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LEARNING MATHEMATICS

WITH MATHEMATICAL SOFTWARE

By

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LEARNING MATHEMATICS WITH MATHEMATICAL SOFTWARE

BY

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Submitted in partial fulfillment of the requirements for the degree

MASTERS IN EDUCATION

(Educational Technology)

In the Faculty of Humanities, University of the Witwatersrand

SUPERVISOR: PROFESSOR I. MOLL

NOVEMBER 2009

ABSTRACT

The study took the form of action research situated in a case study. The participants consisted of sixteen Grade 11 learners who attend a non-profit tutoring organisation called Ikamva Youth on Saturday mornings and who volunteered to learn mathematics with mathematical software. Data were collected from the learners using a structured questionnaire, journals in which learners were encouraged to record their daily experiences and by studying the learners' own written work during the research period. Moreover, the research closed with a focus group session. The study primarily described three aspects, viz. the degree to which learners are able to work with the strands of strategic competence and adaptive reasoning with particular emphasis on interpretation and application, knowledge production and justification and social collaboration; barriers in terms of working with these strands in a digital environment that may interfere with the learning process; and, the personal (affective) response of the students to the availability of technology. Findings suggest that the participants experienced difficulties in accessing these particular strands of mathematical knowledge, and subsequently expressed the desire to revert back to a place where the teacher assumes a more direct teaching style and where the focus of mathematical learning is on technique. In other words, learners preferred routine expertise, whilst appearing to lack in adaptive expertise. With respect to the second aspect of the research, it is suggested that one of the most prominent barriers to integrating technology into mathematics does not necessarily relate to adjustments in terms of the instrumental use of software and the computer environment, but seems to stem from the learners' own epistemologies and beliefs about the nature of what constitutes effective mathematical teaching and learning. In paying attention to the voice of the learners it surfaced that the learners tend to associate computers more strongly with a cultural tool for entertainment than with mathematical learning. Although the study points out that implementing an interactive curriculum integrated with technology into a classroom with learners from low socio-economic backgrounds is not necessarily straightforward, it does suggest that with more frequent exposure certain learners can develop a propensity for working within a cognitively rich problem-solving context and effectively employ the mathematical software as an extension of their own thinking. This study adds to broader research on the role of technology in learning by reporting that the learners felt that the contextualisation of mathematics, followed by the ability to manipulate the graph themselves were the main contributors to their mathematical learning. The learners disregarded the visualisation effect of the computer as they felt that it had very little impact on their mathematical learning.

KEY TERMS:

reform mathematics, modeling, barriers to learning mathematics, teaching with mathematical software

DECLARATION

I, Rina Scott-Wilson, Student no, 0615857K, declare that TEACHING MATHEMATICS WITH MATHEMATICAL SOFTWARE is my own work and that all sources that I have used or quoted have been indicated and acknowledged by means of complete references.



_____ _____26/6/2010_____

SIGNATURE **DATE**
(Rina Scott-Wilson)

ACKNOWLEDGEMENTS

I would like to thank God for his provision and grace during this project.

I am grateful to:

Professor Ian Moll for being my supervisor.

Meg Sperring for her dedicated effort.

And, to Lorne for his support during this time.

Lastly, I would like to thank Ikamva Youth, the learners and their parents for being involved in this research. My life has been enriched through you.

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CHAPTER 1

BACKGROUND AND OVERVIEW OF THE STUDY

1.1 BACKGROUND

In recent history, the world has undergone significant technological change which was largely influenced by the advent of the personal computer, launched in 1981 by IBM (IBM archives, n.d.), and the onset of the World Wide Web in the early 1990's (World Wide Web Consortium, 2001). The digital transformation required the restructuring of international economic structures to the extent where industrial leaders foresee that a 'different kind of learner' is essential in accessing corporate career opportunities (Chapman, 2000). The type of learner envisioned is one who has fluency and competency with computers and the Internet (Chapman, 2000). Moreover, industries' revised portfolio of what constitutes a suitable employee stipulates elements of innovative and adaptive problem solving supported by strong mathematical skill (Dossey, Giordano, McCrone & Weir, 2002; Ramnarain, 2003; Van de Walle, 2007). Hence, a sound foundation in science, mathematics and technology can be seen as the intellectual currency of the 21st century. Simultaneously, behind the international developments on workplace reform and its emphasis on technological aptitude, arises a broader concern that people who already suffer economic and/or social disadvantage are likely to experience even worse forms of exclusion in future as a function of being on the wrong side of the digital divide (Attewell, Suazo-Garzia & Blackwell, 2003). In other words, learners who do not have access to technology during their schooling careers may soon present as 'unemployable' on the basis of their 'digital illiteracy'. However, I argue that digital literacy may be a necessary but not sufficient condition in the evolving economic setup. Rather, digital literacy needs to be enveloped in a cognisance of psychological flexibility that stimulates adaptive interpretation and efficiency, for learners to manage sustainable and progressive adjustment within the digital revolution and its evolving economic systems.

1.2 PROBLEM IDENTIFICATION

In light of this aspect, exposing and equipping learners' with computer skills in the 21st century intuitively comes across as a good idea. Yet, I would like to move beyond a kind of cross-curricular generalisation with regards to computer literacy, by describing a case relating to the teaching and learning of mathematics through the medium of computers within a interactive setting with learners from a low socio-economic background. Part of the description will involve a discourse on the barriers that were experienced.

Describing and discussing learning with mathematical software form part of a larger debate situated around the effect of computers on learning. As late as 2000, Chapman (2000, p.307 – 308) stated that 'still no widespread consensus has been reached on whether computers and the Internet will have a large, small, positive, negative, or inconsequential effect on learning in young people.' In this regard he was speaking of learning in general was not differentiating specific modes of learning. Similarly, Kieran and Drivjers (2006, p.206) indicated that research had 'difficulty in providing evidence of improved learning with technological means, as well as in understanding the influence of technology on learning.'

Hence, this study wants to record the learners' interaction with specific modes of knowledge in a digital environment, but with added emphasis on the learners' own interpretation or expressions of encounters that were salient to them. In other words, apart from monitoring strands of mathematical proficiencies, an additional aim is to establish what learning with technology meant to the learners themselves. This in light of the requirement of outcomes-based education (OBE) that all stakeholders, including learners, be involved in the learning process. Thus far policies such as the White Paper on E-learning (DoE, 2003c) has revealed the government's intentions concerning the role of computers in education, and a growing number of research projects were undertaken to establish the teachers' usage of and beliefs about computers in pedagogy (Windschitl & Sahl, 2002; Somekh, 2003; Ertmer, 2005; Niess, 2005; Judson, 2006). Yet, from searching several academic databases there appears to be a lack of literature on how South African pupils themselves experience learning through computers in a mathematical classroom. Moreover, a need has been

identified in research for analysing the mathematical learning processes in technological environments coupled with a requirement to describe the development of resources for the professional development of new teaching practices (Drivjers & Trouche, 2008). In the light of such occurrences it is deemed appropriate that the education knowledge base be enlarged through additional social science research.

1.3 SIGNIFICANCE OF THE STUDY

Ultimately the discussion over the use of computers in mathematical learning forms part of a larger debate over the proper role of technological tools throughout society and its function in advancing equity. An emerging argument from educational research is that technology has the potential to enhance student teaching and learning, provided that it is used appropriately (Underwood & Underwood, 1990). Underwood and Underwood (1990, p. vi) posit that a computer as a tool becomes 'both an amplifier of human capabilities and a catalyst to intellectual development'. A counter argument is that the critical inquiry, judgement and thinking skills that young people are required to have can be developed in ways other than using computers. Subsequently, the fiscal focus should be on finding solutions to more basic problems confronting the youth in society and not necessarily on expensive installations of hardware (Chapman, 2000). Such an argument could carry significant weight in consideration of the socio-economic conditions and associated hardships of the participants involved in this particular study.

Subsequently, the paper sets out to describe the key cognitive and affective issues that arise when learners from low socio-economic status backgrounds are exposed to an interactive genre of mathematical learning through computer software for the first time. In doing so, I recognise the potential for mathematics learning to be transformed by the availability of technologies such as computers and graphical software. Insights gained from the study can be helpful for both planning teaching and monitoring the progress of students using technology for mathematics. Moreover, it could prove useful for later teacher training.

1.4 CRITICAL QUESTIONS

The idea of the study is to trace the cognitive and affective behaviour of learners using mathematical software by monitoring:

- The degree to which learners are able to work with the five strands of mathematical proficiency as detailed by Kilpatrick et al. (2001). From within this framework, special attention is given to the strands of strategic competence and adaptive reasoning. Emphasis is thus on aspects of mathematical interpretation and application, knowledge production and justification (graphically and numerically), with an additional element of social collaboration being added which has been derived from ultimately placing the study in a socio-constructivist learning paradigm.
- Barriers in terms of working with interactive mathematics that may interfere with the learning processes detailed in the bulleted point above.
- The personal (affective) aspect reflects the response of students to the availability of technology.

1.5 APPROACH TO MATHEMATICS

In this study interactive mathematical problems with a “real world” veneer will be used to motivate and apply mathematical theory. Computer utilities will be used and monitored in an attempt to understand technology’s contribution towards developing conceptual understanding in mathematics. Learners will be asked to express function equations in standardized forms and to check on the reasonableness of the graphs produced by the graphing software. Moreover, learners will be asked to use the concept of functions as models of real world problems and to make connections between a problem situation and its model as a function in symbolic form, and the graph of that function. Hence, one of the mathematical skills being developed and practised in this research is for the learners to recognise and represent quadratic functions in tables, symbols and graphs and to identify the assumptions involved in using these functions as models. This approach involves the application of procedures, processes and conceptual understanding and therefore lends itself suitable

to an objective of the study to monitor learners' engagement with Kilpatrick et al.'s strands of mathematical proficiency.

1.6 DEFINITION OF KEY CONCEPTS

1.6.1 The Five Strands of Mathematical Proficiency

Kilpatrick, Swafford, Findell and Kincheloe (2001) formulated five categories that may be useful in describing mathematical learning

i. *Conceptual understanding,* which relates to the comprehension of mathematical concepts, operations and relations.

ii. *Procedural fluency,* that is, skill in carrying out procedures flexibly, accurately, efficiently and appropriately.

iii. *Strategic competence* refers to the learners' ability to formulate, represent and solve mathematical problems.

iv. *Adaptive reasoning,* which is the capacity for logical thought, reflection, explanation and justification.

v. *Productive disposition* which is possessing the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

The authors of the categories strongly affirm that the five strands are interwoven and interdependent in the development of proficiency in mathematics. Moreover, as recommended by the authors, these strands will be used as a framework for discussing the knowledge, skills, abilities and beliefs that constitute mathematical proficiency (Kilpatrick et al., 2001).

The research has selected certain key aspects of the model to investigate, which broadly fall in the strategic competence and adaptive reasoning strands. These aspects are the ability to interpret and apply knowledge in a localised context; to produce

knowledge and/or creatively reconfigure knowledge to address a complex problem; to justify practical and social relevance in a specific milieu; and, to collaborate with others in order to find possible solutions. Learners' response time and flexibility and reflexivity in tackling these problems, and their organisation and communication patterns in solving or redefining the scenarios, will be taken as additional key indicators of their ability to work within these strands. The reason for the emphasis on the latter strands rather than first two is to gauge whether the students possess an adaptive rather than routine expertise. According to De Bock, Van Dooren and Janssen (2007, p.242), adaptive expertise refers to the ability of students to apply meaningfully the mathematics learnt in creative and flexible manners. In contrast, routine expertise refers to completing school mathematics exercises quickly and effectively but without much understanding.

1.6.2 Model-eliciting Activities

Mousoulides, Sriraman and Christou (2008) cite how Blum (2000) portrays modeling by describing in detail the particular processes and skills involved in modeling.

> Modeling processes are the processes students develop and use during their efforts to solve a real world problem. These processes include describing the problem, manipulating the problem and building a model, connecting the mathematical model with the real problem, predicting the behaviour of the real problem and verifying the solution in the context of the real problem. Student modeling abilities include structuring, mathematizing, interpreting and solving real world problems and the ability to work with models; to validate the model; to analyse it critically and to assess the model and its results, to communicate the model and to observe and to control self adjustingly the modeling process (Mousoulides, Sriraman and Christou, 2008, p.3).

In short, modeling-eliciting activities engage students in more meaningful real-life situations. It is thus a form of applied mathematics (Niss, Blum, & Galbraith, 2007)

which require of students to try to make symbolic descriptions of meaningful situations (Lesh and Doerr, 2003, p.3-4).

Similarly, (Rapoo, 2009) offers a definition of mathematical modeling that reaffirms many of the aspects of Lesh and Doerr's description above, but one that is essentially more useful for the purposes of this paper.

> Mathematical modeling is the basis of applied mathematics. In essence it consists of constructing a mathematical description which will allow us to have at least some understanding of some situation in the world around us. Most "real-life" situations are extremely complex to analyse and understand. Often, formulating a system as a mathematical model, however incomplete, helps us better understand its behaviour, while allowing us to experiment mathematically with the different conditions which might affect its outcome (Rapoo, 2009, p.1).

Hence, what I am trying to achieve through the mathematical exercises (excluding the "creations" activities) is to provide learners with a mathematical description in the form of an equation, which they then turn into a graph through technology. This mathematical representation relates to some concrete socio-economic situation in a real-life setting (I use the word "real" in the context of this paper to denote a meaningful context involving a mathematical problem. Meaningful refers to meaningful to the learners and it essentially involves teaching abstract concepts from within a concrete environment, that is, an environment which is familiar to learners and with which they can identify). The learners are then encouraged to experiment using the technology with different conditions that they might think will help us understand the model's behaviour and that will affect specific outcomes.

1.6.3 Mathematical Software

The mathematical software referred to in this study is *Geometer's Sketchpad.* According to the developers, *Geometer's Sketchpad* is a dynamic form of mathematics visualisation software that can be used to build and investigate mathematical models, objects, figures, diagrams, and graphs. It was produced to serve

as a construction, demonstration and exploration tool in the study of mathematics and to support interactivity in mathematics lessons. (The Geometer's Sketchpad Resource Center, n.d, par 1).

One might argue that mathematical activities should not be combined with technology in so far as that the technology may actually interfere with the learners' cognitive processes. However, the mathematical activities used in this research have been especially designed around technology. Moreover, the technological requirements are mostly limited to producing a graph by inputting a formula followed by a request for key co-ordinates. The cognitive work comes into play after this in so far as learners then have to interpret and apply what the graphs represent by making connections between a problem situation, its model as a function in symbolic form and the graph of that function.

1.7 PROGRESSION OF THE STUDY

Chapter 1 provides a broad introduction to the current tension to produce students that are strong in mathematics and in technology and the benefits of merging the two fields into one as a strategy for mathematical teaching and learning.

Chapter 2 deals with South African government policy on the nature of school mathematics and how it should be taught and learnt. This chapter also provides a conceptual theoretical background, covering literature on the important theories relevant to the teaching of mathematics and the possible roles of technology in this regard.

Chapter 3 is a description of the methodology that was used in the research project. It provides detailed descriptions of the research design, procedure, sample, data analysis and ethical considerations.

Chapter 4 contains a report on the findings and emergent themes are outlined.

Chapter 5 provides a discussion of the results in relation to theoretical frameworks.

Chapter 6 gives a conclusion and provides recommendations on the role of structuring mathematical content and using technology in the mathematics classroom.

1.8 MAKING THE IMPLICIT EXPLICIT

Essentially, the research questions driving this paper are implicit within the critical questions in Section 1.4. This insert therefore attempts to make more explicit the research ideas contained within these questions.

Firstly, this study in particular will look for empirical evidence which can be used to describe the learners' levels or strands of mathematical functioning with a particular focus on the strands of strategic competence and adaptive reasoning. This evidence will be gathered in three ways:

- observing the learners in class as they engage in the given exercises
- monitoring their workbooks (which will contain their solutions in writing)
- analysing their journals (which will contain their own thoughts and evaluations).

It is suggested that evidence that learners who have *conceptual understanding* will be able to recognise salient parabolic concepts such as the equations of the function; the general shape of the graph; its maximum and minimum values; its symmetrical properties; and its translation features.

Learners will show evidence of procedural fluency when they can sketch the parabola; find the turning point through appropriate formulas, find x or y-values of points through substitution and apply transformation properties to given equations.

Strategic competence refers to the learners' ability to formulate, represent and solve mathematical problems. Examples in this study would include being able to construct a "road" across the parabolic bridge or being able to construct a bridge half or double the size of the original bridge. Other examples may include being able to derive the meaning of the answer in terms of the context provided for example to reinterpret an y-value in accordance with the financial scale depicted by the y-value.

Adaptive reasoning, which is the capacity for logical thought, reflection, explanation and justification will be foregrounded when learners have to explain their

interpretations and solutions of problems such as those mentioned in the previous paragraph.
As indicated by Kilpatrick et al. these levels of mathematical proficiency are intertwined and interrelated and this breakdown of these strands are thus artificial and solely for the purpose of data gathering.

The primary aim of data gathering is to provide a description of how the learners engage with the strands of strategic competence and adaptive reasoning of mathematical proficiency framed by Kilpatrick et al. in the context of interactive mathematics in a digital environment. Once the description has been obtained, the discussion will try to identify and analyse potential factors which may contribute to the learners' performance. It is argued that deep descriptions of the different issues that learners deal with during these mathematical experiences may contribute to teachers' gaining more intimate knowledge of how technology combined with an interactive socio-constructivist approach may affect mathematical learning, which could be used to assist in higher level learning achievement.

To more comprehensively understand factors that may hinder (or advance) part of the investigation is also about being able to identify barriers that were encountered by the teachers/researchers and the learners themselves in terms of working with interactive mathematics. By barriers I am referring to dynamics that appeared to interfere with the mathematical learning and teaching processes. These barriers were identified in three ways. The teachers/researchers would have a meeting after each session to discuss the dynamics of the lesson just completed and to brainstorm the progression of the following lessons. Part of the dynamics would be identifying areas where the learners had difficulties in progressing with the lessons. Secondly, the learners themselves would be asked to keep a journal recording aspects of the lessons that helped or hindered them. Lastly, learners were asked to fill in a learner questionnaire at the end of the research period. Some of these questions were much broader than the actual research project in that it asked learners about their schooling environments. As is mentioned in more detail in Chapter 3 the participants were learners from low socio-economic backgrounds with a low average mathematical mark at school. They were attending Ikamva Youth in order to better their results. The questionnaire was designed in a way that would broaden the researchers background knowledge of the

learners and the mathematical environment that is familiar to them at school level. This is discussed in more depth in Chapter 4.
The question to barriers were also inverted by asking what the learners felt supported or advanced their own learning in order to gain feedback from the learners on “what worked” and “what didn’t work” in the mathematical learning/teaching environment.

The third and final critical questions relate to the personal (affective) response of students to the availability of technology. The focus was trying to hear from the learners themselves what they thought of learning mathematics with technology.

CHAPTER 2
LITERATURE REVIEW AND THEORETICAL FRAMEWORK

As was noted, aside from considering important theories relevant to mathematics education and the potential role of technology in this domain, this chapter starts with governmental policy on the nature of mathematics and how it should be taught and learnt, followed by more general academic theories on the learning of mathematics. Governmental policy and the intentions contained therein, however, should not be considered in isolation of the larger contexts that contribute to these contents.

2.1 FACTORS INFLUENCING POLICY

In this paper it is argued that two developments had a strong influence on current governmental outlooks, viz. the international momentum towards a global economy and the socio-economic and educational conditions left in the aftermath of Apartheid. Subsequently, these two aspects will be explored before returning to the stipulation of current educational policy in South Africa**.**

2.1.1 International Globalisation

This paper assumes Castells' (2002) view on the new information age and its impact on the development of a global economy. It is currently posited that for about the last three decades the world has entered into a postindustrial age. In this era older industrial society models are crumbling under the pressure of an 'information age' that requires new cadres of information and knowledge workers (Lyon, 2005). The information age is marked by information becoming central to productive and commercial processes to the point where 'knowledge-based' enterprises have become very profitable (Kumar, 1995). Yet, the production and flow of information cannot be separated from the development of communication and information technologies. These technologies have allowed for information to be digitized and automated and

have resulted in network societies where information, but also goods, people, capital, entertainment and ideas flow around the world and thus develop in increasingly international contexts which in turn advances globalisation (Castells, 2002). Subsequently, the competitiveness of firms now largely depends on them being able to generate and process information electronically. This has given rise to a new kind of worker, viz. 'informational workers', which require 'high knowledge and high skills' (Lyon, 2005, p.226–227). According to Castells (2002) two distinctive characteristics of informational labour are its adaptability and malleability. In other words, these workers are expected to be flexible enough to adapt to constantly evolving situations. Such flexibility extends not only into their ability to continually upgrade their own knowledge and skills, but also incorporates the pliancy of being able to network with others.

Dark (2003, p.228 – 229) expounds further on the concept of what high knowledge and high skills could mean by breaking the skills component into five prominent categories, viz. complex systems, communication, representational fluency, group functioning and self-management.

i. Complex systems requires working with 'a multitude and a variety of dynamic variables to solve problems that frequently have more than one right answer and more than one right solution path' (Dark, 2003, p.228). According to Dark (2003) working within complex systems requires skills such as defining and clarifying problems; sorting and classifying information; evaluating relevance, reliability, validity and sufficiency of information ; understanding complex situations from multiple viewpoints; understanding relationships among variables in a system and using divergent schemes to untangle the problem and being able to monitor, test, revise and document the problem solving process.
ii. Communication skills include being able to communicate problem states, goals, needs, priorities; to work with others who have different skills and to both give and accept suggestions and criticisms.
iii. Representational fluency requires skilfulness in abstraction. Abstraction and representation includes visualising and conceptualizing transformation processes abstractly; understanding systems that do not exhibit any physical manifestations of their

functions; transiting between physical sensory data and symbolic representations; working with patterns and transferring principles appropriately from one situation to the next.

iv. Group functioning includes creating strategies for sharing work tasks with teams; interacting with people in a broad array of functional roles and adapting, co-ordinating, negotiating and co-operating in a team structure for problem resolution.

v. Self-management refers to using independent judgement to make decisions; self-assessing one's own work for revision, refinement and elaboration and adapting quickly to new tools, new tasks, new jobs, new audiences, new teams and new problems.

Taking into account analyses such as Dark's, I see a strong correspondence between her definition of high knowledge and skills, Castells' requirements of an information worker and Kilpatrick et al.'s strands of mathematical proficiency. The main overlap between the three that I wish to emphasise in this paper is the recognition of possessing a self-regulated intelligence that can (re)formulate a problem, adapt to it in an innovative, flexible manner, and reflect on the effectiveness of the adaptation at a relatively fast pace and communicate clearly to others.

The question that emerges from this overlap is how does one prepare South African learners to become informational workers with high knowledge and high skills, who possess the flexibility to adapt to novel situations and who integrate effectively into evolving team setups? The possible solution to this question presents as a particular challenge in South Africa when one considers the many educational inequities left in the wake of Apartheid.

2.1.2 The Emphasis on Redress after Apartheid

In addition to the common global context, much of the large-scale transformations being instituted in the South African schooling system following the official start of democracy in South Africa in 1994 were shaped by the unique circumstances left by Apartheid. Considering the inequity of the educational system that was left behind,

the impetus behind these developments was largely social justice (Muller, 2005). Consequently, the new policies were concerned with political issues of redress, overcoming inequalities, extending participation to previously excluded groups and, expanding and redistributing resources (Bennie, Olivier, Linchevski, 1999). Ultimately the aim of these stipulations is to ensure that all learners progress equitably by having access to credible, quality and efficient teaching.

2.1.3 The Influence of Globalisation and Apartheid on Governmental Policy

In lieu of the above contexts, governmental guidelines (DoE, 2003a, p. 1- 4) foregrounded that all mathematical teaching amongst Grade 10 – 12 learners should adhere to specific criteria. More specifically, in order to make redress after Apartheid, all teaching initiatives should aim at social transformation. Education should be conducted in a context where human rights and social justice are upheld and where indigenous knowledge systems are valued. To address the work requirements demanded by globalization, government policy began to stipulate that educational initiatives should promote high knowledge and high skills together with integration and applied competence.

Despite governmental intentions, pedagogical patterns that manifest in the classroom, however, are influenced by broader mathematical beliefs. Literature supports the notion that practical implementation in the classroom is broadly informed by two alternative epistemologies towards mathematics, viz. the fallibist and the absolutist paradigms.

2.2 MATHEMATICAL EPISTEMOLOGY

The main corollaries of the fallibist and absolutist philosophies are detailed below. These two philosophies represent a form of theoretical polarization when juxtaposed against each other. They are thus two paradigms with varied epistemological natures which underpin different views on how teaching and learning should occur in the classroom context. Drawing from authors and researchers such as Brombacher (1997),

Davis and Maher (1997), Toumasis (1997), Dossey et al. (2002), Markey, Power and Booker (2003) and Kim (2005), a comparison can be offered of how fallibist and absolutist philosophies can direct classroom practice and pedagogy.

2.2.1 The Absolutist Philosophy

In this epistemology mathematics is considered one of the highest and purest forms of reason (Siegel & Borasi, 1994). An absolutist philosophy is based on Plato's ideals in that mathematical knowledge is approached as a pure form of truth that transcends cultures and time. Hence, mathematical knowledge exists 'out there' and needs to be discovered (Niiniloto, 1992). Mathematics is essentially viewed as disembodied and decontextualised. Moreover, it is depicted as a form of knowledge that is objective, universal and abstract (Ernst, 1998).

i) Theory of Learning

The absolutist philosophy emphasis the teacher's role and responsibility in transmitting the appropriate knowledge and skill to the learner by lecturing or reading, which the learner then should absorb and memorise (Toumasis, 1997). Learning is seen as mainly an individual activity, which requires hard work, self-discipline and self-denial (Kim, 2005). Performance is accentuated and achieved through continual practice and drills to the extent where computational procedures are 'automatized'. Ultimately the learner's responsibility is to reproduce objective knowledge (Davis & Mahr, 1997). Hence, the learner becomes locked in a role of listening, memorising and practicing. Errors in mathematics are taken to imply that the important concepts have not been mastered (Toumasis, 1997).

ii) Theory of Teaching

This approach requires the teacher to lecture in a structured manner from a standardised textbook (Kim, 2005). Knowledge is disseminated in a careful manner. Mathematics is identified as a rigid logical structure that rests on the foundation of deductive thinking (Toumasis, 1997). Subsequently, concepts, theorems, proofs, laws

and procedures are correctly and clearly conveyed to learners. The rule-governed behaviour of mathematics is underscored.

iii) Strengths and Weaknesses of an Absolutist Philosophy

- tend to enhance memory-level skills to execute algorithms in self-motivated students
- are time-efficient in terms of explanations and discussion
- present an abstract procedural learning that may result in a lack of meaning-making by learners
- neglects conceptual understanding of relationships and the ability to apply mathematics to real-world situations
- creates the perception that mathematics is a series of algorithms to be memorized
- have the long-term effect that learners tend to forget the algorithm as soon as they are no longer using it

By contrast, an alternative approach to mathematical teaching and learning is found in the fallibist philosophy.

2.2.2 The Fallibist Philosophy

Fallibist or Quasi-Empiricist philosophy considers mathematical essence and meaning as the products of the human mind (Toumasis, 1997) Mathematics is largely produced through the social and rhetorical practices that occur within the mathematical community (Siegel & Borasi, 1994). Since mathematical knowledge is seen as a human construct it is argued that mathematics is historically and culturally embedded (Ernst, 1991); and it is not considered to carry absolute validity. Rather, mathematics is depicted as fallible, tentative and open to revision (Ernst, 1998).

i) Theory of Learning

Due to this theory's key belief that mathematics is a product of the human mind, learners are expected to act as inquirers and insiders of the mathematical community. Through the learner's efforts and operations in solving problems, mathematical knowledge and meaning are created in the mind of the learner (Toumasis, 1997). Learners come to see knowledge as contingent upon the context of the problems, the values embedded in the context, as well as their own choices and decisions (Davis & Maher, 1997). Because of the emphasis on active inquiry, learning incorporates investigation, discovery, discussion, play, co-operative work and exploration. Errors, misconceptions and alternative conceptions play an important role in the construction of mathematical knowledge in that these create conflict (Kim, 2005). In this approach, mental conflict is considered necessary for the development of both critical and innovative thought (Davis & Maher, 1997). In addition, students are encouraged to develop meaning from interactions and discussions with other students or with their teacher (Brombacher, 1997).

ii) Theory of Teaching

The teacher is responsible for creating an environment that will provoke learning. This is accomplished by choosing appropriate resources and carefully structuring these to create situations for explorations (Toumasis, 1997). Resources are selected that will capture the attention of the child; encourage continual engagement; and, will act as a vehicle for thinking rather than being an end in themselves (Dossey et al., 2002). Central to the learning experience is not whether the learners are constructing, but the nature and quality of the mental representation that is being constructed. Once identified, these constructions are elaborated on and expanded by the teacher through guiding, questioning, discussing, clarifying and listening (Toumasis, 1997). Moreover, the teacher needs to establish a nurturing and safe classroom environment which allows learners to take risks and where all contributions are respected and valued (David & Maher, 1997). Within a fallibist framework, it is particularly pivotal that the teacher model meaningful discussions and coach learners through the difficulties they may be experiencing (Markey et al., Date?).

iii) Strengths and Weaknesses of the Fallibist Approach

- intrinsically motivates students to engage in mathematical learning and activities
- focuses on students constructing concepts, discovering relationships for themselves and applying mathematics to solve meaningful problems
- tends to neglect mastery of algorithmic skills
- is more time-consuming in class
- has the long term effect that students tend to retain what they learnt over an extended period of time
- requires teachers to be more creative, understand students as individuals, and be apt at classroom management

Essentially, the fallibist and absolutist claims support contrasting metaphors of learning (Svar, 1998 cited in Kraak & Young, 2005), viz.:

- On the one side is the fallibist belief that learning is participation and which underpins the idea of learner-centredness and 'teacher as facilitator'.
- And on the other side is the opposing absolutist philosophy that sees learning as acquisition and which links pedagogy to the transmission of a given body of knowledge.

These two metaphors are not just a question of improving techniques, rather, as indicated above, they influence two different teaching approaches, each with its own strengths and weaknesses. Adopting a specific claim or metaphor therefore requires the rethinking of assumptions about teaching and learning and the practical implications that follow (Kraak & Young, 2005). To sum up, in broad terms one can refer to the absolutist approach as 'traditional teaching' and the fallibist approach as 'reform teaching'. Aliprantis and Carmona (2003) provide a concise summary of the two perspectives.

> The traditional educators "advocate curriculum standards that stress specific, clearly identified mathematical skills, as well as step-by-step procedures for solving problems"(Goldin, in press). These educators also pay careful attention to the answers that students attain and the level of correctness that they demonstrate. Drill and practice methods constitute a huge portion of the time in the classroom to ensure the correct methods in order to achieve the correct answers. Reform educators, on the other hand, advocate curriculum standards in which higher level mathematical reasoning processes are stressed. These include "students finding patterns, making connections, communicating mathematically, and engaging in real-life contextualized and open-ended problem solving," (Goldin, p.5). It is through this open-minded interpretation of education that different ways of students' thinking are verified and encouraged and where a broader variety of students are acknowledged, especially those that are capable, but considered remedial by traditional standards (Aliprantics & Carmona, 2003, p. 256).

Research indicates that absolutist approach to teaching mathematics is generally adhered to in the South African classroom context (Ramnarian, 2003). Government policy, however, is advocating that education be transformed in accordance with the principles of outcomes based education (DoE, 2003b). To this end government tends to support a fallibist approach to mathematics.

> Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. It is a distinctly human activity practiced by all cultures. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change (DoE, 2003a, p.9).

Moreover, the Department of Education presupposes that an efficient mathematics teacher will engage in active methods, take a constructivist view to learning, and understand learning to be situated in a particular context. Only then are mathematics and the knowledge it communicates expected to make sense to the child (DoE, 2003b).

Both the idea of a child as an active sense-making individual within a social context advocated by the reform approach and the step-by-step learning of procedures through frequent drill and practice session of more traditional proponents can be traced to several psychological positions on learning that emerged. In this regard Tall (1991, p.3) cites Hadamard's (1945) position 'that the subject [of mathematical thinking] involves two disciplines, psychology and mathematics, and would require, in order to be treated adequately, that one be both a psychologist and a mathematician.'

2. 3 LEARNING THEORIES

In this section various learning theories are discussed by first introducing the main proponents and/or principles of the particular learning theory, followed by how these measures affect mathematical teaching and learning, and thereafter demonstrating how the respective corollaries were extended into educational technology. The first orientation that will be addressed is behaviourism.

2.3.1 Behaviourism

Berton and Moore (2004, p.61) define behaviourism as 'the study of the observable, or outward, aspects of behaviour in relation to changes in the environment'. Hence, behaviourism is the belief that behaviour itself is the appropriate object of the study of learning and teaching. In general behaviourism ignores the cognitive processes taking place in the mind (Chen, 2009). Rather, behaviourists posit that it is in studying the cause and effect of behaviour that one is seen to be studying the cause and effect of learning itself. One particular theorist from within this field was B.F. Skinner.

2.3.1.1 B. F. Skinner

Behaviourists such as Pavlov, Thorndike and Watson believed that learning takes place as the result of a response that follows a particular stimulus. By repeating the stimulus-response cycle the organism can be conditioned into specific patterns of behaviour whenever the stimulus is present (Chen, 2009). Skinner took these original ideas of classical conditioning and refined them. His distinct contribution to behaviourism is technically referred to as operant conditioning (Skinner, 1964). Operant conditioning was particularly interested in how learning was influenced by the consequences of behaviour. (For a deeper analysis of how operant conditioning differed from classical conditioning, see Chen, 2009, p.156). Hence, a specific interest of Skinner's was the change of behavioural patterns through properly arranged reinforcements. His argument was that behaviour could be modified by carefully examining and altering the immediate consequences of behaviour (Skinner, 1964). A basic law formulated by Skinner in this respect was that the strength of a behaviour is increased when followed by a reinforcing stimulus (Reynolds, Sinatra & Jetton, 1996). Skinner applied this law to all human behaviour. From within this framework teaching became essentially about structuring opportunities for reinforcement (Berg, 2003). For optimal learning, the frequency and type of reinforcement has to be effectively scheduled.

2.3.1.2 The Influence of Behaviourism on Mathematics

In general, behaviourism emphasises applying rules in order to produce a specific result, without necessarily providing a theoretical attempt to explain why and how these rules work. Hence, attention is directed away from elements of understanding to performance and conduct. Kitchener (1972) refers to the behaviourist approach as providing a type of cookbook knowledge whereby a complex mathematical task is broken down into the form of a recipe or procedure that should be followed in a step-by-step manner to produce a particular product. Additional criteria are necessary to

support learners as they work through each step. In particular, learners need (Berton & Moore, 2004; Bereiter, 2002):

- to progress hierarchically from simple to more complex tasks;

- cues such as prompts or suggestions in their environment as to which behaviours will be effective;

- trial and error feedback after each step;

- frequent small rewards to support their mathematical learning rather than punishment and reprimand; and,

- to behave in an overt way to validate that learning has indeed occurred.

Two other elements underpin the behaviourist assumptions of learning, viz. that learning is an individual process (Berton & Moore, 2004), and that learning is a predictable process (Winn, 2008). Learning is an individual process as each person has a unique history of reinforcement. Behaviour is predictable in so far as there is a reliable link between the stimulus and the response it produces in a learner.

In short, a behaviourist will identify the mathematical subskills the student must master that, in aggregate, permit the intended procedure or process to be learned; formulate observable indicators for assessment; select the strategy for presentation that builds each subskill; and decide on appropriate reinforcers (Bereiter, 2002; Winn, 2008). Hence, mathematical teaching will adhere to highly structured paths, which didactically direct the individual learner through a prescriptive sequence and structure.

2.3.1.3. The Influence of Behaviourism on Educational Technology

In applying Skinner's work to educational technology it is said that Skinner was one of the first psychologists to use computers for educational purposes (Berg, 2003).

Berg (2003) describes Skinner's vision of the role of Educational Technology as follows:

> Skinner argues that the use of technology in teaching can increase learned behaviour by organising learning objectives, increasing the frequency of positive reinforcement, customizing the learning experiences, and freeing teachers from repetitive teaching. (Berg, 2003, p.17)

There are, however, several other ways in which behaviourism has influenced educational technology (Berg, 2003; Burton & Moore, 2004; Chen, 2009) viz.

- The behavioural objectives movement

  The behaviourist design model requires that the objectives of the study be clearly stated in any course; that all objectives are measurable and observable and that there is evidence of a change in the student's behaviour (Burton & Moore, 2004).

- The programmed instruction movement and the teaching machine phase

  Programmed instruction comprises self-administered and self-paced learning materials that contain logically arranged sequences of stimuli and with much repetition of concepts (Britannica Encyclopaedia). The student is generally presented with information in small steps. Thereafter the student is required to answer questions or fill in blank statements relating to the learnt information, and then receives immediate feedback before continuing to the next step. Both programmed instruction and the teaching machine notion subscribed to the same principles.

  Although the concept of a teaching machine was introduced earlier, it was popularised by Skinner's research and used in both instructional research and instructional technology. Skinner believed that the advantage of the teaching machine over programmed instruction was that the teaching machine could provide immediate reinforcement (Berg, 2003). Moreover, he foresaw that the teaching machine had the potential to become an automated private tutor to the

individual learner, thus stimulating the idea of a machine replacing a human teacher.

- Computer-assisted learning

  As personal computers became more popular, the notions of programmed instruction and the teaching machine were transferred to a computer platform resulting in computer-assisted learning (Berg, 2003). The paradigm of computer-assisted e-learning is to explain, practice and test (Chen, 2009). Hence, computer-assisted learning generally consists of drill and practice sessions, tutorials or simulations offered on computer. The idea is that the student can master a subject in his own time and at his own pace. Like programmed instruction and the teaching machine principle, the students' behaviour is shaped by principles such as staged linear progression, reinforcement and repetition built into the software design.

An alternate learning theory with equally strong implications in the field of mathematics and technology is that of constructivism which was formalised initially in the work of Piaget.

2.3.2 Jean Piaget

Piaget developed the theoretical framework of genetic epistemology to explain how intelligence forms in human organisms (Piaget, 1955; Ginsburg, 1985). He likened the development of intelligence to the biological process of an organism adapting to its environment in order to survive (Ernst, 1994).

Piaget (1999, p. 8) defined the adjustment of the individual's cognitive structures to the environment as 'an equilibrium between the action of the organism on the environment and vice versa'. Equilibrium mainly occurs through the processes of assimilation and accommodation. Whereas assimilation involves the interpretation of events in terms of existing cognitive structures, accommodation refers to changing the cognitive structure itself to make sense of the environment (Campbell, 2006). In other

words, assimilation means that people transform information from the environment to fit with their existing way of thinking, whereas during accommodation people adapt their way of thinking to fit new experiences (Siegler, 1995). Hence, assimilation and accommodation are fundamental in learning in that they explain the process of how reality is integrated into pre-existing knowledge structures (Berg, 2003), that is, how concepts are internalised.

Two implications of Piaget's work on equilibrium are emphasized. Firstly, thought is a form of organisation or equilibrium of cognitive structures (Piaget, 1999). In particular, 'human intelligence orders the world it experiences in organising its own cognitive structures' (Ernst, 1993, p.88). This means that intelligence is in part an activity of the organism in organising information effectively by integrating into existing mental structures (Piaget, 1999). And, secondly, there must be a degree of disequilibrium or discrepancy between the organism's existing cognitive structure and some new event in the environment, for cognitive development and change in thought to occur (Ginsburg, 1985).

Piaget (1955) posited that the development of thought progressed through developmental stages representative of four primary cognitive structures namely: sensorimotor, preoperations, concrete operations, and formal operations. The stages are sequential and hierarchical in nature, in so far as each stage signifies a more sophisticated and stable level of thought.

To sum up, the development of mental activity from perception to symbolic, to reasoning and formal thought, is a function of equilibrium. Equilibrium results from the successive adaptations between an organism and his environment through the functions of assimilation and accommodation (Piaget, 1999). Hence, Piaget was considered as a constructivist epistemologist in that his work speaks of the construction of knowledge through cognitive adaptation in terms of the learner's assimilation and accommodation of experiences into an action scheme (Jaworksi, 1994). It is in particular in through the work of Jean Piaget that the idea of the child being 'freed' to build his/her own constructions in a learner-centred environment has become prominent in education and thereafter in educational technology.

2.3.2.1 The Influence of Constructivism on Mathematics

The central tenet of the constructivist metaphor is that humans are knowledge constructors (Mayer, 1996). Since the learners assist in the construction of their own reality, it means that understanding involves individual action (Berg, 2003). In lieu of Piaget's theory, mathematical learning is therefore no longer considered to be simply imposed by environmental forces, that is, by association. Simply put, adopting a 'lecturing style' to 'transmit' mathematical knowledge to learners in the classroom (such as is seen in the absolutist approach to mathematical teaching) is not considered to be an effective form of teaching in a Piagetian model. Rather, learners must take an active role in their mathematical learning and be provided with opportunities to assimilate environmental events into previously acquired knowledge schemas (Ginsburg, 1985). To encourage assimilation and accommodation, learners need to question, experiment and discover mathematical relationships and principles for themselves. In this context, teachers are guides to academic tasks and learners are sense makers (Mayers, 1996). Constructivism is therefore not concerned with the passing of knowledge from one generation to the next in the most efficient way possible. Rather, the emphasis is on giving learners resources through which they can build and refine their own mathematical knowledge in personable and meaningful ways (Riebert, 2008). Moreover, mathematical content in the classroom should not be presented as static and fixed, but learners need to work in ways in which their knowledge is constantly changed and transformed to meet challenges and contradictions (Campbell, 2006). To a large degree fallibism represents a form of constructivism.

Piaget's ideas on constructivism have been further extended into mathematical learning and teaching by authors such as Bereiter (1992).

2.3.2.2 Bereiter's Distinction between Referent-Centred and Problem-Centred Knowledge

Based on Piaget's notion of learners who construct their own knowledge, Bereiter (1992) argued that irrespective of how one teaches, the learners will construct

knowledge. The essence of Bereiter's argument however, lies in his position that two types of knowledge can potentially be constructed by the learner, viz. knowledge that is organised around referents, and knowledge that is organised around problems. He builds his argument by referring back to the more traditional distinction of mathematical knowledge into declarative and procedural knowledge. Declarative knowledge is 'knowing about'. It commonly consists of content that constitutes declarative statements. The sources of declarative knowledge are generally observation and information from authoritative sources. Learners need to read or listen to the content, but are seldom expected to engage in its application. In contrast, procedural knowledge is 'knowing how'. It involves processes and skills that are acquired through imitation, coaching and practice. Bereiter (1992) associates declarative knowledge with referent-centred knowledge. Bereiter (1992) illustrates his comparison between referent-centred and problem-centred knowledge by referring to the concept of gravity. He argues that in a referent-centred format learners can know a great deal about gravity, including potentially useful formulas, and yet have no idea how to apply these. This often results in verbalism, where learners are able to use the word but in a very limited context. In such a case, the higher-order concept of gravity becomes a 'topic'. In contrast, when gravity is seen as problem-centred, one uses gravity to solve problems. In this instance gravity does not stand on its own, for example, a cup falling to the ground is not an instance of gravity. It is rather a phenomenon that gravity tries to explain. There is no abstract thing or referent that gravity is about. Rather, there is a class of gravity problems to which the knowledge applies. To sum up, Bereiter (1992) argues that higher-order mathematical concepts should be centred around problems rather than taught as 'topics'. He provides evidence of how a problem-centred approach increased both motivation and retention in students and enhanced their ability to apply mathematical knowledge. Dossey et al. (2002, p. 256) reiterates that we live in a data-rich society and therefore 'the ability to comprehend the meanings embedded in data and to work with data to answer specific questions is an important educational outcome'.

Apart from Bereiter, who extended constructivism into the need for problem-centred mathematics, Piaget's ideas on constructivism were taken into the field of Educational Technology by Seymour Papert.

2.3.2.3. The Influence of Constructivism on Educational Technology

Papert (1980, p.vii) confesses that he was 'developing a way of thinking that would be resonant with Piaget's'. In particular both started out as constructivists, keeping to the model of children as builders of their own intellectual structures. Papert predicted that the computer has the potential to create a radically different learning environment in the field of mathematics (Riebert, 2008).

Papert was especially interested in how computers may affect the way people think and learn. His interest was not so much concerned with the instrumental role of computers as with the way computers can assist conceptual development in mathematics – even when people are physically removed from a computer (Papert, 1980). Papert felt that children in the mathematics classroom should never be reduced to reactive answering machines – meaning that instruction is not about putting questions and then adjusting answers through right/wrong feedback. Rather, the innate instincts of children to ask their own questions and pursue answers to these should be encouraged. Papert wanted the learners to own the mathematical problem in a way that would make the activity meaningful to them. Subsequently, his ideal was to move beyond computer-assisted instruction where the computer taught the child to the 'child teaching the computer' in the manner of the child being able to program the computer. In other words, Papert wanted the child to control the computer and not the computer to control the child. He built this concept into his programming language for education (called LOGO) and posited that learning how to communicate with the computer through programming would facilitate other forms of learning and simulate a more natural manner of learning than the artificial mode often encountered within the formal curricula.

Despite its profound influence on mathematical teaching and learning, Piaget's form of constructivism, however, has been critiqued for its emphasis on subjective knowledge at the expense of collective knowledge. Although Piaget spoke of social influences as one of the determinants of human behaviour, his constructivism has failed to interact optimally with the larger socio-cultural context and the social nature

of human communities. Social constructivism, however, attenuates that knowledge is constructed socially through negotiation and mediation with others (Jaworski, 1994).

2.3.3 Social Constructivism

Both constructivism and social constructivism highlight the role of activity in learning and development. Whereas constructivists give priority to the individual's sensory–motor and conceptual activity, sociocultural theorists relate activity to participation in culturally organised practices. One of the prominent theorists within social constructivism is Vygotsky.

2.3.3.1 Lev Vygotsky

Wertsch (1988, p. 14-15) argued that Vygotsky's theoretical work comprised three core themes, viz. a reliance on a genetic or developmental model; a claim that the higher mental functions of individuals have their origin in social functions; and, a claim that higher mental functions can be understood only if we understand the tools or signs through which they are mediated. Particular emphasis will first be given to the second claim, that is, that social interaction plays a role in transforming elementary stimulus-response reflexes to higher, conscious cognitive functions. In this regard, Vygotsky (1978, p. 176) posited that 'human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around him". Vygotsky, therefore, did not consider intelligence innate but as a form of psychological development resulting from the cultural-historical experiences of a child interacting with an adult (Pass, 2004).

Hence, in contrast to Piaget who saw development as primarily occurring 'from the inside out', Vygotsky put forward that development was 'from the outside in'. Firstly, the child and adult would converse. Thereafter the child would internalise the conversation with the adult. Later the child would use this form of inner speech to regulate his/her own behaviour. Egocentric speech in young children is considered a

transitional state from communicative speech to inner speech. Vygotsky (1978) articulates this progression from social to inner voice as follows:

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. (Vygotsky, 1978, p. 57)

Vygotsky referred to the above principle the as the genetic law of cultural development. An implication of this law for education is that all higher functions originate as actual relationships between individuals (Vygotsky, 1996). In other words, a cognitive function constitutes a successful transfer from the interpsychological to the intrapscyhological plane. This transfer can occur through cultural sign systems (language, writing and numbers). Yet the primarily medium is through speech (Wertsch, 1988). Vygotsky described speech as a principle psychological tool that can direct thought and behaviour because of the meaning encoded in it.

In contrast to Piaget, Vygotsky held a more positive view of teaching in his argument that the teacher can advance learners' cognitive and metacognitive development through pedagogical operations in the Zone of Proximal Development (ZPD). The notion of the ZPD therefore establishes Vygotsky's position on how instruction can lead to development (Daniel, Cole & Wertsch, 2007).

Vygostsky (1978) defined the ZPD as follows:

> It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978, p. 174).

Simply put, the ZPD is the difference between what a learner can do without help and what he or she can do with help or in collaboration with others. A particular

pedagogical ideal held by Vygotsky was the transformation of a learner's intellect in the ZPD by assisting individuals to learn scientific concepts by connecting them to spontaneous or every day concepts. If the two forms – the abstract and the concrete – (the decontextualised theoretical principles of school and the real experiences from life) do not connect, then true concept development fails to take place (Daniels et al., 2007). Mature concepts thus depend on the merge of scientific and everyday versions of knowledge.

2.3.3.2 The Influence of Social Constructivism on Mathematics

Vygotsky and other social constructivists see similar possibilities concerning the social nature of knowledge and the social formation of the mind in so far as knowledge is mediated, collaborated, culture-bound and contingent on language and other semiotic devices. The congruity between Vygotsky and social constructivism is set in symbolic interactionism, the emergence of intersubjectivity and the context of meaning. Hence, a metaphor employed by social constructivists is that learning is social negotiation and that learners are social negotiators (Mayer, 1996). In line with Social Constructivist thought, learning in the mathematics classroom should take the form of a social process that requires adult guidance and peer collaboration, and an emphasis on dialogue and language (Pass, 2004). Social constructivism confronts the belief that mathematics is a silent activity with each child producing his/her own work. By way of application, social constructivism encourages teachers to help learners create and negotiate meaning through a rich language environment. 'Talking mathematics' as suggested by social constructivism can be particularly beneficial and enriching for learners. Promoting discussions around contextual mathematical tasks may encourage learners to engage in aspects such as hypothesis testing, debating, justifying, and simply expressing a particular view to an audience. In this manner speech can function as a powerful psychological tool in the construction of mathematical thought and consciousness (Daniels, 2005). Such dialogic interaction not only promotes conceptual and cognitive development in the light of Vygotsky's theory, but ultimately presents a form of empowerment for the learners themselves.

2.3.3.3. The Influence of Social Constructivism on Educational Technology

The social constructivist framework allows for several applications to Educational Technology. It is a perspective which emphasises that technology is socially shaped. Subsequently one has to examine the social and cultural factors that may contribute to or detract from the successful integration of computer technology into mathematical environments (Martin, 1999). Martin (1999, p.406) refers to the concept of 'interpretive flexibility' that exists within social constructivism. Interpretive flexibility captures that the meaning and the use of technology is culturally constructed and interpreted. Hence, regardless of the explicit intentions of the original designers, technology can take on multiple meanings for different people. Such an approach safeguards against a more technological determinist drive where the outcome of a technology is considered self-evident, universal and according to a predetermined trajectory. In other words, the social constructivist approach cautions one not to adopt the expectation that the computer in and of itself will bring about a transformation in education. Rather, it reminds us that personal preferences of both teachers and students have a powerful impact on the meaning given to technology (Martin, 1999). It also alerts one to the dialectal influences of learning on culture and culture on learning in a multicultural classroom (Reynolds et al., 1996).

As indicated above, the social constructivist approach emphasises education as an interpersonal participatory activity. It thus became important for computers to facilitate learning in a way that enables relational interchange, intersubjectivity and conversational negotiation. Attention was therefore shifted to seeing how technology could effectively mediate peer-to-peer (or adult-to-peer apprenticeship) learning situations to allow for the co-construction of meaning (Loong, 1998) and to support co-operative learning (Berg, 2003).

Whereas constructivism is described as moving from the individual mind to the social, social constructivism is seen as moving from the social to the individual. In other words, the individual consciousness is built from the outside in and not from the inside out. However, there is another school of thought that entirely abandons the notion of an individual consciousness being constructed to embracing a paradigm

where consciousness is situated solely within the social context. This view is known as Situated Social Cognition.

2.3.4 Situated Social Cognition

In the situated social cognition field learning is described purely in terms of relations pertaining to a specific context or situation (Lave, 1996). Hence the focus is not on the internal or external actions of one or another party. Rather it is on the process of interactivity – the relation, the transaction or the dialect – between people and on how such connections produce particular learning outcomes (Loong, 1998). Within the framework, knowledge is no longer considered a stored artifact in the person or in the environment, but knowledge is constructed-in-action and can thus only exist as a mutually produced whole (Loong, 1998). Subsequently, there is a weaving together of cognition and context in so far as the person acting (cognition) and the social world in which the person acts (the context) cannot be separated from one another (Lave, 1996, p. 5). Learning can therefore be defined as *changing the way one acts* in a particular cultural setting (Lave, 1996) – as an enculturation process. Hence, unlike constructivism where learning is a form of self-organising taking place in the individual, in situated cognition learning is a form of 'eco-social organisation' where the human organises him/herself in terms of the social environment. In other words, the focus of situated cognition is not on how we intrapsychologically internalise a concept, but on how we as novices begin to experimentally imitate the larger culture's use of interpsychological tools and how we eventually adapt ourselves to such usage. Hence, there is a shift in focus from the individual as the unit of analysis towards the social-cultural setting and its practices.

2.3.4.1 The Influence of Situated Cognition on Mathematics

The implications for pedagogy are that mathematical learning is ultimately a social process, and not one contained in the mind of the learner. Hence, learning activities should never be presented or considered in a decontextualised setting (Lave, 1996). In this approach, mathematical cognition without context is not possible. Hence, what is

deemed a more natural approach to learning viz. ‘learning-in-practice’ is promoted in the mathematics classroom. In other words, knowing and doing is linked. How mathematics is learned can therefore not be separated from how it is used in the world. This has given rise to the ‘authentic learning experience’. This means that any mathematical learning activity needs to be placed into its natural mathematical cultural context. Hence the activities of students must resemble the activities of the practitioner in the mathematics field. It is posited that it is only from within such a context that the true meaning of concepts can be negotiated and progressively constructed. If one starts mathematical teaching with abstract, self-contained definitions and general decontextualised principles learners often struggle to make sense of these entities and tend to not apply them appropriately to relevant settings (Lave, 1996). It is thus posited that increasingly rich and implicit understanding of concepts can only be built through learners continually interacting with real-life type of situations. In this respect situated cognition has been criticised for an underlying discourse that is too dismissive of the role of rules and generalisations in learning.

> The situated action of just plain folks comes off as flexible, adaptive, and elegant – in a word, intelligent- whereas action based on formal procedures and principles comes off as brittle, plodding, insensitive to nuance – in a word, stupid. It is time, therefore to look at the other side. (Bereiter, 1997, p. 286)

Moreover, if learning is situated in a particular context it restricts transfer to alternative situations. Krishner and Whitson (1997, p. 9) posit that ‘as long as contexts are seen as isolated units of sociophysical space, there is no adequate explanation for the human ability to move between them.’

Yet, as with the other approaches, situated social cognition has made its mark on educational technology.

2.3.4.2 The Influence of Situated Cognition on Educational Technology

Emerging devices, tools, media and virtual environments offer opportunities for creating new types of learning communities for students and teachers. One particular

application from the field of situated cognition is that of distributive learning tools through technology. Distributed cognition views the combination of people as a cognitive system. Hence, within the distributed cognition framework, computer technologies are not considered as mere conveyors of information, but as cognitive tools and partners in cognition. Moreover, cognition is not the property of individuals, but distributed or "stretched over" an extended cognitive system, which may include the individual, other people, artifacts, and tools.

2.4 THEORETICAL FRAMEWORK

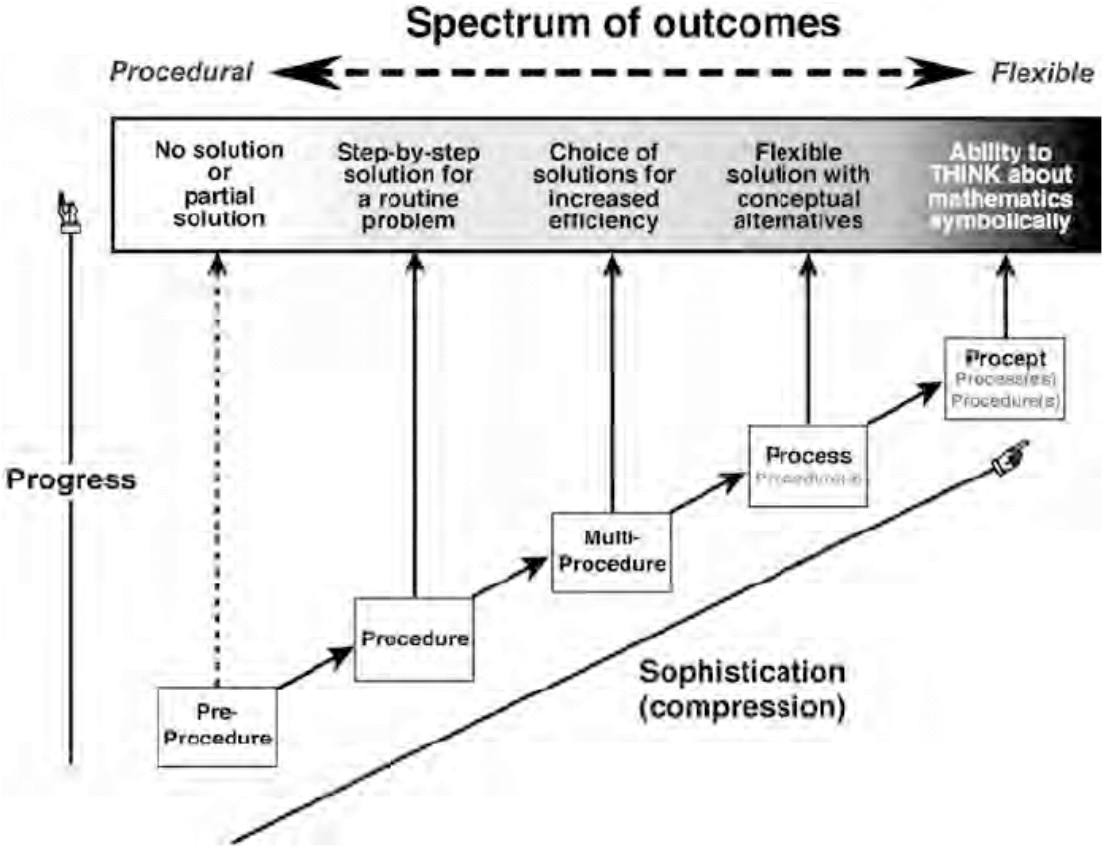
The intent of the research project was to emphasise aspects of the social constructivist and situated cognitive approach to learning. I thus foresaw conceptual change arising out of collaboration. Stated differently, it was assumed that the learners' insights will come not only through placing the mathematics on a digital platform and studying the graphs and its properties within the context of a particular problem, but in particular by talking about it. One primary objective therefore converged on bringing about shared, articulated understanding by conversation. Moreover, the role of technology was to support the conversation and insights by allowing participants to test their hypotheses, illustrate their thoughts, and review and revise their developing understanding thereby emulating the manner of mathematicians at work. However, academic literature (which is discussed in more detail below) informs that when transiting learners in new set-ups, certain "adjustment traits" or "behaviours" from students can be anticipated.

My position is that the interactive exercises up to but not including "Creations" contain elements of emerging model-eliciting activities. The modeling aspects are seen in the real-life setting where a model in the form of the equation and graph is used to represent socio-economic dynamics which can altered through inputting information into the computer. Altering the dynamics using technology offers the students an opportunity to experiment mathematically with different conditions which might affect specific outcomes. By engaging with the technology they can hypothesise, then test their thinking, provide justification of their conclusions whilst

communicating their thinking and their operations with others in the group. Carlson, Larsen and Lesh (2003) cite various studies to argue that conventional curricula have not been successful in promoting such abilities in students. Compared to reform mathematics, traditional teaching styles tend not to offer appropriate exposure and tools to students through which they can develop the abilities to analyze and create complex mathematical models. Similarly Tall (1991, p.3) posits that a logical presentation that tends to give students the 'product of mathematical thought' rather than the process of mathematical thinking, may not be appropriate for the cognitive development of the learner. Subsequently, students' initial interpretations of given goals, and available solution steps tend to be quite 'barren, distorted and unstable' compared to later interpretation (Lesh and Doerr, 2003, p.24). As was noted in Chapter 1, De Bock et al. (2007) discusses how learners who are exposed to the more traditional learning pathways tend to develop a computational and routine rigidity in their thinking, whereas learners who are engaged in emergent modeling activities tend to acquire a stronger from of adaptive intelligence. Learners with routine expertise will carry out procedures, but without much understanding. Adaptive expertise, however, refers to be being able to apply mathematical procedures meaningfully, creatively and in a flexible manner, which again is reflective of Kilpatrick et al.'s third and fourth strands. Yet, Lesh and Doerr (2003) caution that one should not become pessimistic about learners' abilities to engage in productive development activities. Their argument from research is that it is common for learners to make dramatic learning gains when they are put in learning situations where they must express, revise, test and refine their conceptual systems to make sense of mathematical-rich situations. Moreover, these gains include lower achieving students who often begin to show more impressive mathematical performance than their results on standardized tests have previously implied.

Considering the expectation that learners' initial interpretations tend to be barren and distorted, a primary question that must be answered concerns how to move students beyond the limitations of their own initial ways of thinking? Proponents of modeling hold that conceptual development and cognitive tools start germinating as learners work through multiple cycles of revision, testing and expansion of the original model (Lesh and Doerr, 2003). Modeling thus endorses a spiral or cyclical line of development.

In contrast, mathematical researchers such as Tall (2000) work with a more linear progression model to depict the complexity of mathematical development. Figure 1



shows Tall's notion of progression in mathematical development from pre-procedural to procedural to multi-procedural to conceptual to proceptual. Tall (1998, 2000) argues that learners' difficulties in mathematics may be a function of their inability to translate a procedure into a concept and then to code-switch between the two forms depending on the mathematical problem or question at hand. For example, a simple arithmetic example is found in 2 + 3. This symbolism simultaneously represents a procedure (add 2 and 3 to get 5), and a concept (the idea of sum). Using this as a base, he describes the difference in mathematical proficiency amongst students as a divide between those who only perform procedurally and those who have transformed a procedure into a thinkable concept (Tall, 2000). Transforming a procedure into a concept allows for greater cognitive flexibility and this relates to increased mathematical success. Tall was influenced by the work of Piaget in the formulation of his theoretical orientation that learners internalise concepts and build understanding by acting on their environment. Tall (2000) consequently argues that the computer can be used to enhance mathematical thinking in this regard with special reference to the computer's ability to provide visualisation and symbol manipulation. He posits

that one way to symbolically compress a procedure into an object involves the learner operating on visible objects. Using mathematical software to make mathematical ideas visible, the learner is afforded the opportunity to see the effect that actions have on mathematical objects, for instance, when sharing them into equal shares, permuting them into a new arrangement, or translating an object on a plane. The effect in the context of this study is therefore the mathematical change from the initial state to the final state. What is important is that the learner's focus of attention is shifted from the steps of the procedure to the effect of the procedure. Failure to allow the learner room to act on objects in a visible way may result in poor mathematical conception.

> A curriculum that focuses on symbolism and not on related embodiments may limit the vision of the learner who may learn to perform a procedure, even conceive of it as an overall process, but fail to be able to imagine or 'encapsulate' the process as an 'object' (Tall, 2000, p.7).

To sum up, in considering whether one teaches according to a fallibist or an absolutist approach, and the particular learning theories one adapts one's pedagogy in accordance with, should not only depend on one's own epistemology as a teacher, but also on the needs and profiles of the learners one is teaching. It was indicated that the learners in this study come from an area of low socio-economic status in Johannesburg. Authors such as Abadzi (2006) argue that within pedagogy of learners with low socio-economic status (SES) certain learning and teaching elements are more effective than others.

2.4.1 A PEDAGOGY OF LEARNERS OF LOW SOCIO-ECONOMIC STATUS

Abadzi (2006) argues that a 'chalk and talk' lecture approach can be beneficial for low SES learners, provided it is done only for a limited time period. This is because low SES learners generally have lower attention spans due to associated factors such as malnutrition. At the same time, passive listeners in the class generally do not have strong encoding skills. The resulting effect is that learners may fluctuate in their

attention, thus picking up and loosing bits of information, while at the same time insufficiently decoding the information they have succeeded to retain. The absolutist approach is often also associated with individual seat work. Abadzi (2006) feels that individual seat work amongst students from low SES may prove ineffective if it is devoted to 'practice sessions' rather than 'invention' work.

In contrast, a socio-cultural approach involving practice, questioning, feedback and discussion is very effective with low SES learners. It is posited that regular dynamic interactions between students and teachers can help to facilitate recall amongst low SES learners and that it is through students' contributing to the lesson that contemplation and elaboration on the topic is encouraged.

On the other hand, a demand for examples and agitation when these are not offered have been found in the inner city classes amongst African-American students (Solomon, 2006) and lower white socio-economic groups (Lubienski, 2000) when model-eliciting activities were introduced. In contrast, learners from middle to upper white socio-economic groups did not express the same need. Lubienski (2000) argued that it is an issue of communication at home. In lower socio-economic groups the tendency is to communicate with children in a more directive and authoritarian manner, that is, "You do this exactly like I did it or like I told you". However, in middle to higher socio-economic groups parents tend to reason more with their children in respect to particular views and actions. Lubienski's (2000) analysis of data thus showed that reform mathematics could be problematic for the low socio-economic status students. Boaler's (1998) research, however, indicated that reform programmes are beneficial to learners from low socio-economic groups. Subsequently, Franco, Sztajn, and Ortigão (2007) undertook a large-scale investigation in Brazil to determine whether reform teaching was related to increased student achievement for all students, or whether the gains depended on students' social economic status. The study indicated that reform teaching raises all students' achievement levels. Hence, the study suggests that reform mathematics has the potential to narrow the gap between high socio-economic and low socio-economic schools. The picture emerging (Schoen, 1993; Boaler, 1998; Riordan & Noyce, 2001; Clarke, Breed & Fraser, 2004; Kim, 2005) confers that problem-solving students do as least as well, and often better, on standardized tests; are more able to transfer

mathematical ideas into the real world; are more confident in mathematics; value communication in mathematical learning more highly than students in conventional classes; and, developed more positive views about the nature of mathematics than their counterparts. Yet Franco et al. (2007) posit that reform mathematics are more likely to increase inequality in education, in so far as it is often the case that schools with higher SES students provide reform teaching whereas schools with low SES students provide traditional teaching. In other words, learners who could benefit most from reform mathematics are the least likely to receive it.

An additional concern to how low SES learners should be taught mathematics relates to the integration of computer technology into the classroom. As was previously noted, this study seeks to describe how learners from a low socio-economic background respond to and xperience mathematical learning using model-eliciting activities integrated with computer technology. Currently, there is a much academic debate with regards to the use of mathematical software in the classroom. There are those who support the notion that computers can create a mathematical learning space that places less emphasis on manipulative skills and more on conceptual understanding (Dubinsky & Tall, 1991; Tall, 1998) and those who are less supportive of such claims (Artique, 2000). In the following section the South African government's position in this debate will be explored.

2.4.2 CURRENT GOVERNMENTAL POLICY ON INTEGRATION OF TECHNOLOGY INTO MATHEMATICAL LEARNING

Current South African governmental policy supports and encourages the use of technology in teaching and learning in the Further Education and Training phase (DoE, 2003a).

> The computer together with custom designed software packages has become a powerful aid to developing mathematical concepts. The visualisation made possible through the dynamic capabilities of the computer is evident, not only for teaching and learning, but also for the

> advancement of mathematics and its applications. Spreadsheets, graphing packages and dynamic geometry software (for example, Cabri Geometrie, and Geometer's Sketchpad) are particularly useful in this regard. (DoE, 2003a, p. 13)

There is research that further supports the use of computers in mathematical education. Dubinsky and Tall (1991) for example, report that mathematical learning is enhanced by using the computer for explicit conceptual purposes. They argue that appropriate software can aid learners in exploring concepts in directed and meaningful ways that may be more appropriate. Their position (Dubinsky & Tall, 1991, p.231) is that a major purpose of the computer is 'to help students conceptualize, and construct for themselves, mathematics that has already been formulated by others'. Tall (1998) specifically argues that the computer can be used to enhance mathematical thinking in this regard with special reference to the computer's ability to provide visualisation and dynamic symbol manipulation. He posits that one way to symbolically compress a procedure into an object involves the learner operating on visible objects. Using mathematical software to make mathematical ideas visible, the learner is afforded the opportunity to see the effect that actions have on mathematical objects for instance, when sharing them into equal shares, permuting them into a new arrangement, or translating an object on a plane. The effect in the context of this study is therefore the mathematical change from the initial state to the final state. What is important is that the learners' focus of attention is shifted from the steps of the procedure to the effect of the procedure. Failure to allow the learner room to act on objects in a visible way may result in poor mathematical conception.
A curriculum that focuses on symbolism and not on related embodiments may limit the vision of the learner who may learn to perform a procedure, even conceive of it as an overall process, but fail to be able to imagine or 'encapsulate' the process as an 'object' (Tall, 2000, p.7).

Considering the government's positive advocacy with regards to integrating technology into mathematical learning, McDonald and Gibbons (2007) ask why educational technology tends to fail in reaching its promised educational delivery despite strong intentions such as are expressed in the Draft White Paper on E-learning. The answer suggested is that the unexamined assumptions instructional

technologists may hold about a discipline can negatively influence its theory and practice (McDonald & Gibbons, 2007, p.378). Moreover, such assumptions may be powerful enough to prevent theory from translating into practice, thus resulting in a loss of quality originally envisioned or promoted and one which often manifests in some form of reductionist learning approaches. Working on the idea of underlying assumptions, three categories of beliefs were identified, referred to as Technology I, II and III approaches.

> The Technology I criterion presupposes that using media devices automatically leads people to develop quality instruction. The Technology II criterion presupposes that using design formulas or techniques automatically leads people to develop quality instruction. The Technology III criterion proposes that instructional quality is not measured by the technologies or processes used, but rather is measured by the consequences of instruction with students and within the larger system (McDonald & Gibbons, 2007, p.379).

In short, Technology 1 is the assumption that technological devices have an intrinsic ability to solve educational problems. Or, put differently, technology in and of itself is a wonder cure – the panacea for all educational ills. This view entertains the perception that computers "on their own" will bring about a positive education revolution.

In contrast to the Technology I and II beliefs, Stols (2007) puts forth that computers and designs around technology in and of themselves do not guarantee effective learning. However, there are aspects of technology that lend themselves to creating learning environments, which support specific principles of learning.

Despite the good intention of the use of instructional technologies in subjects like mathematics to support active student learning, one questions whether the Draft White Paper is not perhaps too enthusiastic in its assumptions about the role of technology. A thoroughly analysis of the Draft White Paper falls outside the scope of this study, yet one feels compelled to point out that sections of the Draft White Paper seem to promote a 'boosterism' effect of technology on learning, which could be interpreted

as an alignment with the Technology 1 assumptions in McDonald and Gibbon's model.

Taking into account the government's active advocacy, the learners within this study have had no contact with computers, and in particular with mathematical software. This lack of exposure can be interpreted using Valsiner's (1997) extension of Vygotsky's (1979) Zone of Proximal Development (ZPD) to the notions of the Zone of Free Movement (ZFM) and Zone of Promoted Action (ZPA).

2.4.3 Factors Contributing to a Lack of Exposure to Computers

According to Goos and Bennison (2008, p.3) 'the ZFM structures an individual's access to different areas of the environment, the availability of different objects within an accessible area, and the ways the individual is permitted or enabled to act with accessible objects in accessible areas'. Hence the ZFM explores why the learners themselves may not have access to digital technologies. In this study only participants who have not yet had an opportunity to learn mathematics by using mathematical software were selected as participants. A further study is needed to fully explore the reasons for the learners' access (or lack of thereof) to technology as a vehicle for mathematical learning. On face value one could argue that one of the most prominent contributing factors may be their socio-economic condition.

Furthermore, 'the ZPA represents the efforts of a more experienced or knowledgeable person to promote the development of new skills' (Goos & Bennison, 2008, p. 3). Subsequently, the ZPA allows one to question why teachers within schools are not incorporating mathematical software into their pedagogies?

2.4.3.1 Factors that Prevent Optimal Use of Educational Technology amongst Teachers

Both international (Czubaj, 2004; Ertmer, 2005) and national studies (Stols, 2007) reflect a hesitance amongst teacher to fully incorporate technology into their own pedagogical practices and styles. This 'holding back' by teachers has been attributed to various concerns, including teacher training, school budgets, assessment practices

and personal attitudes (Czubaj, 2004). With respect to the latter, teacher anxiety over technology has been reported and labelled ‘cyberphobia’ or ‘technopathology’ (Russell & Russell, 1997).

One specific learning environment underscored by current educational reform as expressed in outcome based education initiatives is student-centered teaching practice. Yet, in spite of governmental regulation, Palak and Walls (2009) have found that even within technology-rich school environments teachers continue to use technology in ways that support their already existing teacher-centred instructional practices. Hence, they found it rare for teachers to use technology in a way that fully support student-centred practice, even amongst teachers who hold student-centred beliefs. In addition, teachers reported most frequent use of technology for preparation, management and administrative purposes rather than for actual teaching.

Hence, what is emerging from these findings is that psychological elements such as beliefs and unexamined assumptions have a very definite influence on the use of technology in the classroom. However, when learners are exposed to technology in the classroom, certain authors suggest that a form of instrumental genesis develops.

2.4.3.2 Instrumental Genesis

Rabardel (1995) (cited in Bussi & Mariotti, 2008) points out the complexity of the relationship between artifacts and knowledge when tools such as information and communication technologies are used in human learning and thinking. Rabardel puts forward an instrumental approach based on the distinction between an artifact and an instrument to describe the context of students’ activity as they interact with technology. In the context of this particular study an artifact would refer to the object, that is, the computer with the mathematical software. The instrument, however, refers to the integration between the object and the subject. More specifically, it relates to the utilisation schemas that are built up in people as they start engaging with the technology. Simply put, learners need to construct personal schemas of the significance and usefulness of the mathematical software in the context of graphing. The evolution of these schemas is, however, an elaborated process that emerges

progressively and one which Rabardel refers to as instrumental genesis. Instrumental genesis includes growing in awareness of the capabilities of the software in terms of its possibilities and constraints for mathematical learning and also developing the utilisation schemas mentioned. The utilisations are ultimately a subjective phenomenon that may or may not relate to the pragmatic utilities of the software.

2.5 SUMMARY

This paper assumes Castells' position that the world is entering into a new era called the information age or the network society age. In this age digital literacy is an indicator of a company's global economic competitiveness and a measure of the individual's marketability for jobs. At the same time I argue that digital literacy in itself is not sufficient, but that it needs to be combined with an evolving cognitive adaptability to knowledge that surpasses the mechanical execution of previously acquired procedures. How then does one combine digital literacy and adaptive knowledge in the field of mathematics education? I propose that a setup for this kind of nexus has been achieved in this study by, firstly, identifying that strategic competence and adaptive reasoning have been recognised as essential parts of learners' mathematical proficiencies in the Kilpatrick et al. framework; secondly, selecting interactive mathematical exercises designed around dynamic mathematical software; and, thirdly, by adopting the tenets from paradigms such as the social constructivist perspective and situated cognition theory which supports social negotiation as a basis for learning.

The primary participants of this study are youth from a particular low socio-economic stratum in South Africa. Learners will be given an equation within the context of a particular problem situation. Thereafter, the learners will have to input the equation into the computer using the dynamic software programme to create the respective graph on the computer and then use their mathematical intuition and knowledge to both interpret and manipulate the information to arrive at a working solution. Learners will be encouraged to work with the contextualised information to derive generalised mathematical principles. An additional aspect is to encourage learners to reflect on the "effect of the general principle" within the domain of mathematics.

Hence, the study affords learners the opportunity to work in a setting of a high level of knowledge and skills, which needs to be integrated and applied. At the same time support is being offered to the learners to help them progress and to add quality to their learning experiences.

Considering the learners' socio-economic background, it is the hypothesis that the majority of the learners will predominantly come from a traditional approach rather than from a more social constructivist context. This assumption is drawn from researchers such as Ramnarain (mentioned in the literature review), stating that although pockets of reform mathematics are being introduced into schools in South Africa, the predominant mode of teaching remains traditional. From an international perspective, Franco et al.'s research predicted that reform mathematics would be more prominent in schools where learners have a higher SES background than in schools where learners are from low SES environments. The concept of working in an interactive manner may thus be a foreign point of engagement. Hence, one anticipates some of the initial cognitive and behavioural characteristics identified by researchers such as De Bock et al. (barren and rigid mechanistic thought), Tall (lack of embodiment of concepts and thus a lack of precepts), and Lubienski's attention towards the demand for examples from the teacher.

An additional anticipation grounded in the ZFM (low socio-economic backgrounds restricting access) and ZPA (teacher's reluctance to incorporate technology into the classroom) aspects of Goos and Bennison's theory is that learners will have had little or no exposure to learning mathematics through computer software.

The study is essentially committed to social transformation in so far as the study (and the larger project that the study forms a part of) is committed to uplifting youth by investigating means by which to improve their mathematical results. Hence, the elements of social transformation, human rights and social justice form part of the overall philosophy and rationale of this project. The rationale is further developed by asking the following key questions during the research.

Primary Research Question

- How do learners engage with the strands of strategic competence and adaptive reasoning of mathematical proficiency framed by Kilpatrick et al. in the context of interactive mathematics in a digital environment?

Secondary Research Questions

- What (if any) are the main barriers learners experience in response to demonstrating strategic competence and adaptive reasoning in a digital environment?
    a) What are the conditions that seem to prevent particular knowledge-based dynamics to emerge in this socio-economic stratum?
    b) What are the conditions that enabled particular knowledge-based dynamics to emerge?
    c) What is the learners' personal response to learning mathematics in a digital environment, which requires an emphasised engagement with strategic competence and adaptive reasoning?

In the next section a fuller description of the research design that was utilised as a means to answering these questions is provided.

CHAPTER 3
RESEARCH DESIGN

In terms of the research design, as well as the different methods of data collection and the data collection process, this study is conducted within a qualitative paradigm. In particular, action research and a case study have been selected from the qualitative continuum as suitable for the objectives of this study in so far as the study aspires to do small-scale research in a bounded setting. Part of the intent was not to produce a 'generalisable result', but to provide a rich description of how certain Grade 11 learners perform according to the strands of mathematical proficiency in a specific context under specific learning conditions. Additional aspects of the description include an identification and discussion of what appears to be barriers experienced in the mathematical learning and teaching environment when an interactive approach is integrated with technology.

3.1 QUALITATIVE RESEARCH

In qualitative research, the researcher generally adopts an inductive approach to uncover and discover themes, categories and patterns within the data that will explain the phenomenon of interest (Lee, 1998). Differing patterns emerging from the same setting enables the researcher to understand the social or human problem from multiple perspectives, yet as an interwoven whole (Maykut & Morehouse, 1994). Moreover, qualitative research contains individual interpretations of events which make it more subjective in nature (Flick, 2006.) Since it focuses on aspects such as contextual detail and individual meaning making, qualitative data are generally more rich, time consuming, and less able to be generalised (Neuman, 2000). In this framework the researcher is considered the primary data-gathering instrument and the research design generally emerges as the study unfolds (Neuman, 2000).

In contrast to qualitative research, Neuman (2000) states that the aim of quantitative research is to develop generalisations that enable the researcher to predict, explain, and understand a phenomenon. The underlying assumption is that social reality, like

natural phenomena, is something that can be studied objectively. Hence, the researcher should remain distant and independent of what is being researched (Neuman, 2000). The researcher seeks precise measurement and analysis of target concepts that could be replicated at a later study (Lee, 1998). Data are expressed through numbers and statistics analysis (Thomas, 2003). Quantitative data are more efficient in their ability to test hypotheses and cause-effect, but may miss the finer nuances of contextual detail (Neuman, 2000).

In light of the primary research question, this study aligns itself with the qualitative paradigm in that it seeks to provide a detailed portrayal of the response of learners to of interactive learning activities integrated with mathematical software. For the sake of completeness the primary research, which is to record a description of how low socio-economic learners engage with the strands of strategic competence and adaptive reasoning of mathematical proficiency framed by Kilpatrick et al. in the context of interactive mathematics in a digital environment, is reiterated.[1] Moreover, the research project not only aims to alert the reader to recurring patterns and themes, but also to capture the subtleties and the subjective responses of the learners to the change in pedagogical circumstances and to engage the readers in a meaning-making discourse of these events. It is argued that deep descriptions of the different issues that learners deal with during these mathematical experiences may contribute to teachers' gaining more intimate knowledge of how technology affects mathematical learning in an interactive context, which could be used to assist in higher level learning achievement. In contrast, a quantitative approach could be helpful in ascertaining the effectiveness of the programme, but may contribute little to preparing teachers and learners for the underlying classroom dynamics that may emerge during the transformation period. Knowledge value is added to the description of the classroom dynamics by exploring the theme of barriers and advances towards the mathematical teaching/learning in the setting, and the theme of learners' own perceptions and likes and dislikes towards learning mathematics with dynamic software.

[1] I am aware of the need for causality in mathematical research. However, I do feel that to establish aspects of causality a much more complex study will be required consisting of separate experimental and control groups including groups that are only exposed to reform mathematics without technology, groups that are exposed to traditional teaching with technology, groups that are exposed to both reform teaching with computers, and groups that are exposed to traditional teaching with computer technology. The complexity of such a study may be pursued at a more advanced level, such as a PhD research undertaking.

3.2 ACTION RESEARCH

On the one spectrum of the research debate, teachers are encouraged to be active simultaneously as teachers and as researchers (Johnson, 1993; Wilson, 1995; Cunningham, 2000; Kincheloe, 2003; Nolen & Vander Putten, 2003).

This means that 'teachers are themselves involved as researchers, or collaborators in the research process, in their own teaching situations' (Nixon, 1987, p.21). In particular, the action-research approach has been selected in this study to allow the teacher to assume the roles of teacher and researcher at the same time.

Several advantages have been cited in the consideration of a teacher as a researcher. It is generally argued that when teachers begin to study their work themselves rather than only having their work being studied by others, teachers' practice and professional development will be enhanced in several ways (Johnson, 1993; Wilson, 1995; Kincheloe, 2003; Nolen & Vander Putten, 2003). For example, research may encourage skepticism in that teachers have the opportunity to question 'common sense' and 'taken-for-granted' assumptions that may enjoy widespread credence (Nixon, 1987). In addition, because teachers are allowed to implement new ideas and reliably assess their effectiveness, their research will most likely inform practice in a more meaningful and lasting manner as it relies on a logic-in-use rather than abstract generalisations (Nolen & Vander Putten, 2007). By way of summary, it is argued that the teacher-researcher role will cultivate teachers' capacities as self-direct professionals by encouraging teachers to build reflective practices and make confident instructional decisions, based on proven techniques (Johnson, 1993; Kincheloe, 2003; Nolen & Vander Putten, 2007). By doing this research, I anticipate that I will be able to apply some of these benefits to my own teaching and thus extend my professional development as a teacher. In other words, by engaging in action research I am afforded the opportunity to learn from the experience of research and to gain insight into curricula developments and dynamics in my own setting.

On the other hand, the possibility of tension or conflict developing when a person has to fulfil the role of researcher/teacher within the complex classroom environment

needs to be assessed from an ethical and a logistical aspect (Wong, 1995). In doing so, a number of methodological issues need to be addressed.

Firstly, the research paradigm that is adopted must allow scope for the teacher to be active as an insider. For the purpose of this study a case study approach has been deemed suitable to allow the teacher to work with the learners in order to present a description of how learners responded to the programme.

Secondly, in the teacher as researcher model the accountability issue needs to be addressed. It is suggested that the teachers/researchers subject their own practices to critical scrutiny by colleagues (Nixon, 1987). This advise was taken to heart in this study by involving two other colleagues as researchers and as member-checks. Subsequently, to alleviate some of the role conflict in this particular study, two other teachers/researchers were engaged in these mathematics lessons as a form of member check to guard against teacher/researcher bias. Hence, a limitation of action-research relates to the element of bias introduced into such a study. Whereas biases can be positive in that they help to focus the research, it is important that elements of bias remain controlled by ensuring quality checks such as the just one mentioned.

Thirdly, there is the issue of validity. In order to ensure validity, the teacher-researcher must be able to trace generalisations made in the research study to clear records. Records such as the learners' journals, mathematical workbooks, learner questionnaire responses, and notes made during observation will be maintained throughout the research period. These are discussed in more detail in Section 3.5.

3.3 CASE STUDY

Gillham (2005, p.1) defines a case as ‘a unit of human activity embedded in the real world which can be studied or understood in a context’. The aim of the case study is then to collate data that can act as sources of evidence to best answer particular research questions. In this study a group of Grade 11 learners as detailed in the Sample section [See Section 3.4] will be the case that will be investigated. It is posited that by adhering to a case study rather than to a more positivistic design, important complexities, embedded character and the specificity of real-life problems that appears when a digital and interactive mathematical platform is introduced are less likely to be missed out or constrained. The case study allows one not only to refer what was planned in terms of the study, but also to describe what happened across the whole situation which may or may not relate to what was planned by the researcher to happen. One of the disadvantages of this particular study is that the findings could not be generalised to other Grade 11 learners, but will remain specific to this project. The lack of generalisability is also linked to the rich subjectivity that marks case study approaches. Although one is not dismissive of the objective evidence, the aim is to find out what lies behind the more objective evidence (Gillham, 2005). Hence, in this study it is the qualitative element that is being pursued, that is, a description how the learners understand and respond to the new setting with computers and software and possible underlying reasons in the learners’ feelings and perceptions. Moreover, the research is also interested in how the learners adjust to the processes of Kilpatrick et al.’s strands and, in particular, the ones that mark strategic competence and adaptive reasoning. From gaining these insights, the researcher can begin to build an understanding of what needs to be done to change things in a mathematical classroom for learners with a history of low mathematical attainment.

Once again, because of the strong element of subjectivity in a case study, the potential of bias remains an issue of concern. Once again, for the sake of completeness it is repeated here that to account for the possibility that the results may be due to the teacher/researcher herself and not necessarily to the mathematical programme, three presenters facilitated the lessons respectively to control for the effect of teacher bias. The presenters constituted of myself, a mathematics teacher and former computer trainer for 15 years; another teacher with one year’s teaching experience; and, a skills

trainer from SETA with 12 years experience in his field. The roles and responsibilities of the research were divided amongst the researchers in that on a rotational basis one researcher would teach, one would walk around and interact with the learners and provide scaffolding for their ideas and support should they get stuck with the technology, and one would take notes. After each session the researchers would meet to discuss the notes, make additions or amendments and discuss the next day's operations.

3.4 SAMPLE

The target population for this study consists of Grade 10 to Grade 12 learners who attend Ikamva Youth. As stated in the *Ikamva Youth Annual Report* (2007), Ikamva Youth is a 'by-youth, for youth' community-based non-profit organisation that drives social change in South Africa by enabling disadvantaged youth to access post-school opportunities in tertiary education.

The organisation achieves its mission by providing supplementary tutoring, career guidance, mentoring, computer literacy training, voluntary HIV counselling and testing, and activities under the media, image and expression programme to learners in grades 10 – 12, free of charge. Ikamva Youth is based in the Western Cape (at the Nazeema Isaacs library in Khayelitsha and the Nyanga library) and Kwa-Zulu Natal (at the Mayville Secondary school in Cato Manor). It has been running in Johannesburg (at the Siyakhula venue in Tembisa) since the beginning of this year (2010). Since its inception in 2003 between 42% and 65% of Ikamva Youth's learners have accessed tertiary institutions (Ikamva Youth Annual Report, 2007). This is a particularly positive achievement considering that the estimates for township youth are generally around 1%. Ikamva Youth runs a supplementary tutoring programme every Saturday morning. Tutors (mostly students from nearby tertiary institutions) provide individual attention to learners in all matric subjects. Learners bring the material they want to cover (sections of the syllabus with which they're struggling, past tests and exam papers, homework etc.), and sit in small groups with the volunteers. Mathematics and physics are the subjects for which assistance is most sought.

One primary objective of the programme is to enable learners to develop active learning skills, whereby they identify the learning areas with which they need help, and take responsibility for their academic progress

The decision to involve the Grade 11 learners incorporates elements of both convenience sampling and of purposive selection. The choice of a social setting, that is, the site where the research is to take place, was taken on the basis of convenience. Since I am already involved in the project it was easier to gain access.

Moreover, involving the Grade 11 learners, as opposed to the Grade 10 or Grade 12 learners, was a purposeful decision. The decision was made after considering that the type of parabola work suggested by this study is more in line with the Grade 11 curriculum covered at school level. Although such material could provide valuable revision for the Grade 12 learners, it was felt that introducing a 'new approach' this late in their school careers posed a risk should unforeseen situations arise that might unsettle or interfere with the Grade 12's normal learning. There was no particular reason for choosing parabolas, as opposed to any of the other graphs the learners have to know in the Grade 11 curriculum, such as hyperbolas, exponential graphs and straight line graphs.

The learners' own studies at the organisation were not disrupted. In other words, after hearing the explanation of the purpose of the project, Grade 11 learners from Tembisa, a large township on the East Rand of Johannesburg, were invited to participate in a holiday time slot outside of their normal tuition time. (The study was initially intended to run on Saturday mornings during term time, but the director of Ikamva Youth requested that the research be moved to the upcoming holiday programme. His decision was approved by the learners and their parents). Only those learners who volunteered were included once parental permission and minor assent had been obtained. They were invited to participate in eight consecutive sessions of 1.5 hours each, over a period of 2 weeks. The demographical features of the learners who volunteered for the research are provided in Section 4.1.1.

3.5 DATA COLLECTION TECHNIQUES

The regular evaluation tools that were used to collect data during this study, and their levels of reliability and validity, are detailed in the following section.

3.5.1 A Group Session

At the outset of the study a group session was held to clarify goals of the research, explain the research procedures to the learners, answer any questions the learners or their parents may have, and to remind the learners of their right to leave the study at any time should they no longer wish to participate. This session was geared towards ethical requisites in preparation for the data collection.

3.5.2 Learners' Journals

After each teaching session the learners were asked to record an honest account of their learning experiences during that session in a journal provided. Essentially they were asked to relate incidences of how the session, and in particular the mathematical software, helped and/or impeded their learning. These journals were collected at the end of the study and analysed according to the frequency of the themes that emerged.

3.5.3 Samples of Learners' Work

The learners' written work during these sessions was analysed to determine evidence of potential symbolic progression (or lack thereof) and mathematical learning that might advance or hinder progress.

3.5.4 The Focus Group Session

At the end of the research a focus group was conducted as an assessment tool for obtaining data concerning the needs and concerns of learners as they worked with model-eliciting mathematics in a computer learning environment. Focused group interviews have the potential to reveal diversity, clarify contradictions, and amplify certain issues and to make amendments. In doing so, the focus group interviews acts as a source of validation (Mouton, 2006). The purpose of this particular focus group was to serve as a vehicle to bring forth the issues the group might consider most prominent at that particular time. In other words, the focus group was designed to sensitively 'listen to the voices of the learners' in order to develop intervention that represents the experiences of those with generally the least power in the learning system (Hall & Hall, 2004). In essence, the objective of the focus group in this research was embodied in Watson and Winbourne's (2008) position on how one goes about identifying mathematical activity.

> To identify mathematical activity we look for ways people talk, what they talk about, what they focus on, how they classify their experience, what levels and kind of generality occur to them, what is varied and what is fixed, what relationships they observe or construct and how they express them (Watson & Winbourne, 2008, p.3)

Stated differently, the focus group concerned itself with how the learners talked about their mathematical experiences, what they spoke about and how they themselves experienced, classified and built relationships based on their own salient experiences and generalisations.

3.5.5 Observation During the Learning Sessions

Additional insights were gained from observing the process learning during the particular sessions. Specific emphasis was given to how the learners construct, interpret and transform the graphs and mathematical knowledge/techniques that facilitate or hinder progress. The objective of recording these insights was to reveal

challenges and future requirements that would need to be addressed for an effective implementation. Moreover, observing the situation provided first hand experience of the issues that develop when computer technology is introduced. During this time comprehensive field notes were taken relating to the critical questions noted previously, viz. the extent to which learners were able to work with aspects of strategic competence and adaptive reasoning with particular emphasis on interpretation and application, knowledge production and justification and social collaboration; barriers in terms of working with technology that may interfere with the learning process; and, the personal (affective) response of the students to the availability of technology.

3.5.6 A learner's questionnaire

An example of the learner questionnaire can be found in Addendum 1.
The information that was collated from the questionnaire foregrounds demographical features such as the age of the learners and their home language; schooling aspects relating to the way they are taught mathematics in their classrooms, the use of computers in mathematical lessons at school, the mathematical textbooks they have, barriers to learning that they are experiencing in mathematics in the classroom and elements of mathematical teaching and learning that they would like to be addressed in order for them to progress in their mathematical attainment at school.

3.6 RELIABILITY AND VALIDITY

Neuman (2000) explains that definitions of reliability are applied differently in qualitative practice compared to quantitative research requirements. The setting and dynamics of qualitative research call for the differences in application. More specifically, qualitative research often takes place in a natural milieu where processes are not stable over time. Rather, continual changes are experienced particularly in the interactions and the relationships between the researcher and the researched. This does not mean that qualitative research is erratic in its approach. It does mean, however, that consistency is achieved through using a variety of techniques (for example, interviews, observation, documents) to record observation, rather than

through the test-retest method (Neuman, 2000). Consequently, reliability in this study will be achieved by triangulation of method. Flick (2006) argues that triangulation can take the form of combining several qualitative methods, or of combining qualitative and quantitative methods. It is conceived that the different methods complement one another and in doing so compensate for the weakness of a single method (Flick, 2006). This study has resorted to a triangulation approach by combining different methods of data collection, both qualitative (observation, focus groups, personal journals) and quantitative (a structured learner's questionnaire), to ensure a form of reliability in the study.

Moreover, Neuman (2000) points out that in qualitative research validity is closely associated with truthfulness. Truthfulness is secured through authenticity. Authenticity means 'giving a fair, honest and balanced account of social life from the view point of someone who lives it everyday' (Neuman, 2000, p.171). In other words, there must be a strong degree to which the social world described by the researcher matches the world of its members. In line with Neuman's suggestions the researcher has made an attempt to provide an insider perspective by being true and candid about the experiences of those being studied.

3.7 ETHICAL CONSIDERATIONS

During the study sensitivity was shown to common research ethics to prevent irresponsible behaviour on the part of the researchers and to protect participants from physical, psychological and legal harm. The learners were not inconvenienced, as they participated in the research outside of formal mathematics classes. Moreover, the content of the mathematics corresponded with the Grade 11 mathematical learning outcomes specified by the Department of Education. Informed consent was obtained in writing from the participants' parents or guardians. Subsequently, only learners whose parents or guardians provided written consent for their children's information to be used were involved.

The ethical integrity of this study was maintained by conducting the study under the auspices of the ethics committee of the University of the Witwatersrand. All research findings are reported with full disclosure of the research methodology and the limitations of research. Moreover, the privacy of the Grade 11 learners and the anonymity and confidentiality of their records have been secured in this research report by substituting their real identities with pseudonyms.

3.8 SUMMARY

By way of summary, it was decided that a qualitative case study embedded in action research would be appropriated to describe how sixteen Grade 11 learners from Tembisa engaged particularly with the third and fourth strands of Kilpatrick et al.'s framework of mathematical proficiency while doing interactive mathematical activities supported by graphing software. The instruments would also be used to describe how these learners related their salient experiences and to identify particular barriers they experienced in the process of learning mathematics with computer technology. Recurring patterns and themes that emerged from the data are detailed in the following chapter.

CHAPTER 4

DATA ANALYSIS

This chapter will consider the data that were collected using the instruments and methodology discussed in Chapter 3. The data were analysed by considering emerging patterns and themes, and by grouping these according to frequency. The first analysis relates to the learner questionnaire, which assumed a structured format and was presented to the learners at the end of the study.

4.1 THE LEARNER QUESTIONNAIRE

Learners were asked to fill in a learner questionnaire at the end of the research period. As is mentioned in more detail in Chapter 3 the participants were learners from low socio-economic backgrounds with a low average mathematical mark at school. They were attending Ikamva Youth in order to better their results. The questionnaire was designed in a way that would broaden the researchers background knowledge of the learners and the mathematical environment that is familiar to them at school level. For example, it was often debated amongst the Ikamva tutors whether the learners were given adequate textbooks at school. Some tutors proposed that the learners were underperforming because they did not have materials such as textbooks to take home for study purposes. Others felt that perhaps seeing that the majority of the learners have English as an additional language and not as their own home language, that they may be struggling with interpreting the English and not necessary with the mathematics as such. Moreover, I was trying to introduce a more interactive approach to mathematics drawing on the theory of socio-constructivism and the importance of communication and mathematical negotiation. It became important to know whether these learners had been exposed to interactive discussion around mathematical problems before or whether this was a new experience for them. As was noted before, these kinds of questions were ultimately concerned with identifying factors that could be used to more comprehensively understand possible contributors to the learners' strands of mathematical performance.

The question to barriers were also inverted by asking what the learners felt supported or advanced their own learning in order to gain feedback from the learners on "what

worked" and "what didn't work" in the mathematical learning/teaching environment. Lastly, learners were asked to evaluate on a scale from 0 – 10 how much (if at all) the intervention advanced their mathematical understanding of parabolas.

4.1.1 Demographical Features of the Learners

The sample consisted of 16 learners from the Tembisa area of which 6 were boys and 10 were girls. The age of the learners ranged from 16 to 19 years, with an average age of 17 years and a mode of 16. The learners represent diverse home language backgrounds. The most common home language indicated was Sepedi, yet other languages recorded were IsiXhosa, Zulu, Xitsongo, Setswana and Venda. None of the learners were from English speaking homes. All the learners were Black. These demographical features provided by the questionnaire is useful in distinguishing the sample of the study more sharply.

4.1.2 Information from the Learners' School Context of Mathematics

The information detailed in this section concerns the language of mathematical instruction and the style of mathematical teaching common to the learners, and their access to mathematical resources.

4.1.2.1 Language of Mathematical Instruction

All the learners stated that they were taught mathematics in English, with the exception of one learner who indicated that his teacher would code-switch between English and Xitshonga during the lessons. This then raised the question as to how well the learners felt that they understood the lessons and the materials in English. Ten learners self-reported that they had an 'average' understanding, whilst one learner said that he 'did not understand English at all' and five learners stated that they were fluent in English. It appears from these comments, that with the exception of one learner, the other learners felt that their comprehension of English was adequate enough to help them achieve a reasonable understanding of the questions or

instruction in their tests at school and in this research project. Yet, on a later set of questions learners indicated that they may have more significant difficulties with English that hindered their mathematics (See Section 4.1.2.3)

4.1.2.2 The Style of Mathematical Teaching

All the learners' descriptions suggested that the teachers appear to follow a more traditional approach to mathematics.

Learner13: The teacher gives us examples, explain them until we understand them, then he gives us a test or an activity.

Learner 10: The teacher shows us the examples and make sure that we understand and we do it on our own.

Learner 5: Our teachers show us problems and solve them for the class, then ask questions, then give us exercises.

This style of teaching was strongly reiterated across the various learner responses. Simply put, none of the learners made reference to any other style of teaching. These findings seem to suggest that teachers are mainly functioning within the absolutist framework. It is important in the light that a more fallibist approach has been introduced in this study. A change in philosophy affects pedagogy and could demand new and complex roles for learners. It can therefore be expected that learners may experience conflict in terms of adjusting to the expected, yet unfamiliar roles that needs to be assumed in a new style of pedagogy.

4.1.2.3 Challenges to Learning

The learners identified a wide range of what they perceived to be barriers to their own learning of mathematics in the classroom context. These included aspects relating to the teachers, the language of instruction, access to resources, actual subject content as well as personal attitudes. The learners wrote about their teacher moving too fast

through the material or not being able to understand their teacher for reasons unspecified; not fully comprehending the English as an additional language learner; having difficulty with specific aspects of mathematics, such as graphs, trigonometry or simultaneous equations; not having access to resources including different kinds of textbooks or extra lessons; not practising enough due to homework overload; writing tests on topics not previously covered in school or being provided with examples that do not match later activities (for example, one learner wrote about doing $(x+2)(x-2) = 0$ in class and then being asked $x + 1/(2x-4) = 0$ in the exam); and, personal attributes such as 'loose concentration', a 'blocked understanding', poor aptitude in mathematics and being a 'slow learner'. Once again, the information given by the learners were used as a source to try and understand from the learners' own perspectives why they are underperforming in mathematics.

4.1.2.4 Ways in which the Teacher is Helping the Learners

The large majority of the learners indicated that they feel that their class teachers help them by 'giving them more examples' and by 'staying for afternoon studies' or alternative remedial sessions.

Learner 5: He gives us more examples and if we still don't understand we stay for afternoon study.

Learner 10: Our teacher opened a study group after school everyday.

Learner 13: The teacher helps by doing lots of examples.

4.1.2.5 Ways in Which the Learners Wish the Teachers Would Help Them

Several learners indicated that they wanted the teacher to adjust the pace of the lessons.

Learner 4: I want the teacher to teach slowly, so that I can understand.

Learner 10: The teacher needs to be specific and patient for slow learners.

Learner 11: I would like her to move slowly and make sure that everyone understands before she moves on.

It seems from these comments that the learners felt that the 'optimal' or 'ideal' mathematics teacher would be a person that slowly and clearly demonstrates the procedures, who works through plenty of examples and who are willing to repeat the information numerous times to individuals who are struggling.

4.1.3 Access to Resources

The following section provides a synopsis of the learners' self-reported access to computers and additional learning resources.

4.1.3.1 Access to Computers

Six of the learners did not have any access to computers during week days nor over weekends. Ten learners did have access to computers. Five learners had access to computers every day of the week – four of which had computers at home, and one who could access a friend's computer. Four of the learners had access to computers at school 3 or 4 times during the week. One learner indicated that she went to the public library whenever she had to use a computer. The learners self-reported that they spent an average of 4 hours on computers per week. Despite many of the learners having a level of access to computers, none of them have used the computers specifically for mathematical activity. There is thus a distinction in this group between having access to computers for games or school projects, and having using the computer as a tool for mathematical learning.

4.1.3.2 Access to Textbooks

Only one learner did not have a school mathematical textbook. The most common textbook was *Classroom Mathematics*, followed by *Study and Master Mathematics*.

One learner indicated that they used a study guide, *System Maths*, as a textbook at school. In contrast to what was anticipated by tutors, access to textbooks does not appear to be one of the primary barriers these learners are facing.

4.1.3.3 Access to Extra Mathematical Lessons

Although this question was not posed in the questionnaire, certain learners indicated that they had access to additional mathematical lessons in answer to the question relating to ways in which the learners felt that the teachers tried to help them to understand mathematics better. Four learners stated that their teachers offer additional mathematical lessons after school on certain afternoons or during weekends. Findings such as these seem to indicate that a proportion of the learners thus have access to some form of remedial intervention offered by their teachers; and, hence although they find themselves in a low socio-economic school setting, they are not entirely isolated and cut off from having access to after hour professional expertise.

4.1.4 Learners' Evaluation of Learning Mathematics Using Mathematical Software

All the learners self-reported that the research programme had a positive impact on their understanding of parabolas. They indicated that their understanding improved by 2 to 5 points (out of a possible 10 points) from the start of the study towards the end. For example, the learners would rate their initial understanding as 4 and their understanding after the research as 8. What is particularly interesting is that the learners felt that the contextualisation of the mathematical problems contributed most towards their improvement, followed by having their own graph to manipulate and with which to test their ideas.

Learner 4: My understanding was helped by the practical questions and how to tackle these.

Learner 7: It helped me in a sense that I now know equations and how to use them and how to change questions in English to maths language.

Learner 11: I understand now that every time we talk about parabolas we are talking about the relationship between x and y. You get to try by yourself and you look what your computer is saying. It helped me to understand both mathematically and computationally.

Learner 5: I understand parabolas because I had my own graph to manipulate.

4.2 IMPLEMENTATION OF THE RESEARCH PROGRAMME

The following modeling-eliciting activities were taken from the book *Graphing Algebra* by Asp, Dowsey, Stacey and Tynan (1998).

4.2.1 Session 1

Before formally starting to work on the computer with the mathematical software, we wanted to establish the learners' abilities to draw the graphs without technology. Hence, we started the session by asking the learners to draw the following three graphs in their workbooks using the procedures they had been taught at school. The idea behind the session was to serve as a type of baseline assessment of established procedural knowledge in the learners.

$y = 2x^2 + x - 1$	$y = 2x(x+1)$	$y = (x-1)^2 + 3$

Only two learners were able to draw the graphs using the more formal procedure by considering the shape of the graph and thereafter by finding the y- and x-intercepts and the turning point respectively. One learner approached the problem by substituting into a table and then by plotting the points derived from the table. The rest did not know how to proceed. They turned to their peers to solicit some form of assistance and gradually the idea of working with a table and substitution diffused from the one learner to the rest of the class. The learners were very slow and hence, although the exercise was planned as an introductory session lasting about 30 minutes, it threatened to consume the entire lesson. Subsequently, the learners were asked to complete their graphs at home and the remainder of the lesson was spent on

teaching learners how to construct the same graphs on the computer using the relevant software. Two learners appeared very unsure and nervous, whereas the rest of the class adapted very readily to the software. Some of the learners started exploring features of the programme on their own which were not demonstrated or even mentioned during the introduction. Three learners minimised the programme and shifted their attention to the possibility of accessing the Internet. Essentially, the learners' difficulties in drawing the graphs appear to suggest that their procedural proficiencies were low.

4.2.2 Session 2

The lesson started by focusing on the key features of a parabola such as the intercepts and the turning point. The learners were then asked to discuss the possible relevance of these points in a context of their own. The intention of the exercise was to gauge whether the learners could relate meaning (or "usefulness") to these features. Simply put, the learners were encouraged to think of the relevance of these features in a realistic or real life context. The following extracts contain the learners' responses and seem to reflect a very narrow awareness of the application and interpretation of the features of parabolas centred around the drawing of the graph. In other words, these salient points of a parabola primarily had meaning to the learners in the context of drawing the graph and not necessarily beyond that. Such findings tend to suggest that the learners' estimate of the value of the graphs were aligned with procedural knowledge and did not necessarily extent into strategic competence or adaptive reasoning as these processes require that specific problems are addressed through mathematical means.

Learner 2: We need the x- and y-intercepts and turning point to get our graphs points where they are turning and x-intercepts.

Learner 7: X-intercept means where the graph cuts x. Y-intercept means where the graph cuts y. Turning point is where the graph turns.

Learner 12: We need to know our co-ordinates so that we are able to know our points.

Learner 13: Turning point to know where our graph turns. Y-axis to see if our graph is up or down and x-axis to see whether it is big or small.

Learner 14: I think they are important because they are the ones that teach you where to plot and it is the ones that show you how to draw the correct graph. They also tell you the co-ordinates that showed your plot.

One learner hinted at the possibility of interpretation by referring to turning point revealing maximum and minimum values.

Learner 6: X- and y-intercepts we need so we can show our sides. The turning point to see how our graph turns and the maximum and minimum.

Moreover, there was one other learner who tried to relate the question in the direction of application and potential practical significance.

Learner 4: I think they can be used in business for graphs and statistics. It can tell you how good or bad a business is doing.

The learners were then introduced to the mathematical activity of session 2 where the parabola was placed in the context of sport and more specifically the distance and height of a shot-put ball travelling through the air.

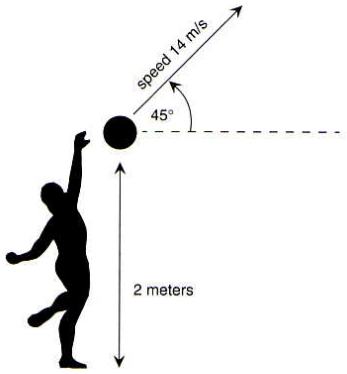
Shot-put

Australia's Olympic athlete Jane Flemming competed in the heptathlon. One of her events was the shot-put. It is known that she released the shot about 2 meters above the ground at a speed of about 14 meters per second and at an angle of 45° to the horizontal.

With this information, scientists calculate that the height above the ground, y meters, of Jane's shot is given by the rule:

$$y = 2 + x - \tfrac{1}{20}x^2$$

where x is the horizontal distance the shot has traveled in meters.

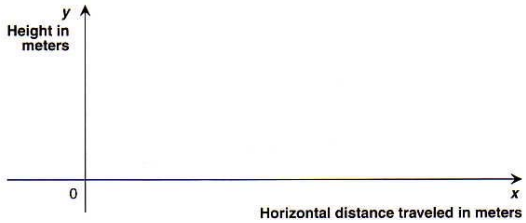


Using your graphing utility, create a blank screen and enter the function given above. You *may* need to use brackets or multiplication signs.

Change the viewing window to obtain a good picture of the graph. A window with these dimensions is suitable.

$$-1 \le x \le 25 \text{ and } -1 \le y \le 10$$

Draw a quick sketch of the graph. Mark the point corresponding to the initial height of the shot.



1 The initial height of the shot at the moment that it is about to leave Jane's hand is 2 meters. Verify that the rule calculates this height correctly by substituting $x = 0$.

Use your graphing utility to help you answer question **2**.

2 **a** **i** How high is the shot after it has traveled 5 meters horizontally from where Jane Flemming released it? _____

ii Check by substituting $x = 5$ in the rule.

b **i** How high is the shot after it has traveled 12 meters horizontally from where Jane Flemming released it? _____

ii Check by substituting $x = 12$ in the rule.

c Find the highest point reached by the shot in its flight and mark this point on your sketch. [*Hint:* Ground level corresponds to $y = 0$.] _____

d Find how far from Jane Flemming the shot will land. Mark this point on your sketch. _____

Several adjustments were made to the lesson shown above. These adjustments followed through into the subsequent sessions. Firstly, the learners were asked to

answer all the questions by considering the information on the graph AND by doing the mathematical calculations to arrive at the same conclusion without reference to the graph. The pen-and-paper work was considered necessary in the light of their classroom requirements and the strong focus on procedures contained therewithin. Hence, for example, Question 2 c in the exercise above requests that learners find the highest point reached by the shot and then mark it on the sketch. Once learners have located this point on the graph, we would ask the learners to arrive at the maximum height using only mathematical calculations, that is, doing the algebra rather than directly reading the information off the graph. Subsequently, learners were encouraged to use the mathematical formulas they were exposed to during class time such as (-b/2a) and subsequent substitution back into the equation or an additional formula to arrive at the height. Once the calculations were completed, learners had to verify their own "paper work" by relating it back to the information provided by the graph itself.

This session proved a very challenging exercise for the learners. Firstly, the notion that x represented the distance and y represented the height and that the graph represented the relationship between the two entities appeared foreign to the learners and had to be explicitly pointed out and emphasized. Secondly, although the learners were familiar with notation such as f(x) when x = 5, they were not able to identify its relevance within the context of the story. In other words, the learners were not able to "translate" or decode from the English to mathematical notation. For example, question 2a speaks of the 'shot traveling 5m in a horizontal direction' which would be rendered as f(x) when x = 5 in mathematics and calculated accordingly. Furthermore, three of the girls in the session needed encouragement to make the link between x and y. In accordance with this question of the shot traveling horizontally, the learners would move 5 units along the x-axis and then stop without proceeding to move up towards touching the graph and then left towards the y-axis in order to gauge the height. In other words, the whole process of reading information off a graph was not yet developed in these learners. Thirdly, the learners did not have strong enough procedural knowledge and memory of the appropriate formulas to carry out the paper work independently. Hence, learners had to be reminded of the formulas, when they should be executed, and thereafter be guided explicitly on how to "do the mathematics" in their workbooks.

Additional mathematical concepts that were raised during this particular session were the domain and the range. It was discussed with the learners that one only needs the information from the time of release to the actual landing of the shot. It was pointed out that extensions below the x-axis ($y < 0$) were not relevant as it was improbable that the ball would travel underground. Once again, although learners understood the practical need for limiting the graph, they were largely unfamiliar with the mathematical language of domain and range and its accompanying notation, that is ,with the ways in which limitations to domain and range are expressed mathematically, for example, $a \leq x \leq b$, or $x \in [a, b]$.

Taking the above mentioned aspects into account, only two of the learners were able to work on the exercises in a more independent manner. The other learners required the researchers to work with them in relating the mathematical notation to the English and in identifying and carrying out the procedures effectively.

4.2.3 Session 3

Whereas the content of the previous session related to shot-put, the context of this session was managing the running costs of a ship relative to its speed.

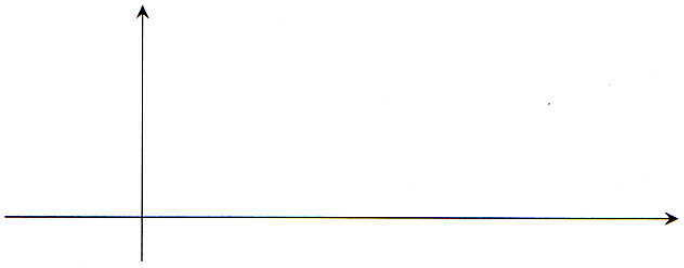
Running a ship

The cost per hour $C(s)$ in thousands of dollars for running a particular ship traveling at a speed of s knots is given by the rule:

$$C(s) = 0.3s^2 - 3s + 12$$

Use your graphing utility to enter the cost function and draw the graph.
You may need to experiment with the viewing window to get a good picture.

1 Draw a quick sketch of the function and label the axes.



Find and mark the point where $s = 0$ and explain what this means.

Use your graphing utility to help you answer question 2.

2 a **i** What is the cost in dollars per hour of running the ship when it is traveling at 3 knots? _____

ii Check by substituting $s = 3$ in the rule.

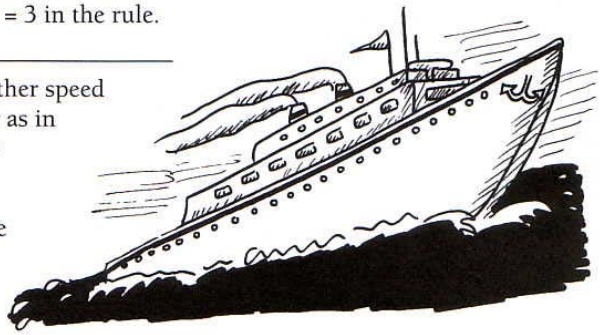
_____

b The ship can travel at another speed for the same cost per hour as in part **a**. What is this speed?

_____

c Find the **best** speed for the ship to run (that is, for cheapest cost per hour).

_____



What then is the cheapest cost per hour? _____
Mark this point on your sketch.

The learning objectives of this activity were very similar to those of the previous day, and so were aspects of the researcher-learner interaction. In other words, time was spent primarily on helping learners relate English depictions into mathematical notation and reminding learners of relevant formulas and their execution. Much of this exercise acted as a form of reinforcement of the previous day's learning. The learners especially struggled with interpreting the turning point's y-value in the light of the context of the ship's running costs. The learners interpreted the cost as $5,7 rather than to re-evaluate the information against its given context and thus more correctly interpret the information as $5 700. Moreover, the learners had difficulty related to their calculator work in using the formula to find the x-intercept. Since they did not use brackets when entering the information into the calculator (with specific reference to the information that resides under the square root sign) the answers that the calculator yielded did not match those in the computer graphic.

4.2.4 Session 4

The objectives and interactive dynamics of this session remained similar to those of the previous two sessions. However, this session started differently in that the learners were shown a video clip of Sydney Harbour bridge from the air, followed by slides in which certain structural properties of the bridge were attenuated. Thereafter, an image

of the bridge was projected onto a screen and the mathematical questions were referenced back to the actual bridge where applicable.

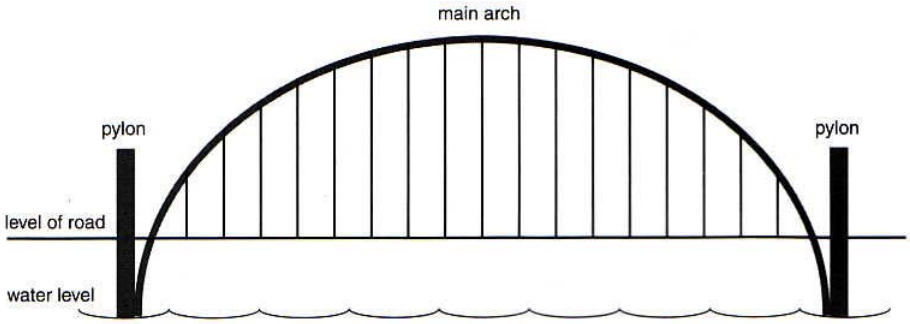


Sydney Harbor Bridge

The main arch of the Sydney Harbor Bridge is approximately parabolic in shape. It is 503 meters between the two pylons. If x is the number of meters measured horizontally from one of the pylons and $H(x)$ is the height of the arch in meters above water level at the point x, it has been found that:

$$H(x) = \frac{x(503 - x)}{475}$$

where x is the number of meters measured horizontally from one of the pylons and H is the height of the arch in meters above the base of the pylons (water level).



Using your graphing utility, define the arch function and draw the graph. You may need to experiment with the viewing window to get a good picture.

6 a **i** Find the height of the arch above the water 100 meters from the pylon. _____

ii Check using the rule for $H(x)$. _____

b **i** Find the height of the arch above the water 300 meters from the pylon. _____

ii Check using the rule for $H(x)$. _____

c Where is the arch 100 meters above the water? _____

d What is the height of the arch at its highest point and how far from the pylon is this? _____, _____

The road which runs across the Sydney Harbor Bridge is 52 meters above the water. On the same set of axes, enter and plot a function corresponding to the road at a height of 52 meters.

7 **a** Find the two points where the arch crosses over the road. _____, _____

b Work out the length of road between the two points where the arch crosses the road. _____

8 Experiment with different functions in your graphing utility to find the function for the height of the arch of a bridge which is as wide as the Sydney Harbor Bridge but:

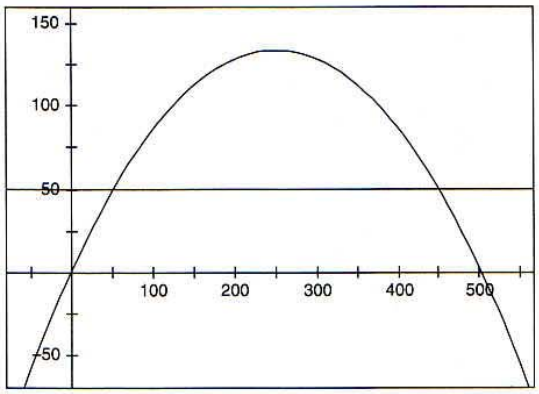
a is only half as high _____

b is twice as high. _____

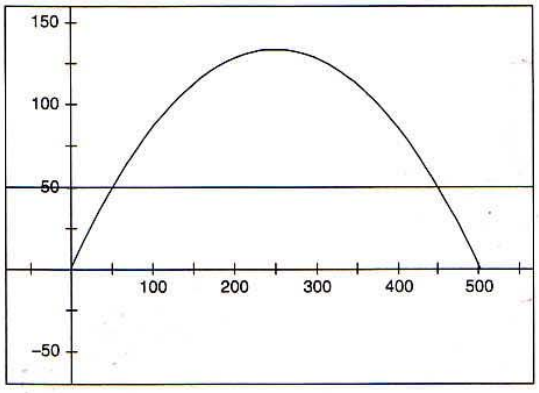
The height function for the bridge only applies for a restricted set of values for x. To take this restriction into account, the height function could be redefined as:

$$\frac{x(503-x)}{475} \quad \text{for } 0 \le x \le 503$$

When the height function with this restricted domain is entered into a graphing utility, what you see changes as shown.



The height function drawn without any restrictions



The height function with restricted domain

9 Restrict the height function further so that the graph you get corresponds to the road together with the portion of the arch above the road.
Write down the domain you have used. _____

One of the key difficulties the learners experienced in this setting was 'constructing the road' 52 meters above the water. Although this is a very basic straight line graph

that learners should be familiar with from around their Grade 9 year onwards, the application proved problematic. None of the learners were able to construct the road without more explicit intervention and a revision session on vertical and horizontal straight lines.

Moreover, many of the learners particularly struggled with question 6c. This question relates to where the height of the bridge is 100 m above the water. The learners tended to only identify one point where the graph meets these criteria. In other words, the concept of the graph being symmetrical and thus having two matching 'height' values was not fully developed. This was also reflected in their paper work. The learners used the formula for finding the x-intercepts, but tended to only choose the positive value and ignore the negative value. Subsequently, when the one point they identified on the computer did not match with the positive value on their calculator, they experienced a sense of confusion relating to why the two tools do not necessarily yield a similar answer.

Question 8 left the learners particularly at a loss. Initially, none of the learners appeared to have any cognitive strategy with which to approach the question and silently and passively withdrew from any attempt at answering. Thereafter, the researcher began to coach the learners by asking them to consider height and how it was represented in the formula and thereafter to double it. One learner began to pick up on the clues given by the researcher and was able to start by showing double the height, that is, 2 H(x) = ….. but he struggled with the mathematical operations in that he was unsure "what to do with the 2 thereafter".

4.2.5 Session 5

The director from Ikamva Youth requested that Session 5 be cancelled due to a prominent HIV/AIDS speaker that the organisation invited to address the teenagers. Unfortunately, the speaker was only able to address the learners in that particular time slot because of his own busy schedule.

4.2.6 Session 6

TRANSFORMATION CREATIONS ON FAMILIES OF QUADRATIC FUNCTIONS

Creations

The creations in this section are based on families of quadratic functions. Use a graphing utility to create the following designs. Record the functions you use. The creations do not need to be exactly the same as the ones shown, but the picture you make should look similar. It will help if you set the viewing window for each before you start.

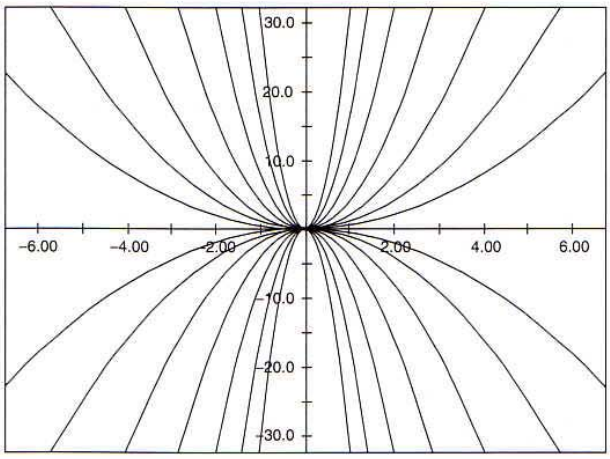
Most of these creations will work well if you enter the quadratic functions using the form $y = a(x - h)^2 + k$, and experiment with the values of a, h and k. For each creation, two of these values will be fixed and the third will change.

1 Fountain reflections

Adjust the viewing window so that you are looking at the region bounded by:

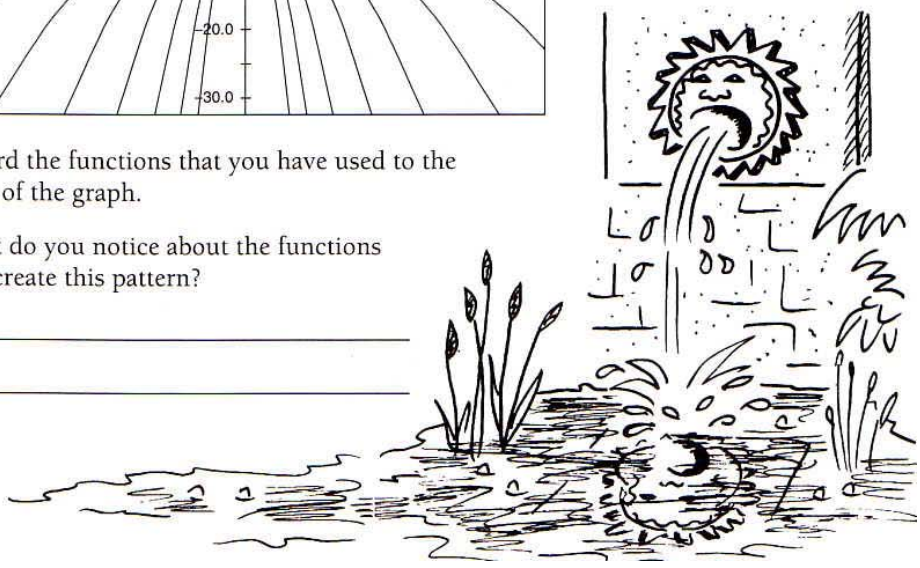
$$-6 \le x \le 6 \text{ and } -30 \le y \le 30$$

Sometimes when water is spurting out of a fountain its reflection can be seen. Enter and plot functions to draw a fountain of water such as this and its reflection.



Record the functions that you have used to the right of the graph.

What do you notice about the functions that create this pattern?



This was a particularly interesting exercise from a case study perspective in so far as it illuminated "what really happened" as opposed to what was "planned by the

researcher to happen". When asked if they were familiar with aspects of graphical transformation from school, all the learners affirmed that they had done the topic at school. The affirmative response was expected by the researchers taken that the research was being conducted late into the school year, and that it was feasible that this topic should already have been covered from within the curriculum at school. Consequently, it was expected that the learners would have some memory of the roles of a, h and k in the transformations of parabolas (or a, p and q as was more standard in South African textbooks – the interchange was made explicit to the learners). Hence, it was planned that the learners would apply the principles when using the graphing software, optimising the freedom to explore as they went along and correcting their mistakes by themselves if and when they appeared, that is, "debugging" their own computer creations while applying the properties of transformations. A mistake in this context would produce a pattern that was not consistent with the given pattern.

Initially, the lesson did not proceed as was planned. In contrast to what was expected by the researcher, this task was met with a "stunned silence". Nobody spoke. Nobody considered the material. Nobody orientated themselves to the computer. Rather, the learners folded their hands on their laps, diverted their eyes away from the computers and towards the researchers and waited. When questioned on their response, the learners expressed that they wanted a researcher to first demonstrate the procedure of how to make the pattern, and thereafter they would continue with the exercise. Despite encouragement to think back to the principles at school and to use the computer as a means of testing these principles, the learners would not proceed. They remained adamant that one of the researchers had to demonstrate.

Trying to "give away as little as possible" I started by coaching them to give me the most basic of parabola graphs and then to "experiment" from there. I was looking for the graph denoted by x^2 or more specifically by $1(x - 0)^2 + 0$. None of the learners responded, which suggests that they were not familiar with the basic graphs. After repeating the question, I then diverted to a more direct question of "Do any of you know what the most basic parabola graph is – or what it looks like – or its equation?" The learners responded by shaking their heads, while some voiced "No" in a chorus like manner. I then explained how the form $1(x - 0)^2 + 0$ equates to x^2 and that this in turn represents the most "basic" form of a parabola. Thereafter, the learners were

encouraged to adapt the basic form $1(x - 0)^2 + 0$ or $a(x - h)^2 + k$ by choosing one aspect (a, h or k) and by substituting different numbers and from there deduce a pattern of change.

After being provided with this cognitive strategy the learners hesitantly proceeded with the instruction, but were in need of consistent affirmation as they continued. Eventually some of the learners began to progress more confidently on their own, which seemed to offer a form of encouragement to the more uncertain learners.

4.2.7 Session 7

After the presenters met and discussed the progress of the research, much concern surfaced over the learners' concept of what a graph represents. The presenters felt that the learners were still unsure in their understanding of the relationship between x and y in a graphing context. It was thus decided to break from the transformation exercises and engage in explicit and systematic teaching to show the relationship between x and y, using the idea of an input/output system, incorporating the notion of restricting input/output (domain and range) and by talking about the impact of continuous and discrete input/output systems. Although conducting an explicit teaching session was never part of the initial research planning, the presenters felt that the learners would benefit from such a move in terms of their school mathematics. Simply put, the presenters did not want the learners to return to school without a session in which concepts they appeared to be struggling with were explicitly addressed, clarified and consolidated. Hence in Session 7, the explicit teaching mentioned above was implemented.

4.2.8 Session 8

The intended plan was for the learners to continue with the pattern making activities based on transformation principles. However, due to unannounced power load shedding in the area, the research could not continue as there was no electricity. Hence, it was decided to allow the learners time to complete the learner's questionnaire and thereafter host the focus group session in this time slot.

One additional aspect to consider is the social collaboration during these sessions.

4.2.9 Social Collaboration

One feature that was visible during the research was the learners discomfort in "talking mathematics" with the presenters and with one another. Learners found communicating mathematics particularly difficult. During these sessions, there was no visible building of student-to-student talk. Mathematical communication was mostly restricted to pointing out errors, copying answers, and requests to borrow calculators. In other words, there was little middle ground where students debated and wrestled with ideas and problems, to reach some common solution based on a mathematical argument. To encourage communication with the learners, the teachers/researchers requested two things from the learners: Firstly, all the learners had to write down their own understanding/perception/explanation of the key questions asked in class in their mathematical workbooks; and, secondly, learners had to express themselves in their journals by commenting on the lessons from a personal/affective view.

4.3 THE LEARNERS' JOURNALS

The learners wrote very short paragraphs in their journals every day. Generally they wrote either that they had a "good" day because they "understood" the mathematics; or that they had a "bad" day, because they felt "confused" and the "maths was hard". There was not too much detail or rich expressions evident in their commentaries. Many of the learners' comments quoted in this paper were taken directly from their journals.

4.4 THE FOCUS GROUP SESSION

The focus group session started by asking learners how they felt about learning mathematics with computers. Some of the learners commented on their initial excitement at the prospect of using computers for the "first time in their lives". Others mentioned that when they sat in front the computer some of the thoughts (or doubts)

that went through their heads related to how they “were going to start this”, “which button to push” and that they initially felt “nervous” about the computer usage.

It was interesting that none of the learners associated computers with mathematics. Rather, the technological tool that was more commonly associated with mathematics was the calculator. When asked to comment on their opinion of the mathematics-computer association, the learners concluded that using a computer makes mathematics easier, but not always in a manner that appears conducive to learning.

Learner 2: The computer makes maths easier because it draws the graphs for you.

Learner 5: The computer helps us to think less.

Learner 8: The computer does everything for you.

Learner 11: There is no computer in the exam. When you have a computer you don’t need to work out the turning point and intersection. That is not good for exam purposes.

Learner 12: The computer does the turning point – it doesn’t really help you do maths.

Learner 16: The computer is just fun, entertainment, a new experience, another skill.

The learners argued that they needed to know the procedures for exam purposes and that the computer was not helpful in this regard. They felt that doing mathematics “on paper” with particular emphasis on practising the procedures was a more productive exercise in light of their classroom and examination needs. The notion voiced by some was that the computer was therefore not particularly valuable in the teaching and learning process of mathematics and that it was best left for playing games or watching a movie. In other words, the entertainment benefits of the computer outweighed its educational significance. Their argument resonated with that of Tall’s

notion that they felt that they had to master the procedures first and therefore the computer would “help them more”. They indicated that the computer was “difficult” for those who did not have the procedural knowledge.

Learner 4: Computers interfere if you don’t know the procedure or concept. It becomes confusing. I am not sure what the computers are doing.

Learner 10: The teachers should start with examples first. We don’t know where to start with the procedures. The procedures must be demonstrated

The learners were then asked to comment on the computer’s ability to “push (advance) their own thinking and conceptual understanding”. No one wanted to comment. The group remained silent.

When asked to comment on the value of the visual aspect of the computer the majority of the learners did not feel that the visual aspect of the computer was in any way superior to the graphs depicted in their textbooks. They stated that seeing and working with the graphs in a visual dynamic environment made “no difference”.

The learners were also asked to comment on the extent to which they used the computer to “test their ideas” and to “check (verify) their workbook answer”. Only four learners indicated that they used the computer for this particular purpose.

4. 5 SUMMARY

The analysis of the data findings showed that learners lacked procedural information and experienced particular difficulties in adjusting to strategic competence and adaptive reasoning forms of mathematical thinking. These difficulties were linked to their difficulty in interpreting information in a particular context, applying information in light of specific socio-economic constraints, and in generating their own knowledge schemes. Another finding was an element of ambiguity concerning the role of mathematical software. Certain learners felt that having their own graph to manipulate aided their conceptual understanding of parabolas, whilst others felt that

the computer was not a worthwhile learning tool in light of their exam requirements and the particular need to be able to execute procedures in the exam context. In the next chapter these findings will be discussed in light of the literature and theoretical orientations presented in Chapter 2.

CHAPTER 5
DISCUSSION

As was indicated in the literature review there exists an ongoing debate between authors who support the computer's ability to produce an alternate learning environment (Stols, 2007) and those who oppose such claims (Artique, 2002). In this study, and in support of the former position, the view of the computer's ability to facilitate a specific type of learning environment came to the fore. However, it is argued in this paper that it is more specifically the merging of the mathematical software, with the nature of the mathematical problems (model-eliciting activities) and the beliefs of the researchers in endorsing an interactive approach that created a very particular role for the learner to emerge. The Technology I view that states that the ability to produce a specific learning environment and type of learning are inherent in computers, is thus rejected. Ultimately, the focal point of this discussion lies within the Technology III realm, that is, considering the consequences of a type of instruction which integrates technology with students and within the larger system.

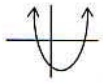
5.1 THE TYPE OF LEARNING ENVIRONMENT CREATED BY THE COMPUTER

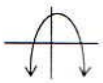
In many aspects the computer in this study allowed for a learner space that resonated with Papert's ideal in so far as it created a domain beyond computer-assisted instruction, evolving from where the computer taught the child to the child teaching the computer, in the manner of the learner being able to manipulate the computer to test or provide support for his/her own thinking. In other words, the Geometer's Sketchpad supports a dynamic interaction between the learner's thinking and the computer, rather than the learner passively receiving feedback or instruction from the computer without being able to alter the instruction through input. Perhaps the most prominent example of such a positioning relates to the section covered during Session 4 where learners had to "reconstruct" Sydney Harbour Bridge imploring a scale of half the height and of double the height of the original structure respectively. In this

environment learners could use the software to hypothesise, test and revise their own mathematical ideas in a more meaningful manner.

This report asserts that the space created by the software technology was different to the one produced in a more traditional classroom where decontextualised procedures are generally taught in the absence of an applied problem-solving context and without technological intervention. Within such a framework learners are expected to learn the procedure to produce the features of a parabola with special emphasis on the shape of the parabola, the turning point, the x- and y-intercepts and the axis of symmetry.

Summary: Sketch graph of a Parabola

1. a pos: 

   a neg: 

2. Determine ***y*-intercept; i.e. *c*-value** $(x = 0)$
3. Determine ***x*-intercepts; Let $y = 0$ and solve** the quadratic equation by means of factors (or the formula).
4. Determine the **turning point**

   4.1 Axis of symmetry: $x = -\frac{b}{2a}$ (x-co-ordinate)

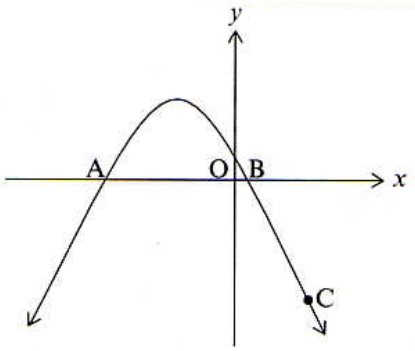
   4.2 Max./Min. Value: $-\frac{\Delta}{4a}$ or substitute the x-value of 4.1 into the (y-co-ordinate) original equation

Commonly, to draw a graph and/or to give the co-ordinates of these salients features are often the objectives of the lesson, that is, the endpoint or the desired outcome. Once the learner has mastered this particular skill, the reverse is then expected in so far as the graph is given to the learner and the learner then has to produce the equation by substituting relevant key co-ordinates back into the general equation. The following two examples, taken from the learners' textbooks currently being used at school, are provided below to illustrate the general objectives during traditional mathematical teaching.

The graph represents the function $f: x \to ax^2 + bx + c$.

The co-ordinates of A, B and C are $(-4; 0)$; and $(1; 0)$ respectively. $(2; -12)$.

Determine a, b and c.



Write $y = -x^2 - 3x + 4$ in the form: $y = a(x - p)^2 + q$.

Determine the co-ordinates of the turning point.

Determine the equation of the axis of symmetry.

Determine the x- and y-intercepts.

Sketch the graph of $y = -x^2 - 3x + 4$.

On the same system of axes, draw the graph of $x + y = 4$.

In contrast, working with technology in the context of the study "freed" the learner from the pressure of following a procedure to produce a parabola or its equation. Simply by inputting a formula, the graph and its salient features were displayed. Moreover, by just clicking on a shortcut menu the co-ordinates of points such as the x- and y-intercepts and turning points were immediately available. The ease of constructing graphs also allow for more complex equations to be endorsed which may serve as a catalyst for more advanced mathematics.

During the first day, time was put aside to teach the learners these skills and by the third day all of the learners, except for one, were inputting and accessing the information with ease. However, within this "automated" learning space, the students were now expected to use the time normally spent on the construction of the graph to interpret, evaluate and apply the information meaningfully in relation to the context of the problem. I think that one way of considering the developments outlined above is that the mathematical software allowed a space to be opened up that allowed more time to concentrate on the specific mathematical proficiencies centred around strategic competence and adaptive reasoning. In light of these proficiencies, learners were expected to engage in the processes of understanding the mathematical factors, interpreting their significance, considering compatibility between their own work and the computer model, and by communicating an argument. Vignettes that arose in the

study foregrounded that the learners were not always comfortable with the nature of this space and the expectations that surrounded this type of mathematical knowledge. Particular challenges that confronted the learners were detailed in the data collection chapter. As was noted in Chapter 4, during Session 1 learners had to be introduced to the idea that x and y had particular meaning in an application context in so far as they relate to particular types of information. Moreover, learners had difficulty in translating the English content, that is, the specifics of the problem to be solved, into mathematical notation. Learners also neglected to reinterpret findings in the light of the social and economic context given. For example, in Session 3 learners ignored the connection between the y-value and what it was meant to represent (thousands of dollars). At the same time, as was discussed in Session 6, learners showed initial discomfort at the requirement to "creatively experiment" with the software in order to anticipate and generate data patterns or to test ideas given by the class. Moreover, active social collaboration was not evident during these sessions even though the researchers encouraged it.

In light of the above observations, I make the following two claims. Firstly, I assert that the Grade 11 learners who participated in this study lacked the necessary cognitive tools, strategies and skills which mark the latter strands of Kilpatrick et al.'s framework (and also the former strands when considering the lack of procedural fluency). And, secondly, I state that two of the principle barriers to the students' learning in a digital environment coupled with interactive mathematical exercises were the learners' beliefs and conceptualisations of the nature of mathematical learning and teaching; and, the cultural significance the learners assigned to the computer as a tool for mathematical learning. These assumptions are expounded in more detail below.

5.2 A PORTRAIT OF LEARNERS' PERFORMANCE RELATING TO STRATEGIC COMPETENCE AND ADAPTIVE REASONING

It can be posited that a crucial element that cuts across strategic competence and adaptive reasoning is the readiness and orientation of the learners to engage in forms of data generation to solve complex problems; it was in particular this aspect that was not well-developed in the learners under consideration. A more visible form of their discomfort, as discussed in Session 7, manifested when the learners were asked to use the computer as a tool to make patterns that drew on the principles of transformation. In this session the focus of the learning activity shifted from finding a solution to a particular problem to creating a system of relationships that are generalisable and reusable. It was noted in Chapter 4 that the learners displayed a strong reluctance to explore structures and their relationships in an attempt to build their own knowledge schema. By this time the learners were familiar with making a graph using software. Hence, it is posited their hesitancy did not stem from the technology, but that the obstacle was psychological in nature in so far as the learners were not comfortable with experimental determination and the deriving of appropriate conjectures from there. When asked during the focus group sessions about their hesitancy to experiment, the learners stated that they struggled with "feeling confused". In other words, to relate this back to Piaget's theory, the learners were not comfortable with knowledge production around a specific problem as it led them to experience deeper forms of cognitive disequilibrium which the learners described as "confusion". Moreover, the learners expressed that these experiences of "confusion", difficulty and struggling to come to a point of an understanding was "bad". They associated their internal dynamics of perplexity and puzzlement with "lack of learning or failure to learn" rather than with Piaget's view as a "necessity" for learning to take place and even a "sign" that learning is taking place. Hence, instead of embracing the feelings of upheaval, turmoil and disorder that the exercises produced in the learners, they immediately rejected the experience, insisting that the researchers remove their discomfort by reverting back to the demonstration and drilling of explicit procedures and by acting as the source of mathematical wisdom and truth. What is seen here, in terms of Piaget's theory, is a transition phase due to the mathematical work requiring more than a simple expansion of the individual's cognitive schemata. Rather, the activities produced cognitive conflict, which required a mental reconstruction. It is

this process of reconstruction which provoked the difficulties learners experienced in the transition phase. Considering that there is conflict between the mathematics that the learner knows (the x- and y- intercepts, and turning points) and the properties of these concepts which must be deduced from the definition (maximum values, initial values, etc.), a period of re-construction and consequent confusion is inevitable. It appeared that the learners did not have effective learning strategies to deal with the conflict and thus found the transition psychologically rather upsetting and threatening, thus requesting that the researchers explicitly "tell them what to do". However, one can also look at this situation from another perspective in that the researchers as teachers should have modeled the situation first as part of the "apprenticeship training" embodied in the situated cognitive approach and thereafter coach the learners until they were willing to engage in the exercises independently. The difficulty lies in finding the balance between creating an environment of knowledge production using the information and the tools at one's disposal and knowledge consumption – copying what the teacher/researcher models. I wanted to evaluate how the learners coped with a measure of self-regulative knowledge production using the tools at their disposal to test their ideas, while being coached by the teachers/researchers along the way, but without being "told what to" do in a procedural fashion and then having the learners merely "following the format". Using Kilpatrick et al.'s terms I could say that I did not want the "creations" activities to become a routine problem where learners only needed to reproduce and apply a known solution procedure. Rather, I wanted the exercises to remain non-routine in that the learners had to engage in productive thinking and invent a way to understand and solve the problem by using the technology at hand to test the feasibility of their ideas. My anticipation was that the learners would engage in a guess-and-check type of reasoning. However, there was little flexibility found amongst the learners in their willingness to pursue this kind of reasoning and approach. Learners were satisfied to stop with well-articulated statements of the procedure from the teachers/researchers, but were reluctant to engage in finding logical patterns themselves or exploring demands for explanation of and connection to the underlying meaning of the procedure.

The question that emerges from the learners' difficulty in transitioning to strategic competence and adaptive knowledge relates to possible barriers or causes that prevent

learners from adjusting more effectively within such a limited time period as was stipulated by the research.

5.2.1 Possible Barriers to Working with Strategic Competence and Adaptive Reasoning

One is reminded of the prediction by Carlson, Larsen and Lesh (2003) that learners' initial thoughts processes with respect to modeling requirements are commonly barren, distorted and unstable. The question emerging is why this is so in relation to these particular students. Two factors emerge. Firstly, the learners are from low socio-economic groups. Are the interactive activities used in this research suitable for learners from impoverished backgrounds? And, secondly, all the learners indicated that their backgrounds may consist of more traditional or absolutist mathematical teaching. Are the delays perhaps harboured by the more traditional forms of teaching that the learners have been exposed to in their current schooling careers?

5.2.1.1 Learners from Low Socio-Economic Backgrounds

With respect to the first point, and as was indicated in the literature section, researchers such as Abadzi argue that low SES learners have different educational needs compared to learners from more affluent economic strata. Yet, in consideration of the criteria discussed in the review, I align myself with Abadzi in that an interactive approach to mathematics is not only effective, but is also necessary as a form of empowerment and advancement. However, when learners are confronted with a strong emphasis on problem posing, data generation and interpretation, and the representation and communication of mathematics it was noted that they are experiencing specific adjustment difficulties. As was further indicated in the literature review, Lubienski's (2000) analysis of data suggested that reform mathematics could be problematic for learners from a low socio-economic status and that these learners would most likely revert to a demand for examples, which was replicated in the context of this research. Lubienski attributed the learners' behaviour to a background of exposure to an authoritarian and directive manner of communication where aspects of social negotiation are limited. The communication patterns of the learners at home

and at school were not directly investigated during this research and hence the link between the learners' call for examples and expected modes of communication remains a plausible hypothesis in need of further investigation. However, such an assumption relates back to research on the relationship between power discourse and learning. The perception of personal power (or lack therefore) could be used as a vehicle to explain the learners' preference towards obedient compliance and toleration of procedural learning at the expense of attributes such as creativity, interpersonal-adeptness and judgement.

In considering the learners' adjustment needs one has to take into account their previous educational experiences.

5.2.1.2 The Effect of Traditional Curricula

As was noted in the data findings, all the learners indicated that they were familiar with the more traditional/absolutist philosophy of teaching. Carlson, Larsen and Lesh's position stated that conventional curricula have not been successful in promoting abilities such as multiple representation, hypothesis, justification and mathematical communication, all associated with mathematical modeling. Perhaps what was most visible in this study was the learners' familiarity with and inclination towards De Bock et al.'s routine expertise at the expense of adaptive accommodation. However, the argument was that with increased exposure to modeling, learners appear to develop the skills and cognitive tools; support for this emerged during the study. There were three learners who, by Session 3, notably began to "pick up" on the concepts and started interacting more effectively with the materials. One could consider these three learners' progress from an alternative perspective such as Tall's.

As was indicated in the literature review, Tall proposed that learners need to advance linearly from pre-procedures to procedures to eventually arrive at flexible conceptual understanding. In accordance with Tall's model, two of these learners were stronger in procedural knowledge than the other learners. Could this perhaps explain why these learners progressed more readily with reference to Tall's proposal that a proficiency in mathematical procedures must be in place before learners can progress to flexible

conceptual understanding? The third learner, however, appeared to have very little procedure knowledge at the onset, yet was one of those who began to grasp the concepts more readily than most of his fellow peers. For example, when initially asked to draw the graphs, as was recorded in Chapter 4 under Session 1 (Section 4.2.1), this particular learner could not proceed, whereas the other two learners mentioned above could draw the graphs without input from the researchers or other learners and by using more advanced procedures than tabling. The learner who could not draw the graphs turned to the participants he was seated next to and followed their suggestion of making a table by substituting numbers into the equation, and from there to construct a graph. However, when consulting the academic record of this particular learner it was interesting to note that on his report he had the highest mathematical score (65%) from amongst the participants, which seems to imply that there has to be some form of procedural knowledge in his mathematical background. Both Tall's theoretical work and the observation that learners with procedural knowledge displayed a quickened response time compared to those who lacked procedural knowledge seem to suggest that a strong continuity between the former and latter strands of mathematical proficiency; and, that a strong knowledge base of the first two strands (concepts and procedures) may be a necessary condition for learners to engage effectively with the later strands. In the background, however, there is Carlson, Larsen and Lesh's argument that it is constant exposure to modeling that facilitates the building of the necessary cognitive tools and strategies to cope, and not necessarily a strong familiarity in procedural knowledge in itself.

However, in further assessing the situation from Tall's model one is confronted with certain anomalies. Tall's model as explained in Section 2.3.5. posits that learners progress from preprocedural to procedural to multi-procedural to conceptual understanding. Yet, these learners began to show conceptual understanding, while at the same time rejected a multiple procedural approach. For example, the one learner who showed strong progression in the study was very reluctant to engage with and explore the notion that the turning point in a parabola could be found in various ways. She would state that she finds the turning point using "df/dy". Yet, when questioned there was no further understanding of what the notation stood for, what it represented and thus why it could be used as a method to find the desired turning point. At the same time, she declined to endorse the –b/2a formula or by "completing the square".

An analysis of such observations seem to suggest that the model of conceptual development is not necessarily linear but may incorporate multiple mini-reiterative cycles within the overall development from pre-procedural to procept. This provides support for Lesh and Doerr's view that each stage of the cognitive development process includes multiple cycles of interpretations, descriptions, conjectures, explanations and justifications that are iteratively refined and reconstructed by the learner. In other words, students need multiple experiences that provide them with opportunities to explore the mathematical constructs, to apply their system in new settings, and to extend their mathematical thinking in new ways. My argument is that these observations merely serve to reaffirm Kilpatrick et al.'s view that mathematical proficiency is not a one-dimensional trait, and that it thus cannot be achieved by focusing on just one or two of the strands. Rather, to help children acquire mathematical proficiency calls for instructional programs that address all its strands in a unified manner.

One cannot help but question why the learners' procedural knowledge was not yet in place, considering that they have already covered parabolas in Grade 11 at school. One possible explanation stems from taking account of the strengths and weaknesses of an absolutist approach to teaching as was detailed in Section 2.2.1. A weakness of absolutism is that it possesses a 'knowledge-in-use' function. In other words, absolutist teaching may have the long-term effect that learners tend to forget the algorithm as soon as they are no longer using it. However, simply assigning a negative outcome to a philosophy may be a gross oversimplification. One has to consider the complexity of the dynamics from a teacher, student and curriculum perspective. One dynamic that may be more relevant here is Bereiter's approach to how learners organise knowledge based on the way information is presented in the classroom. As mentioned previously, a form of empty verbalism was evident with regards to both the execution and the meaning of procedures. There was thus a repetition of words by the learners, which simulated knowledge of corresponding concepts, while actually covering up a vacuum. It was the "verbalism" that Bereiter (1992) described where words have taken over the function of concepts, and although they serve to facilitate communication, they do not carry the characteristics of more fully developed conceptual thought. Bereiter's (1992) argument is that a concept

emerges and takes shape in the course of a complex operation aimed at the solution of some problem. In accordance with Bereiter's argument, although the learners in the study know the terms such as intercept and turning point, they cannot apply or interpret the information, because they may have learnt the concept as a "topic" and not as a "problem".

In addition, what was evident in the study through non-verbal nuances in various forms, and which became explicit when verbalised during the focus group settings, was that the particular space created by the computers incorporated very different assumptions between the learners and the researchers with regards to what "learning and knowing mathematics" meant.

5.2.1.3 Differing Assumptions about the Nature Of Knowledge

The learners largely associated mathematical knowledge with the instrumental learning of formal techniques. They identified a very limited process and product. The process involved the procedure, algorithm and/or formulas involved in learning how to draw the graph. The product was producing the actual graph itself, by plotting the relevant points. Consequently, the learners had particular difficulty in seeing the act of knowing as a participatory activity and thereafter to use their knowledge to characterise a particular situation which is negotiable according to social, political or economic factors (such as the distance/height situation of Session 2 or the speed/profit context of Session 3) or to use their knowledge as a tool for problem-solving in these settings. They seemed to be more comfortable with knowledge as a competence to produce a specific outcome (a correctly drawn parabola) by following a set of rules or a prescribed pattern of participation. Subsequently, they expressed that in order to learn mathematics effectively they needed intentional teaching delivered in a deterministic manner. Simply put, they wanted the teacher to return to an instructivist style of teaching and to assume the role of feeding the information and knowledge to them. They also wanted to learn mathematics by practising how to produce the correct response. It was identified that the learners' view of learning mathematics consisted of the teacher demonstrating a method, procedure or algorithm to be used in the

particular circumstances, followed by the class solving routine problems using the set procedure. The learners were reluctant to attempt to make sense of the parabolas and to explore the structure of the situation by interacting with the English text and the technology.

My own assessment of the situation is that the learners expected only to engage with the first two strands of the framework of mathematical proficiency which focuses on conceptual understanding and building procedural fluency. Their interest was especially rooted in the latter – the need to see procedures demonstrated very clearly. In essence, the learners were seeking a reliable technique to arrive at well-defined ends predictably and efficiently. They were, however, confronted with an approach that encouraged them to intertwine all the different strands in order to arrive at suitable conclusions. Focusing on technique alone clearly does increase the quantity of learning and has a particular place in mathematics. What is questioned, however, is not the effectiveness of the teaching only technique, but its long-term desirability. It is my opinion that by giving in to the students' demands to leave the interactive approach and to teach them how to draw a graph in a step-by-step cookbook fashion will trivialize mathematical thinking by reducing it to instrumental knowledge without necessarily building key relational insights. In excluding exercises that intertwine the different strands of Kilpatrick et al.'s framework and reduce interaction to the memorising of procedures, the learners' schooling would be impoverished and pupils would leave school with an unfortunate view of what constitutes the learning of mathematics. Moreover, I perceive the technical rationality that emerged during the study to be, at heart, a risk-averse orientation to the sudden barrenness of their own cognitive thought that confronts when first exposed to skills typically associated with modeling, and to the dimensions of power shift from teacher to learner. It is thus my position that learners from low socio-economic backgrounds require more frequent exposure to mathematical experiences which engage them in working with meaningful data and where opportunities are provided for understanding the mathematical relationships that underlie procedures and graphical displays. In other words, utilizing mathematics for learning by allowing learners to grapple with meaningful problems and through engaging them in important social processes have a powerful role to play in the context of South African schooling. At the same time, based on this research, one must bear in mind that this type of transition may not be

easy for the majority of learners. Hence, one has to consider appropriate ways to scaffold learners' shift from an instructivist to an interactive approach. Although readings such as those discussed in the literature review present the instructivist approach (with connotations from the behaviourist view of learning and the absolutist philosophy) and the interactive approach (with connotations from constructivist theories of learning and fallibist orientations) of mathematics at length, little is written about effective ways to bridge conceptually and emotionally from the one to the other. More research in thus needed in this area. Another dimension to consider is the cultural value learners attributed to the computer.

5.2.1.4 Instrumental Genesis

As was indicated in Chapter 4, learners associated the computer with entertainment and "fun" activities such as playing games and watching movies. The tool that was culturally associated with mathematical learning was the calculator. Subsequently, one is reminded of the socio-cultural perspective that the significance and the use of any tool are not necessarily inherent in the tool, but are culturally assigned. For many of the learners by-hand techniques and calculators had an elevated status in the domain of mathematics. And although learners adapted themselves to the tool during the first few lessons, within the time period of the research only a small portion of learners showed evidence of adapting the tool to themselves by using the tool as an extension of themselves and their own thinking. Four learners self-reported during the focus group session to using the computer to support their own thinking, three of whom were the learners, as discussed earlier, with a stronger procedural background. In other words, it seems that within the limited time set by the research, only four of the learners began to show evidence of positive personal utilisation schemes developing. As was indicated in the literature, instrumental genesis is an elaborate and progressive process. It is thus expected that with more frequent exposure to mathematical software, some of the other learners will also begin to tailor the tool more effectively to their own thinking.

In the next section I explore factors which the learners felt helped them in their learning and which they perceived as empowering.

5.2.2 Aspects That Facilitated Mathematical Learning and Strategic Competence and Adaptive Reasoning

In analysing the data from the learners' questionnaire (See Section 4.1) the majority of the learners indicated that it was the feature of contextualisation that advanced their mathematical understanding the most. One is reminded that the proponents of situated cognition (Section 2.3.4) advocate that learning mathematics in a context is more meaningful for learners. The findings of this research seem to be in agreement with these movements by suggesting that solving meaningful problems at the outset is easier for learners and more beneficial to their conceptual advancement in mathematics than decontextualised heuristics. One could argue that contextualisation is helpful to the learners in that it allows for the synthesis of knowledge starting with familiar concepts, and building up from experience to more general concepts. In other words, contextualisation could act as a cognitive bridge from the familiar to the abstract and decontextualised. Starting with a specific contextualised case is in contrast to the more traditional approaches which begin with general abstractions and form chains of deductions from there, which can be applied to a variety of contexts. Moreover, the layout of the activities formulated a deep-end principle rather than a step-by-step principle. The advantage of a deep-end principle is that it allows for learners to combine the coherence of mathematical ideas together with the consequences of mathematical ideas in a setting that makes sense to them. Subsequently, the mathematical notation and results may become more meaningful in so far as they leave learners not only with a definition and procedure, but with a fuller cognitive structure that includes mental images of associated properties and processes. Stated differently, the activities used in the study may have the ability to capture a more global structure of a concept by affirming the interdependence of the different mathematical strands. In essence, the learners' appreciation of contextualised learning seems to support the view that an interactive learning experience is suitable for learners' from low socio-economic backgrounds, but that they are likely to need time and support to adjust to the difference in approach.

5.3 Learners' Personal Response to Learning Mathematics with Computers

Much of the learners' personal response to the computer as a tool in mathematical learning was captured during the focus group session in Section 4.4. For the sake of completeness it is repeated here that essentially the learners seemed disappointed in the computer as a tool for their learning. I feel that the learners' perceptions were once again related to their view of mathematical learning as a "needing to know the procedures to pass the examinations", rather than valuing the strategic competence and adaptive reasoning strands of mathematics and how the computer can support one's thinking processes by allowing for the testing of hypotheses and ideas.

5.4 SUMMARY

By way of summary, Chapter 5 provided a discussion of the results in relation to theoretical frameworks. One of the primary ideas involved in Chapter 5 relates to the clash in the philosophical perspectives of instructivist teaching that seems more familiar to the learners from their own educational experiences at school and the associated role of the learner of sitting, watching and practicing what the teacher demonstrated, and the interactive framework supported by the presenters of the research programme where learners are required to explore, conjecture, transform, justify and communicate. The following chapter gives a conclusion and provides recommendations on the role of technology in the mathematics classroom.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 CONCLUSION

This paper concerned Grade 11 learners from low socio-economic backgrounds being exposed to learning mathematics with mathematical software for the first time in their educational histories. Sixteen learners from Ikamva Youth volunteered. Ikamva Youth is a non-profit organisation that provides supplementary tutoring, career guidance, mentoring, computer literacy training, voluntary HIV counselling and testing, and activities under the media, image and expression programme to learners in grades 10 to 12, free of charge. The programme is based in Tembisa, which is a low socio-economic township near Johannesburg, and serves the learners from the surrounding area. The context of the exposure foregrounded interactive mathematical exercises with parabolas in a digital environment, some of which fit the description of modeling given in this paper. The objectives of the research activity were to evaluate the degree to which learners are able to work with aspects of Kilpatrick's et al.'s strategic and adaptive reasoning strands with particular emphasis on interpretation, justification (graphically and numerically) and application; to identify barriers in terms of working with these strands in a digital environment that may interfere with the learning process; and, to measure the personal (affective) aspect which reflects the response of students to the availability of technology. The research took the form of a case study. In order to safeguard against the potential bias associated with the subjective nature of case study, and in order to safeguard against teacher-researcher influence, the eight teaching sessions were distributed amongst three presenters.

What seemed to surface at the onset of the research with respect to the first point is that the learners held symbol manipulation as a collection of routines to be practised, and perhaps not understood. Hence, special attention was given throughout the study to assisting the students in attributing relevant mathematical meaning to the

procedures as they progressed. It was found that learners needed explicit intervention to assist them in applying and interpreting information in light of given socio-economic conditions. The learners were initially also very uncomfortable assuming the role of data generators and needed much encouragement and positive reinforcement to begin to assume this function.

Taking the above paragraph into account, it is argued in this research that the learners' biggest barrier may be their view that mathematical knowledge is something that one can possess and perform. Their desire was thus to acquire absolutes such as facts and procedures through memorising the "teacher's routine of doing the algorithm". The learners were reluctant to see mathematical knowledge as pro-active, that is, a knowledge that can be proactively deployed and be used to reason within disciplines and across context. They became uncomfortable in an interactive environment such as was introduced by the model-eliciting activities and the dynamic graphing software. Moreover, it was found that the learners did not culturally associate computers with mathematical learning. Although all the learners were able to adjust themselves to the computer, only a small number of learners adjusted the computer to themselves by using it as a tool by which to extend their own thinking. In addition, within the limited time frame of the research, it was mostly the learners with a stronger procedural background who were more effective at adjusting the tool to themselves.

In response to what aspect of the programme helped them the most to advance their mathematical understanding, the learners posited that it was learning mathematics within a particular context, followed by being able to manipulate the graph themselves in the dynamic graphing environment. Interestingly enough, the learners felt that the visual aspect of the computer was in no way superior to the visuals found in their textbooks. The learners were thus dismissive of the argument that it was to a large extent the visualisation aspect of a computer that proved helpful in mathematical learning.

6.2 RECOMMENDATIONS

The following incorporates recommendations in terms of future research and in terms of instruction.

The validity of the findings of the study needs to be extended over a broader range of students. As indicated in the methodology, this research took the form of a case study. One of the pertinent effects of a case study is that the data is not necessarily generalisable to the larger population group. Hence, more research with learners from low socio-economic groups is necessary to establish whether the phenomena discussed in this research is localised to this particular group of Grade 11 learners or whether similar findings occur in other cohorts.

Furthermore, this study brings about questions surrounding the arrangement of the contents of mathematics instruction. Its recommendations therefore concern mainly the science of structuring mathematical lessons. Based on the data from the research, and in consideration of the status of the participants, I would recommend that lessons for learners from low socio-economic groups who are struggling with mathematics should initially be structured within a concrete context that is meaningful to the learners. This is in contrast to starting out with abstract heuristics and moving from there into applied mathematics. Rather, one starts with an environment and then starts mathematising in relation to the environment.

It is also interesting that the learners largely considered the first two of Kilpatrick et al.'s strands as important for scholastic purposes. Both their skills and the value of importance they assigned to the strands of strategic competence and adaptive reasoning were low. It is thus important that mathematical content be structured and taught in such a manner that learners are frequently exposed to all five strands in a holistic, intertwined manner rather than limiting focus to competence and procedural fluency.

I further suggest that teachers' use of technology may play a role in facilitating a shift from receiving instruction from the teacher to developing a shared articulated

understanding through conversation and negotiation. This tendency is not inherent in technology, but is ultimately rooted in the epistemological beliefs of the individual teacher. Technology, and in particular mathematical software, could however, act as a catalyst that enables teachers who have a pre-existing dissatisfaction with "lectured"-centered practices, to transform their classrooms into more interactive-centered environments. However, learners may not necessarily be cognitively and emotionally ready to transit from one method to another. Hence, additional measures need to be implemented and evaluated through research to facilitate a smoother transition that is less threatening to the learners than the transition reported in this study. The creation of a modeling-eliciting environment for learning mathematics requires that teachers build an emotionally supportive atmosphere where students feel safe to explore, conjecture, hypothesize, and brainstorm; are motivated to struggle with and keep working on problems which may not have right or wrong solutions and may require extended investment of energy; feel comfortable with temporary confusion or a state of inconclusive results; and are not afraid to experiment with applying different mathematical tools or methods.

Another aspect that warrants further researching is tracing the development of students' thinking particularly in situations where they are required to generate mathematical data and solve more complex problems in realistic settings. In light of certain theoretical foundations and observations described in the previous chapters, I anticipate that multiple cycles of interpretation would emerge as the learners engage with similar mathematical tasks.

Ultimately, this study encourages educators to reconsider curricula, and on the other hand, calls attention to the relationship between learners and computers. It is argued that a key to helping these learners advance mathematically may be to start with contextualised problems and from there advance to decontextualised heuristics, as is proposed the in modeling approach. I also think that the computer as a learning tool may become more significant to learners when frequent exposure facilitates their development along the pathway of instrumental genesis.

My own conclusion is that technology carries the potential to enhance learners' thinking about graphs in a framework that facilitates an interactive approach to teaching and learning. Dynamic software allows students to easily visualize and

manipulate graphs, and enables learners to investigate their properties and form conjectures based on their explorations. However, more needs to be done in order to increase learners' understanding, interpretation and communication of these aspects by investigating the extensive and complex range of socio-cultural factors that interact to influence conceptual development and cognitive strategies in mathematical learning. Software, alone, cannot create a culture of inquiry. It must be an endeavour of teachers and students together and a manner of working with technology as a tool in a way that facilitates questioning, exploration and curiosity.

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Addendum 1

Learner Questionnaire

Please fill in the following.

How old are you? _____

What is your home language? _____

What is the language you get taught in at school?
_____

How well do you understand questions when they are posed in English? Please tick.

Not at all	Average	Fluently

Do you have your own maths textbook at school? Yes/No.
_____

If yes, which one do you use? _____

Describe a typical maths lesson at your school.
_____
_____
_____

What do you wish your teacher would do differently when teaching you maths?
_____
_____

What do you feel is your biggest barrier to understanding maths?
_____
_____
_____

Do you have access to a computer during the week? Yes or No.
_____

If yes:

Where:_____

When:_____

How many hours do you spend on a computer in a normal week?
_____

Have you ever used a computer to learn mathematics before? Yes / No.
_____

If yes, please give some detail.
_____

Rate your understanding of parabolas before and after the research. Use a scale from 1 to 10 where 1 means very poor and 10 means excellent.

Before: _____

After: _____

If you felt that your understanding of parabolas improved, which of the following do you feel helped to improve your understanding of parabolas more during the research?

a) Contextualisation (putting the maths in a story or real life setting such as working with the Sydney Harbour Bridge)

b) Having your own graph to manipulate on the computer

c) Something else that helped you (please give details)_____
_____
_____

Do you believe the course helped you? Yes/No _____

Please give details on how the course helped you or did not help you?_____
_____
_____

Addendum 2

FOCUS GROUP SEMI-STRUCTURED INTERVIEW

1. Tell us how you experienced learning mathematics through computers.

2. What are some aspects of this programme that helped you understand mathematics better?

3. What are some aspects that frustrated you or that you find difficult?

4. Would you prefer learning with mathematical software to not having it? Why or why not?