SECONDARY SCHOOL MATHEMATICS TEACHERS' IDENTITY AND MATHEMATICAL DISCOURSE IN INSTRUCTION

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DECLARATION

I declare that this thesis is my own work, except as indicated in the acknowledgements, the text and the references. It is being submitted in fulfilment of the requirements for the degree of Doctor of Philosophy (Mathematics Education) at the Wits School of Education of the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other university.

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ABSTRACT

More often than not, a disjuncture tends to exist between teaching practices that are encouraged during professional development (PD) interventions and what in-service teachers actually do when teaching mathematics. The study reported in this thesis uses the notion of teacher identity to examine in-service teachers' experiences of learning and their new ways of teaching mathematics after they had participated in a PD intervention called the Transition Maths 1 (TM1) course.

The theoretical framework for the study draws on Wenger's (1998) social theory of learning as a foundational framework, and on Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity frameworks to analyse teachers' mathematics teaching identity. The integration of Wenger's (1998) social theory of learning, Darragh's (2016) performative identity and Sfard and Prusak's (2005) narrative identity frameworks is a key contribution of this study to research teacher identity in the field of mathematics education. The inclusion of Darragh's (2016) performative identity framework harnessed Wenger's (1998) social theory of learning and Sfard and Prusak's (2005) narrative identity frameworks. Drawing on Wenger's (1998) to analyse teachers' identities in relation to what they actually do when teaching mathematics in the classroom was going to be limited for the study. Thus, the study has emphasised the learning and teaching of linear equations (a specific domain of mathematics), whilst researchers in mathematics education who draw from social theories and identity often render mathematics invisible.

The study employs an explanatory mixed methods research design. The use of the explanatory mixed methods research design and its elaboration in this study is another key contribution to researching teacher identity. In the quantitative processes, 45 teachers who participated in the TM1 course completed a closed-ended questionnaire. The questionnaire was analysed using Exploratory Factor Analysis to explore teachers' shared experiences of participating in the TM1 course, which demonstrated that the inclusion of the quantitative processes can be valuable to research teacher identity.

In the qualitative processes, four teachers were selected for observations when teaching learners mathematics and for individual interviews to talk about their learning and teaching of the subject. The observations were analysed using Mathematics Discourse in Instruction framework to understand the teachers' teaching practices. The interviews were analysed using narrative analysis to confirm and expand on the teachers' experiences of learning and teaching mathematics.

The findings of the study revealed that the teachers shared a positive sense of identity towards learning and teaching mathematics. The teachers' positive sense of identity emerged from being conscious of achieving lesson goals through exemplification and explanatory communication. However, the teachers were not paying much attention to how they invite learners to participate in their lessons.

The characterisation of the teachers in how they achieve lesson goals from their mathematical discourse in instruction became their actual teaching identity. The teachers' designated teaching identity highlighted aspects where there was a "mismatch" between their mathematical discourse in instruction and what was promoted in the TM1 course. Nonetheless, the gap between the teachers' actual and designated teaching identities remained relatively narrow when considering that there were fewer aspects where teachers were not competent in their mathematical discourse in instruction.

KEY WORDS

Community of practice, Narrative identity framework, Performative identity framework, Shared experiences of learning, Mathematical discourse in instruction, Teaching practices, Mathematics teaching identities

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ACRONYMS

AA	Alcoholics Anonymous (as a learning community)
ACE	Advanced Certificate in Education
ANA	Annual National Assessments
ATP	Annual Teaching Plan
BEd (Honours)	Bachelor of Education (with Honours)
BSc (Honours)	Bachelor of Science (with Honours)
DBE	Department of Basic Education
EFA	Exploratory Factor Analysis
GDE	Gauteng Department of Education
ICT	Information and communication technology
MDI	Mathematics Discourse in Instruction
ME	Mathematical episodes
MTF	Mathematics Teaching Framework
NA	Narrative analysis
NGOs	Non-government organisations
OoL	Object of learning
PD	Professional development
RQ	Research questions
SACMEQ	Southern Africa Consortium for Monitoring Education Quality
SPSS	Statistical Package for Social Sciences
SMT	School management teams
TIMSS	Trends in International Mathematics and Science Study
TM1	Transition Maths 1

CHAPTER 1

CONCEPTUALISING THE PROBLEM OF THE STUDY

1.1 Background to the study

The study reported in this thesis focuses on understanding teachers' identity that has been enhanced through the learning of mathematics in a professional development (PD) intervention called the Transition Maths 1 (TM1) course. The study seeks to examine secondary school teachers' experiences of learning and their new ways of teaching mathematics (if they have changed) after they had participated in the TM1 course. The study is based on the idea that the notion of identity can provide a fuller story in explaining how and why teachers change (or do not change) when engaging in new learning and teaching experiences (Chronaki, 2013). The majority of teachers who participated in the TM1 course are working in disadvantaged schools, and such schools serve children mainly from poor communities.

With the backdrop of the well-known legacy of apartheid in South Africa, social structures continue to outcast poor communities. Poor communities are closely associated with race and ethnicity, whereby Blacks in general and Africans in particular are mostly affected in the country. Poor communities in townships and informal settlements are facing inequality and poverty, and such woes have an impact on the education system (Adler & Pillay, 2017; Spaull, 2019). Education agents (e.g., policymakers in government departments, non-government organisations (NGOs), teachers, and researchers) are worried about challenges facing the learning and teaching of subjects like mathematics.

Setati-Phakeng (2017) asks the following questions to understand why challenges in mathematics education persist after so many years of research:

> Why is it that children from low socio-economic backgrounds suffer the most? What is the role of research and what can we expect it to contribute to mathematics teaching and learning? Is it reasonable to believe that research will help us solve the problems of teaching, particularly in developing countries such as South Africa?

Adler and Sfard (2017) acknowledge that these critical concerns and questions have been under consideration for decades. For Venkat (2017), institutions – such as universities where research and teacher PD activities are commonly located in many parts of the world – have been criticised for failing to work in ways that bring mathematics education research and teacher PD interventions together.

As a consequence, learners[1] bear the brunt of such challenges with many unanswered questions. For example, learners' performance in mathematics in South Africa remains at the bottom when compared to other countries. The overarching evidence of poor learners performance in mathematics surfaces when South Africa participates in international assessments such as the Trends in International Mathematics and Science Study (TIMSS) and the Southern Africa Consortium for Monitoring Education Quality (SACMEQ) (Spaull, 2019). For instance, Reddy et al. (2019) reported that the performance of Grade 9 learners from South Africa who participated in the TIMSS 2019 remained 'low' when compared to other countries. Whereas, although there were signs of improvements after 2007 (Venkat & Spaull, 2015), the results of Grade 6 learners have remained unsatisfactory in the years 2000, 2007 and 2013 when South Africa participated in the SACMEQ (Van der Berg & Gustafsson, 2019).

In South Africa, before the administering of the Annual National Assessments (ANA) ended in 2014, it confirmed that the standard of learners' performance in mathematics is low. For example, in 2014, the average score on the Grade 9 mathematics assessment in the ANA was 11% (Department of Basic Education (DBE), 2014). The DBE (2014) reported that only 3% of learners from disadvantaged schools obtained more than 50% in the mathematics assessment.

In defining disadvantaged schools for the present study from their functionality standpoint, I include Christie's (1998) arguments that such schools, generally secondary schools, are customarily experiencing:

> ...disrupted authority relations between principals, teachers and students; sporadic and broken attendance by students and often teachers; general demotivation and low morale of students and teachers; poor schools results… (p. 283).

Two teachers who participated in this study blamed the *progression policy* for deepening the problems of poor school results and demotivation of teachers in disadvantaged schools. The progression policy allows learners to progress to the next grades in spite of not having met the minimum requirements of passing the previous grades (DBE, 2011). One teacher elaborated as follows:

> A learner from Grade 5 is progressed to Grade 6 [without having passed Grade 5]. This year he was progressed to Grade 7; then progressed to Grade 8; Grade 8 progressed to Grade 9. Now when you look at our schools, township schools, the majority of learners are in Grade 10, because the chances of them to progress are very slim now (Oena[2], Interview, October 2019).

[1] In this study, I use the term 'learners' to refer to school students.
[2] All names of teachers in this study are pseudonyms.

In this extract, the teacher is talking about the “enrolment bulge in Grade 10” (Van der Berg et al., 2021, p. 6) which reflects high repetition rates in this grade. Van der Berg et al. (2021) also link the enrolment bulge in Grade 10 to the progression policy, whereby learners are not allowed to progress without having complied with all the promotion requirements in the senior secondary phase.

In general, disadvantaged schools are characterised by the continuing shortage of material resources such as libraries, laboratories and classrooms (Christie, 1998; Sedibe, 2011). For instance, NGOs such as *Section27* and *Education for All* that are focusing on reforming basic education in the country had to force the government (even through the Constitutional Court of South Africa) to provide disadvantaged schools with more updated mathematics textbooks that are aligned with the contemporary curriculum (Veriava, 2013). Vinjevold (1999) argues that the worst-case scenario is when available material resources are under-used by teachers in disadvantaged schools.

Whilst I acknowledge that more material resources are required in disadvantaged schools, as they can contribute to improve learner performance in mathematics, this study focuses on understanding how teachers (human resources) can adjust to educational demands when engaging in new ways of teaching the subject. As Magome and Nkosi (2014) argue, teachers can contribute significantly in improving the quality of teaching and learning in disadvantaged schools.

1.2 Teachers’ mathematical discourse in instruction

Researchers (e.g., Adler, 2005; Adler & Davis, 2006) assert that the quality of teaching mathematics can improve if PD interventions concentrate on improving teachers’ mathematics knowledge for teaching. Adler (2005) defines mathematics knowledge for teaching as “what mathematics teachers need to know and know how to do, in order to teach well” (p. 3). As Adler and Davis (2006) elaborate, teachers’ mathematics knowledge for teaching responds to the following two questions: what mathematics content do teachers have to know?; and what teaching practices do teachers have to know, in order to teach the mathematics content effectively? Taylor (2008) states that “teachers cannot teach what they do not know” (p. 24). At the same time, teachers should be able to incorporate different teaching practices from shared frameworks, principles and methods of teaching in order to facilitate effective learning whilst considering both the organisation of content knowledge and its transition when teaching in the classroom (Hoadley, 2012).

When Venkat and Adler (2012) characterised mathematics lessons that they observed as being incoherent for teaching and learning, the authors were concerned with the quality of

mathematics made available to learn in the classroom. On their further research, Adler and Venkat (2014) identified examples and accompanying explanations to be a source of incoherence when teachers are elaborating their mathematical ideas in the classrooms. Adler and Venkat (2014) described these two features of using examples and explanations as teachers' mathematical discourse in instruction.

Later, Adler and her colleagues (Adler, 2017; Adler & Ronda, 2015; Adler & Ronda, 2017; Ronda & Adler, 2017) developed the analytical framework called Mathematics Discourse in Instruction (MDI) – details about MDI follow in Chapter 6 when describing the analysis frameworks of the study. In brief, MDI entailed four features that became sufficiently nuanced to show differences in what is the same and different across the teaching practices of mathematics. Other than the two features of using examples and accompanying explanations, MDI included ideas of differentiating learner participation in a lesson. These features needed to work together for mathematics teaching to be coherent for teachers to achieve the object of learning. The object of learning became the fourth feature of the MDI framework.

Parallel to the research, in 2013, Professor Jill Adler led a team of researchers in the project called Wits Maths Connect Secondary project (details about the project follow in Chapter 4 when providing the context of the study) to adapt the MDI framework to produce a teacher-friendly version called the Mathematics Teaching Framework (MTF). The MTF, and all its features, were introduced to teachers as a central component to the PD intervention – the TM1 course. The MTF became part of a strategy to improve their mathematics knowledge for teaching (Adler, 2013). In brief, the MTF responded to the question: what teaching practices do teachers need to know, in order to teach the mathematics content effectively and coherently? As a result, according to Adler (2013), teachers needed to be inducted on the following four aspects of the MTF:

- The selection and adaption of applicable example sets, tasks and representations (i.e. **exemplification**);
- The promotion of more opportunities for **learner participation**;
- The adoption of appropriate **explanatory communication**; and
- The commitment in ensuring that these all work together to support the **object of learning** when teaching mathematics.

Through the TM1 course, teachers were expected to incorporate the aspects of the MTF in their teaching of mathematics. For example, rather than to make somewhat arbitrary selections of examples from textbooks, teachers were encouraged to deliberately structure the selections of examples in order to promote generalisations of concepts. The incorporation

of the MTF in teachers' teaching practices was intended to be recognised by teachers themselves and by others, which defines teachers' identity.

The TM1 course focused more on mathematics content than on how to teach the content. But the MTF/MDI strongly informed the design of the mathematics component and this was brought to teachers' attention throughout the course. Experiences of learning mathematics content (alongside teachers' goals, beliefs, motivation and attitude) are influential towards teacher identity (McGee & Martin, 2011; Van Zoest & Bohl, 2005), but they should not be considered identities themselves (Goos, 2013). For example, including how teachers were selected to participate in the TM1 course, teachers can gain access to participate in PD interventions by demonstrating some capabilities in their mathematics content knowledge. If teachers can gain new knowledge from the opportunities provided during PD interventions, they can develop stronger relationships with the subject (Boaler, 2002; Kaasila, Hannula & Laine, 2012), which means teachers' mathematical identity.

1.3 Teachers' mathematics teaching identity

Many models of teachers' identities are described in the literature depending on the type of influences and effects foregrounding mathematics contexts. In the TM1 course, teachers were primarily learners of mathematics themselves despite already being professional teachers, which meant that there were opportunities to develop both their mathematical identity and professional identity. Researchers (e.g., Kim, 2018; Ntow & Adler, 2019) have combined teachers' mathematical identity and professional identity, and referred to this model as *teachers' mathematics teaching identity.* Similarly, this model is in focus for the present study.

I use this teachers' mathematics teaching identity model to look at how teachers are expressing their learning experiences after participating in the TM1 course and how their learning of mathematics translates into 'repeated actions' (further meaning of this term becomes apparent in Chapter 2) when they are teaching in the classrooms. Therefore, the teachers' mathematics teaching identity model focuses on the TM1 course as an immediate context of describing learning and teaching of mathematics, which enable the teachers who participated in the TM1 course to be characterised differently from other teachers who have not participated in the course.

Figure 1.1 illustrates my thinking behind teachers' mathematics teaching identity. In thinking about teachers' mathematics teaching identity, I considered the definition of mathematics knowledge for teaching, which suggests that teachers' content knowledge can form a basis of being part of the teaching profession. In this sense, teachers' mathematical identity became a

subset of their professional identity and both models of identity became subsets of teachers' mathematics teaching identity.

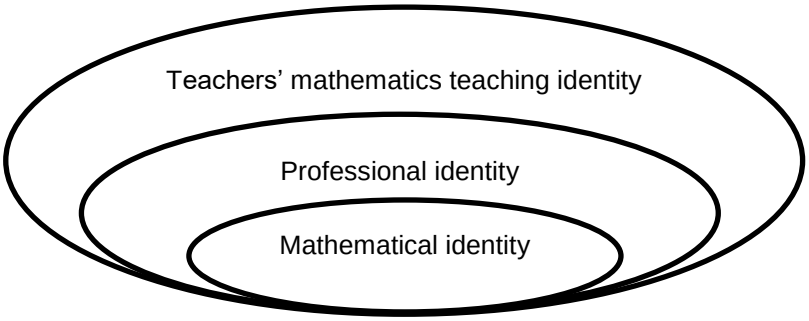


Figure 1.1: Illustrating my thinking of teachers' mathematics teaching identity

Models of identity have mostly received attention independently in the literature. For example, teachers' mathematical identity has been a topic of great interest to educational researchers such as Brown, Jones and Bibby (2004), Kaasila (2007) and Stinson (2009). The context of teachers' mathematical identity resonates from perceptions that mathematics is a difficult subject to learn and to teach (Brodie, 2011; Lutovac & Kaasila, 2018). Despite its perceived challenges, teachers' mathematical identity should form bases of being a mathematics teacher. As Taylor (2008) askes: how can teachers teach something they do not know? Therefore, at the least, teachers can identify themselves to be proficient in mathematics content.

Moreover, the phenomenon of teachers' mathematical identity can be defined as the recognition teachers develop about their capabilities of teaching once they have participated in mathematical activities (McGee, 2014). Teachers' mathematical identity is then referred to how teachers make meaning of mathematical activities in their professional settings (such as professional development groups, schools, classrooms) when addressing the experiences that they endure in their decision-making of learning and teaching of the subject (McGee, 2015). In making decisions that are acceptable to the community of mathematicians and/or mathematics teachers, Nasir and Cobb (2002) stress that teachers should consider social contexts rather than as something they have inside themselves. In this way, teachers can have opportunities to justify and explain themselves in relation to other people with regards to the social contexts they act upon (MacLure, 1993).

In the context of the mathematics teaching profession, for example, teachers' professional identity changes are influenced by a range of factors both internal to the individual, such as beliefs (Rodgers & Scott, 2008), and external to the individual, such as career and life experiences (Flores & Day, 2006). For instance, there are teachers who place limits on their commitment to teaching fearing that the complexities of the profession can take over their lives (Day, Elliot & Kington, 2005), whilst other teachers conduct after-school classes and even

work on weekends. Thus, teachers' professional identity refers to those changes that are measured by how teachers carry themselves, or are expected to carry themselves, within the norms of the teaching profession and how teachers should work as determined by the government policies (Jansen, 2001).

Sachs (2005) places teachers' professional identity at the centre of the teaching profession. The author elaborates as follows:

> It [teacher identity] provides a framework for teachers to construct their own ideas of "how to be", "how to act" and "how to understand" their work and their place in the society. Importantly, teacher identity is not something that is fixed nor is it imposed; rather it is negotiated through and the sense that is made of that experience (Sachs, 2005, p. 15).

Beauchamp and Thomas (2009) endorsed the above definition which encourages teachers to resume agentic positions in "how to be", "how to act" and "how to understand" the norms and standards of the teaching profession. In Sachs's (2005) terms, teachers need to play an active role in negotiating their roles in the profession.

The Sachs's (2005) assertion on teachers' professional identity further highlights the importance of understanding teachers' identity to be a constantly evolving phenomenon. Teachers' identity changes over time and across different social contexts in which teachers operate (Beijaard, Meijer, & Verloop, 2004). For example, in recent times, low performing schools, which are mostly disadvantaged schools, within the Gauteng Department of Education (GDE) in South Africa need to comply with the Annual Teaching Plan (ATP) – I elaborate further on the relevance of the ATP when discussing the significance of the study. In brief, the ATP is a government strategy that compels teachers to follow scripted lesson plans when teaching mathematics. However, even within this culture of bureaucratic compliance towards curricula (Adler, 2017), teachers can evolve (Sachs, 2005), and re-orientate themselves towards an exciting future where they have an equitable opportunity to contribute in the profession alongside other group members of the schooling system (e.g., school management teams, district officials, and researchers that are focusing on PD interventions).

In broad terms, the study looks at the impact of the TM1 course as the PD intervention by understanding teachers' experiences of learning and teaching of mathematics in the profession. However, focusing on the TM1 course does not mean denying the effects of other learning and teaching contextual influences for the teachers. As Wenger-Trayner et al. (2015) argue, practitioners cross boundaries, move between different regimes of competence, and enact their multimembership in different communities whilst their diverse identities are modulated in each context. For example, teachers who participated in the TM1 course brought

along their prior constructed and different selves to the course (personal identity). The same teachers are obligated to teach under the predetermined schooling policies like other teachers (institutional identity), and they are facing the similar social challenges other teachers face when teaching in disadvantaged schools (social identity).

Other forms of social identity such as age, race, sexual orientation, religion and regional backgrounds are also not denied to emerge in this study. Carrim (2001) asserts that looking at teachers as just professional workers without considering other forms of social identity is not enough to research teacher identity. For example, in South Africa, when focusing on teachers' ages and their years of service in the teaching profession, one can identify with teachers who received teaching qualifications during apartheid and those teachers who qualified to teach post-apartheid when teaching subjects like mathematics (Adler & Davis, 2006). In another example, as South African school system is characterised to consist of two different functioning sub-systems (Van der Berg et al., 2011), one can distinguish between White teachers who are mostly teaching in advantaged schools and Black teachers who are mainly teaching in disadvantaged schools when teaching in the classroom. Thereby, as much as this study focuses on examining teachers' mathematics teaching identity, it cannot be ignored that teachers are first and foremost human beings with plural, dynamic, sometimes contradictory and complex forms of identities that are operating simultaneously within single individuals (Carrim, 2001; Lerman, 2012).

1.4 The problem statement

In this study, teachers who had participated in the TM1 course are expected to incorporate the aspects of the MTF as teaching practices when teaching mathematics. The problem is that, more often than not, a disjuncture tends to exist between teaching practices that are encouraged during PD interventions and what in-service teachers actually do in mathematics classrooms (Gainsburg, 2012; Kazemi & Hubbard, 2008; Kim, 2018). From the literature (e.g., Andersson, 2011; Edwards, 2000), teachers become motivated to try out new ways of teaching after participating in PD interventions, but tend to experience difficulties in succeeding to incorporate and to sustain those new ways in their teaching of mathematics.

Evidence of a disjuncture has been more reported in the prevailing discourses of expecting teachers to transition from using traditional methods to using reformed-oriented methods when teaching mathematics (Gainsburg, 2012; Munter, Stein & Smith, 2015). For example, Elin, a mathematics teacher in Andersson's (2011) study elaborated as follows:

> It feels as if it always is there [where] I and the students end up. We start off with, for example, practical mathematics but then, suddenly, I stand there again, with a whole class explanation and think wait, what happened now, how did we get here? (p. 437)

In Edwards' (2000) view, teachers slide back to their old ways of teaching because they are used to teaching in a certain way. According to Darling-Hammond, Hyler and Gardner (2017), many PD interventions appear ineffective in supporting and sustaining changes in teachers' teaching practices.

A disjuncture can be situated in different contexts of learning and teaching mathematics. If one imagines an onion, the problem can be at the bud, of which I can associate the bud with teacher content knowledge. If teachers lack content knowledge (Taylor, 2008), the difficulties of teaching can be associated with how do they feel about learning new content knowledge. For example, if their feeling of learning new content knowledge is manageable, those teachers could continue to strive to gain access to learning more mathematics content (Hodgen & Askew, 2007; Gujarati, 2013). The second layer can be associated with knowledge of teaching practices. If teachers lack knowledge of teaching practices (Hoadley, 2012), and when given opportunities, the successes of teaching can be associated with their willingness to participate in learning more about different teaching practices. The last layer can be associated with PD interventions that are intended to improve teachers' mathematical knowledge for teaching and to change their teaching practices. The PD interventions can aim to support teachers to adjust to educational demands of teaching in the classroom (Gainsburg, 2012). Whichever way, the problem cannot sit in one layer and not cause problems for the whole onion.

In understanding the problem within the layer of PD interventions, studies (e.g., Borko, 2004; Desimone et al., 2002; Edwards, 2000) look at limitations within different kinds of PD interventions. Borko (2004) points to the limitations of using training workshops as learning contexts for in-service teachers. The limitations of training workshops include the following facts:

- teachers do not have means to incorporate what they have learned because of the highly prescriptive teaching contexts (Shalem & Hoadley, 2009);
- the durations of learning are not long enough for teachers to get a full sense of how to incorporate what they have learned in their teaching (Desimone et al., 2002); and thus,
- the interventions are often characterised as being intellectually superficial, farfetched, fragmented, noncumulative and disconnected from deep issues of curriculum and learning (Borko, 2004; Cohen & Hill, 2000).

Despite these limitations, government departments of education in South Africa utilise district officials to conduct different PD interventions in the form of training workshops on mathematics focusing on different topics and concepts to support the teachers.

The impact of training workshops as PD interventions falls outside the scope of this study. However, as one teacher who participated in this study was explaining how the TM1 course was different from other PD interventions, he[3] expressed the following:

> With them [district officials], they concentrate more on the content. “Teachers don’t know this, they say it like that. They just don’t know this. We’re going to teach them this.” Hence, sometimes, if you know it, you feel like you’ve wasted your time because now they’re teaching you [the content knowledge]. I would always go to these trainings and say, I know this, but I want them to show me how to present it to my learners, because I know this but the way I’m presenting it, they [learners] are not getting it (Makwe, Interview, October 2019).

In this regard, as the outcomes of training workshops and short training courses, studies in the literature (e.g., Cohen & Hill, 2001; Desimone et al., 2002; Van der Berg et al., 2011) contend that in-service teachers could learn effectively from PD interventions when the following features are kept in mind:

- The durations of PD interventions are extended to be weeks of participation per year (Van der Berg et al., 2011);
- The learning of mathematics content is intensified and kept central in the intervention (Cohen & Hill, 2001); and
- The teaching practices are specific in their intentions in order to increase teachers’ use of those practices in their classrooms (Desimone et al., 2002).

Indeed, when considering a variety of tensions that exist in realising PD interventions (Adler, 2002; Graven, 2005), Adler (2013) understood how to manage such tensions when deciding on which aspects to take to account when designing the TM1 course within the limited resources. Ultimately, outside the logistics of designing the course, Adler and her research team focused on inducting teachers to the aspects of the MTF and on improving their content knowledge (see Adler, 2013), which meant learning *mathematics-for-teaching*. In the literature (e.g., Adler, 2005; Adler & Davis, 2006; Pournara, 2013), mathematics-for-teaching (MfT) is described as a combination of content knowledge and pedagogic content knowledge without stressing too much about the boundaries between the two notions.

1.5 Purpose of the study

This study examines what the teachers incorporate (or not incorporate) in their teaching of mathematics after they had participated in the TM1 course. In other words, the study aims to determine which aspects of the MTF as teaching practices do the teachers incorporate (or not incorporate) when teaching mathematics in the classrooms. Additionally, the study seeks to

[3] I decided to think of teachers who had participated in the study as males to keep their identity a secret.

determine how the teachers teach mathematics from their improved MfT after participating in the TM1 course. What the teachers incorporate (or not incorporate) when teaching mathematics should emerge from: (1) what they were agreeing or disagreeing with regarding the learning and teaching of mathematics; (2) what they were saying or not saying about learning and teaching mathematics; and (3) what I observed they were doing or not doing when teaching the subject in the classrooms.

Prior to this study, in 2016-2017, Ntow and Adler (2019) worked with two contrasting teachers who participated in the TM1 course to link their teaching practices with their identities. The authors recalled the work of Nasir and Cooks (2009) which focused on research-linked identities and identity resources (ideational, material and relational) to explore how teachers' teaching identities in mathematics develop in PD interventions. The use of Nasir and Cooks (2009) provided insights into describing the *what* and *how* of the PD interventions that can influence mathematics teachers' identities. Ntow and Adler (2019) argued that teachers' participation in the TM1 course motivated the teachers to implement certain teaching practices in their classrooms. That is, the authors reported on the take-up of key components of the MTF, which were exemplification and learner participation.

For this study, I turn to Wenger's (1998) social theory of learning, Sfard and Prusak's (2005) narrative identity, and Darragh's (2016) performative identity frameworks, and ask the same question: what do teachers who have participated in the TM1 course identify with when teaching mathematics in their classrooms? This question followed the question that was asked to teachers towards the end of the TM1 course, which was: What brings you joy in the learning and teaching of mathematics? From reading the responses, teachers were concerned about the following constructs: understanding of mathematics content, confidence in the learning and teaching of mathematics, and accountability towards teaching. Hence, as a start, the study quantitatively explores the extent of teachers' interpretations of such constructs regarding the five interconnected components of learning (identity, practice, confidence, community and meaning) (Graven, 2003; Wenger, 1998) before expanding (or questioning) their conceptions of learning and teaching mathematics using qualitative processes. Thereby, in this sense, the study employs a mixed methods research.

In the qualitative processes, I focus on analysing teachers' identity from two conceptual approaches. Firstly, I analyse teachers' identity as experiences of learning and teaching mathematics. The teachers were expected to draw from their past experiences to clarify their *shared experiences* of learning and teaching mathematics after participating in the TM1 course. The assumption was that *shared experiences* of teachers developed from participating

in the TM1 course, and that those experiences can be understood when teachers as a collective construe views about learning and teaching of mathematics in different contexts.

Secondly, I examine how teachers connect with the four aspects of the MTF (the objects of learning, exemplification, learner participation and explanatory communication) in fostering (or sustaining) coherence in their learning and teaching of mathematics. Here, the analyses focus on how teachers interact with learners when they teach mathematics, how they talk about the MTF, and how they describe themselves as mathematics teachers.

The following research questions guide the study:

1) To what extent do teachers have shared experiences of learning and teaching mathematics after participating in the TM1 course? What changes (if any) have shaped the shared experiences for the teachers?
2) In what ways (if any) does the teachers' learning in the TM1 course connect to their mathematics teaching identities?
3) What are the connecting features (if any) between the teachers' mathematical discourse in instruction and their mathematics teaching identities?

I deliberately kept the first part of research question 1 open and presumed that the teachers' shared experiences of learning and teaching from the TM1 course would include views about their mathematics teaching identity. The research question 2 encapsulated the two objectives of the study (i.e. to determine the extent of the teachers' experiences of participating in the TM1 course and how they incorporate certain aspects of the MTF in their teaching of mathematics). The research question 3 automatically offers an opportunity to describe how the *teachers' mathematical discourse in instruction* (i.e. teachers' competence of teaching from the MTF perspective) connect with their mathematics teaching identity.

To use the phrase 'teachers' mathematical discourse in instruction', I draw on Adler and Venkat (2014). The authors initially used the term to describe teachers' use of examples and accompanying explanations when teaching mathematics. However, in this study, I include how teachers invite learners to participate in lessons and how they ensure that these all work together to support the object of learning when teaching mathematics. Thus, in simple terms, teachers' mathematical discourse in instruction means teachers' competence of teaching mathematics from the MTF perspective as analysed using the MDI framework.

1.6 Significance of the study

The results from this study are significant for a number of ways. In this section, I firstly discuss the significance of the results of the study in relation to the problem statement. Secondly, I

highlight the significance of including the quantitative processes when researching teacher identity. Lastly, I elaborate on how teachers' identities can be connected to practices of teaching mathematics.

Regarding the problem statement, the study contributes in explaining a disjuncture that tends to exist between teaching practices that are promoted in PD interventions and what in-service teachers actually do when teaching mathematics. The notion of teacher identity can shed light on which teaching practices are taken up (or not taken up) by the teachers after their participation in the TM1 course. In order to claim that certain teaching practices are taken up, the teachers are expected to demonstrate aspects of the MTF in their teaching of mathematics. Thus, this study intends to explain *what* and *how* the teachers had learned in the TM1 course that have contributed in their new ways of teaching mathematics, as reported by the teachers themselves.

In explaining *what* and *how* the teachers learned in the TM1 course, I highlight areas of improving teacher knowledge that district officials in the field of mathematics education can focus on in their PD interventions. That is, in the TM1 course, the teachers participated in learning about the MTF, which influenced their teaching practices. The study provides evidence that changing teaching practices changes teachers' mathematics teaching identity. In addition, the teachers attended the TM1 course to learn MfT. The study shows that the teachers' experiences (alongside other emotional factors) of learning MfT have been recognised to contribute in teachers' mathematics teaching identity. Whereas, in Chapter 4, I argue that the teachers' participation in the TM1 course resembles participating in a community of practice[4]. In this regard, the study explains other means of improving learning of mathematics beyond informal and unintended means of learning that Wenger (1998) posits to be central for members of communities of practice.

The inclusion of the quantitative processes when researching teacher identity contributes in providing context to teachers' stories about their experiences of learning and teaching from the empirical results. When characterising teachers from how they participate in a community of practice, the question arises: how common are discourses about the practice to the community of those teachers? For example, the results from the quantitative processes of this study show that the teachers were located in the matrices of how much they are accountable towards teaching mathematics versus their positive sense of identity after learning about the

[4]For now, a community of practice can be understood as a perspective that can be used to view a group of people who are doing similar work (Storberg-Walker, 2008; Wenger, 1998). In the next chapter, I provide further details to define a community of practice.

MTF. Therefore, this background provided an opportunity to further explore relationships of these matrices using the qualitative processes, which implies mixed methods research.

Lastly, the study contributes in illuminating teachers' identities that can be connected to their teaching practices, which can provide in-service teachers with opportunities to learn mathematics. For example, the findings of this study show that the teachers who follow the lesson plans as prescribed in the GDE ATP documents do not change mathematical tasks when teaching mathematics. In the study, I highlight teacher knowledge as an enabling feature for teachers to change mathematical tasks, particularly to include tasks of higher cognitive levels when teaching mathematics. In this regard, district officials can introduce learning interventions that can focus on how tensions between what is mandated in the GDE ATP documents and what secondary school teachers are experiencing in PD interventions like the TM1 course can be minimised for coherent teaching of mathematics.

1.7 Organisation of the chapters of the thesis

In this chapter, I firstly provided the background of the study. Then, I defined the notion of teachers' mathematics teaching identity before subsequently elaborating on the problem statement of the study, its purpose, and its significance towards the field of mathematics education.

In the next chapter, I broadly locate the theoretical framework of the study within a socio-cultural perspective. This leads to the selection of Wenger's (1998) social theory of learning as a foundational theoretical framework of the study. Wenger's (1998) theory is supported by Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity frameworks, particularly to analyse different data sets of the study.

In Chapter 3, I review studies that I selected to highlight different theoretical foci for this study. The review of the selected studies is structured using a theoretical format that was put forward by Randolph (2009). That is, I discuss a group of selected studies that have used a similar theory. The first group of the selected studies used Wenger's (1998) theory, whilst the other two groups used Sfard and Prusak (2005) and Darragh (2016) respectively.

In Chapter 4, I start by describing key features of the TM1 course. Then, I argue that the TM1 course is a community of practice. In this sense, I claim that learning about the MTF informed the teaching practices, whilst the teacher community was undergirded by experiences of learning MfT in the TM1 course.

In Chapter 5, I provide the research methods for the study. The study employs an explanatory mixed methods research design (Creswell & Plano Clark, 2007, 2011). Thus, I give details on how both quantitative and qualitative processes were used to collect data for the study.

In Chapter 6, I present data analysis frameworks for the study. In the quantitative processes, the closed-ended questionnaire was analysed using Exploratory Factor Analysis. In the qualitative phase, the lesson observations were analysed using Mathematics Discourse in Instruction. The interviews were analysed using narrative analysis.

In Chapter 7, I use the results from the quantitative processes to respond to the research question 1 by explaining the extent of teachers' shared experiences of participating in the TM1 course. In the quantitative processes, 45 teachers who participated in the TM1 course completed a closed-ended questionnaire.

In Chapter 8, I discuss the results of what have changed in influencing the teachers' shared experiences of learning and teaching mathematics. The evidence of changes emerged from four teachers who were selected and interviewed to talk about their learning and teaching of the subject before and after they participated in the TM1 course.

In Chapter 9, I present the results on the teachers' mathematical discourse in instruction from the lesson observations. Before participating in the interviews, the four teachers were observed teaching Grade 9 algebraic equations in the classrooms. Four consecutive lessons per selected teacher were observed, at least for three teachers, and the analysis of data focused on one lesson out of the four lessons.

In Chapter 10, I expand (and question) the results from Chapter 9 by including details on what the teachers do when teaching mathematics in the classroom. The results in this chapter rely on what the teachers said they do during the interviews when teaching the subject versus what I had observed they do at the least when teaching algebraic equations.

In the last chapter, I provide connecting features between the teachers' learning in the TM1 course, their mathematical discourse in instruction, and their mathematics teaching identities. The chapter also presents implications and limitations of the study.

CHAPTER 2

THEORETICAL FRAMEWORK FOR THE STUDY

2.1 Introduction

In this chapter, I outline two broad theoretical perspectives that are influential in the literature (e.g., Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Grootenboer, Lowrie & Smith, 2006) when employing the notion of identity to conduct research. Having highlighted socio-cultural perspectives on teacher identity to be prominent in mathematics education, I discuss Wenger's (1998) social theory of learning as a foundational framework for the study. When considering that Wenger (1998) locates his theory within the apprenticeship context, I then discuss the challenges of employing Wenger (1998) within the education context. Given that the study is in education, I extend Wenger (1998) to include the notion of confidence as Graven (2003, 2004) discovered that paying attention to teacher confidence is necessary when examining teacher identity in mathematics education.

Furthermore, given Wenger's (1998) descriptive stance on how identities develop when individuals participate in communities of practice, I elaborate on Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity frameworks respectively. The two frameworks support research analyses for this study as I concur with Essien (2013) that it becomes challenging to use Wenger's (1998) theory without drawing from other definitions of identity that have been operationalised for mathematics education. To end the chapter, I discuss how the theoretical framework for the study (i.e. Wenger, 1998; Sfard & Prusak, 2005; Darragh, 2016) informs teachers' mathematics teaching identity within a socio-cultural perspective.

2.2 Theoretical perspectives on identity

There are two broad theoretical perspectives that are influential in the literature when researching identity in mathematics education: (1) psychological and (2) socio-cultural (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Grootenboer et al., 2006). According to Darragh (2016), the notion of identity has its origins in the work of Erikson (1968) and Mead (1934). For Erikson, identity was grounded on psychological perspectives, and was understood to be more developing through one's life (Graven & Heyd-Metzuyanim, 2019). Whereas for Mead, identity was grounded on socio-cultural perspectives, and was upheld as multiple identities within social and cultural contexts (Darragh, 2016). These broad theoretical perspectives have been characterised in the literature to be distinct and incompatible when researching identity (Darragh, 2016).

Reviewers of the literature (e.g., Beauchamp & Thomas, 2009; Grootenboer et al., 2006; Langer-Osuna & Esmonde, 2017; Lutovac & Kaasila, 2018) acknowledge that creating dichotomies between theoretical perspectives on identity can be problematic as there are many connections and tensions between them. However, due to the complexity of identity research in the field of mathematics education (Grootenboer et al., 2006), Darragh (2016) encourages researchers to employ the two theoretical perspectives independently when conducting studies on identity. Below, I discuss the two theoretical perspectives on identity and connect them to mathematics education, but I elaborate more on socio-cultural perspectives since I adopt this theoretical lens for my study.

2.2.1 Psychological perspectives

Grootenboer et al. (2006) put forward that identity formation stemming from psychological perspectives focus on the individual. In psychological perspectives, identity can be understood as one's perception of one's abilities of what one can do (Grootenboer et al., 2006). Identity becomes a notion of how individuals can compete when searching for their internal integrity and autonomy in life (Grootenboer et al., 2006). From these perspectives, individuals name and characterise themselves based on how they can adapt to fit with norms of certain social groups (Boaler, Wiliam & Zevenbergen, 2000).

Reviewers of the literature (e.g., Graven & Heyd-Metzuyanim, 2019; Lutovac & Kaasila, 2018) point out that there are fewer studies using psychological perspectives to research identity in mathematics education when compared to those using socio-cultural perspectives. Lauriala and Kukkonen (2005), who recognise identity and self-concept as the same notion, elaborate that self-concept comprises of the actual self (the one that currently prevails), the ought self (the one recognised by society or external group as the goal), and the ideal self (the one set by individual as possible target for achievement), whereby these different selves form personal identity. Beauchamp and Thomas (2009) explain that research on personal identity fits well with learners. In this sense, as Gardee and Brodie (2022) postulate, identity of learners can be stable and dynamic at the same time.

Rodgers and Scott (2008) concede that personal identity of teachers in its completeness remain unknowable, but can enable individuals to consciously or unconsciously make meaning out of it when interacting with others in social and cultural contexts. As such, which explain why I shy away from emphasising personal identity, some researchers who drew on both psychological and socio-cultural perspectives (e.g., Hall, Towers & Martin, 2018) have somehow remained in foregrounding their studies on socio-cultural perspectives (Graven & Heyd-Metzuyanim, 2019). Nonetheless, Lutovac and Kaasila (2018) call for a balanced psycho-social approach when researching teacher identity in mathematics education. In

proposing the psycho-social approach, Lutovac and Kaasila (2018) cite Hodges and Cady (2012) who posit for "dual focus on the individual's thinking and the influence of collective experiences in understanding the construction of an identity as a mathematics teacher" (p. 114).

2.2.2 Socio-cultural perspectives

Researchers (e.g., Darragh, 2016; Grootenboer et al., 2006) contend that identity formations emerging from socio-cultural perspectives focus on the interactions between the individual, culture and society. That is, identity gives a "sense of oneself as a participant in the social roles and positions defined by a specific, historically, constituted set of social activities" (Holland & Lachicotte, 2007, p. 104). As such, identity describes individuals within shared cultural experiences and through their social structural contexts (Beauchamp & Thomas, 2009).

Within socio-cultural perspectives, many researchers have accordingly aligned their studies of mathematics teacher identity to Gee (2001), Sfard and Prusak (2005) or Wenger (1998). Heyd-Metzuyanim, Lutovac and Kaasila (2016) clarify that the definitions of teacher identity usually go hand in hand with theoretical frameworks depending on identity constructs and social contexts at play within mathematics education research. From the definition of teachers' mathematics teaching identity in the previous chapter, this study focuses on Wenger's (1998) postulation of participative identity alongside Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity respectively – details of the theoretical frameworks follow in the next sections of this chapter.

Based on a strong emphasis on how much social structures determine individuals' behaviour when learning, Wenger's (1998) social theory of learning has contributed in various studies to theorise about the learning and teaching of mathematics (e.g., Anderson, 2007; Boaler et al., 2000; Chauraya, 2013; Essien, 2014; Graven, 2003, 2004; Grootenboer & Edwards-Groves, 2019; Nel, 2012; Sfard & Prusak, 2005; Solomon, 2007; Van Zoest & Bohl, 2005). In the next chapter, when reviewing the literature of the study, I discuss some studies that have employed Wenger (1998), given that his theoretical framework has proven to be important in researching both in-service and pre-service teachers' identities when they are participating in professional development (PD) interventions. In this sense, Wenger's (1998) social theory of learning became a foundational framework for the present study.

For Wenger (1998), learning can be conceptualised as a product and a process of identity alongside meaning, community and practice. Drawing on Wenger (1998), researchers (e.g., Solomon, 2007; Van Zoest & Bohl, 2005) understood identity as cognitive and emotional

reactions that shift from time to time whenever mathematics is made accessible for learning through active participation that encourages negotiation of meaning and ownership of knowledge. As a consequence, the locus of identity becomes a process of individuals becoming who they are when participating in practices of professional groups (Cobb, Gresalfi & Hodge, 2009). Hence, we define who we are by what we have learned from participating in different communities of practice (Wenger, 1998; Wenger-Trayner et al., 2015), and the literature in mathematics education (e.g., Darragh, 2016; Lutovac & Kaasila, 2018) has referred to this description as a 'participative identity'.

However, many researchers in mathematics education (e.g., Chauraya, 2013; Essien, 2014; Graven, 2003, 2004) have acknowledged that using Wenger's (1998) social theory of learning can present many challenges when applying his theory to the education context, considering that the theory was situated within the apprenticeship context. A main challenge of using Wenger's (1998) theory is his stance of not going beyond describing the central importance of interactions of participants within societal groups – I discuss the challenges in Section 2.4 below after providing an account of Wenger's (1998) theory in detail. Nonetheless, in this regard, as I highlighted in the introduction of this chapter, Essien (2013) argues that it becomes challenging to use Wenger's (1998) theory without drawing from other definitions of identity that have been operationalised for mathematics education when researching mathematics teacher identity. Operationalised definitions of identity in mathematics education necessitate setting out multi-level analytic tools for characterising teachers as both learners of mathematics and teachers of mathematics (Essien, 2013).

In building on the body of knowledge from Wenger (1998) whilst operationalising for mathematics learning and teaching, many studies within the domain of mathematics education (e.g., Andersson, 2011; Bennison, 2015; Bjuland, Cestari & Borgersen, 2012; Gujarati, 2013; Sfard, 2019) adopted Sfard and Prusak's (2005) perspective on identity. Similar to Wenger (1998), I discuss some studies who have employed Sfard and Prusak (2005) in the next chapter. Sfard and Prusak (2005) describe identity as sets of stories that people tell about themselves and others tell about them. Thus, Sfard and Prusak (2005) is necessary to analyse data for teachers' mathematics teaching identity when considering that they get to talk about their experiences of learning and teaching mathematics. In the literature (e.g., Darragh, 2016; Lutovac & Kaasila, 2018), Sfard and Prusak's (2005) framework is referred to as the 'narrative identity'.

Sfard and Prusak's (2005) narrative identity framework contributes in bridging the context of learning mathematics in the TM1 course and the context of teaching the subject in the classrooms when analysing teachers' mathematics teaching identity. During the TM1 course,

as mentioned, teachers were positioned as learners of mathematics. As teachers get opportunity to narrate their stories when reflecting on learning in the TM1 course in the study, they could also talk about their learning and teaching experiences before, during and after the course. In this sense, when teachers talk about their learning after the course, they can be asked to include talking about dynamics of teaching mathematics in the classrooms.

Another part of analysing teachers' mathematics teaching identity was to observe lessons in the context of teaching mathematics in the classrooms. Radovic et al. (2018) argue that individuals are judged from their repetition of actions over time. In this regard, teachers' mathematics teaching identity becomes the observable actions and interactions with learners (Darragh, 2016). The inclusion of Darragh (2016) in particular assists this study to expand (or question) teachers' performances when narrating their teaching practices from the Mathematics Teaching Framework (MTF) perspective within the information that is gathered during the lesson observations.

Ultimately, from the theoretical framework for the study, teachers' mathematics teaching identity becomes how they participated when learning mathematics or when learning to teach mathematics, how they narrate their participations from the TM1 course, and how they narrate their actions when teaching in the classrooms. Darragh (2016) refers to such framework as a 'performative identity'. In the next chapter, I discuss Darragh (2015a, 2015b, 2018) as these studies have employed Darragh's (2016) performative identity framework. Unlike Wenger's (1998) participative identity and Sfard and Prusak's (2005) narrative identity frameworks, which are more frequently used frameworks, Darragh's (2016) performative identity framework has not been frequently used by other researchers in mathematics education.

In summary, as I am arguing from a socio-cultural perspective, this study employs Wenger's (1998) social theory of learning as the foundational framework alongside Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity frameworks as its theoretical framework. This theoretical framework has an ability of connecting learning experiences of individuals and their learning in practice. When connecting teachers' experiences of learning in the TM1 course and their enactment of teaching mathematics in the classrooms, teachers who participated in the course can be characterised differently from other teachers who have not participated in the course.

However, whilst this study does not include Gee (2001) in its theoretical framework, it cannot be ignored that many studies in mathematics education (e.g., Bennison, 2015; Gujarati, 2013; Kasten, Austin & Jackson, 2014; Hodges & Cady, 2012; Spitler, 2012) who employ Wenger (1998) alongside Sfard and Prusak (2005) have included Gee (2001). Gee (2001) defines identity as "being recognised as a certain "kind of person" in a given context" (p. 99). According

to Kasten et al. (2014), the inclusion of Gee (2001) can untangle multiple identities of individual teachers, as his theoretical framework can look more closely at certain issues and less closely at others. For example, with Gee's (2001) framework, researchers can choose to focus on teachers' institutional identity as a position authorised by government departments of education (e.g., district officials) within the predetermined schooling policies. Similar to institutional identity, other models of identity in Gee's (2001) framework (i.e. Nature-identity, Discourse-identity and Affinity-identity) are not in focus for this study. As stated, I employ Wenger's (1998) social theory of learning alongside Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity frameworks as a lens because of their relevant constructs that connect in ways that can guide this study to answer the research questions.

In what follows, I discuss details of each theoretical framework (i.e. participative, narrative and performative identity frameworks) from socio-cultural perspectives starting with Wenger (1998). When discussing Wenger (1998), I attempt to connect it to teacher identity and to this study, but other opportunities of making further connections come when discussing the challenges of using his framework, and when including Graven's (2003, 2004) notion of confidence to his framework. To note, in Section 2.8, I return to elaborate on constructs that are foregrounded from the frameworks of this study.

2.3 Wenger's social theory of learning

Wenger (1998) locates his social theory of learning at the intersection of two axes of "intellectual traditions" (p. 11), as illustrated in Figure 2.1. In the tradition of social theory, the vertical axis has two ends that are described as 'theories of social structure' at the top, and 'theories of situated experience' at the bottom. Theories of social structure put emphasis on institutions, cultural systems, discourses and history, whilst theories of situated experience give primacy to agency of individual actors and their intentions. The social view of learning is thus placed in the middle. In Wenger's (1998) theory, learning takes place through our individual engagement in actions and interactions, but it embeds this engagement in culture and history.

Wenger (1998) also places a horizontal axis and describes each end as 'theories of social practice' on the left-hand side and 'theories of identity' on the right-hand side. Theories of social practice address the production (and reproduction) of learning systems that promote ways of engaging with the world. On the other hand, theories of identity focus on the issues of gender, class, ethnicity, age, and other forms of categorising or labelling particular groups. Along this axis, Wenger (1998) places the social view of learning in the middle again. In short,

Wenger's (1998) social theory of learning is a culmination of conceptions from the surrounding theories.

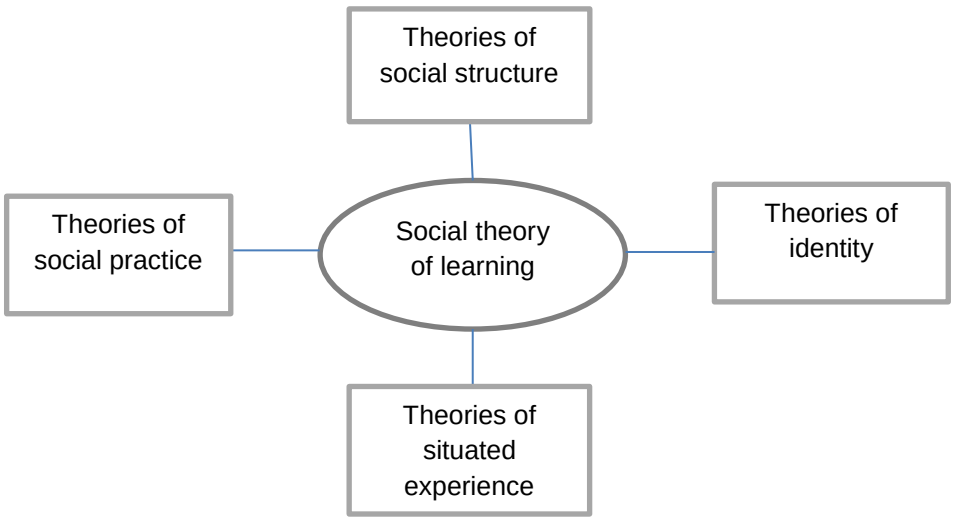


Figure 2.1: Two main axes of relevant traditions (Wenger, 1998, p. 12)

Wenger's (1998) social theory of learning is based on four assumptions, which are:

- people are social beings (Wenger emphasises that this fact is a central aspect of learning);
- knowledge is about competence with respect to valued enterprises;
- people learn by participating in the pursuit of such enterprises; and
- meaning is what learning ultimately produces.

Based on these assumptions, Wenger (1998) states that learning as participation is the "process of being active participants in the practices of social communities and constructing identities in relation to these communities" (p. 4). Thus, such participation contributes in shaping what people do, who they are, "what they affiliate with" (my insertion), and how they interpret what they do (Wenger, 1998).

In characterising participation, Wenger (1998) identifies four components of learning, which are: identity, meaning, practice and community. He describes the four components of learning as follows (p.5):

- **Identity** is a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities;
- **Practice** is a way of talking about shared historical and social resources, frameworks and perspectives that can sustain mutual engagement in action;

- **Community** is a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognisable as competence; and
- **Meaning** is a way of talking about our (changing) ability – individually and collectively – to experience our life and the world as meaningful.

These four "deeply interconnected and mutually defining" (Wenger, 1998, p. 5) components provide a structural model for the social theory of learning. See Figure 2.2 below. Wenger (1998) anticipates that individuals can switch any of the four peripheral components with learning, place it in the centre as the primary focus, and the figure would still make sense. The study foregrounds individuals' learning as *experiencing* (meaning) and learning as *doing* (practice) for the purposes of analysing and synthesising the roles of teachers in the processes of learning and teaching mathematics. These two components contribute explicitly and implicitly to other components of learning (e.g., identity and community) as products of learning and teaching mathematics. Therefore, I discuss all four components starting with learning as *belonging* (community).

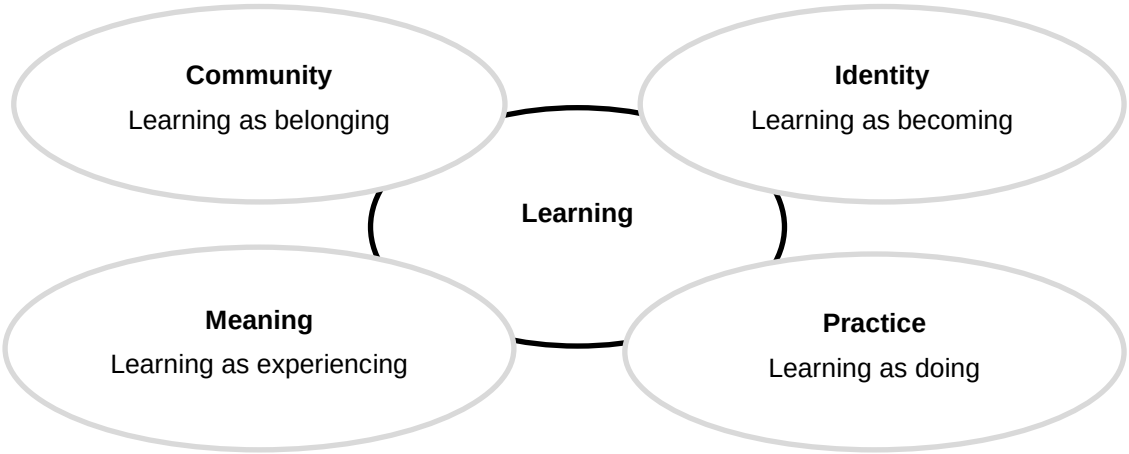


Figure 2.2: Components of Wenger's (1998) social theory of learning (p. 5)

There are three modes of learning as *belonging* (i.e. engagement, imagination and alignment) when individuals participate in the work of communities (Wenger, 1998). Wenger (1998) describes engagement as an active involvement of individuals in mutual processes of negotiating meanings. When negotiating meanings in specific communities (Wenger, 1998), individuals begin to talk about themselves and about each other. That is, individuals narrate stories to give certain meanings that form their identities that they recognise for themselves and by others (Wenger, 1998). For example, when drawing on Lave and Wenger (1991), Alcoholics Anonymous (AA) is considered as a learning community. In this AA community, newcomers learn ways of becoming and remaining sober from old-timers through talking and sharing of personal stories, histories and lived experiences. However, when focusing on how

teachers were engaging with each other in the TM1 course, teachers with long service in the teaching profession were also seen listening to (i.e. learning from) younger teachers with fewer years of teaching mathematics – it was a mutual process of negotiating meanings for the teachers.

The second mode of belonging is imagination. To describe imagination, Wenger (1998) provides an analogy of two stonecutters who were asked what they were doing. One responded: “I am cutting this stone in a perfectly square shape.” The other responded: “I am building a cathedral.” The second stonecutter connected himself to a broader community of builders. In other words, the second stonecutter related his engagement to a broader imaginary scheme of things. Nonetheless, the first stonecutter was not wrong in giving an exact account of what he or she was doing (Wenger, 1998). Similarly, for example, when teachers participate in PD communities, some teachers can remain in solving problems present in front of them on paper, whilst other teachers can further think about how the problem can be integrated into a lesson for learners. In this way, arguably, the latter group of teachers may make more meaning of the problem as they strive to belong to communities of mathematics teachers rather than to communities of mathematics learners.

The third mode of belonging is alignment. Nasir (2002), citing Wenger (1998), describes alignment as “how actions within that community come to be aligned toward a broader common purpose” (p. 219). This mode demands individuals to channel their energies within their professional boundaries (Wenger, 1998). Wenger (1998) warns that alignment can be disempowering as individuals can be subjected to the goals of that profession. As such, individuals learn to navigate the systematic processes in order to remain with the identified profession. For example, during the implementation of Curriculum 2005 in South Africa (Department of Education, 1997), which was to see teachers transitioning from teacher-centred approach to learner-centred approach, teachers developed coping mechanisms as they were not equipped to transition to the curriculum (Jansen & Christie, 1999). In this regard, which relates directly to the problem statement of this study, teachers would claim to be following the curriculum, but researchers who observed their lessons, confirmed that they were doing something different in actual practice (Jansen, 2001).

Learning as *experiencing* (meaning) can be reconciled with the earlier discussions of how individuals can negotiate meanings in their lives. The negotiation of meanings entails talking, thinking, acting and solving problems within the certain practice (Wenger, 1998). For Wenger (1998), more often than not, we may do and say things that have been said and done in the past by various members of the community, “and yet we produce again a new situation, an impression, an experience: we produce meanings that extend, redirect, dismiss, reinterpret,

modify or confirm the histories of meanings of which they are part" (pp. 52–53). As Wenger (1998) elaborates, the process of negotiation of meanings becomes more prominent in communities when members are involved in activities that they 'care about' or when they are presented with problems without certain members imposing one way of finding solutions. Wenger's (1998) features of describing learning as experiencing are relatable to the fundamentals of learning and teaching mathematics that were promoted in the TM1 course.

Learning as *becoming* (identity) entails aspects of lived experiences by considering what we have learned in our pasts. As such, our past experiences shape knowledge and skills that we have accumulated in a process of becoming certain individuals. Certain individuals are characterised by what they know and do not know about the practices when compared to other members of their communities. Learning as *becoming* tends to be continuously changing experiences of what matters and does not matter in the lives of individuals. In Wenger's (1998) terms, the construction of identity is defined "with respect to the interaction of multiple convergent and divergent trajectories" (p. 154). Thus, the construction of present identity is an on-going process of becoming what we are inspired to achieve in the future. In this regard, the features of learning as becoming (teacher identity) are the ultimate measure of the outcomes of participating in the TM1 course. That is, the teachers' characterisation is based on what they were saying or not saying when talking about learning and teaching of mathematics, and on what they were doing or not doing when teaching mathematics.

Learning as *doing* (practice) entails both lived experiences and a display of competence of individuals within their familiar territories (Wenger, 1998). That is, when we engage with other members, we must recognise our areas of competence whilst allowing others to recognise us as being competent in what we do (Wenger, 1998). For Wenger (1998), learning becomes a mutual engagement where we develop certain expectations about how to interact and work together. Wenger (1998) argues that we know who we are by being able to synthesise what is unfamiliar with what is familiar in order to understand the world. For example, mathematics teachers with a strong element of doing mathematics are confident enough to trace back on their work from known to unknown. In this study, broadly speaking, the features of learning as doing (practice) talk more to how the teachers who had participated in the TM1 course can display competences of teaching mathematics from the MTF perspective.

The roles of practitioners are defined within the contexts of learning called communities of practice (Lave & Wenger, 1991; Wenger, 1998). Wenger, McDermott and Snyder (2002) had built on Lave and Wenger (1991) to define a community of practice[5] as "a group of people who

[5] Unlike the initial definition I presented on the footnote in Section 1.6, this definition explains what makes "a group of people who are doing similar work" special.

share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis" (p. 7). In a community of practice, individuals make meanings about the practice within networks of social and cultural relationships (Wenger, 1998). In a community of practice, beginners gain access from experts, and they may either perceive themselves to be members or aspire to membership of a community in which experts' practices are central (Lave & Wenger, 1991). Wenger (1998) identifies three dimensions to consider when characterising a community of practice: *mutual engagement, joint enterprise,* and *shared repertoire.*

Mutual engagement is what bind members together into a social entity (Wenger, 1998). It permits members to take charge of their learning in a community of practice. With mutual engagement, individuals are expected to identify their roles in communities. Boaler (2000) argues that individuals should then be given active roles in communities to prompt a sense of human agency which should allow them to negotiate, shape and reflect upon their participation or non-participation in the community of practice. In turn, individuals can develop certain models of identity in relation to their roles because they will be in good positions to exercise their own freedom and thoughts when engaging with the subject (Boaler et al., 2000). In Chapter 4, I link the notion of mutual engagement to this study, when discussing different roles of individuals in the TM1 course.

A *joint enterprise* is what the community of practice is about. It is characterised by its factors of persuading members in the community of practice to work together towards a common goal (Hughes, Jewson & Unwin, 2007). Directives or constraints influence the understanding of working together (Wenger, 1998). A joint enterprise supports the mutual engagement in the community of practice, as "it creates relations of mutual accountability that become an integral part of the practice" (Wenger, 1998, p. 83). Members become accountable to their roles because they have ownership of the community of practice (Chauraya, 2013). Again, in Chapter 4, I discuss the learning of mathematics-for-teaching (MfT), which was what the TM1 course was about.

A *shared repertoire* gives members access to shared resources which have been produced or adopted in the course of community of practice's existence, and which have become part of its practice (Wenger, 1998). The shared resources comprise of "routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions or concepts" (Wenger, 1998, p. 83). In Wenger's (1998) social theory of learning, the enlisted three dimensions (mutual engagement, joint enterprise, and shared repertoire) of a community of practice, alongside the modes of belonging, are critical sources of identity development given its ability to segregate individuals according how they participate or not participate in conversations and canons of

the practice. Also, in Chapter 4, I link the notion of shared repertoire to how technology and textbooks were accessible to the teachers to contribute in the learning of MfT in the TM1 course.

2.4 Challenges of using Wenger's framework in the study

There are many challenges of using Wenger's (1998) social theory of learning as a framework to research teacher identity in mathematics. Here, firstly, I discuss the fact that Wenger (1998) situates his theory within the apprenticeship context, whilst this study applies it in the education context. Next, I highlight how Wenger (1998) undermines teaching, whereas this study focuses on learning to teach. I follow this with a discussion on the looseness of locating communities of practice as Wenger (1998) maintains that they are everywhere. Lastly, I elaborate on the fact that Wenger (1998) does not provide an operationalised definition of identity to characterise participation of individuals within communities of practice, which creates a space to include other theoretical frameworks when researching identity.

Firstly, Lave and Wenger (1991) and Wenger (1998) do not embrace formal structures of learning within the apprenticeship context. Lave and Wenger (1991) accept that communities could be formally organised for the purposes of learning, but prefer to see most learning as unstructured, unplanned and not taught. Wenger (1998) argues that authentic engagement around an enterprise promotes unexpected community formation.

Within the education context, particularly in the context of teachers teaching in disadvantaged schools in South Africa, where the problems of learning and teaching mathematics are assumed to lie with teachers, in-service teacher PD interventions tend to embrace formal structures of learning mathematics. In other words, PD interventions are gravitated towards predetermined learning outcomes, whereby such learning outcomes are targeted to be achieved within a limited time period. PD interventions seek to promote standardised norms when participating in communities of teachers. In this sense, facilitators in PD interventions have central roles of mediating learning to communities of teachers and they are often deliberate in their teaching when compared to 'masters' in the apprenticeship context (Graven & Lerman, 2003. This is because in-service teachers join PD interventions with varying levels of learning experiences from the teaching profession. In turn, arguably, PD interventions enable teachers to participate in robust learning activities that are transferable to different practical contexts within the teaching profession (Biza, Jaworski & Hemmi, 2014). Hence, as I elaborate on these ideas in Chapter 4, the learning of MfT had targets and the teachers wrote assignments and tests in the TM1 course.

Vangrieken et al. (2017) explain that there are two types of teacher communities that are associated with formal structures of learning: (1) member-oriented teacher communities with pre-set agendas and (2) formal teacher communities. The core characteristics of formal teacher communities include the fact that they are often organised by government officials and aimed at transferring knowledge to the teachers (Elster, 2009). With that aim in mind, formal teacher communities adopt a top-down approach and resemble PD workshops set to achieve standard learning targets by the time the teacher community stops its meeting. Participation in formal communities might be voluntary, but often teachers are compelled to attend such meetings. Most teachers who attended the TM1 course are subjected to these formal teacher communities, and they do not like it.

On the other hand, member-oriented teacher communities with pre-set agendas are often initiated by school principals, teachers or researchers (Pella, 2011). If these type of teacher communities are initiated by researchers, school principals, head of departments in schools and teachers get to align themselves with pre-set agendas. These teacher communities are subjected to pre-set schedules, session formats, and objectives that are set before a start of the sessions (Owen, 2014). When participating in these communities, teachers get to exchange teaching strategies and discuss practical teaching challenges (Vangrieken et al., 2017). The community of teachers who participated in the TM1 course can closely fit into a member-oriented teacher community when considering that the course was initiated by the researchers – Adler and her research team. The researchers needed a buy-in from teachers, school principals and head of departments in schools. In Chapter 4, I link different roles of the community of teachers who participated in the TM1 course to those of the facilitators[6] of the course and the research fellows, and refer to that group as the TM1 community.

Secondly, whilst this study focuses on learning to teach, Wenger (1998) dissociates his theory of conceptualising communities of practice with the teaching profession. Wenger (1998) merely provides a set of guiding principles for maximising learning of community members. In maintaining his position, Wenger (1998) askes the question: “How can we minimise teaching so as to maximise learning?” (p. 267). Wenger (1998) argues that teaching is not a precondition for learning. Graven (2004) states that undermining teaching for individuals who are learning to teach can demoralise the status of their teaching profession despite Wenger’s (1998) intentions of emphasising a reconstituted kind of learning. Kazemi and Hubbard (2008) assert that teachers learn in and from practice, which means that their professional work of teaching mathematics in the classroom contexts influences their learning of the subject in the

[6] I use the word ‘facilitators’ to refer to individuals who mediated the learning of mathematics-for-teaching in the TM1 course. Although they were not used, the words like ‘managers’ or ‘directors’ were applicable in some instances since the facilitators were also running the administrative part of the TM1 course.

PD contexts and vice versa. Thus, at least in mathematics education, undermining teaching cannot be used to promote learning when considering that teachers in formal education structures have a central role in ensuring successful learning of mathematics (Graven & Lerman, 2003).

Thirdly, using Wenger's (1998) social theory relates to the looseness of locating communities of practice (Chauraya, 2013). According to Wenger (1998), communities of practice are everywhere and people belong to different communities of practice at different times. In this sense, communities of practice become pockets of moving targets, and analysing learning of individuals within communities can easily be misinterpreted unless what is taken-as-shared ways of doing and communicating ideas (practice) are kept fixed or identifiable. Thus, when focusing on the formal structure that was set up to improve the learning of MfT including inducting the teacher community to the aspects of the MTF, my study is able to firmly locate the TM1 course as the community of practice when using Wenger's (1998) theory.

Lastly, as alluded to, there is the issue of operationalising Wenger's (1998) four components of learning to characterise participation (Storberg-Walker, 2008). Wenger (1998) does not provide indicators of characterising interactions of participants when learning within communities of practice (Essien, 2014). When learning yields identity, the operationalisation of identity becomes a challenge. For instance, in the case of defining identity as belonging, identity is a sense of belonging to a community (Chauraya, 2013). If so, the following questions arise: how can one analyse individuals' identities without understanding their collective views about a practice? What is the extent of agreements and disagreements about the practice before anticipating individuals to belong to the community? In this sense, as noted in Section 1.5, when discussing the purpose of the study, this study firstly explores the extent of teachers' shared experiences of learning and teaching after they had participated in the TM1 course before embarking on characterising their identities in relation to the teacher community and the teaching practices.

2.5 Graven's extension of Wenger's framework to include confidence

Graven (2003, 2004) employed Wenger's (1998) framework when she was investigating mathematics teacher learning within an in-service community of practice. In her contribution to the field of mathematics education, Graven extended Wenger's (1998) model of interconnected components of learning (as meaning, practice, identity and community) to include confidence. The concept of confidence emerged in relation to how teachers had described their learning of mathematics (Graven, 2002). Graven (2003) referred to this emergent component of learning as 'learning as mastery'. Figure 2.3 represents Wenger's (1998) extended structural model of social theory of learning to include confidence. In this

regard, this study shifts from the original Wenger's model to adopt this model of learning in communities of practice.

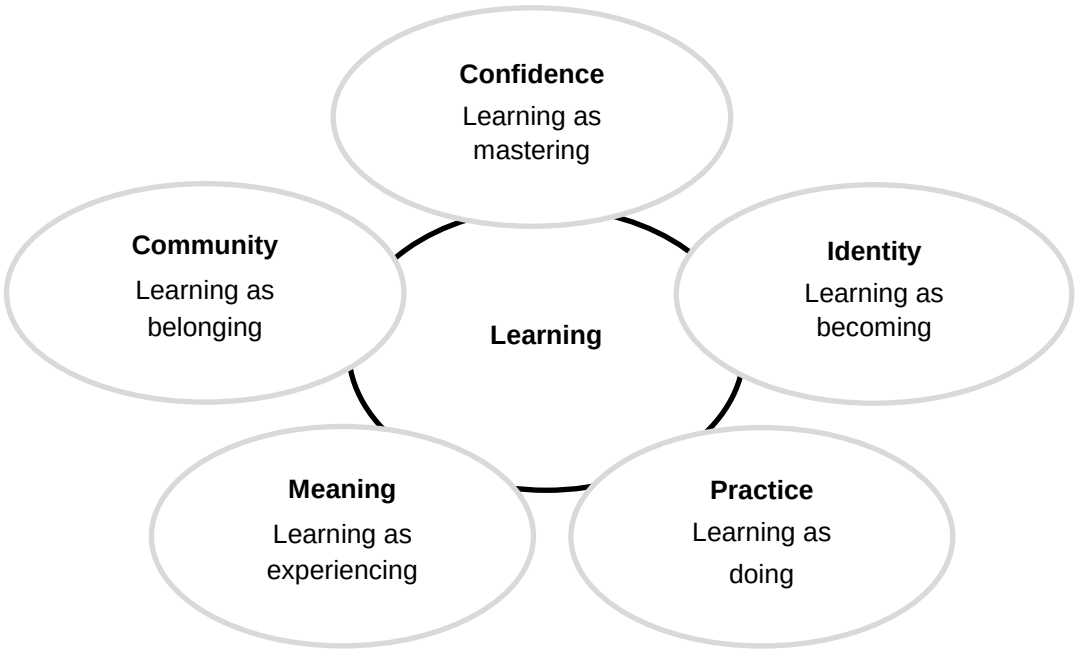


Figure 2.3: Adapted model of Wenger's (1998) theory to include confidence

Graven (2003, 2004) asserts that, like Wenger's (1998) four components of learning (i.e. community, practice, meaning and identity), confidence is both a product and a process inherent in teachers' learning to become professional mathematics teachers. Graven (2003) elaborates on her point of confidence as follows:

> [I]n the same way as *identity involves learning as becoming*, as well as the experience of *being* a person with a particular identity at a particular point in time, *confidence involves learning as mastery*, as well as the production of an experience of the achievement of a particular level of mastery at a particular point in time. In this way, confidence and mastery are both produced by and are productive of learning (p. 36, italics in original).

Thus, the inclusion of Graven's (2003, 2004) notion of confidence in Wenger's (1998) social theory of learning was necessary to theoretically frame this study as it directly concerns learning and teaching of mathematics for teachers having participated in the PD intervention. As briefly mentioned in Section 1.5, teachers who participated in the TM1 course in 2017 expressed that teacher confidence is necessary when learning and teaching mathematics.

Graven (2003) cited the following teachers to characterise confidence in relation to their learning and teaching of mathematics:

> I can expose myself to what I know, I mean to other people and I am willing to say "Okay fine, show me wrong, prove me wrong? What is your idea then? What I say is I am open. Let's learn." That is what that self-confidence is. (Karl, as quoted in Graven, 2003).

> And also knowing that if it doesn't work for this lesson, I can change my method and try something else, it's not a matter of do it or die kind of thing. (Delia, as quoted in Graven, 2003).

Describing confidence this way talks to both the learning of mathematical content knowledge and teaching knowledge within a community of practice. Whether in relation to mathematical knowledge or teaching knowledge, teachers with confidence are able to admit to what they do not know and still need to learn in their teaching profession, whilst they are still confident enough to recognise themselves as mathematics teachers (Graven, 2004). In this way, therefore, confidence becomes an enabling feeling that can assist teachers to understand their own limitations when viewing learning as a life-long project within the teaching profession (Graven, 2003).

2.6 Sfard and Prusak's narrative identity framework

As stated in Subsection 2.2.2 above, Sfard and Prusak's (2005) narrative identity framework is necessary to analyse data for this study when considering that the teachers talk about their experiences of learning and teaching mathematics. The position of Sfard and Prusak (2005) to equate identities to stories about persons has been influential in operationalising identities. Sfard and Prusak (2005) clarify that they are not saying identities find their expression in stories, but they are saying identities are stories. The authors define identities as not just any stories, but only those that are reifying, endorsable and significant to those persons telling them. Sfard (2019, p. 557) explains the terms about stories as follows:

1) reifying – they speak about what a person is and has rather than what she does;
2) significant – they are considered by the storyteller as crucial, indeed, as defining features of the person; and
3) endorsable – they are seen by the storyteller as reflecting the real state of affairs in the world, and thus as reliable guides for their future actions.

Researchers (e.g., Andersson, 2011; McCulloch et al. 2013; Langer-Osuna & Esmonde, 2017; Lutovac & Kaasila, 2018; Sfard, 2019) commended Sfard and Prusak's (2005) stance of framing identities as stories about persons. For example, according to Langer-Osuna and Esmonde (2017), Sfard and Prusak (2005) resolve some contested assumptions about whether identities are internal things we have inside ourselves or unknown discursive constructs that people 'disagree to agree' on in moments of time. As the assumptions about whether identities are internal things or unknown discursive constructs have made the field of researching identities complex (Langer-Osuna & Esmonde, 2017), Sfard and Prusak (2005) clarify that identities as stories are human-made; shaped and re-shaped by society; and change with time, content, context, and narrator. Thus, Sfard and Prusak's (2005) definition

characterises human beings as active agents who play decisive roles in determining the dynamics of their social life and in shaping individual activities.

For Sfard and Prusak (2005), defining identity as a story extends beyond Wenger's (1998) assertions that "The experience of identity in practice is a way of being in the world" (p. 151) or "Learning… implies becoming a different person [and] involves the construction of identity" (Lave & Wenger, 1991, p. 53). Defining identity as a story and spelling out its operational aspects link learning and its socio-cultural context (Sfard & Prusak, 2005). That is, in simple language, as we know what identity is and what it resembles, we can aim at mediating people's experiences towards developing it to contribute in shaping societal activities. In Chapter 4, when linking the learning and its socio-cultural context for the study, I present details on how the learning of MfT in the TM1 course was mediated to influence teachers' experiences of learning and teaching mathematics.

To explain the significance of defining identity as a story, Sfard and Prusak (2005) argue that societal activities promote institutional identities by spelling out 'who one is' and 'what one has' when considering that we are subjected to all kinds of institutions in the modern world. We are described by grades, test results, qualifications, titles, ranks and so on. Sfard and Prusak (2005) assert that "almost any social situation seems to be a good opportunity for reifying" (p. 16). For instance, a teacher that has completed a teaching qualification (e.g., Bachelor of Education) and majored in secondary mathematics education is described as a qualified secondary mathematics teacher. Furthermore for example, if this qualified secondary mathematics teacher attended the TM1 course, he or she is further described as being an in-service teacher of secondary mathematics who participated in the TM1 course. Thus, to overcome the fluidity of change and the use of "*is*-sentences", Sfard and Prusak (2005) explain using a metaphor and proclaim that defining identity as a story collapses a video clip about individuals' identities into a snapshot – I extend this metaphor in the next section when discussing Darragh's (2016) performative identity framework.

As indicators of identities of individuals that can link learning and its socio-cultural context, in this case, societal activities imply teaching of mathematics, Sfard and Prusak (2005) expand on the characterisation of stories using the following adjectives:

1) an identified story could be *reifying* if the words *always, never, usually, can, have* or *be* are used;
2) *endorsable* if the identified person accepts the stories as being about themselves – personal pronouns such as '*I*', '*me*', *'we'* and *'us'* are considered for endorsements; and

3) *significant* if any change to the story was likely to affect the storyteller's feelings about the identified person.

These indicators of identities are necessary to push us to take storytellers' stories seriously (Sfard & Prusak, 2005). From understanding Sfard and Prusak's (2005) framework on narrative identities, there are two grounds of questioning storytellers' stories. Firstly, it must be taken into consideration that every identified story may be represented by the triple ${}_{B}A_{C}$, where A is the identified person, B is the author, and C the recipient. Within this rendering, it becomes clear that multiple identities exist for any person. Stories that are meant to be the same about a same individual may be quite different from one another because the author might not be talking to the same recipient. For example, a teacher (an author) may share a story with a researcher (a recipient) and not clarify to him or her as the recipient that the story is directed to himself or herself (as the author). In that way, the researcher has no ground to question the story to be not true because it might be that the story was not meant for him or her at the first place.

The second rendering is based on the fact that identities can be translated into queries about the dynamics of narratives when considering that the dynamics of narratives are amenable to empirical study (Sfard & Prusak, 2005). That is, "the narrative definition may be expected to catalyse a rich theory of identity" (p. 18). Thus, the reifying and significant narratives about a person can be split into two subsets: actual and designated identity. Actual identity consists of stories about actual state of affairs, and designated identity presents a possible state of affairs (Sfard & Prusak, 2005). The former subset is usually told in the present tense and formulated as factual assertions (e.g., I am a good mathematics teacher). The latter subset of narratives that are believed to have the potential to become a part of one's actual identity in the future (e.g., I want to be a good mathematics teacher).

Because narratives can split into actual and designated identities, I can question teachers' stories that were understood to be untrue when reflecting on the events. Given that I do not intend to double check with storytellers (in this case, the teachers who participated in the study), I can read their stories within the assumed contexts and decide if teachers were not meant to display designated identities, but displayed actual identities in their stories. Designated identities sometimes escape people's rationalisation because they can think that their stories are good enough to be told, given that these are the kinds of stories that seem appropriate for people of their socio-cultural statuses (Sfard & Prusak, 2005). Sfard and Prusak (2005) explain that the scenarios that constitute designated identities are not necessarily desired but always are perceived as binding for people that are telling the stories.

When interrogating the dynamics of narratives, and by distinguishing between actual and designated identities, identities can give direction to one's actions and one's deeds to a large extent (Sfard & Prusak, 2005). Thus, for the study, I bring on board Darragh's (2016) framework on performative identity to contribute to analysing actions of teachers about and when teaching mathematics.

2.7 Darragh's performative identity framework

Darragh (2016) defines performative identity as the stylised repetition of actions. Darragh (2016) situates identity within Mead's (1934) framing, which looks at identity as an action. For Darragh (2016), identity exists in the moment of performance and as it is recognised. It is something we do. Performative identity is not something people have inside themselves, which is the definition that draws from psychological perspectives.

The definition of performative identity can be positioned alongside participative identity (Wenger, 1998) and narrative identity (Sfard & Prusak, 2005) frameworks. For example, Sfard and Prusak (2005) equate identity with telling a story, which in Darragh's (2016) terms that implies action. In another example from Wenger's (1998) social ecology identity, identity can be defined as something we are competent in doing, again an action. Furthermore, as both Darragh (2016) and Beauchamp and Thomas (2009) quote MacLure (1993), the use of terms such as *identity* work in the literature to describe identity as a "resource that people use to explain, justify and make sense of themselves in relation to others, and to the world at large" (p. 311, abstract), which imply action. For example, if teachers are requested (by other teachers, for instance) to conduct after-school classes, some teachers can justify their position of not being able to work outside school hours, by stating that their 'real' lives are with their families.

The emphasis of seeing identity as action harnesses the positions of both Wenger (1998) and Sfard and Prusak (2005), which are central to this study. For example, in order to determine who actors are from what they are saying and doing, as noted above, Sfard and Prusak (2005) used a metaphor and proclaimed that they were collapsing a video clip to a snapshot by defining identities as stories. When extending this metaphor and allow the video clip to play, Darragh (2016) could assert that we know who the actors are by noticing their repetitive actions. Because these actors qualify (or do not qualify) to act in those roles, they themselves should recognise who they are (Darragh, 2016) when considering their display of competence in what they are doing (Wenger, 1998). Thus, in analysing teachers' performance when they are teaching mathematics from the MTF perspective in Chapter 9, I consider how they present mathematical concepts when interacting with learners in the classrooms.

Indicators of performative identity reside in understanding that “we perform ourselves – be it by telling stories, joining groups, acting in a particular way at a particular time, positioning ourselves and others within wider societal discourse” (Darragh, 2016, p. 29). Identity becomes the process of identifying such participatory roles when learning and teaching mathematics, whether as self-identification or identification by others (Darragh, 2016). For instance, our mathematics identity performances might include “putting a hand up to offer an answer during a lesson, persevering to solve a problem, arguing or justifying a solution given” (Darragh, 2015a, pp. 85-86), of which there is power in noticing ourselves that we are capable and willing to play those roles when learning before we are noticed by others. In subsection 5.2.2, I use the same principle to understand how the teachers enact the roles of teaching from the MTF perspective from both self-identification and identification by others (others include the researcher). For instance, the teachers’ performance might mean selecting certain kinds of examples from textbooks (or other resources) to generalise for particular concepts when teaching mathematics.

The context of how teachers were geared up to be competent in teaching mathematics after learning MfT including being inducted to the aspects of the MTF became pertinent when drawing inspiration from Darragh (2016) alongside Wenger’s (1998) notion of community of practice. For Darragh (2015a), the context is a stage where actors perform. A consideration of the stage is necessary to fully understand any performance, as the stage can constrain or hinder performances in significant ways. Arguably, when the stage changes, the actors need to adapt to the new setting to give their audience relevant performances. Thus, the study examines the performances of the teachers when teaching in the classrooms and when talking about their learning and teaching in the interviews.

In addition to the context as a stage, Darragh (2015a) highlights the importance of considering audience when using the metaphor of performance. For example, the type of performance can vary depending on the audience. In this case, the teachers who participated in this study were performers when teaching in the classrooms and during the interviews. As a researcher who was observing and interviewing the individual teachers, and the learners who were being taught mathematics in the classrooms, we were an audience. Darragh (2016) argues that we are simultaneously both performer and audience to others’ performance. The role of audience is to recognise the identity performance of others (Darragh, 2016). For the present study, the notion of audience (Darragh, 2015a, 2016) relates to the fact that stories about the identified person (Sfard & Prusak, 2005) can be told differently depending on the recipients or in this case an audience (the researcher and learners). Thus, the notion of audience must be born in mind as it can present opportunities to question or confirm teachers’ performances when being observed or during interviews.

2.8 The theoretical framework for the study

In this section, I explain how learning as *experiencing* (meaning) and learning as *doing* (practice) from Wenger (1998) are foregrounded in this study. That is, the outcomes of learning as *experiencing* (meaning) include teacher confidence, teacher knowledge, teacher community and teacher identity. When remaining with learning as *becoming* (teacher identity), whilst paying attention to learning as *doing* (practice), the outcomes of learning and teaching mathematics from these two components become teachers' mathematics teaching identity. The outcomes of learning as *doing* (practice) are analysed using Sfard and Prusak's (2005) narrative identity framework and further understood from Darragh's (2016) performative identity framework.

To elaborate on the foregrounded constructs, the theoretical framework for the study is used to explain how the teachers make sense of learning and teaching from their experiences of participating in the TM1 course. Experiences of the teachers are in relation to their meaning, community, practice, confidence and identity within the socio-cultural context of learning and teaching mathematics (Graven, 2003; Wenger, 1998). The socio-cultural context foregrounds the experiences of the teachers when they were learning MfT in the TM1 course. In Chapter 4, I draw on Wenger (1998) to further argue that the TM1 course is the community of practice. Then, the study relies on narratives as the teachers were expressing their experiences of learning and teaching after participating in the TM1 course (Sfard & Prusak, 2005). In this regard, the learning as *experiencing* (meaning) means enhancing the learning of MfT, which is at the centre of Figure 2.4.

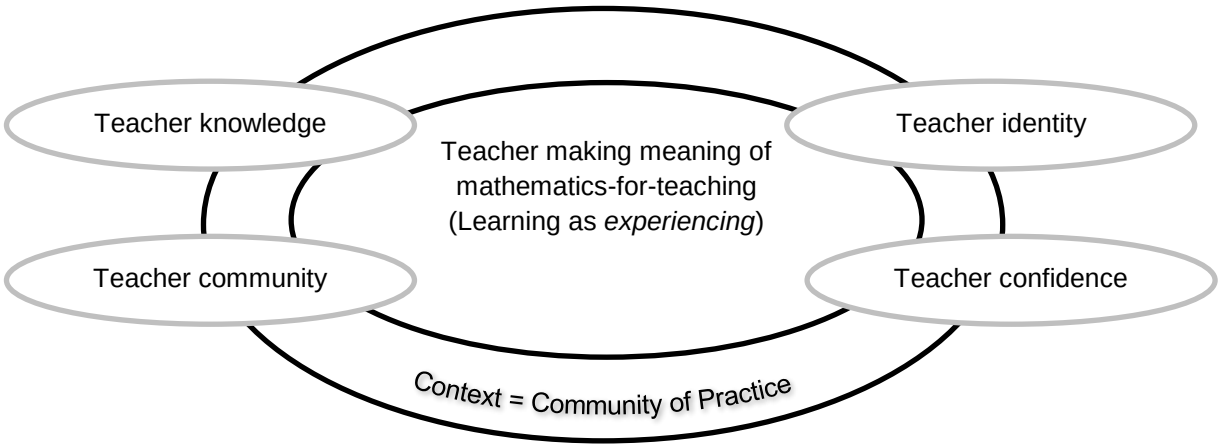


Figure 2.4: Learning as experiencing in the TM1 course as the immediate context

Figure 2.4 resembles the original model of social theory of learning (Graven, 2003; Wenger, 1998). However, in this model, learning about the MTF as teaching practices together with learning how to make meaning when learning MfT in the TM1 course collapse to emerge as teacher knowledge (Hobbs, 2012). As much as the study focuses on examining the outcomes

of participating in the TM1 course, the fact is that the teachers started participating in the TM1 course having gained various experiences of learning and teaching of mathematics from the profession. Their experiences included that they already possessed certain identities; they already belonged to certain communities within the profession; they also possessed knowledge of MfT; and they were somewhat confident teachers. Hence, in Chapter 4, I explain *what*, *how* and *why* the teachers were learning in the TM1 course (Adler, 2013; Adler & Pournara, 2020, Adler, 2021b), whilst the ultimate goal of situating the study within the community of practice is to explore teachers' experiences, particularly on changes of learning and teaching as influenced by the TM1 course.

However, when drawing on Wenger's (1998) social theory of learning, and given that the study seeks to report on the shared experiences of teachers, the idea of describing their mathematics teaching identity whilst foregrounding learning as *experiencing* can be limited to what do teachers think about themselves before and after the TM1 course. That is, what teachers think about themselves before or after participating in the course can vary from one teacher to another. The discrepancies can be due to unlimited possibilities of how far back teachers can go in retrieving their experiences in relation to their statuses of learning and teaching mathematics after the TM1 course. Similarly, when teachers are commenting on what others think about them as mathematics teachers, that information can depend on whom one asks and in what position.

Consequently, when foregrounding learning as *doing* (practice) from Wenger (1998) and expect teachers' mathematics teaching identity to emerge from the frameworks of Sfard and Prusak (2005) and Darragh (2016), one can describe teachers' positions about themselves in relation to learning MfT in the TM1 course. From this perspective, and since the teachers learned about the MTF as teaching practices, the focus can be on what teachers think about themselves and what others think about them in relation to teaching from the MTF perspective. Thus, Figure 2.5 presents the deliberated theoretical frameworks on teachers' mathematics teaching identity in the context of teaching from the MTF perspective.

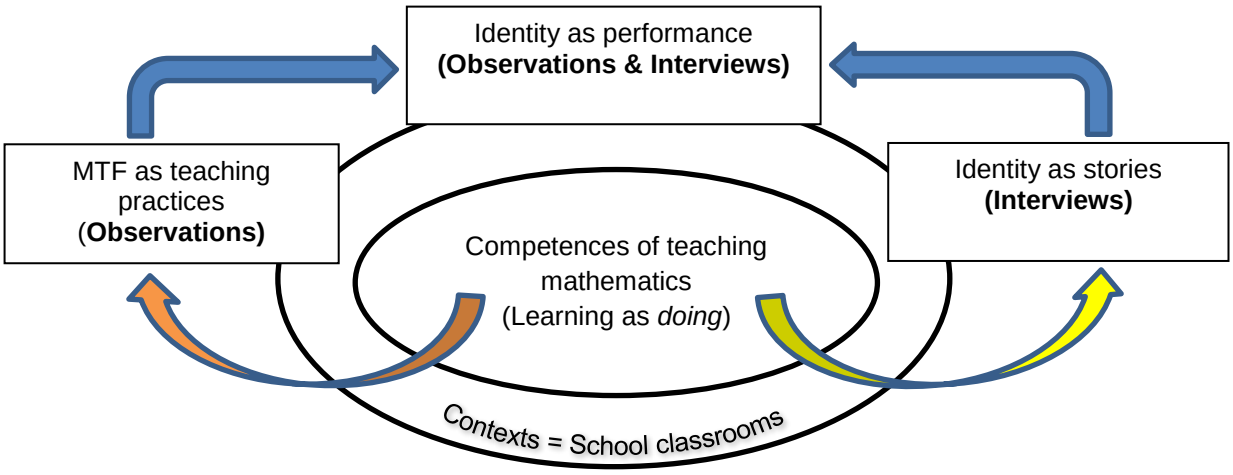


Figure 2.5: Learning as doing when teaching in the classroom

When focusing on learning as *doing* (practice), teachers on the one hand are characterised by their competences of teaching mathematics using the MTF perspective from observations. To indicate the outcomes of learning as *doing* (practice) as observed from the classroom, which are analysed using the Mathematics Discourse in Instruction (MDI) framework, I used the orange arrow in Figure 2.5. In this regard, for example, teachers are expected to consciously select or adapt certain examples when teaching mathematics in the classroom.

On the other hand, teachers are characterised by how they describe their teaching of mathematics during interviews. To indicate the outcomes of learning as *doing* (practice) as expressed by the teachers during interviews, which are analysed using Sfard and Prusak's (2005) narrative identity framework, I used the yellow arrow in Figure 2.5. In this regard, for example, teachers might add other dimensions about their teaching of mathematics such as to elaborate on how they sequence examples during exemplification.

Darragh's (2016) performative identity framework becomes necessary to match teachers' characterisation of learning and teaching mathematics from both observations and interviews. The harnessing of the outcomes of learning and teaching of teachers is indicated by the blue arrows in Figure 2.5.

Using Darragh (2016) to explain teachers' mathematics teaching identity becomes operationalised from Carlone and Johnson (2007), who characterised science teachers' identity from both their competences of teaching and how they recognise themselves and by others as science teachers. Ultimately, how the teachers recognise themselves and how others recognise them as mathematics teachers become understood beyond the descriptions of before and after the TM1 course, but from what they can do when teaching mathematics.

To show how the foregrounded constructs (i.e. learning as *experiencing*, learning as *doing*, identity as *narrative* and identity as *performance*) come together in the theoretical framework of the study, I provide a flow diagram below. This diagram combines the diagrams in Figure 2.4 and Figure 2.5.

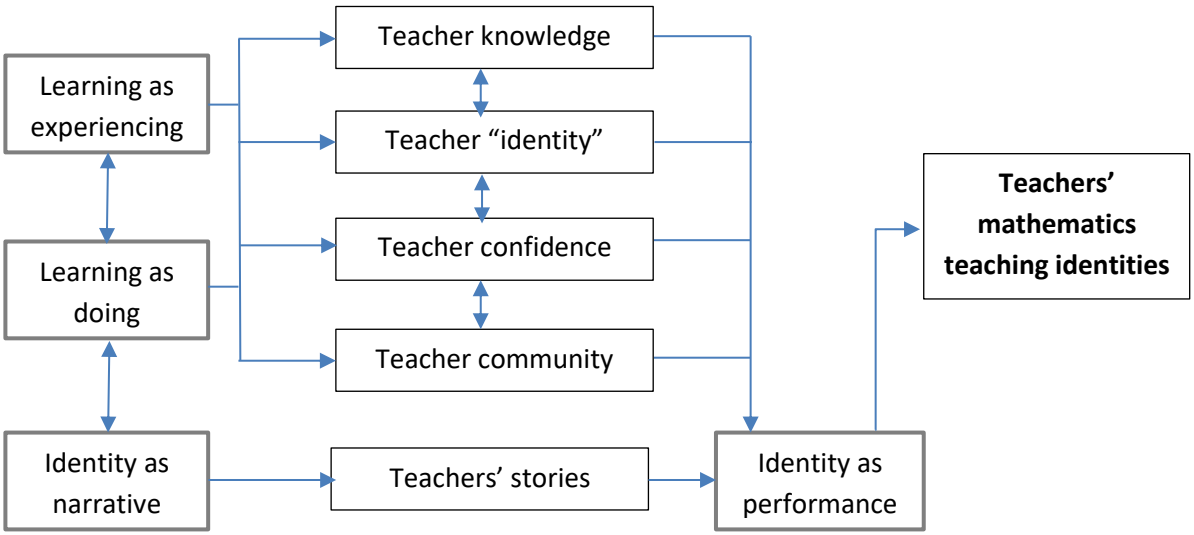


Figure 2.6: A flow diagram showing how identity as performance emerges from learning MfT in TM1

Analysing teachers' learning as *experiencing* from their narratives (Sfard & Prusak, 2005; Wenger, 1998) can yield teacher confidence, community, knowledge and "identity". Here, I have put identity in the quotation marks since teachers can remain in talking about their abilities to experience mathematics as a meaningful subject to learn and to teach (learning as *becoming*) without mentioning any aspects of the MTF. Whereas, drawing from learning as *doing* (Wenger, 1998) can highlight the impact of the TM1 course. The impact of the TM1 course can be explained from how the teachers implement their improved knowledge of teaching mathematics, particularly their performances of teaching the subject from the MTF perspective. As stated, teachers' teaching practices are analysed using MDI, and are discussed in Chapter 9. Thus, as mentioned above, Darragh's (2016) identity as *performative* framework can illuminate teachers' characterisation of learning and teaching to yield their mathematics teaching identities. The teachers' mathematics teaching identities are discussed in Chapter 10.

The theoretical framework for the study (i.e. participative, narrative and performative identity) does not ignore the two other components of learning (i.e. learning as *mastering* (confidence) and learning as *belonging* (community)). However, within the two backgrounded components, since they are not in focus for the study, many learning features can emerge from how teachers have changed (if at all) in their teaching of mathematics after participating in the TM1 course. For example, at the worst, one teacher can easily reflect on the course and talk about

how the course helped him to sharpen his English vocabulary, which in turn, boosted his confidence of teaching mathematics.

2.9 Summary

In this chapter, I established that the theoretical framework for the study draws on Wenger's (1998) social theory of learning as a foundational framework, but includes Graven's (2003, 2004) notion of teacher confidence. I further included Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity frameworks to support and operationalise the notion of teacher identity. Darragh's (2016) performative framework became ideal to aid decisions about teachers' mathematics teaching identity from both observations and interviews. The theoretical framework for the study (i.e. participative, narrative and performative identity frameworks) is situated within the socio-cultural perspective.

CHAPTER 3

THE REVIEW OF THE SELECTED STUDIES

3.1 Introduction

The theoretical framework for this study draws on Wenger's (1998) social theory of learning alongside Graven's (2003, 2004) inclusion of teacher confidence, Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity. In Chapter 2, I explained how learning as *experiencing* (meaning) and learning as *doing* (Wenger, 1998) are foregrounded for the study, whilst Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity frameworks are included to support and operationalise teachers' mathematics teaching identity.

In this chapter, I review selected studies that have employed the theoretical frameworks (whether participative, narrative or performative identity frameworks) which I draw on in this study. As I focus on using a theoretical format to structure the review (Randolph, 2009), and on providing summaries of the selected studies, I tease out similarities and differences between each selected study and the present study when elaborating on different foci of identity in mathematics education. What follows next are the selected studies on Wenger's (1998) social theory of learning.

3.2 Studies on Wenger's social theory of learning

Chauraya (2013), Essien (2014) and Van Zoest and Bohl (2005) were selected to highlight how Wenger's (1998) social theory of learning has been utilised in discussing teacher learning in communities. Two studies (i.e. Chauraya, 2013; Essien, 2014) were conducted in the South African context. Van Zoest and Bohl's (2005) study was conducted in the United States of America. Table 3.1 presents a summary of these studies.

When drawing on Van Zoest and Bohl (2005), I argue that foregrounding learning as *experiencing* (meaning) or learning as *doing* (practice) seems more fitting to research in-service teacher identity, as it is the case for this study. Then, as I draw from Essien (2014), I argue that foregrounding learning as *becoming* (identity) remains relevant to research in-service teacher identity despite having been mostly used in the mathematics literature to research pre-service teacher identity. To end this section, as I draw from Chauraya (2013), I argue that learning as *belonging* (community) can limit in-service teachers to what is attainable in their context of learning and teaching mathematics.

Table 3.1: Selected studies on Wenger's (1998) social theory of learning

Author (year)	Study focus	Participants and data collection methods	Findings
Essien (2014)	The study investigated how teacher educators can develop multiple interacting identities for teachers when learning in communities of practice.	Four mathematics teacher educators were observed when they were teaching mathematics to pre-service teachers.	The study reported a need for teacher educators to induct teachers into becoming teachers of mathematics in certain learning and teaching contexts.
Chauraya (2013)	The study focused on how learning within the professional learning community can influence changes in teaching practices and identities.	Five mathematics teachers participated in a year-long professional learning community. Data was analysed using qualitative methods, particularly interviews, lesson observations and field notes.	During and after interventions, the teachers shifted to identify themselves as members of communities in their mode of teaching, task selection and in their ways of engaging with learners' ideas.
Van Zoest and Bohl (2005)	The study highlighted the theoretical framework for understanding mathematics teachers' learning through community of practise.	The study discussed the theories of how pre-service mathematics teachers can be developed within reform-oriented programmes.	The theoretical framework combined the importance of teacher knowledge and cognitive phenomena within social situations.

The three studies generally presented different kinds of learning that are necessary to change identities of mathematics teachers. The studies analysed teachers' identities from different Wengerian (1998) conceptual approaches. Van Zoest and Bohl (2005) combined theoretical constructs to include constructs from Shulman's theory, which highlight the importance of teacher knowledge and cognitive notions of thinking in one's head, whilst Wenger's (1998) conception of learning as identity in social situations was central in their study. When focusing on Wenger (1998), as mentioned, Van Zoest and Bohl's (2005) study analysed teachers' identities from the perspective of their learning experiences and teaching performances. Essien (2014) analysed teachers' identities as ways of becoming certain kinds of teachers in communities of practice. Chauraya (2013) analysed teachers' identities as ways of belonging to a professional learning community.

Learning as experiencing and learning as doing seem to be more fitting to research in-service teacher identity

As it can be noted, Van Zoest and Bohl's (2005) conceptual approaches on Wenger (1998) were similar to those of the present study. In the present study, teachers were participating in the Transition Maths (TM1) course as the community of practice to make meanings of mathematics-for-teaching (MfT), and then I sought to recognise change in what they were doing in their teaching of mathematics. Wenger (1998) describes these conceptual approaches as learning as *experiencing* (meaning) and learning as *doing* (practice) respectively.

Van Zoest and Bohl (2005) drew from Wenger (1998) to theorise teachers' learning in communities of practice where individuals had to share a common goal of a reformed curriculum. The authors explained ways of understanding learning in the arena of mathematics education using the notion of mathematics teacher identity. In their theoretical framework, they began by incorporating an area of cognitive science, which focuses on the ways individuals make sense of, within their own minds, mathematical processes and concepts. They then moved further outward from the individual minds in terms of analysing learning (i.e. away from a psychological perspective) to account for interactions between groups of individuals learning mathematics together (i.e. toward a more socio-cultural perspective). Thus, drawing directly from Wenger (1998), Van Zoest and Bohl (2005) discussed how groups of teachers, whether pre- or in-service, can learn in communities, as they elaborated on the impact of learning from experiences and relationships that teachers can develop in various communities they act within.

In teasing out the similarities between the studies, firstly, Van Zoest and Bohl (2005) emphasised that individuals' capabilities from cognitive science can contribute towards teacher identity. In the present study, teachers in the TM1 course were participating in improving their personal mathematics content knowledge, and they were being inducted to the aspects of the Mathematics Teaching Framework (MTF), and teachers were assessed as individuals on what they have learned in the TM1 course. Secondly, in Van Zoest and Bohl's (2005) study, teachers were expected to change their identities considering the reformed curriculum that had emphasised learner-centred teaching of mathematics. In disseminating learning of the reformed curriculum, teachers were expected to learn from PD interventions, university learning programmes, short courses, or mentoring mediations from other teachers or teacher educators. In the TM1 course (the PD intervention), teachers were learning MfT including being inducted to the MTF to influence their teaching practices. As such, the present study focuses on the group of teachers teaching in disadvantaged schools, which their learning of mathematics and their learning how to teach mathematics are influenced by participating in the TM1 course. Thus, when drawing parallels between Van Zoest and Bohl (2005) and my study, it confirms that the ideas of learning as *experiencing* (meaning) and

learning as *doing* (practice) are more fitting to research in-service teachers in terms of different contexts that can contribute in influencing the characterisation of their teaching of mathematics.

Learning as becoming remains relevant to research in-service teacher identity

Essien's (2014) study called for a holistic approach in what teachers are to learn in communities of practice. Essien (2014) argued that pre-service teachers should be inducted to multiple interacting identities, including becoming learners of mathematics, becoming learners of mathematical practices, becoming teachers of mathematics, and finally, in the multilingual contexts, becoming teachers of mathematics in multilingual classrooms. The call came as four teacher educators were observed when they were teaching mathematics education in the lecture-rooms. Essien (2014) found out that the teacher educators were mostly inducting the pre-service teachers into becoming learners of mathematics. According to Essien (2014), there were limited practices aimed at inducting the pre-service teachers into becoming teachers of mathematics and even more limited ones that inducted them into becoming teachers of mathematics in the multilingual classrooms. These results emerged on the back of investigating how pre-service mathematics teachers developed multiple interacting identities through engagement support from teacher educators when facilitating learning in a community of practice (Essien, 2014).

When comparing Essien (2014) and the present study, I argue that focusing on improving MfT equip teachers to become learners of mathematics, to become learners of mathematical practices, and to become teachers of mathematics. Of course, inducting teachers to become teachers of mathematics in multilingual classrooms did not feature in the TM1 course. Moreover, the present study does not start by including explanations of learning to become teachers of mathematics other than positive and negative assertions of teachers from the quantitative processes. However, later on when complementing the quantitative results with narrative analysis in Chapter 8, the study provides circumstances of changes of teaching mathematics as influenced by participating in the TM1 course. To conclude this section, after following trends on the studies (e.g. Graven, 2004; Nel, 2012) that are employing Wenger (1998), including Essien (2014), I argue that the learning as *becoming* remains relevant to research in-service teacher identity.

Learning as belonging can limit in-service teachers to what is attainable in their context of learning and teaching mathematics

Chauraya (2013) argued that belonging to professional learning communities (PLCs) promotes long-term, continuous, developmental and collaborative process of teacher learning. The author linked the opportunities for teacher learning in a PLC to be equivalent to learning

in a community of practice. He drew from the three modes of belonging to communities of practice (engagement, imagination and alignment) to analyse changes in teachers' identities, and accredited the community of practice to have contributed directly in changing teachers' identities and practices. Chauraya (2013) analysed identities of five in-service mathematics teachers as they were learning how to belong to the community of practice. Chauraya (2013) collected data using observations and interviews as the teachers were joining the PLC, during the PLC activities and after the PLC intervention. The PLC intervention ran for a year.

For Chauraya (2013), engagement was referred to a process of joint negotiation of new meanings about teaching practices. Through such engagement the teachers developed new understandings of both teaching knowledge and about themselves. As members of the PLC, the teachers expressed visions of progressively learning together to improve their teaching practices (Chauraya, 2013). Wenger (1998) confirms that engagement is "a mode of belonging and a source of identity" (p. 174). Thus, to understand development of teachers' identities, it was important for Chauraya (2013) to analyse teachers' identities before and after teachers participated in the PLC. The analysis was to determine changes in how teachers interacted with other mathematics teachers and learners after the PLC intervention.

Imagination is "a process of expanding our self by transcending our time and space and creating new images of the world and ourselves" (Wenger, 1998, p. 176). In Chauraya's (2013) study, imagination was used to refer to the teachers' images of themselves; the past and future possibilities of participating in the PLC; and their relations with others in the community. He needed to capture the extent of how the teachers imagined their participation in the PLC as a learning trajectory extending from the histories of their teaching experiences into the future, and how they imagined themselves in that trajectory.

Chauraya (2013) referred to alignment as how the teachers related to and acted upon their experiences of the activity of the PLC. This is in line with Wenger's (1998) description of alignment. According to Wenger (1998), alignment involves "coordinating our energy and activities in order to fit within broader structures and contribute to broader enterprises" (p. 174). It entails the discourse and compliance in shaping our ways of belonging within communities of practice. From understanding this mode of belonging, Chauraya (2013) analysed teachers' identity from perceptions of learning in the community and how the teachers saw that learning as part of their professional practice; emphasis on collaboration as part of the teacher professional work; and views of changed practice.

To highlight the shortfalls of the three modes of belonging in relation to the present study, engagement is described to involve empowering teachers by improving their mathematics content knowledge and by influencing their teaching practices. However, when keeping in

mind that the teachers who attended the TM1 course are teaching in disadvantaged schools, the very same power of engagement can limit individuals to what emerges when they interact with others in communities of practice (Wenger, 1998). For instance, amongst other teaching methods, teachers are encouraged to put learners in groups to promote discussions in their classrooms (Makgato, 2007). However, the question arises: how can the teachers who are working in disadvantaged schools with their classrooms full to capacity, promote group discussions? Thus, in reality, these teachers are positioned to develop limited identity trajectories, as they can spend time defending why they do not promote group discussions when teaching mathematics instead of participating in how group discussions can advance learning of mathematics when interacting with other teachers (Britzman, 2003).

Parallel to that, I argue that imagination can remain a comforting factor for teachers who are working in disadvantaged schools. Teachers imagine similar problems to be occurring in other schools and strive to offer solutions to the problems. From reading Wenger (1998), teachers can long for positive changes within their school environments as they imagine the teaching of mathematics to have significant contributions to nation building. Thereby, in the instances of poor results in mathematics, whilst many teachers seem to give up, some teachers become resilient and develop a positive attitude towards teaching the subject (Tsanwani et al. 2014). However, when considering the reality, resilient teachers can remain in envisaging to do better in the future (Wenger, 1998).

Another important mode of belonging is alignment. Teachers tend to respect the profession because of the alignment. The respect of the profession comes with a “controlling energy” (Wenger, 1998, p. 180) of knowing that you belong to a wider community of practice. According to Wenger (1998), alignment within a wider community of practice can be disempowering and sometimes abusive. When taking the example of poor results in mathematics, teachers working in disadvantaged schools are sometimes forced by school management bodies to conduct after-school classes to make up for insufficient teaching time (Tsanwani et al. 2014). In this regard, teachers whose lives and identities lay elsewhere (e.g. in their leisure pursuits, in their families, in their religious activities and in their political affiliations) (MacLure, 1993) can feel disempowered because of the alignment. Thus, as engagement, imagination and alignment constitute learning as *belonging*, and as elaborated in Section 2.8, learning as *belonging* (community) is backgrounded in this study.

3.3 Studies working with Sfard and Prusak’s identity framework

I selected three studies, namely Bjuland et al. (2012), Gujarati (2013) and Bennison (2015) to discuss how Sfard and Prusak’s (2005) narrative identity framework has been utilised in the mathematics education literature. See Table 3.2 for brief descriptions of the selected studies.

When drawing from Gujarati's (2013), I assert that negative teachers' identity does not always imply to not being accountable to teaching practices. Then, when drawing from Bjuland et al. (2012), I highlight indicators of using narrative identity framework. Lastly, when drawing from Bennison (2015), I relate knowledge, affective, social, life history and context to domains of characterising teachers' mathematics teaching identity.

Table 3.2: Selected studies working with Sfard and Prusak's (2005) identity framework

Author (year)	Study focus	Participants and data collection methods	Findings
Bjuland, Cestari and Borgersen (2012)	The study focused on the methodological use of analysing reflective narratives of an experienced primary teacher about discourses and activities of teaching mathematics to provide evidence of his or her professional identity.	Data from the teacher was collected using interviews and lesson observations. The teacher had participated in a professional development and research project, which promoted collaboration between teachers and mathematics educators.	Analysing reflective narratives, particularly using Sfard and Prusak's indicators (i.e. reifying, endorsable and significant narratives), yielded evidence of the teacher's professional identity.
Gujarati (2013)	The study looked at the relationship of early career elementary teachers' beliefs about mathematics and their classroom practices in order to judge who they are as teachers (teacher identity).	Through autobiographical inquiry, reflective practice, classroom observations, interviews, and artefacts, data was collected on the three grade 2 teachers regarding mathematics and their beliefs in learning and teaching the subject.	All the three teachers had an "inverse" relationship between their mathematics identities and their classroom practices due to their accountability to school, students and parents, and to increase student achievement.
Bennison (2015)	The study provided a theoretical model for investigating teacher identity as an embedder-of-numeracy.	The study drew mainly on Gee (2001), Sfard and Prusak (2005) and Wenger (1998) to conclude on the theoretical model.	The theoretical model of teacher identity should include the following five domains: knowledge, affective, social, life history and context.

What is common in these three studies is that teachers were telling stories about themselves and the stories were described to accompany actions (i.e. actions were mainly about teaching practices). Bjuland et al. (2012) used only Sfard and Prusak (2005), while Gujarati (2013) and Bennison (2015) used Sfard and Prusak (2005), Wenger (1998) and Gee (2001) to define and theorise teacher identity. Bjuland et al. (2012) and Gujarati (2013) used reflective narratives from interviews to confirm or question relationships between teachers' identities and their

teaching practices. In Bjuland et al. (2012) and Gujarati (2013), teachers' narratives had shifted from designated to actual identities while their teaching practices had improved. For Sfard and Prusak (2005), closing a gap between designated and actual identities indicates learning. Bennison (2015) provided a theoretical model of how teacher identities can be characterised as an embedder-of-numeracy.

Negative teachers' identity does not always imply to not being accountable to teaching practices

Regarding Gujarati (2013), all the three teachers who participated in the study had negative beliefs about themselves yet they were successful in being accountable to schools, to learners and parents, to increasing learners' achievement in mathematics. Gujarati (2013) reported that the learners who attended mathematics classes of the three teachers were not experiencing negative dispositions towards mathematics, and therefore, the author commended the teachers for the efforts of being accountable to the teaching practices by doing their best of their abilities in teaching the subject. For Gujarati (2013), accountability meant that the teachers were able to reflect openly and honestly about their beliefs of teaching mathematics while ensuring that 'no children are left behind' when learning mathematics. In this sense, Gujarati's (2013) findings provided a new insight considering that most studies about primary teachers do not examine teaching practices in moments when teachers are expressing negative emotions from their past experiences of learning mathematics (see e.g., Hodgen & Askew, 2007). Rather, such studies remain in exploring how teachers' experiences of learning mathematics in different context including teacher education and professional development programmes translate to teacher identity (e.g., Lutovac & Kaasila, 2014; Skott, 2018, 2019; Jong, 2016).

When comparing Gujarati (2013) and this study, I highlight that negative teachers' identity does not always imply to not being accountable to teaching practices. Similarly to Gujarati (2013), the present study examines teachers' learning experiences at the backdrop of their actual teaching practices. When considering teachers' learning experiences for the present study sit firmly within Wenger's (1998) notion of community of practice, which promoted the interactive learning (as becoming, belonging, experiencing, mastering and doing), I expect changes in teachers' identity to have been influenced by their participation in the TM1 course. However, in Chapter 8, if the teachers do not express changes in their identity after participating in the course, I still have grounds in Chapters 9 – 11 to analyse their actual teaching practices and conclude on their identity from performative perspective.

Indicators of using narrative identity framework to reflect on discourses and activities of teaching mathematics in the classrooms

Bjuland at el. (2012) acknowledged that a teacher who participated in their study had gained a sense of belonging to a community of practice after collaborating with university mathematicians in planning her lessons. A 3-year PD project constituted the community of practice. In brief, the PD project included inducting teachers on (1) how to introduce mathematical topics (e.g., algebra and geometry); (2) how to design small-group activities involving working on mathematical problems; and (3) how to cooperate as colleagues teaching in the same school to plan activities to be used in the classrooms (Jaworski, 2007; Bjuland & Jaworski, 2009).

Bjuland et al. (2012) focused on providing methodological evidence of relevance of employing Sfard and Prusak's (2005) identity framework when reflecting on discourses and activities of teaching mathematics in the classrooms. From analysing reflective narratives from discourses and activities, which emerged from the time the teacher was narrating her stories about teaching mathematics to other teachers whilst participating in different phases of the 3-year PD project, Bjuland et al. (2012) presented four identity indicators. The indicators were as follows.

Positioning in relation to pupils. This indicator was about how best the teacher can design tasks that enable learners to make connections between concepts to understand mathematics.

Reflecting on developing a workshop model in teaching. This indicator highlighted the importance of becoming a better teacher that is not shy to learn mathematics with other teachers, as that teacher is not afraid to try out different methods when teaching in the classroom.

Integrating and expanding models of teaching. This indicator was centred around how learners can be invited to fully participate in discussions when exploring and investigating mathematical ideas whilst justifying the concepts when working on problems in small groups.

Challenging positioning in relation to university mathematicians. This last indicator talked to how teachers who participate in PD projects are aware of which mathematical aspects that are implementable when teaching mathematics.

As I use Sfard and Prusak (2005) to reflect on teachers' experiences of participating in the TM1 course, I argue that the first three identity indicators of Bjuland et al. (2012) entail similar features to those of the MTF as teaching practices. The last identity indicator can be linked to the shortfalls of learning as *belonging* (community) as discussed in Chauraya (2013) above. That is, teachers become aware of what is possible (or not possible) to implement in their teaching of mathematics. For example, as teachers were exposed to experiences of working

in small groups themselves in the TM1 course, teachers who are teaching overcrowded classes were aware that implementing group discussions in their classrooms were not always possible in their teaching of mathematics.

Both Bjuland's et al. (2012) study and the present study are similar in terms of the inputs and outputs of learning as experiencing (meaning), but differ in how the outputs of learning were analysed in the studies. In the present study, the outputs of learning are analysed to highlight changes due to learning MfT in the TM1 course, whilst reflecting on how those experiences affect changes in their teaching of mathematics in the classroom. Moreover, the features of the MTF as teaching practices are analysed from the observations when the teachers are teaching in the classroom and then compared with their understanding of the MTF when reflecting on their teaching of mathematics during the interviews.

Domains of characterising teacher identity of teaching numeracy relate to teachers' mathematics teaching identity

Bennison (2015) provided a theoretical model of investigating how teachers' multiple situated identities (i.e. identity as an embedder-of-numeracy) can be characterised for teachers of all subjects including mathematics. By an embedder-of-numeracy, the author meant the need for all teachers teaching in schools to teach mathematical skills and concepts that learners would continue to apply beyond the school years.

Bennison (2015) drew mainly on Wenger (1998), Gee (2001) and Sfard and Prusak (2005) to conclude on the theoretical model of researching identity as embedder-of-numeracy. The theoretical model entailed *knowledge, affective, social, life history* and *context* as the five domains that are necessary to characterise teacher identity in the context of teaching numeracy. Briefly, *context* can include factors hindering or supporting of teacher identity when situating the research; *life history* can relate to teachers' past experiences of mathematics or in-service professional development interventions; *social* can imply professional learning communities or school communities; *affective* can mean teachers' perceptions about themselves (i.e. values, beliefs, and attitudes) towards a subject; and *knowledge* of a subject (i.e. mathematics content knowledge, teaching knowledge and curriculum knowledge).

Bennison's (2015) domains of researching identity can be directly linked to the concepts of the present study. That is, *knowledge* of a subject can be associated with learning MfT. *Context* domain can be associated with learning MfT in the TM1 course and teaching in disadvantaged schools. *Social* domain can imply that teachers are learning together in the TM1 course as the community of practice. *Life history* emerges when comparing what the teachers have learned in the past (e.g., universities or colleges of education) before joining different communities of practice including the TM1 course. However, as noted, this study

emphasises teachers' past experiences of participating in the TM1 course. Whereby, *affective* domain remained backgrounded in the present study, given that teachers' identity for the present study are mainly characterised by what teachers are doing (or can do) when teaching mathematics.

3.4 Studies on Darragh's identity framework

I selected Darragh (2015a, 2015b, 2018) to discuss how Darragh's (2016) used her performative identity framework. As mentioned in Subsection 2.2.2, unlike Wenger's (1998) participative identity and Sfard and Prusak's (2005) narrative identity frameworks, Darragh's (2016) performative identity framework has not been frequently used by other researchers in mathematics education. Thus, I selected Darragh's own work to review the literature that I consider to be relevant for this study. Table 3.3 provides a summary of the selected studies.

Table 3.3: Selected studies on Darragh's (2016) identity framework

Author (year)	Study focus	Participants and data collection methods	Findings
Darragh (2015a)	The study drew on performative identity lens again to understand, analyse and present learners' mathematical identities in the context of secondary school in New Zealand.	The study presented three vignette (i.e. a scene description, a short play, and a monologue) which were analysed to recognise how girl learners learn in the classroom.	As the results, the study highlighted the biases that can influence the recognition of girl learners as being marginalised and thus miss opportunities of recognising their mathematical identities.
Darragh (2015b)	The study employed a performative lens for identity to investigate how learners recognise someone who is 'good at mathematics'.	22 learners were recruited to participate in the study. The study used semi-structured interviews to gather data from the learners, their teachers and their parents. The study also used classroom observations to gather data.	The learners identified the roles of being 'good at mathematics' to include getting high marks, knowing the answer quickly, helping others, and demonstrating the confidence to put up their hand to answer questions.
Darragh (2018)	The study reflected on Darragh (2015a, 2015b) to give further context on how categories such as recognising gender tends to come about in the research.	When reflecting on Darragh (2015a, 2015b), the author acknowledged that recognising gender and ethnicity when using identity lens has a potential to build researchers' identities as they can contribute on wide social and political discourses.	From the study, it became relevant for me to reflect on issues of gender and ethnicity recognition despite my initial position to not include such issues when discussing the present study.

Reviewing Darragh's (2015a, 2018) gave me an opportunity to reflect on why I do not emphasise the issues of gender, ethnicity and race in this study. Darragh (2015a) asserted that audience (researchers, in this case) should foreground recognising mathematics learning identities, and avoid to biasedly recognise gender issues in in the context of learning and teaching mathematics. Darragh (2018) reflected on how to recognise and identify participants and a researcher in mathematics education research through their socio-political acts. Lastly, when drawing from Darragh (2015b), I highlight the importance of observations as another research tool of recognising participants when using performative identity framework.

> ***Opportunity to reflect on why I do not emphasise the issues of gender, ethnicity and race in this study***

Darragh (2015a) employed the performative lens to understand, analyse and present learners' mathematical identities in the context of secondary school in New Zealand. She used the metaphor of a theatrical performance to present the following three vignette: a scene description, a short play, and a monologue. When setting the scene, the author presented a traditional outlook of the class, where desks are spaced in rows and columns, and all the learners are facing a whiteboard. In this scene, teacher's desk is located in the front and in the corner of the classroom.

During the short play, the teacher in the scene had asked learners to join desks and work in groups. When observing the lesson, the researcher noticed that some girls were switching characters when working on tasks from being mathematics learners to being friends to each other. For instance, as mathematics learners, the girls participated by providing solutions to mathematics problems to the teacher, whereby as friends, the girls talked to each other about a movie that played the previous night.

During the monologue, the researcher presented a teacher who was playing the role of the jester. This teacher recognised gender in the performance of his learners by identifying girls to not have natural talent of solving algebraic problems when compared to boys. The teacher stated that "girls are not really good at algebra" because it has abstract concepts.

In the monologue, the researcher argued that the teacher was 'playing the fool' when considering that he had only taught at a single-sex girls' school for only three years – three years was his entire teaching career. The researcher concluded that the teacher had drawn from his experiences of being a mathematics learner and not from being a mathematics teacher to negatively recognise gender in the performance of learning mathematics.

In both the short play and the scene, the researcher questioned if her recognition of gender was not based on her own biases of being once a girl learner in mathematics classes and from

engaging with the research literature that locate girls on periphery in the mathematics learning community. Whilst the author do not wish to be naive, and pretend like there are no circumstances where girls are marginalised, it was also her plea to not allow ourselves to be side tracked by recognising gender everywhere, whilst there are opportunities of recognising other mathematics learning identities (Darragh, 2015a).

Darragh (2018) explained ways of recognising and identifying participants and researchers in mathematics education research during their socio-political acts. Darragh (2018) discussed how she had positioned herself in the communities of mathematics education research by performing her researcher identity in her previous study (i.e. Darragh, 2015a). In this book chapter, Darragh (2018) explained that the sources of biasness when recognising participants can be associated with researcher's identification.

Darragh (2018) explained that she used her researcher identity to shine spotlights on wider social and political discourses by particularly focusing on gender and ethnicity when conducting the previous study (i.e. Darragh, 2015a). Darragh (2018) emphasised that her choice to use Emily (a Samoan girl and an able mathematics learner) in her study (Darragh, 2015a) was deliberate as it had a potential to advance her temporal and multiple identities in mathematics education. However, Darragh (2018) warned us that if researchers' identities are not checked, research studies run the risks of strengthening stereotypes and feed to popular wide statistical and educational discourses instead of contributing in the understandings of the different phenomena when researching mathematics education.

In reading Darragh (2015a, 2018), for the present study, here are some of the questions that I had asked myself as a novice researcher. Do I need to foreground how many female teachers and male teachers had participated in the TM1 course? Is it necessary to highlight which gender group participated in what way during the TM1 course? Would it make any difference to recognise gender in the participants of the present study? These questions come as Darragh (2015a, 2018) contested whether or not it was necessary to recognise gender in her studies. Darragh (2015a, 2018) had asked if it was not enough to recognise Emily as an able mathematics learner for the studies instead of highlighting her gender and ethnicity.

Furthermore, when reflecting on the present study, I had asked myself if it was enough to only recognise the race of the teachers who have participated in the TM1 course, as being mainly Black and teaching in disadvantaged schools, without positioning them amid one or two White teachers who had also participated in the TM1 course. Lastly, I had asked myself if it did matter that a White male, a White female, and a Black foreigner facilitated the TM1 course. The answers to these questions are discussed in the next chapter, but Carlone and Johnson (2007) argue that recognition of individuals' competences about learning and teaching can be

disrupted by gendered, ethnic and racial factors. At the same time, it can be noted that the issues of gender, ethnic and race can easily slide towards post-structural perspectives on identity. Post-structural perspectives on identity talk to "the asymmetricality of power" (Darragh, 2018, p. 78) in the learning and teaching, of which such power dynamics are common in South Africa, but not necessary in focus for the present study.

Observations as an important research tool of recognising participants when employing performative identity framework

Darragh (2015b) examined how learners recognise someone who is 'good at mathematics'. The study revealed many ways of enacting the roles of being 'good at mathematics', including "getting high marks, knowing the answer quickly, helping others, and demonstrating the confidence to put up their hand to answer questions" (Darragh, 2015b, p. 83, abstract). Despite the variety of ways in which to demonstrate 'good at mathematics', the author argued that most learners did not recognise themselves in their own descriptions, of which in some way, goes towards explaining why learners may opt out of further study in mathematics. The context for Darragh's (2015b) study was at the time when learners were transitioning from primary to secondary school.

It is useful to note that, when the learners did not recognise themselves in their own descriptions of being 'good at mathematics', the researcher drew from observing the lessons and from how teachers of the learners commented about the learners and then questioned the learners' positions of not recognising themselves. The idea of questioning learners' conceptions about learning mathematics relates directly to how I (as the audience) intend to question the participants of the present study when they are explaining their teaching of mathematics. In other words, as I elaborate on this point in Chapter 5, the present study intends to triangulate the results by focusing on both data from individual interviews and data from lesson observations.

The obvious difference between Darragh (2015a, 2015b, 2018) and the present study is that Darragh reported on learner identity while I report on teacher identity. However, the idea of drawing from Darragh (2015a, 2015b, 2018) was based on how Darragh had conceptualised her studies on learners' learning, whereas the present study looks at the learning of teachers in the TM1 course and in their teaching of mathematics. The other trivial fact is that the contexts between the studies are different. In Darragh's (2015a, 2015b) studies, the context was during the time when the learners were transitioning from primary to secondary school. Whereas, the present study examines teachers' identities few years after they had participated in the TM1 course.

3.5 Summary

In this chapter, I presented the review of the selected studies that have employed the theoretical frameworks of this study (i.e. Wenger's (1998) participative identity, Sfard and Prusak's (2005) narrative identity and Darragh's (2016) performative identity).

Having employed Wenger (1998) as the foundational framework for the present study, I highlighted Van Zoest and Bohl (2005) to be the study that advocate for the analysis of teacher identity from the learning as *experiencing* and learning as *doing* conceptual approaches. When drawing from Essien (2014), I highlighted that learning as *becoming* was relevant (but less fitting) to research in-service teacher identity. Whereas, when drawing from Chauraya (2013), I argued that learning as *belonging* can limit teachers to what is attainable in the context of learning and teaching mathematics.

When discussing Bjuland et al. (2012), Gujarati (2013) and Bennison (2015), which are the studies working with Sfard and Prusak's (2005) identity framework, I highlighted that teachers tell stories about themselves, and those stories accompany actions about their teaching practices.

Lastly, I reviewed Darragh (2015b) as the selected study to highlight observations as an important research tool of recognising participants from their actions when employing a performative lens to research teacher identity. What remained central in reviewing Darragh's (2015a, 2018) studies was to get the opportunity to explain why I do not emphasise gendered, ethnic and racial factors in this study as they can derail discussions on teachers' competences of learning and teaching mathematics.

CHAPTER 4

THE TM1 COURSE: THE CONTEXT OF THE STUDY

4.1 Introduction

I begin this chapter by referring to the Transition Maths 1 (TM1) course as a community of practice. As the first step in building the argument, in Section 2.4, I highlighted that the TM1 course was aligned with the "member-oriented teacher communities with pre-set agendas" formalised structure of learning. As will be seen in this chapter, I draw on Adler and her colleagues (e.g., Adler, 2013; Adler, 2021b; Adler & Ronda, 2015; Adler & Pournara, 2020; Adler & Venkat, 2014) to provide evidence that the pre-set agendas in the TM1 course were informed by empirical data of working with teachers rather the adoption of a top-down approach. In this regard, the TM1 course is the context of this study.

This chapter is organised around *what, how* and *why* did the teachers learn in the TM1 course. As such, I express *what* the teacher community learned in the TM1 course as the joint enterprise. The joint enterprise was about the learning of mathematics-for-teaching (MfT), which included inducting the teachers to the aspects of the Mathematics Teaching Framework (MTF). Then, I argue that different roles of individuals in the TM1 community (i.e. the facilitators of the TM1 course, the research fellows including myself, and the teachers) relates greatly to mutual engagement, which was *how* the teachers learned in the TM1 course. The description on the shared repertoire follows as the subsection on its own, whereby technology and textbooks were accessible to the teachers to further contribute in *how* they learned in the course. Lastly, I contend that the assessment tasks that were used in the TM1 course enabled the teachers to judge themselves and by others as being competent teachers of mathematics, which was *why* they learned in the TM1 course.

4.2 The TM1 course (The community of practice)

I characterise the TM1 course to be a community of practice despite its formalised structure of learning MfT. The TM1 course was part of the Wits Maths Connect Secondary (WMCS) project. The WMCS project was a ten-year research and development project aimed at researching solutions to the "mathematics crisis" in South African schools and developing models to improve the quality of mathematics teaching in secondary schools, particularly in low-income communities. The improved quality of teaching was judged by learners' opportunities to learn mathematics and their performance on both national and project-specific assessments. The project began in 2010 working in 10 secondary schools in one school district. After five years, the project expanded to work with teachers from more schools in a wider range of districts.

When describing the TM1 course in its existence during the years 2016, 2017 and 2018 for the study, teachers attended the TM1 course for sixteen days organised in 2-day units for a year. These teachers attended the course on campus at the University of the Witwatersrand venue. Considering that the classes were sequenced to coherently move from known to unknown, the teachers were expected to fully participate in all the classes. In instances where some teachers needed more support on certain topics, those teachers could come to campus to receive one-on-one assistance for few hours on the arranged days during public school holidays.

The TM1 course was formally structured and deliberate in achieving the predetermined learning outcomes. In the course, the teachers were required to complete individual and group tasks on content knowledge and about the MTF. Tasks enabled the teachers to reflect on their mathematics and teaching practices. Most short tasks, like quizzes and assignments, qualified as formative assessments to establish continuous portfolios for the teachers. The teachers also sat for summative assessments. Summative assessments were conducted under examination conditions. I return to say more on the assessments towards the end of this chapter.

To connect other features of the TM1 course and the community of practice, as described in the social theory of learning (Lave & Wenger, 1991; Wenger, 1998; Wenger et al., 2002), I discuss the three dimensions: *mutual engagement*, *joint enterprise* and *shared repertoire*. I elaborate on these features in the next subsections, but for now, I briefly describe how each dimension connect to the TM1 course.

The TM1 course was about learning and improving teachers' MfT (*i.e. joint enterprise)*. In the course, the teachers spent three-quarters of their time learning and doing mathematics content, and a quarter further learning about the MTF to influence their teaching practices.

The different roles of individuals in the TM1 community (i.e. the facilitators, the research fellows, and the teachers) were geared towards the learning of MfT in the TM1 course *(i.e. mutual engagement*). The TM1 community had established the standardised norms, expectations, ways of attending the classes and social relationships between the teachers, the research fellows, and the facilitators in order to enhance participation in the TM1 course.

The teachers had full access to resources that were part of learning in the TM1 course such as the GeoGebra software and relevant mathematics textbooks. In addition, the TM1 community had embraced different learning tools of mathematics that are accessible to teachers such as definitions, theorems and mathematical symbols *(i.e. shared repertoire*).

Of importance, I describe the community of practice as if there were no power dynamics in the mediating and learning in the TM1 course. For example, other than the issues of race, as explained when discussing Darragh (2015a, 2018) in Section 3.4, I talk about the different roles of members of the TM1 community as if all the members held an equal say in the running of the TM1 course. The fact is that there were directives that steered the running of the course (Wenger, 1998). The facilitators, as they were assisted by the research fellows, decided on *what* and *how* the teachers were to learn MfT. As described in Section 2.4, despite Wenger's (1998) preferences of unstructured, unplanned and not taught learning, the teacher community was subjected to pre-set schedules, session formats and objectives that were set before and during the classes (Owen, 2014). However, to me, the teacher community took directives and engaged in mostly prescribed interactions with the facilitators being aware of the benefit of the TM1 course, which included improving their competence of learning and teaching mathematics.

What follows next are the discussions on *what* (i.e. the joint enterprise), *how* (i.e. mutual engagement and the shared repertoire) and *why* (i.e. competence of learning and teaching mathematics) the teachers learned in the TM1 course (i.e. the community of practice). To note, I attempt to link the components of learning, mainly teacher identity, to the enlisted dimensions (i.e. the joint enterprise, mutual engagement, and the shared repertoire) of the community of practice, given that these dimensions are considered to be critical sources of identity development (Wenger, 1998). I start by elaborating on the learning of MfT, which was *what* the TM1 course was about.

4.2.1 Mathematics-for-teaching (The joint enterprise)

In Chapter 1, drawing on the literature (e.g., Adler, 2005; Adler, 2013; Adler & Davis, 2006), teachers' MfT was described as a set of knowledge that is constituted by levels of proficiency in mathematics content and necessary teaching practices. Consequently, in the TM1 course, the teachers were learning the mathematics content, and they were also learning about the MTF to influence their teaching practices.

The mathematics content of TM1

When learning the mathematics content, the teachers were given a variety of tasks to promote their learning of mathematics for understanding (Hiebert & Carpenter, 1992), to encourage meaning-making of mathematics, to deepen their knowledge of familiar mathematics and to extend their knowledge to further aspects of each topic (Kilpatrick, Swafford & Findell, 2001). The mathematics content was from Grade 8 to 11 curriculum, and it covered topics on algebra, functions, geometry and trigonometry. For Adler (2005), teachers with deep knowledge of

mathematics are able to decide on the mathematical worth of learner productions when teaching the subject. Moreover, as alluded to in Chapter 1, learning mathematics content on its own can mean developing positive teachers' experiences towards mathematics, which can be linked to positive teachers' mathematical identities.

On the one hand, the idea of deepening teachers' knowledge of familiar mathematics was aided by the sequencing of topics that were covered in the TM1 course. Arguably, the covered topics in the TM1 course support what Wenger (1998) describes as "routines, words, ways of doing things, stories, symbols, actions or concepts" (p. 83) that transcend across other topics when learning mathematics. For example, as Makonye (2011) argues, most concepts of algebra are common in other mathematical topics such as functions, geometry and trigonometry. As such, for example, when solving the trigonometric equation $2(1 - \cos^2 x) + 3\cos x = 3$, one may require the fundamentals of solving an algebraic equation, which are to collect like terms and find factors.

On the other hand, the idea of deepening teachers' knowledge of familiar mathematics was aided by "revisiting school mathematics" (Pournara, 2013; Zazkis, 2011). By revisiting school mathematics, the teachers were re-learning the subject (Pournara, 2013). In the TM1 course, the facilitators would take a step back and provide details of general definitions of words and concepts that have been routinely used in the learning and teaching of mathematics. For example, in one of the earlier sessions in 2018, which was focusing on linear equations, the facilitators dealt with the following questions: What is an equation? What is a solution to an equation? What are the differences between equations, expressions and identities? What are the meanings of the equal sign? When do we find mathematical statements to be always true, sometimes true or never true? When responding to these questions, one or two examples from other topics (e.g., trigonometry) were provided to contrast a variety of examples on linear equations to highlight the applicability of the concepts even in other topics. Indeed, since they were re-learning the subject, the general definitions of words and concepts were followed by workable tasks, whereby the teachers were actually solving the linear equations.

The mathematics tasks that were given to the teachers in the TM1 course became pertinent on two ends. On the one end, the tasks were well structured to highlight key mathematical concepts, procedures, representations or practices for the teachers. Adler and Pournara (2020) refer to such processes of structuring and mediating mathematical tasks as a modelling strategy. As such, there was value for the teachers to think beyond their own learning of mathematics when engaging with the mathematical tasks. During the mathematics sessions, teachers would mention that they intended using some tasks in their teaching. One teacher who participated in the interviews of the present study stated that "From the material that we

were given [during the mathematics session], you know, I would use [some activities] for the Grade 10s" (Makwe, Interview, October 2019).

On the other end, the tasks were orientated for the teachers to learn mathematics for understanding. In general, when learning for understanding, one has to access his or her knowledge of mathematics and reconnect different aspects and concepts to make meanings (Barmby et al., 2007; Carpenter & Fennema, 1988; Hiebert & Carpenter, 1992; Kilpatrick et al., 2001; Skemp, 1976). When accessing knowledge, one might need to ask oneself a series of questions that can aid in deciding which aspects or concepts to connect in order to solve a problem. Succeeding in making the connections can enable individuals to make meaning of mathematics, which leads to understanding mathematics. Below, I share two types of tasks that promoted the learning of mathematics for understanding in the TM1 course.

In this type of tasks, the teachers were required to make connections across mathematical representations (Kilpatrick et al., 2001). In one task, for example, the teachers got different representations of linear functions (i.e. equations, graphs, function machines and tables) with the same meaning. At the low cognitive level, the teachers could match the representations. The focus in this task was for the teachers to find characteristics of linear functions in the representations that were conventionally used for certain purposes. For instance, tables are commonly used to read points to plot graphs. However, in this task, the teachers were required to determine a domain of the function from a table representation. This task mitigated an impression that a domain of a function can merely be defined from input values that are listed in the table.

The teachers were also exposed to a variety of open-ended tasks in the TM1 course (Boaler, 1998). For example, in the task illustrated in Figure 4.1, the teachers were required to write possible equations for two straight-line graphs and to discuss whether the graphs intersect each other.

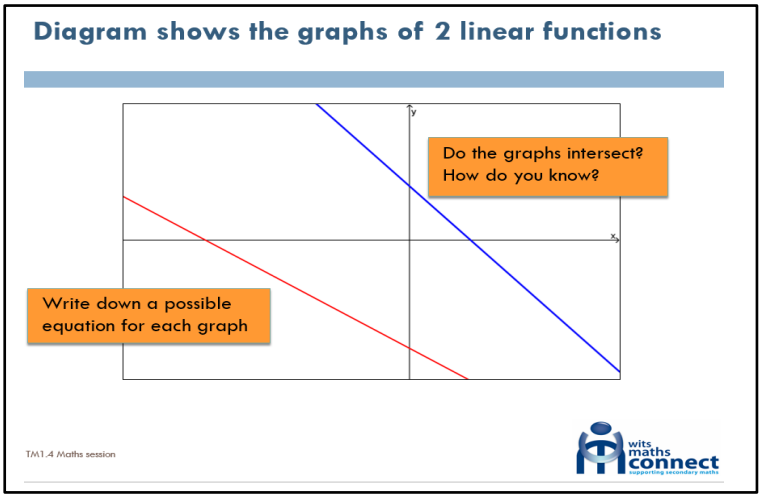


Figure 4.1: An open-ended task from a maths session

The coordinates, where for example the graphs cut axes in the Cartesian plane, were not provided in the exercise. In this way, it was not a matter of finding the equations of the graphs using the coordinates, equate the equations, solve them using algebraic manipulations, and get correct answers. The task provided little information but made substantial demands in terms of teachers' engagement with the mathematical ideas to justify their answers (Van Zoest & Bohl, 2005).

The mathematics teaching content of TM1

As noted, teachers were inducted to the aspects of the MTF in the TM1 course. The MTF was designed to guide the teachers in planning and reflecting on their lessons when teaching mathematics (Adler, 2013). In order for the teachers to gain traction when learning about the MTF (Adler & Ronda, 2017), teachers' knowledge of mathematics content was incorporated in the teaching sessions. However, the learning of the MTF in the TM1 course was explicitly mediated for the teachers.

The MTF responded to the findings by Venkat and Adler (2012), which provided evidence of teachers' lessons that were incoherent across examples and its accompanying explanations. Adler and Venkat (2014) asserted that teachers needed to learn how to keep the object of learning in focus, which could be learned through exemplification and explanatory communication. Subsequently, as mentioned in Section 1.2, the results of learning about the object of learning, exemplification, explanatory communication including learner participation became the four aspects of the MTF. The four aspects of the MTF interrelated as illustrated in Figure 4.2.

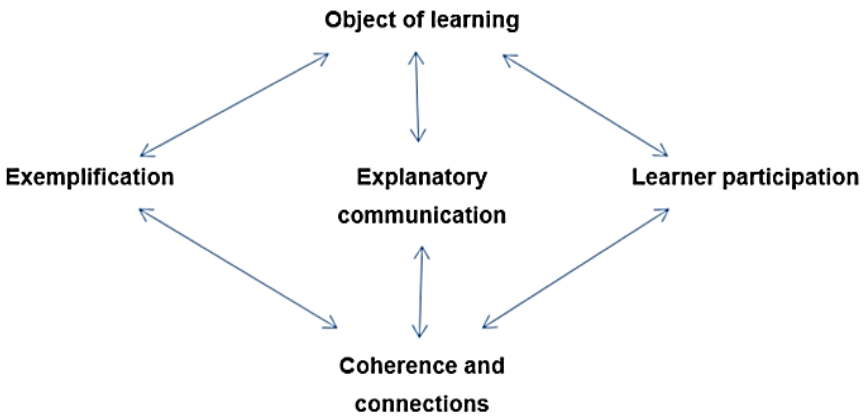


Figure 4.2: The four aspects of the MTF (Adler & Ronda, 2015)

Each aspect of the MTF is defined as follows:

1) **the object of learning:** a focus on what we want learners to know and be able to do by the end of the lesson;

2) **exemplification:** concerns the selected examples, tasks and representations and how these illuminate the object of learning;
3) **explanatory communication:** the talk and boardwork used when mediating learning to explicate the mathematical object and to justify what counts as mathematical; and
4) **learner participation:** interactions between learners and a teacher, and amongst learners, and how teachers can increase opportunities for more active participation by learners.

The object of learning is a lesson goal in the context of teaching in the classroom. However, instead of looking at the object of learning as a lesson goal, it becomes useful to connect both content that learners need to know with what learners are expected to be able to do with respect to that content at the end of the lesson (Adler & Ronda, 2015). If a goal of a lesson is to get learners to be proficient in a mathematical procedure or being able to make connections between specific concepts, for instance, when solving equations using laws of exponents, Adler and Ronda (2015) emphasise the importance of keeping that lesson within that specific focus. The lesson goal[7] is commonly achieved by using examples and explanations through instruction.

The aspect of using example sets (i.e. examples, associated tasks and representations) in the lesson is referred to **exemplification** in this teaching framework. From Zodik and Zaslavsky's (2008) observations, some teachers do not pay attention in their strategies of selecting and using examples for their example sets. Looking back, my high school mathematics teacher preferred giving us examples that were evenly numbered from textbooks as tasks to do at home for practice. That is, the tasks were not selected to illuminate certain aspect from the object of learning that the teacher wanted to bring to focus. The tasks were selected at random, and we easily missed out on using a variety of representations if they were in examples with odd numbers. Adler and Pournara (2020) assert that the appropriately constructed example set increases likelihood that learners will know and be able to do what was intended by the end of the lesson.

The explanatory communication aspect in the MTF orientates teachers to implicitly or explicitly prepare in their planning of the lesson what they will say and write for the learners. Teachers are central in presenting and explaining mathematics to learners in the classrooms. The preparation moves from the kind of language that will be used to name and legitimate what is focused on and talked about in relation to examples and tasks (Adler & Ronda, 2015). The concept of legitimising through explanatory communication requires teachers to draw

[7] 'The lesson goal' and 'the object of learning' wordings are used interchangeably in this study, but ideally 'the object of learning' should relate to the MDI whilst 'the lesson goal' should relate to the MTF.

distinctions between criteria related to "*properties of mathematical objects, accepted conventions* and *derived procedures, instances or empirical cases*, and then *the general case or proof"* (Adler & Ronda, 2015, pp. 5-7, italics in original). In this sense, teachers' explanations add more value in how they justify the mathematics.

Learner participation becomes an additional aspect (other than exemplification and explanatory communication) where, for example, learners can learn from each other (Wydeman, 2015). The overarching interest in discussing this aspect lies in what learners are invited to say and write in the learning of mathematics (Adler & Ronda, 2015). As Adler and Ronda (2015) argue, the emphasis should be whether learners have opportunity to speak mathematically and to verbally display mathematical reasoning through justifications. The observation of this description resonates more in the South African context where mathematics is taught in English while English is a second language for the majority of learners.

All four aspects of the MTF (the object of learning, exemplification, explanatory communication and learner participation) should **connect and be coherent** when describing the lesson. Particularly, a combination of examples that are not carefully selected and incoherent explanations have the potential to confuse learners (Venkat & Adler, 2012), if not leave them with poor experiences of learning the subject. In addition, teachers should attempt to connect different concepts of mathematics in order to obtain coherence from one part of the lesson to the next.

For the mainstay of the MTF, and when using Wenger's (1998) terms, I argue that the framework is sensitive enough to move teachers from their familiar territory to unfamiliar terrains. For example, supposedly teacher talk exceeds learner talk, which is what Cuban (1993) refers as a traditional teacher-centred approach, the MTF provides teachers with opportunities to account for what they say to learners. In consequence, learners are invited to participate with an understanding of what they could say, write or do in the lesson. It becomes a specific kind of interaction between teachers and learners, which prioritises quality over quantity in order to support learning (Esmonde, 2009). This is beyond a learner talk that exceeds teacher talk without anticipating what learners could talk about or write. In another example, supposedly the teacher relies heavily on the textbook, the MTF supports the process of deciding how to select and sequence examples when examples are taken directly from the textbook. Then, teachers are able to justify and explain their choice of example sets for their teaching approaches.

In sum, I have argued that learning MfT in the TM1 course was constituted by learning the mathematics content in order to be used when learning the MTF. I described learning MfT as

the joint enterprise. However, as I mentioned when concluding Chapter 2, the teachers joined the TM1 course with certain mathematics knowledge for teaching, but the evidence of learning to improve their MfT is what teachers would become. In fact, their new ways of teaching (if they have changed) constitute their mathematics teaching identity.

4.2.2 Roles of the TM1 community members (Mutual engagement)

In this subsection, I discuss the effectiveness of the TM1 community in improving MfT for teachers, and classify it as a mutual engagement. The effectiveness of the TM1 community resonates with the different roles members held in the community, which resulted in the participation of the teachers in the TM1 course. Here, I expand on the following: (1) the role of the facilitators; (2) how the teachers were engaging with each other as colleagues; and (3) the role of the research fellows.

In what follows, I describe the TM1 sessions from my perspective as the researcher. I focus on the role of the research fellows, on how the learning of MfT was mediated, and ultimately on how the teachers engaged with each other. My description of the TM1 sessions can be different from those of the facilitators, other research fellows or the teachers. Later, I draw on Adler (2021a, 2021b) to discuss four principles for designing tasks to mediate the MTF specifically.

Mediating the learning of mathematics-for-teaching

As a research fellow, I joined three other doctoral and three postdoctoral fellows in mid-year 2017 to become a member of the TM1 community. Our main role as the research fellows was to support the processes of mediating the learning of MfT. As the research fellows, we attended the TM1 sessions. In the sessions, we were involved in assisting the teachers with their tasks while we observed the processes of mediating learning. We also attended meetings to plan and reflect on the TM1 sessions. In the meetings, we contributed in improving tasks that were designed to mediate the learning of MfT. In 2017 and 2018, we contributed in adapting tasks since the groundwork of the PD intervention had already started in 2012. I expand on how tasks were adapted when discussing the principles for designing tasks to mediate the MTF specifically.

The mediation strategy of the TM1 sessions focused on how the teachers were encouraged to work together in small groups and to further engage in the whole class discussions. I offer three examples to illustrate how discussions were encouraged in the sessions. As the first example, individual teachers were organised in groups of threes to fives per table in a big enough venue. As the teachers were asked to work on tasks (write up solutions) as individuals and then verbally convince each other as group members in their relative tables, the research

fellows and the facilitators would walk around engaging the teachers on how they were explaining tasks and solutions to each other. The teachers got opportunities to discuss in pairs or in their groups, with the research fellows, with the facilitators or with the whole class. The teachers that I interviewed for this study confirmed that sitting and working in groups encouraged discussions amongst the teachers.

As the second example, the facilitators were mindful in making sure that the teachers were getting enough time to share their ideas of learning and teaching in their groups before discussing them with the whole class. For instance, there were individual teachers in the TM1 sessions who were quick to provide solutions to mathematics tasks even if the expectation was for the teachers to first discuss solutions in their groups. One teacher that I interviewed when piloting the interview questions for the study recalled such moments and shared the following:

> I remember at one point Moneoang [the facilitator] gave us a sum and I said, “no this can’t be solved”. She just laughed and said, “think carefully about what you’re saying”. That was when I had to go back, the more I thought about it, that’s when I realised... oh eish! I missed something. That’s when I had to go back and fix it (Modeni, Interviews, September 2019).

The teacher had not written down the solution before communicating that the activity cannot be solved. In this sense, the facilitators were persistent in asking the teachers to write down their responses (whether their solutions were correct or incorrect) as they carefully think about their offerings before discussing them with the class.

Lastly, the facilitators were acknowledging teachers’ ideas during discussions to endorse the learning of their MfT as valid and important although they were not always correct. This was based on the understanding that we all bring mathematical knowledge and teaching ideas to bear when doing tasks together. As such, in most instances, the facilitators would request one member of a particular group to share a solution with the whole class by writing it on the whiteboard or verbally communicating it to others. Other groups often had to identify with the solution from what they have done, and this process provided opportunities for further discussions amongst members of the community. The TM1 sessions became a ‘safe space’ for the teachers to discuss mathematics as community members without being exposed to any form of judgement or prejudice.

Mediating the learning of the MTF specifically

The structure of mediating the learning of the MTF focused on dealing with one aspect of the teaching framework at a time within the duration of the TM1 course. Of course, in the first session, the facilitators talked broadly about the four aspects of the MTF to the teachers and the benefits when teaching mathematics. Whereas, during the last session, the facilitators

connected all the aspects of the MTF for the teachers to highlight what it means to have a coherent lesson.

In the sessions, tasks were used to mediate the MTF. Adler (2021a, p. 1213) provided four principles that guided the design of tasks, which are:

1) Connect the *modelling* of teaching practices and the *explicit mediation* of aspects of the MTF (my description).
2) Select/construct *mediating tasks* that
   a. distinguish the *learner task* from *teacher task*
   b. root the tasks in records of teaching/learning practices
   c. focus on a specific mathematical content and specific curriculum level
   d. ensure the mathematics in the task can be quickly and easily done by teachers.
3) Elicit and externalise teachers' spontaneous ideas for collective discussion.
4) Communicate explicit evaluative criteria for improving the quality of example sets, word use and mathematical justifications.

Principle 1. The learning of the MTF began from what was *modelled* during the mathematics sessions before moving to the *explicit mediation* of aspects of the MTF in the teaching sessions (Adler, 2021b). That is, for example, the teachers would be asked to reflect on what they needed to know and be able to do by the end of the mathematics sessions. In the example when the teachers had learned about linear equations, they were asked to explain the examples, tasks and representation that they used during those sessions. In response, the teachers would highlight that they needed to know the definition of equations, the definition of a linear equation, the meaning of the equal sign as an equivalence, and the like, and how to solve linear equations (i.e. object of learning) using different examples, tasks and representations (i.e. exemplification). In this regard, the explicit mediation was on structuring what the teachers had learned during the mathematics sessions, whereby the introduction of terminology such as 'the object of learning' and 'exemplification' (in this case) came to effect.

When expanding on the *explicit mediation* using the exemplification case, for example, teachers' attention was drawn on the sequence and the nature of the examples that they can use when teaching mathematics. In relation to the nature of the examples in the example set, teachers would be asked what was the same and different in the examples within example sets and across different tasks (Adler & Pournara, 2020). Adler and Pournara (2020) emphasise that the illustration of "what is the same" amidst of "what is different" became a tool of highlighting generality of mathematical structures and concepts. For example, teachers could be given Figure 4.3 in the teaching session. Figure 4.3 is the *learner* task. As the *teacher*

task, teachers could be asked to describe “what changes” and “what stays the same” in the example set.

Determine the values of x:

a) $2x - 3 = 6$

b) $2x - 3 = 2x - 6$

c) $2x - 3 = 4x - 7 + x$

d) $2(x - 3) = 4x - 4 + x$

Figure 4.3: Examples of linear equations in an example set

In this example set in Figure 4.3, what stays the same illuminate the nature of the linear equations, whereby the expression 2x – 3 is kept unchanged for examples a) to c). What is changing, unlike example a), which remains in the domain of arithmetic, examples b) to d) have variables on both sides of the equations, which require that learners operate using the variables (Filloy & Rojano, 1989). That is, when solving example b) to d), learners need to understand that the expressions on both sides of the equal signs are of the same structure, and that there are actions which give meaning to the equality of the expressions (Filloy & Rojano, 1989). Whereas, example d) is of the form A(x + B) = Cx + D. That is, although it has variables on both sides of the equal signs, it also has brackets. Hoch and Dreyfus (2004) explain that the product of the monomial and the binomial bring different interpretations of the structure of the equations, as it demands more cognitively.

To learn about sequencing the examples during exemplification, and using the same example set in Figure 4.3, the teachers were not only expected to notice when planning the lesson that the examples are ordered from simple to difficult according to cognitive demands, but they were also expected to notice the nature of solutions of the examples. For example, it can be trivial to keep the order of the examples as it is in Figure 4.3. However, when considering the nature of the solution of example b), one can notice that it has ‘no solution’, which demands more cognitively than to explain ‘consistent solutions’ of examples c) and d). Thus, applying in-depth knowledge of linear equations provided access to the teaching framework, whilst solving example b) should be easy for the teachers (Adler, 2021b), the point is that teachers needed to be conscious about the nature of the examples in order to bring into focus the meaning of the solutions to the learners.

Principle 2. As mentioned, the mediating tasks were adapted from Grade 8 to 11 curriculum. In 2018, for example, in the last session, the mediating task was based on a lesson plan taken from Grade 9 Mathematics curriculum (GDE ATP, 2018, Term 3, Week 3, Lesson 5, pp. 82-88). The lesson focused on factorising algebraic expressions that involve trinomials of $x^2 + bx$

+ c and $ax^2 + bx + c$, where a is a common factor. To adapt the lesson plan, we took out the mental maths component and some examples on the revision component. The removed examples in the revision component were on the factorisation of the difference of two squares. In this revision component, we retained two examples, one on the expansion of a monomial and a binomial, and another on the expansion of two binomials. The rest of the example sets in other three components remained the same. The idea was to get the lesson to focus on factorising algebraic expressions that involve trinomials. See Appendix A for the adapted lesson plan.

This mediating task assisted the teachers to decide on whether the lesson had coherent connections when describing using the aspects of the MTF. In this lesson, the teachers had to work with imaginary learners to explain to them how to factorise the expressions, and thus work out the *learner* tasks. The *teacher* tasks required the teachers to think about how to improve the quality of examples in the example sets, word use and mathematical justifications.

Principle 3. This principle talks directly to how I have described the learning of MfT in the previous subsection, where I claimed that much mediation strategies encouraged discussions. When connecting to this subsection, Adler (2021b) asserted that learning needs to start from teachers' own offerings on practice-based tasks that can be solicited during discussions. Adler (2021b) provided evidence when discussing explanatory communication that some teachers relied on colloquial language, which is often problematic if not coupled with mathematical justifications. As an example, when solving linear equations, it emerged early in the sessions that some teachers were inclined to using the word 'transpose' other than to use 'additive inverses' to collect like terms. 'Transpose' implies taking a term on the one side of an equation and move it to the other side whereby the term changes its sign when collecting like terms. When eliciting teachers' ideas, whether in written forms or speech, the facilitators got opportunities to intervene in teachers' responses and move them from 'where they were' to more nuanced ways of teaching. The opportunities came as the TM1 community could reflect and interrogate teachers' ideas for improvement.

Principle 4. When communicating explicit evaluative criteria of the explanatory communication case, for example, teachers' attention would be drawn to use mathematical language instead of focusing on reading and manipulating symbols when working out solutions (Adler, 2021b). When using the example set in Figure 4.3, but now focusing on word use, teachers would be asked to explain how they could explain to the learners the procedure of solving the examples. For example, when solving example a), which is $2x - 3 = 5$, a teacher could provide the following justification and explanation:

> We transpose negative 3 to the other side to collect like terms. When it get to the other side, it changes the sign to positive 3 to get 2x = 8 as it adds to 6. We then divide both sides by 2 to get the answer x = 4.

Adler (2021b) explained that such justification and explanation are not always helpful to learners. Whereas, the procedure of finding that number for x when using mathematical language could be explained as follows:

> In order to collect like terms, whilst maintaining balance between the left-hand side and the right-hand side of the equation, we need to add the additive inverse of negative 3, which is 3, on both sides to get 2x = 8. We then multiple both sides by the multiplicative inverse of 2, which is half, on both sides to get the answer x = 4.

The idea that one needs to use mathematical language each time when solving linear equations is not the point. However, the evaluative criteria draws attention of both the teacher and learners to the importance of the discursive elaborations when explaining how to solve the equation 2x – 3 = 5 in this case. That is, the emphasis is that we want to find a number for x that makes the equation true, that when we multiple it by 2 and subtract 3, the answer is 5", and not remain in manipulating symbols to get x = 4.

To conclude this section, when using group-learning activities to discuss mathematics, where individuals become motivated to learn more mathematics alongside different ways of teaching the subject, according to Hima et al. (2019), teachers can change their mathematics teaching identity. As the facilitators in the TM1 course were using the mediating tasks, which focused on inviting teachers to participate in the TM1 community, it can be argued that the teacher community can remain with certain (positive or negative) experiences of learning and teaching the subject (Huillet, Adler & Berger, 2011). In that way, the teachers could continue (or not) to engage with teaching from the MTF perspectives. For Vangrieken et al. (2017), the pre-set agenda of this type of teacher community tends to continue to exist even when teachers are working in isolation. Consequently, evidence of the continuation of the teacher community emerge in the data of the study as one interviewed teacher talked about how they are backing each other as teachers who had attended the TM1 course when participating in other PD programmes.

4.2.3 Mathematics learning tools and resources (The shared repertoire)

As alluded to, the teachers in the TM1 course had full access to mathematics resources such as the GeoGebra software and relevant textbooks. In learning both the mathematics content and the MTF, the teachers were further provided with written materials. At the same time, the teachers could make additional notes from explanatory communication during class

discussions. In addition to physical resources, different learning tools of mathematics such as definitions, formulations, theorems and different forms of representations were embraced in the TM1 course to mediate the learning of mathematics content and the learning of the MTF.

In relation to GeoGebra, each teacher downloaded the software on his or her laptop. The teachers were orientated to the features of GeoGebra that were necessary to enable them to sketch graphs. The graphs from GeoGebra were ordinarily compared with graphs that were sketched on paper in the traditional ways. The purpose of comparing graphs was meant to improve teachers' procedural proficiency (Kilpatrick et al., 2001). For example, in one task, the teachers sketched graphs on paper from formulae and those graphs were further shifted vertically up or down, and horizontally to the right or to the left depending on the given units. However, when using GeoGebra, the teachers only sketched original graphs and then dragged them to shift by the expected units. In that way, GeoGebra yielded precise formulae as outputs for those graphs at any given moment. In that task, the teachers could develop procedural fluency as they were becoming more accurate in producing lines of graphs that were equally spaced apart from each other when drawn on paper, and lines of graphs would span evenly across the axes.

To add value in the traditional ways of sketching graphs, the teachers were supplied with 'big grids'. 'The big grids' are A2 pages made out of plastic and resemble graph papers. The teachers used non-permanent markers to sketch graphs with some level of accuracy. Once graphs were sketched, the teachers could display their work to compare with those of others for discussions. One teacher who was interviewed for this study expressed to have enjoyed working with 'the big grid' to sketch graphs. See Figure 4.4 for some trigonometric graphs that were sketched and displayed in the TM1 venue.

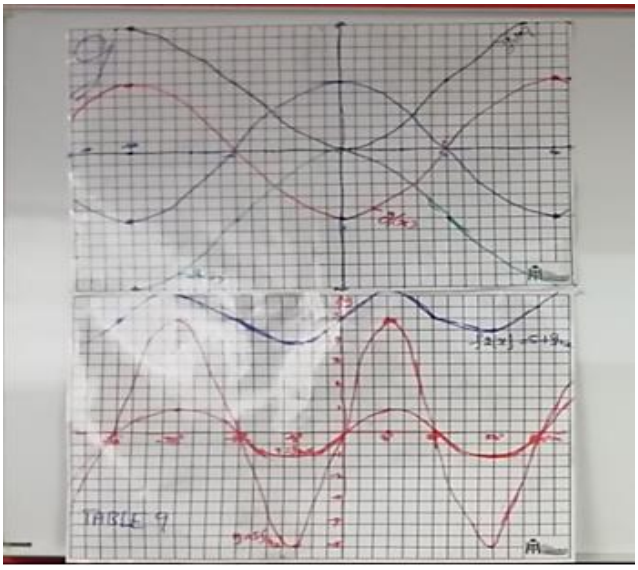


Figure 4.4: Graphs sketched on 'the big grids' and displayed in the TM1 venue

Other than to write and display teachers' work in the TM1 venue, the teachers would be encouraged to write up their offerings when responding to tasks. The tasks themselves for the teachers were written on paper. In the mathematics sessions, some examples were copied from textbooks to construct mediating tasks, and of course, sources were acknowledged accordingly. However, most tasks were constructed, particularly for the teaching sessions.

The learning tools that were used in the TM1 course included different definitions, formulations, theorems and different forms of representations. For example, some teachers use definitions that are sometimes not broad enough for those concepts. For instance, some textbooks remain in defining a function as a special relationship between input and output values where every input value has only one output value (e.g., Classroom Mathematics Grade 10 Learners' Book, p. 139, by Pike et al., 2011). Whereas, in the TM1 course notes, the teachers were also introduced to modern definition of a function, which states: Given two sets, A and B, a function f is a collection of ordered pairs (x ; y) where x ϵ A and y ϵ B and every element in A is associated with a unique element in B by the function f. As noted, the idea was to deepen teachers' knowledge of mathematics.

4.3 Performance when learning to teach in the TM1 course

The other important feature of learning in the TM1 course was that the teachers wrote and submitted assessment tasks. As noted in Section 4.2 above, these assessment tasks took the form of quizzes, assignments and summative tests.

The assessment tasks were designed to locate teachers' competence when learning mathematics in the TM1 course. From reading Wenger (1998), being competent in what one is doing could be *why* people participate in different communities of practice. The teachers could use the results of the assessments to recognise their efforts of participating in the TM1 course. In addition, other members of the TM1 community could also recognise teachers' capabilities from the results of the assessments. Thus, the assessments were central in positioning teachers' learning in the course.

How were the assessments designed in the TM1 course? Considering the idea of moving teachers from familiar concepts of mathematics to those concepts that were unfamiliar to them, the assessments themselves built on each other. For example, the teachers wrote quizzes towards the end of each session, except on the sessions that were scheduled for summative assessments such as 'mid-course test' (i.e. a test written in the middle of attending the course) and final examination. The quizzes were short and focused on one concept. The quizzes assisted teachers to prepare for the assignments. The assignments contributed

towards preparing for the mid-course test, and all assessments accumulated to prepare teachers for the final examination.

In preparing teachers for the mid-course test or the final examination, the teachers were advised to revise the quizzes and the assignments. Furthermore, information on how marks would be distributed per section in the summative assessments was made available to teachers. In sections that were focusing on the MTF in the summative assessments, the teachers were sometimes provided with *learner* tasks in advance before sitting and writing those assessment tasks. Giving teachers *learner* tasks in advance was necessary to enable teachers to think beyond mathematics content and begin to reflect and perhaps talk to their colleagues on how such *learner* tasks could be used during the teaching of mathematics before the assessments. As Adler (2021b) explained, *learner* tasks were distinguishable from *teacher* tasks. See Appendix B for the example of a *learner* task.

Marking of the assessments were mostly shared amongst the research fellows. The facilitators mainly moderated the marking, particularly for the summative assessments. The teachers received their assessments' scripts in due time. Immediately after handing back scripts for a particular assessment, mostly a pair consisting of a doctoral fellow and a post-doctoral, who had marked the assessment, would present what was expected in that assessment and discuss the tasks with the teachers. At that time, the teachers could identify with their mistakes and ask questions for clarity.

Immediately after those sessions of providing feedback regarding assessments were concluded, and the sessions were usually before the end of day, some teachers would sometimes remain behind to explain how and why they responded to certain questions in particular ways. These spontaneous after-class sessions became interesting for me because teachers were presenting how they engaged with questions to learn from what they did not get it correct. These teachers were indicating what probed them to respond in particular ways and what to look for in order to not repeat the same mistakes. Some teachers in these sessions were drawing from their teaching experiences, particularly when talking about the MTF, to state how the learners could benefit from the insight of particular questions. The fact that these discussions were outside the formal sessions, it said to me, the assessments motivated teachers to strive to move to the centre of the TM1 community (Lave & Wenger, 1991). In Wenger's (1998) terms, the assessments became something teachers 'cared about', which then encouraged them to negotiate for meanings.

When using assessments as a tool to encourage teachers to think and talk about the practice (i.e. the learning of mathematics and how to teach mathematics from the MTF perspective), it can be argued that other components of learning such as identity and confidence emerge in

the process. Starting with confidence, it can be accepted as a given that getting good results improves participants' confidence. Equally, getting poor results in the course motivated some teachers to do more and strive for better results. Teachers who had not done well in assessments also attended the one-on-one sessions.

In another related argument, assessments meant that the teachers could track their progress of learning by looking at their past results and project where they wanted to be in the TM1 community. The mid-course test came about at the request of the teachers in the early years of implementing the TM1 course as they did not want to have only the final examination, and then realise that they were not on track. When tracking their progress, the teachers could notice and talk about what has changed in their practices. Furthermore, the teachers could decide on what mattered and did not matter in what they were learning in the course. In Wenger's (1998) terms, being part of the TM1 community could enable teachers to identify or not identify with certain parts of learning and teaching mathematics. Ultimately, for Wenger (1998), changing teachers' practices meant changing their identity.

What remains to be seen are teachers' competences when they are teaching mathematics from the MTF perspective. As noted in Chapter 1, teachers' competences of teaching mathematics, particularly from the MTF perspective, are characterised using Mathematics Discourse in Instruction (MDI) in this study. However, the context of teaching from the MTF perspective would be different than when the teachers were responding to assessment tasks in the TM1 course. The teachers who participated in the observations and the interviews had passed the course. Therefore, it is anticipated that the teachers would perform their learning of how to teach mathematics in the classroom or school context, and then they could justify the implementation of the MTF in their teaching of mathematics.

4.4 Summary

In this chapter, I showed that that the TM1 course was the community of practice in which the teachers participated. I highlighted *what*, *how* and *why* the teachers were learning in the TM1 course. The teachers were learning mathematics-for-teaching in the TM1 course, which included learning about the MTF. In describing *how* the teachers learned in the course, I focused on how the learning of mathematics-for-teaching was mediated in the TM1 course, and how *learner* and *teacher* tasks were used to mediate the MTF. Lastly, I claimed that the assessment tasks encouraged the teachers to participate in the TM1 course in order to be competent in teaching mathematics.

CHAPTER 5

RESEARCH METHODS

5.1 Introduction

In this chapter, I firstly describe a mixed methods research as it is employed for this study. Secondly, after explaining why it was necessary to use the mixed methods research, I discuss the explanatory mixed methods research design to give details on how data was collected using both quantitative and qualitative processes for the study. Thirdly, I highlight how reliability and validity were taken into account in the study. Lastly, I explain ethical considerations for the study.

5.2 Mixed methods research

Some research studies (e.g., Lasky, 2005; Woolhouse & Cochrane, 2015), as it is the case with the present study, see value in employing mixed methods research to explore and examine participants' identity. Mixed methods research investigates the world from the social and cultural perspective that ideally involves human beings with a diversified interpretation of knowing (Johnson, Onwuegbuzie & Turner, 2007). Mixed methods research broadly contributes to knowledge (theory or practice) by considering multiple viewpoints, perspectives, positions and standpoints from both quantitative research and qualitative research (Johnson et al., 2007). Johnson and Onwuegbuzie (2004) refer to mixed methods research as a methodological approach "where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study" (p. 17).

What value is there in employing mixed methods research for this study? Most studies that are researching participants' identity have typically employed qualitative approaches where interviews, narratives and classroom observations are prevalent sources of data (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Lutovac & Kaasila, 2019). One of the methodological features that emerged from reviewing the selected studies in Chapter 3 was that all the studies employed qualitative methods to research participants' identity (e.g., Chauraya, 2013; Darragh, 2015b; Gujarati, 2013). For example, in the studies that employed Sfard and Prusak (2005), individual participants used their own words to tell their own stories about themselves in relation to teaching and learning mathematics, which required a use of qualitative methods since individuals' stories were changing according to whom they were narrated to. Even the studies that were only providing theoretical frameworks (i.e. Bennison, 2015; Van Zoest and Bohl, 2005) or reflecting on their previous studies (i.e. Darragh, 2018) remained in encouraging researchers to use qualitative methods. The prevalence of

qualitative methods is appropriate when considering wide scopes of possibilities when individuals are narrating their experiences of learning and teaching mathematics (e.g., Darragh, 2015a, 2015b), their journeys of becoming members to communities of practice (e.g., Essien, 2013), or their sense of belonging to communities of practice (e.g., Chauraya, 2013). However, one cannot disregard including quantitative methods when exploring the extent of participants' shared experiences regarding the learning and teaching of mathematics (Kaspersen, Pepin & Sikko, 2017).

Systematically, as Teddlie and Tashakkori (2009) argue, researchers can look at the nature of research questions and judge on the appropriateness of employing mixed methods research. In this thesis, I respond to research question 1 of the study using both quantitative and qualitative methods. The first part of the question, which was 'To what extent do teachers have shared experiences of learning and teaching mathematics after participating in the TM1 course?', attempts to confirm (or question) social theory of learning as described by Wenger (1998) and Graven (2003) using the case of teachers who had participated in the Transition Maths 1 (TM1) course. The second part of the question, which was 'What changes (if any) have shaped the shared experiences for the teachers?', expands the emergence of shared experiences by elaborating on what features have contributed in shaping such experiences. The research question 2 was 'In what ways (if any) does the teachers' learning in the TM1 course connect to their mathematics teaching identities?'. The research question 3 was 'What are the connecting features (if any) between the teachers' mathematical discourse in instruction and their mathematics teaching identity?'. Subsequently, the research questions 2 and 3 are explanatory questions, and therefore, I answer them from qualitative methods.

Practically, choosing mixed methods research depends on a research design of that study and what a researcher intends to achieve. For example, Lasky (2005) sought to understand teachers' identity, agency and professional vulnerability in the context of secondary school reform (SSR) across the province of Ontario, Canada. Within the two-phase design called exploratory mixed methods research design (Creswell & Plano Clark, 2007), Lasky (2005) analysed both the teachers' descriptions of the early influences on their identity and agency, followed by the extent of how the SSR shaped their experiences of vulnerability. In the present study, I start by using quantitative methods to explore teachers' shared experiences (entailing their identity) before employing qualitative methods to examine changes that have contributed in their learning and teaching and ultimately, elaborate on their mathematics teaching identity. In accordance with Creswell and Plano Clark (2007), the present study also uses a two-phase design called explanatory mixed methods research design given its order of events (i.e. data collection, analysis and reporting).

5.3 The explanatory mixed methods research design

The explanatory mixed methods research design is characterised broadly within a sequential mixed methods research design. Paraphrasing Creswell and Plano Clark (2011), a sequential mixed methods research design occurs when an individual researcher or a team of researchers alternate qualitative and quantitative methods across different phases of a research study. As Tashakkori and Teddlie (2010) assert, a mixed methods research design should provide a detailed plan for conducting a research study. As noted, this study firstly uses a self-designed closed-ended questionnaire to collect data (quantitative processes). Once data sets from the quantitative processes had been collected and analysed, the results informed the qualitative processes of the study where I used observations and interviews to collect data (Greene, Caracelli & Graham, 1989).

Greene et al. (1989) provide five broad rationales that researchers should consider when using different models of mixed methods research design, namely: a) triangulation, b) complementarity, c) development, d) initiation, and e) expansion. Johnson et al. (2007, pp. 115-116) define these terms as follows:

a) Triangulation – seeking convergence and corroboration of results from different methods studying the same phenomenon;
b) Complementarity – seeking elaboration, enhancement, illustration, clarification of the results from one method with results from the other method;
c) Development – using the results from one method to help inform the other method;
d) Initiation – discovering paradoxes and contradictions that lead to a reframing of the research question; and
e) Expansion – seeking to expand the breadth and the range of inquiry by using different methods for different inquiry components.

This study considers the following four rationales to justify for the explanatory mixed methods research design: triangulation, complementarity, development and expansion. About initiation, I elaborate on some paradoxes and contradictions when discussing the results in Chapters 10 and 11 by drawing on the complementarity feature to avoid reframing the research questions. The power of navigating through such variations of employing mixed methods research design lies in understanding their limitations and contributions to research knowledge (Denscombe, 2007).

Development. The idea of using the quantitative analysis to contribute in selecting participants that are suitable to participate in the qualitative processes was from the initial plans of this study. The results from the quantitative processes inform the selection of participants for

inclusion in the qualitative processes. Collins, Onwuegbuzie and Sutton (2006), who complemented Greene et al's. (1989) rationales, refer to this purpose of using the results to inform the selection of participants as *participant enrichment*. For Collins et al. (2006), this rationale considers that the developed analysis yields participants that can contribute to a richer data.

Expansion. For this study, the expansion feature comes in Chapter 8 where I use the results from the quantitative analyses and ultimately the qualitative analyses to answer the second part of the research question 1. In this sense, I argue how the teachers' experiences were shaped by participating in the TM1 course, but expand to include what has changed in shaping their experiences of learning and teaching mathematics.

Triangulation. In dealing with the across-method triangulation, which involves the use of both quantitative and qualitative processes (Johnson et al., 2007), I corroborate certain closed statements as teachers had agreed or disagreed to them with how they explain them from interview analysis. For example, in the closed-ended questionnaire, teachers were asked to agree or disagree with the following statement: 'Since learning about the MTF, the mathematics I teach is more valued by other teachers'. Whilst during the interviews, teachers were asked to respond to the following question: 'In what way do other teachers identify with you as a mathematics teacher?'. The convergence of the results emerges in Chapters 10 and 11 when discussing teachers' mathematics teaching identity.

Complementarity. The within-methods triangulation relates to complementarity in this study, as I collect data from the lesson observations and teacher interviews. The within-methods triangulation refers to using either multiple quantitative or multiple qualitative processes for different parts of the study (Johnson et al., 2007). In this case, analysis of data collected from the observations are further explained using the interview data. As we see in Chapters 9 and 10, what remain central is that teachers' narratives from different sources seek to elaborate and enhance the results of the study.

The overall mixed methods research design for this study emphasises qualitative over quantitative processes. Considering that the explanatory mixed methods research design starts from the quantitative processes and move to the qualitative processes, Johnson and Onwuegbuzie (2004) describe such design using "quan → QUAL" model. The word 'quan' means quantitative but with less emphasis in a research, and the word 'QUAL' means QUALITATIVE with more emphasis. The arrow '→' indicates that the research processes are sequential, starting from quantitative processes to qualitative processes. Figure 5.1 provides a summary of a sequence of events for the explanatory mixed methods research design in the study.

Phase	Process	Product
Quantitative Data Collection	• Designing closed-ended questionnaire and piloting with TM1 teachers from 2018 course ($n = 42$) • Administrating closed-ended questionnaire to TM1 teachers from 2016 & 2017 courses ($n = 45$)	• Questionnaire with 28 closed statements • Coding of teachers' responses • Numeric data loaded on SPSS software
Quantitative Data Analysis	• Data screening (missing values, data suitability and adequacy) • Exploratory Factor Analysis	• Data was appropriate and adequate • Factor loadings • Descriptive statistics to respond to research question 1
Connecting Quantitative and Qualitative Phases	• Purposefully selecting one participant from each position ($n = 3$) based on group responses • Selecting lessons for observation • Structuring interview questions	• Cases ($n = 3$) – three teachers from 2016 & 2017 groups • Lesson observation protocols • Interview protocols
QUALITATIVE Data Collection	• Piloting interview questions with two teachers from TM1 2018, which necessitated lesson observations of these teachers • Lesson observations of the three teachers, including field notes • Individual face-to-face interviews of the three teachers, including field notes	• Decision to move one teacher from the pilot (2018 group) to the study • Text and videos data (lesson transcripts with images) • Text data (interview transcripts)
QUALITATIVE Data Analysis	• Coding and narrative analysis (context from teachers' shared experiences) • Coding and MDI analysis (content from observations on the MTF) • Coding and narrative analysis (content from interviews on the MTF)	• Respond to research questions 1, 2 and 3
Integration of the Quantitative and Qualitative Results	• Within-case and across-case theme development • Interpretation and explanation of the quantitative and qualitative results	• Discussion • Implications • Future research

Figure 5.1: Model for the explanatory mixed methods research design (Ivankova et al., 2006)

What follows next are the processes within the explanatory mixed methods research design starting with how I designed the closed-ended questionnaire (i.e. quantitative process), to how I selected participants, to how I collected data using lesson observations and teacher interviews. Additionally, I discuss how I used field notes to enhance data collection from observations and interviews.

5.3.1 Designing the closed-ended questionnaire

The purpose of collecting data using the closed-ended questionnaire was to explore shared experiences for teachers after they had participated in the TM1 course. Hence, knowing that teachers were afforded opportunities to learn mathematics whilst influencing their teaching practices, it was anticipated that their shared experiences may emerge to include teachers' mathematics teaching identity.

Designing the questionnaire followed three steps, which promotes validity (Cohen, Manion and Morrison, 2002). Firstly, I built on information that was collected with teachers who attended the TM1 course in 2017, and the information was adapted to fit within Wenger (1998) and Graven's (2003) interconnected learning components. Then, I piloted the statements by giving the draft questionnaire to other people to read and make suggestions. Finally, I piloted the questionnaire with teachers who participated in the TM1 course in 2018.

As noted in Section 1.5, when building on previous information, teachers who had participated in the TM1 course in 2017 were asked the following question towards the end of the course: "What brings you joy in the learning and teaching of mathematics?" Teachers provided written responses, which were concise and mostly to the point. For example, one teacher wrote: "I enjoy assisting learners to understand mathematics". As observed by Gillham (2011), people can be more economical with written responses than when they talk. I then used the basic process of inductive analysis by underlining wordings from the responses to obtain categories. From tallying the underlined wordings, teachers emphasised the following three constructs: (1) understanding of mathematics, (2) accountability towards learning and teaching mathematics, and (3) confidence in learning and teaching mathematics.

The 'understanding of mathematics' construct emerged from two fronts. One, teachers were attributing understanding of mathematics to learning in the TM1 course. For example, one teacher stated that he "got a deeper understanding of certain concepts" from attending the course. Two, teachers were expressing their intentions of teaching learners mathematics for understanding. Regarding the 'accountability towards learning and teaching mathematics' construct, teachers were stating that they are expected to improve learner performance in mathematics. In the 'confidence in learning and teaching mathematics' construct, teachers were mentioning that the TM1 course brought confidence in them. These constructs were used to develop a 5-point closed-ended questionnaire, comprising 36 statements.

The statements were oriented towards the five interconnected components of learning: identity, meaning, community, practice and confidence (Graven, 2003; Wenger, 1998). The statements were structured into four categories in relation to the three determined constructs.

1) Statements focused on how much responsibility teachers as a community can take about the learning and teaching of mathematics, e.g., 'Teachers are accountable for learners' performance in mathematics';
2) Statements that attributed understanding of mathematics to learning from the TM1 course, e.g., 'Since the TM1 course, I can teach learners mathematics for understanding';
3) Statements related to the impact of teachers' confidence on learning and teaching mathematics, e.g., 'Because of confidence, one is not afraid to grapple with mathematical tasks'; and
4) Statements that accredited learning about the MTF to encourage teachers to reflect on their teaching of mathematics, e.g., 'Since learning about the MTF, I feel an increased sense of being a mathematics teacher'.

In the last category, and looking at the MTF as teaching practices, I closely linked the statements on teacher identity to the impact of learning about the MTF because changing practice directly affects identity (Wenger, 1998).

At this stage, I had developed the questionnaire based on the theoretical framework and from the relevant literature (e.g., Graven, 2004; Lasky, 2005; Wenger, 1998). The research questions were borne in mind when developing the questionnaire. Consequently, the results themselves from the questionnaire analysis were to demonstrate if the proposed statements were suitable for teachers. At the same time, the exercise of developing the questionnaire was not to be taken lightly considering the complexities of researching teachers' shared experiences of learning and teaching mathematics (Skott, 2018, 2019). As Skott (2019) opines, teachers are more likely to express different learning experiences from the same professional development initiative. However, according to Gillham (2011), putting together a list of statements that are relevant to participants' learning context whilst considering the research questions is a good starting point to developing a questionnaire.

The next step to developing the questionnaire meant that the researcher should give the list of statements to other people to read and make any suggestions about the statements. Gillham (2011) refers to this as piloting the statements, which is different from piloting the questionnaire. One of my supervisors read the statements and assisted to minimise obstructive language. For example, one of my initial statements in the 'accountability towards learning and teaching mathematics' category read 'Learners come to secondary school with inadequate content knowledge which limits the progress when teaching'. The supervisor suggested that we shorten this statement to 'Learners come to secondary school with inadequate content knowledge'. Each statement from each category needed to be assigned

to a single idea to avoid ambiguities (McMillan, 2012). Moreover, a post-doctoral fellow who mediated the learning of mathematics content in the TM1 course read the statements, and provided valuable suggestions on how the statements could be re-ordered according to the learning components for logical development when teachers are responding to the statements. After editing and re-ordering the statements, the questionnaire was ready for piloting.

The questionnaire was piloted with 42 teachers who participated in the TM1 course in 2018. As it was a pilot, the teachers were encouraged to write short notes next to statements to indicate their concerns regarding those statements. I subjected the questionnaire data to Exploratory Factor Analysis – details about the analysis are discussed in the next chapter. The results from the quantitative analysis assisted me to remove some statements that were not relevant to the participants. For example, the 'Because of the TM1, I have more workload' statement was removed from the questionnaire. The side notes from the teachers assisted me to re-arrange the order of some constructs on the questionnaire, where I moved up the group of statements attributing understanding of mathematics to learning from the TM1 course to come first on the questionnaire. Thus, 28 statements were retained for the study. The 28 statements were considered to be adequate to enable teachers to express their experiences from the restricted standpoint about learning and teaching mathematics after participating in the TM1 course. See Appendix C for the final questionnaire.

5.3.2 Selecting the participants

There are two samples in this study. Firstly, I reached out to all 86 teachers who participated in TM1 in 2016 and 2017, and 45 of them responded by completing the questionnaire. There were 44 teachers who attended the course in 2016 whilst there were 42 teachers in 2017. The groups were selected on purpose considering that the development of the MTF had reached a certain level of maturity when compared to its initial stages since 2012. McMillan and Schumacher (2010) state that purposeful sampling of participants can provide pertinent information about the researched phenomenon.

Secondly, after analysing data from the closed-ended questionnaire, I selected three teachers to participate in the qualitative phase where I gathered data from lesson observations and individual interviews. To connect the quantitative and qualitative phases, the selection of the three teachers was informed by the closed-ended questionnaire analysis. In this case, I considered the three teachers to have had different sense of participating in the TM1 course. The teachers were special cases, and McMillan and Schumacher (2010) refer to this sampling strategy as "sampling by case type" (p. 327). When using the 'sampling by case type' strategy, three participants can be adequate to contribute to an in-depth analysis of a phenomenon of

a study if a researcher considers selecting extreme-case, intensive-case and typical-case (McMillan & Schumacher, 2010). In this study, I selected one teacher with average characteristics (i.e. typical-case), one teacher with dramatic, but below-average characteristics (i.e. intensive-case), and one teacher with recommendable characteristics (i.e. reputational-case) of participating in the TM1 course. It so happened that all the three teachers selected attended the TM1 course in 2017. In Chapter 7, when discussing the results of the questionnaire analysis, I provide specific details of this selection of the three teachers.

I added a fourth teacher in the selected participants for the study. McMillan and Schumacher (2010) explain that researchers can propose a minimum sample size and then continue to add to the sample as the study progresses. I included the fourth teacher after he had participated in the piloting of the interview questions (i.e. qualitative phase) alongside another teacher. These two teachers had participated in the piloting of the closed-ended questionnaire from the 2018 group (i.e. quantitative phase). The two teachers had recommendable characteristics (i.e. reputational-cases) of participating in the TM1 course. I also provide specific details about the two teachers in Chapter 7. Originally, the selection of the two teachers from the analysis of the closed-ended questionnaire was intended to assist in piloting the interview questions. However, to prepare for the piloting of the interview questions, the two teachers were observed when teaching mathematics in the classrooms. Because of the insight that emerge during the lesson observations and the interviews, that one teacher was moved from the pilot to contribute directly in the study. During the interviews, for instance, this teachers was clearly mentioning words such as TM1 and MTF, which was the information with potential to strengthen a case of the three initially selected teachers.

Table 5.1 shows a summary of the four mathematics teachers' biographical information. The information about the teachers can be understood in relation to their positions as special cases, but importantly, when they teach somewhat differently than other teachers or when they present certain views during interviews.

Table 5.1: The teachers' biographical information

Characteristic	**Makwe**[8]	**Mbose**	**Pandor**	**Oena**
Qualification(s)	BSc	BEd (Honours)	ACE[9]	BSc (Honours)
Years of teaching	6	7	26	29
Years of teaching Grade 9 mathematics	6	7	26	21

[8]Makwe was the fourth teacher that I selected and invited to participate in the study.

[9] ACE stands for Advanced Certificate in Education. It is a professional qualification. As published by the South African Qualifications Authority, and within the National Qualification Framework (NQF), ACE is at NQF Level 6. ACE can afford teachers without degree qualifications who wish to further their studies at post-graduate level with the opportunity of gaining access to the Bachelor of Education (BEd) (Honours) degree. BEd (Honours) is at NQF Level 8.

All the four teachers have done mathematics as their major teaching subjects. Both Makwe and Oena did Bachelor of Science (BSc) degrees, although Oena has BSc with Honours. Mbose has a Bachelor of Education (BEd) with Honours, whereas Pandor has an Advanced Certificate in Education (ACE). Makwe and Mbose each has less than 10 years of teaching Grade 9 mathematics, whilst Pandor and Oena each has over 20 years of teaching Grade 9 mathematics. However, when connecting the teachers' mathematical discourse in instruction and their mathematics teaching identity in Chapter 11, I do not return to compare the teachers despite the fact that Pandor and Oena have taught mathematics for many years when compared to Makwe and Mbose.

5.3.3 Collecting data using lesson observations

As the researcher, I needed to see and hear what occurs in the mathematics lessons of the teachers, which was the purpose of the observations. The plan was to observe four consecutive lessons per selected teacher when teaching Grade 9 algebraic equations. Upon requesting permission, the teachers and the schools allowed me as the researcher to observe whilst video-recording the four lessons per teacher during 2019 academic year. Later, I transcribed all the lessons from the videos to obtain text and image data for each teacher.

The lesson observations protocol

Before collecting data from the lesson observations, given good relationships between myself and the teachers, which were established at the time the teachers were attending the TM1 course, I needed to firstly visit teachers' classrooms for trial observations. I used the trial observations to gain entry into the mathematics classrooms and to establish rapport (McMillan & Schumacher, 2010). Likewise, the learners needed to be familiar with me as the researcher. The trial observations were also about familiarising myself with the angles and range of visuals when using a camera to video-record the lessons. So, I video-recorded one lesson per teacher when a teacher was teaching a mathematics topic of his choice. These trial observations took place in the second term[10] of 2019.

During the trial observations, I identified and communicated my roles to the teachers and to the learners accordingly. I was a nonparticipant observer (Marshall & Rossman, 2016; McMillan & Schumacher, 2010), and did not have a formal role in the classroom setting. However, during mathematical activities, I was commenting when approached by individual learners or by the teachers in the classrooms. For instance, there were a few instances where individual learners asked for clues and validations when they were working on their

[10] In 2019, almost similar to other years, the school calendar was as follows: 1st term:- 09 January – 15 March; 2nd term:- 02 April – 14 June; 3rd term:- 09 July – 20 September; and 4th term:- 01 October – 04 December.

mathematical activities. There were also a few instances where the teachers passed general comments about what they had intended to do (or not to do) when they included or excluded certain mathematical activities during their lessons.

The lesson observations

The reason I chose to observe the teachers teaching algebraic equations to Grade 9 classes was because algebra was amongst the topics given most attention in the TM1 course. As the researcher, I focused on what the teachers were writing on the chalkboard and words they were using when explaining and justifying mathematics to the learners. The observations of the four consecutive lessons were achieved with the three teachers. Only one lesson was observed for the fourth teacher, and this was the teacher who participated in the pilot. This teacher was comfortable to repeat teaching one lesson on algebraic equations for the observations and the interviews. His lesson was on the same topic as the first lesson of the other three teachers, which was on algebraic equations using additive and multiplicative inverses. Out of the four lessons, I analysed only the first lesson of each teacher – details on why I chose to analyse the first lesson and how the lesson was analysed are in Chapter 6. Table 5.2 shows details of the mathematics content of the lessons.

Table 5.2: The teachers' video-recorded lessons

<table>
<tr><th>Teacher</th><th>Topic and Lesson number (L)</th><th>Examples of tasks on the topic</th></tr>
<tr><td rowspan="4">Pandor, Oena and Mbose</td><td>Algebraic equations using additive and multiplicative inverses (L1)</td><td>Solve for x: $2x + 2 = 6x - 14$</td></tr>
<tr><td>Algebraic equations with fractional terms (L2)</td><td>Solve for x: $\frac{2x}{5} + 1 = \frac{13}{5}$</td></tr>
<tr><td>Exponential equations (L3)</td><td>Solve for x: $2^{2x+2} = 16$</td></tr>
<tr><td>Use of substitution in equations to generate tables of ordered pairs (L4)</td><td>Determine the value for y for the given values of x if $y = 2x^2$
<table>
<tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr>
</table></td></tr>
<tr><td>Makwe</td><td>Algebraic equations using additive and multiplicative inverses (L1)</td><td>Solve for x: $2x + 2 = 6x - 14$</td></tr>
</table>

The teaching of algebraic equations was scheduled for the third term on the school calendar (GDE ATP, 2019). There were no special arrangements in requesting the teachers to be observed outside the teaching schedule. Ideally, teachers are expected to teach the same topics on the same days when following the schedule. Given several factors in disadvantaged schools, including for example, the issues of broken attendance by learners and teachers (Christie, 1998), teachers are often lagging behind the schedule. Therefore, dates for the lessons for the teachers did not clash. The teachers informed me about when they were going

to cover the four lessons on algebraic equations. The observations of each teacher happened on each week from Week 3 onwards in the third term. The lesson observations contributed mainly in discussing teachers' teaching practices.

5.3.4 Collecting data using teacher interviews

The teachers were interviewed individually, and the interviews were face-to-face. The interviews were intended mainly for two purposes. One, for the teachers to reflect on their ways of teaching mathematics from the MTF perspective and other teaching practices. Two, for the teachers to reflect on their learning experiences after they had participated in the TM1 course. To enable the teachers to talk flexibly when reflecting on these issues, I used a semi-structured type of interviews. Semi-structured interviews use open-ended questions, which are fairly specific in their intents (McMillan & Schumacher, 2006). See Appendix D for the interview questions. I audio-recorded the interviews and later transcribed them to obtain text data for each teacher.

The interview protocols

The purposes of collecting data using interviews necessitated me as the researcher to select the interview questions in advance, group them into topics and decide on the sequence and the wording of the questions to guide the interviews (McMillan & Schumacher, 2010). When discussing the piloting of the interview questions in Subsection 5.3.6 below, I explain how my supervisors assisted me to decide on the sequence of the questions. For the purposes of discussing teachers' ways of understanding the MTF, the sequencing of the questions depended on verifying and expanding the ideas developed by myself as the researcher during the lesson observations. For instance, discussing the object of learning preceded the other three aspects of the MTF considering that the three aspects can be derived from it.

For the purposes of discussing teachers' experiences, the sequencing of the questions followed a logical order to complement information obtained from the closed-ended questionnaire analysis. For example, in following logical order, once teachers had taken certain positions about their participations in the TM1 course, which were either on the periphery or towards the centre of the community of teachers who attended the course, I probed whether and how they moved towards the centre of the community, and how they maintained full membership in the community. The effectiveness of sequencing of the questions depended on efficient probing to elicited elaborations, further explanations, and clarifications of responses (McMillan & Schumacher, 2010). However, sequencing the questions did not deny the teachers to flexibly return to previously asked questions and voluntarily provide further details on their earlier responses.

The teacher interviews

To understand teachers' ways of teaching mathematics from the MTF perspective and other teaching practices, the teachers were encouraged to reflect on the observed lessons and on other lessons in general, and provide examples on how they normally plan and conduct their teaching of mathematics. The interviews took place in October 2019, which was a month gap after the lesson observations. Given that the focus of analysing the observation data was on the first lesson (i.e. on algebraic equations using additive and multiplicative inverses), I gave each teacher a snapshot from the video-recordings of their boardwork of the lesson to assist them to reflect on their observed lessons, and asked them to confirm if that was their work. The snapshots remained in front of the teachers on the table during the interviews. Consequently, the teachers were asked about different aspects of the MTF in relation to the observed lessons, their views in general about which aspects of the MTF they can say have influenced their planning of different lessons, and how they find those aspects of the MTF to be useful in the teaching of mathematics. The analysis of this part of the interviews, whereby teachers performed in describing their teaching of mathematics from the MTF perspective, explain their mathematics teaching identity.

To expand on teachers' experiences after they had participated in the TM1 course, the teachers were asked to express their general sense of participating in the TM1 course, and to explain if they had a sense of belonging to the community of teachers who participated in the TM1 course, or if they felt marginalised. Furthermore, the teachers were asked to comment on the importance of understanding mathematics and teacher confidence, as they reflected on their statements from the closed-ended questionnaire. The analyses of data contribute in explaining what has changed in shaping the extent of shared experiences of teachers that emerge during the closed-ended questionnaire analysis.

To end the interviews, the teachers were asked to comment on how they think of themselves and how others think of them as mathematics teachers. For example, the opening question in this set of questions was 'Have you changed how you think about yourself as a mathematics teacher after participating in the TM1 course? Please explain, in what way?'. These types of questions were meant to consolidate information on teachers' experiences of participating in the TM1 course and their understanding of the MTF to contribute directly in answering the research questions 2 and 3 in Chapter 11.

5.3.5 Collecting data using field notes

Making field notes was part of the qualitative data collection process. The field notes contributed in both the lesson observations and the interviews. In planning the engagements

with the teachers, I relied on a field log where I recorded names of teachers, dates, times, places and activities. Most of these records were easy to verify given that the teachers and I used SMSes to confirm our meetings.

During the lesson observations, as the video camera was capturing what the teachers were writing on the chalkboard and communicating to the learners, I was also taking notes to capture what could not be video-recorded. For example, Figure 5.2 shows some notes that I wrote in my journal during and after the lesson – I have hidden the teacher's identity in the notes.

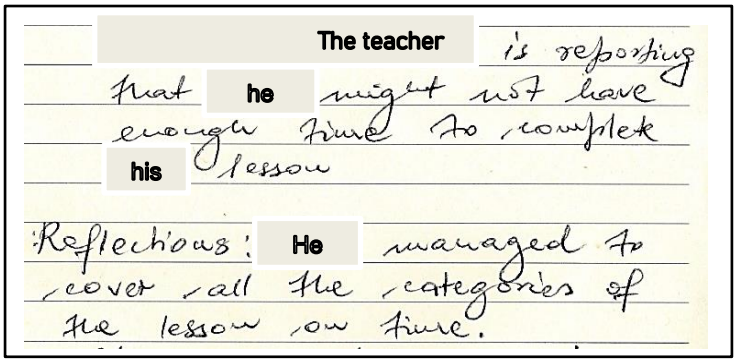


Figure 5.2: Reflections on Oena's first lesson: Scanned notes from my journal

During this first lesson, the teacher had passed a comment to me that he was worried about not finishing the lesson on time. This teacher was determined to follow the lesson plan to the letter. After the lesson, during reflections, I further stated that the teacher kept the learners involved in the last example despite being pressed for time.

During the interviews, I made notes on which aspects to return to or omit after diverting from the script and the wording of the questions. For example, in responding to the opening question of the interviews, the teachers provided overarching statements that touch on several aspects of learning mathematics-for-teaching in the TM1 course. As such, it was important for me to note the points that they have touched on when I eventually delve into asking them to expand on those aspects.

5.3.6 Piloting the interview questions

When discussing the interview questions with my supervisors, I was not clear about the sequence of questions – whether I should start with questions on teachers' experiences of learning and teaching mathematics or I should start with questions on their understanding of the MTF. In the meeting with my supervisors, we decided that I needed to start by getting a sense of teachers' overall experiences of participating in the course before asking specific questions about the aspects of the MTF, and thereafter I can return to get more information about their experiences of the course. As such, the supervisors encouraged that I pilot the

questions before continuing interviewing the three teachers that I had observed. As noted, in order to pilot the questions, I firstly needed to observe the two teachers. In consequence, the piloting of the questions came in between the lesson observations and the interviews of the study.

In line with explanations on pilot studies (e.g., Connelly, 2008; van Teijlingen & Hundley, 2001), the purpose of piloting the interview questions sought to test the design of the interview protocols, the adequacy of the research instrument, and the sampling strategies. In addition, the piloting of the interview questions aided in collecting preliminary data (Connelly, 2008). From getting a broad sense of how the two teachers were answering the questions, it became clear to me that little adjustments were required in how I phrased the questions, and that I could use the collected data (Klymko et al., 2008). From the preliminary data, it was indicative that the interview questions were adequate and appropriate for the study, and were further confirming the suitability of the 'sampling by case type' strategy. Moreover, as explained, Makwe was moved from the pilot to contribute directly in the study.

The lessons learned in the pilot were on how I should phrase certain questions to the teachers. For example, in the questions where I had relied on Wenger's (1998) language of 'belonging to a community' or of 'being situated in the periphery of a community', the two teachers during the pilot had asked me to rephrase such questions. As such, with the other three teachers, I would pose such questions from the script and then explain succinctly what I mean by a community, for instance. In addition, I encouraged the teachers to respond to questions even though they feel like those questions were a repeat of previous ones. To encourage the responses, I would repeat briefly what the teachers had explained previously and continue to ask them to say more regarding those questions. That was another reason for keeping the journal for field notes, as I would write down different components of our conversations.

In the following, I discuss reliability and validity in the research, which links directly to reasons of piloting the research instruments, but extends to give details on other factors to consider when conducting a research study.

5.4 Reliability and validity in the research

Both quantitative and qualitative research should present the truth when discussing and operationalising the research processes, which in turn should lead to meaningful findings of the study. It is even paramount in mixed methods research to present the truth in the research, given its flexibility nature of combining quantitative and qualitative processes, whereby researchers can take 'middle of the road' positions (Onwuegbuzie & Johnson, 2006) that can lead to meaningless findings. In Long and Johnson's (2000) words, "meaningless findings may

result in wasted time and effort, while findings that are simply wrong could result in the adaptation of dangerous or harmful practices" (p. 30). Thus, researchers are expected to take measures to promote reliability and validity in order to observe the norms of presenting the truth in their research studies.

The language of reliability and validity is traditionally prevalent in quantitative research (Golafshani, 2003; Winter, 2000). However, many researchers (e.g., Bashir, Afzal & Azeem, 2008; Maxwell, 1992; Long & Johnson, 2000; Shenton, 2004) have explained the importance of reliability and validity in qualitative research. Researchers promoting mixed methods research (e.g., Onwuegbuzie & Johnson, 2006) have used the language of reliability and validity that is acceptable from both quantitative and qualitative paradigms.

From reading Golafshani (2003), discussing reliability and validity as used in quantitative processes can became a springboard to discussing how these concepts are tested in the qualitative processes for a single study. That is, whilst the language of reliability and validity is definable in the world of observable and measurable facts, it then became necessary to redefine those concepts for their use in a naturalistic approach (Golafshani, 2003). In this regard, I discuss reliability and validity separately, but relate each concept to the quantitative and qualitative processes of this study. However, to connect quantitative and qualitative phases, I conclude this section by discussing how biases (or limitations) from various factors including my perceptions as the researcher were dealt with to maximise the reliability and validity of the study (Bashir et al., 2008; Brink, 1993; Onwuegbuzie & Johnson, 2006).

5.4.1 Validity in the research

Validity in quantitative research answers the question of "whether the research truly measures that which it was intended to measure or how truthful the results are" (Golafshani, 2003, p. 599). That is, validity encourages researchers to verify whether the means of measurement are accurate in enabling them to measure what they are intended to measure (Cohen et al., 2002). As such, researchers become critical in determining which data is to be collected and how it is to be collected, how participants are selected, and what are the appropriate statistical measures to follow when analysing data (Cohen et al., 2002; Golafshani, 2003). For such questions, researchers often look for answers in the research of others.

The language of measuring limits the work of qualitative researchers. Therefore, when conducting a qualitative research, according to Bassey (1999), it becomes necessary to

> explore significant features of the case, create plausible interpretations of what is found, test the trustworthiness of these interpretations, construct a worthwhile argument, … convey

> convincingly to an audience this argument, provide an audit trail by which other researchers may validate or challenge the findings or construct alternative arguments (p. 65).

When focusing on the concept of validity in qualitative research, many researchers (e.g., Davies & Dodd, 2002; Lincoln & Guba, 1985; Stenbacka, 2001) have developed different concepts of validity in order to adopt what they consider to be more appropriate terms, including quality, rigour and trustworthiness. In appropriating such terms, as Bashir et al., (2008) explain, researchers should have some kind of qualifying checks and balances for their studies.

Validity in the quantitative processes

In the quantitative processes of the study, and as mentioned above, the closed-ended questionnaire was piloted with one group of teachers. Piloting the instrument ensured that the closed statements made sense to the teachers and ambiguities were minimised (McMillan & Schumacher, 2010). Moreover, a significant number of closed statements were adapted from other studies such as Graven (2003) and Lasky (2005) in order to increase the validity of the closed-questionnaire. However, the content of statements accommodated teachers' views as a community. That is, statements were adjusted to be relevant to (a) teachers who have learned about the MTF, (b) teachers with certain experiences of teaching mathematics, and (c) teachers who have attended the TM1 course. In this way, the statements had a fair representation of the wider issues under investigation. Cohen et al. (2002) refer to this type of validity as *content validity*.

What further warranted the validity of the closed-ended questionnaire was the strategy of clustering seven statements of the same construct into four categories for the teachers to agree or disagree. As discussed in Subsection 5.3.1, the four categories included statements that were:

1) focusing on how much responsibility teachers take about learning and teaching mathematics;
2) attributing understanding of mathematics to learning from the TM1 course;
3) relating the impact of teachers' confidence to learning and teaching mathematics; and
4) accrediting learning about the MTF to developing teacher identity.

This strategy of aligning the same statements with the components of learning as described by Wenger (1998) and extended by Graven (2004) constitutes *construct validity.* C*onstruct validity* is "the extent to which a particular measure or instrument for data collection conforms to the theoretical context in which it is located" (Cohen, Manion & Morrison, 2011, p. 188).

Validity in the qualitative processes

In this study, the notion of validity received adequate attention because of overlapping research processes starting with multiple methods of data collection, which were: (1) video-recorded observations, (2) audio-recorded interviews and (3) field notes. As explicated in the previous section, utilising multiple data collection methods, and using different instruments, facilitated different focuses of data analyses which can improve the quality of the research. For example, given the collection of data using the observations, I had an opportunity to relate or question some of the teachers' responses during the analysis of the interviews. That is, as much as the observations were intended to focus of the mathematics lessons, I noted that the classrooms of two teachers were over-crowded and there was no space for them to see what the majority of learners were writing in their notebooks other than to look at those of few learners in the front row. From such an observation, I could corroborate teacher's assertions when they were discussing limitations about what learners wrote in their notebooks during the interviews. The processes of using different instruments to collect data promoted the within-methods triangulation in the study, which should improve the quality of the research.

The act of confirming or questioning the results from overlapping research processes seems trivial, but it has been a concern in the identity literature. Gujarati (2013) and Lutovac and Kaasila (2018) point out that researchers use too little data from observations of practice, particularly video-data to triangulate the results when researching teacher identity. For instance, from the reviewed studies in Chapter 3, we saw that Darragh (2015b) triangulated the results on how the learners did not recognise themselves as being 'good at mathematics' using data from different learners during interviews, but did not contest (or confirm) what the learners had listed as indicators of being 'good at mathematics' from the observations. Nevertheless, the present study does use multiple data sources to confirm or question the results from the lesson observations using interviews and vice versa.

5.4.2 Reliability in the research

In the quantitative research, Bashir et al. (2008, p. 36) cite Joppe (2000) who defines reliability as:

> The extent to which results are consistent over time and an accurate representation of the total population under study is referred to as reliability and if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable.

The Joppe's (2000) definition of reliability espouses on the opportunities of repeating or replicating the research with less attention on who the researchers are, but focusing on the degree of consistency within the abilities of instruments.

However, given that qualitative researchers are actively involved in the research processes (Long & Johnson, 2000), on the one hand, they view the definition of reliability from quantitative terms as adequate. On the other hand, some qualitative researchers (e.g., Lincoln & Guba, 1985; Shenton, 2004; Stenbacka, 2001) avoid the term 'reliability' and use the term 'dependability'. Dependability implies that the researchers should take their audience into confidence when explaining the research processes for the purposes of generating understanding of the phenomena.

Reliability in the quantitative processes

In teasing out issues that are raised in defining reliability in quantitative research, starting with the question of whether the results of the study can be reproduced using a similar methodology. As noted, the closed-ended questionnaire was initially administered to 2018 group of teachers as a pilot and then to 2016 and 2017 group in 2019 for the main study. This demonstrated the reliability of the closed-ended questionnaire considering that the results of the pilot were consistent with those of the main study. Cohen et al. (2002) refer to this type of reliability as *reliability as stability*.

Further evidence of the reliability of the closed-ended questionnaire in this study spans across Chapter 6 and 7. In Chapter 6, I explain why the numbers of teachers who participated in the quantitative phase were enough for the study. In Chapter 7, I present the results of the quantitative analysis, which complied with standard measures of *reliability as internal consistency* called Cronbach Alpha (Field, 2009).

Reliability in the qualitative processes

Part of persuading an audience that the research findings are worth paying attention to emanates from providing details of how the initial plans of the study evolved to yield the reported thesis (Golafshani, 2003; Long & Johnson, 2000). In this case, using the theoretical framework and the research design needed to be coherent to show some level of consistency in the study. For example, in Section 5.3 above, I presented the explanatory mixed methods research design as the detailed plan for conducting this study (Tashakkori & Teddlie, 2010). This plan has aided in collecting relevant data for the study in the context of examining teachers' experiences of learning and teaching mathematics after they had participated in the TM1 course.

In the next chapter, I illuminate the dependability of the study by providing details of the analytic frameworks, which highlight step-by-step descriptions of different categories. In turn, clearly describing categories in the analytic frameworks are necessary to heighten recognition of codes and boundaries that I set out to analyse qualitative data in order to answer the

research questions, particularly to answer the second part of the research question 1 in Chapter 8 and the research questions 2 and 3 in Chapter 11.

Ultimately, reading into the definitions of reliability and validity, there can be no reliability without validity. In other words, reliability is a consequence of the validity in a study (Bashir et al., 2008).

5.4.3 Maximising reliability and validity in the research

The idea of maximising reliability and validity in the research implies minimising risks or threats towards the rigour of the study. According to Brink (1993), researchers need to be attuned to the multiple factors that pose risks to the reliability and validity on their studies, and thus are required to plan and implement tactics to minimise them. In other words, researchers must be watchful of biases (or limitations) when planning and implementing their studies (Brink, 1993). If biases are left unchecked, the reliability and validity of the findings of the study can become questionable and indefensible (Bashir et al., 2008). If the reliability and validity can be maximised, then credible and defensible results may lead to generalisability. For Stenbacka (2001), generalisability becomes the structure for both doing and documenting high quality research.

The very presence of the researcher to conduct the study may affect the reliability and validity of the data provided by participants (Bashir et al., 2008; Brink, 1993). The fact that I was both a member of the TM1 community, where I was positioned as a research fellow, and a researcher in the processes of conducting the present study, can be a source of biases. In simplistic terms, I can be biased when explaining teachers' experiences of participating in the course since I was part of shaping their experiences of learning and teaching as much as they were shaping my experiences of participating in the course.

However, when I was part of the Wits Maths Connect Secondary project from mid-year 2017 until the end of the TM1 course in 2018, I was focusing on observing how the learning of mathematics-for-teaching was mediated and how the teachers were participating in the course. During this time, other than to pilot the closed-questionnaire with the 2018 group, which happened on the last day of the course, I did not formally collect data using field notes or interviews, for example. As such, when I was asking a teacher a question about a task during the course, I was genuinely asking about the task, and not making notes on the nature of responses for my research study. In 2019, I re-introduced myself to the participants as a researcher. At this time, as noted in Chapter 1, the focus was on examining teachers' experiences of participating in the TM1 course from their perspectives. In this sense, I had switched from being the research fellow (i.e. a worker) to become the researcher (i.e. a

student). According to Graven (2002), switching 'hats' this way can be challenging, but not impossible. In Chapter 11, when discussing the limitations of the research, I elaborate on some awkward moments during the interviews because the participants continue to some degree to see me as part of the TM1 course.

Ultimately, when reading Darragh (2018) and Graven (2002), many of my biases would have easily emanated from being Black whilst working with Black teachers who are teaching in disadvantaged schools in South Africa. That is, many of my biases would have been based on what I had hoped for as a novice researcher who is a Black South African, who had attended primary and secondary school in disadvantaged schools, and had taught secondary mathematics in a disadvantaged school for three years. The reality is that I had hoped that the TM1 course had contributed significantly in changing teachers' identity, after all, teacher PD interventions are about achieving change in teachers' ways of teaching and learning mathematics (Venkat & Graven, 2017). However, the position on the social theory of learning restricts the study and my perceptions from espousing what more the 'theories of identity' can offer. As described in Section 2.5, the 'theories of identity' addresses different forms of categorising groups of people such as race, class and ethnicity, where for example, one can debate why some groups of society are perceived as better achievers than other groups. However, the study remains within the designated position of the social theory of learning as explained by Wenger (1998).

When questioning the adequacy of using Wenger's (1998) social theory of learning as the foundational framework, the theory triangulation can be noted in this study. For example, as much as the notion of learning as *doing* from Wenger (1998) has a potential to yield the results for the study, from determining what the teachers are competent in doing when teaching mathematics, I also drew from Darragh (2016) who clarify that participants' identities can be accounted for when they perform their competences. The theory triangulation promoted the methodological triangulation, which has dominated the discussions on maximising the reliability and validity of this study, particularly due to the mixed methods research design.

The participants can also risks achieving maximum reliability and validity in the research study. Brink (1993) argues that selecting different cases when sampling can offer evidence from different perspectives. To get evidence from different perspectives is necessary when considering that a proposition deserves some degree of truth only when it has survived attempts to falsify it (Brink, 1993). For this study, when discussing the four teachers who participated in the qualitative processes in Chapter 7, it becomes evident that the "sampling by case type" strategy has a potential to produce diverse but complementary views about learning and teaching mathematics.

In the next section, I discuss the ethical considerations of the study. The ethical considerations also connect directly to the issues of maximising reliability and validity in the research, particularly when the participants understand the nature of the study.

5.5 Ethical considerations

Research involving human beings can be intrusive, inconveniencing and uncomfortable (McMillan & Schumacher, 2010). Researchers are therefore compelled to consider ethics in order to protect and respect the rights of participants. Ethical considerations require that researchers take into account informed consent, confidentiality, anonymity, privacy and fairness when negotiating access to research sites and when conducting the study (McMillan & Schumacher, 2006).

Before undertaking the study, I informed all teachers who attended the TM1 course in 2018, 2017 and 2016 about the nature of the proposed research. In the formal letters that were directed to the teachers, I firstly explained what the study was about. I then highlighted that the study entails two phases of data collection, which were (1) all teachers were requested to complete the closed-ended questionnaire, and (2) three teachers (the fourth teacher was added later) would be selected for observations when teaching mathematics in the classrooms and for individual interviews to talk about their learning and teaching of the subject. To add the fourth teacher later in the main study did not have any adverse implications to him when considering that all the teachers had given consent to participate in the study in its totality. These data collection processes, particularly classroom observations and interviews, can be intrusive, inconveniencing and uncomfortable in nature. Thus, when concluding the letters, I assured all the teachers that there were plans in place to minimise adverse experiences when they are participating in the study. Below, I spell out some of the measures, as I discuss each step of data collection.

With regards to completing the closed-ended questionnaire, teachers' responses to statements were not judged based on having 'right or wrong' answers (McMillan & Schumacher, 2010). Rather, the interest was on how individual teachers interpreted the statements when they are correlated with those of other teachers.

Regarding the observations, and in taking unobtrusive measures (McMillan & Schumacher, 2010), the video camera was placed at the back of the classroom. The video camera focused on the teacher and the chalkboard. As mentioned, I did the video-recording myself and no other persons came along with me for the observations, which further ensured confidentiality and anonymity.

During the observations, learners were in the classroom. Despite that the learners were not participants in the study, I needed to inform them and their parents (or guardians) about the nature of the research. In the formal letters, I explained to the learners and their parents (or guardians) that the video camera will focus on the teacher and the chalkboard. The videos mainly captured the backs of the learners. In cases where some learners were called to write solutions on the chalkboard or had turned to face the camera, their faces were pick-selected and blurred out to protect their identities.

Regarding the interviews, which were also audio-recorded and conducted by me, the teachers were treated with respect to maintain mutual objectives of the research (Davies & Dodd, 2002). For example, I was careful not to interrupt the teachers during the interviews.

In all these processes of data collection, I had received consent from the teachers including to video- and audio-record during the observations and the interviews respectively. In the end, the data collected during the research would be discarded in five years' time after submitting the thesis, and that was communicated to the teachers.

In addition, I kept in mind that teachers have rights to anonymity, protection and confidentiality when participating in the research study (Osler, 2005). Thus, the identities of the teachers were protected by not revealing their characteristics or use their real names at any given moment of the study. As noted, I used pseudonyms (false names) to represent them and their participation in the study and in other academic writings from this study. These measures were also communicated to the teachers before conducting the research, and they were afforded opportunities to accept or reject participating in the study. The teachers were requested to consent to the research by signing the informed assent forms. The teachers were aware that participating in the research was voluntary, which meant that they had other opportunities to withdraw from the research at any point of the study. Appendix E shows the information sheet with the consent form that I used for this study.

Lastly, I also informed both the schools (i.e. school principals and governing bodies) and the head of departments of the schools about the research. I further requested written permissions from the schools to conduct the research. The written permissions were submitted to the University of the Witwatersrand Ethics Committee. The University Committee issued an ethics certificate to conduct research after approving my application (Protocol number H19/03/18). See Appendix F for the ethics certificate. Parallel to the university application, I had applied to the Gauteng Department of Education for their approval to have access to conduct the research in the schools. See Appendix G for the Gauteng Department of Education approval letter.

5.6 Summary

In this chapter, I discussed the mixed methods research design that was employed for the study. I further explained the research methods that were used in the study, which included how I designed the closed-ended questionnaire, how I selected the participants, how I collected data using the lesson observations, the interviews and the field notes, and how I piloted the interview questions. The other methodologically matters discussed in this chapter included reliability and validity in the research, and how ethics were considered for the study.

CHAPTER 6

DATA ANALYSIS FRAMEWORKS

6.1 Introduction

In this chapter, I provide frameworks on how the data sets are analysed. The analyses of the data also followed the explanatory mixed methods research design, which was discussed in the previous chapter. In the quantitative phase, the closed-ended questionnaire was analysed using Exploratory Factor Analysis (EFA) to explore teachers' shared experiences of participating in the Transition Maths 1 (TM1) course. In the qualitative phase, the lesson observations were analysed using Mathematics Discourse in Instruction (MDI) as the analytic framework to understand teachers' teaching practices, particularly when teaching from the Mathematics Teaching Framework (MTF) perspective. The interviews were analysed using the narrative analysis (NA) to verify and expand the ideas developed by the teachers and the researcher during the questionnaire and the lesson observations analyses.

6.2 Analysing teachers' shared experiences of participating in the TM1 course

As stated, the closed-ended questionnaire was structured using the five interconnected components of learning: identity, meaning, community, practice and confidence (Graven, 2003; Wenger, 1998). The learning of teachers was warranted by influencing their experiences when participating in the TM1 course, which meant influencing stories of becoming members in, and of belonging to the TM1 community of practice (Wenger, 1998). Thus, agreeing and disagreeing with the statements in the closed-ended questionnaire illuminate the extent of what matters and does not matter for the teachers in relation to the learning and teaching of mathematics after having participated in the TM1 course.

It is not common to use a closed-ended questionnaire to get collective views about what matters (and does not matter) for participants in a community of practice when researching identity. Typically, researchers rely on small case studies to show how certain individuals belong (or not belong) to communities of practice. We saw when reviewing the selected studies in Chapter 4 that Bjuland et al. (2012) drew data from one teacher per study, and Darragh (2015b) collected data from 22 learners, which was the highest number of participants in these selected studies. The reality is that researching identity can yield many themes from different concepts that can be linked (or not linked) together. This was evident in Darragh (2015b) as the author initially generated 35 codes that translated into many themes. The present study uses data from 45 teachers who completed the closed statements, and relies on quantitative processes within the mixed methods research to reduce the data, whilst complementing the results using qualitative processes.

Responses from the completed questionnaires were entered into a spreadsheet to prepare them for the analysis. That is, the first column on the spreadsheet had teachers' names. The rest of the columns were allocated answers of the statements (questions), from question 1 to question 28. The answers were numerically coded as follows: 'Strongly agree' was coded as '1', 'Agree' was '2', 'Neither agree nor disagree' was '3' and so on. Therefore, each teacher had 28 answers to his or her name, which made it easy to calculate frequencies of teachers' responses per question from the spreadsheet.

The information was imported to the Statistical Package for Social Sciences (SPSS) programme for EFA (Yong & Pearce, 2013). Firstly, the negatively stated statements received reversed coding. For example, selecting 'Strongly disagree' to 'Teachers cannot change example sets because of the scripted lesson plans' received code '1'. Secondly, the 'missing values' search application was used to check whether the imported data missed any entries during the initial coding in the spreadsheet or whether participants responded to all the statements. Teachers had responded to all the statements. Thirdly, Anti-image Matrices was run to ensure that the data was suitable for EFA. Bartlett's Test of Sphericity was used to test the adequacy of data for EFA. Both tests indicated that the items were appropriate and adequate. Then, the data was subjected to principal components factor analysis and oblique rotation. Lastly, scree testing was used to determine the number of factors to retain, and thereafter the statements were presented to establish coefficients of Alpha for the factors.

There are two advantages for using EFA in this study. Firstly, using EFA lies in its flexibility to draw conclusions in an exploratory and descriptive, rather than inferential, manner (Beavers et al., 2013). Unlike the vast majority of statistical methods, EFA provides meaningful conclusions that are beyond considering a quantitative process because of its advantage of providing 'quick numerical results'. The exploratory and descriptive conclusions from EFA manifest when researchers make appropriate technical choices. For example, if the decision to select oblique rotation did not yield 'interpretable results' about teachers' experiences, I had an option of varimax rotation. The other available technical choice in using EFA is to reject statements after the analysis if such statements had contributed to a low coefficient of Alpha for the retained factors. Below, I discuss these technical terms.

EFA is not a singular statistical process, but a group of statistical analyses that share similar functionality (Beavers et al., 2013). Interpretable results come from the many methodological decisions that a researcher has to make to reach the most meaningful solution. For example, choosing oblique rotation means measuring the relationships of items in a factor (i.e. group of items) whilst considering how that factor relates to other factors. In this sense, if the relationships of items make sense on their own, but became meaningless to relate to other

factors, one can opt for varimax rotation. Varimax rotation does not consider significant correlations between factors. The items that are rejected can be understood as outliers that have potential to distort the relationships of items within a factor or the relationships of factors.

Secondly, when using the closed statements, there is a level of tidiness during EFA. For example, I decided to provide the participants with limited choices of responses to choose from. In this case, the teachers were agreeing or disagreeing with the statements. Other researchers even exclude the choice of 'Neither agree nor disagree'. All that I needed was to gauge which answers the teachers selected in order to make claims about their learning experiences within the theoretical framework. Of course, as mentioned in the previous chapter when discussing how I designed the closed-ended questionnaire, this process can be frustrating for some participants if appropriate steps in developing the questionnaire are not taken seriously, and the statements are irrelevant to them.

The main challenges associated with using EFA are the sample size and the emergence of narrow or unexpected results. Generally, sample size should be 'large enough' particularly when using EFA. Beavers et al. (2013) ask how large should the sample be to know if it is 'large enough' to produce a reliable factor analytic solution? In answering the question, the authors drew from recent studies which showed that "the needed sample size is conditional upon the strength of the factors and the items" (Beavers et al., 2013, p. 2-3). That is, for Costello and Osborne (2005), a new criterion of operationalising EFA is that, the factors should have four or more statements with loadings of 0.40 or higher, and then the size of the sample is not relevant. In the study and the pilot, I had 45 and 42 participants respectively, and as it is evident in Chapter 7, the results of the data analysis held to what Costello and Osborne (2005) characterise as a relatively 'strong data' because each emergent factor out of two factors has over four statements with loadings that are higher than 0.40.

In relation to the emergence of narrow or unexpected results, particularly because of research instruments (Gillham, 2011), using the closed-ended questionnaire to explore teachers' experiences could be limited to categories that are not in focus for the research. The intention of the study was to examine teachers' recognitions of their teaching of mathematics (identity) after participating in the TM1 course and having learned more mathematics-for-teaching (MfT). The challenge was that, in South Africa, for example, the dominant discourse when seeking to improve teachers' MfT has been to increase teachers' mathematical content knowledge (Essien, 2013). Thus, it was not going to be surprising if the teachers' shared experiences to focus on understanding mathematical content. Nonetheless, the qualitative processes of data collection were in place to directly open opportunities for teachers to talk about different features of the MTF and other benefits of learning MfT.

6.3 Analysing changes of teachers' shared experiences of participating in TM1

The narrative analysis (NA) was used to complement what emerged from quantitative analysis about teachers' shared experiences of participating in the TM1 course. The narrative analysis provided details on what has changed in teachers' shared experiences of the learning and teaching of mathematics, and on what has worked (or not worked) in effecting those changes. Squire et al. (2014) refer to such focus as the analysis of the narrative context.

The analysis of teachers' responses from the semi-structured interviews was not about the narrative structure. The analysis of the narrative structure concentrates on the sequence and the organisation of events, which can excite researchers (e.g., de Freitas, 2008) who are interested in what fragmentary stories such as of those teachers with 'troubled' identities can offer research. In this case, teachers remembered some details of events whilst responding to specific questions, decided to divert to include such responses, and then returned to respond to directed questions. These occurrences were expected during the interview given that I was not managing teachers' responses by posing validating (or disapproving) prompts to produce coherent events (Squire et al., 2014).

The focus on analysing *events* was intended to address *what* and *how* teachers' experiences of participating in the TM1 course influenced their teaching of mathematics in the classrooms. By *events*, I mean moments teachers highlighted when talking about their learning in the community of practice. The *what* part describes which components of learning (identity, meaning, community, practice or confidence) teachers recognised to have shaped their experiences of learning in the TM1 course. For example, in discussing teachers' experiences of learning in the community, a teacher could acknowledge to have learned from other teachers and could further elaborate on what he or she had learned from such interactions.

The *how* part follows components of learning that contributed in the teaching of mathematics, and further amplifies effects that emerged due to participating in the TM1 course. When expanding on the example of teachers' learning in the community, the teacher could have enjoyed how teachers respected each other during class interactions, and begin to want learners to respect each other as well when interacting in his classroom. In addition, the teacher could further describe how the effectiveness of the learners' learning from each other gets to be sustained during the teaching of mathematics.

The transcriptions became central in taking information and putting it together in order to make sense of the components of learning for analysis. The components of learning became segments of analysis that contributed into shaping teachers' experiences of participating in the TM1 course. Direct responses to the interview questions were an ideal place to start looking for descriptions of such segments. For example, responses to the "Have you changed

how you think about yourself as a mathematics teacher after participating in the TM1 course?" question provided direct access to describing changes that contributed into shaping teachers' identity. However, as mentioned above, if responses to such questions were elsewhere in the transcriptions, I also considered those responses as a relevant data for analysis. For that reason, analysing components of learning were less about which questions provided which descriptions.

The characterisation of teachers' experiences of participating in the TM1 course were developed from guiding questions in order to form categories. The guiding questions were based on the components of learning mathematics (i.e. practice, meaning, confidence, identity and community), and were informed by both the theory and the literature. The list of guiding questions to characterise teachers' experiences of participating in the course was referred to the analysis framework. See Figure 6.1.

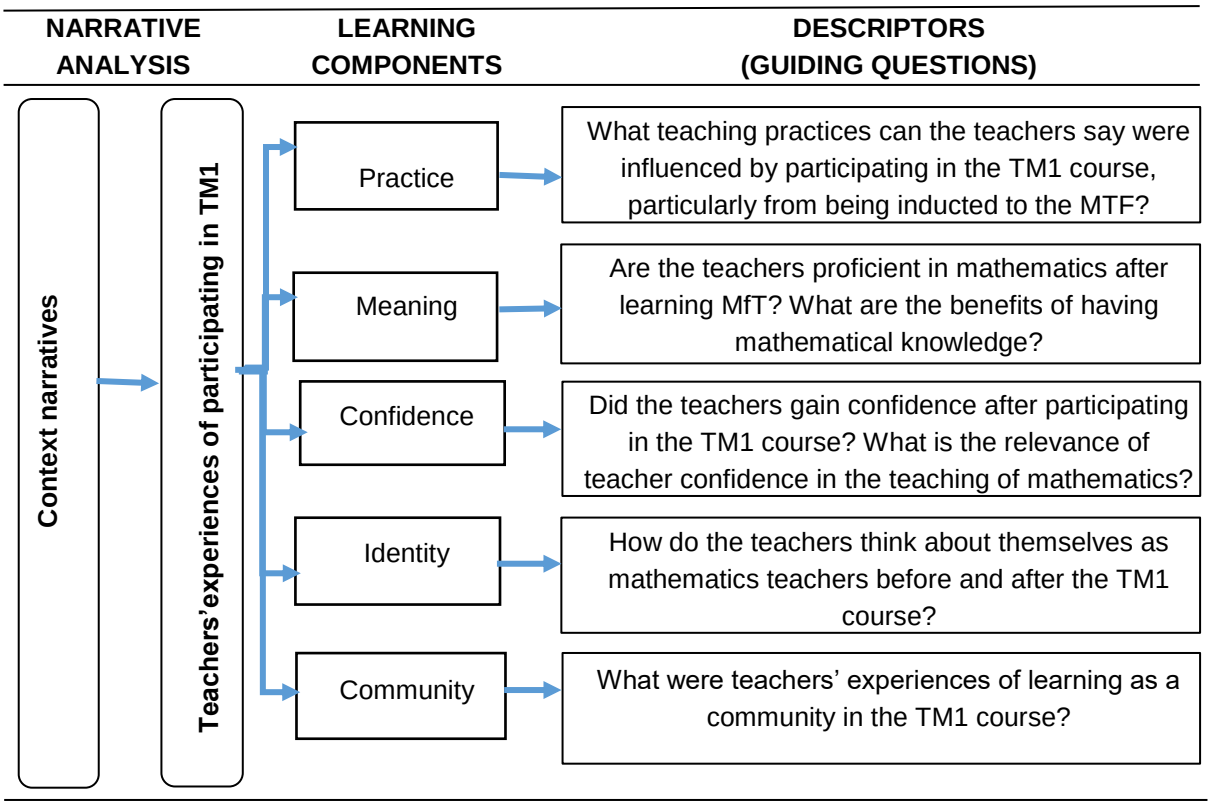


Figure 6.1: Analysis framework: List of guiding questions to describe teachers' experiences

To be systematic and consistent when characterising experiences for the teachers, I coded information from the transcriptions for each category. The first step of the analysis concentrated on *what* the teachers experienced in the TM1 course. In this step, I looked for words that are directly used in the components of learning (i.e. practice, confidence, identity, meaning and community). For example, picking on practice, words such as "selecting and sequencing examples" form part of exemplification, which is one of the aspects of the MTF.

Other than direct words that were used to categorise the components of learning, I looked for alternative words with similar meaning given a particular context in responses, and thus fitted such information in a relevant category. For instance, when discussing mathematical language, which forms part of explanatory communication, one teacher talked about "terminology that we use in mathematics". Table 6.1 provides guiding questions and examples of words that were used to code information per learning component.

Table 6.1: Guiding questions and codes for analysing teacher learning experiences

Learning Component	**Guiding questions (Descriptors)**	**Codes (Indicators)**
Practice	What aspects of the MTF have influenced teachers' ways of learning and teaching mathematics?	Selecting and sequencing examples; Considering cognitive levels of examples; Appropriating mathematical language; Enabling lesson goal for the lesson
Meaning	What enables teachers to learn mathematics-for-teaching and to teach mathematics for understanding?	Understanding mathematics; Knowing mathematics used in teaching; Using different representations and/or technology; Having self-confidence
Confidence	In what way is teacher confidence relevant in the learning and teaching of mathematics?	Having self-confidence; Trying out new things; Less fearful or scared; Mastering familiar things; Open to criticism; Encouraging discussions; Being well-prepared
Identity	How do teachers' think of themselves as mathematics teachers? How do others (e.g., other mathematics teachers) think of the teachers?	Being a mathematics teacher; Having emotional experiences (good or bad) about learning and teaching mathematics
Community	Did teachers feel part of a community of teachers who attended the TM1 course? Did they feel marginalised at any point during the course?	Learning with others; Learning to learn with others; Not wanting to learn with others

There were codes that overlapped across categories. For example, the teachers were shifting from talking about teacher confidence to talking about understanding mathematics and vice versa. In this example, the indicators of teacher confidence could be switched with those of enablers of learning MfT and teaching mathematics for understanding. However, the overall reading of the responses for meanings, and being familiar with different contexts teachers were referring to during the interviews, aided in deciding which words in which sentences to consider for which meanings, and what meanings to consider for which category or categories.

The second step of analysis expanded what the teachers had indicated to have shaped their experiences of participating in the TM1 course, and to include *how* such learning experiences contributed in their teaching of mathematics. Thus, the guiding questions were extended to now include guiding sub-questions to understand the effect of such learning experiences. The

sub-questions yielded sub-categories. For example, in the category of teachers' learning in the community, the sub-questions sought to find out the extent to which such learning experiences influenced their teaching of mathematics. That is, did teachers change to promote (or discourage) learners to learn from each other in the classrooms? How is learning in a community for learners sustained? Table 6.2 provides guiding sub-questions, which extend the guiding questions.

Table 6.2: Guiding sub-questions for analysing change in teachers' experiences

Learning Component	**Guiding questions (Descriptors for categories)**	**Guiding sub-questions (Descriptors for sub-categories)**
Practice	What aspects of the MTF have influenced changes in the learning and teaching of mathematics?	How are those aspects contribute in learning and teaching mathematics in the classroom?
		How are those contributions sustained?
Meaning	What enables teachers to learn mathematics-for-teaching and to teach mathematics for understanding?	How are those enablers contribute in the teaching for understanding or for meaning-making in the classroom?
		How are the effectiveness achieved? How does the effect work?
Confidence	In what way is the teacher confidence important in the learning and teaching of mathematics?	How has teacher confidence contributed in the learning and teaching in the classroom?
		How is teacher confidence get to be achieved for learning and teaching?
Identity	What has changed in how teachers' think about themselves as mathematics teachers before and after the course?	What does the teacher identity contribute to learning and teaching in the classroom?
		How is the contribution of teacher identity achieved?
Community	Did teachers feel part of a community of teachers who attended the TM1 course? Did they feel marginalised at any point during the course?	Did the TM1 community contribute in the learning and teaching of mathematics for teachers? Did teachers change to promote (or discourage) learning in the community of learners?
		How is learning in a community for learners sustained (or achieved), if at all?

Certain utterances that provided timelines were considered to indicate how teachers have changed after participating in the TM1 course. For example, utterances such as "prior to me going to the course" or "after attending the course" meant that teachers' experiences were influenced by participating in the TM1 course. Thus, when looking for the effects of learning that get to be sustained or maintained in the teaching of mathematics in the classroom, words such as "I realised…" or '"because of the course, I now…" in teachers' responses were coded from the transcriptions.

Below, I provide an extract from one teacher who discussed practice, particularly on one aspect of the MTF, when responding to the opening question during the interviews, to illustrate the coding of information. The opening question was 'What did you like the most about the TM1 course?'. Here is the relevant part of the teacher's response.

> Okay, in the Transition Maths 1 course, the terminology that we use in mathematics, I found out there that it's very very important because prior to me going to that course, I just used any term that I saw was suitable at that time. But I've realized in that course that terminology can have a way of making learners understand something.

The phrase "terminology that we use in mathematics" that is underlined by a solid line matches with the wording "mathematical language", which is part of the explanatory communication in the MTF perspective. The teacher highlighted this dimension as his main takeaway at this early stage of the interview. The phrase "prior to me going to that course" that is underlined by a dotted line indicates change in the teacher's experiences from participating in the TM1 course. The wording "But I've realized in that course" that is underlined by double lines indicates the contribution of learning in the course to teaching of mathematics in the classroom. However, the extract does not include justifications on how the effects of learning in the TM1 course are sustained in the teaching of mathematics.

6.4 Analysing teachers' mathematical discourse in instruction

In analysing teachers' mathematical discourse in instruction, I use the original Mathematics Discourse in Instruction (MDI) framework (Adler & Ronda, 2015). MDI was designed to analyse the quality of teaching, particularly to analyse mathematics made available to learn (Adler & Ronda, 2015). Whereas, as explained in Section 4.3, I use MDI to decide on teachers' competences of teaching mathematics in this study. As noted in Chapter 1, MDI has four interacting components (i.e. exemplification, explanatory communication, learner participation and the object of learning) that relate directly to the MTF. Figure 6.2 illustrates the MDI components.

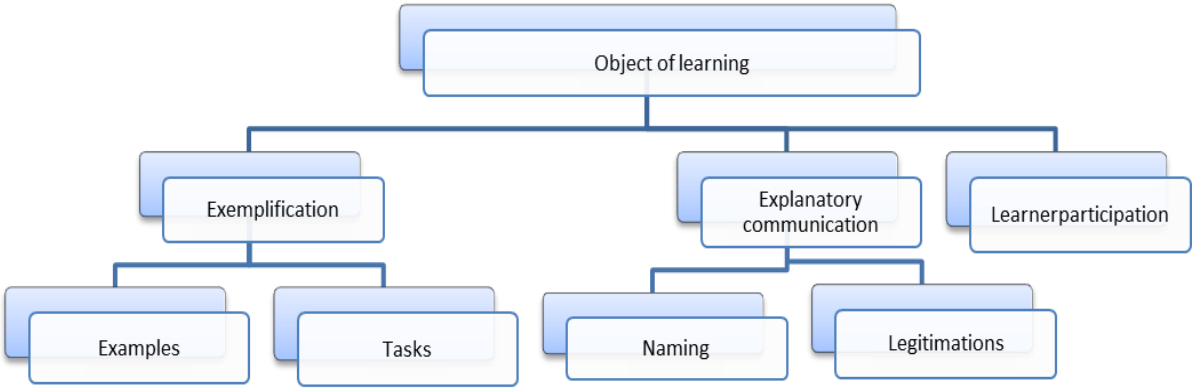


Figure 6.2: Constitutive elements of MDI and their interrelation (Adler & Ronda, 2015, p. 3)

In Section 4.3, I explained that the TM1 teacher community wrote assessments about the MTF. However, the context of performing in the assessments about the MTF was different to that of actually teaching mathematics from the MTF perspective in the classrooms. Specifically, Adler (2021b) describes teaching from the MTF perspective as a resource. Adler (2021b) explains that the MTF gets to be utilised together with other teaching practices, and to segregate its 'take up' from other teaching practices is never trivial. In this case, drawing on Darragh (2016), the researcher (an audience) was observing to characterise teachers (actors) when teaching (acting) in the classroom (stage). Consequently, I did not intend to discuss teachers' performance of teaching mathematics from a deficit end, rather teachers were discussed and characterised from what they had done during the observed lessons.

The four observed lessons per teacher were the sources of data. As noted, all the lessons were video-recorded to capture what the teachers were saying and doing. When preparing to analyse data, I considered how the words on the transcriptions were structured. The transcriptions followed the order of when the teachers were saying something to the learners and when they were writing on the chalkboard. Here are the four scenarios that were prominent in capturing the order of actions as a teacher would:

- write words in symbolic form on the chalkboard, and read those symbols to the learners using spoken words;
- write in symbolic form on the chalkboard, and mention specific areas using verbal words as a form of drawing learners' attention to those specific areas;
- mention mathematical statements in words, and re-write them in symbolic form on the chalkboard; and
- mention mathematical statements in words, and write in symbolic form to complete his or her sentences.

Whether the words were uttered or written, I considered that act as 'words used' during teachers' explanatory communication in the lesson. The use of 'gestures' was noted when teachers were pointing, underlining or highlighting words that they were focusing on during their explanatory communication. There were instances where when teachers were using gestures implied that they were using different mathematical representations. For example, there were instances where putting a box to highlight the variable x in algebra was understood as treating x as a placeholder that needs to be replaced by something else other than to draw learners' attention to focus on x.

I needed to divide the data into analytical units (McMillan & Schumacher, 2010). As a start, I looked at the transcription of one lesson at the time across the four teachers. Usually each lesson focuses on its object of learning. Within a lesson, I further divided the object of learning

into sub-units to accommodate specific tasks. For instance, typically, teachers firstly introduce the lesson. Secondly, teachers develop the lesson. Thirdly, teachers give learners classwork activities. Lastly, learners get homework activities.

Adler and Ronda (2015) refer to sub-units as mathematical episodes. Mathematical episodes are identified by shift in focus of attention with respect to content, typically marked by a task that encompasses selected examples (Adler & Ronda, 2015). According to Adler and Ronda (2015), mathematical episodes become supportive in revealing nuanced interpretations of teaching practices, particularly exemplification, explanatory communication and learner participation, and how all these practices work together towards the object of learning.

As mentioned in Subsection 5.3.3, I analysed only the first lesson (L1) out of the four lessons across the four teachers to conclude on the teachers' mathematical discourse in instruction. The first lesson (L1) was on solving algebraic equations using additive and multiplicative inverses. Choosing L1 offered some opportunities to make general claims about the objects of learning across the four lessons. For example, to solve an exponential equation 2^2x – 2 = 16 from the third lesson (L3) implies being able to solve 2x – 2 = 4 if using the principle of equal bases. In other words, analysing L1 can form basis of drawing conclusions about other types of algebraic equations, particularly on how the teachers connect L1 to other three lessons.

In addition, I encouraged the teachers to provide examples from L1 during the interviews when reflecting on their teaching of mathematics. Choosing L1 also fitted the analysis process given that the fourth teacher was only observed when teaching this lesson. The structure of L1 was the same for the four teachers. As a result, the mathematical episodes were similar for the four teachers (i.e. teachers started with the introduction, followed by developing the lesson, and then gave learners classwork activities). The differences between the teachers were that the fourth teacher used different examples, and only had two mathematical episodes (i.e. the teachers developed the lesson and gave learners classwork activities).

Codes of the MDI components

To be consistent in analysing each lesson using descriptions of the MDI in a lesson, Adler and Ronda (2015) provide codes of different categories that describe the quality of teaching. Table 6.3 provides a summary.

Table 6.3: Codes of MDI to decide on teachers' competences of teaching (Adler & Ronda, 2015, p. 6)

Object of learning				
Exemplification		**Explanatory Communication**		**Learner Participation**
Examples	Tasks	Naming	Legitimating criteria	
Examples provide opportunities within an episode or across episodes in a lesson for learners to experience variation in terms of *similarities* **(S)**, *contrast* **(C)**, *fusion* **(F)**.	Across the lesson, learners are required to: *carry out known operations and procedures* **(K)**, e.g., solve for x, multiply, factorise; *apply known skills, and/or decide on operation to use* **(A)**, e.g., compare/classify/match representations; *use multiple connections* **(C/PS)**, e.g., solve problems in different ways, use multiple problems, prove, reason, etc.	Within and across episodes word use is: *colloquial* **(NM)**, e.g., everyday language and/or ambiguous pronouns such as this, that, thing, to refer to objects in focus; *maths words used as name only* **(Ms)**, e.g., to read string of symbols; *mathematical language used appropriately* **(Ma)** to refer to other words, symbols, images, procedures, etc.	Legitimating criteria: *non-mathematical* **(NM)** *visual* **(V)**, e.g., cues are iconic or mnemonic; *positional* **(P)**, e.g., a statement or assertion, typically by the teacher, as if 'fact'; *everyday* **(E).** Mathematical criteria: *Local* **(L)**, e.g., a specific or single case (real-life or maths), established shortcut, or convention; *general* **(G)** equivalent representation, definition, previously established generalisation, principles, structures, properties, which can be *partial* **(GP)** or *'full'* **(GF).**	Learners answer *yes/no questions or offer single words* to the teacher's unfinished sentence **(Y/N)**. Learners answer (what/how) questions in phrases/sentences **(P/S).** Learners answer why questions; present ideas in discussion; teacher revoices/confirms/asks questions **(D)**.

The codes are explained in the table, and in other instances, examples of using codes are provided. However, for this study, three categories were not perfectly matching the codes from the table. Firstly, learner participation was analysed mainly from what teachers were saying to the learners, whether when asking questions, revoicing and confirming their responses. In addition, and when taking into account that learners could be heard when responding to the teachers' questions in the video, learners' responses were considered from what they were saying. The responses were not followed through from what they had written or done during the observations. As an example, let us look at the following short insert from the transcription.

Teacher:	What do we notice on the left-hand side? Like terms?
Learners:	Yes.
Teacher:	We have like terms so we can subtract them because they are the same. What is $5x$ subtract $3x$?
Learners:	$2x$

This teacher started by asking a 'what' question, and then reduced the question to ask the learners to provide 'yes or no' answer, which could be coded as **(Y/N)** in the table. The learners were heard providing the 'yes' answer. The teacher confirmed the learners' response to be correct, and then moved on to ask another 'what' question, which is "What is 5x subtract 3x?". The learners were heard providing a single answer, which was 2x. Thus, from what the learners were saying, it can be concluded that the learners remained in answering *yes/no questions* and *offer single words* to the teacher's questions.

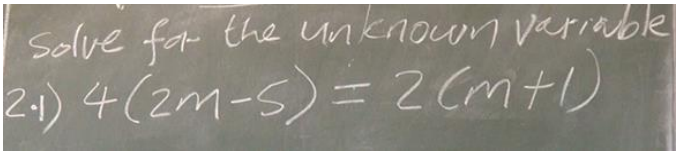
Secondly, when teachers participated in the TM1 course, they were not oriented in all the three variation aspects of exemplification (i.e. *similarity* **(S)**, *contrast* **(C)** and *fusion* **(F)**) when selecting and sequencing examples for teaching. In the course, teachers were mostly exposed to discussions on what *similarity* and *contrast* can constitute in achieving generality when selecting and sequencing examples for an example set. Adler and Ronda (2015) had backgrounded *fusion. Fusion* meant that more than one aspect of an object of learning is simultaneously varying/invariant across an example set. Therefore, in this study, teachers were not expected to consciously provide learners with an opportunity to experience *fusion* when looking at an example set within a mathematical episode or across episodes in a lesson.

Lastly, for this study, I was curious on whether a teacher kept the object of learning (OoL) in focus during the lesson. That is, I wanted to hear whether a teacher could consciously draw learners' attention to what they are doing in a task. A teacher could write and/or mention the OoL during the course of a lesson. Indeed, starting a sentence with 'we are going to…' could give an indication of the OoL. The mention of the OoL can give the researcher an opportunity to confirm what the lesson is about from the teacher in order consider that all other categories work together to inform the OoL.

Example of using the MDI codes

Here is an example of how the MDI codes were used to analyse the collected data from observations in order to decide on the teachers' practices of teaching mathematics. For this example, I start by showing a bit of what this teacher wrote on the chalkboard from a raw data taken from one lesson during the trial observations. The main OoL of his lesson was to teach learners how to solve algebraic equations using additive and multiplicative inverses. To avoid providing a long extract for this example, I summarise the first two steps of solving the equation 4(2m – 5) = 2(m + 1). Then, I return to provide the rest of the extract to complete the example of coding data (Times of 'words used' are indicated at a start of each segment in 'minutes:seconds').

08:36 (The teacher writes)



From 09:01, as I summarise the first two steps, the teacher and the learners talked about "multiplying brackets" to get the next step, which is 8m – 20 = 2m + 2. The teacher asked the learners what to do next. The learners were talking about 8m subtract 2m. The teacher mentioned that they were "collecting like terms". The teacher mentioned that the learners were using "transposition method" instead of "additive inverse method", which was also fine for him. As such, the learners responded in chorus to say 8m – 2m = 2 – 20. The teacher said, "Come again?" to draw learners' attention to the incorrect answer when transposing –20. Below, I provide the rest of the interactions between the teacher and the learners from the transcription with the MDI codes. In the extract, I use square brackets '[ ]' for learners' utterances, round brackets '( )' for teachers' description of actions, codes are in bold, and nothing for teachers' utterances.

11:46 [Learners: minus 20, negative 20, positive 20] The sign here is? (He points at -20 at $8m - 20 = 2m + 2$ in the previous step) **(Y/N)** So it will be? **(Y/N)** [Learners: positive 20] (He continues to write $8m - 2m = 2 + 20$) Now add the two (He points at $8m - 2m$) 8m subtract 2m **(Ms)**, what do you get? **(Y/N)** [Learners: 6m] (He writes $6m =$) What do you get? (He points at $2 + 20$) **(Y/N)** [Learners: 22] (He completes $6m =$ to get $6m = 22$) Okay, 22.

12:38 Now remember, I said to you, write down the step the second time (He points at 6m = 22) (He writes $6m = 22$ again) before you do your multiplicative inverse **(Ma)**. Then from there? **(P/S)** [Learners: divide by 6] What is your multiplicative inverse? **(Y/N)** [Learners: divide by 6] (He divides 6m by 6 and 22 by 6) Here on the left-hand side? **(Y/N)** [Learners: m] m is equal to? (He writes $m =$) **(Y/N)** [Learner: 11/3] 11/3 (He completes $m =$ to get $m = 11/3$) Then, we have found the value of m **(Ma)**.

At the start of the task, the teacher drew learners' attention to a fact that they want to "solve for the unknown variable m". The example and the task support the OoL, which was to solve the linear equations using additive and multiplicative inverses. The starred example could not be compared with other examples for similarities and contrast. The tasks itself could be coded as **(K)** given that the learners were carrying out known procedure of solving the linear equation. Hence, the tasks was titled "Solve for the unknown variable" and that was what the teacher focused on in finding the solution.

Regarding explanatory communication, the teacher was using mathematical language appropriately, but the learners were using mathematics words to name and read the symbols. For example, I mentioned that the teacher and the learners were multiplying out brackets, which replaced talking about the distributive law. Thereafter, the teacher mentioned that he had taught the learners how to use the "additive inverses method" although they were using the "transposition method". As noted, transposition method focuses on manipulating symbols. Hence, we saw the debate on whether the sign of 20 was positive or negative when moved to the other side of the equal sign. Then, the teacher was talking about using "multiplicative inverse". The learners were dividing the terms. Lastly, the teacher mentioned that the task seeks to determine the value of m, which was coded as mathematical language used appropriately **(Ma)**. However, the teacher did not justify to say that the value of m needs to bring balance between the left-hand and the right-hand sides of the equation. The general principle of bringing balance between the left-hand and the right-hand sides of the equation can be legitimating criteria, which can be coded as **(G)**, which can be 'partial' **(GP)** or 'full' **(GF)**.

Lastly, regarding learner participation, the teacher remained in leading the leaners to answer *yes/no* questions and to offer *single words* when participating in this task. The questions like "The sign here is?", "So it will be?", and "Here on the left-hand side?" were evident that the teacher was inviting the learners to provide yes/no answers or single words. These invitations were coded as **(Y/N)** throughout the teacher and the learners' interactions on the task.

Summative judgements across episodes

Once the coding was completed on the mathematical episodes, the next step was to make judgements on the lessons. According to Adler and Ronda (2015), researchers need to look across mathematical episodes to produce summative judgements and express different levels with respect to the four categories (i.e. object of learning, exemplification, explanatory communication and learner participation) that can accumulate over the lesson. See Table 6.4.

Table 6.4: A framework for describing MDI and interpreting differences in teaching (Adler & Ronda, 2015, p. 7)

The set of examples provide opportunity in the lesson for learners to experience: Level 1 – one form of variation, i.e. S or C; Level 2 – both forms of variation, i.e. S and C; Level 0 – Simultaneous variation with no attention to similarity and/or contrast	Tasks provide opportunities for: Level 1 – carrying out known procedures only K; Level 2 – K and/or some application A; Level 3 – L and/or A and C/PS L2→L1: A→K or C/PS→K is assigned to tasks set up at level 2 or 3 but then reduced to 1 when it unfolds	Use of colloquial and mathematical words: Level 1 – NM, there is no focused maths talk, all colloquial/everyday; Level 2 – movement between NM and Ms, and some Ma; Level 3 – movement between colloquial NM or maths words Ms and formal maths talk Ma	Criteria for what counts as mathematics that emerge over time in a lesson and provide opportunity for learning geared towards scientific concepts. Level 0 – all criteria are NM, i.e. V, P, E; Level 1 – criteria include L, e.g., single case; Level 2 – criteria extend beyond NM and L to include generality, but this is partial GP; Level 3 – GF maths legitimation of a concept or procedure is principled and/or derived/proved	Opportunity for learners to speak and so use maths discourse is at: Level 1 – Y/N only (single words only); Level 2 – at least some P/S in more than one episode (phrases and sentences); Level 3 – P/S and at least some D (discussion) in more than one episode

In summary, as I show in Chapter 9, I firstly analyse the lessons when the teachers are teaching how to solve linear equations using additive and multiplicative inverses. I then look across each lesson to make summative judgments using the MDI framework. See Table 6.4 above for different interpretations of teachers' teaching. Lastly, I also analyse the other three lessons to merely show if the teachers were connecting their lessons considering the expectations of coherent teaching. For instance, teachers who are conscious about the importance of coherent teaching tend to draw on learners' previous knowledge when teaching the subject. The indicators of whether a teacher connects the lessons relied on terms such as 'as we saw in the previous lesson' or 'as you know from the previous section'. Thus, as much as one can expect teachers to make connections at any given moment during their lessons, much of connections can be spelled out when introducing the lessons.

6.5 Analysing teachers' mathematics teaching identity

As alluded to in Chapter 2, the narrative analysis (NA) was used to analyse teachers' mathematics teaching identity (Darragh, 2016; Sfard & Prusak, 2005; Squire et al., 2014; Wenger, 1998). The analysis of teachers' mathematics teaching identity followed Wenger's (1998) notion of learning as *doing* in order to understand their ways of teaching mathematics from the MTF perspective from the interviews. As noted, Wenger (1998) defines learning as *doing* to entail both lived experiences and a display of competence within familiar territory. In this case, when teachers engage with different communities (e.g., learners, other teachers and school management team members), they can make sense of mathematics teaching whilst being able to communicate what is familiar from what is unfamiliar (Carlone & Johnson, 2007; Wenger, 1998). At the same time, teachers can recognise their areas of competence and allow others to recognise them as competent mathematics teachers when teaching the subject (Carlone & Johnson, 2007; Darragh, 2016; Sfard & Prusak, 2005; Wenger, 1998).

The lived experiences of teachers participating in the study were centred firmly around the experiences of participating in the TM1 course. However, in analysing teachers' mathematics teaching identity, the study does not exclude experiences of teachers when they were learners of mathematics themselves in schools or in colleges and universities. For example, drawing on their past experiences can highlight if the teachers felt marginalised as high school learners or as student teachers, or they enjoyed their journey of learning mathematics. Thus, tapping into teachers' journeys with mathematics can provide some context when discussing the results from the analysis.

Drawing on Carlone and Johnson (2007) to operationalise Darragh's (2016) performance identity and Wenger's (1998) identity as *doing*, two dimensions were considered for this study: (1) recognition of teachers themselves and by others as teachers of mathematics; (2) display

of competence when they are talking about teaching from the MTF perspective and other teaching practices. I refer to a display of competence as teacher performance considering that teachers could teach from the MTF perspective, which were the learning objectives promoted in the TM1 course. See Figure 3.3 below. Thus, as a prototype, teachers with a strong mathematics teaching identity can mean open recognition of themselves and by other as mathematics teachers, and high ratings in displaying competence about their teaching.

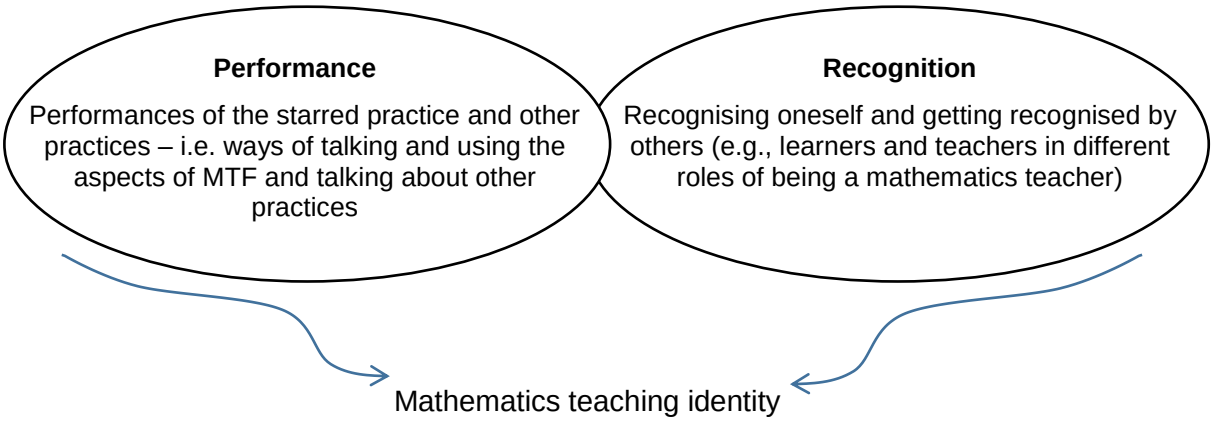


Figure 6.3: Prototype of teachers' mathematics teaching identity (Carlone & Johnson, 2007)

The two dimensions of mathematics teaching identity (i.e. performance and recognition) were analysed further using Sfard and Prusak's (2005) perspective of treating stories as identities. The process of pulling together performance of teaching mathematics from the proposed perspective begins when a teacher consciously attempts to use the tools and resources when teaching from that perspective. The next step includes *what* and *how* a teacher can recognise and communicate openly in relation to that teaching perspective. Ultimately, other people get a window to positively or negatively recognise those individuals as mathematics teachers. Squire et al. (2014) refer to engaging with this kind of data as content-based analysis.

However, the content-based analysis is not mutually exclusive to the context of individuals' narratives (Sfard & Prusak, 2005; Squire et al., 2014). In order to allow adequate reasoning about the matter, it is important to understand content with some context in mind. The context in this case becomes teachers' reasons for acting (or not acting) in a particular way when teaching mathematics in the classroom settings (Darragh, 2016; Andersson, 2011). These are natural occurrences embedded in the school policies or due to the social interactions (e.g., what other teachers are doing or saying, and the nature of the learner population) whereby teachers can draw upon to justify their display (or lack thereof) of competence in the learning and teaching of mathematics (Beauchamp & Thomas, 2009; Pulvermacher & Lefstein, 2016). In addition, teachers could cite their participation in the TM1 course as another reason for acting (or not acting) in a particular way when teaching mathematics (Darragh, 2016). Thus,

this can be classified as teachers' mathematics teaching identity in school context. In Darragh's (2015a) language, school context is the stage where teachers (actors) perform.

This part of the interviews, which needed to understand teachers' way of teaching mathematics from the MTF perspective and other teaching practices, was directed to obtain teachers' stories. The focus of the analysis was on the explanations that were provided by the teachers about their teaching of mathematics. Where there was contrary evidence between teachers' competence of teaching mathematics from the lesson observations and their explanations, the stories are labelled as being envisaged future actions. Teachers' stories were presented in transcriptions. The transcriptions became the process of taking information and converting it into a formal source of data to facilitate analysis.

As mentioned above, the characterisations of mathematics teaching identity emerged from the presented analysis of the narrative content, particularly to amplify interpretations of teaching mathematics from the MTF perspective. To characterise teachers' mathematics teaching identity, four categories were taken into consideration. From the performance dimension, (1) teachers needed to communicate how they teach mathematics from the MTF perspective, and if possible, (2) teachers could express how using MTF relate to other teaching practices. From the recognition dimension, (3) teachers needed to talk about themselves in relation to teaching mathematics from the MTF perspective, and lastly, (4) teachers could share what others say about them as mathematics teachers. Some categories have subcategories. Below, I discuss criteria of categories, with subcategories where applicable, in order to characterise teachers' mathematics teaching identity.

The **first category** related to the teachers' performance when talking about teaching from the MTF perspective. This category has four subcategories that are drawn from Adler and Ronda (2015). The subcategories are as follows: (1) exemplification, (2) learner participation, (3) explanatory communication, and (4) the object of learning. These are the predetermined MDI components known to the researcher.

The first subcategory relates to exemplification. In this subcategory, teachers could explain different kinds of examples they use in their teaching of mathematics. Here are other questions that should assist me to characterise the teachers. Do teachers provide examples that exhibit progression, such as from easy to difficult? Do teachers consider similarity and contrast when selecting and sequencing examples? Do teachers use multiple connections or representations in their examples? What are the sources of their examples? Do they change examples to draw learners' attentions to the object of learning? What are the reasons for using (or not using) certain kinds of examples in their lessons?

The second subcategory relates to learner participation. In this subcategory, teachers could explain what learners say, do and write during mathematics lessons. The teachers were encouraged to use the observed lessons to espouse learner participation in their teaching. For example, as the teachers walked around the classrooms to interact with the learners during mathematics activities, what were the learners writing or doing. In explaining what the learners were doing, the teachers could confirm if the learners get opportunities to discuss mathematics activities in the classrooms. Here are other sub-questions for this subcategory. Do learners discuss with each other in groups or in a whole class? Do learners discuss with teachers in question-and-answer sessions? What mathematical activities enable discussions in the classrooms? Lastly, with what learners are saying, teachers could confirm if learners talk as individuals or sometimes as a whole class. Why do learners talk as individuals or in groups? Do learners use appropriate mathematical language when talking in the classrooms? What are the reasons for learners to participate in certain ways? For instance, teachers can claim that learners often say that 'mathematics is a difficult subject' as one of the reasons of poor learner participation.

The third subcategory involves how teachers incorporate explanatory communication when teaching mathematics. What do teachers consider to be central when explaining mathematics to learners? Do teachers strive for generalisation when connecting concepts? Do teachers get opportunities to justify mathematics to learners using formal proofs? In answering these questions, the teachers were expected to provide reasons for acting (or not acting) in particular ways when explaining mathematics to the learners.

The last subcategory is on how teachers saw all the other subcategories working together to enhance the object of learning. Do teachers see other lessons connecting to the starred lesson? Do teachers use more than one explanation to connect same mathematical concepts? Do teachers connect sub-topics to the main object of learning to produce a coherent lesson? In other words, do teachers point to previous examples to explain the starred examples? How do teachers justify the fact that sometimes they fail to adhere to the object of learning?

The **second category** relates to teachers' explanations on how they were teaching mathematics using other kinds of teaching practices. On the one hand, teachers could explain the components of the MTF from a different perspective, outside Adler and Ronda's (2015) conceptions. On the other hand, teachers could expand on their performance of teaching mathematics from the MTF perspective. These explanations can promulgate those teachers to be characterised to have developed a new mathematics teaching identity. Indeed, reasons for 'winging' the performance should be spelled out for this study.

The **third category** related to the teachers' recognition of themselves. The next questions are assisting me to decide on this category. What do the teachers recognise about themselves in relation to other mathematics teachers when teaching from the MTF perspective? How do the teachers feel about being different (if they are) to others teachers? In what way do the teachers feel that teaching from the MTF perspective add value in their teaching of mathematics, if at all? What are the prospects of wanting to learn more about teaching mathematics? How do the teachers recognise self in some statutorily terms because of teaching from the MTF perspective?

The **last category** was about how others recognise the teachers in their teaching of mathematics. The 'others' refers to individuals that are involved directly or indirectly with the learning and the teaching of mathematics in the school setting such as learners, teachers of mathematics, teachers of other subjects, members of the school management team (e.g., head of departments, school principals and district officials) and parents of learners. Darragh (2016) can refer to the 'others' as audience. The recognitions by others can include (1) positive or negative emotional disposition towards teachers; (2) whether they have confidence in their teaching of mathematics; and (3) whether they single teachers out in honour (e.g., promotions). Ultimately, it is whether others can recognise teachers positively or negatively in their quest of teaching mathematics. Table 6.5 shows examples of judgments of utterances.

Table 6.5: Kinds of recognitions from others

Emotional disposition about mathematics e.g., "They [other maths teachers] feel that I love maths. This one enjoys maths. He won't survive if he's not teaching maths" {Positive}
Confidence in teachers e.g., "He [district official] has much confidence in me" {Positive}
Singled out with academic roles e.g., "I'm moderating Grade 8 papers now" {Positive}

Ultimately, the characterisation of teachers' mathematics teaching identity, meant matching what teachers understood about teaching from the MTF perspective and their competence of teaching mathematics. That is, for example, if a teacher confirmed that the words used in his lesson moved between non-mathematical language to formal mathematical language, whilst that was evident in his or her mathematical discourse in instruction, that teacher could be characterised to poses actual teaching identity, whilst the opposite could mean designated teaching identity. In this sense, Darragh (2016) performative identity framework comes to play to harness Wenger's (1998) notion of identity as *doing* and Sfard and Prusak's (2005) notion of identity as *narratives*.

6.6 Consolidating data analysis

The effect of employing mixed methods research lies in how researchers consolidate data analysis to answer the research questions. The research questions are answered when the researchers discover patterns across the results of different categories (McMillan & Schumacher, 2010). For this study, patterns reveal three strands. Firstly, when looking across the quantitative analysis and qualitative analysis, certain closed statements from the analysis of the closed-ended questionnaire should support the analysis of the interview, particularly on what teachers claimed to have experienced when they attended the TM1 course. That should answer the second part of the research question 1 in Chapter 8. Secondly, to answer the research question 2 in Chapter 11, it would become necessary to trace and elaborate on teachers' mathematics teaching identity in relation to components of learning, which are community, practice, meaning and confidence (Wenger, 1998). Lastly, to answer the research question 3 also in Chapter 11, after establishing teachers' mathematical discourse in instruction (what they do or not do) in Chapter 9, the next step would be to look for patterns that connect teaching practices to their mathematics teaching identity. It should not be surprising to get closely related results from these analyses. The reason of anticipating close results includes the fact that the study does not use the solipsism argument of believing in only what you can see during performances (Darragh, 2016; Wenger, 1998), but it also draws from Sfard and Prusak's (2005) framework, which advocate that teachers' stories should be taken seriously, and as being true.

6.7 Summary

In this chapter, I discussed the frameworks of analysing the data sets for the study. Firstly, I elaborated on how EFA was employed to analyse teachers' shared experiences of participating in the TM1 course. In these elaborations, it was noted that EFA does not constitute a singular statistical process, but rather a group of statistical analyses that share similar functionality for meaningful results. Secondly, I explained how the narrative analysis was utilised to analyse changes in teachers' shared experiences of participating in the TM1 course. The changes in teachers' shared experiences of participating in the course included how learning experiences contribute in the teaching of mathematics in the classrooms. Thirdly, I discussed how MDI as presented by Adler and Ronda (2015) was taken up to analyse teachers' competences of teaching mathematics from the MTF perspective. Lastly, I highlighted the use of teachers' narratives again to analyse teachers' mathematics teaching identity. This analysis implied matching what the teachers were doing when teaching in the classrooms against what they understood about teaching from the MTF perspective.

In order to respond to the research questions, I explained the need to look for patterns across the results of the study. I referred to the exercise of looking for patterns across the results as the process of consolidating data analysis.

CHAPTER 7

THE TEACHERS' SHARED EXPERIENCES OF THE TM1 COURSE

7.1 Introduction

In this chapter, I discuss the results of analysing teachers' shared experiences of participating in the Transition Maths 1 (TM1) course to answer the first part of the research question 1 (RQ1): 'To what extent do teachers have shared experiences of learning and teaching mathematics after participating in the TM1 course?'. I examine the results after conducting Exploratory Factor Analysis (EFA) on the closed-ended questionnaire.

The details of the technical choices for data analysis when using EFA are discussed in Section 6.2. What to note about the data is that 45 teachers who participated in the 2016 and 2017 TM1 course responded to 28 statements of the questionnaire. The details of how the questionnaire was designed and administered are in Subsection 5.3.1.

What follows next is the overview of the findings to answer the first part of the RQ1. In other words, I provide a summary of the findings from the results. The results are presented in the rest of the sections of the chapter. Whereas, before ending the chapter, I return to discuss details on how the four teachers were selected from the quantitative results in order to participate in the qualitative processes. The last section shows interconnection of Wenger's (1998) learning components from the quantitative results.

7.2 Overview of the findings to answer the first part of the RQ1

Teachers shared strong experiences of learning and teaching mathematics after participating in the TM1 course. The teachers were mostly consistent in responding to the closed-ended questionnaire, which means that they were agreeing with each other on many statements. Thus, the analysis reveals that the 45 teachers can be characterised as follows:

1) They are teachers who are understanding mathematics for themselves and for teaching after participating in the TM1 course;
2) They are teachers who value teacher confidence to be necessary for learning and teaching of mathematics;
3) They are teachers who have a positive sense of identity towards the teaching of mathematics after learning about the MTF; and
4) They are teachers who have mixed views about accountability towards teaching mathematics.

The teachers were located in the matrices of how much they are accountable towards teaching mathematics versus how much they value learning and teaching in relation (1) to their positive sense of identity after learning about the MTF, (2) to their understanding of mathematics and (3) to teacher confidence. The presented results suggest that those teachers who valued learning and teaching more also have strong views about being accountable towards teaching mathematics. The characterisation of the teachers who had participated in the TM1 course confirms Wenger's (1998) position that the components of learning are interconnected for members of a community of practice such as the TM1 course.

7.3 The extent of the teachers' shared experiences of participating in the TM1 course

Within EFA, and upon subjecting the data to oblique rotation, two factors emerged for retention. The two factors had 7 and 13 statements each with reliability measure (Cronbach Alpha) of 0.702 and 0.869 respectively. When considering the group of statements loading on each factor, I named the factor with 7 statements 'accountability towards teaching' and named the other factor with 13 statements 'mathematics teachers' identity and confidence'. See Table 7.1 for a summary.

Table 7.1: Subsets of teachers' shared experiences of learning and teaching

Identified factor (No. of statements)	Exemplar Statements	Reliability measure	No. of statements per group
Accountability towards teaching (7)	Teachers are accountable for learners' performance in mathematics / Learners come to secondary school with inadequate content knowledge	0.702	6 community 1 meaning
Mathematics teachers' identity and confidence (13)	Since learning about the MTF, I feel an increased sense of being a teacher / I have enough confidence to allow other teachers to observe my lessons	0.869	6 identity 4 meaning 3 confidence

The resulting statement loadings on the two factors

Table 7.2 below shows the resulting statement loadings obtained from EFA, which further provides exemplar statements used for the study. Ideally, the emergent statements present consistent views, which means that teachers were agreeing with each other on many statements. Seven statements did not load on either of the two identified factors. For example, the following statement did not load from the 'teacher confidence' category: "Confidence helps me not to stress about knowing everything". The statement that did not load from the 'understanding of mathematics' category was "Understanding mathematics enables teachers to make connections between representations".

Table 7.2: Statements loading on each of the two factors

	Mathematics teachers' identity and confidence	**Accountability towards teaching**
Since the TM1 course, I can teach learners mathematics for understanding.	.669	
Since the TM1 course, I understand mathematics.	.531	
Since the TM1 course, one can now teach mathematics with ease.	.577	
I will have to learn with others to understand mathematics.		.643
The more I understand mathematics, the more I become a better teacher.	.598	
Teachers are accountable for learners' performance in mathematics.		.639
Scripted lesson plans restrict teachers from using correct mathematical language when explaining mathematics in the classroom.		.424
Learners are to be blamed for their performance in mathematics.		.510
Learners come to secondary school with inadequate content knowledge.		.647
Mathematics is not interesting for learners.		.612
Teachers cannot change example sets because of the scripted lesson plans.		.643
It is easy to transfer mathematics knowledge to learners' minds when you are confident.	.605	
Learners identify with teachers who have confidence to be more capable than other teachers.	.666	
I have enough confidence to allow other teachers to observe my lessons.	.607	
I am hoping to be the best mathematics teacher in the future after learning about the MTF.	.664	
Since learning about the MTF, I am more conscious on how I choose examples for a lesson.	.665	
Since learning about the MTF, I see mathematics as a subject that needs a lot of strategic thinking.	.496	
I feel empowered as a teacher since learning about the MTF.	.652	
Since learning about the MTF, I feel an increased sense of being a mathematics teacher.	.830	
Since learning about the MTF, the mathematics I teach is more valued by other teachers.	.566	

One statement from the 'understanding of mathematics' category, which was expected to load to the 'mathematics teachers' identity and confidence' factor, attached itself to the 'accountability towards teaching' factor. The statement was "I will have to learn with others to understand mathematics". With hindsight, this statement overlapped to focus on both the work of a community of teachers who envisage to learning together and the benefits of understanding mathematics.

In what follows, I provide details of the two factors to support how I characterised the teachers from frequencies of their responses to the statements. Appendix H presents all the frequencies of teachers' responses including frequencies of the statements that did not load to the factors.

The 'mathematics teachers' identity and confidence' factor

The 'mathematics teachers' identity and confidence' factor had statements that were orientated towards the three groups: identity, meaning and confidence. In this factor, within the first group of statements, all the teachers agreed or strongly agreed to understanding mathematics since attending the TM1 course (meaning). All the teachers also agreed or strongly agreed that they are teaching "learners mathematics for understanding" after participating in the TM1 course. Furthermore, all the teachers were agreeing or strongly agreeing that they "can now teach mathematics with ease" since participating in the TM1 course.

Whilst in the same factor, within the second group of statements, teachers value teacher confidence in the learning and teaching of mathematics. For example, 96% of the teachers agreed or strongly agreed to the statement that "it is easy to transfer mathematics knowledge to learners' minds when you are confident." Also, 82% of the teachers agreed or strongly agreed to the statement that "Learners identify with teachers who have confidence to be more capable than other teachers." Moreover, 96% of the teachers were agreeing or strongly agreeing to "have enough confidence to allow other teachers to observe their lessons." The other teachers who were not agreeing or strongly agreeing in those three statements had selected the 'neither agree nor disagree' option.

In the last group of statements in this factor, teachers expressed a positive sense of identity towards the teaching of mathematics after learning about the MTF. For example, 96% of the teachers were agreeing or strongly agreeing to this statement that they are "hoping to be the best mathematics teachers in the future after learning about the MTF". The remaining 4% of teachers were 'neither agreeing nor disagreeing' with this statement. Also, 100% of the teachers were agreeing or strongly agreeing to feeling empowered as teachers since learning about the MTF, and 76% of the teachers were agreeing or strongly agreeing to be more valued

by other teachers since learning about the MTF. The remaining 24% of teachers in this last example selected ‘neither agree nor disagree’ option.

The ‘accountability towards teaching’ factor

The other factor named ‘accountability towards teaching’ emerged with teachers’ mixed views about accountability towards teaching mathematics. For example, 67% of the teachers were disagreeing or strongly disagreeing that “teachers cannot change example sets because of the scripted lesson plans” when teaching mathematics. Whereas, 24% of the teachers agreed or strongly agreed to not being able to change example sets because of the scripted lesson plans. In another example, 44% of the teachers were agreeing or strongly agreeing to the following statement: “Scripted lesson plans restrict teachers from using correct mathematical language when explaining mathematics in the classroom”. Whereas, 31% of the teachers disagreed or strongly disagreed with this statement.

The results of piloting the close-ended questionnaire

The results of the study resonated with those of piloting the closed-ended questionnaire. As noted in Subsection 5.3.1, 42 teachers who participated in the TM1 course in 2018 responded to 36 statements in the pilot study. Two factors were also retained from EFA. The factors were named ‘teachers’ identity and confidence’ and ‘understanding of mathematics’, and they had reliability of 0.840 and 0.703 respectively. The ‘teachers’ identity and confidence’ factor had 15 statements mainly oriented towards the two groups: identity and confidence. In the first group of statements, for example, teachers had a sense of becoming the “best mathematics teachers in the future” (identity). In the second group of statements, teachers illustrated the importance of confidence in the learning and teaching of mathematics after participating in the course. The ‘understanding of mathematics’ factor had five statements. In this factor, for example, teachers attributed their increased sense of understanding mathematics content to the TM1 course.

7.4 The selection of the four teachers from the quantitative analysis

Considering the results from EFA, what exactly did I mean by special cases in Subsection 5.3.2 when discussing the selection of the four teachers who participated in the qualitative processes? The selection of teachers was achieved from the quantitative analysis by exploring the relationship between the two factors using a scatter plot (Figure 7.1). The figure illustrates different positions of the teachers in their levels of agreement with the questionnaire statements. The lower values on the vertical axis indicate a higher sense of ‘accountability towards teaching’ by teachers given the negative statements in the group. The positive statements in the group, for example, “Teachers are accountable for learners’ performance in

mathematics", received reverse coding to maintain balance. On the horizontal axis, the higher values indicate positive teachers' identity and confidence towards learning and teaching mathematics. Thereby, from the relationship between the two factors in the scatter plot, as this claim was made in the overview of the findings, it was fitting to suggest that teachers who valued learning and teaching more also have strong views about being accountable towards teaching mathematics.

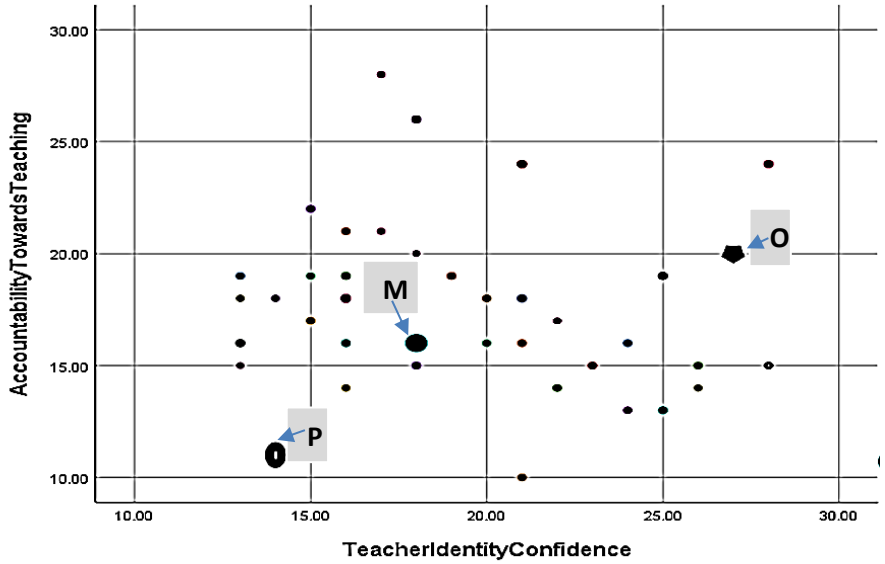


Figure 7.1: Scatter plot of 'teacher identity and confidence' against 'accountability towards teaching'

Different markers with alphabets pointing at them on the graph indicate positions of the three teachers who were initially selected and invited to participate in the qualitative processes. Mbose is located within average of both axes, and letter 'M' points at his position. This teacher seems to have a balanced view on accountability towards teaching and what the MTF has contributed to his teaching. According to McMillan and Schumacher (2010), this case can be classified as a *typical-case*.

Pandor is located at the bottom left of the graph, and letter 'P' points at his position. This teacher seems to be highly accountable towards teaching whilst his identity and confidence towards mathematics from the MTF perspective did not emerge to be strong. In McMillan and Schumacher's (2010) terms, this case can be classified as an *intensive-case*, as this teacher can be characterised as being dramatic when considering his negative sense of identity after learning about the MTF.

Oena is located just below average on accountability towards teaching, and letter 'O' points at his position. This teacher seems to have strong views on the impact of the MTF towards learning and teaching mathematics. From McMillan and Schumacher (2010), this can be classified as a *reputational-case*. At this stage, I argue that this position can be recommended as the appropriate example for teachers' mathematics teaching identity. According to Sachs

(2005) and Boaler (2000), when teachers have some level of agency in the profession, those teachers can contribute meaningfully towards teaching mathematics.

The fourth teacher (Makwe) was selected using similar processes, but I used the scatter plot for teachers who participated in the piloting of the closed-ended questionnaire in 2018. In the analysis, whereby the two factors (i.e. 'teachers' identity and confidence' and 'understanding of mathematics') had emerged, the two teachers I selected to participate in the piloting of the interview questions had a positive sense of 'identity and confidence' towards teaching mathematics and directly attributed their increased 'understanding of mathematics' content to the TM1 course. In this context, these two teachers were classified as *reputational-cases,* which had a potential to further diversify the teachers' views about learning and teaching mathematics since these teachers attended the course in 2018. As noted, I chose Makwe for the study because of his insight during the interviews. Thus, considering that the four teachers were selected for the qualitative processes, I discuss the results from their assertions of what has changed to bring about the shared experiences of learning and teaching after they participated in the TM1 course in the next chapters.

7.5 Interconnection of Wenger's components of learning

The results of the quantitative analysis confirm Wenger's (1998) position that the components of learning are interconnected for members of the community of practice. For example, the results illustrated how teachers' identity interconnects with other components of learning. That is, as we saw in the 'mathematics teachers' identity and confidence' factor, the extent of teachers' identity emerged to be closely connected to understanding mathematics whilst teachers are considering teacher confidence to be necessary for the learning and teaching of the subject. The extent of teachers' shared experiences extends in the 'accountability towards teaching' factor to include a strong accountability towards learning and teaching of mathematics.

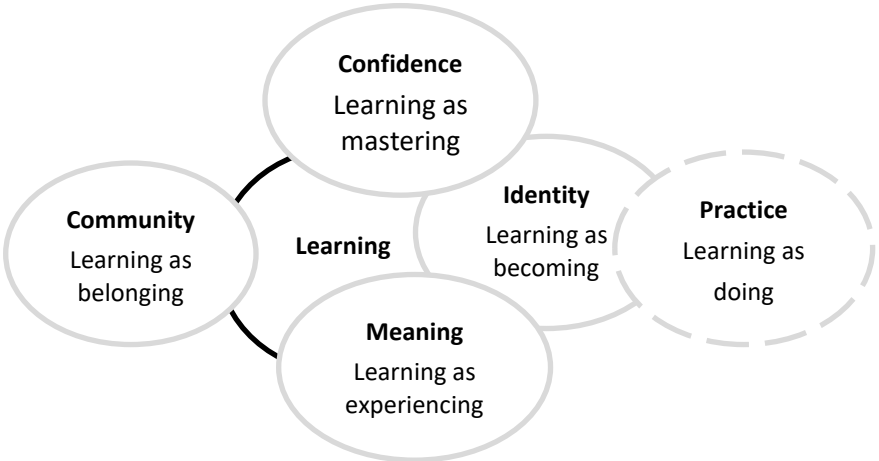


Figure 7.2: Wenger's (1998) social theory of learning for the TM1 teachers

Figure 7.2 shows how the components are interconnected for the teachers who attended the TM1 course. As noted in Subsection 5.3.1, and when considering an artefact of the closed-ended questionnaire design, practice was already linked to identity. In Figure 7.2, I located practice using a dotted line to link practice directly to identity when considering the causal effect proposition. Moreover, from the results, identity is located closer to both confidence and meaning. Community remained directly connected to learning. As much as the impact of the TM1 course may not yield the same results for other groups of teachers, this approach allows teachers' mathematics teaching identity to be viewed from where participants locate themselves in the context of being mathematics teachers within a community of practice (Kaspersen et al., 2017).

7.6 Summary

In this chapter, I presented the results of analysing the teachers' shared experiences of participating in the TM1 course. In this sense, I was able to answer the first part of the research question 1, which was 'To what extent do teachers have share experiences of learning and teaching mathematics after participating in the TM1 course'. What became apparent was that the teachers who participated in the TM1 course value learning and teaching mathematics in relation (1) to their positive sense of identity after learning about the MTF, (2) to their understanding of mathematics, and (3) to teacher confidence. The teachers presented mixed views about accountability towards teaching mathematics. For example, the teachers were not in agreement whether (or not) they can change example sets in the scripted lesson plans.

CHAPTER 8

THE CHANGES SHAPING THE TEACHERS' SHARED EXPERIENCES

8.1 Introduction

In this chapter, I discuss the results of analysing changes in teachers' shared experiences of participating in the Transition Maths 1 (TM1) course to answer the second part of the research question 1 (RQ1). The research question was 'What changes (if any) have shaped the shared experiences for the teachers?' As noted in Subsection 5.3.4, to answer this research question, the data was collected and analysed from the interviews with four teachers.

The interview data complemented the findings from the quantitative phase, which were discussed in the previous chapter. In brief though, the extent of the teachers' shared experiences of participating in the TM1 course mainly highlighted positive sentiments about 'mathematics teachers' identity and confidence' alongside their 'accountability towards teaching' the subject.

The nature of the interview questions were mainly directed to solicit teachers' learning experiences from participating in the TM1 course, particularly on what changes have shaped the shared experiences for the teachers. For example, the first broad questions during the interviews were 'Please share with me what did you like the most about the TM1 course' and 'What did you not like about the course?'. These were followed by more specific questions such as 'What changes in your teaching practices (if any) can you say are attributed to your participation in the course?'.

The details about the data analysis from the interviews are in Section 6.3. Briefly, the analysis of the narrative context was used to complement what emerged from the quantitative analysis about teachers' shared experiences of participating in the TM1 course. When drawing from Sfard and Prusak (2005), the teachers' narratives about the dynamics of learning and teaching mathematics would be their identities. However, at this stage, the shared experiences were centred around *what* and *how* the teachers understand to have changed their teaching of mathematics in relation to community, practice, confidence, meaning and identity (Graven, 2003; Wenger, 1998).

Below, I start by providing the overview of the findings on what changes have shaped the teachers' shared experiences after participating in the TM1 course. In this overview, I summarise the results that have emerged from the interview analysis. The evidence from quotations are presented in the rest of the sections on the changes (1) of learning in the TM1

community, (2) of learning mathematics-for-teaching (MfT), (3) of teacher confidence, (4) of learning about the MTF, and (5) of teachers' teaching identities.

8.2 Overview of the findings to answer the second part of the RQ1

Firstly, the interviewed teachers have conceived that participating in the TM1 community changed their learning experiences considering that it encouraged teamwork. The teachers appreciated the seating arrangement as they were seated in groups of threes to groups of fives, and they could work with teachers from other schools. Three teachers (Mbose, Oena and Makwe) commended how the facilitators of the TM1 course gave them time to engage with each other in their different groups before discussing tasks with the whole class.

In being accountable to the teaching profession, the interviewed teachers were embracing the spirit of learning and working with other teachers in the TM1 community to develop a better understanding of mathematics and to share different strategies of teaching the subject. However, as the effect of the changed learning experiences, Mbose expressed that getting learners to sit and work together in groups is not possible in his class because of time pressures. For Oena and Pandor, it was also not always possible to enable learners to work together in groups as their classes were over-crowded. Makwe expressed to have adopted certain strategies of mediating group work from the experience of attending the TM1 course including being sensitive and polite to the learners when teaching mathematics.

Secondly, learning MfT meant that, when the interviewed teachers were engaging to understand mathematical content in the TM1 course, they were also imagining how best mathematical knowledge can be taught to learners in the classrooms. The teachers recognised themselves as being proficient in content knowledge (Kilpatrick et al., 2001). However, Makwe and Pandor acknowledged that their content knowledge was deepened when learning MfT, which also boosted their confidence of teaching mathematics to learners. Ultimately, the changes of teaching mathematics to learners for the teachers incorporated the recognition of learning mathematics content (for some teachers) whilst focusing on the benefits of improving their strategies of teaching mathematics.

The teachers' ways of teaching mathematics changed as they expressed improvement in their teaching knowledge on the following aspects:

1) knowledge of how to introduce new concepts to learners;
2) importance of justify procedures to learners in terms of why and how those procedures work when solving mathematical tasks;
3) what to consider when answering learners' questions;
4) how to engage with learners' prior knowledge when teaching mathematics; and

5) knowledge of teaching across different grades.

From learning MfT, three teachers (Makwe, Mbose and Pandor) expressed to have changed how they teach mathematics to learners. For example, Makwe expressed to have changed from using teacher-centred approach (whereby learners merely listen to him explaining mathematics in the chalkboard) to using learner-centred approach (whereby learners are expected to "explore things on their own" and produce different solutions to mathematical tasks in accordance to their grades).

Thirdly, the interviewed teachers commented on how teacher confidence contributes in their learning and teaching of mathematics, particularly with an understanding that confidence emerges when teachers are proficient in mathematics. From the analysis of the closed-ended questionnaire, teachers claimed to have gained more confidence in their teaching of mathematics after learning about the MTF. During the interviews, according to Makwe, teachers with confidence have more time to concentrate on "unpacking" mathematical concepts for learners when teaching mathematics. For Pandor, teachers with confidence are able to find a way of going back and make corrections when working out examples for learners in class. For Oena and Mbose, teacher confidence comes when teachers are well-prepared for the lessons. Ultimately, for Oena and Mbose, teachers with confidence are able to think about learners' questions when planning the lessons and anticipate learner offerings when teaching in the classroom.

Fourthly, due to learning about the MTF, the findings from the interview analysis resembled similar trends to those of Ntow and Adler (2019). That is, the teachers highlighted different aspects of the MTF to have influenced changes in their learning and teaching of mathematics. In other words, the teachers did not elaborate on all the aspects of the MTF to the same degree. From the analysis, the teachers highlighted that learning about exemplification has mostly influenced their learning and teaching of mathematics, particularly in how they now pay attention in selecting and sequencing examples. The second aspect of the MTF that emerged from the analysis to have influenced teachers' teaching of mathematics was explanatory communication. For instance, Mbose explained that words used should focus on connecting the mathematical concepts by using established and laws when teaching mathematics. At the least, the third aspect of the MTF that emerged was on the lesson goal. Three teachers (Mbose, Oena and Pandor) limited their definition of the lesson goal to answer the "What learners should be able to know at the end of the lesson?' question, whilst Makwe had shifted to understand the lesson goal to be achievable when teachers consider appropriate explanatory communication and relevant learner participation. However, the teachers did not express learner participation as one of the aspects of the MTF.

Lastly, it was only Makwe and Mbose who had expressed different takes about themselves when commenting on their experiences of participating in the TM1 course. Makwe reported to have shifted from an individual characterisation of what does it mean to be a mathematics teacher to embrace social-identification. That is, before the TM1 course, Makwe was agreeing with a view that "not everybody can do maths". After the course, Makwe explained that he is now aware of the challenges that are faced by other mathematics teachers and learners in general (social-identification). For Mbose, the learning about different aspects of the MTF became central in characterising him after participating in the TM1 course. That is, the teacher saw himself as a teacher who adopts applicable example sets and uses appropriate explanatory communication in order to achieve lesson goals.

8.3 The changes of learning in the TM1 community

In this section, I report on how the interviewed teachers benefitted from teamwork when participating in the TM1 community. All teachers were expected to have gained both mathematical and teaching knowledge from learning in the TM1 community. As a result of discussing the changes of learning in the TM1 community, the interviewed teachers described their learning experiences before, during and after the TM1 course.

Teachers benefiting from teamwork in the TM1 community

From the analysis of the interview data, it emerged that the teachers liked teamwork when engaging with mathematical tasks, and they did not feel marginalised from working with other teachers in the TM1 course. The teachers explained that the facilitators of the course promoted teamwork, although at the beginning one teacher (Makwe) thought that they were going to be lectured to. The teachers described *what* they experienced in the course as follows:

> Makwe: At the beginning, like first day, you feel that everyone is there to do their own thing… but as the course progresses you see that, no, we're actually a team. Even the way she [the facilitator] would say team 1, team 2.
>
> Mbose: Another thing that I liked, the group work. Like when I came we grouped ourselves, but I was with the person that I came with, then from there after some time they grouped me with different people.
>
> Pandor: At some stage, [we would] sit in groups of people whom you know… Ultimately, we became a family.
>
> Oena: It helped us to work as a team.

Thus, as mentioned in the overview, the teachers appreciated the seating arrangement as they were seated in groups of threes to groups of fives, and they could work with teachers

from other schools. In consequence, the teachers have conceived that participating in the course changed their learning experiences, and the words underlined with the dotted lines provide evidence of the changes in teachers' learning of mathematics.

Within this theme of teamwork, I now discuss three factors that contributed towards *how* teachers were encouraged to participate in the group work. Firstly, according to Mbose, mathematical tasks themselves forced teachers to work together in their respective groups. From Mbose's elaboration, most of the mathematical tasks that were used in the course had many components to them. Mbose's point is in line with what I mentioned in Section 4.2 that open-ended tasks were used to encourage discussions in the course.

Secondly, three teachers (Mbose, Oena and Makwe) commended the strategies that the facilitators used to encourage group work in the TM1 community, which included giving teachers time to engage with each other in their different groups before the discussions were consolidated for the whole class. The teachers explained,

> Mbose: The facilitators, as we were busy solving problems, they were busy moving around, going to different groups first to ensure that people were able to do the work. Then, [the facilitators were] monitor[ing] if we were doing it correctly, then they were being helpful where we were experiencing problems. Then from there, we will also remain [in our groups], then from there, continue to interact and solve the problem according to the way we were understanding it.
>
> Oena: We were given sort of a chance to say whatever you feel or you want to do. There was nothing that sort of channeled us to say, "hey, but this one [a facilitator], I must not say this", they [the facilitators] were so open with everyone.
>
> Makwe: I love the way, even if your answer is incorrect or even if you asked it rudely or unmanneredly, she [the facilitator] had a way of neutralizing things. Even when we were arguing, like "no, no, no", [she would say] let's look at it like this, you know. I think it's one of the things that I liked most, that [the facilitators] were not talking too much, they were actually letting us make our mistakes and then rectifying it, and how she [the facilitator] would let us argue amongst ourselves so that you have a clear understanding.

Mbose's comment emphasised that the teachers were given space to interact with each other whilst engaging with the tasks. Oena echoed what I mentioned when describing the roles of the TM1 community members in Subsection 4.2.2 that it was a safe space for the teachers to freely express their thoughts to the facilitators, to the research fellows, and to each other. Makwe agreed to the sentiment about the safe space, and further expressed that the facilitators were sensitive towards them. Furthermore, as underlined in the comments, it can be noted that the teachers used the learning space to make sense of mathematics on their

own. In Section 8.4 below, I link the point of understanding mathematics with the discussion on how the teachers learned mathematics-for-teaching in the course.

The last factor that contributed to learning mathematics in the TM1 community talks to the social nature of human beings, whereby the engagement around an enterprise promotes unplanned community formations (Lave & Wenger, 1991). Teachers in the TM1 community were not always waiting for the facilitators to endorse ideas of working out mathematical tasks or to show them how certain topics could be taught in the classrooms. The teachers elaborated on how they shared ideas of learning and teaching mathematics as follows:

> Oena: We were supposed to discuss amongst ourselves, and that thing did help us a lot because you could sense hey "[Oena] is saying this", but then I didn't think of this.
>
> Makwe: We used to share ideas, like [the other teacher would say] "in my class I say this", oh brilliant. I'm going to use that.
>
> Mbose: Teachers, we were working in groups then we could share ideas, listen to how other teachers are solving certain problems that they encounter and even the teaching methods. So, they will tell you, "this is how I teach or I approach this topic".
>
> Pandor: …as a family, you know whatever the hiccups that comes in mathematics, at some stage we need to sit down together and say what's happening? What is wrong exactly? How can we assist?

The questions that are asked by Pandor above relate to how they would identify teachers who are struggling in the course amongst themselves to offer assistance. The teachers confirmed that the sharing of ideas continued beyond the formal structure of attending the TM1 course. Pandor mentioned that belonging to the TM1 community becomes useful when teachers are attending other professional development interventions, and "[i]f the things are not addressed correctly you are able to stand to say "no, this is not what we are supposed to be doing" and then being backed by others".

Effects of learning in the TM1 community

What are the effects then of learning from other teachers, particularly from how teamwork was promoted in the TM1 community? In other words, what did teachers take from their experiences of learning in the TM1 course that can be implemented in their teaching of mathematics? As highlighted in the overview, Mbose mentioned that it was not possible to implement group work in his teaching. That is, as much as he appreciated the group work when learning mathematics, and agreed with the statement from the closed-ended questionnaire that 'Mathematics is interesting for learners', Mbose argued that he just does not have time for group work in his classrooms. The teacher gave the following details:

> From that [teamwork] experience [in the TM1 course], I can also show learners that they can work with any people that I can group them with. They can work as members of a group because it's very very important. They can share ideas. In those groups there will be peer teaching. There will be, learners will learn to work together. They will solve certain problems, and they will learn to exchange ideas. Maybe come up with different methods of solving one and the very same problem, but for now that is not happening, especially in strategy [Annual Teaching Plan], it's not… it's not happening. So as much as I've learnt that. That actually you can work with any people as members of a group but so for that, in strategy [Annual Teaching Plan] that is not possible.

From the lesson observations, group work was not regularly happening in the classrooms of two other teachers (Oena and Pandor). Oena preferred moving the learners to a school hall whenever I was scheduled to observe his lessons because learners were over-crowded to capacity in their classroom. In Pandor's classroom, it was overwhelming to observe the lessons whilst learners were literally taking turns to sit on top of each other. In Pandor's classroom, there was no space to move between desks or to add more desks and chairs to accommodate learners who were standing during the lesson.

Makwe expressed to have adopted certain strategies of mediating group work from the experience of attending the TM1 course. These strategies included being sensitive and polite to the learners when teaching mathematics. The teacher commented as follows:

> I also took that from her [the facilitator] because sometimes also as educators, you're in class, a child says a wrong answer, you get frustrated and, "no, where you getting that now?" …I took that also from the training that even if the kids are wrong, don't discourage them to participate, just show them where their mistake is.

The other strategy that Makwe adopted talks to the importance of coordinating group work in order to leave learners with no confusion. That is, for Makwe, as Mbose also alluded to, learners can be given the same question, but each group can be expected to present their solutions using different methods. Additionally, according to Makwe, learners need to be given time to discuss in their groups before teachers intervene or open discussions to the whole class. Regarding the whole class discussion, Oena mentioned that he wants learners to be free and be in a position to say, "hey… you have made a mistake" there or to say "start afresh, I didn't hear what is it that you're saying" when teaching mathematics.

8.4 The changes of teaching mathematics to learners

In this section, the analysis of interview data illuminates changes of teaching mathematics to learners as influenced by learning MfT. As mentioned in the overview, from the analysis, teachers acknowledged that learning MfT improved their mathematics teaching knowledge.

From the analysis of the closed-ended questionnaire, teachers were agreeing to the importance of understanding mathematics for themselves and for teaching the learners. Therefore, the assumption is that teachers who have learned MfT become better equipped to teach mathematics for understanding.

Teachers' improved teaching knowledge

Makwe and Pandor felt that the TM1 course had improved their teaching knowledge. For example, when the teachers were asked to share their experiences of participating in the course, Makwe and Pandor stated the following:

> Makwe: …you say that I know maths, what am I going to do there [in the TM1 course], there's nothing new I'm going to learn. But once you're there, [you realise that] it's not like you're learning how to solve mathematical problems. You're learning how to explain it easier, unpack it to learners… because we know it, the main thing is how do you explain it to the learners.
>
> Pandor: [The TM1 course] was not just theoretical, it was not like when you're furthering your studies, where you're… developing an individual without developing the learners. In this instance, [the course] was [about] how can you develop the learners.

On the one hand, the teachers were confident in being proficient in mathematics (Kilpatrick et al., 2001) at the time when they were learning how to teach mathematics. For instance, Makwe explained that before attending the TM1 course, he was concerned about what the facilitators of the course were planning to do with them considering that he knew mathematics and wondered what was new to learn from the course.

On the other hand, as we see in the comments, both Makwe and Pandor felt that the TM1 course was about (amongst other things) developing learners through their improved teaching knowledge. Makwe described *what* was learned in the TM1 course to include how to explain certain mathematical concepts to learners. Makwe's comment is in line with Ball et al. (2004) who asserted that explaining mathematical concepts means unpacking those concepts to make mathematics learning accessible to learners.

From teachers' improved teaching knowledge, the other impact of developing learners was on how to introduce certain topics when teaching mathematics. For example, the teachers explained

> Mbose: Then another thing that they emphasised there is when you introduce the topic, they showed us the better ways of introducing a topic in that course. Because sometimes we start solving certain variables while we must start with the language of mathematics.
>
> Makwe: But when you go through the programme like [the TM1 course], they show you how to introduce a topic to learners. They show you how to make it easier for the learners and you also gain so much knowledge that you're comfortable that if they ask me any question, I'm

> ready for it. Unlike sometime us teachers we block the question part, we waste so much time on teaching what we know, teach what we know, and look at the time so that they don't ask questions, you're leaving the class already.

For Mbose, better ways of introducing a topic hinges around 'words used' that are not fixated to reading symbols in order to solve algebraic problems, but appropriate "language of mathematics". Elsewhere in the interview, Mbose elaborated on language of mathematics to include defining mathematical concepts that will be in focus for the lesson. For Makwe, the better way of introducing a topic involves dealing with learners' conceptions and challenges when learning a concept. In turn, when teachers have knowledge of how to organise learners' first encounters with concepts (Huillet, 2007; Pournara, 2013), they can engage the learners in answering their questions during a lesson. Furthermore, when drawing from the MTF perspective, teachers can be comfortable to entertain learners' questions (and not block them) if they incorporate in their lesson plans what the learners are going to say, do and write in the lessons.

The other aspects of developing learners include "knowing that it's not only about a certain [mathematical] procedure" (Makwe, during the interview) that can be used to answer the questions. According to Makwe, teachers need to justify mathematics to learners as to why they are doing what they are doing. In Huillet's (2007) term, teachers need knowledge of how to justify procedures to learners. In the TM1 course, mathematical proofs were promoted as the highest level of justifying how and why procedures work. Whereas, the question of whether "Showing correct answers is as important as justifying how and why mathematical procedures work" remains unanswered as the majority of teachers who completed the closed-ended questionnaire agreed or strongly agreed to this statement – I highlight this question again in Subsection 10.4.4.

Furthermore, Makwe explained that justifying the procedures could "bring mathematics to life" for learners. For Mbose, incorporating information and communication technology (ICT) when teaching mathematics, like when using 'smart boards', is one way of bringing mathematics to life. Pandor uses additional examples to bring certain mathematical procedures to life. The notion of bringing mathematics to life implied that teachers could connect different concepts for learners in their explanatory communications and in the mathematical tasks, which I defined as teaching learners mathematics for understanding.

Effects of learning mathematics-for-teaching when teaching

It emerged in the previous section that learning in the TM1 community contributed in how teachers learned mathematics for understanding. Therefore, as mentioned in the overview, from learning MfT, the question is, what did the teachers take from the course that they can

say contributes to teaching for understanding in their practice? From the analysis of the closed-ended questionnaire, the interviewed teachers also agreed to being determined to "teach learners mathematics for understanding", and attribute their determination to having participated in the TM1 course. This synthesis of the data demonstrates that the four teachers understood teaching for understanding to mean connecting learners' knowledge of mathematics across different grades. For example, Makwe commented on the importance of teaching mathematics across grades as follows:

> Because again, it's what I saw with the training [the TM1 course]. I thought I would have problems because I'm only teaching 8 and 9 and it was my first time teaching grade 10 because of the training now. Then I feared that they're going to do grade 12 things and I'm not familiar with it, I'll be all lost. But she [the facilitator] would move from 9 all the way to 11. So, she would move with you, so that's what I was trying also to apply that, don't panic, let's work together.

The teacher stated that when he teaches the topic, he connects what learners did in previous grades in order to get learners to relax and say, "we have done this before. This is just the grade 9 way of doing it now." The teacher used the example from the starred lesson to say that he explains to learners that "a letter [or a variable] has been added now and we must solve it in a different way now", but it is nothing new as learners would have seen the concept of solving linear equations before. The teacher emphasised that often learners panic as they are quick to say "maths is difficult" or ask "what is this topic now?"

Two teachers strongly concurred with Makwe's assertion that learners are quick to say "maths is difficult". Although all the three teachers demonstrated knowledge of how to explain and justify the key techniques and procedures in a way that takes learners' prior knowledge to account (Huillet, 2007), as mentioned in Section 1.1, two teachers (i.e. Oena and Pandor) were concerned with how learners are 'pushed' from one grade to the next. Their concern tied with the closed-ended statement that the teachers agreed on to say, "Learners come to secondary school with inadequate content knowledge". Here are the comments from the interviews regarding teachers' concerns about teaching across grade.

> Oena: If you teach your learners, if you know your story, your learners will always pass at the end of the year, but then they have got that mentality of saying hey, "this content subject is difficult", and at the same time, they progress learners all the way from grade 6, it's a problem.
>
> Pandor: But the unfortunate part is we, it's frustrating, it's demoralizing sometimes when as I said before that after all the efforts and then suddenly… let them [learners] all pass.

Pandor was venting his frustrations about grade 10 learners who are 'pushed' from grade 9, and according to the teacher, both learners and teachers fail to cope when going forward.

Makwe expanded on how he is teaching Grade 9 mathematics for understanding. In essence, the teacher drew his inspiration from some of the learning tools and resources that were used in the TM1 course. As stated in Subsection 4.2.3, this is the teacher who enjoyed working with 'the big grid', and expressed his experiences as follows:

> I loved how we used different models. We'd have strings there or triangles, using them for a lesson. The graphs, we'd have graphs…
>
> That's what I'm doing right now also. I have models now where they [learners] actually see things, especially the 3D topics, you know, on shapes.
>
> So, back then you just had in paper that this is a cube, it has so many sides. But now… I wasn't even aware, we get materials for mathematics, even with probability we get dices that they can use instead of you saying, "OK a dice has sides", you know.

By "models", the teacher was referring to the physical objects of learning and teaching mathematics. In the course, mostly in trigonometry, paper cuts were used to illustrate and measure angles of different shapes, including those of triangles. In another instance in the course, a piece of string was used as part of inclinometer to measure angles of elevation and depression. About "the graphs", the teacher was referring to working with 'the big grids'. 'The big grids' were described in Subsection 4.2.3.

The teacher highlighted how he did not pay attention to using different learning tools to teach certain topics before attending the course. The teacher claimed that using different learning tools to teach for understanding became an integral part of his teaching of mathematics. In order to achieve teaching for understanding, the teacher discouraged using the chalkboard as the only teaching tool and promoted a "landscape of investigations" (Skott, 2013). Makwe is quoted saying the following:

> let them [learners] explore things on their own, let them sit in groups and you say, you guys calculate this, you guys calculate that and then move around and you find out what they are doing wrong, why are they saying this.

This approach has led the teacher to highlight that "at least while you're teaching you can still see that ok, they're missing this, they don't know what's happening there and there." According Makwe, when he was only using the chalkboard in his teaching, he could only discover during tests that the learners did not understand what he was teaching in the classroom.

Makwe and Pandor confirmed that gaining mathematics knowledge for teaching boosted their confidence of teaching mathematics to learners. Makwe attributed his ability to teaching Grade 10 mathematics to the teaching material that he received during the TM1 course. The teacher was referring to the learning material that was directly used with teachers during role-playing as teachers would now and then be asked to pretend to be learners themselves to

demonstrate the teaching of mathematics in the classroom. Furthermore, Makwe mentioned that he was ready to teach grade 12 mathematics. When Pandor was responding to “How is the understanding of mathematics important for you?” during the interview, he said, “Look… for teaching especially. For teaching and above all, you know, with the confidence in me, I know for a fact that whatever that I do in class, it’s a right thing.”

8.5 The boosted confidence after participating in the TM1 course

The discussion in this section highlights how the notion of confidence in learning and teaching mathematics has contributed in shaping teachers’ shared experiences of participating in the TM1 course. From the analysis of the closed-ended questionnaire, teachers claimed to have gained more confidence in their teaching of mathematics after learning about the MTF. Insofar, it has emerged from the analysis of this interview data that the teachers gained some level of confidence from learning in the TM1 community and because of their improved mathematics knowledge for teaching. In the next subheadings, I discuss disadvantages of teachers without confidence, whilst later, I elaborate on sources of teacher confidence that could have been achieved by teachers after participating in the TM1 course.

Teachers without confidence

The four teachers commented a great deal on the issue of teachers without confidence. It was not always clear when analysing teachers’ comments about teacher confidence whether they were drawing directly from their experiences of attending the TM1 course or merely talking from other observations. In other words, I cannot always claim that teachers’ boosted confidence (i.e. change in their confidence) was directly due to attending the TM1 course. For example, Makwe shared the experiences of attending trainings that are provided by the Department of Basic Education, and stated the following:

> Starting with attending the trainings that are provided by the department, we avoid them because you’re going to write a pre-test. So, if you don’t have any knowledge, you’re going to get zero. And now you’re scared that people will say, what is [he] teaching in class if [he] is getting zero, you know?
>
> But when you have the knowledge you have the confidence to attend those [departmental trainings] because you know that, I know how to do it [mathematics], I just need to know how to implement it for the learners.

In this extract, despite the fact that I cannot make any inferences about teacher confidence in relation to attending the TM1 course, Makwe is however acknowledging that at times he needs more mathematical knowledge about how to teach that knowledge to learners.

On the contrary, Oena confirmed that the TM1 course contributed to some degree to his boosted confidence. Oena explained that when he did not have confidence, he would make excuses when district officials were arranging to visit his school the next day and observe lessons. Oena shared the following story:

> I will make some scapegoat to say, tomorrow I won't be in. But now after attending all these workshops, especially Transition Maths 1, it helped me, in such a way that, I'm confident in everything. When somebody says to me, I will be visiting you tomorrow… I will be ready for that… I will make it a point that I sit down and do the things that I normally do, and prepare for the topics that I will be teaching. I am not afraid. adopt

In another similar example, Makwe mentioned that teachers without confidence can avoid attending to all the classes. Makwe claimed that teachers without confidence act like learners by skipping some classes because they do not believe in themselves. Reading Makwe's comment, and relating it to Essien's (2014) perspective of describing identity as *becoming*, teachers without confidence are often required to adopt learner identity by having to struggle to do *learner* tasks alongside the learners. Here is the teacher's story.

> Let's just say teachers without confidence are the same as that you're part of the learners. What do I mean by that is? You know how learners bunk classes, neh? If you're a teacher without confidence, you'll also bunk classes because you're not confident in what you want to do, you don't believe in yourself. And again, if you don't believe in yourself then learners will not believe in you, they'll also doubt, is she sure we must do that? They'll end up leaving the class with uncertainty, is it really like this, you know. Then they come back with another textbook tomorrow and say but this textbook says this, and you can't say, no it's also correct because you don't have the confidence, you don't believe in yourself, meaning you don't have the knowledge. So, you end up saying, let me look at it see, maybe I did it wrong it must be like this, only to find out they're both correct but they are using different methods, you know.

When teasing out more from Makwe's quote, teachers with confidence believe in themselves, and they have mathematical knowledge of recognising multiple methods of teaching mathematics. These points tied with a statement in the closed-ended questionnaire where teachers agreed to not being "afraid to grapple with mathematical tasks because of confidence".

Teacher confidence in learning and teaching mathematics

The assertions from the interviewed teachers about teacher confidence are not new in the literature (see Graven, 2003, 2004). Teachers can "avoid" interacting with other community members (i.e. learners, other teachers, school management bodies, district officials) when they lack confidence. They become "afraid" of being judged about not knowing everything as that may reflect on their capabilities of teaching the subject (Graven, 2003). From Makwe's

quote, when teachers cannot readily do learner tasks from different textbooks, and make connections using different methods, they begin to doubt themselves and further lose confidence. It is a predicament, whereby teachers cannot make themselves available to learn mathematics or fulfill their professional duties because they are not sure about what they know or what they need to learn to improve their teaching knowledge.

The four teachers commented on how teacher confidence contributes towards the learning and teaching of mathematics, particularly with an understanding that confidence emerges when teachers are proficient in mathematics. According to the teachers, learners take teachers with confidence seriously. If learners see that a teacher has confidence in what he is doing, they take the learning of mathematics seriously. In turn, as Makwe explained, the work of teachers with confidence becomes easy because learners are able to follow their instructions. Makwe stated that "[i]f I have confidence, I know I can just say, do this, this and this, unpack it for them and they understand, then I have time for that learner that doesn't understand".

Makwe further mentioned that teachers with confidence cover all aspects of different topics during mathematics lessons. For Makwe, covering all aspects when teaching means the inclusion of tasks of higher cognitive levels. Here are the teacher's comments in this regard.

> You're willing to teach any question in that topic because sometimes we are so selective when we don't understand. We select only the easy questions that you know you'll be able to answer the questions if they say, why is it 6 now, why, what happened to…? But we exclude the difficult ones because we don't have the knowledge of them also ourselves, you know. It becomes difficult when you are learning with the learners because now there will come a point where they're asking something and you say, uuuhhhmmm let's look at it tomorrow and they lose confidence in you, ma'am doesn't know maths. But when you have that knowledge it's easy.

Makwe further explained that teachers with confidence do not spend time worrying about messing up when teaching. Even if a teacher misses a step when teaching, Pandor added that he or she finds a way of going back and make a correction – he or she does not "surrender quickly". Oena and Mbose further stated that teachers with confidence tend to achieve what they want to achieve at the end of their lessons, particularly for teachers who are proficient in mathematics.

Mbose took a step back when explaining what makes teacher confidence important, and asserted that teacher confidence comes when teachers are well-prepared for the lessons. Here is his explanation.

> For me, it's what makes the teacher to be confident?... What makes me as a teacher when I go to class to be confident it's when I am well-prepared. I know that, I know the concept. Whatever

> questions that may be asked by learners I will be able to answer. Because if I'm not well-prepared, I'm scared that they shouldn't ask some questions because I might be exposed that I'm not like well-prepared.

From Mbose's comment, I describe being well-prepared as having taken time to think about what questions learners would ask (about which concepts) when planning the lessons. For Oena, being well-prepared means having thought about different ways of explaining the same concepts to learners.

Oena and Pandor also saw teacher confidence to be important when planning the lessons with other mathematics teachers. For the two teachers, again, confidence starts from being proficient in mathematics, and then individual teachers become confident enough to contend their positions as mathematics teachers, which connects to their mathematics teaching identities. For example, when Oena and his teaching team are planning mathematics lessons, and because Oena is a Head of Department (HOD) in his school, he would contend his positions knowing that he is "a maths educator" and not because of his seniority position. On the other end, Pandor commented on being able to stand his ground when he is disagreeing with his HOD because of confidence.

Lastly, the interviews revealed that teachers gained confidence of teaching mathematics after learning mathematics-for-teaching. Reading Makwe's comments, teachers with confidence have more time to concentrate on "unpacking" mathematical concepts for learners when teaching mathematics. It appears to me that the main aspects of unpacking mathematical concepts requires teachers to use appropriate explanatory communication. As it came out from Mbose and Oena, teachers with confidence are able to think about learner questions when planning the lessons and anticipate learners' offerings when teaching in the classroom.

8.6 The changes due to learning about the MTF

This section discusses the teachers' perception of changes that was due to learning about the MTF in the TM1 course. The discussion focuses on what aspects of the MTF (if not all) have contributed in the teachers' perception of changes of teaching mathematics. In Ntow and Adler (2019), it emerged that teachers pick select certain aspects of the MTF as their main take away when expressing their performance of teaching mathematics. For example, one teacher in Ntow and Adler (2019) identified strongly with exemplifying in the MTF as she could control her selections, whilst another teacher was strong in inviting learners to participate in her lesson.

Similarly, when analysing teachers' improved mathematics knowledge for teaching in Section 8.4, the interviewed teachers drew on different aspects of the MTF depending on what they

were focusing on when discussing their teaching in the classroom. In the analysis of the closed-ended questionnaire, teachers agreed that learning about the MTF contributed to improving their knowledge for teaching, to gaining confidence in their teaching, and to developing positive teacher identity. Thus, in this interview analysis, I provide evidence of the changes as experienced directly by the four teachers when learning about the MTF, which are experiences of learning about exemplification, explanatory communication, learner participation and the lesson goal. This should begin to describe how those teachers understand the MTF when they elaborate on how they implement its teaching aspects in the classrooms.

Experiences of learning about exemplification

Three teachers (Makwe, Pandor and Mbose) commented on the impact of exemplification in their learning and teaching of mathematics. The three teachers drew on their past experiences of using some arbitrary selection of examples from textbooks to clarify the current effect of selecting and sequencing examples when teaching mathematics in the classrooms. In the next comments, I have underlined the wordings such as "before the training" and "after the training" to highlight evidence of change in the teachers' experiences of participating in the TM1 course.

> Makwe: Before the training [the TM1 course], I would just look at the textbook and say, 'Do all the questions, Exercise 1', thinking that you are helping the learners, but after the training, now I know that they don't have to do the same thing over and over again to get it.
>
> Pandor: Before I enter into the course, I was just planning. Even the examples were just general. And you simply say… it was a textbook method. I'm doing equations now. After I've done one or two, as to which ones I'm teaching there, even the sequencing, it was not important.
>
> Mbose: Okay, in the course that we did, I never really thought about the examples that I was using. I was simply taking the example that was written on the textbook, not checking if the example is appropriate at that level for the learner.

Mbose explained that he now checks if the examples are appropriate for the learners' grades. Additionally, Makwe explained that not expecting learners to do "the same thing over and over again" creates space for him to select tasks that are necessary to bridge learners' mathematical knowledge across different grades. The created space also accommodates learning of diverse groups of learners as, according to Makwe, "there'll always be those two thirds of learners who are forward or quick to do things" when learning mathematics.

To end discussing exemplification, Makwe had mentioned that he informed the learners about what he had learned in the TM1 course regarding the selection and sequencing of examples for tasks. Makwe commented as follows:

> Like I said, even the selection of questions, I remember coming back to the Grade 10s and I said, “for tomorrow’s homework you’re not doing all the questions.” They said we must not give you all. You’re doing a), d) and f) and they were like, “yes, thank you to the training” [laughs].

Sharing information with the learners kept the teacher in check with how he selects and sequences examples in his teaching of mathematics. From the teacher’s comment, the learners were grateful. I can add to say, more often than not, learners feel empowered when teachers occasionally share information with them that is not only related to the content of mathematics (Boaler, 2000).

Experiences of learning about explanatory communication

The second contributor of learning about the MTF when teachers were participating in the TM1 course talked to explanatory communication. Two teachers (i.e. Oena and Mbose) compared their experiences before and after participating in the TM1 course as they commented on the importance of using appropriate mathematical language when teaching the subject. The teachers elaborated as follows:

> Oena: To me it [the course] was informative because when we’ve got a lot of years teaching the subject you think now we are experienced, whereas you don't use the mathematical language that is supposed to be used in the presentation of the lesson. And then to attend this [course] we managed to know exactly what is it that you must do when you teach mathematics, there are concepts that we are supposed to use in teaching the subject or the content.

> Mbose: Okay, in the Transition Maths 1 course, the terminology that we use in mathematics, I found out there that it’s very very important because prior to me going to that course, I just used any term that I saw was suitable at that time. But I’ve realized in that course that terminology can have a way of making learners understand something.

As noted, I used this Mbose’s quote to exemplify the coding of data in Section 6.3. Nonetheless, Mbose realised from attending the course that, sometimes the way you put it in words, learners can understand mathematics. However, depending on the words used, learners might not “understand it the way you want them to understand it” (Mbose, during the interview). Mbose further argued that teachers need to be conscious about which words they use in order to be “on the same page with the learners”. For Oena, words used in teaching mathematics should focus on connecting the concepts. By concepts, Oena means using previously established principles such as the distributive law as legitimating criteria.

Experiences of learning about the lesson goal

The last aspect of the MTF that the teachers commented on during the interviews related to the lesson goal. On the one hand, and due to participating in the TM1 course, Makwe claimed

to have shifted from thinking about the lesson goal to merely answer the 'What learners should be able to know at the end of the lesson?' question. Makwe explained that

> …looking at the way they do their [TM1's] lesson plans. With us, you just say "I'm going to teach this, we never plan what we want to say, what we want the learners to do, what's the goal of this lesson. We just say ok, by the end of the lesson I want them to know linear equations, that's it, you know. But when you get there [in the course], they've brought in everything they made it this nice bigger picture... So, that also helps.

Apparently, by everything, which is underlined in the quote, Makwe argues that teachers need to think about what they are going to say (explanatory communication) and what they want learners to do during the lessons (learner participation) when planning the lessons. Because of participating in the TM1 course, Makwe envisaged lesson plans that show "this nice bigger picture".

On the other hand, two teachers (i.e. Mbose and Pandor) defined the lesson goal as "what do you want to achieve at the end of the lesson" (Pandor, during the interview). Pandor saw examples as an essential feature in achieving the lesson goal. Here are the teacher's comments in this regard.

> When I could have, in the olden time I could have made an example on equations without brackets and equations with brackets and equations with fractions or miss one and then when I give them an activity, the activity is going to be general. Then I say, these are equations, you understand. …when they don't execute, I become so impatient [because]… I did equations.

In this quote, the teacher is explaining that the type of examples needs to bring to focus the lesson goal. For Pandor, whatever that you want to achieve at the end of the lesson depends on what you have exemplified to the learners when teaching mathematics. "When I teach something, I'm expecting to get what I taught" and not "what I did not teach" (Pandor, during the interview). In short, the learners might struggle to comprehend the concepts when the selection and sequencing of examples are out of focus (Adler & Ronda, 2015).

Mbose explained that the next important thing to achieving the lesson goal is being cognizant of learners' prior knowledge. Mbose stated that "I have to know, if learners have enough prior knowledge for us to continue with the problems that we have to do" before proceeding to connect such prior knowledge to the concept or topic for that lesson.

8.7 The changes in teachers' teaching identities

This section discusses identities of teachers from the context analysis, which emanate from learning as experiencing (Wenger, 1998). That is, this section discusses the changes in how the teachers perceive themselves in relation to other mathematics teachers after participating

in the course. From the closed-ended questionnaire, the teachers were agreeing to statements that learning about the MTF has elevated them to become better mathematics teachers. Therefore, like in the previous sections, I start by looking for evidence where the teachers are comparing their experiences before and after participating in the TM1 course.

However, as explained in Section 2.8, what teachers think about themselves in relation to other teachers after they had participated in a professional development can vary from one teacher to another. In general, most people shy away from expressing what they think about themselves when compared to other people. In Section 3.4, when discussing learners who are 'good at mathematics', most learners did not recognise themselves in their own descriptions (Darragh, 2015a). In this case, as mentioned in the overview, the findings showed only Makwe and Mbose expressing different takes about themselves in relation to other teachers when commenting on their experiences after participating in the TM1 course. Oena and Pandor did not express what they think about themselves in relation to other mathematics teachers after participating in the TM1 course. In Chapter 11, I reflect on these findings when connecting the teachers' learning in the TM1 course and their mathematics teaching identities. In the next subheadings, I discuss the changes of teacher identity case by case, starting with Makwe.

Makwe's teaching identity

As mentioned in the overview, Makwe explained that he is now aware of the challenges that are faced by other mathematics teachers and learners in general. According to Makwe, when "people think you're the intelligent one" because you teach mathematics, he would agree to such sentiments, and say to himself, "not everybody can do maths, you're intelligent". Makwe experienced a turning point as a teacher considering that learners were underperforming in mathematics and suddenly he was blamed for making learners to fail Grade 9 because they are failing mathematics. The teacher elaborated on what has changed in this regard:

> But after the training, I saw that [mathematics] is just another subject. We must stop personalising it as "maths, the difficult subject". I'm not the intelligent maths educator. I'm just another educator of a subject that learners find it challenging.

The comment provides a strong evidence of the teacher's shift in identity from an individual characterisation on what does it mean to be a mathematics teacher (i.e. self-identification) to embrace social-identification (Lerman, 2001; Sfard & Prusak, 2005; Sfard, 2019). Viewing learning and teaching of mathematics from the social perspective takes away unnecessary 'weight on teachers' shoulders' when being aware of the challenges of learning and teaching mathematics. Thus, as learners find mathematics to be challenging, and according to Makwe, teachers can begin to worry about how to help learners to deal with such challenges.

Mbose's teaching identity

Mbose asserted that he has "learned more" about teaching mathematics in terms of the explanatory communication and exemplification aspects of the MTF after participating in the TM1 course. Regarding explanatory communication, the teacher stated that he "learned the importance of knowing the [appropriate mathematical] words, and of being careful with the words" he uses when teaching. In this sense, Mbose can be characterised as a mathematics teacher who uses appropriate mathematical language when teaching mathematics.

About the exemplification, Mbose elaborated as follows:

> The importance of the examples that you're selecting, because there are certain examples that the Prof [the facilitator of the TM1 course] there showed that, okay, they were selected but for Grade 8 learners, those examples were not really good.

The teacher linked exemplification to knowledge of learners' conceptions and difficulties when learning different concepts at different grades (Huillet, 2007). Thus, Mbose can also be characterised as a teacher who is now conscious in how he select and sequence examples in his teaching of mathematics.

8.8 Summary

In this chapter, I presented the findings on the changes shaping the teachers' shared experiences of participating in the TM1 course. In the findings, the teachers expressed mainly to have:

- benefitted from learning in the TM1 community;
- improved their teaching knowledge after learning MfT; and
- developed understanding of teaching mathematics from the MTF perspective.

Although it was not always clear from the analysis that the teachers gained confidence from attending the TM1 course, the teachers linked teacher confidence to their improved teaching knowledge.

It was only Mbose who directly expressed the recognition of newly developed understanding of exemplification and explanatory communication when teaching mathematics, which became part of his mathematics teaching identity. However, in the next chapter, I provide the findings from the observations on how the four teachers are teaching mathematics, and then in Chapter 10, I characterise their actions from the content-based narratives as their mathematics teaching identity.

CHAPTER 9

THE TEACHERS' MATHEMATICAL DISCOURSE IN INSTRUCTION

9.1 Introduction

In this chapter, I present the results on the teachers' mathematical discourse in instruction from their teaching of mathematics in the classroom. In other words, I present the results of the analysis of the four aspects of the Mathematics Teaching Framework (MTF) (i.e. exemplification, explanatory communication, learner participation and the object of learning) from observing the teachers' lessons. The results sought to expand (confirm or question) what had emerged when the teachers were talking about their experiences due to learning about the MTF in the Transition Maths 1 (TM1) course, which was discussed in Chapter 8.

In Chapter 8, briefly, we saw when analysing the teachers' experiences of participating in the TM1 course from the interviews that the teachers identified with the three aspects of the MTF. Exemplification was prevalent in what the teachers described to have influenced their teaching of mathematics. The second aspect that the teachers identified with was explanatory communication. Lastly, the teachers identified with the object of learning. The teachers did not identify with learner participation to form part of the MTF.

As mentioned in Subsection 5.3.3, when discussing the purpose of collecting data using lesson observations, I focused on what the teachers were writing on the chalkboard and communicating to the learners. As explained in Section 6.4, the analysis of the observation data focused on the first lesson out of the four lessons. In brief, I argued that analysing the first lesson which focused on solving algebraic equations using additive and multiplicative inverses formed the basis of drawing conclusions about other types of algebraic equations and their general teaching of mathematics.

The analysis of the observation data was to contribute directly in explaining the teachers' mathematical discourse in instruction. In Section 6.4, I detailed how the data was analysed using the Mathematics Discourse in Instruction (MDI) framework (Adler & Ronda, 2015). The details included how the data was divided into sub-units and coded for analysis. The teachers' mathematical discourse in instruction meant the teachers' competences of teaching from the MTF perspective, particularly in deciding whether (or not) they achieved the lesson goals. The decisions on whether the teachers achieved the lesson goals (or not) were based on their incorporation of exemplification, explanatory communication and learner participation when they were actually teaching mathematics in the classroom.

Below, I provide the overview of the teachers' mathematical discourse in instruction. That is, I provide a summary of the results that are based on the analysis of lesson observations from the four teachers. The results of each teacher are presented in the rest of the sections of this chapter.

9.2 Overview of the teachers' mathematical discourse in instruction

During the observations, three teachers (Mbose, Pandor and Oena) had used examples directly from the scripted lesson plans when teaching Grade 9 mathematics with no changes. As we shall see in the analyses, Oena used all the examples for his lesson. Pandor excluded three examples in his example set for his lesson, whilst Mbose used the scripted lesson plans from the 2018 GDE ATP document to include all the examples. As mentioned in Subsection 5.3.3, Makwe was re-teaching the topic of solving linear equations using additive and multiplicative inverses, and he selected and sequenced the examples from different resources including textbooks. Nevertheless, in analysing the examples, the four teachers had offered the learners opportunities to experience both similarities and contrasts in accordance with the MDI framework – the evidence of this claim is presented in the results of the analysis of exemplification per teacher in the next sections.

However, the mathematical tasks that were used by the four teachers in the lesson remained at low level when described using the MDI framework. As we shall see when looking at the example sets in the next sections, the main tasks in the scripted lesson plans (and those tasks that were used by Makwe) required learners to solve the linear equations using additive and multiplicative inverses. In order to solve the linear equations, the teachers were firstly using additive inverses and sometimes they were using the 'transpose method' to collect like terms. Then, they were using multiplicative inverses to isolate the variables to get the solution. Thus, the results showed that the teachers were focusing on teaching known procedures.

Regarding explanatory communication, it emerged that three teachers (Mbose, Oena and Pandor) were moving between the manipulation of symbols (which included reading strings of symbols in few instances) and using mathematical words appropriately when naming mathematical objects. Such naming of the objects led the three teachers to use legitimating criteria beyond single cases when teaching mathematics. However, Oena was mostly using the general principles partially, which meant that he was not referring to mathematical principles in some instances when justifying mathematics to the learners. Whereas, Pandor and Mbose were inclined to using the general principles fully. Makwe had remained on non-mathematical words and the manipulation of symbols with an inadequate shift to using mathematical words appropriately. The results of the analysis of the teachers' explanatory communication follow in the next sessions.

When looking at learner participation from the observations, on the one hand, three teachers (Makwe, Mbose and Pandor) were characterised to have paid less attention to how they invited learners in the lessons. The three teachers were mainly asking *what* questions in the lessons, which encouraged the learners to complete unfinished sentences or provide single answers. There were fewer instances where the teachers were inviting the learners to provide phrases or sentences. The phrase that emerged strongly when the teachers where asking *what/how* questions was “additive inverse”. Where the learners had opportunities to discuss mathematics or at least respond in sentences, as we shall see in the analyses, Pandor and Mbose were reducing *what/how* questions to *what* questions, particularly when the learners were indistinct as they were responding in choruses or they were not responding to questions.

On the other hand, one teacher (Oena) succeeded to encourage discussions in his lesson. Oena’s approach to opening discussions were noticeable in more than one episode when the learners were called upon to write the solutions on the chalkboard whilst explaining the solutions to the class. At the same time, the results showed that Oena had also asked *what/how* questions to invite learners to provide phrases or sentences to his questions. However, when the teacher was questioning (or confirming) answers from the learners, he was asking *what* questions for the learners to complete unfinished sentences or to provide single answers. Oena mostly questioned the learners when they had committed errors in their answers. However, in some instances, learner errors were not questioned, but ignored.

Lastly, on the one hand, the results showed that three teachers (Pandor, Oena and Mbose) had presented coherent lessons when considering that they used relevant examples and appropriate explanations when solving the equations that involve using additive and multiplicative inverses. In addition, the three teachers were connecting the lessons of solving the equations to the notion of simplifying expressions that they did before these lessons. However, the teachers did not connect the introduction sections to the main object of learning, which was to solve the equations using the known procedures of balancing the left-hand and the right-hand sides of the equations.

On the other hand, as we shall see in the results, Makwe’s lesson was characterised to be incoherent when compared to those of the other three teachers. The shortfall in Makwe’s lesson was his incline to using non-mathematical words and the manipulation of symbols with little shift to using mathematical words appropriately.

To note, the results of the analysis of the four teachers were based on the three sub-units of the lesson (e.g., introduction, lesson development and classwork or homework), which formed mathematical episodes for this study. It was only in Mbose where the three episodes were analysed within the same lesson since he managed to have time to do the classroom activities.

Pandor and Oena pushed the classwork activities to the following lesson, and the analysis of those episodes was conducted from the second lesson, but were considered to form part of the first lesson. Makwe did not introduce his lesson from scratch, but he revised the topic. The revised topic was based on classwork and homework activities. Makwe's homework activities also came in the following lesson, but were only analysed for the exemplification aspect of the lesson. Moreover, for technical reasons, codes are appearing in teachers' quotations when I discuss the results in the next sections of this chapter as they formed part of presenting evidence of analysing data.

9.3 Pandor's mathematical discourse in instruction

Table 9.1 shows the examples and the tasks that were presented in Pandor's lesson. The lesson had three sub-units (i.e. the introduction, the lesson development and the classwork activities). These sub-units formed mathematical episodes. Each mathematical episode (whether ME1, ME2 or ME3) was analysed to accommodate each aspect of the MTF, starting with how Pandor selected and sequenced the examples during exemplification.

Table 9.1: Pandor's mathematical episodes during the first lesson

Introduction (ME1)	**Lesson Development (ME2)**	**Classwork Activities (ME3)**
1.1 Building $3x + 2 = 11$ from $x = 3$ 1.2 Solving $3x + 2 = 11$ to reverse the process of 1.1	2.1 Solving $2b + 2 = 6b - 14$ 2.2 Solving $3(2r - 3) = 5 - 2(r - 5)$	3.1 Solving $4(2m - 5) = 2(m + 1)$ 3.2 Solving $3(x - 1) - 4x = 5 - 2(x + 1)$

Exemplification

In the mathematical episodes (i.e. ME1, ME2 and ME3), the learners were mainly tasked to solve linear equations that involved using additive and multiplicative inverses. When focusing on the ME1, in Example 1.1, the learners were required to build equation 3x + 2 = 11, whereby they firstly multiply x = 3 by 3, and then add 2 to get the equation. Example 1.2 required the learners to reverse the process of Example 1.1 in order to find the value of x, which is equal to 3. The rest of the examples required the learners to find the values of the variables by carrying out known procedures to solve the equations. All the examples involved using additive and multiplicative inverses. The example set showed progression, whereby the structure of the examples were from simple to complex (i.e. simple because of one variable in the equation; and complex because of variables on both sides and more terms in the equation). However, Example 3.1 is easier than Example 2.2 when considering that, for example, Example 2.2 has an extra term.

There were opportunities for the learners to experience similarities in Examples 1.1 and 1.2 since the equation is the same, although the processes of building the equation and of solving were in reverse. Example 1.2 further resembled Example 2.1 except that it had variables and constants on both sides of the equation. Examples 2.2 – 3.2 entailed brackets, but consisted of variables and constants on both sides of the equations. At the same time, Examples 2.2 and 3.2 were similar in structure on the right hand side of the equations (i.e. it was 5 – 2(r – 5) and 5 – 2(x + 1)). Moreover, the learners had an opportunity to experience contrast in Example 1.1 when the teacher was introducing the lesson. Therefore, in accordance with the MDI framework, the teacher's example set was at Level 2 since both forms of variation (i.e. similarities and contrasts) were in place in the lesson.

Explanatory Communication

When using the same structure of presenting the results like that of the teacher's exemplification to discuss explanatory communication, I look at how Pandor was naming the objects in focus when teaching in the classroom alongside different criteria that the teacher was using to legitimate the mathematics to the learners. Hence, I present the results of all the three mathematical episodes for the teacher's first lesson.

In Example 1.1, Pandor moved between the manipulation of the symbols and using appropriate mathematical language. For example, at the beginning of the sentence of building the equation, the teacher stated that "when they say x equals to 3 **(Ms)**, they mean "the value of x is 3 **(Ma)**", which was to use mathematical language appropriately. In the next steps, the teacher was showing learners how to manipulate the symbols in order to build the equation. For example, after getting 3x = 9 by multiplying each side of x = 3 by 3, Pandor stated the following:

> So whatever that I had there (he points underneath $x = 3$) still holds (now he points at $3x = 9$) because if I decide I must also divide by 3 there (he points at $3x = 9$) **(Ms)**, I must also divide by 3 there. It is going to give me x equals to 3 **(Ms)**.

The focus of the explanation in this extract has shifted to show the learners how they can get x to be equal to 3, which is different than to state that the value of x will still be 3 even for 3x = 9.

When legitimating the process of manipulating the symbols in Example 1.1, the teacher focused on using the general principles fully. For example, when the teacher asked what was "3x plus 2 **(Ms)**", and learners had responded to say it was 5x, Pandor said "Unlike terms cannot be added **(GF)**." In this sense, the teacher had managed to explain using appropriate mathematical principle of 'unlike terms'.

In Example 1.2, Pandor also moved between the process of manipulating the mathematical symbols and using appropriate mathematical language. For example, the teacher stated the following as his open statement to the task:

> Now (he writes $3x + 2 = 11$) **(Ms)**, I have been given an equation 3x plus 2 is equal to 11 **(Ms)**, they say solve [the equation]... with equations, we solve, we find the value of x (he points at $3x + 2 = 11$) that is going to make this statement true **(Ma)**. That is going to make the left-hand side to be the same as the right-hand side **(GF)**.

In these excerpts, Pandor presented the mathematical symbols that were meant to be manipulated to find the solution. For example, the teachers stated that "I have been given an equation 3x plus 2 is equal to 11 **(Ms)**" to solve. The teacher went on to explain to the learners the end goal of manipulating the symbols, which was to "find the value of x" that was "going to make this statement true **(Ma)",** which was appropriate mathematical language. In this example, the teacher mostly legitimated the process of manipulating the symbols with an intention of finding the value of x using the general principle in full as the class sought to balance the left-hand side and the right-hand side of the equation.

Similar to the ME1, in the ME2, Pandor was moving between the process of manipulating the mathematical symbols and using mathematical words appropriately during calculations, whilst he was using mostly the general principles to legitimate the mathematics. In Examples 2.1 and 2.2, the teacher mostly manipulated the symbols to find the solutions, but sometimes he used appropriate mathematical words such as "additive and multiplicative inverses". However, in Example 2.2, the teacher was providing fewer reasons of why the procedures were working than in Example 2.1, but there were instances where he also used the general principles in full to legitimate mathematics. For example, at the beginning of Example 2.1, the teacher was clear to express that they "want to find the variables, the value of the variable **(Ma)**, which is going to make the left-hand side [of the equation] the same as the right-hand side **(GF)**".

Examples 3.1 and 3.2, which were intended as the classwork activities, were converted to be the homework activities. In these examples, which were done in the next lesson, the learners were called to write the solutions on the chalkboard. The learners did not discuss their solutions. Pandor explained the solutions to the whole class. When explaining, the teacher remained more in manipulating the symbols and sometimes in reading the strings of symbols when correcting or clarifying steps in the learners' solutions. For example, when the teacher needed to use the distributive law to expand the equation 4(2m – 5) = 2(m + 1), the following communication took place:

> I got two brackets. 4 into 2m minus 5 equal to 2 into m plus 1 **(Ms)**. Removing the brackets **(L)**, I get 8m minus 20 equal to 2 times m, which is going to be 2m, and 2 times 1, which is going to be 2 **(Ms)** (he points at the terms of the equation $8m - 20 = 2m + 2$).

The evidence of reading strings of symbols is as follows: "4 into 2m minus 5 equal to 2 into m plus 1". However, when starting the examples, the teacher was inclined towards using mathematical language appropriately. For example, when he started to discuss Example 3.2, the teacher said "We are suppose to solve [the equation] **(Ms)** by finding the value of x **(Ma)**. That is, to make the left-hand side to be the same as the right-hand side **(GF)**".

In Examples 3.1 and 3.2, Pandor mostly legitimated the process of manipulating the symbols by focusing on finding the value of x using the general principle in full as he was emphasising balancing the left-hand side and the right-hand side of the equation. However, in both examples, the teacher was talking about removing brackets, which was an actual act of manipulating symbols in those specific cases other than the process of multiplying a monomial and a binomial.

Thus, as the summative judgement of how Pandor explained mathematics in the three episodes, the teacher was moving between manipulating the mathematical symbols and using mathematical words appropriately when explaining how the procedures work. From the MDI framework, the naming of mathematical objects by Pandor was at Level 3. The teacher did use mathematical words appropriately to succinctly explain the purpose of the examples and to explain certain procedures in between the manipulation of symbols. At the same time, the teacher was legitimating the mathematical procedures of solving linear equations using mainly the general principles fully to maintain balance between the sides of the equal sign of the equation, which is at Level 3 according to Adler and Ronda's (2015) criterion when considering that the use of language was beyond everyday language and local explanations.

Learner participation

In the first mathematical episode (ME1), particularly in Example 1.1, Pandor started the lesson by encouraging the learners to respond to *what* questions, whereby the learners were completing unfinished sentences and offering single words to the class. In Example 1.2, the teacher moved between asking *what* questions for the learners to complete unfinished sentences or to offer single answers and asking *what/how* questions for the learners to provide phrases.

In Example 1.1, Pandor asked *what* questions in 11 turns, whereby the learners only completed the sentences and provided single answers. For example, in one turn, the teacher wanted the learners to provide the answer of the sum of 3x and 2, so he asked: "3x plus 2 is

going to give me what? **(Y/N)**". The learners responded as a chorus to state that it was 5x. As 5x was an incorrect answer, the teacher repeated the *what* questions in trying to get the learners to complete the sentences. The more the teacher repeated the questions, the more the learners were providing different single answers. In this case, since the offered answers were incorrect, the teachers eventually provided the correct answer and moved on to explain why his answer was correct. His explanation included the fact "[u]nlike terms cannot be added".

Out of 12 turns in Example 1.2, six questions enabled the learners to complete sentences and other six enabled the learners to provide phrases. Out of the 12 questions, two of them presented the learners with an opportunity to discuss ideas, but the teacher reduced one question to want the learners to provide a single answer and another question was reduced for the learners to respond in a phrase. The expected phrases from these questions were mostly about additive inverses or multiplicative inverses. For example, the teacher asked the learners "what should I do? **(P/S)**" to find the value of x in the equation 3x = 9. Of course, the learners had an opportunity to state that the teacher should multiply the equation by its multiplicative inverse, which is one thirds, to get x equal to 3, but in this case, they stated the teacher should "divide by 3".

In the second mathematical episode (ME2), Pandor remained mostly in asking *what* questions to allow the learners to complete the unfinished sentences although there were few other parts during this mathematical episode where he asked *what/how* questions to solicit phrases. For instance, out of 15 questions in Example 2.1, the teacher had asked three questions where the learners had an opportunity to answer with phrases. Even with those three question, the teacher reduced one question to want the learners to provide a single answer. The case was as follows: when the teacher needed to move 6b to the left side of the equal sign to collect like terms, his first question was "How do I remove it (he points at the term 6b)? **(P/S)**", before the learners could respond, he went on to say "What is the additive inverse of 6b? **(Y/N)**". Again, in Example 2.2, there was clear evidence that the teachers had remained mostly in asking *what* questions, but there were those fewer areas during the example where he asked for phrases.

Like in the ME1 and the ME2, Pandor mainly invited the learners to participate in what questions, although there were few instances at the beginning of each step where the teacher had asked "what should I do? **(P/S)**". In Example 3.2 (they started by working out Example 3.2 and not Example 3.1), there were 20 questions posed to the learners. Out of 20 questions, there were 13 'what' questions which encouraged the learners to respond with 'yes or no' answers and with single answers. In the seven other questions, the teacher was asking

what/how questions such as ‘what should I do?’ or ‘how do we remove the brackets?’. In the beginning of working out Example 3.2, the learners would wait for the teacher to reduce the *what/how* question to a ‘what’ question and then they would complete unfinished sentences, whilst late in working out the example they became aware that the teacher wanted phrases like “additive inverse”. It was the same case with Example 3.1, where the learners remained mainly in responding with single answers or ‘yes or no’ answers, whilst the phrases like “additive inverse” and “divide” became prominent to respond to questions like ‘what should I do?’.

Therefore, in this lesson, Pandor was moving between asking *what* questions for the learners to complete the unfinished sentences or to offer single answers and asking *what/how* questions for the learners to offer phrases. Drawing on the MDI framework, the summative judgement for this lesson was at Level 2. The judgement was supported by the importance of bringing phrases such as ‘additive and multiplicative inverses’ to focus when solving linear equations. These phrases address the issues of collecting liked terms and isolating variables in the procedures of solving linear equations, and such phrases were promoted in the TM1 course. However, there were concerns as the teacher had reduced some of the *what/how* questions back to wanting the learners to offer single words answers during the lesson.

The object of learning

Pandor’s lesson was coherent. The teacher needed to draw learners’ attention on how to solve equations that involve using additive and multiplicative inverses. The results showed that the three aspects of the MTF (i.e. exemplification, explanatory communication and learner participation) worked together for the teacher to carry out known operations and procedures. For instance, the teacher used appropriate mathematical words such as additive and multiplicative inverses, which were central in explaining the concepts of solving equations. However, the quality of teaching remained at middle-low levels when analysed using the MDI framework, particularly when considering the fact the learners were required to merely carry out known operations whilst they were mostly invited to offer single answers or to complete unfinished sentences in the lesson.

When looking across the lesson, Pandor only connected Example 1.1 to Example 1.2 in terms of the final answer, without considering that in Example 1.1, he was building the equations while in Example 1.2, he was reversing the process of building the equations to solving the equations. In other words, there was an opportunity to compare the steps of the examples since it was the same equation.

The other three lessons for the three teachers (Pandor, Oena and Mbose) did not confirm (or expand) the results of the first lesson since they were not analysed for the four aspects of the

MTF (i.e. exemplification, explanatory communication, learner participation and the object of learning). However, the transcriptions of the four lessons of the three teachers were available when I needed to confirm whether the teachers were connecting certain mathematical concepts across the lessons. The results revealed that Pandor was making some connections across the lessons. For example, there is evidence in the transcriptions where Pandor was connecting the concepts of simplifying mathematical expressions and what it means to solve equations. See an extract below with the underlined wordings which indicate such connections. The extract was from the first lesson.

> I have been given an equations 3x plus 2 is equals to 11 **(Ms)**, they say, solve, remember with expressions, we simplify, that's what we have been doing, but with equations, we solve, we find the value of x (he points at $3x + 2 = 11$) that is going to make this statement true **(Ma)**.

These explanations of connecting concepts took place mostly when Pandor was introducing his lessons, whilst in between the lessons, the teacher was pointing out how certain known operations were applicable when carrying out known procedures. Therefore, the teacher had ensured coherent connections between the lessons.

9.4 Oena's mathematical discourse in instruction

Table 9.2 shows the examples and the tasks that were presented by Oena in his first lesson. There were similar examples to those of Pandor except that Oena had three additional examples – two more examples in the introduction and another one in the classwork activities. Oena's examples were a complete version of the example set that was prescribed in the scripted lesson plan. In the next segment, I present the results of the analysis of the aspects of the MTF, starting with exemplification.

Table 9.2: Oena's mathematical episodes during the first lesson

Introduction (ME1)	Lesson development (ME2)	Classwork activities (ME3)
1.1 Building $3x + 2 = 11$ from $x = 3$ 1.2 Solving $3x + 2 = 11$ to reverse task of ME 1.1 1.3 Building $3(x + 2) = x - 12$ from $x = -9$ 1.4 Solving $3(x + 2) = x - 12$ to reverse task of ME 1.3	2.1 Solving $2b + 2 = 6b - 14$ 2.2 Solving $3(2r - 3) = 5 - 2(r - 5)$	3.1 Solving $4(2m - 5) = 2(m + 1)$ 3.2 Solving $3(x - 1) - 4x = 5 - 2(x + 1)$ 3.3 Solving $(2x + 1)^2 = (2x - 3)(x + 5) - 3$

Exemplification

The examples used by Oena in the lesson were similar to those that were in Pandor's mathematical episodes, even the classwork activities were converted to be the homework activities. However, the fact that Examples 1.3 and 1.4 were included in the ME1 extended the notion of building and solving the equations. Whereas, the inclusion of Example 3.3 required the learners to find the value of x where the graphs of the two quadratic equations meet, if they intercept each other. Since the lesson was focusing on solving linear equation, Example 3.3 had a potential to provide the learners with a contrasting experience.

The other crucial difference was that Examples 1.3 and 1.4 prepared the learners for Example 2.2 because of their similar nature of inviting learners to apply the distributive law when dealing with the brackets. In other words, Examples 1.3, 1.4 and 2.2 were similar in structure, since they all have variables on both sides of the equal sign, but had varying number of terms and utilisation of minus when applying the distributive law. For instance, in Example 2.2, the learners were expected to multiply the negative number (– 2) with terms that are separated by a minus sign (r – 5), which is understood in the literature to be more challenging than to apply the distributive law on positive terms. Thus, in accordance with the MDI framework, the teacher's example set was at Level 2 since both forms of variation (i.e. similarities and contrast) were present for the learners to experience in the lesson.

Explanatory communication

At the beginning of the first mathematical episode (ME1), Oena relied more on using mathematics symbols to show learners how procedures work when building or solving linear equations. However, towards the end of the ME1, the teacher had moved to use mathematical words appropriately to explain how to build or solve linear equations. At the same time, the teacher was mostly using general principles to legitimate the mathematics for the learners although there were instances where the teacher did not justify his or the learners' actions when they were manipulating the symbols by referring to the mathematical principles.

In Example 1.1, which was at the beginning of the ME1, the teacher was building the equation $3x + 2 = 11$ from $x = 3$, as we saw above. The emphasis at this stage was that the learners must "multiply 3 on both sides" and then "add 2 on both sides" to get the solution. When the learners were conjoining 3x and 2 to get 5x, the teacher provided the answer and did not explain that the unlike terms do not add to one term. Here is evidence when the teacher did not provide an explanation: "Is it possible to say 3x + 2 = 5x? They say add 2 on both sides. 3x you add 2, 9 you add 2. It means 3x + 2 = 11 **(Ms).**" There were other areas in Examples 1.2 and 1.3 where the teacher could have referred to the established principles to legitimate the algebraic operations even though the learners had provided correct answers. For example, one learner was asked to "add x on both sides" of 2x + 6 = –12, and a learner's response was

“3x plus 6 is equal to x minus 12”. The teacher re-voiced the response and went on to confirm that “2x plus x is equal to 3x”, but did not explain why these terms are adding to one term given that they are like terms.

In Example 1.4, Oena was explaining fully to the learners using established principles and using mathematical words appropriately when he was inviting them to participate in discussing ideas. For example, here is an extract when the teacher presented to the learners.

> Now, we’ve got two expressions, the one on the left and the one on the right. We want to know the value of x or unknown value of x (he points at $3x + 6 = x - 12$) **(Ma)**. I want us to group the variables on the left-hand side of the equal sign and all the constants on the right-hand side of the equal sign **(GF)**. Angithi? (Is it?) But you must not forget to use the additive inverses and the multiplicative inverses **(GF)**. Now solve that problem, quickly (he points at $3x + 6 = x - 12$) **(D)**.

In this extract, the teacher talked about finding the value of x in the equation, which is more explicit than solving the equation, and further legitimate the mathematics by communicating to the learners that they need to maintain the equivalence of the equation using additive inverses and multiplicative inverses. Moreover, after the learners had provided the solution, the teacher substituted the value of x in the equation to prove the equivalence of the expressions on either sides of the equal sign to legitimate the mathematics.

Oena started the second mathematical episode (ME2) by switching between using mathematical words appropriately and using the symbols to show the learners how to find the variables. In between using mathematical words and the manipulation of the symbols, the teacher would sometimes use everyday language such as to say “Negative 4 and negative 4, they cancel each other **(NM)**” when he was implying that the numbers divide each other. In this episode, there were only four instances where the teacher was legitimating the mathematics. In all those four instances, the teacher was emphasising the general principles of balancing “the left-hand and the right-hand sides” when solving the equations.

Towards the end of the ME2, particularly during Example 2.2, when Oena was showing the learners how to solve the complex equation (i.e. equation with brackets and more terms), the teacher got fixated on manipulating symbols to solve the equation. Here is an extract of using the distributive law to operate on brackets in Example 2.2:

> 3 times [indistinct] open bracket, multiply 3 by 2r? **(Ms) (Y/N)** [Learners: 6r]. 6r. Okay? [Learners: Yes.] [indistinct] open bracket, multiply 3 by negative 3? **(Ms) (Y/N)** [Learners: negative 9] Negative 9. 2 open the bracket multiply by r? **(Ms) (Y/N)** [Learners: 2r]. Negative becomes negative. Negative times negative? **(Ms) (Y/N)** [Learners: positive]. Positive

[Learners: negative 5, 10]. 10, angithi? (is it?) **(Ms) (Y/N)** [Learners: Yes.]. (He writes $3r - 9 = 5 - 2r + 10$)

In this extract, we see that the teacher remained in manipulating symbols without justifying his steps to the learners. To this end, the teacher would sometimes use everyday language such as to say we want "to get rid of the coefficient of r **(NM)**".

For the ME3, some learners were asked to come to the front to provide written solutions of Examples 3.1 – 3.3. In explaining the written solutions from the learners, Oena remained in manipulating the symbols to the class. For example, a learner had provided the correct solution of Example 3.1, and the teacher read out how the symbols were used to work out the solution as he confirmed to the learners that the solution was indeed correct. Similarly, a learner had provide a correct solution for Example 3.2, which enabled the teacher to read out the symbols with fewer attempts to legitimate mathematics. Lastly, the teacher asked another learner to replace a learner who had provided 4x + 1 when attempting to expand $(2x + 1)^2$ for Example 3.3 on the chalkboard. When the second learner struggled as well to expand $(2x + 1)^2$, the teacher intervened to provide the correct solutions. However, in this last example, on the one hand, the teacher was moving between manipulating symbols to using mathematical language appropriately. On the other hand, Oena was providing reasons that were related to manipulating steps for single cases.

When looking across the three episodes for the lesson, Oena was at Level 3 (Adler & Ronda, 2015) regarding his naming of mathematical objects. The teacher started with showing the learners how to manipulate symbols whereby the emphasis was on how to do the next step to solve the equations. Towards the middle of the lesson, the teacher deepened the use of mathematical words in an appropriate manner by highlighting the importance of finding the value of the variables in order to balance the left-hand side and the right-hand side of the equations. The teacher ended his lesson by returning to showing the learners how to respond to algebraic questions with little legitimation of mathematics. Consequently, the legitimation was at Level 2 because the teacher had missed opportunities to use mathematical principles to refer to, for example, 'unlike terms' when the learners were conjoining 3x and 2 to get 5x.

Learner participation

In the first mathematical episode (ME1), Oena prioritised using *what/how* questions to invite learners to provide phrases or sentences to his questions. However, when the teacher was questioning (or confirming) answers from the learners, he would switch to expect the learners to complete unfinished sentences or to provide single answers. Towards the end of the ME1, there were cases where the learners were invited as individuals to write full solutions on the chalkboard whilst explaining those solutions, which meant that they were discussing the ideas.

In Example 1.1, there were six turns where Oena was inviting the learners to participating in building the equation 3x + 2 = 11 from x = 3. Out of six turns, there were four turns where the learners provided mathematical sentences to Oena's questions. For example, here is an extract from Oena when he wanted the learners to add 2 on both sides of the equation 3x = 9.

> Action on both sides. Add 2. Add 2. Add 2 on both sides. We continue with our equivalent equation (he points at $3x = 9$). Add 2 on both sides. ("yes": he identifies a learner) **(P/S)** [Leaner: $5x = 11$]. You add 2 on both sides. 5x equal to 11? **(Y/N)** [Learner: yes]

In this extract, the teacher invited the learner to add 2 to 3x and add 2 to 9 in order to get 3x plus 2 equal to 11. When the learner conjoined 3x and 2 to get 5x, the teacher reduced the question and asked a 'yes or no' question to indicate to the learner that his response was incorrect when operating on the left-hand side of the equal sign. Oena asked other individual learners to intervene in providing the correct mathematical statement. When another learner insisted on conjoining as well whilst another learner did not respond to the question, the teacher asked if it was "possible to say 3x plus 2 is equal to 5x? **(Y/N)**" before responding to his own question by providing the correct solution. For both instances of conjoining, the teachers did not attempt to ask learners to explain why they were convinced that 3x plus 2 was 5x and not 3x + 2.

In Example 1.2, there were two turns where the learners were expected to operate on both sides of the equation to reverse the processes of Example 1.1. In both instances, the learners provided the correct mathematical statements. In Example 1.3, there was a strong evidence of *what/how* questions that were followed with 'yes or no' questions when the learners provided the correct mathematical statements. Here an example of such occurrence.

> Add 6 on both sides (she points at $2x = -18$) (he identifies a learner) **(P/S)** [Learner: 2x plus 6 is equal to –12] 2x plus 6 is equals to? **(Y/N)** [Learner: –12] –12. Good. Because –18 plus 6 is equals to how much? **(Y/N)** [Learners: –12]

In the extract, the teacher invited a learner to add 6 on both sides of the equation 2x = –18. When the learner offered the correct answer, the teacher then asked 'yes or no' questions to confirm the solution.

In Example 1.4, Oena moved from asking *what/how* questions to expect the learners to discuss the procedures by writing the solutions on the chalkboard. For example, the teacher stated the following after providing mathematical explanations of how to find the value of x: "Now, solve that problem (as he points at $3x + 6 = x - 12$). Solve that problem, quickly **(D)**". In this case, two learners did not provide the correct solutions. They committed one or two errors in certain steps that led to incorrect solutions. As a result, the teacher picked on the

final step, and asked one learner if “x is equal to –2?” and another learner if “x is equal to –3?” respectively and stated that these were incorrect answers. A third learner provided a correct solution and was asked in the similar fashion, if “x is equal to –9?” before the teacher could confirm that indeed this was a correct solution. In this case, the teacher used a mathematical proof to convince the learners that the solution was correct.

In the second mathematical episode (ME2), Oena started by expecting the learners to present full solutions to Example 2.1, which was coded as discussions, as some learners were asked to explain their solutions to the class. In discussing Example 2.1, four learners went to the front to offer their solutions on the chalkboard. Three learners could not proceed from the first step of removing positive 2 by adding its additive inverse which was negative 2. One learner provided the correct solution. The correct solution was kept on the chalkboard as the teacher re-wrote the solution to explain each step to the class. It was during the explanation when the teacher moved from asking *what/how* questions for the learners to offer phrases such additive inverses to asking for *single words* questions in order for learners to complete unfinished sentences. In Example 2.2, the teacher remained mostly in asking *what* questions whereby the learners provided single answers and completed unfinished sentences.

Again, in the third mathematical episode (ME3), some learners were given an opportunity to provide solutions to Examples 3.1 – 3.3 on the chalkboard. However, when some of these learners were struggling to provide correct solutions, particularly for Examples 3.2 and 3.3, the teacher was asking *what* questions to attempt to point the learners to the right direction. These learners who were providing the solutions on the chalkboard were not explaining much mathematics when compared to those learners who explained the examples in the ME1 and ME2.

Oena had offered the learners an opportunity to communicate mathematics at Level 3 when considering that some discussions took place in the ME1 and ME2. As we can notice when comparing the four teachers, how Oena invited learners to participate in the lesson was one level higher than those of Pandor, Mbose and Makwe. Oena expected the learners to respond with phrases such as additive and multiplicative inverses in most parts of the lesson. The teacher also opened discussions when the learners were providing solutions on the chalkboard during Examples 1.4 and 2.1, whereby some of those learners were asked to explain their solutions to the class. However, the teacher switched back to allowing chorus ‘yes or no’ answers from the learners when he was rushing to end the ME2, whilst he did not encourage much discussion in the ME3.

The object of learning

Oena's lesson was also coherent. The goal of the lesson was based on the teachers' explanatory communication, exemplification and how the learners were invited to engage in solving equations that involved using additive and multiplicative inverses. For example, the teacher had encouraged the learners to respond to *what/how* questions to solicit sentences on how additive and multiplicative inverses contribute in the steps of solving equations.

When looking across the lesson, Oena drew the learners' attention to the fact that they were participating in building the equation and in solving the equation. However, the teacher did not highlight that solving the equations was the reverse process of finding the value of the variable if the equation was built from scratch. For instance, when two learners provided incorrect answers for Example 1.4, the teacher did not point out that the value of x was known since the same equation was built from scratch in Example 1.3.

Nevertheless, Oena was conscious in connecting parts of the lessons to the learners' prior knowledge of mathematics. For example, at some point during the first lesson, the teacher talked about two expressions that formed the equation, the one on the left-hand side and other on the right-hand side of the equal sign. Thus, the teacher's teaching was also coherent across the lessons.

9.5 Mbose's mathematical discourse in instruction

Table 9.3 shows the examples and the tasks that were presented in Mbose's lesson, whereby the second and the third mathematical episodes were similar to those in Pandor and Oena's lessons. Of course, as we saw, Pandor did not do Example 3.3. However, as noted, Mbose used the scripted lesson plans from the 2018 GDE ATP document. In this version, the first episode required that the learners decide on the value of x which will make the equations true.

Table 9.3: Mbose's mathematical episodes during the first lesson

Introduction (ME1)	Lesson Development (ME2)	Classwork Activities (ME3)
1.1 Given $3x - 4 = 11$, is $x = 4$ or $x = 5$? 1.2 Given $2x + 7 = 19$, is $x = 4$ or $x = 5$? 1.3 Given $13 - 5x = -7$, is $x = 4$ or $x = 5$?	2.1 Solving $2b + 2 = 6b - 14$	3.1 Solving $4(2m - 5) = 2(m + 1)$ 3.2 Solving $3(x - 1) - 4x = 5 - 2(x + 1)$ 3.3 Solving $(2x + 1)^2 = (2x - 3)(x + 5) - 3$

Exemplification

Examples 1.1 – 1.3 in the first mathematical episode (ME1) kept the option of the values of x the same, as either x = 4 or as x = 5 for the given equations. The structure of the given

equations were the same because each has the variable on the left-hand side of the equation sign. However, the examples themselves were varying in terms of how the algebraic terms were separated. As mentioned when discussing Oena's exemplification, examples with terms that are separated by a minus sign have higher cognitive levels of difficulty when compared to examples with positive signs separating terms.

The teacher did not use Examples 1.1 – 1.3 to connect the ME1 and the ME2 (or the ME3) by providing the learners with an experience of similarity or of contrast across the mathematical episodes. For instance, the solution of Example 2.1 was b = 4, which created an opportunity for the teacher to draw on Examples 1.1 – 1.3 given the sameness in terms of the solutions. Examples with variables on both sides and with brackets in the ME1 or in the ME2 had a potential to present the learners with experience of coherent across the lesson, whilst presenting contrast across the examples. Nonetheless, when drawing on the MDI framework, Mbose can also be at Level 2 given that the learners had an opportunity to experience both forms of variation (i.e. similarities and contrast) during the teacher's exemplification in the lesson.

Explanatory communication

Mbose started the first mathematical episode (ME1) by using colloquial language. The teacher then moved to manipulate mathematical symbols. When the manipulation of symbols did not work in what Mbose had intended to communicate, the teacher switched to use mathematical words appropriately to explain the tasks to the learners before returning to manipulate symbols to find the solutions. The teacher mostly legitimated mathematical processes by using general principles of balancing the left-hand side and the right-hand side of the equal sign.

When using colloquial language at the start of Example 1.1, Mbose expressed that he needed the learners to participate in doing the introduction. The teacher stated that "the introduction says complete the table and together we are going to complete the table **(NM)**". The teacher did not attempt to explain using appropriate mathematical words what the task was about before going ahead with processes of manipulating symbols to get the answers. Here an extract when the teacher was checking if x = 4 for the equation 3x – 4 = 11.

> They say the left-hand side, find the left-hand side if x is equals to 4 (he points at LHS if $x = 4$) **(Ms)**. Now the left-hand side, we are speaking about the left-hand side of the equation (he points at the left-hand side of the equal sign of the equation $3x - 4 = 11$) **(Ms)**. That is the right-hand side **(GP)** (he points at 11). Now the left-hand side, if the x equals to 4. What do I mean by that? Go to the left-hand side and then from there, where you see x, you substitute x with the value of 4, and then from there you find your answer **(Ma) (L)**.

In this extract, Mbose was asserting that the substitution of the value x was for the left-hand side of the equal sign. Such legitimating accommodated only this specific case since variables can be on either sides of the equal sign or on both sides. Nonetheless, the notion of balancing the sides of the equation remains crucial as the general principle of solving linear equations. The coding of the general principle was kept at partial because the left-hand side of the equation in this case can further be described as a mathematical expression.

In the middle of Example 1.1, when the reading of strings of symbols was creating a misunderstanding between the teacher and the learners, Mbose resorted to using mathematical words appropriately. For example, the teacher was saying "The first one [meaning Example 1.1] says 3x subtract 4". The learners completed the sentence and said "equals to 11", and here is how the teacher responded to this misunderstanding:

> No! No! 3x subtract 4**,** we write that expression, and from that expression, we substitute the value of x **(Ma)**, which is equal to, we are given as 4 **(Ms)**, and see that the answer that we are going to find is going to be equals to the one that is on the right-hand side **(Ma)**.

The teacher became explicit that the goal of the task was to substitute the value of x (either as x = 4 or as x = 5) in the given expression before deciding on the answer, and the outcomes was going to be compare to the value given on the right-hand side. For the rest of Example 1.1, Mbose remained more on manipulation of the symbols, where he was expecting the learners to operate on numeric values.

Mbose used the similar approach to explain Examples 1.2 and 1.3. The teacher would start by using colloquial words, then clarify appropriately using mathematical language, and move to manipulate mathematical symbols where learners participate by operating on numeric figures. For instance, here is the first part of Example 1.2 where the class was given 13 – 5x = – 7 and required to check if x = 4 was a solution:

> Let's do the last one under introduction **(NM)**. OK, quickly! [Learners: 13 minus 5] I'm writing the expression on the left-hand side (he writes $13 - 5x$) **(Ma)**. Subtract 5x. Substitution. 13 subtract 5 (he writes $13 - 5( )$) [Learners: 4] 4, close bracket **(Ms)**. This will be equal to 13? [Learners: minus 20] Subtract 20 [Learners: negative 7] Negative 7 **(Ms)**.

The general principle in this task was to check if x = 4 would yield negative 7 (i.e. the right-hand side) when substituted in the expression 13 – 5x. However, the learners' attention was drawn to how to operate on the mathematical symbols.

In the second mathematical episode (ME2), Mbose started with colloquial language. For instance, the crux of the matter in Example 2.1 was to solve the equation 2b + 2 = 4b – 14, but the teacher had mentioned that "Now we are going to do one problem under lesson development **(NM)**". The teacher then relied on manipulating the symbols, although there were

clear instances where he used mathematical words appropriately. The teacher was using mathematical words appropriately when describing why was it important to use additive inverses, whereby he was talking about collecting the like terms to be on the same side of the equation.

Parallel to describing the importance of additive inverses, the teacher was switching back to talking about transposing terms when collecting the like terms. The teacher was communicating to the learners that transposing terms is another way of collecting the like terms. For example, in the following extract, the teacher was explaining the first step of how to solve the equation 2b + 2 = 4b – 14:

> This will be 2b minus 6b **(Ms)**. Alright, when we do transpose, that's what happen. I have b when the sign is not written it's positive. When I bring it to the other side of the equal sign, the sign changes so that means, its term is positive, and you change the sign, not the term but the sign of the term will change**.** If it is positive, it will become negative. If it is negative, it will become positive **(Ms) (V)**.

In this extract, we are seeing the teacher's reliance on manipulating the symbols when collecting like terms. The teacher was providing visual cues to legitimate the mathematics. The teacher continued to rely on manipulating the symbols for the rest of the example 2.1, although he returned briefly to describe the processes of isolating the variable using the multiplicative inverse.

In the third mathematical episode (ME3), Mbose had again started from colloquial language, and then moved to rely on the manipulation of the symbols, whilst applying the notion of additive and multiplicative inverses to solve the equations. For example, in the following extract, the teacher was isolating m after applying two steps from 4(2m – 5) = 2(m + 1) including distributive law to get 6m = 22.

> Actually, we multiply by 1 over 6. That's our multiplicative inverse, 1 over 6 **(Ms)**. That results in what you're telling me that you divide by 6, but I had to emphasise that. In that, you're actually talking about the multiplicative inverse of 6 **(GF)**.

I coded these instances of describing how additive or multiplicative inverses are important to solving linear equations be a full legitimating criterion of balancing the left-hand and the right-hand sides. Whether this general principle was solicited from the learners or it only mentioned by the teacher, the fact is that it was presented during the lesson at appropriate times.

Using MDI framework, Mbose was at Level 3 because of his approach of moving from colloquial to formal mathematics talk, although he relied more on manipulating the symbols when naming the object in focus to solve the equation. As the teachers was using formal mathematical talk, he was also using general principles fully to legitimate the concepts of

balancing the left-hand and the right-hand sides of the equations, for example. Hence, the teacher had given the learners an opportunity to gear towards scientific concepts, which his legitimating criteria was also at Level 3 (Adler & Ronda, 2015).

Learner participation

At the start of the first mathematical episode (ME1), Mbose solely invited the learners to participate in answering *what* questions, whereby they completed unfinished sentences or provided single words only. Here is an example of the extract taken from Example 1.1.

> They are saying, find the left-hand side if now x is no longer 4 but x is what? **(Y/N)** [Leaners: 5] on the very same equation. Alright let's do that. Now I substitute? **(Y/N)** [Learners: 5] 5. Now what is 3 times 5? **(Y/N)** [Learners: 15] 15. 15 subtract 4? **(Y/N)** [Learners: 11] you get 11. When I substitute x equals to 5, what is the answer that I get? **(Y/N)** [Learners: 11] It's 11. Alright. Alright. So now is the left-hand side equals to the right-hand side? **(Y/N)** [Learners: yes]. The answer is 'yes'**.**

In this extract, there are six turns where the teacher was expecting the learners to respond as a chorus to complete unfinished sentences, to operate on numeric figures and algebraic terms, or to provide single answers.

When working out Examples 1.2 and 1.3, Mbose started by requesting the learners to explain how they could find a solution to the examples. The teacher expected the learners to tell him what he needed to write on the chalkboard. For example, here is an extract as the teacher started to work out Example 1.2

> I need you to tell me how are we gonna do the next one **(D)**. Let's start... Tell me. 2x plus 7, you write down the left-hand side, because they are talking about the left-hand side. [Learners: Indistinct] I don't see 9. 2, open brackets **(Y/N)** [learners: Indistinct], 4, close brackets, plus 7, is equal to? **(Y/N)** [Learners: 8]. 8 plus 7, you get 15. Alright? What is my answer here? (he fills the first column with 15 in the table). What is my answer here? (he points at the second column in the table) **(Y/N)** [Learners: No] It's a 'no'.

When the learners responded as a chorus and became indistinct, the teacher reduced the questions to expect the learners to complete the unfinished sentences. The same approach of starting with an open statement was used in Example 1.3 except that the learners led in providing single words and the teacher was aiding the processes by writing on the chalkboard.

At the start of the second mathematical episode (ME2), Mbose did not invite the learners to participate in responding to questions. As the episode progresses, the learners did get an opportunity to mention the phrase 'additive inverse' on two occasions as a chorus. For instance, the teacher asked "How are we gonna go about solving this problem? **(D)**" when he wanted the learners to provide a first step to solve the equation $2b + 2 = 6b - 14$. The learners

responded by saying “transpose” whilst others said “additive inverse”. The question itself had a potential to invite the learners to discuss in details what were the additive inverses that they needed to add to the equation and why they were adding those additive inverses. However, considering that the teacher was not following through on his *how* questions, discussions were not taking place. In the rest of Example 2.1, the teacher returned to encourage the learners to provide single answers when operating on numeric or algebraic terms.

Mbose ended the ME3 in the same fashion, whereby he was asking *how* questions and the learners were providing single phrases or sentences. However, as soon as the learners struggled to respond to *how* questions, the teacher would switch to expect the learners to complete unfinished sentences. For instance, here is an interaction between the teacher and the learners in the start of Example 3.1.

> So how am I going to start, how am I going to start solving this problem **(D)**. [Learner: By removing brackets] Alright. How do we remove the brackets? **(P/S)** [Learners: Indistinct] We are multiplying, so it will be? **(Y/N)** [Learners: 4 times 2]

In this extract, the learners responded as a chorus and became indistinct when the teacher had asked “How do we remove the brackets? **(P/S)**”. The next moment the teacher immediately reduced the how question to expect the learners to offer single answers. The teacher continued the same way in answering Examples 3.2 and 3.3 in the lesson.

When considering that Mbose did not adequately provide the learners with opportunities to speak mathematics, from Adler and Ronda’s (2015) perspective, the lesson was at Level 2. At the start of the lesson, the teacher was mostly expecting the learners to provide ‘yes or no’ answers. In the rest of the lesson, the teacher was switching between asking *how* questions to *what* questions. However, the challenge was, in most of the cases when the teacher had asked a *how* question, and when the learners became indistinct as they responded in a chorus, he would reduce the question to expect the learners to provide single phrases or to complete unfinished sentences. This occurrence of reducing the questions before getting clear responses from learners was also noticed when analysing how Pandor was inviting the learners to participate in the lesson.

The object of learning

The results of analysing Mbose’s mathematical discourse in instruction showed that the lesson was coherent and focused on solving equations that involved additive and multiplicative inverses. The teacher was conscious in using the phrases additive and multiplicative inverses to legitimate the concepts of mathematics. At the same time, although he was inclined in manipulating mathematical symbols, the teacher used mathematical words appropriately.

The shortfall of Mbose's lesson was a disconnect between the examples presented in the first mathematical episode (ME1) and the other two mathematical episodes (ME2 & ME3). The teacher missed the opportunity of checking the answers of the examples in the ME2 and ME3 using the substitution method that was used in the ME1. For Adler and Ronda (2015), learners are required to use multiple connections when solving problems. Moreover, whilst there was evidence of talking about mathematical expressions in the analysed lesson, Mbose did not explicitly refer to other lessons to make connections across lessons.

9.6 Makwe's mathematical discourse in instruction

Table 9.4 shows the examples and the tasks that were presented in Makwe's lesson. Similar to those of the other three teachers, Makwe's lesson had three sub-units, although his sub-units were 'revision', 'classwork activities' and 'homework activities'. These sub-units formed the three mathematical episodes (ME1, ME2 and ME3). Next, like in other three teachers, I discuss each aspect of the MTF, starting with exemplification.

Table 9.4: Mathematical episodes from the Makwe's lesson

Revision (ME1)	**Classwork Activities (ME2)**	**Homework Activities (ME3)**
1.1 Solving $2x - 1 = -5$ 1.2 Solving $5x = 40 + 3x$ 1.3 Solving $3(x + 1) = 8x - 2$ 1.4 Solving $\frac{2x}{3} - 1 = 15$	2.1 Solving $3x - 6 = 9$ 2.2 Solving $7 - 3x = 2x - 3$ 2.3 Solving $2(x + 4) + 2 = 5x + 1$ 2.4 Solving $\frac{3x-1}{2} = 4$	3.1 Solving $2x + 2 = 10$ 3.2 Solving $3x - 2 = x + 4$ 3.3 Solving $8x - 3 = 3x - 22$ 3.4 Solving $4(x + 2) = 16 + 2(x - 1)$ 3.5 Solving $\frac{2x+7}{6} + \frac{x-5}{3} = 0$

Exemplification

There were three main differences in Makwe's example set when compared to those of the other three teachers. One, as noted, Makwe was using examples from different resources outside the prescribed lesson plans. Two, in all the examples, the teacher was only solving the equations that are involving using additive and multiplicative inverses – there were no examples that used different ways of preparing the learners to solve the equations. Three, the teacher included examples of solving fractional equations in his example set, which required the learners to use the highest common denominator to bring the structure of the equation to its normal form.

When looking across the examples that are presented in each mathematical episode (whether ME1, ME2 or ME3), there are no substantial similarities other than varying structures. For instance, when looking across the ME1, Examples 1.1 and 1.4 have one variable on the left-

hand side of the equation, but Example 1.4 has a fraction formation. Examples 1.2 and 1.3 have variables on both sides of the equation, but solving Example 1.3 requires multiplying a monomial and a binomial on the left-hand side of the equation. Therefore, the tasks presented across each mathematical episode have different cognitive levels.

However, when looking across the lesson for the presented example set, the examples display obvious similarities. The first examples in the mathematical episodes have the variable on the left-hand side of the equations. The second and the third examples in the episodes have variables on both sides of the equations, including Example 3.4 in the ME3. The third examples in the ME1 and the ME 2 require multiplying monomials and binomials on the left-hand side of the equations, whilst it is the same case with Example 3.4 in the ME3. The fourth examples in the ME1 and the ME2 are in a fraction form, whilst that is the case with Example 3.5. That is, the examples with varying signs and terms in the equations supported different cognitive levels when working across the episodes of the lesson. Therefore, the learners experienced both forms of variation (i.e. similarities and contrast) during the lesson, and the example set was at Level 2 (Adler & Ronda, 2015).

Explanatory communication

In the explanatory communication, particularly naming, Makwe started his lesson by moving mostly between using non-mathematical language and using mathematics words as names only to explain what learners needed to *know* or *do* through his explanations. When Makwe was using mathematics words as names only, the teacher was focusing on local assertions such as stating that 'here we divide by' to legitimate relevant steps of solving linear equations. In few parts of the lesson, the teacher was using mathematical language appropriately. For example, the teacher did talk about using additive inverses to collect like terms. The notion of collecting the like terms were embedded in the general principle of balancing the left-hand and the right-hand sides to legitimate the mathematics.

At the start of the ME1, when looking at Example 1.1, which was $2x - 1 = -5$, the teacher firstly reminded the learners about the meaning of x, and said: "Remember 2x, simply means 2 multiply by x, x is an unknown number that we want **(Ma)**". This assertion was coded as an appropriate use of mathematical language. Secondly, the teacher moved to use non-mathematical language. He highlighted the variable x with a heart shape, and said: "We want x our lovely girlfriend **(NM)**." When moving to the next step of adding 1 on both sides of the equation, the teacher continued with his non-mathematical language as follows: "We want x to be isolated and we don't want her friends **(NM)** (he points at the constant -1 in the equation) and we don't want the constant **(Ms)**". Thirdly, he switched back to use a mathematical language, as he added the additive inverse of negative 1, which is positive 1, on both sides of

the equation. He uttered the following: "I have $2x - 1$, I'm going to add 1, the inverse of it is plus 1, and what we do on one side we must also do on the other side [he writes $2x - 1 + 1 = -5 + 1$] **(GF)**". Then, from the next step, which was 2x = – 4, the teacher manipulated the symbols to get to the solutions. That is, as I quote him, "So we divide. What am I dividing with? [Learners: 2] (he writes $\frac{2x}{2} = \frac{-4}{2}$) **(L)**", and ended by saying, "so we just write x (he writes $x = -2$), therefore my x is equal to negative 2 **(Ms)**".

In Examples 1.2 and 1.3, the teacher also used non-mathematical language, whilst relying on the manipulating of symbols. The manipulating of symbols was exacerbated by the inclusion of the word 'transpose' in his explanation. Here is the extract from Example 1.2 to elaborate on the teacher's explanatory communication.

> Remember we said x there is our lovely girlfriend (he highlights the x's with the heart shape) **(NM)**. You want to solve for x (Ms). The two girlfriends will eventually meet, Jubujubu will make sure that they meet **(NM)**. This girlfriend [he points at $3x$], remember we need to isolate, and make sure that the variables are, and constants are on the other side **(Ms)**. This girlfriend will meet with the other one, she should move to the other side (he draws an arrow showing that $3x$ can be moved to the other side) **(V)**. What happens to my sign now? [Learners: it changes] (he write $5x - 3x = 40$) **(Ms)**.
>
> And I still want x there [he highlights x in the equation $2x = 40$]. I don't want 2 next to x, which is a coefficient next to x **(Ms)**. So how do I get rid of that? [Learners: divide by 2] divide by 2 **(Ms)**. But we said, what you do on one side, you must also do it on the other side **(GF)**. So my x there is equal to 20 [he writes $x = 20$] **(Ms)**.

When discussing the extract, I zoom in on four aspects that can be associated with how the teacher had used the language in this example. Firstly, the teacher used the word 'isolate' to move the constant to the other side of the equal sign whilst the word 'isolate' refers to removing a coefficient of a variable by multiplying the coefficient of the term with its multiplicative inverse. Secondly, when explaining how to 'move' the terms with the same variables to one side of the equal sign whilst the sign changes, the teacher used visual clues by drawing an 'arrow'. Thirdly, instead of using the word 'isolate', the teacher used the words 'get rid' when he needed to multiple the coefficient by its multiplicative inverse. Lastly, instead of multiplying the term by its multiplicative inverse of 2, which is half, the teacher 'divided by 2' as an action of manipulating the symbols in this specific case. Nonetheless, the teacher legitimated the division of symbols using the general principle of balancing the left-hand side and the right-hand side of the equation as he stated that "what you do on one side, you must also do it in the other side **(GF)**".

Before the ME1 ended, there were also problems of not using the general principle of multiplying terms with 'multiplicative inverses' in order to isolate coefficients when solving linear equations. Throughout the lesson, the teacher was focusing on using specific mathematics words such as 'divide by 2' or 'multiply by 2' when isolating coefficients. As such, working out Example 1.4, which was 2x/3 – 1 = 15, became tricky for the learners as the term 2x had already been divided by 3. Here is an extract when the teacher was explaining Example 1.4 using Example 1.1:

> We have fractions (he points at the first term). [Learners: Eish!] Yeah, eish… these ones are a killer. What is the first thing that we must do there? [Learners: Indistinct]... (he points at $2x$ in Example 1.1) here I had 2, which you had no problem with, but now instead of the coefficient being 2, now it is 2 thirds and people are seeing flames because of the 2 out 3. [Learners: Indistinct]... So now how do I get rid of that 2 out of 3? **(NM)** I don't want it there. How do I get rid of it?

In getting to the next step, the teacher resolved to telling the learners that they have to multiply the term by 3 in order to get 2x and then divide by 2 to isolate the variable x, or just multiply this term by 3 over 2 because mathematics can have more than one method to getting the same solution.

In the ME2, when the teacher was revising what the learners were doing during the classwork activities, there was a mismatch between using the transpose technique and using additive inverses to collect like terms. According to the teacher, when doing Example 2.1, the learners added 1 on both sides of the equation as a next step to solving 3x – 6 = 9. Here is the extract:

> The mistake that I saw, as you guys like 'copy and paste', here I added 1 (he points at positive 1 in the step $2x - 1 + \mathbf{1} = -5 + \mathbf{1}$) and now in question 1 [Example 2.1], people are adding 1. Copy and paste! You have 3x minus 6 which is equal to 9 **(Ms)**. Just to correct you the ones who added 1, the reason why I added 1 is because negative 1 plus 1 will give me zero. It is what we call an additive inverse. So here (he points at $3x - \mathbf{6} = 9$) in order for me to have a 0 [zero] there (he points at $3x - \mathbf{6} = 9$ and then he adds 6 as an additive inverse) I must add 6 because negative 6 plus 6 will give me zero **(Ms)**.

This mismatch pushed the teacher to further emphasised the process of adding additive inverses when collecting like terms. However, in this case, the teacher was using the phrase 'additive inverse' as a process of adding two numbers together to get zero.

We have seen that Makwe had used non-mathematical language to introduce his lesson, whilst relying on manipulating the symbols to find solutions. Considering that using mathematics words as names only hinder the developments of generality, the teacher remained in using specific single cases to legitimate the mathematics. Therefore, Makwe was

at Level 1 in terms of how he was naming the objects in focus during the lesson, and also at Level 1 in his legitimating criteria.

Learner participation

In the first mathematical episode (ME1), Makwe was mostly encouraging the learners to operate on numeric or algebraic formation as part of writing steps of the known procedures to solve the equations. For example, in Example 1.1, there were ten turns where the teacher had invited the learners to participate in solving the equation $2x - 1 = -5$. Eight out of ten turns, the learners responded in a chorus and mostly operated on numbers such as to add or multiply two integers. For instance, when the teacher wanted to isolate x as the next from $2x = -4$, the teacher asked, "2 divided by 2? **(Y/N)**", and the learners would respond by saying "1". The teacher would re-voice the correct response from the learners; write it appropriately on the chalkboard; and move on to the next step. In the other two turns, the learners were expected to provide phrases or sentences, whereby in one turn, the teacher did not wait to get a response from the learners. The question was "So how do we get rid of the constant? **(P/S)"**. The other question was "So what I do on one side? **(P/S)**" and the learners completed the sentence to say "you must also do it on the other side". This last question was coded as **(P/S)** since it enabled the learners to state a sentence although it can be coded as **(Y/N)** since they were completing the sentence.

For the rest of the examples in the ME1, when considering that throughout Example 1.1 Makwe had used additive inverses to collect like terms, the learners had invoked the notion of transposing terms. That is, one learner had asked why can we not just move a term to the other side and change the sign. Whilst the manipulation of the symbols was still dominant, in the instances where the teacher was expecting the learners to offer phrases or sentences, the learners were mostly mentioning in a chorus, "it changes" or "divide". The learners were saying "it changes" when responding to the question "what happens when we move [the number] to the other side? **(Y/N)**", or they were saying "divide", when the teacher wanted to isolate the variable by multiplying the term with its multiplicative inverse.

In the second mathematical episode (ME 2), Makwe had given the learners some time to write solutions of the tasks on their own before discussing how they have solve the equations with the whole class. The teacher walked around the classroom looking at what learners were writing in the notebooks. Towards the end of the class, the teacher returned to share with the learners what he saw as he was walking around and went on to discuss the correct solutions on the chalkboard. The teacher expected the learners to contribute in different steps towards solving the equations.

Similarly to the ME1, the learners in the ME2 were mostly responding as a group chanting single answers or phrases. For example, when Makwe was doing Example 2.2, to solve the equation $7 – 3x = 2x – 3$, there were seven turns of questions. Out of seven turns, there were two instances where the learners were saying “it changes” as they were referring to the sign after the term had been transposed to the other side of the equal sign. In one instance out of seven, the learners had said “divide” as they were talking about multiplying a term using its multiplicative inverse. In the other four instances, the learners were either operating on the numbers or confirming that the positive sign changes to negative or positive when its term had been moved to the other side of the equal sign.

Did the teacher give the learners opportunities to talk mathematics in the lesson? The summative judgement of how the teacher had invited the learners to participate in the lesson was based on the ME1 and the ME2 since I only observed just one lesson. When considering that Makwe did at least invite the learners to provide phrases and sentences on both episodes of his lesson, the teacher was at Level 2 when analysed using the MDI framework.

The object of learning

Makwe’s lesson was incoherent when compared to those of the other three teachers. The analysis of exemplification, learner participation and explanatory communication showed that the lesson was aimed at operating on known procedures of solving linear equations that involve using additive and multiplicative inverses. However, as much as additive inverses were occasionally used to collect like terms, multiplicative inverses were not featured in the lesson. As such, Makwe had relied more on single cases of ‘dividing by specific numbers’ when isolating variables, although he did use general principles of balancing the left-hand and the right-hand sides to legitimate the procedures of solving the equations.

When looking across the lesson, the strength of connecting the three mathematical episodes was how the teacher had selected and sequenced the examples. As explained above, the same structures of the examples were used from the ME1, to the ME2, and to the ME3. For instance, Makwe was seen comparing Examples 1.1 and 2.1 when the learners were copying and pasting what they did in Example 1.1 as they were struggling to apply the same logic to Example 2.1.

9.7 Table summarising the teachers’ mathematical discourse in instruction

In this section, I provide a table to summarise the teachers’ mathematical discourse in instruction. The table provides the levels of analyses of the lessons in relation to exemplification (i.e. examples and tasks), explanatory communication (i.e. naming and legitimating) and learner participation of each teacher using the MDI framework.

Table 9.5: Levels of the teachers' mathematical discourse in instruction (Adler & Ronda, 2015)

Name	**Exemplification**		**Explanatory communication**		**Learner participation**
	Examples	Tasks	Naming	Legitimating	Type of questioning
Pandor	2	1	3	3	2
Oena	2	1	3	2	3
Mbose	2	1	3	3	2
Makwe	2	1	1	1	2

As noted in the results, I have discussed all the analyses of the lessons other than those of the teachers' tasks. The tasks for all the teachers' lessons were at Level 1 (Adler & Ronda, 2015). Indeed, the object of learning was to solve linear equations using additive and multiplicative inverses, but questions can be asked: how else the teachers could have changed the tasks while using the same examples from the scripted lesson plans? For example, what was going to happen if the teachers replaced one example with a worked example with errors, and then ask the learners to (1) discuss the errors in the incorrect solution; and (2) work out the correct solution? As such, in Chapter 11, I return to highlight teacher knowledge as a key feature that can enable the teachers to change mathematical tasks to be of higher cognitive levels when teaching the subjects.

9.8 Summary

In this chapter, I presented the results of analysing one lesson of each teacher using the MDI framework. In the results, regarding exemplifications, the teachers had offered the learners opportunities to experience both similarities and contrasts when selecting and sequencing examples for their lessons. Regarding explanatory communication, it emerged that two teachers (Mbose and Pandor) were moving between the manipulations of mathematical symbols and using mathematical words appropriately when naming mathematical objects, whilst Oena further succeeded to encourage discussions in his lesson. When looking at learner participation, three teachers (Makwe, Mbose and Pandor) were characterised to have paid less attention to how they invited learners in the lessons. However, three teachers (Pandor, Oena and Mbose) presented coherent lesson when considering how they used relevant examples and appropriate explanations when teaching algebraic equations.

The presented results in this chapter have shed some light on the teachers' competences of teaching mathematics in the classroom, particularly from their mathematical discourse in instruction as advocated by Adler and Ronda (2015). In this sense, the teachers had an opportunity to demonstrate how they actually teach mathematics in the classroom. In the next chapter, I get to characterise the teachers' teaching of mathematics from both their actions (Darragh, 2016) and their narratives (Sfard & Prusak, 2005), which becomes their mathematics teaching identities.

CHAPTER 10

THE TEACHERS' MATHEMATICS TEACHING IDENTITIES

10.1 Introduction

In this chapter, I look back at the teachers' mathematical discourse in instruction to look forward at how they explain their teaching of mathematics from the Mathematics Teaching Framework (MTF) perspective and other teaching practices. The teachers' mathematical discourse in instruction was concluded from observing four lessons of three teachers (Pandor, Oena and Mbose) when they were teaching Grade 9 algebraic equations, although I focused on analysing one lesson per teacher on solving linear equations using additive and multiplicative inverses. One lesson of Makwe (other teacher) was observed and analysed when he was teaching the same topic as the ones that were analysed for the three teachers. The results of the analyses were reported in the previous chapter.

In the previous chapter, it emerged from the observation data that three teachers (Pandor, Oena and Mbose) had presented coherent lessons as they used applicable example sets and appropriate explanatory communication when teaching the algebraic equations. Whereas, in Chapter 8, when the four teachers were talking about what has changed in characterising themselves after participating in the Transition Maths 1 (TM1) course, only one teacher (Mbose) included mentioning that he now adopts applicable example sets and uses appropriate explanatory communication in order to achieve lesson goals.

In this chapter, I draw from Sfard and Prusak (2005) to analyse the teachers' stories about their actions of teaching mathematics. Within the findings of the teachers' shared experiences of learning and teaching mathematics after participating in the TM1 course, I further consider how the four teachers explain their actions of teaching learners mathematics from the MTF perspective. The teachers' explanations are their mathematics teaching identities, and the parameters of what it means to display competence when teaching from the MTF perspective are in Section 6.5 in Chapter 6. The parameters are based on answering questions that were set out to describe two dimensions (i.e. performance and recognition) of operationalising the teachers' mathematics teaching identities from Darragh's (2016) performance identity framework.

In the next section, I explain why the presented findings are the teachers' mathematics teaching identities. That is, I use indicators from Sfard and Prusak's (2005) narrative identity framework to distinguish actual identity and designated identity of the teachers' mathematics teaching identities. That section is followed by the overview of the teachers' mathematics teaching identities. Then, I present the actual findings with evidence on the teachers'

mathematics teaching identities from Darragh's (2016) performance identity framework, which interrogate their display of competence when talking about teaching from the MTF perspective. To end the chapter, I discuss the teachers' self-recognition in relation to other mathematics teachers and recognition of the teachers by 'others'. These categories of recognitions form part of teachers' mathematics teaching identities. As noted, 'others' are other teachers, learners or school management team (SMT) members. As the 'other' who analysed the observation data and was a member of the TM1 community, I drew from Sfard and Prusak (2005) to distinguish between the teachers' actual and designated identities when considering that I read their stories within the different contexts of learning and teaching mathematics.

10.2 Why the presented findings are the teachers' mathematics teaching identities?

Parallel to Wenger's (1998) notion of learning as doing (practice) and Darragh's (2016) notion of identity as performances, Sfard and Prusak (2005) insist on the need to check the teachers' stories if they were reifying, endorsable and significant in order to classify them as actual or designated identities. It can be emphasised at this stage that these indicators were not used in the study to distinguish the teachers. However, each indicator is discussed in this section to authenticate the findings as being the teachers' mathematics teaching identities.

In the first instance, all the teachers' stories were endorsable. That is, when the teachers were asked to share their understandings of teaching from the MTF perspective, as they participated in the TM1 course, they were not expected to speak about other teachers and those teachers' understandings of learning and teaching mathematics. The teachers' learning to teach from the MTF perspective was purposefully mediated to advance particular outcomes for the TM1 community. At the same time, both the facilitators and the teachers drew from the same context of learning and teaching mathematics in the classrooms. Thus, the teachers were telling stories from their first hand understandings of teaching mathematics and used personal pronouns such as '*I*', '*me*' or '*we*'.

As much as the teachers' stories were endorsable, it was interesting to note that there were distinctions in how the teachers used *'I'* and *'we'* as the personal pronouns of their choice. The *'I'* pronoun was used mainly when the teachers were agreeing with what was promoted in the TM1 community and what they are implementing in their teaching of mathematics. On the other hand, other than when the teachers were inviting the learners to participate in the lesson, if the teachers were agreeing with what was promoted in the TM1 community, but they are not implementing it in their teaching for whatever reason, the teachers were mostly using *'we'* in their defence. For example, as we shall see in the findings in Subsection 10.4.1, Makwe used *'we'* when he was talking about not changing examples, as he would rather search for other relevant examples in previous examination papers and other textbooks. It was the same case

with Mbose when he commented on not promoting group discussions in his classroom, the teacher used *'we'* in his explanations. As a consequence, the 'we' pronouns were mostly situated in the teachers' designated identities.

The other indicator that assisted to distinguish between the teachers' actual and designated identities was that of reifying. In a continuum of less endorsing to more endorsing terms when referring to the teachers' stories, the teachers were using adverbs like *always, must* and *should* to discuss their actual identities. For example, three teachers (i.e. Makwe, Oena Pandor) explained that they *always* tell the learners to substitute the calculated values back into the original equations to check for the correctness of their answers when solving linear equations. At the same time, the teachers were using adverbs like *sometimes, not always* and *can* to discuss designated identities. For example, the teachers do *not always* achieve the lesson goals when teaching mathematics. In this regard, the use of adverbs in the data did not only confirm that the teachers' stories were reifying, but it further contributed in distinguishing between actual and designated identities.

The last indicator of identity talked to how the teachers judged their stories as being significant to them. The teachers were presenting sub-textual stories such as *laughs, sighs* and *frowns* with their explanations during the interviews, but these actions were not considered for the analysis of the study. However, when the teachers were expressing their feelings such as their levels of *frustration, happiness* and *honesty*, those words contributed to the significance of the stories about the teachers. For example, the teachers expressed that they were being truthful when explaining about not always achieving lesson goals because learners lack prior knowledge. When the teachers were expressing their honest feelings during the interviews, despite whether they were being honest or not, they started their sentences with 'if we can be honest' or 'the truth is'. In another example, one teacher (Mbose) expressed that it was 'unfortunate' that he has to use the Annual Teaching Plan (ATP) document as the source of example sets instead of being allowed to also use textbooks. Ultimately, from the phrases they used, the teachers were expressing more feelings of happiness and satisfaction in the actual identities and more feelings of sadness and frustrations in the designated identities.

10.3 Overview of the findings on the teachers' mathematics teaching identities

The findings on the teachers' mathematics teaching identities are presented to match with the four aspects of their mathematical discourse in instruction, which are exemplification, learner participation, explanatory communication and the lesson goal. In addition, as noted, the findings include the descriptions of how the teachers explained their sense of being recognised by others as teachers of mathematics.

Regarding exemplification, the four teachers agreed on the importance of selecting and sequencing examples for example sets. Although we saw in the previous chapter that the teachers use the scripted lessons plans as their source of examples, they still prefer to work with examples that are sequenced from easy to difficult. If the teachers intend to explain certain concepts further using other examples during the lessons, they prefer drawing those examples from textbooks and other resources rather than to create their own. Mbose, Oena and Pandor claimed to be able to adapt (or change) examples when varying examples during the lessons. Makwe is fearful of adapting (or changing) examples as he could struggle to find solutions of those examples if they were not properly developed. As such, when considering that Makwe is fearful of adapting (or changing) examples, one can argue that the effect of similarity and contrast can be depleted when considering their power of highlighting generality of mathematical structure and concepts when teaching the subject.

Regarding learner participation, three teachers (Makwe, Mbose and Pandor) were mainly inviting learners to respond as chorus when telling them what to write on the chalkboard during the lesson. Oena, on the other hand, was encouraging learners to speak as individuals. Whether the learners were participating as chorus or as individuals, the three teachers (i.e. Mbose, Oena and Pandor) expressed that the learners should be encouraged to use appropriate mathematical language. For example, according to the teachers, the phrases that were appropriately used by the learners when solving linear equations included additive inverses, multiplicative inverses and distributive law. Parallel to what the learners were saying, as we shall see in the results, the teachers were not clear in explaining what the learners were writing in their notebooks. With regards to what the learners were doing, as we saw in the previous results, the fact that the three teachers (i.e. Makwe, Mbose and Pandor) were accepting ‘chorus’ responses from learners further lowered the levels of learner participation in their teaching of mathematics. However, Makwe, Mbose and Pandor envisage seeing learners raising their hands to provide responses or even coming to the chalkboard to do peer teaching. Lastly, the four teachers envisage seeing learners engaging more in group discussions.

In the case of teacher explanatory communication, the four teachers agreed on the importance of using words that capture concepts, rather than to read symbols in order to act on a single case when teaching mathematics. Three teachers (Mbose, Oena and Pandor) promoted mathematical concepts such as a use of additive and multiplicative inverses or distributive law, whilst Makwe was envisaging to use such naming of the objects in his explanatory communication. All the teachers promoted mathematical criteria beyond single cases as they use general principles to legitimate mathematics. For example, the teachers were clear in the

fact that learners need to balance the left-hand and right-hand sides of the equal sign when solving linear equations to legitimate a use of additive and multiplicative inverses.

The four teachers were envisaging to always achieving their lesson goals. Thus, in most instances, the teachers carefully select and sequence examples from easy to difficult to achieve lesson goals alongside appropriate explanatory communication. In the findings, three teachers (i.e. Makwe, Oena and Pandor) expressed the importance of using different explanations to communicate the same concepts when teaching how to solve the equations. For example, whilst the observed lesson for Makwe was incoherent to some degree, the three teachers (Makwe, Oena and Pandor) expressed to be using 'inverses method' and 'trial and error method' to explain how to solve the equations. At the same time, the teachers struggled to connect different sections when teaching how to solve the equations. For example, as it was the case for Oena and Pandor's lessons, whereby the first section was to 'build equations' and the second section was to 'solve equations', the two teachers did not highlight that the processes are a reverse of each other.

Lastly, two teachers (Mbose and Makwe) were recognising themselves to have moved from using single cases to manipulate symbols to show learners how to get solutions using appropriate mathematical language. Mbose's was consistent in recognising himself to have improved his explanatory communication. As noted when discussing the teachers' explanatory communication above, Makwe envisaged giving learners opportunities to use appropriate mathematical language when explaining their methods or procedures of finding solutions to mathematics problems. As noted when analysing his experiences of teaching, Oena was recognising himself to have gained confidence when working with other mathematics teachers. Pandor did not recognise himself in relation to other mathematics teachers.

However, as we shall see in the actual findings below, the four teachers expressed to have been recognised by 'others' to have positively changed after learning about the MTF or due to participating in the TM1 course. For Oena, who is the HOD, he felt that his subordinates recognise him as someone with more knowledge of mathematics. Makwe and Mbose gained new roles of assisting other mathematics teachers, although their roles were unofficial. Pandor has been recognised by learners as they ask him to help them more with mathematics when compared to other mathematics teachers.

10.4 The findings on the teachers' mathematics teaching identities

When reporting the findings, the focus is on responding to the questions that were not answered during the lesson observations. For example, during the lesson observations, I established what the learners were saying in the classrooms, but did not establish what they

were writing in their notebooks. Thus, in the next subsections, I discuss each aspect of the teachers' mathematical discourse in instruction including the agreements and disagreements between what was observed in the classrooms and the teachers' explanations during the interviews about the teaching of mathematics.

10.4.1 Exemplification

Only Makwe selected and sequenced the examples from different sources for his lesson. The other three teachers used the examples that were taken directly from the scripted lesson plans. The examples from the scripted lesson plans offered the learners with experiences of similarities and contrast. It was also evident that the four teachers presented the tasks that required the learners to use known procedures when solving equations. Pandor confessed that maybe teachers "rely too much on the lesson plans that are already been made" and the examples that are there. Consequently, the questions such as 'What are the sources of the teachers' examples?' somewhat were answered from both observation and interview data, although the teachers envisaged using textbooks and examination papers as additional sources. Thus, in this subsection, I start by looking at how the teachers are selecting, sequencing and adapting (or changing) examples before looking at what they envisage in their teaching of mathematics.

The four teachers commented on the importance of selecting and sequencing of the examples to include the levels of difficulty. Pandor admitted to not have always been careful in selecting and sequencing the examples, as he would sometimes start with difficult examples, whilst now he understands that "the learners must work from the known and then they can move to the unknown". Oena was adamant to have mastered the strategy of sequencing the examples from easy to difficult. Oena and Makwe agreed that one or two challenging questions are important when sequencing examples because they accommodate learners who are quick to solving problems using standard procedures. For Mbose, one or two examples should "include real-life situations". Makwe added that he now incorporates "his style" when selecting and sequencing examples for different tasks. By his style, he was referring to including or excluding certain examples in different tasks to accommodate different ways examiners ask questions during examinations.

The teachers were considering similarities and contrasts by looking across an example set when presenting lessons. This means, for example, when looking at clusters of examples in the introduction, those examples must resemble examples in the lesson development, and the examples in the lesson development must again resemble examples in the classwork activities. This was the case in how Makwe selected and sequenced the examples for his lesson. In explaining similarity, Makwe stated the following:

> I tried also to make the classwork and the homework look similar to what they were doing except for the last question in the homework, because now that's what I wanted to choose in the next lesson, that's why I added that question.

This comment followed Makwe's explanation about the importance of examples that move from easy to difficult. In this comment, the teacher expressed his conscious decision in making sure that the cluster of examples in the classwork are the same as those in the homework, whilst the last example in the homework was meant for the learners to experience a contrasting example. The other teachers articulated the same logic. For instance, Pandor explained that he cannot start with the equation without brackets and then from there, when assessing learners he gives them brackets. By the word 'assess', the teacher was referring to the work that learners are expected to present from classwork or homework activities.

Considering that the three teachers (i.e. Mbose, Pandor and Oena) did not change the examples in their lessons, although Pandor excluded the three examples in the lesson, the question of when do the teachers adapt (or change) examples was categorised as what the teachers were envisaging in the future when teaching the subject. The three teachers (i.e. Mbose, Pandor and Oena) stated that they were able to change examples, but outrightly, expressed that it is unnecessary to change examples as they could get more examples from the textbooks and other resources. The teachers explained that they get more examples in order to supplement their explanatory communication when learners are having difficulties with the presented examples. Pandor elaborated in the following extract:

> ...if a learner can say, I did not understand this. You are able to make another example... [if] you have one example... then you give a classwork and only to find that they don't understand, now if you want to explain further, it's either you go back to the very same example, which they did not understand or go to what you want to assess them with.

On the other hand, Makwe stated that for him changing examples can be problematic. The teacher elaborated as follows when he was commenting in this regard:

> No. no, I didn't change anything, I took it as it is because sometimes we have this fear of you creating your own question then you can't even solve it yourself. So, I try to avoid such. I took the questions as they are and if I can't find the question that I want to use, then I go to other resources because I ended up looking at the past Grade 9 question paper for November to get examples from there.

The above comment can be linked to the teacher's understanding of solving linear equations. The understanding of solving linear equations can begin from 'building equations', like what we saw in Pandor and Oena's lessons. The idea of building equations focuses on the notion of equivalence whereby one can start by presenting a certain value of a variable, and then act

on both sides of the equal sign to create equations. A reverse process of building equations is solving equations.

When reading more into not being able to adapt (or change) examples for example sets as it is the case for Makwe, one can argue that the effect of similarity and contrast as advocated in the TM1 course can be depleted when teaching mathematics. Whilst Mbose, Oena and Pandor claimed to be able to adapt (or change) examples for example sets, there was no evidence where the teachers were explicitly referring to the effect of similarity and contrast in their lessons. As alluded to in Subsection 4.2.2 when discussing the mutual engagement of the members of the TM1 community, examples with similarity and contrast highlight generality of mathematical structures and concepts when teaching the subject.

10.4.2 Learner participation

When looking back at how the learners were invited to participate in the lessons during observations, we saw the three teachers (i.e. Mbose, Pandor and Makwe) allowing the learners to respond in chorus when mostly operating on numbers to provide single answers and completing unfinished sentences with phrases. At times, the four teachers were asking *how* questions, but Pandor and Mbose were sometimes reducing those questions to *what* questions when learners were indistinct in their responses. Other than being able to move between *what* and *how* questions, Oena also provided the learners with an opportunity to explain their work to each other.

When looking forward, the three teachers (i.e. Mbose, Pandor and Oena) confirmed that the learners were using appropriate mathematical language when participating in the classrooms. For example, Mbose explained that the learners were using phrases such as ‘distributive law’ when they were multiplying monomials and binomials. In another example, Pandor and Oena raised the same point that the learners used mathematical language appropriately, in their examples, the teachers were referring to how the learners were using additive inverses to collect like terms when solving linear equations.

The other teacher, Makwe, declared that the learners in his lesson were focusing on what action to take when manipulating the equations. In Makwe’s words, “They [learners] said lots of things. Things like, ‘we must divide’, ‘what you do on the one side, you must do on the other side’”. However, when critically looking back at the teacher’s lesson, the learners were mostly completing the sentences, whereby for instance, the teacher would ask “what you do on the one side?” and the learners would say “you must do on the other side”. At the same time, Mbose and Pandor commented on the fact that the learners were also operating mentally on numeric terms to provide single answers towards steps when solving the equations. For

example, Mbose stated that the learners were providing “answers that they were getting when using a calculator or... if the problem did not have huge numbers, they were using their minds”.

What did the learners do in the teachers’ lessons? In Mbose, Pandor and Makwe’s lessons, the learners were mainly telling the teachers in a chorus what to write on the chalkboard as they were responding to the questions. In Mbose’s terms, “they [learners] should talk to me. As I’m busy teaching, they also participate by telling me what to write” on the chalkboard. Pandor and Makwe confirmed that getting learners involved in answering questions as a chorus is one form of learner participation. Pandor explained that he does not mind when learners are ‘singing’ the phrases such as additive inverses when participating in the lesson, whilst Makwe encouraged chorus when learners are operating on numbers. That is, according to Makwe, “Sometimes I want them [learners] to be involved as a class where I say three times two, then everybody screams “six””. In this regard, the inclusion of what the learners are doing during the teaching of mathematics for the three teachers (Makwe, Mbose and Pandor) further lowers the levels of learner participation from the observation data when considering a mismatch between the forms of learner participation that they are encouraging when teaching mathematics and what was promoted in the TM1 course.

In Oena’s lessons, the learners were participating as individuals when they were telling him what to write on the chalkboard or when given an opportunity to write on the chalkboard themselves. The teacher stated the following:

> Now one learner... one learner managed to make sense of what is really happening by getting rid or trying to arrange, the like terms to be on the right-hand side... the terms with variable to be on the left-hand side and all the constants to be on the right-hand side.

In this comment, Oena was identifying with individual learner participation as he expressed that one learner was managing to use additive inverses to collect like terms when solving the equation. The teacher was using an example from his example set to highlight what the learner was doing. The learner was solving the equation $3(x + 2) = x – 12$, and he was at the second step (i.e. $3x + 6 = x – 12$) after using the distributive law to remove the brackets.

When learners are providing verbal answers to teachers, according to the teachers, that becomes an opportunity to identify correct and incorrect responses from the learners. For Mbose and Makwe, some of the correct responses are based on the learners’ prior knowledge. For instance, Mbose acknowledged that the learners must have known about the distributive law since they learned about it in Grade 8. Otherwise, Mbose and Pandor explained that teachers can use more examples to remind the learners the mathematical procedures. For the four teachers, the objective of getting verbal responses when interacting with learners

contributes towards achieving the lesson goal, whereby the learners get to produce correct responses to demonstrate their understanding of mathematics.

What did the learners write in the teachers' lessons? The teachers were not explicit in explaining what the learners wrote in their lessons. What came up was that the teachers provided example sets for the learners to copy in their notebooks. That is, as the teachers were explaining different steps for each example on the chalkboard, whilst being backed by the learners, the learners were copying down those examples, especially when the teachers were introducing and developing the lessons. Then, the learners produced solutions of other examples on their own when doing the classwork activities. Mbose explained that "They [the learners] were writing down the answers that they were getting... when they were using their minds". Whereas, Pandor and Makwe argued, the learners were required to follow similar procedures that were presented when solving the examples during the introduction and the development of the lessons. Thus, if the learners were able to use the procedures appropriately, and got correct answers, according to the teachers, the learners understood what was happening in the lessons.

There are different kinds of learner participation that the teachers are envisaging in their teaching of mathematics. Mbose had listed the following three kinds:

> I would say [1.] learners must raise their hands whenever they want to speak, they want to say something. Then, 2. learners can, provided we have enough time, can group themselves so that they do whatever problems that I've given to them. Then, from there, [3.] they could have a peer teaching... So peer teaching, learners teaching another learner or we could have a learner coming in front. Then, the learner writing down whatever answer that they think it's correct but obviously interacting with the other learners who will be in class.

The idea of wanting learners to participate as individuals emerged strongly from other teachers' comments as well. Makwe stated the following:

> Sometimes I want them to be involved individually where I ask a question and they raise their hands, they give me different answers. Even if a person gives me the correct answer, I still say, do you agree with him? Is there anybody with a different answer? So that I can know what they are thinking, how are they solving this, so that we can rectify those mistakes.

For Makwe, when learners are participating as individuals, there are opportunities to solicit different answers from the learners, whilst embracing learner errors and misconceptions. Pandor and Oena shared the same sentiments. For example, Pandor expressed that most of the time when individual learners are quiet, you won't know exactly what is happening in their minds. In another example, Oena stated that he wants individual learners to feel free to say "let me do the sum for you, eeh... you have made a mistake". Oena intentionally makes

mistakes now and then by changing numbers, for example. In that way, the teacher gets to see whether the learners are paying attention in the lesson. If learners are paying attention, according to Oena, individual learners get an opportunity to jump in and correct mistakes from teachers.

Secondly, the idea of group discussions emerged as well with the other teachers (Makwe, Mbose and Pandor). The teachers explained that, due to time constraints, they do not encourage group discussions. Mbose elaborated as follows:

> The truth is, in mathematics strategy [Annual Teaching Plan], we don't get enough time, but prior to mathematics strategy [Annual Teaching Plan], we used to get that time. Because just grouping the learners alone, it takes a lot of time because we have to tell this one to come to this one. This one does not want to be a part of the group with these ones...

Makwe, Mbose and Pandor expressed that if they would have time for group discussions, they could walk around visiting all groups in the classroom to observe if the learners are doing everything correctly. In the groups, as noted above, Mbose had imagined peer teaching, whereby "the stronger ones are teaching the other ones who are weaker" using different methods.

On the same idea of group discussions, Pandor and Makwe talked about the importance of using physical objects (i.e. models) to teach the learners practical problems. Makwe expressed that now and then he is able to bring things like dice to the classrooms to teach topics like probabilities when he had managed to save time in other lessons. At the same time, although Pandor admitted that group discussions were not happening in his classrooms, the teacher imagined using a geoboard to teach translation, for example. In Pandor's terms, "you give them [learners] translation that they're suppose to take, you watch and then you see them... I'm going to the left, which means it's going to be negative".

Lastly, whilst this was not a problem for Oena, the idea of wanting learners to come to the front emerged strongly as well in Makwe's comments. Makwe expressed that he "would love to have one or two learners coming to [the front to] explain how they understand it [mathematical concepts]". However, Makwe explained that "due to time [constraints], it's very difficult for such things. And behaviour again, once you bring a learner to the chalkboard, then it becomes a mess". In this regard, a substantial mismatch can be highlighted as well when considering what the teachers valued when participating in the TM1 course, which included going to the front of the class to share solutions with their peers, and what they are envisaging in their teaching practice.

10.4.3 Explanatory communication

Similarly in this subsection, I look back at how the teachers used explanatory communication when teaching algebraic equations, and look forward to describe what they consider to be central in explanatory communication when teaching mathematics from the interview data.

As reported in the previous chapter, it was only Makwe who had remained on using non-mathematical words and the manipulation of symbols with an inadequate shift to using formal mathematical words appropriately. In contrast, during the interviews, Makwe explained that he puts effort in connecting non-mathematical words and formal mathematical words to invite learners to understand mathematics.

The four teachers were agreeing on the importance of using words that capture mathematical concepts, rather than to read symbols in order to act on a single case. The teachers referred to the lesson of solving algebraic equations that I observed to highlight that the words such as additive and multiplicative inverses were central when explaining mathematics. The other mathematical words that the teachers put forward included 'distributive laws' instead of talking about 'removing brackets', 'expressions' to be on either side of the equal sign of equations, 'isolation of variables' instead of 'get rid of'. Mbose and Oena acknowledged that such words were emphasised in the TM1 course, whilst Pandor stated that the ATP document highlights the importance of using such words.

Mbose, Oena and Pandor provided examples to explain how teachers can use words like additive and multiplicative inverses appropriately. For instance, Pandor explained:

> They must know what is the multiplicative inverse as compared to 'divide by 3'. Some would say, if I have 3x equal to 6, then you say, I divide by 3. You have to interject that 'division by 3', is not just 'divide by 3', it's multiplicative inverse.

The three teachers went on to describe what multiplicative inverses are – the fact that the product of a number and its multiplicative inverse must yield one. Oena was agreeing with Pandor on the need to interject when learners are using single cases, such as to say 'divide by', instead of using a multiplicative inverse. For me, what remains crucial is that teachers should make learners aware that the process of dividing a term by a number to isolate its variable emanates from the mathematical principle of multiplying that term by its relevant multiplicative inverse.

In the case of additive inverses, Mbose, Oena and Pandor were also in agreement on the fact that such inverses aid the process of collecting like terms whilst maintaining the balance between the left-hand and the right-hand sides of equations. The three teachers were also clear in describing what additive inverses are – the fact that the sum of a number and its additive inverse must be zero.

However, two teachers, Mbose and Makwe, explained instances when they use words like ‘transpose’ instead of additive inverses to collect like terms. Makwe explained that when planning his lessons, he includes words like additive inverse. However, his plans of using formal words tend to fall apart when he gets to the classroom, and gets to be drawn to use words like transpose when teaching the subjects. Here are his explanations.

> But once you’re there in the classroom and you see that this ‘transpose’ now, they [learners] are in an English class. They’re concentrating on it more than what we’re doing, and you end up diverting, but yeah, I actually wanted to use mathematical words.

Mbose argued that transposing terms can help learners to work out solutions with fewer steps which in turn can reduce chances of errors, and that learners can draw from their prior knowledge to solve the equations. The teacher explained his reasons as follows:

> What happens is that when you do additive inverse, the sum becomes so long such that after you’ve written everything on the left-hand side and the right-hand side during that balance, you find that learners after that make a mistake. But with transposition, you find that you’re dealing with only one term or two terms. It doesn’t become, you don’t have many terms, even equations so I also used the transposition.
>
> Number 2, the reason I used the transposition is because they have already been taught that. They understand that method. So I am continuing in what they already know... you observe if they are doing a correct thing, they’re getting the final answer. Because, remember that the learners they don’t come empty in their minds.

As Oena and Pandor were sharing same concerns about the ‘transposition’ processes, Pandor provided an efficient technique to help learners to work out solutions with minimal mistakes while using additive inverse. Pandor explained that sometimes he ignores to include the sum of a number and its additive inverse once the learners are in tune with the notion of collecting like terms. This is his explanation.

> If I put an additive inverse, automatically it tells, it tells me that I’m going to have a zero, which means that you’re not gonna have anything. Because I’m not gonna have anything, maybe with a first example, I can put the additive inverse, the second one... the third one, I simple say, when I add the additive inverse, what do I get, I get a zero, which means, I don’t write anything, which means that this additive inverse is on the other side. Yes. It becomes easy.

Pandor’s technique of collecting like terms becomes transposition, but he does not dwell on talking about moving terms from one side of the equal sign to the other. Oena asserted that learners get confused and ask themselves “what is it that you take to the other side”. Oena and Pandor prefer to use additive inverses as they say it becomes easy for learners to solve linear equations.

So far, the discussions on the teachers' explanatory communication, particularly on the naming of objects, have dwelt on the different methods the teachers prefer (or not prefer) to use when teaching how to solve linear equations using additive and multiplicative inverses. However, which was the case for Mbose, Oena and Pandor in the observed lessons, it becomes imperative for teachers to shift between the manipulation of the symbols to neatly find solutions and the use of appropriate mathematical language to connect concepts when teaching the subject.

When looking back at whether the teachers were using mathematical proofs to legitimate the mathematics, we saw that only Mbose did not check with any example if the worked out answer was correct. That is, the other three teachers substituted values of variables back into the equations to check if the left-hand and right-hand sides were equal. During the interviews as well, the three teachers (i.e. Makwe, Oena and Pandor) commented on the importance of encouraging learners to check their answers. For example, Makwe stated the following:

> What I was concentrating on was that they must always check their answers because sometimes they are so quick to find the unknown but they're not sure whether it's correct or incorrect. So, when they check, at least they'll know that the right-hand side must be equal to the left-hand side because it's an equation.

Although Makwe did not see this as a mathematical proof, and had linked this comment to how teachers can realise to have achieved lessons, the three teachers stated they use this technique to convince learners on the correctness of their answers, of course if all other workouts are correct and accurate.

Lastly, Mbose envisaged including mathematical proofs in his lessons. The teacher understood mathematical proof to be equivalent to using another method to get to the same answer. The teacher contended that, in topics like geometry, teachers can always show learners different methods of arriving at the same answer. However, since they started using the scripted lesson plans, the teacher expressed his concerns as follows: "there's no time for that [using different methods]. So, there's no time to justify using other methods and show them [learners] that in the end, they're going to give you one and the very same answer".

10.4.4 The lesson goal

When reflecting on the questions that were set to guide this subcategory of teachers' mathematics teaching identity, the questions that were not answered during lesson observations are as follows:

1) Did the teachers use more than one explanation to connect same mathematical concepts?

2) Why the teachers did not connect the introduction sections to the main object of learning?
3) How do the teachers justify failing to achieve lesson goals at times?

The second question was daunting for Mbose who did not encourage learners to substitute the unknown values back to the original equations to check the correctness of the answers. Consequently, below, I provide details of how the teachers responded to these questions alongside their explanations of acting (or not) in a particular way to achieve lesson goals in their lessons.

Did the teachers use more than one explanation to connect same mathematical concepts? The four teachers explained that they used the inverse operations method to solve the equations. That is, as noted, suppose you are dealing with the equation of the form $Ax + B = C$, where B and C are constants, and A is a coefficient of a variable x, then the teachers firstly added the additive inverse of B on both sides of the equations to collect like terms. Then, the teachers multiplied Ax with the multiplicative inverse of A on both sides of the equation to isolate x and get the solution. For example, Oena provided the following explanation:

> When dealing with solving these equations, the aim was to isolate if maybe we do have a variable x, maybe with that variable. Then we had to use the additive inverses and the multiplicative inverse and explain to them [learners] the method of using those inverses and then so that at the end of everything you will remain with x, the value that is unknown to them.

Makwe, Oena and Pandor expressed that they go on to encourage the learners to check the solution by substituting the value of the variable back to the original equation to see if the statements are true on both sides of the equal sign.

Makwe and Pandor also presented the trial and error method when solving the equations. In the trial and error method, learners are invited to guess the unknown value and test it by substituting it back to the equation to check if the left-hand side of the equal sign is equal to the right-hand side. A starting point to this trial and error method start with describing an unknown value within the mathematical statement. For instance, if asked to solve for $2x + 6 = 12$, an unknown value can be found by asking 'what is the number that if multiplied by two and add six, you get twelve?'. When responding to that question, there is a potential of guessing that number before substituting it in the statement for its validation. In this regards, Pandor explained the method as follows:

> The best way, I think prior to the one that we did [inverse operations], learners could on their own, be able to substitute the correct value without algorithmic. To say, OK fine, if, in order for, the left-hand side should be the same as the right-hand side.

Makwe shared the same thinking with Pandor. For Makwe, he wanted the learners to understand that solving equations can be associated with “boxes we used to have in primary school” as there is nothing difficult to it other than to find the unknown in the equation. Moreover, both teachers were emphasising that once learners have decided on a number, they need to substitute back into the original equation to check for its correctness.

The other two teachers, Mbose and Oena, were seeing the known procedures that involved ‘transposing’ terms instead of using additive inverses in order to collect like terms as another method that can aid in explaining how learners can find the unknown. However, as we saw, Oena prefers to use inverse operations, whilst Mbose switches between the two explanations in his lessons.

As Mbose and the other two teachers (i.e. Oena and Pandor) had the introduction sections, the question that remained unanswered was how come they did not connect those introduction sections to the main object of learning. If we can recall, in Mbose’s introduction section, the learners were required to substitute the given values to identify with the ones that will make the statements (equations) true. When Mbose was responding to this question, the teacher argued that he was following the scripted lesson plan, and the scripted lesson plan did not articulate any need to connect the sections. The teacher elaborate as follows:

> Okay, remember it’s different. We have mental maths. We have lesson development. We have classwork. We have homework... We are writing mental maths. They were writing the lesson development, and they were writing the classwork, and they were also writing the homework as prescribed by the strategy [Annual Teaching Plan].

Mbose continued to point out that it is the way they have written these things, and you must follow their methods to the letter. The other teachers did not comment in this regard although for them it was about not using the available answers since the introduction sections were about ‘building the equations’ whilst the main object of learning was about ‘solving the equations’.

How do the teachers justify failing to achieve lesson goals at times? Prior to answering that question, the following question can be asked: how do the teachers know if they have achieved the lesson goal? For the teachers, the indicators of knowing to have achieved lesson goals are through how learners engage with examples. Makwe and Pandor explained that they prefer walking around the class to see what learners are writing in their notebooks when doing classwork activities. For example, on the one hand, Pandor elaborated as follows when he was expressing how he determines if he had achieved the lesson goals:

> After we’ve shown them the steps, this how you go about and then, with correct examples that are similar to the ones that you have made an example with, if you give them and you go around

> and check, if they are able to do the same way. Yes. They're getting the correct answer, and then you know very well that, which means they have, the lesson was a success.

Mbose asserted that he knows that the lesson goals are achieved whenever he asked learners questions based on what he was teaching in the lesson, and the learners are able to interact with him by providing correct answers when dealing with different steps in examples – I finally get to address the question of the importance of correct answers in Section 11.3 when answering the research question 3. On the other hand, Oena considers that the lesson goal has been achieved when learners are able to discuss the mathematical concepts when working out examples. When Oena was explaining his position of how he achieves the lesson goals, he used the lesson that I observed as an example, and repeated the inverse operations method of solving linear equations.

The reality is that the teachers envisage achieving lesson goals in all their lessons. In justifying why they sometimes fail to achieve lesson goals, the teachers highlighted that learners do not always have necessary prior knowledge in certain areas of mathematics. For example, Mbose explained:

> On some occasions, I do achieve the lesson goals, but on some topics, you find that I don't achieve the lesson goals. It's because… you find that the prior knowledge of learners is not good. So, now you start going back and do something that they were supposed to do in grade 8.

Pandor further explained that learners treat each topic like they are seeing it for the first time, and even argue that "we never did this" before.

The other point that Pandor raised of not achieving the lesson goals was the language barrier. Earlier, when Pandor was talking about the importance of explanatory communication, the teacher argued that code-switching can hinder an appropriate use of mathematical words. The teacher only uses English in his teaching of mathematics. The teacher understands code-switching as an attempt to translate mathematical words, whilst Setati and Adler (2000) describe code-switching as a movement across languages and discourses in the context of a single conversation. Perhaps that is the reason the issue of language barrier arose in Pandor's comments.

When reading Mbose and Pandor's comments, the teachers are blaming learners for not achieving the lesson goals in some of their lessons. However, as noted in Section 7.3, teachers had mixed views about accountability towards teaching mathematics, and they neither agreed nor disagreed to whether "learners are to be blame for their performance in mathematics". In this sense, the teachers can be characterised to resist the urge to openly blame learners for their performance in mathematics.

10.5 Self-recognition in relation to other mathematics teachers

In this section, I explain what the teachers recognise about themselves after participating in the TM1 course in relation to other mathematics teachers. Like in the previous sections, as I extend or confirm their stories about their competences of teaching mathematics, I continue to characterise the teachers from their actual and designated identities.

When Makwe was positioning himself in relation to other mathematics teachers, he believed that they were all "on the same page", and that they had the same goal of "just wanting learners to pass" mathematics. For learners to pass the subject, Makwe understood their teaching to entail showing learners how to manipulate algebraic symbols using known procedures to find correct answers. The teacher commented as follows:

> We had the same mindset, that if they [learners] just know… we had the same sort of teaching, you know, thinking that it's not us the problem, it's the learners. Maths is difficult for these learners but let them just get marks here [and there]. If I just show them this, if you get a question like this, just know, neh, you do this, without explaining to them why they do that. I think we had the same mentality before training.

Makwe explained his identification with mathematics teaching as a teacher who wanted to "just show" learners how to respond to questions and learners will be able to score marks in the assessments. Given that his attempts were not always successful, the teacher had understood mathematics as a difficult subject for learners.

Mbose agreed with Makwe that as mathematics teachers, they were focusing on showing learners how to manipulate algebraic symbols when working out solutions of questions. For example, he stated that "we were using transpose only" to show learners how to collect like terms when solving algebraic equations. The evidence of the teacher's assertions is as follows:

> So it's what we were using with other educators, because it's what we were using, but when you [learn about the MTF], you get words like the multiplicative inverse, you have to do your additive inverse, and most of the times those are the words that we were not using. We were using transpose only, you see.

The two teachers, Mbose and Makwe, commented on being very different from other mathematics teachers now. Makwe emphasised that other teachers are "still stereotyped on the position that learners just need to know this". Makwe stated the following:

> Now, I'm at a point where it's about understanding, you know. It's not about just doing it, they [learners] need to understand why is it like this, why do we do it like this. I'm not, I'm no longer teaching for an assessment that I'm going to teach you so that when the exam comes, you get a 100%, no. Now I'm teaching more for understanding.

When teasing out from Makwe's comment, the teacher envisages including questions like "why is it like this" or "why do we do it like this" when inviting learners to participate in his lessons. As we saw, Makwe remained mostly in inviting learners to complete unfinished sentences or to provide 'yes/no' answers. Thus, for me, a starting point to teach for understanding will be for teachers to be conscious in inviting learners to make and see connections between different concepts and representations. For example, teachers can draw parallels between building equations and solving equations.

For Makwe, as he elaborated on what he meant by understanding, the teacher stated that presenting random examples from the textbook was not enough for teaching mathematics. Makwe mentioned that he saw a big gap on how he selects examples when planning for his lessons. Makwe stated that he perceives other teachers to "just pick a page on the textbook" and say to learners, "do all the questions there". The teacher emphasised that teachers should use additional teaching resources like physical mathematical objects (i.e. models) and instruments in order to teach for understanding. In addition, when picking from Makwe's previous comment, teachers should justify to learners why they are doing what they are doing. Importantly, the teacher mentioned that when learners understand mathematics, they tend to "enjoy it, and not see it as a difficult subject but seeing it as a nice, fun, and challenging subject" to learn, which define learner identity.

Pandor did not comment directly to how he is now different as a mathematics teacher when comparing his teaching with those of other mathematics teachers. The teacher merely expressed that other teachers do not consider themselves as mathematics teachers, and thus, they do not teach all the topics. Pandor explained that a teacher would say "[a]rgh, we did not do this [topic in class with the learners]". In reacting to such comments, Pandor explained that when other mathematics teachers tell him that they do not teach all the topics, he has to understand their positions of not being capable to teach all the topics. Pandor clarified that some teachers are recruited to teach Mathematical Literacy, but once they get to schools, they suddenly do not feel comfortable to teach the same Mathematical Literacy. Those teacher then ask to teach pure mathematics, and then they "compromise the very same mathematics" they have asked to teach by considering themselves as "auxiliary mathematics teachers".

Lastly, Oena saw himself to have gained confidence when teaching from the MTF perspective and when compared to other teachers. In explaining the challenges with other teachers, he argued that some teachers are proficient in mathematics, but struggle to see how all fit together for learners when teaching in the classroom. Because of his boosted confidence, Oena is able to sit with those teachers and assist them.

10.6 Recognition of the teachers by others

Considering that the teachers had expressed changes in their teaching of mathematics, I am assuming that they were following through in expressing their mathematics teaching identities as recognised by others in relation to changes due to their experiences of learning in the TM1 course and due to teaching from the MTF perspective.

The four teachers felt that they were being recognised in honour after participating in the TM1 course. All the recognition by others was characterised to contribute positively in the teaching of mathematics. Makwe explained that in his school, he is not an HOD, but he assumed the role of an assistant HOD for grades 8 and 9 mathematics. Makwe elaborated on what he does to assist other mathematics teachers:

> I'm the one who runs around to tell them [other teachers], "we have equipment like this here, I'm sharing this with you, here are the protectors for the boards. Give them [learners] individually and use this", you know. [He further tells other teachers,] "If we are more practical, they understand".

The role of an assistant HOD is not an official position in schools. However, Makwe explained that the HOD in his school had expressed confidence in his teaching of mathematics. Hence, the teacher was given such roles.

Oena is the HOD in his school, but he expressed that he now feels that other SMT members have gained more confidence in him. Oena stated the following: "I'm an HOD, but if there is something that has to do with maths, I'm the first one to be called. They say, [Sir] Oena, come." Oena is convinced that other SMT members call him to genuinely get his views about matters relating to mathematics.

In Mbose's case, a subject adviser recognised change in him after attending the TM1 course. The subject adviser had confidence in Mbose's teaching of mathematics and asked him to be a lead teacher of a cluster in the district. Being a lead teacher is not a formal position. Lead teachers get to share their experiences with other teachers and lead sessions during teacher professional development workshops that are organised by subject advisors for different clusters in the district. Mbose expressed his position:

> They [subject advisor] came this year and asked me to be one of the lead educators when they are training... so that they can use the knowledge that I've gained in the TM1 course so that I also share that information with other educators.

In the workshops, Mbose explained that other teachers begun recognising him and approaching him for additional teaching materials and for other teaching methods. According to Mbose, other teachers now ask him questions about teaching. Here are the kinds of questions other teachers approach him with now: "They are using this method and the learners

they don't understand, what do you think is the best method that we can use to teach our learners when it comes to problems like this?"

Pandor talked about how learners have gained confidence in his teaching. Pandor explained that the learners tend to rely on him as that teacher who can assist them. The teacher claimed that the learners do talk about different teaching styles to each other and begin to compare how different teachers teach mathematics in the classrooms. In that way, even the learners who do not attend Pandor's classes tend to approach him for help. According to the teacher, in some cases, "it's frustrating" if a teacher feels "overstretched" and "cannot assist" all the learners.

The four teachers appreciate the exposure that comes with more roles in the teaching of mathematics. For the teachers, getting more roles creates opportunities for them to learn and teach more mathematics with confidence. For example, Mbose commented on the new roles as follows:

> I will say that they [new roles] are helping me, okay. In that, they are building my confidence. At first I wasn't like that, but now I'm doing this [leading sessions in teacher workshops]. Because of the TM1 course that I attended, I am able to successfully do [participate in] the programmes. It's making me to want to do more, to learn more when it comes to mathematics.

When analysing teachers' comments about the recognitions by others, it was clear that being entrusted with new roles in the teaching of mathematics is empowering for the teachers. The teachers have frequently expressed that being recognised by other teachers after participating in the TM1 course have boosted their confidence to further participate in the learning and the teaching of the subject. In this sense, the four teachers expressed that they are comfortable with teaching Grade 9 mathematics.

10.7 Summary

In this chapter, I have built on the teachers' mathematical discourse in instruction from the observed lessons to conclude on their teachers' mathematics teaching identities. When concluding on the teachers' mathematics teaching identities, I argued that the three aspects of their mathematical discourse in instruction (i.e. exemplification, explanatory communication and the object of learning) were mostly matching how they teach mathematics (or how they talk about teaching mathematics), which was their actual teaching identities. However, regarding learner participation, the teachers were mainly inviting learners to respond as a chorus when telling them what to write on the chalkboard during the lesson although they envisage seeing learners raising their hands to provide responses and seeing learners engaging more in group discussions.

CHAPTER 11

CONNECTING LEARNING, PRACTICE AND IDENTITY IN THE COMMUNITY OF PRACTICE

11.1 Introduction

In the previous four chapters, I have presented the findings explaining the four fundamental objectives of this study as guided by the research questions. The first objective was to determine the extent of shared experiences of teachers who had participated in the Transition Maths 1 (TM1) course. The findings on that determination were discussed in Chapter 7. The findings emerged from the quantitative processes, particularly from using Exploratory Factor Analysis (EFA).

The second objective was to examine what has changed to bring about shared experiences of teachers who had participated in the TM1 course. The findings regarding that objective were discussed in Chapter 8. The context of analysing stories in that chapter was on providing evidence of *what* and *how* the teachers experienced changes in relation to the five learning components (community, practice, meaning, confidence and identity) as advocated by Wenger (1998) and Graven (2003, 2004). However, in this final chapter, I draw from Sfard and Prusak (2005) to claim that the teachers' stories in relation to changes in their learning connect to their mathematics teaching identities, but I focus on how such connections emerged from the consolidated data analysis.

The third objective was to present the findings on the teachers' mathematical discourse in instruction. The findings on the teachers' mathematical discourse in instruction were discussed in Chapter 9. The findings emerged from the analysis of the lesson observations. For that analysis, I drew from Adler and Ronda (2015) to decide on the teachers' competences of teaching mathematics, particularly to conclude on which aspects of the Mathematics Teaching Framework (MTF) the teachers incorporate (or not incorporate) to achieve the lessons goals in their teaching of the subject.

The last objective was to elaborate the teachers' mathematics teaching identities based on their stories about teaching mathematics from the MTF perspective. That elaboration was made in Chapter 10. The teachers' stories about teaching mathematics were differentiated according to what they actual do (actual identity) and what they envisage to do in the future (designated identity) as posited by Sfard and Prusak (2005).

In this chapter, I start by drawing from the findings in Chapters 7, 8 and 10 to answer the research question 2 (RQ2), which is 'In what ways (if any) does the teachers' learning in the

TM1 course connect to their mathematics teaching identities?'. Then, I consolidate the findings from Chapters 8, 9 and 10 to answer the research question 3 (RQ3), which is 'What are the connecting features (if any) between the teachers' mathematical discourse in instruction and their mathematics teaching identities?'.

Furthermore, in this chapter, I discuss the implications of the findings to teachers in communities of practice followed by the implications of the findings for further research. Then, I discuss the limitations of the study. To end the thesis, I provide final concluding remarks about the study.

11.2 Connecting the teachers' learning and identity to answer the RQ2

From the explanatory mixed methods research design of this study, as noted in Section 6.6, four key components of the teachers' learning in the TM1 course connect to their mathematics teaching identities. The four components are practice, community, meaning and confidence. The four components were expected to connect to the teachers' mathematics teaching identities as the research design inductively relied on Wenger's (1998) social theory of learning. Wenger's theory formed the foundational framework of the study. Wenger's (1998) theory emphasises the contiguity of learning and identity in the process of participating in a community of practice. The TM1 course, as I have argued in Chapter 4, was the community of practice. The teaching practices in this community of practice were informed by learning about the MTF. The teacher community was undergirded by experiences of learning in the TM1 course, particularly in learning mathematics-for-teaching (MfT). In understanding the learning of MfT, although I had distinguished the learning of mathematics content and the learning of mathematics teaching content in the TM1 course when describing the joint enterprise, the study highlighted that *how* and *why* the teachers learned in the course had shaped their experiences (including their identities) of teaching the subject.

How does the teachers' learning in the TM1 course connect to their mathematics teaching identities? From the quantitative processes, drawing from Wenger's (1998) theory, it was put to the teachers that learning about the MTF meant changing their identity. It emerged from the findings that *since* the teachers learned about the MTF, *then* they are valuing learning and teaching more in relation to their positive sense of identity, to their understanding of mathematics and to teacher confidence. In this causal effect proposition, the teachers provided dominant views about the importance of valuing learning and teaching when responding to the closed-ended questionnaire, whilst they had mixed views about being accountable for the teaching of mathematics. The findings presented in Chapter 7 suggest that teachers who valued learning and teaching more also have strong views about being accountable towards teaching mathematics.

From the qualitative analyses, particularly from what has changed in the teachers' shared experiences as discussed in Section 8.7, one teacher (Mbose) saw the learning about the MTF to have *directly* changed how he identifies himself as a mathematics teacher. For example, the teacher identifies himself as someone who has "learned the importance of knowing the [appropriate mathematical] words, and who is careful with the words" (Mbose, Interview, October 2019) he uses when teaching mathematics. Although two other teachers (Oena and Pandor) did not identify themselves as changed teachers because of learning about the MTF, I can confirm from their stories that their identity changed to now include using appropriate mathematical words and selecting and adapting relevant examples when teaching mathematics. Identifying Oena's explanatory communication and exemplification did not come out from his stories, but on his actions when teaching in the classroom. I can also identify Makwe to have changed in how he plans his lessons to now include thinking more about what he is going to say to the learners (explanatory communication) and what he wants the learners to do in the lesson (learner participation) in order to achieve the object of learning. Lastly, in terms of learner participation, the findings showed that three teachers (Makwe, Mbose and Pandor) did not consider different ways of inviting learners to participate in the lessons to form part of learning about the MTF as it was promoted in the TM1 course.

Other sequences of characterising the teachers from how they have changed their ways of teaching learners mathematics seem to have started from the learning in the TM1 community and the learning of MfT to understanding mathematics for themselves from participating in the TM1 course. In highlighting these sequences, I do not deny reverse influences of the outcomes of understanding mathematics. For example, some evidence points to the fact that understanding of mathematics for teachers is influenced by teacher confidence.

From understanding mathematics for teachers for themselves, the findings showed further connections to include changes in teacher knowledge, in teacher confidence and in teaching learners' mathematics for understanding. Furthermore, when Oena was expressing his self-recognition in relation to other mathematics teachers, the teacher confirmed that learning about the MTF contributed directly to his boosted confidence.

To explain the connections, when the teachers were learning in the TM1 community, the emphasis was on group work. For three teachers (Makwe, Mbose and Oena), group work in the TM1 community gave them opportunities to engage with each other in their different groups when working on mathematical tasks to make sense of mathematics. For example, two teachers (Makwe and Mbose) highlighted that the teachers were given space to make their own mistakes in order to understand mathematics. However, group work was not always promoted in Mbose, Oena and Pandor's teachings due to time pressures and over-crowded

classrooms. Makwe talked about the importance of coordinating group work in order to leave learners with no confusion. That is, for example, and according to Makwe, learners can be given the same question, but each group can be expected to present their solutions using different ways.

The learning of MfT contributed directly to improving teachers' understanding of mathematics and their teacher knowledge. Two teachers (Makwe and Pandor) acknowledged that their content knowledge were deepened when learning to teach despite being proficient in mathematics. However, the four teachers saw the learning of MfT to have contributed in improving their teaching knowledge to include knowledge of how to introduce new concepts to learners; knowledge of justifying procedures to learners; knowledge of teaching across grades; what to consider when responding to learners' offerings; and how to engage with learners' prior knowledge when teaching mathematics.

The findings on changes in teachers' understanding of mathematics for themselves showed direct connections to teachers' improved teacher knowledge, to their boosted teacher confidence, and to how teachers saw themselves teaching learners mathematics for understanding. In improving teacher knowledge, for example, Makwe claimed that understanding of mathematics contributes in developing better strategies of explaining and unpacking mathematical concepts to the learners.

Furthermore, the interviewed teachers mainly attributed teacher confidence to their understanding of mathematics. However, it was not always clear from the interview analysis that the teachers' boosted confidence was directly due to participating in the TM1 course. Nonetheless, for example, Makwe explained that when teachers have mathematical content knowledge, they have confidence to learn more mathematics. In the quantitative analysis, the teachers had agreed or strongly agreed that having enough confidence can enable them to allow other teachers to observe their lessons.

The findings in Section 8.4 showed a direct connection between the teachers' understanding of mathematics for themselves and their ability to teach learners mathematics for understanding. On the one hand, for example, learning in the TM1 community included sketching graphs using GeoGebra alongside 'the big grids' as the learning resources. On the other hand, at least one teacher (Makwe) became aware that he needed to also use different learning tools and resources in order for the learners to understand mathematics. This connection can be corroborated from the quantitative analysis as teachers were consistent in agreeing to statements of understanding mathematics with those of teaching learners mathematics for understanding in the 'mathematics teachers' identity and confidence' factor.

Lastly, as we saw in Section 10.5, one teacher (Oena) claimed that the learning about the MTF contributed directly to his boosted confidence of being a mathematics teacher, particularly when planning lessons with other mathematics teachers. When planning lessons, teachers need to consider all four aspects of the MTF, which are learner participation, explanatory communication, exemplification and the object of learning. In this sense, for Oena, teacher confidence became necessary to engage in all these aspects without fear of contradictions when working with other mathematics teachers who mostly attended the TM1 course.

11.3 Connecting the teachers' mathematical discourse in instruction and their identities to answer the RQ3

The premise of discussing the connecting features between the teachers' mathematical discourse in instruction and their mathematics teaching identities relates to the four aspects of the MTF, which are exemplification, explanatory communication, learner participation and the object of learning. However, I look closer to identify features that connect each aspect of mathematical discourse in instruction and how the teachers are characterised from their actions. When drawing from Darragh (2016) in this case, teachers' actions include how they participated when learning about the MTF, how they narrated their participation of learning about the MTF, what were their observable actions when teaching in the classrooms, and how they narrated their actions of teaching the subject.

Exemplification

There are two main features that connect exemplification and how the teachers are characterised from their actions. The first feature, which is the understanding of mathematics, relates to how the teachers select, sequence and adapt examples. The other feature, which is teacher knowledge, relates to prospects of adapting mathematical tasks in the teachers' designated identity.

Firstly, the findings in Chapters 8 – 10 showed that the understanding of mathematics is a key feature that connects the teachers' competence in selecting, sequencing and adapting examples and their characterisation of teaching mathematics. In Chapter 8, I presented that the four teachers recognised learning about exemplification to have mostly influenced their learning and teaching mathematics, particularly in how they now pay attention to selecting and sequencing examples. In Chapter 9, whilst interacting with the learners in the classrooms, we saw three teachers (Mbose, Oena and Pandor) executing coherent lessons of solving linear equations using additive and multiplicative inverses. The three teachers had used the examples directly from the scripted lesson plans, whilst Makwe had selected and sequenced

different examples for his lesson. In Chapter 10, the three teachers asserted that they are able to select, sequence and adapt (or change) examples when teaching mathematics. Makwe mentioned that he avoids adapting examples after selecting them from resources like previous examination papers and textbooks. If he changes examples, the teacher fears that there are chances that he might struggle to find solutions of those newly adapted examples.

From this background, for example, Makwe's characterisation of being competent in adapting examples when teaching mathematics connects directly to his understanding of mathematics. The findings in Chapters 9 and 10 highlighted that the concept of building linear equations reverses the concept of solving linear equations, which talk to the understanding of mathematics. Furthermore, as we saw in the previous section, the understanding of mathematics connects to teacher confidence and to teaching learners mathematics for understanding. As noted in Chapters 6 and 8, the indicators of the lack of teacher confidence include avoiding formulating and representing examples of certain mathematical concepts when teaching mathematics, which was the case with Makwe regarding the adaptation of examples.

In connecting the understanding of mathematics to teaching learners mathematics for understanding, there is evidence that the three teachers (i.e. Mbose, Oena and Pandor) were not connecting the introductory sections to the lesson development sections of their lessons. For example, in Mbose's introductory section for the observed lesson, the teacher was substituting the given values of x in the linear equations to establish the balance between the left-hand and the right-hand sides of the equations, but he did not use the same logic to check the answers after working out the examples in the lesson development section. The idea of checking answers is a way of convincing learners why the answers are correct (or not correct), and that idea speaks to the strand of promoting adaptive reasoning when teaching learners mathematics for understanding. By adaptive reasoning, I mean capacity for logical thought, reflection, explanation and justification as described by Kilpatrick et al. (2001).

Lastly, the findings in Chapters 8 and 9 highlighted teacher knowledge as another key feature that connects the characterisation of teachers in relation to being competent in adapting mathematical tasks during exemplification. In Chapter 8, there was evidence at least from one teacher (Makwe) that the learners do not have to do the same task over and over again to comprehend the concept. From the observation analysis in Chapter 9, it emerged that the four teachers focused on carrying out known operations and procedures of solving linear equations using additive inverses to collect like terms and multiplicative inverses to isolate variables to get solutions. As alluded to in Section 9.7, the teachers did not include mathematical tasks of higher cognitive levels in their teaching. For example, Adler and Ronda (2015) consider

applying known skills such as comparing representations or providing solutions upfront to deal with learner errors to be mathematical tasks of higher cognitive levels. In Chapter 10, at least one teacher (Mbose) argued that the methods provided in the scripted lesson plan need to be followed to the letter. These findings reflected the results from the quantitative analysis in Chapter 7, which provided mixed views about whether teachers can change examples and tasks from the scripted lesson plans. However, the questions remain unanswered, what if the teachers changed the mathematical tasks to include tasks of higher cognitive levels without changing the examples themselves. Would that have meant digressing from using the examples from the scripted lesson plans?

Explanatory communication

Teacher knowledge is a key feature that connects explanatory communication and how the teachers are characterised from their actions. In Chapter 8, the findings were that the adoption of appropriate explanatory communication by the teachers was the second prevalent aspect that contributed in changing their mathematical discourse in instruction. In Chapter 9, I highlighted that three teachers (Mbose, Oena and Pandor) were able to move between the manipulation of the symbols (and sometimes included reading strings of symbols) and the use of mathematical words appropriately when naming mathematical objects. In Chapter 10, it emerged that Makwe envisaged the idea of using mathematical words appropriately in his designated identity. Whereas, all the teachers demonstrated to be competent in legitimating mathematics beyond single cases.

From this background, for example, Mbose connected his actual identity of including the manipulation of symbols and use terms like ‘transposition’ when collecting like terms to his teaching knowledge. That is, the teacher mentioned that knowing learners’ prior knowledge tells him when to use short-cut methods and when to use mathematical words appropriately. Oena and Pandor insisted that one can use mathematical words appropriately during explanatory communication, but use short-cut methods when writing down the symbols. Makwe mentioned that he includes how he intends to use mathematical words appropriately in his lesson plans, but slides back to focusing on manipulating symbols when interacting with learners. From the literature (e.g., Kennedy, 2016), teachers like Makwe are likely to change and include what they envisage to do in their lessons in due time.

Learner participation

A teaching style is a key feature that connects learner participation and how the teachers are characterised from their actions of interacting with learners. I elaborate on a teaching style after this background. When discussing learner participation in Chapter 8, I highlighted that the teachers did not identify with this aspect to form part of the MTF despite it being discussed

in the TM1 course. From the analysis of the lesson observations in Chapter 9, which was limited to what the learners were saying, three teachers (Makwe, Mbose and Pandor) remained mostly in asking *what* questions where the learners completed sentences and provided *yes/no* answers, although sometimes they invited the learners to provide *what/how* questions where the learners provided mainly phrases. The other teacher (Oena) managed to include asking *why* questions in his lessons to invite the learners to discuss mathematical ideas, of which Adler and Ronda (2015) had described asking *why* questions as the higher level of learner participation. In Chapter 10, where the teachers were accounting for what the learners were doing and writing, the findings confirmed that the three teachers (i.e. Makwe, Mbose and Pandor) were not paying much attention to higher levels of inviting learners to participate in the lessons.

In Subsection 10.4.1, when discussing how the teachers were justifying their understanding of exemplification, I described Makwe's teaching style as a strategy of including or excluding certain examples when teaching mathematics. By teaching styles in relation to learner participation, I am implying different strategies that a teacher can include or exclude when inviting learners to participate in lessons whilst that teacher considers environmental factors such as the over-crowded classrooms.

When connecting the teachers' teaching styles to their mathematics teaching identities, for example, Makwe, Mbose and Pandor were not engaging learners as individuals when teaching. Evidence shown in the findings highlighted that the three teachers (i.e. Makwe, Mbose and Pandor) have accepted getting chorus responses from the learners, irrespective that the TM1 community encouraged coordinated responses from the teachers – In the TM1 course, teachers were taking turns to speak to each other or to the facilitator. Nevertheless, as we saw in Subsection 10.4.2, as a start, the teachers envisage including strategies where learners raise their hands and respond to questions as individuals in the lessons. Ultimately, as envisaged by Mbose, teachers can promote peer-teaching strategies where, for example, learners as individuals can get opportunities to explain and discuss mathematical solutions to each other in class.

The object of learning

The findings highlighted a nature of examples and tasks (alongside appropriate explanatory communication) to be a key connecting feature between the object of learning and their mathematics teaching identities. In Chapter 9, the findings showed that the teachers had achieved the goal of solving linear equations using known procedures to operate on the left-hand and the right-hand sides of the equal sign. Indeed, what aided the teachers to achieve their goal of solving equations was the relevance of examples they used in the lessons and

the appropriateness of explanatory communication. In Chapter 10, when the teachers were explaining what it means for them to achieve lesson goals, three teachers (Makwe, Oena and Pandor) understood using different explanations to communicate the same concepts to the learners to be effective in achieving the lesson. However, the teachers did not emphasise all the necessary connections across the lessons. For Mbose as echoed by Pandor, achieving a lesson goal entails seeing that learners are getting correct answers. On that note, in Chapter 7, one closed statement, which was 'Showing correct answers is as important as justifying how and why mathematical procedures work', affected the reliability of the quantitative findings. The statement loaded in the 'accountability towards teaching' factor, and it was removed for the data analysis. In that analysis, 93% of the teachers had agreed and strongly agreed to that statement, whilst the rest had disagreed and strongly disagreed to the statement.

It follows logically when picking on the last point that the nature of examples and tasks can contribute to achieving (or not achieving) the lesson goal. To explain, as it was the case in the teachers' lessons, if a mathematical task is limited to using known procedures to find answers to examples, finding correct answers can mean that the lesson goal is achieved. If a task is of higher level, and it has many components to it, finding correct answers to examples can be part of achieving the lesson goal. In this regard, justifying how and why mathematical procedures work can supersede showing correct answers to achieve the lesson goal.

At this point, it became interesting to note that the connecting features between the teachers' mathematical discourse in instruction and their mathematics teaching identities emerged mainly from their designated identities. In other words, the findings on the teachers' designated identities mainly contributed in highlighting the (dis)connecting features between what they say they do when teaching mathematics and their actually actions.

11.4 Implications of the findings for teachers in communities of practice

In this section, I draw on Sfard and Prusak's (2005) narrative identity and Wenger's (1998) participative identity frameworks alongside the relevant studies that were reviewed in Chapter 3 to elaborate on the implications of the findings to teachers in communities of practice. When discussing the findings, I firstly elaborate on the gap between the teachers' actual and designated identities, particularly on how the gap can be closed during PD interventions. Then, I provide general recommendations on the need for teachers to participate in communities of practice such as the TM1 course, professional learning communities and community of practice square. Lastly, I highlight the significance of developing social identity for teachers when learning and teaching mathematics.

The gap between the teachers' actual and designated identities in this study was relatively narrow given that they were competent in most aspects of the teaching practices. As noted, the study did not investigate the gap before the teachers' participation in the TM1 course, which would have meant reflecting on that determination to highlight the contribution of "learning" from the course. I have put the word 'learning' in the quotation marks given the assertion by Sfard and Prusak (2005) that closing a gap between actual and designated identities indicates learning. However, the teachers' stories from what has shaped their shared experiences gave a sense of where the teachers were before learning about the MTF and learning MfT and where they are when teaching mathematics after the course. The position of 'where the teachers are' was further answered using the MDI analytical framework (Adler & Ronda, 2015). Thus, it was apparent from the data analyses that the teachers have shifted in their teaching practices, which yielded a narrow gap between their actual and designated identities. In this sense, the analyses revealed that the teachers' actions of teaching mathematics were aligned with their mathematics teaching identities in most aspects of the MTF. Below, I provide examples to highlight the findings that illustrate more of the teachers' actual identities than that of the designated identities.

On the one hand, for example, when taking the teachers' stories as being true (Sfard & Prusak, 2005), the findings revealed that the teachers are able to select and sequence examples in their lessons. In another example, when considering how the teachers were expressing their competences of using mathematical words appropriately and their understanding of the importance of legitimation (Adler & Ronda, 2015; Sfard & Prusak, 2005; Darragh, 2016), it was evident that the teachers have shifted towards higher levels of explanatory communication. Ultimately, the teachers are able to determine how to achieve the lesson goals, and to determine why they sometimes fail to achieve other lesson goals.

On the other hand, for example, a mismatch between what was promoted in the TM1 course and how the teachers are inviting learners to participate in the lessons was evident in their performance of teaching mathematics (Darragh, 2016). In the TM1 course, different forms of how teachers can invite learners to participate in the lesson including encouraging coordinated responses from learners were modelled and promoted for the teachers. In another example, there was evidence that the teachers are rarely adapting mathematical tasks (if they do) despite the opportunities of learning for learners and for teachers that come with adapting tasks of higher cognitive levels. The highlighted mismatch in the findings implied the existence of a critical gap between the teachers' actual and designated identities in those aspects of their mathematical discourse in instruction.

Sfard and Prusak (2005) maintain that “learning is often the only hope for those who wish to close a critical gap between their actual and designated identities” (p. 19). The learning in this case can be directed to changing certain teachers’ teaching practices, such as how teachers can invite learners to participate in higher levels of mathematical tasks. When learning, teachers can be made aware of the contradictions between their teaching practices and what can provide learners with opportunities to learn mathematics. For example, in the findings, some teachers saw nothing wrong with allowing learners to respond in choruses to complete unfinished sentences or to provide ‘yes/no’ answers, whilst research in mathematics education (e.g., Adler & Ronda, 2015) understand those kinds of teaching practices to provide minimal opportunities to the learning of mathematics. In this sense, when reading Sfard and Prusak (2005), the object of learning needs to be crafted for teachers in accordance with their shortfalls when learning in teacher professional development (PD) interventions.

The types of contradictions that emerged in the findings of this study are different than the “mismatch” between teachers’ identities and their teaching practices in Gujarati’s (2013) findings. Like in other research with similar trends in their findings (e.g., Heyd-Metzuyanim, 2019), the teachers’ identities in Gujarati (2013) are misplaced whilst their teaching practices can encourage learners to have positive dispositions toward learning mathematics. In this regard, teachers’ identities can be addressed by changing their affective relationships with mathematics, which are said to be difficult to change (Heyd-Metzuyanim, 2019).

However, as we saw in Bjuland et al. (2012), closing the gap between actual and designated identities can require teachers to participate in communities of practice. Chauraya (2013) argued that professional learning communities (PLCs) could offer teachers a sense of belonging to communities of practice, as PLCs can promote long-term, continuous, developmental and collaborative process of teacher learning. In this sense, positive relationships between teachers’ mathematics teaching identities and their teaching practices can emerge.

Furthermore, the gap between actual and designated identities can be closed when the teachers participate in communities of practice on the TM1 community of practice. Storberg-Walker (2008) refers to a community of practice on another community of practice as a community of practise square (CPsquare). The CPsquare can promote what Pandor (one teacher who participated in this study) expressed as a need for teachers to back each other when participating in PD interventions, particularly if they see that the process of learning and teaching mathematics are not being addressed correctly. It is within the CPsquare where the teachers can be asked which teaching practices are implementable (or not implementable) in the classrooms (Bjuland et al., 2012). For Hodges and Cady (2012), when looking for evidence

of learning in the CPsquare, it remains central to focus on (1) a joint enterprise that focuses on shared goals for mathematics learning and teaching; (2) mutual engagement between teachers; and (3) a shared set of ways of interacting with one another and making sense of their work as teachers of mathematics. Thus, as CPsquares and PLCs can accommodate small numbers of teachers and can be school-based (Chauraya, 2013), they can be recommended as a follow up intervention to closing the gap between teachers' actual and designated identities for members of communities of practice.

Lastly, the idea of developing social identity can be explored when looking at different ways of closing the gap between the teachers' actual and designated identities. One teacher who participated in this study (Makwe) expressed a shift from self-identification to social-identification after attending the TM1 course. That is, the teacher became aware of the challenges that are faced by other teachers of mathematics in other schools. When drawing from Carrim (2001), the premises of developing social identity can mean to understand that teachers are not the same. For instance, teachers can have different religion and regional backgrounds (Carrim, 2001) as much as they have different gender identities (Darragh, 2018), which can all influence their teaching styles. In this regard, one way of accommodating teachers' differences is to encourage a culture of learning together in PLCs (Chauraya, 2013). For Chauraya (2013), when teachers are learning together, they tend to belong together to the teaching profession, which can account for their social identities.

11.5 Implications of the findings for further research

In this section, I start by recommending the learning in communities of practice as the 'fertile ground' of working with in-service teachers in mathematics education. Then, I highlight the benefits of employing the theoretical framework for this study, particularly the inclusion of Darragh's (2016) performative identity framework as it harnessed Wenger's (1998) participative identity and Sfard and Prusak's (2005) narrative identity frameworks. Lastly, I discuss the implications of the findings in relation to the methodology of this study.

PD interventions that can promote the learning of mathematics to preserve the teaching profession include inviting in-service teachers to participate in communities of practice. In Chapter 4, I argued that the TM1 course was the community of practice. The findings that are discussed in this chapter revealed that the problem of disjuncture between teaching practices and teachers' mathematics teaching identities cannot be located in the processes of participating in the TM1 course. The findings in Chapter 7 confirmed that the TM1 teachers developed shared experiences of learning of mathematics after participating in the community of practice. The extent of the shared experiences of participating in the course included valuing the learning and teaching of mathematics more by the teachers whilst they are being

accountable to the teaching profession. In this chapter, I argued that the gap between the teachers' actual and designated identities was relatively narrow. The fact that the gap in the teachers' identities was relatively narrow implied positive relationships between their understanding and competence of teaching mathematics from the MTF perspective and what can be recognised by the teachers themselves and by others (including the researcher). Thus, there was evidence of improving mathematics teachers' knowledge for teaching, of which Adler (2005) and Adler and Davis (2006) assert to be central in the teaching profession.

The findings further revealed the teachers' willingness to learning more about different teaching practices. For example, the teachers were envisaging getting coordinated responses when learners are participating in discussions in the classrooms, which meant designated identities. According to Bjuland et al. (2012), designated teaching identity is an indication that teachers want to improve their work in order to become better mathematics teachers.

I attribute the emergence of the argument on the teachers' willingness to learn different teaching practices to the theoretical framework of the study. To explain, Wenger's (1998) participative identity framework became central in giving a sense of where the teachers were before participating in the TM1 course and where they are in terms of their experiences of learning and teaching mathematics, after completing the course. The idea of foregrounding learning as *experiencing* alongside learning as *doing* in the study begun to unlock what the teachers can identify to have changed in their actual teaching of mathematics after participating in the TM1 course. For example, as highlighted in this chapter, one teacher (Mbose) saw the learning about the MTF to have *directly* changed how he identifies himself as a mathematics teacher. Thus, as recommended by Van Zoest and Bohl (2005), and discussed when reviewing the selected studies on Wenger's (1998) social theory in Chapter 3, the elaboration of how learning as *experiencing* (meaning) alongside learning as *doing* (practice) from Wenger's (1998) social theory in this study may contribute to research identity of in-service teachers.

Furthermore, as explained in Chapter 2, drawing on Wenger's (1998) social theory of learning to analyse teachers' identities in relation to what they actually do when teaching mathematics in the classroom was going to be limited for the study, unless other operationalised definitions of identity were incorporated in the study (Essien, 2014; Graven & Heyd-Metzuyanim, 2019). Graven and Heyd-Metzuyanim (2019), citing Jorgensen (2014), argue that researchers in mathematics education who draw from social theories and identity often render mathematics invisible. As we saw, Darragh's (2016) performative identity framework harnessed Sfard and Prusak's (2005) narrative identity and Wenger's (1998) participative identity frameworks for the study. From the findings, future research on teachers' participation in the learning of

mathematics can be addressed from two fronts, which are on teacher knowledge of solving algebraic equations (i.e. a specific domain of mathematics) and on certain aspects of teachers' mathematical discourse in instruction (i.e. teaching practices). Thus, the theoretical framework for the study, which integrated Wenger's (1998) participative identity, Darragh's (2016) performative identity and Sfard and Prusak's (2005) narrative identity frameworks, became a key contribution of this thesis to research teacher identity in the field of mathematics education.

Methodologically, the mixed methods research design of this study can be recommended for researching teacher identity when teachers participate in communities of practice. As mentioned in Subsection 5.2.1, the vast majority of studies on teacher identity have relied on qualitative tools, where open-ended methods such as interviews, narratives and classroom observations are prevalent sources of data (Lutovac & Kaasila, 2019). The findings, particularly on the evidence of teachers' positive relationships between teaching practices and their identities from their shared experiences of learning and teaching mathematics in Section 11.2, demonstrated that the inclusion of the quantitative processes can be valuable to research teacher identity. The quantitative processes formed part of the mixed methods research design of the study. In this sense, the elaboration of the mixed methods research design for the study became another key contribution to research teacher identity.

11.6 Limitations of the study

Generally, there can be a number of limitations in the research study. The limitations that I discuss for this study relate to the following four issues:

1) The method of accounting for the changes in the teachers' experiences of learning and teaching mathematics since the data was collected after the PD intervention;
2) My involvement in the TM1 community as the research fellow made it somewhat awkward for the teachers to be too critical of the TM1 course;
3) The fact that the observation data was based on one mathematics topic, which was solving linear equations using additive and multiplicative inverses; and
4) The generalisability of the study.

When discussing these issues, I keep in mind the fact that the limitations of the research have to do with validity of the findings (McMillan & Schumacher, 2006). That is, I provide explanations of how those limitations were looked at (or minimised) for this study.

Firstly, to account for the changes in the teachers' experiences of learning and teaching mathematics, I relied on indicators such 'before' and 'after' the TM1 course when teachers were narrating their stories of participating in the PD intervention. As noted, I did not collect

data before the TM1 course in order to compare the impact of learning mathematics after the course. For example, in Chauraya's (2013) study, different shifts in how the teachers perceived their membership in the community (and how they were perceived by others) before, during and after the PLC intervention were insightful in supporting the work of communities of practice. However, getting a sense of what the teachers used to do, what they do and what they envisage to do explained the different research phases of the study. For instance, we saw the causal effect between learning about the MTF and the emergent of the teachers' identity in the quantitative processes of the study. The findings from the quantitative processes were expanded using the teachers' stories (i.e. qualitative processes) when teaching mathematics since learning about the MTF.

Secondly, as mentioned in Subsection 5.4.3, my involvement in the TM1 community as the research fellow made it somewhat awkward for the teachers to be too critical of the TM1 course. On reflecting on some of the responses I received during the interviews, there were few instances where I felt the teachers were careful to not talk negatively about the TM1 course. For instance, when the teachers were asked to express what they did not like about the course, they provided brief responses that were logistically related and not about how the course was offered. One teacher stated that they had "to leave the learners unattended so that [they] can be able to attend the course". However, there were only few instances where the teachers were expected to be critical or 'sing praises' of the course. The nature of almost all the questions was meant to solicit teachers' experiences of learning mathematics and their sense of teaching the subject from the MTF perspective after participating in the course.

Thirdly, the observation data was based on one mathematics topic, which was solving linear equations using additive and multiplicative inverses. The findings of the study may not be generalised to all other mathematics topics for teachers when learning in communities of practice. However, to some extent, evidence of consistence in other three topics (i.e. algebraic equations with fractional terms, exponential equations and use of substitution in equations to generate ordered pairs) indicated that the teachers' identities could be consistent with the findings of other mathematics topics. In addition, in Subsection 4.2.1, I provided evidence that learning how to solve algebraic equations can be influential when solving equations of other areas of mathematics.

Lastly, this study was a case of the teachers who participated in the community of practice, which was the TM1 course. The context of their teaching of mathematics was in disadvantaged schools. As such, the findings may not be generalised to all communities of practice. However, the study provided details on different types of learning in the community of practice. Importantly, from the different strategies of data collection (i.e. closed-ended questionnaire,

lesson observations and teacher interviews), the data was subjected to different phases of the mixed methods research design which promoted triangulation and other forms of consolidating data analyses. Ultimately, the study offered in-depth analyses of the contiguity between teachers' learning and their identities of teaching from the MTF perspective in the community of practice. The operationalising of the study means that the findings can contribute to knowledge of learning and teaching mathematics in other communities of practice. Such knowledge includes *what*, *how* and *why* teachers can learn when participating in communities of practice.

11.7 Conclusion

This study was set up to research teacher identity and mathematical discourse in instruction of the teachers who had participated in the TM1 course. The teachers' identity emanated from the experiences of learning about the MTF and of learning MfT when they were participating in the course. The findings on learning about the MTF revealed that the teachers shared a positive sense of identity towards the teaching of mathematics. The positive sense of identity resonated from being aware of the importance of selecting and sequencing relevant examples (exemplification) and of using appropriate mathematical words (explanatory communication) to achieve the lesson goal (the object of learning) when teaching mathematics. However, the teachers did not recognise learner participation to form part of their experiences of learning about the MTF.

When observed teaching mathematics and from their own recognitions, the findings showed that these teachers can be characterised to achieve their lesson goals through exemplification and explanatory communication. The characterisation of the teachers in how they achieve lesson goals from their competence of teaching mathematics from the MTF perspective (i.e. their mathematical discourse in instruction) became their actual teaching identity. The teachers' designated teaching identity highlighted aspects where there were "mismatch" between their mathematical discourse in instruction and what was promoted in the TM1 course when learning about the MTF. For example, the teachers are not paying much attention to how they invite learners to participate in their lessons. However, the gap between the teachers' actual and designated teaching identity remained relatively narrow when considering that there were fewer aspects where teachers were not competent in their mathematical discourse in instruction.

There were several other features that became influential in characterising the teachers who had participated in the TM1 course. Predominantly, learning MfT influenced teacher knowledge and their understanding of mathematics, which are features necessary for teaching learners mathematics for understanding. Lastly, the findings revealed how learning in the TM1

community influenced the teachers' characterisation regarding their understanding of mathematics and their boosted confidence of teaching the subject.

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APPENDICES

Appendix A: Adapted lesson from the GDE ATP Grade 9 Mathematics

SUBJECT: MATHEMATICS GRADE 9	**WEEK 3 LESSON 5 (Adapted from DBE Lesson plan)**
1. TOPIC: Algebraic Expressions: Factorise algebraic expressions	
2. CONCEPT AND SKILLS TO BE ACHIEVED: By the end of the lesson learners should know and be able to factorise algebraic expressions that involve trinomials of $x^2 + bx + c$ and $ax^2 + bx + c$, where a is a common factor. (Note: if a is a common factor then b and c are divisible by a with no remainders)	
3. PRIOR KNOWLEDGE: Trinomials, variables, coefficient, laws of exponents, factors, HCF, distributive law	
COMPONENTS	**TASKS/ACTIVITIES**
4. REVISION	Revise the following with learners: Expand a) $2(x^2 + 4x + 8)$ b) $(x + 2)(x + 5)$
5. LESSON PRESENTATION DEVELOPMENT	Complete the following activity with learners: Factorise the following trinomial: $x^2 + 7x + 12$
	How many terms are there in the above expression? — 3 terms
	What are the factors of the first terms? — $x \quad x$
	List the pairs of factors of the last term / constant. — $12 \quad 6 \quad 4$ $1 \quad 2 \quad 3$
	Which pair of factors when added gives the coefficient of the middle term? — 4 $3 \qquad\qquad 4 +$ 3
	Put both factors of the first term and last term into each of these brackets $(x \pm ?)(x \pm ?)$ — $(x + 3)(x + 4)$ NB: The signs may not always be positive
	Expand the factors to check if they give you the original trinomial — $x \times x + 3 \times x + 4 \times x + 4 \times 3$ $= x^2 + 3x + 4x + 12$ $= x^2 + 7x + 12$
	NB: Factorisation of an expression is the reverse of expanding an expression
6. CLASSWORK	Factorise the following expressions a) $x^2 + 5x + 6$ b) $x^2 - 5x + 6$ c) $2x^2 + 10x + 12$ d) $-2b^2 + 6b - 4$
7.CONSOLIDATION CONCLUSION and HOMEWORK	Factorise the following trinomials a) $x^2 - 18x + 17$ b) $a^2 - 2a - 63$ c) $b^2 + 15b + 56$ d) $x^2 - 5xy - 24y^2$ e) $a^2 - ab - 30b^2$

GDE ATP Grade 9 Maths, Term 3, Week 3, Lesson 5, pg. 82-88

Appendix B: Example of a learner task to prepare teachers for a teaching task

TRANSITION MATHS 1.5

Teaching task for mid-course assessment: Preparation

The Task

Teacher Tee Emwon wants her learners to focus on different **kinds** of solutions to linear equations.

She produces the following task with 5 examples to give to her learners.

Solve for x in each of the following equations:

a) $2x + 6 = x + 3$
b) $2x + 6 = x + 7$
c) $2x + 6 = 6 + x$
d) $2x + 6 = 2(x + 2)$
e) $2x + 6 = 2(x + 3)$

We recommend that you work through each of these equations in preparation for the assessment task. However, please note that you will **NOT** be allowed to refer to your preparation work while doing the assessment.

Appendix C: Final closed-ended questionnaire for the teachers

First Name…… **Surname……...**

Which grades are you teaching mathematics? ☐$_{8}$ ☐$_{9}$ ☐$_{10}$ ☐$_{11}$ ☐$_{12}$

Please tick one box for each statement to show your level of agreement

		Strongly agree	*Agree*	*Neither agree nor disagre*	*Disagree*	*Strongly disagree*
1	Understanding mathematics enables teachers to make connections between representations.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
2	Showing correct answers is as important as justifying how and why mathematical procedures work.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
3	Since the TM1 course, I can teach learners mathematics for understanding.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
4	Since the TM1 course, I understand mathematics.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
5	Since the TM1 course, one can now teach mathematics with ease.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
6	I will have to learn with others to understand mathematics.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
7	The more I understand mathematics, the more I become a better teacher.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
8	Teachers are accountable for learners' performance in mathematics.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
9	Scripted lesson plans restrict teachers from using correct mathematical language when explaining mathematics in the classroom.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
10	Learners are to be blamed for their performance in mathematics.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
11	Learners come to secondary school with inadequate content knowledge.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
12	Mathematics is not interesting for learners.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
13	Less gifted learners struggle to do mathematics.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
14	Teachers <u>cannot</u> change example sets because of the scripted lesson plans.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
15	Mastering how to teach mathematics broadens learners' understanding of the subject.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
16	It is easy to transfer mathematics knowledge to learners' minds when you are confident.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
17	Because of confidence, one is <u>not</u> afraid to grapple with mathematical tasks.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$
18	I have more confidence in my teaching of mathematics after learning about the MTF.	☐$_{SA}$	☐$_{A}$	☐$_{N}$	☐$_{D}$	☐$_{SD}$

Please turn over the page

Please tick one box for each statement to show your level of agreement

		Strongly agree	*Agree*	*Neither agree nor disagree*	*Disagree*	*Strongly disagree*
19	Learners identify with teachers who have confidence to be more capable than other teachers.	☐SA	☐A	☐N	☐D	☐SD
20	Confidence helps me <u>not</u> to stress about knowing everything.	☐SA	☐A	☐N	☐D	☐SD
21	I have enough confidence to allow other teachers to observe my lessons.	☐SA	☐A	☐N	☐D	☐SD
22	I am hoping to be the best mathematics teacher in the future after learning about the MTF.	☐SA	☐A	☐N	☐D	☐SD
23	Since learning about the MTF, I am more conscious on how I choose examples for a lesson.	☐SA	☐A	☐N	☐D	☐SD
24	Since learning about the MTF, I see mathematics as a subject that needs a lot of strategic thinking.	☐SA	☐A	☐N	☐D	☐SD
25	I feel empowered as a teacher since learning about the MTF.	☐SA	☐A	☐N	☐D	☐SD
26	Since learning about the MTF, I feel an increased sense of being a mathematics teacher.	☐SA	☐A	☐N	☐D	☐SD
27	Since learning about the MTF, the mathematics I teach is more valued by other teachers.	☐SA	☐A	☐N	☐D	☐SD
28	I enjoyed doing maths when I was a learner myself.	☐SA	☐A	☐N	☐D	☐SD

THANK YOU FOR COMPLETING THIS QUESTIONNAIRE

Appendix D: Guiding interview questions for the teachers

Individual teacher interview questions

- Please share with me, what did you like the most about the Transition Maths 1 (TM1) course?
- What didn't you like about the course?

Purpose of Section A: To understand teachers' ways of teaching mathematics from the Mathematics Teaching Framework (MTF) perspective (and other teaching practices).

1. Focusing on your first lesson that was titled 'algebraic equations', please explain, what was the lesson goal that you needed to achieve?
    a. How do you know that it was achieved?
    b. Other than from this particular lesson, in general, do you achieve lesson goals that you set to achieve when teaching mathematics? Please explain
2. When you were selecting (or adapting) the example set, please explain
    a. what influenced your choice of examples? Did you consider any similarities or differences in your example set?
    b. When selecting or adapting the example set, did you make any changes on the examples themselves? Please explain why (or why not)?
    c. Did you change the sequence of the examples in anyway? Please explain why (or why not)?
    d. Other than from this lesson, what influence your choice of examples when teaching mathematics?
4. When explaining your examples, please explain
    a. what words did you plan to use? Why was it important to use those words? Would you make any changes to any words you used?
    b. Would you make any changes in how you explained different steps?
    c. Did you have opportunities to justify different methods with mathematical proofs?
    d. Other than from this particular lesson, what words do you use when explaining mathematics in the class?
5. Reflecting on learner participation during your lesson, please explain
    a. What did the learners say? what did the learners do? What did the learners write?
    b. What kind of a learner participation do you want when planning for mathematics lessons?
    c. Do you have opportunities to open discussions during your lessons?

6. What changes in your teaching practice, if any, can you say are attributed to your participation in the Transition Maths 1 (TM1) course? In other words, what aspects of the MTF have influenced your planning and teaching of mathematics?

    a. In the closed questionnaire, you agreed to statements on how the Mathematics Teaching Framework (MTF) has contributed to your teaching of mathematics. In what way?

    b. When I was observing your lesson on solving linear equations, particularly when you were presenting the lesson, on one hand, you were manipulating algebraic symbols using established mathematical laws such as additive inverses and distributive laws. On the other hand, during the introduction of the lesson, you included the use of word-representation, where for example, you wanted the learners to imagine the number that they can substitute to find equivalence on both sides of the equations. Why was that important?

7. From what we have just talked about, what stands out the most about the MTF?

    a. What did you find to be useful about it?

    b. What did you find not to be useful?

    c. How much of that influence your own teaching?

Purpose of Section B: To explore teachers' experiences of participating in the Transition Maths 1 (TM1) course.

8. Could you please tell me about your experiences of participating in the TM1 course?

    a. Did you feel part of a community of teachers that attended the course?

    b. Did you feel marginalised at any point?

    c. When did you begin to feel that you were being brought to the centre of that community?

    d. What enabled those experiences to be meaningful in order for you to becoming a member of the community?

    e. I know, for example, that in most instances during the course you were working in smaller groups, but sometimes you will be asked to share your solutions with the whole class. Where do you see those experiences contributing to your teaching at the moment?

9. With regards to the closed questionnaire,

    a. for example, you agreed to the importance of understanding mathematics. How is the understanding of mathematics important for you? How does this experience help with your mathematics teaching?

    b. In another example, you also fairly agreed to statements on teacher confidence in the learning and teaching of mathematics. In what way is the teacher confidence important? How does that help with your teaching of mathematics?

    c. How do you think these experiences relate to you as a mathematics teacher?

10. Have you changed how you think about yourself as a mathematics teacher after participating in the TM1 course? Please explain, in what way?

11. Did you get additional roles after completing the TM1 course?

    a. Do such roles enable or compromise you as a mathematics teacher? In other words, are these roles limiting or helping you as a mathematics teacher?

12. Before participating in the course, how can you describe yourself in relation to other mathematics teachers?

    a. Now that you have attended the course, how can you describe yourself in relation to other mathematics teachers?

    b. In what way do other teachers identify with you as a mathematics teacher? And why do you think this is the case? Please explain.

13. Outside the experiences from the TM1 course can you think of other learning experiences that consolidate your role as a mathematics teacher?

    a. For example, I understand that there other PD interventions that are organised by districts, and teachers attend cluster meetings and generally receive support from the subject advisors. How do such experiences help you as a mathematics teacher? Can you give examples?

The end!

Appendix E: Teacher participation information sheet with consent form

Teacher Information Sheet – All the teachers

Date:

Dear Sir/Madam

My name is Wanda Masondo and I am a PhD student in Mathematics Education at the University of the Witwatersrand, Johannesburg. The topic for my doctoral study is **Secondary School Mathematics Teachers' Identity and Mathematical Discourse in Instruction.**

My research study consists of two phases. In the first phase, I am exploring teachers' experiences of learning after participating in a professional development course called Transition Maths 1 (TM1). In the second phase, I am further seeking to gain an understanding of teachers' new ways of teaching (if they have changed) considering that they were introduced to the Mathematics Teaching Framework (MTF) in the TM1 course.

To explore teachers' learning from the TM1 course, I will use a closed questionnaire that I have developed to collect data. The closed questionnaire entails 28 statements in which you will have to tick boxes to show your levels of agreement with the statements. The statements cover four aspects, namely: i) understanding of mathematics content; ii) accountability towards the schooling system; iii) confidence in the learning and teaching of mathematics; and (4) impact of the Mathematics Teaching Framework (MTF) to teachers' identity.

In seeking to understand *what* and *how* teachers' have changed in their teaching (if at all), I will observe mathematics lessons and interview the teachers. This second phase of the study directly focuses on how the teachers connect four components of the MTF (objects of learning, exemplification, learner participation and explanatory communication) in sustaining coherence in their teaching of mathematics.

As you have participated in the TM1 course, I would like to invite you to participate in the first phase of the study. I will like you to complete the closed questionnaire. Please find the closed questionnaire attached, respond to the statements, scan the questionnaire and email it back to me. It should take you about 10 minutes to complete. I request that you provide your name in the questionnaire in order to identify your questionnaire. Depending on your views about learning from the TM1 course, there are chances that I will select you, and thus invite three teachers to participate in the second phase of the study.

The second phase will involve observing four consecutive Grade 9 lessons. Videotape will be used to record lessons for observations. I will be assisted by another PhD peer from the Wits Maths Connect project to observe lessons. In these observations, the video recorder will focus on you, the teacher and not the learners, although the learners will be present. The learners will not be expected to do anything else but attend to the lesson in the normal way.

After observations, the selected teachers will be invited for an interview. The individual interview will be based on the observations of the lessons. The interview will be conducted at a time and place (possibly after school hours and at the schools) that is convenient for

teachers and will take about 40 minutes. With teachers' permission, I would also like to audiotape the interview using a digital device.

I will ensure your anonymity and confidentiality when participating (responding to the closed questionnaire and/or during classroom observations and an interview) in this research study. This means that your name and identity will be kept confidential and your individual privacy will be maintained. You will remain anonymous at all times. I will be using a pseudonym (false name) to represent you and your participation in my final research report and in other academic writings from the study.

The gathered information through the questionnaire and the videos will be contained within the WMCS project. That is, I will not use the videos in any publication or conference. I will only use the information for analysis. Thus, there are no foreseeable risks in participating in this study whereas it can provide opportunities to reflect on your teaching practices. You will not be paid for participating. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty.

This study will be written up as a research report which will be available online through the university library website. If you wish to receive a summary of the report, I will be happy to send it to you upon request. All research data will be destroyed between 3-5 years after completion of the project.

If you have any questions about this research, please feel free to contact me and/or my supervisors. The contact details of my supervisors are as follows: Prof Craig Pournara, telephone 011 717 3253, email Craig.Pournara@wits.ac.za and Prof Nazir Carrim, telephone 011 717 3025, email Nazir.Carrim@wits.ac.za

If you have any queries, concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (non-medical), telephone 011 717 1408, email Shaun.Schoeman@wits.ac.za

Yours sincerely,

Wanda Masondo
Email: wanda.masondo@wits.ac.za
Physical: M88 Marang Building, Wits School of Education
Cellular: 082 257 9635 / WhatsApp: 074 733 4067 / Telephone: 011 717 3449

Teacher Consent Form – All the teachers

Title: Secondary School Mathematics Teachers' Identity and Mathematical Discourse in Instruction.

Name of researcher: Wanda Masondo

Please fill in the reply slip below. I will like to receive it from you together with the completed closed questionnaire if there are no questions regarding the study.

I …….. agree to participate in this research study. The research has been explained to me. I understand what my participation will involve. I give my consent for the following:

	Circle one
I agree that the researcher may use anonymous quotes* in his research report.	YES/NO
I agree to complete the closed questionnaire.	YES/NO
I agree to be observed in class.	YES/NO
I agree to be videotaped during class observations.	YES/NO
I agree to be interviewed for this study.	YES/NO
I agree to be audiotaped during the interview.	YES/NO

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed. YES/NO
- the videotapes will be used for this study only. YES/NO
- I can stop the videotaping at any time during observations. YES/NO
- the audiotapes will be used for this study only. YES/NO
- I can stop the audio recording at any time during interviews and don't have to answer all the questions asked. YES/NO
- all the data collected during this study will be destroyed within 3-5 years after completion of the study. YES/NO

* Anonymous quotes: Words/statements/sentences that you use in the questionnaire, in the mathematics lessons or in the interview that I may use directly in all presentations or writings resulting from this study but will not contain your name.

Name: _____ Surname: _____

Sign: _____ Date: _____

Appendix F: Ethics certificate issued by the Wits Ethics Committee

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Research Office

HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)
R14/49 Masondo

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: H19/03/18**
PROJECT TITLE	Secondary school mathematics teachers' identity and mathematical discourse in instruction
INVESTIGATOR(S)	Mr W Masondo
SCHOOL/DEPARTMENT	School of Education/
DATE CONSIDERED	15 March 2019
DECISION OF THE COMMITTEE	Approved
EXPIRY DATE	10 April 2022

DATE 11 April 2019 **CHAIRPERSON** (Professor J Knight)

cc: Supervisor : Prof N Carrim and Dr C Pournara

DECLARATION OF INVESTIGATOR(S)

To be completed in duplicate and **ONE COPY** returned to the Secretary at Room 10004, 10th Floor, Senate House, University. Unreported changes to the application may invalidate the clearance given by the HREC (Non-Medical)

I/We fully understand the conditions under which I am/we are authorized to carry out the abovementioned research and I/we guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to submit an amendment of the protocol to the Committee. **I agree to completion of a regular progress report. For Minimal and Low studies, this is due annually on 31 December. For Medium and High Risk studies, this is due twice annually on 30 June and 31 December.**

W Masondo
Signature

11/04/2019
Date

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

Appendix G: Gauteng Department of Education approval letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	23 February 2021
Validity of Research Approval:	08 February 2021– 30 September 2021 2021/43
Name of Researcher:	Masondo W
Address of Researcher:	26 Condor Road
	Sharon Park Lifestyle
	Nigel
Telephone Number:	011 717 3449 / 082 257 9635
Email address:	wanda.masondo@wits.ac.za
Research Topic:	Secondary school mathematics teachers' identity and mathematical discourse in instruction
Type of qualification	Doctor of Philosophy
Number and type of schools:	47 Secondary Schools
District/s/HO	Ekurhuleni North, Ekurhuleni South, Gauteng East, Gauteng West, Johannesburg South and Sedibeng East

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

23/02/2021

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of COVID 19 pandemic researchers can ONLY collect data online, telephonically or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate. The approval letter will then indicate the type of arrangements that have been made with the school.***
4. ***The Researchers are advised to make arrangements with the schools via Fax, email or telephonically with the Principal.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 23/02/2021

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix H: Frequencies of the teachers' responses to the closed statements

		Percentage (%) of teachers' responses to the statements		
		Strongly agreeing or agreeing	***Neither agreeing nor disagreeing***	***Disagreeing or strongly disagreeing***
1	Understanding mathematics enables teachers to make connections between representations.	100%	0%	0%
2	Showing correct answers is as important as justifying how and why mathematical procedures work.	93%	0%	7%
3	Since the TM1 course, I can teach learners mathematics for understanding.	100%	0%	0%
4	Since the TM1 course, I understand mathematics.	100%	0%	0%
5	Since the TM1 course, one can now teach mathematics with ease.	100%	0%	0%
6	I will have to learn with others to understand mathematics.	78%	11%	11%
7	The more I understand mathematics, the more I become a better teacher.	98%	2%	0%
8	Teachers are accountable for learners' performance in mathematics.	71%	22%	7%
9	Scripted lesson plans restrict teachers from using correct mathematical language when explaining mathematics in the classroom.	44%	25%	31%
10	Learners are to be blamed for their performance in mathematics.	27%	46%	27%
11	Learners come to secondary school with inadequate content knowledge.	71%	18%	11%
12	Mathematics is not interesting for learners.	20%	33%	47%
13	Less gifted learners struggle to do mathematics.	56%	18%	26%
14	Teachers cannot change example sets because of the scripted lesson plans.	24%	9%	67%
15	Mastering how to teach mathematics broadens learners' understanding of the subject.	98%	0%	2%
16	It is easy to transfer mathematics knowledge to learners' minds when you are confident.	96%	4%	0%
17	Because of confidence, one is not afraid to grapple with mathematical tasks.	93%	2%	5%
18	I have more confidence in my teaching of mathematics after learning about the MTF.	96%	2%	2%

Please turn over the page

		Strongly agreeing or agreeing	***Neither agreeing nor disagreeing***	***Disagreeing or strongly disagreeing***
19	Learners identify with teachers who have confidence to be more capable than other teachers.	82%	18%	0%
20	Confidence helps me <u>not</u> to stress about knowing everything.	71%	7%	22%
21	I have enough confidence to allow other teachers to observe my lessons.	96%	4%	0%
22	I am hoping to be the best mathematics teacher in the future after learning about the MTF.	96%	4%	0%
23	Since learning about the MTF, I am more conscious on how I choose examples for a lesson.	98%	0%	2%
24	Since learning about the MTF, I see mathematics as a subject that needs a lot of strategic thinking.	98%	2%	0%
25	I feel empowered as a teacher since learning about the MTF.	100%	0%	0%
26	Since learning about the MTF, I feel an increased sense of being a mathematics teacher.	100%	0%	0%
27	Since learning about the MTF, the mathematics I teach is more valued by other teachers.	76%	24%	0%
28	I enjoyed doing maths when I was a learner myself.	87%	9%	4%