

Evolution of Anisotropy in Charged Fluids

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Declaration

I declare that this dissertation is my own, unaided work, except where otherwise acknowledged. It is being submitted for the degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

Signed this ____ day of _____ 20____

Dale Wayne Alderton.

Abstract

A computer program has been written to simulate the conditions of the early universe and to test a new idea in the mechanism of structure formation observed in our universe today. The model utilises Newtonian hydrodynamic equations including gravitational and electromagnetic forces in two spatial dimensions. It is proposed that augmenting gravitational forces with plasma forces will complement the problematic Big Bang theory of structure formation which relies on gravity alone. Two sets of initial conditions are tested and the products of the simulation are analysed in a statistical way using power spectra and the two-point correlation function. Differences in the initial conditions were not seen to produce significantly different results. The results show that the Hubble expansion term significantly reduces power in the gravity models but plasma forces can retain power better than similar gravitation-only models. Initial velocity perturbations significantly modify the power spectrum gradient in the higher modes. Some power spectra displayed a definite bend in gradient at a scale which is verified by galaxy survey observations. Plasma forces also appear to cluster matter on smaller scales more efficiently than gravity alone. Thus, this simulation lays a foundation for a more detailed and realistic model that may be compared with real matter distribution observations.

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1 Introduction

Man's search for his origins has always been an important and fascinating part of his life. Throughout history, theories and hypotheses have been put forward as suggestions of how man came to be, with the issues often being hotly contested between the scientific and religious realms. History has shown [23] man's theories of the heavens to be intimately related to his patterns of thought which in turn was a significant indication of the state of prosperity and development of his nation. One's place and purpose in the universe, therefore, are not merely of intense scientific interest, but also have far-reaching consequences for man's wealth of civilisation and spiritual well-being. Although science and knowledge have progressed very far over the centuries, the current answers in this sphere of science called cosmology are far from satisfactory.

The dominant cosmological theory at present is the Big Bang theory - the idea that the universe began in a single cataclysmic explosion. However, it has increasingly been brought to attention over recent years [23] that the Big Bang theory has many problems associated with it. One of the more important of these is the great difficulty the Big Bang has in producing the vast supercluster complexes of galaxies seen in our universe today in the time that it claims the universe has existed - ten to twenty billion years. There is just not enough time to produce these structures through gravity in the time given. As is usual in science, one theory is not abandoned, no matter what problems it may have, unless there is a clear alternative in sight. This can be seen in the repeated attempts to patch up the Big Bang's shortcomings with new hypothetical concepts such as cosmic strings and dark matter - all of which have yet to produce satisfactory results and none of which have any firm evidence to show they exist.

How self-consistent is the Big Bang with the present day universe? Cosmologists started applying statistical methods to predict the distribution of galaxies in the sky about two decades ago. The results sparked immediate controversy over the existence or non-existence of large-scale structures (superclusters, filaments, giant

voids) in the distribution of galaxies. Over the past twenty years or so, virtually conclusive evidence has built up [1] - [6], supporting the claims that these large-scale structures (such as the Great Wall and the Great Attractor) really exist. It was discovered that the function which measures the difference between the observed galaxy distribution and a strictly random one - called the two-point correlation function - diminishes smoothly for increasing distance between galaxies, suggesting that the mechanism whereby the galaxies were formed operated on small scales, but dwindled away at large scales. As a result, this function predicts that large-scale structures would become the conglomerates of small-scale clustering. There has even been fairly recent evidence suggesting periodicity in the galaxy distribution on a 100-megaparsec scale [7].

Many purely gravitational cosmological models have been proposed to predict the observed galactic structures of today, the best of these - for a few years - seeming to be the cold dark matter models (CDM). Dark matter is the invisible matter that dynamical studies of galaxies and clusters of galaxies indicate must be there, but which cannot be seen. The fact that it is cold means that the hypothetical particles that make it up are slow-moving, a property that enables them to clump together into galaxy-sized lumps. Evidence has been presented for [8, 9] and against [1, 6, 10, 11, 12, 13] the existence of dark matter. Various models have been proposed using this hypothesis [14, 15, 16], but they appear to contradict observations of the galaxy distribution on the largest scales given by the Infrared Astronomy Satellite (IRAS) [1]. The CDM model fails because it does not correctly predict the observed galaxy distribution on scales of about twenty megaparsecs and above [12]. The real sky has less randomness than CDM can provide. Quantitative evidence of this shortcoming was provided in 1991 [1]. It remains to be seen whether more complex theories (such as mixtures of cold and hot dark matter) can succeed where others have failed. That there is no unequivocal proof of its existence, is still a major stumbling block for dark matter.

There is an alternative, however, which utilises less exotic means in an effort to claim superiority over the dark matter theories of the Big Bang in explaining universal structure formation - plasma cosmology. This concept uses electromagnetic forces with gravity (rather than gravity alone) as its main structure generator and is based on laboratory studies of plasmas (hot gases in which the electrons are stripped off by the heat, allowing the gases to be electrically conductive). Plasma physics has extrapolated these studies to the cosmos, proposing vast networks of interplanetary and interstellar electric currents and magnetic fields which have enjoyed much

observational confirmation from probes like Pioneer and Voyager [23]. Stellar system and galactic formation models based on plasma cosmology have also met with considerable success [23].

Much knowledge of plasma cosmology has been expounded in literature over recent years, although it is not well known because it does not conform to mainstream cosmological theories. There have been comments by disciples of the Big Bang to the effect that there is no observational evidence to show electric and magnetic forces are important on cosmological scales. Nevertheless, after studies of universal scale invariants (quantities that don't change as the scale of the phenomenon changes) and fundamental constants (such as the gravitational constant and masses of electron and proton) [24], laboratory plasma processes have been applied on a greater scale - particularly to double radio galaxies, quasars and extragalactic jets [25] as well as the formation of systems of galaxies [26]. The latter models also account for the explosive jets of material ejected from active galactic nuclei. Evidence for magnetic filaments at the galactic core has been found to support this [27, 28]. Furthermore, evidence has been provided by simulations [29] to solve the long-standing mystery of the 'flat' rotation curves of the galaxies - the enigma for which dark matter was created in an attempt to rationalise it. The observed rotation curves appear to emerge naturally in a galaxy governed by electromagnetic fields.

The Big Bang and plasma theories differ in two main respects. In the first instance, the Big Bang universe is ruled by gravity alone (Einstein's theory of general relativity in particular) whereas the plasma universe is formed and controlled by electricity and magnetism, not just gravity. Plasma cosmologists argue gravity is such a weak force compared to electromagnetism (the ratio of the magnitudes of the forces exceeds 10^{36} [30]) that its effect only becomes noticeable when dealing with masses of the order of the earth. In addition, electromagnetism is $\approx 10^7$ times stronger than gravity even in neutral hydrogen regions with a minute degree of ionisation of 10^{-4} [30]. On the other hand, electromagnetism is intrinsically more of a dynamic force than a static one and as such, velocity plays a significant role in determining strengths of electromagnetic fields.

All purely gravitational theories which have attempted to explain large clusters of galaxies have met with varying degrees of failure. Peebles [31] attempted a pressureless fluid model to predict the evolution of large-scale structure with no viscosity and Gaussian initial conditions, assuming conventional relativistic Friedmann-Lemaitre cosmology. His model could not produce a consistent picture of the development of large-scale structure out of Gaussian fluctuations assuming mass clusters as galaxies on large scales. The major contributing factor in these failures is that in Big Bang

cosmology the primordial density fluctuations do not grow fast enough under the influence of gravity alone to explain the formation of vast superclusters in the fifteen to twenty billion year time limit imposed by the Big Bang. This implies the existence of real inhomogeneities at all stages of the evolution of the universe [34]. Bayin [40] introduced anisotropic pressures in response, generated by two non-interacting perfect fluids (possibly representing decoupled matter and radiation). MacCallum [34] has also discussed various methods of generating anisotropic stresses. Besides the presence of a viscous term and stresses due to the anisotropic expansion of a cloud of collisionless particles, he also lists electromagnetic fields which may, in part, justify plasma cosmology.

Secondly, the Big Bang hypothesises a beginning to time and therefore a finitely aged universe. On the contrary, due to plasma cosmology's strict adherence to only phenomena that can be observed and studied in the laboratory, it maintains that nowhere is something seen to emerge from nothing and thus precludes such concepts as an origin of the universe *ex nihilo*, a beginning to time or a Big Bang [23]. Instead, plasma cosmology observes an evolving universe and assumes this has always taken place and will continue to do so for an infinite time to come.

There is much evidence for short-range clumping effects in the sphere of plasma cosmology. From observations of X-rays, gamma rays and radio waves, it has been established that more than 99% (by volume) of the universe consists of plasma [18] - a fact which has been neglected by the Big Bang. Combine this with the observed structure of the two-point correlation function and the suggestion emerges that plasma forces may be a significant but short-range ordering while gravity is dominant on the larger scales. Plasma cosmology, however, contends plasma forces are still significant on large scales and that supercluster complexes were built up from plasma processes and gravity subsequently broke the structures up into smaller clouds and galaxies [24]. The effect of gravity on large neighbouring bodies could also have shaped the universal structure into that indicated by the two-point correlation function.

Plasma cosmology, instead of attempting to improve on certain areas of contemporary cosmology which may be erroneous or incomplete, has endeavored a complete separation from the standardised concepts of today. Consequently it has rather alienated itself from the very same established society of cosmological study that it wishes to impact upon. It is probably more prudent to employ a much slower and more systematic addressing of inconsistencies in the present theory. Plasma science is certainly well-founded in the scientific realm and as discussed above has a contribution to make to cosmology too. It is in this vein that this study is motivated;

to assist to reconcile observations of supercluster complexes of galaxies with cosmological theory by introducing elements which have been thoroughly studied and are everyday phenomena, not exotic concepts - namely electromagnetic forces.

Thus, a hypothesis is proposed: that a simple synthesis of the Big Bang universe and electromagnetic forces on cosmological scales can produce the vast galactic structures inhabiting the universe today and also provide a platform for a better understanding of astrophysical processes.

Unfortunately, a full validation of the hypothesis is beyond the scope of this study. Therefore, the objectives of this work are to create a simplified (two space and one time dimension) mathematical model and to study its results to enable better understanding of the mechanics of a universe driven by gravity and electromagnetism together. This is achieved as follows:

1. Fluid partial differential equations are derived for an electromagnetic and gravity environment and subsequently non-dimensionalised;
2. These equations are converted into ordinary differential equations using spectral methods;
3. A computer program is developed in C++ and Visual Basic, incorporating the above equations, and used to produce results for analysis.

The theory and equations are independently developed and implemented. Results are plotted using the Matlab computer package and subsequently analysed.

The next chapter deals with the assumptions of the model and the derivations of the partial differential equations. All spectral analysis and solutions to the equations are described in Chapter 3. Chapter 4 contains details of the computer simulation programming while the results and their discussion are provided in Chapter 5. Conclusions and recommendations follow in Chapter 6.

2 Derivation of Equations

In 1970, based on his and others' work, Gerard DeVaucouleur [33] showed that as objects in the universe become larger, their density decreases - approximately as the square of the object's size. Therefore an object ten times larger would be about one hundred times less dense. This relation has far-reaching consequences. Ω , the ratio of the average density to that needed to halt the expansion of the universe is an important concept in standard cosmology. But in the light of this relation, defining an average density for the whole universe becomes problematic at best. Even within the observable universe, the density will be overestimated if too small an area (even up to a billion light years across) is measured because the measuring will always be taking place inside some large agglomeration, be it a cluster, supercluster or larger structure.

Essentially, the DeVaucouleur relation sets an upper limit on the escape velocity of at most one or two thousand kilometres per second for any gravitating body [23]. This is far below relativistic velocities. Should DeVaucouleur's relation (proven up to distances of a hundred thousand light years [23]) be valid up to the scale of the observable universe, the true density could be up to one hundred times less [23]. Gravitation would be very weak with so little matter. Thus both these points indicate general relativity to be no more than a subtle correction to Newtonian gravitation in the universe except near ultradense regions such as neutron stars [23].

Since this model is primarily concerned with the overall patterns of cosmic structure, in the light of the above discussion, it was considered justifiable to deal exclusively with a simpler Newtonian model and exclude all relativistic effects.

The philosophy used in approaching this problem was to find equations which described interactions between the density, velocity and force fields and then evolve these in time to form a dynamic system. The classical conservation equations were deemed suitable for this purpose. When combined with the appropriate initial conditions, it should be possible to get a good idea of the evolution of a universe governed

by gravitational forces and plasma forces in the form of electrostatic and electrodynamic interactions between positively and negatively charged matter streams.

2.1 The One Dimensional Model ($\mathcal{R} \times \mathcal{R}$)

For the sake of clarity, the equations are initially derived for one space and one time dimension. This would have been the desirable scenario for the entire model due to its simplicity. Unfortunately, the minimum number of dimensions in which electromagnetic forces make sense is two spatial dimensions and one time dimension, which is how the model will be developed. The one space dimension is modelled on the real line. The classical conservation equations for a fluid are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0, \quad (2.1)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial (\rho v^2 + p)}{\partial x} = F. \quad (2.2)$$

for mass and momentum respectively where ρ represents density, v represents velocity in the x direction, p is the pressure and F depicts force (consisting of gravitational and plasma forces). It is assumed that the fluids involved are ideal pressureless ones (i.e. $p=0$). Observations of mildly non-linear density fluctuations on scales of a few tens of megaparsecs can be used as fairly direct indicators of the recent dynamical evolution of the universe. Peebles [31] claims that, if the net velocity dispersion is as small as that of the galaxies, then dynamics on these scales is well approximated by an ideal pressureless fluid. Equation 2.2 can be further simplified using Equation 2.1. The momentum equation can be rewritten as

$$\frac{\partial \rho}{\partial t} v + \rho \frac{\partial v}{\partial t} + \frac{\partial (\rho v)}{\partial x} v + \rho v \frac{\partial v}{\partial x} = F$$

Substituting in Equation 2.1 yields

$$\begin{aligned} -\frac{\partial (\rho v)}{\partial x} v + \rho \frac{\partial v}{\partial t} + \frac{\partial (\rho v)}{\partial x} v + \rho v \frac{\partial v}{\partial x} &= F \\ \rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} &= F \end{aligned}$$

Dividing through by ρ , one obtains the non-linear advection equation which is a Burgers' equation:

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{F}{\rho} \quad (2.3)$$

and describes how the velocity changes with time. This in turn affects the matter density. At this stage, the equations do not take into account the Hubble expansion present in the universe. They need to be rederived to extract the correct additional term/s. The approach adopted is to use a reference frame co-moving with the Hubble expansion and treats the peculiar velocity of a particle as perturbations of the Hubble velocity. Figure 2.1 below represents a volume element expanding at the Hubble rate. Beginning by examining the element's mass conservation, expansion begins at some arbitrary time t .

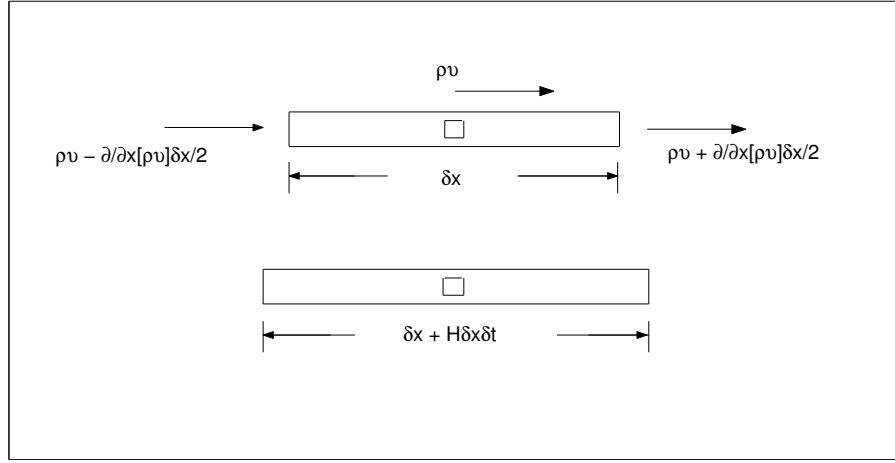


Figure 2.1: Mass Transfer in 1D Expanding Element.

At some later time, $t + \delta t$, note how the volume has changed due to Hubble expansion. In this figure, H is the hubble constant. The reference point for the mass flux through the element is taken at the centre as shown and the mass in the element at time $t = \rho \delta x$. At time $t + \delta t$ it is

$$\left(\rho + \frac{\partial \rho}{\partial t} \delta t\right)(\delta x + H \delta t \delta x)$$

Therefore the change in mass over δt is

$$\rho \delta x + \rho H \delta t \delta x + \frac{\partial \rho}{\partial t} \delta x \delta t - \rho \delta x$$

to first order approximation in small quantities of δx and δt . The change in flux is calculated by taking the difference across the boundaries of the element.

$$\begin{aligned} \rho v - (\rho v + \frac{\partial \rho v}{\partial x} \delta x) \\ = -\frac{\partial \rho v}{\partial x} \delta x \end{aligned}$$

Attributing the change in mass in the element to the flux gradient...

$$(\rho H + \frac{\partial \rho}{\partial t}) \delta t \delta x = -\frac{\partial \rho v}{\partial x} \delta t \delta x$$

The resulting equation is thus

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} + \rho H = 0 \quad (2.4)$$

This extra Hubble term represents the expected density decay of a finite amount of matter in an expanding volume. Note that this is *not* the result obtained by letting the velocity v equal a peculiar velocity plus the Hubble velocity ($u + HR$) in Equation 2.4 where R is some Hubble length. Using this substitution yields an additional term $HR \frac{\partial \rho}{\partial x}$ which causes difficulties when trying to solve these equations.

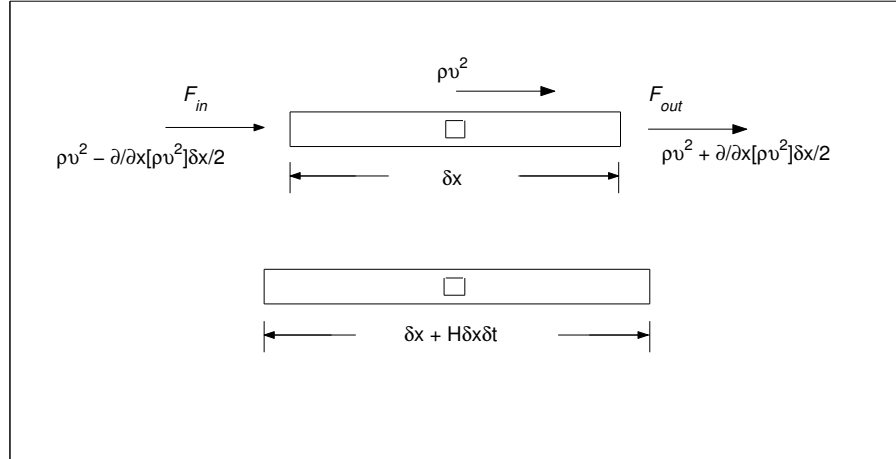


Figure 2.2: Momentum Transfer in 1D Expanding Element.

A similar derivation is observed for momentum in Figure 2.2. Momentum, however, may not appear as simple to define for the element as mass because it is expanding. Observers at either end of the element would observe different velocities for a freely moving particle because they are traveling at different speeds relative to one another. This is of course due to the fact that they are moving in different co-ordinate systems [44]. So momentum is referred back to the centre of the element which is

unambiguous, rather than to the ends. This avoids confusion which might arise as to which end is being referred to. Additionally, one must take into account extraneous forces which form part of the momentum flux here.

The momentum in the element at time t is $\rho v \delta x$. At time $t + \delta t$ it is

$$(\rho v + \frac{\partial \rho v}{\partial t} \delta t)(\delta x + H \delta t \delta x)$$

Therefore the change in momentum is

$$\begin{aligned} & (\rho v + \frac{\partial \rho v}{\partial t} \delta t)(\delta x + H \delta t \delta x) - \rho v \delta x \\ &= (\frac{\partial \rho v}{\partial t} + \rho v H) \delta t \delta x \end{aligned}$$

again to first order approximation. The flux gradient is

$$\begin{aligned} & (\rho v^2 - \frac{\partial \rho v^2}{\partial x} \frac{\delta x}{2} + F_{in} \delta x) \delta t - (\rho v^2 + \frac{\partial \rho v^2}{\partial x} \frac{\delta x}{2} + F_{out} \delta x) \delta t \\ &= (-\frac{\partial \rho v^2}{\partial x} + F_{in} - F_{out}) \delta x \delta t \end{aligned}$$

Let $F_{in} - F_{out} = F$. Equating the flux with the momentum change gives:

$$(-\frac{\partial \rho v^2}{\partial x} + F) \delta t \delta x = (\frac{\partial \rho v}{\partial t} + \rho v H) \delta t \delta x$$

which yields

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v^2}{\partial x} + \rho v H = F \quad (2.5)$$

Once again, this equation may be simplified by substitution of the new mass equation (2.4). It now becomes:

$$\begin{aligned} \rho \frac{\partial v}{\partial t} + v(-\frac{\partial \rho v}{\partial x} - \rho H) + \frac{\partial \rho v^2}{\partial x} + \rho v H &= F \\ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} &= \frac{F}{\rho} \end{aligned}$$

which is identical to Equation 2.3. Thus the Hubble decay term disappears, demonstrating that density would decrease inside an expanding volume as would be expected, but momentum would also decrease only because it is the product of density and velocity. Velocity itself would not deteriorate in an expanding model.

2.2 The Two Dimensional Model ($\mathcal{R}^2 \times \mathcal{R}$)

It was stated in the Introduction that in order to accommodate electromagnetic forces, the model needs to be expanded to at least one time and two space dimensions. Since simplicity is desirable in studying a new model to better understand its features, two spatial dimensions are used. The standard conservation equations are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \quad (2.6)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v^2}{\partial x} + \frac{\partial \rho v v}{\partial y} = F_x, \quad (2.7)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v v}{\partial x} + \frac{\partial \rho v^2}{\partial y} = F_y. \quad (2.8)$$

The last two involve changes in the velocity in the x and y directions respectively. They may also be simplified using Equation 2.6 in an analogous fashion to Equation 2.2 in one dimension. Substituting for $\partial \rho / \partial t$ from Equation 2.6 into Equation 2.7...

$$\begin{aligned} -\left(\frac{\partial \rho v}{\partial x} + \frac{\partial \rho v}{\partial y}\right)v + \rho \frac{\partial v}{\partial t} + \frac{\partial \rho v}{\partial x}v + \rho v \frac{\partial v}{\partial x} + \frac{\partial \rho v}{\partial y}v + \rho v \frac{\partial v}{\partial y} &= F_x. \\ \rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} &= F_x \end{aligned}$$

is obtained. One can now divide through by ρ again to yield a two-dimensional form of Burgers' equation:

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{F_x}{\rho}, \quad (2.9)$$

and similarly with equation 2.8,

$$\frac{\partial \nu}{\partial t} + v \frac{\partial \nu}{\partial x} + \nu \frac{\partial \nu}{\partial y} = \frac{F_y}{\rho}, \quad (2.10)$$

The conservation equations for a two-dimensional expanding fluid are now derived. Figure 2.3 shows the dynamics of mass conservation for an infinitesimal two-dimensional element.

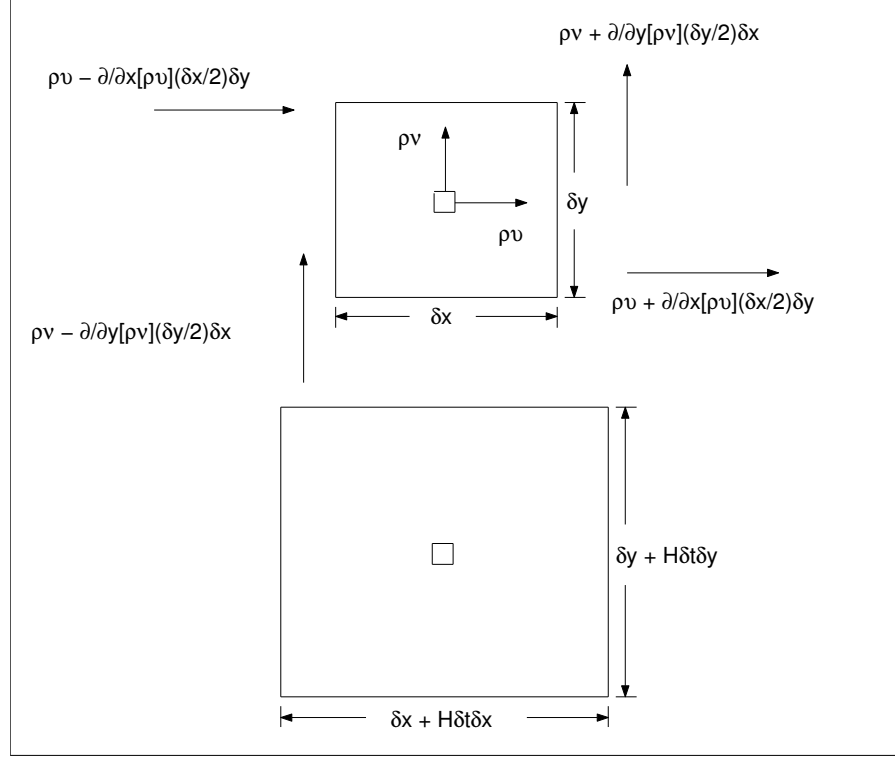


Figure 2.3: Mass Transfer in 2D Expanding Element.

Here H is the Hubble constant as above. Again the reference is taken with respect to the centre of the element with the mass fluxes on the edges of the element shown in both the x and y directions. The mass in the element at time $t = \rho \delta x \delta y$. At time $t + \delta t$ it is

$$\left(\rho + \frac{\partial \rho}{\partial t} \delta t\right) (\delta x + H \delta t \delta x) (\delta y + H \delta t \delta y)$$

Therefore the change in mass over δt is

$$\rho \delta x \delta y + 2\rho H \delta x \delta y \delta t + \frac{\partial \rho}{\partial t} \delta x \delta y \delta t - \rho \delta x \delta y$$

to first order approximation. The flux gradient across the boundaries of the element is

$$\begin{aligned} & \left(\rho v - \frac{\partial \rho v}{\partial x} \frac{\delta x}{2} \delta y + \rho v - \frac{\partial \rho v}{\partial y} \delta x \frac{\delta y}{2} \right) - \left(\rho v + \frac{\partial \rho v}{\partial x} \frac{\delta x}{2} \delta y + \rho v + \frac{\partial \rho v}{\partial y} \delta x \frac{\delta y}{2} \right) \\ &= -\frac{\partial \rho v}{\partial x} \delta x \delta y - \frac{\partial \rho v}{\partial y} \delta x \delta y \end{aligned}$$

Attributing the change in mass in the element to the flux gradient...

$$(2\rho H + \frac{\partial \rho}{\partial t}) \delta x \delta y \delta t = -\left(\frac{\partial \rho v}{\partial x} + \frac{\partial \rho v}{\partial y} \right) \delta x \delta y \delta t$$

The resulting equation is thus

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} + \frac{\partial \rho v}{\partial y} + 2\rho H = 0 \quad (2.11)$$

A similar derivation exists for the momentum equation. It is shown in Figure 2.4. There are two equations involved since momentum is a vector. However, only the derivation for momentum in the x-direction is performed here.

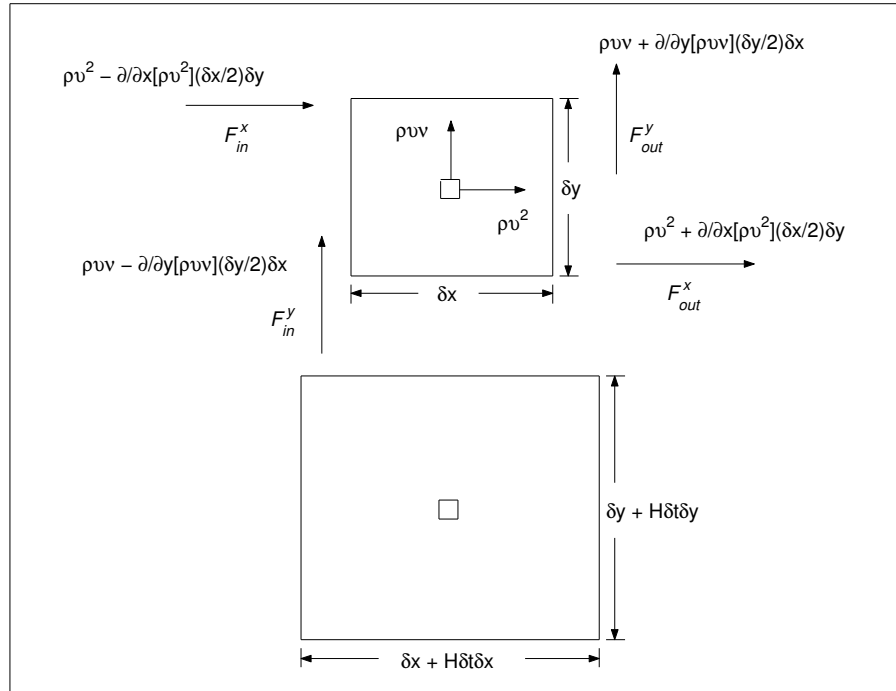


Figure 2.4: Momentum Transfer in 2D Expanding Element.

The change in momentum in the element over δt is

$$\begin{aligned} \rho v \delta x \delta y + 2\rho v H \delta x \delta y \delta t + \frac{\partial \rho v}{\partial t} \delta x \delta y \delta t - \rho v \delta x \delta y \\ = (2\rho v H + \frac{\partial \rho v}{\partial t}) \delta x \delta y \delta t \end{aligned}$$

and the change in flux is

$$\begin{aligned} (\rho v^2 - \frac{\partial \rho v^2}{\partial x} \frac{\delta x}{2} \delta y + \rho v \nu - \frac{\partial \rho v \nu}{\partial y} \delta x \frac{\delta y}{2} + F_{in} \delta x \delta y) \delta t \\ - (\rho v^2 + \frac{\partial \rho v^2}{\partial x} \frac{\delta x}{2} \delta y + \rho v \nu + \frac{\partial \rho v \nu}{\partial y} \delta x \frac{\delta y}{2} + F_{out} \delta x \delta y) \delta t \\ = (-\frac{\partial \rho v^2}{\partial x} - \frac{\partial \rho v \nu}{\partial y} + F_{in} - F_{out}) \delta x \delta y \delta t \end{aligned}$$

On this occasion, let $F_{in} - F_{out} = F_x$ to distinguish it from forces in the y -direction. Equating the momentum change and flux gradient,

$$(2\rho v H + \frac{\partial \rho v}{\partial t}) \delta x \delta y \delta t = (-\frac{\partial \rho v^2}{\partial x} - \frac{\partial \rho v \nu}{\partial y} + F_x) \delta x \delta y \delta t$$

the result is the following:

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v^2}{\partial x} + \frac{\partial \rho v \nu}{\partial y} + 2\rho v H = F_x, \quad (2.12)$$

and similarly

$$\frac{\partial \rho \nu}{\partial t} + \frac{\partial \rho \nu \nu}{\partial x} + \frac{\partial \rho v^2}{\partial y} + 2\rho \nu H = F_y. \quad (2.13)$$

When one again substitutes equation 2.4 into equation 2.12, the following is obtained:

$$\begin{aligned} \rho \frac{\partial v}{\partial t} + v(-\frac{\partial \rho v}{\partial x} - \frac{\partial \rho v}{\partial y} - 2\rho H) + \frac{\partial \rho v^2}{\partial x} + \frac{\partial \rho v \nu}{\partial y} + 2\rho v H &= F_x \\ \rho \frac{\partial v}{\partial t} - v \frac{\partial \rho v}{\partial x} - v \frac{\partial \rho v}{\partial y} + v \frac{\partial \rho v}{\partial x} + \rho v \frac{\partial v}{\partial x} + v \frac{\partial \rho v}{\partial y} + \rho v \frac{\partial v}{\partial y} &= F_x \\ \rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} &= F_x \end{aligned}$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \nu \frac{\partial v}{\partial y} = \frac{F_x}{\rho}$$

which is identical to equation 2.9. In a similar manner, the momentum in the y- direction reduces to equation 2.10. Once again the Hubble term disappears, demonstrating that momentum only decays by virtue of the density expanding in a fixed volume and not due to the velocity component, which is conserved.

The forces may now be formulated, beginning with gravity.

$$F = -\rho \nabla \phi \quad (2.14)$$

where ϕ is the gravitational potential. Dividing through by ρ to make it consistent with the right hand side of Equations 2.9 and 2.10, one obtains

$$\frac{F}{\rho} = -\nabla \phi$$

where

$$\nabla^2 \phi = 4\pi \rho G. \quad (2.15)$$

Here, G is the gravitational constant. The gravitational force components are easily realised - $\partial \phi / \partial x$ for the x-component and $\partial \phi / \partial y$ for the y-component.

Plasma forces may now be resolved in two-dimensional space. It should be noted that the electromagnetic forces described below are initially for a non-expanding fluid. They are then adjusted for an expanding fluid to illustrate the features of the expanding model. The equation used to represent the plasma forces is the Lorentz force equation [43]:

$$F_E = q(E + v \times B) \quad (2.16)$$

where q is the electric charge, E is the electric field strength, v is velocity and B is the magnetic field strength. The unknown individual components of this equation (electric and magnetic field strength) are computed from Maxwell's equations:

$$\nabla \cdot E = 4\pi K_1 \varrho \quad (2.17)$$

$$\nabla \times B = 4\pi K_2 J + \frac{K_2}{K_1} \frac{\partial E}{\partial t}. \quad (2.18)$$

where ϱ is the electric charge density. Here, J is the current density which is equal to ϱv where v is the velocity vector. The symbols K_1 and K_2 are arbitrary constants. No specific values are given because after non-dimensionalisation, they are adjusted for the simulations. The normal values of the constants cause the simulations to become unstable, thus they are reduced by several orders of magnitude in the electromagnetic models.

Firstly, the problem of charge density is addressed. One can only resolve this if the differently charged matter fields are separately calculated and updated. This necessitates three matter streams (neutral, positively and negatively charged). However, in this way, the charge densities can be related back to the matter densities.

$$\varrho = \frac{q}{m} \rho \quad (2.19)$$

Here, q/m is the charge to mass ratio of the particular particle involved. The charge density is needed to compute the electric field. If one compares equation 2.14 with the electrostatic term of equation 2.16, one can see that they are completely analogous and that their determination using potentials is also very similar. Thus the electric field is calculated in the same way as the gravity field, using charge density instead of matter density.

The force terms are now integrated into the momentum conservation equations. However, the electromagnetic force still needs to be divided by ρ as was gravity. This is easily achieved by combining Equations 2.16 and 2.19 to produce

$$\frac{F_E}{\rho} = \frac{q}{m} (E + v \times B)$$

where the electric field potential for determining E is represented by ψ . Let the gravitational potential be ϕ .

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \nu \frac{\partial v}{\partial y} = \frac{\partial \phi}{\partial x} + \frac{q}{m} \left(\frac{\partial \psi}{\partial x} + Bv \right) \quad (2.20)$$

$$\frac{\partial \nu}{\partial t} + v \frac{\partial \nu}{\partial x} + \nu \frac{\partial \nu}{\partial y} = \frac{\partial \phi}{\partial y} + \frac{q}{m} \left(\frac{\partial \psi}{\partial y} - Bv \right). \quad (2.21)$$

The magnetic field strength is required for the electrodynamic term of Equation 2.16. In order to calculate the magnetic field, the change in electric field with respect to

time is needed. This is done by first calculating the electric field and then taking the derivative with respect to time. As shown below, this involves an equation in terms of the time rate of change of the density which has already been calculated above. Along with the current density, these are all the terms necessary to obtain the magnetic field.

There are a few points that should be clarified before proceeding further. Because the model is only in two dimensions, only currents in the x and y directions are possible which means that the magnetic field lines they produce will be pointing in the z direction where they intersect the x - y plane. Normally this would constitute a third spatial dimension and thus be left out of the model. If this were the case, no electrodynamic forces would be possible in two dimensions. However, the magnetic field lines in this z direction interact with the velocity of the charged matter fields to produce forces in the x and y directions. Therefore if the magnetic field strength is calculated and then the cross product with the different velocities (multiplied by the charge) performed, the electrodynamic forces would be obtained without having to introduce a third spatial dimension because it would always be known that the magnetic field lines are in the z direction. Since all the system's properties are confined to the x - y plane, it is not possible to have magnetic field lines in the x or y directions. Thus, $\nabla \times B$ would look as follows:

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & 0 \\ 0 & 0 & B_z \end{vmatrix}$$

This vector cross product provides two different equations:

$$\frac{\partial B_z}{\partial y} = \frac{K_2}{K_1} \frac{\partial E_x}{\partial t} + 4\pi K_2 J_x \quad (2.22)$$

and

$$-\frac{\partial B_z}{\partial x} = \frac{K_2}{K_1} \frac{\partial E_y}{\partial t} + 4\pi K_2 J_y \quad (2.23)$$

That either of these equations may be used to calculate the magnetic field, is demonstrated by using the fact that mixed partial derivatives are equal. By differentiating equation 2.22 with respect to x and equation 2.23 with respect to y and adding them, the left hand side of the new equation will clearly be zero. The right hand side will be

$$\begin{aligned}
& \frac{\partial}{\partial x} \frac{\partial}{\partial t} \left(\frac{K_2}{K_1} E_x \right) + \frac{\partial}{\partial x} (4\pi K_2 \varrho v) + \frac{\partial}{\partial y} \frac{\partial}{\partial t} \left(\frac{K_2}{K_1} E_y \right) + \frac{\partial}{\partial y} (4\pi K_2 \varrho \nu) \\
&= \frac{K_2}{K_1} \frac{\partial}{\partial t} \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right) + 4\pi K_2 \left(\frac{\partial \varrho v}{\partial x} + \frac{\partial \varrho \nu}{\partial y} \right) \\
&= \frac{K_2}{K_1} \frac{\partial}{\partial t} (4\pi K_1 \varrho) + 4\pi K_2 \left(\frac{\partial \varrho v}{\partial x} + \frac{\partial \varrho \nu}{\partial y} \right) \\
&= 4\pi K_2 \left(\frac{\partial \varrho}{\partial t} + \frac{\partial \varrho v}{\partial x} + \frac{\partial \varrho \nu}{\partial y} \right) \\
&= 4\pi K_2 (0) = 0
\end{aligned}$$

from the continuity equation for charge which is analogous to that for matter (Equation 2.6). This shows the two sides of the equation to be equal and proves that either of Equations 2.22 or 2.23 are valid for calculating the magnetic field.

Now the expanding model is considered. The magnetic field equation (2.18) requires adjustment to make it consistent with the expanding continuity equation. To formulate the expanding magnetic field equation, the proof just used above to show Equations 2.22 and 2.23 are equally valid, is exploited. This is because the original continuity equation participates in that proof. One now needs to add an extra expansion term $\frac{K_2}{K_1} 2EH$ to the right hand side of Equation 2.18 to make it compatible with the expanding system:

$$\nabla \times B = \frac{K_2}{K_1} \left(\frac{\partial E}{\partial t} + 2EH \right) + 4\pi K_2 J \quad (2.24)$$

Its correctness is shown below. The mixed partial derivatives are still zero while on the right hand side one now has

$$\begin{aligned}
& \frac{\partial}{\partial x} \frac{K_2}{K_1} \left(\frac{\partial E_x}{\partial t} + 2E_x H \right) + \frac{\partial}{\partial x} (4\pi K_2 \varrho v) + \frac{\partial}{\partial y} \frac{K_2}{K_1} \left(\frac{\partial E_y}{\partial t} + 2E_y H \right) + \frac{\partial}{\partial y} (4\pi K_2 \varrho \nu) \\
&= \frac{\partial}{\partial x} \frac{\partial}{\partial t} \left(\frac{K_2}{K_1} E_x \right) + \frac{K_2}{K_1} \frac{\partial}{\partial x} 2E_x H + \frac{\partial}{\partial x} (4\pi K_2 \varrho v) + \frac{\partial}{\partial y} \frac{\partial}{\partial t} \left(\frac{K_2}{K_1} E_y \right) \\
&\quad + \frac{K_2}{K_1} \frac{\partial}{\partial y} 2E_y H + \frac{\partial}{\partial y} (4\pi K_2 \varrho \nu) \\
&= \frac{K_2}{K_1} \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right) \left(\frac{\partial}{\partial t} + 2H \right) + 4\pi K_2 \left(\frac{\partial \varrho v}{\partial x} + \frac{\partial \varrho \nu}{\partial y} \right) \\
&= \frac{K_2}{K_1} 4\pi K_1 \varrho \left(\frac{\partial}{\partial t} + 2H \right) + 4\pi K_2 \left(\frac{\partial \varrho v}{\partial x} + \frac{\partial \varrho \nu}{\partial y} \right) \\
&= 4\pi K_2 \left(\frac{\partial \varrho}{\partial t} + \frac{\partial \varrho v}{\partial x} + \frac{\partial \varrho \nu}{\partial y} + 2\varrho H \right)
\end{aligned}$$

This is consistent with the newly derived continuity equation (2.11) and thus equals zero which is the same as the left hand side. So either of the two equations below may be used:

$$\frac{\partial B_z}{\partial y} = \frac{K_2}{K_1} \left(\frac{\partial E_x}{\partial t} + 2E_x H \right) + 4\pi K_2 J_x \quad (2.25)$$

$$-\frac{\partial B_z}{\partial x} = \frac{K_2}{K_1} \left(\frac{\partial E_y}{\partial t} + 2E_y H \right) + 4\pi K_2 J_y \quad (2.26)$$

2.3 Scaling and Non-Dimensionalisation

The velocity in the model was non-dimensionalised by dividing it by the Hubble velocity HR (R being the characteristic Hubble length in megaparsecs) and the density by dividing it by the average density.

$$\frac{\rho}{\rho_0} = \hat{\rho}; \quad \frac{v}{HR} = \hat{v}; \quad \frac{\nu}{HR} = \hat{\nu}$$

where ρ_0 is the average density, $\hat{\rho}$ is the non-dimensional density and $\hat{v}$ and $\hat{\nu}$ are the non-dimensional velocities in the x and y directions respectively. In the following equations R and T will be used to represent length and time scales respectively. Note: this is different from the usual density contrast of $(\frac{\rho}{\rho_0} - 1)$ because it allows much simpler non-dimensionalisation. The usual density contrast has zero mean but that used in this study has a mean of one.

Non-dimensionalising Equation 2.11:

$$\frac{\rho_0}{T} \frac{\partial \hat{\rho}}{\partial \hat{t}} + \frac{\rho_0 HR}{R} \frac{\partial \hat{\rho} \hat{v}}{\partial \hat{x}} + \frac{\rho_0 HR}{R} \frac{\partial \hat{\rho} \hat{\nu}}{\partial \hat{y}} + 2\rho_0 H \hat{\rho} = 0$$

Dividing through by $\rho_0 \dots$

$$\frac{1}{T} \frac{\partial \hat{\rho}}{\partial \hat{t}} + H \left(\frac{\partial \hat{\rho} \hat{v}}{\partial \hat{x}} + \frac{\partial \hat{\rho} \hat{\nu}}{\partial \hat{y}} + 2\hat{\rho} \right) = 0$$

The factor $1/T$ above has units consistent with the Hubble constant. One can therefore let the inverse Hubble constant represent the time scale (i. e. $T = 1/H$).

It is now possible to divide through by H to obtain the non-dimensional form of equation 2.11.

$$\frac{\partial \hat{\rho}}{\partial \hat{t}} + \frac{\partial \hat{\rho} \hat{v}}{\partial \hat{x}} + \frac{\partial \hat{\rho} \hat{v}}{\partial \hat{y}} + 2\hat{\rho} = 0 \quad (2.27)$$

The force terms are non-dimensionalised first for insertion into the momentum equations. From Equations 2.14 and 2.15, the non-dimensional gravitational field is as follows:

$$\left(\frac{\hat{F}_G}{\rho}\right) = \left(\frac{F_G}{\rho}\right)/4\pi\rho_0 GR\hat{\rho}$$

The plasma forces consist of multiple parts. Firstly, the electric field is similar to the gravitational field.

$$\hat{E} = E/4\pi\frac{q}{m}\rho_0 K_1 R\hat{\rho}$$

Equation 2.25 for example becomes

$$\begin{aligned} \frac{1}{R} \frac{\partial B}{\partial \hat{y}} &= \frac{K_2}{K_1} \left(H \frac{\partial E_x}{\partial \hat{t}} + 2E_x H \right) + 4\pi K_2 \frac{q}{m} \rho_0 H R \hat{\rho} \hat{v} \\ \frac{1}{R} \frac{\partial B}{\partial \hat{y}} &= \frac{q}{m} \frac{K_2}{K_1} (4\pi\rho_0 K_1 H R \frac{\partial \hat{\rho}}{\partial \hat{t}} + 8\pi\rho_0 K_1 H R) + \frac{q}{m} 4\pi K_2 \rho_0 H R \hat{\rho} \hat{v} \\ \frac{\partial B}{\partial \hat{y}} &= \frac{q}{m} 4\pi K_2 \rho_0 H R^2 \left(\frac{\partial \hat{\rho}}{\partial \hat{t}} + 2\hat{\rho} + \hat{\rho} \hat{v} \right) \end{aligned}$$

Therefore

$$\hat{B} = B/\frac{q}{m} 4\pi K_2 \rho_0 H R^2 \quad (2.28)$$

This leads to the non-dimensionalisation of the electric force equation.

$$\begin{aligned} \frac{F_E}{\rho} &= \frac{q}{m} (E + v \times B) \\ &= \frac{q}{m} \left(4\pi \frac{q}{m} K_1 \rho_0 R \hat{\rho} + H R \hat{v} \frac{q}{m} 4\pi K_2 \rho_0 H R^2 \hat{B} \right) \\ &= \left(\frac{q}{m} \right)^2 4\pi \rho_0 R [K_1 \hat{\rho} + (H R)^2 K_2 \hat{v} \hat{B}] \\ &= \left(\frac{q}{m} \right)^2 4\pi \rho_0 R [c^2 K_2 \hat{\rho} + (H R)^2 K_2 \hat{v} \hat{B}] \end{aligned}$$

since $c^2 K_2 = K_1$. Thus

$$\left(\frac{\hat{F}_E}{\rho}\right) = \left(\frac{F_E}{\rho}\right)/\left(\frac{q}{m}\right)^2 4\pi K_2 \rho_0 H^2 R^3 \left[\left(\frac{c}{H R}\right)^2 \hat{\rho} + \hat{v} \hat{B}\right] \quad (2.29)$$

The full momentum equation is therefore

$$H^2 R \frac{\partial \hat{v}}{\partial \hat{t}} + H^2 R \hat{v} \frac{\partial \hat{v}}{\partial \hat{x}} + H^2 R \hat{v} \frac{\partial \hat{v}}{\partial \hat{y}} = 4\pi \rho_0 G R \hat{\rho} + \left(\frac{q}{m}\right)^2 4\pi K_2 \rho_0 H^2 R^3 (\hat{\rho} + \hat{v} \hat{B})$$

$$\frac{\partial \hat{v}}{\partial \hat{t}} + \hat{v} \frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{v}}{\partial \hat{y}} = \frac{4\pi \rho_0 G}{H^2} \hat{\rho} + \left(\frac{q}{m}\right)^2 4\pi K_2 \rho_0 R^2 (\hat{\rho} + \hat{v} \hat{B}) \quad (2.30)$$

From this it can be seen that the gravitational non-dimensional group is not influenced by an independent length scale but does rely on a time scale. The length scale is concealed in the Hubble constant and cannot be independently changed. However, there are length and time scales in the non-dimensional grouping of the electromagnetic forces. One can take the ratio between the two groups to get an idea of the relationship between the two forces. It is

$$\frac{\left(\frac{q}{m}\right)^2 4\pi K_2 \rho_0 R^2}{4\pi G \rho_0 / H^2} = \frac{\left(\frac{q}{m}\right)^2 K_2 (HR)^2}{G}$$

Since

$$\frac{\left(\frac{q}{m}\right)^2 K_2 (HR)^2}{G} = \frac{\left(\frac{q}{m}\right)^2 K_1}{G} \left(\frac{HR}{c}\right)^2$$

this can be made to be the same as

$$\frac{\left(\frac{q}{m}\right)^2 K_1}{G}$$

if a factor $(c/HR)^2$ is taken out of equation 2.29. This ratio could be quantified by using the properties of a proton ($q/m \approx 9.56 \times 10^7$). It would then be

$$\frac{\left(\frac{q}{m}\right)^2 4\pi K_1 \rho_0 L}{4\pi \rho_0 G L} = \frac{\left(\frac{q}{m}\right)^2 K_1}{G} = \frac{(9.56 \times 10^7)^2 (9 \times 10^9)}{6.67 \times 10^{-11}} \approx 1.2 \times 10^{36}$$

which is the figure for the comparison between the electrical and gravitational forces as given in the Introduction. This would suggest that gravitational forces have relatively little bearing in a realistic model. This may depend somewhat on the densities

involved because all matter succumbs to gravity. Although with a 99% volume of plasma in the universe, there is very little matter not subject to some electromagnetic influence, and any influence of this type appears to outweigh the force of gravity many times over. It now becomes almost inconceivable to formulate cosmological theories without taking plasma forces into account. However, in the simulations, the magnitude of plasma forces relative to gravity had to be substantially reduced in order to prevent numeric instabilities.

In addition, the ratio of the forces can be made to depend on the Hubble velocity - the greater the velocity, the more dominant are the plasma forces. This is equivalent to saying that on greater scales, electromagnetism does not become negligible in comparison to gravity, from which one may be able to infer that plasma forces certainly have an influence on the development of the larger structures. Looking at the electric force term in equation 2.29, it is noted that its contribution will decline as the scale increases and thus magnetic forces will become more prominent.

3 Solution of Equations

The equations derived in the previous chapter are now solved using spectral analysis. It is a useful method for gauging relative contributions to the energy of the system from the various frequencies as well as converting partial differential equations into ordinary differential equations. First, the continuity equation is considered as a simple example. The solution is assumed to take the form of an infinite complex Fourier series. This consequently implies a periodic universe which may place a serious restriction on the results. However, only a single period whose dimensions are dependent on the largest mode chosen and the length scaling may be studied. Thus let

$$\rho = \sum_{j=-\infty}^{\infty} r_j(t) e^{ijx} \text{ and}$$

$$v = \sum_{k=-\infty}^{\infty} u_k(t) e^{ikx}.$$

where each positive and negative pairing of indices ($\pm 1, \pm 2 \dots$) represents one mode of vibration with the zero index value being the steady state uniform value. The product ρv is then

$$\rho v = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} r_j(t) u_k(t) e^{i(j+k)x}.$$

For the sake of brevity, the expressions for ρ, v and subsequent variables or terms will take on the following shortened form:

$$\rho = r_j e^{ijx}$$

and

$$v = u_k e^{ikx}$$

where it is understood that summation occurs over repeated indices.

Equation 2.4 then becomes

$$\frac{\partial}{\partial t}(r_j e^{ijx}) + \frac{\partial}{\partial x}(r_k u_l e^{i(k+l)x}) + r_p e^{ipx} = 0$$

$$\frac{dr_j}{dt}e^{ijx} + i(k+l)r_k u_l e^{i(k+l)x} + r_p e^{ipx} = 0$$

Now let $m = k + l$ such that

$$\frac{dr_j}{dt}e^{ijx} + imr_k u_{m-k} e^{imx} + r_p e^{ipx} = 0$$

This equation can be multiplied by e^{-inx} and integrated to exploit the orthogonality of the exponential functions. Thus:

$$\frac{1}{2\pi} \int_0^{2\pi} \left[\frac{dr_j}{dt} e^{ijx} + imr_k u_{m-k} e^{imx} + r_p e^{ipx} \right] e^{-inx} dx = 0$$

Now, since

$$\frac{1}{2\pi} \int_0^{2\pi} e^{ikx} e^{-ilx} dx = \delta_{kl},$$

where δ_{kl} is the Kronecker delta, Equation 2.1 is transformed to a set of ordinary differential equations:

$$\frac{dr_j}{dt} + ij \sum_{k=-\infty}^{\infty} r_k u_{j-k} + r_j = 0, \quad j = -\infty, \dots, \infty$$

which describe how ρ changes with time. Clearly it is impractical to try and work numerically with infinite series'. Therefore the Fourier series is truncated to a certain appropriate value, N , determined by practicality restrictions. The above equation is then programmed as

$$\frac{dr_j}{dt} + ij \sum_{k=-N}^N r_k u_{j-k} + r_j = 0, \quad j = -N, \dots, N$$

Having provided a brief insight into the spectral process, the non-dimensionalised equations given in Section 3.3 are now solved. Equation 2.27 is solved first.

$$\frac{\partial \hat{\rho}}{\partial \hat{t}} + \frac{\partial \hat{\rho} \hat{v}}{\partial \hat{x}} + \frac{\partial \hat{\rho} \hat{v}}{\partial \hat{y}} + 2\hat{\rho} = 0$$

To accommodate two dimensions, the solutions now take the form

$$\begin{aligned} \rho &= \sum_{j=-N}^N \sum_{k=-N}^N r_{j,k}(t) e^{i(jx+ky)} \\ v &= \sum_{m=-N}^N \sum_{n=-N}^N u_{m,n}(t) e^{i(mx+ny)} \\ \nu &= \sum_{p=-N}^N \sum_{q=-N}^N v_{p,q}(t) e^{i(px+qy)} \end{aligned}$$

or in abbreviated form

$$\rho = r_{j,k} e^{i(jx+ky)}$$

and likewise with the velocities in the different directions. Equation 2.6 is now solved. Applying the assumed solutions to the equation, one obtains:

$$\begin{aligned} & \frac{\partial}{\partial t}(r_{j,k}e^{i(jx+ky)}) + \frac{\partial}{\partial x}(r_{g,h}u_{m,n}e^{i[(g+m)x+(h+n)y]}) \\ & + \frac{\partial}{\partial y}(r_{s,t}v_{p,q}e^{i[(s+p)x+(t+q)y]}) + 2r_{c,d}e^{i(cx+dy)} = 0 \\ & \frac{dr_{j,k}}{dt}e^{i(jx+ky)} + i(g+m)r_{g,h}u_{m,n}e^{i[(g+m)x+(h+n)y]} \\ & + i(t+q)r_{s,t}v_{p,q}e^{i[(s+p)x+(t+q)y]} + 2r_{c,d}e^{i(cx+dy)} = 0 \end{aligned}$$

Let $g+m=a$, $h+n=b$, $s+p=w$ and $t+q=z$.

$$\frac{dr_{j,k}}{dt}e^{i(jx+ky)} + iar_{g,h}u_{a-g,b-h}e^{i(ax+by)} + izr_{s,t}v_{w-s,z-t}e^{i(wx+zy)} + 2r_{c,d}e^{i(cx+dy)} = 0$$

Using orthogonalisation again,

$$\frac{dr_{j,k}}{dt} + ijr_{g,h}u_{j-g,k-h} + ikr_{s,t}v_{j-s,k-t} + 2r_{j,k} = 0$$

There are free indices in this last equation in the form of s and t and these can thus be set equal to g and h to simplify the equation to its final form:

$$\frac{dr_{j,k}}{dt} + i \sum_{g=-N}^N \sum_{h=-N}^N (jr_{g,h}u_{j-g,k-h} + kr_{g,h}v_{j-g,k-h}) + 2r_{j,k} = 0, \quad j, k = -N, \dots, N. \quad (3.1)$$

The gravitational potential must initially be calculated before the force can be solved for. Let the potential $\phi = p_{j,k}(t)e^{i(jx+ky)}$. From Equation 2.15, $\nabla^2\phi = 4\pi\rho G$, thus

$$\begin{aligned} & \frac{\partial^2}{\partial x^2}(p_{j,k}e^{i(jx+ky)}) + \frac{\partial^2}{\partial y^2}(p_{s,t}e^{i(sx+ty)}) = 4\pi Gr_{m,n}e^{i(mx+ny)} \\ & i^2j^2p_{j,k}e^{i(jx+ky)} + i^2t^2p_{s,t}e^{i(sx+ty)} = 4\pi Gr_{m,n}e^{i(mx+ny)} \\ & -j^2p_{j,k}e^{i(jx+ky)} - t^2p_{s,t}e^{i(sx+ty)} = 4\pi Gr_{m,n}e^{i(mx+ny)} \\ & -(j^2+k^2)p_{j,k} = 4\pi Gr_{j,k} \\ & p_{j,k} = -\frac{4\pi G}{j^2+k^2}r_{j,k}, \quad j, k = -N, \dots, N. \end{aligned}$$

To avoid division by zero, both j and k cannot be zero simultaneously. This condition corresponds to the average uniform value for the density field. Since no gravitational field is produced from a uniform density field, this particular index for ϕ will be zero and thus can be ignored in the programming. This can also be seen if F is calculated.

$$\begin{aligned} F_G &= -\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right) \\ &= (ijp_{j,k}e^{i(jx+ky)}, ikp_{j,k}e^{i(jx+ky)}), \quad j, k = -N, \dots, N. \end{aligned}$$

Here, F_G is only zero if both j and k are zero so that this particular term of the series may also be ignored. Thus

$$F_G = (4\pi G \frac{ij}{j^2 + k^2} r_{j,k}, 4\pi G \frac{ik}{j^2 + k^2} r_{j,k}), \quad j, k = -N, \dots, N, \quad j, k \neq 0.$$

The electrostatic field is very similar in form with an extra q/m term involved to relate charge density to matter density as discussed in Chapter 2. Letting the electric potential be $\psi = p_{j,k}(t)e^{i(jx+ky)} \dots$

$$-j^2 p_{j,k} e^{i(jx+ky)} - k^2 p_{s,t} e^{i(sx+ty)} = -4\pi K_1 \rho$$

$$(j^2 + k^2) p_{j,k} = 4\pi K_1 \frac{q}{m} r_{j,k}$$

$$p_{j,k} = \frac{4\pi K_1}{j^2 + k^2} \frac{q}{m} r_{j,k}, \quad j, k = -N, \dots, N, \quad j, k \neq 0$$

after orthogonalisation where $r_{j,k}$ still represents the matter density coefficients.

Now, $E = -\nabla\phi \dots$

$$E = (4\pi K_1 \frac{q}{m} \frac{-ij}{j^2 + k^2} r_{j,k} e^{i(jx+ky)}, 4\pi K_1 \frac{q}{m} \frac{-ik}{j^2 + k^2} r_{j,k} e^{i(jx+ky)}), \quad j, k = -N, \dots, N, \quad j, k \neq 0. \quad (3.2)$$

Equation 3.2 can be used to solve for Equation 2.22. First, $\partial E_x / \partial t$ is determined:

$$\frac{\partial E_x}{\partial t} = -ij \frac{4\pi K_1}{j^2 + k^2} \frac{q}{m} \frac{dr_{j,k}}{dt} e^{i(jx+ky)}, \quad j, k = -N, \dots, N.$$

Now let the magnetic field $B = b_{j,k} e^{i(jx+ky)}$.

$$\begin{aligned} \frac{\partial}{\partial y} b_{j,k} e^{i(jx+ky)} &= -\frac{K_2}{K_1} (4\pi i K_1 \frac{q}{m} [\frac{l}{l^2 + n^2} \frac{dr_{l,n}}{dt} e^{i(lx+ny)} + 2H \frac{c}{c^2 + d^2} r_{c,d} e^{i(cx+dy)}]) \\ &\quad + 4\pi K_2 \frac{q}{m} r_{g,h} u_{s,t} e^{i[(g+s)x + (h+t)y]} \end{aligned}$$

Let $g + s = w$ and $h + t = z$.

$$\begin{aligned} ikb_{j,k} e^{i(jx+ky)} &= -4\pi i K_2 \frac{q}{m} (\frac{l}{l^2 + n^2} \frac{dr_{l,n}}{dt} e^{i(lx+ny)} + 2H \frac{c}{c^2 + d^2} r_{c,d} e^{i(cx+dy)}) \\ &\quad + 4\pi K_2 \frac{q}{m} r_{g,h} u_{w-g, z-h} e^{i(wx+zy)} \\ ikb_{j,k} &= 4\pi K_2 \frac{q}{m} (\frac{-ij}{j^2 + k^2} [\frac{dr_{j,k}}{dt} + 2r_{j,k}]) + \sum_{g=-N}^N \sum_{h=-N}^N r_{g,h} u_{j-g, k-h}, \\ &\quad j, k = -N, \dots, N. \end{aligned} \quad (3.3)$$

Once again there is a problem with division by zero should k assume the zero index. But if this index is ignored, what will that particular value of B be? This question

will be answered by first proving that the right hand side of the above equation is also zero when k is zero. If k is zero, the right hand side becomes:

$$\begin{aligned} & 4\pi K_2 \frac{q}{m} \left[-\frac{i}{j} \left(\frac{dr_{j,0}}{dt} + 2r_{j,0} \right) + r_{g,h} u_{j-g,-h} \right] \\ &= \frac{4\pi K_2}{ij} \frac{q}{m} \left(\frac{dr_{j,0}}{dt} + 2r_{j,0} + ijr_{g,h} u_{j-g,-h} \right) \end{aligned}$$

Comparing the terms in brackets with the continuity equation for $k = 0$:

$$\frac{dr_{j,0}}{dt} + ijr_{g,h} u_{j-g,-h} + i0r_{g,h} v_{j-g,-h} + 2r_{j,0} = 0$$

it is found that the terms in brackets and the equation above are identical, showing that the right hand side of equation 3.3 is indeed zero when k is zero. L'Hospital's rule is used to find the limit as k tends to zero. Here, top and bottom are differentiated with respect to k .

$$\begin{aligned} & \lim_{k \rightarrow 0} \frac{4\pi K_2 \frac{q}{m} \left[-\frac{ij}{j^2+k^2} \left(\frac{dr_{j,k}}{dt} + 2r_{j,k} \right) + r_{g,h} u_{j-g,k-h} \right]}{ik} \\ &= \lim_{k \rightarrow 0} \frac{4\pi K_2 \frac{q}{m} \left[-ij \left(\frac{dr_{j,k}}{dt} + 2r_{j,k} \right) + (j^2 + k^2)(r_{g,h} u_{j-g,k-h}) \right]}{(j^2 + k^2)(ik)} \\ &= \lim_{k \rightarrow 0} \frac{4\pi K_2 \frac{q}{m} (2kr_{g,h} u_{j-g,k-h})}{ij^2 + 3ik^2} \\ &= \frac{0}{ij^2} = 0. \end{aligned}$$

Thus the magnetic field at this particular index is zero and may be ignored.

The momentum equations (2.20 and 2.21) can now be solved in their entirety with spectral methods. To avoid confusion occurring when variables and indices assume the same symbols, uppercase latin letters are assigned to the variables in the following four equations (e. g. U is velocity in the x direction and Q is charge). Let $\phi = P_{a,b} e^{i(ax+by)}$, $\psi = S_{c,d} e^{i(cx+dy)}$ and $B = B_{p,q} e^{i(px+qy)}$. Thus Equation 2.20 becomes

$$\begin{aligned} & \frac{dU_{j,k}}{dT} e^{i(jx+ky)} + igU_{e,f} U_{g,h} e^{i[(e+g)x+(f+h)y]} + inV_{l,o} U_{m,n} e^{i[(l+m)x+(o+n)y]} \\ &= iaP_{a,b} e^{i(ax+by)} + \frac{Q}{M} (icS_{c,d} e^{i(cx+dy)} + B_{p,q} V_{s,t} e^{i[(p+s)x+(q+t)y]}) \end{aligned}$$

Let $\gamma = e + g$, $\delta = f + h$, $\theta = l + m$, $\lambda = o + n$, $\zeta = p + s$ and $\omega = q + t$. This leads to:

$$\begin{aligned} & \frac{dU_{j,k}}{dT} e^{i(jx+ky)} + igu_{g,h} U_{\gamma-g,\delta-h} e^{i(\gamma x+\delta y)} + i(\lambda - o)V_{l,o} U_{\theta-l,\lambda-o} e^{i(\theta x+\lambda y)} \\ &= iaP_{a,b} e^{i(ax+by)} + \frac{Q}{M} (icS_{c,d} e^{i(cx+dy)} + B_{p,q} V_{\zeta-p,\omega-q} e^{i(\zeta x+\omega y)}) \end{aligned}$$

which, after orthogonalisation and elimination of free indices, simplifies down to

$$\begin{aligned} & \frac{dU_{j,k}}{dT} + i \sum_{m=-2N}^{2N} \sum_{n=-2N}^{2N} [mU_{m,n}U_{j-m,k-n} + (k-n)V_{m,n}U_{j-m,k-n}] \\ &= \sum_{m=-2N}^{2N} \sum_{n=-2N}^{2N} [ijP_{j,k} + \frac{Q}{M}(ijS_{j,k} + B_{m,n}V_{j-m,k-n})], \quad j, k = -N, \dots, N \end{aligned}$$

where N represents the number of terms in the series. Similarly,

$$\begin{aligned} & \frac{dV_{j,k}}{dT} + i \sum_{m=-2N}^{2N} \sum_{n=-2N}^{2N} [(j-m)U_{m,n}V_{j-m,k-n} + mV_{m,n}v_{j-m,k-n}] \\ &= \sum_{m=-2N}^{2N} \sum_{n=-2N}^{2N} [ikP_{j,k} + \frac{Q}{M}(ikS_{j,k} - B_{m,n}U_{j-m,k-n})], \quad j, k = -N, \dots, N \end{aligned}$$

These equations were programmed into the computer using the C++ and Visual Basic computer languages. The system is evolved through time using a Runge-Kutta method, the order of which may be specified before each run. Below are the Runge-Kutta equations. Coefficients for the method are calculated first, based on the order chosen. Let the order be l . The coefficients are calculated thus:

$$j = 1 \text{ to } l \quad k_j = 1/(l - j + 1)$$

New values were found a time step (h) later as follows:

$$x_n = \sum_{j=1}^l x_0 + k_j h f(x)$$

where $f(x)$ is the function describing how a particular property (e.g. density or velocity) changes with time.

4 Computer Simulation

The computer program was written in C++ and Visual Basic. Visual Basic was found to be easier to use and implement than C++. Both languages were used because the electromagnetic model appeared to function better in C++. Visual Basic tended to produce overflow errors regularly with this model whereas C++ ran without problems. The program consisted of three files in each language: the starting file containing the model parameters, the initial conditions file and the calculation engine. The various inputs to the program comprised the number of vibrational modes to be used, the order of the Runge-Kutta numerical method, the time increment and the number of iterations required. These last two together determine the period of time elapsed from the beginning of the simulation. Different sets of results were calculated for different times throughout universal evolution so that it could be seen how the structures came into being and developed. As each set of results was obtained (i. e. the densities and velocities for each individual matter stream), the density coefficients were written to a results file for processing while a more comprehensive set of results (including all velocities) were written to a second results file. This file could be used in case the simulation was prematurely interrupted. In this case it is possible to proceed to the next set of iterations without having to go through the previous calculations again.

4.1 Initial Conditions

The initial condition is a fairly important consideration as the final structures are determined by the primordial spectrum to a large extent. There are various forms the primordial spectrum may take. A common practice is to assume a spectrum of a power-law form

$$P(k; t_p) = A_p k^{n_p} \quad (4.1)$$

where P represents the spectral power as a function of wave number and time, A_p is a constant, k is the wavenumber (mode) and n_p is called the spectral index.

This came about because in the 1970's the primordial spectrum was assumed in an ad hoc fashion as the requirements for structure to originate via a adiabatic or isothermal process [41]. In most models n_p is effectively constant over the entire range of scales relevant to the observable universe [41] and thus one of its known names is the scale-invariant spectrum. There seem to be considerable problems with structure formation on the scales of galaxies with certain values of n [41]. These problems have appeared as a result of using spectra obtained by reshuffling matter within a cosmological horizon volume. The amplitude of the spectrum for $n = 0$ (white noise spectrum) is too high while that of the spectrum with $n = 2$ (particles-in-boxes spectrum) is too low. These problems led many cosmologists to resort to some process which apparently lies outside the horizon to produce an appropriate spectrum. Hence the value for n was set at one. This spectrum was independently suggested by Peebles, Harrison and Zel'dovich [41].

Because the universe is considered homogeneous and isotropic on the larger scales, as discussed above, a statistically homogeneous field (a Gaussian distribution) with the above power spectrum was the initial condition set used for the simulations. Gaussian fields are also strongly motivated by inflation as the quantum fluctuations have random phases which should be Gaussian in virtually all realistic inflationary models [41]. This, together with easier computation makes the Gaussian random field a good choice. Note that as this study uses a slightly different density contrast, $\frac{\rho}{\rho_0}$, as explained in Section 2.3, the mean for this distribution will be one instead of zero.

Two different forms of primordial spectra were run through the simulations to observe the effects different values of n_p have on the model. The values of n_p were chosen to be one (as discussed above) and minus one - as a contrasting measure. Equation 4.1 was used as the starting point with the relevant values substituted in. An inverse Fourier Transform was performed on this equation using the Matlab *IFFT2* function to obtain its equivalent in the real domain. Subsequently, values were generated in Matlab using the *Randn* function which produces a normal Gaussian distribution with a mean of zero and a variance and standard deviation of one. A value of one can be added to the results to give the required mean of one for the simulation's density contrast. The values generated by the inverse Fourier Transform of the equation were multiplied by the *Randn* function values so that the initial condition set has a Gaussian distribution. Finally, the Matlab *FFT2* function - a two-dimensional Fourier Transform function - was used to perform a Fourier Transform on the initial condition set to get the power spectrum required by the simulation. The zero'th element of the spectrum was made zero as this is

just the sum of all the other Fourier co-efficients and may be ignored.

4.2 Results

The simulation programs produce the coefficients of the complex Fourier series for the matter density. These results were subsequently converted into a power spectrum and normalised by multiplying by the complex conjugate of each value and dividing by the number of elements in the matrix. The power spectrum was then further normalised by multiplying it by the constant A , which was calculated from the COBE results to be $\approx (24h^{-1}Mpc)^4$ [52]. All calculations were performed and power subsequently plotted as functions of scale in log-log space with the Matlab mathematical software package. The scale axis still inherently has dimensions of volume (or in the case of this two dimensional model - area) [41]. Thus, as a final adjustment, each power data point was divided by the square of its scale. The resulting power spectra were compared with spectra representing other times in the simulation and the actual density distribution power spectra from CfA and IRAS redshift surveys as produced in [50]. These are printed in Chapter 5.

4.3 Programming

The algorithm is basically one large, iterative procedure. Most variables are stored in two-dimensional arrays of complex numbers which can become very large data structures if there are many modes (e. g. a variable 100 hundred modes long requires an array consisting of 40401 elements). These variables are calculated completely in one procedure for each iteration. All the variables are real, which means that the components of the two-dimensional arrays are radial complex conjugates of each other with respect to the matrix, the middle entry being completely real. Thus, only half of the matrix is calculated in order to save time and the value of the entry radially corresponding to the value just calculated is set equal to computed value's complex conjugate. For this reason, the computations are compartmentalised into various sets of iterative procedures. Firstly, the incremental values for the lower, right-hand corner of the density array (from row 0;column 0 to row n ;column n) are calculated but the density is not updated yet as some increments elsewhere in the array (not yet calculated) depend on the old density values. The procedure is repeated for the slightly smaller lower left-hand corner (from row 1;column $-n$ to row n ;column -1). Included in these calculations are those for the magnetic field values,

which rely on the time rate of change of the charged density fields. Because there are an uneven number of rows and columns in the matrix, quadrants of slightly different size need to be used if calculations are not to be repeated for certain elements. By avoiding repeating these calculations, considerable time is saved after a large number of iterations especially if many modes are used. Once the values for these two quarters are calculated, they are updated and the corresponding conjugates are set. The electromagnetic force is now determined using the magnetic field values and the velocity increments are subsequently found in the same manner as for the density. Once the velocities are updated, the next iteration proceeds.

Changing the number of modes of vibration was the biggest influence on computing time. Increasing the modes adds considerable complexity to the calculation of variables. It was also necessary to reduce the time step when increasing the modes to keep numerical instabilities to a minimum. The smaller time step meant more iterations needed to be performed to reach the same endpoint which prolonged the simulation still further. A larger number of modes also tended to become unstable much quicker, limiting the number of modes possible in the simulations, both from a time and stability point of view.

The models for the simulations without plasma forces were reduced to one fluid since gravity does not distinguish between charge. This saved a vast amount of calculation time.

5 Results and Discussion

The following are the results of the simulations. There are three different models with gravity alone and one model that includes plasma forces. Model 1 has a matter density parameter of $\Omega = 0.02$, estimated as the amount of visible matter [48], and no velocity perturbations. Model 2 is similar but has a higher density: $\Omega = 0.3$ which is the current estimate of luminous and dark matter [45]. The third model (Model 3) maintains the higher density and includes velocity fluctuations in the initial conditions. Results for the gravity models are presented first and the plasma model follows. Each model was calculated with two different sets of initial conditions as explained in Section 4.1 and two different numbers of vibrational modes (ten and fifteen modes). Observational results from the redshift surveys of IRAS galaxies are shown in figure 5.1. Results of the simulations were compared to these surveys to try and gauge the degree of realism of the simplified models, and also compared to each other in order to determine the effects of varying the different model parameters. The first eighteen figures show the modal power levels for each simulation. The following six plots illustrate the progression of the power spectra data for the three gravity models with each figure depicting to one stage of evolution. There are three figures each for the different initial conditions. Similar graphs are subsequently shown for the plasma model except that all phases of evolution for this simulation are shown together since there are no other models for comparison. Finally, two-point correlation plots are presented to indicate degree of matter clustering compared with a random sample, the last of which draws clustering comparisons between the plasma model and Gravity Model 1.

5.1 General Comments

This model is a relatively simple non-linear one with several different parameters needing consideration. Firstly, a set statistical model of initial conditions was used throughout. The initial velocities in the x and y directions were made equal because the Big Bang postulates a homogeneous and isotropic universe, so it should not be possible to distinguish one direction from another. The fluctuation amplitudes of the velocities were set to zero in some simulations to see how they would develop in time from purely density perturbations. In other simulations an initial velocity perturbation magnitude of the order of the shortest modal wavelength was used to study their effect too. This choice of velocity fluctuation maintains mathematical consistency with the spectral model. The magnitude of the fluctuation was reduced by two orders of magnitude to prevent the simulations from becoming unstable. Only the density parameters, velocity parameters, initial conditions and number of spectral modes were altered in different simulations since manipulating all possible parameters would result in an overwhelming set of data.

There are two non-dimensional numbers, each relating to the two different forces respectively (gravitational and electromagnetic). Both have certain factors in common and also unique properties. Factors common to both are the average density and the respective force constants. The Hubble constant appears in the gravitational number while the Hubble length scale appears in the plasma force number. This can be altered mathematically, however, as pointed out in Section 2.3, so that both numbers have the Hubble constant and no length scale is involved. This is very useful as it allows one to see (in taking the ratio of these two numbers) that the real difference in these two forces is the charge-to-mass ratio of the particles involved and the value of the fundamental force constants. Since the electric constant is much greater than the gravitational constant, the plasma forces are naturally stronger and have more influence. But the charge-to-mass ratio may make an even greater contribution to this. For fundamental particles such as the electron and proton, this ratio is very large. It is then squared in the plasma non-dimensional number. So on the scales of these and similar particles, gravity is virtually not even a consideration. However, as matter clusters together, the charge-to-mass ratio will decrease due to a greater and greater contribution from neutral matter until on cosmological scales, gravity can compete with plasma forces because it is always attractive and has infinite range. It just remains to be seen whether plasma forces are indeed as prominent on these grand scales as they are on smaller scales. Much evidence for this has been presented in Chapter 1 and together with the major constituent of the universe being plasma, the argument seems convincing. In the plasma simulations

the plasma force constants were reduced by several orders of magnitude because the higher values repeatedly caused numeric instabilities, even with extremely small time steps. The plasma force terms were still considered strong enough to produce noticeable effects.

The average density parameter and the Hubble constant are present in both numbers in the same proportions and thus are factors which could affect overall rates of structure formation and not favour one force above the other. For this reason, the values of these two parameters were not varied as they would only serve to change the overall magnitudes of the forces at work. Besides the fact that this could be easily achieved by adjusting the force constants, the objective of this study was rather in observing the effects created by the different forces by changing their strengths relative to one another. Should a general increase in magnitude of the forces quicken the rate of structure formation in the model (as seems evident from the results), then a greater average density would achieve the same result, as would be expected. One interesting point is that the Hubble constant has the opposite effect on structure formation, but more acutely since the square of the constant is involved. So in these models, the Hubble expansion appears to serve as a significant retardant to general structure formation and this can be observed in the results. Of course this is due to the fact the expansion tends to decay the density (Equation 2.4) which is the vital factor in structure formation.

The average density of luminous matter was calculated using the formula [48]:

$$\rho_0 = \frac{3H_0^2}{8\pi G} \approx 1.9 \times 10^{-29} h^2 \text{gcm}^{-3}$$

where, after recent results of observations of globular clusters from the Hipparcos satellite, a value for the Hubble constant of $70 \text{km s}^{-1} \text{Mpc}^{-1}$ [49] is used. Different values here would only really affect the rate of decay of the density in this model.

Relative quantities of different charge in the three fluid models was a parameter that could also have been varied. However, the conservation of charge demands equal amounts of positive and negative charge. While the results of changing this ratio would probably have been interesting, it was not very relevant to the objectives. This just leaves the ratio of charged matter to uncharged matter to be resolved. Since plasma forces have nothing to do with uncharged matter, it only affects the comparison between gravity and electromagnetism slightly. One could draw comparisons between adding or subtracting neutral matter and introducing dark matter. The problematic nature of this matter has also been discussed in Chapter 1 and this dissertation attempts to produce structure without such exotic concepts. This left the natural choice of using the present element abundance estimates [51] and

deriving the approximate ratios from that. The ratio of neutral matter to charged matter is competently addressed by adjusting the strengths of the two forces with respect to one another.

A good feature of the method used was that it was quite easy to tell if and when the numerical method had become unstable. The velocity and, subsequently, other variables would increase exponentially and produce a floating point overflow error in the computer in a three or four iteration time period. This was independent of the actual magnitude of the velocity and the rate at which it was changing before instability occurred. Some cases with certain velocity values that were very slowly increasing, would suddenly overflow while others with velocities a few magnitudes greater and increasing much faster would proceed to the end of the simulation without incident. The model stability appeared also to be independent of the order of Runge-Kutta method used to evolve the equations in time. Experiments with a simulation which became untenable at a known point showed that different orders of the Runge-Kutta method did not seem to change the point of sudden instability nor the manner in which the values accelerated to floating point overflow. Only the time step had a clear influence on this.

There were no dissipation terms other than the Hubble decay included in the model. The structures in the results should thus be viewed as features of constructive interference between the different wavelengths present. These various components then move on in time to form other features with other components without fading away in any sense. So one may expect some structures to be larger or sharper than they would otherwise be in a more realistic model. As expected, when there were simultaneously no velocity or density perturbations in the initial conditions, none were created during the simulation. The average density merely decayed as a result of the Hubble expansion. In fact, this was the case no matter what forces were introduced into the model. This showed clearly that some sort of fluctuations, be it velocity or density, are essential to begin producing structure of any sort as uniform gravitational and electric fields are useless at achieving this.

The simulations took “snapshots” during the evolution of each model, breaking it up into three stages to observe progress. A characteristic time scale of 10 billion years was chosen so that approximately 15 billion years have elapsed at the end of each simulation which is slightly more than the estimated age of the universe of 12-14 billion years [49]. A time step of 0.008 was used which approximates to a time step of about 80 million years. The characteristic length scale used was 250 Mpc. This allowed comparison with the redshift survey results on the largest observed scales and also provided for the possibility of the largest structures observed today.

One of the shortcomings of the model was the relatively low number of modes used. This was mainly due to numerical instabilities occurring more frequently with more modes and practical limits in CPU time. The simulations were run on a Pentium 3 GHz processor. As a result power spectrum resolution on the smaller scales (less than $20h^{-1}Mpc$) was poor or non-existent. This is something that can be improved.

5.2 Results

The results of the simulations were treated in a power spectrum form. The power spectrum of the density field is a particularly important statistic because it provides a complete statistical characterisation of the density field as long as it is Gaussian (which is another reason a Gaussian field was chosen for the initial conditions). This is because the power spectrum is closely connected with the two-point correlation function or covariance function. The two-point correlation function measures the excess fluctuation with respect to a uniform distribution. In fact, the power spectrum is the Fourier transform of the two-point correlation function (the Wiener-Khintchine theorem) [41].

Measurement of the power spectrum of galaxy density fluctuations provides a strong test of cosmological models with initially Gaussian density fluctuations. For this class of models the initial power spectrum completely specifies the fluctuations from which structure grows via gravitational (and electromagnetic) instability [50]. The power spectrum measures the lowest order clustering in the galaxy distribution. Proposed models for the formation of large-scale structure must reproduce the observed power spectrum [50]. As test cases, two spectra in particular were chosen to compare the results of this model against: the power spectrum of the galaxy distribution in the CfA redshift survey and redshift surveys of IRAS galaxies (figure 5.1) [50].

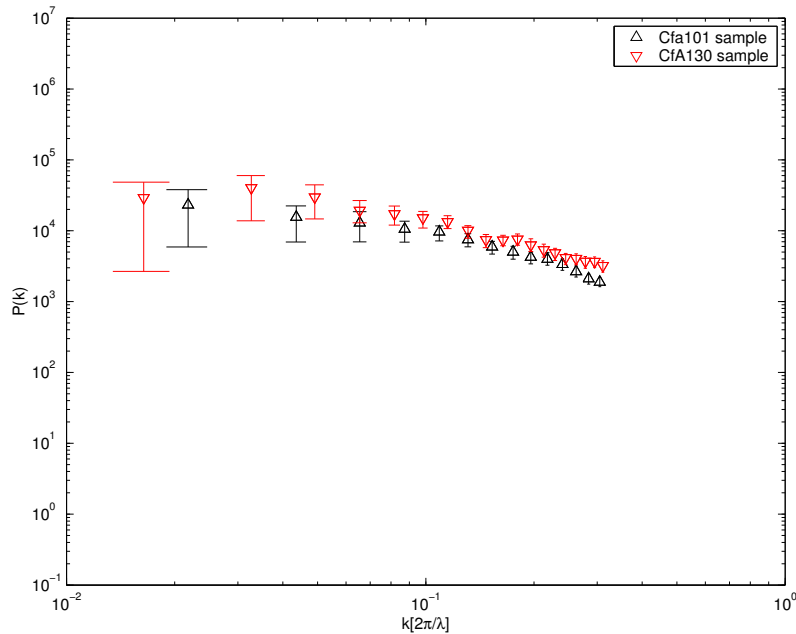


Figure 5.1: CfA Samples

Various cold dark matter models have been compared to these surveys [50] and none

of them have completely satisfied all the requirements of these survey power spectra. The standard CDM model is found to have too large a ratio of small- to large- scale power. An open CDM model (with density ratio $\Omega h = 0.2$) is consistent with the CfA survey and to a lesser degree with the IRAS survey.

How much can be gained by comparing the simulation results with real survey results? What would it mean if the model produces similar results to the CDM models and matches the survey power spectra? Clearly the model is far from realistic and in some areas quite oversimplified. So, the results are not expected to match up to more realistic models such as CDM. Rather, it is anticipated that the model may give an indication of whether a more detailed and realistic plasma and gravity model can be pursued and possibly build on some of its features. Relativistic effects may be added as well as more detailed plasma force constructs and possible luminosity biasing, peculiar velocity fields and dissipation processes. Another possibility is for an improved model to try and predict specific types of structures (such as filaments and voids) characterised by certain statistical fields. Results from this simplistic two-dimensional model in the vicinity of real galaxy survey results surely shows promise for this type of concept.

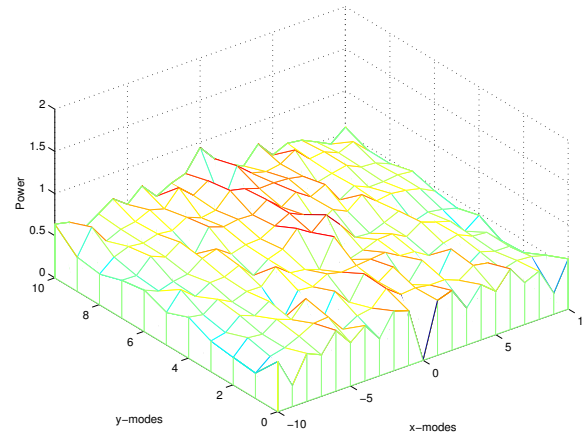
5.3 The Gravity Models

Three types of gravity models were experimented with to observe differences in results and thus behaviour of the simulation in general. The first model was used as a kind of benchmark, having $\Omega = 0.02$ (as this is the approximate observed amount of luminous matter) and starting with no velocity fluctuations - only average velocities in both directions. The second model was similar to the first, only with $\Omega = 0.3$ to account for the so-called cold dark matter as this is observed to be the estimated total matter density after recent X-ray observations [45]. Clusters of galaxies are visible tracers of networks of matter in the universe, marking high density regions where filaments of dark matter join together. When observed at X-ray wavelengths, these clusters stand out due to hot gas trapped in their gravitational potential wells. X-ray emission is linked directly to the total mass of the cluster and so can be used to investigate mass distribution for a sizeable part of the universe. These recent studies have shown good agreement with CDM models with $\Omega = 0.3$ [45]. The third model had the same matter density as the second one but this time included initial velocity perturbations of the order of the smallest spectral wavelength to see what effect that would have on the power spectrum. Two sets of these simulations were produced for differing mode numbers (ten modes and fifteen modes). The following eighteen figures show the raw power data for all three evolution stages of each of the gravity models.

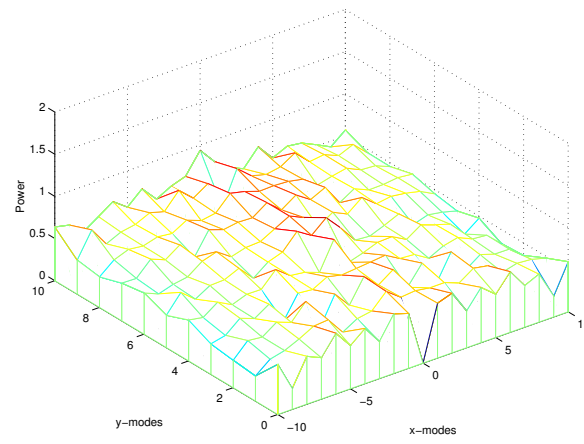
As the models evolve one can see that power increases from Model 1 through to Model 3 - although only marginally between Models 1 and 2. This is expected as adding matter density and subsequently initial velocity fluctuations increases the energy of the system, thus driving the power up and hence increasing clustering potential. However, as the models evolve it is seen that the power spectra tend to decrease slightly. This is due to the Hubble expansion term which acts as an energy sink. Gravity as an energy source is not sufficient in this case to maintain the power spectra at their previous levels. It might have been expected that there would be a significant disparity between the data points of the first two models, but in fact there were only minor differences, showing that the increase in density seems to have marginal effect on the power spectra while the addition of velocity fluctuations has a much more marked effect. This is probably due to the fact that the non-dimensional gravity term (of the order of 10^{-8}) is so small that a tenfold increase in the average density only produces significant differences in the fourth decimal place in the results. In addition to this, adding velocity fluctuations involved altering the initial conditions while the difference in the first two models was merely a consequence of adjusting the average density parameter. Performing the same

simulation with half the time step and double the iterations to give the higher density more time to assert itself, showed that significant differences now occurred in the first and second decimal places but overall power was still not considerably higher, possibly because the Hubble decay also had a greater effect and tended to reduce the spectral power levels.

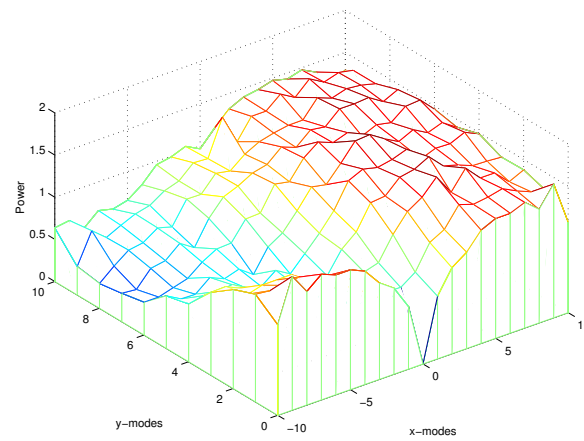
Another interesting observation is the shape of the power graphs. Looking first at the negative gradient initial condition models (figures 5.2 - 5.4 and labelled as 'Negative Slope'), the first two models demonstrate a seemingly random set of spikes and valleys throughout the evolution of the simulation as the power decreases but the third model possesses a much more structured pattern with almost a wave type structure at an angle of approximately 45 degrees as well as generally higher power than the first two models. Most of the power for this third model seems to be driven into the higher modes although there is also a significant peak at the opposite end of the three dimensional spectrum. In the earlier stages of evolution the power spectrum shows several tiny fluctuations but these tend to get smoothed out over time with the surface becoming much flatter. Since the only difference between the second and third models is the velocity perturbations, it is interesting to note that the appearance of some order is introduced when this kinetic energy is added to the system. The Hubble expansion reduces the overall power quite dramatically, making new features more difficult to discern. Although, by viewing the graphs of subsequent stages of evolution at a reduced scale they can be clearly seen and it appears that the first two models retain their overall randomness albeit at a lower power. The excess power in the higher modes of the third model would tend to produce more filamentary structures because higher frequencies with more power will create more and sharper peaks and troughs, and this together with the other modes results in filaments within filaments which strongly suggests clustering.



(a) Spectral Power: Gravity Model 1

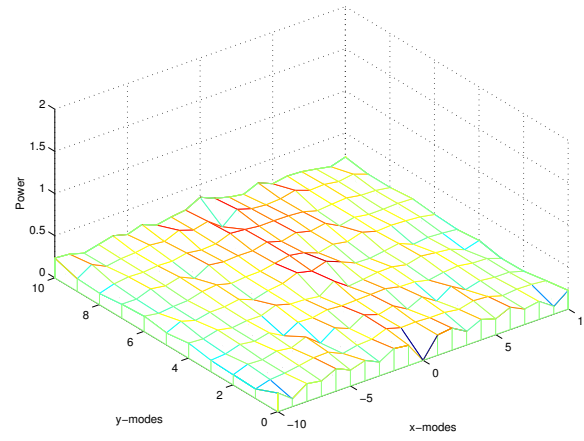


(b) Spectral Power: Gravity Model 2

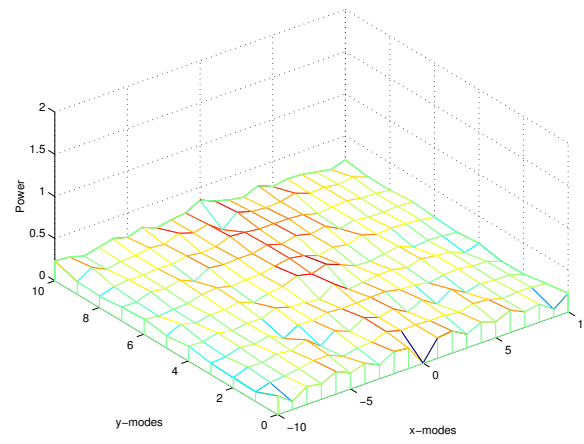


(c) Spectral Power: Gravity Model 3

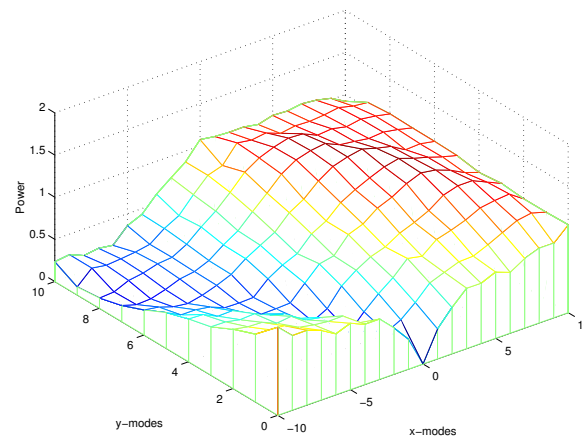
Figure 5.2: Gravity Models, First Stage Evolution, Negative Slope



(a) Spectral Power: Gravity Model 1

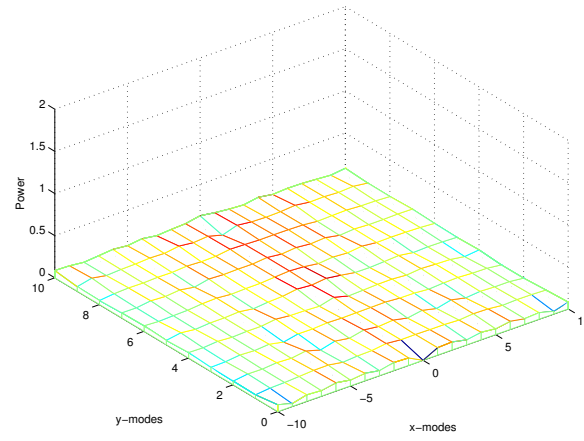


(b) Spectral Power: Gravity Model 2

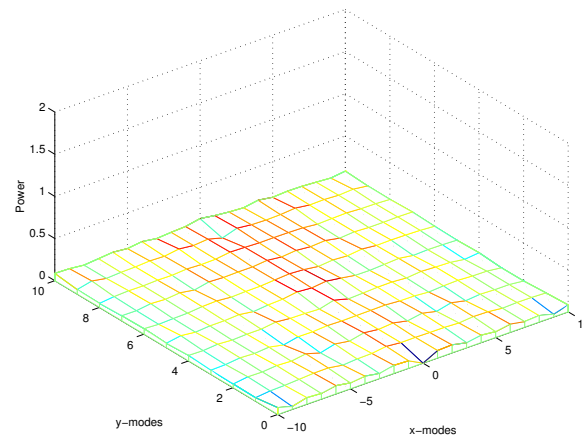


(c) Spectral Power: Gravity Model 3

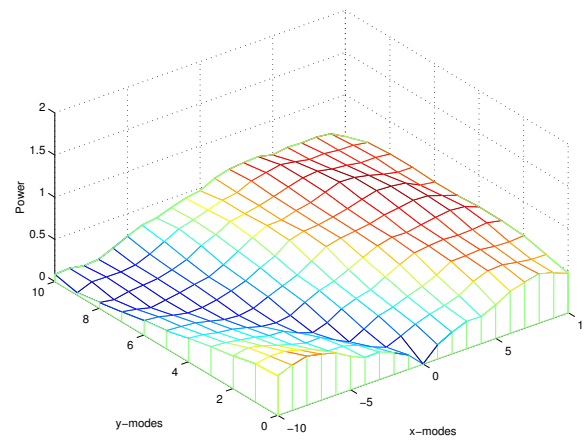
Figure 5.3: Gravity Models, Second Stage Evolution, Negative Slope



(a) Spectral Power: Gravity Model 1



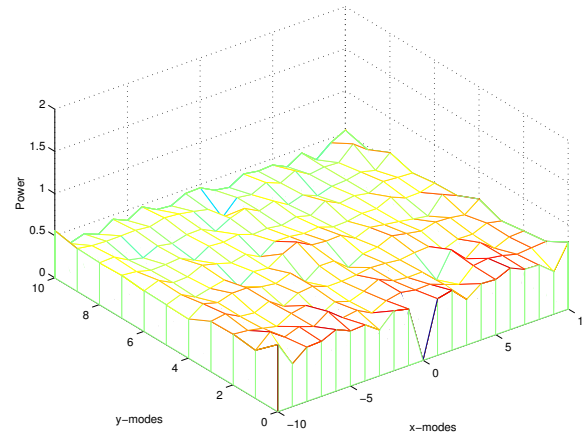
(b) Spectral Power: Gravity Model 2



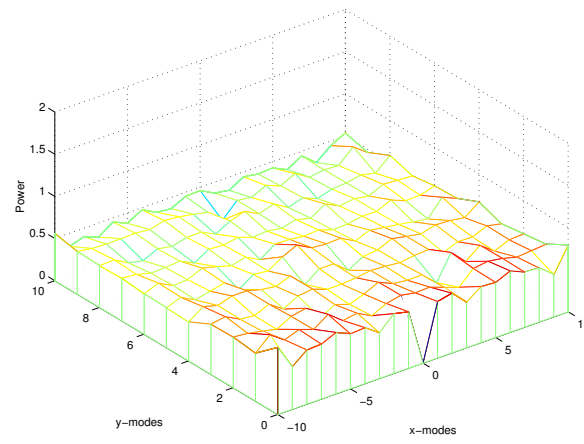
(c) Spectral Power: Gravity Model 3

Figure 5.4: Gravity Models, Third Stage Evolution, Negative Slope

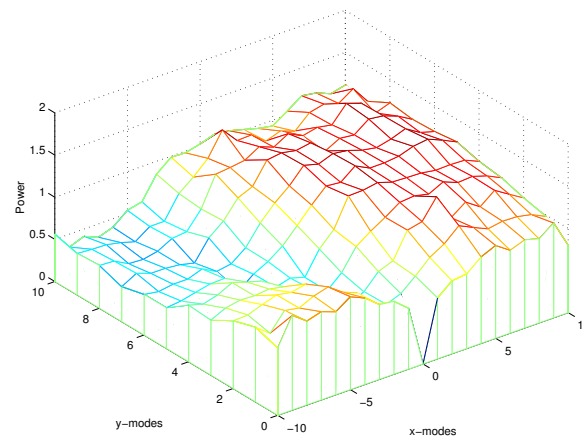
The models with positive gradient initial conditions display similar properties to those models just discussed. These graphs can be seen in figures 5.5 - 5.7. Again, the first two models have fairly chaotic and random power profiles which are maintained throughout their evolution. These models seem to have a slightly lower overall power level than the previous two which is just a consequence of the different initial conditions and their random nature. The model with the velocity fluctuations has a much more structured pattern like its counterpart discussed above. The same kind of diagonal wave structure can be seen here with the peak power persisting in the higher modes through the evolution. Once more, the profile smooths itself out as it evolves until the end product of both sets of initial conditions look virtually identical.



(a) Spectral Power: Gravity Model 1

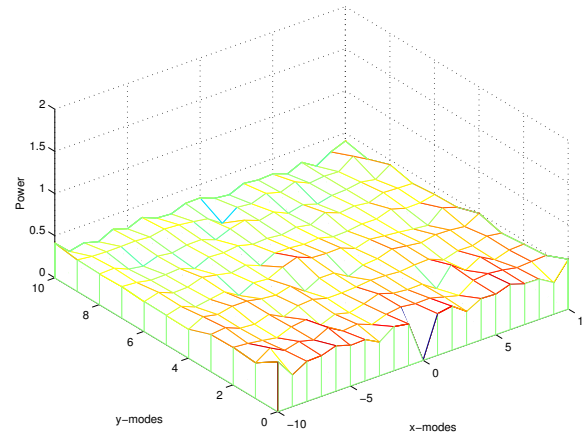


(b) Spectral Power: Gravity Model 2

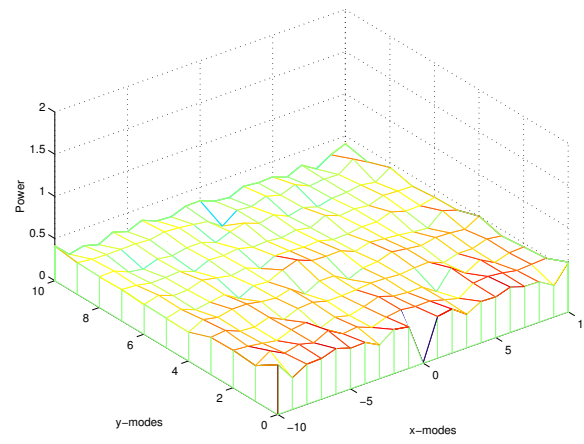


(c) Spectral Power: Gravity Model 3

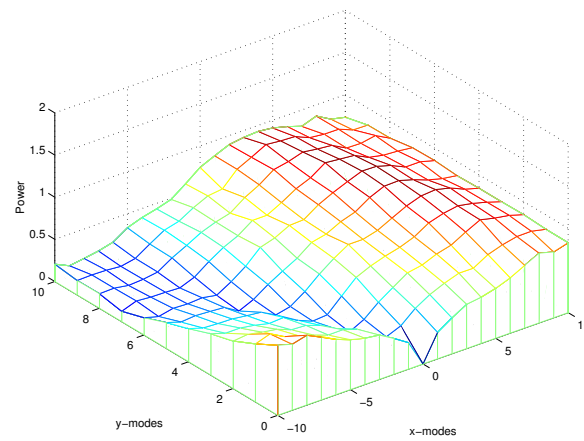
Figure 5.5: Gravity Models, First Stage Evolution, Positive Slope



(a) Spectral Power: Gravity Model 1

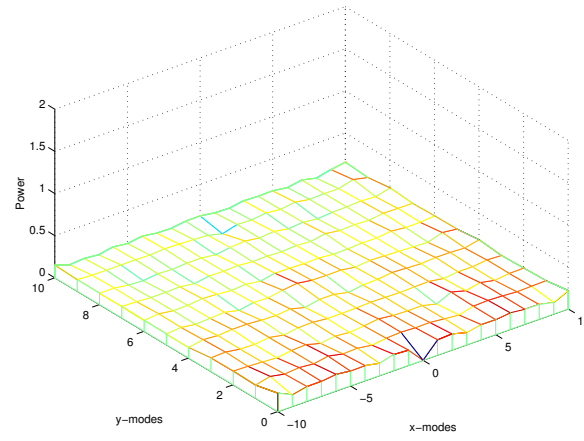


(b) Spectral Power: Gravity Model 2

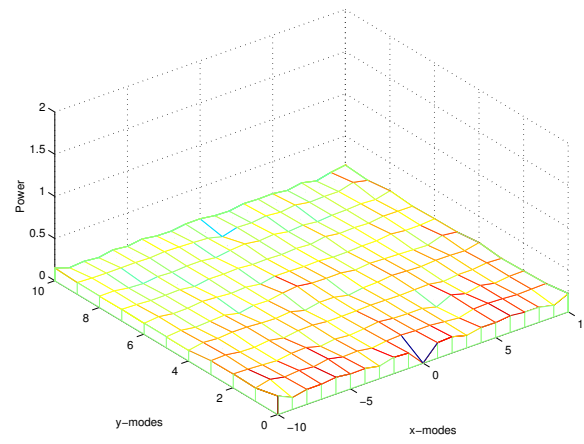


(c) Spectral Power: Gravity Model 3

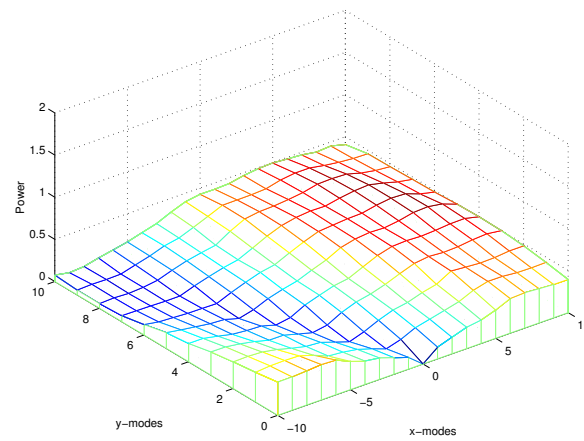
Figure 5.6: Gravity Models, Second Stage Evolution, Positive Slope



(a) Spectral Power: Gravity Model 1



(b) Spectral Power: Gravity Model 2



(c) Spectral Power: Gravity Model 3

Figure 5.7: Gravity Models, Third Stage Evolution, Positive Slope

In figures 5.8 - 5.13, one can gain a better idea of the magnitude of the power spectra. Here, one figure is devoted to each of the three evolution stages, the first three for negative gradient initial conditions and the second three for the positive gradient.

After obtaining the power from the spectral values, the data was subsequently normalised to arrive at their magnitude using the formula:

$$P(k) \approx Ak \approx (24h^{-1}Mpc)^4$$

[52] which is a value arising from study of the COBE data on the cosmic background radiation. The data points represent an average value with the error bars showing the upper and lower values for each point. Such variation in value for each point arises because several different modal values can represent the same wave number as the two-dimensional matrix is collapsed into the single dimension of the power spectrum. The random Gaussian initial conditions and velocity perturbations ensure there are potentially several different values for certain wave numbers.

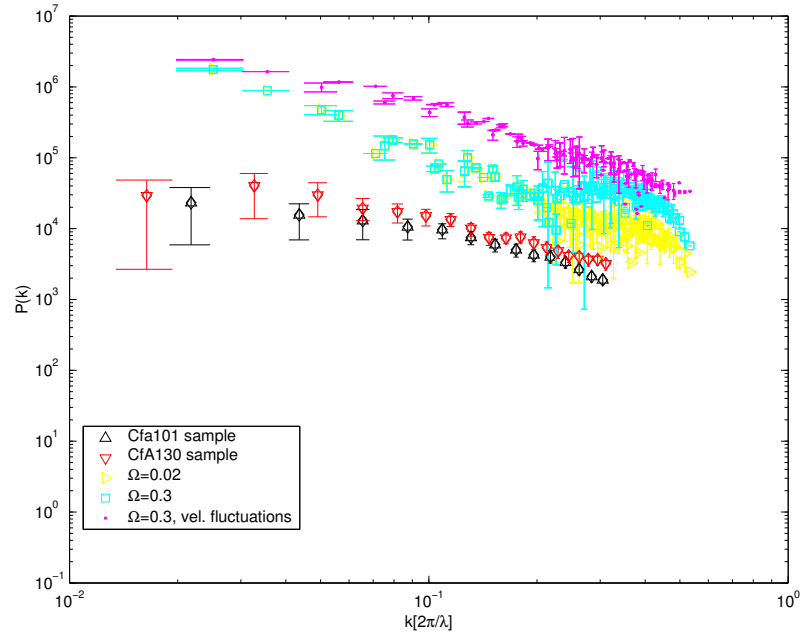


Figure 5.8: Gravity Models: First Evolution Stage, 10 Modes, Negative slope

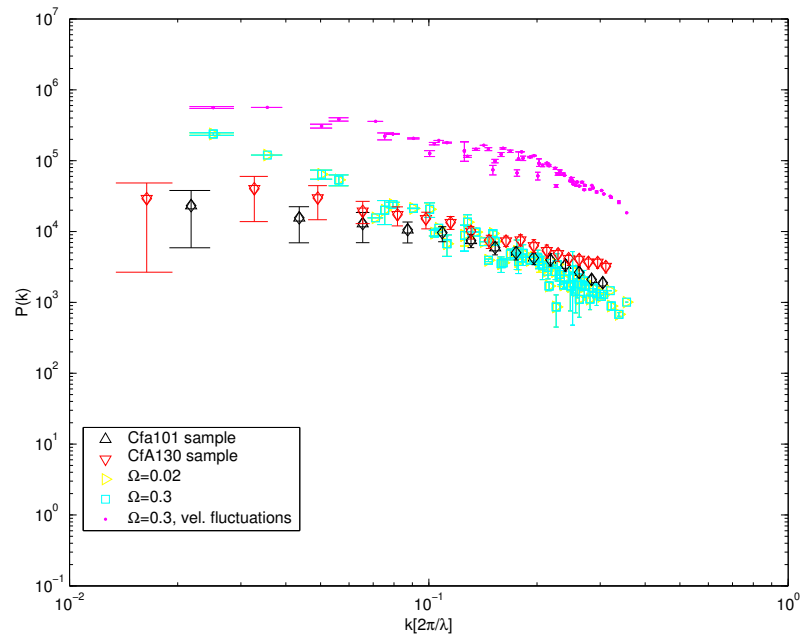


Figure 5.9: Gravity Models: Second Evolution Stage, 10 Modes, Negative slope

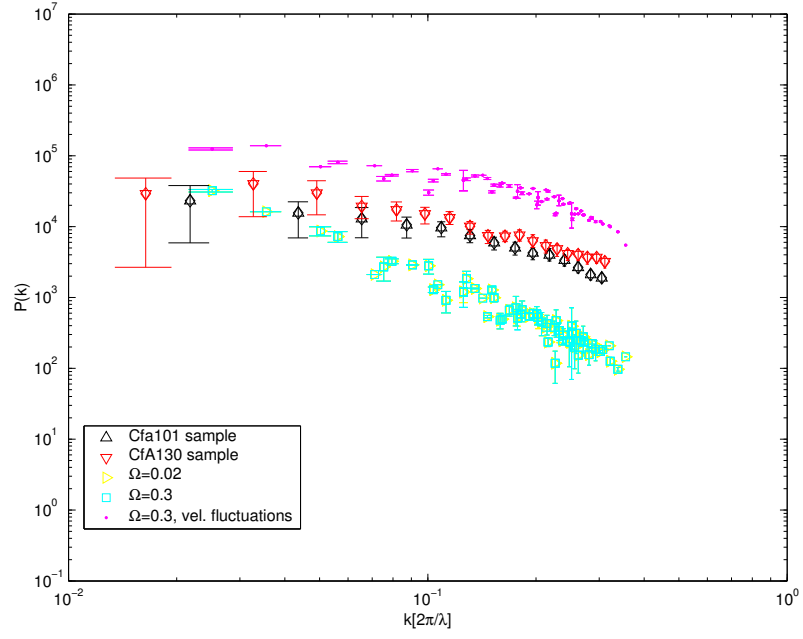


Figure 5.10: Gravity Models: Third Evolution Stage, 10 Modes, Negative slope

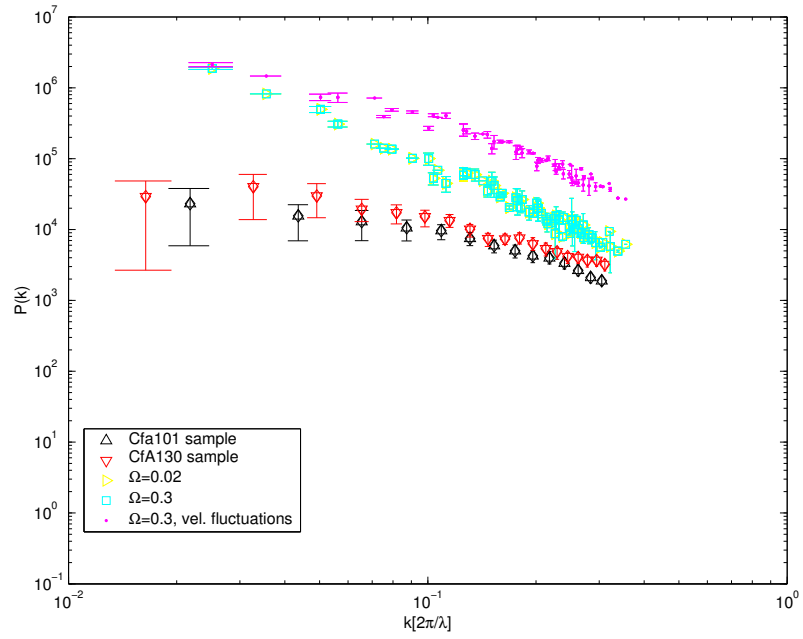


Figure 5.11: Gravity Models: First Evolution Stage, 10 Modes, Positive slope

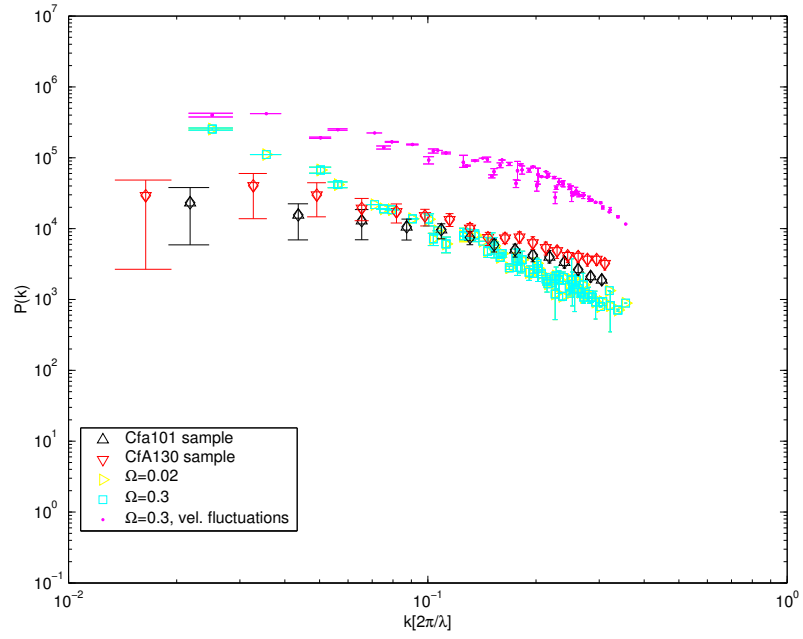


Figure 5.12: Gravity Models: Second Evolution Stage, 10 Modes, Positive slope

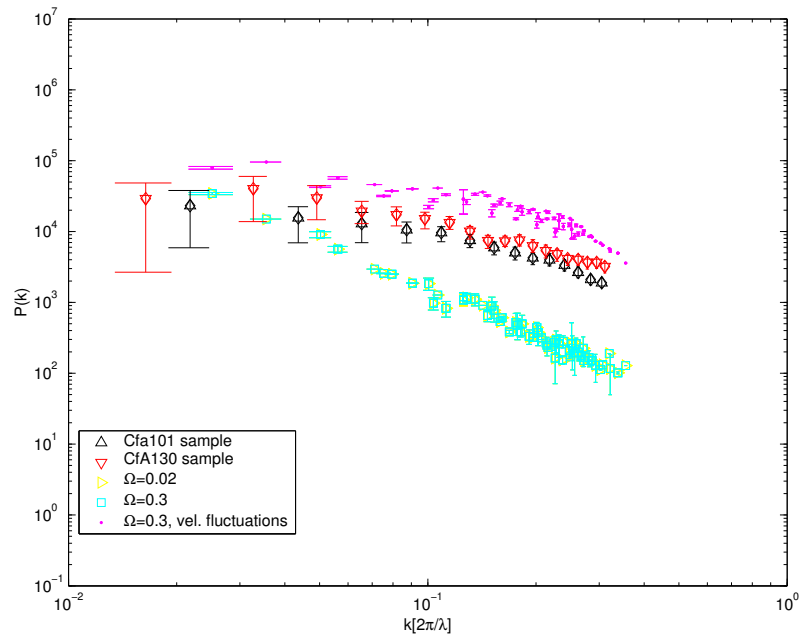


Figure 5.13: Gravity Models: Third Evolution Stage, 10 Modes, Positive slope

Initially the simulation power spectra magnitudes are similar but slightly below the observational power spectra. Then the Hubble expansion tends to degrade this quite significantly but consistently. As a result, the spectra without velocity fluctuations end up in the third stage of evolution substantially below the observations in terms of magnitude. Despite the contrasting initial conditions the slopes of the graphs for both sets of simulations are virtually identical from the start through all stages of evolution. This indicates the random nature of the conditions and the way the simulation acts on them is more important than the gradient.

The following observations will apply to both sets of initial conditions as they display the same behaviour. Initially the purely gravitational simulations demonstrate quite a steep slope compared to the observations while the added velocity fluctuations clearly add more power to the lower modes, resulting in a shallower gradient. The negligible difference in raw power spectra between Models 1 and 2 is confirmed in these figures where they trace each other through the evolution. Their gradient also remains constant over time. Model 3, however, continues to receive more power into the lower modes, flattening out its gradient and matching the observations' gradient very well in the final stages of evolution as can be seen in figures 5.10 and 5.13. Only the magnitude is too high but the shape of the graph is more important because parameters can be adjusted much easier to account for magnitude. Clearly Model 3 is the simulation with the best potential to match observations. Models 1 and 2 maintain too steep a gradient with too much power in the lower modes and too little power in the higher modes. The slope begins like this but does not change over time like Model 3 does. In Model 3, the higher modes lose energy to the expansion less rapidly than the lower modes, giving a flatter gradient - one that closely follows the gradient of the observations. It does appear as if the energy gets transported from the lower modes to the higher modes because the lower modes drop in power at a constant rate over time that is greater than the rate at which the higher modes lose power throughout the evolution.

The two-point correlation function was mentioned earlier as a measure of excess clustering with respect to a random sample. According to analyses of galaxy distributions it has been found that at small separations (up to about $10h^{-1}Mpc$ [45]), the dimensionless or reduced two-point correlation function $\xi(r/r_0)$ is well approximated as a power law [51]

$$\xi = (r_0/r)^\gamma$$

where the parameters are

$$\gamma \approx 1.77, \quad r_0 \approx 5.4h^{-1}Mpc$$

This relation tells roughly that galaxies are strongly clustered within that scale

range while tending to a more homogeneous distribution at larger separations [45]. Although the data collected does not fall within the distance confines mentioned above, figures 5.14 and 5.15 are useful to show that the simulations do produce clustering - more than is random and therefore creates structure. The above power law is plotted for comparison with the two-point correlation values of the power spectra which were obtained by taking the Fourier transform of each spectrum. Model 1 and Model 3 were plotted together to show that adding velocity fluctuations does increase the non-randomness of the clustering over no velocity perturbations at all. Note how the data points for Model 1 cluster quite strongly about the region $15-40 Mpc$ while Model 3 tends to be more scattered in this region, particularly with greater magnitude. This indication of less randomness leads to the conclusion that clustering is taking place and structure is produced and this is more recognisable with the added kinetic energy of Model 3.

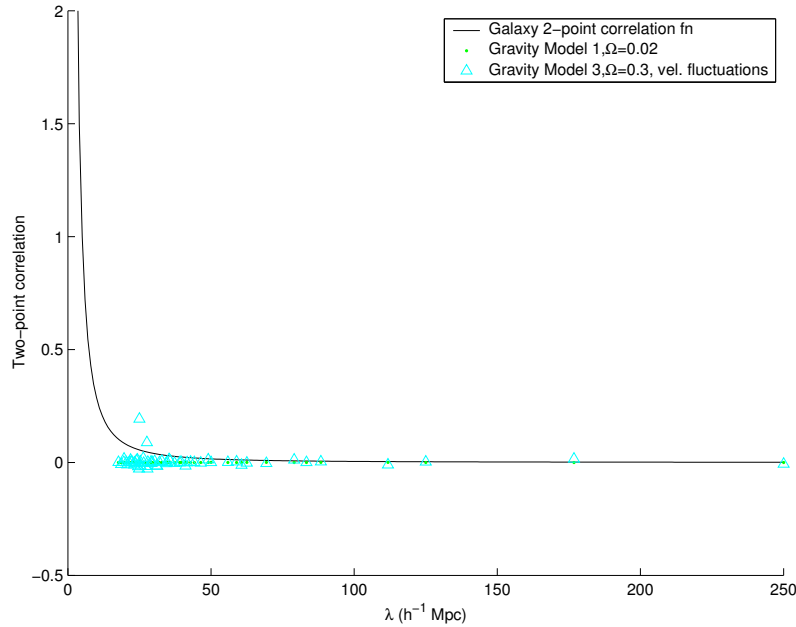


Figure 5.14: Gravity models: 2-point correlation functions, 10 Modes, Negative slope

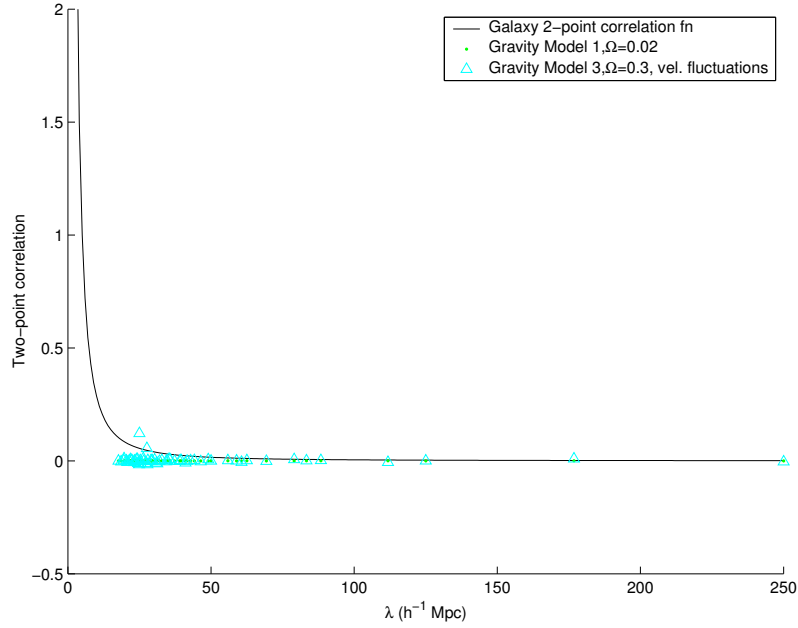
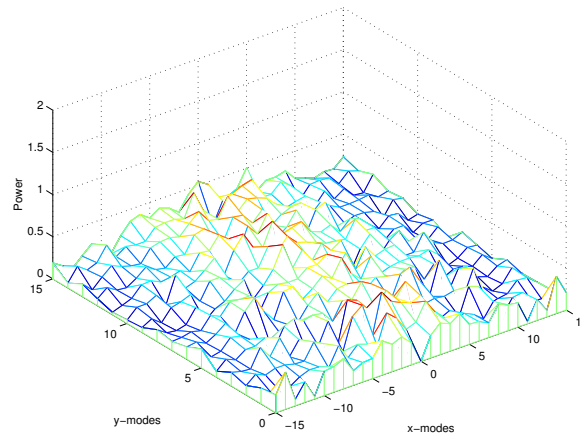
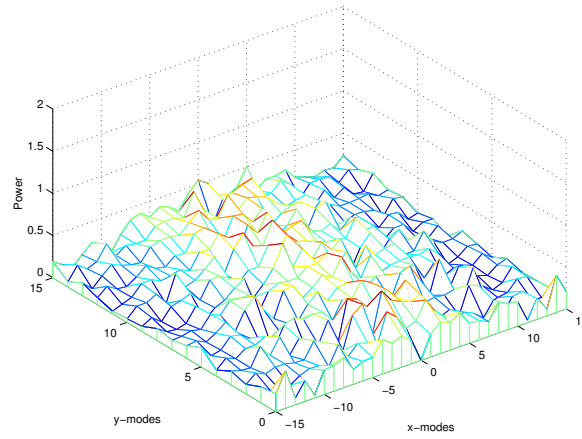


Figure 5.15: Gravity models: 2-point correlation functions, 10 Modes, Positive slope

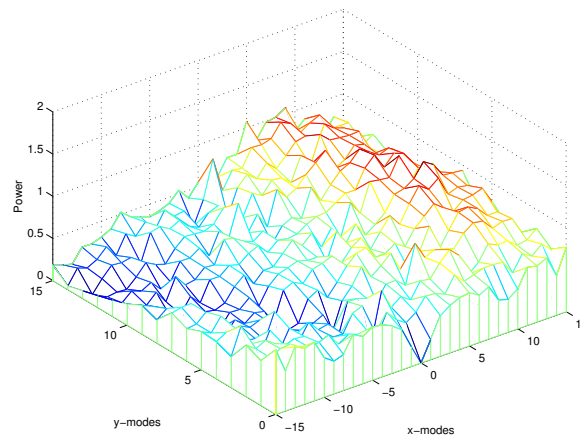
Simulations were also conducted using fifteen modes, models of greater detail to compare with the ten mode models to see what effect this would have on the results. The simulations discussed above are reproduced below using fifteen modes. Firstly the 3-D plots of the power for the gravity models are reproduced for both sets of initial conditions. The first two models are very much the same as the ten mode models. A smooth power profile is maintained through the evolution with a slightly flatter region in the higher y -modes while the entire profile degraded significantly from the Hubble expansion. Once more, Model 3 shows a more chaotic pattern of peaks and troughs of power from the velocity fluctuations as well as the retention of power in the higher modes. It displays the same overall profile pattern as ten modes (a trough along the line of $x = -y$ and rising on either side with most power in the higher modes) but with a more granular structure provided by the extra modes. Both sets of initial conditions demonstrate this behaviour.



(a) Spectral Power: Gravity Model 1

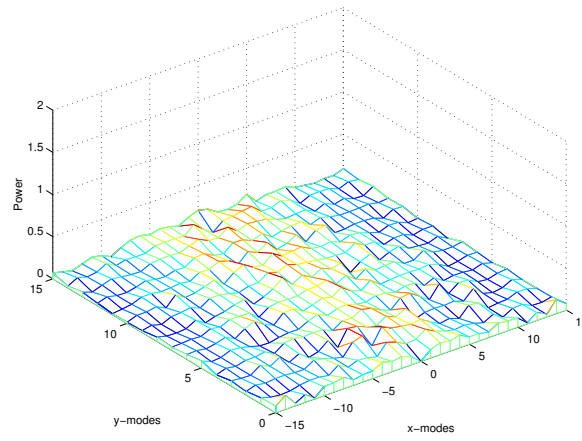


(b) Spectral Power: Gravity Model 2

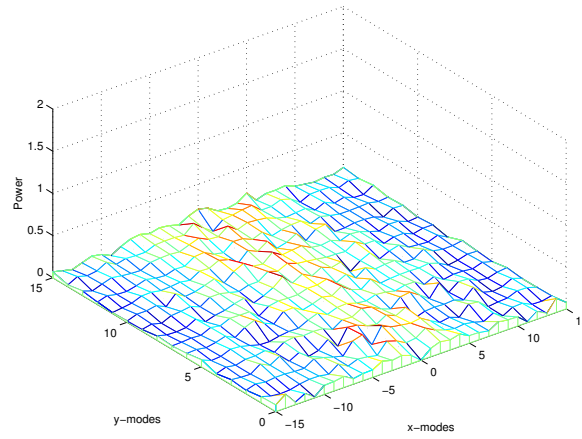


(c) Spectral Power: Gravity Model 3

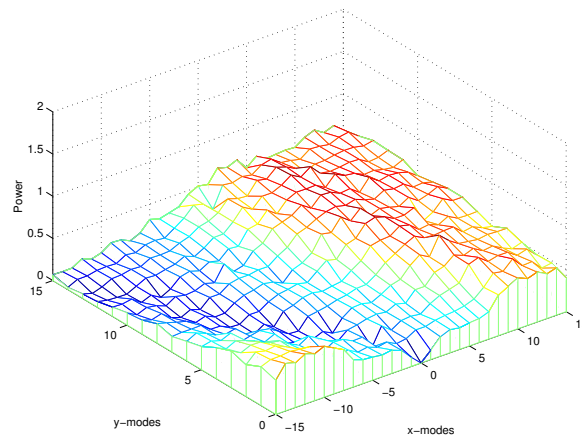
Figure 5.16: Gravity Models, First Stage Evolution, Negative Slope, 15 Modes



(a) Spectral Power: Gravity Model 1

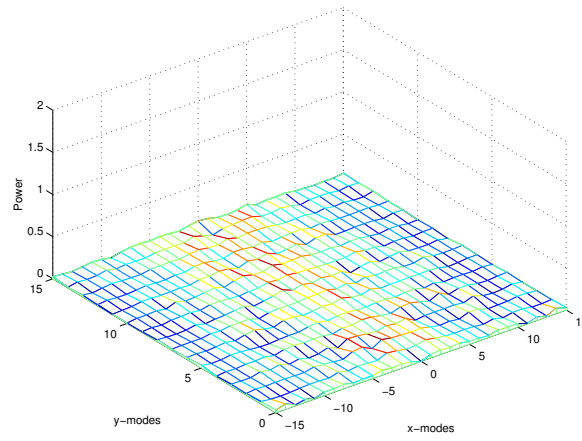


(b) Spectral Power: Gravity Model 2

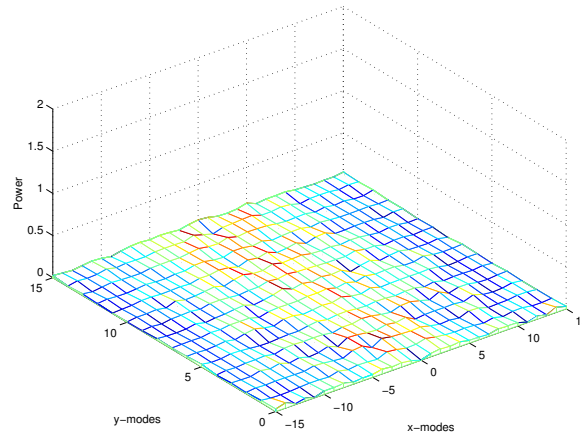


(c) Spectral Power: Gravity Model 3

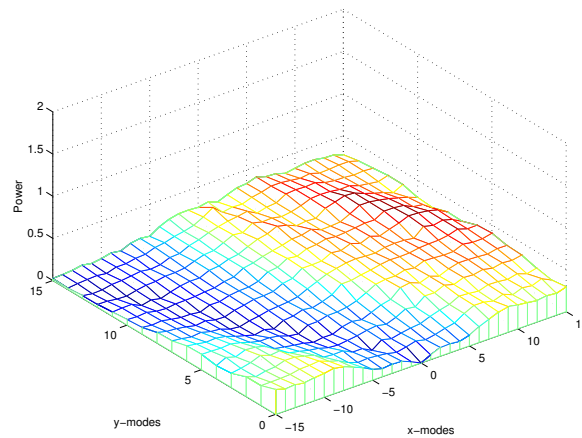
Figure 5.17: Gravity Models, Second Stage Evolution, Negative Slope, 15 Modes



(a) Spectral Power: Gravity Model 1

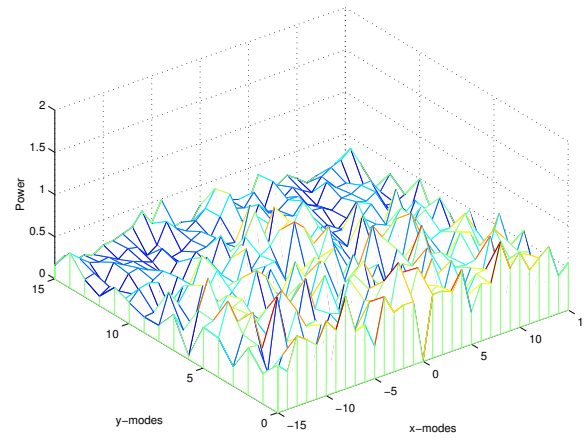


(b) Spectral Power: Gravity Model 2

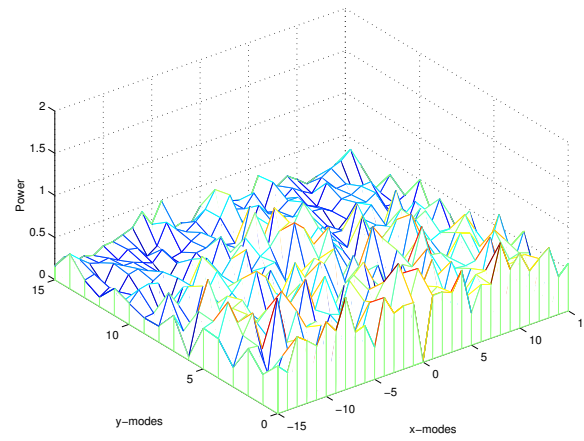


(c) Spectral Power: Gravity Model 3

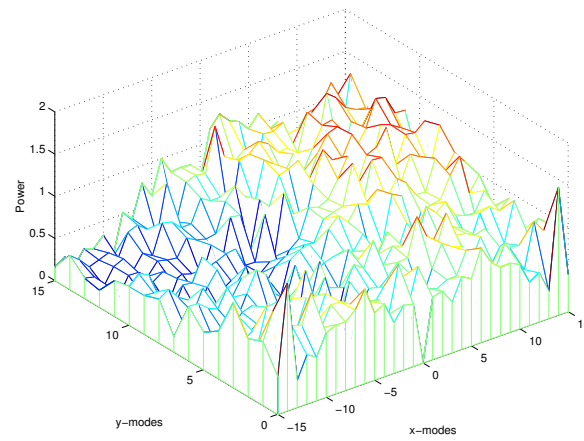
Figure 5.18: Gravity Models, Third Stage Evolution, Negative Slope, 15 Modes



(a) Spectral Power: Gravity Model 1

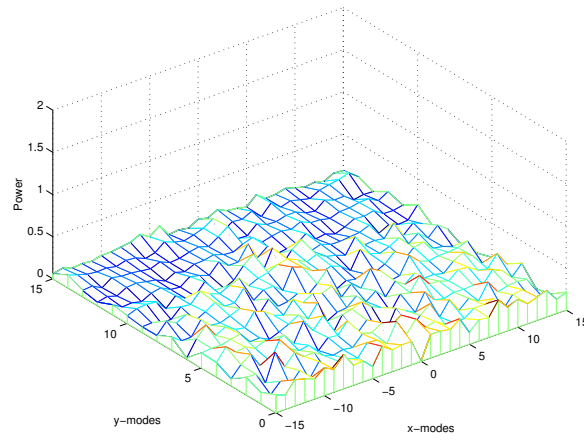


(b) Spectral Power: Gravity Model 2

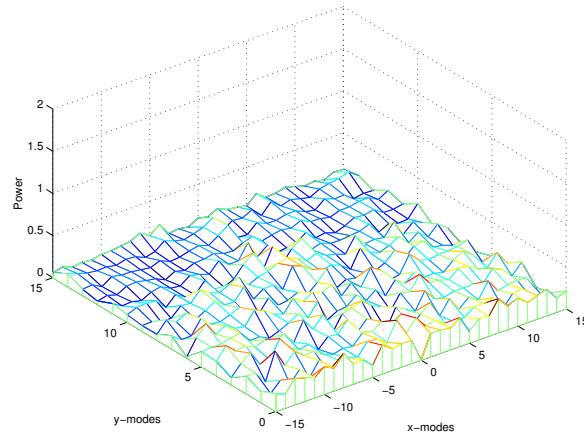


(c) Spectral Power: Gravity Model 3

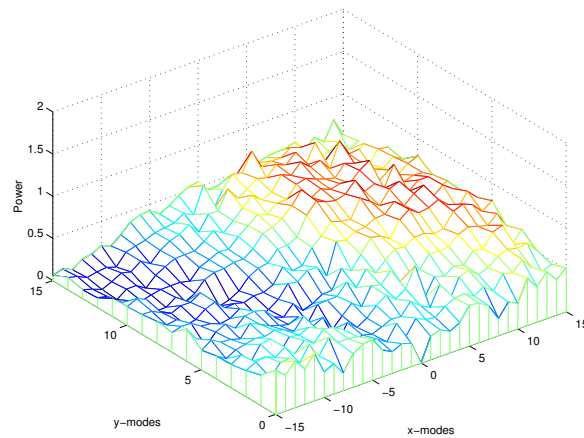
Figure 5.19: Gravity Models, First Stage Evolution, Positive Slope, 15 Modes



(a) Spectral Power: Gravity Model 1

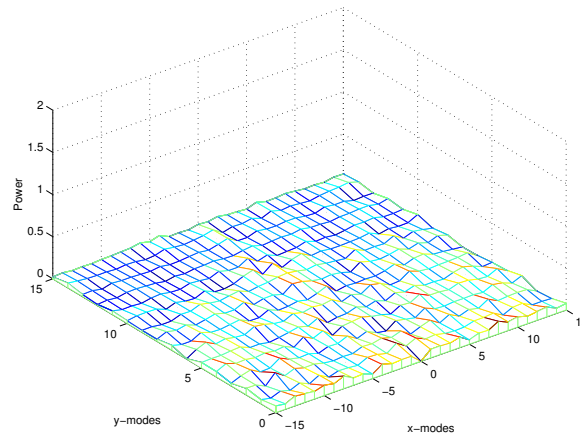


(b) Spectral Power: Gravity Model 2

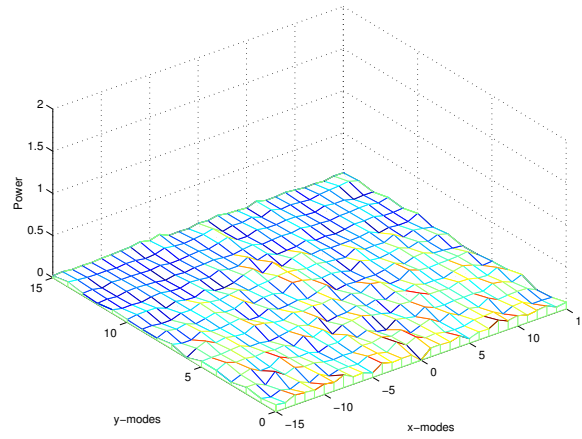


(c) Spectral Power: Gravity Model 3

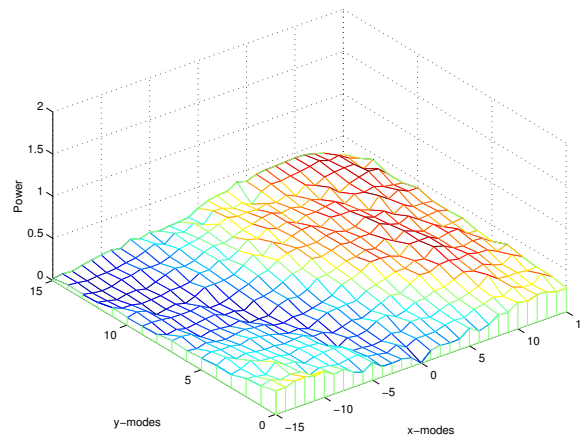
Figure 5.20: Gravity Models, Second Stage Evolution, Positive Slope, 15 Modes



(a) Spectral Power: Gravity Model 1



(b) Spectral Power: Gravity Model 2



(c) Spectral Power: Gravity Model 3

Figure 5.21: Gravity Models, Third Stage Evolution, Positive Slope, 15 Modes

The following six figures compare the power spectra for fifteen modes against the observational spectra. Their behaviours compared to the previous models (with ten modes) are similar in general shape and evolution. The magnitudes of the spectra begin slightly above the observations and decline as expected over time although their shape is of more importance. Both sets of initial conditions display very similar shaped spectra. Models 1 and 2, again, are virtually identical with power that decreases smoothly with k according to a slope that is relatively steeper than the observations because of too little power in the higher modes. Model 3, with its extra initial kinetic energy, shows a much closer agreement to the shape of the observational spectra, matching power in both the lower and higher modes. Once again this demonstrates the tendency of the initial nonzero velocity conditions to maintain power in - or possibly transport power to - the higher modes. Looking at the spectrum for Model 3 it can be seen that the overall gradient does flatten slightly over time, indicating power may indeed be being transported as the lower modes lose power in greater amounts than the higher modes, albeit constantly.

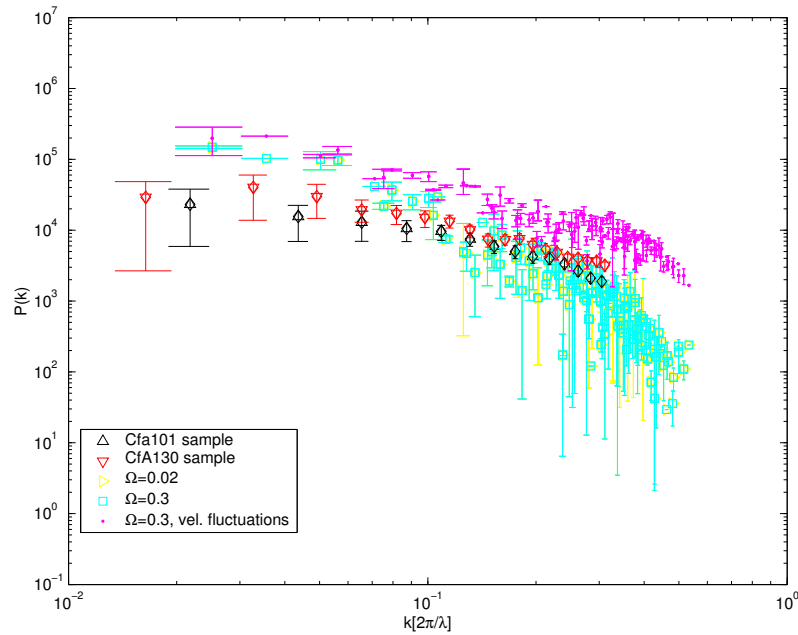


Figure 5.22: Gravity Models: First Evolution Stage, 15 Modes, Negative slope

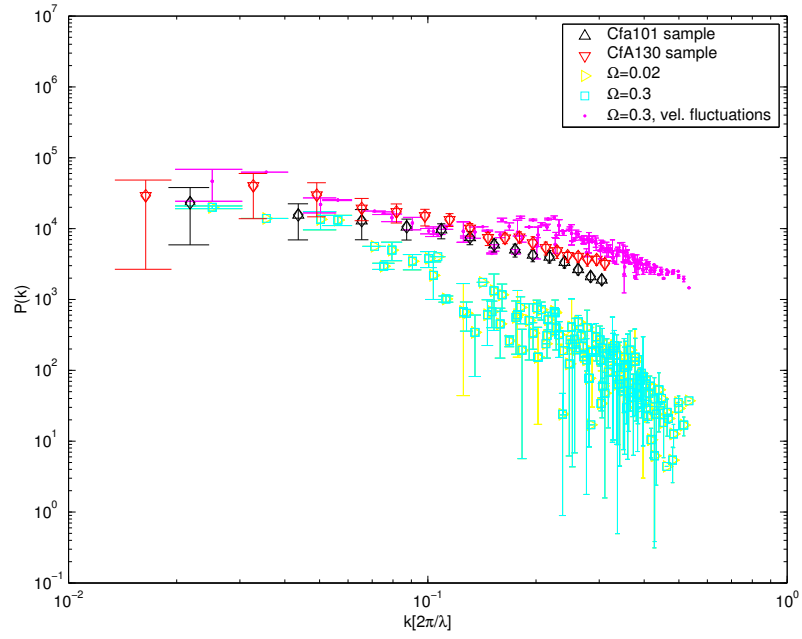


Figure 5.23: Gravity Models: Second Evolution Stage, 15 Modes, Negative slope

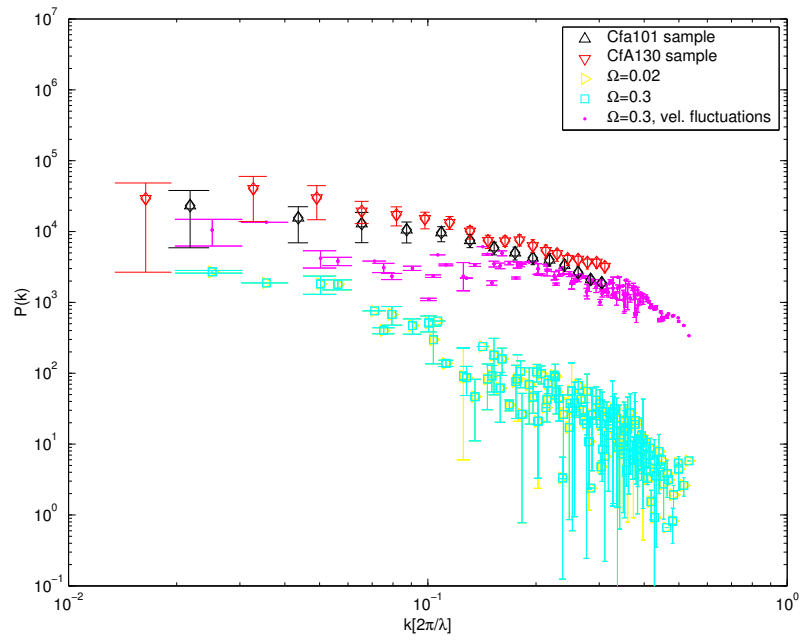


Figure 5.24: Gravity Models: Third Evolution Stage, 15 Modes, Negative slope

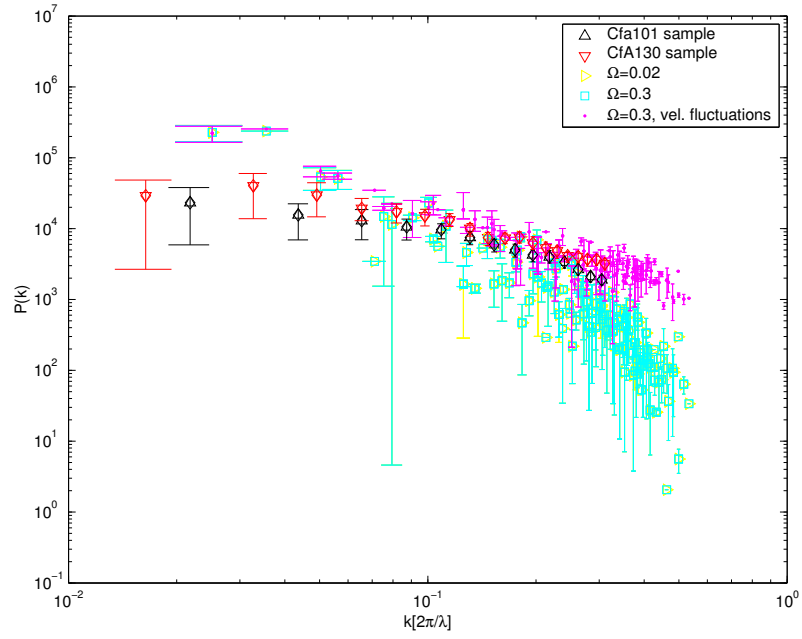


Figure 5.25: Gravity Models: First Evolution Stage, 15 Modes, Positive slope

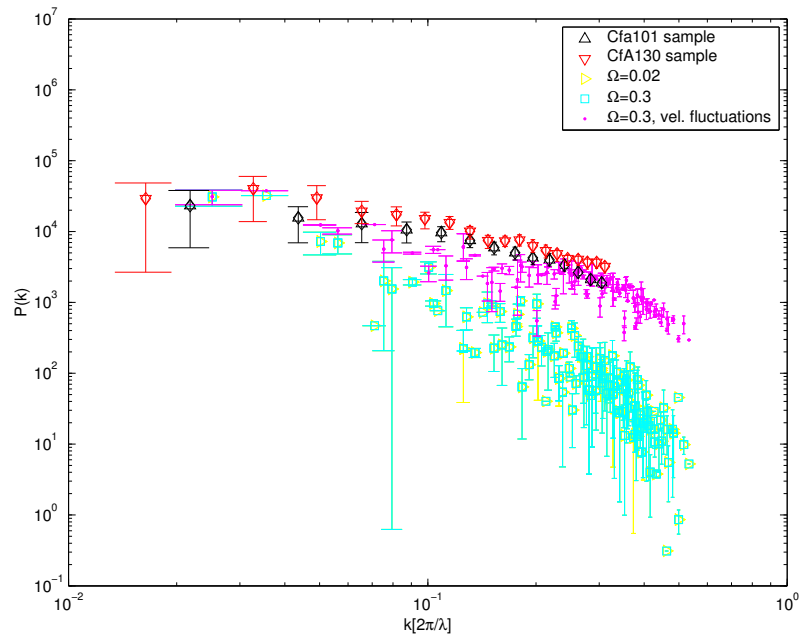


Figure 5.26: Gravity Models: Second Evolution Stage, 15 Modes, Positive slope

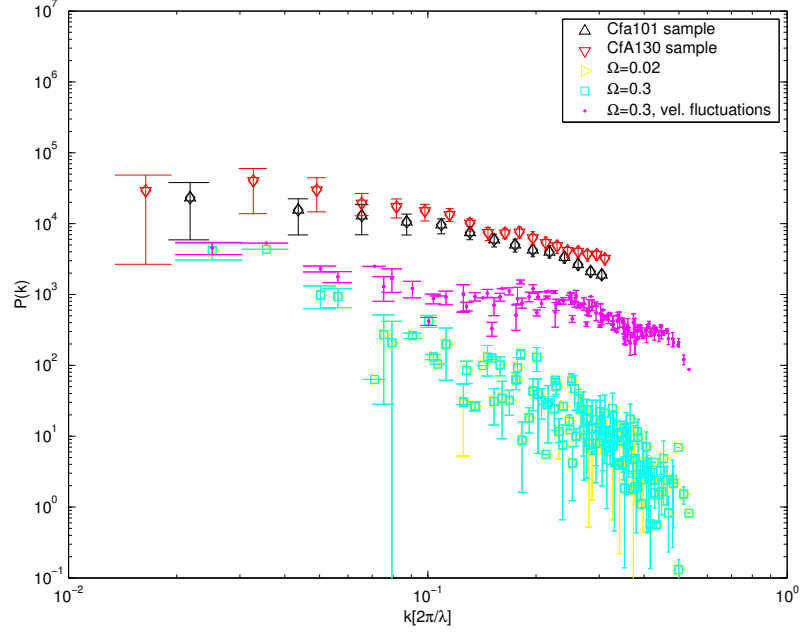


Figure 5.27: Gravity Models: Third Evolution Stage, 15 Modes, Positive slope

The two-point correlation functions for fifteen modes are a bit different to their ten mode counterparts. The former appear to be clustered in a tighter pattern than the latter, especially in the region $15 - 40 Mpc$. Overall, Model 3 still has noticeably larger scatter and magnitude than Model 1, indicating greater clustering potential when velocity perturbations are added initially.

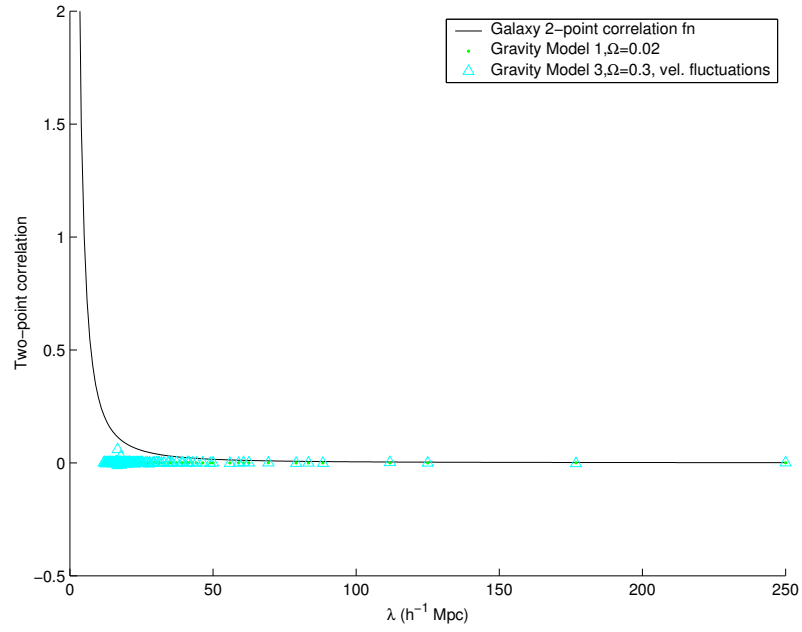


Figure 5.28: Gravity models: 2-point correlation functions, 15 Modes, Negative slope

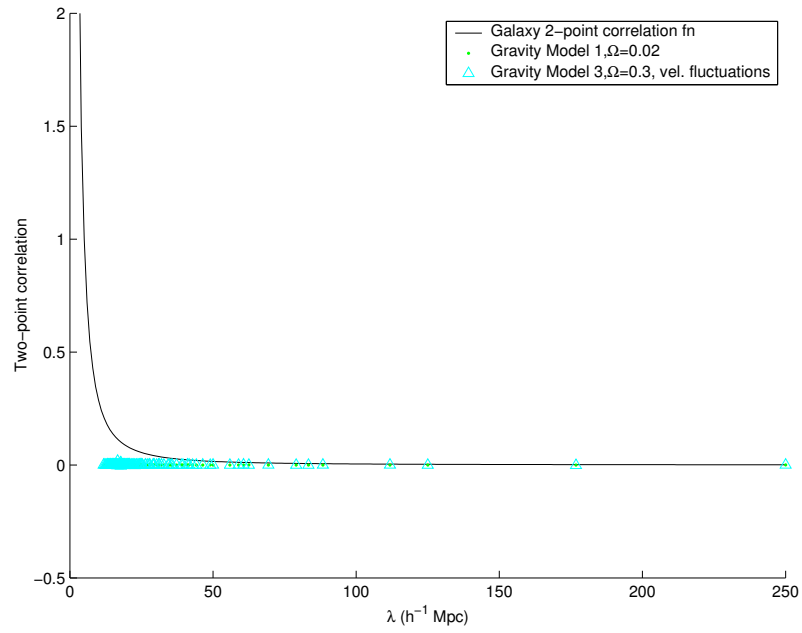


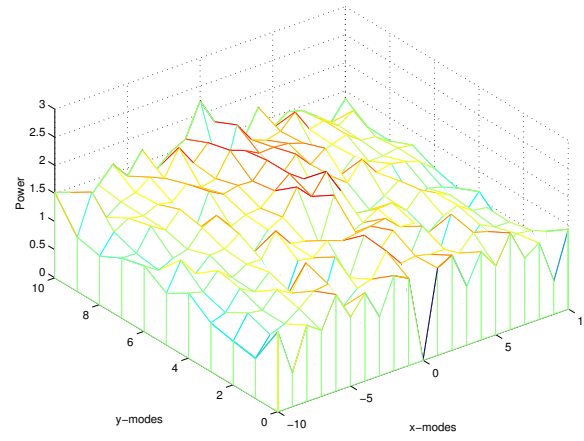
Figure 5.29: Gravity models: 2-point correlation functions, 15 Modes, Positive slope

5.4 The Plasma Model

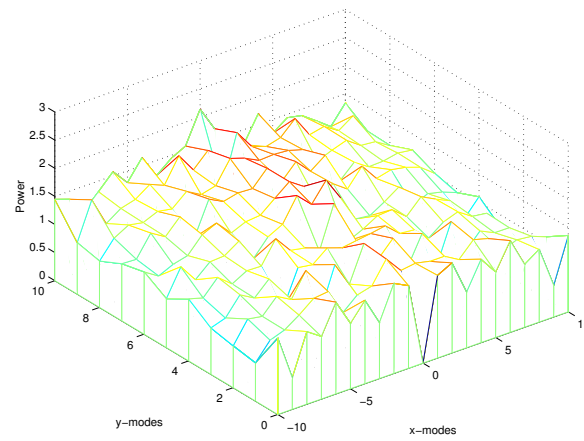
Only two models for the plasma forces were used. Like the first model of the pure gravity simulations, one plasma model has a matter density of $\Omega = 0.02$ and no initial velocity fluctuations while the other does include initial velocity fluctuations. The following six figures show the raw power spectrum data for the first plasma model, firstly with negative slope initial conditions and followed by positive slope initial conditions. This time each figure shows all three stages of evolution for this model in the same figure. The first striking feature is that it is extremely difficult to discern the three evolutionary stages from one another. They are nearly identical with only a very marginal decrease in power with time. This shows that, whereas the gravity models' power spectra were noticeably reduced with time, the plasma model more or less maintained its power level, indicating the combined energy source of gravity and plasma are compensating for the Hubble expansion energy sink. Thus, clearly, plasma forces appear to be an important source of cosmic energy.

Figure 5.30(a) shows the first stage power for negative slope initial conditions. The power profile is very similar to the gravity models without fluctuations. There is virtually no change in the map as the model evolves through figures 5.30(b) and 5.30(c) and modal power relative to the other modes remains constant indicating no transfer of power. This is similar to the first two gravity models, but here, power is maintained through the energy of the plasma medium. It seems that initial velocity perturbations is the only feature to impose a significantly different shape on the power spectrum by transferring power through the various modes and thus the various scales. The same observations are evident for the positive gradient initial condition model too. Figures 5.31(a) - 5.31(c) again display a very similar map to the first two gravity models. This shape is maintained throughout the evolution with only minor losses in power magnitude.

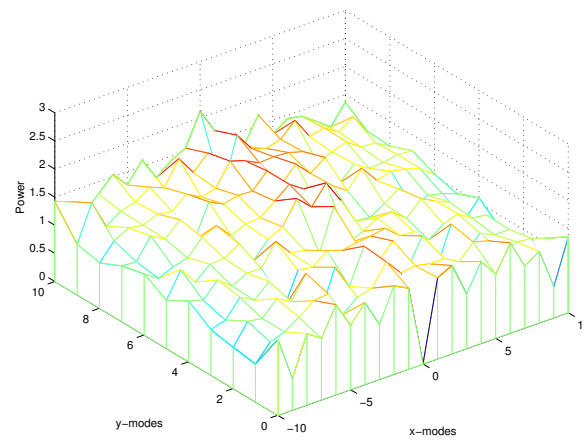
The plasma power spectrum was normalised in the same fashion as the gravity models and remains remarkably stationary.



(a) Spectral Power: First Stage

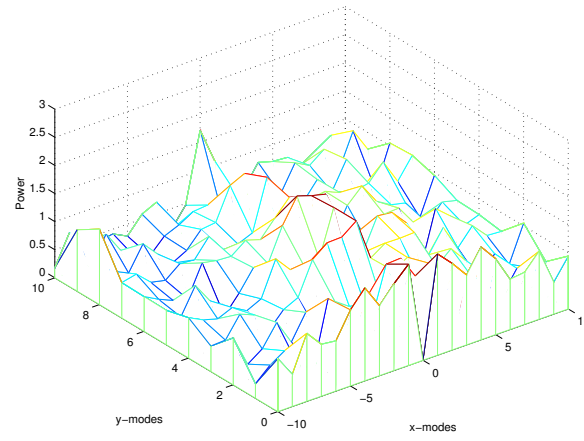


(b) Spectral Power: Second Stage

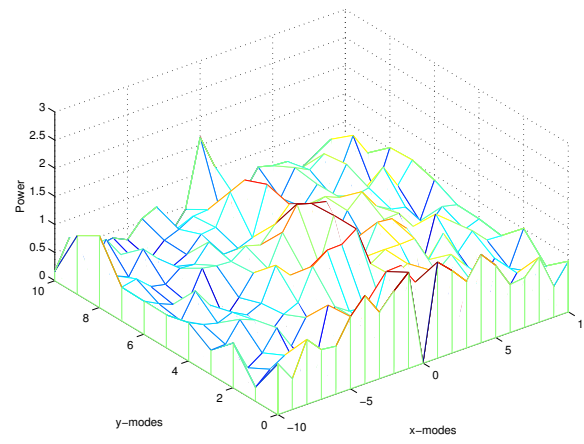


(c) Spectral Power: Third Stage

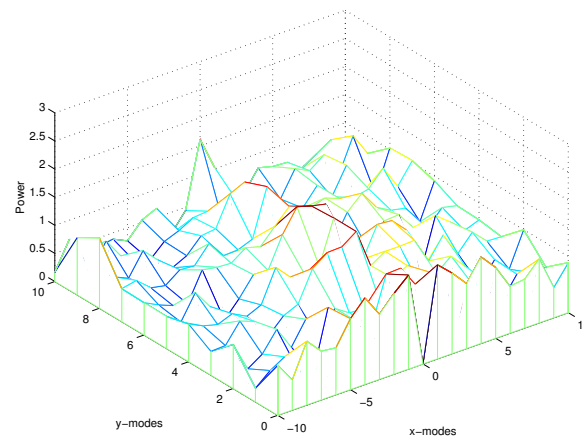
Figure 5.30: Plasma Model 1: Three Stages of Evolution, 10 Modes, Negative Slope



(a) Spectral Power: First Stage



(b) Spectral Power: Second Stage



(c) Spectral Power: Third Stage

Figure 5.31: Plasma Model 1: Three Stages of Evolution, 10 Modes, Positive Slope

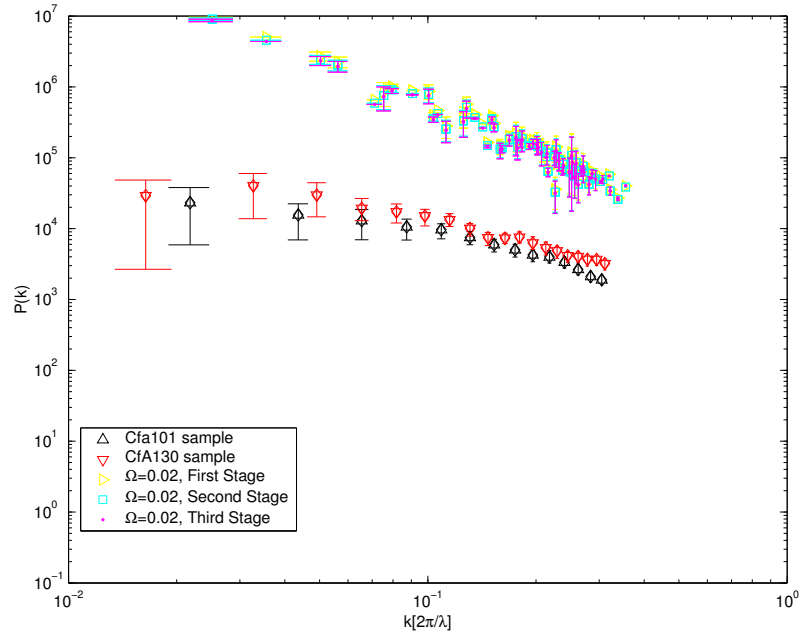


Figure 5.32: Plasma Model 1: Three Stages of Evolution, 10 Modes, Negative Slope

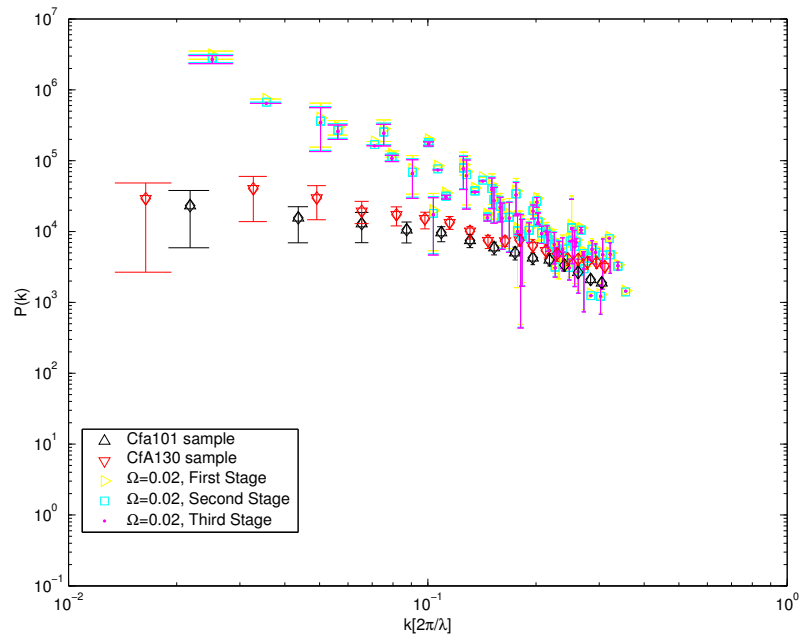
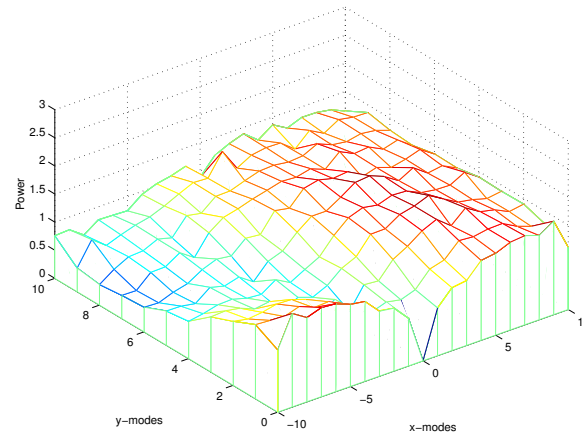


Figure 5.33: Plasma Model 1: Three Stages of Evolution, 10 Modes, Positive Slope

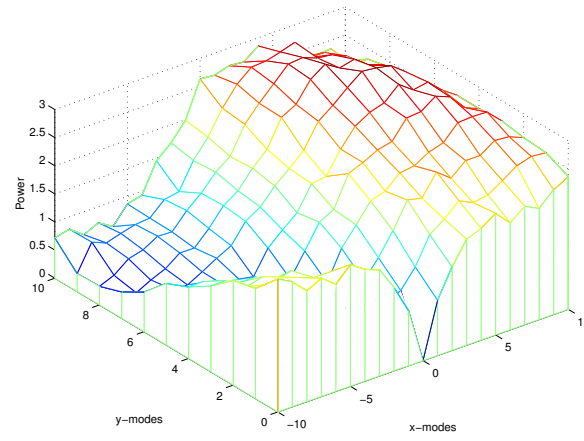
Negative and positive initial conditions both display very similar results in the above graphs. Compared to the purely gravitational simulations the plasma models arguably show better agreement with observations from a magnitude perspective while being similar in shape. All the spectra representing the different stages of evolution are virtually on top of each other and it is difficult to distinguish them, confirming the results seen in figures 5.30 and 5.31. The Hubble term appears to have minimal effect on the power spectrum or, an alternative viewpoint would be that possibly it prevents the power from growing dramatically through the evolution. Keeping in mind the severe reduction in magnitude of the plasma constants in the simulation - which could represent, at least in part, the relative rarity of charged matter compared to uncharged matter - the plasma forces show vast differences over their purely gravitational counterparts.

In figures 5.32 and 5.33, the shape of the plasma spectra appear very similar to that of Gravity Models 1 and 2, albeit at higher power levels. Power levels are comparable at the first stage of evolution but move apart substantially through subsequent stages. Model 3 of the gravitational model has a different shape altogether and is comparable with the second plasma model discussed below.

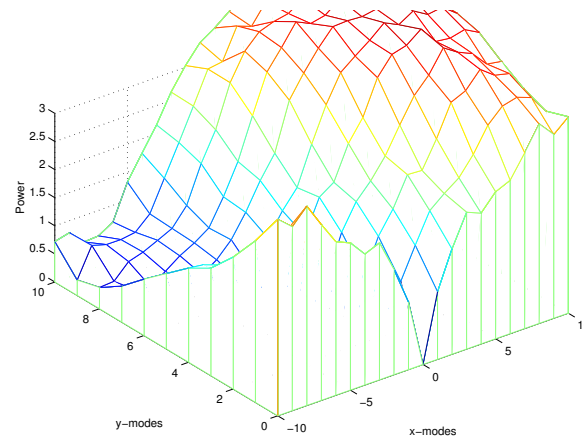
As mentioned previously plasma models were also conducted with velocity perturbations. The following two figures show power for the included velocity perturbations for negative and positive slope conditions respectively. Following that are the power spectrum/scale plots for the same two simulations. Once again, a dramatic transfer of power into the higher modes can be seen in the two dimensional power profiles as the evolution progresses when the extra kinetic energy is added.



(a) Spectral Power: First Stage

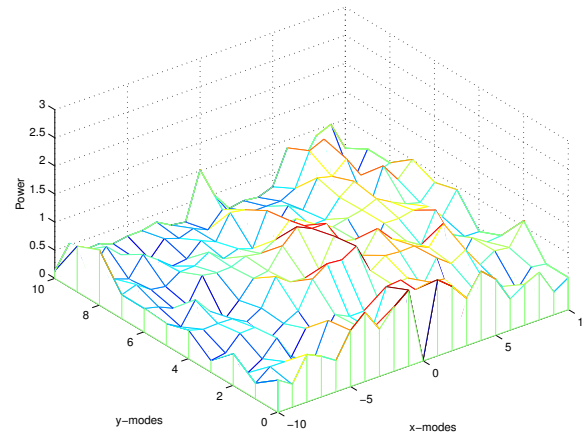


(b) Spectral Power: Second Stage

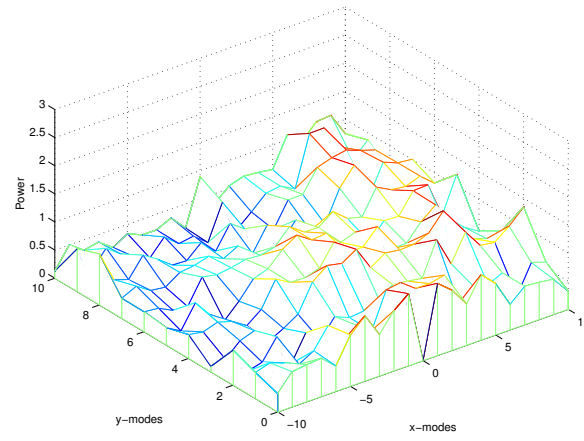


(c) Spectral Power: Third Stage

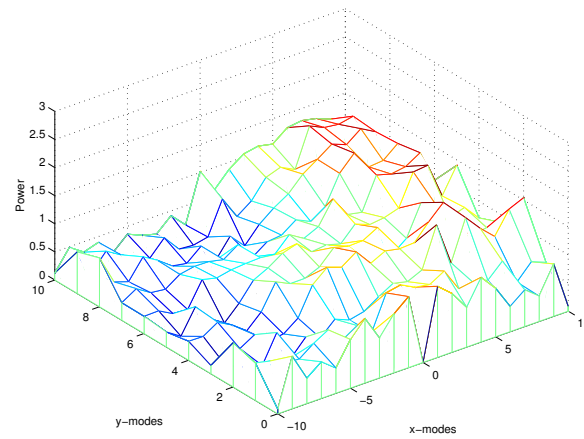
Figure 5.34: Plasma Model 2: Three Stages of Evolution, 10 Modes, Negative Slope, Velocity Fluctuations



(a) Spectral Power: First Stage



(b) Spectral Power: Second Stage



(c) Spectral Power: Third Stage

Figure 5.35: Plasma Model 2: Three Stages of Evolution, 10 Modes, Positive Slope, Velocity Fluctuations

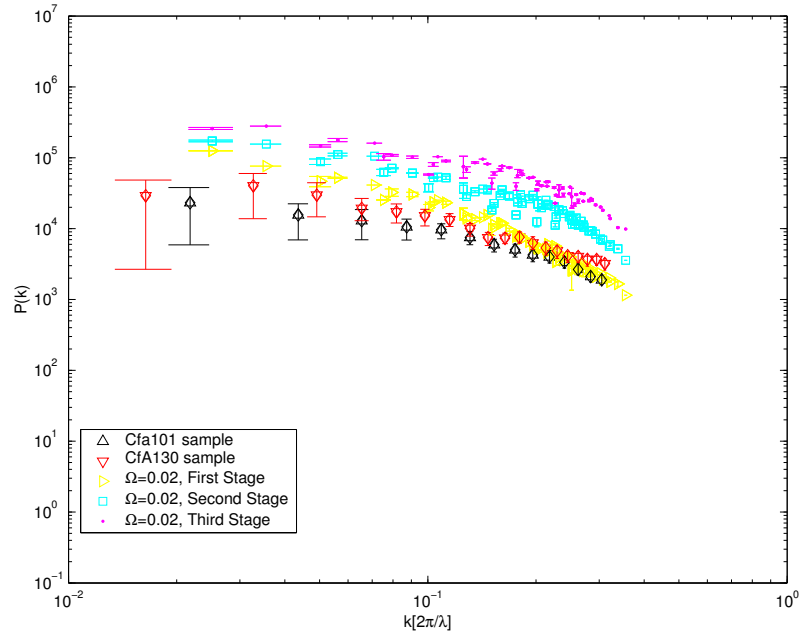


Figure 5.36: Plasma Model 2: Three Stages of Evolution, 10 Modes, Negative Slope, Velocity Fluctuations

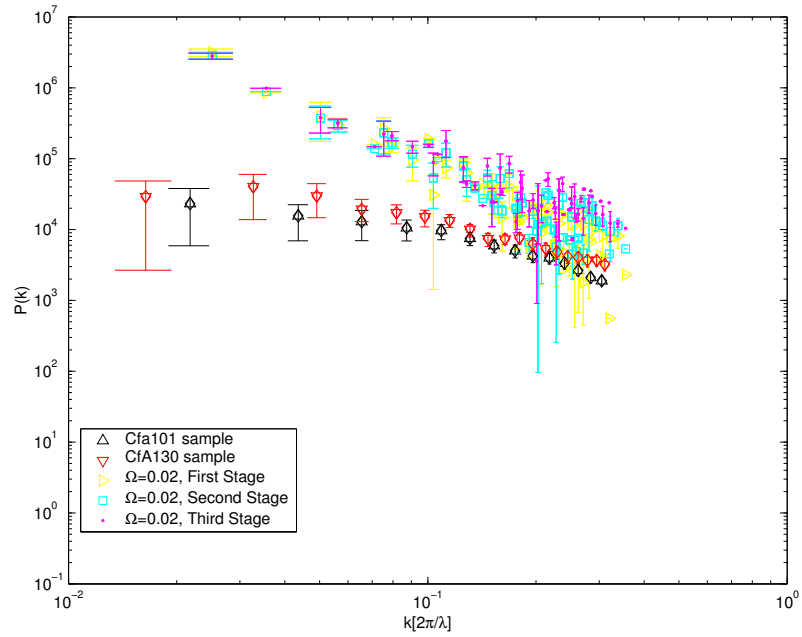


Figure 5.37: Plasma Model 2: Three Stages of Evolution, 10 Modes, Positive Slope, Velocity Fluctuations

In figures 5.36 and 5.37 the effects initial velocity perturbations have on the plasma model are clearly seen. There is some difference between the negative and positive gradient initial conditions due to a slightly greater spectrum magnitude for the negative conditions but the effect is the same - to transfer power from the lower modes to the higher modes as was the case with the gravitational models. In the case of the positive slope initial conditions the result is just noticeable whereas with the negative initial conditions it is clear. In fact if the velocity perturbations factor was increased by an order of magnitude for the positive conditions then the result was as apparent as the negative conditions. Combining plasma forces and initial velocity conditions produces a power spectrum not unlike the observations, if a little high in magnitude; but the shape has more or less the correct balance of power in the lower and higher modes.

Figures 5.38 and 5.39 revisit the two-point correlation function for the two sets of initial conditions without velocity perturbations. Figure 5.38 plots the Fourier transform of the power spectra for the first and last evolution stages of the negative slope initial conditions together with the observed small scale two-point correlation function. Since their power spectra are so similar clearly ξ will also be much alike for both sets of data. In both graphs, data from the 3rd phase of evolution follows the extrapolated correlation slightly more closely than the first phase, indicating a shade more randomness and less clustering as the simulation evolves, albeit still more structured than a purely random arrangement. Following on this, figures 5.40 and 5.41 show the correlation function or clustering comparison between Plasma Model 1 and Gravity Model 1 (the equivalent model without plasma forces) without velocity perturbations for ten modes. It clearly demonstrates that plasma forces have a significant advantage in aggregating matter on smaller scales over gravity alone while on larger scales they are very similar. In comparison to the plasma model, the gravity model is virtually a straight line of data points. The negative initial conditions show a bit more clustering in the plasma model, again because of its slightly higher magnitude. While the values are mainly below the observed power law, the comparison of the models is the important factor and indicate that plasma forces in clustering matter should not be ignored.

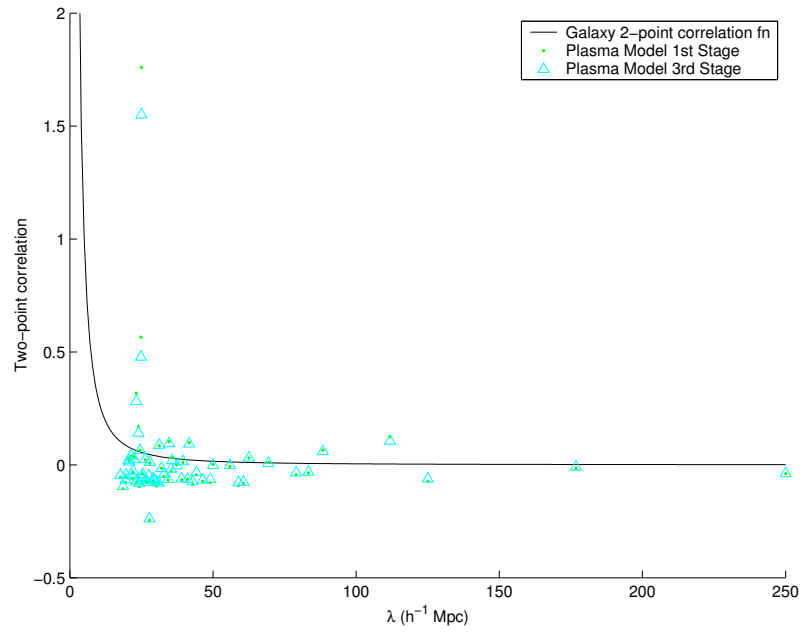


Figure 5.38: Plasma Model 1: 2-point correlation functions, 10 Modes, Negative slope

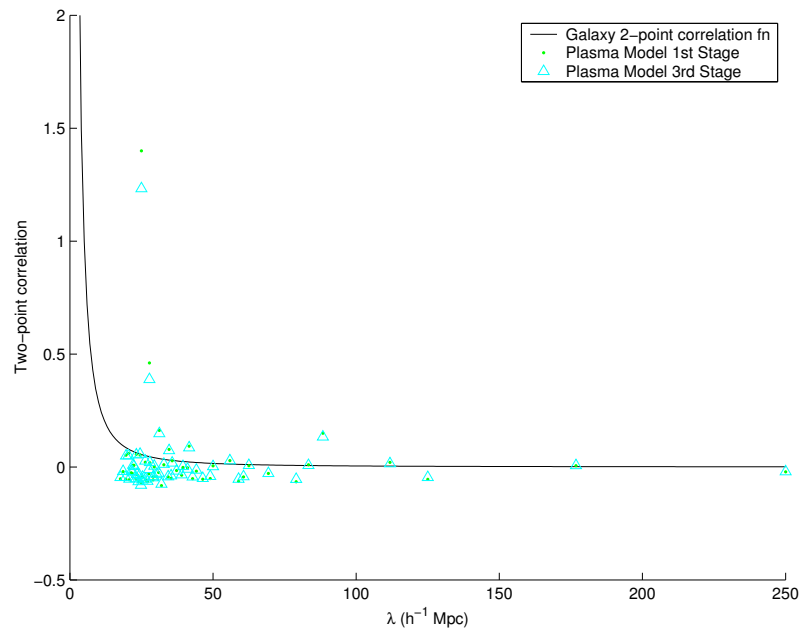


Figure 5.39: Plasma Model 1: 2-point correlation functions, 10 Modes, Positive slope

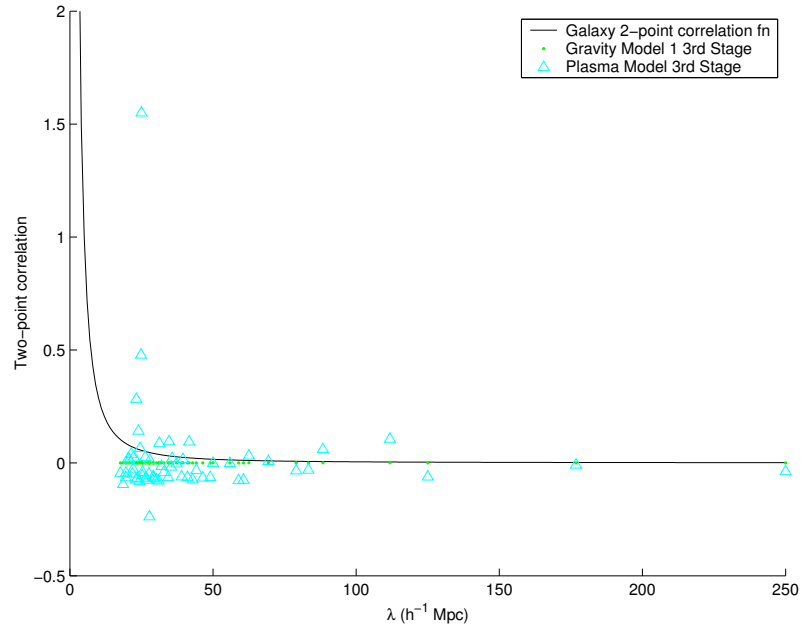


Figure 5.40: Gravity Model 1 vs Plasma Model 1: 2-point correlation functions, 10 Modes, Negative slope

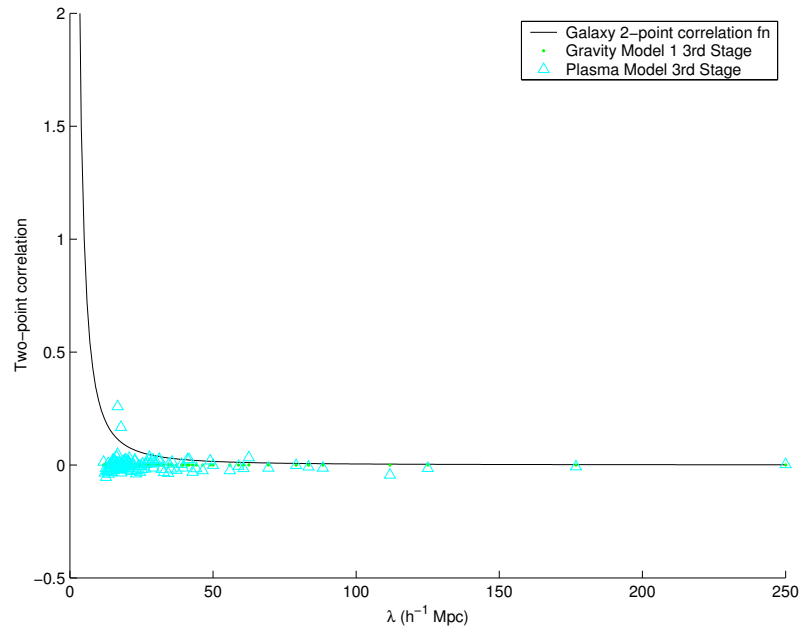


Figure 5.41: Gravity Model 1 vs Plasma Model 1: 2-point correlation functions, 10 Modes, Positive slope

Next, the comparable two-point correlation functions are shown for the initial conditions with velocity perturbations. These show somewhat of a reversal in the results of the previous set of graphs where clustering seemed to diminish slightly over time. Here, there is a small but significant increase in clustering during evolution. Figures 5.42 and 5.43 compare the first and last evolution stages of each set of initial conditions. To properly compare the initial conditions with and without velocity fluctuations, figures 5.44 and 5.45 are plotted, matching the last evolution stage of each condition set against the other. It is noticeable that the additional kinetic energy does not have a substantial effect on clustering potential, particularly with the positive slope initial conditions, remembering that the overall magnitude of these coefficients is lower than that of the negative slope conditions. Indeed the simulations without velocity appear somewhat more effective in generating structure whereas the reverse holds true when applied to the negative slope conditions. Most importantly, these figures show that the driving force of the increased clustering potential is predominantly the initial matter density and plasma forces and not the added kinetic energy.

Finally, figures 5.46 and 5.47 compare gravity and plasma models with velocity fluctuations in their initial conditions, namely Gravity Model 3 and Plasma Model 1. Even though matter density in the gravity model is 15 times greater it is still a relevant contrast because this work seeks to establish the feasibility of replacing additional matter from exotic sources with everyday plasma forces. Thus, these two diagrams demonstrate that plasma forces could create the non-random structure formation generated initially by a higher mass density since their data points are very similar and, in many cases, further from the approximated two-point correlation function in the plasma model.

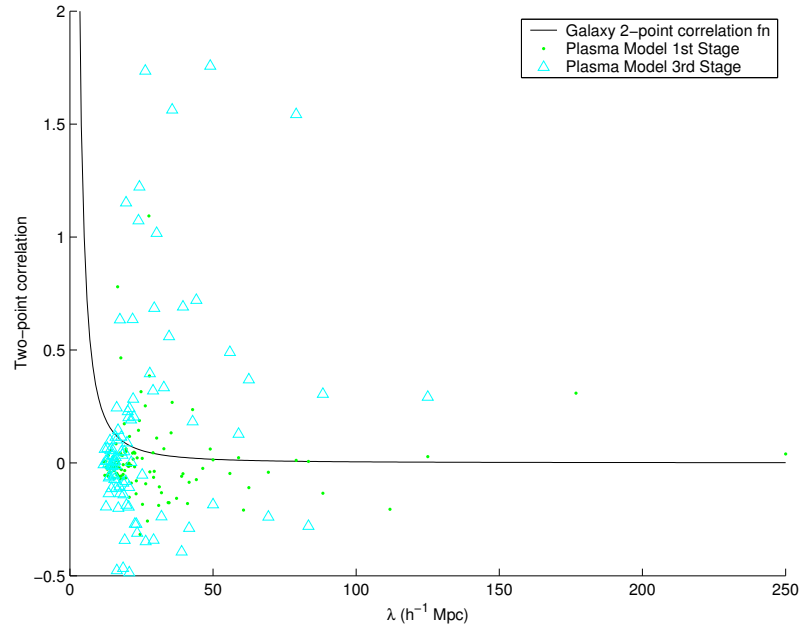


Figure 5.42: Plasma Model 2: 2-point correlation functions, 10 Modes, Negative slope, Velocity Fluctuations

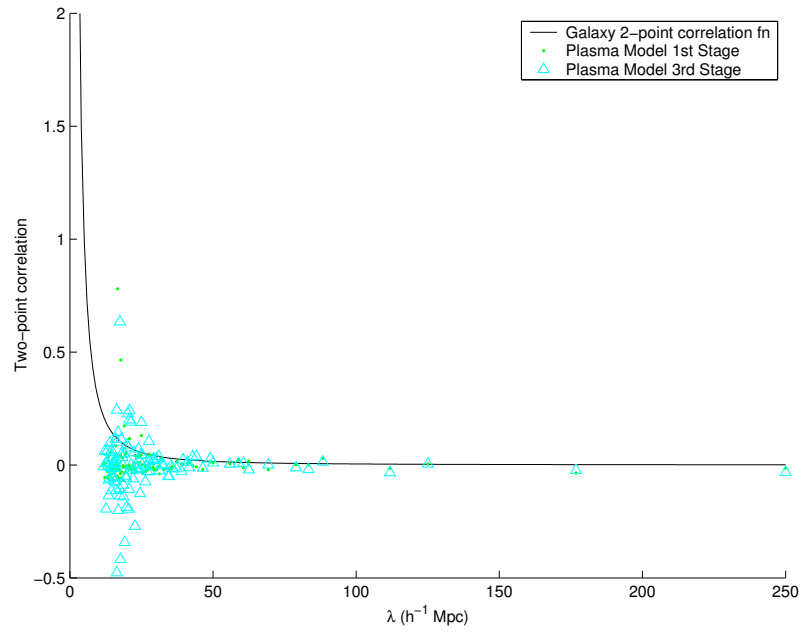


Figure 5.43: Plasma Model 2: 2-point correlation functions, 10 Modes, Positive slope, Velocity Fluctuations

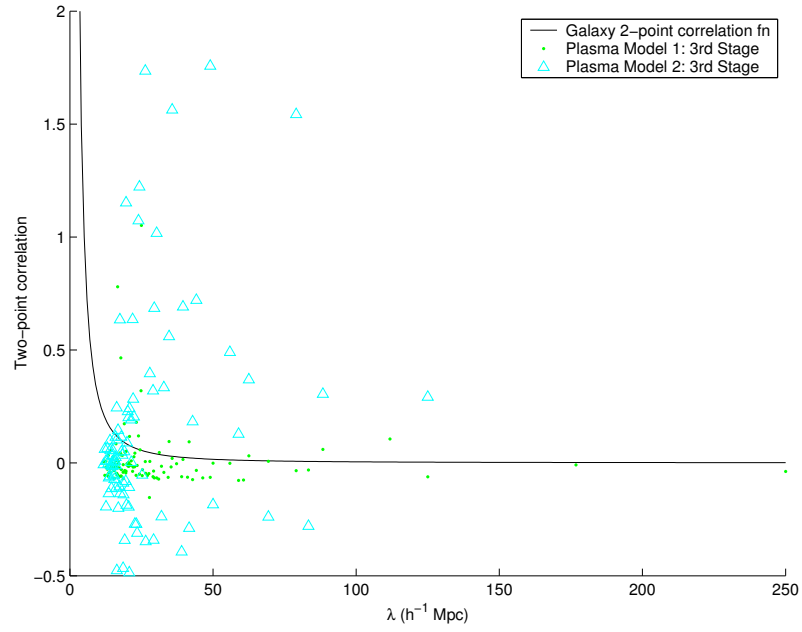


Figure 5.44: Plasma Model 1 vs Plasma Model 2: 2-point correlation functions, 10 Modes, Negative slope, Velocity Fluctuations

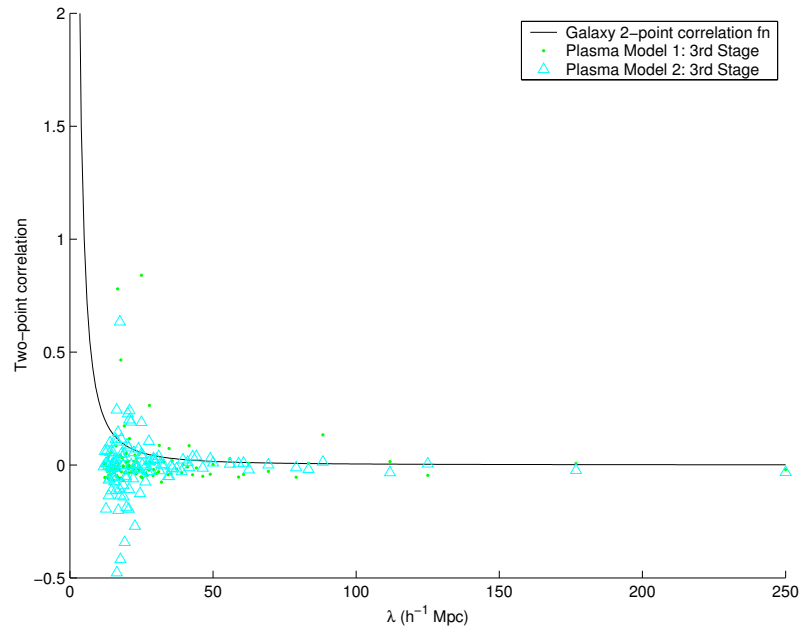


Figure 5.45: Plasma Model 1 vs Plasma Model 2: 2-point correlation functions, 10 Modes, Positive slope, Velocity Fluctuations

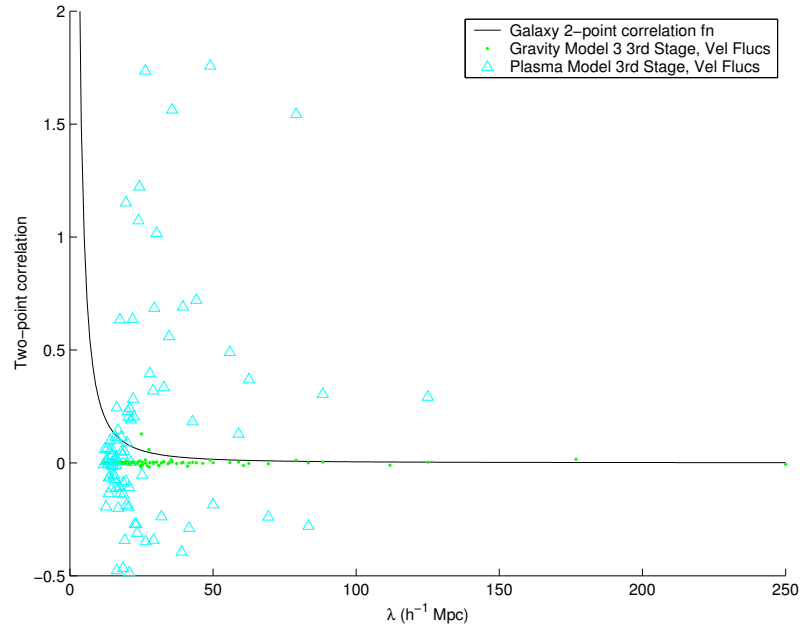


Figure 5.46: Gravity Model 3 vs Plasma Model 2: 2-point correlation functions, 10 Modes, Negative slope, Velocity Fluctuations

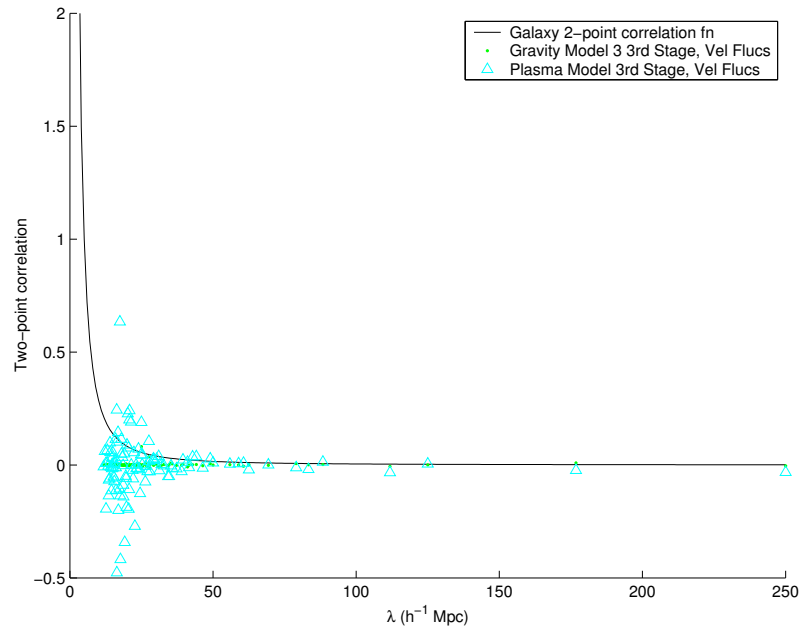
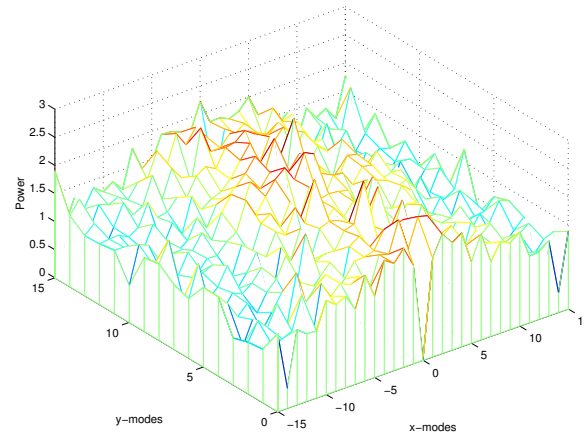
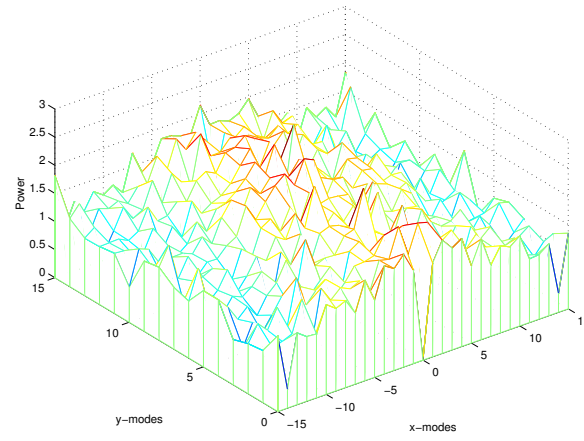


Figure 5.47: Gravity Model 3 vs Plasma Model 2: 2-point correlation functions, 10 Modes, Positive slope, Velocity Fluctuations

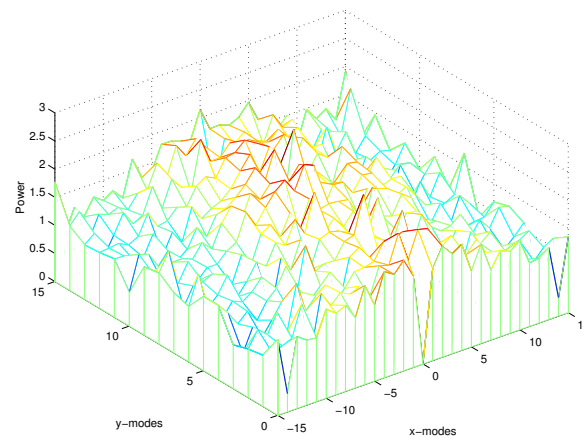
The three dimensional power profiles of the fifteen mode plasma models are virtually identical to the ten mode models. As before, power patterns remain practically constant throughout the evolution with just slight decreases in overall power from the Hubble expansion. When comparing magnitudes of the power spectra, there are again similarities between the two sets of modes while the shape is shifted slightly over to the higher modes in the 15 mode models. There is more of a difference between the two sets of initial conditions with the negative gradient values producing a much flatter pattern at a slightly higher magnitude than the positive gradient conditions which produce a spectrum which tends to taper off towards the higher modes as the galaxy survey data does. Figures 5.48 and 5.49 below show those 3-D power profiles and figures 5.50 and 5.51 demonstrate the comparable power spectra.



(a) Spectral Power: First Stage

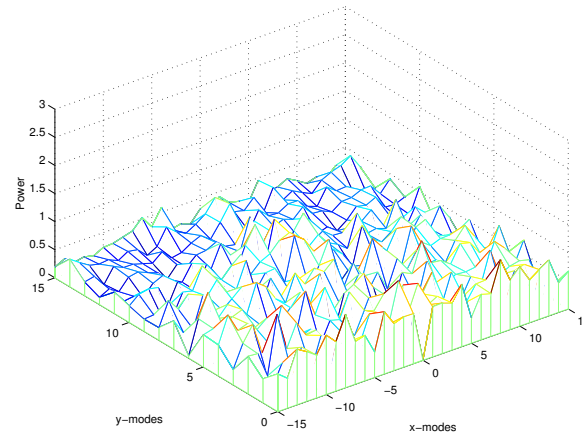


(b) Spectral Power: Second Stage

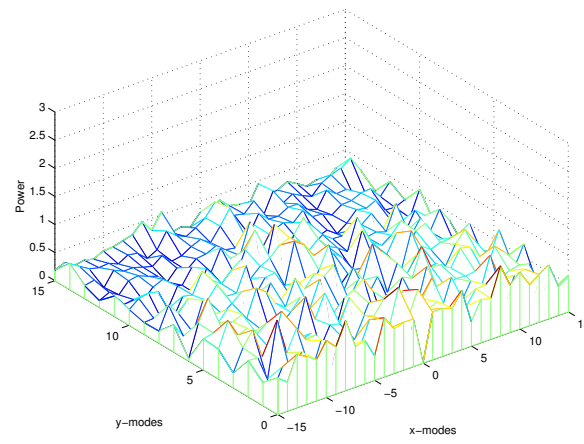


(c) Spectral Power: Third Stage

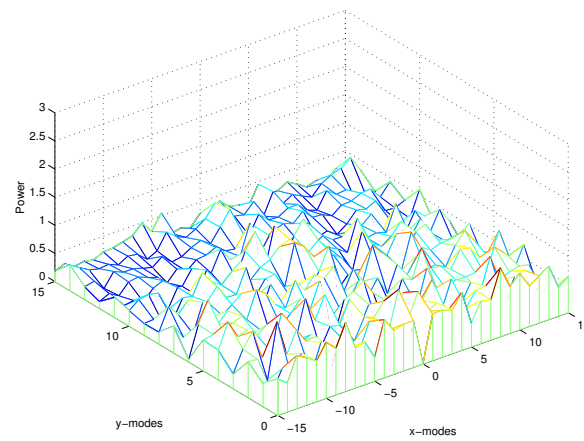
Figure 5.48: Plasma Model 1: Three Stages of Evolution, 15 Modes, Negative Slope



(a) Spectral Power: First Stage



(b) Spectral Power: Second Stage



(c) Spectral Power: Third Stage

Figure 5.49: Plasma Model 1, Three Stages of Evolution, 15 Modes, Positive Slope

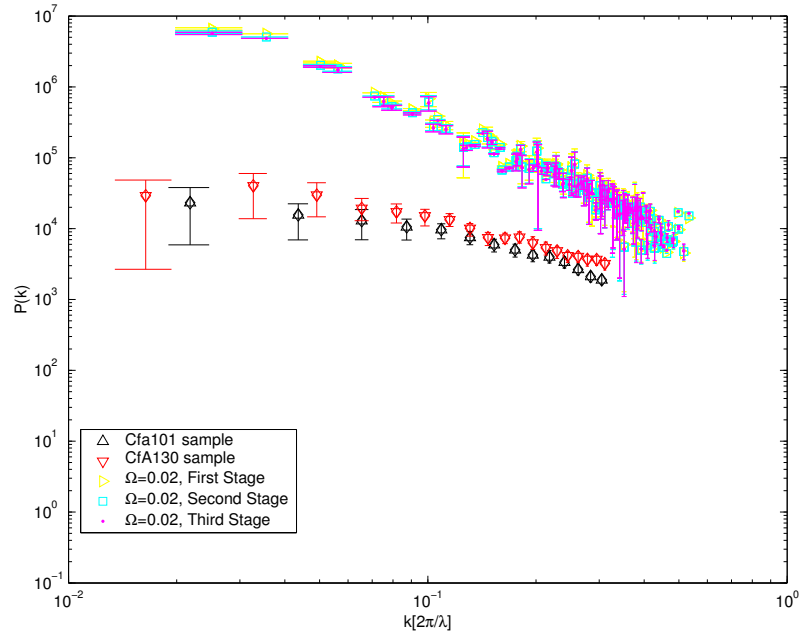


Figure 5.50: Plasma Model 1: Three Stages of Evolution, 15 Modes, Negative Slope

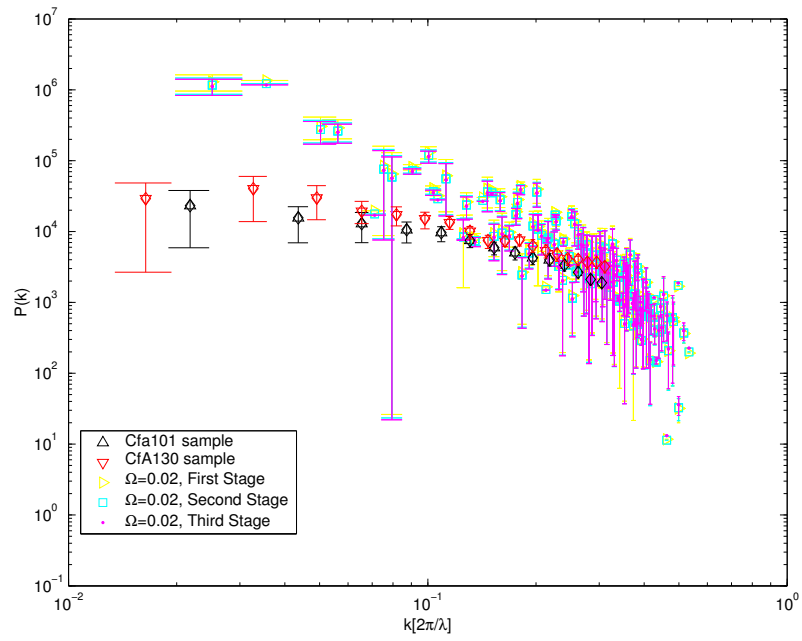
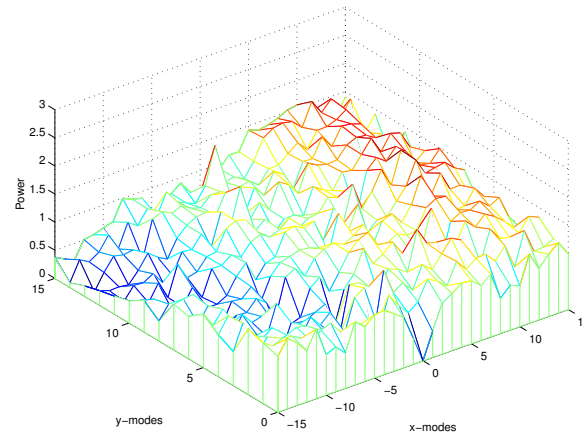
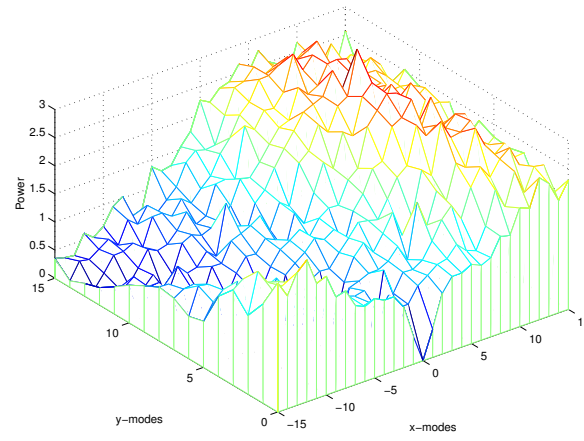


Figure 5.51: Plasma Model 1: Three Stages of Evolution, 15 Modes, Positive Slope

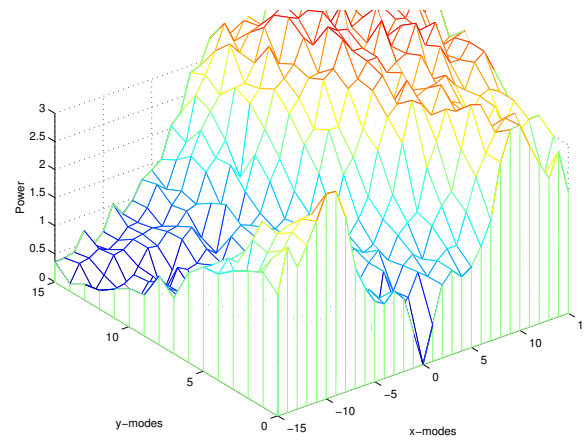
When velocity perturbations are included in the initial conditions, the results, pictured below in figures 5.52 and 5.53 in raw power terms and figures 5.54 and 5.55 as power spectra, display the same effects as the ten mode models. Predictably, the spectra show an upturn in power towards the higher modes. In fact, the power in the higher modes seems to grow significantly as the evolution progresses, transporting power into the larger scales. The negative slope initial conditions have slightly greater magnitude and thus show more pronounced results.



(a) Spectral Power: First Stage

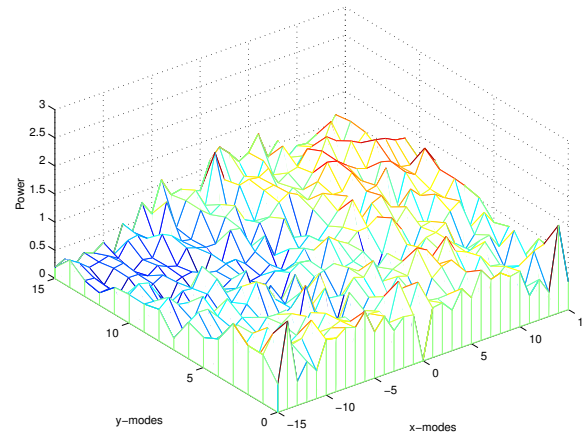


(b) Spectral Power: Second Stage

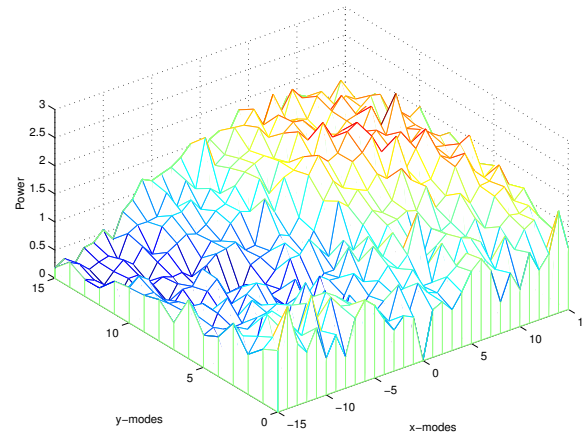


(c) Spectral Power: Third Stage

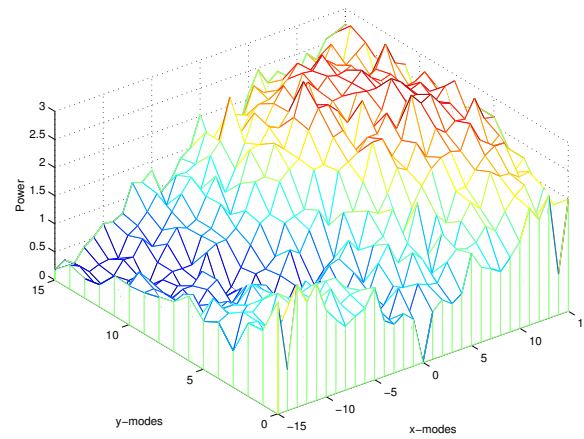
Figure 5.52: Plasma Model 2: Three Stages of Evolution, 15 Modes, Negative Slope, Velocity Fluctuations



(a) Spectral Power: First Stage



(b) Spectral Power: Second Stage



(c) Spectral Power: Third Stage

Figure 5.53: Plasma Model 2: Three Stages of Evolution, 15 Modes, Positive Slope, Velocity Fluctuations

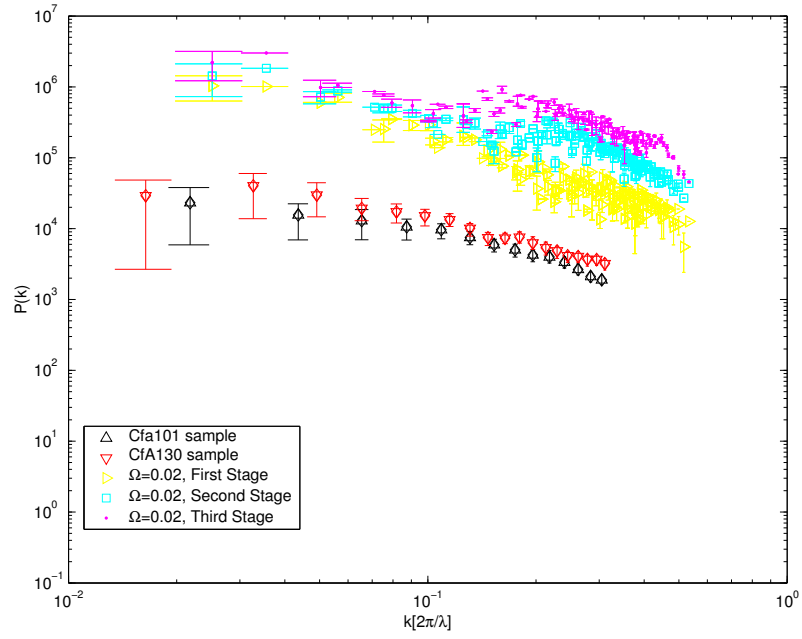


Figure 5.54: Plasma Model 2: Three Stages of Evolution, 15 Modes, Negative Slope, Velocity Fluctuations

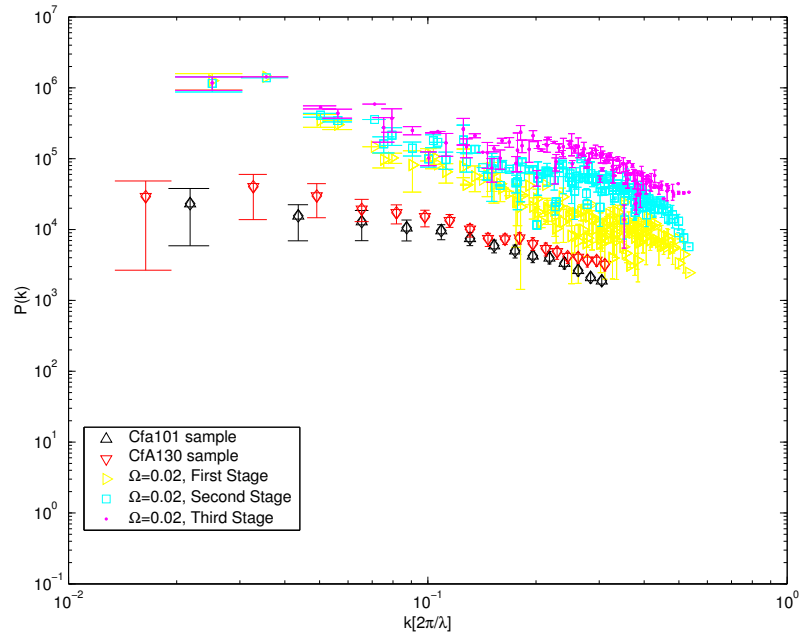


Figure 5.55: Plasma Model 2: Three Stages of Evolution, 15 Modes, Positive Slope, Velocity Fluctuations

Following that, the two-point correlation functions for all initial conditions of the fifteen mode models are plotted. Figures 5.56 and 5.57 show results without velocity perturbations and figures 5.58 and 5.59 include the velocity perturbations. They are similar in magnitude to the ten mode models but just tend to be slightly steeper in gradient. There are minimal differences between the first and last stages of evolution here except perhaps slightly greater clustering in the initial stages in the absence of extra velocity as witnessed with the ten mode models. Similar observations can be made with regard to the comparison of the models with and without initial velocity fluctuations as seen in figures 5.60 and 5.61. Once again there is very little to choose between the two, confirming that any additional clustering that may be present in plasma models is predominantly due to the plasma forces and not additional kinetic energy.

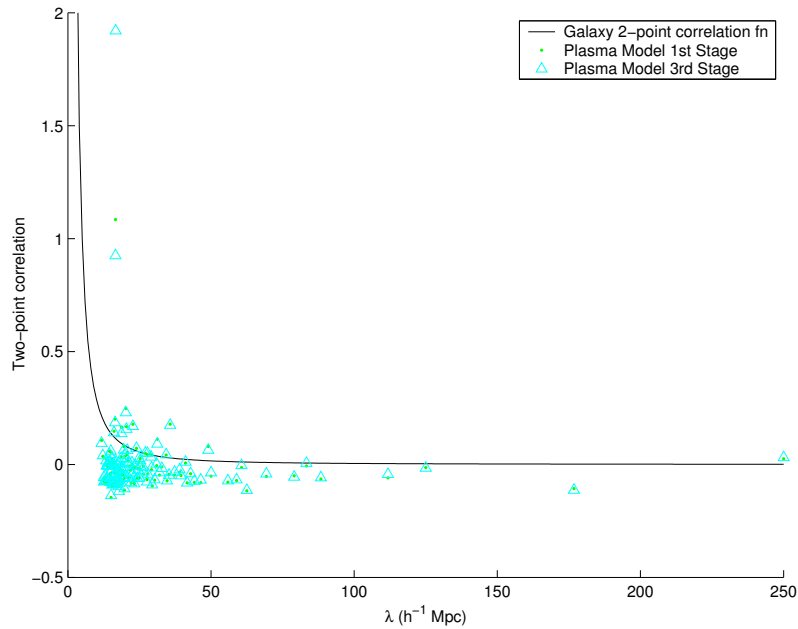


Figure 5.56: Plasma Model 1: 2-point correlation functions, 15 Modes, Negative slope

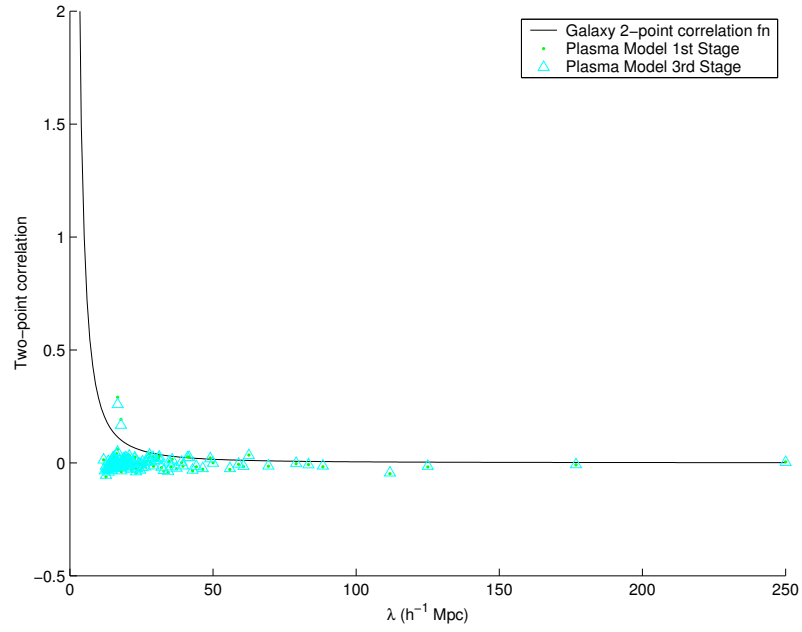


Figure 5.57: Plasma Model 1: 2-point correlation functions, 15 Modes, Positive slope

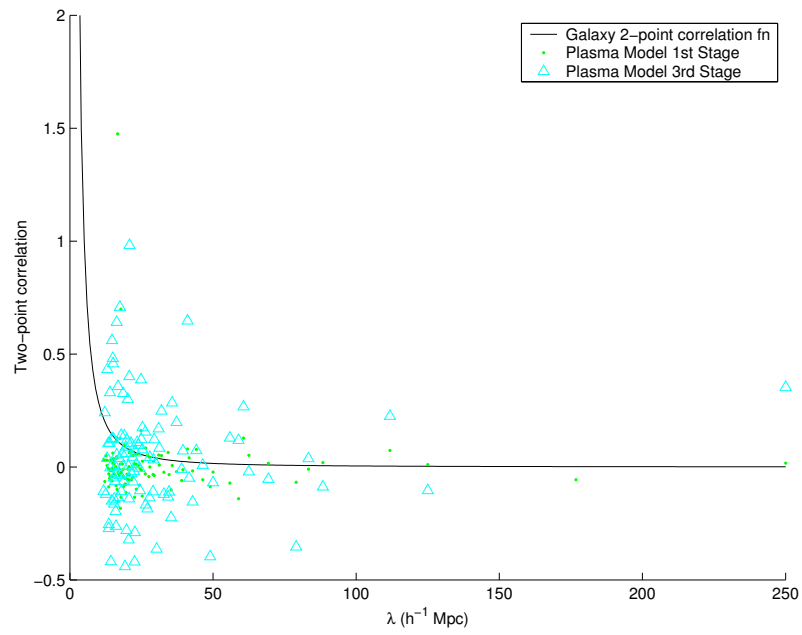


Figure 5.58: Plasma Model 2: 2-point correlation functions, 15 Modes, Negative slope, Velocity Fluctuations

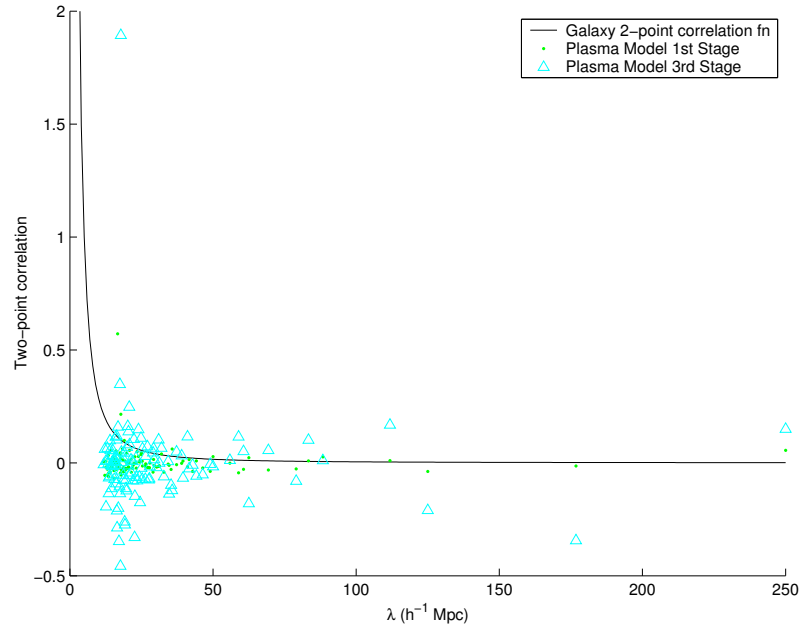


Figure 5.59: Plasma Model 2: 2-point correlation functions, 15 Modes, Positive slope, Velocity Fluctuations

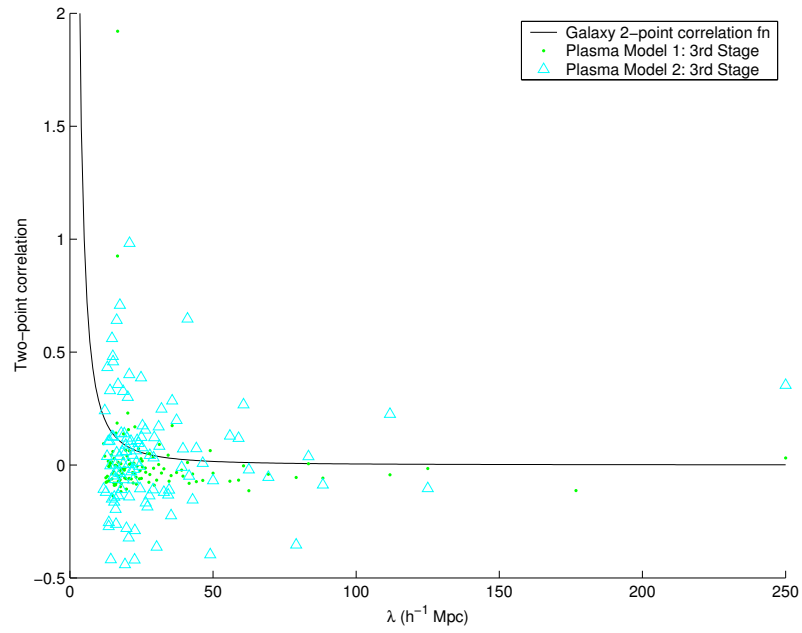


Figure 5.60: Plasma Model 1 vs Plasma Model 2: 2-point correlation functions, 15 Modes, Negative slope, Velocity Fluctuations

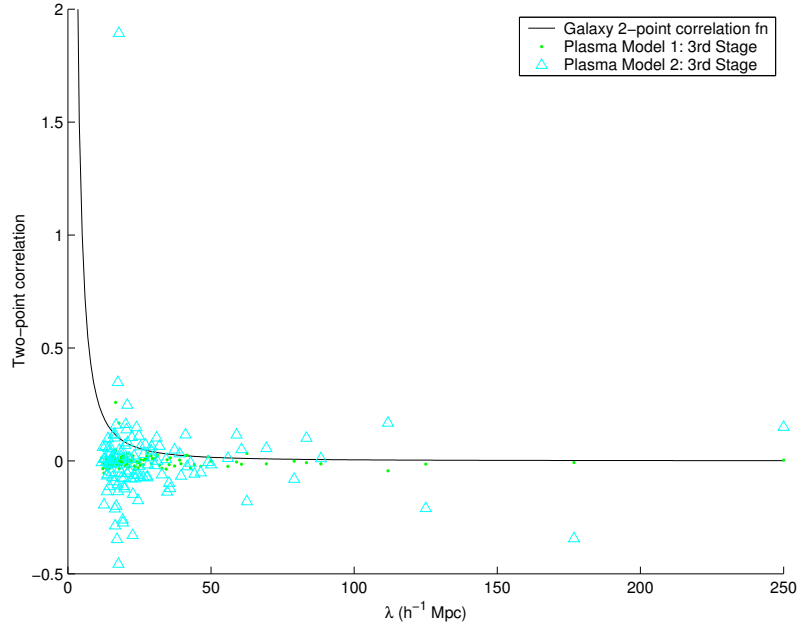


Figure 5.61: Plasma Model 1 vs Plasma Model 2: 2-point correlation functions, 15 Modes, Positive slope, Velocity Fluctuations

The final four graphs (figures 5.62, 5.63, 5.64 and 5.65) contrast the two-point correlation functions of the fifteen mode plasma and gravity models, the latter two having initial conditions with velocity perturbations. In comparison to the plasma model the gravity model shows virtually no non-randomness, once again exhibiting the superior structure-forming potential of plasma forces combined with gravity. This is confirmed in the last two diagrams, which, like their ten mode counterparts, show that plasma forces with lower matter density and velocity perturbations more than match the equivalent gravitational model with the higher matter density when it comes to structure formation and that the extra kinetic energy certainly influences the shape of the power spectra more than the clustering potential.

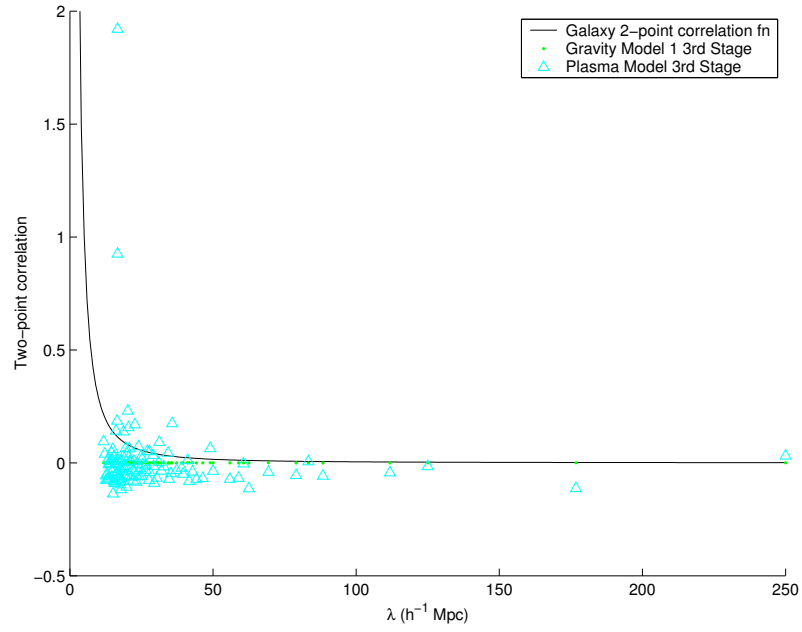


Figure 5.62: Gravity Model 1 vs Plasma Model 1: 2-point correlation functions, 15 Modes, Negative slope

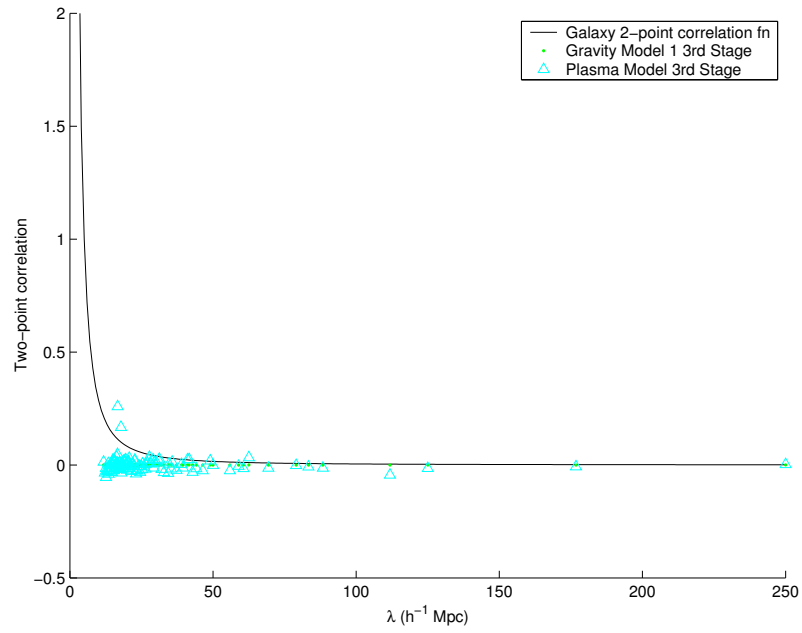


Figure 5.63: Gravity Model 1 vs Plasma Model 1: 2-point correlation functions, 15 Modes, Positive slope

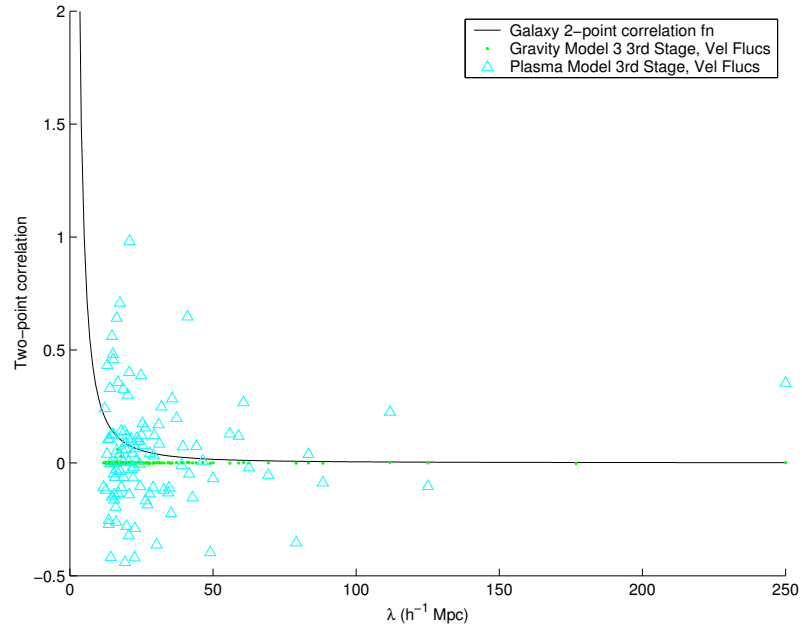


Figure 5.64: Gravity Model 3 vs Plasma Model 2: 2-point correlation functions, 15 Modes, Negative slope, Velocity Fluctuations

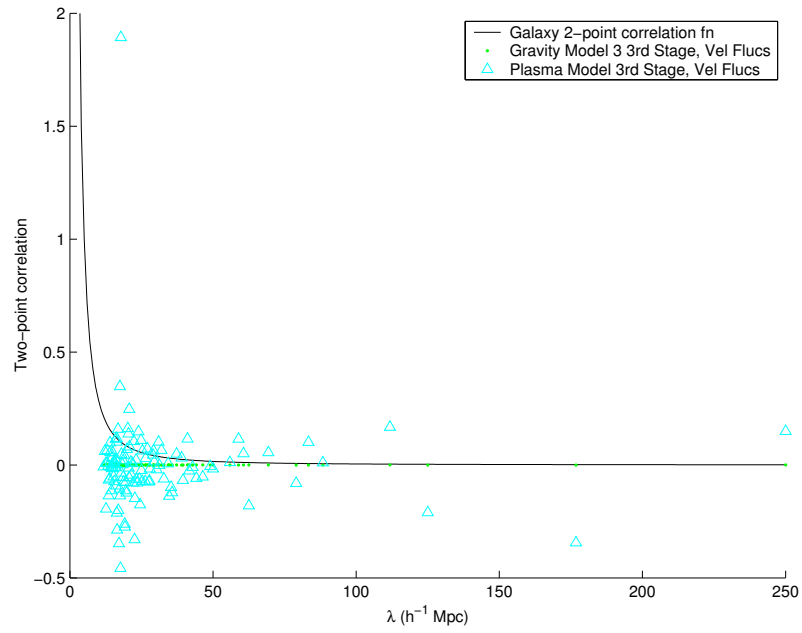


Figure 5.65: Gravity Model 3 vs Plasma Model 2: 2-point correlation functions, 15 Modes, Positive slope, Velocity Fluctuations

6 Conclusions

A computer numerical simulation was developed to establish the anisotropic nature of charged perfect fluids. The idea of using charged fluids was to determine if plasma or electromagnetic forces could create anisotropy in initial Gaussian random fields. This was applied in a cosmological context to the Big Bang cosmology with added plasma forces. It is documented in Chapter 1 that the Big Bang theory has problems reproducing the amount of matter clustering observed in the universe today. Credibility for interstellar and intergalactic plasma has been provided by many scientists in the field of plasma cosmology and thus by adding plasma forces to the existing theory, it was proposed that they could augment the matter structure formation process in the universe.

This model is a relatively simple Newtonian hydrodynamic model in two-dimensional space and makes several assumptions such as ignoring pressure and relativistic effects. Thus it was never attempted to draw serious parallels with real observations although they are added for interest. Rather this model shows that it is feasible to add plasma forces to complement gravity in galaxy clustering and thus possibly anticipates a more realistic - and three-dimensional - simulation where processes could be modelled in greater detail to, subsequently, be compared with observational results. It does this in two main ways. Firstly, the extra energy of the plasma helps prevent matter density and structure decay due to Hubble expansion and maintains power in the power spectrum. The plasma simulations displayed considerably more power than the gravity models over all modes, even after reducing the magnitude of the electromagnetic constants quite considerably to avoid instabilities in the simulations. Secondly the plasma forces show that they can create anisotropy and cluster matter more efficiently than gravity alone, especially at scales of $50h^{-1}Mpc$ and less while gravitational forces appear much more equivalent at scales greater than $50h^{-1}Mpc$. This demonstrates how shorter range structures may be synthesised much quicker with plasma forces and subsequently, through both gravity and long-range plasma, large scale structures could become compositions of the smaller structures, built up in significantly less time than gravity alone - addressing the

main problem discussed in the Introduction.

Several simulations were produced by altering a few key parameters. Firstly, two sets of initial conditions were used - one with a power spectrum approximately linearly proportional to wave number and the other with power inversely proportional to wave number. The former set is the one suggested by cosmologists such as Peebles, Harrison and Zel'dovich [41]. However, in most cases, the differences between the results these two sets of initial conditions produced were seen to be negligible. Both produced very similar spectra after only the first stage of evolution suggesting that the simulations did not find much difference between the two sets. In fact, looking at all the simulations, it is apparent that smaller changes in initial conditions did not make much impression on the simulation results as is evidenced by increasing the average gravity and the changes in slope of the initial values. This would be another area of potential future study - determining the sensitivity of the simulation to the initial conditions. A further parameter that was varied was the number of modes, from ten to fifteen. The two sets of simulations produced largely the same results with some more detail emerging from the larger number of modes.

A remarkable observation that warrants further research is the shape of the resultant power spectrum from the initial conditions with velocity perturbations - both gravity and plasma. Without the velocity perturbations there is an abundance of small scale power but too little large scale power. Velocity fluctuations provide this missing power. There is a fairly distinct bend in the 15 mode power spectrum at a certain scale which is confirmed in Padmanabhan [52] by galaxy surveys. The location of this bend according to Padmanabhan ($\approx 21h^{-1}Mpc$) was also reasonably reproduced in these particular simulations at a scale of approximately $30h^{-1}Mpc$.

Overall, the simulations where initial velocity perturbations were included in the initial conditions matched the observations the best. The main difference between the gravity and plasma simulations was power magnitude. Gravity models without modal kinetic energy dissipated substantially over time whereas the plasma models maintained their power very well throughout with only slight energy decreases. Gravity Model 3 clearly stands out from the other gravitational models and looks more like a plasma model. However, despite its additional kinetic energy it also succumbs to the Hubble expansion more than plasma models. As a result it develops a slightly undulating power spectrum, meaning the plasma models fit the observations marginally better in terms of shape. In addition to this, the plasma models would build and maintain cosmic structure much more efficiently than pure gravitation alone, and together with appropriate initial velocity values, provides a reasonable match for real observations, even with such a simplistic model.

It is envisaged that this “hybridisation” can benefit cosmology, even taking into consideration the breakneck speed at which new discoveries are occurring. There are two major relatively recent discoveries. One is the study of distant supernovae which seems to indicate that the universe is expanding at an increased rate in these areas, contrary to the idea that gravity is slowing the Hubble expansion [47]. The other is a more detailed study of the cosmic background radiation by the BOOMERANG project [46]. This study reveals a peak in the power spectrum of the radiation that indicates a flat universe or that $\Omega = 1$. Both these discoveries require further verification but both hint at an excess of unknown energy that may satisfy these observations. It is not unrealistic to propose that plasma forces may either be, or more likely compose a significant part of, this missing universal energy that has not really been taken into account yet in the framework of the Big Bang. This work shows there is a reasonable basis for this conclusion.

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