Developing practices of mathematics teachers through a Professional Learning Community

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A thesis submitted to the Faculty of Humanities, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements of the degree of Doctor of Philosophy

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……..…… ……...

Nicholas Molefe

…….8th day of March 2022……...

Abstract

Professional learning communities (PLCs) are investigated as an approach to in-service professional development of teachers. The common argument for professional learning communities is that they are long-term, sustainable, and involve collaborative processes. This study explored how mathematics teachers learned as a professional learning community as they engaged with various activities, and how their engagement contributed to their knowledge base and shifts in their teaching practices. The teachers' learning happened under the guidance of a university-based facilitator, who is the author of this thesis and the researcher of both the PLC and his facilitation.

This study was part of a bigger project, the Data Informed Practice Improvement Project (DIPIP) based at the University of the Witwatersrand. The study was influenced by a situated perspective and data-informed practice. The case study involved a total of seven mathematics teachers from four schools in Gauteng, South Africa and the researcher working together as a professional learning community for a period of two years. Data was generated over a period of two years and comprised classroom observation of four mathematics teachers and community data of all the seven mathematics teachers' participation in the PLC. Analysis focused on the shifts in the teachers' practices, how they talked about the learners' errors and their practices, the kind of knowledge created during their conversations, and how the facilitator guided the process.

The results show changes in the teachers' practices and their participation for all the teachers. Two teachers managed to sustain the shifts in their teaching practices over time and the other two did not. The teachers' participation in the PLC show that their conversations in the community raised opportunities to contribute to their PCK in various ways over time. The development of PCK was mostly enhanced by disagreements among the teachers; the community taking up a teacher's input to create a rigorous conversation; and probing opportunities that emerged in the community. The results also show that there was less focus on the misconceptions underlying the learners' errors. The analysis details the kind of support that facilitation gave to the teachers, and reflects on the missed opportunities during facilitation, which occurred as a result of the facilitator's dominance in the community due to his enthusiasm in wanting to see the project succeeding. These missed opportunities afforded the

facilitator the space to reflect on and think deeply about how else he might have supported and guided the teachers.

The analysis results have shown that participation of the teachers in the PLC enhanced teacher ownership through reflective practice. As teachers inquired into their classroom data, they developed ways of taking ownership of artefacts of practice and the community. This study has also shown the importance of good facilitation moves, which analysis has shown that they can provide important guidelines on PLC facilitation.

Key words

Professional Learning Community, Teacher Conversations, Facilitation

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Abbreviations

DBE Department of Basic Education
DoE Department of Education
DIPIP Data Informed Practice Improvement Project
TIMMS Trends in International Mathematics and Science Study
ICAS International Competitions in Assessment of Schools
MLA Monitoring Learner Assessment Study
IEA International Association for the Evaluation of Educational Achievement
SACMEQ Southern Africa Consortium for Monitoring Educational Quality
HSRC Human Sciences Research Council
NGO Non-governmental organisation
PLC Professional Learning Community
PD Professional development
PCK Pedagogical content knowledge
CK Content knowledge
STAAR Supporting the Transition from Arithmetic to Algebraic Reasoning
SP Senior Phase
FET Further Education and Training
HOD Head of Department
BA (Ed) Bachelor of Arts in Education
PTC Primary Teachers Certificate
SEC Senior Education Certificate
SED Senior Education Diploma
BSc Bachelor of Science

HDip Ed	Higher Diploma in Education
STD	Senior Teachers Diploma
B. Tech	Bachelor of Technology
ICCAMS	Increasing Competence and Confidence in Algebra and Multiplicative Structures
AS	Assessment Standard
NLC	Networked Learning Community
IPL	Individually Planned Lesson
JPL	Jointly Planned Lesson
MQI	Mathematical Quality of Instruction
IRE	Initiate-Respond-Evaluate structure

Chapter 1: Introduction

1.1 Background to the study

Performance of learners in mathematics in South Africa is poor, and has been for over twenty years, despite the fact that a new curriculum has been in place for more than twenty years since the advent of a democratic dispensation (Botha, 2016; DBE, 2017; Taylor, 2008). Before 1994, poor achievement was exacerbated by a system of education that was intended to enforce a difference in learner performance between black and white learners (Jansen & Christie, 1999). The new democratic dispensation from 1994, ushered in transformation in education through the Department of Education (DoE), whose intention was to help improve learner achievement (DoE, 1997, 2001, 2003). Despite this transformation, there remains a national outcry every year about learners' mathematics results in the country as these results continue to disappoint. Institutions of higher learning complain about their learner intake from high schools every year because of the learners' poor mathematics results and the performance of learners in their entry level programmes (De Clercq, Galand, & Frenay, 2017; Eddaif et al., 2017; Frick, 2008; Lemmens, 2010; Vasuki, 2009).

South African learners continue to perform badly in international standardised tests such as Trends in International Mathematics and Science Study (TIMMS), International Competitions in Assessment of Schools (ICAS), Monitoring Learner Assessment Study (MLA), and the Southern Africa Consortium for Monitoring Educational Quality (SACMEQ) (Beaton & Robataille, 1999; DBE, IEA, & HSRC, 2015; Howie, 2003; Taylor & Vinjevold, 1999). Recent results on TIMSS 2019 showed that there was improvement in South African learners' performance in mathematics. However, general performance was still below both the benchmark of 550 and the set target of 420, which has been the case over the years that South Africa had participated. Explanations for the continuing poor results often focus on: resources in schools, teacher shortages, learner socio-economic status, overcrowding in schools and homes, teacher unfamiliarity with the curriculum and instructional practices (Graven, 2014; Taylor, Muller, & Vinjevold, 2003).

1.2 Rationale for the study

Research has shown that improved instructional practice is key to improved learner performance (Roberts & Pruitt, 2003). Part of the transformation agenda in education since 1994 has been teacher development that is intended to empower teachers, who are central in bringing improvement in learner achievement. Teacher development in the new curriculum is often offered by the Department of Education (DoE) as well as some non-governmental organisations (NGOs). However, some of these professional developments have not brought improvement in learner achievement as South African learners still continue to perform badly in mathematics (Beaton & Robataille, 1999; Howie, 2003; Taylor & Vinjevold, 1999).

Since the inception of new curricula in the post-apartheid era, the training of teachers in preparing them for new instructional practices is still a cause for concern (Sethole, 2004). These trainings are often not substantive, as they are of short duration and use a one-cap-fits-all approach and their content is often imposed without addressing the real needs of teachers. In some cases, specific teachers are identified so that they become what are known as lead teachers. These lead teachers are then taken through one-day training, usually on a Saturday, on particular content, and the following Saturday they are distributed across various districts to train the general teacher population on the same content. The possibility of shifting many teachers' practices through these kinds of in-service programmes is limited. Therefore, there is a need to develop and research in-service programmes which might substantially shift teachers' practices (Scribner & Cole, 1981) and consequently improve learner performance. The quest for developing and researching professional knowledge and collaborative approaches to instructional practice is therefore of increased importance.

Internationally, learner achievement is often used as an instrument to measure school performance (Earl & Katz, 2006), and teachers find themselves directly accountable for learners' achievement. Teachers are under tremendous pressure to offer quality instruction in their classrooms (Boudett, City, & Murnane, 2008). Katz, Earl & Ben Jafaar (2009) articulate that this accountability "depends on there being people in schools with the knowledge, skill and judgment to make the improvements that will increase learner performance" (p. 15), what Darling-Hammond (1989) calls professional accountability, rather than bureaucratic accountability. Teachers' roles in relation to learner performance and professional

accountability for their performance necessitate the existence of good professional development programmes.

Another related concern is that despite all the effort taken for teacher development by the DoE and NGOs, many teachers still spend most of their time working as individuals (Horn, 2004; Little, 2002). Many teachers often don't work collaboratively partly due to lack of sustainable intervention programmes supporting collaboration. Where opportunities for working as groups are created, there is often no attention given to sustainability of such opportunities and teachers often go back to their initial practices in the classroom – they go back to the same school, same classroom, same learners and same conditions of teaching and learning (Earl & Katz, 2006). Sustainable programmes for collaborative teacher development are therefore needed for improved teaching and learning.

Local and international research suggests professional learning communities (PLCs) as a possible model to address the problems stated above: unsustainability of teacher professional development programmes, teachers working primarily as individuals, limited professional accountability, needs for improved instructional practices and weak learner performance (Brodie, 2013; Brodie & Borko, 2016; Cereseto, 2016; Chauraya & Brodie, 2017, 2018; Koellner-Clark & Borko, 2004a). The purpose of this research was to investigate whether and how a functional PLC could be established in a school(s) and whether and how that PLC could shift the teachers' practices in mathematics[1]. In addition, this study looked at how facilitation of a PLC could assist participants within a PLC to inquire into their practices by collaboratively sharing and having conversations about their classroom data.

1.3 What are professional learning communities?

There are a number of definitions of PLCs, which include: "a professional community where teachers work collaboratively to reflect on their practice, examine evidence about the relationship between practice and learner outcomes, and make changes that improve teaching and learning for particular learners in their classes" (McLaughlin & Talbert, 2006, p. 3), and "a group of connected and engaged professionals who are responsible for driving change and improvement within, between and across schools that will directly benefit learners" (Harris &

[1] Investigating learner performance was not part of my study as it would make the study too large.

Jones, 2010, p. 173). Most of the definitions focus on collaborative work, long-term inquiry, reflective practice and changing practices, for the benefit of learners. Feldman (2017) argues for "PLCs that place teacher professionalism, collaborative learning and a more enriched pedagogical approach to student learning, as central to the on-going dialogical PLC process" (p. 77). Robutti et al. (2016) assert that "across education systems, mathematics teachers work and learn through various forms of collaboration, which contribute to learning and development in differing ways" (Robutti, et al., 2016, p. 651). This study focuses on a PLC as an example of teachers' collaborative work to show the extent to which mathematics teachers collaboratively inquire into their practice in order to learn through that collaboration.

From a research perspective, there are two forms of PLCs that can be established in a school. First, there can be a PLC that focuses on whole school development where everyone, including the principal and all the teachers, is involved. In this type of a PLC, school improvement becomes everyone's business in the school where all subjects and issues of administration are looked at. The other type of a PLC is one that focuses on a specific subject in a school and works towards teacher development in that subject as they professionally learn together in a community. This research investigated the latter type of a PLC, where mathematics teachers in a school worked collaboratively to learn from and with each other to try and improve their practices. The focus of this kind of a PLC then becomes how teachers collaboratively engage in activities with the hope of improving their instructional practices.

This study acknowledges that over the period that it took to produce this thesis, the field on collaborative teacher professional development has grown substantially (Robutti, et al., 2016). A discussion on this development is detailed in the next section as well as in Chapter 2. Developments in the local context regarding the establishment of functional collaborative teacher communities continue to be limited.

1.4 How PLCs operate

A report by Te Maro, Averill, Higgins, & Tweed (2008) on participation of Maori mathematics teachers in a professional development (PD) project provides a relatively good example of how functional learning communities operate. The report focuses on three aspects: the gains and constraints that were experienced in the project; supportive structures for the project; and how a facilitator encouraged and developed networks amongst teachers.

The gains experienced in the communities included increased opportunities for pedagogical and content knowledge development. This knowledge development was enhanced through face-to-face meetings with the facilitator, reflections on the videotaped lessons, teachers taking responsibility for identifying and making changes to their practices, continuing professional development within and across clusters, and shared ownership of internet-based communication which allowed instant access and meetings when necessary. Dialogue was promoted in the project as a means to "challenge beliefs and support the efficacy of competing ideas" (Te Maro, et al., 2008, p. 35), and developing teacher knowledge and practice within their networks. The project afforded participants opportunities to "work together to evaluate their practice, problem-solve issues, and collectively plan for mathematics teaching" (p. 41).

Te Maro, et al. (2008) also describe factors that impacted negatively on the communities. These factors related to a range of school activities like sports, teacher isolation, national assessment that put pressure on teachers, availability and stability of staffing and movement of teachers between schools due to transfers and promotions. Other constraining factors related to school events such as camps, lack of established networks, and a small number of people who supported teachers in their roles. The facilitators in the project also noted the issue of time available to manage more than one cluster as a challenge. These challenges have a potential to disturb continuity and sustainability of a learning community.

Findings from the same project showed that there was a range of supportive structures for the project that existed within the schools and encouraged participation of teachers in their professional development. Those factors related to support systems that facilitated the professional development: support from school managers, boards of trustees, and learners' families. Discussions with school principals were used to promote the functionality of the networks. These support systems were enablers for teacher attendance of community meetings and adjustment of timetables when necessary.

Teachers also identified characteristics of the facilitator which supported the development of their content knowledge and teaching practice. These were the facilitator's humility, the creation of authentic relationships, time spent with the principal during school visits, knowing when to participate and when to step back, having the knowledge to ask and answer the right questions; and enhancing teacher ownership of what they learned. These characteristics put emphasis on the role that a facilitator of a PLC has on the functionality of a PLC.

Te Maro, et al. (2008) recommend that professional development be a long term project, which is also a premise of this study. This study was conducted over two years, focusing on the relationships between learners' learning needs and teachers' learning needs and how these inform the discussion and learning in the PLC (Boudett, et al., 2008; Brodie, 2013; Katz, et al., 2009). Figure 1.1 below shows this relationship.

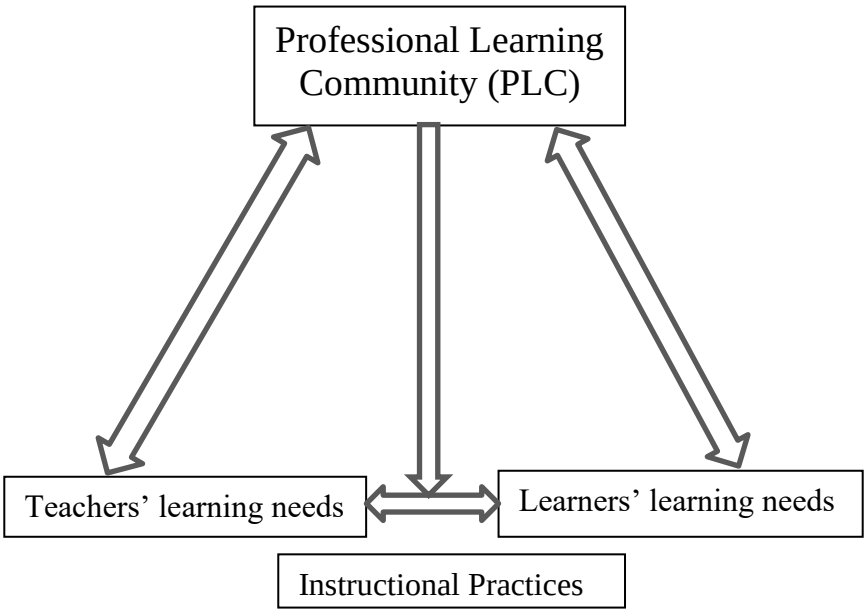

Figure 1.1: Teachers' learning needs and learners' learning needs

For teachers to work on their learning needs, it is important that they understand what their learners' learning needs are. A PLC provides a forum for teachers to make their (teachers) learning needs known to their community and talk about them in order to establish what informs those needs and how to go about addressing them. This means that the PLC conversations need to be deep enough in order to reveal what the learning needs of the teachers and of the learners really are. Learners' learning needs are closely linked to what they learn in the classroom, which makes classroom data key for professional development of teachers. When teachers bring classroom data to the PLC for inquiry, they contribute to the knowledge base of the community and the school. The process is further enhanced when the participating teachers try out in their classrooms ideas that they learned through their PLC conversations and subsequently come back to the community for reflection (Kazemi & Hubbard, 2008). Professional learning is thus a result of teachers engaging with the PLC activities in their learning community, engaging this learning in their classrooms and reflecting on their classroom practice in the PLC. This learning emphasises strong collaboration between and among teachers within a learning community.

Vescio, et al. (2008, p. 81) discusses five important characteristics of PLCs:

- The development of shared values and norms regarding views about learners and the learners' ability to learn
- Clear and consistent focus on student learning
- Reflective dialogue – leading to extensive conversations about curriculum, instruction and student development
- Deprivatising practice to make teaching public
- Focusing on collaboration

Vescio, et al. (2008) assert that the above characteristics are critical in "promoting and sustaining professionals in a school community with the collective purpose of enhancing student learning" (p. 81). The above characteristics are not easy to achieve and research identifies challenges that schools may face in establishing successful and sustainable PLCs (Cormier & Olivier, 2009, p. 20). These challenges primarily include:

- Development and application of shared knowledge;
- Sustaining the effort and function of the PLC beyond its initiation stage; and
- Attempting to transform school culture from an autonomous mindset to one of shared values and collaboration.

Given the promise of PLCs and the challenges in establishing and sustaining them, this study set out to understand how a PLC of high school mathematics in South Africa would experience both success and constraints in working in a PLC.

1.5 Purpose Statement

The purpose of this research was to explore the workings of a Professional Learning Community (PLC) within a school-based professional development programme that focused on mathematics teaching and learning. The PLC was part of a larger project, the Data Informed Practice Improvement Project (DIPIP), which aimed to develop teachers' pedagogical content knowledge in relation to the reasoning underlying learners' errors (Brodie, 2014, 2019; Brodie & Chimhande, 2020; Brodie, Molefe, & Lourens, 2014; Chauraya & Brodie, 2017; Molefe, 2016).

The primary objective of the PLC that this study researched was to support teachers as they worked with classroom data in novel ways in order to improve their practices. This support included guiding the teachers to embrace learners' errors and misconceptions in mathematics teaching and learning, and for teachers to view those learners' errors as being reasonable and reasoned (Ball & Bass, 2003; Brodie, 2013, 2014). For teachers to embrace the learners' errors, they needed to see learners as reasoning and reasonable thinkers. The study was premised on the notion that errors are a result of "a consistent conceptual framework based on earlier acquired knowledge" (Borasi, 1987, 1994; Brodie, 2014; Clements, 1980; Nesher, 1987) and make sense to learners in terms of their current thinking. Smith, diSessa & Roschelle (1993) explain a misconception as "a learner's conception that produces a systematic pattern of errors" (p. 119). This explanation helps in understanding misconceptions – that misconceptions are not simply incorrect concepts that do not a have place in the learners' learning but show the partial validity of learners' thinking and therefore can be used to improve teaching and learning. Misconceptions provide evidence that learners have learned something and what they learned cannot be easily ignored as it makes sense to them, and often is partially valid mathematically.

Understanding the reasons for learners' errors is a key element of teachers' pedagogical content knowledge (PCK). Shulman (1986) states that PCK includes "the most useful forms of representation of topics, the most powerful analogies, illustrations, examples, explanations, and demonstrations; the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Teachers' discussions of their practice afford them opportunities to learn about their learners' thinking, their learners' learning needs and their own mathematical content knowledge (Brodie, 2013, 2014).

1.6 Research Questions

There were two main questions that this research sought to address.

1. To what extent do mathematics teachers' practices shift as they participate in a PLC?
2. How do mathematics teachers work as a professional learning community to learn with and from each other as they work on learners' errors and misconceptions?

The main questions were elaborated by the following sub-questions:

1.1 How do teachers engage with the learners' errors in their classrooms?
1.2 How do teachers create conditions for enhancement of learning in their classrooms?

2.1 What do teachers in a PLC talk about and at what depth do they engage with different activities?
2.2 How is pedagogical content knowledge (PCK) amongst teachers created within the professional learning community?
2.3 How is a space for engaging with the different activities in a PLC created by the facilitator?

A key focus of this study was the teachers' conversations about learners' errors and misconceptions in a PLC (Choppin, 2011). This study explored how teachers reflected on their practices and how their reflections demonstrated what they learned from the PLC conversations regarding their practices. Talking about what happened in their classrooms and in the PLC may support teachers to support each other's professional development. This development is informed by what and how teachers continuously learned from working with their learners in the classrooms and what and how they learn from analysing their classroom data in PLCs. The analysis within a learning community helps teachers to learn about their learners' thinking and what they need to learn themselves from this understanding, including how to think about what works and what does not work for their instructional practices.

1.7 Significance of the study

Findings from this study are significant in various ways. First, the findings will show how participation in a professional learning community can support teacher learning (Brodie, 2016) and the contribution that this learning can have in the teachers' instructional practice. One of the aims of a PLC in a school is to help discern practices that teachers need in order to meet the learners' learning needs. Teachers come to learn of their practices at their own schools or sites without standard prescriptions of how their practices should shift. The teachers' shifts are therefore informed by the professional learning that happens within a PLC. PLCs go beyond mere acquisition of knowledge and skills for teachers (Vescio, et al., 2008) – teachers can use

PLCs as models for inquiring into their own practice and identifying areas that need attention in their practice. Research that shows whether and how the PLC model contributes to teaching practice can add to a much needed gap in the research – there is not much current research on PLCs that shows shifts in teaching practice (Hairon, Goh, Chua, & Wang, 2017; Vangrieken, Meredith, & Kyndt, 2017).

Second, my analysis will show the extent to which the teachers' talk about the learners' errors and misconceptions deepened their pedagogical content knowledge. This study investigated the extent to which the teachers' engagement with the different activities in the PLC focused on content and how those conversations deepened in various ways. My analysis will show how the teachers talked about the learners' errors; the content that their talk focused on and at what depth; the shifts that occurred in the teachers' participation; and the contributions that their talk made to their PCK in various ways. The analysis results will show how the knowledge that is created in the community connected to the teachers' instructional practices.

Third, the analysis results from this study show the key role that facilitation, as leadership in learning communities, plays in the teachers' conversations within a professional learning community. The facilitator is regarded as a 'critical friend' in a learning community (Katz, et al., 2009), which necessitates the dual role of a facilitator: that of being seen as a friend in the learning community, but also being able to critique the teachers' participation in the community, especially assumptions that need to be professionally challenged. Facilitation of a PLC is not easy, and the findings from this aspect of the study may encourage further research in this area and help support PLC facilitators in their work.

1.8 Overview of the study

In this chapter I focused on the rationale as well as the background of the study. I also discussed the statement of the problem and the associated research questions that guided this study and the different ideas that illuminated how this study could contribute to the body of knowledge on professional development of mathematics teachers and professional learning communities. In the next chapter I focus on the theoretical and the conceptual frameworks for the study, and also review relevant literature linked to this research study. In Chapter 3 I describe the research design methods used for this study, including discussions on the research participants, the

research sites, data collection, coding and analysis procedures, ethical considerations as well as the attainment of rigour in this study.

In Chapter 4 I present the results on the shifts identified in four of the participating teachers' practices. I discuss how the participating teachers worked with learners' errors and how this contributed to their practices. In Chapter 5 I present the results pertaining to the conversations that the PLC had in the different activities of the project, and the extent to which these conversations on content and depth of the conversation had on all the seven the teachers' participation – how the PLC talked about learners, mathematics, learner thinking, and their own practices. In Chapter 6 I present the analysis of my facilitation of the professional learning community and how this facilitation enhanced or inhibited the teachers' participation and consequently their practices. Chapter 7 summarises the results of this research, answers the research questions, presents the limitations and implications of the study and proposes areas for future research.

CHAPTER 2: Theoretical Perspectives and Review of Literature

2.1 Introduction

In this chapter I discuss the theoretical perspective within which this study is located, drawing from Wenger's (1998) theory of learning as a social practice. The theoretical perspective is appropriate because my focus is on investigating the practices and participation of the mathematics teachers as they learn in two contexts: the professional learning community and their classrooms, and how working between the two contexts supports their professional development.

To frame my study, I draw on different concepts in relation to professional learning communities: teacher conversations; teacher participation and learning; teacher knowledge; instructional practice; and facilitation. In my discussion of these concepts, I focus on the critical role that teacher conversations play in the PLC to inform the teachers' ongoing participation and learning in the PLC and also their instructional practices. Since research argues for a "clear and defensible focus" (Katz, et al., 2009), I also elaborate on the role that a PLC facilitator can play to improve focused conversations in the PLC and the creation of new knowledge for the teachers.

This chapter also reviews literature that I found relevant to this study, and I draw from this literature to make links to what this study sought to understand about professional development of mathematics teachers. In doing so, I look at what research literature says about how teachers participating in professional learning communities can learn as a community by engaging in conversations that can shift their instructional practices. In addition, I look at the extent to which teachers' knowledge can be developed as they participate in PLCs, and how a facilitator can work with teachers to achieve these goals.

2.2 Theoretical perspective

The study was guided by the theory of learning as a social practice (Wenger, 1998), and this theory is located within situated perspectives, which regard learning as participation in communities of practice (Borko, 2004; Borko & Koellner, 2008; Horn, 2005; Kazemi & Hubbard, 2008; Lave, 1993, 1996; Lave & Wenger, 1991; Wenger, 1998). The nature of

teacher professional learning communities as established in school contexts was a strong motivation for selecting this theoretical perspective. Teacher learning in this context is situated in professional learning communities and practice, and increased learning is viewed as becoming a better participant in communities and practice (Borko, 2004).

Borko's (2004) work on helping teachers to "enhance their knowledge and develop new instructional practices" (p. 3), and Kazemi & Hubbard's (2008) work on learning being situated between two environments: the community and the classroom, are the key ideas underlying this research study. Borko's (2004) ideas resonate with Adler's (2000, p. 37) in viewing teacher learning as "a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching". According to Kazemi & Hubbard (2008), "learning to improve teaching entails developing both knowledge and knowing", where knowledge is viewed as that which is possessed and knowing as that which is deployed in action (p. 429).

Supporters of situated perspectives argue that since learning occurs in many situations: in classes, in discussions with other colleagues and individually, such as teacher planning, for teacher learning to be understood it must be studied within these multiple contexts (Peressini, Borko, Romagnano, Knuth, & Willis, 2004). This study explored teacher learning within two contexts: the PLC and the classroom. It is assumed that teachers engage with community ideas individually and communally, in the PLCs and in their classrooms. These contexts are interrelated, and teachers can draw from these contexts as they engage in various activities that contribute to their learning.

A situated perspective provides us with opportunities to focus on "the practices and participation structures of the situations in which knowledge is used" (Peressini, et al., 2004, p. 82). It is important to understand how teachers learn from each other's practices as they engage in conversations about the curriculum and about mathematics; to understand the practices that are enacted in class when teachers teach their mathematics lessons; and to understand the distinction between the different contexts and how each contributes to teacher learning.

2.2.1 A situated perspective on teacher learning

Situated perspectives view learning as participation in communities of practice (Borko & Koellner, 2008; Lave & Wenger, 1991). The unit of analysis in a situated perspective is the social collective (Peressini, et al., 2004), which differs from cognitive perspectives' arguments that learning is an individual's acquisition of knowledge and can be independent of the context in which it occurs (Anderson, Reder, & Simon, 1997). A situated perspective is derived from an assertion that learning is situated (Greeno, Collins, & Resnick, 1996; Timperley, 2009) – how a person learns and the situation in which a person learns form important parts of that learning (Peressini, et al., 2004). Teacher learning thus becomes a process of participation in communities, and better participation in their communities, including the classroom, is increased learning (Horn, 2005; Wenger, 1998).

According to Peressini et al. (2004, p. 69), "for teachers, learning occurs in many situations of practice": university mathematics courses, teacher-preparation courses, field experiences, and schools where they are employed. Situated perspectives argue that teacher learning should be studied within multiple contexts if it is to be understood well. In the process of learning within communities, it is important that participants should not only support each other's ideas, but rather challenge each other's ideas when necessary (Brodie & Shalem, 2011). Males, Otten, & Herbel-Eisenmann (2010) refer to this interaction among participants as "critical colleagueship". The notions of situated learning and critical colleagueship provide sound meaning to professional learning within a community of practice.

Grossman, Wineburg, & Woolworth (2001) argue that there has been widespread use of the term "community", wherein different kinds of teacher groups were seen as communities, with little emphasis or clarity on learning being the goal in those communities. More clarity is thus needed to understand what constitutes a learning community, which is what this research study sought to explore.

2.2.2 A situated perspective on teacher knowledge

Situated perspectives argue that knowledge cannot be seen as owned by an individual but rather is found in the interaction between people and activity (Greeno, 1997; Mousley, 2003). Teachers can therefore develop knowledge when they interact with each other. It was important

for this study to illuminate what constitutes teacher knowledge and how this knowledge is created and recreated through conversations within a PLC and becomes part of classroom practice. The new knowledge that teachers create as a result of their participation in learning communities is often derived from the teachers' own classroom experience and their learning communities, and this knowledge is usually meaningful to teachers (Calderhead, 1988; Cochran-Smith & Lytle, 1999).

Several scholars have identified the kinds of teacher knowledge that matters for instructional practice: subject matter knowledge, pedagogical knowledge, and pedagogical content knowledge (Ball, Thames, & Phelps, 2008; Hill & Ball, 2004; Hill et al., 2008; Shulman, 1986). This study's focus on teachers' practices and participation, and how they engaged with learner errors necessitated a focus on teachers' mathematical knowledge that is not generic, but situated within the teachers' collaborative inquiry into their practices. This study therefore focused on how participating teachers could work with classroom data to build on their pedagogical content knowledge through the different activities that they engaged with in the PLC.

Engagement with the PLC activities helped illuminate the depth at which the teachers' understanding of the content embedded in the activities could be developed and the extent to which teachers could make this content comprehensible to the learners. Shulman (1986) describes this comprehensibility as pedagogical content knowledge (PCK), which this study investigated through the teachers' participation in the PLC. Shulman (1986) states that PCK includes "the most useful forms of representation of topics, the most powerful analogies, illustrations, examples, explanations, and demonstrations; the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). The depth of the conversations in the PLCs determines the richness of the knowledge base within a community. It is these conversations that inform on the extent to which teachers can share ideas; how they can be open in their talks; and how they can collectively develop their PCK as a community. Good conversations are important for shaping the teachers' PCK, which they need for improving teaching and learning in a classroom. PLCs afford opportunities for teachers to have a deeper understanding of why certain mathematical procedures work, what those procedures mean, and how generalisable those procedures are to other problems of mathematics. This deeper understanding is part of teachers' PCK, which is closely related to their practice (Brodie & Sanni, 2014).

It is important to note that inasmuch as teachers can contribute knowledge to the PLC, teachers also benefit from the new knowledge that is created in the PLC. Koellner & Jacobs (2014) argue that "PD models have similar end goals": enhanced teacher knowledge, improved instructional practices and increased learner knowledge (p. 2). In developing knowledge for mathematics teaching and learning, teachers do not necessarily have to always agree on everything. In fact, disagreements bring an important element of rigour in the production of new knowledge and improved practices. Reasonable and healthy conflicts therefore comprise necessary elements for professional learning in a community. Uline, Tschannen-Moran & Perez (2003) argue that constructive conflict promotes individual and organisational learning and growth (p. 783). As teachers professionally disagree on certain ideas, a platform is created for them to learn from each other within that process of their disagreements. Some may agree with each other and others may disagree. Brodie & Shalem (2011) use the notions of solidarity and challenge in describing these agreements and disagreements between/among teachers in the process of developing "accountability to each other and the profession", which are key in developing new professional knowledge.

While the focus of this study is on PCK, Brodie, Marchant, Molefe, & Chimhande (2018) and Brodie (2014) found that conversations in PLCs on PCK also lead to a focus on CK. The key objective of working with the learners' errors in the project was intended to shape the teachers' PCK. It was from conversations about the learners' errors and misconceptions that important results emerged regarding the teachers' knowledge development: both PCK and CK (Brodie, 2014), and my analysis in Chapter 5 focuses on the extent to which PCK is developed in the community.

2.2.3 A situated perspective on teacher practice

Teacher education has experienced the emergence of teacher learning communities in the last two decades, and the emphasis has been on sustaining these communities (van Es, 2012). This study investigated the link between PLC conversations and shifts in the teachers' knowledge and practice. According to Borko, Jacobs, Eiteljorg, & Puttman (2008), "a situative perspective suggests that strong professional learning communities can foster the enhancement of teachers' professional knowledge and improvement of practice" (p. 418). A situated perspective suggests "how knowledge and practice can be negotiated in context" , and this often occurs through interactions with colleagues and experiences in the workplace (Horn, 2005, p. 210).

Interactions among teachers participating in a PLC illuminate crucial conversations that can have a positive influence on the practices of participating teachers. It is important that these conversations have a "clear, defensible focus" (Katz, et al., 2009). Brodie (2013) argues that "a focus on learner errors in professional learning communities can develop powerful conceptual knowledge of mathematics among teachers at the same time as developing teachers' knowledge for teaching and teaching practices" (p. 5). Conversations that focus on learners' mathematical errors and misconceptions create platforms for practising teachers to learn from and through them.

Wenger (1998) uses the term "community of practice" to emphasise the notion of shared practices. This study adopted the term professional learning community as defined in Chapter 1 of this thesis, which is a special case of community of practice (Brodie & Borko, 2016) and situates learning within the community of teachers participating in this study. The definition in Chapter 1 emphasised collaborative effort by teachers in reflecting on their practice and learning from that process. PLCs afford opportunities to practising teachers to engage in cycles of professional learning. A key feature of this learning is working with classroom data to analyse their own practices and in the process of that analysis, learning new instructional practices. The decisions that teachers take on what and how they learn are linked to their professional experience, evidence from their classrooms and knowledge from research (Brodie, 2012). In their conversations, teachers collaboratively identify a learning focus and deepen their knowledge around that learning focus to inform shifts in their practice.

The intention of the DIPIP project was not to specify particular teacher practices but rather suggest innovative approaches of looking at current teaching practices and inquiring into those in order to contribute to the teachers' future practices. It was important for the participating teachers to see value in participating in the project and building on their experience and expertise supported their valuing of the project. The study investigated how the participating teachers collaboratively engaged with the PLC activities to inform their participation in the PLC and their practices in class. It was important for this study to establish the consistency of patterns of participation in activities across situations (Borko & Koellner, 2008; Peressini, et al., 2004). These patterns of participation related to the shifts that the teachers made in the community and to their knowledge base.

My first research question focused on the shifts in the teachers' practices as they participated in the PLC. A PLC provides a social context in which the participating teachers can engage in activities which relate directly to their practice and which supports them to generalise beyond particular instances of practice. This study investigated the extent to which the teachers' engagement with the PLC activities contributed to their instructional practices. My second research question focused on how the participating teachers worked as a professional learning community to learn from and with each other as they tried to understand their learners' errors and misconceptions. Teacher learning may result from community conversations as teachers engage with activities and the teachers' pedagogical content knowledge, and perhaps content knowledge, may be shaped through this learning. This emerging new knowledge can then be drawn on for classroom instruction. The creation of new knowledge for practice reinforces the idea that professional development of the teachers is situated within the community, a subject that this study investigated.

2.3 Conceptual Framework

The figure below illustrates how the various concepts framing this study relate to each other. I discuss each of the concepts in detail in the next sections and in the process show how they relate to my theoretical framework.

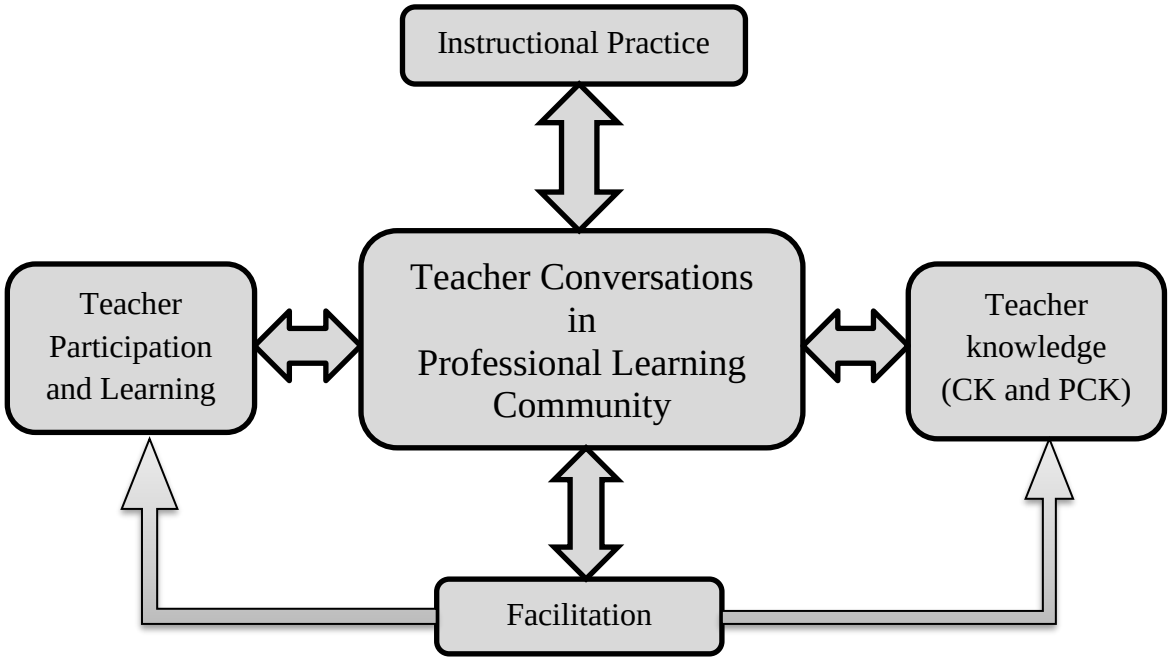

Figure 2.1: Illustration of concepts central to this research study

As shown in the figure, each of the conceptual components in the figure is directly connected to the key component: teacher conversations in a professional learning community. The centrality of the PLC in the figure affirms the importance of PLCs as important vehicles for professional development of teachers (Brodie, 2013; Koellner-Clark & Borko, 2004a). Central to this framework are conversations which are embedded within a learning community – it is through rich conversations that teacher learning within a community can be realised.

Guiding features of a PLC include: shared goals; clear and defensible focus; collaborative inquiry; professional development; and collective responsibility (Brodie, 2013; Earl & Katz, 2005; Katz, et al., 2009). Overarching these constructs is the notion of teacher learning, which happens through conversations in the PLC. In essence, PLCs are about taking collective responsibility as professionals to learn through an inquiry-based approach in order to improve practice. Improvement of practice is linked to creation of new knowledge for PLC participants, and this knowledge creation often happens within the ambits of the professional community using data from participants' own working environments (Brodie, 2013; Earl & Katz, 2006).

Working with data in meaningful ways for teachers needs support and guidance from expert leadership (Brodie, 2016; Katz, et al., 2009). Support involves the creation of a safe environment in which teachers can work with difficult content (Koellner-Clark & Borko, 2004a), while guidance involves working with teachers in ways which support knowledge creation. Knowledge creation in the community is often a result of teachers looking collaboratively at their learners' work and talking about it in meaningful ways (Boudett, et al., 2008). The support and guidance needed in the community required that facilitation also be a focus of this study. The analysis will show the extent to which facilitation of the community contributed to the teachers' participation and their PCK.

2.3.1 Conversations in professional learning communities

Every activity that the PLC engages in involves conversations within that community, which are central to all the other concepts used in this study. This section focuses on the importance of those PLC conversations and the impact that they have for a developing community. When teachers talk about their practice, it can benefit them more if their conversations are open, focused and professional. Teachers are then able to share their good practices and challenges through these professional conversations.

Brodie & Shalem (2011) use the notions of challenge and solidarity in developing accountability conversations among teachers. Professional accountability (Darling-Hammond, 1989) requires that members of a professional community account to each other through professional conversations (Earl & Katz, 2005). Depending on the context of the discussion, community members can challenge each other's ideas in professional ways, but they can also build solidarity with each other (Brodie & Shalem, 2011), without necessarily compromising the richness of the conversations.

Males (2009) worked with the idea of "critical colleagueship" in the development of mathematics teachers participating in their study group. She argues that critical colleagueship also involves confronting unproductive practices and "pushing each other to confront such practices" (p. 932). Other researchers refer to these necessary conflicts as collegial interactions (Collinson & Sherrill, 1996; Males, et al., 2010). This means that a balanced conversation within a professional learning community is one which supports productive practices but also challenges unproductive ones. Meaningful and rigorous conversations provide opportunities for teachers to distance themselves from what Boudett, et al. (2008) describe as "happy talk". Examples of "happy talk" could be comments like 'lesson was well-planned' or 'learners were involved in the lesson'. Such statements lack the substance that is needed for professional learning. In other words, when teachers talk about a well-planned lesson they should explicate and justify why it is well-planned. It may be possible that they may not think the lesson was well-planned but are being polite to keep everyone "happy". In Yonemura's (1982) terms, teachers' conversations should not be mere 'extemporaneous chats', but rather serious examination of and reflections on their practices. Supporting teachers to this level of conversations requires time and patience and is not often achieved (Horn, Kane, & Garner, 2018).

Kazemi & Hubbard (2008) argue that what teachers do in the PLC prepares them for the classroom, and what happens in the classroom can prepare them for participation in the PLC. Teachers' talk within a PLC can produce professional knowledge which can be employed in a classroom, and interactions in the teachers' classrooms produce rich and reliable data for the PLC. Different studies have shown that teachers often talk about content knowledge and practice in communities (Brodie & Chimhande, 2020; Hindin, Morocco, Mott, & Aguilar, 2007; Rowland, 2012). The level of depth in the teachers' conversations and the focus of the conversation are contributory factors in the creation of professional knowledge within a PLC.

2.3.2 Teacher participation and learning

This study adopted a view that defines learning as participation in learning communities (Borko, 2004; Borko & Koellner, 2008; Horn, 2005; Kazemi & Hubbard, 2008; Lave, 1993, 1996; Wenger, 1998). In Chapter 1 of this thesis, I discussed how teachers often work as individuals in their different schools (Hindin, et al., 2007; Horn, 2005, 2012). I also discussed how some PD programmes do not benefit teachers much because teachers often go back to their old practices as soon as they go back to their schools and continue to work as individuals. PLCs provide a community space for teachers to participate in and engage with other teachers teaching the same subject and learners.

Communities of practice emphasise coherence among community members (Brown, Collins, & Duguid, 1989; Chauraya & Brodie, 2017; Lave, 1993; Lave & Wenger, 1991; Wenger, 1998), which is key for professional development of teachers. This study adapted Peressini, et al.'s (2004) notion of practice, which they define as: "the patterns of thought and action that have been established by participants in particular contexts or settings" (p. 69). Peressini, et al.'s (2004) notion of practice resonates with Kazemi & Hubbard's notion of coevolution of practice – teachers learn as they talk about their practice in a PLC, and they continue to learn as they enact their ideas in a classroom (Kazemi & Franke, 2004; Kazemi & Hubbard, 2008). This learning is extended when they go back to their communities to reflect on what they experienced in their classrooms and consequently developing new ideas about their practice. The learning becomes cyclical as teachers move between the PLC and the classroom. Teacher learning is therefore an ongoing process. This process often brings a shift in the teachers' participation (Kazemi & Franke, 2004) which is consequently accompanied by more learning and knowledge creation.

Borko (2004) argues that teacher learning is a complex process and "meaningful learning is a slow and uncertain process for teachers" (p. 6). This kind of a learning process therefore necessitates the presence of an "ongoing and supportive, yet rigorous and critical discussion" (Brodie & Shalem, 2011). As teachers learn, they come to understand their subject, their learners and their practice in new ways. How teachers think about their practice can shift their participation in a learning community. Also, how they employ their community ideas in practice can help develop new ways of thinking about pedagogical content knowledge and their practice. In both cases, teachers' participation is situated within two contexts (Kazemi &

Hubbard, 2008), and this study investigated how the two contexts could contribute to the teachers' learning and practices in different ways. Kazemi & Franke (2004) use Rogoff's (1997) description to refer to this kind of participation as "transformation of participation" – teachers learn in the PLC as they participate, and they also learn in the classroom as they employ the PLC learning.

2.3.3 Instructional practice

Establishing the nature of current practices of teachers is important for every professional development and research project. The process may entail learning from analysing the teachers' lessons and the teachers reflecting on those lessons in individual spaces as well as in community settings. This reflective practice can assist with identification of the teachers' learning needs and constitutes rich data which can be used to develop the teachers' instructional practices in various ways. Research refers to two kinds of practices: mathematical practices and teaching practices (Molefe & Brodie, 2010). Mathematical practices include symbolising, representing, justifying and communicating mathematical ideas (Rand, 2003) and teaching practices involve particular methods that teachers employ in their classrooms to teach mathematics and/or develop new practices. This study focuses on how the latter incorporates elements of the former within the context of professional development of mathematics teachers.

Various studies have shown how practising teachers learn through their teaching experiences (Leikin & Zaskis, 2010; Timperley, 2013). These teaching experiences provide rich data to engage with in learning communities. Teachers participating in PD programs bring along their own knowledge, experience and expertise to these communities in order to share and collectively create practical knowledge that can be incorporated into future mathematics teaching. This process may require various data from the teachers' classrooms and the learning community working with that data in innovative ways to inform future classroom practices and potential shifts in the teachers' practices.

2.3.4 Teacher knowledge

This study explored the extent to which the teachers' pedagogical content knowledge (PCK) was developed as teachers engaged with the PLC activities and conversations. Content knowledge is defined by Shulman (1986) as: "the amount and organisation of knowledge *per*

se in the mind of the teacher" (p. 9). According to Shulman, content knowledge goes beyond mere understanding of facts and concepts, but includes understanding not only "that something is so, but why it is so" (p. 9). PCK refers to how teachers use their content knowledge in ways in which it can be comprehensible to learners – how good their explanations are and how they represent mathematical ideas (Shulman, 1986, 1987). Different teacher development programmes emphasise CK and PCK and the relationship between them in different ways. In the DIPIP project, we took PCK as the entry point into teachers' knowledge, as it is closer to their practice. We hoped that a focus on PCK would support teachers to also talk about CK and have found this in other studies from the project (Brodie, 2014; Brodie, et al., 2018).

As discussed previously, the focus of the PLC in this study was to understand the underlying reasoning behind learners' errors, a key area of PCK. The focus was chosen because all mathematics teachers notice learner errors, yet very few work with them productively (Brodie, 2014). In the DIPIP project we defined errors as "systematic, persistent and pervasive mistakes performed by learners across a range of contexts" (Brodie, 2014, p. 223). We used Olivier's (1989) distinction between slips and errors, where slips are easily corrected, whereas errors are pervasive because they are usually supported by misconceptions, which are underlying conceptual structures that make sense in terms of learners' current knowledge (Smith, et al., 1993). Brodie (2014) distinguishes the different ways in which teachers often work with learners' errors: avoiding the errors, correcting the errors, or embracing the errors. The DIPIP project supported the third approach, which resonates with Nesher's (1987) argument that errors should not be condemned but sought-after in order to support mathematics learning.

Key to the PLC conversations was how the teachers viewed learners' errors. Did they see errors as something to be avoided, corrected or embraced (Brodie, 2013)? DIPIP emphasised embracing errors so that teachers might understand learners' learning needs more deeply (Brodie, 2013) and on this basis mediate content better in class. As teachers embrace learner errors, they are in a position to understand the misconceptions behind those errors. When they understand the misconceptions, they can then develop approaches to assist learners in learning mathematics better.

In working with the constructs of avoiding, correcting and embracing errors, Brodie (2013) sees avoiding errors as "fear that bringing errors into the public realm will support a 'spread' of errors among learners and create more obstacles to learning". Correcting errors is seen as

"making the correct knowledge available to learners but depending on how the errors are corrected, may not illuminate the criteria by which something is judged to be an error". Embracing errors as focus on learners' thinking creates points of conversations about mathematical ideas (p. 8). Learners' errors are produced by their misconceptions (Nesher, 1987; Olivier, 1989; Smith, et al., 1993) and misconceptions suggest a gap in the learners' current knowledge which teachers need to explore for two reasons: first, to establish what the learners' learning needs are; and second, to think of mechanisms that they need to transform the learners' misconceptions.

Working with learners' errors as described above forms part of the teachers' PCK because the teachers develop ways of understanding the source of those errors. Errors are not easy to deal with because "they are the *symptoms* of the underlying conceptual structures that are the *cause* of errors" (Olivier, 1989, p. 4) and they make sense to those who produce them. Engaging with learners' errors means drawing on the learners' reasoning underlying their errors, and in so doing teachers can build their PCK in helping learners develop their conceptual understanding.

DIPIP worked with teachers in the error analysis activity to understand learner reasoning, through their errors. Teachers analysed the learners' errors to explore learner thinking – it was important for the teachers to understand the reasoning behind the errors that learners produced. The inquiry process helped the teachers to establish what the learner(s) knew and did not know and how partially valid knowledge produced errors. This analysis process built on the teachers' PCK in making sense of the learners' answers. The activity involved determining the essential components of the task such as the grade level and the content description, and this helped them in positioning the task within their current curriculum. In addition, the teachers worked out the different methods that the learners might use in order to answer the question, indicating for each method whether it made sense and in what ways it might do so. The process of analysis in DIPIP was concluded by teachers thinking of ways of working with the learners' errors to help surface and transform the misconceptions.

As the teachers in this study analysed their learners' errors, they found that a number of errors were related to learners' misconceptions relating to the notion of a variable. It was important therefore for the teachers to understand how learners viewed variables in the teaching and learning of mathematics. Research has shown that the transition from arithmetic to algebra has always been a problem for learners in mathematics, and this continues to affect the learners'

performance in their later years of learning (Booth, 1988; Chazan, 2000; Hodgen, Brown, Kuchemann, & Coe, 2010; Liebenberg, 2016; MacGregor & Stacey, 1997). Learners are often comfortable with arithmetic problems but encounter problems when numerals are mixed with letters – an expression like 5 + 3 may be too easy to compute for learners but 5 + k often becomes a challenge for learners to work with. Hart (1981) posits six categories in which learners view letters in algebra:

- Letter evaluated.

  This is the case where a letter is given a numerical value instead of being treated as an unknown or generalised number. In a study that Hart looked at, when learners were given $e + f = 8$ and asked to use the expression to work out $e + f + g =$ …., they gave an answer of 12. Leaners saw the expression as 4 + 4 + 4 because they thought e and f in the first expression represented equal numbers and g represented the same number as well.
- Letter as object.

  This often happens when learners use letters to denote an object. For example, $2a + 3b$ is seen as 2 apples and 3 bananas.
- Letter not used.

  When the letter is acknowledged or simply ignored. For example, $2a + 3$ is treated as $5a$ (acknowledging the letter a) or simply as 5 (when ignoring the letter a).
- Letter as specific unknown.

  Here a letter is treated as a particular unknown number. This is the case when, for instance, letters of the English alphabets are treated as specific numbers according to the position they occupy in the alphabetical order list, e.g. $a = 1, b = 2, c = 3, d = 4$, etc.
- Letter as generalised number.

  Here the letter takes several values rather than just one. A question like: what can you say about a when $a + b = 12$ and $a < b$. Learners get attracted to only natural numbers like $a = 1, 2, 3, \& 4$, and $b = 7, 8, 9, 10, 11, \& 12$.
- Letter as variable.

  The letter here is seen as representing a range of values.

Research posits two contrasting views on teacher knowledge. One view argues that teachers' content knowledge is primary in improving the quality of instruction and that a teacher's content knowledge influences their PCK and the quality of instruction (Ball, 1991; Hill & Ball,

2004; Sherin & van Es, 2009). The other view argues that while CK is important, PCK is crucial and primary for classroom instruction, because teachers see their practice and knowledge through PCK (Brodie & Sanni, 2014). Current research argues for both: that CK influences PCK and that PCK and instruction strongly shape CK (Brodie & Sanni, 2014). In resonance with Brodie & Sanni's (2014) view, Mavhunga & Rollnick's (2013) study showed a two-way relationship between the two types of knowledge – showing that the two types of knowledge are interdependent. Potari's (2014) study of three teachers in a design experiment aiming to support teacher learning found that teachers developed both pedagogical knowledge and subject matter knowledge. However, Potari's study did not show how the one knowledge contributed to the development of the other. Working with both views as a basis for professional development eases the tension between them and can generate what Hiebert, Gallimore, & Stigler (2002) refer to as practitioner knowledge – "the kinds of knowledge that practitioners generate through active participation and reflection on their own practice" (p. 4).

According to Hiebert, et al. (2002), practitioner knowledge is not constituted by one form of knowledge but consists of different forms of knowledge that are intertwined. Arguing from a situated perspective on knowledge, learning and practice, this study supports the notion that two kinds of knowledge - CK and PCK - mutually shape one another as participants in a learning community engage with different community activities and how this engagement contributes to the knowledge base of the learning community.

The studies by Brodie (2013, 2014) found that teachers developed new ways of looking at errors as they reflected on their practice within a learning community. These reflections helped deepen the teachers' pedagogical content knowledge and content knowledge which consequently could lead to new teaching practices. Brodie (2013) ascribes the success of PLCs to three aspects: professional learning, collaborative learning, and good facilitators. Embracing learners' errors as resource for teacher learning is not a simple task for teachers and needs professional guidance from a facilitator.

In addition to Brodie, (2013, 2014), there were a number of publications also produced from DIPIP data during the course of this study (Brodie, Molefe & Lourens, 2014; Brodie & Sanni, 2014; Molefe, 2016; Chauraya & Brodie, 2017; Brodie, et al., 2017; Brodie, 2016; Chauraya & Brodie, 2018; Brodie & Chimhande, 2020). All the publications focused on how teachers collaboratively engaged with learner errors in order to inquire into their practices. Chauraya &

Brodie (2017) connected teacher-learning activities in a PLC to shifts in teaching practices, with teachers showing differential shifts in their teaching. Their differential shifts were linked to their differential participation in the PLC, with teachers who engaged more showing more substantive shifts. Brodie & Chimhande (2020) showed how different activities supported different content and depth in teacher conversations, and they argue that "a range of activities is necessary for opportunities for teacher learning to be available in different areas of important content for teachers" (p. 128). They identified the depth of the teachers' conversations in PLCs as an important area for further development, as the teachers' conversations in their study did not show substantive shift in depth.

2.3.5 Facilitation

The success of a professional learning community is closely connected to facilitation – a facilitator can support or constrain PLC activities and conversations. A facilitator of a professional learning community plays a critical role in the functioning of the community and ensuring that knowledge creation occurs within accountable principles of critical colleagueship (Earl & Katz, 2006; Males, 2009; Males, et al., 2010). In the process of talking about their practice, participating teachers critique each other for purposes of developing each other's knowledge and practice. Accepting critique from each other within a community can help the growth of a learning community. The facilitator's role becomes crucial in encouraging this critique. Katz, et al. (2009) define this role of a facilitator as that of a "critical friend", who can enhance rich and robust discussions among community members (Katz, et al., 2009). A critical friend is not afraid to challenge assumptions, beliefs or simplistic interpretations (Brodie, 2016), and this may be a boost for substantive conversations within a community.

Participating teachers in a learning community often consider a facilitator as a problem-solver (Te Maro, et al., 2008). They see a facilitator as someone with solutions to the challenges that they continuously face in their classrooms and in their profession. This perception of a facilitator conflicts with the notion of a PLC taking a "bottom-up" approach, where teachers take ownership for their professional learning. In other words, teachers determine their own developmental trajectories by collaboratively sharing ideas to contribute to their collective professional learning. The role of a facilitator is therefore one of guiding the learning process and promoting a collaborative and reflective inquiry in the learning community.

It is expected of facilitators to be empathetic: to be caring and considerate of the teachers' situations. In addition, it is important for a facilitator to create an atmosphere that is "responsive to the ways in which teachers learn" (Timperley, 2009). Good facilitation of PLCs creates a significant partnership between the facilitator and the participating teachers, based on trust, mutual respect and reciprocity (Hudson, Hudson, Gray, & Bloxham, 2013). The reciprocity of the partnership lies in the fact that both parties can learn from each other: teachers learn from the facilitator with particular expertise in knowledge and practice, and the facilitator learns from the teachers about their contexts and their learning needs. Teachers spend more time with learners in class, and they may have rich data that they can avail to PLCs for collective inquiry. This study positions both teachers and facilitators within a PLC as both learners and experts (Brodie, 2014).

Any intervention that encourages teachers to shift their thinking about what they have been practising for a period of time is often viewed with caution by teachers (Timperley, 2013). This then necessitates the guidance of a good facilitator in getting teachers to understand the importance of the role that they play in their professional development. Brodie (2016) stresses that "negotiating and maintaining the focus of the community is an important part of the community and the facilitator's work" (p. 157).

Feldman & Fataar (2016) use the notion of "pedagogically emptying reflexivity" when describing challenges which muddle the PLC conversations – they relate these to "the teachers' lack of access to a productive language to have a dialogue about, and engage in, pedagogical adaption" (p. 19). It is possible to have incidents which do not relate to the focus of the PLC, but such incidents still being relevant for professional learning. Brodie, Molefe, & Lourens (2014) found that diversion from the focus of a conversation in a community can bring an important aspect for understanding facilitation.

2.3.6 Summary so far

I have shown how this study was guided by Wenger's (1998) theory of learning as a social practice, within situated perspectives (Horn, 2005; Peressini, et al., 2004). I have elaborated on how research views teacher learning within situated perspectives, and that within the context of this study, teacher learning is situated in participation in professional learning communities

and classrooms, and increased learning is viewed as becoming a better participant in practice in these contexts.

I have discussed the different concepts which this study is based on and shown how these concepts link to each other: the conversations that PLCs engage in to develop a knowledge base for instructional practice and how the knowledge that is created within a PLC is found in the interaction between participants within a community. In my discussion of teacher knowledge, I have elaborated on how this study took PCK as the entry point into teachers' knowledge and how a focus on PCK could support teachers to also talk about CK.

I have also shown the critical role that a facilitator plays in a PLC. Research views a facilitator as a "critical friend" (Katz, et al., 2009). In other words, inasmuch as a facilitator should be seen as a friend, the facilitator should not be afraid to challenge ideas and assumptions that do not develop rigour in the PLC conversations. In the next section, I review literature on professional learning communities and show how my study contributes to filling some gaps in this literature.

2.4 Professional Learning Community: A review of literature

2.4.1. Why we need PLCs

For decades, professional development in the education sector involved workshops, courses, and other related programs meant to provide teachers with skills for their practices. Fullan (2007) argues that "The notion that external ideas alone will result in changes in the classroom and school is deeply flawed as a theory of action", even if these ideas are very good (p. 35). Guskey (2002) describes PD programs as "systematic efforts to bring about change in the classroom practices of teachers" (p. 381). He further argues that many programs do not take two key elements into account: motivating factors for teachers to be involved in professional development and processes by which change occurs. This implies that teachers should see the need for their participation and learning in a PD program and their participation should bring shifts in both their participation as well as their practices.

A model of professional development that Guskey (2002) proposes is predicated on three major goals: change in teachers' practices, change in their attitudes and beliefs, and change in the

learning outcomes of students. These changes link to a school culture that develops due to teachers' participation and learning in PD programs. This study explored the first two changes suggested by Guskey. Although Guskey argues that it is important for teachers to see changes in their practice in order for their attitudes and beliefs to shift, the DIPIP project imagined these happening together and informing each other. Changes in teachers' beliefs and attitudes through participating in learning communities may inform changes in their instructional practices and vice versa.

In the ICME-13 survey on mathematics teachers working and learning through collaboration, Robutti et al. (2016) define collaboration as:

> co-working (working together) and can also imply co-learning (learning together). It involves teachers in joint activity, common purpose, critical dialogue and inquiry, and mutual support in addressing issues that challenge them professionally (p. 652).

In the survey, Robutti et al. (2016) cite three examples of teachers' collaborative work: The Lesson Study in Japan; Collaborative Teacher Projects (CTPs) in England; and the Learning Communities in Mathematics (LCM) in Norway. All three involved an inquiry-based approach to professional development of mathematics teachers, and also involved experts (referred to as 'others' or 'didacticians') to guide the teachers' engagement in the activities.

In a seminal paper, Borko (2004) argues that we need PD programmes that are long-term, developmental, practice focused and well-facilitated. Katz, et al. (2009) argue for PD to take account of teachers' learning needs, which are developed from learners' learning needs. PLCs have been proposed as a means for teachers to take ownership of their learning, to collaborate in inquiring into their own practices and to develop coherent and sustainable changes in their schools (Borko, 2004; Curry, 2008; Hindin, et al., 2007). In this literature review, I will examine the extent to which current research shows the successes and challenges of PLCs, and indicate how this study contributes to the literature.

2.4.2 Collaboration and inquiry

Participation of teachers in learning communities is intended to show the extent to which they develop a knowledge base which they can use to contribute to their participation in community conversations and their instructional practices. Kazemi & Hubbard (2008) talk about

coevolution of participation in professional development and classroom practice. They describe this as teachers bringing "representations of their classrooms into the PD context, and aspects of PD travel into teachers' classrooms and back again" (p. 432). In a study that spanned more than five years and involving several hundred elementary school teachers from six districts, Kazemi & Hubbard (2008) found that "what teachers were able to do in their classrooms was a function of how closely coupled the PD experience was with their classroom experience and how their classroom experiences were treated within the context of PD" (p. 435).

The ICME-13 survey on teachers working and learning through collaboration (Robutti, et al., 2016) reviewed more than 300 papers published between 2005 and 2015 and found that: 1. collaborations created an environment in which the teachers felt safe to talk and valued; 2. shared goals and interests seemed to be important for the development and sustainability of the collaboration; 3. opportunities for teachers to interact with other teachers were highly valued; 4. studies in which collaboration was initiated by teachers and sustained by them built on their initial confidence; 5. analysis of lesson videos provided valuable opportunities for teacher learning; 6. teacher learning could not be consistently explicated, and where it occurred, it could not be easily linked to collaboration; and 7. The notions of collaboration and community were taken for granted and not theorised, and where any relation to theory occurred, Wenger's (1998) theory of Community of Practice dominated.

The survey also identified three main barriers to effective collaborations: teachers' lack of ownership, time, and "institutional constraints such as curriculum and assessment structures" (p. 678). These findings prompted the survey team to initiate an ICMI study (Borko & Potari, 2020) which could focus on "these missing areas" (Jaworski et al., 2017).

Building on the work of the survey, The International Commission on Mathematical Instruction (ICMI) Study 25 (Borko & Potari, 2020) was organised under four study themes: 1. Theoretical Perspectives on Studying Mathematics Teacher Collaboration, 2. Contexts, Forms and Outcomes of Mathematics Teacher Collaboration, 3. Roles, Identities, and Interactions of Various Participants in Mathematics Teacher Collaboration, and 4. Tools and Resources Used/Designed for Teacher Collaboration and Resulting from Teacher Collaboration. What emerged as key from various papers presented were: possible frameworks that can be used to promote teacher collaborations in PD programs (Aldon, 2020; Ding & Jones, 2020; Krainer & Spreitzer, 2020; Prediger, 2020); the extent to which tools and resources, which included

teacher knowledge, can be used in PD programs to enhance/inhibit teacher learning and practices (Brodie, 2020; McKie, 2020); the extent to which collaborative reflections can be used in PD programs to recognise potentialities for teacher learning (Canavarro & Serrazina, 2020; Climent, Codes, & Carrillo, 2020); and recognising PD leadership as a crucial resource in teacher collaborative communities (Pöhler, 2020; Schwarts, 2020; Sikko & Ding, 2020).

Kooloos, Oolbekkink-Marchand, Kaenders, Heckman (2020) argue that "teaching mathematics involves getting students to think, figuring out what they think, and supporting them in developing their thinking" (p. 372). In a project which involved collaboration among five mathematics teachers and one researcher over nine meetings within one year, the authors investigated how the group learned to develop and orchestrate classroom discourse. The meetings took place at the researcher's university. Each teacher designed, taught and individually reflected on the taught lesson. The group would later do a collaborative reflection by watching a section of the teacher's lesson video and developed guidelines from that process which they shared with their colleagues in the mathematics community during a workshop. Three main shifts were identified in Koolos et al.'s analysis: 1. The focus of collaborative reflection shifted from the teacher's thinking and actions towards the learners' thinking, 2. The discussion of guidelines for the teacher's work shifted from being mainly about work outside the classroom, such as lesson preparations, towards the teacher's work inside the classroom, and 3. The discussion of learners' thinking shifted from being general towards being content-specific and from individual learners' thinking towards the thinking of the group. The shifts were boosted by the researcher stopping the video after the learners' utterances and allowing the teachers to make input, focusing on learner thinking and linking possible teacher actions to learner thinking.

In a study that involved teachers at a school meeting in monthly workgroups throughout the year to collectively examine student work, Kazemi & Franke (2004) identified two major shifts in the teachers' participation: First, the manner in which "they attended to the details of students' mathematical thinking" improved, and they related this shift to attempts by the teachers to elicit their students' thinking. Second, the shift in teachers' participation consisted of "developing possible instructional trajectories in mathematics that emerged because of the group's attention to the details of student thinking" (p. 213). These shifts were a result of collaborative conversations that teachers had in the PD program and applying ideas from the PD program in their classrooms. Kazemi & Franke (2004) describe the learning process as

transformation of participation, which describes how teacher learning is evidenced in "shifts in participation in discussions centred on student work" (p. 203).

McLaughlin & Talbert (2006) found that PLCs provided social interaction and learning opportunities in which teachers took collective responsibility for learners' performance. Teachers were able to discuss what practices worked well for them, what learners struggled with, and what needed improvement. Little & Horn (2007) refer to such talks as "consequential talks" because as McLaughlin & Talbert (2008) put it: they "led teachers to see themselves as responsible for all the school's students" (p. 160). These conversations are an attribute of what McLaughlin & Talbert (2006) describe as a "culture of collaborative instruction". Teachers share ideas on instructional practices that can help bring improvement in learner achievement. Enactment of these common ideas in classrooms enables them to then reflect on the extent to which their ideas worked with or impacted learners. McLaughlin & Talbert's study involved teachers observing each other in practice and giving feedback to the learning community. The researcher did not observe the teachers in practice but relied on information that teachers brought to the learning community.

Superfine & Pitvorec (2020) argue that "teachers and researchers working together towards changing teaching practices offers new conceptualizations of teacher learning, and affords new opportunities and designs for studying teacher learning over time" (p. 254). In a project that focused on enhancing teachers' formative assessment (FA) practices in algebra, Superfine & Pitvorec focused on a group of ten teachers which met after school on a monthly basis for three years. The aim of the project was to enhance the teachers' understanding of FA practices more broadly, and also to support them in analysing learner thinking. For three years, the project "tended to be more about implementing specific FA techniques and less about engaging students" (p. 256). This finding suggests that engaging with learner thinking can be a complex and difficult process. In the fourth and fifth years, Superfine & Pitvorec reduced the teachers' group size to three. Findings showed enhancement of "opportunities for teachers and students to listen to, evaluate, and act on student thinking and performance" (p. 260). The authors argue that "involving teachers as co-researchers and co-designers, we believe that we are moving in a direction of designing professional learning experiences *with* teachers in ways that hold promise for flexibly adapting to different contexts and situations" (p. 261). In my study, participating teachers inquired into their practices not as co-researcher but in ways in which they could collaboratively learn from their own practices.

In a study that involved Grade 1 to 7 teachers participating in a PLC that ran for six months in a rural school in South Africa, Liebenberg (2016) found that one teacher's acknowledgment of his self-doubt helped him to recognise that "this was probably what his learners experienced". This acknowledgment "foregrounded the importance of building the learners' self-confidence" as they worked on mathematics problems (p. 70). This finding resonates with what Brodie & Shalem (2011) found, where teachers acknowledged that their own learning experiences helped them to understand the reasonability of their learners' errors. Although Leibenberg's (2016) study focused on learners' errors, her study did not look at how teachers engaged with these errors in their classrooms. The PLC discussions on learners' errors were based on the errors that learners produced in a June examination.

In a similar study to the above, Kazemi & Hubbard (2008) analysed two teachers' participation in a PD program and found that one teacher benefited more from the PD program than the other. They accounted for the one teacher's success to that teacher spending "the bulk of his instructional time eliciting and trying to understand his students' strategies or finding ways to bring their ideas in more purposeful engagement with one another" (Kazemi & Hubbard, 2008, p. 433). As they put it: "there was a tighter coupling between her participation in PD and her classroom experiences" (p. 433). Though Kazemi & Hubbard argue for interconnections between settings, however, their study is not explicit on the presence of the PD leader inside the classroom so that he or she is familiar with the two settings that they argue for.

A study by Brodie & Shalem (2011) involved groups of about 50 Grade 3-9 teachers who learned through the project to embrace learners' errors and misconceptions. Teachers designed lesson plans in their small grade groups (Grade 3-9, each grade having two groups), and presented their work to two bigger groups (Grades 4-6 and Grades 7-9) with the aim of providing opportunities for each of the small groups to account for their planning and their reflections. Findings from this activity showed that in both the small and the bigger groups' conversations, teachers accounted for "their current and shifting practices, for the constraints on their practice and their learning, for learner thinking, and for the links between their practices and learner thinking" (p. 18). The teachers in Brodie & Shalem's (2011) study acknowledged that learners' errors are reasonable to learners and that they (teachers) accounted for their own responses to learners' errors. Results from the study showed how accountability was built through teachers challenging each other and building solidarity with each other (p. 427).

Brodie & Shalem's (2011) notion of solidarity and accountability link very closely to what Smith (2016) found in his study of examining the key roles that "relational affordances" – collaboration, trust, and a sense of community – play in building a sustainable PLC. First, the study found that "collaboration became a vehicle for sharing ideas and resources as well as for providing support" (p.147). This result resonates with what Brodie & Shalem (2011) found on building of solidarity between/among participating teachers and also challenging each other as they worked within the context of networked communities. Second, participants in Smith's (2016) study acknowledged that there was a culture of openness in the community and a safe space for them to learn together, which constituted trust among them. Third, data analysis revealed that members "commented on the caring aspect of the community" (p. 151).

A similar study by Males, et al. (2010) using a "critical colleagueship" framework found that challenging interactions were valuable for teachers to change their practice. In another study on interactions among PLC participants, Males (2009) found that three patterns of interaction surfaced in the teacher study group: praising colleagues, advising colleagues, and challenging colleagues (p. 931). Praise was usually directed at a colleague presenting his/her work and often involved clarification questions; advice often involved little questioning and was also directed at the presenting teacher; and challenge involved asking questions that required teachers to elaborate, justify their reasoning, and reflect on their own thinking. The notions of praise, advice and challenge make up a critical construct for building community in PLCs.

In a comparative study design that involved two high schools, Horn (2005) found three community-based resources for teacher learning: collegial interactions between the participants, informal category systems in their conversations, and making different aspects of classroom events available (p. 211), which support "*transformative learning* – learning that challenged existing assumptions" (p. 217). The above discussion on the three studies by Brodie & Shalem, Males, et al., and Horn illuminate a critical aspect of teacher learning: the importance of rigorous conversations between and among participants within a community of practice. However, similarly to Kazemi & Hubbard (2008), Brodie & Shalem's (2011) study does not place the facilitator inside the teachers' classrooms in order to observe their practices.

Kazemi & Franke (2004) expand on teacher learning to show how this learning evolves over time as a result of shifts in the teachers' participation, which confirms Brodie's (2013) argument on professional learning being time consuming and demanding. Increased teacher

learning requires regular, extended participation of teachers in learning communities (Horn, 2005; Peressini, et al., 2004; Wenger, 1998). In a study that focused on teachers' identities in a PLC, Chauraya (2016) found that "engagement with others in the community, imaginations of joint learning extending into the future, and alignment with community practices such as joint lesson planning and problem solving supported the development of their identities as members of the community" (p. 211). Chauraya's finding links individual identity development to participation in a community, which made him emphasise the point that identity development is not homogenous.

Kazemi & Franke (2004) use the notion of transformation of participation, a concept which carries a different meaning from Horn's (2005) transformative learning as discussed above, to describe teachers' learning trajectories. While both concepts relate to teacher participation, transformation of participation focuses on shifts in participation and transformative learning focuses on challenging existing assumptions. Kazemi & Franke (2004) outline key indicators of teacher learning as teachers participate in learning communities. These indicators affirm the importance of a collaborative effort by teachers in inquiring into classroom data with the goal of improving their instructional practices. The indicators reflect what should actually happen within PLCs in order to realise a transformation of practice.

Table 2.1: Teacher learning in communities of practice.

Attributes	Indicators
Collaboration	Teachers working together to support the development of each other's thinking and the practices they use in their classrooms.
Sharing strengths and weaknesses	Teachers asking each other for help and contributing to the discussions in the workgroup because of their own experimentation in the classroom.
Classroom data	Teachers looking at the strategies that learners in other classrooms use and then using those as markers of what to expect from their own learners.

The three indicators outlined by Kazemi & Franke (2004) situate teacher learning in two contexts: the learning community and the classroom. In my discussion of the concepts framing this study as well as my review of the literature on teacher learning, professional development and facilitation, all the three attributes by Kazemi & Franke emerged as being critical in all the afore-mentioned constructs of my review.

2.4.3 Using videotapes in PLCs

Brodie & Borko (2016) argue that a key element of teachers' work in PLCs is artefacts of practice, including learners' work, lesson plans and videotapes of teachers' lessons, all of which were used in the DIPIP project. Artefacts of practice help to situate the conversation in teachers' practices and their PCK. Much of the research has been done with videotapes, so I review this literature here.

In a study that investigated how video analysis influences the process of teacher change, Tripp & Rich (2012) found that the change process consisted of four steps: "recognising the need for change, brainstorming ideas for change, implementing the ideas, and evaluating changes that were implemented" (p.732). The study took place over two months and involved teachers from different educational backgrounds: Special Education, Religious Education, and English Language Learner teachers. Teachers in that study found that "they were more likely to change their practices when they used video analysis" (p. 732) for reflection on their teaching. The facilitator's interventions in the study group was very limited – the teachers were given a tool to use for their analysis and the facilitator observed how the tool was used and listened to the teachers' conversations without becoming what research refers to as a "critical friend". In a similar but earlier study that was conducted by Sherin & van Es (2005), it was found that video became an important tool for working with both novice and veteran teachers. Teachers shifted from a focus on pedagogy to a focus on student thinking, and were better able to identify significant features of the video segments that related to their teaching. The limitation in Tripp & Rich's (2012) study was that the researcher did not analyse the teachers' videotaped lessons – it was the teachers who analysed the lessons. This means that the researcher did not have insightful information from the analysis except from what he heard during discussion groups. In addition, the study took a very short period and could have compromised the generalisability of the results.

In a two-year mathematics PD program called STAAR (Supporting the Transition from Arithmetic to Algebraic Reasoning), Borko, et al. (2008) explored how classroom video could be used as a tool for fostering productive discussions. Their analysis focused on the teacher's role in the classroom as well as the learners' thinking as they worked on activities. Findings show that when participants discussed the teacher's role, they initially "empathised with one another", and were rarely critical of their colleagues (Borko, et al., 2008). However, the

facilitator's intervention helped them to make sense of the classroom events and studying the video clips afforded an opportunity to participants to new insights about "better ways of asking questions to uncover student thinking" (p. 429). Borko, et al. (2008) found that teachers found discussions of their classroom videos very valuable in seeing what they did well and also in identifying areas of improvement. In Borko, et al.'s (2008) study professional development leaders rather than teachers selected episodes from videotaped lessons for discussion in the PD program. In addition, the facilitator's voice is almost silent and not elaborated on except for analysing the teachers' discussions of their lessons. In this study, the participating teachers individually selected the episodes, and the investigation in this study was to see the extent to which the teachers could shift their participation as they engaged in PLC discussions and the extent to which the facilitator supported these shifts.

van Es (2011a) and van Es & Sherin (2002, 2005, 2008) use the concept of "noticing" to describe how teachers are alert to what learners do in a mathematics classroom. In one of their earlier studies, van Es & Sherin (2002) worked with teachers in monthly meetings through what they called 'video clubs' to share and discuss excerpts of the teachers' classroom videos. They found that "teachers developed new ways of noticing and interpreting classroom interactions" (p. 572). They reported that "there were changes in *what* the teachers noticed as they discussed the video excerpts, and there were changes in *how* the teachers discussed these events" (p. 481). Their discussions included shifts from discussing pedagogy to learners' thinking as well as "the ability to identify significant interactions". Subsequent studies of Sherin & van Es (2005) contributed to the development of a framework for teacher learning communities in video clubs (van Es, 2012), which this study adapted.

In the study by Tripp & Rich (2012) discussed above, teachers were trained to use a video analysis tool and met as a group with their facilitator to discuss their analyses, with the researcher remaining a non-participant observer during the discussions. The teachers "felt accountable to the group to implement some of their suggestions" in their classroom and that brought a shift in their practices. Tripp & Rich, though, do not detail specific foci for the teachers' reflections – teachers merely selected three to four codes to use for their analysis, e.g. pacing, active participation, student cues and checking for understanding (p. 731). The study does not elaborate on in-depth analysis or specific indicators of active participation to detail how the teachers worked as a community to reflect on their practices. This study investigated

how the participating teachers talked about their practices as they reflected on their videotaped lessons.

In a similar study to Tripp & Rich (2012), van Es (2012) illuminates the importance of "building relationships with colleagues to look in-depth and inquire into each other's practice" (p. 183). In the study, van Es emphasised the importance of "evidence from teacher practice used to draw conclusion about the impact of teaching" (p. 185). van Es' (2012) study looked across ten meetings to see how teachers developed over that period. Findings from that study show that teachers in the first two meetings expressed their own ideas, rather than attending to each other's, and provided limited detail" (p. 187). In the third and fifth meetings, the community showed signs of collaborating and supporting "each other's analyses and the conversations become more distributed among the group" (p. 188). The fourth meeting as well as subsequent meetings after the fifth meeting showed teachers going to an extent of participants not only listening and supporting each other, but also "probing each other and elaborating one another's thinking" (p. 189). In support of in-depth analysis, van Es (2012) elaborates interpreting the meaning of classroom interactions and deciding what to do next as key aspects of analysis.

Adler (2016) reports on how her team operationalised the Mathematics Discourse in Instruction (MDI) framework in their Wits Maths Connect Secondary (WMCS) project using lesson study that involved teachers, learners and researchers. The framework uses four elements: object of learning, exemplification, explanatory talk and learner participation to guide the planning and reflection of lessons. Teachers in the project came from a cluster of schools and met one afternoon a week for three consecutive weeks in one school. In the first week, the teachers and one researcher jointly planned a lesson on a topic selected by the teachers. In the second week, one teacher taught a lesson to learners (their own or other colleagues') after school hours and the other teachers observed. Afterward, the learners would leave, and the teachers would collaboratively reflect on the taught lesson and replan the lesson. In the third week, a different teacher taught the lesson, and reflection and replanning followed, and the revised lesson plan followed and would be shared with other teachers outside the project. Results relating to this lesson study were not reported in this paper, but a hint was provided on a future report of the results regarding how "LS provides a learning space for all participants (the teachers, their students and ourselves as researchers) (Alshwaikh & Adler, 2017a), and one too where conflicts arise, within and between teachers and researchers" (p. 130).

2.4.4 Summary so far

The review of the research on teacher learning in PLCs illuminated four critical elements from several findings of the literature: First, teacher learning is multidirectional – the classroom data that teachers bring to a PLC avail the necessary affordances for teachers to learn between two environments. Second, teacher learning includes collegial interactions among participants, which subsequently shift as teachers focus on significant features of their lessons. Third, the focus of the teachers' conversations on key artefacts of practice like lesson videotapes can help them to account for their current and shifting practices. Fourth, the teachers' conversations help build a sense of community among them, which is critical for functional professional learning communities. These emerging four elements resonate with what this study sought to understand about teacher learning in a PLC: First, this study investigated how the participating teachers worked as a learning community to learn with and from each other; Second, it investigated the extent to which the teachers' conversations focused on learner thinking, which can contribute to the teachers' PCK in working with learner errors; Third, it investigated depth in the teachers' conversations, which can assist in establishing the generalisability of the conversations in a PLC; and fourth, it accommodated the teachers' critique of each other's contributions as they talked about practices in professional ways.

2.4.5 Facilitation in a PLC

Brodie (2016) argues that since professional learning should be deliberate and systematic, it needs leadership and guidance, through resources such as protocols and through facilitation.
The guidance that a facilitator offers in a PLC is meant to help the teachers understand what the data is telling them regarding their learners and their practices (Earl & Katz, 2006). Earl & Katz argue that "leaders contribute to a culture of inquiry by providing opportunities for others to become inquiry-minded" (p. 20). In my discussion of Tripp & Rich's (2012) study above, I reported on teachers using video analysis to improve their practices in the classroom but facilitation, as an important aspect of that analysis process, receiving less attention. While we have some studies on facilitation in PLCs (Earl & Timperley, 2009; Katz & Dack, 2013), there is still much to learn about how facilitators work in PLCs.

The expertise of a PLC facilitator can be important in the PLC conversations. Earl & Katz (2006) assert that facilitators ask questions, probe for justification and help reformulate

interpretations – this role of a facilitator can help in giving substance to the conversations in a learning community and promote professional development. In a study that focused on insights into students' mathematical reasoning that participating teachers were able to gain from observing learners' mathematical activities, Francisco & Maher (2011) found that "several instances of rich mathematical reasoning from students often go unnoticed to teachers in mathematics classrooms" (p. 62), and this necessitated the researchers' (facilitators) intervention in helping the participating teachers to improve their noticing of the learners' reasoning. This finding resonates with Sherin & van Es' (2005) finding discussed above on teachers developing new ways of noticing and interpreting classroom interactions because the facilitator in the video club made critical interventions when teachers' inputs were not substantive. Good facilitation can assist with conceptual understanding of the mathematical content as well as the different ways in which this content could be made accessible for learners in instruction.

Pöhler (2020) identified three facilitation moves for supporting group collaboration in a PD course: standing back, distributing participation and validating participant ideas. These moves are similar to the ones employed in this study. In Pöhler's study, teachers chose tasks, modified the tasks to include conceptual understanding, and provided written vocabulary scaffolds so that learners could cope with the tasks (p. 519). Findings showed that the facilitation move of validating participants' ideas occurred frequently, which means that the facilitator interjected more and afforded the teachers fewer opportunities to share ideas. Pöhler argues that: "for understanding restrictions of teachers' learning uptake, the case study shows that identifying facilitation moves can be insufficient to capture the role of facilitators of focusing the attention to a specific PD content while supporting teacher collaboration" (p. 522). This study used facilitator moves in its analysis of PLC facilitation to see the extent to which participating teachers could be supported and guided in creating opportunities for PCK development as they engaged with the project activities.

In a study that involved how teachers learned from practices of other unfamiliar teachers through a lesson video, Schwarts (2020) characterises challenges in leading a collaborative video-based professional development. The process of observing the teacher-facilitator revealed how the facilitator's practice and identity evolved over time. The PD program aimed to "foster teachers' reflective skills and mathematical knowledge for teaching, and develop a shared language for discussing practice through peer-analysis of lessons taught by unfamiliar

teachers" (p. 541). Findings reported two prominent challenges: the activities of the second sessions were not connected to the project goals, where the video lesson was only discussed towards the end of the session, and managing a discussion where the teachers could reflect and collaborate was not done well, with discussions being dominated by the facilitator. The facilitator's practice improved after one-on-one sessions were arranged between the facilitator and a mentor.

Sherin & van Es (2009) found that teachers made critical omissions when discussing the learners' reasoning. These omissions prompted the facilitator to probe the teachers into thinking deeper about the learners' reasoning. When the teachers interpreted the learner's thinking as learners being confused by the concept, the facilitator asked them what the learner thought the question was asking. The facilitator's question then helped the teachers to "look in-depth at the learner's comments" and discuss "substantive mathematical ideas related to the video clip" (p. 221). The two studies by Francisco & Maher (2011) and Sherin & van Es (2009) illuminate critical interventions of a facilitator when comments by the PLC participants were not substantive enough. van Es & Sherin (2017) argue that "facilitators need to press teachers' thinking in order to cultivate critical conversations and move away from surface-level judgments that limit teachers' opportunities to learn" (p. 3).

One of the roles of a facilitator is to "create a safe environment for teachers to grapple with difficult content and pedagogical issues" (Koellner-Clark & Borko, 2004b, p. 223). A study by Coles (2013) found that teachers often used evaluative comments in their discussions of the lesson videos. Coles (2013) and Sherin & van Es (2009) both emphasise the importance of teachers overcoming the tendency to use evaluative talk when discussing teaching, and facilitators can be on the lookout for such talk and try to shift it. Coles (2013) argues that "if teachers begin in an evaluative mode of discourse, it appears it is hard to shift discussion to any other mode" (p. 167). Brodie (2016) argues that PLCs create environments for teachers to "explore their strengths and weaknesses with colleagues" , which requires facilitator support (p. 155).

Teachers are often viewed as experts in their fields and it is hoped that they would bring a wealth of knowledge and their different experiences into a PLC in order to learn from one another (Sherin, 2002). Acknowledging teachers as experts in their fields does not remove the prospect of continuous teacher learning or the contributions that a PLC facilitator can make in

that learning. For example, teachers are aware that learners make errors as they work on mathematics problems but they are often unfamiliar with the idea that learners' errors are reasoned and reasonable (Ball & Bass, 2003; Brodie, 2014). Understanding the reasoning behind the learners' errors is not easy and needs facilitator support. In a study that examined how mathematics teachers interacted as critical colleagues in a PD program that examined how teachers discussed their learners' engagement with mathematical tasks, Males, et al. (2010) found that "challenging interactions", which are interactions that probed for substantive comments from teachers, assisted participants in the program to push each other "to consider alternative explanations for what was happening in their mathematics classrooms" (p. 7). Participants initially praised each other but hardly challenged each other's practices, engaging in "happy talk" (Boudett, et al., 2008). Challenging interactions were evidenced by the use of, among other things, modal verbs (could, would) and hypothetical language (if, wonder) that were employed in their interactions.

In a study by Francisco & Maher (2011), the researchers worked with teacher interns, middle school mathematics teachers, and mathematics teacher coaches to "learn about the processes involved in students' mathematical reasoning" (p. 52). The researchers facilitated the learners' mathematical activity in a manner depicting how they would facilitate teacher conversations in the study groups: posing questions such as "Tell me more about it", "Why?", "I am confused" or "I am not convinced". This was done to listen more to the learners' reasoning and encourage their thinking. Such probing methods can push teachers to engage in substantive conversations in PLCs, create comfortable space for teachers to be critiqued, and engage in substantive discussions that discourage "happy talk". This study explored the kinds of moves that a facilitator can make to encourage teacher participation in the PLC and consequently increase the depth of the PLC conversations. This study also investigated the extent to which participating teachers could engage with learners in order to understand the learners' reasoning behind their mathematical errors.

In a detailed description of her own and other facilitators' comments, Brodie (2016) found that facilitators were able "to return to previous points, to relate ideas and to make important points in relation to what teachers brought up in the meeting" (p. 173). The facilitator moves that Brodie's (2016) study shares were adapted for analysis in this study.

The teachers' perception of a facilitator is often that of an 'expert' and therefore puts implicit demands on the facilitator for the teachers' learning needs. In a study that focused on 27 experienced mentors (facilitators), Hudson, et al. (2013) share the following phrases that teachers used to describe facilitators and facilitation: "relationships", "sharing", "guiding", "explicit modelling", "constructive helpful feedback" and "facilitate and nurture; critically self-reflect" (p. 1295). These descriptions reinforce the role of a facilitator as both an expert and as a learner, and maintaining a balance between the two. Infused within these roles is the facilitator becoming a "critical friend" in a PLC (Katz, et al., 2009) – becoming both a friend and a critic within a PLC. Brodie (2016) expands on the notion of a critical friend and argues that: "A friend provides supports and care, tolerates weaknesses and celebrates strengths; a critic challenges, points to shortcomings and makes suggestions for improvement" (p. 158). This notion of critical friendship resonates with Males, et al.'s (2010) notion of "critical colleagueship" which was discussed above. For teachers to allow their practices to be critiqued is never easy but affords them "learning opportunities for themselves and their colleagues" (Borko, et al., 2008). Hudson, et al. (2013) saw mentoring as not a "one way street but a reciprocated arrangement where both parties can learn from each other" (p.1296).

2.4.6 Summary so far

The review of literature on facilitation raised important issues which this study sought to pursue. The two studies by Francisco & Maher (2011) and Sherin & van Es (2009) illuminate critical interventions that a facilitator made to improve the teachers' discussions of the learners' reasoning. However, their studies do not include linking the learners' reasoning to the misconceptions in the teachers' discussions, which may be important for addressing learner errors, and therefore the learners' learning needs as well as the teachers' learning needs. Linking learner errors to misconceptions can provide opportunities for teachers to develop a culture of inquiry in their practices. In addition, the study by Francisco & Maher (2011) is not explicit on the kinds of moves that facilitators can make to probe the teachers.

The study by Hudson, et al. (2013) emphasises the role of the facilitator as an expert and a learner. However, facilitator learning in that study is limited to learning within a PLC. Though the investigation of the facilitator's role in this study was on how he facilitated the PLC, but the facilitator was exposed to two settings which he could draw from to engage the teachers in PLC conversations.

2.5 Chapter Summary

This chapter focused on the theoretical perspective which this study is located within, the relevant concepts framing this study, and the review of literature on professional learning communities. I adopted a situated perspective because it emphasises that learning is situated (Borko & Koellner, 2008; Horn, 2005; Lave, 1996; Peressini, et al., 2004), which resonates with what this study investigated through teachers' participation in a PLC over a period of two years as they learned through the PLC with the hope of improving their instructional practices.

Five conceptual constructs were linked to my theoretical framework: teacher conversations in PLCs, teacher participation and learning, instructional practice, teacher knowledge, and facilitation. Each of these concepts were discussed in detail, with teacher conversations providing a thread that holds the other concepts together because it is inherent in each of the concepts.

In reviewing the literature related to this study, I focused on what local and international research explored regarding professional development of teachers within the contexts of teacher's practices, teacher conversations within the PLC, and PLC facilitation. The review illuminated areas of research which this study sought to explore. First, it was important to look at how teachers in this study worked with learners' errors and misconceptions and how this contributed to their practices. Second, this study investigated the extent to which the participating teachers could talk about their practices in collaborative ways with the hope of contributing to their PCK. Third, this study focused on how the facilitator guided and supported the teachers through their professional development.

The next chapter focuses on the research design and methodology for this study. I provide details of how the design is linked to my theoretical perspective and conceptual framework. I also describe the research methods that I used, my data collection techniques and the forms of analysis strategies I adopted for this study.

Chapter 3: Research Design and Methodology

3.1 Research Approach

This research takes the form of a qualitative case study where the case is four mathematics teachers from two schools working as a learning community with a facilitator. Cohen, Lawrence, & Morrison (2000) argue that "case studies penetrate situations in ways that are not always susceptible to numerical analysis" (p. 181) and that the primary focus of qualitative research is on understanding the nature of the research phenomenon rather than the quantity of the observed characteristics or phenomena. In the context of the working of a PLC, a case study can portray a "*thick description* of participants' lived experience of, thoughts about and feelings for, a situation" (Cohen, et al., 2000, p. 182). Through my data collection I accessed the participating mathematics teachers' lived experiences: observing each of the teachers in practice; hearing the teachers' thoughts about their practice and their learners' learning; observing their participation and learning in the PLC; and engaging in formal and informal discussions with the teachers as individuals or as a group to listen to their perceptions of their classroom situations and their learning in the PLC. In addition, I worked with the teachers through the PLC with the hope of enhancing their professional knowledge in mathematics teaching. This research investigated how the process of working with the teachers unfolded in the two years that I worked with them.

My research questions focused on the extent to which mathematics teachers shifted their practices as they participated in a PLC; how they engaged in the different PLC activities; how they talked about the learners' errors and misconceptions and what they learned from those conversations for instruction in the classroom; and how the whole process of teacher participation was facilitated in the learning community. The first research question: To what extent do mathematics teachers' practices shift as they participate in a PLC, required my analysis of the teachers' instructional practices before they worked on the project activities, and how their practices shifted during and after their participation in the PLC.

The second research question: How do mathematics teachers work as a professional learning community to learn with and from each other as they work on learners' errors and misconceptions, addressed the teachers' actual participation in the community. What their conversations focused on as they engaged with the PLC activities, and at what depth; how they

talked about the learners' errors and misconceptions; and the extent to which the teachers' participation in the PLC contributed to their knowledge base. In my investigation of the teachers' participation in the PLC, I also focused on the extent to which my facilitation of the PLC supported the teachers' conversations, looking at what worked well and what did not.

Given these two research questions, a qualitative case-study methodology was appropriate. Yin (2014, p. 70) frames case study research around six elements, and this research study drew from these elements:

- Planning – understanding the research question(s) and rationale for choosing a case study as well as understanding the strengths and limitations of such a study.
- Designing – this stage focuses on the development of a research design, detailing all the necessary procedures governing the research, including data collection and analysis, as well as defining the unit of analysis, which may be an event, an individual or group, or a process.
- Preparing – developing a case study protocol, which refers to: a clear purpose of the study, procedures and research instruments to be used, outline of the study, consent from the participants and other relevant stakeholders, and establishment of the research sites.
- Collecting – the use of multiple sources of evidence to allow for data triangulation and maintaining a chain of evidence, comprising data collected or generated through documents or records, direct observations, or interviews.
- Analysing – involving the use of relevant tools and techniques for analysing the data collected or generated. The process involves paying careful consideration to aspects that may lead to enhancing or inhibiting teacher participation and contribution to the development of the teachers' knowledge base.
- Sharing – writing up the results of the research process in order to share with the interested parties. The sharing may be textual, audio-visual or both. Enough evidence should be displayed for a reader to reach his/her own conclusions.

My study incorporated all the six stages described above as discussed below. The whole process of observing the development in the teachers' practices and their participation for the entire duration of the project required that I carefully plan, design, and make important preparations before collecting relevant data for this research study, and this process justified my selection of a case study method. Planning involved carefully choosing participants who

would be interested to participate in this kind of long-term study and the selection of the research sites appropriate for this study. The design of this study involved appropriating the whole process of data collection: what methods to use, how those methods were to be implemented, how often and at what level of engagement were the methods employed in the participants' learning community and their classrooms. The preparation stage defined how the first two stages were to be implemented at the research sites – the outline of the study as well as the procedures and approved instruments to be used at the research sites. Collecting data included use of multiple sources of evidence, which contributed to the validity of the study. The analysis stage was the critical stage in this research process because it determined the extent to which the research results could be interpreted in ways in which those results could make sense. The stage of analysing and producing results that make sense justified the last stage: sharing the results with the research community, in journal articles, book chapters and this thesis (Brodie & Molefe, 2014; Brodie, et al., 2014; Molefe, 2016).

An important attribute of a case study is its focus on "a real situation, with real people in an environment familiar to the researcher" (Opie, 2004, p. 74). As a researcher for this case study, I am a former high school mathematics teacher and very familiar with the area in which the schools were located. Going to the two schools to collect the data from the teachers' classrooms helped me to be not only familiar with the environment outside the classroom but with the one also inside the teachers' classrooms. Familiarity with both contexts afforded me opportunities to understand teacher learning in these two contexts, which Kazemi & Hubbard (2008) define as co-evolution of practice.

Being in the teachers' classrooms to observe them in practice increased the level of familiarity with the school and classroom environment. In addition, my presence in the PLC meetings; facilitating the PLC; listening to how the teachers talked about their practices in the PLC; and how they promoted a spirit of community among themselves contributed to my familiarity with the teachers' environments. One of my research questions focused on how the mathematics teachers worked as a PLC to learn with and from each other. This question raised on my part the importance of observing and listening to how the teachers in the PLC talked about their practice and what they learned from such talks. Lastly, my previous experience of working in a classroom as a mathematics teacher as well as of working with a group of teachers previously in a professional development project contributed a lot in increasing my familiarity with the environment in which I conducted my study.

My research study formed part of a bigger project that involved a total of twelve schools. This is the DIPIP project mentioned in the different sections of this thesis, and details of the activities of the project are described in Section 3.3 below. This study focused on four of the schools, which are discussed in detail below.

3.2 Research sites and participants in the study

This section describes the schools in which the research study was conducted as well as the mathematics teachers who participated in the professional learning community investigated in this study. I discuss the process and criteria that were followed for the selection of the participating schools and the teachers. Biographical data of the participating teachers is given in Table 3.2 below.

3.2.1 Research sites

The DIPIP project established one learning community per high school with a minimum of four to five teachers teaching mathematics in grades 8-10. In the case of this community, there were only two or three teachers in each of four schools so the community was established across the four schools to allow for a sufficient number of teachers in the PLC. Some teachers taught in Senior Phase (SP) schools and others in Further Education and Training (FET) schools[2]. Initially there were three schools and later a teacher from a fourth school (SP + FET school) joined the community. Table 3.1 below shows how the participating teachers were spread across the four schools. For purposes of anonymity, I refer to the two SP schools as A and C, the one FET school as B and the fourth school as D. Schools B and C were directly next to each other, with C being a feeder school for B.

[2] Senior Phase schools, also called junior secondary, had class grades 7-9 and Further Education and Training school had class grades 10-12.

Table 3.1: Research sites and teachers' participation.

Name of teacher[3]	School	Year of participation	School type
Khumo	A	2011 & 2012	SP
Mapula	A	2011 & 2012	SP
Funeka	B	2011 & 2012	FET
Dimpho	B	2011 & 2012	FET
Mandla	B	2012	FET
Cikiswa	C	2011	SP
Cele	D	2012	SP & FET

The criteria that the project used for selecting the schools was to have teachers who were interested in working towards improving their instructional practices, together with a supportive principal and mathematics HOD in the school, and that the school was functional[4]. All the four schools mentioned above satisfied these criteria for selection. The District Office also requested that the project worked with these schools. All the schools were located in townships[5] and had been declared no-fee schools[6] by the Department of Education. The schools had limited resources for teaching and learning. For example, they did not have libraries and science laboratories, and some did not have photo copying machines.

The project team, including myself, met with the principal and the teachers of each school to explain the project and our research interest. Based on discussions from these meetings, I decided to work with the above-mentioned four schools for my research study. The choice of schools in the project was primarily based on the functionality of the schools as well as the commitment that the teachers showed towards the learning community.

3.2.2 Research participants

Funeka, Mapula, Khumo and Dimpho participated in the project for the full two years and data on their instructional practices were analysed over the two years. Cikiswa had serious health

[3] For purposes of anonymity, pseudonyms have been used for the teachers' names.
[4] Some schools in South Africa are termed "dysfunctional", meaning that classes are not attended regularly and lessons often disrupted.
[5] A township is a South African term that refers to an often underdeveloped urban living area initially reserved for black residents during the apartheid era.
[6] No-fee schools do not charge learners any fees due to their low socio-economic status. The main funding for these schools comes from the government.

problems, so her instructional data is not analysed. Mandla and Cele joined the community in 2012 and we did not have enough data for their practices to be analysed. The community data were analysed for the teachers' conversations and PLC facilitation, so all the seven participants' contributions in the PLC meetings were analysed, regardless of how long they were in the community.

The participating teachers had varying experiences of teaching mathematics and Table 3.2 below shows a detailed account of their biographical data. The teaching experience is as at the second year of data collection.

Table 3.2: Biographical data of the participating teachers.

Teacher's Name	School	Qualifications	General Teaching Experience (in years)	Mathematics Teaching Experience (in years)	Major Subject(s)
Mapula	A	BA (Ed)	28	23	Sepedi; Business Economics; Economics
Khumo	A	PTC[7]; SEC[8]; SED[9]	29	18	Biology; Zulu
Funeka	B	BA (Ed)	5	4	Sepedi; Life Orientation
Dimpho	B	BA (Ed); BSc (Hon)	19	19	Mathematics; Setswana
Mandla	B	BSc; HDip Ed(PG)	19	17	Biochemistry; Mathematics; Physical Science
Cikiswa	C	STD (Senior Teachers Diploma)	25	22	Accounting; Economics
Cele	D	Dip in Educ; B.Tech (Ed)	17	17	Mathematics

Only Dimpho, Mandla and Cele did mathematics as a major for their pre- and/or in-service training. This meant that from the four teachers whose classroom practices were analysed for this study, only Dimpho qualified professionally to teach mathematics. The other three teachers taught mathematics because the school had a vacancy for a mathematics teacher and they agreed to take up that post. The teacher with the longest, general professional teaching experience among the four was Khumo, followed by Mapula, with Mapula also having the

[7] PTC (Primary Teachers Certificate) is an old qualification which could be pursued immediately after completing Junior Certificate at school, which was an equivalence of grade 10.
[8] SEC (Senior Education Certificate) qualified teachers to teach Senior Phase classes, i.e. grades 7-9.
[9] SED (Senior Education Diploma) as a diploma was a higher qualification than SEC.

longest experience in teaching mathematics. Khumo and Cikiswa were the only two teachers who did not have a degree but had a teaching diploma.

Each teacher chose one class to work with in the project in each year of their participation, and these were the classes whose lessons were analysed for this study. Table 3.3 below shows the classes that the teachers chose for the project. All the teachers, except for Mapula, worked with the same grade for the two years of participation.

Table 3.3: Classes taught over the two-year data collection period.

Name	Class taught in 2011	Class taught in 2012
Mapula	9	8
Khumo	7	7
Funeka	10	10
Dimpho	11	11
Mandla	N/A	10
Cikiswa	7	7
Cele	N/A	11

The Grade 7 class that Khumo worked with in 2011 was the same one that Mapula worked with in Grade 8 in 2012 – most of those learners were the same for the two grades. Similarly, the Grade 10 class that Funeka worked with in 2011 was the same one in Grade 11 that Dimpho worked with in 2012. This kind of class selection from one year to the next by the participating teachers supported the teachers to implement what they learned and to build collective learning in the school.

3.2.3 DIPIP Project

The Data Informed Practice Improvement Project (DIPIP) was a research project which ran in three phases from 2008-2014. Phases 1 & 2 of the project worked with learning communities of teachers at the University of the Witwatersrand from 2008-2010. Teachers came from their schools once a week in the afternoon to work on various activities of the project. Phase 3 of the project was taken to different schools across Gauteng province from 2011-2014, and the schools were not the same as the ones which participated in Phases 1 & 2 of the project. In Phase 3, mathematics teachers from twelve schools participated in the project, and the PLCs at

these schools were facilitated by the project team members from the university. In 2013 and 2014, lead teachers were trained in order to facilitate their communities in those two years and the university facilitators monitored the processes in the PLCs of the different schools. It was hoped that the PLCs would continue to function beyond the data collection period, hence the importance of teachers taking ownership of the community. However, teacher leadership and ownership were not analysed for this study.

Data for this research study was generated from Phase 3 of the project. Teachers worked with facilitators to understand the importance of embracing learners' errors in the teaching and learning of mathematics. My facilitation of the PLC was also a subject of my study in order to understand how facilitation worked in the PLC and how that facilitation could be used in other PLCs. The project team met on a weekly basis to reflect on activities of all the PLCs in the project, and these meetings helped to improve facilitation (Brodie, 2016; Brodie, et al., 2014). The meetings also helped the project team to develop an important document on facilitating PLCs, whose codes are discussed in Section 3.6.2 below.

There are two key challenges that were anticipated for the PLCs: First, the retention of teachers in the PLC for the duration of the programme. Second, teacher ownership was key as it could encourage regular participation in the PLC. The project team worked hard to support the teachers in their learning and encourage them to stay, and in this community four out of the seven teachers were retained in the first two years of the project, while three stayed for all four years, and three for three years. Regarding the second challenge, DIPIP worked with teachers in "placing more responsibility for the development of the programme" with the teachers involved (Te Maro, et al., 2008). Teachers committed to their participation and were supported in various ways to increase and sustain their participation, and this also somewhat contributed to addressing the first challenge. Brodie (2019) reports on the reasons why teachers may have remained committed to or left the project.

The support that the participating teachers received helped them to reflect on their practice as individuals and also as a community. To maximise the teachers' participation, teachers were given laptops to watch their videotaped lessons and reflect on them. The laptops also served as appropriate resources, more than an incentive, for the general needs of the teachers. Teachers would use them to reflect on their practices and also for their teaching and administrative tasks in their schools.

Principals of the participating schools were also involved from the beginning of the project by supplying them with essential information about the project and they were updated regularly over the two-year period of this research study and beyond. Research argues for the establishment of a strong relationship between school management (through the principal) and a PLC as an aspect that can contribute to the sustainability of a professional development programme (Bolam, McMahon, Stoll, Thomas, & Wallace, 2005; Collinson & Sherrill, 1996).

3.3 Activities

All the PLCs for the DIPIP project met regularly, once a week for two hours during school terms, except when other school commitments intervened. In those meetings, the participants engaged in a sequence of activities which were developed by the project team (see Appendix 1 for the project activities).

Curriculum mapping

Learners wrote an algebra test which has been used over a number of years in England[10], and which has been shown to reveal interesting errors that learners make when they respond to questions in mathematics. The project decided to use only the algebra part of the test and selected questions that we felt were relevant for use with learners in South African schools, considering the contextual and curricular differences that exist between South African and English learners.

Curriculum mapping helped teachers work actively with the curriculum in relation to the tests and understand the depth at which concepts were being developed in the curriculum and the tests (Brodie, Shalem, Sapire, & Manson, 2010a). In addition, in engaging with this activity the PLC was able to see the fairness of the test in relation to the South African context. In this activity, we used a template that guided the PLC to look at each item of the test and see how that item fitted with the South African curriculum. The template also encouraged teachers to find assessment standards[11] (ASs) that related to particular test items and justify their selection of those ASs. They then discussed the mathematical concepts embedded in the test item and

[10] The test came from the "Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) project" at King"s College, London.
[11] Assessment Standards provided specific criteria for assessing learners according to certain levels that learners were expected to reach in accordance with the curriculum outcomes.

the ASs and discussed whether they taught those concepts and when. This activity supported a deeper understanding of the mathematics curriculum both within and across grades (Brodie, Shalem, Sapire, & Manson, 2010b).

Error analysis

In this activity the teachers noted all the common and interesting errors that learners had made in the test and captured them on a template for analysis. The main focus of the error analysis activity was for the teachers to learn about the reasoning behind the errors that learners produced when they answered questions in the test. The template guided the PLC to look at the content of the test item; agree on the different methods of getting the correct response to a test question; discuss the type of errors that learners made; get an understanding of how and why learners could have made those errors; and agree on the critical concepts embedded in the test question under discussion. The conversations that ensued in the PLC when discussing the learners' errors and the possible reasoning behind those errors illuminated the teachers' understanding of the errors and how learners learn algebra. This understanding assisted the PLC in developing the professional knowledge that the participating teachers needed for their practice.

Chapter 2 of this thesis discusses literature on errors and misconceptions and its relevance to this study. Error Analysis helped the teachers to understand that errors and misconceptions are a normal part of learning mathematics and can be embraced rather than ignored or avoided (Brodie, 2014). Error Analysis provided us with an opportunity to look across the learners' errors and see how they linked to key concepts in mathematics. This process of linking errors to key mathematical concepts assisted teachers in understanding the deeper meaning of the learners' errors, and contributed to the choosing of a leverage concept. Brodie & Shalem (2011) define a leverage concept as "a concept which should be central to the curriculum and should support teachers to work in a range of topics across the curriculum, and in all the grades" (p. 6). In focusing on how participating teachers shifted their practices by learning through a PLC, Chauraya & Brodie (2017) show how the error analysis activity helped the teachers to identify "that learners did not fully understand the conceptual meanings and connections in the concepts" (p. 230).

Learner Interviews

Teachers identified test items from the Error Analysis activity to use in interviews with selected learners. The interviews were used to obtain more information from learners for items whose analysis was not fully exhausted in the Error Analysis activity. It was hoped that through those interviews learners would assist in providing the reasons behind their errors as they were interviewed. As teachers analysed common errors that learners made in the test, they identified learners whom they thought were interesting to follow up in respect of the errors that they made in the test. Particular learners were selected for particular items of the test and these learners were then interviewed by their own teachers.

An interview protocol for learner interviews was developed by the project team and discussed in the PLC. The interview protocol was prepared in order to assist the participating teachers with: ways of settling the learners during the interviews; keeping the learners interested and open to talk about their errors; and probing learners' ideas, rather than teaching towards the correct answer. The PLC role-played the learner interviews in the community meeting before conducting the actual interviews with learners: one teacher played the role of a learner and another of a teacher, and the other teachers gave feedback. The reason for role-playing the interviews was to prepare teachers for the activity and build their confidence as they had never conducted learner interviews before.

The PLC reflected on the learner interviews in order to get common understandings on the similarities and the differences in the learner responses in the interviews. The reasons that learners gave in the interviews for producing errors helped teachers to better understand the deeper misconceptions that accompanied the learners' learning of new concepts. Other than helping the teacher who interviewed the learner understand his/her learner's thinking and reasoning, the learner interviews also helped members of the PLC to have meaningful conversations about learners' thinking and substantiate why learners make particular errors when they learn mathematics or answer mathematics questions.

Readings and discussion on leverage concepts

This activity built on the findings of the curriculum activity, the error analysis activity and the learner interviews. The idea was to identify a mathematical concept which had an effect across

a number of mathematical topics and errors in the curriculum, referred to as a leverage concept (Brodie & Shalem, 2011). The facilitator then found research literature that the community could read to extend the teachers' understanding of the identified concept and common learner errors related to the concept. Two concepts were identified in this PLC: the notion of a variable and the language of algebra. The teachers were given probing questions to guide them through the reading of the research papers.

Discussions of the literature in the PLC informed the next activity: joint lesson planning. The activity required that teachers think of ways in which important aspects of the lessons were considered and anticipated. For example, teachers were encouraged to anticipate the type of errors that learners would produce when answering particular questions and this helped them to devise strategies in advance to deal with some of the errors as they came up.

First round of lesson planning and teaching

With this activity, it was hoped that the PLC would incorporate ideas emanating from the conversations on the curriculum mapping, the error analysis, the learner interviews and the discussions of the literature into the lessons they planned. The first concept that the PLC agreed to develop with their learners: the notion of a variable, was found to bring serious misconceptions to the fore that teachers and learners could learn from and address. After these lessons were planned, each teacher taught the lessons to his/her class and was videotaped.

Lesson Reflection

The lesson reflection activity was three-fold: individual teachers reflected on their lessons; the community reflected on the teachers' lessons as a group; and the different PLCs in the project met in a bigger meeting with other PLCs in the project to reflect on the different teachers' lessons, referred to as Networked Learning Communities (NLCs) (Katz & Earl, 2010; Katz, et al., 2009).

Teachers were given their lesson videotapes to watch at their convenience, using laptops given to them by the project. Teachers were asked to identify episodes in their lessons where they thought they dealt well with the learners' errors, and episodes in which they thought they did

not deal well with the learners' errors. Only the teachers' selected episodes were reflected on in the community as well as in the bigger meeting.

Each teacher's selected episodes were played back in the PLC meeting and the other teachers helped the teacher whose lesson was watched to reflect on her practice. A guiding template was availed to the teachers for this lesson reflection. There were important norms and values that were to be observed during the lesson reflection activity (these were also practised in the other activities):

- Focus of the discussion was to be on the lesson and not on the person.
- Challenging each other's practices and ideas did not mean challenging the person.
- The discussion was to focus on the learners' errors and how the teachers dealt with those errors in class.
- Listening to and respecting each other's ideas was crucial.

It was acknowledged that it was not easy for anyone to open up their practice to other teachers through lesson reflection and it could have been unsettling for some teachers to have other teachers watch their lesson videotapes in their presence and to critique their lessons. It was my primary responsibility as the facilitator to create an atmosphere of trust for rich and rigorous inputs during these conversations.

The process of reflecting on the lessons was intended to help the teachers to inquire into their practice and allow PLC members to provide suggestions regarding their instructional practice. The process allowed for a pedagogic movement between the classroom and the PLC, wherein teachers brought data form their classrooms to the PLC for inquiry, and they took ideas resulting from the PLC conversations to their classrooms. This process became a continuous learning cycle for the participating teachers. A lesson videotape provides a powerful artefact of practice for teacher development (Borko, et al., 2008), as teachers can use this artefact to reflect on their practices in different ways and learn from that process. The PLC focused on teachers shifting their practices in order to take account of and work with their learners' mathematical errors. Reflecting on their lessons provided a space for teachers to be critical of themselves and account for what happened in their classrooms.

The bigger meetings (NLCs) were organised in order to allow the different communities to present their findings from the different activities that the PLCs had worked on. This platform

created opportunities for teachers to learn through the project activities more deeply: teachers heard different views from teachers of other schools; new practical knowledge was created during the discussions; and the teachers were afforded opportunities to reflect in depth on the findings of the PLC they belonged to. In addition, the meeting was a confidence-builder for teachers and contributed towards extending good social and professional relations between teachers and schools.

Second round of lesson planning and teaching

Conversations that teachers had while reflecting on their lesson videotapes helped them in knowing what their strengths were and what needed attention in relation to how they embraced errors and misconceptions in their teaching. Based on these conversations the participating teachers were in a better position to see what aspects of their practices could be developed, individually and collectively. These conversations informed how teachers could plan subsequent lessons which they would teach in future.

The teachers were given a chance to re-teach the same concept of the notion of a variable at the beginning of the second year of the project. This was done after the project team and the teachers realised there were common issues that surfaced during the PLC and NLC meetings. The PLC took the same lesson plan on the notion of variable which they used previously and incorporated feedback ideas from their lesson reflection activity as well as from the NLC meeting.

Third round of lesson planning and teaching

Teachers went through a similar process as in the first round of lesson planning: they identified a concept which they thought was critical for learners to learn, which was the use of mathematical language in the teaching and learning of algebra. In choosing the concept, the PLC considered: the error analysis activity they did the previous year, and looked at what they had captured for the learners' errors in that activity; their reflection sessions from the first concept they taught; and the reflections they did on the last network meeting. Research papers were sourced for the PLC to read and discuss, and ideas from the literature and the PLC conversations were incorporated into the lesson planning activity. All the teachers taught the lessons, and those lessons were videotaped and reflected on in depth.

3.4 Data collection methods and instruments

Data collection for this research took place over the two years: 2011 and 2012, at the same time as the development of the learning community. The following methods were used to collect data:

- Biographical data and interviews with teachers
- Audiotapes of PLC conversations
- Videotapes of teachers' lessons.
- Field notes

Biographical data

All the teachers who participated in the project were given a form to capture their biographical information. Included in this form were teachers' qualifications and where these were obtained, the grades and all subjects that they taught, how long they had taught, how many learners they taught in that year, and how they travelled to school. Part of this information is discussed in Section 3.2 above.

PLC conversations

All meetings of the PLC were audio recorded and transcribed for analysis. Table 3.6 below shows the PLC activities for the different PLC meetings that the teachers engaged in and from which data was collected.

Table 3.6: PLC meetings conducted over two years.

Date	Activity
14 March 2011	Introduction meeting
11 April 2011	Outline of PLC activities
18 April 2011	Curriculum Mapping
5 May 2011	Curriculum Mapping
9 May 2011	Curriculum Mapping
12 May 2011	Curriculum Mapping
16 May 2011	Curriculum Mapping
6 June 2011	Error Analysis
13 June 2011	Error Analysis
20 June 2011	Error Analysis
25 July 2011	Error Analysis
11 August 2011	Preparations for learner interviews
22 August 2011	Reflection on learner interviews
29 August 2011	Reflection on learner interviews
5 September 2011	Reading and Discussion
23 September 2011	Lesson Planning (First Round) – lessons referred to as cycle 1 of JPL1 below
10 October 2011	Lesson Panning (First round)
13 October 2011	Lesson Planning (First round)
24 October 2011	Preparations for Lesson Reflection
27 October 2011	Lesson Reflection (First round)
6 February 2012	Lesson Reflection (Second round + new lesson planning) – lessons referred to as cycle 2 of JPL1 below
13 February 2012	Lesson Reflection (Second round + new lesson planning)
12 March 2012	Lesson Reflection (Second round + new lesson planning)
19 March 2012	Lesson Reflection (Second round + new lesson planning)
7 May 2012	Reflection on learner interviews
11 June 2012	Lesson Planning (Third round) – lessons referred to as JPL2 below
25 July 2012	Lesson Planning (Third round)
20 August 2012	Lesson Planning (Third round)
10 September 2012	Preparation for network meeting
11 October 2012	Lesson Reflection (Third round)
15 October 2012	Lesson Reflection (Third round)

The table shows that the PLC had different number of sessions for the different activities of the project, and this difference depended on the teachers' level of comfort with the activity as well

as their pace of completing the activity. The timeline for the second year excluded certain activities from the first year. For example, Error Analysis was replaced by the re-planning of the concept lesson of the previous year, and the curriculum mapping activity was left out as teachers saw it as time-consuming and not very useful. There were fewer sessions in 2012 because of the exclusion of the activities which were not helpful for my analysis. 2012 was generally more focused than 2011 in as far as the project activities were concerned.

Teachers' lessons

Five sets of lesson videotapes were collected for each teacher at various points during the two-year data collection period. In each set about three to four lessons were videotaped, depending on the lengths of lessons, except towards the end of the data-collection period when teachers indicated lack of time and only two lessons were videotaped per teacher. The videotaping of lessons was done at various points during the two years beginning in early 2011 before project activities started and ending in late 2012. Table 3.7 below shows the different types of lessons which were videotaped and analysed for this study.

Table 3.7: Three types of videotaped lessons over the two-year data collection period.

Individually planned lessons (IPL)	**Jointly planned lessons (JPL)**	**Individually planned lessons (IPL)**
Normal teaching lessons taught at the beginning of the intervention period	Lessons based on the lesson plan that the teachers worked on together.	Lessons that were taught at the end of the two years' data collection period
Teachers had not started working on DIPIP activities	Teachers had analysed learners' errors, interviewed selected learners, chosen a concept and planned lessons together.	Teachers had completed two cycles of all the DIPIP activities

The table shows two sets of individually planned lessons (IPL): one in the first column and the other in the third column. The first set of IPL was videotaped before teachers worked on any activity of the project and the second was videotaped after teachers had worked on various activities of the project. There were three sets of jointly planned lessons over the two-year data collection period. I refer to these lessons as jointly planned lessons one (JPL1) and jointly planned lessons two (JPL2), with JPL1 having two cycles of teaching and reflection.

For all the JPL lessons, the PLC decided on the concept that they wanted to teach as discussed above: for JPL1 – the notion of a variable, and for JPL2 – the language of algebra. As described

above, the facilitator found research papers that dealt with the chosen concepts and gave those to the teachers to read, discuss in the PLC meetings, and use in their lesson planning. For JPL1, teachers taught the lessons and reflected on their teaching of the concept. This teaching is referred to in Chapter 4 as JPL1 cycle one. After the teachers had reflected on their teaching, they re-planned the lessons using their reflection as a community and re-taught the lessons. This teaching is referred to in Chapter 4 as JPL1 cycle two.

Once the second cycle of teaching had been reflected on, the teachers focused on a new concept for lesson planning to be developed with learners, whose lessons are referred to as JPL2 in Chapter 4. There was only one cycle of planning, teaching and reflecting for JPL2.

Field notes

I took field notes at different stages of my data collection: during my lesson observation, during PLC and NLC meetings, as well as during informal discussions with the teachers regarding their practices and their participation. These notes were used to supplement data obtained through video or audio recording. The notes helped capture what the video and audio could not capture. An example of these notes was: how different teachers informally discussed their strengths and weaknesses with me as the facilitator. In the next section, I show how my data collection methods link to my research questions.

3.5 Link between methods and research questions

All methods that have been discussed above to collect data helped me to address my research questions. Table 3.8 below provides a summary of which data collection method helped in answering which research question(s). The letter "Y" has been used to indicate that the data collection method corresponds to the question. A blank implies that the method was not used for the question. The numbering used in the table, i.e. 1.1 – 2.3 corresponds directly to the numbering used in my research questions in Chapter 1.

1. To what extent do mathematics teachers' practices shift as they participate in a PLC?
2. How do mathematics teachers work as a professional learning community to learn with and from each other as they work on learners' errors and misconceptions?

The main questions were elaborated by the following sub-questions:

1.1 How do teachers engage with the learners' errors in their classrooms?

1.2 How do teachers create conditions for enhancement of learning in their classrooms?

2.1 What do teachers in a PLC talk about and at what depth do they engage with different activities?

2.2 How is pedagogical content knowledge (PCK) amongst teachers created within the professional learning community?

2.3 How is a space for engaging with the different activities in a PLC created by the facilitator?

Table 3.8: Data collection methods and the related research questions.

Data collection methods	**Research Questions**				
	1.1	**1.2**	**2.1**	**2.2**	**2.3**
Audiotaping of PLC conversations	-	-	Y	Y	Y
Videotaping of lessons	Y	Y	-	-	-
Field notes	Y	Y	-	Y	-

3.6 Data Analysis

This section focuses on the strategies that I used to analyse the different data sets I have for this study. My main data sources were classroom video footage and PLC meeting conversations. PLC meetings included the transcripts from those conversations, and these together with PLC audio data were used for analysis of PLC conversations and PLC facilitation. Field notes were used to substantiate discussions in the PLC conversations where the need arose. I discuss the data analysis procedures for the three key foci of my research: teacher change, PLC conversations and PLC facilitation.

3.6.1 Teacher change

Each teacher selected a class in which she could be videotaped during the teaching of the two individually planned sets of lessons and the three sets of jointly planned lessons (see Table 3.7 above & Table 3.10 below for more detail). My analysis focused on all these lessons, except for a few that were excluded due to technical problems with the video camera, about two-three lessons per teacher that did not have good sound quality. Analysis of the videotaped lessons ascertained the extent to which the shifts in the teachers' practices occurred.

To code the teachers' lesson videotapes, I used Hill's (2010) Mathematical Quality of Instruction (MQI) coding scheme (see Appendix 2). The instrument has five key dimensions, with corresponding sub-dimensions as shown in Table 3.9 below:

Table 3.9: MQI coding scheme.

Dimensions	Sub-dimensions	Levels		
		L1	L2	L3
Mode of Instruction	Direct instruction			
	Whole-class discussion			
Richness of the Mathematics	Linking or connections			
	Explanations			
	Multiple procedures or solution methods			
	Developing mathematical generalisations			
	Mathematical language			
Working with Students and Mathematics	Thorough remediation of student errors and difficulties			
	Responding to student mathematical productions in instruction			
Errors and Imprecision	Major mathematical errors or serious mathematical oversights			
	Imprecision in language or notation (mathematical symbols)			
	Lack of clarity			
Student Participation in Meaning-Making and Reasoning	Students provide explanations			
	Student mathematical questioning and reasoning			
	Enacted task cognitive activation			

The dimensions in Table 3.9 fitted with the kinds of instructional improvements DIPIP was interested in, as reflected in my research questions. The dimensions enabled me to describe the teachers' practices during the videotaped lessons and give an account of what was happening in the teachers' mathematics classrooms. Within each sub-dimension are three levels: 1, 2 & 3, where level 1 is considered as 'low' and level 3 is considered as 'high', except in the dimension of *Errors and Imprecision* where level 1 is 'high' and level 3 is 'low'. More details on the levels are given below.

Hill's (2010) MQI coding scheme sub-divides a lesson into seven minutes for each segment of a lesson. I did not adopt the seven-minute segment division because of the difference in the length of the teachers' lessons in the two schools: one school used 55-minute periods and the

other used 40-minute periods. For purpose of convenience, I used eight-minute segments since the number eight is a multiple of 40, and 55 also being close enough to 56 as the next multiple of 8. This was done in order to get as many full episodes as possible. The coding was therefore done according to the sub-dimensions for every eight-minute segment.

Each eight-minute segment in my analysis is referred to as an episode. The table below shows the number of episodes that were analysed for each teacher per set of lessons. Appendix 3 exemplifies how the lessons were coded.

Table 3.10: Number of episodes analysed per teacher.

Name of Teacher	IPL	JPL1, cycle 1	JPL1, cycle 2	JPL2	IPL	Total episodes
Funeka	14	26	12	9	20	81
Dimpho	14	30	21	22	17	104
Mapula	15	14	13	8	16	66
Khumo	9	18	19	9	15	70

Funeka and Dimpho had the 55-minute periods while Mapula and Khumo had 40-minute periods, hence the difference in the total number of episodes between Funeka and Dimpho from one school and Mapula and Khumo from the other school. There were also more lessons recorded for some teachers in some cases. Some periods were shortened due to delays in the school beyond the teachers' control.

I started coding four lesson videotapes with members of the project team for purposes of inter-rater reliability – two with the whole team, and two as individuals and compared our coding results. After reaching agreement of over 80 percent, I then continued to code the rest of the videotapes. As the videotape was played back, it was stopped every eight minutes to record the specific level observed for the episode for each sub-dimension.

These records were then converted to percentages that described proportions for each level within a particular sub-dimension, so as to compare across the different sets of lessons with different numbers of episodes. The coding was done to illuminate the shifts in the teachers' practice from the IPL1 lessons, through the JPL1 & JPL2 lessons, to the IPL2 lessons. A shift in the teacher's practice was viewed as improvement if it moved from a lower level towards a higher level, e.g. from level 1 to level 2 or from level 2 to level 3, except in the dimension of

Errors and Imprecision as explained above, where a decrease in level for the sub-dimensions meant that the teacher made fewer errors or taught less inaccurately and was therefore an improvement.

3.6.2 Teacher conversations in PLC meetings

The coding scheme for the PLC conversations was developed by the project team through a number of meetings in order to use them for analysing all the PLC meetings. All the PLC meetings investigated in this research study were audio recorded and there was an accompanying transcript for each audio file that supported the analysis. To analyse the PLC audio data, I used StudioCode™ software to code all the meetings analysed for this study and did an analysis of the results of this coding focusing specifically on the coded times, which were then converted to percentages.

The teacher conversations during the PLC meetings were analysed using three main codes:

- Activity – defines the activity which the PLC worked on in a meeting: Curriculum Mapping; Error Capturing; Error Analysis, Readings and Discussion, Lesson Planning, Lesson Reflection; Preparation for Network Meeting; Preparation for Learner Interviews; Reflection on Learner Interviews. Many of the activities were done more than once over the two years.
- Content – describes the specificity of the PLC discussion, whether the substance of the conversation was on the learner, on mathematics, on instructional practice, or on DIPIP priorities[12] (see Table 3.11 below for more details).
- Depth – describes the depth of the conversation and uses four levels, L1-L4, where L1 represents a low and L4 represents a high in the richness of the conversation (see Table 3.12 below).

Activity

For purposes of this study, out of the six activities that the PLC worked on, only four activities: Error Analysis, Readings & Discussion, Lesson Planning, and Lesson Reflection, were

[12] DIPIP priorities relate to what the DIPIP project encouraged, for example, understanding the learner thinking behind learner errors or not blaming learners for their errors.

considered for analysis. Curriculum mapping was only done in 2011 and was not included in 2012 because it was time-consuming, and the participating teachers found it less worthwhile than the other activities. Learner interviews were excluded because the teachers did not provide all the necessary data for the learner interviews. Teachers were given audio recorders to conduct learner interviews, but some of the teachers did not use them when interviewing the learners and data was therefore not captured. Error Analysis was not done in 2012 but it was included in the analysis because it cut across all the activities of the project.

Content

The analysis of content drew on codes for content developed by Borko and her colleagues (Borko, et al., 2008). We modified these to reflect DIPIP goals (Brodie, 2016), and settled on four content codes: learner, mathematics, practice and DIPIP priorities. The PLC conversations were analysed in order to see the extent to which the community focused on specific content. Table 3.11 gives the description for each content code.

Table 3.11: Content codes.

Learner	Mathematics	Practice	DIPIP Priorities
Talk where teachers focus on learners' understanding, error, strategies, language, attitudes or participation.	Teachers talk about their own mathematical knowledge.	Focus on what the teacher does in class in relation to instruction and use of resources, more general aspects of practice (i.e. not focused on learner or engaging with learner thinking).	Talk about learner thinking: goes into reasons for learner thinking, anticipate or predict learner thinking. (other priorities were identified but they are not discussed here)

An important difference should be noted between learner understanding under "Learner" and learner thinking under "DIPIP priorities" - learner understanding refers to a description of the learner's knowledge or strategies, while learner thinking refers to talk about the reasoning behind the learner's error.

Depth

The PLC conversations were also analysed to establish the depth of the conversation, and Table 3.12 below shows the different levels at which the depth of the conversation could be coded

for mathematics and for the other three content codes. To develop these codes, we drew on levels from van Es' (2011b) work and modified them to reflect DIPIP goals (Brodie & Chimhande, 2020). We developed different indicators of depth for discussions on mathematics and for the other three codes.

Table 3.12: Depth in community conversations (adapted from van Es, 2011)

For Learner, Practice and DIPIP Priorities		For Mathematics
Level 1 • Form general impressions of what occurred • provide descriptive and evaluative comments • Provide little or no evidence to support analysis	When interpretive/analytical questions are asked but there is no follow up the cognitive depth is at level one.	Calling out of answers, terms, etc. No engagement with mathematical ideas.
Level 2 • Form general impressions and highlight noteworthy events/examples. • Provide primarily descriptive with some interpretive or analytic comments • Begin to refer to specific events or examples or interactions as evidence (may not be relevant to all DIPIP activities)	In order for an event to be noteworthy an example needs to be discussed but it does not necessarily mean the code is Level 2. If a discussion about the example/event takes place where teachers interrogate or work with the example, code level two. Interpreting is **not** about how the individuals interpreting each other's comments. If interpretive/analytical questions are asked **and** followed up with a discussion the cognitive depth may be level 2	Some engagement with mathematical ideas and attempts to explain -mathematically.
Level 3 • Highlight noteworthy events • Provide interpretive/analytical comments • Refer to specific events/examples and interactions as evidence • Elaborates on events/examples and begins to generalise		Engagement with ideas with some new understandings for some of the participants, Examples used embody general principles
Level 4 • Level 3 comments plus: • Generalising • Making connection between events/examples and principles of teaching and learning	When the discussion links concepts, we may consider Level 3 or Level 4. Generalising means if there is a build-up in a sequence of terms to a generalised statement or someone generalises the conversation Generalising is **not** stating a general principle	General mathematical ideas are discussed. Substantially new ideas are discussed.

Coding of depth was done to investigate if the different PLC activities influenced the depth of the conversation and if the depth of the conversation changed over time. In looking at the relationship between activity, content and depth of community conversations in a different

community, Chimande & Brodie (2020) regarded "the quality of the teachers' talk as a measure of the depth of the teachers' engagement with mathematical ideas, issues of practice and learners' thinking" (p. 122).

Coding time

Table 3.6 showed that there was a total of 31 PLC meetings. Table 3.13 below shows that of these, 22 of them, or about two thirds were analysed. Meetings that were not analysed included: discussions on capturing of the learners' errors; preparations for and reflections on learner interviews; and preparation for the network meeting. These did not provide substantive data that could answer the research questions.

Table 3.13: Community meetings which were analysed for this study.

Date	Activity
20 June 2011	Error Analysis
25 July 2011	Error Analysis
5 September 2011	Readings and Discussion
23 September 2011	Lesson planning (First round)
13 October 2011	Lesson planning (First round)
27 October 2011	Lesson Reflection (First round)
6 February 2012	Lesson Reflection (Second round + new lesson planning)
13 February 2012	Lesson Reflection (Second round + new lesson planning)
12 March 2012	Lesson Reflection (Second round + new lesson planning)
19 March 2012	Lesson Reflection (Second round + new lesson planning)
26 March 2012	Lesson Reflection (Second round + new lesson planning)
4 June 2012	Readings and Discussion
11 June 2012	Readings and Discussion
28 May 2012	Lesson Planning (Third round)
11 June 2012	Lesson Planning (Third round)
25 July 2012	Lesson Planning (Third round)
13 August 2012	Lesson Planning (Third round)
20 August 2012	Lesson planning (Third round)
11 October 2012	Lesson Reflection (Third round)
15 October 2012	Lesson Reflection (Third round)
22 October 2012	Lesson Reflection (Third round)
29 October 2012	Lesson Reflection (Third round)

Table 3.14 below shows the amount of time that was coded for the conversations. These times were generated using StudioCode™ as stated above.

Table 3.14: PLC activities that were coded.

Activity	2011	2012	Total time over 2 years
Error Analysis	2 (02:59:11)	-	2 (02:59:11)
Readings and Discussions	1 (01:10:02)	2 (02:44:55)	3 (03:54:57)
Lesson Planning	2 (02:16:18)	4 (06:01:38)	6 (08:17:56)
Lesson Reflection	1 (00:45:46)	9 (09:28:51)	10 (10:14:37)
Total time per year	**(6) 07:11:17**	**(15) 18:15:24**	**(21) 25:26:41**

The number of sessions and time coded (hr:min:sec) for each activity is shown in Table 3.14. For example, there were two meetings on Lesson Planning in 2011 and the total time coded for the two meetings was two hours, sixteen minutes and eighteen seconds. The table shows that more meetings were coded in 2012 than in 2011 in all activities except for Error Analysis which was only done in 2011. Lesson Reflection had more meetings coded than any other activity over the two-year data collection period, with Error Analysis having the least number of meetings.

Coding procedure

The coding was using StudioCode™. To code the PLC conversations, each PLC meeting was divided into conversation units, and these units of conversations were created on a timeline for each activity. A conversation unit comprises a sequence of speaking turns on an identifiable topic within the main conversation. When the topic shifts a new conversation unit is identified. A few exceptional cases were made where the topic was too long, and the topic was then divided into shorter, manageable conversation units. This division was purely for convenience. Each conversation unit would then be coded for activity, content and depth of the conversation, using the tool created in StudioCode (see Figure 3.1 below).

Merriam (1992) describes ‘units of data’ to refer to any meaningful segment of data, be it a word used for a particular description, or some large information describing particular incident(s). This description required of me to sort my data in accordance with continuous comparisons of occurrences, comprising what is described above as conversation units. These occurrences illuminated the regularities and patterns that emerged from the conversation units. Figure 3.1 shows the tool which was used to code the audio and the movie packages.

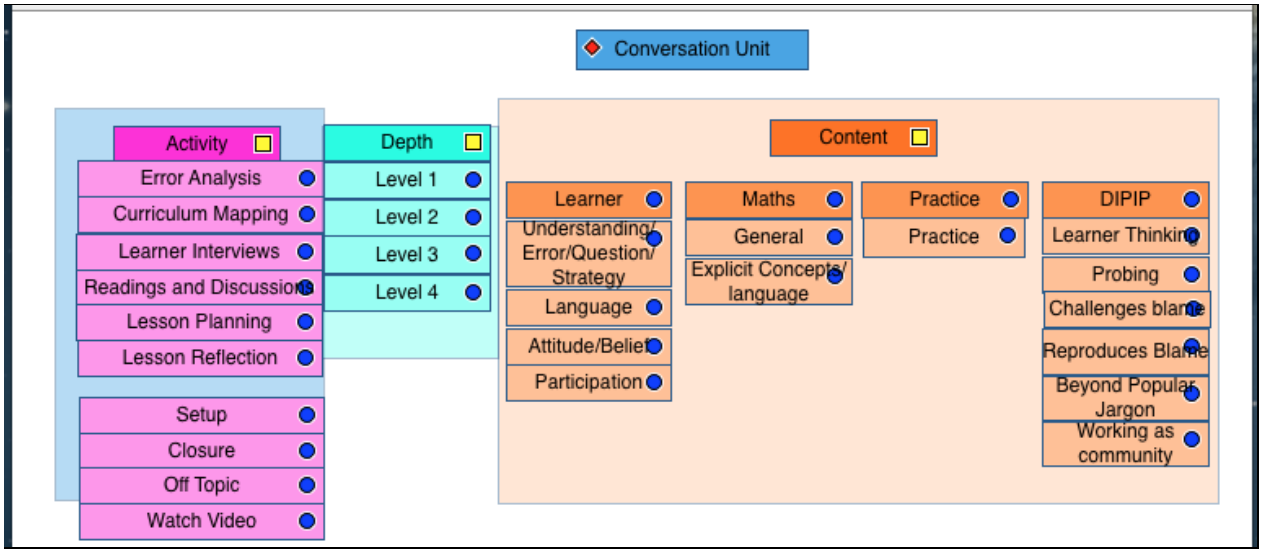

Figure 3.1: Coding tool which was used in coding audio/movie packages.

Setup, closure, off-topic, and watch video were not coded for depth or content and therefore did not provide substantive data for analysis and were left out in the analysis. The subcategories for learner were fused into one and coded 'learner' for any conversation that referred to learner, except for learner thinking which was coded separately. The two subcategories of Maths were fused into one code for Mathematics and coded accordingly. Except for learner thinking and probing, the other codes under DIPIP were also fused and coded 'other codes' as they did not feature prominently as individual codes. In order to analyse the coded data, a scripting report based on time was generated using StudioCode™. This report showed the time spent by the PLC in a conversation on each of the specified codes: Activity, Content and Depth.

The time generated through the scripting report was converted to percentages in order to determine the foci of the PLC conversations as well as the shifts in conversations. I worked with two colleagues to reach inter-rater reliability for the conversation coding using four PLC conversations – we first coded two meetings together, one after the other and discussed our differences and similarities. I then coded a meeting with each of them and compared the results. We achieved inter-rater reliability of 75% through this process, and the process also enhanced my familiarity with the coding scheme. All the PLC conversations were then coded. Transcripts of the conversations were used to help with the interpretation of the codes and exemplify them.

3.6.3 Framework for analysing the PLC conversations

The previous section discusses the coding and the quantitative analysis of the teacher conversations, and in this section, I discuss the framework for the qualitative analysis. In analysing the episodes that exemplified teacher conversations, I developed a framework drawn from different authors' ideas (Brodie, 2013; Hart, 1981; Heritage, Kim, Vendlinski, & Herman, 2009) and DIPIP's framework of analysing learner errors. The framework was used to show the extent to which the teachers' PCK was developed in the PLC conversations.

In their investigation of measures of teacher knowledge for teaching mathematics, Heritage, et al. (2009) focused on how teachers evaluated learner responses for each mathematical concept embedded in a problem. The teachers' evaluation of the learners' responses linked to their PCK because the process had potential to illuminate how they made sense of the learners' understanding of the concepts embedded in the questions using their current knowledge of their learners and of mathematics. Understanding how learners understood the concepts could shape the teachers' PCK and prepare them for appropriate instruction in class. After reviewing a learner's response for each concept, teachers were asked to answer the following questions:
(1) What is the key principle[13] that these assessments address? Why do students need to understand this principle for algebra?
(2) What inferences would you draw from this student's responses? What does this student know? What does this student not know?
(3) If you were this student's teacher what written feedback would you give to this student?
(4) If this student was in your class, based on your responses to questions 2 and 3, what would you do next in your instruction? (p. 25)

These four questions capture the elements of the DIPIP analytic framework as discussed above and so proved useful as a foundation for a framework for further analysis of PCK. Heritage et al. (2009, p. 26) further refined the questions to the following three teacher activities which I adapted for analysis of the PCK in PLC conversations in this study:

(1) Identifying key concepts.
(2) Evaluating student understanding.

[13] Heritage et al. (2009) use the word principle synonymously with concept. This study uses the word concept instead of principle because that is the more commonly used term in mathematics education.

(3) Planning the next instructional step based on the evaluation of student understanding.

It is important for teachers to figure out what learners have done to produce certain responses, and this process has potential to produce PCK. Inquiring into the learners' responses enhances the teachers' PCK because teachers get to know what learners do or do not do in mathematics. The inquiry process can also enhance the development of new knowledge on how to implement what the teachers have learned through their inquiry. Communities of practice provide a space for teachers to develop their PCK of working with learners' responses – teachers come to understand their learners' learning needs better through conversations with colleagues and also from practice. What they learn in practice inform their conversations in the community, which is linked to Heritage et al.'s, (2009) first two constructs and DIPIP's analysis of learners' errors. Their conversations in the community then inform their next instructional step, which resonates with Heritage et al.'s third construct and DIPIP's activity of lesson planning.

Some of the PLC conversations in this study involved how learners understood letters as used in algebraic problems. For example, learners are often comfortable with arithmetic expressions such as $5 + 3$ but encounter problems when faced with expressions such as $5 + k$. Learners tend to conjoin the terms because the expression looks incomplete with two terms.

Hart (1981) posits six categories which describe how learners interpret letters in algebra (discussed in more detail in Chapter 2):

- Letter evaluated – this is the case where a letter is assigned a numerical value.
- Letter not used – when the letter is acknowledged without giving it a meaning or it is simply ignored.
- Letter used as an object – when learners use letters to denote an object, which includes regarding the letter as shorthand for an object.
- Letter used as a specific unknown – when a letter is treated as a particular unknown number.
- Letter used as a generalised number – when the letter takes several values rather than just one.
- Letter used as a variable – when a letter is seen as representing a range of unspecified values.

Hart (1981) argues that for learners to have "any real understanding of even the beginnings of algebra they need to be able to cope with items that require the use of a letter as a specific unknown", but most learners used one of the first three interpretations instead. The first three categories "all describe ways of avoiding generalised arithmetic" because letters are not used as genuine unknowns (p. 108). In addition, the "structural complexity" in an item was also a determinant in learner responses: the nature of the elements used in an item such as the use of small numbers, large numbers, letters or what the elements used meant revealed difficulties in learner responses. For example, the number of letters used in a question showed a difference in learner responses – when more letters were used, the difficulty level often increased. How learners interpret letters in algebra comprise a learning need for teachers because it is directly related to how learners learn mathematics. Teachers can help learners with correct interpretation of letters in algebra if they (teachers) understand the kind of errors that learners produce when working with the letters. A PLC provides a space for teachers to inquire into the learners' understanding of algebra and affords the teachers opportunities to understand the learners' challenges. Hart's finding in linking learners' interpretation of letters to the first three categories is a source of inquiry for mathematics teachers, which is important for mediating algebra in class.

In my qualitative analysis of the teachers' conversations, I use the three afore-mentioned activities of Heritage, et al. (2009) and incorporate DIPIP's and Hart's (1981) ideas to establish the extent to which opportunities for PCK are created in the PLC. A summary of the framework is provided below in Table 3.15. The subcategories used in the framework are not mutually exclusive but build on each other to reinforce the possibility of PCK development. This framework is used to establish the extent to which PCK is developed in the PLC conversations. A detailed analysis of the PLC conversations is done in Chapter 5 of this thesis.

Table 3.15: Framework for analysing the teachers' conversations.

Category	Subcategories
Identifying key concepts	Teachers understand what the concepts in the activity are
	The community talks about the key concepts embedded in the activity or the key building blocks of the activity
	The community talks about the appropriateness of the task for their learners
	Teachers make links with other tasks and concepts or with the learner's work
Evaluating learner understanding	The community investigates/focuses on the learner's reasoning
	Teachers can establish what the learner(s) know and/or do not know, which is linked to exploring learner thinking
	The community makes inferences from the learners' work, which includes learners' interpretation of letters in algebra
	Teachers talk about the anticipated performance of the learners, or how learners learn
	The community talks about the kind of feedback that they could give to learners, which includes correcting and embracing learner errors
Planning the next instructional step	Teachers talk about how the learner(s)' error(s) can be addressed and the misconceptions overcome, i.e. how they might embrace learner errors, rather than correct or avoid them.
	Teachers link learner errors to their practices
	Teachers share their best practices and acknowledge their challenges
	Teachers talk about other issues that may be embedded in the task and relating to practice

3.6.4 Facilitation

The coding tool used to analyse facilitation was developed by the DIPIP project team, with the procedure outlined in Brodie (2016). To understand the facilitation in the community, the 22 PLC meetings were analysed with two different sets of main and sub-codes. The main codes were:

- Process – used when the facilitator manages the conversation, when he makes a move to maintain the discussion, or when he gives his own input in the PLC
- Content – the same as teacher conversation coding

Community codes (Brodie, 2016) did not provide substantive output for analysis and were therefore not included in the analysis of PLC facilitation.

Tables 3.16 and 3.17 below provide descriptions of the codes used in my study. The codes and their respective subcategories are given as they appear in the project document. For purposes of this study, certain modifications were made to the content codes, and this is explained below.

Table 3.16: Process codes

Code	Subcategory	Description
Manages	Introduces	Introduces a session.
	Calls to present	Something that has been prepared before the session, or during the session during small group work.
	Asks for comment	Different from "calls to present", and includes asking questions or for agreement/disagreement or opening up of discussion.
	Ends topic	Ends a session.
	Allocates	Allocates turns for who should speak, especially when people are talking at once.
	Repeats	Includes paraphrasing the idea being made in the contribution or asking someone to repeat their contribution. This may also include the facilitator repeating himself.
	Chooses contribution	Decides which of a number of contributions should be presented or discussed, or not discussed.
	Brings back	Doesn't interrupt the flow of the PLC conversation, but notes key inputs made, and later brings discussion back on track, or simply reminds community about focus.
Maintains	Re-voices	Includes reformulating/extending a contribution, which is more than repeat or paraphrase. Facilitator changes the contribution in order to help make a point.
	Clarifies	Seeks clarification/elaboration from a participant(s).
	Pushes	Includes challenging participants to think more deeply and/or justify their inputs.
	Gives time	Giving a participant(s) time to think.
	Makes links	Explicitly makes links with earlier contributions made, in the current session or others.
	Draws together	Draws together ideas to bring out the main points in a conversation.
Input	Explains	Explains an idea to the community.
	Answers	Answers a question or gives an answer to a mathematical question.
	Clarifies	Clarifies confusion emanating from the discussion.
	Research	Brings in general principles from research, and needs to specify what a general principle is.

Table 3.17: Content codes.

Code	Subcategory	Description
Learner	Understanding/Error/Question/ Learner Strategy	Focuses on learner understanding, or error, or question, or a strategy that a learner uses in a mathematics problem
	Language	Focuses on how a learner uses language in learning.
	Attitude/Belief	Focuses on the attitude or belief of a learner.
	Participation	Focuses about how a learner(s) participate in the teaching and learning of mathematics.
Maths	General	Explains a procedure or definitions under discussion.
	Explicit Concepts/Language	Uses appropriate inquiry-oriented language and also supports participants to be aware of how use of language can hinder learners' thinking.
Practice	Practice	Focuses on what the teacher does in class in relation to instruction.
DIPIP	Learner Thinking	Pushes for participants to understand the reasoning behind the learners' thinking.
	Probing	Focuses on how teachers use probing in class as a questioning technique.
	Challenges blame	Brings out the point that errors are a normal part of learning mathematics and should not be blamed on teachers or learners.
	Beyond popular jargon	Encourages participants for explicitness, and asks them to explain what they mean.
	Working as community	Makes a deliberate move to build community.

The subcategories of the content codes, except for learner thinking and probing, were collapsed into one category as discussed under coding procedures in section 3.6.2 above because they did not provide substantive output from the coding when looked at individually. For analysis purposes, learner thinking was positioned at the same level as the main codes of "learner", "mathematics" and "practices". This is because learner thinking was emphasised in the project and was thought to be key in the development of the teachers' PCK.

I worked with two colleagues to reach inter-rater reliability for facilitator coding using a few PLC conversations – these were from the same data that were used to analyse teacher conversations as discussed in section 3.6.2 above. More than 75% of inter-rater reliability was achieved in this process, and this enhanced my familiarity with the coding scheme. All the data were then coded afterward focusing solely on facilitator utterances.

Coding was done for each facilitator utterance in each PLC meeting, except for certain utterances indicated below. The coding tool is represented in Figure 3.2 below from StudioCode™. An audio recording of each meeting, which was on a movie package in

StudioCode™, was played back and paused every time the facilitator said something substantive. Process codes were allocated first and each facilitator turn was coded with one or more process codes. Process codes were categorised into 'manages', 'maintains', and 'input' as above. For each process code: 'maintains' or 'input', a content code was allocated. 'Manages' did not have a content code because it involved management of the conversation and did not provide any substantive content for analysis.

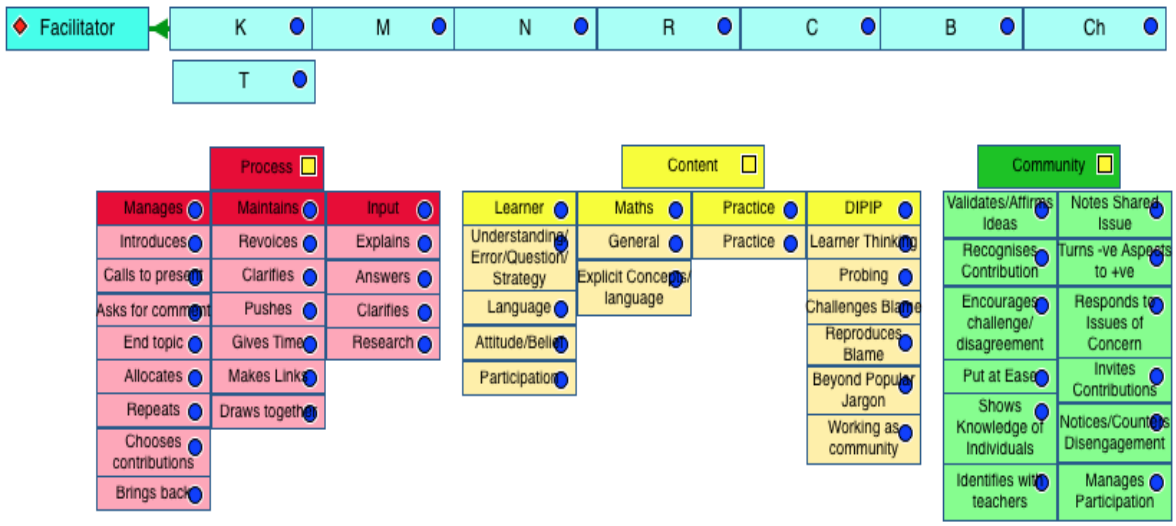

Figure 3.2: Facilitator coding tool.

As stated above, each facilitator turn was coded for process, except for certain utterances which were taken as "no code", such as 'okay', 'mmh', 'ehh', as they did not make substantive inputs. If the facilitator turn was briefly interrupted and then he continued afterward, it was counted as one turn and the second utterance was not coded unless if it was more detailed, and would be the one to be coded. Facilitator moves that were off-topic or were unclear were also taken as "no code". Table 3.18 below gives the time coded for facilitator utterances in each activity over the two years.

Table 3.18: Coded times for facilitator utterances in the different activities.

Activity	2011	2012	Total time over 2 years
Error Analysis (EA)	2 (01:00:54)	-	2 (1:00:54)
Readings and Discussion (RD)	1 (00:47:52)	2 (02:20:04)	3 (3:07:56)
Lesson Planning (LP)	2 (02:06:41)	4 (03:07:05)	6 (5:13:46)
Lesson Reflection (LR)	1 (00:32:35)	9 (05:26:12)	10 (5:58:47)
Total coded time	4:28:02	10:53:21	15:21:23

The coded times generated through the scripting report were subsequently converted to percentages in order to conveniently observe the extent to which the facilitator's utterances could have enhanced or inhibited the teachers' conversations in the PLC.

After the quantitative analysis, I did a qualitative analysis which focused on episodes from the conversations in order to illuminate the kind of support and guidance I provided in the community in the different activities. The qualitative analysis revealed further strengths and weaknesses of my facilitation, which provided opportunities for a reflection on my facilitation and identified areas that needed further research on PLC facilitation. Tables 3.16 and 3.17 also comprise a framework for qualitative analysis of my facilitation.

3.7 Rigour in the research

Rigour in a research study of this nature is meant to improve the trustworthiness of the results as well as the confidence of a reader in the results (Opie, 2004). Since I was both the researcher and the facilitator of the PLC whose learning is analysed in this research, it was important that the strategies that I used to reach my findings were not biased and that they also enhanced the validity, reliability and generalisability of my research findings. This section interrogates the credibility of this study by looking at aspects that can help improve the trustworthiness of the results coming from this study. In order to this, I focus on aspects of validity, reliability and generalisability.

3.7.1 Validity and Reliability

Three important strategies were used in this research study to enhance validity (Merriam, 1998):

- Triangulation – working with multiple investigators, multiple sources of data, or multiple methods to confirm the emerging findings, and strengthen reliability.
- Long-term observation – gathering data over a period of time in order to increase the validity of the findings.
- Peer examination – asking colleagues to comment on the findings as they emerged.

I used multiple sources of data for this study, and these data sources have the potential to increase the validity of my findings. For triangulation, this study used multiple methods that

help confirm the findings that emerged from this study. The classroom observations done through videotaping of the teachers' lessons were used to describe the teachers' practices. These descriptions were important to determine the shifts in the teachers' practices. For the PLC conversations, audiotapes and transcripts of community data were used to analyse teacher conversations and PLC facilitation. The teachers' conversations were analysed to determine the extent to which these conversations contributed to the teachers' knowledge base. Where possible, field notes were consulted in my discussion of the teachers' practices, their participation and my facilitation to support my analysis. All these processes of analysis were replicated because they were applied to more than one set of data in each case: more than one set of lessons for teaching practices and more than one episode for the teachers' conversations and for facilitation.

My data collection took two years, where I worked with almost the same teachers for the two years. These two years afforded me an opportunity for long-term observation of the teachers' practices and participation in the project. Many of the participating teachers continued with the project beyond the two years but data for this extended period is not analysed for this study.

With respect to peer examination, my study formed part of a bigger study on establishing PLCs in different schools (Brodie, 2016; Brodie, et al., 2014; Chauraya & Brodie, 2017, 2018). I regularly interacted with the project leader (also my supervisor) and my peers from the university who facilitated PLCs in the other schools and did research with them. Our conversations focused on both the facilitation and research aspects of the project and supported a strongly collaborative project for all of the researchers.

The range of activities over a long period of time allow for different sets of data to be put into conversation with each other, which builds validity. Merriam (1998) argues that "the length of data collection provides opportunities for continual data analysis and comparison to refine constructs and ensures a match between the researcher's categories and participant realities" (p. 203). An extended period of data collection can create space for a sustained critiquing and re-evaluation through the phases of the research study. The access that I had through videotaping the teachers' lessons and the conversations I had with them brought me closer to the participants' reality. My physical presence in the teachers' classroom during lesson observation as well during PLC meetings, the methods and the instruments I used to collect the data (audio-visual equipment) contributed to the validity of the data collection process. In my

analysis of facilitation data, I was also alert to the critical missed opportunities during my facilitation of the PLC. These missed opportunities are acknowledged in Chapter 6 and discussed in detail in my analysis as opportunities to learn in my facilitation of the PLC.

The use of the MQI instruments which had been validated in other contexts, and the development of coding schemes for PLC conversations and facilitation, drawing on the literature, contributed to the reliability of the study. Reliability refers to the extent to which similar results can be produced "across a range of settings", and by "a range of researchers" (Opie, 2004). A key element of the study was inter-rater reliability which was achieved for all the coding schemes.

Research results in qualitative research are "usually in the form of knowledge claims, which are supported by evidence from the data analysis" (Chauraya, 2013, p. 80), and are not easily generalisable. The results from qualitative research can only be generalised to a particular population, which involves a process of induction (Dey, 1993). In my case, the results are based on empirical evidence gathered from participants in this study, and these results may differ if different participants or facilitator is involved for any given period. However, since qualitative data usually refer to a limited number of cases, this scenario created an opportunity for me to provide a solid basis for inference regarding my research results because of the accuracy of the data collection procedure. In addition, the results from this research can be combined with others in the project to build a bigger picture.

3.7.2 The search for disconfirming evidence

It was important for my analysis in this research study that I focused both on aspects that met my expectations and those that did not. For claims and ideas to be tested well, it is important that we look for disconfirming evidence. In looking at the results I therefore focused intently on the kind of shifts I observed in the participants' practices and participation, and show how those shifts occurred, and most importantly, compare them to what I had initially imagined. In addition, I focused intently on my facilitation of PLC and explored how it enhanced or inhibited teacher participation and development, which was most difficult for me as it related to my own participation in the community. In doing all these, I focused on any evidence that I came across that could have contradicted what I was expecting, and also findings that possibly contradicted what is already known about PLCs.

3.8 Ethical Issues

Any study conducted with human beings requires ethical clearance from relevant authorities as well as ethical action on the part of the researcher. In this case, ethical clearance was obtained for the larger project from the University of the Witwatersrand and permission was granted by the Gauteng Department of Education to conduct research for the bigger project in the schools. I also submitted my personal application for ethical clearance, as required, to the Ethics Committee of the University of the Witwatersrand, which was approved (Protocol Number 2012ECE151). Participants in the study were assured of their anonymity and confidentiality at all times. Pseudonyms were used in this study for purposes of academic writing. The teachers' names and the schools' names used in this research study are therefore not the real names.

Since teachers and learners, as participants in my study, played a major role in my research project, I had ethical obligations towards them. This meant that they had to be protected from any malpractices, information distortions, or biases that could arise as a result of this study. This also included protecting them from whichever practices that could infringe on their rights as participants in my study, or as human beings.

In their articulation of the rights of participants, Verma & Mallick (1999) put emphasis on the Ethical Guidelines outlined by The British Educational Research Association (BERA, 1992) in outlining the responsibility to participants. One of the points BERA argues for is that:

> "Participants in a research study have the right to be informed about the aims, purposes and likely publication of findings involved in the research and of potential consequences for participants, and to give their informed consent before participating in research" (Verma & Mallick, 1999, p. 147).

As a facilitator, I worked with teachers and learners for the two years of the data collection period. Learners comprised a vulnerable population due to their age and they were therefore to be protected and offered the necessary protection when the results of this study were presented and published in the research community. I considered being inside the learners' classroom as a researcher and videotaping their lessons as an invasion of their space. It was therefore important for me to establish which learners did not prefer to be videotaped or did not prefer that their participation in class during videotaping be shown in academic forums.

Confidentiality in disseminating the research results will always be an important aspect for this research.

It was important that in my interaction with the participating teachers in and outside the community meetings that I carefully brought up critical issues to the attention of the community in order to support the teachers in protecting their learners. Learners, as human beings and as children, comprise a vulnerable group and it was important that teachers reflected on how they used the research instruments such as tests with them. I also addressed every class that teachers selected for this study to try and settle the learners during videotaping of the teachers' lessons.

Power relations between researchers and teachers are unavoidable. A long period such as the one taken to collect the data for this study required that I, as the researcher, understood both the participants' academic and their social needs. Maintaining the interest and involvement of the teachers in the project throughout the project was important as it had a direct impact on the sustainability of the PLC. The interest of the teachers in the PLC sustained the community and the professional development program in the schools, and this sustainability went beyond the two years of data collection for this study (for example, Marchant & Brodie (2016b) analysed the conversations in this community for their 2013 activities). Use of data collected from teachers regarding their practices and participation in the project and presenting analysis results at various forums such as academic conferences have the potential to pose some challenges for the teachers as participants in the study. To try and circumvent those challenges, the participants were encouraged to affiliate to mathematics organisations and attend relevant conferences. This participation was meant to expose them to environments in which analysis of teachers' data is shared with local and international communities with the hope of allaying the participating teachers' uncertainties and discomfort. In addition, the conversations that the PLC had on research literature also had a role to play in exposing the participating teachers to the different ways in which research works with practising teachers.

Some of the teachers belonged to labour unions that had standing policies regarding involvement of teachers in projects that could impact on them as employees as well as professionals. One of those policies referred to activities that required teachers' involvement outside the normal school hours. The labour union put in place a moratorium on teachers being involved in any extra-curricular activities because of the stand-off that the union had at the time over the teachers' salaries. Teachers posed questions based on that moratorium and expressed

their fears of being viewed as sell-outs and deviating from the union's position. I found it appropriate to discuss the project with the union leadership, indicating that the project had both a professional development and a research focus, and explained the independent nature of the project. The teacher union indicated interest in the project and asked us for feedback on the results obtained from the research.

The results of my meeting with the labour union were reported to the PLCs and this helped in allaying the teachers' uncertainties regarding their participation in the project. As a researcher and facilitator of a PLC, it was important for me to know how to create an atmosphere wherein teachers could voluntarily participate in the PLC and maintain their participation throughout the data collection period without fear of being negatively labelled or been made unnecessarily vulnerable.

3.9 Summary

This chapter dealt with the research design of my study. I have discussed the research methods that I used for this study and provided justification for these methods. My research followed a qualitative case study. I posited the six-stage process that Yin (2014) uses for a case study research and showed how my study links to that six-stage process. I discussed how the data for this research study were generated and how these data were analysed.

Ethical issues related to this research were considered and adhered to. Aspects related to the reliability, validity and generalisability as well as how rigour for this research was obtained were also discussed. In the next three chapters I discuss the results of this research in order to respond to the research questions set for this study. The next chapter, Chapter 4, focuses on the shifts of the participating teachers' practices over the two years.

CHAPTER 4: Observed Shifts in the Participants' Instructional Practice

4.1 Introduction

In this chapter, I explore how the participants in my study shifted their practices. The data that I use to analyse these shifts come from the five sets of lesson videos that I collected over a period of two years (2011-2012) for each participant (see Table 3.7 in Chapter 3 for more detail on the description of these lessons). I also explained in Chapter 3 that this research is a case study of four mathematics teachers, and in this chapter I discuss each teacher's practice in detail. The four teachers whose lessons were analysed are Funeka, Mapula, Dimpho and Khumo. In my discussion I show how all the four teachers shifted their practices but only two of them, Funeka and Mapula, sustained their shifts in the last set of the individually planned lessons (IPL2) while the other two did not. I use the sub-dimensions of the MQI coding tool (Hill, 2010) (see Chapter 3 and Appendix 2) to document the teachers' shifts and then I provide analyses of episodes from their classrooms to illuminate the shifts.

My analysis of the teachers' lessons addresses the following research question and sub-questions:

To what extent do mathematics teachers' practices shift as they participate in a PLC?

- How do teachers engage with the learners' errors in their classrooms?
- How do teachers create conditions for enhancement of learning in their classrooms?

To address these questions, I use the MQI (Mathematical Quality of Instruction) coding tool. The tool has five key dimensions which are discussed in detail in Chapter 3. In applying these dimensions in my analysis, I divided the five dimensions into two groups according to the resonance I found in the dimensions. Working with these dimensions helped me to establish how each teacher's instruction contributed to the depth of the mathematics taught and learned in the classroom; how the instruction contributed to the teacher's engagement with the learners in the classroom; and also how instruction contributed to the learners' learning of mathematics. Establishing the extent of these influences makes a case for the shifts in the teachers' instructional practice. Foregrounded in my analysis of the teachers' instructional practices is how the teachers worked with the learners' errors in their mathematics classrooms. Episodes

from the teachers' classes are also provided to support the arguments I make in the teachers' shifts or non-shifts.

My analysis will show the shifts or non-shifts that I observed in each teacher's practice based on the five sets of lessons that each teacher taught. I will give an overview of each teacher's instructional practice; discuss the shifts or non-shifts observed from each teacher as analysed through the MQI tool; evidence the shifts or non-shifts through examples from classroom episodes; and give my interpretation of these shifts or non-shifts.

4.2 Coding and analysis

I used the MQI tool to code all the participating teachers' videotaped lessons (See Chapter 3 for details on how the coding was done). The results of this coding for each teacher are captured in Tables 4.2-4.5. These tables show all the five dimensions of the MQI tool and the percentage scores associated with the relevant sub-dimensions. To analyse the teachers' practices over the two years of data collection I divided the five key dimensions into groups as shown in Table 4.1 below.

Table 4.1: Grouping of the key MQI dimensions.

Group 1 Dimensions	Group 2 Dimensions
Mode of Instruction	*Richness of the Mathematics*
Working with Students and Mathematics	*Errors and Imprecision*
Student Participation in Meaning-Making and Reasoning	

I discerned certain resonances among the key dimensions and categorised the dimensions as Group 1 and Group 2. Group 1 dimensions: *Mode of Instruction, Working with Students and Mathematics,* and *Student Participation in Meaning-Making and Reasoning* go together because they all involve how the teacher engages learners in a lesson, and Group 2 dimensions: *Richness of the Mathematics* and *Errors and Imprecision* go together because they both illuminate how the teachers work with content. My analysis focuses on the teachers' practices in each group.

In my discussion of the teachers' shifts[14], I provide episodes with video snapshots from the teachers' classrooms in order to evidence what happened in the teachers' classrooms and to account for the arguments I make about the teachers' shifts. I also show how these examples link to the MQI dimensions in order to argue for the observed shifts, no shifts, or decline in the teachers' practices.

4.3 Overview of Funeka's shifts: Groups 1 and 2 dimensions

Funeka's lessons were initially dominated by a high amount of teacher talk. In IPL1, she often started her lessons by first giving an explanation of the topic in class and doing a few examples on the board for the learners. She would then give learners classwork and walked around the class to check on the learners' work. She often would not deal with issues that she picked up from learners as she walked around. When she called a learner to do a problem on the board and the learner got it wrong, Funeka would call on other learners to take over and to do the same problem until they got it right. The learner who got it correct would be applauded, and Funeka would then say to the class: "now you understand", and proceed to the next problem. The statement "now you understand" suggested that Funeka assumed that when the correct answer was obtained learners understood the mathematics in the question.

When the class corrected homework or classwork, Funeka often asked for volunteers to write down their answers on the board. Four learners would be at the board if there were four problems to be dealt with. After all the solutions were written down, learners often went back to their seats and Funeka would ask the class if the solutions were correct. The class would shout "yes" if the solution was correct and "no" if it was not correct. Correct solutions received praise and the incorrect ones were corrected by learners who knew the correct solutions.

Funeka's practice began to shift in cycles 1 and 2 of the JPL1 lessons. In these lessons, Funeka still called learners to the board to show their solutions but dealt with their solutions differently: she often asked each learner, one at a time, to explain her solution to the class and asked for inputs from the class. She did this for both the correct as well as the incorrect learner responses. Funeka managed to sustain her shifts in JPL2 as well as in IPL2. Her changed instructional

[14] I use the word 'shift' to refer to positive shifts in the teachers' practice, and the word 'decline' to refer to negative shifts. A shift resulting in smaller percentage scores in the MQI scale is considered slight, and a shift resulting in a bigger percentage score is considered significant. Any shift in-between is referred to just as a shift, with no emphasis on slight or significant been made. A similar description is made for decline.

practice remained similar throughout the two years of data collection, often inquiring into the learners' thinking in order to understand the learners' responses and she did that without making learners aware that they were correct or incorrect. When a correct solution was obtained, she would ask a learner who produced the correct to explain it to the class, instead of assuming that the class understood as she did in her IPL1 lessons. When a learner produced an error she would often probe the learner to try and help the learner understand why s/he was incorrect. Table 4.2 below shows the MQI scores of all Funeka's videotaped lessons over the two-year data collection period.

Table 4.2: Frequency of codes – FUNEKA

Dimension	Sub-Dimension	IPL1 lessons (14 episodes)			JPL1, Cycle 1 (26 Episodes)			JPL1, Cycle 2 (12 episodes)			JPL2 (9 episodes)			IPL2 lessons (20 episodes)		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3 3
Mode of Instruction	Direct instruction	0	79	21	4	96	0	8	92	0	0	100	0	5	95	0
	Whole-class discussion	100	0	0	35	65	0	42	58	0	22	78	0	25	75	0
Richness of the Mathematics	Linking or connections	100	0	0	92	8	0	83	17	0	100	0	0	90	10	0
	Explanations	86	14	0	81	19	0	75	25	0	22	78	0	60	40	0
	Multiple procedures or solution methods	100	0	0	84	12	4	75	17	8	100	0	0	100	0	0
	Developing mathematical generalisations	100	0	0	92	8	0	92	8	0	100	0	0	100	0	0
	Mathematical language	100	0	0	100	0	0	100	0	0	78	22	0	90	10	0
Working with Students and Mathematics	Thorough remediation of student errors and difficulties	100	0	0	65	31	4	50	50	0	56	44	0	35	60	5
	Responding to student mathematical productions in instruction	100	0	0	54	46	0	25	75	0	44	56	0	30	65	5
Errors and Imprecision	Major mathematical errors or serious mathematical oversights	43	0	57	54	35	11	75	25	0	78	22	0	80	20	0
	Imprecision in language or notation (mathematical symbols)	86	7	7	54	42	4	67	33	0	89	11	0	75	20	5
	Lack of clarity	71	29	0	42	47	11	50	50	0	67	33	0	80	15	5
Student Participation in Meaning-Making and Reasoning	Students provide explanations	71	29	0	15	81	4	25	67	8	33	67	0	25	70	5
	Student mathematical questioning and reasoning	100	0	0	73	19	8	58	42	0	89	11	0	75	25	0
	Enacted task cognitive activation	100	0	0	69	31	0	58	42	0	33	67	0	90	10	0

4.4 Analysis of Funeka's practice in the Group 1 dimensions

Group 1 dimensions in Table 4.2 show a substantial shift in the dimension *Mode of Instruction*. This shift is mostly linked to whole-class discussion, where Funeka gave the learners the chance to discuss each other's ideas about mathematics problems. The IPL1 lessons show Levels 2 and 3 score on the MQI scale for direct instruction with more Level 2, and these lessons were mostly dominated by a high amount of teacher talk in the classroom. When a learner did a problem on the board and got it wrong, Funeka would often ask other learners to do it until one of them got the correct answer. When a learner got a problem correct, she would often ask the class to applaud and said: "now you understand" without any attempt to ascertain that the class understood.

Whole-class discussion shows a level 1 score on the MQI scale in IPL1 as seen in Table 4.2. However, cycles 1 and 2 of JPL1 show a shift in whole-class discussion in Funeka's lessons, with percentage scores of 65% and 58% in level 2 as shown in Table 4.2 above. This shift was sustained in the JPL2 and in the IPL2 lessons. Table 4.2 shows a recording of 78% at level 2 in the JPL2 lessons and 75% at level 2 in the IPL2 lessons for the sub-dimension of whole-class discussion. At the same time direct instruction decreased slightly, suggesting an interesting mix of direct instruction and whole-class discussion.

There is a substantial shift observed in *Working with Students and Mathematics,* which increased over time and reached level 3 in the IPL2 lessons. The IPL1 lessons show only level 1 of this dimension. Working with learners and mathematics is a key element of DIPIP's work. I will provide evidence below of Funeka's interaction with learners in the classroom to show how she worked with learners' inputs. Funeka's instructional practice from the IPL1 lessons to the IPL2 lessons generally shifted from initially not giving learners the chance to externalise their thoughts to encouraging learners to explain their answers and involving them in classroom discussions in order to inquire into their thinking. Allowing her learners to explain their answers and their thinking after answering mathematics problems or questions suggests that Funeka was creating conditions that were conducive to get learners involved in their learning.

A recording of level 3 of the MQI scale in this dimension is seen in the first cycle of JPL1 lessons as well as in the IPL2 lessons. This shift suggests that Funeka began to listen more to learners' inputs and was responsive to those as soon as she started teaching the cycles 1 and 2

of JPL lessons, and Funeka sustained this practice in the IPL2 lessons. In addition, sustainability of these shifts suggests that Funeka continued to learn how she could work with learners and mathematics in her classroom through her teaching.

There is a substantial shift in *Student Participation in Meaning-Making and Reasoning*, which is mostly linked to learners providing explanations during the lessons. In this dimension, the coding scheme does not distinguish between the correct and incorrect explanations as it does in the dimension of *Richness of the Mathematics*. Funeka afforded learners opportunities to make contributions in class and also to explain their thinking about the contributions they made. These opportunities to explain and share their thinking with the whole class put the learners' ideas in the public realm for critique. The affordances for learner engagement resulted in the class often having interesting discussions about the mathematics.

Student Participation in Meaning-Making and Reasoning recorded a level 3 in cycles 1 and 2 JPL1 as well as in IPL2, and this means that Funeka featured at least four aspects of the criteria that describe this dimension in order for her to reach a score of level 3: learners provided counter claims; learners asked mathematically motivated questions; learners explained what an answer meant; learners engaged in reasoning about a general case. Funeka had changed from asking learners to say or chorus whether a problem written on the board was correct or incorrect as in her IPL1 lessons to asking learners to explain their answers, and engaging with their errors.

Teachers often talk about time constraints to justify why they cannot employ the practice of engaging learners in the lesson as often as they could. However, Funeka rarely used time as a factor to discourage her from employing these practices in her classroom. She would often allow learners to come show their responses on the board as she did in the IPL1 lessons but she did this differently in cycles 1 and 2 of the JPL1 and JPL2 lessons as well as in the IPL2 lessons. In cycles 1 and 2 of JPL1, JPL2 as well as in IPL2, Funeka asked learners to explain their answers and often probed them to try and understand the deeper meaning of their explanations. In addition, she allowed other learners to comment on what the learner at the board had written and/or explained. The comments and questions brought up interesting discussions among learners as well as between the teacher and the learners. In the IPL1 lessons, learners' inputs were restricted to affirming or rejecting what was written on the board.

Below I give two episodes that illustrate how Funeka shifted her practice in the Group 1 dimensions regarding how she engaged learners in her lessons. This engagement includes how Funeka embraced learners' errors in her classroom and how she created conditions for such engagements. The first episode comes from one of her IPL1 lessons and the second comes from cycle 2 of JPL1 lessons.

An earlier episode (Grade 10 class: Feb 2011, IPL1)

In this episode the class was working on multiplication of two binomials. The episode was dominated by a high amount of teacher talk evidenced by Funeka explaining the procedure and using a method which school teachers often refer to as the FOIL[15] method for multiplication of binomials. She did two problems on the board as examples for the class: First, she worked on $(a + b)(c + d)$, thereafter on $(x - 3)(x + 3)$, and explained the procedure to learners for each problem. She then gave the class $(x + 2)(x + 3)$ to work on. As learners worked on the problem, Funeka walked around the class looking at what learners were doing and marked their solutions. In the process of marking the learners' work, Funeka spoke softly to individual learners at their desks. She then gave learners a second problem to work on: $(3x - 5y)(2x + y)$ and she asked for a volunteer to the board. The first learner came to the board to do the problem. Funeka asked him to explain as he wrote and the other learners laughed. This laughter suggested that learners were not used to such practice of having to explain themselves. The learner wrote $3x.2x - 3x.y - 5y$ and before he could finish writing Funeka asked for another volunteer who could assist the learner at the board.

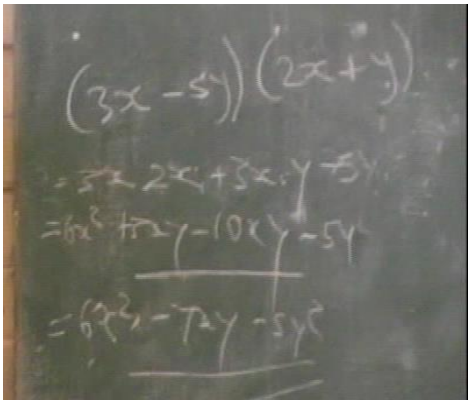

Figure 4.1: Learner solution to a problem.

[15] An approach used in the teaching of multiplication of two binomials, where each letter of the word FOIL represents multiplication of particular terms in the two binomials. F – First terms; O – Outer terms; I – Inner terms; and L – Last terms.

A second learner approached the board and wrote $3x.2x - 3x.2x$ and the teacher asked for help from the class without engaging with what the second learner had written on the board. A third learner came and got the problem correct (see Figure 4.1 above for the third learner's solution). The teacher said: "Good", and the class applauded. The teacher then said to the class: "now you understand" and she gave another problem to the class to work on at their desks. She then walked around the class and later decided to do the problem on the board for learners because she thought the problem was difficult for them.

This episode is typical of many episodes in Funeka's IPL1 lessons and it shows how Funeka did not take up the learners' errors in those lessons, and was interested only in the right answers. The first two learners' answers were not engaged with. Instead, she asked the other learners to help those who got the problems incorrect. This means that Funeka did not embrace the learners' errors but rather ignored or avoided them ((Brodie, 2013). The third learner's answer was accepted without discussion and the learner was applauded. The manner in which Funeka handled the correct answer showed that she did not engage the learners on the correct solutions either.

This episode exemplifies the sub-dimension of direct instruction on the MQI scale. This mode of instruction was a key feature in the episode and was therefore coded level 3 because mathematical content was supplied in a teacher-directed manner. The teacher was in control of delivery of mathematical content in a somewhat superficial manner. She did not engage the learners in a whole-class discussion and that resulted in level 1 on the MQI scale for that sub-dimension. *Working with Students and Mathematics* and *Student Participation in Meaning-Making and Reasoning* featured level 1 scores in this episode. Because Funeka did not respond to the learners' written work or utterances, and the learners' explanations were brief and pertained to the specific problems. The next episode shows how Funeka shifted in the JPL1 lessons.

A second episode (Grade 10 class: Mar 2012, JPL1, cycle 2)

In this episode, the class was going through a True/False worksheet[16] that Funeka gave them (see Appendix 4). Funeka wanted learners to explain why the statements were true or false.

[16] The worksheet was part of joint lesson planning in the PLC.

The statement that they were busy with in this episode was: $(3 + 1)^2 = 3^2 + 1^2$. One learner was convinced that the statement was true because when the 3 in the bracket was squared then it gave 3^2 and squaring the 1 gave 1^2. Learners were divided – some of the learners agreed with this learner and others disagreed. Funeka did not indicate whether she agreed or disagreed with the learner's answer. After a whole-class discussion, with some learners showing their work on the board, Funeka asked the first learner who said the statement was true if he was convinced. The learner agreed in a very low voice which could have meant that he was not convinced. This episode occurred in the last period of the day and the class had a discussion on this problem until the bell rang.

The following day Funeka started the lesson by revisiting the problem, and she wrote: $(3 + 1)^2 = 3^2 + 1^2$ on the board. She asked the same learner who said the statement was correct the previous day if he had changed his mind, and the learner said he still thought that the statement was correct because the problem was about expanding. She asked the learner to come and do the problem on the board and the learner worked it out as in Figure 4.2 below. She then gave him $(2a + 3)^2$ to do and the learner approached it in a similar way of trying to square each term inside the brackets (see Figure 4.2), which shows that the learner's reasoning was consistent.

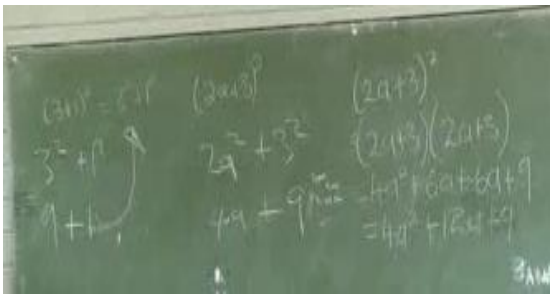

Figure 4.2: Some of the learners' work on the board.

Funeka emphasised the square in the expression and asked the class how a binomial should be squared. A second learner approached the board and got the problem $(2a + 3)^2$ correct (see the right part of Figure 4.2 above). Funeka asked the class what the difference was between the first two solutions done by the first learner and the third one done by the second learner. The first learner stated that $(3 + 1)^2 = 3^2 + 1^2$ was not a product but factorisation, whereas $(2a + 3)^2$ was a product. When Funeka tried to probe the learner for factorisation the learner did not respond. A third learner said he agreed with the first learner because the calculator would give the same answer. When Funeka allowed him to use the calculator he got 16. The class laughed.

Funeka then asked the class to show how they can get that 16 on the board and a fourth learner approached the board and showed it correctly (see Figure 4.3 below).

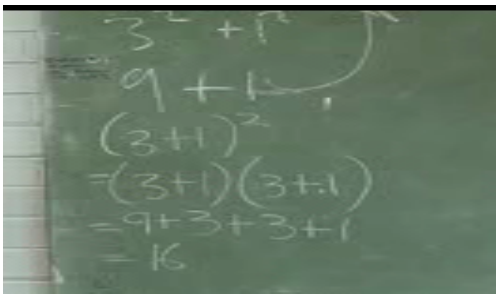

Figure 4.3: Another learner's work.

After some discussion and drawing all the learners' ideas together, Funeka asked the learner who was not convinced at the beginning as to whether he agreed that the given statement was false and the learner said maybe. The learner's response suggested that he was still not happy that his answer was not accepted, and Funeka then asked the learner what still confused him and she eventually pushed the learner to say whether he agreed that the statement was false or whether he still thought it was true. It was clear that the learner was not convinced from the other problems that Funeka asked the other learners to do. The learner finally indicated that he agreed that the statement was false, though he did not explain why he agreed. Funeka also did not ask the learner why he agreed in order to engage him further.

Funeka gave the class a similar problem on squaring a binomial: $(a + b)^2$. This suggests that Funeka was trying to make a link with the first problem, especially after the first learner seemed doubtful. A few learners struggled with it until one learner got it correct. Funeka probably created this link in order to emphasise a method that was needed to solve problems of that nature without leading the learners to the correct answer. When learners asked questions on problems on the board, Funeka redirected them to the class. In doing so, Funeka involved the learners in a whole-class discussion, which signalled a shift in her practice because previously she would respond directly to the learners' questions. The key feature in this episode was that learners commented on each other's contributions. The interaction between Funeka and the learners raised an opportunity for Funeka to establish why learners do what the first learner did when given the square of a binomial, and Funeka did not succeed in establishing that.

Regarding *Mode of Instruction*, Funeka was still in control of delivery of mathematics content, but she did not dominate the discussions as in the previous episode. There were more

affordances for learners in this episode to "share their thinking, to explain the steps in their reasoning, and to build on one another's contributions" (Hill, 2010), and this brought a shift in Funeka's practice. Her practice in this dimension rated level 1 for direct instruction and level 2 for whole-class discussion because of the manner in which she engaged the learners in class.

This episode also exemplifies a shift in the sub-dimensions of thorough remediation of student errors as well as responding to student mathematical productions. Funeka's shift showed that she understood the learners' mathematical productions and errors, and she embraced those in this episode, something that she did not do in the previous episode. The first learner's production was an error, but it offered opportunities for discussing and addressing pertinent mathematical ideas. This episode reached level 2 in both sub-dimensions of *Working with Students and Mathematics* because of the manner in which Funeka engaged with the learners' ideas. The episode reached level 3 in *Student Participation in Meaning-Making and Reasoning* in the sub-dimension of students provide explanations because of the learners' engagement with the tasks.

The different methods that the learners used in this episode evidenced multiple procedures or solution methods and these rated level 3 on the MQI scale, though this is relevant for Group 2, which is analysed in the next section. The two episodes provided illustrate how Funeka's practice shifted from not showing interest in the errors that the learners produced to showing great interest in embracing the learners' errors. In addition, the two episodes illustrate how Funeka worked with the learners' meanings in ways in which learners understood why they were incorrect and subsequently shifting their thinking.

4.5 Analysis of Funeka's practice in the Group 2 dimensions

Funeka's Group 2 dimensions show an inverse relationship between the two dimensions: as *Richness of the Mathematics* improves, *Errors and Imprecision* also increases. An increase in *Errors and Imprecision* means that the teacher produced more errors, implying that the errors worsened. Inasmuch as Funeka worked hard to make the mathematics rich in class, she also produced more errors.

Richness of the Mathematics attempts to capture the depth of the mathematics offered to learners ((Hill, 2010). Learners should be able to build a conceptual mathematical base from

the different components of rich mathematics. These components of rich mathematics are reflected from the scores obtained from the different sub-dimensions of *Richness of the Mathematics*. Funeka's scores in cycles 1 and 2 reflect an attempt to reach the depth required to enable learners to build a conceptual mathematical base, though her MQI scores are at lower percentages.

A slight shift in the teacher's explanations in cycle 1 of JPL1 is observed in *Richness of the Mathematics*. The shift is linked to all the sub-dimensions except mathematical language which remained at level 1. The sub-dimension of multiple procedures or solution methods shows a small level 3 score on the MQI scale in cycle 1 of JPL1. These shifts are sustained in cycle 2 of JPL1 with small percentage increases. The sub-dimension of mathematical language remained at level 1. Mathematical language was emphasised during DIPIP meetings while multiple procedures and developing generalisations received less attention. *Richness of the Mathematics* declines in IPL2.

The sub-dimension of explanations shows an increase in the JPL2 lessons, which records 78% at level 2. The coding scheme emphasises two important aspects: the correctness of the explanations as well why a particular procedure or solution method works. This occurrence of 78% is a result of Funeka pushing her learners to give thoughtful explanations in her lessons. However, the sub-dimensions of multiple procedures and developing mathematical generalisations completely declined in JPL2. This could have been a result of the two sub-dimensions receiving less attention during DIPIP meetings.

There is an increase in *Errors and Imprecision* from the IPL1 lessons to cycle 1 of JPL1. This dimension focuses on the teacher's and uncorrected learners' errors. Funeka's errors were mostly related to the mathematics content. I have discussed previously that a level 3 score on this dimension shows that the teacher is making major errors in her teaching or there are serious uncorrected learners' errors in the lesson, whereas level 1 score means that the lesson is error free. JPL2 and IPL2 lessons show a decrease in *Errors and Imprecision*, suggesting that Funeka became better at co-ordinating her own knowledge with the learners' ideas. A similar result was found by Chauraya & Brodie (2017) when they argued that as the teachers began to interact more with learners, the teachers' mathematical errors and imprecision increased. In the case of Funeka, her errors increased more in cycle 1 of JPL1 but decreased from cycle 2 of JPL1 onward.

The sub-dimension of lack of clarity shows the lowest achievement in this dimension for almost all the lessons. One contributing factor was Funeka often misinterpreting a mathematics task. For example, when the class was working on a worksheet in cycle 1 of the JPL1, they were looking at a set of algebraic statements which required that learners explain how the next statement was obtained from the previous one. Funeka's presentation of the content in this lesson was unclear to learners, especially due to her misinterpretation and eventually distorting the mathematical essence of the task. I will explain in the first episode below how Funeka misinterpreted the task. Due to this misinterpretation and not ascertaining the learners' understanding of the content, the sub-dimension of lack of clarity recorded a score of level 3 in cycle 1 of JPL1 lessons. However, her practice shifted in later lessons and she started showing more confidence in her teaching, and her openness in class allowed learners to freely produce more ideas for discussion. This confidence and openness meant that learners could come up with aspects of mathematics content that she did not prepare for in advance. It was thus possible for her to make mistakes in spontaneously dealing with such content, leading to less clarity. A similar result was found by Chauraya & Brodie (2017). I provide two episodes below to exemplify Funeka's practice over two years in the Group 2 dimensions.

An earlier episode (Grade 10: Oct 2011, JPL1, cycle 1)

As stated above, Funeka misinterpreted a worksheet that she dealt with in cycle 1 of JPL1, which led to lack of clarity (see Appendix 5). The worksheet was on explaining how mathematical statements followed each other, but Funeka dealt with each statement separately as if they were a list of equations to be solved, which was a complete misinterpretation of the task. Funeka rewrote the statements from the worksheet on the board (see Figure 4.4), except for the last statement which should be $a + 2 = \frac{30+a}{3}$. Learners only needed to explain, for instance, how the second statement: $a + 5 = 17$, could be obtained from the first statement: $a = 12$. The expected response was that: you add 5 to both sides of the first statement in order to get the second statement.

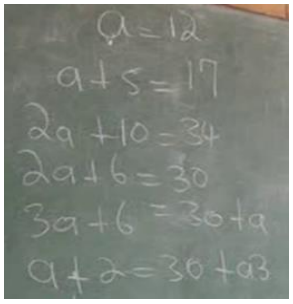

Figure 4.4: First learner's solution.

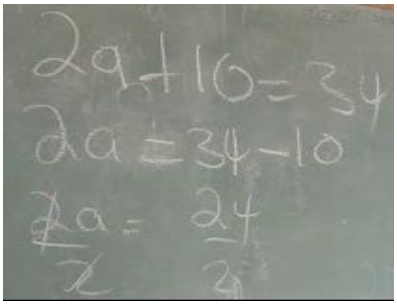

Figure 4.5: Second learner's solution.

Funeka started the discussion by pointing at the first statement in Figure 4.4 to show learners that a = 12 was given, and emphasising that they were "to take it as it was given". She then pointed at the second statement: a + 5 = 17, and asked learners the question: "What are we going to do to get the number for a?" Learners talked at the same time and it was unclear as to what they were saying, and Funeka proceeded to treat this second statement as an equation to be solved instead of explaining how the statement followed from the first one. She got an answer of 12 and told the class that it meant the statement was true since they got 12. This was the first stage of misinterpreting the question.

Funeka went on to treat the next statements in a similar way and referred to them as if they were numbered (See Figure 4.5 for how she treated the third statement, solving it as an equation). She solved each statement to get 12 and every time concluding that each of the statements was true because of that answer, except the last one of: $a + 2 = 30 + 3a$, which gave her 14. For this answer she told the class that the statement was false since "a was not equal to 14". For each of the statements she dealt with, Funeka engaged learners with questions that channelled the learners into treating the task as a collection of equations to be solved, and the learners responded accordingly.

This episode was coded level 1 in *Richness of the Mathematics* due to the distortion in the mathematics that was offered to the learners and the lack of depth in that mathematics. The MQI tool emphasises that elements of an episode should be substantially correct and clear to count as level 2 or level 3, which was not the case in this episode. The explanations given were descriptions of steps of a procedure but the explanations were contextually incorrect due to an incorrect procedure in engaging with the task. For this reason, the episode was coded level 3 on the MQI scale for *Errors and Imprecision*.

A second episode (Grade 10: Aug 2012, IPL2)

In this episode, the class was correcting homework. The problems that they dealt with were on simple interest and compound interest. Funeka called learners to the board to write down and explain their solutions. She then asked the class to look at those solutions and to comment on them. The incorrect ones were corrected by the same learners who wrote them after receiving feedback from the other learners. Below I give two figures depicting the first and the second solution of the same learner. The problem was:

> An amount of R40 000 is invested for five years at an interest of 7% p.a. What will be the accumulated amount after five years if the interest is compounded annually?

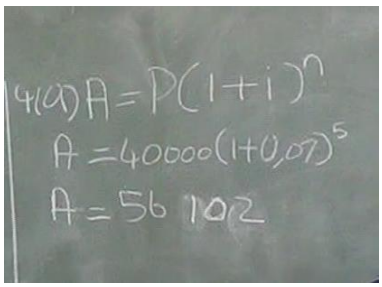

Figure 4.6: Learner's initial solution.

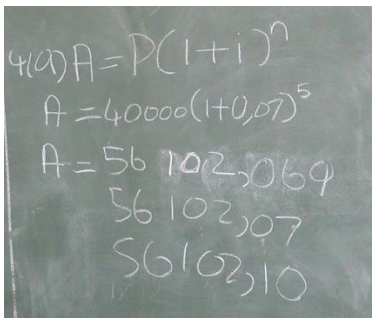

Figure 4.7: Learner's second solution.

The learner who wrote the solution in Figure 4.6 discussed with other learners while he was seated at his desk and immediately stood up afterward and went back to the board to change the final answer of his solution to 56 102,1. Funeka asked the learner: "What is happening there", and the learner said: "Zero seven, ten, I don't understand". Funeka asked him to explain what he did not understand but the learner could not explain. Funeka then asked the class whether the problem was correct or not but there was disagreement among the learners. Some said it was correct and others said it was not. She then asked the class what the mistake was. What could be heard in the background was that "only the decimals were missing" in the solution.

Funeka called the first learner back to the board. The learner modified his solution as depicted in Figure 4.7. Some learners grumbled, and the first learner explained that "comma zero six nine" would give "comma zero seven", which would then give "comma one". A learner was overheard saying "one zero", and the first learner put a zero so that his final answer became

56 102,10. Funeka instructed the first learner, who was still at the board, to stop writing, and asked the class what the problem was. Funeka asked the learner whose hand was up to explain and he said:

> "It should be zero seven because after the comma you consider the third number, and if that number exceeds five you add one, then it's gonna be zero seven".

The first learner wiped off the last part of his solution: 56 102,10 and went back to his desk. The phrase: "if the number exceeds five you add one" was not picked up as an error though it was not completely correct to say that. Funeka then explained how an answer that was obtained from a calculator was rounded off to two decimal places and made the same error, and it was still not picked up. For this reason, the episode was coded level 2 in *Errors and Imprecision* because the teacher neglected to fully discuss a key aspect of a step: if the third digit exceeds four (and not five), a one is added to the preceding digit. Another common error that was made in class and Funeka did not pick it up was on misreading the digits after the decimal point. For example, some learners read 'comma one zero' as 'comma ten', and such errors were coded as uncorrected learner errors on the MQI tool.

This episode was coded level 2 in *Richness of the Mathematics* within the sub-dimensions of explanations and mathematical language because it focused on the meaning of facts and procedures. This was a shift from the previous episode.

4.6 Summary of Funeka's shifts

In my discussion of Funeka's practice, I have referred to Funeka's MQI scores in Table 4.2 to show how she shifted her practice and managed to sustain her shifts in the IPL2 lessons. The shifts in engaging more with learners were accompanied by an increase in teacher errors, which I have argued were a result of teaching in new ways. As she opened up her practice to afford the learners opportunities to participate more in class, she ended up making more errors. I have exemplified the observed shifts by supplying episodes from Funeka's classroom and linked these episodes to the sub-dimensions of the MQI tool. The examples that I gave typified what happened in Funeka's lessons.

My analysis has shown how *Mode of Instruction* shifted in IPL1 lessons from high levels of direct instruction and an absence of whole-class discussion to significant MQI scores in whole-class discussion in the subsequent lessons. *Working with Students and Mathematics* has shown significant shifts in all lessons from cycles 1 and 2 of JPL1 to IPL2. Similarly, *Student Participation in Meaning-Making and Reasoning* shifted significantly from cycles 1 and 2 of JPL1 to IPL2, which was mostly linked to students providing explanations.

There were shifts in *Richness of the Mathematics* in Funeka's cycles 1 and 2 of JPL1. Findings have shown that these shifts were ascribed to the sub-dimensions of explanations, multiple procedures and developing mathematical generalisations. I have also discussed that multiple procedures and developing mathematical generalisations were not emphasised in DIPIP meetings, and these sub-dimensions declined in JPL2. *Errors and Imprecision* has shown Funeka making more errors in cycle 1 of JPL1 as she shifted her practice but the errors were reduced in cycle 2 of JPL1 as she got used to teach in new ways. Findings have shown an inverse relationship between these two dimensions.

An interesting finding in Group 1 was that improvement in *Mode of instruction* was often accompanied by improvement in *Working with Students and Mathematics* as well as in *Student Participation in Meaning-Making and Reasoning*. For Group 2, improvement in *Richness of the Mathematics* was accompanied by an increase in *Errors and Imprecision*, suggesting an inverse relationship.

In the next section, I discuss the practice of the second teacher, Mapula, over two years. I will show how she gradually improved her practice as she continued to participate in the project, and also how she managed to sustain that improvement. I will also provide episodes to support my findings regarding her shifts in her practice.

4.7 Overview of Mapula's shifts: Groups 1 and 2 dimensions

Mapula's lessons were initially dominated by a high amount of teacher talk. A distinct feature in her lessons was giving learners opportunities to explain their answers, though she did not work with those answers in order to understand the reasoning underlying the learners' answers. She would often affirm or reject those answers, which made her practice somewhat similar to Funeka's. When she gave learners opportunities to present their solutions on the board, she

would often go from one learner's solution to the next, asking the class if it was correct or not. In that process, the class would chorus their answers to agree or disagree with the solution on the board. When a solution was correct Mapula would often put a tick next to the final statement to affirm the learner's solution, but when it was incorrect she would rub off the incorrect part and correct it herself, explaining the procedure for the solution to the class.

Mapula began to show shifts in her practice in cycles 1 and 2 of JPL1. She still gave learners opportunities to explain their answers but she listened more to their input. When the class could not get a solution to a particular problem, Mapula would often leave that problem unfinished and got to the next problem. It is not certain if the problem was ever revisited to get its solution. The most significant shifts in Mapula's practice were observed in JPL2, and similarly to Funeka, these shifts were sustained in IPL2. In JPL2 and IPL2, Mapula continued to listen more to learners' inputs and also inquired into the reasoning behind the learners' answers. In addition, she gave learners opportunities to critique each other's solutions. Table 4.3 below shows the MQI scores of all Mapula's videotaped lessons over the two years.

Table 4.3: Frequency of codes – MAPULA

Dimension	Sub-Dimension	IPL1 lessons (15 episodes)			JPL1, Cycle 1 (14 Episodes)			JPL1, Cycle 2 (13 episodes)			JPL2 (8 episodes)			IPL2 lessons (16 episodes)		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Mode of Instruction	Direct instruction	13	67	20	7	79	14	8	77	15	0	100	0	0	94	6
	Whole-class discussion	67	33	0	43	50	7	62	38	0	12	63	25	31	63	6
Richness of Mathematics	Linking or connections	100	0	0	100	0	0	100	0	0	100	0	0	69	31	0
	Explanations	87	13	0	64	36	0	69	31	0	54	46	0	31	69	0
	Multiple procedures or solution methods	100	0	0	100	0	0	100	0	0	100	0	0	100	0	0
	Developing mathematical generalisations	100	0	0	100	0	0	100	0	0	100	0	0	94	6	0
	Mathematical language	80	20	0	64	36	0	77	23	0	8	92	0	100	0	0
Working with Students and Mathematics	Thorough remediation of student errors and difficulties	73	27	0	57	43	0	54	46	0	38	62	0	31	69	0
	Responding to student mathematical productions in instruction	80	20	0	64	36	0	69	31	0	50	50	0	13	74	13
Errors and Imprecision	Major mathematical errors or serious mathematical oversights	87	13	0	65	21	14	77	23	0	38	62	0	75	19	6
	Imprecision in language or notation (mathematical symbols)	87	13	0	71	29	0	92	8	0	54	46	0	81	19	0
	Lack of clarity	74	13	13	65	21	14	77	7	16	63	37	0	87	13	0
Student Participation in Meaning Making and Reasoning	Students provide explanations	53	47	0	50	50	0	54	46	0	0	100	0	0	100	0
	Student mathematical questioning and reasoning	100	0	0	100	0	0	77	23	0	75	25	0	94	6	0
	Enacted task cognitive activation	100	0	0	100	0	0	100	0	0	25	75	0	38	62	0

4.8 Analysis of Mapula's practice in the Group 1 dimensions

Mapula's lessons in IPL1 were dominated by a high amount of teacher talk. Her practice shifted in cycles 1 and 2 on JPL1. She did not necessarily affirm or reject the learners' answers as she did in IPL1 – she listened more to their explanations, though she still dominated the talk. A significant shift in direct instruction is observed in JPL2 which is sustained in IPL2. The shift in JPL2 shows an absence of level 3 scores in direct instruction. Mapula's involvement of learners in JPL2 and IPL2 included her engagement with both the learners' correct and incorrect solutions.

A significant shift in whole-class discussion is observed in cycle 1 of JPL1 with an occurrence of a level 3 score. This shift slightly declines in cycle 2 of JPL1. Though Mapula still listened to learners' inputs in cycle 2 of JPL1, she often took over the explanation of the learners' answers, which was a feature somewhat similar to her IPL1 lessons which demonstrated a high amount of teacher talk. A significant shift for whole-class discussion is observed in JPL2 and this is sustained in IPL2, though it goes down a bit in level 3. This shift is a result of Mapula affording her learners more opportunities to explain their answers and the learners engaging with each other's thinking. With these affordances, she carefully listened to their explanations without taking over the learners' explanations as she did in cycle 2 of JPL1. An interesting feature in the JPL2 and IPL2 lessons was Mapula drawing the learners' inputs together to conclude the whole-class discussion.

Similarly to Funeka's results, the shifts observed in this dimension show an interesting mix of direct instruction and whole-class discussion, particularly in JPL2 and IPL2 – as direct instruction decreases, whole-class discussion increases. In an attempt to get the reasoning behind the learners' errors, Mapula often probed learners in order to get them to explain how they obtained their answers. She used her probing questions in order to establish the source of the misconceptions in the learners' thinking.

Working with Students and Mathematics shows mostly level 2 scores from IPL1 to IPL2 with one sub-dimension in IPL2 recording a level 3 score. Cycles 1 and 2 of JPL1 look similar on the MQI scale for this dimension. Mapula did not embrace the learners' errors in IPL1 – when a learner produced an error, Mapula would often reject the error or correct it. There are slight shifts in JPL1 lessons, with these shifts becoming somewhat significant in JPL2 and sustained

in IPL2. Mapula began to engage more with the learners' responses in cycles 1 and 2 of JPL1 – she afforded learners opportunities to externalise their thoughts and involved them in whole-class discussions as discussed in *Mode of Instruction*. Her learners were comfortable to publicly air their views and raise arguments about what they agreed or disagreed on, and Mapula created space for such engagements. This demonstrated that Mapula began to know her learners better and her learners also understood what was required of them in class, which subsequently brought a shift in pedagogy (Vescio, et al., 2008).

Significant shifts for *Working with Students and Mathematics* are observed in JPL2, recording percentage scores of 62% and 50% at level 2 in the two sub-dimensions. The most shifts in Mapula's practice are observed in IPL2, which shows scores of 69% at level 2 in the sub-dimension of thorough remediation and 74% at level 2 in the sub-dimension of responding to student mathematical productions. A level 3 score in IPL2 is also observed within the sub-dimension of responding to student mathematical productions. The shifts in *Working with Students and Mathematics*, especially in JPL2 and IPL2, show that Mapula changed from correcting the learners' errors in IPL1 to embracing errors and being responsive to the learners' contributions.

Student Participation in Meaning Making and Reasoning shows a percentage score of 47% at level 2 in the sub-dimension of students provide explanations in IPL1, which shows that Mapula gave learners more opportunities to provide explanations in IPL1 than Funeka did. Funeka's score in IPL1 was 29% at level 2 for the same sub-dimension. However, as discussed above, Mapula did not embrace the errors that learners produced in IPL1. No shift is observed within the same sub-dimension until JPL2. A significant shift is observed in JPL2, and this is sustained in IPL2. Both JPL2 and IPL2 show percentage scores of 100% at level 2 within the same sub-dimension on the MQI scale. Student mathematical questioning and reasoning shows only level 1 scores in IPL1 and cycle 1 of JPL1. A slight shift is observed in cycle 2 of JPL1 with a percentage score of 23% at level 2, which remains almost the same in JPL2. A decline is observed IPL2 within the same sub-dimension. Almost similar to student mathematical questioning and reasoning, enacted task cognitive activation shows level 1 scores from IPL1 to cycle 2 of JPL1. A significant shift is observed in JPL2, with a percentage score of 75% at level 2. The observed shift is sustained in IPL2.

There is an observable link between *Mode of Instruction* and the two sub-dimensions of students provide explanations and enacted task cognitive activation in *Student Participation in Meaning Making and Reasoning*. The link shows a direct relation between the two key dimensions: when *Mode of Instruction* improves, *Student Participation in Meaning Making and Reasoning* also improves. I provide two episodes below to illustrate Mapula's instructional practice in the Group 1 dimensions.

An earlier episode (Grade 8: February 2011, IPL1)

Mapula had previously given learners work to do, and in this episode she asked them to write their solutions on the board. She partitioned the board for the learners so that each learner could use the partitioned space for his/her solution. The class was dealing with simplification of algebraic expressions. Learners came to write their solutions on the board but none of them explained their solutions. As the learners were writing their solutions on the board Mapula asked the class to focus on what was written on the board and to check if there was anything to be corrected. When the learners had finished writing their solutions they moved back to their desks and waited for feedback.

After all the learners had finished writing their solutions Mapula moved from one learner's solution to the next to give feedback. She first worked on each problem on her own to check if it was correct or not by checking each term of the expression. When she was convinced that the solution was correct she asked the class the question: "Is this correct?" Learners chorused the response and when the solution was correct Mapula would put a tick next to the final answer to affirm its correctness (see Figures 4.8 and 4.9 below). This practice was typical of how Mapula handled the learners' solutions in her class in IPL1.

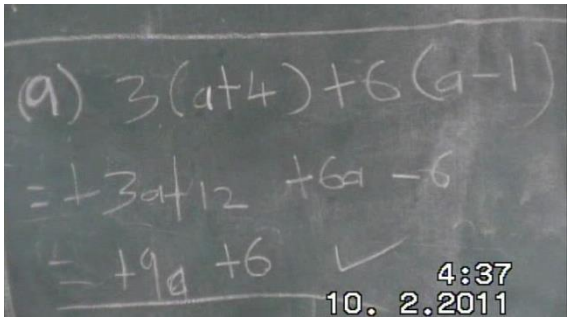

Figure 4.8: Learner's work that is affirmed.

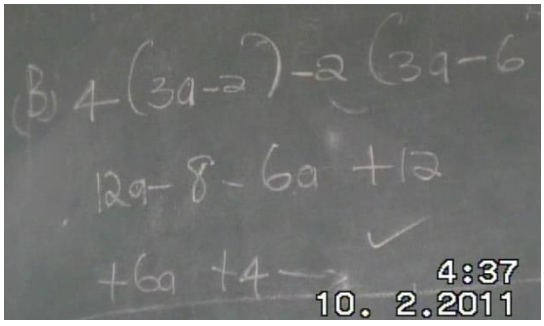

Figure 4.9: Learner's work that is affirmed.

The two problems in Figures 4.8 and 4.9 above were correct. When Mapula checked these problems, she pointed at every term of each problem and affirmed its correctness. She then put a tick next to the final answer. When she asked questions, her questions were often closed and often promoted chorusing from learners. At no stage did Mapula ask the learners to fully explain their solutions nor did she try to understand how a learner got an incorrect solution. She often corrected the incorrect answers herself and asked the class if anyone got the problem correct and proceeded to the next question afterward. She treated all the incorrect solutions in a similar way. Figure 4.10 below shows an example of an incorrect solution by a learner before Mapula corrected it and Figure 4.11 shows the same problem after Mapula had corrected it.

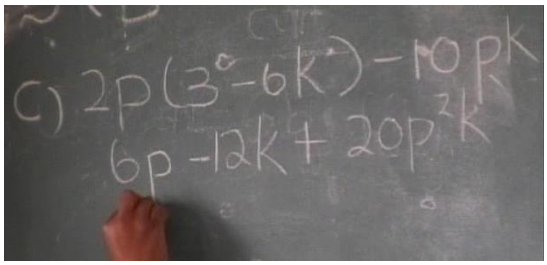

Figure 4.10: An incorrect learner's work being checked.

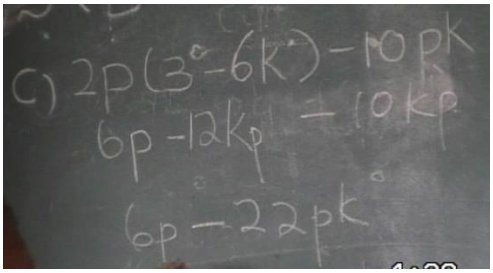

Figure 4.11: Corrected learner's work.

In Figure 4.10, Mapula first checked each problem on her own and explained to the class what was written; what was incorrect in the learner's solution; and what had to be done to correct the problem. In Figure 4.10 she told the class that the last term $+20p^2k$ in the learner's solution was incorrect, wiped it off and replaced it with $-10kp$ as seen in the second statement of Figure 4.11. She then wrote $6p$ in the third statement and thereafter asked the class, facing the board herself at that time, what the answer to $-12kp - 10kp$ was, and the class chorused "$-22pk$", which she then wrote in the solution to give the final answer of $6p - 22pk$ as seen in Figure 4.11 above.

At no stage did Mapula try and check with learners how the errors in the learner's solution came about. This showed that Mapula did not embrace the learners' errors and she did not inquire into the reasoning behind the learners' errors. This episode is a typical example of Mapula's teaching in the sub-dimension of direct instruction, where there is a high amount of the teacher's talk. In *Mode of Instruction*, Mapula's practice was coded level 3 for direct instruction and level 1 for whole-class discussion in this episode because she limited the learners' participation in class by dominating classroom discussions. For remediation of student errors her practice was coded level 2, and level 1 for responding to student productions

in *Working with Students and Mathematics*. These levels were due to Mapula asking closed questions when correcting the learners' incorrect answers and not inquiring into the reasoning behind the learners' errors. All the sub-dimensions of *Student Participation in Meaning-Making and Reasoning* were coded level 1 because the task was a low level task and Mapula did not prompt for learners' reasoning in the manner in which she dealt with it. The next episode exemplifies how Mapula shifted her practice in cycle 1 of JPL1.

A second episode (Grade 8: October 2011, JPL1, cycle 1)

In this episode, Mapula introduced linear equations in her class by first discussing a scale balance. She drew its rough sketch on the board and advised the class that to balance the scale you could add or subtract a mass piece. Afterward Mapula wrote three equations on the board. The equations were: $12 = x + 4$; $4 + x = 12$; and $12 = 4 + x$. She then reminded learners that when they worked on equations they should balance both sides so that "the left hand side was equal to the right hand", and wrote: L.H.[17] = R.H.S. on the board. She explained to the class that to balance equations they could "add on both sides, or subtract or multiply or divide". My discussion of this episode is at a time when Mapula was working on the equation: $4 + x = 12$, and Figure 4.12 below shows how Mapula did the problem on the board.

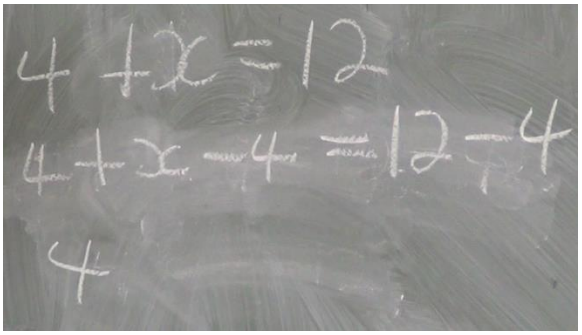

Figure 4.12: First learner's suggested method.

Mapula interacted with learners in doing the problem: she listened more to learners' contributions than in IPL1, which indicated a shift in her practice. In IPL1 she corrected the problems without taking suggestions from the learners. In doing the problem in Figure 4.12 above, one learner suggested that they subtract 4 from both sides. Mapula wrote what the learner suggested: $4 + x - 4 = 12 - 4$. The same learner suggested that they add $4 + x$ on the left

[17] The L.H. is given as it was written on the board instead of L.H.S.

to get 5, but Mapula disagreed and pointed out 4 and −4 to the class that they were like terms. The learner then suggested that they "bring the 4 down" which Mapula did, and thereafter subtract $x - 4$ to get −3. Mapula disagreed again and said they could not add or subtract unlike terms because "x is a variable and 4 is a number". Mapula gave a second learner an opportunity to give his method. The second learner suggested the method shown in Figure 4.14 below.

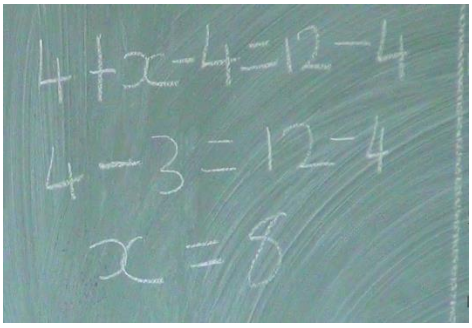

Figure 4.13: First learner's solution.

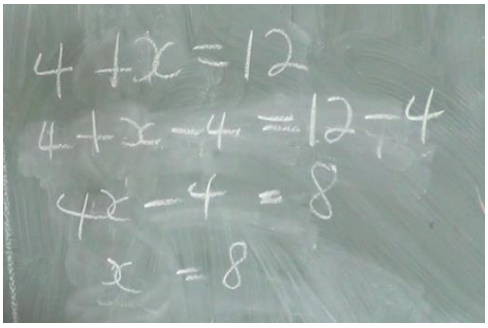

Figure 4.14: Second learner's solution.

The first learner disagreed with the solution in Figure 4.14 and said $4x - 4$ should give zero, but some of the learners grumbled in protest. None of the learners was given the chance to explain why they protested, and Mapula reprimanded them for grumbling. The first learner repeated that $4x - 4$ should give zero. It was clear at this stage what the first learner's misconception was: he wanted to conjoin the terms, irrespective of whether those terms were like or unlike. Mapula had earlier tried to point that error out to him but the learner still wanted to do that. When Mapula asked him if she should write zero from $4x - 4$, he said they could not write zero because the second learner's method was wrong.

Mapula rewrote the problem $4 + x = 12$ on the board and asked the first learner to give the class the correct method. The first learner said something inaudible and Mapula asked the class to give him two minutes to work out a solution and share that solution with the class, and in the meantime the class would continue with the next problem. Mapula did not give learners similar opportunities in IPL1. Figure 4.13 above shows how the first learner worked out the equation, with $x - 4$ giving him -3, which means that he took x as 1.

The affordances that learners had in this episode exemplify a shift in Mapula's practice within the sub-dimension of whole-class discussion, and recorded 50% at level 2 on the MQI scale. Remediation of student errors and responding to student productions in *Working with Students and Mathematics* scored higher percentage scores at level 2 than in IPL1. Mapula embraced

some learners' errors in cycle 1 of JPL1 as demonstrated by the manner in which she handled the first learner's error in this episode. In IPL1, Mapula noted the errors and corrected them immediately without engaging the learners on those errors.

In this episode, after the first learner was given the opportunity to work on the problem, Mapula asked him how he got the second statement (see Figure 4.13 above). The learner argued that since Mapula had once told them that when x was written it was just the same as $1x$, he then took x as 1. Hence, the statement: $4 - 3 = 12 - 4$. Mapula's interaction with the first learner illuminated the learner's misconception. Mapula allowed the discussion to continue and gave other learners opportunities to give their input. No-one in the class was able to come with a convincing argument for the first learner. The first learner still insisted that x and 4 cannot be left separated. To support his argument he used a metaphor, saying to the learner who argued with him: "you cannot allow your hand to be removed from your body as your hand should form part of your body", suggesting that the x and the 4 should be conjoined. Mapula realised the first learner's misconception and promised him that she would sit down with him at some stage to try and resolve the matter. This meant that the other learners would not be part of the discussion between Mapula and the first learner. After her interaction with the first learner, Mapula asked the class to continue with a new problem.

Mapula listened to the first learner's explanations in this episode even though he was incorrect, which justified a level 2 score on the MQI scale within the sub-dimension of students provide explanations. Her listening showed that she was interested in the learners' errors, despite the fact that she did not conclude the discussion on the errors well. The episode evidences how embracing learner errors and giving learners the chance to make contributions in a whole-class discussion environment led Mapula to be confronted with a situation which she never planned for, which was typical of what happened in Mapula's class as she began to listen more to the learners' contributions and teach in new ways. The next section focuses on Mapula's practice in the Group 2 dimensions over two years.

4.9 Analysis of Mapula's practice in the Group 2 dimensions

Similarly to Funeka, observations on the MQI scale do not show any significant shifts in *Richness of the Mathematics* for Mapula for most of the sub-dimensions. The shifts observed on the MQI scale are mostly linked to the sub-dimensions of explanations as well as

mathematical language, which show slight shifts in cycle 1 and cycle 2 of JPL1. In cycle 1 and cycle 2 of JPL1, Mapula afforded learners opportunities to explain their work but she sometimes took over the lesson before she could understand the reasoning behind the learners' thinking. She often focused more on a learner's error in order to correct it, than on the reasoning behind the error. Also, she sometimes left problems without finding closure to the discussion in class as in the second episode above where she indicated she would explain after class to resolve the learner's confusion.

The sub-dimension of explanations shows a slight shift in JPL2 with a percentage score of 46% at level 2 and a significant shift in IPL2 which is at level 2 at a percentage score of 69%. Mathematical language shows a significant shift in JPL2 with a percentage score of 92% on the MQI scale at level 2. The observed shifts suggest that Mapula was using more mathematical explanations to convey content to learners. The other sub-dimensions do not show shifts except in IPL2 where slight shifts related to the sub-dimensions of linking or connections and developing mathematical generalisations can be observed. These links or connections and generalisations were at level 2 as they did not deal with deeper conceptual ideas in mathematics. I discussed in Funeka's results that multiple procedures and developing generalisations received less attention in the DIPIP project. Observations from JPL2 and IPL2 illustrate an attempt by Mapula to help learners build a conceptual mathematical base from the different components of rich mathematics.

Similarly to Funeka, *Errors and Imprecision* shows Mapula's errors increasing from IPL1 to JPL2 as she taught in new ways, with JPL2 showing a significant increase. Mapula's errors from IPL1 to cycle 2 of JPL1 were a result of mathematical content or uncorrected learners' errors, with occurrences of level 3 emerging on the MQI scale. Mapula's errors increased significantly in the JPL2 lessons, recording 62% at level 2 on the MQI scale for the sub-dimension of major mathematical errors. JPL2 lessons comprised tasks on how word problems were used in Algebra. Mapula engaged learners more in JPL2 than she did in IPL1 and in cycles 1 and 2 of JPL1. Engaging learners opened her up to challenges from learners which at times resulted in her committing more errors. The second episode above exemplified these challenges. When Mapula was often confronted with situations where she could not find an acceptable solution to a problem in under discussion, she would often leave the problem incomplete and continue with the next problem.

IPL2 shows that Mapula still continued to make mathematical errors in her lessons. However, these errors were significantly reduced compared to the other lesson sets, and observations on the MQI scale show that IPL1 and IPL2 produced somewhat similar levels of errors. The reduction of errors in IPL2 suggests that Mapula's coordination of content was better in IPL2 than in both cycles of JPL1 and in JPL2. Her presentation of content was also somewhat clearer in IPL2. Overall, an increase in Mapula's errors suggests an interesting mix between *Errors and Imprecision* and *Working with Students and Mathematics*. A similar result was found in Funeka's practice – as the two teachers' practices shifted, they started making more mathematical errors in their teaching. For both teachers, the errors decreased over time. I provide two episodes below which evidence how Mapula's practice shifted in the Group 2 dimensions over two years.

An earlier episode (Grade 8: March 2012, JPL1, cycle 2)

This earlier episode was taken from cycle 2 of JPL1 because Table 4.3 shows that Mapula's practice was similar from IPL1 to JPL1. The two cycles of JPL1 are almost the same. In the episode, Mapula gave learners a worksheet[18] (see Appendix 4 for the worksheet). The worksheet consisted of statements which learners had to state whether they were true or false and to explain their choices. The first statement on the worksheet was: $\frac{6+10}{2} = 3 + 10$.

Mapula wrote the statement on the board and asked for volunteers to the board. When the first learner approached the board Mapula said to the class: "Remember we are balancing the two sides ne (*right*). The left hand side we balance with the right hand side". This statement by Mapula has potential to influence misinterpretation of the question and could contribute to the learners' misunderstanding because some of the learners might find it difficult to discern a critical point in the question: whether the statement was true or false, and why. Mapula's statement exemplifies the sub-dimensions of imprecision in language and lack of clarity. Figures 4.15 and 4.16 below show the learners' work as they responded to the question.

[18] The worksheet was part of joint lesson planning.

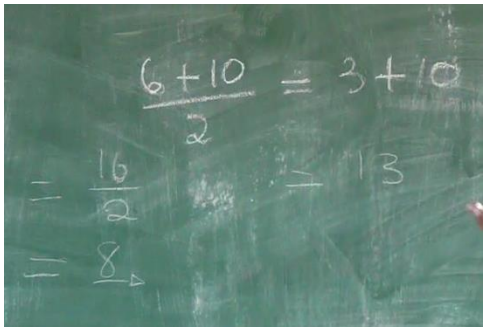

Figure 4.15: First learner's solution.

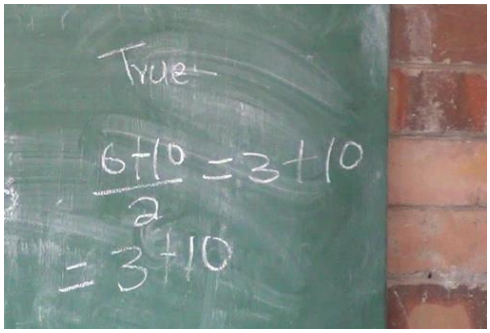

Figure 4.16: Second learner's solution.

The first learner wrote her solution and said “it is false”. Mapula asked her to repeat her answer and the learner said it was false because the two statements did not balance. Mapula then explained what the learner had done. Mapula asked for other solutions from the learners. A second learner approached the board and wrote her solution, and said the statement was true. Mapula asked the second learner to explain and the learner said she divided six by two to get three and added the three to ten. Mapula took the chalk from the learner and added other things to the second learner’s solution. Figure 4.17 below shows the second learner’s solution with Mapula’s additions: the “= 13” in the second statement and “13 = 13” in the last statement”. The second statement had two equal signs and this was not dealt with in class, an example of *Errors and Imprecision*.

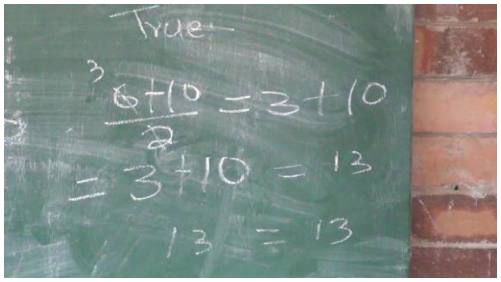

Figure 4.17: Mapula's additions to the second learner's solution.

The first learner disagreed with the second learner’s solution and argued that they should not only divide the six by two but the ten as well. Mapula in her response said the first and the second learners’ solutions were both correct depending on what you did first. It was evident at this stage that Mapula was not aware of the second learner’s error, which was a mathematical oversight and it was coded level 3 on the MQI scale. The episode was coded level 1 for all the sub-dimensions of *Richness of the Mathematics* as it did not “capture the depth of the mathematics offered to students” (Hill, 2010, p. 5).

This episode is both unique and typical: It is unique in the sense that Mapula accepted two different solutions which she accepted as both being correct when only one was correct. It is typical because Mapula asked the class to look at the next question without finding appropriate closure to the first problem, which was what she often did. Opening up the discussion to the class for more engagement might have given the first learner, and maybe others, opportunities to understand why the statement was false and build a conceptual mathematical base for the class on division.

A second episode (Grade 8: August 2012, JPL2)

In this episode, Mapula gave learners a worksheet to work on the previous day and the class was discussing the solutions to the problems on that worksheet (see Appendix 6 for worksheet). The episode is from a lesson on the second concept that the PLC wanted to develop with learners: the use of mathematical language in Algebra. Mapula's class was discussing the following problem:

> Given that $a = 28 + b$. Which of the following do you think would be true?
>
> - a is greater than b
> - b is greater than a
> - $a = 28$
>
> Please provide an explanation of your choice for the answer.

The most popular response in class was the first option of a is greater than b. In this episode Mapula did not call learners to the board as she often did. She asked the learners to say their answers loud enough for the class to hear and she engaged them in a whole-class discussion. The first learner chose the first option of 'a is greater than b'. The learner's reason was that "a is the first letter". Mapula asked the learner if "a variable would always be greater than the next one", but the learner did not respond to this question. The second learner chose the third option of '$a = 28$'. Mapula repeated what the learner said but the learner changed her answer to 'a is greater than b', and told Mapula that she made a mistake with her first choice. When Mapula asked her why she said she made a mistake she said she was confused by the statement of the question which said: $a = 28 + b$ because the way it was written it looked like $a = 28$.

The reason that the second learner gave links to what research says on the operational meaning of the equal sign: that the equal sign means what follows is the answer (Essien & Setati, 2006; Kieran, 1981). In the statement, the number 28 followed immediately after the equal sign and the learner took it as the answer. Mapula did not deal with this misconception. When asked to explain her second choice of *a* is greater than *b*, the second learner said:

> "Because ma'm, when you add 28 and *b* it is equal to *a*, so *a* can subtract *b*", referring to the given statement in the problem.

According to the learner, *a* was greater than *b* because something needed to be added to *b* in order to make it equal to *a*. A third learner objected to the addition of 28 and *b* since they were unlike terms, which suggested that she misunderstood the second learner's point and the meaning of the equation. Mapula did not clear up the third learner's confusion. Mapula gave other learners a chance to share their thoughts. A fourth learner said:

> "You cannot add 28 and *b*, and you also cannot say *a* is greater than *b* because we don't know what the value of *b* is".

The fourth learner's contribution shows the learner's confusion regarding the notion of a variable, which was the first concept that the community developed with the learners in cycles 1 and 2. The fourth learner's response was similar to the third learner's and resonated with one of the categories that Hart (1981) used when arguing about how learners often view variables. He did not see *b* as a variable. According to Hart, learners sometimes avoid operating on the unknown, something that the fourth learner did because the value of *b* was not known in the given statement. In an attempt to address this misconception, Mapula said 28 + *b* could also represent an answer – an important aspect that needs to be emphasised when focusing on the notion of variables (Booth, 1988), although it is not clear whether Mapula's explanation was adequate.

In the process of finding a solution, Mapula did not make learners aware whether their answers were correct or incorrect, which demonstrated Mapula's interest in working with learners' errors to try and understand the learners' reasoning – she did not avoid or correct the learners' errors. She allowed the learners to make inputs in a whole-class discussion and pushed them to justify their answers. The questions that Mapula asked were open-ended and helped her in

getting at the learners' reasoning. This episode typifies Mapula's practice in IPL2 and was coded level 2 in the sub-dimensions of explanations and mathematical language in the MQI tool. The episode was not filled with the teacher's errors, and Mapula made an attempt to address the learners' errors. All the sub-dimensions within *Errors and Imprecision* were thus coded level 1 on the MQI tool.

4.10 Summary of Mapula's shifts

Similarly to Funeka, Mapula's shifts in engaging more with her learners were often accompanied by more teacher's errors. I made an argument in my analysis of Funeka's and Mapula's practice in the Group 2 dimensions regarding the prevalence of these teachers' errors: as the teachers shifted their practice they started making more mathematical errors in their teaching.

Mapula's lessons in IPL1 had a strict pattern of first giving an explanation of the content to be dealt with, doing a few examples on the board and giving learners classwork to do afterward. Mapula often affirmed or rejected the learners' answers in IPL1 without engaging with the reasoning behind the learners' answers. When learners were called to the board to show their solution, which was a practice Mapula often employed in her practice from IPL1, she did not let them explain their solutions but would instead explain the problems herself.

Mapula began showing shifts in cycles 1 and 2 of JPL1 lessons but still taught in ways in which she wanted to avoid or ignore errors in the cycle 1 of JPL1. There were instances in which a learner's serious misconception emerged and Mapula was not able to locate the source of that misconception because she did not fully embrace the learner's error.

Significant shifts in Mapula's practice became evident in cycle 2 of JPL1, where she began to engage learners more and afforded them opportunities to explain their thinking. When learners explained their answers she ensured that the class listened and made contributions. In the process of inquiring into the learners' thinking, she often employed the notion of cognitive conflict (Limon, 2001; Piaget, 1975) to work with learners' thinking, as she demonstrated in the second episode of August 2012.

In the next section, I discuss the shifts and non-shifts in Dimpho's practice, and show how she managed to improve her practice over two years but did not sustain this practice in the end. The shifts and non-shifts observed will be evidenced with episodes from Dimpho's class.

4.11 Overview of Dimpho's shifts: Groups 1 and 2 dimensions

Dimpho's lessons were initially dominated by a high amount of teacher talk. Her approach in IPL1 was often reminiscent of a question-and-answer approach which took the form of an IRE structure (Brodie, 2004; Sinclair & Coulthard, 1992). She often asked a question, listened to a learner's response, and evaluated the learner's response. A distinct feature in her IPL1 lessons was that of engaging learners in ways in which they explained their answers, but she never inquired into the reasoning behind the errors that learners produced. This is similar to Mapula's practice in IPL1.

When Dimpho called learners to the board to present their answers in IPL1, they often wrote the answers and went back to their seats. Learners were not afforded opportunities to explain their work to the class. Dimpho often discussed each learner's solution on the board using the question-and-answer method, and correct answers were affirmed while incorrect answers were rejected.

Dimpho's lessons in cycle 1 of JPL1 were not from the jointly-planned lessons as with the other teachers. She had requested that she should not teach the joint lessons and the community agreed. However, she had been participating in the PLC meetings and possibly learning from the conversations. Her lessons do show shifts in cycle 1 of JPL1 for some of the Group 1 dimensions, and these shifts were sustained in cycle 2 of JPL1 as well in JPL2. The shifts were a result of Dimpho listening more to learners' inputs and responding appropriately to learners' mathematical productions. She often embraced the learners' errors and tried to understand the reasoning behind those errors. Her engagement with learners in a whole-class discussion from cycles 1 and 2 of JPL1 afforded learners the opportunities to critique each other's contributions and build on one another's mathematical productions. Dimpho's practice declined in IPL2 and was characterised by similar features that were observed in IPL1. Table 4.4 below shows the MQI scores of all Dimpho's videotaped lessons over the two-year data collection period.

Table 4.4: Frequency of codes – DIMPHO.

Dimension	Sub-Dimension	IPL lessons (14 episodes)			JPL1, Cycle 1 (30 Episodes)			JPL1, Cycle 2 (21 episodes)			JPL2 lessons (22 episodes)			IPL lessons (17 episodes)		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Mode of Instruction	Direct instruction	14	0	86	7	73	20	14	86	0	36	64	0	35	53	12
	Whole class discussion	64	36	0	44	53	3	43	57	0	50	50	0	29	71	0
Richness of Mathematics	Linking or connections	100	0	0	100	0	0	100	0	0	100	0	0	100	0	0
	Explanations	57	43	0	77	23	0	76	24	0	64	36	0	71	29	0
	Multiple procedures or solution methods	100	0	0	100	0	0	100	0	0	100	0	0	100	0	0
	Developing mathematical generalisations	100	0	0	100	0	0	100	0	0	100	0	0	100	0	0
	Mathematical language	93	7	0	100	0	0	86	24	0	77	23	0	88	12	0
Working with Students and Mathematics	Thorough remediation of student errors and difficulties	57	43	0	90	10	0	43	57	0	45	55	0	76	24	0
	Responding to student mathematical productions in instruction	79	21	0	90	10	0	28	67	5	22	69	9	82	18	0
Errors and Imprecision	Major mathematical errors or serious mathematical oversights	93	7	0	67	30	3	86	14	0	86	14	0	76	24	0
	Imprecision in language or notation (mathematical symbols)	79	21	0	80	20	0	90	10	0	86	14	0	82	18	0
	Lack of clarity	93	7	0	80	17	3	90	10	0	77	23	0	94	6	0
Student Participation in Meaning-Making and Reasoning	Students provide explanations	57	43	0	30	70		24	76	0	27	73	0	71	29	0
	Student mathematical questioning and reasoning	100	0	0	80	17	3	71	29	0	86	14	0	100	0	0
	Enacted task cognitive activation	100	0	0	70	27	3	62	38	0	45	55	0	100	0	0

4.12 Analysis of Dimpho's practice in the Group 1 dimensions

Mode of Instruction shows a level 3 score of 86% on the MQI scale for direct instruction in IPL1. The score was a result of Dimpho's lessons being dominated by a high amount of teacher talk. Though Dimpho invited the learners to the board to show their solutions, the class was not afforded opportunities to engage with those solutions. A shift was observed in cycle 1 of JPL1 where level 3 of direct instruction was reduced to 20% on the MQI scale. Dimpho allowed the learners to engage with their solutions in cycle 1 of JPL1 which indicated a shift in her practice within this sub-dimension of direct instruction, but she still dominated the discussions in class. This shift was sustained in cycle 2 of JPL1 but declined slightly in JPL2 and IPL2. A level 3 score for direct instruction is observed in IPL2, which is an indication that Dimpho's practice in IPL2 somewhat resembled her practice in IPL1 – she did not engage learners as much as she did in cycle 2 of JPL1 and JPL2.

Whole-class discussion is observed in IPL1, though it was not the key feature in the taught lessons in IPL1, and the discussions did not engage in deeper mathematical thinking. A shift is observed in cycle 1 of JPL1 with a small percentage score at level 3 and this shift was sustained in cycle 2 of JPL1 as well as in JPL2. Whole-class discussion was a key feature in these lessons. Dimpho listened more to the learners' inputs and gave them opportunities to share their thinking and to build on each other's contributions (Hill, 2010).

IPL2 showed a significant shift in whole-class discussion with a percentage score of 71% at level 2 on the MQI scale. A similar interesting mix as in Funeka's and Mapula's lessons is observed in Dimpho's lessons: A decline in direct instruction is accompanied by an increase in whole-class discussion. Though Dimpho encouraged whole class discussion in IPL1, she did not embrace the learners' errors. Her IPL2 lessons were also characterised by a similar feature of not embracing the learners' errors, and this will be illustrated in the episodes below. She worked with learners in a whole-class discussion in IPL2 but also combining that with a high amount of teacher talk without embracing the errors that the learners produced in the process.

The level 2 percentage scores observed in *Working with Students and Mathematics* in IPL1 show that Dimpho identified the learners' errors and worked with those. However, Dimpho's practice in working with the learner's errors demonstrated extensive procedural remediation (Hill, 2010). A decline in her practice for this dimension is observed in cycle 1 of JPL1, which

is an interesting finding as the other teachers showed shifts in cycle 1 of JPL1 for this dimension. Dimpho did not teach the jointly planned lessons in cycle 1 of JPL1, which may be the reason for the decline. A significant shift in cycle 2 of JPL1 is observed for both sub-dimensions of thorough remediation of student errors and responding to student mathematical productions in instruction. Scores of 57% and 67% were attained for these sub-dimensions at level 2 on the MQI scale. The shifts in these sub-dimensions were sustained in JPL2. Level 3 scores on the MQI scale, though very small, can be observed in cycle 2 of JPL1 and in JPL2 for the sub-dimensions of responding to student mathematical productions in instruction. The small percentage scores at level 3 in cycle 2 of JPL1 and in JPL2 show that Dimpho commented on the learners' mathematical ideas and asked the other learners to do so. It also shows that she elicited further learner clarification of ideas. Dimpho was now giving learners opportunities to explain their answers and engage with each other's' inputs. She listened more to learners' utterances; became more familiar and alert to what learners said and/or wrote; and she responded appropriately.

Dimpho's engagement with the learners' errors created opportunities for her to try and understand the reasoning behind the learners' errors, which is a key principle that DIPIP encouraged. I provide an episode below to illustrate how Dimpho managed to work with the learners' errors. A decline is observed in IPL2 for both sub-dimensions, and this is similar to what was observed in *Mode of Instruction*. Observations in Table 4.3 are an indication that Dimpho's practice in IPL2 was reminiscent of what was observed in IPL1: her lessons were dominated by a high amount of teacher talk and little working with learners and mathematics.

Students provide explanations shows level 2 score *in Student Participation in Meaning-Making and Reasoning* in IPL1, while the other two sub-dimensions scored level 1. Observations from Table 4.3 show a mixture of significant and slight shifts in cycles 1 and 2 of JPL1 as well as in JPL2. A significant shift, which is linked to the same sub-dimension of students provide explanations, is observed in all the subsequent lessons, but declines in IPL2. The observed shifts in students provide explanations indicate that Dimpho gave the learners better opportunities to explain themselves, afforded the learners space to interact with their ideas, and consequently embraced the errors that the learners produced.

The other two sub-dimensions: student mathematical questioning and reasoning as well as enacted task cognitive activation show level 1 scores in IPL1. Slight shifts are observed in

cycle 1 of JPL1, and these shifts are sustained in cycle 2 of JPL1. Level 3 percentage scores are observed in cycle 1 of JPL1, and this suggests three features listed in the MQI tool were attained: students provided counter-claims to a proposed statement; students asked mathematically motivated questions; and students formed conclusions based on the patterns they identified. Student mathematical questioning and reasoning declines in JPL2 and in IPL2, while enacted task cognitive activation shifts significantly in JPL2 and significantly declines to Level 1 in IPL2.

The observed decline in IPL2 shows that Dimpho's practice in IPL2 reverted back to her initial practice in IPL1. The decline in student mathematical questioning and reasoning as well as in enacted task cognitive activation are an indication that Dimpho limited the affordances that her learners had in the previous cycles 1 and 2 of JPL1 as well as in JPL2 and was less embracive of the learners' errors. I provide two episodes below to illustrate the shifts in Dimpho's practices.

An earlier episode (Grade 11: February 2011, IPL1)

In this episode, Dimpho introduced the topic of solving triangles using trigonometric formulae. She began the lesson by asking learners to name the different types of triangles. The discussions in this episode were purely verbal. Dimpho followed an IRE-structured approach (Brodie, 2007; Molefe & Brodie, 2010; Sinclair & Coulthard, 1992) in her questions: She asked a question, listened to a learner's response, evaluated the learner's response, and the next cycle of questioning started. This cycle of question-and-answer continued for most of Dimpho's engagement with the learners. Though Dimpho engaged the learners through question-and-answer, she still remained in control of delivery of mathematical content. She did not afford the learners opportunities to build on each other's ideas for most of the episode. The episode became a level 3 engagement for direct instruction on the MQI scale as the mode of instruction featured prominently in the episode. Learners did not share each other's thinking in the episode or explain their reasoning and whole-class discussion was therefore at level 1 on the MQI scale.

Dimpho made an attempt at getting different opinions from learners in her engagement but at no stage did she try to understand the reasoning behind the learners' answers. In responding to the question on the types of triangles, one learner gave an answer of a right-angled triangle. The learner described the triangle as one that did not have equal sides. A second learner said

the sides of the triangle should be equal. Dimpho listened to the two learners' answers but she did not engage with any of the learners' answers. Dimpho's reaction shows that she did not embrace the learners' inputs, and this was coded level 1 on the MQI scale for both sub-dimensions of *Working with Students and Mathematics*. In addition, Dimpho did not acknowledge the validity in the first learner's answer regarding the possibility of a right-angled triangle having unequal sides. The second learner's response that a right-angled triangle should have equal sides could also have been discussed to develop deeper understandings of the mathematics.

Hill (2010) defines learners' mathematical errors as "those incorrect student productions that offer opportunities for discussing pertinent mathematical ideas" (p. 11). The affordances to discuss mathematical ideas were there from the learners' productions but these were not used in class. Dimpho's evaluation of the learners' responses was often based on affirming the correct answers they gave, or rejecting the incorrect answers. Learners in this episode did not participate much in deeper mathematical discussions and reasoning, and the episode was coded level 1 for both sub-dimensions of student mathematical questioning and reasoning as well as enacted task cognitive activation. The episode focused primarily on recalling and applying well-established facts and rules, and the class engaged with the activities at a low level. Conceptual engagement was not emphasised in the activities discussed in the episode.

A second episode (Grade 11: August 2012, JPL2)

In this episode, the class was discussing a question from a worksheet from the joint lesson planning of the PLC (see Appendix 4). The worksheet dealt with true/false statement and in this episode the class was discussing whether the statement: $-\frac{1}{2} = \frac{-1}{2}$ was true or false, and to give an explanation for their choice. Dimpho invited learners to the board and engaged the class in a whole-class discussion. Figure 4.18 shows the first learner's response. What looks like an equal sign in the first statement is actually not an equal sign[19], but a minus sign. This made the statement: $-\frac{1}{2} = \frac{-1}{2}$, look like: $= \frac{1}{2} = \frac{-1}{2}$.

[19] There was a mark of a minus sign on the board before the first learner started writing her response.

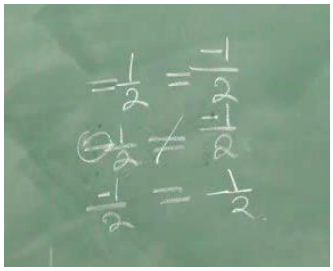

Figure 4.18: First learner's response.

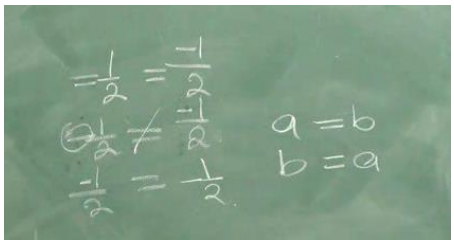

Figure 4.19: First learner's work with the teacher's probing.

The first learner said the statement was false. Her argument was that: "They are not the same because in $-\frac{1}{2}$ the whole fraction is to the negative sign, but this (pointing to $\frac{-1}{2}$), the numerator is the only one which is negative, but the denominator is positive". The learner further said if the two sides of the statement were swopped so that it became $\frac{-1}{2} = -\frac{1}{2}$, then the statement would be true. At this stage, Dimpho did not understand the meaning behind the learner's statement and she asked the learner to write *a* = *b* and *b* = *a* on the board (see Figure 4.19). Dimpho then asked the learner if the two statements were the same and the learner said they were not the same. Dimpho asked the learner to write $\mathbf{3} = \frac{9}{3}$ and $\frac{9}{3} = \mathbf{3}$ and asked her if the two were the same. The learner said they were not the same because only the second statement was true, and the first one was false. The learner's misconception was on treating the equal sign as a symbol that preceded an answer or as a 'do something' signal (Essien & Setati, 2006; Kieran, 1981; Knuth, Stephens, & Alibali, 2006; McNeil et al., 2006). When Dimpho asked the class who agreed with the first learner, some learners agreed and others disagreed.

Dimpho called a second learner to the board. The second learner came to the board and emphasised the notion of equivalence to explain how the two statements: $3 = \frac{9}{3}$ and $\frac{9}{3} = 3$ were the same. Dimpho emphasised the notion of equivalence and went back to the statements on negative half. She asked the second learner to explain it. The learner explained it using division of signs, and Dimpho wrote notes on the board as the second learner explained (see Figure 4.20). When the second learner was done, Dimpho asked the class if they understood. Some said they understood but a third learner used the first learner's argument to argue that $-\frac{1}{2} = \frac{-1}{2}$ was false. The second learner was still on the board and he tried to explain why the statement was true, but the third learner was not convinced. A fourth learner approached the board to argue that the statement was false.

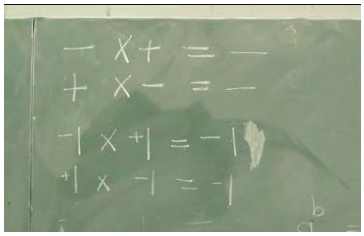

Figure 4.20: Dimpho's notes on the board.

Figure 4.21: Dimpho's notes on the board.

Dimpho explained using multiplication of signs (see Figure 4.20) to demonstrate that multiplication of numbers of different signs yields a negative answer and multiplying numbers of the same signs yields a positive answer. The third learner still referred to the fractions and insisted that $-\frac{1}{2} = \frac{-1}{2}$ was not correct. Dimpho demonstrated with division of positive and negative numbers, but the third learner was still not convinced. Figure 4.21 shows what Dimpho wrote on the board to try and address the learners' confusion. The fourth learner approached the board to explain the third learner's confusion. She said the third learner took $-\frac{1}{2}$ to mean $\frac{-1}{-2}$, and that was why she said the whole fraction was negative. The third learner agreed with what the fourth learner said, and Dimpho understood what the key misconception was. Dimpho then addressed this misconception by using division of signs to explain that $\frac{-1}{-2}$ would have a positive answer.

This episode shows a shift from the previous episode. Dimpho engaged the class in a whole class discussion and learners also commented on each other's inputs. Dimpho was still in control of delivery of content but in this episode she afforded the learners opportunities to share their thinking. This emerged when the second learner approached the board to help clear up the confusion that the first and the third learner had, and the fourth learner also assisted in explaining the first and the third learner's misconception. For *Mode of Instruction*, the episode was coded level 2 for both direct instruction and whole-class instruction as Dimpho engaged the learners in ways in which they could build on one another's contributions.

The episode was coded level 2 on the MQI scale for both sub-dimensions of *Working with Students and Mathematics*. The first and the third learner's responses were incorrect, but Dimpho used their thinking to build up a discussion that addressed a misconception. Opening up the discussion to the whole class created room for thorough remediation of learners' errors.

The sub-dimensions of *Student Participation in Meaning-Making and Reasoning* were also coded level 2 for this dimension because of the manner in which learners expressed themselves without fear. Though explanations were given to correct their thinking, the first and the third learner still continued to argue why their reasoning made sense to them.

4.13 Analysis of Dimpho's practice in the Group 2 dimensions

The dimension of *Richness of the Mathematics* shows momentary shifts that are linked to the sub-dimensions of explanations and mathematical language, and these shifts recorded small percentage scores on the MQI scale. IPL1 shows the highest percentage score in the sub-dimension of explanations at level 2 compared to the other sub-dimensions on the same level: 43%, but this score declined in cycle 1 of JPL1 and shows a non-shift in cycle 2 of JPL1. A slight shift is observed in JPL2 for explanations, recording a score of 36% at level 2, from the 24% of cycle 2 of JPL1 at level 2. This shows that Dimpho made an attempt to give mathematical meaning to ideas raised or to solution methods that were produced in class. This shift declined in IPL2, showing that Dimpho reduced the level of providing mathematical meaning in class.

Another slight shift to 24% at level 2 on the MQI scale is observed in cycle 2 of JPL1 in the sub-dimension of mathematical language. This shift shows that the teacher made an attempt to use mathematical language as a vehicle for conveying content in class. The use of mathematical language will be exemplified in an episode below. The observed shift in the sub-dimension of mathematical language was sustained in JPL2. However, both sub-dimensions of explanations and mathematical language show a decline in IPL2, with level 2 scores of 29% and 12% respectively. This decline shows that Dimpho's explanations and her use of mathematical language did not illuminate the depth of the mathematics that was done in class in IPL2. The other sub-dimensions remained at level 1 on the MQI scale for all the lessons.

Errors and Imprecision registered level 2 scores, and some level 3, on the MQI scale for all the lessons from IPL1 to IPL2. Dimpho's errors increased in cycle 1 of JPL1 with small occurrences of level 3 scores within the sub-dimensions of major mathematical errors and lack of clarity. These errors could be similarly attributed to the shifts observed in the teacher's practice as discussed in Funeka's and Mapula's analyses. An increased interaction with the learners resulted in an increase in the teacher's errors. Dimpho's errors were reduced in cycle

2 of JPL1, and this reduction in errors was sustained in JPL2 except in lack of clarity which showed a slight increase in errors: 23% at level 2 from 10% in cycle 2 of JPL1. However, lack of clarity improved in IPL2: a shift to 6% at level 2. The other two sub-dimensions, major mathematical errors and imprecision in language or notation, show increases in IPL2 at level 2. On the overall, the errors observed in Dimpho's practice were less than those observed for the other teachers.

The sub-dimension of lack of clarity generally appears to be relatively better among the sub-dimensions of *Errors and Imprecision,* except in JPL2 where it is slightly higher. This result shows that Dimpho was better at co-ordinating her own knowledge with the learners' ideas and she also embraced the learners' errors better. However, as Dimpho opened up her practice to engage with the learners' errors, she created more space for whole-class discussion and she consequently produced errors herself. The affordances she created for the learners to engage in class opened her up to dealing with subject matter that she had not planned for in class. I provide two episodes below to exemplify Dimpho's shifts in dealing with the mathematics and the errors that were produced in class as she interacted with her learners.

An earlier episode (Grade 11: October 2011, JPL1, cycle 1)

In this episode, Dimpho introduced trigonometric problems on heights and distances. She asked learners to name and define the different types of triangles. A learner gave an obtuse-angled triangle and defined it as one 'with at least one obtuse angle'. Dimpho affirmed this learner's response through re-voicing. The affirmation was reminiscent of her IRE-structured practice in IPL1 though in this case she re-voiced the learner's response. Dimpho's re-voicing suggested that she accepted the learner's response without realising the error in that response. A second learner challenged the first learner (which implied that he also challenged the teacher) and asked if the first learner's definition suggested that there were triangles with more than one obtuse angle. Dimpho realised the mistake in the first learner's definition and agreed with the second learner that it was not possible to have two obtuse angles in a triangle. She then corrected the first learner's definition and acknowledged the second learner's input.

The challenge by the first learner exemplifies the sub-dimension of student mathematical questioning and reasoning on the MQI tool. This episode was coded level 3 for this sub-dimension because it "featured three student behaviours listed under that code": the second

learner provided a counter-claim to the first learner's statement; he asked a mathematically motivated question; and he also engaged in reasoning about a general case of obtuse angles in triangles (Hill, 2010). This challenge by the second learner is evidence that he listened attentively to the first's response and the teacher's affirmation, and used reasoning to challenge them. Dimpho afforded the learner the space to raise his challenge.

The question that the second learner posed was mathematically motivated and thought-provoking. The use of the phrase "at least" by the first learner triggered the discussion of a mathematical error and awareness of being careful in the use of mathematical language. In addition, the second learner's challenge pushed the discussion in ways in which the class could build a conceptual mathematical base, which was key for rich mathematics. These two aspects qualified this episode to be coded level 2 on the MQI scale within the sub-dimensions of explanations and mathematical language in *Richness of the Mathematics,* and was also coded level 2 within the sub-dimension of imprecision in language in *Errors and Imprecision* because Dimpho did not correct the first learner's error on the use of the phrase "at least", which Hill (2010) describe as a momentary error in mathematical language.

A second episode (Grade 11: August 2012, JPL2)

This episode focuses on how Dimpho dealt with a problem that led to a discussion in class, though the learners' response was not in line with what the question required. In the episode, Dimpho was working on a worksheet that the PLC had developed and agreed to use with learners in their classes (see Appendix 6). The question on the worksheet was:

> Some money is shared between Mark and Jan so that Mark gets R5 more than Jan gets. Jan gets R*x*. Use algebra to write Mark's amount. The money to be shared is R47. Use Algebra to work out how much Jan and Mark would get.

A learner approached the board and used an arithmetic method to show her solution to the class. This learner's solution was similar to how a learner in Funeka's class responded to the same question. Figure 4.22 below shows the learner's arithmetic solution on the board.

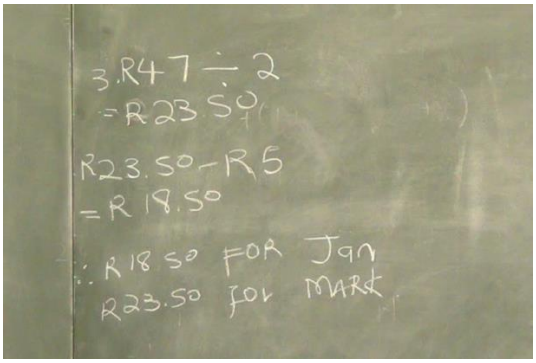

Figure 4.22: A learner's arithmetic solution on the board.

That learners in two different classes and different grades produced the same solution to the same problem signals the existence of a misconception, which may also be more widespread. In this episode, Dimpho first asked the learner who wrote her solution on the board to explain to the class how she worked out the problem. When the learner finished Dimpho asked her if the two amounts she got at the end: R18.50 and R23.50, would give a total of R47. The learner looked at her solution and said they would not add up to R47. Dimpho asked for comments from the class.

A second learner suggested that the R5 should have been added to the R23.50 instead of subtracting it. Dimpho repeated what the second learner had said and allowed a third learner to contribute. The third learner said the R47 should not have been divided by 2. Dimpho asked the third learner to give a reason for his objection to the division but he indicated that he did not have a reason why it should not have been divided. A fourth learner was given an opportunity and she said the R5 should be subtracted first before the amount could be divided by 2. Though the fourth learner was correct, Dimpho continued to ask the class if she was correct.

A fifth learner commented and indicated that she got an answer of 21 and 26 for the two amounts that were needed for a solution, and said she used a different method. When she was asked what was wrong with the method that the first learner had used she said the problem was to be done algebraically. Dimpho emphasised what the fifth learner said. The fifth leaner volunteered to show an algebraic method on the board, and this is shown in Figure 24 below. Dimpho asked this learner to explain his solution and the learner focussed on the procedural algebra rather than on the meaning of the problem.

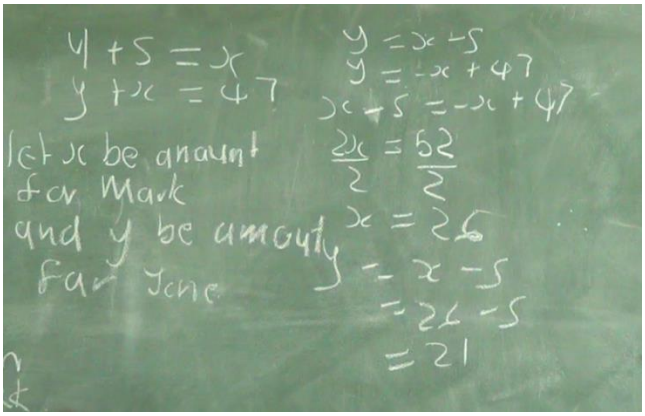

Figure 4.23: A learner's algebraic solution.

In this episode, Dimpho handled the learners' responses differently from how she handled them in the previous episode. In the previous episode, she closed the discussion after getting the correct answer from the second learner, whereas in this episode she gave learners more time to explain both their correct and incorrect answers, which signalled a shift in her practice. In all the four cases of the learners' arithmetic methods, Dimpho did not affirm or reject their responses. She asked for an explanation or allowed for more contributions from other learners in class. Dimpho closed the discussion after the fifth learner gave his algebraic solution and explained it on the board. She did not go back to the arithmetic solution, which was a missed opportunity to explain how the first and fourth learners were also thinking mathematically.

The algebraic solution and the arithmetic solutions provided a platform for multiple solution methods. However, the class did not look at the multiple methods to see how those methods linked to each other. In light of working with learner thinking, the arithmetic approaches were useful. There was something valid in the arithmetic methods that learners used, especially the first and the fourth learners. Dimpho did not work with both the arithmetic and the algebraic methods to show the link and the reasonableness in the two methods. Accepting the algebraic method in the end and not engaging more with both methods could have the following consequences:

- First, learners who used the arithmetic method would leave the classroom thinking that their methods were completely wrong. Also, the teacher's feedback, as is the case with the first four learners, did not afford her opportunities to get at the learner's thinking.
- Second, not acknowledging the validity in the other learners' solutions did not build upon the learners' conceptual understanding and may have left the learners confused.

Working with both methods, despite the problem requiring an algebraic method, would have helped learners know what was valid and what was not valid in their answers, and that would have enriched the mathematical discussion in class. This episode exemplified a slight shift in Dimpho's practice within the sub-dimensions of explanations and mathematical language in *Richness of the Mathematics* because she gave learners more time for their explanations without necessarily making it obvious to them that their answers were incorrect. There would have been a greater shift if Dimpho embraced the arithmetic methods to show that arithmetic methods are not necessarily mathematically incorrect. The episode was coded level 2 on the MQI scale and it typified how Dimpho often closed the discussions in class when a correct solution was found. She often opened up the discussions for incorrect answers but not for correct answers, an attribute which was different from Funeka's and Mapula's practice. Funeka and Mapula often opened up the discussion in class for both the correct and the incorrect answers. The episode also exemplified a reduction in Dimpho's errors though this was accompanied by the teacher's omission in embracing the learners' errors in a substantive way.

4.14 Summary of Dimpho's lessons

Dimpho's practice in IPL1 was dominated by the question-and-answer method which was characteristic of an IRE approach. She somewhat continued with the IRE approach in cycle 1 of JPL1, though she asked better questions in cycle 1 of JPL1 than in IPL1 lessons. When she gave learners work to do in IPL1, she asked them to write their answers on the board and allowed them to go back to their seats without letting them explain their answers. She then began to ask the class questions on the learners' answers which were on the board. Dimpho often tried to conduct a whole-class discussion but her engagement with learners did not focus on the reasoning behind the learners' answers. Her practice shifted in cycle 1 of JPL1 for *Mode of Instruction* as she opened her practice to whole-class discussion, allowing learners to make input on her own productions as well on other learners' productions. Dimpho sustained this practice on whole-class discussion in cycle 2 of JPL1 and in JPL2. Her practice in engaging learners declined in IPL2.

There were significant shifts in *Working with Students and Mathematics* in cycle 2 of JPL1 as well as in JPL2. In cycle 2 of JPL1, Dimpho allowed the learners not only to write their answers on the board but she also asked them to explain their answers. In those explanations, the

interaction was initially between Dimpho and the learner on the board and later it opened up to the whole class. There was a decline in this dimension in IPL2.

Student Participation in Meaning-Making and Reasoning shifted in cycle 1 of JPL1. The shift was more significant within the sub-dimension of students provide explanations. Those shifts were sustained in cycle 2 of JPL1 as well as in JPL2, except for the sub-dimension of student mathematical questioning and reasoning which slightly declined in JPL2. All the sub-dimensions declined in IPL2.

For the Group 2 dimensions, Dimpho's practice showed less shifts in *Richness of the Mathematics*. The slight shifts which were found in cycle 2 and JPL2 were linked to the sub-dimensions of explanations and mathematical language. However, these shifts declined in IPL2. Though Dimpho increased her interaction with learners in cycle 2 of JPL1, this interaction resulted in her producing errors in the process. The production of errors in Dimpho's practice resonated with what was observed in Funeka's and Mapula's results. However, Dimpho's practice produced fewer errors than Funeka's and Mapula's in the two years of data collection. In the next section, I discuss the shifts and/or non-shifts of the fourth teacher, Khumo, and show how her practice managed to improve over a certain period of time but this improvement not been sustained.

4.15 Overview of Khumo's shifts: Groups 1 and 2 dimensions

Khumo's IPL1 lessons often began with examples that she did on the board for learners. When she asked questions, learners often chorused their answers, and this practice recurred in all her lessons. Her practice was also characterised by a high amount of teacher talk. When she called learners to the board to do problems, Khumo often wrote the problem statement on the board and allowed learners to continue with their solutions. Once a learner had completed a problem, Khumo often asked the class if the learner's solution was correct, and learners would chorus a 'yes' or a 'no'. If the learner's solution was incorrect, Khumo would often rub it off and correct the problem for learners.

Khumo made an attempt to engage learners in her lessons and to work with the errors that they produced, but she rarely focused on understanding the reasoning behind the learners' errors. Her practice began to show shifts in cycles 1 and 2 of JPL1. A typical feature that emerged in

cycles 1 and 2 of JPL1 was that when a learner at the board struggled to get an answer, Khumo would often approach that learner and engaged him/her in a one-on-one interaction. This interaction did not involve the other learners in class and Khumo would ask the learner leading questions that funneled the learner to the correct answer. The observed shifts in cycles 1 and 2 were only sustained in *Student Participation in Meaning Making and Reasoning* in JPL2. The other dimensions showed a decline in JPL2. IPL2 showed shifts in *Mode of Instruction* with some slight shifts in *Working with Students and Mathematics.* The other dimensions declined in IPL2.

Table 4.5: Frequency of codes – KHUMO

Dimension	Sub-Dimension	The IPL1 (9 episodes)			JPL1, Cycle 1 (18 Episodes)			JPL1, Cycle 2 (19 episodes)			JPL2 (9 episodes)			IPL2 (15 episodes)		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Mode of Instruction	Direct instruction	11	33	56	5	90	5	0	95	5	0	55	45	6	94	0
	Whole class discussion	100	0	0	77	23	0	89	11	0	89	11	0	100	0	0
Richness of Mathematics	Linking or connections	100	0	0	100	0	0	100	0	0	100	0	0	100	0	0
	Explanations	100	0	0	67	33	0	63	37	0	89	11	0	100	0	0
	Multiple procedures or solution methods	89	11	0	100	0	0	89	11	0	100	0	0	100	0	0
	Developing mathematical generalisations	100	0	0	100	0	0	100	0	0	100	0	0	100	0	0
	Mathematical language	100	0	0	94	6	0	53	47	0	100	0	0	87	13	0
Working with Students and Mathematics	Thorough remediation of student errors and difficulties	100	0	0	77	23	0	63	37	0	89	11	0	80	20	0
	Responding to student mathematical productions in instruction	100	0	0	83	17	0	68	32	0	100	0	0	87	13	0
Errors and Imprecision	Major mathematical errors or serious mathematical oversights	44	12	44	77	23	0	74	2	24	33	55	12	67	6	27
	Imprecision in language or notation (mathematical symbols)	33	45	22	83	17	0	58	42	0	22	56	22	60	40	0
	Lack of clarity	67	33	0	83	0	17	53	32	15	22	56	22	53	33	14
Student Participation in Meaning Making and Reasoning	Students provide explanations	100	0	0	60	40	0	42	58	0	45	55	0	73	27	0
	Student mathematical questioning and reasoning	100	0	0	100	0	0	100	0	0	100	0	0	100	0	0
	Enacted task cognitive activation	22	78	0	100	0	0	100	0	0	45	55	0	100	0	0

4.16 Analysis of Khumo's practice in the Group 1 dimensions

Khumo's scores in the dimension of *Mode of Instruction* are somewhat erratic. Observations show that instruction in IPL1 was in a teacher-directed manner. The sub-dimension of direct instruction has a percentage score of 56% at level 3. Whole-class discussion in IPL1 has a percentage score of 100% at level 1, which suggests that there were no affordances for learners in class to share their thinking or explain their reasoning in the mathematics they did in class. A significant shift is seen in cycle 1 of JPL1 within the sub-dimension of direct instruction which is evidenced by a reduction of the 56% score at level 3 from IPL1 to 5% in cycle 1 of JPL1. Whole-class discussion shows a slight shift in cycle 1 of JPL1.

The observed shifts in direct instruction and whole-class discussion are ascribed to an attempt by Khumo to involve learners in discussions by allowing them to explain the steps as they worked out the problems on the board and thereby sharing their thinking in class. In her attempt to engage with her learners, Khumo did not focus on the reasoning behind the learners' answers.

Observations in Table 4.5 show that the shift in direct instruction is sustained in cycle 2 of JPL1 while whole-class discussion declines, which shows that learners' engagement in Khumo's class featured less in those lessons. Direct instruction shows a reduction in percentage score in JPL2 at level 2: from 95% in cycle 2 of JPL1 to 55% in JPL2. A sharp increase to 94% at level 2 for direct instruction is seen in IPL2. Whole-class discussion shows a non-shift in JPL2 and a complete decline in IPL2 evidenced by 100% at level 1 on the MQI scale. This decline in whole-class discussion shows that Khumo's practice in IPL2 reverted back to her practice in IPL1 which was dominated by a high amount of teacher talk.

The decline in IPL2 on whole class discussion is a result of Khumo resorting to a one-on-one interaction with a learner on the board, instead of opening up the discussion to the whole class. Khumo often allowed a learner at the board to explain his/her solution to the class. However, when she noticed an error on the learner's solution, she often approached the learner and engaged him/her on one-on-one discussion using leading questions. The dialogue between Khumo and the learner at the board often did not involve the other learners in class in the discussions.

The IPL1 lessons show only level 1 scores on the MQI scale for both sub-dimensions of *Working with Students and Mathematics*. This observation shows that Khumo did not embrace the learners' inputs in IPL1, and this resonates with what was discussed above in *Mode of Instruction*. She often wrote the question for learners on the board, called learners to the board to write their answers, asked the learner on the board to explain his or her solution to the class, and asked the class if the learner's solution was correct. When a learner's solution was incorrect, Khumo often rubbed it off and asked leading questions that directed the class to the correct answer, with learners often chorusing the answers. This approach of leading the learners to the correct answer did not help Khumo in getting at the learners' thinking.

Table 4.5 shows slight shifts in both sub-dimensions for cycles 1 and 2 of JPL1: 23% in thorough remediation of student errors and 17% in responding to student mathematical productions, each at level 2. Cycle 2 of JPL1 also shows slight shifts for both sub-dimensions. The observed shifts evidence an attempt by Khumo in implementing discussions from the PLC relating to understanding the reasoning behind learners' errors in their teaching. There was significant decline in JPL2 within the sub-dimension of responding to student productions, reaching a level 1 score of 100% on the MQI scale. IPL2 shows slight shifts for both sub-dimensions at level 2: 20% for thorough remediation and 13% for responding to student mathematical productions. The observed occurrences in IPL2 signal that Khumo did not shift in responding to the learners' mathematical productions. Her practice was mostly erratic, showing slight shifts in some lessons and declines in other lessons.

Student Participation in Meaning Making and Reasoning featured only the sub-dimension of enacted task cognitive activation at level 2 in IPL1 at 78% on the MQI scale, but this declined to 100% at level 1 in cycle 1 of JPL1 and a non-shift in cycle 2 of JPL1. The other sub-dimensions featured level 1 on the MQI scale in IPL1. Enacted task cognitive activation was featured again at level 2 in JPL2 at 55% on the MQI scale but declined in IPL2 to 100% at level 1. These observations within the sub-dimension of enacted task cognitive activation contrast with what was observed in the analyses of Funeka's and Dimpho's practice, which was interesting given the fact that the tasks they dealt with were the same and planned together in the PLC. The observed occurrences in cycles 1 and 2 of JPL1 show that Khumo lowered the cognitive demand of the task (Stein, Grover, & Henningsen, 1996). In addition, the observed occurrences illustrate inconsistencies in Khumo's practice over the two years: a level 2 score

in IPL1, followed by level 1 scores in cycles 1 and 2 of JPL1, and a Level 2 score in JPL2, which is followed by a level 1 score in IPL2.

A shift is observed in cycle 1 of JPL1 within the sub-dimension of students provide explanations. This shift is significant in cycle 2 of JPL1 and it is sustained in JPL2: 58% in cycle 2 of JPL1 and 55% in JPL2, both at level 1 on the MQI scale. Observations show a decline within the same sub-dimension in IPL2. The sub-dimension of student mathematical questioning and reasoning stayed at 100% at level 1 for all the lessons from IPL1 to IPL2, thereby indicating a non-shift in Khumo's practice on this sub-dimension. This observation shows that learners in Khumo's class did not engage in mathematical questioning and reasoning over two years. Below I provide episodes that exemplify Khumo's practice for the Group 1 dimensions over the two-year data collection period.

An earlier episode (Grade 7: February 2011, IPL1)

In this episode, Khumo was dealing with calculations of areas and perimeters of shapes. The lesson focused on regular and irregular shapes. The tasks that Khumo dealt with in that lesson had potential to challenge the learners' thinking, especially for a Grade 7 class. Khumo used free-hand sketches in these tasks. The diagram in Figure 4.24 below is from one of the problems that Khumo gave to learners as homework as an example to calculate area of an irregular shape. In this episode, Khumo was going through the corrections of that homework.

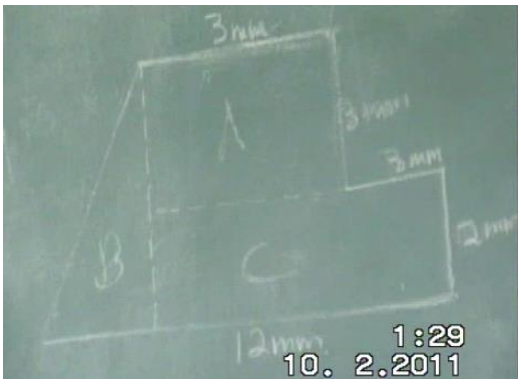

Figure 4.24: Example of an irregular shape.

Khumo began by asking a volunteer to come to the board to sub-divide the figure in order to get regular shapes from the given figure. A learner got three shapes after partioning the figure, and the shapes were then labelled A, B and C as shown in Figure 4.24. Khumo followed an

IRE-structure in posing questions to learners. She first focussed the learners on shape A and asked them what kind of shape it was. The class correctly identified the shape as a square, and Khumo accepted that response. She then asked learners to give a formula for calculating the area of a square. One learner gave the formula as four lengths. Khumo told the class that they wanted the area and not the perimeter. A second learner was inaudible and Khumo said "area, area", indicating that Khumo did not accept the learner's answer, which was reminiscent of ignoring the learner's error. A third learner said "length multiplied by length multiplied by length…" but Khumo interrupted her and again said "area, area". A fourth learner gave an answer of "length times breadth" but Khumo rejected this answer as well. When a fifth learner gave an answer of length times length, Khumo affirmed the fifth learner's answer and asked the class to repeat what the fifth learner said. The class chorussed the fifth learner's answer.

For the entire discussion of this task, there was a high amount of teacher-talk, with Khumo leading the class in a question-and-answer method. The questions that Khumo used were closed, and they did not promote whole-class discussion. She asked the learners to identify the shapes, to give relevant formulae for specific shapes, to give the corresponding dimensions such as length and breadth, and to substitute and give the answers. Khumo's practice in this episode was coded level 3 for direct instruction and level 1 for whole-class discussion. Khumo's remediation in the episode was procedural and non-substantive, and instruction was routine. In light of the two aspects on remediation and instruction, the episode was coded level 1 on the MQI scale for both sub-dimensions of *Working with Students and Mathematics*. Since there were no student explanations or mathematical questioning and reasoning in the episode, Khumo's practice was coded level 1 on the MQI scale for the sub-dimensions of students provide explanations and student mathematical questioning and reasoning. Enacted task cognitive activation was coded level 2 for reasons already explained above.

In the process of engaging with the learners, Khumo often missed out on the opportunity to make connections in mathematical concepts. For example, when the fourth learner gave the formula for calculating the area of a square as length times breadth, Khumo rejected this answer instead of using it to make connections with rectangles. It was clear at this stage that Khumo was looking for a particular answer as she continued to ask the other learners for a correct answer. The rejection of the fourth learner's response denied Khumo opportunities to work with the learners' mathematical productions in order to build on their conceptual knowledge.

It was not clear at that stage whether Khumo was aware that the fourth learner's answer was mathematically correct – this is similar to Dimpho's handling of arithmetic solutions when she dealt with algebraic expressions. In Khumo's episode, a link between a square and a rectangle would have helped the class to see a relationship between the two, and deepened learner understanding in line with *Richness of the Mathematics*. This episode typifies Khumo's practice in IPL1.

A second episode (Grade 7: March 2012, JPL1, cycle 2)

This episode focuses on a small shift observed in Khumo's practice – the shift was from not getting learners involved when she worked on examples to getting them involved. In this episode, Khumo was introducing linear equations in class, and she started by first discussing inverse operations with the class. She then put an example on the board and engaged learners in a question-and-answer method. Learners often raised their hands when they wanted to give answers, and they still chorused when answering. Figure 4.25 below shows the example that Khumo did with the class. When a learner could not answer a question, Khumo often redirected the question to another learner. After the correct answer was obtained, Khumo asked the first learner who did not get the correct answer to repeat that correct answer for the class to hear and to ascertain whether the learner had been attentive.

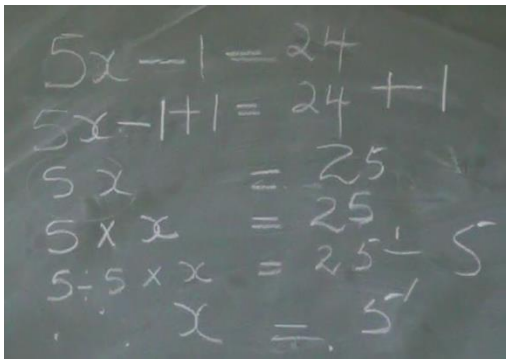

Figure 4.25: Khumo's example on linear equations.

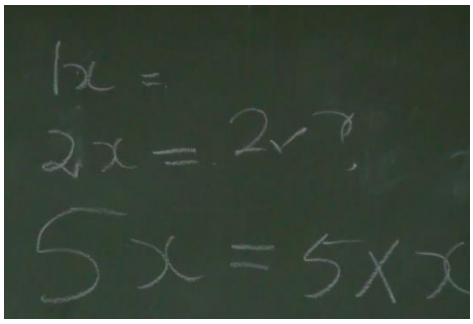

Figure 4.26: Khumo's emphasis on products.

For every statement of the example she dealt with, Khumo emphasised the inverse operation. For example, she asked learners what the inverse of negative was from the first statement in Figure 4.25 in order to get the second statement: $5x - 1 + 1 = 24 + 1$. When she got to the third statement: $5x = 25$, Khumo asked the class to explain what $5x$ meant to them. When only a few learners raised their hands, Khumo separated the $5x$ from the example and wrote it on the

board. She then focused her question on one learner, but that learner could not give the correct answer. She then wrote 1x and 2x (see Figure 4.26) and the learner could still not give the correct answer. When a second learner gave the correct answer she then let the class chorus that answer and asked the first learner to repeat that answer. The first learner repeated it correctly and thereafter correctly explained what 2x meant when Khumo asked him. Separating the 5x from the example helped the class to focus on a key mathematical concept: that 5x means 5 multiplied by x, and that helped the class to continue applying the inverse operations in the example.

This episode exemplifies a small shift in Khumo's practice in cycle 2 of JPL1. In IPL1, Khumo often did the example for the class but in cycle 2 of JPL1 she put the example on the board and engaged the class. This episode was coded level 2 on the MQI scale for both direct instruction and whole-class discussion. Khumo shifted from not being accommodative of the learners' inputs to being accommodative. When a learner gave an incorrect answer, she did not dismiss or reject it as in IPL1, but rather gave other learners in class opportunities to make their contributions. She made an attempt to be more accommodative of the learners' errors and increased her engagement with the class. However, Khumo did not inquire into the reasoning behind the learners' errors, which means that though she tried to be accommodative of the learners' inputs, however she did not embrace their errors.

Khumo's remediation of the learners' errors often focused on getting the correct answer than understanding the learners' misconceptions. Remediation was often brief and non-substantive, though she somewhat responded to the learners' mathematical productions as in the case of explaining the mathematical meaning of 5x. The episode was therefore coded level 2 on the MQI scale for both sub-dimensions of remediation of student errors and responding to student mathematical productions. Khumo built her instruction on the learners' ideas in this episode but her remediation was not thorough or systematically engaging the learners' errors.

The dimension of *Student Participation in Meaning-Making and Reasoning* in this episode did not have features of student mathematical questioning and reasoning, and the level of engagement with the task was not highly cognitive. Engagement with the task was more procedural than conceptual. However, Khumo created a platform for learners to make contributions. The episode was therefore coded level 2 on the MQI scale for the sub-dimension

of students provide explanations and level 1 for both student mathematical questioning and reasoning as well as enacted task cognitive activation.

4.17 Analysis of Khumo's practice in the Group 2 dimensions

Khumo's results in *Richness of the Mathematics* interestingly had features of multiple procedures or solution methods at level 2 in IPL1. The other three teachers' scores for this sub-dimension were at level 1. However, the sub-dimension of explanations featured level 2 scores in the other three teachers' practices in IPL1 but level 1 in Khumo's practice, because Khumo's explanations and the learners' explanations in IPL1 were often descriptions of steps of a procedure with no mathematical meaning. The descriptions made did not explain why a procedure worked or why a solution method made sense.

Observations show a shift to 33% at level 2 in cycle 1 of JPL1 within the sub-dimension of explanations and a further slight shift to 37% at level 2 in cycle 2 of JPL1, which means Khumo sustained the shift in her practice within the sub-dimension of explanations. A decline to 11% at level 2 within the same sub-dimension is observed in JPL2. The observed shifts in cycles 1 and 2 of JPL1 are an indication that the explanations made in Khumo's class included why a procedure worked and why certain solution methods made sense. Observations show that Khumo's practice in IPL2 is similar to IPL1. Her practice in IPL2 scored level 1 on the MQI scale, which shows that explanations made in Khumo's class in IPL2 reverted back to descriptions of procedures with no mathematical meaning.

Mathematical language shifted significantly in cycle 2 of JPL1, reaching 47% at level 2. There were some features of "linguistic sloppiness" (Hill, 2010) in the episode. For example, when dealing with addition and subtraction of terms, Khumo often said "a negative and a positive give a negative", which had potential of creating a misconception for learners. A significant decline to 100% at level 1 is observed in JPL2, with a slight shift to 13% at level 2 in IPL2, showing that the teacher's talk was characterised by use of non-mathematical terms in JPL2 with some improvement in IPL2. The other sub-dimensions in *Richness of the Mathematics* are at level 1 on the MQI scale for IPL2.

The dimension of *Errors and Imprecision* shows prevalence of errors in all Khumo's lessons. This prevalence of errors is an indication that not only did Khumo produce errors in her lessons, but she often did not address the learners' errors and misconceptions appropriately. IPL1 shows level 3 scores of 44% and 22% on the MQI scale in the sub-dimensions of major mathematical errors and imprecision in language, which shows that Khumo made major errors in spoken or written work. A shift to 23% and 17% at level 2 is observed in cycle 1 of JPL1 within the same sub-dimensions. These improvements in the two sub-dimensions show that Khumo was getting better at working with errors in her class, allowing learners to explain their incorrect answers. The sub-dimension of lack of clarity, though, shows a score of 17% at level 3, showing that "the teacher's presentation of the mathematical content was *unclear, vague* or *incomplete* for most of the segment" (Hill, 2010).

Observations show that Khumo's errors increased in cycle 2 of JPL1, and became significant in JPL2 and IPL2. All the sub-dimensions in this category recorded level 3 scores: 12% in major mathematical errors and 22% each in the other two sub-dimensions. The corresponding level 2 scores are 55% in major mathematical errors and 56% each in imprecision in language and lack of clarity. In addition, Khumo produced more errors in her practice than the other three teachers analysed in this study. I discussed above that Khumo, in an attempt to address the learners' errors, often created a one-on-one dialogue between her and a learner at the board. This dialogue was subsequently narrowed to funnelling the learner to a correct answer and ignoring the reasoning behind the learners' error. The one-on-one dialogue and the funnelling suggest that Khumo neglected to deal with key aspects of a problem, which could have helped the class address the misconceptions that emerged in instruction. I provide two episodes below to exemplify the findings in Khumo's practice within the Group 2 dimensions.

An earlier episode (Grade 7: March 2012, JPL1, cycle 2)

This episode is from an introductory lesson for cycle 2 of JPL1. The task was based on one learner thinking of a number, and not telling the class what that number was. A second learner was then to say a number that s/he wanted to add to/subtract from the first learner's number, and the first learner was supposed to give an answer after that operation. The activity would end when the class could correctly say the number that was in the first learner's mind. The dialogue between the first and the second learner was basically meant to produce a linear

equation which the class would solve. Khumo began the lesson by emphasising inverse operations which she wrote on the board: addition with subtraction and division with multiplication. In this episode, Khumo was initially the one writing what learners said. Learners did not write on the board until later. See Figures 4.27 and 4.28 below for some of what Khumo wrote on the board as she interacted with learners.

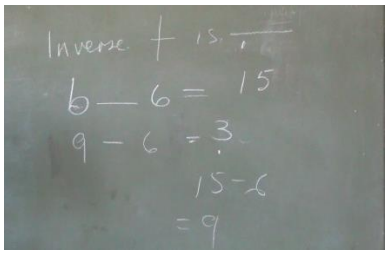

Figure 4.27: Khumo's work on an answer of 9.

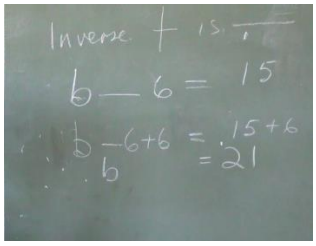

Figure 4.28: Khumo's work on an answer of 21.

After the first learner told Khumo that she had a number in mind, a second learner subtracted six from that unknown number, and the first learner said the answer was fifteen. Khumo wrote $b - 6 = 15$ on the board and asked the class what the first learner's number was, which was the value of *b*. A third learner said the *b*-value was nine and Khumo asked him how he got nine. The learner said fifteen minus six is nine, and Khumo wrote $15 - 6 = 9$ on the board. Khumo then substituted 9 for *b* and showed that the answer was not nine (see Fig. 4.27 above). Though Khumo asked the third learner how he got nine, she did not ask him why he subtracted six from nine in order to get at the reasoning behind his error.

Khumo redirected the question on the value of *b* to the class and a fourth learner gave an answer of twenty-one. Khumo invited her to the board in order to show how she got twenty-one. The learner approached the board and explained that she substituted *b* by twenty-one. Khumo was somewhat unhappy about the learner's explanation and told the learner that twenty-one was not there in the problem, that the learners came with that answer and should explain how she got it. The learner could not explain. Khumo then asked a fifth learner to the board. The fifth learner wrote $15 + 6 = 21$ and $21 - 6 = 15$ on the board and read what he wrote on the board. The class applauded. Khumo then showed an algorithm on the board for getting twenty-one (see Figure 4.28).

Though the fourth learner's answer was correct, Khumo did not acknowledge it and the learner was not clarified on why her explanation was not considered valid. This episode has shown

how Khumo interacted with learners on both the correct and the incorrect answers. She was still looking for the correct answer, and this episode was scored level 2 on the MQI scale. Similar practices of interacting with learners on both the correct and the incorrect answers were found in some of the lessons of the other three teachers' practices.

When a learner was at the board in this episode, Khumo kept her distance from that learner in order to involve the class in a whole-class discussion. The explanations that surfaced in class tried to address the extent to which certain procedures worked and others did not. In the process, Khumo also used mathematical language as a vehicle to convey content, as was the case with the use of inverse operations in solving linear equations. The episode was therefore coded level 2 within the sub-dimensions of explanations and mathematical language.

A second episode (Grade 7: August 2012, JPL2)

In this episode, the class was working on a problem that came from a worksheet that the PLC developed (see Appendix 6), and the problem read:

> The perimeter of a rectangle is 48 *cm*. If the length of the rectangle is three times the width, determine the dimensions of the rectangle.

Khumo gave learners some time to work on the problem. Learners struggled with the problem and could not find a solution. Khumo then decided to do the problem on the board for the class. Khumo began by asking for volunteers to the board but none came up. She then asked a learner for the perimeter of a rectangle and he could not give it. Khumo asked the class about the formula and a second learner said length times breadth. Khumo corrected the learner and explained that length times breadth was for the area. A third learner said "two lengths add two lengths". Khumo asked if there were two lengths. She then directed the learner to the board at an example of a rectangular shape. She pointed to a fourth learner who said "two lengths add two breadths", and she repeated this statement to the first learner who was still at the board. The learner could still not do much about that information except to write an incomplete statement: "48 *cm* = 2(3". Khumo then asked for a volunteer to assist the first learner. A fifth learner stood and approached the board to write what is depicted in Figure 4.29 below.

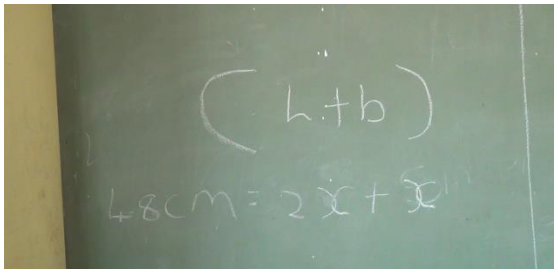

Figure 4.29: The fifth learner's work on the board.

Khumo asked the learner what the *x* represented in the expression that he wrote and he said it represented the breadth. Khumo rejected that answer. She then referred to the classroom board as an example of a rectangular shape and asked the class the question: "How many lengths do we have on a rectangular shape," (still referring to the board). A sixth learner gave an answer of four, and Khumo rejected this answer. When a seventh learner gave an answer of two, she accepted that answer and wrote 2*l* on the board. She asked a similar question about breadth and a learner gave an answer two and she wrote that on the board. She then made the class chorus a formula for the area of a rectangle. By this time, Khumo made a call that learners were unable to get the correct solution, and she decided to do the problem for the class on the board. Figures 4.30a and 4.30b show how Khumo did the problem on the board – these are two parts of the same detail.

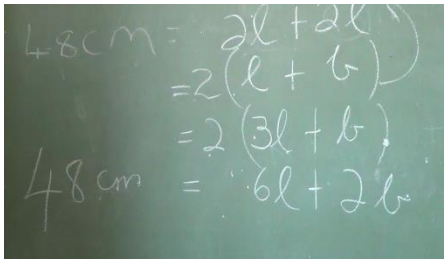

Figure 4.30a: Khumo's solution (part 1).

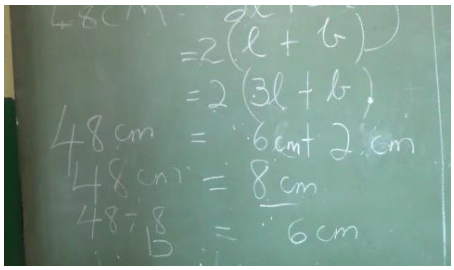

Figure 4.30b: Khumo's solution (part 2).

In this episode, Khumo followed an IRE structure. Khumo's questions prompted for explanations in class and the use of mathematical language in conveying content. The episode was therefore coded level 2 on the MQI scale for the sub-dimension of explanations. Explanations showed a decline though in JPL2 because some of the explanations given were not correct. All the sub-dimensions, including multiple procedures and mathematical language which featured in cycle 2 of JPL1, were not evident in this episode. The episode was therefore coded level 1 for these sub-dimensions on the MQI scale.

Khumo did not try to understand the learners' errors and misconceptions, and made some of her own. In the first statement of Figure 4.30a, Khumo wrote an expression for area as $48\ cm\ = 2l + 2l$. She then took out a common factor of 2 to get $48\ cm = 2(l + b)$ in the second statement. I took the $2l + 2l$ in the first statement as a slip (Olivier, 1989) because the second statement was correct. In showing that the length was three times the width, Khumo wrote $2(3l + b)$ in the next statement, which was the first major error that she produced. The next major error that Khumo produced was in saying that l and b were in centimetres and replacing each letter by cm to get: 48 *cm* = 6 *cm* + 2 *cm*, which "distorted the mathematical essence of the content" (Hill, 2010). Every other statement that followed thereafter was erroneous. The third major error that Khumo produced was treating a unit of measurement: cm, as a variable. Khumo made major mathematical errors when doing the problem. The errors that Khumo produced had potential to create serious misconceptions in learners. This episode was therefore coded level 3 for all the three sub-dimensions of *Errors and Imprecision* because of major mathematical errors that Khumo produced and lack of clarity in her explanations.

4.18 Summary of Khumo's shifts

Khumo's IPL1 lessons started with examples which she used to introduce new content in class. Her style of questioning often encouraged the learners to chorus answers, which recurred in subsequent lessons. Khumo's practice was also characterised by a high amount of teacher talk. Her results on the MQI scale as observed from Table 4.5 were erratic looking across all her set of lessons.

Khumo's practice showed shifts in cycle 1 of JPL1, which was sustained in cycle 2 of JPL1 for all the three dimensions of Group 1. However, her practice declined in JPL2 for *Mode of Instruction* and *Working with Students and Mathematics*, but was sustained in *Student Participation in Meaning-Making and Reasoning*. In cycles 1 and 2 of JPL1, Khumo allowed learners to explain their work but she rarely inquired on the reasoning behind the errors which learners produced. The one distinct feature in Khumo's lessons was to approach a learner who was at the board when she noticed an error and created a dialogue between her and that learner. She used this dialogue to guide the learner towards the correct answer by asking him/her leading questions. Khumo continued with this practice in JPL2 as well as in IPL2. Her response

to learners' answers often suggested that she was looking for particular answers in the learners, which stifled opportunities for whole-class discussion.

Group 2 dimensions revealed similar erratic shifts as in Khumo's Group 1 dimensions. Though Khumo's practice appeared better in cycle 1 of JPL1 for both dimensions of Group 2, her practice declined in *Errors and Imprecision* in both cycle 2 of JPL1 as well as in JPL2. A further decline was found in IPL2 for all the sub-dimensions of *Richness of the Mathematics* and the two sub-dimensions of *Errors and Imprecision*. Findings have shown that Khumo produced more errors than the other three teachers analysed in this study.

4.19 Conclusion

In this chapter, I looked at the practices of four teachers who participated in the DIPIP project and were participants in my research study: Funeka, Dimpho, Mapula and Khumo. The four participants worked in two different schools. This chapter dealt with analyses of the teachers' practices using the MQI instrument. The instrument had five key dimensions which I divided into two groups for purposes of analysis.

The analysis has shown that all the four teachers shifted their practices in cycle 1 of JPL1. Funeka and Mapula managed to sustain their shifts in IPL2 while Dimpho and Khumo did not. Among the four teachers, Funeka's practice shifted the most while Khumo shifted the least. The most shifts in Funeka's practice occurred in cycle 2 of JPL1 and IPL2; for Mapula it was in JPL2 and IPL2; and in Dimpho's and Khumo's practices it was in cycle 2 of JPL1.

In my analysis of the teachers' practices, I also discussed similarities and differences between and among the four teachers. All the four teachers embraced learner errors from the second set of lessons that were videotaped. Funeka and Mapula often probed for the reasoning behind the learners' errors while Dimpho and Khumo rarely probed for that reasoning. Chauraya & Brodie (2017) showed how the teachers' engagement with the learners' errors and the reasoning behind the errors led to the shifts in the teachers' practices. All the four teachers produced errors as they began to interact with learners in new ways. *Richness of the Mathematics* showed the least shifts in all the four teachers. However, the sub-dimensions of explanations and mathematical language emerged prominent within this dimension.

The next chapter focuses on community data to investigate how the four teachers participated in the PLC. The key focus is primarily on how the different PLC activities related with different content and the depth of the PLC conversations, and the extent to which the community conversations contributed to the teachers' PCK.

Chapter 5: PLC Conversations

5.1 Introduction

This chapter looks at the conversations in the professional learning community in 2011 and 2012. I first look at the extent to which the community engaged with the different activities for content and depth in their conversations. A coding tool developed by DIPIP, discussed in Chapter 3, was used for the coding and to guide the analysis of the data collected from PLC meetings. I also look at the extent to which the community conversations increased opportunities to develop pedagogical content knowledge (PCK) within the community in the two years of working with this community.

My analysis of the conversations focused on the relationships between content and activity and depth and activity. First, I looked at the focus of the PLC conversations in relation to the different activities and related the content of the conversations to the activities. Second, I looked at the level of depth in the conversations for specific content in each activity. The analysis shows that different PLC activities focus on specific content at different levels of depth in the PLC conversations. My results also show how some content which received less attention in the project design emerged as central to the PLC conversations.

The activities that the PLC worked on provided a platform for the participating teachers to inquire into their practice and consequently create new knowledge for themselves which they could use in their classrooms. By new knowledge I mean knowledge which was not developed yet in the community or teachers were not aware of until they talked about it in the PLC. This knowledge was both pedagogical content knowledge (PCK) and content knowledge (CK), but I focus on PCK. As teachers talked about their practice in the PLC, implemented the learning from their talk in their classrooms, and came back to the PLC to reflect on their practice, they created new knowledge, more usually in the form of PCK, which then does shape their developing CK (Marchant & Brodie, 2016a).

This chapter therefore focuses on how seven participating teachers learned from and with each other through the conversations they had in the PLC about their practice. The two research questions that this chapter addresses are:

- What do teachers in a PLC talk about and at what depth do they engage with different activities?
- How is pedagogical content knowledge (PCK) amongst teachers created within the professional learning community?

In answering the first question, I focused on how the participants talked about their practice; how they worked on and discussed the different activities in the PLC; how their conversations deepened; and how this depth was influenced by the activity and the content that was discussed in their PLC meetings. This focus included how the participating teachers talked about: the mathematical tasks they designed; the influence that those tasks had on the learners' thinking; how learners responded or would respond to the tasks; and how to work with the errors that the learners produced as they worked on tasks. In answering the second question, I use extracts from episodes of the PLC conversations to show how the participating teachers built on their pedagogical content knowledge through community conversations about the learners' errors. All the claims related to teacher learning are about what was created in the PLC, and so opportunities for developing or enhancing the teachers' PCK, rather than actual learning. The next section focuses on the results of coding the community conversations and how these conversations were analysed.

5.2 Analysis of the PLC conversations

Tables 5.1 to 5.10 below show the percentage scores of the coded times for content and depth in the different activities in the PLC conversations over two years. The activities in the tables appear in the order in which they were engaged with in the PLC meetings in 2011: Error Analysis, followed by Readings and Discussion, then Lesson Planning and lastly Lesson Reflection. In 2012, some of the activities were engaged with in a different order based on what happened in 2011 and PLC decisions. Error Analysis, which focused on analysing an international Algebra test, was not included in the 2012 activities and was replaced by in-depth lesson reflection on the 2011 lessons. The PLC decided to do an in-depth reflection on the previous year's lessons after realising that there was more to reflect on after the 2011 reflections. There was a total of nine sessions on Lesson Reflection in 2012, five of which happened at the beginning of 2012.

Error Analysis: Content and Depth

In the Error Analysis activity (only in 2011), the PLC identified common errors that learners had produced and engaged in discussions to determine the reasoning behind those learners' errors in order to identify the source(s) of the learners' misconception(s). A table of percentages is given below to show the focus of the PLC conversations in the activity in 2011.

Table 5.1: Percentage scores for content in Error Analysis in 2011.

	Error Analysis	2011
Content	Learner	42
	Mathematics	11
	Practice	2
	Learner Thinking	45
	Other codes	0

The focus of the PLC conversations in Error Analysis in 2011 was more prominent on learner (42%) and learner thinking (45%), with some focus on mathematics (11%). The focus on learner and learner thinking means that teachers identified the learners' errors and the strategies that learners used in their solutions; talked about those errors; and talked about the reasoning behind the errors (see chapter 3 for more detail of these codes). The practice of focusing on the learners' errors and misconceptions helped the teachers to think about how they viewed the learners' errors and what they could learn from that process.

The focus on learner entailed what the actual errors that learners produced were, the different strategies that the learners used to produce the errors, and how the learners understood the mathematical content and context of the question. This focus helped the teachers to be considerate of what often happens in their classrooms regarding their learner's learning, and to also acknowledge that a focus on learner errors was important to determine what needs further inquiry about their practices. The focus on learner thinking entailed talking about the reasoning behind the learners' errors and seeing the sense and logic in the errors that the learners produced. The focus on mathematics shows that sometimes teachers talked about the mathematics embedded in the errors and how that mathematics influenced the learners in producing the mathematical errors.

As explained above, Error Analysis was not done in the project in 2012 and this analysis does not talk about the shifts in content in this activity. Table 5.2 below shows the level of depth that the PLC conversations reached in Error Analysis in 2011.

Table 5.2: Percentage scores for depth in Error Analysis.

	Error Analysis	**2011**
Depth	Level 1	54
	Level 2	35
	Level 3	11
	Level 4	0

The PLC spent most of the coded time at level 1 (54%), showing that the PLC conversations in Error Analysis mostly focused on general impressions of what occurred when learners produced the mathematical errors, with little or no evidence to support analysis of the learners' errors (see chapter 3 for more details of the coding system). This result means that the PLC conversations provided little or no evidence to exemplify their ideas or explanations. The 35% percentage score at level 2 shows that the PLC conversations began to refer to specific events or examples as evidence, with some engagement with ideas and attempts to provide explanations. The 11% at level 3 shows that there were some new understandings for some of the participants in the PLC, signifying some greater depth in the PLC conversations. Analysis of the episodes will show how the PLC conversation brought some engagement with mathematical ideas and symbols, with new understandings for some of the participants.

Readings & Discussion: Content and Depth

In this activity, the conversation focused on the literature that participants had read in order to get more ideas and understanding regarding the concepts that they identified to develop with their learners: the notion of a variable in 2011 and the language of Algebra in 2012. Table 5.3 below shows that the focus of the conversations in 2011 was on learner (45%) and mathematics (27%), with some practice (8%). This means that the PLC talked about the learners' errors as well as the strategies that learners used to produce those errors, without focusing on the thinking behind those learners' errors. Teachers' understanding of the learners' strategies as they worked on mathematics problems did not help them to understand why the learners used those strategies.

Table 5.3: Percentage scores for content in Readings & Discussion.

	Readings & Discussion	2011	2012
Content	Learner	45	27
	Mathematics	27	45
	Practice	8	5
	Learner Thinking	0	17
	Other codes	19	5

The content focus shifted in 2012. The PLC conversations focused on learner (27%), mathematics (45%) and learner thinking (17%), with some practice (5%). An increase is observed in mathematics and learner thinking, while a decline is observed in learner and the content of other codes. The increase in talk about mathematics included the PLC's focus on the key concepts in the literature as they talked about those for their practice. The focus on learner means that the PLC talked about the type of mathematical errors that learners made, the learners' understanding, the strategies that learners used and their participation in doing mathematics. The substantial increase in talk about learner thinking shows that the PLC talked more than in 2011 about why learners made those errors so that they could understand the reasoning behind the learners' errors.

The research paper that the community read in 2011 gave an example of the use of metaphors in supporting or limiting teaching and learning algebra, where letters are often viewed as objects in algebra instead of as variables representing numbers. The participating teachers viewed the use of letters by learners differently, with some focusing on the mathematical correctness/incorrectness of the solution and others focusing on the reasonableness of the solution. The PLC conversation promoted rich mathematics among the teachers and could for learners also as it focused on why metaphors were helpful or not helpful to use in class. The example that was focused on was on adding algebraic expressions $2a$ and $5b$ and taking the letters a for apples and b for bananas. The teachers' conversation raised critical concerns of how metaphors can contribute to the learners' misconceptions. It was important for the teachers to understand that the letters a and b stood for numbers, not objects, and $2a$ does not mean 2 apples, but rather 2 multiplied by another number. Analysis of the episodes shows that this was not the case for all the teachers, with one teacher treating letters as objects. It was important for the participating teachers to see letters as representing numbers because many teachers use this analogy of objects because they believe it helps learners to simplify expressions.

Table 5.4 below shows the depth of the conversations in Readings & Discussion. The table shows that the depth of the conversations in this activity in 2011 was similar to Error Analysis, mainly at level 1 (52%), with 30% at level 2 and 19% at level 3. In 2012, the depth of the conversations increased to 50% at level 2 and 27% at level 3.

Table 5.4: Percentage scores for depth in Readings & Discussion.

	Readings & Discussion	**2011**	**2012**
Depth	Level 1	52	23
	Level 2	30	50
	Level 3	19	27
	Level 4	0	0

The coding results mean that the PLC began to refer to some specific examples or interactions as evidence and provided some interpretive or analytic comments in their discussions. They engaged with ideas with some new understandings for some of the participating teachers.

Table 5.4 indicates an important shift in the depth of conversations in this activity, and more generally, over time. Readings & Discussion influenced the depth of the conversation in the PLC as participating teachers interacted more with research literature.

Lesson Planning: Content and Depth

In this activity, the discussion of the community was on planning joint lessons on the concept that the PLC had identified for teaching: the notion of a variable in 2011 and the language of Algebra in 2012. This activity followed substantive discussion on well-known errors in the literature as well as the learners' particular errors in Error Analysis. Table 5.5 below shows that the PLC spent most of the coded time on practice (58%) in 2011, with some time spent on learner and mathematics. The PLC conversations in 2012 focused predominantly on mathematics (43%) and practice (33%), with some focus on learner (16%).

Table 5.5: Percentage scores for content in Lesson Planning.

	Lesson Planning	2011	2012
Content	Learner	22	16
	Mathematics	20	43
	Practice	58	33
	Learner Thinking	0	4
	Other codes	0	4

The focus on practice in 2011 shows that the PLC incorporated key elements of the teachers' practices into the joint lesson plan. They talked about the type of mathematical tasks they needed to use; when and how to increase the level of the tasks; and the type of questions they needed in engaging learners.

The PLC conversations somewhat focused also on learner (22%) and mathematics (20%) in 2011. The focus on learner means that the PLC anticipated the possible errors that learners might produce, the methods that learners might use to produce those errors, with no time spent on understanding why they might produce the errors, which would have been coded as learner thinking. The conversations did not necessarily focus on the reasoning behind the errors (learner thinking), and this practice is reminiscent of avoiding or correcting the learners' errors, which limits the development of PCK in the community. The focus on mathematics shows that the PLC discussed the mathematics in the tasks. The combined focus on learner and mathematics gave the participating teachers the opportunity to anticipate the learners' mathematical strategies, what learners would be able to do and what they would struggle with.

In 2012, the focus was on mathematics and practice at 43% and 33% respectively. The teachers worked out the tasks in the PLC as individuals and shared their solutions in the community, talking about their own mathematical knowledge, including the different methods that they used in their solutions. Those different methods informed the community on the possible methods that they could expect from learners in class and suggested possible errors that learners might produce as they worked on the tasks in class. This anticipation of the learners' errors helped the community to think about the possible methods that the teachers would use to address those errors with learners, which raised opportunities to develop the teachers' PCK. The anticipations account for the PLC's focus on practice in this activity. This finding is supported by research with other communities in the project (Brodie & Chimhande, 2020) and

suggests that the original design, that a focus on pedagogical content knowledge would support talk about content knowledge, was justified.

Table 5.6 below shows that the PLC spent most of the coded time in 2011 at level 1 (76%) followed by level 2 at 24%, with a shift to Level 2 in 2012 at 45%, suggesting higher level talk in 2012. The PLC conversations included some engagement with mathematical ideas and attempts to explain mathematically as evidenced in the 2012 coding in Table 5.5 above.

Table 5.6: Percentage scores for depth in Lesson Planning.

	Lesson Planning	**2011**	**2012**
Depth	Level 1	76	53
	Level 2	24	45
	Level 3	0	1
	Level 4	0	0

Lesson Reflection: Content and Depth

This activity was guided by the lessons that the participating teachers taught and were videotaped. The teachers were given videotapes of their lessons to reflect on as individuals and choose interesting episodes from those lessons for a group reflection in the PLC meeting (see criteria for selecting episodes in Chapter 3). The purpose of each reflection was to inform future instructional practices for the teachers. The reflection was meant to focus on how the teachers engaged with the learners' errors in the classroom and how they transformed the learners' incorrect or partially correct thinking in order to develop correct conceptual ideas among learners.

Table 5.7 below shows that the focus of the PLC conversations in this activity was predominantly on learner (53%) and practice (47%) in 2011 and on learner (34%), mathematics (23%) and practice (26%), with some focus on learner thinking (10%), in 2012. These observations show a shift in mathematics and learner thinking in 2012. These conversations show a better spread in 2012 across the different content codes than in the other activities.

Table 5.7: Percentage scores for content in Lesson Reflection.

	Lesson Reflection	2011	2012
Content	Learner	53	34
	Mathematics	0	23
	Practice	47	26
	Learner Thinking	0	10
	Other codes	0	8

The focus of the PLC conversations on learner and practice in 2011 means that the teachers identified the learners' errors during the reflection process and talked about those errors in order to understand the strategies that learners used to produce the errors and how they could work with learners' errors in their classrooms. The focus shows an omission on talking about the reasoning behind the learners' errors as evidenced by the percentage score on learner thinking, a key aspect which was emphasised in DIPIP. The score for learner thinking in 2011 means that the PLC conversations did not contribute much to the teachers' PCK in understanding the learners' misconceptions. Brodie & Chimhande (2020) argue that, in viewing their videotapes, teachers tended to focus on the learners and themselves – what each was doing, rather than on the underlying reasoning of the learners or the mathematical concepts.

The spread of content observed in 2012 shows that the community talked about the learners' errors, some of the reasoning behind the errors, and they engaged in the mathematics that the learners used to produce those errors. Their conversations also focused on how the teachers worked or could work with those errors in class. This means that the PLC talked about how the teachers could employ the conceptual ideas they developed through the PLC discussions in the classroom. The increased, but still limited focus on learner thinking in 2012 shows that teachers did talk about learners' reasoning to some extent.

Table 5.8 below shows that the depth of the PLC conversations was mostly at level 1 (85%) in 2011 and shifted significantly in 2012, with the depth of the PLC conversations featuring all the four levels, and the most time spent at level 2 (42%) in 2012. This is the only activity that reached a depth of level 4 over the two years.

Table 5.8: Percentage scores for depth in Lesson Reflection.

	Lesson Reflection	2011	2012
Depth	Level 1	85	32
	Level 2	15	42
	Level 3	0	18
	Level 4	0	8

The observed, unprecedented depth in the PLC conversations in 2012 means that the community engaged with ideas and some new understandings for some participants. They also used examples that embodied general principles for mathematics, for learners and for practice, as the content spread in Table 5.7 above shows.

5.3 Overview: Content and Depth

Table 5.9 and Table 5.10 summarise the extent to which particular content featured in the PLC discussions and the depth of the conversations across all the activities over the two years. These shifts enable a view of the teachers' PCK across the activities.

Table 5.9: Content of the conversations across all the activities.

	Activity	Error Analysis		Readings & Discussion		Lesson Planning		Lesson Reflection	
		2011	2012	2011	2012	2011	2012	2011	2012
Content	Learner	42	-	45	27	22	16	53	34
	Mathematics	11	-	27	45	20	43	0	23
	Practice	2	-	8	5	58	33	47	26
	Learner Thinking	45	-	0	17	0	4	0	10
	Other codes	0	-	19	5	0	4	0	8

Table 5.10: Depth of the conversations across all the activities.

	Activity	Error Analysis		Readings & Discussion		Lesson Planning		Lesson Reflection	
		2011	2012	2011	2012	2011	2012	2011	2012
Depth	**Level 1**	54	-	52	23	76	53	85	32
	Level 2	35	-	30	50	24	45	15	42
	Level 3	11	-	19	27	0	1	0	18
	Level 4	0	-	0	0	0	0	0	8

Trends that emerged in the content of PLC conversations show the following:

1. The focus on learner thinking featured prominently in Error Analysis in 2011, which suggests a strong link between Error Analysis and teachers' discussions of learner thinking. The focus of the PLC conversations in the other activities in 2011 suggests that other aspects of teachers' practice took precedence for them and that the focus on learner thinking in these activities was non-existent in 2011 and minimal in 2012. The reappearance of some discussion of learner thinking in 2012 was because of the in-depth re-reflection on the 2011 lessons that the teachers had already reflected on in 2011, supporting stronger inquiry into the reasoning behind the learners' errors instead of merely correcting or ignoring those errors.
2. A focus on learner was high in all activities, lowest in Lesson Planning of 2012, but still higher than learner thinking. The focus on learner affirms two aspects of the teachers' talk: First, the importance of the teachers' acknowledgement of the learners' inputs in class, evidenced in teachers identifying the errors that learners produced and talking about the different methods that learners used to produce those errors. Second, the PLC conversations focused on how the errors were produced without necessarily inquiring into the reasoning behind the learners' errors – they focused on the learner strategies, learner understanding and language.
3. Mathematics was highest in Readings & Discussion and Lesson Planning in 2012. In Readings & Discussion, the community interacted with key mathematical concepts that research raised in the literature that they read. This interaction resulted in critical arguments that helped in discerning the teachers' PCK. In Lesson Planning they worked on the tasks and discussed the mathematics of the tasks.
4. Practice was highest in Lesson Planning and Lesson Reflection. This means the PLC conversations focused more on how the tasks in the joint lesson plans could be mediated for learners in the classroom. When the community reflected on their lessons, they focused primarily on what the teachers did instead of on the learners' reasoning. The slight decline in 2012, though still high, was because the focus of the PLC conversation to some extent shifted to learner and mathematics. The teachers talked about how learners understood the mathematical tasks and the errors that learners produced as they worked on the tasks.
5. The 19% score in 'other codes' is mainly for community talk about probing, which featured in Readings & Discussion in 2011 – the community talked about how they could push learners to explain how they get their answers, without leading them too much. Codes

which received less scores throughout were merged into “other codes” for the analysis. The other content codes such as ‘reproduces blames’ received less attention over the two years. Less prominence of ‘reproduces blames’ emerged as a positive reflection as it showed that teachers did not blame themselves or the learners for the errors that learners produced. The project team emphasised that there were conceptual, mathematical explanations for learners’ errors instead of putting blame on learners or teachers, and this narrative resonates with what research says on learner errors (Brodie, 2013; Nesher, 1987).

The analysis has shown that mathematics emerged as central to the PLC conversations in two activities though it received less attention in the project design than learner thinking, learner and practice. This means that the PLC focused their talk on: what the learners did in the classroom; the mathematics involved in the teachers’ talk in order to enhance their mathematics content; and how they (teachers) could mediate the mathematics content in class. This is unexpected in that the project designed for conversations around PCK, rather than CK. But it seems that the teachers were able to bring in their CK. In analysis of this same community in 2013, Marchant & Brodie (2016a) show that PCK conversations often led to CK conversations, suggesting that teachers can work with both, and the mathematics can emerge from other discussions. Brodie & Chimhande (2020) also showed how a substantial decline in conversations about learner thinking in a community was “accompanied by an increase in talk about mathematics” (p. 125).

The above discussion shows the prominence of learner and mathematics in the different activities, with practice featuring in Lesson Planning and Lesson Reflection, and Learner Thinking featuring mainly in the early Error Analysis sessions. The analysis mirrors analyses of other communities in the project (Brodie & Chimhande, 2020; Chauraya & Brodie, 2017). This finding revealed opportunities for the development of the teachers’ PCK in the community in relation to learners’ strategies and teacher’s practice, although not strongly in relation to learner thinking. These opportunities emerged as the teachers focused on their understanding of the learners’ errors: how those errors were produced and how they could work with those errors in their classes without correcting or ignoring them. The prominence of mathematics supports the narrative that errors are not the fault of teachers or learners, but it is about the process of learning mathematics (Smith, et al., 1993).

The overview for depth suggests the following:

1. The depth of the conversation increased over time – from 2011 Error Analysis to 2012 Lesson Reflection, shifting to level 2 and Level 3 in Readings & Discussion and Lesson Reflection. Error Analysis, done only in 2011, shows depth of level 3, which can be attributed to PLC conversations on learner and learner thinking: a focus on the errors that learners produce and the reasoning behind those errors, the strategies that they use in producing those errors, and the learners' understanding of the tasks.
2. There were more level 1 conversations in all the activities over the two years except for 2012 Readings & Discussion and Lesson Reflection. The 2011 scores mean that the community was still grappling with what was expected of them, and their inquiry into the reasoning behind the learners' errors revealed less focus, signaling correction or ignorance of the learners' errors for the most times. This became different in 2012 as the community began to accommodate errors in their talk in Readings & Discussion and Lesson Reflection.
3. All the activities showed shifts to levels 2 and 3 in the PLC conversations in 2012, with Lesson Reflection extending this shift to level 4 in 2012. The observed shifts mean that the PLC conversations went deeper than providing mere descriptive and analytic comments and engaged with mathematical ideas in different ways.

From the initial claims made in analysis of content and depth above and the trends that emerged from this analysis, the next section looks more deeply at how PCK featured in the community conversations for the different activities to complement the results of the above analysis. Episodes from conversations in the PLC meetings are used to evidence the extent to which the PCK of the participating teachers developed in the different activities of 2011 and 2012.

5.4 PCK in the community

This section focuses on the PCK discussed and the opportunities that were raised for its development in the PLC. PCK includes teachers' knowledge of how learners learn (Van Driel & Berry, 2012a). It is important for teachers to understand what learners are capable of or struggle with as they do mathematics. When learners struggle with algebra, an opportunity arises for teachers to understand the source of the problem and think of ways of assisting learners. The process demands that teachers have the necessary knowledge to interpret their

learners' conceptions and/or misconceptions (Hiebert, et al., 2002). This knowledge defines the teachers' PCK in devising effective strategies that can help learners to understand algebra. Extracts of transcripts of the PLC meetings are provided to evidence how the PLC engaged with the different activities in 2011 and 2012. PLC affords teachers the space to learn *from*, *about* and *for* practice. The teachers in this study used the PLC to talk about and learn from their practices. In doing so, they created opportunities to contribute to their PCK and developed mechanisms needed for enhancement of their instructional practices.

Opportunities for PCK development in the community involved having meaningful conversations about the learners' errors and trying to understand the reasoning behind those errors instead of avoiding or correcting those errors (Brodie, 2013). DIPIP emphasised embracing errors so that teachers might understand the learners' learning needs more deeply (Brodie, 2014) and on this basis mediate content better in class. When teachers understand the learners' misconceptions, they can then develop approaches to assist learners in better mathematics learning, which can contribute to the teachers' PCK.

My qualitative analysis is informed by the DIPIP ideas of working with learners' errors as well as Heritage, et al.'s (2009) ideas as to how teachers can work with learner responses. Drawing from the two, I could describe how the teachers' talk raised opportunities to contribute to PCK in the community. Since the teachers' conversations also involved how they viewed letters in algebra and their evaluation of how learners viewed letters in algebra, Hart's (1981) ideas of looking at letters in algebra also became necessary in analysing the teachers' talk: letter evaluated, letter not used, letter used as an object, letter used as a specific unknown, letter used as a generalised number, letter used as a variable. A detailed discussion of the framework is in Chapter 3. The framework focuses on three constructs: identifying key concepts, evaluating learner understanding, and planning the next instructional step based on the evaluation of learner understanding.

The next section focuses on analysis of episodes to show the extent to which the learning community engaged in conversations to learn from and with each other, and how these conversations raised opportunities for the development of the participating teachers' PCK.

5.5 Analysis of episodes for PCK

In selecting the episodes for the qualitative analysis of PLC conversations, I needed extracts which focussed primarily on teacher conversations about learner errors, learner thinking or learner. I needed to see the extent to which the teachers were talking about embracing, correcting or ignoring the learner errors. This is because embracing, correcting, and ignoring errors involved an analytic frame for looking at the teachers' PCK in their conversations.

Episode 1: Using metaphors for teaching algebra

Readings & Discussion (September 2011)

In this episode, the PLC was discussing a paper that they had read: Children's Difficulties in Beginning Algebra (Booth, 1988), in preparation for planning a joint lesson on the concept of a variable. The PLC's discussion focused on how a lesson could be crafted using ideas from the paper. The teachers were given guiding questions in reading the paper and the conversation in the PLC concentrated on two of those questions: 1. the general issues that were raised in the paper; and 2. the main causes of learners' errors when answering algebraic questions. The paper raised a contentious issue regarding the use of metaphors when teaching the notion of a variable: using letters in algebraic expressions to represent objects (Hart, 1981).

The PLC conversation focused on why a learner gave the response of $7ab$ to the question: simplify $2a + 5b$, which was in Booth's paper. The participating teachers indicated earlier in the conversation that their learners made a similar error, which suggests that the error is common among learners – this issue is discussed further in the analysis below. Booth (1988) explains that learners worked from a metaphor of taking the variable a to represent apples and b to represent bananas, with $7ab$ then meaning seven apples-and-bananas (p. 302). Learners tend to conjoin the terms when working with variables. As learners have "a cognitive difficulty in accepting lack of closure" (Booth, 1988, p. 301), they do not see $2a + 5b$ as complete and want to do something to it.

In the episode, the PLC discussed whether the use of metaphors was helpful or not to use in class when teaching algebra. The discussion followed after two teachers suggested the use of "tangible things" in helping learners with addition of variables, and gave an example of adding

three pairs of shoes and three pairs of socks. The teachers' views differed in this discussion, with two teachers thinking that metaphors helped to make sense of algebra and the other two thinking that they did not.

1	Facilitator	Does this learner mean that he's got seven apples and seven bananas? Is that what the learner means?
2	Funeka	No.
3	Facilitator	And look at how apples and bananas have been written there. Hyphenated. To show that it's one thing. Is there anything wrong with the statement?
4	Dimpho	Seven apples. Then what is and? And is plus.
5	Facilitator	Apples plus bananas.
6	Mapula	The hyphenation
7	Dimpho	Seven apples plus bananas.
8	Funeka	Plus bananas. That's what they are saying
9	Dimpho	So it means that you say that apples are seven. That's what it means. Seven apples.
10	Facilitator	Not in the manner in which it is written.
11	Khumo	(*Inaudible*) seven apples.
12	Mapula	Or will it be fine if he said, seven…seven apples bananas...mm… (*laughter*) Seven apples and…seven apples bananas.
13	Funeka	And bananas.
14	Facilitator	That is what the learner is saying, seven apples and bananas.
15	Khumo	But it won't give, it won't have seven apples and bananas.
16	Dimpho	But if a person says seven apples, ja (**yes**) you don't have seven apples.
17	Mapula	But to me it makes sense.
18	Facilitator	It makes sense, ne?
19	Dimpho	No, it gives me the impression that the learner is saying the apples are seven.
20	Facilitator	No, but talk to what is written there.
21	Khumo	And the bananas also are seven, according to that answer.
22	Facilitator	The way it has been written it says, the learner here indicates that all in all he has seven things, what are those things? Apples and bananas.
23	Mapula	In one bag. You must carry in one bag. I'm carrying…
24	Funeka	Seven bananas.
25	Facilitator	You've got seven units, of apples and bananas combined. (*Pause*) You're thinking about something, Dimpho. What? (*laughs*)
26	Dimpho	Hayi [no], it doesn't give me sense.
27	Facilitator	Tell us, why does it not give you sense (*silence*)
28	Khumo	You cannot add something which are different. Apples and bananas. Two apples, five bananas, seven.
29	Facilitator	You cannot add things that are?
30	Khumo	Not the same.
31	Facilitator	You can't add three and five?

32	Khumo	Three and five are numbers only. But you are talking about variables, terms of the variables.
33	Facilitator	So you can't add *a* and *b*?
34	Khumo	No, you cannot add *a* and *b*, because they are different variables.
35	Facilitator	Why not? What if *a* is the same as *b*?
36	Khumo	*a* is the same as b? They are not the same.
37	Facilitator	No, I'm telling you, what if *a* is equal to *b*? Can't we add *a* and *b*?
38	Dimpho	If ever *a* is equals to *b*? You can because…you can. Because *a* will be the same as *b*. It will be the same thing.
39	Facilitator	You have to explain. Ma'am doesn't agree. We have *a* plus *b*, can we add it, if *a* is equal to *b*
40	Dimpho	If ever *a* is equals to *b*, a solution will be two *a* or two *b*. One of them. But it must be in terms of one variable. Because you'll have equated the other one to the other.
41	Facilitator	It still doesn't make sense to you, ne?
42	Dimpho	So if *a* is equals to *b*, then the solution for a plus *b* will be two *a* or will be two *b*. It depends which one do you substitute.
43	Facilitator	So the *a* plus *b* now also becomes *a* plus *a*? Because *b* is the same as *a*. Or it can become *b* plus *b* because *a* is the same as *b*. Then you get seven *a* or seven *b*. So there are two key things here when it comes to problems of this nature. When you deal with variables. The one using the letters in the (*inaudible*) they can use the number five of the probing questions. But if you have *a* then it's apples, if you have *b* then it's bananas. If you have *m*, it's mangoes. It creates misconceptions for learners. Learners end up not seeing or understanding what variable means. Because according to them variable stands for something that they know, like an apple or a banana or anything like that. So they end up losing all meaning of what actually does a variable mean. They end up not knowing that variable can stand for any number. That's the one thing. So it's not a good thing for us that when we teach things like these, two *a* plus five *b*, then you want to try and emphasise to learners that you cannot add like, unlike terms. Because *a* is not the same as *b*. That is what we tell them, ne? That *a* is not the same as *b*, so you can't add them, and you put a full stop to that. But two *a* plus five *b* can be added, depending on the conditions that we have for *a* and *b*. Remember *a* and *b* in that case are variables. So if you put numbers here, learners must know that variables stand for numbers. So if you put numbers there, then you'll be able to add two *a* plus five *b*, and come with an answer. But in that case, if you leave it as *a* and *b*, then leave it as two *a* plus five *b*. But we must not close shop and say, two *a* plus five *b* cannot be added, because those are variables. When you say that to learners you are telling the learner that you cannot add numbers. Or else we tell them that the variables stand for numbers. Can you see that it's a contradiction there? But you say to learners, this can't be added, but at the same time you say variables stand for numbers. That's the one key thing there. And that other one that I started to say, using letters for things like apples and bananas and whatever. Again it creates misconceptions. And again we can add them if *a* is the same as *b*. Because if *a* is the same as *b* there, say maybe we take any number, if *a* is three and *b* is three, if they are the same this means they have the same number. So if *a* is three and *b* is three then we'll

		be able to add and get an answer. So you must always look at things like this in a broader sense. And then there was which other one that you said to me?

Extract 5.1: Using metaphors for teaching algebra.

Identifying key concepts

Teachers understand what the concepts in the activity are

The teachers' conversation showed that they understood what the learner had done in the task, though their deliberations on the learner's response differed, with two teachers seeing the learners' analogy as making sense while it did not make sense to the other two. Dimpho and Khumo focused on the incorrectness of the learner's answer, that $2a + 5b \neq 7ab$, suggesting that they were more interested in the content (CK). Funeka and Mapula focused on learner thinking, particularly on how the learner saw and reasoned about the expression: $2a + 5b = 7ab$, which raised an opportunity to develop the teachers' PCK. Mapula suggested that it would have been better if the answer was stated as seven apples-bananas (emphasising the use of a hyphen). Her suggestion supported how the learner's answer was interpreted in the paper, that the analogy of apples and bananas misleads the learner in seeing $7ab$ as the correct answer.

The community talks about the key concepts embedded in the activity or the key building blocks of the activity

All the teachers understood that the problem involved addition of variables. Dimpho and Khumo emphasised what the error was without dealing with the source of the learner's error. Mapula and Funeka saw the learner's answer as constituting a single entity in real life (turns 6, 8, 17 & 23) as suggested by Booth (1988). Hart (1981) argues for learners' understanding of at least a letter as a specific unknown in order for them to acquire some basic understanding of early algebra, an issue which was important for the teachers' PCK in developing the concept of a variable with learners but was never raised. Mapula & Funeka showed an attempt to develop their PCK in focusing on how the learner interpreted the letters in his response, whereas Dimpho and Khumo did not.

The community talks about the appropriateness of the task for their learners

The learner in the literature paper interpreted the letters *a* and *b* in the problem as objects (Hart, 1981) because he substituted them for apples and bananas in his response. The contention between the teachers on the learner's response raised possibilities for the creation of new knowledge in the community on how learners interpret letters in algebra, with Dimpho assisting

the community to see letters as numbers to be operated on. The teachers did not take up this opportunity on their own, so the facilitator used the opportunity and gave a long explanation which highlighted the use of letters in algebra. The facilitator's intervention will be discussed in more detail in Chapter 6.

Teachers make links with other tasks and concepts or with the learner's work

Funeka noticed that many of the errors in Booth's paper were common among her learners. She had earlier compared the learner's answer to what they experienced with their own learners: "what I saw is that the errors that learners are making, the same errors there that our learners are making". In this case, she was using what she experienced in class to show an interest in working with learner errors and to build on her argument of looking at the learner's answer. Funeka's input affirmed the prevalence of learners' errors in algebra and created an opportunity for teachers to increase their PCK in helping to address the learners' misconceptions.

Evaluating learner understanding

The community investigates/focuses on the learner's reasoning

In engaging with the learner's error, the community identified the learner's current knowledge. They focused on what the learner had done and tried to understand why the learner did that. As discussed in analysis of the first category, Dimpho and Khumo focused on the learner's conjoining of the terms. Their focus on the incorrectness of the learner's response did not create opportunities for PCK development in the community. Though Mapula's and Funeka's argument raised opportunities for PCK development in trying to understand the learner's error, the community was not able to take the idea further and the facilitator intervened.

Teachers can establish what the learner(s) know and/or do not know

Dimpho and Khumo established that the learner could not work with variables, which was evidenced by conjoining of terms and their argument suggested that their interest was in correcting the learner's error. This means they wanted to make the correct knowledge available for learners, without inquiring into the learner's reasoning (Brodie, 2013). Mapula and Funeka were not as elaborate as Dimpho and Khumo on the mathematical incorrectness of the learner's answer. They were not specific on whether they agreed or not with the learner's conjoining of

terms, which comprises a key aspect for teachers' content knowledge. The difference in the teachers' arguments raised opportunities to build PCK among the teachers, although this was not realised in the conversation.

The community makes inferences from the learners' work, which includes learners' interpretation of letters in algebra

Mapula's and Funeka's inferences from the learner's response were more specific than Dimpho's and Khumo's. In interpreting the learner's analogy of seven apples-and-bananas, Mapula and Funeka saw that as comprising one thing, which gave them, and perhaps the whole community, an idea on how metaphors contribute to misconceptions in algebra. Dimpho's and Khumo's argument may help teachers in enhancing learners' conceptual understanding, whereas Mapula's and Funeka's argument may assist teachers in trying to understand why learners produce certain errors. As discussed above, the two arguments raised opportunities for the teachers to work with both ideas and reach deeper PCK and CK, but this was not realised.

Teachers talk about the anticipated performance of the learners

Nothing in the conversation came up on anticipation of learner performance except at the end of the episode when the facilitator summarised the discussion. In his summary, he discouraged the use of metaphors which might encourage learners to see letters as objects in algebra, which is a point that teachers could take home knowing the challenges that metaphors can bring in algebra learning. Nothing in the teachers' conversation showed that they learned from the facilitator's input at the end.

The community talks about the kind of feedback that they could give to learners

Learning about learner understanding can inform teachers about the kind of feedback that they could give to learners when learners produce errors similar to the one discussed in this episode. The teachers' arguments revealed the confusion that metaphors may bring in learning algebra. Dimpho's explanation on making different letters to be the same enhanced Khumo's, and the other teachers' PCK on the addition of different letters in algebra. The facilitator's summary also assisted the community in seeing that $2a$ and $5b$ can be added: "So if you put numbers there, then you'll be able to add two *a* plus five *b*, and come with an answer", which can contribute to the teachers' PCK in working with different letters in algebra. The teachers'

understanding of this idea could not be ascertained, and this will be discussed as a missed opportunity in Chapter 6.

Planning the next instructional step

Teachers talk about how the learner(s)' error(s) can be addressed and the misconceptions overcome

The teachers were not explicit on how the learner's error could be addressed or the misconception overcome. The different views of the teachers in the episode suggest different understandings of the notion of a variable by the teachers, which illuminated uncertainty of the teachers' PCK in addressing the learner's misconception. Though Khumo did not see variables as representing numbers, Dimpho's explanation of *a* being equal to be *b* had potential to shift Khumo's thinking on working with variables, and maybe others. Mapula and Funeka saw the letters as objects and thought that the metaphor made sense, but their views created opportunities to engage with the reasonableness in the learner's thinking. The community was not able to reconcile the differences in their views, which may have led to deeper understandings of the mathematics and the learner's difficulties.

Teachers link learner errors to their practices

As stated earlier, Funeka had earlier compared the learner's answer in Booth's paper to what she experienced with her own learners when teaching algebra. Her input raised an opportunity to develop the teachers' PCK, especially in knowing how to help learners interpret letters in algebra. The learner's response exemplifies loss of the intended meaning of the letter as he regarded the letters *a* and *b* as objects (Hart, 1981), which is similar to how Khumo also viewed letters in the conversation. Hart argues for learners to work with letters as specific unknowns, or generalised numbers or variables instead of viewing them as mere objects, which was key for the teachers' PCK in this conversation and in algebra.

Teachers share their best practices and acknowledge their challenges

The conversation did not bring up anything explicit to suggest that teachers shared their best practices or acknowledged their challenges, except implicitly when Khumo insisted that the letters *a* and *b* could not be added because they were not the same. Khumo's input suggested that she saw variables as objects. The facilitator's probing and Dimpho's explanation built up

an opportunity for the community to see that the use of different letters does not necessarily mean that variables cannot be added, which raised opportunities for the enhancement of the teachers' PCK on addition of variables. Hence, Dimpho's explanation on $a = b$ which eliminated the general notion that when letters are different then they cannot be added.

Teachers talk about other issues that may be embedded in the task and relating to practice

The conversation among the teachers focused only on the learner's error and did not raise any additional issues for practice. It was the facilitator's input in the end which brought key aspects that were intended to assist the teachers in working with variables: 1. that the learner made an error in conjoining the terms, something which Dimpho and Khumo pursued in the conversation; 2. that metaphors are a source for misconceptions; and 3. why the learner's response was mathematically incorrect and how they could transform learner thinking for instructional practice. One issue in the conversation relating to practice was the misconception on addition of different letters in algebra, which through the facilitator's probing allowed Dimpho to help the community think about the issue.

Summary of the analysis

The teachers' focus on the learner's error brought different views that unfortunately were not reconciled in the discussion. Two teachers, Dimpho and Khumo, focused on the incorrectness of the learner's answer and the other two, Mapula and Funeka, focused on the reasonableness of the learner's answer. As the community tried to identify the key concepts, the two views created an opportunity to increase the teachers' PCK in making sense of learners' answers, as well as their CK in understanding variables. I have argued that one teacher's explanation of the use of different letters could have enhanced the other teachers' PCK on the addition of variables in algebra, but this did not happen. Although an opportunity arose for the teachers to explicitly talk about the expression $2a + 5b$ as an answer, this was not realised, except in the one case of $a = b$, which Dimpho tried to help the community to understand. Seeing $2a + 5b$ as an answer would have contributed to the teachers' PCK in working with different letters in algebra, and perhaps helped them to realise the misconceptions that metaphors create in algebra.

Episode 2: Validity in numerical reasoning

Lesson Planning (August 2012)

This episode focuses on the PLC's discussion of the strategies that learners often use when they work on word problems which involve mathematical language. The community was selecting tasks for developing algebraic language with learners. The process involved how to engage learners with the task, to anticipate the possible learner responses to the task and how the teachers might handle those learner responses. The problem that the community worked on was:

> Three consecutive natural numbers add up to 24. Find the numbers.

The anticipated solution that the community was discussing was when a learner might give an answer of: seven, eight and nine, a numerical solution, without using algebra, which is what they wanted from the task. The PLC discussed the pros and cons of this solution and talked about what they thought might work best for instruction, which was important for building PCK in the community. The discussion started being predominantly between the facilitator and Cele, but it later included the other participants in the community. Cele had tried this problem with a class different from the one he selected to teach for research because he wanted to see the typical errors that learners might produce.

1	Facilitator	The solution, what do you do about that, where is it? Three consecutive natural numbers add up to twenty four, find the numbers. Suggestions for a solution?
2	Cele	Trial and error, seven eight nine is the numbers that would give us that. If you use trial and error you will get seven eight nine.
3	Funeka	So how, how would you end up landing on seven eight nine, when you use trial and error?
4	Cele	They are saying the three numbers that follow one another, that follow each other, should add up to twenty four.
5	Facilitator	If a learner does that and say seven eight nine. How would you deal with it in class?
6	Cele	It's correct, those are the solutions, those numbers. Yes they said twenty four.
7	Funeka	Twenty four?
8	Facilitator	So if the learner writes that you say okay it is correct and you continue.
9	Cele	They said find what
10	Cikizwa	Find the numbers.

11	Cele	Ja, the numbers are seven eight and nine. That is the solution. Thirty six, I think there was another one, she talked about thirty six.
12	Facilitator	Ja. So are you okay with that one? Trial and error. So the learner writes seven eight nine, you accept it, you continue
13	Cele	And then we say how can we solve that without trial and error, these form an algebraic statement. Then you can apply it as an option.
14	Facilitator	Mm, how
15	Cele	If we have to define the first number, then you say let the first number be *x*, the next one *x* plus one, the other one would be *x* plus one plus one which is *x* plus two. Add those, form an equation to get to twenty four. Solve for *x*. *x* is seven.
16	Facilitator	So you wouldn't want to know why the learner started at seven
17	Cele	For them to be consecutive any other, any other three will fail, if you are not starting from seven. Anything greater than that it would not work, as long as they (*interrupted*)
18	Facilitator	I am not asking this question because I want us to talk as teachers. As teachers say, you can make that sense. I am talking about the situation in class, where a learner gives seven, eight, nine.
19	Cele	Okay.
20	Facilitator	Wouldn't you be interested in knowing how that learner ended up with seven, eight, nine
21	Cele	Okay.
22	Facilitator	Did the learner just go straight to seven, eight, nine and know that seven, eight, nine will work and got them right away, correctly. Or the learner will still juggle around with a few numbers…
23	Cele	Alright
24	Facilitator	And lands on seven, eight, nine. That is my input.
25	Cele	Okay, maybe a learner can think like this to say okay these numbers must add up to twenty four. Obviously if you start from two, the next one will be three and then four, it is too little. I jump slightly at six, seven, and eight, it doesn't reach. You must keep on shifting going that side.
26	Mapula	By trial and error.
27	Cele	Yes, the numbers must be following each other. So it means if you take this one and it is not, you move it.
28	Facilitator	Until you reach seven
29	Cele	Ja, seven would be the one that (*interrupted*).
30	Facilitator	So you are taking it, what the learner will be explaining to you
31	Cele	Yes.
32	Facilitator	That I started with two, three, four, it didn't work, then I said three, four, five then four, five, six, …five, six, seven, just like that
33	Cele	Yes, then you see even after, if you take nine, ten, eleven it will not give you.
34	Facilitator	And what do you tell that learner after that learner, after that learner had said that? He said two three four, three four five.
35	Funeka	No, it's correct
36	Facilitator	Four, five six…
37	Funeka	Ja, it's a trial and error method

38	Facilitator	Until the learner lands on that
39	Funeka	That he's been trying to get the answer.
40	Facilitator	So
41	Funeka	It's the right method
42	Facilitator	And you'll be happy with that
43	Funeka	Mm
44	Facilitator	And going forward.
45	Funeka	Going forward like, after …
46	Facilitator	After the learner has explained that, you will say no you are happy, you'll get to the next problem
47	Cele	No, okay if I may take over.
48	Funeka	No, I can answer
49	Cele	I'd say, now what's …(*interrupted*).
50	Facilitator	No, let's listen to Ma'am Mosala first.
51	Funeka	Mmh, okay yes, I will say there is another method that I can use. And you can also use eh, eh… the other method of using *x*, *x* plus one, *x* plus two, to get, I think you will get the answer of seven. Then after seven what is the number after seven. Like three numbers after seven. Then you will get the solution.
52	Facilitator	Ma'am Maisela, ma'am Molemela, how will you deal with that? The learner writes seven eight nine, then you ask the learner how did you come to seven eight nine, then he gives that explanation. After that I do two three four, because they are consecutive, they didn't give me twenty four. And I tried three four five, they still did not give me twenty four and I went on like that until I got to seven eight nine. Which gave me twenty four
56	Khumo	I will try and show them after getting this as an eight, seven eight… seven eight nine, you get twenty four. Then I will try the next one after nine to show that if you add the consecutive one, after nine, you are not going to get twenty four.
54	Facilitator	Mm. And after that
55	Khumo	Mmm…
56	Facilitator	You are still thinking of something, okay.
57	Khumo	I will try to do the algebraic, but I am not sure about that, using the variables…is it…mmm, no-no-no, I am still thinking.
58	Facilitator	Okay, Ma'am Mathebula, seven eight nine thing, how would you deal with it?
59	Cikizwa	If the learner says seven eight nine
60	Facilitator	Mm. Then you want to find out from the learner, how did he arrive at seven eight nine, and he gives that explanation that Cele just gave us now.
61	Cikizwa	I think it is straight forward. Whereby, there is no other number lesser than those three, seven eight nine, that will give twenty four, and bigger than seven eight nine, that will give us twenty four.
62	Facilitator	There's, there is no numbers. Remember my question is how would you deal with it in class, if a learner gives you that? Ma'am Maisela, have you thought of something that you want to share with us
63	Mapula	I am trying to work out something.

64	Facilitator	Oh okay, no that is fine. You can give a shout as soon as you have something. Cele, you wanted to say something
65	Cele	Yes, I wanted to say however I would not run (*inaudible*) that is a method that should be used in trial-and-error. What if somebody says the numbers add up to one sixty eight, some numbers you try them for the, the whole day and you still fail to get them. It is time-consuming this trial and error. So you will have to see if there are clever methods that can lead us to the answer without suffering. Then you emphasise that one.
66	Facilitator	That is what learners, they should know, that these easy steps that they think they can help them, if you get to more complex problems they will not work. And in mathematics we want to come with something that is more general. And if you just concentrate on numbers, you will never be able to generalise. Remember that first discussion we had, the network meeting, that in mathematics we generalise, and algebra helps us to generalise. So if you work arithmetically you never reach that stage.
67	Cele	You say one sixty eight, find two numbers, that three numbers consecutive numbers, that add up to one sixty eight, now you try and try, it is a problem.
68	Facilitator	So where do you start, in this case they gave twenty four, so the learner started at two? Then he tried the two three four five six seven, so he tried six times and then he got the answer. So if they add up to hundred and sixty eight where does the learner start, two three four, three four five, just like that until he reaches sixty eight? You can't do that.
69	Cele	It means, you add maths for the rest of your life.

Extract 5.2: Second episode of Lesson Planning.

Identifying key concepts

Teachers understand what the concepts in the activity are

The teachers selected the problem under discussion because they saw it as being appropriate for developing mathematical language. In their conversation, they initially supported the trial-and-error method, which gave the correct answer, but limited the development of working with algebraic language. They chose the concept because they agreed that mathematical language was a barrier for many learners, but they did not see that the trial-and-error method may reinforce that barrier for learners. Persistent probing from the facilitator shifted the conversation to algebraic concepts, but this shift did not help in developing ways of dealing with numerical solutions which also involve trial and error.

The community talks about the key concepts embedded in the activity or the key building blocks of the activity

The teachers talked about two methods that could be used to get a solution: trial-and-error method and the algebraic method. In talking about the trial and error, the teachers reasoned that numbers other than seven, eight and nine would not give the correct answer, and they gave examples of those numbers. They were initially comfortable with the method until later after persistent probing from the facilitator.

The community talks about the appropriateness of the task for their learners

When selecting the task the teachers may not have anticipated the trial-and-error solution. The discussion in the community supported them to see this and shift to talking about the algebraic possibilities in the task and ways of encouraging the algebraic method through introducing larger numbers. The teachers worked on the task in order to anticipate the possible learner responses and work out how they could respond to learner ideas in class. Task discussion assisted the teachers with anticipating learner responses, which is key for the teachers' PCK in understanding the challenges faced by learners and how best to deal with such challenges.

Teachers make links with other tasks and concepts or with the learner's work

Though the teachers were initially comfortable with the trial-and-error method, persistent probing from the facilitator shifted their thinking about the method. They began to think of how the problem could be modified in order to discourage the trial-and error method (turns 65 and 67), and this involved avoiding small numbers such as 24, which was key for their PCK on task selection and task engagement. This caused the conversation to shift from supporting the trial-and-error method to seeing its limitations.

Evaluating learner understanding

The community investigates/focuses on the learner's reasoning

In support of the trial-and-error method, the teachers talked about ways of trying to validate the method. They said learners might start from lower numbers such as two, three, and four to see if they would get 24, and continue to do so including also numbers giving a sum greater than 24, until they are satisfied with a sum of 24. The teachers' conversation at this stage did not help them shift towards generalising, for themselves or their learners. The teachers' later

realisation of those limitations raised opportunities to develop their PCK in promoting generalisability, which was evidenced by a shift to an algebraic method and the use of bigger numbers in such problems (turns 65 and 67).

Teachers can establish what the learner(s) know and/or do not know, which is linked to exploring learner thinking

The teachers were convinced that the learners would use the trial-and-error method, and supported it: "It's the right method" (turn 41). Though the trial-and-error method was correct, it was not the most useful for the teachers' purposes and the teachers did not engage with the limitations of the method until after the facilitator probed them several times. An opportunity arose through persistent probing, to enhance their PCK in knowing how to handle shifts from numerical to algebraic approaches. This opportunity helped them to understand that numerical approaches are not always generalisable.

The community makes inferences from the learners' work, which includes learners' interpretation of letters in algebra

Knowing the correct answer from the trial-and-error method somehow restrained the teachers from thinking more deeply about alternative strategies, which inhibited the development of PCK in working with multiple-solution methods and thinking about the strengths and weaknesses of different methods. As Cele said: "any other three will fail, if you are not starting from seven", the teachers insisted that no other numbers would give the correct solution, which was true but not addressing the bigger picture – they did not work with learners' answers in ways in which the richness of mathematics could be improved.

Teachers talk about the anticipated performance of the learners, or how learners learn

Since the teachers worked on the problem themselves before taking it to learners, except for Cele who had already tried it with some learners, they could anticipate how learners would respond to the problem. The teachers expected that the question needed to be answered algebraically but realised that the trial-and-error method, which was purely numerical, might be the most common to get from learners. They accommodated the method, and through probing by the facilitator, they talked about ways of probing the learners if the learners used the trial-and-error method in class. Probing, as a skill for pushing for deeper thinking, was

important for the teachers' PCK in their questioning techniques in order to help learners become mathematically competent.

The community talks about the kind of feedback that they could give to learners

Trying the tasks first in the community gave the teachers "some time before the lessons to allow the teacher time to consider the students' approaches prior to teaching" (Hodgen, Coe, Brown & Küchemann, 2014). Their conversation assisted them in knowing that using small numbers in such problems does not contribute to rich mathematics, and that using bigger numbers could help reveal the limitations of the trial-and-error method. This was an interesting opportunity to shift the teachers' PCK in working with alternative solution method.

Planning the next instructional step

Teachers talk about how the learner(s)' error(s) can be addressed and the misconceptions overcome

The conversation in this episode was not on learner errors, but on possible methods that learners might use to solve a word problem, and how teachers might respond to develop mathematical language. The trial-and-error method became a likely method that the learners might use, which might be encouraged by use of small numbers in the question. The community agreed on using the same task with learners and to include bigger values in order to encourage algebraic methods. The shift to bigger values raised opportunities for development of the teachers' PCK in the type of tasks to use in order to promote rich mathematics.

Teachers link learner errors to their practices

This episode, as stated above, did not bring up any conversation on learner errors. The conversation was mostly on the trial-and-error method and how to handle it in class. Though the teachers were initially comfortable with the method, but they later shifted their thinking after the facilitator pushed them to think deeper about its limitations. Cele initiated this shift (turn 40) by suggesting algebra as an alternative method, which helped the other teachers' thinking to shift. He supported this by saying that: "there are clever methods that can lead us to the answer without suffering".

Teachers share their best practices and acknowledge their challenges

Though the teachers were comfortable with trial-and-error – a method which helped to produce the solution more quickly with small numbers – they realised afterward that the method was not helpful with bigger numbers. They then decided to use big numbers when giving similar problems to learners – learners often feel comfortable with methods which get them to their answers quicker, and it is important that teachers develop PCK in working with a variety of methods in mathematics. Working on the problem in this episode helped the teachers develop ways of engaging learners to show the limitations of the trial-and-error method and the use of generalisable methods.

Teachers talk about other issues that may be embedded in the task and relating to practice

Though the facilitator tried to engage Cele on how he would deal with the learner response of seven, eight, and nine in class, Cele kept focusing on the correctness of the solution rather than engagement with the solution. The other teachers also supported him. The correct solution restrained the teachers from thinking about rich mathematics, which inhibited the development of the teachers' PCK for instruction in class. Although the community expected the problem to be done algebraically, they realised that the numerical approach would likely be common with learners. They included it in the lesson plan to see what the learners would do so that they engage them appropriately. This happened after they shifted their thinking in evaluating the method, which raised an opportunity to build their PCK in evaluation of learners' methods.

Summary of the analysis

I have argued that the PLC conversation created opportunities for the teachers to enhance their PCK in responding to learners' answers. Their conversation helped them realise that anticipating learner responses is key in understanding learner challenges and how best to deal with such challenges. A shift in one teacher's thinking, Cele, created a space for a shift in the other teachers' thinking in the community. This shift created an opportunity to increase their PCK in working with more than one method in mathematics and evaluating those methods to promote rich mathematics. Probing became a technique that teachers could use to show why a certain method works better than the other.

In evaluating learner understanding, an opportunity arose for the teachers to enhance their PCK in knowing how to respond to correct numerical approaches in mathematics, even when they wanted to teach algebra. They had an opportunity to learn that the use of small numbers in word problems can encourage non-generalisable methods in learners. Though the teachers initially supported each other on the numerical approach to solve the problem, Cele's shift to a generalisable method could have led the community to realise the importance of using bigger numbers instead of small numbers to promote both generalisable and trial-and-error approaches to solving word problems in mathematics. However, take-up by the other teachers could not be ascertained. The teachers' support of trial-and-error is reminiscent of learners' contentment in methods that give the correct answer, which makes it not easy to persuade them to think deeper about their method or alternative approaches.

In planning the next instructional step, the teachers' shift to use bigger numbers created an opportunity to the improve their PCK in working with trial-and-error as well as in task modification. The shift contributed to the opportunity to build their PCK in evaluation of learners' methods – that numerical methods do help build rich mathematics in learners, but it is also important to understand their limitations, hence the importance of employing a variety of methods.

Episode 3: Mathematical language

Lesson Reflection (March 2012)

This episode focuses on a learner's error that Dimpho brought to the PLC for a group reflection. Her class worked on a list of true/false statements that the community included as tasks for their joint lesson plan. One of the questions included: $a^b = ab$, and learners were to state whether the statement was true or false and give a reason for their choice. One learner said the statement was true because a and b were variables. When Dimpho asked the learner about 2^3, the learner said it would give $2 \times 2 \times 2 = 8$, which shows that she was comfortable with the numeric expression but experienced problems with the algebraic one of a^b. In her explanation, the learner told the class that the expansion of 2^3 was possible because 3 is a number, but with a^b you ought to multiply $a \times b$ because b is not a number and you don't know how many times to multiply. This means the learner struggled with variables as representing numbers, something similar to what emerged in analysis of the teachers' conversation in the first episode. The

learner's reasoning linked to letter as object because she saw the letters as objects that should be operated upon. She acknowledged the letters' existence without according them their appropriate meaning (Hart, 1981, p. 104).

The PLC focused on how the learner's challenge could be addressed in class. The key point in the conversation was on how to work with the expressions 2^3 and a^b in class in order to address the learners' challenges. The conversation for this episode was very repetitive and unnecessarily long, and it was divided into two extracts and excluded repetitive parts in-between.

1	Dimpho	I don't think they know what it means. They take it as variable and they don't actually relate variables to numbers until such time that they can understand what it means maybe if ever we come up with those examples, two to the power of one, two to the power of two, two to the power of three, you know. So maybe they can actually now generalise. We can try giving these examples so that we can maybe give the definition of what *a* to the exponent *b* is. That if you're having two to the exponent three, we multiply two three times. Two to the exponent five, you multiply two how many times? Five times. So they can see, oh, which means if ever this to the exponent, meaning that you multiply *a b* times.
2	Mandla	Multiplying *a b* times, is it not the same as like *ab*?
3	Facilitator	I'm listening.
4	Dimpho	Not…
5	Facilitator	I'm listening. (*laughs*)
6	Mandla	Multiplying *a* to the same…if I say it in words, multiply *a b* times, is it not the same like *a* times *b*? Which is *ab*.
7	Dimpho	If ever I say, multiply two three times.
8	Mandla	Two three times?
9	Dimpho	Ja [**Yes]**.
10	Mandla	Okay, then…okay, I'm saying, it's not the same as like two times three?
11	Dimpho	Two three times, no, it's not two times three
12	Facilitator	It's open to everyone
13	Dimpho	If ever you multiply two three times, it's not the same as two times three.
14	Mandla	I'm just asking.
15	Khumo	Two three times, is it not six?
16	Cikizwa	It is.
17	Dimpho	Two three times…
18	Facilitator	That is what Mandla is asking, two three times.
19	Cikizwa	Two three times
20	Dimpho	You multiply two three times. It will be two times two times two. You multiply it three times.
21	Mapula	Eight.

22	Dimpho	Multiply two four times. So it will be two times two times two times two. How many times? Four times.
23	Facilitator	Ma'am Molemela understands it to be two times three. Multiply two three times.
24	Khumo	And what are you going to say when you want to multiply two by three, if you are giving that, a sum, two multiplied by three and two to the exponent three? How are you going to explain that?
25	Dimpho	Explain what?
26	Facilitator	Two times three.
27	Khumo	Two times three and two to the second exponent
28	Dimpho	This one you multiply two by three.
29	Khumo	Which one? Two…
30	Dimpho	Two by three. Akere [isn't it] two times three.
31	Khumo	Yes.
32	Dimpho	Yes, you multiply two by three.
33	Khumo	Two by three. And then two to the exponent (*interrupted*)
34	Dimpho	You multiply two three times. The first time, the second time, and the third time.
35	Khumo	Okay (*softly*).
36	Facilitator	You're convinced?
37	Khumo	Yes, yes (*laughing*).
38	Facilitator	Why are you convinced (*laughs*)
39	Khumo	She has explained that.
40	Facilitator	Hah….
41	Khumo	She has explained that to me.
42	Facilitator	Okay. It makes sense to you?
43	Khumo	Yes.
44	Facilitator	Okay. Mandla, you are the one who put the cat inside the…
45	Mandla	I'm trying to think
46	Facilitator	In the what, bag of pigeons (*laughing*).
47	Mandla	I'm trying to think. It's like, it's like, if you say, two to the power of, two exponent three, when we teach we say we multiply two three times. This is what you say to our learners.
48	Facilitator	Underline that statement, to our learners.
49	Khumo	To our learners.
50	Facilitator	Okay, that is what I'm reading from what he is saying (*laughs*).
51	Khumo	You're saying, two to…
52	Mandla	Two to the exponent three.
53	Khumo	Yes, you say…
54	Mandla	Means two to the exponent three, and what we say now to learners, learners look on the board, this means what? Two, multiply how many times? Three times.

Extract 5.3a: A variable is not a number.

The part that was excluded from these two extracts made 64 turns of similar interactions as in the above extract, wherein the teachers argued repetitively about the similarity or difference between "two multiplied three times" and "two times three", without coming to any conclusion.

The next extract is taken at a point where the conversation was beginning to take shape, after Funeka had earlier inserted the word “itself” to modify Dimpho’s statement to become: “two multiplied itself three times”.

55	Mandla	You know, if you say, multiply by three, times, that’s where the problem is. Two multiplied by three is a straightforward answer. Two multiplied by three…
56	Mapula	Times.
57	Dimpho	No, you cannot say that. You cannot say two multiplied three times.
58	Facilitator	You’re saying two…
59	Mandla	Multiply.
60	Facilitator	Multiply…
61	Dimpho	Three times. I said it.
62	Mandla	Multiplied three times.
63	Facilitator	Two multiplied three times.
64	Mandla	Ja, the two is multiplied three times.
65	Teachers	Three times.
66	Facilitator	Okay. You put the word itself.
67	Cikizwa	If you multiply itself three times.
68	Facilitator	You simply add one word to what you’re saying.
69	Cikizwa	Ja.
70	Facilitator	It’s the same as what you’re saying.
71	Funeka	Ja.
72	Facilitator	So why are you putting the word itself in this case?
73	Funeka	No, it’s just because there were some ideas, saying maybe…
74	Facilitator	Clarifying something.
75	Funeka	Ja, because he was saying two it’s like, it’s the same as two multiplied by two. And then I was trying to make it simple to say, it’s multiplied itself three times, that two. So to clarify that statement, it’s the same.
76	Facilitator	You were saying ma’am?
77	Khumo	About what Ms Mosala said, it also helped the learners because always when they say two to the exponent three, they always say two multiplied by three. So when you include the word itself, now they can get a difference between two multiplied by three and two multiply itself three times, they know they are going to repeat the base three times. Multiplying the base three times.
78	Facilitator	Multiplying the base.
79	Khumo	Yes. It’s a base two angithi [isn’t it]
80	Facilitator	You are multiplying the base
81	Khumo	Two, it’s a base, and you start from the two, it’s a base. Three it’s an exponent. And from there you expand, so that you can see.
82	Facilitator	And how do you expand when you do? Can you repeat that?
83	Khumo	You say…I agree with Mrs Mosala, by saying that two multiply itself three times, I’m emphasising that what are you multiplying? You are multiplying the base three times. Itself. Itself.
84	Mandla	Oh (*laughing*), because you’re now agreeing with Dimpho (*laughing*).

85	Dimpho	Ja, because (*inaudible*).
86	Mandla	Ja, it's like my eyes were wide open, now
87	Facilitator	What is it, she's agreeing with Dimpho. And disagreeing with who?
88	Khumo	Ma'am Mosala.
89	Mandla	No, she tried to explain what Ma'am Mosala, but at the same time, during her explanation, she was agreeing with Dimpho. The statement of Dimpho, which is interesting.

Extract 5.3b: A variable is not a number.

Identifying key concepts

Teachers understand what the concepts in the activity are

The teachers' conversation shows that they understood the concepts embedded in the problem that the learner struggled with: the learner could work with numbers in computing exponents but struggled when letters were used. She could do 2^3 correctly but took a^b as $a \times b$. The critical argument in the teachers' conversation was interpreting 2^3, with the most interaction at the beginning being between Dimpho and Mandla, and important for raising opportunities to build the teachers' PCK on defining exponents.

The community talks about the key concepts embedded in the activity or the key building blocks of the activity

When Dimpho indicated that "if you're having two to the exponent three, we multiply two three times", and that a^b means "you multiply *a b* times", Mandla raised a concern and asked if the statement "you multiply *a b* times" did not mean *a* times *b*. The other teachers were also uncertain about the language used to describe 2^3, arguing that it could mean two times three. This uncertainty created an opportunity for the teachers to engage with the two statements and improve their PCK on interpretation of mathematical expressions, and the difficulties that learners might have with such interpretations and the language used to describe them.

The community talks about the appropriateness of the task for their learners

Dimpho at the beginning of the episode highlighted learner challenges with literal exponents – she stated that "they don't actually relate variables to numbers", a statement which necessitated the teachers' conversation in finding ways of helping learners to interpret mathematical expressions. In addition, Mandla later reminded the community to think about the learners in talking about interpretation of 2^3. Mandla was pushing the teachers to think deeply about an

appropriate way of interpreting the same or a similar expression in class, and this raised the potential to produce important PCK for the teachers in mediating mathematical terms or expressions in class.

Teachers make links with other tasks and concepts or with the learner's work

The teachers' discussion of Dimpho's statement created disagreement in the community in how the language of exponents might be understood by learners, which emerged as a learning need for the teachers (Katz, et al., 2009). When Mandla challenged Dimpho's statement, an environment was created in the community to differentiate between 'two to the power three' and 'two times three', linguistically. The word "times" confuses learners in knowing how to use it in exponents because of their previous knowledge of how it is used in multiplication. Engaging deeply with this distinction linked to the learner's response of $a^b = a \times b$. Mandla's challenge suggested that Dimpho's statement might create misconceptions for learners because of how she used the word "times".

Evaluating learner understanding

The community investigates/focuses on the learner's reasoning

The learner reasoned that $2^3 = 2 \times 2 \times 2$ because it involved numbers, whereas $a^b = a \times b$ because it did not involve numbers, which means that the learner saw the letters a and b as objects (Hart, 1981). In the teachers' conversation, it was only Dimpho who explicitly related variables to numbers, which occurred at the beginning of the episode and that was important to help the teachers focus on a critical issue that the learner struggled with: not seeing letters as numbers (Hart, 1981). The other teachers did not take this issue up and focused rather on how Dimpho interpreted the exponential expression, an issue which was also important to help the teachers with an approach to the teaching of exponents but changed the focus of the conversation. What started as an opportunity to enhance the teachers' PCK in dealing with the learner's reasoning shifted to a discussion to modify Dimpho's statement.

Teachers can establish what the learner(s) know and/or do not know, which is linked to exploring learner thinking

The teachers established that the learner knew that $2^3 = 2 \times 2 \times 2$ but did know what a^b meant. The learner's error on $a^b = a \times b$ triggered a conversation among the teachers which raised

opportunities for the teachers to improve their PCK on mathematical language. What turned out to be a learner's error triggered the teachers' engagement with the statement made by Dimpho to develop a language that they could use in working with learners' misconceptions in class. The use of the correct mathematical language was important for the development of the teachers' PCK.

The community makes inferences from the learners' work, which includes learners' interpretation of letters in algebra

As discussed earlier, Dimpho stated that learners do not see letters as representing numbers, a point similar to what was found in the first episode and important to trigger a discussion on the learner's error. The other teachers did not take this up and took Dimpho's statement instead to divert the discussion to interpreting the exponential expressions: 2^3 and a^b. Dimpho's statement on the learner's error was used as a springboard to talk about mathematical language. The community did not talk about the learner's interpretation of letters in algebra, which constrained contribution to their knowledge of working with letters in algebra.

Teachers talk about the anticipated performance of the learners, or how learners learn

In explaining his challenge of Dimpho's statement, Mandla emphasised that learners could confuse the statement 'multiplying two three times' with 'two times three' (turn 54), and Dimpho concurred by saying: "Oh! I get your point". Her agreement though was on how learners might interpret her statement, but she still maintained that 'multiplying two three times' was not the same as 'two times three'. For Dimpho, it is a tension between CK and PCK, she sees what is correct mathematically rather than what the learner sees.

The community talks about the kind of feedback that they could give to learners

Though the community changed the focus of the conversation from learner thinking about variables to learners' understanding of the language involved in exponential expressions, there was still another opportunity in the community for teacher learning. The conversation focused mainly on refining a definition for exponents, which would make more sense to learners. Funeka's suggestion of modifying Dimpho's statement to: "two multiplied itself three times", brought some clarity to how exponents should be understood, and this raised the opportunity to enhance the teachers' PCK on mathematical language.

Planning the next instructional step

Teachers talk about how the learner(s)' error(s) can be addressed and the misconceptions overcome

Dimpho started by elaborating on the learner's error, but the community did not take this up. The facilitator allowed for the diversion because it was also important to help in dealing with learner errors in class. After the teachers had long argued around Dimpho's statement, and demonstrated that it might confuse the learners, Funeka suggested that Dimpho's statement be changed to "two multiplied itself three times', and this shifted the teachers' thinking in talking about exponents. This shift evidenced opportunities for the enhancement of the teachers' PCK in mathematical language, language that was important to improve learner understanding in exponents, though the community did not deal with the actual issue that Dimpho brought to the group.

Teachers link learner errors to their practices

Though the conversation was not on the learner's error, Mandla became more specific in turn 47 to make the necessary link to the teachers' practices: "when we teach, we say we multiply two three times. This is what you say to our learners." This input pushed the teachers to focus on how learners could be helped to understand the mathematical language involved in exponents because PCK also includes how learners learn (Van Driel & Berry, 2012b).

Teachers share their best practices and acknowledge their challenges

The teachers in the episode did not explicitly share their practices. When Dimpho stated that: "they don't actually relate variables to numbers", referring to learners' confusion with variables, this was an opportunity for the community to focus on the learner's error and share their best practices, but this did not happen. Khumo later stated that: "always when they say two to the exponent three, they always say two multiplied by three", but she did not share with the community how she works with such statements in class. The best practice that emerged in the teachers' conversation was in probing each other to refine interpretation of exponential expressions. Diverting from Dimpho's statement did not help the teachers in understanding how learners interpret letters.

Dimpho acknowledged that her statement might create confusion for learners. In addition, though Mandla challenged Dimpho's statement, but he also acknowledged that: "I don't think I know the answer hey". The two teachers' acknowledgement necessitated the PLC engagement with Dimpho's statement in order to reinforce it for the enhancement of the teachers' PCK.

Teachers talk about other issues that may be embedded in the task and relating to practice

Though Dimpho's statement was initially perceived as having potential to confuse learners, the teachers did not outrightly reject it. They embraced the statement to talk about the correct mathematical language. Their conversation focused on ways of thinking about the correct mathematical language to use in class, language which might not contribute to the learners' misconceptions. In addition, the manner in which the teachers embraced Dimpho's statement is indicative of how they might embrace the learners' errors in their teaching.

Summary of the analysis

I have argued that in this episode one teacher's input was taken to raise opportunities to build the teachers' PCK on mathematical language. When the community looked at a learner's error of $a^b = ab$, Dimpho made an input that learners do not see letters as numbers, which means that the learner treated letters as objects (Hart, 1981). Instead of engaging with the learner's error within that context, the participating teachers used the learner's error as a springboard to discuss the mathematical language associated with exponents. The community focused the conversation on finding a better way of interpreting a^b or 2^3. They realised that learners could be committing errors in exponents due to how the mathematical language is used in that context, and might be confusing to learners. The crux of the argument was that "two multiplied three times" may sound the same as "two times three" to learners. The teachers' conversation signified that they were very attuned to this error, and that talking about it was an opportunity for "deepening the teachers' knowledge of the subject, knowledge of their learners, and ways of teaching" (Chauraya & Brodie, 2018, p. 8).

What started as an explanation by Dimpho to show what the learner's challenge was, developed into a contestation that resulted in the teachers trying to find a way of how best they can

describe the exponential expression to learners. This suggests that teacher learning was limited on the specific matter that was on the table, which diverted to a different matter. Despite the diversion, the conversation in the community created an opportunity for the teachers to enhance their PCK in knowing the extent to which mathematical language can contribute to learners' misconceptions in exponents.

I have argued that in planning for the next instructional step, the teachers did not engage with Dimpho's input on the learner's error. Instead, they focussed on a subject which was also important for the teachers' PCK by addressing relevant mathematical language that they needed for instruction on exponents. The community shifted the conversation because they thought that Dimpho's interpretation of expression: $2^3 = 2 \times 2 \times 2$ as "two multiplied three times" could confuse learners. After a long discussion, they decided that getting the interpretation right for 2^3 or a^b might assist in helping learners get the full meaning of such expressions.

In the ensuing discussion, Funeka modified Dimpho's statement to "two multiplied itself three times", which the teachers agreed to. This move became appropriate for the teachers to clarify the uncertainties that emerged earlier in the discussion, and consequently an opportunity in the teachers' PCK to interpret exponential expressions was enhanced. Whether the teachers' agreement was not enough to address the learner's confusion of a^b is not conclusive. What started as one teacher's input to deal with the learner's error became a subject of contention in the PLC and built on how best to use mathematical language in class. The original issue of difficulty with letters representing numbers was not fully addressed and it is not clear how the teachers might deal with this in their future practices.

5.6 Conclusion

This chapter focused on the teachers' conversations within the professional learning community. The analysis sought to explore the extent to which these conversations contributed to the teachers' PCK, guided by the key research question, supported by two sub-questions:

- How do mathematics teachers work as a professional learning community to learn with and from each other as they work on learners' errors and misconceptions?

- What do teachers in a PLC talk about and at what depth do they engage with different activities?
- How is pedagogical content knowledge (PCK) amongst teachers created within the professional learning community?

The first sub-question sought to investigate the content and depth of the teachers' talk within the professional learning community. Analysis of these conversations showed a difference in the time that the community spent on the different content for the different activities, with some content receiving more prominence than others. The depth of the community conversations also showed a difference across the different activities. The analysis revealed that the content of "learner", followed by "mathematics" and "practice", was prominent in all the activities over the two years, with some decline on "Learner" and "Practice" in 2012. "learner thinking" showed prominence in Error Analysis, done only in 2011. Practice was prominent in Lesson Planning and Lesson Reflection in 2011 and mathematics was prominent in all the activities in 2012. Table 5.11 below shows the prominence that the different contents received in the PLC conversations in both years.

Table 5.11: Content of PLC conversations.

ACTIVITIES	2011	2012
Error Analysis	Learner (42%)	Not Applicable
	Learner Thinking (45%)	
Readings & Discussion	Learner (45%)	Learner (27%)
	Mathematics (27%)	Mathematics (45%)
Lesson Planning	Learner (22%)	Learner (16%)
	Mathematics (20%)	Mathematics (43%)
	Practice (58%)	Practice (33%)
Lesson Reflection	Learner (53%)	Learner (34%)
	Practice (47%)	Practice (26%)
		Mathematics (23%)

The PLC conversations were intended to focus primarily on identifying the kind of errors that learners produce; the reasons behind the learners' errors: and how to work with these errors in lesson planning and in class. Results obtained show that the focus of the PLC conversation fell short of talking about the reasoning behind the learners' errors, except for learner thinking in Error Analysis in 2011. This focus in Error Analysis is not surprising because it is most removed from the teachers' practice. However, when the teachers needed to apply what they learned in Error Analysis in the other activities, inquiring into learner thinking seemed more

challenging. The focus in the other activities was more on talking about the learner strategies that led to the error and the mathematics involved in the tasks.

The emergence of mathematics as a central focus of the teachers' talk was an unexpected result because it received less attention in the project design compared to learner thinking. The focus of the teachers' conversations on mathematics means that for the most times, the community talked about the mathematical content, concepts and/or language. As they talked about mathematics, the analysis showed that some teachers shared important mathematical content and raised opportunities to contribute to the teachers' PCK in the community. This was exemplified in the first episode when Dimpho explained the possibility of addition when letters are different in algebra after Khumo seemed to be treating letters as objects (Hart, 1981), and also in the third episode when the community discussed Dimpho's statement of "two multiplied three times". The finding on mathematics and on learner may reflect what might be emphasised in the participating teachers' classrooms – the teachers possibly focused on understanding the learner strategies as well as the mathematics embedded in the learners' errors. Understanding the reasoning behind the learners' errors possibly received less focus, which resonates with findings from Chapter 4 of this thesis, where two teachers spent little or no time on the learners' reasoning behind the errors in their lessons. In addition, the prominence of practice in the teachers' talk suggests that the participating teachers in the community conversations could link their talk with what happened, or intended to happen, in class, which can be linked to their PCK in working with learners' errors.

Analysis of the coding results also focused on the depth of the teachers' conversations. Table 5.12 below shows the depth of the community conversations in the different activities in 2011 and 2012. The general trend in the table shows that the community conversations were deeper in 2012 than in 2011.

Table 5.12: Level of depth in the PLC conversations.

ACTIVITIES	Level	2011	2012
Error Analysis	1	54	
	2	35	
	3	11	
	4	0	
Readings & Discussion	1	52	23
	2	30	50
	3	19	27
	4	0	0
Lesson Planning	1	76	53
	2	24	45
	3	0	1
	4	0	0
Lesson Reflection	1	85	32
	2	15	42
	3	0	18
	4	0	8

Only Error Analysis and Readings & Discussion reached a depth of level 3 in 2011, and this depth being maintained in 2012 in Readings & Discussion. The other two activities reached a depth of level 2 in 2011. The greater depth observed in Error Analysis and Readings & Discussion in 2011 means that for the most times, the teachers' talk focused on understanding the reasoning behind the learners' errors, elaborating on the examples under discussion and engaging with issues raised in the literature as was the case when they discussed the problem: $2a + 5b = 7ab$ from one of the papers. Depths of level 2 and level 3 were reached in all the activities in 2012, with Lesson Planning and Lesson Reflection reaching level 3 only in 2012. From 2011 to 2012 there was a shift across all activities from level 1 to level 2 and in two activities to level 3. This increased depth in the conversations was a result of participation of the teachers shifting over time and suggests that extended participation in the PLC can contribute to the depth of the conversation.

The shift observed in 2012 means that the teachers' engagement with the activities was different from their engagement in 2011. For example, in Lesson Planning, they worked out the tasks they wanted to give to learners and discussed them in detail among themselves prior to doing them in class – this was discussed in the second episode above. This process helped the teachers to think about how they viewed the learners' errors and what they could learn from

that. In the two years that the community had been meeting, only Lesson Reflection reached a depth of level 4 in 2012, implying that teachers in 2012 could generalise and make connections between events/examples and principles of teaching and learning. Their conversations went deeper than providing mere descriptive and analytic comments, and they engaged with mathematical ideas in different ways.

In responding to the second sub-question on how PCK was created in the community, three episodes of the PLC meetings were analysed to exemplify the extent to which the PLC conversations contributed to the participating teachers' PCK. In analysing the episodes, three key points stood out: 1. Disagreement between and among the teachers created opportunities for the development of the teachers' PCK in the community; 2. One teacher's input created an opportunity for the community to engage in rigorous conversations which raised opportunities in PCK development; and 3. Probing the teachers' inputs by the facilitator was an important strategy to encourage the conversations in the community. A general trend that emerged in the analysis of the episodes revealed that the teachers' conversations rarely focused on the reasoning behind the learners' errors, supporting the initial coding.

In the first episode, when a learner treated letters as objects in the problem $2a + 5b$ to get $7ab$, Dimpho and Khumo focused on the incorrectness of the learner's mathematical content. The other two teachers, Mapula and Funeka, focused on the reasonableness of the learner's answer but did not deal with the underlying misconception(s) in the learner's answer. This disagreement illuminated the difference in which the four teachers approached the learner's error and contributed to the discussion that the community had. An opportunity for the development of the teacher's PCK emerged when the community discussed the addition of $2a$ and $5b$: first, it occurred when Dimpho, and later the facilitator stated that putting numbers for a and b made addition possible. Second, it occurred when Dimpho explained how making $a = b$ would make addition of $2a$ and $5b$ possible. The community though did not deal with the key aspect of seeing $2a + 5b$ as an answer, which had potential to contribute to the teachers' PCK in working with letters in algebra.

Analysis of the second episode showed how one teacher's input raised an opportunity in the development of the teachers' PCK, which also occurred in the third episode with a different teacher. In the second episode, the community worked on a problem that they planned to use

in developing the concept of language in algebra with the learners. In their initial discussion of how learners would approach the problem, they were initially content with a purely numerical approach and ignored the limitations of the method as well as other alternative methods. Persistent probing by the facilitator brought a shift to thinking about an additional algebraic method and comparing the two methods. Analysis of the episode showed how a shift in one teacher's thinking created a space for a shift in the other teachers' thinking, which was an opportunity to enhance their PCK in working with more than one method and larger numbers in mathematics.

It was interesting to find that the teachers' thinking in this episode was not easy to shift to methods that promoted rich mathematics when they knew the correct answer, something similar to what usually happens with learners. A push for deeper thinking was needed to shift the teachers' views to thinking about a variety of methods and evaluating those methods. These pushes also created opportunities in improving the teachers' PCK in probing learner responses.

In the third episode, the community planned to discuss an error that a learner had made in Dimpho's class, that $a^b = ab$ because b is not a number and you don't know how many times to multiply. When Dimpho began to engage with the error in the community, Mandla challenged her on the mathematical language that she used, and the other teachers joined the conversation. The PLC conversation was then diverted to finding a better language to use with learners when teaching exponents and did not engage with the learner's error. As in the second episode, one teacher's input became a springboard for the development of the teachers' PCK in the community. The main argument in the community was that "two multiplied three times" may sound the same as "two times three" to learners. Despite this diversion, an opportunity was created for the development of the teachers' PCK in mathematical language. It is acknowledged though that teacher learning on learner thinking was compromised at the expense of mathematical language, which supports the coding results in Table 5.11 above, with mathematics emerging as prominent in the PLC conversations. What started here as one teacher's input in the PLC evolved into a subject of contention that the participating teachers engaged with, and this contention subsequently raised opportunities to develop the teachers' PCK.

My role as the facilitator of the PLC was key in how the PLC conversations discussed above unfolded in the community, which made it important to investigate how facilitation played out over the two years. The next chapter focuses on facilitation of the professional learning community in 2011 and 2012.

Chapter 6: Facilitation of a Professional Learning Community

6.1 Introduction

This chapter focuses on my role as a facilitator of the PLC and how I worked with the participating teachers over the two years with the DIPIP activities. My analysis focuses on the extent to which I, as the facilitator of the PLC, created a space for engaging with the different activities in the PLC.

I first look at the coding results on facilitation to show my facilitation of the PLC in relation to the different content. I then provide four episodes from the data to exemplify how facilitation of the PLC played out and the extent to which this facilitation enhanced or inhibited teacher participation in the community. My analysis focuses on what my facilitation of the PLC in the two years meant for the teachers' conversations regarding their professional knowledge, and I also show the strengths and weaknesses in my facilitation.

As the researcher and facilitator of the PLC, I had to wear more than one cap. As a facilitator, it was important for teachers to see me as a colleague working closely with them for purposes of improving their practices. Teachers needed to view me as a co-learner and a critical friend, learning from their practice (Roberts & Pruitt, 2003) as well as a friendly critic of their thinking and practices with the intention of developing better practices for mathematics teaching. The teachers' expectation of me was that of a resource person for their professional development and their instructional knowledge. As a researcher, I was exploring how the participating teachers worked with classroom data to learn from and with each other. This research focused on how the participating teachers inquired into their practice with the guidance of the facilitator with the hope of improving their mathematics teaching, by learning to work with and embrace their learners' mathematical errors. These two roles created possible tensions in my facilitation and my research.

There are also power issues at play in relation to facilitation. First, any facilitation role creates power relations between the facilitator and the participating teachers, especially when the facilitator comes from a university, because the facilitator has expertise and access to resources that school teachers may not have. Second, these power relations exist when the facilitator of

the PLC is also the researcher of that PLC because the participating teachers might be nervous or concerned that their practices are being researched, and they may not always be comfortable with this – research is sometimes perceived as invasion of private space. Third, power relations also existed among the teachers because of: different grades that the teachers taught in their different schools, their teaching experience, and their positions at their schools. Fourth, subject matter knowledge is also connected to the power relations among teachers, and teachers who are thought to know more than others may dominate discussions.

It was important that I created in the PLC an atmosphere that allowed teachers to engage with each other about their classroom data as freely as possible. This meant that trust and care needed to be fostered in the PLC. In addition, I needed to "create a safe environment for teachers to grapple with difficult content and pedagogical issues" (Koellner-Clark & Borko, 2004b, p. 223). It was important for the participating teachers to feel safe enough in the PLC meetings to talk openly and honestly about their practice, which could afford them the opportunities to share both their challenges and their best practices. How a conducive atmosphere was created within the community was an important research focus to explore.

Learning to become a good facilitator is not an easy task and needs constant reflections, individual reflections as well as reflections with a peer group. Careful management was key regarding the tensions between my two roles, and the power relations in the PLC. Appropriate skills and strategies for facilitation need to be learned and mastered, especially in managing the power relations inherent in the PLC (Brodie, 2016; Brodie, et al., 2014).

In this chapter, I analyse the community data (the same data analysed in Chapter 5) to understand the extent to which my facilitation of the PLC created a space for participating teachers to develop new knowledge through engagement with classroom data in meaningful ways. I also analyse this data to show the extent to which this facilitation enhanced or inhibited the teachers' interactions and knowledge in the PLC. The critical question that this chapter answers is:

- How is a space for engaging with the different activities in a PLC created by the facilitator?

My analysis focuses on the extent to which my facilitation enhanced or inhibited the teachers' conversations in the PLC. Enhancement of the teachers' conversations is exemplified by teachers embracing the learners' errors, and mutually adopting collaborative strategies they could employ in their instructional practice. Inhibiting teachers' practices is exemplified by not creating the space for teachers to talk about their best practices and challenges, and create new knowledge for themselves.

6.2 Analysis of facilitation

My analysis of facilitation of the PLC in this chapter proceeded in two ways. First, I coded the facilitator utterances, using the codes described in Chapter 3 and I analysed the results quantitatively. Second, there is a qualitative analysis which focuses on episodes from the PLC meetings to show how I interacted with the participants as the facilitator. The quantitative analysis focuses on my contributions to the PLC conversations in relation to the different PLC activities over the two years, while the qualitative analysis explores further the extent to which my contributions guided the teachers' conversations about their instructional practices.

6.3 Quantitative analysis

Tables 6.1 and 6.2 below show the relationships between the activity-process and activity-content relationships for 2011 and 2012. In Chapter 3, I discussed the coding system for facilitation which has three main categories: process, content, and community, each with subcategories. The process codes have three main sub-categories: manages, maintains, and input, each with further subcategories. Managing the conversation dealt with non-content related tasks such as introducing a topic for discussion, allocating a turn to a participant, asking for comment, or repeating what someone said or what I said. Maintaining the conversation involved moves such as pushing the teachers for teacher deeper thinking, re-voicing someone's input, seeking clarification, making links or drawing ideas together. Making input involved explaining, clarifying confusions, answering questions, or drawing from research. The process moves of maintains and making input were further coded for content. The content codes used in this chapter are the same ones used in Chapter 5: Learner, Mathematics, Practice, Learner thinking, and other codes. Other codes refer to those codes which featured less in the PLC conversations. See Tables 3.16 and 3.17 in Chapter 3 for detailed descriptions of the codes.

The numbers used in the two tables below are percentages of the coded times for facilitation for each of the given activities, rounded off to the nearest whole number. Percentage scores were convenient to use in tracking the foci of my facilitation inputs in the two years, to explore the activity-process and activity-content relationships, and they are also convenient for reporting on the findings of my quantitative analysis.

Activity-Process Relationship

Table 6.1 below shows percentage scores for the process category in the two years of facilitating the different activities: 2011 and 2012.

Table 6.1: Percentage scores for the process category in the different activities.

ACTIVITIES		**Error Analysis**		**Readings & Discussion**		**Lesson Planning**		**Lesson Reflection**	
		2011	**2012**	**2011**	**2012**	**2011**	**2012**	**2011**	**2012**
Manages	Introduces	4	-	3	3	2	5	8	5
	Calls to present	0	-	0	3	0	1	4	0
	Asks for comment	25	-	8	11	3	7	5	6
	Ends topic	5	-	1	2	2	1	1	5
	Allocates	16	-	0	2	2	3	8	2
	Repeats	19	-	7	3	0	2	0	6
	Chooses contributions	1	-	0	3	1	3	2	0
	Brings back	0	-	0	0	0	0	0	1
Maintains	Re-voices	4	-	0	1	0	1	0	1
	Clarifies	1	-	1	4	1	3	2	2
	Pushes	2	-	3	3	0	6	1	7
	Gives time	4	-	3	5	0	2	0	1
	Makes links	0	-	0	3	8	1	0	1
	Draws together	0	-	0	4	0	2	0	1
Input	Explains	14	-	73	46	77	57	67	48
	Answers	5	-	0	0	4	4	0	2
	Clarifies	0	-	2	4	0	3	1	9
	Research	0	-	0	2	0	0	0	3
	Totals	**100**	**-**	**101**	**99**	**100**	**101**	**99**	**100**

Table 6.1 shows that the focus on “manages” is more prominent in Error Analysis in 2011, showing that I focused more on management issues in this activity than maintaining the discussion or making input. These were early stages of the PLC, and I was developing ways of working in the PLC to develop safety and inclusion for all the participants, by ‘asking for

comment', 'allocating turns', and 'repeats' contributions. These occurrences decreased substantially across the other activities in both years. The decrease in these three codes made room for my input in the community to increase over time as observed in the table above, with my explanations taking prominence in the other three activities.

Table 6.1 shows that "explains" is extremely prominent in all the activities for both years, with less focus in Error Analysis in 2011 because manages was somewhat higher than in the other activities and I was also overly conscious of my facilitation. A facilitator contribution was coded "explains" when I made an input on an idea that was already on the table and thought that it was important to elaborate further on that idea. This happened when I made a claim or explained something from my own experience or an idea about teacher knowledge or good practice. If my idea was from research, then it would have been coded "research". My explanations often focused on ideas that I shared with the participating teachers regarding mathematical content knowledge, and/or pedagogy, and/or pedagogical content knowledge; on research; and on learner thinking. This focus will be discussed in detail in later sections.

Though the code of "explains" declined in 2012, its coded time is still significantly more than all the other codes combined. Facilitator explanations were not emphasised in the project and this unexpected result needs elaboration on its prominent occurrence. As a researcher, I need to take this result seriously, and do further analysis and reflection to try to understand why I explained so much. My reflections on this prominence were:

1. As a researcher and a facilitator, I wanted to see the project succeeding. My expectations going into the PLC meetings was that teacher participation would be in the kind of practice that I learned from academic settings as well as in the discussions we had in project meetings regarding what we envisaged as a project. However, when things turned out differently, frustration crept in. This often happened when I thought that teachers were missing key points in the discussions, and I ended up taking over the discussion.
2. Silence from the teachers in the PLC discussions also contributed, especially when research literature was to be discussed, and I ended up doing the most talking in the PLC.
3. When I made a call that teachers were missing the key points in the discussions, and I took over the discussion to clarify those points.

4. Responding to my own questions when I could not get a response from the teachers, or responding directly to the questions which the teachers posed in the community, rather than giving other teachers the opportunity to respond.

Activity-Content Relationship

Table 6.2 below shows percentage scores for the content category over the two years of facilitation. The table shows that the focus of facilitation was more prominent on "learner" and "mathematics", with less focus on "learner thinking" except in 2011 Error Analysis and 2012 Readings & Discussion. "Practice" also shows prominence in Lesson Planning and Lesson Reflection. The other codes recorded low scores in all the activities in the two years. The table shows a number of similarities with the analysis of the community conversations in Chapter 5. This makes sense because I was both responsive to and contributed to these conversations.

Table 6.2: Percentage scores of content codes.

ACTIVITIES	Error Analysis		Readings & Discussion		Lesson Planning		Lesson Reflection	
	2011	**2012**	**2011**	**2012**	**2011**	**2012**	**2011**	**2012**
Learner	45	-	49	30	8	18	60	29
Mathematics	23	-	50	37	40	38	2	27
Practice	7	-	1	8	52	37	38	24
Learner Thinking	25	-	0	22	1	6	0	10
Other codes	0	-	0	2	0	0	0	10
Totals	**100**	**-**	**100**	**99**	**101**	**99**	**100**	**100**

The code of "learner" was prominent in all the activities in 2011 except in Lesson Planning. These observations mean that my input often focused on the errors that learners produced in their mathematics classes, how learners understood the mathematics and the solution strategies that learners employed to produce the mathematical errors. A focus on "learner" declined in 2012 although it was still prominent.

"Mathematics" was prominent in Readings & Discussion and Lesson Planning in both years, and less prominent in Error Analysis and Lesson Reflection. As in the analysis of community conversations in Chapter 5 for these two activities, the PLC discussed the mathematical tasks they needed to do with learners in class and also focused on the key mathematics concepts that

emerged in the literature. The focus on mathematics shows that I paid special attention to explicit mathematical concepts/language as well as mathematical procedures in these two activities. This focus contributed to the development of the teachers' knowledge: both CK and PCK, two concepts which can build on each other as teachers inquire into their practice in the PLC. This focus is exemplified in the qualitative analysis below.

"Practice" was prominent in Lesson Planning and Lesson Reflection in both years and showed very small occurrences in Error Analysis and Readings & Discussion in both years. The small occurrences show that I focused more on talking about learner errors in those activities without situating these errors in the classroom, except for a few cases. The prominence of "practice" in Lesson Planning and Lesson Reflection means that my focus was on what the teachers did in class in relation to instruction and more general aspects of practice. An example is given in the episodes below to show the extent to which I focused on practice in ways in which I related it to the development of the teachers' PCK. While Error Analysis and Readings & Discussion allowed for the PLC to consider learner errors by situating them away from practice, Lesson Planning and Lesson Reflection were intended to bring back the PLC to focus on practice that embraced learner errors and I maintained this focus of the project.

Learner thinking was less prominent than the other codes in all activities in the two years, except 2011 Error Analysis and 2012 Readings & Discussion. This is an unexpected observation since learner thinking was a key focus of the project. This means that even though a focus on learner thinking was a key goal, I did not focus on this aspect beyond 2011 Error Analysis and 2012 Readings & Discussion. Learner thinking includes understanding the reasoning behind the learners' errors and anticipating or predicting the errors that learners might make. Learner thinking and mathematics received similar foci in Error Analysis in 2011. This means that I discussed how the learners' errors related to the mathematics discussed, as exemplified in the episode of apples and bananas metaphor in Chapter 5 and later in this chapter. The focus also included how mathematics contributed to the learners' errors. Linking learner errors to mathematics removes the blame for these errors from both the teachers' prior teaching and the learners' prior knowledge and helps in seeing learners' errors as "a normal part of learning mathematics" (Borasi, 1994; Brodie, 2014).

In the next section I provide episodes from community data to exemplify how facilitation of the PLC led to the results discussed above. The episodes provided here evidence how the different activities focused on the different process and content codes.

6.4 Qualitative analysis: Episodes from PLC facilitation

In the previous section I provided the beginnings of an analysis of how I created a space for the teachers to engage with the different activities in the PLC and showed how my engagement with the different activities of the project related to the process and content categories. In this section I provide analyses of episodes from the data. The four episodes I provide link to the different activities discussed in the study. Two of the episodes were analysed in Chapter 5, and they are re-analysed here to show how my facilitation contributed to the teachers' conversations.

This section complements the previous one in answering the question: How is a space for engaging with the different activities in a PLC created by the facilitator? The question relates to the extent to which facilitation of the PLC guided the teachers' participation in their conversations as well as how it enhanced or inhibited the development of the teachers' PCK. In responding to this question, I use the same framework as above in order to investigate PLC facilitation in the different activities. In order to help my analysis, I also focus on the missed opportunities in my facilitation. The criteria used to identify the missed opportunities focused on identifying gaps in my facilitation, which I considered to be opportunities to learn regarding my facilitation. These opportunities were not picked up during my facilitation but only after the PLC meeting(s) or during my analysis of the data. For each missed opportunity, I identified learning opportunities which informed me on how else I could have facilitated the discussions in the PLC.

Key aspects in my facilitation were maintaining and giving input into the conversation. Moves that related to maintaining the conversation were (see detailed discussion in Chapter 3):

1. Re-voicing, reformulating or extending a contribution to make a point.
2. Seeking clarification or elaboration.
3. Pushing or challenging the teachers to think deeper and/or justify their inputs.
4. Giving the teacher time to think.

5. Making explicit links with earlier contributions.
6. Drawing ideas together to bring out the main points.

Moves that were related to making input in the conversation as a facilitator were (see detailed discussion in Chapter 3):

1. Explaining an idea
2. Answering a question or giving an answer to a mathematical question.
3. Clarifying confusion.
4. Bringing in important general principles from research into the conversation for purposes of relating ideas under discussion to the principles.

Moves for managing the conversation, were not critical in contributing to the teachers' participation in the community. These moves did not need a content code. The most occurring moves were:

1. Introducing a topic in the community.
2. Calling on someone to present.
3. Asking for comments.
4. Allocating turns.
5. Repeating an idea made in the community.

In facilitating the PLC, I aimed to create an environment that supported productive learning conversations, and pushed the teachers beyond their current thinking – a skill that is recommended for a critical friend (Katz, et al., 2009). I pushed the teachers to think about whether they were avoiding, correcting or embracing learners' errors (Brodie, 2013). I encouraged the teachers to build both challenge and solidarity (Brodie & Shalem, 2011) with each other, as they inquired into the learners' errors.

Four episodes were selected to show key elements of my facilitation. The presentation of the extracts below includes two columns which show how my utterances were coded as discussed above and in Chapter 3. The inclusion of these last two columns in the extract helps in supporting the claims I make in my analysis about facilitation of the PLC in order to show the extent to which the coding supports my analysis of the facilitation practices (Brodie, 2016).

My analysis is guided by four themes, which I use as subheadings in my analysis of the four episodes below:

1. The importance of pushing the teachers in conversations, which is also related to the notion of challenge as they both elicit deeper thinking.
2. The results of my coding in Section 6.2 above, which showed the prominence of explaining in the process codes.
3. The prominence of learner, mathematics and practice in the content codes.
4. The missed opportunities in my facilitation, which revealed what I needed to learn.

Episode 1: Seeing $2a + 5b$ as a possible solution

(Readings & Discussion, September 2011)

The example here is the same one discussed under the same activity in Chapter 5 (see Section 5.5 of Chapter 5). The example is used to show the extent to which I worked with the teachers in using a literature paper to guide their Lesson Planning activity. The paper that the PLC discussed was: *Children's difficulties in beginning Algebra* (Booth, 1988). The paper was read in order to get ideas that the community could use in planning a joint lesson on the notion of a variable. The conversation was on whether metaphors were useful or not in the teaching and learning of mathematics, and the paper gave an example of a learner's response to exemplify this narrative: $2a + 5b = 7ab$. The learner had said the statement was correct because when a stands for apples and b for bananas, then $7ab$ would mean seven apples-bananas. The PLC discussion was on the relevance or irrelevance of using the metaphor of apples and bananas in teaching such problems.

Before the PLC discussion ensued, I realised that the teachers did not read the paper as requested, which prompted me to allow them the opportunity to read it during the meeting, so that the discussion could continue. The main argument in the PLC was on whether a solution to the given problem would be easily worked out if the letter a represented apples and b represented bananas. According to the learner, $7ab$ represented seven apples-bananas.

	Speaker	Utterance	Code	Sub-code
1	Facilitator	Does this learner mean that he's got seven apples and seven bananas? Is that what the learner means?	Manages	Asking for comment
2	Funeka	No.		
3	Facilitator	And look at how apples and bananas have been written there. Hyphenated. To show that it's one thing. Is there anything wrong with the statement?	Maintains	Pushes/ Learner
4	Dimpho	Seven apples. Then what is and? And is plus.		
5	Facilitator	Apples plus bananas.	Maintains	Pushes/ Learner
6	Mapula	The hyphenation		
7	Dimpho	Seven apples plus bananas.		
8	Funeka	Plus bananas. That's what they are saying		
9	Dimpho	So it means that you say that apples are seven. That's what it means. Seven apples.		
10	Facilitator	Not in the manner in which it is written.	Maintains	Pushes/ Learner
11	Khumo	(*Inaudible*) seven apples.		
12	Mapula	Or will it be fine if he said, seven…seven apples bananas. (*laughter*) Seven apples and…seven apples bananas.		
13	Funeka	And bananas.		
14	Facilitator	That is what the learner is saying, seven apples and bananas.	Maintains	Pushes/ Learner
15	Khumo	But it won't (*inaudible*) seven apples and bananas.		
16	Dimpho	But if a person says seven apples, (*inaudible*) seven apples.		
17	Mapula	But to me it makes sense.		
18	Facilitator	It makes sense, ne?	Maintains	Pushes/ Maths
19	Dimpho	No, it gives me the impression that, that learner is saying the apples are seven.		
20	Facilitator	No, but talk to what is written there.	Maintains	Pushes/ Maths
21	Khumo	And the bananas also are seven, according to that answer.		
22	Facilitator	The way it has been written it says, the learner here indicates that all in all he has seven things, what are those things? Apples and bananas.	Input	Clarifies/ Maths
23	Mapula	In one bag. You must carry in one bag. I'm carrying…		
24	Funeka	Seven bananas.		
25	Facilitator	You've got seven units, of apples and bananas combined. (*Pause*) You're thinking about something, Dimpho. What? (*laughs*)	Manages	Allocates

26	Dimpho	It doesn't give me sense.		
27	Facilitator	Tell us, why does it not give you sense?	Manages	Asking for comment
28	Khumo	You cannot add something which are different. Apples and bananas. Two apples, five bananas, seven.		
29	Facilitator	You cannot add things that are?	Manages	Repeats
30	Khumo	Not the same.		
31	Facilitator	You can't add three and five?	Maintains	Pushes/ Maths
32	Khumo	Three and five are numbers only. But you are talking about variables, terms of variables.		
33	Facilitator	So you can't add *a* and *b*?	Maintains	Pushes/ Maths
34	Khumo	No, you cannot add *a* and *b*, because they are different variables.		
35	Facilitator	Why not? What if *a* is the same as *b*?	Maintains	Pushes/ Maths
36	Khumo	*a* is the same as *b*? They are not the same.		
37	Facilitator	No, I'm telling you, what if *a* is equal to *b*? Can't we add *a* and *b*?	Maintains	Pushes/ Maths
38	Dimpho	If ever *a* is equals to *b*? You can because…you can. Because *a* will be the same as *b*. It will be the same thing.		
39	Facilitator	You have to explain. Ma'am doesn't agree. We have *a* plus *b*, can we add it, if *a* is equal to *b*	Maintains	Seeks clarity/ Maths
40	Dimpho	If ever *a* is equals to *b*, a solution will be two *a* or two *b*. One of them. But it must be in terms of one variable. Because they have equated the other one to the other.		
41	Facilitator	It still doesn't make sense to you, ne?	Maintains	Seeks clarity/ Maths
42	Dimpho	So if *a* is equals to *b*, then the solution for *a* plus *b* will be two *a* or will be two *b*. It depends which one do you substitute.		
43	Facilitator	So the *a* plus *b* now also becomes *a* plus *a*? Because *b* is the same as *a*. Or it can become *b* plus *b* because *a* is the same as *b*. Then you get seven *a* or seven *b*....Remember *a* and *b* in that case are variables. So if you put numbers here, learners must know that variables stand for numbers. So if you put numbers there, then you'll be able to add two *a* plus five *b*, and come with an answer. But in that case, if you leave it as *a* and *b*, then leave it as two *a* plus five *b*. But we must not close shop and say, two *a* plus five *b* cannot be added, because those are variables. When you say that to learners you are telling the learner that you cannot add numbers….	Maintains	Draws together/ Maths

Extract 6.1: Seeing 2*a* + 5*b* as a possible solution.

The episode featured the three main process categories: manages, maintains and input. Of the 18 process moves, 4 were managing the conversation, 13 were maintaining the conversations and 1 was my input. Maintaining the conversation helps in opening up the conversation to the teachers, which is key in PLC facilitation. Of the 14 content codes, 4 were learner and 10 were mathematics, a result which reflects the prominence of mathematics in the quantitative analysis above. The episode shows that each facilitator turn often follows every teacher's turn, less at the beginning of the episode and more towards the end of the episode. Following every teacher's turn does not open up the conversation in the community and can limit the teachers' contributions and productive discussions in the community. However, the opposite is observed in this conversation as the analysis shows below.

Push/Challenge

Pushes showed more prominence in the conversation than the other moves, taking ten out of the eighteen facilitator moves. These pushes occurred at the beginning of the conversation as well as towards the end. In pushing the teachers to think more deeply about learner thinking, I referred the teachers to the learner's answer and asked them if the learner's answer of seven apples-bananas meant that there were seven apples and seven bananas. The next six facilitator moves were all pushes which may look like I was agreeing with the learner's answer, but that was not the case. Instead, I was pushing the teachers to engage with the learner's answer, though it was incorrect, in order to understand the reasoning behind the learner's answer.

I used this move to encourage the teachers to think deeper about their instructional practices, which included learning to challenge ideas raised. The teachers' contributions show that they engaged with one another and even challenged one another, with Mapula and Funeka challenging Dimpho's idea in turns 6 & 8, and Mapula and Funeka building solidarity with each other in turns 12 & 13 (Brodie & Shalem, 2011). My challenge started at the beginning of the episode, and was thereafter taken up by Funeka and Mapula in trying to show how they understood the learner's answer instead of challenging it as Dimpho and Khumo did.

There were times when I did not necessarily follow after every teacher's turn, where I took a (small) step back and allowed the teachers' conversation to continue, a move which is important for maintaining the conversation in the community. I allowed the teachers to engage with each other, especially after realising that the two pairs of teachers looked at the learner's

answer differently: Funeka and Mapula tried to make sense of the learner's answer and Dimpho and Khumo challenged the learner's answer. At no stage did I show or indicate that I sided with one pair. Instead, I allowed this engagement to continue with the aim of seeing all the teachers linking the learner's answer to learner thinking. In the conversation, I made short inputs to avoid dominating the discussion, except in the end where I drew together ideas from the teachers' inputs.

Facilitator explanations

There were no explicit explanations from the facilitator in this episode. The lengthy utterance which occurred was when I drew together the teachers' ideas at the end of the episode, emphasised the ideas which were made in the conversation, especially on the issue of adding different letters in algebra. Khumo insisted that it was not possible to add when different letters were used: "No, you cannot add *a* and *b*, because they are different variables." This happened even after I said: "What if *a* is the same as *b*?" Khumo's focus was on the different letters used in the problem, which showed that she treated letters as objects instead of generalised numbers (Hart, 1981). Looking at my explanation at the end shows that there was a missed opportunity to enhance the teachers' PCK on working with letters in algebra, which is elaborated below.

Learner, mathematics and practice in the conversation

My input on the learner's answer, followed by Funeka's and Mapula's views on the learner's error, directed the discussion to 'learner'. This happened at the beginning of the episode when I focused the teachers on how the learner had expressed his answer (turn 3). Understanding the learner's answer of $2a + 5b = 7ab$ was important for the teachers' PCK in responding to learners' answers in class – learners tend to conjoin the terms when different letters are used in algebraic expressions. Askew & William (1995) argue that "the presence of the operator symbol "+" makes the 'answer' appear incomplete" (p. 14), which explains why learners conjoin terms such as $2a + 5b$. The teachers' interpretations of the learner's answer differed because Dimpho and Khumo interpreted it as seven apples and seven bananas and Mapula and Funeka thought that the learner's reasoning made sense to them. This prompted me to focus them on how the learner represented his answer with a hyphen to show that all in all there were seven things combined: apples and bananas, which may have been the reason why Mapula stated that: it made sense to her (turn 17). My move was meant to clarify the confusion in the community regarding interpretation of the learner's answer, and I tried to redirect the teachers

to the main goal of the activity – understanding the reasoning behind the learner's answer – instead of challenging the learner's answer. The conversation though did not go towards linking the learner's error to learner thinking.

Dimpho and Khumo challenged the learner's answer and this directed the conversation towards mathematics content, with Khumo insisting that different letters cannot be added: "No, you cannot add *a* and *b*, because they are different variables." This happened even when I said: "What if *a* is the same as *b*?" Dimpho took over the discussion to explain how it was possible to add letters which are assigned equal values. The discussion at this stage was on mathematics content and I allowed it to continue, despite not dealing with learner thinking. I further continued to encourage interaction between Dimpho and Khumo on mathematics content with statements such as: "You have to explain. Ma'am doesn't agree. We have *a* plus *b*, can we add it, if *a* is equal to *b*". My move here was meant to seek more clarity from Dimpho on why it is possible to add even when letters were different, which Dimpho explained but focusing only on letters being assigned equal values, which I had encouraged. The issue of adding when letters do not have equal values was not dealt with in the discussion, except at the end of the episode when I drew together ideas made in the discussion. This issue is discussed below as a missed opportunity, and had potential to enhance the teachers' PCK in working with answers consisting of more than one term.

Challenges from the facilitator in encouraging dialogue in the community are important for the development of the teachers' PCK in handling learners' contributions in class. My direct question to Khumo on letters being equal was meant to get Khumo to answer her own question. However, Dimpho took it over and I allowed Dimpho to provide the response. This was another missed opportunity to probe Khumo about her insistence of seeing *a* and *b* as being different and it points to tensions between developing the conversation in the group and developing the mathematics.

Missed opportunities

In this episode I acknowledged inputs from the teachers. First, I acknowledged Mapula's and Funeka's inputs when they both showed that they understood the learner's reasoning in saying 7*ab* represented seven apples-bananas when *a* stood for apples and *b* for bananas (turns 14 and 18). Learner thinking was important for the project in striving to encourage the teachers'

interpretation of the learners' answers, both correct and incorrect. Second, I acknowledged Khumo's insistence that different letters cannot be added in algebra, and thereafter acknowledged Dimpho's explanation in showing how that was possible when the letters were assigned equal values (turns 38, 40, and 42). I encouraged Dimpho to explain to Khumo to support the teachers to learn from and with each other (Katz, et al., 2009), but in doing so I missed the opportunity of probing Khumo on her thinking that different letters cannot be added. What I could have done was ask Dimpho to give Khumo time to think carefully about the question and share her thoughts in the community. Depending on Khumo's response, I could then have allowed Dimpho to share her response.

When I drew together ideas made in the discussion, an opportunity arose for me to deal with the notion of seeing the expression $2a + 5b$ as a possible answer for addition of $2a$ and $5b$. Though I tried to talk about it in the summary, I was not explicit when I said: "So if you put numbers there, then you'll be able to add two *a* plus five *b*, and come with an answer. But in that case, if you leave it as *a* and *b*, then leave it as two *a* plus five *b*". This may sound like supporting Khumo's argument that: "you cannot add *a* and *b*, because they are different variables", but that was not what I meant. Allowing the teachers more time for the discussion may have clarified the issue.

The issue of not conjoining $2a + 5b$ needed more engagement in the community and I should have opened it up so that the teachers could understand that variables are generalised numbers and can be added, regardless of the symbols used in the algebraic expression. Not seeing variables as numbers might mean that they are treated as objects, which may lead to difficulties in making sense of algebra (MacGregor & Stacey, 1997). In addition, it was important for me to have dealt with what could have prompted the learner to conjoin the terms $2a$ and $5b$. This is an important error to deal with in algebra and can contribute to the teachers' PCK in working with learners' misconceptions. It was important to establish whether my summary was well understood by the teachers so that knowledge creation in the community was enhanced. Giving teachers more time to think, a move important for maintaining the conversation, can help in ensuring that what needs to be covered in the conversation receives the necessary attention.

The contention in the community started with the learner's analogy of apples and bananas and not elaborating whether it was appropriate or not to encourage metaphors in algebra. It could

have helped the discussion in the community if I created space to go back to the issue and establish the teachers' views on the appropriateness or inappropriateness of using metaphors in algebra. A discussion on this could have dealt with the misconceptions that metaphors may bring in learning algebra.

Episode 2: Facilitator dominance

(*Lesson Planning, July 2012)*

In this episode, the PLC was discussing one of the tasks that they had developed for their joint lesson plan. Each teacher was assigned to get mathematical tasks that they thought would be good to include in the lesson plan. My role was to facilitate discussion, to help the teachers see the strengths and weaknesses of the tasks. This episode is from the same task analysed in Chapter 5 but a different extract is used in this chapter in order to illuminate the dominance of the facilitator.

> The perimeter of a rectangle is 48 cm. If the length of the rectangle is three times the width, determine dimensions of the rectangle.

The community looked at how the teachers could work with the task in class to promote learner understanding. The individual teachers had worked on the task in the PLC and the episode follows after the community had discussed the teachers' solutions. In this episode, I focus on how I worked with the teachers in supporting them to work with the task in class. As in the previous episode, my analysis shows how I focused on particular codes of process and content and the extent to which facilitation contributed to the development of the teachers' knowledge.

	Speaker	**Utterance**	**Code**	**Sub-code**
1	Facilitator	Okay. Any other input?	Manages	Asking for comment
2	Mandla	So, the question actually, if I can put is a variable, you can start from the prior knowledge of saying can you write *t* in terms of *x*, if there is information about *x* and *t*. So, there are questions which are like that, whereby you say if so (*inaudible*) you can never express this in terms of that.		
3	Facilitator	Okay. So, when you deal with the question in class, I agree with you, but I wouldn't start it here. I would start it from a very basic form. And I would insist that when the learners do it they must actually write it down to say, so that they	Input	Explains/ Practice

		can show that they are dealing with a rectangle. In this case when you look at it this way, it is implicit, but it is not coming out very clear to anyone that here we are dealing with a rectangle. So, they should actually write it down to say that the perimeter of a rectangle is equal to two times the length plus two times the breadth. They should have that statement for conceptual understanding. When they deal with area then they should write the area of a rectangle is equals to length times breadth, so that they can be able to say the difference between that and that. That when you work with perimeter you will get to two *l* plus two *b*. When you work with area you work with *l* times *b*. So that statement, it should always be there when we deal with area or with perimeter, so that it can be internalised.		
4	Dimpho	You will start from there, you won't start from (*interrupted*).		
5	Facilitator	No, you will first assume the variables, that the length be that, that the breadth be that. Then from there when you start to the actual (*inaudible*) now where you do the calculations, then you actually start it from that statement that gives you the formula of calculating the perimeter of a rectangle. The perimeter of a rectangle is two *l* plus two *b*. And they will also need to explain what they understand by perimeter, even before, they don't have to write the explanation yet, but just to talk about it. What do you know about perimeter, so that they know that they add all the sides?	Input	Explain/ Maths
6	Mandla	Ja [Yes], this is what I was about to say, that before you come to this formula it comes as the explanation or the meaning of a perimeter, is to add all the sides. So, for instance, you can say it is this length, actually you are going to draw the rectangle and say this length plus the breadth plus this length plus the breadth. Then from there you can go to that equation so that they can see where does it come from. Because even if they forget about the equation, but the concept of perimeter is to add all the sides.		
7	Facilitator	They can have mistakes in the calculation because of some silly things. But at least they work from the correct thing. And the background, thank you for the background. You insist that they draw a diagram. And when they draw the diagram they will have that rectangle there then they will have the length, they will have the breadth and then in line with the assumed variables there, because you said let the breadth be *b*, so we will have *b* then there. Then under the length on where they have length they should write it, is equals to three *b*. There in that diagram. So that when they come to substitution they see exactly where this three B	Input	Explains/ Practice

		comes from. You see? So that everything does not just happen in the end and they can actually see oh it comes from here. Or it comes this way. So, it is very clear up to there?		
8	Dimpho	Yes.		
9	Facilitator	Ma'am Molemela?	Manages	Allocates
10	Mandla	Yes.		
11	Facilitator	Are you okay now, okay. So, from here then we know that they will be substituting according to what Mandla gave us before. Perimeter is forty-eight then two times three *b* plus two times *b* then you get six *b* plus two *b*, which is forty-eight. And when you come to that eight *b* that Dimpho gave us and we divide both sides by eight, then your *b* becomes six. But you know what learners will do, *b* equals and they will stop. Just another thing, so that is one other thing that you talk about tomorrow. Because usually learners what they do when they solve these types of problems, after they get this answer they get so excited to say I am done. And they don't go even further and answer the question.	Input	Explains/ Learner
12	Mandla	And the term, that term the dimensions.		
13	Facilitator	The dimensions, I was coming to that to say, so you tell them in class that after you think you have finished the problem go back to the question and read the question again. And see if you have answered the question. What was the question saying? The question was saying determine the dimensions.	Maintains	Re-voices/ Practice
14	Dimpho	The dimensions.		
15	Facilitator	So, what are the dimensions, did they answer the question.	Manages	Repeats
16	Mandla	That is the biggest problem with our learners seriously, that one.		
17	Facilitator	After having a solution, you should always go back to your question. To say what was the question, is this the solution? What was the question if there is a solution? So that is what you should train them to do. Not only in word problems, in any other topic that you deal with in your class, after they finished the problem they should go back to the question. What was the question asking me? Have I answered the question? So, they will do likewise with this one. So now that we have the breadth which is six and the question was saying the length is three times the breadth, then it means our length then will be three times the breadth, three times six. And the length will give us eighteen. So usually with word problems, what I would encourage is after they have written all the values of the variables like *b* is six and is add is eighteen, then the concluding statement right at the bottom, they should answer that question, find the	Input	Explains/ Practice

		dimensions. Then they can actually have to say okay the dimensions, then they write length is equals to how much, here it is here, eighteen. What were the units?		
18	Dimpho	Centimetres.		
19	Facilitator	Centimetres. Then the breadth or the width, how much is it, it is six centimetres. They should write it so that there is logic in what they have done. Alright let's talk about anticipated difficulties. Okay the first one from Dimpho, using area.	Maintains	Re-voices/ Learner
20	Dimpho	Area formula instead of perimeter. Another difficult one.		

Extract 6.2: Facilitator dominance.

The episode featured the three main process categories: manages, maintains and input, with input contributing the most moves. Of the 10 process moves, 3 were managing the conversation, 2 were maintaining the conversations and 5 were my input. All the input codes were my explanations, which dominated the discussion in the community. Of the 7 content codes, 2 were learner, 1 was mathematics, and 4 were practice. In the episode the facilitator utterances follow after every teacher's utterance and reveal my dominance in the discussion.

Push/Challenge

There is no push or challenge in the conversation. I do not challenge the teachers to think deeper about the task, which does not promote rigorous discussions in the community. This is different from the previous episode in which push and challenge moves were prominent, despite the facilitator turn following after every teacher's turn.

Facilitator explanations

My explanations in the episode dominated the discussions in the PLC, and they involved taking over the discussion at times when I thought that the teachers were missing key points in the discussions. For example, in turn 3, I reinforced Mandla's suggested approach to the problem to try and make it relevant for instruction in class. Mandla's explanation assumed that learners already knew how to manipulate letters in algebra, which made his explanation somehow abstract for the level of the learners. I then suggested an approach that could build up the

solution in ways in which it could be used in class. In doing so, I contributed to the teachers PCK in working with learner strategies of doing word problems.

A similar occurrence related to explanations is in turn 17, where I explain the importance of teaching learners to evaluate their own solutions. This happened after Dimpho and Mandla agreed that learners often do not answer such questions fully and did not elaborate further. I then elaborated on how teachers can assist the learners in giving a full response to the question.

Learner, mathematics and practice in the conversation

I first referred to learners when we discussed how the problem should be done, and I alerted the teachers to how learners often do such problems, that they usually calculate the length of one side of the rectangle and not the other (turn 11). In cautioning the teachers, I was hoping to assist them in tracking what the learners do as they work on such a problem. The second reference to learners is in turn 19, where I emphasised the importance of the unit of measurement. This also links to the first point as they both deal with alerting the teachers on the learner strategies. Mathematics featured in the episode when I explained how the problem should be done (turn 5). This occurred when Dimpho suggested an approach to do the problem, and I took over the discussion to suggest how I thought the problem could be approached.

There were four occurrences on practice and they all dealt with supporting the teachers in how they could work with the problem in class. The first two occurrences focused on the approach I suggested for the teachers and the last two focused on how they could help the learners to evaluate their responses. All these occurrences had potential to contribute to the teachers' PCK in working with word problems in class. Each of the occurrences reflects the prominence of my explanations in the discussion.

Missed opportunities

The episode shows my dominance in the discussion and not creating space for the teachers to share their ideas – the discussion was dominated by two teachers, Dimpho and Mandla. There were opportunities to probe the teachers' inputs as in turn 2 when Mandla suggested how the problem could be done. Instead of outrightly coming with a different approach, I could have asked Mandla to elaborate why he thought his suggestion would improve learner understanding. Alternatively, I could have asked the other teachers to comment on Mandla's

idea. In doing so, I could have created an opportunity for the creation of new knowledge in the community instead of suggesting for the teachers how they could work with the problem.

Another missed opportunity occurred towards the end of the episode when I took over the discussion to explain how the teachers could work with the learners to track their own (learners) responses to the question (turns 17 and 19). First, I explained how the teachers could work with the learners to help them answer the question in full. The teachers raised this concern and this was an opportunity to ask them how they thought they could help the learners with that challenge. Second, I reinforced the importance of the measurement unit to assist the teachers in how they could work with the learners to raise the significance of working with units of measurements. Allowing the teachers to talk about how they could work with the learners may have created additional opportunities for learning in the community.

Episode 3: Conjoining terms

(Readings & Discussion, June 2012)

This episode analyses the extent to which my inputs encouraged the participating teachers' ideas on the generalisability of mathematical arguments and how they could work with such arguments in class. The PLC was discussing a paper that they had read to prepare for a joint lesson plan on the language of algebra. The PLC conversation was on an issue that the paper had raised about common errors that learners often produce when letters are involved, an issue which is similar to the one which occurred in the first episode. The problem given in the paper was:

> Dineo is 10cm taller than Khotso. Khotso is *h* cm tall. What can you write for Dineo's height?

One of the errors that the paper gave from a learner's response was $h10$. The PLC was discussing how the learner could have got to this answer so that they could devise strategies for dealing with such errors if they came up in their classes. The expected correct answer to the question was: $h + 10$ or $10 + h$.

	Speaker	Utterance	Code	Sub-code
1	Dimpho	The use of conjoining. Say, *h* ten. Meaning that they think that it is h plus ten.		
2	Facilitator	h plus ten, then they write *h*10.	Manages	Repeats
3	Dimpho	*h*10.		
4	Facilitator	But why should they do that? Why should they say h plus ten is *h*10	Maintains	Pushes/ Learner Thinking
5	Dimpho	They put them together.		
6	Facilitator	Why should they do that?	Maintains	Pushes/ Learner Thinking
7	Funeka	Because of the plus sign. Always they should add. They see the additional sign in between, they should add. Because if it was the…if there was a number there on *h*, they were going to add the number plus ten. So now because it's *h*…		
8	Facilitator	Okay, let's say that number…let's say that number is three, we have three plus ten. So, should you write three ten?	Maintains	Pushes/ Maths
9	Funeka	No, they were going to add them to get *h*10.		
10	Facilitator	Because they wrote *h*10 here.	Manages	Asking for comment
11	Funeka	Ja. And then there is *h*. They will just say *h*10.		
12	Facilitator	You know the term conjoining, co? Togetherness.	Input	Explains/ Maths
13	Teachers	Togetherness.		
14	Facilitator	Something ma'am Maisela said	Maintains	Makes links/Maths
15	Dimpho	You put them together		
16	Facilitator	So you put them together. So three plus ten, can you put them together?	Maintains	Pushes/ Maths
17	Dimpho	Three ten.		
18	Facilitator	Then you get three ten.	Manages	Repeats
19	Mapula	Mm.		
20	Facilitator	Mm? You can have three ten? Three plus ten?	Maintains	Pushes/ Maths
21	Funeka	No.		
22	Facilitator	You are not listening ma'm Mosala. (*laughter*) I'm saying three plus ten is three ten, you're saying, mm, you see, conjoining. Conjoining means you take a three and a ten and you put them together.	Input	Explains/ Maths
23	Funeka	But that one, they won't say three ten, they will just add them.		

24	Facilitator	And get thirteen.	Manages	Repeats
25	Funeka	Ee [yes], and get thirteen.		
26	Facilitator	Anybody else? Why should they write *h* ten? You missed it, it's in the paper, on the first page.	Manages	Asking for comments
27	Mapula	Because they will see this…		
28	Facilitator	The very first paragraph, the last three lines. Of the first paragraph, on the first page. Read those last three lines and you'll see why do they have to put them together.	Input	Clarifies/ Learner Thinking
29	Funeka	A letter next to the numeral indicates addition.		
30	Dimpho	The teacher says that five *x* means…		
31	Facilitator	Ma'am, on the very first page.	Input	Clarifies/ Maths
32	Funeka	Oh the first page, oh.		
33	Dimpho	The teacher…first page. Page number one.		
34	Funeka	Oh.		
35	Dimpho	The teacher says that five *x* means five multiplied by *x*, where a student might think that it signifies addition like five and a half. Or place value like fifty-three.		
36	Facilitator	Do you remember?	Manages	Asking for comment
37	Teachers	Mm.		
38	Facilitator	Five plus half, what do you get?	Manages	Asking for comment
39	Khumo	Five and a half.		
40	Facilitator	Huh?		
41	Teachers	Five and a half.		
42	Facilitator	You get five and a half, five plus half, which gives us five and a half. You take the five and a half, then (*claps hands once*) you put them together. And you can still see them. There is a five there and there is the one. Remember it was five plus half. So you get the answer. Five and a half. This five is there, then the half is still there. And that's the reason why when they have *h* plus ten they write *h* ten. Or they write ten *h*. The *h* is still there, the ten is still there. You simply put them together because that is what they did here. I mean, five plus half is five and a half, is correct. That's something that they learn, it is correct. Put them together. Now they encounter *h* plus ten. They still put it together because of the previous learning. You see? So that is how we link these mistakes that learners make to particular concepts in	Maintains	Draws together/ Learner Thinking/ Practice

		mathematics. Any other example? Let's see the last example. We're limiting the time now.		

Extract 6.3: Conjoining of terms

This episode featured all the process codes of manages, maintains and giving input, where out of the 18 process codes, 7 were managing the conversation, 7 were maintaining the conversation and 4 were my input. Of the 12 content codes (this includes a double code in turn 42 for content), 7 were mathematics, 1 was practice and 4 were priorities of the DIPIP project which all involved learner thinking. The occurrence of mathematics supports the prominence of mathematics in the coding results in Table 6.2 above. In maintaining the conversation at the beginning of the episode, I used mostly pushes. As in the previous episodes, the facilitator turn follows almost after every teacher's turn.

Push/challenge

In discussing the learner's error, I pushed the teachers for learner thinking to explain why the learner got *h*10 as an answer for $h + 10$. Dimpho responded by saying learners "put them together", meaning that learners conjoin *h* and 10, actually repeating what she stated earlier when she mentioned conjoining in turn 1 and showing that she knew what the term meant. Conjoining letters was also discussed in the first episode on metaphors, which evidences the prevalence of such errors in mathematics. Funeka supported Dimpho by saying that the addition sign prompted the learner to put *h* and 10 together: "They see the additional sign in between, they should add. Because if it was the…if there was a number there on *h*, they were going to add the number plus ten". Pushing the community for learner thinking prompted Funeka to elaborate on the reasoning behind the learner's error, which helped to articulate how learners work with letters in algebra.

My follow-up on asking what would happen if there was 3 and 10, would learners still put the numbers together to get 310, emphasised the differences between algebra and arithmetic. This move had potential to contribute to the teachers' mathematical language. Both Dimpho and Funeka disagreed with an answer of 310, and had I taken it further it may have contributed to the teachers understanding why learners struggle with conjoining. I referred the teachers to the literature paper, which I hoped would help.

Facilitator explanations

There were two occurrences where I gave explanations, and they both dealt with conjoining. First, I explained what the term conjoining meant (turn 12). Second, I introduced a counter example to generate further discussions on the notion of conjoining. In addition, I made a lengthy discussion at the end of the episode to draw together the community ideas by using the examples from the literature paper: $5 \times x = 5x$ and $5 + \frac{1}{2} = 5\frac{1}{2}$. These examples were used to explain what could have contributed to learners putting together *h* and 10 to get *h*10. This explanation created opportunities for the development of the teachers' PCK in working with misconceptions in algebra.

Learner, mathematics and practice in the conversation

Mathematics featured in the conversation after I pushed the teachers to explain why the learner conjoined *h* and 10. Though Funeka's reason that learners conjoin: "because of the plus sign" related to learner thinking, but I did not consider it that substantive, which prompted me to bring a counter-example of conjoining 3 and 10. I then realised that the example was mathematically inappropriate, and I referred the teachers to the literature paper so that they could get a better example that could help them to understand the term conjoining. I then used the two examples from the paper to draw the community ideas together, which was important for the teachers' instructional practice. The use of good examples in mathematics is important in promoting learner understanding.

Missed opportunities

There were no serious missed opportunities in the episode, except for one case where I referred the teachers to the paper and yet did not allow them to say what they understood from the text. Instead, I elaborated on the examples myself to explain the reason behind the learners conjoining the terms. In addition, the episode shows me engaging mostly two teachers in the community, Dimpho and Funeka, with the other two only making brief contributions in the conversation. This is an occurrence similar to the one found in the previous episode where the dialogue was with Dimpho and Mandla.

Episode 4: Taking a small step back

(Lesson Reflection, March 2012)

This is the same episode that was analysed in Chapter 5, which focuses on a learner's error that Dimpho brought to the PLC for a group reflection. The learner had stated that $2^3 = 2 \times 2 \times 2 = 8$ because the expressions has numbers, whereas $a^b = ab$ because b is not a number and you don't know how many times to multiply. The learner's reasoning suggested that she did not see variables as representing numbers (Hart, 1981). This conversation is divided into two extracts for purposes of analysis, as in Chapter 5.

1	Dimpho	I don't think they know what it means. They take it as variable and they don't actually relate variables to numbers until such time that they can understand what it means maybe if ever we come up with those examples, two to the power of one, two to the power of two, two to the power of three, you know. So maybe they can actually now generalise. We can try giving these examples so that we can maybe give the definition of what *a* to the exponent *b* is. That if you're having two to the exponent three, we multiply two three times. Two to the exponent five, you multiply two how many times? Five times. So they can see, oh, which means if ever this to the exponent, meaning that you multiply *a b* times.		
2	Mandla	Multiplying *a b* times, is it not the same as like *ab*?		
3	Facilitator	I'm listening.	Maintains	Push/Maths
4	Dimpho	Not…		
5	Facilitator	I'm listening. (*laughs*)	Maintains	Push/Maths
6	Mandla	Multiplying *a* to the same…if I say it in words, multiply *a b* times, is it not the same like *a* times *b*? Which is *ab*.		
7	Dimpho	If ever I say, multiply two three times.		
8	Mandla	Two three times?		
9	Dimpho	Ja [Yes**]**.		
10	Mandla	Okay, then…okay, I'm saying, it's not the same as like two times three?		
11	Dimpho	Two three times, no, it's not two times three		
12	Facilitator	It's open to everyone	Maintains	Push/Maths
13	Dimpho	If ever you multiply two three times, it's not the same as two times three.		
14	Mandla	I'm just asking.		
15	Khumo	Two three times, is it not six?		
16	Cikizwa	It is.		
17	Dimpho	Two three times…		

18	Facilitator	That is what Mandla is asking, two three times.	Input	Clarifies/ Maths
19	Cikizwa	Two three times		
20	Dimpho	You multiply two three times. It will be two times two times two. You multiply it three times.		
21	Mapula	Eight.		
22	Dimpho	Multiply two four times. So it will be two times two times two times two. How many times? Four times.		
23	Facilitator	Ma'am Molemela understands it to be two times three. Multiply two three times.	Maintains	Push/Maths
24	Khumo	And what are you going to say when you want to multiply two by three, if you are giving that, a sum, two multiplied by three and two to the exponent three? How are you going to explain that?		
25	Dimpho	Explain what?		
26	Facilitator	Two times three.	Manages	Repeats
27	Khumo	Two times three and two to the second exponent		
28	Dimpho	This one you multiply two by three.		
29	Khumo	Which one? Two…		
30	Dimpho	Two by three. Akere [isn't it**]** two times three.		
31	Khumo	Yes.		
32	Dimpho	Yes, you multiply two by three.		
33	Khumo	Two by three. And then two to the exponent (*interrupted*)		
34	Dimpho	You multiply two three times. The first time, the second time, and the third time.		
35	Khumo	Okay (*softly*).		
36	Facilitator	You're convinced?	Maintains	Push/Maths
37	Khumo	Yes, yes (*laughing*).		
38	Facilitator	Why are you convinced (*laughs*)	Maintains	Push/ Probing
39	Khumo	She has explained that.		
40	Facilitator	Hah….		
41	Khumo	She has explained that to me.		
42	Facilitator	Okay. It makes sense to you?	Maintains	Push/Maths
43	Khumo	Yes.		
44	Facilitator	Okay. Mandla, you are the one who put the cat inside the…	Manages	Allocates
45	Mandla	I'm trying to think		
46	Facilitator	In the what, bag of pigeons (*laughing*).	Manages	Asking for comment
47	Mandla	I'm trying to think. It's like, it's like, if you say, two to the power of, two exponent three, when we teach we say we multiply two three times. This is what you say to our learners.		
48	Facilitator	Underline that statement, to our learners.	Maintains	Re-voices/ Practice

49	Khumo	To our learners.		
50	Facilitator	Okay, that is what I'm reading from what he is saying (*laughs*).	Maintains	Makes links/ Learner
51	Khumo	You're saying, two to…		
52	Mandla	Two to the exponent three.		
53	Khumo	Yes, you say…		
54	Mandla	Means two to the exponent three, and what we say now to learners, learners look on the board, this means what? Two, multiply how many times? Three times.		

Extract 6.4a: **Taking a small step back.**

I explained in Chapter 5 that in this episode the teachers argued repetitively about the similarity or difference between the statements: "two multiplied three times" and "two times three, without coming to any conclusions. This repetitive part is excluded here. The next extract is taken at a point where the conversation was beginning to take shape.

55	Mandla	You know, if you say, multiply by three, times, that's where the problem is. Two multiplied by three is a straightforward answer. Two multiplied by three…		
56	Mapula	Times.		
57	Dimpho	No, you cannot say that. You cannot say two multiplied three times.		
58	Facilitator	You're saying two…	Maintains	Seeks clarity/ Maths
59	Mandla	Multiply.		
60	Facilitator	Multiply…	Manages	Repeats
61	Dimpho	Three times. I said it.		
62	Mandla	Multiplied three times.		
63	Facilitator	Two multiplied three times.	Manages	Repeats
64	Mandla	Ja, the two is multiplied three times.		
65	Teachers	Three times.		
66	Facilitator	Okay. You put the word itself.	Manages	Asking for comment
67	Cikizwa	If you multiply itself three times.		
68	Facilitator	You simply add one word to what you're saying.	Maintains	Seeks clarity/ Maths
69	Cikizwa	Ja.		
70	Facilitator	It's the same as what you're saying.	Maintains	Seeks clarity/ Maths
71	Funeka	Ja.		
72	Facilitator	So why are you putting the word itself in this case?	Maintains	Push/ Maths

73	Funeka	No, it's just because there were some ideas, saying maybe…		
74	Facilitator	Clarifying something.	Maintains	Seeks clarity/ Maths
75	Funeka	Ja, because he was saying two it's like, it's the same as two multiplied by two. And then I was trying to make it simple to say, it's multiplied itself three times, that two. So to clarify that statement, it's the same.		
76	Facilitator	You were saying ma'am?	Manages	Allocates
77	Khumo	About what Ms Mosala said, it also helped the learners because always when they say two to the exponent three, they always say two multiplied by three. So when you include the word itself, now they can get a difference between two multiplied by three and two multiply itself three times, they know they are going to repeat the base three times. Multiplying the base three times.		
78	Facilitator	Multiplying the base.	Manages	Repeats
79	Khumo	Yes. It's a base two angithi [isn't it]		
80	Facilitator	You are multiplying the base	Maintains	Seeks clarity/ Maths
81	Khumo	Two, it's a base, and you start from the two, it's a base. Three it's an exponent. And from there you expand, so that you can see.		
82	Facilitator	And how do you expand when you do? Can you repeat that?	Maintains	Seeks clarity/ Maths
83	Khumo	You say…I agree with Mrs Mosala, by saying that two multiply itself three times, I'm emphasising that what are you multiplying? You are multiplying the base three times. Itself. Itself.		
84	Mandla	Oh (*laughing*), because you're now agreeing with Dimpho (*laughing*).		
85	Dimpho	Ja, because (*inaudible*).		
86	Mandla	Ja, it's like my eyes were wide open, now		
87	Facilitator	What is it, she's agreeing with Dimpho. And disagreeing with who?	Maintains	Makes links/ Maths
88	Khumo	Ma'am Mosala.		
89	Mandla	No, she tried to explain what Ma'am Mosala, but at the same time, during her explanation, she was agreeing with Dimpho. The statement of Dimpho, which is interesting.		

Extract 6.4b: Taking a small step back.

The noticeable difference that appears in this episode compared to the other episodes is when I took a small step back to allow the teachers to engage with the activity. Of the 26 facilitator codes in this episode, 8 were managing the conversation, 17 were maintaining the conversation, and 1 was my input. Maintaining the conversation came out much stronger in this episode than

the previous three episodes. Maintaining the conversation involved mostly pushes and seeking clarity. Of the 18 content codes, 1 was learner, 15 were mathematics, 1 was practice, and 1 was DIPIP priority.

Push/challenge

When Dimpho started a conversation on the learner's answer of $a^b = ab$, Mandla challenged her on the statement that she made: "that if you're having two to the exponent three, we multiply two three times", and that a^b means "you multiply *a b* times". Mandla challenged the statement by asking if it does not sound like *ab*, and when he said that, he looked at me to try to get my opinion and my response was: "I'm listening". I later repeated a similar statement twice to affirm that I was taking a step back and allowing the teachers to challenge one another. My response was meant to encourage the argument between the two teachers, which was subsequently joined by the other teachers.

When Khumo later raised a similar argument as Mandla's, that multiplying two three times may sound the same as two times three, Dimpho tried to explain how they were different and I pushed Khumo to continue with her argument by asking her: " Are you convinced", and I wanted her to also say why she was convinced. Similar pushes which encouraged argument among the teachers continued, with a slightly different push appearing later when Funeka inserted the word "itself" to modify Dimpho's statement to "two multiplied itself three times", with Cikizwa supporting her. I asked Funeka why she was putting that word, trying to elicit further elaboration from her. This push was different because it focused on Funeka's correct statement which had potential to close the discussion. The last challenge in the conversation occurred when I asked Khumo if she was multiplying the base, a question meant to address the mathematical language. My question was meant to push Khumo, and perhaps the other teachers, to think deeper about Khumo's statement of multiplying the base. I did this in order to support the teachers in developing rich mathematical language when working with exponents in class.

Facilitator explanations

Though there were no explicit moves on explanations, clarifying Mandla's argument to Dimpho served that purpose (turn 18). This happened after Khumo asked if the statement "multiplying two three times" does not mean six and it appeared to me that Dimpho was

uncertain about Khumo's statement. Khumo's statement signals building solidarity with Mandla, and challenging Dimpho. My explanation of what the other person said was important for the teacher's PCK in working with learner responses by making them public. Putting an idea in the public domain can help other participants to make contributions and be heard, and this may generate further discussions.

Learner, mathematics and practice in the conversation

Focus on learner occurred after Mandla explained in the community that his challenge of Dimpho's statement was because he was thinking of learners. I then asked the community to note that statement, which was meant to remind the teachers that PLC conversations were about learners, how learners learn, the type of errors learners make and why, and how the teachers work with the learners' errors and misconceptions. This facilitator move was important for the development of the teachers' PCK in how learners learn. Though the conversation in the community was on getting the mathematical language correct for learners, most of what was discussed was about the mathematics and I continued to encourage the discussion of mathematics content in the community. Practice, like learner, featured briefly in the conversation, and this occurred when I re-voiced Mandla's statement on prioritising learner understanding in their conversation. This move was important in putting the learner at the centre of the community conversation in order to emphasise the importance of understanding how learners learn in mathematics.

Missed opportunities

There was one missed opportunity in the conversation, which related to going back to Dimpho's statement on why learners made errors such as $a^b = ab$. I allowed the conversation to divert to a discussion on getting the correct mathematical language and it produced a useful conversation among the teachers. The conversation on mathematical language may have been relevant for the teachers' PCK, but understanding the learner's error and misconception was also key. Creating space to go back to the learner's error would have helped in making links to learner thinking, which was one of the priorities in the DIPIP projects and could develop the teachers' PCK in working with learners' errors and misconceptions.

6.5 Conclusion

The focus of this chapter was on facilitation of the PLC in 2011 and 2012. The question that this chapter explored was:

- How is a space for engaging with the different activities in a PLC created by the facilitator?

The quantitative analysis gave an overview of how I facilitated the teachers' engagement with the different activities for process and content, across all the sessions. Episodes from the PLC meetings were used to extend the analysis of facilitation.

The quantitative analysis showed that I focused on specific process and content codes in the different PLC activities. During Error Analysis, I focused prominently on the process codes of "manages" and "input" (particularly explains), and on "learner", "mathematics" and to some extent also "practice" and "learner thinking" for content codes. Facilitator dominance in Error Analysis was observed in management of the conversation as well as in the explanations that I made. The analysis also showed that there was reasonable distribution of the process codes in the other three activities, except for "explains" which continued to receive prominence.

The qualitative analysis has shown that a high focus on explanations involved mostly the following:

- When I made a call that teachers were missing the key points in the discussions, and I took over the discussion to clarify those points. My analysis revealed my dominance in the community, which reflects a weakness in facilitation because it did not create space for maximum participation of the teachers.
- When I introduced the teachers to knowledge drawn from research or a context which was unfamiliar to them or when I clarified terms such as re-voicing and conjoining for the benefit of the teachers' mathematical language and practice. My explanations did not open up the conversation enough to hear the teachers' views and this inhibited the creation of new knowledge in the community.
- When emphasising strategies that the teachers could employ as they work with learners in class. The emphasis related to how they could help the learners to evaluate

their work, especially when learners do word problems. These explanations limited the teachers' contributions in the community. When the affordances for the teachers to make input were not created, the potential to develop their knowledge was inhibited.

Mathematics received a higher focus than learner thinking in all the activities for the two years. Practice featured prominently in Lesson Planning and Lesson Reflection and received a low focus in Error Analysis and Readings & Discussion. Although learner thinking was important in the project my analysis has shown that the focus of my facilitation moved away from that priority to other content.

My analysis showed that my moves in the community were dominated by pushes, which involved mostly mathematics and encouraging deeper thinking and constructive critique among the teachers. Pushes/challenges also involved pressing the teachers to put their ideas in the public domain, including their incorrect responses, which afforded the teachers the space to learn from each other's mistakes as they anticipated learner errors and worked with learner responses.

The qualitative analysis showed that though the facilitator turns in some of the episodes followed after almost every teacher's turn, I dealt with the situation differently in the episodes by engaging the teachers differently – I became accommodative of the teachers' inputs, and this contributed to creating space for the development of teacher knowledge in the community.

There were missed opportunities which occurred in the different episodes. First, I often rushed to conclude the discussion in the PLC, as exemplified in the discussion of $2a + 5b$, which resulted in not helping the teachers to see the non-conjoined $2a + 5b$ as a possible solution for addition. This missed opportunity suggests for the teachers' input to be invited after my summary, which could help end the conversation on the same level of understanding for all the teachers. Second, I did not allow sufficient input from all the teachers. This occurred in the second episode when I encouraged dialogue with only two teachers, and did not even consider or follow up on the inputs made by the other participants. This is seen as a weakness in facilitation, and it has potential to discourage the other teachers from participating in the PLC conversations, whether in the current or in future PLC meetings. Third, I allowed the conversation to divert and never came back to the initial conversation. This happened when the

community was meant to discuss the learner's error of $a^b = ab$ and the conversation diverted to a discussion on mathematical language, which I allowed because I saw it as contributing to the teachers' PCK on mathematical language. While these are issues in all facilitation, in this case they may have contributed to my over-reliance on explanation as a facilitation move. I state these three points to acknowledge the limitations in my facilitation, which impacted the functioning of the PLC. These limitations denied the teachers the space to engage constructively with the matter at hand, which I consider a hindrance to knowledge creation in the PLC.

My analysis in this chapter supports the idea that PLC facilitation is a process that takes time and needs careful facilitation skills, but given time, good facilitation of a PLC can bring good results. The results of my analysis have shown that I used mostly pushes in my facilitation of the PLC to create a space for the teachers' engagement with the different activities. In doing so, I created opportunities for the participating teachers to create new knowledge for themselves. My facilitation though also featured some missed opportunities, which if they could have been dealt with might have contributed positively to the teachers' participation in the community as well as to their knowledge base. The missed opportunities involved not keeping track of what still needed to be discussed or not probing the teachers further in order to get more ideas for the development of new knowledge in the community. There were occurrences where my explanations dominated the PLC discussions, and this limited the teachers' contributions in the community and inhibited the creation of new knowledge in the community. This means therefore that my facilitation somewhat impacted the functioning of the PLC. The next chapter focuses on the summary of my findings, my conclusions about this research study, and the implications associated with this research.

Chapter 7: Summary of Findings, Implications, and Recommendations

7.1 Introduction

This research study investigated how practices of a community of mathematics teachers were developed through a PLC, focusing on the teachers' instructional practices, their participation in the PLC, and the guidance of the facilitator. The study was guided by a situative perspective, which argues that learning is becoming a better participant in practice (Borko, 2004; Brodie, 2005). The results of my analysis showed the extent to which the participating teachers shifted their teaching practices over time, how they engaged with the activities to create new knowledge in the community and the extent to which my facilitation of the PLC supported the participating teachers in the community conversations.

Inquiry into the teachers' professional development was built around three constructs: instructional shifts, teacher conversations, and PLC facilitation. Teachers are seen as being central in a teaching and learning environment, and it is vital that their instructional practices be responsive to their environments. PLCs are different to short-term in-service trainings or workshops that teachers are often exposed to and involve collaborative work, long-term inquiry and reflective practice, for the benefit of learners (McLaughlin & Talbert, 2006).

Instructional practices and shifts

My analysis in Chapter 4 showed the extent to which the four participating teachers interacted with learners in lessons, showing how the teachers worked with the learners' errors and the misconceptions associated with those errors, and the extent to which their responsiveness developed over time. The analysis involved a description of the shifts that the teachers made in their practices, looking at their individually planned lessons (IPL) and their jointly planned lessons (JPL).

Hill's (2010) MQI instrument provides a comprehensive way of looking at the quality of mathematics teaching and helps to illuminate the shifts in teaching practices. In looking at the teachers' practices, my analytical lens was on how the participating teachers worked with the learners' errors in their teaching, how they acknowledged those errors and how they worked

with the reasons behind those errors. Brodie (2014) argues that a focus on learner errors can develop teaching practices, and this supports Horn's (2005) argument that professional development of teachers is situated within their practices. Katz, Earl, & Ben Jaafar (2009) and Brodie (2013) argue that in PLCs learners' learning needs inform teachers' learning needs, so teachers' knowledge of what the learners know in mathematics and where and when they can make interventions in the learners' learning, is important.

Teacher participation

In Chapter 5 I looked at the seven participating teachers' participation in the PLC meetings. Situated perspectives regard learning as participation in communities of practice (Borko & Koellner, 2008; Horn, 2005; Kazemi & Hubbard, 2008; Lave, 1993, 1996; Lave & Wenger, 1991; Wenger, 1998). This study investigated the consistency of patterns of participation by the teachers as they engaged with the project activities across two situations: the PLC and their classrooms. I used a tool developed by DIPIP to look at how the teachers engaged with the project activities in the community, what they talked about in the community and at what depth. The key focus of their conversations was "to better understand the nature of and the reasoning behind learners' mathematical errors and how they might respond to these" (Brodie, 2016). In the PLC, the teachers' participation patterns helped me to account for how teacher knowledge developed in the community.

My analysis of the teachers' conversations in the PLC showed the extent to which they engaged with the project activities to create new knowledge for their instructional practices. Their engagement with the activities involved analysis of learner errors, planning joint lessons, teaching the lessons, and reflecting on the taught lessons to inform their subsequent classroom teaching.

Facilitation

In Chapter 6 I investigated my facilitation of the PLC to show the extent to which I supported and guided the teachers in the creation of new knowledge. Research refers to good facilitators as "critical friends" (Katz, et al., 2009; Roberts & Pruitt, 2003), and this involves being both a friend and a critic within a PLC. It is important that PLC facilitation finds a balance between

creating an atmosphere of support and care in the PLC and challenging the teachers' ideas in community conversations. Brodie (2016) argues that becoming a good facilitator requires appropriate techniques and strategies to lead a community, and this supports the creation of a supporting and caring atmosphere in the community that can enhance teacher participation. Relevant episodes were used in the analysis to show how I, as the facilitator, supported and guided teacher participation. I used a tool developed by DIPIP, with various categories of facilitation to analyse my facilitation of the PLC. My support included guiding the teachers in embracing the learners' errors and misconceptions in mathematics teaching and learning, and for the teachers to view those learners' errors as being reasonable and reasoned (Ball & Bass, 2003; Brodie, 2014). In my analysis, I also acknowledged the missed opportunities in my facilitation and discussed how else I could have supported and guided the teachers.

7.2 Summary of findings

This study addressed two main research questions with sub-questions for each to investigate teacher development through the PLC:

1. To what extent do mathematics teachers' practices shift as they participate in a PLC?
2. How do mathematics teachers work as a professional learning community to learn with and from each other as they work on learners' errors and misconceptions?

The questions were guided by the following sub-questions:

1.1 How do teachers engage with the learners' errors in their classrooms?
1.2 How do teachers create conditions for enhancement of learning in their classrooms?

2.1 What do teachers in a PLC talk about and at what depth do they engage with different activities?
2.2 How is pedagogical content knowledge (PCK) amongst teachers created within the professional learning community?
2.3 How is a space for engaging with the different activities in a PLC created by the facilitator?

7.2.1 Teaching practices

1.1 How do teachers engage with the learners' errors in their classrooms?

1.2 How do teachers create conditions for enhancement of learning in their classrooms?

An important shift in the teachers' practices was in mode of instruction, from more teacher-talk to whole class discussions, where learners were afforded opportunities to participate in the lesson by explaining their answers, correct or incorrect. In doing so, the teachers created conditions for enhancement of learning in class as they afforded the learners the opportunities to explain and learn from their own (learners') contributions. As learners wrote their answers on the board the teachers engaged the learner on the board or the whole class in a discussion. This practice of engaging learners allowed the teachers to be responsive in identifying and working with the learners' errors. All the teachers were successful in working with learner responses in this way in their first set of jointly planned lessons, with differences emerging in their second set of jointly planned lessons. Two of the teachers, Funeka and Mapula, continued with this practice and the other two, Dimpho and Khumo, did not. In the last set of individually planned lessons, Funeka and Mapula managed to sustain their shifts while Dimpho and Khumo did not. Funeka and Mapula deepened their inquiry into the reasons behind their learners' errors, while Dimpho and Khumo did not. Although Khumo continued to be interested in the learners' errors, she led the learners to the correct answer through funnelling. Dimpho also continued to be interested in the learners' errors, though she engaged the learners in extensive procedural remediation.

Richness of the mathematics remained consistently low for all the participating teachers, a result which is similar to Chauraya & Brodie's (2017) finding in a different community in the DIPIP project. As in Chauraya & Brodie's study, this finding can be attributed to the challenges that the teachers encountered in working with the learners' ideas in their instructional practices, as they were more responsive in the moment, they struggled with some of the mathematics. The process of working with learners' ideas was also accompanied by an increase in the teachers' errors, and these errors decreased over time for some teachers.

7.2.2 Teacher participation

2.1 What do teachers in a PLC talk about and at what depth do they engage with different activities?

2.2 How is pedagogical content knowledge (PCK) amongst teachers created within the professional learning community?

The quantitative analysis of content in the community data showed that teacher talk about learners, followed by mathematics and practice, was prominent in all the activities, with some decline in learner and practice in 2012. Learner thinking showed prominence in Error Analysis in 2011 and did not feature much in the other three activities. The low focus on learner thinking is inconsistent with some of the teachers' shifts in working more deeply with the learners' errors in class. This study cannot account for this result, which necessitates further research in investigating the transition from teacher conversations in the community to how the teachers' ideas were enacted in the classroom.

The prominence of mathematics in the conversations was an unexpected result because the project gave it less emphasis, and also because there was no evidence of increased richness of mathematics in the classrooms. In a study that focused on the diagnostic competence in the teachers' conversations, Brodie, et al. (2018) found that teachers spent a third of their time talking about their own mathematical knowledge in the PLC. This confirms Brodie & Sanni's (2014) argument that talks about PCK can contribute to CK.

The quantitative analysis of depth showed significant shifts in all the activities over the two years, with the greatest shift occurring in Lesson Reflection. This result means that the teachers' conversations in 2012 linked more closely to their instructional practices because they had substantive discussions on the learners' work – they identified the learners' errors and engaged with those in a collaborative way. This result also shows that though the content of the teachers' talk focused less on understanding the reasoning behind the learners' errors, the teachers did develop depth in their talk, which may account for, or be a result of, their improved practice. This result somewhat supports Kazemi & Hubbard's (2008) notion of coevolution of practice: bringing classroom data to the PLC for inquiry, developing new strategies for practice

and employing them in classroom teaching, and coming back to the PLC to reflect on what transpired in class.

The qualitative analysis supported and deepened the quantitative analysis. The four key findings which emerged from analysis of the episodes were: 1. Disagreements among the teachers created opportunities for the development of their PCK because when the teachers challenged each other, there was greater depth in the their conversations; 2. More than once, an individual teacher's input created an opportunity for rich conversations in the community which contributed to the teachers' PCK; and 3. The depth in the PLC conversations in 2012 showed some richness in mathematics as evidenced in the qualitative results in Chapter 5, and this depth was accompanied by some new understandings for some of the participating teachers, which contributed to PCK in the community; and 4. The teachers' conversations evidenced that probing became a strategy that the teachers learned in the PLC for instruction in class.

The qualitative analysis supports the quantitative analysis, showing that the teachers hardly inquired into the reasoning behind the learners' errors during their conversations. The teachers identified the learners' errors in their engagement with the activities, talked about how the learners produced those errors, and anticipated the learners' errors with less focus on the misconceptions underlying the learners' errors. The qualitative results suggest that the teachers worked best with learners' errors in Error Analysis but did not apply ideas from this activity in the other activities.

7.2.3 Facilitation

2.3 How is a space for engaging with the different activities in a PLC created by the facilitator?

The quantitative analysis showed that I focused on managing rather than maintaining the conversation and when I did make substantial inputs it was in the form of explanations. Although management of the community conversations was important in guiding the teachers to engage with the project activities, too much managing the conversation adversely affected the teachers' participation in the community. Similarly, the prominence of my explanations,

though important in guiding the teachers to have focused discussions in the community, limited the teachers' input and inhibited the creation of new knowledge in the community. These results revealed a weakness in facilitation in relation to the development of PCK.

The quantitative analysis also showed that the focus of my facilitation was prominent on learner, mathematics and practice, which was accompanied by a low focus on learner thinking except in Error Analysis in 2011 and Readings & Discussion in 2012. These results are similar to those in Chapter 5 on the teachers' conversation, which makes sense because I was a participant in the conversations. The focus of my facilitation was mostly on identification of learner errors; how learners understood the mathematics; and the learners' solution strategies in doing mathematics, which raised opportunities to the teachers' understanding of how learners learn and was important for instruction in class.

The qualitative analyses of the four episodes revealed my dominance in two episodes, which explains the prominence of explanations in the quantitative analysis. This dominance related to my over-alertness in facilitation and being overly anxious about the project succeeding. Push emerged as a prominent move in my facilitation, which was important to elicit the teachers' participation in the PLC conversations. Most of these pushes involved mathematics content or mathematical language, which created opportunities for the teachers to develop their mathematics content as in the fourth episode in Chapter 6. This is the same episode in which I took a small step back to afford the teachers to engage with each other, a move which created the space for the teachers to challenge and show solidarity with each other (Brodie & Shalem, 2011) and for deepening the richness of the mathematical discussions in the community.

The analysis results also showed the occurrence of missed opportunities in my facilitation. These missed opportunities restricted the conversations for rich mathematics, as was the case in the first episode when I did not create a space for the teachers to deal with the notion of seeing $2a + 5b$ as a possible answer for addition. The missed opportunities revealed a weakness in my facilitation which may need further research in how this practice can be circumvented in contribution to PLC facilitation. On reflection, I discussed in detail in Chapter 6 how else I could have dealt with the situation for each of the missed opportunities that occurred in the different episodes.

7.2.4 Relationships among teaching practices, teacher participation in the PLC, and facilitation

My analysis focused separately on the three constructs: teaching practices, teacher participation, and PLC facilitation, and there is no direct claim that can be made to connect all three. However, there are two claims that I can make based on my analyses in Chapters 4 and 5 as well as in Chapter 6. First, there was a relation between teacher practices and teacher participation as discussed in my analysis in Chapters 4 and 5. This relationship was exemplified in the JPL1 lessons. The participating teachers looked at their classroom data to plan joint lessons in the PLC and taught those in their different classes. The qualitative analysis in Chapter 4 showed how the four teachers shifted their practices in JPL1.

Second, there was a relation between teacher participation in the PLC and facilitation and this relationship was evidenced in Chapter 6. Quantitative analysis showed how I used the facilitator moves to guide the teachers in the PLC conversations. The qualitative analysis also showed though how I missed on opportunities for further engagement in the PLC, which consequently in some instances did not create opportunities for PCK development. It is in this context that I cannot make a conclusive argument on the relationship between PLC facilitation and the teachers' practices, which results in no link among all the three constructs, but it is something that can be considered for further research.

7.3 The significance of the study

This study is original because I focused on researching the PLC as well as researching my facilitation – not many studies are conducted in this way. In doing so, this study reinforces Kazemi & Hubbard's (2008) notion of coevolution of participation by bringing in facilitation as an additional element. Kazemi & Hubbard (2008) focused on "multidirectional analyses of teacher learning across the PD and classroom contexts" (p. 429) and did not analyse facilitation of the PD. My presence in the PLC created affordances for critical facilitator's interventions which led to the teachers' improvement of their noticing (Francisco & Maher, 2011; van Es & Sherin, 2008) – the guidance I provided in the community contributed to the teachers' discussions about their instructional practices.

This study investigated the extent to which opportunities were created for the teachers to deepen their pedagogical content knowledge as they engaged with the learners' errors. Working on learner errors in the different activities brought different shifts which were accounted for, for different teachers: for both their practices and their participation.

The PLC in this study enhanced teacher ownership through reflective practice. Teachers selected episodes from their own lessons, reflected on them and brought them to the PLC for group reflection. This process increased teacher ownership of artefacts of practice and the community by inquiring into their classroom data and engaging with the project activities. When teachers reflect on their practices in a group, this puts their practice in the public realm, which might not be easy for some teachers. This study provided evidence of how the participating teachers agreed to their practices being discussed in a PLC and how knowledge was created through such community processes.

This study explored how the facilitator created a space for the teachers to inquire into their instructional practices, which raised the importance of the facilitator's familiarity with the two settings in which the teachers operated: the classroom and the PLC. This familiarity contributed to the facilitator's knowledge of both the teachers' practices and their participation, which can increase opportunities for the facilitator's support and guidance of the teachers' learning through identifying with the teachers' learning needs. In addition, acknowledgement of the missed opportunities in facilitation illuminated opportunities for enhancement of learning about PLC facilitation.

7.4 Limitations of the study

There were a number of limitations of working with and researching this PLC as well as facilitating the PLC. The limitations discussed in this study relate to external and internal validity of the findings, which was enhanced by data triangulation, long-term observation of the teachers' practices and participation, and peer examination of the results. The PLC researched in this study was a case of one PLC comprising three schools, and the results of my analysis may not be generalised to all professional learning communities. These results do contribute to knowledge on professional learning communities since the study focused on in-depth analysis of the teachers' practices, their participation in the PLC and PLC facilitation.

The shifts in the teachers' practices and participation that were discussed in my analysis are limited to the specific factors that the instruments focussed on, such as listening to and engaging learners more in class; challenging each other in community conversations; and probing each other. There could be other factors that might have an influence on the teachers' practices and their participation in learning communities. For this reason, the results should not be over-generalised to elements of practice that were not studied. That the participating teachers began to show shifts in their practices and their participation in the second year of their participation supports the notion that PLCs are characterised by long periods of time as platforms for teacher professional development.

A second limitation was on the omission of the teachers' individual or group perceptions of their own instructional practices and their participation, which can be done through teacher interviews. Teacher interviews are important in illuminating the teachers' views of their practices and their participation, and what they could ascribe the shifts in their practices and participation to. Interviews can also provide the space for what Merriam (1998) refers to as "member checks", where participants can be given opportunities to confirm or disconfirm the analysis results. In addition, interviews can contribute to the teachers' reflection of their participation in the project, and can therefore provide important data to explore how professional development of teachers through PLCs could be improved. Interviews though may restrict the teachers' freedom and openness in talking about the strengths and weaknesses of the project, and the benefits and disadvantages of their participation.

The third limitation was on facilitating a PLC and researching it at the same time, which created conflicts of interest for me. My interests were on both supporting and guiding the participating teachers to develop their practices through their participation and also on getting results for my thesis and for the project. Focusing only on facilitating the PLC without researching my facilitation would have diminished the pressure and the anxiety related to research. On reflection of my facilitation, I found that there were instances where I could have facilitated the conversations differently and I noted these instances as missed opportunities in my facilitation. Acknowledgement of the missed opportunities created an awareness of areas that needed improvement on PLC facilitation. In addition, I acknowledge through my reflection, that these limitations clearly impacted the functioning of the PLC.

The fourth limitation was my involvement as an external facilitator, which may have adversely affected the frequency of the PLC meetings. The participating teachers were full-time employees in their schools, which meant that they could have met more frequently without me. I was not employed in any of the schools, which meant that my understanding of the schools' context was limited. A school-based facilitator could have brought a better focus on the professional development of the teachers, and perhaps lessened the pressure of facilitation and the functioning of the PLC. The presence of an external facilitator, though, has potential to reinforce the depth of the community conversations and knowledge creation in the community.

7.5 Recommendations for further research

I identified seven points that may be considered for further research:

1. Consideration for a school-based facilitator. The DIPIP project did work with school-based facilitators in the years following the data collection for this study. Brodie (2015) has investigated differences between school-based and university-based facilitators but more research is needed on this.
2. Composition of the PLC, to investigate whether communities teaching the same or different grades will optimise collaborative knowledge creation. Together with the previous point, careful composition of the PLC can be explored to see the extent to which this kind of composition can reinforce teacher ownership of what and how they need to learn.
3. Building a strong relationship between school management (through the principal) and the PLC, which may need investigation on the extent to which curriculum processes in schools incorporate in-service professional development of the teachers. This relationship can be explored to establish the extent of sustainability of a teacher development program (Bolam, et al., 2005; Collinson & Sherrill, 1996).
4. Considering that learner thinking did not feature as much as was expected in this community's conversation, it is important to explore ways of bolstering its focus in the community conversations. This may need investigating how teachers can reinforce how they work with learners' misconceptions, and explore processes to put in place to achieve this.
5. Reducing facilitator dominance in the community, which may involve a dual approach. First, it is important to look at how less facilitator talk can reinforce community conversations. Second, it is important to look at how more facilitator talk can be controlled in ways in which it can contribute to knowledge creation.

6. Related to the previous point are the missed opportunities identified in my analysis of my facilitation, which I considered to be a weakness in facilitation. It is important to explore the kind of moves that are needed in order to take account of such opportunities in the actual practice of PLC facilitation. This may need exploring how facilitators can keep track of what has been discussed in the community and what still needs to be done.
7. The unexpected, limited focus on learner thinking may also be connected to the facilitation, as well as the community's diversion from the main objective of each activity. It is important to explore how the facilitator can find a balance in carefully guiding the discussions in the PLC without suppressing the teachers' contributions, especially in cases where the community diverts from key objectives.

Both ICME-13 survey and ICMI Study 25 found collaborative reflections as important for teacher learning in professional development settings (Borko & Potari, 2020; Jaworski, et al., 2017; Robutti, et al., 2016). Ideas that they raised for further research concerned: issues of scalability and sustainability, especially for very large projects; the need to get studies focusing on primary and secondary schools collaboratively learning together; and teachers working on mathematics and their learning of mathematics; and usage of interactive digital resources as contribution to teachers' practice, teachers' learning and to teaching development. This last point is also necessitated by the challenges of remote learning due to the COVID-19 pandemic.

7.6 Conclusion

This study investigated how mathematics teachers' practices developed through a professional learning community. This development entailed enhancement of the teachers' professional knowledge through the DIPIP project's lens of how teachers work with learner errors in their mathematics classrooms (Gardee & Brodie, 2021). Teachers were afforded opportunities to engage with the project activities and inquire into their practices. In doing so, the four participating teachers in this study opened up their practice to afford the learners opportunities to engage with the mathematics in class. The teachers shifted their practices, with two teachers, Funeka and Mapula, sustaining the shifts in their practices and the other two, Dimpho and Khumo, not sustaining them.

This study also investigated how the participation of teachers in the PLC contributed to their PCK as they learned with and from each other and showed that the participating teachers negotiated what Wenger (1998) refers to as a joint enterprise – collaboratively working on activities for purposes of building a knowledge base for mathematics teaching and learning. The quantitative results on community data showed the prominence of learner, mathematics and practice in the community conversations and PLC facilitation. The prominence of mathematics was an unexpected result as it did not receive emphasis in the project. The results on community conversations showed that the teachers' talk focused less on understanding the reasoning behind the learners' errors, which may need further investigation. The quantitative results on facilitation showed the prominence on managing the conversation and explanations.

The qualitative analysis supported the quantitative analysis. There was less focus on the misconceptions underlying the learners' errors. The results showed that the teachers contributed to their PCK in various ways, with challenge amongst the teachers contributing to depth in the community conversations and enhancing the knowledge base in the community. On more than one occasion, the substance of the community conversations contributed to the richness of the conversations due to an individual teacher's contribution. The qualitative analysis of facilitation showed that the facilitator moved between dominating the discussion and creating affordances for the teachers' participation. I also acknowledged the missed opportunities on facilitation and discussed how else I could have facilitated the PLC differently.

The analysis results showed that participation of teachers in a professional learning community can support their instructional practices (Kazemi & Hubbard, 2008), especially when there is appropriate guidance and support. Evidence was provided that good facilitation can promote in-depth conversations in the PLC, and this can contribute to the teachers' PCK in a positive way. It is important that this study be understood in the context of other studies related to the project to which this study is linked and also to other studies in South Africa. This understanding may contribute to and influence a focused and concerted effort in getting functional professional learning communities in South Africa and elsewhere, and consequently leading to better practices in mathematics teaching and improved learner performance in mathematics.

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Appendix 1: DIPIP activities

Activity	Tasks
Curriculum mapping	Map test items to the South African curriculum, identify concepts, find relevant Assessment Standard(s), give reasons, state if they do or do not teach the Assessment Standard(s) in their classes.
	Identify concepts
	Find relevant Assessment Standard(s), give reasons
	State if they do or do not teach the Assessment Standard(s) in their classes
Error analysis	Discuss the correct responses and the different methods to be used in solving mathematical tasks.
	Record all the errors that learners produced in the test.
	Discuss the incorrect responses focussing on learners' mathematical thinking and reasoning.
	Identify key concepts embedded in the mathematical problem
Learner interviews	Select and interview particular learners on the errors they made in the mathematics test.
Planning and teaching the identified critical mathematical concept	Identify critical mathematical concepts.
	Readings and discussion of mathematical concepts.
	Lesson planning.
	Teaching the concept.
Lesson reflection	Identify two episodes from the taught lessons where the teacher dealt well/did not do so well in engaging with the learner error.
Network meetings	Presentation on error analysis and learner interviews.
	Presentation on lesson planning.
	Presentation on lesson reflection.

Appendix 2: MQI coding scheme for teaching practices.

MATHEMATICAL QUALITY OF INSTRUCTION (MQI)

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SEGMENT CODES

Format of the Segment

Indicate the main format in which students worked.
Please code both formats if at least one minute of each type of instruction occurs.
If class splits into two or more groups, please code all applicable formats.

Active Instruction	Small Group/Partner/Individual Work
Teacher leads discussion or presentation of mathematical material. May be moments where students work individually to solve a problem, but these are brief interludes (under a minute) before return to whole-group work. Whole group does not necessarily imply whole class; class can be split into halves or other fractional components. Key element is extended teacher presentation of mathematical material or posing of mathematical problems to a group.	Teacher divides students into small groups or pairs for work on mathematical problem or task OR students work individually on mathematical problem or task. Typically, teacher circulates among groups or pairs, checking progress. However, teacher may be working on administrative issues, etc. during this time.

Classroom Work is Connected to Mathematics

Code here for whether the focus is on *mathematics content* during half or more of the segment (3.5 minutes or more total).

No	Yes
Focus for majority of the segment (3.5 minutes or more) is on non-mathematical topics, or student activities that have no clear connections to developing mathematical content (e.g., cutting and pasting). Examples: □ Gathering or distributing materials, other administrative issues □ Disciplinary issues that severely impinge upon instructional time □ Students doing an activity (cutting, pasting, coloring) that is not clearly connected to mathematics ("bad reform") □ Any combination of the above that sums to 3.5 minutes	Focus is on mathematical content for majority of the segment. Examples: □ Teacher reviews content from a prior lesson; teacher introduces content □ Students practice content □ Students work on a warm-up problem while teacher takes attendance

Note: "Majority" means 3.5 minutes or more, unless otherwise noted

Mode of Instruction

Used to code whether mathematical content is supplied in a teacher-directed manner or whether alternative methods are used to develop content. These modes are not exhaustive; that is, segments can receive a "none" for all in some cases.

Direct Instruction

Teacher is in control of delivery of mathematical content. Mathematical content can be correct or incorrect; major feature is high amount of teacher talk and/or control relative to other activities (e.g., student talking, practice, etc).

Can include, e.g., teacher going over homework, going over problems, presenting new material, reviewing, launching a task, etc. Can be in small-groups.

None	Some	Most/All
No direct instruction occurs.	This mode of instruction occurs only for part of the segment or supplements other work featured in the segment – hence, it is *not* the key feature of the segment.	This mode of instruction occurs for the majority of the segment and/or is the *key* feature of instruction featured in this segment.

Whole-Class Discussion

"The teacher is in charge of the class, just as in direct instruction. However, the teacher is not primarily engaged in delivering information or quizzing. Rather, he or she has students share their thinking, explain the steps in their reasoning, and build on one another's contributions. …. [This mode of instruction] gives students the chance to
engage in sustained reasoning." (Chapin et al., 2003, p. 17) **Key feature is that students comment on mathematics of one another's contributions, not just say, "we did it another way."**

Also, most of transcript will be student voices.

Whole-class discussion is applied only to "active" instruction time.

None	Some	Most/All
No whole-class discussion.	This mode of instruction occurs only for part of the segment or supplements other work featured in the segment – hence, it is *not* the key feature of the segment.	This mode of instruction occurs for the majority of the segment and/or is the *key* feature of instruction featured in this segment.

Working on Applied (Real-World) Problems

Teacher and/or students work on applied (real-world) problems.

Examples of applied (real-world) problems:

- Figuring out among four recipes the proportion of orange juice and water that makes a mixture more orangey
- Figuring out which is the best phone-call plan among three plans representing a linear, a proportional, and a stepwise function.

Non-examples:

- Mentioning a past contextualized problem but not working on it
- Story problems used to illustrate a situation, but that are not actively worked on during the segment.

Note: Short word problems from math texts DO count in this code and could result in either a mid or a high code, depending on how they are used in the segment.

None	Some	Most/All
No work on applied (real-world) problems is featured in this segment.	This mode of instruction occurs only for part of the segment or supplements other work featured in the segment – hence, it is *not* the key feature of the segment.	This mode of instruction occurs for the majority of the segment and/or is the *key* feature of instruction featured in this segment.

Richness of the Mathematics

This category attempts to capture the depth of the mathematics offered to students. Rich mathematics is either a) focused on the meaning of facts and procedures OR b) focused on key mathematical practices. Rich mathematics allows students to build a conceptual mathematical base and build connections within and among different components of rich mathematics.

For all, element must be substantially correct and clear to count as "mid" or "high." "Mid" constitutes good use of many of these elements; "high" constitutes extraordinary use.

Note: All codes in this category are a quality codes. A teacher can get a high, even if the aspect of instruction you are coding for occurs for only a portion of the segment.

Linking or Connections

This code refers to teachers' and students' explicit linking and connections:

- Among different representations of mathematical ideas or procedures (e.g., a linear graph and a table both capturing a linear relationship)
- Among different mathematical ideas (e.g., proportionality and linearity; fractions and ratios, etc)
- Across representations and mathematical ideas /procedures (e.g., discussing how linearity is captured in any of the following: a graph, a table, or a mathematical equation)
- Note: If links are made but underlying representation/idea is incorrect, do not count as linking and connections.

Low	Mid	High
No linking or connections occur. Also code low when connections are **completely** pro forma, e.g. "Yesterday we did adding fractions with like denominators, today we will do subtracting fractions with like denominators."	Links or connections are present, but do not have the features included in "high," or feature these only momentarily (e.g., "You can compare ratios the same way you compare fractions" or "You can see that each step in the computation can be seen in this array model here.").	Links and connections are present with sustained, careful work on one or more of the following: - Detailed discussion of how two mathematical ideas are related to one another - Explicit linking of a representation to the underlying idea it is meant to represent - Explicit linking between multiple representations, showing how they correspond

Explanations

Mathematical explanations explain why. This includes giving mathematical meaning to ideas or procedures, meaning of steps, or solution methods, e.g.:

- Why a procedure works
- Why a solution method makes sense
- Why an answer is true

Do NOT code "*how*" e.g., descriptions of steps here (first I did x, then I did y) or simply providing definitions unless meaning is also attached.

Examples: the reason for steps in simplifying fractions (dividing by 2/2, for example, is same as dividing by 1; anything divided by 1 is still itself); why particular steps in a complex problem are justified or work to achieve the solution.

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Note: Do not count incorrect or incomplete explanations as explanations.

Low	Mid	High
No mathematical explanations are offered by the teacher or students or the "explanations" provided are simply descriptions of steps of a procedure.	The explanations offered by the teacher or students meet *any* of the following criteria: - They meet criteria under high, but they are not detailed; OR - They are not generalizable (they pertain only to the specific task/problem under consideration) - They are definitions that contain elements of "why" but that do not fully explain	The explanations offered by the teacher or students meets the following criterion: - They move beyond particular problems (i.e., they are general explanations, not for particular problems) AND meets *one or more* of the following criteria: - They give meaning to steps and procedures - They are detailed

Multiple Procedures or Solution Methods

Multiple procedures or solution methods

- Multiple solution methods for a single problem (including shortcuts)
- Multiple procedures for a given problem type

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Defined as, e.g.:

- Taking different mathematical approaches to solving a problem (e.g., comparing fractions by finding a common denominator AND comparing fractions by finding a common numerator)
- The teacher/students may solve a (word) problem using two different strategies.

If the initial strategy(ies) occurred in a prior interval, code the second (or subsequent) interval at this code (e.g., no need to go back and code the initial interval).

Note: Here you could also include teacher/student mentions multiple different procedures/solution methods.

Note: Do NOT code incorrect procedures or solution methods

Low	Mid	High
No evidence of multiple procedures/solution methods for a given problem type.	Multiple procedures/solution methods present, but do not have the features included in "high," or feature these only momentarily (e.g., "this method is easier than the other" without explicit discussion of why).	Multiple procedures or solution methods occur with special features: - Explicit comparison of multiple procedures/solution methods for efficiency, appropriateness, ease of use, or other advantages and disadvantages - Explicit discussion of features of a problem that cues the selection of a particular procedure - Explicit links between multiple procedures/solution methods (e.g., how one is like or unlike the other)

Developing Mathematical Generalizations

Teacher and/or students **develop** mathematical generalizations by examining instances or examples, then making a general statement (e.g., drawing parabolas x^2, $2x^2$, $4x^2$ then making a generalization about shape as coefficient changes).

Examples of this activity include:

- Generalizations of mathematical facts
- Generalization of mathematical procedures
- "Building up" a mathematical definition or deriving a mathematical property after considering different examples and non-examples (e.g., defining "polygons" after considering different examples and non-examples of polygons)

Notes:

1. Requires at least two examples (either explicitly worked or referred to) from which generalization emerges.
2. Code at generalizations for only the clip in which generalization emerges / becomes explicit.
3. Do NOT code incorrect generalizations.
4. Do NOT code: When teachers *state* generalizations without first developing them from examples.

Low	Mid	High
No generalizations are developed in this segment. Also code "low" for non-mathematical generalizations (e.g., drawing a picture helps solve a word problem).	Students/teacher *develop* a generalization, but the generalization developed is not complete, clear or detailed.	Teacher and/or students *develop* a generalization. The generalization should contain the *mathematical* essence of the work done with regards to a particular task and should be complete and clear.

Mathematical Language		
□ Fluent use of technical language □ Explicitness about mathematical terminology □ Supporting students' use of mathematical terms		
Low	**Mid**	**High**
Teacher does not demonstrate fluency in mathematical language. Teacher uses non-mathematical terms to describe mathematical ideas and procedures AND/OR teacher talk is characterized by sloppy use of mathematical terms. If there is little mathematical language used, code here.	Teacher uses the mathematical language as a vehicle for conveying content, but has few or none of the special features listed under "high." Also code here when segment has both features of "high" but includes some linguistic sloppiness.	Teacher uses mathematical language *fluently*. May include explicitness about terminology, reminding students of meaning, pressing students for accurate use of terms, encouraging student use of mathematical language. Density of mathematical language is high during periods of teacher talk.

Overall Richness of the Mathematics		
Depth of the mathematics offered to students. Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of richness		
Low	**Mid**	**High**
Segment is not rich in any sense, or has only a brief instance of a richness element (brief "mid" for explanation). May also be used for error-filled segments.	No major errors occur and segment offers students some opportunity to learn beyond rote memorization of rules or standard procedures. May be characterized by many "mid" codes above, one well-developed "high" along with substantial procedural focus, etc.	No major errors occur and segment offers students accurate and significant opportunities to learn mathematics as evidenced by one or more "high" codes. Two general ways: □ Focus is on meaning via representations linked to one another or to underlying ideas; explanations that generalize; generalizations that are developed from specific examples □ Focus is on explicit comparing of solution methods or procedures (e.g., most efficient) but without necessary focus on meaning during this discussion

Working with Students and Mathematics

This category captures whether teachers can understand and respond to students' mathematically substantive productions (utterances or written work) or mathematical errors. By mathematically substantive productions, we mean questions, claims, explanations, solution methods, ideas, etc that contain substantial mathematical ideas. By students' mathematical errors, we mean those incorrect student productions that offer opportunities for discussing and addressing pertinent mathematical ideas.

Thorough Remediation of Student Errors and Difficulties

With this code, we mean to mark instances of remediation in which student misconceptions and difficulties with the content are *substantia*lly addressed.

Note: Can be during active instruction or student work time.

Low	Mid	High
No thorough remediation occurs for any of the following reasons: • There are no student misunderstandings or difficulties with the content • Remediation is procedural and brief or otherwise non-substantive • The teacher chooses not to remediate • The teacher remediation is confusing or off-track or not thorough	Teacher engages in thorough remediation *briefly* or *occasionally*. Also includes very *extensive* procedural remediation.	Teacher engages in thorough remediation *systematically* and *at length*. High is characterized by *conceptual* remediation: • Identifying the source of student errors or misconceptions • Discussing how student errors illustrate broader misunderstanding and then addressing those errors. It also includes teacher skill at: • Anticipating common student errors and providing instruction that helps avoid error • Parsing student productions to separate correct and incorrect thinking.

Responding to Student Mathematical Productions in Instruction		
Student evokes new (non-routine) mathematical thought for the class, and teacher understands and responds to it during instruction in mathematically appropriate ways. Student ideas will tend to have features of student explanation/generalization/why question, or a complex description of a solution method. Teacher: □ Identifies mathematical insight in specific student questions, comments, or work □ Builds instruction on student ideas or methods Note: By "ideas" we do not mean simply answers to problems or pointed questions where teacher has sought a specific, bounded piece of information. They tend to be student explanation, questioning, reasoning.		
Low	**Mid**	**High**
Routine instruction with no student productions. There are student productions, but no evidence that student ideas are "heard," taken up or used in instruction. Teacher may provide evidence that he/she does not understand student productions. Teacher uses student productions but in a way that muddles or confuses the mathematics of the lesson.	Student productions are present. Teacher may engage in features listed under "high" briefly, but instruction generally proceeds *without* strong use of student mathematical ideas. Also use in cases where there is evidence the teacher understands student thinking but chooses not to use it at that time.	Student productions are present. Teacher "hears" what students are saying, mathematically, and responds appropriately during instruction. Students' mathematical ideas are woven *at length* into the *development* of mathematical ideas during the lesson. Teacher may comment on student's mathematical ideas, elicit further student clarification of ideas, ask other students to comment on ideas, expand on and reinforce student utterances, etc. Other markers include: □ Indentifying key ideas in student statement ("Mark had an interesting idea…") □ Highlighting key features of student questions ("Do you see Mark asked a question about whether this would work in all cases?") □ Identifying a student with an idea ("Mark's method")

Overall Working with Students and Mathematics		
Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the teachers' interactions with the students around the content.		
Low	**Mid**	**High**
Few substantive interactions between teacher and student or teacher may overlook information about students and/or student insight into mathematics.	A combination of strong and weak features (e.g., some high-level remediation but teacher ignores students' contributions).	Strong and significant teacher understanding and use of student ideas and errors around the content as evident by *outstanding* performance in one area or consistently solid performance in several others.

Errors and Imprecision

This category is intended to capture teacher errors or imprecision in language and notation, uncorrected student errors, or the lack of clarity/precision in the teacher's presentation of the content.

Do not code errors if these errors are captured and addressed within the segment or chapter (in this case, code as “low").

Major Mathematical Errors or Serious Mathematical Oversights

- Solving problems incorrectly
- Defining terms incorrectly
- Forgetting a key condition in a definition
- Equating two non-identical mathematical terms, etc.

Low	Mid	High
Instruction is *clean* of major errors in spoken or written work OR errors that occur are captured and corrected within the segment.	Teacher makes *major errors* either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem). The errors occur in ***part*** of the segment.	Teacher makes *major errors* either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem). The errors occur in ***most*** of the segment.

Imprecision in Language or Notation (mathematical symbols)

- Errors in notation
- Errors in mathematical language
- Errors in general language

Low	Mid	High
Instruction is *clean* of errors in mathematical language, general language and notation. Any errors made and quickly corrected should also be coded here.	Teacher makes a *small number* of momentary errors in notation, mathematical or general language.	Instruction is characterized of linguistic and notational *sloppiness across the segment* and/or by *major* notational and linguistic errors in even a small number of mathematical terms.

Clarification:

- ***Notation*** includes conventional mathematical symbols, such as +, -, =, or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents. Errors in notation might include inaccurate use of the equals sign, parentheses, or division symbol. By "conventional notation," we do not mean use of numerals or mathematical terms.
- ***Mathematical language*** includes technical mathematical terms, such as "angle," "equation," "perimeter," and "capacity." If a teacher uses these terms incorrectly, code as an error. When the focus is on a particular term or definition, also code errors in spelling or grammar.
- Teachers often use "***general language***" to convey mathematical concepts (i.e., explaining mathematical ideas or procedures in non-technical terms). General language also includes analogies, metaphors, and stories. Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. If teacher is unclear in his/her general talk about mathematical ideas, terms, concepts, procedures, code as an error.

Lack of Clarity

- Teacher utterances cannot be understood, e.g.:
  - Mathematical point is muddled, confusing or distorted
  - Language or major errors make it difficult to discern the point
  - Teacher neglects to clearly solve the problem or explain content
- Teachers' launch of a task/activity lacks clarity (the "launch" is the teacher's effort to get the mathematical tasks/activities in play)

Low	Mid	High
Teacher's presentation of the mathematical content and/or launching of tasks is *clear* and *unambiguous*.	Teacher's presentation of the content and/or launching of tasks is *not clear* for *portions* of the segment.	Teacher's presentation of the mathematical content is *unclear*, *vague* or *incomplete* for most of the segment. Teacher's work is *muddled* or *confusing* and *severely distorts* the mathematical essence of the content. Also, teacher conveys mathematical tasks or problems *incompletely* or in a *confusing* manner.

Overall Errors and Imprecision

Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the errors and imprecision in instruction.

Low	Mid	High
No errors occur. Do not use this code if "mid" or "high" is marked in any category above.	Brief error or errors generally not serious enough to indicate teacher may lack mathematical knowledge.	Either multiple small errors, **consistent lack of clarity** or one large error to suggest that teacher may lack key mathematical knowledge.

Student Participation in Meaning-Making and Reasoning

This code attempts to capture evidence of students' involvement in tasks that ask them to "do" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. During active instructional segments, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking. During student work times, this mainly occurs through work on a non-routine task

Students Provide Explanations

Students provide a mathematical explanation for an idea, procedure, or solution.

Examples:

- Students explain why a procedure works
- Students explain the procedure they used to solve a particular problem by attending to the meaning of the steps involved in this procedure rather than simply listing those steps
- Students explain what an answer means
- Students explain why a solution method is suitable or better than another method
- Students explain an answer based on an estimate or other number-sense reasoning

Notes:

- Explanations could be initiated by the teacher or self-initiated; they could also be co-constructed with the teacher or constructed individually
- Explanations do not have to be complete or correct

Low	Mid	High
No student explanations are featured in the segment.	The explanations offered are: - Brief - Pertain to a specific problem/task - Do not generalize to key mathematical ideas of the content under consideration Examples: - A student explains that ¾ is larger than 3/5 because the denominator in the first fraction is smaller than the denominator in the second fraction. - A student explains that a cube net has 24 right angles because it has 6 square faces.	The explanations offered generalize past specific problems to address key mathematical ideas of the content under consideration. Examples: - A student explains that ¾ is larger than 3/5 because the denominator in the first fraction is smaller than the denominator in the second fraction and fractions with smaller denominators correspond to smaller pieces. - A student explains that a cube net has 24 right angles because it has 6 square faces and each square has 4 right angles.

Student Mathematical Questioning and Reasoning		
☐ Students provide counter-claims in response to a proposed mathematical statement or idea (whether from another student, the teacher, or a text) ☐ Students ask mathematically motivated questions requesting explanations (e.g., "why does this rule work?" "What happens if all the numbers are negative?") ☐ Students provide examples of a phenomena ☐ Students make conjectures about the mathematics discussed in the lesson (e.g., "A triangle cannot have two obtuse angles") ☐ Students form conclusions based on patterns they identify or on other form of evidence (e.g., "Because the sum of the angles of any triangle is 180 degrees, a triangle cannot have two obtuse angles") ☐ Students engage in reasoning about a hypothetical or general case ("Because the sum of the angles of any triangle is 180 degrees, a triangle should have at least two acute angle") Note: Students' productions do not have to be complete or correct.		
Low	**Mid**	**High**
The segment does *not* feature any of the student behaviors related to this code.	The segment features *one or two* of the student behaviors listed under this code.	The segment features *three or more* of the student behaviors listed under this code.

Enacted Task Cognitive Activation

This code refers to the enactment of the task, regardless of the initial demand of the curriculum/textbook task or how the teacher sets up the task for students

Notes:

- Student confusion does not necessarily suggest that students are engaging with the content at a high cognitive level
- Working on review tasks or on ideas discussed in previous lessons does not necessarily mean that students use lower order thinking skills
- This code should not be confounded with the difficulty of the task or whether it is appropriate for a certain grade-level.

Low	Mid	High
Students engage with the content at a *low* cognitive level. Examples of cognitively *undemanding* activities include: • Recalling and applying well-established procedures • Recalling or reproducing known facts, rules, or formulas • Listening to a teacher presentation with limited student input • Going over homework with little additional student work (e.g., reporting numerical answers) • Unsystematic exploration (i.e., students do not make *systematic and sustained progress in developing mathematical strategies or understanding*)	Students engage with content at *mixed level* of cognitive activation. May also include tasks with variable enactment (high and then low during segment).	Students engage with content at *high* level of cognitive activation. Examples of cognitively *activating* activities include when students: • Determine the meaning of mathematical concepts, processes, or relationships • Draw connections among different representations or concepts. • Make and test conjectures • Look for patterns • Examine constraints • Explain and justify

Overall Student Participation in Meaning-Making and Reasoning		
This code attempts to capture evidence of students' involvement in "doing" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. • During ***active instruction segments***, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking. • During ***student work time***, this mainly occurs through work on a non-routine task. Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the student participation in meaning-making and reasoning.		
Low	**Mid**	**High**
There are only a few or no examples of student participation in meaning making and reasoning. Tasks are largely procedural in nature. Students might also engage in unsystematic explorations.	Students engage with content at *mixed level*. Students may provide substantive explanations or ask a mathematically motivated question. May also include tasks with variable enactment (high and then low during segment).	Students contribute substantially to the building of mathematical ideas through the activities listed in this section as "high."

OVERALL LESSON CODES

Overall MQI and MKT		
These codes are intended to capture the overall mathematical quality of instruction (MQI) and the teacher's mathematical knowledge for teaching (MKT) as suggested by the teacher's work during the lesson.		
Overall MQI		
Low	**Mid**	**High**
Instruction is characterized by combinations of the following: □ Systematic errors (mathematical errors, in notation, in language) □ Student confusion or misunderstandings □ Unproductive teacher-student interactions around the content (e.g., teacher cannot understand student utterances) □ Lack of directionality	Instruction is relatively error-free but lacks the mathematical richness, appropriate use and discussion of procedures, and the sharp mathematical focus detailed under "high."	Instruction is characterized by combinations of the following: □ Mathematical richness in terms of explanations, generalizations, use and connections of representations or multiple solutions □ Appropriate use and discussion of procedures as well as of their applicability □ Instruction is mostly error-free and has a clear and sharp mathematical focus and directionality that allows students develop the important mathematical ideas under consideration □ Instruction is also characterized by positive teacher-student interactions around the content

Lesson-Based Guess at MKT		
How do you think the teacher would score on our MKT assessment? Note: Differs from overall MQI in that raters may use more judgment to estimate generally what teacher MKT should be. For instance, lesson captured may be mostly practice, but there may be some evidence of very strong teacher MKT.		
Low	**Mid**	**High**
MKT is estimated as low, as suggested by combinations of the following: □ Inappropriate use of representations, mathematical notation and language □ Mathematical errors □ Inappropriate explanations, descriptions of procedures, or discussion of student ideas and contributions □ Teacher puzzlement and incorrect or distorted presentation of the content	MKT is estimated as mid as suggested by the following: □ Instruction is mostly error-free but there is no evidence teacher has capacity to provide rich instruction, understands lesson material deeply, or has capacity to work with students' thinking or errors.	MKT is estimated as high, as suggested by combinations of the following: □ Accurate and rich instruction □ Strong use of mathematical language □ Ability to follow and build on mathematical ideas □ Ability to unpack the content and make it accessible to students □ Ability to identify and remediate student errors and misconceptions

Lesson Level Binary Codes
All codes should be scored as **Present** or **Not Present**
Orienting
The teacher states the lesson objectives at the beginning of the lesson in a way that is clear and understandable to the students. The teacher may not present objectives before reviewing homework from prior assignment. This can be coded present as long as the objectives are presented prior to beginning the primary instruction of the lesson.
Summarization
At the end of the lesson, the teacher (or student designate) reflects on what the class has accomplished and/or describes how the day's objectives connect to future goals (e.g., teacher provides a summary that restates or paraphrases the objective of the lesson, the work achieved during the lesson and/or previews related concepts that prepare students for the next phase of the learning process
Checking Broadly for Understanding
Evidence suggests that teacher has systematically gathered information about student understanding in either this or a past lesson. Includes surveying a large percentage of student about answer (thumbs up/thumbs down; whiteboard), several comments on student non-verbal cues, such as puzzled looks; commenting on common student errors; reporting results of self-assessment back to students.
Differentiated Instruction
The teacher provides differentiated instruction, such that different groups of students engage in tasks focused on different content (e.g., one small group works on geometry problems while another works on solving algebraic equations), or similar tasks of differing level of difficulty (e.g., all students find the area of shapes – some shapes are more difficult to work with than others).

Appendix 3: Exemplifying MQI coding on segments of a lesson

<table>
<tr><th colspan="3">First segment: 0-8 min</th></tr>
<tr><th>Speaker</th><th>Utterance</th><th>Coding</th></tr>
<tr><td>Teacher</td><td>Let's do the corrections. Let's do two or three sums from this worksheet I gave you yesterday. Then I said, go home and do the sums. I want one, two or three people, to come do the homework on the checkboard, on the chalkboard. And try and explain to us, how did they get the answers. You take out this paper (holds papers up), I'll come through to your desk and I'll give to you (talks to an individual learner). Right the first one is question ehhh...question. Okay question four, who volunteers to come and do the corrections for question 4. Who will come and explain to us, how did you get the answer? Did you see question four? Four. is it question four? Anyone who volunteer, any volunteer? Yes, Mafa (referring to a learner).</td><td rowspan="6">Direct Instruction was at level 2 because the teacher’s practice featured a high amount of teacher talk for the most part of the segment. She started the lesson by allowing the learners to the board to give their answers, but she still continued to lead the discussion. Whole-class discussion was at level 1 because interaction was mostly between the teacher and the learner at the board. The segment did not show any richness of the mathematics and was coded level 1 in all the subcategories. All the subcategories of working with students' productions were coded level 2 because the teacher's remediation was occasional, without strong use of learners’ mathematical ideas. The segment did not have any teacher errors, and only uncorrected learner errors regarding usage of the equal sign were identified, which was coded level 2 and the other subcategories were coded level 1. Student participation was coded level 2 only</td></tr>
<tr><td>Learner 1</td><td>Ma'am</td></tr>
<tr><td>Teacher</td><td>Okay Mafa, go to the board. And let us look at Mafa and see how did she get her answer. Write it fast, Mafa. (Learner writes on the board: ($3 + 1)^2 = 3^2 + 1^2$. Right, Mafa. Tell us, is this statement true or false?</td></tr>
<tr><td>Learner 1</td><td>False</td></tr>
<tr><td>Teacher</td><td>Let us listen to Mafa. Tell us, why do you see the statement as false? Explain to us.</td></tr>
</table>

Learner 1	Because three add one is equal to four to the exponent two (*learner writes* $3 + 1 = 4^2$), and four to the exponent two, four multiplied by four is sixteen (*learner writes* $4 \times 4 =$ *underneath the first statement, then writes 16 underneath*). Then three to the exponent two, is three multiplied by three, add one multiplied by one (*learner writes* $= 3 \times 3 + 1 \times 1$ *next to* $3 + 1 = 4^2$). Three multiplied by three is nine, then plus, add one multiplied by one is one is still one (*learner writes* $= 9 + 1$ next $4 \times 4 =$), then equals to ten (*learner writes* = 10 *next to* 16).	for student explanations because the teacher pushed the learners to give explanations. The other subcategories were coded level 1 because learners were not engaged in meaning-making in this segment.
Teacher	Is she correct	
Class	Yes (*learners clap hands*)	
Teacher	Thanks, Mafa. Let's go to the next one. The next one is question. We said we are going to do question six. Okay, okay (*inaudible*). Yes, you may go (*second learner walks to the board*). Here's chalk.	
Learner 2	(*learner writes* $2^3 = 2 \times 3$ *on the board*)	
Teacher	Right, butiza [brother]. (*Learner stops writing*). What is your answer first before you tell us?	
Learner 2	False	
Teacher	False, and why do you say it's false, explain to us	
Learner 2	Two to the second, second, third, two to the third exponent, give us four (*learner writes* $((4)^3$), and then four (*learner writes* 4 *underneath) ….* (*teacher interrupts*).	
Teacher	Wait, before you go on, my son. How did you get this four?	
Learner 2	Two, two….	
Teacher	What does two to the third exponent three means? Kushok'thini [what does it mean], two to the third exponent means what?	
Learner 2	Two to the third exponent means to the third power	
Teacher	What does this two to the exponent, you know that this is the base (*pointing to the two*), this is the exponent, angithi [right] (*pointing to the three*). So tell us, what does this means, two to the third exponent. You, you get four, now I want you tell me, how did you get this four? How did you get this four?	
Learner 2	I divided by two	
Teacher	Why did you divide by two?	
Learner 2	Two divided by two	
Teacher	Two divide by two give us one	
Learner 2	Two multiplied by two	
Teacher	It's two times, two multiplied by two it's two times, that means it's two to the exponent two, but look at this one, it's two to the exponent three, yes. You saying you got this by multiplying two by two, how many times	
Learner 2	Two	
Teacher	But this one is, this one is	
Learner 2	Three	
Teacher	But this one is two, is not three times (*pointing at the* 4, *and wiping off what the learner had written*).	
Learner 2	Two to the third exponent, it will give us two (*learner writes* 2)	

Teacher	Okay, before you give us (*inaudible*). You say this two to third exponent is not two. You say the first one, two to the second exponent, you say two multiplied by two, but this one is two to the third exponent. What are you going to say?	
Learner 2	Two, to the third exponent…(*interrupted*)	
Teacher	Okay, you say two to the exponent two, you say it's four (*teacher writes* $2^2 = 4$ *on the side*). Now this two to the exponent two, you say it is two multipied by two (*writes* 2×2). Do you understand that? Do you understand, class?	
Class	Yes	
Teacher	Right, but let's go back to this one (*pointing at* 2^3). You say this one is two to the third exponent, but this one is two to the second exponent (*pointing at* 2^2), that's why it says two multiplied by two, how many two's did I multiply?	
Learner	Two (*learner raises two fingers*)	
Teacher	Two. This side, how many two's are you supposed to multiply (*pointing again at* 2^3)	
Learner 2	Three (*raises three fingers*)	
Teacher	Do it so, good, write it down (gives learner the chalk and th elearner writes $2 \times 2 \times 2$ underneath 2×3 (*teacher interrupts*)	
Teacher	Work out this side first (*pointing underneath* $2 \times 2 \times 2$). Then talk bhuti [brother], talk, talk	
Learner 2	Two multiplied by two is four, four multiplied by two is eight (*writes* = 8 *underneath*). Two multiplied by three is six (*writes* + 6 *next to* 8)	
Teacher	This is the sign (*pointing at the equal sign*), this is the sign (*learner wipes off the + sign and replaces it by the = sign*)	
Teacher	Is it two third equal to two multiplied by six?	
Learner 2	No (*shakes his head*)	
Teacher	Is it false or true?	
Class	False	
Teacher	False, so that's why we say the statement is…	
Class	False	
Teacher	False, because of this one. The last one and then we go onto (*inaudible*).	

Second segment: 8-16 min		
Speaker	**Utterance**	**Coding**
Teacher	The last one is that, number eight. Eight, anyone who would like to come and do number eight. (*Silence*) Mr Mukanzi, number eight. Shesisa, sisi [hurry up, sister]	The coding in this segment was not that different from the previous segment, except for teacher explanations which shifted to level 2 when she assisted a learner with how division should be done. Also, lack of clarity jumped from level 1 to level 3 when she dealt with equivalent statements to explain how one statement was obtained from the previous one – she instead substituted in the equation and simplified to show that the left side is the same as the right, which suggested that she misunderstood the task.
	(*Learner goes to the board and writes* $\frac{2^a}{2} = a$)	
Learner 3	The statement is true because (*teacher interrupts*)	
Teacher	Okay, thank you sweetheart, just face the class and tell us, is this statement true or false, and tell us why	
Learner 3	The statement is true because two, two multiplied by two (*pointing at the denominator* 2 *and the* 2 *in the numerator. Teacher interrupts*)	
Teacher	Is that two multiplying, my baby?	
Learner 3	(*inaudible*)	
Teacher	Okay, what does that line means?	
Learner 3	Divide	
Teacher	Divide. Okay, you can go on	
Learner 3	Two divide by two is one (*crosses the* 2 *in the denominator, and puts* 1 *next to the crossed* 2), and one divide by *a* is *a.*	
Teacher	Come again	
Learner 3	Two divide by two is one	
Teacher	You divide which two	
Learner 3	This two (*pointing at the denominator*)	
Teacher	It is one, where is one? (*Approaches the board*. I*naudible)* (*learner wipes off the* 1)	
Learner 3	Two divide by two, it's one (*crosses the* 2 *in the numerator and puts a small* 1 *next to it*, one divide by *a* it's *a*	
Teacher	Why do you divide this one, one divide by *a* (*points at what the learner has written*)	
Learner 3	Because two divide by two is one	
Teacher	Which two, are you dividing this and that one or are you dividing the one two? Are you dividing both of them or are you dividing only one?	
Learner 3	Both of them	
Teacher	Show us ow do you divide them. Two into two how many times	
Learner 3	Once	
Teacher	And the other one?	
Learner 3	One	
Teacher	One, go on and then, what's the next step? So, you got one and *a*, and here you got one. What are you going to say?	
Learner	One divide by *a* (*teacher interrupts*)	
Teacher	This one does not divide (*pointing at the learner's work*), one and *a*, does not divide, on top, the numerator.	
Learner 3	(*Inaudible, writes* 1 – *a below her work. Teacher interrupts*)	

Teacher	One and *a* does not divide (*Holds the learner to the side to expose the work on the board and wipes the* 1 – *a*). This one and *a* does not divide (*pointing at the* 1 *and the a*). If you say one divide by *a*, the *a* was supposed to be under this line (*pointing at the division line*), this one does not mean that we are dividing. What does this one and *a* do in this sum? Are they dividing? Yes	
Learner 4	They are multiplying	
Teacher	They are multiplying	
Class	Multiplying	
Teacher	One and *a* they are multiplying. So, we are going to say, one multiply by *a*	
Learner 3	One multiplied by *a* = 1 is a (*writes a underneath*)	
Teacher	And what (*inaudible*). Is she correct?	
	(*Silence*)	
Teacher	Okay, may I explain that for you (*takes the chalk from the learner and writes* $\frac{2a}{2}$ = *a*). Right, here is the sum. The line means dividing (*pointing at the division line*). Let us stop writing and listen because we are going to do the same mistake. Right, we have two in teh numerator and the denominator two (*pointing at the* 2's *in expression*). It means, this line, we are dividing the top number and the below, and the below number. Naniki, then we say, two here it goes once, and here it goes once (*crossing the* 2's *in the numerator and the denominator and putting small 1's next to them*). We are remaining with one and *a*, one multiplied by *a* is *a* (*writes a)*, and beneath here is one (writes $\frac{a}{1}$), you see this one	
Class	Yes	
Teacher	Hey, this side, we have one, and this side we have *a* (*writes = a next to* $\frac{a}{1}$). One divide, one into *a* is equals to *a* (*writes a underneath*). This one we say *a* is equals to *a* (*writes = a*). Right, now we go on (*inaudible*) papers. We are going to do this one (*hands out papers to the learners*). Heh...stop, stop, stop talking, (*inaudible*). Right. Ndithe msila kuphi [what did I say about the tail]	
Learners	(*Inaudible*)	
Teacher	We have (*inaudible*), we have the following statement (*reads from the paper she is holding, inaudible*), and explain how each statement is obtained from the previous. Sibahle, you are sleeping, Sibahle. The first one they say *a* is equals to twelve (*class choruses. Teacher writes a* = 12 *on the board*). It means when I'm looking at this sum, it means that, the value of *a* is twelve. Now the next they say, *a, a* add five is equals to seventeen. Then we have the variable, stop writing (*claps her hands*). Lesego, stop writing. The value of *a* is twelve, and the sum which we are given is *a* add five. So, what are we going to want to know, we want to know, how did they get this seventeen? So, we want to know that this *a* is going to be replaced by...	

Class	Twelve	
Teacher	*a* is going to be replaced by twelve	
Class	Twelve	
Teacher	When we don't say replace, we say substitute, substitute (*writes the word "substitute" on the board*. Right, *a* is going to be substituted by twelve, right	
Class	Yes, ma'am	
Teacher	Now, let's look at this, then we have this, *a* is going to be twelve add five, is equals to seventeen (*class choruses*). Then we add this, twelve add five is equals to seventeen (*class choruses*). Is that correct?	
Class	Yes	
Teacher	Let us look at the next one. The next one, the next one it's two *a*, add ten, is equals to thirty four (*writes* $2a + 10 = 34$). Are we together?	
Class	Yes	
Teacher	Are we together? Mr Malamba, come back. Naniki, Dimples.	

Third segment: 16-24 min		
Speaker	**Utterance**	**Coding**
Teacher	Two *a* add ten is equals to thirty four. Right. Now, before you calculate, before you solve this one, you must know, what does two *a* means. Simplify two *a*, what does two *a* means to you? (*Silence*) Or two *a* is the same as what, two *a* is the same as what? 2*a* is the same as what, yes	The coding in the most subcategories remained the same as previously, except for only a few cases. Direct instruction declined as the teacher took over the discussion, and that was coded level 3. Teacher explanations also declined as her presentation was mostly erroneous, and this was coded level 1. Mathematical language shifted to level 2 as the teacher emphasised the order of operations in BODMAS. The category on working with students and mathematics declined as the teacher did not substantively engage with the learners' productions – learner responses were either rejected or accepted without explanations. Lack of clarity continued to be at level 3 as the task was still misunderstood and the class still encouraged to use substitution instead of focusing on the notion of equivalence. The category on student participation in meaning-making and reasoning remained at a low and was coded level 1 for all the subcategories as there was no evidence of learners being engaged in meaning-making and reasoning.
Learner 4	As multiplying	
Teacher	What?	
Learner 4	We are multiplying *a* from two	
Teacher	We are multiplying *a* from two, uthini	
Learner 4	Two by *a*	
Teacher	Come again	
Learner 4	Two by *a*	
Teacher	What is that, two, I want that…	
Learner 4	Two multiplied by *a*	
Teacher	It means	
Class	Two multiplied by *a*	
Teacher	Two *a* means	
Class	Two multiplied by *a*	
Teacher	Two means two multiplied by…	
Class	*a*	
Teacher	We are writing two multiplied by *a*, add ten, is equals to…	
Class	Thirty four (teacher writes $2 \times a + 10 = 34$	
Teacher	Right, now you can substi…	
Class	Substitute	
Teacher	Now you can….	
Class	Substitute	
Teacher	Right, let's substitute. Substitute *a* is…	
Learners	*a*	
Teacher	*a* is equals to…	
Class	Twelve	
Teacher	Add ten, is equals to thirty four. Right, now we have two multiplied by twelve (writes $2 \times 12 + 10 = 34$. Right, then we come back, we go back to the BODMAS, we have the multiplication sign, and then addition…	
Class	Sign	
Teacher	Right, according to the BODMAS, which one comes first, which sign comes first? Multiplication, and then addition sign, those two signs (*raises two fingers*). Yes	
Learner 5	Addition	
Teacher	Uthi sisi [the lady says], addition. Uthini [what are you saying]	
Learner 6	Multiplication	
Teacher	Uthi multiplication, now I want to go to the right one. This says addition, this one says multiplication, which one comes first, yes	
Learner 7	Subtraction	
Teacher	Do we have any subtraction sign on that sum?	

Class	No ma'am	
Teacher	Do we?	
Class	No	
Teacher	Yes	
Learner 1	Multiplication	
Teacher	The first sign which comes first is...	
Class	It's multiplication	
Teacher	We have a multiplication and a addition sign. According to the BODMAS, multiplication comes first. Which two numbers are we multiplying? Which two numbers we are multiplying? Two numbers we are multiplying, Two numbers we are multiplying on the chalkboard, two numbers, two numbers we are multiplying on the chalkboard. Yes, Sello	
Learner 8	Two multiplied by twelve	
Teacher	We are multiplying two multiplied by twelve, which is equals to	
Class	Twenty four	
Teacher	Twenty four add ten is equals to	
Class	Thirty four (*teacher writes* 24 + 10 = 34).	
Teacher	(*Inaudible*) you're going to do, is this the one. Please do the next one, it's one, two, three, four. Do this quick, number four, number four, and number five for me now, please. Make it snappy. Vuyo, five minutes, sheshisa, five minutes (*inaudible*), I'm giving you five minutes, I have given you two sums (*raises two fingers*). In your book, in your book. Nolu, in your book. Ten seconds, niphambili, nisakhuluma [you are in front, you are still talking]. Ten seconds in your book. Naniki. Shesha tu, shesha [rush please, rush], twenty seconds (*inaudible*). In your book, Vuvu, in your book. (*teacher goes from desk to desk talking to individual learners*). And we do two *a* add six is equals to thirty, two *a* add six is equals to thirty. Yes, work it fast. Two *a* add six is equals to thirty, and don't forget, *a* is equals to twelve, we are substituting, work it fast	
	(*Learners work on the problems and the teacher goes from desk to desk emphasising what the learners should do*)	

KHUMO'S LESSON, MAR 2012, JPL1, CYCLE 2			
Categories/Sub-categories			
Mode of Instruction	**LEVELS**		
Direct instruction	2	2	3
Whole-class discussion	1	1	1
Richness of the Mathematics			
Linking or Connections	1	1	1
Explanations	1	2	1
Multiple procedures or solution methods	1	1	1
Developing mathematical generalizations	1	1	1
Mathematical language	1	1	2
Overall richness of the mathematics	1	1	1
Working with Students and Mathematics			
Thorough Remediation of Student Errors and Difficulties	2	1	1
Responding to Student Mathematical Productions in Instruction	2	2	1
Overall working with students and mathematics	2	1	1
Errors and Imprecision			
Major mathematical errors or serious mathematical oversights	1	1	1
Imprecision in Language or Notation (mathematical symbols)	2	2	1
Lack of clarity	1	3	3
Overall errors and imprecision	2	3	3
Student Participation in Meaning-Making and Reasoning			
Students provide explanations	2	2	1
Student mathematical questioning and reasoning	1	1	1
Enacted task cognitive demand	1	1	1
Overall student participation in meaning-making and reasoning	1	1	1

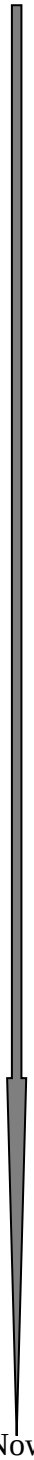
Appendix 4: True/False worksheet

Say whether each of the following statements is true or false and give reasons for your answer:

Statement	**True/False**	**Explanation**
1. $\frac{6+10}{2} = 3 + 10$		
2. $\frac{x+10}{2} = \frac{x}{2} + 10$		
3. $-(5)^2 = (-5)^2$		
4. $(3 + 1)^2 = 3^2 + 1^2$		
5. $(a + b)^2 = a^2 + b^2$		
6. $2^3 = 2 \times 3$		
7. $a^b = ab$		
8. $\frac{2a}{2} = a$		
9. $\frac{1}{3}x = \frac{x}{3}$		
10. $\frac{2}{5}y = \frac{2y}{5y}$		
11. $(xy)^2 = xy^2$		
12. $-\frac{1}{2} = \frac{-1}{2}$		
13. $a + a = a^2$		
14. -2 + 2 = 0		
15. -3 – 3 = 0		

Appendix 5: Worksheet on explaining the operational process

Look at the following statements and explain the operational process from one statement to the next. Simply explain how each statement is obtained from its previous one until you get to the last statement.



$a = 12$	
$a + 5 = 17$	
$2a + 10 = 34$	
$2a + 6 = 30$	
$3a + 6 = 30 + a$	
$a + 2 = \frac{30 + a}{3}$	

Now try to explain the process of the inverse operation, i.e. start from the last statement and move back explaining each step until you get to the first statement.

Appendix 6: Tasks on language of Algebra

Please do the following problems. The aim is to try to work with you and also understand your approach and thinking when answering mathematics questions that have language embedded in them.

1. Dineo is 10cm taller than Khotso. Khotso is *h* cm tall. What can you write for Dineo's height?

2. Three consecutive natural numbers add up to 24. Find the numbers.

3. Some money is shared between Mark and Jan so that Mark gets R5 more than Jan gets. Jan gets R*x*. Use algebra to write Mark's amount. The money to be shared is R47. Use algebra to work out how much Jan and Mark would get.

4. Cakes cost *c* cent each and buns cost *b* cent each. If I buy 4 cakes and 3 buns, what does $4c + 3b$ stand for.

5. Sally is ten years older than her brother. How old will the brother be in six years' time? **Hint: Assume a variable for Sally's age.**

6. A bus took students on a 3-day tour. The distance travelled on Day 2 was 85 km farther than on Day 1. The distance travelled on Day 3 was 125 km farther than on Day 1. The total distance was 1410 km. Let *x* stand for the number of km travelled on Day 1.
   Use algebra to work out the distance travelled each day.

7. Given that $a = 28 + b$. which of the following do you think would be true?
   7.1 *a* is greater than *b*
   7.2 *b* is greater than *a*
   7.3 $a = 28$

   Please provide an explanation on your choice for the answer.

8. The perimeter of a rectangle is 48 cm. If the length of the rectangle is three times the width, determine dimensions of the rectangle.