

https://dspace.org/

Intervening to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems

Kanyane, Mphokane Hellen

2016

http://hdl.handle.net/10539/21744

Kanyane, Mphokane Hellen (2016) Intervening to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems, University of Witwatersrand, Johannesburg, <http://wiredspace.wits.ac.za/handle/10539/21744>

Downloaded from a DSpace repository. https://dspace.org/

Intervening to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems

Name: Hellen Kanyane

Student number: 445397

Supervisors:

Mr. Corin Mathews & Professor Hamsa Venkatakrishnan

A research report submitted to the Faculty of Science, University of the Witwatersrand, in partial fulfilment of the requirements for the degree of Master of Science.

Johannesburg, October 2016

PLAGIARISM DECLARATION

I declare that this research is my own work, submitted for the Degree of Master of Science to the University of the Witwatersrand, Johannesburg, and no part of it has been copied from another source (unless indicated as a quote). All phrases, sentences and paragraphs taken directly from other work have been cited and the reference recorded in full in the reference list.



_____

Hellen Kanyane

Protocol Number: 2015ECE043M

Acknowledgements and dedication

I would like to express my sincere gratitude to my supervisors, Mr. C. Mathews and Prof. Venkatakrishnan for the invaluable guidance and support throughout the study. They gave their unwavering support and thorough guidance that enabled me to become better at research and to complete this study.

I would also like to thank all the learners who participated in this research study.

I am also grateful for the support and encouragement from my beloved husband, Mpapa and our children, Kgato, Mothiba and Mashadi. Le ka moso diTau!

Abstract

This research study makes an attempt at intervening in the Grade6learners' use of models and strategies in solving addition and subtraction word problems based on Realistic Mathematics Education (RME) theory. RME theory advocates for the provision of understandable contexts that learners can relate with to support them in developing models and strategies, with specific reference to the empty number line model in assisting learners to develop an understanding of the structure of number and to work flexibly in solving addition and subtraction word problems.

It is in understanding the models and strategies learners are using that we can begin to understand how the learners need to be supported in order to operate at the appropriate mathematics levels for their grade. Participants in this research study, forty boys and girls doing grade six, all with a weaker mathematical background, wrote the same tests in the form of pre test, post test and the delayed post test. After writing the pre test, the learners attended a series of six intervention lessons before writing the post tests. The intervention lessons encouraged learner engagement with word problems and the development of models as representations of problem situations and strategies which represent learner's manipulation of models in an RME-advocated approach.

Learner responses were analysed aiming at the identification of models and strategies they employed, as well as the correctness and success in solving the problems. The analysis found out that mainly there have been some improvements in the repeat sittings from predominantly using the column model with a lot of incorrect answers to using the empty number line with more correct answers. I would therefore encourage the maximum participation of teamwork amongst teachers for identifying and using efficient models and strategies in order to promote performance levels in mathematics through developing an understanding of the structure of number and working flexibly in solving addition and subtraction word problems.

Contents

CHAPTER 1: INTRODUCTION 1

1.1 Study objectives 1

1.2 Problem statement 1

1.3 Research questions 3

1.4 Classification of word problems 4

1.5 The role of models 5

1.6 The empty number line model 5

1.7 Addition and subtraction strategies 7

1.8 Theoretical Framework 8

1.9 Structure of the research report 9

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK 11

2.1 Introduction 11

2.2 Problems associated with addition and subtraction 11

2.3 Properties related to addition and subtraction 12

2.4 What research says could improve 13

2.5 Classification of additive relation word problems 14

2.6 The role of models 15

2.7 Addition and subtraction strategies 16

2.8 Stages of number learning 20

2.8.1 The emergent number concept 20

2.8.2 Counting and calculating 21

2.8.3 Calculating 22

2.8.4 Advanced calculating 22

2.9 Models of number 23

2.9.1 Line models 24

2.9.2 Group models 26

2.9.3 Combination models 28

2.10 Theoretical framework 28

2.10.1 The nature of Realistic Mathematics Education (RME) 29

2.11 Other solutions 32

CHAPTER 3: RESEARCH METHODOLOGY 33
3.1 Introduction 33
3.2 Research design 33
3.3 Research instrument (test) design 34
3.4 The intervention session 35
3.5 Sampling 40
3.6 Data analysis, reliability, validity, confidentiality and anonymity 42
3.7 Data sources 41
3.8 Analysis 44
3.10 Reliability, validity and limitations 45
3.11 Research report 46

CHAPTER 4: ANALYSIS OF RESULTS 47
4.1 Introduction and rationale 47
4.2 Pre-test results 48
4.2.1 Multiple choice questions results of the pre test 48
4.2.2 Pre test "working out" questions (8 to 14) results 50
4.3 The intervention session 55
4.3.1 Lesson 1 58
4.3.2 Lesson 2 61
4.3.3 Lesson 3 65
4.3.4 Lesson 4 67
4.3.5 Lesson 5 67
4.3.6 Lesson 6 68
4.4 The post test results 69
4.4.1 Post-test multiple choice questions (1 to 7) results 69
4.4.2 Post test "working out" questions (8 to 14) results 70
4.5 The delayed post test 76

CHAPTER 5: THE DISCUSSION 80
5.1 Introduction 80
5.2 Key findings 81

5.3 Conclusion 88
5.4 Limitations 89
5.5 Recommendations for future research 90

Reference list 91

Appendix 1: The test 96
Appendix 2: Intervention lessons questions 99
Appendix 3: Information sheet for learners 102
Appendix 4: Information sheet for parents 104
Appendix 5: Information sheet for SGB 106
Appendix 6: Principal information sheet 108
Appendix 7: GDE approval letter 110

List of Tables:

Table1.1: Exploratory assessment results 2
Table 1.2: Classification of word problems (Carpenter, et al, 1999) 4
Table 1.3: Counting type and solution type methods 6
Table 1.4: Mental addition and subtraction strategies up to 100 7
Table 2.1: Classification of Word Problems 14
Table 2.2: Efficient addition and subtraction strategies 19
Table 3.1: Category of questions in the tests 35
Table 3.2: Schedule of intervention lessons 38
Table 3.3: Intervention lesson structure 39
Table 4.1: Pre-test multiple choice questions (1 to 7) results 50
Table 4.2: Pre test *working out* questions (8 to 14) 52
Table 4.3: Intervention lessons 68
Table 4.4: Post-test multiple choice questions (1 to 7) results 77
Table 4.5: Post test *working out* questions (8 to 14) 79
Table 4.6: Delayed post-test multiple choice questions (1 to 7) results 84
Table 4.7: Delayed post test models and strategies for questions 8 to 14 85

Table 5.5: Comparison of the tests multiple choice questions results 88

Table 5.6: Comparison of tests *working out* questions results 90

List of Figures:

Figure 1: Horizontal and VerticalMathematization 32

Figure 2: Multiple choice questions results 88

Figure 3: Working out questions results 90

CHAPTER 1: INTRODUCTION

1.1 Study objectives

This research study focuses on the outcomes of an intervention that attempted at addressing the difficulties learners experience in solving addition and subtraction word problems within my grade 6 class, in a government township primary school in Gauteng. Research in primary schools has indicated that one reason for the difficulty with additive word problems relates to the development of appropriate and efficient models and strategies (van den Heuvel-Panhuizen, 2008). The intervention approach was based on key tenets of Realistic Mathematics Education (RME) theory, which argues that the providing contexts that learners can relate with (Treffers, 1987) and supporting learners to develop models and strategies can assist learners to develop an understanding of number relations and to work flexibly in solving addition and subtraction word problems.

1.2 Problem statement

In an exploratory assessment test conducted with 45 grade 6 learners in my school consisting of certain of the categories of additive word problems devised by Carpenter, Fennema, Franke, Levi & Empson (1999) with the number range from tens (two digit numbers) to thousands (four digit numbers) I found that the key models learners initially employed were column (vertical) algorithms, horizontal number sentence, grouping with tally marks, or no models, with all of these commonly associated in my experience and in the literature with incorrect calculation strategies and incorrect answers.

Following RME definitions, described later in chapter 2, I investigated the 'models of' problem situations indicated in learners' responses and linked these to key categories that are described in the literature base on models for solving additive relation problems. The literature includes categorizations based on group models in the form of unit tally counts (Schollar, 2008), which are described as common in broader South African (SA) literature, line models - which are number-line based models, and combined models, which incorporate elements of

line and group models (van den Heuvel-Panhuizen, 2008), with traditional column algorithms or double column addition (Kamii & Joseph, 1988;Thompson, 1995) frequently described as problematic in the literature when used in routine ways (with no understanding), as are horizontal number sentence models, if introduced before learners have a strong understanding of place value. I looked also at whether learners' use of their selected models for calculation (i.e. strategies used) produced the correct answer. Results of this initial exploratory assessment, carried out as part of my normal teaching in early 2015, are summarized in Table 1.1. Where children used more than one model in their response, all models were coded.

Table 1.1: Exploratory assessment results

	Join Categories			Separate categories		
	Question 1 Result unknown	Question 2 Start unknown	Question 3 Change unknown	Question 4 Change unknown	Question 5 Start unknown	Question 6 Result unknown
Group model (tallies, circles)	**11**	**12**	**11**	**12**	**6**	**8**
Correct model	8	8	6	9	5	8
Strategy leading to correct answer	3	1	0	5	1	4
Horizontal number sentence model	**4**	**1**	**3**	**0**	**0**	**0**
Correct model	1	1	3	0	0	0
Strategy leading to correct answer	1	0	2	0	0	0
Column (vertical) algorithm model	**44**	**42**	**39**	**34**	**29**	**30**
Correct model	23	12	14	30	4	25
Strategy leading to correct answer	16	10	10	14	4	13
No model	**0**	**3**	**4**	**8**	**15**	**15**
Strategy leading to correct answer	0	0	0	0	0	0
No answer	0	2	4	8	15	15
Proportion with correct answers in overall group	20	11	12	19	5	17

Table 1.1 above points to serious challenges in that while the column (vertical) algorithm model form was most commonly selected, it was only appropriately formulated in many cases on Q4, with further reductions in terms of correct calculation strategies and correct answers. The group model, with tallies and circles was also commonly used in all questions. While models here were often accurate, predictably due to its lack of efficiency, there were often calculation errors. The horizontal number sentence model was the least used, only applied in questions 1, 2 and 3, with no models indicated mainly for questions 4, 5 and 6, all of which led to incorrect calculation strategies and incorrect answers. Overall, less than half the learners answered any

question correctly, yet the Curriculum and Assessment Policy Statement (CAPS) at grade 6 level indicates that learners are expected to flexibly apply a variety of addition and subtraction models and strategies in providing correct answers (Doe, 2011b, p.213).The majority of learners struggled with all six join and separate word problems – explained later (Carpenter et al., 1999) in all three categories: result, change and start unknown as indicated in table 1 above. Gravemeijer & Stephan (2002) suggest that learners be supported to develop models in making sense of situations in order to create better problem solving.

In this study, an intervention lesson sequence encouraging engagement with word problems and the development of models (as representations of problem situations) and strategies (learner's manipulation of these models) will be investigated. An RME-advocated approach in which children learn mathematics by developing mathematical concepts and tools for solving problems that make sense to them (van den Heuvel-Panhuizen, 2003) will be emphasised. In this approach, children will be supported to develop models in a bottom-up approach (invented by learners themselves) as active participants within the classroom (Gravemeijer & Doorman, 1991). With pre- and post-testing around the intervention, the research questions are detailed below.

1.3 Research questions

- What models and strategies do grade six learners employ in solving addition and subtraction word problems prior to an intervention sequence?
- What models and strategies are developed and discussed by these learners in solving addition and subtraction word problems during the intervention sequence?
- What kinds of changes are seen (if any) in the models and strategies that learners employ after the intervention as compared to before the intervention?
- Can changes in extent of success (if any) in solving join and separate-type addition and subtraction word problems be associated with shifts in models and strategies?

1.4 Classification of word problems

A framework on addition and subtraction of word problems identifies four categories of join, separate, part-part-whole and compare word problems (Carpenter, Fennema, Franke, Levi, & Empson, 1999, p.7; Clements, et al., 2009, p.12). Word problems are classified into these categories because Carpenter et al (ibid) suggest that young children view and act upon these different empirical situations in different ways.

Table 1.2: Classification of word problems

Problem type			
Join	(Result Unknown) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether	(Change Unknown) Connie has 5 marbles. How many more marbles does she need to have 13 altogether?	(Start Unknown) Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	(Result Unknown) Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?	(Change Unknown) Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	(Start Unknown) Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with
Part-Part-Whole	(Whole Unknown) Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?	(Part Unknown) Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?	
Compare	(Difference Unknown) Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	Compare Quality Unknown) Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have?	(Referent Unknown) Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

Word problems are designed to help learners make sense of mathematical concepts and applications, solving problems by initially using informal strategies (van den Heuvel-Panhuizen, 1994). Contexts therefore are an aid for learners to develop insight and apply mathematical tools through modelling (van den Heuvel-Panhuizen, 2005) based on their own individual understanding. However, often learners appear to answer with scarce regard for whether their solutions make sense from the real-world view point – using what Greer (1997) refers to as superficial solutions of word problems by not reflecting on what they have done. The focus of my study is on models and strategies within addition and subtraction of word problems.

1.5 The role of models

The use of pre-designed models is replaced in RME by the activity of modelling (Gravemeijer, Lehrer, van Oers, & Verschaffel, 2002, p.145), that is, the role of models is different from the "didactic models that embody the mathematics to be taught" in that they are not derived from the intended curriculum to be taught rather they are grounded in the contextual problems (ibid) that are to be solved by learners. In RME models are related to modeling the contextual problem so as to reinvent the more formal mathematics.

The primary role of models is to bridge the gap between the phenomenological appearances of mathematics in reality on the one hand and the formal mathematics on the other (Doorman, 2002), that is like bringing two worlds together. The models "of" informal context bound work "for" formal standardised manner of operations through learner contributions (van den Heuvel-Panhuizen, 1994, p.4), as they construct own knowledge and understanding. Learners develop formal mathematics by mathematizing their informal mathematics as "formal mathematics is not something out there, but something that grows out of learner activities" (Gravemeijer, Lehrer, van Oers, & Verschaffel, 2002, p.148), which is the essence of learning mathematics. Learners should experience formal mathematics no differently than informal mathematics "mathematics starting within common sense and staying within common sense" (ibid) so as to become useful for the learner.

Formal mathematics reasoning builds on arguments that are located in the newly formed mathematical reality, and the distinction between informal and formal mathematics is relative. In addition, van den Heuvel-Panhuizen (2008, p.23) indicates that models support the process of mathematical growth and reality is attenuated (weakened, reduced in force, intensity) into a model, a model of.

1.6 The empty number line model

The empty number line is proposed in RME as a particularly appropriate means to support flexible mental computation strategies for addition and subtraction up to 100 by marking the

number involved and drawing jumps that correspond with the partial calculations (Gravemeijer, 1999). The empty number line sequence fits well with counting-type (involving the use of counting strategies) solution methods rather than collection-type (putting together numbers for group addition or group subtraction) solution methods. The example below illustrates the differences between counting type solution methods and collection type solution methods: 64 – 29 =, and explains RME's preference for the counting-based approach:

Table 1.3: Counting type and solution type methods

The empty number line (counting-type solution methods)	The base ten block (collection-type solution methods)
64 – 4 = 60 – 20 = 40 – 5 = 35	60 + 4 and 20 + 9; 60 – 20 = 40 and to subtract 9 from 4, the learner needs to view a ten as ten units and 'borrow' which is cumbersome

With the counting-type solution methods, learners increase or decrease directly from one number (usually the first) as in the example above as in counting. The empty number line fits and supports the counting type solution methods by scaffolding both partial calculations and partial results (Gravemeijer 1999). The collection-type solution methods encourage breaking the number into tens and ones before adding or subtraction (which works well for addition but has limitations with subtraction).

Advantages of using the empty number line are that it can be used with flexibility to model the contextual problem. Learners could adapt the empty number line to their thinking in terms of working in more, or fewer, jumps (Gravemeijer 1999), unlike the base ten blocks that are pre-structure and meant to be broken down into tens and ones. This empty number line model can be adapted to fit learner thinking. It can be used to depict more sophisticated strategies such as compensation. This gives indications of the shift from "model of" to "model for". The empty number line worked as a catalyst for the way the RME community thought about models. It helped the researchers realize the shift in learner thinking from thinking about the modeled context situation "model of" to a focus on mathematical relations "model for".

1.7 Addition and subtraction strategies

Clements & Sarama (2009) explains that addition and subtraction can best be understood through a basis in counting, as opposed to primitive, mechanical and meaningless calculating (Anghileri, 2001) activity. RME advocates for the use of realistic contexts promoting bottom-up processes starting with informal strategies(van den Heuvel-Panhuizen, 1994), moving on to more sophisticated counting strategies (Clements & Sarama, 2009). Table 1.4 below illustrates some addition and subtraction mental strategies up to 100, using the examples of 38 + 26 and 64 – 26 (Foxman & Beishuizen, 2002, p.46). The strategies in column 1were derived from the work of Beishuizen, et al. (1997) and Fuson et al. (1997) as cited in Foxman & Beishuizen (2002), and the strategies in column 2 were derived from Thompson (2000) also as cited in Foxman & Beishuizen (2002), similar but with different names, with some strategies more efficient than others for particular questions, and seen below in the differing number of steps.

Table1.4: Mental addition and subtraction strategies up to 100; adapted from Foxman & Beishuizen (2002, p.46)

Beishuizen	Thompson	Examples of strategies for 38+26 and 64-26
N10	Sequencing (jump method)	38 + 20 ⏭58; + 6 ⏭ 64 64 – 20 ⏭44; - 6 ⏭ 38
N10C	Compensation	38 + 30 ⏭68; - 4 ⏭ 64 64 – 30 ⏭ 34; + 4 ⏭ 38
A10	Complementary addition	38 + 2 ⏭40; + 20 ⏭ 60; + 4 ⏭ 64 26 + 4 ⏭ 30; + 30 ⏭60; + 4 ⏭ 64 (answer: 4 + 30 + 4 ⏭38
10S	Mixed method	30 + 20 ⏭ 50; + 8 ⏭ 58; + 6 ⏭64 60 - 20 ⏭ 40; + 4 ⏭ 44; - 6 ⏭38 (invalid: 40 – 4 ⏭ 36 – 6 ⏭ 30
1010	Partitioning (Split method)	30 + 20 ⏭ 50. 8 + 6 ⏭ 14. 50 + 14 ⏭ 64 60 – 20 ⏭ 40. 4 – 6 ⁉carry 10 from 40 ⏭ 30; Then 4 – 6 becomes 14 – 6 ⏭ 8. 30 + 8 ⏭ 38 (often invalid: 4 – 6 ⏭ 2. 40 + 2 ⏭ 42)

According to Foxman & Beishuizen (2002) N10 and 1010 are the main mental calculation strategies for use with larger numbers. The N10 or sequencing strategy (the first addend is kept as is, the other is split and its parts added separately, one at a time) allows learners to initially jump either forwards or backwards mainly in tens. It is regarded as connected well to the counting movements on the empty number line with the first number remaining whole and forward or backward jumps for the addend to subtrahend. The other strategies, N10C or

compensation, A10 (complementary addition) and 10S (mixed methods) also support efficient counting. Compensation involves adding more than the required amount and subtracting the extra amount at the end as a way to compensate for it, effective for numbers closer to tens. A10 or complementary addition involves moving quantities from the one addend to the other in order to create numbers easier to work with whereas 10S or mixed methods combines the sequencing, complementary and partitioning methods in that the numbers are first split into tens and units, but the tens are added together first, then the units are added one at a time All the strategies listed above are well connected to the counting movements on the empty number line with the exception of the 1010 strategy. The 1010 or partitioning could be regarded as a less efficient strategy as the numbers are split separately with no sequence or ordering in a way that supports the column model.

The development of addition and subtraction strategies in the learning – teaching trajectory is called "counting – and – calculating" (van den Heuvel-Panhuizen, 2001, p.21), characterized by progression from untaught, self-invented informal strategies (Foxman & Beishuizen, 2002) to formal, symbolic, abstract ways of working with number (Ensor, Hoadley, Jacklin, Kühne, Schmitt, Lombard & van den Heuvel-Panhuizen, 2009).

1.8 Theoretical Framework

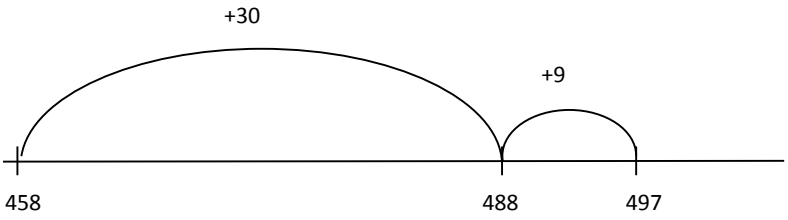
RME is a constructivist based theory that aims to promote learner knowledge and understanding through the use of realistic contexts and didactical models (van den Heuvel-Panhuizen, 2003). Realistic contexts refers to what learners can imagine and is real in the learner's mind (Treffers, 1987) and models are the representations of the problem situation (Gravemeijer & Stephan, 2002) within the given context. The goal of RME is for learners to develop a framework of number relations that offers a basis for flexible mental computations. RME is a domain-specific instructional theory advocating learner support towards the development of informal situated solution strategies into more formal mathematical insights and procedures (Gravemeijer & Stephan, 2002).

RME points out that learners be given an opportunity to re-invent mathematics through "horizontal mathematization," [going from the world of life (the essence of context) into the world of symbols (formal mathematics)] and "vertical mathematization," as moving within the world of symbols (Freudenthal, 1991) as cited in (van den Heuvel-Panhuizen, 1994, p. 12).For example in the exploratory assessment, the problem: "In a school there are 458 boys. During the year 39 boys were admitted. Find the number of boys in the school now" was given. Horizontal mathematization involves the creation of an appropriate model; in my class a great number of learners chose the column model and modelled the situation appropriately as:

458

\+ 39.

Vertical mathematization involves the using the column model to arrive at the answer; in my class many learners used the carrying strategy with errors and hence ended up with incorrect answers. One avenue in my intervention lessons would be to use the empty number line as a model with some of the varying jump strategies described earlier, one of these being the N10 involving adding tens and ones as indicated below to alleviate errors:



1.9 Structure of the research report

Chapter 1, the current chapter on introduction to the study, discusses the study objectives, the problem statement, research questions, classification of word problems, the role of models, the empty number line model, addition and subtraction strategies, the theoretical framework and overall structure of the research report.

Chapter 2, literature review, discusses the literature reviewed on Problems associated with addition and subtraction, properties related to addition and subtraction, what research says

could improve, classification of additive relation word problems, the role of models, addition and subtraction strategies, stages of number learning, models of number, theoretical framework, other solutions. Solving problems involves the classification of word problems, the role of models, in particular the empty number line model, and addition and subtraction strategies. More broadly on models, I discuss line models, group models and combination models. Further, within addition and subtraction strategies I discuss the emergent number concept, counting and calculating, advanced calculating. The theoretical framework discusses the introduction and the nature of Realistic Mathematics Education (RME).

Chapter 3, research methodology, discusses the methods applied in this research by discussing methodology, sampling, data collection and intervention, data analysis, reliability, validity, confidentiality and anonymity, research instrument (test) design, data sources, the intervention sessions and intervention lesson structure, analysis, ethical considerations, reliability, validity and limitations within the research report.

Chapter 4, analysis of results, discusses the analysis of the data from the pre test, post test and delayed post test as well as events of the intervention lessons

Chapter 5 discusses key findings from the research, conclusions and implications thereof, limitations and recommendations for further research.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

The purpose of this chapter is to review some of the existing literature on the use of models and strategies in solving addition and subtraction word problems. Reviewing literature affords me an understanding of how extensive the problem is so as to position my contribution relevantly. It also serves to illuminate my understanding of models and strategies, upon which my research study is based. According to the South African (SA) Curriculum and Assessment Policy Statement (CAPS), "Number, Operations and Relationships" is the major content area (DoE, 2009, p.10), and especially so in the Foundation Phase (FP) and Intermediate Phase (IP). This content area emphasizes the importance of number knowledge and the strategies required, with stipulation that "... there is expectation for learners to move from counting reliably to calculating fluently in all four operations ..." in its specific content focus (ibid). This chapter discusses problems associated with addition and subtraction, properties related to addition and subtraction, classification of additive relation word problems, the role of models, addition and subtraction strategies, stages of number learning, models of number, theoretical framework and other solutions.

2.2 Problems associated with addition and subtraction

Traditionally, as has been the practice in many parts of the world including SA, learners were orientated to addition and subtraction through the use of the column model (column algorithms). The column model together with its associated strategies of column addition / column subtraction have been described as stifling the development of number sense and structure (understanding of the meaning of numbers and their relationship) as they frequently encourage learners to give up their own thinking and to embrace memorisation of algorithms (Kamii & Dominic, 1998) and computation procedures. The use of column strategies undermines the learner's ability to fully grasp the role of addition and subtraction (Thompson, 1995) and their understanding of place value. As such learners find it difficult to make an estimation of the actual answer prior to calculation or check how reasonable their answers are after calculating.

> "The point about the column addition/subtraction model is that children only ever need to operate on single digit numbers at a time, tens and units independently. As a result, learners do not need to have a sense of the relative size or position of numbers." (Thompson, 1994, p.325)

Column strategies are often resorted to in the face of evidence that learners exhibit poor levels of abstraction due to over-dependency on counting strategies as compared to more efficient calculation strategies and other contextual factors (Askew & Brown, 2000; Wright, et al., 2006). The fact that learners mainly rely on unit counting prevents them from calculating efficiently (Schollar, 2008) in particular, using the base-ten number system and progression towards the place value of higher numbers. At the same time though, as noted earlier, column strategies work against the development of more efficient counting strategies or understanding of place value. In addition, there is wide evidence that learners struggle with word problems, and hence they prefer 'straight' computations (Murray, 2012, p.55) where the calculation operation is given and leads to attempts at memorisation of the procedure rather than understanding of the number relations in the situation and the additional need to select the relevant operation.

In the exploratory addition and subtraction assessment test conducted with 45 grade 6 learners in my school, learners were found using mainly the column model, at times grouping with tally marks, and very few of them used horizontal number sentence. All of these were commonly found to be associated with incorrect calculation strategies and incorrect answers. Further, research indicates that learners are subjected to memorization of concepts, with emphasis on procedures and computations rather than meaningful understanding (Kamii, Kirkland & Lewis, 2001; Kilpatrick, Swafford & Findell, 2001).

2.3 Properties related to addition and subtraction

Making estimations and checking for reasonableness in solutions is an important step towards successfully solving problems and so is understanding of number properties such as identity, commutative and associative properties (DoE, 2011b). Properties related to addition and subtraction serves, according to Carpenter, Moser and Bebout (1998) to bring forth an understanding to learners of the different number sentences could provide legitimate

representation of word problems. Knowledge of number properties is essential for the development of number relations. These are the identity, commutative and associative properties. The identity property explains that adding zero or subtracting zero from any number does not in any way change the value of the number; for example 3 + 0 = 3 and 8 – 0 = 8. The commutative property explains that a series of numbers could be added in any order without affecting their sum; for example, 4 + 7 = 7 + 4. The commutative property is only applicable to addition. The associative property explains that the order of addition has no influence on the sum; for example 5 + 2 + 6 = 2 + 6 + 5. The associative property is also not suitable for subtraction. For the sake of this research, it is important to note implicitly how learners work with the different number properties. Formally, as referred to in most textbooks, the components in addition sums are identified as addends (the numbers added together) and sum (the result) whereas the components in subtraction are identified as minuend (the number from which another number is to be subtracted), subtrahend (the number that is to be subtracted) and difference (result).

2.4 What research says could improve

The early arithmetic strategies are most basic, and lay a foundation for the acquisition of all other strategies. A deliberate attempt of presenting learners with situations that allow them to acquire a series of interrelated and comprehensive number strategies (Wright, et al, 2006) is necessary, hence the focus on word problems for this study. Further word problems in South Africa and some other countries are usually confined to the back of the textbooks, as application problems for learnt procedures, yet they provide powerful entry points to mathematical reasoning (Koedinger, Alibali, & Nathan, 2004). A substantial body of research argues that beginning with rich contexts rather than with abstractions and definitions (van den Heuvel-Panhuizen, 2005) will enable learners to apply mathematics in realistic ways.

Often learners experience difficulties in learning mathematics due to the existing gaps between everyday life and formal mathematics (Gravemeijer, 2009). The use of efficient models and contexts fosters the development of formal mathematics as a natural extension of the learners' everyday experience (Gravemeijer & Stephan, 2002) through building learner understanding. That means that effective learning of mathematics needs to be based on what learners already

know and understand and therefore further exploration into the use of efficient models and strategies is necessary.

2.5 Classification of additive relation word problems

A framework on the classification of addition and subtraction situations identifies four categories, namely, join, separate, part-part-whole and compare word problems (Carpenter et al., 1999, p.10; Clements & Sarama, 2009, p.33). Join and separate problems involve change actions over time, whereas part-part-whole and compare situations are usually viewed as static situations. The change action has been found to make join and separate easier to work with as the action involved is directly implied (Carpenter et al., 1999). Given that the results from the exploratory assessment test indicated that half of the class could not cope with join and separate word problems, my study is restricted to these categories.

Table 2.1: Classification of Word Problems according to Carpenter et al. (1999, p.12)

Problem type	Result Unknown	Change Unknown	Start Unknown
Join	(Result Unknown) Connie had 35 marbles. Juan gave her 18 more marbles. How many marbles does Connie have altogether 35 + 18 = ☐	(Change Unknown) Connie has 5 marbles. How many more marbles does she need to have 53 altogether? 35 + ☐ = 53	(Start Unknown) Connie had some marbles. Juan gave her 35 more marbles. Now she has 53 marbles. How many marbles did Connie have to start with? ☐ + 35 = 53
Separate	(Result Unknown) Connie had 53 marbles. She gave 35 to Juan. How many marbles does Connie have left? 53 – 35 = ☐	(Change Unknown) Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan? 53 – ☐ = 35	(Start Unknown) Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with? ☐ - 35 = 18

Table 2.1 above exemplifies join and separate problems and their variations. Carpenter et al. (1999) and Clements & Sarama (2009) note that variations in the position of the missing value

introduce a range of difficulty within both join and separate categories, with change unknown and start unknown problems being harder than result unknown problems and becoming more complex with higher number ranges. In a South African context Venkat (2013) argues that at grade 2 level in one of the lessons the majority of learners were unable to provide correct answers to missing addend tasks. Takane (2013) further indicates that learners familiar in working with number sentences, for example in the form of (a + b = ? and a – b = ?), do not struggle with result unknown problems as they are based on simple joining and separating actions, but would struggle to represent the join change unknown problems usually represented for example in the form of (a + ? = b or a - ? = b) which are generally harder (Clements & Sarama, 2009).

Progression in strategies according to Wright et al. (2006)indicate that learners operating at stage three and four of the early arithmetical strategies (SEAL)are already using the strategies identified in structuring of number. They work with combining (addition of numbers), for example "3 + 1 = 4" and partitioning (breaking up numbers), for example, "3 = 2 + 1" and "5 = 2 + 2 + 1". McIntosh, Reys & Reys (1992, p.7) explains the combining and partitioning strategies as the "recomposing and decomposing" of number properties respectively.

2.6 The role of models

Traditionally mathematics was taught through the use of manipulative and concrete models, which RME opposes in that models are not derived from the intended mathematics; instead, models are grounded in the way that contextual problems are solved by students as

> "... external representations do not come with intrinsic meaning but rather what knowledge and understanding exists to make a meaningful interpretation."(Gravemeijer, 2002, p.13)

The successful use of the model in relaying mathematical understanding is derived from the meaningful interaction of the learner with the model. Didactical models are described to fulfil the bridging function between the informal (ordinary ways of life) and the formal (more scientific ways) level due to their ability to shift from being a model of that is tied to a contextual situation, to a model for problem solution using more formal mathematical strategies (Gravemeijer, 2002). These models (didactical) are the representations of the problem situation (Gravemeijer & Stephan, 2002) within the given context.

Transmission of knowledge with the help of symbols that function as carriers of meaning is being replaced by the image of students constructing their own ways of symbolizing (Gravemeijer, 2002) as part of their mathematical activity. The use of didactical models support learners in the creation and development of own knowledge productions and constructions; initially, informal and later more formal and scientific. The use of context enables learners in creating own knowledge that learners have a better understanding and interpretation of mathematics and the world around them.

A notable difference between RME and the traditional approach to mathematics education is the rejection of the mechanistic, procedure-focused way of teaching such as the use of the column model in which the learners are the receivers of ready-made mathematics. In RME learners are considered as active participants in the teaching-learning process, in which they develop mathematical tools and insights, an approach deemed essential for my intervention lessons. The interactive character of this teaching process involves learners actively involved in creating their own knowledge rather than memorising readymade facts (Sembiring, Hadi, Dolk, 2008).

The starting point is the contextual problem that has to be solved. The problem is solved with the help of a model, hence, modelling (as an organising activity). The idea is that subsequent (successive) acting with these models will help students (re)invent more formal mathematics with the use of mathematical symbols and efficient strategies (explained later) that is aimed for. RME defines formal mathematics as something that grows (develops) out of the student's activity (not something that is out there to be found) so that "students should experience formal mathematics as no different from informal mathematics" (Sembiring, Hadi, Dolk, 2008, p.930).

2.7 Addition and subtraction strategies

Addition and subtraction are the basic operations according to CAPS. The actions associated with these operations include putting numbers together or taking a number away from the larger, and rely on efficient counting and the application of number knowledge (van den

Heuvel-Panhuizen, Kühne & Lombard, 2012). When children begin schooling the literature suggests that they already have their own understanding of working with the addition and subtraction of word problems in ways that could be regarded as informal or spontaneous (Vygotsky, 1978; Gravemeijer, 1994) knowledge. This knowledge provides a foundation towards formal or scientific knowledge (ibid), based on what the child understands.

Traditionally, aligned with the column model or double-column addition (Kamii & Joseph, 1988), learners were compelled to use algorithms in the form of column addition and column subtraction as referred to in most mathematics textbooks. Column addition involves aligning the numbers vertically, one on top of the other for addition of tens and units separately, working from the right to the left, one at a time, often involving carrying over numbers from the units to the tens, creating a strategy of column addition with carrying. Column addition with carrying and tallies combines column addition, with carrying over of number and the use of objects in the form of sticks or circles. In the same way column subtraction involves borrowing from the tens creating a strategy of column subtraction with borrowing. Column subtraction with borrowing and tallies, involves the subtraction, borrowing of number and the use of object like symbols in the form of circles or sticks. Column multiplication / division arranging the numbers in column form for the multiplication / division operation, other strategies such as stringing, splitting and varying are explained in paragraph 2.10. Some learners in this study used a combination of the column addition or column subtraction strategies with explicit partitioning or splitting strategies (explained later), which I classified as column addition with place value.

With more counting children become fluent and experienced as they derive more number facts, such as 6 + 6 = 12 to derive 7 + 5 or 3 + 5 = 8 means 5 = 8 – 3; 3 = 8 – 5 reversing the operation. The derivation of number facts is a reflection of learner understanding on how the numbers are related (number relations). Number facts are very useful particularly when working with large numbers as bigger numbers require the use of more complex strategies (Thompson, 2000) therefore literature does not suggest the need for children to be taught specific number facts or procedures as their understanding allows them to construct their own strategies of basic number concepts. Further learners become flexible in their working to come up with alternative strategies to justify their computational procedures. What children need is motivation and encouragement to construct their own strategies to promote them learning number facts and

higher recall levels of number facts. More experience with number facts minimises the need for traditional column modelling and actions, promoting number relations eliminating the use of trial and error methods by modelling the action or relationship based on the structure of the problem.

Table 2.2 below highlights some of the commonly used strategies for carrying out the addition and subtraction operations. The addition strategies are partitioning, cumulative, adding-on (sequencing), compensation and moving (complementary addition) whereas for subtraction the strategies are partial subtraction, compensation, constant difference and complementary addition as adapted from Thompson (2000, p.571). These strategies were selected due to their efficiency and flexibility (even though not always aligned with RME) in promoting number relations and simplifying the action. In addition all these strategies are compatible with the empty number line.

Addition strategies include partitioning, cumulative, compensation and moving. With the partitioning strategy, the numbers to be added are partitioned or separated into multiples of hundreds, tens and ones or in any other way convenient for the learner so that the number could be added easily. The partitions will be regrouped later to find their sum. This strategy is also referred to as split, decomposition or 1010, even though not so efficient, it is most favoured by the majority of learners as it could also be used for subtraction. The cumulative strategy is also called the adding-on or sequencing strategy or N10, whereby the first addend is kept as is, the other is split and its parts added separately, one at a time. The compensation strategy of addition involves adding more than the required amount and subtracting the extra amount at the end as a way to compensate for it. This strategy is effective for numbers closer to tens. The moving or complementary addition or A10 involves moving quantities from the one addend to the other in order to create numbers easier to work with.

Table 2.2 Efficient addition and subtraction strategies adapted from Thompson (2000, p.571)

Addition strategies	Subtraction strategies
Partitioning (Partial sums) 63 + 56 = (60 + 3) and (50 + 6) 110 + 9 119	Partial subtraction 384 – 168 (168 becomes split into 100 + 60 + 8) 384 - 100 = 284, then 284 – 60 = 224 and finally 224 – 8 = 216 (alternatively 8 could be subtracted in two parts, 4 and 4
Cumulative (adding-on or sequencing) 125 + 38 30 + 8 5 3 125 + 30 = 155 + 5 = 160 + 3 = 163	Constant difference 130 + 200 = 230 + 300 (Add 100 to both numbers) Any number can be added or subtracted from both
Compensation 278 + 390 (390 is rounded off to 400 by adding 10) = 278 + 400 = 678 – 10 = 668 (remove the extra 10)	Compensation 565 - 287 (287 is rounded off to 300 by adding 13) 565 – 300 = 265 265 + 13 = 278 (Add back the extra 13)
Moving 396 + 267 (7 in 267 is split into 4 & 3 so that 4 can be added to 396 to make 400 and 3 is added back to 267) = 396 + 4 = 400 +263 = 663	Complementary addition 73 – 68 = Add 2 to 68 = 70, to come to 73 you need to add another 3. Therefore 73 – 68 = 5

Subtraction strategies include partial subtraction, constant difference, compensation and complementary addition. Partial subtraction involves decomposing the number to be subtracted and subtracting one part at a time. Compensation for subtraction involves subtracting more than the required amount and then adding back the extra amount at the end as a way to compensate for it. This strategy is effective for numbers closer to tens. The constant difference

strategy is important where the difference between two numbers remain the same after adding or subtracting the same quantities to both numbers. This strategy is effective for working with very big numbers. Complementary addition is a very odd strategy as it involves addition in the subtraction process for numbers that are close together. Some of the strategies such as the adding-on, partitioning and compensation could be used interchangeably for both addition and subtraction

2.8 Stages of number learning

The stages of number learning could be divided into four stages of emergent number concept, counting and calculating, calculating and advanced calculating (van den Heuvel-Panhuizen, Kühne & Lombard, 2012). Venkat (2013) emphasizes the necessity for the stages of number learning as learners need to move on from concrete counting progressing to more complex mathematics with the alleviation of errors (theoretical model of progressive in early number learning).

2.8.1 The emergent number concept

The essence in teaching mathematics lies in the identification and support of learning as children construct their own knowledge (Cobb, 1996) throughout the different learning stages. According to Wright, et al. (2006) learners at the emergent stage cannot count but can work with physical objects, what Carpenter; et al. (1999) refers to as modelling the action and relations. Initially learners would model (construct and represent) with physical objects the action or relation that is portrayed in the problem. The learner acts using physical objects to directly show their understanding of what is being portrayed and thereafter count the objects one by one (count-by-ones) as indicated by Wright (2006).

2.8.2 Counting and calculating

At a particular point or as they grow older, children graduate away from relying on physical representations (direct modelling strategies) to more abstract strategies of counting and calculating (Askew & Brown, 2000). Initially the learner might use a combination of modelling and counting (Carpenter, et al., 1999) that is, counting strategies that are consistent with modelling strategies, beginning with small numbers as they gradually shift from modelling to counting. This move is an indication of learner's work by abstraction and recall of number facts.

Further, Wright (2013, p.29) outlines advanced count-by-ones strategies as count-up-from (to solve additive tasks), counting-up-to (to solve missing addend tasks), counting-down-from (to solve take away, subtraction tasks) and count-down-to (to solve missing subtrahend tasks).Young children's ability to count is governed by certain principles (though some principles might not be applicable to certain children), they are one-one principle, staple-order principle, cardinal principle, abstraction principle and order-irrelevance principle (Gelman & Gallistel, 1978; Ensor, Hoadley, Jacklin, Kühne, Schmitt, Lombard & van den Heuvel-Panhuizen, 2009).

The one-one principle involves ticking off items in a count with distinct ticks (tags, numerons, numerlogs). Two processes are involved, partitioning and tagging. Partitioning involves the maintenance of two categories of items, those that are to be counted and those that have already been counted. Partitioning and tagging starts and ends together. The tendency of children to point when counting, provides an assessment information, that they recognise that counting co-ordinates tagging and partitioning. The staple-order principle involves the tags (numerons) that the child uses and should correspond to items in an array that must be chosen or arranged in a stable (repeatable) order. The cardinal principle represents the cardinal number of a set, that is, the tag that applies to the final item in the set, represents the number of items in the set.

The three principles on how to count above are applicable to any arrangement of numbers, seen or not seen. Young children (up to age 2) mainly count visible items, but later (at age 3 to 4)

begin to abstract. This principle is permissive and not restrictive as to what items may be counted. The order-irrelevance principle explains that it does not matter how you count, the order of enumeration (the order in which the items are tagged) is not important. As long as no count word is used more than once. The learner should note that the counted item is a thing and not a one, or a two (abstraction principle), verbal tags are arbitrary, and are temporarily assigned to objects (do not adhere) and the same cardinal number results regardless of the order of enumeration. The order-irrelevance principle involves our ability to count but also our understanding of number properties.

2.8.3 Calculating

Wright, et al (2006), outlines early arithmetical and base ten strategies, forward / backward number word sequences and number word after / before and structuring number in the Learning Framework in number (LFIN) as key stages in the development of counting and further calculating in working with number. Counting is more efficient than just modelling the action as children reflect on numbers as abstract entities; they count on, forward and / or backwards but having the ability to count-down-to (counting down to find the difference between two given numerals) and to count-down-from, which means subtracting a particular number of items from the given collection and ordinarily difficult to notice (Gelman & Gallistel, 1978;Wright, et al., 2006) which are more advanced strategies of counting by ones.

2.8.4 Advanced calculating

At the advanced calculating strategies learners are past the stage of counting by ones (non-count-by-ones stage), they use strategies such as combining (addition of two numbers, first in the range one to five, for example, four and three is seven), partitioning (the complement of combining), breaking up the number, for example, seven is four and three and grouping (arrange items in a particular order, serves as introduction to early multiplication and division), compensation (a strategy useful especially in the addition and subtraction of larger numbers whereby the addend is rounded off to multiples of ten and the added extra subtracted to keep the balance) and other procedures (their own way of working based on their understanding).

Furthermore, children create their own strategies as they become more efficient and fluent with number improving towards the development of derived number facts.

Over a longer period modelling and counting strategies gradually give rise to learners' development of number facts (Carpenter, et al., 1999; Murray, 2012). The LFIN framework(Wright, et al., 2006)gives a description of the stages of development of number as the learner moves from counting (lower stage) to calculating (advanced stage), as indicated by Ensor, et al. (2009). The strategies on the LFIN framework are divided into four parts, A (Early arithmetical strategies and base-ten arithmetical strategies), B (Forward number word sequences and number word after; Backward number word sequences and number word before), C (Structuring number) and D (Early multiplication and division), but this study focuses on the strategies in part C, which are combining and partitioning, spatial patterns and subitizing, temporal sequences; finger patterns and five-based (quinary based) strategies. Stages of number learning are important for my study in that unless a learner develops more advanced calculation skills they would not be able to access the structuring of number when using particular strategies with particular models.

2.9 Models of number

Van den Heuvel-Panhuizen (2008) indicates that whole numbers can be structured according to line models (schematic form of representing number in the form of a string of beads or the number line), group models (numbers grouped into ones, tens and hundreds using tallies, diagrams or sticks) and combination models (using both line and group models for representing larger numbers, for example, using arithmetic racks or one hundred squares). Further, each model is important in the development of calculations skills over three levels, calculation by counting (perform operations through naming and ordering numbers sequentially), calculation by structuring (operating by first breaking out each number into units, tens and hundreds) and flexible, formal calculations (involves sophisticated ways of counting and structuring such as counting sequentially in tens and hundreds rather than ones). Operation at the flexible, formal calculations is at a higher level as it involves efficient and sophisticated ways of counting and structuring. Line and group models support the development of longitudinal counting and structuring towards the use of combination models (van den Heuvel-Panhuizen, 2001).

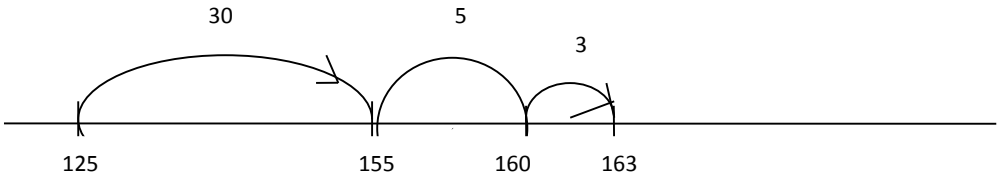
2.9.1 Line models

The empty number line fast tracks the development of informal models to more formal models and strategies in a bottom-up approach (van den Heuvel-Panhuizen, 2003) in line with learner understanding. Gravemeijer & Stephan (2002) describe the empty number line model an emergent model, an organizing activity which emerges from the learner's informal representations of the problem situation. One avenue in my intervention lessons would be to use the empty number line as a model with stringing, splitting and varying strategies (van den Heuvel-Panhuizen, 2001, p. 25), explained below, adding tens and ones to alleviate learner errors.

Counting and skip counting strategies are related to the empty number line. Counting strategies for example, count-by-ones (reading off numbers one-by-one), count-all (reading all the numbers from each individual set), counting-on by abstractly reading the first set and only counting off elements from the second set (Wright, et al., 2006) are useful in creating an understanding of the relationship that exists between numbers. Skip counting strategies overtime lead to splitting strategies (breaking the number into two or more parts). Awareness of the number sequence assists in marking off and making jumps on the empty number line, initially in ones, and later in other different sophisticated strategies such as stringing, splitting and varying (van den Heuvel-Panhuizen, 2008).

The stringing strategy keeps the first (or bigger) number when working on the ENL as is whereas the other is split and its parts added or subtracted separately to the first number, one at a time as in the following example:

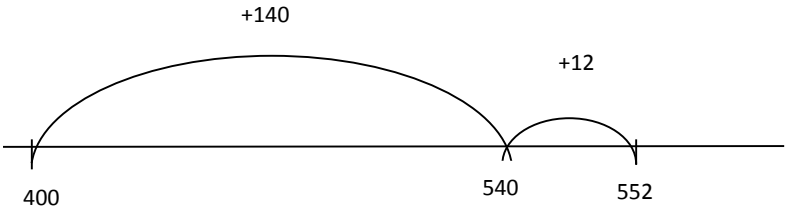
125 + 38 = 125 + 30 155 + 5 = 160 + 3 = 163



With the splitting strategy (commonly used with the column model) both addends or minuend and subtrahend become restructured by splitting or partitioning them into multiples of hundreds, tens and ones or in any other way convenient for the learner so that the number could be added or subtracted easily. The partitions will be regrouped later to find their sum or difference such as in the following example:

384 + 168 =

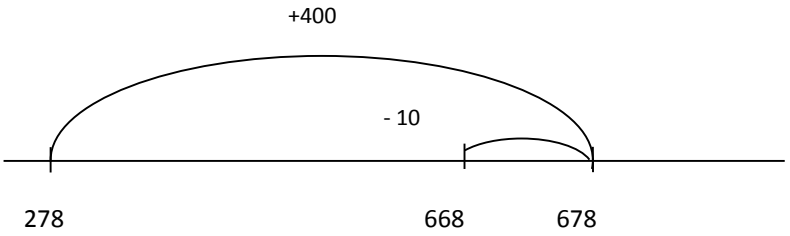
(300 + 80 + 4) + (100 + 60 + 8) gives 300 + 100 = 400; 80 + 60 = 140 and 4 + 8 = 12; therefore 400 + 140 + 12 = 552



Varying strategies are mainly used when learners have acquired good knowledge of number relationships, properties of operations (van den Heuvel-Panhuizen, 2008) and the use of number facts (Carpenter et al., 1999). The use of number facts is for example, when a learner uses 6 + 6 = 12 to derive 7 + 5. Similarly 3 + 5 = 8 means 5 = 8 – 3 and 3 = 8 – 5 by reversing the operation and extending the number relations.

An example of varying strategies is the use of the derivation of number facts for example by using compensation which is a higher strategy than counting whereby the learner operates at a higher level of abstraction and understanding of number relations and uses the splitting strategy conveniently. The compensation strategy for addition or subtraction involves adding or subtracting more than the required amount and subtracting or adding the extra amount at the end as a way to compensate for it. This strategy is effective for numbers closer to tens and

hundreds. The example 278 + 390 is treated as 278 + 400 = 678, then compensating by subtracting the extra 10 gives 668.



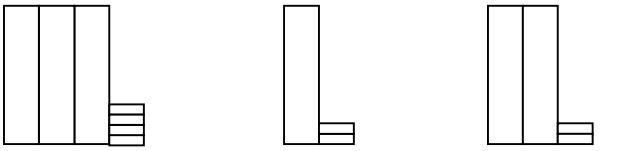
Askew (2004) proposes the use of classroom discussion of the appropriateness and efficiency of different emergent models and strategies as a way of supporting moves towards more formal and efficient symbolic representations over time. Further he points out that the significance of the empty number line model is in shifting from being "models of" to being "models for" depicting solving problems. As a "model of" learners might use models to solve specific problems, whereas with "model for" learners develop insight and a more general understanding; from working with one simple situation to more complex ones. A strategy is when learners manipulate the model in order to determine the answer. Where models and strategies are emergent and discussed widely, there is evidence that they enable learners to make estimations and to check for whether the solution is reasonable.

In conclusion, the empty number line model could be used to support learners to develop their calculation skills from calculating by counting to structuring and more flexible formal calculations. This study aims to explore the possibilities of shifting learners to using the empty number line as an alternative model to support them in making sense of word problems and arriving at correct answers. Given the prevalence of column models in the early assessment, attention to discussion of splitting strategies will also be incorporated, as these fit well with column based models.

2.9.2 Group models

Group models involve the use of a group of objects such as counters, tallies or coins in structuring numbers into ones, tens and hundreds (van den Heuvel-Panhuizen, 2008). South African evidence shows learners widely using tallies (Scholar, 2008) as a strategy along with

the vertical column model rather than as group model. Group models are associated with the stringing and splitting strategies as explained above ranging from counting individual objects, to structuring and expected to improve operation towards becoming varying strategies which are more flexible and formal. The example, 34 – 12, means 34 is grouped into 30 and 4 whereas 12 is grouped into 10 and 2; (30 -10) + (4 – 2) = 22



(34) - (12) = (22)

The column addition / subtraction, which is a formal model (due to its complexity), commonly used by learners in the exploratory assessment, associated with the traditional column algorithmic method (Thompson, 1994) could be linked to the group model as it involves operating with ones, tens or hundreds separately. Kamii & Dominic (1998) indicate that column algorithms encourage learners to give up their own thinking and to embrace memorization and procedures as learners operate with ones, tens, hundreds, etc, independently as if they were all single digits with the application of the carrying strategy. This is noticeable when, for example, to add eight tens (80) to five tens (50) the learner would say eight plus five; the answer is 13, write 3 and carry one without awareness of the quantities associated with the digits they are referring to (Tshesane, 2014).

Thompson (1994) explains that the traditional algorithms compel working from right to left as opposed to the more natural inclination work horizontally from left to right. On the other hand learners can be assisted to provide correct answers with the use of the column method in working out addition problems through the application of the horizontal splitting strategy (ibid), for example, with this strategy being one of the strategies advocated in the CAPS curriculum (DoE, 2011b), and sometimes asked for explicitly in the Annual National Assessment paper memos: 469 + 378 =

400 + 60 + 9

+ 300 + 70 + 8

= 700 + 130 + 17 = 847.

However, using the horizontal splitting strategy with the column method proves to be problematic in determining answers to subtraction calculations (Thompson, 1995), for example, in the question: Mother has R425. She gives Tebo R169 to buy a book. How much money does she have left? 425 – 169 =

400 + 20 + 5

- 100 + 60 + 9

The horizontal splitting strategy produces the need for the carrying strategy or exchange, with confusions about what needs to be added or subtracted. This leads to some research proposing the use of the empty number line as a better option, in that it is a flexible model that works efficiently for both addition and subtraction problems.

2.9.3 Combination models

The application of formal models indicates development towards higher abstraction levels from counting procedures to calculating (Ensor, et al., 2009) which is more flexible and formal. Strategies associated with the combinations models includes stringing, splitting, compensating as indicated above.

2.10 Theoretical framework

RME is a theory about "knowledge construction" hence it is a theory of constructivism, through the use of everyday-life context that is "experientially real" for the learner and can be used to enhance "progressive mathematization" and not just motivation (Gravemeijer, 2002, p.8). The reinvention principle renders RME a dynamic theory with learners' informal solutions as the source of inspiration, for example, dividing three pizzas among four children.

2.10.1 The nature of Realistic Mathematics Education (RME)

RME is a constructivist based theory that aims to promote learner knowledge and understanding through the use of realistic contexts and didactical models (van den Heuvel-Panhuizen, 2003). The RME theory, developed by the Dutch, has been successfully in operation in many different countries of the world for almost four decades. RME is a constructivist based theory founded upon the notion that mathematics should be taught in order to be useful, that is connected to reality (Freudenthal, 1987 as cited in Gravemeijer & Terwel, 2000) and not just the transmission of knowledge. Three main principles of guided re-invention, didactical phenomenology and mediating models (Gravemeijer, 1994; 1999; Sembiring, et al., 2008) are essential in guiding instructional design to promote effective mathematics learning.

2.10.2 RME principles

Guided re-invention principle indicates that mathematics should not be presented as readily discovered facts (ready-made), but rather learners should be allowed to re-discover (re-invent) it (Freudenthal, 1991) through their active participation and teacher support. This principle will play a major role in the intervention lessons in that learners will be given an opportunity to make sense of word problems through discussions and sharing knowledge. The goal of RME is for learners to develop a framework of number relations that offers a basis for flexible mental computations (Gravemeijer, 1999) such as the use of combined strategies and other advanced calculation strategies as mentioned earlier.

A learning route has to be mapped out along with learners for them to discover the intended mathematics for themselves. However, learners cannot reinvent everything by themselves, hence the notion of guided reinvention with emphasis on the learners' ownership of the knowledge they acquire as their own private knowledge for which they are responsible. The teaching should allow learners to build their own "mathematical knowledge store" (Gravemeijer, 1999) based on own understanding. It is important to note that in supporting and guiding this process of knowledge construction care should be taken not to interfere with

students' initiative and intellectual autonomy (Gravemeijer, Lehrer, van Oers, & Verschaffel, 2002) but to allow learning to happen at learner's pace, contrary to our CAPS, unfortunately.

The didactical phenomenology principle advocates for the use of context towards the creation of more formal mathematics that can be generalised. Realistic context refers to what learners can imagine and is from reality (what they experience). Therefore RME aims to promote learner knowledge and understanding through the use of realistic contexts and didactical models (van den Heuvel-Panhuizen, 2003), hence the focus on word problems for this study. RME is a domain-specific instructional theory advocating learner support towards the development of informal situated solution strategies into more formal (symbolic) mathematical insights and procedures (Gravemeijer & Stephan, 2002). The tests in this study are structured into multiple choice questions (where learners select the appropriate symbolic representation) and working out questions (where learners decide on the appropriate model and strategies to use).

With the mediating models principle learners construct models that support them in bridging the gap from informal knowledge to formal notations (Gravemeijer, 2004). In RME models emerge from learner's own activities. Models are the representations of the problem situation (Gravemeijer & Stephan, 2002) within the given context. The goal of RME is for learners to develop a framework of number relations that offers a basis for flexible mental computations. RME can be described by means of the following five characteristics: the use of contexts, the use of models, the use of students' own productions and constructions, the interactive character of the teaching process and the intertwinement of various learning strands (Treffers, 1987) as indicated above.

Therefore, mathematics must be connected to reality, be valuable and relevant to the child's life and society at large. The use of context that is familiar places the learning of mathematics to be relevant to the learners' everyday life to promote better understanding. RME, as opposed to the mechanistic traditional approaches to learning, focuses on the growth in learner knowledge and mathematical understanding, through the use of context. Mechanistic traditional approaches use bare numbers (van den Heuvel – Panhuizen, 2005) and the

application of previously learnt rules (algorithms) to solve problems. Context, mainly in the form of word problems, is valuable in that it provides familiar scenario that learners can relate with, easily understand and can readily interact with. Therefore, within our South African (SA) context, applying the Curriculum and Assessment Policy Statement (CAPS) word problems might be easier and more beneficial for learners when presented at the beginning and not at the end of a mathematics a lesson (van den Heuvel – Panhuizen , 2005).

2.10.3 Solving contextual problems

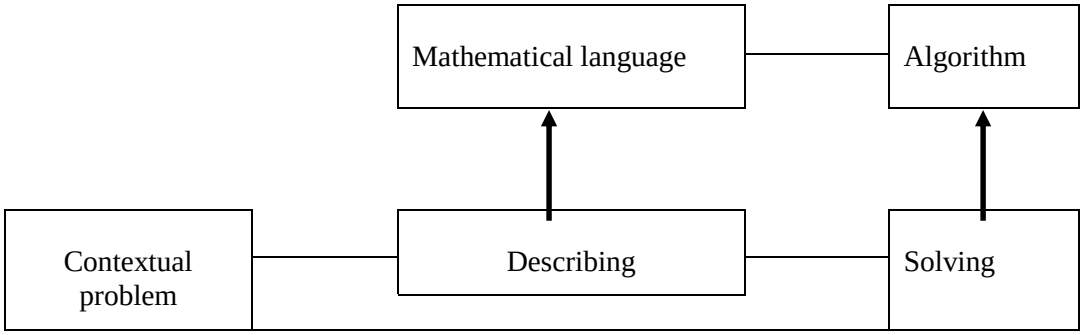
RME points out that learners be given an opportunity to re-invent mathematics through horizontal mathematization, [going from the world of life (the essence of context) into the world of symbols (formal mathematics)] and vertical mathematization, as moving within the world of symbols (Freudenthal, 1991) as cited in (van den Heuvel-Panhuizen, 2003). Later on, Treffers (1987) mentions that in horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. Vertical mathematization is the process of reorganization within the mathematical system itself, like, for instance, finding shortcuts and discovering connections between concepts and strategies and then applying these discoveries.

Figure 1 below on horizontal and vertical mathematization has been adapted from Gravemeijer (1994). The diagram explains the stages in the solution of contextual problems, termed mathematization. Mathematization means that learners become actively involved to create their own knowledge and understanding. Contrary to the traditional, pre-designed hierarchical view of mathematics instruction (Doorman, 2002) learner constructions are an essential element towards mathematizating (Gravemeijer & Terwel, 2000) for mathematics to remain useful. Mathematization takes place in two phases, through horizontal mathematization, with learners representing the problem situation (the use of models) and vertical mathematization with learners applying strategies to solve the problem (doing calculations).

Although this distinction seems to be free from ambiguity, it does not mean, as Freudenthal said, that the difference between these two worlds is clear cut. Freudenthal also stressed that

these two forms of mathematization are of equal value. Furthermore one must keep in mind that mathematization can occur on different levels of understanding.

Figure 1: Horizontal and vertical mathematization adapted from Gravemeijer (1994)



Horizontal mathematization (———); Vertical mathematization (⟶)

In conclusion, RME points out that learners be given an opportunity to re-invent (make self discovery) mathematics through horizontal mathematization, going from the everyday world of life through the use of context into vertical mathematization, the world of symbols and more formal mathematics (Freudenthal, 1991 as cited in van den Heuvel-Panhuizen, 2003). Horizontal mathematization involves the creation of an appropriate model; and intertwinement of various learning strands. Vertical mathematization involves the use of this model with its associated strategies to solve problems. In my class many learners used the carrying strategy with errors and hence ended up with incorrect answers.

2.11 Other solutions

Askew (2004) devised the "Big Books of word problems" as a resource and pedagogic approach based on RME principles to encourage and a support child's working with word problems (ibid). Further, they encourage meaning making as they do not focus on key words but on all key operations such as addition and subtraction. These big books are structured to assist learners from novice (searches for words rather than meaning) to expert problem solvers (deductive thinkers). They are relevant for my study as they also focus on addition and subtraction of word problems.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

The aim of this study, which is largely qualitative in its approach (Opie, 2004; Creswell, 2012) is to identify and develop a better understanding of how to rectify the existing gaps in the learner's poor performance on additive relation word problems. Drawing from the literature, I developed a sequence of – intervention lessons focused on learners' developing efficient models and strategies in solving word problems. The initial focus was on how learners worked with the column model and the strategies of column addition with carrying and column subtraction with borrowing and later on introducing learners to the empty number line model with the strategies of forward and backward jumping.

3.2 Research design

Participants in this research are all the 45 learners, boys and girls, within the age range 11 to 16, in one grade 6 class from a previously disadvantaged school that I currently teach. Participants first wrote a pre-test, post test and delayed post test, made up of fourteen word problems. There were two questions on each category, join and separate (result, change and start unknown) one with a higher whole value number range (hundreds and thousands) and another one on a lower whole number range (two digits), making a total of twelve of the problem on addition and subtraction with two distracter questions requiring a different operation for solution. The intervention lessons followed the pre test, before learners wrote the post test and later the delayed post test. The tests were written under normal class test conditions with learners being provided sufficient working space. Questions in the multiple choice section are of a lower whole value number range (less than 150), and in the working out section with a higher whole number range (up to 1 500) making a total of twelve problems on addition and subtraction with two distracters.

3.3 Research instrument (test) design

The tests (Appendix 1), qualitative in its nature exposes learners to answering the same category of question in two different scenarios where they choose and where they calculate. Two parts are identifiable; the multiple choice section made up of questions 1 to 7 and the 'working out' section made up questions 8 to 14. The multiple choice questions were aimed at learners identifying the most appropriate number sentence representing the word problem among the given four possibilities through selecting the model without being distracted by the given distracters. 'Working out' questions were aimed at learners making a decision on the appropriate model and strategies to solve the given word problem and fully showing their calculation on how they arrived at the answer. Questions in the multiple choice section are of a lower whole value number range (less than 150), and in the 'working out' section with a higher whole number range (up to 1 500) making a total of twelve problems on addition and subtraction with two distracters.

Separating the test into two sections accommodated two forms of assessment styles as required by the Curriculum and Assessment Policy Standards (CAPS) for grade 6 (DoE, 2009), and the RME theory as indicated in paragraph 2.10.2 above. The inclusion of two questions on each category of questions serves for easy comparison in the analysis. It was expected that learners perform uniformly regarding similar question categories in the multiple choice and 'working out' questions. In addition, separating the test into multiple choice and 'working out' questions would give a more appropriate assessment on the categories of questions learners work well or struggle with.

The test, with a duration of one hour focuses on the addition and subtraction categories of word problems according to Carpenter, et al. (1999), join and separate (result, change and start unknown). The questions were randomly mixed to eliminate redundancy or the funnel approach (Opie, 2004), meaning to avoid channelling the respondents towards a particular pattern. There was no learner feedback on all question types, as the purpose of testing is to establish the shifts or improvements in the knowledge acquired over time. Table 1 below explains how the test questions 1 to 14 in columns 3 and 5 have been spread out according to the join and separate

category of questions in columns 1 and 2. Number sentence in columns 4 and 6 represents the symbolic representation of the word problem.

Table 3.1: Category of questions in the tests

		Multiple choice section		**Calculation section**	
Category of questions		Questions	Number sentence	Questions	Number sentence
Join	Result unknown	1	79 + 149 = ☐	13	548 + 257 = ☐
	Change unknown	5	28 + ☐ = 110	8	55 + ☐ = 1 500
	Start unknown	3	☐ + 19 = 43	12	☐ + 153 = 327
Sepa-rate	Result unknown	7	136 – 79 = ☐	14	235 – 76 = ☐
	Change unknown	2	121 - ☐ = 27	10	840 - ☐ = 598
	Start unknown	6	☐ – 14 = 57	9	☐ - 265 = 178

3.4 The intervention session

The aim of this intervention, more qualitative in nature (Opie, 2004; Creswell, 2012) is to identify and develop a better understanding to rectify the existing gaps in the learner's poor performance in the pre-test. Learners were found using mainly the column model with its associated strategies with a lot of unconceivable errors, in the exploratory assessment and in the pre test, a quest for alternative efficient knowledge. RME emphasizes an approach in the learning of mathematics for learners solve problems in a way that make sense to them (van den Heuvel-Panhuizen, 2003) for better understanding.

The planning and implementation of the intervention lessons took into consideration three main guiding principles of RME according to Sembiring, et al. (2008) as explained in chapter 2.10 above. The first guiding principle, guided re-invention was applied to present mathematics such that learners should find the intended mathematics for themselves and not as readily discovered facts. Throughout the intervention lessons, learners were first presented with three contextual problems and were allowed some time (five minutes) to work alone in solving the problems without any prescriptions as to how to solve them. Thereafter they were allowed to work with a partner for another five minutes, to share their working with whole classroom discussion that followed.

The second principle, didactical phenomenology, emphasizes the use of familiar context as important towards the creation of more formal mathematics that can be generalised. All problems presented to learners are word problems adapted to context suitable to them. Problems in the "Big books" (Askew, 2004) were adapted to situations relevant to learner's everyday life. For example, changing the names and currency (question contexts) such as in the question:

> *Brooklyn went out shopping with £30. When he got home, he had £9.50 left. How much did he spend?*

- was changed to:

> *Thabo went out shopping with R300. When he got home he had R96 left. How much money did Thabo spend?*

The third, the mediating models principle, emphasizes the use familiar context so that learners construct models that support them in bridging the gap from informal knowledge to formal mathematics. Familiar contexts approach promotes classroom discourse and supports learners to develop models in a bottom-up approach as active participants within the classroom (Gravemeijer & Doorman, 1991) engaging in discussions and sharing ideas. In the process of adapting though, some elements of the original problem are changed – for example, my numbers do not require understanding of numbers in decimal notation as the original numbers do.

Classroom discourse played a major role in the intervention lessons as a tool for promoting sense making as learners shared their knowledge and ideas. Dialogic discourses allow learners to share their knowledge, negotiate their meaning, come to agreements and extend their scientific knowledge (Scott, Mortimer & Aguiar, 2006). To further assist in determining learners models and strategies 'Big Books' approach was encouraged.

The intervention sessions consisted of 6 lessons of one hour and fifteen minutes each conducted twice per week on Wednesdays and Fridays, during term three, from 2nd to 18th September with one recap lesson (the seventh) before the delayed post test.

Table 3.2: Schedule of intervention lessons

Schedule of intervention lessons					
Lesson 1 10/08/15	Lesson 2 17/08/15	Lesson 3 24/08/15	Lesson 4 31/08/15	Lesson 5 07/09/15	Lesson 6 14/09/15
All join problems, result, change and start unknown.	All join problems, result, change and start unknown.	All separate problems, result, change and start unknown	All separate problems, result, change and start unknown	Join and separate, a combination	Join and separate, a combination

Looking at the addition and subtraction categories of word problems according to Carpenter, et al. (1999) the lessons have been structured according to the join and separate categories. Lessons 1 and 2 on the join problems across all missing value positions, result unknown, change unknown and start unknown. Lessons 3 and 4 on the separate problems, across all missing value positions, result unknown, change unknown and start unknown. Lessons 5 and 6 on a combination of both join and separate word problems, across all missing value positions, result unknown, change unknown and start unknown.

The lessons were conducted roughly from 14:15 to 15:15 (for 1 hour), developed and discussed the empty number line model (ENL) along with its associated strategies of forward / backward jumping (FJ and BJ) as learners count and skip count, initially in ones, and later in other different sophisticated strategies such as stringing (keeps the first or bigger number as is whereas the other is split and its parts added or subtracted separately to the first number, one at a time), splitting (both addends or minuend and subtrahend become restructured by splitting or partitioning them into multiples of hundreds, tens and ones or in any other way convenient for the learner so that the number could be added or subtracted easily) and varying (other more sophisticated strategies based on learner understanding). The column model (CM) was, to a certain extend also discussed along with special emphasis on the column addition with carrying (CAC) and column subtraction with borrowing (CSB) strategies. The pre test indicated an absolute learner orientation towards the column model hence the need to perfect CAC and CSB strategies.

The success of the intervention lessons lies in the provision of a new thinking approach both for teaching and learning, away from the passive, mechanistic, traditional skill based

transmission of knowledge as was witnessed in the pre-test. It is assumed that prior to the intervention lessons learners had not been introduced to the empty number line as none of them used the empty number line in the pre-test or in the preliminary assessment. Naturally some resistance surfaced due to learner discomfort and challenge of changing towards unfamiliar operational modes; hence only three problems were given for working in class and the other three in their own time. Intervention lessons served to foster a problem solving classroom culture where learners take the initiative to think and reason for themselves.

> "When comparing and discussing their solution methods, for instance, some students may realize that other solution methods have advantages over their current method." (Gravemeijer & Terwel, 2000, p.783)

This has been noticeable with learners fully sharing their knowledge though it started slowly especially with the first lesson but improved with more lessons. Further the intervention lessons should serve to make mathematics more relevant in dealing with real life problem situations (the word problems that depicts learners' everyday experience) as a way to emphasize the connected comprehension of mathematical concepts and ideas to reality (Scott, Mortimer & Aguiar, 2006).

Advantages of using the empty number line are that it can be used with flexibility to model the contextual problems (Gravemeijer, 2002). The word problem is modelled as the learner is busy reading. Learners could adapt the empty number line to their thinking unlike the base ten blocks that is meant to be broken down into tens and ones. This model can be adapted to fit learner thinking as numbers are sequenced from smaller to bigger. Further it could be used to depict sophisticated strategies and number facts such as compensation with more efficiency. This gives indication the shift from model of to model for and no longer require the use of the model. The empty number line worked as a catalyst for the way the RME community is thinking about models.

At the end of lesson 6 radical shifts towards the use of the empty number line became evident with minimal errors. It helped the researchers realize the shift in learner thinking from thinking about the modeled context situation model of to a focus on mathematical relations model for. It also supported linear counting-type methods (the development of number relations) and scaffolds partial calculations and partial results.

The theoretical underpinning of RME resonates well with Askew's "Big Books" resources and pedagogic approaches (Askew, 2004) on word problems as they provide realistic context. The content on the Big Books is separated into part A (similar to the multiple choice questions) and part B (similar to the working out questions) according to level of difficulty with part B problems numerically easier. Assistance to learners is provided through a series of intervention lessons with lessons one to four with a lower number range (less than one hundred) and lessons five and six with a higher number range (hundreds and thousands).

Each lesson is composed of four parts as indicated in table 3.3 below. In the opening part, the teacher introduces the lesson by posing a question and allows learners to respond individually or in pairs. There is no mention of correct or incorrect answer but an encouragement to contribute to a whole class discussion for everyone to reveal their knowledge by contributing to the discussion (classroom discourse as mentioned above). Classroom discussion creates a platform for learners to air their views and to clear their misconceptions as they learn from others with the teacher steering the conversation in the correct direction through clarifying questions (Gravemeijer, 2004). The opening part is planned to last for between ten and fifteen minutes after which learners are ready to move to the next part.

Table 3.3: Intervention lesson structure

Part	Description
1. Opening	As introduction teacher poses a contextual problem and learners become meaningfully involved. (whole class discussion)
2.Learners working	Teacher poses another contextual problem, learners work individually and then in pairs. Teacher encourages them to elaborate on their solutions
3. Discussion	Learners discuss their solutions and share ideas. Teacher facilitates the discussion so as to capture vital knowledge. Ends with whole class discussion.
4. Closing	Teacher poses summary question, teacher and learners discuss conclusion, finding links between problems

In the second part learners work individually on three contextual problems for between fifteen and twenty minutes. Problems given here are of a similar nature as the one discussed in part

one above and learners are encouraged to show their working. The teacher makes rounds to capture how learners work and to provide support where required. Thereafter for the next ten to fifteen minutes learners share their work with a partner (pair work) or in a group (group work) of up to four people, after which learners will be ready for the next part. Some learners work slowly and could not manage to finish all three problems in twenty minutes.

Part three is the discussion for the whole class and it is expected to last between fifteen and twenty minutes linking up the problems, finding out what is common, what is different. The role of the teacher was to facilitate the discussions so as to capture vital knowledge. Factors that seemed to negatively affect lesson outcomes are the learners' orientation to traditional ways of working that seems to hold them back from sharing and participating fully in class. However, learners seemed to be enthusiastic about the new learning approach.

In the closing part planned to last for five minutes the teacher poses summary question, teacher and learners discuss conclusion, as a summary of the entire lesson by highlighting important key points, links, and wrapping up.

On average the lessons lasted between one hour five minutes and one hour fifteen minutes, less than the budgeted one hour thirty minutes. Keeping the lessons shorter was advantageous for the learners as the intervention lessons happened at the end of the school day and learners might have been exhausted.

3.5 Sampling

Participants in this research were all the 45 learners, boys and girls, within the age range 11 to 16, from five grade 6 classes, from a previously disadvantaged school that I currently teach. The learners were selected based on their non-use of transport home after school and the first fifty to return consent forms from parents or guardians. The sample could be regarded as convenient (Opie, 2004) as these are participants that I have easy access to. This research is a

case study as its reporting is based on empirical enquiry; with a limited focus (addition and subtraction of word problems) and working with a limited sample (Denzin & Lincoln, 2005).

3.6 Data sources

The data for the research is derived from forty (initially forty five) grade six learners in a previously disadvantaged school, and hence the research becomes a case study. A case study is a report that is based on empirical enquiry, with limited number of events on a limited sample (Denzin & Lincoln, 2005). In other words a case study works with a huge problem within a small scale, thus increasing the focus. A case study is advantageous in that it deals with issues in real life (Denzin & Lincoln, 2005). In this research, upon investigation into the models and strategies grade 6 learners employ in solving word problems, the findings as it is a case study are derived from empirical evidence, learner handwritten scripts and hence are reliable even though they cannot be used to generalize the performance across all the grade six classes in Johannesburg (Baxter & Jack, 2008), one of the disadvantage of a case study, but could rather could be used to generalize the performance of grade six learners within my school.

The weakness of the case study is that only one person does it. With the viewpoint from one perspective there might be some biasness in the end reporting. The research objectives serve to minimize the bias as the focus is on the models (as representations of problem situations) and strategies (learner's manipulation of these models). The researcher identifies the model used, the calculations thereof, whether correct or incorrect as well as whether the calculations led to the correct or incorrect answers. Learners were found using mainly one approach to answering questions, that is, either vertical column or the empty number line model. The recording of models and strategies used therefore was straight forward and less biased. In addition, fortunately my supervisors have been very objective, directing my viewpoints to stay focussed on the actual results so as to minimise my biasness.

Data sources for this research study comprise of learner test scripts from the pre-, post and delayed post tests as well as the intervention workbooks. The tests were written under normal class test conditions with learners being provided sufficient working space. In achieving the

expected outcomes of addressing the difficulties learners experience in solving addition and subtraction word problems within my grade 6 class, both quantitative and qualitative approaches were used. Quantitative research is mainly expressed in numerical form, employing mathematical models such as percentages, rate, ratio and many others as was expressed especially for part A of the test. A qualitative research is one that is open-ended with no limits or restrictions as to how the respondents should respond to questions (Denzin & Lincoln, 2005) as expressed in part B of the test. The following research processes were undertaken: literature review, research instrument (test) design, ethics considerations, data collection, data analysis, focussed intervention study and the research report.

Participants first wrote the pre-test, made up of fourteen word problems, scheduled for one hour. The post test and delayed post tests were repeat sittings of the pre-test. Learner solutions were analysed as to how they model the problems together with strategies they employ in solving the problems in the pre-test, then the intervention and later the post test and the delayed post test.

The pre-test served to answer my first research question:

> *What models and strategies do grade six learners employ in solving addition and subtraction word problems prior to an intervention sequence?*

Learners were found predominantly using the column model in solving addition and subtraction problems, with its associated strategies of column addition / subtraction of carrying / borrowing, often using tallies in the form of sticks or circlesin the exploratory assessment results.

The correct column model for question 1 would be represented as follows:

$$\begin{array}{r} 149 \\ +\ 79 \\ \hline \end{array}$$

and the incorrect model would be represented with a minus sign in the place where the plus sign is indicated: 149

$$\begin{array}{r} -79 \\ \hline \end{array}$$

At times the plus / minus sign would not be indicated. In these instance, the strategies (how learners arrived at the answer by either adding or subtracting determines the correct / incorrect model) were used to infer the model and whether it was appropriate or not, e.g.

149

79

228 indicates the addition model.

The second research question:

- *What models and strategies are developed and discussed by these learners in solving addition and subtraction word problems during the intervention sequence?*

was answered through an intervention session (data was collected from learner books), informed by the pre-test to promoting the use of the relational model advocated in the literature. My observation was that initially, learners either used the vertical column model or the horizontal number sentence model.

The third and fourth research questions:

- *What kinds of changes are seen (if any) in the models and strategies that learners employ after the intervention as compared to before the intervention?*
- *Can changes in extent of success (if any) in solving join and separate-type addition and subtraction word problems be associated with shifts in models and strategies?*

was answered through the post and delayed post test, which were repeat sittings of the pre-test, conducted on 23rd September and 24th November respectively. Learners wrote the post test with an option on the use of models four weeks after the pre-test and with six intervention lessons. As in the pre-test, learners maintained the predominant use of the vertical column model and its associated strategies with very minimal changes in results with only two learners employing the empty number line model in some questions (three and four). Four weeks after the post test and an additional two intervention lessons, learners wrote the delayed post test with only one option of using the empty number line model. The decision to impose the use of this specific model followed the finding of limited move to the number line, and findings reported in Tshesane's (2014) study of an intervention located in Grade 4 on additive relations, where a

similar pattern was found, with increased improvements in the delayed post-test with the instruction to use the number line model.

3.7 Analysis

Similar categories of problems in the pre-test were given in the post and delayed post test. Data analysis is aimed at the identification of models and strategies as used by learners, as well as their correctness and success in solving the problem. The data was analysed through employing both the qualitative and quantitative methods, the mixed approach of research (Denzin & Lincoln, 2005). The initial test analysis is of quantitative nature for the multiple choice questions, showing the number of correct answers also as percentages. Later test analysis will be tackled qualitatively recording from learner scripts the models and strategies employed and classifying them according to the dominant model and the number of correct answers it produced also as percentage. In other instances where learners used a combination of two or more models, only the major one was considered, that is, either the first or the one that led to the answer.

Learner solutions were analysed as to how they model the problems together with strategies they employ in solving the problems. The data collected was arranged in tables according to the models / strategies employed the number of correct models / strategies as well as the number of correct answers in order to correctly identify the shifts from one phase to the other. Multiple choice questions were recorded and analysed separately from the working out questions. Two distinct models, the column and empty number line dominated.

3.8 Ethical considerations

Consent to conduct the study was sought and granted before data collection by Wits University Ethics Committee, Gauteng Department of Education, the principal and the school governing body of the school. An informed consent for writing tests and participation in the intervention lessons was also sought and granted by parents on behalf of the learners involved in the project as the participants are under the age of 18. Learners targeted for participation were also consulted to grant their consent. Consenting participant who showed signs of anxiety and

decided not to continue with the project during the data collection phase, were not forced to carry on. Confidentiality and anonymity are guaranteed as learner names are kept confidential, and pseudonyms are used. After five years data scripts will be destroyed.

Learners were informed that their participation was voluntary and that their parents were approached and provided consent. Further they were informed that their refusal to participate will involved no penalties or loss of benefits to which they as school children in grade 6 are otherwise entitled to and that their participation could be discontinued at any time with no penalty or loss of benefits. In addition, they were re-assured that their participation incurred no foreseeable risks, discomforts, side effects or benefits. Finally their confidentiality and anonymity were guaranteed as their names and the name of their school will be kept a secret and other fictitious names (pseudonyms) will be used instead and were therefore encouraged to participate. After five years data scripts will be destroyed.

3.9 Reliability, validity and limitations

The tests were written under normal test conditions within the stipulated time with no learner feedback, as the purpose of testing was to establish the shifts or improvements in the use of models and strategies acquired over time. All data sources provide empirical evidence (from learner scripts) and are therefore reliable and verifiable. Evidence from learners' scripts is important to this research as it determines the credibility of the study and the trustworthiness of the findings. 'Validity refers to the degree to which a method, a test or a research tool actually measures what it is supposed to measure" (Wellington, 2000, p.201, as cited in Opie, 2004, p.68), meaning that I need to be sure that the data collected gave me reliable evidence relating to learners' models and strategies for additive relations problems. The specific models and strategies employed witnessed from learner responses validates the study.

The limitation might be that the teacher is also the researcher at the same time which might in a way cloud the objectivity with other information going unnoticed. Therefore, in the intervention lessons, care was taken to allow more talk (in the form of explanations or answering of questions) from the learners rather than the teacher. As the teacher I mainly

stepped in to clarify or probe for more discussion, otherwise I spend some time taking notes on the events within the classroom. This research is a case study as the data consists of a limited number of learners (forty) from only one class in one school and therefore "generalization from a few qualitative cases" (Creswell, 2012, p.550) might not be appropriate.

3.10 Research report

The research report will be in a narrative form, detailing an exposition of learner's use of models and strategies in the tests as indicated above. A comparison will be drawn among individual learners and as a group. The report will give evidence of the shifts if any, traced through the research questions in the test sittings. This report will consist of five chapters. Chapter one is the introduction which gives an overall summary of the research study. In chapter two the literature provided guidance as to the efficient models and strategies and defining the conceptual framework as well as the theoretical framework. This chapter 4 outlining the methodology, steps taken towards achieving the study objectives. Chapter five is for analysis, pronouncing and interpreting the results, the discussion chapter which follows here after summarises the findings and evaluated the study.

CHAPTER 4: ANALYSIS OF RESULTS

4.1 Introduction and rationale

This chapter aims to present findings to my investigation on the models and strategies employed by learners in solving addition and subtraction word problems with the aim of answering my research questions indicated below:

Research questions

- What models and strategies do grade six learners employ in solving addition and subtraction word problems prior to an intervention sequence?
- What models and strategies are developed and discussed by these learners in solving addition and subtraction word problems during the intervention sequence?
- What kinds of changes are seen (if any) in the models and strategies that learners employ after the intervention as compared to before the intervention?
- Can changes in extent of success (if any) in solving join and separate-type addition and subtraction word problems be associated with shifts in models and strategies?

The results presented here are of the 40 learners (girls and boys) who wrote the pre test, took part in the six intervention lessons, the post test and later attended one recap intervention lesson before writing the delayed post tests. In addition, this analysis does not separate or compare the performance of boys from girls. The main focus in this analysis is to find out "What kinds of changes are seen (if any) in the models and strategies that learners employ after the intervention as compared to before the intervention?" by writing out what has been achieved through capturing the kind of models and strategies employed in the pre-test and later in the post and delayed test. This analysis also captures whether these models were correctly employed and the strategies used are leading to the correct answers. The models and strategies employed will be recorded as numerals and as percentage for the multiple choice questions, with the quantitative analysis, at the same time being more descriptive, and noting the qualitative working out of questions as described in the methodology chapter above.

In the multiple choice question, the correct answers were identified by the reflection of the correct number sentence. In instances where learners showed their working in the multiple

choice questions the results will be analysed by examining the selected model. "Working out" questions refers to questions which are not multiple choice and learners need to show how they solved the problem. There are two phases in the analysis of the working out questions, whether the model used is correct and whether the strategies applied lead to the correct answer. A correct answer for the working out question would mean the correct use of the model as well as the correct use of the strategy. The analysis is of the pre, post and delayed post tests will follow a similar kind of format with separate analysis tables for the multiple choice and working out questions for easy comparison between the tests. The analysis will be structured so as to answer the research questions as given above in the order: pre-test, intervention, post test and delayed post test.

What models and strategies do grade six learners employ in solving addition and subtraction word problems prior to an intervention sequence (in the pre-test) and how did they perform?

4.2 Pre-test results

The analysis for the multiple choice questions (1 to 7) will be done separately, from the analysis of the working out questions (8 to 14). In the multiple choice questions learners answered by choosing and circling out one answer out of the given four, with one correctly modelled option in the list. Learners were encouraged to show their working on the sides if they needed to.

4.2.1 Multiple choice questions results of the pre test

Table 4.1 below with five columns details pre test multiple choice questions results. The questions are grouped into the join and separate categories and within each of the categories a further division becomes visible in relation to result, change and start unknown (column one). Column two names the questions and sequences them according to the categories to keep join / separate together. In column three the word problems which are the test questions are recorded. The number sentence (column four) is a symbolic representation of the word problem. The overall total correct answers (column 4) indicates the total number of learners

(out of the total forty that wrote) who answered the question correctly, also expressed as a percentage (within the brackets).

Table 4.1: Pre-test multiple choice questions (1 to 7) results

1		2	3	4	5
Category of questions		**Questions**	**Word sentence**	**Number sentence**	**Total correct answers**
Join	Result unknown	1	79 ants crawled into a cave. Inside, they found another 149 ants. How many ants were inside the cave?	79 + 149 = ☐	33/40 (83%)
	Change unknown	5	Tumi's taxi travelled 28 km. He was travelling to Polokwane, which was 110 km away. How much further does Tumi's taxi have to travel?	28 + ☐ = 110	11/40 (28%)
	Start unknown	3	Some friends arrived for Shadi's birthday party. 19 more arrived later and now there are 43 friends at the party. How many friends were first to arrive?	☐ + 19 = 43	20/40 (50%)
Separate	Result unknown	7	136 grade 6 children went on a school trip to the zoo. 79 of them came back early to catch a taxi home. How many learners stayed behind?	136 – 79 = ☐	31/40 (78%)
	Change unknown	2	Romeo had R121. He bought a book. Now he has R27. How much was the book	121 - ☐ = 27	22/40 (55%)
	Start unknown	6	Thakane weighs 57 kg after losing 14 kg in the past three months. How much did Thakane weigh before?	☐ – 14 = 57	6/40 (15%)

Recorded in table 4.1 above are pre-test results showing how forty learners answered and performed in the multiple choice questions 1 to 7 (4 excluded as is the distracter). Reading the table from left to right for question 1, the join result unknown, with number sentence (79 + 149 = ☐), thirty three out of forty learners (83%) answered correctly. Performance is higher on result unknown problems, than on the other types. Change and start unknown problems show variable performance across the join and separate categories. Questions 2 (separate change unknown) and 3 (join start unknown) indicate a moderate performance of 55% and 50% respectively, with very few learners answering questions 5 and 6 correctly.

In summary, learner performance in the multiple choice questions is summarized in table 4.1 above. It is notable from the table above that the highest learner performance was in question 1, the result unknown question where 33 learners (83%) answered correctly, followed by question 7, separate, result unknown with 78% and poor performance was in question 6,

separate, start unknown problem where only 6 learners (15%) answered correctly and question 5, join change unknown with 11 learners (28%).

4.2.2 Pre test working out questions (8 to 14) results

Table 4.2 below presents the different types of models and strategies employed by the grade six learners in solving addition and subtraction word problems (which are not part of the multiple choice questions), questions 8 to 14 in the pre test (11 excluded as is the distracter). Column 1 records the different models used and the strategies aligned to the model. Column 2, the join categories of questions and column 3 records the separate category of questions. The join and separate category of questions (columns 2 and 3) are further subdivided into three sub-columns each, of result, change and start unknown sub-categories. The questions, recorded in the sub-columns are arranged according to the join and separate categories of result, change and start unknown and not in their chronological order.

The first row records the join and separate categories with the questions in row 2. The word problem sentence are given in row 3 with the number sentence (records the symbolic representation of word problems) in row 4.Proportion with correct answers in overall group records correct answers per question (worked out using all models together). The models used are highlighted in grey with all strategies used with the model recorded under. In the case of the Column Model the models are recorded as either "correct column model of addition" or "correct column model of subtraction". Strategies leading up to the correct answer records the number of correct answers that resulted from the correct use of the model.

The models used (highlighted in grey) are the column, horizontal number sentence (horizontal splitting of numbers into units, tens, hundreds, etc), no model (no working out is shown) and group model (involve the use of a group of objects such as counters, tallies or coins in structuring numbers into ones, tens and hundreds). In other instances where learners used a combination of two or more models, only the major one was considered, that is, either the first or the one that led to the answer.

2: Table 4.2: Pre test working out questions (8 to 14)

Categories	Join Categories			Separate categories		
Questions	Question 13 Result unknown	Question 8 Change unknown	Question 12 Start unknown	Question 14 Result unknown	Question 10 Change unknown	Question 9 Start unknown
Word Problems	548 turtles were swimming. Another 257 turtles joined them. How many turtles were swimming?	Big elephant weighs 1 500 kg. She weighed 55 kg at birth. How much has she grown?	Jim cycled 327 km for two days. He cycled153 km in the second day. How far did Jim cycle the first day?	A giant caught 235 zebras, but then decided to let 76 go free. How many zebras was the giant left with?	There were 840 spectators at the soccer stadium some left at half time. There are now 598 spectators at the stadium. How many spectators left at half time?	Vusi saved some money. He used R265 to buy soccer boots. Now he has R178 left. How much money did Vusi save?
Number sentence	548 + 257 = ☐	55 + ☐ = 1500	☐ + 153 = 327	235 - 76 = ☐	840 - ☐ = 598	☐ - 265 = 178
Total number of learners with correct answers out of forty (in overall group)	**32/40 (80%)**	**6/40 (15%)**	**13/40 (33%)**	**21/40 (53%)**	**15/40 (38%)**	**20/40 (50%)**
Number of learners that used the column model out of forty	**38/40 (95%)**	**38/40 (95%)**	**39/40 (98%)**	**40/40 (100%)**	**39/40(98%)**	**40/40 (100%)**
Number of learners that usedcorrect model of addition / subtraction out of forty	34/40 (85%)	8/40 (20%)	21/40 (53%)	37/40 (93%)	35/40 (88%)	26/40 (65%)
Number of learners and the strategies they used (listed below), though not all are leading to the correct answer						
Correct answers from using column strategies	30/40 (75%)	6/40 (15%)	13/40 (33%)	20/40 (50%)	15/40 (38%)	18/40 (45%)
Column addition with carrying	30/40 (75%)	30/40 (75%)	16/40 (40%)	1/40 (3%)	4/40 (10%)	21/40 (53%)
Column addition with carrying and tallies			1/40 (3%)	2/40 (5%)		4/40 (10%)
Column addition with place value	4/40 (10%)					1/40 (3%)
Column subtraction		2/40 (5%)	3/40 (8%)	6/40 (15%)	4/40 (10%)	10/40 (25%)
Column subtraction with borrowing	4/40 (10%)	6/40 (15%)	16/40 (40%)	27/40 (68%)	30/40 (75%)	4/40 (10%)
Column subtraction with borrowing and tallies			4/40 (10%)	4/40 (10%)	1/40 (3%)	
Number of learners that used horizontal number sentence model	**2/40 (5%)**	**2/40 (5%)**				
Correct horizontal number sentence model	2/40 (5%)	1/40 (3%)				
Horizontal addition strategy with place value leading to correct answer	2/40 (5%)	0/40 (0%)				
No model			**1/40 (3%)**		**1/40 (3%)**	
Group model in the form of tallies or circles (included under column model)			5/40 (13%)	6/40 (15%)	1/40 (3%)	4/40 (10%)
Column addition with carrying and tallies			1/40 (3%)	2/40 (5%)		4/40 (10%)
Column subtraction with borrowing and tallies			4/40 (10%)	4/40 (10%)	1/40 (3%)	
Correct column addition / carrying / tallies						2/40 (5%)
Correct column subtraction / borrowing / tallies				1/40 (3%)		

Strategies used along with the column model are column addition with carrying, column addition with carrying and tallies, column addition with carrying with place value, column subtraction, column subtraction with borrowing and column subtraction with borrowing and tallies. Strategies used along with the horizontal number sentence are horizontal addition (place value). There were no strategies portrayed along with the no model as only the answer has been provided with no working shown. Strategies used along with the group model are addition with carrying and tallies and column subtraction with borrowing and tallies, both incorporated under the column model.

In question 13, join result unknown question, thirty two out of forty (80%) answered correctly. There were thirty eight learners (95%) that used the column model, thirty four learners (85%) who used the correct column model out of which thirty (75%) used the column addition with carrying correctly to arrive at the correct answer. Further, two learners (5%) used the correct horizontal model with the horizontal addition (place value) strategy, also arrived at the correct answer. None of the learners used the no model or the group model for question 13.

In question 8, join change unknown question, only six out of forty learners (15%) answered correctly. There were thirty eight learners (95%) that used the column model, eight learners (20%) used the correct column model out of which six (15%) used the column subtraction with borrowing correctly to arrive at the correct answer. Further, two learners (5%) used the correct horizontal model with the horizontal addition (place value) strategy, also arrived at the correct answer. As was the case with question 13, none of the learners used the no model or the group model for question 8.

In question 12, join start unknown question, thirteen out of forty learners (33%) answered correctly. There were thirty nine learners (98%) that used the column model, twenty one learners (53%) used the correct column model out of which sixteen (40%) used the column subtraction strategy with borrowing correctly to arrive at the correct answer. None of the learners used the horizontal number sentence model and one of the learners used the no model, giving the incorrect answer. Five learners (13%) used the group model with the column subtraction strategy with borrowing and tallies, but could not work up to the correct answers.

In question 14, separate result unknown question, twenty one out of forty learners (53%) answered correctly. All forty learners (100%) used the column model, thirty seven learners (93%) used the correct column model out of which twenty seven (68%) used the column subtraction strategy with borrowing correctly with only twenty arriving at the correct answer. None of the learners used the horizontal number sentence model or the no model. Six learners (15%) used the group model with the column subtraction strategy with borrowing and tallies with one working up to the correct answer

In question 10, separate change unknown question, fifteen out of forty learners (38%) answered correctly. There were thirty nine learners (98%) that used the column model with thirty five learners (88%) used the correct column model out of which thirty (75%) seemed to use the column subtraction strategy with borrowing correctly but only fifteen arrived at the correct answer. None of the learners used the horizontal number sentence model and one used the no model, giving the incorrect answer. One learner (3%) used the group model with the column subtraction strategy with borrowing and tallies, but could not work up to the correct answer.

In question 9, separate start unknown question, twenty out of forty learners (50%) answered correctly. All forty learners (100%) used the column model; twenty six learners (65%) used the correct column model out of which twenty one (53%) used the column addition strategy with carrying correctly, and eighteen arrived at the correct answer. None of the learners used the horizontal number sentence model or the no model. Four learners (10%) used the group model with the column addition strategy with carrying and tallies, with two working up to the correct answer.

It is clear the column model is the most popular, used by the majority learners (95%) in questions 13 and 8, 98% in questions 12 and 10 and 100% in questions 14 and 9. The number of learners working up to the correct answer is recorded as always lower than the number using the correct model across all questions. For example, in question 13 (with the highest correct answers), the difference between the numbers of learners working up to the correct answer and

those using the correct model is five percent (85% - 80%). The difference could be an indication of the inefficiency in the application of strategies. Strategies identified with the column model are column addition, column addition with carrying, column addition with carrying and tallies, column addition with place value, column subtraction, column subtraction with borrowing, column subtraction with borrowing and tallies, column multiplication and column division. The inefficiencies regarding the application of strategies were therefore looked at during the intervention session. The first and second intervention sessions mainly worked with the column addition / subtraction along with carrying / borrowing strategies, working up to the correct answer.

The horizontal number sentence model is used in a limited way as it was only used by two learners (5%) out of a total of forty. Just as was with the column model there is no certainty that learners working with the horizontal number sentence would work up to the correct answer, again citing inefficiencies regarding the use of the model and strategies. In addition, inefficiencies regarding the use of the group model and the associated column subtraction strategy with borrowing and tallies as well as column addition strategy with carrying and tallies are noticeable. Under the horizontal number sentence model the strategies used are the horizontal addition with place value and column addition with place value. Strategies associated with the group model are column addition with carrying and tallies as well as the column subtraction with borrowing and tallies. No strategy was use with the no model, and only the answer was provided. Surprisingly, the kind of problems that learners were faced with appeared much bigger than just word problems as they also experienced problems with the use of strategies in solving the problems.

In summarising the pre test results, performance in the multiple choice questions appear to be slightly higher than that of the working out questions. Proportions using correct strategy are low within the groups selecting the correct model, and much lower is the proportions arriving at the correct answers highlighting the need for more efficient models that would effectively lead up to the correct answers. Particularly it is the subtraction with borrowing strategy that creates more problems for learners. The blank spaces indicate that the specific model or strategy was not used for that particular question. All the different models and strategies are discussed in chapter 2.

What models and strategies are developed and discussed by these learners in solving addition and subtraction word problems during the intervention sequence and how did they perform?

4.3 The intervention session

The second research question is answered through the intervention session. The intervention session consisted of six intervention lessons and one recap lesson (the seventh) before the delayed post test. The seventh lesson will not be reported on as it did not follow the structure of lessons one to six. The lessons were conducted from 14:15 to 15:15 (for 1 hour), developed and discussed the empty number line model (ENL) along with its associated strategies of forward / backward jumping (FJ and BJ) as learners count and skip count, initially in ones, and later in other different sophisticated strategies such as stringing (keeps the first or bigger number as is whereas the other is split and its parts added or subtracted separately to the first number, one at a time), splitting (both addends or minuend and subtrahend become restructured by splitting or partitioning them into multiples of hundreds, tens and ones or in any other way convenient for the learner so that the number could be added or subtracted easily) and varying (other more sophisticated strategies based on learner understanding). The column model (CM) was, to a certain extend also discussed along with special emphasis on the column addition with carrying (CAC) and column subtraction with borrowing (CSB) strategies. The pre test indicated an absolute learner orientation towards the column model hence the need to perfect its carrying / borrowing strategies.

Table 4.3 below with eight columns records in brief how the intervention lessons were conducted. Column 1 helps to identify the rows in the table. Column 2 describes the lesson events; columns 3 to 8 record the lessons one to six. In column 1, row 1 identifies the different lessons, row 2 the actual date when the lesson happened, row 3 the total number of learners per lesson. Number sentences for the first three lesson word problems are listed in row 4 (a, b and c). The models advocated and used in each lesson such as column and empty number line and the level of engagement spent working on that model as a percentage are shown in row 5.

Learners who worked with correct models for questions 1, 2 and 3 are shown in row 6 (a, b and c). The strategies advocated as per the model (ENL and CM) are shown in row 7 and the learners who worked correctly with the strategies are given in row 8 (a, b and c). Row 9 (a, b and c) records the numbers of learners with correct answers. The table only records the outcomes for questions one to three as these were the questions reliably completed by all learners before the class discussion and corrections. The data in table 4.3 was recorded from learner working books. At the end of each lesson learner books were collected for checking and recording as to what models and strategies they used, whether the models and strategies were used correctly.

Key to abbreviations used in table 4.3 below:

ENL= empty number line model;

FJ = forward jumps

BJ = backward jumps

CM = Column model;

CAC = Column addition with carrying strategy

CSB = Column subtraction with borrowing strategy

Table 4.3: Intervention lessons

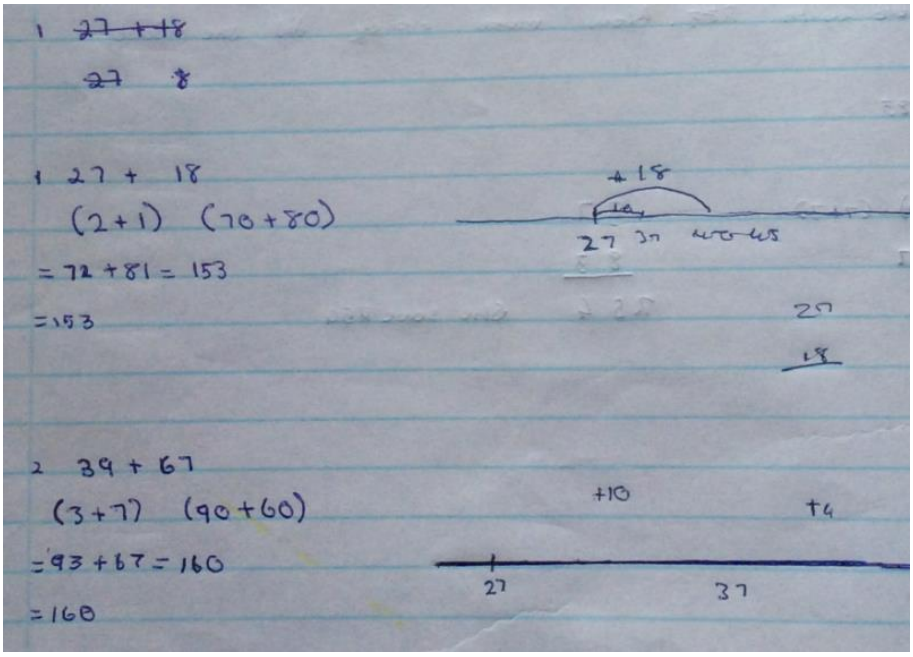
1	Lesson	Lesson 1		Lesson 2		Lesson 3		Lesson 4		Lesson 5		Lesson 6	
2	Lesson date	02 / 09		04/ 09		09 / 09		11 / 09		16 / 09		18 / 09	
3	Learners	43		42		38		45		43		39	
4	Number sentences												
a	Question 1	27 + 18 = ☐		42 – 19 = ☐		☐ - 23 = 51		12 – 5 = ☐		178 + ☐ = 432		☐ - 139 = 464	
b	Question 2	39 + ☐ = 67		32 - ☐ = 15		☐ - 18 = 43		☐ - 7 = 18		124 - ☐ = 77		321 - ☐ = 197	
c	Question 3	☐ - 16 = 52		☐ - 8 = 23		24 + 29 = ☐		☐ - 16 = 37		7100 – 146 = ☐		☐ + 168 = 301	
	Models advocated	Empty number line	Column model	Empty number line	Column model	Empty number line	Column model	Empty number line	Column model	Empty number line	Column model	Empty number line	Column model
5	Models use	ENL 20%	CM 80 %	ENL 50%	CM 50 %	ENL 80%	CM 20 %	ENL 95%	CM 5 %	ENL 100%	CM 0 %	ENL 100%	CM 0 %
6	Learners with correct models												
a	Question 1	0/0	43/43(100%)	9/12 (75%)	30/30(100%)	28/32(88%)	6/6(100%)	43/43(100%)	2/2(100%)	35/35(100%)	8/8(100%)	38/39(97%)	0/0
b	Question 2	0/0	40/43 (93%)	10/12(83%)	29/30(97%)	27/32(84%)	6/6(100%)	43/43(100%)	2/2(100%)	36/36(100%)	7/7(100%)	39/39(100%)	0/0
c	Question 3	0/0	35/43 (81%)	7/9 (78%)	25/33(76%)	32/32(100%)	6/6(100%)	43/43(100%)	2/2(100%)	30/30(100%)	13/13(100%)	39/39(100%)	0/0
7	Strategies advocated	FJ + BJ = 20 % CAC + CSB = 80%		FJ + BJ = 50 % CAC + CSB = 50%		FJ + BJ = 80 % CAC + CSB = 20 %		FJ + BJ = 95 % CAC + CSB = 5 %		FJ + BJ = 100 % CAC + CSB = 0 %		FJ + BJ = 100 % CAC + CSB = 0 %	
8	Learners with correct strategies												
a	Question 1	0/0	37/43 (86%)	9/12 (75%)	30/30(100%)	27/32(84%)	6/6(100%)	42/43(98%)	1/2(50%)	35/35(100%)	8/8(100%)	38/39(97%)	0/0
b	Question 2	0/0	36/43 (83%)	9/12 (75%)	27/30 (90%)	26/32(81%)	6/6(100%)	43/43(100%)	1/2(50%)	36/36(100%)	7/7(100%)	39/39(100%)	0/0
c	Question 3	0/0	33/43 (77%)	6/9 (67%)	22/33 (67%)	32/32(100%)	6/6(100%)	42/43(98%)	1/2(50%)	30/30(100%)	13/13(100%)	39/39(100%)	0/0
9	Learners with correct answers												
a	Question 1	0/0	34/43 (79%)	9/12 (75%)	30/30(100%)	27/32(84%)	4/6(67%)	42/43(98%)	0/2(0%)	35/35(100%)	8/8(100%)	38/39(97%)	0/0
b	Question 2	0/0	32/43 (74%)	8/12 (67%)	25/30 (83%)	26/32(81%)	4/6(67%)	43/43(100%)	0/2(0%)	36/36(100%)	6/7(86%)	39/39(100%)	0/0
c	Question 3	0/0	30/43 (70%)	4/9 (44%)	20/33 (61%)	32/32(100%)	6/6(100%)	42/43(98)	0/2(0%)	30/30(100%)	1/13(8%)	39/39(100%)	0/0
	Errors committed												
	On ENL			Making FJ in the place of BJ and vice versa; increasing numbers from right to left and splitting numbers as in horizontal number sentence		Making FJ in the place of BJ and vice versa; increasing numbers from right to left; expanding as a way of splitting numbers		Careless mistake of not executing all jumps					
	On CM	Working from left to right; not add the number carried over; Subtracting the minuend small digit from subtrahend big digit instead of borrowing		Incorrect CM; not add the number carried over; Subtracting the minuend small digit from subtrahend big digit instead of borrowing; unrealistic answers		Incorrect addition / subtraction due to other carrying / borrowing problems;				Addition / subtraction error			

4.3.1 Lesson 1

The objective for lesson 1, conducted on the 2nd of September 2015 with forty three learners in attendance, was to assist learners in working with the join problems across all three sub categories of result, change and start unknown. It was evident from the pre test that the column model dominated learner work so it was deemed beneficial to lead learners from what they were familiar with to something unfamiliar. Therefore eighty percent (80%) of lesson time (about 45 minutes out of 1 hour) was spend discussing learner column models and its associated strategies such as column addition and column subtraction, the remaining twenty percent was used to introduce learners to the empty number line model and its associated strategies of forward / backward jumps.

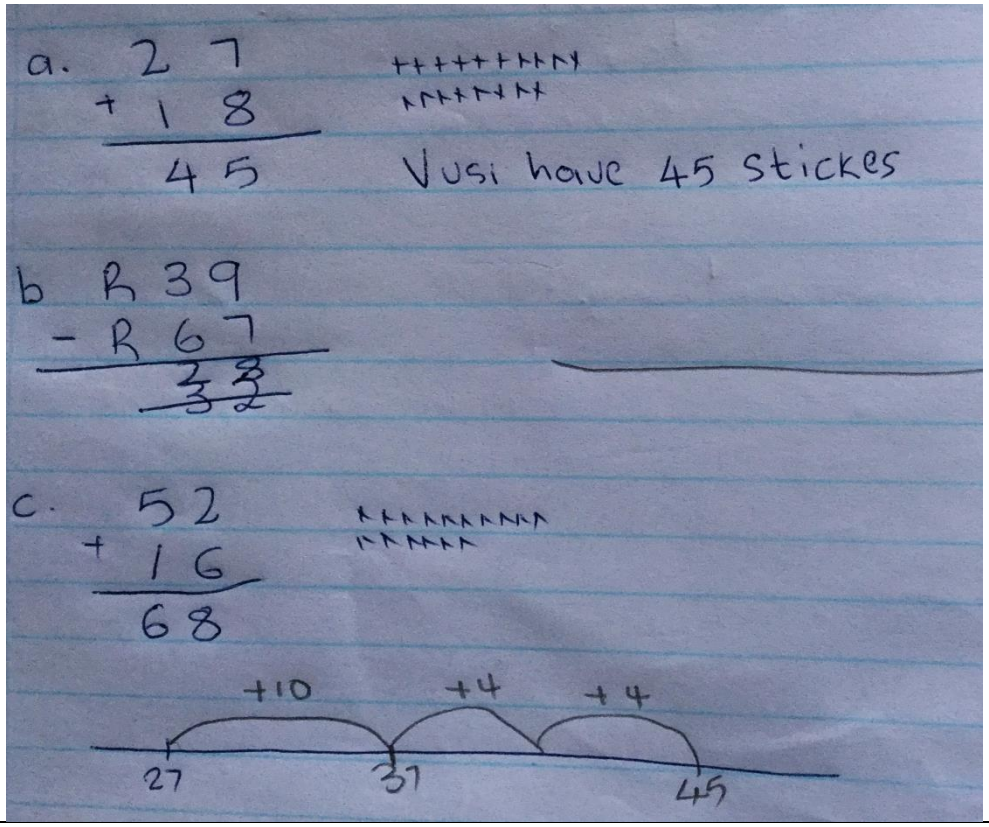
Three word questions were given and all learners used the column model to solve the problems and none used the empty number line as was in the pre test. As was in the pre test, a lot of errors could be noticed from learner books, preventing them from working up to the correct answers. Critical errors noticed were such as working from left to right instead of from right to left on the column model and hence it became difficult for learners to carry or borrow correctly where necessary. Other errors involve not adding the number carried over during the column addition with carrying strategy or merely subtracting the minuend small digit from subtrahend big digit instead of borrowing.

Lesson 1 questions:	Lesson 1A questions:
1. Vusi had 27 football stickers. His friend gave him another 18 stickers. How many stickers does Vusi have? 2. Toni had R39 in her purse. He sister gave her more money. She now has R67 in her purse. How much money did her sister give her? 3. Thabo bought a pair of jeans in the sale. At sale price the jeans cost R52. There was R16 off in the sale. How much did the jeans cost before the sale?	4. Vusi had 36 football stickers. His friend gave him another 28 stickers. How many stickers does Vusi have? 5. Toni had R29 in her purse. He sister gave her more money. She now has R52 in her purse. How much money did her sister give her? 6. Thabo bought a pair of jeans in the sale. At sale price the jeans cost R55. There was R18 off in the sale. How much did the jeans cost before the sale?
Palesa's work: A few learners such as Palesa did not seem to know where to begin with word problems. Upon assistance she attempted to use the horizontal number sentence whereby she split "27 + 18" as "2 + 1" and "70 + 80". She then decided to add "70to 2" and "80 to 1" to obtain "72" and "81" respectively.	



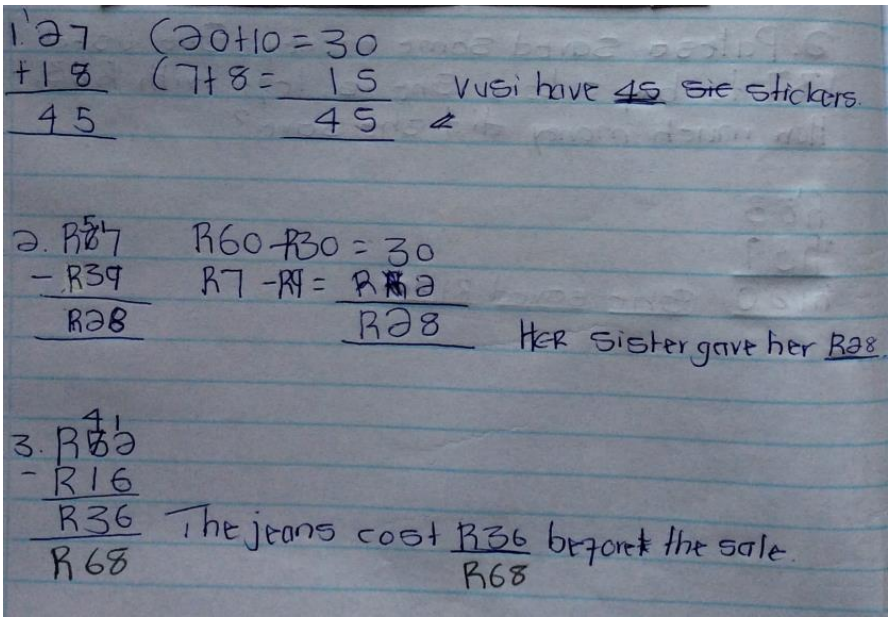
Jabu's work:

Jabu worked out the first problem correctly with the help of tallies but went on to subtract the minuend small digit from subtrahend big digit instead of borrowing with the numbers wrongly arranged. After the discussion he simply just cancelled the wrong answer and wrote the correct one without considering the working. However, Jabu was later willing to try use the empty number line successfully for question "a".



Rebontsheng's work

Rebontsheng was not willing to try working with the empty number line and was confident using the column model.



As a result eighty percent of lesson time was spend discussing the column model and its associated strategies of addition with carrying / subtraction with borrowing and these results are captured in table 4.3 above. Out of a total of forty three learners, all (100%) used the correct column model for questions 1(join result unknown), with thirty seven (86%) attempting the correct column addition strategy with carrying and thirty four (79%) appropriately working up to the correct answer. In question 2, (join change unknown) forty (93%) used the correct column model, with thirty six (84%) attempting the correct column subtraction strategy with borrowing and thirty two (74%) working up to the correct answer. In question 3 (join start unknown), thirty five (81%) used correct model, thirty three (77%) used correct strategies and thirty (70%) worked up to the correct answer.

The remaining 20 % of lesson time was used to introduce learners to working with the empty number line. Some learners were eager to re-do questions 1 to 3 using the empty number line but the results were not captured. It is recorded in table 4.3 above that none of the forty three learners used the empty number line model and strategies with nil correct answers recorded because none of the learners had used the empty number line in their initial working.

In the third part of the lesson some learners that depicted various ways of working were requested to write their work on the board so as to share their working with the rest of the class. A whole class discussion followed, with learners sharing their knowledge, highlighting the correct carrying / borrowing and splitting strategies. Working with the same three questions (one at a time), learners were introduced to working with the empty number line model. A string of beads was placed up on the board, across the wall to remind learners on the order of numbers. Initially a lot of the learners were uncomfortable using the empty number line but seemed to improve with time.

An additional three problems of the same wording but with different number values were given for individual homework at the end of the lesson. Learners were encouraged to use the empty number line model. A few learners worked using the empty number line with the majority sticking with the column model, others used the empty number line model first, and the column model later to verify the correctness of their answers. In a way learners seemed to find it difficult to switch to the new model.

In summary, lesson 1 of the intervention discussed all join word problems, result, change and start unknown. There was more emphasis on learners sharing their knowledge and being actively involved, starting with what they know was important. The solutions were first discussed using the column model for learners to be aware of their difficulties and thereafter the empty number line. About fifteen learners were interested and eager to work using the empty number line model.

4.3.2 Lesson 2

Lesson 2 was conducted on the 4th of September 2015 with forty two learners in attendance, with the aim of assisting learners on working with the separate category of word problems, namely, result, change and start unknown. Further the lesson was aimed at encouraging them to use the empty number line as their main model in doing addition and subtraction. As was with lesson 1, three word problems were given. Problem 1 was tackled together as a class using the empty number line but then erased. Learners were encouraged to work individually using

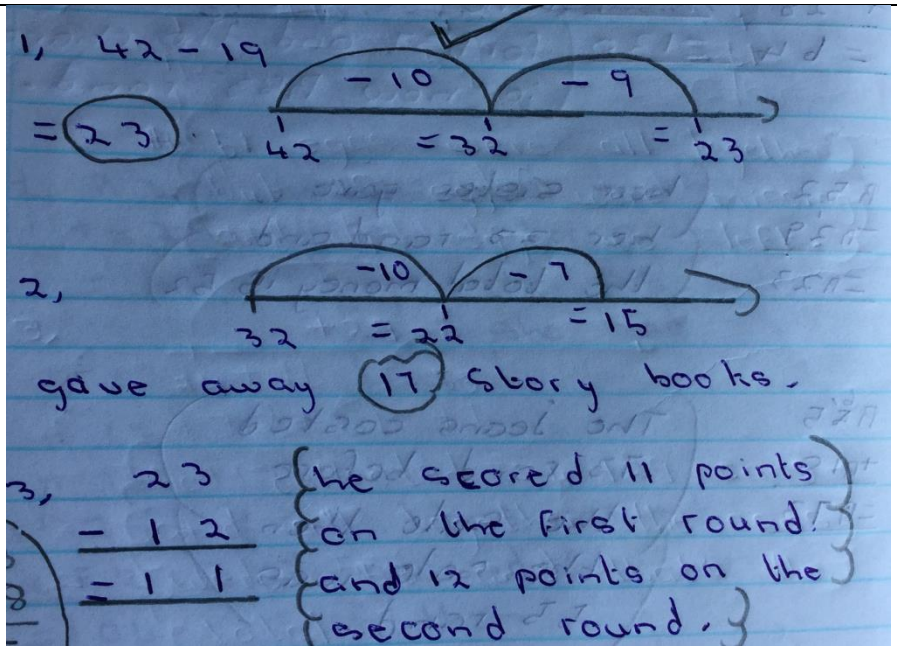
the empty number line to do problems 1, 2 and 3 for about ten minutes, thereafter worked in pairs for some time to share their solutions. This was followed by another whole class discussion.

Half of the discussion time was spend on the empty number line model with its forward / backward counting strategies and the other half on the column model with its addition / subtraction strategies of carrying and borrowing as more than half of the learners were still using the column model so as to find a balance between what was more familiar to the learners and moving to what is not so familiar. Further it was highly anticipated that some learners would not easily switch to using the empty number line model hence the gradual approach.

Twelve learners worked out their solutions for questions 1 and 2 using the empty number line only, with nine in question 3. Common errors notice was making forward jumps in the place of backward jumps and vice versa; drawing the number line with numbers increasing from right to left or splitting numbers in hundreds, tens and units as in horizontal number sentence, rigid at times. The rest first used the column model before the empty number line model.

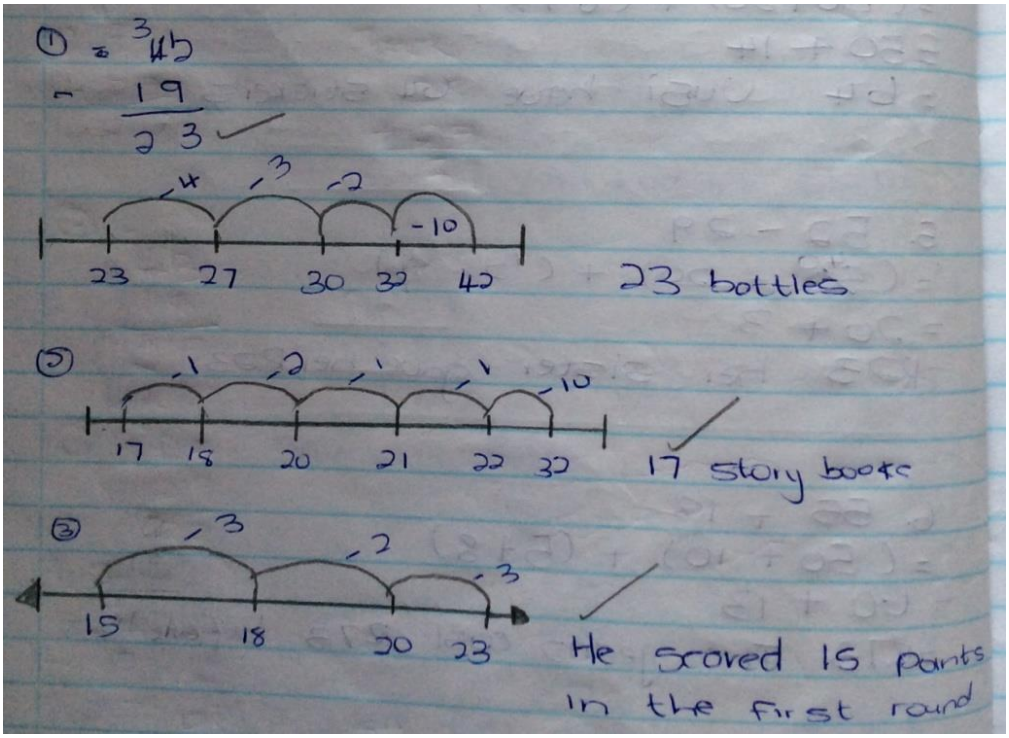
Thato's work

Thato seems to be making forward jumps in the place of backward jumps and vice versa. Further his work indicates an increase in numbers from right to left on the number line and a split of numbers as in horizontal number sentence rather than convenient jumps for easy calculation. But conveniently he works up to the correct answers.



Nelsa's work

Some learners such as Nelsa preferred to use the column model first, then the empty number line model. To a greater extend their number of correct answers seemed improving.



Nine out of twelve learners (75%) the correct empty number line model for question 1, with all of them applying the forward / backward jumps correctly and arriving at the correct answer. Ten out of twelve (83%) used the correct empty number line model for question 2, with nine applying the forward / backward jumps correctly and eight of them arriving at the correct answer. Seven out of nine (78%) also used the correct empty number line model for question 3 with six (67%) applying the forward / backward jumps correctly and four (44%) of the nine working up to the correct answer.

Thirty learners (71 %) used the column model for question 1, with all (100%) applying the column addition / subtraction correctly and working up to the correct answer. In question 2, out of thirty learners twenty nine (97%) used the correct column model, twenty seven (90%) used the correct strategies and twenty five (83%) worked up to the correct answer. In question 3, out of thirty three, twenty five (76%) used the correct column model twenty two (67%) used the correct strategies and twenty (61%) worked up to the correct answer.

At the end of the lesson in wrapping up the various ways of making jumps on the number line were discussed to highlight more strategic and efficient jumps on the number line. Additional three problems with the same wording but different numerical values were given as homework. Comparing lesson 2 with lesson 1, nine, ten and seven learners used the empty number line model correctly with nine, nine and six making the correct jumps and nine eight and four worked up to the correct answers for questions 1, 2 and 3, a great improvement from having started with nothing.

In summary, both the column model and empty number line were discussed and more learners came to understand why it is important to work with the number line than the column model. In addition more learners (twelve – 28%) seemed to gain confidence and became more interested in working with the little stories within the word problems.

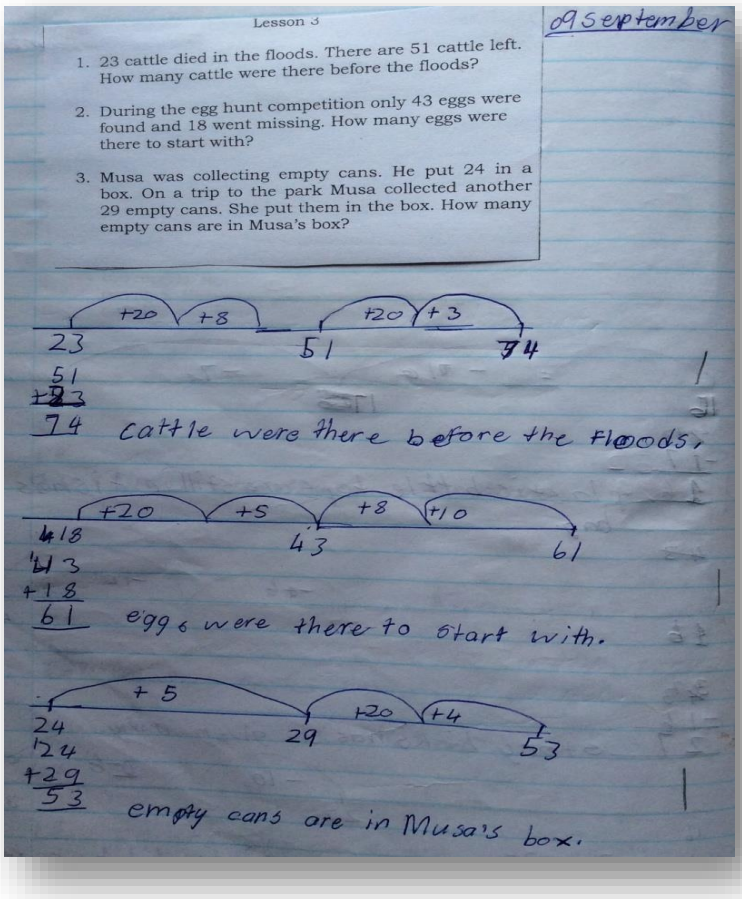
4.3.3 Lesson 3

Lesson 3 was conducted on the 9th of September 2015 with thirty eight learners in attendance was aimed at improving learner working with the separate, start unknown questions with special emphasis on the use of the empty number line. As was with lessons 2 and 3, three word problems were given for working out within the class and an additional three for homework. It was noticeable that learners mainly used the empty number line model accompanied by the column model. It is evident that learners are well orientated to the use of the vertical column model for the addition and subtraction as they continued to use it even when they had other options. And it is therefore highly unexpected that the column model could be discarded completely. Hence it became essential for learners to improve their strategies associated with the column model as an addition to the research objectives by emphasising the relationship amongst numbers as exists on the empty number line. Eighty percent of the lesson was spend discussing the working on the empty number line model with its strategies of splitting, stinging and varying and twenty percent on the vertical column and its related strategies of column addition and column subtraction.

Thirty two out of thirty eight learners only worked using the empty number line, another six started with the column model before working with the number line. The table indicates that of the thirty two learners, twenty eight, twenty seven and thirty two used the number line model correctly, with twenty seven, twenty six and thirty two executing the correct forward and backward jumps and finally arriving at the correct answers for questions 1, 2 and 3 respectively. For the other six the focus was how they worked with both models even though column was their main model. All six used the two models correctly, and went further to apply the strategies correctly, but two of them made mistakes with the backward count and in a way manipulated the answers on the number line to resemble those of the column model.

Palesa's work

The majority of learners used the empty number line but some such as Palesa used the column model to verify their answers. Verification of answers was one good practice that was adopted by a large number of learners.



On the overall learners seem to be developing a good sense on the use of strategies even though the number range in the word problems is still lower (units and tens). The development appears in the form of learners improving from showing numbers increasing from right to left on the number line and no longer from left to right or showing unrealistic numbers as answers. More learners became challenged to use the empty number line alone. In comparing learner performance in lesson 3 with 2, twenty eight, twenty seven and thirty two out of thirty two learners used the empty number line model, with twenty seven, twenty six and thirty two doing the correct jumps and working up to the correct answers for questions 1, 2 and 3 respectively, an increase of sixteen (27 – 9), eighteen (26 – 8) and twenty eight (32 – 4).

4.3.4 Lesson 4

Lesson 4, conducted on the 11th of September 2015 with forty five learners in attendance was aimed at learners improving on the separate start unknown problems using only the empty number line model; hence the number range is kept at ones and tens. Three word questions were given and more learners used the empty number line before using the vertical column model to solve the problems. As an endeavour to emphasize the value in the use of the empty number line no time was spend discussing the column model; all the time was spend on the empty number line with its associated strategies. Forty three learners(96%) worked with the empty number line, modelled the problems and worked on the strategies correctly and worked up to the correct answer for all three problems, 1 (separate result unknown), 2 and 3 (separate start unknown) with the exception of one learner who miscalculated the jumps in question 3 and hence got the incorrect answer. The other two learners started with the correct column model, but one of them just worked quickly and committed some careless mistakes, and both ended up with three incorrect answers each.

Table 4.3 reveals that almost all forty three learners answered using the correct model, strategies and answer using the empty number line when compared with the two that opted to use the column model and ended up with all the incorrect strategies and answer. It appears that learners have started performing better with the separate result unknown questions as was given in questions 1 and 3 due to the use of the empty number line model.

4.3.5 Lesson 5

The aim of lesson 5, conducted on the 16th of September 2015 with forty three learners in attendance, was aimed at improving learner work with join and separate, change unknown word problems with a higher number range (in the hundreds). Problems given in questions four to six were not of a wording repetition of the problems in one to three as learners seemed to be more comfortable working with word problems. Question 1 is on join change unknown, question 2 on separate change unknown and question 3 on separate result unknown. More learners went back to using the column model which was highly unexpected. Only thirty five, thirty six and thirty worked with the empty number line and eight, seven and thirteen went back

to using the column model for the first three problems. All those who used the empty number line used the correct model and strategies and all worked up to the correct answers. Those using the column model also worked with the correct model and strategies but one of them failed to work up to the correct answer in question 2 and another two in question three.

Learners seemed more comfortable working with smaller numbers (ones and tens) on the empty number line as was seen in the previous questions but as the numbers got bigger (hundreds as in question 5) learners tended to use the empty number line with another type of model mainly column. Even though a lesser number of learners (thirty five, thirty six and thirty) as opposed to forty three in lesson 4, used the empty number line; but most important, all of them used the correct model with correct strategies and worked up to the correct answer.

4.3.6 Lesson 6

Lesson 6 was conducted on the 18th of September 2015 with thirty nine learners in attendance. Problems remained at six with the number range still up to the hundreds as was in lesson 5; learners seemed to be more used to working with word problems as they worked faster. Question 1 is on separate start unknown, question 2 on separate change unknown and question 3 on the join start unknown. Learners were compelled to use the empty number line only. All thirty nine learners worked well with the empty number line and the forward and backward jumps up to the correct answer, with only one learner who modelled the problem in question 1 incorrectly, hence made jumps in the opposite direction even though her splitting of "100, 30, 5 and 4" are correct, ended with the incorrect answer.

Learners displayed an improvement in the use of the empty number line model and its associated strategies of jumping forward and backwards, counting in two's, three's and five's and splitting bigger numbers in tens and fives especially across all categories of questions. They did not seem deterred by bigger numbers of hundreds and thousands as was given in lesson 6, but improved from the performance in lesson 5.

In summary, learners became introduced to the use of the empty number line during the intervention lessons. Based the results from learners' books there experienced a radical shift from the predominant use of the column model, column addition with carrying and column subtraction with borrowing strategies to the predominant use of the empty number line, backward and forward jumps in the form of splitting, stringing and varying strategies. The intervention session could be regarded as having been successful for two main reasons. First, learners were challenged to work with a model and strategies new to them, but became willing to work with the empty number line, welcomed the knowledge and championed it. Learner performance improved as very few errors were committed with the empty number line, learners split and stringed the numbers and made jumps convenient to suite their understanding. Second, significant improvement is realisable when one compares learner work from the first to the last intervention lesson, ranging from so many incorrect models / strategies with incorrect answers to very few errors with correct models / strategies and more correct answers.

What kinds of changes are seen (if any) in the models and strategies that learners employ after the intervention as compared to before the intervention? Are these changes in extent of success in solving join and separate-type addition and subtraction word problems associated with shifts in models and strategies?

4.4 The post test results

4.4.1 Post-test multiple choice questions (1 to 7) results

Table 4.4 below, similar to table 4.1 above, with five columns details the post test multiple choice questions results. The questions are grouped into the join and separate categories and within each of the categories a further division becomes visible in relation to result, change and start unknown (column one). Column two names the questions and sequences them according to the categories to keep join / separate together. In column three records the word problems which are the given test questions. The number sentence (column four) is a symbolic representation of the word problem. The overall total correct answers (column 4) indicates the total number of learners (out of the total forty that wrote) who answered the question correctly, also expressed as a percentage (within the brackets).

Table 4.4: Post-test multiple choice questions (1 to 7) results

1		2	3	4	
Category of questions		**Questions**	**Word sentence**	**Number sentence**	**Total correct answers**
Join	Result unknown	1	79 ants crawled into a cave. Inside, they found another 149 ants. How many ants were inside the cave?	79 + 149 = ☐	38/40 (95%)
	Change unknown	5	Tumi's taxi travelled 28 km. He was travelling to Polokwane, which was 110 km away. How much further does Tumi's taxi have to travel?	28 + ☐ = 110	22/40 (55%)
	Start unknown	3	Some friends arrived for Shadi's birthday party. 19 more arrived later and now there are 43 friends at the party. How many friends were first to arrive?	☐ + 19 = 43	30/40 (75%)
Separate	Result unknown	7	136 grade 6 children went on a school trip to the zoo. 79 of them came back early to catch a taxi home. How many learners stayed behind?	136 – 79 = ☐	35/40 (88%)
	Change unknown	2	Romeo had R121. He bought a book. Now he has R27. How much was the book	121 - ☐ = 27	32/40 (80%)
	Start unknown	6	Thakane weighs 57 kg after losing 14 kg in the past three months. How much did Thakane weigh before?	☐ – 14 = 57	9/40 (23%)

As was with the pre test in table 4.1 above, performance is still higher on result unknown problems, but also on the join start unknown and separate change unknown, an improvement is visible. Join change unknown and separate start unknown problems show variable performance across the join and separate categories. In summary, learner performance in the multiple choice questions is detailed in table 4.4above indicates a higher learner performance in questions 1 (95%), 7 (88%), 2 (80%) and 3 (75%), lower in questions 5 (55%) and 6 (23%).

4.4.2 Post test "working out" questions (8 to 14) results

Table 4.5 below, similar to table 4.2 above presents the different types of models and strategies employed by the grade six learners in solving addition and subtraction word problems (which are not part of the multiple choice questions), questions 8 to 14 in the pre test (11 excluded as is the distracter). Column 1 records the different models used and the strategies aligned to the model. Column 2, the join categories of questions and column 3 records the separate category of questions. The join and separate category of questions (columns 2 and 3) are further

subdivided into three sub-columns each, of result, change and start unknown sub-categories. The questions, recorded in the sub-columns are arranged according to the join and separate categories of result, change and start unknown and not in their chronological order.

The first row records the join and separate categories with the questions in row 2. The word problem sentence are given in row 3 with the number sentence (records the symbolic representation of word problems) in row 4.Proportion with correct answers in overall group records correct answers per question (worked out using all models together). The models used are highlighted in grey with all strategies used with the model recorded under. In the case of the Column Model the models are recorded as either "correct column model of addition" or "correct column model of subtraction". Strategies leading up to the correct answer records the number of correct answers that resulted from the correct use of the model

Table 4.5: Post test working out questions (8 to 14)

Categories	Join Categories			Separate categories		
Questions	Question 13 Result unknown	Question 8 Change unknown	Question 12 Start unknown	Question 14 Result unknown	Question 10 Change unknown	Question 9 Start unknown
Word Problems	548 turtles were swimming. Another 257 turtles joined them. How many turtles were swimming?	Big elephant weighs 1 500 kg. She weighed 55 kg at birth. How much has she grown?	Jim cycled 327 km for two days. He cycled153 km in the second day. How far did Jim cycle the first day?	A giant caught 235 zebras, but then decided to let 76 go free. How many zebras was the giant left with?	There were 840 spectators at the soccer stadium, some left at half time. There are now 598 spectators at the soccer stadium. How many spectators left at half time?	Vusi saved some money. He used R265 to buy soccer boots. Now he has R178 left. How much money did Vusi save?
Number sentence	548 + 257 = ☐	55 + ☐ = 1500	☐ + 153 = 327	235 - 76 = ☐	840 - ☐ = 598	☐ - 265 = 178
Total number of learners with correct answers out of forty (in overall group)	26/40 (70%)	19/40 (48%)	23/40 (58%)	25/40 (63%)	26/40 (65%)	26/40 (65%)
Number of learners that used the column model out of forty	**37/40 (93%)**	**40/40 (100%)**	**38/40 (95%)**	**39/40 (98%)**	**40/40 (100%)**	**39/40 (98%)**
Number of learners that used correct model of addition / subtraction out of forty	31/40 (78%)	23/40 (58%)	27/40 (68%)	33/40 (83%)	37/40 (93%)	29/40 (73%)
Number of learners that used the correct strategies (listed below) leading to correct answer out of those that used the column model correctly						
Correct answers from using column strategies	23/40 (58%)	19/40 (48%)	23/40 (33%)	24/40 (60%)	26/40 (65%)	25/40 (63%)
Column addition		13/40 (33%)	2/40 (5%)	6/40 (15%)		1/40 (3%)
Column addition with carrying	23/40 (56%)	2/40 (5%)	7/40 (18%)	3/40 (8%)	5/40 (13%)	28/40 (70%)
Column addition with carrying and tallies	2/40 (5%)				2/40 (5%)	2/40 (5%)
Column addition with place value	1/40(3%)					
Column subtraction	4/40 (10%)	5/40 (13%)	4/40(10%)	1/40 (3%)	2/40(5%)	3/40 (8%)
Column subtraction with borrowing	7/40 (18%)	20/40 (50%)	24/40 (60%)	25/40 (63%)	26/40 (65%)	6/40 (15%)
Column subtraction with borrowing and tallies			1/40 (3%)	4/40 (10%)	2/40 (5%)	
Horizontal number sentence model	**3/40 (8%)**		**2/40 (5%)**	**1/40 (3%)**		1/40 (3%)
Correct horizontal number sentence model	3/40 (8%)		2/40 (0%)	1/40 (3%)		1/40 (3%)
Strategies leading to correct answer	3/40 (8%)		0/40 (0%)	1/40 (3%)		1/40 (3%)
Horizontal addition with place value	3/40 (8%)		0/40 (0%)	1/40 (3%)		1/40 (3%)
No model						
Group model	2/40 (5%)		1/40 (3%)	5/40 (13%)	4 /40 (10%)	2/40 (5%)
Correct column addition / carrying / tallies	2/40 (5%)					2/40 (5%)
Correct column subtraction / borrowing / tallies			1/40 (3%)	5/40 (13%)	2/40 (5%)	
Column addition with carrying and tallies	2/40 (5%)					2/40 (5%)
Column subtraction with borrowing and tallies			1/40 (3%)	4/40 (10%)	2/40 (5%)	

The models used (highlighted in grey) are the column and horizontal number sentence as was in the pre test, with the group model used more as a strategy than the model, unlike in the pre test, the no model was not used in the post test. Only the major model was considered, that is, the one that led to the answer. Column models are the most common across all problems, 93% in questions 13, 95% in questions 12, 98% in questions 14 and 9 and 100% in questions 10 and 8. The horizontal number sentence was used in more questions than it was in the pre test, namely, questions 13, 12, 14 and 9. The group model was used in all questions except question 8, that is, 5%, 3%, 13%, 10% and 5% in questions 13, 12, 14, 10 and 9 respectively. The number of learners who used the group model has been included within the number of learners who used the column model as the group model was mainly used as a strategy to the column model rather than as a model (not as the main model).

Strategies used along with the column model are column addition, column addition with carrying, column addition with carrying and tallies, column addition with carrying and place value, column subtraction, column subtraction with borrowing and column subtraction with borrowing and tallies. Strategies used along with the horizontal number sentence are horizontal addition (place value). Strategies used along with the group model are addition with carrying and tallies and column subtraction with borrowing and tallies, both incorporated under the column model.

Strategies leading up to the correct answers in question 13 are the column addition with carrying and column addition with place value and carrying addition with carrying. Thirty seven learners out of forty (93%) who used column model for question 13, thirty one (78%), used the correct column model of addition but only twenty three (56%) worked up to the correct answer using the column addition strategy with carrying and two (5%) using column addition strategy with carrying and tallies. In question 8, join change unknown question, eight of the twenty three learners (58%) used the correct column model of subtraction with nineteen (48%) working up to the correct answer using the column subtraction strategy with borrowing.

Similar trends are noticeable throughout the questions. As was in the pre test, proportions using correct strategy are low within the groups selecting the correct model, highlighting the need for more efficient models.

In question 12, join start unknown question, twenty three out of forty learners (58%) answered correctly. There were thirty eight learners (95%) that used the column model, twenty seven learners (68%) used the correct column model out of which twenty two (56%) used the column subtraction strategy with borrowing correctly, and the other one (3%) used the column subtraction strategy with borrowing and tallies to arrive at the correct answer. Two learners (5%) used the horizontal number sentence model with the place value addition strategy but could not arrive at the correct answer. Question 14, separate result unknown question, twenty five out of forty learners (63%) answered correctly. All forty learners but one (98%) used the column model, thirty three learners (83%) used the correct column model out of which twenty four (60%) used the column subtraction strategy with borrowing correctly to arrive at the correct answer. One of the learners (3%) used the horizontal number sentence model with the horizontal addition (place value) strategy to arrive at the answer. Five learners (13%) used the group model with the column subtraction strategy with borrowing and tallies with all working up to the correct answer

In question 10, separate change unknown question, twenty six out of forty learners (65%) answered correctly. All learners (100%) used the column model with thirty seven (93%) used the correct column model out of which twenty six (65%) used the column subtraction strategy with borrowing correctly and arrived at the correct answer. None of the learners used the horizontal number sentence model. Four learners (10%) used the group model with the column subtraction strategy with borrowing and tallies, but only two could work up to the correct answer. The last question (9), separate start unknown question, twenty six out of forty learners (65%) answered correctly. Thirty nine learners (98%) used the column model; twenty nine learners (73%) used the correct column model out of which twenty six (65%) used the column addition strategy with carrying correctly to arrive at the correct answer. One of the learners used the horizontal number sentence model and worked correctly up to the answer. Two learners (5%) used the group model with the column addition strategy with carrying and tallies up to the correct answers.

As was in the multiple choice questions above the predominant use of the column model is evident as indicated above. A higher performance (70%) is noticeable for the join result unknown (question 13), with twenty six learners out of thirty seven answering correctly. A moderate performance 61%, 64%, 65% and 67% has been recorded in questions 12 (join start unknown), 14 (the separate result unknown) 10 (separate change unknown) and 9 (separate start unknown) respectively. A low performance of 48% was recorded in question 8 (join change unknown). According to tables 4.2 and 4.5 above the column model is by far the dominant model in the pre-test as well as in the post test. The other models least used are the horizontal number sentence as well as the group model. A variety of strategies predominantly used more successfully along with the column model are, column addition with carrying, column addition with borrowing. Other strategies used include column addition, column addition with carrying and tallies, column addition with place value, column subtraction, column subtraction with borrowing and tallies, column multiplication and column division.

In summarising performance in the post test is higher than the pre test results, though similar performance trends appear, with the multiple choice questions slightly higher than that of the working out questions, however, the drop in question 13 is highly unprecedented. Similar kinds of models and strategies were used, with performance on the join result unknown questions higher and the join start unknown questions low. The rate of correct answers achieved is much lower compared to the correct model usage and the strategies leading to correct answers. The comparison seems to corroborate the need for more efficient models and much more the need for more efficient strategies.

In conclusion, over all, the post test results indicate an improvement from the pre test in terms of models, strategies and understanding the word problems which could be attributed to the success of the intervention lessons. Apart from the decline of nine percent for question 13, increases of 32%, 28%, 11%, 27% and 14% percentages for questions 8, 12, 14, 10 and 9 respectively have been realised indicating learner improvement with models and strategies, working better with word problems additive relations.

4.5 The delayed post test

The delayed post test was written on 24th November, almost two months after the post test. An additional one recap intervention lesson was conducted before the delayed post test, to check if learners still affiliate to the use of the empty number line. The lesson was roughly a repeat of lessons 1(all join problems) and 2 (all separate problems). Learners were required to show their working on all the multiple choice questions as well as on the working out questions making use of only the empty number line. The results are recorded hereunder.

4.5.1 Multiple choice questions results of the delayed post test

Table 4.6 below, similar to tables 4.1 and 4.4 above, with five columns details the post test multiple choice questions results. The questions are grouped into the join and separate categories and within each of the categories a further division becomes visible in relation to result, change and start unknown (column one). Column two names the questions and sequences them according to the categories to keep join / separate together. Column three details the word problems which are, the test questions. The number sentence (column four) is a symbolic representation of the word problem. The overall total correct answers (column 4) indicates the total number of learners (out of the total forty that wrote) who answered the question correctly, also expressed as a percentage (within the brackets).

Table 4.6: Delayed post-test multiple choice questions (1 to 7) results

1		2	3	4	5
Category of questions		**Questions**	**Word sentence**	**Number sentence**	**Total correct answers**
Join	Result unknown	1	79 ants crawled into a cave. Inside, they found another 149 ants. How many ants were inside the cave?	79 + 149 = ☐	40/40 (100%)
	Change unknown	5	Tumi's taxi travelled 28 km. He was travelling to Polokwane, which was 110 km away. How much further does Tumi's taxi have to travel?	28 + ☐ = 110	34/40 (85%)
	Start unknown	3	Some friends arrived for Shadi's birthday party. 19 more arrived later and now there are 43 friends at the party. How many friends were first to arrive?	☐ + 19 = 43	34/40 (85%)
Separate	Result unknown	7	136 grade 6 children went on a school trip to the zoo. 79 of them came back early to catch a taxi home. How many learners stayed behind?	136 – 79 = ☐	35/40 (88%)
	Change unknown	2	Romeo had R121. He bought a book. Now he has R27. How much was the book	121 - ☐ = 27	40/40 (100%)
	Start unknown	6	Thakane weighs 57 kg after losing 14 kg in the past three months. How much did Thakane weigh before?	☐ – 14 = 57	32/40 (80%)

According to table 4.6, unlike the pre test and post test, tables 4.1 and 4.4 above, performance is high (more than 80%) on all questions (result, change and start unknown problems). Learner performance in the multiple choice questions as detailed in table 4.6 above indicates that in questions 1 (100%) and 7 (88%) for the result unknown, 5 (85%) and 2 (100%) for the change unknown and 3 (85%) and 6 (80%) for the start unknown questions, a great improvement across all questions.

Table 4.7: Delayed post test models and strategies for questions 8 to 14

Categories	Join Categories			Separate categories		
Questions	Question 13 Result unknown	Question 8 Change unknown	Question 12 Start unknown	Question 14 Result unknown	Question 10 Change unknown	Question 9 Start unknown
Word Problems	548 turtles were swimming. Another 257 turtles joined them. How many turtles were swimming?	Big elephant weighs 1 500 kg. She weighed 55 kg at birth. How much has she grown?	Jim cycled 327 km for two days. He cycled153 km in the second day. How far did Jim cycle the first day?	A giant caught 235 zebras, but then decided to let 76 go free. How many zebras was the giant left with?	There were 840 spectators at the soccer stadium some left at half time. There are now 598 spectators at the stadium. How many spectators left at half time?	Vusi saved some money. He used R265 to buy soccer boots. Now he has R178 left. How much money did Vusi save?
Number sentence	548 + 257 = ☐	55 + ☐ = 1500	☐ + 153 = 327	235 - 76 = ☐	840 - ☐ = 598	☐ - 265 = 178
Total number of learners with correct answers out of forty (in overall group)	**40/40 (100%)**	**32/40 (80%)**	**32/40 (80%)**	**37/40 (93%)**	**35/40 (88%)**	**38/40 (95%)**
Number of learners that used the empty number line model out of forty	**37/40 (93%)**	**39/40 (98%)**	**32/40 (80%)**	**37/40 (93%)**	**38/40 (95%)**	**38/40 (95%)**
Number of learners that used correct empty number line model	37/40 (93%)	34/40 (85%)	28/40 (70%)	37/40 (93%)	37/40 (93%)	36/40 (90%)
Number of learners that used the correct strategies (listed below), mainly leading to correct answer out of forty						
Number of learners with correct number line strategies and correct answers	37/40 (93%)	32/40 (80%)	28/40 (70%)	37/40 (93%)	37/40 (93%)	36/40 (90%)
Forward jumps	32/40 (80%)	18/40 (45%)	4/40 (10%)		25/40 (63%)	21/40 (53%)
Backward jumps		3/40 (8%)	19/40 (48%)	26/40 (65%)	11/40 (28%)	
Column addition	1/40 (3%)	5/40 (15%)				2/40 (5%)
Column addition with carrying	2/40 (5%)		1/40 (3%)			10/40 (25%)
Column subtraction		3/40 (8%)			1/40 (3%)	
Column subtraction with borrowing	2/40 (5%)	5/40 (13%)	4/40 (10%)	11/40 (28%)		3/40 (8%)
Number of learners that used the column model out of forty	**3/40 (8%)**	**1/40 (3%)**	**8/40 (20%)**	**3/40 (8%)**	**2/40 (5%)**	**2/40 (5%)**
Number of learners with correct column strategies and correct answers	3/40 (8%)	0/40 (0%)	4/40 (10%)	0/40 (0%)	0/40 (0%)	2/40 (5%)
Number of learners that used correct model of addition / subtraction out of forty	3/40 (5%)	1/40 (3%)	6/40 (15%)	0/40 (0%)	0/40 (0%)	2/40 (5%)
Horizontal addition strategy with place value	3/40 (5%)					
Column addition		1/40 (3%)	2/40 (5%)	2/40 (5%)		
Column addition with carrying	3/40 (5%)		2/40 (5%)	1/40 (3%)		2/40 (5%)
Column subtraction with borrowing			4/40 (10%)		2/40 (5%)	

Table 4.7 above indicates that in the delayed post test two types of models were used, namely, the empty number line model and the column model. The empty number line is the dominant model used by thirty nine learners (98%) in question 8, thirty eight (95%) in questions 10 and 9, thirty seven (93%) in questions 13 and 14 and thirty two (80%) in question 12. Learners that used the correct model are thirty seven (93%) with all (100%) correct answers in questions 13 and 14; thirty four (85%) in question 8 with thirty two (80%) correct answers, twenty-eight in question 12 with all correct answers. All the forward / backward jumps were executed correctly (100%). In addition to the forward / backward jumps some learners used column addition / subtraction along with carrying / borrowing strategies.

Few learners continued to use vertical column model, they are three (8%) for questions 13, one (3%) for questions 8, eight (20%) for questions 12, three (8%) for questions 14, two (5%) for both questions 10 and 9. All three used the model correctly, working up to the correct answer in question 13, the one in question 8 could not work up to the correct answer. In questions 12, six of the eight learners (15%) used the correct model and four (10%) worked up to the correct answer. In questions 14, two of the three learners (5%) used the correct model but none worked up to the correct answer. In questions 10, both learners used the correct model but could not work up to the correct answer and in question 9 both learners used the correct model and worked up to the correct answers.

In summary, significant changes in terms of the models and strategies learners use in solving addition and subtraction word problems have been realised from the pre test to the delayed post test. The empty number line is the predominant model, used by on average more than 90% of learners across all questions. Learners displayed a good knowledge of splitting numbers and making appropriate jumps which contributed to a lot of the correct answers. The strategies employed along with the empty number line were a combination of forward / backward jumps along with strategies associated with column model such as column addition / subtraction. Most interesting is the fact that using the empty number line resulted with more than ninety percent correct answers. A few learners (around five percent) continued to use the column model with its associated strategies, but with a lesser success working up to the correct answers.

CHAPTER 5: THE DISCUSSION

5.1 Introduction

The objective of the discussion chapter is to discuss and highlight the key findings of my research study on the use of models and strategies employed by Grade 6 learners in solving addition and subtraction word problems. These findings will briefly discuss the events in the pre, post and delayed post test and the conclusions arrived at. In addition, the limitations and future recommendations will be given.

This research study was undertaken in an attempt at addressing the difficulties learners experience in solving addition and subtraction word problems within my grade 6 class. Realistic Mathematics Education (RME), that advocates for the provision of 'realistic' contexts (Treffers, 1987) in supporting learners to develop efficient models and strategies (Gravemeijer, 2004) proved to be beneficial as learners seemed to have developed an understanding of number relations and to working flexibly (Carpenter, et al., 1999) in solving addition and subtraction word problems.

The research was conducted in four phases: the pre test, intervention session, post test and delayed post test. The pre test was instrumental in the identification of the models and strategies that the learners were using prior to the intervention lessons. The intervention session was used to support learners in developing appropriate models and strategies, an understanding of number relations and working flexibly (van den Heuvel-Panhuizen, 2008), in solving addition and subtraction word problems. The post and delayed post test were aimed at identifying the shifts with respect to the models and strategies that learners would have adopted at the end of the intervention session and to indicate if the shifts could be associated with the use of the empty number line model.

5.2 Key findings

5.2.1 Multiple choice questions

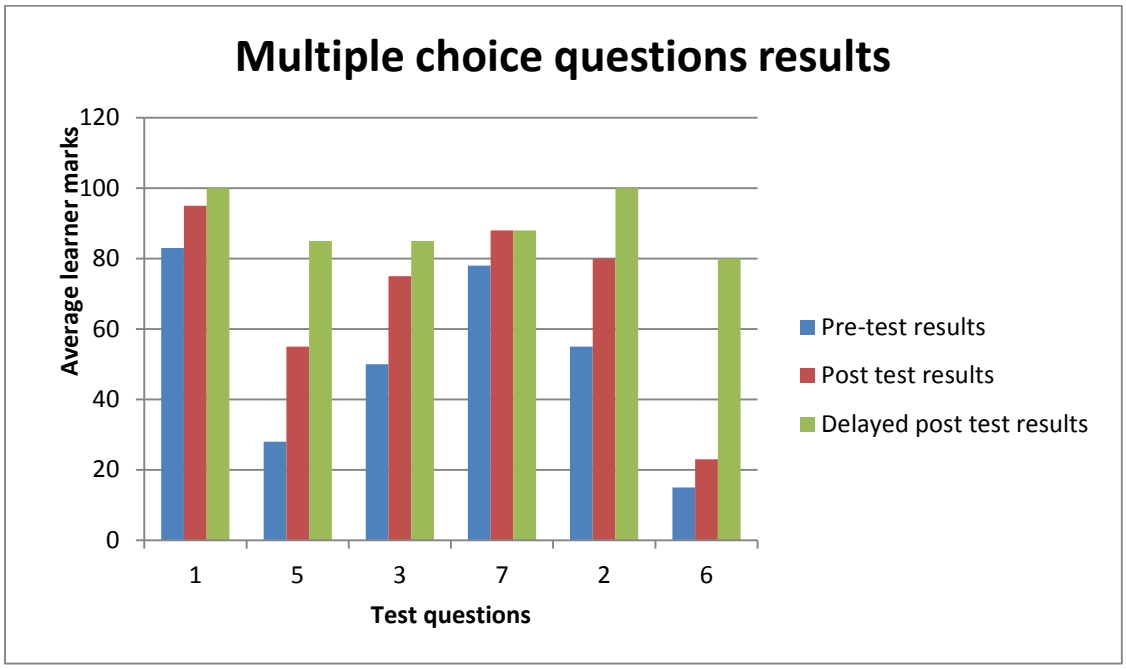
Table 5.1: Comparison of pre test, post test and delayed post tests multiple choice questions (1 to 7)

		Pre-test results	**Post test results**	**Delayed post test results**
Questions	Number sentence	Total correct answers	Total correct answers	Total correct answers
1	79 + 149 = ☐	33/40 (83%)	38/40 (95%)	40/40 (100%)
5	28 + ☐ = 110	11/40 (28%)	22/40 (55%)	34/40 (85%)
3	☐ + 19 = 43	20/40 (50%)	30/40 (75%)	34/40 (85%)
7	136 – 79 = ☐	31/40 (78%)	35/40 (88%)	35/40 (88%)
2	121 –☐ = 27	22/40 (55%)	32/40 (80%)	40/40 (100%)
6	☐ – 14 = 57	6/40 (15%)	9/40 (23%)	32/40 (80%)

Table 5.1 above, with five columns details learner performance in the multiple choice question across all the three test sittings which are the pre test, post test and delayed post test. Column 1 records the test questions, arranged according to the join and separate categories of result, change and start unknown. column 2, the number sentence which is the symbolic representation of the word problem, columns 3, 4 and 5 records the pre test, post test and delayed post test results respectively.

Figure 2 below illustrates the multiple choice questions results comparison of the pre test, post test and delayed post test in the form of bars. The test questions are sequenced similar to those in table 5.1 above, according to the join and separate categories of questions, join and separate. Looking at the three bars over all the questions there is an upward movement starting from the pre test (left hand side) to the delayed post test (right hand side) indication increases in learner performance.

Figure 2: Multiple choice questions results



Detailed in table 5.1 and figure 2 above, is a success story on the progression in learner performance ranging from lower in the pre test, moderate through the post test up to much higher in the delayed post test of the multiple choice questions. Looking at the individual questions, learner performance was higher in the pre test for the result unknown questions due to what is referred to as the direct action involved (Carpenter et al., 1999; Clements & Sarama, 2009) as they could easily be portrayed (acted). Further performance was varied for the other questions but lower with the join change unknown problems as was indicated by Carpenter et al., (1999) and Tshesane (2014) that generally learners tend to find the change and start unknown problems more difficult as the direct action involved is cumbersome. In the post test significant increases in learner performance (on average more than ten percent across all questions) were realized, with much higher increases in the delayed post test. Similar increases in performance are noticeable in the *working out* questions as indicated in table 5.2 and figure 3 below.

5.2.2 Working out questions

Table 5.2: Comparison of pre, post and delayed post test *working out* questions

Categories	Join Categories			Separate categories		
Questions	Question 13 Result unknown	Question 8 Change unknown	Question 12 Start unknown	Question 14 Result unknown	Question 10 Change unknown	Question 9 Start unknown
Pre test results						
Correct column model	34/38 (89%)	8/38 (21%)	21/39 (54%)	37/40 (93%)	35/39 (89%)	26/40 (65%)
Proportion with correct answers	30/38 (79%)	6/38 (16%)	13/39 (33%)	21/40 (53%)	15/39 (38%)	21/40 (53%)
Post test results						
Correct column model	31/37 (84%)	23/40 (58%)	27/38 (71%)	33/39 (85%)	37/40 (93%)	29/39 (74%)
Proportion with correct answers	26/37 (70%)	19/40 (48%)	23/38 (61%)	25/39 (64%)	26/40 (65%)	26/39 (67%)
Delayed post test results						
Correct empty number line model	37/40 (93%)	39/40 (98%)	32/40 (80%)	37/40 (93%)	38/40 (95%)	38/40 (95%)
Proportion with correct answers	37/37 (100%)	32/39 (82%)	28/32 (88%)	35/37 (95%)	34/38 (89%)	36/38 (95%)

Table 5.8 above, with seven columns details learner performance in the working out questions across all the three test sittings which are the pre test, post test and delayed post test. Column 1 records details pertaining to the questions in a number of rows for the different tests. These are the correct column model used, and the proportion with correct answers for both the pre test and post test and the correct empty number line model and proportion with correct answers for the delayed post test. The test questions, arranged according to the join and separate categories of result, change and start unknown are recorded in columns 2 to 7.

Figure 3: Working out questions results

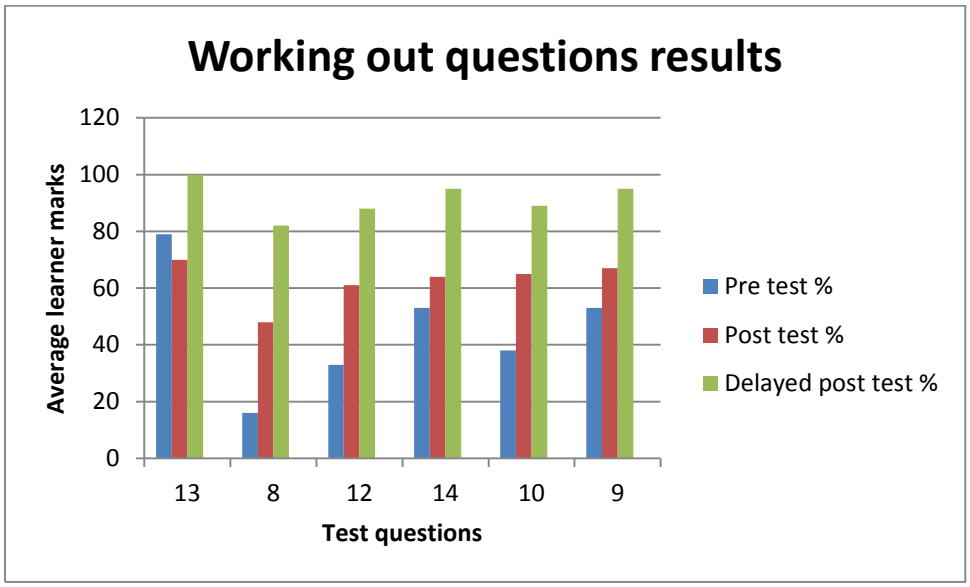


Figure 3 above illustrates the working out questions results comparison of the pre test, post test and delayed post test in the form of bars. The test questions are sequenced similar to those in figure 2 above, according to the join and separate categories of questions, join and separate. Looking at the three bars over all the questions there is an upward movement starting from the pre test (left hand side) to the delayed post test (right hand side) indication increases in learner performance, similar to what is recorded for the multiple choice questions above.

The pre test was characterized by the predominant use of the column model with the column addition with carrying and column subtraction with borrowing strategies. A higher performance is recorded for the join result unknown problems with the lowest recorded in the join change unknown with the others recorded as moderate. In the post test, the predominant use of the column model was pursued, again with the dominant strategies of column addition with carrying and column subtraction with borrowing. However, significant increases in learner performance (on average more than ten percent across all questions) were realized with the join result unknown question realizing a higher performance and the join change unknown remaining low but ten percent higher. The significant increase could be attributed to the use of the empty number line (successfully introduced during the intervention lessons).

There appears to be similar achievements trends in the multiple choice and working out questions across all test sittings. As was with the multiple choice questions, evidence from table 5.2 and figure 3 indicates that in the pre test, learners appeared to work better with the both join and separate result unknown questions, and struggled with both join and separate change and start unknown questions. According to Carpenter, et al. (1999) the start unknown problems harder as learners do not know where to begin, but equally so change unknown problems also posed difficulties as was observed, especially from the pre test. In the post test, achievement ratings improved, learners appeared to work better with the result unknown questions as well as with the start unknown questions. Also in the delayed post test, with much improved rating, outstanding achievement across all questions learners appeared to work best with the result unknown questions and less best with the start unknown questions.

Learners predominantly used the column model in the pre test, but became introduced to the use of the empty number line model in the intervention lessons. For some learners the shift from the column model to the empty number line was instant, but for others it took much longer. In the pre test the column model appeared to be the default model and learners seemed to struggle mainly with modelling the problems. The use of column model of addition and / or subtraction as was highlighted in chapter 4, was problematic, hence learners resorted to guess work to arrive at the answer. Even though some time was spend working on the column model and its associated strategies in the first and second intervention lessons, the focus of the lessons was on creating a platform for learners to confidently engage in a discussion and sharing of knowledge. The reason why it took longer for other learners appears to be the use of the column model (as was indicated in chapter 2), together with its associated strategies of column addition / column subtraction tends to stifle the development of number sense, structure of numbers and their relationship (Kamii & Dominic, 1998).At the end of lesson six (the last intervention lesson) almost all learners appeared to have successfully shifted.

In the post test learner had an option on the models and strategies to use; it became such a shock when the post test appeared just like the pre test with the predominant use of the column model and no empty number line model. Thompson (1995) indicates that the use of column strategies undermines the learner's ability to fully grasp the role of addition and subtraction and their understanding of place value. Seemingly learners become stuck with the use of the column

model and are independently intimidated to use other ways of working such as the empty number line despite implementing the empty number line successfully over six weeks. As a result learners were required to use the empty number line in the delayed post test. Further increases of about one hundred percent achievement levels in the delayed post test could further be attributed to the use of the empty number line along with its strategies of forward and backward jumps.

It was highly unexpected that learners would revert back to predominantly using the column model in the post test as was in the pre test, especially after demonstrating success in the use of the empty number line model. The fact that some learners were seen reverting to column methods in the post test despite having experienced more success with the number line, points out the harmful effects on the reliance on column algorithms as indicated by Kamii & Dominic (1998). Therefore children need to be supported to develop models in a bottom-up approach (invented by learners themselves) as active participants within the classroom as indicated by Gravemeijer & Doorman (1991). One could explain reverting back to the column model as an indication of rote ways of working with word problems. Six weeks of intervention was a very short time to can undo the use of the column model, learners needed more time, and probably one year could yield better results. If as teachers we could learn to build a good foundation for our learners as advocated through RME, we would then encourage confidence in learners working with word problems.

Even though learners continued to use the column model in the post test, improvements of on average ten percent across all questions were realised in both the multiple choice as well as working out questions. The improvement could be attributed to learners having a better understanding of word problems (Murray, 2010). The use of the column model requires learners to use the column addition and column subtraction strategies as was noticed in the pre and post tests. Even though learners appeared to use the correct column model the application of the column addition and column subtraction strategies prevented learners from working up to the correct answers. In addition learners appeared to experience difficulties on computations and procedures (Kamii, Kirkland & Lewis, 2001), that is, working with column addition and column subtraction strategies involving carrying and borrowing (see chapter 4), which indicates the lack of meaningful understanding. These difficulties seemed to prevent learners from arriving

at the correct answers (Thompson, 1994), as witnessed by the lower performance, hence the necessity for more efficient models and strategies.

The intervention session, based on RME, a domain-specific instructional theory promoting the use of the empty number line as an emergent model of and for learning (Gravemeijer, Lehrer, Van Oers & Verschaffel, 2002), away from algorithms and computations (Greer, 1997; Kilpatrick, Swafford & Findell, 2001) and knowledge. Throughout the intervention session, learners were afforded an opportunity to work with word problems in authentic contexts (Gravemeijer, 2002) so as bring the mathematics home for the learner. In addition learners were asked to draw pictures or set up their models in ways they could understand so as to make sense of the worded problems through the use of Big Books approach (Askew, 2004). Askew encourages teachers to support the learners to make sense of the problem through asking clarifying questions, and becoming interested in their models and strategies, hence the interactive class discussions.

In lesson 1, learners were predominantly using the column model, characterized by a lot of errors as was also hinted by Tshesane (2014). Errors such as not adding carried number over to the next digit and just subtracting the minuend small digit from the subtrahend big digit instead of borrowing were noticeable that prevented learners from working up to the correct answers. In addition learners appeared to experience difficulties on operating on single digit numbers at a time, with tens and units as independent from other digits, forgetting to add the numbers carried over or reduce the number borrowed from. As a result, learners did not seem to have a sense of the relative size or position of numbers (Thompson, 1994) looking at the nature of errors. These difficulties seemed to prevent learners from arriving at the correct answers and hence the necessity for more efficient models and strategies. It became evident at the end of the second intervention lesson that some learners, after working out their solutions using the empty number line, had become interested in verifying the correctness of the their answers by using other ways, something they did not do while working exclusively with column addition. There was also an indication that the use of the empty number line encouraged a new way of solving word problems correctly.

In the delayed post test learners predominantly used the empty number line model with increased performance which could be explained as an improvement in development an understanding of number relations through the use of efficient models and strategies (Gravemeijer, 2004) attributed to the use of the empty number line model. Research in primary schools, according to by van den Heuvel-Panhuizen (2008), has indicated that one reason for the difficulty with additive word problems relates to the development of appropriate and efficient models and strategies as was indicated in chapter 2. Hence the advocacy for the use of the empty number line in the intervention lessons. As in Tshesane's (2014) study, the results improved significantly; however, several problems were noticeable regarding the setting up the model. In certain instances learners seemed to experience difficulties with strategies related to the line model, and hence reverted back to the column model and column addition / subtraction strategies.

In summarizing the key findings, the delayed post test portrayed a radical shift in terms of the models and strategies employed and the increase in performance. The shifts recorded are in terms of the models and strategies, from the use of the column model along with its associated strategies of column addition / subtraction in the pre test and post test to the use of the empty number line model in the delayed post test along with its strategies of forward / backward counting, splitting, stringing and varying of numbers which resulted in the significantly improved achievement of up to one hundred percent in some of the questions.

5.3 Conclusion

In the pre test learners predominantly used the vertical column model along with its column addition / column subtraction strategies and the achievement was lower, on average forty five percent. The intervention lessons introduce learners to the use of the empty number line model with the aim of improving additive number relations. The lessons could be regarded as successful as all learners were using the empty number line model by the end of lesson six, an improvement from the use of the vertical column model. In the post test, written a week after the last intervention lesson learners continued the predominant use of the vertical column model along with its column addition / column subtraction strategies, but the achievement was higher, with an average of about sixty percent. The delayed post test, characterized by the predominant

use of the empty number line model along with its strategies of forward / backward counting, splitting, and stringing of numbers recorded a radical shift in achievement. The achievement, significantly higher, with an average of ninety percent could be attributed to the predominant use of the empty number line model as promised in the RME theory.

The pre test results indicated an absolute learner orientation towards the vertical column model hence the need to perfect its carrying / borrowing strategies but the strategies associated with the empty number line proved to be more sophisticated, advanced and produces much better results as indicated by Gravemeijer (2004). These strategies involve counting forward and / or backward, in a variety of patterns towards the development of higher mental strategies (Carpenter, et al., 1999) as was witnessed in the post and delayed post tests.

This research has clearly demonstrated to me the harmful effects of algorithms without appropriate contexts (Kamii & Dominic, 1998).Thus, learners were confined to limitations in the form of only using column addition / subtraction strategies regarding additive relations. It is therefore important for us as mathematics teachers to teach mathematics in ways that engage learners and promote mathematical reasoning (Koedinger, Alibali, & Nathan, 2004) and understanding through making sense of the word problem. I have come to the conscious realization that how the child is learning today has direct implications to their future learning and progress. It is therefore necessary that teachers at grades six and below revisit the use of the vertical column model so as to improve learner performance in mathematics.

5.4 Limitations

The limitations as was indicated in chapter 5, the methodology, above might be that as the teacher I am also the researcher at the same time which might in a way cloud the objectivity with other information going unnoticed or becoming biased. The strength of this research study lies in its research questions, which continuously draws my attention to specific issues, to remain objective so as to eliminate the bias. As was indicated in paragraph 3.10 above, learners were actively involved in the discussions to allow more note taking by the teacher and researcher and in addition the lesson events were reported on immediately after the lesson.

Further, this research is a case study as the data consists of a limited number of learners (forty) from only one class in one school and therefore "generalization from a few qualitative cases" (Creswell, 2012, p.550) might not be appropriate. The time for conducting the research was limited, and that impacted on the limited sample size. However, all these limitations would not take away the gains and positive contributions imparted by the study.

5.5 Recommendations for future research

This research provides strong motivation for the use of efficient models and strategies in solving addition / subtraction word problems. Even though RME and the use of the empty number line might not solve all the mathematics problems associated with addition / subtraction, it could prove to alleviate major problems. Recommendation is therefore made on the for further research on the use of the empty number line in lower grades (1 to 5) due to its optimal impact as it might assist in the improvement of mathematics results. The use of the control group could also be helpful in solidifying the findings and for better comparison.

Reference list

Anghileri, J. (Ed.), (2001). Principles and practices in arithmetic teaching: innovative approaches for the primary classroom. Buckingham. Open University Press.

Askew, M. (2004). BEAM's Big Book of Word Problems Years 3 & 4. BEAM Education. London: BEAM.

Askew, M., & Brown, M., (2000). How do we teach Children to be numerate? A professional User Review of UK research undertaken for the British Educational Research Association. King's College: London.

Barnes, H. (2004). Realistic mathematics education: eliciting alternative mathematical conceptions of learners. *African Journal of Research in Mathematics, Science and Technology Education,* 8, 53-64.

Baxter, P. & Jack, S. (2008). Qualitative Case Study Methodology: Study Design and Implementation for Novice Researchers. *The Qualitative Report* Volume 13 Number 4. pp 544-559.

Carpenter, T., Fennema, E., Franke, L.M., Levi, L. & Empson, S.B. (1999). Children's mathematics. The National Council of Teachers of Mathematics, Inc. Heinemann.

Carpenter, T. P., Moser, J. M., & Bebout, H. C. (1988). Representation of Addition and Subtraction Word Problems. *Journal for Research in Mathematics Education*, *19*(4), 345-357.

Clements, D.H. & Sarama, J. (2009). Learning and teaching early maths: The learning trajectory approach. Routledge, New York.

Cobb, P. (1996). Theories of mathematical learning and constructivism: A personal view. In G. Kadunz, H.Kautschitsch, G. Ossimitz, & E. Schneider (Eds.), *Shriftenreihedidaktikdermathematik (Vol. 23): Trendsundperspectiven* (pp. 61–84). Vienna: Verlag Holder-Pichler-Tempsky.

Creswell, J. (2012). W. 1994. Research Design: Qualitative and Quantitative Approaches. *Thousand Oaks*.

Denzin, N.K. & Lincoln, Y. S., (Eds), (2005). The Sage Handbook of Qualitative Research, Thousand Oaks, CA.

Doorman, M. (2002). How to guide students? A reinvention course on modeling motion. In *Common Sense in Mathematics Education. Proceedings of 2001 The Netherlands and Taiwan Conference on Mathematics Education* (pp. 97-114).

Doorman, L. M., & Gravemeijer, K. P. E. (2009). Emergent modeling: discrete graphs to support the understanding of change and velocity. *ZDM*,*41*(1-2), 199-211.

Ensor, P., Hoadley, U., Jacklin, H., Kuhne, C., Schmitt, E., Lombard, A., & van den Heuvel – Panhuizen, M., (2009). Specialising in pedagogic text and time in Foundation Phase numeracy classrooms. *Journal of Education,* No 47.

Freudenthal, H. (1968). Why to Teach Mathematics so as to Be Useful. Educational Studies in Mathematics, 1, 3-8.

Freudenthal, H. (1991). Revisiting Mathematics Education. China Lectures. Dordrecht: Kluwer Academic Publishers.

Foxman, D., & Beishuizen, M. (2002). Mental calculation methods used by 11-year-olds in different attainment bands: A reanalysis of data from the 1987 APU survey in the UK. *Educational Studies in Mathematics*, *51*(1-2), 41-69.

Gelman, R., & Gallistel, C. (1978). Young children's understanding of numbers.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education.* The Netherlands, Utrecht: Freudenthal Institute.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical thinking and learning*, *1*(2), 155-177.

Gravemeijer, K. (2002). Pre-amble: From models to modelling. p. 7 – 22.Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K., & Doorman, M. (1991). Context problems in Realistic Mathematics Education: A calculus Course as an example. *Educational Studies in Mathematics*, *39*(1-3), 111-129.

Gravemeijer, K., Lehrer, R., Van Oers, B. & Verschaffel, L. (2002) (Eds.) Symbolizing, modelling and tool use in mathematics education; p. 145-169. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. & Stephan, M. (2002). Emergent models as an instructional design heuristic. In Gravemeijer, K., Lehrer, R., Van Oers, B. & Verschaffel, L. (Eds.), Symbolizing, modelling and tool use in mathematics education; 145-169. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K., & Terwel, J. (2000). Hans Freudenthal: a mathematician on didactics and curriculum theory. *Journal of curriculum studies, 32*(6), 777 - 796.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and instruction*, *7*(4), 293-307.

Haylock, D. & Cockburn, A. (2008). Understanding Mathematics for Young Children. Sage Publications.

Kamii, C. K., & Dominick, A. (1998).The harmful effects of algorithms in grades 1-4. In L. J. Morrow (Ed.).*The teaching and learning of algorithms in school mathematics: 1998 Yearbook* (pp. 130-140). Reston, VA: National Council of Teachers of Mathematics.

Kamii, C., & Joseph, L. (1988). Teaching Place Value and Double-Column Addition. *Arithmetic Teacher, 35*(6), 48-52.

Kamii, C., Lewis, B. A., & Kirkland, L. (2001). Manipulatives: When are they useful? *The Journal of Mathematical Behavior, 20*(1), 21-31.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington DC: National Academy Press. Chapter 8 (pp 255 – 282).

Koedinger, K.R., Alibali, M., & Nathan, M.J. (2004). Trade-offs between grounded and abstract representations: Evidence from algebra problem solving. *Cognitive Science.*32 (2), 366-397.

Mcintosh, A., Reys, B.J., Reys, R.E. (1992). A Proposed Framework for Examining Basic Number Sense: *For the Learning of Mathematics*, Vol. 12 No. 3, pp 2 – 8, 44.

Murray, H. (2012). Problems with Word Problems in Mathematics. Research Unit for Mathematics Education, University of Stellenbosch. *Learning and Teaching Mathematics*, No. 13, 2012, pp. 55-58

Opie, C. (2004). Doing educational research – a guide to first time researchers. London: Sage Publications.

Schollar, E. (2008). Towards evidence based educational development in South Africa. Final report. The Primary Mathematics Research Project 2004-2007.

Scott, P. H., Mortimer, E. F., & Aguiar, O. G. (2006). The tension between authoritative and dialogic discourse: A fundamental characteristic of meaning making interactions in high school science lessons. *Science Education, 90*(4), 605-631.

Sembiring, R., Hadi, S. & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME.*ZDM Mathematics Education.*Vol 40:927–939

South African Department of Basic Education (DBE). (2014). Report on the Annual National Assessments 2014: Grades 1 to 6 & 9. Pretoria: DBE.

South African Department of Basic Education (DBE). (2011a). Curriculum and Assessment Policy Statement Grades 1-3: Mathematics Policy. Pretoria: DBE.

South African Department of Basic Education (DBE). (2011b). Curriculum and Assessment Policy Statement Intermediate Phase Grades 4-6. Pretoria: DBE. Doi: 10.1787/9789264030206-4-en.

Streefland, L. (Ed.). (1991). *Realistic mathematics education in primary school*. Utrecht, The Netherlands: CD-ß Press/Freudenthal Institute.

Takane, T.B. (2013). Investigating addition and subtraction strategies of Grade 3 learners within the context of a Maths Club. A research report submitted to the Faculty of Humanities, in partial fulfillment of the requirements for the degree of Master of Education University of the Witwatersrand, Johannesburg.

Thompson, D. (2000). Learning Mathematics Vocabulary: Potential Pitfalls and Instructional Strategies. *The Mathematics Teacher, 93*(7), 568-574. Retrieved from http://www.jstor.org/stable/27971502

Thompson, I. (1999). Calculation strategies for addition and subtraction. Part 1.*Mathematics Association*. Vol 28 No. 5, pp. 2-4.

Thompson, I. (1995). Calculation Algorithms: Narrowing the gap between mental and written methods. *Mathematics in School*, Vol 28 No. 4, pp. 3-13.

Thompson, I. (1994). Young children's idiosyncratic written algorithms for addition. Education Studies in Mathematics, Vol. 26 No. 4, pp. 323-345.

Treffers, A. (1987). Three Dimensions.A Model of Goal and Theory Description in Mathematics Instruction the Wiskobas Project. Dordrecht: Reidel Publishing Company.

Tshesane, H. (2014). Exploring Grade 4 learners' use of models and strategies for solving addition and subtraction problems. Research report submitted to the Faculty of Science, University of the Witwatersrand, in partial fulfillment of the requirements for the degree of MSc.

Van den Heuvel-Panhuizen, M. (1994). Van Den Heuvel-Panhuizen, M. (1994). Improvement of (didactical) assessment by improvement of problems: An attempt with respect to percentage. *Educational Studies in Mathematics, 27*(4), 341-372.

Van den Heuvel-Panhuizen, M. (1998). Realistic Mathematics Education as work in progress. *Theory into practice in Mathematics Education. Kristiansand, Norway: Faculty of Mathematics and Sciences*.

Van den Heuvel-Panhuizen, M. (2001). Van den Heuvel-Panhuizen, M. (2001). Realistic Mathematics Education as work in progress, in F.L. Lin (ed.), *Common Sense in Mathematics Education. Proceedings of 2001 The Netherlands and Taiwan Conference on Mathematics Education, Taipei, Taiwan*, National Taiwan Normal University, Taipei, Taiwan, 1–42.

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9-35.

Van den Heuvel-Panhuizen, M. (2005).Van Den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the learning of mathematics, 25*(2), 2-23.

Van den Heuvel-Panhuizen, M. (2008). Van den Heuvel-Panhuizen, M. (2008). Learning from "Didactikids": An Impetus for Revisiting the Empty Number Line. *Mathematics Education Research Journal,* Vol. 20, No. 3, 6-31.

Van den Heuvel-Panhuizen, M., Kühne, C., & Lombard, A. P. (2012).*Learning Pathway for Number: In the Early Primary Grades*. MacMillan.

Van den Heuvel-Panhuizen, M., & Treffers, A. (2009). Mathe-didactical reflections on young children's understanding and application of subtraction-related principles. *Mathematical Thinking and Learning, 11*, 1, 102–112.

Venkat, H. (2013). Using temporal range to theorize early number teaching in South Africa. *For the Learning of Mathematics* 33, 2 (July, 2013) FLM Publishing Association, Fredericton, New Brunswick, Canada

Venkat, H. & Naidoo, D. (2012) Analyzing coherence for conceptual learning in a Grade 2 numeracy lesson. *Education as Change* 16(1), 21-33.

Vygotsky, L. (1978). *Mind in Society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press. Chapters 4 and 6.

Wright, J., Martland, J. & Stafford A.K. (2006). Teaching number Advancing children's skills and strategies. Paul Chapman Publishing (SAGE) London.

Appendix 1: The test

Name of learner: _____ Grade: _____ Date: _____

Question 1: Circle the number sentence that matches the story

1. 79 ants crawled into a cave. Inside, they found another 149 ants. How many ants were inside the cave?
A. 79 + ☐ = 149 B. ☐ + 79 = 149 C. 79 + 149 = ☐ D. 149 = ☐ + 79
2. Romeo had R121. He bought a book. Now he has R27. How much was the book?
A. 121 - ☐ = 27 B. 27 + 121 = ☐ C. ☐ - 27 = 121 D. 121 + 27 = ☐
3. Some friends arrived for Shadi's birthday party. 19 more arrived later and now there are 43 friends at the party. How many friends were first to arrive?
A. 43 x 19 = ☐ B. ☐ + 19 = 43 C. 43 + 19 = ☐ D. 43 = ☐ - 19
4. Percy put tiles up on a kitchen wall. The wall was 9 tiles long and 12 tiles high. How many tiles did Percy need?
A. 9 x 12 = ☐ B. 9 + 12 = ☐ C. ☐ x 9 = 12 D. ☐ - 9 = 12
5. Tumi's taxi travelled 28 km. He was travelling to Polokwane, which was 110 km away. How much further does Tumi's taxi have to travel?
A. 110 + 28 = ☐ B. 28 + ☐ = 110 C. ☐ - 110 = 28 D. ☐ - 28 = 110

6. Thakane weighs 57 kg after losing 14 kg in the past three months. How much did Thakane weigh before?

A. 57 + 14 = ☐
B. 14 + ☐ = 57
C. 57 – 14 = ☐
D. ☐ - 14 = 57

7. 136 grade 6 children went on a school trip to the zoo. 79 of them came back early to catch a taxi home. How many learners stayed behind?

A. ☐ - 136 = 79
B. 136 + 79 = ☐
C. 136 – 79 = ☐
D. 79 x ☐ = 136

Question 2: Choose the correct answer

8. Big elephant weighs 1 500 kg. She weighed 55 kg at birth. How much has she grown?

Show your working in this space	Answer

9. Vusi saved some money. He used R265 to buy soccer boots. Now he has R178 left. How much money did Vusi save?

10. There were 840 spectators at the soccer stadium. Some left at half time. There are now 598 spectators at the soccer stadium. How many spectators left at half time?

11. Every time Marge left a kiwi fruit in her cupboard for a month, the kiwi fruit turned into 8 worms. One month, Marge left 14 kiwi fruit in her cupboard. How many worms did she find?	
12. Jim cycled 327 km for two days. He cycled153 km in the second day. How far did Jim cycle the first day?	
13. 548 turtles were swimming. Another 257 turtles joined them. How many turtles were swimming?	
14. A giant caught 235 zebras, but then decided to let 76 go free. How many zebras was the giant left with?	

Summary of questions

	Join Questions	Separate Questions
Result unknown	1, 13	7, 14
Change unknown	5, 8	2, 10
Start unknown	3, 12	6, 9
Distracters	4, 11	
Lower number range questions: 1 – 7 Higher number range questions: 8 – 14		

Appendix 2: Intervention lessons questions

Lesson 1	Lesson 2
1. Vusi had 27 football stickers. Her friend gave her another 18 stickers. How many stickers does Vusi have? 2. Toni had R39 in her purse. Her sister gave her more money. She now has R67 in her purse. How much did her sister give her? 3. Thabo bought a pair of jeans in the sale. At sale price the jeans cost R52. There was R16 off in the sale. How much did the jeans cost before the sale?	1. Masi collects plastic bottle tops. She put 42 plastic bottle tops in a bag. On the way home the bag split and 19 plastic bottle tops fell out. How many plastic bottle tops are still in Aisha's bag? 2. Naledi has 32 story books. She gives some books away, but keeps 15 books. How many story books has she given away? 3. Matome played a car game and scored 23 points in two rounds. He had lost 8 points in round 2 in penalties. How many points did he score in the first round?

Lesson 3	Lesson 4
1. 23 cattle died in the floods. There are 51 cattle left. How many cattle were there before the floods? 2. During the egg hunt competition only 43 eggs were found and 18 went missing. How many eggs were there to start with? 3. Musa was collecting empty cans. He put 24 in a box. On a trip to the park Musa collected another 29 empty cans. She put them in the box. How many empty cans are in Musa's box?	1. On her twelfth birthday party, Lebo wanted to blow out all candles on her birthday cake. 5 candles did not blow out. How many candles did Lebo blow out? 2. The zoo has 18 rabbits. 7 rabbits have died in the floods. How many rabbits did the zoo have? 3. 37 maidens came to take part in the annual reed dance. 16 other maidens did not arrive for the annual reed dance. How many maidens were to take part in the annual reed dance?
Lesson 1 A	Lesson 2 A
4. Vusi had 36 football stickers. Her friend gave her another 28 stickers. How many stickers does Vusi have? 5. Toni had R29 in her purse. Her sister gave her more money. She now has R52 in her purse. How much did her sister give her? 6. Thabo bought a pair of jeans in the sale. At sale price the jeans cost R55. There was R18 off in the sale. How much did the jeans cost before the sale?	4. Masi collects plastic bottle tops. She put 33 plastic bottle tops in a bag. On the way home the bag split and 17 plastic bottle tops fell out. How many plastic bottle tops are still in Aisha's bag? 5. Naledi has 43 story books. She gives some books away, but keeps 16 books. How many story books has she given away? 6. Matome played a car game and scored 34 points in two rounds. He had lost 16 points in round 2 in penalties. How many points did he score in the first round?

Lesson 3 A	Lesson 4 A
4. 17 cattle died in the floods. There are 44 cattle left. How many cattle were there before the floods? 5. During the egg hunt competition only 36 eggs were found and 17 went missing. How many eggs were there to start with? 6. Musa was collecting empty cans. He put 25 in a box. On a trip to the park Musa collected another 19 empty cans. She put them in the box. How many empty cans are in Musa's box?	4. On her thirteenth birthday, Lebo wanted to blow out all candles on her birthday cake. 5 candles did not blow out. How many candles did Lebo blow out? 5. The zoo has 18 rabbits. 7 rabbits have died in the floods. How many rabbits did the zoo have? 6. 41 maidens came to take part in the annual reed dance. 18 other maidens did not arrive for the annual reed dance. How many maidens were to take part in the annual reed dance?

Lesson 5		**Lesson 6**
1. Andile has been saving some money to buy a bicycle. His grandmother gave him R178. Now Andile has R432 required to buy the bicycle. How much money did Andile save? 2. Aunt Daphne gave Hloni a bag of 124 gums on her birthday. Hloni gave some gums to her friends at school. Now Hloni has 77 gums left in the bag. How many gums has she given to her friends? 3. 7 100 children joined the marathon running. 146 of them could not finish running the marathon. How many children finished the marathon running? 4. Lesedi has finished reading a book. She read 278 pages in August and some pages in September. The book has 560 pages. How many pages has Lesedi read in September? 5. Kabelo had 178 model dinosaurs. His sister gave him 166 model dinosaurs for his birthday. How many model dinosaurs does Kabelo have now? 6. The grade 6 learners collected money in fund raising to buy more computers for the school last week. This week all the other grades together collected R1 758. How much money did the grade 6 learners collect if a total of R5 113 was collected?		1. After losing 139 sheep in the drought, a farmer is left with 464 sheep. How many sheep did the farmer have before the drought? 2. Piggy weighed 231 kg. She lost some weight and now she weighs 197 kg. How much weight has piggy lost? 3. Elephant gained weight of 168 kg in six months. Elephant now weighs 301 kg. How much did Elephant weigh in six months before she gained weight? 4. The caterer brought chairs to put inside the tent for the party. Only 745 chairs could fit inside the tent and 76 chairs were turned back. How many chairs had the caterer brought? 5. There were 1 052 spectators watching the soccer match. 817 spectators came early and 19 more came later. Other spectators came after halftime. How many spectators came after half time? 6. A crate contains 1 235 peaches. Some of the peaches were found to be rotten. Only 987 were good to be eaten. How many peaches were found rotten?

Additional questions A	Additional questions B
7. 1 500 ants went up and down our stoep. Mother sprayed them and many died. Only 155 were left. How many ants died? 8. Uncle Tom bought new tyres, seat covers and petrol for his car and spends R3 005. The seat covers cost R149 and the petrol R678. What was the cost of the new tyres? 9. A shop sold 118 sweets in the morning and some more sweets throughout the day. 400 sweets in one day. How many sweets were sold throughout the day?	7. 5 768 litres of drinking water is wasted when water drops from leaking taps daily in Gauteng. 4 975 more litres of water is wasted daily from burst water pipes. How much water is wasted daily in Gauteng? 8. A lorry delivers some bags of cement to Nathi's hardware. Later that day the lorry delivered 572 bags of cement to Lerato's hardware. How many bags of cement were delivered to Nathi's hardware if a total of 1260 bags were delivered in all? 9. There are 630 birds in a bird sanctuary. Some birds died during winter and 196 survived. Find the number of birds that died.

Appendix 3: Information sheet for learners

INFORMATION SHEET: LEARNERS DATE: 20 August 2015

Dear Learner

My name is Hellen Kanyane and I am a Masters student in the School of Education at the University of the Witwatersrand.

I am conducting research on how to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems using the empty number line.

I write to ask you to consider giving your consent for me to collect data in the form of a written pre-test, scheduled for one hour, six intervention lessons and a post-test, also for one hour, focused on the use of models and strategies. I envisage that the pre-test will happen early September in the current year, the six intervention lessons to be conducted once a week in late September and October and the post-test in late October.

Remember, this work is not for marks and it is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way. I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

Please return the signed slip indicating your consent for participation in the written tests and the six intervention lessons.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

_____

SIGNATURE

NAME: Hellen Kanyane

EMAIL : hellenk59@gmail.com

TEL NUMBER: 011-930-1930(office) or 082 647 2025 (cell)

Learner consent form

Please fill in the reply slip below if you agree to participate in my study called: How to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems

My name is: _____

Permission to review/collect documents/artefacts	**Circle one**
I agree that the pre-test can be used for research purposes only.	YES/NO
I agree that the post-test can be used for research purposes only.	YES/NO

Informed Consent

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____ Date_____

Appendix 4: Information sheet for parents

INFORMATION SHEET: PARENTS DATE: 20 August 2015

Dear Parent

My name is Hellen Kanyane and I am a Masters student in the School of Education at the University of the Witwatersrand.

I am conducting research on how to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems.

My research involves using the empty number-line as an alternative model to develop understanding of number relations in solving addition and subtraction problems. I hereby request your permission to work with your child in Grade 6 in this investigation into learners' use of models for addition and subtraction.

This research mainly uses qualitative research techniques which contain the pre-test, intervention and post-test for both the collection and analysis of data. Participants will first write a pre-test, made up of fourteen word problems scheduled for one hour. The Intervention will be made up of 6 lessons of 1 hour and 30 minutes each, conducted once per week. Similar categories of problems in the pre-test will be given in the post test as well.

The lessons will be conducted at normal times and the topic is part of the normal school curriculum. My research will not disrupt any school activity**.** Your child will not be disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study. Should the learner choose not to participate, they will still do the tests as it is part of the curriculum with feedback but their results will not be analyzed. In cases, where permission is not given for learners to participant they will be placed in another class and the lesson will be taught to them at a convenient time.

I am assuring you that the learners' names and identity and the school's name and identity will be kept confidential at all times and in all academic writing about the study. All research data will be securely stored and will be destroyed between 3-5 years after completion of the research.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

_____

NAME: Hellen Kanyane

EMAIL : hellenk59@gmail.com

TEL NUMBER: 011 866 2856 (home) or 082 647 2025 (cell)

Parent's consent form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called "how to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems".

I, _____ the parent of _____

Permission to review/collect documents/artefacts	**Circle one**
I agree that my child's written work can be used for research purposes only.	YES/NO

Permission for test

I agree to, my child writing the pre-test for this study.	YES/NO
I agree to, my child writing the post-test for this study.	YES/NO

Informed Consent

I understand that:

- My child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- He/she does not have to answer every question and can withdraw from the study at any time.
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____ Date_____

Appendix 5: Information sheet for SGB

INFORMATION SHEET: SGB CHAIRPERSON 20 August 2015

Dear SGB Chairperson

My name is Hellen Kanyane and I am a Masters student in the School of Education at the University of the Witwatersrand.

I am conducting research on how to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems. The use of models and strategies is an attempt to assist learners to develop an understanding of number relations and to work flexibly in solving addition and subtraction word problems. I will particularly focus on the use of the empty number-line as an alternative model for solving addition and subtraction problems. I hereby request your permission to work with your learners in Grade 6 in this investigation into learners' use of models for addition and subtraction.

This research mainly uses qualitative research techniques which contain the pre-test, intervention and post-test for both the collection and analysis of data. Participants will first write a pre-test, made up of fourteen word problems scheduled for one hour. The Intervention will be made up of 6 lessons of 1 hour and 30 minutes each, conducted once per week. Similar categories of problems in the pre-test will be given in the post test as well.

The lessons will be conducted at normal times and the topic is part of the normal school curriculum. My research will not disrupt any school activity. I am assuring you that the learners' names and identity and the school's name and identity will be kept confidential at all times and in all academic writing about the study. All research data will be securely stored and will be destroyed between 3-5 years after completion of the research. All learners who will participate in the study will do it on a voluntary basis. Should the learner choose not to participate, they will still do the tests as it is part of the curriculum with feedback but their results will not be analyzed. In cases, where permission is not given for learners to participant they will be placed in another class and the lesson will be taught to them at a convenient time.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

_____

NAME: Hellen Kanyane

EMAIL: hellenk59@gmail.com

TEL NUMBER: 011 866 2856 (home) or 082 647 2025 (cell)

SGB Consent form

SGB Chairperson
I consent / do not consent for my school to participate in your study.
School name: _____ SGB Chairperson's name: _____ SGB Chairperson's signature: _____ Date: _____

Appendix 6: Principal information sheet

LETTER TO THE PRINCIPAL DATE: 20 August 2015

Dear Principal

My name is Hellen Kanyane and I am a Masters student in the School of Education at the University of the Witwatersrand.

I am conducting research on how to improve the grade 6 learners' use of models and strategies in solving addition and subtraction word problems. The use of models and strategies is an attempt to assist learners to develop an understanding of number relations and to work flexibly in solving addition and subtraction word problems. I will particularly focus on the use of the empty number-line as an alternative model for solving addition and subtraction problems. I hereby request your permission to work with your learners in Grade 6 in this investigation into learners' use of models for addition and subtraction.

This research mainly uses qualitative research techniques which contain the pre-test, intervention and post-test for both the collection and analysis of data. Participants will first write a pre-test, made up of fourteen word problems scheduled for one hour. The Intervention will be made up of 6 lessons of 1 hour and 30 minutes each, conducted once per week. Similar categories of problems in the pre-test will be given in the post test as well.

The lessons will be conducted at normal times and the topic is part of the normal school curriculum. My research will not disrupt any school activity. I am assuring you that the learners' names and identity and the school's name and identity will be kept confidential at all times and in all academic writing about the study. All research data will be securely stored and will be destroyed between 3-5 years after completion of the research.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

_____

NAME: Hellen Kanyane

EMAIL: hellenk59@gmail.com

TEL NUMBER: 011 866 2856 (home) or 082 647 2025 (cell)

Principal
I consent / do not consent for my school to participate in your study.
School name:_____ Principal's name: _____ Principal's Signature_____ Date: _____

Appendix 7: GDE approval letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

For administrative use:
Reference no: D2016 / 207
enquiries: Diane Buntting 011 843 6503

GDE RESEARCH APPROVAL LETTER

Date:	**11 August 2015**
Validity of Research Approval:	**11 August 2015 to 2 October 2015**
Name of Researcher:	**Kanyane M.H.**
Address of Researcher:	**59 Heather Road; Leondale; 1401**
Telephone / Fax Number/s:	**011 866 2856; 011 901 1930; 082 647 2025**
Email address:	**hellenk59@gmail.com**
Research Topic:	**Intervening to improve the Grade 6 learners' use of models and strategies in solving addition and subtraction word problems**
Number and type of schools:	**ONE Primary School.**
District/s/HO	**Ekurhuleni South.**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved. A separate copy of this letter must be presented to the Principal, SGB and the relevant District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted. However participation is VOLUNTARY.

The following conditions apply to GDE research. The researcher has agreed to and may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

CONDITIONS FOR CONDUCTING RESEARCH IN GDE

1. The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter;
2. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB);

2015/08/12

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

3. A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned;
4. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, SGBs, teachers and learners involved. Participation is voluntary and additional remuneration will not be paid;
5. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal and/or Director must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage;
6. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year;
7. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
8. It is the researcher's responsibility to obtain written parental and learner consent;
9. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources;
10. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations;
11. On successful completion of the study the researcher must supply the Director: Education Research and Knowledge Management with an electronic copy (and a Hard copy if possible) as well as a Research Summary of the completed Research Report;
12. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned; and
13. Should the researcher have been involved with research at a school and/or a district office level, the Director and school concerned must also be supplied with the Research Summary of the study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Dr David Makhado

Director: Education Research and Knowledge Management

DATE: 2015/08/12

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za