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EXPLORING EVALUATIVE CRITERIA AND MODES OF REPRESENTATION IN EARLY NUMBER TEACHING ACROSS ENGLISH AND SEPEDI MEDIUM CLASSROOMS

A thesis submitted to the Wits School of Education, Faculty of Humanities, and University of the Witwatersrand in fulfilment of the requirements for the degree of Doctor of Philosophy

By

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ABSTRACT

In South Africa, Hoadley (2007 & 2008) has investigated evaluative criteria at the classroom level and focused on differences between teachers' use of evaluative criteria in working class and middle-class schools. Such studies have not investigated how language(s) of teaching and learning might interact with the ways in which evaluative criteria are transmitted or taken up. In the field of primary mathematics, a lack of progression from counting to calculation strategies and reliance on concrete modes of number representations during the teaching and learning of number has been noted in earlier research (e.g. Ensor, Hoadley, Kühne, & Schmitt, 2009; Schollar & Schollar, 2008). Given the reliance on concrete modes of representation and Hoadley's (2007) evidence pointing to the absence of evaluative criteria, this study sought to investigate what teachers transmitted as evaluative criteria and how modes of representation featured in what was transmitted in the context of language.

Two English and two Sepedi medium grade 3 mathematics teachers participated in this study. A total of 16 lessons (Four lessons from each teacher) on the teaching of early number, additive and multiplicative relations were observed, and video recorded. A grounded theory analysis of the 16 lessons led to the development of a framework that combined attention to evaluative criteria (EC) and moves between modes of representation (MoR) considered in relation to multilingual (ML) and mathematical (MM) moves: the EC/MoR – Framework. The EC/MoR framework is the major contribution of this study.

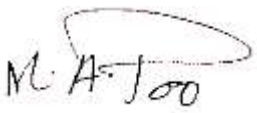
Using EC/MoR framework, I report on a range of levels of evaluative criteria teachers transmitted and variations in how ML and MM representations were used across the two language groups. General finding of this study showed that lower levels of the evaluative criteria were associated with more limited moves between modes of representation across ML and MM while higher levels of evaluative criteria more often included a network of moves between modes of representation. An important aspect of difference between the two language settings was evidence of much greater prevalence of very basic

‘restatement’ moves between the oral and the symbolic modes of representation with teaching in the Sepedi medium classrooms in comparison to the teaching in the English medium classrooms where there was evidence of moves between representations involving concrete, iconic and number-based modes alongside oral working.

Keywords: Evaluative criteria, modes of representation, number, additive relation, multiplicative relations, language, moves between representations, multilingual moves, mathematical moves

DECLARATION

I declare that this thesis is my own work. It is submitted for the degree of Doctor of Philosophy at the University of Witwatersrand. It has not been submitted before for any qualification or examination at any other institution.



14 May 2020

DEDICATION

To the Almighty God for the strength and wisdom he granted me.

PUBLICATIONS AND PRESENTATIONS EMANATING FROM THIS THESIS

Book chapter

Mdluli, M. (2017). Language in Early Number Learning in South Africa: Linking Transparency and Explicitness. In M. Graven & H. Venkat (Eds), *Improving Primary Mathematics Education, Teaching and Learning: Palgrave Studies in Excellence and Equity in Global Education n* (pp. 115–128). London: Macmillan.

Conference presentation

Poo, M. (2019). Evaluative criteria and modes of representation in South African grade 3 mathematics teaching. Oral Presentation: *Proceedings of the 43rd Annual meeting of the International Group for the Psychology of Mathematics Education*,Vol 2, pp.36

GLOSSARY

Number names in Sepedi

Numbers 1-24 & 100

1	2	3	4	5	6	7	8	9	10
Tee	Pedi	Tharo	Nne	Hlano	Tshela	Šhupa	Seswai	Senyane	Lesome
11	12	13	14	15					20
Lesome tee	Lesome pedi	Lesome tharo	Lesome nne	Lesome hlano	Lesome tshela	Lesome šhupa	Lesome seswai	Lesome senyane	Masome pedi
21	22	23	24						
Masomepedi-tee	Masomepedi-pedi	Masomepedi-tharo	Masomepedi-nne						
									100
									Lekgolo

Numbers 100-104

101	102	103	104						
Lekgolo- tee	Lekgolo -pedi	Lekgolo-tharo	Lekgolo-nne						

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CHAPTER 1: INTRODUCTION

1.1 BACKGROUND TO THE STUDY

The focus of this study is the communication of evaluative criteria and modes of representation when grade 3 mathematics teachers focus on early number, additive and multiplicative relations in Sepedi and English medium classrooms. The participants were four grade 3 teachers who, within the broader Wits Maths Connect-Primary project, had indicated willingness to be part of this study.

Bernstein (2000) suggests that evaluation is central to pedagogic situations. In his language of description, the notion of 'evaluative criteria' refers to the messages that are communicated about what learners are expected to do in order to produce a legitimate text. Evaluative criteria involve communicating to learners what is expected from them. These criteria include clarifications of concepts and ideas, and communication of principles and competences required to produce a legitimate text. While evaluative criteria refer to all the ways in which teachers transmit what they consider to be 'acceptable' in classrooms, responses to learner offers and moves between representations during classroom interactions provide a particularly important space in which these messages are communicated. In the next section, I provide three rationales that prompted for this study. Firstly, the evidence of absence of evaluation criteria in the working-class settings in Foundation phase in South Africa; the complexity of language of instruction in mathematics classrooms in South Africa, and lastly, the nature of progression and the modes of representations in early number learning in South Africa and internationally.

1.2 RATIONALE FOR THIS STUDY

1.2.1 Evaluative criteria within South African classroom contexts

While evaluative criteria are central to classroom teaching contexts across the world, evaluative responses to learners' offers are particularly important in South African classrooms given the absence of communication of evaluative criteria in instruction in Foundation Phase classrooms evident in research studies such as Hoadley's (2007 & 2008) work. Hoadley investigated evaluative criteria across two social class settings and

suggested that the ways in which they were communicated could contribute to variable learner achievements across classrooms in different social contexts. The study comprised of eight grade 3 teachers located in four schools in South Africa. Two of these four schools were located in working class townships. These township schools were historically disadvantaged schools largely attended by African children from poor socio-economic backgrounds. The other two schools were located in middle-class suburbs in Cape Town attended by children of all racial groups. Analysis of the data sourced from this study showed that evaluative criteria were explicitly communicated in the middle-class contexts. In Hoadley's findings the rules to produce the legitimate text were often made explicit for learners in middle class schools. However, in working class contexts, the evaluative rules were generally not communicated, or were unclear or implicit. This unclear or implicit communication of the evaluative rules suggested that learners in working class schools often were not guided in the production of answers to given tasks. Hoadley extended Bernstein's (1990) notions for strong, or explicit, communication of evaluative criteria (F^+) and weak, or inexplicit, communication of these criteria (F^-) by creating the F^0 code to point to the phenomenon of absence of evaluative criteria rather than strength or weakness of the criteria offered to learners. Though Hoadley's study was small and conducted within the context of social class, her model suggested that exploring evaluative criteria could be useful for understanding the differences in mathematics learning outcomes between English and African home language foundation phase classrooms given that lower educational outcomes for those learning in African home language media of instruction in South Africa rather than in English or Afrikaans continue to persist (Department of Education, 2008). This phenomenon has not been studied using Bernstein's evaluative criteria.

Political developments in South Africa since 1994 have led to shifts in school policies and practices in terms of language(s) of instruction. The following section provides reasons for taking medium of instruction (language of learning and teaching) into account when studying teachers' communication of evaluative criteria.

1.2.2 Rationale for choosing language as a contextual variable

Since the introduction in 1997 of the Language in Education Policy (LiEP), that allows the use of all eleven South African languages as official languages, there have been increases in the number of learners in the foundation phase (grades R-3) learning through the medium of an African language (Department of Basic Education, 2010). Whilst some schools in South Africa have chosen to teach through the medium of African languages in the foundation phase and switch to English at grade 4, others have chosen to teach through the medium of English right from the foundation phase. These differences in the Language of Learning and Teaching (LoLT) raise questions about differences in learner performances. In 2007 the Department of Basic Education conducted a Systemic Evaluation in the foundation phase at grade 3 level in Literacy and Numeracy (Department of Basic Education, 2008). The results showed that learners obtained 36% in numeracy and 36% in literacy. These results suggested that learners performed below the expected levels in both numeracy and literacy at grade 3 level (Department of Education, 2008) Further analysis of these results suggested that African language mother-tongue speakers generally performed lower when compared to English or Afrikaans speakers in this 2007 assessment (Green, Parker, Deacon & Hall, 2011). Whilst social class analyses like Hoadley's study provide some reasons for differences in learner performance, given that language background overlaps with social class background in South Africa, Bernstein (1990) reminds us that learning outcomes are produced or reproduced in classrooms, so studying the pedagogic relay across different language settings provides the possibility of understanding differences in evaluative criteria that can extend Hoadley's findings.

1.2.3 Rationale for investigating progression in early number

The focus of this study is on the range of evaluative criteria four grade 3 mathematics teachers transmit during the teaching of early number which aims to lead to understanding of number, additive and multiplicative relations, in the context of two Sepedi and two English medium classrooms. The position I have taken is that learning early number and additive relations are foundational to young learners' success in

learning other mathematical concepts. There are a number of South African studies that point to problems relating to the teaching of early number and additive relations. The works of Ensor, Hoadley, Kùhne, Schmitt, Lombard and van den Heuvel-Panhuizen (2009) and Schollar (2008) are critical in providing a context for these problems.

Schollar and colleagues (Schollar, 2008) studied the mathematics tasks and responses in the work of 4483 learners in South African classrooms and investigated strategies these learners used to solve tasks. Their findings indicated that the majority of learners were performing below the expected levels, with number problems often solved by unit counting in which learners reduced all numbers to single tallies. These findings showed that almost 80% of Grade 5 and 60% of Grade 7 learners relied on unit counting approaches to solve addition, subtraction, multiplication and division problems. They also noted that multiplication and division problems were reduced, at best, to repeated addition and subtraction. Schollar (2008) argues that poor learner performance in primary mathematics within the South African context is caused by learners' inability to progress in number development as learners in his study continued to rely on concrete counting strategies rather than more abstract calculation strategies when working with number. Similarly, Ensor et al (2009) observed similar patterns of working with number in South African classrooms.

Ensor et al (2009) studied mathematics teaching in eighteen classrooms across the foundation phase in three schools serving poor communities in South Africa. These authors investigated eighteen foundation phase teachers' pedagogical practices when teaching early number. Their findings showed teaching practices that lacked an understanding of progression from concrete ways of representing number to more abstract symbolic ways of representing number. Findings of both Schollar (2008) and Ensor et al (2009) point to a pedagogy that is characterised by a lack of moves beyond unit counting and the lack of a move from more concrete to more abstract ways of representing number. Ensor et al (2009) suggest that this progression can be attained firstly through specialization of content which refers to teaching that focuses on moves from calculation by counting to calculation by structuring and then to formal calculation,

(van den Heuvel-Panhuizen, 2001, 2003). Secondly the move from concrete to more abstract ways of representing number can be attained through a pedagogy that focuses on specialising representational modes which refers to moves from more concrete to more abstract ways of representing number. Both these types of progression (from calculation based to formal calculation and from concrete to more abstract ways of representing number) have support in the mathematics education literature. The mathematics education literature also points to moves between representations rather than moves in the direction of abstraction (Duval, 2006).

A further aspect of progression in the mathematics education literature points to the importance of generalisations in mathematics teaching. Watson and Mason (2005) suggest that generalization is achieved by moving from particular examples to general properties of connected sets of examples and rules that can be applied to these sets of examples. These three avenues for progression in early number teaching informed this study of evaluative criteria. In the evaluative criteria framework developed in this study, attention to generalisation informed by the work of Warren (2004) was incorporated. Specialization of content as described by van den Heuvel-Panhuizen (2001, 2003) was also incorporated in the framework. Specialization of modes of representation was developed as a separate dimension, with multilingual modes of representation incorporated into the study.

1.3 THEORETICAL UNDERPINNINGS OF THE STUDY

The work of Dowling (1998) and Hoadley (2007) on pedagogic strategies has been useful for examining, describing and analysing strategies teachers deploy to support learners' progress when teaching early number. I turned to the work of Duval (2006) to understand the theory of modes of representation and of moving between representations to enhance mathematical understanding. The theories of Dowling, Hoadley, Duval, Watson and Mason have provided a useful insight into the strategies teachers deploy to transmit what they view as the evaluative criteria during the teaching of early number and how they move between representations to support learners' progression from using approaches for counting to using more abstract approaches.

1.4 AIMS OF THE STUDY

This present study has sought to take forward investigation into factors that contribute towards poor learner performance and variations in learner achievement by investigating aspects of pedagogy in mathematics teaching at the grade 3 level with a specific focus on language and early number learning. The aim of this investigation has been to:

- Understand ways in which the language of instruction features in ways teachers communicate evaluative criteria
- Understand the range of the evaluative criteria communicated within instructional talk during the teaching of early number, additive and multiplicative relations in grade 3 mathematics classrooms where the LoLT is either English or Sepedi.
- Identify the differences in the pedagogic strategies teachers in the different language settings deploy to communicate the evaluative criteria
- Understand ways in which modes of representations feature within the evaluative criteria presented in instructional talk in different language settings.

1.5 RESEARCH QUESTIONS

This exploratory comparative study responds to the following main question:

What do teachers transmit as evaluative criteria in grade 3 English and Sepedi medium mathematics classrooms in their teaching of number involving additive, multiplicative relations and modes of representation?

Related sub-questions:

- What is transmitted as evaluative criteria in Grade 3 mathematics classrooms during the teaching of early number?
- What pedagogic strategies do teachers use to communicate the evaluative criteria in the two different language settings?
- How do modes of number representation feature in the transmission of the evaluative criteria?

1.6. RESEARCH DESIGN

The study was conducted following qualitative case study research design methods. In 2014 the Wits Maths Connect project in 2014 involved a content knowledge course in which 33 teachers participated. Given the positive relationships established with teachers in this work, an opportunistic selection of four teachers from this cohort was made, based on their willingness to participate in a study focused on classroom language use in mathematics, and the language of instruction used in their classrooms. Data were collected for this study from observation and video recording of four lessons per teacher over a period of three school terms. A total of 16 lessons were video recorded in 2014. These lessons were transcribed and separated into lesson excerpts, based on what teachers transmitted in each excerpt. A total of 104 excerpts were generated. Eighty eight percent or 92 of these excerpts (those involving early number and modes of representations), were analyzed and reported on in this study. The analysis of these excerpts focused on teachers' talk and specifically on what they transmitted as evaluative criteria in the context of learners' incorrect or correct offers and how they used modes of representation to respond to these offers. I identified the responses, compared these and categorized these following Dowling (1998) and Hoadley's (2007, 2008) notions of strategies and Duval's (2006) notion of moves between representations. Through a grounded theory-based process of categorizing (Glaser & Strauss, 1967), a range of evaluative criteria emerged together with various moves between representations. These categories led to an emergent framework with five levels of evaluative criteria. These five levels of evaluative criteria and accompanying representational modes are discussed in detail in Chapter Five.

1.7 THE STRUCTURE OF THE THESIS

Having introduced the study, I now outline the content and structure of the remaining chapters of this study.

Chapter Two discusses and explicates the theory that underpins the study.

Chapter Three provides a review of the literature that is relevant to the study.

Chapter Four explicates the methodology and research methods central to the study.

Chapter Five discusses the emergent framework and contributions of this study to the field of research.

Chapter Six focuses on data analysis.

Chapter Seven discusses the findings and recommendations emanating from the study.

CHAPTER 2: THEORETICAL FRAMING OF THE STUDY

2.1 INTRODUCTION

In the introduction, I have pointed to some of the differences in mathematical performance associated with whether or not learners are learning mathematics in their home language, and also discussed some of the more general problems described as common in South African Foundation Phase mathematics teaching. Some of these problems are directly related to the ways in which language is used in teaching, while others are more generally at the level of pedagogy. However, as noted in the introductory chapter, the ways in which evaluation and evaluative criteria play out in classrooms where either English or an African language is the LoLT has not been the focus of prior studies. An exploration of this issue is at the heart of this study, and notions of evaluation and evaluative criteria therefore form the theoretical base.

My focus on evaluation and evaluative criteria are drawn from the foundational work of Basil Bernstein, which was elaborated by Paul Dowling, and then applied and further elaborated in the work of Ursula Hoadley. In this chapter, I introduce and discuss the concept of evaluative criteria and the ways in which evaluation and evaluative criteria are discussed and developed across the work of these three key authors.

2.2 PEDAGOGICAL DISCOURSES AND THEORY OF CODES, FRAMING AND CLASSIFICATION

In his theory of codes and orientation to meaning Bernstein (1964) introduced two types of orientation to organizing experience and making meaning: restricted and elaborative codes. Codes refer to principles regulating meaning systems. In Bernstein's writing, restricted code refers to the recognition and realization of what Bernstein describes as 'context dependent meanings', whilst elaborated code refers to the recognition and realization of 'context independent meanings. For Singh (2002) context dependent meanings are related to everyday knowledge. By contrast, context independent meanings are exemplified by school, or formal, knowledge, the kind of knowledge embedded in disciplines like mathematics.

Bernstein's argument for the usefulness of these two codes is predicated on his view that the purpose of schooling is to socialize learners into the 'school code', which he describes as fundamentally linked to the elaborated code. Bernstein (2000, p. 200) cites Holland's (1981) empirical study, among others, in support of the claim that this theory offers an explanation of some of the socio-cultural causes of underachievement by working class children. In Holland's study, a group of seven-year-old children were tasked to categorize food. The working-class group predominantly categorized the food according to their everyday personal experiences and used words like "these are what my mom cooks at home", "these are some of the things I eat for breakfast". The middle-class group of these seven-year-old learners predominantly categorized the food according to properties of food and not according to their everyday personal experiences. These learners described their food categories as "these are a group of vegetables", "these foods contain animal products, and these are sea foods". In their responses to the task, the learners from working class homes exhibited a way of organizing food which demonstrated a restricted code for meaning making. Their orientation to meaning seemed to be based on their direct, personal everyday experiences or everyday knowledge of food. In contrast, the middle-class group more often used generalisable categories, e.g. "these are vegetables, and these are sea foods". When asked to group foods in a different way, the working-class group presented different everyday experience categories while the middle-class group subsequently moved to offering everyday experience-based categories. The interpretation of this data was that middle-class learners demonstrated elaborated orientations to meaning (Hoadley& Muller 2010) in being able to work with both restricted and elaborated codes. The argument Holland (1981) makes with this experiment is that when middle-class seven year olds enter formal schooling; they enter the schooling system more advantageously positioned in relation to school knowledge, with this advantage helping to explain their relative success in learning trajectories through school in comparison to working class groups.

The theory also helps with understanding social learning and forms of cultural discontinuity between the home and the school. Bernstein (1975) argued that mastery of the elaborated code is necessary for learners to be successful in their schooling. A

number of scholars have used this theory of codes to explicate how particular pedagogies may perpetuate the gap in schooling for learners coming from different social class groups.

The differential pattern of outcomes between African home language medium contexts and English medium contexts in South Africa raised questions about whether learners in these two language contexts might be differently oriented towards meaning making as suggested by Bernstein's theory or restricted and elaborated codes. While evaluative criteria came to be the focus through which this question was studied, a brief overview of some more basic Bernsteinian concepts is required to get to the focus on evaluative criteria. The subsequent section discusses framing and classification. These two terms provide the language to describe ways in which knowledge is relayed through teachers' pedagogic practices.

2.3 CLASSIFICATION AND FRAMING

2.3.1 Classification

The focus of this study is not on knowledge, classification and framing but on evaluative criteria. However, to explicate the concept of evaluative criteria and how these are transmitted through pedagogy, it is important to understand knowledge, framing and classification. Bernstein (1975) analyzed ways in which knowledge is recontextualised within pedagogy. He introduced the concepts of classification and framing rules. His theory of classification and framing enables researchers to explicate and analyse the effects of curriculum, assessment and pedagogy on learning. This theory provides a language for researchers to use in analysing classroom interactions in terms of transmission, acquisition and evaluation of knowledge.

Classification refers to the way in which power relations are created in discourses. It refers to the strength of boundaries. Classification is also defined as "*the relations between and the degree of maintenance between categories and these include the boundaries between agents, spaces and discourses"* (Hoadley, 2008). Bernstein uses the strengths of boundaries to distinguish two key classification categories. Strong classification C^+ refers to a situation where the contents of each subject or discourse are strongly bounded

or separated. The type of knowledge or the message within a category is explicit, C^{+}or unambiquous, in contexts where strong boundaries exist. Strong classification specialises the pedagogic discourse. Weak classification C^{-} refers to situations where content or the "voice" of the subject or discourse is less insulated or is not bounded. In these contexts, messages of the category are implicit and may be ambiguous (Bernstein, 1975). Classification can be expressed in terms of a continuum from C^{++} to C^{--} . Simply put, classification determines the extent to which knowledge types, content of a subject and concepts within a subject are kept apart or integrated. Classification regulates the voice of the category (Hoadley, 2008). Specialising the voice refers to making the legitimate text explicit and evaluating learners' responses following the criteria of the discourse field. In the context of this study, classification provides the language to talk about what is communicated as the evaluative criteria related to the teaching of number, additive and multiplicative relations. For instance, strong classfication in the context of this study could be used to describe a situation where a teacher privileges one particular type of additive relation problem or a particular approach or procedure over others. Weak classification can be used to describe a situation where teachers transmit many different approaches/ procedures or types of additive/multiplicative relations problems at a time without a focus on a particular approach or type/structure or procedure. The notion of classification figured within this present study in identifying the boundaries between types of additive/multiplicative relations problems, approaches or procedures that teachers focused on when they transmitted evaluative criteria.

Whilst classification specialises the voice of the category, I cannot talk about classification of evaluative criteria without referring to framing because the way these two concepts (classification and framing) are combined brings about variations in the pedagogy of the practice (Hoadley, 2008). Bernstein's core argument is that variations in pedagogy, related to teachers' classification and framing practices, may result in inequalities in learning outcomes for different categories of students. In a nutshell, framing and classification can lead to differences in what is transmitted through pedagogy; hence it is important for this study to explain and understand the relevance of these two concepts in the context of English medium and Sepedi medium grade 3 mathematics classrooms.

2.3.2 Framing

Framing refers to the degree of control the teacher and learners have over the pacing, selection and evaluation of the knowledge transmitted (Bernstein, 2000). This author argues that when the framing is strong the transmitter has explicit control over the selection, pacing and the evaluative criteria. The acquirer has less control over the selection, pacing and the evaluative criteria. Weak framing F^{-}, refers to a context where acquirers have more control over the sequencing, selection and the evaluation of knowledge (Bernstein, 2000). In contexts where the framing is strong F^{+}, the evaluative criteria are explicitly transmitted and the teacher has more control over how she or he communicates the criteria whilst acquirers have limited control over how to produce the legitimate text. Like classification, framing is also expressed on a continuum from F^{++} to F^{--} . Simply put, framing refers to the relationship of control between the teacher and learners in terms of what is taught, how it is taught and when it is taught. In the context of this present study, framing was useful for describing what learners were given access to during the teaching of additive relations, how this content was sequenced and how representational modes were used in the transmission of evaluative criteria.

The strengths of both classification and framing create variation in terms of what is transmitted and acquired. Hoadley (2008) emphasises that the clarity and 'separateness' of the concepts of classification and framing does not play out in any straightforward way in empirical data: "There are, however, difficulties in working with the concepts of classification and framing empirically. Because they are dialectically linked, to 'see' them separately poses a challenge for the researcher. Classification cannot maintain itself without framing" (Hoadley, 2007, p683). This author also states that "it is through interaction (framing) that boundaries between discourses, spaces and subjects are defined, maintained and changed. Based on Hoadley's view of the interrelated nature of these two concepts of classification and framing, this study brings the two concepts together within the descriptions and analysis of what teachers transmit as evaluative criteria and how modes of representations feature in what is transmitted.

The extent to which the evaluative rules are communicated relates to the concepts of classification and framing. Explicit communication of evaluative rules and evaluative criteria strengthens the boundaries (classification). Where boundaries are strong, teachers specify what learners should know, do or understand in order to produce the legitimate text. Implicit communication of evaluative rules weakens the boundaries; this can lead to learners being unable to recognize the legitimate text (Hoadley, 2007).

The concepts of classification and framing assisted in this study to develop the language to describe “what” were transmitted as evaluative criteria and “how” evaluative criteria were transmitted in Sepedi and English medium grade 3 classrooms. The next section of this chapter discusses the pedagogic discourse which is a term drawn from Bernstein (1990) work to describe goals, purposes, content and the people involved in the process of teaching and learning.

2.3.3 Pedagogic discourse

This study focuses on Bernstein’s theory of pedagogic discourse (Bernstein,1990). Pedagogic discourse is concerned with the production, distribution and reproduction of official knowledge (Sadovnik, 1991). For teachers, this involves selecting what knowledge to teach and transforming that knowledge for use in the classroom and it also involves evaluating learners’ work and responses. Preparation for this evaluation involves making learners aware of how to produce correct answers. Bernstein (2000) suggests that all pedagogic discourses are complex and goal directed.

The purpose of pedagogic practices is to communicate principles and competences in which evaluative criteria play a role. Bernstein argues that for acquirers to produce the legitimate text, they should be able to recognise the text (recognition rules) and select the relevant meaning (realisation rules) in order to produce the text according to this meaning (Morais, 2002). Realisation rules are transmitted by the teacher and acquired through the framing of the pedagogy. Bernstein’s theory of the pedagogic device suggests that evaluative criteria can be implicit or explicit. In the context where the rules of how to produce the legitimate text are explicit, the acquirer has a chance to know what is

expected from her. Bernstein refers to the type of pedagogy where the evaluation is made explicit as a visible pedagogy (VP). Explicit communication of the evaluative criteria assists learners to recognise what counts as valid knowledge in the subject or discipline. According to Sadovnik (1991) Bernstein describes implicit pedagogy as pedagogy in which the rules of the discourse are implicit and the acquirer has more freedom to create his or her individual criteria for evaluation.

While Bernstein takes no 'position' on whether implicit/explicit evaluative criteria are good or bad, some research suggests that through making evaluative criteria explicit, teachers give learners, particularly those from disadvantaged backgrounds, more possibility of learning about the legitimate text and how to go about producing correct answers in future (Slonimsky & Shalem, 2010; Morais, 2002). A number of authors have argued that making the evaluative rules explicit and telling learners what is expected of them is an important part of the pedagogic practice, as explicit transmission of the evaluative rules promotes high levels of learning amongst the learners and it improves achievement gains for learners, particularly for those coming from working class backgrounds (Hoadley, 2007; Morais, Neves & Pires 2004). In earlier work, at lower levels, these criteria can be connected to replicating procedures (procedural criteria) and at higher levels, criteria can be about principles for how to construct procedures or about what counts as an efficient procedure or argument (principling criteria) (Morais et al, 2004). Both the evaluation feedback and the evaluative criteria can be more visible or less visible (framing) depending on the extent to which the teacher communicates the recognition and realization rules.

This present study focuses on evaluative criteria because evaluative rules are central to any pedagogic practice. The study also focuses on evaluative criteria because evidence from the studies of scholars listed above suggests that strong classification and framing of evaluative rules contribute to learner achievement, and particularly so, for disadvantaged groups. As mentioned in the introductory part of this study, while the language of learning and teaching in the foundation phase in South Africa remains linked to socio-economic background, with English/Afrikaans media associated with higher

socio-economic resources and African language media associated with lower socio-economic resources, it is of interest to investigate for any differences in what is transmitted in these classrooms as evaluative criteria and how this information is communicated.

Bernstein's theory of codes, framing, classification and evaluative criteria is at a general level that is hard to directly apply to early mathematics classroom practice. Interim research though, provides some useful concepts for application to mathematics. I now turn to the work of Dowling (1998) which focuses on domains and strategies within the context of mathematics.

2.4 DOWLING'S THEORY OF DOMAINS AND STRATEGIES

2.4.1 Knowledge domains

Dowling took forward Bernstein's conceptualisation of classification and linked this conceptualisation to the practice of mathematics. Dowling classified mathematical content into four knowledge domains: expressive, descriptive, esoteric and public (Dowling, 1998). For purposes of this study, two of these domains, the esoteric and the public, are discussed in detail. Drawing on Bernstein's (1990) notion of different types of knowledge, Dowling argues that school mathematical activity varies according to the extent to which forms of expression and content are specialized. Strengths of institutionalization of modes of expression and of content relate to Bernstein's theory of strong classification. Strong classification refers to a situation where the content of subjects or discourses are strongly bounded or separated from each other. The mathematical activities for which forms of expression (language) and of content are strongly institutionalized form the non-negotiable part of school mathematics. Dowling refers to such mathematical activity as the esoteric domain of the practice (Dowling, 2013). Mathematical activities where forms of expression/ language and of content are weakly institutionalized are regarded as located in the public domain of school mathematics. Dowling suggests that mathematical activities for which forms of expression and of content are weakly institutionalized can be recontextualised and brought into alignment with the esoteric domain of school mathematics through what he describes as

‘specializing strategies’. This author suggests that mathematical knowledge is distributed through strategies (Dowling, 1998) and these strategies may expand or limit the range of evaluative criteria being transmitted. The next section discusses Dowling’s strategies.

2.4.2 Pedagogic strategies

Dowling (1998) distinguishes between two pedagogic strategies, localising and specialising, and links these strategies to the public and esoteric domains (Dowling, 1998). Specialising strategies distribute abstract messages, methods or concepts located in the esoteric knowledge domain. Dowling suggests that specialising strategies can involve examining different examples or cases of a method to distinguish common principles and making these principles explicit, a point I return to later. He also suggests that only specialising strategies offer learners access to a principled mathematical knowledge which is located in the esoteric domain of mathematics. Localising strategies distinguish particular examples or methods without making underlying principles of these examples or methods explicit. These localising strategies do not offer learners access to principled mathematical discourses (Dowling, 2009). Dowling’s theory of domains and strategies can be used to identify what evaluative criteria teachers transmit and the strategies teachers deploy to transmit the knowledge they want learners to access.

2.4.3 Application of theory of domains and strategies

Hoadley (2008) focused on the distribution and acquisition of knowledge between working class and middle class students. In this study she applied Bernstein’s theory of classification and framing as well as Dowling’s theory of domains and strategies to analyse mathematics tasks across grade 3 middle class and working class schools in Cape Town, South Africa. She disaggregated Dowling’s strategies into ‘specialising proceduralising’ and ‘specialising principling’ strategies. Hoadley (2007) refers to specialising principling strategies as strategies that distribute principles of the discipline. These strategies are often complex and involve teaching activities that belong to the esoteric domain. Learners must have special knowledge of mathematics and have developed reasoning, justification, and explanation to successfully complete tasks distributed through specializing principling strategies. On the other hand, she suggests

that specializing proceduralising strategies distribute procedures. The focus of specializing proceduralising strategies is on the procedure required for the construction/production of legitimate texts for evaluation. These strategies allow learners to practice a concept or perform an operation without necessarily engaging in the principles for the generation of texts. Specializing proceduralising strategies do not require learners to understand why certain procedures work to produce certain answers (Zacharos, Koustourakis & Papadimitriou, 2014; Hoadley, 2007)

Hoadley further disaggregated localising strategies into mathematical and non mathematical localising strategies to account for different forms of localising strategies that were observable in her study. She states that localising non-mathematical strategies distribute activities that have non-mathematical content whilst localising mathematical strategies distribute activities such as those involving instances where learners are asked to name mathematical symbols or objects. Another category of localising mathematical tasks includes what Hoadley refers to as ritual tasks which involve repetition of words or phrases after the teacher and mechanical tasks involving copying numbers or colouring a drawing with mathematical content.

Findings from Hoadley (2007) study revealed that localising strategies involving non-mathematical tasks were observable in the grade 3 working class group. In this cohort, teachers privileged the distribution of everyday knowledge. The discussions that took place in these classrooms did not specialise the “voice” and did not require learners to employ specialised mathematical knowledge and skills to produce what counted as the legitimate text. The discussions required learners to chant words and discuss non-mathematical and familiar themes that had no mathematical basis. There was also evidence in the same contexts of specialising procedural strategies where the pedagogy privileged the discussion of rules and procedures for producing the legitimate text. The study found no evidence of deployment of specialising principling strategies in working class contexts. In contrast, these latter specialising principling strategies were sometimes observable in some of the middle class contexts. Interestingly though, the study noted the

dominance of specialising proceduralising strategies in both working class and middle class contexts.

Hoadley (2007) also introduced an additional category to indicate the absence of evaluative criteria (F^0). This coding was used to denote the phenomena of the absence of the evaluative criteria.

Hoadley's framework of specializing and localizing strategies and her introduction of an of evaluative criteria (F^0) at the base level, provided initial categories in this study for the exploration of the strategies that English and Sepedi medium teachers in grade 3 mathematics classroom employed to distribute evaluative criteria.

In this present study, the notion of domains is expanded further to identify and describe the range of evaluative criteria teachers transmitted in Sepedi and English medium grade 3 mathematics classrooms. The expansions introduced are linked to the literature on early number, additive and multiplicative relations progressions that I detail in the next chapter. In specific terms, linking to this literature base and to South African evidence of lack of progression beyond counting based approaches, localizing pedagogic strategies are used to describe strategies where teachers transmit different counting-based proceduralising strategies for learners to use in order to produce answers. These strategies are deployed without a focused teaching trajectory that encourages learners to move from counting-based procedures to using more efficient calculation strategies and number facts. These localizing proceduralising counting approaches can be related to everyday ways of counting in that they require no awareness of particular numbers as part of a systematic number system with properties that can be used to leverage more efficient calculation. These strategies are deployed without generalizing approaches across sets of examples (linking back to Dowling's point that focus on more general principles forms one part of specializing strategies). There is also no pointing out the underlying principles involved in each counting approach.

Specializing proceduralising strategies are those strategies that deploy general procedures or step by step methods for producing answers without using counting-based approaches. Specializing principling strategies describe those strategies where teachers

allow learners to use number facts and efficient calculation strategies to produce answers. In deploying these specializing principling strategies, teachers provide different examples of a method or strategy and point out the underlying principles of the method or strategy. These strategies are often coupled with requests for learners to explain and justify how they produced answers.

2.4.4 Specialisation of representations

While Dowling referred to specialising as involving attention to common principles across different instances, neither his work nor Hoadley's work pays specific attention to instances commonly occurring in mathematics across multiple representations. Literature in the field of primary mathematics education provides strong support for exposing learners to mathematical concepts through multiple representations (Doerr & Lesh, 2011; Lesh & Lehrer, 2003; Noble, Nemirovsky, Wright &Tierney 2001). It is argued that successful mathematical thinking involves understanding of different representations of the same concept and the ability to move between representations. Using multiple representations to represent the same mathematical idea and moving between these representations is a way of specializing. However, literature on multiple external representations (MER), reports that learners find it very difficult to translate between various representations as they often struggle to make connections between multiple representations, understanding their similarities and differences (Noble et al., 2001; Ainsworth, 1999).

The work of Dreher, Kuntze and Lerman (2016); Doerr and Lesh (2011) and Duval (2006) on representations brings an additional perspective on specialising strategies in mathematics, noting that the use of multiple representations and moves between the various representations is considered as an expansion of mathematical knowledge. Moves between languages formed one aspect of moves between representations. This perspective on specialising, while not foregrounded in Dowling's theory of domains and strategies, was incorporated in the analytical framework developed in this study, and formed an important part of the differences in the evaluative criteria communicated across English and Sepedi early number teaching.

CHAPTER 3: LOCATING THE STUDY IN THE LITERATURE

3.1 INTRODUCTION

This chapter consists of six related sections:

- the context of the study, literature and theory relating to language in mathematics teaching in South Africa;
- the concept of evaluative criteria and its centrality to teaching mathematics;
- early number as the content focus of the study; ways of supporting progression in the development of these topics;
- representations, connections between representations, representational moves and progression from concrete to more abstract representations
- research findings on problems of early years mathematics teaching in South African classrooms;

  .

3.2 LANGUAGES AS A CONTEXTUAL VARIABLE IN THIS STUDY

3.2.1 Language and mathematics teaching

Research findings have shown that the language of instruction and the formal language of mathematics are important in the teaching and learning of mathematics (Moschkovich, 2002; Setati & Adler, 2000). The importance of the language(s) of teaching and learning, in the work of Setati & Adler, as well as Moschkovich, is emphasized to support learners to use both the language of instruction and the language of mathematics to negotiate meaning and to make sense of mathematics. Challenges in teaching mathematics through the medium of African mother tongue have been noted in South Africa. These include the fact that many resources or materials are still written in English and that many teachers who teach in early grades were trained in English and not in the languages they are expected to teach mathematics in (Mostert, 2019). However, benefits to teaching mathematics through African Languages in the early grades are also noted. In earlier writing emanating from this study, I pointed out that number names in most African Languages are very logically constructed (Mdluli, 2017), as has been noted about

languages like Japanese and Chinese (Aunio, Aubrey, Godfrey, Pan & Liu, 2008; Nunes & Bryant, 1996; Miura, Okamato, Kim, Steere, & Fayol, 1993) . After learning the number words for 0-10 the construction of subsequent number names is built through the combination of this first set of number names (0 -10) in a logical order and reflecting the base 10 structure that is central to the learning of place value. For instance, in Setswana and Sepedi, the number word for seventeen is 'lesome šupa' – literally 'one ten seven'. This is a more straightforward building up than seventeen in English, where the construction of the number as 'a ten and seven' is less transparent. Hence, authors suggest that the irregularity of English number words makes them harder to learn. Conversely, the regular structure of number names in African languages like Sepedi and Setswana can be useful for learners to develop knowledge of the base ten system (Mostert, 2019; Mdluli, 2017; Ho & Fuson,1998; Fuson & Briars, 1990). This evidence suggests that there is substantive value for young learners from learning mathematics in an African language.

Setati and Adler, (2000) differentiate between 'informal' and 'formal' mathematical language. These authors link the everyday language that people use to express mathematical ideas to informal mathematics language whilst the standardized forms of expressing mathematics, usually learnt in formal situations, constitute the formal language of mathematics. Pimm (1987) points out that often learners come to school with a wealth of everyday mathematical language which is sidelined in school formal learning. He suggests that it is important for teachers to support learners to understand formal and informal types of mathematical languages and to make a transition from using informal to formal mathematical language.

Pimm (1987) further proposes that learning mathematics involves been able to use the mathematics register and to talk the language of mathematics. Whilst Setati and Adler, (2000) acknowledge that it is possible for children to learn formal written mathematics language, they caution that the mathematics register is not fully developed in some African languages. The absence of a fully developed mathematical register in these African languages presents a challenge to teachers teaching through these languages.

This challenge cannot be ignored if one is investigating how teachers use the language of instruction to support learners to make a shift from using every day or informal language to using formal mathematics language.

Whilst it is important to acknowledge the challenges associated with teaching through African mother tongue, it is equally important to acknowledge the distinction between the language of mathematics and everyday language and the fact that Mathematics has its own set of complex rules and system of writing which is unique to this practice (Pimm, 1987). The complex rules and system of writing embedded in mathematics and the inappropriate use of everyday language may present a challenge to both teachers and children learning mathematics.

The challenges that result from mathematics language are not only syntactic, semantic or pragmatic nature. These challenges include formal definitions, multiple meanings of words, homophones and words with similar sounds, numerals, symbols and the importance of order in mathematics learning (de Oliveira & Cheng, 2011; Adams, 2003; Spanos, Rhodes, Dale & Crandall, 1988).The fact that mathematics uses its own language, signs, symbols and words which are different from everyday language and the language of other disciplines requires teachers of this discipline to be explicit about what they say and do with the linguistic (and other) representations they use.

3.2.2 Language and variations in pedagogy

As already noted, there is both international (Bernstein, 1990) and local (Hoadley, 2007) literature that focuses on differential performance in the context of social class. In working class contexts, Hoadley presents evidence of teaching that focuses on context dependent messages which are restricted to personal experiences and which use “everyday codes”. Furthermore, Hoadley & Ensor (2009) question whether teachers’ own social class and professional dispositions have an impact on teachers’ pedagogic practices. Their study found differences in professional dispositions between teachers from middle class backgrounds and those from working class backgrounds. The differences are reflected in what teachers in these two classes prioritize in the classroom. Hoadley and Ensor et al’s findings indicate that teachers from working class backgrounds prioritize the child and the

importance of discipline whilst teachers from middle class backgrounds prioritized children's schooling experience and subject matter knowledge.

Hoadley's (2007) and Ensor et al's (2009) findings are important for this study because they study how teachers' pedagogic choices of evaluative criteria tend to perpetuate social class divisions in schooling. The present study focuses on how studying different language settings can add to our understandings of how pedagogy might contribute to the differential outcomes for learners seen in South Africa.

Language plays a significant role in mathematics classrooms some of which could be understood as political. Setati (2005) acknowledges the political role of language in mathematics classrooms but also argues that in multilingual classrooms, different languages are used for different purposes. This author identifies different types of mathematical Discourses. A Discourse, in the context of Setati's work, refers to ways in which people belonging to the same social group have agreed to use language so as to identify themselves as members of the same social network. Setati identifies four types of Discourses observable in multilingual mathematics classrooms. The two Discourses specific to mathematics are:

- the procedural Discourse, which focuses on procedural steps required to solve a mathematical problem;
- conceptual Discourse, a discourse in which reasons for calculating or solving a mathematical problem in a particular way are discussed. In such discourses, learners are often asked to share, discuss and reflect on their understanding of mathematics.

Over and above these mathematical Discourses, Setati identifies two non-mathematical Discourses. She refers to the first of these as the regulatory Discourse which is used mainly by teachers to regulate learners' behavior or to call on to learners to pay attention in the classroom. Setati identifies the second nonmathematical Discourse as the contextual Discourse. She states that contextual Discourses are mainly observable in context where teachers and learners work with word problems and focus on the context of the word-problem rather than the mathematical aspects of the problem. In her study,

Setati observed one grade 4 teacher in a multilingual mathematics classroom in South Africa over a period of 10 one-hour lessons to understand how different languages were used and for which purposes. In her study, Setati found that the Home language of learners was mainly used to regulate learners' behavior whilst English was mainly used as the language of authority to give instructions and as the language of procedural Discourse to communicate procedures or steps for solving mathematical problems. The point of Setati's study is that in this multilingual classroom English and procedural Discourses were often used together. Setati proposes that this could be the case because often in South Africa, learners learn numbers, arithmetic procedures and multiplication tables in English even where the LOLT in the school is not learners' first language.

The findings of Setati's study are important for this study as they reveal complex relationships between language of instruction and mathematics teaching particularly in multilingual contexts. Embedded in my motivation for undertaking this study was the notion that children in English and multilingual classrooms may be exposed to different kinds of pedagogic relays and messages during the teaching of early number. Hence it was important to focus on language as a contextual variable in this study. In the next section I discuss and review literature on evaluative criteria.

3.3 THE FOCUS OF THE STUDY

3.3.1 Evaluative criteria

Having introduced and discussed evaluative criteria in Chapter 2 as rules that regulate the extent to which legitimate text is made explicit to acquirers (Morais, 2002) I proceed in this chapter to introduce Bernstein's (1990) explication of different strengths of evaluative criteria (see Table 1). To recap, he argues that framing is strong when evaluative criteria are made explicit to the acquirer and framing is weak when evaluative criteria are implicit.

Table 1: Framing of the evaluative criteria

F^{++}	F^{+}	F^{-}	F^{--}
Evaluative criteria very clear and explicit	Evaluative criteria quite clear and explicit	Evaluative criteria quite unclear and implicit	Evaluative criteria very unclear and implicit

Table 1 shows various degree of framing of evaluative criteria in the classroom. The F^{++} represents the strongest framing of the evaluative criteria whilst the F^{--} represents very weak framing of the evaluative criteria (Hoadley, 2007). Morais (2002) using Bernstein's theory of pedagogic discourse (1990), argues that making the evaluative criteria explicit to learners who have limited access to the school code (elaborated code), may assist these learners to acquire the recognition and realization rules to produce the legitimate text. Morais points out that a teacher who does not make the evaluative criteria explicit does not provide learners with an opportunity to learn the legitimate text and does not support learners to give correct answers in future. Explicit communication of the evaluative criteria provides learners with an opportunity to self-evaluate and to produce the correct answer in future (Morais, 2002). Given the location of this study in classrooms serving historically disadvantaged learner populations across Sepedi and English language LOLT settings, Morais' privileging of explicit evaluative criteria was useful to bear in mind in my analysis. As outlined already, I was also alert, not just to the possibility of different strengths of transmitting the evaluative criteria, but also to Hoadley's (2007) evidence of instances in within the South African context where no evaluative criteria were transmitted at all (the F^0 classification).

Overall, this range of strengths of framings of evaluative criteria meant that it was useful, in South African classrooms, to study instruction for the ways in which 'correctness' of an answer or 'appropriateness' of a solution approach in terms of efficiency were communicated. It was also useful to look for the extent of attention to the progressions underlying communications of better and worse solution approaches.

Having discussed evaluative criteria, the next section discusses the context within which evaluative criteria are transmitted in classroom interactions between teachers and learners. This provided me with insights into the context of teacher-learner interaction, which formed the empirical base for this study.

3.3.2 Evaluative criteria within Initiation-Response-Evaluation/Feedback (I-R-E/F) patterns of classroom interaction

Evaluative criteria are offered or made available in classroom interactions. This present study pays attention to teachers' evaluation of learners' responses and analyzes these responses to understand what teachers make available for learners to learn during the teaching of number and additive relations. The study focuses on teachers interacting with learners in whole class in Initiation- Response- Evaluation/Feedback (I-R-E/F) interactions. The evaluation or feedback section of this model is seen as important to this study because it is within this part of the I-R-E/F model that teachers are likely to provide evaluative feedback and transmit the evaluative criteria relating to the mathematical ideas, concepts or strategies they want learners to develop as they teach number and additive relations (Schollar, 2008). Sinclair and Coulthard (1975) introduced the model or structure referred to as the Initiation-Response-Evaluation/Feedback (I-R-E/F) to explain the nature of classroom interactions. The I-R-E/F structure involves the teacher initiating (I) learning often by way of asking a question, followed by learners responding (R) to the question and the teacher evaluating (E) or providing feedback in various ways which could include repetition of the learner's answer, revoicing, rephrasing, summarizing, translating or stating the response in a different language.

The work of Lyster and Ranta (1997) which originates in the teaching of language is important in this study for understanding the various feedback moves teachers make when they respond to learners' offers. These authors define six types of feedback moves that teachers often use in classroom interactions. They define 'recasts' as a feedback move where the teacher reformulates all or parts of the learner's response excluding the part where the learner made an error. Clarification requests involve moves where the teacher indicates to a learner whether their response has been misunderstood by the

teacher or whether the response is ill-informed. Metalinguistic feedback refers to a combination of various ways of providing feedback such as comments or information that suggests that there is an error in the learner's response without explicitly providing the correct answer. Explicit correction refers to a situation where the teacher explicitly provides the correct answer. Elicitation strategies include a strategic pause by the teacher to allow the learner to "fill the blank" or complete a sentence. Repetition is a teacher move where the teacher repeats the learner's response/offer with a particular intonation so as to highlight the error in the response. Lyster (1998) also states that repetition can be used to show agreement, appreciation or understanding of the learner's answer.

The discussion on various Evaluation/ Feedback moves provided useful lenses for the present study to examine what teachers do or say following learners' responses. This focus on the Evaluation or Feedback section of the I-R-E/F is critical for this study as the ways in which teachers responded to learner offers served as the central unit of analysis.

While the evaluation/feedback categories worked well with the evaluative criteria theory to examine how teachers responded to learner offers, these literature bases did not provide insight on what might count as explicit evaluative criteria in the context of early number learning. In the next section, I discuss literature on mathematical content relating to early number and its learning and teaching, as this body of writing provided insights into the kinds of evaluative criteria that could usefully feature in instruction, and particularly in relation to South African evidence of lack of progression in children's early number working.

3.4. MATHEMATICAL CONTENT FOCUS OF THE STUDY

The literature base on important demarcations in early number points to two key features that are of particular interest to this study. Firstly, given the prevalence of unit counting-based approaches in South Africa, the literature on what progression looks like in children's work with additive relations and multiplicative relations is useful. An important part of the progression in additive working in particular involves working with the base ten relationships in the decimal number system, so this aspect is incorporated into the additive relations discussion. The second feature of interest relates to breadth, in terms

of the range of additive and multiplicative situations that children need to be given access to. This second feature relates to word problem tasks where situations are incorporated. Research in this second area suggests that access to the full range of situations is important if learners are going to develop flexible problem-solving skills. This research base includes attention to keyways in which problems can be made increasingly difficult. Progression and breadth of problems are both dealt with in this section

3.4.1 Progression in early number and additive relations

The literature base on early number and additive relations includes attention to the breadth of realistic situations with an additive base, and to progression in the efficiency of approaches to problem-solving.

In relation to progression, several South African studies point to problems in the teaching of early number that does not support learners in making a shift from basic counting procedures to more abstract ways of working with number Schollar (2008) and Ensor et al (2009). This section of work discusses various authors' views on how learners progress in the development of early number, additive and multiplicative relations knowledge beginning with the development of counting strategies. Steffe (1994) suggests that counting approaches are central in learning early number as the counting approaches learners use influence the choices, they make relating to problem solving later as they develop additive and multiplicative skills.

Of relevance to this section of work is the work of Wright, Martland and Stafford (2006) as they have developed frameworks that describe early number progression.

Wright et al (2006) developed the Learning Framework in Number (LFIN). This Framework comprises of four parts which are Counting, Forward Number Word Sequences, Backward Number Word Sequences and Numeral Identification. At the centre of the LFIN, Wright et al (2006) present Stages in Early Arithmetic Learning (SEAL) which make up part A of this framework. The SEAL stages outline a progression in the development of counting approaches that learners follow in the development of early arithmetical strategies – detailed in Table 2:

Table 2: Stages of early arithmetical learning (SEAL)

Stages	Names	Stage descriptors
0	Emergent counting	A learner who operates at this stage cannot count visible objects
1	Perceptual counting	Can count perceived objects
2	Figurative counting	A learner can count screened objects but when asked to combine objects from two sets, counts from one. 'Counting all' approaches characterize this stage.
3	Initial number sequence	A learner who operates at this stage is able to 'count on' when confronted with the task of combining two sets of objects and 'count down from' the first number when solving subtraction tasks. However, a learner at this stage cannot 'count-down to' the second number in order to solve subtraction tasks like 17-14 more efficiently.
4	Intermediate number sequence	A learner who operates at this level can use count-down to and count down from approaches as appropriate for efficiency in order to solve subtraction tasks
5	Facile number sequence	Can use partitioning by 5 or 10 and compensation and other approaches that do not include counting by ones to solve additive relation tasks.

Wright et al (2006)

In addition to the progression and development of counting approaches in the SEAL outlined in Table 2, Wright et al (2006) identify a domain of number development which includes working with base 10 relationships in the decimal number system. In this part of

the framework, Wright et al identify three levels of Base Ten Arithmetical approaches that learners go through detailed in Table 3:

Table 3: Base Ten arithmetical approaches

Level	Names	Level descriptors
1	Initial concept of ten	A learner at this level does not see ten as a unit but sees ten as ten individual items. When confronted with addition or subtraction tasks involving ten, these learners can count backward or forward by ones
2	Intermediate concept of ten	Ten is understood as a unit. A learner at this level can perform addition and subtraction tasks using strips of tens and ones. However, learners at this level cannot solve addition and subtraction tasks involving tens and ones in written number sentence format.
3	Facile concept of ten	A learner at this level can solve addition and subtraction tasks involving tens and ones without using materials or representations.

Wright et al (2006)

This discussion deals with progression in the context of working with number through attending to base ten thinking and place value in early additive situations via increasingly efficient strategies that make use of number relationships. The work of Wright et al (2006) provides a framework to understand the various approaches that learners progress through as they learn early number, but more importantly for this study, this work provided indications of the progression that instruction should focus on, and therefore, emphases on evaluative criteria that supported moves beyond counting.

Carpenter and Moser (1984) provided some ideas of the different types of addition and subtraction problems that teachers are likely to choose from when they teach early number. In terms of breadth Carpenter and Moser (1984) identify four types of additive relation problems whilst Askew (2005) identified sub-categories of each of these problem types. These authors also identify different strategies relating to these problem types in a progression that includes moves to more abstract representations and into the more efficient problem-solving strategies described above. The different problem types are the join or change increase type, separate change decrease type, part-part whole or combine type as well as the compare type of problems. Each problem type has sub-categories based on the position of quantities referred to as unknown.

Join change increase

The action of joining increases the number in the set or a situation where two sets are joined together to create a new set. For instance, □ + 7= 12 e.g. Kate has some apples. Her mom gave her 7 more apples. Now she has 12 apples. How many did she start with?

Separate change decrease

The separate type of problem involves a context where the quantity is removed from the given quantity or set. This type of problems depicts the act of taking away or decreasing a given quantity. □ - 8 =7. E.g. Corin bought some marbles. He gave away 8 marbles. Now he has 7 marbles left. How many marbles did he buy?

Part–part whole or combine type:

Part-part–whole are problem types where the whole and one part are known. The part-part whole type of problems are characterised by “no” action. Unlike the join and the separate types of problems involving actions of increasing or decreasing quantities, part-part whole type of problems involve a static relationship between a whole and its two parts, e.g. □= 17 + 8.

There are apples in a fruit basket. 17 are red and 8 are green. How many apples are in the fruit basket altogether?

Compare type

The compare type problems involve a situation where two quantities are compared. These problem types require one to find the difference between two quantities. For instance, 8 – 3 =□ e.g. Herman has 8 apples. Lawan has 3 apples less than Herman. How many apples does Lawan have?

The trajectory of strategic efficiency in solving word problems across these situation types is linked to van den Heuvel-Panhuizen's trajectory described above. Carpenter, Moser and Bebout (1988) argue that learners use three levels of strategies to solve tasks involving addition and subtraction problems. Firstly, they use Direct Modeling as an approach (DM). This approach involves the physical use of concrete objects or counters to represent numbers in the problem followed by physical counting of the objects to arrive at the answer. Learners who use modelling will solve the task 9+5 by taking out 9 counters and a set of 5 counters, joining these together and counting the two sets combined. Carpenter suggests that the modeling strategy is the most rudimentary strategy often used by young learners in kindergarten (Carpenter & Moser, 1984) . Following from modelling, learners progress to use various counting based approaches. Researchers such as Thompson (1999) examined addition and subtraction approaches learners use to solve mathematical problems. Thompson argues that early number teaching supports learners to progress from counting to addition and subtraction. In his earlier work Thompson (1999) identified more counting approaches in line with van den van den Heuvel-Panhuizen (2001) and Wright et al (2006). Thompson separates approaches learners use into counting and calculation approaches. He suggests a hierarchy of addition approaches which begin with counting all, counting on, counting from a larger number to using recall and derived facts. Whilst Askew draws a distinction between recall and derived facts. Askew (2013) points out that known facts could be spontaneous whereas derived facts are scientifically developed. Spontaneous number facts could mean that learners come to school knowing facts like two plus two is four or five doubled is ten. This knowledge is regarded as spontaneous because learners have developed it through their everyday interactions. Derived facts are referred to as

scientifically developed number facts because learners require some deliberate instruction in order to develop and use these derived facts. Thompson (1999) emphasizes a lack of linearity in moving from counting to calculation approaches as learners who are regarded as competent in using calculation approaches often revert to using counting when solving certain mathematical problems. For clarity, strategic efficiency within the counting based approaches is separated for addition and subtraction below before summarizing the calculation and derived based approaches.

Addition counting based approaches

Carpenter, Fennema, Peterson and Carey (1988) and Thompson (1999) identify three counting approaches learners use to solve addition problems. The first counting-based approach is counting all. This approach is regarded as the most elementary counting approach that involves counting out two sets of objects and counting all objects in both sets to determine the total number of objects. This approach is similar to what Carpenter et al (1988) refers to as direct modelling. The second counting-based approach learners use is called counting on. This approach involves counting objects in two sets by counting on objects from the second set without beginning from one. The third approach is called counting on from a larger number. This approach involves identification of the larger number in the expression and beginning to count on from that number in order to solve additive tasks. Different counting-based approaches for subtraction are taught in the early years of studying number.

Subtraction counting based approaches

Four subtraction counting based approaches are identified by Carpenter et al (1988). The first subtraction counting approach is direct modeling, followed by counting back from. When using this approach learners count back from the first number. E.g. 8 – 3. A learner would count from 8…, 7, 6, and 5 in order to arrive at 5 as the answer. Clements and Sarama (2009) argue that this way of counting relates to the process of “taking away” which is a notion of subtraction. Thompson (2010) reports that counting back from approaches demand learners to be able to count backwards from a given number, and to

be able to keep track of the steps. The third subtraction approach these authors highlight is counting back to. In order to solve a problem such as 8 – 5 learners would count back from 8…., 7, 6, 5, and count the numbers they have counted in order to arrive at the answer. In this context, subtraction is viewed in terms of difference between two terms rather than taking away the second term. The fourth subtraction counting based approach is called counting up from. When using this approach to solve a problem like 8 – 6, learners begin by counting up from the smaller number e.g. 6: 7, 8 in order to arrive at 2 as the answer. Kilpatrick, Swafford and Findell (2001) suggest that it is not only important for learners to learn and remember procedures or approaches required to produce answers. Learners need to know whether the approach or procedure they chose yielded the correct answer or not. This implies that learners need to know how to check and verify their answers. These authors suggest that knowledge of working with number, which approaches and procedures to use and when to use them would lead to the development of procedural fluency.

Calculation & derived approaches

Ellemor-Collins and Wright (2009) sugget that when learning number and additive relations, learners progress from using calculation by counting approahes to use calculation by structuring to a point where they can use formal calculations and automatisatopn or derived approaches. These authors argue that structuring number serves as a bridge from calculation by counting approahes to formal facile calculations. In addition, these authors suggest that structuring number refers to organizing of number in more formal ways, it involves establishment of a sense of regularity in numbers and relating numbers to other numbers. Two structuring models are presented by Ellemor-Collins and Wright, (2009). These structuring approaches include base 10 or decade based approaches starting from the range of 1 to 10 then to 20. This approach involves working with decade numbers like 10, 20, 30, and 40 as reference points. The second structuring model is referred to as collection-based number structuring which involves constructing numbers in terms of collections of ones, tens or hundreds. For instance, a number like 38 can be seen as thirty and eight or 3 tens and 8 ones.

Whilst many low attaining learners struggle to progress from counting based approaches to more efficient calculation approaches (Ellemor-Collins & Wright, 2009), learners experience other challenges when working with number. Clements & Sarama suggest that the number range used in a problem, the type of a problem or structure and the position of the unknown variable in the problem determine the level of complexity of addition and subtraction problems. Extending the number range to include multidigit numbers has been found to make problems more difficult for young learners (Clements and Sarama; 2009). Anghileri, Beishuizen and Van Putten (2002) describe the different sub-categories of types of addition and subtraction problems. These sub-types include problems where the position of the unknown variable is varied for instance a type where the start is unknown □ + 5 = 12, the second sub-type involves a context where the change is unknown 7 +□ =12. The third sub-type involves tasks where the result is unknown such as 7 + 5 =□. These authors suggest that the number range used in a problem, the type of a problem or structure and the position of the unknown variable in the problem determine the level of complexity of addition and subtraction problems. For instance, the result unknown types of problems are usually easier whilst change unknown are moderately difficult and start unknown types has been found to be the most difficult for learners to solve. This sequencing of different types of problems suggests that teachers need to give learners access to the breadth of problems and allow them to progress from easier to harder versions, pointing out the differences between them through the evaluative criteria. In additive situations, working with structure involves recognizing and working with part-whole relationships, rather than only with counted quantities (Björklund & Runesson, 2019). The work of van den Heuvel Panhuizen (2001) overlaps with the progression in number learning outlined by Wright et al (2006), and offers the following broad sequence: counting, calculation by counting, calculating by structuring and flexible calculation as follows:

Level1: Counting.

This author describes various levels of counting that learners go through as they develop number knowledge. These levels of counting include context-based counting where

learners need to see and understand the context so they can tell how many objects. Learners at this level cannot tell “how many objects” without seeing and understanding the context.

Level 2: Calculating by counting

The second level (van den Heuvel-Panhuizen, 2001) describes is referred to as calculation by counting. At this level learners demonstrate the ability to use count all strategies. For instance, when given a problem like 3 + 4 = □. The learners will layout 3 counters and 4 counters. The learner will then count the counters one by one to produce the answer. Learners progress within this level 2 to use the count on strategy. For instance, when confronted with the problem 3 + 4 = □. The learner might say that they will keep 4 in their head and count on three more to derive 7 as the answer.

Level 3: Calculating by structuring

In this level, learners use their knowledge of the decimal structure, in particular, to derive answers. For instance, when given a problem like 46 + 17 =□, the learner can breakdown the two numbers into tens and units 40 + 10; 6 + 7. She will then add the tens together and the units together as follows: 50 + 13

$$50 + 10 + 3 = 63$$

Level 4: Flexible calculating

Learners who operate at this level use knowledge of number relations and properties of operations to derive answers efficiently. For instance, when confronted with a problem like 64 – 29 a learner can round up 29 to 30, and then subtract 30 from 64 to get 34 and add back the one to arrive at 35.

The work of van den Heuvel –Panhuizen (2001) and Wright et al (2006) provided this study with the ideas on how learners progress when developing approaches to solve additive and multiplicative relations tasks, and the kinds of efficiencies and generalisations that instruction needed to distinguish through evaluative criteria.

The subsequent section discusses breadth and progression in the context of multiplicative relations tasks.

3.4.2 Multiplicative relations

Multiplication is seen in the literature as different from addition and subtraction, in spite of the fact that repeated addition can be used to solve multiplication problems. Addition and subtraction are unary operations with each input representing the same kind of element such as 5 marbles and 3 marbles. Unlike addition, multiplication is a binary operation with two different inputs (multiplier and the multiplicand) representing the size of a set and the number of times that set is replicated (Askew, 2018).

Four types of multiplicative reasoning problems are identified in the literature in relation to early multiplication/division in the context of whole number working (Anghileri, 2006).

Table 4: Types of multiplicative relations problems

Type of problem	Example
Equivalent groups	4 groups of 5 marbles. How many marbles altogether
Multiplicative comparisons	Jim is 13 years old. His father is four times older than him. How old is Jim's father?
Rectangular arrays	4 rows of 3 apples. How many apples altogether
Cartesian product	Kate has 4 skirts and three tops in her bag. How many different outfits can she make from her back?

In addition to these multiplication problem types outlined in Table 4, the literature identifies two situations of division which are partitioning and equal groups, also described as quotitive or subtractive situations (Haylock & Manning, 2014).

Table 5: Division situations as described by Anghileri (2006).

Division situation	Example
Partitive	20 marbles are shared equally between 4 children. How many will each child get? (how many in each portion)
Subtractive	How many groups of 4 marbles are there in a set of 20 marbles? (How many times can you subtract 4 from 20)

Multiplication and division approaches can be developed by first teaching learners to share and group numbers into equal groups. Wright et al (2006) discuss a framework for developing early multiplication and division knowledge which has 5 levels as follows:

Initial grouping: A learner who operates at this level uses perceptual counting to establish the total number of a collection of equal groups or to share items into groups of given sizes. At this stage of development, a learner needs perceptual counting involving counting in ones to produce a group or collection.

Perceptual counting in multiples: A learner who operates at this level is able to produce the answer through the use of multiplicative counting approaches, i.e. counting in multiples of fives or twos.

Figurative composite grouping: A learner who uses this composite grouping is able to use a multiplicative counting strategy to count items arranged in equal groups even when the individual items are not available. For instance, when presented with a screened collection of four groups of five counters, the learner can produce the answers by counting in multiples of five, 5, 10, 15, and 20. There are 20 counters.

Repeated abstract composite grouping: A learner who operates at this stage can use repeated addition to solve multiplication tasks and repeated subtraction to solve division related tasks and can do so in the absence of visible or screened objects.

Multiplication and division as operations: A learner who operates at this fifth level can recall or quickly derive basic facts for multiplication and division. Additionally, a learner who operates at this level is able use commutative principle of multiplication and the

inverse relationship between multiplication and division that is within his level of development.

Across early number additive and multiplicative situations, the work of van den Heuvel-Panhuizen (2001) and Wright et al (2006) discussed in this chapter suggests the need for learners to progress from using simple counting strategies to use more efficient and effective calculation approaches and number facts to produce answers. Learners' ability to progress from counting to calculation-based approaches and from concrete to more abstract ways of representing number has been identified as a challenge in South African classrooms. The work of Schollar (2008) and Ensor et al (2009) attest to this problem. The importance of learners' ability to develop and use counting and calculation approaches to solve tasks is emphasized in the literature (Askew, 2005; Wright et al., 2006) and in the CAPS (Department of Basic Education, 2011) which states that children should:

> *"solve different types of problems and record their responses in a systematic way, record calculations using plus (+), minus (-) and equals sign (=), explain their answers and describe their methods, work with formal ways to record addition and subtraction calculations, choose strategies from a range of other strategies in order to solve problems",* (Department of Basic Education, 2011).

It is important to note that both breadth and progression in approaches feed into the idea of increasingly 'general' approaches in mathematics. This is because being able to deal with a wider range of problems and being able to work with more efficient problem-solving approaches allows a wider range of problems to be solved effectively and efficiently. To bring generality into play, evaluative criteria would need to mark distinctions between less and more efficient approaches and between different problem types. Of importance, once again, in this progression is that the move towards more efficient calculation approaches is accompanied by moves towards increasingly abstract symbolic representations. Moves between representations and progression of representations are considered in the next section of this chapter.

3.5 MATHEMATICAL MODELS AND REPRESENTATIONS

3.5.1 Defining mathematical models and modeling

A mathematical model is a representation of some situation or object of interest. The model captures, simulates or represents selected features or behaviours of the thing without being the thing. A mathematical model functions as an instrument and a predictive tool. As an instrument, a mathematical model enables users to see different aspects that they could not see before. As a predictive tool, mathematical models enable users to predict what they will see next (Lesh, Post & Behr, 1987). Models and representations are not the same things but the two are related. Models are described as ways of thinking about abstract concepts and representations are described as various forms of models (Warren & Cooper, 2009). Models and modeling perspectives in the mathematics education literature suggest that to have sufficient power for dealing with complex problem solving situations, concepts should be expressed using a variety of interacting media including spoken language, written symbols and diagrams.

Various authors suggest different ways of working with models and representations. The next section discusses key ways in which representations can feature in primary mathematics teaching, encompassing the need for connections between representations, moves between them, and progressions towards more abstract modes of representation.

However, in the multilingual context of most Gauteng classrooms, moves between representations also occurred through moves between Sepedi and English (the sociopolitical environment described in Chapter 1 meant that moves from English to African home languages were very rare). Therefore, I use literature on the ways in which moves between languages are described as resources for learning, to point to further avenues for the productive use of evaluative criteria to support meaning-making in early number.

3.5.2 Connections between experiences

Haylock and Manning (2014) suggest that understanding mathematics requires teachers to make connections between various experiences. These experiences include language, concrete experiences, pictures and symbols. Connections between mathematical

experiences require that teachers use language and other forms of communication such as gestures and scribbles to explicitly communicate these mathematical connections and relationships to learners. Haylock and Cockburn's (2008) view of connections, as reflected in their model, emphasizes the importance of making connections between multiple representations.

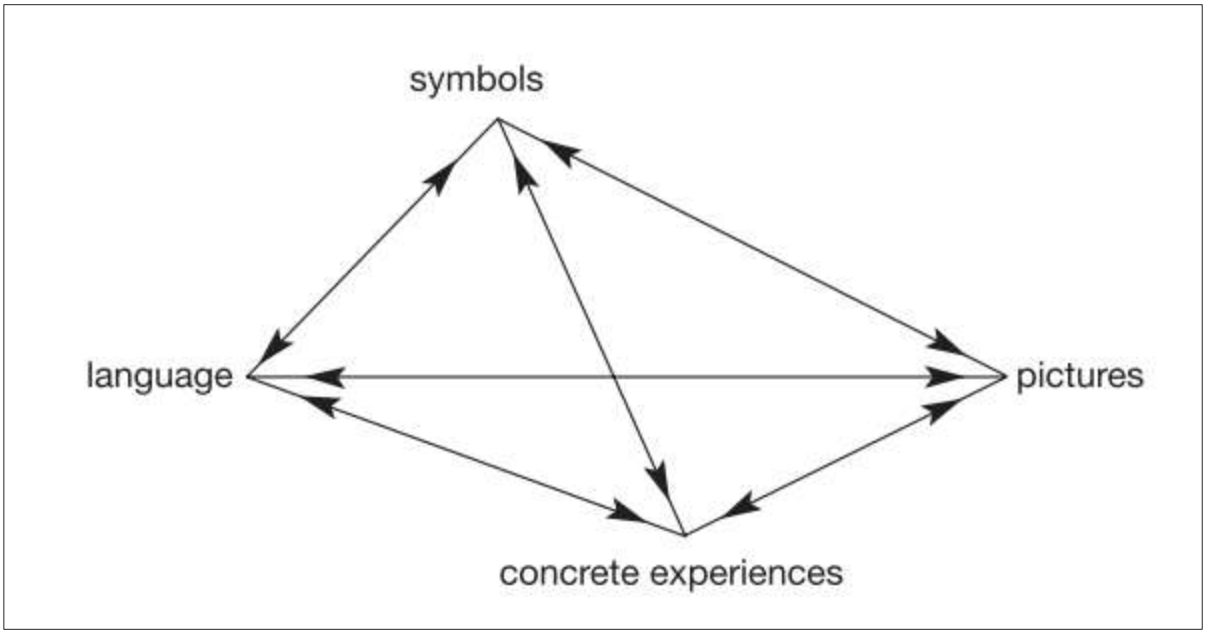


Figure 1: Mathematical connections (Haylock & Cockburn, 2008)

Experiences with concrete apparatus, pictures and symbols as reflected in Figure 1 have been widely used in the foundation phase to support learners to make sense of mathematics. This model suggests that language is critical in making explicit the connections between the various experiences. The development of abstract mathematical thinking requires careful guidance in which teacher talk and writing play an important role (Sfard, 2008) . The point of interest here is that language can be used with other verbal and nonverbal forms of language to communicate meaning in mathematics teaching in order to support learners develop mathematical knowledge. This study seeks to understand how teachers use language representations, and other forms of nonverbal communication in the relay of the evaluative criteria during the teaching of number and additive relations. Whilst the literature discussed in in this section has focused on making

connections between various experiences, there is a cluster of literature that focuses on the importance of moves between representations. The next section discusses literature on using multiple representations and on making moves between these representations.

3.5.3 Moves between representations

Mathematical modeling is a process of translating between the real world and mathematics (Lesh et al., 2003). Scholars in this field suggest that moving between representations is an important criterion in and of itself and it is an intrinsic part of the legitimate text in mathematics classrooms where this idea is seen as important. Warren and Cooper (2009) argue that mathematical understanding is reflected through the number and strength of connections in learners' internal network of models and representations. Like Haylock & Cockburn (2008) the argument made by Warren and Cooper (2009) is that it is not enough for learners to simply use various models and representations, but it is important for them to move between representations and make connections. However, these authors argue that moving between representations and making connections between multiple representations requires careful instruction.

Duval (2006) and Gagatsis, Elia and Mousoulides (2006) argue that the presence of multiple representations and moves between representations is not a guarantee that learners will identify the target mathematical knowledge the teacher wants them to acquire. These authors investigated the abilities of 79 grade 6 learners' in Cyprus to move between multiple representations. They were interested to test two models and the assumptions of these models relating to representations and moves between representations. The first model was based on the assumption that multiple representations and moves from one representation to another can be sufficiently explicit for learners to abstract the target mathematical knowledge and use representations to produce expected answers to tasks. The second model was based on the assumption that different representations show different aspects of mathematical knowledge that may not assist learners to abstract the targeted mathematical knowledge if the moves between representations were not taught explicitly. In their investigation, these authors provided learners with mathematical problems using one mode of representation and asked

learners to solve the problem using a different representational mode. The problems they created for their study involved tasks where graphs, tables, verbal and symbolic representations were used. The findings of their study supported the second model revealing that learners struggled with moves between visual representations/iconic modes particularly where they were required to move from the iconic to graphical and symbolic modes of representations. Based on the findings of their study, Gagatsis et al (2006) argued that mathematical representations can be placed in a hierarchy because some representations have characteristics of other representations and can be used to assist learners to move between representations and to understand more complex representations. For instance, verbal and tabular representations have some characteristics of graphs and can be used initially to help learners move between these representations to use graphical representations. These authors further argued that that moves between representations required special attention during teaching and learning.

Gagatsis et al (2006) and Duval (2006) argue that mathematical understanding is developed through moves between mathematical representations or registers. Duval identified four types of registers which are natural language, diagrams, symbols and graphs. This author further identified two types of moves between registers which are treatment and conversion moves. Treatment moves refer to when a learner stays within one register, e.g solving a mathematical task like 24 + 8 = A learner can solve the task by staying within the symbolic number register, either in horizontal or column algorithm format.

Treatment moves mainly involve manipulation of symbols to represent the same idea while remaining within the same symbolic mode of representation. In contrast, Duval defines conversion moves as moves between registers e.g solving a mathematical problem like 24 + 8 a learner can solve the problem by using s number line and bring about a transformation between the symbolic and diagrammatic registers. This type of move is referred to as the conversion move. Duval (2006) suggests that conversion moves are harder than treatment moves because conversion moves require one to recognise ‘sameness’ or structural similarity in two representations which very often do

not look similar. Bringing to the fore the centrality of moves between representations and the complex nature of conversion moves suggests that the two moves can be placed in a hierarchical order with the conversion moves on a higher level than treatment moves. In classrooms, instruction observed in this study often involved moves between symbolic and oral linguistic registers. In multilingual classrooms there was an additional layer available involving moves between languages, an issue that is dealt with separately later in this section.

Duval's claim about the importance of moves between representations and the complex nature of conversion moves is supported by a study conducted by Berg (2013) with 52 primary mathematics student teachers in Norway where this researcher wanted to test Duval's theory of moves between representations and in particular the claim that conversion moves are more complex moves. In this study Berg introduced the students teachers to a task that required them to represent the idea of $3n^2$ using a sentence with words and with geometric shapes. Student teachers worked in groups to solve the task and were later interviewed about their views of the task. Three groups of responses emerged. Two groups of students were positive about the task. The first group of 39 student teachers, who were able to move between representations, stated that that the task was interesting because it allowed them to move between different representations which offered them opportunities to see connections between diffent represenational modes. They also noted that the task allowed them to use various representations to check and justify why their answers were correct. The second group comprising of 11 students, had difficulties with making moves between representations and particularly geometric representations, and generally had lower mathematical knowledge than the first group. The third group comprising of 2 students pointed out that the task was difficult but could not offer an explanation of what made the task difficult for them.

Berg's study is important for the present study because its findings suggest that conversion moves are interesting but challenging. The findings also suggest that conversion moves between verbal expressions (words) and symbols are not as difficult as moves between verbal expressions, symbols and geometrical shapes (iconic or

drawings). In addition, the findings of this study suggest that multiple representations and conversion moves between these representations provide opportunities for learners to see connections between representations and ideas. Futhermore, these findings suggest that robust mathematical knowledge is required for one to see connections between different representations that may not look the same but represent the same mathematical idea (Berg, 2013).

Whilst the previous section discussed moves between representations within the context of mathematics, the next section discusses moves between representations within the context of multilingual classrooms.

3.5.4 Multilingual moves between representations

The present study focuses on evaluative criteria and how representations feature in what teachers transmit as they teach addition and subtraction in Sepedi and English medium classrooms. For this reason, it became important for the study to understand the role of language in representational moves. Two types of multilingual move, namely translation and translanguaging were identified from the language literature. The literature on multilingual pedagogic practices acknowledges the usefulness of these two ways of moving between languages. Makalela (2016), Hornberger and Link (2012) and Creese and Blackledge (2010) provide concepts that are useful for identifying and describing these practices and their usefulness in learning a language. This literature refers to the simultaneous use of two or more languages or language varieties as a resource available to multilingual teachers for making meaning available to learners. These authors make a distinction between these two multilingual practices. Here, I distinguish between the two practices to understand how they feature in what teachers transmit as the evaluative criteria in multilingual classrooms.

Translation/substitution is a practice used by multilingual individuals to move between two languages. This practice entails the substitution of one word or phrase in one language with a phrase/word in another language (Baker, 2011; Childs, 2016). It entails repetition of an oral or written text in more accessible home language of learners. Adler (1999) defines this move between languages as code switching. She defines code switching as

an alternation in use of more than one language in a single speech act. It is often responsive and unplanned. On the other hand, translanguaging centrally involves purposeful alternation of languages in spoken and written forms (Garcia & Wei 2014).This definition of translanguaging suggests that translanguaging moves include moves between languages, spoken and written registers and focuses on how the language user draws upon different linguistic, cognitive and semiotic resources to make meaning and sense (Garcia & Wei, 2014). It is a systematic and planned use of the home language of the learners with the language of the classroom in order to foster teaching and learning (Childs, 2016).

Makalela (2016) argues that translanguaging can be used as a systematic tool to promote learning in multilingual classrooms. He conducted a study with 60 student teachers at Wits School of Education in order to investigate the effectiveness of translanguaging practices. The home languages of these students included IsiZulu, IsiSwati and IsiNdebele. These 60 students were registered to study Sepedi because the university required students to study a new language they had not learned at school or that was not their home language. He separated the students into an experimental group involving 30 students and a control group involving 30 students. The experimental group was exposed to translanguaging practices using Sepedi, IsiZulu, IsiXhosa, IsiSwati and IsiNdebele. The students in the experimental group were encouraged to move between different languages. This group was also encouraged to write language blogs and to join a Facebook group where they communicated with others in Sepedi and English. The control group on the other hand was not exposed to the translanguaging practice available to the experimental group. The two groups wrote pre and posttests in Sepedi. 12 students were also interviewed. The analysis of the findings showed that students in the experimental group outperformed the students in the control group in vocabulary learning and reading proficiency.

Translanguaging moves involved moving between oral languages (English and Sepedi) coupled with the offer of written or diagrammatic representations, offering both bilingual and multimodal access. In the context of mathematical moves, a move between oral and

written representations is classified as a conversion move. In the context of this study where translanguaging is understood as purposeful alternation of languages in spoken and written forms coupled with a variety of modes of representations. I argue that translanguaging moves are more like conversion moves because they involve elaborated explanations which bring about connection between the written and the oral language. The following text provides an example of a translation move and a translanguaging move in the context of Sepedi and English languages seen in this study (square brackets are used in transcript sections to provide translation/explanation of verbatim speech from Sepedi to English):

Example 1: Translation move between English and Sepedi.

1. T: Ten take away four? Lesome ntsha nne? [Task is stated orally in English and immediately translated into Sepedi].
2. L1: Four.
3. T: Nne? [Teacher repeats the offer of 'four' in Sepedi with an inflection]. Ten take away four? Can it be four? Lesome ntsha nne? We continue. [T moves to next task.]

In line one, the teacher stated the task in English. She then immediately stated the task Sepedi. In line two, the learner offered four in English as the answer. In line three the teacher substituted the number word four in English with the corresponding number word "nne" in Sepedi. In this interaction, moves where the teacher substitutes one word or phrase in one language with another in a different language (translation moves) are observed.

Example 2: Translanguaging moves

1. T: {takes out number cards 100, 30 and 5 from a container on her table. Gives each learner a number card with a symbol written on it. 30 to Jabulani, 5 to Relebogile and 100 to Thabo. She rearranges the learners from the left to the right as 100, 30 and 5} Lekgolo- masometharo- hlano {'Hundred, tens three, five.' Verbalises the number name in Sepedi}. What number is this? {Moves her hand to point to all three children}

2. Ls: Lekgolo masometharo- hlano. [Hundred tens three five]

3. T: All I wanted to explain to you with this activity is the place value of numbers. Place value of numbers. {Points to 3 in the number 135 on the board}. This is not just 3. Ke masometharo, ['It is tens three', pointing to 3 in the number 135 on the board]. It stands for 3 tens. {Holds Jabulani who has the tag masome/tens}. The one stands for hundred. Ke lekgolo. {Points to 1 in 135 and moves to hold Thabo who has the hundred/ makgolo

name tag}. It is not just one. It is one hundred. Five stands for units. It is five. Do we understand each other?

In the example above, the teacher is explaining the concept of place value. She called three learners to the front of the class and allocated a number card to each learner. She asked learners to read out the number name in line one as she moved her hand to point to all three learners. A learner provided the answer in Sepedi as “lekgolo -masome tharo-hlano” (135). The teacher moved to explain the purpose of the task in English and in Sepedi. In this explanation the teacher does not only substitute a word or phrase with a corresponding word or phrase in another language. She makes connections between the learners representing the numbers, the number cards and the language as she explains how the numbers are structured and the principle of place value. This multimodal way of moving between the two languages, involving pre-prepared resources, suggests planned instruction and goes beyond substitution. In the context of this study, I refer to such moves as translanguaging moves which, as noted earlier, are more akin to conversion moves in Duval’s theory of mathematical moves between registers.

In addition to the importance of connections between multiple representations (Haylock & Cockburn, 2008; Klein, Beishuizen & Treffers, 1998) and of moves between representation (Duval, 2006) there is the literature base that focuses on progression in representational modes in moves from concrete to more abstract ways of representing number (Hoadley, 2008; Piaget, 1964).

3.5.5 Progression from concrete to more abstract ways of representing number

Moves to more efficient approaches of working with early number are accompanied by moves to more abstract ways of representing number. Hiebert and Wearne (1993) and Klein et al., (1998) provide guidelines on how the use representations or models to support learners to progress from using basic counting approaches to use more sophisticated approaches of solving additive and multiplicative relations problems. These authors suggest instruction that maps progression from basic counting approaches to structuring number and eventually working with tens whilst at the same time learners are

supported to move from using concrete objects such as unifix cubes, to Base –Ten blocks or pictures of these blocks, and 100 charts and empty number lines to written symbols.

Ensor et al (2009) outline what this progression from concrete unit counting to more sophisticated calculation approaches and from concrete number representation to more abstract ways of representing number consists of in a study considering the nature of the extent of specialisations across approaches and modes of representations. Ensor et al (2009) argue that specialization of content and modes of representation is important for supporting the shift from context dependent concrete ways of working with number to more abstract ways of working with number. These authors suggest that specialization of content involves shifts from counting through to calculation without counting. Secondly, they argue that specialization of modes of representations require increased specialization of verbal teacher communication or teacher talk, inscriptions on the board, gestures and specialization of materials or apparatus used. They describe specialization of modes of representation in terms of moves from more concrete representations towards symbolic and syntactical representations of number (Ensor et al, 2009). These authors outline various forms of number representations used in some of the classrooms they observed in. The first form of number representation they observed is the concrete mode of representation. This form of representation involves manipulation of concrete apparatus or physical objects. The second form of number representation involves the iconic form of representation which includes the use of pictures, drawings or cartoons to represent number. The indexical form of number representation involves the use of drawings, dots or sticks to represent everyday objects. For instance, the use of dots to represent the number of sweets is referred to as the indexical form of representation. The fourth form of number representation is referred to as the symbolic- number based. This form of representation involves the use of number lines, number charts and number cards to represent number. The fifth form of number representation is referred to as the symbolic syntactical which refers to the use of mathematical notations and statements. The final and sixth form of number representation is the no representation used where learners have internalized the representation (Ensor et al., 2009).

The discussion about the progression in the development of early number and moves from counting based to more sophisticated calculating approaches as well as the progression from concrete to more abstract ways of representing number is valuable for this study to understand what teachers could usefully transmit as the evaluative criteria during the teaching of early number to support increasingly general, and therefore more widely applicable, problem-solving approaches. Therefore, the literature provided a perspective from which to consider the actual evaluative criteria that were transmitted and how ideas about generality and increasingly abstract modes of representation actually figured in the empirical data.

Whilst the previous section discussed mathematical content, concepts and progression relating to early number and modes of representations, the subsequent section discusses problems relating to the teaching of early number within the South African context that led to a focus on evaluative criteria incorporating attention to generality and modes of representation. The problems in the teaching of early number within the SA context are clustered and discussed in 3.6 under the central topics of this study which are evaluative criteria, progression in early number development and representations.

3.6 MATHEMATICS TEACHING IN SOUTH AFRICA

3.6.1 Evaluative criteria

Hoadley (2008) and Hoadley (2007) study discussed in this study pointed to the absence of the evaluative criteria and the prevalence of procedural rather than principling evaluative criteria in the South African context. Venkat and Naidoo (2012) studied connections and coherence in mathematics teaching within the South African context. These researchers took one grade 2 lesson from each of the ten schools in their study and analyzed the lesson using the notions of coherence and connections as defined in their work. In this investigation these authors noted disconnections between a range of classroom aspects involving teacher talk, activities, teacher explanations and materials being used to support learning. The findings of this analysis pointed to instances of ambiguity in teacher talk. Secondly, Venkat and Naidoo noted the use of various representations to represent concepts. They identified instances of teacher talk that

lacked a focus on ideas of equivalence presented by different modes of representation during teaching. Furthermore, Venkat and Naidoo found a lack of attention to mathematics embedded in activities. This lack of attention to the mathematics embedded in tasks was coupled with a lack of structure in ways in which examples were sequenced. In addition, these authors observed a lack of connections across episodes as well as between episodes. Finally, these researchers observed pedagogy where links were not made between what learners had learned previously and the new tasks introduced. Venkat and Naidoo (2012) noted that every example or task that entered the classroom space was treated as a separate task or example with no connections being made to the previous tasks or ideas dealt with. These authors referred to the practice where each example that entered the instructional space was treated as a separate example and the use of concrete counting approaches to produce or verify answers as "extreme localization" (Venkat & Naidoo, 2012). Similarly, Ekdahl, Venkat, Runesson and Askew (2018) investigated the teaching of three grade 3 teachers with a view to exploring the nature and the extent of connections teachers made during the teaching of additive relations. Findings from Ekdahl et al's (2018) study point to a lack of pedagogy that focuses on connections within and between examples and representations.

Findings of Hoadley (2007, 2008), Venkat and Naidoo (2012) and Ekdahl et al (2018) point to a problem of pedagogy relating to the evaluative criteria, a lack of coherence and connection as well as a lack of connections within and between examples and representations. These problems of pedagogy within South African classrooms could lead to a disruption in the transmission of the evaluative criteria.

3.6.2. Progression: Structure, generality and representations

In the South African context teaching that is underpinned by a lack of pedagogy that focuses on number relationships, structure and progression was noted. For instance, Kuhne, van den Heuvel- Panhuizen and Ensor (2004) pointed out that teachers do not have a common conception of how learners progress in the development of early number. Kuhne et al's (2004) undertook an exploratory study with 6 Foundation Phase teachers (GradeR-3) in the Western Cape South Africa. The aim of the study was to encourage

teachers to reflect on and express their perceptions about aspects of teaching and learning early number. The findings of this study showed that teachers had some understanding of number development. However, this understanding lacked an understanding of specific levels or stages that learners go through when they learn early number. In addition, this study showed that teachers had some understanding of the importance of using various forms of representations in order to facilitate the move from concrete to abstract ways of number representations. This understanding showed an understanding of progression in terms of moves from concrete ways of working with number but lacked an understanding of progression in terms of moving from particular understandings to more generalized ways of thinking and working with number. Development of learners' abstract thinking requires careful planning and sequencing of materials and models from which learners can abstract the target mathematical knowledge the teacher wants them to learn. The work of Warren and Cooper (2005, 2009) suggests a teaching and learning trajectory about representations and models that supports abstraction and generalization.

The findings of Kuhne at al (2004) study point to a need for teachers to understand the progression in learning number in terms of a pedagogy that is underpinned by a focus on structure, generality so they can make a long term plan for supporting learners to progress from one level of understanding to another.

Watson and Mason (2005) argue that mathematical thinking is developed through teaching approaches that focus on mathematical structure and generality. These authors advocate the use of variation theory to plan, select and sequence examples for the development of mathematical structure and generality in both teaching and learning. The practice advocated by these authors involves a choice of mathematical examples including contexts where certain aspects of an example are changed (varied) and others are kept the same (unvaried). The principle of variation theory and a focus on structure and generality can be applied in the teaching of additive relations using part-part-whole relationships as shown by the example taken from the work of Ekdahl et al (2018). For instance, 10- □ =7 can be varied to 10 =□ -7. In these two examples the same quantities

are used but the quantities are positioned in various parts of the example. For instance, in the first example, 10 is positioned as a whole with one part whilst another part is unknown. In the second example, 10 is presented in a position where it is not a whole but as a part where the whole is unknown (Ekdahl et al, 2018). These authors suggest that such examples can aid learners to discern the relationship by focusing on the structure of the two examples and identifying the underlying rule or procedure to solve examples of this nature. Attention to structure and generality involves teaching that focuses on specific examples, conjecturing, justifying and generalising ideas. This type of teaching supports learners to think about relationships connections in learning about number. Structure and generality underpin progression in early number development.

Similarly Jurow (2004) suggests that representations are useful in making generalization as these can be used to foreground what is important to pay attention to and background what needs to be ignored across given mathematical examples in order to identify the general pattern. Jurow further suggests that generalizations make it possible to derive general principles or laws of mathematics.

The work of Blanton and Kaput (2004) echoes the same idea that learners in the early grades have the capacity to develop functional thinking when given the opportunity to use various modes of representations such as tables, graphs, pictures, words and symbols to make sense of a task and express mathematical relationships. These authors conducted a study where they asked young learners across grades to identify the number of eyes and tails for a number of dogs, starting with 1 dog how many eyes, two dogs, how many eyes and increasing the number of dogs so learners could identify the pattern by using various modes of representations to express varying quantities. Blanton and Kaput (2011) propose that it is important for teachers to begin by scaffolding the use of representation and later allow learners to take responsibility for their own recording and making sense of the recorded information. The ability to use multiple representations and to move between these representations is regarded as an important aspect of developing conceptual understanding (Blanton & Kaput, 2004, 2011; Lesh, et al., 1987).

3. 7. CONCLUSION

The first part of this chapter discussed the role of language in mathematics teaching within the South African context. I argued in this section that learners who learn mathematics in African-mother tongue classrooms might be exposed to different pedagogies and messages from those who learn through the medium of English. In the second section of this chapter, the concept of evaluative criteria was explicated. In this section it was argued that evaluative criteria are central to the teaching of mathematics and it is important to understand what teachers transmit as evaluative criteria when they teach number and additive relations as what they transmit exposes what they think to be the legitimate text when teaching number and additive relations. The third section of this chapter focused on the content knowledge relating to early number, additive relations and related strategies. In this section I discussed a typology of additive relation problems as well as the trajectory in terms of strategies that could be used to solve these problems. In the fourth section, representational modes and various possible moves between representations that could be observed in various mathematics classrooms were discussed. The fifth section focused on mathematics teaching and problems observed by different researchers. The final section focused on approaches for the teaching of mathematics in primary schools and I argued for approaches that focus on mathematical structure, generalization and abstraction. The central idea emanating from the review of the literature in this chapter is that there are problems in the teaching of early number in primary schools in South Africa. These problems range from a lack of focused instruction on connections and coherence, poor selection of examples and a lack of focused instruction on structure, generality and representations that underpins the development of progression in early number teaching.

In the next chapter I discuss the research design and methods that informed this study.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

This chapter deals with the research design and methods of the study. The discussion is presented in five sections. The first discusses the research design and the justification of the choices made in undertaking the study. The second discusses data collection methods linked to the research design. The third discusses the context of the study and research participants. The fourth provides a discussion of data analysis methods. The final section discusses trustworthiness and ethical considerations related to the study.

4.2 RESEARCH DESIGN AND METHODOLOGY

The study adopted a qualitative interpretative research design. Such a design is concerned with learning how individuals experience and interact with their social world. According to Merriam (2002) the primary aim of this type of research is to understand a phenomenon from the perspective of participants. This author, together with Creswell (2014) further suggests that phenomenology underpins qualitative research as the design allows the researcher to gain an understanding of the complex nature of people's experiences and actions (Creswell, 2014). There are different types of phenomenological approaches (Padilla-Diaz, 2015). For the purposes of this study, a descriptive interpretative phenomenology approach was chosen since this study seeks to describe and interpret classroom practice experiences of teachers in terms of what they communicate as valuable knowledge, cognitive competences and processes in the teaching of early number, additive and multiplicative relations at grade 3 level and how modes of representation feature in what is transmitted.

The theoretical assumption underpinning this study is based on the researcher's view that teachers transmit varied evaluative criteria and use modes of representation in different ways to explain and elaborate on what they want learners to know and use to produce the legitimate text. The phenomenological approach was chosen because it enables a researcher to interpret the individual classroom instructional practices of each participant from the perspective of the particular teacher (phenomenology), and to use analyses

based on comparisons across them to understand what they might have in common and key ways in which they differ in their practice. In order to explore an in–depth understanding (case study approach) of what each teacher transmitted as the evaluative criteria and how they worked with representational modes, I undertook to observe and video record classroom lessons where each teacher interacted with learners in whole class teacher- led lessons focusing on the teaching of early number, additive and multiplicative relations. For this reason, both a case study approach and the phenomenological approach were chosen.

4.2.1 Case study research design

A case study is an intensive investigation of a phenomenon that provides the researcher with an opportunity to describe the phenomenon in–depth (Merriam, 2002). Additionally, a case study is defined as an empirical enquiry that investigates a phenomenon in its real-life context over a period of time (Yin, 2003). The reason for undertaking this study with a focus on teachers' enactment of tasks was to identify what each teacher transmitted as evaluative criteria and how each one moved between representational modes in their own respective classroom environments. This transmission of evaluative criteria and the involvement of modes of representation in this transmission constituted the phenomena being explored within each case. This case could only be studied within the context of the classroom. It would have been impossible for the researcher in this present study to have a full understanding of what teachers transmit as the evaluative criteria without considering classroom contexts and interactions with learners. Yin (2003) argues that the boundary between the phenomenon under study and the context within which the phenomenon is studied is often unclear. For this reason, the researcher undertook to study transmission of the evaluative criteria at the level of the classroom.

4.2.2 Multiple case study approach

Since the study investigated transmission of the evaluative criteria and representational modes across two language settings (Sepedi and English medium classrooms), a multiple case study design was adopted. A multiple case study approach involves studying several cases to understand the similarities and differences between the cases (Baxter & Jack,

2008). Using multiple case studies enables the researcher to gain a more in-depth understanding of cases as a unit. In addition, evidence emerging from multiple cases rather than from a single case is regarded as stronger and more reliable (Yin,2003).

Having defined a case study and multiple case studies, I made a decision to use multiple case studies and to place a boundary on what would be investigated and where in the lessons and across the cases this phenomenon would be bounded. In order to create a boundary around the phenomenon under study, I had to develop a proposition or a number of propositions. According to Baxter and Jack (2008) a proposition refers to the boundaries of the study. These propositions are found in the literature and can guide the scope of the data collection and discussion of the data leading to the development of a conceptual structure or framework. Baxter et al further suggest that propositions in qualitative studies are useful because they place a limit on the scope of the study and increase the chances of focusing the investigation and completing it.

I needed to establish a place in lessons where teachers were likely to explicitly or implicitly transmit evaluative criteria. I therefore turned to the IRE/F model of patterns of interaction (Sinclair & Coulthard, 1992) as a place in the lesson where, by definition, teachers were likely to transmit criteria about the knowledge, cognitive competences and strategies they wanted learners to acquire in order to produce the legitimate text. The IRE/F slot within this model was the place where teachers would explicitly or implicitly transmit evaluative criteria (with the caveat relating to evidence of absences of evaluative criteria in some South African contexts). The decision to focus the investigation on the Feedback/Evaluation segment of the IRF/E interaction was based on Singh (2002) view that “continuous evaluation of learners’ responses through processes of identifying what is present or absent from their responses could be useful for learners to produce the legitimate text”. Based on this view, an assumption was made that teachers could transmit the evaluative criteria in the IRE/F move of the IRF-E model. In the coding of data I focused on this aspect of lessons as the unit of analysis.

4.3 PARTICIPANTS

Participants in a phenomenological research study are generally purposively selected. A purposive sample is made up of a group of participants who meet common criteria (Padilla Diaz, 2015). Four teachers who participated in this present study were drawn from a group of 33 teachers from 10 primary schools in one district that participated in the Wits Maths Connect-primary Project. This WMC-P project included a Teacher Professional Development course designed to support teachers to develop mathematical content knowledge from pedagogical perspective. The empirical data collection in this study happened just after the initial pilot study of the WMC-P project, which began with Grade 2 teachers. Given that the teachers worked with in this study were all from Grade 3, there was a minimal impact from the WMC-P project on the teachers involved in this study beyond being drawn from the same schools.

The four teacher participants were selected on the basis that they taught mathematics in English or Sepedi and they could speak another South African official language in case they needed to move between two or more languages whilst teaching mathematics at grade 3 level. This is because in the Foundation Phase (FP- Grades R-3) within the South African context, learners are allowed to learn mathematics in English or in their African languages (Language-in-Education Policy,1997). The four teachers were selected from four different schools. Two teachers taught through the medium of Sepedi and they could also speak English. The two Sepedi medium teachers in this study, Nkele and Mirriam (Pseudonyms) taught Sepedi which is their first language. The other two teachers were selected from two English medium schools because they taught mathematics through the medium of English, and they could speak another South African language. The two English medium teachers, Laura and Flora (Pseudonyms) taught in a language that is not their first language. For instance, Laura's first language is Afrikaans whilst Flora's first language is IsiXhosa. This language criterion was included in the selection of participants to ensure that all participants could move between two languages whilst teaching if they needed to do so.

Creswell (2014) suggests that for a purposive sample to work well all participants must have a common experience or characteristic. Therefore, a second criterion for selection of participants was based on teachers' years of teaching experience in the Foundation Phase. I sought teachers who had been teaching in the Foundation Phase for 5 years or more. In addition to the teaching experience, these teachers were selected because they all participated in the WMC-P pilot project. Furthermore, all four teachers had matriculated and held a three-year teacher's diploma specific to Foundation Phase teaching, (grade R-3). All four participants were females between the ages of 35 to 50.

Given that teachers had to be willing to participate in the study, I was guided by my supervisor on teachers who had responded positively to prior interactions with the WMC-P team in their school-based work. Thus, while the selection of participants was criterion-based, it was also an opportunity because selections were aimed at the likelihood of in-depth insights rather than representativity and generalizability.

Table 6: Participants' data

Biographical details of participants	Qualification	Experience in FP	Language of instruction	Grade taught
Nkele	Junior Prim Teachers Diploma	8 years	Sepedi	3
Mirriam	Junior Prim Teachers Diploma	15 years	Sepedi	3
Laura	Junior Prim Teachers Diploma	6 years	English	3
Flora	Junior Prim Teachers Diploma	9 years	English	3

4.4 DATA SOURCES AND PROCEDURES

The purpose of data collection in a research study is to provide data for analysis of the research questions being investigated. Mulhall (2003) argues that observations provide access to interactions and are useful because they capture contexts and processes which could not be captured through the use of other methods such as interviews. Creswell (2014) argues that observations are a qualitative data collection method that allows the researcher to access open-ended firsthand data at the research site. This author advocates the use of field notes for recording observations.

In the present study, evidence was collected through observations and digital recording of lessons which were later transcribed into texts and field notes. The data collection process happened in 2014 over a period of three school terms whilst lesson transcription and checking started soon after the first lesson was observed. This process of transcribing and checking, with initial analysis, continued from 2014 to 2016.

A total of 16 lessons were observed and video recorded. Lessons on the teaching of number, additive and multiplicative relations were targeted in each of the tree school terms. The decision to observe over a period of time was based on the researcher's view that reality changes over time. As a result of such changes, lessons observed in the first term may not be the same as lessons observed in the second and third terms and this was important because the study was interested in progression in the development of number, additive and multiplicative relations as well as representational modes. The length of lessons observed differed from one teacher to the other as the duration of lessons was dictated by the teacher, with the observation ending when the teacher completed the whole class teaching segment of the lesson. Each teacher was asked to identify a date and time in each of the three school terms in 2014 where they would teach number, additive or multiplicative relations and allow the researcher to come in on that date to observe and record the lesson. It was the teachers' choice of which four lessons they wanted the researcher to observe. Below, I detail the topics, dates and times of each lesson observed.

4.5 SUMMARY OF LESSONS OBSERVED ACROSS THE FOUR TEACHERS

The next section provides a brief summary of the content focus of each lesson observed across the four participants.

4.5.1 Nkele's lessons

Four lessons were observed in Nkele's grade 3 Sepedi medium classroom.

Lesson 1 – Adding to 20 (27 minutes 27 seconds on the18.02.2014)

Nkele began the first lesson with ten number sentences. These additive relation tasks were presented as individual tasks to be completed by learners working with the teacher in a whole class context. Following the completion of these 10 number sentences, Nkele gave learners a task on counting South African coins from the DBE workbook on page 62. This task was an individual task where each learner worked independently in their workbook.

Lesson 2 - Counting, ordering numbers and addition (53 minutes 21 seconds on the 05.05.2014)

Nkele began the second observed lesson by asking learners to chant number words in ones from 100 to 200 and in fives from 100 to 200. This chanting exercise lasted for five minutes with learners holding up a finger for every number word they chanted. Following the chanting task, the teacher handed out 7 number cards in the 10 to 90 range with cards 70 and 80 missing to seven learners and asked them to arrange the numbers on the board. A similar task of sequencing numbers between 115 and 135 was presented immediately after the first sequencing task. The subsequent task involved arranging numbers from 115 to 135 in a sequence of counting in fives. Nkele placed number cards on the board including 115, 110, 120, 117, 130, 135, and 125. The number card with the number symbol "117" was included in the sequence as a distractor.

Lesson 3 - Counting, place value and addition (45 Minutes and 16 seconds on the 09.06.2014)

Nkele began the third observed lesson by asking learners to chant numbers in multiples of ten from 200 to 400. She then asked them to repeat the chanting and count back in

tens from 400 to 200. As in the previous lessons, learners chanted the number words in English only. In the second part of this third lesson, the teacher focused on identifying the place value of digits in numbers 25 and 135 respectively. In the last part of this lesson, Nkele moved to ask learners to add 10 to 54. Once the correct answer was offered, she moved to adding three-digit numbers and two digit numbers involving 323 + 32 and 323 + 136. In this last part of the lesson she instructed learners to work out the task by way of breaking down the second number and adding the hundreds, tens and units of this second number to the first number. The final task in this lesson was an independent work task where each learner was asked to solve the task 341+141 and record their solutions in the mathematics books.

Lesson 4 Counting, mental maths, addition and subtraction (45 minutes and 19 seconds on the 15.09.2019)

In the first part of this lesson, Nkele instructed learners to count in multiples of 5 from 100 to 200. Learners chanted the number words sequence holding up 5 fingers every time they mentioned a number word. In the second part of this fourth lesson, Nkele orally presented subtraction tasks and learners were required to produce answers immediately. The position of the unknown variables was varied to include start unknown or the change unknown type of tasks. In addition to varying the position of unknown variables, Nkele moved between the English and Sepedi number names in contexts where learners could not produce the required answers. Correct or incorrect offers were not interrogated. The final part of this fourth lesson was an independent task where learners were asked to work on rounding off money tasks involving shopping items lists pasted on the board.

4.5.2 Mirriam's lessons

Four lessons were observed in Mirriam's Sepedi medium classroom.

Lesson 1: Addition of 2-digit numbers (31 minutes, 01 second on the 19.02.2014)

The first lesson focused on addition of two-digit numbers. Mirriam started this lesson by asking learners to take out their DBE workbooks and open them on page 87. This page presented a strategy of breaking down numbers into tens and units in order to first add the tens together then the units together to produce the answer. Page 87 of the DBE

workbook presented 4 addition problems involving 86 + 62, 72 + 63, 81 + 57 and 69 + 71. Though the teacher asked learners to open on this page, she wrote her own addition problems on the board involving 56 + 73, 22 + 34, 47 + 52, and 38 + 51 and 75 + 12. In her instruction, she suggested that learners break down the numbers into tens and units in order to produce answers. In the last part of this lesson, the teacher asked learners to work independently to solve tasks in their workbook using this breaking down strategy.

Lesson 2: Counting, number identification, ordering of numbers (35 minutes & 08 seconds on the 06.05.2014)

In the first part of this lesson Mirriam asked learners to take out the number chart from 301 to 400 and to count in ones whilst pointing to the numbers on their number charts. The second part of this lesson focused learners on the number 376 as the teacher asked learners to find this number on the number chart. The subsequent part of this lesson focused on identifying numbers on the number chart that are “bigger” than 376. Subsequently, Mirriam then wrote numbers 315, 355, 305, 315, and 350 on the board and asked learners to arrange these numbers in order from the biggest to the smallest and from the smallest to the biggest. The last part of this lesson was an individual seatwork task taken from page 76 of the DBE workbook. The task focused on filling in missing numbers on 101 to 200 number chart.

Lesson 3: Addition and subtraction of two- and three-digit numbers (31 minutes and: 49 seconds on the 10.06.2014)

Mirriam handed out one abacus to every second learner in class and asked learners to work in pairs. She had a list of 10 number sentences written on a piece of paper on her table. She continuously referred to this piece of paper every time she introduced a new task. Her first task required learners to add 75 and 5. The subsequent tasks involved working with numbers that are more than 100. For instance, the second task involved 175- 25.The three subsequent tasks involved adding a set of different numbers not involving adding with multiples of 5.The last five tasks of this lesson involved adding or subtracting 10.

Lesson 4: Addition and subtraction of two- and three-digit numbers (35 minutes and 41seconds on the 16.09.2014)

In her fourth lesson, Mirriam focused on addition and subtraction of three- and two-digit numbers. In this lesson Mirriam introduced a structured number line in multiples of 10 from 100 to 200 and in multiples of 5s as a tool to represent numbers and produce answers. Multiples of 5 and 10 were not clearly marked on the number line. In addition to the use of the structured number line, Mirriam used the standard written algorithm to verify answers following the production of answers on the number line. In this lesson, 6 number sentences were presented one after the other. Each task in the set was completed independently with no reference to the working or outcomes on the other tasks. A structured number line was used as a tool to solve these 6 tasks.

4.5.3 Laura's lessons

Four lessons were observed in Laura's English medium classroom.

Lesson 1: Counting in multiples of 10, 5 and 3 (25 minutes, 18 seconds on the 20.02.2019)

Laura began her first lesson with what seemed like warm up activities involving different counting activities in multiples of five, and three. In the first counting activity, learners were asked to show ten fingers and to count each finger as a representation of 5. Laura then wrote 10 x 5, the board. The second counting task involved counting three fingers in multiples of 5s. Laura wrote 3 x 5 on the board. The third counting task involved counting 10 fingers in 3s. Laura then wrote 10 x 3 on the board. Lastly Laura asked learners to take out their DBE workbooks and open up on page 4 and independently work on the activities on pages 4 to 7. The activities on these pages were counting activities leading to repeated addition and multiplication.

Lesson 2: Counting in 3s and in 4s, multiplication problems (26 minutes and 33 seconds on the 07.05.2014)

Laura's second lesson focused on multiplication types of problem. Laura began the first part of this second lesson with counting based activities as starters where learners were asked to count in 3s and in 4s on their number charts. Each learner had a set of A4

number charts consisting of numbers in the range 1-100, 101 to 200, 301 to 400 in a file. Laura then focused on multiplication problems where 4 was a multiplier. She started with problems involving pictures of squares and learners counting in multiples of 4 to establish the number of corners in each set of squares. This section was followed by an oral activity where Laura gave learners numbers of chairs and learners multiplied the numbers by 4. Finally, she gave learners a word problem task where she wanted them to work out the number of sweets required for 40 people if each person was given 4 sweets at a party. In this task Laura focused on doubling and halving strategies.

Lesson 3: Bonds of ten, counting in 3s and in 4s. Addition and subtraction of two digit numbers (29 minutes and 57 seconds on the 12.06.2014)

Laura began her third lesson by focusing on bonds of ten. The teacher orally stated a number less than ten and learners stated the corresponding number to make 10. Subsequent to this section, Laura moved to focus on counting in threes and in fours where learners used their fingers to track their counting sequence. The main part of this third lesson focused on the revision and teaching of additive and subtractive tasks involving the split strategy where each number was split into two parts and parts were added separately to produce answers. In the section of this lesson Laura asked learners to open up on page 45 of their DBE book and to work on the tasks on that page.

Lesson 4: Counting in 2s, making up number 251, counting in fives starting from 2. Counting in 5s starting from 36 (25 minutes and 17 seconds on the 17.09.2014)

Laura's fourth lesson started with her instructing learners to take out their number charts and to count on in 2s from 40. Following this counting on task, learners were instructed to count back in twos from 40 to 2 whilst pointing to the relevant numbers on their number charts. This section of lesson 4 was followed by Laura asking learners to determine the place value position of digits in the number 251. Laura then moved to contrast 251 and 521 to show that the value of a digit in a number is determined by the place value position of the digit. The subsequent part of this fourth lesson focused on counting in fives starting from two and starting from 36 on the number chart.

4.5.4 Flora's Lessons

Lesson 1: Counting in 1s from 1 to 100, number recognition, identification of missing numbers in a sequence, counting in fives from 105 (23 minutes and 16 seconds on the 21.02.2014)

The first lesson started with Flora instructing learners to count from 1 to 100 on their number chart and to point to the number symbols. In the subsequent section of this first lesson, Flora asked learners to identify the numbers 98 and 77 on the number chart. Following this number identification task, Flora presented a set of numbers in multiples of 5 from 150 back to 100 and wanted learners to identify the missing five numbers in the sequence presented. This task was followed up with another task involving identification of missing numbers in a sequence. In the last section of this first lesson, Flora wrote the individual seatwork activity on the board. It consisted of number patterns activities where learners were asked to identify and complete patterns.

Lesson 2: Counting forwards and backwards in 4s, multiplication and arrays (33 minutes and 22 seconds on the 08.05.2014)

Flora's lesson two focused on multiplicative relations and the use of arrays. Flora began the second lesson by asking learners to count forwards and backwards in multiples of 4 on their number charts. This counting forward and backwards was followed by an introduction of multiplicative relation number sentences involving 4 x 5, 5 x 4, 6 x 4 and 4 x 6 and representation of these problems using arrays. This lesson was concluded with an individual seat work activity involving multiplication word problems and arrays to represent the problem.

Lesson 3: Number identification and place value (30 minutes and 34 seconds on the 12.06.2014)

Flora's third lesson focused on numerical identification and place value of numbers. In the first part of this lesson, Flora asked learners to take out their number charts from 501 to 600. She then wanted learners to identify numbers 501 and 600 on the chart. Following the identification of these two numbers on the number chart, Flora asked learners to count in ones from 501 to 600 and to point to the number symbols on the number chart. The subsequent section of this third lesson focused on identifying numbers and their place

value position. In this part of the lesson Flora wrote the place value structure of Hundreds, Tens and Units on the board. She then asked learners to pick numbers from a box on her table. Once a learner had picked a number, they were asked to identify the number name and the place value position of a digit in a number. Flora then moved to ask learners to identify numbers that come before 540 on the number chart and the number of tens in numbers between 540 and 550.

Lesson 4 Adding to 20 on a number line, counting in 5s starting from 3 (45 minutes and 41 seconds on the 18.09.2014)

Flora's fourth lesson focused on additive relation problems and the use of a number line. Tasks presented involved part-part whole relationship tasks where one part and a whole were given, and learners were required to identify the missing part in order to make 20 as the whole. Following this section, Flora moved to ask learners to take out their number charts from 1 to 100. Learners were instructed to count in 5s starting from 3. This lesson was concluded with individual seatwork where learners were asked to add numbers on the structured number line from 1 to 20 written on the board.

4.6 DATA ANALYSIS APPROACH

4.6.1 Grounded theory analysis approach

One principle of grounded theory (GT) analysis is that data collection and data analysis processes are two interrelated processes and therefore data collection and data analysis processes in qualitative research should happen simultaneously (Corbin & Strauss, 1990). Categories and patterns in the data emerge from the data set rather than the researcher imposing a predetermined theory on the data (Creswell, 2014; Glaser & Strauss, 1999). While earlier studies had outlined indicators for evaluative criteria in general pedagogic terms, these studies had not focused on the particularities of evaluative criteria transmitted during the teaching of early number. I then consulted the literature on early number development and modes of representation in order to make sense of the grounded data. Therefore, a grounded theory approach and the literature were chosen for developing concepts and indicators of evaluative criteria and representational modes emanating from, and applicable to, this mathematical focus.

There are differences and similarities in ways in which Glaser, Corbin and Strauss present GT coding methods. Glaser (1978) identifies two levels of coding which are open and theoretical coding. Whilst Corbin and Strauss (1990) identify three levels of coding which are open coding (study of words and line segments from the raw data), axial coding (making connections between segments of data from the open coding process) and selective coding (selection of core themes). Glaser's theory of open coding is similar to open coding as discussed by Strauss and Corbin. However, Glaser's (1978) levels of coding do not include axial coding which is the second level of coding suggested by Corbin and Strauss (1990). Kendall (1999) suggests that the differences between the works of these GT theorists are as a result of the inclusion of axial coding by Corbin and Strauss.

Whilst these similarities and differences were noted, I made a choice to use Corbin and Strauss' three levels of coding and the literature on evaluation criteria, early number and modes of representation.

4.7 STEPS IN GT ANALYSIS

The analysis begins with transcription of all the lessons. Transcriptions involve producing a verbatim record of what was transmitted and how representational modes featured in each lesson. In this process of transcribing lessons, I included thick descriptions of events and utterances in order to keep the flavour of the original data and to report direct phrases and sentences as these were originally presented by participants. Gestures and comments about tone/inflection are included in lesson transcripts in curled brackets to point to nonverbal actions that accompanied verbal communications in each excerpt. Translations from one language to the other are shown by square brackets and are written in italics.

In the context of the present study, 16 lessons were observed, transcribed and separated into lesson excerpts. A lesson excerpt in this study is defined in similar ways to that used by Ensor et al (2009). These authors define a task as a segment of a lesson which focuses on a single idea, goal or theme. Whilst most tasks focused on one idea at a time, a few tasks were noted where the teacher simultaneously presented two tasks within the same

excerpt for instance: where the teacher asked learners to chant the number word sequence in ones and in fives from 100 to 200 or instances where the teachers asked learners to solve two problems simultaneously on the board such as 38 + 51 =□ and 75 + 12 =□. Such tasks were analysed as a single task. The start of a task or lesson segment is defined by a teacher's introduction of a task, topic or theme and it concludes with the teacher's announcement of a new topic or change of classroom interaction pattern. This breaking down of lessons into lesson excerpts led to the generation of 102 excerpts across the 16 lessons. Ten of these lesson excerpts were not coded as they involved individual learner seatwork whilst the study was interested in whole class teaching excerpts. Each of the 92 lesson excerpts were examined to identify what teachers communicated or did at the introduction of the task, how they responded to learner offers in the context of incorrect, correct or no offers made by learners and how they used representational modes in the transmission of the evaluative criteria..

Below I describe how I used the three levels of coding, open coding, axial coding and selective coding.

4.7.1 Open coding

Open coding is regarded as the initial stage of data coding in qualitative data analysis. At this level, the researcher reads every line in the transcript and allocates a label or colour to a piece of text, segment or phrase in order to describe and categorise the text (Corbin & Strauss, 1990). In this level of coding, the data is broken down analytically into smaller segments. These segments are then investigated and compared to identify similarities and differences. Similar segments are later grouped together to form categories (Cohen, Manion & Morrison, 2011).

In the initial stage of analysis, I took all excerpts from the first lessons observed across the four classrooms and open coded these excerpts. In this initial level of coding I examined each line or segment of an excerpt and created summary phrases that remained close to the raw data. In the subsequent section, I include an excerpt of transcript with an exemplification of the first level 'close' codes that was created.

Excerpt 1: Drawn from lesson 1 of Nkele's Sepedi medium classroom



Figure 2: Number sentence with the change unknown

1. T: I only have two 1 numbers in each number sentence *(two known numbers*). [moves her hand downwards from the first task to the last one]. You must tell me which other number to add *(operation is suggested*) in order to make 20. I have 6. [Points to 6] What can I add to six in order to make twenty (*missing number)*? [Some children were seen counting from 1 to 6 on their fingers *(concrete counting from one*) and others were seen counting from 6 to 20 *(concrete counting on)* Johannes [calls out a specific learner] -

2. L: Twenty-six *(incorrect offer*)

3. T: Twenty-six? (*Repetition/*revoicing) Masome pedi tshela? (Translating) What can we add to six to make twenty? (*Restatement of task*) What is that number? (*Unknown number)* [Points to another learner], Count from 6. *(Counting)* Use your fingers (concrete counting)-

4. L: Seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty. [*Counting on in ones)*

5. T: [writes 14 on the board to complete the sum 6+14 =20. (*Symbolic representation*) Points to the sum and verbalizes the complete number sentence (moves from oral to symbolic)] six plus fourteen is twenty. Tshela le lesome nne ke masome a mabedi *(move between English and Sepedi).*

In the excerpt above, I investigated each line segment and allocated a code or a word or phrase to describe what was observed in that line segment. The words or phrases are written in italics and in brackets to describe a concept or an idea in the line segment. This process of coding line by line is referred to as open coding (Kendall, 1999) . This process of line by line coding is regarded as useful because it allows the researcher to remain close to the raw data. Open coding was carried out with all the other excerpts generated from the sixteen lessons. The initial codes were kept open until no new codes emerged. Following the initial coding of all excerpts various categories and dimensions of these emerged. This lead to the second level of coding, the axial coding explained in the subsequent section.

4.7.2 Axial coding

Axial codes describe events and activities leading to the occurrence or phenomenon under study (Cohen, Manion & Morrison, 2011). This process of coding involves an intense examination of a category at a time and its relationship to other categories. At this

axial coding level all open codes generated in the initial coding process were compared and clustered. Similar categories were grouped together and examined to establish relationships or differences in and between categories. Corbin and Strauss (1990) suggest that making comparisons between and across categories supports the researcher to develop an analytical framework. Following the initial coding, 21 codes emerged as outlined in Table 7:

Table 7: Conceptual codes

Codes from the raw data	Conceptual labels
1. Symbolic and oral representation of tasks	Representations
2. Oral representation of tasks	Representations
3. Concrete representations	Representations
4. Moves between symbolic and oral	Translation/moves
5. Repetition of incorrect learner offers	Repetition
6. Requests for learners to repeat their answers	Repetition
7. Restatement of tasks	Repetition
8. Teacher repeats the correct offer	Repetition
9. Draw pictures	Representations
10. Suggestion to count in multiples	Approach
11. Move to ask another learner for a response	Feedback
12. Writing down offers on the board	Acceptance
13. Teacher offers the correct answer	Feedback
14. Moves between languages accompanied by symbols and explanations	Moves/translation
15. Suggestion of a strategy to count in ones	Approach
16. Suggestion to use own methods	Approach
17. Rejection of the offer	Feedback
18. Suggestion to count on	Approach

19. Substitution of one language by another	Moves/translation
20. Suggestion to breakdown numbers	Approach
21. Moves to the next task	Feedback

21 codes emerged from the initial coding process as reflected in Table 7. Each code was allocated a conceptual label.

This process enables the researcher to be consistent in ways in which categories are grouped to develop theory.

Corbin and Strauss (1990) use the term “dimension” to refer to variations between same categories. The codes that belonged together were grouped into categories. Four categories emerged which are representations, moves/translations, feedback and approaches. These different categories and their dimensions signaled different ways teachers used representations, different ways teachers used to provide feedback to learners in the context of correct, incorrect or no learner responses, different ways teachers moved between representations and different additive and multiplicative relations approaches teachers offered or accepted.

Table 8: Categories and dimensions

Categories	Dimensions
Representations	Symbolic and oral representation of tasks, oral representation, concrete/physical manipulatives, drawing of pictures
Moves/translation/transformation	Translation moves, moves between languages, moves between languages accompanied by symbols
Feedback	Repetition of learner incorrect offers, asking learners to repeat, restatement of tasks, asking another learner to help, move to another learner, move to another task, writing down of learner offers, outwards rejection of offers, acceptance of offers, provision of the correct answer,
Approaches	Asking learners to count in ones, counting in multiples of 2s, 3s, 4s, 5s, and 10s. asking learners to break down numbers using place value, asking learner to use own strategies and known facts

Table 8 provides a summary of the main categories and dimensions of each category

In the context of this present study I compared categories of excerpts by examining the conditions, contexts, strategies and consequences leading to incidents in each excerpt. In comparing and clustering these excerpts into categories of excerpts I asked myself the following questions using the four characteristics of axial coding (Glaser, 1978):

Conditions: What does this category of excerpts tell me about what the teacher does or communicate as feedback to learners' correct or incorrect offers?

Context: What does this category of excerpts tell me about ways teachers use representations to respond to learners' offers?

Approaches: What does this category of excerpts tell me about approaches teachers offer or accept to help learners produce required answers?

Consequences: What does this category of excerpts tell me about the implications of what teachers do or say at the close of each excerpt?

Table 9: Examples of axial coding

Evidence		Analysis			
Task	Excerpt:	Condition	context	approaches	consequences
Excerpt 19: Lesson 3	T: Now let us do the next one. It is very easy. Next year you are going to do more sums up to 800 because it is easy this way. [Writes 62 + 15 = □on the board]. Sixty-two plus fifteen. What is my first step? L: Minus. T: No! We are not going to minus. What are we doing when we write sixty and two? [Points to 62 on the board] Ls: Breaking down the number. T: What number are we breaking down here? [Points to the expression 62 +15 on the board] Ls: sixty plus two. T: [Writes 60 + 2 on the board. What do I do after this? [Takes a green piece of chalk and writes the + just below the expression 60 + 2]. I wrote the plus sign because there is a plus in the sum. What do we do next children? Ls: Breakdown the second number. T: Fifteen is ten plus five. [Writes 10 + 5 and draws a line below the number sentence]. Where do we start, at the tens or units? L: units. T: [points to 2 and 5. Holds up her fingers to show 5 and two] Ls: Seven. T: [writes 7]. What do I write now? [Points to the space just before 7] L: Plus.	Task is symbolically written and orally presented.	Incorrect offer. Teacher rejects the offer: Feedback is provided Correct offer Feedback is provided	Suggests breaking down numbers	At the close of the excerpt correct answer is produced through breaking down numbers

	T: [writes +]. What is six plus one? Reduction L: Seven T: What is sixty plus ten? L: Seventy. T**:** [writes 70 on the board]. What is seventy plus seven? L: Seventy T: Seventy plus seven is seventy-seven. Am I finished? [Points to the next learner] L: No. T: What is next? Ls: Write the answer at the sum. T: [Writes 77 and the writing looked as follows				

All excerpts which seemed similar to the one exemplified in table 9 were grouped together. Memos were generated at different levels of coding to examine and reflect on how particular categories related to others and to identify relationships between codes, categories and sub-categories. This generation of analytical memos allowed me to discuss and reflect on what I was learning about different categories of excerpts and to make constant comparisons of excerpts and events within and across categories (Charmaz, 1995).

Table 10: Examples of an analytical memo

Tasks in the lesson	Of interest in observing lessons across the four teachers is that the codes revolved around teacher moves between representations and languages during the statement or restatement of tasks.
Teacher responses and feedback within excerpts	Incorrect offers, Teachers repeat such offers or ask learners to repeat their responses. Teachers repeat or restate tasks. Repetition or re-voicing plays a role in ways in which teachers respond to learner offers Teachers write the offers on the board, they repeat the offers, they acknowledge or accept the offers No offers: Teachers moved to the next learner or restate the tasks, pointed to the task, suggested that other learners offer some help, move to a new task without providing the answer.
Feedback at the close of excerpts	I noted that at the close of each excerpt, different teachers did different things. In some excerpts I noted that learners were made aware of correct answers to tasks, in other instances they were made to move to another task without knowing the correct answers to tasks dealt with. In some instances, teachers offered or accepted answers without an explanation.
Approaches offered/accepted	I noted that in contexts where learners could not suggest a strategy or produce answers, teachers asked them to count in various ways such is counting in ones, 2s, 3s, 4s, 5s, 10s, count on in ones from the first number, breakdown numbers using place value knowledge, or to use known number facts.
Language moves	The teacher orally presented tasks in both Sepedi and English whilst pointing to the number symbols on the board or teachers used substitution of words or phrases from one language to another
Mathematical moves	I noted that gestures were used to make connections between the oral and the symbolic representations No connections were made between the various tasks even through the task focused on the same topic or related numbers

At this level of coding, variations between excerpts was the focal point of analysis. Comparing and clustering categories to identify similarities and differences in teachers'

enactment of tasks surfaced various categories of teacher enactment of tasks. The one category that emerged was representational moves and various ways of moving between representations leading to the creation of four sub-categories of moves between representations. All excerpts which exhibited similar moves between representations were grouped together to form a category of representational moves. Similarly, teachers offered or accepted different approaches they wanted learners to use in order to produce answers. These variations in approaches resulted in the emergence of three sub-categories of approaches. Excerpts where teachers suggested or offered similar approaches such as counting based, calculation by counting based approaches, structuring and derived facts approaches to produce answers were grouped together to form a category named approaches. This clustering of approaches was informed by the literature and progression path defined by van den Heuvel-Panhuizen (2001). The axial coding process also surfaced different ways in which teachers offered feedback in response to learners' offers. The various feedback moves observed during enactment of tasks led to the emergence of subcategories of feedback (implicit/explicit) which contributed towards the third category of excerpts. The different ways in which teachers concluded excerpts resulted in the emergence of five different ways in which excerpts were concluded. All excerpts concluded in the same way were grouped together to form the fourth category of excerpts. The third and the fourth categories were collapsed to form one category referred to as evaluation/feedback category. As more excerpts were categorised similar patterns of teacher enactment of tasks began to emerge repeatedly as categories began to repeat themselves over and over. This repetition of similar categories is referred to as saturation of categories (Saldaña, 2013). It is at this point that I stopped comparing and clustering categories.

Table 11 : Categories and sub-categories

Categories and conditions	Subcategories				
Conditions of moves between representations in the context of correct or incorrect learner offers	Moves between oral, symbolic, pictures, or concrete	Moves between symbols and symbols, concrete objects and concrete objects	Substitution of words or phrases	Elaborated moves involving two languages coupled with oral and symbolic moves	
Approaches offered/accepted in the context of correct or incorrect learner offers	Counting based Chanting number approaches- Chanting number words unit counting, skip counting in 2, 3s, 4s, 5s, 10s Counting on	Structuring based approaches Breaking down the first number into Hundreds, Tens and Units Breaking down both numbers into Hundreds, Tens and Units	Use of number facts to produce answers		
Feedback to learner offers during enactment of tasks	Reject offers	Accept offers/write them down/offer answers	Move to the next leaner	Move to the next task	Re-state the task
Feedback at the close of excerpts	No answers offered or accepted	Correct answers offered or accepted with no explanations of how answers were produced	Answers produced through different ways of counting	Answers were produced through strategies involving breaking down numbers	Answers were produced through the use of known facts and number relations

Table 11 shows that four different categories of moving between representations emerged from the data set. This table further shows that teachers offered or accepted different counting strategies, structuring and number facts as ways to assist learners produce answers to tasks. In addition, the table shows five different ways of transmitting

the evaluative criteria emerged from the data set. The next section presents the third level of coding.

4.7.3 Selective coding

Corbin and Strauss (1990) describe selective coding as the stage of data coding where categories are linked with the core category which is a representation of what the study is about. In addition, these authors suggest that selective coding is at a higher level of abstraction as it requires the researcher to have a deep understanding of the topic or the phenomenon under study. They further suggest that at the selective level of coding, the researcher needs to ask a set of questions in order to identify the core-category and how other categories link to this core category. These authors propose that the researcher ask the following set of questions in the process of selecting the core category:

What is the main analytical idea presented in this research?

What do all the actions/interactions seem to be about?

It is through this process of questioning the relationship between the core category and other categories that I began to attach meaning to what the data presented. I began this study with an interest in what teachers transmitted as the evaluative criteria during the teaching of number, additive and multiplicative relations and how modes of representations featured in what was transmitted. The empirical data implicitly surfaced that the idea of evaluative criteria is located in conceptions of evaluative feedback. I then selected feedback/evaluation as the core category, and I investigated how other categories integrated with this core-category. The data showed that different categories of transmitting the evaluative criteria existed. The data further revealed that different moves between representations featured in what teachers transmitted. Additionally, the data surfaced that teachers transmitted or accepted different additive and multiplicative strategies as ways to assist learners produce the legitimate text. After selecting the core-category and integrating them with other categories, I revisited the literature on evaluative criteria and modes of representations. Revisiting the literature after focused coding helps the researcher to clarify, sharpen and integrate the emerging theory. It is at this level of analysis that I noted levels of different moves between representations as informed by

the literature through the work of Duval (2006), the literature in language about translations and translanguaging practices informed by the work of Garcia and Wei (2014), Makalela (2016), Childs (2016) featured at different levels of transmission of the evaluative criteria. These hierarchies of levels of evaluative criteria and levels of moves between representations are elaborated in the emerging framework chapter.

The next section discusses broader validity considerations relevant to the study.

4.8 VALIDITY OF THE STUDY

Creswell and Miller (2000) define validity as the level of accuracy with which the researcher accounts for and represents participants' abilities or experiences of the social phenomena. In these authors' view validity does not refer to the data itself but to the inferences the researcher makes from the data. In addition, these authors posit that there are various ways a qualitative researcher could promote validity or credibility of their studies. These ways include member checking, triangulation, thick descriptions and peer reviews.

Denzin (1978) suggests that time triangulation involves collecting data from the same research participants over a period of time. This author also suggests that triangulation can be conducted through a process of member checking to establish whether participants' viewpoints were faithfully interpreted. In this present study three ways of promoting validity were built in. These procedures involved time triangulation, member checking and a sustained period of return to data. Firstly, data was collected over a period of three school terms in 2014 to ensure time validity of the study and to ensure that changes in what teachers transmit in the transmission of the evaluative criteria are captured over a period of time if such changes were noticed. Secondly, following the transcription of each lesson, participants were invited to view the video recorded lessons and read each related lesson transcript. This step was undertaken to ensure that lesson transcripts captured exactly the situation as shown in the videos. In this process of triangulation through member checking, particular attention was paid to the language used in the transcripts and the descriptions of teachers' non -verbal actions as shown in video lessons. Thirdly, during the process of analysis, I continually returned to the data

to check whether the codes, categories and sub-categories related to the explanations and interpretations I made throughout the process of coding and analysis. Field notes and memos were kept to ensure coherent and explicit data capturing. These processes were followed with a view to providing confirmation or to strengthen the findings.

4.8.1 Transferability

Transferability refers to giving consideration to whether the findings of a study can be applied to other contexts, settings or groups. To promote transferability in this study, I have provided information to the reader about participants and their contexts through provision of thick descriptions of incidents and a detailed description of procedures and process of data collection and analysis, while recognizing that transferability is not central to case study research given that it involves in-depth study of a bounded phenomenon.

4.8.2 Dependability

Dependability in qualitative research refers to consistency within the employed procedures in the data analysis process (Cope, 2014). Dependability refers to when one or more researchers agree with the decisions made at each stage of another researcher's study. In order to promote reliability and dependability of the present study, I continually engaged my supervisor and other PhD students within my university in discussions about my analysis methods including sharing of lesson transcripts. Furthermore, I presented and shared my data analysis methods and transcripts with colleagues from other universities in three years during the stages of analysis at the Research School held in 2015, 2017 and 2019.

4.8.3 Confirmability

Confirmability refers to the researcher's ability to demonstrate that the data used in the study represent participants' responses and not the researcher's viewpoints. Cope (2014) suggests that this criterion can be achieved by describing how conclusions and interpretations were established in a study. These descriptions can by enriched through provision of examples and rich quotes from the empirical data. In order to promote confirmability in this present study, I presented examples of excerpts from the empirical

data with descriptions of incidents in each example before I provided my interpretations of each excerpt.

4.9 ETHICAL CONSIDERATIONS

Ethical considerations in qualitative research refer to the researcher's respect and recognition of participants' rights. These rights include the right to be informed about the study, the right to decide whether to participate in the study or not and the right to withdraw at any time without penalty (Orb, Eisenhauer & Wynaden, 2001). In this present study, the four schools that participated in the research study were selected from a list of ten schools that participated in the Wits Maths Connect Project. Principals of the four schools were approached to obtain permission for their schools to participate in the study. Principals then called a meeting with the grade 3 teachers in their respective schools and the researcher. The aim of the study and the criteria was explained to all grade 3 teachers in the four schools. Teachers who satisfied the criteria and were willing to participate volunteered to participate in the study.

The researcher provided each teacher with the information letter (Annexure 1)about the study and two letters of consent. The first letter of consent was for permission to observe the teachers (Annexure 2) in the classroom and the second letter was for the video recording of lessons (Annexure 3) . Though the study focused mainly on teachers, learners and parents were given information letters (Annexure 4) and the consent forms to state whether parents permitted their learners to participate in the study or not (Annexure,5). All parents allowed their children to participate in the study with an understanding that no participants' names would be used in the study. Only pseudonyms were used. The participants were informed that there would be no payment for participating in the study and they had the right to withdraw from the study at any given time should they wish to do so. Copies of the information letter and the consent forms are attached as annexures. A general ethics letter of approval was granted by the Department of Education for the Wits-Maths Connect Project (Annexure 6) underwhich this research study was conducted.

4.10 CONCLUSION

This chapter has discussed the research design, methods, participants and analysis procedures followed. The chapter provided reasons for choices and decisions made during the study. Furthermore, the chapter discussed the validity and reliability of the study and how these considerations were taken into account in the study. The subsequent chapter presents the emerging framework and how this framework was used to code and analyse empirical data.

CHAPTER 5: EMERGENT FRAMEWORK

5.1. INTRODUCTION

In chapter one I explained that the lack of explicit transmission of evaluative criteria evident in Hoadley's studies of foundation phase teachers at work (2006, 2007) motivated this investigation into what four teachers in grade 3 mathematics classrooms, using either English or Sepedi as medium of instruction, transmitted as evaluative criteria during the teaching of early number, additive and multiplicative relations. In the literature review, I noted that Morais and Neves (2001) and Morais (2002) have argued that in contexts where there is an imperative to improve low attainment, strong framing of evaluative criteria (EC) that involves explicit communication of what counts as legitimate text is more helpful than weakly framed evaluative criteria. In all pedagogic contexts, there are criteria that learners need to acquire and apply. Understanding of these criteria contributes to the production of the legitimate text. Morais (2002) argues that a teacher who does not make the evaluation criteria explicit does not provide learners with opportunities for learning what is involved in producing the legitimate text and how to construct correct answers in the future. She points out that in order to understand differential achievements in schools it is necessary to analyze specific pedagogic discourses as sets of rules that regulate the transmission of and acquisition of knowledge. In South African classroom contexts, in which language of instruction continues to be a marker of differential outcomes, the work of Morais provides a rationale for investigating the nature and extent of EC offered across language settings, building on Dowling's (1998) work on strategies and Hoadley's (2007) work on proceduralising and principling strategies.

This chapter commences by considering categories of EC and representational moves across mathematical and multilingual representations separately, explaining the levels that were developed within each of these categories by drawing on distinctions in the empirical data and linking these to the theoretical bases. All mathematical offers are necessarily presented in some representational form, so EC in relation to these offers implicitly or explicitly include feedback on the mode of representation. However, to enable a more nuanced analysis, I developed a two-dimensional initial coding system: EC level

and Modes of Representation (MoR) level, with the latter broken down into Mathematical Moves (MM) of representation and Multilingual (ML) moves of representation. This sub-division enabled me to identify differences in the nature and range of EC interwoven with MoR moves across MM and ML types between English and Sepedi LoLT settings.

In the presentation of the EC and MoR categories and hierarchies, illustrative excerpts are incorporated to provide the reader with insight into how the framework was developed and the kinds of insights into pedagogy that could be derived from its use. I conclude the chapter with a commentary on the framework and the insights provided, before proceeding, in the next chapter, to applying the framework to the analysis of lesson sequences in the two Sepedi and two English LoLT classrooms.

5.2. EVALUATIVE CRITERIA

As previously stated, this study investigated the pedagogic practices of four teachers in terms of what they transmitted as evaluative criteria during the teaching of early number and additive/multiplicative relations. Variations were observed in these teachers' transmission of evaluative criteria and also in the way modes of representation featured in what was transmitted. These variations were categorized into levels using (Dowling, 1995) and Hoadley (2007) relating to strategies. As explained in Chapter 2 Dowling draws on Bernstein (1964) work on restricted codes and elaborated codes (the latter being the language associated with much of the language of school subjects). Dowling (1998) introduced the notion of knowledge domains relating to the two language codes identified by Bernstein and also introduced the idea of localizing and specializing pedagogic strategies. Through his analysis of textbook tasks, Dowling (1995) drew attention to tasks belonging to the everyday public domain which were characterized by low discursive saturation practices in which generalizable mathematical principles and strategies were not made explicit. Such tasks tended to be context-dependent in that learners were required to produce a particular answer specific to the task without attention to a generalizable mathematical strategy that could be used to solve similar mathematical tasks in the future. In contrast, tasks belonging to the esoteric domain of mathematical practices were characterized by high discursive saturation practices and generalizable

foci. Mathematical knowledge and tasks included generalising strategies where symbolic language was explicitly discussed, and general principles of the discipline were made explicit in the esoteric domain. In this category of tasks, meanings were far more context–independent with generalisable mathematical definitions, principles and propositions made more visible.

Attention to mathematical generalisations, efficiency, mathematical relationships and properties were therefore among the key characteristics of tasks that promote specializing, and through these generalizations and efficiencies, greater access to mathematical practices. In thinking about the early number context of this study, the literature pointed to specializing/generalizing occurring through two key routes:

- progression in terms of moves from counting based approaches to calculation by counting, structuring and derived facts approaches (van den Heuvel-Panhuizen, 2001, 2003) and (Wright et al., 2006). Ensor et al's work (2009) point to a lack of progression from concrete to more abstract ways of working with number as one of the factors contributing to poor mathematical attainment within the South African context.
- moves between multiple representations (Duval, 2006; Lesh & Lehrer, 2003) and moves towards increasingly abstract representations over time.

Looking at progression in terms of moves from counting to more efficient calculation approaches when teaching number, additive relations and multiplicative relations suggests looking for acceptance of, or explanations involving, moves beyond concrete counting. Concrete one-by-one counting approaches would fall within lower levels of EC given their 'localized', highly inefficient approaches, and more efficient approaches, involving more general procedures based on calculation-by-structuring indicating higher EC levels.

It also suggests looking at moves between representations in MM and ML terms, as these moves also represent expansions or generalisations in how an idea is understood. While Ensor et al (2009) focused on progression from concrete to increasingly abstract representations; the mathematics education literature pointed more to the importance of

flexible moves between representations. Duval's (2006) writing provided empirical evidence that moves within particular representational registers (treatments) were generally easier than moves between representational registers (conversions).

Given the focus on different language settings in this study, moves between representations were also considered in terms of multilingual moves. Here, the literature provided a hierarchy comprised of translation (Baker, 2011; Childs, 2016) at the lower level and translanguaging (García & Li, 2014; Makalela, 2016) at the upper level.

In the sub-sections that follow, I explain my operationalizing of the literature-based hierarchies in each of the above categories into the analytical framework used for this study. I pay particular attention to the changes made to earlier models and explain the reasons for these changes.

5.2.1 Evaluative criteria categories

In Chapter 2, I detailed Hoadley's studies of teacher talk and task enactment in the classroom environment. Hoadley (2007) draws on the work of Bernstein (1971) and Dowling (1998) in investigating whether the pedagogic relay communicates context independent (esoteric) or context dependent (public) knowledge messages.

Hoadley uses the concepts of knowledge domains and strategies to describe the unequal distribution of knowledge to learners in a range of social class contexts. She identifies two types of pedagogic strategies that deploy mathematical messages in the classroom: localizing and specializing. She proposes that localizing strategies transmit mathematical tasks and explanations that involve solutions that do not involve specialised mathematical knowledge and notes that such localizing strategies often require learners to do things they enjoy, such as games they already know how to play (Hoadley, 2007). Examples of teacher talk in this type of tasks were generally concrete and context dependent.

In this present study, I adapted Hoadley's notion of localising to refer to excerpts where counting related procedures such as reciting numbers and calculating-by-counting approaches were encouraged through teacher talk. This adapted notion of localizing was useful in this study to code and categorise teacher talk observed within the context of

grade 3 classrooms where teachers encouraged learners to count to produce answers without pushing learners to move to use more sophisticated approaches defined in the progression of early number learning.

In contrast to localizing tasks, Hoadley (2007) classified some enactments of mathematical tasks and teacher talk as involving specializing procedural and specializing principling strategies. For Hoadley, specialising strategies were required when learners needed to use particular mathematical elements or concepts to solve tasks. Specializing procedural mathematical strategies required learners to construct or produce the legitimate text through following a mathematical procedure without necessarily engaging with principles for its use. A parallel in this study involved the task 65 + 15 where the teacher talk focused on use of a more efficient structure-based approach involving place value properties like the split strategy (breaking down numbers into tens and units, then adding the tens together and the units together) to produce the answer without discussion on underlying principles of how number is structured in the decimal system would be categorized as a specializing procedural strategy.

In the same paper, Hoadley (2007) describes specializing-principling tasks and strategies as entailing a rule based or rule governed performance on the part of the learner, where broader application of knowledge, operations or skills is required. She notes that this level of working with tasks can apply to oral work where learners are required to reason, justify or explain their answers. As an example of specializing principling strategies, Hoadley describes grade 3 learners being provided with a word-problem task involving 20 sparkles (sweets) and being required to find the number of green sparkles in a jar of 20 green and red sparkles with 4 more red sparkles than the green ones. Though the task was presented in the form of a contextualised everyday problem, teacher talk in this task focused learners' attention on the general technique she wanted them to use to solve the problem and led to a move from context dependent to context independent meanings. Hoadley categorizes the teacher's explanation of the technique in the example discussed as specialising-principling because although the task was context-dependent at the beginning, in the end, teacher attention came to focus on the mathematical rule for solving

the problem. For the purposes of this study though, the example provided by Hoadley would be categorized as a specializing procedural teacher talk because the teacher talk focused on the step by step procedure to produce the answer to the particular task. In the context of this study, specializing principling teacher talk would involve instances of teacher talk focused on approaches that are more efficient, non-count based approaches and derived fact approaches to produce answers, or more general, rather than particular to the task at hand, or involve rationales or justifications. In the context of this present study a task like 174 + 12 where teacher talk focused on encouraging learners to use the number facts, they already know to produce the answer and to justify and explain their answers would be categorized as specializing-principling teacher talk.

Looking across specialising strategies, Dowling and Hoadley's work in comparison to the ways in which I came to integrate the idea of specializing strategies in this study brought interesting contrasts to light. In particular, the sociological base in Bernstein's work leads to specialising considered in terms of a move from everyday emphases to disciplinary emphases. This boundary was very salient to the first wave of South African post-apartheid curriculum reform where the idea of 'relevance' to everyday life was strongly marked (Taylor & Vinjevold,1999). However, constituting specialising in these terms was less useful in the present study, located in the context of a curriculum specification that has moved away from the emphasis on everyday relevance. Instead, in this study, specializing is marked by progressions within the discipline of mathematics. As indicated earlier in this chapter, this involves moves towards generality, which can be achieved through two avenues: the moves towards increasing attention to structure and more efficient strategies in evaluative criteria that have been considered in this section; and the moves towards generality via moves between representations, mathematical and multilingual, that I come to in section 5.3.

5.2.2 Evaluative criteria levels

The language of 'strategies' was used to analyse the nature of the EC teachers transmitted during the enactment of tasks focused on early number, additive relations and

multiplicative relations. Four broad theoretically derived and empirically anchored EC levels were generated in this study.

In this framework the first level of transmitting the evaluative criteria is referred to as Level EC0, and following Hoadley work (2007) involved no evaluative criteria being offered. To recap, Hoadley identified instances in which learners were not made aware of whether their offers were mathematically correct or incorrect and referred to this level as an absence of framing in EC: F^0. At the close of EC0 excerpts learners were left unaware of whether offers they made were correct or incorrect. One level above EC0 became evident from the empirical data. This data set presented a level where teachers offered evaluative criteria relating to correctness or incorrectness of offers.

At the close of each excerpt categorized at Level EC1, the interactions provided learners with access to the correct answers making these excerpts qualitatively different from the EC0 excerpts. The subsequent EC levels emerged from the empirical data and describe what teachers in the context of this study transmitted. The literature relating to strategies (localising proceduralising and specialising proceduralising) is used to describe what emerged from the empirical data set.

At Level EC2, two different types of procedures emerged which led to EC Level 2 being separated into two sub-categories labelled as Level EC2A (localising procedural strategies) and EC2B (specialising procedural strategies). As noted above, this sub-categorization is slightly different from Hoadley's work on localizing and specializing strategies, and instead, tends to follow Venkat and Askew (2018) notion of localizing as involving strategies that provide learners with a base to answer the immediate examples provided by the teacher but not beyond. In this study I interpreted Venkat and Askew's notion of localization by foregrounding counting based procedures encouraged through teacher talk without a push to more efficient approaches that are then applicable more generally, and to larger number range to help learners to produce answers to similar tasks in future. In this study, localized procedural enactment of tasks involves excerpts where the teacher offered or accepted calculation by counting procedures. Following van-den Heuvel-Panhuizen (2001) categories, counting procedures observed at EC2A were

varied and included sub-levels of counting such as chanting number words, unit counting, counting all, counting on and skip counting in different intervals. While counting procedures can be seen as general procedures because they can lead to production of correct answers, these kinds of counting procedures are highly inefficient to work within the context of two- and three-digit numbers. In subcategory EC2B more general procedures based on calculation by structuring involving place value and breaking down of numbers were offered or accepted. Here specialising procedural strategies characterised teacher talk.

Level EC3 was marked by teacher talk that offered or required learner to use known facts to derive answers. Specialising principling strategies characterised teacher talk at Level EC3. A clear boundary emerged between excerpts in the EC2 category and those in EC3. In EC3 excerpts, teacher talk focused on the use of known number facts, and requests for justification from the learners.

Table 13 provides an overview of the EC levels and categories used to classify the various excerpts, and an illustrative excerpt for each level and category drawn from my empirical data.

Table 12: Indicators of the evaluative criteria

Code	EC0	EC1	EC2A	EC2B	EC3
Label	No evaluative criteria offered.	EC offered relating to correctness or incorrectness	EC involve localized counting procedures	EC involve specialising through calculation by structuring	EC involve specialising through the use of known facts to derive answers.
Description	At the close of EC0 excerpts, learners are left unaware of whether offers are correct or incorrect.	At the close of EC1 excerpts, learners are made aware of whether offers are correct or incorrect.	At the close of EC2A excerpts, offers based on calculating by counting have been offered or accepted.	At the close of EC2B excerpts, offers based on calculation by structuring have been offered or accepted.	At the close of EC3 excerpts, offers are produced based on the use of known facts to derive answers and requests have been made for learners to justify and

					explain their answers
Illustrative excerpt	The teacher asked “what is ten take away four. In the context of an incorrect learner offer she announced a move to the next task	The teacher asked “what plus five will make 20? In the context of the correct offer of 15, the teacher accepted the offer and announced a move to the next task.	The teacher wanted learners to add 120 + 40 on a number line. In the context of a few incorrect offers, the teacher instructed learners to find 120 on the number line and count forwards in tens	The teacher wanted learners to add 26 + 32 together. She suggested that learners break down each number into tens and units. First add the tens together, add the units together, then add the two totals to produce the answer	The teacher wanted learners to add 174 + 12 on a number line. A learner solved the task by taking 1 from 12 to add to 174. He as 175 + 11. Then moved to add 10 from 11 to 175 to have 185 he then added 1 to produce 186 as the answer.

5.3 MODES OF REPRESENTATION

While Hoadley focused on the enactment of tasks and strategies teachers deployed, she did not focus on representations and how these featured in the production of answers to tasks. This present study has also investigated how modes of representation feature in what teachers transmitted as the evaluative criteria, thus extending Hoadley’s research.

As noted already, research in mathematics education and early mathematics teaching points to the central role in mathematical learning of mathematical communication through moves within and across various modes of representation (Duval, 2006; Lesh et al 2003). There is also a literature base in mathematics education that focuses on the use of multiple modes of representation in mathematics teaching Haylock and Cockburn (2008) and Haylock and Manning (2014). In this present study the role of representations and moves between and across representations were investigated within the context of the evaluative criteria because evaluative criteria in mathematics are transmitted through mathematical representations. Prior South African work has highlighted problems in

teachers' work with representations. Coherence between representations and explanations was flagged in teachers' work with representations (Venkat & Adler, 2012) and reluctance to move beyond concrete representations was flagged in Ensor et al's (2009) work. The latter study, while located in Foundation Phase, did not focus on representation *moves* (MM), a feature that the literature has highlighted as critical. Given this gap, a focus on moves between modes of representation provides a second important aspect of investigation into the transmission of evaluative criteria.

In view of my specific interest in differences in the transmission of evaluative criteria as a possible explanation for ongoing differences in attainment in mathematics in English and African Home Language classrooms, I have also considered shifts in modes of representation based on Multilingual (ML) moves as well as Mathematical (MM) moves within and between representational registers. The literature on language in mathematics education, detailed in Chapter 3 suggests that teachers' and learners' use of translation and translanguaging, within the ML moves category, are important for learning of mathematics in multilingual classrooms.

The MM and ML literature base and hierarchies are drawn on here in combination to allow a transversal focus on representations while looking at EC. Following the evidence there on the MM side led to the rationale for two levels of MM: treatment moves at the first level and conversion moves at the second level. Duval (2006) emphasized that construction of connections between registers has to be made explicit for learners to develop understanding of concepts or of the mathematical activity at hand. Empirically, I was therefore interested in the kinds of explanations and rationales offered/accepted in instructional talk around teachers' work with treatment and conversion moves. In this part of the study, my interest lay in describing the different types of moves between representations and how these contributed to progression in learning and solving place value, additive and multiplicative relations tasks.

On the ML side, in the context of this study, I categorized translanguaging moves as shifts between languages where explicit links were made between the spoken and written forms in Sepedi and / or in English. In contrast, I term instances where teachers simply

substituted one word or phrase in one language with a word or phrase in another as a translation move.

The nature of elaboration via translation/substitution or via translanguaging provided an important avenue for identifying differences between ways in which teachers moved between representations in multilingual classrooms. The literature on bi- and multilingualism in education does not include a hierarchy of levels when referring to the two pedagogic strategies. In this study I chose to make a clear distinction between translation and translanguaging moves and to place these pedagogic moves in hierarchical positions. This was justifiable theoretically, given that translanguaging moves offered more explicit pointers across languages to alternative ways of expressing mathematical ideas, and as such, offered the more explicit framing that Morais (2002) has described as necessary for supporting learning.

In summary then, mathematical moves (MM) and multilingual moves (ML) are presented in a hierarchy because conversion moves are regarded as more complex than treatment moves since conversion moves involve moving between registers. This move between registers requires one to recognize two objects that are represented in two different ways as the same even where the commonality between the two objects is not so obvious (Duval 2006). For this reason, I placed these moves in hierarchy.

Multilingual literature suggests that translation moves involve substitution of one word or phrase by another in a different language. On the other hand, translanguaging is defined as purposeful pedagogical moves between languages in spoken and written form. Translanguaging moves are defined as more complex than translation moves as the former involve both written and spoken moves (García & Li, 2014). For this reason, I placed these two languages moves in a hierarchy to show the levels of complexity they present. The hierarchy in MM and ML is underpinned by moves towards generality as conversion and translanguaging moves in mathematics allow for a broader and more varied understanding of the ideas under discussion.

In Table 13 categories and hierarchies across the MM and ML moves between modes of representation (MoR moves) are summarized, described and illustrated as done previously for the EC.

Table 13: MoR moves

	Mathematical moves		Multilingual moves	
Code: Label	MM1: Treatment moves	MM2: Conversion moves	ML1: Translation moves	ML2: Translanguaging moves
Description & illustration	Moves between representations that happen within the same representational register. For instance: In responding to learner offers, the teacher moved within the same mathematical register writing 5 + 5 =10 and 4 + 6 =10. This shows moves between two symbolic representations of 10.	Conversion refers to moves between representations that involve changing a register without changing the object represented. For instance: In responding to learner offers the teacher moved between mathematical representations for instance, 5 + 5 and the iconic  +  to show that the two representations represent the same idea.	Translations are moves between two languages where one word or phrase in one language is substituted with another in a different language. For instance: In responding to learner offers the teacher moved between English and Sepedi languages by substituting one word or phrase in English or in Sepedi with another one. For instance: the teacher asks “Tharo plus lesome?” [‘*Three plus ten?*’] A learner answered: ‘Lesome tharo.’ [‘Ten three.’] Here the teacher substituted the	Translanguaging moves involve moves between two languages using oral language supported by written or diagrammatic representations. For instance: In responding to learner offers, the teacher moved between English and Sepedi languages by way of using full sentences and written language to explain and elaborate on learner offers. For instance, the teacher writes an expression symbolically on the board, orally represents the expression in another language while she points to the symbolic representation on the board.

			Sepedi number word with the English number word saying: 'Yes, thirteen.'	

5. 4. THE EC-MOR FRAMEWORK

I combined the ideas presented above on EC and MoR to formulate the 'Evaluative criteria/modes of representation' framework (EC/MoR framework) that I have used to analyze the evaluative criteria evident in the classroom data collected. This framework represents a key contribution of this study as it brings bodies of work on evaluative criteria in monolingual settings together with work on mathematical moves within/between representations and work on multilingual moves within representations which have stayed largely separate in South African research, a point acknowledged by Hoadley (2012) in her overview of research in primary classroom-based studies. The combined EC/MoR framework is illustrated in Table 14

Table 14: EC/MoR Framework

EC Levels	Multilingual moves		Mathematical moves	
	Translation (ML1)	Translanguaging (ML2)	Treatment (MM1)	Conversion (MM2)
EC0: At the close of EC0 excerpts, learners are left unaware of whether offers are correct or incorrect	Translations are moves between two languages where one word or phrase in one language is substituted with another in a different language.			
EC1: At the close of EC1 excerpts,	Moves between two languages where one word or phrase in one		Moves between representations that happen within the same	Moves between representations that involve changing a register without

learners are made aware of whether offers are correct or incorrect.	language is substituted with another in a different language.		representational register.	changing the object represented.
EC2A: At the close of EC2A excerpts, offers based on calculating by counting have been offered or accepted.	Moves between two languages where one word or phrase in one language is substituted with another in a different language.	Moves involve moves between two languages using oral language supported by written or diagrammatic representations	Moves between representations that happen within the same representational register.	Moves between representations that involve changing a register without changing the object represented.
EC2B: At the close of EC2B excerpts, offers based on calculation by structuring have been offered or accepted.	Moves between two languages where one word or phrase in one language is substituted with another in a different language.	Moves involve moves between two languages using oral language supported by written or diagrammatic representations	Moves between representations that happen within the same representational register.	Moves between representations that involve changing a register without changing the object represented.
EC3: At the close of excerpts, offers are produced based on the use of known facts to derive answers	Moves between two languages where one word or phrase in one language is substituted with another in a different language.			Moves between representations that involve changing a register without changing the object represented.

This EC/MoR framework allowed me to consider instructional excerpts in the dataset for how MoR interplayed with EC across the observed lessons within early number teaching. As noted already, moves to higher EC levels, and to the higher level of either MM or MR,

both represent moves towards increasing generality, and therefore into increasingly 'powerful' mathematical working in terms of the range of examples that can be dealt with through the procedures and/or principles offered.

5.5 CONCLUSION

Four broad levels of transmitting the evaluative criteria with one level sub-divided into two categories emerged. These levels ranged from no transmission of the evaluative criteria, F^0 to the level where the EC focus was on the use of number facts to solve additive and multiplicative relation tasks in efficient ways, and sometimes involving rationales and principles. Modes of representation were considered through MM and ML moves, each with a lower and higher level viewed as possible.

Analysing all the relevant data excerpts across the empirical base of this study in terms of this framework provided a way of exploring and comparing the pedagogic relay in a sample of early number, additive and multiplicative relations lessons for the four teachers involved in this study, two working in Sepedi medium and two in English medium. While, in some ways, the lower levels of the EC/MoR framework in both dimensions can be considered as rather 'basic' in comparison to examples in the international literature, these levels were necessary for considering differences in the pedagogic relay between classrooms and between language settings within the South African context. The framework presented in this chapter provided the language and tools for categorizing and describing what teachers in the four schools transmitted as evaluative criteria and how modes of representation featured in what they transmitted. In this chapter, I have shown how the evaluative criteria and representational modes framework is useful for categorizing and analyzing lesson excerpts in two Sepedi and two English medium classrooms respectively. The next chapter presents the outcomes of this application of the FWK to the data.

CHAPTER 6: FINDINGS AND DISCUSSIONS

6.1 INTRODUCTION

In this chapter, the EC/MoR framework, introduced in the last chapter, is used to analyse what teachers transmitted as evaluative criteria and how modes of representation featured in what was transmitted. My particular interest in this study was focused on exploring whether the EC/MoR framework might be of use in identifying any patterned differences in instruction between the English and Sepedi medium classrooms.

As stated in the methodology chapter, four lessons on number, additive and multiplicative relations were observed in each teacher's classroom across the first three school terms in 2014. These lessons were broken down into a number of lesson excerpts following Ensor et al (2009) methodology focused on shifts in tasks marking demarcations of excerpts. An excerpt is defined as a part of a lesson which focuses on a single mathematical task or problem. The start of an excerpt is marked by the teacher presenting a task or problem. The end of an excerpt is marked by the teacher's announcement of a new task or a move to a new task. For example, the move from the task 3 + □ = 20 to 9 + □ = 20 indicates a shift to a new task, and therefore, a new excerpt.

In this chapter, I present the outcomes of the analysis of all the instruction-oriented excerpts for each teacher across evaluative criteria and modes of representation. In some instances, during lessons teachers called on specific learners names to respond to their questions. For purposes of this study, these learners' names are replaced with pseudonyms. Each teacher's story begins with a tabular overview of that teacher's lesson excerpts and how these were coded using the EC/MoR framework. These lesson excerpts have been numbered consecutively for each teacher. The kinds of instruction underpinning the overview EC/MoR profile for each teacher is then detailed using illustrative excerpts drawn from the pool of excerpts across EC levels. In discussing the EC/MOR, the MM2 moves observed across the four teachers were separated into MM2 and MM2* where MM2* moves involve basic moves between the oral and the symbolic number representation. MM2 moves involve more sophisticated moves that include moves between the oral, concrete, pictures, symbols and the symbolic number based

(number charts, number lines) modes of representation. At the end of the analysis sections dealing with each teacher, I explore for overlaps and contrasts based on comparing the pool of excerpts across the Sepedi and English language medium lessons. 57 excerpts were generated from the Sepedi medium classrooms and 47 from the English medium classrooms. Of these 104 excerpts, 92 were coded and analysed based on instruction occurring within them. The languages are separated in this chapter to explore what teachers in the two languages transmitted as evaluative criteria during the teaching of early number, additive and multiplicative relations, and to investigate whether there was evidence of any patterned differences in the occurrence of EC/MoR on the basis of the classroom LOLT.

6.2 SEPEDI MEDIUM LESSONS

6.2.1 Nkele's teaching

An overview of the content of each teacher's lessons is presented prior to the discussion of each teacher's story.

Table 15: Overview of excerpts from Nkele's lessons

Lesson1	Task	Timing	EC/MoR
1	On board: 6+□ =20	00:23-23:24	EC2A/ ML2, MM2*
2	On board: 3+□ = 20	23:25 -23:46	EC2A/ ML1,MM2
3	On the board: 2 +□ =20	23:46-24:10	EC1/ ML1,MM2*
4	On the board:1+□ =20	24:10-24:34	EC1/ ML1,MM2*
5	On the board 7+□ =20	24:34- 24:58	EC2A/ ML1,MM2*
6	On the board 5+□ =20	25:00-25:25	EC1/ ML1,MM2*
7	On the board 9+□ =20	25:25- 25:48	EC1/ ML1,MM2*
8	On the board 4+□ =20	25:49-26:08	EC2A/ ML1,MM2*
9	On the board 0+□ =20	26:10-26:50	EC1/ MM2*
10	On the board 8+□ =20	26:52-27:27	EC1/ ML1,MM2*
11	Individual seatwork: Worksheet on South African money from p.18 &19, DBE workbook	27:28-59:05	
Lesson 2			
12	Oral instruction: Chant number word sequences in ones and in 5s from 100 to 200	00:14-04:15	EC2A/MM2*
13	Number cards handed out to learners to arrange on the board from smallest to largest in multiples of 10 from 10 to 90	06:35-18:00	EC2A/MM2*

14	Arrange numbers from smallest to largest 115, 110, 120,117, 125 Later oral explanation indicated that teacher wanted class to arrange numbers in multiples of five, removing the 117.	18:10-25:21	EC2A/ ML1,MM2*
15	Written on the board:104 +71 =□	27:00-53:21	EC2B/ ML1,MM1,MM2*
Lesson 3			
16	Recite number words in multiples of 10 from 200-400 and back	00:06-08:00	EC2A/ MM2*
17	Oral instruction: Identify the place value of digits in 25, with 25 written on board	14:15-21:09	EC2B/ MM2*,ML1,ML2
18	Identify the place value of digits in 135. With 135 written on the board.	21:30-50:21	EC2B/ MM1, MM2*, ML1, ML2.
19	Add 10 to 54 with 54 + 10 written on the board	52:16-05:21	EC1/ MM2
20	Add by breaking down the second number 323 +32 written on the board	06:00-10:45	EC2B/ MM1,MM2*
21	323+136	15:36-25:55	EC2B/ ML2,MM1,MM2*
22	(345+141) Individual seat work	30:00-45:16	Not analyzed
Lesson 4			
23	Recite numbers words in multiples of 5 from 100 to 200	00:04-02:06	EC2A/MM2*
24	Oral task: Ten take away one	02:08-03:16	EC1/ ML1
25	Oral task: Ten take away four is what?	03:29-05:16	EC0/ML1
26	Oral task: Fourteen take away mmm --- I am left with ten	06:20-08:45	EC0/ ML1
27	Oral task: I have mmm I take away four. I am left with ten.	09:00-11:34	EC1/ML1
28	Oral task: I have fifteen. I take away mmm. I am left with four	11:34-11:36	EC0/ ML1
29	Oral task: I have eleven. I take away mmm. I am left with four	11:60-12:22	EC1/ ML1
30	Rounding off money: Individual seatwork	13:56-45:19	Not analyzed

The four lessons observed in Nkele's class could be broken down into the thirty excerpts shown in Table 15. Three excerpts were not analyzed as they involved individual seat work which was not the focus of this study. Nkele's teaching across the twenty-seven analyzed excerpts could be located as follows in the EC/MoR framework (see Table 16):

Table 16: Coded excerpts from Nkele's teaching

EC Level	No of excerpts	Multilingual moves		Mathematical moves	
		Translation (ML1)	Translanguaging (ML2)	Treatment (MM1)	Conversion (MM2)
0	3	L4: E25 L4: E26 L4: E28			
1	10	L1: E3 L1: E4 L1: E6 L1: E7 L1: E10 L4: E24 L4: E27 L4: E29			L1: E3* L1: E4* L1: E6* L1: E7* L1: E9* L1: E10* L3: E9*
2A	9	L1: E1 L1: E2 L1: E5 L1: E8 L2: E14	L1: E2		L1: E1 L1: E2 L1: E5* L1: E8* L2: E12* L2: E13 L2: E14* L3: E16* L4: E23*
2B	5	L2: E15 L2: E17 L3: E18	L2: E17 L3: E18	L2: E15 L3: E18 L3: E20 L3: E21	L2: E15* L2: E17* L3: E18* L3: E20* L3: E21*

At the overview level, Table 16 indicates 16/27 excerpts characterized by ML1 and MM2 moves across EC levels 0 to 2A. A further 3/27 excerpts indicated only ML1 moves at EC0 level. 3/27 excerpts indicated ML1 moves at EC2B, 2/27 involved ML2, 4/27 involved MM1 moves and 5/27 MM2 moves at EC2B level. In the analysis that follows, the excerpts within each EC level are considered and compared, with particular attention to how they overlap and contrast, and how ML and MM moves feature across the levels.

EC0

Three excerpts (25, 26 and 28) in Nkele's teaching were categorised as EC0 level excerpts. All these excerpts only contained translation (ML1) moves. Excerpt 28 from Nkele's lesson 4 provides a useful illustration of Nkele's EC0 level excerpts where evaluative criteria were not offered.

L4, E28: EC0/MoR –ML1

1. T: I have lesome hlano [ten five]. I take away mmm. I am left with nne [four]. What did I subtract? I have fifteen. I take away mmm. What did I subtract?
2. L: Tshela [six]
3. T: Tshela? Lesome hlano ntšha mmm ke tshela? Six? [Ten five take away mmm is six?] You have to be quick and give me answers quickly. We move to the next task

As with the other two excerpts, this task ends with no resolution on the missing subtrahend value and the teacher moves to the next task without a correct offer stated or accepted. In this excerpt, presented in what the teacher referred to as a mental maths lesson, in the context of an incorrect offer of six from a learner the teacher repeated the incorrect offer in Sepedi, restated the task in English, urged learners to be quick whilst she moved to the next task. While evaluative feedback that the answer is incorrect is implicitly provided through repetition of the incorrect offer (Lyster & Ranta, 1997) this is not followed through into a production of the correct answer, or discussion of why the given offer is incorrect. It is noteworthy that even with an incorrect answer offered, no attempts are made to move beyond the oral task representation into some gestural, written or diagrammatic format that can be used to show how an answer might be produced.

Translation between English and Sepedi number words and phrases (ML1) is observed in the context of an incorrect offer in line 3, noted as useful for supporting understanding by (Childs, 2016; Baker, 2011)

While not the central focus of this study, the EC0 coding is interesting in relation to the low level of this task in the context of Grade 3 curricular specifications, particularly as this excerpt appeared in the fourth observed lesson in term 3. The fact that learners could not

answer this low level task makes the EC0 Level and EC Level 1 (EC1) codings with limited, and often no, moves between languages or representations to support meaning-making even more problematic, while providing insight into the ways in which instruction may be failing to support independent problem-solving through such moves.

EC1

In EC1 excerpts, correct answers were produced by learners or offered by the teacher. Ten excerpts in Nkele's Sepedi medium class were categorised at EC1 level. Three of these excerpts contained only ML1 moves, and so were similar in their playing out to EC0 excerpts except that in these EC1 excerpts correct answers were produced by learners or the teacher. Five EC1 excerpts included the ML1 and MM2* phenomena pointed out above as common. Two EC1 excerpts included only MM2*

An illustrative example of EC1 with ML1 and MM2* representational moves is drawn from Excerpt 10 in lesson 1 and illuminates the ongoing association of lower level ML moves with lower EC levels.

L1, E10: EC1/ ML1 & MM2*



Figure 3: Number sentence with the change unknown

{Standing in front of the class, the teacher presents the last in a sequence of missing addend tasks with total 20, writing 8 + ☐ =20 on the board}.

1. T: {Pointing to 8 on the board} Re ka hlakantšha nomoro e le eng go bona masomepedi? [What can we add to this number to make twenty]. We need to add something to it so that we make masomepedi [tens two], {Calls out a learner's name for a response}

2. L1: Lesomepedi [tens two] {said immediately without counting}

3. T: We can add eight and twelve, lesome pedi. [Tens two], {fills in 12 in number sentence}. Do you understand how you counted to get masome pedi [tens two]?

4. Ls: Yes

Here, the task of 8 + □ = 20 presented symbolically on the board was answered correctly by a learner saying: 'lesome pedi' [*t*welve]. The teacher acknowledged the answer

repeating it in English and then in Sepedi in line 3 (ML1). The teacher then moved the orally offered number into symbolic form by writing in ‘12’ into the number sentence on the board. Moves between symbolic and oral language number forms therefore occurred in this excerpt, with evaluative feedback provided on the correctness of the offer, allowing for an EC1 code. While ‘counting’ was mentioned as the procedure that led to the production of the answer, there was no request for clarification from the learner as to how she produced the answer through counting. The teacher’s move to accept the offer without requesting further clarification meant it could not be coded above the EC1 level.

Excerpt 9 in Lesson 1 from the same set of missing addend tasks also involved a learner offer accepted as correct coupled here only with moves between the oral and the symbolic forms of representations, without any translation between Sepedi and English.

As noted in other writing, no connections are made between the set of examples that focused on bonds of 20 in Lesson1, thus negating possibilities for focus on structure and general procedures or principles to emerge through looking for common aspects across examples (Askew,2019; Venkat & Adler, 2012).

EC2A

In EC2A category excerpts teacher talk focused on counting based procedures to produce answers. While the offer or demonstration of a procedure moves beyond the statement of the answer at level EC1, the limitations of counting based procedures to small number ranges for effective use has been discussed in the literature chapter, making these procedures ‘localised’ in their efficient applicability (Ellemor-Collins & Wright, 2008).Nine excerpts in Nkele’s Sepedi medium classroom were categorised at EC2A level. Four of these excerpts include the ML1 and MM2* coding and are therefore similar to the largest sub-category of EC1 excerpts detailed above involving translation and moves between oral language and symbolic number representations. Another four contain only MM2* with the teacher moving between oral language and the symbolic forms of representation as before without translation. One EC2A excerpt - Excerpt 2 from Nkele’s Lesson 1 - includes ML1, ML2 and MM2, and is described below as it illuminates EC2A counting procedures with elaborations in language and written representations.

L1, E2: EC2A/ ML1, ML2 & MM2



Figure 4: Number sentence with the change unknown

1. T: What can we add to three to make twenty? What number is that? {Calls out L1 name. Points to the second task on the board, 3+□ =20}, what can we add to 3 to make twenty? What number is that? {L1 name}

2. L1: seventy

3. T: What? {Teacher paused, looking surprised} Say that again.

4. L1: Seventy

5. T: Say the number in Sepedi so you can hear what you are talking about.

6. L1: Lesome supa. [Ten seven]

7. T: Seventeen. Not seventy [masome -šupa]. If you say seventy then you are talking about masome šupa. [Tens seven]. There is a difference between masome šupa [tens seven] and lesomešupa [tens seven] {as she writes 17 and 70 on the board and points to each number}. Lesome šupa [ten seven] is one ten. Masome šupa is many tens, seven tens. Do you see it?

8. Ls: Yes, mam

9. T. Now count from this number to twenty. {Points to 3 in the number sentence on the board}

10. Ls: Four, five, six, seven, eight nine, ten, eleven twelve, thirteen, fourteen, fifteen sixteen, seventeen, eighteen, nineteen, twenty {learners count as they raise one finger at a time to track the number of fingers}

11. T: How many did you count?

12. L: Seventeen fingers.

13. T: Yes. Thank you,

A move between symbolic and oral language representations (MM2*) occurs early in this excerpt. In the context of the incorrect offer of 70, Nkele introduces what appears to be an intentional use of translation between the more marked oral language difference

between the number names for 17 and 70 in Sepedi compared to English (line 5) to support awareness of the number distinctions. The correct answer was offered through this translation move. The move from the English number name to the Sepedi number name was coded as a translation move (ML1) where one word or phrase in one language is substituted by another in another language (Baker, 2011; Childs 2016). Following the production of the correct offer, the teacher contrasted the English and the Sepedi number words for the numerals 17 and 70 (line 7). This explanation of the difference between seventeen and seventy was coded as ML2 a translanguaging

García and Wei, (2014) and Makalela (2016) move because this explanation involved the move between the two languages accompanied by a symbolic representation on the board. Both ML1 and ML2 were therefore evident.

Of interest in the teacher's explanation of the difference between 17 and 70 in English and in Sepedi is the absence of attention to the broader system of how number names are constructed in Sepedi, a language with a highly regular system of number naming linked closely to the base ten structure of the number system (Mdluli, 2017) that contrasts with the irregularity of English number words (Fuson & Briars, 1990). The teacher's explanation therefore lacked a focus on this general structure and how the base 10 number system is structured.

Following the production of the correct answer, the teacher instructed learners to count from 3 in the expression (line 9) and a communal counting in ones was accepted. Here, a calculation by counting procedure was used to confirm the answer. Localising proceduralising strategies characterised teacher talk in this category of excerpts, supported by learners use of concrete finger representations to keep track of their counting. The connection between the oral and the concrete representation indicated a different type of MM2 phenomena here. This excerpt is the only one of Nkele's EC2A excerpts showing any support for making sense through elaborating either in language or in other representations apart from the symbolic.

In the EC2A excerpts in Lessons 2-4, calculating by counting procedures (sometimes involving counting in multiples) were similarly offered or accepted. The fact that learner

answers were often correct following the presentation of a localizing procedure suggests a possible reason for the absence of attention to procedures for producing answers. However, given this offer of correct answers, the instruction to use counting-based procedures instead of more efficient calculating by structuring procedures can be called into question – as it has been in earlier South African writing noting the phenomena of 'holding back' or 'pulling back' into less efficient procedures (Venkat & Askew, 2018; Ensor et al., 2009).

EC2B

Five excerpts were categorised as EC2B excerpts where teacher talk focused on number structuring approaches to produce answers. These excerpts contain more varied patterns of moves between representations when compared to moves observed at EC1 and EC2A excerpts discussed previously. While the MM2 moves observed in these five excerpts were similar to the MM2 moves between symbolic and oral representations discussed previously, MM1 moves came into play in Nkele's teaching only in this category in 4/5 excerpts. 2/5 excerpts also indicated ML2 phenomena. Two excerpts are presented to exemplify these 'new' aspects in the EC2B category of excerpts, with the first example chosen to exemplify EC2B with ML1, MM1 & MM2* moves.

L2, E15: EC2B/ML1, MM1 and MM2*

1. T: {Writes 104 + 71 =□ on the board}. Lekgolo le nne plus masome- šupa tee [Hundred four plus tens seven, one], One hundred and four plus seventy-one will give us what as the answer? {Points to the number sentence on the board}. Give me the answer quickly. Give me the answer quickly. We put tens together and units together.
2. L1: One seventy-five.
3. T: One seventy-five?
4. L1: Lekgolo masome- supa hlano [Hundred, tens seven, five]
5. T: Lekgolo masome- supa hlano [hundred, tens seven, five] One hundred and seventy-five. Thank you. Do you see how it works? You put seven here {points to the space below the zero in 104} and one here {points to the space below the four in 104 and re-writes the task in vertical column algorithmic form – see Figure 3}. You are done!



Figure 5: Vertical representation of the task

As previously, the symbolic form of the task of adding 104 and 71 was orally presented, leading to MM2* coding. A procedure involving partitioning the numbers into tens and units were overtly communicated in line 5 following the production of the correct answer. This move was coded as the MM1 move where the teacher moved within the same register of symbolic representation. Within the same line, the teacher repeated the correct offer in Sepedi and in English whilst she explained where learners should place the numbers. This second move between representations was coded as ML1 move where the number word “lekgolo masome supa hlano” was substituted with the number word one hundred and seventy-five. The approach the teacher offered to produce the answer in line 5 was coded as EC2B level involving partitioning or structuring of number into hundreds, tens and ones. This excerpt was coded as EC2B where a procedure with general applicability was observed coupled with ML1, MM1 and MM2*.

Whilst these moves were evident, there was a lack of focused attention on principles in the move to column format and a lack of focus on number relationships whilst moving between the various representations, and thus a ‘principling’-oriented EC code could not be given.

Excerpt 21 from L3 provides a further illustration of the teacher offering structuring of numbers as a way to produce answers coupled with language moves to elaborate on the answer. In this excerpt, following an incorrect response for how to ‘break down the second number only’ in the expression 323 + 136, the teacher offers an elaborated response that once again, suggests the intentional use of ML moves to support understanding as follows:

L3, E21: EC2B/ML2, MM1, MM2*

1. T: We break down that number {points to 136 and points to another child to offer a response}

2. L1: It is hundred plus thirty plus six

3. T: {Writes 100 + 30 + 6 below the 136 in the original expression. Points to 136 pauses and points to the 1 in 136, and then to 3 in 136 and then points to 6}. Say this number {points to 136} in Sepedi so that you can hear it better. When you say the number, it tells you the place it represents. What number is that? {Moves her hand under the number 136}

4. Ls: Lekgolo masome- tharo tshela [Hundred, tens three, six].

5. T: Lekgolo masome tharo tshela. [Hundred one, tens three six]. {Points to 100 in 100 + 30 + 6}. I am hundred {pointing to 1 in 136}. I am 3 tens [points to 30 in the same expression], I am six {points to 6 in the same expression}. It is not one, three and six. One hundred and thirty-six, lekgolo masome tharo tshela. [Hundred one, tens three, six]. We broke it down. Now we will add it to the first number.

Here, the teacher explicitly communicated the rule she wanted learners to follow in order to produce the answer – breaking down the second number, the lead in to working with what have been described as 'jump strategies' in the literature (Sarama & Clements, 2009; Wright et al., 2006). This procedure makes use of base ten relations and therefore has some generality, so EC2B was noted. Following the correct offer of breaking down 136 into one hundred, three tens and 6 units the teacher requested learners to say the number word in Sepedi and moved to explain the value of each digit in the number 136 whilst pointing to each digit on the board. The move to breakdown the number into hundred, tens and units was coded as MM1 move where a different representation of the quantity in the same symbolic register was noted in line 3. A move between English number words and Sepedi coupled with the written symbolic and oral explanation of values of numbers (line 5) was coded as ML2. This explanation indicated again the teacher's awareness of the advantage of the regular structure of names in Sepedi over the irregular English number names (Mostert, 2019; Mdluli,2017) but once again, this regular structure was not generalised to use with other examples as a principle.

The procedure was subsequently followed through:

6. T: In three hundred and twenty-three {points to 323} we start with hundreds. Three hundred and twenty-three plus hundred. {Points to the 100 in the expression 100 + 30+6} What is the answer?

7. Ls: Four hundred and twenty-three.

8. T: {writes 423 below the expression 100 + 30 + 6}. It is one hundred plus three hundred and twenty-three. Plus, thirty plus six. And then we come back and add tens and tens. When we add tens, we will have ….?

9. Ls: {shout out two different answers} 459. 450.

10. T: {pauses and looks at learners. She separates the sum by placing brackets around as in Figure --}

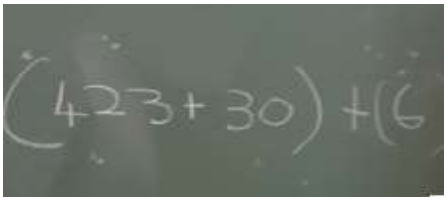


Figure 6: Mid-stream addition

11. Ls: Some: 'Four hundred and fifty-three.' Others: 'Four hundred and fifty-nine.' {Learners continue to shout out two different answers}

12. T: Four hundred and fifty---? I did not hear well. Look at the tens only.

13. Ls: Four hundred and fifty-three

14. T: I only put the numbers that we must add in brackets.

15. Ls: Four hundred and fifty-three

16. T: We will get four hundred and fifty-three {writes 453 under the expression 423+30}. Plus, six. We are left with units now. In the first instance we added hundreds, we then added tens. Now we add units. Units and units will give us--

17. Ls: Four hundred and fifty-nine

18. T: You see. I started by adding hundreds, tens and units. We only break down the second number and add it to the first number, right?

The correct answer of 453 was produced through the general rule of breaking down numbers and adding hundreds, tens and units together. This excerpt was coded as EC2B where answers were produced through approaches involving structuring of numbers coupled with ML2, MM1 and MM2* moves. Specialising proceduralising strategies were observed in this category of excerpt where the teacher transmitted the general rule of place value to produce answers.

Of interest in Nkele's EC2B category excerpts is the manner in which she dealt with the knowledge of place value and number relationships. Excerpt 15 of lesson 2 and excerpts 17, 18, 20 and 21 of lesson 3 all focused on knowledge of place value and addition. Excerpts 17 and 18 focused on establishing the value position of digits in numbers 25 and 135. The MM2* moves observed across these EC2B involved basic moves between the oral and the symbolic modes of representations. Across these excerpts, there is relatively widespread offer of procedures with general applicability, but with each example continuing to be treated separately. This leaves no space for discussions of what could become general principles – that two-digit numbers can be broken down into tens and units, while three digit numbers can be broken into hundreds, tens and units for example.

In relation to the fact that all Nkele's EC2B excerpts are focused on place value, it is worth noting that the literature notes two principles as central to the teaching of place value in the early years. The first is the principle of being able to exchange units for tens, tens for hundreds and hundreds for thousands. The second is the principle of "carrying over" which refers to exchanging 1 in any place for ten in the next place on the right (Haylock & Manning, 2014). In all five excerpts in Nkele's class which were concerned with place value, no focused attention was paid to these two principles. In each excerpt, the focus was on the general procedure of establishing the position of numbers and breaking down numbers using the base ten structure to produce the required answer.

Considered overall, conversion moves in Nkele's teaching relate to connecting oral and written symbolic representations, with relatively limited instances of either treatment or conversion moves beyond the oral-symbolic connection in Nkele's teaching. Such moves, in many ways, do not require the kinds of 'structure preservation' that writers like Duval (2006) associate with moves between registers, even though Haylock and Manning, (2014) and Haylock, (2010;) have emphasised the importance of linking oral language situations with representations in other registers in early mathematics learning. Additionally, Nkele seemed to work with each example task separately and her working lacked a focus on structural representations. Hence her teaching did not incorporate structure. I return to this point in reflecting on the framework in Chapter 7. While Nkele's

data indicates that she did purposively use ML2 moves to sort misconceptions relating to number names and the structure of numbers, her instruction showed limited attempts to move beyond lower level EC for producing solutions, even when learner responses suggested relatively fluent working with the tasks presented.

6.2.2 Mirriam's teaching

Table 17: Overview of excerpts from Mirriam's lessons

Lesson1		Timing	EC/MoR
1	On the board: 56+73 =□	00:04-01:59	EC2B/MM1, MM2*
2	On the board:22 + 34= □	02:01 -04:31	EC2B/MM1, MM2*
3	On the board: 47 + 52 = □	04:39 –08:00	EC2B/ MM1,MM2*
4	On the board: 38+51 and 75 +12	08:07- 17:03	EC2B/MM1, MM2*
5	Individual seatwork: Addition tasks page 87 of the DBE book	17:05-31:01	Not analyzed
Lesson 2			
6	Oral instruction: Chanting in ones from 301 to 400 on a number chart	00:09-07:13	EC2A/ MM2
7	Oral instruction: Identifying 376 on a number chart.	07:19-10:17	EC1/ML1,MM2
8	Oral instruction: Identifying numbers that are bigger than 376 on a number chart	10:46- 11:07	EC1/ MM2
9	Numbers written on the board 315, 355, 305, 351,350: Arranging numbers from the biggest to the smallest	11:09-27:18	EC2A/ ML1,MM2*
10	Numbers written on the board: 315, 355, 305, 351,350 Arranging numbers from the smallest to the biggest	27:33-31:15	EC2A/ MM2*
11	Individual seatwork on page 76 of the DBE workbook	31:40-35:08	Not analyzed
Lesson 3			
12	On the board:75+ 5= □ Number line	00:01-00:08	EC2A/MM2
13	On the board: 175- 25= □ partitioning numbers	00:56-02:33	EC2B/ML1,MM1, MM2*
14	On the board:150+20-100= □	02:39-05:20	EC1/ML1, MM2*
15	On the board: 271+10=□ partitioning numbers	07:24-08:02	EC2B/MM1, MM2
16	On the board:234+23=□ partitioning numbers	11:24-13:16	EC2B/MM1, MM2*
17	On the board: 542+10=□ partitioning numbers	14:00-15:37	EC2B/ MM1,MM2*
18	On the board: 552 +10= □ partitioning numbers	15:50- 18:19	EC2B/MM1, MM2*
19	On the board:552-10=□	18:45- 19:19	EC1/ MM2*
20	On the board: 326+10 =□ counting	20:20- 24:09	EC2A/MM2*
21	On the board:198-10= □	24:41- 31:49	EC1/MM2*
Lesson 4			

22	On the board 120+40=□ using number lines	00:08 -03:08	EC2A/MM2
23	On the board 180-80 =□ using number lines	03:10-04:41	EC2A/ML1,MM2
24	On the board 135+45= □ on a number line	04:55- 11:33	EC2B/ML1,MM1, MM2
25	On the board 175-30 =□ using number lines	11:53- 13:15	EC2A/MM2
26	On the board 126+14=□ using number lines	13:28-27:56	EC3/ML1,MM2
27	On the board 174+12=□ using number lines	28:28-35:41	EC3/MM2

Table 17 shows that four lessons were observed in Mirriam's class over a period of three school terms in 2014. Of interest, is the generally higher number range of additive working in Mirriam's class in comparison to Nkele's class. Twenty-five of the twenty-seven excerpts were analysed. Two excerpts were not analysed because they involved individual seatwork.

Table 18: Coded excerpts from Mirriam's lessons

EC Levels	No of excerpts	Multilingual moves		Mathematical moves	
		Translation (ML1)	Translanguaging (ML2)	Treatment (MM1)	Conv (MM2)
1	5	L2: E7 L3: E14			L2:E7 L2:E8 L3:E14* L3 E19* L3:E21*
2A	8	L2:E9 L4:E23			L2:E6 L2:E9* L2:E10* L3:E12 L3:E20* L4:E22 L4:E23 L4:E25

2B	10	L3:E13 L4:E 24		L1:E1 L1:E2 L1:E3 L1:E 4 L3:E13 L3:E15 L3:E16 L3:E17 L3:E18 L4:E24	L1:E1* L1:E2* L1:E3* L1:E 4* L3:E13* L3:E15* L3:E16* L3:E17* L3:E18* L4:E24
3	2	L4:E26			L4:E26 L4:E27

Table 18 provides a summary of the spread of excerpts from Mirriam's teaching in relation to what she transmitted as evaluative criteria and how modes of representation featured in what was transmitted across excerpts and across lessons. In the analysis that follows, the excerpts within each EC level are considered and compared, with particular attention to how they overlap and contrast, and how ML and MM moves feature across the levels. Table 18 shows that 5/25 excerpts in Mirriam's Sepedi medium classroom were categorized at EC1 level, marked by incidents where learners offered correct answers, or the teacher offered correct answers without offering a rule or a procedure used to produce the answer. Two of these five excerpts were characterised by ML1 and MM2 moves. The remaining 3 excerpts were categorised by MM2 only moves.

EC1

An illustrative example of EC1 with ML1, MM2 is drawn from Excerpt 7 in lesson 2 to demonstrate the ongoing association of lower level of ML1 moves with lower EC levels.

L2, E7: EC1/MoR-ML1, MM2

301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350
351	352	353	354	355	356	357	358	359	360
361	362	363	364	365	366	367	368	369	370
371	372	373	374	375	376	377	378	379	380
381	382	383	384	385	386	387	388	389	390
391	392	393	394	395	396	397	398	399	400

:

Figure 7: Number chart from 301 to 400

1. T: {Learners have the 301-400 number charts on their desks.} On your charts, there are smaller numbers. Put your fingers on makgolo tharo masome šupa tshela [hundreds three, tens seven, six.]
2. Ls: {Look at the teacher}
3. T: Point to three hundred and seventy-six. Say the number and point.
4. Ls: Makgolo tharo masome šupa tshela [hundreds three, tens seven, six].
5. T: Yes, makgolo tharo- masome šupa tshela [hundreds three, tens seven, six] {Points to hundreds three tens seven six on the number chart} Let us move to the next number.

In this excerpt, learners had their number charts on their tables when the teacher instructed them to put their fingers on number hundreds three, tens seven, six [376]. A translation move was evident in line 1 when the teacher first stated the number name orally in Sepedi and in English. In line 5 pointing was used to make connections between oral and symbolic representation resulting in an MM2 coding as in Nkele's teaching, but here, a number chart representation - a symbolic number system based representation in Ensor et al (2009) terms was also incorporated, pointing to a wider repertoire of work with conversion moves in comparison to Nkele's teaching. Learners pointed to the correct answer on their number charts. The teacher accepted the correct answer by pointing to the number on the chart and orally presenting the number word. This offering and acceptance of the answer was coded as EC1. The excerpt was coded as EC1/ ML1, MM2.

As with the previous example, Excerpt 19 is chosen to exemplify EC1 category excerpts where MM2* moves are evident and a lack of elaboration or unpacking of correct offers.

L3, E19: EC1/MoR-MM2*

1. T: {writes 552 -10 on the board}. We take away ten from five hundred and fifty two. Do not touch your abacus. Leave the abacus. Face the front. We are taking away ten here. {Pointing to the number symbol 552 on the board}. What is the answer? We are taking away ten.

{Teacher moves from learner to learner as incorrect answers are offered until}

2. L: Makgolo hlano- masomenne-pedi. [Hundreds five, tens four, two]3. T: Good people, we are moving back now. The answer is makgolo hlano- masomenne- pedi. [Hundreds five, tens four, two].

In this excerpt, the teacher introduced the task symbolically on the board as 552-10 = □ and then orally stated the task as five hundred and fifty-two take away ten. MM2* was observed. No procedure for learners to follow to produce the answer was stated, in spite of several incorrect interim offers. A learner provided the correct answer in line 2. The teacher repeated the correct answer in Sepedi and suggested a move to the next task. This move was coded as EC1 where at the close of the excerpt a learner produced the correct answer and the teacher accepted the offer.

Three excerpts at EC1 in Mirriam's teaching involved a context in which MM2* contained moves between the symbolic and the oral representations with no elaboration on learner offers. Whilst two excerpts at this level involved the use of the number chart- symbolic-number based form of representation. The EC1 level excerpts observed in Mirriam's classroom focused on identifying numbers on the number chart and adding or subtracting 10 from a given number as shown in the overview table above. The use of structured symbolic representations in Mirriam's teaching involving number charts was observed in 2/5 EC1 excerpts. The use of this symbolic number-based form of representation provided access to the decimal system. This use of structured or number base symbolic representation was not seen in Nkele's teaching. While excerpts 7 and 8 in lesson 2 in Mirriam's teaching, focused on identification of the numeral 376 and on identifying numbers that are "bigger than" 376 on the number chart, suggested openings for relational thinking, there was no focused attention on structure and relationship in her

enactment of these two excerpts. Each excerpt was treated separately with no explicit talk about the structure of the number chart and how this tool could enable the development and generalizing rule relating to movement on the number chart when identifying numbers that are more or less than 376 or any number. A focus on structure and appreciating structure involves careful selection of examples and making explicit the relationships and general properties observable in selected examples (Mason et al., 2009).

Whilst Mirriam worked with examples separately her working with number charts (structured representations) incorporated attention to structure. This incorporation of structure was not observable in Nkele's teaching. The overview table shows that in excerpt 18 Mirriam focused on adding 10 to 552. In excerpt 19 she focused on subtracting 10 from 552. Whilst she had dealt with 552 + 10 in excerpt 18 and learners produced the answer to this task, in excerpt 19 no reference was made to the previous task of adding or subtracting ten. The relationship between adding ten and subtracting (inverse relationship) was not made explicit in the way in which the Mirriam dealt with the two tasks. Each task was treated as a separate task. A lack of teacher talk that focused on connections within and between examples (Ekdahl et al., 2018) is noted in this category of excerpts.

The subsequent section is presented to exemplify EC2A category of excerpts where teacher talk focused on offering or accepting counting-based procedures to produce answers. 8/25 excerpts in Mirriam's Sepedi medium classroom were categorised as EC2A level excerpts. Two excerpts were categorised as ML1, MM2. Six excerpts in this category were categorised as EC2A/MM2. Three of these 8 excerpts were characterised by MM2* whilst the other five were characterised by MM2 moves. In this category of excerpts translation and conversion moves between representational modes were evident. Similar to conversion moves observed in EC1, conversion moves in this category were dominated by moves between the oral and the symbolic modes of representation, but there were some instances of conversion moves involving symbolic number-relation

based representations, as seen in the example below where the number line was used as a form of representation.

EC2A

Example 1 below is provided to illuminate EC2A category of excerpts with translation and conversion moves where counting based procedures are privileged.

L4, Excerpt 23: EC2A/ ML1 & MM2

1. T: {cleaned the board retaining a number line from 100 to 200 in multiples of 10s. She then wrote the number sentence 180-80}. Lekgolo- masomeseswai ntsha masomeseswai [Hundred, tens eight take away tens eight].

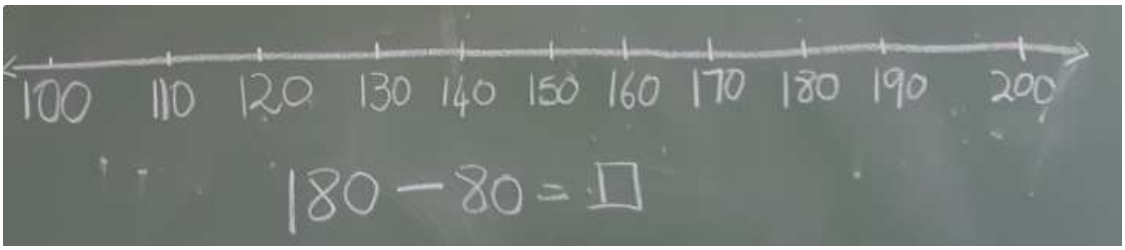


Figure 8: Number line subtraction task

{Points to the number sentence on the board} You always look at your first number and that is where you should start. Re na le lekgolo masome seswai [We have hundred tens eight]. You have to look at where that number is on the number line. {Point to 180 on the board}. Do you hear me?

2. Ls: Yes

3. T: Our number is hundred and eighty. Lekgolo- masomeseswai. Where are you going to start?

4. Ls: Lekgolo masome seswai [hundred, tens eight]

5. T: From lekgolo masome seswai. [Hundred, tens eight] what are we going to subtract?

6. Ls: Masome seswai [tens eight].

7. T: How are we going to count?

8. Ls: tens.

9. T: We count in tens because there are no fives. {Points to 180 on the board}. Because we are subtracting; are we going forward or backwards?

10. Ls: Backwards others forward.

11. T: Ko pele or ko morago? [Backwards or forwards].

12. Ls: Backwards

13. T: {makes the jumps on the number line backwards from 180 to 100 as she counts in multiples of ten}. One hundred and seventy, one hundred and sixty, one hundred and fifty, one hundred and forty, one hundred and thirty, one hundred and twenty, one hundred and ten and one hundred. What is our answer? Count from one hundred and eighty.

14. Ls: lekgolo masome šupa, lekgolo masome tshela, lekgolo -masomehlano, lekgolo-masomenne, lekgolo- masometharo, lekgolo -masomepedi, lekgolo-lesome, lekgolo {chanting the numbers in multiples of ten as the teacher made the jumps on the number line}. [Hundred, tens seven, hundred, tens six, hundred, tens five, hundred, tens four, hundred, tens three, hundred, tens two, hundred, ten one, hundred]

15. T: What is our answer?

16. Ls: Lekgolo [Hundred]

17. T: Hundred. Lekgolo. Did you understand?

18. Ls: Yes.

In this task learners were asked to subtract 80 from 180 on the number line. The teacher made it explicit in her statement of the task that she wanted learners to look for the first number and start from that first number. Conversion moves were observed at the statement of the task when the teacher presented the task symbolically on the board and orally represented the same task (line 1). This move was coded as MM2 involving the use of a number line. This use of a number line (symbolic number based form) in Mirriam's teaching shows a different type of MM2 teaching to Nkele's teaching which simply involved moves between the oral and the symbolic form of representations. Translation moves were observed in two parts of this excerpt. In line 3, the teacher stated the number name one hundred and eighty in English and moved to state the same number word in Sepedi and thus this move was coded as ML1. Evaluative criteria were transmitted in line 9 when the teacher told learners to count in tens, and then produced the answer through counting backwards with 8 jumps of ten whilst she counted in multiples of ten from 180 to 100. While the reasoning in Line 9 stating counting in tens because there are no fives is mathematically problematic, a figurative composite grouping approach (Anghileri, 2006)

is noted as the way to count. This way of producing the answer was coded as EC2A where the answer was produced through counting in multiples of ten. The excerpt was coded as EC2A/ML1, MM2. Localising proceduralising strategies were observed in the enactment of this task where learners were encouraged to rely on counting (in multiples here) to produce the answer.

The first three excerpts categorized at EC2A level in Mirriam's class focused on counting and arranging numbers from the biggest to the smallest and from the smallest to the biggest. It was interesting to note that in Mirriam's EC2A there were no attempts made across the three excerpts to encourage learners to look for relationships between numbers given in any of the three tasks. Teacher talk in Mirriam's EC2A excerpts lacked a focus on number relationships. In one example where learners were asked to sequence numbers 315, 355, 305, 351 and 350 from the smallest to the biggest, Mirriam focused on placing the numbers in a sequence without a focused attention on the values of numbers and relationships between the numbers involved. Each number presented had a 3 and a 5 in different place value positions. This salient feature of the numbers was not focused on to help learners to rearrange the numbers and produce the required sequence. Mirriam suggested that learners use their number chart and count. In this approach, numbers that come up later in the count are bigger although this is not demonstrated nor made explicit through teacher talk.

Four excerpts in this EC2A category focused on addition and subtraction of numbers on number lines whilst one excerpt, excerpt 6 focused on counting on the number chart. The symbolic number base form of number representation was noted (Ensor et al., 2009) in these five excerpts. Another representation apart from oral to symbolic is noted. Similarly, in these excerpts, learners were instructed to count on and back in tens or in fives to produce answers with no push to move learners to use known number facts to produce answers. There was no focus on the use of known number facts that learners could retrieve such as splitting one or both numbers to use known facts to produce the answer. For instance, one of the tasks in this category required learners to subtract 30 from 175. One efficient way, other than counting in fives, would be to split 30 into 15 and 15 and

make one jump of 15 from 175 to land on 160 and make another jump of 15 to land on 145. Instead of only making the two quick jumps, the teacher insisted that learners take away five at a time from 175 to arrive at 145. A lack of focused teacher talk on number relations, structure and progression from counting to flexible calculation approaches characterized Mirriam's teaching at this EC2A level coupled with moves between the oral and the symbolic representations. Localising proceduralising strategies characterised teacher talk in all the EC2A excerpts observed in Mirriam's Sepedi medium classroom where calculation by counting based approaches were offered or accepted at the close of each of the eight excerpts that categorised EC2A.

Whilst the previous section presented examples of EC2A category excerpts in Mirriam's class, the next section consists of examples of EC2B category excerpts. 10/25 excerpts from Mirriam's Sepedi medium classroom were categorised at EC2B where teacher talk focused on number structuring to produce answers to tasks. These excerpts exhibited a variety of moves between representations. Seven excerpts in this category were dominated by MM2 involving moves between the oral and the symbolic representations and the symbolic number base form of representation involving the use of number lines. Two excerpts contained MM1 and MM2 moves. The tenth excerpt was characterised by ML1, MM1 and MM2 moves.

EC2B

The subsequent example has been chosen to exemplify EC2B category excerpts where a network of moves involving, ML1, MM1, and MM2 were observed.

L4, Excerpt 24: EC2B/, ML1, MM1, MM2

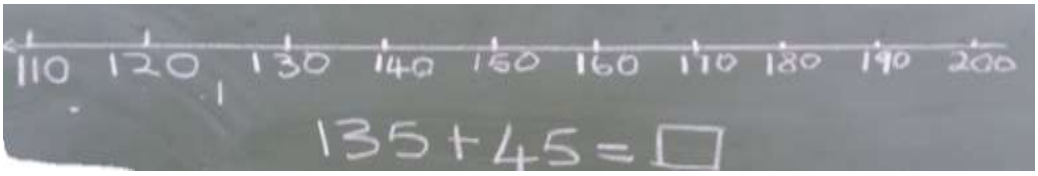


Figure 9: Addition on a number line

1. T: {Writes 135 + 45 below the number line on the board}. Lekgolo- masometharo- hlano plus masomenne- hlano ['Hundred, tens three, five plus tens four, five], {points to the number sentence on the board}. Do we have this number in our number sentence? {Points to 135

on the board below the number line} What do we have? What must we start with? What do we have that we must begin with before we do everything else?

2. L: {points to the space between 130 and 140 representing 135 on the number line}.

3. T: Yes, we then make our marks of fives and add masomenne -hlano to lekgolo- masome tharo-hlano [tens four five to hundred three tens five]. {Makes a mark between 130 and 140. Writes 135 below the mark on the number line}. We start with lekgolo masome tharo hlano [hundred, tens three, five] and add masomenne- hlano [tens four five]. We start here. {Points to 135 on the board} What do we count in?

4. Ls: Fives

5. T: We will count fives that will give us four tens five. Do you hear me? We start here. {Points to 135 on the number line}. Let us start, hlano [five] {continues to point between the tens in fives and chanting along the numbers}

6. Ls: {Count along 45 jumps in 5s from 135}. Five, ten, fifteen twenty, twenty-five, thirty, thirty-five, forty, forty five {as learners were chanting numbers, the teacher traced around the 9 jumps of 5 on the number line}

7. T: What is our answer?

8. Ls: Lekgolo masome seswai, [Hundred, tens eight]

9. T. Yes. One hundred and eighty. Lekgolo- masomeseswai. Now we check if our sum is right. When we check, we do not check in our books. We write on the side. Lekgolo- masometharo-hlano plus masomenne- hlano. [Hundred tens three five plus tens four five {she writes this sum in vertical column form as shown in Figure 10}. We look at our answer if it is the same. Hlano plus hlano? Five plus five? {Pointing to the vertical calculation on the board}

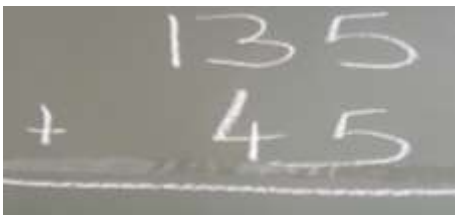


Figure 10: Vertical representation of the task

10. L: Lesome [one ten].

11. T: {writes 0 below 5 and 5 on the board}. What do we do?

12. Ls: We carry one

13. T: I cannot hear you. Can one person tell us?

14. L: We write zero. We carry one.

15. T: We write zero. We carry one. What is three and four? T: What do we do with the one that we carried? We add it. Šupa [seven] plus tee [one] is what?

16. Ls: Seswai. [Eight]

17. T: Seswai. [Eight]. {Writing 8 on the board underneath 4 and 5}. One plus zero?

18. Ls: Tee [One]

19. T: {[Writes the answer 180 below the numbers written in the vertical column forma}. This means our answer is correct. We did the correct thing on our number line. {Points to the number line above the horizontal algorithmic task she wrote to verify the answer}. Did you understand it?

20. Ls: Yes

The teacher wrote the task on the board symbolically as 135 + 45 and orally presented the task in Sepedi in line 1. This move was coded as MM2 involving the move from the symbolic number sentence to the number line representation. Following the production of the answer, the teacher made a suggestion to verify the answer and wrote the task vertically on the board. . This move was coded as MM1 where moves within the same symbolic register were observed. In the same line 8 two instances of translation moves were observed when the teacher repeated the correct answer in Sepedi and in English and when she asked learners in Sepedi to add 5 and 5 and repeated the same instruction in English. This move was coded as ML1. In her method of verifying the answer, all values of digits in the two numbers 135 and 45 were reduced to single digit values, for instance, 3 in the tens position was referred to as three and 4 in the tens position was referred to as four. The place value procedure of carrying numbers from units to tens was observed in this excerpt. Hence it was coded as EC2B given that structuring of numbers into hundreds, tens and units is configured into the conventions of traditional column addition. Whilst the principle of exchanging a ten from units to the tens column underlies this calculation, this principle was not focused on. This excerpt was coded as EC2B level because general procedures: plotting the first number and jumping the second number and column addition underpinned by place value were transmitted.

Ten tasks in Mirriam's teaching were categorised at EC2B level. Nine of these tasks were characterised by MM2* moves whilst one task (L4:E24) was characterised by MM2. Mirriam's talk in the enactment of tasks categorised at EC2B level included some focus on more general procedures. However, examples continued to be dealt with separately (Ekdahl et al, 2018) and mathematical principles underlying the procedures were not in

focus. Additionally, excerpts at this EC2B level involved MM1 moves. This is a treatment move – a move to a different representation of the quantity in the same symbolic register. These treatment moves are regarded as less complex moves than the conversion move (Duval, 2006) as they do not involve moving between registers.

EC3

The subsequent section discusses excerpts in Mirriam's teaching that made up EC3 category. Two excerpts (excerpts 26 and 27) in Mirriam's teaching were empirically different to excerpts across the first three levels of the EC. In this level of the EC, Mirriam's enactment of tasks focused on allowing learners to use their own approaches to produce answers. In these two excerpts, the use of known facts to derive answers was noted. One example is presented to exemplify EC3 category of excerpts coupled with ML1, MM2 moves. Excerpt 26 in the subsequent example is used to exemplify EC3 where teacher talk focused on the use of known number facts to produce answers

L4, Excerpt 26: EC3/ ML1, MM2.

1. T: We are moving on now. {Writing 126+14 =□ on the board}. Add the two numbers on the number line like we did before. Lekgolo- masomepedi -tshela plus lesome nne. [Hundred tens two six plus ten four]. Can someone come and do this one? One hundred and twenty-six plus fourteen. {Pointing to the number sentence on the board}. We count forwards when we do what? {Calls a learner to the front}

2. L1. {Makes three jumps of 5 from 125, to 130, to 135 then to 140. He completed the number sentence and wrote 126 + 14 =140}.

3. T. Let's us check his answer now. Is there another way? Come show us. {Calling on another learner}

4. L2. {Makes a jump of ten from 125 to 135 and a small jump of 5 to land on 140}

5. T. I do not follow what you are doing here.

6. L2. {Points to the number line he made on the board}. I took the one from one hundred and twenty-six and added it to fourteen to make it fifteen. I then added ten to hundred and twenty-five to have one hundred and thirty five. I plussed a five to have one hundred and forty.

7. T. {looked at learner for a while}. You took that one and put it here? {Pointing to 126 and 14}.

8. L: Yes. I took one from lekgolo- masomepedi- tshela [hundred tens two six] and added it to lesome nne. [Ten four] to have lesome hlano [ten five]. Then I added ten first then five. My answer is one hundred and forty. It is the same.

9. T: Do you see it?

10. Ls: Yes

In this excerpt learners were required to add 14 to 126 on a number line. A structured number line from 100 to 150 in multiples of fives was available on the board. In line 1 the teacher presented the task symbolically and orally presented the task (MM2). The phrasing in line 1 “add the two numbers on the number line like we did before” suggests the use of a previously established general procedure. In the same line the teacher orally presented the number sentence in Sepedi and moved to state the same number sentence in English (ML1). A learner produced the answer by making three jumps of 5 without visibly counting (line 2). At this point the excerpt was coded as EC2A. The learner then wrote the correct answer to complete the number sentence. Following the production of the correct answer the teacher suggested that learners verify the answer. A learner came up and made one jump of ten and a small jump of 5 on the number line without any visible act of counting. This learner explained his approach which was devoid of counting and involved awareness of the principle of equivalence in addition: reducing one quantity by one while increasing the other quantity by one produces the same total. There was also evidence within this working, evaluated by the teacher as correct, of learner work with known facts relating to adding fives, and the derived fact that 126 + 14 would be equivalent to 125 + 15 (Askew, 2013). A treatment type move is also evident in this (move within register). Additionally, this approach requires learners to demonstrate the ability to partition any single number in different ways and to borrow from one number in order to make another number a five or a ten (Threlfall, 2002; Thompson, 1999). This excerpt was coded as EC3 where number facts and knowledge of number relations were used to produce the answer. This excerpt was coded as EC3/ML1, MM2. Specialising principling strategies were observable in this excerpt.

This experience where learners were asked to explain their approaches was only observed in these two EC3 category excerpts. The use of known facts to derive unknown

number facts is regarded as an important aspect of learning and understanding mathematics. It is argued that derived facts can lead learners to move away from being reliant on counting based approaches to produce answers to addition and subtraction tasks (Thompson, 2011; Wright et al., 2006).

While some progression was evident in Mirriam's teaching with lesson one focused on addition and subtraction of two digit numbers and lessons three and four focused on addition and subtraction of two and three digit numbers, and while there was inclusion of general procedures and principles, examples were dealt with separately with predominant focus on production of correct answers and not on drawing learners' attention to structure and generalizing of mathematical ideas. Mirriam's lower level excerpts though, unlike Nkele's lower level excerpts, contained MM2 moves that included more than simply oral restatements of symbolic number sentences. Instead, her instruction commonly included number lines/100 squares (symbolic number based modes of representation in Ensor et al's (2009) terms with more substantive conversion moves between symbolic number sentences, symbolic number base and oral language modes of representation.

Section 6.2 focused on analysis of excerpts observed in the two Sepedi medium classrooms. Section 6.3 focuses on analysis of excerpts in the English medium classrooms.

6.3 ENGLISH MEDIUM CLASSROOMS

6.3.1 Laura's teaching

Table 19: Overview of excerpts from Laura's lessons

Lesson1	Task	Timing	EC/MoR
1	Oral instruction: Counting 10 fingers in multiples of fives	00:15-01:29	EC2A/ MM2
2	Oral instruction: Counting three fingers in multiples of fives	01:50- 03:44	EC2A/ MM2
3	Oral instruction: Counting 10 fingers in multiples of three.	03:55- 04:23	EC2A/MM1, MM2

4	Individual seatwork: DBE pages 4 to 7 (counting activities)	04:29 -25:18	Not analyzed
Lesson 2			
5	Oral chanting numbers in multiples of 3s from 3 to 100	00:00-01:39	EC2A/ MM2*
6	Oral chanting numbers in multiples of 4 from 4 to 100 in 4s	01:45-04:06	EC2A/ MM2*
7	Drawing of 3 squares on the board: Three squares how many corners altogether? Counting 3 squares in groups of 4	04:10 -03:20	EC2A/ MM2
8	Written on the board: 3 x 4 corners	03:24- 07:45	EC2A/ MM2
9	Written on the board:5 x 4 =20	07:51-10:18	EC2A/ MM2
10	Oral instruction:10 squares x 4 corners	10:20- 11:27	EC2A/ MM2
11	Oral instruction accompanied by the symbolic representations of numbers in multiples of 4. Number of chairs with 4 legs each with. There are 4 chairs in class. Each chair has 4 legs. How many legs altogether?	11:28: -11:30	EC1/ MM2*
12	Oral instruction: Party and sweets in class	11:30-12:27	EC1/ MM2*
13	Oral instruction with multiples of 4 from 4 to 20 written vertically on the board. Forty people 4 sweets each	13:49- 16:56	EC3/ MM2*
14	Individual seat work (doubling)	16:59-26:33	Not analyzed
Lesson 3			
15	Oral instruction: Friends of ten	00:04-01:25	EC1MM2
16	Chanting numbers in threes from 301 to 400 on a number chart	01:40 -01:54	EC2A/ MM2
17	Chanting numbers fours from 401 to 500 on a number chart	01:58 -02:26	EC2A/ MM2
18	Written on the board: Breaking down numbers 26+32 =□	02:34 -09:45	EC2B/MM1, MM2*
19	Written on the board: 62+15 =□	09:56 -14:05	EC2B/MM1, MM2*

20	Written on the board: 89 -25 =□	14:08- 18:09	EC2B/MM1, MM2*
21	Written on the board: 93 – 61 = □	18:20 -22:04	EC2B/ MM1,MM2*
22	Individual seatwork: p45 of the DBE workbook for Terms 3 and 4.	22:18 -29:57	Not analyzed
Lesson 4			
23	Oral instruction: Chanting numbers in twos from 2 to 40	00:13- 00:49	EC2A/MM2*
24	Number cards: 251 which number cards do I need to make this number?	01:12- 02:55	EC2B/MM1, MM2
25	Oral instruction: Counting in 5s starting from 2 on the number chart	03:11- 05:29	EC2A/MM2
26	Oral instruction: Counting in 5s starting from 3 on the number chart.	05:57- 08:13	EC2A/MM2
27	Individual seat work: Completion of number patterns on page 31 of the DBE workbook term 3 and 4.	08:34- 25:17	Not analyzed

Four lessons were observed in Laura's classroom across the three school terms in 2014. These lessons were broken down into excerpts. Twenty-seven excerpts were generated from Laura's four lessons, of which 23 were coded as shown in Table 19.

Table 20: Coded excerpts from Laura's lessons

EC levels	No of excerpts	Multilingual moves		Mathematical moves	
		Translation (ML1)	Translanguaging (ML2)	Treatment (MM1)	Conv (MM2)
1	3				L2: E11* L2:E12* L3:E15
2A	14			L1:E3	L1:E1 L1:E2 L1:E3 L2:E5*

					L2:E6* L2:E7 L2:E8 L2:E9 L2:E10 L3:E16 L3:E17 L4:E23* L4:E25 L4:E26
2B	5			L3:E18 L3:E19 L3:E20 L3:E21 L4:E24	L3:E18* L3:E19* L3:E20* L3:E21* L4:E24
3	1				L2:E13*

At an overview level, Table 20 indicates that Laura's teaching was dominated by conversion moves. This kind of move between representations is regarded as more complex than a treatment move because it requires recognition of the sameness between two representations that are different (Duval, 2006), although Nkele's excepts also indicate possibilities for moves between representations that involve only oral restatement. Conversion moves observed in Laura's enactment of tasks commonly involved more than just moves between oral and symbolic representations. Ten excerpts were characterised by MM2* where Laura moved between the oral and the symbolic representations whilst thirteen excerpts were characterised by MM2 involving moves between the oral, concrete, pictures and number based modes of representation. The characteristics of her conversion moves will be illustrated in the subsequent discussion.

3/23 excerpts observed in Laura's lessons were categorized as EC1 excerpts where correct offers were produced by learners and accepted by the teacher or offered by the teacher. All excerpts in this EC1 category involved MM2 moves. Two of Laura's MM2

excerpts involved moves between oral and symbolic representations. The third excerpt involved moves between oral and concrete representations. Both types are illustrated below.

EC1

L2, Excerpt 11: EC1/ MM2*

1. T: Lets' say there are chairs with four legs in our classroom. I give you the number of chairs; you give me the number of legs. One chair, how many legs?
2. Ls: Four legs
3. T: {Writes 4 on the board}. Right, two chairs?
4. Ls: Eight legs
5. T: Three chairs?
6. Ls: Twelve legs:
7. T: Four chairs, how many legs:
8. Ls: Sixteen
9. T: {Writes 16 below 12}. Now five chairs, how many legs altogether?
10. L: Twenty
11. T: {Writes 20 below 16 on the board}

In this excerpt, Laura accepted learners' responses for the number of legs for a given number of chairs. This excerpt was coded as EC1. As learners offered correct answers orally, Laura wrote the answers vertically on the board as shown in figure 13. This move between the oral and the symbolic was coded as MM2.The excerpt was coded as EC1/MM2.

L3, Excerpt 15: EC1/MM2

1. T: Today we are going to do friends of 10. The easiest one! Friend of five? [Points to a learner]
2. L: Ten.
3. T: Five plus five is? {Showing five and five fingers using both her right and left hands at the same time}. So, five's friend is five not ten. Four's friend {holding up four fingers}? Four's friend is?

4. Ls: Six.

5. T: Six's friend?

6. Ls: Four.

7. T: What is seven's friend?

8. Ls: Three.

9. T: Three. Do you understand? And seven's friend. Okay! One's friend?

10. L: nnnnnn.

11. T: One's friend? {Stares at the class}

12. Ls: Nine

13. T: Nine. And nine's friend?

14. Ls: One.

15. T: Don't shout! You are very restless! I want to move to something else now!

The task, task involving partitioning of ten or bonds of ten, was presented orally with the teacher wanting learners to identify "friends of ten". In line 1 the teacher asked learners for a "friend of five". A learner provided an incorrect offer of ten. In response to this incorrect offer, the teacher offered the correct answer of 5 plus 5 in line 3 whilst she held up five fingers on each hand. This move between the oral and the concrete representation was coded as MM2. A similar move between the concrete and the oral was observed in the same line when the teacher asked about 4s friend and showed 4 fingers. No rules for producing the answer were stated. The teacher offered the correct answer. EC1 was noted. The excerpt was coded as EC1/MM2. Repetition of correct offers was used to acknowledge the answers and to show agreement (Lyster & Ranta, 1997). Acknowledgement of the correct response was followed by the teacher's request for a friend of that number. Though the sequence of the teacher's questions presented opportunities to talk about the commutative principle of addition, this principle was not explicitly foregrounded in the enactment of this task, as there was no reference to the previous combinations of ten already stated. A lack of explicit talk about the mathematical principle of commutativity and connections between examples (Ekdahl et al., 2018) is

evident in this excerpt. In the context of this task, learners were made aware of the correct responses and thus this excerpt was coded as EC1.

14/23 excerpts in Laura's English medium class were coded as EC2A where teacher talk focused on the use of counting based approaches to produce answers. Eleven of these excerpts involved MM2 where the teacher moved between the oral, concrete, iconic and symbolic modes of representations (Haylock, 2010). Three excerpts in this category involved MM2 moves between the oral and the symbolic modes of representations were noted similar to the MM2 codings observed in the EC1 excerpts discussed previously. Eleven of these 14/23 involved moves between the concrete, iconic and symbolic modes of representation. Two excerpts are chosen to exemplify EC2A category of excerpts. Whilst the first example was chosen to illuminate MM2 where the teacher moved between the oral, iconic and symbolic representations, the first example is chosen to exemplify Laura attempt to move learners from learners from concrete counting to more abstract ways of counting on a number chart.

EC2A

L2, Excerpt 7: EC2A/ MM2

1. T: What about if I had 3 squares. {Drawing ☐ ☐ ☐}. One, two and three squares. How many corners does this square have? {Pointing to the first square on the left}.
2. Ls: Four
3. T: I have got three. [Writes 3 below the three squares]. Times how much? How many corners each? Times how many corners? {Writing x after the 3. Now the writing on the board is 3 x}
4. L: Four
5. T: {writes 3 x 4}. Let's count. Do we have to count one, two, three, four? {Points to the four corners in the first square} No. We can count these squares in groups of four
6. Ls: No.
7. T: What do have to do? No do not shout! {Calls out a learner to respond}

8. L1: It's twelve

9. T: Yes, we do not have to say, one, two three, four. Let's count altogether in fours. {Pointing to each square as learners count}

10. Ls: Four, eight twelve

11. T: {Completes the 3 x 4 number sentence with = 12}. Let's count again. {Points to each square starting from the left to the right}

12. Ls: Four, eight, twelve

13. T: How many squares altogether? {Using hands to show altogether}. If we have three squares {Holding up three fingers} how many corners altogether?

14. Ls: Four, eight, twelve.

15. T: Let's count again. {Raising one finger at a time, learners chant 4, 8, 12}. How many corners altogether, Tumi?

16. L: Twelve

17. T: {completes the sum on the board to read as 3 x 4 = 12} Well done. Three is the number of squares {pointing to 3 in the expression 3 x 4 = 12} then 4 {pointing to 4 in the expression 3 x 4 = 12} is the number of corners in each square. Do you understand now?

Here, the iconic representation of 3 squares on the board accompanies an oral mode of representations were observed. Once learners identified the number of squares and the number of corners in one square, the teacher wrote a number sentence to represent the three squares each with four corners on the board. This move between the iconic, the oral and the symbolic was coded as MM2. Following the establishment of the number of squares and the number of corners in each square, learners were instructed to count to produce the answer. A deliberate instruction is noted in line 9 to discourage learners from counting in ones to produce the answer. The answer was produced by counting in multiples of four, with the teacher using her fingers to help them keep track of the number of fours counted. This way of counting was coded as EC2A. The excerpt was coded as EC2A/MM2.

L4, Excerpt 26: EC2A/ MM2

A 100-number chart from the previous excerpt was on the board.

1. T: What if we count in fives but starting at three? It will be three and eight. Are you okay with three and eight? Count! {Uses her finger to point to the sequence of numbers as learners chant}

2. Ls: {Silently count on in ones using five fingers from the first number and say the last number}. Eight, thirteen, eighteen, twenty-three, twenty-eight, thirty-three, thirty eight, forty three, forty eight, fifty three, fifty eight, sixty three, sixty eight, seventy three, seventy eight, eighty three, seventy eight, eighty three, eighty eight, ninety three ninety eight.

3. T: {Writes, 8, 13, 18, 23, 28, 33, and moves to place a sticker on 36}. What if we start at thirty-six? {Holding up 5 fingers}. Show your fingers and count in 5s.

4.Ls: {Silently count on five fingers from the given number} Thirty-six, forty-one, forty six, fifty one, fifty six, sixty one, sixty six, seventy one, seventy six, eighty one, eighty six, ninety one.

In this excerpt, the teacher wanted learners to count in fives but starting from three. The number chart she presented started from 1 to 100. As learners started counted five on their fingers and chanted number words from eight, the teacher wrote the number symbols on the board thereby making a move from the concrete, oral to the symbolic mode of representation. This move was coded as MM2. An interesting move away from the number chart was noted in the subsequent part of the excerpt when the teacher introduced bigger numbers not represented on the number chart as seen below:

5. T: What if I asked you to start with one hundred and twenty-three and one hundred and twenty-eight? [Writes 123 and 128 on the board and points to a learner]

6. L: {silently counts on five fingers from one hundred and twenty-eight} One hundred and thirty-three

7. T: Well done! {Writing 123, 128, and 133}. What will the next number be?

8. L: {silently count on five fingers from one hundred and thirty-three} One hundred and thirty-eight.

9. T: Well done. You do not have to worry about the numbers, you need to remember the pattern and will only worry when we get to two hundred. What if I asked you to start at {writing 567, 572, and 577? and points to the last two digits in the numbers 567, 572, 572, 577, she wrote above}. What will the next number be?

10. L: {Silently count five on whilst raising one finger at a time to track the counting}. Five hundred and eighty-two.

11. T: What will the next number be?

12. L: {No response}

This excerpt was coded as EC2A because the answer was produced through calculating by counting on approaches which the literature describes as more sophisticated than

counting from the first number (Ellemor-Collins & Wright, 2009; Van Den Heuvel-Panhuizen, 2001). While a move away from the number chart was evident when the teacher extended the number range to include numbers not visible on the number chart, no connections were made to generalizing the earlier number pattern so learners could produce the answers without counting. Teacher talk made no reference to values underlying the numbers or to number relations. Instead the teacher focused on the physical appearance of the numbers when she asked learners to look at the units in each number. A more general procedure was therefore not in focus. This excerpt was coded as EC2A/MM2 where counting was used to arrive at the answer.

14/23 excerpts from Laura's lessons were coded as EC2A level of the EC/MoR model. Several of these excerpts involved counting on in various intervals 'off' the multiple (e.g. counting on in 4s starting at 3). Learners were encouraged to count on using their fingers to track the amount they have counted. Thompson (1999) points out that counting on is the first approach learners use after they have mastered the counting all approach and is more advanced because it teaches learners that counting can begin at any point and not always from one. Though the examples used in excerpts such as excerpts 25 and 26 had potential for seeing more general procedures or principles with a base in known and derived facts, instruction remained largely tied to particular examples (Warren & Cooper, 2005). Learners were not guided to purposefully generalize the identified similarities and differences between the numbers and the representations involved in the counting pattern in order to identify subsequent numbers without counting.

Conversion moves between symbolic, iconic (drawing of squares, circles or other shapes to represent everyday objects) and concrete representations were evident across the fourteen excerpts categorized at the EC2A level in Laura's lessons. The use of oral, iconic, symbolic and concrete (fingers) representations is evident in 11/14 excerpts. Though moves between representations are evident across the counting excerpts, these moves were not connected across the examples to show similarities and differences between counting in different multiples and the representations. A number chart is a useful model for teaching sequential patterns (Klein et al., 1998). Whilst a number chart

was used frequently in Laura's EC2A excerpts, this tool was not exploited to engage learners in identifying patterns and generalising its structured patterns.

Though multiple representations were used in these excerpts, a lack of focused attention on affordances of each representation and connections between these representations was noted in these EC2A excerpts. This finding has been noted in prior research (Duval, 2006; Gagatsis et al., 2006; Lesh & Lehrer, 2003) pointing to limited coordinating of multiple representations of the same concept in their respective studies. These authors refer to this lack of coordination of representations as compartmentalization. Compartmentalization reveals difficulties in accomplishing flexible and competent moves back and forth between representations (Duval, 2006). Localising procedural strategies were deployed in the way in which Laura dealt with tasks at EC2A level.

The previous section discussed examples from excerpts coded as EC2A level in Laura's classroom. The next section presents and discusses excerpts from Laura's teaching which were coded at EC2B level. 5/23 excerpts were categorised at EC2B level. These five excerpts involved moves between the oral and the symbolic modes of representation. One example is presented to illuminate a category of excepts where teacher talk focused on partitioning of numbers into tens and units in order to produce answers.

EC2B

L3, Excerpt 21:EC2B/MM1, MM2*



Figure 11: Subtraction task

1. T: Let us do one more minus then you try a few in your books. {Writing 93 -61=□} Ninety-three take away sixty-one. What now? Maki, what is our first step?
2. L: You breakdown the number ninety-three
3. T: Yes, you break down the number ninety-three. If I break down the number ninety-three, what do I get?
4. L: Ninety plus three

5. T: Well done {Writing 90 + 3 below 93 -61}. What do I write now? Anybody tell me what to write now. What do I write next? Here?

6. L: Minus

7. T: Minus {writes – below 90+3}. And sixty-one? How are we going to breakdown sixty-one? {Looks at a learner}

8. L: Sixty plus one

9. T: Well done. Sixty plus one. {Writes 60 +1 below the minus sign}. Now where do I start?

Various moves between oral and symbolic modes of representation are evident across this excerpt. At the start of the task, the teacher wrote the task symbolically on the board and orally stated the task in line 1. This move was coded as MM2 move. A treatment move (MM1) was observed in line 9 where the teacher split sixty-one into sixty and one and symbolically wrote this as 60 + 1. This move involved a move to a different representation of the quantity in the same symbolic register. Following the partitioning of the two numbers, the teacher suggested that learners subtract as follows:

10. T: Three take away 1 is two. {Writes 2 below the sum. Points to 90}. Nine take away six. {Pointing to 60 on the board}. Nine take away six.

11. L1: Three

12. T: Well done. If nine take away six is three, what is ninety take away sixty?

13. L: Thirty

14. T: Well done. Thirty plus two,

15. L2: Thirty-two

16. T: Well done. {Writing 32 below the sum}. What must I remember: What must I remember? {Writes 32 at the top where she initially wrote 93-61}

After splitting numbers into tens and units, a sequence of steps to produce the answer is communicated. This task was coded as EC2B because a general rule to produce the answer by way of partitioning numbers was explicitly provided. While difficulties with split strategies when exchange between place values is required have been noted in the literature (ref), the procedure remains more generally applicable, justifying this coding.

5/23 excerpts from Laura's lessons were categorized as EC2B level because the general procedure for breaking down numbers was offered. These excerpts involved MM1 and MM2 moves between representations. Four of these EC2B excerpts involved basic MM2* moves where the teacher moved between the oral and the symbolic representations. One excerpt in this category of excerpts involved the use of number cards which seemed useful to develop knowledge of the number system and structuring of numbers. In addition to transmission of the general rule for producing the answers, Laura frequently restated steps within tasks, for instance, in the excerpt discussed here (excerpt 21) she initially asked learners to subtract 6 from 9 and then stated this as subtracting 60 from 90. While this can be interpreted as the teacher supporting learners to use what they know about small numbers or single digit numbers to solve problems involving two digit numbers (Haylock & Manning, 2014), such restatements were not explicitly explained, remaining somewhat ad hoc in Laura's use. While the excerpts from Laura's lessons at EC2B level involved the transmission of the general rule of breaking down numbers and some ad hoc use of number relations to produce the answer, these methods were not generalized beyond the examples provided in each excerpt, nor were reasons for their use offered. Specialising proceduralising strategies were deployed in the way in which the teacher dealt with tasks at the EC2B level.

1/23 excerpt from Laura's English medium classroom was categorized as an EC3 level excerpt in which the teacher enabled learners to produce the answer through the use of known number facts coupled with moves between the oral and the symbolic modes of representation. Excerpt 13 illustrates the use of known facts to produce an answer to the task.

EC3

L3, Excerpt 13: EC3/ MM2*

1. T: We are forty at a party. We are getting four sweets each. Are you ready? How many sweets are needed altogether? {Pointing to the numbers 4, 8, 12, 16, 20 to 40 that she wrote initially on the board} Count! {She pointed at each number on the board}.
2. Ls: Four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, thirty two, thirty six, and forty. {Pointing to numbers}.

3. T: Stop! For ten children {Pointing at number 40 on the number sequence}, Busi is going to need how many sweets?

4. Ls: Forty {without counting}

Laura then asked learners the number of sweets required for ten people if each person was given 4 sweets. Getting '40' as the answer she continued as follows:

5. T: {Writes 10 = 40 on the board}. For another ten is?

6. Ls: Forty

7. T: {Writes another 10 = 40 below where the previous line} Ten, ten is twenty. Forty plus forty is eighty. For twenty of us it is eighty sweets, but we are not twenty. We are forty in total. {Writing 20 = 80}. If we are forty, how many sweets does she need? How many is she going to need for twice as much? Double of twenty?

8. Ls: Forty

9. T: Double twenty is? {Pointing to 20 and 20 on the board}

10. Ls: Forty

11. T: Double four is eight. Double eight is? Shhhhhh! What is double eight? {Points to 4 on the board where she initially wrote 4, 8, 12, 16, 20}. Double eight.

12. Ls: Sixteen

13. T: Sixteen. So, what is double eighty then?

14 Ls: One hundred and sixty

15. T: One hundred and sixty. Not sixteen. {Writing 40 = 160 on the board}. So Busi is going to buy one hundred and sixty sweets. So, you can give each one of us four. Do you understand how we worked that out?

16. Ls: Yes

While the initial counting in 4s was coded as EC2A, the teacher recorded the result of the number of sweets needed for ten people and then proceeded to work with this result to derive further results. Once learners established that 40 sweets would be required for ten learners in line 4, the teacher worked out the number of sweets for twenty learners (line 7) and suggested doubling (line 9) as an approach to produce the answer. A move to using known facts to derive new results (Thompson, 2011) was noted. This was coded as EC3 where a calculation-based approach rather than a counting-based approach was suggested as the way of producing the answer. In this excerpt, the connections between 'ten times bigger' calculations are more direct (lines 11 and 13), in her connecting

between 8 and 80 to help learners produce the answer without counting, in spite of the principle not being explained. Given the use of doubling using known facts to produce the answer, the excerpt was coded as EC3/MM2. Specialising principling strategies were deployed to support learners to produce the answer through the use of number facts.

Laura's teaching showed four levels of transmitting evaluative criteria and various teacher feedback moves. In her transmission of evaluative criteria, Laura moved between the symbolic, oral, concrete and iconic modes of representation in the first two levels of the EC. Conversion (Duval, 2006) features across all the excerpts observed in Laura's teaching. Literature argues that mathematical representations and translations between these representations make clearer the meaning of mathematical relationships (Lesh et al., 2003). What is interesting is that there is inclusion of concrete and iconic modes in Laura's teaching, in ways that were not seen in Nkele and Mirriam's teaching.

This section has discussed excerpts from Laura's lessons and how these were coded using the EC/MoR-Framework. The next section discusses excerpts from Flora's teaching.

6.3.2 Flora's teaching

Table 21: Overview of excerpts from Flora's lessons

Lesson1	Task	Timing	EC/MoR
1	Oral instruction: Chanting numbers in 1s from 1 to 100	00:21--02:24	EC2A/MM2*
2	Oral instruction: Number identification 98 on a number chart	02:26-02:40	EC1/MM2
3	Oral instruction: Identification of the numeral 77 on a number chart	02:60-03:27	EC1/MM2
4	Written on the board. Identifying missing numbers in a sequence from 150 to 100 on a number chart	03:56-11:09	EC2A/MM1, MM2*
5	Chanting numbers in 5s from 105 and back from 105 to 5	11:49-12:00	EC2A/MM2*
6	Individual seat work: complete the pattern: DBE workbook, page 9	12:35- 23:16	Not analyzed
Lesson 2			
7	Chanting forwards and backwards number word sequence in 4s on a number chart	00:18 -03:49	EC2A/MM2

8	Oral and written instruction: Making arrays of 4x5 and 5x4 using counters on the board.	04:00-17:14	EC3/MM1,MM2
9	Oral and written instruction: Making four array representations of 6 x 4 and 4 x 6 using counters on the board.	17:39 -22-39	EC3/MM1, MM2
10	Individual seatwork: Practice with 5, How many altogether: DBE workbook page 54	23:06-33:22	Not analyzed
Lesson 3			
11	Identification of missing numbers on the number chart from 501 to 600	00:09 -08:03	EC2A/ MM2
12	Using number cards to identify number 900 and its place value position.	08:09-10:08	EC2B/MM1, MM2
13	Identification of digits in the tens position from numbers 540 to 550 on a number chart.	10:15-15:50	EC2B/MM1, MM2
14	Identifying a number that comes before 540 on a number chart.	17:39-26:00	EC2A/MM2
15	Identifying tens in numbers between 540 and 550	26:05-30:34	EC2A/MM2
Lesson 4			
16	12+□ =20 on a number line	08:00 -09:27	EC2A/MM2
17	□ + 5 =20 on a number line	10:15-13:45	EC2A/MM2
18	6+ □ =20 on a number line	14:20-17:59	EC2A/ MM2
19	Counting in 5s starting from 3	18:18-20:29	EC2A/ MM2
20	Individual seat work: adding numbers to 20 on a number line	22:40- 45:51	Not analyzed

Table 21 shows that twenty lesson excerpts were generated from Flora's teaching. Four levels of transmitting evaluative criteria are evident in Flora's teaching. Three excerpts were not included in the analysis as these excerpts involved individual seatwork

Table 22: Coded excerpts from Flora's lessons

EC Level	No of excerpts	Multilingual moves		Mathematical moves	
		Translation (ML1)	Translanguaging (ML2)	Treatment (MM1)	Conv (MM2)
1	2				L1:E2 L1:E3
2A	11			L1:E4	L1:E1* L1:E4* L1:E5*

					L2:E7 L2:E11 L3:E14 L3:E15 L4:E16 L4:E17 L4:E18 L4:E19
2B	2			L3:E12 L3:E13	L3:E12 L3:E13
3	2			L2:E8 L2:E 9	L2:E8 L2: E9

Table 22 provides a summary of the spread what Flora transmitted as evaluative criteria and how modes of representation featured in what was transmitted. Three of the seventeen excerpts were characterised by MM2* moves where the teacher moved between the oral and the symbolic modes of representation whilst fourteen excerpts involved moves between the oral, concrete, iconic and symbolic number-based modes of representation. 2/17 excerpts were categorized as EC1 excerpts in which correct answers were offered or accepted. Both these excerpts involved moves between the oral and the symbolic representation. One example, excerpt 3 is chosen to exemplify EC1 with MM2 moves.

EC1

L1, Excerpt 3: EC1/ MM2

1. T: Who can show me seventy-seven on the number chart? {Learners had their number charts in front of them, walks around to check}.

2. Ls: {Most learners were able to point to 77 on their number charts}

3. T: Very good! Right! Say that number. Point to the number. {Walks around from one to the other to look at the numbers they were pointing to}.

4. L: {pointing to 17 on their number charts}

5. T: This is not what I am looking for. It is right there in front of you. {Pointing to 77 on the number chart}

6. L: {Points to 77} Seventy-seven.

7. T: Good, good, good! Point with your fingers. Say the number, seventy-seven.

8. Ls: Seventy-seven, seventy-seven

9. T: Good. Now we are moving.

Learners had their number charts from the previous task in which they were asked to identify 98. In this present task they were asked to identify number 77 on the same number chart. A move from the oral to the symbolic number base ten form of representation was noted as a MM2 move. Whilst many learners were able to point to this numeral, one learner pointed to the numeral 17 on the number chart as opposed to 77 (line 4), suggesting the possibility, once again, the difficulties in English of the similarity of the sounds of the numerals seventy and seventeen noted earlier (Fuson & Briars, 1990). However, the teacher did not focus her instruction of the learner's incorrect offer and the morphological construction of number names in English. Instead, she pointed to the correct numeral on the number chart and asked learners to say the number name whilst they pointed to the number symbol on their charts. EC1 was noted in the teacher offering the correct answer. The move between the oral and the symbolic was noted in line 7, with the symbolic form occurring here symbolic number base representation and was again coded as a conversion move or MM2.

In the two excerpts that made up EC1, the teacher focused on the topic of numeral identification. In excerpt 2 learners were asked to point to the numeral 98 and in excerpt 3 to point to the numeral 77. In both instances some learners made an error in identifying the numerals. In excerpt 2 a learner pointed to 89 instead of 98 – potentially, a reversal error (Wright et al, 2006). In the case of excerpt 3, a learner pointed to 17 instead of 77. Whilst the literature suggests different likely reasons for the two errors in learners' responses, these errors were not addressed in instruction showing learners how number words are constructed and where each numeral is located in relation to the other numbers on the number chart. The way the teacher dealt with learner responses in the two excerpts categorized at EC1 showed a lack of support for developing learners' knowledge of early

number names, structure, and conventions of the numeration system (Fuson & Briars, 1990). While occurring within a structured symbolic number representation, restatement moves between the oral and the symbolic modes of representation dominated the two excerpts.

The next section discusses excerpts categorized at EC2A. 11/17 excerpts were categorised as EC2A where counting based procedures were offered or accepted at the close of each excerpt.

Two examples have been chosen to show how Flora privileged counting procedures in her teaching. Example 1 below exemplifies EC2A category excerpts accompanied by treatment and conversion moves.

EC2A



Figure 12: Number sequencing task

L1, Excerpt 4: EC2A/ MM1, MM2*

1. T: {Looking at the preparation file, Flora writes numbers in a descending order from 150 to 100 as shown in the figure 12. We are busy with 'five' arrays this week, nhe! We have some missing numbers here. What are the missing numbers?
2. Ls: {No response}
3. T: {Re-wrote the same number symbols as the ones in figure 1-, this time with spaces to show missing numbers as follows 150, 145, 140, 135, -, 125, -, - , 110, - , 100}. Count!

In her statement of the task the teacher recalled five's arrays worked on previously. The teacher recalls a previous task involving 'five' arrays, presumably, arrays involving groups of five, but the connection between that task and this one is not made explicit at this stage. In the context of no response, the teacher added five tiny marks between the first few consecutive numbers in her sequence. While there was still no explicit direction on how to produce the intermediate numbers, this move was classified as an MM1 move in that

a different representation of the task in the same register was offered. In this move, a task involving ‘skip’ counting backwards in jumps of fives was restated with a cue to work with five jumps between consecutive numbers. Skip counting was reduced to unit counting in this move. At this stage learners were able to produce the correct number sequence, and the explicit rule for producing the sequence was offered and acknowledged by the teacher

4. Ls: One hundred and fifty, one hundred and forty-five, one hundred and forty, one hundred and thirty-five-----

5. T: {Walks to the board and writes 130 in the space between 135 and 125} the number sequence looks like this: 150,145, 140, 135, 130, 125, - , - , 110, - , 100} .Who can tell me, what are we counting in here?

6. Ls: Fives

7. T: We are counting in fives, nhe! But we are counting backwards. Count!

8. L: {Learners went on to collectively chant the correct version of the full sequence}

A rule for producing the answer was eventually offered (lines 5 and7) where the teacher alluded to counting backwards in fives. While counting in multiples was used to produce the answer, this only occurred after some cueing of five jumps. This excerpt was coded as EC2A. Whilst learners chanted the number words in in 8 and10, the teacher chanted along and pointed to the number symbols on the board. This move was coded as MM2. The excerpt was coded as EC2A/MM1, MM2.

The second excerpt was chosen to exemplify the EC2A category of excerpts characterized by MM2 moves involving moves between the oral, symbolic number base (number line) and concrete representations.

L4, Excerpt 16: EC2A/ MM2



Figure 13: Number line

1. T: {Draws a number line on the board starting from 1 to 20. She puts marks on numerals 12 and 20}. Look at the number line. I have two numbers which are twelve and twenty. {Making bigger marks on these two numbers}. You must add that number

to twelve to make twenty. {Pointing to 12 and 20 on the number line} What is that number?

2. L: Twenty

3. T: No, No! You need to add a number to 12 in order to have 20. [She points to 12 then to 20 on her number line]. This is what you must do. You have to count the jumps from twelve to twenty. {Makes jumps of 1 from 12 to 20 as learners count the jumps in 1s}.

4. Ls: One, two, three, four, five, six, seven, eight {learners count along. Some raise their fingers as they count whilst others chant the numbers}

5. T: So! This is what we do {writing 12 + 8 = 20 underneath the number line she had on the board}. You are going to say twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen and twenty. {Raising one finger as she counts on from 12 to twenty}

6. Ls: Eight

7. T: It will give us---

8. Ls: Twenty

9. T: Are we together? You just count the number of jumps.

10. Ls: Yes!

11. T: Let's do another one.

In this excerpt, moves between the oral and the symbolic number base (number line) forms of representation were coded as MM2. The teacher clarified that she wanted learners to identify a number that could be added to twelve in order to have twenty as the answer. An incorrect offer of twenty was made by a learner and was rejected by the teacher (line 3). Evaluative feedback was followed by the restatement of the task and the rule for producing the answer. Counting on in ones from the first number was recommended as an approach to produce the answer (line 3 and 5). This offering of a counting procedure was coded as EC2A. The move between concrete jumping actions on the number line and the oral tracking of number of jumps by learners (sometimes with fingers) was observed in line 5 when the teacher counted on using her finger to add 8 to 12. This move was coded as MM2. The excerpt was coded as AC2A/MM2.

11/17 excerpts were categorized at EC2A level in Flora's lessons involving moves between the oral and the symbolic number-based forms of representations and moves

between the oral, the symbolic and the concrete modes of representations as exemplified in the two examples presented in this section. All excerpts coded as EC2A were characterized by teacher talk that privileged counting procedures to produce answers to tasks. Tasks ranging across multiplication, place value and additive relations including 'change unknown' and 'start unknown' problems, there was no explicit talk about number relations and connections between representations, concepts and tasks. While there were allusions to connections between tasks as in reference to arrays of fives in excerpt 4 discussed previously, talk did not systematically expose mathematical structure and enable learners to make connections and generalizations.

2/17 excerpts in Flora's English medium classroom were coded as EC2B where the teacher offered or accepted structuring of numbers as general procedures to produce answers. These excerpts involved moves between the oral and the symbolic representations. One of the three excerpts included MM1 move. Example 1 has been chosen to exemplify category EC2B with MM1 and conversion moves involving moves between the oral and the symbolic representations.

EC2B

L3, Excerpt 12: EC2B/ MM1 &MM2.

{With H T U written on the board, teacher asks a learner to take a number out of a box with numbers written on pieces of paper. Although the number 900 is shown, the first learner – when asked what number is shown – responds with 'Hundred and sixteen'}.

1. T: One hundred and sixteen? {Points to the number card in the learner's hand}. Okay, how many tens or hundreds are in this number? Who can help? {Points to another learner and hands over the same number card to this learner}. What is this number? Is it made up of units, tens or hundreds?

{T then writes the number '900' on the board and points to it}. This number is the same as the number that you have. What is this number?

2: L2: Nine hundred

3. T: Where can we put it then?

4. L: {With the paper smaller than the H T U on the board, the learner places the paper with 900 under the H on the board}.

5. T: Very good. What number is this? {Pointing to the number card on the board}. What does it stand for? It is nine hundred. {Gesturing to the two zeros :} The two zeros tell us that there are no tens and no units but only nine hundred. How many hundreds?

6. Ls: Nine hundred. Hundreds!

7. T. Good. It stands for hundreds.

While Flora's talk in this excerpt did not distinguish between focus on any particular digit and focus on the overall number, her explanation in Line 5 started to draw attention to the positions of numbers being related to place values. In the teacher's acceptance of the second learner's correct verbalization of the number, a move from symbolic to oral language register is noted (MM2). While this learner placed the whole number card (900) under the hundred's column, the teacher orally explained the value of each digit in the numeral 900 whilst she pointed to the number symbol on the board in line 5. The teacher's explanation pointed to a focus on place value and partitioning of numbers into hundreds, tens and units, and through this, to attention to number structure in the base ten system. This excerpt was therefore coded as EC2B because the teacher offered a number structure-based approach to produce the answer. In this restatement of the number 900 as 'nine hundred, no tens and no units', there is a move to a different verbal representation of the same quantity – an MM1 move. The excerpt was coded as EC2B/MM1, MM2.

The teacher's work in the episode following this one continued to attend to place value relationships in similar ways. Two excerpts at EC2B level focused on the idea of place value. In this category of excerpts, the focus of teacher talk was on identifying the place value position of numbers i.e. whether a digit is in the units' place, tens place or hundreds place. However, the teacher talk did not focus explicitly on the fact that the position of a digit in a number shows its size, and therefore, there was some of the kinds of ambiguity in teacher talk noted in Venkat and Naidoo's (2012) analyses. Learners were not asked to state the number of ones, tens or hundreds in this number. Similarly, in excerpt 13 learners were asked to identify digits in the tens position from numbers between 540 to 550, without reference to the value conferred to the digit or to the overall number through its place value position. While not the focus of this study, it is important to remember that

Flora taught through the medium of English whilst English was not her first language – a point stated in Chapter 4.

2/17 excerpts in Flora teaching were coded as EC3 level excerpts. Both these excerpts focused on making arrays. One excerpt below is presented to show how Flora implicitly dealt with the commutative principle of multiplication.

EC3

L2, Excerpt 9: EC3/MM1, MM2

This excerpt followed immediately after excerpt 8 in learners had been asked to construct arrays showing 4 x 5 with sets of counters. In that excerpt, some learners had constructed arrays with 4 rows each containing 5 counters and others had constructed arrays with 5 rows each containing 4 counters. Having accepted '20' as the answer for the total number of counters, the teacher had written: 4 + 4 + 4 + 4 + 4 = 20 on the board to conclude Excerpt 8. Following up on the 5 rows of 4 array in Excerpt 9 (though not explicit about her choice), Flora told learners to make another row just below the fifth row of four counters they had from the previous excerpt. The subsequent conversation indicates some attention to the principle of commutativity in multiplication through array representations in the transmission of the EC, although it also points to some of the kinds of slips in coherence that (Mathews, Venkat and Askew (2018) have noted previously in the context of division instruction:

1. T: Now make another row of four counters. {Writes the symbolic representation 4+ 4+ 4+ 4+ 4+ 4 = on the board}. Now make the sixth row. Work together to make your own arrays. Use the same counters {gestures to the arrays learners have made on their desks in the previous excerpt}. Who can tell me what the answer is? {Pointing to the symbolic representation on the board}.

3. L: {Pointing to the symbolic representation 4+ 4+ 4+ 4+ 4+ 4 = on the board}: Twenty-four.

4. T: We have four groups of six. What is the answer?

5. Ls: Twenty-four

6. T: Very good. Do you understand? Who can write the multiplication number sentence?"

7. L: {Writes 6 x 4 =24}

8. T: Very good. Who can write another multiplication number sentence? The last one? Nhe!

9. L: {Writes 4 x 6 =24}

10. T: Well done! We can write the twenty-four in three different ways

In this excerpt, there are several instances of moves between the concrete counter array representations and symbolic representations: for example, the initial construction of 6 rows of 4 counters array is linked with the expression: 4+ 4+ 4+ 4+ 4+ 4 =. Later, learner input produces both 6 x 4 = 24 and 4 x 6 = 24 from the completed array. The moves between representations were coded as MM2 with MM1 moves incorporated in the acceptance of different symbolic representations of the array. Whilst the repeated addition symbolic representation on the board represented 4 repeated six times, the teacher's Line 4 utterance read the representation as four groups of 6 (line 4). It was unclear here whether the teacher was responding to versions of this latter form of the array that some learners had made or misreading her symbolic number sentence based on her own awareness of a commutativity-based equivalence of the two expressions. Her lack of clarity on the referent for her statement does create ambiguity in any case. In spite of this though, her acceptance in lines 7 and 9 of two learners' offers of 6 x 4 =24 and 4 x 6 = 24 as possible representations of the same array points towards the start of incorporation of the commutative property of multiplication. In this acceptance EC3 was observed, even though the teacher did not generalize this principle. This excerpt was coded as EC3/ MM1, MM2.

These two excerpts focused on the general principle of commutativity using arrays. An array is an image model used to make the commutative property of mathematics explicit to learners (Haylock & Manning, 2014). This mathematical property teaches learners that when we add or multiply two numbers the order in which the two numbers are added or multiplied does not change the result. For instance, in addition 6 + 4 = 4 + 6 or in multiplication, 6 x 4 = 4 x 6. In excerpts 8 and 9 as shown in Table 22, Flora's teaching focused on creating concrete arrays. While there was no explicit pointing to groups made up of rows of counters or of columns of counters, the acceptance of both multiplication expressions did point towards instruction geared towards the commutative property.

6.4 SIMILARITIES AND DIFFERENCES OBSERVED ACROSS THE 4 TEACHERS

While this study set out to probe for possible differences between Sepedi and English-medium instruction, the analysis of the four teachers indicated, at the first level, differences at the individual level, in their teaching. The next section provides a summary of each teacher's teaching

6.4.1 Nkele's teaching

Four levels of the EC were observed in Nkele's teaching. Interestingly, EC0 where no evaluative criteria were offered at the close of excerpts was only observed in Nkele's Sepedi medium classroom. This EC0 level in Nkele teaching was characterised by translation moves only. This observation suggests that translation moves could be associated with lower levels of the EC. More translation moves were observed across EC levels in Nkele's teaching than in the other Sepedi medium classroom. In these moves, Nkele substituted words or phrases in one language with other words or phrases in the other language. In addition to these translation moves, Nkele's moves between representations involved translanguaging moves where she used words and symbols together with pointing gestures whilst she moved between the two languages to explain mainly how numbers are constructed in Sepedi to help learners understand number names. These translanguaging moves were observed only in the higher levels of the EC in Nkele's teaching and were not observed in the other Sepedi medium classroom. Treatment moves observed in this teacher's teaching were only observed at EC2B level. Whilst conversion moves featured in three of the four EC levels observed in Nkele's teaching, it was interesting to note that conversion moves observed in Nkele's Sepedi medium classroom were mostly characterised by restatement moves between oral and symbolic modes of representations.

6.4.2 Mirriam's teaching

Four levels of transmission the EC were observed in Mirriam's Sepedi medium classroom. Mirriam's teaching showed higher EC profile than Nkele because Mirriam's teaching included the use of structured resources. This was not observed in Nkele's teaching. Whilst EC0 was not noted in Mirriam's teaching, EC3 was observed where she focused

on the use of known number facts to derive answers. EC3 was noted in two excerpts in Mirriam's classroom and not in the other Sepedi medium classroom. Fewer instances of translation moves were observed in Mirriam's teaching than were observed in the other Sepedi medium classroom. Whilst translanguaging moves were observed in the other Sepedi medium classroom, no such moves were observed in Mirriam's teaching. Similar to the other Sepedi medium classroom, treatment moves were observed in Mirriam's teaching particularly at EC2B level. Conversion moves observed in Mirriam's teaching involved moves between the oral and the symbolic as well as moves between the symbolic number base where Mirriam used structured number representations such as number charts and number lines as well as symbolic modes of representation to represent number. This type of move made explicit references to more efficient ways of calculating which were not observed in the other Sepedi medium classroom.

6.4.3 Laura's teaching

Four levels of the EC were observed in Laura's teaching. Similar to Mirriam, one excerpt in Laura's teaching was coded as EC3 where the teacher offered or accepted known number facts to derive answers. It was interesting to note that more of Laura's excerpts were coded as EC2A level excerpts where, at the close of excerpts, the teacher offered or accepted counting based approaches to produce answers to tasks. Whilst Laura could speak another language (Afrikaans) other than English no translation or translanguaging moves were observed in her teaching. She made no moves between languages. Similar to the two Sepedi medium teachers discussed previously, treatment moves in Laura's teaching were mainly observed at EC2B level. Another interesting point to note about Laura's teaching was her incorporation of structured symbolic number-based representations (MM2) involving the use of number charts which was also observed in Mirriam's teaching. Similar to Mirriam's teaching, Laura's teaching involved an explicit focus on efficient ways of calculation through the use of these structured symbolic number base forms of representation. In addition to the use of number charts, Laura incorporated iconic and concrete mode of representations involving drawings on the board and allowed

learners to use their fingers to derive answers. This occurrence was not observed in either of the two Sepedi medium classrooms discussed previously.

6.4.4 Flora's teaching

Four levels of EC were observed in Flora's teaching with two excerpts coded as EC3 level where she implicitly focused on the principle of commutativity. Similar to Laura, Flora could speak another language (IsiXhosa) other than English but she remained with English as the medium of instruction throughout lesson observation. No translation or translanguaging moves were observed in her teaching. There were more issues with clarity in Flora's teaching than in Laura's teaching, perhaps linked to English not being Flora's first language. This struggle with the language was mainly evident in her excerpts categorised at EC2A, EC2B and EC3 discussed in this chapter. Like Mirriam and Laura, Flora incorporated number charts and number lines - symbolic number base-ten system forms of representation. Though Flora made use of these forms of representations, she did not make explicit reference to more efficient ways of counting

The similarities and differences about teaching in the four teachers' classrooms discussed provides an interesting exploratory picture of what the four teachers transmitted as the evaluative criteria and how modes of representations featured in what they transmitted. In order to explore for any differences in performance across the two language groups, the subsequent section provides a synthesis of the similarities and differences observed across the combined Sepedi and English language excerpts for all four teachers.

6.5 SUMMARY COMPARISONS AND INSIGHTS INTO SIMILARITIES AND DIFFERENCES OBSERVED ACROSS SEPEDI AND ENGLISH MEDIUM CLASSES

The narrative summaries above allowed me to consider each teacher individually. Below, I provide a quantitative summary of each teacher in relation to the EC/MoR categories. This summary facilitates comparison and commentary of the EC profile and patterned differences in work with MoR across English and Sepedi medium excerpts in the four classrooms, Table 23 presents a summary of similarities and differences observed across the four teachers.

Table 23: Comparison of similarities and differences

Nkele						Mirriam						Laura				Flora			
EC Lev	No of Ex	ML 1	ML 2	MM 1	MM 2	EC Lev	No of Ex	ML 1	ML 2	MM 1	MM 2	EC Lev	No of Ex	MM 1	MM 2	EC Lev	No of Ex	MM 1	MM 2
0	3/27 11.1 %	3				0						0							
1	10/27 37.0 %	8			7; 7*	1	5/25 20%	2			5; 3*	1	3/23 13.0 %		3; 2*	1	2/17 11.8 %		2
2A	9/27 33.3 %	5	1		9; 7*	2A	8/25 32%	2			8; 3*	2A	14/23 60.9 %	1	14; 3*	2A	11/17 64.7 %	1	8; 3*
2B	5/27 18.5 %	3	2	4	5: 5*	2B	10/25 40%	2		10	10; 9*	2B	5/23 21.7 %	5	5; 4*	2B	2/17 11.8 %	2	2
3						3	2/25 8%	1			2	3	1/23 4.3%		1; 1*	3	2/17 11.8 %		2
Lower EC profile than Mirriam 19/27 MM2 instances are MM2* (70%) Some translanguaging seen in higher EC excerpts						Higher EC profile than Nkele with broad inclusion of structured resources. 15/25 MM2 instances are MM2* (60%) Greater incidence of treatment moves than Nkele (10/25 = 40% in comparison to 4/27 = 14.8% of excerpts)						Higher EC profile than Mirriam with broad inclusion of structured resources at EC2B level. Higher incidents of MM2* (10/23 = 43%) and treatment (22%) moves when compared to Flora.				Higher EC profile than Laura with 2 excerpts at EC3 level whilst Laura only has one excerpt at this level. Similar in terms of EC2A as more than 50% of both their excerpts are at this level. Fewer instances of MM2* (3/17 excerpts) or (17.6.%) than Laura (43%) and 17.6% of treatment moves in comparison to Laura's 22%% treatment moves			

Predictably, and particularly given that G3 is widely seen as the transition year to English, there is relatively wide prevalence of translation from Sepedi to English, though more so in Nkele's teaching than in Mirriam's teaching (19/27 of Nkele's excerpts vs 7/25 of Mirriam's excerpts. But given that learners are still learning English, very low incidence of translanguaging is problematic.

Broadly similar levels of treatment moves across the two language settings are observed, although there is variance between individual teachers within this, and overall occurrence of treatment was quite low, and lower than might be expected given that working with building up and breaking down of numbers is a large part of grade 3 number curriculum. While there was broad evidence of MM2 integration, the MM2* phenomena were the key area of patterned difference between the two languages, with a much higher incidence of MM2* within MM2 incidence in Sepedi in comparison to English. Of the 46 excerpts containing MM2 phenomena across the two Sepedi classrooms, 34/46 = 73 % contained MM2* restatements between oral and symbolic registers. This contrasted with 13/37 of the MM2 excerpts in English medium classes marked by MM2*. What this means is 35 % of the English medium excerpts were marked by MM2*moves. The MM2* seemed more prevalent in the Sepedi medium classrooms than in English medium classrooms.

6.6.2 Findings relating to MoR and EC

Findings from this study show that different multilingual and mathematical moves were observed across the ninety-two excerpts analyzed. Translation and translanguaging moves were observed only in the Sepedi medium classrooms. These translation moves were characterized by moves where the teacher substituted one word or phrase in Sepedi with another word or phrase in English following learners' offers. Translation only moves occurred at EC0 level while other translation moves were observed across the other EC levels in the Sepedi language classrooms coupled with other moves. This finding suggests that translation or ML1 only moves are more associated with the EC0 level.

In addition, findings from this study show that incidents of translanguaging moves were observed at EC2A and EC2B levels. Translanguaging moves between the languages were more noticeable in Nkele's teaching as she made systematic connections between the languages and the symbolic representations that went beyond replacing one word or phrase in one language by the other in a different language. This finding suggests that translanguaging or ML2 moves are associated with higher levels of the EC as these were not observed at EC0 and EC1 levels.

Two types of mathematical moves, treatments and conversion, were observed across the four classrooms. These moves involved context where teachers moved between the same representational registers and across different registers. The incidents of treatment or MM1 moves involved teachers substituting mathematical signs and symbols with other signs or symbols within the same symbolic register. This type of move provides one-way access to mathematical concepts and does not make explicit connections and relationships between concepts. These MM1 moves were more observable across the EC2B level in both language groups.

Table 23 shows that there were variations of conversion moves observed across the two language groups. In these instances, teachers moved between different representational registers across EC1, EC2A, EC2B and EC3 levels of the EC/MoR-Framework. One category of MM2 moved involved basic moves between the oral and the symbolic. 46 instances of these conversion moves were observed across the two language groups- see Table 23. However, more instances of these basic MM2* moves were observed in the Sepedi medium classrooms with 34 instances of MM2* noted (see Table 23). Whilst these basic moves were mainly observed in the Sepedi medium classrooms, majority of these moves were observed in Nkele's Sepedi medium classroom. Conversion moves observed across the two English medium classrooms included mainly moves between the oral, symbolic, concrete and iconic modes of representations with only 13 instances of MM2* across the two English medium classrooms.

6.6 CONCLUSION

In the final chapter, I suggest implications of findings of this study for teaching, for teacher education and for further research, noting the limitations.

CHAPTER 7: CONCLUSION

7.1 INTRODUCTION

This chapter begins by providing a summary of the focus of this study followed by a summary of main findings. The chapter discusses the contribution to knowledge made by the present study and the implications of its findings for primary mathematics teaching in South African contexts. The chapter also outlines some limitations of the study and suggests ways in which it could be extended. Finally, it offers suggestions for possible research studies emanating from this present study and concludes with a reflection on my experiences of conducting the research.

7.2 MAIN FOCUS AREAS OF THE STUDY

The first main area of focus in this study was evaluative criteria. Evaluative criteria, as noted in the theoretical chapter and the literature review chapter, are linked to the instructional rules used to transmit the knowledge, cognitive competences and processes that learners need to acquire in order to produce a legitimate text. Teachers use evaluative criteria to continuously evaluate what is present or absent in learners' responses in order to assist them to produce a legitimate text. This focus on evaluative criteria was chosen in order to explore and identify variations in what four grade 3 teachers transmitted during the teaching of number, additive and multiplicative relations, using either Sepedi or English to do so. A model for identifying and describing what the teachers transmitted was developed and described in Chapter 5, (See Table 13). This model and its indicators were developed from analysis of transcribed excerpts based on whole class teaching in the videotaped lessons and back-up field notes.

The second area of focus in this study was on modes of representation and moves between representations. This focus was chosen to explore and describe how representations feature in the transmission of evaluative criteria. Literature suggests that representations are central to building conceptual understanding. Given the centrality of representations in mathematics teaching, a main focus of this study was motivated by the mathematics literature that suggests that moving within and between representational registers and making connections between representations is useful for learners'

understanding of mathematical concepts. The study investigated how moves among and within several modes of representation featured in the transmission of evaluative criteria. A model to describe the various moves within and between representations was developed in Chapter 5-- (see Table 23)

The main research questions that guided this study were:

What do teachers transmit as evaluative criteria during the teaching of number, additive and multiplicative relations? How do modes of representation feature in this transmission?

A grounded theory approach informed by literature was used to analyse the data collected. This allowed me to describe what teachers transmitted as evaluative criteria in multilingual teaching and learning contexts, thereby bringing together foci that have remained largely separate in the literature base – namely sociological studies of instruction focused on evaluation and language-based studies on access to learning. After coding the data, a number of categories and sub-categories emerged which pointed to the transmission of various levels of evaluative criteria across the four classrooms in which either English or Sepedi was the main language of instruction. The literature pointed to two broad categories of representational moves and sub-moves: Mathematical Moves (MM) and Multilingual Moves (ML). Two types of moves were suggested within MM moves in the literature: Treatment and Conversion moves. Two moves were suggested within ML moves in the literature: Translation and Translanguaging moves. The findings, considered below, led to nuances within how these categories could be considered in relation to the data.

7.3 SUMMARY OF MAIN FINDINGS OF THE STUDY

The purpose of this present study was to understand what teachers transmitted as evaluative criteria during the teaching of number, additive and multiplicative relations and how modes of representation featured in this transmission. A model that combines both the evaluative criteria and representational modes (EC/MoR- Framework) was developed (see Table 15). Five different levels of evaluative criteria and four different moves between representations emerged.

Applying the categories of this model to the data led to the results and findings of this study. Table 23 shows that three excerpts from one Sepedi medium classroom were categorised as EC0 level excerpts where no evaluative criteria were communicated. This level of no evaluative criteria transmitted supports Hoadley's (2007) conceptualization of F^0 discussed in chapter 2.

Teaching in which no evaluative criteria were transmitted was characterized by translation moves only where the teacher substituted one word or a phrase in one language with another in a different language. This finding suggests that translation moves could be associated with no transmission of evaluative criteria.

The second level of transmission of evaluative criteria was termed EC1 which comprised of excerpts where evaluative criteria were transmitted through offering or acceptance of correct answers without specifying the rules of how these answers were produced. 15/52 excerpts from the two Sepedi medium classrooms were categorised at EC1 level with a combination of moves between the translation moves and moves between oral and the symbolic modes of representations observed mainly in Nkele's teaching. Mirriam's teaching at EC1 level involved translation and conversion moves similar to Nkele's teaching. However, Mirriam was also noted moving between the symbolic number base (number charts) and oral forms of representation which was not observed in Nkele's teaching.

5/40 excerpts observed across the two English medium classrooms were categorised as EC1 level excerpts. These 5/40 excerpts were marked by MM2 moves where the teachers moved between the symbolic number base and the oral as well as between the concrete and the symbolic modes of representation. Within this EC1 level teachers transmitted the evaluative criteria implicitly by way of offering or accepting correct answers without being specific about how these answers were produced. As a result of this way of responding to learners they would be likely to know the correct answer to a task but not likely to know how to produce the correct answer in future tasks that are similar to the tasks they had worked on previously. Excerpts where teachers offered, or accepted answers were predominantly observed across the two Sepedi medium classrooms.

The third level of EC was termed EC2A level. In this category of excerpts, teachers across the two languages offered or accepted counting-based approaches as ways learners could use to produce answers to additive or multiplicative relations tasks. 17/52 excerpts from across the two Sepedi medium classrooms were categorised as EC2A level excerpts. These excerpts were dominated by moves between the oral and the symbolic modes of representation and a small percentage of translation and translanguaging moves in Nkele' Sepedi medium classroom whilst in Mirriam's Sepedi medium classroom, conversion moves that involved the use of number lines and number cards (number based symbolic) representations were noted. These number-based forms of number representation were not observable in Nkele's EC1 and EC2A level excerpts.

25/40 excerpts across the two English medium classrooms were categorised at EC2A level. These excerpts were characterised by moves between the oral, symbolic, concrete and iconic modes of representations. Whilst this EC2A level was dominated by basic moves between the oral and the symbolic representations in the Sepedi medium classroom (Nkele's teaching) and number based forms of representations as noted in Mirriam's teaching, in the English medium classrooms the EC2A excerpts were dominated by MM2 moves involving moves between the oral, symbolic number base and oral, symbolic, concrete and iconic modes of representation. The concrete and the iconic modes representation were not observed in any of the two Sepedi medium classrooms.

This level of EC2A category of excerpts made up 45.5% of all the excerpts observed and analysed in this study. This finding suggests that counting-based approaches were dominant in what teachers transmitted as evaluative criteria in this present study. This finding confirms findings by Schollar (2008) and Ensor et al (2009). Many of these EC2A excerpts were observable across the two English medium classrooms.

The fourth level of the EC was termed EC2B. In this category of excerpts teachers offered or accepted breaking down of numbers or partitioning of numbers using knowledge of place (structuring) as a way to produce answers to additive and multiplicative relations tasks.

Twenty-two excerpts across the two language groups made up this category of excerpts. 7/40 English medium classroom excerpts were categorised at EC2B level whilst 15/52 Sepedi medium excerpts were categorised at this level. In the Sepedi medium classrooms, this level of EC was characterised by a network of translation, translanguaging, treatment and conversion moves. The conversion moves observed in the Sepedi medium classrooms at EC2B involved moves between the oral and the symbolic modes of representation. In the English medium classrooms, the EC2B category of excerpts were dominated by treatment moves (MM1) and MM2 moves involving basic moves between the oral and the symbolic modes of representation similar to MM2 moves observed across the Sepedi medium lessons. It was important to note that treatment moves where teachers moved within the same representational register were observable across EC2B level excerpts in both language groups.

The final level of the EC was termed EC3. In this category of excerpts teachers offered or accepted the use of known number facts to derive answers to additive or multiplicative relations tasks. Five excerpts across the two language groups made up this category. 2/5 excerpts were observed in one Sepedi medium classroom. The two excerpts categorised at EC3 level in the Sepedi medium classroom were characterised by translation and conversion moves involving moves between the oral and the symbolic modes of representation. 3/5 excerpts were observed in one English medium classroom and were characterised by conversion moves involving moves between the concrete, oral and the symbolic modes of representations.

Whilst conversion moves between representations were observed across the two language groups, conversion moves observed in the Sepedi medium classrooms involved mainly moves between the oral and the symbolic modes of representations. Conversion moves observed in the two English medium classrooms, in contrast, involved moves between the oral, symbolic, concrete and iconic modes of representations mainly observable at EC2A level across the two English medium classrooms. This patterned difference across Sepedi and English medium instruction was a key finding of this study. The use of symbolic number-based forms of representations (number lines and number

charts was observable mainly in both Flora's and Laura's teaching and quite widely present in Mirriam's teaching. Secondly, these findings suggest that teachers mostly transmit counting-based approaches as ways to produce answers, with few efforts to push learners to use more efficient and effective approaches such as structuring and number facts to produce answers to additive and multiplicative relations problems.

These findings demonstrate that various levels of transmitting evaluative criteria were used across the four classrooms. The findings also indicate that there are variations in terms of what teachers in Sepedi medium classrooms and teachers in English medium classrooms transmitted as evaluative criteria. Unsurprisingly perhaps, given the ongoing sociopolitical nature of language choice in South Africa, the findings pointed to multilingual moves (translation and translanguaging moves) only occurring in Sepedi medium classrooms. Furthermore, the findings suggest that translanguaging moves are associated with higher levels of transmitting the EC. Conversion moves involving only restatements from oral to symbolic forms or symbolic to oral forms were much more prevalent in the Sepedi classrooms than the English classrooms, and this occurred in spite of the very different EC profiles of the two Sepedi classroom teachers. These differences provide insights into possible underlying factors in the differential achievement that continues to prevail in learners' mathematical performance across language settings.

7.4 CONTRIBUTIONS OF THE STUDY

7.4.1 Content and representational modes

The EC/MoR framework is a key contribution of the study that brings language and learning of mathematics together into a single model. While mathematics education literature talks about treatment and conversion moves, this present study found evidence of low levels moves between the oral and symbolic forms of representation that did not require engagement with mathematical structure of the object being considered. These moves involved simple restatement. While the need to learn to move between the oral and symbolic registers has been noted as very important in early mathematics learning, the prevalence of this feature in Sepedi grade 3 classrooms pointed to limited inclusion

of iconic and concrete modes and also limited elaborations via translanguaging moves. Thus, breaking conversion moves into MM2 and MM2* was a useful contribution.

Secondly, the literature in primary mathematics education has mainly focused on the use of mathematical representations and moves between representations. This study identified the existence of multilingual moves between languages during the teaching of number, additive and multiplicative relations in African language mother-tongue instruction classrooms. These multilingual moves involve translation and translanguaging. In the teaching of languages, the two moves are referred to as multilingual practices for enhancing language learning. Both practices are regarded as effective though each has different affordances. In the context of this study a distinction in terms of these moves between languages (translation and translanguaging) was required because each language move brought about different mathematical results in the transmission of evaluative criteria. It was noted in this study that simply substituting one word or phrase in English with another in Sepedi or the other way around (translation/substitution) often did not lead in constructive ways to the transmission of evaluative criteria. The study identified that moving between Sepedi and English while employing various mathematical modalities such as pointing to representations, and incorporating written symbols and pictures/diagrams supported the explicit transmission of evaluative criteria, with Morais (2002) noting that this kind of explicit work with evaluative criteria was important in supporting learning in contexts of disadvantage. This type of elaborated move (translanguaging) between languages featured only in excerpts involving higher levels of the evaluative criteria as opposed to translation/or substitution moves which mainly featured in the lower levels of the EC/MoR-Framework of this study.

This study suggests that language moves in multilingual classrooms contribute differently to the transmission of EC. Translation moves are more observable at low levels of EC whilst translanguaging moves are more associated with higher levels of EC. This points to a ‘double advantage’ of incorporating attention in instruction to more powerful general procedures and/or principles for Sepedi children in this study. Teachers who presented these more general approaches were also the teachers who incorporated more extended

repertoires of representational moves across the ML and MM categories. Conversely, lower levels of evaluative criteria involving counting based procedures or simply stating the answer or F^0 were associated with more basic translation moves, and conversion moves that involved only the oral 'reading' of symbolic statements. Thus, such excerpts indicated a double disadvantage.

7.4.2. Theoretical contributions

This study found that various levels of transmission of the EC were observable in two Sepedi and two English medium classrooms. In an earlier study of the framing of evaluative criteria, Hoadley (2007; 2008) noted the absence of any evaluative criteria in some of the lessons she observed. Building on this work, this present study noted this phenomenon on a very small scale. In addition, this present study identified a level of transmission of evaluative criteria which suggests an extension of Hoadley's $F^{0.}$ Whilst in Hoadley's study no evaluative criteria were transmitted, in the context of this study evaluative feedback was transmitted but this was devoid of the knowledge, competences or approaches learners needed to know in order to produce the legitimate text. This notion was categorized as EC1 level, representing a limited extension of $F^{0.}$

Earlier studies of mathematics teaching have alluded to problems in mathematics teaching in South Africa related to shifts from basic counting-based approaches to more flexible calculating approaches (Schollar, 2008 & Ensor et al, 2009). The findings of this study confirm this phenomenon where teachers, across both language settings, predominantly transmitted counting-based approaches as opposed to more flexible calculation or number facts approaches.

Additionally, previous studies in South Africa pointed to teacher talk in mathematics teaching that lacked a focus on connections and coherence across and between episodes (Venkat & Naidoo, 2012) and a pedagogy that lacked a focus on connections within and between examples and representations. This phenomenon was noted in this study with several instances where every task or example presented was treated as a separate example and counting based approaches were privileged. This lack of focus on connection and coherence sidelines teaching of structure and generality, which in turn

limits transmission of higher level EC and progression in learning number, given that teacher talk that focuses on connections, coherence, structure and generality is essential for progression in the development of number, additive and multiplicative relations. Furthermore, studies in South Africa pointed to a lack of progression in the development of number knowledge (Schollar 2008). This lack of progression was noted in this study as more counting based approaches were privileged than structuring approaches and number facts. Structuring is regarded as important in progression of early learning because a key aim of progression into structuring by working with properties and relationships is that it enables learning of larger numbers and therefore generalizations of number to come into play.

7.4.3 Methodological contribution

The study makes two methodological contributions to the field of mathematics education. Firstly, the framework developed in this study could be useful for teacher professional development programmes which focus on evaluative criteria and representational modes. Previously, literature has tended to focus on evaluative criteria and modes of representation separately, with searches indicating no literature on EC that presented findings relating to a range of levels of EC transmitted in the context of multilingual teaching and learning contexts. Secondly, this study developed a framework that combines both EC and MoR to show how these two concepts can be used together to facilitate the effective teaching of number and progression in early number, additive and multiplicative relations development. Finally, the framework shows that various levels of transmission of evaluative criteria exist in African language and English medium classrooms which could account for variations in learner performance in mathematics classrooms in South Africa.

7.5 IMPLICATIONS OF THE STUDY FOR MATHEMATICS TEACHING

The findings of this study point to teacher talk that predominantly transmitted or accepted counting-based approaches to produce answers to additive and multiplicative relations task. These findings suggest a need for mathematics teaching of number, additive and multiplicative relations that focuses on progression in the development of approaches to

solving mathematical problems and moves between representations to support this development. The findings have implications for how pre-service and in-service teachers are supported to effectively teach early number, additive and multiplicative relations in grade 3. They call for an approach to teacher professional development programmes that focuses on progression in the development of number (Van den Heuvel-Panhuizen, 2001) and progression from concrete to abstract ways of representing number (Ensor et al,2009).

The language literature base already provides detail on these two kinds of moves. What the study provides are detail on how these moves played out in two Sepedi medium classrooms. The data provides instances of translanguaging moves that show the affordances of African languages such as Sepedi in the development of powerful and general principles involved in early number naming and how the number system is constructed to facilitate the base ten structures or development of place value knowledge. This language-related finding is potentially useful and important to inform training of teachers who are being prepared to teach in multilingual mathematics classrooms across South Africa. The data also provides instances of excerpts of specialized procedural and principling discourses containing a number of ML and MM moves. Excerpts like this are useful to inform and incorporate into teacher development going forward.

7.6 LIMITATIONS OF THE STUDY

The present study focused on what four teachers transmitted as evaluative criteria and how modes of representations featured in what was transmitted in the context of two Sepedi and two English medium classrooms. Teachers who participated in the study were chosen from a pool of schools regarded as under-performing and that taught through the medium of Sepedi and English. Although there are many schools with similar contextual circumstances, the findings of this study cannot be generalized to all schools across the spectrum with similar contexts. However, the findings point to avenues to explore further in terms of links between EC levels and the nature of ML and MM moves.

Secondly, this study had no follow-up or intervention component to specifically support teachers to understand the importance of evaluative criteria and moves between

representations in the development of early number. It is possible that such an intervention could lead to an extension or refinement of levels of EC and moves between representations being observed in future teaching of number, additive and multiplicative relations.

7.7 DIRECTIONS FOR FURTHER RESEARCH

The teachers who participated in the study were chosen from four primary schools which were regarded as under-performing schools at that time. It is possible that different findings would be obtained if teachers in “well-performing” or “high performing” schools were studied. Studying the teaching in these schools would be of interest to ascertain whether the finding of connection between higher level EC and higher level MoR continues to prevail in those settings. It is also possible that if data were collected from classrooms in which teachers used other languages (e.g. Afrikaans and IsiZulu) the findings of this study could be extended or refined.

7.8 SELF REFLECTIONS

7.8.1 Physical access to early schooling vs epistemological access to mathematics knowledge

This study has contributed considerably to my professional development as a foundation phase specialist teacher educator. Through this research study I have come to understand two things about teaching in the early years. Firstly, I have understood that teaching young children is a challenging task that requires an understanding of the contexts from which learners come, what they should be able to do, and how their linguistic and mathematical backgrounds can be supported. I have also come to understand that some teachers in the early years have limited understanding of what learners know, what they need to learn and be able to do in mathematics that will enable them to succeed in future mathematics learning. This understanding of education in the early years in South Africa is carefully captured in an essay written by Hoadley (2013). This author refers to three important aspects of early schooling in South Africa: the current status of primary schools, the key sources of under-performance in early schooling and what needs to be done to enable learning in the early years of schooling. What stood out

for me in Hoadley's essay is that physical access to schooling, which was not available to majority of children in South Africa previously, is now addressed to some extent by the government through the focus on the Early Childhood Education and the introduction of grade R classrooms. This focus on early childhood (years 0-4) and grade R has opened opportunities for many children to access schooling. However, this author argues that physical access to early childhood education is not necessarily epistemological access to reading, writing, counting and calculation skills and knowledge. She points out that the socio-economic backgrounds of children have an influence on how well they are prepared for schooling and how well they can succeed academically. Learners' levels of preparedness and their home backgrounds affect them even before they set their feet in the schooling environment and hence there are different levels of performance within early years of schooling. This challenge impacts on teachers of young children and demands that they are better prepared to teach in ways that close the gaps that children come to school with. This present study pointed to variations in what teachers transmitted as evaluative criteria during the teaching of number, additive and multiplicative relations in two grade 3 English medium and two grade 3 Sepedi medium classrooms. The study also pointed to various ways teachers moved between representations when transmitting evaluative criteria. The fact that teachers transmitted the EC and moved between the oral and the symbolic modes of representations did not show that learners accessed knowledge of number, additive and multiplicative relations because much of the teaching privileged counting and calculation by counting approaches in grade 3 from the beginning of the year up to the third school term. In grade 3 learners are expected to at least be efficient and flexible when working with number (DBE, 2011). This shift to flexibility and more efficient ways of working with number was rarely observed across the four mathematics classrooms. The findings of this study show that the teachers' instruction provided few indications of their awareness of what learners needed to know in relation to early number teaching. In her essay, Hoadley asked a question: "what needs to be done to enable learning in the foundation phase". Her answer is a move to a more intensive and ongoing teacher training programme in the foundation phase that focuses on salient knowledge, skills and competences for teaching reading, writing and number

concepts. My experiences of conducting this research in grade 3 mathematics classrooms have led me to agree with Hoadley's suggestion, with the findings suggesting the usefulness of broader repertoires of translanguaging and conversion moves coupled with attention to more general procedures and principles in early number instruction. The way the four teachers taught number, additive and multiplicative relations showed limited attention to enabling learners to progress to more flexible and sophisticated ways of dealing with number. In addition to the need for quality education in early schooling in South Africa, I have come to appreciate the value of using African languages in the teaching of mathematics in foundation phase classrooms.

7.8.2 Affordances of African languages in mathematics classrooms

Conducting this research in two English medium and two Sepedi medium classrooms has contributed to my understanding of how African languages can be useful for advancing mathematical learning in contexts where learners are not yet fluent in English. Two main ideas stood out for me about the usefulness of African languages in the teaching of mathematics in the foundation phase. Firstly, I have come to appreciate the forms of number names in African languages such as Sepedi and Setswana because the way number names are constructed in these languages makes it easier for young learners of mathematics to understand the concept of place value and the number value which makes up a big part of the number sense development curriculum.

Secondly, this study compelled me to go outside my comfort zone of reading the mathematics literature to also consult the language literature. In particular, I needed to read literature on the language practices of translation and translanguaging as these two practices surfaced in the multilingual mathematics classrooms in which I conducted observations. Through observing teachers move between languages, I noted practices where teachers substituted words or phrases from one language with words or phrases in another language and I also noticed teachers move between languages and mathematical symbols to explain and provide mathematical feedback to learners' responses to tasks. The two practices provided different levels of making meaning and providing feedback to learners. Translation (substitution) only indicated to learners what

they said or could have said that was correct or incorrect. However, moving between the two languages and connecting to symbols in mathematics (translanguaging) provided more meaningful explanations and feedback to learners than simply translating or substituting words or phrases between the languages. These moves between languages were a new learning experience for me.

7.8.3 Conducting research in primary mathematics teaching within the South African context

Finally, I have come to appreciate the challenges that came with conducting a research study in mathematics teaching in South Africa where the quality of teaching is not the same as in many 'developed' countries. Whilst I found volumes of literature on what is central in the development of number and the use of representations in developing this concept, these did not seem to assist in making sense of the empirical data that I gathered in this study. The data presented unique and low levels of teaching number that I could not compare to standards in the international literature. It was for this reason that I decided to develop a new analytical framework in order to make sense of my findings.

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ANNEXURE 1: INFORMATION LETTER FOR TEACHERS

My name is Manono Angeline Mdluli. I am a PhD student at the University of the Witwatersrand (Wits School of Education). My PhD study focusses on the links between the language of instruction and patterns of interaction during the learning of number in Grade 3. The focus of this study is mainly on teachers and how they use the language of instruction to help support learners to make a transition from concrete to abstract ways of working with number.

This study involves observation and video-taping lessons where teachers teach place value. I request that you participate in this study and allow me to observe, videotape and interview you on your teaching. I have chosen you and your class because I need teachers who can use the language of teaching and learning like Sepedi or English to help me understand the links between the language of instruction and patterns of interaction prevalent in a grade 3 African mother-tongue or English medium classroom. Your participation in this study is vital for me to understand the link between these phenomena.

Please note that participation is voluntary. You are at liberty to withdraw from the study at any time without penalty or pressure from the researcher to provide reasons. The researcher will undertake to ensure that participants are not caused any detriment by participating in this study. Your name will not be used in this study. Only pseudo names will be used. All information about you will be kept confidential. Outcomes of the research will be made available to you on request.

I am looking forward to working with you. Please feel free to contact me should you require further information relating to this study.

Yours faithfully

Manono Angeline Mdluli

Address: Unit 100 Cedar Creek Ormonde

Tel: 011 717-3052

Cell: 071 895 3569

ANNEXURE 2: CONSENT TO BE OBSERVED: TEACHERS' OBSERVATION

Dear teacher

My name is Manono Angeline Mdluli. I am a doctoral student at the University of Witwatersrand. The aim of my study is to investigate the nature of the relationship between the language of instruction teachers use and the interactions that happens when they teach early number in grade 3 mathematics classrooms. Part of this study requires that I conduct observation with teachers involved. In order to capture your understandings of how you use the language of teaching and learning to support children to learn, need to observe you teach.

Please fill in the reply slip below if you agree to be observed. These observations will be used for purposes of my research study entitled:

Exploring the links between the medium of instruction and patterns of interaction in relation to the learning of early number in Grade 3

Only myself and my supervisor will have access to this observation data

Permission to be observed:

I, ----------

Please delete that which IS NOT applicable

Agree/do not agree to be observed in class for this research study

- I know that I may withdraw from the study at any time and that I will not be advantaged or disadvantaged in any way.
- I am aware that the researcher will keep all information confidential in all academic writing.
- I know that the observation will only be used for this research study.

Teacher's signature ---------- Date ----------

Contact person:

Name: Manono Angeline Mdluli

Address: Unit 100 Cedar Creek Ormonde

Tel: 011 717-3052

Cell: 071 895 3569

ANNEXURE 3: CONSENT FORM: TEACHERS' AUDIO RECORDING

Dear teacher

My name is Manono Angeline Mdluli. I am a doctoral student at the University of Witwatersrand. The aim of my study is to investigate the nature of the relationship between the language of instruction teachers use and the interactions that happens when they teach early number in grade 3 mathematics classrooms. Part of this study requires that I conduct post observation interviews with teachers involved. In order to capture your understandings of how you use the language of teaching and learning to support children to learn, I am likely to need to audio-record the interview conversations. Audi-recording allows me to listen to you during the interview and to back and listen again afterwards.

Please fill in the reply slip below if you agree to be audio recorded in the post observation interview. These audio recordings will be used for purposes of my research study entitled:

Exploring the links between the medium of instruction and patterns of interaction in relation to the learning of early number in Grade 3

Only myself and my supervisor will have access to this data

Permission to be audio recorded

I, ----------

Please delete that which IS NOT applicable

Agree/do not agree to be audio recorded in class for this research study

- I know that I may withdraw from the study at any time and that I will not be advantaged or disadvantaged in any way.
- I am aware that the researcher will keep all information confidential in all academic writing.
- I know that the audio recordings will only be used for this research study.

Teacher's signature ---------- Date ----------

Contact person:

Name: Manono Angeline Mdluli

Address: Unit 100 Cedar Creek Ormonde

Tel: 011 717-3052/ Cell: 071 895 3569

ANNEXURE 4: INFORMATION LETTER FOR PARENTS AND LEARNERS

Dear Parents and learners,

My name is Manono Angeline Mdluli. I am a doctoral student at the University of the Witwatersrand. My study aims to investigate the nature of the relationship between the language of teaching and learning that teachers use to teach mathematics in Grade 3 and the way they interact with children. The focus of this study is mainly on teachers and how they use the language of instruction to help support you to learn.

This study involves observation and video recordings lessons of mathematics teachers teaching in your class. I request that you participate in this study and allow me to observe and video record your teacher while teaching you. The reason I am requesting your permission is that your teacher has agreed to participate in this study.

Remember, this is not a test, it is not for marks and it is voluntary. You do not have to be involved if you do not want to. If you decide to participate and you decide along the way that you do not want to continue, I will understand and this will not affect you negatively in any way.

I will not use your own name in the study. All information collected will be stored safely. Only my supervisor and I will have access to this information.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Manono Angeline Mdluli

Address: Unit 100 Cedar Creek Ormonde

Tel: 011 717-3052

Cell: 071 895 3569

ANNEXURE 5: CONSENT FORM: LEARNERS' VIDEOTAPING

Please fill in the reply slip below if you agree to be videotaped in the lessons that I will observe your teacher teaching. These videotapes will be used for purposes of my research study called:

Exploring the links between the medium of instruction and patterns of interaction in relation to the learning of early number in Grade 3.

Permission to be videotaped

I, ----------

Please delete that which IS NOT applicable

Agree/do not agree to be videotaped in class for this research study

- I know that I may withdraw from the study at any time and that I will not be advantaged or disadvantaged in any way.
- I am aware that the researcher will keep all information confidential in all academic writing.
- I know that the videotapes will only be used for this research study.
- All participation in this study is voluntary

ANNEXURE 6: ETHICS APPROVAL



UMnyango WezeMfundo
Department of Education

Reference No: D2011/69
Lefapha la Thuto
Departement van Onderwys

Enquiries: Diane Buntting (011) 843 6503

Date:	**31 January 2011**
Name of Researcher:	**Professor H. Venkatakrishnan**
Address of Researcher:	**Room 2,**
	WMC corridor,
	Marang Block;
	WITS School of Education
Telephone Number:	**011 717 3742 / 082 099 1967**
Fax Number:	**0865088324**
Email address:	**hamsa.venkatakrishnan@wits.ac.za**
Research Topic:	**Improving the teaching and learning of primary school numeracy/mathematics**
Number and type of schools:	**TEN PRIMARY Schools**
District/s/HO	**Johannesburg East**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. ***The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.***
2. ***The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.***
3. ***A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.***

Office of the Chief Director: Information and Knowledge Management
Room 501, 111 Commissioner Street, Johannesburg, 2000 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0809 Fax: (011) 355-0734

4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

31 January 2014

Shadrack Phele MIRMSA
[Member of the Institute of Risk Management South Africa]
CHIEF EDUCATION SPECIALIST: RESEARCH COORDINATION

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	