Explanations: A comparison of novice and experienced teachers' explanations in Grade 9 linear functions

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Abstract

This study examines how novice and experienced teachers explain linear functions in Grade 9 using a qualitative case study approach. Hargie's (2006) classification of explanations into interpretive, reason-giving, and descriptive categories served as the theoretical framework. The study involved four teachers, two novice and two experienced, each delivering an introductory lesson on Grade 9 linear functions, which were video recorded for analysis. The research identified key themes in teachers' explanations: concept, structure, and procedure, which informed further subcategories. Findings reveal that teachers initially rely on their subject matter understanding when explaining concepts. Over time, experienced teachers refine their explanatory practices, enhancing learners' grasp of the concepts. This study addresses a key aspect of mathematics education by examining how novice and experienced teachers explain linear functions, a fundamental concept in the curriculum. In addition, this study makes a significant contribution to the field by offering a detailed analysis of teacher explanations, a relatively underexplored area in research. The study contributes to the broader discourse on mathematics education by highlighting the role of experience in refining teaching practices. The insights gained suggest that teacher development programs should emphasise not only content knowledge but also the evolution of effective explanatory techniques.

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Plagiarism Declaration

I declare that this research report is my own work and no part of it has been copied from another source (unless indicated as a quote). All phrases, sentences and paragraphs taken directly from other works have been cited and the reference recorded in full in the reference list.

S. Mbhele

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List of acronyms:

ANA: Annual National Assessment

CAPS: Curriculum Assessment Policy Statements

CCK: Common Content Knowledge

CK: Content Knowledge

DBE: Department of Basic Education

DC: Descriptive Concept

DS: Descriptive Structure

DP: Descriptive Procedure

HCK: Horizon Content Knowledge

IC: Interpretive Concept

IS: Interpretive Structure

IP: Interpretive Procedure

KCC: Knowledge of Content and Curriculum

KCS: Knowledge of Content and Students

KCT: Knowledge of Content and Teaching

MKT: Mathematical Knowledge for Teaching

NCS: National Senior Certificate

PCK: Pedagogical Content Knowledge

PK: Pedagogical Knowledge

RC: Reason Giving Concept

RS: Reason Giving Structure

RP: Reason Giving Procedure

SCK: Specialized Content Knowledge

SMK: Subject Matter Knowledge

TIMSS: Trends in International Mathematics and Science Study

CHAPTER 1: INTRODUCTION TO STUDY

1.1 Background

Shulman (1986) presented a knowledge domain of specialized teacher knowledge called pedagogical content knowledge. Pedagogical content knowledge (PCK) is knowledge that is constituted of content and pedagogy, it is the knowledge to translate subject matter into usable knowledge that learners can easily understand. PCK involves the integration of content knowledge (CK) and pedagogical knowledge (PK). Therefore, teachers must not only understand the content (mathematical principles, relationships, etc.) but also possess effective pedagogical strategies to convey this content to learners. Explanations provided by teachers should demonstration this integration. Effective explanations demonstrate a deep understanding of the subject matter and an ability to communicate it in a way that is accessible and meaningful to learners. It would be expected that experienced teachers carry a more developed PCK because they typically have a wealth of teaching experience, which allows them to refine and develop their PCK over time. On the other hand, novice teachers have a less developed PCK because of their lack of experience. Thus, making experienced teachers have better or more nuanced explanations in the classroom compared to novice teachers.

Within the context of mathematics education, Ball, Thames & Phelps (2008) worked with the category of knowledge with is PCK and developed mathematical knowledge for teaching (MKT). Their study identified two subdomains within pedagogical content knowledge: knowledge of the content and students, and knowledge of content and teaching. The study acknowledged the existence of a subdomain termed "pure" content knowledge unique to teaching. This specialised content knowledge, which was identified as distinct from the common content knowledge needed by both teachers and those who are not teachers. Ball et al.'s (2008) work acknowledges the importance of teachers possessing strong content knowledge in mathematics. When teachers provide explanations, they draw on their understanding of the subject matter. Within the MKT framework, one of the subdomains is knowledge of content and students. This aligns with the idea that effective explanations take into consideration the background, abilities, and potential challenges of the students. Teachers need to know how to tailor their explanations to the specific characteristics of their audience. Another subdomain identified by Ball et al (2008) is knowledge of content and teaching. Effective explanations require not only a deep understanding of the content but also the ability to translate that understanding into effective teaching strategies. This involves knowing how to present material, elicit student understanding, and address misconceptions. Mathematical Knowledge for Teaching (MKT) can depend on various factors. However, it would be expected that experienced teachers have better or more developed MTK thus making more effective explanations in the classroom compared to novice teachers.

Baki (2012) asserts that effective mathematics teaching requires a multifaceted knowledge encompassing a deep understanding of operational processes, awareness of learner needs, and proficiency in various methods, strategies, and techniques. Essentially, Baki (2012) is highlighting the intricate processes integral to delivering a comprehensive explanation in the context of teaching mathematics. Exploring further, it becomes intriguing to investigate how both novice and experienced teachers perform in providing explanations that embody the nuanced knowledge outlined by Baki (2012). Building on Baki's insights, Baumert et al. (2010) underline the crucial role of teachers' Pedagogical Content Knowledge (PCK) in delivering effective explanations. This perspective aligns with the emphasis placed by Lachner & Nückles (2016) on the contribution of teachers' pedagogical content knowledge to the formulation of coherent and meaningful mathematical explanations that facilitate learner understanding. In essence, both Baumert et al. (2010) and Lachner & Nückles (2016) underscore the importance of a specialised knowledge blend that intertwines pedagogical strategies with content understanding to enhance the quality of explanations in mathematics education.

Efforts to assess teachers' proficiency in delivering explanations have been made, with Guler & Celik (2016) contending that more research is necessary to obtain a comprehensive understanding of the associated issues. Perry (2000) underscores the importance of viewing explaining as a crucial skill within the mathematics classroom, advocating for in-depth analysis and comprehension of this pedagogical aspect. Consequently, there is a call for increased research focused on teacher explanations to enhance the teaching and learning processes. Teachers employ explanations for various purposes, and Wittwer & Renkle (2008) assert that the effectiveness of these explanations is contingent upon the manner and purpose for which teachers utilize them. In a study comparing expert and novice teachers, Borko and Livingston (1989) discovered that teachers with substantial experience exhibited greater proficiency in delivering effective explanations. Given these findings, our study is particularly interested in conducting a comparative analysis of experienced and novice teachers' explanations, with a specific focus on the teaching of linear functions in Grade 9. This interest is rooted in the notion that understanding how teachers of varying experience levels approach the explanation of mathematical concepts can provide valuable insights for improving teaching practices.

1.2 Problem statement

In 2014, findings from the Annual National Assessment (ANA) report indicated that 90% of Grade 9 students scored below the average in mathematics. Additionally, only 3% of Grade 9 students who took the ANA mathematics exam managed to achieve a score of 50% or higher (DBE, 2014). Furthermore, the average marks per content area in patterns, functions, and algebra were notably low at 37.9%. According to the Trends in International Mathematics and Science Study (TIMSS, 2015), South African students' performance in the patterns, functions, and algebra content area was the lowest at 20.2% compared to the international average performance. Moreover, diagnostic reports from the NSC 2020 highlighted that:

> *While calculations and performing well-known routine procedures form the basis of answering questions in a Mathematics paper, a deeper understanding of definitions and concepts cannot be overlooked. Candidates did not fare well in answering questions that assessed an understanding of concepts.*

DBE, (2020, p184)

This implies that leaners were lacking in the knowledge of explanations of definitions and concepts. Although the Grade 9 Annual National Assessment diagnostics have been discontinued, they provide guidance into understanding the performance trends in the National Senior Certificate (NSC) result for Grade 12 (DBE, 2017; DBE, 2018; DBE, 2019; DBE, 2020) because these have similar results as they also identify patterns, functions, and algebra as an area of weakness in South African school mathematics. We can then argue that the performance patterns of the learners have not changed and may continue in the same manner if there is no intervention to provide quality explanations. At the same time, the National Development Plan sets a goal that by 2030, the academic achievement of South African students should match that of students from nations with comparable levels of development and access to international standardized assessments (NDP, 2012). Learners are struggling in mathematics, particularly with the topic of functions which that has been identified as difficult for them in the above diagnostics. It might be possible that the poor performance in the topic is the result of the teachers finding it very challenging to explain concepts in the topic.

The challenges identified in the South African mathematics education system, particularly concerning the comprehension of linear functions in Grade 9, align with the research conducted by Kinch (2002), Toluk-Uçar (2011), and Baki (2013). These studies collectively highlight a widespread of issues wherein teachers tend to provide inadequate explanations that hinder learners' understanding of fundamental concepts. Kinch's (2002) investigation into subject-matter knowledge in explanations involving procedures with integers revealed that prospective mathematics teachers often exhibited instrumental pedagogical content knowledge, potentially contributing to a preference for procedural over conceptual understanding. Similarly, Toluk-Uçar's (2011) findings suggested that prospective teachers leaned towards instrumental explanations, indicating a potential gap between theoretical understanding and practical articulation. Additionally, Baki's (2013) study, uncovering insufficient explanations of the division operation by primary school teachers, implies a broader challenge in effectively communicating mathematical concepts. In essence, these studies collectively underscore a prevalent trend among teachers to prioritize procedural knowledge at the expense of conceptual understanding. Recognising these insights, it becomes crucial to investigate how South African teachers explain linear functions in Grade 9 and assess whether similar challenges in explanations contribute to the persistent struggles in mathematics education.

1.3 Research questions

From the literature on teacher explanations in the mathematics classroom, this research study compares two novice and two experienced teachers’ explanations in Grade 9 linear functions. This study is aware of the several factors contributing to the performance and does not associate it purely with the years that teachers have been in the field. However, its sole concern is to further understand how novice and experienced teachers' explanations compare in the teaching of concepts in Grade 9 linear functions. In doing so, the following research questions are applied:

1. What types of explanations are preferred by the two novice teachers when teaching Grade 9 linear function?
2. What types of explanations are preferred by the two experienced teachers when teaching Grade 9 linear function?
3. What are the predominant types of explanations used by teachers when delivering instruction on linear functions to Grade 9 learners?

4. How do the explanations provided by novice teachers compare to those offered by experienced teachers in the teaching of Grade 9 linear functions instruction?

1.4 Rationale

1.4.1 Why Linear Functions:

The decision to focus on linear functions in this study is driven by several compelling factors. Firstly, scholarly research, as highlighted by Makonye (2014), emphasizes the crucial role of functions within mathematics. According to Leinhardt et al. (1990), functions and relationships serve as fundamental concepts where learners first encounter the interplay between dependent and independent variables. Furthermore, Knuth (2000) positions functions as a centralizing principle in mathematics, suggesting not only its connection to other mathematical concepts but also its inherent interdependency with them. Secondly, the significance of functions is evident in their substantial presence in the National Senior Certificate (NSC) final Grade 12 mathematics examination, underscoring their pivotal role within the broader curriculum. Furthermore, historical diagnostic reports on Grade 12 examinations consistently highlight functions as a challenging topic for learners. These findings, both national and international, underscore functions as a recurring area of weakness in South African school mathematics. The foundation of understanding functions is laid in the senior phase mathematics curriculum (Grades 7–9). The specific focus on linear functions in Grade 9 is crucial; ineffective teaching of this topic may result in learner frustration and confusion. The study acknowledges that difficulties in grasping linear functions at this stage may persist into the Further Education and Training (FET) phase (Grades 10–12), potentially impacting students' decisions to discontinue mathematics in Grade 10. With an increasing demand for students to translate real-life scenarios into graphical representations, proficiency in linear functions becomes essential. Investigating how teachers explain linear functions in Grade 9, an introductory grade for functions in the Curriculum and Assessment Policy Statement (CAPS), is thus imperative. Consequently, this study aims to shed light on the explanations provided by both novice and experienced teachers regarding the topic of linear functions.

1.4.2 Why Grade 9:

The concept of functions undergoes development throughout Grades 7 to Grade 9 within the South African CAPS curriculum, where they are integrated into the broader framework of Patterns, Functions, and Algebra in the senior phase (DBE, 2011a). As learners progress into

Grades 10-12, functions become a distinct and central topic in the Further Education and Training Phase (FET). In Grades 10-11 of the FET phase, learners are introduced to various types of functions such as linear, quadratic, hyperbolic, exponential, and trigonometric functions as separate topics, followed by log and cubic functions in Grade 12. Functions constitute a significant portion of the Grade 12 mathematics final examination and have consistently posed challenges for learners, as evidenced by previous diagnostic reports.

Given its foundational role in senior phase mathematics, there is a compelling rationale to investigate how functions, particularly linear functions, are explained to learners. Insufficient explanations of linear functions in Grade 9 may result in learners carrying misconceptions into the FET phase, contributing to the perpetuation of low performance in this area. Thus, exploring the quality of explanations provided to learners becomes crucial for addressing the persistent challenges observed in their understanding and application of functions.

The literature presented and diagnostic reports, indicate the topic of functions as one of the topics that learners find difficult to pass. As such, it should be a topic of investigation in research. The instruction of linear functions in senior phase serves as the foundation for the FET topic of functions. The topic advances as students proceed through FET, and concepts from lower grades are important for the progression of learner knowledge in the topic. The investigation of teacher explanations of linear functions in Grade 9 is therefore essential if we are to challenge the pattern of learner performance in the country. Guided by the results of the national assessment diagnostic reports, this study investigates Grade 9 linear functions as a foundation of functions in other grades and a persistent area of weakness in South African school mathematics.

1.4.3 Why explanations:

Hargie (2006) defines explanation as an endeavour to foster understanding. Furthermore, Lachner & Nückles (2016) argue that a crucial aspect of teaching and learning mathematics is for educators to prioritize supporting learners' comprehension of mathematical concepts. Perry (2000) outlines the purpose of explanations, stating that they should elucidate both how and why to undertake tasks. Schoenfeld (1988) further contends that teachers' explanations should facilitate learners in developing a profound understanding of mathematical concepts and procedures. Considering the current trends in learner performance related to functions, literature (Lachner & Nückles, 2016; Hargie, 2006; Kinch, 2002; Taluk-Uçar, 2011; Baki,

2013) suggests that there is a compelling rationale to investigate the quality of teacher explanations due to their pivotal role as a standard classroom practice. Zaslavsky & Zodik (2007) also assert that explanations play a critical role in the teaching and learning process.

On the other hand, Richland, Stigler, & Holyoak (2012) argue that explanations should also offer learners the ability to transfer and generalize their knowledge and understanding flexibly to situations. Lastly, Perry (2000) highlighted the importance of explanations as a skill that needed to be analysed and understood. The above literature is suggestive of the idea of investigating how teachers explain functions to Grade 9 learners would be an activity that is worthwhile both for research and pedagogical practice. This study investigates expert and novice teacher' explanations in Grade 9 algebra. This investigation will be informed by Hargie's (2006) types of Explanation: Interpretative explanations (what), Descriptive explanations (how), and Reason-giving explanations (why) as the analytical framework.

1.4.4 Why novice and expert teachers?

Kinach (2002), in a study on subject-matter knowledge in explanations involving procedures with integers, found that most of the participant (prospective mathematics teachers) had instrumental pedagogical content knowledge, and their explanations focused on procedures. Kinach (2002) contextualized these observations within Skemp's (1978) framework of instrumental and relational understandings in mathematics. Kinach (2002) asserted that the content level aligns with instrumental understanding, highlighting proficiency in procedural knowledge. Conversely, levels of concept, problem-solving, and epistemic have been identified as indicators of relational understanding, suggesting a more comprehensive grasp that involves connecting mathematical concepts, effective problem-solving abilities, and an awareness of the epistemological aspects inherent in the discipline. Similarly, Toluk-Uçar (2011) discovered that prospective teachers exhibited understanding of the content and provided instrumental explanations. Consistent with this trend, Baki (2013) observed that most of his participants (primary school teachers) offered inadequate instructional explanations of the division operation. These studies provide significant insights into the overall quality of teachers' explanations, especially in relation to their years of teaching experience. Consequently, the comparison between the explanations provided by novice and experienced teachers emerges as a valuable area for further research.

1.5 Significance of the study

My experience as a South African mathematics teacher has made me aware that many learners in Grade 9 have difficulties with patterns, functions, and algebra. This seems to persist from Grade 9 through to Grade 12, and these topics seem to not make sense to several learners I have encountered. My observation of learner difficulties with the topics is further endorsed by several national diagnostics findings, such as the Annual National Assessment (ANA, 2015) results for Grade 9 and the National Senior Certificate (NSC) results for Grade 12 (DBE, 2017; DBE, 2018; DBE, 2019; DBE, 2020). Functions as a topic in mathematics has been mentioned in past studies and diagnostic reports as a subject where students struggle to pass. This may be a result of teaching techniques, such as how teachers explain the topic, that do not support learner understanding. Learner understanding of functions in FET (Grades 10–12) builds on how linear functions were taught in senior phase (Grades 7–9). As students go through FET, there is a progression in the topic from lower grades. Guided by national assessment results, it is therefore imperative to investigate the teaching and explaining of linear functions in Grade 9 as it is a foundation of functions in upper grades, which is a recurring weakness in South African school mathematics.

This study is particularly valuable for me, my teaching practice, and my colleagues at my school. The study meant a deeper understanding of how novice and experienced teachers' explanations compare. From the best practice of both, it assists us in making informed choices in our teaching practice. Furthermore, this is an attempt to contribute to knowledge in the research field of explanation and the broader research community. I also benefited because taking up this study grew my capacity as a researcher and helped create and sculpt the academic in me.

1.6 Conclusion

This chapter has delineated the study's focal point, along with its purpose and significance within the field. Based on the diagnostic assessments and literature provided, it becomes apparent that to enhance learner performance and meet the skills requirements of the South African economy, considerable attention and emphasis are required in elucidating the teaching and learning of functions. Furthermore, the quality of teacher explanations in linear functions significantly influences learners' comprehension of mathematics as a subject. Hence, it is imperative to scrutinize and comprehend teacher explanations. Consequently, the subsequent chapter will closely examine theories and literature pertinent to the focus of this study.

CHAPTER 2: CONCEPTUAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

This chapter commences with an introductory section that delineates the purpose and structure, laying the groundwork for subsequent discussions. The literature review is then organized into several subsections, each addressing distinct facets pertaining to the teaching and learning of functions in the mathematics curriculum. This encompasses an exploration of functions within the Curriculum Assessment Policy Statements (CAPS), followed by an in-depth examination of linear functions. Moreover, the literature review delves into the concepts of Pedagogical Content Knowledge (PCK) and Mathematical Knowledge for Teaching (MKT), scrutinizing how teachers' grasp of mathematical content shapes their instructional methodologies, particularly in explanations. The significance of explanations in the teaching and learning process is underscored, alongside a comparative analysis of teaching practices between experienced and novice educators. Concluding the literature review is a synthesis of pivotal findings, paving the way for the conceptual framework. In the subsequent section, the conceptual framework will be expounded upon, integrating insights from the literature review to steer the analysis and interpretation of research findings. Finally, Chapter 2 culminates with a recapitulation of the key points elucidated in the literature review and conceptual framework, setting the stage for methodological deliberations in the ensuing chapters.

2.2 Literature review

In this section of the literature review, I explore the complex terrain surrounding the teaching and learning of functions within the mathematics curriculum. I examine various critical aspects, including the integration of functions in the mathematics curriculum (CAPS), students' conceptions of functions, the use of multiple representations to illustrate linear functions, the fundamental importance of linear functions and slope, effective strategies for teaching and learning functions, the essential role of pedagogical content knowledge and mathematical knowledge for teaching, the significant impact of explanations on facilitating teaching and learning processes, a comparative analysis between experienced and novice teachers, and finally, a comprehensive summary encapsulating the key findings and insights drawn from the literature.

2.2.1 Functions in the mathematics curriculum (CAPS).

In the South African CAPS school curriculum, the Senior Phase (Grades 7 – 9) serves as the initial encounter for learners with linear functions. This topic falls under the content area of Patterns, Functions, and Algebra (DBE, 2011a). As outlined by the Department of Basic Education (DBE, 2011b), this content area is subsequently divided into three distinct topics during the Further Education and Training (FET) phase (Grades 10 – 12). Within the senior phase curriculum, learning objectives encompass various tasks such as drawing linear graphs from provided equations, determining the equation of a linear function using linear graphs, and computing the gradient, x-intercept, and y-intercept of a linear function (DBE, 2011a). Furthermore, learners are exposed to numeric and geometric patterns, functions and relationships involving input and output values in flow diagrams, tables, formulae, and equations, algebraic expressions including expanding and simplifying them, and algebraic equations. In Grade 8, learners begin graphing by plotting points from a table, progressing by Grade 9 to drawing straight line graphs using algebraic equations with a comprehension of gradients and intercepts. Additionally, they are expected to ascertain the equations of linear functions from provided graphs.

From Grades 10 to 12, these concepts evolve into distinct topics. In the Further Education and Training (FET) Phase, functions are treated as independent and central topics. During this phase, learners are taught and evaluated on various types of functions, starting with linear and progressing through quadratic, hyperbolic, exponential, and trigonometric functions in Grades 10 and 11. Finally, they cover logarithmic and cubic functions in Grade 12.The goal for conceptual development in Patterns, Functions, and Algebra is for learners to progress from arithmetic thinking (understanding of numbers) to algebraic thinking (understanding of variables), and to understand a variable not just as an unknown but also as a generalized value. Learners transition from patterns and relationships to functions. Thus, changing the view of Mathematics as a collection of memorized facts and separate topics to seeing Mathematics as interrelated concepts and ideas represented in different models (DBE, 2011a). in the next section I explore learner conceptions of functions.

2.2.2 Learner conceptions of functions.

Understanding linear functions is fundamental in mathematics education as they serve as building blocks for more advanced concepts and applications. Theories of learner conceptions of functions offer a framework for comprehending how learners perceive and engage with these fundamental mathematical entities. From action-oriented views focused on computation to object-oriented perspectives emphasizing relationships and properties, learners navigate through various conceptualizations of functions as they progress in their mathematical journey. However, despite the framework, learners encounter significant challenges in understanding linear functions Dubinsky, Hawks, & Nichols (1992).

The theories of learner conceptions of functions offer a comprehensive framework for understanding how individuals perceive and engage with mathematical functions. Initially, learners often develop an action-oriented view of functions, as described by Breidenbach, Dubinsky et al (1992). In this perspective, functions are seen as computational tools, akin to algorithms or rules, used to generate numeric outputs based on given inputs. For instance, a function like $f(x) = 2x^2 - 1$ might be perceived solely as a means of computation without necessarily considering broader patterns or relationships between inputs and outputs. As students' progress, they may transition to more object-oriented views of functions, as discussed by Sfard (1991) and Sfard & Linchevski (1994). Object-oriented perspectives encompass correspondence (relational) views and covariance views, which emphasize understanding functions as more permanent entities with inherent properties and relationships. The correspondence view, as articulated by Briedenbach et al. (1992), centers on identifying and generalizing causal and dependency relationships between input-output pairs. For instance, students may grasp that a function expression signifies a transformational process involving causal and dependency relationships between dependent and independent variables.

In contrast, the covariance view, as explored by Schwarz & Dreyfus (1995), underscores comprehending how dependent and independent variables alter in relation to each other. This perspective entails analyzing relationships between varying quantities and investigating patterns of change or growth in functions. For instance, students may examine linear equations to discern how alterations in the independent variable impact the dependent variable, such as recognizing the effect of doubling within a specific equation. Finally, the property-oriented view, as articulated by Schwarz & Dreyfus (1995), focuses on identifying invariant properties characteristic of functions and understanding the physical makeup of graphs. This perspective emphasizes recognizing function properties and understanding relationships between different representations, such as graphical, numerical, and algebraic. These theories provide a nuanced understanding of students' conceptions of functions, highlighting the progression from action-oriented views to more object-oriented perspectives, including correspondence, covariance, and property-oriented understandings.

Understanding linear functions presents numerous challenges for learners, as noted by scholars in the field of mathematics education. One significant hurdle is the fragmented nature of students' concept images of linear functions, a concept described by Tall and Vinner (1981). Concept image encompasses the entire cognitive structure associated with a concept, including mental images, properties, and processes. Research conducted by Elia, Ambrosini, and Berrettini (2008) indicates that students often struggle to construct a cohesive understanding of linear functions, which can hinder their ability to effectively apply mathematical concepts. Another notable difficulty arises from students' difficulty in connecting various representations of linear functions. For instance, Teuscher & Reys (2010) observe that students may struggle to relate algebraic expressions, such as $y = mx + b$, with graphical representations of lines on a coordinate plane. This lack of coherence between representations restricts students' ability to visualize and interpret linear functions across different contexts, leading to misconceptions and errors in problem-solving (Lobato & Siebert, 2002).

Additionally, students frequently encounter difficulty when attempting to apply the concept of slope in real-world scenarios. Lobato and Thanheiser (2002) discovered that students struggle with comprehending the practical implications of slope, including interpreting the significance of a slope value within a given context, such as its representation as a rate of change or a measure of steepness. Lacking a clear grasp of how slope relates to real-world phenomena, students may encounter challenges in establishing meaningful connections between mathematical concepts and their practical applications (Teuscher & Reys, 2010). Another obstacle lies in the prevalence of procedural knowledge superseding conceptual understanding among learners. Carlson, Moses, & Breton (2002) contend that many students resort to relying on memorized algorithms and formulas for tasks such as calculating slope or graphing linear functions, without fully comprehending the underlying principles. This procedural approach often results in surface-level learning and difficulties in transferring knowledge to novel situations. Moreover, the multifaceted nature of slope poses challenges for learners. Slope can be interpreted and defined in various ways, such as a rate of change, a measure of steepness, or an indicator of line behavior (Nagle & Moore-Russo, 2012). The diverse conceptualizations of slope contribute to students' confusion and hinder their ability to develop a cohesive understanding of linear functions (Stump, 1999).

The literature on learner conceptions of functions and the challenges of understanding linear functions are deeply interconnected, offering valuable insights into learners' perspectives and difficulties. The theories of learner conceptions provide a framework for understanding the diverse ways in which learners engage with mathematical concepts, from action-oriented views to more object-oriented perspectives. These theories align with the challenges identified in understanding linear functions, such as fragmented concept images and difficulties in making connections between representations. By recognizing common challenges, such as reliance on procedural knowledge and struggles in applying slope to real-world situations, teachers can anticipate areas of difficulty and tailor their explanations accordingly. Effective teaching strategies informed by this literature include scaffolding, incorporating multiple representations, and fostering critical thinking skills. Moving forward, the discussion shifts to the various representations of linear functions, highlighting their significance in fostering a deeper comprehension of mathematical concepts.

2.2.3 Multiple representations of linear functions.

Multiple representations of linear functions play a crucial role in fostering a deep understanding of mathematical concepts among students. According to Elia et al. (2008), the use of multiple representations has been strongly associated with promoting better understanding in the process of learning mathematics. This sentiment is echoed by Kaput (1992), who emphasizes that employing more than one representation aids students in gaining a comprehensive picture of mathematical concepts. Stylianou and Kaput (2002) illustrate the significance of connecting graphical representations and physical models within a dynamic environment to enhance students' understanding of complex phenomena and underlying concepts. Furthermore, Blanton and Kaput (2004) suggest that different representational tools, such as tables, graphs, pictures, words, and symbols, should evolve with students' age development to facilitate sense-making and expression of functional relationships.

In the context of linear functions, multiple representations are particularly valuable. Graphical representations, such as plotting points on a Cartesian plane, provide a visual depiction of linear relationships between variables. This graphical approach allows students to observe patterns, trends, and the behavior of linear functions (Elia et al., 2008). In addition to graphical representations, algebraic representations are essential for understanding linear functions. Algebraic expressions, such as $y = mx + c$, provide a symbolic representation of linear relationships, where m represents the slope and c represents the y-intercept. Understanding the algebraic form of linear functions enables students to manipulate equations, solve problems, and make predictions (Kaput, 1992). Moreover, real-world situations can serve as meaningful contexts for exploring linear functions. By contextualizing linear relationships in scenarios such as distance-time graphs or cost-revenue analysis, students can connect abstract mathematical concepts to practical applications (Stylianou & Kaput, 2002).

The integration of multiple representations allows students to approach linear functions from different perspectives, reinforcing their understanding and facilitating transfer of knowledge across contexts. By engaging with graphical, algebraic, and contextual representations, students develop a more robust conceptual framework for comprehending linear functions (Elia et al., 2008; Blanton & Kaput, 2004). The utilization of multiple representations, as advocated by scholars such as Stylianou, Kaput, and Blanton, is essential for promoting deep understanding of linear functions among students. Graphical, algebraic, and contextual representations complement each other, providing diverse entry points for learning and reinforcing connections between abstract mathematical concepts and real-world phenomena. Additionally, the exploration delves into the concept of slope within linear functions, elucidating its fundamental role in describing relationships between variables. The discussion that follows focuses on the idea of slope as a central feature of the linear function.

2.2.4 Linear functions and the slope.

The concept of slope serves as a cornerstone in mathematics, its importance transcending mere numerical computation. Scholars emphasize its critical role in discerning the covariational behavior of linear and non-linear functions (Teuscher & Reys, 2010). Linear functions heavily rely on slope to articulate the relationship between two variables. When depicted graphically, slope signifies the rate of change of the dependent variable concerning the independent variable. This fundamental characteristic of slope is paramount for comprehending the behavior of linear functions and their graphical representations.

Furthermore, slope holds pivotal significance in advanced mathematical domains like calculus. A robust grasp of slope is indispensable for understanding derivative concepts and instantaneous rates of change (Stump, 2001a). In calculus, slope forms the bedrock for comprehending the tangent line to a point on a curve, crucial for derivative calculations. Absent a thorough understanding of slope, learners may grapple with apprehending these advanced mathematical principles, underscoring the importance of a solid grounding in slope within linear functions. Apart from calculus, slope finds applications in statistical analysis. In statistics, slope aids in determining the line of best fit in regression analysis (Casey & Nagle, 2016). Mastery of slope is pivotal for interpreting the relationship between variables and making predictions based on data. Hence, a profound comprehension of slope is imperative for learners to effectively apply mathematical concepts in real-world scenarios, accentuating its significance beyond the confines of the mathematics curriculum.

In the South African mathematics curriculum, slope is prominently featured across various topics, spanning functions, analytical geometry, calculus, and statistics (DBE, 2011). The curriculum introduces slope as a characteristic of linear functions in the early grades, gradually expanding on this concept to delve into more advanced topics like derivatives and linear regression in later grades. This structured progression ensures that learners cultivate a comprehensive understanding of slope and its applications across diverse mathematical domains. However, Nagle, Moore-Russo, Viglietti, & Martin (2013) contend that slope's nature is intricate and multifaceted, presenting varied interpretations across different contexts within the mathematics curriculum. Researchers have identified numerous conceptualizations of slope, including geometric ratio, algebraic ratio, functional property, parametric coefficient, and trigonometric conception (Moore-Russo, Conner, & Rugg, 2011). These diverse interpretations underscore the depth and complexity of slope as a mathematical concept, posing challenges for learners in grasping its multifaceted nature within linear functions. Below is a comprehensive table adapted from Stump (1999, 2001a, 2001b), who proposed 11 conceptualizations of slope, later supported by (Moore-Russo, Conner, & Rugg, 2011). The table delineates various conceptualizations and descriptions of slope/gradient properties, spanning from geometric and algebraic ratios to real-world applications and behavioral indicators.

Table 2.1 Conceptualisations of the gradient.

Conceptualisation	**Description**
Geometric ratio	The ratio between vertical displacement and horizontal displacement, often expressed as 'Rise over Run'
Algebraic ratio	The ratio of change in y to change in x, typically represented as $\Delta y/\Delta x \ or \ \frac{(y2 - y1)}{(x2 - x1)}$

Functional property,	The rate of change between the independent and dependent variables, sometimes encountered in problems involving related rates.
Parametric coefficient	A parametric coefficient m in the equation: $y = mx + c$.
Trigonometric conception	A property associated with the angle a line forms with a horizontal line, involving the tangent of the line's angle of inclination/decline and the directional aspect of a vector.
Physical steepness	A characteristic of a line, often described with terms like incline, pitch, steepness, slant, or tilt, indicating its vertical rise.
Calculus conception	Referring to the tangent line to a point on a curve.
Real-world situation	A tangible object (e.g., a ramp) or a dynamic scenario with practical implications.
Determining property	Characteristics defining parallel or perpendicular lines.
Behaviour indicator	A property reflecting the trend of a line—whether it's increasing, decreasing, or maintaining a constant slope—and its rate of change.
Linear constant	A property invariant when a line is translated.

Despite its critical role, learners and even teachers often possess limited views of slope, hindering effective teaching and learning experiences (Nagle et al., 2013). Learners may struggle to make connections between different representations of slope, such as graphical, algebraic, and numerical representations (Stump, 2001b; Teuscher & Reys, 2010). This lack of coherence in understanding slope can lead to misconceptions and errors in problem-solving, highlighting the need for instructional strategies that promote a deeper conceptualization of slope within linear functions. Concerns arise regarding teachers' inadequate comprehension of slope, which further exacerbates students' challenges in understanding the concept. Studies have indicated that teachers often struggle to interpret slope as a uniform rate of change, articulate its meaning, establish links between conceptualizations, or interpret graphs using slope effectively (Nagle & Moore-Russo, 2013). Teachers' fragmented understanding and instructional approaches may inadvertently lead students to rely on operational conceptions of slope, limiting their ability to develop a multidimensional understanding of the concept. Moreover, secondary teaching tends to focus on procedurally based conceptualizations of slope, neglecting its broader implications and applications (Nagle et al., 2013). Teachers emphasize the procedurally based conceptualizations of slope, such as the 'rise over run' method or the computation of slope values using specific formulas, without delving into the deeper meanings of slope as covarying quantities or its real-world applications. This procedural approach therefore hinders learners' preparedness for advanced mathematical concepts like calculus, which rely heavily on a conceptual understanding of slope. Students may memorize algorithms and formulas for calculating the slope without fully understanding the underlying principles, further exacerbating their challenges in grasping the concept within linear functions. The following section investigates literature on the teaching and learning of functions.

2.2.5 The Teaching and learning of functions.

The literature on functions, as described by Knuth (2000), emphasizes its pivotal role as a unifying concept in mathematics. This emphasises the overarching importance of functions in mathematics, that when explaining mathematical concepts, particularly functions, it is crucial to highlight their role as a fundamental and unifying element in the discipline. Makonye (2014) further underscores the significance of the function concept, describing it as one of the fundamental ideas that form the basis of discipline. Building upon this, Makonye (2014) not only reaffirms the crucial nature of the function concept but also advocates for active learner involvement in presenting functions. This underscores the imperative for educators to incorporate the function concept as a fundamental cornerstone in their explanations, ensuring that learners grasp its pivotal role in the realm of mathematics. To achieve this, educators are encouraged to integrate active learner participation when presenting functions, employing interactive and participatory teaching methods. This engagement strategy proves especially valuable when introducing or explaining functions, contributing to a more dynamic and effective learning experience for learners.

Moreover, Birgin (2006) posits that functions hold a central role not only within mathematics but also across other disciplines, prompting educational reforms advocating for changes in teaching approaches to various topics, including functions. Particularly concerning linear functions, studies aim to define the concept and illuminate its nature and significance. This insight allows educators to emphasize the interdisciplinary relevance of functions in their explanations, showcasing their applicability beyond the boundaries of the mathematics classroom. Laridon, Barnes, Kitto, Myburg, Pike, Scheiber, Sigabi, & Wilson (2004) define a linear function as any correlation between two variables represented by a straight line, focusing on its graphical representation. In contrast, Webster (2015) offers a definition centered on the symbolic aspect of a linear function, emphasizing mathematical functions with first-degree variables multiplied by constants and combined using addition and subtraction. These varying definitions contribute to a comprehensive understanding of the multifaceted nature of linear functions. Laridon et al. (2004) and Webster (2015) enrich the comprehension of linear functions by presenting diverse definitions, which educators can leverage when explaining linear functions, providing learners with a comprehensive view encompassing both graphical and symbolic representations. Hence, this approach caters to diverse learning styles and enhances learners' overall understanding.

The prevailing consensus within the research community suggests that challenges in teaching and learning linear functions stem from learners' difficulties in grasping the concepts and the employed teaching methods. Leinhardt et al. (1990) assert that linear functions pose considerable complexity, primarily due to their intricate connections to other mathematical ideas. Consequently, learners often exhibit disorganized understanding of the subject, leading to the susceptibility to developing misconceptions about linear functions. Moschkovich's (1996) investigation delves into the difficulty learners face in comprehending the functions of x-intercept, y-intercept, slope (m), and y-intercept (b), as well as understanding the relationships between these components. The findings highlight the significance of context in contemplating linear functions. The identified issues seem rooted in learners' challenges in correlating various representations of functions and a potential incomplete understanding of fundamental mathematical concepts. Lobato and Siebert's (2002) research further explores challenges in learners' understanding, focusing specifically on the concept of slope. Learning to establish connections between the physical and functional components of a rate of change proves to be a notable challenge for learners, as indicated by their findings. Overall, these research insights highlight the intricate nature of learners' struggles with linear functions, emphasizing the importance of addressing conceptual gaps and fostering a more integrated understanding of the subject matter.

The research conducted by Leinhardt et al. (1990), Moschkovich (1996), and Lobato and Siebert (2002) collectively underscores the challenges associated with teaching and learning linear functions. Leinhardt, Zaslavsky, & Stein (1990) emphasize the complexity of linear functions due to their intricate connections to other mathematical ideas, resulting in learners' disorganized understanding and potential development of misconceptions. Therefore, educators should recognize the multifaceted nature of learners' struggles with linear functions. When introducing concepts in linear functions, emphasizing the interconnectedness of mathematical ideas can provide a more organized framework for learners. Moschkovich (1996) investigates learners' difficulties in comprehending various components of linear functions, highlighting the importance of context and the challenges in correlating different representations. This means that contextualizing concepts, using multiple representations, and explicitly addressing common misconceptions are crucial strategies to enhance comprehension. Lobato and Siebert (2002) concentrate on learners' difficulties in grasping the concept of slope, observing challenges in establishing connections between the physical and functional components of rate of change. The specific challenge outlined by Lobato and Siebert (2002) underscores the need for pedagogical strategies that aid learners in bridging the gap between the physical and functional aspects of rate of change. Real-world contextualization and demonstrating the relevance of slope in various applications may enhance learners' understanding more effectively. Expanding on the literature, the focus shifts towards the teaching and learning processes associated with functions, underscoring the significance of pedagogical content knowledge and mathematical knowledge for teaching.

2.2.6 Pedagogical Content Knowledge and Mathematical Knowledge for teaching

In the realm of educational research, there is widespread acknowledgment of the distinction between teachers' knowledge about teaching within a specific domain, commonly referred to as Pedagogical Content Knowledge (PCK), and their domain-specific subject-matter knowledge, known as Content Knowledge (CK) (Shulman, 1986, 1987). Shulman (1986) defines PCK as teachers' understanding of "the most useful forms of representation of the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject … that make it comprehensible to others" (p.9). This definition underscores the importance of comprehending how to effectively represent and articulate subject matter to enhance understanding, while also being cognizant of students' subject-specific conceptions and misconceptions. In contrast, CK pertains to a teacher's understanding of the structures within their domain, necessitating a profound grasp of the material to be imparted to students (Shulman, 1986).

Pedagogical Content Knowledge (PCK) constitutes a pivotal dimension in teachers' expertise, highlighting the unique fusion of content and pedagogy that distinguishes them from subject matter experts. Shulman (1986) drew attention to the integration of content and pedagogy in teacher development, with a focus on understanding the conceptions and preconceptions students from diverse backgrounds bring to their learning experiences. This includes an awareness of challenges such as misconceptions and errors that learners may encounter. PCK represents a synthesis of content knowledge and pedagogical insights, with a specific emphasis on how to structure, modify, and present subject matter effectively for instruction. Shulman's (1986) perspective underscores the thoughtful consideration and proactive measures involved in transforming how subject matter knowledge is taught. This transformative process encompasses reflection, preparation, and action to deliver high-quality mathematics lessons.

In the classroom, PCK becomes the lens through which teachers adapt their subject matter knowledge to facilitate effective explanations. This adaptation involves various teaching methods, verbalization strategies, and demonstration techniques aimed at enhancing learners' understanding of mathematics. The awareness of students' conceptions and potential challenges, as highlighted by Shulman (1986), plays a pivotal role in tailoring explanations to address individual learning needs. By integrating pedagogical insights into content delivery, teachers with robust Pedagogical Content Knowledge (PCK) can formulate explanations that resonate with diverse learners, fostering a deeper and more meaningful understanding of mathematical concepts in the classroom. PCK extends to the capacity to translate one's comprehension of a particular concept into instructional strategies that learners can grasp (Shulman, 1986). Mayer (2004b) contends that while Shulman's definitions offer a generic understanding of pedagogical content knowledge and content knowledge, domain-specific approaches provide more insightful perspectives on instructional processes. The term "mathematical knowledge for teaching" (MKT) is introduced by Ball, Hill, and Bass (2005) to contextualize Shulman's (1986) Pedagogical Content Knowledge within the realm of mathematics.

Explanations in teaching inherently revolve around knowledge. In their reinterpretation of Shulman's categories of teacher knowledge, Ball et al. (2005) emphasize what they term as the 'works of teaching'. Mathematical knowledge for teaching, as defined by Ball et al. (2005), is conceptualized as the specific knowledge required for the practice of teaching mathematics, distinct from the knowledge demanded by other mathematically intensive professions. The authors redefine Shulman's (1986) categories by providing subject (mathematics) specific sub-categories for Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK). Pedagogical content knowledge is further segmented into three categories: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum. Similarly, SMK is divided into three categories: common content knowledge (CCK), horizon content knowledge, and specialized content knowledge (SCK), Ball et al. (2005). The authors derive these sub-categories by actively observing classrooms and documenting the actual occurrences within teaching practices, focusing on incorporating the activities inherent in teaching, which they term as the 'works of teaching.'

Pedagogical Content Knowledge (PCK) and Mathematical Knowledge for Teaching (MKT) are closely intertwined concepts pivotal in shaping the expertise of mathematics educators. While PCK emphasizes the fusion of content knowledge and pedagogical insights tailored to teaching, MKT offers a comprehensive framework encompassing various facets of knowledge essential for effective teaching in the mathematics domain. Within the MKT framework, PCK is delineated into three distinct categories: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). KCS entails grasping mathematical content vis-à-vis students' characteristics, perceptions, and potential hurdles. KCT focuses on translating mathematical content into pedagogical strategies, aligning closely with Shulman's (1986) original concept of PCK. KCC relates to the integration of mathematical content into the broader curriculum, underscoring the interconnectedness of subject matter and instructional approaches.

PCK relies on a solid groundwork of subject matter knowledge (SMK), recognizing that effective teaching necessitates profound comprehension of both content and instructional methodologies. SMK within MKT is further subdivided into three categories: Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), and Specialized Content Knowledge (SCK). CCK encompasses general mathematical knowledge shared among educators, HCK involves understanding the broader spectrum of mathematical knowledge beyond common content, and SCK entails detailed knowledge specific to mathematical topics. PCK, can be regarded as a subset of MKT, and underscores the application of content knowledge in instructional contexts. It guides teachers in selecting appropriate teaching methods, anticipating student difficulties, and designing effective learning experiences. MKT, in contrast, provides a comprehensive view of various knowledge domains essential for successful teaching, extending beyond the classroom to include considerations related to curriculum, students, and the broader educational context. PCK, as part of MKT, directly shapes how teachers explain mathematical concepts. It involves tailoring explanations to students' needs, addressing misconceptions, and selecting instructional strategies that foster understanding. MKT, by incorporating PCK, emphasizes the importance of teachers' subject-specific knowledge in crafting explanations that are not only accurate but also pedagogically effective. The integration of PCK within MKT forms a cohesive framework that guides teachers in navigating the complex interplay between content and pedagogy, ultimately enhancing their ability to deliver impactful mathematics education.

The investigation into the theoretical differentiation between Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) has generated diverse findings, offering a nuanced perspective on their interaction. Some studies advocate for merging CK and PCK into a unified knowledge domain termed Mathematical Knowledge for Teaching (MKT) (Hill, Schilling, & Ball, 2004), while others highlight a correlation between CK and PCK, accentuating their distinct dimensions (Krauss, Brunner, Kunter, Baumert, Blum, Neubrand, & Jordan, 2008). Moreover, the intricate relationship between CK and PCK demonstrates variability across various teacher cohorts (Krauss et al., 2008). The theoretical differentiation between CK and PCK, as explored in numerous studies, influences the way teachers deliver explanations in the classroom.

The significance of teachers' subject-specific knowledge in shaping professional competence has been underscored by various studies (Ball, Lubienski, & Mewborn, 2001; Shulman, 1986). When delivering explanations, teachers draw upon their CK to convey accurate and comprehensive content information. Simultaneously, PCK influences how this content knowledge is translated into pedagogically effective explanations that cater to students' understanding. The effectiveness of explanations in the classroom is closely intertwined with teachers' proficiency in both Content Knowledge (CK) and Pedagogical Content Knowledge (PCK). Recent studies underscore the impact of teachers' knowledge on student learning outcomes. For instance, Hill et al. (2005) discovered a significant correlation between elementary teachers' Mathematical Knowledge for Teaching (MKT) and students' progress in mathematical comprehension. In a German study, Baumert et al. (2010) utilized data from the Program for International Student Assessment (PISA) to emphasize the influence of both Pedagogical Content Knowledge (PCK) and Content Knowledge (CK) on student learning.

Although there was a robust correlation, CK exhibited lower predictive capability for student advancement compared to PCK. PCK, however, emerged as a pivotal factor in critical aspects of instructional quality.

These findings underscore the importance of comprehending how teacher education contributes to the development of subject-specific knowledge (Ball et al., 2001). The intricate interplay among CK, PCK, MKT, and effective teaching practices becomes central to effective teaching. Tailoring teacher education programs to foster a balanced and integrated knowledge base is crucial for enhancing instructional quality. Consequently, this directly influences the clarity, effectiveness, and depth of explanations provided in the classroom, thereby enriching the learning experience for students across diverse educational settings. Subsequently, attention turns to the role of explanations in teaching and learning, exploring how different types of explanations contribute to conceptual understanding.

2.2.7 Explanations in teaching and learning

Among numerous classroom practices, explanations emerge as a critical element in content delivery, signifying educators' efforts to cultivate understanding among learners (Hargie, 2006). Teachers, as the primary conveyors of explanations, play a central role in effective learning, a perspective strongly emphasized by Baumert et al. (2010). Recognizing the pivotal role of explanations in learning underscores the importance of developing effective explanatory skills among teachers. The assertion by Lachner & Nückles (2016) regarding the indispensability of profound content knowledge for crafting compelling explanations carries implications for teacher training and professional development. It suggests that efforts to enhance teaching effectiveness should prioritize the development of teachers' in-depth content knowledge. Furthermore, the disruptive impact of inadequate or unclear explanations on the learning process, as highlighted by Lachner & Nückles (2016), underscores the need for pedagogical strategies that ensure clarity and coherence in explanations. In alignment with Hargie's (2006) definition, Lachner & Nückles (2016) further argue that an explanation not only serves to elucidate specific concepts but also facilitates generalization. These insights have implications for instructional design and teaching methodologies, suggesting that teachers should structure explanations in a way that goes beyond immediate comprehension, fostering the ability of learners to apply knowledge to varied tasks and contexts. Perry's (2000) research, emphasizing the multifaceted use of explanations in the mathematics classroom, suggests that educators should be equipped with versatile explanation skills to address diverse objectives within the mathematics teaching domain.

A comprehensive examination of instructional explanations in the broader literature review (Wittwer & Renkl, 2008) emphasizes key principles for effective teaching. Adaptability to learners' knowledge levels, a focus on fundamental concepts and principles, integration with ongoing cognitive skills, and the avoidance of replacing learners' knowledge construction activities are identified as crucial considerations. That teachers should recognize the diverse knowledge levels of learners and tailor their explanations accordingly. The adaptability of explanations to individual learners, as highlighted by Wittwer & Renkl (2008), ensures that students receive content at an appropriate level, fostering better comprehension. Richland et al. (2012) argue that the use of explanations is to empower learners to flexibly transfer and generalize their knowledge to diverse situations. While Schoenfeld (1988) argues that the role of teachers' explanations is to facilitate the understanding of mathematical concepts and procedures among learners. As advocated by Richland et al. (2012), teachers should design explanations that enable students to transfer and generalize their knowledge to different situations. This ensures that students can apply what they've learned in diverse contexts, promoting a more robust and flexible understanding of the material. Effective explanations, as supported by Schoenfeld (1988), should prioritize fundamental concepts and principles. Teachers need to ensure that their explanations go beyond procedural knowledge and delve into the underlying principles, promoting a deeper understanding of the subject matter.

The studies conducted by Wittwer & Renkl (2008), Richland et al. (2012), and Schoenfeld (1988) offer valuable insights into the principles and purposes of instructional explanations, shedding light on commonalities and distinctions among their perspectives. Both Wittwer & Renkl (2008) and Richland et al. (2012) stress the importance of adapting explanations to learners' knowledge levels. The flexibility of explanations is deemed crucial for ensuring that students receive content at an appropriate level, thereby enhancing comprehension. The shared emphasis on prioritizing fundamental concepts and principles in instructional explanations is notable in Wittwer & Renkl (2008) and Schoenfeld (1988). Both studies argue that effective explanations should surpass procedural knowledge, delving into the underlying principles to foster a deeper understanding. Wittwer & Renkl (2008) and Richland et al. (2012) converge on the idea that explanations should be seamlessly integrated with ongoing cognitive skills. Aligning explanations with students' current cognitive processes ensures a smoother assimilation of new information into their existing mental frameworks. Richland et al. (2012) and Schoenfeld (1988) share a common perspective on the role of explanations in facilitating transfer and generalization of knowledge. Both studies posit that effective explanations empower learners to apply their understanding flexibly across diverse situations, promoting a more robust comprehension.

However, Wittwer & Renkl (2008) focus on general principles for effective teaching and the adaptability of explanations, while Richland et al. (2012) emphasizes the broader goal of empowering learners to transfer and generalize knowledge. Schoenfeld (1988) specifically highlights the role of teachers' explanations in facilitating the understanding of mathematical concepts and procedures. The notion of avoiding the replacement of learners' knowledge construction activities is addressed by Wittwer & Renkl (2008). This perspective underscores the complementary nature of explanations to students' active engagement in constructing their understanding. These studies collectively advocate for adaptable, concept-focused, and cognitively integrated explanations that empower learners to transfer and generalize knowledge. While there are commonalities in their emphasis on adaptability, conceptual understanding, and integration with cognitive skills, the distinct contributions lie in the specific purposes of explanations and the caution against replacing knowledge construction activities. These nuanced perspectives contribute to a comprehensive understanding of the principles guiding effective instructional explanations.

The inquiry into effective instructional explanations in mathematics classrooms has been a pivotal area of investigation, with Gaea Leinhardt making significant contributions to this domain. Leinhardt's extensive body of work distinguishes four types of locations where explanations unfold, with instructional explanations specifically designed for the communication of subject matter knowledge, particularly in the realm of mathematics and algebraic expressions (Leinhardt, 1987, 1989, 1997, 2009, 2010). This categorization underscores the specificity and targeted nature of instructional explanations within the broader landscape of explanatory contexts. In the realm of instructional explanations, Leinhardt delineates clear goals and associated actions, providing a structured framework for effective communication. A crucial aspect highlighted by Leinhardt is the identification of the nature of the problem, establishing a criterion for explanation related to the overarching goals of the lesson (Leinhardt, 1997). Leinhardt's framework for effective instructional explanations in mathematics classrooms emphasizes key criteria such as goals, actions, coherence, and access to disciplinary principles. The literature echoes these points, drawing attention to the intricate connection between knowledge of the representational base, exemplified through the selection of examples and their representations, and effective pedagogical practices (Leinhardt, 1997). Notably, the ability to navigate diverse representational forms is highlighted as a critical aspect of quality explanations (Brown and Daines, 1981; Sevian and Gonsalves, 2008).

However, a closer examination reveals certain gaps in Leinhardt's model, particularly in the aspects of achieving coherence and accessing disciplinary principles. While Leinhardt emphasizes the need for completeness and connectedness to the problem's nature and its representation, the specifics of achieving coherence are not fully elaborated in her model (Leinhardt, 1997). Findeisen's work (2017) underscores the importance of providing explicit details and strategies for achieving coherence in instructional explanations, advocating for a more detailed exploration to guide educators practically. Similarly, the importance of accessing the principles of the discipline for explanation integrity is highlighted by Leinhardt, yet the model lacks specific guidance on the actions required to achieve this access (Leinhardt, 1997). Here, Findeisen's framework (2017) emphasizes the necessity of explicit articulation of disciplinary principles and strategies for their incorporation into instructional explanations, providing clarity crucial for educators aligning their explanations with foundational principles.

Furthermore, Leinhardt's model outlines actions involved in effective explanations, such as selecting examples, representation, problems, and tasks, recognizing that these actions are primarily observable in the planning phase and may vary during execution (Leinhardt, 1997). The literature, supporting this insight, acknowledges the different considerations required for planning and execution phases, emphasizing the need for meticulous planning and flexibility in adapting approaches based on real-time cues and student responses during lessons (Duffy, Roehler, Meloth, Vavrus, Book, Putnam, & Wesselman, 1986). Leinhardt's framework offers a structured foundation for understanding instructional explanations, but the literature, particularly Findeisen's framework, highlights the importance of addressing specific challenges. Detailed guidance on achieving coherence and accessing disciplinary principles within instructional explanations is necessary for optimal learning outcomes. Recognizing the dynamic nature of planning and execution phases underscores the importance of flexibility and adaptability in delivering high-quality explanations in educational settings (Duffy et al., 1986).

Expanding on the significance of effective instructional explanations, van de Sande and Greeno (2010) underscore a crucial factor contributing to their efficacy: shared expectations between teachers and learners. Their research suggests that instructional explanations are more likely to be effective when both teachers and learners share expectations regarding patterns of interaction and information organization. The alignment of perspectives between the explainer (teacher) and the recipient (learner) is deemed essential for ensuring that the explanation is meaningful and understandable (van de Sande & Greeno, 2010). This emphasis on shared expectations resonates with the broader framework of instructional explanation quality outlined by Findeisen (2017). According to Findeisen's framework, effective instructional explanations are unique communicative forms facilitating learning and understanding, characterized by interaction between the explainer and the audience, differing levels of knowledge between them, and the intention to clarify something for the audience (Findeisen, 2017). The notion of shared expectations aligns with the necessity for interaction and alignment in the instructional process.

Moreover, the importance of shared expectations highlights the learner-teacher interaction aspect of the explanation quality framework (Findeisen, 2017). This dimension underscores the significance of considering students' prior knowledge and characteristics when crafting explanations, actively involving students in the explanatory process, and adapting explanations based on cues from students (Findeisen, 2017). The alignment of perspectives proposed by van de Sande and Greeno (2010) can be viewed as part of the broader dynamics of interaction between the explainer and the audience, reinforcing the concept that effective instructional explanations are inherently linked to learners' engagement and understanding. The literature provided by van de Sande and Greeno (2010) complements the discourse on effective instructional explanations by stressing the necessity for shared expectations, aligning with the broader framework of explanation quality advanced by Findeisen (2017). This alignment underscores the interconnected nature of various dimensions, including student-teacher interaction, in shaping instructional explanations that genuinely facilitate learning and understanding.

To evaluate the effectiveness of instructional explanations, Findeisen (2017) presents a comprehensive framework comprising five key dimensions: Content, Student-Teacher Interaction, Process Structure, Representation, and Language. This framework, synthesized from a review of literature on various quality aspects, allows for in-depth analysis and targeted feedback on different facets of explanation. The first dimension, Content, underscores the fundamental requirement that explanations should present subject matter appropriately, ensuring accuracy, completeness, and logical coherence (Findeisen, 2017). Emphasizing the provision of multiple approaches to the same content and adaptability in explanatory approaches aligns with the notion that effective explanations should cater to diverse student needs (Geelan, 2013).

The second aspect, Student-Teacher Interaction, underscores the interactive nature of explanations. Teachers are encouraged to consider students' prior knowledge, actively involve them in the explanatory process, and respond to students' cues (Findeisen, 2017). This dimension resonates with the work of Wittwer and Renkl (2008), which emphasizes the importance of understanding students' characteristics and engaging them in the learning process. Process Structure, the third dimension, focuses on the logical sequencing of the explanatory process, including clarifying the explanation's aim, outlining its structure, assessing prior knowledge, reinforcing key points, and checking for understanding (Findeisen, 2017). This dimension aligns with insights from Charalambous, Hill, & Ball (2011), stressing the significance of structured and well-sequenced explanations.

Representation, the fourth dimension, highlights the importance of employing various forms such as examples, visualizations, and analogies in explanations. Teachers are encouraged to blend different representational forms and elucidate the similarities and differences between representations and the explanatory content (Findeisen, 2017). This aligns with the work of Sevian and Gonsalves (2008), emphasizing the role of diverse and illustrative representations in effective explanations.

Finally, the fifth dimension, Language, emphasizes the linguistic elements of explanations, including considerations of speech levels, clarity, and the supportive use of gestures and movement (Findeisen, 2017). This dimension resonates with previous research by Charalambous et al. (2011), which highlights the importance of clear and appropriate language in instructional explanations. Findeisen's (2017) framework provides a robust and multifaceted approach to evaluating the quality of instructional explanations. Each dimension draws upon insights from various sources, offering a nuanced understanding of the elements contributing to effective instructional explanations. This framework serves as a valuable tool for researchers and teacher educators, enabling detailed analyses and targeted feedback on different aspects of explanation quality.

In the field of mathematics education, the efficacy of instructional explanations offered by teachers holds significant sway over students' comprehension of mathematical concepts. An investigation led by Korkmaz (2021) delved into the instructional explanations provided by both class teachers and primary school mathematics educators concerning the division operation. Through qualitative research, this study provided insights into the extent of conceptual understanding and pedagogical approaches employed by educators in the realm of division instruction. Findings from Korkmaz's study revealed that while teachers demonstrated operational knowledge of division, it often remained superficial and lacked conceptual depth (Korkmaz, 2021). This aligns with previous research indicating challenges in developing teachers' conceptual understanding and instructional skills (Toluk Uçar, 2009; Baki, 2013). Despite possessing procedural knowledge, many teachers struggled to articulate the underlying rationale behind division operations, relying primarily on rules and procedures in their instructional explanations (Korkmaz, 2021).

Furthermore, Korkmaz's study highlighted that teachers often failed to address the multiple meanings and interpretations of division, limiting their teaching approaches and impeding students' comprehension (Korkmaz, 2021). This finding resonates with the notion that effective mathematics instruction should encompass a variety of representations and interpretations to foster students' conceptual understanding (Charalambos et al., 2011). Korkmaz's (2021) study provides valuable insights into the instructional practices of class teachers and primary school mathematics teachers regarding division instruction. By addressing the gaps in teachers' conceptual understanding and instructional skills, educators can create more meaningful learning experiences for students and foster a deeper appreciation for mathematics. The study underscores the importance of addressing teachers' conceptual understanding and instructional practices in mathematics education. Inadequate teacher training and professional development may contribute to the prevalence of superficial instructional explanations in division instruction. Therefore, efforts to enhance teacher education programs and support teachers in developing deeper conceptual knowledge and pedagogical skills are essential for improving student learning outcomes in mathematics.

Studies also emphasize the learnability of the act of explaining, as demonstrated by pre-service teachers, Charalambous et al. (2011). This perspective aligns with the notion that explaining is a practice that can be cultivated and refined over time through intentional efforts. In studies focused on pre-service teachers, Charalambous et al. (2011) outline specific criteria for what constitutes good explanations. These criteria serve as valuable guidelines for educators and contribute to a more comprehensive understanding of effective instructional practices. Some of the key criteria highlighted in these studies include: (1) Establishing Central Questions: The act of posing central questions within explanations is identified as a crucial criterion for effective teaching (Charalambous et al., 2011). This aligns with the notion that a well-structured explanation should be centered around a key question, fostering a clear focus on the main topic or concept. (2) Building on Learners' Existing Knowledge: The literature emphasizes the importance of connecting new information to learners' prior knowledge (Charalambous et al., 2011). This criterion recognizes the role of building upon students' existing understanding, promoting coherence and continuity in the learning process. (3) Considering Difficulties and Misconceptions: Effective explanations should consider potential challenges and misconceptions that learners may encounter (Charalambous et al., 2011). Addressing these difficulties proactively contributes to the overall clarity and comprehensibility of the explanation. (4) Providing Precise Information: Clarity and precision in the information presented are highlighted as critical criteria for good explanations (Charalambous et al., 2011). This involves articulating concepts in a concise and accurate manner, ensuring that students receive information that is both meaningful and relevant.

These criteria collectively contribute to a framework for assessing the quality of explanations, particularly within the context of pre-service teacher training. By recognizing the importance of central questions, connecting to prior knowledge, addressing potential challenges, and ensuring precision, educators can guide pre-service teachers in developing effective explanatory skills. Furthermore, Andrews' (2009) observations of teaching practices in European countries reveal that explaining is one of the observable didactic strategies employed by teachers. This highlights the practical relevance of understanding the learnability of the act of explaining, as it is a fundamental component of effective teaching strategies in diverse educational settings. Effective instructional explanations demand a balance between well-defined goals, thoughtful actions, coherence, and access to disciplinary principles. The learnability of the act of explaining, the importance of shared expectations, and a robust framework for quality assessment contribute to the evolving landscape of research in instructional explanations. Teachers' explaining skills, defined as the ability to provide correct, coherent, and engaging explanations, necessitate a solid content knowledge coupled with an understanding of students' learning processes and misconceptions (Brown, 2006).

Furthermore, a study conducted by Findeisen, Deutscher, and Seifried (2020) aimed to assess the efficacy of a training module intended to enhance the explanatory abilities of prospective teachers during their university education. Employing a treatment and control group design, Findeisen et al. (2020) focused on scrutinizing five dimensions of explanation quality: Content, Student-teacher interaction, Process structure, Representation, and Language. The results revealed an overall enhancement in explaining skills over time, yet the degree of improvement did not significantly differ between the treatment and control groups. Nevertheless, distinct effects were observed across various quality dimensions of explanations. Particularly, enhancements were evident in Student-teacher interaction and Process structure, with the latter being the sole aspect significantly impacted by the treatment. This study emphasizes the need for targeted training and practice opportunities for prospective teachers to develop their explaining skills effectively. It underscores the importance of considering different quality aspects of explanations, such as student-teacher interaction and process structure, in designing interventions aimed at improving explaining skills. Additionally, the study highlights the value of incorporating theoretical knowledge and reflective practices into teacher education programs to optimize learning outcomes in this area. Furthermore, the discussion addresses the differences between experienced and novice teachers, shedding light on the implications of teacher expertise for instructional practices.

2.2.8 Experienced and novice teachers

Comparative studies of expert and novice teachers' explanations have been prevalent in the literature, highlighting that effective explanations often stem from experienced educators (Duffy et al., 1986; Borko & Livingston, 1989). Leinhardt's pivotal work on instructional explanations, particularly her in-depth exploration of eight consecutive lessons from an expert teacher, serves as a foundational cornerstone in the realm of understanding how expert teachers cultivate and refine their explanation skills (Leinhardt, 1987). By closely examining the patterns, strategies, and decision-making of an expert teacher during the act of explanation, Leinhardt shed light on the dynamic nature of pedagogical skills. The exploration delves into the teacher's ability to adapt explanations to the evolving needs of the students, demonstrating a responsiveness that goes beyond mere adherence to a predetermined plan. Leinhardt's work lays the groundwork for understanding how expert teachers develop their explanation skills with experience. These findings are aligned with the notion that years of teaching experience contribute to the refinement of instructional skills. However, this generalization requires careful consideration in diverse educational landscapes, as highlighted by the cautionary note regarding the South African context. The challenges posed by historical and socio-economic factors in

South Africa demonstrate the need for a context-specific understanding of the correlation between teaching experience and effective explanations.

In parallel, studies focusing on pre-service teachers, such as Charalambous et al. (2011), offer a different perspective, emphasizing that the act of explaining can be learned. These studies suggest that during training, novice teachers can develop the skills needed to provide effective explanations. Charalambous et al. (2011) identify several criteria for good explanations, including establishing central questions, building on learners' existing knowledge, addressing potential difficulties, and offering clear information. These criteria serve as important guidelines for teaching and underscore the adaptable nature of instructional practices. The development of these skills among pre-service teachers highlights the critical role of teacher education programs in fostering effective teaching strategies.

Furthermore, the literature advocates for a shift in focus from experience-based expertise to shared expectations between teachers and learners (van de Sande & Greeno, 2010). This perspective suggests that effective instructional explanations arise from a collaborative understanding between the teacher and the learner, moving beyond the novice-expert dichotomy. The focus on shared expectations emphasizes the interactive and dialogic nature of teaching, bridging the experiences of novice and expert teachers.

Collectively, these studies contribute to our understanding of instructional explanations by examining teachers at different career stages and identifying key factors that influence the effectiveness of their explanations. The literature reveals a nuanced view of effective instructional explanations, shaped by teaching experience, the ability to learn, and the importance of shared expectations. By comparing findings across different contexts, it becomes clear that effective teaching practices require a flexible and context-specific approach. This section concludes with a synthesis of the key insights from the literature, providing an integrated overview of the themes discussed.

2.2.9 Summary of the literature

In accordance with the South African Curriculum and Assessment Policy Statement (CAPS), the mathematics curriculum assigns considerable importance to functions, recognizing them as foundational components. Across various grade levels, learners are introduced to functions, with linear functions occupying a central role due to their foundational nature and relevance to more advanced mathematical concepts. As learners progress, their conceptions of functions evolve, transitioning from action-oriented views focused on computation to object-oriented perspectives that emphasize relationships and properties. Multiple representations, including graphical, algebraic, and contextual, are pivotal in nurturing a deeper understanding of functions, offering diverse entry points for learning and reinforcing connections between abstract mathematical concepts and real-world phenomena.

Additionally, within the realm of linear functions, the concept of slope holds paramount importance, serving as a foundational element in distinguishing between linear and non-linear relationships. While slope extends beyond numerical computation to encompass its significance in calculus, statistics, and real-world applications, learners often face challenges in fully grasping its multifaceted nature. Effective instructional approaches are needed to contextualize slope, enhancing learners' comprehension and application abilities. Moreover, the teaching and learning of functions necessitate contextualized instruction, incorporating multiple representations and addressing common misconceptions. Crucial for fostering dynamic and effective learning experiences are pedagogical strategies that encourage active learner participation and collaborative understanding between educators and students.

At the heart of successful teaching and learning lies the integration of pedagogical content knowledge (PCK) and mathematical knowledge for teaching (MKT). Teachers must blend their expertise in subject matter with pedagogical insights to craft explanations tailored to the diverse needs of learners and select suitable instructional approaches. Teacher education programs are instrumental in cultivating effective teaching methodologies and honing the instructional capabilities of future educators. Moreover, scholarly literature emphasizes the significance of adept explanations in the teaching-learning process, highlighting the importance of adapting to learners' varying levels of knowledge, prioritizing fundamental concepts, and facilitating the transfer and application of knowledge.

Comparative studies shed light on the factors influencing the effectiveness of instructional practices among experienced and novice teachers. While experienced teachers demonstrate adaptability and sophisticated understanding in delivering explanations, pre-service teachers can develop effective explanation skills through training and guided practice. The emphasis on shared expectations between teachers and learners underscores the interactive nature of instructional interactions and the importance of context-specific understanding of effective teaching practices. The literature underscores the intricate interplay between content knowledge, pedagogical insights, and instructional practices in shaping effective explanations in teaching and learning. By understanding and integrating these dimensions, teachers can cultivate a set of skills making them capable of delivering impactful explanations that enhance learning outcomes for all learners. This section summarized the literature discussed in the review; the subsequent section provides the conceptual framework for this study.

2.3 Conceptual framework.

Originating from a linguistics background, Brown (2006) aimed to construct a comprehensive conceptual framework focusing on explanation. This involved synthesizing studies spanning from various professions to create a foundational structure that could be utilized for both the analysis and provision of explanations. The basis for this exploration stemmed from evidence indicating the paramount importance of clear explanations for students, patients, and clients. Clear explanations were found to contribute to enhanced learning outcomes in educational settings and improved understanding, satisfaction, and health outcomes in doctor-patient interactions. Brown (2006) suggests that professionals can improve their explanatory skills through training but underscores the necessity of considering the specific contexts and cultures in which they operate. In Hargie (2006), Brown categorizes explanations into three distinct types: Interpretative explanations, Descriptive explanations, and Reason-giving explanations. Building on this framework, Murtafiah, Sadijah, Candra, Susiswo, & Asari (2018) conducted a study examining the explanations provided by pre-service teachers during mathematics teaching practice. They argue that these types of explanations are observable in the mathematics classroom, providing a more in-depth examination of mathematical concepts, procedures, and principles compared to previously explored explanation types (Murtafiah et al., 2018).

In Hargie (2006), these explanations, as described by Brown (2006), encapsulate the essential questions of 'What? Why? and How?' and are further enriched by considerations of ‘Who?’, ‘When?’ and ‘Where?’. Hargie (2006) describes the types of explanations as follows:

- *Interpretive explanations*– “address the question, ‘What?’. They interpret or clarify an issue or specify the central meaning of a term or statement.” (p 186)
- *Descriptive explanations*– "address the question, 'How?' These explanations describe processes, structure, and procedures" (p 186)

- *Reason-giving*– "explanations address the question 'Why?' They involve reasons based on principles or generalizations, motives, obligations or values." (p 186)

Interpretive explanations, as defined by Hargie (2006), are crucial for defining and understanding mathematical concepts, focusing on the fundamental question of 'what.' Luxomo (2019) emphasizes the investigative nature of the term 'what,' guiding inquiries into the facts, identities, and qualities of mathematical objects. Within the mathematics classroom, interpretive explanations serve to elucidate the nature of knowledge and provide definitions for various mathematical elements, including concepts, symbols, diagrams, and procedures. A concrete example of the power of interpretive explanations lies in the concept of a gradient, explained as the steepness of a slope. This approach connects the abstract mathematical notion to real-world experiences, allowing learners to relate the gradient to their everyday knowledge, such as walking up a hill. This engagement with a real-world scenario aid in a deeper understanding of what the gradient represents. For instance, a high gradient on a hill corresponds to a steep path, necessitating extra effort to climb. Similarly, a high gradient in a linear function is likened to a steep line with rapid value changes. Conversely, a low gradient on the hill suggests a gentle path, requiring less effort to climb, and in a linear function, it indicates a more gradual change in values. The interpretive explanation extends to the mathematical representation of the gradient in the linear equation $y = mx + b$, where the gradient (m) is a crucial factor. In the equation $y = 2x + 1$, the gradient is 2, signifying that for every one-unit increase in the x values, the y values increase by 2 units.

The above examples not only assist in interpreting and clarifying the concept of a gradient but also attribute its central meaning to the steepness of a slope. Therefore, interpretive explanations, particularly illustrated through the concept of the gradient, bridge the gap between abstract mathematical ideas and real-world scenarios, making mathematical concepts more tangible and meaningful for learners. Various studies emphasize the importance of clarifying mathematical concepts and definitions to enhance understanding. For example, Stylianou and Kaput (2002) illustrate the significance of connecting graphical representations and physical models within a dynamic environment to enhance learners' understanding of complex phenomena and underlying concepts.

By framing slope as a representation of the rate of change, educators can facilitate learners' comprehension of this core mathematical concept. Elia et al. (2008) stress the importance of interpretive explanations in elucidating linear functions for learners. They propose that by interpreting slope as the rate of change in the dependent variable concerning the independent variable, instructors can offer learners a lucid and cohesive understanding of how linear functions operate. This approach aids learners in linking abstract mathematical concepts to real-world situations, thereby rendering the concept of slope more tangible and relevant.

Descriptive explanations, as elucidated by Hargie (2006), serve to elucidate the structure, process, and procedure of phenomena, detailing their inherent qualities and operational mechanisms. Luxomo (2019) underscores that the term 'how' in descriptive explanations prompts an exploration into the methods, actions, and procedures entailed within a process. In the realm of mathematics education, descriptive explanations are frequently employed to articulate procedural steps and furnish comprehensive descriptions of mathematical processes and entities. For instance, consider the process of determining and computing the gradient within the context of a linear function. Explaining how to find the gradient given the equation $y = 2x + 1$, one can highlight that the gradient is represented by the coefficient of x, in this case, 2. This means that for every one-unit increase in the x-value, the y-value increases by 2 units. The explanation further delves into practical calculations, such as finding the corresponding y value for a given x value, exemplified by substituting $x = 1$ into the equation, resulting in $y = 3$.

Additionally, descriptive explanations extend to the gradient formula, expressed as $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$, which involves dividing the change in y-values by the change in x-values. To calculate the gradient of a linear function, exemplified by $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$, two points (x_1, y_1) and (x_2, y_2) on the line are needed. For instance, if the points $(3, 7)$ and $(5, 11)$ are on the line $y = 2x + 1$, the calculation $\frac{((11) - (7))}{((5) - (3))}$ results in a gradient of 2. This signifies that in the equation $y = 2x + 1$, the gradient is 2, and the y value increases two times for every single increase in the x value or input. These examples effectively engage learners by providing clear explanations of procedures, elucidating the nature of structures, processes, and procedures related to the gradient. They articulate the methods used to calculate the gradient through the gradient formula and explain how the gradient is represented in various mathematical forms, including equations, formulas, graphs, and arithmetically through ratio. The use of multiple representations of linear functions is crucial for teacher explanation, especially concerning descriptive explanations.

Research underscores the significance of offering precise and elaborate elucidations of mathematical procedures and operations. For instance, Blanton and Kaput (2004) propose that varying representational tools, such as tables, graphs, visuals, textual descriptions, and mathematical symbols, should evolve in tandem with learners' developmental stages to facilitate comprehension and expression of functional relationships. Descriptive explanations play a pivotal role in guiding learners through the interpretation of graphs, the analysis of tables, and the comprehension of algebraic expressions pertinent to linear functions. Moreover, Elia et al. (2008) underscore the importance of descriptive explanations in fostering learners' procedural fluency in linear functions. They contend that by delineating the sequential steps for problem-solving and equation manipulation, educators can aid students in acquiring the requisite skills to navigate linear functions proficiently. Descriptive explanations furnish learners with a clear grasp of the structure and sequence involved in resolving tasks related to linear functions, thus augmenting their procedural fluency. Furthermore, Murtafiah et al. (2018) noted that pre-service educators frequently employ descriptive explanations to elucidate the execution of mathematical procedures and furnish comprehensive descriptions of mathematical processes. This underscores the pivotal role of descriptive explanations in mathematics education, guiding students through problem-solving endeavors and cultivating a deeper understanding of linear functions.

Reason-giving explanations, which delve into the 'why' question, provide essential rationales for actions, drawing on principles, causes, and generalizations (Hargie, 2006). Luxomo (2019) aligns with this perspective, asserting that the term 'why' prompts considerations of reason, purpose, justification, or motive. In the mathematics classroom, these explanations are employed to elucidate concepts, articulate underlying principles, and justify various procedures. To enhance learners' logical thinking about concepts and procedures, the examples discussed earlier, such as the calculation of gradients and the use of the gradient formula, necessitate reason-giving explanations. Explaining why gradients and their formulas are employed contributes to a deeper understanding of their significance and functionality. For instance, linear functions play a vital role in modeling real-world events, allowing the representation of relationships like distance and time or cost and quantity. This modeling is crucial as it enables predictions of future outcomes or the examination of historical data by assessing the connections between input and output values. For instance, a company might use a linear function to predict its profit based on how much it sells a product.

Furthermore, the reason-giving explanations extend to clarifying why the gradient and its formula work effectively. The gradient measures the rate of change of a function, with the formula $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ expressing the average rate of change between two points on the line. This formula serves as a generalized representation of the gradient concept, encapsulating the slope of the line connecting any two points—a foundational characteristic of linear functions. Notably, the formula's consistency in yielding the gradient value persists irrespective of the selected points along the line, underscoring its robustness and practical utility. Reason-giving explanations within the realm of mathematics instruction offer invaluable insights into the 'why' behind various concepts and procedures. By elucidating the principles, causal factors, and overarching principles that govern mathematical processes, these explanations nurture learners' logical reasoning abilities and cultivate a deeper appreciation for the significance and real-world applications of mathematical concepts, such as gradients and their everyday lives.

Reason-giving explanations, as outlined by Hargie (2006), provide essential rationales for actions, drawing on principles, causes, and generalizations. In the context of linear functions, reason-giving explanations play a crucial role in elucidating underlying principles, justifying various procedures, and articulating the significance of mathematical concepts. Support for reason-giving explanations in the literature on linear functions can be found in studies that emphasize the importance of providing students with a deeper understanding of the rationale behind mathematical concepts and procedures. For example, Stylianou and Kaput (2002) illustrate the significance of connecting graphical representations and physical models within a dynamic environment to enhance students' understanding of linear functions. By explaining why certain mathematical relationships exist and how they are derived, educators can help students develop a deeper appreciation for the underlying principles of linear functions.

Furthermore, Nagle et al. (2013) advocate for a shift beyond mere procedural explanations in teaching linear functions, suggesting that educators should provide students with opportunities to engage in reasoning and justification. Reason-giving explanations enable students to grasp not only how to solve mathematical problems but also why specific strategies prove effective. This approach fosters adaptive reasoning, empowering students to apply logical thinking and reflective processes to their problem-solving endeavors. Additionally, Lobato and Siebert (2002) underscore the importance of assisting students in establishing connections between the physical and functional aspects of linear functions. Through reason-giving explanations, educators can elucidate why particular mathematical relationships hold true and how they manifest in real-world contexts. This contextualization enriches students' comprehension of linear functions and their applicability across diverse scenarios.

When considering Hargie's (2006) three categories of explanations—Interpretative explanations, Descriptive explanations, and Reason-giving explanations—in conjunction with Kilpatrick et al.'s (2001) five strands of mathematical proficiency, a rationale emerges for incorporating Hargie's explanation types into Grade 9 linear functions instruction. Kilpatrick's strands underscore the interconnected nature of mathematical learning, with each explanation type serving distinct roles in fostering conceptual understanding, procedural fluency, strategic competency, adaptive reasoning, and productive disposition. This amalgamation offers a holistic framework for the effective teaching and learning of mathematics.

Firstly, Interpretive explanations, as delineated by Hargie (2006), focus on elucidating concepts and delving into the "what" of mathematical ideas. This corresponds with Kilpatrick et al.'s (2001) emphasis on Conceptual understanding, which underscores the need for a comprehensive and functional comprehension of mathematical concepts. Interpretive explanations serve to foster conceptual understanding by emphasizing the interconnectedness of mathematical ideas, treating them as integral components rather than disparate elements. This approach establishes a sturdy groundwork for learning.

Second, Descriptive explanations play a pivotal role in the mathematics classroom by shedding light on the "how" of mathematical processes. Procedural fluency, as defined by Kilpatrick et al. (2001) as the mastery of procedures and adeptness in executing them accurately and efficiently, aligns with the use of descriptive explanations. Similarly, Descriptive explanations elucidate the methods and procedures involved, emphasizing the structure, process, and sequence in mathematical operations. By detailing how tasks are performed, Descriptive explanations contribute to enhancing Procedural fluency. This underscores the interconnectedness of conceptual understanding and procedural fluency, emphasizing the significance of comprehending procedures and their effective utilization for gaining a deeper understanding of concepts through practical application.

Thirdly, Strategic competency, as defined by Kilpatrick et al. (2001), entails the capacity to formulate, represent, and resolve mathematical problems, with an emphasis on employing reason-giving explanations. Reason-giving explanations, addressing the 'why' aspect, foster logical reasoning concerning both concepts and procedures, thus supporting learners' strategic competency in problem-solving. Additionally, Adaptive reasoning, as delineated by Kilpatrick et al. (2001), involves logical thinking, reflection, explanation, and justification. This mirrors the logical thought process inherent in both reason-giving and interpretive explanations, highlighting the ability to reason logically about the connections between concepts and scenarios. Together, reason-giving and interpretive explanations contribute to adaptive reasoning, forming a unified framework for logical problem-solving.

Productive disposition (Kilpatrick et al.,2001), which involves seeing mathematics as sensible, useful, and worthwhile, aligns with the motivation behind interpretive and reason-giving explanations. Every type of explanations offered to learners, whether interpretive, descriptive, or reason-giving, contribute to making mathematics sensible, meaningful, and applicable. These explanations collectively foster a positive attitude (productive disposition) toward mathematics by showcasing its real-world relevance and utility. The integration of these strands and types of explanations creates a robust foundation for fostering a positive and meaningful learning experience in mathematics.

Although different and serving different purposes, the types of explanations can be intertwined and used simultaneously (Hargie, 2006). Interpretative explanations are associated with the interrogative particle 'what', which takes on a conceptual nature. Alternatively, Descriptive explanations are associated with the interrogative particle ‘how’ which is procedural. Luxomo (2019) makes two claims regarding the interrogative particles ‘What’ and ‘How’. First, the what and the how could have a when and why component for legitimization. Second, the typical separation of the interrogative particles, what and how as conceptual and procedural, is misleading. Those procedures rely on a conceptual base. In my exploration of explanation types, I saw it essential to understand how interpretive, descriptive, and reason-giving explanations intersect and overlap. To visually represent this concept, I have developed a diagram illustrating the relationships among these explanation types. This diagram will provide insight into how teachers combine interpretive, descriptive, and reason-giving explanations to deliver comprehensive and effective explanations to learners. Let's delve into the diagram to explore the interconnected nature of these explanation types.

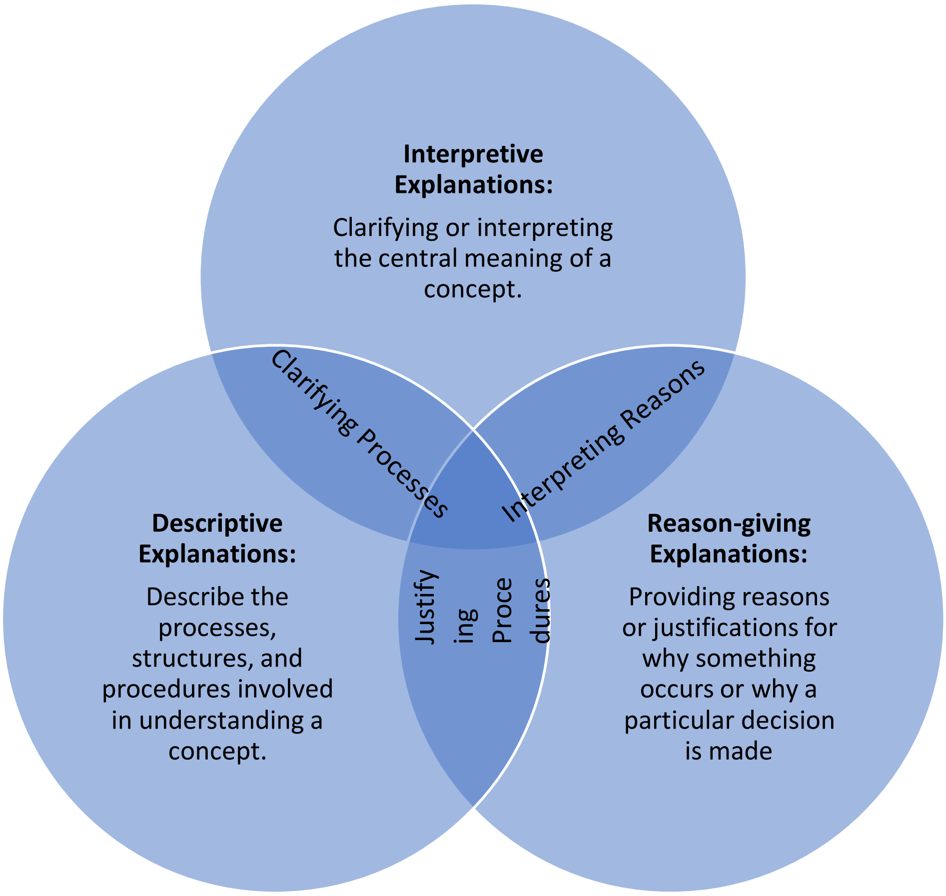


Figure 2.1 *Hargie (2006) typology of explanations and their interconnectedness.*

In Hargie's framework of explanation types, namely interpretive, descriptive, and reason-giving explanations, there is significant overlap among these categories. Interpretive explanations, which focus on clarifying or interpreting the central meaning of a concept, often intersect with descriptive explanations. While interpretive explanations aim to provide insight into the meaning of terms or statements, descriptive explanations delve into the processes, structures, and procedures involved in understanding a concept. Therefore, when explaining a concept, teachers may often find themselves offering both interpretive explanations to elucidate the meaning and descriptive explanations to outline the procedural aspects, leading to an overlap between the two types. When interpretive explanations overlap with descriptive explanations, I referred to it as "clarifying processes.". This occurs when explanations not only interpret or clarify the central meaning of a concept (interpretive) but also describe the processes, structures, and procedures involved in understanding that concept (descriptive). This overlap helps provide a comprehensive understanding by offering both the meaning and the practical aspects of the concept being explained.

Similarly, reason-giving explanations, which involve providing reasons or justifications for why something occurs or why a particular decision is made, can overlap with both interpretive and descriptive explanations. In providing reasons, educators may need to interpret the underlying principles or concepts guiding their explanations (interpretive), as well as describe the procedural steps or logical sequence of actions (descriptive) that support their reasoning. When interpretive explanations overlap with reason-giving explanations, it typically involves providing reasons or justifications that are grounded in an interpretation of the central meaning of a concept. I referred to this overlap as "interpreting reasons". In this scenario, teachers not only clarify the central meaning of a concept (interpretive) but also offer reasons or justifications for why something occurs or why a particular decision is made (reason-giving), with these reasons being informed by their interpretation of the concept. This integration ensures that explanations are both insightful and logically supported, enhancing understanding, and promoting critical thinking. When descriptive explanations overlap with reason-giving explanations, it involves providing reasons or justifications that are grounded in the description of processes, structures, or procedures. I termed this overlap as "justifying procedures". In this context, educators not only describe the processes, structures, or procedures involved in understanding a concept (descriptive) but also offer reasons or justifications for why these processes occur or why certain procedures are followed (reason-giving). This integration ensures that explanations are both informative and logically supported, enhancing understanding, and facilitating problem-solving. The integration of interpretive, descriptive, and reason-giving explanations ensures that explanations are comprehensive, logical, and contextually relevant.

When interpretive explanations, descriptive explanations, and reason-giving explanations all intersect, it reflects a comprehensive and multifaceted approach to explanation. This intersection can be conceptualized as "holistic explanation" or "comprehensive elucidation." In this scenario, teachers provide interpretations to clarify the central meaning of a concept, then describe the processes or structures involved in understanding that concept and offer reasons or justifications for why those processes occur or why certain procedures are followed. This holistic approach ensures that explanations address the "what," "how," and "why" of a concept, catering to different aspects of learners' understanding and promoting a deeper comprehension of the subject matter. This section discussed the conceptual framework of the study, in the next section I conclude chapter 2.

2.4 Conclusion

This chapter has provided a comprehensive exploration of the teaching and learning of linear functions, focusing particularly on the role of explanations in instructional practices. By examining learner conceptions, multiple representations, the concept of slope, and teaching processes, it has underscored the complexities inherent in understanding linear relationships. Through Hargie's framework, the chapter elucidated how interpretative, descriptive, and reason-giving explanations play pivotal roles in clarifying concepts, procedures, and underlying principles within linear functions. Additionally, it emphasized the significance of addressing the differences between experienced and novice teachers and the importance of ongoing professional development to enhance instructional practices. This groundwork sets the stage for further investigation into the methodologies employed in studying these concepts and their implications for mathematics education in the subsequent chapter.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter offers an in-depth exploration of the research design, and methodologies employed in the study. It commences with an introduction that underscores the importance of the research design concerning the study's context, objectives, and inquiries to be tackled. Following this, the chapter delves into a discussion and justification of the chosen research design, elucidating the rationale behind its selection. Subsequent sections detail the methods of data collection and participant selection, outlining the tools utilized for gathering data and the criteria for participant inclusion. The method of analysis is then elucidated, clarifying the procedures employed to analyse the collected data. The focus later transitions to data coding and analysis, providing comprehensive explanations of the transcription process, unit of analysis, and coding of explanations utilizing Hargie's (2006) types of explanation. The chapter also addresses the trustworthiness of the study, discussing matters related to research rigor, and navigates ethical considerations, outlining the ethical principles and protocols adhered to throughout the research process. Lastly, the chapter summarizes the key points discussed and sets the stage for the subsequent chapters, ensuring a thorough exploration of the research design and methodology, thereby enhancing clarity and coherence in the study's methodological framework.

3.2 Research Design

This study employed a qualitative approach to analyse and interpret data. McMillan and Schumacher (2010) define qualitative research as a method focusing on analysing data in the form of words rather than numerical data, with an emphasis on exploring various meanings to gain a deeper understanding of the data. Denzin and Lincoln (2003) argue that qualitative research involves interpretive and naturalistic methods for data analysis. Given the study's aim to explore how both novice and experienced teachers use explanations to teach linear functions in Grade 9, qualitative research methods are deemed appropriate. The study seeks to interpret and derive meaning from the data utilizing Hargie's (2006) types of explanation.

For its research design, this study adopted a Case Study approach, influenced by Mertens (1998), who highlights its effectiveness in educational and psychological research, particularly for investigating instructional strategies. Schumacher and McMillan (2010) define a case study as a research design that extensively examines a bounded system or case over time, utilizing multiple data sources within the setting. Stake (1995) further elaborates on three types of case studies: intrinsic, instrumental, and collective. Intrinsic case studies are chosen when researchers aim to understand a specific case itself and explore its unique characteristics. On the other hand, instrumental case studies serve external interests beyond understanding the situation. Collective case studies involve the examination of multiple cases with shared characteristics, often for the purpose of comparison.

This study aligns with Stake's typology, taking the form of an Instrumental Case Study. In this context, an instrumental approach is justified as it allows for the strategic selection of a sample of participants (novice and experienced teachers) to closely examine and discern specific behavioural patterns. This choice is in line with the research goal of gaining nuanced insights into instructional practices within the given context, going beyond understanding individual cases to uncover broader patterns and implications. For this research, the instrumental design serves as a powerful tool to illuminate and understand the targeted how novice and experienced teachers teach linear functions to Grade 9 learners.

Yin (2003) categorizes case studies into three distinct types: explanatory, exploratory, and descriptive. Explanatory case studies are valuable for unravelling causal links in real-life interventions deemed too intricate for conventional survey or experimental approaches. Exploratory case studies find application in scenarios characterized by a lack of clear, single outcomes, allowing for an in-depth exploration of complex situations with multiple possibilities. Descriptive case studies serve the purpose of meticulously describing an intervention or phenomenon, along with its real-life context, aiming to provide a comprehensive and detailed account of the subject under scrutiny.

For this study, the adoption of a descriptive case study design aligns with its aim. The emphasis on description is crucial as the study seeks to elucidate the characteristics of populations, specifically novice and experienced teachers, within the selected sample from a school in Gauteng. The research investigation focuses on the comparative analysis of how two novice and two experienced teachers articulate explanations while teaching Grade 9 linear functions. The chosen case centres on the mathematics department of one school in Gauteng province, offering a nuanced exploration of how explanations are utilized in the teaching of Grade 9 linear functions within this educational context. The descriptive nature of the case study seamlessly aligns with the study's objectives, providing a detailed and context-specific account of instructional practices within the selected school setting. Tellis (1997) argues that case studies are not representative, but their emphasis is on what we can learn and understand from the case. The results from this case study should not be extrapolated to reflect the teaching and learning of linear functions in the entire county. Instead, they provide insights into the teaching and learning practices of the mathematics department in one school. However, they can be used to illustrate how novice and experienced teachers' explanations compare.

3.2.1 Methods of data collection

Data collection for this study was conducted through a meticulous process of lesson observations, which involved the video recording of lessons. The research encompasses a series of detailed lesson observations, specifically concentrating on the approaches employed by both novice and experienced teachers when explaining linear functions in Grade 9. This method of data collection proves potent as it allows direct engagement with the subject of study, affording the opportunity to witness firsthand how teachers interact with both the content and the learners. The video recordings serve as a valuable resource, enabling a comprehensive exploration of the comparative nuances in explanations provided by novice and experienced teachers. However, it is acknowledged that one potential drawback of observational methods is the likelihood that teachers may alter their behaviour due to the awareness of being observed. This inherent limitation underscores the need for a thoughtful interpretation of observed behaviours within the broader context of the study. The overall methodological approach for the study is qualitative descriptive statistics were used to summarise the data.

3.2.2 Selection of participants

The selected participant pool for this study were drawn from a school in Gauteng, chosen for its convenience as it aligns with the researcher's workplace. This choice is motivated by the enhanced accessibility and convenience offered by the familiarity with both learners and staff, facilitating a smoother study process due to the participants' willingness to engage. The participants specifically constitute the entire mathematics department of the chosen school, with the researcher maintaining a non-participant role throughout the study. The decision to focus on a school in Gauteng is driven by considerations of accessibility and convenience, streamlining the research process for the investigator. Within the identified school, the sample comprises four teachers, consisting of two novice teachers and two experienced or expert teachers. For the purposes of this study, a novice teacher is defined as someone with five or less years of teaching experience, whereas an experienced or expert teacher is characterized as having more than five years of teaching experience or more. The demographic details of the participants are concisely outlined in Table 3.1 below:

Table 3.1 Participant demographic information

Participant	Age	Gender	Experience as a teacher	Experience teaching grade 9 mathematics
Teacher A	25	Male	1	1
Teacher B	28	Male	5	5
Teacher C	43	Male	14	10
Teacher D	52	Female	19	19

3.3 Method of analysis

This research employed Hargie's (2006) classifications of explanations as a framework to scrutinize the comparative aspects of explanations provided by novice and experienced teachers in the instruction of linear functions in Grade 9. The data collection involved lessons from four different teachers, each responsible for planning their own sessions on Grade 9 linear functions. I personally observed all the lessons conducted by these teachers, with each one delivering a single lesson. In addition to direct observation, the lessons were recorded on video, facilitating thorough post-collection analysis. A total of four lessons were observed, recorded, and subsequently transcribed. The study focused on observing one lesson per teacher to ensure an in-depth, consistent analysis of their teaching strategies in a specific context. Furthermore, practical constraints, such as time and availability, limited multiple observations. However, the chosen lesson reflected the teachers' broader practices. When I used Hargie's (2006) as my analytical framework for this study, I found that Hargie (2006) proposed three main categories of explanations. However, through engagement and the dialogic process during the analysis, each category revealed subcategories. The following table discusses the descriptors (recognition rules) for this study's analytical framework. These descriptors were meticulously developed through an extensive exploration of the literature pertaining to Hargie's (2006) types of explanation:

Table 3.2 Analytical framework, taken from Hargie (2006).

Explanation	***definition***	***Descriptor***	***Analytic framework***	***Codes***
Interpretive	*Interpretive explanations*– “address the question, ‘What?’. They interpret or clarify an issue or specify the central meaning of a term or statement.” (p 186)	The teacher’s explanations involve clarifying concept and definitions of mathematical concepts, symbols, diagrams, and procedures using examples or analogies.	Interpretive Concept consists of clarifying concepts and providing definitions, using analogies and examples.	IC
			Interpretive Structure consists of clarifying concepts using the arrangement of symbols, and notation, diagrams.	IS
			Interpretive Procedure would consist of clarifying and or deriving procedures	IP
Descriptive	*Descriptive explanations*– "address the question, 'How?' These explanations describe processes, structure, and procedures" (p 186)	The teacher’s explanations address when and how to perform procedures, and describe mathematical concepts and procedures using step-by-step instructions, or various representations.	Descriptive Concept consist of providing the characteristics of a concept.	DC
			Descriptive Structure consists of describing the different structural representations of concepts. These explanations involve describing how symbols, notation, and diagrams are used to convey information about a concept.	DS
			Descriptive Procedures consists of providing learners with the details of a procedure. These explanations break down the steps involved in a process or procedure, highlighting the sequential order and specific actions required.	DP
Reason-giving	*Reason-giving*– "explanations address the question 'Why?' They involve reasons based on	The teacher explains concepts, outlines the underlying principles of a	Reason Giving Concept – Justifies and outlines underlying principle of a concept	RC
			Reason Giving Structure justifies the structural	RS

	principles or generalizations, motives, obligations or values." (p 186)	concept, and justifies procedures.	arrangements within a concept or procedure	
			Reason Giving Procedures – justifying procedures and outlining the rationale behind specific steps or processes.	RP

Hargie (2006) underscores the significance of explanations in educational settings, clarifying how they serve as vehicles for conveying understanding and facilitating learning. Within this overarching framework, three primary types of explanations emerge: interpretive, descriptive, and reason-giving, each with its own distinct characteristics and subcategories. First, Interpretive explanations, within this category, subcategories such as Interpretive Concept (IC), Interpretive Procedure (IP), and Interpretive Structure (IS) further refine the focus, offering insights into how educators navigate the terrain of conceptual understanding, procedural clarity, and structural organization. Second, Descriptive explanations, subcategories such as Descriptive Concept (DC), Descriptive Procedure (DP), and Descriptive Structure (DS) provide a nuanced lens through which educators can dissect the complexities of concepts, procedures, and structural elements, shedding light on the mechanics of mathematical reasoning and problem-solving. Lastly, Reason-giving explanations, Reason Giving Concept, Reason Giving Procedures, and Reason Giving Structure offer avenues for educators to clarify the rationale behind mathematical concepts, procedures, and structural arrangements. Through these explanations, educators can instil a deeper understanding of the underlying principles guiding mathematical inquiry, fostering critical thinking and analytical reasoning skills among learners.

Interpretive Explanations:

Expanding upon the foundational distinctions observed within interpretive explanations, I delve deeper into this framework to identify three distinct subcategories: Interpretive Concept (IC), Interpretive Procedure (IP), and Interpretive Structure (IS). Each subcategory offers a unique perspective on how educators convey information to learners, providing a comprehensive framework for delivering interpretive explanations tailored to different classroom contexts.

Interpretive Concept (IC):

IC explanations focus on clarifying the central meaning, characteristics, and properties of a particular concept. Educators employing IC explanations strive to deepen learners' understanding by providing clear definitions, utilizing analogies, and offering illustrative examples. For example, teaching Grade 9 learners about the concept of slope in linear functions. A teacher could explain that slope represents the rate of change between two variables. To clarify this concept, you might use the analogy of a hill. A steeper hill represents a higher slope, indicating a faster rate of change, while a gentler slope represents a lower slope, indicating a slower rate of change. The teacher could then provide examples, such as comparing the slopes of different lines on a graph to illustrate this concept. By clarifying concepts and providing relatable comparisons, IC explanations enhance learners' conceptual grasp and facilitate meaningful connections to real-world scenarios.

Interpretive Structure (IS):

IS explanations aim to convey information about a concept or procedure through various structural representations. Educators employing IS explanations leverage symbols, notation, and diagrams to visually depict the arrangement and relationships within a concept or procedure. In teaching linear functions, one could use graphs and equations to illustrate the relationship between variables. For example, a teacher might draw a graph showing a linear function, such as $y = 2x + 3$, and then represent the same function using an equation. This structure helps learners visualise how changes in one variable affect the other and understand the graphical representation of linear functions. By utilising graphical representations and symbolic notation, IS explanations facilitate learners' comprehension and promote deeper understanding of complex structures and relationships. They bridge the gap between abstract ideas and concrete representations, allowing learners to visualize and make connections between different aspects of the concept.

Interpretive Procedure (IP):

IP explanations are centred on clarifying the steps, and processes involved in a specific procedure or task. Educators utilizing IP explanations guide learners through the derivation or clarification of procedural knowledge, ensuring comprehension of the underlying principles guiding the process. When teaching grade 9 learners how to find the equation of a line given two points, a teacher can clarify the use of the two points $(x_1; y_1)$ and $(x_2; y_2)$ in the formula for calculating slope (change in y over change in x). Also, the use of one of the points and the slope to write the equation of the line in slope-intercept form, $y = mx + b$. The teacher is not necessarily breaking down the procedure into steps but clarifying the procedure of how to find the equation of a linear function systematically. Through logical reasoning and clarification, IP explanations empower learners to apply procedural knowledge effectively in diverse contexts.

Descriptive explanations:

Expanding upon the foundational distinctions observed within descriptive explanations, I delve deeper into this framework to identify three distinct subcategories: Descriptive Concept (DC), Descriptive Procedure (DP), or Descriptive Structure (DS). Each subcategory offers a unique perspective on how educators provide descriptions for concepts, procedures, and structural arrangements, enhancing learners' understanding and engagement in the teaching and learning process.

1. **Descriptive Concept**

Explanations classified as DC focus on providing learners with the details of a concept. These explanations aim to describe and provide the characteristics of a concept. They help learners understand the nature, features, and properties of a particular concept. Examples of DC explanations may include explaining key attributes or discussing relevant properties of a concept. The purpose of DC explanations is to enhance learners' conceptual understanding. For example, in teaching Grade 9 linear functions, a Descriptive Concept explanation might focus on the concept of slope. A teacher could explain that slope represents the rate of change between two variables in a linear function. The teacher would describe its key attributes, such as how a positive slope indicates an upward trend, a negative slope indicates a downward trend, and a slope of zero indicates a horizontal line. By providing these characteristics, learners develop a deeper understanding of what slope represents in the context of linear functions.

2. **Descriptive Structure**

Explanations classified as DS focus on providing details of how different representations are used to convey a concept or procedure. These explanations describe how various visual, symbolic, or diagrams are used to convey information. DS explanations enhance learners' comprehension and facilitate the understanding of the use of mathematical symbols, diagrams, charts, graphs, or other visual aids to convey complex concepts or processes. When teaching Grade 9 linear functions, a Descriptive Structure explanation could involve describing how different representations, such as graphs and equations, are used to convey the concept of linear functions. For example, a teacher could explain how a graph visually represents the relationship between variables, showing how changes in one variable affect the other. Additionally, you could describe how equations, such as $y = mx + b$, provide a symbolic representation of linear functions, with each component representing specific characteristics of the function. By understanding these different representations, learners can interpret and analyse linear functions more effectively.

3. **Descriptive Procedure**

Explanations classified as DP concentrate on providing learners with the details of a procedure. These explanations break down the steps involved in a procedure, highlighting the sequential order and specific actions required. DP explanations aim to enable learners to grasp the procedural knowledge necessary to perform a particular task or achieve a desired outcome. They may include detailed instructions, step-by-step breakdowns, or algorithmic explanations. A Descriptive Procedure explanation in teaching Grade 9 linear functions might focus on the steps involved in graphing a linear function. A teacher would break down the procedure into sequential steps, such as identifying the slope and y-intercept from the equation, plotting the y-intercept on the graph, and using the slope to find additional points to plot. Each step would be explained in detail, highlighting the specific actions required and the rationale behind them. By following this procedure, learners gain the procedural knowledge necessary to graph linear functions accurately and confidently.

Reason-Giving Explanations:

Expanding upon the foundational distinctions observed within reason-giving explanations, I go deeper into this framework to identify three distinct subcategories: Reason Giving Concept (RC), Reason Giving Procedure (RP), and Reason Giving Structure (RS). Each subcategory offers a unique perspective on how educators provide justifications and rationale for concepts, procedures, and structural arrangements, enhancing learners' understanding and engagement across diverse situations.

Reason Giving Concept (RC):

RC explanations focus on justifying and outlining the underlying principles of a concept. Educators employing RC explanations provide reasoned explanations for the fundamental principles and theories that govern a concept. By providing the rationale behind key concepts, RC explanations enhance learners' understanding of the foundational principles underlying mathematical concepts and foster critical thinking skills. Explaining the concept of parallel and perpendicular lines in Grade 9 linear functions, A teacher would justify the fundamental principles behind these concepts. For parallel lines, the teacher would explain that they have the same slope, emphasizing that lines with different y-intercepts can still be parallel. For perpendicular lines, you would justify that their slopes are negative reciprocals of each other, highlighting the geometric relationship between the slopes. By providing the rationale behind these principles, learners understand the foundational concepts underlying parallel and perpendicular lines.

Reason Giving Structure (RS):

RS explanations aim to justify the structural arrangements within a concept or procedure. Educators employing RS explanations provide reasoned justifications for the organizational structures, symbolic representations, and notational systems used to convey information. By clarifying the rationale behind structural arrangements, RS explanations enable learners to comprehend the underlying logic and relationships within mathematical concepts and procedures. In teaching Grade 9 linear functions, a Reason Giving Structure explanation might focus on justifying the structural arrangement of a linear function graph. A teacher would explain why the x-axis represents the independent variable (input) and the y-axis represents the dependent variable (output). Additionally, the teacher could justify why the slope of the line represents the rate of change between the variables and why the y-intercept represents the initial value of the dependent variable. By clarifying the rationale behind these structural arrangements, learners develop a deeper understanding of the logic and relationships within linear functions.

Reason Giving Procedure (RP):

RP explanations centred on justifying procedures and outlining the rationale behind specific steps or processes. Educators utilizing RP explanations provide reasoned justifications for the procedural knowledge required to perform a task or solve a problem. A Reason Giving Procedure explanation in teaching Grade 9 linear functions could involve justifying the procedure for finding the slope of a line given two points. A teacher would outline the rationale behind each step of the procedure, such as subtracting corresponding coordinates to find the change in y and x, and then calculating the ratio to find the slope. By providing reasoned justifications for each step, learners understand the significance of each calculation and how it contributes to finding the slope accurately. Through logical reasoning and systematic justification, RP explanations empower learners to understand the significance of each step in a procedure and apply procedural knowledge effectively in diverse contexts.

My investigation into the subcategories of interpretive, descriptive, and reason-giving explanations potentially yields valuable insights for literature and educators seeking to enhance their instructional practices. Through an in-depth analysis of instructional strategies and educational literature, I have identified distinct subcategories within each explanation type, shedding light on the nuanced ways in which educators convey mathematical concepts to learners. This highlights the importance of integrating diverse subcategories of explanations in teaching and learning. By embracing a multifaceted approach to explanations, educators can create dynamic learning environments that cater to the diverse cognitive needs of learners, foster deeper conceptual understanding, and promote meaningful engagement with mathematical concepts.

3.4 Data Coding and Analysis

3.4.1 Transcripts

The data for this study was obtained through lesson observations. I video recorded each of the four lessons and later transcribed each teacher's lesson. The transcripts are a record of what both teacher and learners were saying throughout the lesson. I further used brackets to indicate the teachers' gestures, and to give a description of the teacher's actions, for instance, walking around, and writing on the board. I took a snapshot of what the teacher was writing on the board and included it in the transcript.

3.4.2 Unit of analysis

The unit of analysis for this study is constituted by what the teacher is explaining. This study draws its attention to one explanation from beginning to end, focusing on what the teacher is doing. Therefore, the researcher has segmented the transcript according to what is being explained, mainly focusing on concept development, worked examples, and assessments. In each segment, I used a coding system that classifies the explanations in three: interpretive, descriptive, and reason giving, Hargie (2006). I focused on the teachers' practices and utterances and excluded what the learners were doing or saying in the lesson in the coding because the interests of this study are focused on the teachers' explanations.

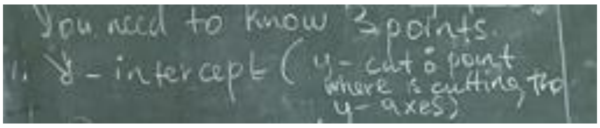
3.4.3 Coding of explanations

I started with coding each teacher's lesson transcript using the analytical framework mentioned in chapter 3. Each of the teacher's utterances were examined using Hargie's (2006) typology of explanations: descriptive explanations, interpretive explanations, and reason-giving explanations. The code for descriptive explanations was given 'DE', interpretive explanations as 'IE', and reason-giving explanations as 'RG'. Any utterance that did not constitute as an explanation was given the code 'NE'. After categorising of the explanations using Hargie's (2006) typology, I further used this study's analytical framework to categorise each explanation using the subcategories: Under Interpretive explanations, Interpretive Concept (IC), Interpretive Structure (IS), and Interpretive Procedure (IP). The subcategories under Descriptive explanations, Descriptive Concept (DC), Descriptive Structure (DS), and Descriptive Procedure (DP). Under Reason-giving explanations, Reason-giving Concept (RC), Reason-giving Structure (RS), and Reason-giving Procedure (RP). From Hargie's (2006) typology of explanations and the analytical framework, I was able to code each teacher's utterances, below are tables with descriptions for each code below with examples from the data. The first column is the description of the code; the second column shows the data source from which the example was found, for example, the source TD/SG1/U7 represents the example taken from Teacher D's lesson, TD is a code for teacher, SG1 refers to Unit of analysis number 1 in the lesson transcript, and U7 refers to Utterance 7; The third column is the example taken from the data.

Interpretive explanations

The first explanation I codes were Interpretive explanations, these are explanations involving the nature of knowledge and definitions of mathematical concepts, symbols, diagrams, and procedures. This includes definitions, providing the characteristics of a concept, or providing learners with instances or analogies that can help them connect the concept to actual circumstances or explain how the concept relates to other topics and disciplines. These types of explanations were coded as interpretive explanations (IE). The following table is an example of the coding of and explanations as an interpretive explanation followed by the rational for the coding:

Table 3.3 An example of the coding of an interpretive explanation.

Explanation		Source	Example
Interpretive		TD/SG1/U7	y-intercept is the point where your graph cuts the y-axis, it’s the y-cut, where your graph is cutting the y-axis. 

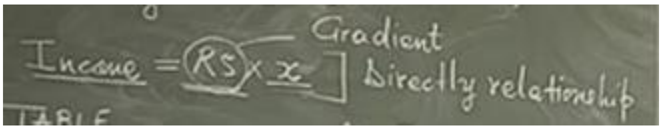
The explanation provided is an interpretive explanation. This is because it interprets or clarifies the meaning of the term "y-intercept" by specifying its central meaning or defining it in simpler terms. It addresses the question "What?" by explaining what the y-intercept represents in the context of graphing. It does this in three ways, Interpretation of the Term, Simplification for Understanding, and Contextualization within Graphing. Firstly, interpretation of the term, the explanation begins by addressing the question "What?"—it interprets or clarifies the meaning of the term "y-intercept" by specifying its significance in graphing. By stating that the y-intercept is the point where the graph intersects the y-axis, it offers a fundamental interpretation of this mathematical concept. Second, simplification for understanding, the use of language like "y-cut" serves to simplify the concept, making it more accessible for learners who may not be familiar with technical mathematical terminology. This simplification aids in breaking down the concept into more digestible components, enhancing understanding, especially for those who might find traditional mathematical terminology intimidating or challenging. Thirdly, contextualization within graphing, the interpretation is contextualized within the broader framework of graphing, which is crucial for understanding the concept's relevance and application. By explaining that the y-intercept is where the graph intersects the y-axis, learners can grasp how this concept relates to the graphical representation of mathematical functions or relationships.

The excerpt provided falls under the category of Interpretive Concept (IC). It focuses on providing learners with the details of a mathematical concept, specifically the y-intercept, by interpreting and clarifying its meaning. The explanation offers a fundamental understanding of the concept by specifying its significance in graphing, addressing the question "What?" through interpretation of the term. This classification aligns with the characteristics of Interpretive Concept explanations, which aim to provide learners with the details and characteristics of a concept to enhance their conceptual understanding. While continuing with the coding of explanations, I then followed by coding descriptive explanations.

Descriptive explanations

All explanations that address when and how to perform the procedure and provide descriptions for mathematical objects were coded as descriptive explanations (DE). These explanations concentrate on giving learners the details of a concept or procedure. Descriptive explanations include a breakdown of a procedure using step-by-step instructions, or various representations. Teachers use descriptive explanations to explain how to complete procedures using sample problems or examples. The following table is an example of the coding of and explanations as a descriptive explanation followed by the rational for the coding:

Table 3.4 An example of the coding of a descriptive explanation.

Explanation	**Source**	**Example**
Interpretive	TB/SG2/U13	We need to have the gradient, right and X intercept. Now in this case my gradient will be what will be 5, right?  The five Rands will represent the gradient, so the five rands will then show me how the graph On the Cartesian plane will increase right because obviously income and the number of go slows it needs to increase as the number of go-slows that I'm selling increases.

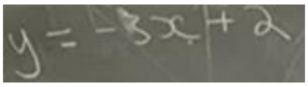
The explanation describes the relationship between the gradient (in this case, 5 Rands) and its representation on the Cartesian plane. This is done by Describing the Gradient, Representation on the Cartesian Plane, and Contextual Application. (1) Describing the Gradient: The explanation states, "Now in this case my gradient will be what will be 5, right?" This part clarifies that the gradient, which is a measure of the steepness of the graph, is 5. It specifies the numerical value of the gradient, which is essential for understanding how the graph behaves. Additionally, it explains how the gradient of 5 Rands represents the rate of increase in income as the number of go-slows sold increases. (2) Representation on the Cartesian Plane: The explanation continues, "The five Rands will represent the gradient, so the five rands will then show me how the graph on the Cartesian plane will increase..." Here, the explanation describes how the numerical value of the gradient (5 Rands) corresponds to the slope of the graph on the Cartesian plane. It indicates that each unit increase in the x-axis (e.g., each additional go-slow sold) will result in a 5 Rands increase along the y-axis (e.g., in income). This description helps visualize how changes in one variable (number of go-slows sold) affect another variable (income), illustrating the concept of gradient in action. Contextual Application: Additionally, the explanation contextualizes the relationship between the gradient (numerical value) and the Cartesian plane (graphical) within the scenario of selling go-slows. It explains that as the number of go-slows sold increases, the income needs to increase proportionally, which is reflected in the steepness of the graph.

The explanation provided falls under the category of Descriptive Concept (DC). Descriptive Concept explanations focus on providing learners with the details of a concept by describing its characteristics, features, or properties. In the given excerpt, the teacher explains the concept of gradient and its representation on the Cartesian plane. Firstly, the explanation clarifies the numerical value of the gradient, stating that in this case, it is 5 Rands. This specification of the gradient's value provides learners with a clear understanding of this concept. This aspect of the explanation aligns with Descriptive Concept because it describes the characteristic or property of the concept (gradient). Secondly, the explanation elaborates on how the gradient of 5 Rands is represented on the Cartesian plane. It explains how each unit increase in the x-axis corresponds to a 5 Rands increase along the y-axis. This description provides learners with a visual understanding of how the gradient is graphically represented. This aspect of the explanation aligns with Descriptive Structure because it describes the arrangement or representation of the concept (gradient) on the Cartesian plane. As I concluded with descriptive explanations, the next type of explanation was the reason-giving explanation.

Reason-giving explanations

When the teacher explains concepts, outlines the underlying principles, and justifies procedures, the explanation is coded as reason-giving explanations (RG). This explanation gives justifications for a specific idea or method. Reason-giving explanation involve providing justifications for why a certain formula or procedure is used or why one method is more effective than another. The following table is an example of the coding of and explanations as a reason-giving explanation followed by the rational for the coding:

Table 3.5 An example of the coding of a reason-giving explanation.

Explanation	Source	Example
Reason-giving	TA/SG6/U146-149	Looking at this function is it going to increase or decrease? Decrease? Why decrease? Because the gradient is negative, right? 

Reason-giving explanations entail offering reasons or justifications for the occurrence of events or the decision-making process. They aim to answer the question "Why?" and usually involve presenting rationales grounded in principles, generalizations, motives, obligations, or values. The given explanation provides a reason-giving rationale by explaining the anticipated decrease in the function, citing the negative gradient as the basis for this expectation. It does so in four ways, Justification Based on Principle, Clarification of Reasoning, Explanation of Rationality, and Contextual Application. (1) Justification Based on Principle: The explanation offers a reason for why the function is expected to decrease by citing the fact that "the gradient is negative." This is an appeal to a fundamental principle in mathematics: in a linear function, a negative gradient indicates a decreasing trend. By referencing this principle, the explanation provides a rationale for why the function is expected to decrease. (2) Clarification of Reasoning: The statement "Because the gradient is negative, right?" serves to clarify the reasoning behind the conclusion. It explicitly connects the negative gradient to the expected behaviour of the function, demonstrating the logical progression of thought. (3) Explanation of Rationality: By explaining the reason for the expected decrease in the function, this explanation helps to elucidate the rationality behind the conclusion. It shows that the prediction is based on a reasoned analysis of the function's characteristics, specifically its gradient. (4) Contextual Application: The explanation is tailored to the context of analysing a function and predicting its behaviour. It utilizes mathematical principles (such as the relationship between gradient and trend) to provide a reasoned explanation within this specific context. Furthermore, it demonstrates how reasoning based on mathematical principles can be used to explain and predict the behaviour of functions.

Under Reason-giving explanations, I found the following subcategories of explanation, Reason Giving Concept (RC), Reason Giving Structure (RS), or Reason Giving Procedures (RP). The explanation provided falls under the category of Reason Giving Concept (RC). Reason Giving

Concept explanations involve justifying and outlining the underlying principles of a concept. In the excerpt, the teacher justifies the expected decrease in the function based on the negative gradient. Firstly, the explanation offers a reason for why the function is expected to decrease by citing the fact that "the gradient is negative." This justification is based on a fundamental mathematical principle: in a linear function, a negative gradient indicates a decreasing trend. This aspect of the explanation aligns with Reason Giving Concept because it outlines the underlying principle (negative gradient) that supports the conclusion.

Secondly, the statement "Because the gradient is negative, right?" serves to clarify the reasoning behind the conclusion. It explicitly connects the negative gradient to the expected behaviour of the function, demonstrating the logical progression of thought. While this aspect of the explanation does not fit squarely into the category of Reason Giving Procedures or Reason Giving Structure, it contributes to the overall justification of the concept by reinforcing the reasoning behind the conclusion. The explanation provided primarily encompasses elements of Reason Giving Concept. It justifies the expected decrease in the function based on a fundamental mathematical principle and clarifies the reasoning behind the conclusion, aligning with the characteristics of Reason-giving explanations. Following the coding of explanations, the next section outlines the trustworthiness of the study.

3.5 Trustworthiness of the study

Ensuring the trustworthiness of a study involved assessing whether its research findings accurately reflect the phenomena under investigation. Cohen et al. (2007) contend that establishing trustworthiness can pose challenges in qualitative research, primarily because researchers inevitably shape meanings during data observation and analysis (Gall, Gall & Borg, 2007). Nonetheless, Krefting (1991) proposes four strategies for bolstering the trustworthiness of qualitative studies: credibility, transferability, dependability, and conformability. Lincoln and Guba (1985) advocate for member checking as a method to enhance credibility in research. Consequently, this study underwent peer review to ensure credibility. This was achieved through a thorough evaluation by my supervisor and research colleagues in my master's supervision group. They critically assessed the study’s methodology, data collection, and analysis, ensuring that the research accurately reflected the phenomena under investigation. By providing feedback and identifying any potential biases or misinterpretations, they helped refine the study’s conclusions. This collaborative process contributed to the study's credibility by ensuring the research was rigorous, reliable, and aligned with academic standards.

To enhance transferability, Seale (1999) suggests providing a comprehensive description of the study's methods, contexts, and assumptions, enabling readers to assess the applicability of the findings to other settings. Therefore, this chapter offers a detailed exposition of the research framework and contextual factors. Seale (1999) further recommends auditing to bolster a study's conformability, achieved by offering a methodologically self-critical account of the research process. In line with this, the analysis process in this study adheres to a theoretically and analytically derived framework, minimizing subjectivity. Moreover, the data generated from this study will be archived in the Wits library for five years, ensuring transparency and accessibility for potential future scrutiny. This aligns with the principle of construct validity, which Cohen et al. (2007) assert is achieved through a thorough literature review presenting diverse perspectives on the subject matter.

This study's depth and rigor are evident in its meticulous adherence to established methodologies and principles outlined in the literature. Drawing upon insights from scholars such as Lincoln and Guba (1985), Cohen et al. (2007), and Seale (1999), the study demonstrates a commitment to credibility, construct validity, transferability, dependability, and confirmability. The study's commitment to credibility is exemplified through the rigorous peer-review process, inspired by Lincoln and Guba (1985) and Cohen et al. (2007). Three colleagues were involved in the analysis of the data, acknowledging potential discrepancies. Despite minor differences, most of them coded the data in a manner consistent with my own coding. Any discrepancies identified were openly discussed, leading to adjustments in the coding system. By subjecting the research findings to scrutiny from peers, the study ensures that interpretations and conclusions accurately reflect the data.

Additionally, the thorough literature review conducted in Chapter 2, following the guidance of Cohen et al. (2007), this study establishes construct validity by presenting a comprehensive overview of different perspectives on the subject matter. This ensures that my understanding aligns with accepted norms within the field. The study providing a detailed account of research methods, contexts, and assumptions, enhancing transferability as suggested by Seale (1999). This allows readers to assess the applicability of the research findings to other contexts. By transparently documenting the research process, the study provides readers with the necessary information to judge the relevance and generalizability of the findings. To ensure dependability and confirmability, the study employs techniques, as recommended by Seale (1999). A methodological self-critical account of the research process is provided, offering insights into the methodological decisions made and potential biases encountered. This self-reflexive approach enhances the dependability of the study by demonstrating a commitment to transparency and accountability in the research process.

Furthermore, the utilization of a theoretically and analytically derived framework to guide the analysis process, as advocated by Cohen et al. (2007), ensures that the analysis is not subjective to the researcher's opinions but is grounded in a rigorous theoretical framework. This enhances the confirmability of the research findings by minimizing researcher bias and subjectivity. The study demonstrates a commitment to triangulation and peer involvement, drawing on insights from Cohen et al. (2007). By involving three colleagues in the analysis process, the study acknowledges potential discrepancies and openly discusses adjustments, thereby enhancing the dependability and credibility of the findings. This form of triangulation adds depth to the study's analysis by incorporating multiple perspectives and ensuring that interpretations are not solely reliant on the researcher's viewpoint.

In essence, the study's depth and rigor are evident in its meticulous adherence to established methodologies and principles outlined in the literature. By integrating multiple strategies to ensure credibility, transferability, dependability, and confirmability, the study demonstrates a comprehensive approach to research that contributes to its trustworthiness and reliability.

3.6 Ethical consideration

This study meticulously adhered to all ethical protocols to safeguard the rights and well-being of its participants. Ethical clearance for conducting the study was obtained from both the University of the Witwatersrand Research Ethics Committee and the Gauteng Education Department. The necessary administrative procedures, including completing the GDE ethics application form and obtaining supporting documents, were meticulously followed. Permission to access the school was formally requested from the principal through a letter of request, and consent was obtained from all participants before their involvement in the study. Consent forms were provided to teachers, learners, and their parents, ensuring informed consent was obtained from all parties involved.

Throughout the study, no participants were subjected to any form of risk or danger, and they were reassured that they could withdraw from the study at any time without facing negative consequences. The confidentiality, anonymity, and privacy of participants were rigorously upheld, with pseudonyms used to protect their identities. Additionally, all data collected during the study will be securely stored in the Wits library archives for five years and on a password-protected laptop, with access restricted to only the researcher and their supervisor.

The comprehensive documentation, including consent forms, information sheets, and ethics certificates, has been included in the appendices for transparency and accountability. With these ethical safeguards in place, the study proceeded with integrity and respect for all participants involved.

3.7 Conclusion

Chapter 3 served as a comprehensive presentation of the methodology adopted for this study, complemented by an exposition of the conceptual framework guiding the analysis. The methodology drew upon Hargie's (2006) taxonomy of explanation types, specifically Interpretative explanations, Descriptive explanations, and Reason-giving explanations. These categories were strategically employed to dissect and understand the distinctions in how novice and experienced teachers articulate explanations concerning Grade 9 linear functions, thereby illuminating their implications for teaching and learning quality. The upcoming chapter will delve into a meticulous analysis and discussion of the ways in which teachers deploy explanations and the consequential effects on the quality of teaching and learning within their instructional contexts. Through this exploration, a nuanced understanding of the interplay between explanation types, teaching practices, and student learning outcomes will be uncovered, shedding light on effective pedagogical strategies and areas for potential improvement.

CHAPTER 4: ANALYSIS AND DISCUSSION

In the preceding chapter, a thorough discussion was presented regarding the research design, methodology, sample selection, and sampling techniques. The study's objective was to examine the disparities between novice and expert teacher explanations in the instruction of Grade 9 linear functions at a school situated in the Johannesburg North district of the Gauteng Province. The research questions guiding this investigation were as follows: How do explanations provided by novice teachers compare to those delivered by experienced educators? What varieties of explanations do teachers employ when elucidating linear function concepts to Grade 9 students? How do teachers structure their explanations concerning Grade 9 linear functions?

This chapter conducts an in-depth analysis of how both novice and expert teachers convey concepts pertaining to linear functions to Grade 9 learners. The primary approach to data collection for this analysis involved observing lessons conducted by educators with diverse levels of experience in teaching mathematics within the General Education and Training (GET) phase, spanning Grade 8-9. The chapter critically examines how these teachers delivered their lessons on linear functions to Grade 9 students to address the research questions. Employing the analytical framework outlined in Chapter 3, this chapter discusses the findings derived from the observations. The primary focus of the data analysis is on comparing the explanations provided by experienced and novice teachers. Through this study, an exploration is conducted into both the commonalities and discrepancies in the types of explanations offered by expert and novice teachers, with the aim of investigating how understanding of linear functions is cultivated and can be enhanced among Grade 9 learners.

4.1 Overview

This study took lesson observations as a method of data collection, it focused on how novice and experienced teachers explain linear functions in Grade 9. The method used is powerful because of its direct contact with the participants of the study, and it allowed me to be able to witness how teachers engage with the content and the learners. By observing how they explain the concept to learners, I was able to explore how novice and experienced explanations compare. The sample of participants was from a school in Gauteng, the choice was informed by how accessible and convenient it is to conduct this study for the researcher. The four teachers comprise of two novice teachers and two experienced or expert teachers. In this study, a novice teacher is a teacher with five or less years of experience, and an experienced or expert teacher is a teacher with more than five years of experience. Each of the teachers taught one lesson. The decision to base the research on only one observed lesson per teacher was made to ensure a focused, in-depth analysis of their instructional strategies within a specific context. By observing a single lesson, the study aimed to capture a detailed snapshot of how novice teachers approach the teaching of linear functions, ensuring consistency and comparability across participants. This approach minimized variability that might arise from observing different lessons under varying conditions. Additionally, practical constraints such as limited time, the availability of teachers, and school schedules made multiple observations challenging. However, the selected lesson was representative of the teachers' broader pedagogical practices, allowing for meaningful insights into their instructional methods. Nonetheless, including additional observations in future research could further enhance the richness and depth of the findings.

Conducting this study with Grade 9 teachers was informed by the fact that Grade 9 is an important grade for learners because it is a bridging grade. In grade 9, learners are introduced to the concept of the linear function. Their comprehension and understanding of linear functions and the concept of a function is built in this grade and how it is explained to them may affect how they understand the concept in other grades. Furthermore, Grade 9 is the grade where learners choose FET subjects and whether they will do pure mathematics in grade 10. Thus, affecting the number of learners doing mathematics in later grades.

4.2 An analysis of the explanation codes

In this section, I provide a detailed analysis of the explanation codes utilized in the four different teachers’ lessons on linear functions for Grade 9 learners. The aim is to scrutinize how these educators’ explained concepts to the learners, employing Hargie's (2006) typology of explanations as a framework. Each teacher's lesson transcript was meticulously reviewed, and a systematic tabulation of the explanation codes was conducted. This section outlines the methodology employed, including the construction of tables delineating the types of explanation codes utilized by each teacher and a subsequent summary of these codes. The focus is on how novice and experienced teachers compare. The findings provide insights into the prevalence of descriptive, reason-giving, and interpretive explanations in each teacher's pedagogical approach, shedding light on the varied instructional strategies employed in teaching linear functions. For each lesson transcript, I constructed a table as shown below for types of explanations codes used in the lesson, utterances that are classified under Hargie's (2006) typology of explanations. The table enabled me to count how many times each of the types of explanation were used.

Table 4.1 Explanation codes for each participant

TEACHER A		TEACHER B		TEACHER C		TEACHER D	
UTTERANCE	CODING	UTTERANCE	CODING	UTTERCANCE	CODING	UTTERANCE	CODING
9	IC	2	DS	1	DS	5	DP
10	IC	3	DS	3	DP	6	DP
11	IC	4	DS	4	DP	7	DS
12	IP	10	IC	5	DP	8	DS
13	IP	11	IS	6	DP	9	DS
14	IP	12	RC	7	DP	10	DS
16	DS	13	IC	8	DP	11	DS
26	DS	15	DC	9	DP	12	DS
28	DS	17	DC	10	DP	13	DC
29	RS	18	DC	11	DP	14	DS
39	RS	19	DS	12	DP	15	DS
41	DP	20	DS	13	DS	16	DS
46	DP	21	DS	14	DS	17	DP
48	DP	22	DS	15	DS	18	DP
54	DP	23	DS	20	DS	19	DS
57	DP	24	DS	21	DS	21	DS
59	DP	25	DS	22	DS	25	DS
62	DS	25	DS	23	DS	29	DP
65	DS	27	IC	24	RP	30	DP
75	IC	32	RC	25	DP	32	DP
79	IS	33	DS	26	DP	33	DS
80	IC	34	DS	27	DP	37	DP
87	DS	35	DS	28	DP	38	DP
88	DS	36	DS	29	DP	39	DP
90	DS	37	DS	36	DP	40	DP
95	DS	38	DS	37	IP	41	DP
96	DC	40	DP	38	DS	42	DP
98	DC	42	DP	39	IS	43	DP
99	DC	44	DP	40	DP	44	DP
100	DP	46	DS	42	DS	48	DP
105	DS	47	DP	43	DP	49	DP
127	DP	50	DP	45	DP	50	DS
130	DC	51	DP	46	DP	51	DS

132	DS	53	DP	49	DP	52	DP
133	DS	55	RP	53	DS	54	DP
134	DS	57	DS	55	DS	57	DP
135	DS	58	DS	57	DS	59	DC
149	RS	62	DP	58	DS	68	IC
153	RS	63	DP	59	DS	79	RP
168	DC	66	RP	60	DS	80	DP
172	DP	67	DS	61	DS	85	DP
177	DS	68	DS	62	RS	86	DP
183	DS	69	RS	64	RS	90	DP
		76	IP	67	DS	95	RP
		77	IC	69	DS	96	RP
		79	RS	70	DS	98	DP
		80	DS	72	DP	99	DP
		82	DS	73	DP	100	DP
		83	DS	74	DP	101	DP
		84	DS	77	DS	102	DP
		88	DS	78	DP	103	DP
		89	DS	79	DP	104	DP
		90	RS	81	DP	106	DS
		93	DS	82	DP	107	DS
		95	DS	83	DP	109	DP
		97	DS	84	DP	110	DP
		98	DP	85	DP	112	DP
		99	DP	86	DP	113	DP
		100	DS	87	DP	118	DP
		101	DC	88	DP	126	DP
		102	DS	90	DS	128	DP
		103	DS			129	DP
						134	DP
						136	DP
						137	DP
						138	DP
						143	DP
						144	DP
						151	DP
						152	DP
						154	DP

From each of the teachers' explanation code tables, I constructed a summary of explanation codes for each teacher as shown on the table below. Table 4.3 provides a concise breakdown of the percentage of different explanation types utilized by each teacher. It offers a quick overview of the instructional strategies employed by Teachers A, B, C, and D in terms of Descriptive, Reason-giving, and Interpretive explanations. This table serves as a snapshot of the distribution of explanatory approaches among the four educators, highlighting variations in emphasis and utilization across different explanation categories.

Table 4.2 Percentage of Explanation Types by each Teacher.

Teacher	**Descriptive**	**Reason-giving**	**Interpretive**
Teacher A	69.77%	9.30%	20.93%
Teacher B	70.03%	11.29%	9.68%
Teacher C	93.44%	4.92%	1.64%
Teacher D	94.37%	4.23%	1.41%

The data presented in the table above represents the results of the analysis of how four different teachers explained linear functions to Grade 9 learners. The teachers were observed during four lessons, and the number of utterances coded as explanations using Hargie's (2006) typology of explanations were recorded. The percentages of descriptive, reason-giving, and interpretive explanations for each teacher were then calculated and presented. My focus proceeded to analysing the teachers' explanations using the analytical framework developed in chapter three, which will be the focus of my analysis. The following is a summary of the subcategories of Hargie's types of explanations:

Table 4.3 Percentage of Explanation subcategories by each Teacher.

Explanation Category	**Teacher A**	**Teacher B**	**Teacher C**	**Teacher D**
Interpretive Concept (IC)	21.28%	6.45%	0.00%	1.41%
Interpretive Structure (IS)	2.33%	1.61%	1.64%	0.00%
Interpretive Procedure (IP)	6.98%	1.61%	1.64%	0.00%
Descriptive Concept (DC)	21.28%	6.45%	0.00%	2.82%
Descriptive Structure (DS)	37.21%	54.84%	37.70%	23.94%
Descriptive Procedure (DP)	20.93%	17.74%	54.10%	67.61%
Reason Giving Concept (RC)	0.00%	3.23%	0.00%	0.00%
Reason Giving Structure (RS)	9.30%	4.84%	3.28%	0.00%

In analysing the introductory lessons on grade 9 linear functions delivered by four different teachers, a comprehensive understanding emerges through examining both the frequency and percentage distribution of various explanation categories. Teacher A employed a total of 43 explanations, with a pronounced emphasis on Descriptive Structure (16), Descriptive Procedure (9), and Descriptive Concept (5), representing 37.21%, 20.93%, and 21.28% of the total explanations, respectively. Teacher B's instructional approach comprised 62 explanations, with a significant focus on Descriptive Structure (54.84%), Descriptive Procedure (17.74%), and Descriptive Concept (6.45%). Meanwhile, Teacher C's delivery encompassed 61 explanations, with Descriptive Procedure (54.10%), Descriptive Structure (37.70%), and Interpretive Concept (1.64%) being the primary focus. Lastly, Teacher D incorporated a total of 71 explanations, where Descriptive Procedure (67.61%), Descriptive Structure (23.94%), and Descriptive Concept (2.82%) predominated. While Interpretive and Reason Giving explanation categories were less prevalent across all teachers, Teacher A utilized Interpretive Concept (21.28%), and Reason Giving Structure (9.30%) more prominently compared to the other teachers. Teacher B demonstrated a slight emphasis on Reason Giving Concept (3.23%) and Reason Giving Procedure (3.23%). Teacher C showcased a minimal utilization of Interpretive Procedure (1.64%) and Reason Giving Procedure (1.64%). Lastly, Teacher D incorporated Reason Giving Procedure (4.23%) among the other categories. This variation in emphasis of explanations highlights nuanced approaches to engaging learners in the topic of linear functions. In the following section I give an in-depth analysis of both novice and experienced teachers using the analytical framework. The following section is the analysis of novice teachers and how they employ the different types of explanations in their lessons.

4.2.1 An analysis of Novice teachers.

In this analysis, we explore the varieties of explanations utilized by two novice teachers, designated as Teacher A and Teacher B, in their instruction on linear functions. The classification of these explanations aligns with Hargie's (2006) framework, which encompasses descriptive explanations, reason-giving explanations, and interpretive explanations. Upon scrutinizing the instructional approaches of Teacher A and Teacher B during their introductory sessions on Grade 9 linear functions, discernible differences emerge in the subcategories of explanations employed. Teacher A utilized a total of 43 explanations. Predominantly employed Descriptive Structure (16), Descriptive Procedure (9), and Descriptive Concept (5), comprising 37.21%, 20.93%, and 21.28% of the total explanations, respectively. Also utilized Interpretive

Concept (5) and Reason Giving Structure (4) to a notable extent. Teacher B employed a total of 62 explanations. Showed a significant focus on Descriptive Structure (34), Descriptive Procedure (11), and Descriptive Concept (4), representing 54.84%, 17.74%, and 6.45% of the total explanations, respectively. Additionally, used Reason Giving Concept (2) and Reason Giving Procedure (2) to some extent.

Both teachers heavily relied on Descriptive explanations, with Teacher B exhibiting a more pronounced emphasis on this category compared to Teacher A. Teacher A incorporated a broader range of explanation subcategories, including Interpretive and Reason Giving, while Teacher B primarily focused on Descriptive explanations. Teacher A demonstrated a more balanced approach by incorporating a variety of explanation subcategories, potentially enhancing conceptual understanding and engagement. In contrast, Teacher B's emphasis on Descriptive explanations might suggest a more procedural approach, potentially limiting conceptual depth. Both teachers utilized Descriptive Procedure as the most frequently employed explanation subcategory, indicating a focus on guiding learners through procedural knowledge.

Interpretive Explanation (IE)

From the lessons of both Teacher A and Teacher B, Interpretive explanations were employed least frequently. Teacher A incorporated 9 interpretive explanations out of 43 explanations, constituting approximately 20.93% of their explanations, while Teacher B used 6 interpretive explanations out of 62 explanations, accounting for approximately 9.68% of their explanations. In a further examination the of two novice teachers' interpretive explanations during their introductory lessons on grade 9 linear functions, notable disparities emerge in their utilization of interpretive explanations within the explanation subcategories. Teacher A employed Interpretive Concept and Interpretive Procedure to varying degrees, with Interpretive Concept accounting for 21.28% of their explanations, and Interpretive Procedure making up 6.98%. Conversely, Teacher B exhibited a more limited incorporation of interpretive elements, with Interpretive Concept comprising only 6.45% of their explanations and Interpretive Procedure totalling 1.61%. This discrepancy suggests that Teacher A placed a greater emphasis on clarifying and defining key concepts and procedures, potentially fostering a deeper conceptual understanding among students. In contrast, Teacher B's less frequent use of interpretive explanations may indicate a more procedural-oriented approach, which could potentially hinder students' comprehension of the underlying principles of linear functions. I now proceed with an in-depth analysis of how these teachers use the subcategories under interpretive explanations, starting with Interpretive Concept followed by Interpretive Structure, then Interpretive Procedure.

1. Interpretive Concept

IC explanations clarify the central meaning, characteristics, and properties of a concept through clear definitions, analogies, and illustrative examples. They aim to deepen understanding and foster connections to real-world scenarios. In the excerpt below, Teacher A is explaining the concept of using straight-line graphs to predict trends, using the analogy of starting a business and predicting profits based on different levels of capital investment.

Table 4.4 Teacher A explaining linear functions using an analogy.

Source	
TA/SG2/U9-12	**Teacher A** So usually, the straight-line graphs they give out trends neh, uhm how so? Maybe you have a business, in a business you know that you have to have capital to start a business, right? When you start a business, you must know how much you need and what profit you gonna get right? Yes, so it’s not only that you can also predict using a graph. Probably you have only R20 to start your business, right? And the profit its R10, which means you’ve made R30 right? Then you will probably want to predict, whether how much you’ll have if you had R100, how much profit would you make, it’s easy to use a straight-line graph you get me?

The provided explanation employs Interpretive Concept (IC) explanation to clarify the concept of a straight-line graph. Initially, Teacher A asks, "So usually, the straight-line graphs they give out trends neh, uhm how so?" This question sets the stage for exploring the broader applications of straight-line graphs beyond just trends. Next, they expand on this idea, stating, "Yes, so it's not only that you can also predict using a graph." This broadens the understanding to include predictive capabilities. To illustrate further, the speaker uses a relatable analogy: "Probably you have only R20 to start your business, right? And the profit it's R10, which means you’ve made

R30 right?" This analogy demonstrates predictive analysis, aligning with the use of straight-line graphs for forecasting. Additionally, they say, "Then you will probably wanna predict, whether how much you'll have if you had R100, how much profit would you make." This concrete example reinforces the concept's practical relevance.

Through clear explanations and relatable examples, the example effectively navigates the complexities of Interpretive Concept, offering a deeper understanding of using straight-line graphs for prediction. It demonstrates how analogies and examples can facilitate comprehension, making abstract concepts more accessible and relatable. It does not involve detailing procedural steps or structural arrangements, which are characteristic of Interpretive Procedure (IP) and Interpretive Structure (IS), respectively. Instead, the emphasis is on conceptual understanding and interpretation of mathematical terms, aligning with the criteria of Interpretive Concept (IC). Ultimately, the example showcases the power of Interpretive Concept in clarifying complex ideas in a comprehensible manner.

2. Interpretive Structure

Interpretive Structure (IS) explanation clarify concepts using symbols, notation, and diagrams. In the excerpt below, Teacher B is explaining the equation of a linear function and the concept of a gradient using income for a selling snack.

Table 4.5 Teacher B explaining linear functions using income for a selling snack.

Source	
TB/SG2/U10-12	I will be expecting income in fact. Alright, let me just rephrase that. OK, yeah, I'll be having my income, and my income depends on how many go-slows that I have sold. Right now, let me say that now x should Represent what the number of go slows Sold in a month, are we fine, X will represent the number of go slows that I would have sold. now the income equation. There it will be what? It's going to be 5 * X. Right now, it means that now there is every relationship Between the income and the number of go slows that I will be selling. Right, then, we know that now under functions and relationships. We need to have the gradient, right and X intercept. Now in this case my gradient will be what will be 5, right? The five Rands will represent the gradient, so the five rands will then show me how the

> graph On the Cartesian plane will increase right because obviously income and the number of go slows it needs to increase as the number of go-slows that I'm selling increases.

In the excerpt, Teacher B's explanation exemplifies Interpretive Structure (IS) through its meticulous clarification of the relationship between income and the quantity of "go-slows" sold, employing the arrangement of symbols, notation, and graphical representation to foster comprehension. Initially, Teacher B verbally introduces the concept, stating, "I will be expecting income... depends on how many go-slows that I have sold." This verbal explanation serves as a precursor to the subsequent structural clarification, laying the groundwork for the transition to a more formal representation. Transitioning to the mathematical realm, Teacher B introduces the variable x to represent the number of "go-slows" sold, stating, "Now x should represent what the number of go-slows Sold in a month." By assigning a symbolic representation to the quantity sold, Teacher B begins the process of structural clarification through mathematical notation. Subsequently, they formulate the income equation, stating, "It's going to be $5 \times x$," thus explicitly expressing the income as a function of the quantity sold. This step marks a pivotal moment in the explanation, as it translates the verbal concept into a structured mathematical equation, aligning with the principles of Interpretive Structure.

Furthermore, Teacher B delves into the graphical representation of the income equation, emphasizing the importance of understanding functions and relationships. They mention the necessity of determining the gradient and x-intercept, sign align an intent to visualize the concept on a Cartesian plane. This transition to graphical representation underscores the interpretive aspect of the explanation, as it utilizes diagrams to enhance understanding. Continuing, Teacher B clarifies the significance of the gradient, affirming, "Now in this case my gradient will be what will be 5." By contextualizing the gradient within the symbolic representation, Teacher B clarifies the structural arrangement within the concept, elucidating how income varies concerning the quantity of "go-slows" sold. Throughout the explanation, Teacher B maintains a focus on clarifying relationships within the concept. They emphasize the correlation between income and the number of "go-slows" sold, stressing the intricate interplay between these variables. This emphasis on relationships further solidifies the interpretive aspect of the explanation, as it aims to clarify how different elements interact within the structural framework of the concept. In essence, Teacher B's explanation embodies Interpretive Structure by meticulously clarifying the concept of income dependence on the quantity of "go-slows" sold, employing the arrangement of symbols, notation, and graphical representation to facilitate comprehension.

3. Interpretive Procedure

Interpretive Procedure (IP) explanations clarify and/or derives procedures. In the excerpt below, Teacher A is explaining how to find the dependent variable using a profit analogy.

Table 4.6 Teacher B explaining how to find the dependent variable using a profit analogy.

Source	
TB/SG1/U42-44	When we plot a linear function, how many points do I need to plot A linear function. How many points do I need to plot a linear function. given the points that are in the table? I cannot plot every point one by One. These points, all these points come from one equation. So, if I have two points on my cartesian plane, now let me show you Something. in order to draw a linear function, we only Need what two points? Two exact points? On one on our Cartesian plane. now if I Have two points here, I can join these two points and have a straight line.

In the excerpt provided, Teacher B's explanation aligns with Interpretive Process (IP) as it focuses on clarifying procedures involved in plotting a linear function rather than providing a step-by-step breakdown. The explanation begins with a series of rhetorical questions, prompting learners to consider the procedural aspect of plotting linear functions: "When we plot a linear function, how many points do I need?" This question initiates a discussion aimed at clarifying the procedure rather than detailing every step. Moreover, Teacher B emphasizes the minimal requirement of two points to plot a linear function, stating, "In order to draw a linear function, we only need what two points?" This statement reinforces the procedure involved in plotting linear functions by highlighting the essential components necessary for graphing the function. Teacher B's statement, "I cannot plot every point one by one," acknowledges a practical limitation in the plotting process while emphasizing the efficiency of the method being discussed. By stating this, Teacher B highlights the importance of identifying a simpler method that can still accurately represent the linear relationship between variables. This recognition of practical constraints aligns with the aim of the interpretive procedure (IC) explanations, which focus on clarifying procedure, thus providing learners with more practical strategies for graphing linear functions efficiently.

Moreover, Teacher B's explanation concludes with a practical demonstration of the procedure: "Now if I have two points here, I can join these two points and have a straight line." This emphasises the significance of identifying key points that capture the essence of the linear relationship. Instead of plotting every single point, which could be time-consuming and unnecessary, Teacher B emphasizes the importance of selecting key points that can effectively represent the linear function. The clarification of the procedure reinforces learners' understanding of plotting a linear function and emphasises how conceptual knowledge can be translated into procedural knowledge to achieve a specific outcome. Teacher B's explanation falls under Interpretive Process (IP) as it primarily focuses on clarifying the procedure involved in plotting a linear function rather than providing a step-by-step breakdown. Through questioning, clarification, and practical demonstration, Teacher B aids learners in understanding the procedural aspect of plotting linear functions on a Cartesian plane.

Descriptive Explanations

From the lessons of both Teacher A and Teacher B, descriptive explanations emerged as the predominant form of explanations. Specifically, out of a total of 43 utterances made by Teacher A, 30 of these, constituting approximately 69.77%, were categorized as descriptive explanations. Likewise, Teacher B, out of 62 utterances, conveyed 49, accounting for approximately 79.03%, as descriptive explanations. In the excerpt below, Teacher B is providing an explanation of the representation of linear functions using a table. Analysing the usage of descriptive explanations by novice teachers, Teacher A and Teacher B, reveals distinctive patterns in their instructional strategies.

Teacher A demonstrated a well-rounded approach to descriptive explanations. Across the Descriptive Concept (DC), Descriptive Structure (DS), and Descriptive Procedure (DP) subcategories, Teacher A exhibited a balanced distribution, indicating versatility in their instructional methods. With approximately 45.45% of their explanations falling under Descriptive Concept, Teacher A focused on providing detailed descriptions to enhance learners' understanding of concepts. Moreover, with 35.56% of their explanations categorized as Descriptive Structure, Teacher A effectively utilized various visual and graphical representations to facilitate comprehension. Additionally, Teacher A allocated 19.91% of their explanations to Descriptive Procedure, indicating a commitment to guiding learners through procedural knowledge with step-by-step instructions. This holistic approach suggests Teacher A's intention to cater to diverse learning styles and promote comprehensive understanding among students.

In contrast, Teacher B's instructional approach leaned heavily towards Descriptive Structure. With approximately 63.00% of their explanations categorized as Descriptive Structure, Teacher B relied predominantly on visual or graphical representations to convey information. While this emphasis may facilitate visualization and comprehension of complex structures or relationships, it suggests a potential limitation in providing detailed conceptual explanations. Furthermore, Teacher B allocated a comparable percentage of explanations to Descriptive Procedure (20.37%), indicating a focus on guiding learners through procedural knowledge. However, Descriptive Concept was less emphasized, with only around 10.53% of explanations falling under this category. This suggests a potential gap in providing detailed explanations of concepts and attributes, which could impact learners' depth of understanding. While both Teacher A and Teacher B effectively utilized descriptive explanations in their instructional practices, Teacher A demonstrated a more balanced and versatile approach across the Descriptive Concept, Descriptive Structure, and Descriptive Procedure subcategories. In contrast, Teacher B exhibited a stronger reliance on Descriptive Structure, potentially at the expense of providing detailed conceptual explanations. This analysis highlights the importance of considering the balance between visual representations and detailed conceptual explanations in instructional practices to promote comprehensive understanding among learners.

1. **Descriptive Concept**

Descriptive Concept (DC) explanations Provides the characteristics of a concept. In the excerpt below, Teacher A is explaining the concept of relationships between variables, specifically focusing on directly proportional and inversely proportional relationships. Discussing how the sign of the gradient indicates the nature of the relationship.

Table 4.7 Teacher B explaining the relationships between variables using proportion.

Source	
TB/SG2/U16-18	**Teacher B:** It will also increase, right? Then? What type of Relationship is shown there? When we are talking about relationships, have you ever heard of directly proportional relationship? Inversely Proportional relationship, right? have we ever heard of that? now in this case I'm going to have an increasing function, right? And it Is going to be Shown by what, it is going to be shown by the sign of My gradient.

> In this case is. Going to be positive, right? Like in this case, it's going to be positive, hence I'll be having an increasing Function, are we fine? So, meaning that now the relationship that will be shown that it's going to be a directly proportional relationship. It's going to be a directly proportional relationship. Why? Because of my positive gradient. Am I making sense?
> The one with the negative gradient is going to show what an inverse relationship, meaning that now when one quantity increases, the other quantity decreases, right? So that will be shown with Our what? with our graduate.

In this explanation, Teacher B describes the characteristics of a gradient with different signs, particularly in the context of linear functions and their relationship to directly proportional and inversely proportional relationships between variables. Teacher B states, "Now in this case, it's going to be positive, hence I'll be having an increasing function." Here, Teacher B associates a positive gradient with an increasing function, indicative of a directly proportional relationship between variables, where both quantities increase together. Furthermore, Teacher B explains the implication of a negative gradient, stating, "The one with the negative gradient is going to show what? An inverse relationship, meaning that now when one quantity increases, the other quantity decreases." This highlights how a negative gradient suggests a decreasing function, characteristic of an inversely proportional relationship, where one quantity increases as the other decreases.

While the primary focus of this explanation is on the characteristics of the relationship (DC), there are also elements of Descriptive Structure (DS). Teacher B refers to the sign of the gradient, which is a structural representation of the relationship between variables. By explaining how a positive or negative gradient symbolizes different types of relationships (direct or inverse), Teacher B describes how symbols (in this case, the gradient sign) convey important information about the concept.

Teacher B also engages in Interpretive Concept (IC) explanations by asking, "What type of relationship is shown there? Have you ever heard of directly proportional relationship? Inversely proportional relationship?" This shows an effort to clarify these concepts, helping learners understand the meaning of directly and inversely proportional relationships. Teacher

B defines these relationships by relating them to the gradient's sign, thus interpreting the concept for learners. There is also a minor element of Interpretive Procedure (IP) when Teacher B explains how to use the gradient's sign to determine the type of relationship (direct or inverse). This process of interpreting the gradient's meaning can be seen as clarifying how to apply a procedural understanding of relationships.

Overall, this excerpt is a strong example of a Descriptive Concept (DC) explanation, as Teacher B focuses primarily on describing the characteristics of a concept, the gradient, and its role in identifying proportional relationships between variables. By providing detailed descriptions of how the gradient's sign affects the relationship between quantities, Teacher B helps learners grasp the concept's key characteristics. However, elements of Descriptive Structure (DS), Interpretive Concept (IC), and a minor aspect of Interpretive Procedure (IP) are also present, making this excerpt rich with different types of explanations that deepen learner understanding.

2. **Descriptive Structure**

Descriptive Structure (DS) explanations describe different structural representations of concepts, using symbols, notation, and diagrams. In the excerpt below Teacher A is providing an explanation of the components of a straight-line graph equation, defining the gradient as the slope of the graph and the y-intercept as the point where the graph intersects the y-axis.

Table 4.8 Teacher A explaining the components of a straight-line graph equation.

Source	
TA/SG2/U15-16	**Teacher A:** This is the equation of a straight-line graph neh. Whereby m is the gradient, and c is the y-intercept. The gradient is the slope guys, the slope of the graph and the y-intercept is where the graph touches the y-axis, right?

Teacher A's explanation exemplifies Descriptive Structure (DS) as they clarify the components of a straight-line graph equation using symbols and notation, thereby enhancing learners' understanding. Firstly, Teacher A introduces the equation of a straight-line graph, stating, "This is the equation of a straight-line graph neh." By contextualizing the discussion within the framework of an equation, Teacher A establishes a structured approach to explaining the concept. They proceed to define the variables within the equation, stating, "Whereby m is the gradient, and c is the y-intercept." This clear delineation of symbols and their corresponding meanings demonstrates the use of symbols and notation to convey information, a characteristic of Descriptive Structure. Furthermore, Teacher A elaborates on the significance of these variables, stating, "The gradient is the slope guys, the slope of the graph, and the y-intercept is where the graph touches the y-axis, right?" Here, Teacher A provides a detailed explanation of the symbols within the equation, using notation to describe their roles and functions. By equating the gradient to the slope of the graph and the y-intercept to the point of intersection with the y-axis, Teacher A enhances learners' understanding of the structural representations of the concept.

Moreover, Teacher A reinforces their explanation by providing contextual examples. This application of symbols and notation in real-world scenarios demonstrates the practical relevance of the structural representations of the concept, further solidifying learners' comprehension. Through these examples, Teacher A effectively employs Descriptive Structure to clarify the concept of a straight-line graph equation, using symbols, notation, and contextual examples to convey information in a structured and comprehensible manner.

3. Descriptive Procedure

Descriptive Procedures (DP) explanations provide learners with the details of a procedure, breaking down steps involved in a process. In the excerpt below, Teacher A is guiding students through the process of substituting values into an equation to determine corresponding y-values and understand coordinate pairs. They demonstrate this process using specific values for x and the equation $y = 2x - 1$, providing step-by-step calculations and explanations.

Table 4.9 Teacher A explaining the process of substituting values into an equation.

Source	
TA/SG2/U46-59	So, for x you can take any values, I like to choose: -2; - 1; 0; 1; and 2. So those values we can substitute them in our equation to get what? The y values neh. So, I'll show you one example using our equation there, y = 2x - 1. For this value alone, it’s gonna be y = 2(-2) - 1. Is four, minus one, this will equals to negative 4 minus 1. Negative five. This will be minus five neh. Uhm, for the second one, is it fine if I write it here? Don’t get confused guys uhm, its 2(-1) – 1, this will give us?

> Negative three neh.
> Now it's time to substitute 0. So, guys this negative 2 and negative 5 they create what we call a coordinate. Yes, I'll show you what a coordinate is, we write a coordinate like this.

In this excerpt, Teacher A engages in a detailed explanation that aligns with the category of Descriptive Procedure (DP) as they guide students through the step-by-step process of substituting values into an equation and using coordinate pairs. Teacher A begins by inviting students to select specific values for x, stating, "So, for x you can take any values, I like to choose: -2; -1; 0; 1; and 2." This initial step sets the stage for the subsequent procedural explanation by allowing students to actively participate and select values for analysis. Following this, Teacher A proceeds to demonstrate the substitution process using the equation $y = 2x - 1$ as an example. They calculate the corresponding y-values for selected x-values, such as -2 and -1, and explain each step along the way. For instance, Teacher A computes the expression "$y = 2(-2) - 1$," resulting in "-4 $- 1 = -5$," providing a clear and detailed breakdown of the calculation. This meticulous approach to the process of substitution demonstrates the procedural nature of Teacher A's explanation, as they systematically guide students through each step of the procedure.

Moreover, Teacher A emphasizes the significance of the resulting pairs of x and y values by introducing the concept of coordinates. They state, "So guys this negative 2 and negative 5 they create what we call a coordinate." This statement not only reinforces the procedural understanding gained through the explanation but also connects the abstract mathematical concept of equations with concrete visual representations. By introducing the term "coordinate" and illustrating its representation, Teacher A expands students' understanding of the relationship between x and y values as points on a coordinate plane. Teacher A's explanation falls under Descriptive Procedure as they provide a detailed and systematic breakdown of the process of substituting values into an equation and understanding coordinate pairs. Through clear explanations, step-by-step calculations, and the introduction of mathematical terminology, Teacher A effectively guides students in navigating mathematical concepts related to functional relationships and coordinate geometry.

Reason-Giving Explanations

Reason-giving explanations were employed less frequently by both Teacher A and Teacher B. Teacher A utilized reason-giving explanations in a mere 4 out of 43 utterances, comprising approximately 9.30% of their explanations. Similarly, Teacher B incorporated reason-giving explanations in 7 out of 62 utterances, representing approximately 11.29% of their explanation. Analysing the utilization of reason-giving explanations by novice teachers, Teacher A and Teacher B, provides valuable insights into their instructional approaches and the emphasis placed on justifying conceptual principles, structural arrangements, and procedural choices. Teacher A's instructional strategy primarily emphasized providing reasons for structural arrangements within concepts or procedures. With 4 explanations falling under the Reason-giving Structure (RS) category, Teacher A demonstrated a concerted effort to elucidate the rationale behind organizational choices or configurations. By offering insights into the reasons for specific structural arrangements, Teacher A aimed to deepen students' understanding of how concepts and procedures are organized. However, Teacher A did not allocate any explanations to Reason-giving Concept (RC) or Reason-giving Procedure (RP), indicating a potential gap in justifying underlying principles of concepts or procedural choices.

In contrast, Teacher B's utilization of reason-giving explanations was more limited across all categories. However, Teacher B demonstrated some effort in providing reasons for structural arrangements (RS) and procedural choices (RP) with 3 and 2 explanations, respectively, the overall utilization of reason-giving explanations was relatively low. Moreover, Teacher B employed only 2 explanations categorized under Reason-giving Concept (RC), suggesting a limited focus on justifying the underlying principles of concepts. This indicates a potential missed opportunity to foster deeper understanding and critical thinking among learners by providing rationales for conceptual principles and procedural choices. Both Teacher A and Teacher B demonstrated varying degrees of emphasis on reason-giving explanations in their instructional practices. While Teacher A exhibited a stronger emphasis on providing reasons for structural arrangements, their reason giving explanations covered only one of the subcategories. On the other hand, Teacher B's utilization of reason-giving explanations covered all subcategories but was more limited overall. This analysis highlights the importance of incorporating reason-giving explanations into instructional practices to promote deeper understanding, critical thinking, and engagement among learners, emphasizing the need for a balanced approach across all categories to effectively support students' learning and development.

1. Reason-giving Concept (RC)

Reason Giving Concept (RC) explanations justify and outlines underlying principles of a concept. It provides reasons or explanations based on principles, generalizations, motives, obligations, or values and aim to elucidate the rationale behind a concept or decision. In the excerpt below, Teacher B is discussing the representation of a linear function in a tabulated format and explaining why it corresponds to a linear function. They emphasize the presence of a gradient as evidence of linearity in the relationship.

Table 4.10 Teacher B explaining the representation of a linear function in a tabulated format.

Source	
TB/SG2/U32-31	**Teacher B** so, this is now representing our relationship in a tabulated format. whereby we have your independent variables and your dependent variables, are we fine. All right. Now let's come back. This is our relationship represented in a templated format, right? Now this relationship is a linear function, right? It shows a linear function, why? because we have our gradient.

Teacher B's explanation aligns with the category of Reason-giving Concept (RC), as they provide justification and reasoning behind the classification of the relationship as a linear function. Teacher B begins by introducing the tabulated format as a representation of the relationship between independent and dependent variables, stating, "so, this is now representing our relationship in a tabulated format whereby we have your independent variables and your dependent variables." Here, Teacher B lays the groundwork for the subsequent reasoning by establishing the format used to analyse the relationship. Teacher B then asserts that the relationship is a linear function, attributing this classification to the presence of a gradient. They state, "Now this relationship is a linear function, right? It shows a linear function, why? Because we have our gradient." This statement exemplifies the Reason-giving Concept as Teacher B provides a clear rationale for their classification. By identifying the gradient as the defining characteristic of a linear function, Teacher B offers a reasoned explanation for why the relationship under consideration fits this classification.

Moreover, Teacher B's use of the term "gradient" demonstrates a deeper understanding of the mathematical principles underlying linear functions. By invoking this term, Teacher B connects the concept of linearity to its mathematical representation, further strengthening their reasoning. This aligns with the Reason-giving Concept category, as Teacher B not only classifies the relationship but also provides a deeper understanding of the underlying principles driving this classification. Teacher B's explanation falls under the Reason-giving Concept category as they justify the classification of the relationship as a linear function by highlighting the presence of a gradient. Through clear reasoning and the use of mathematical terminology, Teacher B provides a comprehensive explanation that enhances learners' understanding of the concept.

2. Reason-giving Structure (RS)

Reason Giving Structure (RS) explanation justify the structural arrangements within a concept or procedure. It focuses on providing reasons for the organization or arrangement of elements within a structure and aim to clarify the rationale behind structural choices and configurations. In the excerpt below, Teacher A is explaining how to recognize a linear function based on its equation. They emphasize that when the highest exponent of x is 1, it indicates a linear function, contrasting this with higher powers of x, which do not produce linear functions.

Table 4.11 Teacher A explaining how to recognize a linear function based on its equation.

Source	
TA/SG2/U26-29	So, guys this the standard uhm, standard equation for a straight-line graph neh. How do you see if it’s a straight-line graph? The highest power of x is? One, right? Yes, when the highest power of x is one, it means it’s a straight-line graph. Once it becomes 2, going above its no longer a?... It’s no longer a straight-line graph neh.

In this excerpt, Teacher A is engaging in Reason-giving Structure (RS) by providing justification and reasoning behind the identification of a straight-line graph based on its equation. Teacher A initiates the explanation by introducing the concept of a standard equation for a straight-line graph, stating, "So guys this the standard uhm, standard equation for a straight-line graph neh." By establishing the equation as a standard representation, Teacher A lays the foundation for further discussion on how to recognise a straight-line (linear function) graph. Teacher A then employs an interactive teaching approach by prompting students to consider how to determine if a graph is a straight line. They ask, "How do you see if it’s a straight-line graph?" This question encourages critical thinking among students and sets the stage for Teacher A to provide reasoning behind the identification criteria. Teacher A follows up by confirming the criterion for a straight-line graph, stating, "The highest power of x is?

One, right?" By focusing on the highest exponent of x in the equation, Teacher A simplifies the identification process, making it accessible to learners.

Subsequently, Teacher A offers a logical explanation for why a linear function is characterized by the highest power of x equal to one. They assert, "Yes, when the highest power of x is one, it means it's a straight-line graph." This statement serves as the reasoning behind the identification criterion, providing clarity on why this condition indicates a linear function. Furthermore, Teacher A contrasts this criterion with the scenario where the highest exponent of x is two, stating, "Once it becomes 2, going above it's no longer a straight-line graph neh." This comparison reinforces the reasoning behind the identification criterion and helps solidify students' understanding of the concept. Teacher A's explanation falls under Reason-giving Structure (RS) as they provide justification and reasoning behind the identification of a linear function graph based on its equation. Through clear explanations, interactive questioning, and logical reasoning, Teacher A enhances students' understanding of the criteria for recognizing a straight-line graph, making the concept accessible and comprehensible.

3. Reason-giving Procedure (RP)

Reason Giving Procedures (RP) explanations justify procedures or methods, outlining the rationale behind specific steps or processes. They provide reasons for why certain actions or methods are chosen over others and aim to explain the reasoning behind procedural choices and decision-making. In the excerpt below, Teacher B is explaining the concept of scaling the x-axis in a graph and choosing appropriate increments to accommodate the range of values in the dataset. They emphasize the need for a systematic approach to scaling and demonstrate how to select multiples of a specific number, such as two or five, to create a scale that accommodates all values in the dataset.

Table 4.12 Teacher B explaining the concept of scaling in the axis in a graph.

Source	
TB/SG3/U38-40	All right now, the scale that I will use on my X axis. It needs to Be a multiple of something. If I'm saying I will use multiples of two, what are multiples of two? 2,4,6,8,10,12 and so forth, right? Now since we are just looking at Even and what you call it even Numbers, I can come up with any scale, but now that scale it needs to accommodate whatever numbers that I have there on my X. What you call my x Row, which is my independent row, right? So, we need to accommodate every number. Let me just say. You use one and Then 1,2,3,4,5,6,7,8,9, I can continue. alright now here I'm looking at a gradient of five. Right, and I noticed that now this year I'm using what multiples of what of five, 5,10,15,20,25 and so forth.

In this excerpt, Teacher B's explanation aligns with the category of Reason-giving Procedure (RP) as they provide justification and reasoning behind the procedure of how to choose the correct scaling the x-axis and y-axis in a graph. Teacher B initiates the explanation by discussing the necessity of using a scale that is a multiple of a specific number, stating, "All right now, the scale that I will use on my x-axis. It needs to Be a multiple of something." This statement serves as the basis for the subsequent discussion, establishing the rationale behind the procedure of selecting a scale for the x-axis. Using the data given, Teacher B then illustrates the concept by suggesting the use of multiples of two for the x-axis scale. They explain, "If I'm saying I will use multiples of two, what are multiples of two? 2,4,6,8,10,12 and so forth, right?" This example demonstrates the application of the procedure, showing how to choose increments based on a data set. By providing this reasoning, Teacher B offers insight into the method behind selecting a scale, enhancing learners' understanding of the procedure.

Furthermore, Teacher B emphasizes the importance of accommodating every number in the dataset on the x-axis scale. They state, "So, we need to accommodate every number," underscoring the necessity of selecting a scale that includes all values from the dataset. This rationale behind the procedure ensures that students grasp the significance of choosing an appropriate scale to accurately represent the data on the graph. Moreover, Teacher B provides a specific example by discussing the y-values and choosing multiples of five for the scale on the y-axis. They state, "Here I'm looking at a gradient of five. Right, and I noticed that now this year I'm using what multiples of what of five, 5,10,15,20,25 and so forth." This example demonstrates how the chosen increments align with the dataset, reinforcing the reasoning behind the procedure of scaling the x-axis. Teacher B's explanation falls under Reason-giving Procedure (RP) as they provide justification and reasoning behind the procedure of scaling in a graph. Through clear explanations, illustrative examples, and emphasis on accommodating all values from the dataset, Teacher B enhances students' understanding of the procedure and its significance in accurately representing data on a graph.

Following the analysis of the provided excerpts of novice teacher explanations, it is pertinent to introduce a graph summarizing the types of explanations used by the novice teachers. This graph will serve to visually depict the distribution and frequency of different types of explanations employed by these teachers. By presenting this graphical representation, we can gain further insights into the predominant types of explanations utilized in their lessons. Additionally, the graph will facilitate a comparative analysis, allowing us to observe any patterns or trends in the usage of interpretive, descriptive, and reason-giving explanations among the novice teachers. This visual aid will complement the verbal explanation provided earlier and provide a comprehensive overview of the explanatory practices employed by the novice teachers.

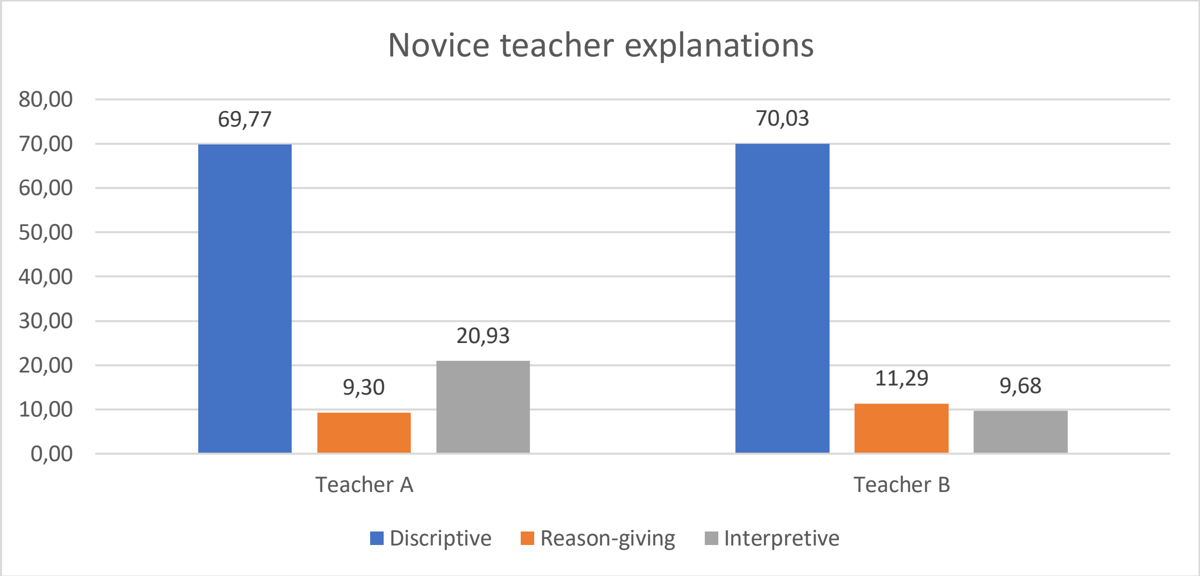


Figure 4.1 A graph summarising the types of explanations used by two novice teachers.

The results indicate that the two novice teachers used descriptive explanations most frequently, with Teacher A and B using 69.77% and 70.03% descriptive explanations in their explanations, respectively. Reason-giving explanations were used less frequently, with Teacher A at 9.30%, Teacher B at 11.29%. Interpretive explanations were used even less frequently, with Teacher A (20.93%) and Teacher B (9.68%). These results suggest that descriptive explanations are the most used type of explanation by all four teachers when teaching linear functions to Grade 9 learners. To also represent the distribution of explanation subcategories visually among Teacher A, Teacher B, I present the following graph. Which illustrates the percentages of each explanation category for each of the novice teachers. The graph allows for a comparison of their instructional strategies and emphases on different types of explanations.

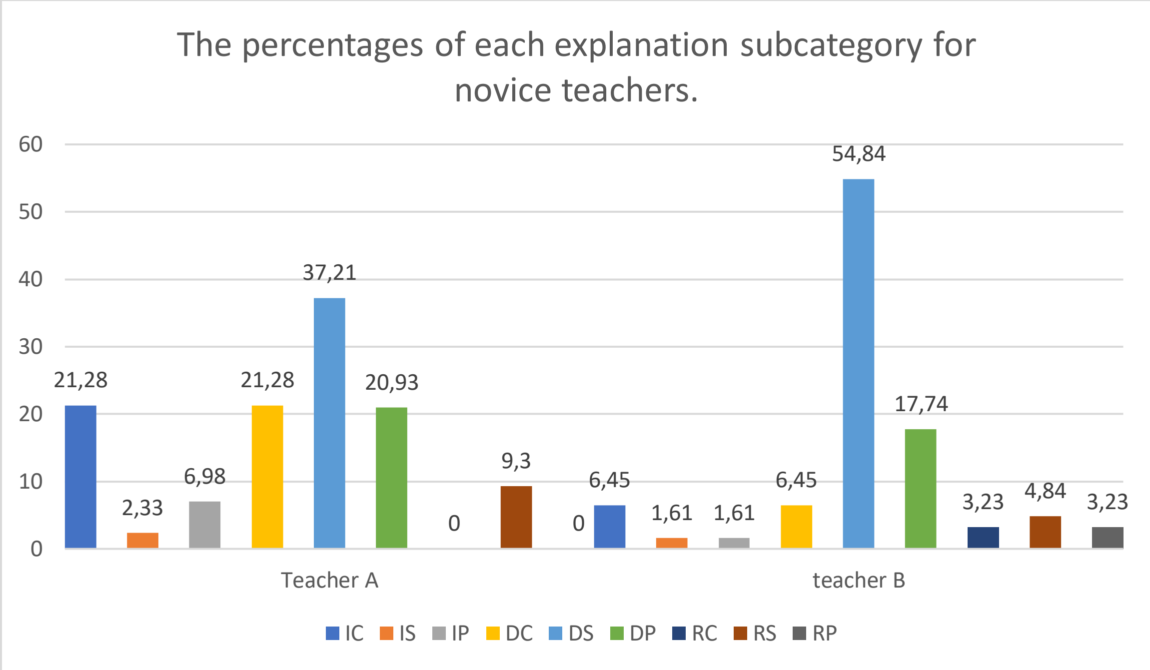


Figure 4.2 A graph summarising the types of explanations used by two novice teachers.

Apon analysing Teacher A and Teacher B, the graph shows that Teacher A uses balanced distribution of explanation categories, with a notable emphasis on Descriptive Structure and Descriptive Procedure. This indicates a focus on providing detailed descriptions, visual representations, and procedural guidance to enhance learners' understanding. Additionally, Teacher A shows a moderate utilization of Interpretive Concept and Interpretive Procedure, suggesting a commitment to clarifying conceptual understanding and procedural steps. Teacher B exhibits a different instructional strategy characterized by a strong reliance on Descriptive Structure and a considerable emphasis on Descriptive Procedure. This suggests a preference for providing visual representations and procedural guidance to facilitate learners' comprehension and application. However, Teacher B demonstrates a limited utilization of other explanation categories such as Interpretive Concept and Interpretive Procedure, potentially indicating a lesser emphasis on clarifying conceptual understanding and procedural steps compared to Teacher A. Following my analysis of novice teachers, in the next section I analyse experienced teachers’ use of explanations.

4.2.2 An analysis of Experienced teachers.

In this comparative analysis, we scrutinize lessons from two experienced teachers, denoted as Teacher C and Teacher D, with a focus on the categorization of their explanations into three distinct types: descriptive explanations, reason-giving explanations, and interpretive explanations, Hargie (2006). For Teacher C, in her introductory lesson, out of 61 explanations; 57 constitute as descriptive explanations, 3 reason-giving explanations, and 2 interpretive explanations. For Teacher D, in her introductory lesson, out of 71 explanations; 67 constitute as descriptive explanations, 3 reason-giving explanations, and 1 interpretive explanation.

Interpretive Explanations

Interpretive explanations were the least frequently employed type of explanation by both Teacher C and Teacher D. Teacher C incorporated 2 interpretive explanations out of 61 explanations, constituting approximately 1.64% of their explanations. Similarly, Teacher D employed 1 interpretive explanation out of 71 explanations (1.41%). Analysing the utilization of interpretive explanations by Teacher C and Teacher D reveals distinct instructional approaches and priorities in fostering conceptual understanding and clarity among learners. Teacher C demonstrated a moderate utilization of interpretive explanations, with a notable emphasis on Interpretive Structure and Interpretive Procedure. Approximately 1.64% of Teacher C's explanations fell under Interpretive Structure, highlighting a focus on organizing and presenting information in a structured format. This indicates a deliberate effort to facilitate comprehension and visualization of complex structures or relationships among learners. Furthermore, Teacher C allocated a similar percentage of explanations (1.64%) to Interpretive Procedure, suggesting a commitment to clarifying or deriving procedural methods. This emphasis on procedural knowledge aims to enable learners to grasp procedural knowledge necessary for performing tasks effectively. However, Teacher C did not allocate any explanations to Interpretive Concept, potentially indicating a lesser focus on clarifying the central meaning of concepts or terms compared to the emphasis on structuring information and procedural knowledge.

In contrast, Teacher D's approach to interpretive explanations differed significantly from Teacher C. Teacher D exhibited minimal utilization of interpretive explanations, with no explanations allocated to Interpretive Structure or Interpretive Procedure. While approximately 1.41% of Teacher D's explanations fell under Interpretive Concept, indicating a marginal effort to clarify the central meaning of concepts or terms, the overall emphasis on interpretive explanations was notably low. This suggests a limited focus on enhancing learners' conceptual understanding or organizing information in a structured format, potentially impacting the depth of comprehension among learners. While both Teacher C and Teacher D utilized interpretive explanations in their instructional practices, they exhibited differences in emphasis and approach. Teacher C demonstrated a moderate utilization of interpretive explanations, with a notable focus on Interpretive Procedure and Interpretive Structure, indicating a commitment to enhancing learners' conceptual understanding and comprehension. In contrast, Teacher D showed minimal utilization of interpretive explanations, with a limited focus on clarifying conceptual meanings or organizing information in a structured format. These differences underscore the importance of considering the balance between conceptual understanding and procedural knowledge in instructional practices to promote comprehensive learning experiences for students.

1. **interpretive Concept**

IC explanations clarify the central meaning, characteristics, and properties of a concept through clear definitions, analogies, and illustrative examples. In the following excerpt, Teacher D is explaining the concept of a gradient by equating it with slope and emphasizing its role in indicating the steepness or gentleness of a graph. They use the analogy of "climbing a hill" to make the concept more relatable to students.

Table 4.13 Teacher D explaining the concept of a gradient by equating it with slope.

Source	
TD/SG3/U67-68	**Teacher D** …It's the gradient, what is the gradient? The slope, the slope is telling us what? How steep the graph is neh, how steep or gentle the graph is, like climbing a hill.

In the provided excerpt, Teacher D's explanation exemplifies the concept of Interpretive Concept (IC) by employing various strategies to clarify the abstract concept of gradient in the context of climbing a hill. First, Teacher D initiates the explanation by asking a question, "What is the gradient?" This engages learners in active thinking and prompts them to recall or consider their existing knowledge of the concept, setting the stage for deeper exploration. Subsequently, Teacher D provides a clear definition of gradient, stating, "The slope, the slope is telling us what?" By equating gradient with slope, Teacher D simplifies the concept for students by using familiar terminology, thus making it more accessible and understandable. This approach aligns with the core tenet of Interpretive Concept, which emphasizes clarifying concepts through relatable explanations. Moreover, Teacher D enhances students' comprehension by employing an analogy, stating, "How steep the graph is neh, how steep or gentle the graph is, like climbing a hill." The teacher uses an analogy of "climbing a hill" to illustrate the concept of gradient, which goes beyond a mere description of the concept's characteristics. This analogy provides a tangible visualization for students, enabling them to conceptualize the abstract idea of gradient as a measure of the graph's inclination. By linking the concept to a familiar scenario, Teacher D makes it more relatable and easier for students to grasp, further exemplifying the principles of Interpretive Concept.

Furthermore, Teacher D prompts students to reflect on the practical implications of gradient by asking, "How steep or gentle the graph is neh?" This encourages critical thinking and analytical skills, as students consider the significance of gradient in interpreting graphs and understanding their visual representations. By fostering deeper reflection, Teacher D reinforces the conceptual understanding of gradient and promotes meaningful learning experiences, which are central to Interpretive Concept. Teacher D's explanation primarily focuses on explaining the underlying principle and significance of gradient in graphing linear equations. It goes beyond simply describing the characteristics of the concept and instead emphasizes understanding its practical implications and significance. The teacher's explanation of gradient in the excerpt exemplifies Interpretive Concept by employing strategies such as engaging questions, clear definitions, relatable analogies, and prompts for reflection. Through these approaches, Teacher D facilitates students' comprehension of the abstract concept of gradient, thereby promoting deeper understanding and meaningful learning experiences.

2. Interpretive Structure

Interpretive Structure (IS) explanation clarify concepts using symbols, notation, and diagrams. In the excerpt below, Teacher C is explaining the concept of intercepts using the graphing linear function. They define the x-intercept as where the graph intersects the x-axis and the y-intercept as where it intersects the y-axis, constantly pointing at the graphical representation on the board. Teacher C also emphasizes the importance of intercepts, stating that only two points, the x and y intercepts, are needed to draw a graph.

Table 4.14 Teacher C explaining the concept of intercepts using the graphing linear function.

Source	
TC/SG4/U39-40	What is the y intercept? We have to find the intercepts, but we don't know the intercepts. So, the x intercept is where? Ok, in the cartesian plane, here is the cartesian plane (pointing at the board). The x intercept is where the graph intersects with the x-axis. This is the x axis (pointing), if a graph intersects with the x-axis we call it the x intercept. if a graph intersects with the y-axis, we call it the y intercept. So, to draw a graph you only need two points. So, we are using the x and y intercepts…

In the provided excerpt, Teacher C effectively utilizes the visual aid of the Cartesian plane drawn on the board to clarify the concept of intercepts within the context of graphing linear equations, thereby aligning with the Interpretive Structure (IS) category. By physically pointing at the Cartesian plane while explaining, "Here is the Cartesian plane," Teacher C establishes a tangible reference point for learners, enhancing their understanding of the structural framework in which linear equations are graphically represented. This visual representation serves as a structural scaffold upon which the explanation unfolds, enabling students to grasp the spatial relationship between intercepts and the coordinate axes. Furthermore, Teacher C leverages the visual representation of the Cartesian plane to define the x and y-intercepts in relation to their intersections with the x-axis and y-axis, respectively. They state, "The x-intercept is where the graph intersects with the x-axis," and "If a graph intersects with the y-axis, we call it the y-intercept." By directly correlating intercepts with specific points on the graph drawn on the board, Teacher C provides learners with a concrete understanding of intercepts' structural significance within the context of linear equations.

Moreover, Teacher C's explanation culminates in a clear demonstration of how intercepts contribute to the structural composition of a linear graph. They emphasize, "To draw a graph you only need two points. So, we are using the x and y-intercepts." This statement underscores the efficiency of intercepts in representing linear equations graphically, while also reinforcing their structural importance in the overall graphing process. By integrating the visual representation of the Cartesian plane with explanations of intercepts, Teacher C enhances students' comprehension of the structural framework underlying linear equations and graphing. Teacher C's utilization of the graph on the board serves as a pivotal element in clarifying the concept of intercepts within the context of graphing linear equations, thereby exemplifying the Interpretive Structure category. Through the strategic integration of visual aids with clear explanations, Teacher C effectively reinforces the structural aspects of intercepts and their relationship with the Cartesian plane, facilitating students' understanding of linear equations' graphical representation.

3. Interpretive Procedure

Interpretive Procedure (IP) explanations clarify and/or derives procedures. In the excerpt below, Teacher C is introducing the "dual intercept method" for drawing a straight line and explains why it is called "dual intercept." They draw an analogy with a dual-SIM cell phone to reinforce the concept.

Table 4.15 Teacher C explaining the "dual intercept method".

Source	
TC/SG1/U36-38	**Teacher C** Yes. Ok, you can draw a straight line using another method. Another method we call it the dual intercept method. Why do we call it dual intercept? Because we use two intercepts. Even if a cell phone is a dual sim, what does it mean? So dual intercept, two intercept. Which is the x and the y. do you know the x intercept?

In this provided excerpt, Teacher C is engaging in an interpretive procedure (IP) by outlining a specific method, the dual intercept method, for drawing a straight-line graph. The excerpt begins with Teacher C introducing the alternative method for drawing a straight-line graph, stating, "Ok, you can draw a straight line using another method." By highlighting the availability of another method, Teacher C prompts learners to consider alternative approaches to achieve the same outcome, fostering critical thinking and exploration of different problem-solving strategies. Teacher C proceeds to explain the name "dual intercept method," stating, "Why do we call it dual intercept? Because we use two intercepts." This explanation provides insight into the methodology behind the dual intercept method, indicating that it involves utilizing two intercepts to draw the straight-line graph. By clarifying the reasoning behind the method's name, Teacher C enhances learners' understanding of the procedure and its underlying principles.

Moreover, Teacher C reinforces the concept of dual intercepts by drawing a parallel with the dual sim functionality of cell phones, stating, "Even if a cell phone is a dual sim, what does it mean? So dual intercept, two intercepts." This analogy helps learners relate the mathematical concept to a familiar real-world scenario, making it more relatable and understandable. By drawing connections between the mathematical concept and everyday experiences, Teacher C facilitates learners' comprehension and retention of the procedure. Furthermore, Teacher C prompts learners to recall specific intercepts, asking, "do you know the x intercept?" This question serves to assess learners' prior knowledge and understanding of intercepts while also reinforcing the key components of the dual intercept method. By engaging learners in active participation and prompting them to apply their knowledge, Teacher C encourages deeper learning and mastery of the interpretive procedure. Teacher C's explanation exemplifies interpretive procedure (IP) by introducing a specific method for drawing a straight-line graph, providing rationale for the method's name, drawing connections to real-world scenarios, and prompting learners' engagement and application of knowledge. Through these interpretive procedures, Teacher C enhances learners' understanding of the mathematical concept and fosters critical thinking and problem-solving skills.

Descriptive Explanations

Both Teacher C and Teacher D predominantly employed descriptive explanations during their lessons on linear functions. For Teacher C, approximately 93.44% of her explanations were categorized as descriptive explanations. Similarly, Teacher D was coded for 94.37% of explanations as descriptive explanations. Analysing the utilization of descriptive explanations by Teacher C and Teacher D offers insight into their instructional methods and priorities, allowing for a comparative understanding of their teaching strategies. Teacher C displayed a balanced approach to descriptive explanations, although with distinct emphases across the Descriptive Concept, Descriptive Structure, and Descriptive Procedure categories. With approximately 1.64% of their explanations dedicated to Descriptive Concept, Teacher C demonstrated a minimal emphasis on providing detailed descriptions of concepts. This suggests a potential gap in clarifying the nature and significance of concepts compared to other teachers. However, Teacher C exhibited a significant reliance on Descriptive Structure, allocating around 37.70% of their explanations to this category. This emphasis on visual or graphical representations indicates a strong commitment to facilitating comprehension and visualization of complex structures or relationships among learners. Additionally, Teacher C allocated approximately 54.10% of their explanations to Descriptive Procedure, highlighting a notable focus on providing step-by-step instructions and procedural guidance. This suggests an emphasis on enabling learners to understand and apply procedural knowledge effectively, potentially at the expense of detailed conceptual explanations.

On the other hand, Teacher D's approach to descriptive explanations differed significantly from Teacher C. Teacher D demonstrated a stronger reliance on Descriptive Procedure, with around 67.61% of their explanations falling under this category. This emphasis on procedural guidance suggests a primary focus on guiding learners through tasks or activities, prioritizing practical application over conceptual understanding. Moreover, Teacher D allocated approximately

23.94% of their explanations to Descriptive Structure, indicating a moderate reliance on visual or graphical representations to convey information. This suggests a commitment to facilitating learners' comprehension and visualization, although to a lesser extent compared to Teacher C. Notably, Teacher D allocated approximately 2.82% of their explanations to Descriptive Concept, indicating a minimal emphasis on providing detailed descriptions of concepts. This suggests a potential gap in clarifying the nature and significance of concepts, potentially impacting learners' depth of understanding. while both Teacher C and Teacher D utilized descriptive explanations in their instructional practices, they exhibited differences in emphasis and approach. Teacher C demonstrated a balanced approach, with a strong focus on providing visual representations and procedural guidance. In contrast, Teacher D showed a stronger emphasis on procedural guidance, potentially at the expense of detailed conceptual explanations. These differences underscore the importance of considering the balance between conceptual understanding and procedural knowledge in instructional practices to promote comprehensive learning experiences for students.

1. **Descriptive Concept**

Descriptive Concept (DC) explanations Provides the characteristics of a concept. In the excerpt below, Teacher D is explaining the concept of slope within the context of graphing. They define slope as indicating the steepness or gentleness of the graph and explain how positive slope leads to an increase while negative slope leads to a decrease. This concise explanation helps learners understand the characteristics of a slope and graph behaviour.

Table 4.16 Teacher D explaining the concept of slope within the context of graphing.

Source	
TD/SG3/U67-68	The slope, the slope of the graph. What is the slope? The slope is showing us the steepness or gentleness of the graph, the steepness how steep the graph is, how gentle or is it positive. If it’s positive, it will increase. If negative, it will decrease.

In this excerpt, Teacher D's explanation aligns with the Descriptive Concept (DC) category as they provide a clear definition and description of the concept of slope within the context of graphing. Teacher D begins by defining slope, stating, "The slope is showing us the steepness or gentleness of the graph." This definition describes the fundamental characteristic of slope, namely its role in indicating the degree of inclination or steepness of a graph. By providing this clear definition, Teacher D lays the groundwork for learners to understand the concept in a descriptive manner. Furthermore, Teacher D continues to elaborate on the concept by explaining its practical implications for graph behaviour. They state, "If it's positive, it will increase. If negative, it will decrease." This explanation highlights how the sign of the slope directly influences the direction of the graph: positive slope corresponds to an upward trend or increase, while negative slope corresponds to a downward trend or decrease. By describing these outcomes in relation to the slope's sign, Teacher D enhances learners' understanding of how slope affects the behaviour of graphs.

Moreover, Teacher D's explanation employs simple language and straightforward examples to convey the concept of slope. By using terms like "steepness" and "gentleness," Teacher D breaks down the abstract notion of slope into more tangible and understandable components. Additionally, the use of the phrases "positive" and "negative" slope further clarifies the concept by providing learners with clear markers for identifying different types of slopes in graphs. Teacher D's explanation exemplifies the Descriptive Concept category by providing a comprehensive and detailed description of the concept of slope. Through clear definitions, practical examples, and accessible language, Teacher D enhances learners' understanding of slope and its role in graphing, thereby facilitating their learning process in a descriptive manner.

2. **Descriptive Structure**

Descriptive Structure (DS) explanations describe different structural representations of concepts, using symbols, notation, and diagrams. The excerpt below shows Teacher D explaining the gradient-intercept form of a linear function, $y = mx + c$, and its components. They clarify that 'c' represents the y-intercept, where the graph intersects the y-axis.

Table 4.17 Teacher D explaining the gradient-intercept form of a linear function.

Source	
TD/SG1/U15-17	**Teacher D** Then we are having a standard form, the standard form for a line function or graph, we have what we call a standard form of the linear graph, which is $y = mx + c$, what is our standard form? $Y = mx + c$, where your c stands for the, this is your y cut, that is where your graph is going to cut, the what, the? y-axis. The c is for the y cut and your x…

In the provided excerpt, Teacher D's explanation fits within the category of Descriptive Structure (DS) as they provide a structured breakdown of the standard form of a linear graph equation, $y = mx + c$, along with its key components. Teacher D begins by introducing the concept of the standard form, stating, "Then we are having a standard form, the standard form for a line function or graph," thereby establishing the topic of discussion. This initial statement sets the framework for a structured explanation of the equation and its components. Teacher D proceeds to define each component of the equation systematically, focusing on 'c' as the y-intercept. They elucidate, "$y = mx + c$, where your c stands for the, this is your y cut, that is where your graph is going to cut, the what, the?" This statement not only identifies 'c' as the y-intercept but also provides a visual representation by describing it as the point where the graph intersects the y-axis. By offering this structured breakdown, Teacher D provides learners with a clear understanding of the role and significance of each component in the equation. Teacher D's explanation falls under the Descriptive Structure category as they provide a structured breakdown of the standard form of a linear graph equation, $y = mx + c$, along with its key components. Through clear descriptions, and the introduction of related concepts, Teacher D enhances students' understanding of the equation in a structured manner, facilitating effective learning and comprehension.

3. Descriptive Procedure

Descriptive Procedures (DP) explanations provide learners with the details of a procedure, breaking down steps involved in a process. In the excerpt below, Teacher C is explaining how to substitute values into a linear equation and calculate the corresponding output. They also explain the concept of operations with numbers of different signs, demonstrating arithmetic operations within the context of the equation.

Table 4.18 Teacher C explaining how to substitute values into a linear equation.

Source	
TC/SG1/U3-5	**Teacher C** Yeah, yeah. So, you take the x values and substitute here, and you find one. So, we have y= 3x+1. Instead of x we put negative two. So, we will have y= 3(-2) +1. Negative six plus one? Ok, here we are having two numbers with different signs. If you have two numbers? With different signs. You subtract those numbers and take and take the sign of the bigger number. The bigger number here is six, which is negative. Then six minus one is five, so it will be negative five. Then number two. OK, here the answer will be minus five. Then We have $y = 3(-1) + 1$. Three times minus one?

In this provided excerpt, Teacher C's explanation fits within the Descriptive Procedure (DP) category as they methodically guide students through the process of substituting values into a linear equation and computing the corresponding output. The teacher begins by establishing the procedure, instructing, "So, you take the x values and substitute here, and you find one." This clear directive sets the stage for a step-by-step explanation of how to carry out the task, emphasizing the importance of substituting values for 'x' in the given equation. Teacher C proceeds to demonstrate the procedure by substituting a specific value into the equation $y = 3x + 1$. They provide an example, stating, "Instead of x we put negative two. So, we will have $y = 3(-2) + 1$. Negative six plus one?" This step-by-step demonstration illustrates the application of the procedure, ensuring that students understand how to carry out the substitution effectively. By offering a concrete example, Teacher C makes the procedure more tangible and accessible to students.

Furthermore, Teacher C reinforces the procedure by explaining how to perform arithmetic operations with numbers of different signs. They state, "Here we are having two numbers with different signs. If you have two numbers? With different signs. You subtract those numbers and take and take the sign of the bigger number." This explanation provides additional clarity on how to handle calculations involving numbers of different signs, enhancing students' understanding of arithmetic operations within the context of the procedure. By incorporating multiple examples, Teacher C reinforces the procedure and demonstrates its applicability in different scenarios. This approach allows students to see how the procedure can be used in various contexts, further solidifying their understanding. Teacher C's explanation falls under the Descriptive Procedure category as they provide a systematic and detailed explanation of how to substitute values into a linear equation and compute the corresponding output. Through clear directives, step-by-step demonstrations, and explanations of arithmetic operations, Teacher C facilitates students' understanding of the procedure and enables them to apply it effectively in problem-solving situations.

Reason-Giving Explanations

Both Teacher C and Teacher D employed reason-giving explanations to a lesser extent. Teacher C used 3 reason-giving explanations out of 61 explanations, constituting approximately 4.92% of their instructional discourse. Teacher D also incorporated 3 reason-giving explanations out of 71 explanations, representing approximately 4.23% of their explanations. Comparing Teacher C and Teacher D's use of reason-giving explanations provides insights into their instructional strategies and priorities in justifying principles, motives, or decisions to learners. Experienced teachers, Teacher C and Teacher D, both demonstrate a relatively limited utilization of reason-giving explanations in their instructional practices, although with slight differences in emphasis on specific subcategories. Teacher C allocates a modest percentage of explanations to reason-giving categories, with a focus primarily on Reason Giving Structure (3.28%) and a minor allocation to Reason Giving Procedure (1.64%). This suggests a moderate effort in providing reasons for the organization or arrangement of elements within a structure and justifying procedural methods. However, there are no instances of Reason Giving Concept, indicating a lack of emphasis on providing explanations based on underlying principles or motives. While Teacher C shows some engagement with reason-giving explanations, the overall allocation remains limited compared to other explanation categories.

Teacher D also exhibits a minimal utilization of reason-giving explanations, with no instances of Reason Giving Concept or Reason Giving Structure. However, Teacher D allocates a slightly higher percentage of explanations to Reason Giving Procedure (4.23%), indicating a modest effort in justifying procedural methods or decisions. Despite this, the overall emphasis on reason-giving explanations remains limited compared to other explanation categories. Both Teacher C and Teacher D demonstrate a minimal emphasis on reason-giving explanations in their instructional approaches. While Teacher C exhibits a moderate effort in providing reasons for structural arrangements and justifying procedural methods, Teacher D shows a slightly higher engagement with justifying procedural methods. However, both educators display limited utilization of reason-giving categories overall, highlighting the importance of integrating comprehensive reasoning and justification into instructional practices to foster deeper understanding and critical thinking skills among learners.

1. **Reason Giving Concept**

The absence of instances of Reason Giving Concept in the lessons of both experienced teachers, Teacher C and Teacher D, is a notable observation that warrants an in-depth analysis. Reason Giving Concept involves providing justifications or explanations based on underlying principles, motives, obligations, or values. Its absence suggests a potential gap in the teachers' approaches to connecting abstract concepts with their underlying principles or broader theoretical frameworks. One possible explanation for the lack of Reason Giving Concept could be attributed to the nature of the subject matter or instructional objectives. If the lessons primarily focus on procedural knowledge or skill development, teachers may prioritize Descriptive or Procedural explanations over Reason Giving Concept. In such cases, educators may deemphasize theoretical justifications in favour of practical application or task completion.

Another consideration is the teachers' pedagogical strategies and instructional preferences. Some educators may feel more comfortable delivering concrete explanations or step-by-step procedures rather than delving into abstract theoretical concepts. This preference could stem from their own training, teaching experiences, or perceived effectiveness in engaging students. As a result, they may unintentionally neglect the incorporation of Reason Giving Concept in their instructional practices. Furthermore, contextual factors such as time constraints, curriculum requirements, or institutional expectations may influence the extent to which teachers incorporate Reason Giving Concept in their lessons. If educators feel pressured to cover a broad range of topics or adhere strictly to standardized assessments, they may prioritize content coverage over deeper conceptual understanding, leading to a diminished emphasis on theoretical justifications in their lessons. However, the absence of Reason Giving Concept in instructional practices is not without consequences. Without providing students with opportunities to explore the theoretical underpinnings of concepts, educators risk fostering a surface-level understanding of the subject matter. Learners may struggle to make meaningful connections between concepts, apply their knowledge in novel contexts, or engage in higher-order thinking tasks such as problem-solving or critical analysis.

To address this limitation, educators could consider incorporating more opportunities for students to explore the theoretical foundations of concepts through inquiry-based activities, discussions, or reflective tasks. By explicitly highlighting the rationale behind concepts or principles, teachers can help learner develop a deeper appreciation for the subject matter and enhance their ability to transfer their learning to diverse contexts. While the absence of Reason Giving Concept in instructional practices may reflect various pedagogical considerations or constraints, it also underscores the importance of fostering a balance between practical application and theoretical understanding in education. By incorporating more opportunities for learner to explore the underlying principles or motives behind concepts, educators can empower them to become critical thinkers and lifelong learners.

2. Reason Giving Structure

Reason Giving Structure (RS) explanation justify the structural arrangements within a concept or procedure. It focuses on providing reasons for the organization or arrangement of elements within a structure and aim to clarify the rationale behind structural choices and configurations. Teacher C explains why the sign of the gradient affects the graph's behaviour.

Table 4.19 Teacher C explaining why the sign of the gradient affects the graph's behaviour.

Source	
TC/SG5/U62-64	**Teacher C** So, my graph will be increasing. It will be the increasing function. Why? The gradient is two, and two is greater than zero. So, the graph will be? Will be an increasing function because the gradient is positive. It means the y values are increasing as the x values increase. If m is less than zero, which means it is negative. If it is negative, your graph will be a decreasing function. It will decrease, it will be decreasing. Anyways, it will go down because of the negative gradient.

In this excerpt, Teacher C's explanation falls under the Reason Giving Structure (RS) category as they provide a rationale for the relationship between the gradient of a linear function in equation form, and a graph and its behaviour. Additionally, Teacher C's explanation, the use of the words "why" and "because" highlights the reasoning behind the relationship between the gradient of a graph and its behaviour, thereby exemplifying the Reason Giving Structure (RS) category. The teacher begins by stating, "So, my graph will be increasing. It will be the increasing function. Why? The gradient is two, and two is greater than zero." Here, Teacher C lays out the reasoning behind the graph's increasing behaviour (graphical) by directly linking it to the positive value of the gradient(symbolic). This explanation forms the structural backbone of their argument, providing a logical basis for understanding the subsequent conclusions about the graph's behaviour.

Furthermore, Teacher C poses the question, "So, my graph will be increasing. It will be the increasing function. Why?" This prompts learners to consider the underlying rationale behind the graph's behaviour. Teacher C continues to reinforce their reasoning by stating, "Will be an increasing function because the gradient is positive. It means the y values are increasing as the x values increase." This statement elaborates on the rationale provided earlier, emphasizing the cause-and-effect relationship between the positive gradient and the increasing trend of the graph. By explicitly stating how the positive gradient influences the behaviour of the graph, Teacher C strengthens the structure of their argument and reinforces the logical coherence of their explanation.

Moreover, Teacher C extends their reasoning to cover the scenario of a negative gradient, stating, "If m is less than zero, which means it is negative. If it is negative, your graph will be a decreasing function." Here, Teacher C provides a clear rationale for why a negative gradient lead to a decreasing function, further enhancing the structure of their explanation. By systematically presenting the reasons behind the relationship between the gradient and the graph's behaviour, Teacher C facilitates learners' understanding of this fundamental mathematical concept. Teacher C's explanation exemplifies the Reason Giving Structure category by providing a logical rationale for the relationship between the gradient in a symbolic form to that of a graph and how the sign affects the behaviour of the graph. Teacher C's use of the words "why" and "because" effectively emphasizes the reasoning behind the relationship between the gradient of a graph and its behaviour, thereby enhancing the structural coherence of their explanation and facilitating students' comprehension of this fundamental mathematical concept.

3. Reason Giving Procedure

Reason Giving Procedures (RP) explanations justify procedures or methods, outlining the rationale behind specific steps or processes. They provide reasons for why certain actions or methods are chosen over others and aim to explain the reasoning behind procedural choices and decision-making. In this explanation, Teacher D is guiding learners through a mathematical procedure, specifically solving for a variable in an equation. They emphasize the rationale behind each step, explaining that dividing by -2 eliminates the coefficient in front of the variable, ensuring the result aligns with the desired outcome of solving for y. Additionally, Teacher D corrects a misconception about the sign of the resulting variable, reinforcing the importance of accuracy in mathematical operations.

Table 4.20 Teacher D explaining how to solve for a variable in an equation.

Source	
TD/SG1/U95-96	**Teacher D** Minus $2y$, why? Because you want to get rid of negative 2, now you want to get rid of negative 2 so that you remain with y. So, you going to do what, divide by minus 2. 2 is not correct, because if you leave that minus behind, your answer going to be minus y, and we are not looking for minus y, we are looking for y. Together, are we?

In this explanation, Teacher D engages in a Reason Giving Procedure (RP) by providing a rationale for the steps involved in solving the equation. They begin by stating, "Minus $2y$, why? Because you want to get rid of negative 2, now you want to get rid of negative 2 so that you remain with y." Here, Teacher D clarifies the reason behind isolating the variable y by emphasizing the necessity of removing the coefficient -2. By posing the question "why," Teacher D prompts learners to consider the underlying logic behind the procedure, encouraging critical thinking and understanding. Furthermore, Teacher D continues to provide reasoning for their actions by explaining the consequence of neglecting the negative sign. They assert, "2 is not correct, because if you leave that minus behind, your answer going to be minus y, and we are not looking for minus y, we are looking for y." In this statement, Teacher D underscores the importance of accuracy in mathematical operations. By pointing out the potential error and clarifying the desired outcome, they guide learners towards a correct understanding of the procedure.

Moreover, Teacher D's use of the word "because" throughout the explanation further reinforces the rationale behind each step. By explicitly linking each action to its purpose, Teacher D enhances learners' comprehension of the underlying principles governing the solution process. This approach not only facilitates understanding of the specific problem at hand but also equips learners with a deeper understanding of mathematical concepts and problem-solving strategies. Teacher D's explanation exemplifies a Reason Giving Procedure by providing a clear rationale for the steps involved in solving the equation. Through prompting learners to consider the "why" behind each action and emphasizing the importance of accuracy, Teacher D fosters a deeper understanding of mathematical procedures and encourages critical thinking among learners.

Following the explanations provided in the excerpts, it is pertinent to introduce a graph summarizing the types of explanations used by the experienced teachers. This graph will serve to visually depict the distribution and frequency of different types of explanations employed by these teachers. By presenting this graphical representation, we can gain further insights into the predominant types of explanations utilized in their lessons. Additionally, the graph will facilitate a comparative analysis, allowing us to observe any patterns or trends in the usage of interpretive, descriptive, and reason-giving explanations among the experienced teachers. This visual aid will complement the verbal explanation provided earlier and provide a comprehensive overview of the explanatory practices employed by the experienced teachers.

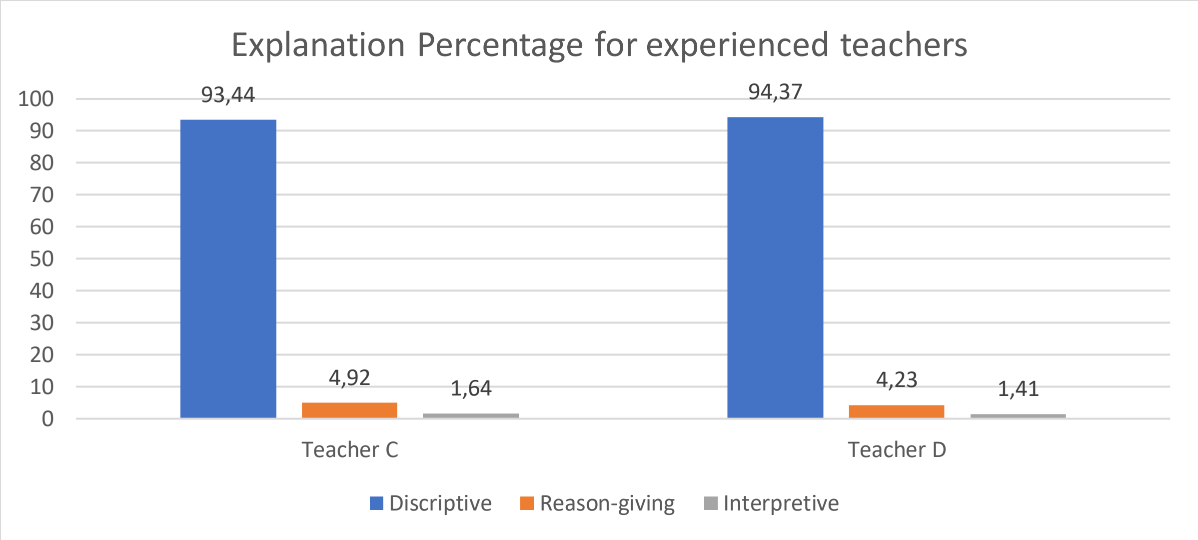


Figure 4.3 A graph summarising the types of explanations used by expert teachers.

The results indicate that the two experienced teachers used descriptive explanations most frequently, with Teacher C and D using descriptive explanations occupying 93.44%, and 94.37% of their explanations, respectively. Reason-giving explanations were used less frequently, with Teacher C at 4.92%, and Teacher D at 4.23%. Interpretive explanations were used even less frequently, with Teacher C at 1.64%), and Teacher D at 1.41%. These results suggest that descriptive explanations are the most used type of explanation by the two teachers when teaching linear functions to Grade 9 learners. To also represent the distribution of explanation subcategories visually among Teacher C and Teacher D, I present the following graph. Which illustrates the percentages of each explanation category for each of the novice teachers. The graph allows for a comparison of their instructional strategies and emphases on different types of explanations.

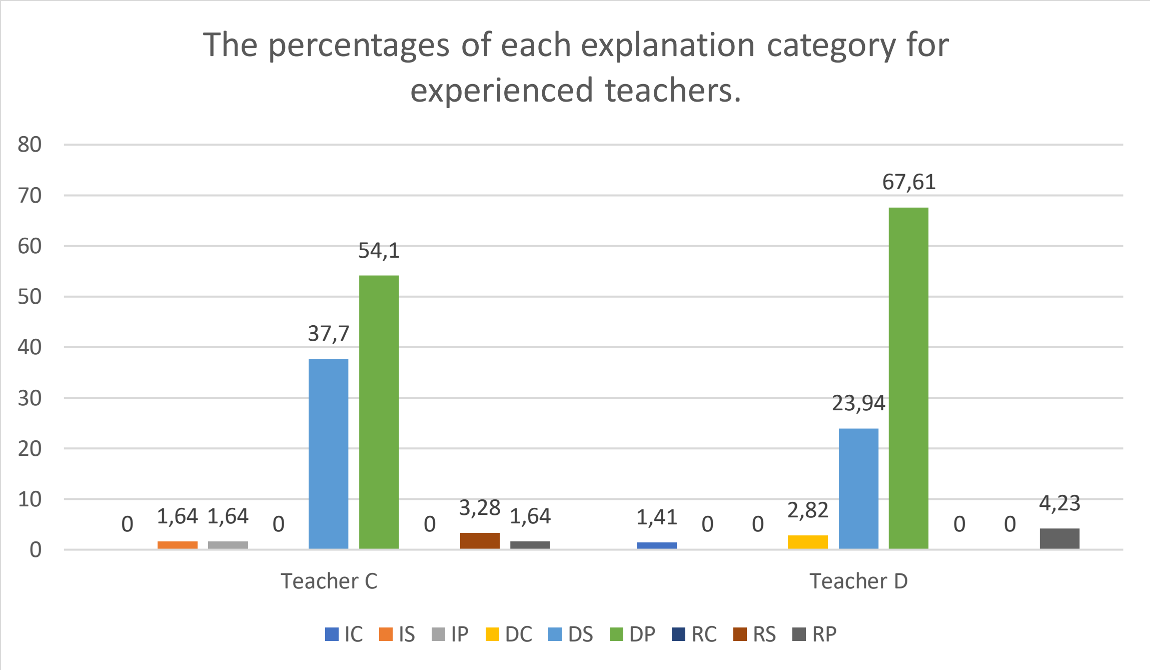


Figure 4.4 A graph summarising the explanations subcategories used by expert teachers.

In comparing the instructional practices of Teacher C and Teacher D, it's evident that both educators employ a diverse range of explanation categories to convey concepts to their learners. However, notable differences exist in the emphasis placed on certain categories, reflecting variations in pedagogical approaches and instructional priorities. Teacher C demonstrates a balanced distribution across explanation categories, with a notable emphasis on Descriptive Structure and Descriptive Procedure. This suggests a focus on providing detailed descriptions and procedural guidance to facilitate learners understanding. While there is a modest allocation to interpretive categories, such as Interpretive Concept and Interpretive Procedure, the overall emphasis appears to be on practical application and procedural knowledge. Additionally, Teacher C allocates a limited percentage to reason-giving categories, indicating a potential gap in providing justifications or explanations based on underlying principles or motives.

In contrast, Teacher D exhibits a more focused distribution, with a predominant emphasis on Descriptive Procedure. This suggests a primary focus on providing procedural guidance to support students in task completion. While there is also a significant allocation to Descriptive Structure, indicating a utilization of visual representations to convey information, there is a lesser emphasis on interpretive categories compared to Teacher C. Additionally, Teacher D demonstrates a slightly higher engagement with reason-giving explanations, particularly Reason Giving Procedure, suggesting a modest effort in justifying procedural methods or decisions. Teacher C's instructional approach appears to prioritize a balanced combination of descriptive, interpretive, and procedural explanations, with a limited emphasis on reason-giving explanations. In contrast, Teacher D's approach leans towards a more procedural focus, with a greater emphasis on descriptive explanations and a slightly higher engagement with reason-giving explanations compared to Teacher C. These findings highlight the importance of considering the balance between different explanation categories in instructional practices to foster comprehensive understanding and critical thinking skills among learners. In the next section I provide a comparison of novice and experienced teachers use of explanations.

4.2.3 A comparison of novice and experienced teachers

This section compares the explanations used by the four teachers, novice (Teachers A and B) and experienced (Teachers C and D), in teaching linear functions to grade 9 learners. Through a graph, we observe distinct patterns in the average percentage use of explanations by the two groups, shedding light on the similarities and differences between novice and experienced educators. The findings highlight the reliance on descriptive explanations by both groups, with novices exhibiting a higher frequency in reason-giving explanations. Interpretive explanations are used more sparingly, particularly by experienced teachers. This analysis sets the stage for further exploration into the interplay between teaching experience and instructional preferences.

Using the data from the four teachers’ lessons, I found the average percentage use of the explanations for novice (teacher A and B) and experience teachers (teacher C and D). The results are shown in the graphs below.

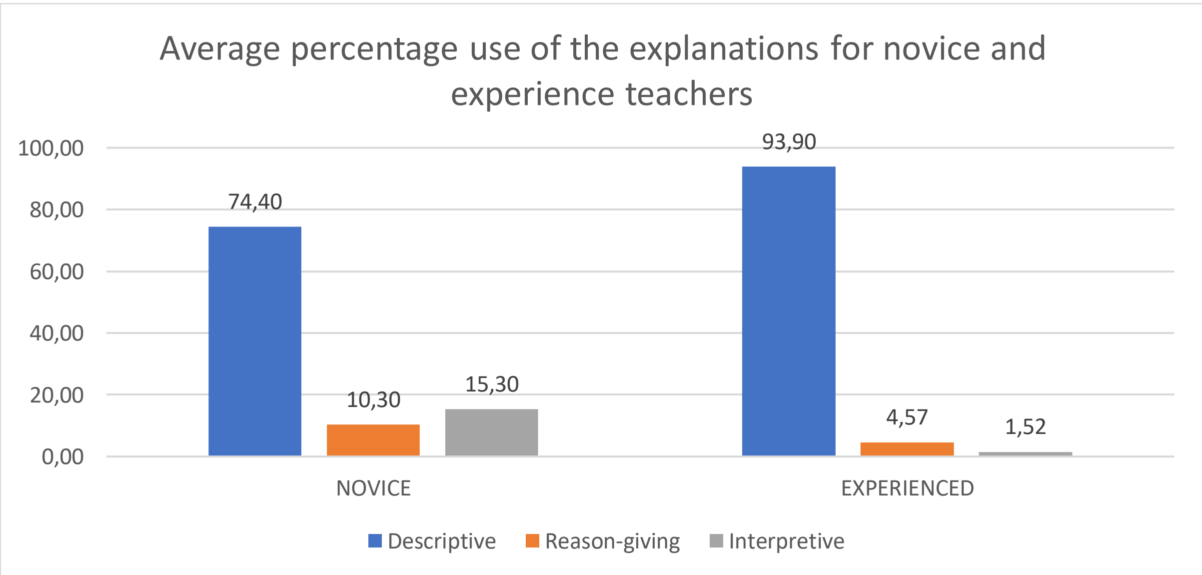


Figure 4.5 A graph showing the average percentage use of the explanations for novice and experience teachers.

The data presented in the graph provides insight into how the four different teachers explain linear functions to grade 9 learners. As previously indicated teachers A and B are classified as novice, and teachers C and D as experienced. When looking at the percentage use of the three types of explanations (descriptive, reason-giving, and interpretive), we can see that novice teachers (A and B) tend to rely less heavily on descriptive explanations compared to experienced teachers (C and D). However, both groups show a significant reliance on use of descriptive explanations.

Interpretive Explanations:

Novice teachers used interpretive explanations, but Teacher A employed them more often (20.93%) compared to Teacher B (9.68%). Experienced teachers used interpretive explanations to a significantly lesser extent. Teacher C used them in 1.64% of their explanations, while Teacher D employ 1.41% interpretive explanations in their explanations. This suggests that interpretive explanations are less common among experienced teachers. The table below goes in depth in comparing how novice and experienced teachers use these explanations. In comparing the utilization of interpretive explanation categories between novice teachers (Teacher A and Teacher B) and experienced teachers (Teacher C and Teacher D), nuanced differences in instructional strategies and emphasis become apparent.

Firstly, novice teachers, Teacher A and Teacher B, demonstrate a notable focus on interpretive procedures. Teacher A allocates 6.98% of their explanation efforts towards clarifying and deriving procedures, while Teacher B allocates 1.61%. These percentages suggest a deliberate effort by novice teachers to elucidate procedural aspects, potentially aiming to scaffold learners' understanding of complex processes through step-by-step elucidation. Conversely, experienced teachers, Teacher C and Teacher D, exhibit minimal to negligible involvement in providing interpretive procedures. This discrepancy suggests a departure from the tendency observed among novice teachers and reflects a potential shift towards alternative instructional strategies, wherein experienced educators may prioritize conceptual understanding over procedural elaboration. In terms of interpretive concepts and structures, novice teachers also demonstrate active engagement, albeit with varying degrees of frequency. Teacher A allocates 11.63% of their explanations to interpretive concepts and 2.33% to interpretive structures, while Teacher B allocates 6.45% and 1.61%, respectively. These percentages underscore novice teachers' efforts to clarify concepts and provide definitions, often supplemented by analogies and examples, alongside limited utilization of symbolic representations.

On the other hand, experienced teachers exhibit minimal involvement in interpretive concepts and structures, as evidenced by negligible percentages across these categories. This discrepancy may reflect experienced educators' reliance on alternative pedagogical approaches honed over years of teaching practice, wherein they leverage a broader repertoire of instructional techniques to facilitate learners' conceptual understanding. While novice teachers (Teacher A and Teacher B) demonstrate a proactive approach towards employing interpretive explanation categories, experienced teachers (Teacher C and Teacher D) exhibit a more reserved stance, with minimal emphasis on procedural elucidation and symbolic representation.

Analysing the teachers' interpretive explanation, it's evident that both novice and experienced teachers demonstrate varying degrees of utilization of interpretive explanation categories. While experienced teachers may not uniformly allocate percentages to all interpretive categories, their explanations often reflect a deeper understanding of concepts and reasoning, contributing to a more comprehensive learning experience for learners. However, it's essential to recognize that novice teachers also show potential in utilizing interpretive explanations, although to a lesser extent. For instance, while Teacher C did not allocate any percentage to Interpretive Concept (IC), their explanations demonstrating interpretive reasoning to justify underlying principles. Similarly, Teacher D, despite not allocating percentages to Interpretive Structure (IS) and Interpretive Procedure (IP), employs interpretive reasoning to derive procedures for graph interpretation, contributing to student understanding.

Comparatively, novice teachers may allocate percentages to interpretive categories, but their explanations may lack the depth and nuance observed in those of experienced teachers. However, this does not imply that novice teachers are incapable of utilizing interpretive explanations effectively. With further development and support, novice teachers can enhance their interpretive reasoning skills and provide more comprehensive explanations to foster student learning. Therefore, while experienced teachers may exhibit a more refined approach to interpretive explanations, both novice and experienced teachers have the potential to facilitate learner understanding through the effective use of interpretive reasoning and explanation strategies. The key lies in providing ongoing support and professional development opportunities to help teachers refine their instructional practices and enhance learning outcomes.

Descriptive Explanations:

In analysing how novice (Teacher A and Teacher B) and experienced (Teacher C and Teacher D) teachers employ descriptive explanation categories, we can draw insights from the data percentages provided in the table. The data indicates the distribution of each teacher's explanations across different descriptive categories, shedding light on their instructional strategies. Novice Teachers (A and B): 69.77% of Teacher A's explanations and 79.03% of Teacher B's explanations were categorized as descriptive explanations. This indicates that novice teachers heavily rely on descriptive explanations, with Teacher B relying on them even more prominently. Similarly, experienced teachers also made extensive use of descriptive explanations. Teacher C had 93.44% of their explanations categorized as descriptive, while Teacher D had 94.37%. This suggests that both novice and experienced teachers emphasize descriptive explanations as a primary means of conveying information about linear functions.

In comparing the utilization of descriptive explanation categories between novice teachers and experienced teachers, nuanced differences in instructional strategies and emphasis become apparent. Firstly, novice teachers demonstrate a notable emphasis on descriptive structures. Teacher A allocates 37.21% of their explanation efforts towards describing different structural representations of concepts using symbols, notation, and diagrams, while Teacher B allocates 54.84%. These percentages suggest a concerted effort by novice teachers to utilize visual aids and structural representations to convey information about concepts effectively. On average, experienced teachers exhibit relatively lower engagement in providing descriptive structure explanations. Teacher C allocates 37.70% of their explanations to descriptive structures, while Teacher D allocates 23.94%. Despite this, their percentages remain notably lower compared to those of novice teachers. This suggests that while experienced teachers acknowledge the importance of structural representations, they may employ them to a lesser extent in their instructional practices, possibly relying on alternative teaching methodologies honed through their teaching tenure.

Secondly, experienced teachers demonstrate an active involvement in descriptive procedures. Teacher C allocates a considerable 54.10% of their explanations to this category, while Teacher D allocates a substantial 67.61%. These percentages indicate a pronounced focus by experienced teachers on detailing procedural steps, suggesting a commitment to ensuring thorough understanding among learners. Conversely, novice teachers exhibit varying levels of engagement in providing descriptive procedures. Teacher A allocates 20.93% of their explanation efforts to providing learners with detailed procedural information, while Teacher B allocates 17.74%. These percentages suggest a deliberate effort by both groups of teachers to break down procedural steps, emphasizing sequential order and specific actions to facilitate learner understanding, with experienced teachers putting a larger focus on these explanations.

Lastly, novice teachers also exhibit active engagement in providing descriptive concepts but to a lesser extent compared to other descriptive explanations. Teacher A allocates 11.63% of their explanation efforts to offering characteristics of concepts, while Teacher B allocates 6.45%. These percentages imply that novice teachers aim to provide comprehensive characterizations of concepts, potentially enhancing learners' understanding by highlighting key attributes. Conversely, experienced teachers display varying levels of involvement in descriptive concepts and structures. While Teacher C allocates 0.00% of their explanations to descriptive concepts, Teacher D allocates 2.82%, indicating some degree of engagement, although minimal.

This analysis underscores distinct instructional approaches between novice and experienced teachers regarding descriptive explanations. Novice teachers demonstrate a proactive stance in utilizing both descriptive structures and concepts, potentially aiming to provide comprehensive explanations to enhance learners' understanding. In contrast, experienced teachers, while still acknowledging the importance of descriptive structures, may rely more on alternative instructional strategies. Experienced teachers rely more on Descriptive Concept and Procedure, as they provide step-by-step reasoning for solving a mathematical equation, enhancing understanding through a systematic breakdown of the procedure. Additionally, the discrepancy in the utilization of descriptive concepts between novice and experienced teachers highlights the evolving nature of instructional practices and the impact of teaching experience on pedagogical approaches.

Reason-Giving Explanations:

Both novice teachers used reason-giving explanations, but to a lesser extent. Teacher A employed reason-giving explanations in 9.30% of their explanations, while Teacher B used them in 11.29%. Interestingly, the experienced teachers also used reason-giving explanations, though at a lower rate. Teacher C utilized them in 4.92% of their explanations, while Teacher D used them in 4.23%. This indicates that both novice and experienced teachers provide justifications or reasons, but experienced teachers do so relatively less frequently**.** When comparing the utilization of reason-giving explanations between novice and experienced teachers, distinct patterns emerge from the data.

Novice teachers, on average, allocate a higher percentage to reason-giving concepts and reason-giving structure compared to their experienced counterparts. Experienced teachers, represented by Teacher C and Teacher D, do not allocate any percentage to the reason-giving concept category, indicating a lesser emphasis on providing reasons for concepts. This suggests that novice teachers compared to experienced teachers, place greater emphasis on justifying underlying principles and outlining the structural arrangements within concepts or procedures. In doing so, they aim to provide students with a clear rationale behind the theoretical foundations and organizational frameworks of the subject matter.

Additionally, Novice teachers also show some engagement in justifying the structural arrangements within concepts or procedures. Teacher A allocates 9.30% of their explanation efforts to this category, while Teacher B allocates 4.84%. This indicates a concerted effort by novice teachers to elucidate the reasoning behind structural elements. However, experienced teachers, represented by Teacher C and Teacher D, allocate 3.28% and 0.00% respectively, suggesting a relatively lower emphasis on providing reasons for structural arrangements.

Furthermore, novice teachers exhibit varying levels of engagement in justifying procedures and outlining the rationale behind specific steps or processes. Teacher A allocates 0.00% to this category, while Teacher B allocates 3.23%. In contrast, experienced teachers, represented by Teacher C and Teacher D, demonstrate more consistent involvement in this aspect. Teacher C allocates 1.64% of their explanation efforts to reason-giving procedures, while Teacher D allocates 4.23%. This suggests that experienced teachers may prioritize providing rationale for procedural steps, potentially aiming to enhance learners' understanding of the underlying logic behind various processes.

This analysis indicates that novice and experienced teachers differ in their utilization of reason-giving explanation categories. Novice teachers exhibit varying degrees of engagement in providing reasons for concepts, structures, and procedures, with some demonstrating efforts to justify these elements. On the other hand, experienced teachers generally allocate fewer percentages to reason-giving categories, suggesting a lesser emphasis on providing explicit reasoning in their instructional practices. These findings underscore the importance of understanding the nuanced approaches adopted by educators in conveying the rationale behind concepts and procedures, which can significantly impact learner comprehension and engagement.

Following the comparative analysis, I summarise the use of the explanation subcategories by novice and experienced teachers using bar graph below. The graph illustrates the distribution of explanation subcategories between novice and experienced teachers based on the average percentage allocation of each category. These explanation categories encompass various instructional approaches utilized by teachers to convey concepts and facilitate learning in the classroom. By comparing the percentages allocated to different categories, we can gain insights into how novice and experienced teachers prioritize and emphasize different aspects of teaching, ranging from interpretive concepts and descriptive structures to reason-giving procedures. This analysis provides valuable insights into the instructional practices employed by teachers at varying experience levels, shedding light on their approaches to delivering effective and comprehensive explanations in educational settings.

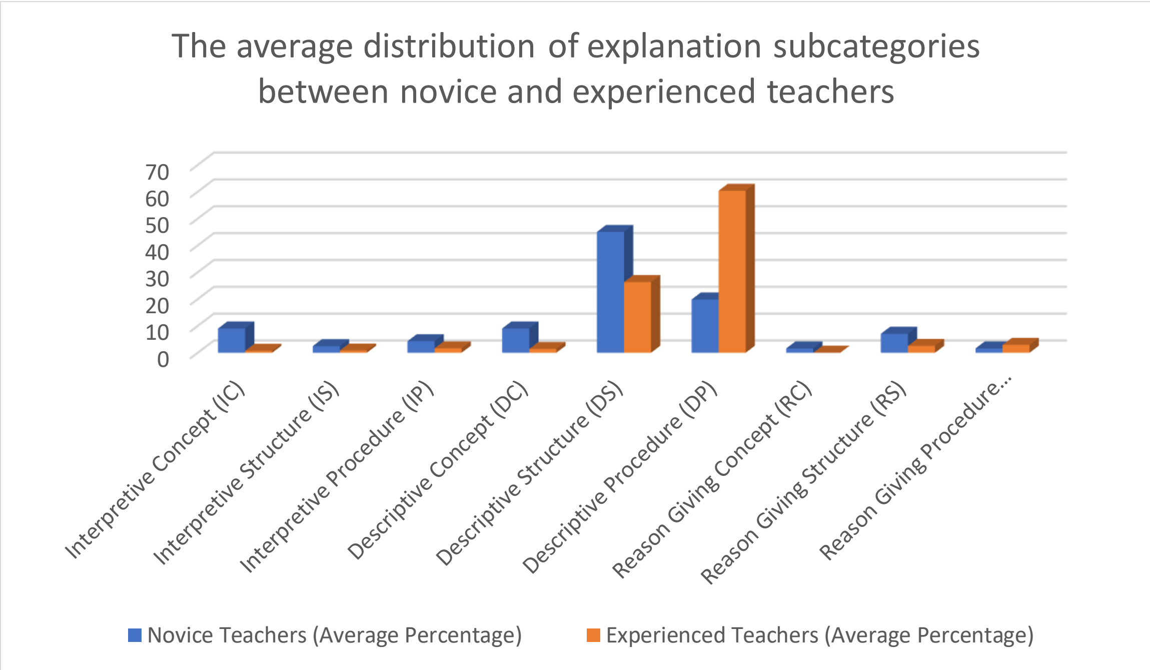


Figure 4.6 A graph showing the average percentage use of the explanation subcategories for novice and experience teachers.

The graph showcases notable disparities in the distribution of explanation categories between novice and experienced teachers. Novice teachers allocate a significantly higher percentage of their instructional time to descriptive categories, with descriptive structure and descriptive procedure comprising most of their explanation strategies. This emphasis suggests that novice teachers may prioritize providing detailed descriptions and procedural explanations to scaffold students' understanding of concepts. On the other hand, experienced teachers exhibit a more balanced distribution across explanation categories, with a notable focus on descriptive procedures. While descriptive structure remains important, experienced teachers allocate a larger proportion of their instructional time to reason-giving procedures, indicating a shift towards emphasizing the rationale behind specific steps or processes.

Furthermore, it's intriguing to observe that experienced teachers allocate minimal to no percentage to reason-giving concepts, unlike novice teachers who allocate a small percentage to this category. This discrepancy suggests that experienced teachers may rely less on explicitly justifying underlying principles or theoretical concepts in their explanations, instead emphasizing procedural knowledge and application. Conversely, novice teachers may place greater emphasis on providing comprehensive justifications and conceptual explanations to scaffold students' understanding. Overall, the data from the graph underscores varying instructional approaches between novice and experienced teachers, reflecting their differing levels of expertise and pedagogical strategies in delivering effective explanations in educational settings. The next section concludes the chapter.

The following picture depicts the framework I that I started with at the beginning of the study. The analysis process discussed in the methodology and in this chapter provides a demonstration of how the framework expanded as it went into a dialogic process with the data. And so, the framework that came as result of the analysis is also shown in the diagram.

The study's analytical framework developed from Hargie (2006)

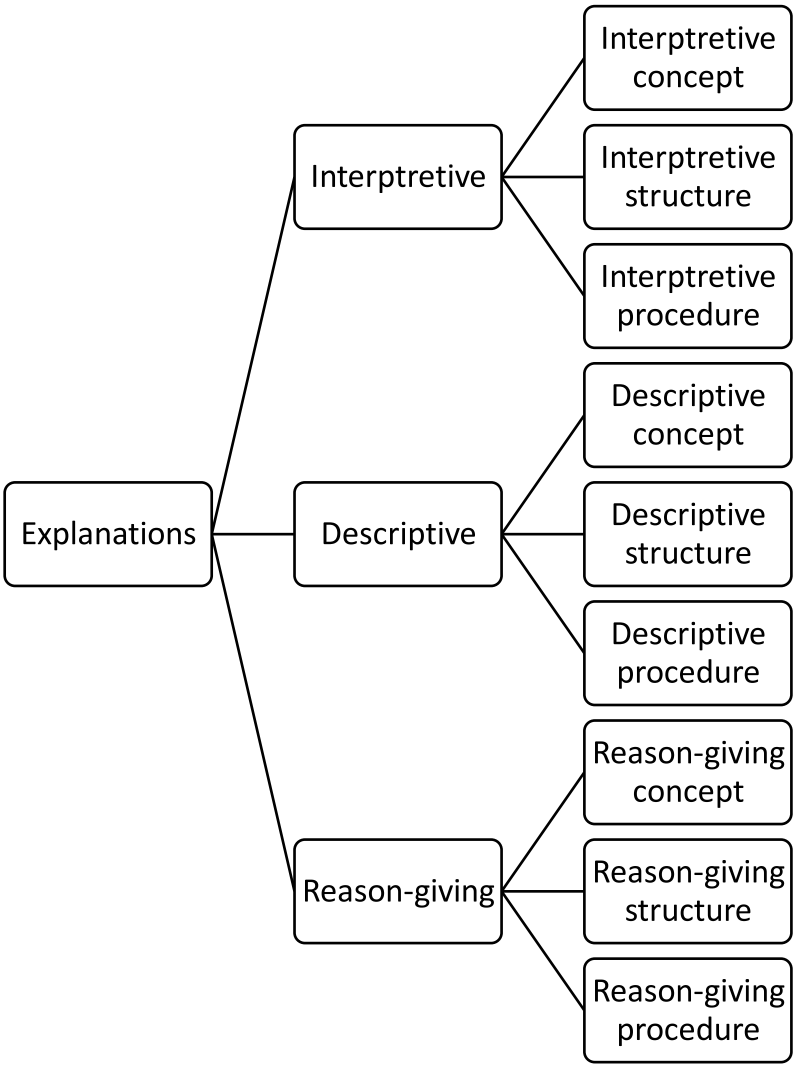


Figure 4.7: Analytical Framework of the Study Adapted from Hargie (2006)

The next section concludes the chapter.

4.5 Conclusion

In this chapter, I have presented the findings obtained from analysing how teachers convey linear functions to Grade 9 learners. Through this examination, I discussed and compared the methods used by both novice and experienced teachers when explaining linear functions. By observing and categorizing the types of explanations used by the four teachers in four separate lessons, we gained valuable insights into the instructional techniques employed by novice and experienced educators. The next chapter will provide a comprehensive summary and discussion of the findings, highlighting their implications, acknowledging the study's limitations, and suggesting potential directions for future research.

CHAPTER 5: FINDINGS AND CONCLUSIONS

The aim of this study was to examine how both novice and expert teachers articulate concepts related to linear functions to Grade 9 students. The driving force behind this inquiry was a genuine desire to comprehend the disparities between novice and expert explanations to enhance my own teaching practices, as well as to contribute insights that could benefit other educators. At the school I teach in, the teaching staff is made up of novice and experience teachers. Grade 9 linear functions serve as the foundation for functions in future grades. The topic advances as students proceed through FET, and concepts from lower grades are important for the progression of learner knowledge in the topic. It thus became necessary for me to investigate how teachers explain linear functions in Grade 9 if we are to challenge the pattern of learner performance in the country on the topic. To develop an awareness and understanding of teacher explanations in Grade 9 linear functions, I started with the following research questions:

1. What types of explanations are preferred by the two novice teachers when teaching Grade 9 linear function?
2. What types of explanations are preferred by the two experienced teachers when teaching Grade 9 linear function?
3. What are the predominant types of explanations used by teachers when delivering instruction on linear functions to Grade 9 learners?
4. How do the explanations provided by novice teachers compare to those offered by experienced teachers in the teaching of Grade 9 linear functions instruction?

To address the research questions thoroughly, I conducted an analysis of how both novice and experienced teachers explain linear functions to Grade 9 students. This analysis involved observing and categorizing the types of explanations used by four teachers across four different lessons. As the study concludes, I provide a summary of the findings aimed at addressing the research questions. Furthermore, I reflect on the limitations and implications of this study, offering recommendations based on the findings and identifying potential avenues for further research.

5.1 Summary of the Findings

In this section, I provide a comprehensive overview of our research findings. I begin by summarizing the observations regarding both novice and expert teachers, highlighting their instructional strategies and approaches. Following this, I conduct a comparative analysis between the two groups, emphasizing the differences and similarities in their teaching methodologies. Finally, I conclude this section with a synthesis of our findings, offering insights and implications for linear function instruction.

Initially, the study began with three broad types of explanations: interpretive, descriptive, and reason-giving explanations, as outlined by Hargie's typology. However, as the research progressed and the explanations offered by teachers were analysed in detail, it became evident that each of these broad categories encompassed several subcategories with distinct characteristics and roles in instructional practice. For interpretive explanations, the initial focus was on explaining the meaning and significance of mathematical concepts. However, upon closer examination, it became apparent that interpretive explanations could be further divided into interpretive concepts, interpretive structures, and interpretive procedure. Interpretive concepts involved clarifying the meaning and attributes of mathematical concepts, interpretive structures focused on elucidating the structural arrangements within mathematical procedures or representations, and Interpretive procedure focused on clarifying procedures. Similarly, descriptive explanations initially encompassed providing detailed descriptions of mathematical concepts. However, through analysis, it was discovered that descriptive explanations could be disaggregated into descriptive concepts, descriptive structures, and descriptive procedures. Descriptive concepts involved elucidating the characteristics and properties of mathematical concepts, while descriptive structures focused on providing visual or symbolic representations of mathematical relationships. Descriptive procedures, on the other hand, emphasized breaking down mathematical processes into sequential steps to facilitate understanding.

Lastly, reason-giving explanations initially centred on justifying mathematical principles or decisions. However, further examination revealed that reason-giving explanations could be categorized into reason-giving concepts, reason-giving structures, and reason-giving procedures. Reason-giving concepts involved providing justifications based on underlying principles or theoretical frameworks, while reason-giving structures focused on explaining the structural arrangements within mathematical concepts or procedures. Reason-giving procedures emphasized providing rationales for specific steps or processes involved in solving mathematical problems.

Through this iterative process of analysis and refinement, the initial three types of explanations expanded into nine subcategories, each capturing a distinct aspect of instructional practice in mathematics education. This expansion allowed for a more nuanced understanding of how teachers convey mathematical concepts and provided a comprehensive framework for analysing explanation practices in the classroom. The findings were obtained from observations and coding of four teachers' explanations during four different lessons. The data was analysed using an analytical framework adapted and developed from Hargie's (2006) typology of explanations, and the percentages of descriptive, reason-giving, and interpretive explanation subcategories were calculated for each teacher. Later, an average percentage was calculated for each group: Novice teachers (Teacher A and Teacher B) and Experienced teachers (Teacher C and Teacher D). The following tables summarize the percentages of each type of explanation for each teacher:

Table 5.1 Percentage of Explanation subcategories by each Teacher.

Explanation Category	Teacher A	Teacher B	Teacher C	Teacher D
Interpretive Concept (IC)	21.28%	6.45%	0.00%	1.41%
Interpretive Structure (IS)	2.33%	1.61%	1.64%	0.00%
Interpretive Procedure (IP)	6.98%	1.61%	1.64%	0.00%
Descriptive Concept (DC)	21.28%	6.45%	0.00%	2.82%
Descriptive Structure (DS)	37.21%	54.84%	37.70%	23.94%
Descriptive Procedure (DP)	20.93%	17.74%	54.10%	67.61%
Reason Giving Concept (RC)	0.00%	3.23%	0.00%	0.00%
Reason Giving Structure (RS)	9.30%	4.84%	3.28%	0.00%
Reason Giving Procedure (RP)	0.00%	3.23%	1.64%	4.23%

5.1.1 Findings on novice teachers

Novice teachers, represented by Teacher A and Teacher B, exhibit distinct patterns in their utilization of interpretive, descriptive, and reason-giving explanations, as revealed by the statistical analysis of their instructional practices in teaching linear functions to grade 9 learners.

Interpretive Explanations:

The analysis indicates that both Teacher A and Teacher B incorporate interpretive explanations into their instructional discourse, albeit with varying degrees of frequency. Teacher A allocates a higher percentage of their explanations to interpretive explanations compared to Teacher B. Specifically, Teacher A allocates 20.93% of their explanations to interpretive explanations, demonstrating a proactive engagement in elucidating the underlying reasoning behind concepts and procedures. In contrast, Teacher B allocates a slightly lower percentage of 9.68% to interpretive explanations, indicating a relatively lesser emphasis on providing interpretive reasoning to justify concepts and procedures. A closer examination of the data reveals that novice teachers, particularly Teacher A, demonstrate a notable focus on interpretive procedures. Teacher A allocates 6.98% of their explanation efforts towards clarifying and deriving procedures, indicating a deliberate effort to elucidate procedural aspects and scaffold learners' understanding through step-by-step elucidation. Conversely, Teacher B allocates a smaller percentage of 1.61% to interpretive procedures, suggesting a lesser emphasis on detailing procedural steps compared to Teacher A.

Furthermore, novice teachers also exhibit engagement in interpretive concepts and structures, albeit to varying extents. Teacher A allocates 11.63% of their explanations to interpretive concepts and 2.33% to interpretive structures, suggesting active efforts to clarify concepts and provide definitions, often supplemented by analogies and examples. In contrast, Teacher B allocates 6.45% to interpretive concepts and 1.61% to interpretive structures, indicating a relatively lesser emphasis on providing comprehensive characterizations of concepts and structural representations.

Descriptive Explanations:

Novice teachers, Teacher A and Teacher B, demonstrate a pronounced reliance on descriptive explanations, particularly in conveying the structural aspects and procedural steps related to linear functions. Both teachers allocate a substantial percentage of their explanation efforts to descriptive categories, with descriptive structures and procedures comprising most of their instructional strategies. Teacher A allocates 69.77% of their explanations to descriptive explanations, with a notable emphasis on descriptive structures, accounting for 37.21% of their explanation efforts. This suggests a deliberate effort by Teacher A to utilize visual aids and structural representations to convey information about concepts effectively. Similarly, Teacher B allocates 79.03% of their explanations to descriptive explanations, with an even higher emphasis on descriptive structures, accounting for 54.84% of their explanation efforts. This indicates a concerted effort by Teacher B to provide detailed descriptions and procedural breakdowns to scaffold learners' understanding.

Additionally, novice teachers demonstrate active engagement in providing descriptive procedures, aiming to detail procedural steps and enhance learners' understanding through systematic breakdowns. Teacher A allocates 20.93% of their explanation efforts to providing learners with detailed procedural information, while Teacher B allocates 17.74%. These percentages suggest a deliberate effort by both teachers to break down procedural steps, emphasizing sequential order and specific actions to facilitate learner understanding. Furthermore, novice teachers also exhibit engagement in providing descriptive concepts, aiming to offer comprehensive characterizations of concepts and highlight key attributes. Teacher A allocates 11.63% of their explanation efforts to descriptive concepts, while Teacher B allocates 6.45%. These percentages underscore novice teachers' efforts to provide learners with detailed explanations of concepts, potentially enhancing their understanding through clear and comprehensive descriptions.

Reason-Giving Explanations:

Novice teachers, represented by Teacher A and Teacher B, also utilize reason-giving explanations to justify concepts and procedures, although to a lesser extent compared to descriptive explanations. Teacher A allocates 9.30% of their explanations to reason-giving explanations, while Teacher B allocates 11.29%. This indicates that both teachers provide justifications or reasons, albeit with varying frequencies. A closer examination reveals that novice teachers allocate a higher percentage to reason-giving concepts and reason-giving structure compared to their experienced counterparts. Teacher A allocates 4.65% of their explanation efforts to reason-giving concepts and 4.65% to reason-giving structures, indicating a concerted effort to justify underlying principles and outline structural arrangements within concepts or procedures. Similarly, Teacher B allocates 6.45% to reason-giving concepts and 4.84% to reason-giving structures, suggesting active engagement in providing reasons for concepts and structural arrangements. Furthermore, novice teachers also demonstrate some engagement in justifying procedures and outlining the rationale behind specific steps or processes. Teacher A allocates 0.00% to this category, while Teacher B allocates 3.23%. This suggests a deliberate effort by both teachers to break down procedural steps and provide learners with a clear rationale behind various processes, aiming to enhance understanding through explicit reasoning.

In summary, novice teachers, Teacher A and Teacher B, demonstrate distinct patterns in their utilization of interpretive, descriptive, and reason-giving explanations. While both teachers actively engage in providing comprehensive explanations to enhance learners' understanding, they exhibit nuanced differences in the emphasis placed on different explanation categories and subcategories. These findings provide valuable insights into the instructional practices employed by novice teachers in teaching linear functions, offering implications for teacher training and curriculum development initiatives aimed at enhancing teaching effectiveness and learning outcomes in mathematics education.

5.1.2 Findings on experienced teachers

Experienced teachers, represented by Teacher C and Teacher D, exhibit distinct instructional strategies in their utilization of interpretive, descriptive, and reason-giving explanations, as evidenced by the statistical analysis of their instructional practices in teaching linear functions to grade 9 learners.

Interpretive Explanations:

The analysis reveals that experienced teachers allocate a lower percentage of their instructional time to interpretive explanations compared to novice teachers. Both Teacher C and Teacher D utilize interpretive explanations to a significantly lesser extent, focusing more on other types of explanations. Teacher C allocates only 1.64% of their explanations to interpretive explanations, while Teacher D allocates 1.41%. This indicates that experienced teachers provide justifications or reasons less frequently, opting for alternative instructional approaches.

Moreover, when examining the distribution of interpretive explanation categories, experienced teachers exhibit minimal involvement in interpretive concepts and structures. Teacher C allocates 0.00% of their explanations to interpretive concepts, while Teacher D allocates 2.82%. Similarly, Teacher C allocates 0.00% to interpretive structures, while Teacher D allocates only 0.00%. These percentages suggest that experienced teachers may prioritize other instructional strategies over providing comprehensive explanations of concepts and structural arrangements. Furthermore, experienced teachers demonstrate negligible engagement in interpretive procedures, with both Teacher C and Teacher D allocating minimal percentages to this category. Teacher C allocates 0.00% to interpretive procedures, while Teacher D allocates only

0.59%. This suggests that experienced teachers may rely less on detailing procedural steps and may prioritize other aspects of instruction.

Descriptive Explanations:

Experienced teachers, Teacher C and Teacher D, exhibit a pronounced reliance on descriptive explanations, particularly in conveying procedural steps related to linear functions. Both teachers allocate a substantial percentage of their explanation efforts to descriptive categories, with descriptive procedures comprising a significant portion of their instructional strategies. Teacher C allocates 93.44% of their explanations to descriptive explanations, with a notable focus on descriptive procedures, accounting for 54.10% of their explanation efforts. Similarly, Teacher D allocates 94.37% of their explanations to descriptive explanations, with descriptive procedures comprising a substantial 67.61%. These percentages indicate a deliberate effort by experienced teachers to provide detailed descriptions and procedural breakdowns, aiming to scaffold learners' understanding through systematic explanations.

Additionally, experienced teachers exhibit active engagement in providing descriptive structures, aiming to utilize visual aids and structural representations effectively. Teacher C allocates 37.70% of their explanations to descriptive structures, while Teacher D allocates 23.94%. These percentages suggest a concerted effort by experienced teachers to convey information about concepts through structural representations, enhancing learners' understanding through visual aids. Furthermore, experienced teachers also demonstrate some engagement in providing descriptive concepts, aiming to offer comprehensive characterizations of concepts and highlight key attributes. Teacher C allocates 0.00% of their explanations to descriptive concepts, while Teacher D allocates 2.82%. These percentages underscore experienced teachers' efforts to provide learners with detailed explanations of concepts, potentially enhancing their understanding through clear and comprehensive descriptions.

Reason-Giving Explanations:

Experienced teachers, represented by Teacher C and Teacher D, also utilize reason-giving explanations to justify concepts and procedures, although to a lesser extent compared to descriptive explanations. Both teachers allocate minimal percentages of their instructional time to reason-giving explanations, focusing more on other types of explanations. Teacher C allocates only 4.92% of their explanations to reason-giving explanations, while Teacher D allocates 4.23%. This indicates that experienced teachers provide justifications or reasons less frequently, opting for alternative instructional approaches.

Moreover, when examining the distribution of reason-giving explanation categories, experienced teachers exhibit minimal involvement in reason-giving concepts and structures. Teacher C allocates 0.00% of their explanations to reason-giving concepts, while Teacher D allocates only 0.00%. Similarly, Teacher C allocates 3.28% to reason-giving structures, while Teacher D allocates only 0.00%. These percentages suggest that experienced teachers may prioritize other instructional strategies over providing explicit justifications of concepts and structural arrangements. Furthermore, experienced teachers demonstrate some engagement in reason-giving procedures, aiming to outline the rationale behind specific steps or processes. Teacher C allocates 1.64% of their explanation efforts to reason-giving procedures, while Teacher D allocates 4.23%. These percentages suggest that experienced teachers may prioritize providing rationale for procedural steps, aiming to enhance learners' understanding of the underlying logic behind various processes.

In summary, experienced teachers, Teacher C and Teacher D, demonstrate distinct patterns in their utilization of interpretive, descriptive, and reason-giving explanations. While both teachers actively engage in providing comprehensive explanations to enhance learners' understanding, they exhibit nuanced differences in the emphasis placed on different explanation categories and subcategories. These findings provide valuable insights into the instructional practices employed by experienced teachers in teaching linear functions, offering implications for teacher training and curriculum development initiatives aimed at enhancing teaching effectiveness and learning outcomes in mathematics education.

5.1.3 Conclusion of findings

The examination of instructional practices among novice and experienced teachers in teaching linear functions to grade 9 learners unveils subtle distinctions in their application of interpretive, descriptive, and reason-giving explanations. Novice teachers, exemplified by Teacher A and Teacher B, exhibit a proactive approach in employing interpretive explanations, particularly in elucidating procedures and concepts. Their instructional strategies prioritize providing comprehensive explanations to scaffold learners' understanding, emphasizing the rationale behind structural arrangements and procedural steps. In contrast, experienced teachers, Teacher C and Teacher D, exhibit a more reserved approach to interpretive explanations, allocating minimal percentages to this category. They rely more heavily on descriptive explanations, particularly in conveying procedural steps and structural representations, suggesting a shift towards alternative instructional approaches honed through teaching experience.

Furthermore, novice teachers demonstrate a notable emphasis on descriptive explanations, particularly in providing detailed procedural breakdowns and conceptual characterizations. Their instructional strategies prioritize conveying information through descriptive structures and procedures, aiming to enhance learners' understanding through visual aids and systematic explanations. On the other hand, experienced teachers exhibit a pronounced reliance on descriptive explanations, allocating a substantial percentage of their instructional time to conveying procedural steps and structural representations. They also demonstrate some engagement in providing descriptive concepts, aiming to offer comprehensive characterizations of concepts and highlight key attributes.

Moreover, novice teachers exhibit varying degrees of engagement in reason-giving explanations, particularly in justifying concepts and procedures. Their instructional strategies prioritize providing reasons for underlying principles and procedural steps, aiming to foster deeper understanding and critical thinking skills among learners. In contrast, experienced teachers also utilize reason-giving explanations, albeit to a lesser extent, focusing more on descriptive explanations. They prioritize providing rationale for procedural steps, aiming to enhance learners' understanding of the underlying logic behind various processes.

In conclusion, the findings highlight the diverse instructional approaches adopted by novice and experienced teachers in conveying mathematical concepts to grade 9 learners. While novice teachers demonstrate a proactive stance in utilizing interpretive explanations and justifying concepts and procedures, experienced teachers exhibit a more reserved approach, relying more heavily on descriptive explanations. These findings underscore the importance of understanding the nuanced differences in instructional practices between novice and experienced teachers and offer implications for teacher training and curriculum development initiatives aimed at enhancing teaching effectiveness and student learning outcomes in mathematics education.

5.2 Limitations of the Study

In this section, I critically examine the limitations encountered during our study. I discuss the impact of sample size constraints on the generalizability of our findings and consider how contextual factors may have influenced the observed outcomes. Furthermore, I acknowledge the inherent subjectivity involved in coding data and reflect on how the observer effect may have affected the reliability of our results. By transparently addressing these limitations, I aim to provide a nuanced understanding of the scope and implications of our research findings.

5.2.1 Sample Size:

The study's sample size was relatively small, consisting of only four teachers. Despite efforts to include educators with diverse levels of experience, caution should be exercised when generalizing the findings to a larger population.

5.2.2 Contextual Factors:

The study specifically targeted grade 9 learners and their understanding of linear functions, limiting the generalizability of the findings to other grade levels or mathematical topics. Therefore, when applying the results, it is crucial to consider the distinct attributes of various contexts.

5.2.3 Subjectivity of Coding:

The coding process for identifying and classifying explanation types did not involve subjectivity. The coding system was taken from Hargie's framework and each type was further developed into three categories. Thus, having started with 3 types, through engagement and dialogic process with the data, the result for the framework was that we had nine categories in total because each type had three subcategories. This is the contribution this study made to our knowledge of explanation types as presented by Hargie.

5.2.4 Observer Effect:

The presence of observers during the lessons may have influenced the teachers' behaviours and explanations. This potential observer effect should be acknowledged as a limitation of the study.

5.3 Potential Implications of the Findings

In this section, we delve into the potential implications arising from our findings. I consider the pedagogical implications, examining how our insights can inform teaching practices and instructional strategies. Additionally, I explore the implications for learning and engagement, considering how our findings may impact student comprehension and participation. Furthermore, I discuss the potential implications for teacher professional development, highlighting opportunities for growth and enhancement in educator training programs. By delving into these implications, I seek to add to the ongoing discussion on effective teaching methodologies and tactics. It's important to note that the implications outlined here are suggestions, as they haven't been empirically proven within this paper and extend beyond its scope. Nevertheless, the study offers intriguing insights that could guide future research on teacher explanations and shed light on the pedagogical approaches of novice and expert teachers.

5.3.1 Pedagogical Implications:

The findings highlight the importance of providing a balance between different types of explanations when teaching linear functions. While descriptive explanations are crucial for introducing concepts, reason-giving explanations foster logical reasoning, and interpretive explanations connect concepts to real-world applications. Teachers should be intentional in selecting and combining these types of explanations to promote a holistic understanding of linear functions among Grade 9 learners. Both novice and experienced teachers can benefit from professional development programs that focus on enhancing their use of reason-giving and interpretive explanations. Providing training and support in constructing logical justifications and connecting mathematical concepts to practical contexts can help them develop more nuanced and effective explanation strategies. Experienced teachers, while proficient in providing descriptive explanations, should be encouraged to incorporate more interpretive explanations into their instructional practices. This can be achieved through ongoing professional development programs that emphasise the value of linking abstract concepts to real-life applications, fostering deeper conceptual understanding and engagement among students.

5.3.2 Learning and Engagement:

The incorporation of different types of explanations can significantly impact learning outcomes and engagement. By utilizing descriptive explanations, teachers ensure that learners have a solid foundation of conceptual knowledge. Reason-giving explanations promote critical thinking and logical reasoning skills, enabling learners to understand the underlying principles of linear functions. Interpretive explanations, when used appropriately, can enhance learners' motivation and application of mathematical concepts to real-world contexts. Teachers should consider the diverse learning needs and preferences of Grade 9 learners when selecting and delivering explanations. Some learners may benefit from explicit, step-by-step descriptions (descriptive explanations), others may thrive when given opportunities to explore logical connections and justifications (reason-giving explanations). While others become motivated by the applicability of mathematical concepts to real-world contexts (interpretive explanations). By catering to different learning styles and preferences, teachers can foster a more inclusive and engaging learning environment.

5.3.3 Teacher Professional Development:

The results underscore the significance of continuous professional development for educators. Offering avenues for reflection, collaboration, and training in proficient explanation strategies can enrich teachers' pedagogical expertise and competencies. Collaborative learning communities or professional learning networks can serve as platforms for sharing best practices, discussing challenges, and refining instructional approaches related to explaining mathematical concepts. These platforms can provide a supportive environment where teachers can learn from one another and implement evidence-based strategies. School administrators and policymakers should prioritize offering professional development opportunities tailored to the specific requirements of teachers in delivering effective explanations. By investing in the professional growth of educators, educational institutions can ultimately enhance the quality of mathematics education and improve learning outcomes.

5.4 Recommendations

In this section, I provide recommendations based on our findings to enhance instructional practices in teaching linear functions. I draw insights from the observed patterns and strategies employed by both novice and expert teachers to offer actionable suggestions for educators. Additionally, I propose avenues for further research to deepen understanding and address any remaining gaps in the field of linear function instruction. These recommendations aim to support continuous improvement in teaching methodologies and contribute to the advancement of pedagogical knowledge.

5.4.1 Recommendations based on the findings.

The implications of this study underscore the pedagogical importance of integrating a variety of explanation types when teaching linear functions to Grade 9 learners. By adopting a well-rounded approach that incorporates descriptive, reason-giving, and interpretive explanations, teachers can promote deeper conceptual understanding, foster critical thinking skills, and enhance learner engagement. Moreover, the findings highlight the potential advantages of professional development initiatives aimed at helping teachers diversify their range of explanation strategies and achieve a harmonious balance between different types of explanations.

It is recommended that both novice and experienced teachers receive targeted support and training to enhance their use of reason-giving explanations and strike a balance between interpretive and descriptive explanations. This can help them develop a repertoire of strategies that promote deeper understanding and critical thinking among students. Experienced teachers can benefit from opportunities for professional development that emphasize incorporating more interpretive explanations to encourage learners' connections between mathematical concepts and real-life applications. By focusing on these areas for improvement, teachers can elevate their instructional approaches and thereby bolster students' learning and attainment in mathematics. By harnessing the unique advantages of each type of explanation, teachers can enrich their instructional methods and cultivate deeper comprehension and involvement among Grade 9 learners when studying linear functions. By putting these implications into action, educators can forge more impactful and stimulating learning environments for students, ultimately fostering enhanced outcomes in mathematics education.

5.4.2 Recommendations for further research

This study examines the teacher explanations in Grade 9 linear functions. The findings of this study create room for further research in mathematics education. As such, future studies can explore the long-term effects of different types of explanations on learning outcomes and the understanding of mathematical concepts. In additions, studies can investigate the impact of explanation on different learners with diverse backgrounds, learning difficulties, and learners of varying levels of prior knowledge. This can provide valuable insights into how explanations play a role in differentiated learning. Furthermore, researchers can investigate more into the cognitive processes involved when learners receive different types of explanations. This can involve examining learners’ interpretations of teacher explanations, problem-solving strategies, and conceptual development.

5.5 Conclusion

In this chapter, the study's findings have been thoroughly examined to address the research questions at hand. Limitations inherent in the study have been acknowledged, and the potential implications of the findings have been explored. The recommendations presented encompass actionable suggestions for classroom practice and assessment, along with proposed directions for future research endeavours.

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Appendix A: Lesson observations coding data

The following table presents a sample of the coding protocol used in the analysis of the transcript. It categorises explanations based on their nature, definitions, and the specific analytic framework they correspond to, along with the associated codes. The coding protocol differentiates between three types of explanations: interpretive (Red), descriptive (Green), and reason-giving (Yellow). Any utterance that did not qualify as an explanation was labeled as a non-explanation utterance 'NE'(white). After classifying the explanations according to Hargie's (2006) framework, I applied this study's analytical framework to further categorize each explanation into subcategories.

Interpretive explanations (IC, IS, IP) address the question, "What?" They interpret or clarify an issue or specify the central meaning of a term or statement. These explanations involve clarifying concepts and providing definitions, often through analogies, symbols, or procedures.

- **Interpretive Concept (IC)** focuses on defining or explaining the central meaning of a concept.
- **Interpretive Structure (IS)** involves clarifying concepts using symbols, notation, or diagrams.
- **Interpretive Procedure (IP)** derives or clarifies procedures related to the concept.

Descriptive explanations (DC, DS, DP) address the question, "How?" They describe processes, structures, or procedures, breaking down the steps or elements involved.

- **Descriptive Concept (DC)** provides the characteristics or features of a concept.
- **Descriptive Structure (DS)** describes how symbols, notation, and diagrams are used to represent concepts.
- **Descriptive Procedure (DP)** breaks down the steps involved in a process or procedure, explaining the sequence and specific actions required.

Reason-giving explanations (RC, RS, RP) address the question, "Why?" They justify a concept or procedure based on underlying principles, values, or motives.

- **Reason Giving Concept (RC)** justifies the principle underlying a concept.
- **Reason Giving Structure (RS)** explains and justifies the structural arrangement of a concept or procedure.

- **Reason Giving Procedure (RP)** provides the rationale behind specific steps or processes within a procedure.

This categorisation provides a framework for understanding the different ways information is explained and justified within the data.

TEACHER A		TEACHER B		TEACHER C		TEACHER D	
UTTERANCE	**CODING**	**UTTERANCE**	**CODING**	**UTTERANCE**	**CODING**	**UTTERANCE**	**CODING**
SG1		**SG1**		**SG1**		**SG1**	
1.	NE	1.	NE	1.	DS	1.	NE
2.	NE	2.	DS	2.	NE	2.	NE
3.	NE	3.	DS	3.	DP	3.	NE
4.	NE	4.	DS	4.	DP	4.	NE
5.	NE	5.	NE	5.	DP	5.	DP
6.	NE	6.	NE	6.	DP	6.	DP
7.	NE	7.	NE	7.	DP	7.	DS
8.	NE	8.	NE	8.	DP	8.	DS
9.	IC	**SG2**		9.	DP	9.	DS
10.	IC	9.	NE	10.	DP	10.	DS
11.	IC	10.	IC	11.	DP	11.	DS
12.	IP	11.	IS	12.	DP	12.	DS
13.	IP	12.	RC	13.	DS	13.	DC
14.	IP	13.	IC	14.	DS	14.	DS
SG2		14.	NE	15.	DS	15.	DS
15.	NE	15.	DC	**SG2**		16.	DS
16.	DS	16.	NE	16.	NE	17.	DP
17.	NE	17.	DC	17.	NE	18.	DP
18.	NE	18.	DC	18.	NE	19.	DS
19.	NE	19.	DS	19.	NE	20.	NE
20.	NE	20.	DS	20.	DS	21.	DS
21.	NE	21.	DS	21.	DS	**SG2**	
22.	NE	22.	DS	22.	DS	22.	NE
23.	NE	23.	DS	23.	DS	23.	NE
24.	NE	24.	DS	24.	RP	24.	NE
25.	NE	25.	DS	25.	DP	25.	DS
26.	DS	26.	DS	26.	DP	26.	NE
27.	NE	27.	IC	27.	DP	27.	NE
28.	DS	28.	NE	28.	DP	28.	NE
29.	RS	29.	NE	29.	DP	29.	NE
30.	NE	30.	NE	30.	NE	30.	DP
31.	NE	31.	NE	31.	NE		DP
32.	NE	32.	RC	32.	NE		
33.	NE	33.	DS	33.	NE	31.	NE
34.	NE	**SG3**		34.	NE	32.	DP
35.	NE	34.	DS	35.	NE	33.	DS
36.	NE			**SG3**		34.	NE
37.	NE			36.	DP	35.	
38.		35.	DS	37.	IP	36.	NE
39.	RS	36.	DS	38.	DS	**SG3**	
40.	NE	37.	DS	39.	IS	37.	DP
41.	DP	38.	DS	40.	DP	38.	DP
42.	NE	39.	NE	41.	NE	39.	DP
43.	NE	40.	RP	42.	DS	40.	DP
44.	NE	41.	NE	43.	DP	41.	DP
45.	NE	42.	IP	44.	NE	42.	DP

SG3	
46.	DP
47.	NE
48.	DP
49.	NE
50.	NE
51.	NE
52.	NE
53.	NE
54.	DP
55.	NE
56.	NE
57.	DP
58.	NE
59.	DP
60.	NE
61.	NE
62.	DS
63.	NE
64.	NE
65.	DS
66.	NE
67.	NE
68.	NE
69.	NE
70.	NE
71.	NE
72.	NE
73.	NE
74.	NE
75.	IC
76.	NE
77.	NE
78.	NE
79.	IS
80.	IC
81.	NE
82.	NE
SG4	
83.	NE
84.	NE
85.	NE
86.	NE
87.	DS
88.	DS
89.	DS
90.	DS
91.	NE
92.	NE
93.	NE
94.	NE
95.	DS
96.	DC
97.	NE
98.	DC
99.	DC
100.	DP
101.	NE

43.	NE
44.	IP
45.	NE
46.	DS
47.	DP
48.	NE
49.	NE
50.	DP
51.	DP
52.	NE
53.	DP
54.	NE
55.	RP
56.	NE
57.	DS
58.	DS
59.	NE
60.	NE
61.	NE
62.	DP
63.	DP
64.	NE
65.	NE
66.	RP
SG4	
67.	DS
68.	DS
69.	RS
70.	NE
71.	NE
72.	NE
73.	NE
74.	NE
75.	NE
76.	IP
77.	IC
78.	NE
79.	RS
80.	DS
SG5	
81.	NE
82.	DS
83.	DS
84.	DS
85.	NE
86.	NE
87.	NE
88.	DS
89.	DS
90.	RS
91.	NE
92.	NE
93.	DS
94.	NE
95.	DS
96.	NE

45.	DP
46.	DP
47.	NE
48.	NE
49.	DP
50.	NE
51.	NE
52.	NE
53.	DS
54.	NE
55.	DS
56.	NE
57.	DS
58.	DS
59.	DS
60.	DS
61.	DS
62.	RS
63.	NE
64.	RS
65.	NE
66.	NE
67.	DS
68.	NE
69.	DS
70.	DS
71.	NE
72.	DP
73.	DP
74.	DP
75.	NE
76.	NE
77.	DS
78.	DP
79.	DP
80.	NE
81.	DP
82.	DP
83.	DP
84.	DP

43.	DP
44.	DP
45.	NE
46.	NE
47.	NE
48.	DP
49.	DP
50.	DS
51.	DS
52.	DP
53.	NE
54.	DP
55.	NE
56.	NE
57.	DP
58.	NE
59.	NE
60.	NE
61.	NE
62.	NE
63.	NE
64.	NE
65.	NE
66.	NE
67.	NE
68.	IC
69.	NE
70.	NE
71.	NE
72.	NE
73.	NE
74.	NE
75.	NE
76.	NE
77.	NE
78.	NE
79.	RP
80.	DP
81.	NE
82.	NE
83.	NE
84.	
85.	DP
86.	DP
87.	NE
88.	NE
89.	NE
90.	DP
91.	NE
92.	NE
93.	NE
94.	NE
95.	RP
96.	RP
97.	NE
98.	DP
99.	DP
100.	DP

102.	NE
103.	NE
104.	NE
105.	DS
SG5	
106.	NE
107.	NE
108.	NE
109.	NE
110.	NE
111.	NE
112.	NE
113.	NE
114.	NE
115.	NE
116.	NE
117.	NE
118.	NE
119.	NE
120.	NE
121.	NE
122.	NE
123.	NE
124.	NE
125.	NE
126.	NE
127.	DP
128.	NE
129.	NE
130.	DC
SG6	
131.	NE
132.	DS
133.	DS
134.	DS
135.	DS
136.	NE
137.	NE
138.	NE
139.	NE
140.	NE
141.	NE
142.	NE
143.	NE
144.	NE
145.	NE
146.	NE
147.	NE
148.	NE
149.	RS
150.	NE
151.	NE
152.	NE
153.	RS
154.	NE
155.	NE
156.	NE
157.	NE

97.	DS
SG6	
98.	DP
99.	DP
100.	DS
101.	DC
102.	DS
103.	DS
104.	NE
105.	NE

85.	DP
86.	DP
87.	DP
88.	DP
89.	NE
90.	DS
91.	NE
92.	NE
93.	NE
94.	NE

101.	DP
102.	DP
103.	DP
104.	DP
105.	NE
106.	DS
107.	DS
108.	NE
109.	DP
110.	DP
111.	NE
112.	DP
113.	DP
114.	NE
115.	NE
116.	NE
117.	NE
118.	DP
119.	NE
120.	NE
121.	NE
122.	NE
123.	NE
124.	NE
125.	NE
126.	DP
127.	NE
128.	DP
129.	DP
130.	NE
131.	NE
132.	NE
133.	NE
134.	DP
135.	NE
136.	DP
137.	DP
138.	DP
139.	NE
140.	NE
141.	NE
142.	NE
143.	DP
144.	DP
145.	NE
146.	NE
147.	NE
148.	NE
149.	NE
150.	NE
151.	DP
152.	DP
153.	NE
154.	DP
155.	NE
156.	NE

158.	NE			
159.	NE			
160.	NE			
161.	NE			
162.	NE			
163.	NE			
164.	NE			
165.	NE			
166.	NE			
167.	NE			
168.	DC			
169.	NE			
170.	NE			
171.	NE			
172.	DP			
173.	NE			
174.	NE			
175.	NE			
176.	NE			
177.	DS			
178.	NE			
SG7				
179.	NE			
180.	NE			
181.	NE			
182.	NE			
183.	DS			
184.	NE			

Appendix B: Learner consent form

Learner Consent Form

Explanations: A comparison of novice and experienced teachers' explanations of linear functions

Saneliso Mbhele

Informed Consent:

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped, photographed and/or videotape
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

I, ………………………………………………………, agree to participate in this research project.

I agree to the following:

(Please put a tick on the relevant options below)

I agree to be observed in class.	YES	NO
I agree to be videotaped in class	YES	NO

I know that the videotapes will be used for this project only.	YES	NO

Signature: ………………………………………………
Name of learner: ……………………………………
Date: ……………………………………………………

Appendix C: Parent consent form

Parent Consent Form

Explanations: A comparison of novice and experienced teachers' explanations of linear functions

Saneliso Mbhele

Informed Consent:

I understand that:

- My child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- He/she does not have to answer every question and can withdraw from the study at any time.
- he/she can ask not to be audiotaped, photographed and/or videotape
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

I, ……………………………………, agree that my child………………………… participate in this research project.

I agree to the following:

(Please put a tick on the relevant options below)

I agree that my child be observed in class.	YES	NO
I agree that my child be videotaped in class	YES	NO

I know that the videotapes will be used for this project only.	YES	NO

Signature: ………………………………………………
Parents full name: …………………………………
Date: ……………………………………………………

Appendix D: Teacher consent form

Teacher Consent Form

Explanations: A comparison of novice and experienced teachers' explanations of linear functions

Saneliso Mbhele

Informed Consent:

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped, photographed and/or videotape
- All the data collected during this study will be destroyed within 3-5 years after completion of my project.

I, ……………………………………………………………, agree to participate in this research project.

I agree to the following:

(Please put a tick on the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I understand that I can volunteer to take part in the study.	YES	NO

I agree that the interview may be audio recorded.	YES	NO
I agree that the lesson may be video recorded.	YES	NO
I agree that direct quotations from my interview and lesson may be used by the researcher in their research report.	YES	NO
I agree that my participation will remain anonymous (my name will not be used by the researcher in their research report)	YES	NO
I agree that other researchers may use the information I provide in my interview and lesson depending on their own ethics clearance being obtained) but my name and any personal information will not be used or passed on.	YES	NO

Signature: ……………………………………………
Name of participant: ……………………………………
Date: ……………………………………………………

Signature: ……………………………………………
Name of researcher: ……………………………………
Date: ……………………………………………………

Appendix E: Learner Information Sheet

Learner Information Sheet

Dear learner

My name is Saneliso Mbhele, I am a master's student in Mathematics education at the University of the Witwatersrand, Johannesburg. My supervisor is Dr Nontsikelelo Luxomo. I am conducting a research study about how teachers' explanations of linear functions affect the quality of their teaching in Grade 9. The study title is "Explanations: A comparison of novice and experienced teachers' explanations of linear functions".

I am interested in observing a series of lessons focused on the teaching of linear functions in a Grade 9 class. I intend to videotape the lessons because my focus is on the notion of explanations and how the quality explanations affect quality teaching. Additionally, to the videotaped data I also intend to conduct and audiotape a follow up interview with the teacher to discuss the lessons. The process of the research in the school should not take more than a week.

Would you mind if I observe lessons instructed by your Mathematics teacher when they teach algebra? I need your help with attending classes where I will observe and videotape the lessons

Remember, this is not a test, it is not for marks, and it is voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

I will not be using your own name, but I will make one up so no one can identify you should your voice be captured in the videotape. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

I look forward to working with you!

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely

S. Mbhele

Researcher:
Saneliso Mbhele
835211@students.ac.za

Supervisor:
Nontsikelelo Luxomo
Nontsikelelo.luxomo@wits.ac.za

Appendix F: Parent Information Sheet

26 August 2022

Parent Information Sheet

Dear Parent

My name is Saneliso Mbhele, I am a master's student in Mathematics education at the University of the Witwatersrand, Johannesburg. My supervisor is Dr Nontsikelelo Luxomo. I am conducting a research study about how teachers' explanations of linear functions affect the quality of their teaching in Grade 9. The study title is "Explanations: A comparison of novice and experienced teachers' explanations of linear functions".

I am interested in observing a series of lessons focused on the teaching of linear functions in a Grade 9 class. I intend to videotape the lessons because my focus is on the notion of explanations and how the quality explanations affect quality teaching. Additionally, to the videotaped data I also intend to conduct and audiotape a follow up interview with the teacher to discuss the lessons. The process of the research in the school should not take more than a week.

Would you mind if I observe lessons instructed by your child's Mathematics teacher when they teach algebra? I need your help with attending classes where I will observe and videotape the lessons

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be always kept confidential and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Thank you very much for your help.

Yours sincerely

S. Mbhele

Researcher:
Saneliso Mbhele
835211@students.ac.za

Supervisor:
Nontsikelelo Luxomo
Nontsikelelo.luxomo@wits.ac.za

Appendix G: Participant Information Sheet

26 August 2022

Participant Information Sheet

Dear Sir / Madam

My name is Saneliso Mbhele. I am a master's student in Mathematics education at the University of the Witwatersrand, Johannesburg. My supervisor is Dr Nontsikelelo Luxomo. I am conducting a research study about how teachers' explanations of linear functions affect the quality of their teaching in Grade 9. The study title is "Explanations: A comparison of novice and experienced teachers' explanations of linear functions".

I am inviting you to take part in a video recorded lesson that will be followed by a group interview. If you decide to take part, your participation in this research study will last about 40 minutes. The video recording will take place at Blue Eagle High School at 7h00. With your permission, I would like to audio and video record the lesson and interview. This data will be stored in the university of the Witwatersrand library archives for a period of 5 years and deleted after the period has ended. Only I the researcher will have access to the data. When I share the results of the research study, I will not include your name or anything else that could identify you. With your permission, other researchers may use the data collected from this research study, but your name and any personal information will not be used or passed on.

If you decide to take part in the research study, it should be because you want to volunteer. You do not have to take part. You can stop being in the study at any time. You do not have to answer any questions if you do not want to. You will not get any direct benefits if you choose to join the research study. You will not lose any services, benefits or rights you would normally have if you decided not to join. Taking part in the research study will not cost you anything. You will not be paid for being in this research study.

The risks for this research study are no more than what happens in everyday life. This research study will be written up as a research report. If you would like to receive a summary of this report, I will be happy to send it to you.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely

S. Mbhele

Researcher:
Saneliso Mbhele
835211@students.ac.za

Supervisor:
Nontsikelelo Luxomo
Nontsikelelo.luxomo@wits.ac.za

Appendix H: Letter To the Principal

26 August 2022

Letter To the Principal

Dear Madam

My name is Saneliso Mbhele, I am a master's student in Mathematics education at the University of the Witwatersrand, Johannesburg. My supervisor is Dr Nontsikelelo Luxomo. I am conducting a research study about how teachers' explanations of linear functions affect the quality of their teaching in Grade 9. The study title is "Explanations: A comparison of novice and experienced teachers' explanations of linear functions".

I am interested in observing a series of lessons focused on the teaching of linear functions in a Grade 9 class. I intend to videotape the lessons because my focus is on the notion of explanations and how the quality explanations affect quality teaching. Additionally, to the videotaped data I also intend to conduct and audiotape a follow up interview with the teacher to discuss the lessons. The process of the research in the school should not take more than a week.

The reason I have chosen your school is because it is a school that upholds the dignity of the of the Gauteng department of education and strives for excellence through a great team of dedicated teachers and learners. These values will allow my research to be carried out in a conducive learning environment.

I am inviting your school to participate in this research because I would like gain insights into how teachers use explanations when teaching algebra. The results of the research will be beneficial to me as a young mathematics teacher and aspiring researcher. Furthermore, the results of the study will be discussed with the participating teacher from which we can both learn and advance our understanding in relation to the teaching of inequalities.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be always kept confidential in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely

S. Mbhele

Researcher:
Saneliso Mbhele
835211@students.ac.za

Supervisor:
Nontsikelelo Luxomo
Nontsikelelo.luxomo@wits.ac.za

Appendix I: GDE Research Approval Letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**14 September 2022**
Validity of Research Approval:	**08 February 2022– 30 September 2022** **2022/436**
Name of Researcher:	**Mbhele S**
Address of Researcher:	**11 Albania Street**
	Cosmo City
	Ext 8
Telephone Number:	**011 5782247/071 059 5058**
Email address:	**sirmbhele@gmail.com**
Research Topic:	**Teacher expalnations in Grade 9 Algebra : A comparison of nivice and experienced teachers**
Type of qualification	**Masters**
Number and type of schools:	**1 Secondary School**
District/s/HO	**Johannesburg North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

16|09|2022

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below are met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The letter would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools, or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.***
4. ***The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s has been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter/document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs, and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort to obtain the goodwill and cooperation of all the GDE officials, principals, and chairpersons of the SGBs, teachers, and learners involved. Persons who offer their cooperation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school program is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her research resources, such as stationery, photocopies, transport, faxes, and telephones, and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers, and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study, the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings, and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a summary of the purpose, findings, and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr. Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: ....16/09/2022.................................

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix J: Ethics Certificate

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

CERTIFICATE OF COMPETENCE IN RESEARCH ETHICS

Name: Saneliso Mbhele

Student/Staff No: 835211

Date of Certification: 17 August 2022 - 16 August 2025 (This certificate is valid for a period of three years)

TRAINED BY:

PROFESSOR JASPER KNIGHT
(RESEARCH ETHICS)

SIGNATURE

ISSUED BY:

DR ROBIN DRENNAN
(DIRECTOR: RESEARCH OFFICE)

SIGNATURE

This certificate is confirmation of successful completion of a training course in Research Ethics for Non-Medical human research, based upon achieving a minimum level of competence in different assessment tasks.

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2022ECE059M**
PROJECT TITLE	Teacher explanations in Grade 9 algebra: A comparison of novice and experienced teachers.
INVESTIGATOR	SANELISO MBHELE
SCHOOL/DEPARTMENT OF INVESTIGATOR	WSoE
DATE CONSIDERED	05 September 2022
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	MINIMAL RISK
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	16 September 2024
CHAIRPERSON	Mbokane Dr. Batseba Mofolo-Mbokane

cc: Dr. Nontsikelelo Luxomo

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

BM

Signature **Date** 18/03/2024

27 St Andrews Road, www.wits.ac.za/education