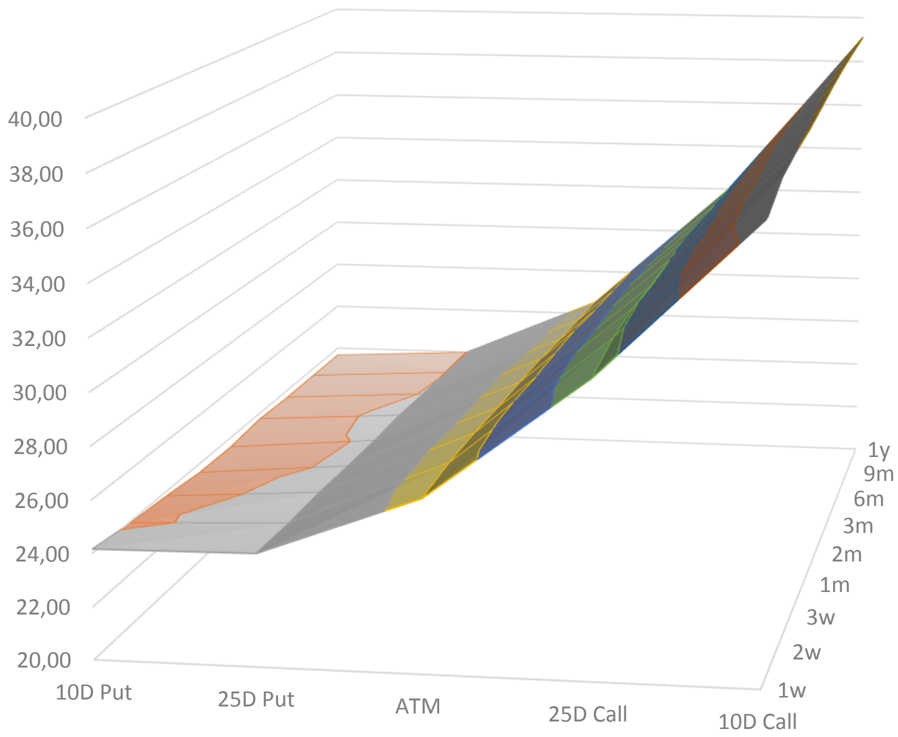

A volatility focus: The appropriateness of Black-Scholes Modelling in hedging ZAR volatility.

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Abstract

The Black-Scholes Model infers preconditions for its application and this paper refers to them as the Black-Scholes assumptions. The model makes the following assumptions, volatility and interest rates are constant, the underlying instrument does not pay dividends, the options can only be exercised at expiry, there are no transaction costs in buying or selling of the underlying instrument and that the price follows random walk theory (Black & Scholes 1973a).

Implied volatility is an important variable as it forms the volatility smile and terms structure which together derive the implied volatility surface. The shape of the volatility surface enables the paper to test certain aspect of the Black-Scholes Model assumptions. The study focuses on USDZAR currency pair and the literature review section of the paper investigates the history of the South African Rand from 1961 to 2005. The value of an option premium can be better understood with the aid of two option dynamics i.e. intrinsic value and time value. Intrinsic value is the amount of money that is realized by exercising the option, when the price of a call or put option is greater than its intrinsic value, it is said that the option has time value. The moneyness of an option is a crucial concept that describes the potential gain or loss emanating from exercising an option, an option can be in-the-money, out-the-money and at-the-money (Chisholm, 2010) and (Hull, 2015).

The study finds that the original Black-Scholes Model is not an appropriate valuation model for hedging against USDZAR volatility purely based on its assumptions and its inability to incorporate domestic and foreign interest rates. The Black-Scholes Model assumes an important foundation in the understanding of option valuation. Since its inception, there have been extensive expansions of the Original Black-Scholes Model but the most significant one (for this study) is the Garman Kohlhagen Model which was specifically derived to deal with two interest rates (foreign and domestic).

The paper suggest that the Garman Kohlhagen Model is a more appropriate alternative to the Original Black Scholes Model, it is worth noting that this model has similar limitation by assuming that volatility remains constant during the life of the option. It is more appropriate (not perfect) in the sense that it alleviates the Black-Scholes Model assumption that risk-free rates of the domestic currency are the same as the risk-free rate of the foreign currency, this is not true (Garman & Kohlhagen 1983).

Declaration

I declare that this research project is my own work. This report is being submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the Wits Business School. It has not been submitted before for any degree or examination in any other University. I further declare that I have obtained the necessary authorization and consent to carry out this research.

Tebogo Mekgwe

28/09/2018

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CHAPTER 1

Introduction

1.1 Chapter Introduction

This chapter discusses the outlay of the paper by assessing the background of the original Black-Scholes Model, option sensitivities and the history of the South African Rand. The problem statement is based on the assumption that implied volatility remains constant throughout the life of the option, the problem statement is discussed in Section 1.3 of the chapter. The paper has identified multiple research objections and questions that are discussed in Section 1.4 and 1.5 respectively. The significance of the study is discussed in Section 1.6 and limitations of the study are highlighted in Section 1.7.

1.2 Background

Pioneers of option valuation models, Fischer Black and Myron Scholes argued that it is not possible to make a profit by creating a synthetic position consisting of options and their underlying instrument. This argument is an important building block in formulating the theoretical framework for the valuations of options. Black & Scholes (1973) conducted empirical investigations and found that buyers of vanilla options paid higher premiums than their valuation model had predicted. Geske et al. (1983) attributed the discrepancy to imperfect protection of dividends in over-the-counter markets.

The Black-Scholes Model infers preconditions for its application and the paper refers to them as the Black-Scholes assumptions. The model makes the following assumptions, volatility and interest rates are constant, the underlying instrument does not pay dividends, the options can only be exercised at expiry, there are no transaction costs in buying or selling of the underlying instrument and that the price follows random walk theory (Black & Scholes 1973a). The assumptions and the limitations of the Black-Scholes Model are discussed in the Literature review section of the paper. Cox & Ross (1976) used jump-diffusion model to calculate the value of European options. Merton (1976) argued that the stochastic process with a continuous simple path can describe the return dynamics of stock instruments.

The paper will derive mathematical formulas for option sensitivities and interpret their output values, option sensitivities are delta, gamma, theta, vega and rho. Delta measures the sensitivity of an option given the price change of the underlying instrument, gamma measures the change in the delta for a given change in the price of the underlying instrument and the abovementioned sensitivities form important building blocks in understanding delta and gamma hedging. Theta measures sensitivity in the value of the option given a change in the time to expiry of the underlying instrument. Vega measures the sensitivity of the value of the option given the change in the volatility of the underlying instrument and rho measures the sensitivity of the value of the option given the change in the interest rates (Hull 2015).

The paper back tests out-the-money and in-the-money call and put vanilla options. Chisholm (2010) and Hull (2015) argued that call options are in-the-money when spot rate is higher than the strike price, are out-the-money when the spot rate is lower than the strike price. Similarly put options are in-the-money when the spot rate is lower than the strike price and are out-the-money when the spot rate is higher than the strike price. Call and put options are at-the-money when the spot rate is the same and the strike price. Vanilla options with different strikes or deltas will assume different implied volatilities and options with greater time values will assume higher implied volatilities.

Implied volatility is an important variable as it informs the volatility smile and terms structure which together form the implied volatility surface. The shape of the volatility surface would enable the paper to test certain aspect of the Black-Scholes Model assumptions. The study focuses on USDZAR currency pair and the literature review section of the paper investigates the history of the South African Rand from 1961 to 2005.

1.3 Problem Statement

The Black-Scholes Model has become one of the more appreciated option valuation models in modern literature, the model is predominantly used in equity (stock), currency and fixed income markets. Financial Market practitioners such as option structurers and traders rely immensely on the Black-Scholes Model to market make and structure vanilla options. The Black-Scholes Model makes significant assumptions which are discussed at length in the next chapter, it is the application and appropriateness of these assumptions that has drawn significant criticism from other scholars. The statement of the problem lies in making use of the Black-Scholes Model to price vanilla options with the objective of reducing the risk associated to currency volatility.

The assumption that volatility is constant is of significant concern as this has wider-reaching implications for this study. The constant volatility assumption will inevitably impact the prices of call and put options, impact the shape of the term structure, volatility smile and ultimately the volatility surface. The implication of this assumption is that the volatility surface and vega output flat through the life of the option. Volatility measures the spread in the movements of the underlying instrument and a constant implied volatility would mean that there are no movements in the price of the underlying instrument. If true, this would negate the need to use currency options to hedge against exchange rate volatility.

1.4 Research Objectives

The objectives of this research are as follow:

1.4.1 To determine the impact of the 2008 Global Financial Crisis on USDZAR option implied volatility.

1.4.2 To determine the impact of the 2008 Global Financial Crisis on USDZAR option premiums.

1.4.3 To explain how Black & Scholes (1973) derived the equation for option valuation.

1.4.2 To use the shape of the volatility surface to determine if USDZAR volatility is constant.

1.4.3 To use OVML Bloomberg Function to determine the performances of short-dated and long-dated vanilla option during the 2008 Global Financial Crisis.

1.4.4 To use OVML Bloomberg Function to interpret the USDZAR option sensitivities for out-the-money and at-the-money vanilla options.

1.5 Research questions

1.5.1 Given its assumptions, is the original Black-Scholes Model an appropriate option valuation model to hedge against USDZAR volatility?

1.5.2 What is the performance differences in out-the-money and at-the-money vanilla options?

1.5.3 What does the shape of the volatility surface infer in relation to the Black-Scholes Model assumption that implied volatility remains constant throughout the life of an option?

1.5.4 How did vanilla options perform relative to the implied forwards during the 2008 Global Financial Crisis?

1.6 Significance of Study

There is extensive literature on Black-Scholes Model theory, a majority of scholars have investigated the differences in valuation techniques between the Black-Scholes Model, the Binomial tree and the Monte-Carlo Simulation. These studies focus largely on stock (equity) market of developed economies because their sophistication and liquidity. Papers focusing on the South African currency are limited, the topic is significant in that it assesses the implied volatility patterns of an emerging market currency to model the appropriateness of the Black-Scholes Model and the paper also makes use of a particular global event to model this.

1.7 Limitations of the study

1.7.1 The paper focuses on vanilla options and does not extend its analysis to exotic options, the vanilla options are exercised at expiry.

1.7.2 The paper does not investigate compare the Black-Scholes Model to other valuation models such as the Binomial tree and the Monte Carlo Simulation.

1.8 Chapter Summary

The chapter provides the outline for the theoretical and mathematical frameworks of the study, the Black-Scholes Model is derived and discussed at greater length in the literature review section of the paper. The chapter emphasizes the importance of implied volatility as a variable of the Black-Scholes Model and as a key determinant in assessing the appropriateness of Black-Scholes Model is hedging against USDZAR volatility.

CHAPTER 2

Literature review

2.1 Introduction

This Chapter introduces the theoretical framework for this paper, the appropriate terminologies and derivation of the Black-Scholes Model. Section 2.2 discusses option nuances such as the differences between a call and put option, European and American options and the diverse types of option moneyness. Section 2.3 introduces volatility and the fundamental differences between implied and historical volatility while Section 2.4 discusses the Volatility Risk Premium. Section 2.5 discusses the various shapes that the volatility curve has such as the volatility smile or smirk. The term structure and volatility surfaces are discussed in Section 2.6 and 2.7 respectively, the Black-Scholes Model is derived and interpreted in Section 2.8, 2.9 and 2.10. The Put-Call Parity and the option bounds are explained in Section 2.11 and the assumptions of the Black-Scholes Model are discussed in Section 2.12. The options sensitivities are introduced in Section 2.13 and Section 2.14 discusses the history of USDZAR as a currency pair from 1961 to 2005. Chapter 2.15 highlights the theoretical framework of the Garman Kohlhagen.

2.2 Option Basics

Options are traded as exchange-traded and over-the-counter (OTC) instruments, they are classified into two types of vanilla options i.e. call options and put options. A call option gives the holder of the option the right to purchase the underlying instrument at a predetermined price and purchase date. Similarly, a put option gives the holder of the option the right to sell the underlying instrument at a predetermined price and purchase date. The predetermined price and purchase date are also known as the strike price and expiry date respectively. The seller of call/put option has the obligation to sell/buy the underlying instrument when the holder of the options exercises the right (Hull 2015).

Options can be differentiated into multiple categories but the commonly known categories are European, American and Bermudan style options, this paper will only focus on the European style options (Hull 2015). A European style option is an option that can only be exercised on the expiry date while an American style option is an option that can be exercised at any trading day during the life of the option (usually with exchange-traded options). A Bermudan style option can only be exercised at certain dates during the option period, it is a hybrid of an European and American style option (Chisholm 2010).

An option can be regarded as insurance against unfavourable financial market conditions. When financial markets are unfavourable, the market participants are protected against adverse price movement and similar when the markets are favourable, the market participants have the option to participate in yield enhancing price movements. Options remain attractive to market participants because of this flexibility, however; this flexibility or optionality comes at a cost to the option purchaser in the form of a premium. The seller of the option receives a premium for giving this right to the buyer, the premium has to be paid on the day the option is traded and is a percentage of the value of the underlying instrument and represents compensation to the seller for the risk involved with the option contract over the life of the option (Chisholm 2010).

2.2.1 Option Moneyness

The value of an option premium can be better understood with the aid of two option dynamics i.e. intrinsic value and time value. Intrinsic value is the amount of money that could be realized by exercising the option, when the price of a call or put option is greater than its intrinsic value, it is said that the option has time value. The moneyness of an option is a crucial concept that describes the potential gain or loss emanating from exercising an option, an option can be in-the-money, out-the-money and at-the-money (Chisholm, 2010) and (Hull, 2015).

Options that are in-the-money have intrinsic value because profits are realized upon exercise. Call options are said to be in-the-money when the prevailing spot rate is higher than the strike price of the option, and for a put option when the prevailing spot rate is below the strike price. A call option is said to be out-the-money if the prevailing spot rate is less than the strike price of the option and similarly put option is out-of-the-money if the prevailing spot rate is currently more than the strike price of the option. An option that is out-of-the money at expiry will have no value, and the holder of the option will allow it to expire worthless (Chisholm, 2010) and (Hull, 2015).

When the prevailing spot rate and the strike price are the same, the option is said to be at-the-money. Effectively, options that are in-the-money have both intrinsic value and time value while options that are in the-money and at-the-money only have time value. The time value of an option will diminish over the life of the option and will be zero at expiry, the time value portion of an option is at its greatest when the option is at-the-money because the entire premium is equal to the time value, as the option has no intrinsic value (Chisholm, 2010) and (Hull, 2015).

2.3 Understanding Volatility

The volatility of an option measures the spread in the movements of the underlying instruments; the volatility of the underlying instrument has a positive relationship with the time value of the option. The greater the volatility the greater the time value of the option will be. Black & Scholes (1973) described volatility as the variance rate on the return of the underlying stock, whereas Hull (2003) described it as the "measure of the uncertainty of the return realized on an asset".

As new information is received into the financial market, market participants adjust their expectations of future volatility accordingly and this results in changes in implied volatility. Rational market participants will consider existing and new information before revising their expectations about future volatility (Stein, 1989). This notion was also supported by Krylova et al. (2005) who argued that option prices contain information about investor's volatility expectations, therefore implied volatility can be considered as one of the better estimates for market uncertainty.

The study of volatility has become an important area in the field of financial mathematics. Firstly; volatility is important for market participants such as asset and portfolio managers because it assists them in determining the level of volatility associated with their investments, this enables them to make plausible portfolio management decisions. Lastly; Mitra (2009) argued that volatility assists market participants to better understand option pricing dynamics because volatility is an unobservable variable when it is modelled in the Binomial tree or the Black Scholes equation. Volatility can be used to make sense of extreme market events such as the 1987 Stock Market Crash which was dubbed as "Black Monday".

2.3.1 Implied volatility and historical volatility

Implied volatility is a critical component of this paper as it will aid in enhancing the understanding of option price dynamics. For the purpose of completeness, this paper will also study historical volatility albeit not extensively as implied volatility.

Historical volatility is also known as realized volatility and this paper describes it as the daily movement magnitude of an underlying instrument over a specific time horizon. Although both implied and historical volatility are used to measure volatility, implied volatility assumes that the underlying instrument has a continuous price with historical volatility making use of the underlying instrument's historical data.

Implied volatility can be described as the risk that is inherent with the price movements of an instrument and as a parameter, implied volatility is calculated using the Black-Scholes Model. Implied volatility is an indicator of investor's expectations regarding future volatility of an instrument and whenever new information enters the market, these expectations also change to reflect this. Krylova et al. (2005) argued that if investors are rational, implied volatility should contain all the readily available information and any new information. An increase in implied volatility could be interpreted as investors are expecting the underlying instrument to be volatile in the future.

In theory, implied volatility should be regarded as an informationally efficient and an unbiased predictor of future realized volatility. The study of the predictive ability of implied volatility was brought to the fore by Latan & Rendleman (1976) followed by Beckers (1981). Both papers argued that historical volatility was an inferior forecaster of future volatility, this was also reiterated by Christensen & Hansen (2002) who found that implied volatility of in-the-money call options is an efficient and unbiased predictor of future volatility.

In support, Szakmary et al. (2003) conducted their research on futures options and found that implied volatility (in relative sizes) outperforms historical volatility as an estimator of future volatility. In an earlier paper, Christensen & Prabhala (1998) argued that not only does implied volatility outperforms historical volatility but the former, in some of its specifications contains information regarding historical volatility. Jorion (1995) argued that although implied volatility was informationally efficient, it displayed biased properties for currency options. Fleming (1998) found that implied volatility has the appropriate information regarding future volatility, Fleming also argued that implied volatility may be regarded as an upward biased forecast and that implied volatility has more predictive properties than historical volatility.

In contrast, Day & Lewis (1992) found that implied volatility does not necessarily contain more information regarding future volatility than historical volatility. Lamoureux & Lastrapes (1993) also concurred that implied volatility does not contain more information about future realized volatility. In the same year Canina & Figlewski (1993) found that implied volatility has no correlation with future return volatility and that implied volatility is in fact a poor predictor of realized volatility as it does not contain any information about future realized volatility. Della et al. (2011) found implied volatility to be a biased predictor of future volatility as it tends to overestimate future volatility.

2.4 Volatility Risk Premium

Volatility risk premium is the difference between implied volatility and realized volatility, recent literature suggests that negative volatility risk premiums are synonymous with financial markets. Jackwerth & Rubinstein (1996) argue that implied volatilities for at-the-money options were consistently higher than realized volatilities, volatility risk premiums can be interpreted as the cost of insurance for hedging against volatility. When realized volatility is higher than the option-implied volatility, the cost of insurance is low (Corte et al. 2013).

Bakshi & Kapadia (2003) found volatility risk premiums to be negative due to the following reasons; market participants (buying volatility) are prepared to pay a premium for protection against a downward movement in the price of the underlying instrument, this results in an increase in the option price. In support, Carr & Wu (2009) found that volatility risk premiums were "strongly negative" for the Dow Jones Industrial Average and S&P 500 indices returns which also generated a highly negative beta. They also found evidence of a systematic volatility risk factor in the equity market that requires a risk premium to be highly negative.

Carr & Wu (2009) also argued that the negative risk premium is indicative of market participants perceiving an increase in volatility as a negative shock and for this, they are prepared to pay a premium to hedge against the increased volatility. This is consistent with the findings by Ammann & Buesser (2013) who argued that the average volatility risk premium is significantly negative, time varying, inversely correlated to implied volatility and that currency market participant are prepared to pay significant premium to hedge the underlying risk. Huang et al. (2013) also found volatility risk premiums to be time-varying while Coval (2000) found an inverse relationship between returns and volatility risk premium.

Della et al. (2011) and Ornelas (2016) documented the presence of volatility risk premiums in the currency market with later finding a positive relationship between volatility risk premiums and future returns. Londono & Zhou (2017) argue that currency volatility risk premiums have the ability to predict future currency returns, a finding also supported by Bollerslev et al. (2009) and Bollerslev et al. (2014) with the latter finding that volatility risk premium predicts aggregate equity market returns.

Dumas & Solnik (1995) found that equity and currency markets support the existence of currency risk premiums. The presence of volatility risk premium is attributed to exposure to a global risk factor such as the sub-prime crisis and often, the exposure is often reflected in the volatility term structure of the particular currency pair (Corte et al. 2016). Volatility risk premium is negative for over-the-counter currency options which means that option buyers should pay a premium to compensate option sellers for taking on the volatility risk (Low & Zhang 2005).

2.5 Volatility Smile and Volatility Smirk

Dupire (1994) describes implied volatility across strikes as the volatility smile while Dumas et al. (1998) and Rubinstein (1994) referred to it as the volatility sneer, the volatility smile is regarded as the financial market's perception of risk. Both the volatility smile and smirks are curves that are formed when implied volatility is plotted against the various strikes. When curvature changes from volatility smirk to volatility smile or vise versa, it is referred to as the evolution of volatility.

The shape illustrates that implied volatility increases as options gravitate towards being deeper in-the-money. The smirk is often referred to as a skew and has a downwards sloping curve, the smile is an asymmetrical concave-up parabola with implied volatility on the wings being higher than that of the body (at-the-money).

The "U" shape of the volatility smile is indicative of the market's demand for in-the-money and out-the-money options compared to at-the-money options, this would imply that the at-the-money option would naturally have a lower extrinsic value. Both the volatility smile and the volatility smirk illustrate the implied volatility varies across the different strikes for options with the same tenors. The implied volatility for call options with high strikes tends to have an upward lift as these options represent protection when the market anticipates a significant move upwards.

Rubinstein (1985) found strike price biases to be statistically significant and that short-dated out-the-money call options were significantly higher in price when compared to other call options. It was perhaps his subsequent findings that are of more significance to this paper, he demonstrates that currency options that are at-the-money are less expensive than options that are out-the-money or in-the-money (volatility smile) and that stock options exhibit a volatility skew (Rubinstein, 1994) and (Jackwerth & Rubinstein, 1996).

Put options with lower strikes have higher implied volatilities because market participants are prepared to pay premium for downside protection. There is extensive literature investigating the possible reasons behind the volatility smile, Black & Scholes (1973) argued that the existence of the volatility smile meant that option prices are not what they should be after adjusting for the effects of the valuation model variables such as strike price, risk free rate, time to expiration, volatility and the price of the underlying instrument. This implies that there is a variable that affects the prices of options and this variable is not accounted for but is absorbed in the implied volatility. In an ideal Black-Scholes Model, a single option may not be cheaper or more expensive than any other single options because there are no arbitrage opportunities (Chance, 2017).

Literature suggests that a volatility smile is a result of a violation(s) of the Black-Scholes Model assumptions. Rubinstein (1994) and Das & Sundaram (1998) identified jump processes while Dumas et al. (1998) identified stochastic volatility. Empirical findings suggest that the incorporating stochastic volatility (with and without random jumps) and stochastic interest rates does not eliminate the volatility smile (Corrado & Tie 1996). Other studies such as Vagnani (2009) suggest that the behaviour of traders is one of the key determinants of the volatility smile, the paper also finds that the heterogeneity of traders beliefs and the manner in which they update their expectations has a significant contribution to the formation of the implied volatility smile.

Peña et al. (1999) found that transaction costs emanating from the bid-offer spread was a key determinant in the curvature of the implied volatility smile. Bollen & Whaley (2004) found that changes in implied volatility smile are directly correlated to buying pressures emanating from the public order flow. Shaw (1998) argued that the implied volatility smile or skew results when market participants use one model for the pricing of options but use a different model to obtain the implied volatility.

There are two important rule concerning implied volatility that should be taken into consideration, the sticky strike rule and the sticky delta rule. The former suggests that implied volatility is a function of the strike and no other variable, this means that for a given strike on an option, changes in the prevailing spot rate does not change the implied volatility of the option, and therefore a change in the spot does change the shape of the volatility smile. The latter rule i.e. stick delta rule suggests that the implied volatility is a function of the delta and no other variable or rather, the implied volatility is a function of spot/strike ratio (also referred to as the moneyness ratio). This rule implies that when the underlying spot fluctuates, the smile will shift along the strike axis.

2.6 Term Structure of Volatility

The relationship between implied volatility and their maturities are reflected in the option's volatility term structure, this paper has incorporated sufficient literature that suggest that implied volatility may be considered as a reflection the markets future volatility expectation. Market participants have different perceptions regarding future volatility and this is well reflected in the varying volatilities across surface to expiry.

Stein (1989) investigated the term structure of S&P 100 index options and had the following hypothesis; implied volatility of longer-dated options should fluctuate less than one percent in response once percent fluctuation in the implied volatility of shorter-dated options. He found that expected sub one percent move was grossly understated as options with longer-dated maturities tend overreact in response to changes in the implied volatility of shorter-dated options and that term structure of option's implied volatilities contradicted rational expectations. This was disputed by Diz & Finucane (1993).

Krylova et al. (2005) found that implied volatilities of longer-dated options are on average higher than that of shorter-dated currency options. Heynen et al. (1994) investigated the relationship between longer-dated and shorter-dated implied volatilities based on the behavioural assumptions of stock return volatility. They find that exponential generalized autoregressive conditional heteroscedastic model (EGARCH) model offers the best description of the terms structure of an option's implied volatility and that rational expectation theory holds true as volatility expectation display rational behaviour.

They also found that the overreaction of longer-dated dated options to changes in shorter-dated options was heavily dependent on the model employed. Das & Sundaram (1998) argued that both the stochastic volatility model and the jump-diffusion model are incapable of adequately capturing all aspects of observed implied volatilities but the former model exhibits more pronounced variety patterns of the term structures of implied volatilities (for options that are at-the-money).

Xu & Taylor (1994) found that the term structure slope varied on a frequent basis and that these variations in the expectations of long-term volatility could be used in the pricing of options and to investigate whether there is an overreaction in the options market. Mixon (2002) and Krylova et al. (2005) attributed the variation in the volatility term structure to economic fundamentals that are observable, the latter also argued that the volatility term structure are upward sloping. This was also the finding made by Das & Sundaram (1998). The volatility term structure in the currency market is well researched with Campa & Chang (1995) arguing that the volatility term structure assumes an important role in forecasting future short-term volatility. Xu & Taylor (1994) also found that the volatility term structure of currency options possess fragments of volatility forecasting properties and that currency options term structures tend to be similar across various currencies, this was supported by Krylova et al. (2005).

2.7 Volatility Surface

The relationship between implied volatility and their maturities are reflected in the option's volatility term structure whereas the volatility surface simultaneously depicts the volatility smile and the volatility term structure. The volatility surface is essentially the moneyness of options relative to expiry. Chance et al. (2017) defined the volatility surface as the combined effects of persistent patterns in the implied volatility of options with respect to the time to expiration and strike prices. They further argued that it is inconsequential whether the relationship between the strike price and the implied volatility resembles a smile, smirk or a skew; any deviation from the horizontal line is indicative of a model implying multiple volatilities for the underlying instrument.

This paper makes use of the Black-Scholes Model which assumes constant volatility; this has been argued not to be true as volatility fluctuates across time to expiration of option contracts and therefore form a term structure (Xu & Taylor, 1994). Volatility smile is a term used to describe variation in volatilities across strike prices as argued by Rubinstein (1994) and Mayhew (1995).

2.8 The Black-Scholes Model (BSM)

The Black-Scholes Model remains the most appreciated option valuation models of modern finance study, the model was developed by Fischer Black and Myron Scholes. Black & Scholes (1973) argued that when market participants buy an instrument and the option on the underlying instrument, a hedged position is created. They expressed the value of the option as $g(x, t)$ which was a function of the instrument price and time, x and t respectively. The quantity of options that market participants would need to sell against one purchased share of the underlying instrument is given by equation (1) where the subscript referring to the partial derivative of (x, t) with respect to the first argument.

$$\frac{1}{g_1(x,t)} \tag{1}$$

The value of the position that is being hedged is expressed as $g_1(x,t)$. If the price of the underlying instrument changes by price Δx, the price of the option will change by amount $g_1(x,t)\Delta x$ and the number options as per formula (1) will also change by price Δx. In essence the change in the price of the underlying instrument is offset by the change in the price of the corresponding option (Black & Scholes 1973). The sold option that forms part of the hedged position must be adjusted continuously in order to neutralize the risk and the hedged position is expressed as

$$x - \frac{g}{g_1} \tag{2}$$

The subsequent change in the value of the underlying instrument for a given time frame Δt is represented as

$$\Delta x - \frac{\Delta g}{g_1} \tag{3}$$

Provided that the sold option is adjusted continuously, Black & Scholes (1973) expanded Δg which is in fact $g(x+\Delta x, t+\Delta t) - g(x,t)$. The expansion is done through the wiener process (stochastic calculus) which is also known as the Brownian motion, see formula (18).

$$\Delta g = g_1 \Delta x + \frac{1}{2g_{11}\sigma^2 x^2 \Delta t} + g_2 \Delta t \tag{4}$$

The subscripts on g are the partial derivatives and the volatility (variance rate) is given by σ^2. The change in the value of the hedged position is expressed by substituting the following equation

$$\Delta g = g_1 \Delta x + \frac{1}{2g_{11}\sigma^2 x^2 \Delta t} + g_2 \Delta t \quad into \quad \Delta x - \frac{\Delta g}{g_1} \tag{5}$$

The resultant substitution from equation (5) is

$$-\frac{\left(\frac{1}{2g_{11}\sigma^2 x^2} + g_2\right)\Delta t}{g_1} \tag{6}$$

The continuous adjustment of the sold option increases the certainty in the return on the equity of the hedged position, the return is equal to $r\Delta t$. The change in equity (6) is equal to the value of the equity multiplied by $r\Delta t$

$$\frac{\left(\frac{1}{2g_{11}\sigma^2x^2}+g_2\right)\Delta t}{g_1}=\left(x-\frac{g}{g_1}\right)r\Delta t \tag{7}$$

By dividing both by equations by Δt on both sides of the equation, the resultant expression is

$$\frac{\left(\frac{1}{2g_{11}\sigma^2x^2}+g_2\right)}{g_1}=\left(x-\frac{g}{g_1}\right)r \tag{8}$$

Through rearrangement, the differential equation for the value of the option is expressed as

$$g_2=rg-rxg_1-\frac{1}{2\sigma^2x^2w_{11}} \tag{9}$$

Black & Scholes (1973) expressed t^* and k as the option expiry date and strike price respectively. They further expressed the following *boundary condition*

$$g(x,t^*)=x-k, \qquad x\geq k \tag{10}$$

$$=0, \qquad x<k$$

Formula $g(x,t^*)$ satisfies the differential equation (9) subject to the boundary condition (10), Black & Scholes (1973) argued that this must be the option valuation formula and to solve the differential equation, the authors made the following substitution:

$$g(x,t)=e^{r(t-t^*)}y\left[\left(\frac{2}{\sigma^2}\right)\left(r-\frac{1}{2}\sigma^2\right)\left[\frac{lnx}{k}-\left(r-\frac{1}{2}\sigma^2\right)(t-t^*)\right]\right.$$
$$\left.-\left(\frac{2}{\sigma^2}\right)\left(r-\frac{1}{2}\sigma^2\right)^2(t-t^*)\right] \tag{11}$$

With the above substitution, Black & Scholes (1973) inferred the differential equation as $y2=y11$ (see heat transfer solution below)and the boundary equation becomes:

$$y(u,0)=0, \qquad u\geq 0$$

$$= k\left[e^{\frac{u\left(\frac{1}{2}\sigma^2\right)}{(r-\frac{1}{2}\sigma^2)}} - 1\right], \qquad \geq 0 \tag{12}$$

The differential equation $y_2 = y_{11}$ is the heat transfer. Black & Scholes (1973) express the heat-transfer solution as

$$y(u,s) = 1\sqrt{2\pi}\int_{-\frac{u}{\sqrt{2\delta}}}^{\infty}$$

$$k\left[e^{\frac{u+q\sqrt{2s}}{\left(r-\frac{1}{2}\sigma^2\right)}} - 1\right] e^{-\frac{q^2}{2}} dq, \tag{13}$$

By substituting the heat transfer solution (13) into the differential equation (11), Black & Scholes (1973) found the expression (14) and (15) through simplification. $N(d)$ is the cumulative probability distribution or the cumulative normal density function. The continuously compounded risk-free interest rate and the variance of are given by r and σ^2 respectively (see Section 2.10 for explanations of d_1 and d_2).

$$g(x,t) = xN(d1) - ke^{r(t-t^*)}N(d_2)$$

$$\mathrm{d}_1 = \frac{\frac{\ln x}{k} + (r+\frac{1}{2}\sigma^2)(t^*-t)}{\sigma\sqrt{t^*-t}} \tag{14}$$

$$\mathrm{d}_2 = \frac{\frac{\ln x}{k} + (r-\frac{1}{2}\sigma^2)(t^*-t)}{\sigma\sqrt{t^*-t}} \tag{15}$$

$$\mathrm{d}_2 = \mathrm{d}_1 - \sigma\sqrt{(t)}$$

2.9 Deriving the call and put options

A call option that has a strike price c at expiry date t will have a payoff function (16). The $\mathrm{x(t)}$ is the price of the underlying instrument at expiry date t. The equation for the call option at $\mathrm{t} = 0$ is expressed as $\mathrm{call(0)}$

$$\text{call}(t) = \max[x(t) - k, 0] \tag{16}$$

$$\text{call}(0) = x(0)N(d_1) - e^{-rt}kN(d_2) \tag{17}$$

Similarly, a put option that has a strike price c at expiry date t has payoff function (18). The formula for the put option at $t = 0$ is expressed as $\text{put}(0)$.

$$\text{put}(t) = \max[k - x(t), 0] \tag{18}$$

$$\text{put}(0) = \text{call}(0) + e^{-rt}k - x(0) \tag{19}$$

$$\text{put}(0) = x(0)N(d_1) - e^{-rt}kN(d_2) + e^{-rt}k - x(0) \tag{20}$$

$$\text{put}(0) = e^{-rt}kN(-d_2) - x(0)N(-d_1) \tag{21}$$

Black & Scholes, (1973) expanded their option valuation model through stochastic processes (wiener process with drift and infinitesimal variance). The process is captured with the formula below. The μ is a constant drift

$$x_t = \mu t + \sigma w_t \tag{22}$$

The formulas for the put and call options can also be stated as

$$\text{call} = e^{-r(t*-t)}\left[(x-k)N(d_1) + \frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right] \tag{17′}$$

$$\text{put} = e^{-r(t*-t)}\left[(k-x)N(d_1) + \frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right] \tag{21′}$$

2.10 Interpreting the Black Scholes formula

$N(d_2)$ expresses the probability that the option will be exercised at expiry or the risk adjusted probability that the strike price (k) will be paid at expiry. The paid strike (k) is attained when the prevailing spot price is higher than the strike price at expiry and the $kN(d_2)$ is the expected value of the paid strike. The probability is said to be being risk-adjusted because it is discounted by r or e^{-rt} (risk free rate) in order to calculate the present value (PV) of the cash flow. In essence $\mathrm{e}^{-\mathrm{rt}}\mathrm{kN}(\mathrm{d}_2)$ is the present value of paying the strike price at expiry.

Provided that prevailing spot price is higher than the strike price at expiry; $\mathrm{xN}(\mathrm{d}_1)$ is the present value (PV) of the expected underlying instrument at expiry. The buyer of the call option takes stock of the underlying instrument when the option exercised has a value that is equivalent to the notional multiplied by the prevailing spot rate at expiry. In essence, $\mathrm{N}(\mathrm{d}_1)$ measures the benefit of purchasing the underlying instrument outright. The Black-Scholes Model can be interpreted as the difference between the expected benefit of purchasing the underlying instrument outright at expiry $\mathrm{N}(\mathrm{d}_1)$ and the present value of paying (k) at expiry $\mathrm{e}^{-\mathrm{rt}}\mathrm{kN}(\mathrm{d}_2)$.

2.11 The Put-Call Parity

The Put-Call Parity theorem describes the relationship between European call and put options, this theorem is not applicable to American options as these options may be exercised prior to expiry. The Put-Call parity theory, also known as law of one price is able to determine whether options have been priced correctly as it is mathematically possible to determine the value of a European call by making use of a European put that has an identical strike price, expiry date (provided that the put has been priced correctly).

Using the principles of statistical arbitrage; it is possible to create a synthetic position that replicates the payoff-profile of the underlying instrument but to mitigate any arbitrage opportunities, the synthetic position and the underlying instrument must have identical risk -return profiles. Stoll (1969) was instrumental in investigating the parity between put and call option prices and subsequent to his study, other academics have made significant contributions to this topic, these include Klemkosky & Resnick (1979), Merton (1973) and Cremers & Weinbaum (2010).

The Put-Call Parity formula is $call + ke^{-rt} = put + x$. The equation illustrates that the value of a European call with a given strike rate and expiry date can be derived from the value of a European put with identical strike rates and expiry date. Merton (1973) argued that the put-call relationship implied that European option contracts were risk-free, however; this is not the case for American options.

(Merton 1973) suggests that increased volatility levels in the spot price made early option exercise attractive however one of the key conditions of the Put-Call Parity is that the option contract is to be held to expiry.

Klemkosky & Resnick (1979) argued that American option contracts required higher returns relative to European option contracts (risk free) and this could explain the deviations in literature on American options. It is possible to develop Put-Call Parity formulas that enables the prediction prices of financial instruments and that opportunities for arbitrage are only ever-present when the prices of calls and puts vary significantly. Similarly Cremers & Weinbaum (2010) found that deviations from the Put-Call Parity theorem contained important information about the future returns of stock and stocks with relatively expensive call premiums had a superior performance to relative to expensive puts.

2.11.1 The upper and lower bounds of options

Upper bound of call options ($k \leq x_0$): The value of the call option cannot be greater than the value of the underlying instrument because if that were to occur, it would give rise to arbitrage opportunities as market participants would naturally sell their call options and buy the underlying instrument with the premium.

Lower bound for call options ($c \geq max(x_0 - ke^{-rt}, 0)$): The lowest value that a call option can take is zero as the option can never have a negative value. If $c \leq max(x_0 - ke^{-rt}, 0)$, then market participants would simply buy the call option and simultaneously sell the underlying instrument to earn interest (r) from investing ke^{-rt}. The portfolio can attain two possible values at expiry i.e. $0 - x_t + c > 0 \; when \; x_t \; is \; less \; than \; c$ and $(x_t - c) - \; x_t + c = 0 \; when \; x_t \; is \; higher \; than \; c$. In essence, the lower bound of a call option can be interpreted as the present value (PV) of $x_t - c$.

Upper bound of put options ($p \leq ke^{-rt}$): A put option can never have a value that is higher than the strike price because if that were to occur market participants would simply sell puts for an amount where the put price exceeds the strike price. It is as a result of this arbitrary opportunity that the maximum value (at any given point in the life of the put option) of the put option must equal to ke^{-rt}

Lower bound for put options ($p \geq \max(ke^{-rt} - x_0, 0)$): The lower bound of the put option is difference between the spot price and ke^{-rt}. The lower bound of put options can options be expressed as the present value (PV) of $c - x_t$. Any violation in the boundary conditions could result in arbitrage opportunities.

2.12 Assumptions of Black Scholes Model

Black & Scholes (1973b) set out ideal market conditions for which the valuation formula is most appropriate, these conditions are known as the Black-Scholes Model assumptions.

i. The known rate of interest in the short term is constant through the life of the option
ii. The price of the underlying instrument follows a random walk
iii. The underlying instrument does not pay any dividends
iv. The option can only be exercise at expiry i.e. European option
v. There are no transaction cost relating to the selling/purchasing of either the underlying instrument or the option
vi. The volatility of the underlying instrument is constant

These ideal conditions have not gone without any criticism in as financial markets evolve and become more efficient. In practice, these assumptions result in the model's predictions being unrealistic when tested empirically or when compared to current option prices that are observed in the financial markets. Teneng (2011) argued that while it remained possible to make use of the U.S Government 30-day Treasury Bill (TB), the reality is that a risk-free rate does not exist and that even the 30-day TB does change during periods of increased volatility. The assumption that short-term interest is constant or that a single rate applies to all borrowing and lending transactions is not factual.

Random walk theory suggests that prices of underlying instruments fluctuates randomly, the unpredictability of this path makes it difficult for market participants to predict the future prices of instruments. Random walk is best explained by the Efficiency Market Hypothesis as described by Fama (1970) who identified three forms of market efficiency.

The *Weak Form Efficiency Market Hypothesis* – suggests that all material historic information has been priced into the current price of the stock and that the yields on the stock are independent (past yield has no effect on future yields). Technical analysis cannot be used to assist market participants to make informed trading or investment decisions.

The *Semi-Strong Form Efficiency Market Hypothesis* – suggests that all publicly available information is incorporated in the price of the stock and that the price adjusts rapidly to absorb any new information. Neither fundamental nor technical analysis can be used to assist market participants to make informed trading or investment decisions.

The *Strong Form Efficiency Market Hypothesis* – suggests that both private and publicly available information has been priced into the price of the stock and that market participants cannot outperform the market even if they were given new information. The strong-form combination of the weak and the semi-strong form.

The assumption that the underlying instrument does not pay dividends is unrealistic as most multinational corporates declare and issue dividend payments to their shareholders at the end of each financial year. The premium on call options are lower when companies pay dividends with high yields, Gordon (1959) argued that stock prices are determined by the dividends issued in the preceding year.

Teneng (2011) found that the Black-Scholes Model could be adjusted for dividends if the discounted value of the future dividend is subtracted from the price of the underlying instrument. Investors tend to find stocks that yield dividend as attractive and this affects its price hence the non-dividend-paying assumption inferred by Black & Scholes (1973b) is limiting.

Similarly, is the assumption at all options are European options, this paper models options that can only be exercised at expiry (European options) for simplicity. While this assumption may suit this paper, this assumption is in fact flawed as market participants have the choice of trading American options (options that can be exercised prior to their expiry). American option tend to be more expensive than European options due to their flexibility.

Another assumption made by Black & Scholes (1973b) is that there are no transaction cost or commissions applicable to option trades. Fees in the form of brokerage fees/spreads by stockbrokers are applicable whenever market participants establish and unwind option trades, these fees are added to the prices of instruments and thus affect the intrinsic value of the instrument (Teneng 2011).

2.13 Option Sensitivities

Option price sensitivities are known as option *Greeks*, they measure the change in the calculated option value if the there is a change in one of the variables, with all other variables remaining constant. Greeks are in essence partial derivatives of the underlying instrument with respect to the variables that determine the value of an option. The most commonly used *Greeks* are the delta, gamma, theta, vega and rho although kappa is occasionally used instead vega as the latter is not a Greek letter (Chisholm 2010).

Option sensitivities are used both for the American and European options, Reiss & Wystup (2001) studied the various relationship and properties for European style options. Broadie & Glasserman (1996) published research pertaining to option price sensitivities using the Monte Carlo Simulation, Hakala & Wystup (2007) studied the various formulas for Greeks using foreign exchange exotic options.

2.13.1 Delta

Chisholm (2010) describes the delta (Δ) as the change in the value of an option given the small change of the underlying instrument, with all other variables remaining constant. Delta describes the option's sensitivity to the price of the underlying instrument (Hull 2015). Hull also argued that delta is probably the most important option sensitivity and that a call has a positive (+) delta which indicates that the value of the option increases as the price of underlying instrument increases. The delta of a put is negative (-). The delta is used as a *hedge ratio* because it represents the slope of the option price and the price of the underlying instrument (Chisholm 2010).

$$delta = \frac{\partial option}{\partial underlying\ instrument} \tag{23}$$

A call option is out-the-money when the delta is less than 50% and the more the delta percentage decreases (converging to zero) the deeper the call option becomes out-the-money, this indicates the unresponsiveness of the option to minimal changes to the price of the underlying instrument (Chisholm 2010).

A call option is at-the-money when the derived delta is around 50% and this implies that given a small change in the price of the underlying instrument, the value of the call moves by half as much in the same direction. A long call is in-the-money when the delta is more than 50% and the more that the delta percentage increases (converging to 100%) the deeper the call becomes in-the-money. Increasingly the derivative mimics the behaviour of the underlying instrument (Chisholm 2010).

Call options will derive deltas of 0% to 100%, put options on the other hand will have deltas between -100% and 0%. In-the-money call options converge to 100% and in-the-money put options converge to -100%. Out-the-money options (call and put) have zero deltas.

2.13.2 Delta Hedging

Hull (2015) argued that the delta of an option does not remain the same and that portfolios remain delta-hedged (delta neutral) for a limited time only. Portfolio positions are immunized or neutralized through taking an opposite position (in the underlying instrument) equal to the size of the option delta, the delta must be adjusted periodically though rebalancing or dynamic hedging.

Delta hedging differs from static hedging (hedge-and-forget) where the hedge is put in place initially and never readjusted through the life of the portfolio. Delta hedging works well under the Black-Scholes Model because it assumes no transactions costs and that rebalancing can take place continuously. In practice, rebalancing takes place at discrete time intervals because it is unrealistic to rebalance a portfolio continuously without incurring any transaction costs.

2.13.3 Gamma

Gamma (Γ) measures the change in the delta given a small change in the price of the underlying instrument. Simply stated, the gamma measures the change in change (Chisholm 2010).

The gamma needs to be managed to ensure that it is not too high in a delta-neutral (delta of zero) portfolio. By implication, the gamma illustrates the degree of risk that a hedged position will undertake if the hedge is not properly managed (Ross et al. 2012). Large gammas indicate that the delta is extremely responsive to changes in the price of the underlying instrument and low gammas indicate that delta adjust slowly to changes in the price of the underlying instrument.

$$gamma = \frac{\partial^2 option}{\partial underlying\ instrument^2} \tag{24}$$

Chisholm (2010) argued that the gamma is high when the delta is at its most volatile, this is usually when the option is at-the-money or has a short term to expiry. The delta is most unstable at 50%, another way describing it is to compare the delta to the speed and the gamma to the acceleration as the latter essentially measures the stability of the delta. The gamma is low when the call is deep in-the-money or deep out-of-the-money because the delta is either close to 1 or close to 0 respectively.

The gamma illustrates how often a portfolio must be rebalanced to maintain a delta-neutral position, gamma is always positive for call options. Similar to delta hedging a portfolio, a gamma-neutral position can also be achieved through taking an opposite position (in the underlying instrument) equal to the size of the option gamma.

2.13.4 Theta

Theta (Θ) measures the sensitivity of an option relative to the change in the expiry of the underlying instrument, theta measures the time erosion of an option as it measures the unit change in the price of the option for a 1-day decrease in the number of days remaining until expiry. The longer the term of an option, the more expensive the price of the option will be. Theta is high for at-the-money and short-dated options because of acceleration in the time decay.

$$theta = \frac{\partial option}{\partial time} \tag{25}$$

If the life of the option is reduced by seven days, this will have a minimal impact if the option has three years to expiry. The time value of options with short maturities deteriorates very quickly (Chisholm 2010). A positive theta means that the value of the option increases with the passage of time and inversely for an option with a negative theta (Ross et al. 2012).

2.13.5 Vega

Vega (lambda, kappa or sigma) measures the option's sensitivity to the volatility of the underlying instrument which is normally positive for a long call and a long put, a positive vega means that the value of the option increases as volatility increases. The higher the volatility, the higher the premium is likely to be for both call and the put option, statistical models like the ARIMA and GARCH model predict the implied volatility of options to ascertain if the option is under or over-valued (Fontanills, 2005). Vega does not measure relative changes in volatility (for this we use normalized vega) but instead measures the absolute changes.

$$vega = \frac{\partial option}{\partial expected\ volatility} \tag{26}$$

Vega is at its highest when an option is at-the-money and is positive before expiry, the reason behind this is when an option is in-the-money, the premium is made up of the intrinsic value which is not affected by the changes to the volatility hence it would not affect the value of the option. Normalized vegas for options that are out-the-money and in-the-money are larger than those that are at-the-money. Hull (2015) argued that if an option's vega is close to zero, then changes in the volatility do very little to the value of the portfolio.

2.13.6 Rho

Rho (ρ) describes the options sensitivity to risk free interest. Chisholm (2010) argued that rho measures the change in the option value given the changes in interest rates and is positive for calls options and negative for put options meaning that the value of the call increases (put decreases) during rising interest rates.

$$rho = \frac{\partial option}{\partial risk\ free\ interest\ rate} \tag{27}$$

2.14 History of the South African Rand

2.14.1 1961 – 1978

The South African Rand (ZAR) was formally introduced in 1961 when the country became a republic after its membership from the British Commonwealth of Nation was revoked as a consequence of the country adopting a “whites-only” referendum. The South African Rand replaced the South African Pound as the official currency and from inception, the South African Rand was pegged to the United States dollar (USD) at $1.40 for every R1.00 until 1971.

It was then pegged to the Sterling in 1972 before being re-pegged to the United States Dollar in October of 1972. In June of 1974, the Reserve Bank of South Africa introduced significant monetary dispensations including the implementation a monetary policy independent managed floating regime, a process which involved frequent adjustment of the South African Rand causing it to weaken against the United States Dollar. The dispensation only lasted one year as it was discontinued in June of 1975 before the Rand was re-pegged to the Dollar until 1978 (Eun et al. 2012).

2.14.2 1979 – 1985

In January of 1979, the Financial Rand replaced the Securities Rand and by the early 1980s, sanctions imposed against the country together with rising inflation and increasing political pressure, the South African Rand was beginning to weaken against the Dollar. The Rand traded R1.00 to $1.00 (parity) for the first time in 1982 and at the beginning of 1983, the country policymakers made a conscious decision to cease the use of the Financial Rand and migrate to a single exchange rate system. This decision did little to curb the depreciation of the domestic currency which continued well into 1985 where rand was trading above R2.00 per $1.00.

The acceleration in the currency depreciation was so rapid that in July of 1985, currency trading was suspended for three days in attempt to curb the depreciation (Eun et al. 2012). The infamous Rubicon speech made by then State President P.W. Botha on the 15th of August 1985 collapsed the Rand to almost R2.40 per $1.00. By the end of August, the economy faced a debt standstill as a result of United States banks suspending its lending to South Africa. The rand continued to weaken so much against the Dollar that policymakers re-introduced the financial rand on the 1st of September resulting in both the financial (Eun et al. 2012).

2.14.3 1986-2005

The price of gold started declining at the beginning of the second quarter of 1988 and this resulted in ZAR depreciating against the USD and the spread between the two exchange rates started widening again. In the same year, the South African government started participating in the international debt markets and the foreign markets gradually started increasing their confidence in the domestic economy. Subsequent to this and in conjunction with the release of Nelson Mandela, both the exchange rates started appreciating towards the end of 1990. The uncertainty regarding the political transition process increased the Financial Discount Rand (spread between the financial and the commercial rand) towards the end of 1991. The 3.00 USD/ZAR spot level was breached in November of 1992 (Eun et al. 2012).

In early 1993, the UN lifted most sanctions against the South Africa and the ZAR started strengthening, the 1994 inauguration of Nelson Mandela as president was a further catalyst for the sustained Rand rally and on the 12th of March 1995, the dual exchange rate was abandoned (the Commercial and Financial Rand were unified).

Since independence, South economy has made significant strides to bolster its macroeconomic outlook and as a result, exchange rate pressures experienced in 1998 and 2001 become more difficult to comprehend. Bundia and Ricci (2005) investigated possible factors that could explain the why the Rand depreciated during these years and found that shocks (nominal, real supply and real demand) were the primary force behind the unexpected depreciation.

When Thabo Mbeki took over the reins from Mr. Nelson Mandela in 1999, the USD/ZAR spot had slipped to above R6.00 to the Dollar, the terrorist attacks on the Twin-Towers of the United Stated plummeted the currency to a historic level of 13.84 in December of 2001. This appeared to be a kneejerk reaction as the currency markets gradually corrected themselves (USD/ZAR spot trading under 9.00 and 5.70 in 2004 and 2005 respectively).

2.15 Garman Kohlhagen

The Garman Kohlhagen Model is an extension of the original Black Scholes Model, it was specifically designed to incorporate the domestic and foreign interest rates. Garman & Kohlhagen (1983) also argued that currency options should be regarded stocks that do not pay dividend and the model requires that the option strike and underlying spot be quoted as "units of domestic currency per unit of foreign currency". Like the original valuation model, the Garman Kohlhagen Model makes the following assumptions (Garman & Kohlhagen 1983).

i. The option can only be exercised at expiry
ii. The are no transaction costs and taxes related to buying and selling the currency
iii. The risk-free rate of the domestic and foreign currency is constant
iv. The volatility of the underlying instrument is constant and follows are lognormal distribution
v. There are no arbitrage opportunities
vi. There are no impediments for short selling stock

2.16 Chapter Summary

Volatility is an important variable for this paper as it implies the shape of the volatility surface, similarly the assumption that volatility remains constant throughout the life of the option has immense implications. The literature review on the extension of the Black Scholes Model is limited and even more so in the South African context (ZAR volatility), this study aims to illustrate (descriptively) the appropriateness or inappropriateness of the original Black Scholes Model in hedging against ZAR volatility during the Global Financial Crisis. Intuition suggests that constant volatility would imply a flat ZAR volatility surface. The Garman Kohlhagen Model is an extension of the Original Black Scholes and given some of its theoretical framework, it may be an alternative valuation model to hedge against the volatility of the South African Rand.

CHAPTER 3

Research Methodology

3.1 Introduction

This chapter discusses the Bloomberg OVML Function which will be used to calculate the historical option prices and the option sensitivity outputs. This study does not make use of any econometric and statistical models to calculate the outputs but the paper derives the option valuation and greek equations to assist the reader in calculating the outputs manually or as a “sanity check”. The equations necessary to extract the data, options sensitivities, volatility risk premiums and forward prices are also derived in this chapter.

Section 3.2 discusses the OVML Function, Section 3.3 explains where and how the data will be extracted. Section 3.5 derives the equations for the option sensitivities from the Black-Scholes equation discussed in Section 3.4. The Forward curve is discussed in Section 3.6 and the performances of the vanilla options are mentioned in Section 3.7. Section 3.8 formulates the equation required to calculate Volatility Risk Premium and the Volatility Surface is discussed in Section 3.9. The equations required to extract the data from Bloomberg are discussed in Section 3.10.

3.2 Bloomberg OVML Function

OMVL is a function that is used to calculate current and historical option strategy prices for a given currency pair. MARS (Multi-asset Risk Analysis System) is an additional OVML function that is used for scenario analysis of exchange-traded and over-the-counter currency options. OVML uses the Black-Scholes and Heston model to calculate the prices of vanilla options, this paper will make use of the former valuation model. The option sensitivities outputs are also calculated in OVML but for purposes of this study, the OVML Backtest function is the most important function as it enables the following capabilities.

- Selection of the frequency i.e. rolling 3 months (quarterly), 6 months (semi-annually) and 12 months (annually).
- Selection of the start date i.e. when the backtesting should start. The start date for this paper is

the first working days of 2008 and 2009 respectively i.e. 02/01/2008 and 02/01/2009.

- Modelling against the equivalent forward rate (refer to Section 3.6).

3.3 Data and Data Sources

The paper makes use of the last traded price for each of the selected variables i.e. LAST_PX (as identified in Bloomberg). The sample period ranges from 2008 Q1 to 2009 Q4 which equates to 522 trading days. The sampled period allows the paper to back-test vanilla options strategies where the USDZAR 3-month implied volatility surged above 40%, during this period the underlying USDZAR spot exchange rate was above R10.00 and this was a result of the 2008 Global Financial Crisis (Bloomberg 2016).

The paper makes use of the Bloomberg Generic Composite rate (BGN), which is an index comprising of indicative rates that are contributed by market participants. BGN uses a pricing algorithm to generate indicative bid and offer prices, contributors to the BGN index include white label trading platforms, proprietary traders, interbank dealers, sales-traders, regional banks and brokers. These contributors are selected through consistency their pricing, the frequency and quality of their data. The possibility of biasness is significantly reduced because the BGN Composite rate includes quotes from market takers and market makers. BGN offers the most accurate measure of market liquidity as it designed to track bid and offer rates that are indicative or tradable (Bloomberg 2016).

3.4 The Option Valuation Model

The call and put equations were derived in Section 2.9, the explanations of $\mathrm{N(d_2)}$ and $\mathrm{N(d_1)}$ were explained in Section 2.10 of the paper. As discussed, $\mathrm{N(d_1)}$ measures the benefit of purchasing the underlying instrument outright and $\mathrm{N(d_2)}$ measures the probability that the option will be exercised at expiry or the risk adjusted probability that the strike price (k) will be paid at expiry (t). The continuously compounded risk-free interest rate and the variance of are given by r and σ^2 respectively.

$$\text{call} = e^{-r(t*-t)}\left[(x-k)N(d_1) + \frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right] \tag{17'}$$

$$\text{put} = e^{-r(t*-t)}\left[(k-x)N(d_1) + \frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\right] \tag{21'}$$

$$d_1 = \frac{\frac{\ln x}{k}+(r+\frac{1}{2}\sigma^2)(t^*-t)}{\sigma\sqrt{t^*-t}} \tag{14}$$

$$d_2 = \frac{\frac{\ln x}{k}+(r-\frac{1}{2}\sigma^2)(t^*-t)}{\sigma\sqrt{t^*-t}} \tag{15}$$

$$d_2 = d_1 - \sigma\sqrt{(t)}$$

3.5 Deriving the formulas for Option Greeks

Hull (2015) argued that the option greeks can be derived through differentiation of the Black-Scholes model, the delta for a purchased call option should be expressed as $\Delta(\text{call}) = N(d_1)$ where (d_1) is defined as in equation (14) and $N(x)$ represent the cumulative distribution function for a standard normal distribution. Similarly, the delta for a sold call option should be expressed as $\Delta(\text{call}) = -N(d_1)$, thus the delta hedging for a bought call option involves maintaining a sold position of $N(d_1)$ of the underlying instrument for each call option that is purchased. The delta hedging for a sold call option involves maintaining a bought position of $N(d_1)$ of the underlying instrument for each call option that is sold. He also argued that delta on a put option is negative which implied that delta hedging a bought/sold put option must be maintained by buying/selling of the underlying instrument.

The formulas for the call, put options and (d_1) are computed by formulas (17'), (21') and (14) respectively and through differentiation of these of these formulas, Iwasawa (2001), (Hull 2015) and Yu et al. (2013) computed the derivation of the option greeks. This paper differentiates the formulas by Iwasawa, Hull and Yu et al. with superscripts (i), (h) and (y) respectively. Although the fomulas are expressed differently, they yield the same output and can be used interchangeably.

3.5.1 Delta

$$Delta\ (Call)^i = \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{x}-\mathrm{k})\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\ \frac{1}{\sigma\sqrt{t^*-t}}\right] + \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\mathrm{N}(\mathrm{d}_1)$$

$$+\ \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left[-\frac{\mathrm{x}-\mathrm{k}}{\sigma\sqrt{t^*-t}}\right]\left[\frac{1}{\sigma\sqrt{t^*-t}}\right]$$

$$=\ \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\mathrm{N}(\mathrm{d}_1)$$

$$Delta\ (Call)^h =\ \mathrm{e}^{-\mathrm{r}(\mathrm{t}^*-\mathrm{t})}\mathrm{N}(\mathrm{d}_1)$$

$$Delta\ (Put)^i = \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{k}-\mathrm{x})\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\ \frac{1}{\sigma\sqrt{t^*-t}}\right] - \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\mathrm{N}(-\mathrm{d}_1)$$

$$+\ \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left[-\frac{\mathrm{x}-\mathrm{k}}{\sigma\sqrt{t^*-t}}\right]\left[\frac{1}{\sigma\sqrt{t^*-t}}\right]$$

$$=\ \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\mathrm{N}(-\mathrm{d}_1)$$

$$Delta\ (Put)^h =\ \mathrm{e}^{-\mathrm{r}(\mathrm{t}^*-\mathrm{t})}[\mathrm{N}(\mathrm{d}_1)-1]$$

3.5.2 Gamma

The gamma is the second partial derivative of a portfolio in respect to the price of the underlying instrument. The put call parity theorem necessitates that the gamma formulae for both the put and call options to be identical. The changes in delta do not occur at a rapid rate if the gamma is small but if the gamma is significantly positive or negative, the delta is very sensitive to the price of the underlying instrument (Hull 2015).

$$Gamma\ (Call)^i = Gamma\ (Put)^i = \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\frac{1}{\sigma\sqrt{t^*-t}}$$

$$=\ \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\frac{\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}$$

$$Gamma\ (Call)^h = Gamma\ (Put)^h = \frac{N'(d_1)e^{-r(t^*-t)}}{x_0\sigma\sqrt{t^*-t)}}$$

3.5.3 Theta

The theta of both the put and call option is negative because as the option approaches expiry, the option loses value. This phenomenon is referred to as the time decay of an option (Hull 2015).

$$Theta(Call)^i = -\mathrm{r}\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{x}-\mathrm{k})\mathrm{N}(\mathrm{d}_1)+\frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]$$

$$-\,\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{x}-\mathrm{k})\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left(\frac{\mathrm{x}-\mathrm{k}}{2\sigma(\mathrm{t}^*-\mathrm{t})^{\frac{3}{2}}}\right)\right]-\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[\frac{\sigma}{2\sqrt{2\pi t}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]$$

$$-\,\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[\frac{\sigma\sqrt{\mathrm{t}^*-\mathrm{t}}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left(-\frac{\mathrm{x}-\mathrm{k}}{\sigma\sqrt{t^*-t}}\right)\left(-\frac{\mathrm{x}-\mathrm{k}}{2\sigma(\mathrm{t}^*-\mathrm{t})^{\frac{3}{2}}}\right)\right]$$

$$=\ -\mathrm{r}\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{x}-\mathrm{k})\mathrm{N}(\mathrm{d}_1)+\frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]-\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[\frac{\sigma}{2\sqrt{2\pi t}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]$$

$$Theta(Call)^h = -\frac{x_0 N'(d_1)\sigma e^{r(t^*-t)}}{2\sqrt{t^*-t}} + r x_0 N'(d_1)\sigma e^{r(t^*-t)} - rke^{-r(t^*-t)}N(d_2)$$

$$Theta(Put)^i = -\mathrm{r}\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{k}-\mathrm{x})\mathrm{N}(-\mathrm{d}_1)+\frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]$$

$$-\,\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{k}-\mathrm{x})\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left(\frac{\mathrm{x}-\mathrm{k}}{2\sigma(\mathrm{t}^*-\mathrm{t})^{\frac{3}{2}}}\right)\right]-\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[\frac{\sigma}{2\sqrt{2\pi t}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]$$

$$-\,\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[\frac{\sigma\sqrt{\mathrm{t}^*-\mathrm{t}}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left(-\frac{\mathrm{x}-\mathrm{k}}{\sigma\sqrt{t^*-t}}\right)\left(-\frac{\mathrm{x}-\mathrm{k}}{2\sigma(\mathrm{t}^*-\mathrm{t})^{\frac{3}{2}}}\right)\right]$$

$$=\ -\mathrm{r}\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[(\mathrm{k}-\mathrm{x})\mathrm{N}(-\mathrm{d}_1)+\frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]-\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\left[\frac{\sigma}{2\sqrt{2\pi t}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\right]$$

$$Theta(Put)^h = -\frac{x_0 N'(d_1)\sigma e^{r(t^*-t)}}{2\sqrt{t^*-t}} - r x_0 N'(d_1)\sigma e^{r(t^*-t)} + rke^{-r(t^*-t)}N(-d_2)$$

3.5.4 Vega

The vega of a portfolio will have a be significantly positive or negative if the value of the portfolio is sensitive to negligible changes in volatility (Hull 2015).

$$Vega\ (Call)^{i} = Vega\ (Call)^{i}$$

$$= \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}(\mathrm{x}-\mathrm{k})\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left[-\frac{\mathrm{x}-\mathrm{k}}{\sigma^2\sqrt{t^*-t}}\right] + \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\frac{\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}$$

$$+ \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\frac{\sigma\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}\left[-\frac{\mathrm{x}-\mathrm{k}}{\sigma\sqrt{t^*-t}}\right]\left[-\frac{\mathrm{x}-\mathrm{k}}{\sigma^2\sqrt{t^*-t}}\right]$$

$$= \mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})}\frac{\sqrt{t^*-t}}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}}$$

$$Vega\ (Call)^{h} = Vega\ (Call)^{h} = \mathrm{x}_0\sqrt{(\mathrm{t}^*-\mathrm{t})}\mathrm{N}(\mathrm{d}_1)e^{-r(t^*-t)}$$

3.5.5 Rho

Rho measures the portfolio value's sensitivity to a change in the rate of interest. Yu et al. (2013) and Hull (2015) derived the formula for rho as

$$Rho(Call)^{y} = x_t N(d_1) - ke^{-r(t^*-t)}N(d_2) + (t^*-t)ke^{-r(t^*-t)}N(d_2)$$

$$x_t N(d_1) = ke^{-r(t^*-t)}N(d_2)$$

$$= (\mathrm{t}^*-\mathrm{t})\ ke^{-r(t^*-t)}N(d_2)$$

$$Rho(Call)^{h} = k(t^*-t)ke^{-r(t^*-t)}N(d_2)$$

This paper has noted the put-call parity formula as $call + ke^{-rt} = put + x$ and on the premises of that theory, the rho on the put option can be derived as

$$Rho\ (Put)^{y} = \mathrm{Rho\ (call)} - (\mathrm{t}^*-\mathrm{t})\ \mathrm{ke}^{-\mathrm{r}(\mathrm{t}^*-\mathrm{t})}$$

$$= (\mathrm{t}^*-\mathrm{t})\ ke^{-r(t^*-t)}N(d_2) - (\mathrm{t}^*-\mathrm{t})\ ke^{-r(t^*-t)}$$

$$= (\mathrm{t}^*-\mathrm{t})\ ke^{-r(t^*-t)}[N(d_2)-1]$$

$$= -(\mathrm{t}^* - \mathrm{t})\ ke^{-r(t^*-t)}[N(-d_2) - 1]$$

$$Rho(Put)^h = -k(t^* - t)ke^{-r(t^*-t)}N(-d_2)$$

3.6 Deriving the forward curve

The paper measures the performances of vanilla options relative to the implied forward, the implied forward curve is derived using the United States and South African deposit rates. The implied forward is derived by the following formula

$$Forward = spot \times \frac{\left(1 + US\ interest\ rate \times \frac{days}{annual\ base}\right)}{\left(1 + SA\ interest\ rate \times \frac{days}{annual\ base}\right)}$$

For the South African *interest rate*, the paper makes use of the 3-month JIBAR (Johannesburg Interbank Average Rate) which is a daily fixed rate. The JSE is responsible for calculating the JIBAR rate on a daily basis and it achieves this by obtaining bid and ask quotes from contributing banks that quote Negotiable Certificates of Deposit (NCDs) larger than R100 million in size. The interest rate is quoted on an Actual/365 (fixed) day count convention.

For the United State *interest rate*, the paper will use the USD Overnight Index Swap (OIS) which bases its payments on a fixed versus floating-rate overnight index. The interest rate is obtained from quotes submitted by contributing banks and brokers, it is quoted on an Actual/360 (fixed and floating) day count convention.

3.7 Measuring the performance of the vanilla options

The paper makes use of 3-month, 6-month and 12-month rolling strategies i.e. in a given calendar year, an Institutional Investor will enter into four 3-month call/put options, two 6-month put/call options and one 12-month put/call options. The option are 10delta, 25delta and 50delta (ATM) options.

The options will be compared to 3-month, 6-month and 12-month rolling forward contracts. The pricing dates for both call, put options and forward contracts are the first trading days of 2008 and 2009 i.e. 02/01/2008 and 02/01/2009 respectively.

3.7.1 Option Performances relative to the Forward contract

The performances of the option contracts will be calculated by determining the moneyness of the options at expiry, the paper will make use of the following formula to calculate the call and put options performances, $call(put)\ strike + spot\ at\ expiry - forward\ rate$

Call option performance

$$= \left[\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})} \left[(\mathrm{x}-\mathrm{k})\mathrm{N}(\mathrm{d}_1) + \frac{\sigma\sqrt{\mathrm{t}^* - \mathrm{t}}}{\sqrt{2\pi}} \mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}} \right] + \text{spot at expiry} \right] - \mathrm{x} \times \frac{\left(1 + US\ interest\ rate \times \frac{days}{annual\ base}\right)}{\left(1 + SA\ interest\ rate \times \frac{days}{annual\ base}\right)}$$

Put option performance

$$= \left[\mathrm{e}^{-\mathrm{r}(\mathrm{t}*-\mathrm{t})} \left[(\mathrm{k}-\mathrm{x})\mathrm{N}(\mathrm{d}_1) + \frac{\sigma\sqrt{\mathrm{t}^* - \mathrm{t}}}{\sqrt{2\pi}} \mathrm{e}^{-\frac{\mathrm{d}_1^2}{2}} \right] + \text{spot at expiry} \right] - x \times \frac{\left(1 + US\ interest\ rate \times \frac{days}{annual\ base}\right)}{\left(1 + SA\ interest\ rate \times \frac{days}{annual\ base}\right)}$$

3.7.2 Calculating the effective rates

The effective rates are the intrinsic strike rates that option buyers pay for the option at expiry, the effective rate is calculated adding the cost of the option (when option is in-the-money) or the spot at expiry (when the option is out-the-money).

3.8 The Volatility Risk Premium

The formula for calculating risk premium (negative or positive) is given by

$$VRP = Implied\ Volatility - Historical\ Voltility$$

3.9 Constructing the Volatility Surface

Volatility surfaces are constructed using two approaches; the first approach involves using Risk Reversal (RR) and Butterfly (BF) strategies across the respective deltas and tenors. The second approach involves using call and put options across the respective deltas and tenors, the two approaches will yield identical implied volatilities through substitution as the Risk-Reversals and Butterflies are derived using call and put options i.e. 25delta Risk Reversal = 25delta Call option – 25delta Put Option. The paper also extrapolate the volatility surface to explain the curvatures of the volatility term structure and volatility smile.

3.10 Bloomberg formulas for extracting data

3.10.1 USDZAR Spot
=bdh("zar curncy","px_last","2008/01/02","2009/12/31","cols=2;rows=522")

3.10.2 USDZAR 3-month Historical Volatility
=bdh("usdzarh3mcurncy","px_last","2008/01/02","2009/12/31","cols=2;rows=522")

3.10.3 USDZAR 3-month Implied Volatility
=bdh("usdzarv3mcurncy","px_last","2008/01/02","2009/12/31","cols=2;rows=521")

3.10.4 JIBAR 3-month
=bdh("jiba3m index","px_last","2008/01/02","2009/12/31","cols=2;rows=501")

3.10.5 Forward Points 3-month
=bdh("zar3m curncy","px_last","2008/01/02","2009/12/31","cols=2;rows=520")

3.10.6 Forward Points 6-month
=bdh("zar6m curncy","px_last","2008/01/02","2009/12/31","cols=2;rows=520")

3.10.7 Forward Points 12-month
=bdh("zar12m curncy","px_last","2008/01/02","2009/12/31","cols=2;rows=520")

3.10.8 USD Overnight Index Swap

=bdh("ussoc curncy","px_last","2008/01/02","2009/12/31","cols=2;rows=522")

3.11 Chapter Summary

The data for this paper is extracted from Bloomberg BGN and the analysis will consist of 522 sample points. The Black-Scholes Model derived in Chapter 2 is the premises for deriving the formulas for the option sensitivities, the Forward Curve is formulated using the 3-month JIBAR and the USD Overnight Index Swap. The options are measured relative to the spot at expiry and the implied forward, Bloomberg excel function builder aids the paper in deriving the formulas required to extract the data from Bloomberg. OVML is used to calculate the historical option prices and option sensitivities for backtesting purposes.

CHAPTER 4

Empirical Findings

4.1 Introduction

This chapter discusses the implied forward curves, volatility risk premium, the volatility surface, option performances and the option sensitivities. Section 4.2 constructs and explains the shapes of the 3-month, 6-month and 12-month implied forward curves. Section 4.3 discusses and plots the volatility risk premium. Section 4.4 constructs the volatility surface by plotting the volatility smile and term structure. Section 4.5 interprets the performance of the call and put options by assessing the option strikes relative to the spot at expiry and the implied forward. Section 4.6 discusses the option sensitivities outputs.

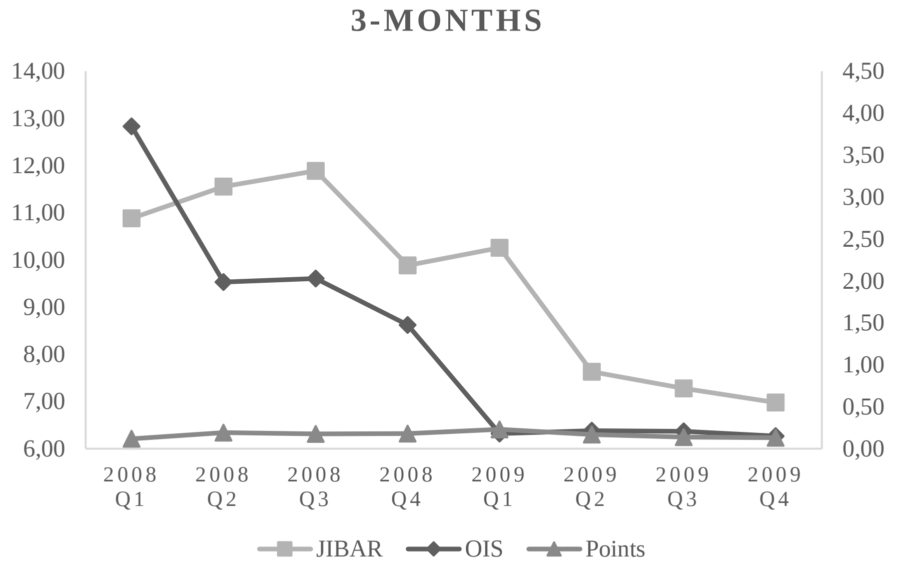
4.2 The Forward Points

The paper has used the 3-month Johannesburg Interbank Average rate and US Overnight Index Swap to derived USDZAR 3-month, 6-month and 9-month forward points

4.2.1 The 3-month Forward Points

As illustrated in Figure 1 below, the shape of the OIS curve is downward sloping and flattens at the beginning of 2009 Q1, the highest (3.8416%) and lowest levels (0.1504%) were at 2008 Q1 and 2009 Q4 respectively. The biggest quarterly decline was between 2008 Q1 and 2008 Q2 when OIS declined by 1.86%. The JIBAR curve increased from 2008 Q1 to 2008 Q3 and reached a high of 11.8883%. JIBAR declined by to 2% to 9.8848 at 2008 Q4 before declining from 2009 Q2 to Q4.

Figure 1: 3-month Forward Points

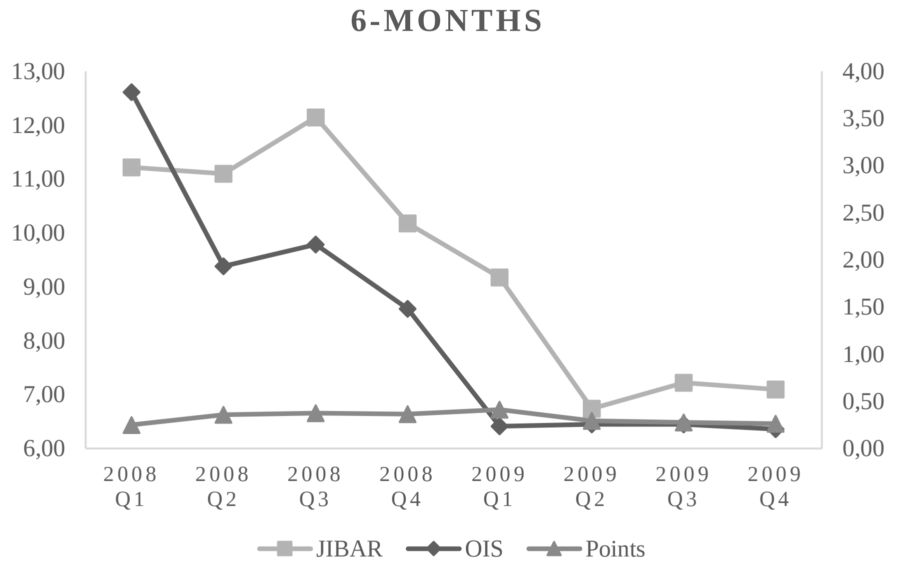


The 3-month forward points were at their highest (0.2310) and lowest levels (0.1179) at 2009 Q1 and 2008 Q1 respectively, the average forward points from 2008 Q1 to 2009 Q4 was 0.1670.

4.2.2 The 6-month Forward Points

As illustrated in Figure 2 below, the shape of the OIS curve is downward sloping and flattens at the beginning of 2009 Q1, the highest (3.7782%) and lowest levels (0.2054%) were at 2008 Q1 and 2009 Q4 respectively. The biggest quarterly decline was between 2008 Q1 and 2008 Q2 when OIS dropped by 1.8355%. JIBAR curve increased from 2008 Q2 to 2008 Q3 and reached a high of 12.1465%. JIBAR declined by 1.9703% to 10.1762% at 2008 Q4 before declining further to 7.0934% at 2009 Q4.

Figure 2: 6-month Forward Points

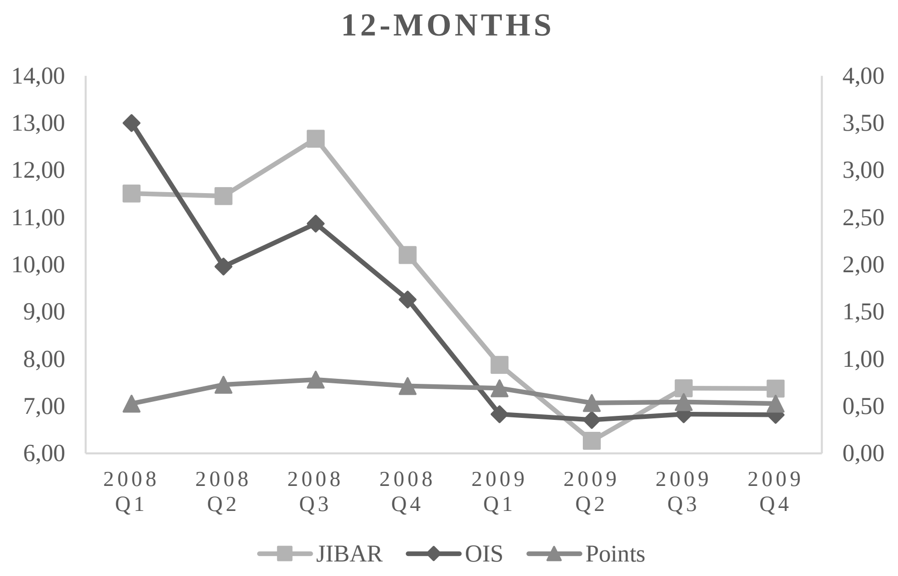


The 6-month forward points were at its highest (0.4120) and lowest levels (0.2508) at 2009 Q1 and 2008 Q1 respectively, the average forward points from 2008 Q1 to 2009 Q4 were 0.3240.

4.2.2 The 12-month Forward Curve

As illustrated in figure 3 below, the shape of the OIS curve is downward sloping and flattens at the beginning of 2009 Q1, the highest (3.5007%) and lowest levels (0.3551%) were at 2008 Q1 and 2009 Q2 respectively. The biggest quarterly decline was between 2008 Q1 and 2008 Q2 when OIS dropped by 1.5217%. JIBAR curve increased from 2008 Q2 to 2008 Q3 and reached a high of 12.6704%. JIBAR declined by 6.4039% to 6.2665% at 2009 Q2 before rising in 2009 Q3 and 2009 Q4.

Figure 3: 12-month Forward Points



The 12-month forward points were at its highest (0.6908) and lowest levels (0.5287) at 2008 Q3 and 2008 Q1 respectively, the average forward points from 2008 Q1 to 2009 Q4 were 0.6315.

4.3 The Volatility Risk Premium

The Volatility Risk Premium is the difference between implied volatility and the realized (historical) volatility. From 2008 Q1 to 2009 Q4, the Volatility Risk Premium was predominantly negative which indicated that the realized volatility was higher than implied volatility as illustrated in Figure 4 below.

Figure 4: Volatility Risk Premium

The average volatility risk premium from 2008 Q1 to 2009 Q4 was -4.2390, the average premium for the first 9-months of 2008 was -0.0634. The month of October 2008 had an average premium of 2.1247 with highest level at 14.8295. The premium averages for November and December were -13.2426 and -23.9918 respectively with the lowest level at -26.1258. The average premium for 2009 was -5.5049 however, the last two quarters had an average of -0.8550.

4.4 The Volatility Surface

The Volatility Surface is a combination of the volatility smile and the term structure, the USDZAR volatility smile resembles a smirk rather than a smile for 2008 and 2009. Lower delta puts have lower implied volatility than the lower delta calls.

Figure 5: USD/ZAR Volatility Smile

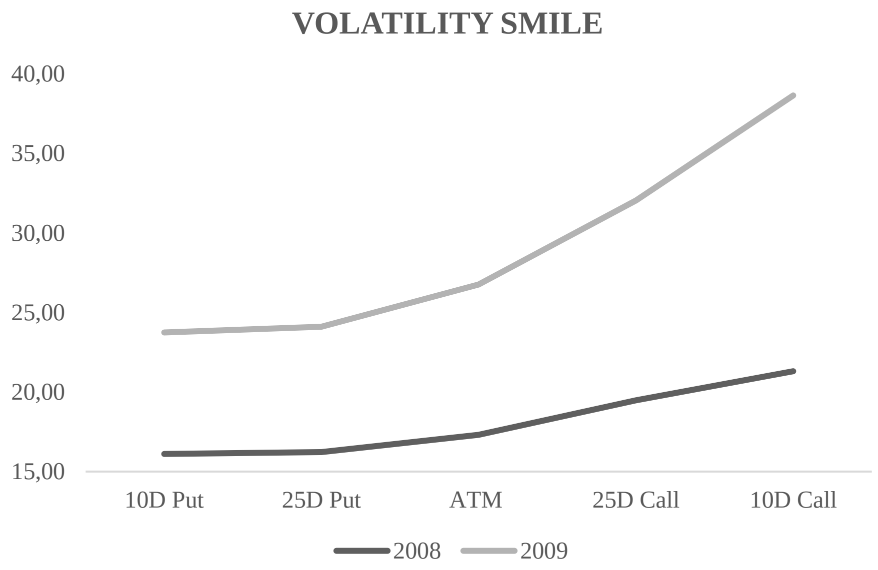
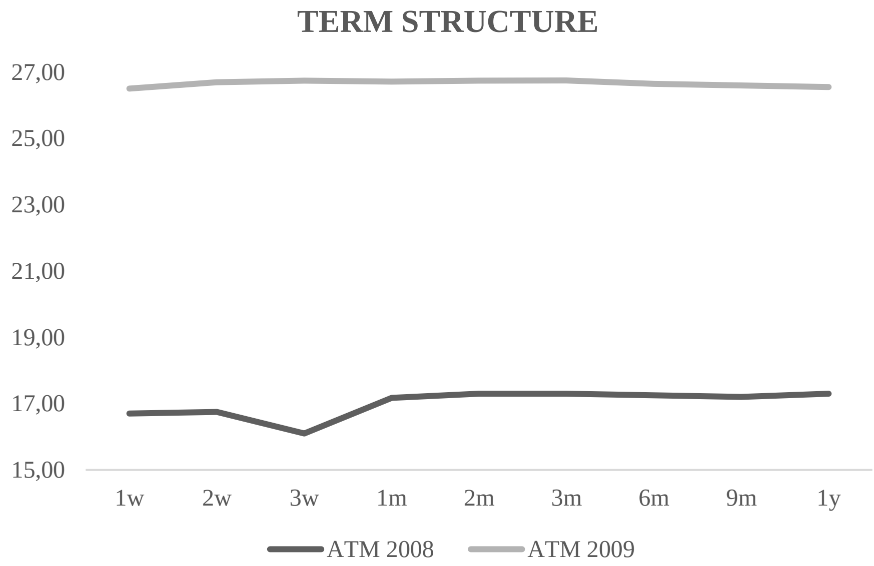


Figure 5 above illustrates that put options have lower implied volatilities than call options. There is upward parallel shift in implied volatility from 2008 to 2009 across the delta strikes. In 2008, the 10-delta and 25-delta puts had implied volatilities of 16.11% and 16.23% respectively, while the at-the-money implied volatility was 17.30. In 2009, the 10-delta and 25-delta puts volatility increased to 23.74% and 24.11% respectively.

The At-the-money call options increased from 17.30% to 26.75%. In 2008, the 10-delta and 25-delta calls had implied volatilities of 21.31% and 19.48% respectively. In 2009, the 10-delta and 25-delta calls volatilities increased to 38.64% and 32.04% respectively.

Figure 6: USD/ZAR Term Structure



The Term Structure given in Figure 6 above is the implied volatility across time, the implied volatility increased from by an average of 9.63% (17.03% to 26.66%), the volatility surfaces for 2008 and 2009 are given by Figure 8 and Figure 9 respectively

Figure 7: 2008 Volatility Surface

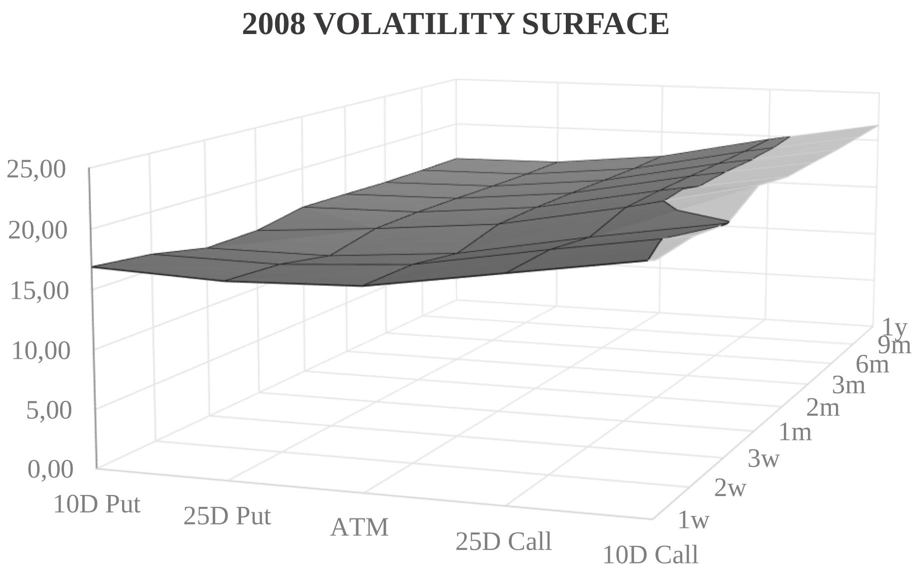
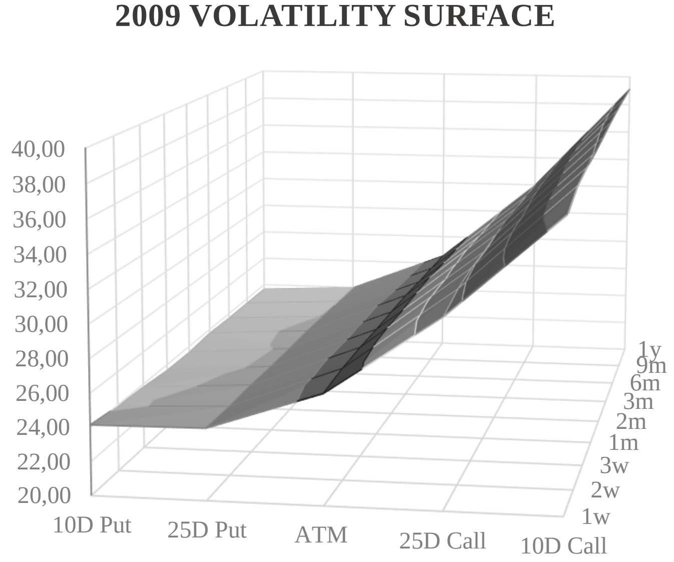


Figure 8: 2009 Volatility Surface

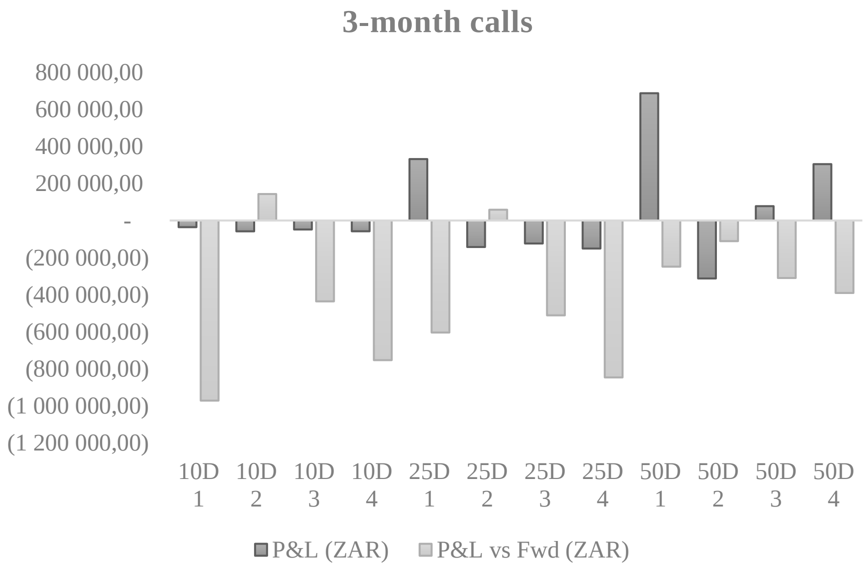


4.5 Vanilla Option Performances

4.5.1 The 2008 Performance of 3-month calls

The 3-month 10-delta call options had net losses against the spots at expiry and against the implied forwards, the generated losses were R201 220.62 and R2 019 568.44 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

Figure 9: 3-month call option performance for 2008



The 3-month 25-delta call options had net losses against the spots at the expiry and against the implied forwards, the generated losses were R84 943.38 and R1 905 291.19 respectively. The first option expired in-the money and the other options expired out-the-money as the spots at expiry were lower than the option strikes.

Table 1: 3-month call options effective rates for 2008

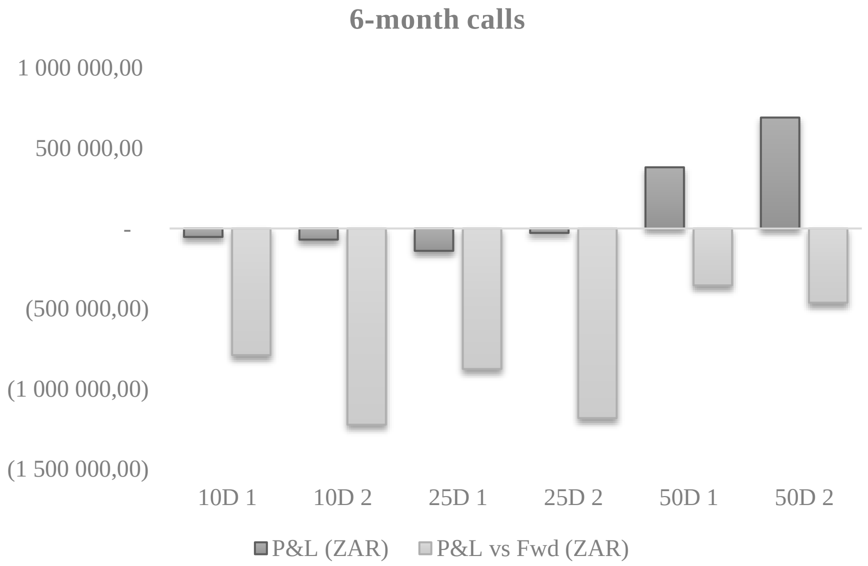
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	8,0075	7,8791	6,9429	36 003,52	7,9151
10D 2	9,5796	7,8124	8,0138	58 470,71	7,8709
10D 3	9,4827	8,4289	8,041	48 513,78	8,4774
10D 4	10,4995	9,3912	8,6957	58 232,60	9,4494
25D 1	7,4491	7,8791	6,9429	98 339,79	7,5474
25D 2	8,7454	7,8124	8,0138	143 190,73	7,9556
25D 3	8,6895	8,4289	8,041	124 055,67	8,553
25D 4	9,5106	9,3912	8,6957	151 350,56	9,5426
50D 1	6,9621	7,8791	6,9429	229 934,03	7,192
50D 2	8,0508	7,8124	8,0138	312 897,85	8,1253
50D 3	8,0709	8,4289	8,041	280 479,22	8,3514
50D 4	8,7411	9,3912	8,6957	345 786,07	9,0869

The 3-month 50-delta call options had a net profit against the spots at expiry and net losses against the implied forwards, the generated profits and losses were R755 972.95 and R1 062 374.86 respectively. Three options expired in-the-money as the spots at expiry were higher than the options strikes, the other option expired out-the-money.

4.5.2 The 2008 Performance of 6-month calls

The 6-month 25-delta call options had net losses against the spots at expiry and the implied forwards, the generated losses were R122 463.66 and R2 013 388.59 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

Figure 10: 6-month call option performance for 2008



The 6-month 25-delta call options had net losses against the spots at expiries and the implied forwards, the generated losses were R167 404.45 and R2 058 329.38 respectively. The first option expired in-the-money and the other option expired out-the-money.

Table 2: 6-month call options effective rates for 2008

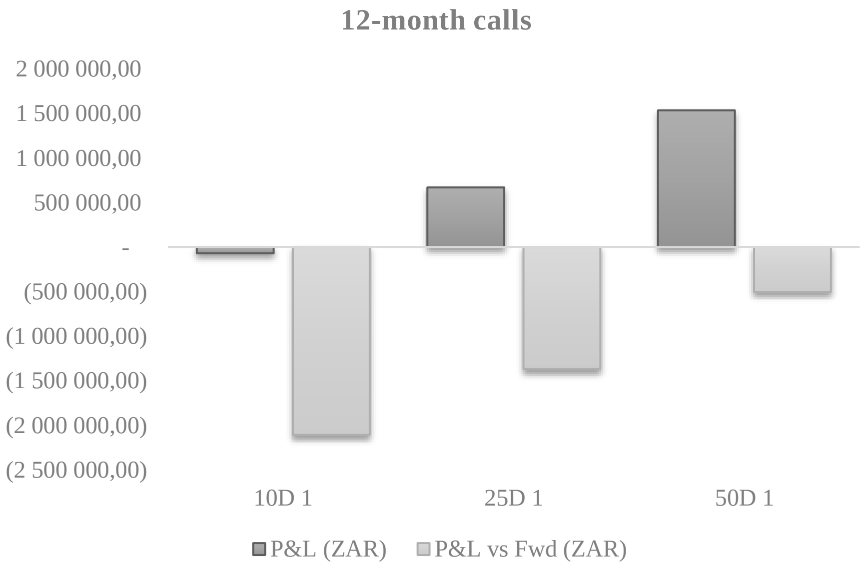
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	8,6781	7,8124	7,0758	53 008,33	7,8654
10D 2	10,5535	9,3912	8,2369	69 455,33	9,4607
25D 1	7,8298	7,8124	7,0758	139 353,42	7,9518
25D 2	9,2374	9,3912	8,2369	181 892,69	9,4193
50D 1	7,1087	7,8124	7,0758	322 020,39	7,4307
50D 2	8,296	9,3912	8,2369	403 274,15	8,6993

The 6-month 50-delta call options had a net profit against the spots at expiry and net losses against the implied forwards, the generated profits and losses were R1 073 593.55 and R817 331.38 respectively. The options expired in-the-money as the spots at expiry were higher than the option strikes.

4.5.3 The 2008 Performance of 12-month calls

The 12-month 10-delta call option had losses against the spot at expiry and the implied forward, the generated losses were R70 206.01 and R2 107 621.46 respectively. The option expired out-the-money as the spot at expiry was lower than the option strike.

Figure 11: 12-month call option performance for 2008



The 12-month 25-delta call option had a profit against the spot at expiry and a loss against the implied forward, the generated profit and loss was R669 707.60 and R1 367 707.85 respectively. The call option expired in-the-money as the spot at expiry was higher than the option strike.

Table 3: 9-month call options effective rates for 2008

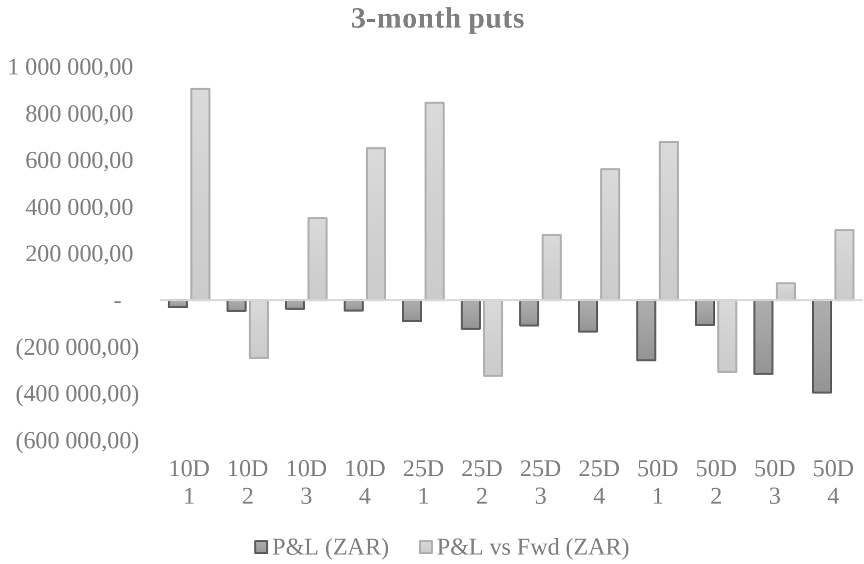
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	9,9119	9,3912	7,3538	70 206,01	9,4614
25D 1	8,5304	9,3912	7,3538	191 099,38	8,7215
50D 1	7,4105	9,3912	7,3538	445 764,78	7,8563

The 12-month 50-delta call option had a profit against the spot at expiry and a loss against the implied forward, the generated profit and loss was R1 534 933.00 and R507 482.45 respectively. The call option expired in-the-money as the spot at expiry was higher than the option strike.

4.5.4 The 2008 Performance of 3-month puts

The 3-month 10-delta put options had net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R155 125.94 and R1 663 221.87 respectively. The options expired out-the-money as the spots at expiry were higher than the option strikes.

Figure 12: 3-month put option performance for 2008



The 3-month 25-delta put options had net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R453 484.90 and R1 364 862.92 respectively. The options expired out-the-money as the spots as expiry were higher than the option strikes.

Table 4: 3-month put options effective rates for 2008

Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	6,2837	7,8791	6.9429	29 802,63	7,8493
10D 2	7,204	7,8124	8.0138	45 199,55	7,7672
10D 3	7,275	8,4289	8.041	36 125,92	8,3928
10D 4	7,7542	9,3912	8.6957	43 997,84	9,3472
25D 1	6,5971	7,8791	6.9429	89 553,20	7,7895
25D 2	7,5739	7,8124	8.0138	121 843,17	7,6906
25D 3	7,6261	8,4289	8.041	10 802,47	8,4181
25D 4	8,1873	9,3912	8.6957	134 026,06	9,2572
50D 1	6,9772	7,8791	6.9429	257 349,89	7,6218
50D 2	8,0617	7,8124	8.0138	355 414,42	7,457

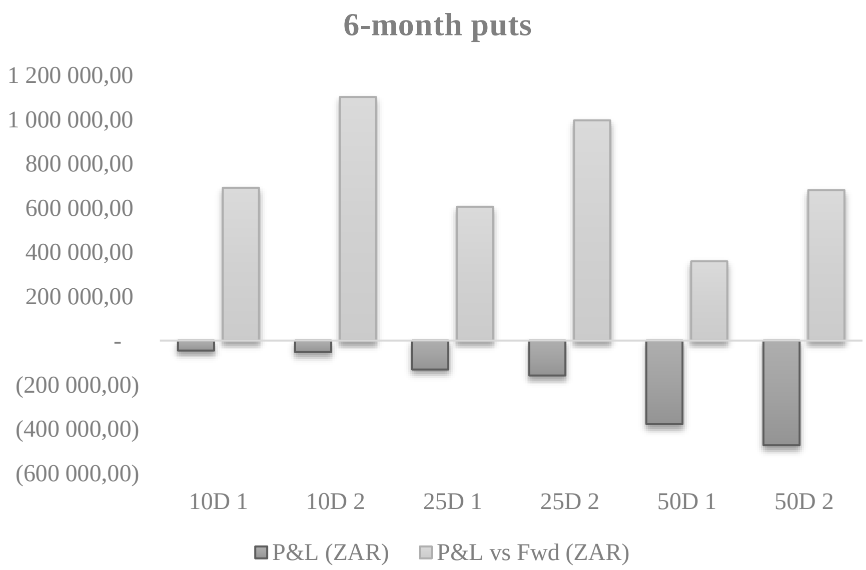
50D 3	8,0807	8,4289	8.041	315 305,38	8,1136
50D 4	8,7501	9,3912	8.6957	395 392,97	8,9958

The 3-month 50-delta put options had net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R1 074 115.18 and R744 232.64 respectively. The second option expired in-the-money as the spot at expiry was lower than the option strike, the other options expired out-the-money.

4.5.5 The 2008 Performance of 6-month puts

The 6-month 10-delta put options had net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R97 422.26 and R1 793 502.67 respectively. The options expired out-the-money as the spots at expiry were higher than the option strikes.

Figure 13: 6-month put option performance for 2008



The 6-month 25-delta put options had net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R289 211.20 and R1 601 713.73 respectively. The options expired out-the-money as the spots at expiry was higher than the option strikes.

Table 5: 6-month put options effective rates for 2008

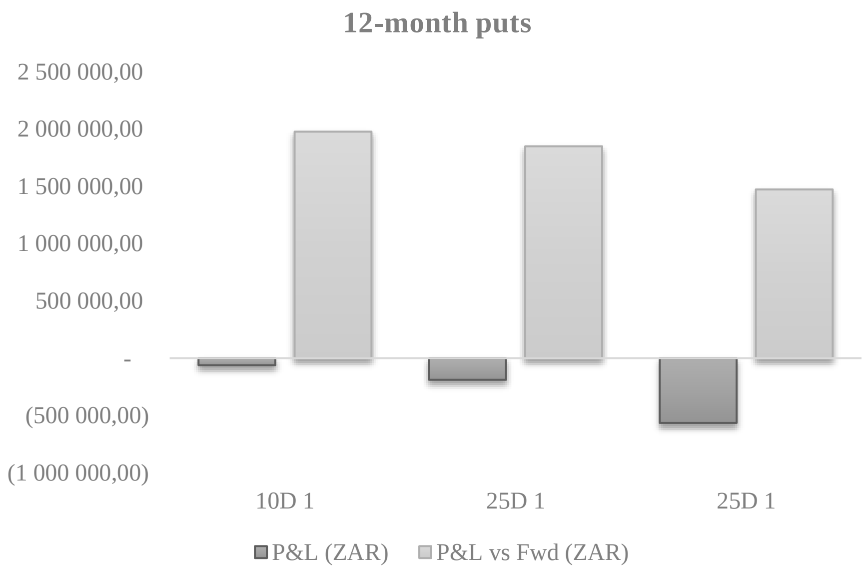
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	6,1684	7,8791	7,0758	45 260,25	7,8338
10D 2	7,1505	9,3912	8,2369	52 162,01	9,339
25D 1	6,604	7,8791	7,0758	131 127,92	7,748
25D 2	7,6542	9,3912	8,2369	158 083,28	9,2331
50D 1	7,1525	7,8791	7,0758	378 132,34	7,501
50D 2	8,3286	9,3912	8,2369	473 510,07	8,9177

The 6-month 50-delta put options had net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R851 642.41 and R1 039 282.52 respectively. The options expired out-the-money as the spots at expiry was higher than the option strikes.

4.5.6 The 2008 Performance of 12-month puts

The 12-month 10-delta put option generated a loss against the spot at expiry and a profit against the implied forward, the generated loss and profit was R61 523.48 and R1 975 891.97 respectively. The option expired out-the-money as the spot at expiry was higher than the option strike.

Figure 14: 12-month put option performance for 2008



The 12-month 25-delta put option generated a loss against the spot at expiry and a profit against the implied forward, the generated loss and profit was R189 025.51 and R1 848 389.94 respectively. The option expired out-the-money as the spot at expiry was higher than the option strike.

Table 6: 12-month put options effective rates for 2008

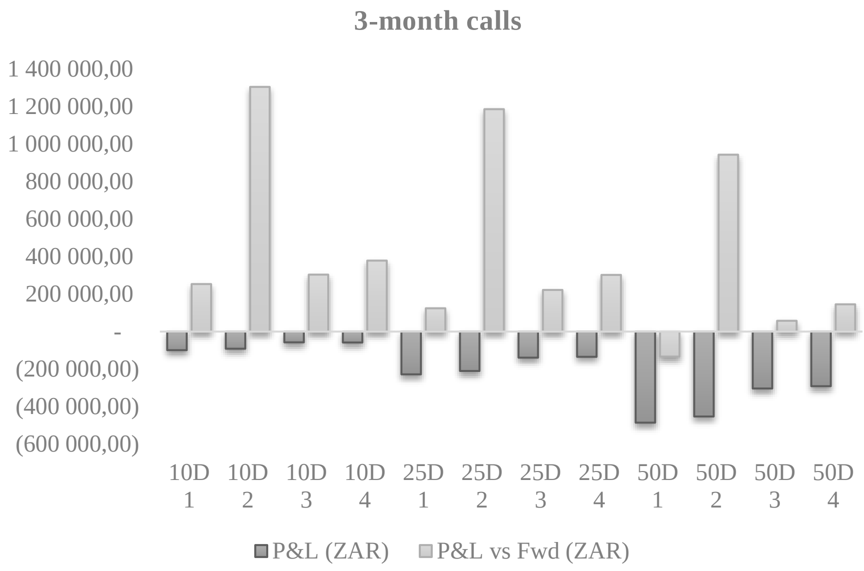
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	6,0786	9,3912	7,3538	61 523,48	9,3297
25D 1	6,7037	9,3912	7,3538	189 025,51	9,2022
50D 1	7,5328	9,3912	7,3538	565 281,83	8,8259

The 12-month 50-delta put option generated a loss against the spot at expiry and a profit against the implied forward, the generated loss and profit was R565 281.83 and R1 472 133 respectively. The options expired out-the-money as the spot at expiry was higher than the option strike.

4.5.7 The 2009 Performance of 3-month calls

The 3-month 10-delta call options generated net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R306 207.78 and R2 239 492.46 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

Figure 15: 3-month call option performance for 2009



The 3-month 25-delta call options generated net losses against the spots at expiry and net profits against the implied forwards. The generated and profits against the implied forwards were R711 939.37 and R1 833 760.87 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

Table 7: 3-month call options effective rates for 2009

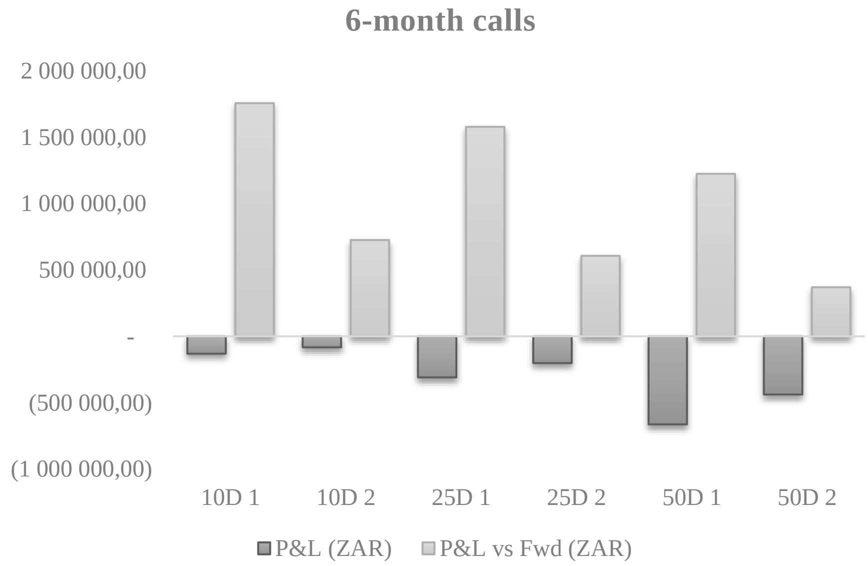
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	12,52	9.1935	9,5459	99 088,60	9,2926
10D 2	11,8806	7.8388	9,2346	91 395,77	7,9302
10D 3	9,6815	7.6441	8,0051	57 408,51	7,7015
10D 4	9,3146	7.3322	7,7689	58 314,90	7,3905
25D 1	10,8018	9.1935	9,5459	228 071,46	9,4216
25D 2	10,3618	7.8388	9,2346	210 044,23	8,0488
25D 3	8,7333	7.6441	8,0051	139 124,20	7,7832
25D 4	8,4454	7.3322	7,7689	134 699,48	7,4669
50D 1	9,6347	9.1935	9,5459	485 570,35	9,6791
50D 2	9,3124	7.8388	9,2346	452 760,43	8,2916
50D 3	8,0448	7.6441	8,0051	303 465,76	7,9476
50D 4	7,8054	7.3322	7,7689	291 236,52	7,6234

The 3-month 50-delta call options generated net losses against the spots at expiry and net profits against the implied forwards, the generated losses and were R1 533 033.07 and R1 012 667.17 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

4.5.8 The 2009 Performance of 6-month calls

The 6-month 10-delta call options generated net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R214 270.05 and R2 482 279.95 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

Figure 16: 6-month call option performance for 2009



The 6-month 25-delta call options generated net losses against the spots at expiry and net profits against the implied forwards, the generated losses and profits were R511 567.73 and R2 184 982.27 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

Table 8: 6-month call options effective rates for 2009

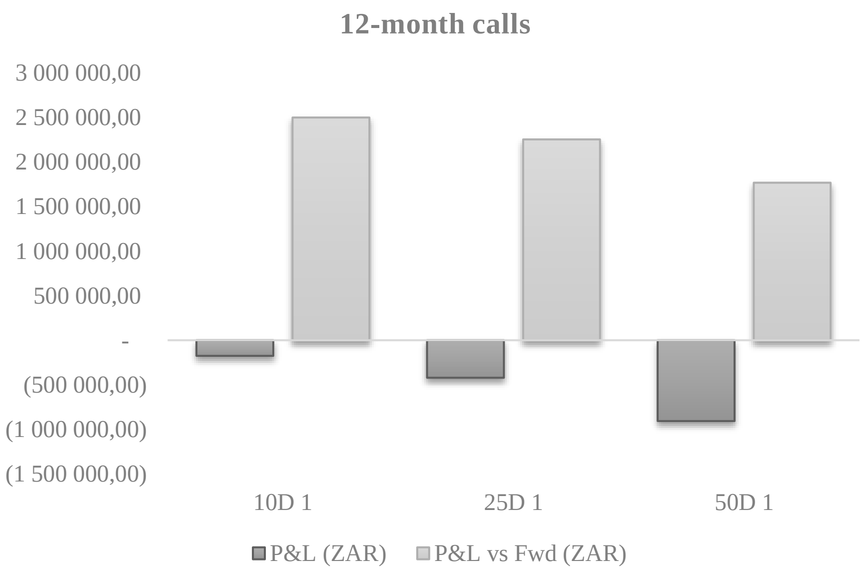
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	14,5051	7.8388	9,7269	131 270.44	7,9701
10D 2	10,9467	7.3322	8,1407	82 999.61	7,4152
25D 1	11,6958	7.8388	9,7269	309 601.98	8,1484
25D 2	9,3285	7.3322	8,1407	201 965.75	7,5342
50D 1	9,9102	7.8388	9,7269	663 712.54	8,5025
50D 2	8,2303	7.3322	8,1407	438 722.96	7,7709

The 6-month 50-delta call options generated net losses against the spots at expiry and net profits against the implied forwards, the generated losses and were R1 102 435.51 and R1 594 114.49 respectively. The options expired out-the-money as the spots at expiry were lower than the option strikes.

4.5.9 The 2009 Performance of 12-month calls

The 12-month 10-delta call option generated a loss against the spot at expiry and a profit against the implied forward, the generated loss and profit was R175 314.07 and R2 498 135.88 respectively. The option expired out-the-money as the spot at expiry was lower than the option strikes.

Figure 17: 12-month call option performance for 2009



The 12-month 25-delta call option generated a loss against the spot at expiry and a profit against the implied forward, the generated loss profit was R420 881.01 and R2 252 568.94 respectively. The option expired out-the-money as the spot at expiry was lower than the option strike.

Table 9: 12-month call options effective rates for 2009

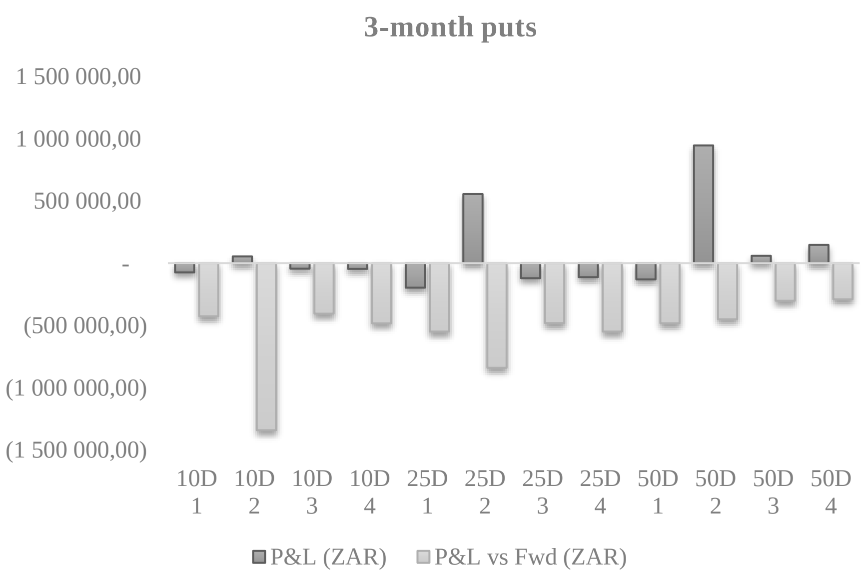
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	18,2421	7,3322	10,0056	175 314,07	7,5075
25D 2	13,2642	7,3322	10,0056	420 881,01	7,7531
50D 1	10,3913	7,3322	10,0056	906 656,90	8,2389

The 12-month 50-delta call option generated a loss against the spot at expiry and a profit against the implied forward, the generated loss and profit was R906 656.90 and R1 766 793.05 respectively. The option expired out-the-money as the spot at expiry was lower than the option strikes.

4.5.10 The 2009 Performance of 3-month puts

The 3-month 10-delta put options had net losses against the spots at expiry and the implied forwards, the generated losses were R111 471.84 and R2 657 172.08 respectively. The second option expired out-the-money other options expired in-the-money as the spots at expiry were higher than the option strikes.

Figure 18: 3-month put option performance for 2009



The 3-month 25-delta put options had net profits against the spots at expiry and net losses against the implied forwards, the profits and losses were R124 489.99 and R2 421 210.26 respectively. The second and fourth options expired in-the-money and the other options expired out-the-money as the spots at expiry were higher than the option strikes.

Table 10: 3-month put options effective rates for 2009

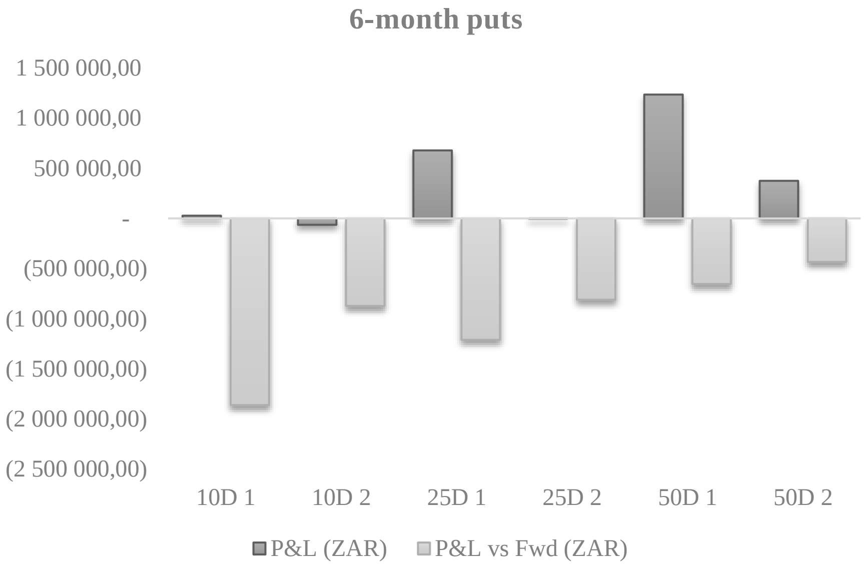
Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	8.2603	9.1935	9,5459	74 159.27	9.1193
10D 2	7.9661	7.8388	9,2346	73 129.26	7.893
10D 3	7.1195	7.6441	8,0051	44 828.55	7.5993
10D 4	6.9421	7.3322	7,7689	46 625.28	7.2856
25D 1	8.8628	9.1935	9,5459	197 054.39	8.9964
25D 2	8.5827	7.8388	9,2346	188 728.35	8.394
25D 3	7.557	7.6441	8,0051	120 813.64	7.5233
25D 4	7.3391	7.3322	7,7689	119 646.75	7.2126
50D 1	9.6363	9.1935	9,5459	573 188.47	8.6203
50D 2	9.3141	7.8388	9,2346	530 233.61	8.7839
50D 3	8.0459	7.6441	8,0051	343 181.90	7.7027
50D 4	7.8062	7.3322	7,7689	327 631.36	7.4786

The 3-month 50-delta put options had net profits against the spots at expiry and net losses against the implied forwards, the generated profits and losses against were R1 019 652.29 and R1 526 047.95 respectively. The options expired in-the-money as the spots at expiry were lower than the option strikes.

4.5.11 The 2009 Performance of 6-month puts

The 6-month 10-delta put options had net losses against to the spots at expiry and against the implied forwards, the generated losses were R37 041.30 and R2 733 591.30 respectively. The first option expired in-the-money and the other option expired out-the-money.

Figure 19: 6-month put option performance for 2009



The 6-month 25-delta put options had net profits against to the spots at expiry and net losses against the implied forwards, the generated profits and losses were R675 943.82 and R2 020 606.17 respectively. The options expired in-the-money as the spots at expiries were lower than the options strikes.

Table 11: 6-month put options effective rates for 2009

Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	7.9654	7.8388	9,7269	99 558.13	7.8658
10D 2	6.8967	7.3322	8,1407	64 079.43	7.2681
25D 1	8.796	7.8388	9,7269	279 451.20	8.5165
25D 2	7.5078	7.3322	8,1407	177 409.05	7.3304
50D 1	9.9162	7.8388	9,7269	842 860.59	6.9959
50D 2	8.2345	7.3322	8,1407	527 827.29	7.7067

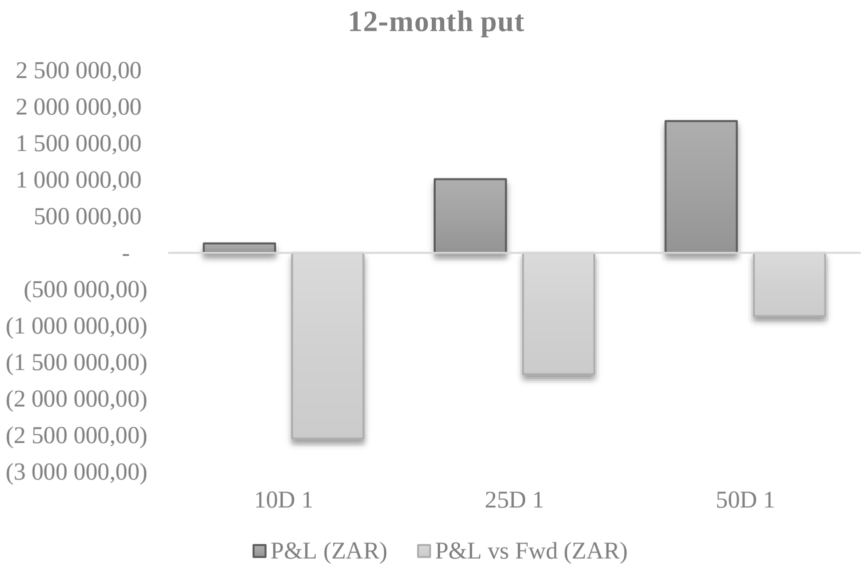
The 6-month 50-delta put options had net profits against to the spots at expiry and net losses against the implied forwards, the generated profits and losses were R1 609 016.55 and R1 087 533.45

respectively. The options expired in-the-money as the spots at expiries were lower than the options strikes.

4.5.12 The 2009 Performance of 12month puts

The 12-month 10-delta put had a profit against to the spot at expiry and a loss against the implied forward, the generated profit and loss was R70 206.01 and R2 107 621.46 respectively. The trade expired in-the-money as the spot at expiry was lower than the option strike.

Figure 20: 12-month put option performance for 2009



The 12-month 25-delta put had a profit against to the spot at expiry and a loss against the implied forward, the generated profit and was R669 707.60 and R1 367 707.85 respectively. The trade expired in-the-money as the spot at expiry was lower than the option strike.

Table 12: 9-month put options effective rates for 2009

Trade	Strikes	Spot on expiry	Implied forward	Premium (ZAR)	Effective rate
10D 1	7,6000	7,3322	10,0056	138 457,86	7,4615
25D 2	8,7504	7,3322	10,0056	408 021,55	8,3424
50D 1	10,4244	7,3322	10,0056	1 285 385,50	9,1390

The 12-month 50-delta put had a profit against to the spot at expiry and a loss against the implied forward, the generated profit and loss was R669 707.60 and R1 367 707.85 respectively. The trade expired in-the-money as the spot at expiry was lower than the option strike.

4.6 The Option Sensitivities

The delta is positive for vanilla call options and negative for put options. The OVML outputs suggests that options with high deltas have higher gammas and that options with longer tenors have lower gammas. The gamma of 3-month call options is larger than that of 6-month and 9-month call options. Put and call options with the same delta had significantly lower gammas in 2009. In 2008, the gamma for the 3-month 10-delta call option was 0.2410 and in 2009, the gamma on the same option was 0.1003. Similarly, the gamma for the 3-month 10-delta put option was 0.3257 and in 2009, the gamma on the same option was 0.1655 (Table 13).

Table 13: Option sensitivities for 3-month options

2008 3-month call options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	0.1043	0.2512	0.4973
Gamma	0.2410	0.4662	0.6526
Vega	0.0062	0.0108	0.0135
Theta Q (1 day)	-0.0009	-0.0015	-0.0019
Rho	0.0017	0.0040	0.0079
2008 3-month put options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	-0.1077	-0.2557	-0.5059
Gamma	0.3257	0.5658	0.6982
Vega	0.0063	0.0109	0.0135
Theta Q (1 day)	-0.0004	-0.0007	-0.0005
Rho	-0.0019	-0.0046	-0.0092
2009 3-month call options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	0.1161	0.2608	0.5014
Gamma	0.1003	0.2042	0.3005
Vega	0.0091	0.0151	0.0186
Theta Q (1 day)	-0.0023	-0.0034	-0.0040
Rho	0.0025	0.0055	0.0105
2009 3-month put options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	-0.1207	-0.2610	-0.4993
Gamma	0.1655	0.2726	0.3003

Vega	0.0094	0.0151	0.0186
Theta Q (1 day)	-0.0010	-0.0014	-0.0015
Rho	-0.0030	-0.0066	-0.0131

The 25-delta and 50-delta options for 6-month and 9-month tenors had similar declines in gamma from 2008 to 2009, at-the-money call and put options have similar gammas. The vega is positive for all options and it increases as the delta of the option increases. The longer the tenor, the higher the vega of the option. The vegas for similar sized delta put and call options are identical. The vega on all the options increases from 2008 to 2009 (Table 14 & 15).

Table 14: Option sensitivities for 6-month options

2008 6-month call options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	0.1077	0.2526	0.5488
Gamma	0.1716	0.3264	0.4579
Vega	0.0089	0.0153	0.0187
Theta Q (1 day)	-0.0007	-0.0011	-0.0015
Rho	0.0034	0.0079	0.0169
2009 6-month put options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	-0.1123	-0.2588	-0.5037
Gamma	0.2344	0.3989	0.4513
Vega	0.0092	0.0155	0.0189
Theta Q (1 day)	-0.0003	-0.0003	-0.0002
Rho	-0.0041	-0.0095	-0.0191
2008 6-month call options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	0.1125	0.2584	0.5012
Gamma	0.0697	0.1440	0.2130
Vega	0.0126	0.0213	0.0262
Theta Q (1 day)	-0.0016	-0.0025	-0.0030
Rho	0.0046	0.0105	0.0200
2009 6-month put options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	-0.1151	-0.2576	-0.4987
Gamma	0.1161	0.1941	0.2129
Vega	0.0128	0.0212	0.0262
Theta Q (1 day)	-0.0006	-0.0008	-0.0008
Rho	-0.0059	-0.0134	-0.0275

The theta is negative for all options, call options with high deltas have more negative thetas. Rho is positive for call options and negative for put options, options with high deltas have higher rhos. The longer the tenor, the higher the rho. The rho on all options increased from 2008 to 2009.

Table 15: Option sensitivities for 12-month options

2008 12-month call options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	0.1048	0.2529	0.5006
Gamma	0.1187	0.2291	0.3163
Vega	0.0123	0.0215	0.0263
Theta Q (1 day)	-0.0005	-0.0009	-0.0012
Rho	0.0065	0.0154	0.0297
2009 12-month put options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	-0.1050	-0.2524	-0.4998
Gamma	0.1596	0.2771	0.3122
Vega	0.0123	0.0214	0.0263
Theta Q (1 day)	-0.0001	-0.0001	0.0002
Rho	-0.0078	-0.0191	-0.0398
2008 12-month call options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	0.1090	0.2562	0.5021
Gamma	0.0480	0.1007	0.1494
Vega	0.0174	0.0299	0.0370
Theta Q (1 day)	-0.0012	-0.0018	-0.0022
Rho	0.0084	0.0197	0.0377
2009 12-month put options			
Option sensitivities	Analytical	Analytical	Analytical
Delta	-0.1095	-0.2535	-0.4979
Gamma	0.0805	0.1368	0.1491
Vega	0.0174	0.0298	0.0370
Theta Q (1 day)	-0.0004	-0.0005	-0.0003
Rho	-0.0116	-0.0277	-0.0592

4.7 Chapter Summary

The 3-month, 6-month and 12-month JIBAR curves have a similar shape, similarly; the 3-month, 6-month and 12-month OIS curves have a similar shape. The highest JIBAR and OIS points were at 2008 Q3 and 2008 Q1 respectively. The average forward points for the 3-month, 6-month and 12-month curves are 0.1670, 0.3240 and 0.6315 respectively. The average Volatility Risk Premium from 2008 Q1 to 2009 Q4 is -4.2390 which indicates that the realized volatility was relatively higher than the implied volatility, there was an increase in implied volatility from 2008 to 2009.

The USDZAR smile resembles a smirk as low delta call options had higher implied volatilities. In 2008, the 3-month 10-delta and 25-delta call options expired out-the-money, the 3-month 50-delta call options expired in-the-money. The 6-month call options had a similar payoff; however, the 12-month 25-delta and 50-delta call options expired -in-the-money. Overall, the 2008 put options expired out-the-money but the options had a contrasting performance in 2009 where the call options expired out-the-money and the put options expired in-the-money.

CHAPTER 5

Conclusion

5.1 Introduction

This is the penultimate chapter in this paper and discusses the OVML outputs relative to the literature review. The chapter also argues why the Garman Kohlhagen valuation model is more appropriate than original Black-Scholes. Section 5.2 concludes by arguing the appropriateness of the Black-Scholes assumptions.

5.2. Discussion

The paper finds that 3-month implied volatility increased significantly from 2008 to 2009 (Figure 5) and this is attributed to the Global Financial Crisis; as a result of the increased implied volatility, the premiums payable for the vanilla options were significantly higher in 2009 compared to the preceding year. This supports the argument made by Corte et al. (2013) that the cost of insurance will be low when the realized volatility of an option is higher than the implied volatility.

The paper finds both the call and put options in 2009 had higher premiums and higher implied volatilities in 2009. This finding was consistent with 10-delta, 25-delta and 50-delta call and put options, Jackwerth & Rubinstein (1996) also found that the implied volatilities of 50-delta options were consistently higher than the realized volatilities.

The paper supports the existence of volatility risk premiums in currency markets as argued by Dumas & Solnik (1995) and that during periods of increased volatility in the currency markets, other market participants are prepared to pay a premium for upside participation rather than downside protection. The bid and offer spread is generally a good indicator of how the market is perceived by market participants, however the paper also supports the argument made by Bakshi & Kapadia (2003) that the negative volatility risk premium is a result of market participant’s preparedness to pay a premium to hedge against an adverse movement in the price of the underlying instrument.

The Global Financial Crisis was largely perceived as a negative shock as the underlying currency and the implied volatility reached unprecedented highs. The paper supports the argument by (Carr & Wu 2009) that the negative risk premium is indicative of market participants perceiving an increase in volatility as a negative shock which they are prepared to pay an increased premium to hedge against.

Our finding suggests during periods of increased volatility such as in 2009, writers of vanilla options will experience varying returns. The 3-month, 6-month and 12-month call options expired out-the-money while the 3-month and 6-month put options had a combination of in-the-money and out-the-money options. The 10-delta, 25-delta and 50-delta 12-month put options expired in-the-money.

Chisholm (2010) and Hull (2015) argued that intrinsic value is the amount of money that could be realized by exercising the option, when the price of a call or put option is greater than its intrinsic value, the option has time value, this argument is consistent with the paper's finding. They also argued that out-the-money options have no value and will expire worthless, this argument is also consistent with the finding of this paper.

The study into the predictive ability of volatility was not extensively investigated in this paper but from the analysis into the behaviour and relationship between implied volatility and realized volatility, the paper finds that implied volatility does not necessarily contain more information regarding future volatility than realized volatility and this argument supports the assertions made Day & Lewis (1992) and Lamoureux & Lastrapes (1993).

The implied volatility curve resembles a smirk or a skew rather than a "U" shape smile, Rubinstein (1985) argued that shorter-dated options were significantly higher in price compared to other options. The findings in this paper argue the contrary as the shorter dated out-the-money call options had the least premium compared to the 6-month and 12-month out-the-money call options, this is true for both 2008 and 2009.

Rubinstein (1985) also argued that at-the-money currency options are less expensive than options that are in-the-money and out-the-money but this paper's finding is contrary as 50-delta (at-the-money) call and put options are more expensive than the out-the-money options (10-delta and 25-delta), this is true for both 2008 and 2009. The paper also finds that put options with lower strikes have lower implied volatilities and that during 2009, the implied volatilities of longer-dated options are on average higher than that of shorter-dated options, this finding is consistent with the argument made by Krylova et (2005).

The change in the term structure from 2008 to 2009 is attributed to the changes in implied volatility as a result of the Global Financial Crisis and other economic fundamentals such as a change in the interest rates and inflation. The terms structure was relatively flat in 2008 and 2009 our finding support the arguments made Mixon (2002) and Krylova et (2005) that the variation in the volatility term structure is a result of economic fundamentals that are observable. Xu & Taylor (1994) argues that the volatility terms structure of currency options possess fragments of forecasting properties but this paper does not find sufficient evidence to support this.

The paper finds that calls options become deeper out-the-money as the delta decreases from 50% to 25% and from 25% to 10%. Similarly, put options also converge to zero delta as they gravitate to being deeper out-the-money as argued by Chisholm (2010). The delta of both the call and the put option does not remain constant throughout the life of the option and is to be managed periodically until expiry, the paper therefore concurs with argument by Hull (2015) that the delta must be adjusted periodically through dynamic hedging.

The paper finds that the gamma increases as the delta increases and this true for both call and put options, the gamma increases as the option converges to the 50% delta threshold and this finding is consistent with the argument made by Chisholm (2010) that the gamma is high when the delta is most volatile, this is normally when the option is at-the-money. Chisholm (2010) also argued that options with shorter term to expiry have higher gammas and this is also consistent with our finding as 3-month option gamma is higher than the gamma of the 6-month and 12-month options.

The theta is negative for both call and put options and the rho is positive and negative for call and put options respectively, this is consistent with Chisholm (2010) and (Ross et al. 2012) arguments. The paper finds that that vega is positive for both call and put options as the value of the option increase when volatility increases, at-the-money (50-delta) options have higher vegas and similarly, options with longer tenors have higher vegas. Fontanills (2005) argued that options with higher implied volatilities have higher premiums, this argument is consistent with the findings in this paper.

5.3 Conclusion and Recommendations

The paper's topic is a volatility focus that investigates the appropriateness of the Black-Scholes Model in hedging against USDZAR volatility using vanilla options. The Black-Scholes Model assumes that the underlying instrument does not pay dividends and that the option can only be exercised at expiry, these two assumptions are true in relation to the topic as currency options do not make any dividend payments and the paper focuses primarily on European vanilla options, thus the two assumptions stated above are applicable and appropriate for this study.

The model assumes that there are no transaction costs, while this may be factual for OTC trades but in reality, exchange traded options incur JSE-related settlement and execution fees. The assumption is thus appropriate for the purpose of this paper given that the focus is on OTC options rather than listed (exchange traded) options.

The paper further agrees with the assumption that the rate of interest is known at inception but refutes that they remains constant throughout the life of the option because interest rates may change as a result of Monetary Policy interventions. The price path of USDZAR during the Global Financial Crisis did not follow a predetermined band or trend and this suggests that the underlying instrument followed a random walk.

The assumption that volatility is constant is problematic because by definition, volatility measures the spread in movements of the underlying instruments. If this assumption were true it would negate the need for continuous rebalancing of the portfolio as there would be no movement in the price of the underlying instrument, the volatility surface would be stale or flat and it would not be possible to plot a smile or a smirk, the volatility risk premium would also be constant throughout the life of the option.

The paper suggest that the Garman Kohlhagen Model is a more appropriate alternative to the Original Black Scholes Model, it is worth noting that this model has similar limitation by assuming that volatility remains constant during the life of the option. It is more appropriate (not perfect) in the sense that it alleviates the Black-Scholes Model assumption that risk-free rates of the domestic currency are the same as the risk-free rate of the foreign currency, this is not true (Garman & Kohlhagen 1983).

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