction so that they may develop into habits of mind or important mental processes. What Kieran (2007) described as Global/meta level activities of school algebra seems to fit with what Driscoll (1999) says are algebraic thinking processes, when generalising. Also, it is clear that Driscoll as a mathematics education researcher is giving processes high preference (this is the legitimate text for Driscoll (1999)). Interesting to notice is the fact that Driscoll is not concerned with specifying or recommending certain content but mathematical processes which should grow into habits of mind.

In the next section I discuss is the research done by Warren and Cooper (2008) and what they call *teaching actions* that bridges the difficulties young learners encounter when representing linear patterns algebraically. I use this study to add additional depth to what is identified as the legitimate text in relation to number pattern in the mathematics education literature.

2.3.3 Teaching actions

Warren and Cooper's (2008) study explores teaching actions and thinking that begins to bridge many of the difficulties young learners experience when it comes to expressing linear patterns as functions and in algebraic terms. This study was carried out with 8-year old learners in Australia. The authors support the claim that mathematics teaching has been focusing on product rather than process. Their main aim was to find teaching actions that begin to assist young learners to view and describe visual growth patterns in terms of their positional relationships. This reveals what they are privileging as the primary and legitimate text (process instead of product, positional way of generalising) already.

The results of the pre-test and post-test study show that there was growth in the learners' ways of describing the relationship between the term and its position, growth in understanding as well. The supporting process or teaching actions that helped learners to produce this growth were:

- Use of concrete materials

The use of iconic signs (the tiles) and indexical (the cards with numbers indicating position) signs. The use of tiles and cards was essential to the introduction of co-variational thinking

- Use of patterns where the relationship between pattern and position is explicit
- Explicit questioning to link the position to the pattern

Specific questioning proved helpful in assisting/ helping students see the relationship between pattern and position. Also, specific questioning assisted students to reach generalisations in relation to unknown positions.

- Generalising from the pattern in small position numbers to large position numbers
- Using colour to represent different growing components of the pattern
- Using visual patterns to mark those that were not in sequence

They also described and noted some of the hindering processes or teaching actions.

- Language used to describe the generalisation. The researchers acknowledged that students lacked some of the mathematical vocabulary needed to provide precise responses. Imprecise language was embellished with gestures by learners.
- Writing the generalisation instead of expressing it orally
- Completing patterns – single variation
- Reversal of cognitive strategy/thinking (identifying the position when given the value of the term), most students in their study found this process very difficult when given the total number of tiles and asked to find the position.
- Expressing the generalisation in language. Learners found it difficult to distinguish between ordinary language and cardinal language when describing the pattern for the nth position.

They concluded that young children are capable of thinking about relationships between two data sets and expressing this relationship in a very abstract, algebraic form. Even though this study was conducted with young learners, what comes through here is that the researchers are advocating for the use of explicit ways of questioning and teaching and thus the creation of access to recognition and realisation rules of the legitimate text. What this study suggests is that there is a need to focus on making the legitimate text explicit for learners through teaching manipulatives and questioning. The next section reports on a recent study conducted by Rivera (2010) who talks about the importance of visual representations.

2.3.4 Visual representations

Rivera (2010) qualitatively assesses the implications of a design research experiment. Rivera studied how 12 and 13-year olds in Grade 7 and 8 in an urban school in California worked with visual patterns and how they used the visual structure observable in them (visual patterns) as templates which give clues towards the general formula. A template in Rivera's article is taken to be a type of sequential knowledge that guides towards the correct answer. Rivera provides empirical evidence of the existence and effectiveness of visual templates named: additive (counting things one at a time without attending to structure and involves single abstraction); multiplicative (multiplicative thinking is constructed out of addition at a higher level of abstraction) and pragmatic (happens in a problem situation where a learner has to combine additive and multiplicative schemes) in dealing with figural patterns that have linear and relatively simple quadratic structures. Rivera emphasises the role of multiplicative schemes in pattern generalisation with a focus on a unit that is central in pattern structure formation and discernment.

Rivera argues that visual templates help learners engage in 'meaningful and purposeful diagram parsing'. Rivera (2010) talks about meaningful pattern generalisation which involves the coordination of two interdependent actions that is: a) abductive – inductive action on objects and b) symbolic action. In the abductive phase, learners offer an explanation of the pattern based on what is given – this is the known stage – and this explanation is often used to extend the pattern. The inductive stage is described as managing the unknown, for example, finding the 10^{th} or 30^{th} term and so on. Meanwhile, symbolic action involves finding the direct formula and representing it in symbolic form. Rivera argues that learners need to know how to bring together their

perceptual and symbolic inferences effectively in relation to an interpreted structure of a pattern that applies both to known and unknown stage.

Rivera (2010) says that the concepts of generalisation, justification and proof are interwoven. In other words, generalisation cannot be accomplished without justification and for something to be justified it has to be proven true. From the iconic images given by the learners Rivera's study it became apparent that the underlying structure of a pattern does not necessarily become immediately obvious simply because the pattern is a visual image, as argued earlier. Learners continued with the diagrams in different ways to show that their perception of the pattern was different even though visual. The learners' uncertainty about how to continue the pattern is not only a problem with numerical pattern only but was also a problem with visual patterns as well. And that is why Rivera talks about 'pattern goodness' or the 'Law of Gestalt', which refers to how ambiguous or not ambiguous the picture is. When the picture is confusing there is a low Gestalt and where the picture is clear on how to continue the pattern there is a high Gestalt law or pattern goodness.

Representing number patterns in various ways is important for learners to grasp concepts in mathematics. For deeper and fuller understanding to develop a range of representations needs to be present as each representation will be highlighting a different aspect of the pattern and relationship. Therefore, learners must be able to deal with multiple representations when put up (by teacher, assessment, textbook or worksheet) and be able to translate, compare and find connections amongst these. For example, learners may experience a variation in representations which means that they should not only encounter a pattern as a whole number but as a word problem or a diagram and different types of numbers. For example, a pattern may not always start with a small number but must sometimes start with a big number and so on. It is important that this kind of variation is made available at the level of different representations in the curricular materials provided for teachers to teach. Also, the exploration of different but equivalent expressions that arise from different ways to generalise pattern proved to be a valuable strategy in Warren and Cooper's (2008) study. They pointed out that comparison of different representations and their common aspects is important. As in all aspects of life, a limited experience is unhelpful. When learners have not had exposure to more complex ways used to start sequences, they may be unaware of the need for critical, reflective thinking and the

value of simplifying and organising data (Watson, 2009). This means that learners who have only seen one simple way used to start a sequence generation have a limited and impoverished outlook because they have not had a chance to experience examples that support observation, critical reflective thinking and so on. In relation to my study, what comes out of this section is the notion of different contexts as a category and as a way of opening access to different ways of teaching and presenting the legitimate text. In the next section I will discuss three processes that Ellis (2007) looked at, these are justification, proof and generalisation.

2.3.5 Justification, Proof and Generalisation

These three words cropped up in Rivera's discussion in the last section. However, Rivera is focusing on visual representations while Ellis(2007) is focusing on the three words: justification, proof and generalisation and is using a particular taxonomy to classify learners' generalisation and justification strategies. Ellis (2007) was interested in how generalising and justifying activities are related to one another when learners work with linear patterns in two real life situations: gear ratios and speed. These were seventh graders (aged 12) who participated in the teaching experiment and Ellis was interested in what learners understood to be general and convincing. These learners were pre-algebra learners. Ellis argues that generalisation and justification are considered important components of algebraic activity. And that the ability to generalise has been linked to what it means to reason algebraically and is fundamental to mathematical activity and thinking. Ellis continues to argue that justification in any form is an important part of algebraic reasoning because it triggers a habit of mind whereby one naturally questions and makes conjectures to create a generalisation. Ellis says the connection between generalisation and justification is bi-directional which means that they influence each other. I agree because as I understand it a generalisation comes into being after going through a process of justification and proving. Concerning the last two, proving and justifying, there is a thin line this is shown in the discussion that will follow on justification as it is discussed by Ellis. For now, the focus is on getting some clarity on generalisation so as to be clear about the two (justification and generalisation).

What is generalization?

Ellis defines generalisation as a process of extending or expanding one's range of reasoning beyond the case or cases considered. This particular definition tallies with one of the habit of mind put forward by Driscoll (1999) earlier. Generalisation is further described as the process of creating a rule; the process of identifying commonalities. Kaput (1999) says this about generalising:

> 'lifting the reasoning or communication to a level where the focus is no longer on the cases or situations themselves, but rather on the patterns, procedures, structures and relations across and among them' (p. 137).

In the typical approach to generalisation, a formal verbal or algebraic description of a correct rule is required as evidence of generalisation. In his study, Ellis used his taxonomy of generalisation which describes the different types of generalisation that learners create when reasoning algebraically.

Using the Ellis taxonomy of generalisation, the learner's activity of (making) generalisation fell into the three major categories:

- Relating – an association that is formed between two or more problems
- Searching – performing a repeated action in order to test if the given solution remains valid for all cases.
- Extending – removing some contextual details in order to establish a global case
    - Reflection generalisations (students final statements of generalisation) – this represents one's ability to identify or use an existing generalisation. This involves the implementation/use of previously developed generalisations.

However, the focus in this study is how the legitimate text is constituted by the teacher in the classroom when teaching number patterns to grade 11. The question that I am asking here is what knowledge is privileged in relation to learning number pattern? From the above review of Ellis (2007), it can be seen that Ellis (2007) is privileging the processes of justification, proof and generalisation when engaged with this the topic of number pattern.

What is justification?

Generalisations and justifications jointly influence one another to support the development of more sophisticated reasoning (Ellis, 2007). To do this, Ellis (2007) argues that learners need to be provided with opportunities to exercise their justifying acts which support the development of powerful generalisations about linearity. As mentioned earlier, the acts of justifying induce a habit of mind where one questions and conjectures to form a generalisation. According to Harel and Sowder's (1998, cited in Ellis (2007)), learners' justifications were categorized according to taxonomy of proof schemes in Ellis's study and the reasons for choosing a proof taxonomy to categorise learners justifications is explained in the definition that the (Harel and Sowder, (1998) gave as proof.

Proof is a process of removing or creating doubt about the truth of an observation. Ellis (2007) distinguishes between **ascertaining** where one removes his or her own doubt and **persuading** where one removes doubts of others. Ellis is using these notions of ascertaining and persuading because he believes that they are compatible with the notion of justification and I agree because when you are required to justify your answer you have probably ascertained and now you are actually required to persuade others to believe or agree with what you are saying or doing. For Ellis (2007) there were five proof schemes from Harel and Sowder's (1998) taxonomy applied to the participants (learners) in his study:

1. Authoritarian
2. Symbolic
3. Inductive
4. Perceptual
5. Transformational

These five belong to three families where different means of conviction/assurance are employed. The first two (authoritarian and symbolic) belong to the external conviction family where conviction is obtained by the word of an authority or the symbolic form of an argument. The last three (inductive, perceptual and transformational) belong to the empirical family of proof schemes where conviction is obtained by validation or invalidation of specific cases or sensory experiences. Furthermore, Ellis (2007) analysed and coded learners' justification and

generalisations using the two taxonomies, his taxonomy of generalisation and Harel and Sowder's (1998) taxonomy of proof.

Ellis (2007) observed that learners' justifications evolved over time from symbolic to empirical and to transformational, through comparing the shift and growth in sophistication in the ways that learners were justifying and generalising their work from the beginning to the end of the experiment.

Learners' results showed that relationship between justification and generalisations were rarely self-contained. Students moved from simple to more sophisticated ways of generalizing and justifying over time. Mechanisms were identified which described how justification and generalising influence each other to support the development of more sophisticated reasoning. So in this section, Ellis (2007) argues that learners need to be provided with opportunities to exercise their justifying and proving acts because these acts induce a habit of mind where one questions and conjectures to produce a generalisation. In other words, Ellis (2007) places emphasis on and privileging mathematical processes of justifying, proving and generalising as the legitimate text when engaged with this topic of number pattern.

2.3.6 Technology and number patterns

Generalisation of numeric and geometric patterns has been proposed by many researchers as one of the approaches that can be used to bridge the gap between arithmetic and algebra (see Kieran, 2007; Watson, 2009). Tabach, Arcavi and Hershkowitz (2008) described and analysed student learning in a computer intensive environment (CIE) with seventh graders. They described the mediating role of spreadsheets from arithmetic to algebra.

Tabach et al. (2008) investigated the kinds of symbolic generalizations these seventh graders engaged in at the beginning of the CIE, and whether learners' generalisations changed during the spreadsheet course and how. These researchers were interested in seeing was whether learners could wean themselves off from the tool and engage in activities without needing to use the spreadsheet.

The types of problems used here were real-life situations which required students to model using numerical, symbolical and graphical representations while keeping in mind the meaning of the

situation. The writers argued that 'spreadsheets support the functional approach to algebra and the envisioning of patterns which lead to generalisations'. (Tabach et al. 2008, p. 56)

The researchers view spreadsheets as a tool for making sense of the dynamic aspect of functional relationships between the variable (independent variable) and the value of the expression which is the dependant variable. However, Tabach et al. (2008) report that the use of spreadsheets to support the symbolic language is questioned by many researchers since the language that is used in Microsoft Excel is different from the language that is used in mathematics and in algebra specifically.

Tabach et al. (2008) found that learners used a variety of strategies to generalize and they are numerical, multi-variable this involves using more than one variable to express generality, recursive which emphasises local relationships between consecutive elements and explicit which involves expressing the general relationship and displaying the full relationships among variables. They also report that 23 percent of the learners continued using the spreadsheet while the rest moved from computer to paper and pencil even when the computer was fully available to them. This computer intensive environment is not available for the majority of learners in South Africa. So the usefulness of the equipment is applicable to a few who have it and can use it. Once again though, the process of generalising in an explicit way is favoured by these researchers.

Radford (2010) also studied the generalisation strategies learners employ when engaged in number pattern tasks. Radford is talking about the 'domestication of the eye' that it is a lengthy process in which we come to see and recognise pattern/ things according to efficient cultural means. He argues that it is this process that converts the eye into a sophisticated intellectual organ – he calls this process 'objectification'. He concludes that the generalisation process requires a transformation (domestication of the eye, by converting the eye into a cultural-theoretical organ of perception) of the eye into a more sophisticated form of perception. He suggests a conceptualisation of learning as a process of objectification. We continue to see mathematics education researchers advocating for processes and giving fancy names to these processes either than specifying content as the legitimate text. What follows is a report on number patterns with older learners.

2.3.7 Recent research on number patterns in secondary school and beyond

Research done in Norway by Carlsen(2010) showed how 17 to 18-year old students who choose the voluntary course called 3MX which is meant to prepare them for university appropriated the concept of geometric series. Carlsen (2010) describes the process of appropriation as the process of learning and making something your own, the process of becoming 'a knower'. Carlsen (2010) worked with the same small group of 6 students, who were selected by their teacher based on their ability – it is reported that they were relatively high achievers – throughout the 13 sessions. The five excerpts used in the article are a selection across the 13 sessions to show how appropriation happens over time. Carlsen (2010) discovered that the process of appropriation happens through three processes that is: a) involvement in joint activity, b) having a shared focus of attention and developing shared meanings which are in accordance with the mathematics community. The results show that the students had to work together and have a shared focus of attention when discussing mathematical objects and make sense of the mathematical signs and their meaning. Learners were involved in activities such as making explicit what the general term is all about ($a_n = ak^{n-1}$), Understanding and discussing the values of k that will satisfy the formula: $S_n = \frac{a(k^n - 1)}{k - 1}$. Carlsen (2010) showed how students were discussing, internalising and appropriating the concept of converging and diverging series given the value of 'k'. This study showed the important part that learners have in acquiring the legitimate text. It shows that for acquisition of the legitimate text is about having an internal knowledge base. The emphasis here was on understanding the mathematical conventions used for expressing generality for a sequence and summation of a series.

Przenioslo (2005) reports that research on secondary school learners and university student's understanding of the concept of limits is distressing. Hence, Przenioslo (2005) proposed a didactic tool that can help students to better understand and be aware of the various aspects of the formal notion of limit in a sequence. Przenioslo argues that conceptions of a sequence in high school, as a rule for producing numbers or a long list of numbers contribute and serve as obstacles to understanding limits. However, Przenioslo does not offer alternative ways of how the concept of sequence can be conceptualized and handled so as to not create obstacles for understanding limit, but he proposes a set of specially designed problems which can be used to

organize classroom discussions for the concept of limit. Therefore, the manner in which the teaching of the concept sequence is structured is important as it may serve as an impediment for future learning of topics developing from it (number sequence) or related to it, like the topic of limits or annuities in financial mathematics.

2.4 Conclusion

In this chapter, I have reviewed literature on algebra and on number patterns. Literature on number patterns over the past five years shows more studies conducted on this topic in the primary school and in grade 8 and 9 which is the GET (General Education and Training) phase within the context of South Africa. From this survey, it is also evident that within the past five years there is very little literature showing studies conducted on FET (Further Education and Training – Grade 10, 11 and 12) band. However there is nothing reported on grade 10 and 11 on this topic of number pattern in the past five years. This review of literature provides insights into how learners of different age groups, from some parts of the world, experienced this topic of number pattern. This review has also provided insight into some of the useful ways that can be employed into the teaching of this topic for learners to get better understanding.

Three themes emerged, that is mathematical processes, mathematical conventions and contexts. I will discuss each of these themes in turn below to summarise what has come through from the literature.

2.4.1 Mathematical processes

This review has also showed beyond reasonable doubt the point that this topic is important as it teaches some of the core mathematical processes which can be impediments when a general understanding mathematical processes is not grasped. These processes, as it has been argued here by various researchers, serve as tools that a learner can employ when faced with a mathematical problems of any type. As Mason (2006) puts it, the entire discipline (mathematics) is about expressing generality. It was also interesting to notice that mathematics education researchers are concerned with the specification/specifying of mathematical processes and put emphasis on processes more than specifying content. Radford (2010) recommends a process of learning called objectification – domestication of the eye. Earlier, Ellis (2007) highlighted how the processes of generalising and justifying as well as proving are dependent on each other and influence each

other. Cooper and Warren (2008) have emphasised the role specific teaching actions play in the process of teaching number patterns to young learners. Moreover, from Kieran's (2007) model it is clear that emphasis is put on all types of activities, but in particular from the literature, there is more emphasis on the third type of activity and that is global/meta-level activity of school algebra. Literature has shown that the topic of number pattern has as its major activity the process of generalisation. This process requires that an algebraic expression be generated; this makes the activity to be more of a generational one than transformational or global/meta-level. The acts of getting to the general term will incorporate some of the transformational and global/meta-level activities but the overarching activity is a generational one.

In summary, this review has detailed what number pattern means/requires for the FET phase at high school level. It has also gives insight into what school algebra is: that it is an activity and in the process of specifying those activities. As seen from the literature reviewed, the content has not been specified instead, the focus has been on the mathematical practices and processes as a way of describing actions that are done in algebra at school level. The literature also highlighted the importance of knowing the advantages and disadvantages of the two ways of generalising (Lannin, Barker, & Townsend, 2006) Literature has also argued for using the explicit way of generalising because of the advantages it comes with in understanding and working with two sets of variables (Warren & Cooper, 2008; Radford, 2001; Tabach et al. 2008;). Also clarified in the review was the vital role these mathematical processes play and the importance of having these processes developed in learners as 'habits of mind' (Driscoll, 1999). The literature has also emphasised the importance of definitions of sequence that fit with mathematical discourse accurately as early as possible so as to not become a hindrance for future learning in the finishing grades and university (Przenioslo, 2005).

2.4.2 Mathematical conventions

The literature review on school algebra has highlighted the importance of the specialised language of mathematics as a symbolic system used to express generality. Researchers like Watson (2009) have advocated for precise teaching of the notation and symbols and so that learners may understand the messages being conveyed and understand the discipline itself.

Carlsen (2010) showed how a process called appropriation helps learners to own and understand the meaning of each letter embedded in the notation $a_n = ak^{n-1}$ and $S_n = \frac{a(k^n - 1)}{k - 1}$.

2.4.3 Contexts and different representations

The literature has given insight into the importance of different representations to introduce the pattern and represent the pattern in different forms. In particular the importance of visual representations, how they can make easy and at the same time can make the generalisation process (Rivera, 2010) more difficult. Finally, literature has shown how the use of technology as a context for teaching and learning enhances learners' ability to grasp and understand the notions of variable, structure, notation (to some extent) and generality within this topic of number pattern. Table 1 shows a summary of what come through as the 'legitimate text' from the literature.

Table 1: Themes emerging from the literature

Processes	**Conventions**	**Contexts**
- Globalising - Extending - Justifying - Proving - Generalising • Explicit • recursive	- Symbol system - notation - different meanings of letters in expressions	- visual representations - different representations - real life contexts - technology environment

These findings from the literature are used in my study to classify and categorise the legitimate text in the curriculum document, in the assessment texts, in the textbook and in the teacher's practice. In the methodology chapter, I am going to use Kieran's (2007) notion of school algebra as an *activity* to segment the transcript into smaller units of analysis within the broader unit of analysis called the *evaluative event*. From here, I will also comment on the findings across the different texts analysed in relation to conventions, processes and contexts. In the analysis chapters, the GTG model (Kieran, 2007) is also used to determine the kind of activity that was dominant in the assessment, textbook and the teachers practice. In the next chapter the theoretical approach used to describe, analyse and understand phenomena in this study will be discussed.

Chapter 3 Theoretical Framework

3.1 Introduction

I chose Bernstein's (2000) theory of the pedagogic device because it provides a language with which to talk about the problem of the 'legitimate text' and so frame the study. The assumptions made about learning in this theory include the transmission and acquisition[1] of values and beliefs which are influenced by social contexts. Bernstein's theory of the pedagogic device provides researchers with tools to describe the structuring of knowledge in pedagogic contexts. Bernstein (2000) argues that most studies have focused in what is carried or relayed and have not studied the constitution of the relay itself. This study is concerned with both finding out *what* is constituted as the legitimate text and *how* the legitimate text is constituted in the National Curriculum Statement, the official assessment texts, and the textbook used in the classroom in particular, how this legitimate text is constituted in the classroom for the topic of number pattern in a Grade 11 class. In other words, Bernstein's theory of the pedagogic device was used as a lens through which to describe and compare the constitution of the legitimate text in the documents and in the classroom. In the classroom, a methodological tool developed by Davis, Adler, Parker and Long(2003)and elaborated by Adler (2009) was used to see and describe this legitimate text.

Bernstein's theory and work focused on social class as a variable that should not be ignored when doing educational research. However, this variable (social class) is beyond the scope of my study. The question this study seeks to answer is what constituted as the legitimate text, how is it constituted, and what opportunities are offered by the teacher to learners for acquiring the legitimate text?

This pedagogic device is a mechanism that describes the sociological nature of pedagogic knowledge. Singh (2002), in her elaboration of Bernstein, says the pedagogic device is composed

[1] Transmission and acquisition are technical terms used by Bernstein. These must not be confused with what is typically referred to as transmission teaching in a pejorative sense in much of mathematics education literature.

of a system of rules which control the conversion, or the pedagogising, of knowledge into classroom talk and curricula. The rules of the device are not ideologically free but reflect on the knowledge preferred and created by dominant groups in society. It therefore follows that the device is not neutral and is possibly biased. However, with respect to social disadvantage, this bias is beyond the scope and focus of my study. I am interested in the '*what'* (the legitimate text) of learning and '*how'* it gets transmitted to learners and hence opportunities to learn. On the other hand it is important in the process of finding out the '*what'* to understand how the '*what'* travels from the original discourse to curriculum to textbook and to classroom and hence the following discussion about Bernstein's theory of the pedagogic device.

3.2 The Pedagogic Device

According to Bernstein (2000), the pedagogic device controls the communication it makes possible and operates selectively, according to preferences of the dominant class, within the knowledge domain. This leads to questions about whether legitimate ways of transmitting and acquiring the text are explicitly communicated in the message controlled by the device. This is one of the reasons that led me to problematise the concept of the legitimate text and its constitution in various documents and the classroom. I will start off by describing the rules of Bernstein's pedagogic device as these are necessary for one to understand how the local communication which the device makes possible comes into being. This device has internal rules: distributive rules, recontextualising rules and evaluative rules which manage the pedagogic communication which the device makes possible. These rules are interrelated in a hierarchic manner to each other. This means that the recontextualising rules are a function of the distributive rules; and evaluative rules are a function of both recontextualising and distributive rules. I discuss each one of them below and indicate where my study is located within each.

3.2.1 Distributive rules

Bernstein says (2000) distributive rules decide on what comes to be the message/communication and who may convey the message, who may get the message and under what conditions. 'Distributive rules translate sociologically into fields of production of knowledge with their own rules of access' (Bernstein, 2000, p. 33). While I am concerned about opportunities for acquiring the legitimate text, as noted above, I do not focus on its distribution.

Recontextualisation rules, which I discuss next, have the responsibility of refocusing this knowledge to be suitable for high school mathematics, for school curriculum, for assessment at the official level and for textbook mathematics at secondary school level. Given that these are the documents I am analysing to see how the legitimate text is constituted for the topic of number patterns my study is therefore located within the field of recontextualisation. However it was necessary for me to discuss the three rules as they stand in a hierarchic manner to each other.

3.2.2 Recontextualising rules

Recontextualising principles serve to transform the original discourse to create an imaginary discourse. Bernstein (2000) argues that in the process of moving/transforming a discourse from its original site, a gap or a space is created over which ideology can play and he stresses the point that 'no discourse ever moves without ideology at play' (2000, p. 33), so every discourse as it moves is ideologically transformed and it is not the same discourse any longer. Therefore, *pedagogic discourse* (PD) is constructed by a 'recontextualising principle which selectively appropriates, relocates, refocuses and relates other discourses to constitute its own order and it cannot be identified with any of the discourses it has recontextualised' (Bernstein, 2000, p. 33).

Bernstein (2000) distinguishes between the Official Recontextualising Field (ORF), which is created and dominated by the state and its preferred agents and organisations, and the Pedagogic Recontextualising Field (PRF). The PRF consists of pedagogues in schools and colleges, in departments of education, specialised journal and private research foundations. Bernstein (2000) says the PRF has an effect on what comes to be PD independently of the ORF. In this study, I am looking at the topic of number pattern from the official National Curriculum Statements for Mathematics (NCSM) and the official grade 12 National Senior Certificate (NSC) examinations for mathematics within the ORF.I am also looking at the most dominant mathematics textbook in South Africa, a text that is within the PRF. Furthermore, and still within the PRF, I explored how patterns in school mathematics is described in such texts. That is, for each one of these texts, I asked what is constituted as the legitimate text for the topic of number patterns.

I now want to elaborate more on the pedagogic discourse because my study, at the level of the classroom, is located within pedagogic discourse. I focus here on instructional and regulative discourses, the notions of classification and framing of knowledge, and within framing, I focus

on evaluation criteria and the distinction Bernstein makes between what he terms recognition and realisation rules. According to Morais (2009), and her elaboration of Bernstein's work in a study of science pedagogy in Portugal, pedagogic discourse refers to the transmission of (mathematical) competences, the acquisition of (mathematical) competences and evaluation of (mathematical) competences. Pedagogic discourse is made up of two discourses: the Regulative Discourse (RD) and the Instructional Discourse (ID). The RD refers to expectations about conduct, manner and character. According to Bernstein (2000) the regulative discourse is responsible for the transmission of rules and values of the dominant society; it also regulates how knowledge is to appear and is transmitted. The instructional discourse refers to the selection, sequence, pacing and evaluative criteria of knowledge. So the instructional discourse refers to what is transmitted. The regulative discourse always dominates the instructional discourse (Bernstein, 1996). In this study, the concern is on the evaluative criteria and so on instructional discourse. The question is asked: "What criteria come to play and less so, how criteria come to play?" What comes to be constituted as criteria points to the legitimate text for the teacher, from this I will then analyse how opportunities for learners acquiring the legitimate text have been opened up.

3.2.3 Evaluative rules

Bernstein (2000) says the key to pedagogic discourse is continuous evaluation. Evaluation is the whole purpose of the device. Bernstein says the evaluative rules transform the discourse into pedagogic practice and any pedagogic practice is there for one purpose: to transmit criteria. The evaluation is defined by Bernstein as what counts as valid realisation of the knowledge on the part of the taught (learner). At the level of the acquirer (learner), Bernstein (1996) says that the evaluation is characterised by 'recognition' and 'realisation rules'.

3.2.3.1 Recognition and classification

The recognition rule depends on the classificatory principle. Classification is described by Bernstein(1982) as the relationship between contents, 'the degree of boundary maintenance between contents' (p. 159). When the boundaries are blurred, classification is weak, whereas strong classification has distinct boundaries. By way of example, this means that if you enter into a mathematics classroom you can immediately discern from the discourse being used, that it is a

mathematics classroom but if one finds a discussion on finance in mathematics classroom then there is likely to be confusion as to whether this is an accounting, business or mathematics class because the boundaries are blurred. In mathematics, the contexts that are used from the everyday, non-academic knowledge can create ambiguity in context recognition and it is a blurring of the boundaries. The classificatory principle therefore, strong or weak, will determine how one context differs from another. The classificatory principle provides the key to the distinguishing feature. It therefore orientates the speaker to what is legitimate, to what is expected. The recognition rule enables the reading of the context and a weakly classified context can create ambiguity in contextual recognitions. Without the recognition rule Bernstein (2000) says 'contextually legitimate communication is not possible' (p.17). The recognition rule enables appropriate realisations to be put together (Bernstein, 2000).

3.2.3.2 Realisation and Framing

Singh argues that:

> Realisation rules enable student to produce the legitimate text within the parameters established by specific pedagogic discourses. Students acquire realisation rules by making inferences about the procedures or principles of selection, organisation and evaluative criteria of pedagogic relations. Thus realisation rules are derived from the framing principle (Singh, 2002,p. 579).

The realisation rule determines how we put meaning together and how we make meaning public. Therefore, the realisation rule is necessary for one to produce the legitimate text.

Framing is described as the control over the selection, pacing, sequencing and evaluation. Where framing is weak, learners have more apparent control over the selection, sequencing and pacing and where framing is strong the teacher has more control over the selection, sequencing and pacing (Bernstein, 2000).

The evaluation criteria are a crucial characteristic of pedagogic practice at the school level. Evaluation criteria are rules which regulate the extent to which the legitimate text is made explicit or implicit to acquirers. Framing is strong when evaluation criteria are made explicit to the acquirer and is weak when evaluation criteria are implicit. Morais (2009) advocates for

strong framing at the level of the evaluation criteria and says it may lead the children to acquire recognition and realisation rules of the school context or subject. On the other hand, Davis (2010) reports that strong framing at the level of evaluative criteria does not necessarily give the learners access to recognition and realisation rules. What is critical, he argues, are 'the grounds', (which serve as reference for the mathematical objects) operated on were not made explicit. Strong framing of evaluative criteria enables access when the mathematical content of instruction is clear and coherent. A focus on framing without attention to what and how grounds reference content being taught, is not enabling.

Davis and Johnson (2010) have noted that learners' failure to produce the legitimate text is a result of the way mathematical objects are treated in class. They noted that there was not enough explication of grounds which serve as references for the mathematical objects, thus providing confusing evaluative criteria. They also noted that framing at the level of pacing was also very weak. On the contrary, Morais (2009) argues that weak framing at the level of pacing is one of the necessary conditions for it, directly or indirectly, allows for the explication of evaluative criteria. Therefore, understanding the evaluation criteria contributes to the production of legitimate text. The process of making the evaluation criteria explicit gives the learner an opportunity to acquire the legitimate text as well as how to give a correct answer when assessed, provided of course, grounds are sufficiently explicated.

Next is a discussion on how the tools provided by the theory are used and I indicate the exact places where my study is located within the theory. The following table (Table 2) is a summary of relevant constructs from Bernstein's theory of the pedagogic device for this study.

Table 2: Relevant constructs from the Pedagogic Device

Field	Construct	Consists of:	My study sources
Recontextualisation field	Pedagogic Recontextualising Field (PRF)	pedagogues in schools and colleges, departments of education, specialised journals, private research foundations	Mathematics education research and mathematics textbook
	Official Recontextualising Field (ORF)	created and dominated by the state and its selected agents and ministries	Curriculum and national assessments at grade 12 level.
Reproduction field	Evaluation	evaluation criteria for recognition and realisation of the legitimate text	a sequence of five lessons based on number pattern in a Grade 11 classroom

Within the recontextualisation field in the ORF I looked at the curriculum document for mathematics in the FET phase, which is Grade 10, 11 and 12. The notion of classification was used to look at the topic of number patterns across the FET phase in the curriculum document.

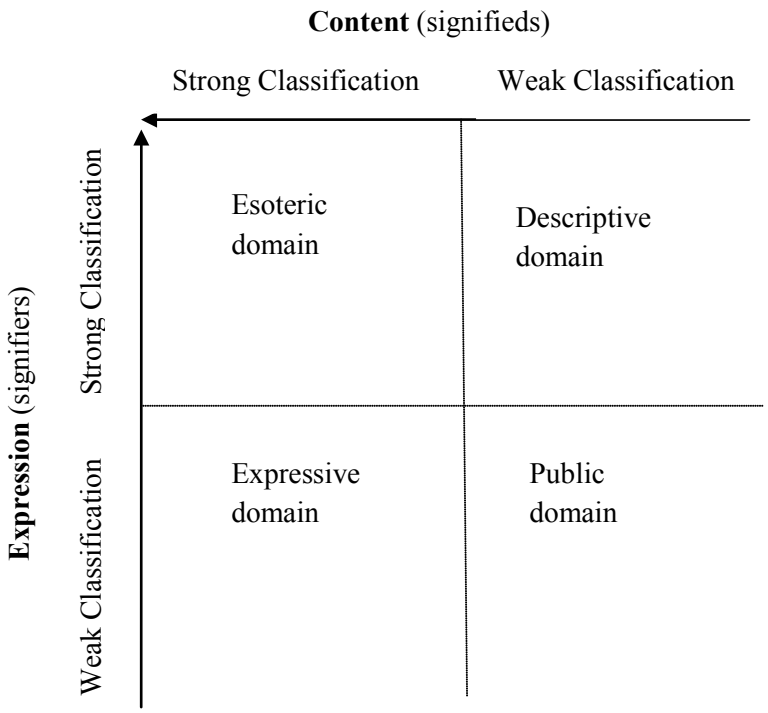
Still within the ORF discourse, I looked at the grade 12 national external examinations and in particular I am looking at how this topic of number pattern is examined because what is examined reflects the actual knowledge or legitimate knowledge learners are supposed or are expected to have acquired.

In the same recontextualisation field I am also looking at the PRF and within the PRF I am looked at the textbook used in the classroom that I observed. From this textbook, 'Classroom Mathematics Grade11', I looked at how the legitimate text is presented for this topic of number pattern. This is important because the textbook serves as an interpreter of the curriculum document for the teacher and suggests possible ways that a teacher can embark on when dealing with the topic of number pattern. And again, the textbook like the curriculum shows what the intended object of learning should contain but in more detail because it gives exercises, definitions and possible strategies and ways of explaining the content.

Still within the PRF discourse, I looked at mathematics education research to see what has been considered as the legitimate text for this topic by mathematics education researchers. As argued earlier in this chapter, the ORF and PRF both have an influence on what comes to be school knowledge independently of each other, therefore, it was necessary to look at both during my explanation of what is constituted as the legitimate text.

A further analytical tool that proved to be useful for comparing legitimate text as they appeared across the curriculum, the assessment, the textbook and the classroom was Dowling's (1998) classification of contexts and forms of expression. Dowling provides an elaboration of classification and so an external language of description. Dowling examined mathematical textbooks using this model. Dowling (1998) says classification for mathematical texts can be determine by examining both the form of expression (symbols, conventions and notation) the mathematical task takes and the content (academic or everyday/non-academic) from which the mathematical task draws from. Dowling uses the words content and context interchangeably; he explains that a non-mathematical content is a different context from mathematics e.g. economic practice of running a cafe, domestic practice of shopping. If both the form of expression and the context are mathematical, then the domain of practice is esoteric and classification at both levels (form of expression and content) is strong. When the form of expression and the context are non-mathematical then the domain of practice from which the teacher draws from is public, this means that the everyday non-academic knowledge and classification is weak. Both the expressive and the descriptive domains of practice draw from the esoteric and the public domain. The descriptive domain is where the form of expression is strongly classified, that means mathematical, and the context from which the text draws from is non-mathematical with weak classification. On the other hand, the expressive domain has strong classification at the level of context and weak classification at the level of form of expression used. See Figure 4 below.

Figure 3: Dowling's (1998, p. 135) domains of practice



Sourced from Dowling (1998, p.135).

The domains of practice were determined for each document and also for classroom data to describe the nature of classification. At the level of framing, I am looking at how the legitimate text is constituted in a sequence of five lessons based on number pattern in Grade 11. Specifically, I looked at what and how evaluative criteria come into play. With the aid of the construct 'evaluative events' as a methodological tool, which will be discussed in the next chapter, I looked at how the mathematics is constituted in the interaction between teacher and learners. Framing, as described earlier, has four components to it and my focus is on the evaluative criteria. The following table (table 3) summarises what I have discussed here.

Table 3: Framing

Framing			
Selection	**Sequencing**	**Pacing**	**Evaluative Criteria**
How are ideas within this topic selected?	How are ideas within this topic sequenced? What comes first and what comes last and what is privileged?	In terms of pacing what is given more priority, more time and less priority less time?	What criteria and how are criteria working?

The first three columns above were useful when it came to comparing what was constituted in the classroom and the curriculum, however, the main focus was on the last column because this addresses the question that I am asking is: what is the legitimate text and how is it constituted?

3.3 Conclusion

To this end, language of description includes both the internal, which are the concepts of classification and framing at the level of criteria from the theory, and the external, which is Dowling's elaboration on classification; Kieran's GTG model from the literature review and the framework from the literature (conventions, process and context). In this study, Davis and Adler (2006) and Adler's (2009) notion of evaluative events and mathematical objects was used as an external language of description, together with analytic resources emerging from the literature review in Chapter 2, and an elaboration of classification provided by Dowling (1998).This is summarised in table 4 below

Table 4: Languages of description

Internal language of description		**External language of description**
➢ Classification ➢ Framing at the level of evaluative criteria - Recognition rules - Realisation rules - Legitimate text		➢ Dowling's domains of practice - Esoteric - Public - Descriptive - Expressive
		➢ Kieran's GTG model - Generational - Transformational - Global/meta-level
		➢ Literature review framework - Conventions - Processes - Contexts
		➢ Evaluative events - Actions

The legitimate text in each document and in the classroom was studied according to its classification and Dowling's (1998) domains of practice were used to determine the classification. Kieran's GTG model (2007) was also used to determine the type of activity for-grounded in each document and the classroom. The notion of school algebra being an activity was also used to further segment the broader unit of analysis named evaluative event into more manageable units of analysis. The categories that came out of the literature review were also used to categorise and present the legitimate text in the documents and in the classroom.

The methodology chapter that follows, details the methods that were used to collect, organise the data into units of analysis, and analyse the data using the tools obtained from the literature and the theory.

Chapter 4 Methodology

4.1 Introduction

In this chapter, I discuss the research design, the methods used to collect the data and the methods used to analyse the data. The data for this study were obtained from different sources documentary and empirical. The data that are described in Chapter 5 come from several documents. The first document that was explored/ interrogated was the official national curriculum (NCS) document and the official national grade 12 assessments. Both of these documents come from the ORF as described in the previous chapter. The last document that I analysed was the textbook that was used by the teacher and it is a document that comes from the PRF.

My concern in this study is about what constitution of the legitimate text within the documents (noted above) as well as a sequence of lessons based on number patterns in a grade 11 class and opportunities created for learners to acquire the legitimate text. The documents mentioned here are analysed in the next chapter (Chapter 5) and the classroom data obtained is analysed in Chapter 6. A brief summary of the lessons is given towards the end of this chapter, together with an example of how these have been analysed, so as to illustrate the methodology in use.

Bernstein's (2000) theory only provides the language for describing phenomenon; it is an abstract language that is not easily operationalised. That means, the language for describing how these are to be seen is not provided in the theory of the pedagogic device. Bernstein (2000) refers to these, respectively, as internal and external languages of description. By internal language Bernstein is referring to theory as I have described it in the theoretical framework. I have used this internal language of description given by Bernstein's theory to discuss the documents analysed and the classroom data, together with the external language developed from Dowling (1998), Kieran (2007) and the literature framework. As Bernstein argued, the external language of description is derived from the internal language of description as data meets the theory. With respect to the classroom data, and in addition to Dowling, Kieran and the literature framework, the external language of description that I am using is derived from Bernstein's theory as described in Adler (2009). Adler & Davis (2006) and Adler (2009), use Davis et al. (2003) notion of an evaluative event as unit of analysis for studying the constitution of particular

knowledge objects, which are elaborated through engagement with both teacher education and school mathematics classrooms as empirical fields. The notion of mathematical objects and evaluative events provides researchers with tools to see evaluation criteria as these are transmitted (within an event). Looking at the evaluation criteria, implicit or explicit, is important for the purposes of describing how the legitimate text comes to be constituted in the classroom and how opportunities for learners to acquire the legitimate text are opened up. The classroom transcripts need to be divided up in some systematic way so that we can see what is unfolding. The methodology and language that I am going to use to this end enables a chunking of the data into more manageable units of analysis and is based on the notion of evaluative event as a unit of analysis; within each event there is an input object written on the board in this classroom or given as homework. This input object is acted upon to generate the algebraic expression which is then tested to see if it works.

Before I explicate further on the notions used to organise classroom transcripts into units of analysis called 'evaluative events' (Davis et al. 2003) I shall first start off by explaining the approach taken the methods used to collect the data, the setting, the sample, the procedures, and then the methods used to organise the data, issues of validity and reliability and ethical issues in that order.

4.2 Approach

The epistemological assumptions that I am making are that knowledge is acquired in social settings through social interactions. People are social human beings and therefore acquire knowledge through social interactions with each other; in this case, learners and the teacher co-constitute what comes to be the legitimate text through interactions. From Bernstein's theory of the pedagogic device, the sociological nature of knowledge and pedagogy has been explained. To recap: this theory explains how knowledge travels through different agents in different fields and comes to be constituted in the classroom. The approach therefore is a 'naturalistic qualitative research' that will produce text that represents real life *in situ* events of teaching, (Hatch, 2002). The focus of my study at this point is to disseminate the actual teaching processes the teacher undertakes with the intention of understanding how the teacher creates opportunities for learners to learn the legitimate text in the classroom. And the relationship of these processes to the legitimate text projected in the ORF and the dominant textbook in use.

4.3 Method

The method of this study is a case study because one classroom is observed with the same teacher teaching a sequence of five lessons. This method was chosen for the purpose of doing an extensive in-depth analysis of the teaching process (Opie, 2004). By focusing on one teacher and one classroom, this has allowed me to analyse what is constituted mathematically and how mathematical objects are structured and handled. Since this is a case study, findings that are obtained from this study will not be used to generalise because they are not representative of all teachers in South Africa. Within the boundaries of case study method, I will be looking at the teacher; the focus of the study is the teaching process and what gets constituted as the legitimate text.

4.4 Setting

This research was conducted in South Africa, in Johannesburg. Data was collected from a high school that serves learners who mostly come from a low socio-economic background in the inner-city part of Johannesburg. Learners in the school are responsible for moving between classes and they spend a lot of time on the way between classes. As a result lesson time is shorter than originally designed to be and it varies from day to day. All the learners (aged between 16 and 17 years) in the class that I observed have been given a mathematics textbook (Classroom Mathematics Grade 11) which they work from and are expected to return at the end of the year.

4.5 Sample

The sample was composed of one qualified mathematics teacher in the school described above who agreed to participate and be observed when teaching number patterns to grade 11. The reason for choosing a qualified teacher is to ensure that the teacher has a reasonable base of mathematical knowledge. Given this, the sampling process was therefore opportunistic because I was depending on the teacher agreeing to participate and be observed. The results obtained from this case study cannot be generalised since the sample is not representative of the population of mathematics teachers in South Africa. However, the depth of description allows readers to make decisions about aspects of the study that may be ‘transferable’ to other similar settings within the context of South Africa.

4.6 Procedures

Cohen, Manion and Morrison (2007) argue that policy documents are social products located in specific contexts and thus have to be interpreted and interrogated rather than simply accepted. Cohen et al. (2007) say understanding documents is a hermeneutic exercise. The focus of the study was to find out what comes to be constituted as the legitimate text in the classroom and this in the coming chapters is contrasted with what has been projected as the legitimate text from the official curriculum, the official assessment and the mathematics education research field

Data was collected over the course of a sequence of lessons on number pattern the topic lasted for five days – Monday to Friday.

The main data collection instrument for the collection of classroom data in this study was an observation that was recorded by videoing these sessions. The video camera focused on the teacher only. Although the learner voices was captured but visuals of the learners was not captured for ethical reasons. Of course, this was not possible to eliminate learners from the video completely as there were scenarios where the teacher required learners to go to the board and write some work there. The reason why a video was used is that I was looking to see how the teacher introduced mathematical ideas, structured his explanations, and responded to and evaluated learner contributions. A voice recorder can only capture the voice; writing on the board which is very important for the identification of the input objects cannot be captured, and what the teacher does (gestures) cannot be captured. To further justify the use of a video as the principal instrument for data collection, Denscombe (2007) says data that is obtained from an interview on its own is limiting in that it puts the researcher in a position where he/she has to rely on what the respondent will be saying. With videotape, the researcher gets first-hand information from observing phenomena rather than hearing what people say they do or think they do and it is for this reason that an interview was not used (Denscombe, 2007). Two researchers were in the classroom during this period. One researcher was taking field notes while I was videoing the lesson. According to Opie there is an advantage with videoed data because it can always be re-examined (Opie, 2004). In summary, the reason why a video is used is that it collects:

- All the talk in the classroom,
- Writing on the board is collected

- Gestures and the body language can be seen

All these are necessary if one wants to analyse what is constituted as the legitimate text and hence the use of the video as the main data collection instrument as opposed to an audio tape because it collects data that goes beyond what is said verbally.

4.7 Limitations of the data collection instrument used

I now discuss the disadvantages of the instrument used and how these were resolved. A disadvantage of observation according to Denscombe (2007) is that it opens up practice for scrutiny and it is not necessarily welcome by those being researched/ observed. To resolve this problem I gave the informants (teacher and Principal) a letter inviting them to participate and telling them that participation is voluntary, that they withdraw their consent for the study anytime they wished to, and this will have no negative consequences on their work or position if they do so. Also in the letter given to the informant it was stated that anonymity and confidentiality of himself and the school would be ensured in all writing and reporting (see appendix A for further details).

Another problem with observations is that they are unavoidably influenced by observer's presence and the people being observed will change the way they behave. This is true because a teacher will prepare to the best of his ability when knowing that they will be observed and this limitation for a study like this one cannot be avoided. The topic lasted for five days and after the first two lessons learners already knew who we were and what we're doing. Therefore, I cannot claim to have a fully representative record of what occurs on a daily basis in this classroom. As a way of eliminating bias and to avoid the researcher's influence on the interpretation that goes into the observation I have developed an external language of description to analyse and interpret the data. I did not have the power to intervene in the lesson; I was merely recording the lessons.

Denscombe (2007) noted that a disadvantage with video data is that a lot of data is gathered and irrelevances are picked up as well. So everything has been reported, meaning all the talk has been transcribed verbatim, but the focus of interpretation and analysis is on the part of the lesson that is mathematical. I now discuss the notion of evaluative events and input objects and how they were used to chunk and organise the data.

4.8 Organization of data

The unit of analysis builds on Davis et al.'s (2003) notion of an evaluative event with underlying theoretical rationale lying in Bernstein's theory of the pedagogic device, particularly the significance he places on evaluation. When following Bernstein's line of thought (that of evaluation being the whole purpose of the device), Adler (2009) elaborates that what comes to be constituted as mathematics in any pedagogic practice is reflected through the evaluation, 'through what and how criteria come to work' (2009, p. 6). How explicit or implicit the criteria for acquisition of the legitimate text are, is of particular importance in my study. Then Davis et al's (2003) notion of evaluative events and input objects as described in Davis (2011) was used to organize the data and identify the evaluation criteria.

Adler (2009) says 'an evaluative event is an evaluative sequence aimed at the constitution of a particular mathematics object' (2009, p. 6), so it is more like an objective the teacher is intending to achieve. For Adler (2009), the shift from one event to the next is marked by a change in the object of learning. In this study, there are numerous *input objects* (described in Davis, 2011) within the same event. The input object was often written on the board by the teacher or announced verbally or found in the textbook as part of the homework exercises. What was done on this input object has been named *actions* to align with Kieran's (2007) notion of school algebra being an activity. Steps enacted on input object are called actions and actions sequences on particular input objects and within particular evaluative events are analysed for the kinds of algebraic activity they represent in relation to Kieran (2007), and domains of practice in relation to Dowling (1998). Davis (2011) says operations are carried out on the input object and the final result is a transformed object. In the case of my study actions are carried out on the input object and the final result is an algebraic expression that represents the numeric pattern. In Kieran's (2007) terms this is more of a generational activity than a transformational one or global/meta-level.

The talk in the classroom for each lesson was transcribed. Following that the transcript was chunked into evaluative events and it became apparent that there was one major evaluative event across the five lessons for this topic that is: finding the general algebraic term/expression. However, there were many sub-events within this one event marked by insertion of new input objects, within each input object there were actions performed. To identify the input object, I

looked at what the teacher and learners were engaged in and focused on, and in most cases, it was written on the board or it was homework items from the textbook. To see how criteria came to work I looked at what the teacher endorsed and legitimised from the actions carried out on the input object and I looked at what the teacher drew the learners' attention towards.

Bernstein's theory of the pedagogic device as an internal language of description helps me describe and talk about this legitimate text, while Davis (2003), Davis and Johnson (2007), Adler and Davis (2006), Adler (2009) and Davis (2011)'s notion of evaluative event, input object as an external language of description help me see this legitimate text. Also, Dowling's (1998) 'domains of practice', and Kieran's (2007) notion of action and the GTG model as well as the literature review framework were the external languages of description which enabled me to categorise and present the legitimate text.

Table5 is a summary of the lessons showing the sub-events under the general evaluative event of finding the general term. The change in the sub-events is marked by a change in the input object. Each input object is acted upon to generate the algebraic expression. From the actions, the legitimating criteria of the teacher emerged. In the chapter on analysis (Chapter 6), I show that the teacher draws on mathematical content, conventions and processes to legitimate what they are doing.

Table 5: Summary of five lessons

Lesson 1	Evaluative Event	Input Object	Actions	legitimating criteria	Time Taken
	1.1 Finding the general term for linear	3; 6; 9 ...	Finding the next number by continuing the sequence from 12 until 18	- justify 12 - Multiples of three - Constant differences between successive terms - Meaning of three dots	00:00 – 18:00
			Giving position numbers for term1 to term 6	-how to write numbers according to position	
			Finding the tenth term	- justify - relationship between term number and subscript, how to generalise	
			Finding the expression for the nth term	- used to generate all other terms - conventions - procedure	
			Testing general term by substituting into the expression	-substitution into expression and procedure made explicit -name of the term given in terms of n and its role -how it will be assessed -not always n, can take any letter -substitute into it to generate the terms of the sequence -given in terms of an unknown -used to predict missing terms -some kind of order identified due to constant	

				differences	
		4; 7; 10 ...	Continuing the sequence by finding the next number after 10		18:00 – 30:00
			Finding the tenth term	- observation of pattern required - justify 31 - justify 41 - proven incorrect thru substitution - observation between subscript &term value required-ways of making observation made explicit	
			Observe subscript and output value		
			Observation tested		
			Observation written in symbolic form		
	1.2 Cubic sequence	1; 8; 27...	Not operated on	-Learners say "This is not a sequence" and the teacher decides to park the sequence. Not revisited over the sequence of five lessons.	
	1.3 Quadratic sequence	1; 4; 9; 16 ...	Finding differences		31:00 – 35:00
			Comparing quadratic and linear sequences	- Comparing linear constant differences and quadratic constant differences - Meaning of constant	
	1.4 Homework given		Finding the general term for linear patterns		35:00 – 37:00

Lesson 2	**1.5**Going over homework	3; 5; 7... -4; -2; 0; 2... 3; 7; 11; 15 ... 1; 4; 7 ...	Learners give their solutions and they are tested	- proving if the general term is correct - substitution	
			Inspection method used to find the solutions to homework	formula used to find the general term is called inspection	
	1.6 General terms for linear and quadratic functions – patterns	$y = mx + c$ $y = ax^2 + bx + c$ $T_n = 2n + 1$ $T_n = 4n^2 - 2n + 1$ 3; 5; 7; 9; 11; ... 3; 13; 31; 57; 91; ...	$T_n = an + b$ $T_n = an^2 + bn + c$ $T_n = an^3 + bn^2 + cn + d$ $T_n = an^4 + bn^3 + cn^2 + dn + e$	- linear function - linear equation - form taken by linear equation - constant differences for quadratic, cubic and quartic - contrasting of constant differences	
			Example of linear general term given by learners	- generating terms – conventions on how to write the staff - generating first differences	
			Example of quadratic general term given by learners	- generating terms – conventions on how to write the staff - generating first differences - generating second differences	
			Comparison of linear and quadratic constant differences3; 5; 7; 9; 11; ... 3; 13; 31; 57; 91; ...	- contrasting linear, quadratic and predicting for cubic constant differences	
Lesson 3	**1.7** Comparing linear and quadratic constant	3; 5; 7; 9; 11; ... 3; 13; 31; 57; 91;	Generating terms of the sequence from T1 to T5 from $T_n = 2n + 1$ and	- forms that can be taken by a general term - generating terms and how to write them	

	differences	...	$T_n = 4n^2 - 2n + 1$	- finding constant differences	
			Finding differences	discussing what is constant for linear, quadratic, cubic and quartic (n^4)-	
	1.8 Linear algebraic general term	3; 5; 7; 9; 11; ... $T_n = an + b$	T1 and T2 from $T_n = an + b$ are generated Equating algebraic T1 and T2 with numeric T1 and T2 A system of two linear equations are solved simultaneously to obtain the values of a and b.	- What is constant for a linear general term? - Constant first differences suggest general form that will be taken by the pattern - justify choice of $T_n = an + b$ - how to find T1 and T2 from general term - method for finding the values of a and b	
	1.9 Quadratic algebraic general term	3; 13; 31; 57; 91; ... $T_n = an^2 + bn + c$	Generating T1 to T5 from algebraic general term		
			First and second differences found	-second constant differences suggest a quadratic general term	
			T1 from algebraic and numeric pattern form an equation, first term in first differences from both numeric and algebraic patterns equated to form an equation Second differences from both numeric and algebraic patterns are equated to make an equation	-how to generate a system of linear equations to solve simultaneously -order and way of solving simultaneously	

			System of three linear equations are solved simultaneously to give the values of a, b and c.		
	1.10 Homework given		Using this method to find the general term for linear and quadratic patterns		
Lesson 4	**1.11** Going over homework	9; 11; 13; 15 ...	Learner finds first differences and concludes pattern is linear and writes $T_n = an + b$	-constant first differences justify the choosing of the following formula: $T_n = an + b$	
			T1 and T2 from $T_n = an + b$ are generated		
			Equating algebraic T1 and T2 with numeric T1 and T2		
			A system of two linear equations are solved simultaneously to obtain the values of a and b.		
			a and *b* are substituted back and general term tested		
			Teacher corrects use of equal sign	-teacher evaluates learner written work on the board by correcting three things: use of equal sign, division by invisible one and testing the general term	
			Teacher corrects division by invisible one		
			Teacher corrects how general term is written		
		1; 4; 9; 16 ...	First and second differences are found and the general term is		

			decided as quadratic because second differences are constant		
			Learner substitutes into the general term and finds the quadratic sequence from the general term, the first and second differences accordingly.		
			Learner equates the algebraic term 1 from the algebraic sequence to the numeric sequence to get equation 1. To get equation 2 the learner equates algebraic term 2 with numeric term 2. For the last equation learner equates algebraic second difference with numeric second difference from the number sequence and solves		
			Learner finds the values of a, b and c simultaneously		
			General term is tested		
			Teacher shows a short way of doing this	-evaluative criteria for a shorter method, that will not take time is shown by the teacher.	
	1.12 Relationship between two successive numbers	3; 6; 9; 12 ...		-a relationship between two successive terms is required	
Lesson 5	**1.13** Relationship between two	3; 6; 9; 12....	Observe something between T2 and T1	-what can you say about this in terms of that?	

	successive numbers		$T_2 = T_1 + 3$		
			Relationship tested if it holds for other terms		
			Relationship written symbolically with emphasis on knowing the starting point	Conventions on how to write	
			Recursive or iterative way of generalizing defined	Recursive and iterative are defined	
	1.14 exponential sequence	1; 2; 4; 8; 16 ...	$T_2 = 2T_1$		
			Written symbolically with emphasis on knowing the starting point	Conventions on how to write	
	1.15 Fibonacci sequence	1; 1; 2; 3; 5; 8; 13 ...	Generalised numerically and then symbolically	Conventions on how to write	
	1.16 Homework				

4.9 Validity and Reliability of the study

As stated before this study is qualitative and it is a case study and thus important to discuss the issues of reliability and validity. Cohen, Manion and Morrison (2007) say that reliability and validity are important for both qualitative and quantitative research. For validity, I address the issue of descriptive, interpretative and theoretical validity and for reliability I address the issue of whether the findings obtained from this study are generalisable. The descriptions of the data provided in Table 5 have been drawn directly from the transcript. In the transcript, the utterances are shown and these utterances are segmented into small units of analysis called 'actions'. These actions are instigated by teacher questions and explanations and they give rise to what I call the 'legitimating criteria' which are the descriptions obtained in Table 5.

The notion of how trustworthy is the study in describing what happened can be further supported by the data transcripts. The interpretation of the teacher's legitimating criteria as the teacher's legitimate text can be sustained by the theory used to analyse the data to enhance validity. The theory explains the research fully and external languages of description from other researchers are recruited to further explain phenomena in this study. How do we know that the research is worthwhile? The trustworthiness of the data collection has been enhanced by mechanical means of video recording the lessons. This instrument is useful because the data obtained can always be revisited as mentioned in the earlier section on procedures. For analysis, the trustworthiness has been enhanced by the amount of empirical evidence given in the analysis. The results obtained in this study cannot be generalised except for cases where similar settings exist then the results may be transferable to those kinds of settings if found useful in understanding them.

4.10 Ethical considerations

I had to approach the principal of the school and the mathematics teacher and request permission to collect data for research in their school and classroom. I am grateful to the school and especially the teacher who allowed me to enter into their space and observe and videotape his lessons based on number patterns. Full disclosure of information was given to the participating teacher in relation to the interests and intentions of the study and how the participant would be involved and what information was needed from him. In the consent form given to the

participant, it was mentioned that the participant had the right to withdraw from the study at any time they feel uncomfortable or threatened. The teacher found no reason to do so and we continued till the end of the topic. The learners in the teacher's classroom were informed verbally by the teacher that there would be two researchers in their classroom who would come with a video to record the lessons for the topic: number pattern, however their faces would not be captured but their voices were going to be captured because the focus of the study is about the teaching process and the constitution of the legitimate text within this topic.

However, for instances where the learners were required to go to the board and make public their work then their faces were captured, but their names were kept confidential. The risk for the teacher was that the informed consent forms were asking him to expose himself and his intellectual property in relation to how he was teaching under the scrutiny of the researcher who is also a mathematics teacher. Protection over this issue was given in the consent form which was promising the participant that the information gathered would not be used for anything else except for the research. To keep anonymity as promised in the consent form the school's name, teacher's name and the learners' names are not mentioned in the transcript. The consent form also mentioned how the findings from the research study would be disseminated (see Appendix A).

4.11 Conclusion

In this chapter, I have discussed the approach and the methodological tool used to organise data. At the moment I move on to the next chapter where I carry out the analysis of documents. The literature review framework, Kieran's GTG model and Dowling's domains of practice are used as tools to analyse the documents. Bernstein's notion of classification is also used to analyse the curriculum document. The analysis of the documents is interested in answering the question: what is constituted as the legitimate text in the documents?

Chapter 5 Analysis

5.1 Introduction

Following Bernstein (2000) this chapter is located within the field of recontextualisation. I analyse the following documents: official curriculum document, official assessments and the textbook. The first part is curriculum analysis which is within the ORF and what I was looking for in this chapter is the legitimate text with respect to number patterns. The next part is analysis of the G12 National Examinations within the ORF and the last thing that is analysed is the textbook within the PRF. These serve as resources from which the teacher draws from and it is important to see what they project as the legitimate text. The framework that came out of the literature (that is conventions, processes and contexts) is used to analyse and present the legitimate text projected in each document.

During the curriculum analysis, in addition to these three categories, an additional category (content) became necessary to add in. This expanded framework is used throughout this chapter together with (1) Dowling's model of the domains of practice reflected in a mathematical text, and (2) Kieran's GTG model, which is also used for the assessment and the textbook to determine the types of activities required by the texts. For both the PRF and the ORF I looked at the topic of number patterns. In the PRF, I looked at the G11 textbook; in the ORF I looked across the FET phase in the curriculum document; and in the Assessment I looked at G12 mathematics national external examination papers from 2008 and 2009.

5.2 Curriculum

Curriculum policy is seen as a key factor in showing the direction for teaching and learning. Taylor (1999) says that in a curriculum policy the aims of the policy makers are put down. Parker (2006) says that school curriculum documents show symbolic images of what the state regards as valuable knowledge and forms of transmission of that knowledge for schooling, thus harmonises with what Taylor (1999) stated. What Taylor (1999) refers to as the aims or intentions of the policy makers are viewed by Parker (2006) as symbolic images of what the state considers valuable knowledge. Following and in sync with Bernstein's theory of the pedagogic device, local communication for schools is thus a result of ideologies of dominant groups in society who decide on what should constitute school knowledge and how it should be transmitted

(Bernstein, 2000). Parker (2006) says that these images/aims are implanted within the formal mathematics curriculum statements, and these statements project what officially counts as legitimate mathematical knowledge, skills and values as well as legitimate pedagogic modes for acquiring those skills, values and knowledge. For this reason the statements from the curriculum document for this topic of number pattern across the FET phase will be taken as the legitimate text at the official level.

In the National Curriculum Statements (NCS)(DoE, 2003)the topic of number pattern is under LO: 1 for the FET phase (Further Education and Training) which is grade 10, 11 and 12. Learning Outcome 1 (LO: 1) states:

> When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions (Department of Education, 2003, p. 18).

In Grade 10 Assessment Standard 3 (AS 10.1.3) under Learning Outcome one states:

> We know this when the learner is able to: Investigate number patterns (including but not limited to those where there is a constant difference between consecutive terms in a number pattern, and the general term is therefore linear) and hence: (a) Make conjectures and generalizations; (b) Provide explanations and justifications and attempt to prove conjectures (Department of Education, 2003, p. 18).

In Grade 11, (the grade in focus for this study), the Assessment Standard (AS 11.1.3) states:

> We know this when the learner is able to: Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence: (a) Make conjectures and generalizations; (b) Provide explanations and justifications and attempt to prove conjectures (Department of Education, 2003, p. 19).

In Grade 12 the Assessment Standard (AS 12.1.3) states:

> We know this when the learner is able to: (a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series. (b) Correctly interpret sigma notation. (c) Prove and correctly select the formula for and calculate the sum of series, including:

$$\sum_{i=1}^{n} = n \sum_{i=1}^{n} = \frac{n(n+1)}{2} \sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}\left[2a + (n-1)d\right] \sum_{i=1}^{n} a.r^{i-1} = \frac{a(r^n - 1)}{r-1}; r \neq 1$$

$$\sum_{i=1}^{\infty} a.r^{i-1} = \frac{a}{1-r} \text{ for } -1<r<1$$

(d) Correctly interpret recursive formulae: (e.g. $T_{n+1} = T_n + T_{n-1}$)

(Department of Education, 2003, p. 19).

The sequencing at the level of content across the grades as seen in the curriculum document is that learners in grade 10 should be conversant with the linear pattern at least (following the statement 'not limited to'. As they progress to Grade 11, they are expected to know their way around the linear pattern and the processes of conjecturing, justifying, generalizing, explaining, and attempt to prove conjectures. In Grade 11, the same processes are expected but the only difference is that the quadratic pattern is included. Once again, the statement stresses the point 'not limited to' what is stated. This means that teachers can go beyond what is stated in the curriculum statements if they wish to in their classes. In Grade 12 we begin to see a more formal way of stating what is expected with exemplification of formulae needed and emphasis on notation for arithmetic and geometric sequences. What follows is a presentation of the legitimate text using the categories from the literature review. The analysis of the curriculum has added one more category and that is content.

5.2.1 Literature/Curriculum framework

From all this, what seems to be projected as the legitimate text from the curriculum document across the FET are mathematical processes related to linear and quadratic patterns, with formal content with conventions and notation for arithmetic and geometric series introduced in Grade 12. This is all summarized in Table 6 below and I give examples of what each sequence mentioned below entails including the mathematical processes involved.

Table 6: Legitimate text from the curriculum statements

	Grade 10	**Grade 11**	**Grade 12**
Content	Linear pattern	Linear and Quadratic sequences	Arithmetic, geometric and recursive sequences
Process	Investigate, make conjectures and generalizations, and provide explanations and justification, attempt to prove conjectures.	Investigate, make conjectures and generalizations, and provide explanations and justification, attempt to prove conjectures.	Identify, solve problems, correctly interpret and select formulae, prove, correctly interpret recursive formulae, calculate sum of series
Conventions			sigma notation, recursive formulae,
Contexts	Mathematical	Mathematical	Mathematical

Zazkis and Liljedahl(2002) have described this topic of number patterns as the heart and soul of mathematics and Driscoll (1999) describe this topic as the proper bridge for early grades between arithmetic and algebra. Previously it was not part of the core curriculum in many places including South Africa but now is. The following discussion is based on common patterns that are dealt with in South African high schools. The quadratic and linear sequences have been classified as constant difference sequences and the explanations for this are given.

5.2.1.1 Common Patterns

Constant difference sequences are ones that have a constant difference between any two consecutive terms. For linear sequences, where the general term is: $T_n = an + b$ or can be represented as: $T_n = a + (n-1)d$, where a is the first term and d is the common difference. However with the former equation ($T_n = an + b$) a is not necessarily the first term as it is the case with the latter ($T_n = a + (n-1)d$) but a is the value of the common difference and a is the value of the term that can be associated with T_0. The constant difference is in the first differences for

linear, while for quadratic sequences ($T_n = an^2 + bn + c$) the constant differences are in the second differences and for cubic sequences (where the general term is: $T_n = an^3 + bn^2 + cn + d$) the constant difference is in the third differences and so on, meaning for a polynomial that has the highest power of *n* being *p* then the constant differences will be the p^{th} differences, as shown in Figure 4 below.

Figure 4: Constant Differences

Sequence	1	3	5	7	9	
1st differences	2	2	2	2		
Sequence	1	4	9	16	25	
1st differences	3	5	7	9		
2nd differences	2	2	2			
Sequence	1	8	27	64	125	216
1st differences	7	19	37	61	91	
2nd differences	12	18	24	30		
3rd differences	6	6	6			

For all these, the value of '*n*' is defined as positive whole numbers or natural numbers and hence the use of the letter '*n*' as a convention that denotes natural numbers. From these sequences, it can be seen that the value of the highest power of the independent variable will be the row number for constant differences, for instance, in the preceding figure the highest power of *n* is three, therefore is a cubic pattern and the third differences are the ones that are constant.

Geometric sequences (where the general term is: $T_n = ar^{n-1}$), also known as the exponential pattern, are sequences where there is a constant ratio between successive terms. When it comes to finding the sum, a geometric sequence can be a converging geometric series or diverging geometric series depending on the value of *r* the common ratio (where *r* is obtained through the following formula: $r = \frac{T_{n+1}}{T_n}$). When the value of *r* lies between negative one and positive one ($-1 < r < 1$) then the sequence is converging and when the value of *r* is greater than and equals to 1 or less than and equals to negative 1 then the sequence is diverging. If the progression is

converging the sum of the series would get closer and closer to a particular number as more and more terms are added, whereas for a diverging series the sum does not approach any one particular value but increases or decreases. For instance the sequence: 1; 2; 4; 8; 16; 32; ... is a diverging geometric sequence because the value of *r* is 2 and the sum of the terms approaches infinity (gets very big), while the following sequence: $1;\frac{1}{2};\frac{1}{4};\frac{1}{8};\frac{1}{16};\frac{1}{32}$ is a converging geometric sequence because the value of *r* is $\frac{1}{2}$ and the sum of terms approaches 1. However, the formula for finding the sum of a diverging geometric sequence ($S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$ or $S_n = \frac{a(1 - r^n)}{1 - r}; r \neq 1$) is undefined when *r* is equal to positive one, therefore *r* may not be equal to positive one. So constant sequences which means patterns of this type: 1; 1; 1; 1; ... or 2; 2; 2; 2; … cannot be summed according to this formula.

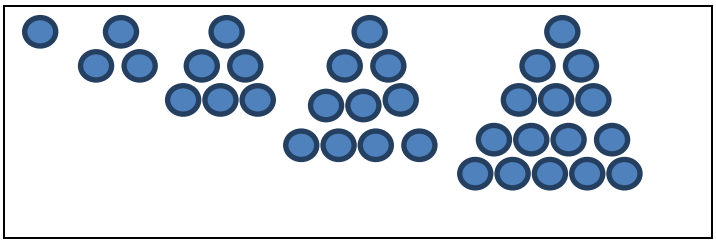
On the other hand, Fibonacci type sequences are sequences where the general term cannot be generalised explicitly in a global way within the context of school mathematics, but can only be generalised in a recursive manner. Consider the following sequence: 1; 1; 2; 3; 5; 8; 13; 21; ... This sequence can only be generalised in the following way $T_n = T_{n-1} + T_{n-2}; T_1 = 1 \& T_2 = 1$. This is a recursive way of generalising and is discussed in more detail in section 2.3.1. An explicit way of generalising this pattern is beyond the scope of school mathematics within the South African context. Interesting to notice is the fact that across all types of sequences there is a focus on deriving generalised terms in a global way where possible, and where it is not possible to generalise globally using the recursive method of generalising. With geometric and linear sequences, there is a focus in deriving the sum of the series S_n, but not in others.

5.2.1.2 Mathematical processes involved

Mathematical processes involved in this kind of work are observing and looking for a pattern, continuing the pattern once the structure has been seen, generalising the pattern algebraically, finding a faraway term and finding the position when given the term value. In Grade 12 this work is taken a step further to determining the sum of the series for arithmetic and geometric sequences. For geometric sequences learners have to discern the difference between converging

and diverging sequences referred to earlier. Learners need to be aware of the many ways that a pattern can be continued when given a certain number of terms. Consequently, number patterns presented pictorially are less ambiguous than number patterns presented as number (Samson, 2008). For instance the pattern 1; 2; 4; … can be continued in different ways: it can be continued as 1; 2; 4; 7; 11; 16; 22; … or 1; 2; 4; 8; 16; 32; 64; … and so on. This makes it necessary to give more than three terms when the pattern is presented as number. Consider the example in Figure 5 below where the numerical pattern is 1; 3; 6; 10; …

Figure 5: Pictorial representation of triangular numbers



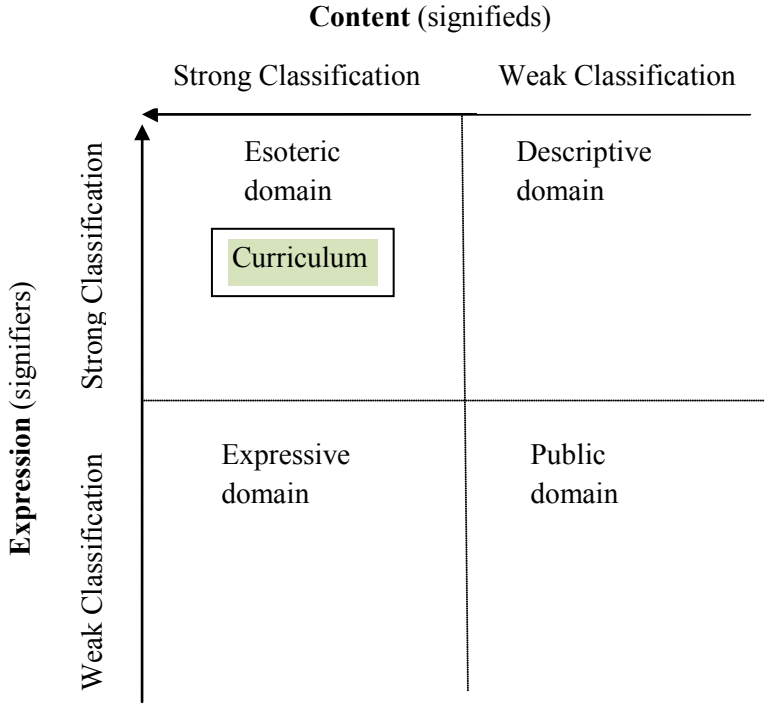
Learners can immediately recognise that they can continue the pattern by adding one more marble to form the next bottom row. The next picture should have six marbles in the first row of the base of the triangle giving a total of 15 marbles and so on.

5.2.2 Classification and Dowling's domains of practice

Curriculum is defined by Bernstein as what counts as legitimate knowledge and classification is important in terms of access to recognition rules of this legitimate knowledge as discussed in the theoretical framework. Strong classification means there are no ambiguities in context recognition and weak classification can cause confusion in terms of context identification and hence legitimate communication within this ambiguous context may not be possible. Classification as defined by Bernstein is the degree of boundary maintenance between contents and strong classification means that the boundaries are clear and weak classification means that the boundaries are blurred. For this study, Dowling's model is used to elaborate and operationalise the notion of classification. From Dowling's domains of practice (see figure 6 below) it is clear that this topic, as seen in the curriculum document, does not draw from everyday non-academic knowledge for any of its formulations of what is to be taught/learned.

The form of expression is specialised and the content is specialised. So the domain of practice taken is esoteric. Classification is strong because the curriculum is interested in specifying content, mathematical processes and conventions. This means that the topic stands on its own in that it has its own unique voice and the boundaries are clear. Access to recognition rules is made possible because there is no ambiguity in context recognition.

Figure 6: Domains of practice in the curriculum



However, it is worth noting that the 'not limited to' phrase in Grade 10 and 11 weakens the classification because it is not clear what then is included and excluded. There is thus a weakening of the boundaries at the level of content (types of patterns that may be included or excluded).

The statement 'we know this when the learner is able to' from the curriculum document suggests a performance model according to Bernstein (2000, p. 44) which is focusing on what the learner can produce and what is absent in the learners product. Learners' inability to produce what is expected of them by the curriculum is failure to achieve the ASs and it (the statement: 'we know this when the learner is able to') suggests an absence of the legitimate things that were supposed to have been in the learners' product. Therefore, one can infer from the phrase: 'we know this

when the learner is able to' that these AS's are specific outputs the learner is expected to produce: the ASs are the legitimate text.

Shulman (1986) in his concern about types of knowledge that are needed to teach in the most effective way counts, among others, curricular knowledge. Here, Shulman (1986) is not referring to the curriculum document as Parker and Taylor have referred; he is referring to curricular knowledge and he says it is represented by the complete variety of materials designed to teach a particular subject and topics at a given grade, the set of instructions that indicate the proper and improper use of the material (e.g. the teacher's guide). Shulman (1986) says that curriculum, and the types of material that categorize it, is the primary resource from which the teacher draws tools of teaching and which give exemplars of particular content and sequencing and evaluations of learner achievement. Within the context of my study these "necessity appeals" as Adler and Pillay (2008) put it, are documents which contain the legitimate text. In describing this knowledge Shulman (1986) gives the analogy of a physician: just as a physician is expected to know and understand the full range of treatments available as well as the range of existing options for particular circumstances of sensitivity, cost, convenience, safety, or comfort (Shulman, 1986), so is the educator expected to know all the materials available including research done and particular didactic strategies recommended by both policy and research. In the case of my study, the full range of available resources is important for a teacher to have a better idea of what legitimate forms of knowledge are available as offered in different types of material. In the next two sections I will look at some of the curricular materials that are available for a teacher. I begin with an analysis and discussion of the G12 National Senior Certificate Examinations for the year 2008 and 2009 and then the textbook.

5.3 Number patterns in the 2008 and 2009 official examination papers

The purpose of a curriculum document is to inform those who produce assessment instruments for measuring what has been learnt at classroom, district, provincial or national levels (Taylor, 1999). Assessment texts, such as past examination papers help teachers to know what is expected and so how to prepare learners. To prove just how important assessment inferences are made about the quality of teaching that learners went through from the results of the assessment, For teachers, assessment is an indication of which topics are highly valued and teachers in most schools use assessment at national level as a guide to decide on the amount of time they spend to

teach each topic depending on how much it is valued in the assessment. In this way, the assessment texts, determines much of the work learners will be expected to do and affects the approaches taken by teachers choose to teach the curriculum content. This shows just how important and crucial a role assessment plays in education in general and particularly in mathematics education. Bernstein (2000) argued that the purpose of the device is to evaluate and most educators are aware of this. Content from grade 10 and 11 is assessed in the national G12 examinations and that is why it was appropriate to look at G12 national assessments. These examinations/assessments have importance as they provide the secondary school exit certificate which is used as entrance into institutions of higher learning. As several authors cited here have noted, the assessment at national level is a projection of the legitimate official forms of knowledge learners are expected to have acquired (Parker, (2006) and Taylor, (1999)).

The first group of Grade 12 learners from this NCS curriculum was assessed in 2008 and which is the reason I only start looking from 2008 to 2009 papers in this study. In mathematics a number of new topics were going to be tested for the first time at a national level. Those topics included: transformation geometry, statistics and probability in paper two. In paper one, number pattern used to be tested as arithmetic and geometric sequences and series in Grade 12 only. The notion of the theory of finite differences was not part of the syllabus, now this topic runs from Grade 1 up to Grade 12 in the curriculum.

What follows is a brief discussion of the preliminary examination papers and the final examination paper for the year 2008 and 2009.

5.3.1 Literature/Curriculum Framework and Assessment

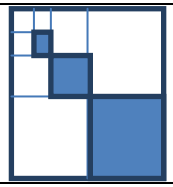
The geometric converging ($-1 < r < 1$) patterns are mostly introduced via a diagram or a pattern with variables instead of numbers. However, the linear, quadratic and geometric diverging patterns are introduced as number. Number patterns in the 2008 preliminary exam (DoE, 2008a) are introduced as mixed sequences, one arithmetic and the other constant. In the November paper (DoE, 2008b) the mixed pattern is geometric and arithmetic. This ‘mixed’ form of testing number patterns did not appeared (again) in the DoE (Department of Education) exams in 2009 (DoE, 2009a, 2009b). Quadratic patterns are introduced in all three papers by giving the first five numbers of the sequence, with the first question asking for a continuation of the pattern and the

second question asking for the 10th term (which pushes for a 'near' generalisation), and then a closed generalisation as the question asks explicitly for what the n^{th} term will be. The last question for quadratic patterns in all three papers asks for 'n' when given the value of T_n. That means that learners have to know that the value of 'n' is always a positive integer, especially in the case of quadratic patterns where one has to, sometimes, choose between two different values.

The exponential/geometric number patterns are introduced as a story or a word problem and similar types of questions as in quadratic patterns are asked. There are two instances where learners are asked to justify by proving/explain their thinking.

The analysis of the examination paper content is summarized in the following table (table 6), identifying content, process, conventions and contexts for questions in each of the examination papers. For more detail for how this topic was examined, see Appendix C.

Table 7: Number pattern in the national papers from 2008 to 2009

	2008 (Prelim)	2008 (Nov.)	2009 (Prelim)	2009 (Nov.)
CONTENT				
Linear	Mixture of arithmetic and constant: 1; 2; 1; 5; 1; 8; 11; …	Mixture of arithmetic and geometric: ½; 4; ¼; 7; 1/8; 10; …	1; 1; 3; 2; 5; 3; 7; 4; ... $\sum_{n=1}^{50}(2n-1)$	T: 5; 9; 13; 17; 21; … Word problem about a sequence that is both arithmetic and geometric with a = 1. $\sum_{t=0}^{99}(3t-1)$
Quadratic	Quadratic: 3; 6; 11; 18; 27; …	Quadratic pattern: 8; 18; 30; 44; …	1; 5; 11; 19; ...	Quadratic: -3; -2; -3; -6; -11; …
Geometric				
- **converging**	Converging geometric series: $8(x-2)^2$; $4(x-2)^3$; $2(x-2)^4$; … $x \neq 2$	Converging geometric series: $8x^2 + 4x^3 + 2x^4 + ...$		Word problem and a table given based on growth of a certain tree.
- **diverging**	Word problem based on sum of money for doing homework.			M: 5; 125; 3125; 78125; 1953125; …
PROCESSES				
Linear	10^{th} term, sum of the first 50 terms	Next two terms, sum to 50 terms	Next two terms, Calculate	n^{th} term for both T and M, justify your answer, first three terms, S_{99},
Quadratic	next two terms, formula for the general term, use formula to find p if the p^{th} term is = 627	which term is = 330, n^{th} term	Next two Find the n^{th} term Which term of the sequence is 2549	
Geometric				
- **converging**	Determine x, sum to infinity if $x = 2{,}5$	Which value of x will the series converge, sum if $x = \frac{3}{2}$	First 3 terms from diagram Calculate the sum	explain, show that 312 is converging height of tree
- **diverging**	Is it worth his time? Learners are required to reason if the proposal is worth Kopano's			n^{th} term for M, justify your answer,

	time.			
CONVENTIONS				
	$8(x-2)^2$; $4(x-2)^3$; $2(x-2)^4$; ... $x \neq 2$ p^{th}	$8x^2 + 4x^3 + 2x^4 + ...$ n^{th}	$\sum_{n=1}^{50}(2n-1)$ n^{th}	$\sum_{t=0}^{99}(3t-1)$ n^{th} S_{99}
CONTEXTS	For linear, quadratic and converging geometric series the context is mathematical, that is the sequences are expressed numerically and symbolically, but for a diverging geometric series the context is nonmathematical and is based on "incentivising performance through money", and so the context is money.	All the contexts used in this paper for this topic were mathematical	The context for all the sequences presented here is mathematical, however the context for the converging geometric series is based on fractal geometry	The geometric converging series has a nonmathematical context based on forestry or gardening. The rest of the sequences have a mathematical context

Content

Questions like these (1; 2; 1; 5; 1; 8; 1; 11; …) have content drawn from G10, (linear sequence) but also incorporate some of the hypothesising and justifying in the process strand. These mixed patterns elevate the level of complexity of the problem in terms of recognition of pattern and continuation. In each paper, for each year, the quadratic pattern is tested, that is, content is drawn from G11 according to the curriculum. The geometric sequences are tested, and to find the sum, learners have to discern if the sequence is a diverging or converging geometric series. Recursive types of sequences have not been tested in these papers.

Contexts

Learners should be familiar with different ways of introducing a pattern. As shown in the last row, contexts do not always appear as number, but also as a word problem or in a diagram that needs to be read and understood in order for one to be able to generate the number sequence. In addition, a range of everyday contexts are used to represent a sequence. Specifically money as a context has been used, such as forestry or gardening, and fractal geometry has been used as contexts. Kieran (2007), as discussed in the literature review, describes these kinds of problems as generational activities of school algebra, that is, activities which involve generating the number sequence from the story (word problem) or diagram. Warren (2000) noted that when a sequence is presented like this, where other contexts are used either than number, the processing load is increased.

Processes

Mathematical processes like justifying, substituting and so on take place during the processes of finding the general term and testing if it works. Learners need to be able to continue the number pattern. Learners need to recognise the kind of pattern formed by the numbers to be able to continue it and then generalise globally by finding the algebraic representation of the pattern which is the general term. A learner needs to be able to solve for an unknown (specifically ‘n’ and they should know that ‘n’ is a positive integer in the case of quadratic equations) from the algebraic representation of the pattern. These, according to Kieran (2007), are the transformational activities of school algebra. There was only one question that I have noted where learners are given a statement and are required to generate and provide proof to support

their answer in the 2009 November paper. This question is an example of a global/meta level activity of school algebra according to Kieran's model of school algebra.

Conventions

I have categorised questions like $\sum_{n=1}^{50}(2n-1)$ and $\sum_{t=0}^{99}(3t-1)$ as conventions. Similarly, questions like $8x^2+4x^3+2x^4+\ldots$ and $8(x-2)^2$; $4(x-2)^3$; $2(x-2)^4$; ... $x \neq 2$ are categorised as conventions as well. There are also conventions for noting the sum to a number of terms (S_{99}) and particular terms (p^{th} and n^{th}). Learners have to know and understand the symbolism embedded in sigma notation. Learners have to know that the (+) sign in-between the terms denotes a series and the (;) mark in-between terms denotes a sequence. The mathematical meaning of: S_{99}, p^{th} and n^{th} have to be known as well. So, the symbol system that Watson (2009) referred to here, if not understood, can be a serious impediment.

5.3.2 Dowling's domains of practice

When looking at the assessments according to Dowling's domains of practice, it is clear that the official assessment draws from all of the domains of practice. This means there are problems from the esoteric; the descriptive; the expressive and the public domain of practice. I will give four examples, an example of each of these domains of practice, from the assessment to support my point.

Figure 7: Example 1 – Esoteric domain of practice

Consider the following sequence: 3; 6; 11; 18; 27; ... 4.1 Determine the 6th and 7th terms of the given sequence, if the sequence behaves consistently. (2) 4.2 Determine a formula for the general term, *p*, of the sequence. (4) 4.3 Use your formula to calculate *p* if the p^{th} term in the sequence is 627. (4) (Taken from the DoE/Preparatory Examination 2008, question 4 page 4 of Paper 1)

The content is explicitly written as 3; 6; 11; 18; 27 ... This is a quadratic number sequence. The first process learners have to go through is extending the sequence by finding the next two terms

of the sequence. The next process is to generate the algebraic expression that represents the pattern in terms of p. The last process is solving for *p* and in the process learners have to know that *p* is a positive number. p^{th} *term* is a convention that is used to denote the general term in this topic. The context from which this problem draws from is specialised mathematical context and the form used to express the problem is specialised and mathematical. The use of 6^{th} and 7^{th}, the use of *p* to denote natural numbers and p^{th} *term* are forms of expression used in mathematics. The demands made by this kind of a problem are fairly straightforward if a learner understands the symbol system and what it means. This shows that when the domain of practice is esoteric, then the demands made by the problem are relatively low because classification is strong. By demands, I am referring to the processing load (Warren, 2000) made by the problem because of the non-academic problem contexts drawn on or used to express the problem. Therefore an esoteric domain of practice is unambiguous when it comes to recognition of the context and has gains for learners because it opens access to the recognition rule of both the context and form of expression. If the symbol system (Kaput, 1989) is mastered by the learner there is a possibility of being able to produce the legitimate text within this domain of practice.

An example of a problem that falls within the public domain of practice is shown in Figure 8 below.

Figure 8: Example 2 – Public domain of practice

5.1 Kopano wants to buy soccer boots costing R800, but he only has R290, 00. Kopano's uncle Stephen challenges him to do well in his homework for a reward. Uncle Stephen agrees to reward him with 50c on the first day he does well in his homework, R1 on the second day, R2 on the third day, and so on for 10 days.

5.1.1 Determine the total amount uncle Stephen gives Kopano for 10 days of homework well done. (5)

5.1.2 Is it worth Kopano's time to accept his uncle's challenge? Substantiate your answer. (2)

(Taken from the DoE/NCS Preparatory Examination 2008 Question 5 of Paper 1 page 4)

The content of this problem is a diverging geometric sequence with a common ratio of 2. Learners have to process the sequence from the word problem. Processes that learners have to go through are expressing the sequence, finding the sum of the first ten terms of the series and

justify their answers by arguing if the offer should be accepted. There are no conventions used in the expression of problem. From the story learners, have to generate the sequence and recognise that it is a geometric diverging sequence. The first question could be expressed as: find the sum for the first ten terms. In this case, the expression would be strongly classified; but the question says: *Determine the total amount uncle gives Kopano for 10 days of homework well done*. So both form of expression and content/context are non-mathematical and therefore this problems falls within the public domain of practice. The context could be thought of as "incentive based performance" where an incentive like money is given provided there is some sort of improvement/achievement; or as a means of encouraging people to be industrious. There are no mathematical symbols and conventions used to express the problem. The demands made by this problem are, therefore, high: the weak classification at both levels implies that the legitimate text is not explicit in this problem.

Example 3 below is an example from the descriptive domain of practice.

Figure 9: Example 3 – Descriptive domain of practice

Question 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increased by 18 cm. In each successive year , the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	Second year	Third year	Fourth year
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)
5.2 Calculate the height of the tree after 10 years (3)
5.3 Show that the tree will never reach a height of more than 312 cm (3)

(Taken from the DoE/NCS November 2009 Question 5 of Paper 1 page 4)

Presented here is a table with data that shows the height of the tree over four years. Differences between successive heights are found and growth is given in centimetres. The content presented

in the table is a geometric converging series. So the first two questions require learners to continue the sequence. This is a process known as near generalisation (Driscoll, 1999) within mathematics education literature which means learners are being pushed towards finding the overall general term by first finding the nearby terms. And the last question requires learners to find the sum. In the process of finding the sum, learners have to notice that this is a converging series and choose the correct formulae. In the phrasing of the problem, there are no mathematical symbols that are used, but a table is used to display the data. The context, which is content in Dowling's terms, is a non-mathematical context based on growth of a tree, and the broader context within this could be gardening or forestry; therefore classification at the level of context is weak. However classification at the level of expression is mathematical because of the table that is used to display the data, so classification at the level of form of expression is strong. The demands for realisation made by this problem are very high especially if one is not familiar with the context of how trees grow. However, the demands for recognition made by this problem are not high because some of the numeric data are presented in table form. In the next example, I discuss example 4 from the expressive domain in Figure 10 below.

Figure 10: Example 4 – Expressive domain of practice

Question 2 2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term of the sequence is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5) (Taken from DoE/November 2009 page 3, question 2.2 of Paper 1)

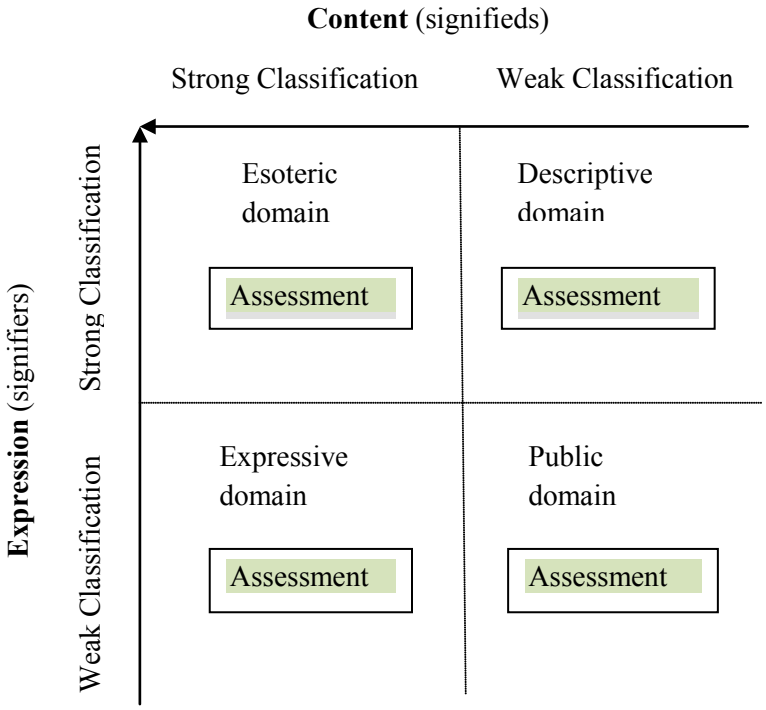
The content here is a sequence which is both arithmetic and geometric and the first term is 1. The processes involved are generating the sequence from the information given and proving that the generated sequence is the one Nomsa is referring to. The process of justification is used to prove as Ellis (2007) mentioned that these two processes are bi-directional. There are no conventions used to phrase the problem. I have categorised this problem as expressive because the context is strongly classified, it is a purely specialised mathematical context that is used to generate the problem, while the form of expression weakly classified. There are no mathematical symbols and notation used in the problem. The demands made on the learners by this problem are very high because it requires learners to do a lot of processing. For example, learners have to know that the

sequence has a difference of zero for it to be arithmetic and a common ratio of 1 for it to be geometric, and if this is the case then the sequence is a constant sequence of 1's (1; 1; 1; 1; 1; 1; ...). To communicate all of this, a learner must be able to write using the language of expression – the symbol system of algebra. The demands made by the problem at the level of recognition are high when the content/context drawn from is non-academic everyday knowledge and the form of expression used is non-mathematical.

The criteria I am using to say the demands are high or low depends on the context and the form of expression used to formulate the problem, if the context is non-mathematical then the processing load is increased thus making the demands high at the level of recognition. If the expression is non-mathematical the processing load is still increased and so the demands are high. Consequently the esoteric domain is the one with relatively low demands because it has neither of these conditions.

From the four problems, two have weakly classified contexts that are the descriptive and the public domains of practice and the demands made by both were high. The expressive had a strongly classified mathematical context with weak classification at the level of expression and the demands made by the problem were high. So we can see here that the legitimate text in the assessment texts takes on different domains of practice with different and obviously this has serious implications for teaching and learning. Figure 11 below illustrates this.

Figure 11: Dowling's model for the national assessment



Sourced from Dowling (1998, p 135)

5.3.3 Kieran's GTG model and the NCS assessment

Within the assessment, different activities are emphasised by different problems. Which activities are dominant and which ones are not, for instance, the problem that lies in the esoteric domain has the transformational activity as the dominant activity. The processes involve extending the sequence by finding the next two terms, generating the algebraic expression that represents the pattern in terms of p, this is a generational act, and solving for p - this is transformational. From the three processes required by this problem, two of the processes can be classified as transformational and one process can be classified as generational and that is why I said the transformational activity is the most dominant activity within this problem. The problem that fell within the expressive domain of practice has the global/meta-level activity as the dominant activity. Learners have to prove and justify by working out if the answer they choose is the correct one. Both these processes (proving and justifying) fall under the global/meta-level activities of school algebra in Kieran's model. So we see that some problems will have all three

activities at play and the dominant activity can be easily determined sometimes and maybe not in some cases.

The analysis across all the FET grades of the national assessment has shown that the types of problems offered cover all four domains of practice that is the esoteric, descriptive, expressive and public. From Kieran's GTG model, it is clear that the there are problems which require global/meta-level thinking as much as there are generational and transformational activities. Also content from G10 and G11 is tested as well as G12 content. The analysis of the national assessments showed that problems that fall within the descriptive, expressive and the public domain actually have higher demands because of the processing load and the type of activity that is fore-grounded is one that requires thinking at a meta-level.

5.4 The textbook

The textbook is created by agents in the recontextualising field and, in particular, the PRF as mentioned in Chapter 3. Taylor (1999) says a curriculum document is a guide, not only for teachers as they plan their day-to-day classroom activities, but for textbook writers as they decide what material to make available to support teachers and learners. An effective textbook therefore serves as a tool that translates curriculum guidelines given by authorities at state level into activities for classrooms and this happens in the field of recontextualisation in the PRF. A textbook is an interactive part within the activities of teaching and learning. Textbooks and the curriculum often determine what school mathematics is for teachers and learners (Ensor, Dunne, Galant, Gumedze, Jaffer, Reeds (2002)). Textbooks provide indications of learners' opportunities to learn depending on what is made available in the textbook in terms of content and the processes involved. Therefore, textbooks are a link between curriculum and pedagogy and textbooks, like assessment, have an impact on what is taught and how it is taught[2].

Across the FET, this topic of number patterns is in the first chapter of the textbook, with the first work starting with number, exponents and surds and then number pattern in G10 and G11. In G12, it is the first chapter learners are expected to start with. Work schedules from the

[2] Textbooks remain the major resource for mathematics teachers see (Askew, Hodgen, Hossain, & Bretscher, 2010)

department of education to schools also put it as one of the first topics that needs to be done beginning of the year and one week is allocated for the teaching of this topic.

The Classroom Mathematics G11 textbook has detailed the topic of number pattern in more depth than seen in the curriculum because for Grade 11 there are more patterns included in the textbook than in the curriculum. The general structure of this topic in this textbook is mainly organized into activity, general discussion and example which lead back to activity and towards the end more lengthy and complex exercises are given which are mixtures of all the types of sequences discussed. See Appendix C.

I begin the textbook analysis with the framework that emerged from the literature (conventions, processes and contexts) and the curriculum (content). A textbook chapter is typically structured to have three parts: there is an introductory section which introduces the topic to be learnt, there is a middle section which they work on it and there is a consolidation/concluding section which sums up what they were working on and sometimes introduces the next idea. So I looked at all these trying to find out what is legitimated in each section. Classroom Mathematics is the most dominant textbook in South Africa and therefore has a significant impact on what is learnt by most learners in the country. In the G11 textbook, the topic of number pattern is a topic within the first chapter starting from page 12 to page 20 which makes it 8 pages of textbook analysis. This is presented in table8 below.

Table 8: Number Patterns in Classroom Mathematics Grade 11

ACTIVITY	CONTENT	PROCESSES	CONVENTIONS	CONTEXTS
Introduction	3; 6; 9; 12; ...; $3n$; ... is given as an ordered list of numbers defining a sequence. Finite and infinite sequences are defined		T_n is explained to indicate terms while n indicates the position of the term. $T_1 = 3$ Is the 1^{st} term $T_2 = 6$ 2^{nd} term $T_3 = 9$ 3^{rd} term and $T_n = 3n$ is an equation expressing the general term, it is a rule for finding all the other terms of the sequence.	The context used in the introduction section is purely mathematical. There are no contexts used from the everyday non-academic domain.
Activity 1.8	2 Linear, 3 quadratic and 1 cubic number sequence	Extend, explain and give a rule or the formula for the n^{th} term	n^{th}	Mathematical context for all sequences given
General discussion and example		This is a discussion about finding differences and the example is given: 2; 5; 10; 17; 26; where first and second differences are found		The example uses a mathematical context
Activity 1.9	3 linear and 3 quadratic sequences.	Find differences, find general formula and conjecture about the type of general term obtained and constant differences		Mathematical context for all sequences given
General discussion and example	This discussion is based on constant differences and the type of general term of the highest power of n The example is given: 2; 5; 10; 17; 26; ...	The instruction is to find the general term 2 5 10 17 1^{st} differences 3 5 7 9 2^{nd} differences 2 2 2 The next thing is comparison of terms of the sequence with squares of natural numbers: 1; 4; 9; 16; 25; ... n^2 ... and first and second differences are found	n^2 $T_n = n^2 + 1$	The general discussion does not draw examples from the non-academic context but the discussion and the example are based on a mathematical contexts

		The general term is found to be $T_n = n^2 + 1$		
Exercise 1.10	16 sequences are given of which half of them is linear and one quarter is quadratic and the remaining quarter is cubic.	Learners are required to extend the sequence, investigate 1st, 2nd and 3rd differences; give reasons for deciding on a linear, quadratic or cubic general term and find the formula for the general term Number two is asking learners to generate sequences which have linear and quadratic general terms and swop with a partner who will investigate between terms and decide whether the general term for each sequence is linear or quadratic.		All the sequences are numeric and the context is mathematical
Using recursion to define sequences	The sequence: 2; 5; 7; 12; … is given. The sequence 1; 1; 2; 3; 5; … is given and it is known as Fibonacci sequence named after the person who discovered it Leornado Fibonnaci.	An explanation on how to continue the sequence (by adding the previous two terms each time).	The expression $T_n = T_{n-1} + T_{n-2}$ ($T_1 = 2; T_2 = 5$) is given and the reader is told that the terms of the sequence can be generated from the definition provided the first two terms are known.	The context used to introduce Fibonacci type sequences is mathematical
Exercise 1.11	The first two terms are given for five sequences	Learners to generate eight terms using the definition: $T_n = T_{n-1} + T_{n-2}$ The last two questions on this activity are word problems, the first one requires learners to show that the sequence maybe Fibonacci while the second one requires learners to find a recursive rule	$T_n = T_{n-1} + T_{n-2}$	The context is mathematical and the last two problems are presented as word problems instead of number sequences
Activity 1.12	A dot pattern is given	Learners are required to study and extend, describe the properties of the		The context is mathematical. The

		sequence, find formula (recursively and explicitly) for generating the sequence. Generate two of their own sequences similar to the one given swop with partner who must give the formula recursively and explicitly.		sequence is represented as number but as a diagram of dots
General discussion	First thing noted is that the sequence does not produce constant differences but a constant ratio. Learners are told that they will learn more about these in grade 12.		$a;ar;ar^2;...;ar^{n-1};...$	Mathematical context
Exercise 1.13	Includes sequences like $x-3;x-1;x+1;...$ $am^4;am^5;am^6;...$ $\frac{8}{x^6y};\frac{16}{x^8y};\frac{32}{x^{10}y};...$ $\frac{1}{2};\frac{2}{3};\frac{3}{4};\frac{4}{5};\frac{5}{6};\frac{6}{7};...$ and a diagram of a bouncing ball and a dot pattern of triangular numbers. A drawing using match sticks, a triangle showing a pattern of odd numbers. The last item is a story about teams and the number of matches they will play.	Learners are required to investigate constant differences and the ratio of successive terms.	$x-3;x-1;x+1;...$ $\frac{8}{x^6y};\frac{16}{x^8y};\frac{32}{x^{10}y};...$	Bouncing ball diagram– playground context , dot pattern diagram– mathematical context, matchsticks diagram– mathematical context , a sports context is used as a context
Exercise 1.14 – CHECK YOUR SKILLS	Number sequences as whole numbers, negative integers, mixed fractions and pentagonal numbers are given as a sequence of numbers followed by a diagram showing the pentagons.	Learners are required to determine whether the sequences are linear, quadratic, cubic, and geometric or none of these. Extend sequence by three terms. Find the expression of the general term.		Pentagons diagram used as a context
Exercise 1.15 APPLY YOUR SKILLS	These kinds of patterns are given: $\sqrt{x};\sqrt{\sqrt{x}};\sqrt{\sqrt{\sqrt{x}}};...$ and this kind of a	Extend the sequence	$\sqrt{x};\sqrt{\sqrt{x}};\sqrt{\sqrt{\sqrt{x}}};...$	Ancestral tree context is used

	sequence is given: $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}...$ A word problem based on a family tree and generations of ancestors.			
Activity 1.16 PROBLEM SOLVING	A spiral of isosceles triangles and the second item is a story about Egyptian fractions called unit fractions: $\frac{2}{3}=\frac{1}{2}+\frac{1}{6};\frac{2}{9}=\frac{1}{6}+\frac{1}{18};\frac{2}{15}=\frac{1}{10}+\frac{1}{30};...$	Explore the sequence that is formed by the length of the successive hypotenuses. Continue the sequence of Egyptian fractions and find the general formula.		Isosceles triangles context used

5.4.1 Literature/Curriculum Framework in the Textbook

The method that is used to find the general term for a quadratic pattern in the textbook can only be used for patterns with the general term: $T_n = an^2 \pm k$. The textbook does not provide an alternative method of finding the general term for a quadratic pattern that takes the form: $T_n = an^2 + bn + c$. However, Exercise 1.10 has quadratic patterns that will take the general form: $T_n = an^2 \pm k$, but in the *check your skills* exercise there are problems like triangular, rectangular and pentagonal numbers which take the general form: $T_n = an^2 + bn + c$. Therefore, the following is legitimated in, and across all the sections for this topic, in the textbook is summarised in Table 9 below.

Table 9: Textbook summary according to literature and curriculum framework

Content	**Process**	**Conventions**	**Contexts**
-Linear/ Arithmetic - Quadratic - Cubic - Exponential/Geometric -Fibonacci sequences	Investigate constant differences Generating own sequences, Determine whether the sequences are linear, quadratic, cubic, and geometric or none of these. Extend sequence by three terms. Find the expression of the general term.	n^{th} $T_n = 3n$ $T_n = an^2 \pm k$ $a; ar; ar^2; ...; ar^{n-1}; ...$ $T_n = T_{n-1} + T_{n-2}$ $T_1 = 2; T_2 = 5$	Number Negative integers Whole numbers Mixed fractions Improper fractions Proper fractions Decimal fractions Picture Bouncing ball Matchsticks Dots Ancestral tree Spiral isosceles triangle Table Triangle with shading but not a fractal. Story – word problems

Recursive as it is used here is referring to Fibonacci type sequences which cannot be generalised explicitly (within the context of school algebra) but recursively; therefore they are recursive types of sequences. While some of the sequences, linear and exponential, can be generalised in both ways. For the recursive formula and the explicit formula, the textbook does not provide a

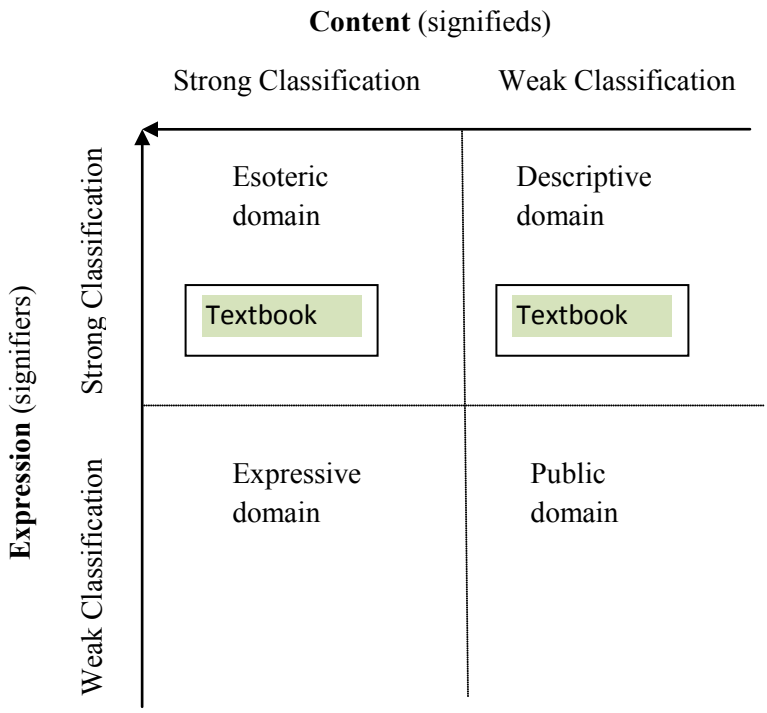
discussion on how to compare the two so that learners may see the disadvantage of defining the general term in recursive terms only. So, it is clear that at the level of content, the textbook is specific. In the case of the exponential or geometric pattern, the textbook specifies that will be learnt in more detail in Grade 12 but gives extensive exercises on this kind of pattern anyway. The textbook is explicit on notation and conventions and the processes learners have to learn.

We see that in the Grade 11 Classroom Mathematics textbook, there are more sequences covered, some of them come from the Grade 12 content as specified in the curriculum, for example: the recursive and exponential or geometric. The textbook has also included the cubic sequence which is not part of the sequences specified in the curriculum. Therefore, the textbook has recontextualised the curriculum and has not limited the content to linear and quadratic sequences.

5.4.2 Dowling's domains of practice and the textbook

The domains of practice in the textbook for this topic are esoteric and descriptive. There were no problems that drew from everyday knowledge with non-mathematical expressions which fall under the public domain of practice. There were no problems that have strong classification at the level of content and weak classification at the level of expression which fall under the expressive domain. For the descriptive domain stories from real life contexts, like the family tree problem (non-mathematical context) were used but the form used to express these is essentially mathematical (the symbol system is used to express the problem), therefore, classification is strong at the level of form of expression and weak at the level of context.

Figure 12: Domains of practice in the textbook



5.4.3 Kieran's GTG model

There are contextualised problems in '*APPLY YOUR SKILLS'* and '*PROBLEM SOLVING'* sections and include all the activities of school algebra that is generational (learners have to do a lot of processing to get to the number sequence), transformational (in the processing stage, the sequence is transformed and represented in another manner – numeric and algebraic) and global/meta level (all the problems involved here require justification, generalisation and proof processes).

The analysis of the textbook has shown that the textbook has problems to solve that covered only the esoteric and descriptive domains of practice. In mitigation, the textbook had specified content in more detail than seen in the curriculum. From Kieran's model, the textbook has problems to solve which covered all three types of the activities in the GTG model. The textbook has recontextualised the curriculum in a useful way, but Dowling's model shows that the textbook can do better by providing problems which cover all of the four domains of practice. For a textbook, this is possible because it is not a referral document like the curriculum document.

Therefore, the textbook is expected to go beyond the minimum requirements stipulated in the curriculum document, and also include problems that fall within the expressive and the public domain.

5.5 Comparison of all three documents using the tools

To end this documentary analysis, I compare all three documents using the literature review framework and curriculum. The next table is a comparison of the three documents analysed in this chapter using the literature review/ curriculum framework.

Table 10: Comparison of all three documents

	Curriculum	**Assessment**	**G11 Textbook**
Content	G10 – linear G11 – linear and quadratic G12 – arithmetic geometric and recursive	Linear, Quadratic, Geometric	Linear/Arithmetic, Quadratic, Cubic, Exponential/Geometric, Fibonacci (recursive) type sequences
Processes	Investigate, make conjectures and generalizations, and provide explanations and justification, attempt to prove conjectures	continuing sequence, finding the general term, finding the position of a term, finding the sum, justifying, proving and conjecturing	continuing sequence, finding the general term, justifying, proving and conjecturing, generating own sequences
Conventions	Sigma notation and recursive formulae	Sigma notation	recursive formulae
Contexts	No contexts suggested	Money, forestry/gardening, fractal geometry	Geometric diagrams, dots, matchsticks family tree, sport context

From the table above, we see that for the context category the curriculum has no suggestions but the assessment and the textbook use contexts from real life and contexts from other topics within mathematics to express the problem. With regard to 'conventions', there was alignment across the documents. Similar processes like justifying, proving and conjecturing are specified for all three documents. However, the textbook did not include the process of finding the position of a specified term value in a sequence where the general term is known. This process is always

tested for the quadratic sequence in the assessments and finding the sum is a process tested at G12 level. For content, the curriculum is specific about content that should be covered in each grade and for G11 linear and quadratic patterns were specified as content. The textbook covered all that is mentioned as content within the phase (FET – G10, 11 and 12) in the curriculum document.

Figure 13: Domains of practice across documents and classroom

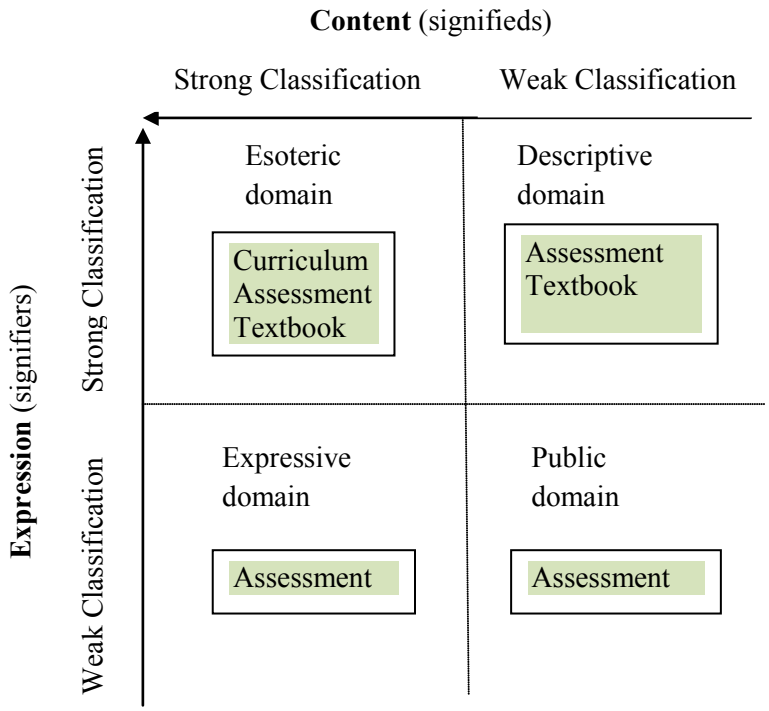


Figure 13 above shows that there is no alignment between the three documents. The assessment draws from all domains of practice as I had shown in the analysis of the assessment. The textbook draws from two domains of practice. The misalignment is creating a noise already for a teacher who needs to be using these and different teachers will have different preferences. However, the nature of the curriculum document makes it hard to go beyond the esoteric domain because the curriculum is a document that specifies minimum requirements for content.

Kieran (2007) across the documents

The following table is a comparison of the documents using Kieran's GTG model.

Table 11: Comparison of all three using Kieran's (2007) GTG model

	Generational	Transformational	Global/meta-level
Curriculum	√	√	√
Assessment	√	√	√
Textbook	√	√	√

This model determines the type of activity and so for the curriculum the activity can be inferred from the types of processes specified. From the curriculum processes like: 'justify and attempt to prove' fall within the global meta-level activities of school algebra. 'Generalising, investigating and making conjectures' make the generational and the transformational activities of school algebra. Therefore, the specified processes suggest different types of activities within Kieran's GTG model. From the assessment, it was clear that problems that fell within the expressive, descriptive and public domain had high demands and the activity was one that required meta-level type of thinking. The textbook also offered a range of problems which cover all three types of activities in Kieran's GTG model. Therefore, the boxes ticked in table 11 indicate a presence of each activity from each document.

5.6 Conclusion

In this chapter I have discussed what the PRF projects as the legitimate text for number patterns in the textbook that was used, which was also the most dominant textbook in mathematics within the context of South Africa. I have also described what the ORF projects as legitimate in the curriculum document and the national assessment texts for the topic of number pattern. It became clear that these three things discussed here do not necessarily speak with one voice, one emphasises a particular aspect while the others are concentrating on another. As for the curriculum, it is simply a policy document that presents the minimum requirements and is not helpful beyond that. It is also of interest at this particular juncture, how this legitimate text that has been projected by these documents is constituted in the classroom. Because these documents

only serve as the intended object of learning and how this intended object of learning is enacted is another story that will be told in the coming chapter.

However, I want to note that the level of incongruity portrayed by these documents is not ideal and definitely not useful for a teacher in the classroom. Because the textbook might be going beyond the curriculum requirements and the teacher may not follow this, because he/she (the teacher) would be under the assumption that the curriculum document is a supreme document, while the assessment concurs with the textbook specifications. On the other hand, teachers have the responsibility of interpreting the curriculum document but phrases like ‘not limited to’ can be confusing because they weaken the classification at the level of content. So this creates additional work load for teachers having to consult and interpret so many documents mentioned here and those not mentioned here for what is legitimate to do, say and transmit to learners. The next chapter focuses on how the legitimate text for the topic of number patterns in a G11 class was constituted.

Chapter 6 Classroom Analysis

6.1 Introduction

This chapter is located within the field of reproduction, to use Bernstein's terms. In this chapter I am going to start by foregrounding the lessons through what the teacher taught and how he has transmitted criteria and then I am going to draw from the transcripts as evidence in support of what I have called the legitimating criteria. In this chapter I show that what has been transmitted as criteria is the legitimate text for this teacher. I will then move on to relate what has been constituted as the legitimate text by the teacher to what was constituted as the legitimate text in the documents. Also using the curriculum framework and Kieran's model I elaborate further on the constitution of the legitimate text for this teacher. Using Dowling's notion of domains of practice I go back to what is constituted as legitimate text in the classroom and how it relates to the curriculum, assessment and textbook.

What follows is the data description, and it shows how criteria are working in relation to the input object and actions carried out on the input object. As this was happening the teacher transmitted criteria about whether what they were doing was correct and incorrect to do, either verbally or written down in book or on the blackboard.

All the events within this topic illuminate how criteria are working; they also illuminate the categories in the curriculum framework and Kieran's model. For evidencing how the legitimating criteria come through I have selected from the first lesson the first input object (3; 6; 9...) and I discuss it from beginning to end. In Lesson 1, the teacher introduced the topic and hence gave a lot of explanations and asked more questions to lay the foundation. Therefore, the first input object (from lesson 1) seemed a good choice to use here because it contained more elaboration than other input objects (lessons) that follow. In all lessons, the actions were instigated by teacher questions/explanations and that is the focus in this chapter, it tells about how the teacher elaborates and legitimates the object.

As elaborated in Chapter 4, the lessons can be summarized as follows:

Table 12: Summary of lessons and legitimating criteria

Lessons		**Main input objects acted on**		**Legitimating Criteria**
lesson 1		3; 6; 9; 12; 15; ... 4; 7; 10; 13; 16; ... 1; 8; 27; ... 1; 4; 9; 16; 25; 36; ... H/W		- justify 12 - Multiples of three - Constant differences between successive terms - meaning of successive - Meaning of three dots -how to write numbers according to position – conventions - justify - relationship between term number and subscript, how to generalise - conventions - used to generate all other terms - conventions - procedure -substitution into expression and procedure made explicit -name of the term given in terms of n and its role – conventions -how it will be assessed -not always n, can take any letter – conventions -substitute into it to generate the terms of the sequence -given in terms of an unknown -used to predict missing terms -some kind of order identified due to constant differences - observation of pattern required - justify 31 - justify 41 - proven incorrect thru substitution and dismissed - observation between subscript &term value required-ways of making observation made explicit – conventions - teacher parks the sequence because learners can't recognize a pattern - Comparing linear constant differences and quadratic constant differences - Meaning of constant h/w
lesson 2		3; 5; 7; ... 3; 6; 9; ... -4; -2; 0; 2; ... 3; 7; 11; 15; ... 3; 5; 7; 9; 11; ... 3; 13; 31; 57;		- proving if the general term is correct - substitution - strategy used to find the general term is called inspection - linear function - linear equation - form taken by linear equation - constant differences for quadratic, cubic and quartic - contrasting of constant differences - generating terms – conventions on how to write the staff - generating first differences - generating terms – conventions on how to write the staff

		91; ...		- generating first differences - generating second differences - contrasting linear, quadratic and predicting for cubic constant differences
lesson 3		3; 5; 7; 9; 11; ... 3; 13; 31; 57; 91; ...		- forms that can be taken by a general term - generating terms and how to write them - finding constant differences - discussing what is constant for linear, quadratic, cubic and quartic (n^4) - What is constant for a linear general term? - Constant first differences suggest general form that will be taken by the pattern - justify choice of $T_n = an + b$ - how to find T1 and T2 from general term - method for finding the values of a and b - second constant differences suggest a quadratic general term -how to generate a system of linear equations to solve simultaneously -order and way of solving simultaneously - h/w
lesson 4		9; 11; 13; 15; ... 1; 4; 9; 16; ... 3; 6; 9; 12; ...		- constant first differences justify the choosing of the following formula: $T_n = an + b$ - a system of two linear equations are generated and solved simultaneously - teacher evaluates learner 1's written work on the board by correcting three things: use of equal sign, division by invisible one and testing the general term - a system of three linear equations are generated and solved simultaneously - teacher evaluates learner 2's written work by showing a shorter method which elaborates on which equations to choose when general term for a quadratic pattern - a relationship between two successive terms is required as a specific way of generalizing
lesson 5		3; 6; 9; 12; 15; ... 1; 2; 4; 8; 16; ... 1; 1; 2; 3; 5; 8; 13; ...		- a relationship between two successive terms is required as a specific way of generalizing - conventions on how to write the recursive definition - testing by substituting input values - recursive and iterative defined - constant differences not a method - conventions on how to write - Fibonacci- conventions on how to write

The dominant practice across the five lessons is finding the general term. This is done in different ways across the five lessons. Firstly, the linear pattern generalisation is derived by inspection; then both linear and quadratic patterns are generalised algebraically. Lastly a recursive method is used to generalise three different patterns - that is linear, exponential and Fibonacci-type patterns.

Now what I am going to do is to draw from the transcript to evidence how criteria are being transmitted within action sequences on particular input objects. As mentioned earlier actions on input object 1 are drawn on to evidence the teachers practice because it is in lesson 1 that more questions and explanations are given which illuminate the criteria for the teacher.

The first seven actions that I show from the transcript are from the beginning of lesson 1 (input object 1: 3; 6; 9; …) up until the end of the input object 3; 6; 9 ... where the teacher is telling learners other ways that the general term can appear in the assessment. This is a continuous chunk that is a sub-event under evaluative event 1, however this chunk has been segmented into actions and these actions form the headings of the episodes.

6.2 Data presentation and discussion

6.2.1 Input object 1.1, Action 1: Finding the next number

00:00-02:39 *T: Right, suppose you are given a list of numbers starting with.* (Teacher writes on the board 3; 6; 9…). *Somebody tell me the next number?*
Lrs: 12
T: Somebody?
Lrs: 12
T: 12? He says the next number will be 12. Anybody who does not agree? (Teacher puts his hand up and pauses, no response from the learners; teacher folds his arms and asks). *But how do we know its 12? Suppose somebody comes in from a distance and says its 13. Why 12? Why not 13? Brian?*
Brian: (inaudible)
Lrs: They are multiples of three 3; 6; 9; 12
T: Okay all the numbers are multiples of three. Okay you are all right. The first number is three (teacher pointing to the list of numbers written on the board 3; 6; 9; 12;)*, the next one is six and a*
Class: 9
T: 9. Somebody has made an observation to say, 'Okay there is a difference of 3 between any two successive numbers ehe'
Lrs: Successive numbers?

T: Eeeh between any two numbers one coming after the other. Any two successive numbers, there is a difference of three suggesting the next term must be
Class: 12
T: 12. And after 12 there must be number...?
Class: 15
T: 15. And after 15?
Class: 18
T: 18. And so on (teacher writes … after writing 18 on the board3; 6; 9; 12; 15; 18; …) *and so on. When we say and so on we indicate by* (teacher points to the three dots written on the board)*?*
Lrs: Dots
T: And there must be three dots. The three dots there, what do they indicate? There are many more terms we have left out there. There are many more terms that we have not written not necessarily three. There are many more numbers that follow the same pattern that may be written there but they are not necessarily three. It's okay? Right!

The sequence 3; 6; 9; is written on the board and the teacher asks what the next number is. In the process of finding the next number learners mention that these are multiples of three. The teacher evaluates this by re-voicing learner's response that an observation of constant differences of three has been made. From this it is clear that the notion of multiples can generate a term but is not sufficient to generate the next term in the sequence of successive numbers. Also the notion of multiples applies to some linear sequences, but constant difference notion applies to all linear sequences, so the teachers' re-voicing has extended generality. So the evaluation criteria around continuing a sequence are the notions of successive numbers and constant differences.

What we saw next is the teacher explaining the meaning of successive terms and the meaning of the three dots after 18. This shows that the teacher is working with mathematical content to explain the meaning of words and mathematical conventions to explain the meaning of the written dots to legitimate what they are doing in class. Successive is defined as a sequence of numbers one following another and the three dots indicate that there are many more terms that have not been written, not necessarily three. Here we see that the teacher was stressing the successiveness of the numbers in the sequence and hence multiples of three is not a completely legitimate justification for the next number being 12, but constant differences of three between successive numbers are and apply to all linear sequences. Also, what is apparent is that the notion of an infinite sequence comes through in the teacher's talk but it was done implicitly. The

next action that follows after the sequence has been extended to 18 is about giving position to values in the sequence.

6.2.2 Input object 1.1, Action 2: Writing numbers in the sequence according to position

02:41-03:16 *T: So we have said the first number we have there we shall call it term one, term number one* (teacher writes on the board T_1). *So what is the first term we have there?*
Class: 3
T: The first term is a three. Second term?
Class: 6
T: Second term is six good. Third term is?
Class: 9
T: Third term is 9, fourth term?
Class: 12
T: Firth term?
Class: 15 (this is written on the board: T_1 T_2 T_3 T_4 T_5
3 6 9 12 15)

Here, the teacher tells the learners that the first number in the sequence is called 'term number 1' and he shows how to write this. In this way, the teacher is drawing on mathematical conventions and makes it explicit to the learners how values in the sequence can be written according to position. Also, the data was originally written as single variational data and now is written as two variational data suggesting there must be some relationship of some sort between these two variables that he expects learners to observe and describe (Warren, 2000). The teacher's push towards generalisation becomes more evident in the next action that follows, which is based on finding the tenth term. Therefore, the legitimating criteria here are that the teacher is making explicit the link between term and its position and transmitting mathematical conventions on how to write terms in relation to their positions.

6.2.3 Input object 1.1, Action 3: Finding the tenth term

03:18-04:21 *T: Right! Okay. Shall somebody tell us what the tenth term will be? Oh somebody tell us the tenth term what will it be? What will the tenth term be? Term number ten?*
L: (shouting) *you multiply ten by 3*
T: Term number ten? Yes
L: 30 Sir
T: 30. How do we know?
L: 10 multiply by three
T: Okay, somebody has made an observation to say if its term number one term number one is a
L: three
T: Three. Two is a six term number three (teacher says all this while underlining the subscript of T and the value of the sequence).
T_1 T_2 T_3 T_4 T_5 ... T_{10} ...
3 6 9 12 15 30
There appears a pattern, a relationship between the term number and (pointing to the subscript and the term value) *good!*

In this activity, the teacher requires learners to find the tenth term now that a conventional way of writing has been discussed. In the process of finding term number ten the teacher wants learners to justify their answers. This gives him a clue into learners thinking about this topic. The teacher re-voices the learner's response by showing a specific way that the rest of the learners should be using to find the tenth term. He does this by underlining the subscript number and the term value and brings learners attention to the appearance of a relationship between the two variables. So the teacher transmits criteria not only on how to write the ideas but also on how to observe a relationship and generalize. The next thing the teacher requires learners to do is find the nth term which is the action that follows.

6.2.4 Input object 1.1, Action 4: Finding term number n

04:22-05:00 *T: What about term number n? What shall term number n be?*
Lrs: 3n (learners shouting)
T: No, no, no,shhhshhshh! Term no n? By now you should know that somebody must raise their hand then they…yes, yes
Lrs: 3n
T: 3n term number n is 3n. Another observation there. It looks like it's simply this (pointing to position number) *by three to get that* (pointing to term value). *This by …*
Class: 3
T: to get that. It's okay? We have got one two three four five six terms in this sequence there are many more that we have not written. Right it's okay? Right. Then there is this term written in terms of n (teacher circles this term on the board). *That term given in terms of n yah we are saying term number n will be equal to…*
L: 3n

Overall, there is a move from 'near-generalisation' (finding 10^{th} term) to overall closed explicit generalisation in this sequencing. Here the teacher was re-emphasizing two things (alongside insisting on order and taking turns when speaking), that those were: (a) observing subscript and term value and (b) revisiting the notion of infinite sequence. The emphasis on these two things illustrates that at this point the criteria that the teacher wants learners to know and remember was the notion of mathematical conventions when it comes to writing and meaning of what is written. This clearly demonstrated by the notion of three dots, the teacher wants learners to write them and know what they mean. The teacher is again transmitting criteria on how to generalize using the functional way/ explicit/global way (Driscoll, 1999; Warren, 2000). So the method that the teacher and learners are using to find the general term for a linear pattern is observation of a relationship between subscript number and term value. Now that the general term has been generated, the next action is to see if it generates the terms of the sequence.

6.2.5 Input object 1.1, Action 5: Using term number n to generate the terms of the sequence

05:01-10:52 *T: 3n. We can use this term to generate all the other terms. It's okay? From this term we can generate all these terms from this term, we can use this term,* (teacher is pointing to T_n = 3n on the board) *term number n to generate all these terms* (pointing to sequence on the board 3; 6; 9; 12; 15; 18…). *How do we do that? We want to find term number one* (teacher writes on the board T_1). *How do we find term one? How do we find T_1? How shall we find term one from this term here? How shall we find term number one? We know what it is but how shall we find it from this term* (pointing to T_n = 3n). *We want to find term number one.*

L: Sir

T: Okay (class discusses and the teacher approaches one learner to listen and reports back). *He suggests we divide by n both sides* (teacher goes to the board and writes what the learner is saying). *Divide by n which side and which side? This side* (pointing to T_n)*?*

$T_n = 3n$

$n \quad n$

$T_1 = 3$

L: yes

T: this side by n, by n. Ok, oh alright (with a surprised tone) *by n and n to say n into n*

L: T_1

T: Oh then we have T_1, ok? is equal to 3, then we have term number one equal to three. Anybody who does not agree with that? Yes, so you agree

L: yes

T: Oh he agrees mhh lets leave it there suppose we want to find T_2 we have got T_n = 3n lets find T_2 we know what it is. We know term number two is six.

L: Sir what if we substitute n by two sir these …

T: substitute

T: Okay you got term number one first time (pointing to the learner who suggested dividing by n) *by dividing both sides by n. what about term number two?*(teacher pauses and waits for a response). *How shall we get term number two? We know it is six. How shall we get six now?* (Teacher keeps quiet and looks at the learner for some time. There is mumbling in the classroom) *you see it fails, it fails. In fact, why is it not correct to divide both sides by n* (teacher writes T_n = 3n on the previous portion of the board)

L: three and n are multiplying Sir

T: What is multiplying what?

L: three and n

T: What is 3n (pointing to 3n on the board)*?*

L: three multiply by n

T: 3n means 3 multiply n. what about this? (pointing to T_n)

L: no Sir
T: What about this. What about this (pointing to T_n)
Class: mumbling in the classroom
T: he says 3n is three multiply by n and I agree to that, what about this (pointing to T_n)
Class: (class discusses, some say T is multiplied by n and others disagree)
T: Okay, okay alright I see what you are meaning. Quiet! Shhh. Alright in this case (underlines 3n on the board). *3 is multiplied by n* (teacher writes on the board $3 \times n = 3n$). *3 by n will give…*
L: 3n
T: And T multiplied by n equal to Tn it's okay. Will give us Tn. This n here (pointing to $T \times n = Tn$) *is different from this n here* (pointing to T_n) *it's okay?*
Class: Yes
T: This n here is indicating the term number it's a subscript (n from T_n = 3n is circled). *There is a difference between this small n here and this* (writing on the board $T_n Tn$). *These two T is multiplied by n* (underlines Tn) *but in this case* (pointing to T_n) *just like when we have a superscript like this* (writes on the board 2^3), *what does it mean? This does not mean 2 is multiplying?*
Class: chorus 3
T: When we have something like this (pointing at T_1) *in our case this does not mean T is multiplying 1. It simply indicates the term number to say this is term number one. It's okay, right, and this one is term number two* (writing on the board T_2). *This one will be term number three* (writing on the board T_3). *Note, note, note where the three is, it's different from this* (writing on the board T.3), *different it's okay, right.* ($T_1 T_2 T_3$ is written on the board)
There is a difference here (teacher erases the board). *So this one* (pointing to: T_n) *is term number n. Not to say T is multiplied by n. So here* (pointing back to the original problem on the board) *T is not multiplying n, it is term number n. Makes sense now? Good. So we cannot divide both sides by n.*

Within this activity, the teacher and learners are trying to generate the terms of the sequence, but notation and its meaning gets in the way. As a result, the teacher sidetracks and tackles the current problem which is notation and its meaning. In doing this the teacher highlights the meaning of Tn and how Tn is different from T_n and what Tn and T_n indicate. This activity shows the advantage of opening up opportunities for learners to say what they are thinking. The teacher did this by asking the question: "*We know what it is but how shall we find it from this term?"* Hence, the teacher could attend to their misconceptions and correct them by making explicit the

mathematical conventions on how to write and their meaning. At first, learners seemed convinced by the method until the teacher required that the learner find term number 2 using the same method. There is silence and the teacher tells the learner that: *'You see it fails, it fails'* and starts correcting the misconception. This is an important part of the lesson because the lesson cannot continue with learners not understanding what the notation and symbolisation mean and how to read it and this activity of making the notation and symbolisation explicit took up almost 6 minutes of the lesson. The teacher towards the end asks if what he is saying makes sense and also tells learners that dividing both sides by n is incorrect because the notations means something else within this topic of number pattern. So from all the actions that have been performed so far, the key concepts being taught here is 'notation' and 'mathematical conventions' and that is what the teacher was busy transmitting as criteria so that learners are able to recognize the mathematical language used to communicate mathematical ideas and hence produce the legitimate text. This focus on conventions and notation is located within the overall idea across all five lessons that the teacher is concerned with - making sure that learners are able to generalise explicitly and recursively and generate the algebraic expression for the explicit generalisation, but this action and the previous one indicate attention to supporting learner's realisation of the notation and conventions that underlie patterning activity and problem-solving.

6.2.6 *Input object 1.1, Action 6: Finding term number one from the general term*

10:53-14:07 *T: How then shall we find T_1 from $T_n = 3n$?* (Teacher writes on the board).
L: Since Sir you …cannot multiply, you substitute the n to the one
T: Substitute to find T_1. Where there was an n there is now a one (pointing to $T_n = 3n$ on the board) *it's okay. Where there was an n there is now a one. So wherever there is an n we shall now be writing one. We are writing one in place of what?*
L: of n
T: Of n (teacher writes $T_1 = 3.1$) *so this will become three by, where there is n write one. Right, so three by one is three. So, term number one becomes three.*

$$T_n = 3n$$

(written on the board $T_1 = 3.1$

$$T_1 = 3$$

T: How shall we find term number two from there? Term number two from there?
L: Sir from term number n if we substitute n with 2 (learner goes up to the

board to write what he is trying to say). *Let's find n =3*
T: To substitute what?
L: Sir if we found this by substituting one all of them must do like this.
T: Okay, we will come back to you.
(class discusses and makes a noise)
T: Okay, alright, we want to find term number two. What is term number two? What is term number two?
L: T_2
T: T_2 *what is* T_2 *in this case?*
L: 6
T: Six, it's okay. We want to find six. We know what it is, but how are we going to find that six (teacher write T_2 below $T_n = 3n$ on the board)*?*
L: Three times two (learners mutter)
T: Ok, alright. Where there was an n there is two now. So wherever there is an n we shall put a?
L: 2
T: So it will be three by?
L: 2.
T: So term number two becomes?
L: 6.
T: Six it'sokay? How shall we find term number five? We know T_n *is equal to three n. How shall we find* T_5*? How shall we find* T_5*? Yes?*
L: We substitute the n with a five.
T: Where there is an n we no longer write n but it's now what?
L: 5.
T: 5. So it's going to be three by five which is equal to?
L: 15.
T: 15.

Now that learners understand the meaning of the subscript there is a smooth transition from general term to generating other terms of the sequence. The teacher here is emphasizing the act of substituting input values into the general term and the notion of putting in place of comes out clearly when he says: "where there was an n we no longer write n but it's now what?" He asks this question. This is done as a way of checking or testing if the general term is correct, and the only way they can know is if it (the general term) generates the terms of the sequence. So the teacher is transmitting criteria about how to test the general term. So terms one, two and five are found and the next action the teacher takes is to define and explains the uses of the general term as a way of concluding the actions done on this input object. In Driscoll's (1999) terms, this involves 'reversing', before from the pattern the rule was generated and now the process is being reversed, the rule or expression is the one generating the pattern.

6.2.7 *Input object 1.1, Action 7: Uses of the general term and how it will be assessed*

14:16-16:36 *T: Alright, okay alright, observe when we've got $T_n = 3n$ we can use this term it's okay to generate all the other terms. From this one we can find term number one, we can also find T_2 we can find T_5 we can find term number 50, 10 or number 100* (teacher points to $T_n = 3n$) *from this term we can generate all the other terms. It's okay, this term that is given in terms of n we call it the 'general term'. This one* (teacher points to $T_n = 3n$) *we refer to it as the general term. So when they say find the general term. Simply they are asking you to find the term in terms of what?*
L: of n
T: It's okay? (Teacher underlines the general term). *It can also be in terms of k. If you got T_k what will it be in this case* (teacher writes on the board $T_k =$)
L: 3k
T: T_k will be equal 3k is still the general term it's okay? Right!
L: Sir can I ask the general term can it also be given as a number
T: The general term is given in terms of an unknown where we substitute to generate the sequence. It's okay? Right! We can be able to predict some missing things there because we have observed a pattern there. There is a pattern there. What pattern is there? There is a difference of three between any two successive terms. It's okay? So there is some kind of an order in this list of numbers. There is some kind of an order. It is that order that enables us to predict some missing terms. We can always predict some forthcoming terms because we have observed a pattern there right? When we have got a list of ordered numbers as in this case, they make what we call a sequence (pointing to the word sequences written as part of the heading for today's lesson). *So a list of ordered numbers creates a sequence. So this one* (pointing to 3; 6; 9; 12; 15; 18…) *is a sequence of numbers. Right? Good!*
T: (board erased)

The last action, the teacher does to conclude this input object within evaluative event 1 is to define the general term and its uses and how it may appear in the assessment, he also defines the word 'sequence'. So he tells learners that missing terms and forthcoming terms can be found using the general term. The teacher stresses the point that a term written in terms of n is called the general term. He refers to the assessment to stress this point. This shows that the teacher is aware of the assessment and its tendency to use different words and symbols for the same thing, so in this case he is stressing that the *nth term* and the *general term* are the same thing. In addition to this, the teacher tells learners that the general term is not always expressed in terms of

n but can be expressed in terms of k as well, so that they may be able to recognize it in the assessment. So the teacher draws on mathematical conventions on the range of ways in which the ideas are written in the assessment to legitimate what they are doing.

Further along in this activity, we see that the learner was not clear about this notion of the general term and the teacher stresses that it is written in terms of an unknown. The notion of a constant difference between any two successive numbers is stressed once more and then a definition of a sequence was given. The concept of constant differences is used to provide order to the list of numbers, therefore, this ordered list creates a sequence.

6.3 Summary of this sub-event

The reason why I chose this sub-event/input object as the basis of my analysis is that the teacher was introducing the topic and therefore had more things to say, was more explicit in his explanations and laying foundations for work that follows. From this the way that evaluation is working across the actions is that the teacher is certainly transmitting criteria on what are conventions in working with this, what it means to generate a general term, how you write it, and how you prove if it is the correct one. He transmitted what is mathematical language when he defines terms like 'successive, constant, ordered list and sequence'. He also transmits the criteria on what the symbols and notations mean. All these are part of what is transmitted here. In Bernstein's terms, what has been transmitted as the legitimating criteria is in fact the legitimate text for this teacher. As noted in Chapter 4, the summary shows the events, input objects, actions and the legitimating criteria across the five lessons, therefore, the kind of messages transmitted, on the whole, are the legitimate text for the teacher.

The next section discusses how this legitimate text relates first to the curriculum framework and secondly to Kieran's activities of school algebra. Later, I contrast what has been constituted as the legitimate text in this classroom with what was constituted as the legitimate text in the documents.

6.4 Looking across the five lessons using literature/curriculum framework

What came through from working with the data and segmenting it into units of analysis called evaluative events was that there was only one event across the five lessons - that of finding the

general term and expressing it algebraically. This means that the activity across the lessons is that of finding the general term with different input objects employed on which actions are carried out.

In the process of finding the general term, the methods differed according to different stages in the topic. The following bullets show the methods used by this teacher from beginning to the end of this series of lessons.

1. Generalise a linear pattern explicitly by observing the relationship between subscript and output,
2. Generalise a linear pattern explicitly through algebraic methods,
3. Generalise a quadratic pattern algebraically,
4. Generalise linear, exponential and Fibonacci type patterns recursively or iteratively.

However, there was no distinction made between sequences that can be generalised explicitly and those that cannot be generalised explicitly within the context of school algebra. By explicit, I mean generalisations that use two variational data or the functional approach to generalising. An explicit discussion addressing the advantages of using the explicit, global way of generalising and contrasting with the recursive way of generalising was not done.

The following table uses the curriculum framework to see what content, process and conventions the teacher has employed in his practice, and so as he transmits the legitimate text. For content I have indicated the types of sequences that were used and the number of times some of them were used.

Table 13: Classroom data in literature and curriculum framework

Content		Process	Conventions	Context
Linear - 3; 6; 9... used four times - 6 linear sequences altogether **Quadratic** - 2types, 1st type (1; 4; 9; 16...) used 2 times **Cubic** - one sequence not acted on **Exponential/Geometric** - one sequence generalised recursively **Fibonacci-type** - One sequence	3; 6; 9; 12; 15; ... 4; 7; 10; 13; 16; ... 1; 8; 27; ... 1; 4; 9; 16; 25; 36; ... 3; 5; 7; ... -4; -2; 0; 2; ... 3; 7; 11; 15; ... 3; 13; 31; 57; 91; ... 9; 11; 13; 15; ... 1; 2; 4; 8; 16; ... 1; 1; 2; 3; 5; 8; 13; ...	Calculate/find next number; justify finding. Explain, substitute, predict missing terms, compare, solve simultaneously, test, generalise by inspection, generalise recursively and explicitly.	Meaning of (...), How to write term values according to their position, Relationship between subscript and term value Notation T_1 and Tn versus T_n Different letters used to express the general term How the general term is expressed in the assessment. How to write a full definition for a recursive general formula	No real life contexts used, only mathematical contexts used

From this table, we can see that only number sequences were dealt with in this classroom and only one sequence used negative integers. Otherwise, there are no fractions in any form used, nor were diagrams or tables or word problems used. Because of this, there was no opportunity created for learners to process the number pattern from diagrams and word problems. Also, the number sequences start with a small number all the time, thus, there were no big numbers used, for example, a sequence where the starting point is 366 or more. Also, a relatively limited number of numerical patterns covered even the ones included in homework tasks. This poses problems because the national official assessments that I analysed used a variety of ways to introduce the sequence. Also, the type of content the teacher uses goes beyond the curriculum's minimum requirements for example there are sequences like: 1; 8; 27; ..., 1; 2; 4; 8; 16; ... and 1; 1; 2; 3; 5; 8; 13; ... The cubic sequence was erased immediately when learners could not recognize a pattern, the exponential and the Fibonacci type were generalised recursively. The recursive way of representing the sequence in the curriculum is a minimum requirement specified in grade 12 but the teacher dedicates one lesson to teach it. At this point it seems as though the teacher is following the textbook that they are using because all these examples used here were from the textbook. Most of the sequences that were used were linear and quadratic, even though most of them were used more than once, this shows that most of the focus was on what was specified as minimum requirements in the curriculum document for this topic. So this tells us something about what the teacher privileges from the curriculum.

The processes used are not too different from those stated in the curriculum document and this will became clearer in the comparison section. And for conventions in the curriculum documents there was no specification for grade 10 and 11. However, what came through across the five lessons is that conventions and processes emerged as the actions were carried out on the input object and so the type of sequence used. As we have seen from the transcript the evaluation criteria were explicit with respect to the conventions and processes needed to generate the general term. This is evidenced by the two learners who volunteered to do corrections on homework in lesson 4. In Bernstein's (2000) terms it shows that learners had acquired the legitimate text. In Carlsen's (2010) terms it shows that learners appropriated and owned the processes and meaning of conventions because they were made explicit. The next discussion shows from the first lesson how categories from Kieran's model of activities of school algebra relate to what has been transmitted as criteria.

6.5 Looking across the lessons using Kieran's GTG model

Briefly, as a way of reminding the reader, the generational activity in school algebra involves the *forming of expressions or equations.* We saw immediately that this was the dominant objective and hence the activities across all five lessons were based on finding the algebraic expression that represents the numeric pattern. Once this expression was generated, numerical values were then substituted into it to verify if it (algebraic expression) generates the terms of the sequence. Substitution is an act that falls within the transformational activities of school algebra. However, transformational activities as they are described by Kieran (2007) are activities that work on expressions to transform them. In the case of this study, there are no transformations that are performed on the expression to change it into something else but transformations that are performed are simply to check if the generated expression generates the terms of the sequence and expanding the sequence in the sense of continuing the pattern is another act that falls within the transformational activities of school algebra. Across the five lessons, these are the only two types of transformational activities that were used: extending the sequence and substituting values into the expression to see if the expression is the correct one. According to Kieran (2007) the global/meta level activities of school algebra involve working with generalisable pattern, justifying and looking for relationships or structure. Kieran (2007) reported that the global/Meta level activities are cross-cutting and overarching, and that it proved more practical to discuss them under the generational or transformational activities in the literature that she reviewed. In the case of my study, the activity of working with generalisable pattern, justifying and looking for relationships were the processes employed across the five lessons. However, the dominant practice and the aim across the five lessons is that of finding the general term and expressing it algebraically. Therefore, I came to the conclusion that the overarching activity across the five lessons is a generational activity, the other two that is transformational and Global/meta-level were drawn to a limited degree to support the forming of the expression within the generational activity. In the table that follows, Kieran's model is elaborated upon with respect to the first lesson only. You will see that I have highlighted in bold the actions that were manifested. For the analysis of the remaining lessons in Kieran's terms, see Appendix E.

Table 14: Kieran's model and classroom data

	Generational	**Transformational**	**Global/Meta level**
	Involves the **forming of** the objects of algebra. Objects of algebra are **equations and expressions from** geometric and **numeric patterns**	Involves activities like factorizing, collecting like terms, **substituting, expanding**	Involves **working with generalisable pattern, justifying and looking for relationships**
Lesson 1	**EE1.1 – finding the general term for linear sequences** **Input Object 1:** 3, 6, 9, 12 T_1 T_2 T_3 T_4 T_5 ... T_{10} ... T_n 3 6 9 12 15 30 $3n$ $T_n = 3n$ $T_k = 3k$ **Input object 2:** 4; 7; 10; T_1 T_2 T_3 T_4 T_5 ... T_{10} ... T_n 4 7 10 13 16 31 $3n+1$	3; 6; 9; **12; 15; 18; ...** $T_n = 3n$ $T_n = 3n$ $T_n = 3n$ $T_1 = 3 \cdot 1$ $T_2 = 3 \cdot 2$ $T_3 = 3 \cdot 3$ $= 3$ $= 6$ $= 9$ 4; 7; 10; **13; 16; ...** Teacher does not go through the substitution stage because learners have seen that the formula works	Justify 12 looking for a relationship between subscript and output value Justify 31 looking for a relationship between subscript and output value
	EE 1.2 – a cubic pattern **Input object 3:** 1; 8; 27; ...		

	EE 1.3 – a quadratic pattern **Input Object 4:** 1; 4; 9; ... 1; 4; 9; 16; 25 36; 3 5 7 9 11 2 2 2 2 3; 6; 9; 12; 15; 3 3 3 3	1; 4; 9; **16; 25; 36**	
	EE 1.4 – giving homework from Classroom Mathematics G11		

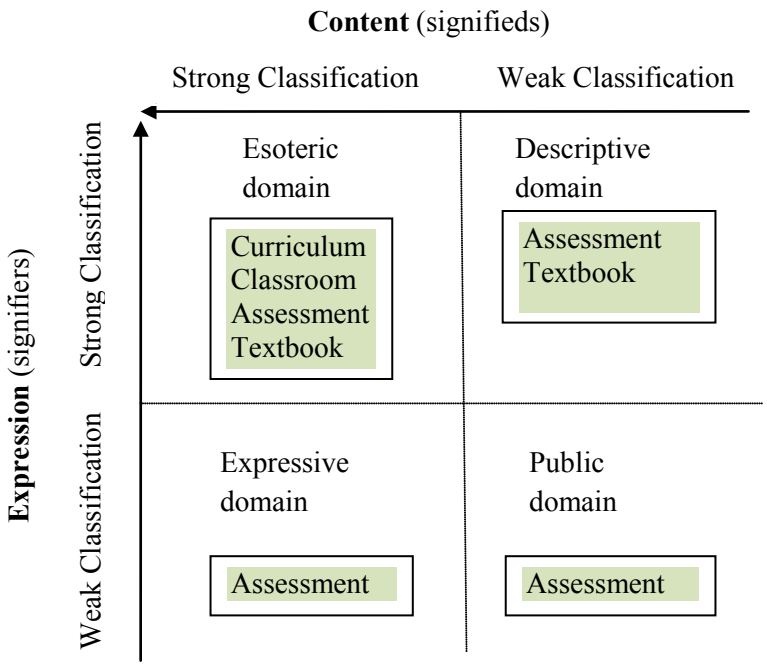
6.6 Comparing documents with classroom using Bernstein's notion of classification and Dowling's domains of practice

I now want to contrast what has been constituted as the legitimate text from the classroom with what has been constituted as the legitimate text in the curriculum document, official assessments and the textbook used in the classroom. Earlier in this chapter I concluded that what has been transmitted as the legitimating criteria by the teacher is the legitimate text for the teacher. Later on, when relating the legitimate text for the teacher to the curriculum framework, it was evident that the teacher aligned with various aspects of the curriculum document in terms of content and the domain of practice but did not align with some of the range that is in the assessment and the textbook. The curriculum document itself when examined using Dowling's domains of practice appeared that it is using both highly classified forms of expression and content. Therefore, the curriculum document for this topic was within the esoteric domain of practice. Parker (2006), examined the same document, and takes this topic of number pattern as an example of an esoteric domain of practice.

However, the assessment and the textbook have examples drawn from other domains - for example, the analysis of the textbook in the previous chapter had shown that the textbook draws from the esoteric as well as the descriptive domain of practice while the analysis of the assessment showed that the assessment draws from all of the (4) domains. I showed one example from the public and another from the expressive domain. It thus becomes clear that the documents are multi-vocal and the teacher aligns with the curriculum statements because they both draw from the specialized domain only. Also, the teacher used four out of five lessons to teach linear and quadratic patterns which are the minimum requirements for grade 11. Staying within the esoteric domain is not altogether disadvantageous for learners because they gain access to the ways of expressing oneself verbally and in written form within the community of mathematics. However, remaining within the esoteric domain is problematic given that the assessment draws from all domains. This means learners might not be able to recognize and produce the legitimate text across all ways in which they will be assessed. This is so because the criteria that were transmitted to learners to acquire were limited to one document only – the curriculum document. So, in this way, opportunities to learn the legitimate text in relation to documents other than the curriculum have been constrained. The following figure (Figure 14)

captures the multi-vocal nature of the documents using Dowling's domains of practice and incorporates the locale of the focal teacher's practice. In the following summary, his practice is aligned with the curriculum document and thus privileging the esoteric domain of practice, at the expense of the other domains.

Figure 14: Summary of all the documents and the classroom



Sourced from Dowling (1998, p. 135)

6.7 Using literature/curriculum framework to compare across documents and classroom

Now I compare the detail of content, processes, conventions and contexts across the curriculum, assessment, Classroom Mathematics G11 textbook as presented in Chapter 5 with what I saw being privileged in the G11 classroom data.

Table 15: Comparison of the classroom with documents using the Literature & Curriculum framework

	Curriculum	**Assessment**	**G11 Textbook**	**G11 Classroom**
Content	G10 – linear G11 – linear and quadratic G12 – arithmetic geometric and recursive	Linear, Quadratic, Geometric	Linear/Arithmetic, Quadratic, Cubic, Exponential/Geometric, Fibonacci (recursive) type sequences	linear, quadratic, cubic, recursive and exponential
Processes	Investigate, make conjectures and generalizations, and provide explanations and justification, attempt to prove conjectures	continuing sequence, finding the general term, finding the position of a term, finding the sum, justifying, proving and conjecturing	continuing sequence, finding the general term, justifying, proving and conjecturing, generating own sequences	continuing sequence, finding the general term, testing the general term, justify
Conventions	Sigma notation and recursive formulae	Sigma notation	recursive formulae	Notation T_1 and Tn versus T_n Meaning of (...)
Contexts	No contexts suggested	Numeric, Money, forestry/gardening, fractal geometry	Numeric, Geometric diagrams, dots, matchsticks family tree, sport context	no everyday contexts used, only numeric contexts used

This table has been presented and discussed in the documentary analysis chapter (Chapter 5), therefore, I will focus this discussion on the last column of the table and compare it with what came through in Chapter 5. In the classroom, the linear and the quadratic sequences were dominant; four lessons out of five were spent on these two. The cubic sequence was written on the board and erased without being acted on. The exponential sequence was given as one of the examples the teacher used to generalise recursively. From this one can conclude that the teacher aligned with the curriculum specifications at G11 for content. Processes employed in the classroom align with some of the processes in the assessment, curriculum and textbook, however, processes like: finding the position of a term, proving and generating own sequence are

not seen in the teacher's practice and also there was no extension to examples beyond numerical contexts that are used.

6.8 Comparison using Kieran's GTG model across documents and classroom

The following table (Table 15) presents the types of activities employed in the assessment, textbook and classroom and the type of activity suggested by the statements in the curriculum as mentioned in the previous chapter.

Table 16: Kieran's (2007) GTG model across the documents and the classroom

	Curriculum	Assessment	Textbook	Classroom
Generational	√	√	√	√
Transformational	√	√	√	
Global/meta-level	√	√	√	

As discussed in chapter 5, the curriculum statements suggest a presence of global/meta-level, transformational as well as generational activities of school algebra. The textbook and the assessment had problems that fore-grounded each activity. From the analysis of the classroom, it became apparent that the teacher worked within the generational activity of school algebra. Kieran's (2007) GTG model therefore shows that the teacher did not align with any of the documents at the level of type of activity of school algebra.

6.9 Conclusion

In this chapter, I have presented a summary that shows the legitimating criteria from the sequence of five lessons based on number patterns. I have also drawn from the transcript to evidence how these criteria come to play. The conclusion that I came to is that what is transmitted as the legitimating criteria in the classroom is actually the legitimate text for the teacher. Then I contrasted this with what came through in the curriculum document, the official assessments and the textbook that was used in the classroom. These were the results of the comparison that the documents are multi-vocal and therefore create a noise for the teacher. This is problematic because it looks as if the teacher is being examined on what is the legitimate text.

From the comparison, it became apparent that the teacher's practice is most aligned closely to the curriculum document specifications given the amount of parallels between classroom practice and the statements in the curriculum document except in the case of the GTG model. In the next chapter, I discuss these findings further in the light of what implications they have for policy and practice. I also return to the research questions and explore how I have responded to the research questions.

Chapter 7 Conclusion

7.1 Introduction

In this chapter, I conclude the research by revisiting the findings and discussing them in relation to implications they have for policy and for practice. In the process I discuss the research questions and how they have been answered.

The aims of the research were framed in the following research questions:

The title of the study was: *An Investigation of the Constitution of the Legitimate Text and Opportunities to Learn Number Pattern in Grade 11*. These were the critical research questions:

1. What is constituted as the legitimate text for the topic of number patterns across:
    a. The key 'official' curriculum (NCS-National Curriculum Statement)
    b. The National Assessments (Matric papers 2008 and 2009)?
    c. Within the mathematics textbook that was used?
2. What is constituted as the legitimate text by a teacher within a sequence of lessons focused on number patterns in grade 11?
3. What is the relationship between what is constituted in the classroom and what is constituted in the official curriculum, assessment texts as well as the mathematics textbook that was used in the classroom?
4. What opportunities for learners discerning number pattern are made available?

7.2 Findings

Findings from the literature survey, documentary analysis and classroom analysis are summarised below.

7.2.1 Tools from the literature review/curriculum and theory

From the literature review, three themes emerged as dominant within the studies reviewed in relation to number patterns (conventions, process and contexts), which were then used as a framework to analyse the data. However, the literature showed that mathematics education researchers placed more emphasis on processes than content. Cooper and Warren (2008)

explicitly advocated for this when they say focus in mathematics education has been put on the product than process. From the curriculum analysis, the same framework emerged, with the exclusion of contexts as a category and with the inclusion of content as a category. This framework was a very useful tool for the analysis of both the classroom and documents. It categorised the legitimate text into four categories (content, process, conventions and contexts) and allowed room for comparison across the documents and the classroom.

The literature review did not only provide an analytic framework for categorising the legitimate text but also provided a way of talking about the unit of analysis across the five lessons. For the methodology, the main unit of analysis was called the 'evaluative event' (Davis et al, 2003; Adler and Davis, 2006; Adler, 2009) because criteria are at play. It was observed that the teacher exercised judgement throughout his pedagogy of teaching number pattern. Within this unit of analysis, Kieran's notion of school algebra being an activity proved to be a useful concept for further segmenting of the data into smaller units of analysis. The GTG model from Kieran also came out of the literature and was useful in terms of classifying the types of activities embedded in a problem. Thus, from the literature three things were useful in the discourse about the legitimate text that is (1) the framework, (2) GTG model and (3) the notion of school algebra being an action.

Bernstein's (2000) theory proved useful for understanding and analysing the legitimate text across the documents and the classroom data. It provided a language of describing and talking about the problem. It showed that the discourse of the legitimate text is not only about the content, processes, conventions and contexts but is also about the classification of the mathematical text. Dowling's extension of this notion of classification came in very handy. He provided a lens for looking at the classification of a mathematical text in two ways based on the context from which it draws from and the form of expression the text employs. Dowling's model as a tool presented and illuminated the legitimate text across the data in a very lucid way. The model showed the misalignment that exists among the different data analysed in terms of what is constituted as the legitimate text for this topic of number patterns.

7.2.2 Findings from the documentary analysis

From the analysis of the documents it was clear that they were different emphases across the different documents at the level of content, process and contexts. The domains of practice engaged by each document were different. The teacher's practice and the curriculum were in the esoteric domain of practice while the assessment had problems which could fall into each domain of practice. By virtue of these documents taking on different domains of practice I then concluded that the documents were multi-vocal. A study that was conducted by Adler and Pillay(2008) showed that these documents are part of the resources the teacher draws from. To put it in the language of my study I could say these documents contained what Bernstein calls the 'legitimate text' and it is a problem that there is such an apparent incongruity in the manner in which they communicate and present the legitimate text.

7.2.3 Findings from the classroom

To answer the second research question, I observed a sequence of lessons based on number patterns. From the classroom, using Kieran's GTG model, it became apparent that there was one dominant activity throughout the five lessons focused on finding the general term. It was also interesting to see that within one evaluative event there were multiple sub-events named input objects and then multiple sub-actions within each sub-event/input object. Some of these actions worked across content, process, conventions and contexts, others were in response to learner misconceptions. The curriculum and literature review framework showed that the teacher spent four days out of five days teaching linear and quadratic patterns. This was evidence that the teacher valued the curriculum as the document that contains the legitimate text and hence the teacher aligned more with the curriculum document at the level of content. However, the GTG model showed that the teacher did not align with any of the documents at the level of type of activity of school algebra. From the analysis of the actions that were performed on the input object the evaluative criteria became apparent. When using Dowling's model it became apparent also that this teacher prefers to work within the esoteric domain. Overall framing was strong at the level of criteria.

7.2.4 Comparison of the data

To answer the third question Dowling's model of domains of practice was used to compare what constituted as the legitimate text in the classroom and in the documents. It became apparent that

the teacher aligned with the curriculum. From this I concluded that the curriculum document was viewed as the primary document and others as secondary.

I therefore argue that the misalignment displayed in the documents creates an unnecessary noise for the teacher. It looks as though the teacher is being assessed for what is the legitimate text. When looking at the assessment it was clear that all domains of practice according to Dowling's model were tested. However, the textbook used in the classroom observed confined itself to two domains of practice that is the esoteric and the descriptive domain. Although the textbook did provide opportunity to engage with other types of sequences present in the assessment in more detail but the teacher chose to spend more time on the ones specified in the curriculum. In this way opportunities for learners to acquire the legitimate text were constrained.

7.3 Implications and limitations of the study

The findings from this study have significant implications for people involved in the writing of the curriculum, assessment and textbook materials. The documents needs to be better aligned and the textbook needs to in cooperate with some of the problems that lie within the expressive and the descriptive domain for this topic of number patterns. Elimination of the noise that is currently seen in the documents will make the work of finding solutions to the many problems in mathematics education easier. Once the legitimate text is explicit and the documents aligned the problem that will be of concern will be efficiency in using these documents so as to ensure that learners acquire the legitimate text. However, the documents analysed in my study are just a few, there are others and teacher guides that have not been analysed here, which a teacher is expected to consult and interpret.

The desired change is that all of the documents should be aligned, however that is not enough, the documents need to communicate in a very explicit manner the legitimate text. It would be desirable to have a limited number of documents which are user-friendly and communicate the legitimate text in the most explicit manner without requiring/placing extra demands by creating a massive workload for the teacher to understand and interpret.

7.4 Limitations of the study

This study did not look into all the mathematics textbooks used in G11 classrooms. This study also did not look into all of the topics in mathematics and the entire curriculum specifications in mathematics FET curriculum. The study did not look into all the curricular materials made available. The study only looked at one teacher in the inner-city Johannesburg school, consequently, the findings cannot be generalised. However, the findings did provide hints for understanding similar settings as described in the methodology section.

7.5 Recommendations and Conclusions

This study has the following recommendations to make for further research:

- Look into all topics within mathematics in the GET and FET phase.
- Look across different textbooks
- Look into all the papers including exemplars.
- Further research is still needed on how these resources are used by teachers and learners and how much are they benefiting from them and how these can be optimised for their benefit.
- What I would suggest for further research to broaden understanding of these findings would be to do a similar study for all the topics in mathematics and see if the same findings will crop up.

Recommendation for policy

- Since this topic stays in the curriculum from Grade 1 through to end of G12, minimum requirements, at the level of content, at G11 needs to go beyond linear and quadratic.
- Curriculum document needs to remove ambiguous statements like ‘not limited to’ when specifying content.
- Textbook writers need to provide an unrestricted range of problems covering all of the domains in Dowling’s model; this is possible at textbook level maybe not at curriculum level. A textbook is different from a reference book and it should meet all the requirements needed.

Recommendation for practice

- Mathematics educators needs to stress the importance of knowing and consulting all relevant curricular material available and provide training in this regard
- Mathematics teachers need to be trained and encouraged to utilize unrestricted, wide range of real-world problems that go beyond the esoteric domain.

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Appendix A Ethical Clearance

07 May 2010

Dear Principal and teachers

Information and invitation to participate in the research project

I am a student at Wits University studying towards a Master of Science degree. One of the requirements in the course is that I do a research project. My intention therefore is to carry out a research study based on the teachers teaching processes.

I am looking to investigate the way the teacher select questions, structure explanations, and respond to learners' contributions in class when focused on teaching Assessment Standards relating to number patterns. My aim is to video tape sequences of lessons focused on number patterns, and then to analyse this data in order to understand how the teacher select, structure and progress the teaching of this topic in grade 11. I hope, through this research, to better understand teachers' classroom choices in relation to this specific topic of the mathematics curriculum.

To this end I would like to observe and videotape a sequence of Grade 11 lessons in which the specific assessment standard I have identified in relation to Number Patterns is taught. I would like to speak to the teacher briefly prior to the lesson sequence to get the teachers' outline of what is to be focused on, and to collect information on tasks/ questions/ plans being used. Similarly, if possible, I would like to informally interview the teacher following the lesson sequence to get a feel for the teachers' reflections on how the lessons proceeded, and to get the teachers' views on choices made during the course of lessons.

In instances where data is shared more broadly for research/ teacher development purposes, I undertake to ensure the anonymity of the school and teacher involved. In reporting my findings, I undertake to maintain anonymity of all participants as well. In this regard I will further ensure that no improper references are made about the school or the teacher. Lessons will continue as normal and as scheduled, with my presence within the classroom.

I must stress that participation is voluntary. The school and the teacher are under no obligation to participate and there are no consequences should you choose not to. All participants also have the right to withdraw from the study at any future point. I would be very grateful for this opportunity however, and if you are agreeable to this process please read and complete the attached consent form and return it to me.

If you have any questions or concerns or would like to discuss the aims of this research in more detail, please do not hesitate to contact me (Nontsikelelo Luxomo 074 915 2952, nluxomo@gmail.com or Millicent.Luxomo@students.wits.ac.za) or my supervisors (Prof Jill Adler, 011 717 3413/Prof Hamsa Venkat, 011 717 37 42).

Yours sincerely

Nontsikelelo Luxomo
Master's Student
Wits School of Education

Consent form for participation in the research project.

(Please delete clearly where applicable)

I have read the above and **give consent / do not give consent** for my school to participate in the research project subject to the conditions laid out in the accompanying letter. These include the use of the video and transcripts for research and teacher development purposes and in articles for publication in academic journals on condition that the school is anonymous and all participants are referred to by pseudonyms.

Name of principal: .....

Signature of principal:.....

Name of teacher: .....

Signature of teacher:.....

Wits School of Education

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STUDENT NUMBER: 0217775M
Protocol: 2010ECE53C

03 May 2011

Ms. Millicent Luxomo
4607 Luruli Avenue
CHIAWELO ext 2
1818

Dear Ms. Luxomo

Application for Ethics Clearance: Master of Science

I have a pleasure in advising you that the Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has agreed to approve your application for ethics clearance submitted for your proposal entitled:

An investigation of the constitution of the legitimate text and opportunities to learn number pattern in Grade 11

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

Yours sincerely

M Mabeta

Matsie Mabeta
Wits School of Education

Cc Supervisor: Prof. H Venkatakrishnan (via email)

Appendix B – 2008 to 2009 National Official Examination Questions on Number patterns

Number Patterns in the Preparatory Examination 2008

QUESTION 3

Consider the following sequence of numbers: 1; 2; 1; 5; 1; 8; 1; 11; ...

3.1 What is the 10th term of the above sequence? (2)

3.2 Calculate the sum of the first 50 terms of the sequence. (4)
[6]

QUESTION 4

Consider the following sequence: 3; 6; 11; 18; 27; ...

4.1 Determine the 6th and 7th terms of the given sequence, if the sequence behaves consistently. (2)

4.2 Determine a formula for the general term, p, of the sequence. (4)

4.3 Use your formula to calculate p if the p^{th} term in the sequence is 627. (4)
[10]

QUESTION 5

5.1 Kopano wants to buy soccer boots costing R800, but he only has R290,00. Kopano's uncle Stephen challenges him to do well in his homework for a reward. Uncle Stephen agrees to reward him with 50c on the first day he does well in his homework, R1 on the second day, R2 on the third day, and so on for 10 days.

5.1.1 Determine the total amount uncle Stephen gives Kopano for 10 days of homework well done. (5)

5.1.2 Is it worth Kopano's time to accept his uncle's challenge? Substantiate your answer. (2)

5.2 Consider the geometric sequence: $8(x-2)^2$; $4(x-2)^3$; $2(x-2)^4$; $x \neq 2$

5.2.1 Determine the value of x for which the sequence converges. (3)

5.2.2 Determine the sum to infinity of the series if $x = 2{,}5$. (3)
[13]

Number Patterns in the November 2008 Examination

QUESTION 2

2.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; \ldots$

2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)

2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

2.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; ...

2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

2.2.2 Calculate the n^{th} term of the sequence. (6)

2.2.3 Which term of the sequence is 330? (4)
[21]

QUESTION 3

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + \ldots$

3.1 Determine the n^{th} term of the series. (1)

3.2 For what value(s) of x will the series converge? (3)

3.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)
[7]

A1

Gauteng
Preparatory Examination Sept 2009
Paper 1
Marks: 150
Time: 3 hours

Gauteng
Voorbereidende Eksamen Sept 2009
Vraestel 1
Punte: 150
Tyd: 3 ure

Question 2

2.1 Consider the sequence:
1; 1; 3; 2; 5; 3; 7; 4 ...
Write down the next two terms in the sequence. (2)

2.2 Calculate: $\sum_{n=1}^{50}(2n-1)$ (4)

[6]

Vraag 2

2.1 Beskou die volgende ry
1; 1; 3; 2; 5; 3; 7; 4 ...
Skryf die volgende twee terme van die ry neer.

2.2 Bereken: $\sum_{n=1}^{50}(2n-1)$

Question 3

Consider the sequence: 1; 5; 11; 19; ...

3.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

3.2 Calculate the n^{th} term of the sequence. (6)

3.3 Which term of the sequence is 2549? (4)

[12]

Vraag 3

Beskou die volgende ry: 1; 5; 11; 19; ...

3.1 As die ry op dieselfde patroon voortgaan, skryf die volgende TWEE terme van die ry neer.

3.2 Bereken die n^{de} term in die ry.

3.3 Watter term in die ry sal 2549 wees?

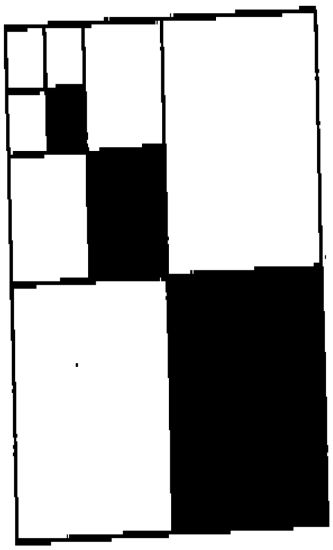
Vraag 4

4.1 Die skets toon 'n vierkant wat telkens in vier kleiner vierkante verdeel word. Die oppervlakte van die oorspronklike vierkant is $4m^2$. Die oppervlakte van die ingekleurde vierkante vorm 'n meetkundige ry.

4.1.1 Gee die eerste DRIE terme (met betrekking tot die oppervlakte van die ingekeurde vierkante) van die ry.

4.1.2 Indien hierdie patroon oneindig aanhou, bereken die som van al die ingekleurde vierkante.

Question 4



4.1.1 Refer to the areas of the shaded squares and write down the first THREE terms of the sequence. (3)

4.1.2 If this pattern is continued without end, calculate the sum of all the shaded squares. (5)
[8]

Number Patterns in the 2009 November Examination

QUESTION 2

2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: 5 ; 9 ; 13 ; 17 ; 21 ; ...
Tebogo's sequence: 5 ; 125 ; 3 125 ; 78 125 ; 1 953 125 ; ...

Write down the n^{th} term (or the rule in terms of n) of:

2.1.1 Matthew's sequence (3)

2.1.2 Tebogo's sequence (2)

2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)
[10]

QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
[5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$$-3\ ;\ -2\ ;\ -3\ ;\ -6\ ;\ -11\ ;\ \ldots$$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the 35^{th} and 36^{th} terms of the quadratic sequence. (2)

4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
[11]

QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	**Second year**	**Third year**	**Fourth year**
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
[8]

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Appendix C – Classroom Mathematics Grade 11 Textbook – topic on Number Patterns

Number sequences

- A sequence is an ordered list of numbers such as 3; 6; 9; 12; ...; $3n$; ... Each number in the sequence is called a term (T_n), where the n indicates the position of the term in the sequence.
- Referring to the sequence 3; 6; 9; 12; ...; $3n$;.... ,
  $T_1 = 3$ is the first (1st) term; $T_2 = 6$ is the second (2nd) term; $T_3 = 9$ is the third (3rd) term, and $T_n = 3n$ is the nth term or the general term of the sequence.
- The general term is often expressed in terms of an equation in $T_n = 3n$ which gives us a rule for finding all the other terms in the sequence.
- A sequence that has a limited number of terms is finite.
- A sequence that has an unlimited number of terms is infinite.

Sequences and differences between terms

Activity 1.8

LO1 AS 11.1.3

- Work in pairs.

1. Extend each of the following number patterns by a further 3 terms:

   a) 4; 8; 12; ... b) 2; 7; 12; 17; ... c) 1; 4; 9; ...

   d) 1; 8; 27; ... e) 2; 8; 18; ... f) 2; 5; 10; 17; ...

2. For each number pattern in 1: (i) Explain how you extended the pattern; (ii) if possible, give a rule or formula for the n^{th} term.

General discussion

Interesting discoveries can be made about number sequences by looking at the differences between the successive terms of the sequence. These are the first differences. The second differences are the differences between the successive terms of the first differences. Similarly we can find the third difference, and so on.

Example

	2		5		10		17		26 ...
1st differences:		3		5		7		9 ...	
2nd differences:			2		2		2	...	
3rd differences:				0		0	...		

Activity 1.9

LO1 AS 11.1.3

Work in pairs or small groups.

1. For each of the following sequences:
   (i) Find the 1st differences between the terms.
   (ii) Find the 2nd differences between the terms.
   (iii) Write down a general formula for the sequence.
   a) 2; 4; 6; 8 ...
   b) 9; 11; 13; 15; ...
   c) 11; 15; 19; 23; ...
   d) 1; 4; 9; 16; ...
   e) 0; 3; 8; 15
   f) –1; 5; 15; 29; ...
2. Write down a conjecture relating the type of general term obtained to the differences which are constant.

The following ... arise out of Activity 1.9:

- When the first differences are constant (or the same number other than 0), then the formula for the general term of the sequence will be linear (the highest power of the variables will be 1).
- When the second differences are constant, then the general term for the sequence is quadratic (the highest power of n in the formula for the general term is 2).
- When the third differences are constant then the general term will be cubic and so on.

Example

1. Decide whether the general term for the sequence 2; 5; 10; 17; 26 ... is linear or quadratic.
2. Find a formula for the general term of the sequence.

Solution

1. Look for the differences between successive terms:

	2	5	10	17	26 ...
1st differences:	3	5	7	9	
2nd differences:	2	2	2	(Constant)	

The sequence will therefore have a general term which is quadratic.

2. To find a formula, compare the terms of the sequence with squares of natural numbers:

1	4	9	16	25	.... n^2 ....
2	5	10	17		

By inspection, the general term for the sequence 2; 10; 17; 26; .. is $T_n = n^2 + 1$.

Exercise 1.10 — LO1 AS 11.1

- Work on your own for number 1.
- Work in pairs for numbers 2 and 3.

1. For each of the following sequences, assuming that 1st, 2nd or 3rd differences are constant:
   (i) Extend the sequence by at least three terms
   (ii) Investigate the 1st, 2nd and 3rd differences
   (iii) Give reasons for deciding whether the formula for the general term will be linear or quadratic or cubic
   (iv) Find a formula for the general term of each sequence.

   a) 0; 1; 2; 3; ... **b)** 3; 5; 7; ... **c)** 3, 6; 9; 12; ...
   d) −4; −2; 0; 2; ... **e)** 3; 7; 11; 15; ... **f)** 5; 8; 11; 14; ...
   g) 1; 4; 7; 10; ... **h)** 6; 11; 16; 21; ... **i)** 2; 8; 18; 32; ...
   j) 3; 12; 27; 48; ... **k)** 3; 9; 19; 33; ... **l)** 1; 7; 17; 31; ...
   m) $-\frac{3}{2}; 0; 2\frac{1}{2}\ 6; 10\frac{1}{2}$ ... **n)** 2; 9; 28; 65; 126 ... **o)** 2; 16; 54; 128; 250; 432; ...
   p) −2; 5; 24; 61; 122; 213; ...

2. Generate sequences which have linear or quadratic general terms and swop with a partner.

3. Investigate between term values and decide whether the general term of each sequence from your partner is linear or quadratic or otherwise.

Using recursion to define sequences

Consider the sequence 2; 5; 7; 12; ...

- We can generate this sequence by adding the previous two terms together each time, starting with 2 and 5.
- This can be expressed as follows $T_n = T_{n-1} + T_{n-2}$ $(T_1 = 2; T_2 = 5)$.
- Using this recursive definition, we can generate the sequence, provided that we know the initial two values (2 and 5).

The sequence 1; 1; 2; 3; 5; ... is known as the Fibonacci sequence after its original discoverer, Leornado Fibonnaci, (He lived in about 1180 – 1250 BCE).

- The sequence can be generated by using $T_n = T_{n-1} + T_{n-2}$ with initial values $T_1 = T_2 = 1$.
- Because each successive term is obtained by adding the previous two terms, the Fibonnaci sequence is another example of a sequence that is derived by using recursion.

Exercise 1.11 — LO1 AS 11.1.3

Work with a friend.

1. Generate eight terms of a sequence using the definition $T_n = T_{n-1} + T_{n-2}$ but for which the initial two values are:

   a) 2; 3; ... **b)** 1; 4; ... **c)** 5; 5 ... **d)** −2; 1; ... **e)** −4; 2; ...

. The terms 34 and 55 appear as successive terms of a sequence defined by means of recursion. Show that the sequence may be Fibonnaci.

. Refer to the "tree" of Exercise 7.1 question 7. The number of new branches that grow on the tree depend on a rule that involves recursion. Find a recursive rule that will predict the number of new branches.

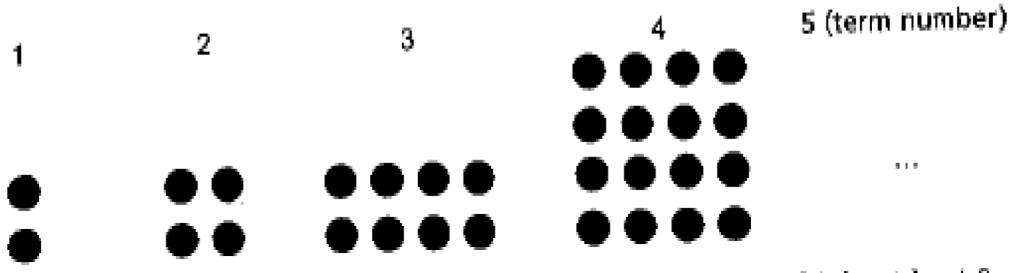
'urther sequences

Activity 1.12

LO1 AS 11.1.3

- Work in pairs.

Study the following pattern of dots:



1. a) Write out the sequence suggested by the dots and extend it by at least 3 terms.
   b) What property has the sequence got?
2. Write down a formula for generating the pattern
   a) that does not use recursion
   b) that uses recursion and give the initial value.
3. Generate at least two of your own sequences of a similar type. Swop with your partner who must then give a formula for T_n that is a) not recursive b) that uses recursion.

- The sequence ... contains terms ...
- Notice that the general term is 2 and that the ratio of ... $\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = \ldots = 2$.
- Sequences of the type; a; ar; ar^2; ...; ar^{n-1}; ... are known as geometric sequences. You will learn more about these in Grade 12.

Exercise 1.13

LO1 AS 11.1.3

1. For each of the following sequences:
   (i) Investigate the 1st, 2nd and 3rd differences.

(ii) Investigate the ratio of successive terms.
(iii) Describe the nature of the general term of the sequence or of the sequence itself.
(iv) Extend the sequence by at least three terms.
(v) Find an expression for the general term of the sequence

a) 7; 14; 21; ...
b) 1; 6; 11; ...
c) 0; −4; −8; ...
d) $x-3; x-1; x+1; \ldots$
e) −2; 4; −8; ...
f) 2; −6; 18; ...
g) 1,1; 2,2; 3,3; ...
h) −4; 9; 5; 14; ...
i) $am^4; am^5; am^6; \ldots$
j) $\frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}; \frac{5}{6}; \frac{6}{7}; \ldots$
k) $1; \frac{1}{4}; \frac{1}{9}; \frac{1}{16}; \ldots$
l) $\frac{8}{x^4y}; \frac{16}{x^3y}; \frac{32}{x^2y}; \ldots$

2.



During each successive bounce, a tennis ball covers 80% of the horizontal distance it covered during the preceding bounce. The first bounce covered 2m.

a) Represent the distance bounced by the ball in successive bounces by means of a sequence.
b) What type of sequence do the distances between successive bounces of the ball represent?
c) Find the general term describing the distances covered by the bounces.
d) Calculate the distance covered during the 8th bounce.

3. Consider the following pattern and answer the questions that follow:

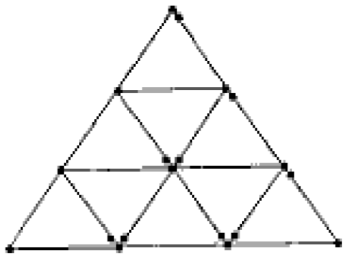


The pattern above suggests a sequence of numbers called the **triangular numbers.**

a) Write down the sequence of triangular numbers suggested by the pattern.
b) Investigate the 1st, 2nd and 3rd differences generated by successive terms of the sequence.
c) Extend the sequence by at least three terms.
d) Find a formula for the general term of the sequence.
e) Use recursion to describe the sequence.

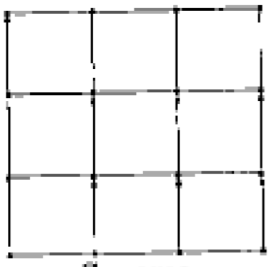
4. For each of the following,
   (i) Determine the sequence of the number of matches required to build the grids shown.
   (ii) Give a formula for and describe the nature of the n-th term of the sequence, i.e., linear, quadratic, cubic or other.

a)



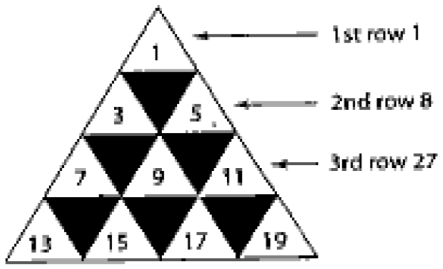
Equilateral triangles

b)



Squares

5. Find the sum of the numbers that will make up the 100th row of this triangle.



6. There are 18 teams involved in a netball competition. Each team plays two matches, home and away.
   a) The organisers want to know how many matches will be played. Copy and complete the following table.

number of teams	1	2	3	4	5	6	...	n
number of matches	0	2	6					

   b) Determine the general term of the sequence generated in 6 a).
   c) How many matches will be played by the 18 teams altogether?

CHECK YOUR SKILLS

Exercise 1.14

LO1 AS 11.1.1; 11.1.2; 11.1.3

1. State whether the following are true or false (If false, give a reason.):
   a) $\frac{a}{0}$ is rational.
   b) $\sqrt{16} + \sqrt{9}$ is an integer.
   c) $\sqrt{9} - \sqrt{3}$ is irrational.
   d) If y is rational and $y \neq 0$, then $y \times \sqrt{2}$ is always real.
   e) If y is rational, then $y \times \sqrt{2}$ is always rational.
   f) $\sqrt{-1} \times \sqrt{5}$ is imaginary.
   g) $\sqrt{-1} \times \sqrt{-1}$ is irrational.

2. Say whether each of the following numbers are real or non-real:

a) -3 b) $1+\sqrt{2}$ c) $1-\sqrt{-1}$ d) $2\times\sqrt{-1}$

e) $\sqrt{2}$ f) $-2-\sqrt{-1}$ g) $\sqrt{2}\times\sqrt{-2}$

3. $\sqrt{\sqrt[5]{-\frac{243}{32}}}$ is rational/irrational/imaginary. Explain.

4. Simplify (all bases are positive.):

a) $\left(\frac{\sqrt{a}}{a^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$ b) $\left(\frac{a}{b}\right)^{x+y}.\left(\frac{b}{a}\right)^{y-x}$ c) $\left(\frac{(3x)^{-2}}{3x^{-1}}\right)^{-\frac{1}{3}}$

d) $\frac{5^{2n+1}.3^{2n-1}}{15^{2n}}$ e) $\frac{(xy)^{\frac{1}{3}}(x^3)^{-\frac{1}{6}}(xy)^{\frac{5}{6}}}{(x^2)^{-\frac{1}{4}}}$ f) $\frac{(a^x)^{x+1}(a^2b)^{-2x}}{(ab)^{-2x}.b^{-x}}$

g) $\frac{5^{2n}.3^x.15^{x-1}}{125^x.3^{x-1}}$ h) $\frac{27^{n-1}.6^n}{162^n}$

5. Simplify:

a) $\frac{4+\sqrt{12}}{2}$ b) $\frac{9-\sqrt{18}}{3}$ c) $\frac{16+\sqrt{32}}{8}$

d) $\left(\sqrt{5}+\frac{1}{\sqrt{5}}\right)\left(2\sqrt{5}-\frac{1}{\sqrt{5}}\right)$ e) $\frac{\sqrt{3}-2}{(1-\sqrt{3})^2}$

f) $\frac{\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$ g) $\frac{\sqrt{2}}{(2-\sqrt{2})(1+\sqrt{2})}$

6. For each of the following sequences:

(i) Determine the nature of the sequence (state whether the sequence has a general term which is linear, quadratic or cubic, or is a geometric sequence or none of these).

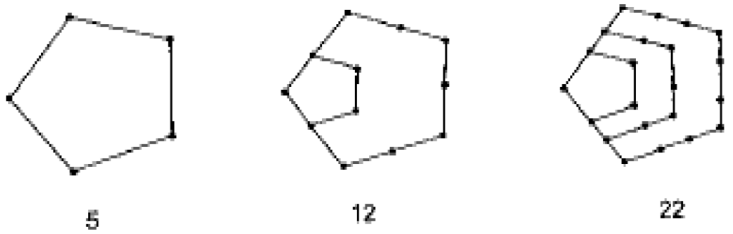
(ii) Extend the sequence by at least three terms and explain your method.

(iii) Find an expression for the general term of the sequence.

a) 2; 4; 6; ... b) $-19; -14; -9; \ldots$ c) $2\frac{1}{2}; 4; 6\frac{1}{2}; 10; 14\frac{1}{2}; \ldots$

d) $1; \frac{1}{2}; \frac{1}{3}; \frac{1}{4}; \ldots$ e) $1; -8; 27; -64; \ldots$ f) $-7; 14; -21; 28; \ldots$

g) $1; \frac{1}{8}; \frac{1}{27}; \ldots$ h) The pentagonal numbers: 1; 5; 12; 22; ...



1 5 12 22

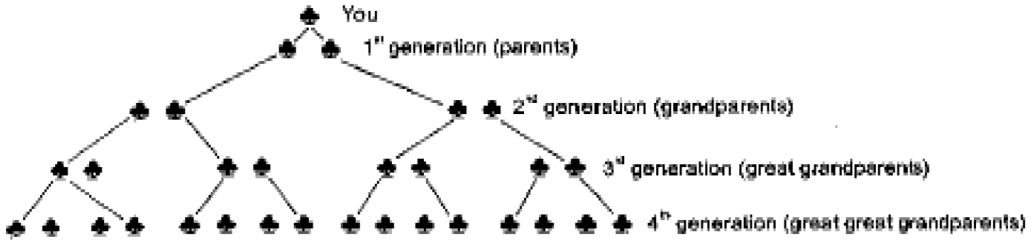
LO1 AS.11.1.1; 11.1.2; 11.1.

1. Given any real number x (use a calculator if necessary):

a) Generate the next three terms of a sequence that is formed as follows:
$\sqrt{x}$; $\sqrt{\sqrt{x}}$; $\sqrt{\sqrt{\sqrt{x}}}$; ...

b) Express the general term T_n in exponent form.

2. Ancestors



Your ancestors consist of your 2 parents (1st generation). Then there are your 4 grandparents (2nd generation). The 3rd generation is your great-grand parents consisting of 8 people.

a) Explore ways in which the successive numbers of ancestors can be expressed as a sequence, paying particular attention to:
(i) the ratio of consecutive terms of the sequence
(ii) the general term of the sequence (iii) the type of sequence obtained.

b) Assume that a generation gap (G_n) is on average 25 years (generation gap means the separation in years from one generation to the next one).
(i) What is the gap between yourself and your grandparents (2nd generation)?
(ii) How many years will come between yourself and your 5th generation grandparents?

c) (i) How many generations will come in a span of 100 years?
(ii) Write down an expression that can be used to calculate the number of generations (n) that will pass in N years.
(iii) Use your expression to calculate the number of generations that have passed from yourself back to the turn of the Common Era (C.E.). (C.E. was formerly referred to as A.D.)

d) One set/pair of your great-grandparents had nine children (one of whom is your grandparent) and each of those children had six children (one of whom is your parent) and each of those children had three children. How many direct descendants did your great-grandparents have?

The formula $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ ... is sometimes used to evaluate the value of π to any required number of digits.

a) Rewrite the formula in your book and extend by at least another 3 terms.

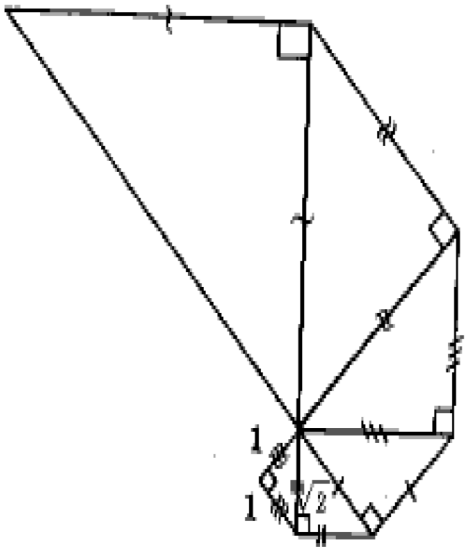
b) Now use the formula to evaluate the value of π to 6 decimal places (Compare your value to the value of π given by the calculator when the π button is used. Remember that the more terms of the expansion for $\frac{\pi}{4}$ you take the better your answers will get.)

PROBLEM SOLVING

Activity 1.16

LO1 AS 11.

1. In the following diagram, a spiral of isosceles right angled triangles is formed by:
   - starting with an isosceles right angled triangle of side 1
   - using the hypotenuse as one of the equal sides for the next triangle.



Explore the sequence that is formed by the lengths of the successive hypotenuses.

2. Egyptians refused to admit fractions with a numerator other than 1. Fractions with a numerator of 1 are called unit fractions. The Egyptians would write a fraction with a numerator that is more than 1 as a sum of unit fractions. Study the following pattern of fractions written as the sum of unit fractions:

$$\frac{2}{3} = \frac{1}{2} + \frac{1}{6};\ \frac{2}{9} = \frac{1}{6} + \frac{1}{18};\ \frac{2}{15} = \frac{1}{10} + \frac{1}{30};\ \ldots$$

   a) Write the next three Egyptian fractions in this sequence
   b) Write a general formula to find the sum of unit fractions that form the fraction $\frac{2}{3k}$.

Appendix D – Kieran's GTG model across the five lessons

	Generational	Transformational	Global/Meta level
	Involves the **forming of** the objects of algebra. Objects of algebra are **equations and expressions from** geometric and **numeric patterns**.	Involves activities like factorising, collecting like terms, **substituting, expanding** and simplifying	Problem solving, modeling, **working with generalisable pattern, justifying and looking for relationships** or structure, studying change in functional situations.
Lesson 1	**EE1.1 – finding the general term for linear sequences** *Input Object 1:* 3, 6, 9, 12 T_1 T_2 T_3 T_4 T_5 ... T_{10} ... T_n 3 6 9 12 15 30 $3n$ $T_n = 3n$ $T_k = 3k$ *Input Object 2:* 4; 7; 10; T_1 T_2 T_3 T_4 T_5 ... T_{10} ... T_n 4 7 10 13 16 31 $3n+1$	3; 6; 9; **12; 15; 18; ...** $T_n = 3n$ $T_n = 3n$ $T_n = 3n$ $T_1 = 3 \cdot 1$ $T_2 = 3 \cdot 2$ $T_3 = 3 \cdot 3$ $= 3$ $= 6$ $= 9$ 4; 7; 10; **13; 16; ...** Teacher does not go through the substitution stage because learners have seen that the formula works	Justify 12 Justify 31
	EE2 – a cubic pattern *Input Object 3:* 1; 8; 27; ...		
	EE3 – a quadratic pattern *Input Object 4:* 1; 4; 9; ...	1; 4; 9; **16; 25; 36** 1; 4; 9; 16; 25; 36; 3 5 7 9 11 2 2 2 2	

		3; 6; 9; 12; 15; 3 3 3 3	
	EE 1.4 – giving homework from Classroom Mathematics G11		
Lesson2	**EE1.5 – going over homework** *Input Object 5:* $T_n = 2n+1$ *Input Object 6:* $T_n = 2n-6$ *Input Object 7:* $T_n = 4n-1$ *Input Object 8:* $T_n = 3n-2$	$T_1 = 2\times 1+1$ $=3$ $T_3 = 2\times 3+1$ $=7$ $T_1 = 3\times 1-2=1$ $T_2 = 3\times 2-2=4$ $T_3 = 3\times 3-2=7$	
	EE1.6 – the day's work – finding general term for linear and quadratic $y = mx+c$ $y = ax^2+bx+c$ *Input Object 9:* $T_n = an+b$ $T_n = an^2+bn+c$ $T_n = an^3+bn^2+cn+d$		

	Input Object 10: $T_n = 2n+1$ *Input Object 11:* $T_n = 4n^2 - 2n + 1$	$T_1 = 3$ $T_2 = 5$ 2 $T_3 = 7$ 2 $T_4 = 9$ 2 $T_5 = 11$ 2 $T_1 = 3$ $T_2 = 13$ 10 8 $T_3 = 31$ 18 8 $T_4 = 57$ 26 8 $T_5 = 91$ 34	
Lesson 3	**EE1.7 – recapping yesterdays last discussion** *Input Object 12:* $T_n = an+b$ $T_n = 2n+1$ *Input Object 13:* $T_n = an^2+bn+c$ $T_n = 4n^2-2n+1$ $T_n = an^3+bn^2+cn+d$ $T_n = an^4+bn^3+cn^2+dn+e$	$T_1 = 3$ 2 $T_2 = 5$ 2 $T_3 = 7$ 2 $T_4 = 9$ 2 $T_5 = 11$ $T_1 = 3$ 10 8 $T_2 = 13$ 18 8 $T_3 = 31$ 26 8 $T_4 = 57$ 34 $T_5 = 91$	Justify choice of linear general term Justify choice of quadratic general term

EE1.8 – finding linear general term

Input Object 12:

$T_n = an + b$

$T_1 = a + b = 3$

$T_2 = 2a + b = 5$

$a + b = 3 \ldots\ldots\ldots\ldots (i)$

$2a + b = 5 \ldots\ldots\ldots (ii)$

Taking (i)

$a + b = 3$

$a = 3 - b$

Sub in (ii)

$2(3-b) + b = 5$

$\Rightarrow 6 - 2b + b = 5$

$-b = -1$

$b = 1$

$a = 3 - b$

$\Rightarrow a = 3 - 1$

$a = 2$

$T_n = 2n + 1$

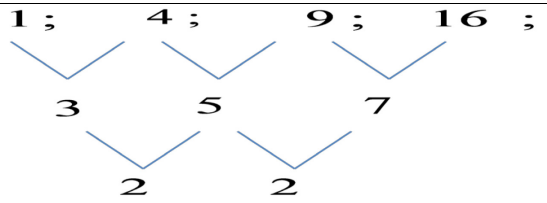
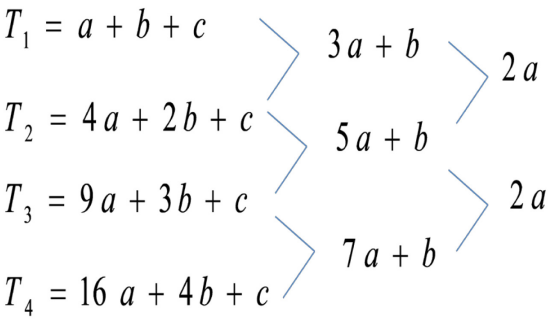
$3;\ \ 5;\ \ 7;\ \ 9; \bullet\bullet\bullet$

$2 \quad 2 \quad 2$

Constant first differences suggest a linear general term.

	EE1.9 – finding the general term for a quadratic pattern *Input Object 13:* $T_n = an^2 + bn + c$ $3a + b = 10 \ldots\ldots\ldots\ldots\ldots (i)$ $a + b + c = 3 \ldots\ldots\ldots\ldots\ldots (ii)$ $2a = 8 \ldots\ldots\ldots\ldots\ldots\ldots (iii)$ $\frac{2a}{2} = \frac{8}{2}$ $a = 4$ *Sub in (i)* $3(4) + b = 10$ $12 + b = 10$ $\Rightarrow b = -2$ *Sub in (ii)* $a + b + c = 3$ $4 + -2 + c = 3$ $c = 3 - 2$ $c = 1$ $\therefore T_n = 4n^2 - 2n + 1$	$3;\ 13;\ 31;\ 57;\ 91;\ \cdots$ $10 \quad 18 \quad 26 \quad 34$ $8 \quad 8 \quad 8$ Constant second differences suggest a quadratic general term. $T_1 = a + b + c$ $T_2 = 4a + 2b + c$ $T_3 = 9a + 3b + c$ $T_4 = 16a + 4b + c$ $T_5 = 25a + 5b + c$ $3a + b$, $5a + b$, $7a + b$, $9a + b$ $2a$, $2a$, $2a$	
	EE1.10 – giving homework from Classroom Mathematics Grade 11		

Lesson 4	**EE1.11 – Learner 1 finding the general term for a linear pattern** *Input Object 14:* 9; 11; 13; 15; ... $T_n = an + b$ $T_1 = a(1) + b$ $a + b = 9$.....(1) $T_2 = a(2) + b = 11$ $= 2a + b = 11$.....(2) Taking $a + b = 9$ $a = 9 - b$ Sub in (2) $= 2(9 - b) + b = 11$ $= 18 - 2b + b = 11$ $= -2b + b = 11 - 18$ $\frac{+b}{+} = \frac{+7}{+}$ $b = 7$ $a + b = 9$ $a + 7 = 9$ $a = 9 - 7$ $a = 2$ $T_1 = 2n + 7$ $= 2(1) + 7$ $2 + 7 = 9$ Teacher corrects the use of the equal sign and recommends that learners can use $\Rightarrow$ which means this follows Teacher corrects division by invisible -1 and writes: $\frac{+b}{+1} = \frac{+7}{+1}$ Teacher tells the class that the general term is always written in terms on n and writes: $T_n = 2n + 7$	9 ; 11 ; 13 ; 15 ; 2 2 2	Justification for choosing a linear general term

	EE1.11 – Learner 2 finding the general term for a quadratic pattern *Input Object 15:* 1; 4; 9; 16; ... $2a = 2$ $\Rightarrow a = 1$.....(1) $T_1 = a + b + c$.....(2) $T_2 = 4a + 2b + c$.....(3) $T_1 = 1 + b + c = 1$ $T_2 = 4 + 2b + c = 4$ $b + c = 0$ $b = -c$ $4 + 2(-c) + c = 4$ $4 - 2c + c = 4$ $44 - c = 4$ $-c = 0$ $\frac{-c}{-1} = \frac{0}{-1}$ $c = 0$ $b = 0$ $a = 1$ $T_n = 1n^2 + 0 + 0$ $T_n = 1n^2$	  Teacher tells class they must not repeat this but need to have the following system of equation to solve simultaneously $2a = 2$.....(1) $3a + b = 3$.....(2) $a + b + c = 1 =$ .....(3)	
	EE 1.12– looking for a relationship between two successive terms		

	Input Object 16: 3; 6; 9; 12; ... $T_1 = 3$ $T_2 = 6 = 2T_1$ $T_3 = 9$ $T_4 = 12$ (End of lesson)		
Lesson 5	**EE1.13 – finding the relationship between successive terms** *Input Object 16:* 3; 6; 9; 12; 15; ... $T_1 = 3$ $T_2 = 6$ $T_3 = 9$ $T_4 = 12$	$T_2 = T_{2-1} + 3$ $= T_1 + 3$ $= 3 + 3$ $= 6$	

	$T_2 = T_1 + 3$ $T_3 = T_2 + 3$ $T_4 = T_3 + 3$ . . $T_{10} = T_9 + 3$ . . $T_{75} = T_{74} + 3$ $T_k = T_j + 3$ $T_k = T_{k-1} + 3$ $T_n = T_{n-1} + 3$ $T_{n-1} = T_{n-2} + 3, T_1 = 3, n = 2;3;4...$		
	EE1.14 – defining the exponential sequence recursively *Input Object 17:* 1; 2; 4; 8; 16; ... $T_1 = 1$ $T_2 = 2$ $T_3 = 4$ $T_4 = 8$ $T_5 = 16$	$T_3 = 2 \cdot 2$ $= 4$ $T_2 = 2T_{2-1}$ $= 2T_1$ $= 2(1) = 2$ $T_3 = 2T_{3-1} = 2T_2 = 2(2) = 4$	

	$T_2 = 2T_1$ $T_3 = 2T_2$ $T_4 = 2T_3$ $T_5 = 2T_4$ $T_{94} = 2T_{93}$ $T_k = 2T_j$ $T_k = 2T_{k-1}$ $T_n = 2T_{n-1}; T_1 = 1, n = 2;3;4...$		
	EE1.15– Fibonacci *Input Object 18:* 1; 1; 2; 3; 5; 8; 13; ... $T_3 = T_2 + T_1$ $T_4 = T_3 + T_2$ $T_5 = T_4 + T_2$ $T_6 = T_5 + T_4$ $T_7 = T_6 + T_5$ $T_n = T_{n-1} + T_{n-2}; T_1 = 1 \& T_2 = 1$		
	EE 1.16 – homework from Classroom Mathematics G11		