THE CONCEPT OF EFFECTIVE STRESS IN PARTLY SATURATED SOILS

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THE CONCEPT OF EFFECTIVE STRESS IN PARTLY SATURATED SOILS

A thesis
presented to the
University of the Witwatersrand
South Africa

In partial fulfilment
of the requirements for the degree of
Master of Science in Engineering

by

J.B. BURLAND

June 1961

D E C L A R A T I O N

I, the undersigned, declare that this thesis :

1. is entirely my own work.

2. has not been submitted to another University for the degree of M.Sc. (Eng)

[signature]

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ACKNOWLEDGEMENTS

The work described in this thesis was carried out in the Department of Civil Engineering at the University of the Witwatersrand during 1960 and the first half of 1961 on a research grant from the South African Council for Scientific and Industrial Research. I wish to acknowledge with gratitude the opportunity provided by the Council for carrying out this research.

Throughout this investigation my supervisors, Professor J.E.B. Jennings and Mr. K. Knight, have always been prepared to discuss the various problems which arose. Their interest, help, guidance and encouragement have been a continual source of inspiration for which I express my sincere gratitude.

The stimulating discussions with Mr. R.H. Mills, Mr. C. Wolhuter and Dr. G.E. Blight on the staff of the department, were also greatly appreciated.

The many lengthy and fruitful discussions with my fellow research students (and in particular Mr. F. Wagener) helped in clarifying many of the problems encountered during the investigation.

My special thanks are due to Mr. J.H. Frew for his help in the design and construction of the apparatus, and to Mrs. K. Dyer for typing this thesis.

Finally I wish to record my gratitude to my parents without whose patience and encouragement this thesis could not have been completed.

SYNOPSIS

The definition of the principle of effective stress and its implications have been examined for saturated and partly saturated soils. The results of a series of oedometer and all round compression tests on partly saturated and fully saturated soils are presented. These results together with additional experimental data indicate that most soils, from sands right through to clays, exhibit behaviour which, below a critical degree of saturation, cannot be accounted for by the effective stress principle. In sands the critical degree of saturation appears to be below 50%. In clays, however, the critical degree of saturation is upwards of 85%.

An explanation for the observed behaviour of partly saturated soils is offered. It is apparent that structural changes resulting from a change of pressure deficiency in a soil are very different from those resulting from an equivalent change in applied stress.

The investigation as a whole indicates that, below the critical degree of saturation, the concept of effective stress in a partly saturated soil is not valid. It is suggested that the term 'intergranular stress' is more suitable than the term 'effective stress' since its use does not imply the validity of the principle of effective stress.

The practical significance of the investigation is discussed briefly and the lines along which further research would prove profitable are indicated.

TABLE OF CONTENTS

page No.

CHAPTER V. THE DEVELOPMENT OF HIGH PRESSURE ALL ROUND COMPRESSION APPARATUS.

CHAPTER VI. EXPERIMENTAL PROCEDURE AND RESULTS.

CHAPTER VII. ANALYSIS AND DISCUSSION OF TEST RESULTS.

INTRODUCTION

The science of soil mechanics developed mainly in and around Europe where the climate is temperate and the land mainly low lying. The predominate characteristic of engineering soils in these regions is that they are nearly always fully saturated. It is not surprising, then, that classical soil mechanics is concerned chiefly with soil in which the voids are filled with water.

It is for these soils that Terzaghi formulated his principle of effective stress. This principle has proved invaluable to the engineer who is concerned with the prediction of soil behaviour. It is true to say that the principle of effective stress lies at the foundation of most modern soil mechanics theory and practice. In fact, it has come to be regarded by most foundation engineers as axiomatic.

In many parts of South Africa, and other semi-arid regions, the engineering soils do not conform to the classical pattern since they are not usually fully saturated. Moreover, even in regions where the soils are fully saturated the water table may be well below the surface - a condition seldom encountered in temperate climates. The engineering problems associated with such soils have recently been discussed by Jennings (1960). Besides involving the more classical questions of consolidation and shear strength these problems also include phenomena not encountered in fully saturated soils, e.g. heave due to swelling of desiccated clays and rapid settlement due to collapse of soil structure on wetting. The need for placing the prediction of the behaviour of unsaturated soils on a sound scientific basis is becoming very apparent.

In extending the science of soil mechanics to cover the case of partly saturated soils the possibility that some of the classical theories might require modification, to account for certain soil phenomena not manifested in fully saturated soils, must not be overlooked. For this reason it is important that theories formulated for fully saturated soils should not be applied to partly saturated soils until their validity has been tested.

In view of the fundamental role played by Terzaghi's principle of effective stress in the study of fully saturated soils the possibility of its extension to partly saturated soils is an important consideration. The purpose of this thesis is to examine the concept of effective stress in partly saturated soils and to carry out a preliminary investigation into the behaviour of such soils in relation to the effective stress principle. The investigation will be confined mainly to considerations of void ratio.

Reference :

Jennings, J.E. (1960) : "A revised Effective Stress Law for use in the Prediction of the Behaviour of Unsaturated Soils."
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ORGANISATION OF THESIS

Before examining the problem of effective stresses in partly saturated soils it is necessary to have a reasonably clear understanding of the principle of effective stress and its implications when applied to fully saturated soils. Chapter I has therefore been devoted to the problem of effective stresses in fully saturated soils.

A partly saturated soil may be considered as having a two phase pore fluid consisting of free air and water. The interface between the air phase and the water phase is the seat of very complex physical phenomena. It was felt that, before proceeding with the enquiry into the behaviour of par[illegible]turated soils, a brief exposition of some of [illegible] fundamental physical properties of the [illegible]ined air-water phase was necessary and chapter II has been devoted to this.

In chapter III some of the more important work on effective stresses in partly saturated soils is discussed. In chapter IV a preliminary examination of the validity of the principle of effective stress in partly saturated soils has been carried out making use of existing experimental data.

The laboratory phase of the investigation involved firstly the development of special apparatus for the study of volume changes in partly saturated soils. This apparatus is described in chapter V and is followed by a description of testing procedures and the presentation of results in chapter VI. These test results are discussed in chapter VII.

The major portion of the investigation is devoted to considerations of volume changes in partly saturated soils. In order to afford some measure of completeness a brief examination of the shear strength characteristics of partly saturated soils is presented in chapter VIII. Chapter IX is devoted to a general discussion of the investigation as a whole.

CHAPTER 1

EFFECTIVE STRESSES IN FULLY SATURATED SOILS

1.1 Introduction

This thesis is primarily concerned with the concept of effective stresses in partly saturated soils. However, it is first necessary to obtain a clear idea of what is meant by the effective stress principle when applied to fully saturated soils before proceeding to the partly saturated case. Much has been written on the subject since the principle was first proposed by Terzaghi in 1923, and there is a need to re-state the original proposition and to examine its implications. A brief survey of the most important work on the subject will be undertaken with a view to assessing the validity of the principle for fully saturated soils.

1.2 The principle of effective stress.

The principle of effective stress was first formulated by Terzaghi in 1923 when he proposed his theory of consolidation. In 1932 Terzaghi indicated the significance of effective stress on the shear strength of a soil. The principle of effective stress as it applies to volume changes and shear strength was stated by Terzaghi (1936) as follows :

"The stresses in any point of a section through a mass of earth can be computed from the total principal stresses σ_1, σ_2 and σ_3 which act at this point. If the voids of the earth are filled with water under a stress u, the total principal stresses consist of two parts. One part, u, acts in the water and in the solid in every direction with equal intensity. It is called the neutral stress. The balance $\sigma_1' = \sigma_1 - u$, $\sigma_2' = \sigma_2 - u$ and $\sigma_3' = \sigma_3 - u$ represents an excess over the neutral stress and it has its seat exclusively in the solid phase of the earth.

This fraction of the total principal stresses will be called the effective principal stresses ...... All the measurable effects of a change of the stress, such as compression, distortion and a change of shearing resistance are exclusively due to changes in the effective stresses σ_1', σ_2' and σ_3'. Hence every investigation of the stability of a saturated body of earth requires the knowledge of both the total and the neutral stresses.'

Thus, the principle of effective stress may be defined in the form of two proportions :

(i) The effective stress σ' in a soil is defined as the excess of the total applied stress σ over the pore pressure (neutral stress) u.

i.e. $\sigma' = \sigma - u$ . . . . (1.1)

(ii) The volume change and shearing characteristics of a soil are dependant purely on the effective stress.

If proposition (ii) is proved to be incorrect for a particular soil then the principle of effective stress is not valid for the soil. However, equation (i) can still be applied to the soil but it has lost much of its practical significance since it cannot be used to predict the behaviour of the soil. For the purpose of this thesis the principle of effective stress is defined by proposition (ii) and the effective stress law for a fully saturated soil is defined by proposition (i).

1.3 The influence of the area of contact between soil particles

In 1950 Bishop and Eldin examined the validity of the effective stress principle for saturated granular soils. They showed that the distortion of the soil grains was due to a pressure $\sigma_c' = \sigma - u$ but that the frictional force mobilized during shearing between grains was due to the pressure $\sigma_s' = \sigma - (1-a)u$ where 'a' is the area of contacts between grains projected onto a plane of unit area. By attributing the compressibility of the soil structure C_s to the action of σ_c' Bishop and Eldin showed that

$$\frac{\partial u}{\partial \sigma} = \left[\frac{1}{1 + n \frac{Cu}{Cs}} \right] (= B) \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (1.2)$$

where B is a factor proposed by Skempton (1954), and that the angle of undrained shearing resistance with respect to applied loads (ϕ_{uu}) is given by

$$\text{Sin } \phi_{uu} = \frac{1}{1 + \frac{1 - \text{Sin } \phi_{cu}}{\text{Sin } \phi_{cu}} \left[\frac{(1 - Du)(1 + n.Cu/Cs)}{a + n.Cu/Cs} \right]} \quad . \; . \; . \; (1.3)$$

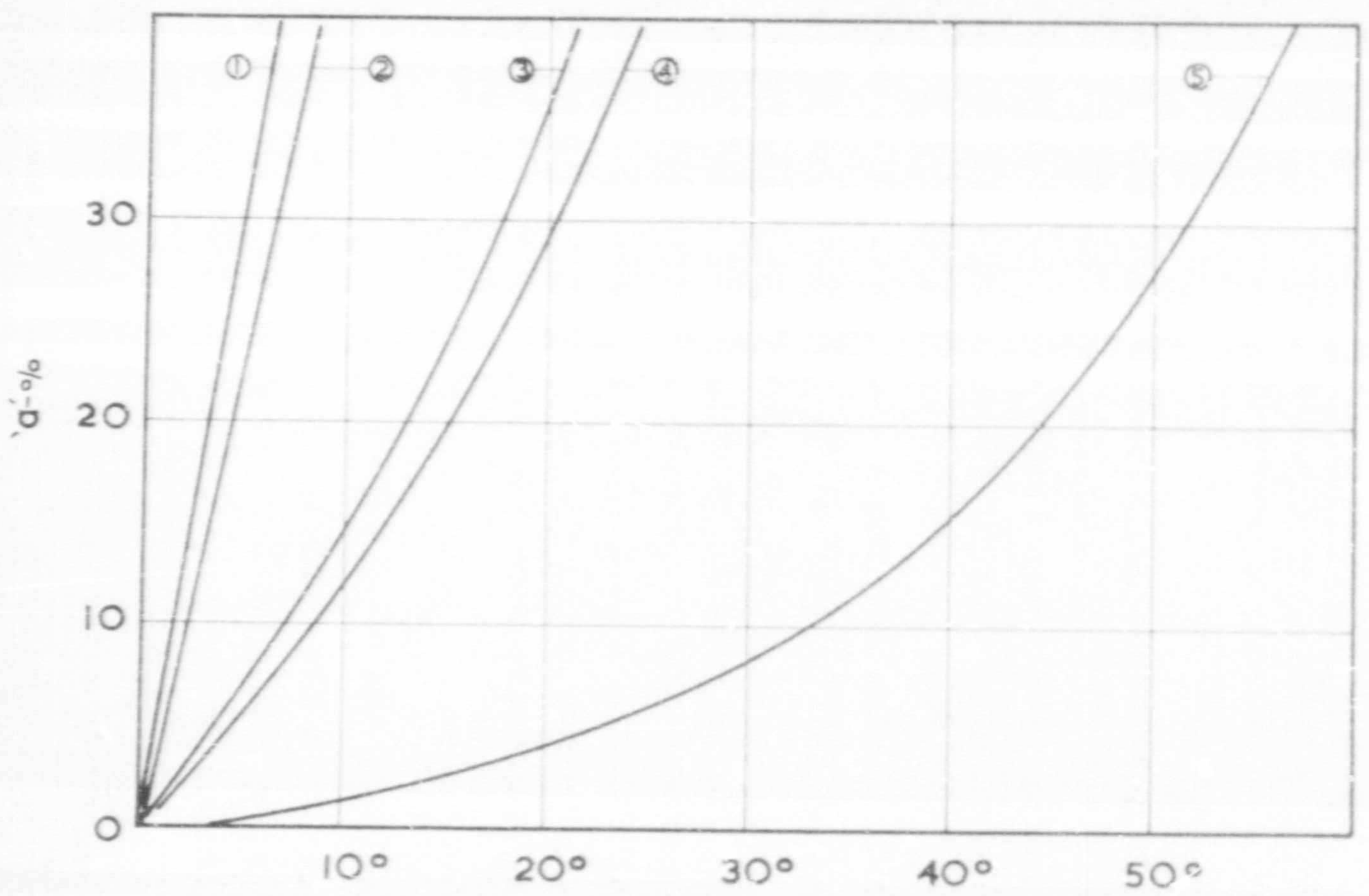
Where C_u is the compressibility of the pore fluid, C_s is the compressibility of the soil structure, 'n' is the porosity, 'a' is the contact area between grains projected onto a plane of unit area and D is a constant given by the expression

$$D = \frac{1}{bS + u}$$

where S is the crushing strength of the grains and b is a constant whose value will depend on the type of surface failure produced. It can be seen from equation (1.3) that the angle of undrained shearing resistance depends on the physical properties of the soil grains, the soil structure and the pore fluid.

In fig. 1.1 the effects of varying 'a' in equation (1.3) is shown for a number of typical soils. It can be seen that for clays, contact areas as high as 15 percent (probably well within the cemented range) give negligible values of ϕ_{uu}. In the case of granular materials, however, even small values of 'a' yield significant angles of undrained shearing resistance.

It is doubtful whether σ_c' applies to the compression of all soils. In the case of densely packed granular materials compression can only occur as a result of grain distortions. Similarly in clayey soils compression will occur due to the bending of flaky particles and the squeezing out of water from the adsorbed layers surrounding each clay particle. In these cases σ_c' governs the com-



ϕ_{UU}

	1	2	3	4	5
	SOFT CLAY	STIFF CLAY	LOOSE SAND	COMPACT SILT	M/DENSE SAND
n %	60	37	46	35	43
ϕ_{CU}	13°	17°	36°	40°	67°
C_S (IN²/LB)	1×10^{-2}	7×10^{-4}	5.7×10^{-4}	2×10^{-3}	3×10^{-4}
C_U (IN²/LB)	3.4×10^{-6}	3.4×10^{-6}	3.4×10^{-6}	3.4×10^{-6}	3.4×10^{-6}
U (LB/IN²)	50	50	50	50	50
D	10^{-4}	10^{-4}	10^{-4}	10^{-4}	10^{-4}
B	·9998	·9982	·9969	·9994	·9951

$$\sin\phi_{UU} = \cfrac{1}{1 + \cfrac{1-\sin\phi_{CU}}{\sin\phi_{CU}} \cdot \cfrac{(-D.U)(1+n.C_U/C_S)}{a+n.C_U/C_S}}$$

$$D = \frac{1}{b.S + U}$$

S = CRUSHING STRENGTH OF ROCK

b = CONSTANT

VARIATION OF ϕ_{UU} WITH 'a' FOR SOME TYPICAL SOILS

FIG. 1.1

pression of the soil. However in the case of loosely packed and medium dense granular materials compression is thought to result from actual displacement of grains which tend to take up a closer pack by slipping and rolling. Evidence presented later in this thesis confirms this view. For these soils compression is governed largely by the frictional resistance between grains and is therefore a function of σ_f'. If σ_f' is used in place of σ_c' the expressions for B becomes *

$$\frac{\partial u}{\partial \sigma} = \left[\frac{1}{n.\frac{C_u}{C_s} + 1-a} \right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1.4)$$

and the expression for the angle of undrained shearing resistance becomes

$$\sin \phi_{uu} = \frac{1}{1 + \frac{1-\sin \phi_{cu}}{\sin \phi_{cu}} \left[1 + \frac{(1-a)}{n.\frac{C_u}{C_s}} \right]} \quad . \; . \; . \quad (1.5)$$

A very significant conclusion concerning the magnitude of 'a' can be drawn from equation (1.4). The factor B must always be less than or equal to one. This means that 'a' (in equation 1.4) must always be less than or equal to $n.C_u/C_s$. The values of n, C_u and C_s used for a medium dense sand in fig. 1.1 give a maximum value for 'a' for the material as 0.5 percent. It is interesting to note that calculations made by Bishop and Eldin (1950) using the crushing strength of the grains yielded a value for 'a' also of approximately 0.5 percent. Substitution of this value for 'a' in equation (1.5) yields a value for ϕ_{uu} of about 3 degrees. In the case of loose sands and compact silts both 'a' and ϕ_{uu} are very much less than for a medium dense sand.

It is important to remember that the above analysis assumes that compressibility of the soil structure results only from grain displacements and not from grain distortion. In general compression will result from the combined action of σ_c' and σ_f'. σ_c' will be more significant in clayey

* See appendix A for the derivation of equations (1.4) and (1.5)

soils and σ_f' will predominate in most granular soils where packing is not tight and the grains are hard. In cases of closely packed granular soils or where the grains are relatively soft (lead shot), compression can be attributed to σ_c'. The conclusion to be drawn from the analyses presented in this section is that the angle of undrained shearing resistance is not greatly affected by the magnitude of 'a' in either clayey or granular soils.

1.4 The validity of the effective stress principle in granular soils

Test results obtained by Bishop and Eldin (1950) confirm that ϕ_{uu} is very nearly zero for saturated granular soils including those possessing a dilatant structure. These tests also show that $\partial u/\partial \sigma$ is equal to unity within the limits of experimental error. Subsequent work by Laughton (reported by Bishop 1959) has confirmed that $\partial u/\partial \sigma$ is equal to one even at very high stresses where the areas of contact are very large.

The theoretical considerations discussed in the previous section show that the relationship between u and σ in the undrained triaxial test cannot be used to measure 'a' directly. Nevertheless, the results of the theoretical analysis and the experimental observations mentioned above indicate that the area of contact between grains does not effect the soil behaviour to any significant extent. It is reasonable therefore to assume that both the volume changes and the shear strength of granular soils are controlled by σ' as defined in equation (1.1).

1.5 The validity of the effective stress principle in clays

Rendulic (1936) was one of the first workers to indicate that the practical application of the principle of effective stress to engineering problems. He developed a generalized principle of effective stress which states that for a clay, with a given initial void ratio, the void ratio after the addition of small increments of effective stress is determined exclusively by the three principal effective stresses. This means that for identical specimens a unique relationship exists between the void ratio and the effective stress irrespective of whether the effective stress results from changes in pore pressure or changes in applied stress.

Rendulic undertook a number of experiments to test the principle stated above. He used a Dutch cell type triaxial apparatus in which both the cell pressures and the pore pressures could be measured. Samples of remoulded clay were subjected to various equal all round consolidating pressures. The samples were then failed in an undrained condition by a change of axial stress either by increasing or decreasing σ_1. During this process σ_1' and σ_3' were carefully measured. Curves of σ_1', against $\sqrt{2}\,\sigma_3'$ were then plotted for different void ratios (see fig.1.2). A number of consolidated drained tests were then performed and the void ratios were measured for varying σ_1 and σ_3. These points were plotted on the same axes as the previous undrained tests and the contours of equal void ratios were sketched in. It was found that the curves of equal void ratio obtained from the drained tests were almost identical to the curves obtained from the undrained tests. Rendulic's work shows that even for large changes in effective stress the void ratio of initially identical clay specimens is controlled exclusively by the three principal effective stresses.

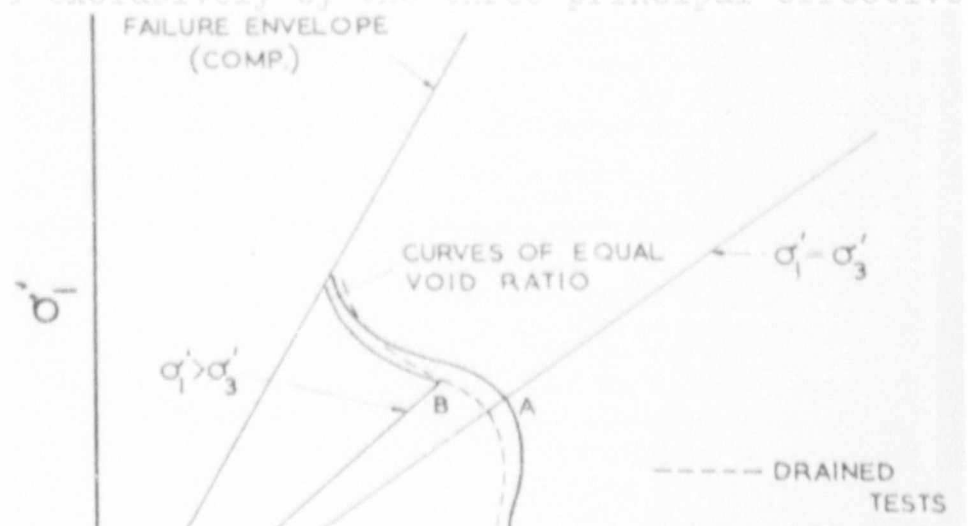


Fig. 1.2

Void ratio contours from drained tests and stress paths in undrained tests

Henkel (1959 and 1960) confirmed Rendulic's results and extended them to the case of preconsolidated clays. He used six types of loading :

(a) Axial stress increased : radial stress constant
(b) Axial stress constant : radial stress decreased.
(c) Average principal stress constant : axial stress increased : radial stress decreased.
(d) Axial stress decreased : radial stress constant.
(e) Axial stress constant : radial stress increased.
(f) Average principal stress constant : axial stress decreased : radial stress increased

Henkel found that the stress paths followed in the undrained tests corresponded very closely to the equivalent contours of constant void ratio obtained from the results of drained tests. This was true for both normally consolidated and overconsolidated clays. Henkel also performed a limited number of tests on samples which were consolidated under an anisotropic stress condition. Thus instead of consolidating along a stress path OA (fig.1.2) with $\sigma_1' = \sigma_3'$ they consolidated along a path OB with $\sigma_1' > \sigma_3'$. On subsequent shearing in an undrained condition the stress paths followed were found to be almost identical to the paths obtained for equivalent samples consolidated under equal all round pressures.

Rendulic's and Henkel's results offer conclusive evidence that the principle of effective stress is valid in fully saturated clays when the volume changes and shear strengths are treated over relatively short periods of time.

1.6 The mechanistic approach to effective stress.

Lambe (1959 and 1960) studied the forces between clay particles with a view to testing the validity of the effective stress principle. He considered a granular material initially and derived an equation for the equilibrium of forces across a wavy plane cutting the points of contact between the mineral particles. This equation is

$$\sigma = \bar{\sigma} . a_c + u (1-a_c) \ldots \ldots \ldots \quad (1.6)$$

where σ is the total vertical stress acting across the plane, $\bar{\sigma}$ is the average vertical stress in the mineral particles at the zones of contact and a_c is the horizontal component of the area of contact between mineral particles divided by the total cross sectional area.

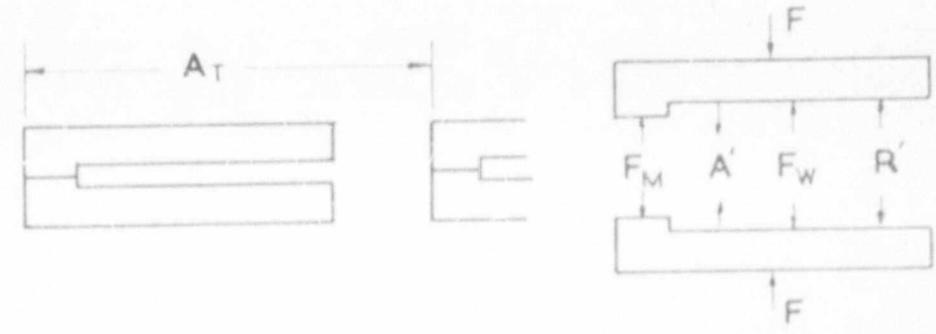
Lambe then proceeded to the case of clay particles. These particles are, in general, separated by layers of adsorbed water. The interparticle forces are transmitted through this water by electrical forces. At the same time it is reasonable to assume that the pore water pressures can be transmitted through this water. Occasionally mineral particles are actually in grain to grain contact due to reverse charges at the edges of the particles. In such cases the area between the contacts cannot transmit pore water pressure. To account for these effects Lambe modified equation (1.6) to

$$\sigma = \bar{\sigma} . a_s + u (1-a_u) \ldots \ldots \ldots \quad (1.7)$$

a_s is the horizontal projection of the area between adjacent mineral particles through which the stress in the soil skeleton can be transmitted, divided by the total area of the surface under consideration.

a_u is the horizontal projection of the area between adjacent mineral particles which are in such contact that the water which can transmit water pressure is excluded, divided by the area of the surface under consideration.

Lambe then considers the various components of the stress $\bar{\sigma}.a_s$ which is the net force transmitted between the adjacent clay particles. Figure 1.3 shows the average interparticle conditions for a saturated clay mass.



F is the externally derived force
Fm is the force where contact is mineral to mineral
Fw is the force where contact is water to water.
R' is the electrical repulsion.
A' is the electrical attraction.

Fig. 1.3

Forces between adjacent clay particles

For equilibrium

$$F = F_m + F_w + R' - A'$$

If A_m = the area of mineral contact and A_w is the area of water contact then

$$F = \bar{\bar{\sigma}}\ A_m + u.A_w + R'-A'.$$

where $F_m = \bar{\bar{\sigma}}\ A_m$ and $F_w = u.\ A_w$.

Dividing through by the area of cross section A_t

$$\sigma = \bar{\bar{\sigma}}.a_m + u.a_w + R-A \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1.8)$$

where $a_m = \frac{A_m}{A_t}$, $a_w = \frac{A_w}{A_t}$, $R = \frac{R'}{A_t}$ and $A = \frac{A'}{A_t}$

It can be seen that a_w is equal to $(1-a_u)$ in equation (1.7). Equation (1.8) may therefore be written

$$\sigma = \bar{\bar{\sigma}}.a_m + u(1-a_u) + R-A \quad . \ . \ . \ . \ . \ . \ . \ . \quad (1.9)$$

Lambe has suggested that a_u will not be greater than about 0.2. This is probably very high even for a stiff clay. Reference to figure 1.1 shows that ϕ_{uu} corresponding to $a = 20$ percent will not be greater than about 4 degrees even for compressibilities of 7×10^{-4} in.^{2} per lb. The effect of a_u can therefore be neglected and equation (1.7) becomes

$$\sigma = \bar{\sigma}.a_s + u \quad \ldots\ldots\ldots\ldots \quad (1.10)$$

and equation (1.9) becomes

$$\sigma = \bar{\bar{\sigma}}.a_m + u + R - A \quad \ldots\ldots\ldots\ldots \quad (1.11)$$

Equation (1.10) is identical to equation (1.1) with $\sigma' = \bar{\sigma}.a_s$. Hence from equation (1.1), (1.10) and (1.11) we get

$$\sigma' = \bar{\sigma}.a_s = \bar{\bar{\sigma}}.a_m + R - A \quad \ldots\ldots\ldots \quad (1.12)$$

where $\bar{\bar{\sigma}}.a_m$ is due to Coulombic attraction forces.*
R is due to Coulombic repulsive forces.*
A is due to van der Waal's attraction forces.*

Lambe suggests that since R, A and $\bar{\bar{\sigma}}.a_m$ cannot be measured separately σ' is not a meaningful quantity. He also points out that soil behaviour depends not only on σ' but also on the maximum previous value of effective stress. Moreover shear strength and void ratio are also determined, to a small degree, by secondary effects such as rate of shear strain, rate of secondary compression and pressure increment ratio. These objections will be examined more closely in the light of equation (1.12).

Consider a soil which is subjected to constant applied pressure σ and a constant pore pressure u. From the definition of effective stress given in equation (1.1) σ' must also remain constant. If for some reason R (say) in equation (1.12) decreases then either $\bar{\bar{\sigma}}.a_m$ or A or

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* See appendix B for a more detailed description of these forces.

both must change in order to maintain equilibrium.

A change in A must be accompanied by a change in interparticle distance and hence the volume and the shear strength of the soil will change. The phenomena described are known as secondary compression, and thixotropy respectively, i.e. change in volume and shear strength at constant effective stress. Clearly the principle of effective stress does not account for this behaviour, however the secondary effects just described are usually small and take place over long periods of time.

The phenomenon of precompression can also be explained by irreversible changes in $\bar{\bar{\sigma}}.a_m$, R and A resulting from the application of an effective stress at some previous time and which has since been removed. In this case the generalized principle of effective stress, defined in section 1.4, can be used to predict soil behaviour.

It can be concluded that the effective stress principle is valid for all saturated soils provided secondary effects such as thixotropy and secondary compression can be neglected.