



**SOURCES OF MACROECONOMIC FLUCTUATIONS AND
STABILIZATION POLICIES IN AFRICAN ECONOMIES**

by

Mutiu Gbade Rasaki

Supervisor:

Professor Christopher Malikane

A thesis submitted to the Faculty of Commerce, Law and Management,
University of the Witwatersrand, Johannesburg, in Fulfillment of the
Requirements for the Degree of Doctor of Philosophy in Economics

15 July, 2015

DECLARATION

I declare that this thesis is my own unaided work. It is submitted for the award of the degree of Doctor of Philosophy in Economics, University of the Witwatersrand, Johannesburg. It has not been submitted to this university or any other university for any other degree or examination.

Name of Student: Mutiu Gbade Rasaki

Signed:

Date: 15th day of July 2015

DEDICATION

This work is dedicated to the glory of Almighty God and to my family.

ACKNOWLEDGEMENTS

First, I give thanks to God for the success of this programme. I am deeply grateful to my supervisor, Professor Christopher Malikane for his helpful comments, consistent encouragement, and guidance. I also thank the authority of the University of the Witwatersrand (WITS), Johannesburg, for giving me Post Graduate Merit Award (PMA). I also express my appreciations to comrades and colleagues at the Financial Markets and Macroeconomics Unit, SEBS.

This thesis would not have been possible without the abundant support of my family, friends and well-wishers. I thank my parents for their unceasing support and prayers. Special thanks to my darling wife for her love, sacrifice, and support. I thank all my brothers and sisters. I am specially thankful to Ibrahim Razaq for holding forth during my absence. I sincerely appreciate my uncle, Alhaji Fola Bello for his support and contributions towards the success of this programme. I am grateful to all members of the department of Economics, Emmanuel Alayande College of Education (EACOED), Oyo, Nigeria. I thank Dr R. A. Adefabi, the Provost, Dr J. O. Adeagbo, the Principal, Mr O. O. Oladokun, the Dean, and Mr G. Salako, the HOD, Economics for their immense support. I specially thank Mrs R. O. Mosobalaje for her numerous support. I sincerely thank all my friends and well-wishers at home and at WITS for their support.

I am also grateful to the academic staff members at WITS who have been very supportive during the Ph.D. programme. I thank Prof. Kalu Ojah of WITS Business school for his maximum support. Abundant appreciations to Prof. Oluyele Akinkugbe of Economics department and Prof. Michael Samuel of Human Resources department for their multifarious support. I also thank Prof. J. Pedro of Aeronautics Engineering department and Dr Iyabo Usman of Physics department for their assistance especially on MATLAB.

ABSTRACT

The thesis focuses on the sources of macroeconomic fluctuations in ten (10) selected African economies over the period 1990-2011. Data for the study were obtained from the International Financial statistics (IFS), the World Bank, and Central Bank database of the selected countries. We formulate a dynamic stochastic general equilibrium (DSGE) model for the thesis. We estimate the model using quarterly time series data. Due to data availability, the sample size differs from one country to the other. First, we investigate the relative contributions of internal and external shocks to economic fluctuations in African economies. Second, we evaluate the significance of the balance sheet channel in African economies. Third, we investigate the effectiveness of sovereign wealth funds in reducing macroeconomic volatility caused by commodity price shocks. The thesis has 5 chapters. Chapter 1 is the general introduction. Chapters 2, 3, and 4 are stand-alone related papers on macroeconomic fluctuations. Chapter 5 is the conclusion.

Chapter 1 introduces the study. We discuss the research problem, the motivation, the objectives, and the research questions. We also explain both our theoretical and empirical contributions to the literature. Moreover, we highlight the significance and the key findings of the study. Finally, we conclude the chapter with a brief outline on the organisation of the study.

Chapter 2 investigates the relative contributions of internal and external shocks to macroeconomic fluctuations in African economies. We formulate and estimate a monetary DSGE model to examine the sources of economic fluctuations in ten African countries. The model is estimated with the Bayesian technique using twelve macroeconomic variables. Generally, the findings indicate that both the internal and external shocks significantly influence output fluctuations in African countries. Over a four quarter horizon, internal shocks are dominant while over eight to sixteen quarter horizons, the

external shocks are dominant. Among the external shocks, external debt, exchange rate, foreign interest rate and commodity price shocks account for a large part of output variations in African economies. Money supply and productivity shocks are the most important internal shocks contributing to output fluctuations in African countries. To ensure macroeconomic stability, African countries need to formulate appropriate exchange rate and external debt management policies, diversify the economies, and create sovereign wealth funds (SWFs) or use hedging instruments.

Chapter 3 evaluates the quantitative significance of the balance sheet channel in African economies. We construct an open economy monetary DSGE model where entrepreneurs finance investment by issuing foreign currency-denominated debt. The model is estimated with Bayesian technique. The evidence suggests that the balance sheet effects are empirically important in African economies. The marginal likelihood results clearly favour the model with financial frictions. Moreover, the findings indicate that the balance sheet effect reduces the effectiveness of monetary policy, raises the sensitivity of the risk premium to external debt, and contracts output. This indicates that exchange rate depreciation is contractionary in African economies. We conclude that African countries should reduce their exposure to foreign currency-denominated debt and also deepen their domestic bond markets.

Chapter 4 investigates the effectiveness of sovereign wealth funds (SWFs) in reducing macroeconomic volatility in commodity exporting African countries. We formulate and simulate a dynamic stochastic general equilibrium (DSGE) model that features SWFs. The simulation results suggest that the creation of SWFs can reduce macroeconomic volatility in commodity exporting countries. Particularly, SWFs can reduce government expenditure, real exchange rate, and external debt volatility. Since these are the channels through which commodity price shocks are transmitted to the African economies, we recommend that African countries should create SWFs to sterilize the inflow of

commodity revenue and to prevent the resource curse problem.

Chapter 5 concludes the study. We summarize the key findings in Chapters 2, 3, and 4. We highlight the policy implications of our findings. Finally, we suggest areas for further research.

TABLE OF CONTENTS

DECLARATION.....	ii
DEDICATION.....	iii
ACKNOWLEDGEMENTS.....	iv
ABSTRACT.....	v
TABLE OF CONTENTS.....	viii
LIST OF TABLES.....	xi
LIST OF FIGURES.....	xii
LIST OF APPENDICES.....	xiii

CHAPTER ONE: Introduction.....1

1.1 Background to the study.....	1
1.2 Motivation for the study.....	3
1.3 Objectives of the study.....	4
1.4 Research questions for the study.....	4
1.5 Contributions of the study.....	5
1.6 Significance of the study.....	6
1.7 Organisation of the study.....	7

CHAPTER TWO: Macroeconomic Shocks and Fluctuations in African Economies.....8

2.1 Introduction.....	8
2.2 Review of literature.....	10
2.3 The Stylised facts.....	12
2.4 The model.....	16
2.4.1 Households.....	16
2.4.2 Firms.....	21
2.4.3 Exchange rate and external debt.....	23
2.4.4 Monetary policy.....	26
2.5 Data and estimation.....	28
2.5.1 Data sources and treatment.....	28

2.5.2 Prior distribution and calibration of the parameters.....	29
2.5.3 Posterior estimates of the structural parameters.....	31
2.5.4 Posterior estimates of the structural shocks.....	35
2.5 Sub-sample estimates.....	39
2.6 Impulse response functions.....	41
2.7 Forecast error variance decomposition of output.....	42
2.8 Historical decomposition of output.....	47
2.9 Conclusion.....	48
3.0 Appendices.....	51
CHAPTER THREE: The Balance Sheet channel in African Economies.....	59
3.1 Introduction.....	59
3.2 Review of literature.....	61
3.3 The model.....	64
3.3.1 Households.....	64
3.3.2 Firms.....	67
3.3.3 Exchange rate and external debt.....	70
3.3.4 The entrepreneur.....	72
3.3.5 Money market.....	73
3.3.6 Monetary policy.....	74
3.4 Data and estimation.....	77
3.4.1 Data source and treatment.....	77
3.4.2 Prior distribution of the parameters and calibration.....	77
3.4.3 Empirical results.....	79
3.5 Impulse response analysis.....	86
3.5.1 Exchange rate shocks.....	86
3.5.2 Monetary policy shocks.....	87
3.5.3 Foreign interest rate shocks.....	89
3.6 Conclusion.....	90

CHAPTER FOUR: Sovereign Wealth Funds and Macroeconomic Stability in African Economies.....93

4.1 Introduction.....	93
4.2 Review of literature.....	95
4.3 Stylised facts.....	98
4.4 The model.....	102
4.4.1 Households.....	102
4.4.2 Firms.....	106
4.4.3 The sovereign wealth funds.....	109
4.4.4 The government.....	109
4.4.5 Exchange rate and external debt.....	110
4.4.6 Monetary policy.....	112
4.5 Calibration.....	114
4.6 Simulation.....	116
4.6.1 Commodity price shocks.....	117
4.7 Conclusion.....	119

CHAPTER FIVE: Conclusion and Policy Implications.....121

5.1 Introduction.....	121
5.2 Macroeconomic shocks and fluctuations in African economies.....	121
5.3 The balance sheet channel in African economies.....	122
5.4 Sovereign wealth funds and macroeconomic stability in African economies...	123
5.5 Policy implications and recommendations of the study.....	123
5.6 Suggested areas for further research.....	124
References.....	125
Appendix A.....	138
Appendix B.....	151

LIST OF TABLES

Table 2.1: Correlation between economic shocks and output.....	16
Table 2.2A: Prior and posterior distribution of structural parameters.....	33
Table 2.2B: Prior and posterior distribution of structural parameters.....	34
Table 2.3A: Prior and posterior distribution of structural shocks.....	37
Table 2.3B: Prior and posterior distribution of structural shocks.....	38
Table 2.4: Sub-sample estimates for structural shocks (Pre-2008 estimates).....	40
Table 2.5A: Forecast error variance decomposition.....	45
Table 2.5B: Forecast error variance decomposition.....	46
Table 3.1: Calibration of parameter.....	79
Table 3.2A: Prior and posterior distribution of structural parameters.....	84
Table 3.2B: Prior and posterior distribution of structural parameters.....	85
Table 4.1: Business cycle properties.....	100
Table 4.2: Correlations between commodity prices and macroeconomic aggregates.....	102
Table 4.3: Calibrated parameters.....	116

LIST OF FIGURES

Figure 3.1: Dynamic response of output to exchange rate depreciation.....	87
Figure 3.2: Dynamic response of output to an expansionary monetary policy shock.....	88
Figure 3.3: Dynamic response of output to positive foreign interest rate shock.....	90
Figure 4.1: Responses of macroeconomic aggregates to positive commodity price shocks.....	118

LIST OF APPENDICES

Appendix 2.1: Impulse response functions.....	51
Appendix 2.2: Historical decomposition of GDP.....	54
Appendix 2.3: Prior vs posterior.....	57
Appendix A: Appendix to chapter one.....	138
Appendix A: Appendix to chapter two.....	151

CHAPTER ONE

Introduction

1.1 Background to the study

Sharp and persistent economic fluctuations are one of the major problems facing developing and emerging economies. Macroeconomic fluctuations negatively affect investment in human capital, reduce private investment, hinder economic growth, reduce societal welfare, and increase poverty rate in developing countries (see Ramey and Ramey, 1995; Hausmann and Gavin, 1996; Flug et al. 1998; Aizenman and Marion, 1999; Pallage and Robe, 2003; Imbs, 2007; Berument et al., 2012). In particular, macroeconomic instability has been reported to adversely affect growth and welfare in African countries (see Guillamont et al, 1999; Loayza et al., 2007).

The findings, however, on the sources of economic fluctuations in developing economies have been rather divergent. For example, Hoffmaister and Roldós (2001) and Raddatz (2007) find that domestic shocks are the main sources of output fluctuations in developing countries. Hoffmaister et al. (1997) find that domestic shocks account for significant output variations in Africa. Jidoud (2012) shows that productivity shocks are the dominant sources of output volatility in Côte d'Ivoire. In contrast, Mendoza (1995), and Agénor et al. (1999) find that terms of trade shocks significantly affect output fluctuations in developing countries. Neumeyer and Perri (2005) and Uribe and Yue (2006) find that foreign interest rate shocks influence economic fluctuations in emerging economies. Similarly in Africa, Bleaney and Greenaway (2001) find that trade shocks significantly explain output fluctuations in Africa. Raddatz (2008) shows that the relative importance of external shocks as sources of output instability in Africa has increased since the 1990s.

Moreover, studies have shown that relatively small shocks can be transmitted and amplified in emerging economies through the financial market imperfection (see Bernanke et al., 1999; Céspedes et al., 2004). In a small open-economy where debt is denominated in foreign currency, exchange rate depreciation can deteriorate the net worth and the balance sheets of firms and banks. Consequently, this increases the debt service payment and the risk premium, lowers investment, and contracts output (see Korinek, 2011; García and González, 2013). African countries borrow in foreign denominated currency and at a premium to finance capital accumulation (see Sy, 2013). This makes their balance sheets vulnerable to exchange rate depreciation and foreign interest rate shocks.

One of the measures proposed to mitigate the adverse effects of commodity price shocks in developing countries is the creation of sovereign wealth funds (SWFs). Given the developing countries limited access to international capital markets and the incompleteness of commodity-linked hedging market, the establishment of SWFs can be a mechanism for reducing developing countries large exposures to commodity price risk. The SWFs can reduce fiscal volatility, smooth consumption, sterilize exchange rate inflows, decouple GDP growth from commodity price swings, prevent the resource curse phenomenon, insure the African economies against commodity price swings, and mitigate macroeconomic volatility (see, Shabsigh and Ilahi, 2007; Astrov, 2007; Mehrara et al., 2012; Tsani, 2013).

Africa experienced high and continuous economic growth in the past decade. Growth average 5.3 percent between 2000 and 2010 and doubled the growth rate between 1990 and 2000 (UNCTAD, 2014). This marked a rapid increase from 1.8 per cent in the period 1980–1989. The Africa impressive growth success has been attributed to both internal and external factors. The internal factors that contribute to the growth performance include high domestic demand, better macroeconomic management, and a relatively more stable

domestic environment. On the external front, rising commodity prices, improved economic cooperation with emerging economies, higher foreign direct investment (FDI), and increase official development assistance contributed to improved growth performance.

1.2 Motivation for the study

Chapter 2 is motivated by the adverse effects of macroeconomic fluctuations on growth and welfare in African countries. Loayza et al. (2007) findings suggest that macroeconomic fluctuations lower growth and reduce welfare in African economies. Existing studies have been inconclusive on the sources of economic fluctuations. While some authors argue that internal shocks are largely responsible for macroeconomic fluctuations (see Sissoko and Dibooglu, 2006), other posit that external shocks are significantly responsible (see Kose and Riezman, 2001). If external shocks significantly influence economic fluctuations in African countries, there is need for African countries to reduce their vulnerability to trade shocks occasioned by the dependence on export of primary commodity and world financial shocks induced by foreign interest rate shocks. Alternatively, if internal shocks are more important, there is the need for African countries to strengthen their monetary and fiscal policies.

Chapter 3 is motivated by the puzzle of how relatively small shocks can induce large real effects in African economies. Negative external shocks seem to have disproportionate effects on the African economies. Empirical studies have shown that in a small open economy where external debt is denominated in foreign currency, the financial market imperfections amplify external shocks in the economy (see Elekdag et al., 2006). If the financial accelerator channel or balance sheet effect is significant for African economies, there is need for African countries to reduce their exposure to foreign currency-denominated debt. It also has policy implication for the choice of exchange rate regime as currency depreciation affect the cost of servicing debt.

Chapter 4 is motivated by the pro-cyclical fiscal policy in commodity exporting African countries and its distortionary effects on the aggregate economy. The reasons for pro-cyclical fiscal policy in African countries include their limited access to international capital market, over-dependence on a single and sensitive source of revenue such as transfers from the state-owned natural resources industries, frequent changes in the discretionary portion of government expenditure, weak budget institutions, and low political commitment (see, Baldini, 2005; Kumah and Matovu, 2007; Villafuerte and Lopez-Murphy, 2010; Spatafora and Samake, 2012). Moreover, there is apparent lack of progress in the management of the boom-bust cycles in commodity prices. Based on this fiscal volatility occasioned by commodity price swings, there is need for African countries to create SWFs that will de-link fiscal operations from the vagaries of commodity price fluctuations.

1.3 Objectives of the study

- (i) To evaluate the relative contributions of internal and external shocks to macroeconomic fluctuations in African economies¹;
- (ii) To empirically investigate the balance sheet channel in African economies; and
- (iii) To assess the effectiveness of sovereign wealth funds (SWFs) in ensuring fiscal and exchange rate stabilities in commodity exporting African countries.

1.4 Research questions for the study

- (i) What are the relative contributions of internal and external shocks to macroeconomic fluctuations in African economies?
- (ii) Does the balance sheet channel exist for African economies?

¹The ten (10) selected African countries are: Egypt, Ghana, Kenya, Malawi, Morocco, Nigeria, South Africa, Tunisia, Uganda, and Zambia

(iii) Can the establishment of sovereign wealth funds (SWFs) in commodity exporting African countries reduce fiscal and exchange rate volatility?

1.5 Contributions of the study

Chapter 2 of the thesis examines the relative contributions of internal and external shocks to economic fluctuations in African countries. This builds on the work of Kose and Riezman (2001). The chapter makes both theoretical and empirical contributions to the existing literature. First, we extend the works of Muhanji and Ojah (2011) by incorporating broader external shocks and internal shocks that are considered empirically relevant for African economies. Second, we formulate a monetary DSGE model incorporating the role of money, inflation and external debt dynamics for ten African economies. Third, we estimate our model with the Bayesian technique for each of the ten African countries in order to capture the heterogeneity that may exist among these economies.

In chapter 3, the thesis evaluates the quantitative significance of the balance sheet channel in African economies. This contributes to the existing literature in three respects. First, building on the works of Bernanke et al. (1999) and Céspedes et al. (2004), we formulate an open economy monetary DSGE model with financial market imperfections. Second, we investigate the quantitative significance of balance sheet channel in dampening the expansionary effects of exchange rate depreciation in African economies. Third, to highlight the heterogeneity of each country, we use the Bayesian technique to estimate our DSGE model for each of the African economies.

In chapter 4, the thesis assesses whether the creation of sovereign wealth funds (SWFs) can reduce fiscal volatility and promote macroeconomic stability in commodity exporting African countries. This chapter contributes to the existing literature on sovereign wealth funds (SWFs) in two respects. First, we construct a DSGE model incorporating the sovereign wealth funds

for commodity exporting African countries. Secondly, we simulate our model to investigate whether the creation of SWFs can cushion the fiscal and macroeconomic volatilities in commodity exporting African countries.

1.6 Significance of the study

Chapter 2 is significant as the understanding of the sources of macroeconomic fluctuations will assist the policymakers in the formulation of appropriate macroeconomic policies that could promote macroeconomic stability. It will assist the policy makers in the formulation of relevant industrial policy such as the diversification of the economy.

Chapter 3 is significant for the conduct of monetary policy particularly, the question of whether monetary policy should react to exchange rate fluctuations or not. Should the monetary authority implement tight or loose policy when the currency depreciates? This chapter is also important for the choice of exchange rate regime. While Cook (2004) advocates for a fixed exchange rate regime when the balance sheet effect is empirically important, Céspedes et al. (2004) argue for a flexible exchange rate system. In addition, the chapter is also important for the conduct of fiscal operation. Findings show that exposure to foreign currency debt constrains the use of fiscal policy instruments to deal with economic shocks (see, Jeanne and Zettelmeyer, 2002). A depreciation of the exchange rate weakens the government net worth and limits its ability to borrow to finance public investment.

Chapter 4 is significant for policy-makers in commodity exporting African countries. Given the strong positive correlation between commodity revenue and government expenditure in commodity exporting African countries (see Baunsgaard, 2003), the chapter provides measures for the de-coupling of fiscal operation from the volatile commodity revenue. Similarly, the chapter provides mechanism for mitigating the effect of export receipt inflow on money

supply on the domestic economy. Raju and melo (2003) show that coffee price shocks significantly affect money supply in Columbia. Lastly, the chapter will assist the policy-makers in the efficient management of commodity revenue.

1.7 Organisation of the study

The rest of the study is organised as follows. Chapter 2 examines the sources of macroeconomic fluctuations in African economies. Chapter 3 investigates the quantitative significance of balance sheet channel in African economies. Chapter 4 evaluates the effectiveness of sovereign wealth funds (SWFs) in mitigating the macroeconomic volatility in African commodity exporting countries. Finally, chapter 5 concludes the study with policy implications and recommendations.

CHAPTER TWO

Macroeconomic Shocks and Fluctuations in African Economies

Abstract

We formulate a monetary dynamic stochastic general equilibrium (DSGE) model to examine the relative contributions of internal and external shocks to economic fluctuations in ten African countries. The model is estimated with the Bayesian technique using twelve macroeconomic variables. Generally, the findings indicate that both the internal and external shocks significantly influence output fluctuations in African countries. Over a four quarter horizon, internal shocks are dominant while over eight to sixteen quarter horizons, the external shocks are dominant. Among the external shocks, external debt, exchange rate, foreign interest rate and commodity price shocks account for a large part of output variations in African economies. Money supply and productivity shocks are the most important internal shocks contributing to output fluctuations in African countries.

2.1. Introduction

This paper quantifies the role of different shocks in driving macroeconomic fluctuations in African economies. Large and recurrent economic fluctuations in developing countries have been a major concern for economists and policymakers. Findings, however, on the most important shocks influencing economic fluctuations in developing countries have been rather inconclusive. A strand of literature attributes the recurrent economic fluctuations in developing countries to external shocks (Mendoza, 1995; Kose, 2002; Hammed, 2003). In contrast, other studies conclude that internal shocks are largely

responsible for output fluctuations in developing countries (Hoffmaister et al., 2001; Raddatz, 2007).

The significance of understanding the sources of macroeconomic fluctuations is that policymakers can formulate appropriate policies to mitigate the effects of adverse shocks on their economies. For example, Guillaumont et al.(1999) find that macroeconomic fluctuations lower economic growth and reduce welfare in African economies. However, as in other studies that focus on the role of different shocks, studies on African economies yield conflicting results. Kose and Riezman (2001) find that external shocks exert a dominant influence on output fluctuations in Africa. On the other hand, Hoffmaister et al. (1997) and Sissoko and Dibooğlu (2006) find that internal shocks largely explain output variations in Africa. Therefore understanding the relative importance of these shocks is crucial for sound macroeconomic management.

The gap that this paper seeks to fill is that existing studies do not use a monetary DSGE model estimated for each of the African economies. Furthermore, the existing studies investigate relatively few shocks in their models. For example, Kose and Riezman (2001) calibrate a non-monetary DSGE model for a typical African economy. They consider the relative importance of terms-of-trade and foreign interest rate shocks in driving macroeconomic fluctuations. Muhanji and Ojah (2011) estimate a monetary DSGE model for several African economies. They, however, only focus on the impact of commodity price and world interest rate shocks on external debt accumulation. Cashin et al.(2004) focus on the impact of commodity price shocks on the real exchange rate of commodity exporting countries. Hoffmaister et al.(1997) and Sissoko and Dibooğlu (2006) use VAR to examine the relative contributions of internal and external shocks to macroeconomic fluctuations in sub-Saharan African countries.

The contribution of this paper is to formulate and estimate a monetary DSGE model for ten African economies. The ten African countries are selected based

on the availability of consistent quarterly time series data. We build on the works of Kose and Riezman (2001) and Muhanji and Ojah (2011) by incorporating the role of money, inflation and external debt dynamics in the DSGE model and considering a broader set of shocks. Our model incorporates eleven structural shocks that are considered empirically relevant for African economies. Moreover, we estimate our model with the Bayesian technique for each of the ten African countries in order to capture the heterogeneity that may exist among these economies.

The paper is structured as follows: section 2 reviews existing literature on different shocks and their impact on macroeconomic fluctuations, section 3 presents stylised facts for Africa in the spirit of Agénor et al.(1999), in section 4 we formulate our DSGE model, section 5 provides data description and estimates the model's parameters and section 6 concludes with some policy recommendations.

2.2. Review of literature

There is inconclusive evidence on the sources of economic fluctuations in developing countries. For example, Basu and McLeod (1992), Mendoza (1995), and Agénor et al. (1999) find that terms of trade shocks significantly affect output fluctuations in developing countries. Using a calibrated DSGE model, Kose and Riezman (2001) find that trade shocks significantly influence macroeconomic fluctuations in Africa during the period 1970-1990. Similarly, Bleaney and Greenaway (2001) employ the fixed effects panel regression to examine the impact of terms of trade shocks in 14 Sub-Saharan African (SSA) countries over 1980-1995. The evidence suggests that trade shocks negatively affect growth in SSA countries. In contrast, Hoffmaister and Roldós (2001), and Raddatz (2007) find that terms of trade shocks have little impact in developing countries. Hoffmaister et al. (1998) and Sissoko and Dibooğlu (2006), using VAR technique, find that internal and not external shocks influence output variations in African countries. Jidoud (2012),

using a DSGE model, finds that productivity shocks are the dominant sources of output volatility in Côte d'Ivoire.

A number of authors have examined the contribution of commodity price shocks to output and inflation dynamics in commodity exporting developing countries. Edwards (1984) finds that a higher price of coffee generates a higher growth of money and a higher rate of inflation in Colombia. Raju and Melo (2003) find that positive coffee price shocks increase real output and inflation in Colombia through the revenue and spending effects. Similarly, Mehrara and Oskoui (2007) find that oil price shocks affect output fluctuations in Iran and Saudi Arabia. Iwayemi and Fowowe (2011), however, find that oil price shocks do not have significant effect on output and inflation in Nigeria.

Similarly, studies have examined the link between commodity prices, exchange rate and external debt in commodity exporting countries. For example, Cashin et al. (2004) show that commodity prices influence the real exchange rate in commodity exporting countries through change in wages in the commodity sector. Frankel (2007) finds that mineral prices affect real exchange rate movements in South Africa. Koranchelian (2005) finds a positive relation between oil price and exchange rate in Algeria. Bodart et al. (2012) and Dauvin (2014) conclude that commodity price shocks drive the real exchange rate movements in commodity exporting countries. In relation to external debt, Muhanji and Ojah (2011) find that positive commodity price shocks lead to external debt accumulation in African countries. This is attributed to increased expenditure and over-borrowing during commodity price boom.

Relatedly, studies have also investigated the vulnerability of developing countries to exchange rate and external debt shocks. For example, Carranza et al. (2003) find that exchange rate volatility negatively affect investment in emerging economies. This is attributed to the balance sheet effects of liability

dollarization. Kamin and Rodgers (2000) find that exchange rate depreciation leads to high inflation and economic contraction in Mexico. In contrast, Bastos and Divino (2009) find that exchange rate volatility has limited impact on output fluctuations in Mauritius. Hsing (2003) finds that external debt shocks negatively affect output in Brazil.

International shocks such as foreign interest rate, US output and monetary policy shocks have been found to affect economic activities in emerging and developing economies. Uribe and Yue (2006) find that foreign interest rate shocks affect output fluctuations in emerging economies through a change in the country's spread. Similarly, Canova (2005) finds that US monetary policy shocks significantly affect Latin American economies through its influence on their domestic interest rate and capital flow. The study, however, shows that US output shocks have insignificant effects on output in Latin American countries. Maćkowiak (2007) finds that US monetary policy shocks influence the price level and output in emerging market economies through its effects on the exchange rate. Sosa (2008) also underlines that US output shocks are the main factors driving economic fluctuations in Mexico.

Many authors have attempted to quantify the effects of domestic interest rate and monetary policy shocks on the real economy. Reinhart and Reinhart (1991) find that monetary policy shocks influence output fluctuations in Columbia. Using DSGE model for Brazil, Kanczuk (2004) findings indicate that output fluctuations in Brazil are quite responsive to real interest rate shocks. Mallick and Sousa (2012) provide evidence that contractionary monetary policy has strong negative effects on output in emerging economies.

2.3. The stylised facts

Except in Malawi and Tunisia, the selected African countries maintain a flexible exchange rate regime. Morocco maintains a tightly managed float against a basket of currencies. Table 1 presents the correlations for macro-economic shocks and output in African countries. The correlations between

foreign input price shocks and output are positive in nine African countries. This possibly reflects the importance of the trade channel in those countries. Positive foreign input price shocks worsen the terms of trade, improve the trade balance, and expand output. The output expansion cuts across both fixed and flexible exchange rate regimes. This suggests that the exchange rate regime does not matter in output variability for African countries. This is consistent with the findings by Flood and Rose (1995) and Ghosh et al. (1997). But for Malawi, the correlation is negative, indicating that output declines following positive foreign input price shocks.

The correlations for nominal exchange rate and output are quite mixed. In eight African countries, there are positive correlations between the nominal exchange rate and output, suggesting that depreciation is expansionary. This implies that exchange rate depreciation decreases imports, increases export and expands output. This is consistent with the findings by Kandil and Mirzaie (2005) for developing countries. But for Malawi and Morocco, the correlations are negative, indicating a contractionary depreciation. This may be attributed to the balance sheet effect where an exchange rate depreciation deteriorates the countries' net worth, increases debt service payments and reduces output (see, e.g., Berganza et al., 2004). Alternatively, it may be the supply shock channel where a depreciation increases the cost of imported intermediate inputs and lowers output.

Similarly, the correlations between real exchange rate and output are also heterogeneous across African economies. The correlations are positive for Ghana, Nigeria, South Africa, Tunisia, and Uganda. This implies that real exchange rate depreciation is expansionary. This is similar to the findings by Kandil and Mirzaie (2005). In contrast, the correlation results are negative for Egypt, Kenya, Malawi, Morocco, and Zambia, indicating contractionary exchange rate depreciation. This is in line with the conclusion by Ahmed (2003), in the context of Latin America.

Moreover, the correlations between commodity prices and output are positive in eight African countries, indicating that natural resource abundance promotes growth in these countries. A rise in commodity prices increases revenues, capital expenditure, and gross capital formation in these countries. This gross capital formation increases investment and output. This is in line with the findings by Collier (2007) for commodity African countries. But for Kenya and Uganda, the correlations are negative, suggesting that natural resource abundance lowers growth in the two countries. This may be evidence of Dutch disease syndrome whereby commodity price booms lead to real exchange rate appreciation and export uncompetitiveness (see, e.g., Sachs and Warner, 2001).

In line with *a priori* expectation, the correlation results are generally negative between interest rates and output for all the countries. This indicates that a tight monetary policy increases the cost of capital, reduces investment and output. This is consistent with the results by Kanczuk (2004) for Brazil. Likewise, there are negative correlations between foreign interest rate and output for nine African countries. In view of the level of external debt of African countries, positive foreign interest rate shocks can increase the cost of borrowing and debt service payments, reduce domestic investment and output. This is in line with the findings by Uribe and Yue (2006) for emerging economies. But for Malawi, the correlation result is positive. This is quite counterintuitive.

In addition, there are positive correlations between foreign output and domestic output in nine countries. This reflects the trade channel and international transmission of business cycles where a rise in foreign output increases demand for African commodity exports. The rising demand for African exports increases export revenue, capital expenditure, and output. This is similar to the findings of Berument and Kilinc (2004) for Turkey. However, the correlation result is negative for Malawi. This is counter-intuitive.

Lastly, there is a high negative correlation between external debt to GDP and output in eight African countries. This may be attributed to the debt overhang hypothesis where high debt acts as a tax on future output and reduces the incentive for savings and investment. Alternatively, it may be explained by liquidity constraint, where the requirement to service debt reduces funds for investment purposes and hence reduces output. This is line with the findings by Fosu (1999) for Sub-Saharan African and Sen et al. (2007) for Latin America. In Malawi and South Africa, on the other hand, there is a positive correlation between external debt to GDP and output.

Table 2.1: Correlation between economic shocks and output

	$\rho_{y_t, p_{it}}$	ρ_{y_t, s_t}	ρ_{y_t, rer_t}	ρ_{y_t, q_t}	ρ_{y_t, r_t}	ρ_{y_t, r_t^f}	ρ_{y_t, y_t^f}	ρ_{y_t, d_t^*}
Egypt	0.98 (36.05)	0.82 (10.3)	-0.34 (-2.6)	0.98 (35.74)	-0.84 (-11.45)	-0.56 (-5.02)	0.92 (17.57)	-0.85 (-11.59)
Ghana	0.98 (47.4)	0.94 (24.55)	0.64 (8.33)	0.63 (7.52)	-0.72 (-9.69)	-0.69 (8.79)	0.95 (28.46)	-0.55 (-6.07)
Kenya	0.99 (60.5)	0.74 (10.06)	-0.95 (-30.02)	-0.12 (-1.09)	-0.67 (-8.55)	-0.67 (-8.27)	0.93 (23.49)	-0.82 (-13.23)
Malawi	-0.90 (-17.27)	-0.99 (-67.03)	-1.00 (-95.14)	0.96 (28.19)	-0.47 (-4.52)	0.50 (4.87)	-0.99 (76.82)	0.04 (0.36)
Morocco	0.98 (40.12)	-0.69 (-7.94)	-0.40 (-3.67)	0.84 (13.14)	-0.77 (-9.99)	-0.69 (-7.95)	0.94 (22.76)	-0.92 (-19.82)
Nigeria	0.84 (13.12)	0.76 (10.09)	0.60 (6.49)	0.94 (24.6)	-0.37 (-3.41)	-0.43 (-4.11)	0.78 (10.59)	-0.85 (-14.0)
S. Africa	0.97 (33.39)	0.77 (10.01)	0.18 (1.53)	0.74 (9.11)	-0.77 (-10.09)	-0.68 (-7.69)	0.96 (28.53)	0.81 (11.36)
Tunisia	0.34 (3.06)	0.80 (11.55)	0.80 (11.55)	0.29 (2.63)	-0.75 (-9.8)	-0.15 (-1.29)	0.72 (8.92)	-0.06 (-0.52)
Uganda	0.97 (33.4)	0.95 (25.93)	0.47 (4.59)	-0.25 (-2.17)	-0.18 (-1.55)	-0.64 (-7.06)	0.96 (30.0)	-0.86 (-14.6)
Zambia	0.91 (16.17)	0.80 (10.06)	-0.91 (-16.04)	0.89 (14.99)	-0.84 (-11.7)	-0.48 (-4.13)	0.96 (24.14)	-0.94 (-21.57)

ρ is the correlation coefficient, y_t is output proxied by the GDP or industrial output ; p_{it} is foreign input price proxied by US producer price index for manufactured goods , s_t is nominal exchange rate defined as domestic currency units per one dollar, rer_t is real exchange rate, q_t is commodity prices, r_t is domestic interest rate, r_t^f is foreign interest rate proxied by LIBOR, y_t^f is foreign output proxied by US GDP, and d_t^* is external debt to GDP. The t-statistics is in parenthesis.

2.4. The model

2.4.1 Households

We employ a DSGE model with non-separable money similar to the one in Andrés et al. (2006) and Castelnuovo (2012). Given the importance of foreign currency holdings by African households as noted by Elkhafif (2002) and

Adom et al. (2008), we assume that households allocate their real holdings between domestic and foreign currencies. For simplicity, we assume this allocation to be in fixed proportions, so that $S_t M_t^* = \varrho M_t$, where S_t is the nominal exchange rate, M_t^* is foreign nominal money and M_t is domestic nominal money. The households' preferences are given by:

$$U_t = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left[\left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi \right] - \frac{N_t^{1+\psi}}{1+\psi} \quad (1)$$

where C_t , P_t and N_t represent aggregate consumption, domestic price level and labour respectively. The parameter h measures the degree of habit formation in consumption. The parameter σ is the relative risk aversion coefficient ; $\beta \in (0, 1)$ is the discount factor; ψ is the inverse of the Frisch labour supply elasticity and ϕ represents interest rate elasticity of money demand.

Households hold their wealth in the form of foreign and domestic currency and domestic bonds. Furthermore, households borrow from foreign capital markets. Therefore, the budget constraint is:

$$\begin{aligned} C_t + \left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} + \frac{B_t}{P_t} - \frac{S_t D_t^*}{P_t} &= \frac{W_t N_t}{P_t} + \frac{B_{t-1}}{P_t} (1 + r_{t-1}) \\ &+ \left(1 + \frac{\varrho}{S_t} \right) \frac{M_{t-1}}{P_t} \\ &- \frac{S_t D_{t-1}^*}{P_t} (1 + r_{t-1}^d) \end{aligned} \quad (2)$$

where D_t^* is the level of foreign debt, W_t is the nominal wage rate, r_t is the domestic nominal interest rate and r_t^d is the interest rate on foreign debt.

Households enter period t with domestic money holdings M_t , bonds B_t , and foreign debt D_t^* .

Households choose the path of C_t, M_t, B_t , and N_t that maximize expected utility. The first order conditions are:

$$\frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi = \lambda_t \quad (3)$$

$$\begin{aligned} \frac{\phi}{1-\sigma} \left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left(1 + \frac{\varrho}{S_t} \right)^\phi \left(\frac{M_t}{P_t} \right)^{\phi-1} \frac{1}{P_t} &= \frac{\lambda_t}{P_t} \left(1 + \frac{\varrho}{S_t} \right) \\ &\quad - \frac{\beta \lambda_{t+1}}{P_{t+1}} \left(1 + \frac{\varrho}{S_t} \right) \end{aligned} \quad (4)$$

$$\frac{W_t}{P_t} = \frac{N_t^\psi}{\lambda_t} \quad (5)$$

$$\beta \lambda_{t+1} (1 + r_t) \frac{P_t}{P_{t+1}} = \lambda_t \quad (6)$$

$$\beta \lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} (1 + r_{t-1}^d) = \lambda_t \frac{S_{t+1}}{P_t} \quad (7)$$

where equations (3)–(7) denote the derivatives of the utility function to its arguments.

To derive our consumption equation, we equate Eq. (3) to Eq. (6) as:

$$\frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi = \beta \lambda_{t+1} (1 + r_t) \frac{P_t}{P_{t+1}}$$

This is linearized to derive our consumption Euler equation:

$$\begin{aligned}\tilde{c}_t = & \frac{h(\sigma - 1)}{\sigma_c} \tilde{c}_{t-1} + \frac{\sigma}{\sigma_c} E_t \tilde{c}_{t+1} + \frac{\phi}{\sigma_c} \tilde{m}_t - \frac{\phi}{\sigma_c} E_t \tilde{m}_{t+1} \\ & - \frac{\varrho\phi}{s_0\sigma_c} \tilde{s}_t + \frac{\varrho\phi}{s_0\sigma_c} E_t \tilde{s}_{t+1} - \frac{1}{\sigma_c} (\tilde{r}_t - E_t \tilde{\pi}_{t+1})\end{aligned}\quad (8)$$

where $\tilde{\cdot}$ denotes percentage deviation from the steady state and $\sigma_c = \sigma + h(\sigma - 1)$. As in Castelnovo (2012), Eq.(8) shows that consumption depends on the weighted average of past and expected future consumption, real interest rate, and real balances. Furthermore, to the extent that households hold domestic currency, a depreciation of the current exchange rate reduces household wealth, which reduces aggregate consumption. Through this channel, the depreciation of the nominal exchange rate may generate a contractionary effect on output as pointed out by Krugman and Taylor (1978) and Edwards (1986).

In order to express the IS curve in terms of output, we first write the macro-balance (market equilibrium) equation as:

$$\tilde{y}_t = \gamma_c \tilde{c}_t + \gamma_x \tilde{x}_t - \gamma_z \tilde{z}_t + \varepsilon_{g,t} \quad (9)$$

where $\tilde{y}_t$ is the percentage deviation of output from the steady state, $\tilde{x}_t$ percentage deviation of export from the steady state and $\tilde{z}_t$ is the percentage deviation of import from the steady state. The parameter γ_j is the steady state ratio of variable j to output, $\varepsilon_{g,t}$ is a demand shock that combines government expenditure and investment shocks. We follow McCallum and Nelson (2000) in formulating our net export function as:

$$\tilde{n}x_t = \gamma_{yf} \tilde{y}_t^f - \gamma_y \tilde{y}_t + \gamma_r \tilde{r} \tilde{e}r_t \quad (10)$$

where $\tilde{y}_t^f$, γ_{yf} , γ_y , and γ_r , are the foreign output, elasticity of net export to foreign output, the elasticity of net export to domestic output, and sum of elasticity of substitution in production for home and abroad respectively . The real exchange rate is defined as $\widetilde{rer}_t \equiv \tilde{s}_t + \tilde{p}_t^* - \tilde{p}_t$. Substituting Eq.(10) into Eq.(9) yields the expression:

$$\tilde{c}_t = \frac{1}{\gamma_c} (1 + \gamma_y) \tilde{y}_t - \frac{\gamma_r}{\gamma_c} \widetilde{rer}_t - \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_t^f \quad (11)$$

Substituting Eq.(11) into Eq.(9) yields the output-based IS equation:

$$\begin{aligned} \tilde{y}_t = & \frac{h(\sigma - 1)}{\sigma_c} \tilde{y}_{t-1} + \frac{\sigma}{\sigma_c} E_t \tilde{y}_{t+1} - \frac{\gamma_c}{\sigma_c (1 + \gamma_y)} (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \\ & + \frac{\phi \gamma_c}{\sigma_c (1 + \gamma_y)} \tilde{m}_t - \frac{\phi \gamma_c}{\sigma_c (1 + \gamma_y)} E_t \tilde{m}_{t+1} - \frac{\phi \varrho \gamma_c}{s_0 \sigma_c (1 + \gamma_y)} \tilde{s}_t \\ & + \frac{\phi \varrho \gamma_c}{s_0 \sigma_c (1 + \gamma_y)} E_t \tilde{s}_{t+1} - \frac{h(\sigma - 1) \gamma_r}{\sigma_c (1 + \gamma_y)} \widetilde{rer}_{t-1} \\ & + \frac{\gamma_r}{(1 + \gamma_y)} \widetilde{rer}_t - \frac{\sigma \gamma_r}{\sigma_c (1 + \gamma_y)} E_t \widetilde{rer}_{t+1} - \frac{h(\sigma - 1) \gamma_{yf}}{\sigma_c (1 + \gamma_y)} \tilde{y}_{t-1}^f \\ & + \frac{\gamma_{yf}}{(1 + \gamma_y)} \tilde{y}_t^f - \frac{\sigma \gamma_{yf}}{\sigma_c (1 + \gamma_y)} E_t \tilde{y}_{t+1}^f + \varepsilon_t^a \end{aligned} \quad (12)$$

We assume that the innovation ε_t^y follows a first-order autoregressive process as $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \mu_t^a$. Output depends positively on the real exchange rate and foreign output. As a departure from McCallum and Nelson (2000), our dynamic IS equation also features output as a function of lags and leads of the real exchange rate and foreign output.

2.4.2 Firms

In line with Batini et al. (2005) and Malikane (2014), we assume that final goods producing firms exhibit non-linear input requirement in the production function such that $X_{i,t} = Y_t^{\delta_i}$ where $X_{i,t}$ is the amount of non-labour input i required in production, Y_t is output, and $\delta_i > 0$ is the elasticity of the input requirement with respect to changes in output. Labour and non-labour inputs are complements as in Smets and Wouters (2002). The production function is given as:

$$Y_t = A_t N_t^\eta \left[\prod_{i=1}^n Y_t^{\theta_i \delta_i} \right] \quad (13)$$

where A_t denotes productivity shocks, N_t is the level of employment, θ_i is the elasticity of output with respect to input i , $\delta_i > 0$ is the input requirement coefficient, and $0 < \eta < 1$. The reduced form of Eq.(13) is:

$$Y_t = A'_t N_t^\alpha \quad (14)$$

where $\chi = \sum_{i=1}^n \theta_i \delta_i$, $\alpha = \frac{\eta}{1-\chi}$, and $A'_t = A_t^{\frac{1}{1-\chi}}$. The productivity shock is assumed to follow a first order autoregressive process: $A_t = p_A A_{t-1} + \varepsilon_t^p$. Using Eq.(14), total real cost is:

$$TC_t = \frac{W_t Y_t^{\frac{1}{\alpha}}}{A_t'^{\frac{1}{\alpha}} P_t} + \sum_{i=1}^n \frac{P_{it}}{P_t} Y_t^{\delta_i} \quad (15)$$

where P_{it} is the price of foreign intermediate input i , P_t is the aggregate price level while W_t denotes the nominal wages. Let p_{it} be the real price of non-labour input, the real marginal cost can be specified as:

$$MC_t = \frac{W_t Y_t^{\frac{1-\alpha}{\alpha}}}{\alpha A_t^{\frac{1}{\alpha}} P_t} + \sum_{i=1}^n \delta_i p_{it} Y_t^{\delta_i - 1} \quad (16)$$

where MC_t represents the marginal cost.

We follow Galí and Gertler (1999) and formulate the hybrid new Keynesian Phillips curve as follows:

$$\tilde{\pi}_t = \gamma_f E_t \tilde{\pi}_{t+1} + \gamma_b \tilde{\pi}_{t-1} + \lambda \tilde{mc}_t \quad (17)$$

where:

$$\begin{aligned} \gamma_f &\equiv \beta \theta \{ \theta + \omega [1 - \theta (1 - \beta)] \}^{-1}; \lambda \equiv (1 - \theta) (1 - \beta \theta) (1 - \omega) \xi \\ \gamma_b &\equiv \omega \{ \theta + \omega [1 - \theta (1 - \beta)] \}^{-1}; \xi \equiv \frac{(1 - \alpha)}{1 + \alpha (\varepsilon - 1)} \{ \theta + \omega [1 - \theta (1 - \beta)] \}^{-1}, \end{aligned}$$

θ measures price stickiness, ω is the degree of price indexation, ε is the goods' elasticity of substitution. We linearize Eq. (16) and substitute Eq. (5) and Eq.(11) to derive the marginal cost written as:

$$\begin{aligned} \tilde{mc}_t &= \vartheta_a \tilde{y}_t - \vartheta_b \tilde{y}_{t-1} - \vartheta_c \tilde{m}_t + \vartheta_d \tilde{s}_t - \vartheta_e \tilde{r} \tilde{e}_t \\ &\quad + \vartheta_f \tilde{r} \tilde{e}_{t-1} - \vartheta_g \tilde{y}_t^f + \vartheta_h \tilde{y}_{t-1}^f + \vartheta_i \tilde{p}_{it} - \vartheta_j \tilde{a}_t \end{aligned} \quad (18)$$

where:

$$\begin{aligned}
\vartheta_a &= \left(\frac{\gamma_{ls}\sigma(1+\gamma_y)}{\gamma_c(1+\psi)} + \frac{\gamma_{ls}}{\alpha} - \frac{\alpha\gamma_{ls}}{(1+\psi)} + \sum_{i=1}^n \delta_i \frac{p_{io}x_0}{y_0} \right); \\
\vartheta_b &= \frac{\gamma_{ls}h(\sigma-1)(1+\gamma_y)}{\gamma_c(\alpha+\psi)}; \vartheta_c = \frac{\gamma_{ls}\phi}{(1+\psi)}; \vartheta_d = \frac{\gamma_{ls}\phi\varrho}{s_0(1+\psi)} \\
\vartheta_e &= \frac{\gamma_{ls}\gamma_r}{\gamma_c(1+\psi)}; \vartheta_f = \frac{\gamma_{ls}h(\sigma-1)\gamma_r(1-\alpha)\gamma_{ls}}{\gamma_c(1+\psi)\alpha} \\
\vartheta_g &= \frac{\gamma_{ls}\sigma\gamma_{yf}}{\gamma_c(1+\psi)}; \vartheta_h = \frac{\gamma_{ls}h(\sigma-1)\gamma_{yf}}{\gamma_c(1+\psi)}; \vartheta_i = \sum_{i=1}^n \delta_i \frac{p_{io}x_0}{y_0} \\
\vartheta_j &= \frac{\gamma_{ls}}{\alpha}; \gamma_{ls} = y_0 a_0^{-1} n_0^{-1}
\end{aligned}$$

2.4.3 Exchange rate and external debt

The interest rate on foreign debt is made up of the foreign risk-free rate and the risk premium. We assume that $r_t^d = r_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^*$. This implies that the sovereign risk premium is driven positively by the burden of external debt, which increases the risk of default and negatively by commodity prices, which increase the dollar liquidity of the commodity-exporting country. In our formulation, a positive shock to the foreign risk-free interest rate increases the country's interest rate spread and the cost of borrowing (see Uribe and Yue, 2006). In contrast, positive shocks to commodity prices reduce the spread and cost of borrowing for commodity exporting countries (see Senhadji, 2003; Muhanji and Ojah, 2011).

We combine Eqs.(6) and (7):

$$\beta \lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} (1 + r_{t-1}^d) = \beta \lambda_{t+1} (1 + r_t) \frac{P_t}{P_{t+1}} \frac{S_{t+1}}{P_t}$$

This is linearized to derive the UIP expression as follows:

$$\tilde{s}_t = E_t \tilde{s}_{t+1} - (\tilde{r}_t - \tilde{r}_t^f) - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^* + \varepsilon_t^d \quad (19)$$

where $\tilde{s}_t$ is the nominal exchange rate, $(\tilde{r}_t - \tilde{r}_t^f)$ is the risk premium, $\tilde{q}_t$ is the commodity price and $\tilde{d}_t^*$ is external debt to GDP ratio. The coefficients ω_q and ω_d represent the sensitivity of the sovereign risk premium with respect to the commodity price and external debt respectively. As noted by García and González (2013), a higher risk premium induces capital outflow and depreciates the exchange rate. Moreover, the exchange rate depends negatively on commodity prices and positively on external debt to GDP ratio, as shown by Cashin et al.(2004).

The dynamics of external debt depends on the current account balance and foreign debt service payment. Thus, the external debt to GDP ratio evolves according to the following equation:

$$\frac{\Delta D_t^*}{P_t Y_t} = \frac{Z_t - X_t}{Y_t} + (1 + r_{t-1}^d) d_{t-1}^* \quad (20)$$

where $\frac{D_t^*}{P_t Y_t}$ is the ratio of external debt to GDP, $\frac{Z_t - X_t}{Y_t}$ represents the current account balance by the private sector and r_t^d is the interest rate on external debt. The change in the debt ratio over time can then be written as:

$$\Delta d_t^* = (z_t - x_t) + d_{t-1}^* (1 + r_{t-1}^d - \pi_{t-1} - \Delta y_{t-1}) \quad (21)$$

Eq.(21) shows that the change in the external debt ratio is a positive function of net import and foreign interest rate. For example, a rise in net imports is financed by borrowing, thereby increasing the level of debt. Similarly, an increase in interest rate on external debt increases the debt service payment and therefore puts upward pressure on the debt ratio. We linearize Eq.(21) and substitute Eq.(10) to derive the equation for the dynamics of the external debt ratio. Note that from Eq.(19) we have $r_t^d = r_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^*$ and we substitute $(z_t - x_t) = \tilde{n}x_t = \gamma_{yf} \tilde{y}_t^f - \gamma_y \tilde{y}_t + \gamma_r \tilde{r}er_t$. Using this fact, we have the debt equation as follows:

$$\tilde{d}_t^* = \beta_a \tilde{d}_{t-1}^* + \beta_b \tilde{r}_t^f - \beta_c \tilde{y}_t - \beta_d \Delta \tilde{y}_t - \beta_e \tilde{q}_t - \beta_f \tilde{r}er_t - \beta_g \tilde{y}_t^f + \varepsilon_t^e \quad (22)$$

where

$$\begin{aligned} \gamma_d &= (1 + g_0)^2 - r_0 \omega_d; \beta_a = \frac{(1 + g_0) + r_0}{\gamma_d}; \beta_b = \frac{r_0}{\gamma_d}; \\ \beta_c &= \frac{(1 + g_0) \gamma_y}{\gamma_d d_0^*}; \beta_d = \frac{r_0 + (1 + g_0)^3}{(1 + g_0)}; \beta_e = \frac{r_0 \omega_q}{\gamma_d}; \\ \beta_f &= \frac{(1 + g_0) \gamma_r}{\gamma_d d_0^*}; \beta_g = \frac{(1 + g_0) \gamma_{yf}}{\gamma_d d_0^*}; \varepsilon_t^e = \rho_e \varepsilon_{t-1}^e + \mu_t^e \end{aligned}$$

Eq. (22) describes the external debt evolution where g_0 is the steady state growth rate, r_0 is the steady state interest rate, and d_0^* is steady state debt-GDP ratio. Positive commodity price shocks generate additional revenue which can be used to reduce the level of external debt (see Arezki and Brückner, 2012). Thus, Eq. (22) shows a negative relation between external debt and commodity prices. In addition, external debt to GDP depends negatively on domestic output and positively on foreign output. Positive domestic output growth reduces external borrowing and hence decreases the level of external

debt. Lastly, external debt to GDP depends positively on foreign interest rate and real exchange rate.

2.4.4 Monetary policy

To derive the money market equation, we equate (4) and (6). This is written as:

$$\frac{\phi}{1-\sigma} \left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left(1 + \frac{\varrho}{S_t} \right)^\phi \left(\frac{M_t}{P_t} \right)^{\phi-1} \frac{1}{P_t} = \frac{\lambda_t}{P_t} \left(1 + \frac{\varrho}{S_t} \right) - \frac{\lambda_t}{P_t (1+r_t)} \left(1 + \frac{\varrho}{S_t} \right) \quad (23)$$

We linearize Eq.(23) and substitute $\tilde{c}_t = \frac{1}{\gamma_c} \tilde{y}_t - \frac{\gamma_x}{\gamma_c} \tilde{x}_t + \frac{\gamma_z}{\gamma_c} \tilde{z}_t$ and $\tilde{x}_t - \tilde{z}_t = \gamma_r \tilde{r} \tilde{e} r_t - \gamma_y \tilde{y}_t + \gamma_{yf} \tilde{y}_t^f$. This yields the money market equation written as:

$$\tilde{r}_t = \eta_a \tilde{y}_t - \eta_b \tilde{m}_t + \eta_c \tilde{s}_t - \varrho E_t \tilde{s}_{t+1} - \eta_d \tilde{r} \tilde{e} r_t - \eta_e \tilde{y}_t^f + \varepsilon_t^b \quad (24)$$

where

$$\begin{aligned} \varepsilon_t^r &= \rho_b \varepsilon_{t-1}^r + \mu_t^b; \eta_a = \frac{\gamma_r (1 + \gamma_y)}{\gamma_m \gamma_c}; \eta_b = \frac{\gamma_r}{\gamma_m}; \eta_c = \frac{\varrho}{s_0} \gamma_r; \\ \eta_d &= \frac{\gamma_r \gamma_r}{\gamma_m \gamma_c}; \eta_e = \frac{\gamma_r \gamma_{yf}}{\gamma_m \gamma_c} \end{aligned}$$

Eq.(24) describes the money market equation. The interest rate depends positively on real output and negatively on real balances. The interest rate also depends positively on current exchange rate and negatively on expected future exchange rate. Lastly, interest rate depends negatively on real exchange rate and foreign output.

Given monetary aggregate targeting in Africa, we follow Muhanji and Ojah (2011) in the specification of our monetary policy reaction function. Money supply is driven by inflation gap, output gap, and commodity price gap. We include the real exchange rate in the monetary aggregate Taylor-type rule. The monetary aggregate Taylor-type rule is:

$$\begin{aligned}\tilde{m}_t &= \rho_m \tilde{m}_{t-1} - (1 - \rho_m)[\rho_\pi \tilde{\pi}_{t-1} + \rho_y \tilde{y}_{t-1} + \rho_{rer} \widetilde{rer}_{t-1} + \rho_q \tilde{q}_{t-1}] \\ &\quad + \varepsilon_t^m\end{aligned}\tag{25}$$

where all variables are in percentage deviations from the steady states. $\tilde{m}_t$ is monetary aggregate, $\tilde{\pi}_t$ is inflation rate gap, $\tilde{y}_t$ is output gap, $\widetilde{rer}_t$ is real exchange rate gap, $\tilde{q}_t$ is commodity price gap. The disturbance follows an AR(1) process: $\varepsilon_t^m = \rho_c \varepsilon_{t-1}^m + \mu_t^c$. The parameter ρ_m is policy rate smoothing, ρ_π is policy reaction to inflation gap, ρ_y is policy reaction to output gap, ρ_{rer} is policy reaction to real exchange rate gap, and ρ_q is policy reaction to commodity price gap. The structural shock processes in the model are given by the following vector:

$$\tilde{\xi}_t = \rho_\xi \tilde{\xi}_t + \varepsilon_{\xi,t}; \quad \varepsilon_{\xi,t} \sim N(0, \sigma_\xi^2)\tag{26}$$

where

$$\varepsilon_{\xi,t} = \left[\tilde{q}_t, \tilde{r}_t^f, \tilde{y}_t^f, \tilde{p}_{it}, \tilde{\pi}_t, \widetilde{rer}_t \right]$$

The equations to be estimated are summarized below:

$$\begin{aligned}
\tilde{y}_t = & \frac{h(\sigma-1)}{\sigma_c} \tilde{y}_{t-1} + \frac{\sigma}{\sigma_c} E_t \tilde{y}_{t+1} - \frac{\gamma_c}{\sigma_c(1+\gamma_y)} (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \\
& + \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)} \tilde{m}_t - \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)} E_t \tilde{m}_{t+1} - \frac{\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)} \tilde{s}_t \\
& + \frac{\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)} E_t \tilde{s}_{t+1} - \frac{h(\sigma-1)\gamma_{rer}}{\sigma_c(1+\gamma_y)} \tilde{r}er_{t-1}
\end{aligned} \tag{27}$$

$$\begin{aligned}
& + \frac{\gamma_r}{(1+\gamma_y)} \tilde{r}er_t - \frac{\sigma\gamma_r}{\sigma_c(1+\gamma_y)} E_t \tilde{r}er_{t+1} - \frac{h(\sigma-1)\gamma_{yf}}{\sigma_c(1+\gamma_y)} \tilde{y}_{t-1}^f \\
& + \frac{\gamma_{yf}}{(1+\gamma_y)} \tilde{y}_t^f - \frac{\sigma\gamma_{yf}}{\sigma_c(1+\gamma_y)} E_t \tilde{y}_{t+1}^f + \varepsilon_t^a
\end{aligned}$$

$$\tilde{r}_t = \eta_a \tilde{y}_t - \eta_b \tilde{m}_t + \eta_c \tilde{s}_t - \varrho E_t \tilde{s}_{t+1} - \eta_d \tilde{r}er_t - \eta_e \tilde{y}_t^f + \varepsilon_t^b \tag{28}$$

$$\begin{aligned}
\tilde{m}_t = & \rho_m \tilde{m}_{t-1} - (1-\rho_m)[\rho_\pi \tilde{\pi}_{t-1} - \rho_y \tilde{y}_{t-1} + \rho_{rer} \tilde{r}er_{t-1} + \rho_q \tilde{q}_{t-1}] \\
& + \varepsilon_t^m
\end{aligned} \tag{29}$$

$$\tilde{s}_t = E_t \tilde{s}_{t+1} - (\tilde{r}_t - \tilde{r}_t^f) - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^* + \varepsilon_t^d \tag{30}$$

$$\tilde{d}_t^* = \beta_a \tilde{d}_{t-1}^* + \beta_b \tilde{r}_t^f - \beta_c \tilde{y}_t - \beta_d \Delta \tilde{y}_t - \beta_e \tilde{q}_t - \beta_f \tilde{r}er_t - \beta_g \tilde{y}_t^f + \varepsilon_t^e$$

$$\begin{aligned}
\tilde{\pi}_t = & \gamma_f E_t \tilde{\pi}_{t+1} + \gamma_b \tilde{\pi}_{t-1} + \lambda \{ \vartheta_a \tilde{y}_{t-1} + \vartheta_b \tilde{y}_t + \gamma_{mc} \phi \tilde{m}_t \\
& - \phi \gamma_{mc} \frac{\varrho}{s_0} \tilde{s}_t - \vartheta_c \tilde{r}er_{t-1} + \vartheta_d \tilde{r}er_t - \vartheta_e \tilde{y}_{t-1}^f + \vartheta_f \tilde{y}_t^f \\
& + \gamma_{mcb} \tilde{p}_{i,t} + \varepsilon_t^\pi
\end{aligned} \tag{31}$$

2.5. Data and estimation

2.5.1. Data sources and treatment

Data for the study were obtained from the International Financial statistics (IFS), the World Bank, and Central Bank database of the selected African countries. We estimate the model using quarterly time series data on twelve macroeconomic variables in ten African countries for the period 1990:1–2011:4. Due to data availability, the sample size differs from one country

to the other. For Egypt, 1998:2 – 2011:4; Ghana, 1990:1 – 2011:4; Kenya, 1990:1 – 2011: 4; Malawi, 1990:1 – 2007:4; Morocco, 1995:4 – 2011:4; Nigeria, 1990:1 – 2008:4; South Africa, 1994:1 – 2011:4; Tunisia, 1994:1 – 2011: 4; Uganda, 1993:2 – 2011:4 ; and Zambia, 1997: 1 – 2011:3.

The twelve macroeconomic variables are: inflation, nominal interest rate, real GDP (industrial output), real money balances, external debt to GDP, real commodity price, nominal exchange rate, real exchange rate, foreign interest rate, real foreign output, foreign inflation and foreign inputs price. The foreign interest rate is proxied by LIBOR, foreign output by US GDP, foreign inflation by US CPI, and price of foreign inputs by US PPI for manufactured goods. Real commodity price is derived by deflating the nominal commodity price with the US CPI. Due to the non-availability of reliable quarterly GDP data for Malawi, Nigeria, and Tunisia, we use industrial output to proxy GDP for the three countries.

2.5.2. Prior distribution and calibration of the parameters

In line with the Bayesian estimation literature (see e.g., Smets and Wouters, 2003), we estimate the model by forming priors distributions and minimizing the posterior distributions of the model parameters (see appendix 2.3 for prior and posterior graphs). As in Smets and Wouters (2007), the persistence of the AR(1) processes follow a beta distribution with mean 0.5 and standard error 0.2. The standard error of the shocks are inverse-gamma distribution with a mean 0.1 and two degrees of freedom. We use the same prior values for all countries in our sample as in García and González (2013). This allows the data to reveal the degree of fit of these values to the realities of the countries. The Metropolis-Hastings (M-H) draws, however, for the convergence of the Markov chain differ among countries².

²The M-H algorithm draws are: Egypt (10,000), Ghana, (10,000), Kenya, (50,000), Malawi, (10,000), Morocco, (5,000), Nigeria, (50,000), South Africa, (10,000), Tunisia, (20,000), Uganda, (10,000), and Zambia, (20,000)

Similar to Castelnovo (2012), we assume the habit parameter h to be a beta distribution with a mean 0.7 and standard deviation 0.1. The price stickiness parameter θ and price indexation parameter ω are assumed to be beta distributed with mean 0.65 and 0.5 and standard errors 0.1 and 0.15 respectively. The calibration of α , σ , and ψ , comes from Smets and Wouters (2007). In line with Benchimol and Fourçans (2012), the inverse elasticity of money holding is assumed to be a normal distribution with mean 1.25 and standard error 0.05. As in Elkhafif (2002), the parameter for currency substitution is a beta distribution with mean 0.3 and standard deviation of 0.14.

The monetary policy reaction function parameters follow the money-based Taylor rule. Following Smets and Wouters (2007), the long run reaction coefficient to output and inflation are assumed to be a normal distribution with mean 0.12 and 1.5 and standard error 0.05 and 0.25 respectively. The monetary smoothing parameter is assumed to be a beta distribution with a mean 0.75 and standard error 0.1. Similar to García and González (2013), the policy reaction parameters to real exchange rate is assumed to be a beta mean 0.5 and standard error 0.1. We assume the same value for reaction to commodity price shocks.

Similar to Steinbach et al. (2009), the calibration for β is 0.99 and in line with Castelnovo (2012), the calibration for ε is 6. The parameter γ_r is calibrated to be 0.66 as in McCallum and Nelson (2000). Following Lombardo and McAdam (2012), our calibration for γ_y is 0.2, γ_{yf} is 0.2, γ_{ls} is 0.5, and for γ_c is 0.58. In line with Batini et al. (2005), we assume that input requirements per unit of output is increasing at the margin because firms tend to use less efficient machines as output rises, we therefore set $\delta_i = 2$. We assume that the steady state share of intermediate input costs in total output $p_{i0}x_0y_0^{-1} = 0.5$. This is a reasonable calibration for African economies, given the evidence by Eifert et al. (2008).

2.5.3 Posterior estimates of the structural parameters

Table 2A and 2B present the prior mean and posterior estimates of the model behavioural parameters. We estimate the posterior distribution by using Metropolis-Hastings algorithm³. Starting from the estimates of behavioural parameters, the habit formation parameter h is estimated to be more than 70 percent of past consumption in Egypt, Kenya, Malawi, and Tunisia. This is similar to the estimates reported in Smets and Wouters (2007). In Ghana, Nigeria, South Africa, Uganda, and Zambia habit formation is around 60 percent of past consumption while it represents less than 50 percent of past consumption in Morocco. The interest rate elasticity of money ϕ estimates revolve around 1.1 to 1.3 for all the countries. This is close to the estimates by Benchimol and Fourçans (2012). Estimates for foreign currency holding, ϱ , range from 0.27 to 0.32. This is similar to the findings by Elkhafif (2002).

Turning to the degree of Calvo pricing, the mean estimate of the degree of price stickiness θ ranges from 0.49 in Ghana to 0.73 in Kenya. This indicates that price is re-optimized within the average of 2-3 quarters in African economies. This is similar to the estimates by Steinbach et al. (2009) for South Africa. On average, the degree of price indexation ω seems to be generally lower than 0.5 in African countries, implying that less than half of price setters are backward looking. This is in line with the estimates by Smets and Wouters (2007). Except in Egypt and Morocco, forward-looking behaviour γ_f dominates inflation dynamics in African economies. This reflects in higher implied estimate of γ_f compare to γ_b in African countries.

In line with the Taylor rule, the parameter estimates for inflation ρ_π indicate strong long run reaction of monetary policy to inflation in African countries. The mean estimates range from 1.24 in Kenya to 1.85 in South Africa. This is higher than the estimates by Steinbach et al. (2009) for South Africa. We

³For the Bayesian estimation and algorithm, we employed Dynare developed by Michel Juillard and others.

also find a substantial degree of policy smoothing ρ_{mag} , ranging from 0.63 in Egypt to 0.98 in Morocco. Policy response to output gap in Africa ranges from moderate in Nigeria to strong in South Africa. Moreover, the estimates indicate a strong response of policy to commodity price shocks ρ_q in few African countries. Lastly, we find that policy responds both moderately and strongly to real exchange rate gap ρ_{rer} in African countries.

Table 2.2A: Prior and Posterior Distribution of Structural Parameters

		Egypt			Ghana			Kenya			Malawi			Morocco		
		Prior			Posterior			Posterior			Posterior			Posterior		
Par.	Distr.	Mean (std)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	
h	Beta	0.70 (0.10)	0.86 (0.01)	0.86 (0.85,0.86)	0.71 (0.01)	0.69 (0.67,0.70)	0.74 (0.00)	0.76 (0.72,0.78)	0.70 (0.00)	0.71 (0.70,0.71)	0.45 (0.01)	0.45 (0.45,0.46)				
ϕ	Normal	1.25 (0.05)	1.15 (0.00)	1.15 (1.14,1.15)	1.23 (0.00)	1.23 (1.23,1.23)	1.29 (0.00)	1.29 (1.29,1.30)	1.22 (0.00)	1.22 (1.22,1.22)	1.10 (0.00)	1.10 (1.10,1.11)				
σ	Normal	1.50 (0.375)	1.56 (0.03)	1.57 (1.57,1.58)	1.04 (0.01)	1.04 (1.03,1.04)	1.07 (0.01)	1.05 (1.04,1.06)	1.32 (0.01)	1.32 (1.32,1.32)	2.61 (0.02)	2.62 (2.58,2.68)				
α	Beta	0.33 (0.10)	0.41 (0.01)	0.42 (0.41,0.42)	0.25 (0.00)	0.25 (0.24,0.26)	0.47 (0.01)	0.51 (0.47,0.55)	0.29 (0.00)	0.29 (0.29,0.29)	0.26 (0.00)	0.26 (0.26,0.27)				
ψ	Beta	2.00 (0.02)	0.87 (0.08)	0.82 (0.78,0.84)	3.43 (0.02)	3.43 (3.37,3.51)	1.93 (0.05)	2.39 (2.27,2.48)	0.27 (0.00)	0.29 (0.29,0.29)	2.19 (0.01)	2.19 (2.11,2.27)				
ϱ	Beta	0.30 (0.02)	0.32 (0.00)	0.32 (0.31,0.32)	0.31 (0.00)	0.31 (0.31,0.31)	0.28 (0.00)	0.27 (0.27,0.27)	0.29 (0.00)	0.29 (0.29,0.29)	0.31 (0.00)	0.31 (0.31,0.31)				
θ	Beta	0.65 (0.10)	0.51 (0.01)	0.51 (0.51,0.51)	0.49 (0.01)	0.49 (0.48,0.49)	0.67 (0.00)	0.73 (0.71,0.75)	0.51 (0.01)	0.51 (0.51,0.51)	0.54 (0.01)	0.55 (0.54,0.56)				
ω	Beta	0.5 (0.15)	0.69 (0.01)	0.69 (0.68,0.69)	0.19 (0.01)	0.19 (0.18,0.20)	0.25 (0.00)	0.14 (0.12,0.17)	0.57 (0.01)	0.56 (0.56,0.56)	0.42 (0.01)	0.41 (0.40,0.42)				
ω_d	Beta	0.2 (0.15)	0.34 (0.01)	0.33 (0.32,0.34)	0.05 (0.00)	0.05 (0.05,0.05)	0.04 (0.00)	0.04 (0.03,0.04)	0.07 (0.00)	0.07 (0.07,0.07)	0.28 (0.01)	0.28 (0.27,0.29)				
ω_q	Beta	0.50 (0.15)	0.06 (0.01)	0.05 (0.05,0.05)	0.91 (0.02)	0.91 (0.90,0.93)	0.64 (0.01)	0.62 (0.60,0.64)	0.33 (0.01)	0.33 (0.33,0.33)	0.31 (0.01)	0.31 (0.29,0.33)				
ρ_y	Normal	0.12 (0.05)	0.08 (0.00)	0.08 (0.08,0.09)	0.12 (0.00)	0.12 (0.12,0.12)	0.07 (0.00)	0.08 (0.07,0.08)	0.15 (0.00)	0.15 (0.15,0.15)	0.19 (0.00)	0.19 (0.18,0.19)				
ρ_π	Normal	1.50 (0.125)	1.55 (0.01)	1.55 (1.55,1.56)	1.58 (0.01)	1.58 (1.56,1.60)	1.37 (0.00)	1.24 (1.20,1.27)	1.66 (0.00)	1.66 (1.66,1.66)	1.59 (0.00)	1.58 (1.58,1.59)				
ρ_{mag}	Beta	0.75 (0.10)	0.64 (0.01)	0.63 (0.62,0.64)	0.66 (0.01)	0.66 (0.65,0.67)	0.84 (0.00)	0.84 (0.83,0.85)	0.93 (0.01)	0.93 (0.93,0.93)	0.98 (0.01)	0.98 (0.97,0.98)				
ρ_{rer}	Beta	0.50 (0.10)	0.78 (0.03)	0.79 (0.78,0.79)	0.64 (0.02)	0.64 (0.63,0.65)	0.99 (0.01)	0.98 (0.96,1.00)	0.97 (0.01)	0.97 (0.97,0.97)	0.57 (0.00)	0.57 (0.56,0.59)				
ρ_q	Normal	0.50 (0.10)	0.27 (0.02)	0.28 (0.27,0.30)	0.36 (0.01)	0.37 (0.35,0.40)	0.03 (0.00)	0.04 (0.02,0.06)	0.38 (0.01)	0.38 (0.37,0.38)	0.52 (0.00)	0.52 (0.51,0.54)				

Table 2.2B: Prior and Posterior Distribution of Structural Parameters

Par.	Distr.	Nigeria			South Africa			Tunisia			Uganda			Zambia		
		Posterior			Posterior			Posterior			Posterior			Posterior		
		Mean (std)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)
h	Beta	0.70 (0.10)	0.66 (0.01)	0.66 (0.66,0.66)	0.61 (0.00)	0.60 (0.58,0.62)	0.79 (0.00)	0.80 (0.78,0.82)	0.69 (0.00)	0.69 (0.69,0.69)	0.61 (0.00)	0.62 (0.61,0.62)	0.61 (0.00)	0.62 (0.61,0.62)	0.61 (0.00)	0.62 (0.61,0.62)
ϕ	Beta	1.25 (0.05)	1.25 (0.00)	1.25 (1.25,1.25)	1.14 (0.00)	1.14 (1.13,1.14)	1.20 (0.00)	1.20 (1.20,1.20)	1.24 (0.00)	1.24 (1.24,1.24)	1.26 (0.00)	1.26 (1.26,1.26)	1.26 (0.00)	1.26 (1.26,1.26)	1.26 (0.00)	1.26 (1.26,1.26)
σ	Normal	1.50 (0.375)	1.28 (0.01)	1.28 (1.27,1.29)	1.06 (0.01)	1.06 (1.05,1.07)	1.57 (0.01)	1.57 (1.56,1.60)	1.56 (0.01)	1.58 (1.57,1.59)	1.24 (0.00)	1.23 (1.23,1.24)	1.24 (0.00)	1.23 (1.23,1.24)	1.24 (0.00)	1.23 (1.23,1.24)
α	Beta	0.33 (0.10)	0.23 (0.00)	0.23 (0.23,0.23)	0.24 (0.00)	0.24 (0.24,0.25)	0.42 (0.00)	0.43 (0.42,0.44)	0.26 (0.00)	0.26 (0.26,0.26)	0.27 (0.00)	0.27 (0.27,0.27)	0.27 (0.00)	0.27 (0.27,0.27)	0.27 (0.00)	0.27 (0.27,0.27)
ψ	Beta	2.00 (0.02)	2.74 (0.03)	2.76 (2.75,2.78)	2.62 (0.02)	2.55 (2.47,2.61)	1.08 (0.03)	1.01 (0.95,1.09)	2.19 (0.01)	2.18 (2.16,2.20)	2.26 (0.03)	2.25 (2.24,2.27)	2.26 (0.03)	2.25 (2.24,2.27)	2.26 (0.03)	2.25 (2.24,2.27)
$\omega_{\mathcal{Q}}$	Beta	0.30 (0.02)	0.27 (0.00)	0.27 (0.27,0.27)	0.28 (0.00)	0.28 (0.28,0.29)	0.27 (0.00)	0.27 (0.27,0.28)	0.30 (0.00)	0.30 (0.30,0.30)	0.27 (0.00)	0.28 (0.28,0.28)	0.27 (0.00)	0.28 (0.28,0.28)	0.27 (0.00)	0.28 (0.28,0.28)
θ	Beta	0.65 (0.10)	0.63 (0.00)	0.63 (0.63,0.63)	0.73 (0.00)	0.72 (0.72,0.73)	0.60 (0.01)	0.58 (0.58,0.59)	0.59 (0.00)	0.60 (0.59,0.60)	0.56 (0.01)	0.56 (0.55,0.56)	0.56 (0.01)	0.56 (0.55,0.56)	0.56 (0.01)	0.56 (0.55,0.56)
ω	Beta	0.50 (0.15)	0.33 (0.00)	0.33 (0.33,0.33)	0.21 (0.00)	0.21 (0.20,0.23)	0.42 (0.01)	0.42 (0.41,0.44)	0.79 (0.00)	0.79 (0.78,0.79)	0.48 (0.00)	0.48 (0.47,0.48)	0.48 (0.00)	0.48 (0.47,0.48)	0.48 (0.00)	0.48 (0.47,0.48)
ω_d	Beta	0.20 (0.15)	0.02 (0.00)	0.02 (0.02,0.03)	0.20 (0.00)	0.20 (0.19,0.21)	0.00 (0.00)	0.00 (0.00,0.01)	-0.03 (0.01)	-0.03 (-0.0, -0.03)	0.16 (0.00)	0.16 (0.16,0.17)	0.16 (0.00)	0.16 (0.16,0.17)	0.16 (0.00)	0.16 (0.16,0.17)
ω_q	Beta	0.50 (0.15)	0.79 (0.00)	0.79 (0.79,0.80)	0.44 (0.00)	0.43 (0.40,0.46)	0.25 (0.01)	0.26 (0.23,0.27)	0.53 (0.00)	0.53 (0.53,0.54)	0.51 (0.00)	0.51 (0.51,0.51)	0.51 (0.00)	0.51 (0.51,0.51)	0.51 (0.00)	0.51 (0.51,0.51)
ρ_y	Normal	0.12 (0.05)	0.05 (0.00)	0.05 (0.05,0.05)	0.19 (0.00)	0.19 (0.18,0.19)	0.17 (0.00)	0.17 (0.16,0.18)	0.07 (0.00)	0.07 (0.07,0.07)	0.08 (0.00)	0.07 (0.07,0.08)	0.08 (0.00)	0.07 (0.07,0.08)	0.08 (0.00)	0.07 (0.07,0.08)
ρ_π	Beta	1.50 (0.125)	1.42 (0.00)	1.42 (1.42,1.42)	1.84 (0.01)	1.85 (1.84,1.85)	1.46 (0.01)	1.46 (1.44,1.47)	1.51 (0.00)	1.50 (1.50,1.51)	1.36 (0.00)	1.36 (1.35,1.36)	1.36 (0.00)	1.36 (1.35,1.36)	1.36 (0.00)	1.36 (1.35,1.36)
ρ_{mag}	Beta	0.75 (0.10)	0.92 (0.01)	0.93 (0.93,0.93)	0.69 (0.00)	0.70 (0.68,0.71)	0.81 (0.00)	0.81 (0.80,0.82)	0.93 (0.00)	0.93 (0.92,0.93)	0.84 (0.00)	0.84 (0.84,0.84)	0.84 (0.00)	0.84 (0.84,0.84)	0.84 (0.00)	0.84 (0.84,0.84)
ρ_{rer}	Beta	0.50 (0.10)	0.69 (0.01)	0.68 (0.68,0.69)	0.87 (0.01)	0.88 (0.84,0.91)	0.50 (0.00)	0.50 (0.48,0.52)	0.25 (0.00)	0.26 (0.25,0.26)	0.96 (0.01)	0.96 (0.95,0.96)	0.96 (0.01)	0.96 (0.95,0.96)	0.96 (0.01)	0.96 (0.95,0.96)
ρ_q	Normal	0.50 (0.10)	0.31 (0.01)	0.31 (0.30,0.31)	0.23 (0.01)	0.23 (0.19,0.27)	0.54 (0.01)	0.57 (0.55,0.58)	0.62 (0.00)	0.62 (0.62,0.62)	0.43 (0.00)	0.43 (0.43,0.43)	0.43 (0.00)	0.43 (0.43,0.43)	0.43 (0.00)	0.43 (0.43,0.43)

2.5.4 Posterior estimates of the structural shocks

The estimated processes for the exogenous disturbances are shown in Table 3A and 3B. The estimated processes reveal some similarities for many African countries. For example, in Egypt the most persistent shocks are foreign input price with mean 1.00, external debt and exchange rate with mean 0.98 each, productivity with mean 0.96, and money supply with mean 0.79. Similarly, in Ghana, the most persistent shocks are foreign interest rate with mean 1.00, external debt with mean 0.96, real exchange rate with mean 0.82, and foreign input price with mean 0.65, and productivity with mean 0.64. In Kenya, the most persistent shocks are external debt and nominal exchange rate with mean 0.93 each, commodity price with mean 0.92, foreign output with mean 0.91, and nominal exchange rate with mean 0.85.

In Malawi, the most important shocks are foreign input price with mean 1.00, productivity with mean 0.98, external debt with mean 0.93, real exchange rate with mean 0.74 and foreign interest rate with mean 0.66. The most persistent shocks in Morocco are nominal exchange rate with mean 1.00, external debt with mean 0.99, foreign interest rate and domestic interest rate with mean 0.96 each, and money supply with mean 0.91. Foreign input price, productivity, external debt, interest rate, and foreign interest rate shocks are estimated to be the most persistent in Nigeria with mean of 1.00, 0.94, 0.93, 0.88 and 0.69 respectively. In South Africa, the most persistent shocks are external debt with mean 0.95, foreign input price and commodity price with mean 0.89 each, nominal exchange rate with mean 0.86, and foreign interest rate with mean 0.84.

Estimates for Tunisia indicate that external debt, foreign interest rate, foreign inflation, productivity, and interest rate shocks are the most persistent with mean coefficient of 1.00, 0.78, 0.71, 0.69, and 0.62 respectively. In Uganda, interest rate, productivity, external debt, foreign interest rate, and foreign input price shocks are the most persistent with mean 0.98, 0.97, 0.89, 0.88 and

0.86 respectively. Lastly, results for Zambia suggest that foreign input price, external debt, productivity, real exchange rate, and interest rate shocks are the most persistent with mean 1.00, 0.97, 0.96, 0.91, and 0.73 respectively.

Table 2.3A: Prior and Posterior Distribution of Structural Shocks

		Egypt			Ghana			Kenya			Malawi			Morocco		
		Posterior			Posterior			Posterior			Posterior			Posterior		
Par.	Distr.	Mean (std)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)
ρ_b	Beta	0.50 (0.20)	0.72 (0.02)	0.72 (0.72,0.73)	0.48 (0.01)	0.46 (0.44,0.47)	0.67 (0.00)	0.82 (0.77,0.86)	0.20 (0.01)	0.20 (0.20,0.20)	0.97 (0.01)	0.20 (0.20,0.20)	0.97 (0.01)	0.96 (0.94,0.97)	0.97 (0.01)	0.96 (0.94,0.97)
ρ_c	Beta	0.50 (0.20)	0.77 (0.03)	0.79 (0.77,0.80)	0.29 (0.01)	0.28 (0.25,0.31)	0.80 (0.01)	0.93 (0.87,0.98)	0.14 (0.02)	0.14 (0.13,0.14)	0.91 (0.01)	0.14 (0.13,0.14)	0.91 (0.01)	0.91 (0.89,0.93)	0.91 (0.01)	0.91 (0.89,0.93)
ρ_d	Beta	0.50 (0.20)	0.95 (0.04)	0.98 (0.97,0.99)	0.62 (0.00)	0.64 (0.63,0.66)	0.76 (0.01)	0.85 (0.78,0.89)	0.53 (0.02)	0.52 (0.52,0.52)	1.00 (0.01)	0.52 (0.52,0.52)	1.00 (0.01)	1.00 (0.99,1.00)	1.00 (0.01)	1.00 (0.99,1.00)
ρ_e	Beta	0.50 (0.20)	0.98 (0.03)	0.98 (0.98,0.99)	0.97 (0.01)	0.96 (0.95,0.97)	0.92 (0.01)	0.93 (0.92,0.94)	0.93 (0.01)	0.93 (0.93,0.93)	0.99 (0.01)	0.93 (0.93,0.93)	0.99 (0.01)	0.99 (0.98,0.99)	0.99 (0.01)	0.99 (0.98,0.99)
ρ_f	Beta	0.50 (0.20)	0.96 (0.01)	0.96 (0.94,0.96)	0.65 (0.01)	0.64 (0.63,0.64)	0.57 (0.00)	0.61 (0.58,0.64)	0.98 (0.02)	0.98 (0.98,0.98)	0.77 (0.01)	0.98 (0.98,0.98)	0.77 (0.01)	0.76 (0.74,0.79)	0.77 (0.01)	0.76 (0.74,0.79)
ρ_g	Beta	0.50 (0.20)	0.53 (0.02)	0.53 (0.52,0.54)	0.26 (0.01)	0.27 (0.23,0.31)	0.75 (0.01)	0.92 (0.90,0.94)	0.45 (0.01)	0.45 (0.45,0.45)	0.51 (0.01)	0.45 (0.45,0.45)	0.51 (0.01)	0.51 (0.49,0.52)	0.51 (0.01)	0.51 (0.49,0.52)
ρ_h	Beta	0.50 (0.20)	0.52 (0.02)	0.51 (0.51,0.52)	0.99 (0.01)	1.00 (0.99,1.00)	0.74 (0.01)	0.78 (0.75,0.81)	0.66 (0.00)	0.66 (0.66,0.67)	0.97 (0.01)	0.66 (0.66,0.67)	0.97 (0.01)	0.96 (0.94,0.98)	0.97 (0.01)	0.96 (0.94,0.98)
ρ_i	Beta	0.50 (0.20)	0.49 (0.01)	0.51 (0.50,0.52)	0.47 (0.01)	0.47 (0.46,0.49)	0.78 (0.00)	0.91 (0.89,0.95)	0.22 (0.01)	0.22 (0.22,0.22)	0.57 (0.01)	0.22 (0.22,0.22)	0.57 (0.01)	0.56 (0.54,0.57)	0.57 (0.01)	0.56 (0.54,0.57)
ρ_k	Beta	0.50 (0.20)	1.00 (0.02)	1.00 (1.0,1.0)	0.63 (0.00)	0.65 (0.64,0.67)	0.46 (0.00)	0.37 (0.34,0.41)	1.00 (0.00)	1.00 (1.00,1.00)	0.46 (0.01)	1.00 (1.00,1.00)	0.46 (0.01)	0.47 (0.45,0.48)	0.46 (0.01)	0.47 (0.45,0.48)
ρ_l	Beta	0.50 (0.20)	0.38 (0.01)	0.38 (0.37,0.38)	0.59 (0.01)	0.59 (0.58,0.60)	0.77 (0.01)	0.77 (0.75,0.79)	0.45 (0.01)	0.45 (0.45,0.45)	0.69 (0.01)	0.45 (0.45,0.45)	0.69 (0.01)	0.69 (0.68,0.70)	0.69 (0.01)	0.69 (0.68,0.70)
ρ_m	Beta	0.50 (0.20)	0.74 (0.02)	0.73 (0.72,0.74)	0.83 (0.02)	0.82 (0.79,0.84)	0.68 (0.01)	0.76 (0.74,0.79)	0.73 (0.01)	0.74 (0.74,0.74)	0.66 (0.01)	0.74 (0.74,0.74)	0.66 (0.01)	0.65 (0.62,0.66)	0.66 (0.01)	0.65 (0.62,0.66)

Table 2.3B: Prior and Posterior Distribution of Structural Shocks

		Nigeria		South Africa		Tunisia		Uganda		Zambia	
Prior		Posterior		Posterior		Posterior		Posterior		Posterior	
Par.	Distr.	Mean (std)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)	Mean (5%,95%)	Mode (std)
ρ_b	Beta	0.50 (0.20)	0.87 (0.01)	0.88 (0.87,0.88)	0.60 (0.00)	0.62 (0.61,0.63)	0.60 (0.02)	0.62 (0.60,0.64)	0.98 (0.00)	0.98 (0.97,0.99)	0.74 (0.00)
ρ_c	Beta	0.50 (0.20)	0.47 (0.00)	0.47 (0.47,0.48)	0.61 (0.01)	0.62 (0.61,0.64)	0.40 (0.01)	0.40 (0.38,0.42)	0.47 (0.01)	0.48 (0.47,0.48)	0.35 (0.01)
ρ_d	Beta	0.50 (0.20)	0.50 (0.01)	0.51 (0.50,0.52)	0.85 (0.01)	0.86 (0.85,0.87)	0.53 (0.01)	0.52 (0.48,0.56)	0.22 (0.01)	0.22 (0.21,0.24)	0.65 (0.00)
ρ_e	Beta	0.50 (0.20)	0.93 (0.02)	0.93 (0.93,0.94)	0.95 (0.01)	0.95 (0.94,0.96)	1.00 (0.01)	1.00 (0.99,1.0)	0.88 (0.02)	0.89 (0.88,0.89)	0.97 (0.00)
ρ_f	Beta	0.50 (0.20)	0.93 (0.01)	0.94 (0.94,0.94)	0.75 (0.01)	0.76 (0.75,0.77)	0.67 (0.01)	0.69 (0.67,0.70)	0.97 (0.01)	0.97 (0.96,0.98)	0.96 (0.01)
ρ_g	Beta	0.50 (0.20)	0.14 (0.01)	0.13 (0.13,0.14)	0.90 (0.02)	0.89 (0.87,0.91)	0.56 (0.01)	0.54 (0.50,0.58)	0.30 (0.00)	0.31 (0.30,0.31)	0.53 (0.00)
ρ_h	Beta	0.50 (0.20)	0.69 (0.01)	0.69 (0.69,0.69)	0.82 (0.01)	0.84 (0.83,0.86)	0.74 (0.01)	0.78 (0.76,0.80)	0.89 (0.01)	0.88 (0.87,0.89)	0.41 (0.00)
ρ_i	Beta	0.50 (0.20)	0.59 (0.01)	0.59 (0.59,0.59)	0.40 (0.01)	0.39 (0.35,0.43)	0.59 (0.01)	0.62 (0.59,0.66)	0.44 (0.01)	0.43 (0.42,0.45)	0.67 (0.01)
ρ_k	Beta	0.50 (0.20)	1.0 (0.01)	1.00 (1.0,1.0)	0.88 (0.01)	0.89 (0.87,0.91)	0.49 (0.01)	0.53 (0.50,0.56)	0.86 (0.01)	0.86 (0.86,0.87)	1.00 (0.01)
ρ_l	Beta	0.50 (0.20)	0.25 (0.01)	0.24 (0.23,0.24)	0.82 (0.01)	0.84 (0.83,0.85)	0.69 (0.01)	0.71 (0.67,0.75)	0.53 (0.00)	0.53 (0.53,0.54)	0.55 (0.01)
ρ_m	Beta	0.50 (0.20)	0.41 (0.01)	0.40 (0.40,0.41)	0.01 (0.02)	0.02 (0.01,0.03)	0.59 (0.02)	0.60 (0.58,0.62)	0.60 (0.01)	0.59 (0.58,0.60)	0.91 (0.01)

2.5 Sub-sample estimates

To examine whether our results are sensitive to the inclusion of global crisis period, (2008-2010), we estimate the sample up till 2007 i.e before the beginning of the global crises. Table 3C shows the estimates of the structural shocks up till 2007. There is no significant difference between the sub-sample estimates and the whole sample estimates. The sub-sample results indicate that external shocks still remain persistent. A notable feature is the persistence of external debt shocks across the countries. This is followed by productivity shocks, foreign input price shocks, commodity price shocks, domestic interest rate shocks, foreign interest rate shocks, and exchange rate shocks.

The persistence of external shocks for the sub-sample and the whole sample estimates suggests that African economies are vulnerable to trade and world financial shocks. This could be attributed to their dependence on the exports of narrow range of primary commodities whose prices are very volatile. Commodity price volatility generate erratic export revenue and instability in foreign exchange and fiscal operations. The volatility in fiscal operations trigger external debt problem. African countries borrow externally to smooth consumption during negative commodity price shocks.

Table 2.4: Sub-sample estimates for structural shocks (Pre-2008 estimates)

Prior			Posterior									
Par.	Distr.	Mean (std)	Egypt	Ghana	Kenya	Malawi	Morocco	Nigeria	S. Africa	Tunisia	Uganda	Zambia
ρ_b	Beta	0.50 (0.20)	0.95 (0.02)	0.71 (0.01)	0.64 (0.00)	0.20 (0.01)	0.61 (0.00)	0.49 (0.00)	0.93 (0.01)	0.57 (0.00)	0.82 (0.01)	0.68 (0.00)
ρ_c	Beta	0.50 (0.20)	0.28 (0.01)	0.38 (0.01)	0.41 (0.00)	0.14 (0.02)	0.40 (0.00)	0.49 (0.00)	0.57 (0.01)	0.51 (0.00)	0.50 (0.01)	0.54 (0.00)
ρ_d	Beta	0.50 (0.20)	0.29 (0.01)	0.54 (0.00)	0.77 (0.00)	0.53 (0.02)	0.53 (0.00)	0.61 (0.00)	0.53 (0.01)	0.52 (0.00)	0.43 (0.01)	0.52 (0.00)
ρ_e	Beta	0.50 (0.20)	1.00 (0.02)	0.97 (0.01)	0.87 (0.00)	0.93 (0.01)	0.93 (0.01)	0.74 (0.00)	0.94 (0.02)	0.66 (0.00)	0.84 (0.02)	0.93 (0.00)
ρ_f	Beta	0.50 (0.20)	1.00 (0.02)	0.58 (0.00)	0.70 (0.00)	0.98 (0.02)	0.74 (0.00)	0.81 (0.00)	0.92 (0.01)	0.76 (0.00)	0.80 (0.01)	0.56 (0.00)
ρ_g	Beta	0.50 (0.20)	0.70 (0.01)	0.95 (0.01)	0.50 (0.00)	0.45 (0.01)	0.89 (0.01)	0.40 (0.00)	0.63 (0.01)	0.42 (0.00)	0.50 (0.01)	0.41 (0.00)
ρ_h	Beta	0.50 (0.20)	0.65 (0.01)	0.62 (0.00)	0.97 (0.00)	0.66 (0.00)	0.61 (0.00)	0.77 (0.00)	0.25 (0.01)	0.55 (0.00)	0.23 (0.01)	0.48 (0.00)
ρ_i	Beta	0.50 (0.20)	0.71 (0.03)	0.38 (0.01)	0.53 (0.00)	0.22 (0.01)	0.69 (0.01)	0.51 (0.00)	0.34 (0.01)	0.45 (0.00)	0.16 (0.02)	0.29 (0.00)
ρ_k	Beta	0.50 (0.20)	1.00 (0.02)	0.51 (0.01)	0.73 (0.00)	1.00 (0.00)	0.67 (0.00)	0.75 (0.00)	1.00 (0.01)	0.48 (0.00)	0.55 (0.01)	0.39 (0.00)
ρ_l	Beta	0.50 (0.20)	0.67 (0.01)	0.86 (0.01)	0.70 (0.00)	0.45 (0.01)	0.54 (0.00)	0.64 (0.00)	0.48 (0.01)	0.65 (0.00)	0.52 (0.01)	0.51 (0.00)
ρ_m	Beta	0.50 (0.20)	0.94 (0.03)	0.76 (0.01)	0.46 (0.00)	0.73 (0.01)	0.58 (0.01)	0.44 (0.00)	0.00 (0.01)	0.46 (0.00)	0.47 (0.01)	0.82 (0.00)

2.6 Impulse response functions

Fig. 1-11 in appendix 2.1 are impulse response functions (IRFs) showing the dynamic response of output to both internal and external shocks in African countries. Starting from Fig. 1, an increase in interest rate causes reduction in output in many of the countries. Fig. 2 indicates that a rise in money supply increases output in all African countries. Moreover, Fig. 3 reveals that following nominal exchange rate depreciation, output initially contracts but later expands in some African countries. But in Morocco and Uganda, nominal exchange rate depreciation seems to have no effect on output after the fifth quarter. As shown in Fig. 4, external debt shocks initially reduce output in African countries. Output later increases over the sixth quarter.

The IRFs in Fig.5 show the impact of productivity shocks on output. Following productivity shocks, output declines on impact but later expands in some African countries. This may be attributed to existence of financial frictions. But in Uganda output increases after productivity shocks. Fig. 6 generally shows that positive commodity price shocks initially lower output on impact in African countries but output later increases. In Fig.7, output declines in African countries after a positive shock to foreign interest rate.

The IRFs in Fig.8 reveal the effects of foreign output shocks on domestic output. In six of the countries, domestic output increases after the positive foreign output shocks. However, in Malawi, Morocco, Tunisia and Uganda, domestic output declines. Fig. 9 indicates that following foreign input price shocks, output increases in all the countries except in Uganda. This largely reflects trade channel effects where a rise in foreign input price worsens the terms of trade, increases the trade balance and output. In Fig. 10, except in Morocco, output declines on impact after foreign inflation shocks but later rise for African countries. Lastly, real exchange rate shocks reduce output on impact but output later rises.

2.7 Forecast error variance decomposition of output at different horizons

The variance decomposition (Table 4A and 4B) is employed to show the relative contribution of each structural shock to output fluctuations in African countries at different horizons. Generally, the variance decomposition shows that both internal and external shocks significantly influence output variations in African economies. The decomposition, however, reveals that internal shocks are mainly dominant in the 4th quarter. External shocks appear to be the most persistent shocks driving output swings in African countries. This reflects in their dominance over the 8th and 16th quarters. The influence of external shocks in the 8th and 16th quarters may be due to African countries exposures to trade and financial shocks. External shocks also have greater impact on output fluctuations in all African countries except in Uganda.

A disaggregation of external shocks reveals the most important external shocks driving output fluctuations in African economies. Nominal exchange rate shocks account for some of the variance of output in African countries. Exchange rate depreciation may increase the cost of capital, reduce investment and output. Depreciation may also worsen country's balance sheet, increase debt service payments and reduce output. Moreover, external debt shocks significantly contribute to output fluctuations in some African countries. High external debt increases the risk premium and the cost of capital and lowers output. This is related to the findings by Hsing (2003) for Brazil.

In addition, foreign interest rate shocks significantly affect output variations in Africa. A positive shock to foreign interest rate may not only increase spread and debt service cost but may also cause capital outflow in African countries. These will lead to a decline in investment and output. Similarly, output fluctuations is affected by commodity price shocks. The effects of

commodity price shocks have been found to be asymmetric in African countries. While negative commodity price shocks have been found to hamper growth, positive price shocks have not promoted growth (see, Dehn, 2000). Also found to influence output swings is foreign input price shocks. A positive shock to foreign input price may increase output through the trade channel or decrease output through the supply shock channel. This reinforces the findings by Kose and Riezman (2001) for African economies. Lastly, foreign inflation and foreign output shocks play minimal role in the variance of output.

Furthermore, among internal shocks, money supply shocks contribute most to output fluctuations in many African countries. This is similar to the findings by Kandil (2014) for developing countries. An increase in money supply reduces the cost of borrowing, increases investment and output. Other dominant internal shock that influence output fluctuations is the productivity shocks. This demonstrates the significance of supply shocks in output dynamics. This is in line with findings by Hoffmaister and Roldós (2001) for Korea. A positive productivity shock lowers the cost of production and increases output. The interest rate shocks only play a minor role in output swings for African economies. This is comparable to the results by Kanczuk (2004) for Brazil.

The dominance of external shocks in output fluctuations in African countries can be attributed to their dependence on exports of few primary commodities, reliance on foreign inputs for domestic production, and exposure to foreign currency denominated debt. These expose them to commodity price fluctuations, exchange rate volatility and world interest rate shocks. The significance of external shocks in influencing output fluctuations in African countries raises the question over their exchange rate system. While many African countries maintain that their national currencies are floating, empirical findings suggest otherwise. Slavov (2011) finds that many African

countries operate soft peg and display "fear of floating". The tendency to defend the exchange rate may be responsible for the greater impacts of external shocks in African economies. The fear of floating may be attributed to liability dollarization, high level of exchange rate pass-through and low level of financial development in African countries.

Table 2.5A: Forecast error variance decomposition

	Horizon		Structural shocks										Average	
	Quarters	r_t	m_t	s_t	d_t^*	a_t	q_t	r_t^f	y_t^f	p_{it}	π_t^f	rer_t	ε_t^i	ε_t^e
Egypt														
Output	4	1.19	49.83	2.01	2.97	2.63	6.31	1.44	0.60	3.02	0.33	0.12	53.65	16.8
Output	8	0.71	27.44	7.34	6.31	20.92	6.30	1.76	3.75	13.91	1.13	1.93	49.07	42.43
Output	16	2.57	22.43	36.81	8.90	8.56	2.13	3.11	2.47	5.57	0.38	1.34	33.56	60.71
Ghana														
Output	4	0.43	0.56	1.00	2.00	0.92	0.45	7.12	0.02	5.78	0.23	0.08	1.91	16.68
Output	8	0.41	2.30	0.63	5.55	1.37	0.95	9.98	0.01	9.68	0.13	0.19	4.08	27.12
Output	16	1.06	2.97	2.08	11.84	2.19	1.58	5.90	0.02	11.71	0.06	0.18	6.22	33.37
Kenya														
Output	4	2.89	1.56	3.35	0.15	1.29	1.00	0.86	0.88	0.56	0.30	0.01	5.74	7.11
Output	8	1.09	3.47	6.38	0.12	2.88	3.90	3.37	1.40	1.71	0.10	0.00	7.44	16.98
Output	16	0.51	10.05	8.87	0.01	3.86	8.64	6.67	2.68	2.52	0.33	0.07	14.42	29.79
Malawi														
Output	4	0.68	0.30	1.34	0.24	4.26	0.86	1.78	0.07	1.28	0.47	0.36	5.24	6.40
Output	8	0.60	0.17	4.32	3.83	19.39	2.92	4.99	0.19	0.82	0.26	2.05	20.16	18.23
Output	16	1.17	0.13	13.49	6.09	17.07	5.67	6.29	0.98	1.37	1.26	3.38	18.37	38.53
Morocco														
Output	4	12.58	14.29	5.14	9.28	2.15	0.90	5.91	1.15	0.29	2.33	2.82	29.02	27.82
Output	8	10.60	12.75	6.46	7.86	2.18	2.65	7.31	1.27	1.71	4.08	2.92	25.53	34.26
Output	16	10.11	12.19	7.05	7.16	2.44	2.70	11.35	0.95	2.04	3.94	2.37	24.74	37.56

ε_t^i is internal shocks and ε_t^e is external shocks.

Table 2.5B: Forecast error variance decomposition

	Horizon Quarters	Structural shocks											Average		
		r_t	m_t	s_t	d_t^*	a_t	q_t	r_t^f	y_t^f	p_{it}	π_t^f	rer_t	ε_t^i	ε_t^e	
Nigeria															
Output	4	0.19	11.51	4.85	0.05	7.39	14.11	0.02	0.00	3.91	1.74	0.19	19.09	24.87	
Output	8	2.30	15.94	6.45	0.81	11.16	21.16	0.13	0.05	7.98	7.41	0.47	29.4	44.46	
Output	16	11.12	12.04	6.32	3.02	10.13	16.62	3.12	0.22	6.90	12.21	0.88	33.29	49.29	
S. Africa															
Output	4	1.97	2.44	1.16	3.00	2.02	1.10	0.58	0.06	0.17	0.03	0.00	6.43	6.1	
Output	8	4.37	5.67	2.35	8.66	2.35	3.80	0.94	0.04	0.11	0.01	0.01	12.39	15.92	
Output	16	4.47	9.02	3.68	15.11	3.85	3.97	2.13	0.16	0.20	0.02	0.05	17.34	25.32	
Tunisia															
Output	4	0.83	0.20	0.85	2.07	0.72	0.32	0.01	0.30	0.64	2.05	0.84	1.75	7.08	
Output	8	0.22	0.07	1.78	0.92	11.42	1.74	2.95	0.65	4.25	1.06	2.50	11.71	15.85	
Output	16	5.98	7.97	12.49	6.49	10.16	2.81	9.38	1.34	3.76	1.53	1.32	24.11	39.12	
Uganda															
Output	4	1.19	10.31	0.11	1.80	2.69	0.22	7.61	0.00	0.00	0.02	0.76	14.19	10.52	
Output	8	2.60	15.37	0.26	3.06	4.47	0.27	16.54	0.00	0.00	0.01	0.92	22.44	21.06	
Output	16	1.83	21.21	0.43	4.09	3.94	0.27	21.07	0.01	0.03	0.02	0.90	26.98	26.82	
Zambia															
Output	4	0.15	0.86	7.84	2.34	5.71	0.35	3.77	0.08	0.34	0.73	0.37	6.72	15.82	
Output	8	0.44	0.28	24.62	7.48	8.00	0.08	2.87	0.07	0.33	0.07	0.26	8.72	35.78	
Output	16	1.18	0.06	28.48	12.82	6.04	0.12	2.07	0.06	1.67	0.54	0.08	7.28	45.84	

ε_t^i is internal shocks and ε_t^e is external shocks.

2.8 Historical decomposition of output

Fig. 12-21 in appendix 2.2 plot the historical decomposition of output for the ten African countries. Fig. 12 represents the historical decomposition of output in Egypt. Productivity, external debt, nominal exchange rate and foreign input price shocks are the most important shocks responsible for output fluctuations. Fig. 13 shows that money supply, external debts, productivity and foreign interest rate shocks are the most important shocks to GDP in Ghana. Fig. 14 indicates that money supply, productivity, external debt and commodity price shocks contribute to GDP swings in Kenya. Fig. 15 reveals that commodity price, productivity, external debt, and nominal exchange rate shocks are the important shocks affecting output variations in Malawi.

Moreover, Fig. 16 reveals that money supply, external debt, productivity, and foreign interest rate shocks influence GDP movements in Morocco. Fig. 17 presents that productivity, external debt, commodity price, and foreign interest rate shocks trigger output fluctuations in Nigeria. Evidence in Fig. 18 suggests that foreign interest rate, external debt, commodity price, and foreign input price shocks are responsible for output fluctuations in South Africa. Fig. 19 displays that commodity price, productivity, interest rate and foreign input price shocks influence output movements in Tunisia. In Uganda, as shown in Fig. 29, money supply, external debt, foreign input price, and commodity price shocks influence output variations. Lastly, Fig. 20 demonstrates that money supply, productivity, commodity price, and nominal exchange rate shocks induce output swings in Zambia.

The dominance of external debt shocks among the shocks indicate the effect of high debt service payment on African economies. The requirement to service debt decreases funds available for investment and consequently contracts output in African countries (see Fosu, 1999). Moreover, the significance of commodity price shocks illustrates the dependence of African countries on a narrow range of primary exports for their fiscal revenue and foreign exchange

earnings. Hence, commodity price shocks affect the budgetary framework and the exchange rate (see Spatafora and Samake, 2012; Iwayemi and Fowowe, 2012). Productivity and money supply shocks also account for much output variations in African countries. This shows the importance of supply shocks in African economies (see Jidoud, 2012).

2.9. Conclusion

This paper investigates the shocks generating macroeconomic fluctuations in African countries. An open economy monetary DSGE model was developed and estimated with the Bayesian technique for ten African countries. Generally, the findings indicate that the structural shocks that cause economic fluctuations are similar across African economies. Our variance decomposition analysis shows that both internal and external shocks are responsible for macroeconomic fluctuations in African countries. However, external shocks account for greater variations in output than internal shocks. Internal shocks only dominate in the fourth quarter. External shocks significantly influence output at longer horizons; in the eighth and sixteenth quarters. Among external shocks, the exchange rate and external debt shocks are the main sources of output fluctuations. Among internal shocks, money supply and productivity shocks also account for significant output fluctuations in African countries.

In terms of persistence, we find that external shocks are the most persistent shocks influencing output fluctuations. Listing specific shocks from high to low persistence, we find that external debt, exchange rate, foreign input price, foreign interest rate, and commodity price shocks have significant persistent effect on output variations in African countries. External shocks induce significant economic fluctuations in African countries under floating and fixed exchange rate regime. The similarity of shocks impacting floaters and non-floaters may be due to the fear of floating which induces the monetary authorities to use reserves to limit exchange rate fluctuations (see Calvo

and Reinhart, 2002). The result is similar to the findings by Flood and Rose (1995) and Maćkowiak (2007) and differs from the findings by Sissoko and Dibooğlu (2006) for African countries. Among internal shocks, we find productivity shocks to be the most persistent in affecting output. Other internal shocks that are found to be persistent in influencing output are money supply shocks and domestic interest rate.

The significance of external shocks in influencing economic fluctuations in African countries since the 1990s may be due to several factors. Since the early 1990s, African countries have implemented series of economic reforms ranging from privatization, commercialization to trade liberalization. There has also been improvement in democratic accountability and economic management. All these might have reduced the influence of internal shocks on economic fluctuations in African countries. Moreover, on the international front, African countries have become more open to trade and FDI and commodity prices have increased more than the previous decades. All these, on the other hand, have increased the significance of external shocks in influencing macroeconomic fluctuations in African countries (see Raddatz, 2008).

The results of the study have important policy implications for African economies. In view of the vulnerability of African economies to external shocks, it is imperative that African countries formulate appropriate exchange rate policies, devise sound external debt management, and use hedging instruments to insulate or mitigate the effects of external shocks. Since many of these economies rely heavily on very few, if not one, commodities for foreign exchange earnings, it is necessary that these countries pursue policies that promote industrial diversification. Another way in which African countries can create a buffer against adverse external shocks is to set up sovereign wealth funds (SWFs). These funds can be used to reduce macroeconomic volatility arising from commodity price boom-bust cycles. Establishment of SWFs with transparent rules of operation can be used to support balanced

fiscal positions and promote macroeconomic stability.

3.0 Appendices

Appendix 2.1: Impulse response functions

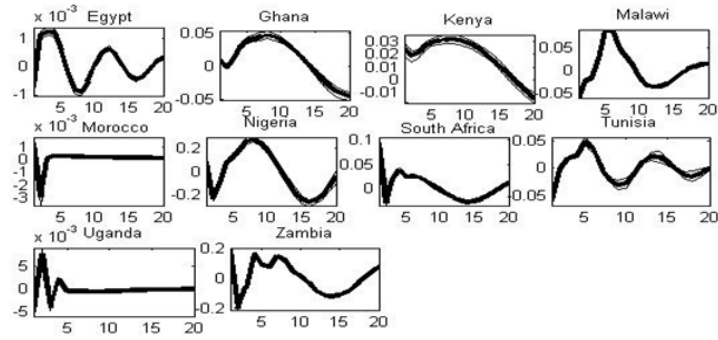


Fig1.Dynamic response of output to interest rate shocks

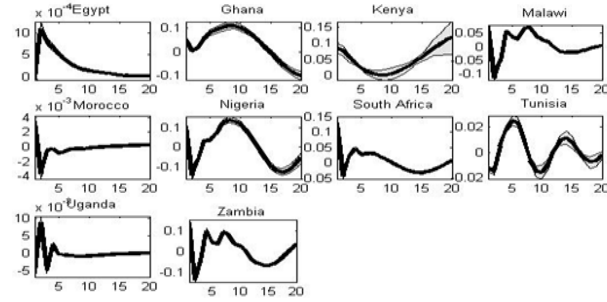


Fig. 2. Dynamic response of output to money supply shocks

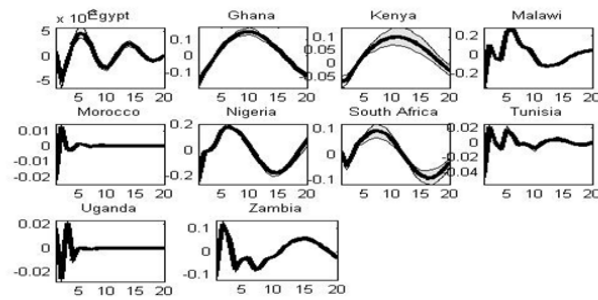


Fig.3. Dynamic response of output to nominal exchange rate shocks

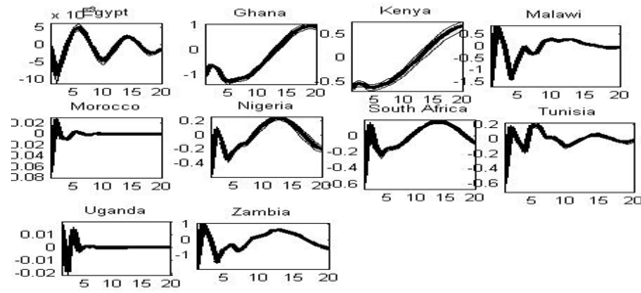


Fig.4. Dynamic response of output to external debt shocks

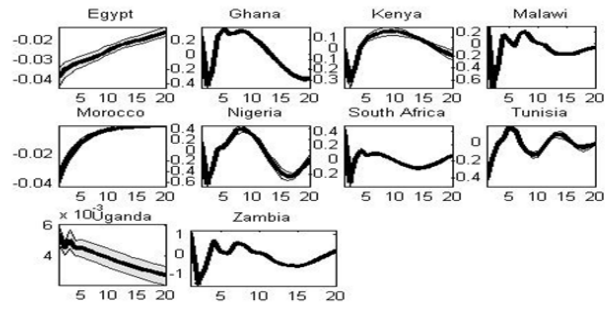


Fig.5. Dynamic response of output to productivity shocks

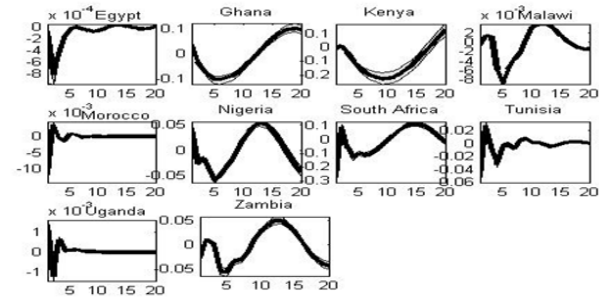


Fig.6. Dynamic response of output to commodity price shocks

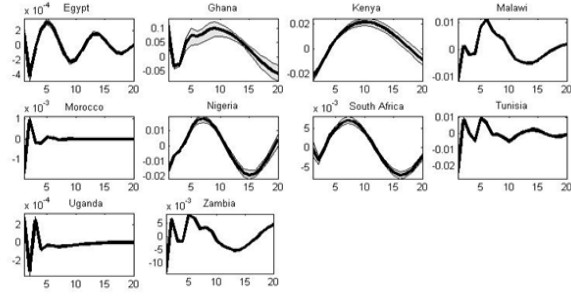


Fig.7. Dynamic response of output to foreign interest rate shocks

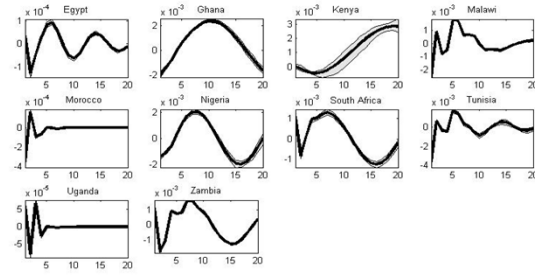


Fig.8. Dynamic response of output to foreign output shocks

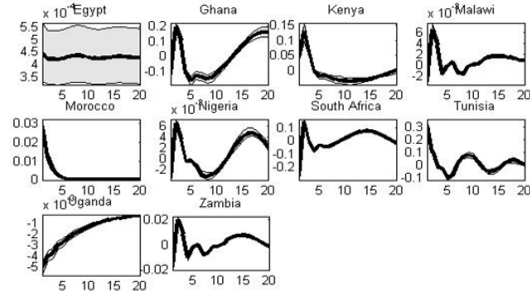


Fig.9. Dynamic response of output to foreign input price shocks

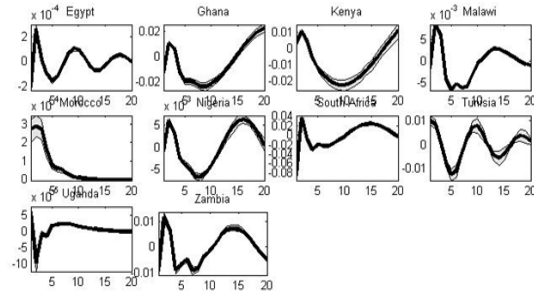


Fig.10. Dynamic response of output to foreign inflation shocks

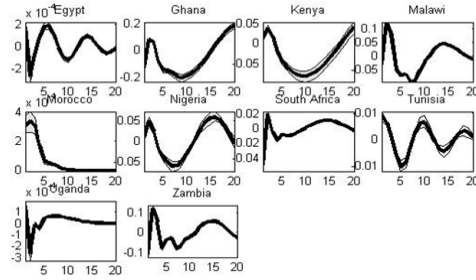


Fig.11. Dynamic response of output to real exchange rate shocks

Appendix 2.2: Historical decomposition of GDP

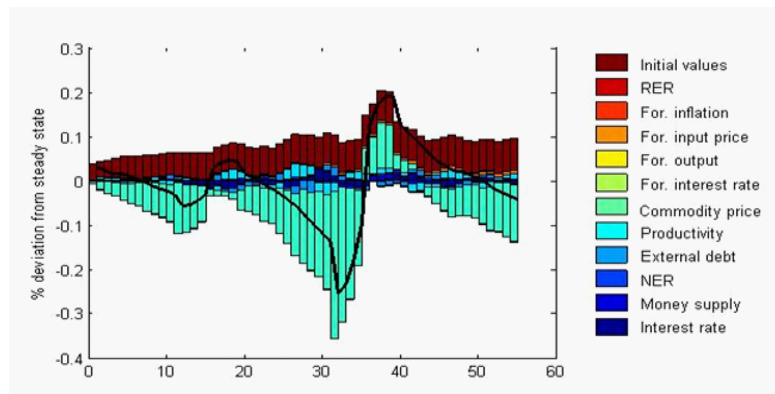


Fig. 12. Historical decomposition of GDP in Egypt

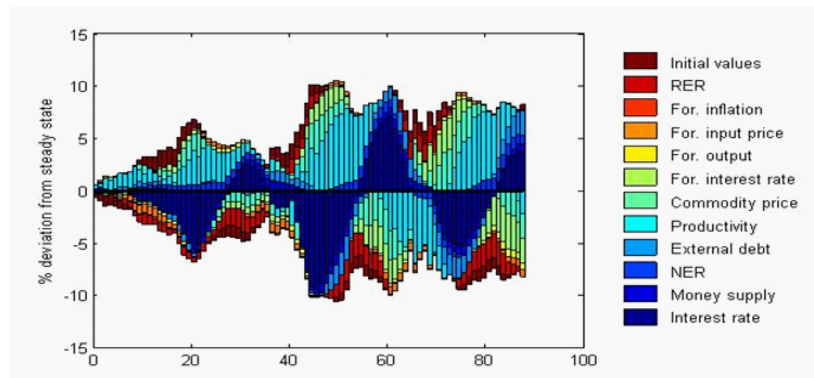


Fig. 13. Historical decomposition of GDP in Ghana

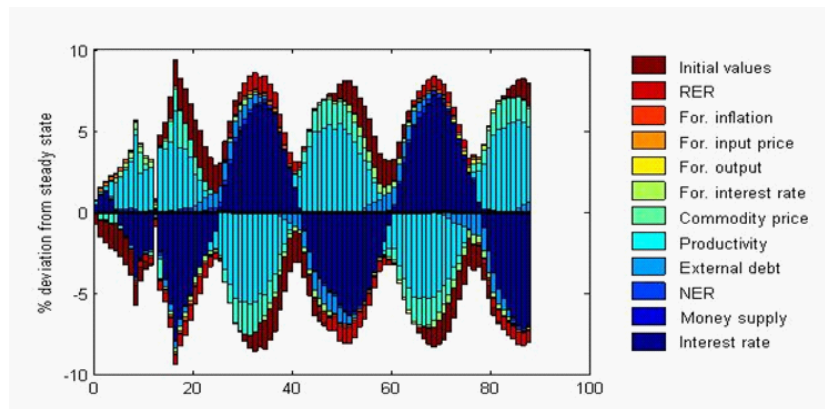


Fig. 14. Historical decomposition of GDP in Kenya

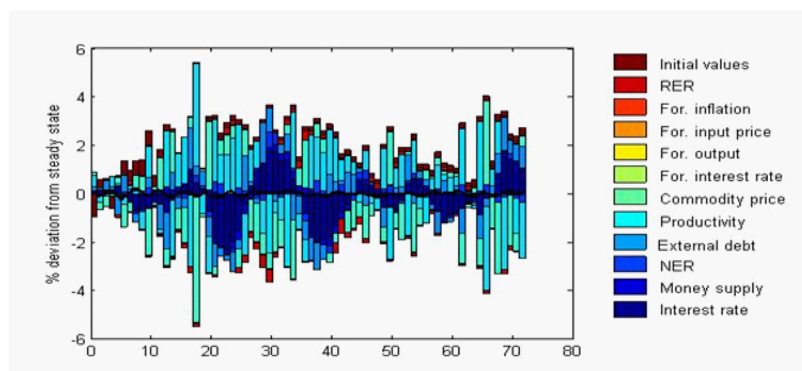


Fig. 15. Historical decomposition of GDP in Malawi

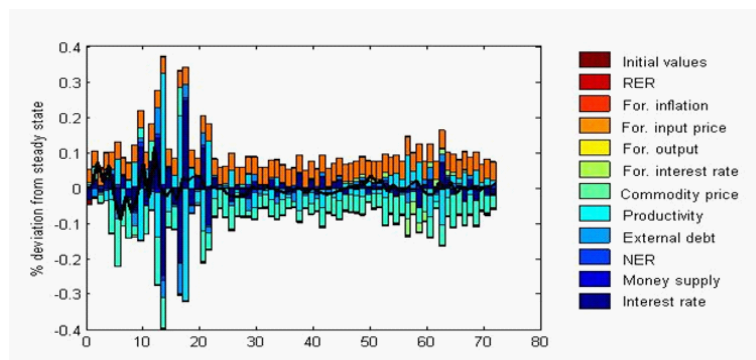


Fig. 16. Historical decomposition of GDP in Morocco

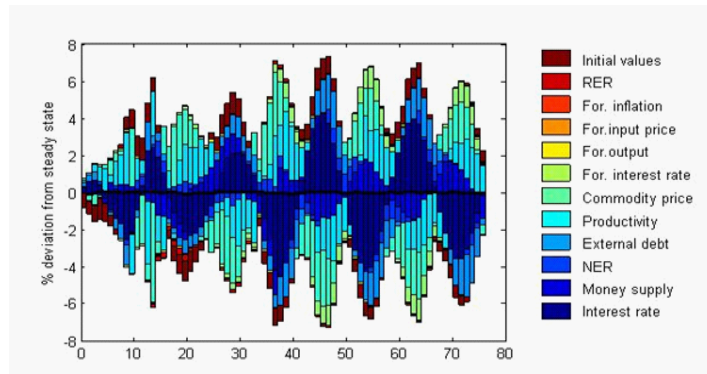


Fig. 17. Historical decomposition of GDP in Nigeria

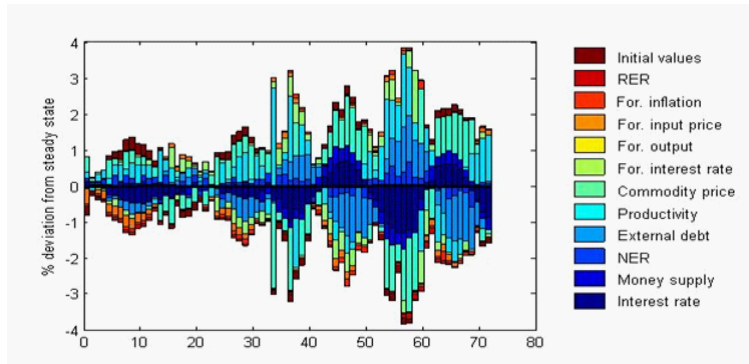


Fig. 18. Historical decomposition of GDP in South Africa

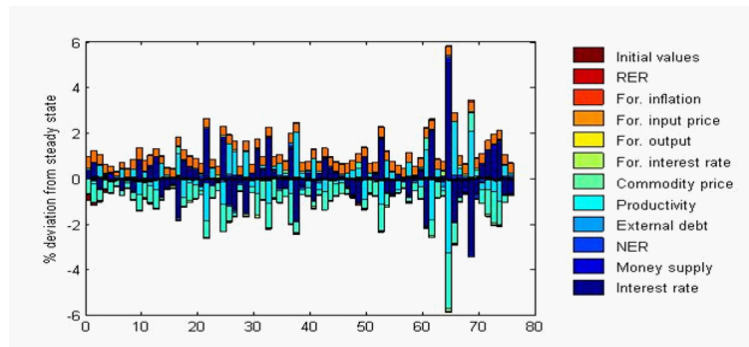


Fig. 19. Historical decomposition of GDP in Tunisia

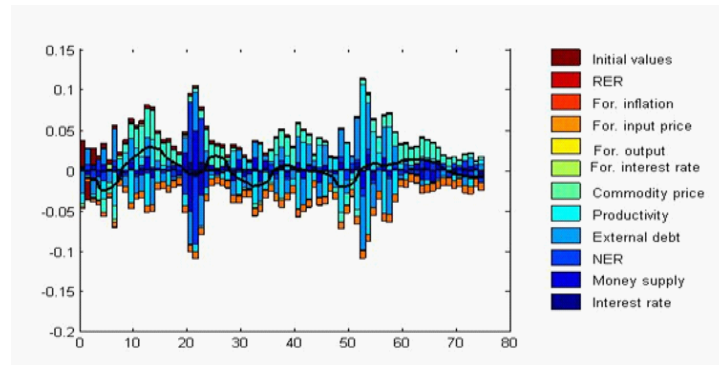


Fig. 20. Historical decomposition of GDP in Uganda

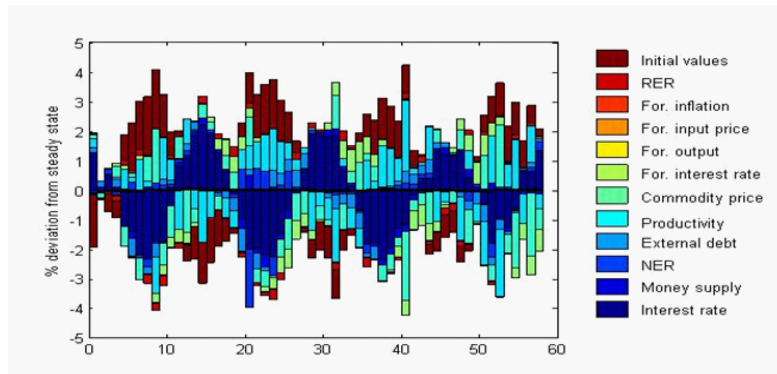


Fig. 21. Historical decomposition of GDP in Zambia

Appendix 2.3: Prior vs posterior

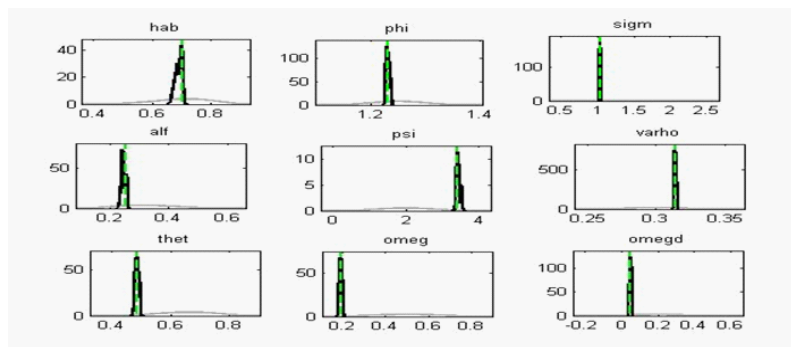


Fig. 22. Prior vs posterior

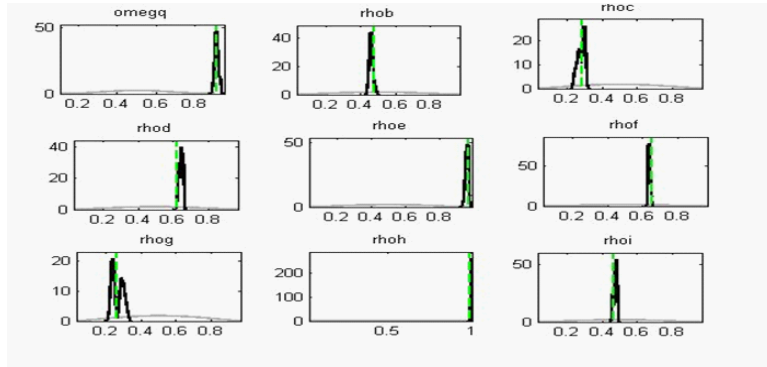


Fig. 23. Prior vs posterior

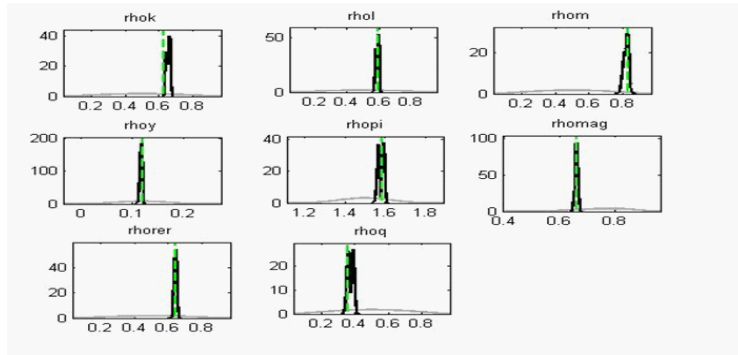


Fig. 24. Prior vs posterior

CHAPTER THREE

The Balance Sheet Channel in African Economies

Abstract

This study investigates the quantitative significance of the balance sheet channel in African economies. We formulate and estimate an open economy monetary DSGE model where entrepreneurs finance investment by issuing foreign currency-denominated debt. We estimate the model with Bayesian technique. Our Bayesian estimation shows that the balance sheet effects are empirically important in African economies. The marginal likelihood results clearly favour the model with financial imperfections. Moreover, the findings indicate that the balance sheet effect reduces the effectiveness of monetary policy, raises the sensitivity of the risk premium to external debt, and contracts output following exchange rate depreciation.

3.1. Introduction

Financial market imperfections have been identified as one of the amplifiers of relatively small shocks to the aggregate economy (see Bernanke et al., 1999; Christensen and Dib, 2008). In a small open economy with foreign currency-denominated debt, financial market frictions dampen the effect of devaluation via the balance sheet channel (see Céspedes et al., 2004; Bebczuk et al., 2006). This balance sheet effect is found to be responsible for declining output in East Asia and Latin America after a currency devaluation (see Krugman, 1999; Aguiar, 2005; Mulder et al. 2012). In addition, evidence suggests that balance sheet weaknesses arising from foreign currency debt triggered financial instability in emerging economies in the 1990s (Eichengreen and Hausmann, 1999; Bordo, et al., 2010).

The gap that this study seeks to fill is to evaluate the quantitative significance of the balance sheet channel in African economies. Given the inability

of African countries to borrow in their own currencies, the balance sheet effect might be an important channel for the propagation of shocks in African countries. Foreign currency-denominated debt exposes the balance sheets of firms and countries to exchange rate volatility. Moreover, due to financial market imperfections, African countries borrow at a premium. This makes their balance sheets equally vulnerable to foreign interest rate shocks. Despite the growing importance of the balance sheet channel, especially in developing countries, there has been dearth of empirical studies focusing on African countries. Rather, studies have focused on Asia and Latin America (e.g., Benavente et al., 2003; Bordo et al., 2010; Bleakley and Cowan, 2008).

This study is significant for the conduct of monetary policy particularly, the question of whether monetary policy should react to exchange rate fluctuations or not. Should the monetary authority implement tight or loose policy when the currency depreciates? The study is also important for the choice of exchange rate regime. While Cook (2004) advocates for a fixed exchange rate regime when the balance sheet effect is empirically important, Céspedes et al. (2004) argue for a flexible exchange rate system. In addition, the study is important for the conduct of fiscal operation. Findings show that exposure to foreign currency debt constrains the use of fiscal policy instruments to deal with economic shocks (see, Jeanne and Zettelmeyer, 2002). A depreciation of the exchange rate weakens the government net worth and limits its ability to borrow to finance public investment.

Our paper contributes to the existing literature in three respects. First, building on the works of Bernanke et al. (1999) and Céspedes et al. (2004), we formulate an open economy monetary DSGE model with financial market imperfection for African economies. Second, we investigate the quantitative significance of the balance sheet effect in the propagation of economic fluctuations in African economies. Third, to highlight the heterogeneity of each country, we use the Bayesian technique to estimate our model for each of the

nine (9) African economies.

The rest of the paper proceeds as follows. Section 2 reviews the existing literature on the balance sheet channel. Section 3 constructs the model for the study. Section 4 describes the data and presents the estimated results. Section 5 concludes and makes policy recommendations.

3.2. Review of literature

Empirical literature on the propagation of economic shocks has identified financial market imperfections as one of the factors responsible for amplification of relatively small shocks. Bernanke and Gertler (1989) and Carlstrom and Fuerst (1996) show that under asymmetric information where borrower's net worth determines the cost of capital, credit market frictions amplify economic fluctuations. Similarly, Bernanke et al. (1999) develop a financial accelerator framework where developments in the credit markets propagate and magnify shocks to the real economy. This works through the external finance premium which is determined by the net worth of the firms.

The financial accelerator model proposed by Bernanke et al. (1999) has been extended to an open economy to evaluate the role of financial frictions when a country debt is denominated in foreign currency. Aghion et al. (2000) show that in the presence of nominal rigidities, a currency depreciation leads to an increase in the firm's debt burden and a decline in profit and net worth. Consequently, this may constrain the firms' access to credit, reduce investment and lower output. In a related study, Aghion et al. (2004) provide evidence that corporate balance sheets play a significant role in the amplification of currency crises. The lower output brought about by the deterioration of firm's balance sheets will result in a fall in money demand, and thus lead to a currency depreciation. Céspedes et al. (2004) show that in an economy with foreign currency-denominated debt, a currency depreciation increases debt service payment and deteriorates the balance sheets of firms and banks.

Studies investigating the empirical significance of the balance sheet channel in emerging economies have reported mixed results. Forbes (2002) finds that following a currency depreciation, firms with foreign sale exposure have higher growth performance, while firms with higher debt ratio have lower growth performance in emerging economies. Carranza et al. (2003) find that for firms having dollar debt in Peru, real exchange rate depreciation leads to a decline in investment. Echeverry et al. (2003) find that firms with liability dollarization exhibit negative balance sheet effects during devaluation in Colombia. Pratap et al. (2003), Pratap and Urrutia (2004), and Aguiar (2005) find that exchange rate depreciation increases the debt burden and reduces investment in Mexico. Bordo et al. (2010) find evidence of balance sheet effect in a sample of firms in 45 countries. Mulder et al. (2012) find that corporate balance sheet and maturity mismatch play significant role in the amplification of Asian economic crises.

In contrast, a number of studies have found the balance sheet effect to be statistically insignificant in emerging economies. The competitive effect of exchange rate depreciation has been found to dominate the balance sheet effect. For instance, the competitive effect dominates when exchange rate depreciation increases the demand for a country's exports, decreases the domestic demand for imports, and increases the trade balance and output. Benavente et al. (2003) find that the competitive effect of currency depreciation dominates the balance sheet effect in Chile. Hence, currency depreciation leads to an expansion of investment. Similarly, Bonomo et al. (2003) find no evidence of the balance sheet effect in Brazil. Bleakley and Cowan (2008) find that competitive effect of depreciation dominates the balance sheet effect in 5 Latin American countries. The results indicate that firms with dollar-denominated debt do not reduce their investment after depreciation.

A related strand of literature employs macro data to assess the importance of the balance sheet channel in an open economy. For instance, Berganza et al.

(2004) find that devaluation increases the country's risk premium, reduces investment and output in emerging economies. Céspedes (2005) finds that the balance sheet effect has a negative impact on output in developed and emerging economies. The impact, however, depends on the level of external debt and financial deepness. Elekdag et al. (2006) find that the balance sheet channel magnified the impact of shocks during the Korean crises. Using a panel of 57 countries, Bebczuk et al. (2006) find that liability dollarization diminishes the expansionary effects of devaluation in the countries where external debt as a proportion of GDP is high.

Given the financial intermediary role of banks, some studies have focused on the role of banks' balance sheet in the propagation and magnification of shocks to the real economy. Choi and Cook (2004) and Chue and Cook (2008) show that exchange rate depreciation deteriorates the bank balance sheet, constraints credit supply and reduces economic activity in emerging economies. Céspedes (2005) finds that real exchange rate devaluation worsens bank balance sheets and negatively affects output in developing countries. Blejer et al. (2002) and Beck et al. (2006) show that through their balance sheet, the bank play active role in the propagation of terms of trade volatility in emerging economies.

Evidently, the empirical literature has been inconclusive on the quantitative significance of the balance sheet effect in an economy. The main determinant of the balance sheet effect appears to be the level of external debt and the level of financial development. High level of foreign currency-denominated debt reduces the expansionary effect exchange rate depreciation on the economy through the deterioration in the net worth and balance sheet of firms. This increases the cost of capital and debt service payment and contracts output. Similarly, low level of financial development exacerbates the impact of exchange rate shocks by further tightening the credit constraint.

3.3. The model

3.3.1 Households

There is a continuum of households that maximize utility subject to a standard budget constraint. Given the increasing holding of foreign currency in Africa by households as noted in Elkhafif (2002), we assume that households allocate a fraction of their real holdings between domestic and foreign currencies, so that $S_t M_t^* = \varrho M_t$, where S_t is the nominal exchange rate, M_t^* is foreign nominal money and M_t is domestic nominal money. Our representative household preferences is defined by the following utility function:

$$U_t = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left[\left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi \right] - \frac{N_t^{1+\psi}}{1+\psi} \quad (1)$$

where C_t , is the aggregate consumption, $\frac{M_t}{P_t}$ is the real balances, and N_t is units of labour. The parameter h measures the degree of habit formation. The parameter σ is the relative risk aversion coefficient ; $\beta \in (0, 1)$ is the discount factor; ψ is the inverse of the Frisch labour supply elasticity and ϕ represents interest rate elasticity of money demand.

The budget constraint is:

$$\begin{aligned} C_t + \left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} + \frac{B_t}{P_t} + \frac{Q_t K_t}{P_t} - \frac{S_t D_t^*}{P_t} &= \frac{W_t N_t}{P_t} + \left(1 + \frac{\varrho}{S_t} \right) \frac{M_{t-1}}{P_t} \\ &+ \frac{B_{t-1}}{P_t} (1 + r_{t-1} + p r_{t-1}) \\ &- \frac{S_t D_{t-1}^*}{P_t} (1 + r_t^d) \\ &+ Q_{t-1} (1 - \delta) K_{t-1} \end{aligned} \quad (2)$$

where D_t^* , is the ratio of external debt to the GDP, W_t , is the wage rate, Q_t , is the price of capital, r_t is the nominal interest rate, pr_t , is the risk premium, and r_t^d is the debt service payment. Households enter period t with domestic money holdings M_t , bonds B_t , and foreign debt D_t^* .

Households choose the path of C_t , M_t , N_t , B_t , and D_t and that maximize expected utility. The first order conditions are:

$$\frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi = \lambda_t \quad (3)$$

$$\begin{aligned} \frac{\phi}{1-\sigma} \left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left(1 + \frac{\varrho}{S_t} \right)^\phi \left(\frac{M_t}{P_t} \right)^{\phi-1} \frac{1}{P_t} &= \frac{\lambda_t}{P_t} \left(1 + \frac{\varrho}{S_t} \right) \\ &\quad - \frac{\beta \lambda_{t+1}}{P_{t+1}} \left(1 + \frac{\varrho}{S_t} \right) \end{aligned} \quad (4)$$

$$\frac{W_t}{P_t} = \frac{N_t^\psi}{\lambda_t} \quad (5)$$

$$\beta \lambda_{t+1} (1 + r_t + pr_t) \frac{P_t}{P_{t+1}} = \lambda_t \quad (6)$$

$$\beta \lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} (1 + r_t^d) = \lambda_t \frac{S_{t+1}}{P_t} \quad (7)$$

where equations (3) - (7) denote the derivatives of the utility function to its arguments.

We equate Eqs.(3) and (4) to derive the intertemporal consumption function which is further linearized to give the aggregate dynamic consumption equation:

$$\begin{aligned}\tilde{c}_t = & \frac{h(\sigma-1)}{\sigma_c}\tilde{c}_{t-1} + \frac{\sigma}{\sigma_c}E_t\tilde{c}_{t+1} + \frac{\phi}{\sigma_c}\tilde{m}_t - \frac{\phi}{\sigma_c}E_t\tilde{m}_{t+1} \\ & - \frac{\varrho\phi}{s_0\sigma_c}\tilde{s}_t + \frac{\varrho\phi}{s_0\sigma_c}E_t\tilde{s}_{t+1} - \frac{1}{\sigma_c}(\tilde{r}_t - E_t\tilde{\pi}_{t+1}) - \frac{1}{\sigma_c}pr_t\end{aligned}\quad (8)$$

where $\tilde{\cdot}$ denotes percentage deviation from the steady state and $\sigma_c = \sigma + h(\sigma - 1)$. Eq.(8) is the standard consumption equation showing that consumption depends on past and expected future consumption and real interest rate. Consumption also depends positively on current real balances and negatively on expected real balances (Castelnuovo, 2012).

In line with McCallum and Nelson (2000), our macro-balance for a small open economy is:

$$\tilde{y} = \gamma_c\tilde{c}_t + \gamma_x\tilde{x}_t - \gamma_z\tilde{z}_t + \varepsilon_{g,t} \quad (9)$$

where $\tilde{y}_t, \tilde{c}_t, \tilde{x}_t, \tilde{z}_t$ are percentage deviations of output, consumption, exports, and imports from their steady states respectively. $\gamma_c, \gamma_x, \gamma_z$ are steady state ratios of consumption, exports and imports to output. $\varepsilon_{g,t}$ is the government expenditure and investment shocks. Our net export function is given by:

$$\tilde{n}x_t = \gamma_{yf}\tilde{y}_t^f - \gamma_y\tilde{y}_t + \gamma_r\tilde{r}er_t \quad (10)$$

where $\tilde{y}_t^f$, γ_{yf} , γ_y , and γ_r , are the foreign output, elasticity of net export to foreign output, the elasticity of net export to domestic output, and sum of elasticity of substitution in production for home and abroad respectively. The real exchange rate is defined as $\tilde{r}er_t \equiv \tilde{s}_t + \tilde{p}_t^* - \tilde{p}_t$. Substituting Eq.(10) in Eq.(9) yields the expression:

$$\tilde{c}_t = \frac{1}{\gamma_c} (1 + \gamma_y) \tilde{y}_t - \frac{\gamma_r}{\gamma_c} \tilde{r} \tilde{e} r_t - \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_t^f \quad (11)$$

Substituting Eq.(11) into Eq.(8) yields a dynamic IS equation:

$$\begin{aligned} \tilde{y}_t = & \frac{h(\sigma - 1)}{\sigma_c} \tilde{y}_{t-1} + \frac{\sigma}{\sigma_c} E_t \tilde{y}_{t+1} - \frac{\gamma_c}{\sigma_c (1 + \gamma_y)} (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \\ & - \frac{\gamma_c}{\sigma_c (1 + \gamma_y)} \tilde{p} r_t + \frac{\phi \gamma_c}{\sigma_c (1 + \gamma_y)} \tilde{m}_t - \frac{\phi \gamma_c}{\sigma_c (1 + \gamma_y)} E_t \tilde{m}_{t+1} \\ & - \frac{\varrho \phi \gamma_c}{s_0 \sigma_c (1 + \gamma_y)} \tilde{s}_t + \frac{\varrho \phi \gamma_c}{s_0 \sigma_c (1 + \gamma_y)} E_t \tilde{s}_{t+1} \\ & - \frac{h(\sigma - 1) \gamma_r}{\sigma_c (1 + \gamma_y)} \tilde{r} \tilde{e} r_{t-1} + \frac{\gamma_r}{(1 + \gamma_y)} \tilde{r} \tilde{e} r_t - \frac{\sigma \gamma_r}{\sigma_c (1 + \gamma_y)} E_t \tilde{r} \tilde{e} r_{t+1} \\ & - \frac{h(\sigma - 1) \gamma_{yf}}{\sigma_c (1 + \gamma_y)} \tilde{y}_{t-1}^f + \frac{\gamma_{yf}}{(1 + \gamma_y)} \tilde{y}_t^f - \frac{\sigma \gamma_{yf}}{\sigma_c (1 + \gamma_y)} E_t \tilde{y}_{t+1}^f + \varepsilon_t^y \end{aligned} \quad (12)$$

We assume that the innovation ε_t^y follows a first-order autoregressive process as $\varepsilon_t^y = \rho_a \varepsilon_{t-1}^y + \mu_t^a$. Eq.(12) is an open economy IS equation where domestic output also depends on nominal and real exchange rate and foreign output. As a contribution, our dynamic IS equation equally features output as a function of external finance premium and as a function of lags and leads of real exchange rates and foreign outputs.

3.3.2 Firms

We adopt the hybrid, Calvo-style New Keynesian Phillips curve proposed by Galí and Gertler (1999) and Galí et al. (2001) of the following form:

$$\tilde{\pi}_t = \gamma_f E_t \tilde{\pi}_{t+1} + \gamma_b \tilde{\pi}_{t-1} + \lambda m c_t \quad (13)$$

where

$$\begin{aligned}\gamma_f &\equiv \beta\theta\{\theta + \omega[1 - \theta(1 - \beta)]\}^{-1}; \lambda \equiv (1 - \theta)(1 - \beta\theta)(1 - \omega)\xi \\ \gamma_b &\equiv \omega\{\theta + \omega[1 - \theta(1 - \beta)]\}^{-1}; \xi \equiv \frac{(1 - \alpha)}{1 + \alpha(\varepsilon - 1)}\{\theta + \omega[1 - \theta(1 - \beta)]\}^{-1}\end{aligned}$$

Similar to Smets and Wouters (2002), we assume a Leontief technology for labour and capital inputs. Labour and capital inputs are used in fixed proportion of output Y_t . The production function is written as:

$$Y_t = \min [A_t\varphi_K K_t^\alpha, A_t\varphi_N N_t^{1-\alpha}] \quad (14)$$

where A_t is technology shocks common to all firms, K_t is the units of capital, N_t is the units of labour, φ_K and φ_N are the proportion of capital and labour used in output production. The total cost is given as:

$$TC_t = w_t N_t + r_t K_t \quad (15)$$

where w_t is the real wage and r_t is the nominal interest rate. We can write the real marginal cost as:

$$mc_t = w_t \left(\frac{1}{(1 - \alpha)} \frac{1}{A_t\varphi_n} \left(\frac{Y_t}{A_t\varphi_n} \right)^{\frac{\alpha}{1-\alpha}} \right) + r_t \left(\frac{1}{\alpha} \frac{1}{A_t\varphi_k} \left(\frac{Y_t}{A_t\varphi_k} \right)^{\frac{1-\alpha}{\alpha}} \right) \quad (16)$$

Linearising Eq.(16) around the steady state and incorporating Eq.(11), we get the following relationship for the marginal cost:

$$\begin{aligned}\widetilde{mc}_t &= \{\kappa_a \widetilde{y}_t - \kappa_b \widetilde{y}_{t-1} + \kappa_c \widetilde{r} \widetilde{e} r_t + \kappa_d \widetilde{y}_{t-1}^f - \kappa_e \widetilde{r} \widetilde{e} r_t - \kappa_f \widetilde{y}_t^f \\ &\quad + \kappa_g \widetilde{m}_t + \kappa_h \widetilde{s}_t + \lambda_b \widetilde{r}_t - \lambda_d \widetilde{a}_t\end{aligned}\quad (17)$$

where

$$\begin{aligned}\kappa_a &= \left(\lambda_a(1 - \alpha) + \lambda_c - \frac{\lambda_a(1 - \alpha)^2}{\psi_m} + \frac{\lambda_a \sigma(1 - \alpha(1 + \gamma_b))}{\psi_m \gamma_c} \right) \\ \kappa_b &= \frac{\lambda_a h(1 - \alpha)(\sigma - 1)(1 + \gamma_y)}{\psi_m \gamma_c}; \kappa_c = \frac{\lambda_a h \gamma_r(1 - \alpha)(\sigma - 1)}{\psi_m \gamma_c} \\ \kappa_d &= \frac{\lambda_a h \gamma_{yf}(1 - \alpha)(\sigma - 1)}{\psi_m \gamma_c}; \kappa_e = \frac{\lambda_a \sigma \gamma_r(1 - \alpha)}{\psi_m \gamma_c}; \\ \kappa_f &= \frac{\lambda_a \sigma \gamma_{yf}(1 - \alpha)}{\psi_m \gamma_c}; \kappa_g = \frac{\lambda_a \phi(\alpha - 1)}{\psi_m}; \kappa_h = \frac{\lambda_a \phi \varrho(1 - \alpha)}{\psi_m}\end{aligned}$$

We can then insert Eq.(17) in Eq.(13) to get the following extended version of the New Keynesian Phillips curve (NKPC) of the following form:

$$\begin{aligned}\widetilde{\pi}_t &= \gamma_f E_t \widetilde{\pi}_{t+1} + \gamma_b \widetilde{\pi}_{t-1} + \lambda(\kappa_a \widetilde{y}_t - \kappa_b \widetilde{y}_{t-1} + \kappa_c \widetilde{r} \widetilde{e} r_t + \kappa_d \widetilde{y}_{t-1}^f \\ &\quad - \kappa_e \widetilde{r} \widetilde{e} r_{t-1} - \kappa_f \widetilde{y}_t^f + \kappa_g \widetilde{m}_t + \kappa_h \widetilde{s}_t + \lambda_b \widetilde{r}_t - \lambda_d \widetilde{a}_t) + \varepsilon_t^\pi\end{aligned}\quad (18)$$

The inflation disturbance is assumed to follow an AR(1) process: $\varepsilon_t^\pi = \rho_f \varepsilon_{t-1}^\pi + \mu_t^f$. In eq.(18), inflation depends positively on past and expected future inflation, past and current output, and the real balances. As our contribution, inflation also depends on nominal exchange and real exchange rates.

3.3.3 Exchange rate and external debt

We equate (6) and (7) to derive the uncovered interest parity condition (UIP) expression. We assume $\tilde{r}_t^d = \tilde{r}_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^*$. This implies that external debt service payment depends positively on the foreign interest rates and the level of external debt to GDP ratio, and negatively on commodity prices. The UIP expression is further linearised to yield:

$$\tilde{s}_t = E_t \tilde{s}_{t+1} - (\tilde{r}_t - \tilde{r}_t^f) - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^* + \varepsilon_t^{er} \quad (19)$$

where $\tilde{s}_t$ is nominal exchange rate, $(\tilde{r}_t - \tilde{r}_t^f)$ is interest rate differential, $\tilde{q}_t$ is commodity price, and $\tilde{d}_t^*$ is ratio of external debt to GDP. The innovation is assumed to follow an AR(1) process with an IID-Normal error term: $\varepsilon_t^{er} = \rho_d \varepsilon_{t-1}^{er} + \mu_t^d$. The coefficients ω_q and ω_d represent exchange rate elasticity with respect to commodity price and external debt respectively. Eq.(19) suggests a positive link between exchange rate and external debt to GDP ratio.

The dynamics of external debt depend on the current account balance and foreign debt service payment. Thus, the external debt to GDP ratio evolves according to the following equation:

$$\frac{\Delta D_t^*}{P_t Y_t} = \frac{Z_t - X_t}{Y_t} + (1 + r_{t-1}^d) d_{t-1}^* \quad (20)$$

where $\frac{D_t^*}{P_t Y_t}$ is the ratio of external debt to GDP, $\frac{Z_t - X_t}{Y_t}$ represents the current account balance by the private sector and r_t^d is the interest rate on external debt. The change in the debt ratio over time can then be written as:

$$\Delta d_t^* = (z_t - x_t) + d_{t-1}^* (1 + r_{t-1}^d - \pi_{t-1} - \Delta y_{t-1}) \quad (21)$$

Eq.(21) shows that the change in the external debt ratio is a positive function of net import and foreign interest rate. For example, a rise in net imports is financed by borrowing thereby increasing the existing level of debt. Similarly, an increase in interest rate on external debt increases the debt service payment and therefore puts upward pressure on the debt ratio. We linearize Eq.(21) and substitute Eq.(10) to derive the equation for the dynamics of the external debt ratio. Note that from Eq.(19) we have $r_t^d = r_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^*$ and we substitute $(z_t - x_t) = \tilde{n}x_t = \gamma_{yf} \tilde{y}_t^f - \gamma_y \tilde{y}_t + \gamma_r \tilde{r} \tilde{e}_t$. Using this fact, we have the debt equation as follows:

$$\tilde{d}_t^* = \beta_a \tilde{d}_{t-1}^* + \beta_b \tilde{r}_t^f - \beta_c \tilde{y}_t - \beta_d \Delta \tilde{y}_t - \beta_e \tilde{q}_t - \beta_f \tilde{r} \tilde{e}_t - \beta_g \tilde{y}_t^f + \varepsilon_t^e \quad (22)$$

where

$$\begin{aligned} \beta_a &= \frac{(1 + g_0) + r_0}{\gamma_d}; \beta_b = \frac{r_0}{\gamma_d}; \beta_c = \frac{(1 + g_0) \gamma_y}{d_0^* \gamma_d}; \\ \beta_d &= \frac{r_0 + (1 + g_0)^3}{(1 + g_0)}; ; \beta_e = \frac{r_0 \omega_q}{\gamma_d}; \\ \beta_f &= \frac{(1 + g_0) \gamma_r}{d_0^* \gamma_d}; \beta_g = \frac{(1 + g_0) \gamma_{yf}}{d_0^* \gamma_d}; \gamma_d = (1 + g_0)^2 - r_0 \omega_d \end{aligned}$$

The external debt shock follows an AR(1) process: $\varepsilon_t^e = \rho_e \varepsilon_{t-1}^e + \mu_t^e$. Eq.(22) describes the external debt evolution where g_0 , and r_0 , represent average

growth rate and average interest rate respectively. There is a negative relation between external debt and commodity prices. Positive commodity price shocks generate more revenue for the government to payoff existing external debt. In addition, external debt to GDP depends negatively on domestic output and positively on foreign output. This indicates that a fall in domestic output increases external debt to GDP while a rise in foreign output leads to a rise in external debt to GDP.

3.3.4 The entrepreneur

Similar to Céspedes et al.(2004) and Cook (2004), the entrepreneur's net worth is defined as assets minus liabilities. Hence, net worth is written as:

$$NW_t = Y_t - r_t^f S_t D_t^* \quad (23)$$

where NW_t is the net worth, Y_t is output, r_t^f is the foreign interest rate, S_t is the exchange rate, and D_t^* is foreign currency debt respectively. From Eq.(23), a rise in the exchange rate (depreciation) reduces the net worth of entrepreneur. This underlines the susceptibility of the firms' balance sheet to exchange rate fluctuations. Eq.(23) is linearized to give:

$$\widetilde{nw}_t = \varphi_y \widetilde{y}_t - \varphi_r \left(\widetilde{r}_t^f + \widetilde{s}_t + \widetilde{d}_t^* \right) \quad (24)$$

where $\varphi_y = \frac{Y_0}{NW_0}$ and $\varphi_r = \frac{r_0^f S_0 D_0^*}{NW_0}$. The denotation $\frac{Y_0}{NW_0}$ represents the steady state ratio of average output to the net worth of the entrepreneur while $\frac{r_0^f S_0 D_0^*}{NW_0}$ is the steady state leverage ratio times the steady state interest rate. Similar to Elekdag et al.(2006) and Elekdag and Tchakarov (2007), the external finance premium can be written as an increasing function of the domestic currency value of debt relative to net worth:

$$E_t(1 + pr_t) = \left(\frac{S_t D_t^*}{NW_t} \right)^{\psi_p} \quad (25)$$

where $E_t(1 + pr_t)$ is the expected external financing premium and ψ_p is the elasticity of external finance premium with respect to the firm's leverage ratio. A depreciation of the exchange rate will increase the leverage ratio, which in turn increases the external finance premium of the firm. This hinders investment and magnifies the effects of exchange rate shocks on output. Eq.(25) is log-linearized to give:

$$\tilde{p}r_t = \psi_p \left(\tilde{s}_t + \tilde{d}_t^* - \tilde{nw}_t \right) \quad (26)$$

Substituting Eq.(24) in Eq.(25), we derive the log-linearized equation for the external finance premium

$$\tilde{p}r_t = \psi_p (1 + \varphi_r) \tilde{s}_t + \psi_p (1 + \varphi_r) \tilde{d}_t^* + \psi_p \varphi_r \tilde{r}_t^f - \psi_p \varphi_y \tilde{y}_t \quad (27)$$

Eq.(27) indicates that the external finance premium is positively related to foreign interest rate, exchange rate, and foreign currency debt, but negatively related to output. A rise in the foreign interest rate can depreciate the exchange rate, increase the debt service payment, worsen the net worth and increase the external finance premium. Similarly, an increase in the external debt may lower the net worth and increase the external finance premium. Lastly, an output expansion increases the net worth and reduces the external finance premium.

3.3.5 Money market

We equate Eqs.(4) and (6) and linearize to derive the money market equation. We then substitute Eqs.(10) and (11) to get the following money market equation:

$$\tilde{r}_t = \frac{\alpha_r (1 + \gamma_y)}{\gamma_c} \tilde{y}_t - \alpha_r \tilde{m}_t - pr_t + \alpha_s \tilde{s}_t - \varrho E_t \tilde{s}_{t+1} - \frac{\alpha_r \gamma_r}{\gamma_c} \tilde{r}er_t - \frac{\alpha_r \gamma_{yf}}{\gamma_c} \tilde{y}_t^f + \varepsilon_t^b, \quad (28)$$

where

$$\alpha_r = \frac{\phi s_0 (1 + r_0 + pr_0)}{m_0 (1 - \sigma)}; \alpha_s = s_0 (1 + r_0 + pr_0).$$

The interest rate shocks follow an AR(1) process: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \mu_t^b$. Eq.(28) describes the money market equation. Interest rate depends positively on real output and negatively on real balances. Also, interest rate depends positively on current exchange rate and negatively on expected future exchange rate. Our money market equation also indicates that interest rate depends negatively on real exchange rate, foreign output, and risk premium.

3.3.6 Monetary policy

Given monetary aggregate targeting in Africa, we follow Muhanji and Ojah (2011) in the specification of our monetary policy reaction function. Money supply is driven by the inflation gap, the output gap and commodity price gap. We also include the real exchange rate in our monetary aggregate Taylor-type rule, which has been found to be empirically relevant for emerging market economies (see, Mohanty and Klau, 2004). The monetary aggregate Taylor-type rule is therefore:

$$\tilde{m}_t = \rho_m \tilde{m}_{t-1} - (1 - \rho_m) [\rho_\pi \tilde{\pi}_t + \rho_y \tilde{y}_t + \rho_{rer} \tilde{r}er_t + \rho_q \tilde{q}_t] + \varepsilon_t^m, \quad (29)$$

where all variables are in percentage deviations from the steady states. $\tilde{m}_t$ is monetary aggregate, $\tilde{\pi}_t$ is inflation rate; $\tilde{y}_t$ is output gap; $\tilde{rer}_t$ is the real exchange rate gap, $\tilde{q}_t$ is the commodity price gap. The uncorrelated monetary disturbance follows an AR(1) process: $\varepsilon_t^m = \rho_c \varepsilon_{t-1}^r + \mu_t^c$. The parameter ρ_m is the policy rate smoothing, ρ_π is policy response to inflation gap, ρ_y is policy response to output gap, ρ_{rer} is policy response to real exchange rate shocks, and ρ_q is policy response to commodity price shocks.

The equations to be estimated are summarised as follows:

$$\begin{aligned}
\tilde{y}_t = & \frac{h(\sigma-1)}{\sigma_c} \tilde{y}_{t-1} + \frac{\sigma}{\sigma_c} E_t \tilde{y}_{t+1} - \frac{\gamma_c}{\sigma_c(1+\gamma_y)} (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \\
& - \frac{\gamma_c}{\sigma_c(1+\gamma_y)} \tilde{p}r_t + \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)} \tilde{m}_t - \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)} E_t \tilde{m}_{t+1} \quad (30) \\
& - \frac{\varrho\phi\gamma_c}{s_0\sigma_c(1+\gamma_y)} \tilde{s}_t + \frac{\varrho\phi\gamma_c}{s_0\sigma_c(1+\gamma_y)} E_t \tilde{s}_{t+1} \\
& - \frac{h(\sigma-1)\gamma_r}{\sigma_c(1+\gamma_y)} \tilde{r}er_{t-1} + \frac{\gamma_r}{(1+\gamma_y)} \tilde{r}er_t - \frac{\sigma\gamma_r}{\sigma_c(1+\gamma_y)} E_t \tilde{r}er_{t+1} \\
& - \frac{h(\sigma-1)\gamma_{yf}}{\sigma_c(1+\gamma_y)} \tilde{y}_{t-1}^f + \frac{\gamma_{yf}}{(1+\gamma_y)} \tilde{y}_t^f - \frac{\sigma\gamma_{yf}}{\sigma_c(1+\gamma_y)} E_t \tilde{y}_{t+1}^f + \varepsilon_t^y
\end{aligned}$$

$$\begin{aligned}
\tilde{r}_t = & \frac{\alpha_r(1+\gamma_y)}{\gamma_c} \tilde{y}_t - \alpha_r \tilde{m}_t + \alpha_s \tilde{s}_t - \varrho E_t \tilde{s}_{t+1} - \tilde{p}r_t \\
& - \frac{\alpha_r\gamma_r}{\gamma_c} \tilde{r}er_t - \frac{\alpha_r\gamma_{yf}}{\gamma_c} \tilde{y}_t^f + \varepsilon_t^b \quad (31)
\end{aligned}$$

$$\begin{aligned}
\tilde{m}_t = & \rho_m \tilde{m}_{t-1} - (1-\rho_m)[\rho_\pi \tilde{\pi}_{t-1} + \rho_y \tilde{y}_{t-1} + \rho_{rer} \tilde{r}er_{t-1} + \rho_q \tilde{q}_{t-1}] \\
& + \varepsilon_t^c \quad (32)
\end{aligned}$$

$$\tilde{p}r_t = \psi_p(1+\varphi_r)\tilde{s}_t + \psi_p(1+\varphi_r)\tilde{d}_t^* + \psi_p\varphi_r\tilde{r}_t^f - \psi_p\varphi_y\tilde{y}_t \quad (33)$$

$$\tilde{s}_t = E_t \tilde{s}_{t+1} - (\tilde{r}_t - \tilde{r}_t^f) - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^* + \varepsilon_t^d \quad (34)$$

$$\tilde{d}_t^* = \beta_a \tilde{d}_{t-1}^* + \beta_b \tilde{r}_t^f - \beta_c \tilde{y}_t - \beta_d \Delta \tilde{y}_t - \beta_e \tilde{q}_t - \beta_f \tilde{r}er_t - \beta_g \tilde{y}_t^f + \varepsilon_t^e \quad (35)$$

$$\tilde{\pi}_t = \gamma_f E_t \tilde{\pi}_{t+1} + \gamma_b \tilde{\pi}_{t-1} + \lambda m c_t \quad (36)$$

$$\begin{aligned}
m c_t = & \{\kappa_a \tilde{y}_t - \kappa_b \tilde{y}_{t-1} + \kappa_c \tilde{r}er_t + \kappa_d \tilde{y}_{t-1}^f - \kappa_e \tilde{r}er_t - \kappa_f \tilde{y}_t^f \\
& + \kappa_g \tilde{m}_t + \kappa_h \tilde{s}_t + \lambda_b \tilde{r}_t - \lambda_d \tilde{a}_t
\end{aligned} \quad (37)$$

The structural shock processes in the model are given as:

$$\tilde{\xi}_t = \rho_\xi \tilde{\xi}_t + \varepsilon_{\xi,t}; \quad \varepsilon_{\xi,t} \sim N(0, \sigma_\xi^2) \quad (38)$$

where

$$\varepsilon_{\xi,t} = \left[\widetilde{q}_t, \widetilde{r}_t^f, \widetilde{y}_t^f, \widetilde{inp}_t, \widetilde{f \inf l}_t \right]$$

3.4. Data and estimation

3.4.1 Data source and treatment

Data for the study were obtained from the International Financial statistics (IFS), the World Bank, Central Bank database of the sample countries. The model is estimated with quarterly time series data on fourteen macroeconomic variables in nine (9) African countries for the period 1990:1–2011:4. Due to data availability, the sample size differs from one country to the other. For Egypt, 1998:2 – 2011:4, Ghana, 1990:1 – 2011:4, Kenya, 1990:1 – 2011: 4; Malawi, 1990:1 – 2007:4; Morocco, 1995:4 – 2011:4; Nigeria, 1990:1 – 2008:4; South Africa, 1994:1 – 2011:4; Uganda, 1993:2 – 2011:4 ; and Zambia, 1997: 1 – 2011:3.

The foreign interest rate, real foreign output, foreign inflation, and price of foreign inputs are proxied by LIBOR, US real GDP, US consumer price index and US producer price index for manufactured goods respectively. The data were taken from Federal Reserve Bank of St. Louis. The commodity price index was taken from World Bank pink sheet. Real commodity price is derived by deflating the nominal commodity price with the US consumer price index. Due to non-availability of reliable quarterly GDP data for Malawi and Nigeria, we use industrial output for the two countries.

3.4.2 Prior distribution of the parameters and calibration

In line with the Bayesian literature, we estimate the model by forming priors distributions. Similar to Smets and Wouters(2007), the persistence of the AR(1) processes is assumed to be beta distributed with mean 0.5 and standard deviation 0.2. The standard errors of the shocks are assumed to be distributed according to inverse-gamma distribution with a mean of 0.1 and

two degrees of freedom. As in García and González (2013), we use the same prior values for all the countries in our sample. This allows the data to reveal the degree of fit of these values to the realities of the countries. However, the sample draws for the convergence of Metropolis-Hastings (M-H) algorithm differ among countries⁴. The countries converge at different M-H draws.

Similar to Castelnovo (2012), we assume the habit parameter h to be a beta distribution with a mean 0.7 and standard deviation 0.1 and money-interest rate elasticity ϕ to be a beta distribution with mean 0.2 and standard error 0.05. The degree of price stickiness θ and price indexation ω are assumed to be beta distributed with mean 0.65 and 0.5 and standard errors 0.1 and 0.15 respectively. Following Elkhafif (2002), the parameter for currency substitution ϱ is assumed to be a beta distribution with mean 0.3 and standard deviation of 0.14. Our prior for external premium elasticity $\varphi_p = 0.05$. This is similar to the estimates by Christensen and Dib (2008).

Finally, the monetary policy reaction function parameters follow the Taylor's rule. The long run reaction to output (ρ_y) and inflation (ρ_π) are assumed to be a normal distribution with mean 0.12 and 1.5 and standard error 0.05 and 0.25 respectively. The monetary smoothing parameter ρ_m is assumed to be a beta distribution with a mean 0.75 and standard error 0.1. Lastly, the monetary policy function parameters to commodity price shocks (ρ_q) and real exchange (ρ_{rer}) rate is assumed to be a beta mean 0.5 and standard error 0.1 each.

Some parameters are calibrated for the study. The model calibration is summarized in Table 1. The calibration of α , σ , and ψ , comes from Smets and Wouters (2007). The value chosen for β is standard in the literature. The calibration for ε comes from Castelnovo (2012). The values chosen for γ_y ,

⁴The M-H algorithm draws are: Egypt (100,000), Ghana, (100,000), Kenya, (50,000), Malawi, (50,000), Morocco, (2,000), Nigeria, (100,000), South Africa, (100,000), Tunisia, (20,000), Uganda, (100,000), and Zambia, (5,000)

γ_r , γ_c , and $\gamma_{yf} = 0.25$ come from McCallum and Nelson (2000).

Table 3.1: Calibration of parameter

$\beta = 0.99$	$\varepsilon = 6$
$\sigma = 1.5$	$\alpha = 0.33$
$\gamma_y = 0.25$	$\gamma_c = 0.58$
$\gamma_r = 0.66$	$\gamma_{yf} = 0.25$
$\gamma_{ls} = 0.5$	

3.4.3 Empirical results

Table 2A and 2B present the posterior mean estimates along with the [5th, 95th] posterior percentile of the estimated structural parameters. We contrast the model with financial frictions (FA) where the elasticity of external finance premium $\varphi_p = 0.05$ with model without the financial frictions (NFA) where there is no external finance premium, $\varphi_p = 0.00$. Firstly, the elasticity of external finance premium φ_p is significantly different from zero for all the countries. This indicates the statistical significance of the financial accelerator channel in African countries. This is similar to the findings by Elekdag et al. (2006) for South Korea. Moreover, this implies that African economies are vulnerable to shocks affecting the aggregate balance sheets. A negative shock that depreciates the exchange rate may deteriorate the net worth and worsen the balance sheets of firms. Consequently, this increases the cost of borrowing, lowers investment and contracts output.

Based on the standard for model comparison in the Bayesian literature (see Coop, 2007; von Heideken, 2009; Castelnovo, 2012), we compare the marginal likelihoods between the models with and without financial frictions. The ratio of the marginal likelihoods (the posterior odds ratio) clearly favours the model with financial frictions. The marginal likelihood estimates for the model with financial frictions are higher than for the model without in seven

African countries. There is a strong evidence of balance sheet channel in Ghana, Kenya, Malawi, Nigeria, South Africa, Uganda, and Zambia. In contrast, the marginal likelihood estimates do not favour model with financial imperfection in Egypt and Morocco.

The presence of the balance sheet channel indicates that African economies are vulnerable to exchange rate and foreign interest rate shocks. Given their large exposure to foreign currency-denominated debt (see Sy, 2013), exchange rate depreciation will worsen their balance sheets, increase the risk premium and cost of debt service and consequently lower output. Through the balance sheet channel, exchange rate depreciation may contract rather than expand output in African economies. As observed by Bebczuk et al. (2006), liability dollarization dampens the expansionary effect of exchange rate depreciation in highly indebted countries. The policy implication is that purely floating exchange rate regime may not be optimal for African countries with foreign currency-denominated debt.

Importantly, our results show that the balance sheet effect weakens the impact of money in business cycle fluctuations. The parameter estimates for the preferences for non-separability between consumption and real balances (ϕ) are lower under the model with financial accelerator. This suggests that the balance sheet effect reduces the effectiveness of monetary policy transmission in African countries. This may be due to the existence of foreign currency-denominated debt which makes the impact of monetary policy on the exchange rate to further exacerbate the debt problem thereby reducing the effectiveness of monetary policy. This is in line with the findings by Kamber and Thoenissen (2012). The authors find that the output response to monetary policy shocks is attenuated by the presence of financial imperfection. A restrictive monetary policy seems to be the optimal response when debt is denominated in foreign currency (see Aghion et al. 2001).

Moreover, the estimates indicate that the response of sovereign risk to exter-

nal debt (ω_d) are higher under the model with financial frictions. This shows that the net worth of the country significantly affects its cost of borrowing. When a country debt is denominated in foreign currency, a depreciation of the exchange rate worsens the country's balance sheet, increases the cost of debt service and the country's risk premium. Consequently, this increases the level of external debt and further increases the risk premium. This is in line with the findings by Berganza et al. (2004) and Korinek (2011) that exchange rate depreciation increases a country's risk premium when the balance sheet effect is significant. The policy implication of this is that African countries should deepen their domestic bond markets so as to reduce their exposure to foreign currency-denominated debt.

In a manner similar to Christensen and Dib (2008), we examine the estimates of the monetary policy parameters especially the coefficient that measures the response of monetary policy to deviation of output (ρ_y). This is to check whether the monetary authority in Africa countries react more aggressively under the model with financial accelerator mechanism given its propensity to amplify shocks. We find this not to be robust for African countries as only three countries' monetary authorities react more aggressively to output deviation under the financial accelerator model. This may imply that the monetary authorities prefer to react more to other variables that affect the balance sheet of firms as shown in their response to real exchange rate deviation. In view of the muted effect of monetary policy on output under the model with financial accelerator mechanism, the monetary authority should respond more aggressively to exchange rate fluctuations.

Given the relative contribution of commodity price shocks to exchange rate and macroeconomic fluctuations in African countries (see Kose and Riezman, 2001; 2003; Collier, 2007; Arezki et al., 2012), we examine the response of monetary policy to deviation of commodity prices in African countries. The estimated value of (ρ_q), the coefficient that measures monetary policy

response to deviation of commodity prices appear to be more aggressive under the model with financial accelerator in six out of the nine African countries. This implies that the monetary authorities in Africa attempt to minimize the adverse effect of commodity price fluctuations on the the real exchange rate. This is similar to the results by García and González (2013) for commodity exporting countries.

On the remaining monetary policy rule, we find evidence of strong monetary policy reaction to inflation in all the countries. This is in line with the findings by Smets and Wouters (2007). The estimates range from 1.25 in Nigeria to 1.67 in Egypt. Our estimates also indicate a considerable degree of policy smoothing ranging from 0.76 in Zambia to 0.98 in Egypt. We also find that monetary authorities in Africa generally react to real exchange rate movements. This is with a view to reducing the impact of real exchange rate fluctuations on inflation and growth volatility. This is in line with the findings by García and González (2013) that monetary authority in emerging economies react to real exchange rate movement to reduce its destabilizing effect.

Regarding the other structural parameters, there is evidence of habit formation in consumption in all the countries. The estimates of habit parameter, h , range from 0.55 in Malawi to 0.85 in Egypt. There is evidence of foreign currency holding in all African countries as the estimates of the parameter for foreign currency holding, ϱ , range from 0.29 in Malawi and Zambia to 0.32 in Egypt. This is close to the estimates by Elkhafif (2002). Turning to Calvo pricing, the mean of the degree of price stickiness, θ , ranges from 0.56 in South Africa and Zambia to 0.6 in Ghana and Malawi. This indicates that prices are re-optimized within every 2-3 quarters. This is similar to the estimates by Steinbach et al. (2009) for South Africa. The degree of price indexation, ω , ranges from 0.49 in Morocco to 0.70 in Malawi and South Africa indicating that average of half to to two-third of the price setters are

backward looking.

Table 3.2A: Prior and Posterior Distribution of Structural Parameters

Par.	Distr.	Egypt		Ghana		Kenya		Malawi		Morocco	
		Prior		Posterior		Posterior		Posterior		Posterior	
		FA	NFA	FA	NFA	FA	NFA	FA	NFA	FA	NFA
		Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean
		(std)									
h	Beta	0.70 (0.10)	0.85 (0.00)	0.87 (0.01)	0.74 (0.00)	0.64 (0.00)	0.75 (0.00)	0.55 (0.00)	0.67 (0.00)	0.74 (0.00)	0.58 (0.00)
ϕ	Normal	0.2 (0.05)	0.06 (0.00)	1.15 (0.00)	1.22 (0.00)	0.04 (0.00)	1.26 (0.00)	0.03 (0.00)	1.19 (0.00)	0.06 (0.00)	1.23 (0.00)
ϱ	Beta	0.30 (0.02)	0.32 (0.00)	0.31 (0.00)	0.27 (0.00)	0.30 (0.00)	0.29 (0.00)	0.29 (0.00)	0.29 (0.00)	0.30 (0.00)	0.32 (0.00)
ψ_p	Beta	0.05 (0.02)	0.04 (0.00)	—	—	0.06 (0.00)	—	0.06 (0.00)	—	0.05 (0.00)	—
θ	Beta	0.65 (0.10)	0.59 (0.00)	0.51 (0.01)	0.68 (0.00)	0.62 (0.00)	0.71 (0.00)	0.68 (0.00)	0.67 (0.00)	0.67 (0.00)	0.54 (0.01)
ω	Beta	0.5 (0.15)	0.56 (0.01)	0.68 (0.01)	0.57 (0.00)	0.65 (0.00)	0.49 (0.00)	0.70 (0.00)	0.47 (0.00)	0.49 (0.00)	0.60 (0.01)
ω_d	Beta	0.2 (0.15)	0.01 (0.00)	0.34 (0.01)	0.20 (0.00)	0.28 (0.00)	0.06 (0.00)	0.39 (0.00)	0.09 (0.00)	0.36 (0.00)	0.10 (0.01)
ω_q	Beta	0.50 (0.15)	0.45 (0.00)	0.04 (0.01)	0.48 (0.00)	0.42 (0.00)	0.44 (0.00)	0.46 (0.00)	0.34 (0.00)	0.56 (0.00)	0.42 (0.01)
ρ_y	Normal	0.12 (0.05)	0.09 (0.00)	0.10 (0.00)	0.13 (0.00)	0.07 (0.00)	0.16 (0.00)	0.13 (0.00)	0.12 (0.00)	0.13 (0.00)	0.10 (0.00)
ρ_π	Normal	1.50 (0.125)	1.67 (0.00)	1.52 (0.01)	1.50 (0.00)	1.47 (0.00)	1.50 (0.00)	1.38 (0.00)	1.50 (0.00)	1.48 (0.00)	1.63 (0.01)
ρ_{mag}	Beta	0.75 (0.10)	0.98 (0.00)	0.65 (0.01)	0.81 (0.00)	0.82 (0.00)	0.85 (0.00)	0.89 (0.00)	0.76 (0.00)	0.75 (0.00)	0.89 (0.01)
ρ_{rer}	Beta	0.50 (0.10)	0.79 (0.01)	0.78 (0.03)	0.39 (0.00)	0.49 (0.00)	0.72 (0.00)	0.57 (0.00)	0.86 (0.00)	0.53 (0.00)	0.99 (0.03)
ρ_q	Normal	0.50 (0.10)	0.44 (0.00)	0.29 (0.02)	0.49 (0.00)	0.59 (0.00)	0.48 (0.00)	0.55 (0.00)	0.20 (0.00)	0.48 (0.00)	0.29 (0.01)
ML			226.83	790.13	62.57	591.18	329.64	765.24	90.19	216.96	650.84

Note: FA stands for model with Financial Accelerator, NFA stands for model

without Financial accelerator, and ML is the Marginal Likelihood

Table 3.2B: Prior and Posterior Distribution of Structural Parameters

Par.	Distr.	Nigeria				South Africa				Uganda				Zambia			
		Prior		Posterior		Posterior		Posterior		Posterior		Posterior		Posterior		Posterior	
		Mean (std)	FA Mean	NFA Mean	NFA Mean	FA Mean	NFA Mean	FA Mean	NFA Mean	FA Mean	NFA Mean	FA Mean	NFA Mean	FA Mean	NFA Mean	FA Mean	NFA Mean
h	Beta	0.70 (0.10)	0.71 (0.00)	0.62 (0.01)	0.72 (0.00)	0.58 (0.00)	0.72 (0.00)	0.74 (0.00)	0.67 (0.00)	0.65 (0.00)	0.77 (0.00)	0.65 (0.00)	0.77 (0.00)	0.65 (0.00)	0.77 (0.00)	0.65 (0.00)	0.77 (0.00)
ϕ	Normal	0.2 (0.05)	0.04 (0.00)	1.22 (0.00)	1.25 (0.00)	0.03 (0.00)	1.25 (0.00)	0.05 (0.00)	1.17 (0.00)	0.06 (0.00)	1.32 (0.00)	0.06 (0.00)	1.32 (0.00)	0.06 (0.00)	1.32 (0.00)	0.06 (0.00)	1.32 (0.00)
ϱ	Beta	0.30 (0.02)	0.31 (0.00)	0.27 (0.00)	0.30 (0.00)	0.30 (0.00)	0.30 (0.00)	0.31 (0.00)	0.32 (0.00)	0.29 (0.00)	0.31 (0.00)	0.29 (0.00)	0.31 (0.00)	0.29 (0.00)	0.31 (0.00)	0.29 (0.00)	0.31 (0.00)
ψ_p	Beta	0.05 (0.02)	0.06 (0.00)	—	—	0.05 (0.00)	—	0.06 (0.00)	—	0.05 (0.00)	—	0.05 (0.00)	—	0.05 (0.00)	—	0.05 (0.00)	—
θ	Beta	0.65 (0.10)	0.64 (0.00)	0.67 (0.00)	0.63 (0.00)	0.56 (0.00)	0.63 (0.00)	0.59 (0.00)	0.56 (0.01)	0.56 (0.00)	0.56 (0.01)	0.56 (0.00)	0.56 (0.01)	0.56 (0.00)	0.56 (0.01)	0.56 (0.00)	0.56 (0.01)
ω	Beta	0.5 (0.15)	0.60 (0.00)	0.38 (0.01)	0.51 (0.00)	0.70 (0.00)	0.51 (0.00)	0.55 (0.00)	0.36 (0.00)	0.50 (0.00)	0.45 (0.01)	0.50 (0.00)	0.45 (0.01)	0.50 (0.00)	0.45 (0.01)	0.50 (0.00)	0.45 (0.01)
ω_d	Beta	0.2 (0.15)	0.25 (0.00)	0.03 (0.01)	0.18 (0.00)	0.27 (0.00)	0.18 (0.00)	0.21 (0.00)	0.31 (0.00)	0.21 (0.00)	0.19 (0.00)	0.21 (0.00)	0.19 (0.00)	0.21 (0.00)	0.19 (0.00)	0.21 (0.00)	0.19 (0.00)
ω_q	Beta	0.50 (0.15)	0.36 (0.01)	0.51 (0.01)	0.51 (0.00)	0.44 (0.00)	0.51 (0.00)	0.40 (0.00)	0.23 (0.00)	0.49 (0.00)	0.41 (0.00)	0.49 (0.00)	0.41 (0.00)	0.49 (0.00)	0.41 (0.00)	0.49 (0.00)	0.41 (0.00)
ρ_y	Normal	0.12 (0.05)	0.14 (0.00)	0.16 (0.00)	0.11 (0.00)	0.08 (0.00)	0.11 (0.00)	0.13 (0.00)	0.11 (0.00)	0.13 (0.00)	0.13 (0.00)	0.13 (0.00)	0.13 (0.00)	0.13 (0.00)	0.13 (0.00)	0.13 (0.00)	0.13 (0.00)
ρ_π	Normal	1.50 (0.125)	1.25 (0.01)	1.53 (0.00)	1.52 (0.00)	1.44 (0.00)	1.52 (0.00)	1.50 (0.00)	1.42 (0.00)	1.53 (0.00)	1.66 (0.00)	1.53 (0.00)	1.66 (0.00)	1.53 (0.00)	1.66 (0.00)	1.53 (0.00)	1.66 (0.00)
ρ_{mag}	Beta	0.75 (0.10)	0.83 (0.00)	0.87 (0.01)	0.73 (0.00)	0.94 (0.00)	0.73 (0.00)	0.88 (0.00)	0.64 (0.00)	0.76 (0.00)	0.82 (0.00)	0.76 (0.00)	0.82 (0.00)	0.76 (0.00)	0.82 (0.00)	0.76 (0.00)	0.82 (0.00)
ρ_{rer}	Beta	0.50 (0.10)	0.26 (0.01)	0.46 (0.01)	0.49 (0.00)	0.14 (0.00)	0.49 (0.00)	0.25 (0.00)	0.73 (0.01)	0.41 (0.00)	0.97 (0.01)	0.41 (0.00)	0.97 (0.01)	0.41 (0.00)	0.97 (0.01)	0.41 (0.00)	0.97 (0.01)
ρ_q	Normal	0.50 (0.10)	0.62 (0.01)	0.29 (0.01)	0.50 (0.00)	0.28 (0.00)	0.50 (0.00)	0.65 (0.00)	0.66 (0.00)	0.57 (0.00)	0.52 (0.01)	0.66 (0.00)	0.57 (0.00)	0.66 (0.00)	0.57 (0.00)	0.66 (0.00)	0.52 (0.01)
ML			669.98	287.68	37.74	101.96	37.74	366.25	216.73	995.82	19.27	366.25	216.73	995.82	19.27	366.25	216.73

Note: FA stands for model with Financial Accelerator, NFA stands for model without Financial accelerator, and ML is the Marginal Likelihood

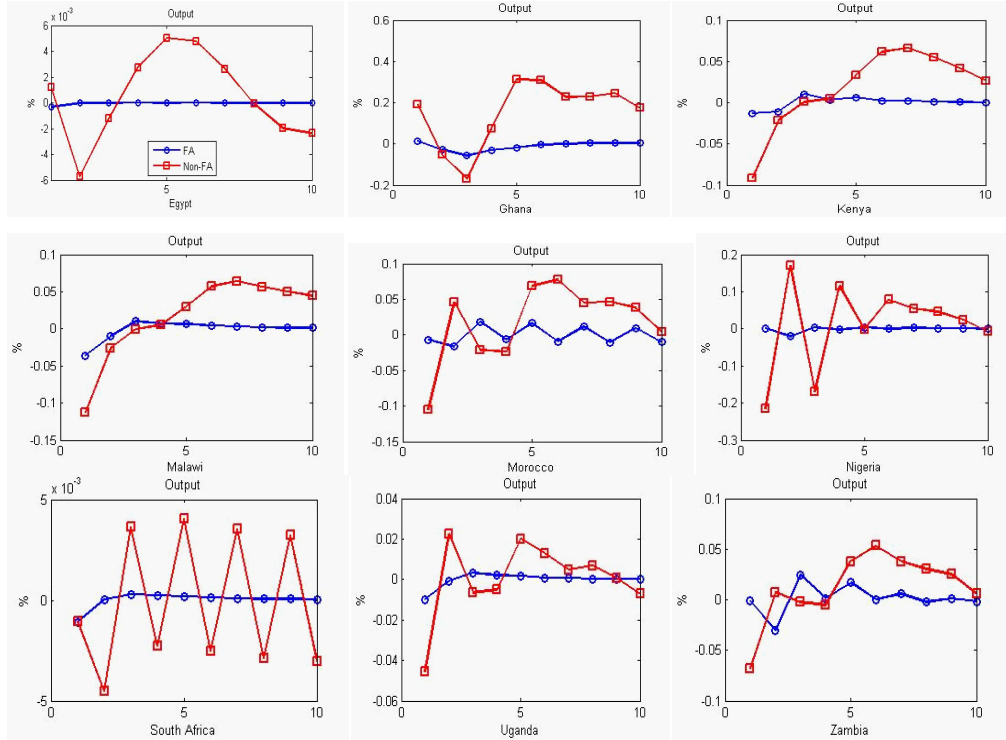
3.5 Impulse response analysis

3.5.1 Exchange rate shocks

Based on the mean of posterior distribution reported in Table 2, Fig. 1 shows the dynamic response of output to exchange rate depreciation under the baseline model (model without financial imperfection) and model with financial imperfection. In the baseline model, exchange rate depreciation is expected to expand output through the competitive effect. Exchange rate depreciation reduces the price of exports relative to the price of imports. This increases export, the trade balance and output. Under the financial accelerator model, however, the balance sheet effect is expected to dominate the competitive effect thereby leading to a contractionary devaluation. For example, exchange rate depreciation deteriorates the balance sheet of a firms, lowers the net worth, increases the cost of capital, lowers investment, and contracts output.

Generally, Fig.1 indicates that output initially declines after depreciation and later increases in all the countries under the baseline model. This is in line with the J-curve hypothesis (see, Stučka, 2006; Hsing, 2008). In contrast, under the model with financial imperfection, output contracts after exchange rate depreciation. This indicates that exchange rate depreciation deteriorates the net worth and the balance sheet of firms. Consequently, this increases the external finance premium and contracts investment and output. The balance sheet effect diminishes the expansionary effects of exchange rate depreciation. This is similar to the findings by Céspedes et al. (2005) and Bebczuk et al. (2006).

Fig. 3.1. Dynamic response of output to exchange rate depreciation.

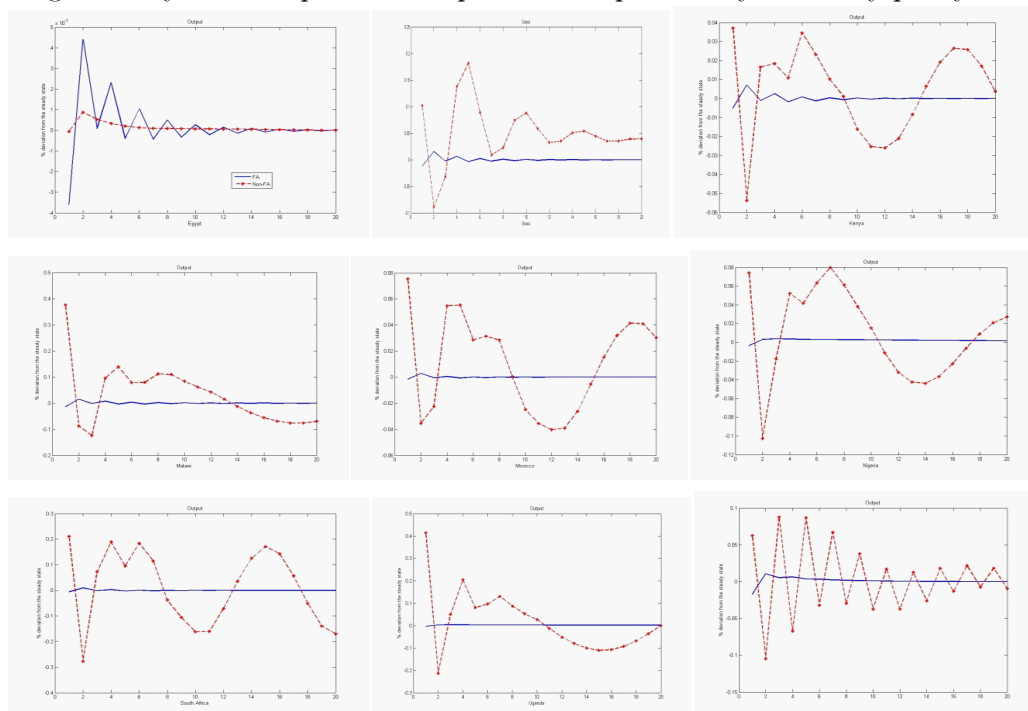


3.5.2 Monetary policy shocks

Fig.2 presents the dynamic response of output to an expansionary monetary policy shock under the baseline model (model without financial imperfection) and model with financial imperfection. Under the model without the financial imperfection, an expansionary monetary policy shock will lead to a decline in the interest rate, an increase in investment and an expansion in output. For the model with financial imperfection, a monetary expansion decreases the interest rate, depreciates the exchange rate, deteriorates the firm's balance sheet by lowering the net worth, increases the cost of capital, reduces investment, and contracts output. An expansionary monetary policy shocks is contractionary under the model with financial friction but expansionary in the baseline model.

Fig 2 shows that under the model without financial imperfection, output initially decreases following an expansionary monetary policy shock but later expands in all the countries except in Egypt. In Egypt, output continues to decline after an expansionary monetary policy shock. In contrast, under model with financial imperfection, an expansionary monetary policy shock leads to a contraction in output for all the countries. This may arise as the decline in the interest rate after a loose monetary policy depreciates the exchange rate. The depreciation worsens the firm's balance sheets, and dampens investment and output. This is line with the findings by Cook (2004).

Fig. 3.2. Dynamic response of output to an expansionary monetary policy shock

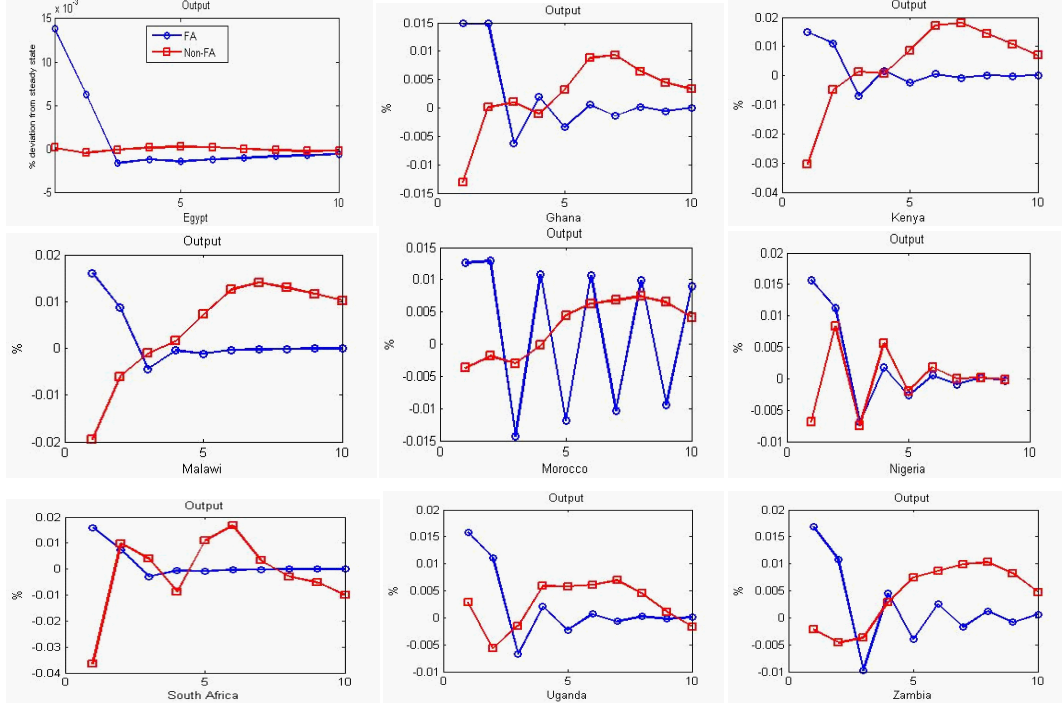


3.5.3 Foreign interest rate shocks

Fig. 3 displays dynamic response of output to positive foreign interest rate shocks under the baseline model and the model with financial imperfection. Under the model with financial imperfection, a positive shock to the foreign interest rate depreciates the exchange rate, increases the debt service obligation, lowers the net worth and thereby deteriorates the balance sheet of firms. Consequently, the worsening of the balance sheet contracts investment and output. Under the baseline model, a rise in foreign interest increases the debt service payment and lowers output. It is expected that output will decline more under the model with financial imperfection.

Generally, output declines under the two models after a positive shock to the foreign interest rate. Output, however, contracts more under the model with financial imperfection in African economies. A positive shock to the foreign interest rate may depreciate the exchange rate. This deteriorates the net worth and balance sheet of the firms, contracts investment, and lowers output (see Céspedes et al., 2004). Through this balance channel, the effect of foreign interest rate shock is amplified in the economy.

Fig. 3.3. Dynamic response of output to positive foreign interest rate shock.



3.6. Conclusion

The study investigates the quantitative significance of the balance sheet channel in African economies. We construct an open economy monetary DSGE model where the entrepreneur's net worth determines the cost of borrowing. Using the Bayesian technique, we estimate structural parameters for two versions of the model: one with and one without the financial market imperfections. In line with Bayesian literature, we employ the marginal likelihood ratio to compare the fit of the two models.

The results indicate that the balance sheet channel is empirically important in African economies. The marginal likelihood ratio favours the model with financial imperfection in seven of the nine selected African countries. However, the estimated parameter of the external finance premium is statistically

different from zero for all African countries. This suggests that the balance sheet channel plays a significant role in business cycle fluctuations in African economies. This is similar to the findings by Elekdag et al. (2006) for South Korea.

The presence of balance sheet channel dampens the expansionary effects of exchange rate depreciation in African economies. Exchange rate depreciation increases the cost of debt service and deteriorates the net worth and balance sheet of firms. Consequently, this increases the cost of capital, lowers investment and output. This is similar to the findings by Céspedes (2006) and Bebczuk et al. (2006). Furthermore, the balance sheet channel reduces the effect of monetary policy expansion on output. A monetary expansion reduces the interest rate and depreciates the exchange rate. The depreciation of the exchange rate deteriorates the balance sheet of firms, reduces investment, contracts output and further exacerbates the external debt problem. This largely reduces the potency of monetary policy in influencing output. As observed by Kramer and Thoenissen (2012), financial frictions reduce the effectiveness of monetary policy.

These findings pose a policy dilemma for monetary authorities in African economies. If loose monetary policy is implemented, the low interest rate reinforces the exchange rate depreciation thereby amplifying the output loss (see Krugman, 1999b). Alternatively, if tight monetary policy is introduced, the high interest rate strengthens the currency but contracts output. Central to the findings also is the appropriate exchange rate policy for African countries when the balance sheet effect is significant. While Cook (2004) recommends fixed exchange rate regime when the balance sheet effect matters, Céspedes et al. (2004) and Gertler et al. (2007) advocate for flexible exchange rate regime. Based on the balance sheet effects of exchange rate depreciation, it will be prudent for African countries to manage their exchange rate fluctuations.

In addition, African countries should reduce their exposure to foreign currency-denominated debt. African countries should also deepen their domestic bond markets. The deepening of domestic bond market will not only reduce their exposure to foreign currency debts but also give African countries the policy space to react to exogenous shocks.

CHAPTER FOUR

Sovereign Wealth Funds and Macroeconomic Stability in African Economies

Abstract

This paper investigates the effectiveness of sovereign wealth funds (SWFs) in reducing macroeconomic volatility in commodity exporting African countries. We formulate and simulate a dynamic stochastic general equilibrium (DSGE) model that features sovereign wealth funds (SWFs). The simulation results suggest that the creation of SWFs can reduce macroeconomic volatility in commodity exporting countries. Particularly, SWFs can reduce government expenditure, real exchange rate, and external debt volatility. Since these are the channels through which commodity price shocks are transmitted to the African economies, the creation of SWFs can reduce the macroeconomic instability in African commodity exporting countries.

4.1. Introduction

Commodity exporting developing countries are quite vulnerable to commodity price shocks. Sharp and unpredictable fluctuations in commodity prices induce exchange rate volatility (see Cashin et al. 2004; Dauvin, 2014) and fiscal uncertainty and macroeconomic instability in commodity exporting countries (see Medina, 2010; Spatafora and Samake, 2012). In view of the developing countries limited and imperfect access to international capital markets and the incompleteness of commodity-linked hedging market, the establishment of sovereign wealth funds (SWFs) is seen as the means of reducing these exposures to commodity price risk. The creation of SWFs may reduce income volatility, de-couple fiscal policy from revenue uncertainty, prevent the occurrence of Dutch disease problem associated with commodity

price boom, and promote macroeconomic stability (see Fasano, 2000; Shabsigh and Ilahi, 2007).

The gap that this study seek to fill is to investigate whether the creation of SWFs in commodity exporting African countries can reduce the macroeconomic instability engendered by commodity price fluctuations. There has been limited empirical studies evaluating whether the creation of SWFs can reduce economic volatility in African economies. Rather, the few existing empirical studies on SWFs as fiscal and macroeconomic stabilization tools have focused on oil exporting non-African countries (see Mehrara et al., 2012).

This study is significant for policy-makers in commodity exporting African countries. Given the strong positive correlation between commodity revenue and government expenditure in commodity exporting African countries (see Baunsgaard, 2003), the study will provide measures for the de-coupling of fiscal operation from the volatile commodity revenue. Similarly, the study will provide the mechanism for mitigating the effect of export receipt inflow on money supply on the domestic economy. Raju and melo (2003) show that coffee price shocks significantly affect money supply in Columbia. Lastly, the study will also assist the policy-makers in the efficient management of commodity revenue.

This study contributes to the existing literature on sovereign wealth funds (SWFs) in two respects. Firstly, we simulate our model to investigate whether the creation of SWFs can reduce macroeconomic instability occasioned by commodity price shocks in commodity exporting African countries. Secondly, we construct a DSGE model incorporating the sovereign wealth funds for commodity exporting African countries.

The rest of the paper is organised as follows. Section 2 reviews the existing literature on commodity price shocks and macroeconomic fluctuations. Section 3 presents the stylised facts on commodity prices, fiscal balances and

macroeconomic aggregates. Section 4 describes the model for the study. Section 5 discusses the simulation results and summarises the findings. The last section draws conclusion and suggests policy implications.

4.2. Review of Literature

Empirical evidence on the impact of resource abundance on growth has been quite inconclusive. A strand of literature, the resource curse hypothesis, argues that the availability of mineral resources promotes corruption, weakens institution and reduces economic growth. For example, Sachs and Warner (1999, 2001) find that natural resource abundance reduces economic growth. This occurs through the increase in the price of non-traded goods which makes manufactured export uncompetitive. Similarly, Gylfason (2001) concludes that resource abundance weakens the incentive to accumulate human capital and therefore lowers economic growth. Sala-i-Martin and Subramanian (2003) find that resource abundance negatively impacts growth by weakening the institutional quality. Papyrakis and Gerlagh (2004) find that resource wealth promotes corruption, weakens institution, reduces the need for savings and investment, and hence decreases economic growth.

In contrast, another strand of literature argues that resource abundance does not retard but rather promote economic growth. For example, Alexeev and Conrad (2004) find that resource abundance promotes economic growth. Similarly, Lederman and Maloney (2007) find that it is not resource abundance that hampers growth but export concentration. Brunnschweiler and Bulte (2008), however, find that resource abundance enhances better institution that positively promotes economic growth. They also find that resource dependence has no effect on growth.

Resource abundance significantly influences fiscal operation in resource rich countries. Thus, a number of authors have examined the impacts of commodity price shocks on fiscal outcome in commodity exporting countries.

For instance, Medina (2010) shows that commodity price shocks exert strong influence on fiscal expenditure in commodity exporting Latin American countries. Kumah and Matovu (2007), Sturm et al. (2009), and Spatafora and Samake (2012) find that fiscal policy is highly procyclical in commodity-exporting developing countries. This is attributed to the positive response of expenditure to commodity prices. Evidence from oil producing countries also indicates that government expenditure is pro-cyclical in oil exporting countries. Villafuerte and Lopez-Murphy (2010) and El Anshasy and Bradley (2012) find strong fiscal expansion during boom and fiscal tightening during bust in oil exporting countries.

In addition, commodity price shocks have been found to affect exchange rate fluctuations in commodity exporting countries. For example, Cashin et al.(2004) find that an increase in commodity price appreciates the real exchange rate through a rise in wages in commodity exporting countries. Arezki et al. (2012) show that gold price volatility induces real exchange rate volatility in South Africa through capital inflows. Löfgren et al. (2002) and Bova (2009) find comovement between cocoa price and exchange rate in Cote d'Ivoire. Olomola and Adejumo (2006) and Iwayemi and Fowowe (2012) report that oil price shocks influence real exchange rate movements in Nigeria.

Given the macroeconomic instability induced by commodity price shocks in commodity exporting countries, different measures have been proposed to cushion the impact of commodity price shocks on the domestic economy. A number of studies emphasize the adoption of fiscal rules and strengthening of fiscal institutions to dampen macroeconomic volatility in commodity exporting countries. For example, Baunsgaard (2003) and Frankel (2011) show that fiscal rules and strengthening of fiscal policy framework promote stability in oil producing countries. Moreover, Schmidt-Hebbel (2012) finds that fiscal rule has contributed to fiscal and macroeconomic stability in Chile. In

contrast, Arezki and Ismail (2013) find that fiscal rules have limited influence on government spending in oil producing countries.

Another strand of literature argues for the establishment of resource funds or SWFs to mitigate the adverse effect of commodity price swings in commodity exporting countries. Fasano (2000) reviews the experience of resource funds in six commodity exporting countries. The author finds that resource funds have contributed to effective management of fiscal policy in all the countries except in Oman and Venezuela. Davis et al. (2001) find that countries with national revenue funds have lower expenditure volatility than countries without national revenue funds. Ashafa (2007) concludes that resource funds is an effective fiscal stabilization tools when strong institution exists.

Similarly, Usui (2007) evaluates the effectiveness of oil funds in Azerbaijan and Kazakhstan. The results indicate that oil funds have reduced fiscal volatility in the two countries. Kalyuzhnova (2011) finds that national revenue funds have been effective in the management of fiscal balance in Kazakhstan. Everhart and Duval-Hernandez (2001) find that oil funds smooth consumption and reduce expenditure volatility in Mexico. Basu et al. (2013) simulate the effects of SWFs in Papua New Guinea. The simulation results indicate that SWFs reduce macroeconomic volatility and enhance a stable fiscal regime. In contrast, Devlin and Titman (2004) report that oil funds have limited success on fiscal stability but are very useful for investment strategy. Le Borgne and Medas (2007) find that SWFs have not provided fiscal stability in the Pacific Island.

The benefits of resource funds have been found to extend beyond the fiscal balances. For instance, Shabsigh and Ilahi (2007) and Mehrara et al. (2012) investigate oil funds and macroeconomic stability in oil producing countries. The results suggest that oil funds significantly lower broad money, reduce inflation and mitigate macroeconomic volatility. Astrov (2007) finds that oil funds sterilize exchange rate inflows and decouple GDP growth rate from oil

price dynamics in Russia. Park (2008), however, reports that SWFs has been a major source of financial instability in Asia.

Moreover, the establishment of sovereign wealth funds has been found to reduce the negative effects of reserve accumulation and improve governance and institution. For example, Aizenman and Glick (2008) find that sovereign wealth funds reduce the opportunity costs of holding large reserves by the central banks. Similarly, Arreaza et al. (2009) report that transfer of excess reserves to sovereign wealth funds reduce the cost of sterilization. On institution and governance, Tsani (2013) finds that resource funds prevent resource curse, improve governance and institutional quality in resource rich countries. Ebert and La Menza (2015) concludes the creation of SWFs has mitigated the Dutch-disease effects in Chile.

4.3. Stylised Facts

High export concentration can generate remarkable instability in the economy of commodity exporting countries. Given the degree of export concentration and commodity export dependence in Africa (see UNDP, 2012), commodity price fluctuations markedly affect the African economies. For example, Bleaney and Greenaway (2001) find that terms of trade instability negatively affect growth in Africa economies. Similarly, Addison and Ghoshray (2013) find that commodity price shocks have effects on per capita income in agricultural commodity exporting sub-Saharan African (SSA) countries. Table 1 shows the volatility of some macroeconomic aggregates in African commodity exporting countries.

The first apparent property from the Table 1 is the high volatility of money supply in all the countries. This might suggest non-sterilization of resource revenue in African countries. The revenue from commodity export expands the domestic money supply. This is similar to the findings by Raju and Melo (2003) in Columbia. The table also shows high volatility of revenue and expenditure. This possibly reflects procyclical fiscal policy in African countries

This is related to the findings by Erbil (2011) for oil producing developing countries. Relative to other commodity exporting countries, output is more volatile in oil exporting countries- Egypt, Nigeria and Tunisia. This suggests that oil exporting countries are more vulnerable to oil price shocks than other commodity exporting countries.

Table 4.1: Business cycle properties

Country	σ_{rgdp}	σ_{Rev}	σ_{Expdt}	σ_{RER}	σ_{MS}	σ_{π}
Egypt						
Mean	4.40	7.09	7.32	1.51	22.04	4.40
Min	3.94	6.85	7.01	1.36	20.01	3.50
Max	4.90	7.50	7.88	1.61	24.10	5.25
Kenya						
Mean	4.29	21.74	21.85	4.39	22.21	3.62
Min	3.62	21.58	21.75	3.77	21.85	1.61
Max	4.82	21.82	21.94	4.61	22.66	5.41
Nigeria						
Mean	25.18	9.94	9.53	4.52	23.81	2.67
Min	24.80	9.47	9.37	3.40	23.38	-0.42
Max	26.05	10.76	10.04	4.89	25.12	5.19
South Africa						
Mean	4.38	21.70	21.82	1.86	22.62	3.88
Min	4.10	21.18	21.28	1.65	22.15	2.13
Max	4.84	22.42	22.55	2.03	23.56	5.04
Tunisia						
Mean	4.40	18.34	18.30	0.18	18.94	4.47
Min	3.95	17.86	17.83	0.001	18.31	4.04
Max	4.85	18.88	18.89	0.26	19.68	4.84

σ denotes standard deviation. All variables are logged and H-P filtered with smoothing parameters 100. Rev is total revenue; expdt is total expenditure; rgdp is real gdp; ms is money supply; rer is real exchange rate; and π is inflation rate.

Table 2 presents the correlation between some macroeconomic aggregates. Firstly, there is a strong correlation between revenue and expenditure in all

the countries, implying pro-cyclical fiscal policy. This suggests that commodity exporting African countries increase expenditure when there is a rise in commodity revenue and vice versa. This is in line with the findings by Spatafora and Samake (2012). Similarly, there is a positive correlation between commodity prices and fiscal expenditure in all the sample countries except in Kenya. This indicates that government expenditure increases when commodity price rises and declines when commodity price falls. This is related to the findings by Kuma and Matovu (2007) and Kaminsky (2010). A notable feature in table 2 is the high positive correlation between commodity prices and government expenditure in the oil-exporting countries- Egypt, Nigeria and Tunisia. This demonstrates that oil revenue significantly affects fiscal balances in oil exporting countries.

Moreover, the correlation indicates that commodity price is positively related to output in all the countries except in Kenya. This implies that a rise in commodity price increases output while a fall in commodity price reduce output in African countries. This is a form of symmetric relation which is similar to the findings by Addison and Ghoshray (2013). However, this is in contrast to the Dehn (2000) who find asymmetric impact of commodity price shocks on economic growth in developing countries. In addition, the correlation reveals a negative comovement between commodity prices and real exchange rate for Egypt, Nigeria, and South Africa. This shows that positive commodity price shocks decrease (appreciate) the real exchange rate. This suggests that these countries' currencies are commodity currencies. This is similar to the findings by Frankel (2007) and Arezki et al. (2012) for commodity exporting countries.

The correlations between commodity price and money supply are quite mixed. The result shows a positive correlation between commodity prices and money supply in Nigeria, South Africa, and Tunisia. This implies that a rise in commodity price increases domestic money supply, suggesting non-sterilization of

the export revenue. However, the correlation is negative between commodity prices and money supply in Egypt and Kenya. This denotes that commodity revenues are sterilized in these two countries. Lastly, the table shows that mixed correlation between commodity prices and inflation in African economies.

Table 4.2: Correlation between commodity prices and macroeconomic aggregates

Country	$\rho_{rev,exp}$	$\rho_{comp,exp}$	$\rho_{comp,rgdp}$	$\rho_{comp,rer}$	$\rho_{comp,ms}$	$\rho_{comp,\pi}$
Egypt	1.00 (0.00)	0.98 (0.00)	0.89 (0.00)	-0.16 (0.49)	-0.88 (0.00)	0.25 (0.27)
Kenya	0.83 (0.00)	-0.78 (0.00)	-0.91 (0.00)	0.04 (0.81)	-0.79 (0.07)	0.06 (0.73)
Nigeria	0.90 (0.00)	0.76 (0.00)	0.38 (0.03)	-0.45 (0.01)	0.75 (0.00)	-0.41 (0.02)
South Africa	0.99 (0.01)	0.43 (0.01)	0.57 (0.00)	-0.16 (0.39)	0.64 (0.00)	-0.19 (0.31)
Tunisia	1.00 (0.00)	0.94 (0.00)	0.89 (0.00)	0.67 (0.00)	0.93 (0.00)	-0.12 (0.61)

ρ denotes correlation; rev is revenue; exp is expenditure; comp is commodity price; rer is real exchange rate; and ms is money supply
All variables are logged and H-P filtered with smoothing parameter 100.

4.4. The Model

4.4.1 Households

We employ a DSGE model with non-separable money similar to the one in Andrés et al. (2006). Given the importance of foreign currency holdings by African households as noted by Elkhafif (2002) and Adom et al. (2008), we assume that households allocate their real holdings between domestic and foreign currencies. For simplicity, we assume this allocation to be in fixed proportions, so that $S_t M_t^* = \varrho M_t$, where S_t is the nominal exchange

rate, M_t^* is foreign nominal money and M_t is domestic nominal money. The households' preferences are given by:

$$U_t = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left[\left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi \right] - \frac{N_t^{1+\psi}}{1+\psi} \quad (1)$$

where C_t , P_t and N_t represent aggregate consumption, domestic price level and labour respectively. The parameter h measures the degree of habit formation consumption. The parameter σ is the relative risk aversion coefficient ; $\beta \in (0, 1)$ is the discount factor; ψ is the inverse of the Frisch labour supply elasticity and ϕ represents interest rate elasticity of money demand.

Households hold their wealth in the form of foreign and domestic currency and domestic bonds. Furthermore households borrow from foreign capital markets. Therefore the budget constraint is:

$$\begin{aligned} C_t + \left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} + \frac{B_t}{P_t} - \frac{S_t D_t^*}{P_t} &= \frac{W_t N_t}{P_t} + \frac{B_{t-1}}{P_t} (1 + r_{t-1}) \\ &+ \left(1 + \frac{\varrho}{S_t} \right) \frac{M_{t-1}}{P_t} \\ &- \frac{S_t D_{t-1}^*}{P_t} (1 + r_{t-1}^d) \end{aligned} \quad (2)$$

where D_t^* is the level of foreign debt, W_t is the nominal wage rate, r_t is the domestic nominal interest rate and r_t^d is the interest rate on foreign debt. Households enter period t with domestic money holdings M_t , bonds B_t , and foreign debt D_t^* .

Households choose the path of C_t , M_t , B_t , and N_t that maximize expected utility. The first order conditions are:

$$\frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi = \lambda_t \quad (3)$$

$$\begin{aligned} \frac{\phi}{1-\sigma} \left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left(1 + \frac{\varrho}{S_t} \right)^\phi \left(\frac{M_t}{P_t} \right)^{\phi-1} \frac{1}{P_t} &= \frac{\lambda_t}{P_t} \left(1 + \frac{\varrho}{S_t} \right) \\ &\quad - \frac{\beta \lambda_{t+1}}{P_{t+1}} \left(1 + \frac{\varrho}{S_t} \right) \end{aligned} \quad (4)$$

$$\frac{W_t}{P_t} = \frac{N_t^\psi}{\lambda_t} \quad (5)$$

$$\beta \lambda_{t+1} (1 + r_t) \frac{P_t}{P_{t+1}} = \lambda_t \quad (6)$$

$$\beta \lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} (1 + r_{t-1}^d) = \lambda_t \frac{S_{t+1}}{P_t} \quad (7)$$

where equations (3)–(7) denote the derivatives of the utility function to its arguments.

Our consumption Euler equation is:

$$\begin{aligned} \tilde{c}_t &= \frac{h(\sigma - 1)}{\sigma_c} \tilde{c}_{t-1} + \frac{\sigma}{\sigma_c} E_t \tilde{c}_{t+1} + \frac{\phi}{\sigma_c} \tilde{m}_t - \frac{\phi}{\sigma_c} E_t \tilde{m}_{t+1} \\ &\quad - \frac{\varrho \phi}{s_0 \sigma_c} \tilde{s}_t + \frac{\varrho \phi}{s_0 \sigma_c} E_t \tilde{s}_{t+1} - \frac{1}{\sigma_c} (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \end{aligned} \quad (8)$$

where $\tilde{\cdot}$ denotes percentage deviation from the steady state and $\sigma_c = \sigma + h(\sigma - 1)$. As in Castelnovo (2012), eq.(8) shows that consumption depends on the weighted average of past and expected future consumption, real interest rate, and real balances. Furthermore, to the extent that households hold domestic currency, a depreciation of the current exchange rate reduces household wealth, which reduces aggregate consumption. Through this channel,

the depreciation of the nominal exchange rate may generate a contractionary effect on output as pointed out by Edwards (1986).

In order to express the IS curve in terms of output, we first write the macro-balance equation:

$$\tilde{y}_t = \gamma_c \tilde{c}_t + \gamma_x \tilde{x}_t - \gamma_z \tilde{z}_t + \gamma_g \tilde{g}_t + \varepsilon_{g,t} \quad (9)$$

where $\tilde{y}_t$ is the percentage deviation of output from the steady state, $\tilde{x}_t$ percentage deviations of exports, $\tilde{z}_t$ is the percentage deviation of imports and $\tilde{g}_t$ is the deviation of government expenditure from the steady state. The parameter γ_j is the steady state ratio of variable j to output, $\varepsilon_{g,t}$ is a demand shock that combines government expenditure and investment shocks. We follow McCallum and Nelson (2000) in formulating our net export function as:

$$\tilde{nx}_t = \gamma_{yf} \tilde{y}_t^f - \gamma_y \tilde{y}_t + \gamma_r \tilde{rer}_t \quad (10)$$

where $\tilde{y}_t^f$, γ_{yf} , γ_y , and γ_r , are the foreign output, elasticity of net export to foreign output, the elasticity of net export to domestic output, and sum of elasticity of substitution in production for home and abroad respectively. The real exchange rate is defined as $\tilde{rer}_t \equiv \tilde{s}_t + \tilde{p}_t^* - \tilde{p}_t$. Substituting eq.(10) into eq.(9) yields the expression:

$$\tilde{c}_t = \frac{1}{\gamma_c} (1 + \gamma_y) \tilde{y}_t - \frac{\gamma_r}{\gamma_c} \tilde{rer}_t - \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_t^f - \frac{\gamma_g}{\gamma_c} \tilde{g}_t \quad (11)$$

Substituting eq.(11) into eq.(9) yields the output-based IS equation:

$$\begin{aligned}
\tilde{y}_t = & \frac{h(\sigma-1)}{\sigma_c} \tilde{y}_{t-1} + \frac{\sigma}{\sigma_c} E_t \tilde{y}_{t+1} - \frac{\gamma_c}{\sigma_c(1+\gamma_y)} (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \\
& + \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)} \tilde{m}_t - \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)} E_t \tilde{m}_{t+1} - \frac{\phi\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)} \tilde{s}_t \\
& + \frac{\phi\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)} E_t \tilde{s}_{t+1} - \frac{h(\sigma-1)\gamma_r}{\sigma_c(1+\gamma_y)} \tilde{r}er_{t-1} \\
& + \frac{\gamma_r}{(1+\gamma_y)} \tilde{r}er_t - \frac{\sigma\gamma_r}{\sigma_c(1+\gamma_y)} E_t \tilde{r}er_{t+1} - \frac{h(\sigma-1)\gamma_{yf}}{\sigma_c(1+\gamma_y)} \tilde{y}_{t-1}^f \quad (12) \\
& + \frac{\gamma_{yf}}{(1+\gamma_y)} \tilde{y}_t^f - \frac{\sigma\gamma_{yf}}{\sigma_c(1+\gamma_y)} E_t \tilde{y}_{t+1}^f - \frac{h(\sigma-1)\gamma_g}{\sigma_c(1+\gamma_y)} \tilde{g}_{t-1} \\
& + \frac{\gamma_g}{(1+\gamma_y)} \tilde{g}_t - \frac{\sigma\gamma_g}{\sigma_c(1+\gamma_y)} E_t \tilde{g}_{t+1} + \varepsilon_t^a
\end{aligned}$$

We assume that the innovation ε_t^y follows a first-order autoregressive process as $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \mu_t^a$. Output depends positively on the real exchange rate and foreign output. As a departure from McCallum and Nelson (2000), our dynamic IS equation also features output as a function of lags and leads of the real exchange rate, foreign output and government expenditure.

4.4.2 Firms

In line with Batini et al. (2005) and Malikane (2014), we assume that final goods producing firms exhibit non-linear input requirement in the production function such that $X_{i,t} = Y_t^{\delta_i}$ where $X_{i,t}$ is the amount of non-labour input i required in production, Y_t is output, and $\delta_i > 0$ is the elasticity of the input requirement with respect to changes in output. Labour and non-labour inputs are complements as in Smets and Wouters (2002). The production function is given as:

$$Y_t = A_t N_t^\eta \left[\prod_{i=1}^n Y_t^{\theta_i \delta_i} \right] \quad (13)$$

where a_t denotes productivity shocks, N_t is the level of employment, θ_i is the elasticity of output with respect to input i and $0 < \eta < 1$. The reduced form of eq.(13) is:

$$Y_t = A_t' N_t^\alpha \quad (14)$$

where $\chi = \sum_{i=1}^n \theta_i \delta_i$, $\alpha = \frac{\eta}{1-\chi}$, and $A_t' = A_t^{\frac{1}{1-\chi}}$. The productivity shock is assumed to follow a first order autoregressive process: $A_t = A_{t-1}^{\rho_A} e^{\varepsilon_t^p}$. Using eq.(14), total real cost is:

$$TC_t = \frac{W_t Y_t^{\frac{1}{\alpha}}}{A_t'^{\frac{1}{\alpha}} P_t} + \sum_{i=1}^n \frac{P_{it}}{P_t} Y_t^{\delta_i}, \quad (15)$$

where P_{it} is the price of non-labour input i , P_t is the aggregate price level while W_t denotes the nominal wages. Let p_{it} be the real price of the non-labour input, the real marginal cost can be specified as:

$$MC_t = \frac{W_t Y_t^{\frac{1-\alpha}{\alpha}}}{\alpha A_t'^{\frac{1}{\alpha}} P_t} + \sum_{i=1}^n \delta_i p_{it} Y_t^{\delta_i - 1} \quad (16)$$

where MC_t represents the marginal cost. We follow Galí and Gertler (1999) and formulate the hybrid new Keynesian Phillips curve as follows:

$$\tilde{\pi}_t = \gamma_f E_t \tilde{\pi}_{t+1} + \gamma_b \tilde{\pi}_{t-1} + \lambda \tilde{mc}_t, \quad (17)$$

where:

$$\begin{aligned} \gamma_f &\equiv \beta \theta \{\theta + \omega [1 - \theta (1 - \beta)]\}^{-1}; \lambda \equiv (1 - \theta) (1 - \beta \theta) (1 - \omega) \xi \\ \gamma_b &\equiv \omega \{\theta + \omega [1 - \theta (1 - \beta)]\}^{-1}; \xi \equiv \frac{(1 - \alpha)}{1 + \alpha (\varepsilon - 1)} \{\theta + \omega [1 - \theta (1 - \beta)]\}^{-1}, \end{aligned}$$

θ is measures price stickiness, ω is the degree of price indexation, ε is the goods' elasticity of substitution. We linearize eq.(16) and substitute eq.(5) and eq.(11) to derive the marginal cost written as:

$$\begin{aligned} \tilde{mc}_t &= \vartheta_a \tilde{y}_t - \vartheta_b \tilde{y}_{t-1} - \vartheta_c \tilde{m}_t + \vartheta_d \tilde{s}_t - \vartheta_e \tilde{rer}_t \\ &\quad + \vartheta_f \tilde{rer}_{t-1} - \vartheta_g \tilde{y}_t^f + \vartheta_h \tilde{y}_{t-1}^f + \vartheta_i \tilde{p}_{it} - \vartheta_j \tilde{a}_t, \end{aligned} \quad (18)$$

where:

$$\begin{aligned}
\vartheta_a &= \left(\frac{\gamma_{ls}\sigma(1+\gamma_y)}{\gamma_c(1+\psi)} + \frac{\gamma_{ls}}{\alpha} - \frac{\alpha\gamma_{ls}}{(1+\psi)} + \sum_{i=1}^n \delta_i \frac{p_{io}x_0}{y_0} \right); \\
\vartheta_b &= \frac{\gamma_{ls}h(\sigma-1)(1+\gamma_y)}{\gamma_c(\alpha+\psi)}; \vartheta_c = \frac{\gamma_{ls}\phi}{(1+\psi)}; \vartheta_d = \frac{\gamma_{ls}\phi\varrho}{s_0(1+\psi)} \\
\vartheta_e &= \frac{\gamma_{ls}\gamma_r}{\gamma_c(1+\psi)}; \vartheta_f = \frac{\gamma_{ls}h(\sigma-1)\gamma_r(1-\alpha)\gamma_{ls}}{\gamma_c(1+\psi)\alpha} \\
\vartheta_g &= \frac{\gamma_{ls}\sigma\gamma_{yf}}{\gamma_c(1+\psi)}; \vartheta_h = \frac{\gamma_{ls}h(\sigma-1)\gamma_{yf}}{\gamma_c(1+\psi)}; \vartheta_i = \sum_{i=1}^n \delta_i \frac{p_{io}x_0}{y_0} \\
\vartheta_j &= \frac{\gamma_{ls}}{\alpha}; \gamma_{ls} = y_0 a_0^{-1} n_0^{-1}
\end{aligned}$$

4.4.3 The sovereign wealth funds

Similar to Basu et al. (2013), we develop a link between sovereign wealth funds and commodity windfall. We assume that the government adopts a fiscal stance where a constant share of the commodity windfall τ^q is saved in the SWFs. The commodity windfall is $(q - q^a)$ where q is the commodity spot price and q^a is the average price. The accumulation of funds in SWFs depends on the deviation of commodity spot price from its average. The SWFs equation evolves according to:

$$f_t = \left(1 + r_{t-1}^f\right) f_{t-1} + \tau^q (q_t^s - q_t^a) \quad (19)$$

where f_t is the SWFs and r_t^f is the interest rate on the existing SWFs.

4.4.4 The government

The government finances its expenditure by issuing one-period domestic bond denoted by B_t , by foreign debt D_t^* , and through the tax revenue denoted by T_t . Both the government and the private hold some foreign debt. The

fraction of government budget deficit financed through foreign borrowing is $(1 - \phi_m)$. The government expenditure consist of spending for the provision of public goods G_t , interest payments on domestic currency debts r_t , and interest payments on foreign currency-denominated debt r_t^f . The proportion of the government financed from the commodity windfall is $(1 - \tau_q)(q_t - q_t^a)$. In real terms, the government budget constraint is:

$$\begin{aligned} b_t + (1 - \phi_m) d_t^* &= (g_t - t_t) + (1 + r_{t-1}) b_{t-1} - (1 - \tau_q)(q_t - q_t^a) \\ &\quad + (1 - \phi_m) (1 + r_{t-1}^f) d_{t-1}^* \end{aligned} \quad (20)$$

where $(g_t - t_t)$ is the government budget deficit.

4.4.5 Exchange rate and external debt

The interest rate on foreign debt is made up of the foreign risk-free rate and the risk premium. We assume that $r_t^d = r_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^* - \omega_f \tilde{f}_t$. This implies that the sovereign risk premium is driven positively by the burden of external debt, which increases the risk of default and negatively by commodity prices and SWFs, which increase the dollar liquidity of the commodity-exporting country. In our formulation, a positive shock to the foreign risk-free interest rate increases the country's interest rate spread and the cost of borrowing (see Uribe and Yue, 2006). In contrast, positive shocks to commodity prices reduce the spread and cost of borrowing for commodity exporting countries (see Senhadji, 2003; Muhanji and Ojah, 2011). Combining Eqs.(6) and (7), we can therefore write the UIP expression as follows:

$$\tilde{s}_t = E_t \tilde{s}_{t+1} - (\tilde{r}_t - \tilde{r}_t^f) - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^* - \omega_f \tilde{f}_t + \varepsilon_t^d \quad (21)$$

where $\tilde{s}_t$ is the nominal exchange rate, $(\tilde{r}_t - \tilde{r}_t^f)$ is the interest rate differential, $\tilde{q}_t$ is the commodity price and $\tilde{d}_t^*$ is external debt to GDP ratio, and $\tilde{f}_t$ is the sovereign wealth funds (SWFs). The coefficients ω_q , ω_d , and ω_f represent the sensitivity of the sovereign risk premium with respect to the commodity price, external debt, and SWFs respectively. As noted by García and González (2013), a higher risk premium induces capital outflow and depreciates the exchange rate. Moreover, the exchange rate depends negatively on commodity prices and positively on external debt to GDP ratio, as shown by Cashin et al.(2004).

The dynamics of external debt depends on the current account balance of the private sector, foreign debt service payment, budget deficit, and on the share of the windfall that is available to finance fiscal deficit. Thus, the external debt to GDP ratio evolves according to the following equation:

$$\frac{\Delta D_t^*}{P_t Y_t} = \frac{Z_t - X_t}{Y_t} + (1 + r_{t-1}^d) d_{t-1}^* + (1 - \phi_m) \left(\frac{G_t - T_t}{Y_t} - (1 - \tau_q)(q - q^a) \right) \quad (22)$$

where $\frac{D_t^*}{P_t Y_t}$ is the ratio of external debt to GDP, $\frac{Z_t - X_t}{Y_t}$ represents the current account balance by the private sector and r_t^d is the interest rate on external debt. The change in the debt ratio over time can then be written as:

$$\begin{aligned} \Delta d_t^* &= (z_t - x_t) + d_{t-1}^* (1 + r_{t-1}^d - \pi_{t-1} - \Delta y_{t-1}) \\ &\quad + (1 - \phi_m) (g_t - t_t - (1 - \tau_q)(q - q^a)) \end{aligned} \quad (23)$$

Eq.(23) shows that the change in the external debt ratio is a positive function of net import and foreign interest rate. For example, a rise in net imports

is financed by borrowing which increases the level of debt. Similarly, an increase in interest rate on external debt increases the debt service payment and therefore puts upward pressure on the debt ratio. We linearize Eq.(23) and substitute Eq.(10) to derive the equation for the dynamics of the external debt ratio. Note that from Eq.(21) we have $r_t^d = r_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t^* - \omega_f \tilde{f}_t$ and we substitute $(z_t - x_t) = \tilde{n}x_t = \gamma_{yf} \tilde{y}_t^f - \gamma_y \tilde{y}_t + \gamma_r \tilde{r}er_t$. Using this fact, we have the debt equation as follows:

$$\tilde{d}_t^* = \beta_a \tilde{d}_{t-1}^* + \beta_b \tilde{r}_t^f - \beta_c \tilde{y}_t - \beta_d \Delta \tilde{y}_t - \beta_e \tilde{q}_t - \beta_f \tilde{r}er_t - \beta_g \tilde{y}_t^f - (1 - \tau_q)(q - q^a) + \varepsilon_t^e \quad (24)$$

where

$$\begin{aligned} \gamma_d &= (1 + g_0)^2 - r_0 \omega_d; \beta_a = \frac{(1 + g_0) + r_0}{\gamma_d}; \beta_b = \frac{r_0}{\gamma_d}; \\ \beta_c &= \frac{(1 + g_0) \gamma_y}{\gamma_d d_0^*}; \beta_d = \frac{r_0 + (1 + g_0)^3}{(1 + g_0)}; \beta_e = \frac{r_0 \omega_q}{\gamma_d}; \\ \beta_f &= \frac{(1 + g_0) \gamma_r}{\gamma_d d_0^*}; \beta_g = \frac{(1 + g_0) \gamma_{yf}}{\gamma_d d_0^*}; \varepsilon_t^e = \rho_e \varepsilon_{t-1}^e + \mu_t^e \end{aligned}$$

Eq. (24) describes the external debt evolution where g_0 , is the steady state growth rate, r_0 , is the steady state interest rate, and d_0^* is steady state debt-GDP ratio. There is a negative relation between external debt and commodity prices. In addition, external debt to GDP depends negatively on domestic output and positively on foreign output. Lastly, external debt to GDP depends positively on foreign interest rate and real exchange rate.

4.4.6 Monetary policy

We equate Eqs. (4) and (6) and linearize to derive the money market equation. We substitute Eqs.(10) and (11) and the money market equation is

given as:

$$\tilde{r}_t = \eta_a \tilde{y}_t - \eta_b \tilde{m}_t + \eta_c \tilde{s}_t - \varrho E_t \tilde{s}_{t+1} - \eta_d \tilde{r} \tilde{e} r_t - \eta_e \tilde{y}_t^f + \varepsilon_t^b \quad (25)$$

where

$$\begin{aligned} \varepsilon_t^r &= \rho_b \varepsilon_{t-1}^r + \mu_t^b; \eta_a = \frac{\gamma_r (1 + \gamma_y)}{\gamma_m \gamma_c}; \eta_b = \frac{\gamma_r}{\gamma_m}; \eta_c = \frac{\varrho}{s_0} \gamma_r; \\ \eta_d &= \frac{\gamma_r \gamma_r}{\gamma_m \gamma_c}; \eta_e = \frac{\gamma_r \gamma_y f}{\gamma_m \gamma_c} \end{aligned}$$

Eq.(25) describes the money market equation. The interest rate depends positively on real output and negatively on real balances. The interest rate also depends positively on current exchange rate and negatively on expected future exchange rate. Lastly, interest rate depends negatively on real exchange rate and foreign output.

Given monetary aggregate targeting in Africa, we follow Muhanji and Ojah (2011) in the specification of our monetary policy reaction function. Money supply is driven by inflation gap and output gap and commodity prices gap. We include the real exchange rate and sovereign wealth funds in our money-type Taylor rule. The monetary aggregate is:

$$\begin{aligned} \tilde{m}_t &= \rho_m \tilde{m}_{t-1} - (1 - \rho_m) [\rho_\pi \tilde{\pi}_{t-1} + \rho_y \tilde{y}_{t-1} + \rho_{rer} \tilde{r} \tilde{e} r_{t-1} + \rho_q \tilde{q}_{t-1} - \omega_f \tilde{f}_{t-1}] \\ &\quad + \varepsilon_t^m \end{aligned} \quad (26)$$

where all variables are in the steady states. $\tilde{m}_t$ is monetary aggregate, $\tilde{\pi}_t$ is inflation rate gap; $\tilde{y}_t$ is output gap; $\tilde{r} \tilde{e} r_t$ is real exchange rate gap, $\tilde{q}_t$ is

commodity price gap and $\tilde{f}$ is the sovereign wealth funds (SWFs). Commodity revenue affects money supply in African countries. Hence, we introduce the SWFs in the monetary policy rule to evaluate policy reaction when the SWFs takes part of the revenue. The disturbance follows an AR(1) process: $\varepsilon_t^m = \rho_c \varepsilon_{t-1}^m + \mu_t^c$. The parameters ρ_m is policy rate smoothing, ρ_π is policy reaction to inflation gap, ρ_y is policy reaction to output gap, ρ_{rer} is policy reaction to real exchange rate gap, and ρ_q is policy reaction to commodity price gap, and ω_f is the policy reaction to the sovereign wealth funds. The structural shock processes in the model are given by the following vector:

$$\tilde{\xi}_t = \rho_\xi \tilde{\xi}_t + \varepsilon_{\xi,t}; \quad \varepsilon_{\xi,t} \sim N(0, \sigma_\xi^2) \quad (27)$$

where

$$\varepsilon_{\xi,t} = \begin{bmatrix} \tilde{q}_t, \tilde{r}_t^f, \tilde{y}_t^f, \tilde{p}_{it}, \tilde{\pi}_t, \tilde{rer}_t \end{bmatrix}$$

4.5. Calibration

Table 3 shows the calibration for the study. The calibration for the habit parameter h , price stickiness parameter θ , price indexation parameter ω , and good elasticity of substitution ε comes from Castelnuovo (2012). The calibration of α , σ , and ψ , comes from Smets and Wouters (2007). Similarly, the calibration for monetary policy reaction coefficient to output (ρ_y), inflation (ρ_π), and monetary smoothing parameter (ρ_m) is from Smets and Wouters (2007). The calibration for the inverse elasticity of money holding ϕ is in line with Benchimol and Fourçans (2012). The parameter for currency substitution is calibrated along the line of Elkhafif (2002). The calibration for policy reaction to exchange rate (ρ_{rer}) comes from García and González (2013). We assume the same value for reaction to commodity price shocks.

Moreover, the calibration of the discount factor β follows the standard in the literature (see Steinbach et al., 2009). The calibration of the sum of elasticity

of home and foreign production γ_r is from McCallum and Nelson (2000). Our calibration for γ_y , γ_{yf} , and γ_c follows Lombardo and McAdam (2012),. In line with Batini et al. (2005), we assume that input requirements per unit of output is increasing at the margin because firms tend to use less efficient machines as output rises, we therefore set $\delta_i = 2$. We assume that the steady state share of intermediate input costs in total output $p_{i0}x_0y_0^{-1} = 0.5$. This is a reasonable calibration for African economies, given the evidence by Eifert et al.(2008).

Table 4.3: Calibrated parameter

Description	Parameter	Value
Habit formation	h	0.70
Elasticity of money holding	ϕ	1.25
Intertemporal elasticity of substitution	σ	1.50
Capital share in production	α	0.33
Inv. of Frisch labour elasticity	ψ	2.00
Elasticity of currency substitution	ϱ	0.30
Goods elasticity of substitution	ε	6.00
Discount factor	β	0.99
Price stickiness	θ	0.65
Price indexation	ω	0.50
Elasticity of risk premium to external debt	ω_d	0.50
Elasticity of risk premium to commodity price	ω_q	0.20
Policy response to output	ρ_y	0.12
Policy response to inflation	ρ_π	1.50
Policy rate smoothing	ρ_{mag}	0.75
Policy response to real exchange rate	ρ_{rer}	0.50
Policy response to commodity price	ρ_q	0.50
Sum of elast. of subst. for home and foreign prod.	γ_r	0.66
Elasticity of net export to domestic output	γ_y	0.20
Elasticity of net export to foreign output	γ_{yf}	0.20
Steady state ratio of consumption to output	γ_c	0.58
Steady state ratio of gov. expenditure to output	γ_g	0.25
Share of budget deficit financed by foreign debt	ϕ_m	0.5

4.6. Simulations

We simulate our baseline model based on the calibrated values for our parameters in table 1. To investigate the effectiveness of sovereign wealth funds

(SWFs) in reducing macroeconomic instability, we simulate our model with SWFs under different values for (τ_q) that goes into the SWFs account. Our hypothetical value of τ_q - the share of the windfall that goes into the funds is 0.5 . Generally, our simulation shows that the creation of SWFs can reduce the macroeconomic volatility associated with commodity price fluctuations.

4.6.1 Commodity price shock

Fig. 1 illustrates the responses of selected macroeconomic variables to a positive commodity price shock under the baseline model (model without SWFs) and the model with sovereign wealth funds (SWFs). Generally, the simulation results indicate that SWFs can reduce the volatility of macroeconomic variables in commodity exporting countries. There is stability in all the macroeconomic variables following a positive commodity price shock. For example, the volatility in government expenditure is lower under the SWFs model compared to the baseline model. This indicates that SWFs can be used to reduce expenditure volatility in commodity exporting African countries.

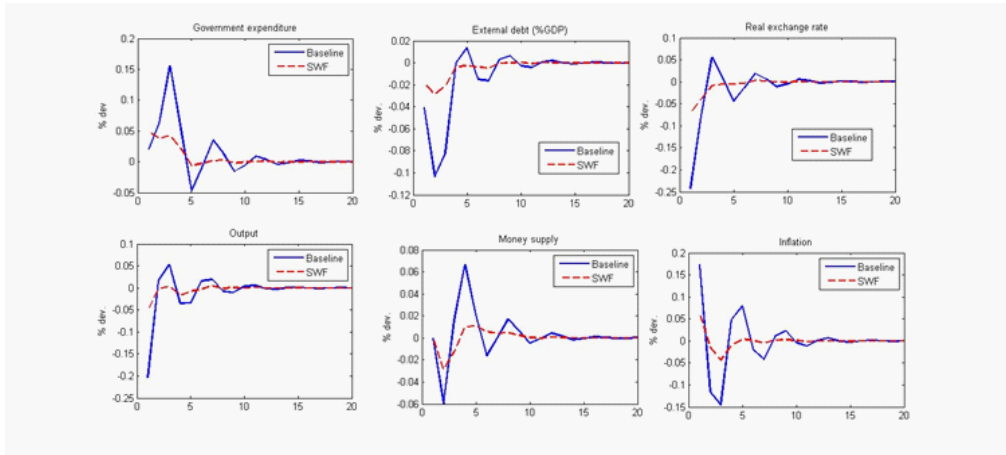
The SWFs prevent the total injection of windfall into the domestic economy thereby insulating the economy from the inflow of commodity revenue. This is in line with the findings by Everhart and Duval-Hernandez (2001) for Mexico. Since fiscal policy is the main channel through which commodity price shocks are transmitted to the domestic economy (see Spatafora and Samake, 2012), SWFs can reduce the macroeconomic instability in commodity exporting African countries.

In addition, the simulation result indicates that the SWFs are effective in reducing external debt and the real exchange rate volatility in commodity exporting countries. Under the baseline model, external debt seems to be more volatile after a positive commodity price shock. The introduction of SWFs reduces the volatility of external debt. Since part of the windfall is

channeled to the SWFs, this prevents using the whole revenue to pay the for all the debt. Moreover, the real exchange rate remains stable under the SWFs model after a rise in commodity price. The SWFs absorb the commodity revenue that could cause real exchange rate appreciation. Through this channel, SWFs can prevent the occurrence of the Dutch disease problem. This suggests that the creation of SWFs can mitigate the adverse effects of commodity price fluctuations in commodity exporting countries.

Furthermore, the simulation results suggest that the introduction of SWFs can stabilize output volatility arising from commodity price shocks. This might be through the effects of SWFs on the exchange rate. While exchange rate appreciation occurs under the baseline and the SWFs model, the appreciation is small under the SWFs model. The minimal size of appreciation under the SWFs model prevents output volatility. Moreover, the introduction of the SWFs reduces the volatility of money supply. By directing part of the commodity windfall into the SWFs, the government reduces the effect of commodity revenue on the monetary base. This indicates that SWFs can be used as a tool of sterilization in African countries. This is similar to the conclusion by Astrov (2007) for Russia. Similarly, the simulation result suggests that inflation is stable under the model with SWFs.

Fig. 3.1. Responses of macroeconomic variables to positive commodity price shocks



The simulation result in fig. 3.1 is however sensitive to the choice of calibrated value for the elasticity of sovereign risk premium (ω_q) to commodity prices. When this elasticity is high ($\omega_q > 0.4$), the effectiveness of SWFs in insulating the economy from external shocks is weakened. This indicates that if the sovereign risk premium for commodity exporters is high, SWFs impact may be weakened.

4.7. Conclusion

African countries derive their export revenues from the export of a narrow range of primary commodities. This exposes the African economies to fluctuations in commodity prices. Moreover, fiscal operations in African countries strongly correlate with commodity prices, hence commodity price volatility is directly translated to fiscal uncertainty. The fiscal policy volatility generates macroeconomic instability in commodity exporting African countries. In this paper, we formulate an open economy DSGE model with sovereign wealth funds (SWFs) to investigate the effectiveness of SWFs in reducing macroeconomic instability in commodity exporting African countries.

We simulate the model for two different scenarios- a baseline model without the SWFs and a model with SWFs. The simulation results indicate that the creation of SWFs can insulate the African economies from commodity price shocks. The creation of SWFs can reduce expenditure volatility in commodity exporting African countries. Given the fact that the government expenditure is an important channel through which the effects of commodity price shocks are transmitted to African economies, the creation of SWFs might protect the African economies against volatile commodity prices. This is similar to the results by Shabsigh and Ilahi (2007). Moreover, through the SWFs impact on fiscal balances, the resource curse syndrome may be reversed in African economies.

In addition, we find that the creation of sovereign wealth funds can stabilize external debt, real exchange rate, money supply, and output of African

countries. SWFs may stabilize the external debt following a commodity price shocks. Similarly, our simulation results suggest that SWFs can insulate the exchange rate from commodity price shocks. This can serve as a mechanism for preventing the Dutch disease problem associated with commodity price boom in African economies. Also, we find that the introduction of SWFs lowers the volatility of output and money supply. This serves to prevent the macroeconomic distortion caused by inflow of commodity revenue.

In view of the results on the effectiveness of sovereign wealth funds (SWFs) on fiscal expenditure, policy makers in African countries should set up SWFs to insulate the economy from the vulnerability to external shocks. Empirical findings have shown that African countries are vulnerable to external shocks (see Kose and Riezman, 2001). Moreover, SWFs should also be created in African countries to prevent the resource curse problem. Furthermore, African countries should establish SWFs to sterilize the inflow of commodity revenue during the period of commodity price boom.

CHAPTER FIVE

Conclusion and Policy Implications

5.1 Introduction

This thesis has 3 objectives. Firstly, we investigate the relative contributions of internal and external shocks to macroeconomic fluctuations in ten African economies. Secondly, we evaluate the quantitative significance of the balance sheet channel in African economies. Thirdly, we assess the effectiveness of sovereign wealth funds (SWFs) in ensuring fiscal and macroeconomic stability in commodity exporting African countries.

5.2 Sources of macroeconomic fluctuations in Africa

In chapter 2, we investigate the sources of macroeconomic fluctuations in ten (10) African countries over 1990-2011. We consider eleven macroeconomic shocks that are considered empirically important for the African economies. We formulate and estimate an open economy monetary dynamic stochastic general equilibrium (DSGE) model. We build on the work of Kose and Riezman (2001) . We employ the Bayesian technique to estimate our model. We also employ variance error decomposition to show the contribution of each shocks to economic fluctuations and impulse response function to illustrate the dynamic response of output to different shocks.

We find that both internal and external shocks account for output fluctuations in African economies. The effects of internal shocks are short-lived; they last up till the 4th quarter. The most important internal shocks influencing output in African economies are productivity and money supply shocks. In contrast, external shocks have long-lasting impacts on African economies. Their effects range from the 8th to 16 quarters. Among the external shocks,

external debts, foreign interest rate, foreign input price, exchange rate and commodity price shocks are found to account for large output fluctuations in African economies.

The dominant effects of external shocks might be attributed to the dependence of many African countries on the export of primary commodities for their export earnings. Given the volatility of the commodity prices and procyclical fiscal stance of African countries, negative commodity price shocks induce fiscal deficits which are financed by external borrowing. Consequently, a positive shock to world interest rate aggravate the debt problem as the cost of servicing debt increases. The rising debt service cost reduces capital expenditure, public investment and economic growth. We recommend that African countries need to diversify the economies from the dependence on export of primary commodities. We recommend the diversification of the economy and creation of sovereign wealth funds (SWFs) to cushion the effects of commodity price fluctuations.

5.3. The balance sheet channel in African economies

In chapter 3, we evaluate the role played by financial market imperfections in the propagation and amplification of relatively small shocks to the real economy in nine (9) African countries. We formulate and estimate an open economy monetary DSGE model with financial market imperfection. In the model, entrepreneur net worth determines the cost of borrowing. The higher the entrepreneur net worth, the lower the cost of borrowing and vice versa. In a manner similar to Elekdag et al. (2006), we estimate our model with Bayesian technique. Also, similar to Castelnovo (2012), we compare the model with and without financial imperfections.

Our results suggest that the balance sheet channel is quantitatively important for African economies. The presence of the balance sheet channel dampens the expansionary effects of exchange rate on output in African countries.

Following exchange rate depreciation, output contracts rather than expands as in the traditional Mundel-Fleming hypothesis. In many countries, output tends to negative following exchange rate depreciations. This can be attributed to the balance sheet effects. When debts are held in foreign denominated currencies, exchange rate depreciation increases the domestic value of debt and deteriorates the country's balance sheet. The worsening of balance sheet increases the cost of capital, lower investment and output. We also find evidence of J-curve effects under the model without financial imperfection. Following exchange rate depreciation, output initially contracts but later expands in the medium to long run.

5.4 Sovereign wealth funds (SWFs) and macroeconomic stability

In chapter 4, we investigate whether the establishment of SWFs can reduce macroeconomic instability in commodity exporting African countries. We formulate and simulate a DSGE model incorporated with the fiscal sector and sovereign wealth funds. Our simulation results suggest that sovereign wealth funds reduce fiscal volatility in commodity exporting countries. Through SWFs impacts on fiscal outcomes, macroeconomic instability in commodity exporting countries can be reduced. This arises from the fact that fiscal operations serve as the main channel through which the impacts of commodity prices are transmitted to the economies of commodity exporting countries (see, e.g., Medina, 2010; El Anshasy and Bradley, 2012).

5.5 Policy implications and recommendations of the study

The results of the study have important policy implications for African economies. In view of the vulnerability of African economies to external shocks, it is imperative that African countries formulate policies such as sound external debt management and hedging strategies to insulate or mitigate the effects of external shocks. Since many of these economies rely heavily on very few, if not one, commodities for foreign exchange earnings,

it is necessary that these countries pursue policies that promote industrial diversification. Another way in which African countries can create a buffer against adverse external shocks is to set up sovereign wealth funds (SWFs). These funds can be used to reduce macroeconomic volatility arising from commodity price boom-bust cycles. Establishment of SWFs with transparent rules of operation can be used to support balanced fiscal positions and promote macroeconomic stability.

The findings pose a policy dilemma for monetary authorities in African economies. If loose monetary policy is implemented, the low interest rate reinforces the exchange rate depreciation thereby amplifying the output loss. Alternatively, if tight monetary policy is introduced, the high interest rate strengthens the currency but contracts output. Tight monetary policy seems to be the optimal response in the face of balance sheet effects. Important policy recommendations for African countries is the reduction in their exposure to foreign currency debts and deepening of the domestic bond markets. The deepening of domestic bond market will not only reduce their exposure to foreign currency debts but also give African countries the policy space to react to exogenous shocks.

Moreover, the simulation results suggest that SWFs reduce volatility in fiscal spending in commodity exporting countries. Fiscal spending is the main channel through which macroeconomic effects of commodity price shocks are transmitted to the domestic economy. Hence, SWFs might promote macroeconomic stability in commodity exporting African countries.

5.6 Suggested areas for further research

Further studies should include fiscal policy shocks among shocks affecting African economies. Furthermore, other studies should also compare fiscal rules and SWFs in the promotion of macroeconomic stability in commodity exporting African countries.

References

- Adom, A. D., Sharma, S. C., & Morshed, A. K. M. (2008). Currency Substitution in Selected African Countries. *Journal of Economic Studies*, 36(6), 616-640.
- Agenor, P.-R., McDermott, C. J., & Prasad, E. S. (1999). Macroeconomic Fluctuations in Developing Countries: Some Stylized Facts. *IMF Working Paper*, 99(35).
- Aghion, P., Bacchetta, P., & Banerjee, A. (2000). Currency Crises and Monetary Policy in an Open Economy with Credit Constraints. *European Economic Review*, 45, 1121-1150.
- Aghion, P., Bacchetta, P., & Banerjee, A. (2004). A Corporate Balance-Sheet Approach to Currency Crises. *Journal of Economic Theory*, 119, 6-30.
- Aguiar, M. (2005). Investment, Devaluation, and Foreign Currency Exposure: The Case of Mexico. *Journal of Development Economics*, 78, 95-113.
- Ahmed, S. (2003). Sources of Economic Fluctuations in Latin America and Implications for Choice of Exchange Rate Regimes. *Journal of Development Economics*, 72, 181-202.
- Andrés, J., López-Salido, J. D., & Vallés, J. (2006). Money in an Estimated Business Cycle Model of the Euro Area. *The Economic Journal*, 116(457-477).
- Arezki, R., Freytag, E. D., & Quintya, M. (2012). Commodity Prices and Exchange Rate Volatility: Lessons from South Africa's Capital Account Liberalization. *IMF Working Paper*, 12(168).
- Arezki, R., & Ismail, K. (2013). Boom-bust cycle, Asymmetrical Fiscal Response and the Dutch disease. *Journal of Development Economics*, 101, 256-267.
- Arreaza, A., Castilla, L. M., & Fernandez, C. (2009). The Coming Age of Sovereign Wealth Funds: Perspectives and Policy Issues within and beyond Borders. *Global Journal of Emerging Market Economies* 1(1), 25-41.
- Ashafa, S. G. (2007). National Resource Funds: Their Efficacy for Fiscal

Stability and Inter-generational equity. *International Institute for Sustainable Development Mimeo*.

Astrov, V. (2007). The Russian Oil Funds as a Tool of Stabilization and Sterilization. *The Vienna Institute for International Economic Studies Mimeo*.

Baldini, A. (2005). Fiscal Policy and Business Cycles in an Oil-Producing Economy: The Case of Venezuela. eloping Economies. *IMF Working Paper (237)*.

Bastos, F., & Divino, J. A. (2009). Exchange Rate and Output Fluctuations in the Small Open Economy of Mauritius. *The World Bank Policy Research Working Paper (5065)*.

Basu, P., & McLeod, D. (1992). Terms of Trade Fluctuations and Economic Growth in Developing Economies. *Journal of Development Economics*, 37, 89-110.

Batini, N., Jackson, B., & Nickell, S. (2005). An Open-economy New Keynesian Phillips Curve for the UK. *Journal of Monetary Economics*, 52, 1061-1071.

Basu, S., Gottschalk, J., Schule, W., Vellodi, N., & Yang, S.-C. S. (2013). The Macroeconomic Effects of Natural Resource Extraction: Applications to Papua New Guinea IMF Working Paper(138).

Baunsgaard, T. (2003). Fiscal policy in Nigeria: any role for Rules? *IMF Working Paper(155)*.

Bebczuk, R., Galindo, A., & Panizza, U. (2006). An Evaluation of Contractionary Devaluation Hypothesis. *Inter-American Development Bank Working Paper(582)*.

Beck, T., Lundberg, M., & Majnoni, G. (2006). Financial Intermediaries Development and Growth Volatility: Do Intermediaries Dampen or Magnify Shocks. *Journal of International Money and Finance*, 25, 1147-1167.

Benavente, J. S., Johnson, C. A., & Morande, F. G. (2003). Debt Composition and Balance Sheet Effects of Exchange Rate Depreciations: A Firm-level Analysis for Chile. *Emerging Markets Review*, 4, 397-416.

- Benchimol, J., & Fourçans, A. (2012). Money and Risk in a DSGE Framework: A Bayesian Application to the Eurozone. *Journal of Macroeconomics*, 34, 95-111.
- Berganza, J. C., Chang, R., & Herrero, A. C. (2004). Balance Sheet Effects and the Country Risk Premium: An Empirical Investigation. *Review of World Economics*, 140(4), 592-612.
- Bernanke, B. S., & Gertler, M. (1989). Agency Costs, Net Worth, and Business Fluctuations. *The American Economic Review*, 79(1), 14-31.
- Bernanke, B. S., Gertler, M., & Gilchrist, S. (1999). The Financial Accelerator in a Quantitative Business Cycle Framework. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics* (Vol. 1, pp. 1341-1393): Elsevier Science B. V.
- Berument, H., & Kilinc, Z. (2004). The Effects of Foreign Income on Economic Performance of a Small Open Economy: Evidence from Turkey. *Applied Economic Letters*, 11, 483-488.
- Bleakley, H., & Cowan, K. (2008). Corporate Dollar Debt and Depreciations: Much Ado about Nothing. *The Review of Economics and Statistics*, 90(4), 612-626.
- Bleaney, M., & Greenaway, D. (2001). The Impact of Terms of Trade and Real Exchange Rate Volatility on Investment and Growth in sub-Saharan Africa. *Journal of Development Economics*, 65, 491-500.
- Blejer, M. I., Feldman, E. V., & Feltenstein, A. (2002). Exogenous Shocks, Contagion, and Bank Soundness: A Macroeconomic Framework. *Journal of International Money and Finance*, 21, 33-52.
- Bodart, V., Candelon, B., & Carpentier, J. F. (2012). Real Exchange Rates in Commodity Producing Countries: A Reappraisal. *Journal of International Money and Finance*, 31, 1482-1502.
- Bonomo, M., Martins, B., & Pinto, R. (2003). Debt Composition and Exchange Rate Balance Sheet Effects in Brazil: A Firm Level Analysis. *Emerging Markets Review*, 4, 368-396.

- Bordo, M. D., Meissner, M., & Stuckler, D. (2010). Foreign Currency Debt, Financial Crises and Economic Growth: A Long-Run View. *Journal of International Money and Finance*, 29, 642-665.
- Bova, E. (2009). Is the Zambian Kwacha a Commodity Currency? The Comovement between Copper Prices and the Exchange Rate. *Swiss National Centre of Competence Research Working Paper*(2009/11).
- Brunnschweiler, C. N., & Bulte, E. H. (2008). The resource curse revisited and revised: a tale of paradoxes and red herrings. *Journal of Environmental Economics and Management*, 55, 248-264.
- Carlstrom, C. T., & Fuerst, T. S. (1996). Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. *Federal Reserve Bank of Cleveland Working Paper*(9602).
- Calvo, G. A. & Reinhart, C. M. (2002). Fear of Floating. *The Quarterly Journal of Economics*, 117(2), 379-408.
- Canova, F. (2005). The Transmission of US Shocks to Latin America. *Journal of Applied Econometrics*, 20, 229-251.
- Carranza, L. J., Cayo, J. M., & Galdón-Sánchez, J. E. (2003). Exchange Rate Volatility and Economic Performance in Peru: A Firm Level Analysis. *Emerging Markets Review*, 4, 472-496.
- Cashin, P., Céspedes, L. F., & Sahay, R. (2004). Commodity Currencies and the Real Exchange Rate. *Journal of Development Economics*, 75, 239-268.
- Castelnuovo, E. (2012). Estimating the Evolution of Money's Role in the US Monetary Business Cycle. *Journal of Money, Credit and Banking*, 44(1), 23-52.
- Céspedes, L. F. (2005). Financial Frictions and Real Devaluations. *Central Bank of Chile Working Paper*(318).
- Céspedes, L. F., Chang, R., & Velasco, A. (2004). Balance Sheets and Exchange Rate Policy. *The American Economic Review*, 94(4), 1183-1193.
- Choi, W. G., & Cook, D. (2004). Liability Dollarization and the Bank Balance Sheet Channel. *Journal of International Economics*, 64, 247-275.

- Christensen, I., & Dib, A. (2008). The Financial Accelerator in an Estimated New Keynesian Model. *Review of Economic Dynamics*, 11, 155-178.
- Chue, T. K., & Cook, D. (2008). Sudden Stops and Liability Dollarization: Evidence from Asia's Financial Intermediaries. *Pacific-Basin Finance Journal*, 16(436-452).
- Collier, P. (2007). Managing Commodity Booms: Lesson of International Experience. *A Paper for African Economic Research Consortium*
- Collier, P. & Goderis, B. (2012). Commodity Prices and Growth: An Empirical Investigation. *European Economic Review*, 56, 1241-1260.
- Cook, D. (2004). Monetary Policy in Emerging Markets: Can Liability Dollarization Explain Contractionary Devaluations? *Journal of Monetary Economics*, 51, 1155-1181.
- Dauvin, M. (2014). Energy Prices and the Real Exchange Rate in Commodity-Exporting Countries. *International Economics*, 137, 52-72.
- Davis, J., Ossowski, R., Daniel, J., & Barnett, S. (2001). Stabilization and saving funds for non-renewable resources: experience and fiscal policy implications. *IMF Occasional Paper*(205).
- Dehn, J. (2000). Private Investment in Developing Countries: The Effects of Commodity Shocks and Uncertainty . *The University of Oxford Working Paper*, WPS/2000-11.
- Devlin, J., & Titman, S. (2004). Managing Oil Price Risk in Developing Countries. *The World Bank Research Observer*, 19(1), 119-139.
- Ebert, L., & La Menza, T. (2015). Chile, Copper, and Resource Revenue: A Holistic Approach to Assessing Commodity Dependence. *Resource Policy*, 43, 101-111.
- Echeverry, J. C., Fergusson, R. S., & Aguilar, C. (2003). "Dollar" Debt in Colombian Firms: Are Sinners Punished During Devaluations? *Emerging Markets Review*, 4, 417-449.
- Edwards, S. (1984). Coffee, Money, and Inflation in Columbia. *World Development*, 12(11/12), 1107-1117.

- Edwards, S. (1986). Devaluation and Economic Activity: An Empirical Analysis of Contractionary Devaluation Issue. *UCLA Working Paper*(412).
- Eichengreen, B., & Hausmann, R. (1999). Exchange Rate and Financial Fragility. In: New Challenges for Monetary Policy. *Federal Reserve Bank of Kansas City*, 329-368.
- Eifert, B., Gleb, A., & Ramachandran, V. (200). The Cost of Doing Business in Africa: Evidence from Enterprise Survey Data. *World Development*, 36(9), 1531-1546.
- Elekdağ, S., Justiniano, A., & Tchakarov, I. (2006). An Estimated Small Open Economy Model of the Financial Accelerator. *IMF Staff Paper*, 53(2), 219-241.
- Elekdağ, S., & Tchakarov, I. (2007). Balance Sheets, Exchange Rate Policy, and Welfare. *Journal of Economic Dynamics and Control*, 31, 3986-4015.
- El Anshasy, A. A., & Bradley, M. (2012). Prices and fiscal policy response in oil-exporting countries. *Journal of Policy Modeling*, 34, 605-620.
- Elkhafif, M. (2002). Exchange Rate Policy and Currency Substitution: The Case of Africa's Emerging Economies. *African Development Bank Economic Research Papers*(71).
- Erbil, N. (2011). Is Fiscal Policy Procyclical in Developing Oil-Producing Countries? *IMF Working Paper*(11/171).
- Everhart, S., & Duval-Hernandez, R. (2001). Management of Oil Windfalls in Mexico: Historical Experience and Policy Options for the Future. *The World Bank Policy Research Working Paper*(2592).
- Fasano, U. (2000). Review of the experience with oil stabilization and saving funds in selected countries, *IMF Working Paper*(112).
- Flood, R. P., & Rose, A. K. (1995). Fixing Exchange Rate: A Virtual Quest for Fundamentals. *Journal of Monetary Economics*, 36, 3-37.
- Forbes, K. J. (2002). How Do Large Depreciation Affect Firm Performance. *IMF Staff Paper*, 49(Special Issue), 214-238.
- Fosu, A. K. (1999). The External debt Burden and Economic Growth in the

- 1980s: Evidence from Sub-Saharan Africa. *Canadian Journal of Development Studies*, 20(2), 307-318.
- Frankel, J. (2007). On the Rand: Determinants of South African Exchange Rate. *South African Journal of Economics*, 75(3), 425-441.
- Frankel, J. (2011). How can commodity exporters make fiscal and monetary policy less procyclical? *HKS Faculty Research Working Paper Series RWP* 11-015, John F. Kennedy School of Government, Harvard University
- Galí, J., & Gertler, M. (1999). Inflation Dynamics: A Structural Econometric Analysis. *Journal of Monetary Economics*, 44, 195-222.
- Galí, J., Gertler, M., & López-Salido (2001). European Inflation Dynamics. *European Economic Review*, 45, 1237-1270.
- García, C. J. & González, W. D. (2013). Exchange Rate Intervention in Small Open Economies: The Role of Risk Premium and Commodity Price Shocks. *International Review of Economics and Finance*, 25, 424-447.
- Ghosh, A. R., Gulde, A- M., Ostry, J. D., & Wolf, H. C. (1997). Does the Nominal Exchange Rate Matter?. *NBER Working Paper Series*, 5874.
- Guillaumont, P., Jeanneney, S. G., & Brun, J.-F. (1999). How Instability Lowers African Growth. *Journal of African Economies*, 8(1), 87-107.
- Hammed, S. (2003). Sources of Economic Fluctuations in Latin America and Implications for Choice of Exchange Rate Regimes. *Journal of Development Economics*, 72(1), 181-202.
- Hoffmaister, A. W., & Roldós, J. E. (2001). The Sources of Macroeconomic Fluctuations in Developing Countries: Brazil and Korea. *Journal of Macroeconomics*, 23(1), 213-239.
- Hoffmaister, A. W., Roldós, J. E., & Wickham, P. (1998). Macroeconomic Fluctuations in Sub-Saharan Africa. *IMF Staff Paper*, 45, 132-160.
- Hsing, Y. (2003). Impacts of External Debt and other Macroeconomic Policies on Output in Brazil: A VAR Approach. *Revista de Analisis Economico*, 18(2), 97-108.
- Hsing, Y. (2008). A Study of the J-Curve for Seven Selected Latin American

- Countries. *Global Economic Journal* 8(4), 1-12.
- Iwayemi, A., & Fowowe, B. (2011). Impact of Oil Price Shocks on Selected Macroeconomic Variables in Nigeria. *Energy Policy*, 39, 603-612.
- Jeane, O., & Zettlemeyer, J. (2002). "*Original Sin*", *Balance Sheet Crises and the Roles of International Lending*. Paper Presented at the Currency and Maturity Matchmaking: Redeeming Debt from Original sin, Washington.
- Jidoud, A. (2012). The Sources of Macroeconomic Fluctuations in Sub-Saharan African Economies: An Application to Côte d'Ivoire. (*TSE-GREMAQ*) *Mimeo*.
- Jumah, A., & Kunst, R. M. (2007). Inflation in West African Countries: The Impact of Cocoa Prices, Budget Deficit, and Migrant Remittances. *Institute for Advanced Studies, Vienna, Economic Series*(219).
- Kalyuzhnova, Y. (2011). The National Funds of the Republic of Kazakhstan (NFRK): From Accumulation to Stress-test to Global Future. *Energy Policy*, 39(6650-6657).
- Kamber, G., & Thoenissen, C. (2012). The Financial Accelerator and Monetary Policy Rules, *Economic Letters*, 115, 309-313.
- Kaminsky, G. L. (2010). Terms of Trade Shocks and Fiscal Cycles. *NBER Working Paper*(15780).
- Kamin, S. B., & Rogers, J. H. (2000). Output and Real Exchange Rate in Developing Countries: An Application to Mexico. *Journal of Development Economics*, 61, 85-109.
- Kanczuk, F. (2004). Real Interest Rates and Brazilian Business Cycles. *Review of Economic Dynamics*, 7, 436-455.
- Kandil, M. (2014). On the Effects of Monetary Policy Shocks in Developing Countries. *Borsal Istanbul Review*, 14, 104-118.
- Kandil, M. & Mirzaie, I. (2005). The Effects of Exchange Rate Fluctuations on Output and Prices: Evidence from Developing Countries. *The Journal of Developing Areas*, 38(2), 189-219.
- Koop, G. (2007). *Bayesian Econometrics*: Cambridge University Press.

- Koranchelian, T. (2005). The Equilibrium Real Exchange Rate in a Commodity Exporting Country: Algerian Experience. *IMF Working Paper*(05/135).
- Korinek, A. (2011). Foreign Currency Debt, Risk Premia, and Macroeconomic Volatility. *European Economic Review*, 55, 371-385.
- Kose, M. A. (2002). Explaining Business Cycle in Small Open Economies: How Much do World Prices Matter? . *Journal of International Economics*, 56, 299-327.
- Kose, M. A., & Riezman, R. (2001). Trade Shocks and Macroeconomic Fluctuations in Africa. *Journal of Development Economics*, 65, 55-80.
- Krugman, P. (1999). Balance Sheets, the Transfer Problem and Financial Crises. *International Tax and Public Finance*, 6, 459-472.
- Kumah, F. Y., & Matovu, J. M. (2007). Commodity price shocks and the odds on fiscal performance: a structural vector autoregression approach. *IMF Staff Paper*, 54(1).
- Maćkowiak, B. (2007). External Shocks, U.S Monetary Policy and Macroeconomic Fluctuations in Emerging Markets. *Journal of Monetary Economics*, 54, 2512-2520.
- Le Borgne, E., & Medas, P. (2007). Sovereign Wealth Funds in the Pacific Island Countries: Macro-Fiscal Linkages. *IMF Working Paper*(297).
- Lofgren, H., Robinson, S., & Thurlow, J. (2002). Macro and Micro Effects of Recent and Potential Shocks to Copper Mining in Zambia. *TMD Discussion Paper*(99).
- Lombardo, G. & McAdam, P. (2012). Financial Market Frictions in a Model of the Euro Area. *Economic Modelling*, 29, 2460-2485.
- Malikane, C. (2014). A New Keynesian Triangle Phillips Curve. *Economic Modelling*, 43, 247-255.
- Mallick, S. K., & Sousa, R. M. (2012). Real Effects of Monetary Policy in Large Emerging Economies. *Macroeconomic Dynamics*, 16(2), 190-212.
- McCallum, B. T., & Nelson, E. (2000). Monetary Policy for an Open Economy: An Alternative Framework with Optimizing Agents and Sticky Prices.

- Oxford Review of Economic Policy*, 16(4), 74-91.
- Medina, L. (2010). The Dynamic Effects of Commodity Prices on Fiscal Performance in Latin America *IMF Working Paper*(10/192).
- Mehrara, M., Karsalari, A. R., & Haghiri, F. (2012). Oil funds and the instability of macroeconomy in oil rich countries. *World Applied Science Journal*, 16(3), 331-336.
- Mehrara, M., & Oskoui, K. N. (2007). The Source of Macroeconomics in Oil-exporting Countries: A Comparative Study. *Economic Modelling*, 24, 365-379.
- Mendoza, E. G. (1995). The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations. *International Economic Review*, 35(1), 101-137.
- Mohanty, M. S., & Klau, M. (2004). Monetary Policy Rules in Emerging Market Economies: Issues and Evidence. *BIS Working Paper*(149).
- Muhanji, S., & Ojah, K. (2011). External shocks and Persistence of External Debt in Open Vulnerable Economies: The Case of Africa. *Economic Modelling*, 28, 1615-1628.
- Mulder, C., Perrelli, R., & Rocha, M. D. (2012). External Vulnerability, Balance Sheet Effects, and the Institutional Framework: Lessons from Asian Crisis. *International Review of Economics and Finance*, 21(16-28).
- Olomola, P. A., & Adejumo, A. V. (2006). Oil Price Shock and Macroeconomic Activities in Nigeria. *International Research Journal of Finance and Economics*(3), 28-34.
- Papyrakis, E., & Gerlagh, R. (2004). The resource curse hypothesis and its transmission channels. *Journal of Comparative Economics*, 32, 181-193.
- Park, D. (2008). Capital Outflows, Sovereign Wealth Funds, and Domestic Financial Instability in Developing. *Asia Asian Development Bank Working Brief Series*(129).
- Pratap, S., Lobato, I., & Somuano, A. (2003). Debt Composition and Balance Sheet Effects of Exchange Rate Volatility in Mexico: A Firm Level Analysis. *Emerging Markets Review*, 4, 450-471.

- Pratap, S. & Urrutia, C. (2003). Firm dynamics, Investment and Debt Portfolio: Balance Sheet Effects of the Mexican Crisis of 1994. *Journal of Development Economics*, 74, 535-563.
- Raddatz, C. (2007). Are External Shocks Responsible for the Instability of Output in Low-income Countries? *Journal of Development Economics*, 84, 155-187.
- Raddatz, C. (2008). Have External Shocks become more Important for Output Fluctuations in African Countries? In D. S. Go & J. Page (Eds.), *Africa at a Turning Point? Growth, Aid, and External Shocks*. Washington, US: World Bank Publication.
- Raju, S. S., & Melo, A. (2003). Money, Real Output, and Deficit Effects of Coffee Booms in Colombia. *Journal of Policy Modeling*, 25, 963-983.
- Reinhart, C. M., & Reinhart, V. R. (1991). Output Fluctuations and Monetary Shocks. *IMF Staff Paper*, 38(4), 705-735.
- Sachs, J. D., & Warner, A. M. (2001). The Curse of natural Resources. *European Economic Review*, 45, 827-838.
- Schmidt-Hebbel, K. (2012). Fiscal Institutions in Resource-Rich Economies: Lessons from Chile and Norway. *Documento de Trabajo*(416).
- Schmitt-Grohé, S. (1998). The International Transmission of Economic Fluctuations: Effects of U.S. Business Cycles on the Canadian Economy. *Journal of International Economics*, 44, 257-287.
- Senhadji, A. S. (2003). External Shocks and Debt Accumulation in a Small Open Economy. *Review of Economic Dynamics*, 6, 207-239.
- Sen, S., Kasibathla, K. M., & Stewart, D. B. (2007). Debt Overhang and Economic Growth- the Asian and Latin American Experiences. *Economic Systems*, 31, 3-11.
- Shabsigh, G., & Ilahi, N. (2007). Looking beyond the fiscal: do oil funds bring macroeconomic stability? *IMF Working Paper*(96).
- Sissoko, Y., & Dibooğlu, S. (2006). The Exchange Rate System and Macroeconomic Fluctuations in Sub-Saharan Africa. *Economic Systems*, 30, 141-

156.

Slavov, S. (2011). *De jure* versus *De facto* Exchange Rate Regimes in Sub-Saharan Africa economy. *IMF Working Paper*, WP/11/98.

Smets, F., & Wouters, R. (2002). Openness, Imperfect Exchange Rate Pass-through and Monetary Policy. *Journal of Monetary Economics*, 49, 947-981.

Smets, F., & Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5), 1123-1175.

Smets, F., & Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *The American Economic Review*, 97(3), 586-606.

Steinbach, M.R., Mathuloe, P. T., & Smit, B. W. (2009). An Open Economy New Keynesian DSGE Model of the South African Economy. *South African Journal of Economics*, 77(2), 207-227.

Sosa, S. (2008). External Shocks and Business Cycle Fluctuations in Mexico: How Important are U.S. Factors?. *IMF Working Paper*, WP/08/100.

Spatafora, N., & Samake, I. (2012). Commodity price shocks and fiscal outcomes. *IMF Working Paper*(112).

Stučka, T. (2004). The Effects of Exchange Rate Change on the Trade Balance in Croatia. *IMF Working Paper*, (65).

Sturm, M., Gurtner, F., & Alegre, J. G. (2009). Fiscal Challenges in Oil-Exporting Countries: A Review of Key Issues. *European Central Bank Occasional Paper Series*(104).

Sy, A. N. R. (2013). First Borrow: A Growing Number of Countries in Sub-Saharan Africa are Tapping International Capital Markets. *Finance and Development*, 52-54.

Taylor, J. B. (1993). Discretion versus Policy Rule in Practice. *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214.

Triki, T., & Faye, I. (2011). Africa's Quest for Development: Can Sovereign Wealth Funds Help? *African Development Bank Working Paper*(142).

- Tsani, S. (2013). Natural resources, governance and institutional quality: the role of resource funds. *Resources Policy*, 38, 181-195.
- Uribe, M., & Yue, V. Z. (2006). Country Spreads and Emerging Countries: Who drives Whom? *Journal of International Economics*, 69, 6-36.
- Usui, N. (2007). How Effective are Oil Funds: Managing Resource Windfalls in Azerbaijan and Kazakhstan. *Asian Development Bank Policy Brief Series*(50).
- Villafuerte, M., & Lopez-Murphy, P. (2010). Fiscal policy in oil producing countries during recent oil cycle. *IMF Working Paper*(28).
- von Heideken, V. Q. (2009). How Important are Financial Frictions in the United States and the Euro Area?. *The Scandinavian Journal of Economics* 111(3), 567-596.

APPENDIX A

Appendix to Chapter One

A. 1. The household's preference is:

$$U_t = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left[\left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi \right] - \frac{N_t^{1+\psi}}{1+\psi} \quad (1A)$$

The budget constraint is

$$\begin{aligned} C_t + \left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} + \frac{B_t}{P_t} - \frac{S_t D_t^*}{P_t} &= \frac{W_t N_t}{P_t} + \frac{B_{t-1}}{P_t} (1 + r_{t-1}) \\ &+ \left(1 + \frac{\varrho}{S_t} \right) \frac{M_{t-1}}{P_t} \\ &- \frac{S_t D_{t-1}^*}{P_t} (1 + r_{t-1}^d) \end{aligned} \quad (2A)$$

The Langrange multiplier function is:

$$\begin{aligned} \ell &= \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left[\left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi \right] - \frac{N_t^{1+\psi}}{1+\psi} \\ &+ \lambda_t \left[\frac{W_t N_t}{P_t} + \frac{B_{t-1}}{P_t} (1 + r_{t-1}) \right] + \left(1 + \frac{\varrho}{S_t} \right) \frac{M_{t-1}}{P_t} - \frac{S_t D_{t-1}^*}{P_t} (1 + r_{t-1}^d) \\ &- C_t - \left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} - \frac{B_t}{P_t} + \frac{S_t D_t^*}{P_t} \end{aligned} \quad (3A)$$

The first order conditions are :

$$U_C : \quad \frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi = \lambda_t \quad (4A)$$

$$U_M : \quad \frac{\phi}{1-\sigma} \left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left(1 + \frac{\varrho}{S_t} \right)^\phi \left(\frac{M_t}{P_t} \right)^{\phi-1} \frac{1}{P_t} = \frac{\lambda_t}{P_t} \left(1 + \frac{\varrho}{S_t} \right) - \frac{\beta \lambda_{t+1}}{P_{t+1}} \left(1 + \frac{\varrho}{S_t} \right) \quad (5A)$$

$$U_N : \quad \frac{W_t}{P_t} = \frac{N_t^\psi}{\lambda_t} \quad (6A)$$

$$U_B : \quad \beta \lambda_{t+1} (1 + r_t) \frac{P_t}{P_{t+1}} = \lambda_t \quad (7A)$$

$$U_D : \quad \beta \lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} (1 + r_{t-1}^d) = \lambda_t \frac{S_{t+1}}{P_t} \quad (8A)$$

A.1. 1 Derivation of the IS equation

Equate (4A) and (7A)

$$\frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi = \beta \lambda_{t+1} (1 + r_t) \frac{P_t}{P_{t+1}}$$

$$\frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi = \beta \frac{(1 + r_t)}{(1 + \pi_t)} \frac{1}{C_t^h} \left(\frac{C_{t+1}}{C_t^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_{t+1}} \right) \frac{M_{t+1}}{P_{t+1}} \right]^\phi \quad (9A)$$

Re-write Eq. (9A) as

$$C_{t-1}^{h\gamma-\gamma} C_t^{-\gamma} \left[\left(1 + \frac{\varrho}{S_t} \right) m_t \right]^\phi = \beta \frac{(1+r_t)}{(1+\pi_t)} C_t^{h\gamma-\gamma} C_{t+1}^{-\gamma} \left[\left(1 + \frac{\varrho}{S_t} \right) m_{t+1} \right]^\phi \quad (10A)$$

Linearize Eq. (10A)

$$\begin{aligned} & (h\gamma - \gamma) c_0 c_0^{-1} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{c}_{t-1} - \sigma c_0 c_0^{-1} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{c}_t \\ & - \phi \varrho s_0^{-1} c_0 c_0^{-1} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{s}_t + \phi c_0 c_0^{-1} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{m}_t \quad (11A) \\ = & (h\gamma - \gamma) \frac{\beta(1+r_0)}{(1+\pi_0)} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{c}_t - \sigma \frac{\beta(1+r_0)}{(1+\pi_0)} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{c}_{t+1} \\ & - \phi \varrho s_0^{-1} \frac{\beta(1+r_0)}{(1+\pi_0)} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{s}_{t+1} + \phi \frac{\beta(1+r_0)}{(1+\pi_0)} \left[\left(1 + \frac{\varrho}{s_0} \right) m_0 \right] \tilde{m}_{t+1} \\ & + (r_t - \pi_{t+1}) \end{aligned}$$

$$\frac{\beta(1+r_0)}{(1+\pi_0)} \approx 1$$

$$\begin{aligned} & \left(\left(1 + \frac{\varrho}{s_0} \right) m_0 \right) \left[(h\gamma - \gamma) \tilde{c}_{t-1} - \sigma \tilde{c}_t - \phi \frac{\varrho}{s_0} \tilde{s}_t + \phi \tilde{m}_t \right] \quad (12A) \\ = & \left(\left(1 + \frac{\varrho}{s_0} \right) m_0 \right) \left[(h\gamma - \gamma) \tilde{c}_t - \sigma \tilde{c}_{t+1} - \phi \frac{\varrho}{s_0} \tilde{s}_{t+1} + \phi \tilde{m}_{t+1} + (r_t - E\pi_{t+1}) \right] \end{aligned}$$

$$\begin{aligned} (h\gamma - \gamma) \tilde{c}_{t-1} - \sigma \tilde{c}_t - \phi \frac{\varrho}{s_0} \tilde{s}_t + \phi \tilde{m}_t &= (h\gamma - \gamma) \tilde{c}_t - \sigma \tilde{c}_{t+1} - \phi \frac{\varrho}{s_0} \tilde{s}_{t+1} \\ &+ \phi \tilde{m}_{t+1} + (r_t - E\pi_{t+1}) \quad (13A) \end{aligned}$$

$$\begin{aligned}
(\sigma + h(\sigma - 1))\tilde{c}_t &= h(\sigma - 1)\tilde{c}_{t-1} + \sigma\tilde{c}_{t+1} + \phi\tilde{m}_t - \phi\frac{\varrho}{s_0}\tilde{s}_t - \phi\tilde{m}_{t+1} \\
&\quad + \phi\frac{\varrho}{s_0}\tilde{s}_{t+1} - (r_t - E\pi_{t+1})
\end{aligned} \tag{14A}$$

Let

$$\begin{aligned}
\tilde{c}_t &= \frac{h(\sigma - 1)}{(\sigma + h(\sigma - 1))}\tilde{c}_{t-1} + \frac{\sigma}{(\sigma + h(\sigma - 1))}\tilde{c}_{t+1} + \frac{\phi}{(\sigma + h(\sigma - 1))}\tilde{m}_t \\
&\quad - \frac{\varrho}{s_0}\frac{\phi}{(\sigma + h(\sigma - 1))}\tilde{s}_t - \frac{\phi}{(\sigma + h(\sigma - 1))}\tilde{m}_{t+1} + \frac{\varrho}{s_0}\frac{\phi}{(\sigma + h(\sigma - 1))}\tilde{s}_{t+1} \\
&\quad - \frac{1}{(\sigma + h(\sigma - 1))}(r_t - E\pi_{t+1})
\end{aligned} \tag{15A}$$

Market equilibrium

$$Y_t = C_t + I_t + G_t + X_t - Z_t$$

Assuming investment and government are exogenous, we re-write the market equilibrium:

$$\tilde{y}_t = \gamma_c\tilde{c}_t + \gamma_x\tilde{x}_t - \gamma_m\tilde{z}_t \tag{16A}$$

$$\tilde{c}_t = \frac{1}{\gamma_c}\tilde{y}_t - \frac{\gamma_x}{\gamma_c}\tilde{x}_t + \frac{\gamma_z}{\gamma_c}\tilde{z}_t$$

Substitute Eq. (17A) in Eq. (15A)

$$\begin{aligned}
\frac{1}{\gamma_c} \tilde{y}_t - \frac{\gamma_x}{\gamma_c} \tilde{x}_t + \frac{\gamma_z}{\gamma_c} \tilde{z}_t &= \frac{h(\sigma-1)}{(\sigma+h(\sigma-1))} \left[\frac{1}{\gamma_c} \tilde{y}_{t-1} - \frac{\gamma_x}{\gamma_c} \tilde{x}_{t-1} + \frac{\gamma_z}{\gamma_c} \tilde{z}_{t-1} \right] \\
&+ \frac{\sigma}{(\sigma+h(\sigma-1))} \left[\frac{1}{\gamma_c} \tilde{y}_{t+1} - \frac{\gamma_x}{\gamma_c} \tilde{x}_{t+1} + \frac{\gamma_z}{\gamma_c} \tilde{z}_{t+1} \right] \\
&+ \frac{\phi}{(\sigma+h(\sigma-1))} \tilde{m}_t - \frac{\varrho}{s_0} \frac{\phi}{(\sigma+h(\sigma-1))} \tilde{s}_t \quad (18A) \\
&- \frac{\phi}{(\sigma+h(\sigma-1))} \tilde{m}_{t+1} + \frac{\varrho}{s_0} \frac{\phi}{(\sigma+h(\sigma-1))} \tilde{s}_{t+1} \\
&- \frac{1}{(\sigma+h(\sigma-1))} (r_t - E\pi_{t+1})
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\gamma_c} \tilde{y}_t &= \frac{h(\sigma-1)}{(\sigma+h(\sigma-1))} \left[\frac{1}{\gamma_c} \tilde{y}_{t-1} - \frac{\gamma_x}{\gamma_c} \tilde{x}_{t-1} + \frac{\gamma_z}{\gamma_c} \tilde{z}_{t-1} \right] \\
&+ \frac{\sigma}{(\sigma+h(\sigma-1))} \left[\frac{1}{\gamma_c} \tilde{y}_{t+1} - \frac{\gamma_x}{\gamma_c} \tilde{x}_{t+1} + \frac{\gamma_z}{\gamma_c} \tilde{z}_{t+1} \right] \\
&+ \frac{\phi}{(\sigma+h(\sigma-1))} \tilde{m}_t - \frac{\phi\gamma_s}{(\sigma+h(\sigma-1))} \tilde{s}_t \\
&- \frac{\phi}{(\sigma+h(\sigma-1))} \tilde{m}_{t+1} + \frac{\phi\gamma_s}{(\sigma+h(\sigma-1))} \tilde{s}_{t+1} \quad (19A) \\
&- \frac{1}{(\sigma+h(\sigma-1))} (r_t - E\pi_{t+1}) + \frac{\gamma_x}{\gamma_c} \tilde{x}_t - \frac{\gamma_z}{\gamma_c} \tilde{z}_t
\end{aligned}$$

$$\begin{aligned}
\tilde{y}_t &= \frac{h(\sigma-1)}{(\sigma+h(\sigma-1))} \tilde{y}_{t-1} - \frac{\gamma_x h(\sigma-1)}{(\sigma+h(\sigma-1))} \tilde{x}_{t-1} + \frac{\gamma_z h(\sigma-1)}{(\sigma+h(\sigma-1))} \tilde{z}_{t-1} \\
&+ \frac{\sigma}{(\sigma+h(\sigma-1))} \tilde{y}_{t+1} - \frac{\gamma_x \sigma}{(\sigma+h(\sigma-1))} \tilde{x}_{t+1} + \frac{\gamma_z \sigma}{(\sigma+h(\sigma-1))} \tilde{z}_{t+1} \\
&+ \frac{\phi\gamma_c}{(\sigma+h(\sigma-1))} \tilde{m}_t - \frac{\varrho}{s_0} \frac{\phi\gamma_c}{(\sigma+h(\sigma-1))} \tilde{s}_t - \frac{\phi\gamma_c}{(\sigma+h(\sigma-1))} \tilde{m}_{t+1} \\
&+ \frac{\varrho}{s_0} \frac{\phi\gamma_c}{(\sigma+h(\sigma-1))} \tilde{s}_{t+1} - \frac{\gamma_c}{(\sigma+h(\sigma-1))} (r_t - E\pi_{t+1}) + \gamma_x \tilde{x}_t - \gamma_z \tilde{z}_t \quad (20A)
\end{aligned}$$

$$\begin{aligned}
\tilde{y}_t = & \frac{h(\sigma-1)}{(\sigma+h(\sigma-1))}\tilde{y}_{t-1} + \frac{\sigma}{(\sigma+h(\sigma-1))}\tilde{y}_{t+1} - \frac{\gamma_c}{(\sigma+h(\sigma-1))}(r_t - E\pi_{t+1}) \\
& + \frac{\phi\gamma_c}{(\sigma+h(\sigma-1))}\tilde{m}_t - \frac{\phi\gamma_c}{(\sigma+h(\sigma-1))}\tilde{m}_{t+1} - \frac{\varrho}{s_0}\frac{\phi\gamma_c}{(\sigma+h(\sigma-1))}\tilde{s}_t \\
& + \frac{\varrho}{s_0}\frac{\phi\gamma_c}{(\sigma+h(\sigma-1))}\tilde{s}_{t+1} - \frac{h(\sigma-1)}{(\sigma+h(\sigma-1))}(\gamma_x\tilde{x}_{t-1} - \gamma_z\tilde{z}_{t-1}) \\
& - \frac{\sigma}{(\sigma+h(\sigma-1))}(\gamma_x\tilde{x}_{t+1} - \gamma_z\tilde{z}_{t+1}) + \gamma_x\tilde{x}_t - \gamma_z\tilde{z}_t
\end{aligned} \tag{21A}$$

Similar to McCallum and Nelson (2001), net export is defined as:

$$\gamma_x\tilde{x}_t - \gamma_z\tilde{z}_t = \gamma_r\widetilde{rer}_t - \gamma_y\tilde{y}_t + \gamma_{yf}\tilde{y}_t^f \tag{22A}$$

Let $\sigma_c = (\sigma + h(\sigma - 1))$. Substitute Eq. (22A) in Eq. (21A) to derive an open economy IS equation:

$$\begin{aligned}
\tilde{y}_t = & \frac{h(\sigma-1)}{\sigma_c}\tilde{y}_{t-1} + \frac{\sigma}{\sigma_c}E_t\tilde{y}_{t+1} - \frac{\gamma_c}{\sigma_c(1+\gamma_y)}(\tilde{r}_t - E_t\tilde{\pi}_{t+1}) \\
& + \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)}\tilde{m}_t - \frac{\phi\gamma_c}{\sigma_c(1+\gamma_y)}E_t\tilde{m}_{t+1} - \frac{\phi\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)}\tilde{s}_t \\
& + \frac{\phi\varrho\gamma_c}{s_0\sigma_c(1+\gamma_y)}E_t\tilde{s}_{t+1} - \frac{h(\sigma-1)\gamma_r}{\sigma_c(1+\gamma_y)}\widetilde{rer}_{t-1} \\
& + \frac{\gamma_r}{(1+\gamma_y)}\widetilde{rer}_t - \frac{\sigma\gamma_r}{\sigma_c(1+\gamma_y)}E_t\widetilde{rer}_{t+1} - \frac{h(\sigma-1)\gamma_{yf}}{\sigma_c(1+\gamma_y)}\tilde{y}_{t-1}^f \\
& + \frac{\gamma_{yf}}{(1+\gamma_y)}\tilde{y}_t^f - \frac{\sigma\gamma_{yf}}{\sigma_c(1+\gamma_y)}E_t\tilde{y}_{t+1}^f + \varepsilon_t^a
\end{aligned} \tag{23A}$$

A.2. Derivation of the New Keynesian Phillips Curve

Equate (4A) and (6A)

$$N_t^\psi \frac{P_t}{W_t} = \frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \frac{M_t}{P_t} \right]^\phi \quad (1B)$$

Linearize Eq. (1B)

$$\psi \frac{n_0}{w_0} \tilde{n}_t - \frac{n_0}{w_0} \tilde{w}_t = \left(\left(1 + \frac{\varrho}{s_0} \right) m_0 \right) \left[(h\gamma - \gamma) \tilde{c}_{t-1} - \sigma \tilde{c}_t - \phi \frac{\varrho}{s_0} \tilde{s}_t + \phi \tilde{m}_t \right] \quad (2B)$$

$$\psi \frac{n_0}{w_0} \tilde{n}_t - \frac{n_0}{w_0} \tilde{w}_t = \left(\frac{n_0}{w_0} \right) \left[(h\gamma - \gamma) \tilde{c}_{t-1} - \sigma \tilde{c}_t - \phi \frac{\varrho}{s_0} \tilde{s}_t + \phi \tilde{m}_t \right]$$

$$\psi \frac{n_0}{w_0} \tilde{n}_t = \frac{n_0}{w_0} \tilde{w}_t + \left(\frac{n_0}{w_0} \right) \left[(h\gamma - \gamma) \tilde{c}_{t-1} - \sigma \tilde{c}_t - \phi \frac{\varrho}{s_0} \tilde{s}_t + \phi \tilde{m}_t \right]$$

$$\psi \tilde{n}_t = \tilde{w}_t + (h\gamma - \gamma) \tilde{c}_{t-1} - \sigma \tilde{c}_t - \phi \frac{\varrho}{s_0} \tilde{s}_t + \phi \tilde{m}_t$$

$$\tilde{n}_t = \frac{1}{\psi} \tilde{w}_t + \frac{\gamma(h-1)}{\psi} \tilde{c}_{t-1} - \frac{\sigma}{\psi} \tilde{c}_t - \frac{\phi \varrho}{s_0 \psi} \tilde{s}_t + \frac{\phi}{\psi} \tilde{m}_t \quad (3B)$$

Given a production function

$$Y_t = A_t N_t \left[\prod_{i=1}^n Y_t^{\theta_i \delta_i} \right] \quad (4B)$$

The reduced form is

$$Y_t = A'_t N_t^\alpha \quad (5B)$$

The total cost is:

$$TC_t = \frac{W_t Y_t^{\frac{1}{\alpha}}}{A_t'^{\frac{1}{\alpha}} P_t} + \sum_{i=1}^n \frac{P_{i,t}}{P_t} Y_t^{\delta_i} \quad (6B)$$

The marginal cost is:

$$MC_t = \frac{W_t Y_t^{\frac{1-\alpha}{\alpha}}}{\alpha A_t'^{\frac{1}{\alpha}} P_t} + \sum_{i=1}^n \delta_i p_{it} Y_t^{\delta_i-1} \quad (7B)$$

Recall that $N_t = \left(\frac{Y_t}{A'_t}\right)^{\frac{1}{\alpha}}$. The marginal cost can be re-written as:

$$MC_t = \frac{w_t N_t}{\alpha Y_t} + \sum_{i=1}^n \delta_i p_{it} Y_t^{\delta_i-1} \quad (8B)$$

Linearize Eq. (8B)

$$\widetilde{mc}_t = \frac{w_0 n_0}{mc_0 y_0} (\widetilde{w}_t + \widetilde{n}_t + \widetilde{y}_t) + \frac{1}{mc_0} \sum \delta_i p_{i0} (\delta_i - 1) y_0 \widetilde{y}_t + \frac{1}{mc_0} \sum \delta_i y_0 p_{i0} \widetilde{p}_{it} \quad (9B)$$

Recall that $w_t = MP_L = \alpha (y_t - n_t)$

$$\begin{aligned} \widetilde{mc}_t &= \frac{w_0 n_0}{mc_0 y_0} [(1 - \alpha) \widetilde{n}_t - (1 - \alpha) \widetilde{y}_t] + \frac{1}{mc_0} \sum \delta_i p_{i0} (\delta_i - 1) y_0 \widetilde{y}_t \\ &\quad + \frac{1}{mc_0} \sum \delta_i y_0 p_{i0} \widetilde{p}_{it} + \frac{1}{mc_0} \widetilde{a}_t \end{aligned} \quad (10B)$$

Recall that

$$\tilde{c}_t = \frac{(1 + \gamma_y)}{\gamma_c} \tilde{y}_t - \frac{\gamma_r}{\gamma_c} \widetilde{rer}_t + \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_t^f \quad (11B)$$

Substitute Eq. (11B) in Eq. (3B)

$$\begin{aligned} \tilde{n}_t = & \frac{\alpha}{\psi} (\tilde{y}_t - \tilde{n}_t) + \frac{\gamma(h-1)}{\psi} \left[\frac{(1 + \gamma_y)}{\gamma_c} \tilde{y}_{t-1} - \frac{\gamma_r}{\gamma_c} \widetilde{rer}_{t-1} + \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_{t-1}^f \right] \\ & - \frac{\sigma}{\psi} \left[\frac{(1 + \gamma_y)}{\gamma_c} \tilde{y}_t - \frac{\gamma_r}{\gamma_c} \widetilde{rer}_t + \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_t^f \right] - \frac{\phi \varrho}{s_0 \psi} \tilde{s}_t + \frac{\phi}{\psi} \tilde{m}_t \end{aligned} \quad (12B)$$

$$\begin{aligned} \tilde{n}_t = & \frac{(1 - \alpha)}{(\psi + (1 - \alpha))} \tilde{y}_t + \frac{h(\sigma - 1)(1 + \gamma_y)}{\gamma_c(\psi + (1 - \alpha))} \tilde{y}_{t-1} - \frac{\gamma_r h(\sigma - 1)}{\gamma_c(\psi + (1 - \alpha))} \widetilde{rer}_{t-1} \\ & - \frac{\gamma_{yf} h(\sigma - 1)}{\gamma_c(\psi + (1 - \alpha))} \tilde{y}_{t-1}^f - \frac{\sigma(1 + \gamma_y)}{\gamma_c(\psi + (1 - \alpha))} \tilde{y}_t + \frac{\gamma_r \sigma}{\gamma_c(\psi + (1 - \alpha))} \widetilde{rer}_t \\ & + \frac{\gamma_{yf} \sigma}{\gamma_c(\psi + (1 - \alpha))} \tilde{y}_t^f + \frac{\phi}{(\psi + (1 - \alpha))} \tilde{m}_t - \frac{\phi \varrho}{s_0(\psi + (1 - \alpha))} \tilde{s}_t \end{aligned} \quad (13B)$$

Substitute Eq. (13B) in Eq. (10B) yields:

$$\begin{aligned} \widetilde{mc}_t = & \left(\frac{\gamma_{ls} \sigma (1 + \gamma_y)}{\gamma_c (1 + \psi)} + \frac{\gamma_{ls}}{\alpha} - \frac{\alpha \gamma_{ls}}{(1 + \psi)} + \sum_{i=1}^n \delta_i \frac{p_{i0} x_0}{y} \right) \tilde{y}_t - \frac{\gamma_{ls} h(\sigma - 1)(1 + \gamma_y)}{\gamma_c (\alpha + \psi)} \tilde{y}_{t-1} \\ & - \frac{\gamma_{ls} \phi}{(1 + \psi)} \tilde{m}_t + \frac{\gamma_{ls} \phi \varrho}{s_0 (1 + \psi)} \tilde{s}_t - \frac{\gamma_{ls} \gamma_r}{\gamma_c (1 + \psi)} \widetilde{rer}_t + \frac{\gamma_r \gamma_{ls} h(\sigma - 1)(1 - \alpha)}{\gamma_c (\alpha + \psi)} \frac{\gamma_{ls}}{\alpha} \widetilde{rer}_{t-1} \\ & - \frac{\gamma_{ls} \sigma \gamma_{yf}}{\gamma_c (1 + \psi)} \tilde{y}_t^f + \frac{\gamma_{ls} h(\sigma - 1) \gamma_{yf}}{\gamma_c (\alpha + \psi)} \tilde{y}_{t-1}^f + \sum_{i=1}^n \delta_i \frac{p_{i0} x_0}{y} \tilde{p}_{it} - \frac{\gamma_{ls}}{\alpha} \tilde{a}_t \end{aligned}$$

where

$$\gamma_{ls} = \frac{y_0}{a_0 n_0}$$

The new Keynesian Phillips curve:

$$\tilde{\pi}_t = \gamma_f E_t \tilde{\pi}_{t+1} + \gamma_b \tilde{\pi}_{t-1} + \lambda \tilde{m} c_t \quad (14B)$$

A.3. Derivation of Debt equation

$$\frac{\Delta D_t}{Y_T} = \frac{Z_t - X_t}{Y_t} + r_t^d \frac{D_{t-1}}{Y_t} \quad (1C)$$

Differentiate with respect to time

$$\Delta d_t = (d_t - d_{t-1}) = (z_t - x_t) + r_t^d (1 + \Delta y_t)^{-1} d_{t-1} - \Delta y_t d_t \quad (2C)$$

$$d_t + \Delta y_t d_t = -(x_t - z_t) + r_t^d (1 + \Delta y_t)^{-1} d_{t-1} + d_{t-1}$$

$$(x_t - z_t) = \gamma_r r e r - \gamma_y y_t + \gamma_{yf} y_t^f$$

$$d_t + \Delta y_t d_t = - \left(\gamma_r r e r - \gamma_y y_t + \gamma_{yf} y_t^f \right) + r_t^d (1 + \Delta y_t)^{-1} d_{t-1} + d_{t-1}$$

$$(1 + \Delta y_t) d_t = d_{t-1} + r_t^d (1 + \Delta y_t)^{-1} + \gamma_y y_t - \gamma_r r e r - \gamma_{yf} y_t^f \quad (3C)$$

Linearize Eq. (3C)

$$\begin{aligned}
d_0 (1 + g_0) \tilde{d}_t + d_0 (1 + g_0) \Delta \tilde{y}_t &= d_0 \tilde{d}_{t-1} + r_0 d_0 (1 + g_0)^{-1} \tilde{r}_t^d \\
&\quad + r_0 d_0 (1 + g_0)^{-1} \tilde{d}_{t-1} - r_0 d_0 (1 + g_0)^{-2} \Delta \tilde{y}_t \\
&\quad + \gamma_y y_0 \tilde{y}_t - \gamma_r r e r_0 \widetilde{r e r} - \gamma_{yf} y_0^f \tilde{y}_t^f \quad (4C)
\end{aligned}$$

$$\begin{aligned}
d_0 (1 + g_0) \tilde{d}_t &= d_0 \tilde{d}_{t-1} + r_0 d_0 (1 + g_0)^{-1} \tilde{r}_t^d + r_0 d_0 (1 + g_0)^{-1} \tilde{d}_{t-1} \\
&\quad - r_0 d_0 (1 + g_0)^{-2} \Delta \tilde{y}_t + \gamma_y y_0 \tilde{y}_t - \gamma_r r e r_0 \widetilde{r e r} - \gamma_{yf} y_0^f \tilde{y}_t^f \\
&\quad - d_0 (1 + g_0) \Delta \tilde{y}_t \quad (5C)
\end{aligned}$$

Assuming

$$\tilde{r}_t^d = \tilde{r}_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t$$

Thus,

$$\begin{aligned}
d_0 (1 + g_0) \tilde{d}_t &= d_0 \tilde{d}_{t-1} + r_0 d_0 (1 + g_0)^{-1} \left(\tilde{r}_t^f - \omega_q \tilde{q}_t + \omega_d \tilde{d}_t \right) + r_0 d_0 (1 + g_0)^{-1} \tilde{d}_{t-1} \\
&\quad - r_0 d_0 (1 + g_0)^{-2} \Delta \tilde{y}_t + \gamma_y y_0 \tilde{y}_t - \gamma_r r e r_0 \widetilde{r e r} - \gamma_{yf} y_0^f \tilde{y}_t^f \\
&\quad - d_0 (1 + g_0) \Delta \tilde{y}_t \quad (6C)
\end{aligned}$$

$$\begin{aligned}
d_0 \left((1 + g_0) - \frac{r_0 \omega_d}{(1 + g_0)} \right) \tilde{d}_t &= d_0 \left(1 + \frac{r_0}{(1 + g_0)} \right) \tilde{d}_{t-1} + \frac{r_0 d_0}{(1 + g_0)} \tilde{r}_t^f - \frac{r_0 d_0 \omega_q}{(1 + g_0)} \tilde{q}_t \\
&\quad - d_0 \left((1 + g_0) + \frac{r_0}{(1 + g_0)^2} \right) \Delta \tilde{y}_t + \gamma_y y_0 \tilde{y}_t \\
&\quad - \gamma_r r e r_0 \tilde{r} e r - \gamma_{yf} y_0^f \tilde{y}_t^f \tag{7C}
\end{aligned}$$

$$\begin{aligned}
\tilde{d}_t &= \frac{(1 + g_0) + r_0}{(1 + g_0)^2 - r_0 \omega_d} \tilde{d}_{t-1} + \frac{r_0}{(1 + g_0)^2 - r_0 \omega_d} \tilde{r}_t^f - \frac{r_0 \omega_q}{(1 + g_0)^2 - r_0 \omega_d} \tilde{q}_t \\
&\quad - \frac{(1 + g_0)^3 + r_0}{(1 + g_0) ((1 + g_0)^2 - r_0 \omega_d)} \Delta \tilde{y}_t + \frac{\gamma_y (1 + g_0) y_0}{d_0 ((1 + g_0)^2 - r_0 \omega_d)} \tilde{y}_t \\
&\quad - \frac{\gamma_r (1 + g_0) r e r_0}{d_0 ((1 + g_0)^2 - r_0 \omega_d)} \tilde{r} e r_t - \frac{\gamma_{yf} (1 + g_0) y_0^f}{d_0 ((1 + g_0)^2 - r_0 \omega_d)} \tilde{y}_t^f \tag{8C}
\end{aligned}$$

Assuming $\gamma_d = (1 + g_0)^2 - r_0 \omega_d$

$$\begin{aligned}
\tilde{d}_t &= \frac{(1 + g_0) + r_0}{\gamma_d} \tilde{d}_{t-1} + \frac{r_0}{\gamma_d} \tilde{r}_t^f - \frac{r_0 \omega_q}{\gamma_d} \tilde{q}_t - \frac{(1 + g_0)^3 + r_0}{(1 + g_0) \gamma_d} \Delta \tilde{y}_t \\
&\quad - \frac{\gamma_y (1 + g_0) y_0}{d_0 \gamma_d} \tilde{y}_t - \frac{\gamma_r (1 + g_0) r e r_0}{d_0 \gamma_d} \tilde{r} e r_t - \frac{\gamma_{yf} (1 + g_0) y_0^f}{d_0 \gamma_d} \tilde{y}_t^f \tag{10C}
\end{aligned}$$

$$\tilde{d}_t = \beta_a \tilde{d}_{t-1} + \beta_b \tilde{r}_t^f + \beta_c \tilde{y}_t - \beta_d \Delta \tilde{y}_t - \beta_e \tilde{q}_t - \beta_f \tilde{r} e r_t - \beta_g \tilde{y}_t^f \tag{11C}$$

where

$$\begin{aligned}\beta_a &= \frac{(1+g_0)+r_0}{\gamma_d}; \beta_b = \frac{r_0}{\gamma_d}; \beta_c = \frac{\gamma_y(1+g_0)y_0}{d_0\gamma_d}; \beta_d = \frac{(1+g_0)^3+r_0}{(1+g_0)\gamma_d} \\ \beta_e &= \frac{r_0\omega_q}{\gamma_d}; \beta_f = \frac{\gamma_r(1+g_0)rer_0}{d_0\gamma_d}; \beta_g = \frac{\gamma_{yf}(1+g_0)y_0^f}{d_0\gamma_d}\end{aligned}$$

A. 4. Derivation of LM equation

Equate (5A) and (7A)

$$\begin{aligned}& \frac{\phi}{1-\sigma} \left(\frac{C_t}{C_{t-1}^h} \right)^{1-\sigma} \left(1 + \frac{\varrho}{S_t} \right)^\phi \left(\frac{M_t}{P_t} \right)^{\phi-1} \frac{1}{P_t} \\ &= \frac{1}{C_{t-1}^h} \left(\frac{C_t}{C_{t-1}^h} \right)^{-\sigma} \left[\left(1 + \frac{\varrho}{S_t} \right) \left(\frac{M_t}{P_t} \right) \right]^\phi \left[\left(1 + \frac{\varrho}{S_t} \right) - \frac{1}{1+r_t} \left(1 + \frac{\varrho}{S_t} \right) \right]\end{aligned}$$

$$\frac{\phi}{1-\sigma} \left(\frac{C_t}{C_{t-1}^h} \right) \left(\frac{M_t}{P_t} \right)^{-1} = \frac{1}{C_{t-1}^h} \left[\left(1 + \frac{\varrho}{S_t} \right) - \frac{1}{1+r_t} \left(1 + \frac{\varrho}{S_t} \right) \right] \quad (2D)$$

$$\frac{\phi}{1-\sigma} C_t m_t^{-1} = \left(1 + \frac{\varrho}{S_t} \right) - \frac{1}{1+r_t} \left(1 + \frac{\varrho}{S_t} \right) \quad (3D)$$

$$\frac{\phi}{1-\sigma} C_t m_t^{-1} = \left(1 + \frac{\varrho}{S_t} \right) \left(1 - \frac{1}{1+r_t} \right) \quad (4D)$$

Linearize Eq. (4D)

$$-\frac{\phi}{1-\sigma} \frac{c_0}{m_0} \tilde{m}_t = -\frac{\phi}{1-\sigma} \frac{c_0}{m_0} \tilde{c}_t - \frac{\varrho}{s_0} \tilde{s}_t + \frac{\varrho}{s_0(1+r_0)} \tilde{s}_{t+1} + \frac{1}{s_0(1+r_0)} \tilde{r}_t$$

$$\tilde{m}_t = \tilde{c}_t + \frac{\varrho}{s_0} \frac{1-\sigma}{\phi} \frac{m_0}{c_0} \tilde{s}_t - \frac{\varrho}{s_0(1+r_0)} \frac{1-\sigma}{\phi} \frac{m_0}{c_0} \tilde{s}_{t+1} - \frac{1-\sigma}{\phi} \frac{m_0}{s_0(1+r_0)^2 c_0} \tilde{r}_t \quad (5D)$$

Recall

$$\tilde{c}_t = \frac{(1+\gamma_y)}{\gamma_c} \tilde{y}_t - \frac{\gamma_r}{\gamma_c} \tilde{r} e r_t + \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_t^f \quad (6D)$$

Substitute Eq. (6D) into Eq. (5D) yields the money market equation:

$$\begin{aligned} \tilde{m}_t = & \frac{(1+\gamma_y)}{\gamma_c} \tilde{y}_t - \frac{\gamma_r}{\gamma_c} \tilde{r} e r_t + \frac{\gamma_{yf}}{\gamma_c} \tilde{y}_t^f + \frac{\varrho}{s_0} \frac{1-\sigma}{\phi} \frac{m_0}{c_0} \tilde{s}_t - \frac{\varrho}{s_0(1+r_0)} \frac{1-\sigma}{\phi} \frac{m_0}{c_0} \tilde{s}_{t+1} \\ & - \frac{1-\sigma}{\phi} \frac{m_0}{s_0(1+r_0)^2 c_0} \tilde{r}_t \end{aligned} \quad (7D)$$

Appendix B

Appendix to Chapter Two

$$Y_t = \min [A_t \varphi_K K_t^\alpha, A_t \varphi_N N_t^{1-\alpha}] \quad (1E)$$

The total cost is:

$$TC_t = w_t N_t + r_t K_t \quad (2E)$$

The marginal cost is:

$$MC = w_t \frac{dN_t}{dY_t} + r_t \frac{dK_t}{dY_t} \quad (3E)$$

$$mc_t = w_t \left(\frac{1}{(1-\alpha)} \frac{1}{A_t \varphi_n} \left(\frac{Y_t}{A_t \varphi_n} \right)^{\frac{\alpha}{1-\alpha}} \right) + r_t \left(\frac{1}{\alpha} \frac{1}{A_t \varphi_k} \left(\frac{Y_t}{A_t \varphi_k} \right)^{\frac{1-\alpha}{\alpha}} \right) \quad (4E)$$

Linearize Eq. 4(E)

$$\begin{aligned} \widetilde{mc}_t = & \frac{w_0 y_0}{a_0^2} \widetilde{w}_t - \frac{w_0 y_0}{a_0^2} \widetilde{a}_t + \frac{\alpha w_0 y_0}{(1-\alpha) a_0^2} \widetilde{y}_t - \frac{\alpha w_0 y_0}{(1-\alpha) a_0^2} \widetilde{a}_t + \frac{r_0 y_0}{a_0^2} \widetilde{r}_t \\ & - \frac{r_0 y_0}{a_0^2} \widetilde{a}_t + \frac{(1-\alpha) r_0 y_0}{\alpha a_0^2} \widetilde{y}_t - \frac{(1-\alpha) r_0 y_0}{\alpha a_0^2} \widetilde{a}_t \end{aligned}$$

$$\begin{aligned} \widetilde{mc}_t = & \frac{w_0 y_0}{a_0^2} \widetilde{w}_t + \frac{r_0 y_0}{a_0^2} \widetilde{r} - \left(\frac{w_0 y_0}{a_0^2} + \frac{\alpha w_0 y_0}{(1-\alpha) a_0^2} + \frac{r_0 y_0}{a_0^2} + \frac{(1-\alpha) r_0 y_0}{\alpha a_0^2} \right) \widetilde{a}_t \\ & \left(\frac{\alpha w_0 y_0}{(1-\alpha) a_0^2} + \frac{(1-\alpha) r_0 y_0}{\alpha a_0^2} \right) \widetilde{y}_t \end{aligned} \quad (5E)$$

Recall

$$w_t = (1-\alpha) (y_t - n_t) \quad (6E)$$

and

$$\begin{aligned} \widetilde{n}_t = & \frac{(1-\alpha)}{(\psi + (1-\alpha))} \widetilde{y}_t + \frac{h(\sigma-1)(1+\gamma_y)}{\gamma_c(\psi + (1-\alpha))} \widetilde{y}_{t-1} - \frac{\gamma_r h(\sigma-1)}{\gamma_c(\psi + (1-\alpha))} \widetilde{rer}_{t-1} \\ & - \frac{\gamma_{yf} h(\sigma-1)}{\gamma_c(\psi + (1-\alpha))} \widetilde{y}_{t-1}^f - \frac{\sigma(1+\gamma_y)}{\gamma_c(\psi + (1-\alpha))} \widetilde{y}_t + \frac{\gamma_r \sigma}{\gamma_c(\psi + (1-\alpha))} \widetilde{rer}_t \\ & + \frac{\gamma_{yf} \sigma}{\gamma_c(\psi + (1-\alpha))} \widetilde{y}_t^f + \frac{\phi}{(\psi + (1-\alpha))} \widetilde{m}_t - \frac{\phi \varrho}{s_0(\psi + (1-\alpha))} \widetilde{s}_t \end{aligned} \quad (7E)$$

Substitute Eq. (6E) and 7E in Eq. (5E) gives the marginal cost equation:

$$\begin{aligned}\widetilde{mc}_t = & \{ \kappa_a \widetilde{y}_t - \kappa_b \widetilde{y}_{t-1} + \kappa_c \widetilde{r} \widetilde{e} r_t + \kappa_d \widetilde{y}_{t-1}^f - \kappa_e \widetilde{r} \widetilde{e} r_t - \kappa_f \widetilde{y}_t^f \\ & + \kappa_g \widetilde{m}_t + \kappa_h \widetilde{s}_t + \lambda_b \widetilde{r}_t - \lambda_d \widetilde{a}_t\end{aligned}\quad (17)$$

where

$$\begin{aligned}\kappa_a &= \left(\lambda_a (1 - \alpha) + \lambda_c - \frac{\lambda_a (1 - \alpha)^2}{\psi_m} + \frac{\lambda_a \sigma (1 - \alpha (1 + \gamma_b))}{\psi_m \gamma_c} \right) \\ \kappa_b &= \frac{\lambda_a h (1 - \alpha) (\sigma - 1) (1 + \gamma_y)}{\psi_m \gamma_c}; \kappa_c = \frac{\lambda_a h \gamma_r (1 - \alpha) (\sigma - 1)}{\psi_m \gamma_c} \\ \kappa_d &= \frac{\lambda_a h \gamma_{yf} (1 - \alpha) (\sigma - 1)}{\psi_m \gamma_c}; \kappa_e = \frac{\lambda_a \sigma \gamma_r (1 - \alpha)}{\psi_m \gamma_c}; \\ \kappa_f &= \frac{\lambda_a \sigma \gamma_{yf} (1 - \alpha)}{\psi_m \gamma_c}; \kappa_g = \frac{\lambda_a \phi (\alpha - 1)}{\psi_m}; \kappa_h = \frac{\lambda_a \phi \varrho (1 - \alpha)}{\psi_m}\end{aligned}$$