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The usefulness of everyday mathematics in the senior secondary curriculum: a controlled experiment

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THE USEFULNESS OF EVERYDAY MATHEMATICS IN THE SENIOR SECONDARY CURRICULUM: A CONTROLLED EXPERIMENT

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A research report submitted in partial fulfilment of the requirements for the degree of Master of Education.

February 1998
University of the Witwatersrand
Johannesburg

Declaration

I hereby declare that this research report is entirely my own work. The research is original as are the conclusions drawn and the activities created. Anything that I have used from another source has been referenced in my bibliography.

C. Gunkel 24/2/98

Researcher's Signature Date

UNIVERSITY OF THE WITWATERSRAND

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Johannesburg

Place

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ABSTRACT

The focus of this research report is on a controlled experiment I performed at a Johannesburg secondary school, which I called Com-Tech High. The experiment was of a quasi-experimental nature. The aims of the experiment were to see if teaching through methods related to the use of materials based on everyday mathematics (realistic/ethnomathematics) improved pupils' academic performance in any significant way, and to observe if teaching in this way had any effect on pupils' attitudes to school mathematics. The experimental material was created from an ethnomathematical perspective bearing Dewey's experiential learning in mind. The pupils in the experimental group worked in a social constructivist manner. Unexpected results showed that although everyday based activities improved pupils' attitudes, unless they had a sound instrumental understanding of related concepts, ethnomathematical methods did not improve their mathematical performance. The results are explained in terms of Kant's epistemology of the *a priori synthetic*, Piaget's constructivism and Skemp's relational understanding.

KEYWORDS

Ethnomathematics	Everyday Mathematics
Relational Understanding	Instrumental Understanding
A priori synthetic	Constructivism

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CHAPTER 1: INTRODUCTION

The idea for this research project arose from eleven heart-rending years of teaching mathematics in a school where bright pupils, who were able to support whole families on their wages, were not able to succeed with the South African school mathematics curriculum. I do not think I succeeded in helping very many of them while I taught at that school even though I put in as much effort as possible. Perhaps my research findings may help future generations of pupils and teachers in a similar position. This research report will be of interest to anyone who has ever felt that the daily lives of the pupils must dictate what is taught in the classroom. Although the findings came as a surprise to me, when looked at in terms of educational psychology and pure philosophy they actually make a lot of sense. This research report is about a passionate mathematics teacher and pupils who need a good mathematics education, but do not have many resources. Even if no one else ever reads this research report or uses the results in their teaching I believe it has achieved at least three important things:

(1) It rekindled a few pupils' interest in mathematics just before they were going to finish school;

(2) It changed my teaching methods considerably;

(3) It demonstrates a real need for teachers to understand and learn epistemology and educational psychology, which are often unfortunately not given their true due respect by teaching students.

I believe that the struggling pupil, the stressed teacher, the ethnomathematician, the epistemologist and the educational psychologist amongst us may find this research of interest.

1.1 Formal Introduction

1.1.1 The Problem

The studies that I have looked at provide insufficient evidence in support of the conclusion that linking everyday mathematics to the school mathematics curriculum will NOT enhance pupil performance[1].

1.1.2 Hypothesis

Everyday mathematics enhances matric pupils' performance and motivation.

1.1.3 Aims

1.1.3.1 Main Aims

(1) To show that there is a gap in ethnomathematics research in that:

(a) Few if any controlled comparative experiments have been performed;

(b) Few if any ethnomathematical studies have been performed with senior high school

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[1] Performance of the pupils in the context of this research means academic performance as well as general ability to use mathematics in their everyday lives as a result of what they have been taught at school.

pupils.

(2) To perform a controlled experiment between two groups of matric groups in the same school, standard and stream to show that everyday mathematics can in fact enhance matric students' performance.

1.1.3.2 Subsidiary Aims

In the course of pursuing these two main aims I will also fulfil a number of subsidiary aims:

(1) To create workable useable ethnomathematical material for the Std 10/Grade 12 curriculum.

(2) To create evaluation tests for the use of ethnomathematics in the matric mathematics curriculum.

(3) To make the matric curriculum more interesting to motivate both pupils and teachers.

(4) To raise the performance of struggling matric mathematics students in the final examination, until a system of continuous assessment is created.

(5) To improve my teaching.

1.2 Rationale

Often when I have put a great deal of effort into teaching a class a new concept, at the end of the lesson or at a point of pupil confusion the question arises: "Where will I use this in my real life anyway?" Then ensues a miserable debate about the value of school mathematics or the pupil is told that to get to the real world she'll have to pass school mathematics and the

lesson continues, without the question being answered. Yet the question is crucial; "It is important for pupils to realise mathematics is something that real people do...and sometimes even die for" (REIMER and REIMER, 1995: 172). Another phenomenon is that several of my pupils will fail dismally at school mathematics tasks; yet when I encounter them serving in the local shops they are able to work efficiently and well with mathematics concepts related to their everyday situations. This leads me to see merit in the following statement: "Involving students in solving constraint-filled problems...engages them in doing mathematics in a meaningful context and encourages the understanding of the mathematical concepts and processes embedded in the problem and problem solving process" (MASNIGILA, 1995: 168).

Richard Skemp says that: "To understand something is to assimilate it into an appropriate schema" (SKEMP, 1979: 46). Further Piaget suggests that the learner will only develop if the learner's cognitive structures undergo the process of assimilation, accommodation, and equilibration (PIAGET, 1970: 34). Assimilation is the fitting of experiences into our present stage of cognitive development. Accommodation is the changing of our present stage of cognitive processing to incorporate new experiences. Equilibration is an emerging cognitive structure that reconciles the thinking conflicts of a prior stage, i.e. the balancing of the old with the new. Part of my motivation for researching this area of mathematics education lies in a belief that this constructivist perception

of how a pupil learns and develops actually is valid. Piaget's many experiments tend to validate if not entirely prove this notion.

Skemp distinguishes two types of understanding. Firstly, instrumental understanding in a mathematical situation, which consists of recognizing a task as one of a particular class for which one already knows a rule. Then applying the rule by rote to the task provided. Secondly, relational understanding, which consists primarily of relating a task to an appropriate schema. If there is already a plan, well and good, but if no such plan exists then the learner can adapt an already existing plan or combine parts of other plans to create a new one that she can use. A pupil who does know the school algorithm for finding the area of a trapezium, may find the area of a trapezium by dividing it into a parallelogram and two triangles, and find the area of each of these and then sum them to find the ultimate area of the trapezium (SKEMP, 1979: 259).

I believe that my task as a mathematics teacher is to provide the pupils with tools so that they are able to construct new knowledge by adapting their already existing knowledge through the use of critical thinking.

A further motivation for research into relating context to content in school mathematics arises from Dewey (1859-1952). Dewey always used the phrase "Some organism in some environment" (SPRINTHALL, 1981: 27). He believed that learning could not

occur in the abstract, the broader context - the environment in which the learning takes place could not be ignored. Dewey thought of the learner as a person within an environment. Hence he advocated careful, guided experience for children guided by their interests and capacities. Like the constructivists Dewey viewed the learner as an active participant in her[2] environment. Hence Dewey promoted a balanced curriculum between experiential learning and careful rational examination. Dewey worked by the dictum that experience should precede or at least be concurrent with educational ideas and concepts. I believe mathematics education should create active learners who engage with their environments in order to develop relational understanding, further their mathematical knowledge and improve their knowledge of the world.

A number of years ago as an undergraduate student I conducted some research on why so many children fail mathematics and concluded that the main reason was lack of interest. It then occurred to me that if pupils at Com-Tech High[3] were to combine their out of school activities with the school curriculum the pupils would be more interested and hence more likely to succeed.

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[2] In this text 'she' will be used as a generic term to indicate both genders.

[3] Com-Tech High is a pseudonym for the school where I taught. This school is in a poor socio-economic area in Johannesburg. It has a history of a large Portuguese population influence and is currently a model C school bordering on model D, with a largely black school population, a few Coloured and Indian pupils and some Portuguese whites. The teachers are largely white Afrikaners with four year post-matric qualifications.

I believe that if teachers can find ways to interest their pupils, the pupils will perform better because they will study more willingly. In addition the classroom atmosphere will improve and hence so will performance. This is important research because mathematics is compulsory for all technical pupils, yet their performance is dismal. We have a mathematics hate attitude in schools in general in South Africa and a very low matric pass rate. Those who do pass generally know how to regurgitate algorithms but have no idea how to reapply their knowledge in the real world. "One of the most crucial aspects of a teacher's role is to motivate young people's interest in mathematics. One way to achieve this is by appealing to the pupils' own experience" (SELLINGER, 1994: 113).

"A person who has studied mathematics should be able to live more intelligently than one who has not...Greater emphasis on applications should offer the student a more balanced appreciation of the place of mathematics in our civilization as a whole" (BUSHAW, 1980: v and vi). If pupils view their school mathematics education as relevant to their interests and lives in general I agree with Bushaw that it is possible to demonstrate that not only their mathematics performance will improve but their lives will be enriched by the mathematics that they now understand and voluntarily use.

Mathematics is a much needed subject in our modern world of technology. We need engineers, doctors, scientists, computer people and many other mathematically literate people in South

Africa. At the Annual Conference of Science and Technology (1996) it was stated that South Africa rates 48th in the world with regard to producing new technology and technologically competent people. Africa as a whole drops off the map when technological ability and advancement is confronted, even though it has vast human and natural resources. Pupils need to succeed in mathematics to enter vocations where there are jobs and to build up the technological resources of Africa, or to survive in the first world. "Every citizen needs increased mathematical literacy to function in the modern world. We must reach out with mathematics to all and consider the mathematics of all. Ethnomathematics, with its broader view of what mathematics is, can help us meet these needs" (HOUSE, 1995: 150).

The research is important because it will fill a gap in ethnomathematics literature by providing a controlled experiment, showing how ethnomathematics can be used and tested in senior classes. Furthermore in creating the experimental material, practical teaching aids will be designed which could later be used by teachers to make our curriculum more relevant. By performing the experiment I will no doubt improve my teaching, because a great deal of reflection has gone into each activity and I may also be in a position to discard some misconceptions or build further activities centred on ethnomathematical teaching.

1.3 Purpose Revisited

The initial purpose of the study was to verify my hypothesis that: Everyday mathematics enhances Std 10 pupils' performance and motivation. At the time I began the research I could find very little research in this specific area and none in the form of a properly controlled experiment. I assumed it my purpose to show through a controlled experiment that there was truth in my hypothesis. Whilst doing my research I discovered that this statement is only conditionally true and for my pupils only held true where they already had a good instrumental understanding of related concepts. This added a new purpose to the study, to find out why this was so and what explanation or justification could be produced for these conclusions.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

In this literature review I will begin by focusing on the title of my research:

The usefulness of everyday mathematics in the senior secondary curriculum: A controlled experiment.

Initially I will unpack the term "everyday mathematics" and see how this is related to research in ethnomathematics[1], and to Dewey's concept of experiential learning, constructivism and Skemp's notion of relational understanding. In doing this I will look at how these theories are related to each other and to my research activities. My review will also include a discussion of the gaps in the existing research in these fields and hence show where my research fits in. More particularly I will indicate that very little if any ethnomathematics research has been done in the senior secondary phase, and that I have been unable to find published records of controlled experiments that have been performed to see if teaching in this way has any

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[1] The term ethnomathematics is used in two ways in this thesis. Ethnomathematics is firstly used to represent a field of mathematics which studies the different types of mathematics arising from different cultures. As the reader will see this is the way that Gerdes (1994) and (Ascher) 1991 use the term. However this is an education thesis where my primary concern is how to teach mathematics in the best possible way; so the second use of the word ethnomathematics is in a pedagogical sense of how to use the mathematics of the pupils' culture to best teach the school curriculum. This sense of using ethnomathematics is more in the line of D'Ambrosio (1985), Carraher (1985), De Abreu (1988) and Bishop's 1988 usage.

effect. For example improved marks or better attitudes, than have been observed when compared to traditional teaching.

2.2 What is Everyday Mathematics?

Everyday mathematics is the mathematics we encounter in our daily lives. It is the mathematics we encounter in the communities we grow up in and live in related to the context of our life in that community. It is not something abstract but directly related to the houses we live in, the food we eat, the work we do, the currency we use and the games we play. Most importantly it is mathematics which has direct bearing on our community and is not superimposed upon us by another community. It is as such the everyday mathematics of the life-world of the learner which is pretheoretical and relates man to his fellow-man and world. Obviously this everyday mathematics will be different for different learners growing up in different cultures, for example in De Abreu's work Tanya's everyday mathematics consisted of purchasing loaves of bread using escudoes in Madeira (DE ABREU, 1990: 4) whereas Carraher's Brazillian street children selling coconuts using cruzeiros is quite a different everyday mathematics (CARRAHER, 1985: 22). This also varies tremendously when comparing a Johannesburg urban child's everyday mathematics to that of a rural child who never transacts with money.

Every learner uses mathematics in a context. While some contexts do overlap and so allow activities created for one

culture[2] to be applied to another, other activities are culture specific and may thus be as abstract to the learner as if no context was used. For example forcing a Johannesburg Zulu to learn sequences and series through Angolan Sand drawings is ridiculous. At no stage am I suggesting this. My research suggests that teachers create activities which are within the realm of their own students' life-worlds and draw upon their everyday mathematics. For example when teaching at Com-Tech High[3] many of my pupils lived in Highveld houses, all my pupils read comic books and most of my pupils enjoy music so I felt justified in creating mathematical activities from these everyday areas.

2.3 How Does Everyday Mathematics Relate to Ethnomathematics?

Ethnomathematics is related to everyday mathematics because the ethnomathematicians derive their mathematics from the social, cultural, anthropological and historical milieu of the learner.

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[2] The term culture is a complex one which must be understood as follows: a rural Zulu in Kwazulu-Natal shares a cultural heritage with an urban Zulu in Hillbrow, Johannesburg. However the urban Zulu in Hillbrow shares an urban culture with the other urban dwellers which is alien to the rural person, ie the urban Afrikaner next door may have more in common with the urban Zulu than the rural Zulu. Furthermore the person concerned also belongs to a specific culture according to his generation, for example teenage pop culture. Hence when culture is spoken of, all cultures will consist of many cultures.

[3] Com-Tech High is the pseudonym for the school where the research for this experiment was performed. I cannot mention the name of the school because of ethical concerns on the part of the school's previous principal.

D'Ambrosio describes ethnomathematics as:

> The evolution of the concepts of mathematics in a cultural and anthropological framework (D'AMBROSIO, 1985: 44).

Ethnomathematics as seen in the educational context is:

> The building of bridges between street and school mathematics (CARRAHER, 1985: 19).

In using ethnomathematics I aim to relate students' everyday experiences to their learning in the classroom at school. The formal academic mathematics is constructed from their daily lives. In addition ethnomathematics may create better daily lives using the mathematics that is learnt through relating the everyday to the formal learning situation. Individual behaviour is homogenized in certain respects through mechanisms such as education and then builds up a social behaviour which can be called culture (D'AMBROSIO, 1985: 46).

A further aim is to conscientize the students through the mathematics they encounter in their daily lives, to emancipate them from the lack of confidence they have in mathematics by showing them how it is used by their own community. Thus the ethnomathematics educator is concerned with the question:

> What kind of knowledge do children bring with them into school, and what are the socio-cultural constraints which allow, or prevent, the use of outside knowledge in school? (BISHOP, 1988: 1)

In the urban South African context where my research was carried out the pupils came from a selection of the 11 official language

backgrounds in addition to a Portuguese immigrant community from Mozambique and Angola. So my activities were focused on the common urban background of familiarity with certain music, comic strips and houses. This research attempted to transcend the diversity of languages and ethnicity to the common denominations of urban life in Johannesburg.

Ethnomathematicians can be seen to share common goals as well as to have divided goals depending on what they understand the term ethnomathematics to mean. All ethnomathematicians are concerned with mathematics in everyday areas of life as well as in ethnic and culturally exclusive areas, such as basket weaving and hut architecture. Ethnomathematics education is grounded in the social milieu of the learner and in my opinion relies on the theory of social constructivism. The ethnomathematically aware teacher aims at constructing mathematics the pupils have to learn in the curriculum - from their daily lives. The dividing line between ethnomathematicians comes in the extent to which the ethnomathematics is used as a tool for political conscientization. There are as two such branches of ethnomathematicians. The politically obsessed such as Gerdes (1994), and those with a broader conception such as D'Ambrosio (1985), Carraher (1985), De Abreu (1988) and Bishop (1988).

Gerdes believed that colonization reduced, negated or reduced to rote memorization the mathematics of third world people (GERDES, 1985: 17). If this is so, how is it that people like the Jews of Israel who only reclaimed their own homeland in 1948 still

managed to retain a sophisticated culture of mathematics and in fact contribute to the development of mathematics and mathematics education in spite of having been colonised not once but over and over again? I believe that it is a convenient and unnecessary excuse to blame a culture's deficiencies solely on colonization. I refuse to have my work seen in this narrow political light and am stressing my use of the broader D'Ambrosio definition of ethnomathematics by calling it everyday mathematics as opposed to ethnomathematics.

However a very important analysis of ethnomathematics is put forward by Gerdes which I would not like to minimize by our political disagreements. Gerdes views ethnomathematics as being *a priori synthetic* in nature as opposed to *a priori analytic* (GERDES, 1994: 19). This is a very constructive way of defining ethnomathematics with respect to creating an epistemology from it. These terms are taken from Kant's *Critique of Pure Reason* where he analyses different types of concepts. Statements which are *a priori analytic* have the predicate entirely contained in the subject and occur on a completely abstract level of thought. For example, 'All bachelors are unmarried men'. Whereas in those which are *a priori synthetic* the predicate is not entirely explained by the subject since the subject contains something more. Some examples are:

> The shortest distance between two points is a straight line.
>
> Seven plus five is twelve.
>
> In all changes of the material world the quantity of

matter remains constant.

Every event has a cause.

Usually the distinction goes *a priori, analytic* and necessary as opposed to *a posteriori synthetic* and sufficient. By creating the *a priori analytic* Kant is allowing for the abstract to be grounded in the life world of the learner, and allowing us to pursue approaches such as ethnomathematics and constructivism which require us to construct the formal out of the informal and the abstract from the concrete of our everyday lives.

2.4 What Research has Already Been Conducted in This Field, What is Missing and What Still Needs to be Done?

Ethnomathematics can be seen as having two related branches of research involved in it. The first branch unfreezes mathematics inherent to specific cultures and can be seen as the anthropological, historical branch (Gerdes, Ascher). The second is the pedagogic branch using ethnomathematical artefact as teaching and learning aids (Gerdes, D'Ambrosio, De Abreu, Bishop).

2.4.1 The Unfreezing of Mathematics Through Ethnomathematics Research

Much of the work of Gerdes (1994; 1995) available to me describes the unfreezing of mathematics from cultures which through

colonization (according to him) have lost their own mathematics. This aspect of Gerdes' work is interesting but not relevant to my work. Angolan sand drawings and Mozambican basket weaving are alien to my culture and indeed to most of my pupils in a multicultural Johannesburg classroom. Further, without the cultural explanation these little booklets are very difficult to understand, even for me as a mathematics teacher.

Gerdes believes that mathematics constitutes an integrated body of those means to understand and change reality (GERDES, 1985: 15). Hence problematizing reality motivates the pupils and makes an entrance to powerful mathematical methods. He sights the example of creating relevant problems for the rural pupil. Cattle are struck by Rickettsiosis and terramycine medicine is available to cure them. How much must each animal be given; what factors can be used to decide this? He then explains how the basic body of a cow can be viewed as cylindrical and that by calculating the volume of the cylinder it is possible to approximate the mass of a cow and hence how much medicine to give the cow. Gerdes believes that by teaching pupils to problematize reality in this way people will be able to improve their own life-styles, i.e. more cows will survive, people will have more milk and meat (GERDES, 1982: 17). I believe that Gerdes is correct in this hypothesis and he is in fact saying what Dewey was saying a long time ago, about the experiential basis of the learner (DEWEY, 1938). Freire proved in Brazil that by problematizing students' real political concerns in material he was teaching them to read, they learnt to read faster (adults in

48 hours) and were thus able to improve their reality (FREIRE, 1972).

With outcomes based education coming into South Africa it should become more and more possible to make the mathematics curriculum sufficiently relevant to pupils to achieve these aims. At the time this research was done although work was being done on a new curriculum, the students undergoing the experiment were still required to follow a core syllabus, so I felt the best way to problematize this reality would be to show the relationship of the current compulsory learning material to the pupils' everyday lives.

In a book *Explorations in Ethnomathematics and Ethnoscience* Cherinda (1994), a student of Gerdes describes children making woven hats as their ancestors did. It is claimed that the traditional approach used here inspired self-confidence in the children, heightening their ability to relate the mathematics learnt at school to their everyday lives. The relationship created between formal and informal mathematics is said to allow the mathematical thinking to come more naturally. What is disturbing here is that we are not shown how this approach was shown to be better. What tests were carried out? How do the results differ to those obtained when the same mathematics was presented in the conventional manner? Hence the research results appear to be at an anecdotal or hypothetical stage rather than the results of proved controlled research. This indicates a gap, that research needs to be done comparing in a controlled manner

ethnomathematical teaching and learning methods to other more conventionally used methods.

Marcia Ascher (1994) has done much work in discovering what mathematics is frozen in which culture. Ascher believes that there are two distinct aspects of ethnomathematics, as I have already mentioned. She has concentrated on the aspect of seeking understanding of the relationship between mathematical ideas and culture. She believes that only once more research has been carried out in these areas can the question of incorporating ethnomathematics into education be addressed. Ascher points out that:

> Modification of education depends largely on the goals of the educator and the setting of the education. This is not a question of research but of clarifying one's goals specifically enough to develop methods or practices that move towards these goals" (ASCHER, 1994: 41).

I believe that after many years of crisis in education in South Africa with the development of an outcomes based curriculum, educators are being forced to clarify their goals and that this is as opportune a time as will ever arise for ethnomathematics to be introduced as a method of making the mathematics curriculum accessible and worthwhile to the learner. The South African educator needs to research mathematical ideas embedded in the South African culture, which has been done to a great extent by

the RADMAST/WITS[4] Ethnomathematics Project - but the work must not stop here. Methods and practices of applying this knowledge in the classroom - which are easily accessible to the overworked teacher - must be developed. This development is as important if not more important than finding the cultural links. Over half the work involved in this research project was not as such research but creation and development of activities in order to present the ethnomathematically linked material in an interesting and constructive way.

Ascher agrees with Ernest (1995) and D'Ambrosio (1994) that in the education of children, by excluding the context of mathematical work, values are being transmitted. She implies that better values would be transmitted if the context was included. Ascher also points out that this may also be a reason for dislike of mathematics amongst young people. Those alienated by mathematics often have a feeling that it is emotionless and lacks feeling. "Even students who like mathematics associate it with a certain inhumanity" (ASCHER, 1995: 40). Ascher adds that the more we ignore the variety of culture in our mathematics teaching the less the students will understand what we are talking about. From earlier research I did into why so many students fail secondary school mathematics, I found that the most significant factor was lack of interest. If we are teaching mathematics as a non-human totally abstract activity it is not surprising

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[4] Reports on the development of this research project can be obtained from the Foundation for Research and Development, and worksheets are currently being published for teacher use by Via Africa Johannesburg.

students lose interest, because it is only the very rare student who enjoys mathematics for the intrinsic beauty of its internal consistency and perfection. The majority of us need to see how things relate to our world in order to appreciate them.

In ethnomathematics teaching, the teaching of subject matter is intertwined with the teaching of culture and humanity. This is particularly true for adult learners, who are already fully enculturated (ASCHER, 1994). These learners are able to make significant contributions to our understanding through their own wealth of life experiences. This observation of Ascher's is particularly relevant to my research since the matric student may be regarded as almost adult and the adult night school students that I teach matric work to are adults.

Very importantly Ascher does not wish to see school mathematics replaced by ethnomathematics, but she sees ethnomathematics as a revitalization of mathematics. I equally do not believe that every section in the school curriculum can or should be taught using ethnomathematics; but I aim to show that in both exponent and analytic geometry the use of an ethnomathematics approach will enhance pupil performance and attitude to mathematics.

2.4.2 Pedagogical Ethnomathematics Research by Interview and Pre- and Post-test in Brazil

A more direct ethnographic approach was used by Carraher, Carraher and Schliemann in a study of youngsters in Recife

Brazil. This study showed that in commercial transactions the children did not use school algorithms in the real life context. Their performance in the real life situation was superior to their performance on context-free problems at school, and to word problems at school which involved the same n mber of operations (CARRAHER, CARRAHER and SCHLIEMANN, 1985: 21).

The children observed in the study usually did mental arithmetic rather than using pen and paper. The study interviewed four boys and one girl aged between nine and fifteen years, giving them both a formal test and an informal test. The same defining criteria were used in both the formal and informal tests; "Context-embedded problems were much more easily solved than ones without a context" (CARRAHER et al, 1985: 24). The subjects performed significantly better on informal tests than on the formal tests even though the level of difficulty was not significantly different. The researchers observed that the children interviewed in the study were concrete in their thinking and thus performed best in concrete situations. "Real-life and word problems may provide the daily human sense which will guide children to find a correct solution intuitively without requiring an extra step namely, the translation of word problems into algebraic expressions" (CARRAHER et al, 1985: 25). At the end of the study the researchers suggest that mathematical systems should not be treated as formal subjects by educators but should instead be introduced where possible as systems in contexts allowing them to be sustained by daily life.

After further research in 1986, 1987 and 1988 the researchers came to the conclusion that classroom mathematics taught students sophisticated symbolic systems which were not readily accessible outside the classroom. Everyday mathematics could then be used to assist the student to grasp these concepts in a more natural way. This appears to advocate that research into the applications of ethnomathematics in the classroom is necessary. "Building bridges between street and school mathematics appears to be a route worth investigating in education" (CARRAHER, 1988: 19).

The work of Carraher et al relates to my situation very well. One of the major reasons that I started to think of doing this research at Com-Tech High was the analogous situation where the majority of my pupils were perfectly competent local shop assistants coping impeccably with financial transactions in the work environment. However when presented with the same type of calculations in the school situation they came unstuck. In addition I have found like Carraher and De Abreu that in school my pupils do not trust their own methods and rely heavily on teacher or textbook taught algorithms and having not mastered them they often do not cope with the mathematics required of them. The few pupils who do have the self-confidence to apply their own out of school methods and come to an answer mentally are often thought to have cribbed and hence their method is not given the correct merit it should be.

2.4.3 Ethnographic Pedagogic Research in Madeira by Interview and Individual Testing

In an ongoing research project carried out by Bishop and De Abreu, conducted in rural Madeira with the hope of improving the mathematics pass rate, the following was found:

(1) Children who show competence in solving mathematical problems in an outside situation could be unsuccessful in the school situation because they believe that their out of school knowledge is inappropriate for school or vice versa;

(2) The kinds of constraints that school imposes could negatively affect children and hence impair their understanding and progress;

(3) Children when faced with school tasks tend to use school methods and do not doubt their results. However they should doubt their results because they are more accurate when using their own non-school methods.

This research indicates that children do not trust their own intuitive methods, possibly because we at school often insist on the use of specific algorithms. Perhaps part of the task of context-based mathematics teaching is to teach the teachers some of the alternate methods which work, so that they do not only encourage the use of algorithms. Children should also be taught to doubt and check no matter what method they use. Children appear to have more self-confidence in everyday situations than in the school situation. There is something fundamentally wrong with this. Children should feel comfortable at school and able

to use whichever method they feel most competent in without fear. No learner should feel that her own method is of any less value than a school-taught algorithm, if it works satisfactorily. No teacher should have the power to undermine a child's ability and self-confidence by insinuating cheating was occurring when it was not. I feel very strongly that if a pupil is suspected of dishonesty in this manner she should be given a fair chance to explain her method. Our system of marking national examinations at no stage allows for this. We are told for an answer to give one mark only. Hopefully the new continuous assessment will allow more space for the acknowledgment of individual thinkers and not again be limited by ignorant teachers.

A part of the exclusion of pupils' everyday methods in the school curriculum is a transmission of a particular value system that we do not want transmitted in the new South Africa. Bishop (1988) argues that a mathematical education can only be called an education if it transmits values, otherwise it is not an education at all but merely a training. Mathematical education should make values explicit and develop the learners' capacity for choosing critically. Teacher education is fundamental here because teachers must be made fully aware of their task in this respect. Much value learning occurs implicitly in the mathematics classroom, and hence if contexts are excluded, positive attitudes are not encouraged often enough. Bishop suggests that in order for these values to contribute positively to the pupils' mathematical education the values need to be made more explicit and meaningful.

A focus for research offered by Bishop is to make the assumption: "All formal mathematics education is a process of cultural interaction, and that every child experiences some degree of cultural conflict in that process" (BISHOP, 1994: 16). In this respect he suggests the question as to which criteria should be used to evaluate an appropriate intended maths curriculum in a culturally pluralistic society? To what extent can a culture-blind intended mathematics curriculum be made less of an obstacle to learning in the classroom? Research is necessary on the cultural framing and hidden assumptions involved in classroom activities. What knowledge about the learners' cultures can help mathematics teachers with their classroom decision-making? (BISHOP, 1994: 17,18)

The research done by Bishop (1991) has shown that context is important for the pupils' understanding of mathematics and the formulation of values in the pupil. His research does not appear to be complete because it is ongoing.

> "Knowing that children and adults from poor backgrounds learn much mathematics in everyday life without the benefit of systematic teaching is certainly a starting point for research in mathematics education. Perhaps the contribution of concrete situations - not concrete materials - to new pedagogical practices seems worth investigating, is what different types of representations may offer to conceptual learning if brought from everyday life into the classroom" (CARRAHER, 1988: 26).

Bishop (1991) points to teacher education as being the key focus for research in ethnomathematics to be used in order for the correct values to go back to the classroom. Once again in theory this is fine but work needs to be done to ensure methods of practical implementation and to create the necessary resources for teacher educators, teachers, students and pupils. I feel that research like mine embodying the creation of teaching materials if successful could be used as an example of what student teachers and practising teachers could try to do in their own teaching situations.

It appears that much research has been carried out in ethnomathematics education from the case study and pupil interview perspective and over and above that a fortune of research into cultural artefact and human history. I keep asking myself how valid this research can be if there is no control group - or has there been research of this nature? One such study has been found which I will now discuss.

2.4.4 A Controlled Experiment

The 1993 Kings College Research Project (KINGS COLLEGE, 1993: 341-373) analyses the performance of two classes, class A and class B, at two different schools from the same socio-economic cultural backgrounds in England. The only apparent difference between these two groups of pupils was the method of teaching.

Class A was taught in a content-based fashion while class B was taught from a context-based perspective. Then class A and B were given the same tasks to do. The results were fascinating in that "The students who had an environment characterized by the integration of process and content (class A in school A) were more able to apply their knowledge across contexts" (KINGS COLLEGE, 1993: 341). Class B in school B seemed to apply various methods on the basis of context insertion and content variation. The students did not seem to know what to apply where.

Very importantly this research points out that it is a fallacy that context aids universal understanding because contexts can act as both bridges and barriers to understanding. The research also cites this interesting quote from Einstein: "As far as the laws of math refer to reality, they are not certain, and as far as they are certain they do not refer to reality" (EINSTEIN in FASHE, 1982). This is of interest here because it ties up the fallibilist position with what we are actually able to learn and teach in mathematics and how reality can help us define the perimeters which absolutism has not really enabled us to do with respect to the real world. I believe that part of the reason pupils find our current theoretical syllabus so difficult is because they cannot find instances where it refers to their real world experience with certainty; and because they are schooled to believe the mathematician must strive for certainty anything else seems inadequate and not worthy of them attaching value to.

The tasks given to the pupils in class B, the experimental group,

were about fashion, woodcutting, fertilizers and fractions. The fashion and wood tasks were gender related. The students who performed best on the abstract questions used school-taught algorithms to do them. However they did not succeed in applying these to the context-bound questions. The students who tried to apply contexts used them incorrectly. These students showed a lack of grounding or anything to hold together or organize their mathematical knowledge. They were unable to generalize from the specific contexts to the general situation and they were unable to find general principles to work with in other specific situations.

I personally feel that for this research to really be valid both classes should have been in the same school, taught by the same teacher in the two different ways. Otherwise school A may have had a good teacher and school B a poor teacher and then the context\content analysis would be irrelevant. In fact probably to do really good research we should still use several schools, but two classes in each and make sure the teachers are also of the same background and calibre.

This article has given me the only piece of research where context vs content research has been evaluated. This is also very much a model for my own research - take a problem; teach two classes in different ways - one with context-based examples, the other from a content exclusive method; compare the performance on a subsequent test. I do not believe the reason for school-B's poor performance was necessarily related to having been taught

in a context-based fashion. Hence in my research I will try to limit the samples' variables more specifically.

Although I can see how the everyday mathematics as I am using it in my activities can be grounded in the broad definitions of ethnomathematics where politics are not the primary concern, I feel my work has much deeper and older roots. I see the roots of my research being grounded in the work of (i) John Dewey, (ii) the (social) constructivists and (iii) Richard Skemp.

2.5 How Does the Work of Dewey, the Constructivists and Skemp Provide Roots for This Research in Ethnomathematics?

John Dewey used the phrase "Some organism in some environment" (DEWEY, 1938) to emphasise that learning does not occur in a vacuum of abstractness. The learner is living in an environment and his learning is affected by the context provided by that environment. In fact I'd go even further than Dewey and state that the learning is affected by the environment, but that the learner in turn affects and changes his environment with his learning again. In my research I have tried to contextualize the learner's mathematics so that it is not learnt as something abstract to the learner but as something that he already has a degree of familiarity with from his environment.

Dewey advocated careful guided experience for learners arranged according to their interests and capacities. These experiences

create a forum for active learning within the learner's own environment. I do not believe people learn very much at all through passive listening and in fact what we remember best is what we have experienced. It was in using this idea of Dewey that my experimental group activities were created.

Dewey was striving for an experience-based curriculum to promote both more effective learning and greater competence. Although Dewey's own experimental school proved to be a failure I believe that he was correct in these two aims and hence have tried to prove to an extent through my controlled experiment that this can in fact happen if one uses Dewey's dictum that experience should precede or at least be concurrent with the development of educational concepts and ideas (SPRINTHALL, 1930: 32).

A further aspect of Dewey's philosophy of education that supports my research is the fact that he was a firm believer in democracy, and because of his belief in the equality and integrity of each individual he was strongly opposed to elitism. A goal of ethnomathematics is to broaden the history of mathematics to make it multicultural and global.

> Not everyone thinks the same. The differences however, are not in the ability to think abstractly or logically. They are in the subjects of thought, the cultural premises and what the situations call forth which thought processes (ASCHER, 1991: 187).

My research assumes that it is wrong to view mathematics as an exclusively Western and Eurocentric activity.

> Ethnomathematics...is the mathematics which is practised among identifiable cultural groups such as national tribal societies, labour groups, children of certain age bracket, professional classes and so on (D'AMBROSIO in LARIDON, 1995: 45).

Ernest uses the fallibilist and connectivist positions to argue for ethnomathematics as being a multicultural, antiracist, and anticlassist mathematics and a tool of society (ERNEST, 1995: 2). If we want African children to learn mathematics we must find the mathematics in the context of their world, which is accessible to their experiences and not present them with something alien and abstract. This idea is supported by ethnomathematicians in general and can clearly be seen in the work of Gerdes (1985) unfreezing the mathematics of Mozambique in activities such as basket weaving and sand drawings.

> The cultural heritage of non-Western groups can be recognised and valued if we draw on the origins of Mathematics from non-Western cultures. This would help pupils from these cultures to respect the cultures of their origin (AMOAH, 1996: 46).
>
> The inclusion of ethnomathematics may serve to reduce the disempowerment of students whose backgrounds are not congruent with Western culture (LARIDON, 1991: 47).

This in turn, "will develop pupils' interest in mathematics" (AMOAH, 1996: 52).

Viewing the learner as an active part of her environment, and her learning as being born out of her environment, I am compelled to look at my research through constructivist spectacles. Piaget believed that there exists a relationship between biology and culture of cognition. He suggests that the learner will only develop if she undergoes the process of assimilation, accommodation and equilibration (PIAGET, 1970: 34). Assimilation is the fitting of experience into our present stage of cognitive development. Accommodation is the changing of our present stage of cognition to incorporate new experiences. Equilibration is an emerging cognitive structure that reconciles the thinking conflicts of a prior stage i.e. the balancing of the old with the new. If we look at the students taking part in my controlled experiment: all the students came to do the experiment while already being a part of a specific environment and being at specific stages of their own cognitive development. Each student was expected to fulfil certain specific learning outcomes by the end of the experiment irrespective of whether they were in the control group or the experimental group. The difference between the two groups was in how they would assimilate, accommodate and equilibrate the knowledge that they were exposed to. It was hoped that the use of familiar learning environments for the experimental students would assist assimilation and hence accommodation and equilibration.

Also in designing the activities both Piaget's developmental stages and Maslow's hierarchy of needs were kept in mind (SPRINTHALL, 1990: 327). The activities were designed so that participating students would move from a concrete stage, through a formal stage, to an abstract stage with each activity on each different section of the work. No student was assumed to be at a specific developmental stage. The students taking part were of the sixteen to twenty year age group and theoretically should have been at an abstract thinking stage. However owing to deficiencies in the socio-economic needs of most of the participants most did not appear to be able to cope on an abstract level even after having moved through a concrete and a formal activity. It is interesting to note that those students who were already abstract thinkers at the formal operational level did not respond well to returning to concrete stages and in fact did worse on post-tests than on pre-tests. However, considering the deficiencies in the backgrounds of the majority of participants this was the exception as opposed to the rule and most of the students responded well to the constructivist approach to building the abstract from the formal and the formal from the concrete.

The experimental group activities made use of social constructivism in their design. Social constructivism as explained by Vygotsky (1993) and others is based in the real-life world situation of the learner. The learner lives within a society which has a certain culture with shared beliefs upon which the objective knowledge of that culture is based. The

learner becomes conscious of these beliefs through language and in particular communication with someone more knowledgeable than herself. Ascher suggests that in ethnomathematics the teaching of subject matter is intertwined with the teaching of culture. She says this is particularly true for adult learners, who are already fully enculturated. These learners are able to make significant contributions to our understanding through their own life experiences (ASCHER, 1991: 187).

Vygotsky (1993) refers to the knowledge through everyday experience as spontaneous knowledge, while he refers to the knowledge acquired formerly at school as scientific knowledge. Children do not learn about numbers as something isolated from their everyday lives and their pasts. D'Ambrosio believes that number is better understood within the context of a civilization. If we abstract number out of this context we are distorting our civilization and the meaning of number. A goal of teaching mathematics should be to close the gap between school mathematics and the modern world. Ethnomathematics can be seen as a pedagogy for addressing individual action as a part of co-operative action. Ethnomathematics in an active manner encourages co-operative socialization amongst the learners, taking cognisance of their common educational goals, simultaneously valuing, and respecting, solidarity and cooperation. This attitude to mathematics teaching is entirely within the realm of social constructivism. The activities performed by the experimental group attempt to bridge the gap between spontaneous knowledge and

scientific knowledge by allowing the scientific knowledge to grow out of the spontaneous knowledge.

In viewing ethnomathematics as an a priori synthetic activity (GERDES, 1994: 19), we are using epistemological questions which are based on Kant's epistemological position. Although the constructivists rejected the a priori synthetic because they believe as biologists that all learning is in fact a posteriori (VON GLASERSFELD, 1988: 255), they still rely heavily on Kant's Epistemology. Both Kant and the constructivists use space and time as necessary components to all our knowledge. It is interesting to note that both Kant and the constructivists also elevate the importance of mathematics and use it to explain how we know. Furthermore Kant's conclusion that we cannot know things-in-themselves but only how they appear to us is echoed in the constructivist perspective that reality is subjective because it is constructed by us, and hence we cannot know anything objectively although we may reach consensus. Of course this idea is re-echoed by ethnomathematicians with statements like:

> All formal mathematics education is a process of cultural interaction, and every child experiences some degree of cultural conflict in the process (BISHOP, 1994: 16).

I thus think it is fair to say that a close link exists between the mathematical epistemology of Kant, the constructivists and the practice of ethnomathematics.

> Ethnomathematics will help pupils link mathematics to their environment, culture and everyday life. It will

> also develop pupils' interest in mathematics (AMOAH, 1996: 52).

This fits with Skemp's definition of relational understanding as compared to instrumental understanding:

> Instrumental understanding, in a mathematical situation consists of recognizing a task as one of a particular class for which one already knows a rule...To find the area of a trapezium multiply half the sum of the parallel sides lengths by the perpendicular distance between them... Relational understanding, in contrast consists primarily in relating a task to an appropriate schema. If there is also a ready-to-hand plan so much better...
>
> But if there is not one, one is not at a loss. One can devise a plan for the particular task in hand; or one can adapt an existing plan or combine parts of two such plans. If he has an appropriate parent schema, someone who does not know the rule can still calculate the area of a given trapezium by dividing it into a triangle and a parallelogram, or into two triangles and a rectangle, or even into three triangles. If he wants to he can also by any of the above paths arrive at a general rule for finding the area of a trapezium (SKEMP, 1979: 259).

I believe that in using ethnomathematics from a constructivist paradigm with the pupils learning more through self-investigation actively than passive listening, the process

is dependent on relational understanding. Not only as Skemp puts it between mathematical concepts but also between other areas of their learning and also with knowledge that the pupils have acquired in their daily lives through a social cultural existence.

2.6 Why Do I Call Research Grounded in Dewey, Constructivism and Skemp's Relational Understanding Ethnomathematics Research?

D'Ambrosio (1985) describes ethnomathematics as "The evolution of the concepts of mathematics in a cultural and anthropological framework" (D'AMBROSIO, 1985: 44). Ethnomathematics as seen in the educational context is, "The building of bridges between street and school mathematics" (CARRAHER, 1985: 19).

The ethnomathematics educator is concerned with the question, "What kind of knowledge do children bring with them into school and what are the social-cultural constraints which allow, or prevent, the use of outside knowledge in school?" (BISHOP, 1988: 1) Ethnomathematics education is grounded in the social milieu of the learner, hence it is grounded in the life experience of the learner and it makes sense to view it as a manifestation of Dewey's 'experiential learning'.

Furthermore, because ethnomathematics is grounded in the social cultural heritage, for example Pythagoras' theorem in the

Mozambican basket weaving, of the learner, it relies in my opinion on the structure provided by social constructivism to operate as a method of learning and teaching as opposed to being merely a study in social anthropology. In ethnomathematics classes that have already been presented, a definite move is made from the concrete, to the formal, to the abstract, for example Cherinda, a student of Gerdes describes children making traditional woven hats (concrete) as their ancestors did. From here moving on to studying the formal mathematics relationships provided by the hats and then moving on to the abstract mathematics which is required to be learnt by the curriculum (GERDES, 1994). A similar approach can be seen in the RADMAST WITS Ethnomathematics Project's worksheets where the formal grows out of the concrete informal and finally the student is exposed to the abstract after investigating the concrete and formal first. D'Ambrosio refers to the recent advances in theories of cognition showing how strongly culture and cognition are connected. Ethnomathematics can be seen as a pedagogy for addressing individual action as a part of co-operative action. Ethnomathematics is active as opposed to passive and encourages co-operative socialization amongst learners, taking cognisance of their common educational goals, simultaneously valuing respect, solidarity and co-operation. This is very obviously an exposition of the theories of Piaget (1993), von Glassersfeld (1991) and Vygotsky (1993).

A task of ethnomathematics is seen to be to take the ad hoc practices and solutions of problems which are developed into

theories and thereafter into scientific inventions. Furthermore, "Ethnomathematics can be seen as a bridge being built between anthropologists, historians of culture and mathematicians in order to recognize that different modes of thought may lead to different forms of mathematics" (D'AMBROSIO, 1985: 44). "Ethnomathematics allows for abstracting from a familiar situation, and the application of previously learnt procedures" (AMOAH, 1996: 46). If one looks at what is actually being advocated here the learner is being forced to create a relational understanding as defined by Skemp. She is not being given an abstract algorithm to work from in a vacuum; but is being asked to use the knowledge she already has through her socio-cultural existence and previous learning she has done, in order to build new knowledge that she can in turn use again to further build her understanding of mathematics, herself and the world she lives in.

Hence I feel perfectly justified in calling my research ethnomathematics, because I believe that ethnomathematics as an educational practice is in fact a manifestation of Dewey's experiential learning having acquired a specific structure from constructivism which uses Skemp's relational understanding and can be used to interpret progress being made towards mathematical abstractions.

2.7 What Motivation Exists for Further Research in Ethnomathematics in South African Senior Schools?

> As South Africa heads towards a multicultural education system the mathematics class the teacher will encounter, will be large and consist of pupils of multilingual and a range of cultural backgrounds. This suggests a need for research in ethnomathematics which would inform the development of new and more appropriate curricula (AMOAH, 1996: 47).
>
> The possibilities of approaching curriculum development from an ethnomathematical perspective in the South African context are exciting and demanding of investigation in spite of the related problems (LARIDON, 1995: 12).

With the advent of Curriculum 2005 and outcomes-based education starting in South Africa in 1998 the necessity for materials which lend themselves to an outcomes-based approach satisfying the learning area outcomes has become fundamental. The textbooks that have been used in the past do not satisfy an outcomes-based approach:

> A learner should be able to develop a familiarity with and intuit about numbers, their history, ways of working with them, ways they are represented and ordered, their patterns and relationships, and how numerical relationships are used to make sense of different and changing world-views, perpetuate

inequalities or promote human rights (LAC, 1996: 2).

Our mathematics curriculum up until this point has been taught in an examination-centred approach concentrating on algorithms to solve particular types of problems and not emphasising the historical, cultural and life-world links for the student. It was also often taught in an abstract instrumental way which did not reach many of the students whom we hope will be reached by a more constructive approach.

> Teachers expressed a need for textbooks and teaching aids with an ethnomathematical orientation. Further work in using ethnomathematical activities in multicultural classroom needs to be done (AMOAH, 1996: 54).

This need is borne out by the fact that even before this research was complete, parts of it were being used by both the Ikaheng Project and the Ethnomathematics Project to create, test and workshop new activities to help teachers cope with outcomes-based education and provide them with ideas for creating their own activities.

Furthermore the research is important because it will fill a gap in ethnomathematics literature by providing a controlled experiment; showing how ethnomathematics can be used and tested in senior secondary classes.

CHAPTER 3: RESEARCH METHODOLOGY AND DATA COLLECTION

This chapter will describe only the methodology and collection of data. The analysis of data will follow in the next chapter.

3.1 Research Methodology

My research was performed by conducting a controlled experiment at Com-Tech High[1].

3.1.1 Theoretical Experimental Framework Design

"A controlled experiment usually involves two groups of subjects, an experimental group and a control group or a comparison group...The experimental group receives a treatment of some sort (such as a new textbook or a different method of teaching), while the control group receives no treatment... The control group is critically important in all experimental research, for it serves the purpose of determining whether the treatment has had an effect, or whether one treatment is more effective than another" (FRAENKEL & WALLEN, 1990: 232).

My research follows a **QUASI-EXPERIMENTAL** design as opposed to a truly experimental design. The experiment is of a quasi-experimental design because in the educational setting it is often impossible to undertake a true experiment, due to problems

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[1] Com-Tech High is the pseudonym for the school where I conducted the research. The name cannot be mentioned because of ethical concerns on the part of the previous principal.

with randomisation of exposures. I as a researcher was unable to randomly select my samples but had to work with classes already created within the school because of the technical/ commercial/academic combination of pupils for timetable purposes.

Diagrammatical Representation of My Quasi-experimental Design

Experimental	O1	A(b)	X	A(a)	O2
Control	O3		Y		O4

The line separating the experimental and control groups shows that they have not been selected by randomisation. X refers to the exposer of the experimental group to my ethnomathematical activities. Y refers to the conventional teaching of the control group, learning the same sections as the experimental group learns, through the activities. The O's refer to the processes of observation and measurement. O1 will represent the results of the experimental group on the pre-test before treatment. O2 will represent the results of the experimental group after treatment on the post-test. O3 will represent the results of the control group before conventional teaching on the pre-test. O4 will represent the results of the control group on the post-test after having been taught in the conventional manner. A(b) represents the results of attitude tests written by the experimental group before doing any of the activities. A(a) represents the results of the same attitude tests written by the experimental group after having performed the activities. Readers must note that the O1, O2, O3 and O4 results will occur

three times over i.e. before and after each of the three activities.

3.1.2 The Actual Experimental Situation

Three ethnomathematical activities were undertaken by experimental groups. At the same time the same three sections of the syllabus were taught to three control groups in the conventional manner without any ethnomathematical input (i.e. in a traditional text-book, chalk-talk fashion). Both groups wrote the same pre-test and post-test. The results were intended to ascertain whether an ethnomathematical approach leads to better academic performance in the senior secondary mathematics syllabus, and if in fact it proved more effective than traditional teaching methods. The experimental group was also required to write an attitude test before and after doing the experiment.

3.1.3 The Subjects

The control and experimental groups came from the same school, Com-Tech High. In order to get representative class sizes the experimental group was taken from the SG/HG Std 10/Grade 12 group of mathematics pupils. The control group was taken from the SG/HG Std 9/Grade 11 group of mathematics pupils. The pupils were chosen randomly as far as possible according to the classes which they normally attend. The Std 9/Grade 11 group had covered graphs and exponents when they were used as a control group.

This means they had covered the same work as any Std 10/Grade 12 pupil would have covered at the point of commencing the sections covered by the experiment. Hence their pre-knowledge was on a par with that of the Std 10/Grade 12 group and equivalent to what one would normally encounter on starting these sections of the syllabus with a matric group.

The pupils were all in the seventeen to twenty age group and come from relatively poor socio-economic areas. Many of the pupils have had deprived educational opportunities, such as poor or no teaching, lack of text books and very poor mathematical results prior to the point of experimentation. Some of the pupils also had a negative attitude towards mathematics because they felt incapable of doing it. The pupils in both the control and experimental groups were co-operative during experimentation.

Both groups were taught by the same teacher (me), which may in fact make this more action research, than quasi-experimental research, although quasi-experimental research was intended. I had no alternative but to collect the data myself, because of lack of co-operation on the part of other teachers. Originally three teachers from Com-Tech High were given the research to do, as well as teachers in six other schools who were interested in doing the research but did not find time to submit results. Each teacher showed immense interest in receiving the material but none handed in results of the pre- and post-tests when asked to. Some felt that they were too pressurised, some lost the papers and I think others were just too lazy or did not care. The only

results I received other than those I collected myself were for the exponential curve activity from the experimental group teacher at Com-Tech High.

3.2 Treatments

3.2.1 The Exponential Curve, Paper Folding and Music Activity

The control group were taught to draw $y=2^x$, $y=2^{-x}$, $y=3^x$ and $y=3^{-x}$ using the table method. The graphs were drawn by a question/answer explanation on the board with the pupils working together with the teacher. Then the pupils repeated the exercise on paper individually. For each graph the domain, range and intercepts were discussed. Then the concepts of domain, range and intercepts were generalised to the graphs of $y=a^x$ and $y=a-x$. Exercises from *Classroom Mathematics for Std 10* were performed to practice the new concepts. Then the pupils looked at graphs of exponential curves which had already been drawn, and interpreted them finding domains, ranges, intercepts, intersections and the determination of equations.

The experimental group performed a paper folding activity.
A piece of paper was folded first in half, then in half again and so on until the paper could not be folded any more. After each new fold the number of rectangles formed by the folding were counted. In this manner a table was completed for the number of folds against the number of rectangles created. The pupils then completed the table and plotted the resulting graph of $y=2^x$.

After the paper folding activity the pupils listened to a tape of increasing octaves of 'C' note, going from a frequency around 'middle C' to the 'topmost C' on the piano keyboard. They then looked at the graphical representations of the notes as seen on an oscilloscope. From listening to the tape and looking at the wave patterns it became obvious that we were dealing with another representation of the function $y=2^x$.

The pupils completed tables for the specific graph relating to the musical notes and then plotted the graph from the table. Questions were also asked and answered, discussing the domain, range, intercepts and the status of the graph as a function.

The pupils then listened to a tape of decreasing octaves of 'C' note, going from a frequency around 'middle C' to the lowest note on the piano keyboard. Again the pupils looked at the graphical representations on the oscilloscope and went through the motions of completing tables, answering questions and plotting the graph of $y=2^{-x}$. Thus the pupils drew the graphs symmetrical about the y-axis.

Subsequently the pupils were given as a homework exercise to take a spoon and beat a coffee tin with it to the timing of a semi-breve, minim, crotchet, quaver and semi-quaver. The beats also follow the graphical representation of $y=2^x$. This provided the pupils with a further example to practice the graph from an everyday life context.

In the next lesson the pupils again performed a paper folding activity. Here they folded the paper in three then in three again and so on. Every time a fold was made the pupils opened out the paper again and counted the number of rectangles present. From this they completed a table comparing number of folds to number of rectangles and plotted the graph of $y=3^x$. They then answered questions regarding the domain, range, intercepts and status of the relation as a function.

This was followed by listening to tapes of increasing octaves of 'G' and decreasing octaves of 'G'. These respectively led to the pupils discovering the graphs of $y=3^x$ and $y=3^{-x}$, after the pupils went through the same kind of exercises as they did with the 'C' octave graphs.

Finally pupils completed worksheets generalising the information learnt from $y=2^x$, $y=2^{-x}$, $y=3^x$ and $y=3-x$; to the functions of $y=a^x$ and $y=a^{-x}$. They were specifically required to discuss domain, range, intercepts and the status of the relations as functions. The pupils then had an opportunity to practice what they had learnt in an exercise revising all the significant mathematics away from any ethnomathematical activity.

The worksheets were performed in groups with plenty of discussion amongst the pupils. The tests however were performed individually under normal test conditions with no discussion.

3.2.2 The Basotho Hut/Distance Formula Activity.

The control group was introduced to the distance formula on the board through an explanation of Pythagoras' theorem. The pupils were then given the standard formula for calculating the distance between two points and told to learn it off by heart. The teacher then went through a couple of examples of using the distance formula in a question/answer teaching method on the board. The pupils then completed exercises out of *Classroom Mathematics for Std 10*. The teacher then went through the examples the pupils found difficult or got wrong, on the board. The pupils were taught in a very standard textbook, chalk-talk manner.

The experimental group was given a comprehension passage to read. The passage described the history of the use of right-angled triangles by the Babylonians, Pythagoras and the Basotho in order to calculate distance and build perfectly rectangular buildings. The passage was followed by questions relating to the passage, basically reinforcing the historical background in the pupils' minds and providing the ethnomathematical cultural background for the activities to follow and the study of the distance formula. The comprehension was completed by the pupils individually.

The pupils were asked to divide themselves into groups of about five pupils in each group. Each group was given string, a board, drawing pins and a set of 'Lego' blocks with a set of worksheets. The pupils made twelve knots in a piece of string equally spaced

apart from each other. They then pinned the string to the board after four knots, turned the string and pinned it again after three knots. At this stage the pupils were required to measure the length of the four-knot piece of string, the three-knot piece of string and the remainder of five knots joining the start to the finish. They were led through a process of squaring the lengths of the three- and four-knot pieces of string respectively and finding the sum of the squares.

Then they squared the five knot piece of string's length and checked if this in fact gave the same result as the sum of the squares of the three- and four-knot strings. In doing this the pupils were proving Pythagoras' theorem for themselves and verifying that the knot-in-rope method of building rectangular structures actually did produce the required right angles.

Each group built 'Lego' houses surrounding their knot in string triangles, doubled to form rectangles. They decided what shape house they had built and how they could prove its rectangular nature. In performing this activity the pupils had actually in a modular scale relived the cultural, historical method of building a rectangular house.

The pupils were then required to take a sheet of square paper and draw a system of axes onto the paper. They then marked a point four squares along the y-axis and another three squares along the x-axis. They had to write the co-ordinates of each of these points as ordered pairs, and calculate the distance between these

two points using Pythagoras' theorem. The pupils were also required to count the blocks between the two points in the shortest way and compare this answer to the calculated answer. At this stage the formal distance formula as we know it as applied to a Cartesian plane was introduced.

Several textbook-type numerical examples were given to the pupils to do individually using the distance formula. Thereafter they were given time to practice their newly acquired knowledge of the distance formula by doing the relevant exercises in *Classroom Mathematics for Std 10* partly in their groups in class and partly as homework.

3.2.3 The Cartoon Strip/Locus Activity

The control group was given a clear explanation as to what a locus is, using the formal definition and examples on the board. The pupils were then shown examples of calculating different loci depending on whether distance formula had to be used or the formulae for gradients and perpendicular lines. The pupils then completed two exercises from *Classroom Mathematics for Std 10*. Examples which the pupils got wrong or were unable to do were discussed and repeated on the board. The pupils then wrote the post-test on locus.

The experimental group wrote the same pre-test on locus as the control group. After writing the pre-test pupils were given an art lesson on creating a cartoon strip using small pieces of

paper clipped together with a bulldog clip. Each group of students was given enough paper, the clip and told to draw exactly the same picture on each page changing only one thing slightly each time. The pupils were then told to turn the pages quickly and see what happened. After establishing that in this way they could create a moving picture the pupils were returned to the mathematics at hand.

Each group was again given a set of twenty small pages and a bulldog clip. The pupils were told to draw a line across the middle of the page horizontally and a dot at the top of the first page. As they moved through the twenty pages the dot moved lower on each page eventually crossing the line and finally going to the bottom of the page. The pupils were then asked to clip their pages together and flick through them quickly.

The pupils then discussed what they thought happened to the dots when flicking through the book quickly. They were asked if this is a single point or a set of points with a defining equation. This led the pupils to the definition of what a locus is, and a clear understanding of what it is not. The pupils were again asked what they were looking for when asked to find a locus. They were also asked to choose from a list including single points, and functions which could also be loci.

The flick-cartoon book made in class illustrated the locus theorem "The locus of points equidistant from the ends of a line segment is the perpendicular bisector of that line segment". The

pupils were asked to make similar flick-cartoon strip books to illustrate the following locus theorems at home:

- The locus of the vertex of a right angle with fixed hypotenuse is a semi-circle with the hypotenuse as diameter;

- The locus of points equidistant from the sides of an angle is the bisector of that angle;

- The locus of points equidistant from two parallel lines is a third line, parallel to the two given lines, and midway between them.

Before any loci were actually calculated, the major portion of time allocated to this section was spent on the pupils acquiring a firm understanding of the definition of locus through the flick animated cartoon strips. Part of the reason for the creation of this ethnomathematical activity was because in the researcher's previous teaching experience, most pupils' calculations went wrong because they were under a misconception about what they were looking for in finding a locus. Rather than trying to find an equation representing locations of a point these pupils try to find an ordered pair representing an actual point.

This was followed by the pupils being taught how to calculate loci. Firstly they were taught how to use the distance formula

to calculate loci. They then practised exercises on this from *Classroom Mathematics for Std 10*. Their mistakes and problems were discussed on the board. The pupils were then taught how to use gradient and perpendicular gradients to find loci. Again they practised exercises from *Classroom Mathematics for Std 10*. The problems and errors were again discussed on the board.

3.3 Literature Connections

3.3.1 Treatment Justification in Relation to the Literature

What meaning does an exponential curve have to us out of any context? If, however, we have experienced a relationship between musical notes (see Appendix 1) we can see the point of the mathematical description of this relationship far more easily. Formulas relating sides of abstract triangles are very boring to me as a learner and have little relevance to my life until I can see that this is how my home is built. If I actually build a house and in its construction verify Pythagoras' theorem, then the knowledge contained in the theorem won't be abstract to me but will actively become a part of my experience (see Appendix 2). When I first encountered locus as a learner I was continuously under the misconception that I was looking for a specific point. The notion that locus is in fact a relationship between points only in fact came to me when all my answers were wrong. What in our daily lives relates points together to form a relationship? An animated cartoon strip. Hence, if I make the pupils draw an animated cartoon strip first and they see how the

individual points form a relationship, won't they understand what a locus is much better than I did (see Appendix 3)?

Specifically with reference to the music/exponential curve activity, (see Appendix 1) the pupils assimilate new mathematical concepts into their already existing conceptual frameworks, containing concepts about paper folding and the different sounds as one goes higher or lower in octaves. The work on exponential curves will then be accommodated through the easy assimilation through the existing real world concrete concepts of paper folding and music. As the new concepts are fitted and matched with previous mathematics the pupils have learnt, such as work on functions, exponential representation of numbers and curve sketching, the pupils will reach stages of conflict between their already existing knowledge and the new exponential curve knowledge. At this point a stage of equilibration should occur where the pupils realise that another function that they may draw, work with and interpret is the exponential curve. At all stages during the experiment the pupils were allowed to discuss the work together and argue about the outcomes.

In the Basotho Hut/Distance Formula activity (see Appendix 2) the pupils were reminded of the theorem of Pythagoras through a comprehension about it and its use in the construction of rectangular-based buildings. Then the pupils constructed their own huts using string and "Lego" blocks. After construction the pupils proved that these structures had right angles between the walls and were thus led into using Pythagoras to work out the

length of the diagonal. They then transferred this information to a Cartesian plane and formalised the distance formula. After working with the specific example from their huts they had to generalise the information to find the distance between any two points. The activity of reaching a stage where generalisation is possible requires abstract thinking, as does the application of the generalisation to other specific cases. It is also interesting to be able to relate Euclidian geometry to Analytic geometry, and the establishment of these connections and the understanding of them require relational understanding and abstract thinking. Again the pupils worked in groups with plenty of discussion and all helping each other. It was interesting to note that different pupils were at different zones of proximal development with respect to different concepts, and thus where Lerato was able to help Lesego with understanding the necessary calculations, she would never have got to the calculation stage if Lesego had not helped her with the construction of her hut.

In the locus/cartoon activity (see Appendix 3), the concrete situation of drawing an animated cartoon was used as a basis for deriving a definition for locus which would avoid the misconception of thinking of it as a point as opposed to as a relation between points. Once the pupils had designed animated cartoons it was an easy step to design a point moving through space using a flick cartoon and to define a relation to describe this point. From there on the pupils moved into abstract calculations of the defining equations of other loci and did not have to construct each and every one to be able to perform the

calculation competently. In this activity pair work was more prevalent than group or individual work but I still saw pupils assist each other to develop through social interaction.

In order to advance in this way pupils were drawing on other areas of their existing knowledge. Pupils were learning their work in such a way that they could apply one area to another and see the connections between different areas of mathematics, mathematics and other subjects and mathematics and their daily lives. Ernes argues for a more connectivist approach to the learning and teaching of mathematics encouraging the learner and the teacher of mathematics to explore the full social context surrounding the mathematics.

If we look at the experimental activity on exponential curves (see Appendix 1), the pupils are required to use what they already know about curve sketching, completing tables and interpreting relations and functions to add to their existing mathematical schema the exponential curve. They arrive at the mathematics of the exponential curve not only through their previous mathematics knowledge but also by relating paper folding and musical octaves to numbers. The pupils were not given a set of algorithms for drawing and interpreting the exponential curve. They were able to generalise their own workings into an algorithm for future exponential curve work from the work they did.

With the Basotho Hut/Distance Formula activity (see Appendix 2), the pupils were not initially given or taught the distance

formula as such. The activity forces the pupils to understand where it comes from by leading them through the Pythagorean hut-building activity and grounding the rectangular building in history through the comprehension. The pupil eventually arrives at the distance formula and practices using this algorithm after having related sides of right-angled triangles, Euclidean Geometry and the Cartesian plane.

The activity of drawing loci through drawing an animated cartoon strip (see Appendix 3) first requires a broader relational understanding than of mathematical concepts only. Here the pupil uses artistic and visual co-ordination skills to create the original cartoon. The progression to a point moving through space follows easily. However to transcend to finding the equation of the locus by relating different points in different ways requires an already tip-top understanding of other analytic geometry concepts like distance, gradient and division of a line segment in proportion.

3.3.2 *Literature Used for Activity Creation*

Activity	*References used*
Exponents/ Logs/Music	*BENADE, Fundamentals of Musical Acoustics* HARRIS, *Write Your Own Pop Song* NOBLE, *The Power of School Mathematics*
Distance Formula	FLETCHER, *Practical Maths Std 6* KLUTZ, *Kids Travel Survival Kit* KRUILIK, *A Handbook of Aids for Teaching Junior Secondary School Maths* McDONALD, *Now Try This* VORDERMAN, *How Mathematics Works*
Locus	BERNSTAIN, *Draw-It.* DOUST, *A Manual of Caricature and Cartoon Drawing* GREEN, *Animation Pack* KLUTZ, *Kids Travel Survival Kit* KRUILIK, *A Handbook of Aids for Teaching Junior Secondary Maths* NOBLE, *The Power of School Mathematics* THOMSON & HEWISON, *How to Draw and Sell Cartoons*
Overall Structure of Activities	LARIDON, *Classroom Mathematics 6-10* RADMAST, *Ethnomathematics Projects: strings, madice, coins, scale*

3.4 Research Instruments

The pupils participating in the experimental group and the

control group all wrote a pre-and post-test based on the generalized content of the work that they were taught in each of the three sections. In addition to the pre- and post- content tests the pupils in the experimental group wrote an attitude test before and after completing the activities. Where peculiarities existed in the pupils' scores or answers they were also interviewed. Throughout the experiment the pupils' actions and reactions were also closely observed.

3.4.1 The Exponential Curve Pre- and Post-tests (see Appendix 1)

For this activity the pre- and the post-tests were slightly different in that for the pre-test pupils were given tables to complete before drawing graphs, but in the post-test they were expected to know that they had to draw a table to construct the graphs. In both tests the pupils answered questions based on the curves $y=2^x$, $y=2^{-x}$, $y=3^x$, $y=3^{-x}$ and the generalizations $y=a^x$, and $y=a^{-x}$. In both tests the pupils had to identify y-intercepts, discuss asymptotes to the x-axis, domain and range. They were also required to discuss the nature of the exponential relation i.e. if it is a function or not and why. The pupils thus had to draw exponential curves and interpret them. The questions for both tests were based on ideas obtained from past matric paper questions. In the post-test one question had a straight line graph in it in addition to an exponential curve. The pre-test was out of forty marks whereas the post-test was out of fifty marks. The comparison was made in terms of percentages. These

tests were set in the manner I would normally set a standard class test. The pupils were given thirty minutes to write each of these tests.

In the activities that follow the pre- and post-tests are in the form of multiple choice tests and identical for before and after the activity. The change in design occurred because the RADMAST WITS project required the tests in the later format for their research and I felt it was not necessary to duplicate work.

3.4.2 The Distance Formula Pre- and Post-tests (see Appendix 2)

Here the test was a multiple choice test with ten questions each counting three marks, giving a total of thirty marks over thirty minutes. The first two questions addressed historical issues relating to square-based houses. Questions 3 and 4 related exclusively to right-angled triangles and the use of Pythagoras' theorem. Question 5 related to the advantages of using square-based houses. Question 6 discussed the definition of a rectangle. Question 7 again refers to Pythagoras' theorem. Questions 8, 9 and 10 ask the pupils to calculate distance using the formula they derived from Pythagoras' theorem. The test progresses according to the derivation of the distance formula in the same pattern as the activity does. It progresses from theoretical, historical type questions through general geometry definitions and theorems to the application in the form of the distance formula to specific situations. These questions were

again adapted from past matric questions, but also made into multiple choice questions. The questions that are more definition specific and relating to Pythagoras and history do not relate to past matric papers but are original and relate to past Std 6 and 8 papers of mine.

3.4.3 The Locus Pre- and Post-tests (see Appendix 3)

Again this was a thirty mark thirty minute test of ten questions for three marks each. The first two questions concentrate on testing the pupils' understanding of the definition of locus. Question 3 and 4 discuss methods of calculating specific loci without the pupils actually being required to do the calculations. Question 5 discusses the relationship between cartoons and locus. Questions 6-10 ask the pupils to calculate different loci in different situations using different formulas.

The questions in this test again were based on past matric questions, but changed to multiple choice. Obviously the definition-type questions and those relating to the specific activity were totally original and bear no relationship to past matric exams.

3.4.4 The Attitude Tests Before and After the Activities (see Appendix 4)

For the first seven questions the pupils marked on a scale of 1-5 between two opposite polls their opinions on mathematics In the

second part of the attitude test the pupils were given a statement and had to mark off the degree to which they agreed with the given statement. In the tables which follow the following codes have been used to indicate responses: strongly disagree (SD), disagree (D), neutral (N), agree (A), and strongly agree (SA). In the third section of the attitude questionnaire the pupils were asked to give two or more examples of where they see mathematics in their daily lives. These questions were answered using sentence responses.

3.5 Data Collection

All the tests were written under test conditions with no communication between pupils seated in rows about half a meter apart from each other. I invigilated all the tests and marked them myself. Where there were strange phenomena pupils were interviewed by me. In addition I lost the Basotho Huts/Distance Formula answers to the post-test of the experimental group and made them write the test again a week later. Upon finding the first set of responses and comparing the marks to those obtained a week later I found the results to be identical which shows a degree of validity. I also wrote down observations I made with regard to attitude changes whilst teaching the pupils. T-tests have also been performed to indicate validity. Their results will be discussed in Chapter 5 together with the research conclusions relating to the literature.

3.5 Conclusion

To conclude I can say that the experiment followed a quasi-experimental nature of a controlled experiment as is described above. The data was collected using the pre-, post- and attitude tests as well as interviews and observations of the subjects. The next chapter will discuss the results of the experiment.

CHAPTER 4: EXPERIMENTAL RESULTS

4.1 The Analysis Procedure

This chapter presents the results of the controlled experiment. Results for each activity will be discussed separately with a comparative table showing pre and post-test results. A line graph is provided for each activity, comparing the pre-test results to the post-test results for both the experimental and the control groups. This is to indicate the difference between the pupils' knowledge prior to any teaching, and then after being taught by either method (control or experimental). A bar graph for each case, compares the differences in the pre- and post-test results of the control and experimental groups. This is to directly compare the performance of the control group to that of the experimental group in accordance with the major intention of the experiment.

The analysis of the three activities was followed by an analysis of the attitude tests performed on the experimental group. The attitude tests written by the pupils before and after completing activities had three sections (see Appendix 4). In the first section the pupils were presented with a continuum of what were thought of as opposites and had to mark between 1 and 5 where they thought the truth lay[1]. The results of these continuums are represented on a bar graph comparing results before activities to those after activities.

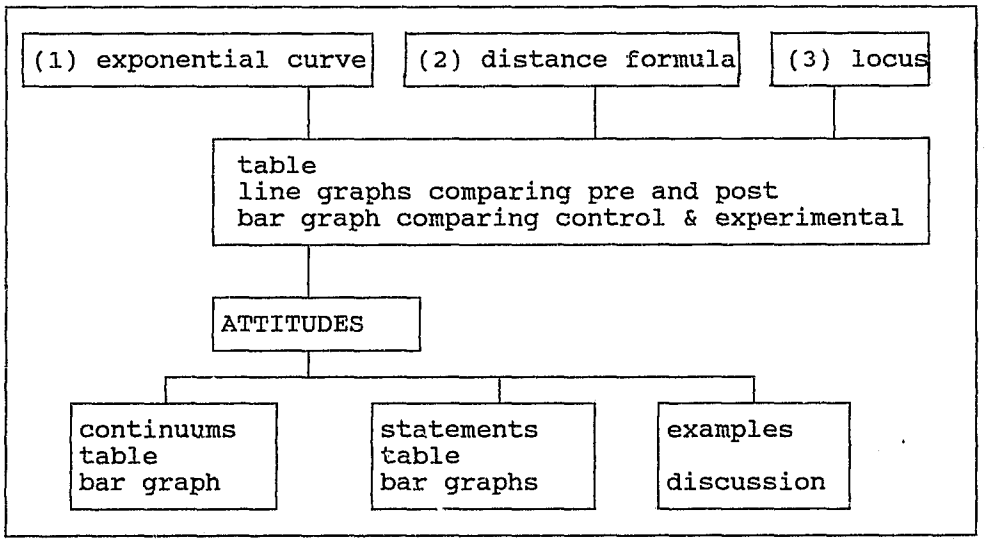
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[1] Some of the opposites were not exclusive opposites, like No 6, where learning rules and laws may in fact be part of a process of understanding.

The second section of the attitude test made statements where the students had to indicate strong agreement, agreement, neutrality, disagreement or strong disagreement. These results are represented by a table followed by a bar graphs of the test written before and after doing the activities.

The third section asked the pupils to give examples of where they see mathematics in their daily lives. Results of this section are discussed in 4.6.

4.2. FLOW CHART OF RESULT ANALYSIS



This chapter does not relate the results back to the literature. That will be discussed in detail in Chapter 5.

4.3 Content-based Results

4.3.1 Exponential Curves

Table 4.3.1 Exponential Curves Control group

Student	Pre-test %	Post - test %	Difference Pre-Post %
1	30	12	-18
2	8	45	+33
3	5	6	+1
4	13	Not Returned	Experimental Mortality
5	0	4	+4
6	8	0	-8
7	20	Not Returned	Experimental Mortality
8	23	8	-15
9	33	14	-19
10	5	Not Returned	Experimental Mortality
11	20	32	+12
12	10	40	+30
13	3	12	+9
14	28	18	-10
15	13	32	+19
16	0	6	+6
17	18	12	-6

Average Pre-Test = 13,9 %
Average Post-Test = 17,2 %
Difference = 3,3 % improvement

Observations

The average for the control group of the post-test was 3,3 % higher than for the pre-test. There was an experimental mortality of three students who were not present for the post-

test. T-test results showed that the pre-test average results were not significantly lower than the post-test results, with a greater than 10 % possibility of the difference having random causes. Six students performed less well on the post-test than on the pre-test and nine students improved from the pre-test to the post-test. Thus only 57 % of students benefitted from conventional teaching on this topic. This has important implications for teaching and will be examined in the next section.

Table 4.3.2. Exponential Curves Experimental Group

Student	Pre-Test %	Post-Test %	Difference Pre-Post %
1	30	52	+22
2	33	Absent	Experimental Mortality
3	60	64	+4
4	60	44	-16
5	33	65	+32
6	45	76	+31
7	40	41	+1
8	5	46	+41
9	15	35	+20
10	25	62	+37
11	25	66	+41
12	73	57	-16
13	7,5	Absent	Experimental Mortality

Average Pre-test 35 %
Average Post-test 55 %
Difference 20 % improvement

Observations

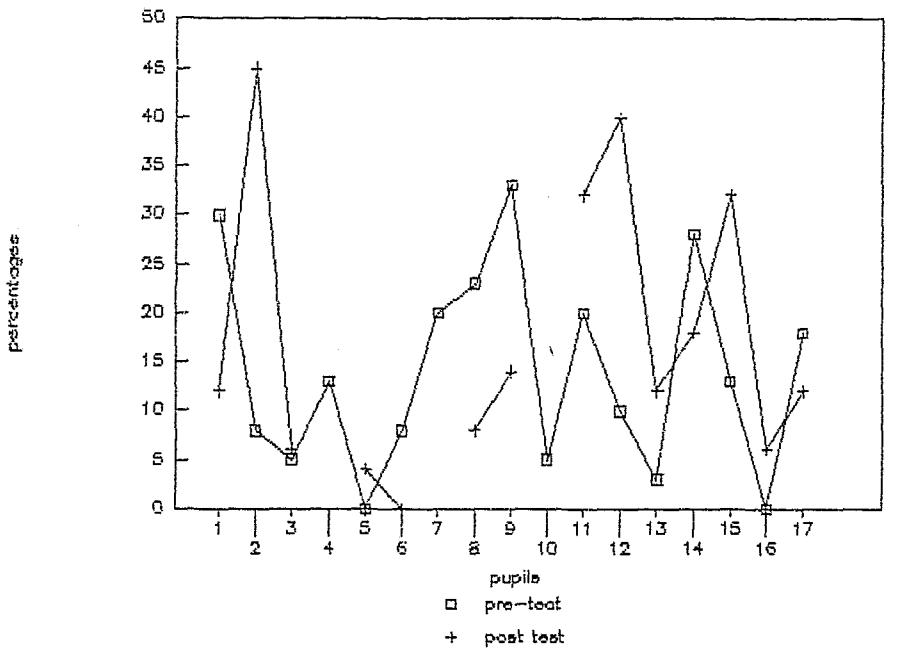
For the experimental group, the average for the post-test was 20 % above the average for the Pre-test. There was an experimental mortality of one student who changed from Higher Grade to Standard Grade for reasons not related to the experiment. For the experimental group the T-test result showed that the post-test average is significantly higher than the pre-test average, as a result of the treatment, to a 0,005 % level of accuracy. One student was absent for the post-test. Two students less well on the post-test than on the pre-test. Nine students improved on the post-test from the pre-test. The two students who achieved less well on the post-test were interviewed. One student said that she was bored with the activities and found them to be a waste of time. The other could not see any relationship between music and mathematics at all. It is interesting to note that these two students normally achieved the highest marks in the class and were the most conscientious.

In stating uses of exponential curves, only one student mentioned musical tuning after having gone through the activities. When asked why, many students said that they thought that they were not allowed to state music because the whole activity had been about music.

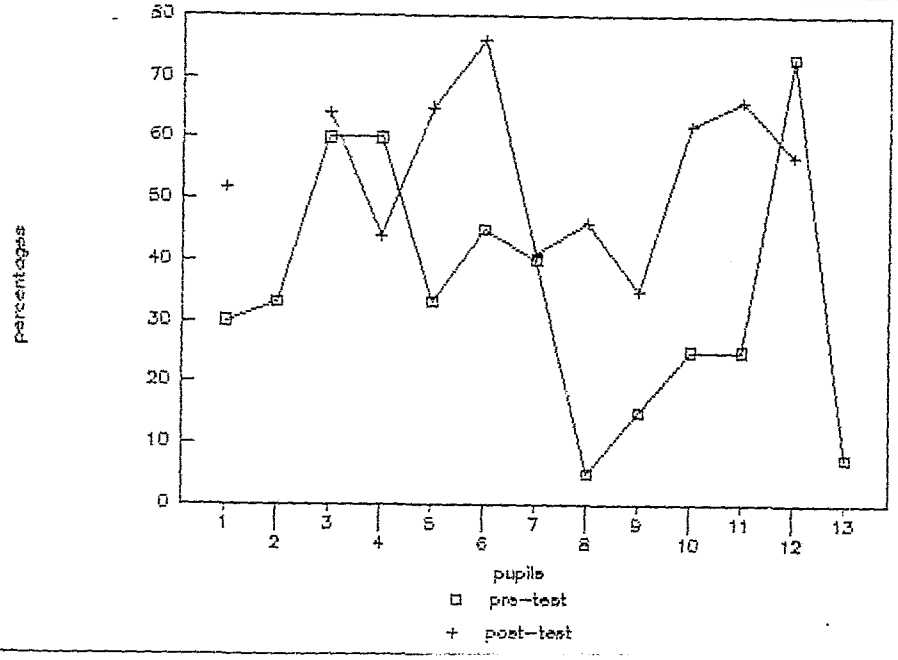
Comparison of Control Group to Experimental Group

The experimental group's performance was 16,7 % better than the control group's performance, considering the difference between pre- and post-test scores. The T-test results show that the control group's results are significantly less than the experimental group's results. It is with a 2,5 % significance level that we can be sure the result is due to the treatment and not random.

Line Graph 4.3.1



Line Graph 4.3.2



Bar Graph 4.3.1

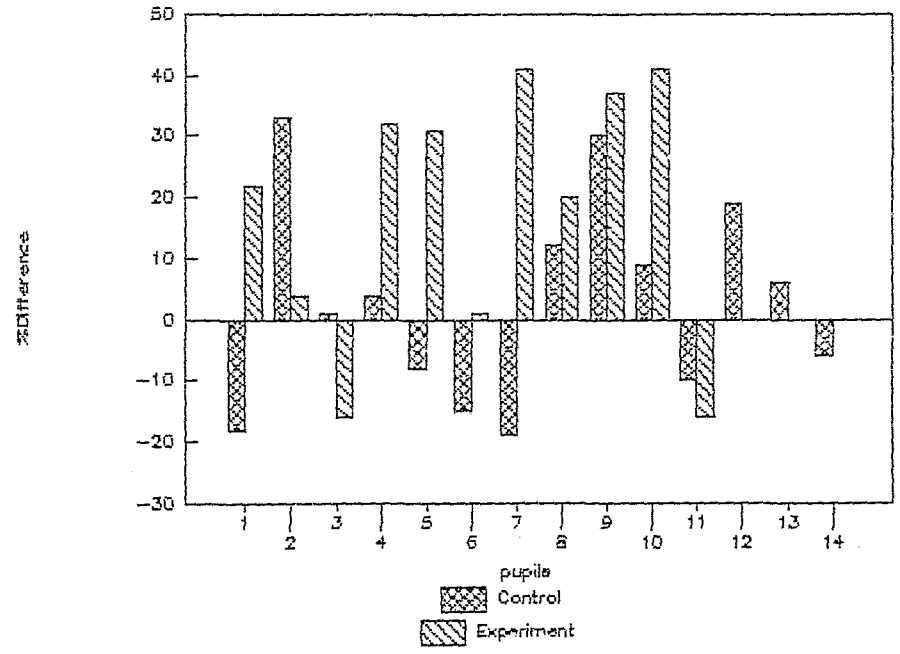


Table 4.3.3 Basotho Huts/Distance Formula Experimental Group

Student	Mark Pre-Test	Mark Post-Test	Difference %
1	9	15	+20
2	12	12	0
3	3	12	+30
4	12	18	+20
5	12	9	-10
6	3	12	+30
7	12	12	0
8	12	12	0
9	12	15	+10
10	9	12	+10
11	3	18	+50
12	9	15	+20
13	12	18	+20
14	6	15	+20
15	6	12	+30
16	3	9	+20
17	15	15	0
18	9	6	-10
19	6	12	+20
20	6	12	+20
21	6	6	0
22	9	21	+40
23	18	15	-10
24	9	12	+10
25	12	12	0
26	12	12	0
27	15	12	-10
28	9	12	+10
29	-	9	-
30	-	9	-

Average Pre-test = 31 %
Average Post-test = 42 %
Difference = 11 % average improvement

Observations

56,6 % of the students did better on the post-test than on the pre-test. 26,6 % of the students obtained worse results for the post-test than the pre-test. There were two students who wrote

the post-test but had not been present for the pre-test, so their results are invalid. The T-test results show that the improvement between the pre- and post-test averages is owing to the treatment at a 5 % level of significance. The four students who performed less well on the post-test than the pre-test said that they guessed the answers in the pre-test and when it came to doing the post-test they had forgotten the activities.

Table 4.3.4. Basotho Huts/Distance Formula Control Group

Student	Mark Pre-test	Mark Post-test	Difference %
1	9	6	-10
2	9	9	0
3	12	18	+20
4	12	18	+20
5	9	18	+30
6	9	3	-20
7	6	12	+20
8	15	18	+10
9	9	21	+40
10	6	9	+10
11	6	18	+40
12	12	12	0
13	6	15	+30
14	9	9	0
15	12	12	0
16	6	9	+10
17	9	24	+40
18	6	15	+30
19	3	6	+10
20	18	12	+20
21	6	15	+30
22	6	15	+30
23	15	18	+10
24	3	9	+20
25	12	12	0
26	6	12	+20
27	12	18	+20
28	6	15	+30
29	12	18	+20

Average Pre-test = 28 %
Average Post-test = 43 %
Difference = 15 % improvement

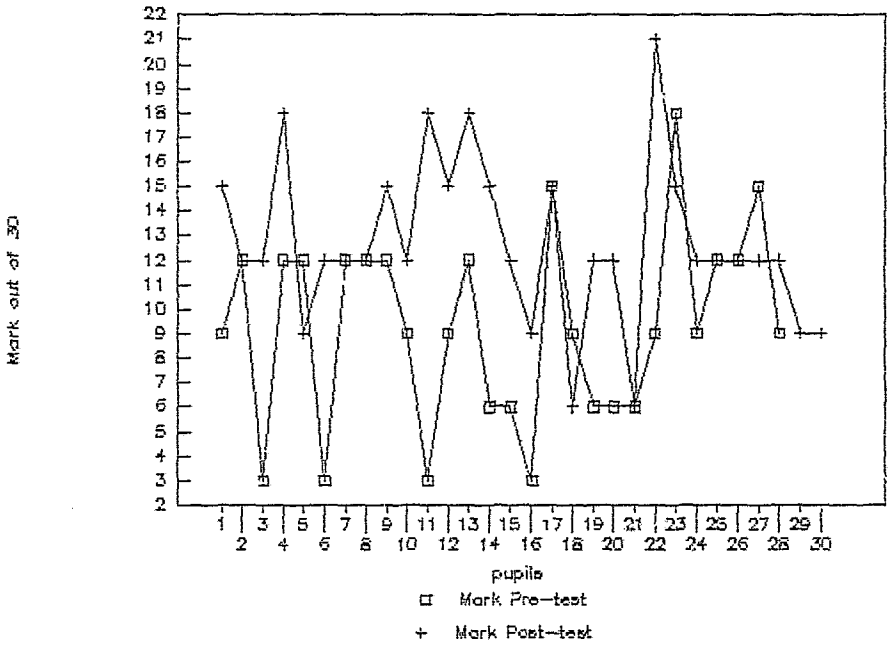
Observations

75 % of students improved on the post-test from the pre-test. 13,8 % of students achieved the same mark on the post-test as on the pre-test 3 % of students performed less well on the post-test than on the pre-test. The T-test showed that the improvement observed between pre- and post-test results for the control group was at a greater than 0,05 % level of significance, due to the treatment and not random.

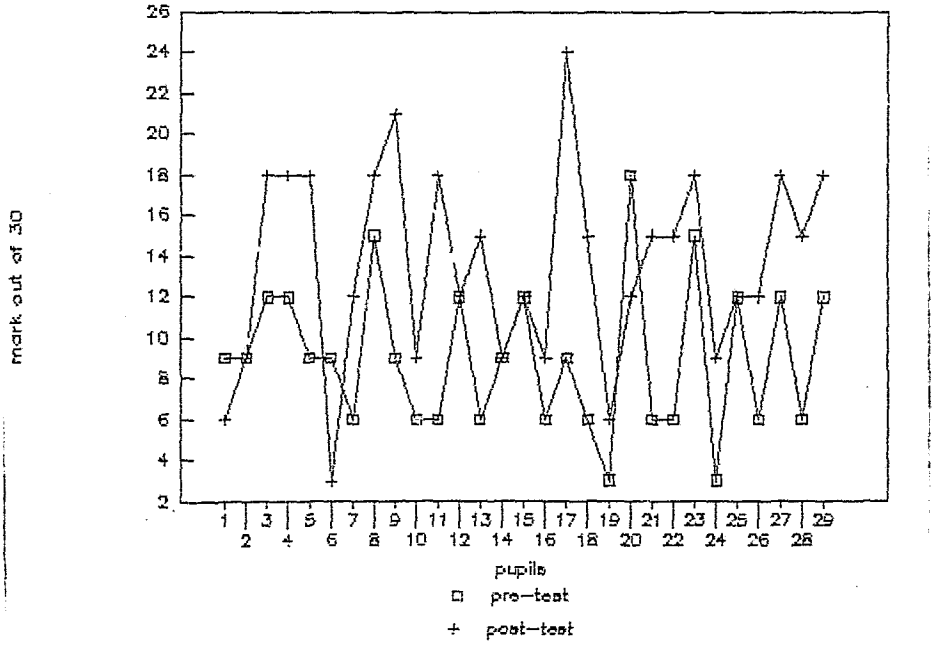
Comparison of Control to Experimental Group

For the control group, the difference between post-test and pre-test results was 4 % better than that for the experimental group. The T-test results, on a comparison between the differences between the control and experimental groups shows that the results were not significant and could have been caused by a greater than 10 % attribution to random causes, as opposed to the treatment. It is obvious that although certain pupils' attitudes to mathematics changed in doing these activities, their marks did not improve, in fact their marks were worse than those of the control group.

Line Graph 4.3.3.



Line Graph 4.3.4.



Bar Graph 4.3.2

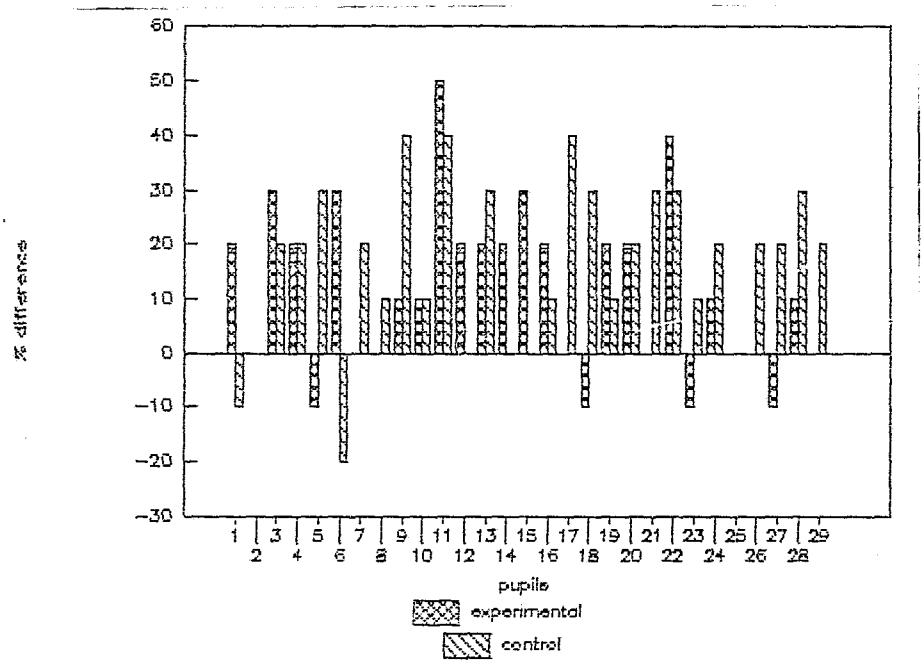


Table 4.3.5 Cartoon Strip/Locus Control Group

Students	Mark Pre-test	Mark Post-test	Difference %
1	3	9	+20
2	9	21	+40
3	18	15	-10
4	6	12	+20
5	3	15	+40
6	3	9	+20
7	3	9	+20
8	6	9	+10
9	3	18	+50
10	0	12	+40
11	12	12	0
12	18	18	0
13	3	12	+30
14	0	12	+40
15	15	12	-10
16	6	18	+40
17	6	15	+30
18	6	15	+30
19	9	15	+20
20	9	15	+20

Average Pre-test = 23,5 %
Average Post-test = 46 %
Difference = 22,5 % improvement

Observations

In the control group 80 % of students achieved better results in the post-test than in the pre-test; 10 % of students achieved the same results and a further 10 % performed worse on the post-

test. The T-test shows that these results were due to the treatment to a 0,05 % level of accuracy and that they were not caused randomly.

Table 4.3.6 Cartoon Strip/Locus Experimental Group

Student	Mark Pre-test	Mark Post-test	Difference %
1	6	9	+10
2	9	18	+30
3	9	9	0
4	9	15	+20
5	21	9	-40
6	12	15	+10
7	9	15	+20
8	12	12	0
9	9	12	+10
10	12	6	-20
11	6	12	+20
12	9	12	+10
13	3	12	+30
14	12	15	+10
15	0	9	+30
16	15	15	0
17	3	15	+40
18	6	15	+30
19	6	9	+10
20	9	18	+30
21	6	9	+10
22	3	18	+40
23	9	12	+10
24	6	9	+10
25	3	12	+30
26	3	15	+40
27	6	15	+30
28	6	3	-10

Average Pre-Test = 26 %
Average Post-test = 41 %
Difference = 15 % improvement

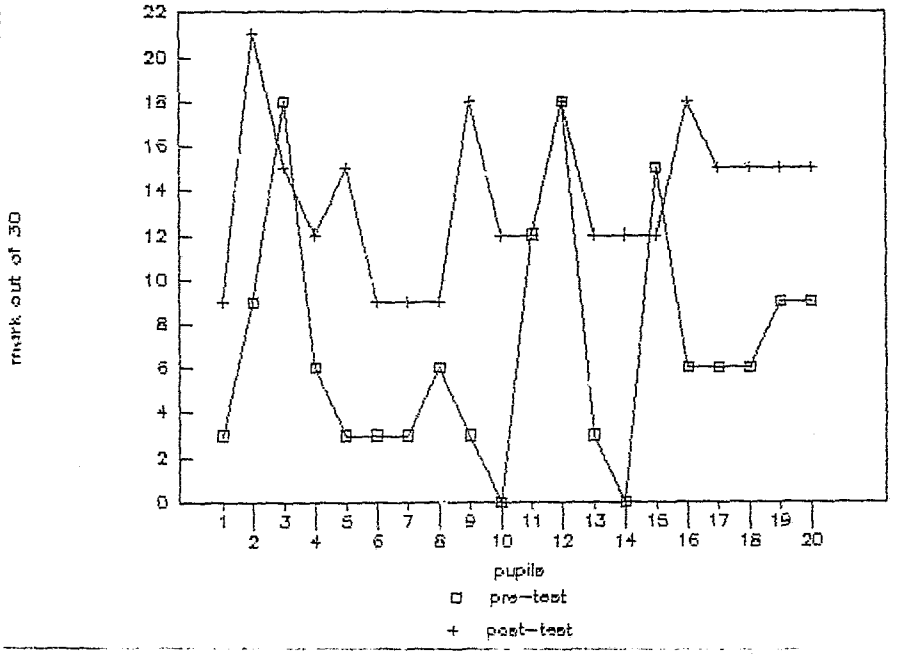
Observations

78,5 % of students improved on the post-test from the pre-test. 7,1 % achieved the same results on both tests. 10,7 % of students obtained worse results for the post-test than for the pre-test. The T-test results show that the experimental group's average improvement from pre- test to post-test was at a greater than 0,05 % significance level, due to the treatment.

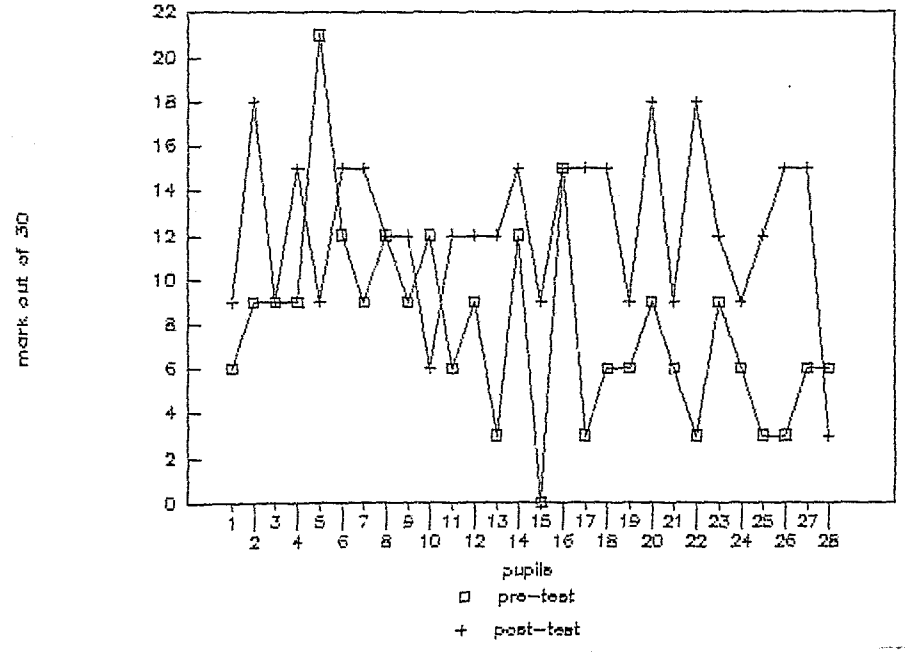
Comparison of Control to Experimental Group

Once again with this activity, the results show that the pupils' marks improved more, using conventional teaching than the ethnomathematics based activities. The T-test results between the experimental and control groups show that the control group's superiority over the experimental group's was due to the treatment at a greater than 0,05 % level of significance, and showed that it was most unlikely to be due to random events. Their attitudes to the work, however, may have been better after performing the ethnomathematics activities than before.

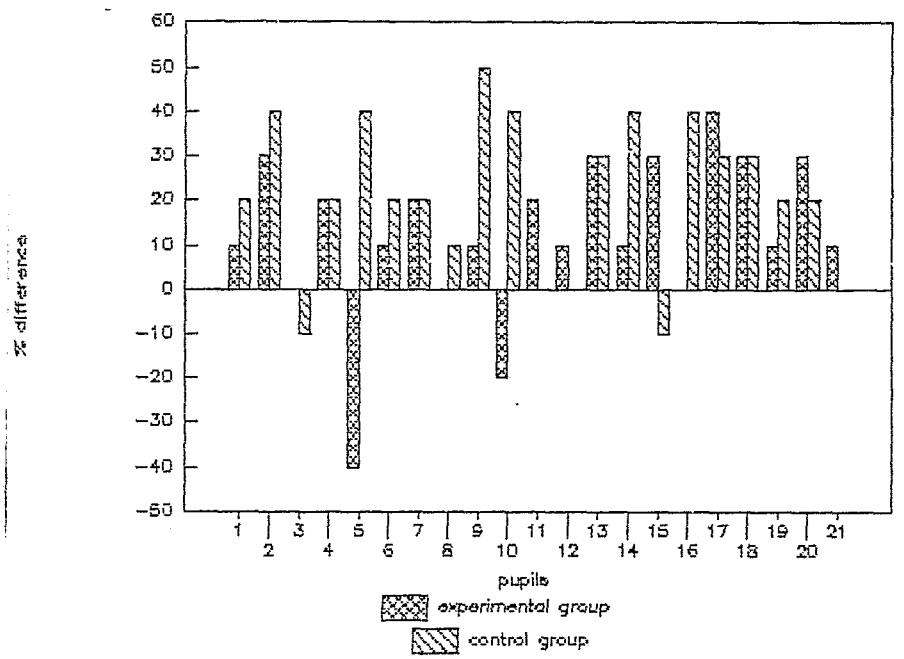
Line Graph 4.3.5



Line Graph 4.3.6



Bar Graph 4.3.3



4.4 ATTITUDE TEST RESULTS

For the first seven questions the pupils marked, on a scale of 1-5 between two opposite poles, their opinions on mathematics before and after performing the ethnomathematics activities[2]. The results for these first five questions will be represented using a cumulative table and a bar graph showing general positive, neutral and negative attitudes before and after the activities[3].

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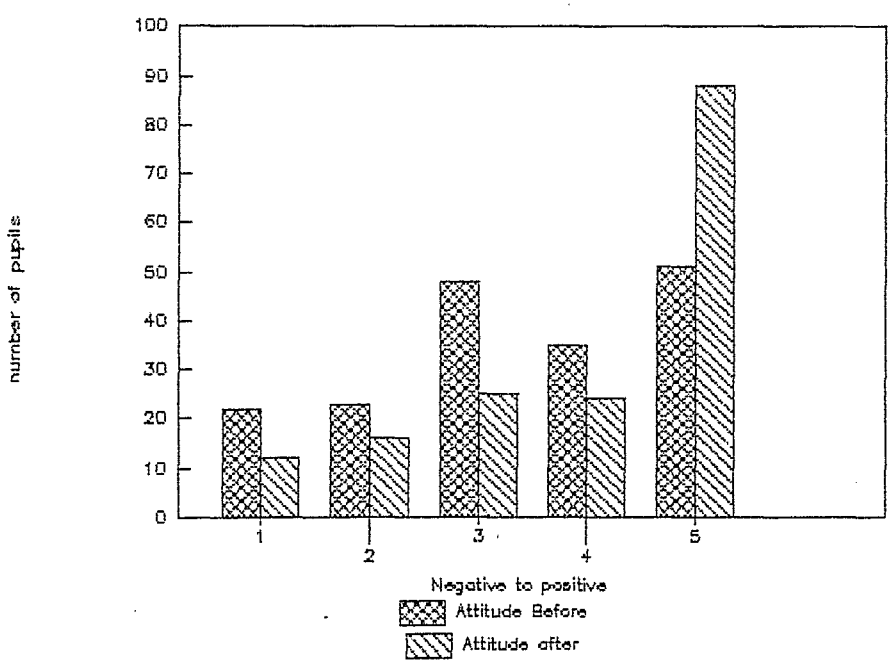
[2] The opposites are not necessarily exclusive, for example No 6 learning rules may be part of a process of understanding.

[3] Some of the scores have been reversed in order to create a general negative to positive analysis. This does not change their meaning at all, for example in the questionnaire fun was rated as 1 and dull as 5, now 1 is very dull, 2 dull, 3 neutral, 4 fun and 5 great fun.

Table 4.4.1 General Attitude Frequency Poles

scale		1		2		3		4		5	
BEFORE/ AFTER	b	a	b	a	b	a	b	a	b	a	BEFOR/ AFTER
USELESS	3	0	2	1	6	1	4	3	13	22	USEFUL
DULL	1	3	8	5	10	5	7	2	5	13	FUN
IRRELEVANT	4	0	1	1	14	6	7	5	6	4	RELEVANT
REMAINS CONSTANT	7	7	7	7	9	7	3	1	2	5	CHANGES CULTURES
UNIMPORTANT	1	0	3	2	4	10	7	6	15	20	IMPORTANT
ABOUT LEARNING RULES & LAWS	6	2	2	0	5	6	7	7	10	14	ABOUT UNDER-STANDING PROCESS
NEGATIVE	22	12	23	16	48	25	35	24	51	88	POSITIVE

4.4.1 Bar Graph: General Attitude Frequency Poles



The graph indicates a much more positive attitude after the experiment than before.

4.4.2 In the second part of the attitude test, the pupils were given a statement and had to mark off the degree to which they agreed with the statement. I have grouped these statements together according to questions relating to similar issues, in order to establish general trends and not bore the reader with too many tables and graphs. In the tables which follow the following codes have been used to indicate responses: strongly disagree (SD), disagree(D), neutral (N), agree(A), strongly agree (SA)[4].

4.4.2.1 Mathematical Self-confidence

The first table refers to the following statements which relate to pupils self-confidence with respect to mathematics:

(1) I have no role in the making of mathematics (statement 1).
(2) I cannot invent my own mathematics (statement 10 reversed).
(3) Only talented people can do mathematics (statement 18).
(4) Not everybody can do mathematics (statement 2 reversed).

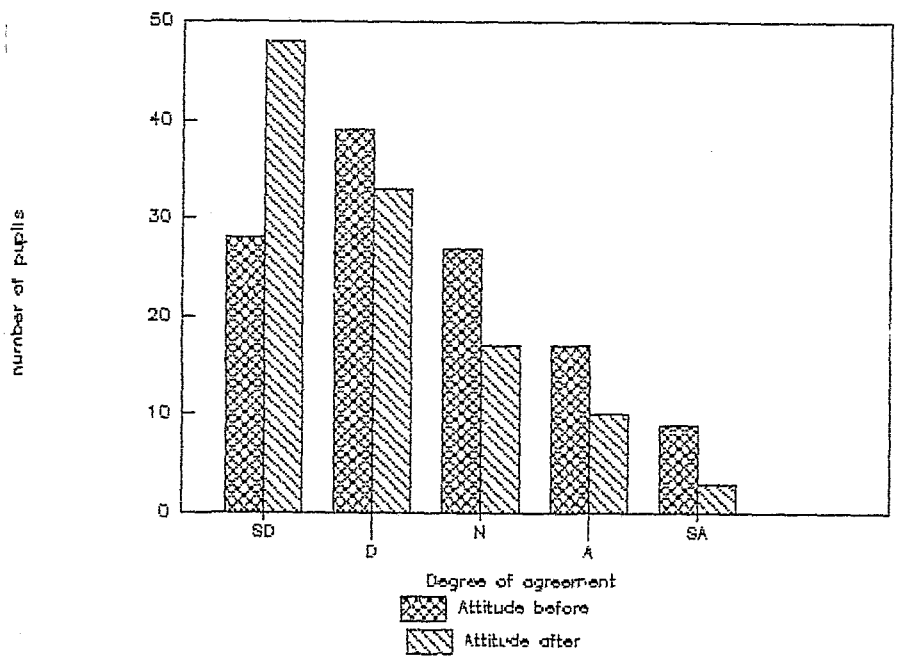
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[4] Sometimes here again I had to change the response order or statement without changing the original meaning in order to group together concepts to form a trend.

Table 4.4.2.1 Mathematical Self-confidence

scale		SD		D		N		A		SA
before/after	b	a	b	a	b	a	b	a	b	a
1	5	9	5	6	11	9	5	3	2	1
2	3	9	8	9	9	4	5	4	5	1
3	15	17	10	9	3	1	2	0	1	1
4	5	13	16	9	4	3	5	3	1	0
Total	28	48	39	33	27	17	17	10	9	3

Bar Graph 4.4.2.1 Mathematical Self-confidence



Pupils showed much higher self-confidence in their mathematics abilities after experimentation.

4.4.2.2 *The Relationship of Mathematics to the Everyday World of the Pupil*

This table refers to the following statements:

(1) I can see mathematics in the daily things I do (statement 15).

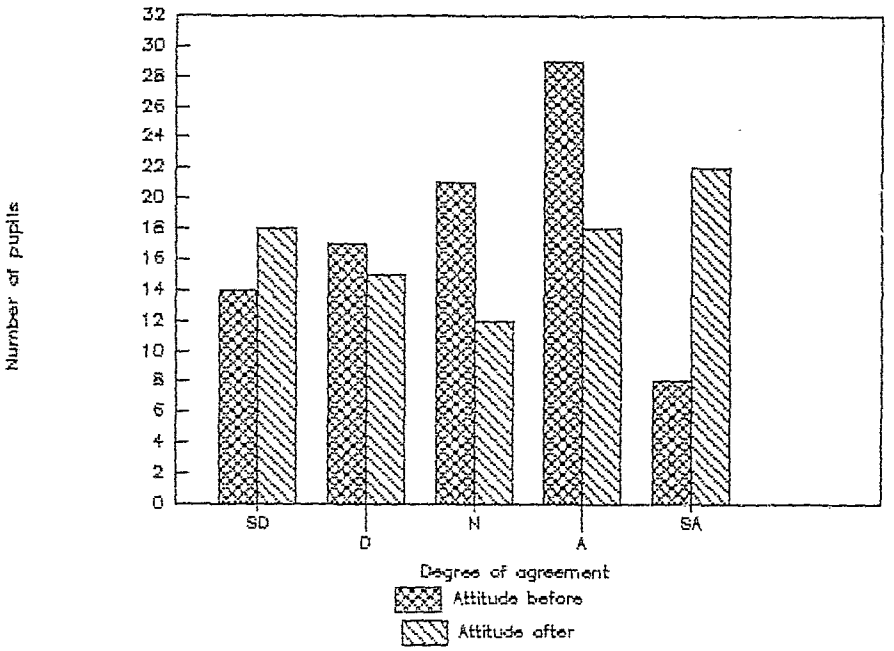
(2) Mathematics makes more sense if it is about real things (statement 4).

(3) There is mathematics in the games that we play (statement 7).

Table 4.4.2.2 The Relationship of Mathematics to the Everyday World of the Pupil

scale		SD		D		N		A		SA
before/ after	b	a	b	a	b	a	b	a	b	a
1	2	3	4	1	8	3	12	7	2	13
2	4	3	4	2	8	4	11	11	4	8
3	8	12	9	12	5	5	6	0	2	1
Total	14	18	17	15	21	12	29	18	8	22

4.4.2.2 *Bar Graph: The Relationship of Mathematics to the Everyday World of the Pupil*



Pupils strongly agreed more often to seeing mathematics in the everyday world after experimentation.

4.4.2.3 *The Relationship of Mathematics to the Pupils' Culture*

This table refers to the following statements:

(1) Mathematics is part of my culture (statement 9).
(2) Mathematics can be found in the building of traditional buildings (statement 12).
(3) There is mathematics in Ndebele Murals (statement 16).
(4) Mathematics is easier if it is familiar to my culture

(statement 13).

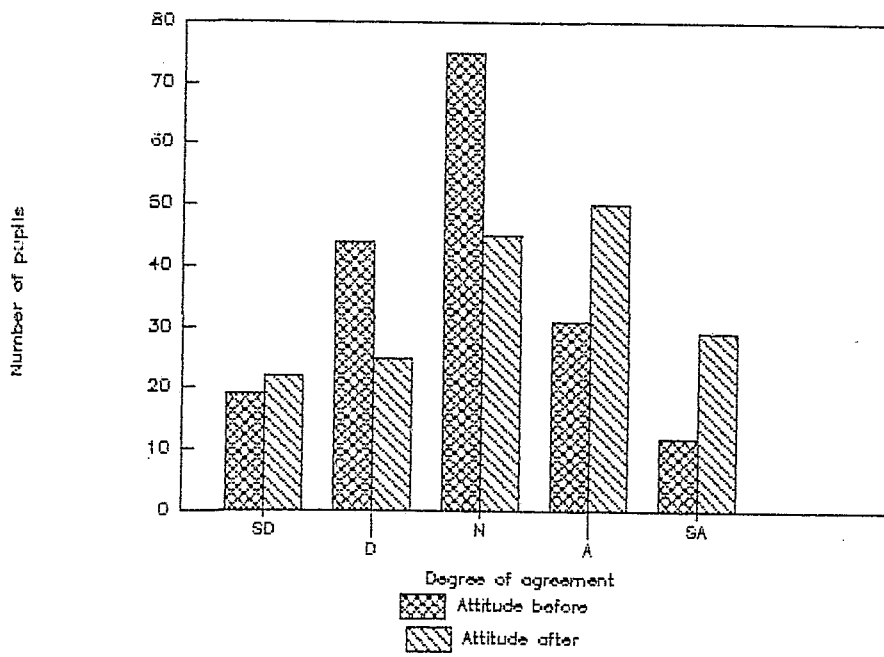
(5) Knowing the history of mathematics is important (statement 17).

(6) The history of mathematics is relevant in class (statement 8 reversed).

Table 4.4.2.3 The Relationship of Mathematics to Pupils' Cultures

scale		SD		D		N		A		SA
before/after	b	a	b	a	b	a	b	a	b	a
1	5	2	10	6	8	6	7	10	0	4
2	1	0	4	0	15	5	4	13	6	10
3	5	6	5	5	17	15	3	1	0	3
4	1	4	10	4	14	10	5	7	2	3
5	3	6	9	5	10	4	6	7	2	5
6	4	4	6	5	11	5	6	12	2	4
Total	19	22	44	25	75	45	31	50	12	29

4.4.2.3 Bar Graph: The Relationship of Mathematics to Pupils' Cultures



More pupils felt that mathematics was related to culture after experimentation.

4.4.2.4 Dependence on Accuracy

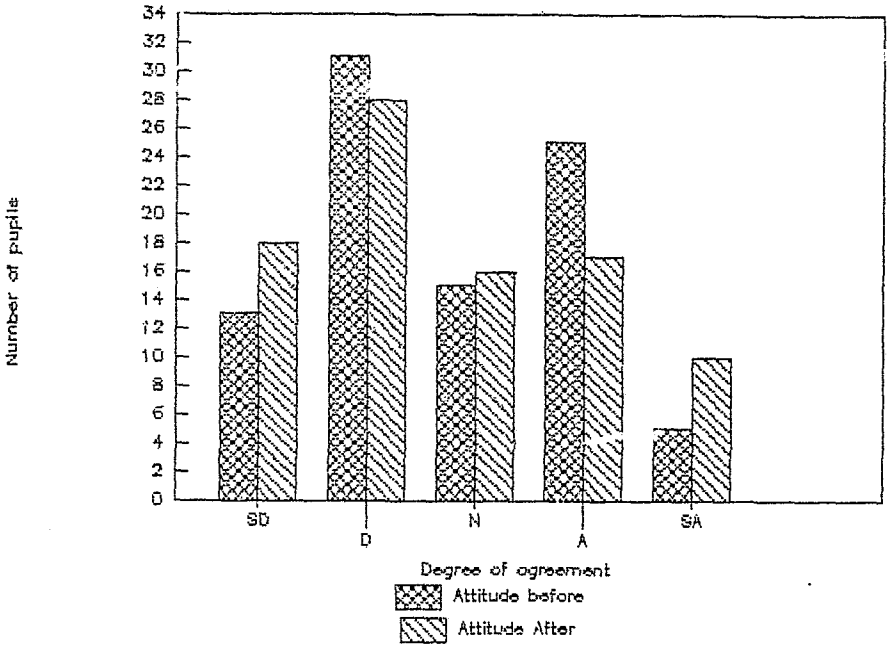
The table deals with the following statements:

(1) I often do mathematics with a calculator (statement 5).
(2) Estimation is never used in mathematics (statement 6 reversed).
(3) There is only one correct method in mathematics (statement 3).

Table 4.4.2.4 Dependence on Accuracy

scale		SD		D		N		A		SA
before/after	b	a	b	a	b	a	b	a	b	a
1	0	1	6	4	5	7	14	12	4	6
2	4	6	14	12	5	7	6	4	0	1
3	9	11	11	12	5	2	5	1	1	3
Total	13	18	31	28	15	16	25	17	5	10

4.4.2.4 Bar Graph: Dependence on Accuracy



Pupils felt mathematics did not have to be calculator dependent, that estimation was a sound method and that there were many correct methods after experimentation. The experiment didn't seem to have much effect on pupils' attitudes here.

4.4.2.5 Working in Groups

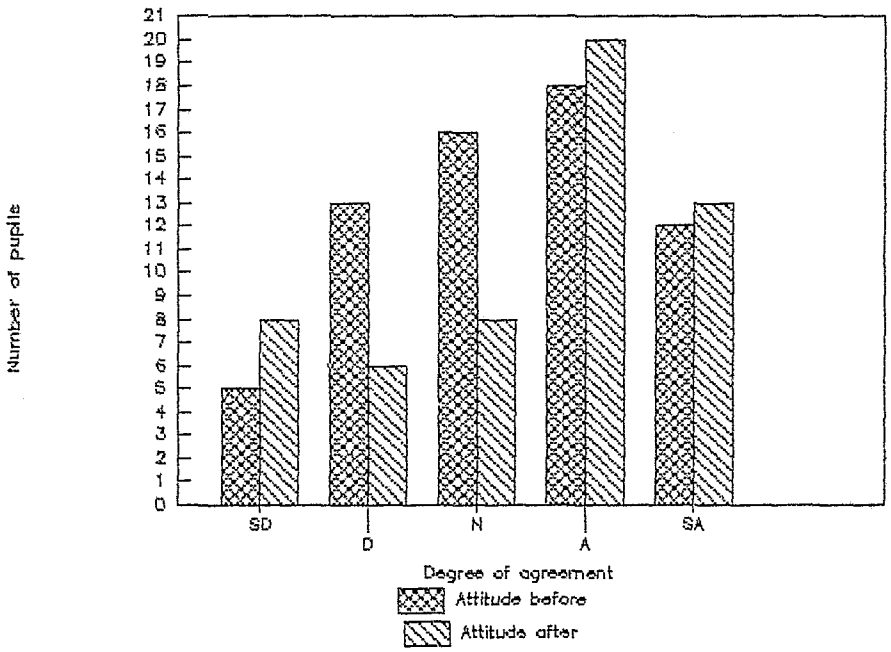
This table refers to the following statements:

(1) I enjoy doing mathematics in groups or pairs (statement 4).
(2) Working in groups is not confusing (statement 14 reversed).

Table 4.4.2.5 Working in Groups

scale		SD		D		N		A		SA
before/after	b	a	b	a	b	a	b	a	b	a
1	4	3	4	2	8	4	11	11	4	8
2	1	5	9	4	8	4	7	9	8	5
Total	5	8	13	6	16	8	18	20	12	13

4.4.2.5 Bar Graph: Working in Groups



After the experiment more pupils found group work tolerable than before.

4.5 Summary of Attitude Test Sections 1 and 2: Important Findings

For the first seven questions, pupils marked on a continuum between two poles between 1 and 5 how they felt about mathematics before and after performing the experiment. On these continuum type questions the attitudes shifted from 29 % extremely positive before the experiment to a 48 % extremely positive response after the experiment.

In the second section of the attitude test the pupils were presented with a statement and had to mark off the extent to which they agreed with it. Again vast differences in attitude were seen before and after the experiment. To name a few examples: Before the experiment 17,9 % of pupils felt they had a role to play in the making of mathematics, whereas after the experiment 32,7 % felt they had a role to play in the making of mathematics.

Before experimentation 16,1 % of pupils felt everyone can do mathematics, but after the experiment 46,4 % of pupils strongly agreed with this statement.

Before the experiment 24,1 % of pupils felt mathematics makes more sense if it is about real things whereas after the experiment 40,7 % of pupils felt this way.

Before the experiment 7,1 % of pupils strongly agreed that they could see mathematics in their daily activities; once the experiment had been done this

figure rose to 48,1 %.

Prior to the experiment 12,9 % of pupils enjoyed mathematics when it related to familiar activities, whereas after the experiment this figure rose to 40,7 % of pupils.

4.6 Third Section Attitude Tests

In the third section of the attitude questionnaire the pupils were asked to give two or more examples of where they see mathematics in their daily lives. Before the activities were performed most pupils saw mathematics only in monetary transactions such as paying taxi fares. However, after the experiment many mentioned other subjects such as accountancy and technical drawing. They also mentioned activities such as cooking, road construction, time, distances, calendars, statistics in newspapers, number of people on a bus, school reports, building, computers, sewing patterns, architecture, car designs and music. This showed that the activities helped the pupils to relate mathematics to the rest of their lives and made it easier for them to pick out mathematics from everyday life.

4.7 Other Interesting Observations

Some pupils participating in the experimental group had not opened a book or lifted a pen to paper in five years of secondary

mathematics education. It was wonderful to observe these very same pupils taking part in the activities and enjoying them. Unfortunately, as can be seen by the test results, even though these pupils may have even improved 30 % between the pre- and post-test results, it still was insufficient to bring them to a reasonable passing average. The same phenomenon was observed with pupils who, although they try consistently, have either poor backgrounds or very weak ability to start with, although both by the experimental teaching method and by the control method they improved, but not sufficiently to pass comfortably.

CHAPTER 5 - CONCLUSIONS

5.1. Introduction

In this chapter I will discuss and analyze the results of the controlled experiment described in Chapter 4. I will try to provide a conceptual explanation for the results, based on my understanding of them and their relationship to established educational and epistemological theories. My conceptual framework is grounded in Kant's 'Meditations', Piaget's constructivism and Skemp's notion of relational understanding. I will also compare my results to those already obtained by other researchers in the area of ethnomathematics in order to establish trends, correlations and differences.

5.2. General Analysis of the Content-based Pre- and Post-tests

The general aim of the researcher in teaching mathematics is to assist the learner to form a relational understanding of mathematics. It is assumed throughout this discussion that ethnomathematics is expecting pupils to form a relational understanding of concepts.

The following patterns emerged from the results of the content based pre- and post-tests. Pupils in the experimental group for the exponential curve activity achieved a 16,6 % improvement

between the pre- and post-test; whereas in the control group an improvement of 3,3 % was observed. However in the distance formula activity 56,6 % of the experimental group improved between the pre- and post-tests, while 75 % of the control group improved. Furthermore in the locus activity 78,5 % of the experimental group showed improvement between the pre- and post-tests as opposed to 80 % of the control group. Overall it appears that the students taught by conventional methods in the control group improved more often than those taught using the contextually based experimental methods with the exception of the exponential curve activity results. What is the cause of this discrepancy? How can I explain it and what does it tell us?

The exponential curve/paper folding/music activity followed a long and involved Grade 11/Std 9 study of functions and relations. This study included much work on curve sketching and interpretation of straight lines, parabolas, hyperbolas, circles and semi-circles. Most of the pupils embarking on the study of exponential curves already had a valuable working understanding of functions and the mathematics of their interpretation (see Chapter 4 results 4.1). If pupils are proficient with straight lines, parabolas, hyperbolas, circles and semi-circles they should be able to apply the same general principles to any curve, once they have isolated what the general principles are. Thus I would conclude that all the pupils taking part in the study had at least a minimum of a proficient instrumental understanding (Skemp, 1971) of functions. With the Distance Formula/Basotho huts and the Locus/Cartoon strip activities (see Chapter 4

results 4.2 and 4.3), the pupils came to the activities with a basic instrumental understanding of Pythagoras' theorem and finding points using simultaneous equations. However their instrumental understanding of these two concepts did not appear to be as fresh in their minds or available to them to access with ease. In fact in teaching these two sections it was almost as though the students had no prior mathematical knowledge to assist the facilitation of the new knowledge.

The pupils in the experimental group were able to recognise the exponential curve as belonging to the class of functions. They already had a set of rules on how to work with functions, for example domain means x-values. They connected the old mathematics that they already knew to the mathematics they discovered through the constructive activities of paper folding and music listening. By combining these different ideas they were relating the tasks at hand to an appropriate schema (Piaget, 1970), making a plan on how to work with exponential curves. In fact they were designing their own algorithms which could then be used to do exercises on exponential curves in an instrumental way. The pupils were able to form a more relational understanding when they moved from being able to work with the exponential curve as a function in isolation to relating it to the rest of their mathematical knowledge.

The pupils in the exponential curve experimental group activity were able to build bridges between street and school mathematics. Carrraher (1985) indicates that this is one of the aims of

ethnomathematics. I believe that the pupils were really only able to do this because their school mathematics foundations in this area were fairly sound. As can be seen with the other activities and the King's College research project (1993) where students had no idea where the contexts fitted in to the greater mathematics schema, they were unable to use their school algorithms to assist them. In addition, where they did not have a foundation of algorithms to apply to specific contexts, the students confused contexts and were unable to make sense of their work.

Bishop (1988) asks an important question, "What kind of knowledge do children bring with them into school?" I would like to argue that although children bring a wealth of knowledge of all types to school, they do not bring sufficient analytic algorithms to allow them to progress in school work at an adequate pace solely with respect to what they know from the outside world. My results indicate that school-taught algorithms, when mastered, assist the pupils in context based work.

I believe that pupils in high school are expected to be able to think in an *a priori analytic* manner as part of their being expected to be at a formal operations stage of thinking where they should be able to think in an abstract manner. The expectations of the school mathematics curriculum require *a priori synthetic* concepts. The pupil's everyday knowledge is *a posteriori synthetic* and conventional teaching is often aimed at acquiring a priori analytic concepts with the hope that they

through abstract thinking pupils will be able to transcend to the *a priori synthetic*. Thus in my view the school curriculum is built upon hidden assumptions about *a priori synthetic* concepts. When pupils learn rules and master algorithms through repeated practice and are able to reproduce the use of these algorithms in tests I would say they have an instrumental understanding of the *a priori analytic* aspects of their school work. When they are able to function well in everyday situations such as selling vegetables and calculating change I would say that they have a good *a posteriori synthetic* understanding which is also instrumental of the task at hand. However in order to succeed in obtaining a relational understanding these need to be married together and then used as a whole.

Let us examine some examples to illustrate the complexity of the proposition I am making. Using Kant's own classic example from *The Critique of Pure Reason* one can present the following example. How do we know what a 'bachelor' is if we do not know what 'unmarried' is? To understand 'unmarried' we actually need to understand 'married'. In life we acquire an understanding of 'marriage' through our synthetic experience of the socialisation of man and the families we were born into and live in. Once we have grasped the difference between 'married' and 'unmarried' it is possible to understand the analytic sentence: All bachelors are unmarried men. In a similar way it is possible to understand the meaning of the analytic sentence: All that is sacred is holy. In both these *analytic a priori* sentences the meaning of the predicate is entirely contained in the meaning of the subject.

Now the student can move to making sense of an a priori synthetic sentence where the predicate's meaning is not entirely contained in the meaning of the subject like: Marriage is a sacred institution. If the student did not understand the two preceding *a priori analytic* sentences, her understanding of the more complex a priori synthetic sentence would be insufficient because she was lacking the necessary conditions for a complete understanding.

An example from the study of school mathematics is as follows: Pupils are required to understand the *a priori synthetic* statement: The only prime number which is even is two.
What are the *a priori analytic* statements leading to this *a priori synthetic* statement?

(1) Prime numbers have only two factors; they are exactly divisible only by one and themselves. (One is not prime itself because it only has one factor.)
(2) All multiples of two are even numbers.

It is only possible to understand the term 'prime' if you understand that 'divisible by only one and itself' means that the number has only those two factors. This statement is entirely *analytic a priori* because the meaning of the predicate is totally encompassed by the meaning of the subject. Similarly the predicate 'even numbers' is entirely explained by the subject 'a multiple of two' and hence this is also *analytic a priori*. The combination of the two statements forms the *a priori synthetic* statement that: The only prime number which is even is two. If

we did not understand that 'even' meant a factor of two or that 'prime' meant divisible by one and itself only, it would be impossible to understand the more complicated *a priori synthetic* statement.

Kant puts the argument far more succinctly in the following quote from *The Critique of Pure Reason*. It is important for the reader to note that my discussion focuses on the second half of this profound argument. "Upon such *synthetical*, that is augmentative propositions, depends the whole aim of our speculative knowledge *a priori*; for although *analytical* judgements are indeed highly important and necessary, they are so, only to arrive at that clearness of conception which is requisite for a sure and extended synthesis and this alone is a real acquisition" (KANT, 1924: 9).

Gerdes (1994) suggested that ethnomathematics is in fact a part of what Kant calls *a priori synthetic* knowledge. Hence it is important for us to understand what Kant is implying by this statement and for us to see how this relates to the constructive nature of the activities in this experiment.

If we look at the matter from a Piagetian constructivist angle, we will observe the following: initially in the intuitive stage the child will learn the meaning of unmarried, bachelor, married, sacred and holy through relationships experienced within her own family.

Later in the concrete stage she will become aware of the rules of society relating married people and holy events. By the formal operations stage the child will be able to make a priori synthetic judgements like 'marriage is a sacred institution' from opinions on their validity and truth (PIAGET, in Sprinthall, 1981: 121).

Consider the example about a prime even number. In the intuitive stage we learn the meaning of two and factors and multiples thereof through *synthetic* experience and we learn about divisibility and primeness through our experiences of sharing things like sweets. Later in the concrete stage we become familiar with the formal mathematical rules classifying different sets of numbers as prime, even, factors of two, divisible by only one and itself etc. In the formal operational stage it is possible to combine knowledge from concrete operations to synthesise new knowledge that is not *analytic* in character but *synthetic a priori*. We can say that two is the only even prime number with conviction and understanding. If we did not have the basis of the solid analytic understanding from the concrete stage we would battle to have sufficient clarity of understanding to claim a formal operations understanding.

It appears that when we learn something new as we are progressing through assimilation, accommodation and equilibration (SPRINTHALL, 1981: 126), we are actually going through the stages Piaget described childhood development as going through. Also

in going through those stages the learner is moving from a *synthetic a posteriori* understanding, relating it to an *a priori analytic* understanding and finally moving to *a priori synthetic* understanding.

Gerdes said that ethnomathematics falls in the realm of the a priori synthetic (1994). More generally Kant says that all mathematics falls in the domain of the a priori synthetic (KANT, I. 1924: 9). The mathematics we try to teach in the senior secondary phase at school definitely falls into this domain and into that of expecting formal operational thinking. If we look at any section of the school mathematics syllabus we are expecting the pupils to understand *analytic* rules about the different concepts and to be able to join them together to form new ones. If we look at the exponential curves section, the pupils must understand the rules for functions and the physical appearance of the curve before they can go on and deduce things about and from exponential curves.

In the exponential curve activity the pupils in the experimental group were able to use my problematization of paper folding and music to improve their mathematical knowledge (see results Chapter 4.1). Taken a step further after the experiment they may have been able to use their mathematics to tune an instrument. This is in line with Gerdes' view (1982) on ethnomathematics. Students' interest in music was engaged by the activity which created a forum together with their *analytic* understanding of functions for them to learn actively new *a priori analytic*

abstract knowledge. With the distance formula and locus activities (see results Chapter 4.2 and 4.3), even though the pupils' interest was engaged by the activities, they were unable to learn effectively actively because they were lacking in background and experience, and hence had concrete and intuitive problems not allowing them to have sufficient *a priori analytic* knowledge to transcend to the *a priori synthetic*. At best they were able to understand on an *a posteriori synthetic* level with some *analytic* connections being made.

5.3. Further Observations on Attitude Tests and Classroom Observations With Respect to the Literature

Much of the research that has been carried out in ethnomathematics indicates a need to unfreeze mathematics inherent to a specific culture (GERDES, 1985). Gerdes believes this is necessary in order to show pupils the mathematics in their own culture. Furthermore enculturating the mathematics will assist the pupil to problematize reality in the most opportune way for learning mathematics and making it of use in the pupils' daily lives. The ethnomathematics attitude tests performed on my students showed that although pupils still felt much the same about the relationship between mathematics and their own cultures, they thought the mathematics much more relevant after completing the activities. Prior to the activities only 23,3 % of the pupils agreed that mathematics was part of their culture. After the activities 35,7 % of pupils

agreed that mathematics was part of their culture and 14,3 % strongly agreed. This in total shows a 26,7 % improvement in pupils' cultural mathematical self-image which supports Gerdes' position. In addition 13,9 % more pupils felt that mathematics was easier if it was familiar to their cultures, after they had done the experiments. Most importantly before the experiment 57 % of pupils could see mathematics in the daily things they did, but afterwards 75 % of pupils were able to do this. The learner learns in an environment partially rooted in her cultural heritage (see results Chapter 4.4.2.4). Gerdes has pointed out that this will influence her learning and appreciation of mathematics.

It is interesting to note that before experimentation pupils could only name uses of mathematics in their daily lives with regard to money, such as paying taxi fares to and from school. After the experiment they saw mathematics in many other areas such as technical drawing, cooking, road construction, buildings, computers, sewing and music. It seems as though the ethnomathematical activities opened the pupils' eyes to other places in their lives where they use mathematics (see results Chapter 4.4.3). This does indicate the beginning of a conscientisation process which probably would occur with more activities of this nature and if they occurred throughout the learner's learning experience and not only as an experiment in the final stages of her school career.

If we accept that the learner's environment is influenced by

cultural heritage we can go further and say that Dewey's notion of the experiential learning of an organism in an environment (Dewey, 1938) is supported by these changes in attitudes of my pupils. However readers must bear in mind that the changes in attitudes had little or no significant bearing on content based tests unless the pupils already had a good instrumental understanding of related concepts. This suggests that Ascher is entirely correct in not wanting school mathematics to be replaced by ethnomathematics (Ascher, 1994). Moreover to view ethnomathematics activities as a **necessary but not sufficient** revitalization of school mathematics appears to be a sensible stand to adopt.

Pupils taking part in my experiment were able to operate with self-confidence in their daily lives with regard to mathematical tasks. Before the experiment they were not able to operate with this degree of confidence in the school mathematics situation. During the experiment it appeared to me that some pupils gained self-confidence (see results Chapter 4.4.2.1). Pupils who had never before participated in my class except to disrupt proceedings suddenly became key members of their groups. This was particularly so with the hut building activity. They enjoyed building the huts immensely and without knowing it, even did some mathematics. When the tasks became less concrete and it was necessary for them to use formal mathematical thinking once again they were not able to cope because they had no background to help them. Readers must remember that these were Grade 12/Std 10 pupils in their last year of school. I believe that if at any

stage prior to this, teachers had managed to grab their attention, they would not be mathematical casualties. It also seems plausible to say that one way of interesting these students is to: "Build bridges between street and school mathematics" (CARRAHER, 1988: 19).

The attitude tests suggest that ethnomathematics methods do make pupils more self-confident about their own roles in mathematics, and provide them with the necessary positive attitude to start learning. Before the experiment only 16,1 % of pupils strongly agreed, and 51,6 % of pupils agreed that everyone can do mathematics but after the experiment these scores shifted to 46,6 % of pupils strongly agreeing and 32,1 % agreeing and a mere 10,7 % of pupils not believing that everyone can do mathematics. The majority of pupils after the experiment agreed (77,7 %) that mathematics makes more sense if it is about real things. After the experiment 71,4 % of pupils thought mathematics was about real-life applications as opposed to only 44,4 % before the experiment. An astounding 81,4 % of pupils enjoyed maths when it related to familiar activities after the experiment whereas only 45,2 % felt that way before having tried to do mathematics in this way. The overall suggestion of these attitude test results is that teachers must indeed try to build bridges between the pupils' daily lives and the mathematics that they teach in the classroom. This however cannot exclude emphasis on process and drill in areas where ideas need to be mastered, because the pupils don't benefit if their mathematical knowledge is only context based as is shown both by the Kings College Research

Project (1993) and by my own content pre- and post-tests (see results Chapters 4.2 and 4.3).

It was interesting to note that the pupils who usually achieve the best results in mathematics were not those who achieved the best results in the ethnomathematics activities. In fact in some of the activities these pupils became bored and achieved worse results on post tests than on pre-tests. De Abreu (1991) found that the child who did best in school mathematics in her class at school was unable to calculate her own change when buying loaves of bread for her family. Although this research was conducted in rural Madeira the results correlate well with my results in Johannesburg South Africa.

Pupils normally shied away from using everyday methods in the mathematics class, because they felt they would be rejected by the teacher. During these activities they were much happier to use their own everyday methods and in fact found it necessary to do this because they had no algorithms to rely on (see results Chapter 4.4.2.2).

Although we do not like to believe it at this stage of educational transition the pupils who had some algorithms to help them were more able to form a relational understanding of the new concepts presented in the activities than those who did not have this foundation. Here I am not speaking about the normally best student but about those who had a working understanding of their mathematics. It appears that these pupils

benefit the most from the activities and those who previously had no success at all now had a measure of success but not sufficient to pass matric with.

Bishop (1994) says that teacher education should be the key focus for research in ethnomathematics. I believe that my research points to this as well for the following reasons:
If teachers are taught how to use ethnomathematics activities earlier in the school curriculum we will not lose so many pupils' interest by the time they are completing their mathematics school curriculum. The fact that students in my classes for the first time ever paid attention and tried to do the activities showed that their interest was sparked. Very careful teacher education is needed to achieve a working balance between ethnomathematical activities and conventional mathematics teaching so that pupils are able to form a relational understanding based on a sound instrumental understanding. It is through good teacher education that teachers will be able to help pupils to form the necessary *analytic a priori* understandings to allow them to transcend to the *synthetic a priori*. This involves teaching teachers to be good facilitators helping them pace their lessons and identifying the essential *analytic a priori* knowledge. It is also through teacher education that teachers can learn of suitable ethnomathematical activities to perform with their classes which are not only specific to the teacher's cultural heritage. The most important part of teacher education is to make the teacher a critical thinker so that she can decide for herself what is appropriate for the students to learn at which stage.

CHAPTER 6: SUMMARY AND RECOMMENDATIONS

6.1. Summary

In this research report I performed a controlled experiment on pupils from Com-Tech High to verify the hypothesis: Everyday mathematics enhances matric pupils' performance and motivation. My experiment arose because of insufficient research to the contrary of my hypothesis. The experiment was of a quasi experimental nature because the subjects could not be selected randomly.

The pupils were divided into a control and an experimental group. The experimental group studied the sections of exponential curves, distance formula and locus using the everyday ethnomathematical activities of music and paper folding, Basotho hut architecture and cartoon strips respectively; while the control group were taught in the conventional manner (see Chapter 3 for methodology).

The results were analyzed using statistical methods (see Chapter 4) and conclusions were drawn from them from a philosophical and psychological perspective on education and current ethnomathematical literature (see Chapter 5). The main thrust of my argument is based on Kant's *a priori synthetic* understanding being dependent on a satisfactory *a priori analytic* understanding of the concepts making up the *a priori synthetic*

(see 5.1). This also leads to an in-depth analysis of the learning of mathematics from a Piagetian perspective and relies heavily on Skemp's notion of instrumental and relational understanding.

Did this research achieve the aims it set out to achieve?
I believe that my literature review does in fact show that few controlled experiments have been performed in ethnomathematics education, and that very little work has been done with senior secondary pupils in this field. I did in fact perform a controlled experiment with two groups of matric pupils in the same school, stream and phase of their education. The results in most cases held an acceptable degree of significance from which to draw conclusions (see T-tests Chapter 4). However I found that everyday mathematics only enhances students' performance who already have a sound instrumental understanding of related *analytic* concepts.

With regard to the subsidiary aims of this research the following was achieved: The ethnomathematical materials used for this research were not only workable but fun as well. The pupils enjoyed the activities. The attitude changes bear this out. In addition to this some of the materials have been adapted and used by both the ethnomathematics project of RADMAST, WITS and the Ikaheng Outcomes Based Education project. Evaluation tests were created and used for each ethnomathematical activity. The matrics who participated in this experiment by and large left the experiment with a more positive attitude to mathematics and were

more motivated to work (see attitude tests results, Chapter 4.4). It was interesting to note that most of the pupils who participated in the experiment passed their final matric examinations even though I left the school shortly after the experiment, and the school was unable to get a replacement teacher immediately. I do not say the pupils did any better ultimately than they would have normally but I think under the circumstances their performance was outstanding. My teaching has changed since doing this research; I have become much more careful in my stress on an instrumental understanding of basic analytic concepts. I do not know if it is for this reason or because I now teach adults, but my results are much better in general and my teaching is a much happier experience. I seem to have a deeper understanding about how we learn, and by putting Kant and Piaget into practice am achieving greater success.

6.2 Recommendations

My conclusions are open to further debate and research since they are philosophical as opposed to pure scientific; however I believe that for these students and this teacher at this time they are valid and useful. This research was conducted during a period of vast educational transition in South Africa and during a period of personal transition to another teaching post. It may be valuable to repeat the research under more stable conditions.

Furthermore I recommend a new study to be performed with three

groups: a control taught conventionally, an experimental group taught with only contextually based work and a Kant group taught contextually with the *analytic a priori* necessary instrumental understanding being emphasised. By performing an experiment in this way more conclusive results may be obtained.

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Appendix 1

MUSIC AND EXPONENTIAL CURVES

INTRODUCTION:

> Music and mathematics have always been closely related. They both involve responses of human reason - rationality - to surroundings. Mathematics was inspired by our recognition of natural regularities and patterns. In the same way,music is made possible because of our awareness (intuitive or rational) of regularity in the behaviour of sound.Because sound relationships are physical ones, they can be described mathematically. This is why the two subjects have always "informed" each other.(MENUHIN,Y.1977:p12)

In this activity music will be used together with paper folding to introduce the idea of exponential relationships.The students will move from hearing a musical relationship, to describing the relationship mathematically in words ,tables and graphs.

The content covered is: (a) Exponential relationships
(b) The exponential curve

Appropriate Level
Although the rules of exponents are covered every year from std7/grade9 until std10/grade12 in varying detail, the exponential curve is only drawn for the first time in std10/grade12.
This activity could be used as an introduction to functions in std8/grade10.However it has been specifically devised for the exponential curve sketching in std10/grade12.

Curriculum related Outcomes
(1) To demonstrate intuitions about ways of working with number.
(2) To generate and describe numerical relationships.
(3)To demonstrate an understanding of how number systems are used in various social and cultural contexts.
(4) To collect and order data.
(5) To summarise,display and communicate data using tables,graphs and words.
(6) To interpret data.
(7) To demonstrate the use of symbols and notations in generalisations and variability.

(8) Using mathematical language to communicate generalisations,concepts,ideas and thought processes.
(9) To set up models to represent real life and abstract situations.

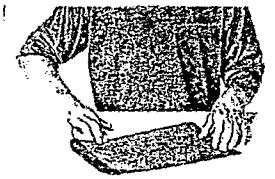
Outcomes related specifically to the paper folding and music listening activities
(1) To identify the pattern in both the paper folding activity and the music listening activity as being an exponential one
(2) To move from the concrete activities of paper folding and music listening to drawing a mapped table of the relationships established through the concrete activity.
(3) To move from the table of the concrete relationship to a graphical representation.
(4) To move from drawing a graph of a concrete function to drawing an abstract graph not specifically related to an activity.
(5) When given a graph of an exponential function, to be able to interpret the graph and describe aspects of it such as domain,range, intercepts,and the relationship both as an equation and in words.

Tips for teachers
For this activity the teacher will require enough paper so that each pupil may have two sheets,graph paper so that each child may have 8 sheets (square paper will do just as well), a coffee tin and stick for each pupil or xylophones per group and a cassette player or a musical instrument for the teacher. The class may work individually ,as a whole class with each pupil performing the activity or in pairs or small groups. I would suggest that the paper folding and music listening are performed individually as part of the class but that the discussion that follows each activity be in pairs or small groups.

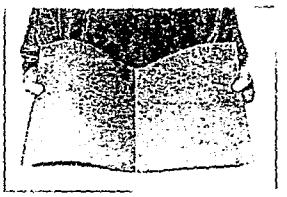
WORKSHEET 1 -PAPER FOLDING

You will need a large sheet of paper.

(1) Lay the sheet of paper on the table in front of you. Fold it exactly in half,either vertically or horizontally. Sharpen the crease with your fingernail so that you will be able to see it clearly.



(2) Unfold the paper.



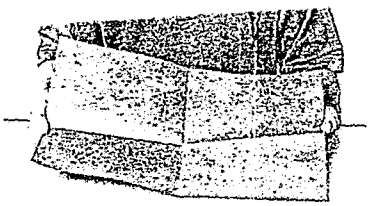
(a) Into how many parts is the paper now divided? _____

(b) Can you write this number in exponential form?_____

(c) Complete the following:

2 = 2 X 1 = _____

(3) Re-fold the paper along the original line and fold it in half again.



(a) Into how many parts is the paper now divided?_____

(b) Write the answer to (a) as a product of 2.

_____

(c) Write the answer to (a) as a power with base 2.

_____

(4) Re-fold the paper along the original lines again and fold it in half again.

(a) Into how many parts is the paper now divided?_____

(b) Write the answer to (a) as a product of 2.

_____

(c)Write the answer to (a) as a power with base 2.

_____

(5) Repeat the activity 3 more times then complete the following table.

No. folds	1	2	3	4	5	6	7
no. divisions							
divisions as a product of 2							
divisions as a power with base 2							

(6) Can you see a relationship between the number of folds, the number of twos in the product and the powers of two?

_____

(7) Describe the relationship observed in (6) in words.

_____

(8) Plot a graph of number of folds (x-axis) against number of divisions (y-axis).

(9) Is this graph a function? Why?

_____

_____

(10) What kind of curve is this?_____

(11) Determine the domain and range of the graph.

(a) domain_____

(b) range_____

(12) Determine the intercepts of the graph.

x-intercept_____

Y-intercept_____

WORKSHEET 2 - OCTAVES

(1) Listen to the notes played on the cassette or by the teacher on the instrument.[1]
(2) Are the notes the same or different?

_____

(3) Is there anything similar about the sounds that you hear?

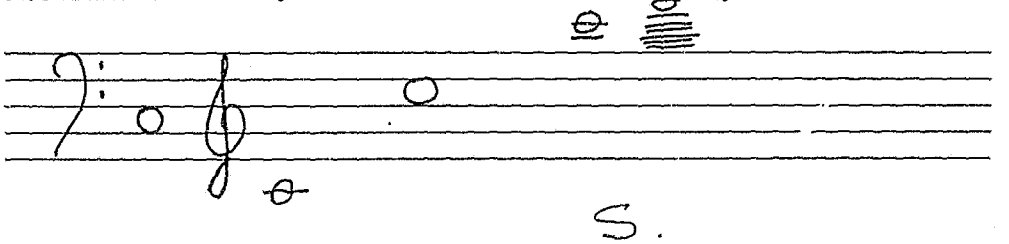
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(4) What do you think is the relationship between the notes played?

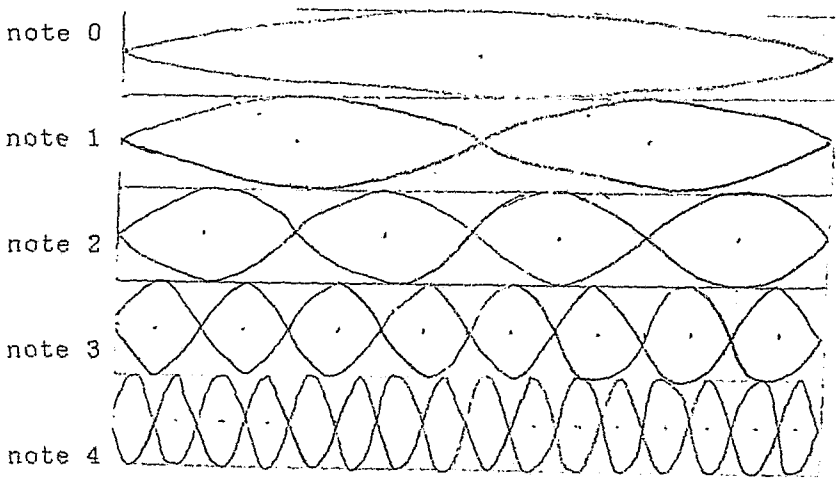
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_____

[1] Teachers should play the following notes on a musical instrument if they do not have a cassette player.



(5) Look at the wave patterns produced by the strings the notes are played on.



What do you notice about the waves ?

_____

_____

(6) Complete the following table with respect to the wave patterns of the notes.

Notes	0	1	2	3	4
No. of waves in given space					
No. of waves as product of 2					
No. of waves as a power with base 2					

(7) Now draw a graph of the note (x - axis) against the number of waves (y - axis).
(8) What kind of graph is this?

_____

(9) Is this graph a function ? Why ?

_____

_____

(10) State the domain and range of the graph.

(a) domain_____

(b) range_____

(11) Determine the intercepts of the graph.

(a) x - intercept _____

(b) y - intercept_____

(12) Now look back to the graph that you drew in worksheet 1 and compare it to the graph that you have just drawn. What do you notice ?

_____

_____

(13) What explaination can you give for this ?

_____

(14) Knowing that the octaves of the note C increasing to higher C's form a harmonic series according to the exponential curve

$$y = 2^{x}$$

how can you make use of this information in the real world ?

_____

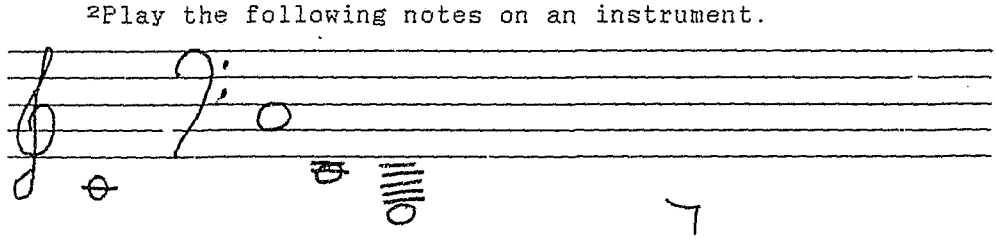
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WORKSHEET 3

(1) Listen to the notes on the cassette or the notes played by your teacher on a musical instrument.[2]

_____

[2]Play the following notes on an instrument.



(2) Are the notes the same or different ?

_____

(3) Is there anything similar about the notes that have been played ?

_____

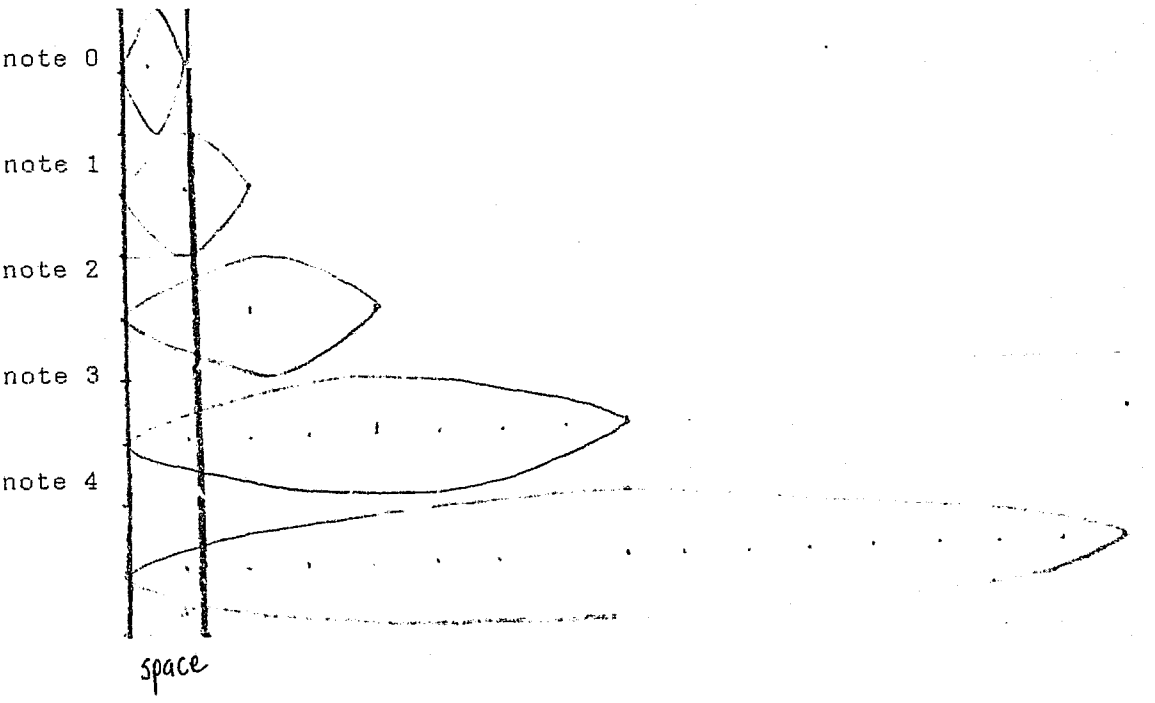
_____

(4) What do you think is the relationship between the notes that have been played ?

_____

_____

(5)Look at the wave patterns produced by the strings the notes are played on.



What is happening to the space taken up by each wave as the note gets lower.

(6) Complete the following table with respect to the wave patterns of the notes.

Notes	0	1	2	3	4
Fraction of wave in given space.					
fraction of waves as a product of 1/2					
Fraction of wave as a power with base 2.					

(7) Now draw a graph of the note (x-axis) against the fraction of wave (y-axis).

(8) What kind of graph is this ?

(9) Is this graph a function ? Why ?

(10) State the domain and range of the graph.

(a) Domain_____

(b) Range_____

(11) Determine the intercepts of the graph.

(a) x - intercept _____

(b) y - intercept_____

(12) Now compare this graph to those that you drew in worksheets 1 & 2 . What do you notice ?

_____

_____

(13) What explaination can you give for this ?

_____

(14) Knowing that the octaves of the note C decreasing to lower C's form a harmonic series according to the exponential curve

$$y = 2^{-x}$$

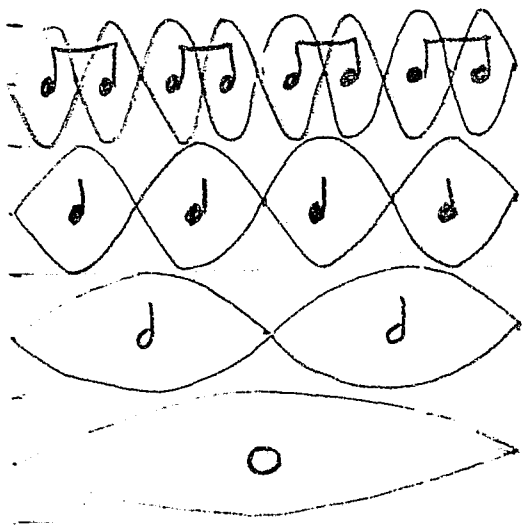
how can you make use of this information in the real world ?

_____

_____

WORKSHEET 4

For this activity each child needs either a musical instrument such as a xylophone or an empty coffee tin and a stick or a spoon.
(1) Play a long note by hitting the coffee tin with the stick and counting to eight evenly before dampening the sound with the palm of your hand.
(2) Hit the coffee tin again but this time only let the note sound to the count of four.
(3) Hit the tin yet again but now only count to two.
(4) Hit the tin again and let the note sound for one count.
(5)The note that sounds for eight counts is the longest and is called a semibreve. The note that sounds for four counts is a minum. The note that sounds for two counts is a crotchet,and the note that sounds for one count is a quaver.



quaver - note 3 - 1 count
⅛ time.

crotchet - note 2 - 2 counts
¼ time

minim - note 1 - 4 counts
½ time

semi-breve - note 0 - 8 counts
Full time

(a) How much longer is a semibreve than a minim ?

(b) How much longer is a minim than a crotchet?

(c) How much longer is a crotchet than a quaver?

(6) Complete the following table:

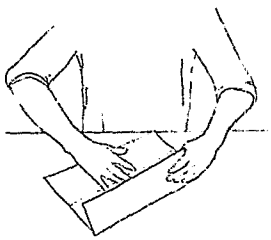
Note Name	0. semi-breve	1. minim	2. crotchet	3. quaver
Time taken for note. (full note = 8 counts)				
How many fit into a semi-breve.				

(7) Plot a graph of notes (x-axis) against counts taken for notes (y-axis).
(8) Plot a graph of notes (x-axis) against how-many fit into a semi-breve (y-axis).
(9) Now compare these graphs to those that you drew in the previous worksheets.
(a) What is the defining equation for the graph of notes against counts taken for notes?

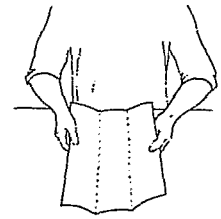
(b) What is the defining eugation for notes against how many fit into a semi-breve?

WORKSHEET 5

You will need a large sheet of paper.
(1) Lay the sheet of paper on the table in front of you. Fold it into three. Sharpen the creases with your fingernail so that you will be able to see them clearly.



(2) Unfold the paper.



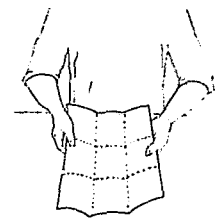
(a) Into how many parts is the paper now divided?_____

(b) Can you write this number in exponential form?_____

(c)Complete the following

3 = 3 X 1 =_____

(3) Re-fold the paper along the original lines and fold it in three again.



(a) Into how many parts is the paper now divided?

_____

(b) Write the answer to (a) as a product of 3.

_____

(c) Write the answer to (a) as a power with base 3.

_____

(4) Re-fold the paper again along the original lines and fold it into three again.(a) Into how many parts is the paper now divided ?

_____

(b) Write the answer to (a) as a product of three.

_____

(c) Write the answer to (a) as a power with base three.

_____

(5) Repeat the activity once more and then complete the following table:

No. of folds	1	2	3	4
No. of divisions				
Divisions as a product of 3.				
divisions as a power with base 3.				

(6) Can you see a relationship between the number of folds, and the number of threes in the product and the powers of three?

_____

(7) Describe the relationship observed in (6) in words.

_____

(8) Plot a graph of number of folds (x-axis) against number of divisions (y-axis).

(9) Is this graph a function? Why?

_____

_____

(10) What kind of curve is this?_____

(11) Determine the domain and range of the graph.

domain_____

range_____

(12) Determine the intercepts.

x- intercepts_____

y - intercepts_____

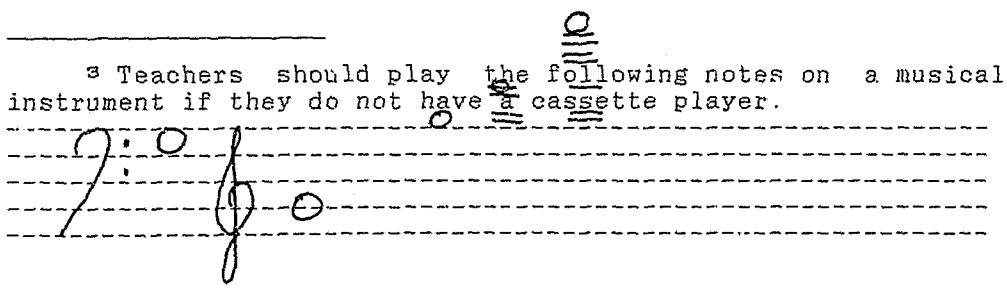
WORKSHEET 6

(1) Listen to the notes played on the cassette or by the teacher on the instrument.[3]
(2) Are the notes the same or different?

(3) Is there anything similar about the sounds that you hear?

(4)What do you think is the relationship between the notes played?

[3] Teachers should play the following notes on a musical instrument if they do not have a cassette player.



(5) Look at the wave patterns produced by the strings the notes are played on.

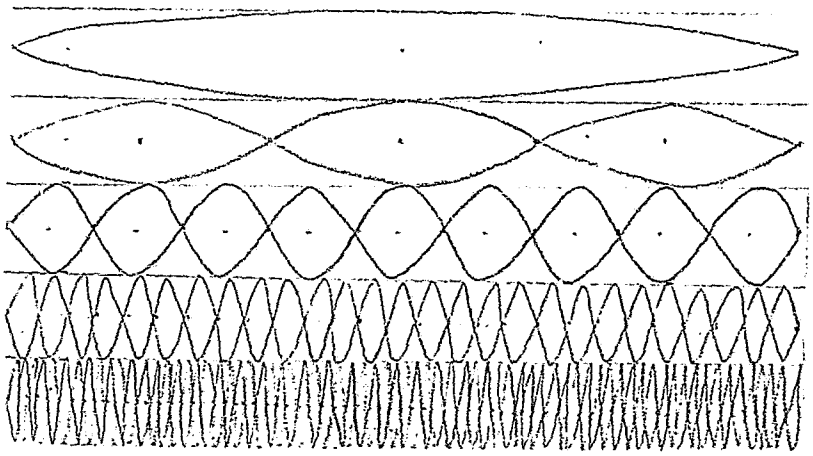
note 0

note 1

note 2

note 3

note 4



What do you notice about the waves?

_____

_____

_____

(6) Complete the following table with respect to the wave patterns of notes.

Notes	0	1	2	3	4
No. of waves in given space.					
No. of waves as a product of 3.					
No. of waves as a power with base 3.					

(7) Now draw a graph of the note (x-axis) against the number of waves (y-axis).

(8) What kind of graph is this?

_____

(9) Is this graph a function? Why?

_____

_____

(10) State the domain and range of the graph.

(a) Domain_____

(b) Range_____

(11) Determine the x and y intercepts of the graph.

(a) x - intercept_____

(b) y - intercept_____

(12) Compare this graph to the graph you drew in worksheet 5. What do you notice?

_____

_____

(13) What explaination can you give for this?

_____

(14) Knowing that the octaves of the noteG going higher form a harmonic series according to the exponential curve

$$y = 3^{x}$$

how can you use this information in the real world?

_____

_____

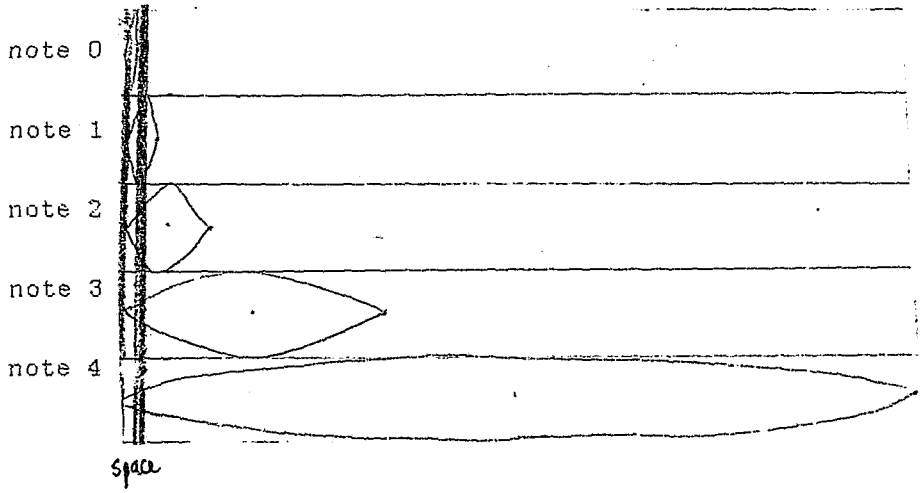
WORKSHEET 7

(1) Listen to the notes on the cassette or the notes played by your teacher on a musical instrument.[4]

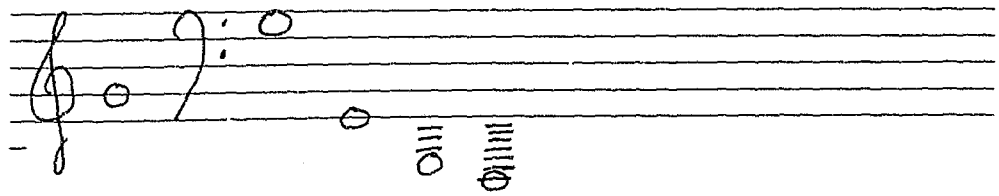
(2) Describe the similarities and differences between the notes.

(3) What do you think is the relationship between the notes?

(4) Look at the wave pattern produced by each note.



[4]Play the following notes on an instrument.



What is happening to the space taken by each wave as the note becomes lower?

_____

_____

(6)Complete the following table with respect to the wave patterns of the notes

Notes	0	1	2	3	4
Fraction of wave in given space.					
Fraction of wave as a product of 1/3.					
Fraction of wave as a power with base 3.					

(7) Now draw a graph of the note (x-axis) against the number of waves (y - axis).
(8) Is this graph a function? Why?

_____

_____

(9) State the domain and range of the graph.

(a) Domain_____

(b) Range_____

(10) State the intercepts of the graph.

(a) x - intercept_____

(b) y - intercept_____

(11) Now look back at the equations you drew in worksheet 3 and worksheet 6 and deduce a defining equation for the graph.

_____

(12) How do you think this information could be practically used in the real world ?

_____

_____

WORKSHEET 8

(1) Name a method of drawing an exponential curve.

_____

(2) Is the exponential curve a function? Why?

_____

_____

(3) Draw the graphs of the following:

$y = 2^{x}$ $\qquad$ $y = 3^{x}$

$y = 2^{-x}$ $\qquad$ $y = 3^{-x}$

(4) For each graph state the domain, range, x - intercept and y - intercept.

_____

_____

_____

_____

(5) In general for the graph $y = a^{x}$

(a) The y - intercept is always_____

(b) As x increases , y_____

(c) THe x - axis is a horizontal_____

(d) The domain is real numbers and the range is_____

(e) The greater the value of a the_____ the curve becomes.

(f) Draw a sketch of the graph $y = a^{x}$.

(6) In general for the graph $y = a^{-x}$

(a) The y - intercept is always _____

(b) As x increases ,y _____

(c) The x- axis is a horizontal_____

(d) The domain is real numbers and the range is _____

(e) The greater the value of a the _____ the curve becomes.

(f) Draw a sketch graph of $y = a^{-x}$.

ANSWERS

WORKSHEET 1 -PAPER FOLDING

(a) Into how many parts is the paper now divided?
2

(b) Can you write this number in exponential form? 2^1

(c) Complete the following:

$2 = 2 \times 1 = 2^1$

(3) Re-fold the paper along the original line and fold it in half again.

(a) Into how many parts is the paper now divided? 4

(b) Write the answer to (a) as a product of 2.

$2 \times 2 = 4$

(c) Write the answer to (a) as a power with base 2.

2^2

(4) Re-fold the paper along the original lines again and fold it in half again.

(a) Into how many parts is the paper now divided? 8

(b) Write the answer to (a) as a product of 2.

$2 \times 2 \times 2 = 8$

(c)Write the answer to (a) as a power with base 2.

2^3

(5) Repeat the activity 3 more times then complete the following table.

No. folds	1	2	3	4	5	6	7
no. divisions	2	4	8	16	32	64	128

divisions as a product of 2	2X1	2X2	2X2X2	2X2X2X 2	2X2X2X 2X2	2X2X2X 2X2X2	2X2X 2X2X 2X2X 2
divisions as a power with base 2	2^1	2^2	2^3	2^4	2^5	2^6	2^7

(6) Can you see a relationship between the number of folds, the number of twos in the product and the powers of two?

Yes_____

(7) Describe the relationship observed in (6) in words.

___They are the same._____

(8) Plot a graph of number of folds (x-axis) against number of divisions (y-axis).

(9) Is this graph a function? Why?

_Yes, because for every x value there is only one y-value____

(10) What kind of curve is this?Exponential

(11) Determine the domain and range of the graph.

(a) domain__Real numbers____

(b) range___positive real numbers_____

(12) Determine the intercepts of the graph.

x-intercept__none_____
Y-intercept_____(0;1)_____

WORKSHEET 2 - OCTAVES

(1) Listen to the notes played on the cassette or by the teacher on the instrument.[1]
(2) Are the notes the same or different?
Similar but not identical

(3) Is there anything similar about the sounds that you hear?
yes

(4) What do you think is the relationship between the notes played?
The notes are the same note played one octave higher each time.

What do you notice about the waves ?

As the frequency increases two waves go in the space of one, then four ,then eight and so on.

(6) Complete the following table with respect to the wave

patterns of the notes.

Notes	0	1	2	3	4
No. of waves in given space	1	2	4	8	16
No. of waves as product of 2	-----	2X1	2X2	2X2X2	2X2X2X2
No. of waves as a power with base 2	2^0	2^1	2^2	2^3	2^4

(7) Now draw a graph of the note (x - axis) against the number of waves (y - axis).

(8) What kind of graph is this?

Exponential curve
(9) Is this graph a function ? Why ?

Yes, for every x value there is only one corresponding y value.

(10) State the domain and range of the graph.

(a) domain_real numbers_____

(b) range__positive real numbers greater than zero___

(11) Determine the intercepts of the graph.

(a) x - intercept _____none_____

(b) y - intercept_____(0;1)_____
(12) Now look back to the graph that you drew in worksheet 1 and compare it to the graph that you have just drawn. What do you notice ?
It is the same graph.
(13) What explaination can you give for this ?
Both graphs represent the relation:

$$y = 2^x$$

---

(14) Knowing that the octaves of the note C increasing to higher C's form a harmonic series according to the exponential curve

$$y = 2^x$$

how can you make use of this information in the real world ?

To tune a musical instrument.

WORKSHEET 3

(2) Are the notes the same or different ?
similar but not identical

---

(3) Is there anything similar about the notes that have been played ?
yes

(4) What do you think is the relationship between the notes that have been played ?
They are the same note played one octave lower each time.

What is happening to the space taken up by each wave as the note gets lower.
The space doubles then is four times as much , then eight times as much and finally is sixteen times as much.
(6) Complete the following table with respect to the wave patterns of the notes.

Notes	0	1	2	3	4
Fraction of wave in given space.	1	1/2	1/4	1/8	1/16
fraction of waves as a product of 1/2	-----	1/2X1	1/2 X 1/2	1/2 X 1/2 X 1/2	1/2 X 1/2 X 1/2 X 1/2
Fraction of wave as a power with base 2.	$\frac{0}{2}$	$\frac{-1}{2}$	$\frac{-2}{2}$	$\frac{-3}{2}$	$\frac{-4}{2}$

(7) Now draw a graph of the note (x-axis) against the fraction of wave (y-axis).

(8) What kind of graph is this ?
exponential curve

---

(9) Is this graph a function ? Why ?
__Yes, for every x value there is one corresponding y value

---

(10) State the domain and range of the graph.

(a) Domain_Real numbers_____

(b) Range_real numbers greater than zero_____
11) Determine the intercepts of the graph.

(a) x - intercept none___

(b) y - intercept_(0;1)____

(12) Now compare this graph to those that you drew in worksheets 1 & 2 . What do you notice ?
The graph has a similar shape but it is a reflection about the y-axis._
(13) What explaination can you give for this ?
IN the previous two graphs as x increased y increased ,here as x increases y decreases, but in each case it is by a power of two- in the first two cases positive in the third negative.

---

(14) Knowing that the octaves of the note C decreasing to lower C's form a harmonic series according to the exponential curve

$$y = 2^{-x}$$

how can you make use of this information in the real world ?
To tune a musical instrument.

WORKSHEET 4
(a) How much longer is a semibreve than a minim ?
twice as long

(b) How much longer is a minim than a crotchet?
twice as long

(c) How much longer is a crotchet than a quaver?
twice as long

---

(6) Complete the following table:

Note Name	0. semi-breve	1. minim	2. crotchet	3. quaver
Time taken for note (full note = 8 counts)	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
How many fit into a semi-breve.	1	2	4	8

(7) Plot a graph of notes (x-axis) against time taken for notes (y-axis).

(8) Plot a graph of notes (x-axis) against how-many fit into a semi-breve (y-axis).

(5) Repeat the activity once more and then complete the following table:

No. of folds	0	1	2	3
No. of divisions	3	9	27	81
Divisions as a product of 3.	3 x 1	3 x 3	3 x 3 x 3	3 x 3 x3 x 3
divisions as a power with base 3.	3^1	3^2	3^3	3^4

(6) Can you see a relationship between the number of folds , and the number of threes in the product and the powers of three?

Yes,it is the same.

(7) Describe the relationship observed in (6) in words.
The powers are identical to the number of folds.
(8) Plot a graph of number of folds (x-axis) against number of divisions (y-axis).

(5) Repeat the activity once more and then complete the following table:

No. of folds	0	1	2	3
No. of divisions	3	9	27	81
Divisions as a product of 3.	3 x 1	3 x 3	3 x 3 x 3	3 x 3 x3 x 3
divisions as a power with base 3.	3^1	3^2	3^3	3^4

(6) Can you see a relationship between the number of folds , and the number of threes in the product and the powers of three?

Yes,it is the same.

(7) Describe the relationship observed in (6) in words.
The powers are identical to the number of folds.
(8) Plot a graph of number of folds (x-axis) against number of divisions (y-axis).

(9) Now compare these graphs to those that you drew in the previous worksheets.
(a) What is the defining equation for the graph of notes against counts taken for note.

$y = 2^{-x}$

What is the defining euqation for notes against how many fit into a semi-breve?

$y = 2^{x}$

WORKSHEET 5

(a) Into how many parts is the paper now divided? 3

(b) Can you write this number in exponential form? 3^1

(c)Complete the following

$3 = 3 \times 1 = 3^1$

(a) Into how many parts is the paper now divided?
nine

(b) Write the answer to (a) as a product of 3.
3 x 3 =9

(c) Write the answer to (a) as a power with base 3.

3^2

(4) Re-fold the paper again along the original lines and fold it into three again.(a) Into how many parts is the paper now divided ?
twenty-seven

(b) Write the answer to (a) as a product of three.
3 x 3 x 3=27

(c) Write the answer to (a) as a power with base three.

3^3

(6) In general for the graph $y = a^{-x}$

(a) The y - intercept is always (0;1)_

(b) As x increases ,y decreases_

(c) The x- axis is a horizontal asymptote_

(d) The domain is real numbers and the range is positive real numbers greater than zero._

(e) The greater the value of a the less steep_ the curve becomes.

(f) Draw a sketch graph of $y = a^{-x}$.

(1)

Exponential Curves Post Test Time ½hour Total 50 marks

(1)(a) On the same set of axes draw sketch graphs of the curves defined below:'
$f=\{(x;y)/\ y = 2^x\}$,
$g =\{(x;y)/\ y = 2^{-x}\}$ (6)
(b) Are these curves functions ? Why? (2)
(c) For each graph state the domain and the range (4)
(d) For each graph state the x-intercept(s) and Y-intercept(s). (4)
(e) What do we call a curve which approaches a line but never actually touches it. (1)
(2)(a) Plot the following graphs from the tables below:

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

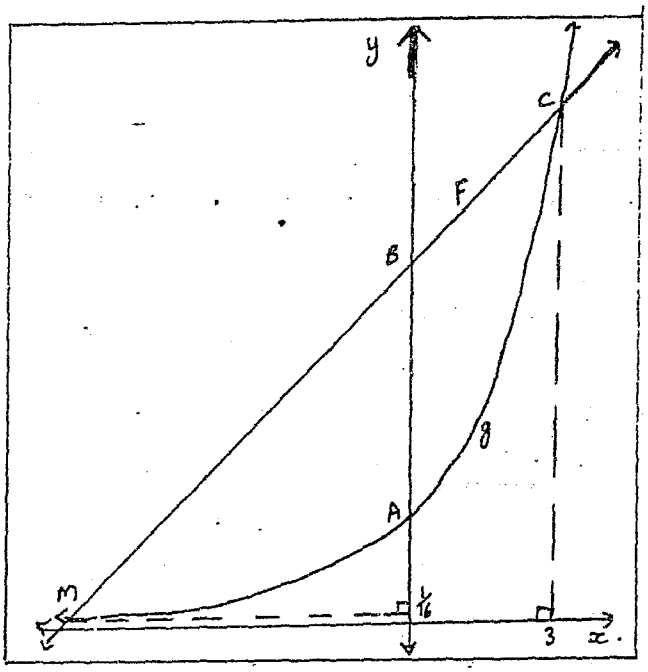
and

x	-3	-2	-1	0	1	2	3
y	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

(b) What are these functions to each other? (2)
(c) What is the y-intercept? (1)
(d) Which axis is an asymptote to the graph ? (1)
(e) State the domain and range of each graph. (4)
(3) In general for the graph of $y = a^x$
(a) The y- intercept is always _____(1)
(b) As x increases what happens to y? (1)
(c) The x- axis is never cut and is thus a_____(1)
(d) The domain is _____(1)
(e) The range is _____(1)
(f) As a becomes larger what happens to the curve ?(1)
(4)(a) Draw a sketch graph of $y = a^{-x}$. (3)
(b) Deliniate the differences between this graph and $y = a^x$ (2)
(c) What are the similarities between the two graphs ?(3)
(4) The figures drawn below represent the graphs of:
$F = \{(x;y)/\ y = 127.x + K\}$

$\frac{}{112}$

$g =\{(x;y)/\ y = 2^x\}$



Use the graphs to determine
(a) the length of AB (4)
(b) the gradient of f (1)
(c) The values where the straight line intersects with the exponential curve.(2)
(5) Explain places where the exponential curve can be used in the world outside the mathematics classroom.(4)

/50/

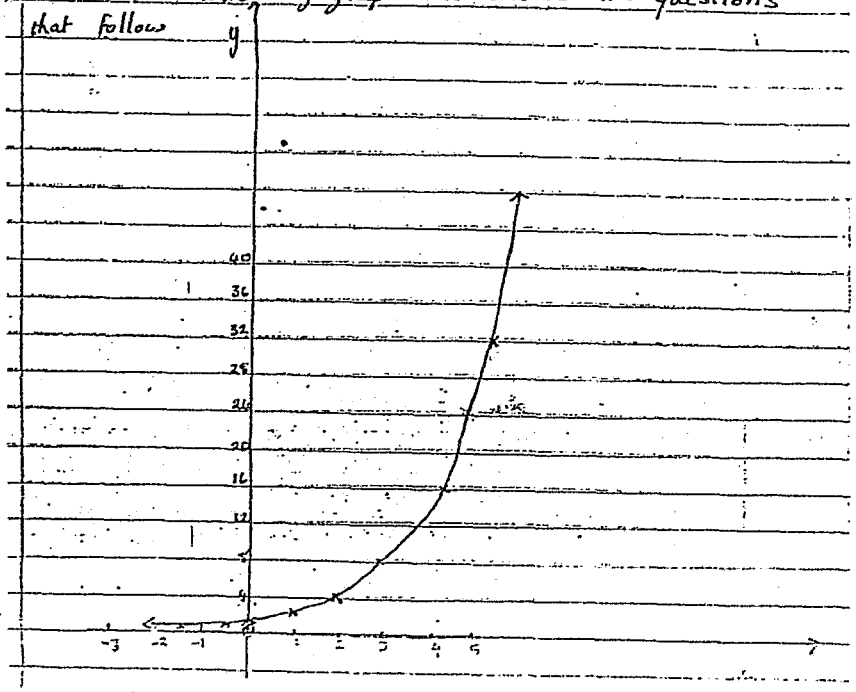
Exponential - Curves - Re - Test

1) complete the following table:

Number	Number as a product of 2	Number as a power with base 2.
1		
2		
4		
8		
16		
32		

(12)

2) Look at the following graph and answer the questions that follow



(2.1) Name (a) the y-intercept (1)

(b) the x-intercept (1)

(2.2) State (a) the domain (1)

(b) the range (1)

(2.3) Is the graph a function? Why? (2)

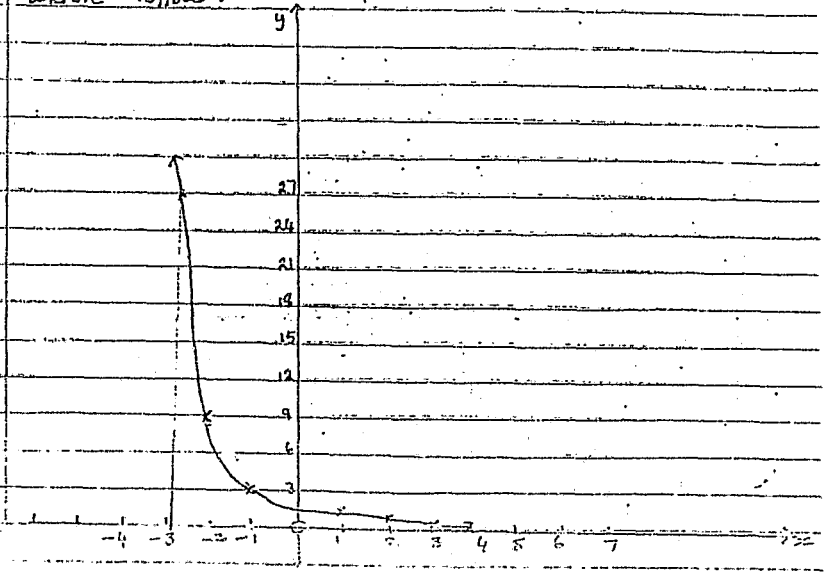
(2.4) What kind of curve is the graph? (1)

(3) Complete the following table

Number	Number as product of $\frac{1}{3}$	Number as power with base 3
$\frac{1}{3}$		
$\frac{1}{9}$		
$\frac{1}{27}$		
$\frac{1}{81}$		
$\frac{1}{243}$		

(10)

(4) Look at the following graph and answer the questions which follow.



1) Name the y-intercept (1)

2) What is the x-axis to the graph (1)

3) State (a) The domain (1)

(b) The range (1)

4) What is the defining equation of the graph (1)

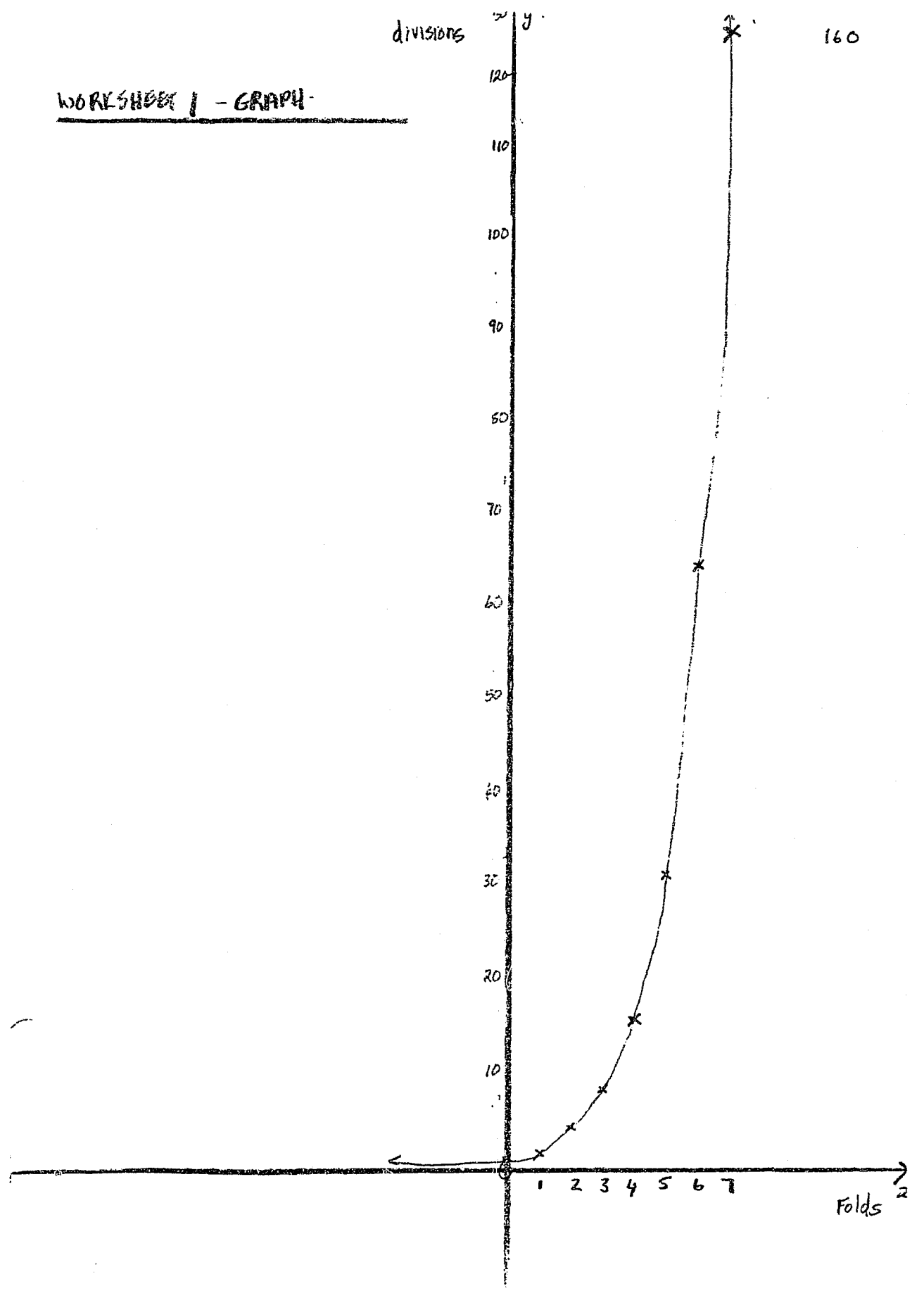
5) Plot a graph from the following table:

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

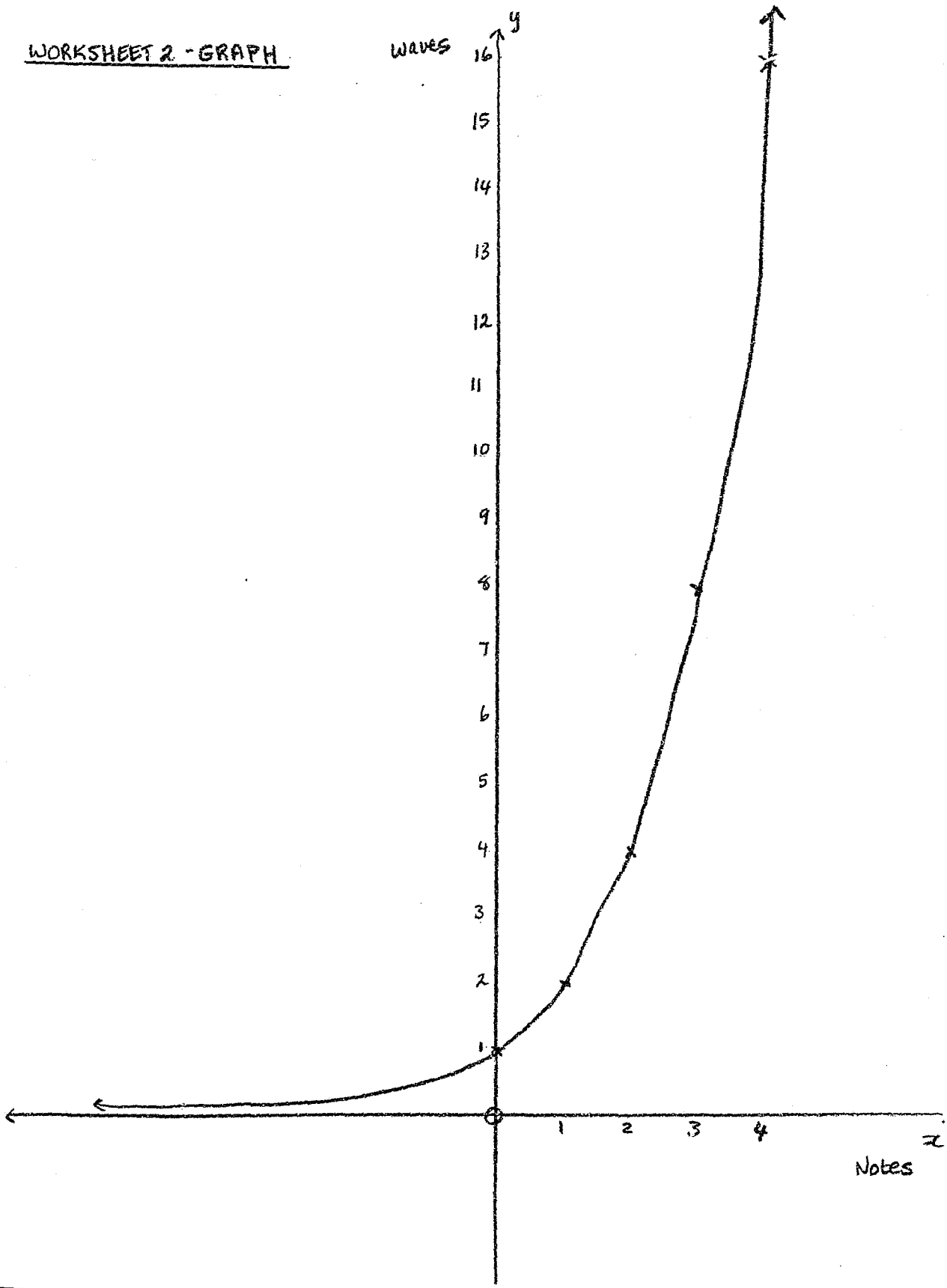
(6)

TOTAL = 40 marks.

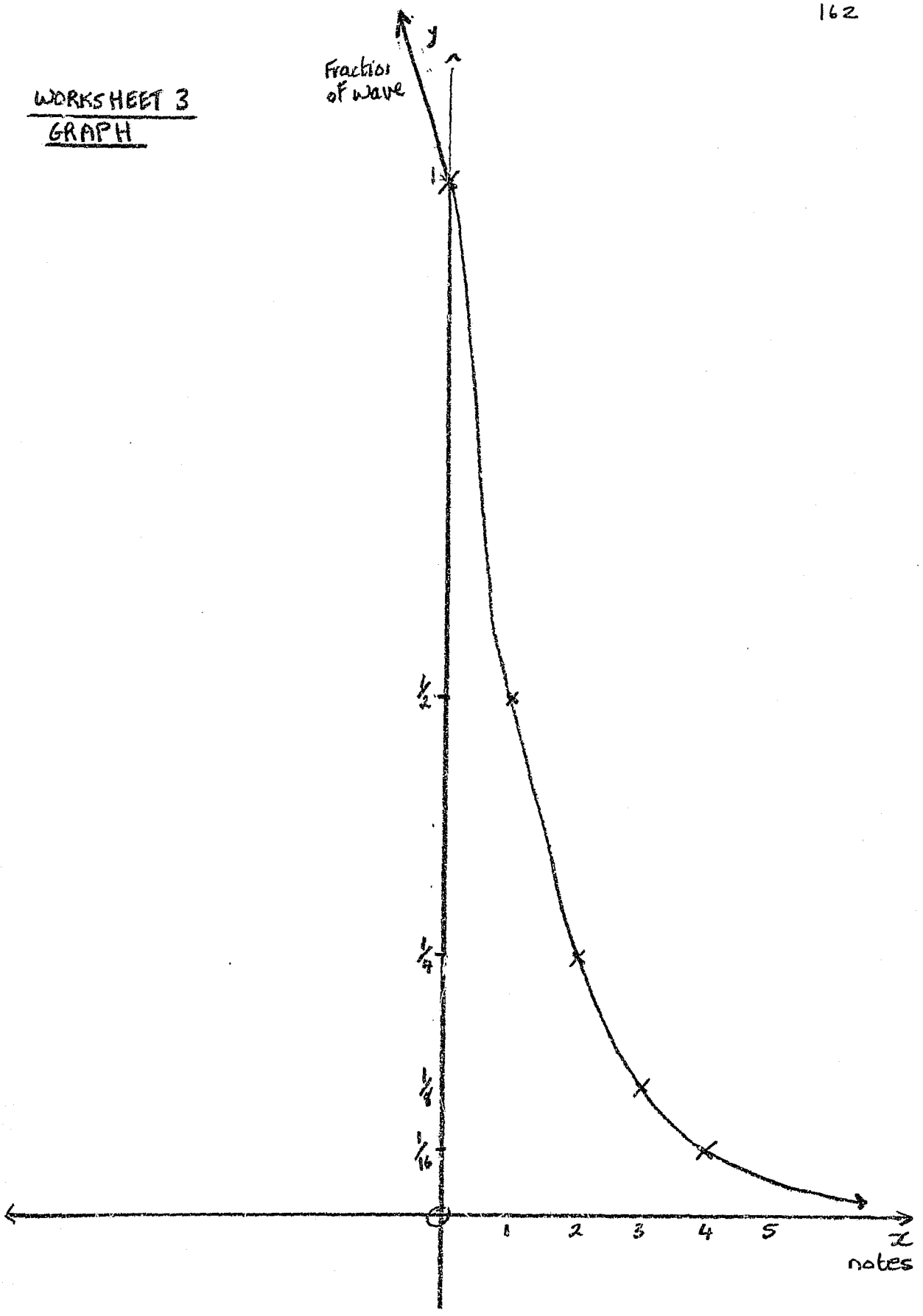
WORKSHEET 1 - GRAPH



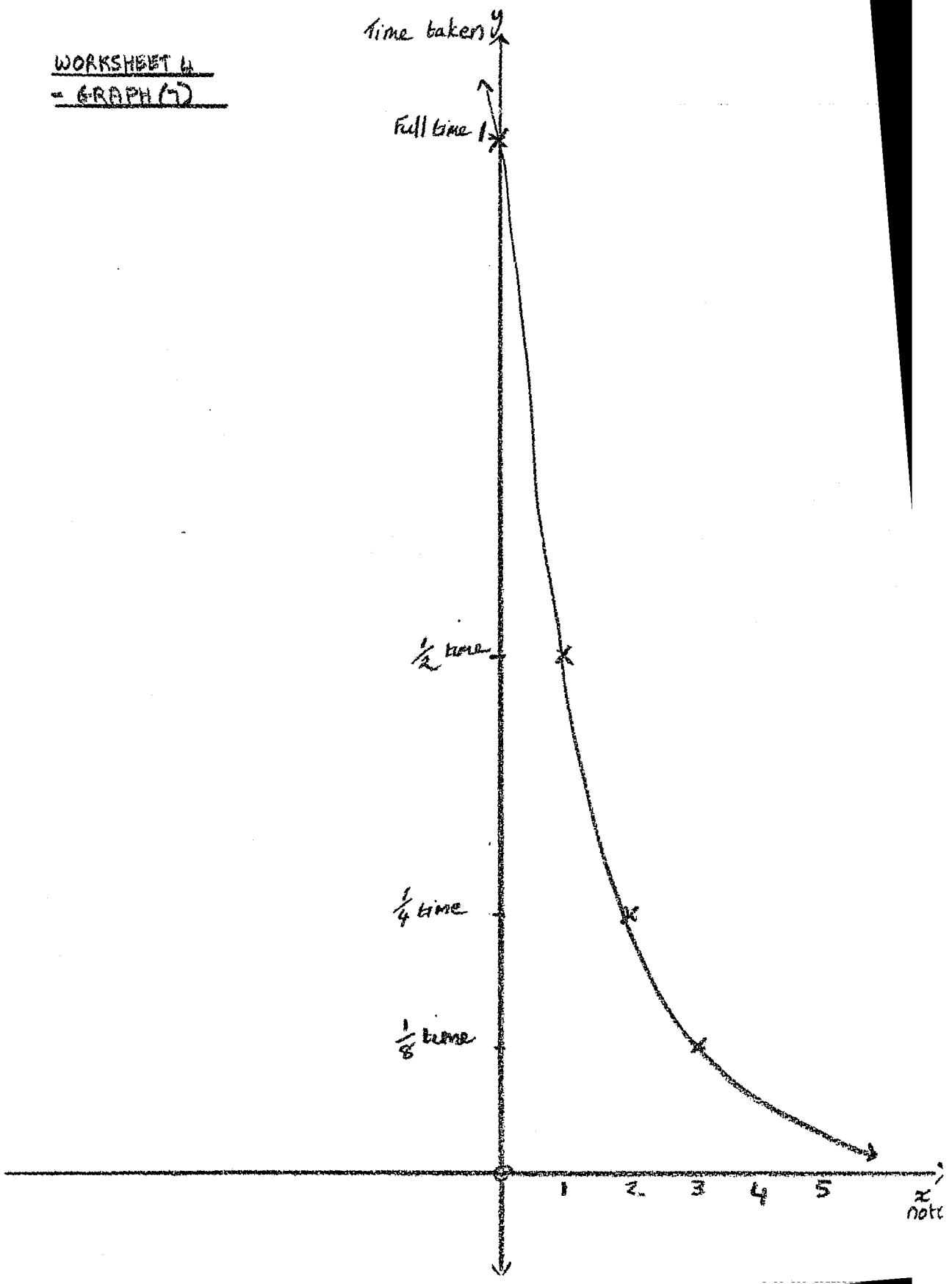
WORKSHEET 2 - GRAPH



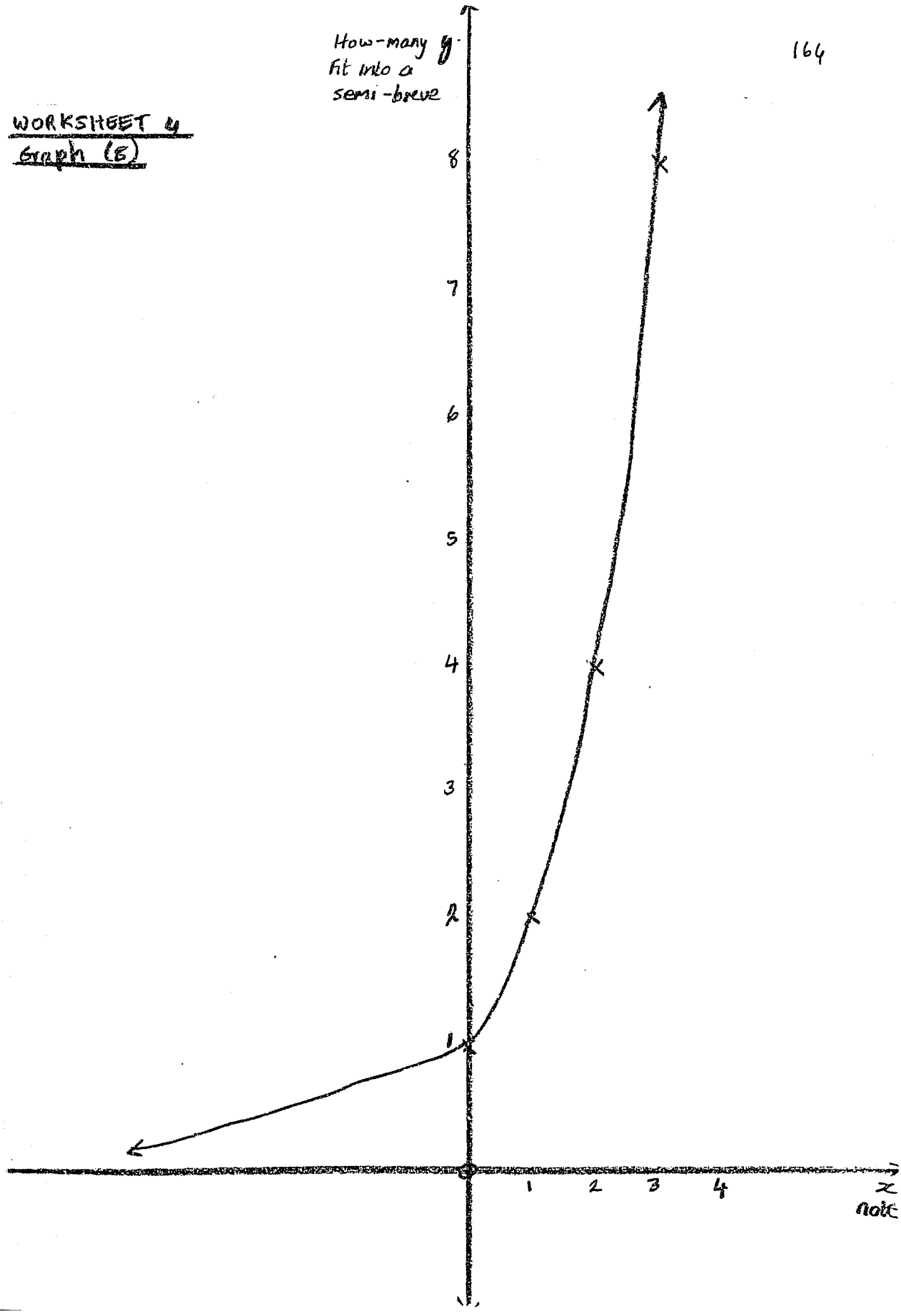
WORKSHEET 3
GRAPH



WORKSHEET 4
- GRAPH (7)

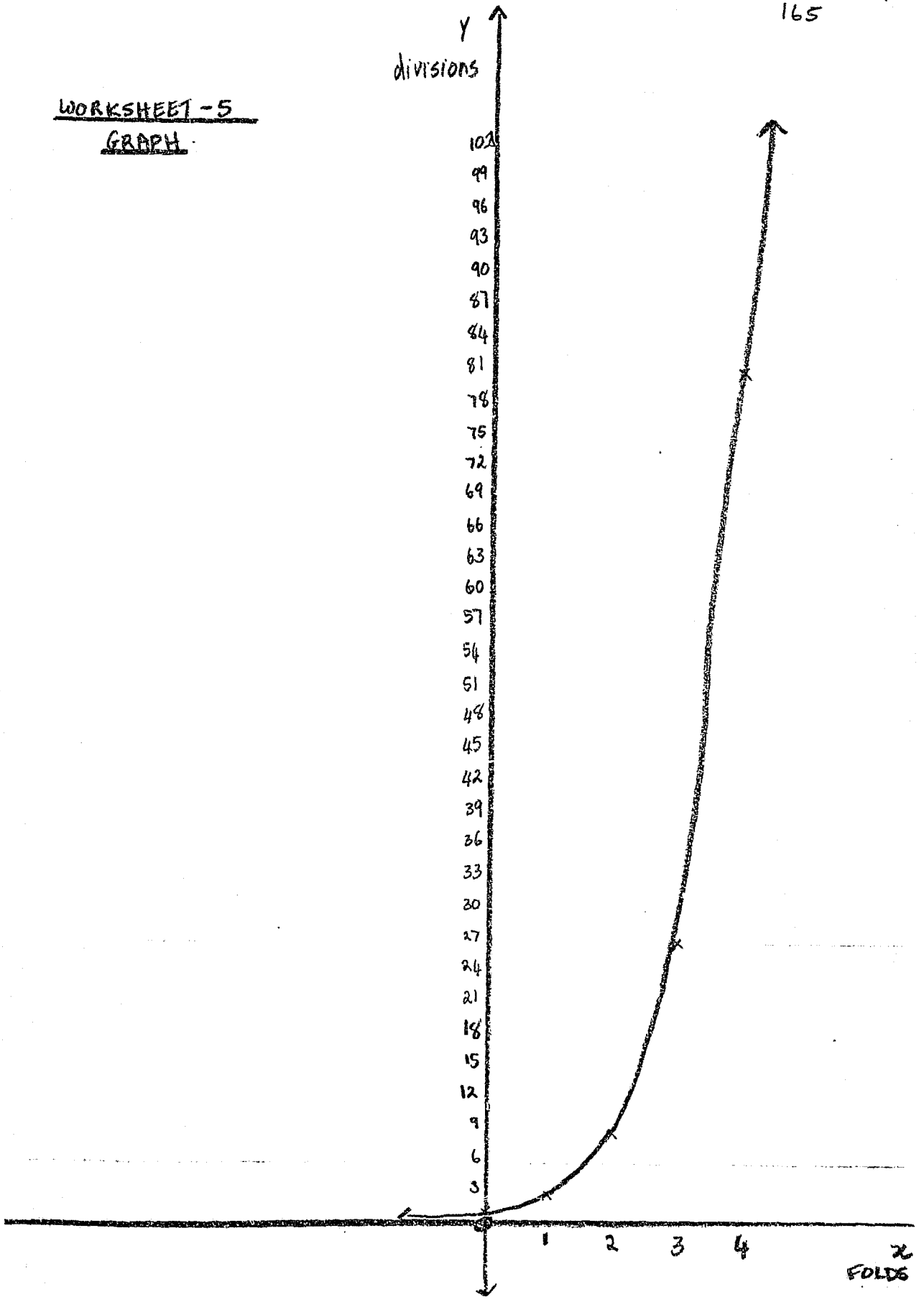


WORKSHEET 4
Graph (6)

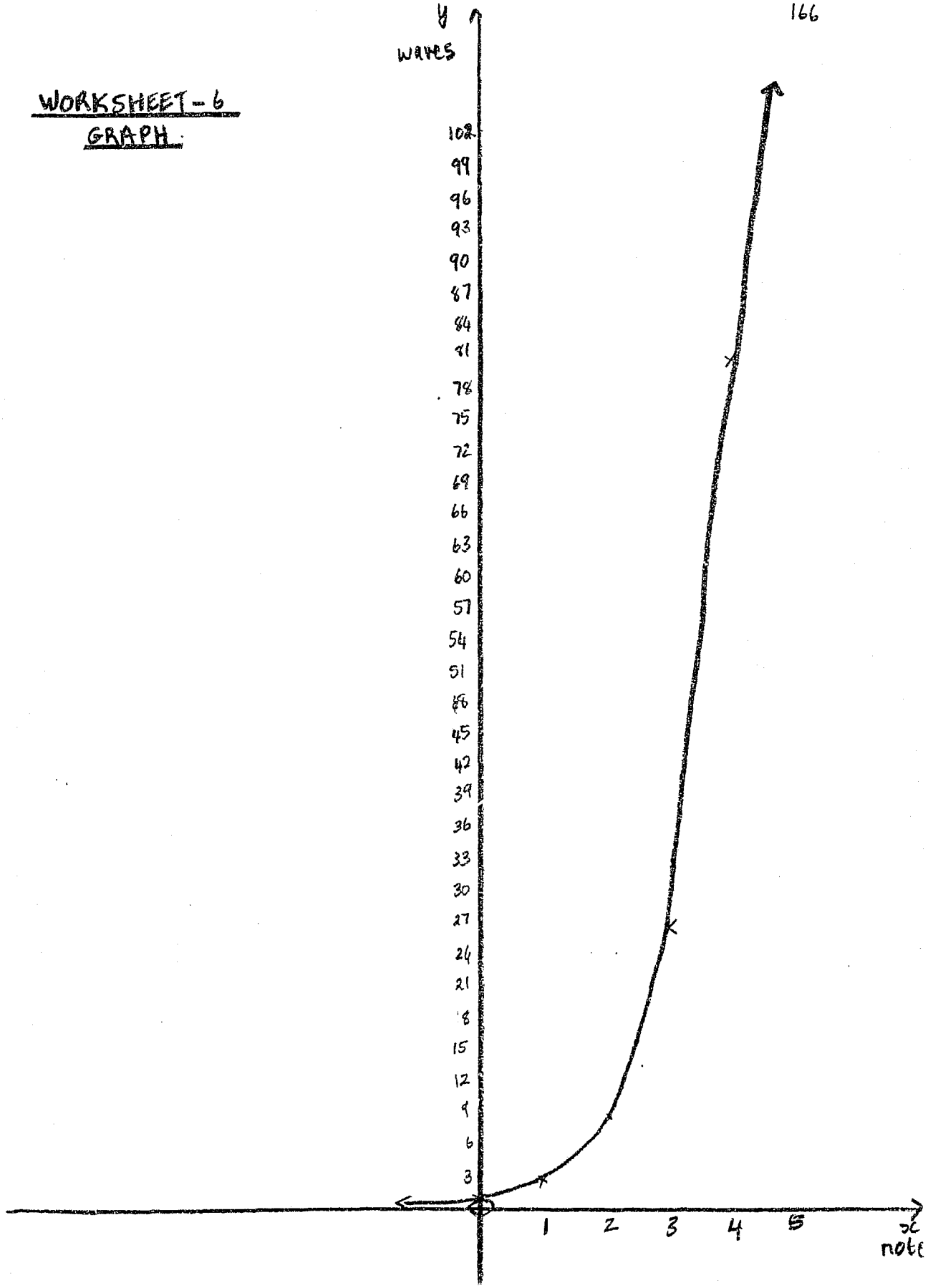


WORKSHEET - 5

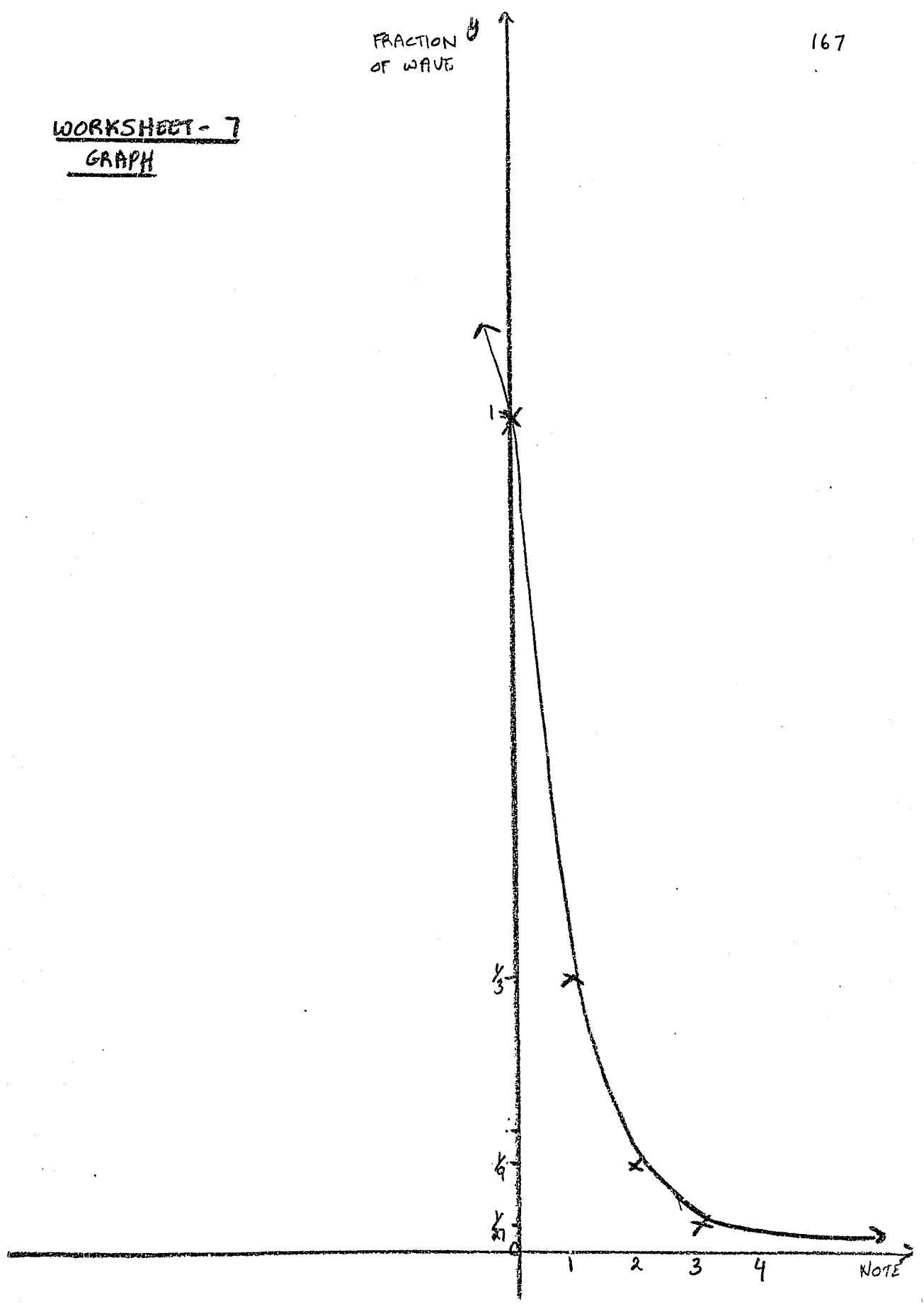
GRAPH



WORKSHEET - 6
GRAPH

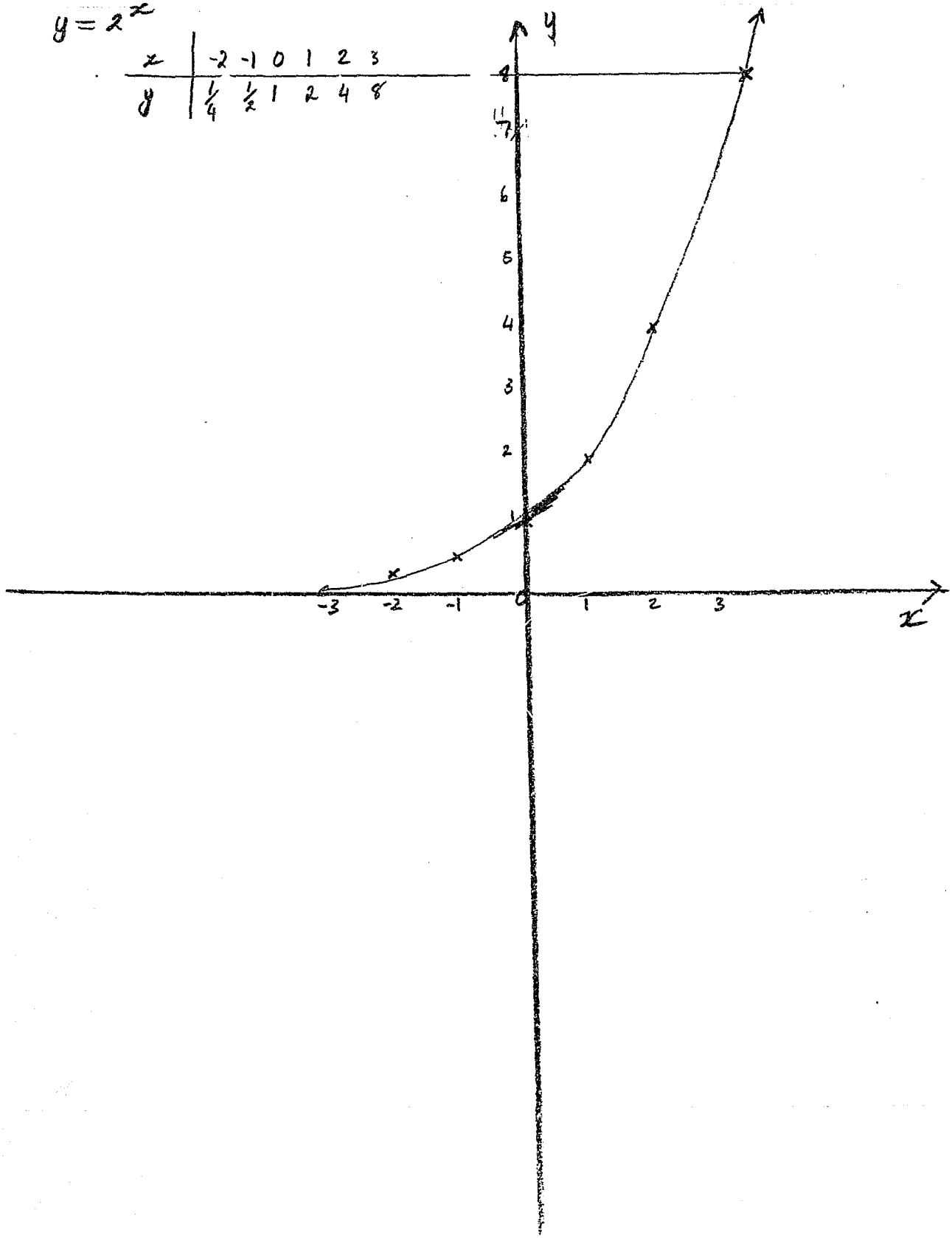


WORKSHEET - 7
GRAPH



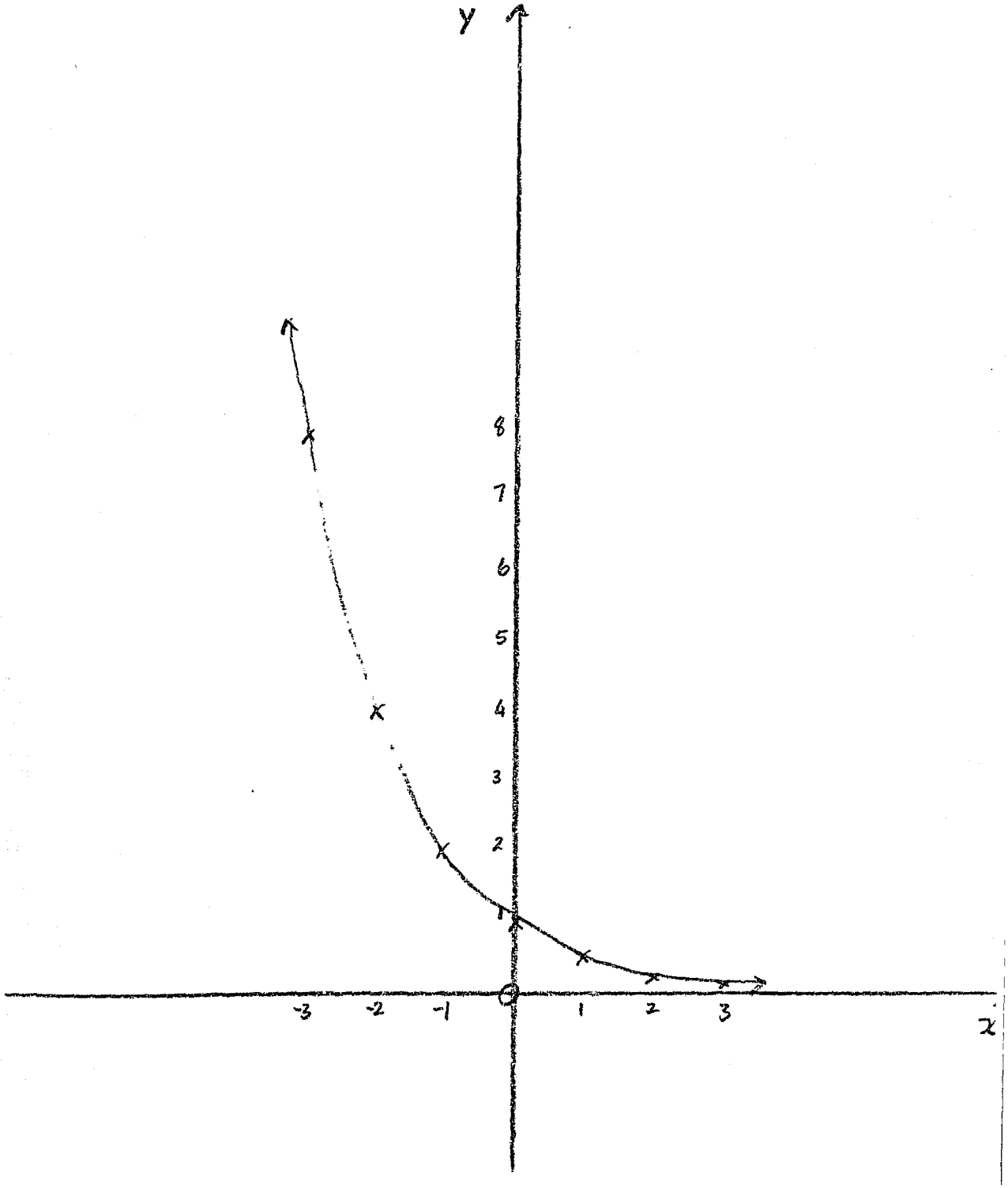
$y = 2^x$

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



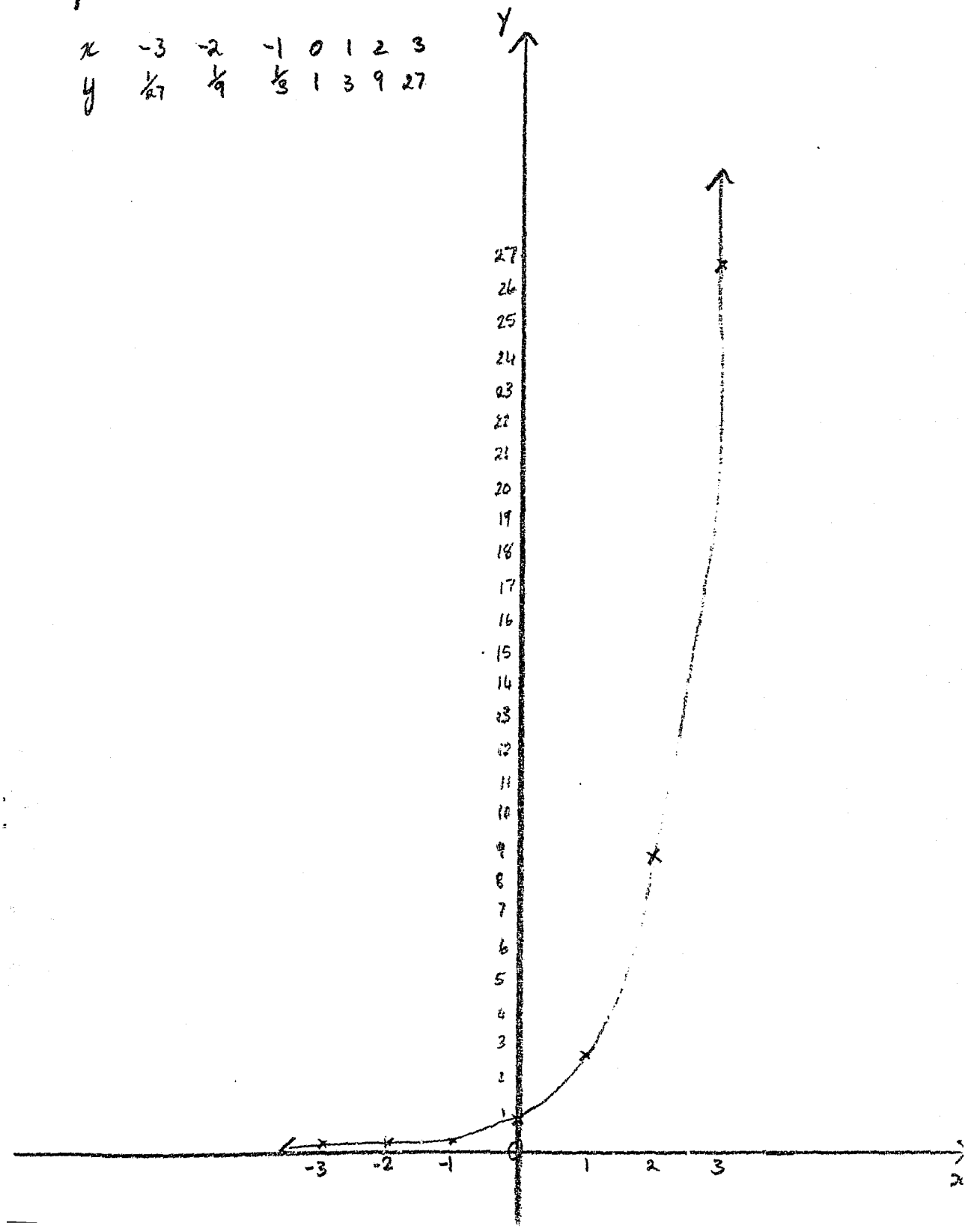
$y = 2^{-x}$

x	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



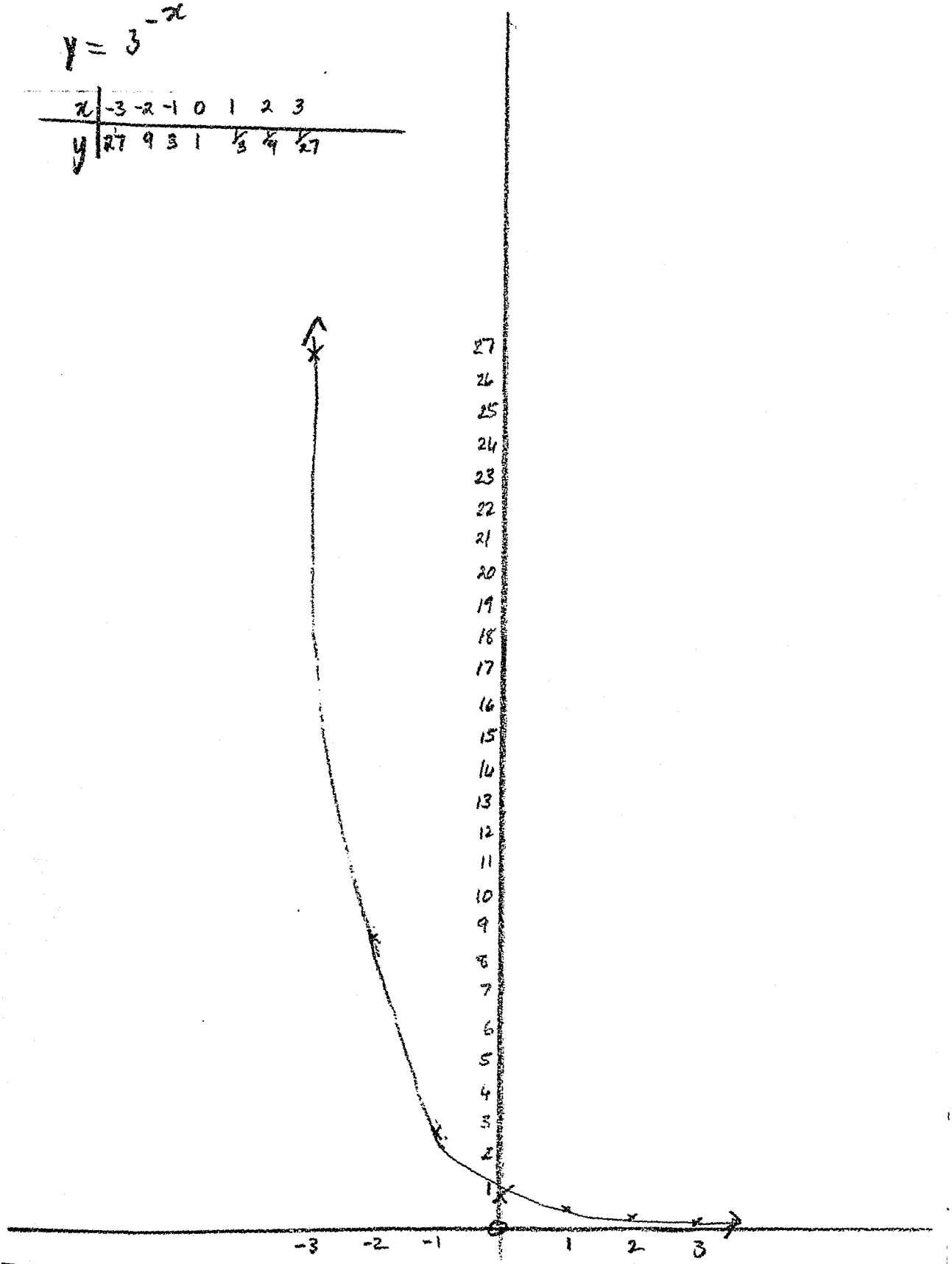
$y = 3^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

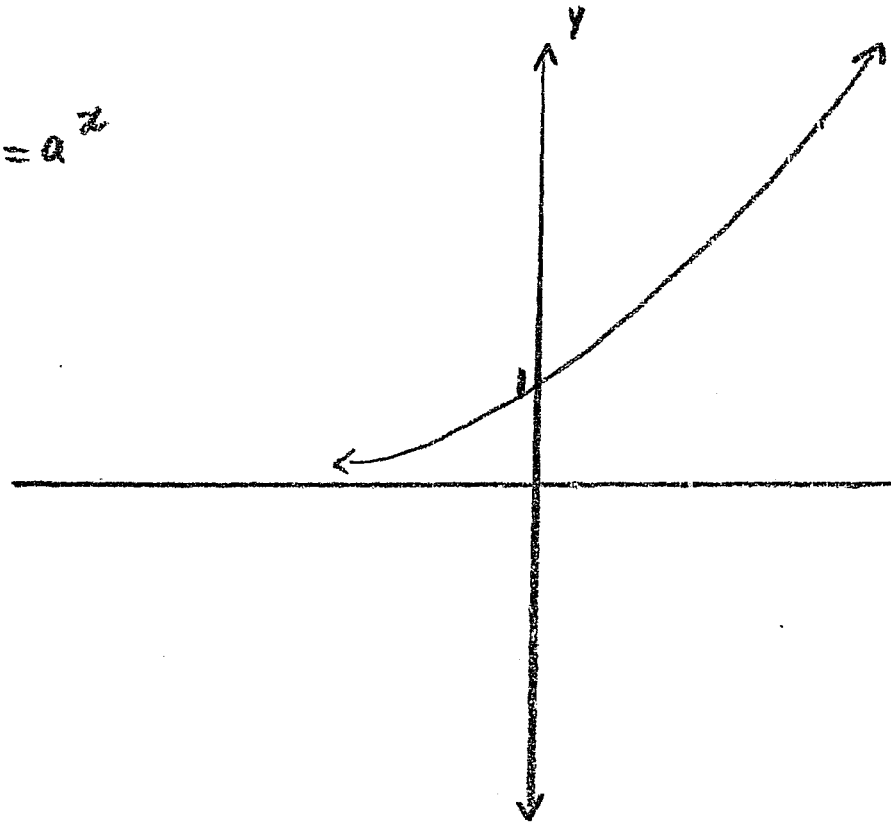


$y = 3^{-x}$

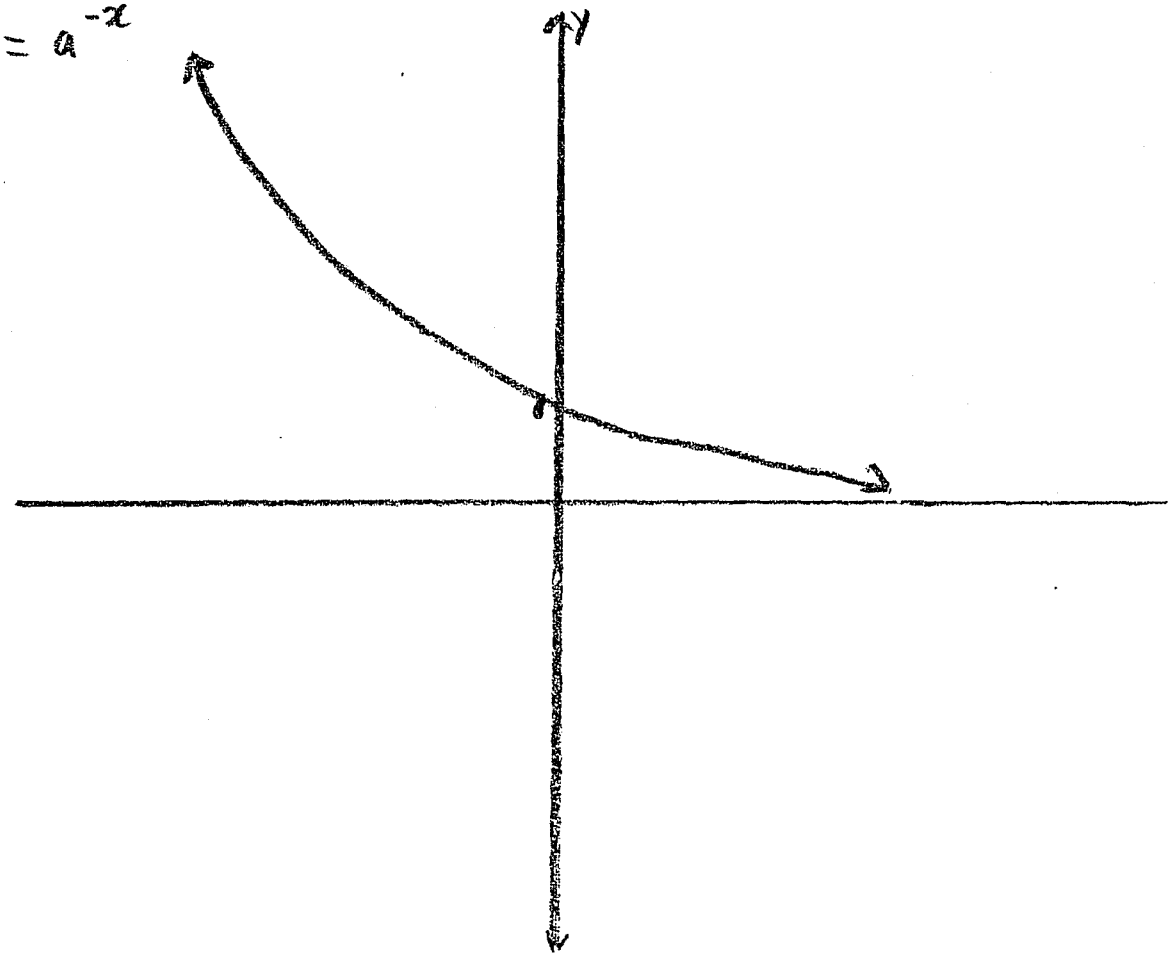
x	-3	-2	-1	0	1	2	3
y	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$



$y = a^x$



$y = a^{-x}$



(9) Is this graph a function? Why?

_Yes, for every x value there is exactly one y value.

(10) What kind of curve is this? exponential
(11) Determine the domain and range of the graph.

domain real numbers
range real numbers greater than zero_
(12) Determine the intercepts.

x- intercepts none_
y - intercepts (0;1)_

WORKSHEET 6

(2) Are the notes the same or different?
Similar but not identical.

(3) Is there anything similar about the sounds that you hear?
They sound like the same note played in octaves going higher.

(4)What do you think is the relationship between the notes played?

_The notes are G played one octave apart going higher.
What do you notice about the waves?
The second note has three times as many waves as the first and each that follows thereafter has three times as many waves as the previous note.

(6) Complete the following table with respect to the wave patterns of notes.

Notes	0	1	2	3	4
No. of waves in given space.	1	3	9	27	81
No. of waves as a product of 3.	-----	3 x 1	3 x 3	3 x 3 x 3	3 x 3 x 3 x 3
No. of waves as a power with base 3.	3^0	3^1	3^2	3^3	3^4

(7) Now draw a graph of the note (x-axis) against the number of waves (y-axis).

(8) What kind of graph is this?
exponential curve

---

(9) Is this graph a function? Why?
Yes,for every x value there is exactly one y value.
(10) State the domain and range of the graph.

(a) Domain real numbers_

(b) Range real numbers greater than zero_
(11) Determine the x and y intercepts of the graph.

(a) x - intercept none_

(b) y - intercept (0;1)
(12) Compare this graph to the graph you drew in worksheet 5. What do you notice?
It is the same.
(13) What explaination can you give for this?
Both graphs are of the curve:

$$y = 3^x$$

---

(14) Knowing that the octaves of the noteG going higher form a harmonic series according to the exponential curve

$$y = 3^x$$

how can you use this information in the real world?

To tune a musical instrument.

WORKSHEET 7

(2) Describe the similarities and differences between the notes. The notes are the same going lower and lower.
(3) What do you think is the relationship between the notes?
The notes are all G played one octave lower each time.
What is happening to the space taken by each wave as the note becomes lower?
It is becoming three times as large as the previous wave.(6)Complete the following table with respect to the wave patterns of the notes

Notes	0	1	2	3	4
Fraction of wave in given space.	1	1/3	1/9	1/27	1/81
Fraction of wave as a product of 1/3.	-------	1/3 x1	1/3 x 1/3	1/3 x 1/3 x 1/3	1/3 x 1/3 x 1/3 x 1/3
Fraction of wave as a power with base 3.	3^0	3^1	3^2	3^3	3^4

(7) Now draw a graph of the note (x-axis) against the number of waves (y - axis).

(8) Is this graph a function? Why?
Yes, for every x value there is exactly one y value.
(9) State the domain and range of the graph.

(a) Domain real numbers_

(b) Range real numbers greater than zero_

(10) State the intercepts of the graph.

(a) x - intercept none_

(b) y - intercept (0;1)_

(11) Now look back at the equations you drew in worksheet 3 and worksheet 6 and deduce a defining equation for the graph.

$y = 3^{-x}$

(12) How do you think this information could be practically used in the real world ?
To tune an instrument.

WORKSHEET 8
(1) Name a method of drawing an exponential curve.
table method

---

(2) Is the exponential curve a function? Why?
Yes, because for every x value there is exactly one y value._
(3) Draw the graphs of the following:

$y = 2^{x}$ $\qquad$ $y = 3^{x}$

$y = 2^{-x}$ $\qquad$ $y = 3^{-x}$

(4) For each graph state the domain, range, x - intercept and y - intercept.
domain - real numbers
range- real numbers greater than zero

(5) In general for the graph $y = a^x$

(a) The y - intercept is always (0;1)
(b) As x increases , y increases_

(c) THe x - axis is a horizontal asymptote_

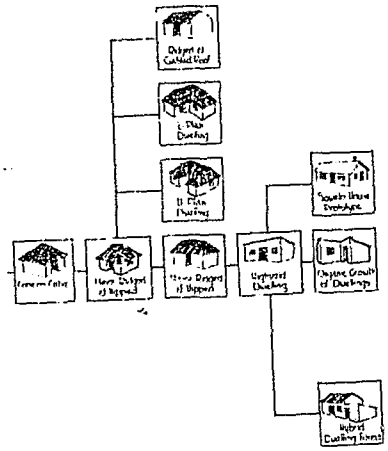
(d) The domain is real numbers and the range is positive real numbers greater than zero_

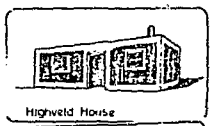
(e) The greater the value of a the steeper_ the curve becomes.

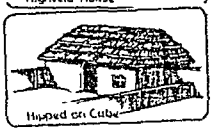
(f) Draw a sketch of the graph $y = a^x$

Appendix 2

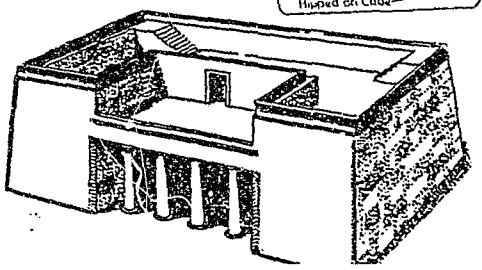
BASOTHO HUTS











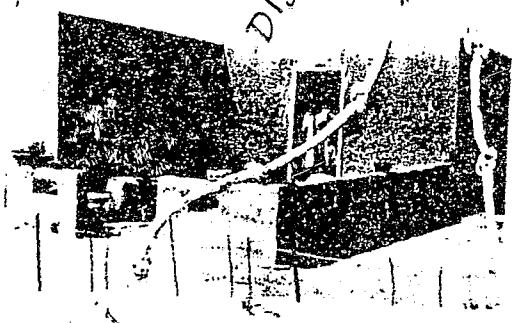
PYTHAGORAS

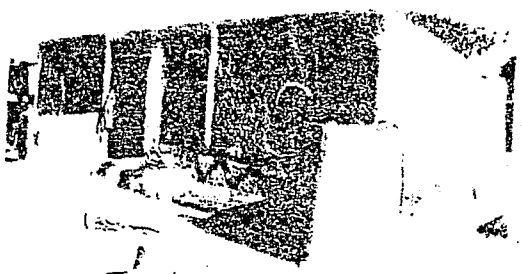


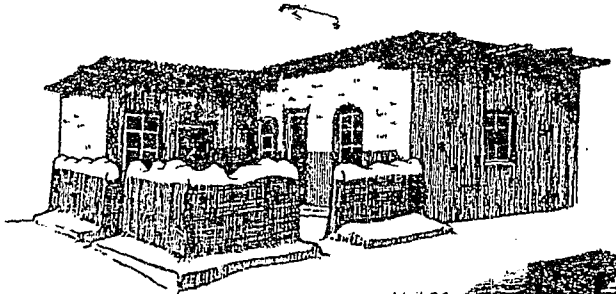
DISTANCE FORMULA

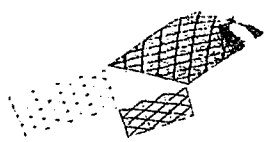


HIGHVELD HOUSES.



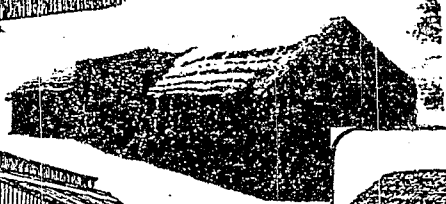


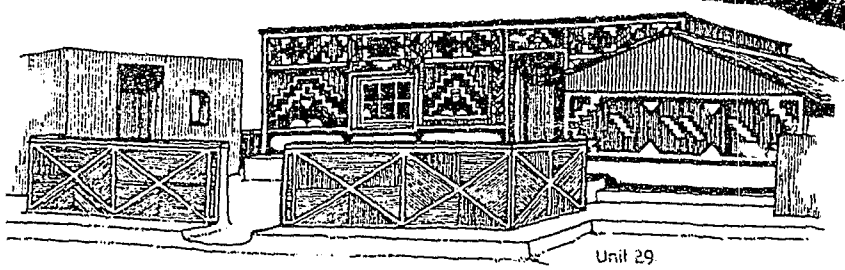




Highveld house: Units 28 and 29.

Unit 28





Unit 29



Basotho huts,Highveld houses ,Pythagoras and Distance Formula

Introduction
Buildings,houses and huts are built with square and rectangular bases all over the world ; but more particularly in South Africa the Basotho huts, Highveld houses and White settlers houses all have a square or rectangular base as opposed to the circular based huts of the majority of other population groups in the region.

In this activity the pupils will:
(1)Learn something about the history of the architecture and mathematics of their home.
(2) Use their knowledge of Pythagorean triangles from their house and classroom to derive the distance formula.
(3) Learn how to use the distance formula to find the distance between any two points.
The content covered will be:
(1) Pythagoras 'Theorem
(2) Plotting points on a coordinate plane.
(3) Distance formula.

Appropriate Level
std 6/ grade 8 Pythagors' theorem
std 8/ grade 10 triangles,rectangles, plotting points
std 10/ grade 12 Annalytic Geometry/ Pythagoras's theorem

Curriculum Related Outcomes
(1) Demonstrate intuitions about ways of working with numbers.
(2) Demonstrate ways of working with numbers.
(3) Demonstrate understanding of the historical development of numerical relationships in various social and cultural contexts.
(4) Generate and describe various numerical relationships.
(5) Show how knowledge of certain numerical relationships is of benefit to a community.
(6) Move between standard and non-standard systems of measurement.
(7) Collate and interpret data.
(8) Describe and represent experiences encountered in everyday life using mathematics.
(9) Use geometric properties to classify , identify and develop technology.
(10) Use knowledge of 2D and 3D in experiential geometry in the modelling of real life contexts.
(11) Formalise and systematise relationships of space, shape, motion and time.
(12) Demonstrate use of symbols and notations in generalisations and variability.
(13) Use mathematical language to communicate generalisations,concepts,ideas and thought processes.
(14) Set up models to represent real life and abstract situations.
(15) Use various logical processes to formulate, test and justify conjectures.

Outcomes related to the hut activity

(1) To acquire an understanding of the history of square and rectangular buildings; and their relationship to mathematics.
(2) To learn and use Pythagoras' theorem practically and more abstractly.
(3) To plot points on a graph.
(4) To derive the distance formula and be able to make use of it.
(5) To move from the practical situation ,to a diagram, to a mathematical relationship expressed as an equation where numerical values may be calculated.

Tips for teachers

Divide the class into small groups of 4 or 5 pupils in each. Provide each group with a set of worksheets, string building blocks, drawing pins and a maisonite or hard board. The pupils will also need scrap paper, square paper or graph paper,protractor, pen,pencil, rubber and ruler.

WORKSHEET 1- HISTORY

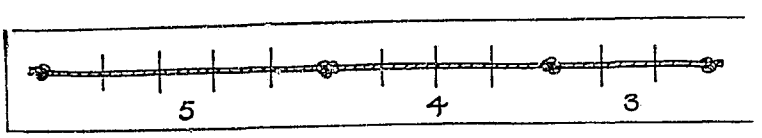
This worksheet may be done as a mathematics comprehension even as a homework activity by each pupil individually or by the groups with discussion.

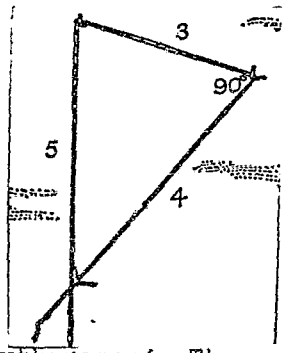
The Basotho nation is part of a larger group of The Sotho tribe. The Basotho tribe now lives on the Southern Highveld in square- based Highveld houses. It appears that after a period of tribal warfare in the early 1800's they escaped to the Maluti mountains where they were lead by Chief Moshweshwe. At this time in the 1830's they met up with Dutch settlers. The two groups of people shared architectural ideas and the Highveld hut with a square- based floor area was the result. A need for square-based huts existed because with the round based-hut it is possible to only have single roomed structures and building on additional rooms is an impossibility.Where did the Dutch settlers obtain the notion of square based buildings ?

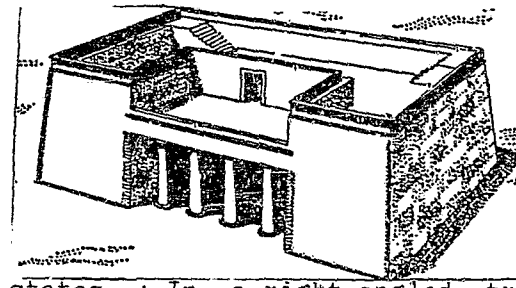
In Europe square- based buildings have been in existence since approximately 539 B.C.. Pythagoras a Greek mathematician/ philosopher was taken to Babylon as a prisoner of Persia (Iraq) in Asia, during the 527 B.C. war between Persia and Cyrus (a part of Greece today).Pythagoras was befriended by the Babylonian Priesthood who ensured that he had a certain degree of freedom in Babylon despite his prisoner status. Pythagoras used the 12 years he was in Babylon to study the way of life in that country. One of the pieces of knowledge he took back to Greece was the 3:4:5 triangle which was known and used in Babylon as early as 2000 B.C.. Since two triangles of equal size put together form a rectangle, it makes it possible to draw a rectangular-based house. Furthermore if the sides of the triangle are equal in length a square-based house can be designed.

The astronomers and priests of antiquity had a method for making a set square using rope which appears to have also been used by the Basotho in their hut construction.A long piece of rope is knotted at both its ends then it is divided into twelve equidistant segments between these two extremes. It is then pegged down after three and four segments, and the remaining five segments form the hypotenuse of a right angled triangle.By placing rope with pegs in such a fashion it is possible to build walls at perfect right angles to each other without any real difficulty.

(Figure 16,1967,Hogben,L. p47)







Pythagoras' Theorem states : In a right angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. ($r^2 = x^2 + y^2$)

Questions

(1) Which South African tribe favours square-based huts?

(2) How was it possible for the Sotho to build a hut with perfect right-angled corners without modern technology?

(3) Where did the idea of a square-based hut come to the Sotho's from?

(4)From which Greek did the Dutch learn about right-angled triangles?

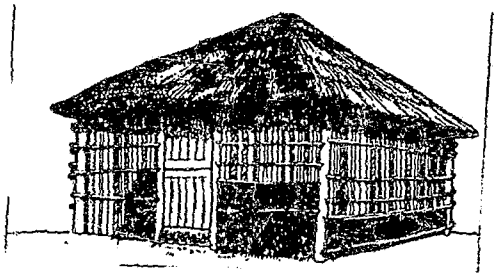
(5) What does Pythagoras' theorem say?

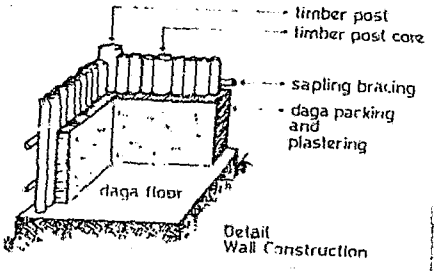
(6) What does this mean to you?

(7) The Babylonians were using the 3:4:5 rope triangle in 2000 B.C. can you name three other Pythagorean triangles?

(8) Do you think the rope,peg, set-square method of building is still used today?

(9) Look at the floor plan for the given hut. Is this hut a rectangular-based hut? Justify your answer with a proof.





CONE ON CUBE
PIET RETIEF.
Pyramid on cube

(10) What advantage does a square-based hut or a rectangular based hut have over a round hut?

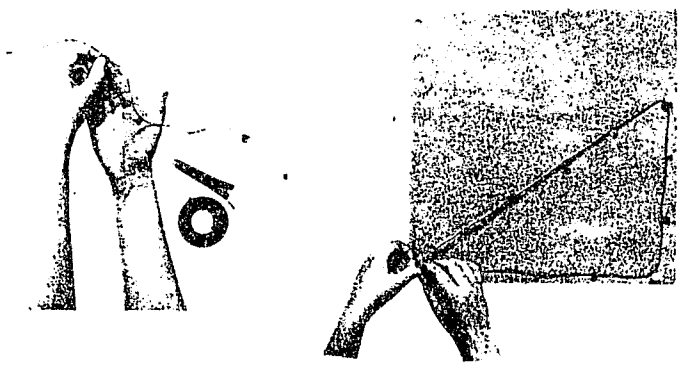
WORKSHEET 2 - CONSTRUCTION OF A HIGHVELD HOUSE

For this activity each pupil needs string, drawing pins, scissors, building blocks and a board to work on.

(1) Take a piece of string and tie a knot on each end.

(2) Tie twelve further knots evenly spaced between these two knots.

(3) Pin your string to your board at one end.
Put another drawing pin after 3 knots in a horizontal direction.
Turn your string to face vertically and pin it to the board after four knots.



(4) Now build these two walls of your house with the blocks by placing them along the string.

(5) Repeat steps (1) to (5) starting opposite the 3 knot wall making another string triangle again and then building again.

Questions

(1) What shape house do you have?

_____

(2) How can you be sure of this?

_____

(3) Let us prove that your house is a rectangle by proving that it is a quadrilateral with angles of 90°.

(a) Measure the length of string used for each side.

horizontal side_____

vertical side_____

(b) Measure the length of string used for the hypotenuse (diagonal of quadrilateral) of the triangle.

_____

(c) Square the length of all the sides.

Horizontal side ^{2}_____

Vertical side ^{2} _____

Hypotenuse^2_____

(d) Add the squares of the vertical and horizontal sides together.

_____ + _____ = _____

(e) What do you notice?

_____

(f) What does this tell you about the two triangles making up your house?

_____

_____

(g) What does this tell you about the quadrilateral making up the base of your house?

_____

_____

(h) Measure the angles of your house using a protractor just to be sure.
What do they measure?

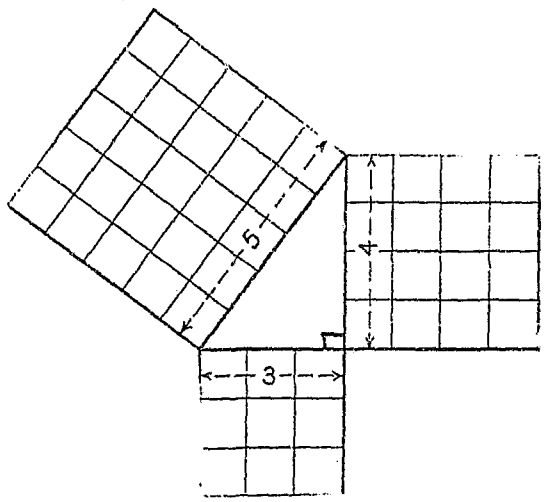
(1)_____

(2)_____

(3)_____

(4)_____

(i) What kind of house have you built?_____



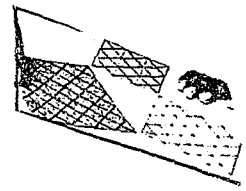
CONTINUOUS ASSESSEMENT WORKSHEET 2

Watch the pupils building in their groups, discussing and answering the worksheet and then complete the table for each group.

Pupils participating in group_____

Can the pupils follow the instructions to build the house ? (Set up models to represent real life).	5	4	3	2	1
Can the pupils take the concrete relationship before them and convert it into a mathematical relationship?					
Are the pupils able to express their findings in mathematical language?					
Are the pupils able to test their practical results mathematically?					

Average percentage_____



Worksheet 3 - Deriving distance formula from the Pythagorean triangle.

For this activity pupils need square or graph paper and their results from worksheet 2.

(1) On your square paper draw a set of axes at the bottom and left hand edges of your page.

(2) Label the axis running along the bottom the x-axis and that running vertically up the left hand side the y-axis.

(3) Mark a point 3 squares along the x-axis to represent the horizontal string you used.

What are the coordinates of this point ? (__;__)

(4) Mark a point 4 squares along the y-axis to represent the vertical string you used.

What are the coordinates of this point ? (__;__)

(5) Join the coordinate on the x-axis to that on the y-axis with a straight line..

From worksheet 2 how long do you think this line is ?

We can work out the length of the line using Pythagoras' theorem since the x-axis and y-axis are at 90° to each other

Let us call (3;0) A and (0;4)B.

Therefore $AB^2 = OA^2 + OB^2$

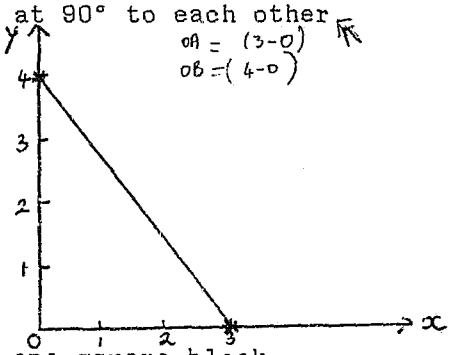
$= (3-0)^2 + (4-0)^2$

$= 9 + 16$

$= 25$

Now $AB = \sqrt{25}$

$= 5$ units



Check this answer.

(a) With your ruler measure the size of one square block.

_____

(b) Now measure the length of AB with your ruler.

_____

(c) Divide the length of AB by the size of one block. What do you get?

_____

(d) Does this correspond to the answer obtained using Pythagoras' theorem ?

_____

(6) In fact we do not need to draw a triangle every time we want to find distance we can merely apply Pythagoras' theorem in the form of what we call the distance fromula.

The distance AB between $A(x_1;y_1)$ and $B(x_2;y_2)$ is:

$$\underline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(7) Use the distance formula to find the distance between the following points and check your answers by plotting them on square paper and following the method indicated in 5a,b,c.

(a) (2;3) and (4;5)

(b) (6;1) and (-6;6)

(c) (3;-7) and (-1;3)

(d) (-4;3) and (0;0)

(e) (-3;-1) and (4;-6)

(f) (-2;1) and (-4;-1)

(8) Remembering that perimeter is the distance around an object find, the perimeter of the lego house your group built.
(a) By measurement.
(b) Using the distance formula a few times over.
(a)_____
(b)_____

WORKSHEET 4 - SUMMARY AND CONCLUSION

(1)Square and rectangular based huts are constructed using _____.

(2) Pythagoras' theorem actually came from

_____

(3) How can knots in a piece of string be used to measure distance?

_____

_____

_____

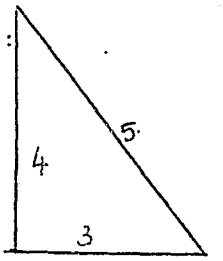
(4) The distance formula is derived from_____

(5) To find any distance AB we can use the following formula.

_____

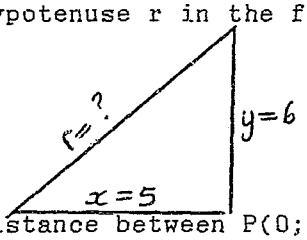
Pre/Post test Basotho huts, Highveld houses,Pythagoras and distance formula. Time 30mins Total 30 marks

(1) Square based huts are built by _____ people.
(a) Xhosa (b) Zulu (c) Sotho (d) Venda
(2) The square-based building originates in_____
(a) Africa (b) Asia (c) Europe (d) America
(3) Pythagoras' theorem states that. In a right angled triangle.....
(a) The hypotenuse is equal to the sum of the other two sides.
(b) The hypotenuse is equal to the difference of the squares on the other two sides.
(c) The hypotenuse is equal to the product of the other two sides.
(d) The square on the hypotenuse is equal to the sum of the squares on the other two sides.
(4) The 3,4,5, triangle is special because:



(a) It is isosceles.
(b) It has a right angle.
(c) It is obtuse.
(d) It is scalene.

(5) What advantage does a square based house have over a circular based house?
(a) You can build additional rooms onto it.
(b) It is warmer in winter.
(c) It is more spacious.
(d) It is cooler in summer.
(6)A rectangle is defined as follows:
(a) An elongated square.
(b) A quadrilateral with two pairs of opposite sides parallel.
(c) A quadrilateral with two pairs of opposite sides equal in length.
(d) A quadrilateral with all angles equal to 90°.
(7) The hypotenuse r in the following triangle is .....



(a) 61 units
(b) 11 units
(c)7,81 units
(d) 30 units

(8) The distance between P(0;2) and Q(0;6) is.....
(a) 8 units (b) 6,36 units (c) -4units (d) 4 units
(9) The distance between R(-2;0) and S(5;0) is.....
(a) 7 units (b) 3 units (c) 5,39 units (d) 4,58 units
(10) If A(-2;3) and B(1;-2), the distance between these two points is.....

(a) $\sqrt{2}$ units (b) 5,83 units (c) 0 units (d) 6,2 units

Answers Pre/Post Test Distance Formula

1c
2b
3d
4b
5a
6d
7c
8d
9a
10b

Appendix 3

Locus Activity

At the time this activity was performed the school photocopier was out of order so it does not follow the worksheet type activity format that the other two activities do.

The pupils were shown an example of a flick cartoon strip of an animal kicking a ball. As they turned the pages quickly they could see how the ball moved. They were then divided into groups and given the rest of that period to create their own flick cartoons. Some pupils even went home and created more technologically advanced ones with pages that could be turned quicker and even rolling pages in a book with a pencil.

The next day the pupils were divided into their groups again and given twenty pages and a bulldog clip. They were told to draw a dot at the top of the first page. Then move the dot a little lower on each consecutive page that followed, so that by the last page the position of the dot was at the end of the page. They then clipped their pages together and taking turns flicked through the pages quickly. The question was what did they see when they flicked through the pages ?

The groups were then introduced to the idea that a locus was like their points in their flick cartoon books 'a point moving through space defined thus by an equation and not an ordered pair'.

The pupils then played with the idea some more by producing flick cartoon books for the following theorems in their groups:
(1) The locus of the vertex of a right angle with fixed hypotenuse is a semi-circle with the hypotenuse as diameter.
(2) The locus of points equidistant from the sides of an angle is the bisector of the angle.
(3) The locus of points equidistant from two parallel lines is a third line parallel to the two given lines and midway between them.

The pupils were then asked again several times in different ways to describe what a locus actually , and to differentiate between a coordinate ordered pair and a locus before moving onto any calculations.

The next lesson pupils were taught how to find loci using the distance formula and they practised exercises on this from Classroom Mathematics. This was followed by another lesson where they found loci using the formula for gradient.

A further lesson followed where they calculated a mixture of loci type examples from past matric examinations using all the different methods. This was followed by the post-test and an analytic geometry test.

Appendix 4

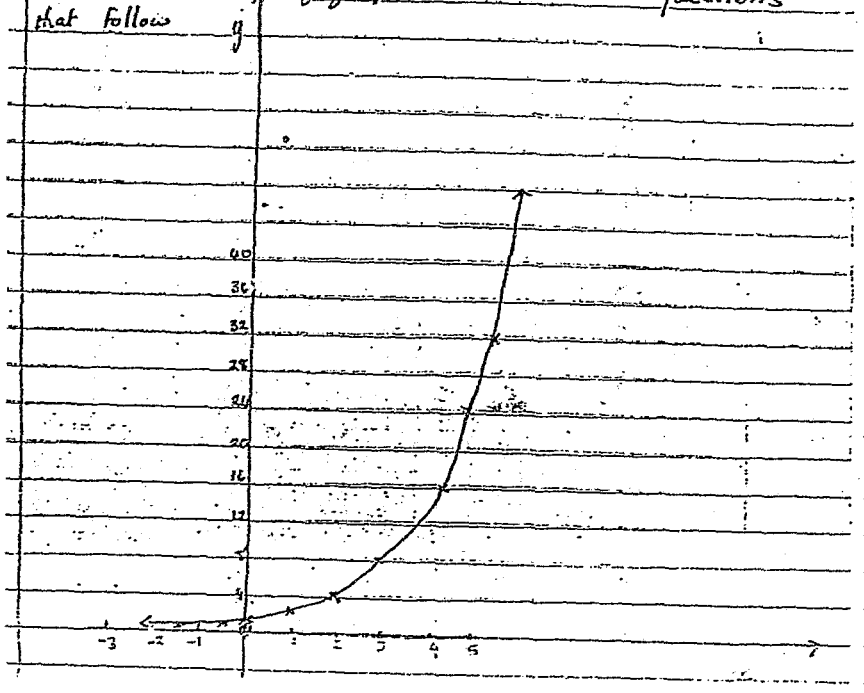
Exponential - Curves - Pre - Test

1) Complete the Following table:

Number	Number as a product of 2	Number as a power with base 2.
1		
2		
4		
8		
16		
32		

(12)

2) Look at the following graph and answer the questions that follow



(2.1) Name (a) the y-intercept (1)

(b) the x-intercept (1)

(2.2) state (a) the domain (1)

(b) the range (1)

(2.3) Is the graph a function? Why? (2)

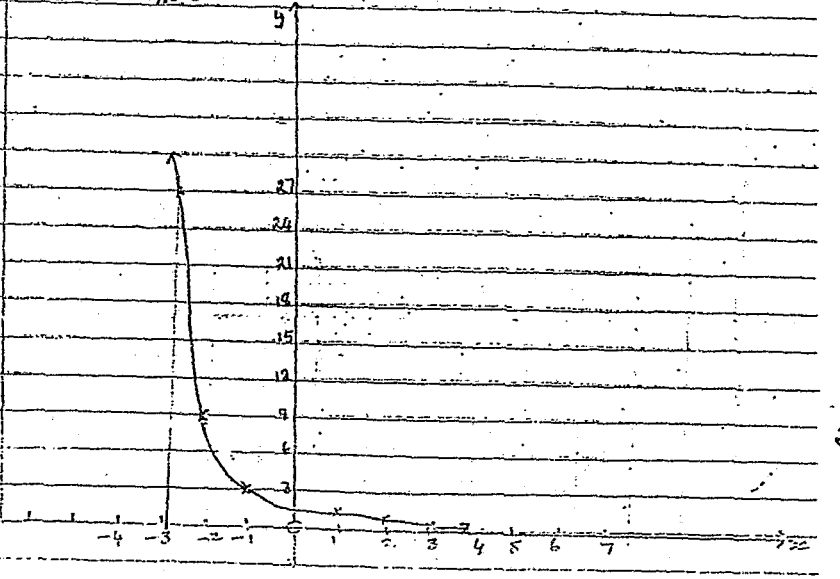
(2.4) What kind of curve is the graph? (1)

(3) Complete the Following table

Number	Number as product of $\frac{1}{3}$	Number as power with base 3
$\frac{1}{3}$		
$\frac{1}{9}$		
$\frac{1}{27}$		
$\frac{1}{81}$		
$\frac{1}{243}$		

(10)

(4) Look at the following graph and answer the questions which follow.



AL1

1) Name the y-intercept (1)

2) What is the x-axis to the graph (1)

3) State (a) The domain (1)

(b) The range (1)

4) What is the defining equation of the graph (1)

5) Plot a graph from the following table:

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

(6)

TOTAL = 40 marks.

Exponential Curves Post Test Time ½hour Total 50 marks

(1)(a) On the same set of axes draw sketch graphs of the curves defined below:

$f = \{(x;y)/\ y = 2^x\}$

$g = \{(x;y)/\ y = 2^{-x}\}$ (6)

(b) Are these curves functions ? Why? (2)

(c) For each graph state the domain and the range (4)

(d) For each graph state the x-intercept(s) and Y-intercept(s). (4)

(e) What do we call a curve which approaches a line but never actually touches it. (1)

(2)(a) Plot the following graphs from the tables below:

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

and

x	-3	-2	-1	0	1	2	3
y	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

(b) What are these functions to each other? (2)

(c) What is the y-intercept? (1)

(d) Which axis is an asymptote to the graph ? (1)

(e) State the domain and range of each graph. (4)

(3) In general for the graph of $y = a^x$

(a) The y- intercept is always _____(1)

(b) As x increases what happens to y? (1)

(c) The x- axis is never cut and is thus a_____(1)

(d) The domain is _____(1)

(e) The range is _____(1)

(f) As a becomes larger what happens to the curve ?(1)

(4)(a) Draw a sketch graph of $y = a^{-x}$. (3)

(b) Deliniate the differences between this graph and $y = a^x$ (2)

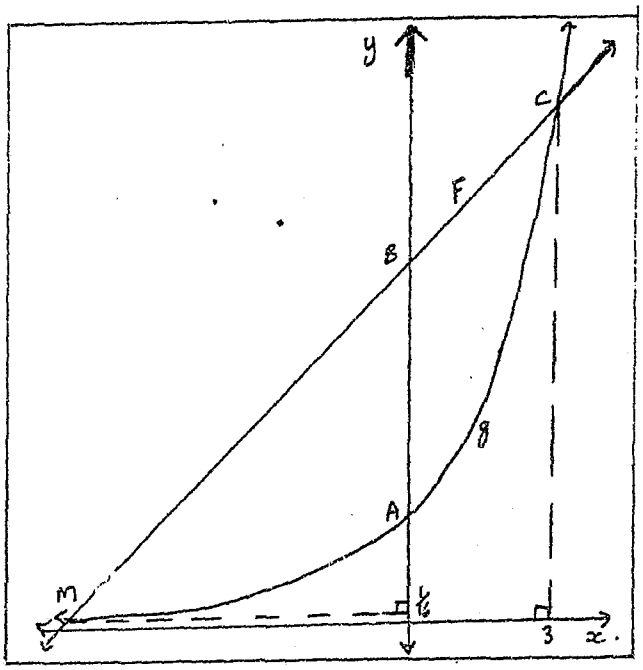
(c) What are the similarities between the two graphs ?(3)

(4) The figures drawn below represent the graphs of:

$F = \{(x;y)/\ y = 127.x + K\}$

112

$g = \{(x;y)/\ y = 2^x\}$

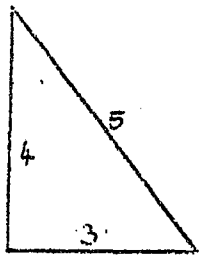


Use the graphs to determine
(a) the length of AB (4)
(b) the gradient of f (1)
(c) The values where the straight line intersects with the exponential curve.(2)
(5) Explain places where the exponential curve can be used in the world outside the mathematics classroom.(4)

/50/

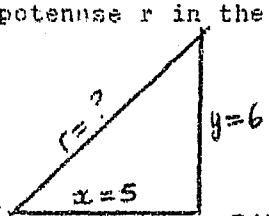
Pre/Post test Basotho huts, Highveld houses,Pythagoras and distance formula. Time 30mins Total 30 marks

(1) Square based huts are built by _____ people.
(a) Xhosa (b) Zulu (c) Sotho (d) Venda
(2) The square-based building originates in_____
(a) Africa (b) Asia (c) Europe (d) America
(3) Pythagoras' theorem states that. In a right angled triangle.....
(a) The hypotenuse is equal to the sum of the other two sides.
(b) The hypotenuse is equal to the difference of the squares on the other two sides.
(c) The hypotenuse is equal to the product of the other two sides.
(d) The square on the hypotenuse is equal to the sum of the squares on the other two sides.
(4) The 3,4,5, triangle is special because:



(a) It is isosceles.
(b) It has a right angle.
(c) It is obtuse.
(d) It is scalene.

(5) What advantage does a square based house have over a circular based house?
(a) You can build additional rooms onto it.
(b) It is warmer in winter.
(c) It is more spacious.
(d) It is cooler in summer.
(6)A rectangle is defined as follows:
(a) An elongated square.
(b) A quadrilateral with two pairs of opposite sides parallel.
(c) A quadrilateral with two pairs of opposite sides equal in length.
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(a) 8 units (b) 6,36 units (c) -4units (d) 4 units
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(a) 7 units (b) 3 units (c) 5,39 units (d) 4,58 units
(10) If A(-2;3) and B(1;-2), the distance between these two points is.....

(a) $\sqrt{2}$ units (b) 5,83 units (c) 0 units (d) 8,2 units

Locus PRE/POST TEST

1) Locus is:
a) an ordered pair
b) a fixed point
c) a set of all possible positions or locations of a point P under a given condition
d) an equation.

2) A locus can not be defined by:
a) a point
b) a line
c) a circle
d) a parabola.

3) If P is equidistants from two points. The locus can be found using:
a) midpoint formula
b) gradient
c) distance formula
d) proportionality formula.

4) If A and B are the points (-4;0) and (4;0)
To determine the equation of locus of C so that $A\hat{C}B = 90°$ the following formula should be used:
a) $m_{AC} \times m_{BC} = -1$
b) $m_{AC} = m_{BC}$
c) Distance AC^2 + Distance BC^2 = Distance AB^2
d) midpoint AB

5) Locus can be compared to an animated cartoon strip because:
(a) a point is found by flicking pages.
(b) a picture is created by a point moving through space.
(c) the locus uses cartoons.
(d) The cartoons make use of the principle of locus

(6) P has coordinates (x;y). Find the relationship between x and y if the ordinate of P is twice the abscisse.
(a) $x = 2y$
(b) $y = 2x$
(c) $x = \frac{1}{2}y + 1$
(d) $y = \frac{1}{2}x$

(7) The distance of P from the origin is 5 units. What is the locus of P?
(a) $x^2 + y^2 = 5$
(b) $y = 5$
(c) $xy = 5$
(d) $x^2 + y^2 = 25$

(8) The locus P is equidistant from A(-1;2) and B(2;6)
(a) (0;4)
(b) $(x+1)^2 + (y-2)^2 = 40$
(c) $6x + 8y - 35 = 0$
(d) $\frac{y-2}{x+1} = \frac{6-y}{2-x}$

(9) PM is perpendicular to MN where M is (2;3) and N (-2;-3). An equation for the locus P is
(a) $(x-2)^2 + (y-3)^2 = (-2-x)^2 + (y+3)^2$
(b) $3y + 2x = 13$
(c) $x^2 + y^2 = 13$
(d) $\frac{y-3}{x-2} = \frac{-6}{-4}$

(10) Q divides OP in the ratio 2:1. If O is the origin and P lies on the line $x + 2y = 4$, determine the equation of the locus of Q.
(a) $|x| + |y| = 3$
(b) $6y + 3x = 8$
(c) $3x + 6y = 8$
(d) $3x^2 + 3y^2 - 16y + 16 = 0$

Student's Questionaire

Thank you for helping us with our research. Please fill in the following. Please ask if there is any part of the questionaire that you do not understand.

Std: Gender (male or female)

Main Home Language:

Instruction: Mark on the scale the option closest to your opinion.
Mathematics is:

		1 2 3 4 5	
1.	useless	(...)(...)(...)(...)(...)	useful
2.	fun	(...)(...)(...)(...)(...)	dull
3.	irrelevant	(...)(...)(...)(...)(...)	relevant
4.	changes with different cultures	(...)(...)(...)(...)(...)	remains constant
5.	unimportant	(...)(...)(...)(...)(...)	important
6.	about learning rules and laws	(...)(...)(...)(...)(...)	about understanding processes
7.	about real life	(...)(...)(...)(...)(...)	about theorems and formulae

Instruction: Please tick the space that shows how much you agree or disagree with the following:

		strongly disagree	dis-agree	neutral	agree	strongly agree
1.	I have no role in the making of maths	()	()	()	()	()
2.	Everybody can do maths	()	()	()	()	()
3.	There is only one correct method in maths	()	()	()	()	()
4.	I enjoy doing maths in groups or pairs	()	()	()	()	()
5.	I often do maths with a calculator	()	()	()	()	()
6.	Estimation is often used in maths	()	()	()	()	()
7.	There is no maths in the games we play	()	()	()	()	()
8.	The history of maths is irrelevant in class	()	()	()	()	()
9.	Maths is part of my culture	()	()	()	()	()
10.	I can invent my own maths	()	()	()	()	()
11.	Maths makes more sense if it is about real things	()	()	()	()	()
12.	Mathematics can be found in the building of traditional dwellings	()	()	()	()	()
13.	Maths is easier if it is familiar to my culture	()	()	()	()	()
14.	Working in groups is confusing	()	()	()	()	()
15.	I can see maths in the daily things I do	()	()	()	()	()
16.	There is no maths in Ndebele murals	()	()	()	()	()
17.	Learning the history of maths is important	()	()	()	()	()
18.	Only talented people can do maths	()	()	()	()	()
19.	Mathematics is not about real life applications	()	()	()	()	()
20.	I enjoy maths when it relates to familiar activities	()	()	()	()	()

Instruction: Write a few sentences on the following:

1. Give 2 or more examples of the maths you see in your daily lives.

Author Grinker C
Name of thesis The Usefulness Of Everyday Mathematics In The Senior Secondary Curriculum: A Controlled Experiment Grinker C 1998

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