



Effects of professional noticing of learners' errors and misconceptions on grade 11 learners' trigonometry performance in the Johannesburg East District, Gauteng South Africa

By Ashton Ndebele

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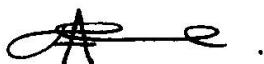
Master Of Education

Student number: 2633966

Supervisor: Professor J P Makonye

Declaration

I Ndebele Ashton, do hereby declare that the research report entitled: "*Effects of professional noticing of learners' errors and misconceptions on grade 11 learners' trigonometry performance in the Johannesburg East District, Gauteng South Africa*" is my own work. All the sources I used have been acknowledged accordingly and listed as references.



Mr A. Ndebele

June 2023

DATE

Dedication

This research report is dedicated to the following special people:

My wife Jane

My son Washington

My daughter Joanna

Acknowledgements

I am so grateful to my creator, our father in heaven, the Almighty God for the gift of life, for touching me always, and for giving me the strength to do this work. He is my everything.

Special thanks to my wife Jane, you became my study partner, and I would not have done this without you. You are special.

To my son Washington and my daughter Joanna, you are the reason for my motivation; this work is dedicated to you.

Thank you to my supervisor Prof J.P Makonye for your guidance, support, and encouragement throughout this journey. You held me up at my weakest point when the going was tough; I greatly appreciate the wisdom in you.

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Abstract

Grade 12 learners perform poorly in trigonometry in the final National Senior Certificate (NSC) examinations. The errors committed by learners in trigonometry lead to a high failure rate in trigonometry and hence in mathematics. The traditional teaching method that has found a home in many South African learning spaces is largely blamed for failing to effectively deal with learner errors in trigonometry. The errors continue to show every year and are believed to persist because the instructional methods being used in the teaching of trigonometry do not confront them. In this study, the aim was to investigate the effectiveness of professional noticing and probing on performance and on learner errors in grade 11 trigonometry.

All the grade 11 learners taking mathematics in the Johannesburg East District South Africa and their teachers, constituted the population. Purposive sampling was used to select two schools that participated in the study. The sample size was of $n = 176$ grade 11 mathematics learners from the two schools plus five teachers. One school was taken as a control group with $n = 97$ learners and the other school became the experimental group with $n = 79$ learners. After the pre-test, the experimental group had an intervention using professional noticing and probing while the control group used a traditional teaching approach.

The study used a mixed method research where a quasi-experimental design with a non-equivalent group design was employed. The learners were not assigned to classes, instead the intact classes that already existed at the schools were used, to avoid unwanted disruptions at the school. All the activities took place within the schools' operating hours to avoid unnecessary strain on the participants. The pre-test scores for both groups were analysed using a t-test and this test revealed that both groups were at the same level in terms of trigonometry performance. The scores from the test at both pre- and post-testing stages were analysed using ANCOVA and the test revealed that the intervention that took place with the experimental group using professional noticing and probing was superior in improving both the performance and in dealing with learner errors.

A questionnaire was used to gather information from the teachers about the errors and misconceptions that have been observed in their classes. Five teachers responded to the questionnaire and their responses were analysed qualitatively.

The constructivist theory was used as the theoretical framework for this study. The professional noticing and probing used in the experimental group followed a constructivist approach as learners were given opportunities to actively construct their own knowledge during the probing process. The probing process enabled learners to use their errors and misconceptions to construct correct mathematical concepts. An error analysis conducted post-intervention revealed that learners managed to overcome their errors and misconceptions.

Keywords: Professional noticing, probing, constructivism, learner errors and misconceptions, mathematics, trigonometry.

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CHAPTER ONE

INTRODUCTION AND BACKGROUND OF THE STUDY

1.1. INTRODUCTION

This section gives the background of the topic, the aim and objectives, research questions, the significance of the study, the problem statement and outlines how the rest of the research report is structured.

1.2. BACKGROUND

Since the first grade 12 final examination under the Curriculum and Assessment Policy Statement (CAPS) in 2014, there has been a high failure rate in mathematics (Department of Basic Education (DBE), 2016). The failure rate has been observed across all grades in mathematics. Poor performance in mathematics hinders learners from pursuing careers that require sound background knowledge of mathematics such as engineering, pharmacy, and medicine. To qualify for admission to these courses at South African universities, students must obtain a minimum of 50% in mathematics. The revised CAPS curriculum for mathematics grade 12 in South Africa has seen trigonometry as a learning area increasing in weighting from 40 marks to 50 marks out of 150 marks in paper 2 for the years 2021, 2022, and 2023 National Senior Certificate (NSC) examinations (DBE Examination Guidelines; 2021). This marks a 6.7 percent increase in the weighting and as such renders the topic one of the top determinants in mathematics success at the grade 12 level.

The NSC is the high school certificate obtained by learners as they exit high school, and it is therefore used as the yardstick for measuring the country's success in educating its citizens. The NSC mathematics results have been a serious concern in the past five years and the number of learners that sit for the NSC mathematics final examinations declined notably from 2016 to 2020 by more than 30 000 learners, from 265810 to 233315 (DBE, 2020).

Trigonometry has the highest weighting in mathematics paper 2 and hence "failure to understand trigonometry concepts on the part of the learners has a ripple effect on the success rate of the learners in Mathematics" (Chikonga, 2017, p. 163). Trigonometry has been identified as one of the topics that learners perform poorly in grade 12 NSC examinations. Most of the trigonometry concepts examined in grade 12 are covered in grade 11 and failure to master these could result in learners

struggling with the topic in grade 12. Grade 11 learners are expected to simplify trigonometric expressions using special angles, reduction formulae, and trigonometric identities. These expressions cascade to grade 12 and learners are expected to master them at the grade 11 level.

Learner errors when dealing with trigonometry have been identified as one of the major causes of poor performance in the topic and hence mathematics (DBE, 2021). The diagnostic reports DBE (2017, 2018, 2019, and 2020) all point to persistent errors committed by learners in trigonometry throughout these years. Simplifying trigonometric functions and expressions, use of the cosine rule, solving trigonometric equations, and failing to use identities are some of the sections of the topic where the errors have been pointed out throughout the four years. Such errors lead to massive loss of marks and hence poor performance in mathematics. It is therefore imperative that educators begin to identify these errors in the classroom and map up strategies to deal with them.

Studies in mathematics education indicate that a focus on errors committed by learners enables educators to better understand the kind of assistance and remedial activities that are required to enhance learner understanding of concepts (Adler, 2005; Brodie, 2014; Venkat & Adler, 2012). Learners need to construct mathematical knowledge through the interaction of prior knowledge with new concepts. In this research, an intervention strategy that deals directly with learner errors was used, in which mathematics lessons are grounded on the identification and probing of learner errors in grade 11.

1.3. The problem statement

Poor performance in trigonometry by grade 12 learners in the final examinations is indicative of the knowledge gaps that most candidates have at the time of writing. Diagnostic analysis reports by DBE (2017, 2018, 2019, and 2020) all point out the failure in trigonometry to errors committed by learners in answering the questions. The behaviourist type of teaching popularly known as traditional teaching, largely characterizes many mathematics classrooms, by rewarding correct answers with marks and punishing wrong responses does not leave much room for analysis and use of learner errors during the teaching process (Borasi, 1994). Traditional teaching

hinges on absolutist philosophies of mathematics that argue that mathematical knowledge exists in itself and therefore must be transferred to learners who are ignorant of it (Ernest, 1998). Mathematics teachers who subscribe to this philosophy of teaching perceive mathematical concepts as rigid (Ernest, 1995). This absolutist approach, which is mainly teacher-centred and does not accept errors by learners, has found a home in many classrooms and has rendered mathematics a special subject that is not meant for everyone. One wonders whether it is possible to use a humanist approach that centres on the learner and accepts the errors in trigonometry they show as learning opportunities, through probing (Jacobs et al., 2011).

1.4. Aim of the study

The aim of the study was to investigate the effects of professional noticing and probing in helping to deepen learners' understanding of trigonometry at grade 11.

1.5. Objectives of the study

The objectives of the study were to:

- Identify the type of errors exhibited by grade 11 learners in trigonometry,
- Interpret learner errors and understand where they are stemming from,
- Determine the effect, if any, of responding to grade 11 learner errors in trigonometry by probing learners about their errors.

1.6. Research questions

The research questions for the study were:

- What types of errors are committed by grade 11 learners in trigonometry and how do teachers attend to them?
- What are the possible causes of learner errors in grade 11 trigonometry and how do teachers interpret them?
- What is the effect, if any, of responding by probing learners about their errors on learners' performance in trigonometry grade 11?

1.7. Significance of the study

Errors committed by learners in mathematics lead to failed knowledge construction of mathematical concepts. Learning mathematics does not occur through knowledge transmission from the teacher but rather through what the learners can construct and

process as mathematical knowledge (Borasi, 1994). During the knowledge construction process, learners are bound to commit errors. It is incumbent upon teachers to create learning environments that enable learners to realise that making errors is part of learning mathematics (Gardee & Brodie, 2021). Noticing these learner errors, interpreting them, and responding accordingly should therefore be the focal point in the teaching process.

Research on errors and misconceptions is dominated by studies that focus mainly on the researchers and the teachers and rarely on the learners, serve for a few current studies. Probing learners on their mathematical errors has also shown positive trends in dealing with learner errors. The enactment of the component of responding, using probing, in dealing with errors portrayed by learners in trigonometric expressions and functions has not received much attention in the South African context and hence the results of the study could potentially be used for teacher development purposes. The results of this study could also help reduce learner errors in trigonometry and hence improve learner performance in mathematics.

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CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This section is a review of relevant literature to point out what other scholars have unearthed concerning professional noticing and the use of probing learner errors in the teaching of mathematics in general. The study hinges on the principles of professional noticing and constructivism and employed probing as a way of responding to emerging errors committed by learners and the underlying misconceptions in grade 11 trigonometry. Challenges in the teaching of grade 11 trigonometry are also explored.

2.2 Trigonometry

The CAPS curriculum for grade 11 mathematics requires the learners to master the concepts of trigonometric expressions as indicated by the insert.

- | |
|--|
| <p>1. Derive and use the identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\theta \neq k \cdot 90^\circ$, k an odd integer; and</p> <p>$\sin^2 \theta + \cos^2 \theta = 1$.</p> <p>2. Derive and use reduction formulae to simplify the following expressions:</p> <p>2.1. $\sin (90^\circ \pm \theta)$; $\cos (90^\circ \pm \theta)$</p> <p>2.2. $\sin (180^\circ \pm \theta)$; $\cos (180^\circ \pm \theta)$ and $\tan (180^\circ \pm \theta)$</p> <p>2.3. $\sin (360^\circ \pm \theta)$; $\cos (360^\circ \pm \theta)$ and $\tan (360^\circ \pm \theta)$;</p> <p>2.4. $\sin (-\theta)$; $\cos (-\theta)$ and $\tan (-\theta)$</p> <p>3. Determine for which values of a variable an identity holds.</p> |
|--|

Extracted from DBE (2020a)

As mentioned earlier, the grade 11 trigonometry concepts are examinable in grade 12. For learners to perform well in trigonometry both in grade 11 and grade 12, they need to master the concepts as per the CAPS requirements in the insert. Trigonometry is a prerequisite for learning calculus in senior mathematics and is crucial for students who want to major in Science, Technology, Engineering, and Mathematics (STEM) courses, it is important to highlight (Ngu & Phan, 2020). Because of the several connected ideas, including algebraic transformation abilities, geometry, and graphical representation, learners have trouble mastering trigonometry problems (Ngu & Phan, 2020). Several studies have been conducted

related to the teaching and learning of trigonometry (Andika, Juandi, & Rosjanuardi, 2017; Dewento, Budiyono, & Pratiwi, 2018; Karthikeyani, 2017; Maknun, Rosjanuardi, & Jupri 2019; Nanmumpuni, & Retnawati, 2021). The challenges faced by learners in the topic are the major driving force for the extensive research. These studies focused mainly on the teaching and learning of trigonometry with the aim of improving learner performance in trigonometry.

Trigonometry, as a topic has the highest weighting in mathematics paper 2 in the South African mathematics curriculum (DBE, 2021), and therefore challenges that learners face in trigonometry have to be tackled head-on to avoid high failure rates in mathematics.

Weighting of Topics per Paper for Grade 12 mathematics

PAPER 1	MARKS	PAPER 2	MARKS
Algebra, Equations and Inequalities	25	Statistics and Regression	20
Number Patterns	25	Analytical Geometry	40
Functions and Graphs	35	Trigonometry	50
Finance, Growth and Decay	15	Euclidean Geometry	40
Differential Calculus	35		
Counting Principle and Probability	15		
TOTAL	150	TOTAL	150

Extracted from DBE examination guidelines (2021, p. 4)

The grade 12 NSC mathematics examination of November 2022 is an example of the implementation of the above policy. The questions on trigonometry (questions 5, 6, and 7) in the November 2022 paper 2 question paper carry a total of 50 marks.

Trigonometry has been identified as one of the topics that learners perform poorly in grade 12 NSC examinations consistently from 2016 to 2021 (DBE, 2021). Most of the trigonometry concepts examined in grade 12 are taught to the learners when they are still in grade 11, and failure to grasp these concepts could have a negative effect when learners are faced with more challenging trigonometry concepts in grade 12. In grade 11, learners are expected to simplify trigonometric expressions using special angles, reduction formulae, and trigonometric identities. These expressions are carried forward to grade 12 and learners are expected to have internalised them at the grade 11 level.

Learner errors in trigonometry have been cited as one of the major causes of poor performance in the topic and hence mathematics in grade 12 (DBE, 2021). The diagnostic reports DBE (2017, 2018, 2019, and 2020) all point to persistent errors and misconceptions that learners show in trigonometry throughout these years. Many of these errors show when learners are expected to simplify trigonometric functions and expressions, use the cosine rule, solve trigonometric equations, and use trigonometric identities throughout the four years. These errors contribute greatly to the loss of marks and hence learners end up performing poorly in mathematics. It is therefore crucial that educators begin to identify these errors in the classroom during lessons and when doing written assessments and map up strategies to deal with them.

Studies on learner misconceptions and errors in trigonometry have intensified with many scholars using Newman's error analysis method to determine the type of errors made by learners (Ahmed, Febryanti, Muthmainnah, Yakin, Al & Sarbi, 2018; Centillas & Larisma, 2016; Fahrudin, Mardiyana, & Pramudya, 2019; Mensah, 2017; Price & Van Jaarsveld, 2017; Sari & Watsqa, 2019; Usman & Hassaini 2017).

The intensity of the study of learner misconceptions and errors was driven by the high failure rate in trigonometry by mathematics learners. According to DBE (2016, 2017, 2018, 2019, 2020, & 2021), grade 12 mathematics candidates attain low marks in trigonometry mainly because of the errors they commit in answering the questions.

2.3 Misconceptions and errors

During the learning process, learners tend to show misconceptions and errors during knowledge construction. Scholars define the term misconception differently with Hammer (1996) defining it as having a perception that differs from that of experts in the subject, while Ojose (2015) believes that misconceptions are misunderstandings and misinterpretations. If a learner derives meaning for a concept that differs from the scientific meaning, a misconception arises (Yasin, 2017). Errors emerge due to such misconceptions. These errors and misconceptions, according to Nesher (1987, as cited in Gardee & Brodie, 2021), are inevitable and should not be avoided or discouraged during lessons. The errors and misconceptions provide some insight into the learners' reasoning. Instruction that stems from the errors and misconceptions has a potential of helping learners overcome the errors.

Misconceptions and errors emanate from prior knowledge and as learners process new information, they tend to exhibit these errors and misconceptions, mainly because of the knowledge construction process. If a misconception exists in a learner's prior knowledge, then it is highly likely that the learner will have misconceptions with the new concepts, that is, the misconception held by a learner has a ripple effect in the acquisition of new concepts (Baki, 1999; Driver & Easley, 1978). Yasin (2017) posits that because misconceptions are consistent and are held strongly to one's experiences, they resist change.

Dewanto et al (2018) uncovered three types of errors and misconceptions and they referred to them as Error type I, Error type II, and Error type III. Error type I is when learners cannot interpret the problem, Error type II is when learners cannot use the correct trigonometric formulae, and Error type III is when learners cannot use special angles. Dewanto's error type II is what was referred to as failure to use reduction formulae in this study, while Sari & Wutsqa (2019) refer to this as a type-T error. Concerning this type of error, Malambo (2021) clarifies that trigonometric expression and functions should not be treated as if they were algebraic functions, there are some conceptual differences.

Learning programs that seek to shift the educational space from focusing mainly on the teacher to one that promotes learner-centredness have received strong advocacy in the teaching of mathematics (Hiebert et al., 1997; Murray et al., 1998; Wessels, 2018).

Studies that place focus on learner thinking, errors, and misconceptions as well as teacher development on the aspect of noticing all point to the need by educators to use the errors (Borasi, 1994; Davis & Renert, 2014; Makonye & Khanyile, 2015) misconceptions (Schoenfeld, 2011; Thomas et al., 2017;), and learner thinking (Hine & Lesseig, 2021; Son & Sinclair, 2010), as opportunities for learning. Learners acquire new concepts through the active interaction between the learner's prior knowledge and the concept to be learnt (Gordon, 2009). Educators need to know the causes of misconceptions and shift their classroom instruction to create efficient learning spaces (Ojose, 2015). This is the basis of the constructivist theory.

2.4 Constructivism

Constructivism is an epistemology in which the “learning of mathematics entails that, learners actively create, interpret and reorganise knowledge in individual ways” (Gordon, 2009, p. 54). Constructivists believe that learning is largely dependent on prior knowledge. Success in mastering new concepts depends on how rich the prior knowledge is (Davis, 1997). Therefore, meaningful learning occurs when “learners construct and give their own meaning to mathematical knowledge based on their prior knowledge and background experience” (Schunk, 2000, p. 73). When confronted with new concepts, learners can either accommodate or assimilate them into their existing schema. Assimilation occurs when the new concepts ‘fits’ into the schemata. Accommodation occurs when the new concepts interact with the existing knowledge during the reorganisation of the schemata. The schemata expand as new knowledge is being added to the existing knowledge during assimilation (Gardee & Brodie, 2021). Misconceptions are a consequence of the simultaneous application of the new and old concepts in new mathematical contexts. It should be noted that misconceptions are constructs that learner may have but are not mathematically sound. Errors on the other hand may be a result of having a misconception a wrong application of a correct concept. Misconceptions, therefore, arise when a learner wrongly uses, applies, or connects prior concepts with the new concepts being presented. For example, a learner who understands that a three-digit number is greater than a two-digit number may incorrectly conclude that 1.23 is greater than 12. This manifests in the form of errors during lessons or assessments. At times the misconception arises because of wrongly constructed prior concepts that then affect future learning. When such concepts relate to new concepts, learners fail to construct meaningful mathematical knowledge. Constructivism, in this study, was used to help learners reconstruct knowledge from the errors that they initially show through probing.

2.5. Professional noticing

Professional noticing can be defined as the ability of a teacher to see learners’ mathematical practices and conceptions as they show and to shape instructional strategies accordingly (Jacobs et al., 2010). In the same line of thought, Hine and

Lesseig (2021) note that “professional noticing is comprised of three interrelated, consequential practices: attending to the mathematics evidenced in the student thinking, interpreting what that thinking reveals about student understanding, and deciding how to respond to the student based on this interpretation” (p. 98). To attend to the learners’ mathematical ways of thinking means to take into consideration every mathematical contribution that learners bring into the classroom, whether wrong or right, and trying to make sense of what the learners write (Jason et al., 2002), but interpreting calls on both the teacher's expert subject-matter expertise and the knowledge of their students.

Teachers of mathematics frequently face the enormous challenge of figuring out how their students feel about the mathematical ideas that are being drilled into them. Mathematics teachers would agree that classes are unique, the teaching style and the explanations and demonstrations that are done in one class are not necessarily the same even for two classes in the same grade at the same school. When students exhibit certain misconceptions, teachers may have prepared the ideas to cover and the pace of the sessions, but these plans may be abandoned. In this case, the teacher must pay attention to the students' thoughts, interpret them, and then react appropriately. This occurs when the teacher is able to observe how the learners reason, how they apply their mathematical ideas and concepts, and how they create their mathematical ideas and concepts.

When interpreting learners’ work the teacher zooms into the work of the learners to gain a deeper understanding of why learners say what they are saying. This is a very crucial stage of professional noticing as it determines how the teacher then responds to the learners’ ways of thinking. Learners may, for example, have a misconception used in a calculation but this may lead to correct answers. Without interpreting such responses from learners, a teacher may conclude that learners have successfully grasped the concepts. It is therefore of paramount importance that teachers use their forensic lenses when they assess learners’ work. Responding is the process of using a teacher's pedagogical content knowledge to come up with solutions to problems that arise in the learners' thinking during classes or during written examinations. The three types of professional noticing are not exclusive to one another; they might all

take place simultaneously during sessions (Jacobs et al., 2011). The teacher's ability to pay attention to students' unpredictable thinking that is veiled by errors and assumptions, reflect on it, and evaluate how that thinking might be used to help students create new knowledge based on the misconceptions is consequently known as professional noticing.

Professional noticing is a skill that one develops because of learning and training. For teachers to effectively use professional noticing in the practice requires them to undergo some training. Therefore, for a teacher to successfully notice learners' thinking, they should undergo training by experts in the field. Content expertise, knowledge of the learners, and pedagogical topic knowledge combined with professional noticing skills can place teachers in a good position to cope with students' errors and thinking in a way that propels them toward mathematical mastery. To this purpose, Schoenfeld (2011); Thomas et al. (2017) claim that professional noticing, teacher knowing of the learners, and pedagogical expertise have a significant correlation. One of the reasons Australian and American colleges decided to integrate professional noticing courses in their teacher preparation programs was because of this (Hine, 2021). By enhancing their abilities to use learners' mistakes and misunderstandings as "springboards" (Borasi, 1994) in the learning process, these courses are designed to prepare preservice teachers for the teaching profession.

Without professional development, instructors find it difficult to use professional noticing in classroom settings. It has been noted that "preservice teachers have consistently found it to be the most challenging of the three components to execute with proficiency" (Hine et al., 2021, p. 98). Despite having a strong understanding of their subject matter, teachers frequently struggle with using this knowledge to respond appropriately to the faults they witness in their students throughout lessons. Son and Sinclair's (2010) study found that "teachers prefer to respond by delivering answers or clarifying methods, rather than posing questions or next tasks that build on student thinking" (p. 37). This effectively robs the learners of the opportunity to learn from their misconceptions and instead it promotes the recurrence of the misconceptions. Hine et al. (2021) note that professors frequently just inform students

what the expected processes were, which has the effect of ritualizing mathematics instruction (Sfard, 1998) by placing more emphasis on proper procedures and accurate answers than on idea development.

Professional noticing practices can be put into action either during or after sessions and examinations. During lessons, learner mistakes may occur, and teachers may be forced to engage in professional noticing without previous planning. In-the-moment decision making, as described by Jacob et al. (2010), relates to the choices teachers make during lessons when students submit their work either vocally or in writing. Since decisions are never made in advance, they require the teachers to draw on their specialized content knowledge (SCK), pedagogical knowledge (PK), and knowledge of content and students (KCS) in order to utilize professional noticing approaches. Nesher (1987, as cited in Gardee & Brodie, 2021) recommends the use of errors and misconceptions to promote successful learning.

2.6 Probing

Probing, as defined by Makonye and Khanyile (2015, p. 56), is “regarded as a teaching method in which the teacher follows-up on the learner’s response with a question or cue that induces a learner to think more deliberately on their response and make a further response”. In a way probing ‘presses’ learners to be on the defensive as it promotes learners’ mathematical argumentation (Gordon-Calvert, 2001). During probing, learners are asked questions that seek to help them understand more about their errors and in the process rectify them. This way, learners begin to construct important mathematical connections and eventually reconcile their errors as they refine their earlier knowledge to suit standard mathematical knowledge. Probing ignites an inquiring mind in the learners which enables them to question their own thinking, make critical judgments that are key ingredients to understanding mathematics concepts. It is argued that:

When learners are probed about their responses to algebraic tasks, they question and explore their current understanding and develop insights on the shortcomings of their current knowledge. During probing open rather than closed questions are often used. Closed questions have one short answer

whereas open questions elicit longer responses which can open up further vistas for probing and thinking (Makonye & Khanyile, 2015, p. 57).

The learner responses in this setting are used as the basis for attending to the errors manifesting during the learning process. Probing can be done in two ways, through funnelling and revoicing. During funnelling, probing questions are sequenced from open to more direct closed ones as learners are made to realise the wrongs in their responses (Makonye & Khanyile, 2015). The questions are structured such that they guide the learners smoothly from their position of misconception to a level where they reconsider their own initial positions and can rectify their own errors. On the other hand, Moschkovich argues that revoicing is “when a teacher re-utters a student’s contribution to draw attention to a specific claim and prompts the speaker to explain and elaborate one’s own claim” (2015, as cited in Makonye & Khanyile, 2015, p. 58). Other researchers successfully used the probing method in their studies on geometry, algebraic fractions, and other mathematical concepts (Borasi, 1994; Makonye & Khanyile, 2015). A success rate of 98% was recorded in the study where probing was used as an intervention in Algebraic fractions by Makonye and Khanyile. This means that 98 percent of the learner errors observed during the pre-test stage could not show after the intervention that used probing. This study focused on the effectiveness of professional noticing and probing in dealing with learner errors and improving learner performance in trigonometry grade 11. A combination of funnelling and revoicing was used during the intervention phase of the study where learners were required to explain their choice of answers.

2.7 Theoretical framework

The theoretical framework that was followed in this study is constructivism. The following were the guiding principles:

- Creating an environment that supports and encourages learner thinking,
- Engaging learners in activities that challenge previous conceptions of their prior knowledge,
- Constantly placing learners on the defensive by asking probing questions about their errors, enabling them to reconsider their initial responses.

The study used the errors that learners show to enable learners to realise their misconceptions and hence reconstruct mathematical knowledge. The procedure

followed during the intervention process hinged lightly on Polya's model. Polya propounded a four-step problem solving model. The four steps are:

- Understanding the problem,
- Devising a plan,
- Carrying out the plan,
- Looking back.

During the intervention phase of this study, learners were probed on their errors to help them gain a deep insight into their responses. The aim was to make them understand the question at hand and re-assess their initial erroneous responses. The procedure used only two, the first and the last, of Polya's four steps in problem solving, during the intervention phase in the experimental class.

To achieve this, learners' errors will be attended to, interpreted, and responded to using probing.

When learners commit errors during lessons and assessments, the errors can only be attended to if the educator is able to identify and interpret the errors and possible misconceptions behind them. Some experienced teachers lack professional noticing skills and hence experience without growth does not lead to increased noticing skills. While it is crucial for teachers to have a solid content knowledge, it is insufficient to effectively interpret and address conceptual blunders and misconceptions. The issue is in using this information to act appropriately. Instructors with a solid background in content knowledge frequently spot pupils' mistakes during lessons and examinations. Son and Sinclair's (2010) research shows that "rather than pose questions or next tasks that build on student thinking, teachers tend to respond by giving answers or explaining procedures" (p. 42). By merely explaining to the students what the expected procedures were, teachers, The teaching of mathematics, in accordance with Hine et al (2021), tends to become ritualized (Sfard, 1998) because of the focus placed on perfect processes and precise responses over conceptual growth. Ritualizing mathematics instead of exploring it undermines the fun of learning and seldom creates a constructive mindset toward learning mathematics (Kilpatrick, Swafford & Findell, 2001; Sfard, 1998). Giving answers as a response may not remedy the situation as that only addresses the outer visible features of learning

instead of the inner and enduring invisible features inherent in a learner (Dewey, 1933) and the learner errors may recur and affect future learning. As stated by Thomas, Jong, Fisher, and Schack (2017) “enacted fluidly within a mathematical interaction between a teacher and a student, the process of attending, interpreting and deciding should result in highly individualised and responsive instructional tactics” (p. 6). In the same breath, Santagata (2005) points out that learner errors provide room for mathematics teachers to use the errors as an opportunity to elaborate mathematics concepts further and to consider the mathematics behind the errors. Usman and Hussaini (2017) conducted a study in which 85 percent of the learners made mistakes in using trigonometric formulae in solving trigonometric equations.

This study used constructivism as a theoretical framework to help learners construct new knowledge through professional noticing of their errors. The learners wrote a pre-test that was used for two purposes, to record the pre-intervention scores of the learners and to analyse the test for errors. As a way of assisting students in overcoming these mistakes and erasing their misunderstandings, the errors that were discovered were subsequently exploited. The following model was followed during the study:

- Learners wrote a pre-test, and the scores were recorded for each learner,
- The observed errors are categorised,
- Intervention was done to both groups, with professional noticing and probing used in the experimental group while traditional teaching approaches were used in the control group,
- Learners were probed about their erroneous answers from the pre-test and teaching continued as usual in the control group,
- Learners in both groups wrote the post-test,
- Error analysis was done for both groups and the post-test scores were recorded.

The model presented was enacted in a week and the results and data collected were analysed statistically for the relationship between the average scores of the two groups and to determine whether there is a reduction in the number of observed errors in both groups.

CHAPTER THREE

METHODOLOGY

3.1. INTRODUCTION

The research design used for the study, the population, the sample, and sampling strategies used, the instrumentation, and the ethical considerations are outlined in this section.

3.2. Research design

The study used a mixed methods research (MMR) approach and was inclined largely toward a quantitative component, where a quasi-experimental design, using a non-equivalent control group design was used. Teachers and learners from two selected schools formed the participated in the study. In quasi-experimental designs, the subjects are not randomly assigned (McMillan & Schumacher, 2010). The non-equivalent control group design used in the study is a type of quasi-experimental design in which subjects are not randomly assigned to groups (Cohen, Manion & Morrison, 2011; Gay, Mills & Airasian, 2011). This design was chosen to avoid unnecessary disruption of the normal running of the two schools used. One school served as the experimental group, and the other served as the control group.

The learners from the other school, the control group, were taught by the current teacher at the school using a conventional teaching strategy whereas the experimental group was instructed by the researcher employing professional noticing and reacting through probing. Traditional teaching approaches (TTA) are teacher-centred instructional approaches. When this methodology is used, learners are considered to be empty vessels (*tabula rasa*), and the teacher is portrayed as a knowledgeable person (Ngu & Phan, 2020). This approach characterises many South African mathematics classes and has been largely blamed for the high failure rate in mathematics (Chigonga, 2017; Dlamini, 2012; Gweshe, 2014).

The two approaches to the teaching of mathematics are different. The traditional teaching approach is mainly teacher centred and the teacher dominates the discussions during lessons. The activities and the pacing of the lesson are

determined by the teacher and the learners have to fit in the setup. The learners are given little or no room for raising their opinions, concerning how their learning should be structured. Professional noticing and probing, on the other hand, centres around the learners and the way they create their own new concepts during interactions in the classroom. Professional noticing and probing offer learners the opportunity to evaluate their own erroneous responses and actively use them to construct new concepts and knowledge. Teachers who use professional noticing and probing in their practices believe in the constructivist ideology in which learners are expected to create their own new knowledge instead of having a knowledge transmission process. All the other conditions in the two schools were kept similar, except for the teaching methodologies, which are subject to comparison.

The study used hypotheses testing to compare the results from the experimental group and control group. To achieve the aim, the null and the alternative hypotheses were stated as follows:

Null hypothesis (H₀): There is no significant difference between the average test scores of grade 11 learners who are taught using professional noticing and probing them about their errors and those that are taught using the traditional teaching methods.

$$H_0: \mu_{\text{professional noticing and probing}} = \mu_{\text{traditional teaching method}}$$

Alternative hypothesis (H₁): There is a significant difference between the average test scores of grade 11 learners who are taught using professional noticing and probing them about their errors and those that are taught using traditional teaching methods.

$$H_1: \mu_{\text{professional}} \neq \mu_{\text{traditional}}$$

The notation “ $\neq$ ”, in the alternative hypothesis, indicates that both teaching methods will have an unequal or non-equivalent effect on learners’ mathematical performance, while “ $=$ ”, in the null hypothesis, indicates that there is no significant difference between the two average scores obtained by learners taught using the two instructional methods.

In addition, a *descriptive survey design* was used to collect data from teachers' questionnaires, which forms part of the qualitative component of the study. In descriptive survey studies, information is assessed on attributes such as attitudes, opinions, and behaviour (Gay, et al., 2011). The questionnaire assessed the opinions, behaviour, and attitudes of teachers towards learner errors. The qualitative component was infused in the study to provide a rich narrative account of the observed errors and how attending to them through probing impacts on the learners' cognitive shift on those errors.

3.3. Participants

This section gives a brief description of the participants in the study.

3.3.1. Population

McMillan & Schumacher (2010) define a population as “a group of elements or cases, whether individuals, objects or events that conform to specific criteria and to which we intend to generalize the results of the research” (p. 129). In other words, a population can be defined as the totality of all subjects to which a research study focuses. For this study, the population was all the grade 11 learners and teachers in all 40 public secondary schools in Johannesburg East district in Gauteng South Africa. It is not always feasible to include all the subjects that constitute the population in a study and therefore a smaller group (sample) that fairly represents the population is used.

3.3.2. Sampling techniques

Purposive sampling was used to select two public schools in Johannesburg East District in Gauteng for the study. In purposive sampling, subjects are selected because they have a certain characteristic that one wants to explore or because it is convenient to use those subjects in a study (McMillan & Schumacher, 2010). Request letters were sent to eight schools that were selected using purposive sampling. These eight schools were selected on the basis that: 1) the researcher already had existing relations with them, 2) they were located a short distance from the researcher's home and some from the researcher's workplace, and 3) their NSC mathematics results were comparable for the three years prior to the study being conducted. Only two out

of the eight schools responded and agreed to have the research being conducted at their schools. These are the schools that were eventually used for the study.

3.3.3. Sample

The sample size in this research was the number of mathematics learners at the two schools. A total of 176 learners participated in the study, with $n = 97$ in the control group and $n = 79$ in the experimental group. Since the learners were already set up in fixed classes, they could not be randomly assigned to classes; the existing classes were used as they were, for both the experimental and control groups. This type of sampling arrangement was followed to avoid disruptions in the school's programs and plans. All the mathematics teachers at the selected schools, who were teaching grade 11 when the study was conducted and those that have taught grade 11 in the previous 3 years responded to the teacher questionnaire. A total of 5 mathematics teachers from both schools responded to the questionnaire. One teacher from the control school participated during the intervention phase of the study. This was the incumbent teacher assigned by the school to teach the two grade 11 mathematics classes. This was followed to avoid disturbing the already existing teaching timetable at the school.

3.4. Research Instruments

The data collection instruments used for this study were: a *performance test*, at both pre-and post-testing stages, as well as a teacher *questionnaire*.

3.4.1. Pre- and Post-test

A trigonometry performance test was set to use at the pre-and post-testing stages of the study. The items of the test were set to expose a variety of learner errors and it also tested different mathematical skills that learners have. Included in the test were questions on proving identities, simplifying trigonometric expressions using special angles, and simplifying trigonometric expressions using the reduction formulae. The same performance test was used as a pre-test and as a post-test. The results of the pre-test were analysed using a t-test to compare the mean performance of the two groups of learners from the two schools before intervention. The mean scores of the pre-test indicated that the two groups' performance was comparable in grade 11 trigonometry. The pre-test was also analysed for the type of errors that learners show

in simplifying trigonometric expressions. The observed errors were recorded against each of the learners in both groups and noted for intervention purposes.

3.4.2. Teacher Attitudes and perceptions on learner errors questionnaire

A questionnaire was used to collect data relating to teachers' qualifications, teaching experience and attitudes toward the grade 11 trigonometry. The questionnaire was responded to by the grade 11 mathematics teachers and the teachers who have taught grade 11 mathematics in the past three years from the participating schools. Some questionnaire items were drawn to source information on the academic performance of the school in mathematics, to get a sense of how the school's culture influences the results. The other questions were structured so that the responses can be used to answer research questions 1 and 2. The items elicited information on the type of errors that the respondents observed during their teaching of trigonometry in grade 11, the possible causes of the observed errors, and the way the teacher dealt with the errors.

3.5. The intervention process

The intervention started in both groups with the researcher teaching the experimental group, using professional noticing and probing, and the control group being taught by the incumbent teacher using traditional teaching approach. All the other conditions were similar in experimental and control groups, with only the teaching approaches being a different item. The lessons were run for one week, teaching the same topic in both groups. The researcher designed all the lessons that were used during this process. During the intervention process, the identified learner errors from the pre-test were responded to, during the lessons. A total of 272 errors were recorded from both groups of learners with 141 errors observed from the control group and 131 from the experimental group. An intervention strategy, in which professional noticing was employed, and responding to learner errors through probing was used for the experimental group. The learners in the experimental group were grouped according to the commonality in the errors observed from the pre-test. Some groups had five members while others had six.

During the intervention, the learners were probed on their erroneous answers from the pre-test. The probing process was aimed at helping the learners discover that

their answers were erratic and to help them use their errors as opportunities for learning. Learners were tasked with explaining their choice of answers and it was during this process that many learners became aware of their errors and managed to overcome them (see section 4.2.3). At the end of the intervention phase, learners in both learning groups wrote a post-test. The total number of errors from both groups decreased from 272 errors in the pre-test to 108 errors in the post-test. Interestingly, only 6 out of the 108 errors post intervention were observed from the experimental group while we had 102 from the control group.

3.6. Analysis of results

The results of the study were analysed qualitatively and quantitatively.

3.6.1. Quantitative analysis

The test scores from the pre-test for both groups were analysed using a t-test to determine whether the two groups of learners were at the same level in terms of their performance in trigonometry. The test revealed that the two groups of learners were indeed at the same level in trigonometry performance. The results of the post-test were used as a performance indicator, to test whether there was a significant difference between the two teaching methods, using hypothesis testing. Analysis of Covariance (ANCOVA) was then be used to measure whether there was a significant difference between comparable teaching methodologies. The ANCOVA was preferred over the Analysis of Variance (ANOVA) because of its ability to statistically adjust group differences initially present and “to increase the likelihood of finding a significant difference between the two means” (McMillan & Schumacher, 2010, p. 309). The ANCOVA was used, and the results revealed that the mean score of the experimental group was much higher than that of the control group.

3.6.2. Qualitative analysis

Results from the questionnaire were analysed qualitatively to give a deeper insight into the type of errors that teachers notice during their lessons and assessments, how they interpret the errors as well as how they respond to the learner errors. The responses from the questionnaire enabled the researcher to gain better understanding on what happens in the mathematics classrooms concerning noticing of errors and to use them assist learners develop their new knowledge.

The pre-test scripts of the learners were analysed for errors and a total of 272 errors were recorded from both groups. During intervention, learners in the experimental group were probed about their answers to the pre-test and they managed to overcome most of the errors. The analysis at this stage involved checking each error identified at pre-testing stage per learner and documenting whether there has been a cognitive shift by the learners in dealing with their errors.

3.7. Validity and reliability

Both the pre and post-tests were set from CAPS compliant textbooks that were being used in schools during the time of writing this research. To ensure validity of the tests, moderation was done by both a mathematics HOD from one of the Secondary schools in Johannesburg East District and by the research supervisor. The moderation process ensured that the test was set such that it covered all the cognitive levels, according to Bloom's taxonomy, as per CAPS stipulations. The items in the questionnaire were drafted such that it was easy for the respondents to understand and answer them. The questionnaire was also submitted for moderation by the supervisor to ensure that the questions are precise, and the language used is easy to understand.

Reliability refers to how consistent responses by an individual or individuals are if the same instrument is administered at different times and settings (Fraenkel & Wallen, 2009). To ensure reliability of the research instruments, they were subjected to a 'trial' before the main study was conducted. This was done for the questionnaire and the performance test. The instruments were deemed reliable and then used in the study.

3.8 Ethical considerations

Letters of consent were issued to all the learners from the sampled schools to request their parents to allow them to take part in the study. The parents of all learners in both schools allowed them to take part in the study. Learners were then issued assent letters that they signed as proof that they are willing to participate. Request letters were sent to school principals and the participating teachers. All the letters explained the reasons for undertaking the study, the importance of the study, confidentiality clause, anonymity, data collection period, the process of data collection and the contacts of the research supervisor (McMillan & Schumacher, 2010). The letters to

the parents clearly indicated that should a learner decide to withdraw from the project, no penalties are involved, that is, participating in the study was not compulsory. Furthermore, applications for ethical clearance were made at the University of Witwatersrand well as at the Gauteng Department of Education (GDE). Both institutions approved the application and issued certificates to that effect.

CHAPTER 4

PRESENTATION AND ANALYSIS OF DATA

4.1. INTRODUCTION

The purpose of the study was to measure the effect of professional noticing and probing on learner performance in trigonometry and on the identified errors. The two dependent variables that were the subject of this study were the learner performance in a trigonometry and their cognitive shift in overcoming errors in trigonometry.

4.2. Quantitative analysis

To analyse the effect on learner performance, of professional noticing and probing, the following null and alternative hypotheses were used:

Null hypothesis (H_0): There is no significant difference between the average test scores of grade 11 learners who are taught using professional noticing and probing them about their errors and those that are taught using the traditional teaching methods.

Alternative hypothesis (H_1): There is a significant difference between the average test scores of grade 11 learners who are taught using professional noticing and probing them about their errors and those that are taught using traditional teaching methods.

The effect of professional noticing and probing on learner errors was analysed through tracking of learner errors from the pretest stage through the intervention to the post test stage. The pretest was analysed for learner errors and the errors were recorded for each learner. During the intervention process, learners were probed about their errors, after which they sat for the post test. The post test scripts were analysed for errors with the intention of checking whether there has been a cognitive shift in dealing with the errors identified at the pretesting stage. This process also helped to identify some persistent errors.

4.2.1. The t-test

The pre-test scores were analysed to determine whether the two groups used were comparable in their performance in trigonometry. The t-test was used to establish

whether the mean scores from the two groups were significantly different prior to the intervention process. The results from the analysis of the pre-test scores are presented in Table 4.1.

Table 4.1: Analysis of the Pre-test scores for the two groups of learners.

t-Test:Two-sample assuming Unequal Variances. Unequal sample sizes							
α = 0.05							
	Control group	Experimental group	Diff	95% conf. interval		Cohen's d	Effect size
Mean	11.58763	12.20253	-0.61	-2.47	1.24	0.092915381	0.046408
Variance	36.41151	40.36871	Cannot reject Null Hypothesis because p>0.05 (Means are not different)				
Observations	97	79					
Hypothesised mean difference	0						
df							
t-stat	-0,653						
P(T<=t) one tail	0,257						
T critical one tail	1.65						
P(T<=t) two tail	0.515						
T critical two tail	1.974						
Cannot Reject Null Hypothesis because p>0.05 (Means are not different)							

Using a t-test to test for the difference between the means of the pre-test marks between the control group and the experimental group, the hypothesis being tested is:

H₀: the average marks for the two groups before any treatment are equal Vs **H₁**: the average marks for the two groups are different.

From the results obtained, the calculated t statistic is -0.653 and the critical value for the two-tailed test at 95% confidence is 1.974. This means the null hypothesis cannot be rejected at the 5% significant level and we conclude that there is no significant difference between the means of the two groups. This implies that before the interventions were enacted, the two groups of learners were comparable in their performance in trigonometry.

Furthermore, the p-value, $p = 0.515$ is greater than 0.05 and we fail to reject the null hypothesis and conclude that the two means are not different. The value of the

Cohen's d (0.092915381) indicates that the two means differ by 0.092915381 standard deviations. The difference is so small that it can be concluded that there is no significant difference between the two means.

4.2.2. Analysis of covariance (ANCOVA)

The test scores of learners for both the pre-test and the post-test were analysed using statistical software called QI Macros. The analysis was done for the $n=97$ learners in the control group and $n=79$ learners in the experimental group at the pre-testing stage. At the post testing stage, the number of learners in the control group remained as $n=97$ while in the experimental group the number decreased by two learners to $n=77$. The two learners could not write the post-test as they were absent from school. The results of the analysis are displayed in Table 4.2.

Table 4.2: Analysis of covariance of the test scores

ANCOVA TABLE

<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>
Factor	43836.2	3	14612.07	82.01036
Covariate	2242.35	1	2242.354	12.58522
Within	61291.7	344	178.1735	

Covariate SUMMARY ANOVA: Single Factor

$\alpha = 0.05$

<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>
---------------	--------------	------------	----------------	-----------------

A	97	1180	12.1649 5	41.66001
B	77	1492	19.3766 2	42.98667
C	97	3933.33 3	40.5498 3	462.889
D	77	3853.33 3	50.0432 9	503.5335
E				

ANOVA: Single Factor

Groups	Count	Sum	Average	Variance
A		1124	11.58763	36.41151
B	79	964	12.20253	40,36871
C	97	3746	38.61856	404.5724
D	77	3213	41.72727	276.1183

Slopes	A	B	C	D
SSx	288.8	126.8	3208.889	1408.8
Slope	0.970914	- 0.2871	0.970914	- 0.2870
SSx*Slope	280.4	-36.4	3115.556	
Bw	0.667455			
Bt	0.763814			

ANOVA				
Source of Variation	SS	df	MS	MS
Between Groups	87961.22	3	29320.41	29320.41
Within Groups	90475.77	345	262.25	262.25
Total	178436.99	348		

ANOVA				
Source of Variation	SS	df	MS	F
Between Groups	60146.6	3	20048.87	105.89
Within Groups	65509.06	346	189.33	
Total	125655.66	349		

Reject Null Hypothesis because $p < 0.05$ (Means are Different)

ANOVA				
Source of Variation	SS	df	MS	F
Between Groups	60146.60	3	20048.87	105.89
Within Groups	65509.06	346	189.33	
Total	125655.66	349		

Reject Null Hypothesis because $p < 0.05$ (Means are Different)

The results of the Analysis of Covariance (ANCOVA) using the data from both groups are summarized in Table 4.2. ANCOVA is used mainly in studies where no random selection and assignment of participants into groups is done. The ANCOVA has the ability to deal with other undesirable variables that can otherwise affect the results of the study or influence some of the variables thereby compromising the results.

The variances for the two groups are not significantly different (41.66001 for the control group and 42.98667 for the experimental group). This implies that there was homogeneity of variances between the two groups. The two schools that were used for the study are 32km apart and hence it is safe to say that the learners forming the control group did not interact with the learners from the experimental group during the intervention period. It is therefore statistically sound to conclude that the analysis of covariance results obtained in the study were reliable. The p-value obtained is very close to zero (recorded as zero in Table 4.2 due to the many zeros after the comma)

and therefore less than 0.05. This indicates that the use of professional noticing and probing is more superior compared to the traditional teaching approaches used for the control group in the teaching of trigonometry in grade 11 mathematics. This means that the null hypothesis as stated in 3.2 was rejected in favour of the alternative hypothesis and it was concluded that the means for the two groups are statistically different and hence the learners taught using professional noticing and probing performed better than those taught using traditional teaching methods.

The means and standard deviations of the two groups are presented in Table 4.3 for comparison.

Table 4.3: Mean and standard deviation of the pre- and post-tests for the two groups

	PRE-TEST			POST-TEST		
GROUP	Number of learners	Mean	Standard deviation	Number of learners	Mean	Standard deviation
CONTROL	97	11.5876	6.0342	97	12.16495	6.454
EXPERIMENTAL	79	12.2025	6.3536	77	19.37662	6.556

The average score of the control group increased by 0.57735, marking a 4.98 percent increase from pre-test to post test. The standard deviation also increased from 6.0342 to 6.454. The average score of the experimental group increased from 12.2025 to 19.37662 (an increase of 7.17412) this means that there was a 56.01 percent increment. The standard deviation increased from 6.3536 to 6.556 for the experimental group.

There was an observed increase in the mean score of both groups from the pre-test to the post-test, however it was noted that the percentage increase of the mean scores for the experimental group (56.01%) was much higher than the percentage increase in the mean scores of the control group (4.98%). This difference suggests that after intervention, the performance of the experimental group improved more than that of the control group. The other conditions at the two schools were deemed to be the same before the intervention except for the teaching methodologies and therefore the difference in the average test scores of the two groups is attributed to the teaching methodologies. Therefore, the analysis of the test scores reveal that

professional noticing and probing used in the experimental group is more effective than the traditional teaching method used for the control group.

4.3. Qualitative analysis

The pre-test learner scripts from both groups were analysed for the type of errors learners show and these were recorded for each learner. The type of errors observed were code-named using the codes used in the error analysis by other scholars (Makonye & Khanyile, 2015; Otten, Figueras, & Males, 2008) with a few additions stemming from the observations made during this study. Error types like cancellation errors, careless errors, and division by zero are some of the error types that were adopted for the study while specific error types like learners' failure to use reduction formula and learners' error in treating trigonometric ratios were coined to explain the errors observed.

4.3.1. Analysis and interpretation of results

The type of errors committed and frequency of the observed errors in the pre-test for both groups are summarised in Table 4.4

Table 4.4: Frequency Table showing errors per group of learners.

ERROR TYPE	CONTROL GROUP	EXPERIMENTAL GROUP	TOTAL
Cancellation error	22	25	47
Replacing a variable with a number	8	12	20
Dropping the denominator	11	8	19
Careless error	17	11	28
Treating a Trig ratio as a variable	23	16	39
Division by zero	6	9	15
Failure to use reduction formula	45	37	82
Inability to factorise trig functions	9	13	22
TOTAL	141	131	272

A total of 272 learner errors were recorded from the pre-test analysis of scripts. From the Table, the most common error among the learners in both groups was their *failure to use reduction formula* in simplifying trigonometric functions and expressions. This error accounted for 30.14% of the total number of errors observed. Many learners who committed this error seem to lack the basics in trigonometry. Most learners struggled to use reduction formula to simplify, for example, $\tan(330^\circ)$, $\cos(450 + \theta)$, $\cos(540 + \theta)$, and $\tan(-\theta - 180)$. Learners from both the experimental and control groups displayed similar errors at the pre-test stage.

4.3.2. Examples of learner errors

A few samples of learner errors observed in this category are discussed below:

Learner A31 (*failure to use reduction formula*)

$$\frac{\cos(540 + \theta) + \tan(-\theta - 180)}{\sin(180 + \theta)}$$

$$\frac{\cos(180 + \theta) + \tan(-\theta - 180)}{\sin(180 + \theta)}$$

$$\frac{\cos \theta + \tan(-\theta - 180)}{\sin \theta} \checkmark$$

Learner A31 is from school A, the experimental school, where the intervention was professional noticing and probing. The learner correctly reduces $\cos(540 + \theta)$ to $\cos(180 + \theta)$ but wrongly simplifies this to $\cos \theta$. The denominator, $\sin(180 + \theta)$ is also simplified to $\sin \theta$, strongly suggesting that these two similar errors may not be a result of carelessness. The learner at this stage did not fully understand the quadrants in relation to trigonometry. The reduction formulae were not properly applied by the learners.

Learner B52 (*failure to use reduction formula*)

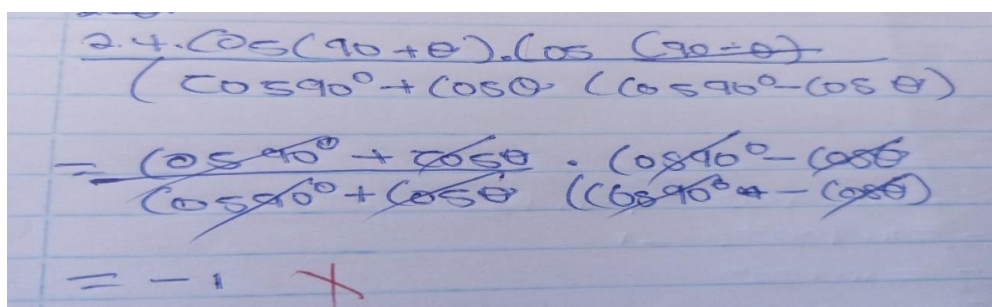
$$\frac{23 \cos(450 + \theta)}{\sin(180 + \theta)} + \cos^2(180 - \theta)$$

$$= \frac{\cos(270 + \theta)}{\sin(180 + \theta)} + \cos^2(180 - \theta)$$

Learner B52 from the control school wrongly simplified $\cos(450^\circ + \theta)$ to $\cos(270^\circ + \theta)$, an indication that the learner has an underlying misconception that informs their choice of simplifying $\cos(450^\circ + \theta)$. With the value of θ restricted to the range $[0; 90]$, it is noted that $\cos(450^\circ + \theta)$ is in the second quadrant while $\cos(270^\circ + \theta)$ is in the fourth quadrant. The learner seems to lack the basic concept of identifying the quadrants where certain angles may lie.

A total of 39 learner errors were observed in which *trigonometric ratios were treated as variables* are treated in algebra. Given, for example, $\cos(270^\circ + \theta)$, some learners would treat the “cos” outside the bracket as a variable to obtain $\cos 270^\circ + \cos \theta$. This error was prevalent in both groups of learners. Learners fail to differentiate, for example $\cos \theta$, from $5a$ in algebra. The ‘5 times a’ in algebra is written as $5a$, using the same understanding, some learners treated $\cos \theta$ as ‘cos times θ ’ and therefore committed an error in trigonometry. This type of error is what Makonye and Khanyile (2015) termed the incorrect application of the distributive law since the learners seem to be distributing the cos over the bracket. This group of learners applied a correct concept from previous interactions wrongly in new situations (Gardie & Brodie, 2021). The concept of algebra is correctly understood by the learners who showed this misconception, but it was wrongly applied in trigonometry.

Learner A15 (Treating trigonometric ratios as variables)



The image shows a handwritten mathematical derivation on lined paper. The first line is $2.4. \cos(90^\circ + \theta) \cdot \cos(90^\circ - \theta)$. The second line shows the expression in brackets: $(\cos 90^\circ + \cos \theta) (\cos 90^\circ - \cos \theta)$. The third line shows the distributive expansion: $= (\cancel{\cos 90^\circ} + \cancel{\cos \theta}) \cdot (\cancel{\cos 90^\circ} - \cancel{\cos \theta})$. The fourth line shows the result: $= -1$, which is crossed out with a red 'X'.

Learner A15 incorrectly used the concept of clearing brackets in algebra in this situation. This error cannot be classified as a careless error since it was observed that the same learner incorrectly used the concept in question 2.3 as shown:

$$\begin{aligned}
 & 2.3. \frac{\cos(450^\circ + \theta) + \cos^2(180^\circ - \theta)}{\sin(180^\circ + \theta)} \\
 & = \frac{\cos(450^\circ + \theta)}{\sin(180^\circ + \theta)} + \frac{\cos^2(180^\circ - \theta)}{\sin(180^\circ + \theta)} \\
 & = \frac{\cos 450^\circ + \cos \theta}{\sin 180^\circ + \sin \theta} + \frac{\cos^2 180^\circ + \cos^2 \theta}{\cos 180^\circ \cos 180^\circ}
 \end{aligned}$$

The learner simplified $\cos(450^\circ + \theta)$ to $\cos 450^\circ + \cos \theta$, which is consistent with the earlier error. Evidently, the learner was using the rules used in algebra to solve trigonometry questions. This error accounts for 14.34% of the observed errors in both groups.

Cancellation errors are made by learners when they incorrectly cancel out a term in the numerator and another term in the denominator disregarding the rules that govern division in such cases (Otten et al, 2008). Such cancellation errors usually manifest when there are two or more terms in the numerator or denominator being added or subtracted and learners cancel out as if the terms are multiplying. There are 47 cancellation errors recorded from the pre-test of both groups. This amounts to 17.28% of the total number of errors recorded. Most of the learners who have this misconception performed below 30% in the pre-test.

Learner B76(cancellation error)

$$\begin{aligned}
 & 2.3. \frac{\cos(450^\circ + \theta) + \cos^2(180^\circ - \theta)}{\sin(180^\circ - \theta)} \\
 & = \frac{\cos(360^\circ + 90^\circ + \theta) + \cos^2(90^\circ + 90^\circ - \theta)}{\sin(90^\circ + 90^\circ - \theta)} \\
 & = \frac{\cos(360^\circ + \theta) + \cos^2(90^\circ + 90^\circ - \theta)}{\sin(90^\circ + \theta)}
 \end{aligned}$$

The learner wrote 450 as $360 + 90$ and 180 as $90 + 90$. She went on to 'cancel out the 90' from both the numerator and the denominator to obtain $\cos(360^\circ + \theta)$ and $\sin(90^\circ + \theta)$.

Learner A14 (cancellation error)

Handwritten work for Learner A14. The student is simplifying the fraction $\frac{1 - \cos^2 \theta}{\cos^2 \theta + 2 \cos \theta + 1}$. They incorrectly cancel the $\cos^2 \theta$ terms in the numerator and denominator, resulting in $\frac{1 - \cos \theta}{1 + \cos \theta}$. A large red 'X' is drawn over the final result, indicating it is incorrect.

$$\begin{aligned} & \frac{1 - \cos^2 \theta}{\cos^2 \theta + 2 \cos \theta + 1} \\ &= \frac{1 - \cos \theta}{1 + \cos \theta} \end{aligned}$$

Learner A14, upon realising that there is $\cos^2 \theta$ both in the numerator and in the denominator, they went on to cancel it out ignoring the addition and subtraction of terms in the fraction. This error is quite prevalent in students as it accounts for 30 percent of the learners who participated in the project.

This error could be a result of an underlying misconception carried over from algebraic fractions. Errors may have been carried over from other topics because trigonometry has many connections to other subjects. 'Cancelling' common terms can only be done when all the terms in the numerator are multiplying as well as in the denominator. This error could result from a correctly constructed concept of simplifying algebraic terms but applied wrongly in this different context.

Learner B23 (Careless error)

Handwritten work for Learner B23. The student is simplifying the expression $\frac{\cos(540^\circ + \theta) \cdot \tan(-\theta - 180^\circ)}{\sin(180^\circ + \theta)}$. They incorrectly write 540° as $360^\circ - 180^\circ$ in the numerator. The final result is $\frac{[\cos(360^\circ - 180^\circ + \theta) \cdot (-\tan \theta - 180^\circ)]}{\sin(180^\circ + \theta)}$, which is then simplified to $\frac{[-\cos(180^\circ + \theta) \cdot (-\tan \theta - 180^\circ)]}{\sin 180^\circ + \theta}$. A red circle highlights the -180° term in the numerator, indicating the error.

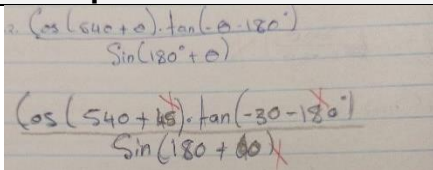
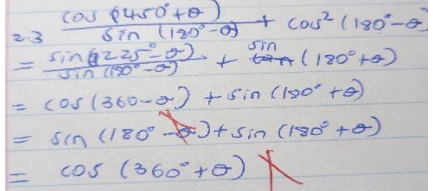
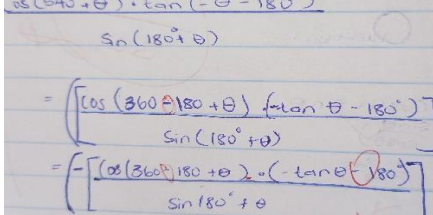
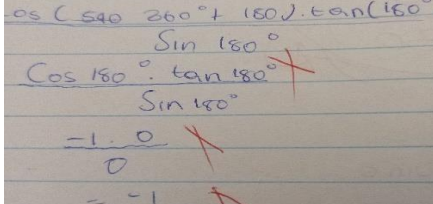
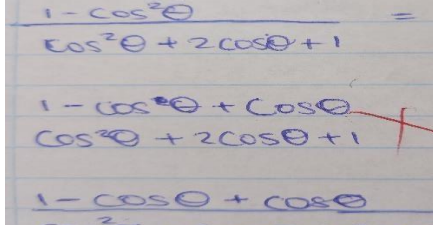
$$\begin{aligned} & \frac{\cos(540^\circ + \theta) \cdot \tan(-\theta - 180^\circ)}{\sin(180^\circ + \theta)} \\ &= \frac{[\cos(360^\circ - 180^\circ + \theta) \cdot (-\tan \theta - 180^\circ)]}{\sin(180^\circ + \theta)} \\ &= \frac{[-\cos(180^\circ + \theta) \cdot (-\tan \theta - 180^\circ)]}{\sin 180^\circ + \theta} \end{aligned}$$

Learner B23 carelessly wrote 540 as $360 - 180$. It is clear that the learner was aware that 540 can be written as 360 plus 180 but carelessly subtracted instead of adding.

This kind of error is sporadic and can rarely be repeated by the same learner. It is not a result of any misconception.

Table 4.5 shows some examples for the rest of the types of errors that were noticed from the pre-test.

Table 4.5: Examples of learner errors from the pre-test

Type of error	Example
Replacing a variable with a number	
Dropping the denominator	
Careless error	
Division by zero	
Inability to factorise	

In Table 4.5, the type of error resulting from both sets of learners' pre-tests, replacing a variable with a number accounted for around 7.35 percent of all errors. In several cases when they were unable to utilize the reduction formulae, the students substituted numbers for the variable. The majority of the numbers used are those for unusual angles, that need to be changed to numbers such as 30° , 45° , and 60° . Although they were aware that some expressions called for the usage of particular angles, learners frequently misapplied these ideas in situations where they were not necessary.

Dropping the denominator in table 4.4 account for 19 errors out of 272. Before appearing in trigonometry, this inaccuracy may have grown and shown in other subjects. It comes from earlier subjects like simplifying fractions and algebraic formulas, thus it is not specific to trigonometry. Until the teacher or feedback from evaluations alerts the student, most learners who make this mistake are unaware that they have a problem. Due to the lack of calculators, the students made casual mistakes like the one in Table 4.5 where they failed to apply the fundamental knowledge of how to write a number as the sum of two numbers.

Grade 11 learners still have harbour misconceptions presumably carried forward from number operations in grade 8 and 9. The errors that show at this level where grade 11 learners are dividing by zero are a result of such misconceptions that were carried over from previous grades and topics. As shown in Table 4.4 there were 18 errors where learners divided by zero noted. These students could not have been concerned when the denominator in the equation that they were simplifying contained zero. Because students perceive zero to signify nothing, they interpret this as diving by nothing when multiplying by zero, hence the numerator is left unchanged. Zero is incorrectly used as the identity for division in this situation. This misunderstanding could therefore be a result of how people conceptualize zero, and it persisted even after they dealt with the algebraic requirements for undefined fractions.

Some students had trouble simplifying the expression in question 1.2 using the rule for factoring the difference of two squares. Table 4.5 shows an example where some learners struggled to simplify the expression. Such errors could have their roots from the misconceptions that learners developed when doing algebra where, for example, x^2 is considered wrongly as $x + x$.

4.3.3. The intervention process

As stated earlier, the intervention process took place at school A for a week, using professional noticing and probing. Due to the high number of observed errors and the number of learners at the school ($n=79$ learners), the learners were divided into 15 groups, with seven groups in the first class and eight groups in the other class. Most groups had five members while others had six. The grouping was done according to the similarities in the type of errors exhibited by the members. This was an attempt to attend to each group of learners with related or similar errors at the same time

during the intervention process. The probing process proceeded while learners were in these groups.

The intervention process proceeded simultaneously in both group with the incumbent teacher at the control school teaching the control group while the experimental group was taught by the researcher. It was convenient to sit the learners in such groups because they managed to interact on their own. When questions were posed to specific individuals in the group, that learner was requested to explain or justify their choice of answers or working that were identified as errors. The rest of the group members would further interrogate the response from the one justifying why they think their way of reasoning is correct. This process of knowledge creation considers the one justifying their reasoning as the constructor of knowledge while the others act as critiquers. In the process of justifying their responses, learners tend to realise their error and most often correct themselves. All the errors observed in that group are dealt with during the probing process.

While interacting with group AA1, learner A31 whose error is shown in section 4.2.2, the researcher asked the following question,

Researcher: Tell us, why did you simplify $\cos(180^\circ + \theta)$ to $\cos \theta$.

Learner A31: Mmmh sir, remember 180° is a straight line, so any angle added after the straight line does not change, it must be acute (*signalling other group members for support*).

Learner A43: Which quadrant are you referring to?

Learner A31: I mean hmmm, oh no, that should be 360 not 180, isn't it sir?

Researcher: What do you mean? (*Learner A31 quickly draws the quadrants in her book and starts explaining*)

Learner A31: I mean here in this quadrant (pointing at the first quadrant). All of them are positive, 180 is here (*pointing at the negative x-axis*) so if you add θ it goes there (*pointing at the third quadrant*).

Learner A18: Yes, it means our 'cos angle' is in the third quadrant.

Learner A31: Oh, now I see my mistake, here only $\tan$ is positive, so my $\cos$ is negative.

Researcher: So, what does $\cos(180^\circ + \theta)$ simplify to?

Learner A31: To $\cos\theta$, now I get it sir.

One member of the group then asked how sometimes the trigonometric ratios change from $\cos\theta$ to $\sin\theta$ and interestingly, 3 members of the group simultaneously said cofunctions and the researcher allowed the group of learners to engage in discussion while he attended to other groups. As learner A31 was trying to justify her answer, the questions raised to her assisted her to shift from what she referred to as a mistake to a correct concept.

Group AA4 consisted of learners who treated trigonometric ratios like variables. During the interaction with the group, the researcher asked learner A15 to explain their second step in question 2.4. The extract with the error was as shown:

$$\begin{aligned} & \frac{2.4 \cdot (\cos(90^\circ + \theta) \cdot \cos(90^\circ - \theta))}{(\cos 90^\circ + \cos \theta) (\cos 90^\circ - \cos \theta)} \\ &= \frac{(\cancel{\cos 90^\circ} + \cancel{\cos \theta}) \cdot (\cancel{\cos 90^\circ} - \cancel{\cos \theta})}{(\cancel{\cos 90^\circ} + \cancel{\cos \theta}) (\cancel{\cos 90^\circ} - \cancel{\cos \theta})} \\ &= -1 \quad \times \end{aligned}$$

Learner A15: I cleared the bracket for both terms in the numerator.

Researcher: What does the bracket in $\cos(90^\circ + \theta)$ mean?

Learner A15: The term outside the bracket must multiply both terms in the bracket, like when you have $3(4x + 5)$.

Researcher: So how about if you have, say, $\cos(30^\circ + 15^\circ)$?

Learner A15: We start by adding the like terms in the bracket to get $\cos$ bracket 45 degrees (meaning $\cos(45^\circ)$). In 2.4, 90 and θ are unlike terms we can't add them.

The other group members begin to argue.

LearnerA39: If you take $\cos(30^\circ + 15^\circ)$ and write it as $\cos 30^\circ + \cos 15^\circ$ is it the same as $\cos 45^\circ$ in the calculator?

Researcher: Good question, what do you think A15?

LearnerA15 and the other group members work with their calculators quickly and A15 says they are not the same.

Learner A15: It means it is wrong to clear the bracket when we have $\cos$, it does not work, but why? Does it mean $\cos$ is not a term like the normal ones?

Learner A42 interjects 'you can't just say $\cos$, it should be $\cos$ of an angle.

Learner A15: Oh, so that means it is different, $\cos$ is not a term and therefore cannot be used to clear the bracket. So, it means we must stick to those formulae all the time?

Researcher: Well, you say the bracket must not be cleared, if you also do not use the formulae, is there another way of doing it?

Learner A15: No sir it looks like we must use the formulae, I must master them...
(A42 interjects)

LearnerA42: What is in the bracket helps you to know which quadrant the angle is, then you know whether it is positive or negative.

Learner A15: Now I see where I went wrong, so $\cos(90^\circ + \theta)$ is in the second quadrant and it must give us $\sin\theta$, and it is positive in the second quadrant. Thank you so much sir, let me try and master the rest of the formulae.

Researcher: You are welcome, A42 gave you advice, do more activities similar to this from your textbook.

Learner A15 could not differentiate the expression $3(4x + 5)$ and $\cos(90^\circ + \theta)$. after the probing session with her and the group, she realised that the two cannot be treated the same and hence shifted from her initial wrong concept of using the rules of simplifying algebraic terms in this part of trigonometry. As the learner attempted to justify her wrong answer, she was probed about her solutions to a point where she dropped her initial position that stimulated the error. As the researcher went on to interact with the rest of the members of the group, most of the group members were

already aware of their misconception and could explain why they think their responses were wrong.

Malambo (2021), in his study of errors in trigonometric equations, he gave the following statement:

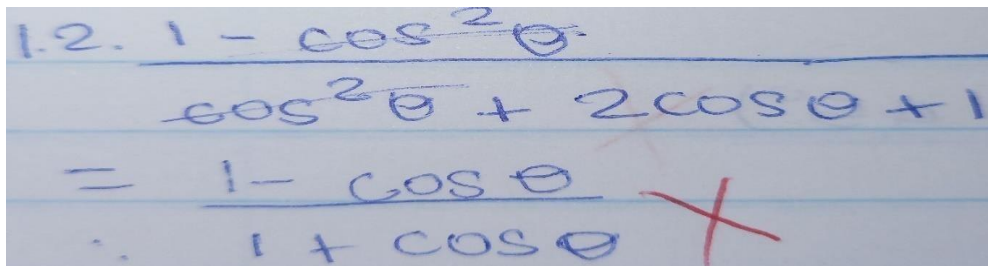
Trigonometric equations are unique in the sense that unlike conventional algebraic equations, their solutions are not mere numbers but angles. The conceptual difference carries a likelihood of students assuming that trigonometric equations could be treated in exactly the same way as algebraic equations such as linear and quadratic equations are solved (p. 8).

Malambo's assertion aligns with the observations made in this study, learners have the correct concepts for simplifying algebraic expressions, but these are translated incorrectly to trigonometric expressions.

Learners in group AA2 were grouped together because of the cancellation error observed in their pre-test scripts, among other errors. All the six learners in the group had a problem in cancelling a term in the numerator with a term in the denominator even when the terms were being added or subtracted. All the six learners explained their responses to questions in which the researcher had identified the errors. Learners responded to the questions posed by the researcher. While this process unfolded, the researcher requested to record the probing sessions so that they can be transcribed later. One learner from a different group was recording the sessions as they proceeded. Learners requested that transcription should be done soon after all sessions and the recordings be destroyed for fear of being identified through their voices if the recordings are kept. This was agreed to so that the interactions can be free, and learners can express themselves without the fear that someone else could know the challenges they have in mathematics.

The following is the extract from the interview between the researcher and learner A14 concerning the cancellation error as shown in the insert.

Learner A14 (cancellation error)



The image shows a handwritten algebraic fraction on lined paper. The numerator is $1 - \cos^2 \theta$ and the denominator is $\cos^2 \theta + 2 \cos \theta + 1$. Below the fraction line, the simplified form is written as $\frac{1 - \cos \theta}{1 + \cos \theta}$. A large red 'X' is drawn over the simplified fraction, indicating it is incorrect. The error is a cancellation of $\cos^2 \theta$ from the numerator and denominator, which is not valid because the denominator is a sum, not a product.

Researcher: Please explain to me how you solved this problem.

Learner A14: I realised that there are terms that are the same for both the numerator and the denominator, (poses)

Researcher: Then what did you do next?

Learner A14: Yes sir the cos squared is in the numerator and also in the denominator, so we need to cancel it as we simplify.

Researcher: when do we get to cancel a denominator with a term in the numerator?

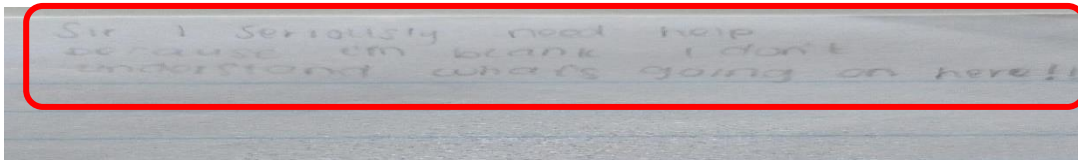
Learner: When the terms are multiplying oh sir, I see the problem now, we can't cancel because they are not multiplying at the top and in the denominator.

Researcher: good.

The cancellation error was observed in the study by Makonye and Khanyile (2015) dealing with errors in algebraic fractions. Similar observations have been made in this study.

Learner A14 had a problem of cancelling terms in the numerator with terms in the denominator even though there were addition or subtraction signs between the terms. Through probing, the learner managed to overcome their initial errors and finally managed to solve the problem. Learners in other groups who had similar challenges were also engaged and most of them managed to correct their own initial errors and grasp the correct concept.

The probing process continued slowly for a week and all the types of errors noticed were confronted during the intervention process. Most of the learners expressed their gratitude to the researcher for helping them understand trigonometry. Learner A10 had written the following on the pre-test script:



After the intervention, the learner indicated that she was ready for a test in trigonometry because the concepts were all 'clear'. The probing process enabled this learner to understand the concepts.

4.3.4. Error analysis: post-intervention

The analysis of the learner errors at the post-testing stage is summarised in Table 4.5.

Table 4.5: Analysis of learner errors at the post-testing stage

Error Type	Control Group	Experimental Group	Total
Cancellation error	18	0	18
Replacing a variable with a number	5	1	6
Dropping the denominator	7	0	7
Careless error	11	2	13
Treating a Trig ratio as a variable	15	0	15
Division by zero	4	0	4
Failure to use Reduction formula	36	2	38
Inability to factorise trig functions	6	1	7
TOTAL	102	6	108

The total number of errors observed after intervention in both groups was 108 down from 272. It is interesting to note the decrease in the total number of errors for the experimental group from 131 to only 6 after the intervention. There is also a decrease in the total number of recorded errors for the control group, from 141 to 102. The decrease in both groups can be due to the intervention that took place for a week after the pre-test was written. Interventions in both the control group and the experimental group led to a decrease in the number of errors. To analyse the comparative effect of the two intervention methodologies, we use the percentage

decrease in the number of errors for the two groups. The number of errors observed in the control decreased from 141 to 102, marking a 27.66 percent decrease. A total of 131 errors were recorded in the experimental group prior to intervention and this decreased to 6 post intervention. This marks a 95.42 percent decrease in the number of errors committed by the learners in the experimental class. This suggests that the professional noticing and probing that was used is more effective than the traditional teaching methods used for the control group.

The probing process helped learners realise their initial errors and therefore assisted in overcoming some misconceptions that were initially prevailing. Interestingly, learners in the experimental group could correctly use reduction formulae which was a huge challenge prior intervention.

4.3.5. Data analysis for the questionnaire

The questionnaire was infused into the study mainly to determine the type of errors that teachers have observed in their mathematics classroom as they teach grade 11 trigonometry. Furthermore, the questionnaire gathered information on what the teachers perceive as the possible reasons for learners to make those errors as well as how they handle the errors that are observed. Five mathematics teachers responded to the questionnaire from both schools. These are teachers assigned to teach grade 11 when the study was conducted and those that had taught grade 11 within three years of the study.

4.3.5.1. Teacher qualifications

Teacher qualifications were included in the questionnaire to help explain, in part, teacher competences in mathematics subject knowledge and the teaching of mathematics. All the 5 teachers were qualified to teach mathematics, and all had a Bachelor of Education degree, with one teacher studying for an Honors degree in mathematics teaching. All the teachers that took part in the study were suitably qualified to teach trigonometry in grade 11 mathematics. The information obtained from the teachers can therefore be considered reliable in as far as the two schools are concerned.

4.3.5.2. Teacher Experience

There was a vast difference between the number of years of experience between the teachers, with only 1 teacher having less than 3 years of teaching experience while the other two had between four- and six-years' experience in teaching mathematics. Two teachers, one from each school had more than seven years' experience. This implies that the teachers at both schools were suitably qualified and experienced to teach grade 11 mathematics.

4.3.5.3. School performance in grade 11 Mathematics

The data collected from both schools indicate that the two are performing at slightly below average both in mathematics and in trigonometry as a topic. The two schools have been performing at between 45 and 50 percent pass rate in matric for the past two years. The performance in grade 11 seem to be the same as that of grade 12 in both schools.

4.3.5.4. Learner errors and possible causes

The five teachers were requested to highlight the type of errors that they have observed during the teaching of trigonometry over the years they have been teaching. Data obtained indicate that learners commit errors when they must use the reduction formulae. According to the teachers, learners tend to mix up the trigonometric ratios and hence struggle with the use of reduction formulae. It was also pointed out that learners make a lot of errors when they must use trigonometric identities.

The causes of learner errors as pointed out by the teachers were, among others, carelessness, lack of focus, and poor mathematics background. Two of the teachers indicated that they believe some of the learners in their mathematics classes made wrong subject choices, they should have opted for mathematical literacy and not mathematics. Mathematical literacy in the South African curriculum is a form of waived mathematics that is offered to learners whose career paths do not require mathematics as a pre-requisite. This curriculum offers less mathematics content knowledge than mathematics. Learners who take mathematical literacy usually manage to pass the subject as compared to mathematics. A follow up was made with the teachers that raised this issue, they believe that not all learners can do

mathematics and that the learners that struggle with mathematics should be channelled to mathematical literacy.

4.3.5.5. Dealing with learner errors.

This part of the questionnaire was responded to by all the five teachers who participated in the study. The five teachers raised 4 ways in which they deal with learner errors, and these are:

- Remedial lessons
- Revision of trigonometry tests,
- Encouraging learners to discuss and help one another,
- Increase the frequency of tests.

Interestingly, none of the respondents mentioned how they help learners overcome the errors. Asked on what they do during remedial lessons to deal with the errors observed from their learners, some teachers would just revise the questions that most learners would have failed to answer in the tests. The understanding is that the concept was not well grasped by the learners so the remedial lessons are actually a form of re-teaching the learners so that the concept can be understood. Some teachers have resorted to giving the learners more tests with the hope that the observed errors can eventually be dealt with during the revision of the tests. Encouraging learners to discuss and help one another is one way some teachers believe can assist the learners to overcome the errors. The teachers encourage discussions among learners with the hope that those who mastered the concepts can assist the learners with errors to reconstruct the concepts and thereby deal with the errors.

4.4. Summary and conclusion of the chapter

The purpose of the study was to determine the effect of professional noticing of learners' errors and misconceptions on grade 11 learners' trigonometry performance. This was done through comparing the performance of learners who were taught through professional noticing and probing and those that received instruction through traditional teaching approaches. Data were collected through a pre-test, post-test, and a questionnaire. The pre-test and post-tests were used to record the

performance scores of the learners as well as to record the type and frequency of errors that learners were showing. The pre-test scores were analysed using a t-test to determine whether the two groups of learners were comparable in trigonometry performance. The t-test revealed that there was no statistically significant difference between the performance of the two groups prior intervention. The pre-test and post-test score were analysed using ANCOVA, with the post-test scores being the dependent variable while the pre-test scores were considered the covariate. Results of this test revealed that at 5 percent level of significance, the mean score of the learners taught using professional noticing and probing was higher than the mean score of the learners taught using traditional teaching methods. Therefore, the null hypothesis was rejected at 0.05 significance level and it was concluded that the performance of the group that received instruction through professional noticing and probing was statistically higher than the performance of the group where the intervention was the traditional teaching method.

Error analysis of the post-test revealed that the probing process used at the responding stage of professional noticing was effective in helping learners realise their errors and hence solve their initial challenges. Professional noticing and probing can be viewed as a constructivist approach where learners are allowed to construct their own knowledge with some guidance from the teacher. Having noticed that learners have errors and attended to them, the researcher went on to respond to them through probing, which according to the analysis done, assisted learners to overcome their errors. Learners were therefore able to construct their knowledge when probed about their erroneous errors.

CHAPTER FIVE

DISCUSSION, RECOMMENDATIONS, AND CONCLUSION

5.1. INTRODUCTION

In this chapter, aspects such as a summary of the literature review, results of the study, recommendations, and a conclusion are presented. The constructivist theoretical framework that guided the study is used to interpret the results, giving special attention to how the research questions were answered. Recommendations for future studies and a conclusion are then presented.

5.2. Summary of the literature review

The NSC Mathematics results in South Africa have been below average since the inception of the CAPS examination in 2014. Trigonometry, as one of the topics examined in mathematics, has the highest weighting in the paper (50 out of 150), and hence more attention ought to be channelled towards the topic. The teachers' pedagogical knowledge has a strong bearing on the success of learners in mathematics and the performance of learners.

Learner errors when answering trigonometry questions is one of the major causes of poor performance in the topic and hence mathematics (DBE, 2021). The diagnostic reports DBE (2017, 2018, 2019, and 2020) all point to persistent errors committed by learners in trigonometry throughout these years as the main reason why learners perform badly in trigonometry. Simplifying trigonometric functions and expressions, solving trigonometric equations, and the inability to use trigonometric identities are some of the sections of the topic where most of the candidates have shown errors throughout the four years in question. These errors lead to massive loss of marks and hence poor performance in mathematics. It is therefore imperative that educators begin to notice these errors in the classroom and find ways to deal with them.

Studies in mathematics education indicate that a focus on errors committed by learners enable educators to better understand the kind of assistance and remedial activities that are required to enhance learner understanding of concepts (Adler, 2005; Brodie, 2014; Venkat & Adler, 2012). When learners construct mathematical knowledge, the already existing concepts interact with the new concept to build new knowledge (Gordon 2009). In this research, an intervention strategy that deals

directly with learner errors was used, in which mathematics lessons are grounded on the identification and probing of learner errors in grade 11.

A constructivist approach was used throughout the study. Learners were given opportunities to process their own reasoning and construct new concepts from their errors. The approach used in the study shifted away from telling to using what the learners already believe is the correct concept to construct new knowledge. When learners were given opportunities to relook and analyse their own errors, they were able to realise their misconceptions and they improved on their errors.

Professional noticing has been used, with great success, in the teaching of mathematics. Studies that place focus on learner thinking, errors, and misconceptions as well as teacher development on the aspect of noticing all point to the need by educators to use the errors, misconceptions, and learner thinking as opportunities for learning (Borasi, 1994; Davis & Renert, 2014; Hine & Lesseig, 2021; Makonye & Khanyile, 2015; Schoenfeld, 2011; Son & Sinclair, 2010; Thomas et al., 2017). In their study Makonye and Khanyile (2015) used probing to help learners overcome their errors with a 98 percent success rate. The current study used professional noticing and probing and concluded that professional noticing and probing is more effective in dealing with learner errors than traditional teaching methods.

5.3. Research findings

The study was conducted in two high schools in the Johannesburg East District in Gauteng South Africa. Grade 11 mathematics learners from these two schools and their mathematics teachers took part in the study. A total of $n = 176$ learners participated in the study with $n = 79$ learners in one school being the experimental group while $n = 97$ learners were in the control group. A total of 5 teachers responded to the teacher questionnaire used in the study.

The following research questions were set for this study.

- What type of errors are committed by grade 11 learners in trigonometry and how do teachers attend to them?

- What are the possible causes of learner errors in grade 11 trigonometry and how do teachers interpret them?
- What is the effect, if any, of responding by probing learners about their errors on learner performance in trigonometry grade 11?

5.3.1. Answering the first research question

Responses from teacher questionnaires provided a rich narrative on the type of errors committed by grade 11 learners in trigonometry. Analysis of the pre-test written by both the experimental and the control groups added more insight into the type of errors that are committed by grade 11 mathematics learners in trigonometry (see also section 4.2). The most common errors recorded from both the teacher questionnaires and the pre-test were:

- Failure to use the reduction formula,
- Cancellation error,
- Treating a trigonometric ratio as a variable,
- Careless error,
- Inability to factorise functions, and others.

The intervention, using professional noticing and probing, helped learners to realise their own errors. The learners, during the intervention phase of the study, used these errors as springboards (Borasi, 1994) for the acquisition of knowledge. Teachers identify these errors in their classrooms when learners write formal and informal assessments,

5.3.2. Answering the second research question

Responses from the teacher questionnaire helped in answering this question. Several causes of learner errors in trigonometry were highlighted by the teachers that responded to the questionnaire. The causes of errors identified were:

- carelessness,
- lack of focus,
- poor mathematics background,
- Wrong subject choice.

Some of the teachers that responded to this question believe that most learner errors in trigonometry are a result of carelessness. If the errors are a result of

carelessness, then there may be no underlying misconceptions behind them. Treating all errors this way may hinder the possibility of uncovering the underlying errors that learners have, and the errors and misconceptions may be carried over to other grades and topics.

Other respondents pointed to lack of focus, poor mathematics background, and wrong subject choice as the causes of errors. There is a general belief among some mathematics teachers that mathematics is not meant for everyone, so when learners struggle with a concept, it is generally believed that the learners are not 'mathematics material'. Teachers who subscribe to this belief may not necessarily dig deeper when encountering errors in the classroom, the errors may simply be attributed to lack of focus, poor mathematics background, and wrong subject choice. This way the learners are denied an opportunity to learn from their mistakes, which is a crucial stage in the development of knowledge.

5.3.3. Answering the third research question

What is the effect, if any, of responding by probing learners about their errors on learner performance in trigonometry grade 11?

To answer this question, a pre-test and a post-test were written by learners and the score were analysed using the analysis of covariance. The following hypotheses were stated:

Null hypothesis (H_0): There is no significant difference between the average test scores of grade 11 learners who are taught using professional noticing and probing them about their errors and those that are taught using the traditional teaching methods.

Alternative hypothesis (H_1): There is a significant difference between the average test scores of grade 11 learners who are taught using professional noticing and probing them about their errors and those that are taught using traditional teaching methods.

The analysis of the pre- and post-test scores, using ANCOVA, revealed that the mean score for the experimental group was significantly higher than the mean score of the control group, at the 5 percent level of significance. The performance of the learners in the experimental group was better than that of the control group. The mean of the control group increased by 4.98 percent from the pre-test to the post-

test, while the mean of the post-test increased by 56.01 percent. The huge difference in the percentage increase is an indication that professional noticing and probing is more superior to the traditional method used for the control group.

Error analysis was also done on both the pre-test and the post-test of learners in both groups. This analysis revealed that professional noticing and probing used for the experimental group helped learners to use their errors as opportunities for learning. The number of observed errors in the experimental group decreased by 125 errors from 131 errors in the pre-test to 6 errors in the post test. The number of errors in the control group decreased from 141 to 102, a decrease of 39. The drop was more pronounced in the experimental group than in the control group, suggesting that the skilled noticing and probing used is more successful in assisting learners deal with errors than the conventional method used in the control group. Similar results were obtained by Makonye and Khanyile (2015) and Borasi (1994).

The two groups were deemed to be comparable in their performance in trigonometry upon analysis of the pre-test scores. All the other conditions were the same for both groups except for the teaching methods and therefore the difference in learner performance and in the reduction of learner errors can be attributed to the approaches used during the intervention process.

The results of the study also indicate that the constructivist approaches of professional noticing and probing used for the experimental group was effective in helping learners create new concepts and knowledge on their own due to probing. The misconceptions initially held by learners were turned into new knowledge during the intervention process. The learners developed conceptual understanding and productive disposition (Kilpatrick et al.; 2001) during the intervention phase.

5.4. RECOMMENDATIONS

Having done the study, the following recommendations were made:

- The study only used two schools, the recommendation is that the study can be done using a bigger sample size and more schools from different provinces so that the results obtained can give better insights into the different teaching methodologies in the whole country.

- Further research could be done on the persistent errors that were recorded post intervention to gain more insight on why they were persistent.
- Teacher development programs should include aspects like professional noticing and probing as well as how to deal with observed learner errors.
- Include courses on professional noticing and probing in teacher education programs and ensure that these courses are compulsory.

5.5. Conclusion of the chapter

The poor performance by learners in mathematics and trigonometry triggered research interest into this part of mathematics education. Studies that focus on learner errors have been conducted (Borasi, 1994; Gardee & Brodie, 2021; Makonye & Khanyile, 2015) and there is a consensus that errors cannot be ignored when learners present them. Professional noticing and probing have a potential helping learners overcome their errors and improve in their performance in mathematics, if enacted diligently. Responding to learner errors through probing allow learners to construct new knowledge from their own errors and hence assists in unearthing the misconceptions behind the errors.

This study was conducted in two schools, one using professional noticing and probing and the other one using traditional teaching approach. Analysis of the results revealed that learners that were taught using professional noticing and probing assists learners to use their misconceptions as springboards to overcome their errors and hence improve on their performance.

Mathematics teachers should strive to get a deeper understanding of professional noticing and probing and apply them in their daily interaction with learners. The use of this method can help learners achieve all the five strands of mathematical proficiency as stated by Kilpatrick et al. (2001). Teachers should be able to notice learner errors during lessons and map out strategies to help learners replace their misconceptions with new constructed knowledge.

Further research with a larger and more representative sample can be done in mathematics so that the results can be generalisable to a larger population. Another recommendation is that the department of education should organise workshops to develop the teachers in the field of professional noticing and probing.

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APPENDIX A: TRIGONOMETRY PERFORMANCE TEST

Topic: Trigonometry

Grade 11

Duration: 40 minutes

Total marks: 30

Answer all questions

You may not use a calculator

Question 1

Prove the following identities

1.1. $\cos\theta (1 + \tan\theta) = \cos\theta + \sin\theta$ (3)

1.2. $\frac{1 - \cos^2 \theta}{\cos^2 \theta + 2\cos\theta + 1} = \frac{1 - \cos\theta}{1 + \cos\theta}$ (3)

Question 2

Simplify the following as far as possible without the use of a calculator

2.1 $\tan^2 330^\circ - \sin 120^\circ \cdot \tan 135^\circ$ (4)

2.2 $\frac{\cos(540^\circ + \theta) \cdot \tan(-\theta - 180^\circ)}{\sin(180^\circ + \theta)}$ (4)

2.3 $\frac{\cos(450^\circ + \theta)}{\sin(180^\circ - \theta)} + \cos^2(180^\circ - \theta)$ (4)

2.4. $\frac{\cos(90^\circ + \theta) \cdot \cos(90^\circ - \theta)}{(\cos 90^\circ + \cos\theta)(\cos 90^\circ - \cos\theta)}$ (4)

2.5. $\frac{\cos(180^\circ + \theta) \cdot \tan(180^\circ - \theta)}{\sin(-\theta)}$ (4)

2.6. $\frac{\tan(180^\circ + \theta) \cos(360^\circ + \theta)}{\sin(180^\circ + \theta) \cdot \cos(90^\circ + \theta) + \cos^2(360^\circ - \theta)}$ (4)



APPENDIX B: TEACHER ATTITUDES AND PERCEPTIONS ON LEARNER ERRORS QUESTIONNAIRE

Teacher Code _____

INSTRUCTIONS

You are requested to respond to all the questions

Your responses to the questions will be treated with strict confidentiality.

The information you provide will be used strictly for the purpose of this study only.

1. What is your highest qualification in mathematics teaching?

2. For how many years have you been teaching mathematics in grade 11? (specify in years only, e.g. 3years, 21years, etc)

3. How has been the performance of grade 11 learners in mathematics? (*choose from below*)

Below average	Average	Above average	Outstanding
---------------	---------	---------------	-------------

4. How has been the performance of grade 11 learners in trigonometry? (*choose from below*)

Below average	Average	Above average	Outstanding
---------------	---------	---------------	-------------

5. What type of errors are committed by learners in trigonometry?

6. What do you think is the cause of these observed errors?

7. How do you usually deal with the errors committed by learners in trigonometry?



APPENDIX C: Participation letter to school Principals

RE: Request to conduct research at your school

Dear Sir/Madam

My name is Ashton Ndebele and I am registered with the University of Witwatersrand for a degree of Master of Education specializing in mathematics education. The title of the study is "*effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance*", under the supervision of Professor J.P. Makonye. The aim of the study is to investigate the effects of professional noticing of learner errors and misconceptions in helping deepen learners' understanding of trigonometry in grade 11. In this research, I am supposed to work with grade 11 mathematics learners.

I hereby request to conduct my research at your school. The study will be carried out during school hours and all the research activities will be done within the school with strict compliance with all Covid 19 protocols in place. For the research I kindly request the following:

- Participation of grade 11 mathematics teacher(s),
- Participation of your grade 11 mathematics learners.

The grade 11 learners are expected to write a 40-minute test, at pre and post-testing stages, and the test scores obtained will be analysed and used as research findings. Privacy, anonymity, confidentiality, and respect for human dignity will be honoured during this research. Participants will be referred to using pseudonyms. There will be no incentives given to those who choose to participate in this research study. Participation in the research is not compulsory, it is voluntary, and you will not incur any costs. If you decide to withdraw from participation at any stage of the research, you are free to do so without any penalties. If you allow your school to participate in this research, the researcher will convene a meeting where the objectives and activities of the research will be fully explained. Participants will be given an opportunity to ask questions. All the research data will be destroyed within 3 to 5 years after the completion of this project.

This research study will be written up as a research report. The report will be available on the university library website. If you would like to receive a summary of this report, I will be happy to send it to you. If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za. Please complete and sign the attached return slip to indicate your decision on this matter. Your cooperation is greatly appreciated.

Supervisor: Professor J P Makonye
Email: Judah.Makonye@wits.ac.za

Contact: 011 717 3086

Yours faithfully

Mr. Ashton Ndebele

Email: 2633966@students.wits.ac.za

079 194 9551



Consent Form (School Principal)

Title of project: Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

Name of researcher: Ashton Ndebele

I, ..., the principal of, secondary school agree to participate in this research project.

I agree to the following: (Please circle the relevant options below)

The research study was explained to me. I understand what this study is about. YES NO

I understand that I can volunteer to take part in the study on behalf of my school YES NO

I agree that my school's participation will remain anonymous (my school name will not be used by the researcher in their research report. YES NO

I agree that other researchers may use the information I provide in my interview/focus group/other activity (depending on their own ethics clearance being obtained) but my school name and any personal information will not be used or passed on YES NO

Signature

Name of Principal

Date

Signature

Name of researcher: Ashton Ndebele

Date



Appendix D: Participation letter to parents

RE: Request for your child to participate in research

Dear Sir/Madam

My name is Ashton Ndebele and I am registered at the University of Witwatersrand for a Master of Education degree. To complete this degree, I am required to do research. My research is entitled "effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance" and my supervisor is Professor J.P. Makonye.

I am requesting you to allow your child to participate in this research. The participation of your child will help in the improvement of mathematics teaching in secondary schools. If you allow your child to participate, he/she will write a test in trigonometry. The results of the test will be analysed as research findings. The test will be written at your child's school and will be monitored by your child's mathematics teacher. This will be done during school hours within the time your child is at school.

Participation in the research is entirely voluntary and children are free to withdraw from participating at any time without any penalties. No incentives of any kind will be given to those that will participate in the study. The names of teachers, learners, and the school that take part in the research will not be revealed at any point during the research. Pseudonyms will be used instead. If you allow your child to take part in this study, a meeting will be held with them at school to fully explain the aim and procedures for the study and all participants will be given a chance to ask questions.

This research study will be written up as a research report. The report will be available on the university library website. If you would like to receive a summary of this report, I will be happy to send it to you. If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za. Please complete and sign the attached consent form to indicate your decision on this matter. Your cooperation is greatly appreciated.

Supervisor: Professor J P Makonye

Email: Judah.Makonye@wits.ac.za

Contact: 011 717 3086

Yours faithfully

Mr. Ashton Ndebele

Email: 2633966@students.wits.ac.za

079 194 9551



Consent Form (Parents)

Title of project: Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

Name of researcher: Ashton Ndebele

I, ..., the parent/guardian of, agree to allow my child to participate in this research project.

I agree to the following: (Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
--	-----	----

I understand that I my child can volunteer to take part in the study	YES	NO
--	-----	----

I agree that the results of the test may be used by the researcher in their research report	YES	NO
---	-----	----

I agree that my child's participation will remain anonymous (my child's name will not be used by the researcher in their research report).	YES	NO
--	-----	----

Signature

Name of Parent

Date

Signature

Name of researcher: Ashton Ndebele

Date



APPENDIX E: Participation letter to mathematics teachers

RE: Request to participate in a research study

My name is Ashton Ndebele and I am registered at the University of Witwatersrand for a Master of Education degree, specializing in mathematics education. To complete this degree, I am required to do research. The title of the study is *“effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance”*, under the supervision of Professor J.P. Makonye. I hereby request you to participate in the research study. If you agree to take part in the study, you will be expected to:

- Help to administer a pre and post-test set by the researcher to your learners, □
- Respond to a questionnaire concerning the teaching of grade 11 trigonometry.

All the activities will be conducted during school hours. Strict confidentiality will be exercised, and all participating teachers, learners, and the school will not be mentioned by name anywhere in the research report. Pseudonyms will be used instead. There are no incentives given to those who will take part in the study and participation is voluntary. You will not incur any cost because of participating in this study. Should you decide to withdraw from participating during the study, you are free to do so without any penalties. Your participation will help in improving mathematics teaching in grade11.

If you decide to take part in the research, you will be invited to a meeting at your school where the aim and objectives of the research will be fully explained. All participants will be free to ask questions during the meeting.

This research study will be written up as a research report. The report will be available on the university library website. If you would like to receive a summary of this report, I will be happy to send it to you. If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee

(Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za. Your cooperation in this regard is greatly appreciated.

Yours sincerely,

Mr. Ashton Ndebele

Email: 2633966@students.wits.ac.za

079 194 9551

Supervisor: Professor J P Makonye

Email: Judah.Makonye@wits.ac.za

Contact: 011 717 3086



Consent Form (Mathematics teachers)

Title of project: Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

Name of researcher: Ashton Ndebele

I, ..., agree to participate in this research project.

I agree to the following: (Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
--	-----	----

I understand that I can volunteer to take part in the study	YES	NO
---	-----	----

I agree that respond to the Questionnaire and I will assist to administer the pre-test and post-test	YES	NO
--	-----	----

I agree that direct quotations from my responses to the questionnaire may be used by the researcher in their research report.	YES	NO
---	-----	----

I agree that my participation will remain anonymous (my name will not be used by the researcher in their research report).	YES	NO
--	-----	----

I agree that other researchers may use the information I provide in my responses to the questionnaire (depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on.	YES	NO
--	-----	----

Signature

Name of Teacher

Date

Signature

Name of researcher: Ashton Ndebele

Date



APPENDIX F: Assent letter to learners

RE: Request to participate in a research study A LETTER REQUESTING ASSENT FROM SECONDARY SCHOOLLEARNERS TO PARTICIPATE IN A RESEARCH STUDY

Title of project: Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

Name of researcher: Ashton Ndebele

Dear _____ Date _____ I

am registered at the University of Witwatersrand for a Master of Education degree, specializing in mathematics education. To complete this degree, I am required to do research. Your principal has given me permission to do this study at your school. I would like to invite you to be a very special part of my study. I am doing this study so that we can understand better the challenges you encounter when solving trigonometry problems.

This may help you and many other learners of your age in different schools. This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

If you take part in the study, you will write a 40-minute test based on trigonometry. We will then do some revision activities in trigonometry for a week, and you will write another trigonometry test. You will not use your name on the answer sheet, I will give you codes to use in place of your names so that no one will know how you perform in the test.

I will write a report on the study, but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you do not want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I have completed the study, I will return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I may also invite you to come and listen to my talk.

There are no potential risks in this study. You will not be paid or receive any incentives for your participation in the research. There will be no costs for you if you participate in the study. If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at 079 194 9551. Do not sign the form until you have all your questions answered and understand what I would like you to do.

Researcher: Ashton Ndebele

079 194 9551

ASSENT FORM

Make sure that before you complete this form, you understand the letter above. Ask me or your mathematics teacher if you have any questions or if there is something that you do not understand.

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about the study, and I know what I will be asked to do. I am willing to be part of the study.

Learner's name (print):

Learner's signature:

Date:

Witness's name (print)

Witness's signature

Date:

(The witness is over 18 years old and present when signed.)

Parent/guardian's name (print)

Parent/guardian's signature:

Date:

Researcher's name (print)

Researcher's signature:

Date:

APPENDIX G: GDE APPROVAL



GAUTENG PROVINCE

Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	07 July 2022
Validity of Research Approval:	08 February 2022– 30 September 2022 2022/286
Name of Researcher:	Ndebele A
Address of Researcher:	41 Doorn Street Observatory Johannesburg
Telephone Number:	079 194 9551
Email address:	magezaashton@yahoo.com; & 2633966@students.wits.ac.za
Research Topic:	The effect of probing learners about their errors on learner performance in trigonometry grade 11
Type of qualification	Master of Education (MEd)
Number and type of schools:	1 Secondary School
District/s/HO	Johannesburg East

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below are met. Approval may be withdrawn should any of the conditions listed below be flouted:

Making education a societal priority

Office of the Director: Education Research and Knowledge Management

7th Floor, 17 Simmonds Street, Johannesburg, 2001

Tel: (011) 355 0488

Email: Faith.Tshabalala@gauteng.gov.za

Website: www.education.gog.gov.za

1. The letter would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. **Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools, or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.**
4. **The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.**
5. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s has been granted permission from the Gauteng Department of Education to conduct the research study.
6. A letter/document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs, and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
7. The Researcher will make every effort to obtain the goodwill and cooperation of all the GDE officials, principals, and chairpersons of the SGBs, teachers, and learners involved. Persons who offer their cooperation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
8. Research may only be conducted after school hours so that the normal school program is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
9. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
10. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
11. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
12. The researcher is responsible for supplying and utilising his/her research resources, such as stationery, photocopies, transport, faxes, and telephones, and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
13. The names of the GDE officials, schools, principals, parents, teachers, and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
14. On completion of the study, the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
15. The researcher may be expected to provide short presentations on the purpose, findings, and recommendations of his/her research to both GDE officials and the schools concerned.
16. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a summary of the purpose, findings, and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind Regards



Mr. Gupani Mukatuni

Acting CES: Education Research and Knowledge Management

DATE: 08/07/2022

2

Making education a societal priority

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APPENDIX H: ETHICS CLEARANCE



SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE

PROTOCOL NUMBER: 2022ECE033M

PROJECT TITLE

Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

INVESTIGATOR

Ndebele Ashton

SCHOOL/DEPARTMENT OF INVESTIGATOR

WSoE

DATE CONSIDERED

22 July 2022

DECISION OF THE COMMITTEE

Approved unconditionally

RISK LEVEL

No Risk

EXPIRY DATE

Date of submission of the Research Report

ISSUE DATE OF CERTIFICATE

CHAIRPERSON

Dr. Batseba Mofolo-Mbokane

cc: Prof Judah Makonye

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.



Signature

Date 25 / 07 / 2022

APPENDIX I: SIGNED CONSENT FORMS

Consent Form (Mathematics teachers)

Title of project: Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

Name of researcher: Ashton Ndebele

I, MOKWALA T.V, agree to participate in this research project.

I agree to the following: (Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.

~~YES~~

NO

I understand that I can volunteer to take part in the study

~~YES~~

NO

I agree that respond to the Questionnaire and I will assist to administer the pre-test and post-test

~~YES~~

NO

I agree that direct quotations from my responses to the questionnaire may be used by the researcher in their research report.

~~YES~~

NO

I agree that my participation will remain anonymous (my name will not be used by the researcher in their research report).

~~YES~~

NO

I agree that other researchers may use the information I provide in my responses to the questionnaire (depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on

~~YES~~

NO

Signature

Name of Teacher

Date

Mokwala

MOKWALA T.V

02/09/2022

Signature

Name of researcher:

Date

Ashton

Ashton Ndebele

2/09/2022

Consent Form (School Principal)

Title of project: Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

Name of researcher: Ashton Ndebele

I, Maloto TC, the principal of Kensington, secondary school agree to participate in this research project.

I agree to the following: (Please circle the relevant options below)

The research study was explained to me. I understand what this study is about. YES ☒ NO ☐

I understand that I can volunteer to take part in the study on behalf of my school YES ☒ NO ☐

I agree that my school's participation will remain anonymous (my school name will not be used by the researcher in their research report. YES ☒ NO ☐

I agree that other researchers may use the information I provide in my interview/focus group/other activity (depending on their own ethics clearance being obtained) but my school name and any personal information will not be used or passed on YES ☒ NO ☐

Signature [Signature]
Name of Principal Maloto TC
Date 12/09/2022

Signature [Signature]
Name of researcher: Ashton Ndebele
Date 12/09/2022



Consent Form (Parents)

Title of project: Effects of professional noticing of learner errors and misconceptions on grade 11 learner trigonometry performance.

Name of researcher: Ashton Ndebele

I, Rachel Mandag, the parent/guardian of Shamillah,
agree to allow my child to participate in this research project.

I agree to the following: (Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.

☒ YES

☐ NO

I understand that I my child can volunteer to take part in the study

☒ YES

☐ NO

I agree that the results of the test may be used by the researcher in their research report

☒ YES

☐ NO

I agree that my child's participation will remain anonymous (my child's name will not be used by the researcher in their research report).

☒ YES

☐ NO

Signature

Name of Parent

Date

Signature

Name of researcher: Ashton Ndebele

Date