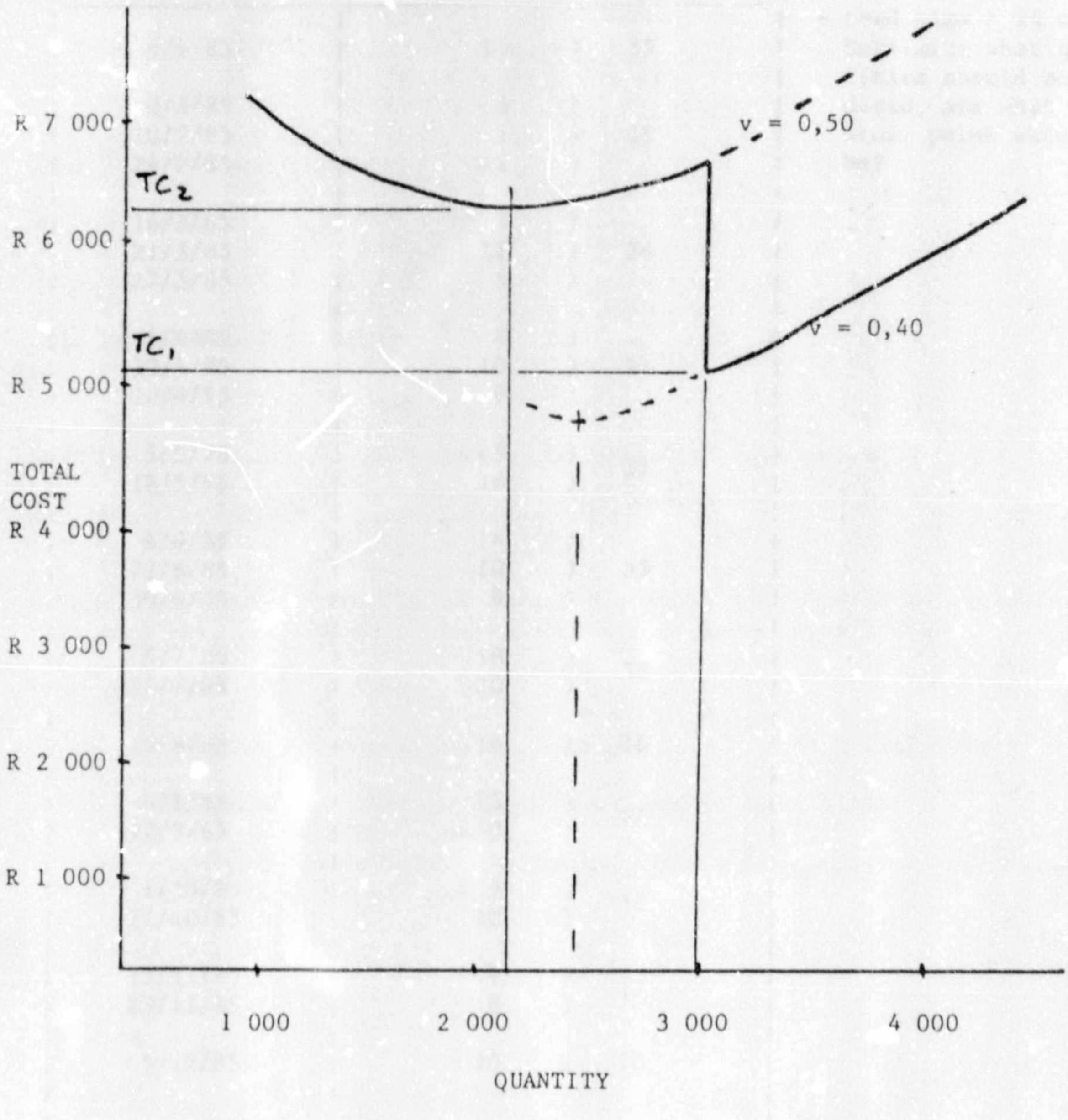
5. Since this is less than the costs calculated for v = R0,50 we must buy this quantity. So order in lots of 3 000.

The scenario is graphically depicted below.

TOTAL COST VS ORDER SIZE



NOTE:

If Q^* for v = 0,40 had been greater than 3000, then this would have been the minimum.

619006

EXAMPLE 4

Assume Mill Balls have the following particulars.

DATE	QUANTITY ISSUED	
2/1/85	15)	15
2/2/85	5)	
10/2/85	1)	18
24/2/85	12)	
16/3/85	3)	
21/3/85	18)	26
27/3/85	5)	
1/4/85	8)	
10/4/85	10)	27
20/4/85	9)	
3/5/85	15)	31
18/5/85	16)	
6/6/85	14)	
22/6/85	10)	33
30/6/85	9)	
5/7/85	18)	38
21/7/85	20)	
15/8/85	16)	16
4/9/85	15)	24
12/9/85	9)	
1/10/85	5)	15
21/10/85	10)	
15/11/85	9)	17
23/11/85	8)	
6/12/85	20)	20
TOTAL	280	280

- Price = R6 325
- A item.
- Lead time = 24 weeks
- Determine what quantities should be ordered, and what the order point should be?

ANSWER

Calculate the value of D.
(It can be assumed that the 1985 issues are reasonably representative of 1986).
A reasonable forecast for January 1986 is say 17.

Now calculate the EOQ

$$Q^* = \sqrt{\frac{2\ DA}{vr}} = \sqrt{\frac{2 \times 17 \times 190}{6325 \times 0{,}3/12}} = 6{,}39 \text{ balls}$$

Calculate the order point

$$OP = D \times LT + SS$$
$$= 17 \times 24/4 + SS$$
$$= 102 + SS$$

Calculate the safety stock

$$SS = k\ (\sigma_D \sqrt{LT} + \sigma_L\ D)$$

So calculate k and σ_D and assume the lead time is constant.

By using the usual formula (see Appendix 2)
get $\sigma_D = 5{,}2$ $n = 25$ $x = 11{,}2$ per issue

Because this is an A item it is definitely worth calculating an optimal service level, but before we calculate k, we need a value of C_S

Determination of C_S

C_S = (MW hrs lost per incident) X (Cost per MW hr lost) X (down time)

- 90 MW is the average output loss due to mill balls
- the lead time is 6 months
- the cost (for that power station) for a continuous 100 MW outage in 1986 is R1 499 350

So $C_S = 1\ 499\ 350 \times \frac{90}{100} \times \frac{6}{12}$ = R674 707 per year.

Calculate k ,

$$k = \frac{1}{b}\left[a - \frac{(4{,}605)(\sigma_D \times \sqrt{LT} \times v.\ r.\ Q^*)}{b.\ C_S.\ D}\right]$$

$$k = \frac{1}{2{,}49}\left[5{,}65 - \frac{(4{,}605)(5{,}2 \times \sqrt{6} \times 6325 \times 0{,}3/12 \times 6{,}39)}{2{,}49 \times C_S \times 17}\right]$$

$$k = \frac{1}{2{,}49}\left[5{,}65 - \frac{1400{,}1}{674\ 707}\right]$$

$$k = 2{,}268$$

This then represents a service level percentage of 98,8%.
(From table 1)

Now $SS = k(\sigma_D . \sqrt{LT})$
$SS = 2,268 \times 5,2 \times \sqrt{6}$
$SS = 28,89$
$SS = 29$ balls

So the OP = 102 + 29
OP = 131

This means that an order of 7 balls should be placed every 9 or 10 days, when the order point of 131 is reached.

To ensure that the optimum Q* is being used a sensitivity analysis can be done.

SENSITIVITY ANALYSIS

Q = 7

$C_1 = \frac{Q^*}{2} \times v \times r + SS \times v \times r$

$C_1 = (\frac{7}{2} + 29) \times 6325 \times 0,3$

$C_1 =$ R61 668 p.a.

$C_2 = \frac{D \times 12}{Q} \times A$

$C_2 = \frac{17 \times 12}{7} \times 190$

$C_2 =$ R5 537 p.a.

$C_3 = \frac{D \times 12}{Q} \times P \times C_S$

$C_3 = \frac{17 \times 12}{7} \times 0,012 \times 674\ 707$

$C_3 =$ R235 954

$C_1 + C_2 + C_3 =$ R303 159

Q = 8

$C_1 = (\frac{8}{2} + 29) \times 6325 \times 0,3$

$C_1 =$ R62 617

$C_2 = \frac{17 \times 12}{8} \times 190$

$C_2 =$ R4 845

$C_3 = \frac{17 \times 12}{8} \times 0,012 \times 674\ 707$

$C_3 =$ R206 460

$C_1 + C_2 + C_3 = R273\ 922$

<u>Q = 9</u>

$C_1 = \frac{9}{2} + 29) \times 6325 \times 0,3$

$C_1 = R63\ 566$

$C_2 = \frac{17 \times 12 \times 190}{9}$

$C_2 = R4\ 036$

$C_3 = \frac{17 \times 12}{8} \times 0,012 \times 674\ 707$

$C_3 = R206\ 460$

$C_1 + C_2 + C_3 = R274\ 062$

So the minimum occurs for Q = 8.

Using the EOQ (refinement) formula we get

$$Q^* = \frac{\sigma_D \sqrt{LT}}{b} + \sqrt{\frac{\sigma_D^2\ LT}{b^2} + \frac{2\ DA}{vr}}$$

$$Q^* = \frac{5,2 \times \sqrt{6}}{2,49} + \sqrt{\frac{(5.2)^2\ 6}{(2.49)^2} + \frac{2(17)(190)}{6325 \times 0,3/12}}$$

$$Q^* = 5,115 + 8,186$$

$$Q^* = 13,3 \text{ say } 13$$

From this example the following points are apparent:

- the optimum Q is 8. This compares poorly with the present system where a Q of 93 would have been used. The holding cost, for the same service level would have been R143 577 as compared with the holding cost of R62 617 for the proposed solution.

- the ordering costs would be R4 845 for the proposed and R572 for the present system. This is a difference of R4 000 as compared with the difference of R81 000 for the holding cost.

- The safety stock is extremely high because:
  - the lead time is excessive
  - the demand is very irregular
  - the service level desired is very high because it is such a critical item

- The simple EOQ formula gives a better result because the "refined" formula over compensated for the high standard deviation.

EXAMPLE 5

A power station used the following quantities of furnace oil.

J	F	M	A	M
750 000	800 000	700 000	720 000	760 000

Furnace oil costs R0,70 per litre - The ordering cost is R45.
If an order of 1 000 000 litres is placed, the price is R0,65 per litre.
The lead time is 1 month.
Calculate the order point and order quantity.

ANSWER

$$Q^* = \sqrt{\frac{2\,D\,A}{vr}} \qquad \text{where} \quad D = \text{Average per month}, \quad D = 746\ 000$$

$$Q^* = \sqrt{\frac{2 \times 746\ 000 \times 45}{0{,}70 \times 0{,}3/12}}$$

$$Q^* = 61\ 940\ \ell$$

No value of C_S so assume value of k.
It is critical (say) that furnace oil is available so choose a service level of 99,5% and hence a k = 2,58.

Now $SS = k\,.\,(\sigma D.\sqrt{LT})$ because $\sigma L = 0$

$SS = 2{,}58 \times 34\ 410 \times \sqrt{1}$ i.e. $\sigma D = 34\ 410$

$SS = 88\ 777\ \ell$

and the probability of a stockout is only 0,005.

$OP = D \times LT + SS$
$OP = 746\ 000 \times 1 + 88\ 777$
$OP = 834\ 777\ \ell$

This means that when the tank level reaches 834 777 ℓ a new order must be placed. The order quantity must be 61 940 ℓ.

PRICE BREAK

for $Q = 61\ 940\ \ell$

$$TC_1 = \frac{Q}{2} \times v \times r + \frac{D \times 12}{Q} \times A + D \times 12 \times v$$

$$TC_1 = \frac{61\ 940}{2} \times 0{,}7 \times 0{,}3 + \frac{746\ 000 \times 12}{61\ 940} \times 45 + 746\ 000 \times 12 \times 0{,}7$$

$$TC_1 = 6\ 503 + 6\ 503 + 6\ 266\ 400$$

$$TC_1 = R6\ 279\ 406 \text{ p.a.}$$

Now for v = R0,65

$$Q = \sqrt{\frac{2 \times 746\ 000 \times 45}{0{,}65 \times 0{,}3/12}}$$

$$Q = 64\ 278\ \ell$$

Since $Q < 1\ 000\ 000\ \ell$

The min cost is at $Q = 1\ 000\ 000\ \ell$

So $$TC_2 = \frac{Q}{2} \times v \times r + \frac{D \times 12}{Q} \times A + D \times 12 \times v$$

$$TC_2 = \frac{1\ 000\ 000}{2} \times 0{,}65 \times 0{,}3$$

$$+ \frac{746\ 000 \times 12}{1\ 000\ 000} \times 45$$

$$+ 746\ 000 \times 12 \times 0{,}65$$

$$TC_2 = R5\ 916\ 702 \text{ p.a.}$$

Since this is less than the R6 279 406 for the 70c/ℓ quantity, it is ordered at the higher volume, i.e. order in quantities of 1 000 000 ℓ

Present system (PA 041)

OQ = 2 to 4 times average demand

= 2 to 4 x 746 000

OQ = 1 492 000 ℓ to 2 984 000 ℓ

Even the smallest value is within the discount range and will yield a cost of R5 964 540 p.a. The larger OQ will yield a cost of R6 109 875 p.a. So, we would have been worse off!

EXAMPLE 6

Determine the order quantity and reorder level for the following situation such that all the relevant costs are minimized:

The demand for a shaft is given as follows:

Quantity used	10	8	7	6	8	11	8	12	9	6	4	7
Month	J	F	M	A	M	J	J	A	S	O	N	D

The shaft costs R5000

The cost of placing the order is R190

The inventory holding cost is 30% of the item price (per annum).

The shaft lead time is 4 months.

The stockout cost is R10 000

ANSWER

1. Calculate Q^*

$$\text{Using } Q^* = \sqrt{\frac{2 \times D \times A}{v \times r}}$$

with
D = 8/month (average of usage per year)
A = R190/order
v = R5000
r = 30% p.a.

$$\text{hence } Q^* = \sqrt{\frac{2 \times 8 \times 12 \times 190}{0,3 \times 5000}} = 4,93 \text{ units}$$

2. EOQ for price breaks

N/A since no discounts are available

3. Optimum service level

$$k = \frac{1}{b} \left[a - \log_e 100 \left(\frac{\sigma_D \times v \times r \times \sqrt{LT} \times Q^*}{b \times D \times Cs} \right) \right]$$

where $a = 5.65$

$b = 2,49$

$v = R\ 5000$

$r = 30\%$ per annum of v

$LT = 4$ months

$Q^* = 4,93$

$D = 8$ per month

$C_s = R10\ 000$

$\sigma_D = 2,25$

$$\text{gives } k = \frac{1}{2,49}\left[5,65 - \left(\log_e \frac{100\ (2,25 \times 5000 \times 0,3 \times \frac{1}{12} \times 4 \times 4,93)}{2,49 \times 8 \times 10\ 000}\right)\right]$$

$k = 2,24$

and from standard normal deviation tables (Table 1), this gives (k) = 0,98745
and hence a service level percentage of 98,7.

4. Safety stock

$$SS = k\ [\sigma_D \sqrt{LT} + \sigma_L D]$$

where $k = 2,24$

$\sigma_D = 2,25$

$LT = 4$

$\sigma_L = 0$

So $SS = 2,24\ [2,25 \times \sqrt{4} + 0]$

$= 10,08$ units

5. Re-order point

$OP = D \times LT + SS$

$OP = 8 \times 4 + 10,08$

$OP = 42,08$ units

6. Costs

Now $Q^* = 4,93$

$SS = 10,09$

$OP = 42,09$

$C_1 = \frac{Q^*}{2} \times v \times r + ss \times v \times r$ (holding cost)

$= \frac{4,93}{2} \times 5000 \times 0,3 + 10,09 \times 5000 \times 0,3$

$= 3697,5 + 15120$

$C_1 = R18\ 817,5$

So the holding cost per year is R18 817,50.

$C_2 = \frac{D \times 12}{Q^*} \times A$ (ordering cost)

$C_2 = \frac{8}{4,93} \times 12 \times 190$

$C_2 = R3\ 699,80$

So the ordering costs per year are R3 699,80

$C_3 = \frac{D}{Q^*} \times P \times Cs$ (stockout costs)

$C_3 = \frac{8 \times 12}{4,93} \times \frac{(100 - 98,7)}{100} \times 10\ 000$

$C_3 = R2\ 434$

So the stockout costs are R2 434 per year.

This gives the total cost of $C_1 + C_2 + C$

$= R18817,50 + R3699,80 + R2434$

$= R24\ 951,30$

7. Sensitivity analysis

$Q^* = 4,93$ $ss = 10,09$ $OP = 42,09$

Now, the EOQ can't be 4,93 so it should be either 4 or 5. From figure 1, where it is shown that the right hand side of the EOQ curve is flatter, it would be wise to try 5. The sensitivity analysis will, however, give the optimum.

<u>So try Q = 5.</u>

Now calculate new k, using Q = 5.

gives k = 2,24 and hence ss = 10,08 = 10.

Then $C_1 = \frac{5}{2} \times 5000 \times 0,3 + 10 \times 5000 \times 0,3$

$C_1 = R3750 + R15\ 000$

$C_1 = R18\ 750$

$C_2 = \frac{8}{5} \times 12 \times 190$

$C_2 = R3\ 648$

$C_3 = \frac{8 \times 12}{5} \times (0,0125) \times 10\ 000$

$C3 = R2\ 400$

Total cost = R24 789.

<u>Next try Q = 4</u>, k = 2,248 = 2,25, ss = 10,1 (say 10)

then $C_1 = \frac{4}{2} \times 5000 \times 0,3 + 10 \times 5000 \times 0,3$

$C_1 = R3000 + R15000$

$C_1 = R18\ 000$

$C_2 = \frac{8}{4} \times 12 \times 190$

$C_2 = R4\ 560$

$C_3 = \frac{8 \times 12}{4} \times (0,0122) \times 10\ 000$

$C_3 = R2\ 928$

Total Cost = R25 488

Now we have

Q	4	4,93	5
Cost (Total)	25 488	24 951	24 789

So try Q = 6 then k = 2,24, ss = 10

then $C_1 = \frac{6}{2} \times 5000 \times 0,3 + 10 \times 5000 \times 0,3$

$C_1 = R4500 + R15\ 000$

$C_1 = R19\ 500$

$C_2 = \frac{8 \times 12}{6} \times 190$

$C_2 = R3\ 040$

$C_3 = \frac{8 \times 12}{6} \times \frac{(100 - 98,75)}{100} \times 10\ 000$

$C_3 = R2\ 000$

Thus total cost = R24 540

Now try Q = 7 then k = 2,23 s[illegible]0

then $C_1 = R20\ 250$

$C_2 = R\ 2\ 605,71$

$C_3 = R\ 1\ 765,[illegible]3$

Total cost = R24 620,70

So, now we have

Q	4	4,93	5	6	7
C TOTAL	R25 488	R24 951	R24 789	R24 540	R24 620

And as can be seen the minimum Cost occurs with Q = 6 and it is also better to order 7 than 5.

So our model is as follows:

Order a quantity $Q^* = 6$ each time the reorder point = 42 is reached.

The SS = 10 and the service level is 98,72%.

619006

<u>So try Q = 6</u> then k = 2,24, ss = 10

then $C_1 = \frac{6}{2} \times 5000 \times 0,3 + 10 \times 5000 \times 0,3$

$C_1 = R4500 + R15\ 000$

$C_1 = R19\ 500$

$C_2 = \frac{8 \times 12}{6} \times 190$

$C_2 = R3\ 040$

$C_3 = \frac{8 \times 12}{6} \times \frac{(100 - 98,75)}{100} \times 10\ 000$

$C_3 = R2\ 000$

Thus total cost = R24 540

<u>Now try Q = 7</u> then k = 2,23 ss = 10

then $C_1 = R20\ 250$

$C_2 = R\ 2\ 605,71$

$C_3 = R\ 1\ 765,03$

Total cost = R24 620,70

So, now we have

Q	4	4,93	5	6	7
C TOTAL	R25 488	R24 951	R24 789	R24 540	R24 620

And as can be seen the minimum Cost occurs with Q = 6 and it is also better to order 7 than 5.

So our model is as follows:

Order a quantity $Q^* = 6$ each time the reorder point = 42 is reached.

The SS = 10 and the service level is 98,72%.

Please note the following:

- The value used for D was simply the arithmetic mean for the year. Had a forecasting technique been used (such as with projection code I) more accurate results would have been obtained.

- Ensure that all your units cancel ie. if the demand is per month and the holding cost is for the year, then you must correct for this.

- The 1st Q^* calculated (only taking the ordering and the holding costs into account) was not the final optimum value. The final optimum was reached once the load loss costs were added.

- There was substantial standard deviation in the demand which we should have allowed for. This would have resulted in the following Q being calculated.

$$Q = \frac{\sigma_D \sqrt{LT}}{b} + \sqrt{\frac{D^2 LT}{b^2} + \frac{2DA}{vr}}$$

$$= \frac{(2,25).\sqrt{4}}{2,49} + \sqrt{\frac{(2,25)^2(4)}{(2,49)^2} + \frac{(2)(18)(12)(190)}{(0,3)(5000)}}$$

$$= 1,807 + \sqrt{3,26 + 24,32}$$

$$= 7,058$$

  Compare this value with the optimum Q obtained in the sensitivity analysis.

- Ensure that the costs calculated are for the whole period. This does not have to be for a year, but this is usually found to be convenient.

- Sensitivity analysis must be applied so that an optimum is reached.

- The cost per order (A) varies with the type of order.

- Once k has been calculated, another method of calculating the service level is:

  if k = 2,24, then this represents a service level of $(100 - e^{(5,65-2,49xk)}) = 98,92\%$.

  This compares with the 98,75% of the other method using the normal tables.

- The calculation of σ_D. This was done by the usual method - most scientific/statistical calculators can do the calculation for you, but you need raw data. The data must be grouped into specific equal time periods.

° Using the "quick and dirty" approach (see Appendix 2) we get

$$\sigma_D = \frac{\text{Max value - Min value}}{3}$$

$$= \frac{12-4}{3}$$

$$= 2{,}67$$

This compares reasonably with the previously calculated value.

8.2 Appendix B

(SPECIMEN PRINTOUT OF COMPUTER PROGRAMS)

ITEM NUMBER	UMC	CAT	HOLDING COST (fract)	ORDERING COST	ITEM PRICE	PRICE BREAK 1 PRICE 2	PRICE BREAK 1 QTY	PRICE BREAK 2 PRICE 3	PRICE BREAK 2 QTY	PRICE BREAK 3 PRICE 4	PRICE BREAK 3 QTY	LT 1	LT 2	LT 3	LT 4	LT 5	LT 6	AVE LT (calc)	MANUAL LT
111111	EA	23	0.30	50.00	91.00	91.00	300.00	85.00	1000.00	80.00	5000.00	4.00	5.00	6.00	7.00	7.00	2.00	5.17	6.00
222222	M	23	0.30	50.00	0.66	0.66	50000.00	0.63	[illegible]000.00	0.63	100000.00	4.00	3.00	1.00	2.00	2.00	2.00	2.33	2.00
333333	EA	23	0.30	50.00	2065.00	2065.00	1.00	2065.00	[illegible]000.00	2065.00	3000.00	10.00	8.00	12.00				10.00	10.00
																		ave 5.83	ave 6.00

ITEM LEAD TIMES IN MONTHS (LT 1 – LT 6)

ISSUES PER MONTH J	F	M	A	M	J	J	A	S	O	N	D	AVE ISSUES (calc)	MONTHLY DEM (manual)	USAGE VAL (year)	QUANTITY ON HAND
1017.00	454.00	885.00	1663.00	964.00	454.00	455.00	296.00	839.00	179.00	378.00	443.00	663.92	650.00	709800.00	7973.00
929419.00	169312.00	268639.00	589717.00	345809.00	402135.00	555084.00	739429.00	469473.00	361117.00	136898.00	117854.00	427909.08	400000.00	3264000.00	733678.00
40.00	90.00	80.00	36.00	44.00	21.00	86.00	45.00	63.[illegible]	83.00	27.00	43.00	54.83	60.00	1486857.60	25.00
														total 5460657.60	

DATE	MNTHS SUPL ON HAND	ON HAND VALUE	OPTIMUM ORDER QTY	OPTIMUM MNTHS SUPPL	DO YOU HAVE SURPLUS ?	SURPLUS QUANTITY	SURPLUS VALUE	RE-ORDER POINT	SERV LVL % (manual)	DESIRED SERV LVL %	STOCKOUT COST	STANDARD DEVIATION	SAFETY STOCK (calc)	SAFETY STOCK (manual)
1/6/86	12.27	725543.00	1000.00	1.54	Y	• 3520.00	582320.00	5353.00	95.00	98.97	34567.00	400.52	1591.98	1453.00
1/6/86	1.83	498901.04	100000.00	0.25	Y	99178.00	67441.04	1334500.00	95.00	98.99	100000.00	232915.97	534511.26	534500.00
5/12/86	0.43	53692.08	10.78	0.13	N	0.00	0.00	700.00	95.00	98.36	10000.00	23.36	119.88	100.00
	average 4.84	total 1278136.12		average 0.66			total 569761.04							

SAFETY STOCK COST (p.a)
37666.90
109038.00
61952.40
total 210657.30

9. REFERENCES

Plossl, G.W. and Wight O.W. (no date) Production and inventory control. Prentive Hall.

Peterson, R. and Silver, E. (1979) Decision systems for inventory management and production planning. John Wiley and Sons, 1979.

Dudick, T.S. and Cornell, R. (1979) Inventory control for the financial executive. John Willey and Sons, 1979.

Hamberg, M. (1977) Statistical analysis for decision making. Harcourt, Brace, Johanovich, INC, 1977.

Woolsey, R. and Swanson, H. (1975) Operations research for immediate application - a quick and dirty manual. Harper and Row, 1975.

Aucamp, D.C. (1986) The evaluation of safety stock. Journal of the South African production and inventory control society. Second quarter, 1986.

I.B.M. (1972) Communications oriented production information and control system. Vol IV, Chapter 5. I.B.M publications.

Bishett, P. (1986) Practical forecasting for stores and consumer products. 8th annual SAPICS conference.

White, G.P. (1986) Some common myths and misconceptions about exponential smoothing.SAPICS journal.

Makridakis and Wheelright (no date) Forecasting.