

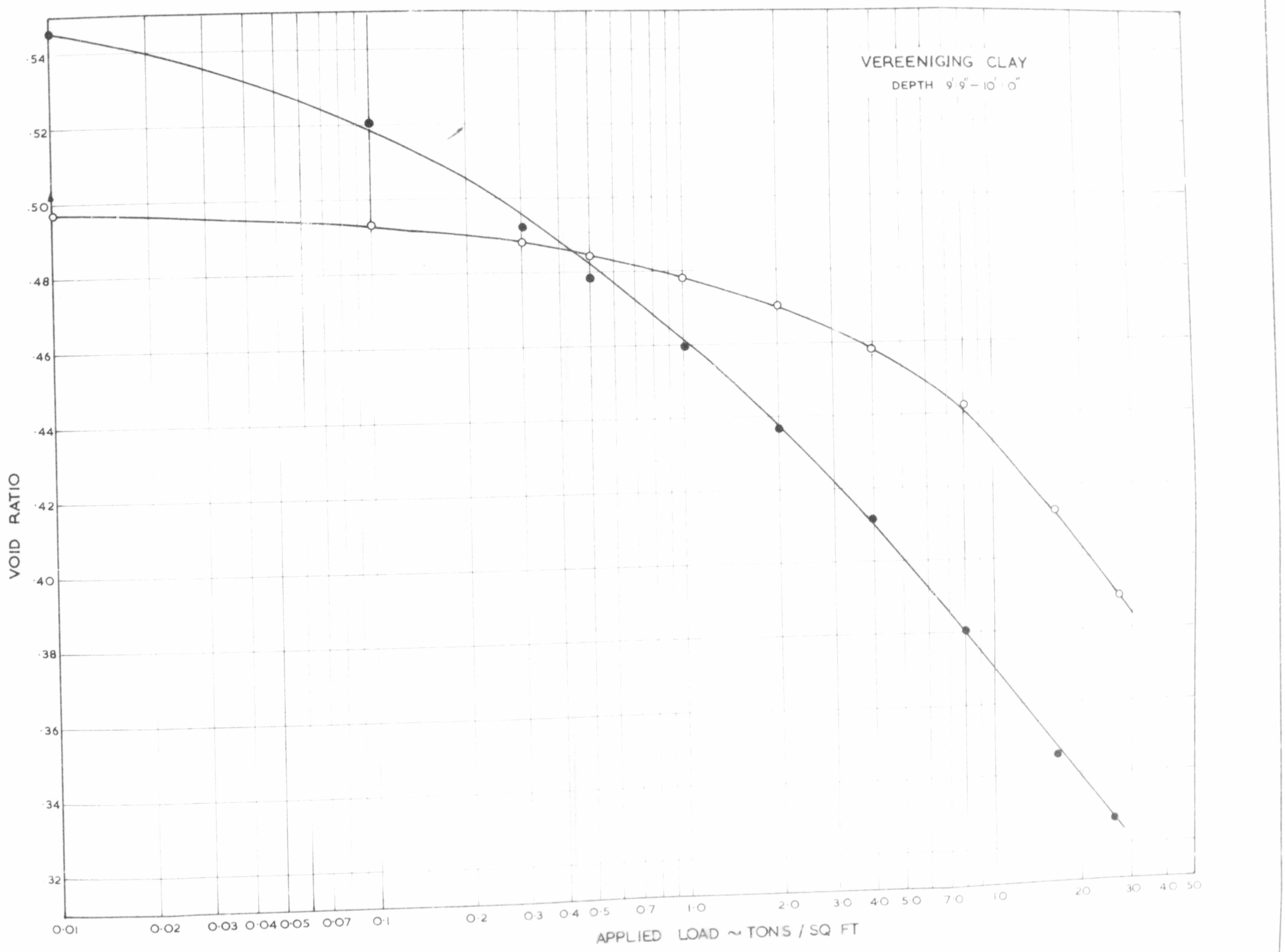
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The concept of effective stress in partly saturated soils

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TYPICAL DOUBLE OEDOMETER CURVES FOR VEREENIGING EXPANSIVE CLAY

FIG. 4.4

Epstein, P.S. and Plesset, M.S. (1950) : "Solution of a stationary bubble including surface tension effects "
J. Chem. Phys. 18, p. 1503.

Fisher, R.A. (1926) : "On the capillary forces in an ideal soil."
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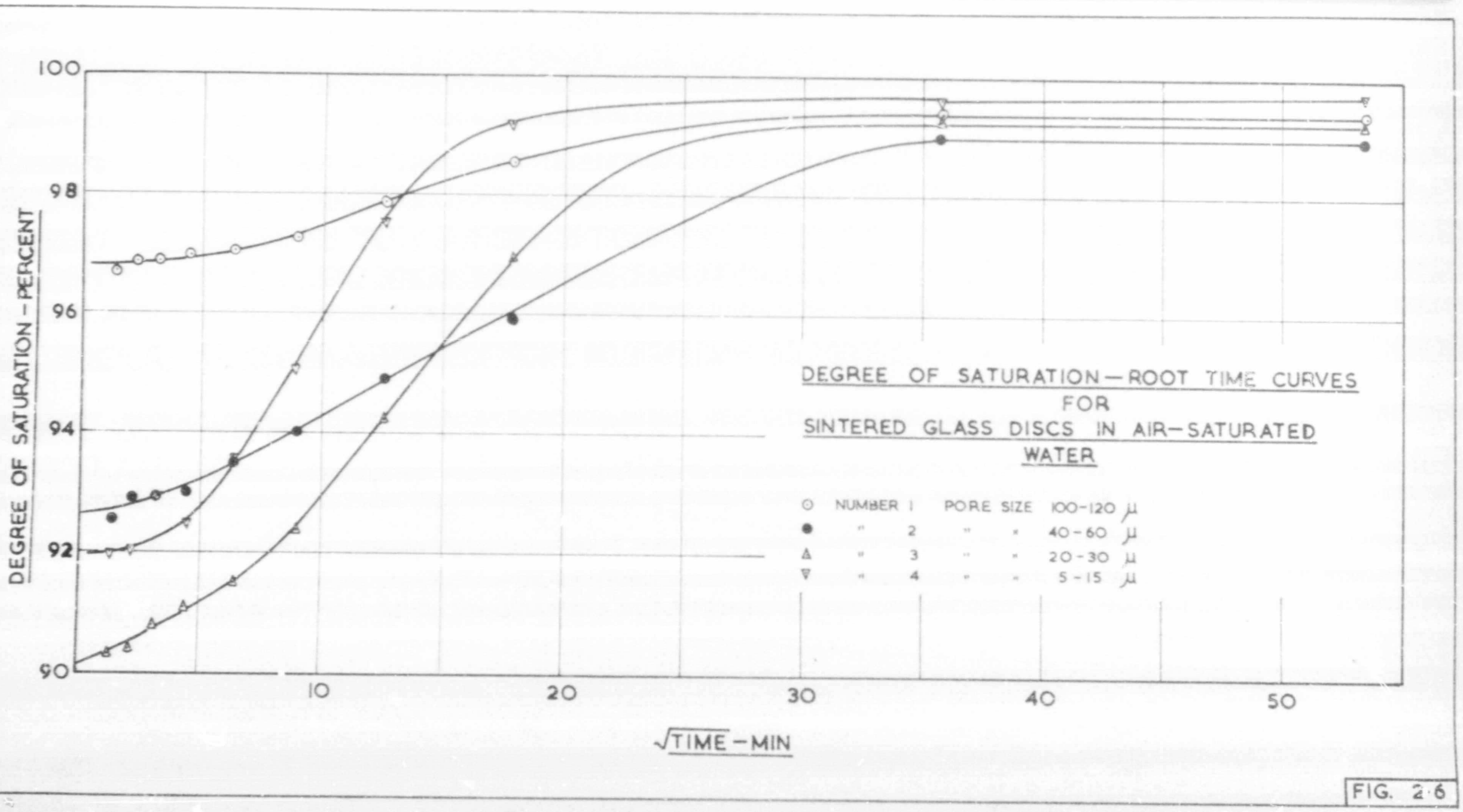

100
98
96
94
92
90
DEGREE OF SATURATION - PERCENT
10
20
30
40
50
√TIME - MIN
DEGREE OF SATURATION - ROOT TIME CURVES
FOR
SINTERED GLASS DISCS IN AIR-SATURATED
WATER
○ NUMBER 1 PORE SIZE 100-120 μ
● " 2 " " 40-60 μ
△ " 3 " " 20-30 μ
▽ " 4 " " 5-15 μ

FIG. 2·6

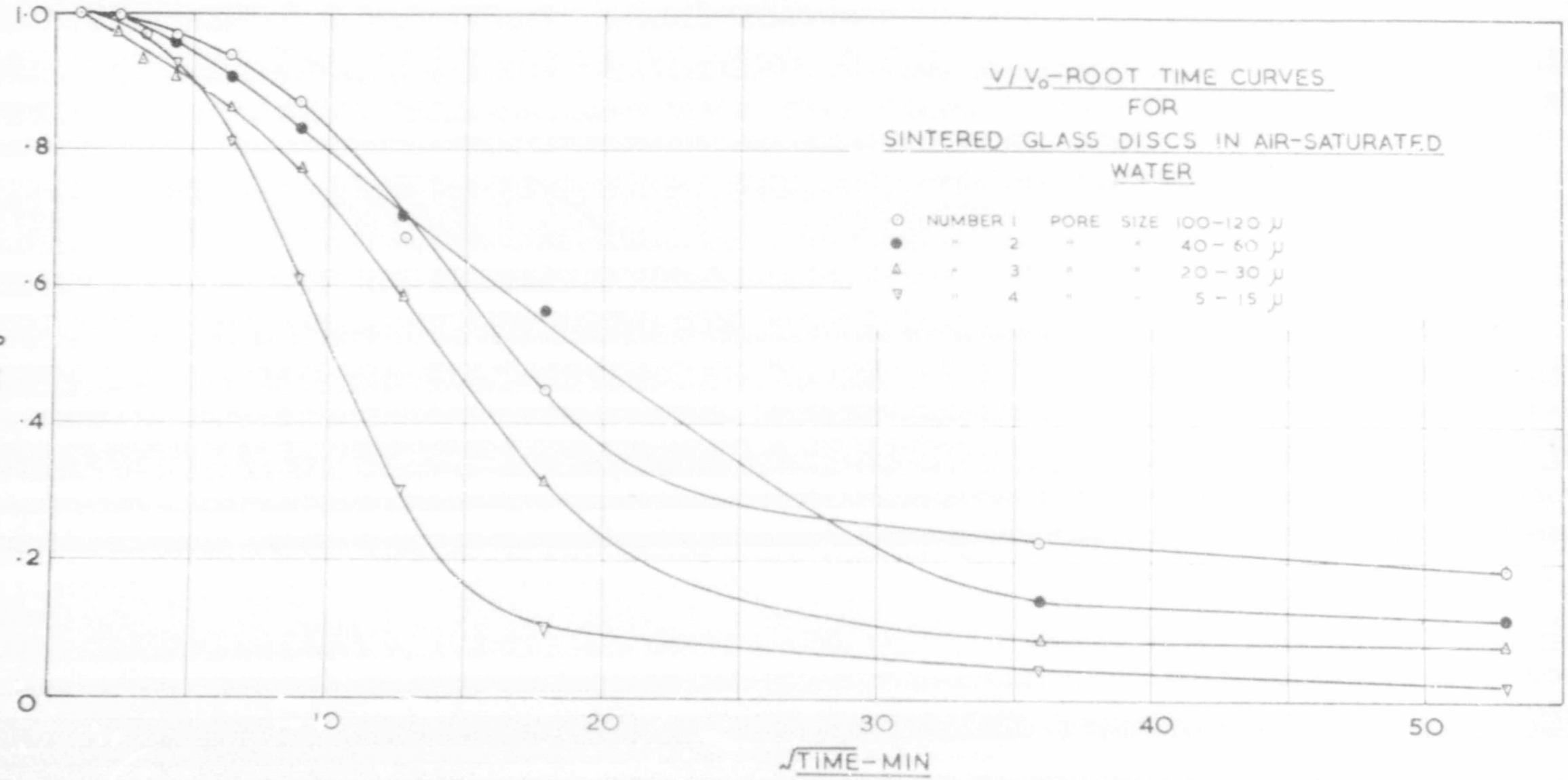

V/V₀–ROOT TIME CURVES
FOR
SINTERED GLASS DISCS IN AIR-SATURATED WATER
○ NUMBER 1 PORE SIZE 100–120 μ
● " 2 " " 40–60 μ
△ " 3 " " 20–30 μ
▽ " 4 " " 5–15 μ
V/V₀
1·0
·8
·6
·4
·2
0
10
20
30
40
50
√TIME–MIN

FIG. 2.7

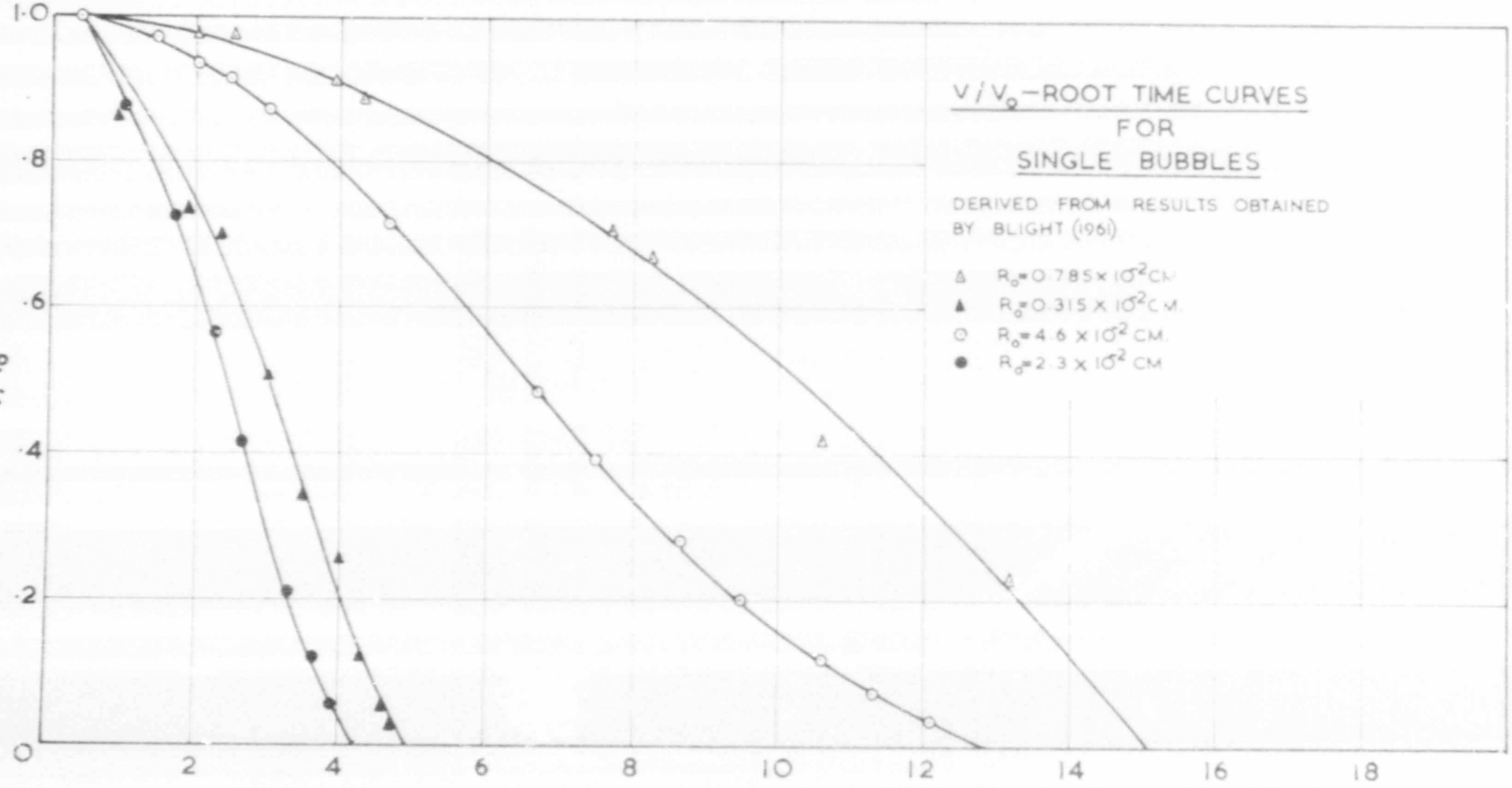
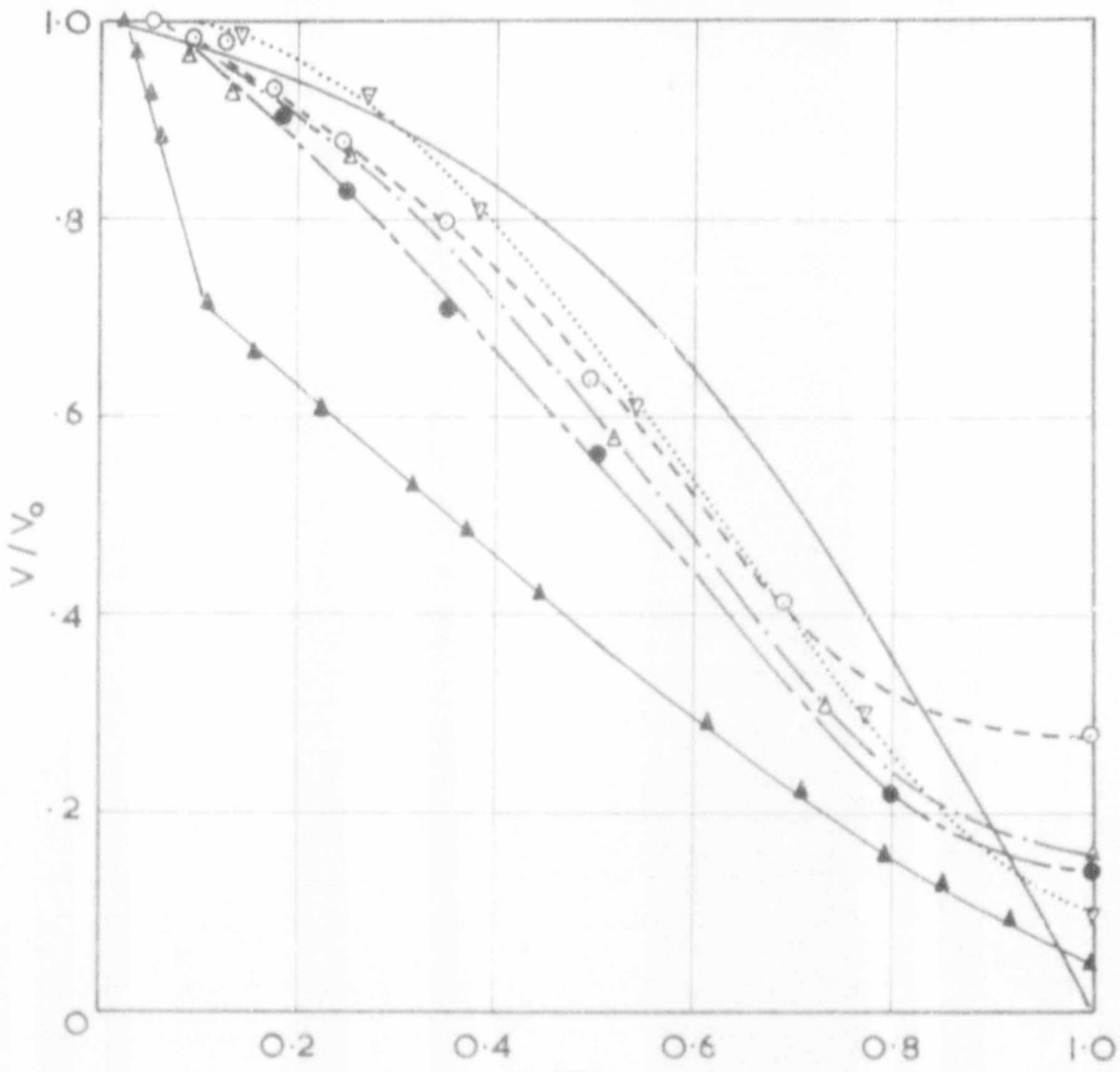

1·0
·8
·6
·4
·2
0
V/V₀
2
4
6
8
10
12
14
16
18
√TIME-MIN
V/V₀—ROOT TIME CURVES
FOR
SINGLE BUBBLES
DERIVED FROM RESULTS OBTAINED BY BLIGHT (1961)
△ R₀=0·785×10⁻²CM.
▲ R₀=0·315×10⁻²C.M.
○ R₀=4·6×10⁻² CM.
● R₀=2·3×10⁻² CM.

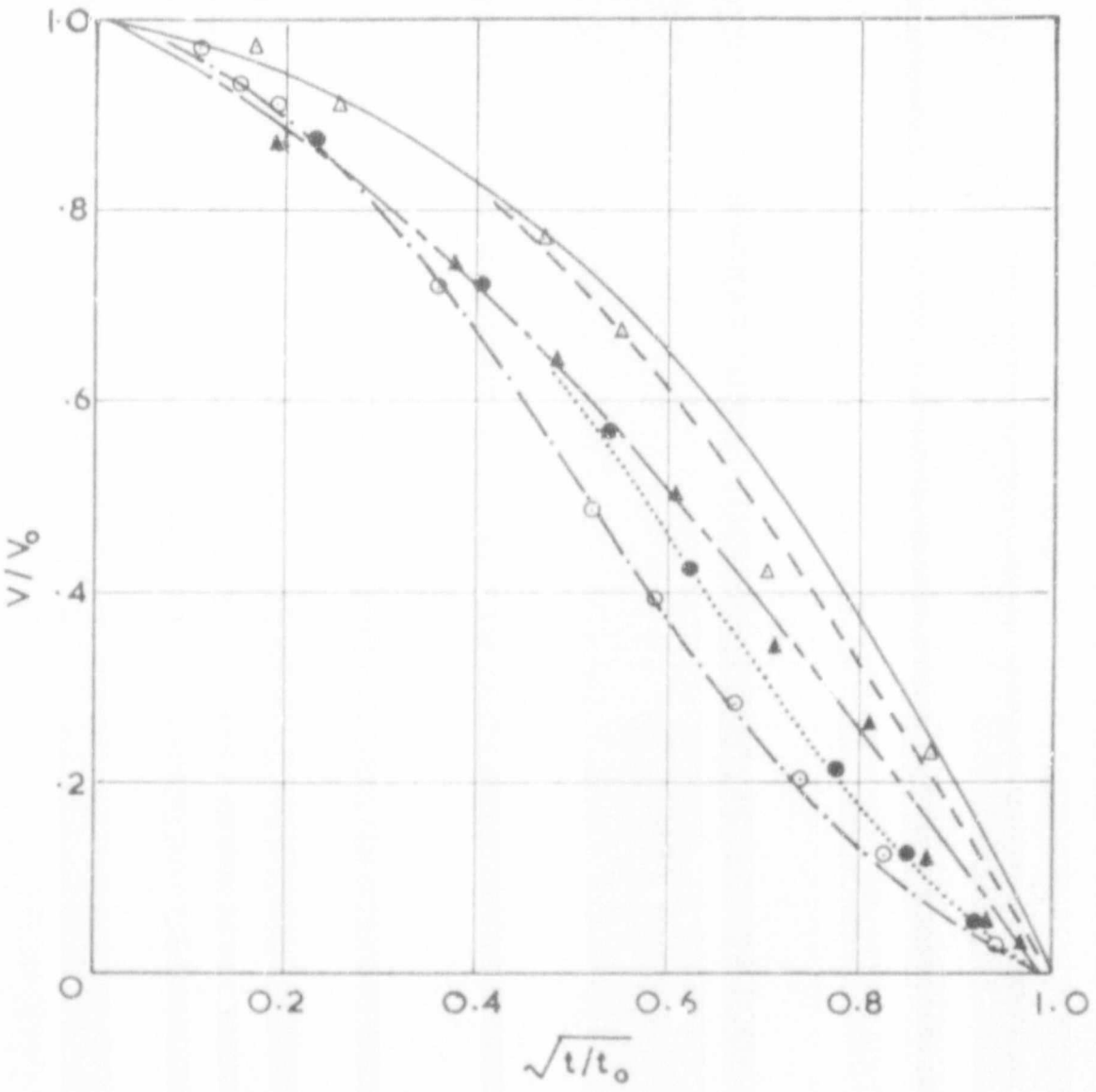
FIG. 2.8



○ NUMBER 1 PORE SIZE 100 120μ
● " 2 " " 40 60μ
△ " 3 " " 20 30μ
▽ " 4 " " 5 15μ
▲ POROUS STONE
—— THEORETICAL FOR SINGLE BUBBLE

NON-DIMENSIONAL PLOT OF VOLUME AGAINST ROOT TIME FOR POROUS DISCS

FIG. 2.9



△ $R_0 = 785 \times 10^{-2}$ CM.

▲ $R_0 = 315 \times 10^{-2}$ CM.

○ $R_0 = 46 \times 10^{-2}$ CM.

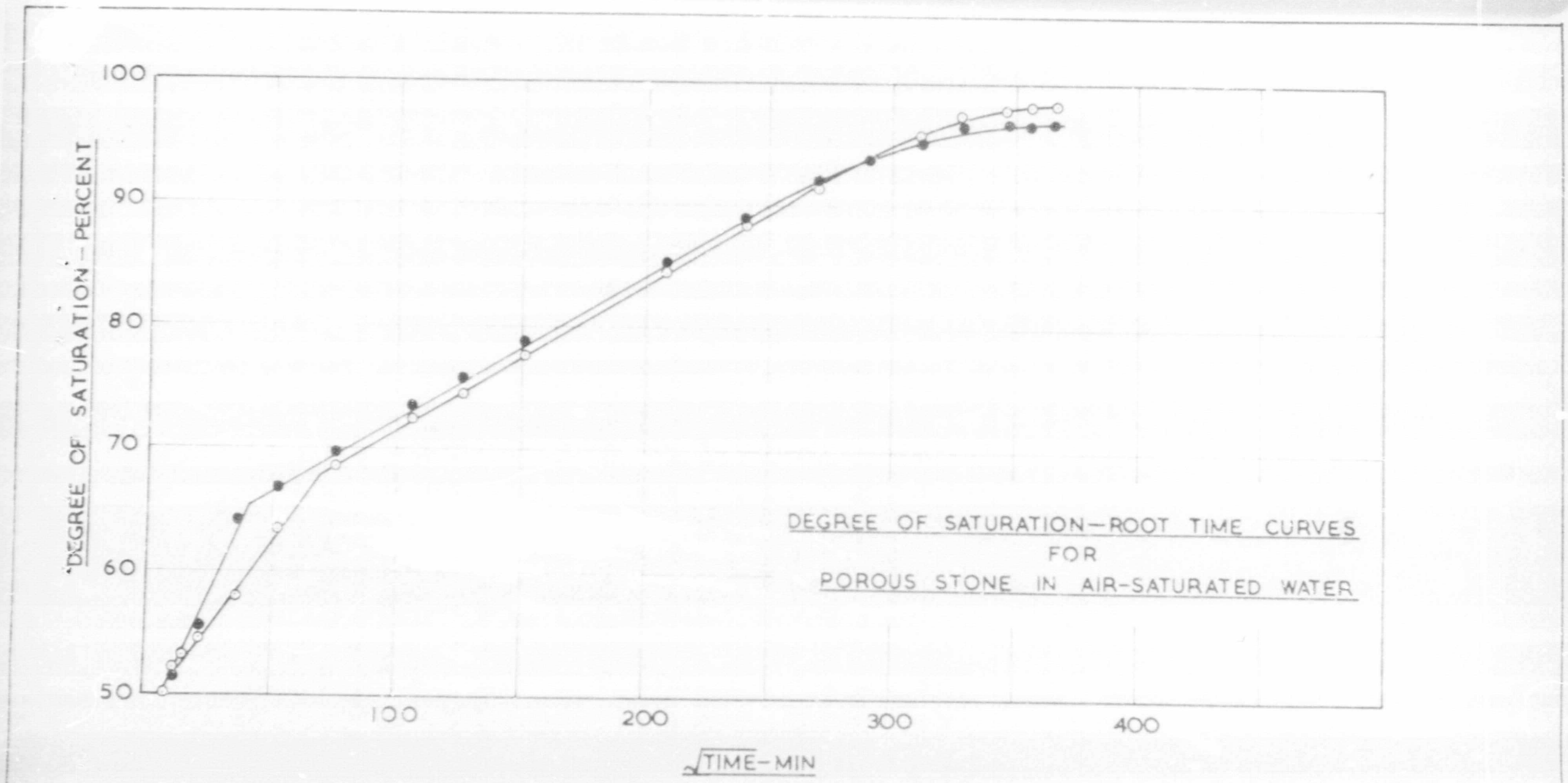
● $R_0 = 23 \times 10^{-2}$ CM.

—— THEORETICAL.

NON-DIMENSIONAL PLOT OF VOLUME AGAINST ROOT TIME FOR SINGLE BUBBLES

DERIVED FROM RESULTS OBTAINED BY BLIGHT (1961)

FIG. 2.10



DEGREE OF SATURATION—ROOT TIME CURVES
FOR
POROUS STONE IN AIR-SATURATED WATER

FIG. 2.11

CHAPTER III

PREVIOUS WORK ON EFFECTIVE STRESSES IN PARTLY SATURATED SOILS

3.1 Introduction

In chapter I the validity of the effective stress principle as it applies to the case of saturated soils was examined. It was shown to be valid for both granular and cohesive soils with the proviso that in the case of cohesive soils secondary effects such as thixotropy and secondary compression could be neglected.

A saturated soil may be regarded as a two phase system, i.e. it is composed of solids and water. In South Africa the soils encountered by the engineer are often three phase systems, i.e. the pores contain both air and water. The pressure in the pore-water in a partly saturated soil is usually much less than the pressure in the pore-air. This is due to surface tension forces acting at the air-water interfaces. The effective stress law defined in chapter I cannot be applied, as it stands, to partly saturated soils because the pressures in the pore fluids do not act over the whole area of cross section of the soil.

A number of workers have recently attempted to modify the effective stress law to cover the more general case of the three phase soil. The argument is that the effective stress principle has been proved valid for saturated soils and there is no apparent reason for doubting its validity when the pore fluid changes from a single phase to a two phase system. This chapter is devoted to examining the various approaches to the problem of effective stresses in partly saturated soils.

3.2 Consolidation of clay due to desiccation

Terzaghi (1943) appears to be one of the first workers to have considered the extension of the effective stress law

$$\sigma' = \sigma - u$$

to soils in which the value of the pore pressure u is negative. Terzaghi stated that the process of drying increases the effective pressure of a clay by $p_s = -u_s$ where u_s is the pore water pressure and p_s is the capillary pressure. Terzaghi maintained that the same ultimate effect can be obtained by consolidating the clay under a surface load $p_s = \gamma_w . h_c$ where h_c is the height of capillary rise. The work just described is based on the assumption that the pores of the soil remain saturated. Terzaghi did very little work on partly saturated soils. He suggested rather tentatively that at a limiting value of $-u_s$ the air-water interface enters the soil structure and thereafter the stress in the water at the surface of the soil remains constant. The problem of evaluating effective stresses in a partly saturated soil was not treated by Terzaghi. In chapter II it was shown that the stress in the water in the pores continues to decrease after unsaturation has occurred and hence Terzaghi is in error in this respect. However his postulate that the pore-water pressure in a quasi-saturated soil contributes directly to the effective stress will be shown in the next section to be valid.

3.3. The work of Aitchison and Donald

In 1956 Aitchison and Donald studied the problem of the two phase pore fluid. They suggested that the air voids were uniformly distributed throughout the total void space. Working from this assumption they were able to show, by means of a statistical model, that the area of water intersected by any plane of unit area passing between the soil particles is equal to the ratio $S_r/100=A$". Following Terzaghi's definition that the pore pressure u

is considered as acting over the whole area of cross section they deduced that the effectiveness of a pressure deficiency p" in contributing to the effective stress is dependent upon S_r. Aitchison and Donald carried out a detailed theoretical and experimental investigation aimed at yielding the relationship between p", S_r, and σ'. This investigation involved mainly the study of granular soils, although some preliminary tests were performed on cohesive materials. This work will be summarized briefly in the following paragraphs.

(a) <u>The theoretical determination of effective stresses in idealised granular soils</u> : An idealised granular soil consists of uniform sized homogenous spherical particles arranged in a known systematic packing. The two packings most amenable to treatment are the open or cubical system and the close packed hexagonal system. The assumption is made that the grains are completely incompressible. Haines (1927) has shown that the force between two spherical particles, of radius 'a', due to a given pressure deficiency p" could be expressed in terms of the corresponding curvature of the air water interface

$$F = \frac{2\pi \cdot a \cdot T}{1 + \tan^2 \theta/2}$$

where T is the surface tension and 2θ is the angle subtended at the centre of the particle by the inter-particle lense of water (see fig. 3.1)

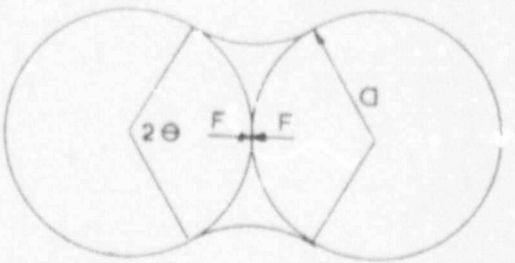


Fig. 3.1

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where T is the surface tension and 2θ is the angle subtended at the centre of the particle by the inter-particle lense of water (see fig. 3.1)

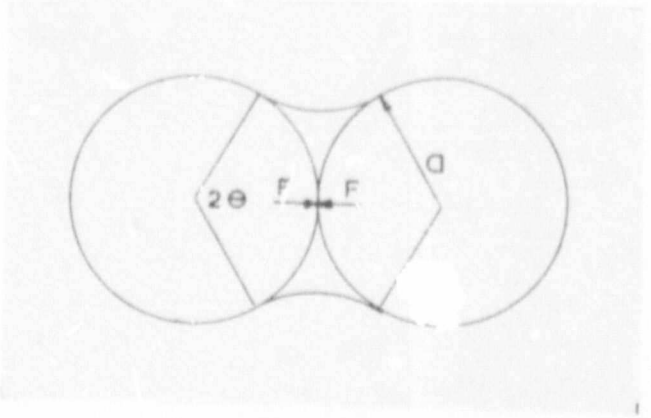


Fig. 3.1

This interparticle force may be transformed into an equivalent stress by considering the number of contacts per unit area. For one interparticle contact the area is $4a^2$ for open packing and $\sqrt{2}.a^2$ for close packing. Further extension of geometrical and statical considerations enabled Aitchison and Donald to calculate the pressure deficiency required to cause drainage of an initially saturated ideal granular soil. This is given by $p_d'' = 12.9$ T/a and $p_d'' = 4.8$ T/a for close and open packing respectively. With this data it is possible to calculate the relationship between pressure deficiency and inter-granular stress for the whole range of undersaturation. Aitchison defined the effective stress resulting from a pressure deficiency as σ'' and the maximum effective stress that can be achieved by this means as σ''_{max}. Figures 3.2(a) and 3.2(b) show the relationship between σ'',p'' and S_r as calculated by Aitchison and Donald for idealised granular soils.

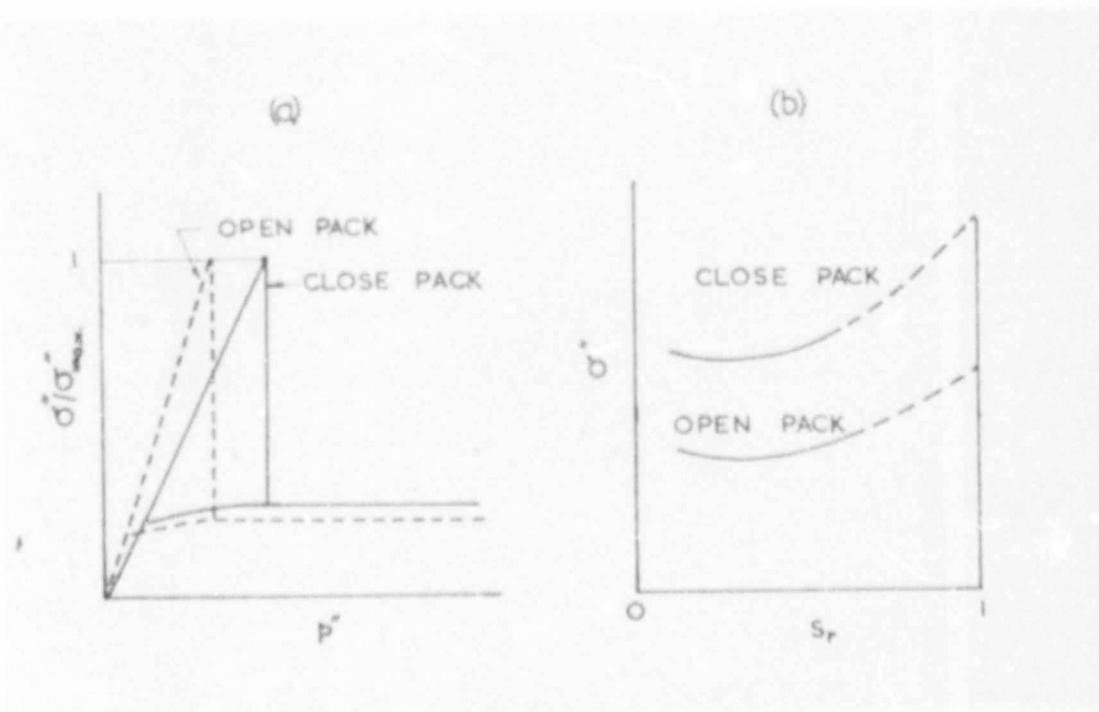


Fig. 3.2

The relationship between σ'',p'' and S_r for idealised granular materials

(b) The theoretical determination of effective stresses in granular soils. By making certain simplifying assumptions Aitchison and Donald have been able to extend the work discussed above to actual granular materials.

The pore water is considered in two phases :

(i) Water filled pores in which S_r=100 and $p" = \sigma"$

(ii) Drained pores in which $\sigma" = 0.31\ \bar{p}"\ S_r$.

where $\bar{p}"$ is the pressure deficiency increment resulting from a change of saturation ΔS_r. Both these values can be obtained from the moisture content : pressure deficiency curve for the material.

The total effective stress at any value of the pressure deficiency p" is given by

$$\sigma" = \frac{1}{100}\left(S_r.p" + \int_0^{p"} 0.31.\bar{p}."\Delta S_r\right) \quad . \quad . \quad . \quad (3.$$

Donald (1956) performed shear te four sands under fully drained and controlled pr deficiency conditions. He found that for complete uration $\sigma" = p"$ as predicted by Terzaghi. At degrees of saturation less than one he calculated the theoretical values of $\sigma"$ from equation (3.1). The values of $\sigma"$ compared very favourably with the values of σ' obtained by using the shear strength of the fully drained soil as a measure of effective stress.

The three main conclusions to be drawn from this work of Aitchison and Donald are :

(i) The equivalent pore pressures may be calculated with an error normally not exceeding about 10 percent. The procedure is, however, very laborious.

(ii) Throughout the range of quasi-saturation the pressure deficiency contributes directly to the effective stress.

(iii) The maximum value of $\sigma"$ for a uniform granular material occurs at a water content slightly below saturation value. For well graded materials maximum effective stress occurs at a water content much less than saturation value.

There are two important observations to be made on the work just described. The first one is that the material used was mainly uniform fine sand containing nothing smaller than 0.01mm grain size, and the major portion of the material greater than 0.05mm. Therefore the material, when placed in the triaxial machine, took up a uniform stable structure which proved to be very incompressible. The importance of this point will be brought out in Chapter IV. The second observation worth noting is that the values of compressive strength resulting from samples subjected to pressure deficiency tests never exceeded more than about 12 lb. per sq.inch. Hence the contribution made by the pressure deficiency in sands and gravels to the total effective stress will, in most practical cases, be negligible.

(c) Effective stresses in partly saturated compressible soils. The work described in sections 3.3(a) and 3.3(b) applies to coarse grained incompressible materials. The theoretical results were checked by using the shear strength of the soil as a measure of the effective stresses. For a highly compressible material there is no fixed relationship between particle size and pore dimensions and therefore the geometric and statistical arguments used in the previous sections cannot be extended to very fine grained soils. Moreover the use of shear strength as a measure of effective stress involves many practical and theoretical difficulties. Another method must therefore be found by which the effective stress in a soil can be measured. Void ratio is, if anything, a more direct manifestation of effective stress than shear strength. Aitchison undertook a limited number of tests on partly saturated compressible materials using the void ratio of the saturated counterpart as a measure of the effective stress. Using laboratory and field data he drew $e : \log \sigma'$ and $e : pF$ curves for five soils. Four of these soils were clays having -2μ fractions varying from 65% to 85%. The fifth soil was a silty sand. For the clays Aitchison found that the curves of pF and $\log \sigma'$ were approximately coincident even over a limited range of unsaturation. The tests carried out

on the silty sand were interesting in that the log σ' curve showed that the soil was fairly compressible whereas the pF curve showed that the soil was completely incompressible as regards changes in pressure deficiency. Aitchison remarks that the void ratio provides no index of effective stress for this case and the methods described previously should be employed for purposes of analysis.

It is not possible to draw any quantitative conclusions from these tests since the degree of laboratory control was not high. The tests are of value, however, in that they indicate lines along which an accurate evaluation of effective stresses in partly saturated compressible soils may be made.

(d) Discussion of Aitchison's and Donald's work. A close examination of Aitchison's and Donald's work will reveal that, in fact, they only studied a limited range of soils, viz. highly incompressible uniform sands and highly compressible clays. The one test performed on a silty sand was rejected because the result was anomalous. For this reason care must be taken in extending their results to soils which lie in the intermediate range. Such soils probably form a large majority of the soils likely to be encountered in engineering practice.

The three main conclusions to be drawn from the work described in this section are :

(i) The normal concept of effective stress as defined by Terzaghi is valid for all quasi-saturated soils.

(ii) The same concept is also applicable in highly compressible clayey soils up to pressure deficiencies of the order of pF4.5.

(iii) In the case of incompressible unsaturated soils the normal concept of effective stress is totally unacceptable. However, consideration of void geometry enables equivalent pore pressures to be calculated from known pressure deficiencies.

Aitchison's and Donald's work is important for a number of reasons, the three main ones being :

(i) They have indicated that the bridge between effective stresses in saturated and partly saturated soils lies in the adjustment of the value of the pressure deficiency to give an equivalent pore pressure.

(ii) They have outlined a way of evaluating effective stresses in unsaturated soils by comparing the behaviour of similar soils in which the effective stresses are known.

(iii) They have demonstrated conclusively that the effective stress principle is valid for all quasi-saturated soils.

3.4 A revised effective stress law proposed by Jennings

Jennings and Knight (1957) proposed a method for predicting the heave due to swelling of desiccated clays when allowed to imbibe water without change in applied loads. The problem involves changes in effective stress which cannot be predicted by means of the normal effective stress equation. This is because the soils are generally unsaturated and very shattered. Jennings (1957) suggested that the final equilibrium effective pressure in an element of soil at depth z beneath an infinite impermeable membrane is given by

$$\sigma z' = \gamma .z + \beta . \gamma_w.(H-z)$$

where H is the depth of the water table and β is a factor which depends on the geometry of the air and water phases. It can be seen that $\gamma_w.(H-z)$ is the pressure deficiency p" under equilibrium conditions. Hence a more general form of the equation may be written

$$\sigma ' = \sigma + \beta .p" \quad \ldots\ldots\ldots \qquad (3.2)$$

On comparing this equation with Terzaghi's effective stress law it can be seen that

$$\beta . p'' = -u = \bar{u}$$

With reference to the work of Aitchison discussed in section 3.3, equation (3.2) is seen to be the logical outcome of his efforts to relate the pore pressure with the pressure deficiency in an unsaturated soil. In fact Aitchison (1960) proposed an identical equation to (3.2) above in which a factor ψ substitutes for Jennings' β.

Jennings (1960) explains the heaving process in terms of his revised effective stress law. He also demonstrates by means of a capillary model that $\beta = 1$ for the case of quasi-saturated and partially saturated soils.

Jennings then outlines two methods for the measurement of β in remoulded soils. Both methods are refinements of the experimental procedures followed by Aitchison and Donald.

In the first method two similar samples are consolidated, one under externally applied pressures and the other under applied pressure deficiencies (see fig.3.3a) The assumption is made that the void ratio at equilibrium is a measure of the effective stress in the soil. Then at any specific void ratio the effective stress in both samples is the same. The relationship between σ', p'' and β is therefore given by

$$\sigma' = \beta\, p''$$

Similarly for shear two parallel tests are run on similar samples, one consolidated under external pressure and the other under applied pressure deficiencies (see fig. 3.3b). The relation between σ', p'' and β is then given by

$$\sigma' = \beta\, p'' (1 + \sin \phi')$$

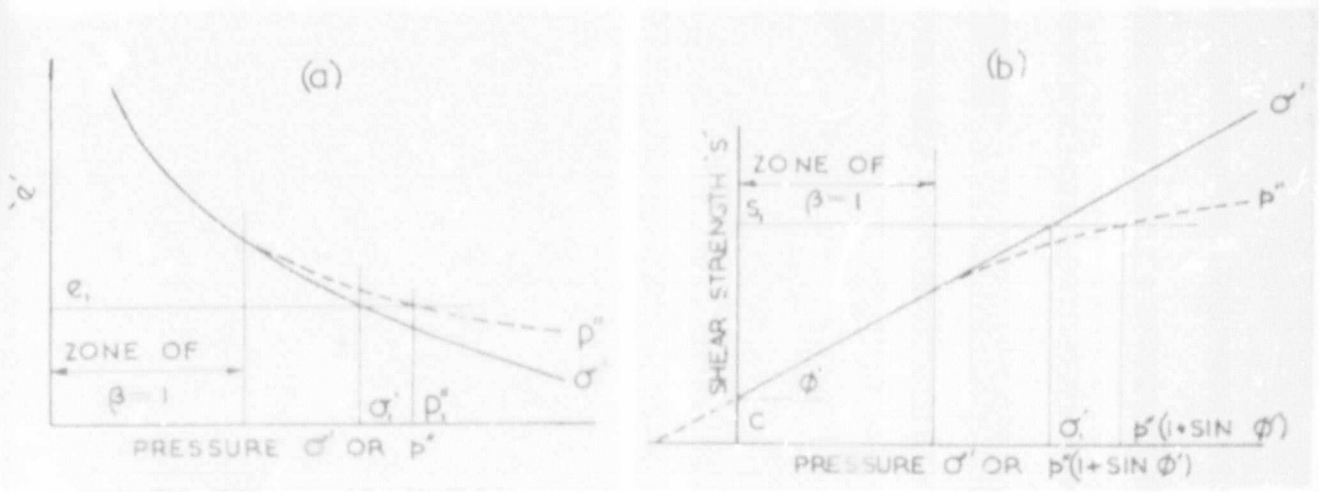


Fig. 3.3

The determination of β from consolidation and shear tests

Both these tests rely on the assumption that the effective stresses in the applied pressure sample are equal to the effective stresses in the applied pressure deficiency sample. This assumption is based on work by Rendulic (1937) and Henkel (1959 and 1960) which was studied in detail in chapter I. These workers have shown that with identical samples of clay, subject to identical stress treatment, the void ratio is a measure of the effective stress in a soil. This proposition reveals one more implicit assumption made in the above experiments. This assumption is that stress application due to change of external loads is identical to stress application due to change of pressure deficiency. The validity of this assumption will be examined in detail in chapter IV.

3.5 Effective stresses derived from energy principles

In 1958 Croney et al approached the problem of partly saturated soils using energy principles. They considered an element of soil having a volume V_s and acted on by an applied pressure p. The volume of water in the

element is V_m and the pressure in the pore water is p".* It must be mentioned that p is an independent variable but p" is a function of V_s and V_m.

p changes by an amount Δp and p" by Δp". These changes are accompanied by a change in the total volume of the soil ΔV_s and a change in the volume of the water in the element ΔV_m. The resulting change in free energy is

$$\Delta F = (p + \Delta p)\, \Delta V_s + (p" + \Delta p")\, \Delta V_m$$

Neglecting second order terms this becomes

$$\Delta F = p . \Delta V_s + p" . \Delta V_m \quad \ldots\ldots\ldots \quad (3.3)$$

If p is allowed to change but p" is held constant then

$$\left(\frac{\partial F}{\partial V_s}\right)_{p"} = p + p" \left(\frac{\partial V_m}{\partial V_s}\right)_{p"} \quad \ldots\ldots \quad (3.4)$$

Croney stated that if for small variations of soil conditions $\left(\frac{\partial V_m}{\partial V_s}\right)_{p"}$ could be considered constant equation (3.4) could be written

$$\sigma = p + \beta p" \quad \ldots\ldots\ldots\ldots \quad (3.5)$$

Equation (3.5) looks identical to Jennings' equation (3.2). If this is the case then, since $\beta = \left(\frac{\partial V_m}{\partial V_s}\right)_{p"}$, the slope of the curve of water volume change against total volume change at any specific pressure deficiency

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* Croney defines u as the pressure in the water in the soil when the soil is under an applied load. u is therefore equivalent to the pressure deficiency p". For the purposes of this thesis the term p" will be used when referring to the pressure deficiency in the pore water and will only be equal to -u in special cases.

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p changes by an amount Δp and p" by $\Delta p"$. These changes are accompanied by a change in the total volume of the soil ΔV_s and a change in the volume of the water in the element ΔV_m. The resulting change in free energy is

$$\Delta F = (p + p)\ \Delta V_s + (p" + \Delta p")\ \Delta V_m$$

Neglecting second order terms this becomes

$$\Delta F = p . \Delta V_s + p" . \Delta V_m \quad \ldots\ldots\ldots \quad (3.3)$$

If p is allowed to change but p" is held constant then

$$\left(\frac{\partial F}{\partial V_s}\right)_{p"} = p + p" \left(\frac{\partial V_m}{\partial V_s}\right)_{p"} \quad \ldots\ldots \quad (3.4)$$

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