

https://dspace.org/

Prediction of the bleaching characteristics of hydraulically placed tailings

McPhail, Gordon Ian

2014-04-07
http://hdl.handle.net10539/14519

Downloaded from a DSpace repository. https://dspace.org/

PREDICTION OF THE BEACHING CHARACTERISTICS OF HYDRAULICALLY PLACED TAILINGS

Gordon Ian McPhail

A thesis submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree Doctor of Philosophy



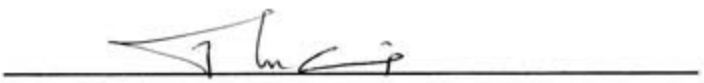
43682 2505011

301069298

BD354113

UNIVERSITY OF THE WITWATERSRAND
The Library

I declare that this thesis is my own, unaided work. It is being submitted for the degree Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University



15 day of July 1994

ABSTRACT

The safety and performance of a tailings dam is dependent on a number of factors the most important of which is the particle size distribution of the deposited tailings within the dam. The particle size distribution determines important geotechnical parameters such as density, permeability and shear strength, parameters that determine the rate of drainage and construction, the slope angle and the maximum height of the tailings dam. The deposited particle size distribution is determined by the mill grind, in the first instance, but thereafter by the method of discharge and the gravitational segregation properties of the slurry as this travels from the discharge point to the pond of supernatant water. Notwithstanding the importance of the particle size distribution little progress has been made with regard to its prediction and correlation with slurry characteristics and method of discharge. This can be ascribed to the difficulty of modelling the unsteady flow regimes of the slurry after discharge and of adequately taking account of the slurry density, viscosity and frictional properties using classical hydrodynamics. The problem is that there are too many variables and too few equations

Observations of the behaviour of the tailings after discharge indicated that the depositional or beaching characteristics of a tailings slurry was highly dependent on the energy with which the flow stream set out down the beach. A scalar parameter of stream power, which is the multiplicand of the slurry density, the flow rate and the square of the velocity over two, was selected as a key parameter since it is the stream power which maintains the tailings particles in suspension and determines the distance they will travel before settling out. More specifically it is the shortfall in stream power that determines the mass of particles that will settle out at any location down the beach. To obtain an estimate of the variation in stream power down the beach and at the same time avoid the problems associated with the classical hydrodynamics approach the variability of the effect of the parameters as opposed to the variability of the individual parameters has been incorporated. This has been achieved by drawing on the entropy analogy from thermodynamics and information theory.

Entropy is a measure of probabilistic uncertainty and it is argued that all natural systems tend to maximum disorder, maximum uncertainty and therefore maximum entropy. The Shannon measure of entropy combined with the notion that this measure will be maximised at any point in a system provides two additional equations for the solution of the flow problem. These equations have been applied to the stream power of the flow stream and estimates of the most probable stream power at selected points along the beach have been determined. The entropy analogy has been extended to the determination of the most probable particle size distribution in the flow stream at any point along the beach from which the most probable particle size distribution of the deposited material has been determined. A spreadsheet model has been developed and applied to beaching data in the literature, flume data and field measurements with reasonable success. The model enables the estimation of the beach profile and particle size distribution at any point along a beach from knowledge of the slurry density, mill grind, viscosity and frictional characteristics of the slurry. The model is able to accommodate a range of discharge methods from point discharges to spray-bar discharges and requires input of the physical arrangement of the discharge.

The model has been applied to two case studies in which the segregational characteristics of the slurry have played a key role in the development and performance of two tailings dams. Changes in discharge technique have been justified on the basis of the model results which in turn have been compared with field measurements. The benefits of the ability to model beaching and depositional characteristics of tailings and optimise tailings dam performance by virtue of controlled deposition practice has been explored in detail

DEDICATION

In loving memory of my Parents who never doubted it was possible.....

To our Lord who met me at every corner and inspired each innovation.
May the glory be His eternally

To Verity, Carole Lee and Bruce. How can I ever make it up to you ?

ACKNOWLEDGEMENTS

I would like to express my gratitude and appreciation to the people and organisations listed below. Without their assistance, both active and passive, the work documented in this thesis would not have been possible.

- my wife, Verity and my two children Carole Lee and Bruce. Their patience was stretched beyond all reasonable limits yet their understanding and longsuffering held out miraculously

- the Directors and staff of my employer, Steffen, Robertson and Kirsten who not only provided essential financial and logistical backing but also moral support and understanding

- the following organisations who gave permission for the use of data gathered on their mining operations:

    - Rössing Uranium Mine Limited
    - Palabora Mining Company Limited
    - Potgietersrust Platinums Limited
    - Rio Paracatu Morro do Ouro
    - Pering Mine Limited

- my friends and colleagues in the Environmental Group, Steffen, Robertson and Kirsten

- my supervisor, Professor G E Blight, who knew just when to provide the ultimate motivator and weathered the exuberance that came with the breakthroughs

PREDICTION OF THE BEACHING CHARACTERISTICS OF HYDRAULICALLY PLACED TAILINGS

Contents

Chapter	Description
1	**Review of current literature referring to the influence of deposition practice on the performance of tailings dams**
2	**Review of literature relating to the prediction of particle size distribution within and along a flow stream**
3	**Development of equations for the prediction of the beaching characteristics of tailings**
4	**Development of a beaching simulation model based on entropy**
5	**Sensitivity analysis**
6	**Application of entropy to flume testing**
7	**Application of entropy to field measurements**
8	**Optimisation of tailings dam performance through controlled deposition practice**
9	**Conclusions**

LIST OF FIGURES

FIGURE 1.1 : TERNARY DIAGRAM FOR SYNCRUDE (After Scott and Cymerman (1984)) 1-6

FIGURE 1.2 : SEQUENCE OF BEDFORMS FOR INCREASING FLOW STRENGTH (After Küpper (1991)) 1-8

FIGURE 1.3 : GRAIN-SIZE SEGREGATION ALONG TAILINGS BEACHES (After Vick (1990) 1-10

FIGURE 1.4 : FRACTIONATION OF TAILINGS (After Bentel (1981) and Blight and Bentel (1983)) 1-12

FIGURE 1.5 : DIMENSIONLESS PROFILES AND n VALUES FOR DIFFERENT TYPES OF TAILINGS (After Bentel (1981)) 1-14

FIGURE 1.6 : SUMMARY OF INFLUENCES ON THE BEACH PROFILE (After Bentel (1981)) 1-16

FIGURE 1.7 : INFLUENCE OF SOLIDS CONCENTRATION ON DIMENSIONLESS BEACH PROFILES AS ESTABLISHED FROM MODEL TESTS COMPARED WITH FIELD TESTS (After Thomson (1984)) 1-17

FIGURE 1.8 : UPSTREAM, DOWNSTREAM AND CENTRELINE METHODS OF TAILINGS DAM FORMATION (After ICOLD (1982)) 1-21

FIGURE 1.9 : SIDE-HILL RING DYKE AND VALLEY IMPOUNDMENTS (After ICOLD (1982)) 1-21

FIGURE 1.10 : RAINFALL EXCESS AND RAINFALL DEFICIENCY AREAS IN SOUTH AFRICA (After Blight et al (1987)) 1-25

FIGURE 1.11 : EFFECT OF SUNDRYING ON VOID RATIO AND CONSOLIDATION CHARACTERISTICS OF SLIMES (After Blight and Steffen (1979)) 1-26

FIGURE 1.12 : SECTIONS THROUGH A TYPCIAL SOUTH AFRICAN GOLD TAILINGS DAM ILLUSTRATING THE METHOD AND SEQUENCE OF DAYWALL, NIGHTWALL AND FLOW DEVELOPMENT UP TO THE 1970's (After National Building Research Institute (1959)) 1-28

FIGURE 1.13 : PLAN ILLUSTRATING THE METHOD OF SOUTH AFRICAN GOLD TAILINGS DAM DEVELOPMENT FROM A SINGLE POINT DISCHARGE UP TO 1970's (After Mrost (1974)) 1-29

FIGURE 1.14 : PARTICLE SIZE DISTRIBUTION FOR SOUTH AFRICAN GOLD TAILINGS DURING THE 1950's (After National Building Research Institute (1959)) 1-30

FIGURE 1.15 : PLAN ON A TYPICAL CURRENT GOLD TAILINGS DAM 1-31

FIGURE 1.16 : TYPICAL SECTION THROUGH A CURRENT GOLD TAILINGS DAM 1-32

FIGURE 1.17 : TYPICAL TAILINGS PARTICLE SIZE DISTRIBUTIONS (After Jennings (1978)) 1-33
FIGURE 1.18 : SECTIONS ILLUSTRATING THE SEQUENCE OF OPERATION OF A TYPCIAL SOUTH AFRICAN TAILINGS DAM CONSTRUCTED USING SPIGOTS ON A SPRAY-BAR 1-34
FIGURE 1.19 : EFFECT OF VARYING PERMEABILITY ON POSITION OF THE PHREATIC SURFACE IN A TAILINGS DAM (After Blight (1987)) 1-36
FIGURE 1.20 : TYPICAL SECTION THROUGH A TAILINGS DAM ILLUSTRATING THE FOUR VARIABLES AVAILABLE TO A DESIGNER TO OPTIMISE GEOTECHNICAL PERMFORMANCE 1-37
FIGURE 2.1 : DISTRIBUTION OF SUSPENDED LOAD IN A FLOW ACCORDING TO THE EQUATION OF ROUSE - Equation 12. (After Raudkivi (1991))2-10
FIGURE 2.2 : SEDIMENT DISTRIBUTION CURVES FOR A MIXTURE OF TWO PARTICLE SIZES, WITH VARYING RATIO OF FALL SPEEDS (After Hunt (1969)) 2-14
FIGURE 2.3 : SAND PARTICLE SIZE DISTRIBUTION (After Cooke and Lazarus (1988)) 2-15
FIGURE 2.4 : MEASURED VELOCITY AND CONCENTRATION PROFILES (After Cooke and Lazarus (1988)) 2-15
FIGURE 2.5 : PARTICLE SIZE DISTRIBUTION AT DIFFERENT VERTICAL POINTS WITHIN PIPE CROSS SECTION (After Cooke and Lazarus (1988)) 2-16
FIGURE 2.6 : VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS (After Cooke and Lazarus (1988)) 2-16
FIGURE 2.7 : VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS - LARGER SCALE (After Cooke and Lazarus (1988)) 2-17
FIGURE 2.8 : THE BAGNOLD-VLUGTER MODEL OF SUSPENSION (After Raudkivi (1990)) 2-20
FIGURE 2.9 : CLASSIFICATION OF SINGLE AND MULTIPHASE FLUIDS . . . 2-40
FIGURE 2.10 : TYPICAL RHEOGRAMS FOR TIME-INDEPENDENT FLUIDS . . 2-42
FIGURE 2.11 : TYPICAL RHEOGRAMS FOR TIME-DEPENDENT FLUIDS 2-42
FIGURE 2.12 : EQUATIONS FOR NON-NEWTONIAN FLOW (After Verkerk (1990))44
FIGURE 2.13: DESCRIPTIVE SKETCHES OF BEACH DEVELOPMENT (After Bentel (1981)) 2-49
FIGURE 2.14 : PROGRESSIVE BEACH BUILD-UP FROM POINT DELIVERY . 2-49
FIGURE 2.15 : HYDRAULIC JUMP - SKETCH ILLUSTRATING FURROWS AND RIDGES (After Bentel (1981)) 2-50
FIGURE 2.16 : ACTUAL PROFILE SHOWING LINEAR SECTIONS OF BEACH

BETWEEN JUMPS (After Bentel (1981)) 2-51

FIGURE 2.17 : ILLUSTRATION OF MEASURED BEACH PROFILE AND TYPICAL BEDFORMS (After Bentel (1981)) 2-52

FIGURE 3.1 : VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS (After Cooke and Lazarus (1988)) WITH CONCENTRATIONS PREDICTED USING HUNT'S EQUATIONS (HUNT (1967)) 3-3

FIGURE 3.2 : VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS - LARGER SCALE (After Cooke and Lazarus (1988)) WITH CONCENTRATIONS PREDICTED USING HUNT'S EQUATIONS (HUNT (1967)) 3-3

FIGURE 3.3 : PARTICLE SIZE DISTRIBUTIONS IN THE FLOW STREAM AND ON THE BEACH 3-19

FIGURE 3.4 : INFLUENCE OF INCREASING CONCENTRATION AND VELOCITY ON THE FRICTIONAL RESISTANCE OF A PICAL TAILINGS SLURRY 3-26

FIGURE 3.5 : PLAN AND SECTIONS THROUGH A TYPICAL PLUNGE POOL AT A TAILINGS DISCHARGE 3-29

FIGURE 3.6 : TYPICAL SECTION THROUGH PLUNGE POOL AND DISCHARGE CHANNEL 3-32

FIGURE 3.7 : TYPICAL SPECIFIC ENERGY DIAGRAM FOR THE PLUNGE POOL AND DISCHARGE CHANNEL 3-33

FIGURE 3.8 : TYPICAL SPECIFIC ENERGY DIAGRAM FOR PLUNGE POOL AND DISCHARGE CHANNEL 3-35

FIGURE 3.9 : PHOTOGRAPHS ILLUSTRATING THE UNSTEADY NATURE OF FLOW FROM A TAILINGS SLURRY DISCHARGE 3-37

FIGURE 3.10 : PHOTOGRAPHS ILLUSTRATING THE UNSTEADY NATURE OF FLOW FROM A TAILINGS SLURRY DISCHARGE 3-38

FIGURE 4.1 : KÜPPER:TEST 1 SECTION 3 - MEASURED BEACH PROFILE . 4-10

FIGURE 4.2 : KÜPPER:TEST 4 SECTION 1 - MEASURED BEACH PROFILE . 4-10

FIGURE 4.3 : KÜPPER:TEST 1 SECTION 3 - PREDICTED BEACH PROFILE . 4-11

FIGURE 4.4 : KÜPPER:TEST 4 SECTION 1 - PREDICTED BEACH PROFILE . 4-11

FIGURE 4.5 : KÜPPER:TEST 1 SECTION 3 - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-12

FIGURE 4.6 : KÜPPER:TEST 4 SECTION 1 - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-12

FIGURE 4.7 : KÜPPER:TEST 1 SECTION 3 - PREDICTED PARTICLE SIZE DISTRIBUTIONS 4-13

FIGURE 4.8 : KÜPPER:TEST 4 SECTION 1 - PREDICTED PARTICLE SIZE DISTRIBUTIONS 4-13
FIGURE 4.9 : THOMSON GOLD DATA - MEASURED BEACH PROFILE 4-15
FIGURE 4.10 : THOMSON GOLD DATA - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-15
FIGURE 4.11 : THOMSON GOLD DATA - PREDICTED BEACH PROFILE 4-16
FIGURE 4.12 : THOMSON GOLD DATA - PREDICTED PARTICLE SIZE DISTRIBUTIONS 4-17
FIGURE 4.13 : BENTEL PLATINUM DATA - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-18
FIGURE 4.14 : BENTEL PLATINUM DATA - MEASURED BEACH PROFILE .. 4-18
FIGURE 4.15 : BENTEL PLATINUM DATA - PREDICTED BEACH PROFILE .. 4-19
FIGURE 4.16 : BENTEL PLATINUM DATA - PREDICTED PARTICLE SIZE DISTRIBUTION 4-20
FIGURE 4.1 : KÜPPER:TEST 1 SECTION 3 - MEASURED BEACH PROFILE . 4-9
FIGURE 4.2 : KÜPPER:TEST 4 SECTION 1 - MEASURED BEACH PROFILE . 4-10
FIGURE 4.3 : KÜPPER:TEST 1 SECTION 3 - PREDICTED BEACH PROFILE . 4-10
FIGURE 4.4 : KÜPPER:TEST 4 SECTION 1 - PREDICTED BEACH PROFILE . 4-11
FIGURE 4.5 : KÜPPER:TEST 1 SECTION 3 - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-11
FIGURE 4.6 : KÜPPER:TEST 4 SECTION 1 - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-12
FIGURE 4.7 : KÜPPER:TEST 1 SECTION 3 - PREDICTED PARTICLE SIZE DISTRIBUTIONS 4-12
FIGURE 4.8 : KÜPPER:TEST 4 SECTION 1 - PREDICTED PARTICLE SIZE DISTRIBUTIONS 4-13
FIGURE 4.9 : THOMSON GOLD DATA - MEASURED BEACH PROFILE 4-14
FIGURE 4.10 : THOMSON GOLD DATA - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-15
FIGURE 4.11 : THOMSON GOLD DATA - PREDICTED BEACH PROFILE 4-16
FIGURE 4.12 : THOMSON GOLD DATA - PREDICTED PARTICLE SIZE DISTRIBUTIONS 4-16
FIGURE 4.13 : BENTEL PLATINUM DATA - MEASURED PARTICLE SIZE DISTRIBUTIONS 4-17
FIGURE 4.14 : BENTEL PLATINUM DATA - MEASURED BEACH PROFILE .. 4-18
FIGURE 4.15 : BENTEL PLATINUM DATA - PREDICTED BEACH PROFILE .. 4-19
FIGURE 4.16 : BENTEL PLATINUM DATA - PREDICTED PARTICLE SIZE DIS-

TRIBUTION 4-19
FIGURE 5.1 : COARSE , MEDIUM AND FINE PRODUCTS USED IN SENSITIVITY ANALYSIS 5-5
FIGURE 5.2 : PREDICTED BEACH PROFILES - COARSE, MEDIUM AND FINE PRODUCTS 5-6
FIGURE 5.3 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - COARSE PRODUCT 5-6
FIGURE 5.4 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - MEDIUM PRODUCT 5-7
FIGURE 5.5 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - FINE PRODUCT 5-7
FIGURE 5.6 : PREDICTED BEACH PROFILES FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY 5-9
FIGURE 5.7 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY - AT DISCHARGE 5-9
FIGURE 5.8 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY - AT 125 m 5-10
FIGURE 5.9 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY - AT 225 m 5-10
FIGURE 5.10 : PREDICTED BEACH PROFILES FOR VARIATIONS IN FLOW RATE 5-12
FIGURE 5.11 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATIONS IN FLOW RATE - AT DISCHARGE 5-12
FIGURE 5.12 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 12,5 m 5-13
FIGURE 5.13 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 25 m 5-13
FIGURE 5.14 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 125 m 5-14
FIGURE 5.15 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 225 m 5-14
FIGURE 5.16 : PREDICTED BEACH PROFILE FOR VARIATIONS IN WATER:SOLIDS RATIO 5-16
FIGURE 5.17 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN WATER:SOLIDS RATIO - AT DISCHARGE 5-16
FIGURE 5.18 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN WATER:SOLIDS RATIO - AT 125 m 5-17
FIGURE 5.19 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN

WATER:SOLIDS RATIO - AT 225 m 5-17
FIGURE 5.20 : PREDICTED BEACH PROFILES FOR VARIATION IN BEACH LENGTH 5-19
FIGURE 5.21 : PREDICTED NORMALISED BEACH PROFILES FOR VARIATION IN BEACH LENGTH 5-19
FIGURE 5.22 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN BEACH LENGTH - AT DISCHARGE 5-20
FIGURE 5.23 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATION IN BEACH LENGTH - AT 125 m 5-20
FIGURE 5.24 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN BEACH LENGTH - AT 225 m 5-21
FIGURE 5.25 : NORMALISED BEACH PROFILE PLOTS AFTER BENTEL (1981) AND BLIGHT (1987) 5-22
FIGURE 5.26 : PREDICTED BEACH PROFILES FOR VARIATION IN VELOCITY EX-PLUNGE POOL 5-24
FIGURE 5.27 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN VELOCITY EX-PLUNGE POOL - AT DISCHARGE 5-24
FIGURE 5.28 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATION IN VELOCITY EX-PLUNGE POOL - AT 125 m 5-25
FIGURE 5.29 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN VELOCITY EX-PLUNGE POOL - AT 225 m 5-25
FIGURE 5.30 : PREDICTED BEACH PROFILES FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope) 5-27
FIGURE 5.31 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope) - AT DISCHARGE 5-27
FIGURE 5.32 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope) - AT 125 m 5-28
FIGURE 5.33 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope) - AT 225 m 5-28
FIGURE 5.34 : PREDICTED BEACH PROFILES FOR VARIATIONS THICKENER OPERATION AND ORE TYPE 5-31
FIGURE 5.35 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATION OF THICKENER OPERATION AND ORE TYPE - AT DISCHARGE 5-32
FIGURE 5.36 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN THICKENER OPERATION AND ORE TYPE - AT 125 m 5-32
FIGURE 5.37 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN THICKENER OPERATION AND ORE TYPE - AT 225 m 5-33

FIGURE 6.1 : PROFILES OF TS TESTS WITH Q_t = 5 l/min AND VARIOUS Cw (After Kùpper (1991) 6-7
FIGURE 6.2 : PROFILES OF TS TESTS WITH Q_t = 10 l/min AND VARIOUS Cw (After Kùpper (1991) 6-8
FIGURE 6.3 : PROFILES OF TS TESTS WITH Q_t = 15 l/min AND VARIOUS Cw (After Kùpper (1991) 6-8
FIGURE 6.4 : PROFILES OF TS TESTS WITH Q_t = 20 l/min AND VARIOUS Cw (After Kùpper (1991) 6-9
FIGURE 6.5 : MEASURED AND CALIBRATED BEACH PROFILES FOR FLUME TEST TS 19 6-10
FIGURE 6.6 : MEASURED AND PREDICTED BEACH PROFILES FOR TEST 4 , SECTION 1 BASED ON FLUME DATA 6-13
FIGURE 6.7 : PHOTOGRAPHS OF THE PILOT-SCALE FLUME 6-15
FIGURE 6.8 : PARTICLE SIZE DISTRIBUTION OF BRAZILIAN GOLD TAILINGS PRODUCT 6-15
FIGURE 6.9 : MEASURED BEACH PROFILE ON THE TAILINGS DAM 6-16
FIGURE 6.10 : MEASURED BEACH PROFILE FOR FLOW RATE OF 2,6 l/s ... 6-16
FIGURE 6.11 : MEASURED BEACH PROFILE FOR FLOW OF 0,44 l/s 6-17
FIGURE 6.12 : CALIBRATED BEACH PROFILE FOR FLUME TEST AT FLOW OF 2,6 l/s 6-19
FIGURE 6.13 : CALIBRATED BEACH PROFILE FOR FLUME TEST AT FLOW OF 0,44 l/s 6-19
FIGURE 6.14 : PREDICTED BEACH PROFILE USING DATA FROM THE FLUME TEST FOR A FLOW RATE OF 2,6 l/s 6-20
FIGURE 6.15 : PREDICTED BEACH PROFILE USING DATA FROM THE FLUME TEST FOR A FLOW RATE OF 0,44 l/s 6-21
FIGURE 6.16 : CONCENTRATION VS INITIAL SLOPE FOR THE RANGE OF TS TESTS DOCUMENTED BY KÜPPER (1991) 6-24
FIGURE 7.1 : PARTICLE SIZE DISTRIBUTION OF SANDS, SLIMES AND TOTAL PRODUCT AT RÖSSING URANIUM 7-2
FIGURE 7.2 : AERIAL VIEW OF THE RÖSSING TAILINGS DAM 7-3
FIGURE 7.3 : HEAD LOSS DETERMINATIONS FROM RHEOLOGY (after Rössing Uranium Limited (1993)) 7-5
FIGURE 7.4 : PHOTOGRAPHS SHOWING THE DISCHARGE OF TAILINGS DURING THE TEST ON PADDOCK ST 7-7
FIGURE 7.5 : MEASURED AND PREDICTED BEACH PROFILES 7-8
FIGURE 7.6: MEASURED PARTICLE SIZE DISTRIBUTIONS 7-8

FIGURE 7.7 : PREDICTED PARTICLE SIZE DISTRIBUTIONS 7-9
FIGURE 7.8 : VARIATION OF PLUNGE POOL VELOCITY WITH HEIGHT OF DISCHARGE PIPE 7-11
FIGURE 7.9 : PREDICTED BEACH PROFILES FOR RÖSSING OLD BEACH . 7-12
FIGURE 7.10 : MEASURED PARTICLE SIZE DISTRIBUTIONS FOR RÖSSING OLD BEACH 7-13
FIGURE 7.11 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR RÖSSING OLD BEACH 7-13
FIGURE 7.12 : PLUNGE POOL VELOCITY VERSUS PIPE HEIGHT ABOVE PLUNGE POOL FOR RÖSSING OLD BEACH 7-14
FIGURE 7.13 : RHEOGRAMS FOR THE BRAZILIAN GOLD TAILINGS 7-15
FIGURE 7.14 : FANNING FRICTION FACTORS 7-16
FIGURE 7.15 : CALIBRATED AND MEASURED BEACH PROFILES 7-17
FIGURE 7.16 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR THE BRAZILIAN TAILINGS 7-19
FIGURE 7.17 : COPPER TAILINGS RHEOGRAMS 7-20
FIGURE 7.18 : BINGHAM VISCOSITY vs MOISTURE CONTENT 7-21
FIGURE 7.19 : YIELD STRESS vs MOISTURE CONTENT 7-22
FIGURE 7.20 : FANNING FRICTION PARAMETERS 7-23
FIGURE 7.21 : MEASURED AND CALIBRATED PROFILES 7-23
FIGURE 7.22 : MEASURED, CALIBRATED AND PREDICTED PROFILES 7-24
FIGURE 7.23 : PREDICTED PARTICLE SIZE DISTRIBUTIONS 7-25
FIGURE 7.24 : MEASURED PARTICLE SIZE DISTRIBUTIONS 7-25
FIGURE 7.25 : COMPARISON OF d_{90} SIZE WITH DISTANCE 7-26
FIGURE 7.26 : COMPARISON OF d_{50} SIZE WITH DISTANCE 7-26
FIGURE 7.27 : COMPARISON OF d_{20} SIZE WITH DISTANCE 7-26
FIGURE 7.28 : PHOTOGRAPH OF A TYPICAL DISCHARGE ON THE MAGNETITE DAM 7-28
FIGURE 7.29 : VARIATION OF NEWTONIAN VISCOSITY WITH VOLUMETRIC MOISTURE CONTENT 7-29
FIGURE 7.30 : CALIBRATED, MEASURED AND PREDICTED BEACH PROFILES 7-30
FIGURE 7.31 : FANNING FRICTION FACTORS 7-31
FIGURE 7.32 : PREDICTED PARTICLE SIZE DISTRIBUTIONS DOWN THE BEACH BASED ON CALIBRATED PARAMETERS 7-32
FIGURE 7.33 : PHOTOGRAPHS SHOWING TYPICAL DISCHARGE STREAMS AND THEIR EFFECT ON THE PLUNGE POOL VELOCITY 7-34
FIGURE 8.1 : INFLUENCE OF UNDERDRAINAGE 8-5

FIGURE 8.2 : INFLUENCE OF THE POND ON THE PHREATIC SURFACE IN THE EMBANKMENT AREA 8-6
FIGURE 8.3 : CONSTRUCTION OF ELEVATED DRAINS WITH CHIMNEYS . 8-10
FIGURE 8.4 : CONSTRUCTION OF ELEVATED DRAINS WITHOUT CHIMNEYS 8-12
FIGURE 8.5 : DRAIN CONSTRUCTION USING THE INNER-OUTER WALL METHOD 8-14
FIGURE 8.6 : DRAIN CONSTRUCTION BY MEANS OF THE BUTTRESS DAM CONCEPT 8-15
FIGURE 8.7 : PLACEMENT OF FINES OVER THE FLOOR OF THE TAILINGS DAM 8-17
FIGURE 8.8 : ILLUSTRATION OF POOL WALL OPERATION 8-18
FIGURE 8.9 : FILLING THE BASIN BY STEPPING IN THE DELIVERY LINES 8-20
FIGURE 8.10 : LAYOUT OF PADDOCKS ON A LARGE TAILINGS DAM 8-21
FIGURE 8.11 PARTICLE SIZE DISTRIBUTIONS OF THE COARSE AND FINE PRODUCTS 8-24
FIGURE 8.12 : CONCENTRATION vs VISCOSITY 8-25
FIGURE 8.13 : 50 m BEACH, COARSE PRODUCT, DENSITY 1,45 t/m^3 8-26
FIGURE 8.14 : MEASURED BEACH 8-26
FIGURE 8.15 : PHOTOGRAPHS SHOWING TYPICAL DISCHARGE CONDITIONS DURING DEPOSITION TRIALS 8-27
FIGURE 8.16 : 200 m BEACH, COARSE PRODUCT, DENSITY 1,45 t/m^3 8-29
FIGURE 8.17 : 700 m BEACH, COARSE PRODUCT, DENSITY 1,45 t/m^3 8-29
FIGURE 8.18 : 700 m BEACH, COARSE PRODUCT, DENSITY 1,75 t/m^3 8-30
FIGURE 8.19 : 700 m BEACH, FINE PRODUCT, DENSITY 1,45 t/m^3 8-30
FIGURE 8.20 : 700 m BEACH, FINE PRODUCT, DENSITY 1,75 t/m^3 8-31
FIGURE 8.21 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,2 m, DENSITY 1,45 t/m^3 8-31
FIGURE 8.22 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,01 m, DENSITY 1,45 t/m^3 8-32
FIGURE 8.23 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,2 m, DENSITY 1,45 t/m^3 8-32
FIGURE 8.24 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,01 m, DENSITY 1,45 t/m^3 8-33
FIGURE 8.25 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,2 m 1,75 t/m^3 8-33
FIGURE 8.26 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,01 m, DENSITY 1,75 t/m^3 8-34

FIGURE 8.27 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,2 m, DENSITY 1,75 t/m³ 8-34
FIGURE 8.28 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,01 m, 1,75 t/m³ 8-35
FIGURE 8.29 : TYPICAL SECTION THROUGH THE DAYWALL 8-38
FIGURE 8.30 : TYPICAL PLAN ON THE DAYWALL 8-38
FIGURE 8.31 : SPECTACLE PLATE ARRANGEMENT 8-39
FIGURE 8.32 : PHOTOGRAPH ILLUSTRATING TYPICAL SPECTACLE PLATE FAN SPRAY OPERATION 8-40
FIGURE 8.33 : SECTION THROUGH HYBRID DEPOSITION SYSTEM ILLUSTRATING COVERING OF THE MAIN DRAIN 8-41
FIGURE 8.34 : PLAN ON HYBRID DEPOSITION SYSTEM ILLUSTRATING COVERING OF THE MAIN DRAIN 8-41
FIGURE 8.35 : TYPICAL SECTION THROUGH TOE EMBANKMENT 8-43
FIGURE 8.36 : SPIGOT PIPE SUPPORT AND OUTER PADDOCK WALL GEOMETRY 8-45
FIGURE 8.37 : ALTERNATIVE METHODS OF DRAIN FORMATION 8-46
FIGURE 8.38 : DESIGN PARTICLE SIZE DISTRIBUTION OF PERING TAILINGS RELATIVE TO THE BLIGHT AND STEFFEN GOLD TAILINGS ENVELOPE (Blight and Steffen (1979)) 8-49
FIGURE 8.39 : PARTICLE SIZE DISTRIBUTIONS OVER THE PERIOD SINCE COMMISSIONING 8-50
FIGURE 8.40 : PARTICLE SIZE DISTRIBUTIONS UTILISED IN BEACHING ANALYSES 8-51
FIGURE 8.41 : CONCENTRATION vs VISCOSITY 8-52
FIGURE 8.42: PLAN OF TAILINGS DAM SHOWING UNDERDRAINAGE STRUCTURES AND DELIVERY LOCATIONS 8-53
FIGURE 8.43 : PHOTOGRAPH OF THE SPRAY-BAR DISCHARGE PIPE AT PERING MINE 8-53
FIGURE 8.44 : FANNING FRICTION FACTORS FROM VERKERK RUNS 8-54
FIGURE 8.45 : PREDICTED BEACH PROFILES FOR THE COARSE AND FINE PRODUCTS 8-55
FIGURE 8.46: PREDICTED PARTICLE SIZE DISTRIBUTIONS - COARSER PRODUCT : POINT DISCHARGE 8-56
FIGURE 8.47: MEASURED PARTICLE SIZE DISTRIBUTIONS IN 1987 : COARSE GRIND 8-56
FIGURE 8.48: PREDICTED PARTICLE SIZE DISTRIBUTIONS - FINE PRODUCT :

POINT DISCHARGE 8-57
FIGURE 8.49: MEASURED PARTICLE SIZE DISTRIBUTIONS IN 1987 - FINE PRODUCT : POINT DISCHARGE 8-57
FIGURE 8.50 : FANNING FRICTION FACTORS FROM VERKERK RUNS 8-59
FIGURE 8.51 : CALIBRATED BEACH PROFILE - COARSER PRODUCT 8-59
FIGURE 8.52 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - COARSER PRODUCT WITH INDIVIDUAL DISCHARGES COMBINING DOWN THE BEACH 8-60
FIGURE 8.53 : MEASURED PARTICLE SIZE DISTRIBUTIONS 8-60
FIGURE 8.54 : SINGLE OPENING BEACH PROFILE - COARSER PRODUCT : SPRAY-BAR DISCHARGE 8-61
FIGURE 8.55: PREDICTED PARTICLE SIZE DISTRIBUTION FROM A SINGLE OPENING 8-62
FIGURE 8.56 : PREDICTED BEACH PROFILE - FINER PRODUCT COMBINED SPIGOT FLOWS 8-63
FIGURE 8.57 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - FINER PRODUCT COMBINED SPIGOT FLOWS 8-63

LIST OF TABLES

TABLE 2.1 FALL VELOCITIES OF QUARTZ SAND OF 0.15 < d < 1.5 mm 3-2

TABLE 4.1 : PRINTOUT OF SPREADSHEET MODEL 4-3

TABLE 5.1 : MATRIX OF VARIABLES ASSUMED IN SENSITIVITY ANALYSIS .. 5-3

TABLE 5.2: SUMMARY OF PRIMARY AND SECONDARY VARIABLES ASSUMED IN CORRELATED SENSITIVITY ANALYSIS 5-30

TABLE 6.1 : SIMILITUDE SCALING RATIOS 6-5

TABLE 6.2 : SCALING FACTORS FOR FROUDE SIMILITUDE 6-10

TABLE 6.3 : SCALING FACTORS FOR REYNOLDS SIMILITUDE 6-11

TABLE 6.4 : SCALING FACTORS FOR 2,6 l/s FLOW RATE 6-17

TABLE 6.5 : SCALING FACTORS FOR 0,44 l/s FLOW RATE 6-18

TABLE 6.6 : IDEALISED SCALING FACTORS FOR KÜPPER'S TEST TS 19 6-22

TABLE 6.7 : IDEALISED SCALING FACTORS FOR THE BRAZILIAN FLUME TEST 6-23

LIST OF SYMBOLS

A	cross-sectional area
c	concentration (by weight)
c_a	reference concentration at height a above the bed level
c_r	concentration of size fraction r
c_w	concentration (by weight)
$\bar{c}$	mean concentration
c_v	concentration (by volume)
d	particle diameter
d_{eff}	effective grain diameter
D_{50}, d_{50}	particle size for which 50 % of a sample is finer
D	diffusivity
D_{ij}	turbulent diffusion
D_M	molecular diffusion
$D(p:q)$	cross-entropy between two distributions P and Q where p and q represent probabilities within P and Q respectively
e	energy of a system
F_r	Froude number
f	Fanning friction factor (equivalent to one quarter of the Darcy-Weisbach friction factor)
f_0	Darcy-Weisbach friction factor
G	specific gravity
g	gravitational acceleration
h	height drop at any distance along a beach
H	total height drop over a beach
$H(X)$	entropy (Chiu)
i_{av}	average slope of beach
k	constant
k	roughness coefficent
L	beach length
m	mass per unit area of suspended sediment
n	Manning's constant
P	stream power
P	rate of transport across a boundary
P'	ratio between gravitational and inertial forces on a grain of tailings
P_N	net power demand
p	probability
Q	total flow rate
q	specific flow rate (per unit width)
q	probability within a distribution Q
R	hydraulic radius
S	slope
S_i	initial slope
S_e	energy gradient
S_{eq}	equilibrium slope
$S(p)$	Shannon's measure of entropy
S_s	particle specific gravity
S_n	Schmidt number
t	time
U	velocity
U_s	velocity of solids in a slurry
U_s	relative velocity of a suspension
u	horizontal component of velocity

$\bar{u}$	horizontal component of mean velocity
u_*	shear velocity $(gy_0S)^{1\backslash 2}$
w	fall velocity of grain
w_0	fall velocity of grain at low particle concentration
x	horizontal coordinate
y	vertical coordinate
y_0	maximum flow depth
y_c	critical flow depth
ϵ	diffusion coeficient of fluid momentum, eddy viscosity
ϵ_w	diffusion coefficient for water
ϵ_s	diffusion coefficient for sediment
α	constant
β	constant
Δ	$(\rho_s - \rho)/\rho$
ν	kinematic viscosity
ρ	density of a suspended sediment (slurry)
ρ_s	solids density in suspended sediment
ρ_w	density of water
λ	constant
λ	Darcy-Weisbach friction factor
μ	constant
μ	dynamic viscosity
κ	von Karman's constant
Φ	non-dimensional sand transport factor
ψ	non-dimensional shear stress
τ	shear stress

CHAPTER 1

CONTENTS

Sec	Description	Page
1	REVIEW OF CURRENT LITERATURE REFERRING TO THE INFLUENCE OF DEPOSITION PRACTICE ON THE PERFORMANCE OF TAILINGS DAMS	1-1
1.1	Introduction	1-1
1.2	Tailings Production, Transport and Disposal	1-3
1.3	Tailings Slurry Depositional Characteristics	1-5
1.3.1	Separation of Water and Solids	1-5
1.3.2	Hydraulic Sorting/Gravitational Segregation	1-9
1.3.3	Layering	1-11
1.3.4	Beaching and Beach Profiles	1-13
1.3.5	Conclusions : Tailings Depositional Characteristics	1-18
1.4	General Description of Tailings Dam Construction	1-19
1.4.1	General	1-20
1.4.2	Water retention type dams	1-22
1.4.3	Raised embankments	1-22
1.4.4	Other tailings dam formation methods	1-24
1.5	Typical South African Tailings Dam Construction	1-24
1.6	Geotechnical Performance of Tailings Dams	1-35
1.7	Justification of Further Research	1-37
1.8	Approach	1-37

CHAPTER 1 REVIEW OF CURRENT LITERATURE REFERRING TO THE INFLUENCE OF DEPOSITION PRACTICE ON THE PERFORMANCE OF TAILINGS DAMS

1.1 Introduction

The failure of Aberfan Coal Waste Dump in Wales in the sixties, the Buffalo Creek Coal Waste dam in West Virginia and the Bafokeng Platinum tailings dam in South Africa in the seventies focused the world's attention on the potential hazard created by some of these structures. In the case of the first two, the failures were responsible for the deaths of more than 100 people who were not miners. In the case of the Bafokeng failure 3 million cubic metres of tailings liquified and flowed out of the tailings impoundment down a water course for some 40 km. Some of the tailings flowed down a nearby shaft killing 13 miners underground.

Smaller scale failures killing only a few miners, or doing only environmental damage had previously been relatively common in world mining, but had usually been dismissed simply as an occupational hazard. The Bafokeng failure showed the extent to which the loss of lives due to tailings dams would be outside of the bounds of occupational hazard - those underground were not even aware of their exposure due to tailings dam failures. With the recognition of the potentially great hazard that could be created by these structures came the need for better design and construction to increase their safety. Recognition of water retention structures as "dams" regardless of the material or methods of construction led to an involvement of the International Committee on Large Dams (ICOLD) and the production of the ICOLD Manual on Tailings Dams and Dumps, (ICOLD (1982)) which forms a land-mark document on tailings dam design and operation.

Notwithstanding the production of the Manual, the failure of the tailings dams at Stava in 1985, during which 268 non-mining people tragically lost their lives, served to once again indicate that while the technology for controlling tailings dam performance and safety had developed considerably since the Buffalo Creek and Bafokeng failures, it is far from complete and comprehensive.

In each of the three tailings dam catastrophes mentioned above, the cause of failure can be ascribed to a lack of understanding by mine personnel of the geotechnical characteristics and behaviour of the tailings at various locations within each impoundment. More specifically there was inadequate understanding of the impact of the depositional practices on the performance of

the tailings material under conditions of high rates of rise and severe hydrology.

ICOLD (1982) notes that tailings dams or dams for the retention of tailings slurries have many features in common with water diversion or retention dams. Therefore, it is natural to start with water dam technology for their design. However, their operational needs are different since construction usually proceeds simultaneously with their operation, construction techniques differ and minerals characteristics usually play a more significant role in the design considerations. Where possible tailings dams do not cross streams, nor do they usually impound water for purposes other than sedimentation. Their ultimate purpose is for solid waste disposal, not water retention, which is only incidental to their operation. Therefore the automatic application of water dam technology is often not recommendable and may prove unnecessarily expensive.

Tailings are one of the most abundant dam building materials in the world. They exist in some quantities in most countries and in some countries they exist in very large quantities. Mine waste are, in fact the most commonly handled materials in the world. The volume of materials handled throughout the world each year exceeds that of any other industry in the work including non mine construction.

Morgenstern (1985) in his Theme Lecture paper presented at XI International Conference on Soil Mechanics and Foundation Engineering states that *although the economics of disposal and the geotechnical integrity of the tailings structures are both intimately related to the behaviour of the tailings during placement, tailings deposition has attracted little attention from a geotechnical perspective*. Vick (1990)[1] summarises most available information on above water beach characteristics from peripheral discharge systems. For most types of tailings, the beach slopes downwards to the pond on an average grade of 0.5-2.0% within the first hundred metres or so. The depositional process produces a highly heterogeneous deposit both vertically and laterally. Mittal and Morgenstern (1977) and Jennings (1979) drew attention to the danger of depositing thin slime layers within the sand if beach and depositional control is poor.

Morgenstern further notes that *in principle, particle sorting should occur along the beach and it would be of value to be able to rely on it when assessing the strength and permeability of the hydraulic*

---

[1] Morgenstern references the 1983 edition of Vick's book, the current edition of which is 1990

fill structure. Blight and Bentel (1983) have developed a theoretical relation for the particle size sorting that occurs on a hydraulic beach and demonstrated at least a consistent fit with observations of beaches in gold and platinum tailings. However, inspection of most available data from tailings beaches reveals a complicated picture. The degree of grain-size segregation ranges from high to almost non-existent. As observed by Soderberg and Busch (1977), initial discharge densities and initial grains size distributions are at least two factors that will influence the degree of sorting that can develop on a beach. There has been only limited progress with understanding the operating procedures that will make optimal use of the above.

When Morgenstern (1985) refers to the strength and permeability he is in effect referring to aspects of "performance" of the hydraulic fill structure, in this case the tailings dam. This study seeks to develop approaches which would make it possible to rely on particle sorting and thereby improve performance which in the context of this dissertation refers specifically to geotechnical performance. It is recognised that other issues have equal impact in assessing tailings dam performance specifically:

- environmental issues arising from dusting and erosion of exposed tailings
- foundation shear strengths and drainage characteristics.

These are not specifically covered in the scope of this study which is therefore not exhaustive but will nonetheless provide the reader with guidelines on appropriate tailings placement procedures.

1.2 Tailings Production, Transport and Disposal

ICOLD (1982) notes that the Oxford International Dictionary defines tailings as the inferior qualities, leavings or residue of any product and that tailings therefore may be defined as the waste product from any number of industries including mining, industrial, chemical, etc. As would be expected, wide variations exist in the type of tailings that are produced by different industries. The large volumes of tailings, comprising finely ground rock and water produced by the mining industry, usually can be utilized at least in part to construct the tailings dams required for their retention. On the other hand the tailings or residue produced by most industrial and chemical operations and by some mineral benificiation processes are fine grained, often fluid or semi-fluid, and in many instances contain highly toxic materials. Such tailings are not suitable for use in the construction of tailings dams. The retention of these tailings should be achieved by the construction of

embankments, designed to conventional engineering standards and satisfying the requirements for dam safety as set out in ICOLD Bulletin No. 59 "Dam Safety Guidelines" (ICOLD (1989)).

Mine tailings consist of the ground up rock that remains after the mineral values have been removed from the ore, The grain size distribution of the tailings depends upon the characteristics of the ore and the mill processes used to concentrate and extract the metal values. A wide range of tailings gradation curves exists for the various mining operations and consequently tailings may range from essentially fine sands to clay-sized material.

The main focus in this dissertation is on Mine, and predominantly mill, tailings production. Disposal of run-of-mine rock and dry disposal of tailings such as in heap leach operations is not considered. Most specifically this study focuses on tailings transported and deposited in slurry form where the slurry is either the result of a hydrometallurgical process or is produced by slurrying tailings for ease of transport pump and pipeline. Tailings are normally transported to the disposal sites as a slurry at concentrations varying from 30% to 55% by weight of solids to liquids. Sometimes mine tailings are the residue not only after milling and reduction but also after desliming of the mill tailings stream to recover sands for placement underground. In this event the residual tailings product is usually finer and poorer draining.

Coal fired power station flash (as opposed to bottom or boiler ash) captured by precipitators, and other industrial residues usually ashes, slags or precipitates, where these are disposed of as a slurry, are industrial tailings products which have been considered in this study. These products are often of very high or very low particle specific gravity or very fine.

Once at the site the tailings slurry is discharged into an impoundment at selected locations either through open ended pipes, spray bars or via cyclones. On discharge the solids and the motive medium, the water, begin to separate depending on slurry characteristics. It is the art of controlling the separation of the water and solids and hydraulic sorting of the solids that takes place as the slurry flows that is the nucleus of this dissertation.

1.3 Tailings Slurry Depositional Characteristics

An important feature of tailings slurry streams is that they are usually hyperconcentrated and tend to deposit solids immediately following discharge forming a beach of settled tailings at the end of which a pool of the residual transport water accumulates. The shape of the beach is a function of the slurry characteristics and remains relatively consistent as long as the slurry characteristics remain constant. Where slurry make-up in terms of solids content, grind or discharge conditions change during deposition or from one deposition to another, there is a tendency for the beach to cut down, notwithstanding the hyperconcentrated state of the slurry, or to build up more rapidly closer to the discharge pipe.

The majority of research on slurry erosion/deposition and sedimentation is in the area of open channel solids transport pertaining to river engineering. In this research the slurries generally studied have low solids concentrations. and flow in relatively deep channels as opposed to the hyperconcentrated state of tailings slurries and the almost sheet flow conditions that develop on tailings beaches. The sections below summarise tailings-specific research on the separation of water and solids, hydraulic sorting and resulting beach geometry.

1.3.1 Separation of Water and Solids

Immediately following discharge of the slurry there is a tendency for the solid tailings particles and the carrying fluid to separate as a result of their differing densities. The extent to which this occurs depends on the concentration of solids, the grain size distribution and particle shape. Scott and Cymerman (1984) recognised the need for the development of planning tools for the evaluation of various waste disposal techniques. They developed a sand-fines ternary diagram which summarises the results of tests to determine the properties of a particular slurried material over a wide range of compositions. The diagram enables the delineation of boundaries between various modes of slurry behaviour. Figure 1.1 illustrates the diagram produced by Scott and Cymerman for Syncrude tailings. Of particular significance on the diagram is the segregating-non segregating boundary; above this boundary, coarse particles will settle through the fines-water slurry and hence segregate, while below the boundary no appreciable settling takes place regardless of the concentration of the coarse particles.

FIGURE 1.1 : TERNARY DIAGRAM FOR SYNCRUDE (After Scott and Cymerman (1984))

The boundary appears to follow a constant fines-water ratio. From this diagram alternative tailings disposal strategies can be developed depending upon whether a tailings stream segregates or not although the extent to which it is practical to adjust the fines content of a tailings product is limited by the tailings production process.

Thomson (1984) notes that general sedimentation theory as originally presented by Stokes and usually applied to settling or thickening tanks relies on a number of factors in order to be applicable:

- a uniform distribution of flow and particles - especially at the discharge end. Most tailings discharge systems rely on point discharges spaced at 1 m to 200 m centres - several orders of magnitude larger than the flow depth.

- no re-suspension of particles. Tailings flow after discharge comprises a series of hydraulic jumps as the tailings stream moves between supercritical and subcritical flow (Bentel (1981) and Küpper (1991)) creating turbulence and a tendency for particles to re-suspend depending on local flow conditions.

- no flocculation. Flocculation of mine tailings is only carried out in specific cases such as for diamond or phosphate tailings where the fine fractions are particularly small and clay-like.

The cost of flocculation in relation to tonnages usually is prohibitive except in the most sever cases.

- no particle interference during settling. Since the tailings slurries are generally hyperconcetrated, interference between settling particles is inevitable. Mc Roberts and Nixon (1976) present analytical methods to take cognisance of this phenomenon. The methods are an extension of an accepted approach to the analysis of hindered settling as previously developed by Kynch (1956) which is based on the assumption that the velocity of fall is dependent only on the local concentration of particles. Mittal and Morgenstern (1976) present data which supports Mc Roberts and Nixon's findings for samples taken in the pool area of tailings dams in Canada.

- no temperature effects - this is generally applicable to slurried mill tailings but not necessarily the case for industrial slurried tailings.

Yong (1984) discusses that for tailings in which surface active solids such as clay minerals predominate as in phosphatic clay slimes and bauxite red muds the classic hindered settling approaches are inapplicable. Surface-active forces predominate over simple gravitational relationships. Yong and Elmonayeri (1984) propose an approach based on convection-diffusion for these tailings.

Küpper (1991) describes in detail the various bedforms that develop as a segregating slurry, typical of most tailings dam situations, flows after discharge and grains of tailings tend either to deposit or to flow close to the surface of the deposit, constituting what is referred to as bedload and creating a situation of a flow of a fluid over an erodible boundary. The erodible boundary is changed by eroding and depositing material. Figure 1.2 is taken from Küpper (1991) and illustrates the transition from ripples to chute and pool conditions with varying Froude number. The importance of this diagram is to illustrate the influence of turbulence in determining the bedform of the deposited material.

As the tailings settle out on the beach they also trap water in the pores of the solids as the settled tailings moves from a state of sedimentation to a state of consolidation (where a transfer of stress between the particles and the pore fluid occurs). This water, referred to as interstitial water can be from 35% up to 200% of the solids mass. Fluid which is not trapped in the pores is termed supernatant and flows on down the beach to a pond.

FIGURE 1.2 : SEQUENCE OF BEDFORMS FOR INCREASING FLOW STRENGTH (After Küpper (1991))

Settling out of the tailings on a typical tailings dam takes place under both subaerial conditions, on the beach area, and subaqueous conditions in the pond. ICOLD (1989) describes subaerial tailings disposal as a method of depositing tailings in thin layers on a gently sloping beach in order to achieve a dense (as a result of desiccation stresses which occur as the placed tailings desaturates), less permeable, higher strength deposit through settling, draining, and air drying. Knight and Haile (1983) present data which indicates that significantly greater unit weights are achieved in the deposited tailings where these are deposited subaerially. The majority of tailings tend to under go hydraulic sorting after discharge onto the beach. The extent to which this occurs and the differences between subaerial and subaqueous conditions is discussed in the following section.

1.3.2 Hydraulic Sorting/Gravitational Segregation

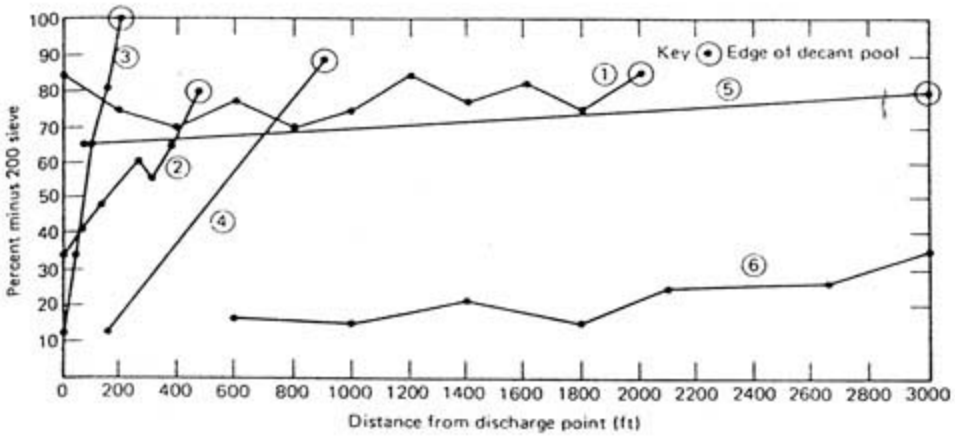
ICOLD (1982) notes that when tailings are discharged over a non submerged beach, the coarsest particles are deposited first, followed by a deposition according to grain size, with the slime settling in the pond. The separation of the material depends on its uniformity coefficient, as well as on the density and quantity of the pulp. It is necessary to predict the separation pattern of the deposited tailings as to coarseness and grain size, density, shear strength and permeability. It is on this basis that the method of construction of a tailings dam is determined. Limited analytical and empirical methods (Melent'ev et al (1973) and Abadjiev (1985)) have been developed to predict these characteristics and separation pattern predominantly by scaling up model test results or by obtaining data from existing operations.

The natural separation of the tailings increases the stability of the tailings dam, since coarser material with a greater potential strength is deposited in the outer zones. The separation also has a favourable effect on the seepage since the coarser material in the outer zones is more permeable, while it is more impermeable in the inner zones. The ratio of the permeability coefficient at the nearby beach to that at its far end near the pond may vary from 100 to 1000. This produces a lower phreatic surface than in the case of a homogeneous tailings dam (Blight and Bentel (1983), Soderberg and Bush (1977)). It is therefore important that the deposition be properly controlled so that a more strongly marked separation can be achieved to advantage, and the dangers of depositing low permeability materials in the outer zones avoided.

ICOLD (1982) states that underwater spigoting separation is much less marked and may sometimes be disregarded. In actual fact what occurs is that the remaining coarse material in the slurry stream is deposited at the edge of the pool as a result of the sudden change in flow velocity (the flow rate of the pool is very small in relation to the slurry stream) leaving clay and fine silt-sized materials in suspension. Deposition in the pond occurs through a process of sedimentation as

opposed to the saltation or particle rolling mechanisms characteristic of beach sorting. Under these conditions the theories of hindered settling become more applicable.

Vick (1990) notes the effect of gravitational segregation on the grain size distribution vertically, along the direction of flow of the tailings after discharge, as well as laterally across it. Vick notes that vertically at any point along the beach, the fines content can vary by as much as 10 to 20% over a few centimetres, while if discharge points are widely spaced, variations of 50 % or more can occur laterally. Moreover, where periodic encroachment of ponded water containing slimes occurs over sandy deposits, thin layers of fine slimes are deposited. Figure 1.3 illustrates measured changes in percentage of minus 75 micron material with distance from the discharge point for several tailings types. Vick notes that most mill tailings have similar grinds and therefore the differences between the tailing products must result from more than just variations in grind. Soderberg and Busch (1977) concluded that pulp density of the discharged slurry controls the degree of beach segregation.



Curve	Tailings type	G	%−200, Mill discharge	Pulp density of mill discharge	Source
1	gold	–	–	–	Blight and Steffen, 1979
2	copper	2.7	45	45	Volpe, 1979
3	lead-zinc slimes	3.4	75	< 40	Unpublished
4	–	2.7	38	30	Soderberg and Busch, 1977
5	–	2.7	60	50	Soderberg and Busch 1977
6	copper	3.0	–	–	Unpublished

FIGURE 1.3 : GRAIN-SIZE SEGREGATION ALONG TAILINGS BEACHES (After Vick (1990))

Küpper (1991) notes the Soviet standard specification SMiP-11-53-73 which recommends consideration of sorting if the slurry solids have a $D_{60}/ D_{10} > 2,5$ and/or $D_{90}/D_{10} > 5$ but notes that actually hydraulic sorting begins at $D_{60}/D_{10} = 1,3$ to 1,6 if the beach is long enough. Küpper (1991) stresses, however, that present ability to forecast and control grain size separation is till limited and a greater understanding of this process is needed.

Melent'ev et al (1973) present empirical equations for the determination of the grain size distribution by means of the parameter of fractionation defined as the ratio (in percent) of the content by mass of a given size fraction at a particular point on the beach to the mass content of the same size fraction in the total tailings. They present distributions of fractionation for natural materials used in the hydraulic fill construction of water storage dams serving hydro-electric power stations. Both Bentel (1981) and Abadjiev (1985) apply the fractionation approach to various tailings types to derive empirical equations for each material type.

Bentel (1981), Blight and Bentel (1983) and Abadjiev (1985) present data on the fractionation of the d_{50} of gold, platinum, diamond and copper tailings along the beach. They pick up on observations by Melent'ev et al (1973) that the d_{50} fraction represents the size of which there is the greatest quantity by mass of particles and therefore represents the dominant size fraction. They apply the fractionation approach to derive empirical equations for each material type. Figure 1.4 presents typical data from Blight and Bentel.

1.3.3 Layering

Bentel (1981), Bentel and Blight (1983) and Blight (1988) describe how the tailings stream flowing down a beach does so in series of hydraulic jumps resulting in changes from supercritical to subcritical flow, and meandering. Moreover, as noted by Blight (1987, 1988) the unsteady nature of the flow results in a changing pattern of rills which, as they vary, often overflow, depositing material that is finer than it should be at that distance from the point of deposition. The fine layers are deposited in the plane of the beach and often have permeabilities two to three orders of magnitude lower than the coarser material above and below the layer.

Jones and van Zyl (1981) present data obtained from piezometer cone probings on platinum tailings which were constructed using a spray bar system of operation. These indicated layers commonly 10 to 50 mm thick at spacings in line with deposition cycles. The layers have significantly lower permeabilities as evidenced by pore pressure spikes induced as the probe moves through the layer.

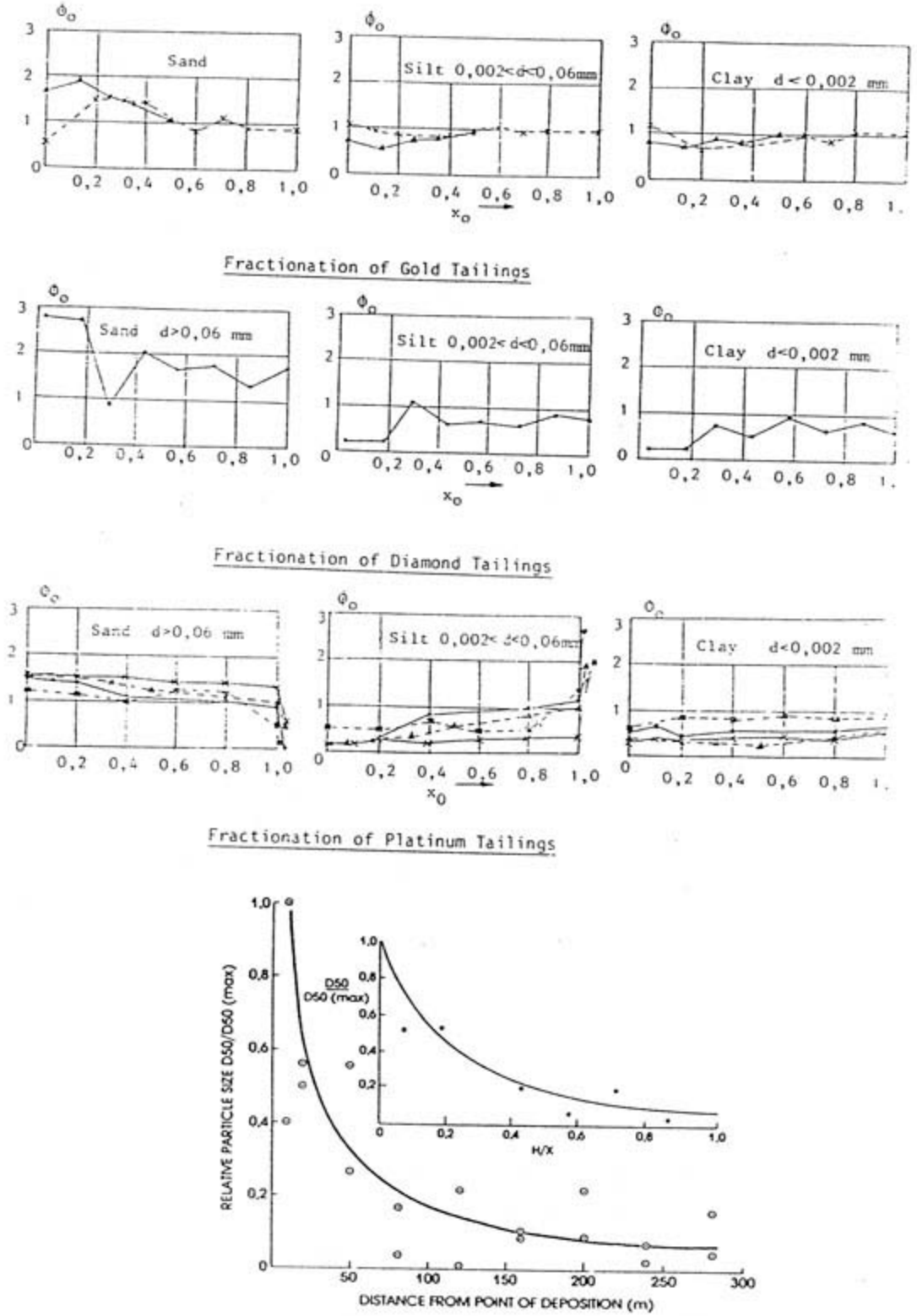


FIGURE 1.4 : FRACTIONATION OF TAILINGS (After Bentel (1981) and Blight and Bentel (1983))

Jennings (1979) describes the cause and mechanism of the Bafokeng failure where the specific method of deposition employed up to the time of the failure created severe layering. This layering coupled with severe pool level variations was found to be root cause of the catastrophic failure.

1.3.4 Beaching and Beach Profiles

The gradation from coarse to fine materials and the rate at which tailings particles deposit results in a concave beach profile from the crest to the pond along the direction of flow. The shape of the beach depends on slurry and discharge characteristics and is particularly important in determining pond storage characteristics.

Melent'ev et al (1973) documented their findings from empirical studies of beached natural alluvial soils used in the placement of hydraulic fills, and of mine tailings, and noted that if beach profile data were plotted on normalised axes, the profiles grouped in such a way that empirical equations could be fitted to the envelopes of the forms:

Subaerial beaches:

$$y = i_{av} L (1 - x_0)^n$$

where i_{av} = H/L is the average slope of the beach, x_0 = x/L and n is a parameter dependent on the particle size distribution of the deposited materials, H is the total drop over the beach from discharge to edge of pool and L the distance from discharge to edge of pool.

Subaqueous beaches:

$$y = h (1 - \exp^{ax/L})$$

where h is the total depth of the pool, and "a" is a dimensionless coefficient. The y axis is positive downwards with origin at the top of the underwater slope. In this equation L is the beach length to the lowest point (maximum depth).

Melent'ev et al gave several examples of n which they determined for various tailings and beach material types. Bentel (1981) reviewed the form of the equations developed by Melent'ev and applied these to South African gold, diamond and platinum tailings. Bentel derived n values for diamond, gold and platinum tailings as shown in Figure 1.5.

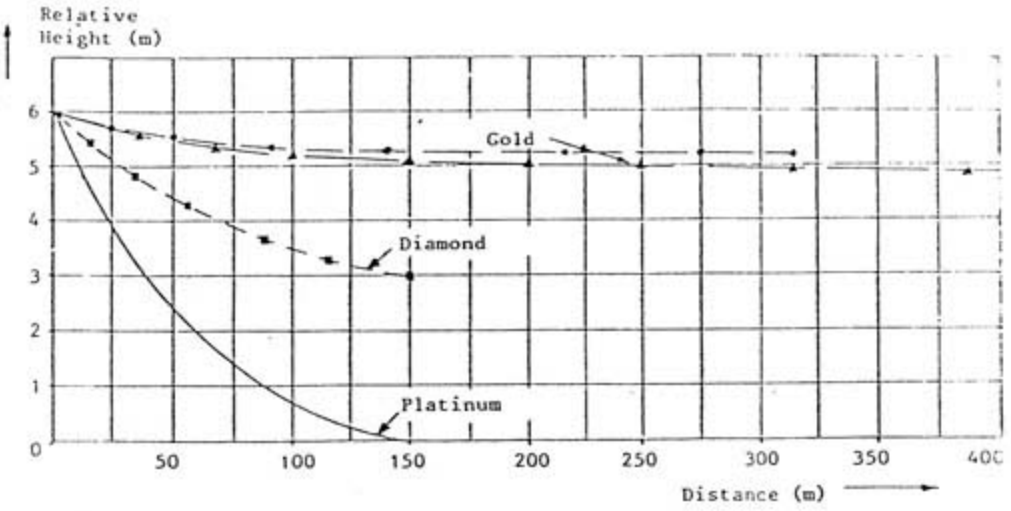
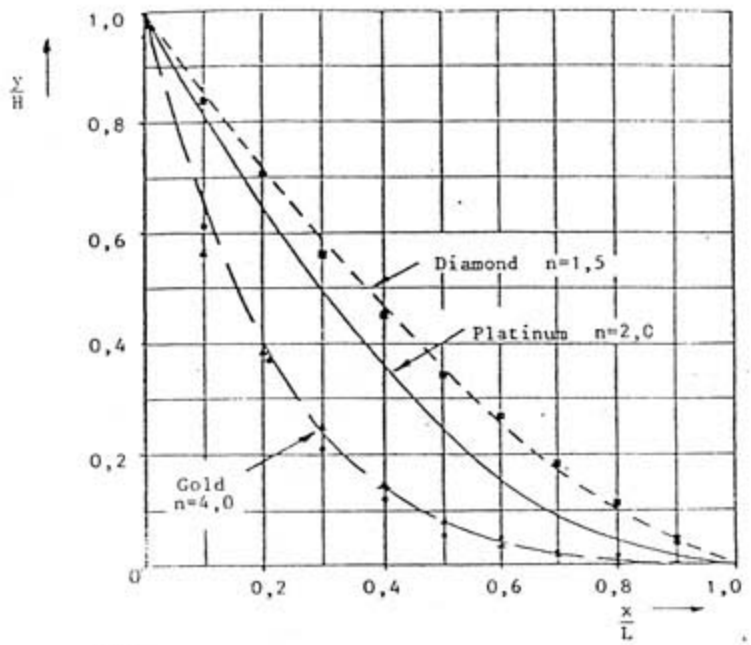


Figure 2.8 (a): Comparison of Typical Measured Beach Profiles



(b): Dimensionless Beach Profiles for the Various Types of Tailings

FIGURE 1.5 : DIMENSIONLESS PROFILES AND n VALUES FOR DIFFERENT TYPES OF TAILINGS (After Bentel (1981))

Bentel noted that the subaerial form of the equations applied only up to the edge of the pond. Bentel describes the influence of incremental rise of the beach, varying beach lengths, varying d_{50} size of the tailings and varying flow rate. Figure 1.6 is reproduced from Bentel (1981) and illustrates the influence of these variations.

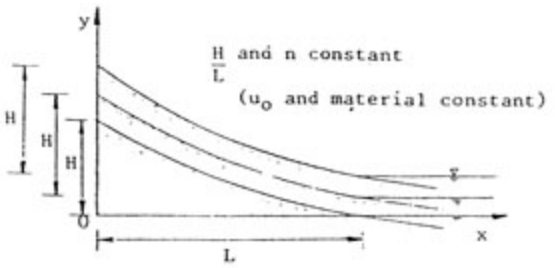
Blight (1987) documented the concept of the Master Profile for a tailings product discharged onto beaches of different lengths and documents dimensionless beach profiles for gold, diamond, platinum and copper tailings. Thomson (1984) describes the effect of solids concentration on the dimensionless beach profile as derived from model tests compared with field tests. Figure 1.7 shows the beach profiles derived by Thomson.

Fourie (1988) documents laboratory sized flume tests conducted on Coal fines, Bauxite and Nickel tailings and concludes that variation of the parameters of concentration, grind proved the most sensitive in dictating the beach profile

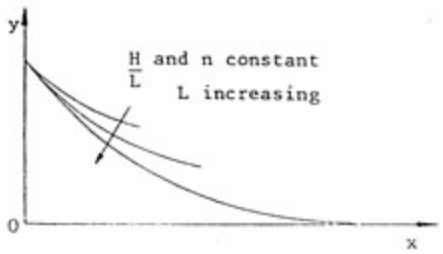
de Groot et al (1988) describe laboratory and field tests on the placement of estuarine sands as hydraulic fill dams across estuaries at deltas in Holland and conclude that the results show considerable sensitivity to slurry flow rate on discharge and grain size. Coarse sands placed at low flow rates show steeper slope profiles. Grain size is of primary importance in determining the slope of the placed sands during under water placement

Wates et al (1987) describe pilot scale flume tests on gold and uranium tailings at varying relative densities and state that they found this parameter to be the most important factor in determining the beach slope.

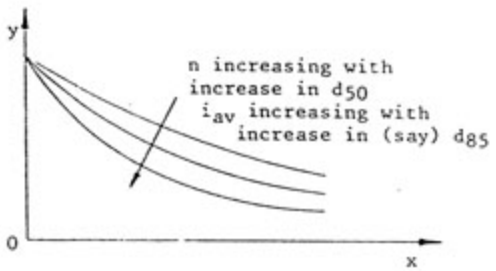
Küpper (1991) defines a new parameter P' which represents the ratio between gravitational forces and inertial forces and is based on the concept that the larger the inertial forces are in relation to the gravitational forces, the farther the grains could be transported and consequently the flatter the average beach slope will be. P' also forms a scaling factor that permits normalisation of the results of tests performed at different scales for example lab versus field scales.



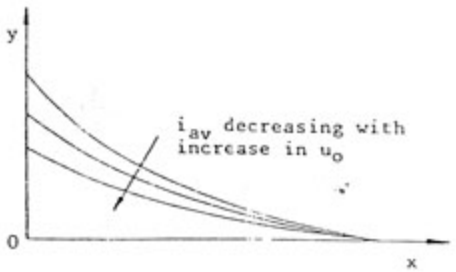
a. Profiles for constant set of system conditions showing incremental rise of beach



b. Profiles for constant set of system conditions but varying beach length



c. Profiles for variation in deposited tailings (u_o constant)



d. Profiles for variation in deposition flowrate, i.e. u_o (initial velocity (d_{50} constant)

FIGURE 1.6 : SUMMARY OF INFLUENCES ON THE BEACH PROFILE (After Bentel (1981))

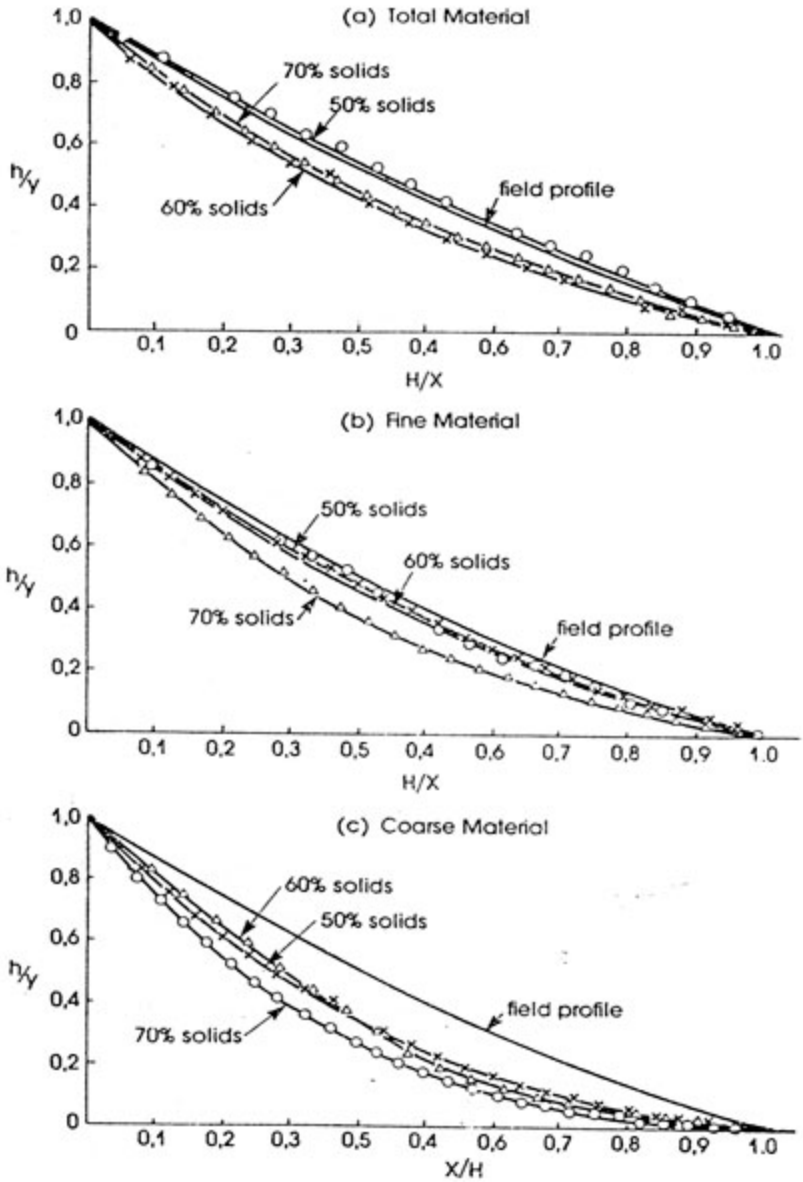


FIGURE 1.7 : INFLUENCE OF SOLIDS CONCENTRATION ON DIMENSIONLESS BEACH PROFILES AS ESTABLISHED FROM MODEL TESTS COMPARED WITH FIELD TESTS (After Thomson (1984))

P' has the following relationship:

$$P' = \frac{A\sqrt{g(G-1)}\sqrt{D_{50}}C_w}{Q}$$

where:

$$G = \frac{\rho_s}{\rho_w}$$

and $i_{av} = f(P')$

P' depends on discharge parameters and therefore does not require a knowledge of the overall beach slope. Küpper notes, however, that it is an empirical equation derived for her specific test results.

Küpper (1991) describes the "deep channelling" phenomenon observed in field beaching trials during which all the tailings slurry flow is conveyed directly to the pond and notes that this phenomenon is caused by three categories of variations:

- increase in flow rate
- decrease in solids concentration of the feed
- artificial over steepening of the original equilibrium slope

Küpper notes observations during which under water beach slope instability induced deep channelling. On the resumption of stable conditions the beach built back from the "nick" point ie the point where the newly cut beach met the original beach and re-establishes itself.

1.3.5 Conclusions : Tailings Depositional Characteristics

The most current study which specifically examines depositional characteristics of hydraulic fills including tailings is Küpper (1991). After presenting the results of an extensive laboratory and field testing programme Küpper concludes that:

....the master profile equation of Melent'ev et al (1973) seems to provide a reasonable approximation of the beach profile shape, however it is not a simple to obtain the master profile from flume tests as it appeared from Blight's tests. The influence of several factors on the value of the exponent n is not well understood at this point. In any case, the utilization of the master profile concept still depends on knowing the overall slope. From the analysis of the equations for overall slope presented in the literature, the conclusion is that there is still no adequate method to predict the beach overall slope at the design stage.

and:

Although the estimate of beach geometry is an important aspect of the design of a hydraulic fill, there are no adequate methods available. The most promising type of approach seems to be one based on the physics of sediment deposition and that incorporates the knowledge of fluid mechanics and sediment transport. However, both methods that follow this approach (Winterwerp et al,. 1990, Fan and Masliyah, 1990) do not compare well with observed values in all cases.

It is evident from the literature and confirmed by observations by the author that the factors affecting the segregation and hydraulic depositional characteristics of a tailings product may be separated into:

- Flow rate
- Velocity
- Slurry and solids density
- Particle size distribution
- Distance from discharge

It is therefore pertinent to consider these variables in detail and develop theoretical and empirical relationships that incorporate them.

1.4 General Description of Tailings Dam Construction

The sections above have covered literature on purely the depositional characteristics of tailings slurry. Historically methods for controlling the placement and containment of the tailings have evolved on the basis of those methods that offer acceptable levels of safety and reliability. Literature on containment structures for tailings slurries, or tailings dams, and their methods of formation are reviewed below.

According to the ICOLD Tailings Dams Safety Guidelines of ICOLD (1989) there are three classifications of tailings structures:

- Structures where the tailings are delivered to the dam as a slurry and stored as both the tailings solids and substantial quantities of free water

- Structures where comprehensive underdrainage is provided and the deposition of tailings is managed so as to drain excess liquid from the solids and recovered

- Structures where mechanical dewatering means are used and the tailings placed dry

This study confines itself to hydraulically transported tailings where drainage of the tailings is managed. Moreover, the study is confined to onshore disposal

1.4.1 General

The simplest, in concept at least, method of confining and storing tailings slurry is by constructing an earth or rockfill retaining embankment across a valley or on the side of a gently sloping hill and utilising the embankment as a containment structure. However, this is frequently the most costly method of tailings containment, both in terms of total cost and cash flow. Consequently, systems for incrementally raising the confining embankment have evolved over the centuries of mining. There are broadly three classes of raised embankment tailings dams classified according to the direction in which the crest of the dam moves in relation to the starter embankment. The classification of tailings dams in this way has been extensively documented (Soderberg and Busch (1977), ICOLD (1981), Vick (1990)) as:

- Upstream - where the crest moves inwards towards the centre of the tailings dam with each new lift

- Centreline - where the crest remains above the centreline of the toe starter embankment

- Downstream - where the crest moves outwards away from the centre of the dam with each new lift

The above are illustrated in Figure 1.8. The topography and location of a tailings dam has resulted in classification of tailings dams still further into side-hill ring dyke and valley as illustrated in Figure 1.9 from ICOLD (1982).

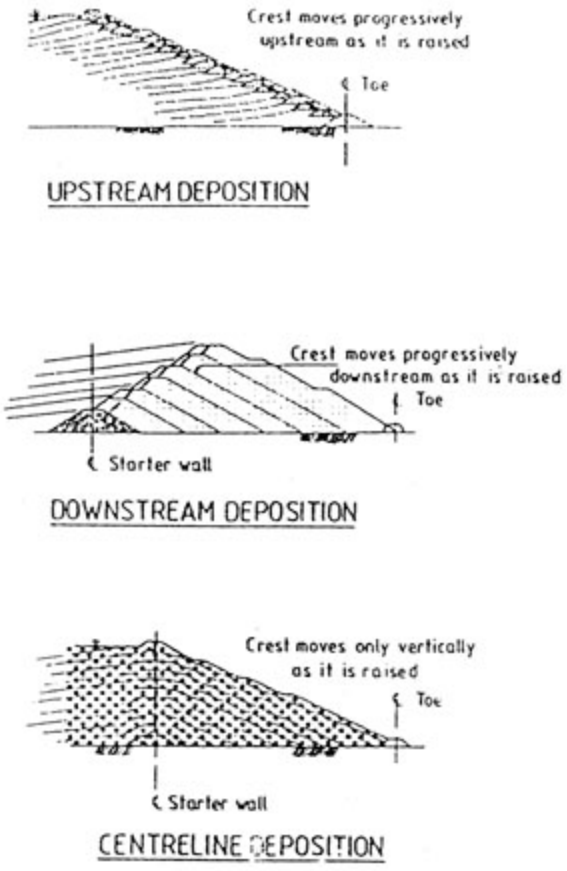


FIGURE 1.8 : UPSTREAM, DOWNSTREAM AND CENTRELINE METHODS OF TAILINGS DAM FORMATION (After ICOLD (1982))

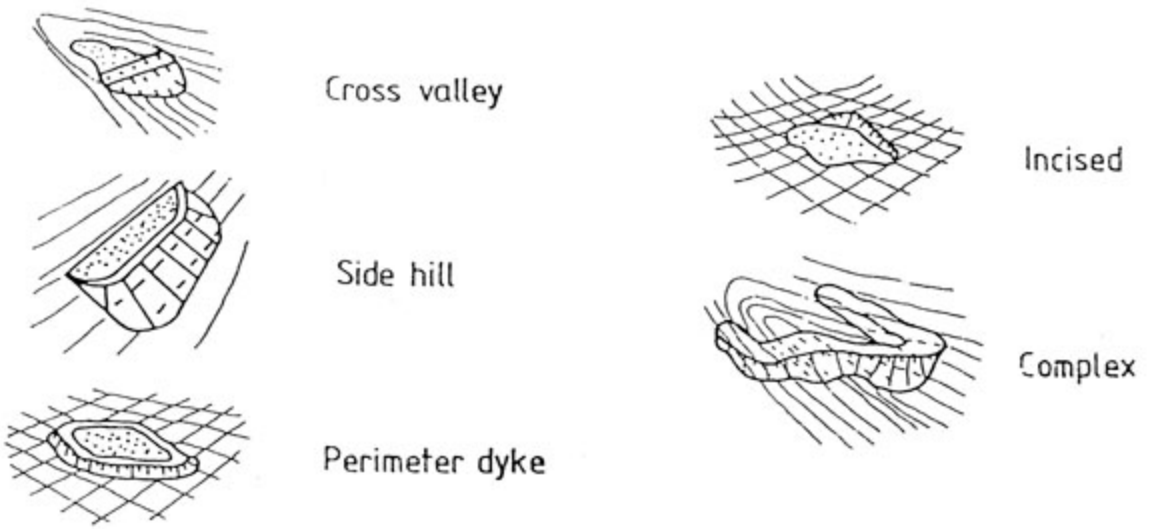


FIGURE 1.9 : SIDE-HILL RING DYKE AND VALLEY IMPOUNDMENTS (After ICOLD (1982))

The embankment material above the starter embankment can comprise natural materials, run of mine rockfill or tailings placed mechanically or hydraulically from point discharges, spray bars or via hydrocyclones. Vick [1990] sets out the relative features and advantages and disadvantages of water retention type dams versus raised embankment dams as follows:

1.4.2 Water retention type dams

These structures are similar to conventional water storage dams including filters, zoning etc. They are built to their full height at the start of development of the tailings dam. These structures are necessary in cases where the slurries are of very high water content, where there is a large storm catchment behind the tailings dam, and/or there are low plant return rates for the supernatant water and there is therefore a consequent need for large surge capacity or evaporation areas. Minns (1988) describes some examples of upstream/downstream water dam-type tailings dam construction in North America and Australia.

1.4.3 Raised embankments

In raised embankment tailings dams the embankment is raised incrementally so as to keep ahead of the deposited tailings. This provides more flexibility on the type of embankment material and allows a spread of capital expenditure over the life of the mine. In this case the embankment is formed by the tailings dam operators or as part of operations. As described above the type of raised impoundment is determined by the direction in which centreline of rising embankment moves, upstream, downstream and centreline. The relative advantages and disadvantages of the upstream, downstream and centreline systems are summarised below.

- **Upstream method of forming raised embankment**

  In this case the beach becomes the foundation for the next level of perimeter bund. The tailings must therefore form a competent beach to support the perimeter bund which implies that the tailings should contain significant proportions of sand. Often the beach tailings itself is used to build the perimeter bunds. The system has the advantages of cost and simplicity. Constraining factors are phreatic surface control, water storage capacity, and seismic liquefaction susceptibility. Upstream embankments are therefore poorly suited to conditions where water accumulation is anticipated due to flooding, long term accumulation of seasonal runoff or high rates of mill water accumulation. Upstream embankments are clearly inappropriate in areas of high seismic potential. Rate of rise of the tailings dam is a further

important limiting consideration due to pore pressure build-up in fine slimes zones and the need to allow the tailings beach to dry sufficiently to facilitate equipment access to raise the perimeter bund.

Vick (1990) states that although cycloning can be used to promote segregation of sands and slimes within the deposit and such measures as underdrains can be used to have the effect of increasing foundation permeability, pond water location is the only factor influencing the phreatic surface that can be controlled during operation. Upstream embankments are therefore subject to a number of very critical constraints and proper use of the method can be justified only when these constraints are thoroughly investigated and satisfied.

- **Downstream method of formation of raised embankment**

In this system successive raises of the perimeter bund are placed on the outer or downstream side of the tailings dam. This system more readily accommodates the storage of water on top of the tailings dam than the upstream system and yet gains the advantage of a spread capital expenditure if the perimeter embankment is raised incrementally. The system results in good liquefaction resistance with little limitation placed on raising rates. However, the system uses a lot of space at the toe, and consequently needs careful planning. Large volumes of fill are required which introduces constraints on the cost and availability of fill material. Moreover, the volume of fill increases exponentially with each successive raising.

- **Centreline method of formation of raised embankment**

This system represents a compromise between the upstream and downstream systems and so shares some of their advantages and disadvantages. As in the case of upstream construction competent beach material on which to found successive raisings of embankment is required although this is usually not a major problem since even high slimes content tailings usually have enough sands. It should not be used for permanent storage of large depths of water, but has acceptable performance during floods and generally good seismic resistance. The volumes in the embankment are still very significant and, as in the case of downstream systems, can be costly and fill shortages can be a problem.

- **Selection between methods of forming raised embankments**

  Historically early-day miners effectively eliminated unsuccessful tailings dam configurations since poor selections failed and thereby eliminated themselves. This empirical process has resulted in historic precedents in different mining districts. Precedents often arise because of the topography of the mining district. Upstream embankments commonly used in South African gold mining districts for example have been employed for over a century with relatively few catastrophic failures and consequently retain their dominance. These districts are usually arid which dictates small pools and they usually have low seismicity.

  Usually the process of natural selection is effective and problems only occur when deviations from these tried and tested systems take place.

1.4.4 Other tailings dam formation methods

Less common is the thickened discharge system of tailings disposal such as that proposed by Robinsky (1979). Tailings solids contents of 65% to 70% solids result in average tailings slopes of between 4% and 8%. The system suffers from problems of pipe wear, thickening difficulties, seismic risk, very large area requirements. Significant problems can occur where thickened discharge systems are formed on existing tailings dams since the thickened tailings use up the storm water holding capacity on the existing dam.

1.5 Typical South African Tailings Dam Construction

With very few exceptions tailings dams in South Africa are formed and operated on an upstream basis. Few of the mining areas in South Africa are in mountainous regions and soil covers are generally thin (less than 2 m). Ring-dyke and side-hill impoundments therefore predominate but since such impoundments are associated with long perimeter bunds severe constraints with regard to cost and availability of embankment fill materials occur. Under these conditions upstream systems are favoured. This is feasible because of two main factors:

- According to Williamson (1987) most South African mines are located in areas of very low seismic activity. Furthermore, extensive testing has revealed that gold mine and other tailings display good stability, without tendency to liquify but with increased pore pressure, under cyclic loading.

- The majority of the mining areas of South Africa fall within areas of rainfall deficiency. Figure 1.10 from Blight (1987) shows the areas of rainfall excess/deficiency. The country is generally arid presenting a high potential for densification through drying and desiccation.

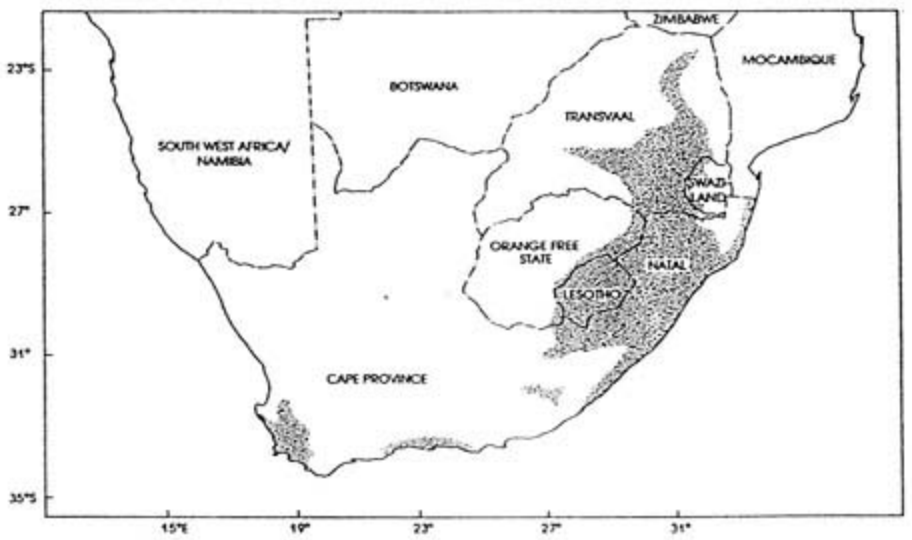


FIGURE 1.10 : RAINFALL EXCESS AND RAINFALL DEFICIENCY AREAS IN SOUTH AFRICA (After Blight et al (1987))

Blight and Steffen (1979) discuss the way in which South African tailings dam operations take advantage of the arid characteristics of the South African gold mining areas to promote desiccation consolidation of the tailings. Swarbrick (1992) presents comprehensive measurements of similar improvements in density and consolidation characteristics for high clay mine tailings both in laboratory and field measurements in Australia, country of similar climate to South Africa. Figure 1.11 is taken from Blight and Steffen's (1979) paper and indicates the changes in void ratio induced through desiccation consolidation on typical gold mine tailings. This method of tailings dam formation is referred to as a "semi-dry" method. The increased density of the tailings as a result of the desiccation consolidation improves several related geotechnical performance aspects (International Atomic Energy Agency (1981)):

- Considerably increased shear strength
- Lower total tailings volume
- Low liquefaction potential
- Lower settlement potential

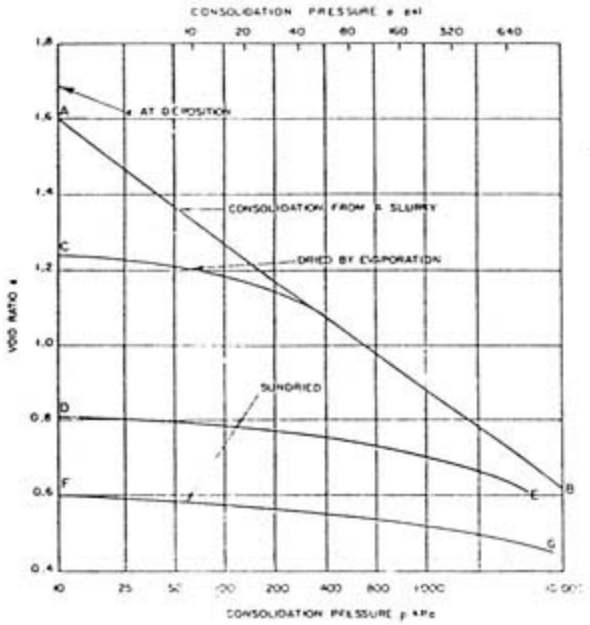


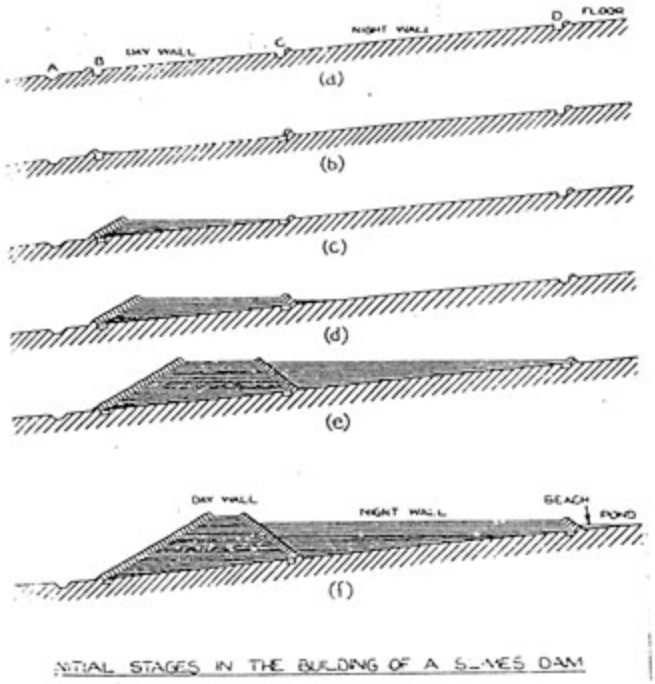
FIGURE 1.11 : EFFECT OF SUNDRYING ON VOID RATIO AND CONSOLIDATION CHARACTERISTICS OF SLIMES (After Blight and Steffen (1979))

Blight (1988) notes that there, however, are some negative features associated with the "semi-dry" method. One specific negative relates to the common formation of desiccation cracks in each layer. The cracks may form a network of interlinked channels within the tailings dam and could later result in piping failures.

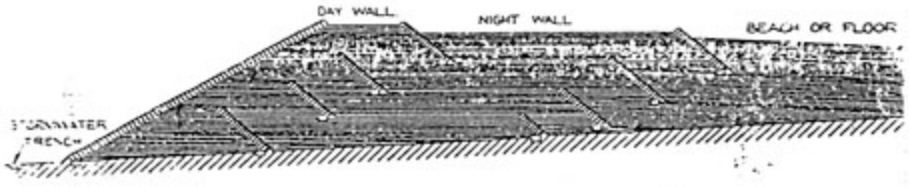
The long perimeters bunds characteristic of the South African tailings dams do not favour cyclone placement of tailings. Demands on underflow materials are unusually large as a result of the long perimeters which make it necessary to operate the cyclones over the entire perimeter. Moreover, the costs of cyclone placement far exceed those of the more common paddock and spigot/spray bar systems that predominate and are described in more detail below. Where rates of rise of the tailings dam exceed 4 to 5 m per year, however, the designers have little choice but to consider cyclone placement and this has been done successfully (Lyell and Prakke (1988)). This study does not consider the depositional characteristics of cyclone underflow material in detail per se. However, many of the developments in the prediction of beaching characteristics described in subsequent chapters are directly applicable to the underflow and overflow products which differ in solids content and particle size distribution only. Lyell and Prakke (1988), Williamson (1987) and ICOLD (1982) provide detailed descriptions on the method of operation of a hydrocyclone when used for tailings placement.

- **Paddock system of gold tailings dam formation**

The paddock system of gold tailings dam formation commonly used in South Africa has its origins in the Cornish mining district of England. Cornish miners who participated in early gold mining introduced tailings disposal methods which were similar to those applied on the mines in the Cornwall area. These entailed (National Building Research Institute (1959), Donaldson (1960) and Mrost (1974)) the formation of a ring-dyke tailings dam by careful deposition of an outer "daywall" area, so called because the tailings placed in this area is placed in thin (150 to 250mm) lifts with negligible freeboard and therefore required detailed supervision. Immediately inside the daywall, a nightwall was formed to improve pond location and to facilitate periodic widening of the daywall as this "pinched out". Figure 1.12 illustrates in section the sequence of development of the day and night walls. The dam was initiated from a toe "trench" as indicated. Inside the nightwall, the "floor" area was created to contain the majority of the tailings discharged both at night and during the day shift, if cycle times which dictate the rate at which the daywall can be raised permit. Figure 1.13 illustrates the method of distribution of the slurry from a single discharge point usually located at the topographic high point on the tailings dam site. At the topographic low point a pond would form within the floor area. Supernatant water would be decanted via "penstock" decants for return to the plant.



INITIAL STAGES IN THE BUILDING OF A SLIMES DAM



SECTION THROUGH WALL SHOWING SUCCESSIVE STEPPINGS BACK OF INNER BOUNDARIES OF DAY AND NIGHT WALLS.

FIGURE 1.12 : SECTIONS THROUGH A TYPCIAL SOUTH AFRICAN GOLD TAILINGS DAM ILLUSTRATING THE METHOD AND SEQUENCE OF DAYWALL, NIGHTWALL AND FLOW DEVELOPMENT UP TO THE 1970's (After National Building Research Institute (1959))

The system was relatively successful during the early gold mining years when the availability of land permitted the development of dams of relatively large area resulting in low rates of rise and generous amounts of time for drainage and drying of the tailings. The system was made possible by the low hydraulic segregation characteristics of the gold tailings resulting primarily from the uniformity of the particle size distribution. The particle size distribution published by National Building Research Institute (1959) re-produced in Figure 1.14 has uniformity coefficients as follows:

D_{60}/D_{10} = 24 to 40, and

D_{90}/D_{80} = 75.

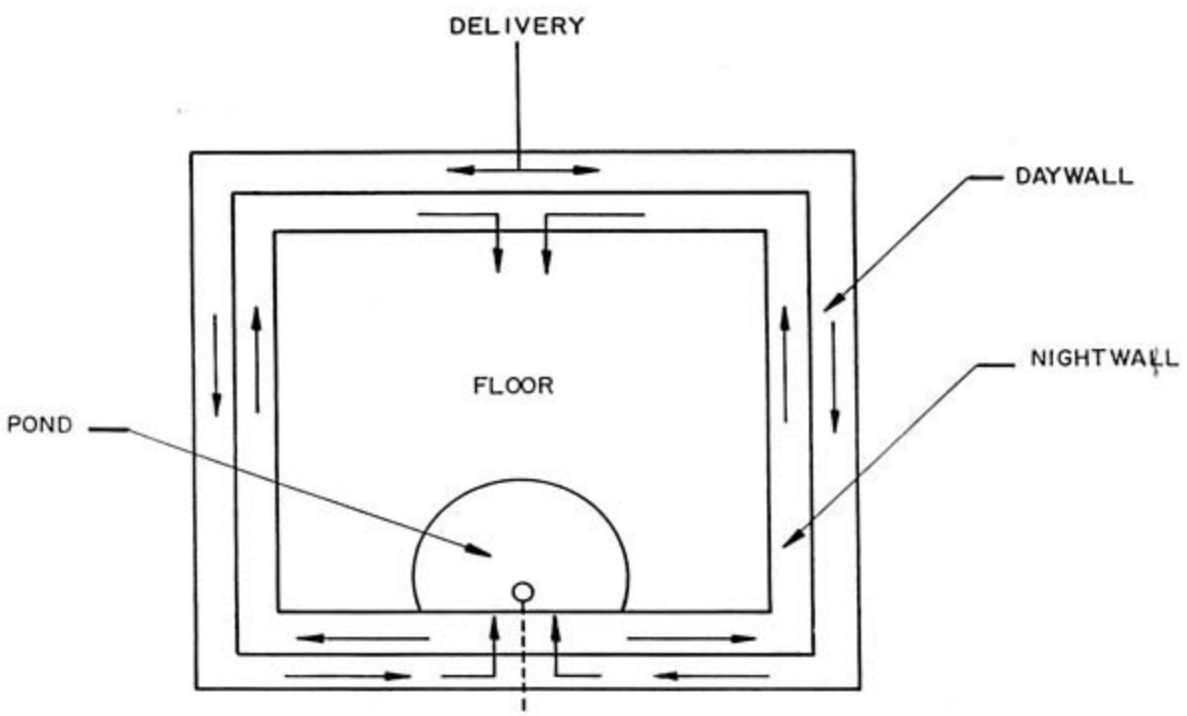


FIGURE 1.13 : PLAN ILLUSTRATING THE METHOD OF SOUTH AFRICAN GOLD TAILINGS DAM DEVELOPMENT FROM A SINGLE POINT DISCHARGE UP TO 1970's (After Mrost (1974))

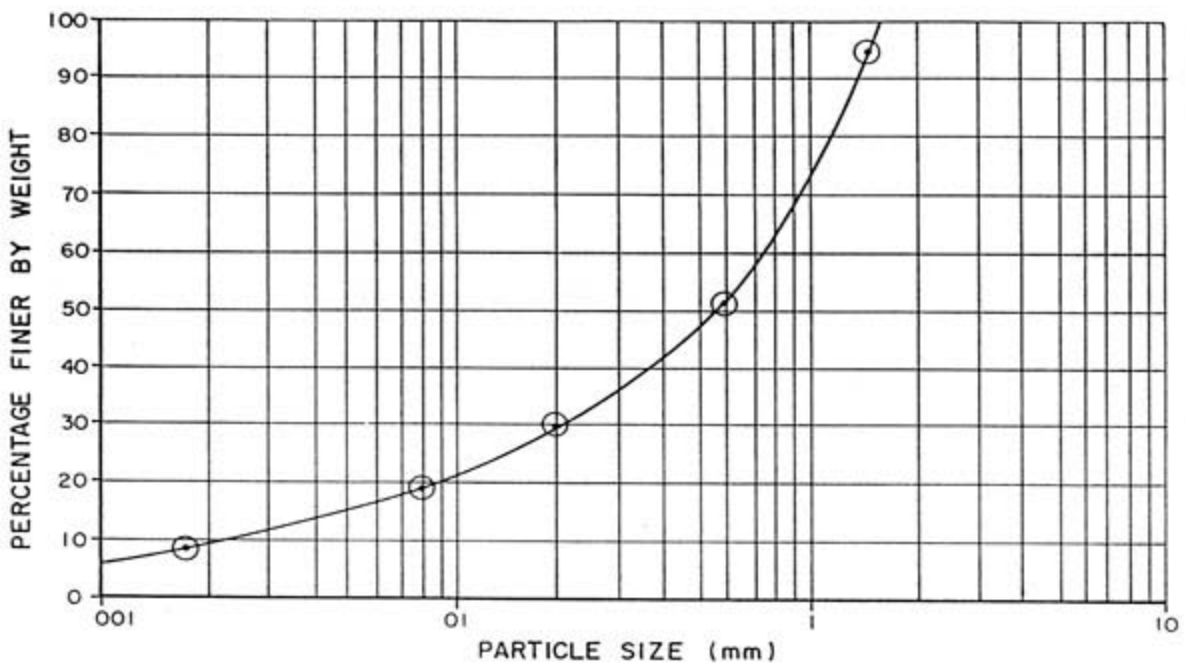


FIGURE 1.14 : PARTICLE SIZE DISTRIBUTION FOR SOUTH AFRICAN GOLD TAILINGS DURING THE 1950's (After National Building Research Institute (1959))

In terms of the Soviet standard specification quoted by Küpper (1991), the above ratios should be less than 2,5 and 5 respectively if hydraulic sorting is to be discounted. It was the extent of hydraulic sorting that did occur that forced the gradual updating of the paddock system. The material at the penstocks proved considerably finer than at the discharge. Drainage characteristics at the most critical location were therefore the poorest and often resulted in slope stability problems.

The paddock system was operated as described until the seventies when improved understanding of tailings behaviour brought about several detail improvements. Specifically multiple discharge points became the norm with distances between the discharge points regulated to 200 to 300 m in order to reduce the effects of hydraulic sorting. A pool wall system for the control of the pond depth and location, and to simplify access to the centrally located decants, was gradually introduced. The night wall was disposed of as it became the norm to equip each tailings dam with a water dam at ground level and the need for the additional freeboard security afforded by the nightwall became less important. Doing away with the nightwall also afforded some tailings dam operation cost savings. Figures 1.15 and 1.16 illustrate a typical plan and section on a current gold tailings dam. Specifically indicated on Figures 1.15 and 1.16 are the following:

- the pool wall to improve pool control and ensure that the deepest part of the pool is around the decant (Plan)

- the "in-wall" day and night deliveries which are embedded in the tailings (Section)

National Building Research Institute (1959) and Blight and Steffen (1979) describe how desiccation of the tailings specifically in the daywall permits the improvement of the geotechnical characteristics of the deposited tailings.

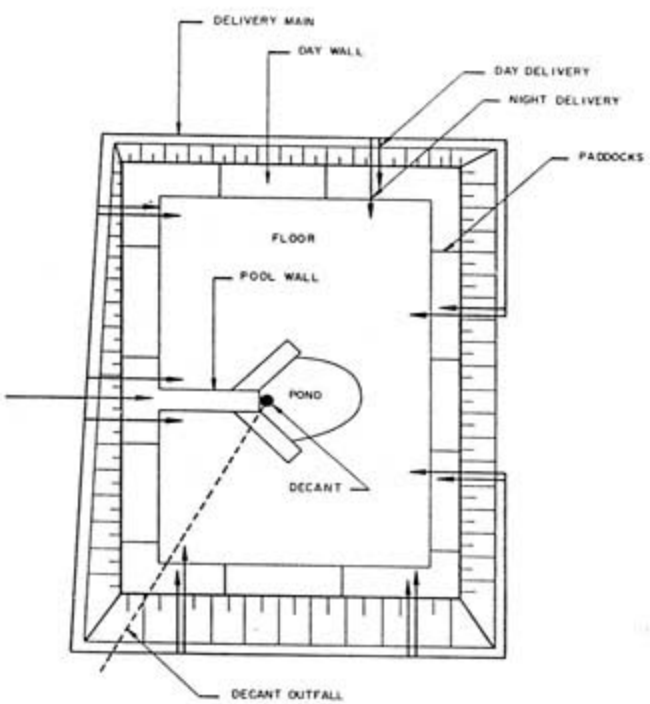


FIGURE 1.15 : PLAN ON A TYPICAL CURRENT GOLD TAILINGS DAM

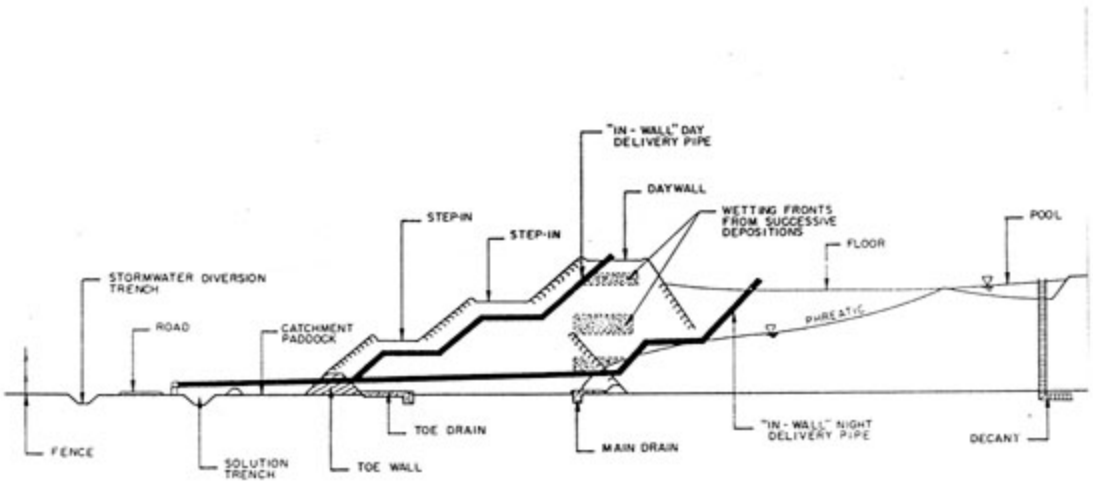


FIGURE 1.16 : TYPICAL SECTION THROUGH A CURRENT GOLD TAILINGS DAM

- **Spray-bar (Spigot) system of tailings dam formation**

  Where a tailings is found to be heavily segregating such as in the case of copper and platinum tailings, typical gradings for which are indicated on Figure 1.17, spray-bar (usually referred to in South Africa as spigot) discharge systems are usually adopted in order to induce the placement coarse material on the outer zones of the dam. Such tailings have uniformity coefficients of:

  $D_{60}/D_{10} = 10$

  and

  $D_{90}/D_{10} = 20$

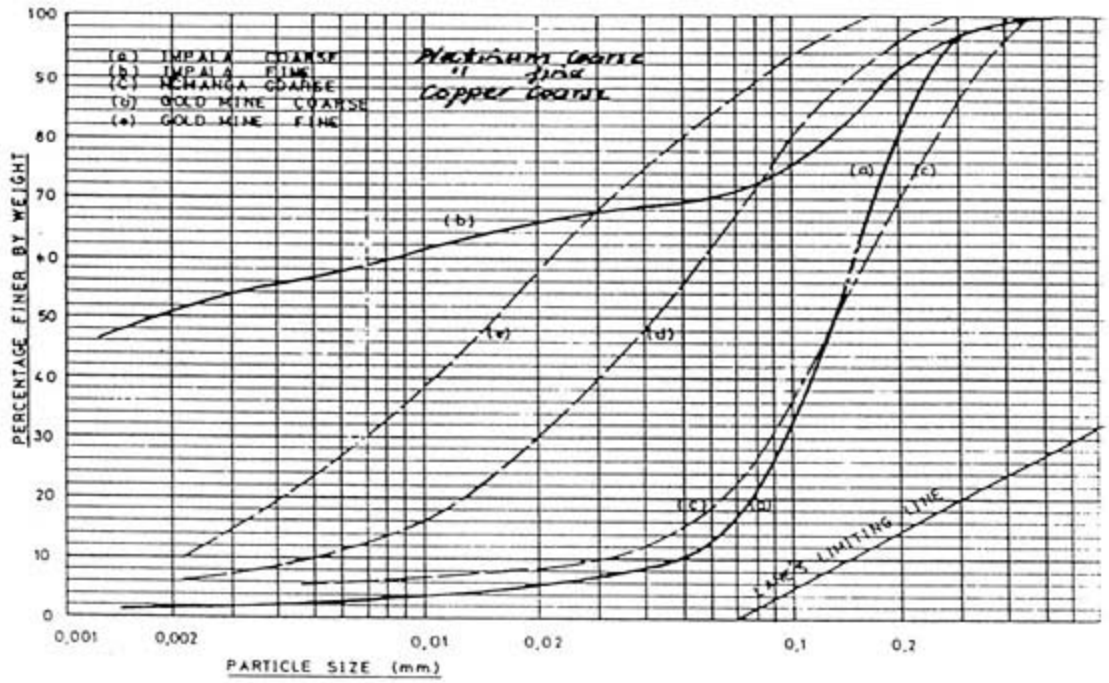


FIGURE 1.17 : TYPICAL TAILINGS PARTICLE SIZE DISTRIBUTIONS (After Jennings (1978))

Typically the spigot line is positioned on the crest of the tailings dam. With each deposition a section of the spigot line is dismantled and moved to one side to permit the raising of the perimeter bund which is usually constructed of the beach tailings. The sequence of operations is illustrated in Figure 1.18 which also indicates that resulting distribution of materials. Spigot openings in the pipe range from 25 mm to 100 mm depending on tonnage and they are usually spaced at 1 m to 2,5 m centres respectively. In the course of a deposition cycle, a batch of adjacent spigots is opened, sufficient to cater for the tailings slurry flow rate. As the beach fills spigots at one end of the batch are opened while the equivalent number at the other end are closed so that the deposition gradually moves along the spigot pipe and around the tailings dam.

- **Importance of Rate of Rise**

  It will be noted from Figure 1-16 that the original toe trench has been replaced with a substantial "toewall" is located at the toe of the slope of the embankment. This has become necessary in order to accommodate the higher rates of rise characteristic of current South African gold mining; monthly tonnage production rates have increased and land is no longer as freely available with the result that the provision of a substantial toewall and underdrainage has become essential.

The upstream method of tailings dam construction which predominates in South Africa therefore necessitates that particular attention be paid to the marginal rate of rise of the crest. This is not only to ensure adequate consolidation of the tailings foundation material but, and in most instances this is the controlling factor on rate of rise, to accommodate equipment access to effect perimeter bund raising.

To permit equipment access it is necessary not only to ensure that there are no excess pore pressures residual in the tailings but also to allow sufficent time for the tailings to desaturate and desiccate since only then will it develop the bearing strength to permit trafficability of the equipment. Constraints on this rate of rise dictate that on even moderately sloping natural topography it is necessary to provide a substantial toe starter embankment. The starter embankment will permit deposition of the tailings without the need to gain access for perimeter bund raising at least until the tailings deposition are has developed to the stage where the marginal rate of rise of the crest reduces to an acceptable level to permit equipment access.

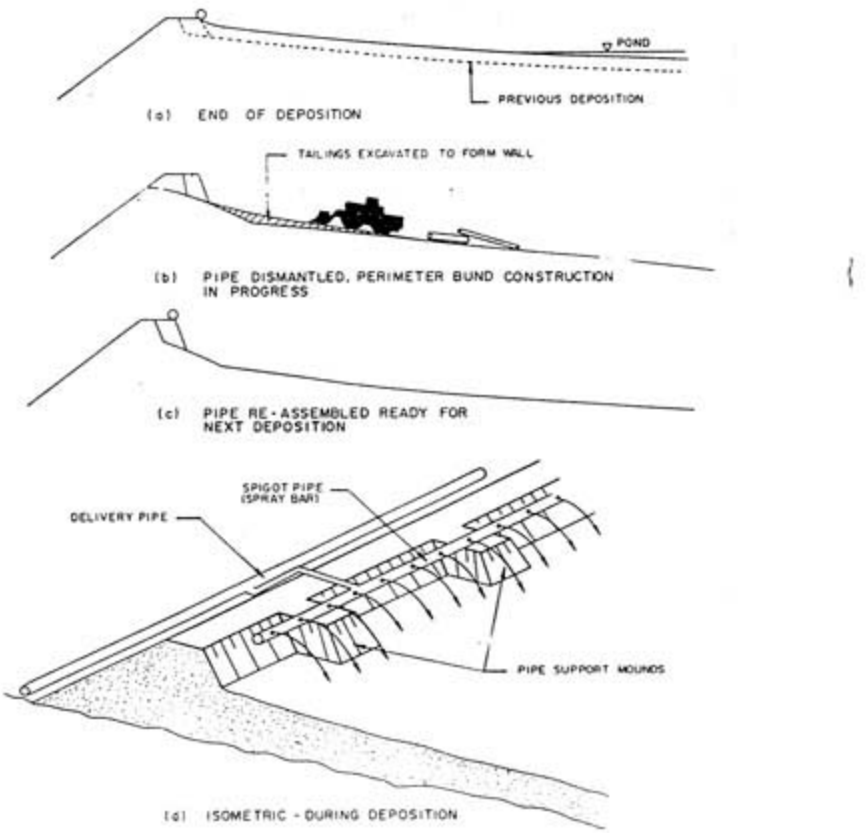


FIGURE 1.18 : SECTIONS ILLUSTRATING THE SEQUENCE OF OPERATION OF A TYPCIAL SOUTH AFRICAN TAILINGS DAM CONSTRUCTED USING SPIGOTS ON A SPRAY-BAR

1.6 Geotechnical Performance of Tailings Dams

In the majority of mining developments in South Africa today the location and site of a potential tailings dam is dictated by the economics of slurry transportation and land availability. Increasingly the tailings dam designer is finding it necessary to take tailings dams higher in order to accommodate tailings arisings.

Once a site has been defined, there are only three parameters that the designer can optimise in order to accommodate the increasing heights:

- Adopt flatter average side slopes

- Equip the site with underdrainage systems which will control the location of the phreatic surface in the embankment area of the tailings dam

- Control and regulate tailings deposition practice so as to:

  - ensure that the coarse materials are placed in areas which will enhance the operation of underdrainage systems and result in the placement of sandy materials on the beaches. Blight (1987) discusses the implication for seepage of variation in particle size distribution of tailings from the outer crest towards the pool. This change in particle size means that the permeability of the tailings will decrease continuously from the point of deposition to the pool. Blight proposes an exponential equation relating permeability to the distance from the point of deposition on the crest of the tailings dam:
    $K = ae^{-bH}$,
    where a and b are characteristic of the beach. The influence of this equation is illustrated in Figure 1.19

  - limit the height of tailings material placed per deposition cycle. This will regulate the intensity of the wetting front that will migrate through the desiccated tailings in the embankment area.

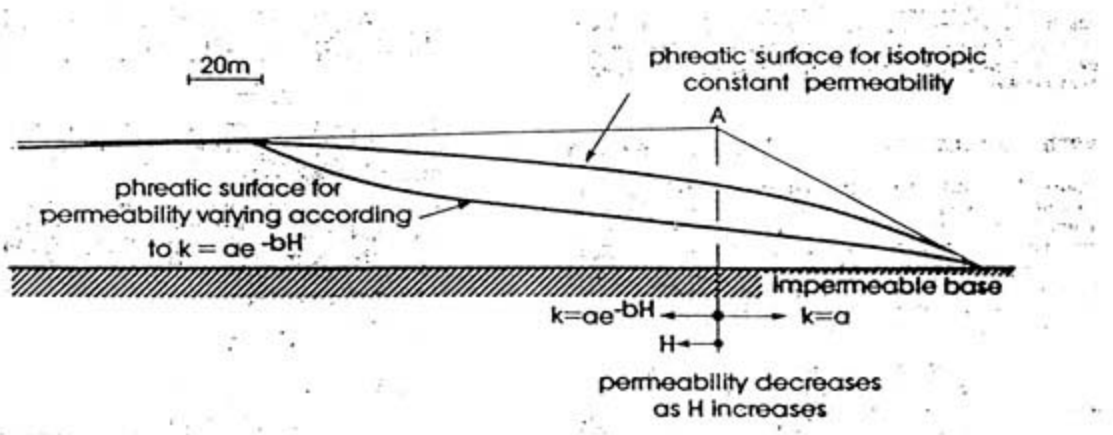


FIGURE 1.19 : EFFECT OF VARYING PERMEABILITY ON POSITION OF THE PHREATIC SURFACE IN A TAILINGS DAM (After Blight (1987))

The above are illustrated on a typical section through a tailings dam as shown in Figure 1.20. The zone of desaturated material in the embankment enhances seismic resistance to the extent that seismic events of 5,4 on the Richter scale have been successfully withstood by gold tailings dams in the Welkom Goldfields.

Geotechnical performance is usually monitored and verified by:

- Measuring drain outlet flow rates to check for unexplainable sustained decreases in flow rate. These are usually as a result of fungal activity or inorganic deposits building up inside the outlet pipes. In the event of a decrease in flow it is possible to rod the drain and clear any obstructions

- Measuring piezometric heads in piezometers located either side of the underdrainage blanket

- Making use of movement monitoring systems such as survey beacons and inclinometers.

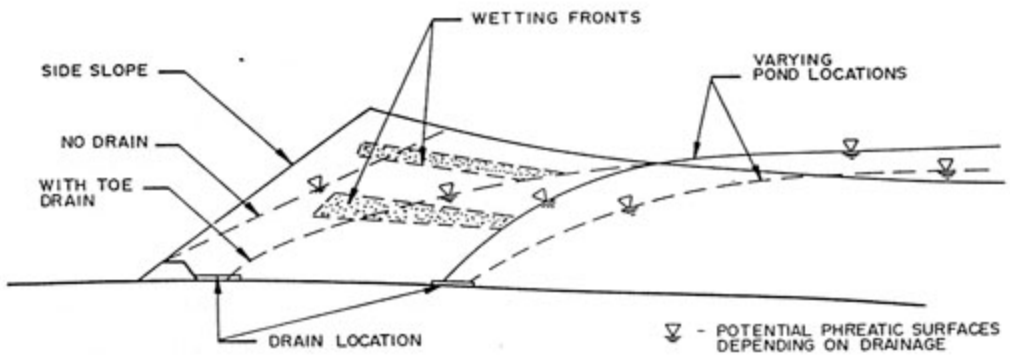


FIGURE 1.20 : TYPICAL SECTION THROUGH A TAILINGS DAM ILLUSTRATING THE FOUR VARIABLES AVAILABLE TO A DESIGNER TO OPTIMISE GEOTECHNICAL PERMFORMANCE

1.7 Justification of Further Research

Morgenstern (1985) in his Theme lecture at the XI International Conference on Soil Mechanics and Foundation Engineering (1985) entitled *Geotechnical Aspects of Environmental Control* notes that although the economics of disposal and the geotechnical integrity of the tailings structures are both intimately related to the behaviour of the tailings during placement, tailings deposition has attracted little attention from the geotechnical perspective. Küpper (1991) concluded in her PhD dissertation produced in 1991 under the supervision of Professor Morgenstern that several aspects of hydraulic fill are still not well understood including the mechanism of fill formation and the factors that affect the properties of the fill. It would therefore appear that additional research into the depositional behaviour of tailings in the broadest sense is warranted and it is against this background that this research study has been undertaken.

1.8 Approach

The author is a practising consulting engineer who for the past 15 years has been involved with the design, analysis construction and operation of tailings dams for Mining and Industry in South Africa covering ash and coal, gold, platinum, copper, lead-zinc, uranium, and chromite tailings. This

dissertation brings together, in a practical form, observations and testing and analysis conducted on tailings and tailings dams with the objectives of identifying why certain operational procedures are successful, how these can be improved and optimised from a predictive basis to yield improved tailings dam performance. The study is broad-based and practically orientated but nonetheless involves the development of theoretical approaches to provide a theoretical basis on which predictions can be made. Much of the data in the study has been drawn from project files of specific tailings dam projects and the author is fortunate that both his employer and their Clients, the various Mining organisations have had the vision and foresight to permit the drawing together of information to form a consolidated study from which all concerned may benefit.

Over the years of designing and observing tailings dams the author has observed four areas in which depositional behaviour of tailings and would significantly affect geotechnical performance of a tailings dam:

- Control of materials distribution with reference to the location of coarse materials placement
- Control of materials distribution with reference to the location of fine materials
- Control of beach profiles to ensure adequate pond control under storm conditions

To date methods developed for predicting the depositional behaviour that would affect the above aspects have been almost entirely empirical. It is essential to extend the empirical (after the fact) methods to predictive (before the fact) methods if an improvement is to be made in the understanding of depositional behaviour. In order to make predictive statements on the above it has been necessary to gain an understanding of, and evaluate in detail tailings depositional behaviour, gravitational segregation, the mechanisms controlling this and methods of prediction. This has been approached by:

- extension of sedimentation hydraulics theory based on energy and probability approaches to hyperconcentrated slurry
- flume testing
- field observations and measurements
- development of a method for predicting the slope, and particle size distribution of deposited tailings at any location on the "beach"

Extensive use has been made of available data from other research studies and in the literature. Where practical actual laboratory testing and large and full scale simulations have been carried out.

The topics remain, however, broad based. To have carried out extensive testwork would have proved onerous in terms of cost and time. At the outset exhaustive testing was envisaged but had to be tempered as the reality of gaining this data within the framework of a practising consulting engineer become more real than apparent.

CHAPTER 2

CONTENTS

Sec	Description	Page
2	REVIEW OF LITERATURE RELATING TO THE PREDICTION OF PARTICLE SIZE DISTRIBUTION WITHIN AND ALONG A FLOW STREAM	2-1
2.1	Introduction	2-1
2.2	Sediment Transport Equations	2-2
2.2.1	General	2-2
2.2.2	Suspended Sediment Transport Relationships	2-5
2.2.3	Total Sediment Transport Equations	2-23
2.3	Models Applied to River Sedimentation	2-30
2.4	Models on Beaching of Tailings	2-34
2.5	Laboratory-scale and Pilot-scale Flume Testing	2-35
2.6	Empirically Derived Beaching Equations	2-38
2.7	Rheology and Slurry Flow Resistance Prediction	2-39
2.8	Discussion	2-48
2.8.1	Comparison of Tailings Beach Flow Characteristics with River Sedimentation	2-48
2.8.2	Sediment Transport Equations	2-52
2.8.3	Flume Testing	2-55
2.8.4	Abadjiev's Empirical Methods	2-56
2.8.5	Rheology and Slurry Flow Resistance Prediction	2-56
2.9	Conclusions	2-57

CHAPTER 2 REVIEW OF LITERATURE RELATING TO THE PREDICTION OF PARTICLE SIZE DISTRIBUTION WITHIN AND ALONG A FLOW STREAM

2.1 Introduction

The literature survey in Chapter 1 indicates that methods for predicting beach tailings profiles and particle size distributions at various locations on the tailings dam are almost entirely empirically based. A predictive approach would need to predict the change in concentration of the slurry stream with distance along the beach, the average vertical concentration in the stream at any point and the distribution of particle sizes at any vertical location. Research of limited applicability to tailings has been recorded in literature on river sedimentation and sedimentary petrology. A number of differences between the conditions commonly encountered in river engineering and those prevalent on the tailings dam should, however, be noted at the outset:

- in river sediments particles range from gravels of 25 mm effective diameter to clay fraction (comprising clay minerals) whereas on tailings deposits particle sizes range from 6 mm to clay fraction (comprising ground hard rock except in the cases of red muds and phosphatic tailings and diamond tailings from Kimberlite)

- concentrations of solids in the rivers are usually considerably lower than in tailings slurries - typically up to 10 % by mass compared with 30 to 65 % on tailings operations. At these concentrations the tailings slurries are strongly depositional.

Theoretical approaches in river sedimentation usually regard particle size distribution as being of secondary importance since the focus of these approaches is the calculation of sediment load, bed aggradation and bed degradation, not particle size distribution. A notable exception is the research conducted by James (1987, 1988) in which the mechanics associated with the distribution of heavy minerals are described. In tailings deposition the particle size distribution is of fundamental importance in suspension since this will determine the particle sizing of tailings deposited at various locations along the stream length.

The focus of this chapter is the review of literature on the estimation of the particle size distribution at any point on the beach of a tailings dam and in the pond. The literature review will identify that an approach which merits detailed consideration would be based on an *entropy*

analogy applied to *stream power.* Chapter 3 will describe the development of an approach which results in theoretical equations and a model. A second approach will also be developed in Chapter 3 and involves *empirical correlation* on a probabilistic basis of the various parameters impacting on tailings deposition such as particle size distribution, slurry density etc. Both approaches will be applied to available beach particle size distribution data in the literature, within the author's project files and as obtained through measurements on existing tailings operations.

2.2 Sediment Transport Equations

2.2.1 General

A large number of approaches and equations have been documented in the literature (Raudkivi (1992), Annandale (1987)) with those of Engelund-Hansen, Ackers-White, van Rijn, Yang, Einstein being the more commonly referenced. These approaches and equations embrace probabilistic, stochastic, diffusion-based, energy based and numerical solutions to the sediment transport problem. There are several publications (White et al (1975),Yang and Molinas (1982), van Rijn (1984)) in which the various equations are applied to large volumes of data for various rivers and catchments around the world with the objective of determining the relative reliability of the equations. Each publication comes forward with a different equation as being the most reliable. The reliability depends on the data because, as a result of the complexity of the subject, most of the equations incorporate empirically based functions determined for specific data one way or another. Moreover, the identification of the "best" model in the literature is a matter of subjectivity on the part of the relevant authors and a function of the method of comparison. In order to select a set of equations for a particular application, the first criterion must be that the these should have been calibrated on data obtained in situations similar to that at hand. Immediately this presents a problem since little or no equations have been developed specifically for tailings beaching. It is, however, evident from the literature (Raudkivi (1991)) that:

- the quality of predictions of the various models generally improves with increasing sediment load transport, and that

- certain of the equations, specifically those of Yang (1982) are not reliable at large flow depths.

In the case of tailings slurries, concentrations are particularly high, and flow depths on the beach very small in comparison to the large flow depths characteristic of the rivers used to calibrate and check most of the above equations and approaches. This bodes well for tailings.

The majority of models developed are confined to a steady state condition in which sediment and fluid are neither lost nor gained during flow. This is clearly inapplicable in tailings deposition processes. For cases where sediment inflow and outflow do not balance, such as in erosion or deposition processes, it is best to treat these with the aid of numerical models.

Raudkivi (1991) developed a rule of thumb which can be applied in river sedimentation studies to define the form of sediment transport according to the ratio of the particle fall velocity (w) to the shear velocity ($u_* = (gy_0S)$ as follows:

$2 < w/u_* < 6$	bed load
$0{,}6 < w/u_* < 2$	saltation
$0 < w/u_* < 0{,}85$	suspension

where g = gravitational acceleration, y_0 is the maximum depth of flow and S the slope of the bed.

Bed load describes sediment transported in sliding and rolling mode along the bed. This mode of transport occurs at relatively low shear stress and the grains generally move intermittently. *Saltation* describes transport where particles adopt a hopping or bouncing mode of movement. This mode would occur at intermediate shear stresses. In the case of *suspension*, which occurs at high rates of shear, the upward diffusion of turbulence from the boundary maintains the particles in suspension against gravity. According to Raudkivi, once the suspension phase of transport has developed the subdivision is less meaningful, although there should still be particles which roll and slide and those which saltate. Observation of the high turbulence of the flow regimes typical of tailings flow down a beach (Bentel (1981)) and by the author suggests that tailings sediment transport would follow the suspension mode.

Sedimentation velocities reported by Mittal and Morgenstern (1976) for various fine tailings, where the maximum size ranged from 75 micron to 100 micron, were of the order 10^{-3} to 3×10^{-3} cm/s. If these are taken as representative of the fall velocity and a typical tailings beach flow depth of 1,5 cm at a slope of 1:500 is taken, the above ratio will range from $5{,}8 \times 10^{-4}$ to $1{,}7 \times 10^{-3}$

. It is therefore evident that the form of sediment transport on tailings dams constructed of fine tailings dams falls within the description suspension. Coarse particles of the size 0,5 mm have a fall velocity of some 6 cm/s. If this is taken as representative of the size in contact with the bed close to the point of deposition where the flow depth could typically be 3 cm and the slope 1:10 the ratio will be 0.34. This ratio too indicates that the transport form would be described as suspension. In reality the sedimentation velocity of the coarse tailings would be considerably lower most likely of the order 0,5 to 0,8 cm/s. These figures further confirm that tailings sediment transport is by suspension.

A further mode of transport which can have particular significance in the case of fine tailings slurries high in clay and fine silt size fraction and their consequent *hyperconcentrated* state is that of wash load. The very fine particles in general tend to be suspended in Brownian motion.

James(a) reports that the physics of the interaction between suspended sediment particles and the bed material and for bed load is poorly understood at present. Any mathematical description of this process must be speculative to a considerable degree. A purely theoretical model for the bed boundary condition of the suspended material concentration distribution is not possible at the present state of research.

The confinement of tailings slurries generally to suspended transport modes means that bed load equations developed for bed load transport such as those of Du Boys, Shields, Meyer-Peter and Muller, Yalin (1972), van Rijn (1984), and to a greater extent Einstein and Bagnold, have only limited relevance. Samaga et al (1986) showed that Einstein's relations did not predict transport rates for four suspended sediment mixtures of varying arithmetic mean diameter and standard deviations in series of experiments in which suspended load transport rates of individual fractions were measured and compared with Einstein's method of calculation. The sediments comprised predominantly sands with maximum size of 2 mm and percentages less than 75 microns from zero to 4 %. Conversely, Low (1989) compared Einstein's relation (as modified by Brown) with measured data on sediments of low particle specific gravity (from 1,17 to 2,46). The ratio of w/u. for the sediments ranged from 2,5 to 4 which, in terms of Raudkivi's guidelines would mean that the mixtures were generally in the bed load range. Reasonable predictions were made by the relation indicating that the landmark research conducted by Einstein and several other researchers who have suggested modifications to Einstein's original equations has application but specifically to bed load situations.

The effect of the presence of one size of particle on the transport rate of another size in the case of a nonuniform sediment should be taken care of through correction factors introduced by Einstein but checks by several researchers notably Misri et al (1984) have shown that even with the corrections, Einstein's method yields unsatisfactory predictions of sediment load. Swamee (1991) presented empirically based data on nonuniform sediments in both the bed load and the suspended load ranges using data originally used to evaluate the Einstein relations and those of subsequent researchers such as Somaga et al (1986). Swamee was evidently pleased to find empirical relationships that predicted transport rates where 80 % of the data fell within the 50 % error bands !.

James(b) discusses three approaches to catering for the interaction between bed load material and suspended load since it is generally accepted that suspended load in low concentration river systems is continually moving into and out of the bed load layer. One approach, the most commonly used for equilibrium conditions, assumes that the bed load and hence the bed layer instantaneously adapts to local conditions and that there is a smooth, continuous transition between the bed layer concentration and the suspension concentration. The bed layer concentration is then assumed to lie on the suspension concentration profile at some specified level and the rest of the distribution is related to this value by application of a diffusion model. An alternative approach where suspended load is considered an advanced stage of bed load has been developed by Somaga et al (1986), but this approach has not found general acceptance. It is unable to allow the computation of a vertical distribution of sediment concentration, a problem with all of the common bed load approaches. Consequently bed load models are not reviewed in detail in this dissertation.

2.2.2 Suspended Sediment Transport Relationships

Raudkivi (1991) notes that observations show that the concentration decreases with distance up from the bed. One of the aims of the theories of sediment suspension is to define this distribution of concentration. In the case of tailings deposition it is necessary to be able to define the distribution of certain particle size groups at a section. Literature on the subject of sediment suspension is voluminous but the physical and analytical description of suspension is still far from complete.

Suspended sediment approaches can loosely be divided into:

(a) diffusion

(b) energy

(c) stochastic

(d) numerical.

The approaches all have as their basis a system for simulating the transportation process which is then extended to determine sediment load.

(a) Diffusion Models

Diffusion models have found the greatest acceptance for practical applications to suspended sediment and are usually also part of the numerical or mathematical models of suspended sediment transport.

In these models mixing of the fluid and solid particles is assumed to be caused by mechanisms of random molecular motion of the fluid, turbulent mixing and velocity differentials over the cross section of the stream. The first two mechanisms are referred to as diffusion or dispersion while the third mechanism is referred to as convective diffusion.

The analysis is based on a combination of Fick's Law and conservation of matter:

Fick's Law:

$$P = - D \frac{\partial c}{\partial y} \qquad (1)$$

where

P = rate of transport across a unit area normal to the y-direction

D = coefficient of diffusion or diffusivity

c = concentration (by weight) of material transported by diffusion or, in the case of probabilistic

theories, the probability of occurrence of a single particle in a given unit volume.

Combined with conservation of matter this yields:

$$\frac{\partial c}{\partial t} = - \frac{\partial P}{\partial y} = \frac{\partial^2 c}{\partial y^2} \qquad (2)$$

Fick's Law can be generalised to:

$$\frac{\partial u}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial x^2} + D\frac{\partial^2 c}{\partial y^2} + D\frac{\partial^2}{\partial z^2} \qquad (3)$$

or ;

$$\frac{\partial c}{\partial t} + u_i\frac{\partial c}{\partial x_i} = D\frac{\partial^2 c}{\partial x_i \partial x_i} \qquad (4)$$

Incorporation of the analogy of turbulent flow to molecular motion and addition of the gravitational fall velocity term leads to the following equation:

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}_i\frac{\partial \bar{c}}{\partial x_i} = \frac{\partial}{\partial x_i}\left[D_{ij}\frac{\partial \bar{c}}{\partial x_i} + D_M\frac{\partial \bar{c}}{\partial x_i}\right] + \frac{\partial}{\partial y}[w_s c] \qquad (5)$$

where D_M refers to molecular diffusion and D_{ij} is the turbulent diffusivity tensor and $\bar{c}$ and $\bar{u}$ represent the mean values of c and u respectively.

Several simplifying assumptions are generally made at this stage in order to render the above equation more useful:

1 generally the turbulent diffusion, D_{ij} is much larger than the molecular diffusion D_M and the latter term can be neglected.

2 the turbulent diffusion is assumed to be homogeneous and isotropic so that D_{ij} reduces to a scalar function D_T. In an analogy to the diffusion coefficient for fluid momentum, (usually

represented by the eddy viscosity ε) the diffusion momentum coefficient for sediment is denoted by ε_s.

3 concentration is assumed to be a function of elevation y only, and independent of lateral or longitudinal coordinates.

4 a steady state of concentration is assumed ie $\delta c/\delta t = 0$.

5 steady flow is assumed.

6 suspension of one particle size only is assumed.

7 low concentration is assumed.

8 constant fall velocity is assumed, ie $\delta w/\delta y = 0$.

Assumptions 5 to 8 are not applicable to tailings slurries. Since tailings slurries are strongly depositional flow can only be regarded as approximately steady over very short distances, mill tailings is made up of a range of particle sizes and the tailing slurry has a high concentration. Coarse and fine particles are variably distributed through the section and it is therefore incorrect to assume that the fall velocity is constant. It is therefore not relevant to pursue the various solutions such as those of Rouse (Raudkivi (1991)) and van Rijn (1984) other than to take note of Rouse's equation and the predicted distribution of concentration with depth.

Under the above assumptions equation 5 reduces to :

$$w\frac{\partial c}{\partial y} + \frac{\partial}{\partial y}\left[\varepsilon_s \frac{\partial c}{\partial y}\right] = 0 \qquad (6)$$

Integrating once yields:

$$wc + \varepsilon_s \frac{\partial c}{\partial y} + A = 0 \qquad (7)$$

The constant of integration is set to zero in line with the assumption that up and down movements must balance. This also implies that the concentration at the water surface must go to zero since ε_s must be zero in the surface.

The solutions then start from:

$$wc + \varepsilon_s \frac{\partial c}{\partial y} = 0 \tag{8}$$

or:

$$\ln \frac{c}{c_a} = - w \int_a^y \frac{dy}{\varepsilon_s} \tag{9}$$

where y = a is a reference level at which c = c_a.

ε_s is usually assumed to be a constant which yields:

$$\frac{c}{c_a} = \exp \left[- \frac{w}{\varepsilon_s} (y - a) \right] \tag{10}$$

ε_s is then taken to be equal to ε, the momentum coefficient in fluid which, in turbulent flow, has a parabolic distribution over depth given by:

$$\varepsilon = \kappa u_* y(1 - y/y_0) \tag{11}$$

which follows from fluid equations for turbulent flow using the Prandtl mixing length concept. κ is the von Karman constant.

The most commonly used solution to equation 10 is that proposed by Rouse:

$$\frac{c}{c_a} = \left[\left(\frac{y_0 - y}{y_0 - a} \right) \frac{a}{y} \right]^z \tag{12}$$

where z = w/κu..

Figure 2.1 illustrates the distributions predicted by equation 12. Specifically, the figure indicates that fine grained sediments for which w is small and therefore z is small, have a relatively uniformly distributed concentration distribution, whereas coarse sediments for which z is larger,

show a large variation in concentration over depth.

The distribution produced by Rouse's equation is based on a logarithmic velocity distribution. This equation is valid only for the lower 15 to 20 % of the flow depth. The value of κ for sediment laden flow is the subject of some debate. Researchers have both established that its value varies considerably with concentration and that it remains essentially constant at 0,433. Notwithstanding the shortcomings of the equation viz a viz the intrinsic assumptions in its derivation, Rouse's equation has been shown to fit observed data quite well with suitable exponents of z.

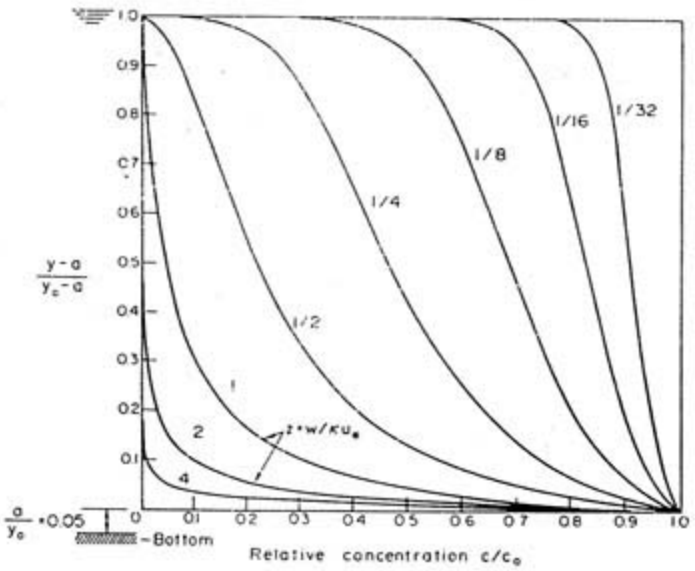


FIGURE 2.1 DISTRIBUTION OF SUSPENDED LOAD IN A FLOW ACCORDING TO THE EQUATION OF ROUSE - Equation 12. (After Raudkivi (1991))

This has led to numerous proposals for evaluation of z which is a function of the effects of the sediment on turbulence, the effect of sediment and turbulence on the fall velocity, the grading of the sediment and bed features and secondary currents. Most of the proposals are based on specific data and involve curve fitting.

James (a) notes that investigations by Taggart et al (1972) to assess the applicability of Rouse's equations to heterogeneous sediment if represented by d_{50} obtained unsatisfactory correlations with measurements. This is because the d_{50} of the sediment varies with depth and at no level

does it correspond to d_{50} of the bed material. Taggart et al suggest computation of the distribution of suspended heterogeneous sediment by superposition of homogeneous fractional distributions. The fractional distribution must be computed with cognisance of the dependence of the ratio of sediment and momentum diffusivities on sediment size and the dependence of the von Karman constant on concentration as well as u. and y.

Hunt (1969) presented a theoretical derivation of particular significance to tailings since it avoids all but one of the simplifying assumptions necessary to arrive at Rouse's equation that are problematic to tailings slurries. His derivation accommodates multiple particle sizes, high concentrations and variable fall velocities. The only assumption retained relates to that of steady flow which implies no flux of sediment in the vertical direction. Use of the equations would require calculations to be carried out over short distances and the assumption of a stepped change in slurry concentration with distance. To simplify the derivation Hunt introduces one further intrinsic assumption - the eddy diffusivity is a function only of height above the bed, ie the known fact that the eddy diffusivity is a function of concentration is ignored.

From the diffusion equations Hunt (1969) derives the general equation:

$$\varepsilon_r \frac{\partial c_r}{\partial y} + c_r \left[w_r - \frac{\varepsilon_w \Sigma (w_r c_r / \varepsilon_r)}{1 - \Sigma c_r + \varepsilon_w \Sigma (c_r / \varepsilon_r)} \right] = 0 \quad r = 1,2,3... \qquad (13)$$

where r represents each of the size fractions. If the simplifying assumption that $\varepsilon_r = \varepsilon_s$ ie that the momentum exchange coefficient is the same for individual particle sizes and equal to the coefficient for the mixture, then

$$\varepsilon_s \frac{\partial c_r}{\partial y} + c_r \left[w_r - \frac{\varepsilon_w \Sigma (w_{rc_r})}{\varepsilon_s + (\varepsilon_w - \varepsilon_s) \Sigma c_r} \right] = 0 \qquad (14)$$

It is of interest to note that for a special case of a homogeneous sediment this equation reduces to:

$$[\varepsilon_s + c(\varepsilon_w - \varepsilon_s)] \frac{\partial c}{\partial y} + (1 - c) c w = 0 \qquad (15)$$

and that for the special case of a homogeneous sediment of low concentration and assuming that ε_w is equal to ε_s the equation reduces to that of Rouse.

The effect of the inclusion of concentration was described as physically implying that downward flux of the heavier (larger size) sediment is balanced by the upward flux of the displaced finer sediment. Hunt refers to this as a "squeezing" process whereby the heavier coarse particles close to the bed use up the available space and displace a lot of the finer particles upwards. Hunt notes that experiments by various researchers has shown that $\varepsilon_s/\varepsilon_w = \beta$ to be approximately constant between 1,2 and 1,5 depending on the concentration of coarse material.

Summation of equation 14 over all size fractions and putting $\varepsilon_s = \varepsilon_w = \varepsilon$ yields:

$$\varepsilon \frac{\partial}{\partial y} \Sigma c_r + (1 - \Sigma c_r) \Sigma w_r c_r = 0 \qquad (16)$$

The non linear system of simultaneous equations represented by equation 16 has an explicit solution:

$$c_r(y) = \frac{c_r(\alpha)\exp[w_r(\psi(\alpha) - \psi(y))]}{1 - \sum_i [c_i(\alpha)[1 - \exp\, w_i(\psi(\alpha) - \psi(y))]]} \qquad (17)$$

where α is a reference level and :

$$\psi(y) = \int \frac{dy}{\varepsilon(y)} \qquad (18)$$

Thus the ratio of the concentration of any two size fractions $c_i(y)$ and $c_j(y)$ at a given y is given by:

$$c_i(y) \exp(w_i \psi(y)) = A_{ij}\, c_j(y) \exp(w_j\, \psi(y)) \qquad (19)$$

where :

$$A_{ij} = \frac{c_i(\alpha)}{c_j(\alpha)} \exp\left[(w_i - w_j)\psi(\alpha)\right] \tag{20}$$

Hunt derived the following equations for the vertical distribution of a heterogeneous sediment :

$$c_r(y) = \frac{c_r(\alpha)\left(\frac{Z}{Z(\alpha)}\right)^{\gamma_r}}{1 - \sum_i \left[c_i(\alpha)\left(1 - \left(\frac{Z}{Z(\alpha)}\right)^{\gamma_i}\right)\right]} \tag{21}$$

where :

$$Z = \frac{y/y_0}{1 - y/y_0}, \qquad Z(\alpha) = \frac{\alpha/y_0}{1 - \alpha/y_0} \tag{22}$$

$$\gamma_r = w_r/\kappa u_*, \qquad \gamma_i = w_i/\kappa u_* \tag{23}$$

and $\varepsilon(y)$ may be represented from the bed to $y_0/2$ by a parabolic equation also proposed by van Rijn (1984):

$$\varepsilon(y) = \kappa y u_* \left(1 - \frac{y}{y_0}\right) \tag{24}$$

van Rijn (1984) proposed a constant ε from $y_0/2$ to y_0 in which case :

$$\varepsilon(y) = constant = 0{,}5\ \kappa y u_* \tag{25}$$

provided κ is taken as constant.

Hunt (1969) applied the equations 21 to 24 to a theoretical slurry containing a mixture of two

particle sizes and obtained the following sediment distribution curves for varying ratio of fall speeds :

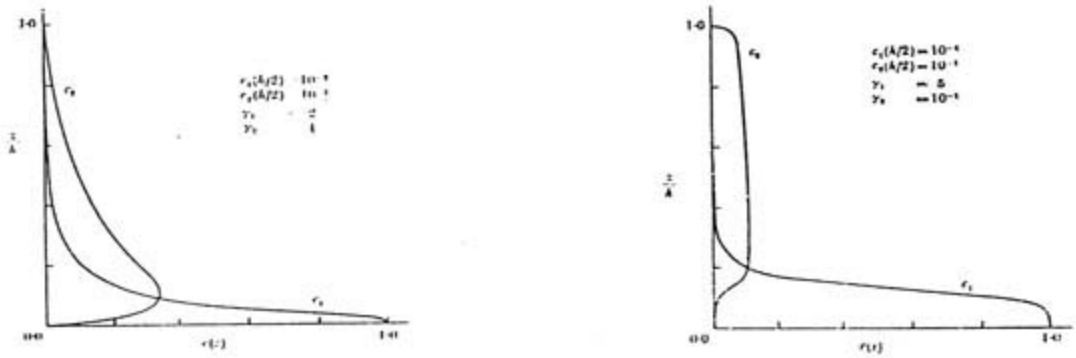


FIGURE 2.2 SEDIMENT DISTRIBUTION CURVES FOR A MIXTURE OF TWO PARTICLE SIZES, WITH VARYING RATIO OF FALL SPEEDS (After Hunt (1969))

Hunt(1969) concludes that when fall velocities of fine and coarse particles differ by several orders of magnitude, there is very little overlap in the vertical distributions; the light particles will be found in the upper layers of the flow and the heavy particles near the bed. When the fall velocities differ only slightly, the distributions overlap to a considerable extent, but again the majority of the heaviest material appears in the lowest layer adjacent to bed level.

The theoretical conclusions developed by Hunt(1969) have not been experimentally checked possibly because of the relative obscurity of the publication but more likely because in river sedimentation the vertical distribution of sizes in a heterogeneous sediment is not of primary concern. Cooke and Lazarus (1988) describe testwork conducted using slurries flowing in a pipeline aimed at proving the reliability of a sampling device referred to as an isokinetic sampler. The device is able to sample sediment from any location within a pipeline while this is flowing and ensures that the sample is extracted through a sampling circuit while the sample is flowing at the same rate as it would in the pipeline. Figures 2.3 to 2.7 below are extracted from the paper by Cooke and Lazarus (1988).

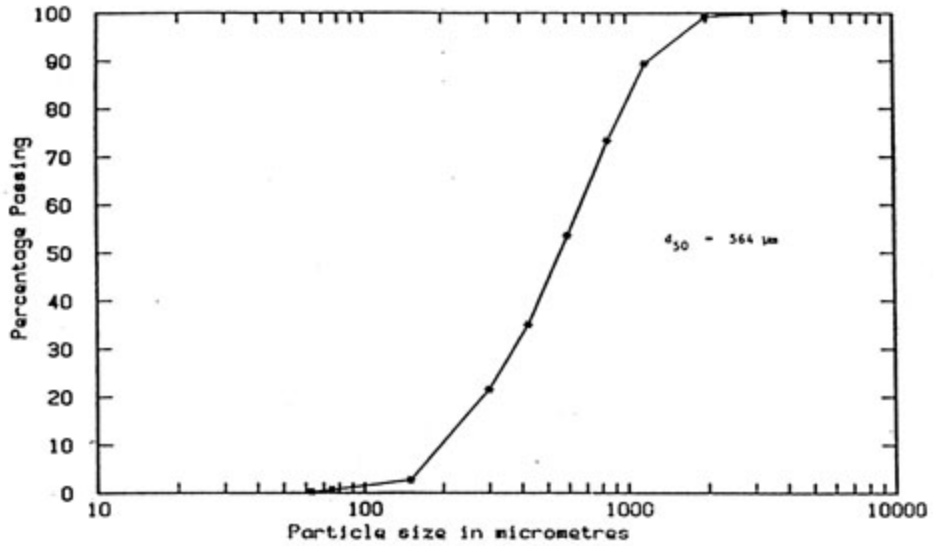


FIGURE 2.3 SAND PARTICLE SIZE DISTRIBUTION (After Cooke and Lazarus (1988))

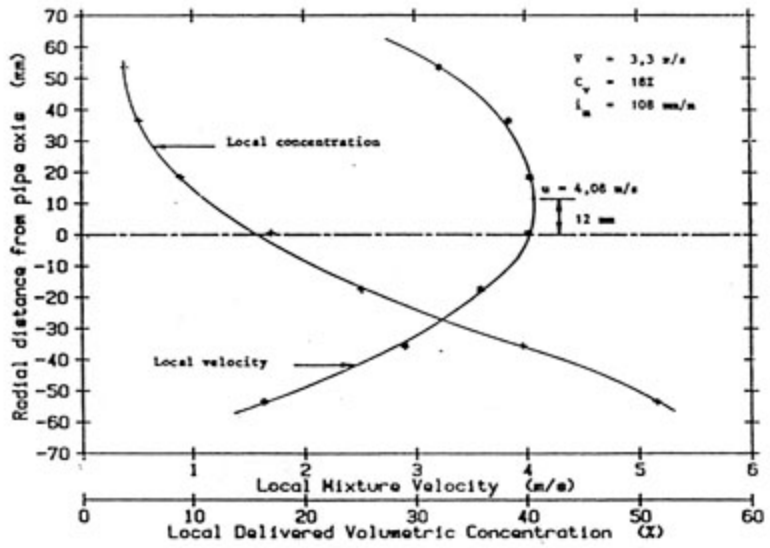


FIGURE 2.4 MEASURED VELOCITY AND CONCENTRATION PROFILES (After Cooke and Lazarus (1988))

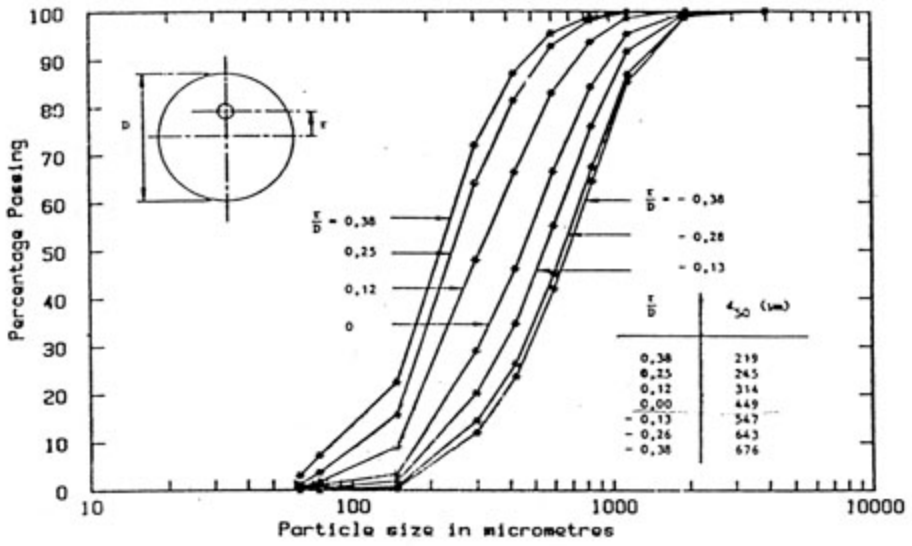


FIGURE 2.5 PARTICLE SIZE DISTRIBUTION AT DIFFERENT VERTICAL POINTS WITHIN PIPE CROSS SECTION (After Cooke and Lazarus (1988))

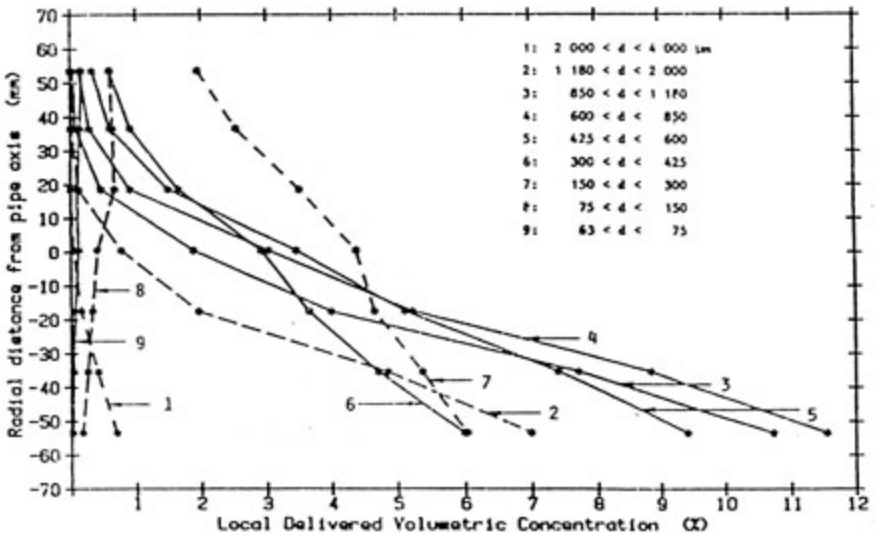


FIGURE 2.6 VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS (After Cooke and Lazarus (1988))

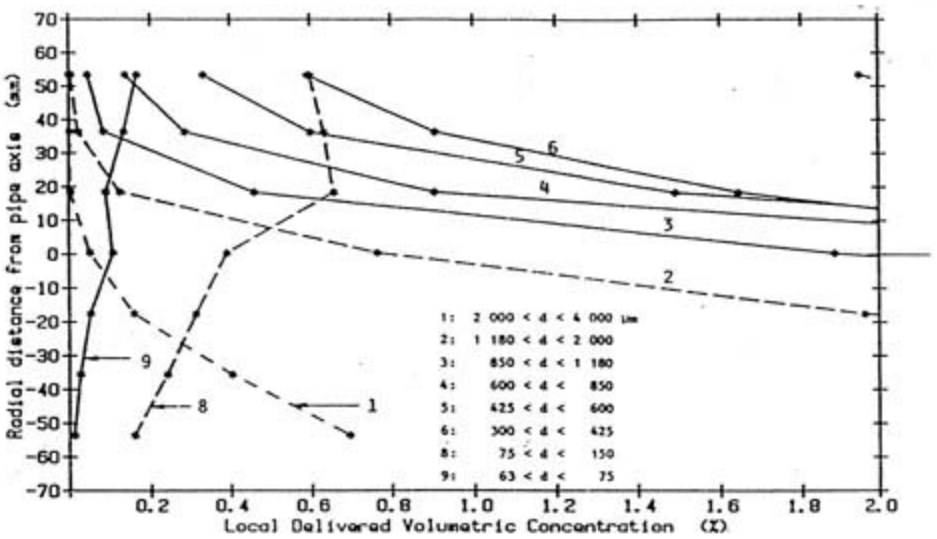


FIGURE 2.7 VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS - LARGER SCALE (After Cooke and Lazarus (1988))

The tendency for the coarse particles to predominate at the bottom of the pipe and the relative shapes of the distributions of the finer and coarser size fractions clearly resembles those predicted by Hunt (1969). This author will attempt to apply Hunt's equations directly to the data of Cooke and Lazarus in Section 2.2.4 in order to further confirm the validity of Hunt's equations.

The influence of hyperconcentration in sand water mixtures has been extensively studied by Winterwerp et al (1988) in a series of laboratory based flume tests. Two sands were tested, one with a d_{50} of 120 microns and the other with a d_{50} of 225 microns, at a range of concentrations from 5 % to 45 % by volume (12,2 % and 68,4 % by weight at a particle specific gravity of 2,65). While the sands are not necessarily representative of tailings in terms of particle size distribution, being free of silt and clay-size fractions, mixture concentrations cover the range applicable to tailings. There is not enough specific data in the paper to enable the calculation of the w/u. ratio and therefore determine whether sediment transport is in the suspended mode but the sediment flow relations applied relate to suspended flow. A number of conclusions that have relevance to tailings slurries were reported:

- at small specific flow rates, the flow is laminar and the grains are supported by dispersive pressures through the exchange of momentum by mutual grain to grain collision. At large specific flow rates the flow is turbulent and the velocity profiles are logarithmic.

- as concentration increases the von Karman constant first decreases to about 50 % of its initial value and then increases up to the clear water value. Measurements are qualitative rather than quantitative and these conclusions may not be generalised or used specifically.

- the vertical sand concentration distribution in turbulent flows is governed by a balance between hindered settling and eddy diffusivity. As a result the sand concentration as a whole can be described fairly well by Rouse's distribution.

- due to the dominant effects of hindered settling, for a given specific flow rate and slope, a two-valued solution exists for the diffusion equation yielding two different mean concentrations depending on the initial conditions.

- the sand concentration near the bed first increases with increasing mean concentration. It is hypothesised that already at relatively low mean concentrations a thin layer of hight concentration is formed near the bed. An increase in mean concentration will result in an increase in thickness of this layer and thus in fuller concentration profiles, with smaller concentration gradients and a decrease in turbulent damping. Quantitatively this effect is strongly dependent on grain size.

A formula is found to describe the sand transport of hyperconcentrated flows. This formula resembles the transport formulation of Engelund-Hansen, modified for the effects of hindered settling. The equations utilised were:

$$\Phi = \frac{0{,}4\ \psi^{2{,}5}}{f_0} \qquad (26)$$

where Φ is the non dimensional sand transport factor:

$$\Phi = q\, c_v \left[d^3\, S_s\, g\right]^{-0,5} \tag{27}$$

(S_s is the particle specific gravity, d the particle diameter, q the flow rate and c_v the volumetric concentration) and where ψ is the non dimensional shear stress:

$$\psi = \frac{u_*^2}{d\, S_s\, g} \tag{28}$$

The parameter f_0 is the Darcy-Weisbach friction factor.

Winterwerp et al define an effective grain diameter:

$$d_{eff} = d_{50}\,(1 - c_v)^2 \tag{29}$$

If this equation is applied to Engelund-Hansen and the equilibrium slope, S_{eq}, solved for, then the following equation is obtained:

$$S_{eq} = \left[S_s^2\, \frac{d_{50}}{0.05}\, c_v\, (1 - c_v)^2\right]^{0,6} \left[\frac{g^2}{8 f_0}\right]^{0,1} q^{-0,4} \tag{30}$$

- the equilibrium slope of a turbulent flow of the mixture is a function of specific flow rate and grain size, and at concentrations below 20 % by volume (39,4 % by weight at a particle specific gravity of 2,65) almost independent of mean concentration. This conclusion is in agreement with experience from slurry transport through pipelines.

- at very high concentration, the increase in apparent viscosity will become dominant, the flow will become laminar and grain stresses will prevail.

(b) Energy-based Models

Energy-based models incorporate three broad approaches:

- **Gravitational Theory Approach :** this is based on a continuity requirement that the time-averaged vertical transport of water and sediment must be zero, and that in the lifting process the flow must lose energy due to friction. This yields an equation of very similar form to that developed in the diffusion approach since most of the simplifying assumptions are similar.

- **Bagnold-Vlugter Approach :** in which the excess weight of solids in motion is assumed to be supported by momentum transfer from solid or from fluid to solid or both. The fluid must lift the solids at the rate that these are falling under gravity.

  The mass per unit area of suspended sediment, m, is acted on by gravity and buoyancy as illustrated in Figure 2.8.

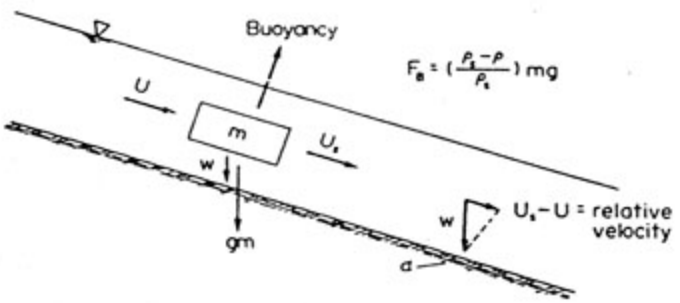


FIGURE 2.8: THE BAGNOLD-VLUGTER MODEL OF SUSPENSION (After Raudkivi (1990))

The maintenance of suspension requires a power input F_B w cosα and the weight component provides the power $[(\rho_s - \rho)/\rho_s]$mg sinα U_s, where U_s is the relative velocity of the suspension. The difference has to come from flow, ie the net power demand is:

$$P_N = \left[\frac{\rho_s - \rho}{\rho_s}\right] mg\ (w \cos\ \alpha\ -\ U_s \sin\ \alpha) \tag{31}$$

$$= \left[\frac{\rho_s - \rho}{\rho_s}\right] mg\ U_s \left[\frac{w}{U_s}\cos\ \alpha\ -\ \sin\ \alpha\right] \tag{32}$$

where P_N is the power and tends to zero when w = U_s tanα ie the power input from the weight component covers the power requirement for suspension. For $P_N \leq 0$ a state of auto-suspension exists.

In terms of the Bagnold-Vlugter derivation, the energy balance requires that the energy of the total mass available through the slope must equal the energy loss

$$(m_s\rho_s + m\rho)gSU_s = m_s\ (\rho_s - \rho)gw \cos\ \alpha\ + mgU_sS\left(\frac{U_s}{U}\right)^2 \tag{33}$$

where $(U_s/U)^2$ accounts for the ratio of energy loss in flow with sediment and without, including the energy transfer from water to sediment. Hence:

$$\rho_s m_s SUs + \rho mSU_s = m_s(\rho_s - \rho)w \cos\ \alpha\ + m\rho\ U_sS\left(\frac{U_s}{U}\right)^2 \tag{34}$$

which reduces to :

$$\frac{m_s}{m} = \frac{\frac{\rho}{\rho_s}\left[\left(\frac{U_s}{U}\right)^2 - 1\right]}{1 - \frac{\rho_s - \rho}{\rho}\frac{w}{S\,U_s}\cos\alpha} \qquad (35)$$

- **Itakura and Kishi Approach :** in which the energy per unit volume is put equal to the rate of power pressure fluctuations, the rate of energy dissipation through the motion and collisions of grains and the dissipation of energy by water flow. The approach again leads to an equation of similar form to Rouse's equation. Raudkivi notes that while the approach has yielded good predictions on laboratory data it has yet to be adequately tested in the field.

(c) Stochastic Models

In the development of stochastic models it is argued that since suspension is maintained by turbulence which is random by nature, the description of suspended sediment should be subject to description by probabilistic methods. Assumptions made in the development of these theories are generally acceptable when the solid and the fluid have similar densities but lead to suspect results in cases, such as in tailings slurries where the density ratio is large. In the approach the force required to accelerate the particle is equated to the viscous resistance force according to Stokes law plus any external potential force. Statistical models have not yet, however, established themselves in practical application.

(d) Numerical Models

Numerical models enable the use of more complex descriptions of turbulent mixing. The basic equations are mass and momentum balance for fluid and sediment in turbulent flow. A hydrodynamic turbulence model, such as the k - ε model, is used together with the application of the Reynolds equations to define the fluid and sediment mixing coefficients. Finite difference techniques are usually applied to solve the equations numerically.

According to Raudkivi (1991), Celik and Rodi (1985) combined the hydrodynamic model with a concentration distribution described by:

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\varepsilon}{S_n} \frac{\partial c}{\partial y}\right] + \frac{\partial}{\partial y}(w\ c) \qquad (36)$$

where S_n is the Schmidt number and was taken as 0,5, and ε_s is taken as equal to 2ε.

At the bed the net sediment flux is set equal to the change in suspended load transport rate d_{qs}/dx as:

$$\frac{d_{qs}}{dx} = -\varepsilon_s \frac{\partial c}{\partial y} - wc = E - D \qquad (37)$$

where E and D represent entrainment and deposition rates respectively.

2.2.3 Total Sediment Transport Equations

The total sediment load is the sum of bed load, suspended load, and wash load. In river sedimentation researchers are more often not specifically interested in the volume of material transported in a particular mode. Frequently total sediment load in river sedimentation amounts to bed load only. The approaches of Einstein, and the other researchers mentioned at the start of this section find application most frequently. In the case of most tailings, however, only suspended load is of specific importance by virtue of the sizing and concentration of the suspension. But certain approaches have been developed for total sediment transport that have application in tailings by virtue of the data used in their calibration or the assumptions intrinsic in their development. Two such approaches are the stream power approach and the entropy approach. These are discussed below.

(a) stream power approach

The energy concept in relation to stream channel geometry and sediment transport has been studied extensively by Yang (1972, 1973,1976, 1977, 1979, 1982, 1984) and by Song and Yang (1976, 1980, 1982). Yang's approach began with dimensional analysis and refinement by multiple

regression on the rate of expenditure of potential energy expressed as unit stream power US. Yang (1972) reasoned that the only source of energy a unit mass of water in a natural stream can have is its potential energy above a datum. For engineering purposes gravitational acceleration can be neglected as (small) compared to the variation of other variables. The unit stream power will be defined as the time rate of potential energy expenditure per unit weigh of water in an alluvial channel. This unit stream power can be expressed mathematically in terms of average water velocity and energy slope by:

$$\frac{dY}{dt} = \frac{dX}{dt}\frac{dY}{dX} = US \qquad (38)$$

in which y = the elevation above a datum which also equals the potential energy per unit weight of water above a datum; X is the longitudinal distance and t = time. For steady uniform flow, the energy slope can be replaced by water surface slope without introducing any error.

For higher concentrations in which incipient motion can be neglected Yang's equation emerged as:

$$\log c_T = 5{,}165 - 0{,}153 \log\left(\frac{wd}{\nu}\right) - 0{,}297 \log\left(\frac{u_*}{w}\right) + \left[1{,}780 - 0{,}360 \log\left(\frac{wd}{\nu}\right) - 0{,}480 \log\left(\frac{u_*}{w}\right)\right] \log US/w \qquad (39)$$

where C_T is the total sediment concentration in parts per million (ppm) by weight, d the mean particle diameter, and ν is the kinematic viscosity of the suspension. The equations were developed from data encompassing mean grain sizes from 0,15 mm to 1,71 mm, slurry densities from 1,07 to 1,85 (in the laboratory tests), channel widths from 0,134 to 532 m, flow depths from 0,01 to 15,2 m, flow velocities from 0,23 to 1,97 and slopes from $4{,}3.10^{-5}$ to $2{,}79.10^{-2}$. These ranges certainly encompass those to be found on tailings dams. A study by van Rijn (1984) using data other than that used by Yang to develop his equation but similar ranges of particle size, channel geometry and flow conditions indicated that Yang's equation underestimated sediment concentrations at large flow depths. It is generally apparent, however, that the quality of

predictions improves with increasing suspended load transport.

Rooseboom (1975) produced a theoretical derivation of the general form of Yang's equation:

$$\log C_T = z_1 \log U S + \log \left[\frac{\alpha_1 \alpha_2 c_a}{\left(S d \frac{\sqrt{2 \pi g d S}}{y_0} \right)^{z_1}} \right] \qquad (40)$$

where c_a is the concentration at a reference point a distance a from the bed, $z_1 = 0{,}822\ z$, ($z = w/\kappa u_*$ and κ is von Karman's constant) and α_1 and α_2 are constants. Yang and Molinas (1982) published a derivation notably similar to that developed by Rooseboom but without acknowledgement to Rooseboom (presumably because they were unaware of Rooseboom's work) and arrived at a similar equation to Rooseboom's. Yang and Molinas simplified the equation to the form:

$$\log c_T = M_1 + N_1 \log \left[\frac{US}{w} \right] \qquad (41)$$

where w is the average fall velocity of the sediment particles and US is referred to as the unit stream power.

It is important to note that the above derivations make very similar assumptions to those of the diffusion approach and therefore have the same intrinsic shortcomings as far as tailings slurries are concerned. Of over-riding importance, however, is the fact that calibrations have been made on materials similar in concentration, size and flow conditions to tailings under laboratory conditions, and the calibrated equations could still find suitable application to tailings.

Equations derived for the stream power approach provide no data on the vertical distribution of concentration.

(b) entropy approach

There is a striking resemblance between a tailings beach and the characteristic sorting of particle sizes from coarse at the point of discharge to fine at the pool and a river profile where the largest boulders are found in the head waters and the finer materials predominantly in the deltas. Using a probabilistic argument and random walk modelling, Leopold and Langbein (1962) predicted the longitudinal river profile. The principle on which the profile equations was developed was that the most probable condition in a river exists when the energy is as uniformly distributed along the river as physical constraints allow. Leopold and Langbein note that there are insufficent hydraulic equations to determine the five most important variables in a river system, namely velocity, depth of flow, width of flow, slope and friction. By drawing an analogy to the second law of thermodynamics which states that there is an increase in entropy in every natural process providing all systems taking part in the process are considered it is possible to circumvent the need to know as many as the five variables to define the river profile. The analogy is expanded on by arguing that the distribution of energy in a geomorphic system is one way of expressing the relative elevation of particles of water and sediment which will gradually move downhill. The longitudinal profile of a river is therefore a spatial distribution of the streambed materials with regard to their elevation and therefore with regard to their potential energy. In thermodynamics the base energy level is absolute zero temperature whereas in the geomorphic situation the base level is sea level.

On the basis of the above analogy, Leopold and Langbein determined that:

- the longitudinal profile of a stream subject only to the constraint of base level (and not subject to a length constraint will exponential with respect to elevation above the base level

- the longitudinal profile of a stream subject only to the constraint of length will be exponential with respect to length (or logarithmic with respect to elevation)

- the profile subject to both length and base level will have decreased concavity.

The most definitive work found by this author on the entropy approach to sediment hydraulics has been done by Chiu (1987). In this paper and subsequent papers (Chiu (1988, 1989, 1991), Chiu describes the concept of entropy, as already well established in statistical mechanics and

$$H(X) = -\int p(X)\ln p(X)dX \tag{43}$$

This equation is applied in conjunction with two equations characteristic of all probability density functions referred to by Chiu as *constraints*:

$$\int_0^{X_{max}} p(X)dX = 1 \tag{44}$$

and

$$\int_0^{X_{max}} Xp(X)dX = \overline{X} \tag{45}$$

where X is the mean value of X.

Chiu applies the method to modelling of the velocity distribution, the shear stress distribution, and the distribution of sediment concentration. In the case of the velocity distribution Chiu derives an equation similar in form to the Prandtl-von Karman universal velocity distribution law is produced. Chiu compares of the equations using available data and shows that the new distribution considerably improves predictions using the Prandtl-von Karman equation. In the case of the shear stress distribution a widely used equation usually derived from a conservation of mass law assuming steady uniform flow in one dimension is derived.

In the case of the distribution of sediment concentration, the derivation begins with the knowledge that the probability density function describing the "random walk" of a sediment particle released from a point source where the particle is at a point (x_1, x_2, x_3) after time t in three dimensional space satisfies the differential equation :

$$\frac{\partial p}{\partial t} = \sum_{j=1}^{3}\left[\varepsilon_j \frac{\partial^2 p}{\partial x_j^2} + \left(\frac{\partial \varepsilon_j}{\partial x_j} - u_j\right)\frac{\partial p}{\partial x_j}\right] \tag{46}$$

in which ε_j is the "diffusion coefficient" of sediment transfer in the x_j direction; u_j is the x_j component of velocity vector **U** which the vector sum of the fluid velocity and the settling velocity

of a sediment particle due to gravity.

Assuming:

- steady uniform sediment transport in the x_1 direction (longitudinal direction)
- a wide channel so that p does not vary in the transverse direction (x_3 direction)

and letting the x_2 direction be the vertical coordinate y, $p(x_1, x_2, x_3, t)$ becomes :

$$\varepsilon_y \frac{d^2p}{dy^2} + \left[\frac{d\,\varepsilon_y}{dy} - u_y\right]\frac{dp}{dy} = 0 \qquad (47)$$

Further simplifying assumptions are then introduced:

- the y component of the flow is taken as negligible (inapplicable under turbulent conditions), in which case $u_y = -w$ (the settling velocity of the particles)
- ε_y which varies over y is replaced with its depth-averaged value, ε_s

Equation 47 can then be simplified as :

$$\varepsilon_s \frac{dp}{dy} + wp = 0 \qquad (48)$$

Equation 48 is carried forward into the method of calculus of variations and the following relations developed:

$$p(y) = p_0 \exp(-wy/\varepsilon_s) \qquad (49)$$

in which p_0 is the value of p at the bed (where y = 0). By applying the *constraints* and expression for p_0 can be obtained as:

Chiu notes that p(y)dy is a probability of a sediment particle being between y and y + dy. Thus

$$p_0 = \frac{w}{\varepsilon_s\left[1 - \exp -\frac{wy_0}{\varepsilon_s}\right]} \qquad (50)$$

if N represents the total number of sediment particles of a given size (having settling velocity w) between y = 0 and y = y_0, Np(y)dy gives the number of particles between y and y + dy. N may also represent the volume or weight of sediment of that size; then, Np(y) is the sediment concentration by volume or by weight depending on whether N is in volume or weight. The vertical distribution of sediment concentration is, therefore:

$$c = c_0\exp(-wy/\varepsilon_s) \qquad (51)$$

in which c = Np(y) and c_0 = Nc_0. Equation 51 was derived in the course of deterministic hydraulics by Rouse in developing his equation. Chiu goes on to apply the entropy principle and the calculus of variations to derive a data specific equation for β where β is the parameter in the relation between the sediment diffusion coefficient and that for water:

$$\varepsilon_s = \beta\varepsilon_w \qquad (52)$$

Chiu uses data collated by Einstein and Chien in the course of their work at the U S Corps of Engineers to correlate the entropy of concentration with the Reynolds number applicable to the settling particles for a fine, a medium and a coarse sand.

2.3 Models Applied to River Sedimentation

The preceding section dealt with relevant sediment transport relations and their potential for application to tailings slurries. However, the relations form only a part of a system required for modelling of sedimentation. Uncertainties in the sediment transport relations are often added to by the vagaries of computer modelling systems and their limitations.

Lu and Shen (1986) present a comparison of various degradation models available at the time. They set out the basic requirements of any degradation model as being:

- the continuity, momentum and sediment continuity equations. Intrinsic in the above equations are the assumptions that flow is one dimensional ie the channel is straight, the pressure distribution is hydrostatic along a vertical, the density of the sediment laden water is homogeneous, friction losses in unsteady non-uniform flow are not significantly different to those in steady uniform flow.

- the diffusion model together with the assumptions that the friction slope can be approximated by the bed slope (kinematic wave assumption), the sediment transport rate can be expressed as a function of mean flow velocity, and the diffusion coefficient is assumed to be constant. The inaccuracies that result from the kinematic wave assumption increase as the bed slope reduces.

The models differ in their approach to the solution of the above sets of equations and achieve varying degrees of success in application to one specific set of data. An important aspect differentiating between them is the fact that some authors use a coupled method of solution of the sediment transport and the continuity/momentum equations while others use uncoupled solutions. The former are more representative but add considerable complexity and sensitivity to the numerical methods for solving the various equations.

Zhang and Kahawita (1987) presented details of a model which, in addition to the usual continuity and momentum equations, made use of a sediment transport relation developed by Soni et al (1984):

$$G = a_0 u_0^{b_0 - b} u^b \qquad (53)$$

where G is the unit sediment discharge in volume per unit width per unit time, the subscript 0 refers to flow under uniform conditions, and a_0 and b are constants determined by Soni et al (1984). A resistance relation of the form :

$$u = \left[8gy \frac{S}{\lambda}\right]^{0,5} \qquad (54)$$

where λ is the Darcy-Weisbach friction factor.

Vollmers (1989), in his State of the Art address at the International Symposium on Sediment Transport Modelling in August, 1989, noted that it is *"..... astonishing that nowadays in many technical areas as for example in astronautics precise projects are being executed, whereas we still do have problems in recognising the physics of our "near field events". This is a fact Galileo noticed already more than 300 years ago. The older Einstein is said to have warned his son strongly of the difficulties in dealing with sediment transport processes. Nevertheless the latter achieved a considerable contribution to the subject."*

At the above symposium several papers describing a number of computer models were presented. Van Rijn (1989) noted that there was considerable progress made on modelling in 3 dimensions - a requirement in large river modelling. Borah and Bordoloi (1989) describe the model STREAM which models both bed evolution and stream bank erosion. The model simulates sediment sorting, aggradation and degradation, armouring and transport of non-uniform sediment in alluvial streams, and predicts time varying sediment discharges, particle size distributions of sediment load and materials on the bed and bed elevation changes. Yang (1989) describes the model GSTARS which allows the computation of both subcritical and supercritical flow conditions and utilises the stream power equation of Yang (1973). Yang notes that more development work is required to enable the model to simulate surface erosion, channel networking, delta development among other possible improvements relevant in river sedimentation studies. Chang (1989) describes the model FLUVIAL_12 which enables engineering design of bank protection and grade control structures and the influence of bedrock outcrops to be catered for. The influence of lateral inflows and curves in the channel can be studied using the model. Holly and Rahuel (1989) describe a model SEDICOUP which accommodates non equilibrium solid transport in unsteady, nonuniform river flow, separation of bed load and suspended load, representation of sediment mixtures as a set of distinct size classes, sudden sediment inflows. This model incorporates coupled solution of the various equations and is able to cater for sediment concentrations in excess of the limit of validity of the Rouse-type solutions. This model is generally regarded as the most comprehensive model available at the time of writing James and van Niekerk (1993).

Van Niekerk et al (1992) and Vogel et al (1992) seem to have made a significant contribution through their development of a moveable bed model for routing of heterogeneous sediments.

The model was developed specifically to treat multiple grain densities and size distributions and therefore provide an insight into the genesis of heavy mineral placers. The model, MIDAS, simulates both bed load and suspended load and is able to handle several sediments of varying grain size distribution and particle density. The model comprises the following basic elements:

- **flow model:** this is based on the standard gradually varied flow and conservation of mass equations with one important limitation for tailings deposition - that flow will always remain subcritical. When flow becomes supercritical, then flow conditions are approximated by the critical conditions. (In tailings deposition, certainly over the initial 30 % of the beach length, the flow comprises a series of hydraulic jumps as described by Blight (1988) and Bentel (1981)).

- **bedload transport :** a modified Bagnold equation was selected over alternatives presented by Ackers and White (1973), Einstein (1950), Bridge and Dominic (1984), Meyer-Peter and Muller (1948), van Rijn (1985) and Yang (1973) because the transport rates can be determined for each grain size separately and it is well suited to numerical modelling.

- **suspended load transport :** suspended sediment concentrations for the individual size-density fractions are calculated from the convection-diffusion equation for vertical distribution of the sediment. It is assumed that steady state conditions apply over a time step.

- **bed treatment :** computation of the sediment mass exchange between the flow and the active layer, and consequently, computation of erosion and deposition at each downstream site is accomplished by using the conservation of mass equation written for each size-density fraction. The relation used was developed from approaches put forward by Lee and Odgaard (1986), Rahuel et al (1989), where the active layer thickness is taken to be a function of the water depth, and Borah et al (1982) and Park and Jain (1987), where the thickness is taken to be a function of particle size. A time step is chosen sufficiently small that the fluid flow and sediment transport rates may be considered constant over the time step.

The model was tested (Vogel (1992)) on laboratory flume data, field data from US rivers and for credibility in simulating the behaviour of dense minerals in the formation of the gold paleoplacers on the Witwatersrand in South Africa and found to operate very satisfactorily.

Consideration was given to using the model to simulate tailings deposition but it was established (van Niekerk and James (1993)) that the model was still in unrefined user unfriendly form and would not easily accommodate tailings disposal due to the sediment concentrations involved (these would be incompatible with the form of the diffusion equation utilised in the model), and the assumption of subcritical flow.

2.4 Models on Beaching of Tailings

Fan et al (1991) present a paper on the laboratory modelling of tailings beach profiles. The paper outlines the basis of a non-linear parabolic numerical model developed for the prediction of the beach profile which was then compared with the beach profiles determined in the course of laboratory-scale beaching tests. The model has several theoretical building blocks:

- the continuity equation for water and sediment
- the momentum equation for the water in a channel of constant width
- Manning's equation for the resistance relationship of the water
- The sediment transport relation developed by Meyer-Peter and Muller with constants as determined by Wiberg and Smith (1989).

The equations relating the above were simplified to facilitate non-linear modelling using a finite difference scheme.

Fan et al show that reasonable agreement is achieved with the numerical models, however, the following limitations on the work need to be noted:

- the solids concentration tested did not exceed 14,1 % by mass
- a single tailings product was tested in a laboratory flume at varying solids concentrations and flow rates but the particle size analysis of the tailings is not given.
- a sediment transport relation originally derived for gravels was found to produce results that

most closely fitted the flume tests.

No other literature relating specifically to modelling of tailings has been found.

2.5 Laboratory-scale and Pilot-scale Flume Testing

Flume testing has proved exceedingly important in the development of design criteria for hydraulic engineering and it is only natural that engineers would turn to flume testing of slurries in an attempt to model the segregation and layering effects characteristic of beached tailings. Kùpper (1991) notes that while the potential contributions of laboratory experiments are yet to be exhausted, any generalisations will ultimately be limited by scale effects. So far flume testing has not made major contributions to hydraulic fill studies. Blight et al (1985), Thomson (1984), Blight (1987), and Fan and Masliyah (1990) have confirmed the concept of a master profile of a tailings dam beach from laboratory-scale flume tests. However, similar confirmation was not obtained by pilot-scale tests by Wates et al (1987).

Kùpper reports having reviewed a number of flume-based experiments whose objectives were to forecast beach angles and fill properties as a function of a certain depositional parameter such as composition of fill and rate of deposition. The results have generally been disappointing in that few general trends consistent with field behaviour have been observed.

Flume tests do however have several significant advantages in that they permit the observation of the mechanics of the deposition process and make it possible to perform a large number of tests and study the effects of several variables at reasonable cost.

Flume tests rely on the fact that equations derived for either the flume or the field test should be independent of scale if they are normalised such that all the algebraic terms are dimensionless. Provided the flume is geometrically similar to the field example the same dimensionless equations would describe both the flume and the field cases and constants in the equations should be scalable. However, there are problems associated with scaling down hydraulic phenomena that involve sediments. Sediment size can only be scaled down to a certain point whereafter cohesive forces are introduced as a result of the fineness of the scaled down particles. These cohesive forces will invalidate the model. Grain shape, and grain surface characteristics affect resistance to flow, sediment transport, equilibrium slope, deposited density and the like and are very difficult

to scale accurately. There are practical limits to the extent to which parameters such as fluid density, fluid viscosity and sediment density can be scaled down.

It is generally accepted that, apart from the scaling of physical dimensions, hydraulic scaling to a reasonable degree can be achieved by a Reynolds-Froude model in which these parameters are kept constant between flume and field. Küpper has developed a new dimensionless parameter P' which can be scaled between a flume and the field tests represents a ratio between gravitational forces acting on the solids and inertial forces. The relationship between beach slope and the parameter P' is based on the concept that the larger the inertial forces are in relation to the gravitational forces the farther the grains could be transported, and consequently the flatter the slope will be. P' is given by:

$$P' = \frac{A\sqrt{g(G-1)}\sqrt{D_{50}\,C_w}}{Q} \quad where\ i_{av} = \frac{\rho_s}{\rho_w} \tag{55}$$

and

$$i_{av} = f(P') \tag{56}$$

where i_{av} is the average beach slope. Küpper deduces that for the Syncrude tailings tested in the flume and observed in the field the relationship between i_{av} and P' is given by:

$$i_{av} = 5\sqrt{P'} \tag{57}$$

Küpper notes, however, that the equation is only applicable to concentrations which exhibit segregating behaviour.

Yang (1980) describes the approach adopted in conducting a series of model studies to model the movement and deposition of sediment at the confluence of the Chippewa and Upper Mississippi Rivers. Apart from the necessary physical dimension scales (which were distorted for reasons of space), dimensional similarity on the basis of the Froude criterion was established so that:

$$F_r = \left(\frac{U_p}{U_m}\right)\left(\frac{y_m}{y_p}\right)^{0.5} = 1 \tag{58}$$

where U is velocity, y is depth of flow is particle diameter and p and m refer to the prototype

(field) and model respectively; equal concentrations. A unit stream power criterion was also introduced in terms of which:

$$\frac{U_r S_r}{w_r} = \left(\frac{V_p}{V_m}\right)\left(\frac{S_p}{S_m}\right)\left(\frac{w_m}{w_p}\right) = 1 \qquad (59)$$

where r denotes ratio and w is the fall velocity of the sediment particles. Yang concludes that:

- the rate of energy dissipation would be related to the rate of sediment transport and the stream power equation (59) is the most reliable of those considered in the study

- the dimensionless stream power (59) can be used as a criterion for moveable bed model studies. With proper adjustment of model parameters there is no need to distort Froude number, sediment size and specific weight, sediment concentration, and the same time scale can be used for water movement

- laboratory and field data seem to suggest that a general principle in river engineering is to work with rivers in accordance with the concept of unit stream power and the theory of minimum rate of energy dissipation.

On the whole, however, the main usefulness of flume tests arises from their ability to provide an understanding of the physical phenomena of hydraulic deposition and a development of basic concepts and formulations with which to study the full scale situation in a qualitative manner. Küpper concludes, after an exhaustive programme of flume tests that flume tests have been proved not to be suitable to predict quantitatively some properties of field scale beaches for while both produce concave beaches that are similar in shape, laboratory beaches tend to be much steeper than their field counterparts. This discrepancy precludes the direct use of flume test results for the prediction of actual beach slopes.

Blight et al (1985) conclude that flume tests could be used to determine the master profile of the beach. However, in order to predict the field profile it is necessary to make a guestimate of the overall beach slope for field conditions which cannot be estimated from flume tests.

Fourie (1988) concludes that the normalised profile does not depend on slurry concentration for non-segregating slurries which exhibited "mud flow" flow characteristics. These conclusions

are at variance with Küpper's findings but this could be attributable to the very low flow rates adopted by Fourie and the particular tailings products tested.

Blight (1987) presented the results of flume tests on power station fly ash which showed that the slope of the beach profile at first increased with concentration to a concentration by mass of 42 % (the relatively low particle specific gravity of 2,2 for the ash should be noted in assessing this concentration). Above this concentration the beach slope decreased to a minimum at concentrations of approximately 60 % whereupon the slope would probably increase once again.

Wates et al (1987) conclude from a series of pilot-scale tests in a 10 m long flume using gold mine tailings residues from backfill plants that water content and not particle size difference is the main determinant of beach angle. This conclusion is supported by Küpper whose research surely represents the most comprehensive flume-based study on the beaching of high concentration slurries of recent times although it should be borne in mind that Küpper's work is based on a virtually single sized material.

2.6 Empirically Derived Beaching Equations

Abadjiev (1984), a Bulgarian researcher, proposed three empirically based methods for estimating the particle size distribution along the beach of a spigotted tailings dam. The data from which Abadjiev derived his equations were from copper and lead recovery operations in Bulgaria and the USSR.

The first method involves extensive sampling along a trial beach and the determination of particle size distributions. The proportions of various particle sizes with distance along the beach are determined and an exponential equation fitted to each size using the method of least squares. The second method involves sampling of a number of tailings dams along representative beaches of each and the determination of particle size distributions. Ratio's of the average particle size for each sample and the average particle size of the source material for each tailings dam are plotted on a common graph and, with the continual addition of new data can be used to provide an indication of the ratio for any similarly deposited tailings. The third method involves fitting an equation to the above plots. What is of relevance to this third method is that the equation derived by Abadjiev is a function of beach length, flow rate and

pulp density.

The approach requires a large amount of data to be meaningful and would apply to specific tailings products and methods of deposition. The equations developed by Abadjiev are therefore unlikely to be suitable for general application but the fact that the equation takes into account several of the most important variables is noteworthy

2.7 Rheology and Slurry Flow Resistance Prediction

The relatively low concentrations that usually occur in river sedimentation mean that it is seldom necessary to consider the effect on the flow resistance properties of the fluid when attempting to model or simulate sediment behaviour; these can usually be approximated by applying a nominal adjustment to the resistance properties of water. This accounts for the fact that only nominal treatment of the rheology of the water-sediment mixture is given in most references on river sedimentation analysis.

The flow resistance characteristics of water can be modelled using equations that have their route in Newton's equations such as the well known Darcy-Weisbach equation. More specifically water has the characteristic that the shear resistance of water is directly proportional to the rate of shear which makes it possible to apply equations developed from Newton's laws. It is therefore referred to as Newtonian. In general the flow characteristics of solid-liquid mixtures particularly at higher concentrations of solids differ from those of Newtonian fluids because:

- superimposed on the properties of the fluid are the properties of the solid particles as well as the effect of the particles on the mixture properties.

- the particular flow conditions and hence the rates of shear influence the flow characteristics

Any consideration of the beaching characteristics of tailings would, in view of the relatively high concentration of solids, need to take into account the flow characteristics of the slurry and the influence of the shear versus rate of shearing characteristics or the rheology of the tailings.

Slurries fall into a broad class of flow forms best classified as complex mixtures usually referred to as multiphase mixtures. Figure 2.9 is an attempt to delineate the behaviour of fluids consistent with gross characteristics (Round (1986)):

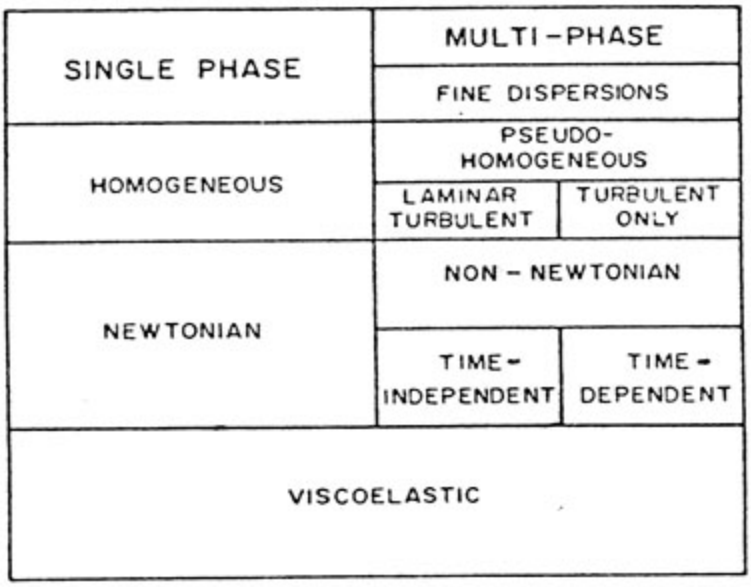


FIGURE 2.9 : CLASSIFICATION OF SINGLE AND MULTIPHASE FLUIDS

Slurries can be further categorised in terms of the influence of the materials carried by the slurry on the carrying medium. The following categories exist (Verkerk (1990)):

1 where the liquid retains its normal flow properties and carries along relatively large or heavy particles

2 where the solids may be small or light and be suspended without seriously affecting the liquid flow properties

3 where very small solids at high concentrations result in a completely homogeneous slurry of properties markedly different from normal liquids

4 where large particles are carried at high concentration by inter-particle contact and carried by a fluid that retains its normal characteristics

5 where large particles are carried in a homogeneous medium of small particles of properties markedly different from the original liquid.

Categories 1 and 2 are conveniently grouped under the heading of settling slurries whilst those in the remaining categories are classified as non/slow settling or mixed regime slurries. Most of the slurries discharged onto tailings dams are settling slurries. These can be further categorised according to the flow regimes which occur as follows:

1 pseudo-homogeneous flow where the slurry is characterised by a uniform concentration and velocity profile across the flow section

2 heterogeneous flow where the concentration increases towards the bottom of the flow section. The slurries exhibit a minimum deposition velocity below which particles settle out and are transported along the bottom of the flow section. In these flow systems particle inertial effects are prominent as opposed to homogeneous mixtures where the rheology of the system is the dominant factor. For large particle slurries the definition of the particular flow regime, heterogeneous or pseudo-homogeneous, is dependent on the flow velocity and for slurries with a range of small to large particles the definition is based on the velocity and the rheology of the system. Mine tailings slurries are regarded as mixed regime slurries because they exhibit flow regimes which are dependent on both velocity and rheology

3 saltation where settled particles move with consecutive bounces along the base of the flow section

4 sliding bed/stationary bed flow where particles slide along the bottom of the flow section (unlikely in loose boundary flow)

Newtonian fluids are characterised by a single variable - viscosity. The governing equation for a Newtonian fluid is given by:

$$\tau = \mu \frac{du}{dy} \qquad (60)$$

where τ is the applied shear stress, μ is the viscosity, u the fluid velocity and y the depth of flow.

The viscosity in a Newtonian fluid is a constant for a particular fluid.

Non Newtonian fluids are fluids which do not have constant viscosity and which may have an intercept on the τ axis. These fluids are further classified into time-independent fluids and time-dependent fluids. Typical graphs of τ versus du/dy, referred to as rheograms are illustrated in figures 2.10 and 2.11 below.

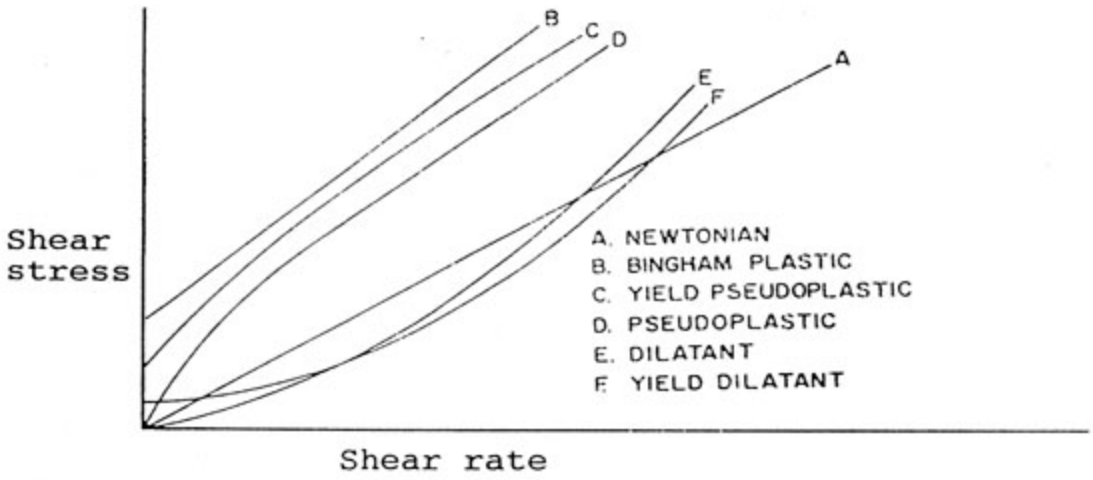


FIGURE 2.10 : TYPICAL RHEOGRAMS FOR TIME-INDEPENDENT FLUIDS

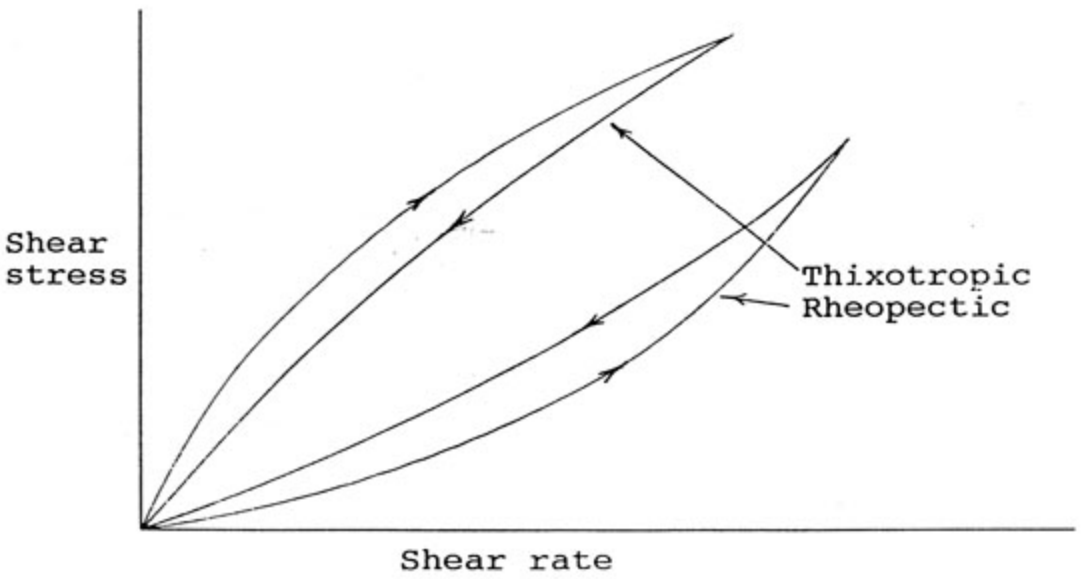


FIGURE 2.11 : TYPICAL RHEOGRAMS FOR TIME-DEPENDENT FLUIDS

Time dependent fluids show hysteretic properties as shear rates are first increased and then decreased. The majority of tailings slurries are time-independent.

Figure 2.12 summarises the common equations used for the analysis of non-Newtonian fluids. The figure was compiled by Verkerk (1990)

Verkerk (1990) documents an extensive assessment of slurry pumping and pipeline characteristics with respect to the particle size distribution and the slurry concentration. Verkerk established that of the parameters which influence the hydraulic transportation characteristics of a slurry in a pipeline, the particle size distribution is the most important because of its direct effect on the slurry rheology and the maximum packing density and thereby on the friction loss along the pipeline and the critical deposit velocity. Verkerk also notes that other factors also influence the frictional characteristics such as particle shape, particle absorption and adsorption characteristics and the attractive or repulsive charges which may be present.

It is evident from Verkerk's work that in general for the tailings slurries characteristic of most South African mining operations the slurries will display a mixture of Newtonian and Non-Newtonian characteristics with the latter becoming more predominant as the particle size distribution of the carrier medium reduces. Specifically, slurries with particle size distributions where $d_{50} > 0{,}13$ mm will generally display Newtonian rheology up to concentrations by mass of 70 %. Slurries with smaller d_{50}'s can be expected to have non-Newtonian characteristics below a concentration of 70 % with the concentration at which non-Newtonian characteristics are evident decreasing as d_{50} decreases.

Verkerk recommends that specific viscometer tests should be carried out on the slurry at varying concentration from which the rheological characteristics of the slurry can be estimated.

Verkerk concludes that the non-Newtonian method of Wasp et al (1977) based on the turbulent flow Torrence equations the Hanks dynamic parameter appear to be suitable for the prediction of the performance characteristics of mixed regime slurries typical of tailings provided the correct carrier rheology is utilised.

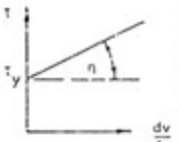
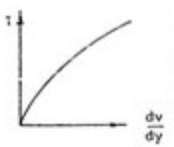
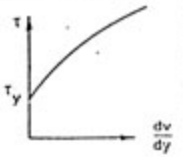
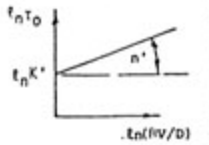
RHEOGRAM	CONSTITUTIVE EQUATION	LAMINAR FLOW		REYNOLDS NO. FOR LAMINAR FLOW	TURBULENT FLOW		
		PRESSURE GRADIENT	FRICTION FACTOR		SMOOTH WALL (SWT)	PARTIALLY-ROUGH WALL (PRWT)	FULLY-ROUGH WALL (FRWT)
NEWTONIAN 	$\tau = \mu\left(\frac{dv}{dy}\right)$ $\tau_r = \mu\left(-\frac{dv}{dr}\right)$	$\frac{D\Delta P}{4L} = \mu\left(\frac{8V}{D}\right)$ HAGEN-POISEUILLE	$f = \frac{16}{R_e}$	$R_e = \frac{\rho VD}{\mu}$ $R_e < 2100$ (Approx)	$\frac{1}{\sqrt{f}} = 4\log\left(\frac{R_e\sqrt{f}}{1,26}\right)$	$\frac{1}{\sqrt{f}} = -4\log\left(\frac{k}{3,70} + \frac{1,26}{R_e\sqrt{f}}\right)$	$\frac{1}{\sqrt{f}} = 4\log\frac{D}{2k} + 3,48$ $R_{ek} = \frac{k}{D}R_e\sqrt{f/2}$ $R_{ek} < 70$
BINGHAM PLASTIC 	$\tau = \tau_y + \eta\left(\frac{dv}{dy}\right)$ $\tau_r = \tau_y + \eta\left(-\frac{dv}{dr}\right)$	$\frac{D\Delta P}{4L} = \eta\left(\frac{8V}{D}\right)B$ $B = 1 + \frac{4}{3}\alpha + \frac{\alpha^4}{3}$ $\alpha = \tau_y/\tau_o$	$f = \frac{16}{R_{eB}}$ (Approx) $\frac{1}{R_{eB}} = \frac{f}{16} + \frac{H_e}{6R_{eB}^2} + \frac{H_e^4}{3^3R_{eB}^8}$ $H_e = D^2\tau_y\rho/\eta^2$	$R_{eB} = \frac{\rho VD}{\eta\left(1 + \frac{\tau_y D}{6\eta V}\right)}$ $R_{eB} < 2100$ (Approx)	$\frac{1}{\sqrt{f}} = 4,53\log(R_{eB}\sqrt{f}) - 2,3$ $+ 4,53\log(1-\alpha)$ (Torrance) $\alpha = \tau_y/\tau_o$	$f = \left(\frac{f_{rough}}{f_{smooth}}\right)_{water} f_{SWT}$ At $R_e = R_{eB}$	$\frac{1}{\sqrt{f}} = 4,07\log\frac{D}{2k} + 3,36$ $R_{ek} = \frac{k}{D}R_{eB}\sqrt{f/2}$ $R_{ek} < 70$
PSEUDO PLASTIC 	$\tau = K\left(\frac{dv}{dy}\right)^n$ $\tau_r = K\left(-\frac{dv}{dr}\right)^n$	$\frac{D\Delta P}{4L} = K\left(\frac{6n+2}{n}\right)^n\frac{V^n}{D^n}$	$f = \frac{16}{R_{epL}}$	$R_{epL} = \frac{8D^nV^{2-n}\rho}{K}\left(\frac{n}{2+6n}\right)^n$ RepL < 2100 (Approx)	$\frac{1}{\sqrt{f}} = \frac{2,69}{n} - 2,95 + \frac{4,53}{n}$ $\log\left\{R_{epLc}\sqrt{f^{2-n}}\right\}$ $+ \frac{0,68}{n}(5n-8)$ $R_{epLc} = \frac{D^nV^{2-n}\rho}{K8^{n-1}}$ (Torrance)	$f = \left(\frac{f_{rough}}{f_{smooth}}\right)_{water} f_{SWT}$ At $R_e = R_{epL}$	$\frac{1}{\sqrt{f}} = 4,07\log\frac{D}{2k} + 6 - \frac{2,65}{n}$ $R_{ek} = \frac{k}{D}R_{epL}\sqrt{f/2}$ $R_{ek} < 70$
YIELD-PSEUDO PLASTIC 	$\tau = \tau_y + \eta\left(\frac{dv}{dy}\right)^n$ $\tau_r = \tau_y + \eta\left(-\frac{dv}{dr}\right)^n$	$\frac{2V}{D} = n\left(\frac{\tau_o}{\eta}\right)^{\frac{1}{n}}\left(1-\alpha\right)^{\frac{n+1}{n}}\left(\frac{(1-\alpha)^2}{(3n+1)} + \frac{2\alpha(1-\alpha)}{(2n+1)} + \frac{\alpha^2}{(n+1)}\right)$ First trial assume n = 1 $\left(\frac{\Delta P}{L}\right)_{\tau} < \left(\frac{\Delta P}{L}\right)_T$		$R_{epLc} = \frac{D^nV^{2-n}\rho}{\eta 8^{n-1}}$ $R_{epLc} < 2100$	$\frac{1}{\sqrt{f}} = \frac{2,69}{n} - 2,95 + \frac{4,53}{n}$ $\log(1-\alpha) + \frac{4,53}{n}$ $\log\left(R_{epLc}\sqrt{f^{2-n}}\right) +$ $0,68\,(5n-8)/n$ (Torrance)	$f = \left(\frac{f_{rough}}{f_{smooth}}\right)_{water} f_{SWT}$ At $R_e = R_{epLc}$	$\frac{1}{\sqrt{f}} = 4,07\log\frac{D}{2k} + 6 - \frac{2,65}{n}$ $R_{ek} = \frac{k}{D}R_{epLc}\sqrt{f/2}$ $R_{ek} < 70$
METZNER AND REED Generalised Approach 	$n' = \frac{d\ln\left(\frac{D\Delta P}{4L}\right)}{d\ln(8V/D)}$ $-\left(\frac{dv}{dr}\right) = \frac{8V}{D}\left(\frac{1+3n'}{4n'}\right)$ Rabinowitsch & Mooney	$\frac{D\Delta P}{4L} = K'\left(\frac{8V}{D}\right)^{n'}$	$f = \frac{16}{R_{egen}}$	$R_{egen} = \frac{D^{n'}V^{2-n'}\rho}{K'8^{n'-1}}$ $R_{egen} < 2100$ (Approx)	$\frac{1}{\sqrt{f}} = \frac{4}{n'^{0,75}} \times$ $\log\left(R_{egen}f^{1-\frac{n'}{2}}\right) -$ $\frac{0,4}{n'^{1,2}}$	$f = \left(\frac{f_{rough}}{f_{smooth}}\right)_{water} f_{SWT}$ At $R_e = R_{egen}$	$\frac{1}{\sqrt{f}} = 4,07\log\frac{D}{2k} + 6 - \frac{2,65}{n}$ $R_{ek} = \frac{k}{D}R_{egen}\sqrt{f/2}$ $R_{ek} < 70$

FIGURE 2.12 : EQUATIONS FOR NON-NEWTONIAN FLOW (After Verkerk (1990))

Wasp (1977) et al developed a method for analysing pipe flow in which they partition solids load into a symmetrically suspended vehicle and an asymmetrically suspended coarse fraction. The pressure drop for the vehicle is calculated using Newtonian homogeneous flow methods. The asymmetrically suspended part of each size cut in the particle size distribution is treated separately using an equation developed by Durand (1952) and his coworkers from extensive tests with sands, gravels and coal ranging in particle size from 0,2 to 25 mm in pipelines from 40 to 580 mm in diameter and volumetric concentrations from 2 to 23 %.

Durand developed the following relation between the energy gradient of the slurry i_s and the energy gradient of water, i flowing at the same velocity in the same pipe:

$$\phi_m = \frac{i_s - i}{i} = 1{,}5\ C_v \left(\frac{gD}{V_m^2} \frac{\rho_s - \rho}{\rho\sqrt{C_D}} \right)^{3/2} \qquad (61)$$

where C_v is the volumetric concentration, C_D the drag coefficient, v_m the mixture velocity, D the pipe diameter, and ρ_s and ρ the densities of the slurry and the carrier fluid respectively. The total pressure drop due to the asymmetrically suspended particles is calculated as the sum of contributions due to the individual size fraction cuts and is added to the homogeneous vehicle pressure drop to obtain the overall pressure drop for the whole slurry. Wasp's method involves the estimation of a concentration profile using an equation similar to that developed by Rouse (equation 12) but attributed by Wasp to Ismail:

$$\frac{e}{C_{iv}} = \left[\frac{\left(\frac{y_r}{y}\right)\left(\frac{D}{2} - y\right)}{\left(\frac{D}{2} - y_r\right)} \right]^{w/\beta\kappa u^*} \qquad (62)$$

where u^* is the shear velocity, β the ratio of turbulent eddy transport coefficient for the solids to the eddy momentum diffusivity, κ the Prandtl - von Karman constant, C_{iv} the volumetric concentration of the ith cut size and w the settling velocity for the representative particle size. The

variable y is the depth in the pipe measured from the invert of the pipe, and y_r is a reference concentration depth. Wasp assumed that β = 0,24, κ = 0,35, y = 0.8D and y_r = 0,3D.

With these assumptions Wasp modified Ismail's equation to be:

$$\log_{10}\left(\frac{C_{iv}}{C_{iv,0.8}}\right) = 5.14\ \frac{w_i}{v_s}\left(\frac{f_v}{2}\right)^{1/2} \qquad (63)$$

where C_{iv} is the volumetric concentration of the ith size cut at y and $C_{iv,0.8}$ is the volumetric concentration of the ith size cut at 0.8D, w_i is the settling velocity of the ith size cut, v_s the velocity of the slurry and f_v the Fanning friction factor for the vehicle flowing at v_s.

C_{iv} is calculated from:

$$C_{iv} = \frac{W_j\ W_s\ \rho_s}{\rho} \qquad (64)$$

where W_j is the mass of the particles in the ith size cut and W_s is the total mass per unit volume.

Verkerk (1990) showed that for the tailings slurries tested and the pipe types utilised (steel) the friction factor could be calculated reasonably reliably using the equations in Figure 2.12 but with one further modification to take into account the non-Newtonian behaviour of the vehicle. This modification is similar to that suggested by Hanks (1978) who established that where the vehicle exhibits appreciable non-Newtonian rheological behaviour Wasp's method predicts much larger values of friction gradient than actually occurs. Hank suggested modifications in respect of :

- vehicle pressure losses. Most frequently the vehicle rheology will be a Bingham plastic in which case the equations for the Bingham plastic in Figure 2.12 should be used

- the drag coefficient which is used to determine the asymmetric particle losses needs to be adjusted to take account of the non-Newtonian rheology of the vehicle in which the large

particles must settle. Hanks suggested the following equations:

$$C_{D_i} = 0{,}4[1 + 6{,}423.10^5 \, P_{D_i} - 2{,}856]^{1/3} \tag{65}$$

where :

$$P_{D_i} = \frac{\left(\frac{y}{C_{D_i}}\right)}{\left[\left(\frac{y}{C_{D_i}}\right)^{1/2} + \frac{7}{24}\pi He_{P_i}\right]} \tag{66}$$

and

$$He_{P_i} = \frac{d_{P_i}^{\,2} \, \tau_{0_v} \rho_v}{\eta_v^{\,2}} \tag{67}$$

In the above equations τ_{0_v}, ρ_v and η_v are the yield stress, density and plastic viscosity respectively of the vehicle. The plastic viscosity for a Bingham plastic is given by the equation (Figure 2.12):

$$\tau = \tau_y + \eta\left(\frac{du}{dy}\right) \tag{68}$$

where τ_y is the yield shear stress of the vehicle.

Verkerk established that slurries at similar concentrations may be transported in the turbulent or laminar regimes depending on the particle size distribution and the extent to which this affects:

- the maximum packing density of the particles
- the surface area for particle wetting (specific surface)
- the Newtonian/non-Newtonian character and the associated absolute values of the rheology.

The maximum packing density determines the maximum concentration at which a slurry can be transported. This is of particular relevance in backfill applications or on thickened discharge of tailings.

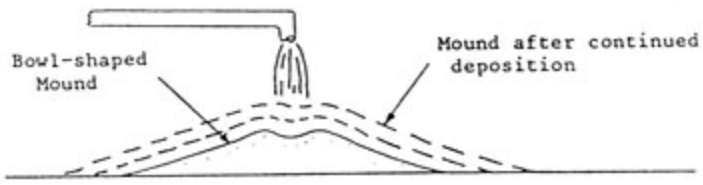
Verkerk has developed a computer program based on Wasp et al's analysis with modifications as effected by the above equations. This program has been made available to this author. The program calculates the slurry-based Fanning friction factor f as well as the head loss per metre within a given pipe of specified roughness. The particle size distribution and the rheological characteristics of the slurry are input to the program which is able to analyse Newtonian, Bingham, Pseudoplastic and Yield-pseudoplastic slurries using the equations in Figure 2.12.

2.8 Discussion

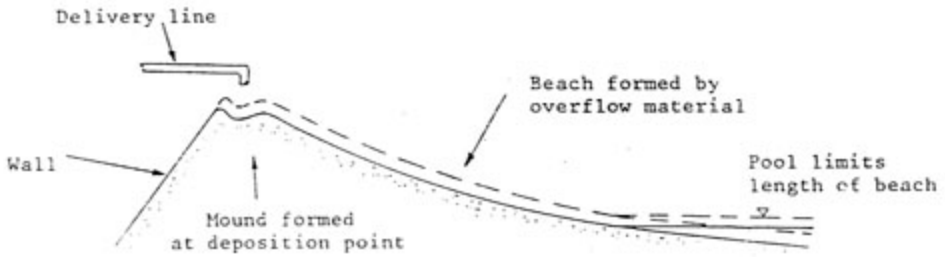
2.8.1 Comparison of Tailings Beach Flow Characteristics with River Sedimentation

Tailings sediment flow comprises series hydraulic jumps over initial 20 - 40 % of beach. Bentel (1981) provided a qualitative explanation of the development of a tailings beach illustrated with Figures 2.13 to 2.16 which, with the exception of Figure 2.14 are taken from his MSc thesis. The figures illustrate important elements and hydraulic conditions prevailing on a typical tailings beach as follows:

- Figure 2.13 illustrates the characteristic development of a "plunge pool" dissipator which develops naturally due to the fact that kinetic energy is dissipated through impact and turbulence so that the velocity of the deposited slurry is far less than the velocity in the pipeline and coarse material which could be transported in the pipe is deposited at this point. The excess slurry overflows the edge of the plunge pool flowing radially outwards, or if the beach is already developed from previous deposition cycles, then towards the pond area as illustrated in Figure 2.14.



a. Deposition on Flat Ground



b. Development of a Beach on a Tailings Dam

FIGURE 2.13 DESCRIPTIVE SKETCHES OF BEACH DEVELOPMENT (After Bentel (1981))

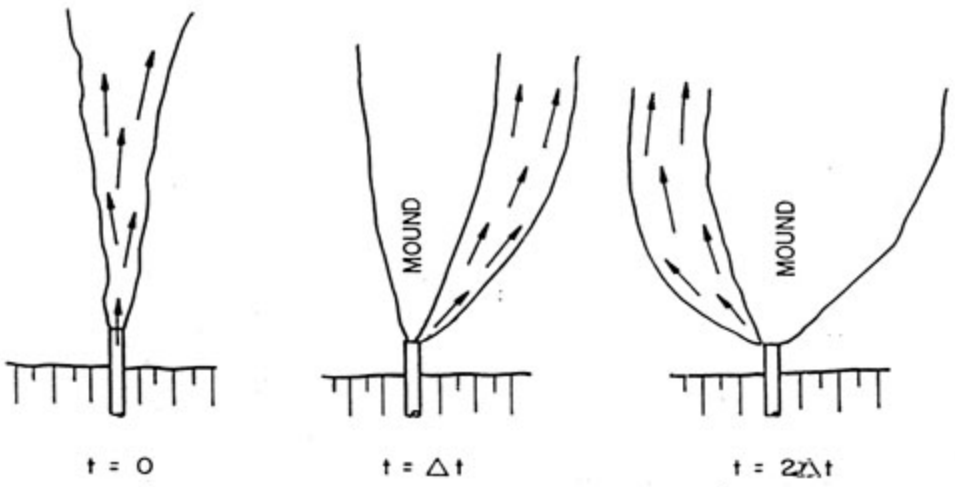


FIGURE 2.14 PROGRESSIVE BEACH BUILD-UP FROM POINT DELIVERY

- Figure 2.15 illustrates the flow characteristics immediately below the plunge pool on the beach where the slurry flow is supercritical. The slurry flows in individual scoured rivulets in which numerous hydraulic jumps occur. The jumps result from the scouring of furrows that cause a local instability of flow, and are accompanied by a decrease in the velocity of flow due to the energy lost in the jump. Ridges are formed after the jumps where material is deposited due to the sudden decrease in velocity. These jumps become less numerous and spaced further apart as the flow proceeds along the slope and the velocity decreases. The slope of the beach becomes more gentle. The rivulets begin to meander and widen and eventually join up, whereafter the flow proceeds as sheet flow with little evidence of scour. This is illustrated in Figure 2.14.

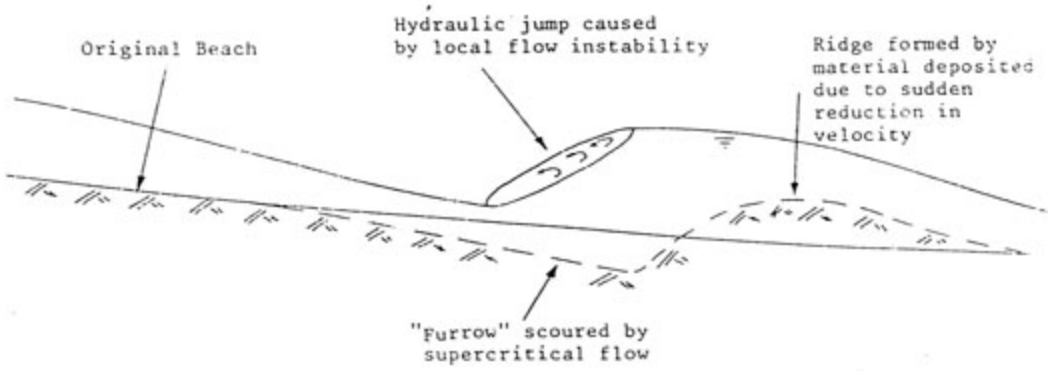


FIGURE 2.15 HYDRAULIC JUMP - SKETCH ILLUSTRATING FURROWS AND RIDGES (After Bentel (1981))

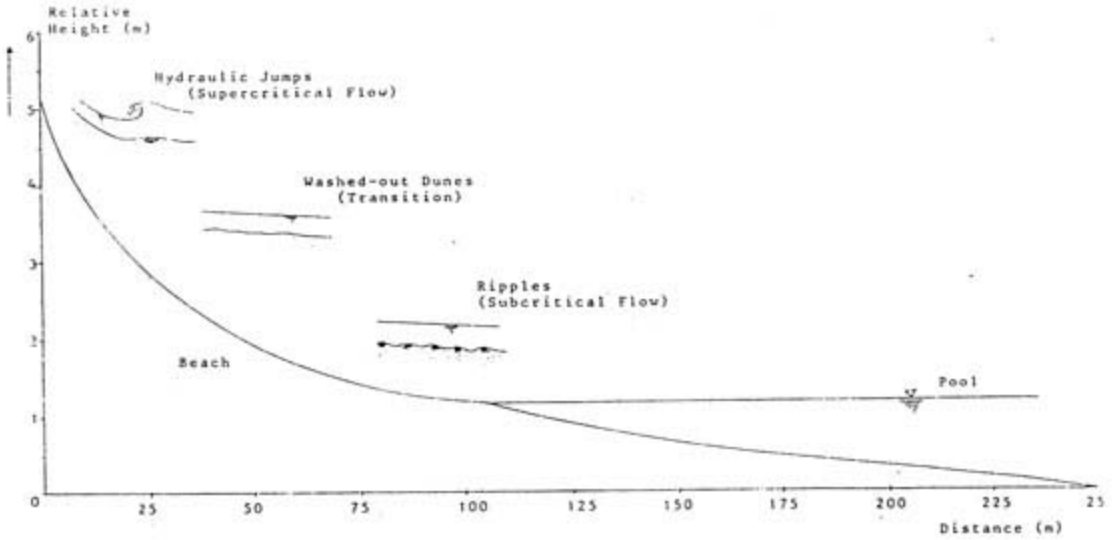


FIGURE 2.16 ACTUAL PROFILE SHOWING LINEAR SECTIONS OF BEACH BETWEEN JUMPS (After Bentel (1981))

- after the high velocity supercritical region in which the flow proceeds along a straight path, the width of the stream increases progressively along the beach due to the fact that the sliming region is higher than the surrounding areas (refer to Figure 2.14). Therefore, although the velocity of flow at the end of the beach is subcritical, the depth of flow may be less than the depth of flow at the top of the beach, depending on the extent to which the stream broadens. As the velocity decreases along the beach bedforms as illustrated in Figure 2.17 develop. Bentel notes the fact that a steepening of the beach occurs at the edge of the pond and the particle size distribution of the deposited material becomes abruptly finer. This is due to the relatively stagnant conditions in the pond. The velocities in the pond are unable to suspend the residual coarse particles which then deposit rapidly at the edge of the pond leaving fine predominantly colloid and fine silt sized fractions to gently flow into the pond.

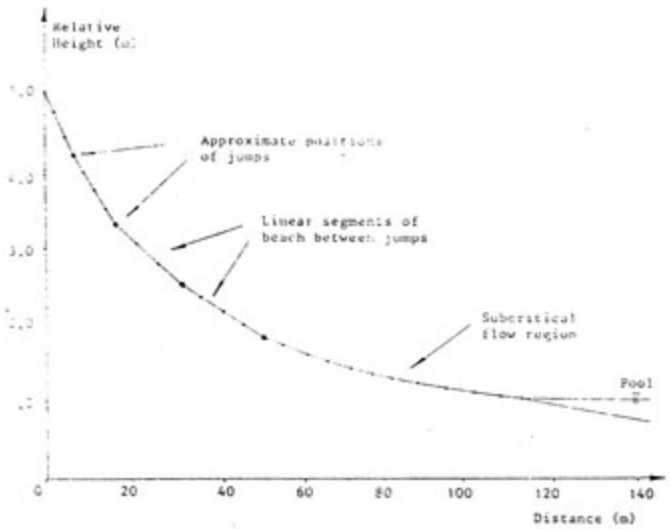


FIGURE 2.17 ILLUSTRATION OF MEASURED BEACH PROFILE AND TYPICAL BEDFORMS (After Bentel (1981))

It will be unrealistic to expect that with the current state of development of sedimentation models it will be possible to develop and all-encompassing model which will cater for the various flow regimes and rivulet developments characteristic of a beach. The theoretical models developed for river sedimentation are centred around assumptions of subcritical flow, and concentrations considerably lower than applicable to tailings. Extension of the river sedimentation models to tailings applications will in itself represent a journey into previously uncharted territory. This dissertation will therefore attempt to advance the state of the art of modelling beach deposition on tailings dams to the extent that averaged approximations of the flow regimes are accommodated.

2.8.2 Sediment Transport Equations

- **General**

The division of the stream into two or more layers representing suspension, mixing between suspended material and bed material, a contact layer or bed load layer and the bed material

is the subject of considerable debate (James (b)). It is not clear whether the inaccuracies near the bed typical of a diffusion approach are a result of simplifications inherent in the derivation of or due to the neglect of bed load effects of saltation rather than by suspended load. There has been insufficient research to check the improvements offered by Hunt's approach. This author suspects that Hunt's more generalised assumptions will overcome inaccuracies. An attempt to provide substance to this assertion is presented in Chapter 3.

- **Suspended Sediment Equations**

(a) Diffusion Equations

The diffusion approach put forward by Hunt (1967) eliminates all but one of the invalidating assumptions inherent in Rouse's solution to the diffusion equation. The remaining assumption, namely that concentration does not change with flow in the longitudinal direction, can be accommodated by conducting step-wise calculations over short distances and assuming that the concentration over the short distance remains constant. Hunt's approach therefore holds promise for development on tailings applications There is, however, only limited research data available to confirm the validity of Hunt's equations due to the difficulty of obtaining representative data. Cooke and Lazarus (1988) provide useful data measured in pipe flow for a sand material to which Hunt's equations may be applied. The applicability of Hunt's equations is tested on the data provided by Lazarus and Cooke (1988) in Chapter 3.

A significant problem area with the evaluation of suspended sediment concentration using the diffusion approach is the need to define a reference concentration c_a. The predictions are directly proportional to c_a and c_a is near the bed where the rate of change in concentration with elevation is high. James (b) summarises three approaches of defining a reference concentration where the suspended load is related to the bed load and/or bed material

(b) energy-based equations

Energy-based equations are based on the premise that it is energy that determines whether a particle remains in suspension and it is the loss of energy through friction that determines the amount of sediment in suspension at any stage. While this approach has been identified for many years it does not seem to have received that same development attention as the

diffusion approach. In the opinion of this researcher, this is unfortunate since most engineering systems have their root in energy utilisation and apportionment.

(c) stream power equations

The energy-based approach that has received considerable development attention, albeit only from a handful of researchers is that based on stream power. Stream power incorporates most of the variables that have an influence on sediment transport into a single function. These variables are:

- the density of the sedimented stream
- the flow rate
- the frictional resistance of the flow stream

Since these are the primary variables in determining the behaviour of a tailings slurry it would appear that the concept of stream power could be a useful parameter in the modelling of tailings beaching.

The concept of stream power has been used to develop an equation for concentration at any point along a stream (but not within a stream). Calibrations of the constants that emerge from an axiomatic treatment of the unit stream power equation of Yang are based on a large body of data. However, that data available for high the concentrations and for particle sizes similar to most tailings is scarce. It is therefore evident that while the concept of stream power as a comprehensive measure of the energy of a sedimented stream flow, it will be necessary to either obtain sufficient data from which to carry out similar regression analyses to those of Yang, or to develop a new set of equations involving stream power

(d) entropy approach

The entropy approach enables a probabilistic concept to be introduced to sediment flow determinations by introducing an expression for entropy, a measure of probabilistic uncertainty. Considering the number of variables that interact in sediment flow calculations and the degree

of uncertainty both in their measurement and estimation, it would appear that the adoption of a probability-based approach is particularly relevant. Using entropy, Chiu is able to derive equations for velocity profile, shear stress profile and concentration that are very similar to those derived using conventional hydrodynamics and the diffusion approach. This demonstrates the adaptability of the approach and if applied to stream power, could be promising for tailings modelling.

- **River Sedimentation Models**

The river sedimentation models described in the literature are particularly large and comprehensive having undergone considerable development mostly for specific applications to large river systems or the study of precious metal deposition. The models are generally designed for low concentration sediments and make the assumption of subcritical flow. Undoubtedly they could be adapted for tailings deposition studies but given that most of the more comprehensive models run on supercomputers at significant expense, it is considered impractical to make use of them for tailings studies for all but the most critical problems.

- **Tailings Sedimentation Models**

Only one sedimentation model for tailings has been discovered in the literature. This model holds promise but has only been tested for a single tailings product at relatively low concentrations. Moreover, the sediment transport relation used in the model, and found to yield the most representative results when compared with flume tests, was originally derived for gravels.

2.8.3 Flume Testing

As a calibrating method, flume testing is an essential way of evaluating the effects of several variables in a practical cost effective manner. Küpper has developed an interesting dimensionless parameter for use in scaling from model to prototype which is based on the Froude law and introduces the concept that beach formation and particle sorting are a function of gravitational and inertial forces. The parameter combines velocity, concentration, and particle diameter into a single relationship. Yang takes this one step further and argues for the use of unit stream power as a scaling function in addition to the Froude law which brings

together similar variables to Küpper's P' parameter but notes that concentration and particle diameter can be kept constant between prototype and model without significant error This author believes that while the forces noted by Küpper are of direct significance it is the energy associated with these forces that actually plays the defining role and therefore would anticipate that Yang's unit stream power criterion could provide a more comprehensive basis on which to conduct and interpret flume test results

2.8.4 Abadjiev's Empirical Methods

The approach put forward by Abadjiev (1984) is significant and important but has limited potential for application in predictive situations. This dissertation has as its objective the development of a theoretically based predictive approach and therefore the methods proposed by Abadjiev are not of direct significance and will not be pursued in detail. However, in the South African situation where a large amount of data exists there would be considerable merit in setting up South African tailings data in the same format as that of Abadjiev and developing similar equations for general application in the gold mining and platinum mining industries, for example. This could form the subject of further research.

2.8.5 Rheology and Slurry Flow Resistance Prediction

Verkerk has conducted particularly relevant research on the transportation characteristics of tailings for pipeline design to the extent where tailings pumping and pipeline systems can be designed with reasonable confidence as long as appropriate consideration is taken of the influence of rheology. Most standard textbooks on fluid mechanics in civil engineering applications set out the parallels between open channel and pipeline head loss equations (for example Webber (1971)) in which the parameter hydraulic radius represented as sectional flow area divided by wetted perimeter is substituted for pipe diameter. Such adaptation should be suitable for loose boundary open channel flow with the added complication that the flow section is not known with certainty. Provided reasonable estimates of the flow section can be make, therefore, it should be possible to make use of slurry characterisation methods developed for pipeline design in the assessment of beaching tailings slurries for which rheology is likely to play a significant role.

2.9 Conclusions

From the review of literature relating to the prediction of particle size distribution within and along a flow stream the following conclusions are drawn:

- there are no equations which have been developed for tailings slurries at concentrations in the range commonly occurring on the majority of mining operations

- it will be difficult to justify the application of classical hydrodynamic equations to the flow of a tailings slurry down a beach in view of the unstable nature of the flow which results in frequent hydraulic jumps and waves. It would appear that a probabilistic approach would be better suited to tailings slurry modelling

- a relatively obscure approach to the application of diffusion theory to sediments documented by Hunt (1969) holds promise in that it is able to cater for a distribution of sizes and avoids many of the simplifying assumptions inherent in the well documented approaches of Rouse (Raudkivi (1991)) and van Rijn (1984). There appears to be no literature documenting tests as to the validity of Hunt's equation. One limitation on the application of the equation is the need to have a reference concentration for each of the particle size groups in a flow stream. Such a concentration will be very difficult to determine with certainty and this difficulty has troubled many researchers as a similar requirement occurs for all diffusion-based equations.

- the parameter stream power brings a number of important variables in tailings slurries, namely, flow rate, flow velocity, slurry density and slurry frictional characteristics, into a single variable. It would be useful to develop the concept of stream power further in tailings slurry applications.

- the concept of entropy (as probabilistic uncertainty), the manner of its application as demonstrated by Chiu (1991), and its adaptability suggest that this approach merits serious consideration in tailings slurry modelling.

Chapter 3 sets out the development of equations for the prediction of the beaching characteristics of tailings beginning with an assessment of the applicability of Hunt's equations and then setting out an entropy-based approach to modelling with stream power as the primary variable.

CHAPTER 3

CONTENTS

Sec	Description	Page
3	**DEVELOPMENT OF EQUATIONS FOR THE PREDICTION OF THE BEACHING CHARACTERISTICS OF TAILINGS**	3-1
3.1	**Introduction**	3-1
3.2	**Test of the Validity of Hunt's Diffusion Approach**	3-1
3.3	**Application of Entropy and Stream Power to the Prediction of Particle Size Distribution along the Beach**	3-4
3.3.1	**Introduction**	3-4
3.3.2	**Outline of Entropy Optimisation Principles**	3-6
3.3.3	**The Variation of Stream Power Down the Beach**	3-9
3.3.4	**Predicting the Beach Profile**	3-12
3.3.5	**The Maxwell-Boltzmann Distribution**	3-14
3.3.6	**Predicting Particle Size Distributions within the Flow Stream using the Maxwell-Boltzmann Distribution**	3-17
3.3.7	**Estimation of the Particle Size Distribution Deposited at any Point on the Beach**	3-18
3.3.8	**Calculation of the Particle Settling Velocity**	3-19
3.3.9	**Application of Minimum Cross-Entropy**	3-21
3.4	**Information Required for Calculation of the Beach Profile and Particle Size Distribution**	3-24
3.5	**Frictional Resistance Characteristics**	3-25
3.6	**Velocity in the Plunge Pool**	3-31
3.7	**Discussion**	3-35

CHAPTER 3 DEVELOPMENT OF EQUATIONS FOR THE PREDICTION OF THE BEACHING CHARACTERISTICS OF TAILINGS

3.1 Introduction

Chapter 2 concluded that three aspects warranted detailed consideration in the development of equations to simulate the beaching of tailings. These were:

- the diffusion approach as extended by Hunt
- the application of stream power as a measure of the energy in a tailings slurry stream
- the entropy approach as applied by Chiu

The above are evaluated in detail in this Chapter beginning with a test of Hunt's equations on the data published by Lazarus and Cooke (1988).

3.2 Test of the Validity of Hunt's Diffusion Approach

Lazarus and Cooke (1988) provide useful data, albeit for pipe testwork, on which to test the validity of Hunt's approach as set out below.

No data on the settling velocity of individual size fractions is given by Lazarus and Cooke, but using the equations (Raudkivi (1991)):

$$w\ (mm/s) = 663\ d^2 \qquad d(mm) < 0.15\ mm \tag{1}$$

and

$$w\ (mm/s) = 134.5\ d^{0.52} \qquad d(mm) > 1.5\ mm \tag{2}$$

with the transition between them as given in Table 2.1 (Raudkivi (1991)) below:

d (mm)	w (mm/s)	d (mm)	w (mm/s)	d(mm)	w (mm/s)
0.15	14.8	0.5	64.0	0.9	110.0
0.2	21.1	0.6	6.4	1.0	121.0
0.3	36.1	0.7	88.6	1.2	137.3
0.4	50.0	0.8	99.0	1.5	166.0

TABLE 2.1 FALL VELOCITIES OF QUARTZ SAND OF 0.15 < d < 1.5 mm

The influence of particle concentration can be accounted for by using the expression (Raudkivi (1991)):

$$w = w_0 (1 - c)^{\beta}$$

where β is a function of the Reynolds number, Re = wd/ν, and the particle shape and for natural grains can be approximated by β = 4.65 for D. < 40 and β = 2.35 for d. > 800 with the transition defined by $\beta = 7.478\ D_*^{-0.129}$. D. is given by :

$$D_* = (S_g g d^3/\nu^2)^{1/3} \qquad (4)$$

The kinematic viscosity for the slurry can be calculated using the equation (Raudkivi (1991)):

$$\frac{\nu_{slurry}\ \rho_{water}}{\nu_{water}\ \rho_{slurry}} = 1 + kc + k^2c^2 + k^3c^3 + ... \qquad (5)$$

The kinematic viscosity of water can be taken as 1 m^2/s and k should be determined from viscosity testing. If viscosity tests are not available then k can be taken as 2.5.

Particle size distributions have been calculated from the data of Lazarus and Hunt (1988). In carrying out the calculations a shear velocity has been calculated for each segment of the pipe stream using the equations quoted by Lazarus and Cooke in their paper. Figures 3.14 and 3.15 show the concentrations calculated using Hunt's equation superimposed on the plots produced by Cooke and Lazarus (1988). The reference concentrations were taken at $y_0/2$ directly from the measurements recorded by Lazarus and Cooke at this depth.

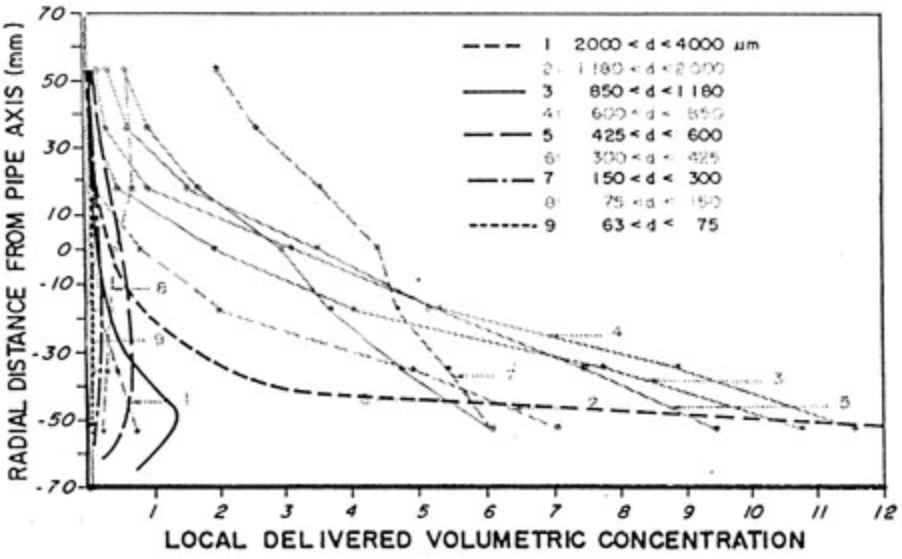


FIGURE 3.1 : VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS (After Cooke and Lazarus (1988)) WITH CONCENTRATIONS PREDICTED USING HUNT'S EQUATIONS (HUNT (1967))

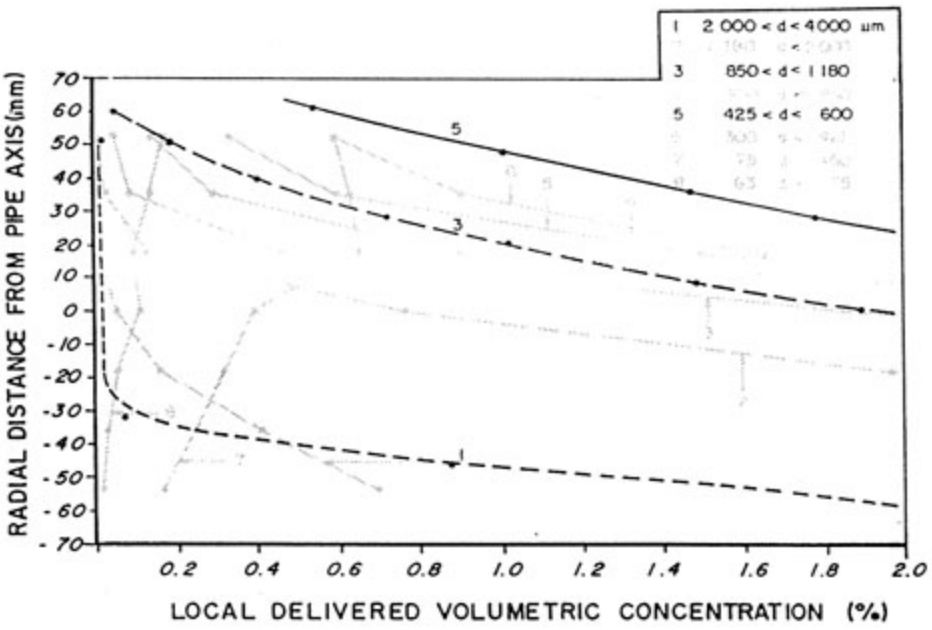


FIGURE 3.2 : VARIATION OF LOCAL CONCENTRATION OF DIFFERENT SIZE FRACTIONS - LARGER SCALE (After Cooke and Lazarus (1988)) WITH CONCENTRATIONS PREDICTED USING HUNT'S EQUATIONS (HUNT (1967))

It is evident from the above plots that there is reasonable correlation between the calculated concentrations of the various particle size distributions and the measurements of Lazarus and Cooke thereby lending credibility to Hunt's equations.

Practical application of Hunt's equations will, however, require the selection of a reference concentration for the various particle sizes. In the above test case the data of Lazarus and Cooke (1988) provided directly the concentrations at any depth to within 20 mm of the pipe boundary. Seldom is such comprehensive data available for pipes. No such data has been found in the literature for tailings beaches and understandably so given the shallow depths at which the tailings streams flow and the associated problems of scale. Consideration was given by this author to constructing an isokinetic sampler for use in the tailings streams but following personal communication with Professor Lazarus who noted that some attempt had been made by his study team with regard to mine backfill placement, this idea was dropped, as was further development of an approach using Hunt's equations.

3.3 Application of Entropy and Stream Power to the Prediction of Particle Size Distribution along the Beach

3.3.1 Introduction

It was noted in the literature survey in Chapter 2 that the primary variables that affect the beaching characteristics of a slurry are:

- discharge conditions (flow rate, velocity, direction of discharge)
- particle size distribution
- slurry relative density
- specific gravity of the tailings particles
- site geometry (length of beach)

Predictive equations of the beaching characteristics of tailings would be of most use in the

following situations:

- a greenfields situation exists such as for a new mining venture, and therefore where no tailings dam exists on which to do any measurements

- a tailings dam exists but changes in grind, ore or slurry relative density are anticipated for metallurgical or mining reasons and the potential effect on the tailings dam needs to be assessed

In both situations the discharge conditions and to some extent the slurry relative density can be varied to suit specific requirements. The remaining variables, notably the balance of those listed above, will be fixed. A parameter which brings together all of the important variables is that of stream power.

The stream power of a slurry stream of relative density ρ flowing at a rate of Q is determined from the expression $\rho Q g S_e$, where g is gravitational acceleration and S_e is the slope of the energy line. In this expression:

- ρ, the slurry relative density incorporates the specific gravity of the tailings particles

- Q entrenches discharge conditions (flow rate and direction of discharge)

- S_e incorporates the combined effects of particle size distribution, slurry relative density and frictional characteristics which determine the frictional characteristics of the slurry as it flows. Since on a tailings beach the energy is given by the velocity head, this parameter also incorporates the variability of the frictional resistance characteristics of the slurry with velocity.

As a result of gravitational sorting and deposition of the tailings the above parameters will vary along the beach with the net effect of a reduction in the stream power with distance. The length of the beach will determine the total reduction in stream power for the particular slurry and therefore the size and volume of tailings particles that can be maintained in suspension at any location along the beach.

It is contended, after the extensive literature review. that the entropy approach applied to stream power represents the most promising means for incorporating the instability of the flow conditions down the beach and accommodating all of the above variables. This statement is clarified in the ensuing sections beginning with an explanation of entropy optimisation principles, moving through the application of entropy to stream power, the development and application of the Maxwell-Boltzmann distribution (an entropy-based statistical distribution of specific relevance to the prediction of particle size distributions) and the estimation of beach profiles.

3.3.2 Outline of Entropy Optimisation Principles

Kapur and Kesavan (1992) provide comprehensive documentation on the background, principles and applications of entropy. The following explanations of entropy, an established and accepted function for entropy, and the principle of maximum entropy are abstracted from Kapur and Kesavan (1992).

In 1948 C E Shannon, a communications engineer, developed a measure of uncertainty which turned out to be identical to the expression for entropy in thermodynamics:

$$S(p) = -k \sum_{1}^{n} p_i \ln p_i \qquad (6)$$

where p is a probability distribution and $p = (p_1, p_2, p_3, .. p_n)$, k is an arbitrary positive constant. This measure is not unique and other measures have been put forward in the period since 1948. It is however particularly pertinent because:

- the function is permutationally symmetric ie it is independent of the order of the outcomes
- the function is dependent on all of the probabilities $p_1, p_2, p_3, .. p_n$
- the function is a continuous function of $p_1, p_2, p_3, .. p_n$ and changes by only a small amount when $p_1, p_2, p_3, .. p_n$ change by small amounts

- the function is a concave function of p_1, p_2, p_3, .. p_n This means that it possesses a globally maximum or minimum value

- when the function is maximised subject to linear constraints the maximising probabilities are all greater than zero

The above and numerous other properties documented by Shannon are satisfied only by (6) which was derived using the axiomatic method of Euclid to quantify the concept of uncertainty of a probability distribution; Shannon first thought of the properties that this measure should have and then put them down as axioms or postulates. He then proceeded to derive a measure having the desired properties.

Entropy is taken to mean *probabilistic uncertainty* or, more specifically, the uncertainty associated with the probability of outcomes. In any system, depending on the constraints governing the system, there may be an infinity of probability distributions consistent with the constraints, that could describe the system. Out of all of these distributions one has maximum uncertainty. Uncertainty can only be reduced through the inclusion of information. Thus the use of any other distribution than the maximum uncertainty distribution implies the use of additional information over and above that which determines the system constraints. It is vital in any situation to use all the information available and to scrupulously avoid using any information not given. This principle of ancient wisdom or scientific objectivity and honesty requires maximum uncertainty, or, maximum entropy. Its application implies that uncertainty about information not available is maximised subject to the use of information given to us.

Application of the principle of maximum uncertainty means that out of all of the possible distributions only one can be used with absolute honesty - that with maximum entropy. Provided a function describing entropy is specified it is possible to maximise this entropy and obtain an additional equation relating the variables in the system.

In 1957 E T Jaynes presented a paper on the explicit enunciation of the principle of maximum entropy and showed that if a probability distribution is chosen with less than maximum entropy, this reduction in entropy must have come from some additional information. The principle can be stated in a number of different forms:

- Choose the distribution that is most random

- Choose the distribution that is least biased (any deviation from maximum entropy will imply a bias)

The validity of the approach is enhanced when it is considered that most of the theoretical probability distributions derived using classical statistical theory can be derived using the Shannon measure (equation 6) combined with Jaynes' Principal of Maximum Entropy (.Kapur and Kesavan (1992)). These include the exponential, normal, gamma, beta, Laplace and Cauchy distributions and, most importantly, a number of distributions from statistical mechanics namely the Maxwell-Boltzmann, the Bose-Einstein and the Fermi-Dirac distributions.

Using Shannon's measure of entropy, and applying Jaynes' principle of maximum entropy together with Lagrange's method of undetermined multipliers several researchers, Reza (1961), Goldman (1968) and Tribus (1969), showed that given a range of constraints or information, the least biased distributions that should be applied in any partial information problem are as follows:

Given:	**least biased distribution**
no information	uniform
expected value (mean)	exponential
expected value, standard deviation	normal
expected value, standard deviation, maximum and minimum	beta

3.3.3 The Variation of Stream Power Down the Beach

Along a tailings beach stream power, given by the expression:

$$P = \rho g Q S_e \tag{7}$$

is a maximum at the point of discharge and a minimum at the centre of the pond. Within the delivery pipeline the stream power is maintained at a level that prevents settling out of particles. On discharge, however, the prime mover of the tailings particles becomes gravity as opposed to pressure head and the stream power immediately beyond the pipe, and all the way to the pool, is always too low to prevent particles from settling out. The flow rate and slurry density therefore vary down the beach as particles settle out. It is postulated that this variation can adequately be described by the variation in stream power using an approach similar to that set out by Chiu (1987):

Let P be the stream power at a distance x from a discharge. Then at any distance less than x the stream power is greater than P. Assuming that all values of x between 0 and L are equally likely, it can be stated that the probability of the stream power being equal to or more than P is (1-x/L), or, since P is a monotonically decreasing function of x/L the cumulative distribution function is:

$$f(P) = \frac{L - x}{L} = (1 - \frac{x}{L}) \tag{8}$$

Since by definition:

$$p(P) = \frac{df(P)}{dP} = \frac{df(P)}{dx}\,\frac{dx}{dP} \tag{9}$$

the probability density function, p(P), (yet to be identified) is:

$$p(P) = -\left(L\,\frac{dP}{dx}\right)^{-1} \tag{10}$$

The following constraint which applies to all probability distributions must be satisfied:

$$\int_0^{P_0} p(P)\,dP = 1 \tag{11}$$

where P_0 is the maximum value of P which occurs at x = 0.

Also:

$$\int_0^{P_0} P\,p(P)\,dP = \overline{P} \tag{12}$$

where P is the distance averaged power.

In terms of Jaynes Principle of Maximum Entropy and using Shannon's measure of entropy:

$$S = -\int p(P)\,\ln p(P)\,dP \tag{13}$$

we have to maximise S.

The Lagrangian is given by:

$$-\int_0^{P_0} p(P)\,\ln p(P)\,dP - (\lambda - 1)\left(\int_0^{P0} p(P)\,dpP - 1\right) - \mu\left(\int_0^{P0} P\,p(P)\,dP - \overline{P}\right) \tag{14}$$

Differentiating with respect to P and using the Euler-Lagrange equations of the calculus of variations gives:

$$p(P) = \exp^{-\lambda}\,\exp^{-\mu P} \tag{15}$$

Substituting into equation 11 :

Solving for P gives:

$$\int_0^{P_0} \exp^{-\lambda} \exp^{-\mu P}\, dP = 1 \tag{16}$$

$$\exp^{-\lambda} = \frac{\mu}{(1 - \exp^{-\mu P_0})} \tag{17}$$

$$\frac{\exp^{-\lambda}}{\mu} - \frac{\exp^{-\lambda}}{\mu} \exp^{-\mu P_0} = 1 \tag{18}$$

Substituting equation 15 into equation 9:

$$\exp^{-\lambda} \exp^{-\mu P_0} = -\left(L \frac{dP}{dx}\right)^{-1} \tag{19}$$

$$dx = -L \exp^{-\lambda} \exp^{-\mu P}\, dP \tag{20}$$

$$x = -L \exp^{-\lambda} \int \exp^{-\mu P_0}\, dP \tag{21}$$

$$= \frac{L \exp^{-\lambda}}{\mu} \exp^{-\mu P} + Constant \tag{22}$$

When x = 0, P = P_0, therefore:

$$Constant = -\frac{L \exp^{-\lambda}}{\mu} \exp^{-\mu P_0} \tag{23}$$

Substituting:

$$x = \frac{L \exp^{-\lambda}}{\mu} \exp^{-\mu P} - \frac{L \exp^{-\lambda}}{\mu} \exp^{-\mu P_0} \tag{24}$$

$$P = -\frac{1}{\mu} \ln \left(\frac{\mu}{L \exp^{-\lambda}} x + \exp^{-\mu P_0} \right) \quad (25)$$

Substituting equation 18:

$$P = -\frac{1}{\mu} \ln \left((1 - \exp^{-\mu P_0}) \frac{x}{L} + \exp^{-\mu P_0} \right) \quad (26)$$

Equation 26 allows the estimation of the stream power at any distance x along a beach of total length L and stream power at the discharge of P_0 provided the constant μ is known. A method for determination of μ and the beach profile is set out below.

3.3.4 **Predicting the Beach Profile**

At any location on the beach, provided the delivery conditions are constant, the slope of the stream power curve will be equal to the energy gradient. From equation 19 it is evident that:

$$\frac{dP}{dx} = \frac{-1}{L \exp^{-\lambda} \exp^{-\mu P}} \quad (27)$$

but from equation 18:

$$\exp^{-\lambda} = \frac{\mu}{(1 - \exp^{-\mu P_0})} \quad (28)$$

therefore:

$$\frac{dP}{dx} = \frac{-(1 - \exp^{-\mu P_0})}{L \mu \exp^{-\mu P}} \quad (29)$$

At the discharge, $P = P_0$ and dP/dx represents the slope of the energy line at this point and will depend on the slurry characteristics, the particle size distribution, the particle specific

gravity and the velocity of the flow stream. It is at this location that μ can be estimated using the equation:

$$initial\ slope\ S_i = \frac{-(1 - \exp^{-\mu P_0})}{L\ \mu\ \exp^{-\mu P_0}} \qquad (30)$$

At any location along the beach, there are only two components of stream energy:

- kinetic energy by virtue of the velocity of flow
- potential energy by virtue of the height differential between the point on the beach and the pool location

Therefore the change in kinetic energy must equal the change in potential energy and the energy line and the beach profile must be parallel.

This means that from equation 29:

$$\frac{dy}{dx} = \frac{dP}{dx} = \frac{-(1 - \exp^{-\mu P_0})}{L\ \mu\ \exp^{-\mu P(x)}}$$

The elevation y at any distance x along the beach is given by:

$$y = H - \int_0^x \frac{dy}{dx}\, dx$$

where H is the elevation of the beach at the discharge. The beach profile can therefore be calculated from the equation:

$$y = H - \int_0^x \left(\frac{(1 - \exp^{-\mu P_0})}{L\ \mu\ \exp^{-\mu P(x)}} \right) dx \qquad (33)$$

Equation 33 can be used in its numerical form:

$$y = H - \sum_{0}^{x}\left(\frac{(1 - \exp^{-\mu P_0})}{L\ \mu\ \exp^{-\mu P(x)}}\right)\Delta x$$

The parameter μ is determined by solving equation 30 using an estimate of the initial slope.

The next section sets out the derivation and interpretation of a distribution of statistical mechanics which is of particular relevance in estimating the distribution of particles at any point along the beach.

3.3.5 The Maxwell-Boltzmann Distribution

Kapur and Kesavan (1992) note that the earliest success of the principle of maximum entropy was in the discipline of statistical mechanics where Jaynes was able to derive all its principal distributions in a manner totally different from the classical derivations. It was this success that set in motion the application of the principle of maximum entropy in several other areas, thus establishing it as a viable discipline in its own right.

The central idea in statistical mechanics distributions is to predict the distributions of the microstates, which are the particles of the system, on the basis of the knowledge of some macroscopic data. The latter information is specified in the form of some simple statistical moment constraints with the distributions differing only in the way in which the moment constraints are specified.

The Maxwell-Boltzmann distribution makes use only of the expected energy of the system. Using only this knowledge and the Shannon entropy measure and by an application of the principle of maximum entropy the Maxwell-Boltzmann distribution may be derived as follows:

Let $p_1, p_2, p_3, .. p_n$ be the probabilities that a particle in a system has energies $e_1, e_2, e_3, ... e_n$ respectively. If the only information about the system is that the *expected energy* of the particles of the system is $\bar{e}$ then:

$$p_1 e_1 + p_2 e_2 + ... p_n e_n = \bar{e} \tag{35}$$

There is also the information:

$$p_1 + p_2 + ... p_n = 1 \tag{36}$$

and $p_i \geq 0$ for each i. There may be an infinity of probability distributions satisfying these constraints. If there is no other information available about the system, then according to Jaynes' maximum entropy principle, the least biased distribution is the one that maximises:

$$-\sum_{i=1}^{n} p_i \ln p_i \tag{37}$$

subject to the above constraints.

Using Lagrange's method of undetermined multipliers the Lagrangian is:

$$-\sum_{i=1}^{n} p_i \ln p_i - (\beta - 1)\left(\sum_{i=1}^{n} p_i - 1\right) - \alpha\left(\sum_{i=1}^{n} p_i e_i - \bar{e}\right) \tag{38}$$

where α and β are constants.

Setting the derivatives of the Lagrangian with respect to p_1, p_2, p_3, ... p_n equal to zero:

$$-\ln p_i - \beta - \alpha e_i = 0 \qquad i = 1,2,3, ... n \tag{39}$$

or:

$$p_i = \exp^{-\beta - \alpha e_i} \qquad i = 1, 2, ... n \tag{40}$$

Now:

$$\sum_{i=1}^{n} p_i = 1 \tag{41}$$

and therefore:

$$\exp^{-\beta} \sum_{i=1}^{n} \exp^{-\alpha e_i} = 1 \qquad (42)$$

so that:

$$p_i = \exp^{-\alpha e_i} \left(\sum_{i=1}^{n} \exp^{-\alpha e_i} \right)^{-1} \qquad (43)$$

The distribution given by equation 40 is referred to as the Maxwell-Boltzmann Distribution of Statistical Mechanics. The Lagrangian multiplier α is determined from the constraint that:

$$\left[\frac{\sum_{i=1}^{n} e_i \exp^{-\alpha e_i}}{\sum_{i=1}^{n} \exp^{-\alpha e_i}} \right] - e_i = 0 \qquad (44)$$

It can be shown (Kapur and Kesavan (1992)) that if $e_1 \leq \bar{e} \leq e_n$, there always exists a unique Maxwell-Boltzmann distribution for a given average energy. This also follows from the knowledge that the entropy function is concave and the constraints are linear, so that there can be only one globally maximum value for the entropy and only one value for this maximum value of entropy.

Kapur and Kesavan (1992) note that the fact that the distribution obtained in this way agrees with the observed distribution shows that Nature has also placed only one constraint, namely an energy constraint, but otherwise has left the system free to equalise the probabilities as much as possible. In the above derivation entropy has been maximised using the constraints represented by equations 31 and 32 because there is no other information. It appears that Nature has also no additional information and the only physical constraint on the system is the energy constraint.

3.3.6 Predicting Particle Size Distributions within the Flow Stream using the Maxwell-Boltzmann Distribution

According to Kapur and Kesavan (1992) the principle of maximum entropy is applicable to proportions as well as probabilities. This means that the variables p_i can represent proportions as for example are measured in particle size analyses in which the proportions of particles passing or retained on various sieve sizes are determined

It is postulated that the energy in the Maxwell-Boltzmann distribution can be represented by the stream power.

The stream power is used up in maintaining particles in suspension through turbulence. This power would be given by:

$$P = force \ x \ \frac{distance}{time} \qquad (45)$$

During turbulence and suspended flow the upward movement of each particle as a result of the power induced eddy currents is balanced by the downward settling out of the particle under gravity. In fact the power may be represented by (Bagnold (1956),(1966) and Wang (1984)):

$$P = \Sigma \ (particle \ \ submerged \ \ weight \ \ x \ settling \ \ velocity) \qquad (46)$$

It is therefore postulated that it is possible to:

- split the particle size distribution into particle size intervals and take the mean of the interval as the representative particle size for the interval

- utilise knowledge of the fluid properties and determine the stream power absorbed by similar sized particles.

If there are n intervals of particle size and the power of the representative particle size in each interval is represented by e_1, e_2, ... e_n, and the proportions of the particle intervals are represented by p_1, p_2, ... p_n, then at any depth in the flow stream:

$$p_1e_1 + p_2e_2 + \ldots + p_ne_n = P(x,y) \qquad (47)$$

And, since there is a probability, no matter how small, of each particle size occurring at any point in the stream flow,

$$p_1 + p_2 + ... + = 1 \qquad (48)$$

The above constraints are identical to those applied in the derivation of the Maxwell-Boltzmann distribution which as noted is unique for each set of constraints. This means that, using the arguments presented in Sections 3.4.2 and 3.4.3, it is possible to calculate the proportions of the various particle size intervals in the flow stream using equations 44 and 45.

3.3.7 Estimation of the Particle Size Distribution Deposited at any Point on the Beach

The particle size distribution of the material deposited on the beach must be representative of the change in the distribution in the flow stream from one point to the next. This follows from the principle of conservation of mass since the mass of material that settles out from the flow stream at distance x along the beach plus the mass of the material remaining suspended in the flow stream at x must equal the mass in the flow stream at a point immediately upstream of the x.

Moreover, the change in proportion of the mass of each particle size interval in the flow stream must be representative of the relative differences between the particle size distribution in the flow stream and that in the deposited material. This is illustrated in Figure 3.3. Assume particle size distribution (a) in Figure 3.3 is representative of that in the flow stream at point x along the beach and the distribution (b) is representative of that in the flow stream at a point immediately preceding x. If Δp_1, Δp_2, Δp_3, ... Δp_n are the changes in the proportions of the various size fractions between (a) and (b), then it is postulated that the distribution of the material deposited in the beach between x and the point immediately preceding x will be represented by (c) where (c) can be determined by subtracting the changes

in proportions Δp_1, Δp_2, Δp_3, ... Δp_n for the n size fractions from (a).

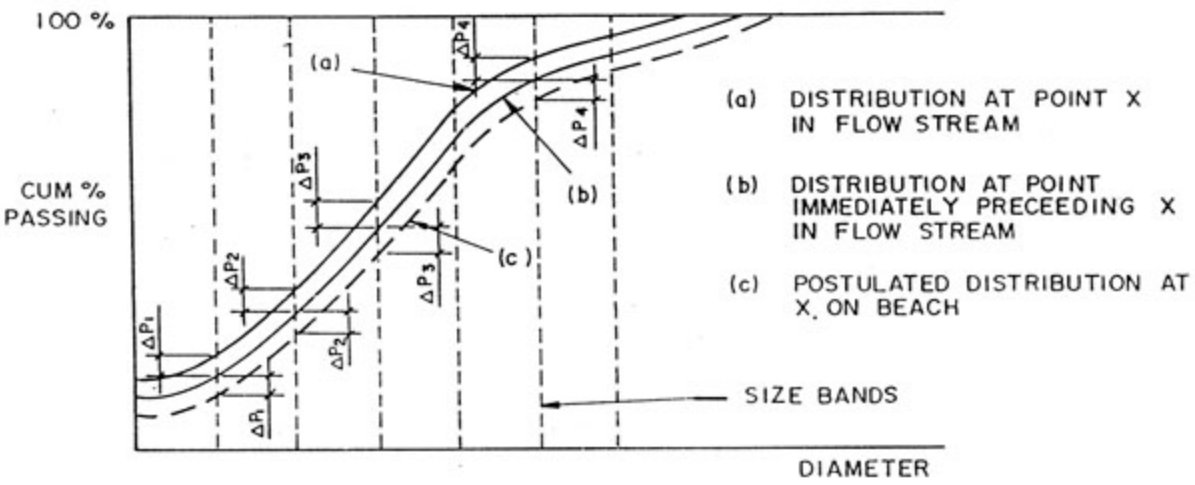


FIGURE 3.3 : PARTICLE SIZE DISTRIBUTIONS IN THE FLOW STREAM AND ON THE BEACH

3.3.8 Calculation of the Particle Settling Velocity

The settling velocity can be estimated using Stokes law in a manner similar to that used in hydrometer analyses to determine the particle size distributions of soils and may be expressed as (Head (1980)):

$$w = \frac{D^2 \, g \, (\rho_s - \rho_L)}{18 \, \nu} \qquad (49)$$

where ρ_s represents the density of the tailings particles and ρ_L represents the density of the liquid. The parameter ν represents the kinematic viscosity of the fluid which is influenced by the particle concentration. Correction for the concentration can be estimated from (Raudkivi

(1991)):

$$\frac{\mu_{susp}}{\mu} = 1 + kC_{vol} + k^2 C_{vol}^2 + k^3 C_{vol}^3 + ... \quad (50)$$

where μ_{susp} is the dynamic viscosity of the slurry, μ is the dynamic viscosity of water (1×10^{-3} kg/m/s), C_{vol} is volumetric concentration and k is a factor which is dependent on the shape of the particle and is determined from viscosity testing or, in the absence of this testing may be estimated from the values below:

Axial ratio	Min value of k	Max value of k
1	2.50	2.50
2	2.17	2.82
5	2.04	3.55
10	2.01	4.49

The kinematic viscosity is calculated from the expression

$$\nu_{susp} = \frac{\mu_{susp}}{\rho_{susp}} \quad (51)$$

where ρ_{susp} is the density of the slurry which can be calculated from the mass concentration using the equation:

$$\rho_{susp} = \frac{100}{\%_{water} + \frac{\%_{solids}}{SG_{solids}}} \cdot 10^3 \ kg/m^3 \quad (52)$$

Since it is stream power that maintains particles in suspension, it is stream power that determines mass concentration, c. It may therefore be argued that at any point along the beach the mass concentration is directly proportional to the stream power. The average mass concentration in the stream at any point on the beach may therefore be calculated from:

$$\bar{c}\ (x) = \frac{\bar{c}_0}{P_0} P(x) \quad (53)$$

where $\bar{c}(x)$ is the average mass concentration at x and $\bar{c}_0$ is the mass concentration at the

discharge point. To obtain the volumetric concentration the following relationship can be used (Raudkivi (1991)):

$$c_v = \frac{c/S_s}{(1 - c) + c/S_s} \qquad (54)$$

where S_s is the specific gravity of the solids particles. Stokes law under-predicts the drag on particles at Reynolds numbers greater than 1. For coarser tailings products, therefore it is better to utilise the relationships described in Section 3.2.

3.3.9 Application of Minimum Cross-Entropy

Straightforward application of the Maxwell-Boltzmann equation to determine the particle size distribution will not make maximal use of available information. Under conditions of consistent discharge of a slurry of constant characteristics (comprising hard rock particles that undergo negligible grinding during flow down the beach) the principle of conservation of mass determines that the numbers of particles cannot increase with distance along the beach. (This would not apply under conditions of erosion such as would occur after a change in slurry characteristics or discharge conditions). There is therefore additional information in that the particle size distribution along the beach must be a function of the particle size distribution of the tailings as discharged. According to the principle of maximum entropy the particle size distribution would tend to remain as close to the original distribution as environmental constraints will permit. This applies, for example, in the delivery pipeline where flow conditions are deliberately maintained such that settling out of the particles does not occur or else the pipeline would choke. On the beach, however, the flow constraints differ and segregation of solids and water and of different sizes of particles occur.

The principle of maximum entropy dictates, however, that the slurry would tend to remain as similar to discharge conditions as flow constraints will allow. The application of maximum entropy concepts and the Maxwell-Boltzmann statistical distribution enables the selection out of an infinity of distributions that distribution which has maximum uncertainty or least bias. In order to retain the requirement of least bias and yet make use of the information that the particle size distribution at any point on the beach must be determined by the original particle size distribution, the distribution selected through application of the principle of maximum

entropy and the Maxwell-Boltzmann equation should be that which has maximum uncertainty but most closely approaches the original.

To achieve this it is necessary to introduce the concepts of *directed divergence, probabilistic distance or cross-entropy* of two distributions. If there are two distributions, P and Q and Q represents the original or *a priori* distribution, then D(p:q) could represent the probabilistic distance or cross-entropy between P and Q. Choosing P such that D(p:q) is a minimum requires that the following conditions be satisfied (Kapur and Kesavan(1992):

- D(p:q) should be greater than zero (ie non negative)
- D(p:q) should be unique ie d(p:q) = 0 if and only if p = q
- D(p:q) should be a convex function of $p_1, p_2,\ldots,p_n$ (in order to have a minimum
- when D(p:q) is minimised subject to known linear constraints using Lagrange's method none of the resulting minimising probabilities should be negative.

The Kullback-Leibler measure of cross-entropy is the most fundamental measure among all other possible measures satisfying the above four criteria (Kapur and Kesavan (1992)). The Kullback-Leibler measure plays the same central role in the discussion of minimum cross-entropy as the Shannon measure does in the case of maximisation of entropy. Kapur and Kesavan (1992) provide an axiomatic treatment for the derivation of the Kullback-Leibler measure which is defined as:

$$D(p:q) = \sum_{i=1}^{n} p_i \ln\frac{p_i}{q_i} \qquad (55)$$

where it is assumed that whenever $q_i = 0$, $p_i = 0$. This measure is a continuous function of $p_1, p_2,\ldots,p_n$.

The method of minimising the cross-entropy is similar to that applied for maximisation of entropy. In this case the following function is minimised:

$$\sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i}$$

subject to:

$$\sum_{i=1}^{n} p_i = 1 \tag{57}$$

and:

$$\sum_{i=1}^{n} p_i e_i = \bar{e} \tag{58}$$

where, as in the Maxwell-Boltzmann derivation in Section 3.3.5 p_1, p_2, ...p_n are the probabilities that a particle in a system has energies e_1, e_2,....e_n respectively, and the only information about the system other than the values of q_1, q_2,...q_n is that the expected energy of the particles in a system is ė. The following relation is obtained through application of Lagrange's method of undetermined multipliers in a manner similar to that applied in the derivation of the Maxwell-Boltzmann equation:

$$p_i = q_i \exp(-\beta - \alpha e_i) \tag{59}$$

The constants α and β can be determined by substituting into equation 54 and 55 to obtain:

$$\exp(\beta) = \sum_{i=1}^{n} q_i \exp(-\alpha e_i) \tag{60}$$

and

$$\bar{e} \exp(\beta) = \sum_{i=1}^{n} q_i e_i \exp(-\alpha e_i) \tag{61}$$

and therefore:

$$\frac{\sum_{i=1}^{n} q_i e_i \exp(-\alpha e_i)}{\sum_{i=1}^{n} q_i \exp(-\alpha e_i)} - \bar{e} = 0 \qquad (62)$$

Equation 59 is solved by iteratively selecting α until the relation is satisfied. Solution of this equation at any point on the beach a distance x from the discharge such that $\bar{e}$ is equal to P(x), the stream power at x, will yield the least biased estimate of the particle size distribution at x.

3.4 Information Required for Calculation of the Beach Profile and Particle Size Distribution

To estimate the beach profile and the particle size distribution at any point along the beach using the equations derived in the preceding sections, the following information is required:

1 the particle size distribution ex-mill

2 the slurry relative density, or the water:solids ratio, or the concentration by weight or by volume, and the particle specific gravity

3 the volumetric flow rate or the dry tonnage rate

4 the discharge pipe diameter and number of simultaneously operating discharges to be able to estimate the discharge velocity and therefore the discharge velocity head

5 an estimate of the friction gradient characteristic of the total tailings slurry at the velocity of the discharge from the plunge pool.

6 an estimate of the velocity of the flow stream exiting the plunge pool that will develop at the point where the discharge meets the beach

Data items 1 to 3 are the basic details for any specific tailings disposal operation and are process-related. Item 4 could be a matter of detailed design and could be the reason for

initiating a detailed estimation of beaching characteristics. For example, in the case of a new mine, it may be desirable to choose the discharge detail to suit specific beaching requirements, or, in the case of an existing mining operation, where a grind change, or an ore type variation is to be evaluated, it may be necessary to modify the existing discharge detail in order to suit beaching requirements.

The data required in item 5 poses a special problem since in the words of Raudkivi(1990) :

The description of the resistance to flow as a function of sediment and flow parameters is one of the unsolved problems of loose boundary hydraulics. The methods in current use are essentially empirical and have a low level of accuracy for prediction purposes. Not even in a steady two-dimensional flow can the roughness height and the dimensions of bed forms be predicted reliably.

Estimates of the velocity in item 6 will depend on the discharge rate and can be made from calculations, and from observations of existing operations.

The methods of estimating items 5 - frictional resistance characteristics of the slurry and item 6 - velocity in the plunge pool are discussed in more detail in the ensuing two sections.

3.5 Frictional Resistance Characteristics

Raudkivi (1992) points out the dearth of reliable equations and information on which to base estimates of the frictional resistance of slurries in open channel flow. Raudkivi makes his statements in reference to river sedimentation for which there has been extensive research effort over the past century.

With respect to tailings slurries there is almost no information on the subject except with regard to the pumping and pipeline transportation characteristics of slurries. Technically the frictional characteristics of the slurry at the pipeline velocity should be no different to those on the beach at an equivalent velocity. What will be different is the boundary shear. Generally the velocities on the beach will be lower than those in the pipeline which are deliberately kept above the settling velocities to prevent choking of the pipelines. Figure 3.4 illustrates a typical relationship between the velocity of a tailings slurry and the frictional resistance in metres per metre in a pipeline. The influence of increasing concentration manifests itself as a family of

curves increasing in frictional resistance.

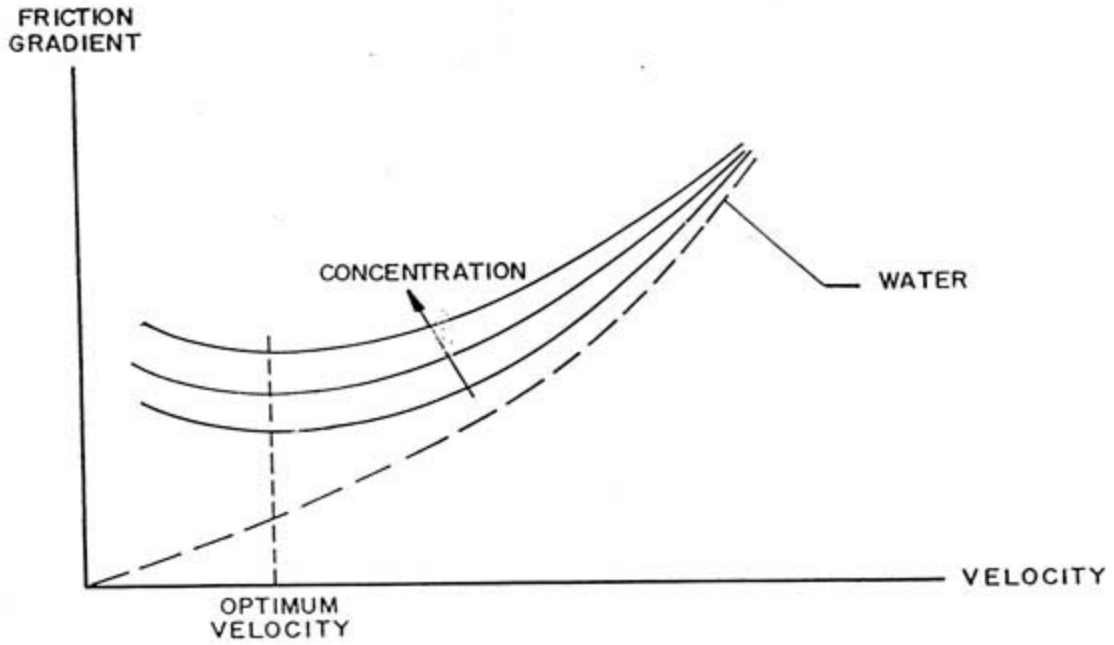


FIGURE 3.4 : INFLUENCE OF INCREASING CONCENTRATION AND VELOCITY ON THE FRICTIONAL RESISTANCE OF A TYPICAL TAILINGS SLURRY

Velocities down the beach are such that the main area of interest in a graph such as illustrated in Figure 3.4 is to the left of the minimum resistance velocity. The curves in Figure 3.4 reflect an increase in frictional resistance as velocity decreases. This is because, in the case of pipeline flow, as the velocity decreases, settling out of the larger particles begins to occur. The invert of the pipe begins to silt up and the area of the pipeline decreases resulting in an effective increase in the velocity for a constant flow. In loose boundary flow, as the base of the flow channel silts, the channel meanders and if the stream power at a given distance down the beach is constant the channel will remain relatively constant in section. It is therefore postulated that the curves will not increase as severely as is the case for pipelines.

Verkerk's (1990) program (see Chapter 2 Section 1.7.5) has been applied iteratively to the slurry stream exiting the plunge pool at the point of critical velocity by substituting $y_c/2$ for the

diameter in the input. The program output contains estimates of the Fanning friction factor, f which can be converted to the Darcy-Weisbach friction factor, λ using the equation:

$$\lambda = 4f \tag{63}$$

In the majority of cases the tailings stream exiting the discharge pipe is at supercritical velocity and at Reynolds numbers in the highly turbulent flow regime. The stream impinges on the beached tailings and, as a result of the stream power, initially erodes a plunge pool to a depth where the resistance of the beached material forces the flow stream through almost 180 degrees to the extent that the flow stream wells upwards within the plunge pool and the flow stream goes subcritical.. This is illustrated in Figure 3.4 (a). As deposition progresses the beach builds up and height h reduces as the plunge pool gradually increases in elevation. An outflow develops on the rim of the plunge pool and the stream flows out of the pool down the beach to the pond as illustrated in Figure 3.4 (b).

Depending on the discharge conditions and the slurry characteristics along the initial part of the beach, immediately following the plunge pool, the flow again goes subcritical and turbulent and generates a series of standing waves as illustrated in Figure 3.4 (c) until flow rates and the beach slope decrease to a level where the stream splits and braids and ultimately goes subcritical. The location of the outflow point continually changes as the rim of the plunge pool rises and tailings builds up on the beach. If the stream remains subcritical and flow is essentially laminar the stream will meander as deposition takes place down the beach with the plunge pool forming a pivot point.

If the stream exiting the delivery pipe remains intact as a jet stream then the velocity v_2 in Figure 3.4 (a) will be given by

$$v_2 = \sqrt{v_1^2 + 2gh} \tag{64}$$

The specific energy of this stream will be given by

$$E_s = d\ Cos^2(\theta) + \frac{v^2}{2g} \tag{65}$$

where θ is the angle of entry of the jet stream in Figure 3.4 (a). Energy will be lost in the plunge pool as flow moves from supercritical to subcritical. At the overflow flow will again move from subcritical to supercritical and in doing so pass through the critical velocity. At this point the Froude number will, by definition, be unity:

$$F_r = \frac{v_c}{\sqrt{\Delta g y_c}} = 1 \qquad (66)$$

where $\Delta = (\rho_s - \rho_w)/\rho_w$, and ρ_s is the slurry density, ρ_w the density of water and y_c is the critc ical depth..

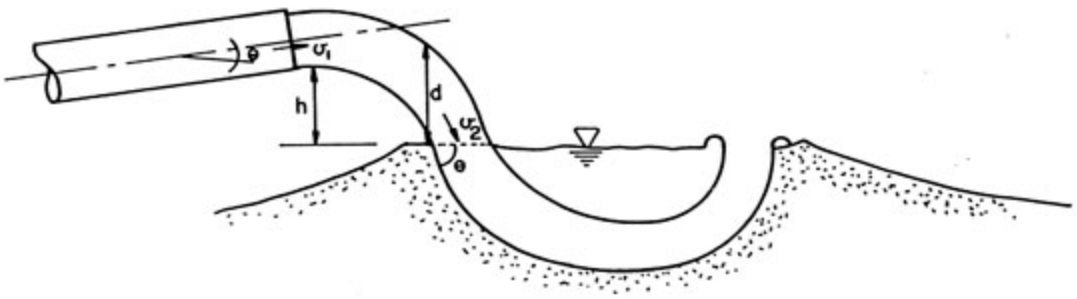
The principle of maximum entropy determines that the flow stream leaving the plunge pool will adopt the most efficient section ie that section that maximises the outflow. It can be shown (Weber (1971) that the most efficient section is that of a semi circle. It is therefore reasonable to make the assumption that the section at the point of critical velocity may be approximated by a semi-circular section of radius y_c.

Equation 62 may therefore be manipulated to yield :

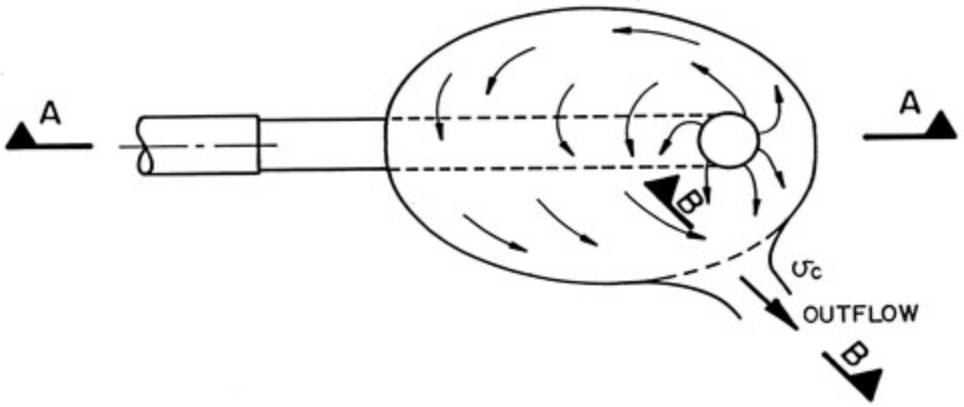
$$y_c = \left(\frac{2Q}{\pi\sqrt{\Delta g}}\right)^{2/5} \qquad (67)$$

where Q is the total flow rate out of the delivery pipe. The critical velocity will be given by

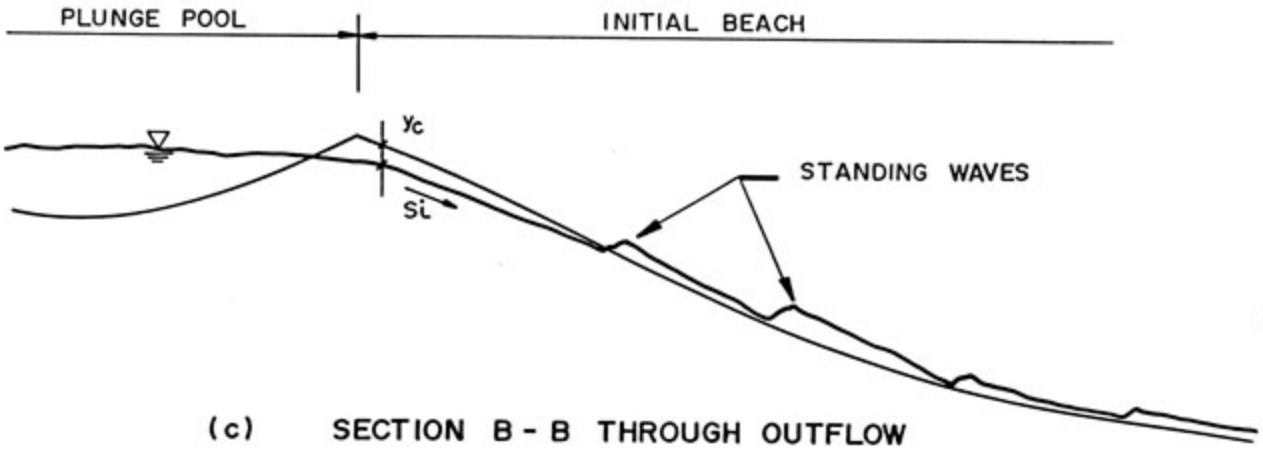
$$v_c = \frac{2Q}{\pi y_c^2} \qquad (68)$$



(a) SECTION A-A THROUGH PLUNGE POOL



(b) PLAN ON PLUNGE POOL



(c) SECTION B - B THROUGH OUTFLOW

FIGURE 3.5 : PLAN AND SECTIONS THROUGH A TYPICAL PLUNGE POOL AT A TAILINGS DISCHARGE

The specific energy will be given by:

$$E_{sc} = y_c \, Cos^2[Arctan(S_i)] + \frac{v_c^2}{2g} \tag{69}$$

The slope S_i may be determined from Manning's equation:

$$v_c = \frac{1}{n} R^{2/3} S^{1/2} \tag{70}$$

where n is Manning's constant, R is the hydraulic radius and is equal to the area of flow divided by the wetted perimeter and S is the slope, provided an estimate of n can be made. The parameter n will depend on the frictional resistance characteristics of the slurry. It is postulated that the most practical method of estimating n will be from rheological calculations as set out in the next section. At this juncture it is necessary only to note that n may be estimated from estimates of the Darcy-Weisbach friction factor as follows:

The Darcy-Weisbach equation for frictional resistance in a pipe of diameter D, length L at velocity v is

$$\frac{h_f}{L} = \frac{\lambda v^2}{2gD} \tag{71}$$

where h_f is the friction head loss. The term h_f/L is equal to the slope of the energy line which, as noted previously, on the beach will be equal to the slope of the beach. Also, it is reasonable to make the approximation that D = R. For a semi circular section flowing full the hydraulic radius is half of the depth of flow. At the point of critical velocity on the outflow $R = y_c/2$ and the slope is S_i. Equation 68 may therefore be written:

$$S_i = \frac{\lambda v_c^2}{g y_c} \tag{72}$$

Substituting for v_c in equation 67 yields:

$$n = \frac{(y_c/2)^{2/3}}{\sqrt{g y_c/\lambda}} \qquad (73)$$

Verkerk's (1990) program has been utilised and the frictional resistance predicted using the program applied to beaching slurries by means of the following algorithm to determine the initial slope of the beach:

- An estimate of the roughness coefficient is obtained by taking k, the roughness depth initially as d_{50} (Webber (1971) quoting Strickler), and the Verkerk program is run

- A value of λ, the Darcy-Weisbach friction factor, corresponding to the critical velocity is used to calculate the manning n. According to Webber (1971), the Manning equation is particularly appropriate to the rough turbulent zone in which most open channels operate and the roughness. the variation of the roughness depth with n has been checked against the equation

$$n = \frac{k^{1/6}}{26} \qquad (74)$$

- The slope S_i is calculated from equation 67 with $R = y_c/2$

In cases where supercritical velocity is likely to be exceeded, therefore, it is possible to calculate directly the flow depth and critical flow velocity with knowledge of the frictional characteristics of the flow. In cases where the slurry characteristics and/or discharge conditions are such that flow remains subcritical from the plunge pool it is still possible to make use of Manning's equation and the continuity equation but it is necessary to iterate the velocity through estimates of the frictional characteristics until a balance of the equations is found. From the estimates of the frictional characteristics and velocity it will be possible to calculate a Reynolds number and confirm that flow remains subcritical.

3.6 Velocity in the Plunge Pool

The stream power that determines the formation of the beach is determined by the power in

the plunge pool. As noted in 3.5 the stream from the delivery pipe undergoes a change from supercritical to subcritical flow in the plunge pool. But even within the plunge pool there is an additional reduction in velocity as a result of the geometry of the plunge pool and the secondary currents which cause this geometry. There are also additional frictional losses as a result of this flow. In addition part of the stream (10% to 20 % or more if the discharge is at very high pressure) laps over the rim of the plunge pool and does not find its way to the main channel. Ultimately it is the stream power in the flow within the plunge pool that is of importance and needs to be estimated.

Figure 3.6 shows the plunge pool and discharge channel in section. A number of section locations numbered 1 to 11 have been indicated on the sections.

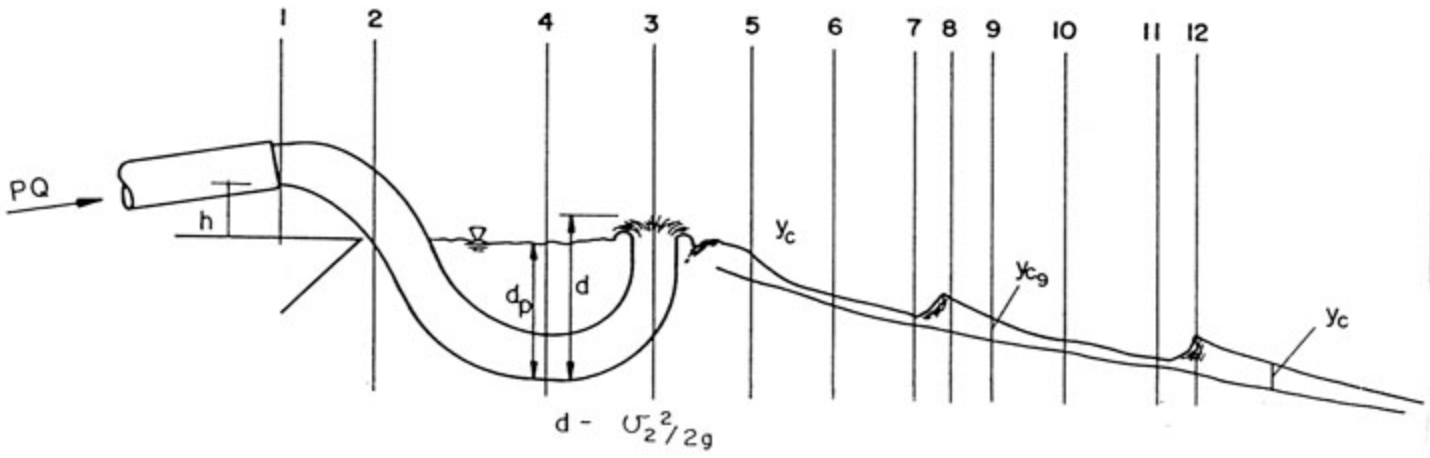


FIGURE 3.6 : TYPICAL SECTION THROUGH PLUNGE POOL AND DISCHARGE CHANNEL

Figure 3.7 shows the specific energy diagram that will apply to the flow stream into the plunge pool and down the channel.

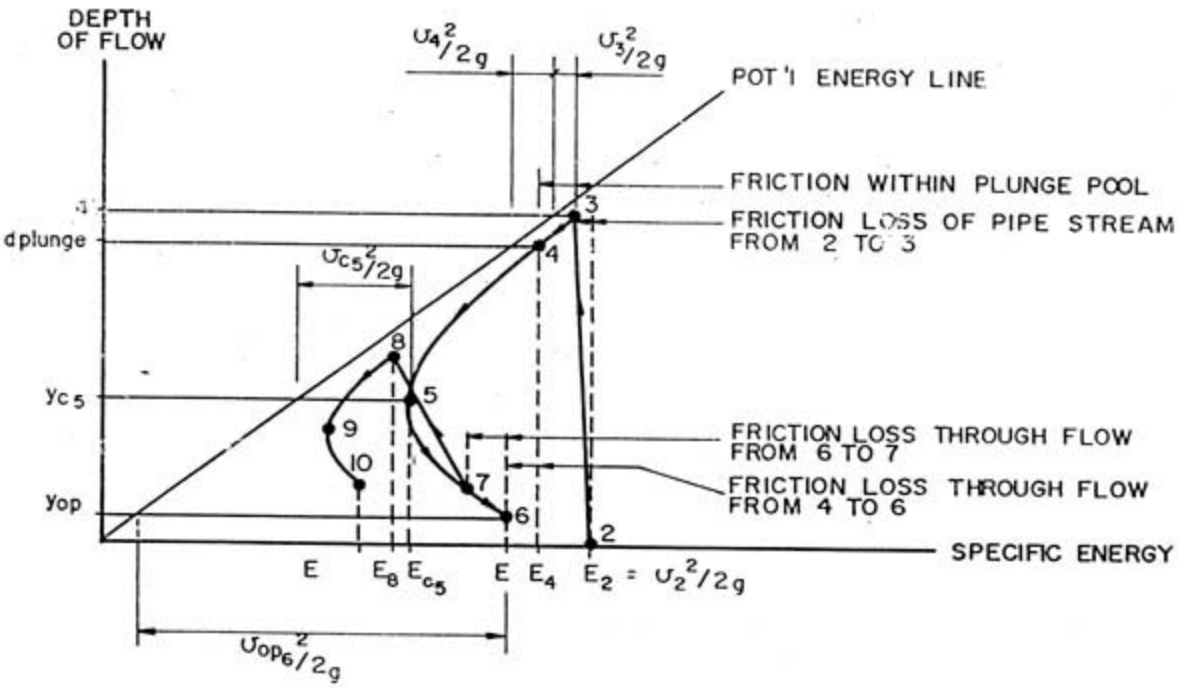


FIGURE 3.7 : TYPICAL SPECIFIC ENERGY DIAGRAM FOR THE PLUNGE POOL AND DISCHARGE CHANNEL

The following notes with respect to cross-sections 1 to 12 in Figure 3.6 are noteworthy:

1 flow from the pipe is supercritical and above the settling velocity to prevent siltation of the pipeline

2 the velocity of the discharge stream increases as a result of falling through h and may be conservatively approximated by equation 61. On the specific energy diagram the specific energy is approximately $v_2^2/2g$

3 as a result of the resistance of the beach the flow stream is deflected to near vertical. At this stage the depth d may be estimated by the from $d = v_2^2/2g$. - frictional losses. The frictional losses in moving from 2 to 3 can be estimated from the friction head loss versus velocity diagram which can be developed from the equations in Section 3.5.1. A

typical diagram is illustrated in Figure 3.8.

4 the plunge pool depth d_p will be slightly less than d due to frictional resistance to flow in the pool. The plunge pool is cup shaped and d_p varies across the section in all directions. It will be acceptable to approximate d_p by d. Observations of plunge pools suggest that the surface diameter of the cup-shaped plunge pool could be approximated by 2d. The velocity at 4 can be estimated by taking a cross-sectional area at 4 and dividing this into the flow rate. This cross-sectional area could be estimated as $\pi d/2$. The flow rate should be reduced by 10 % to 20 % to take into account plunge pool rim losses. The velocity head at 4 can then be estimated and the plunge pool stream power calculated as :

$$P = \rho Qg\left[\frac{v_4^2}{2g} - \textit{pool friction head loss}\right] \tag{75}$$

$$= \rho Qg\left[\left(\frac{2Q}{\pi d^2}\right)^2 \frac{1}{2g} - \textit{pool friction losses}\right] \tag{76}$$

where Q is the pipe flow rate and ρ is the slurry density.

5 the outflow the stream moves through critical velocity with an approximately semi-circular flow section

6 the stream stabilises at the optimum velocity indicated on Figure 3.8 since at this velocity flow is maximised relative to frictional loss.

7 although flow is at optimum velocity there is still appreciable frictional loss and the velocity decreases and flow depth increases with distance

8 the velocity has decreased to a depth where an hydraulic jump occurs and the flow depth increases. This is observed in the field as a standing wave

9 the stream again moves through critical depth due to the stream power

10 Flow again approaches optimum velocity which is lower than that at 6 due to the

deposition of solids that will have occurred at the hydraulic jump

11,12 ... the above cycles are repeated down the beach until sheet flow is developed.

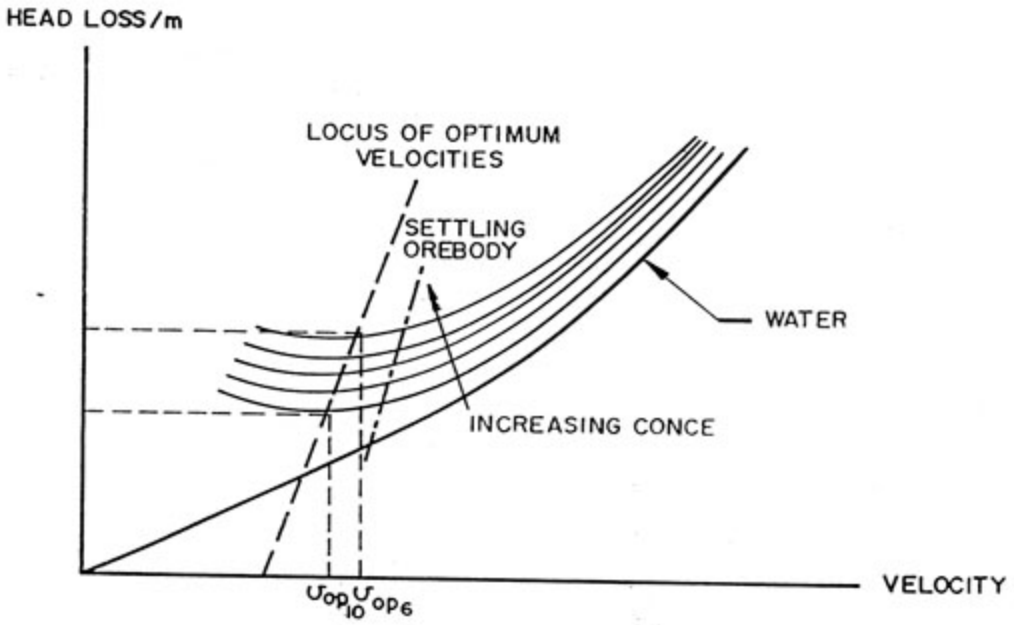


FIGURE 3.8 : TYPICAL SPECIFIC ENERGY DIAGRAM FOR PLUNGE POOL AND DISCHARGE CHANNEL

Chapter 4 describes the development of a spreadsheet-based model using the entropy approach and the application of this model to data in the literature. This data pertains to several tailings dams and products. Chapter 5 will use the model to check on the influence of variations of the six data inputs. It is evident at this stage, however, that since the inputs all combine into a single measure, the stream power P, where $P = \rho Qg(v^2/2g) = \rho Qv^2/2$ (v represents the velocity of the flow in the plunge pool) variation of any one of these variables affects each of the others and the beaching characteristics are likely to be extremely sensitive to variations in any of the parameters.

3.7 Discussion

The probabilistic approach described above differs markedly from those in classical hydrodynamics and the application of diffusion theory which form the bases for the majority of the

equations in the literature. Observations of tailings flows by this author and as documented in the literature survey indicate that the flow down a beach is unsteady in the extreme, dominated by sequences of hydraulic jumps and varied flow states which, at first, appear to be random. Typical examples of the unsteady flow regimes are illustrated in Figures 3.9 and 3.10.

In the opinion of this researcher the probabilistic approach takes into account both the apparent disorder of the deposition process as well as the extent to which the process is, in fact, ordered. The imposition of too much order, as is the case with classical hydrodynamics, is avoided by utilising only the information available and fundamental principles regularly observed in nature; nature always optimises itself by minimising energy lost and maximising randomness. The probabilistic approach therefore represents an attempt to make the least biased estimates of the beaching characteristics about which there is only partial information. It can never be exact nor is that the intention.

Through the introduction of entropy and entropy maximisation techniques additional information has effectively been made available to the analysis. This has overcome the problem in the Hunt-based approach, which requires knowledge of a reference concentration.

The beaching equation is exponential as are the majority of empirically-developed equations. Unlike these equations, however, the stream power-entropy equation:

- is fully predictive

- incorporates all of the variables known to affect the behaviour of the slurry as it beaches, namely:

    - the method of discharge - the velocity and flow rate and the slurry properties. This is taken into consideration in the stream power approach.

    - the specific gravity of the tailings particles. This is taken into account in density determinations and in calculation of the settling velocity





FIGURE 3.9 : PHOTOGRAPHS ILLUSTRATING THE UNSTEADY NATURE OF FLOW FROM A TAILINGS SLURRY DISCHARGE

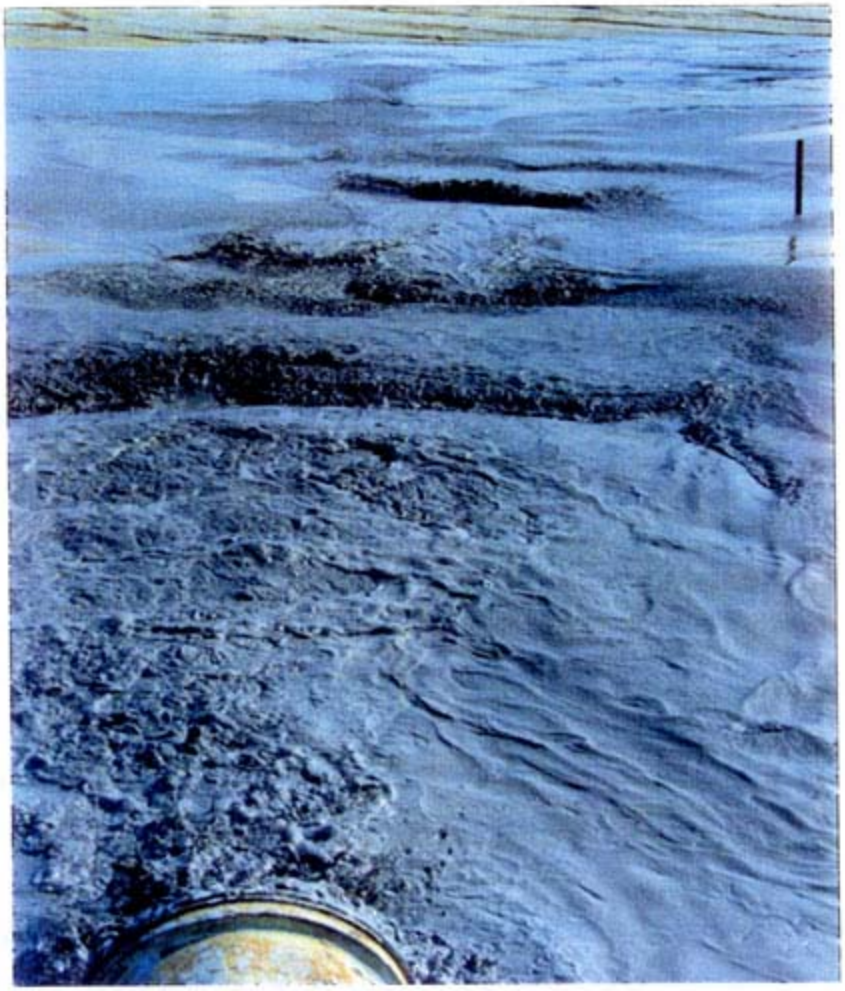




FIGURE 3.10 : PHOTOGRAPHS ILLUSTRATING THE UNSTEADY NATURE OF FLOW FROM A TAILINGS SLURRY DISCHARGE

- particle size distribution and its influence on particle segregation, the viscosity and the frictional flow resistance of the slurry

- the concentration of solids in the slurry and their effect on viscosity. This is taken into account in the calculation of the settling velocity

- the length of the beach and the relative elevations of the pool and the discharge. This is taken care of in the exponential equations proposed as the least biased estimate of the energy and beach profiles.

- takes account of the entire flow stream from a discharge but does not require detailed knowledge of the flow depths and flow regimes, parameters that are almost impossible to estimate due to the unsteady flow conditions. The principle of conservation of mass is adhered to through continuous accounting of the settled out mass and the mass remaining in suspension with distance down the beach.

An important aspect of the approach is the ability to take account of the rheology of the slurry. Research by Verkerk (1990) showed that the most significant influence on the flow resistance characteristics of a slurry is the particle size distribution. In this approach the influence of viscosity on the following properties is taken into consideration:

- frictional resistance, through its incorporation into the determination of the frictional characteristics of the slurry as calculated by the methods described by Verkerk (1990) in which the particle size distribution and its influence on the viscosity are accounted for

- settling velocity through its incorporation with the volumetric concentration into the calculation of settling velocities of the individual particle sizes in the slurry

The stream power-entropy equation assumes that the stream power is a maximum at the point of discharge and approaches zero at L the distance to the centroid of the pond. In cases where the pond is extensive and the influence of the inflowing stream relatively small, the stream power of the stream at the edge of the pond will be largely dissipated before the stream reaches the centroid of the pond. The effective beach length may therefore be less than that to the centroid of the pond and should be adjusted. It is suspected that in the case

of a massive pond the stream power would have reduced to a negligible value within 10 to 20 metres of the edge of the pond and the beach length L should be taken as the distance to the edge of the pond. In cases where estimates of the underwater profile need to be made, it should be possible to extend the stream power-entropy approach into the pond using the stream power at the entrance to the pond as the initial stream power and making adjustments to the slope of the stream power curve at the edge of the pool to take into account the increased flow resistance of the still water. This aspect has not been evaluated in detail in this dissertation.

Since the stream power-entropy equation is a monotonously decreasing function it can be expected that even slurries which do not exhibit settling characteristics would have a concave (or convex) beach profile with the maximum elevation at the discharge. The steepness of the beach profile would be dependent on the initial stream power since all of the stream power has to be lost by the time the pond is reached. The concavity (or convexity) of the beach profile will depend on the frictional characteristics of the slurry which, in turn, will determine the initial slope of the profile.

The next chapter sets out an algorithm for the development of a beaching simulation model which is based on the stream power-entropy based equations for beach profile and for the particle size distribution at any point along the beach.

CHAPTER 4

CONTENTS

Sec	Description	Page
4	DEVELOPMENT OF A BEACHING SIMULATION MODEL BASED ON ENTROPY	4-1
4.1	INTRODUCTION	4-1
4.2	PROGRAM ALGORITHM AND EQUATIONS	4-2
4.2.1	Input Data	4-2
4.2.2	Calculate the Beach Profile	4-4
4.2.3	Calculate Slurry Properties along the Beach	4-5
4.2.4	Determine Average Characteristics for Particle size Bands	4-6
4.2.5	Determine the Proportion of Each Particle Size Band	4-7
4.3	INITIAL TEST RUNS TO CHECK ON THE OPERATION OF THE BEACHING MODEL	4-8
4.3.1	TEST RUN ON THE DATA OF KÜPPER (Syncrude Tailings)	4-8
4.3.2	TEST RUN ON THE DATA OF THOMSON (1984) (Gold Tailings)	4-14
4.3.3	TEST RUN ON THE DATA OF BENTEL (1981) (Platinum Tailings)	4-17
4.4	DISCUSSION	4-20

CHAPTER 4 DEVELOPMENT OF A BEACHING SIMULATION MODEL BASED ON ENTROPY

4.1 INTRODUCTION

In Chapter 3 the following six items were identified as being required in order to carry out predictive assessments of beach profiles and particle size distributions along the beach:

1 the particle size distribution ex-mill

2 the slurry relative density, or the water:solids ratio, or the concentration by weight or by volume, and the particle specific gravity

3 the volumetric flow rate or the dry tonnage rate

4 the discharge pipe diameter and number of simultaneously operating discharges to be able to estimate the discharge velocity and therefore the discharge velocity head

5 an estimate of the losses that will occur in the plunge pool that will develop at the point where the discharge meets the beach

6 an estimate of the friction gradient characteristic of the total tailings slurry at the velocity of the discharge from the plunge pool.

In this Chapter the above items are taken as known and the equations developed in Chapter 3 are programmed into a spreadsheet model. Thereafter, to test the model, available data in the literature on the measured beach profiles of several tailings products and the results of particle size distribution tests along the beaches, will be compared with predictions made by the model as a first check on the operation of the model.

It will be necessary to make certain assumptions with regard to the data in the above items due to the fact that in the studies described in the literature not all of the above parameters were measured. These assumptions will be set out in detail under each test run.

4.2 PROGRAM ALGORITHM AND EQUATIONS

In this Section the broad sequence of calculation of the beach profile and particle size distribution is set out with the relevant equations as developed in Chapter 3. Table 4.1 contains a printout of the spreadsheet model

4.2.1 Input Data

The following details of slurry flow etc are entered:

- Particle size distribution by specifying the percentage passing each standard sieve size and hydrometer time steps
- Slurry water:solids ratio or concentration by weight
- Pipe diameter
- Tonnage per hour flowing through the discharge point
- The estimated velocity of the stream exiting the plunge pool at the discharge point. This velocity is calculated using the equations set out in Chapter 3, Section 3.5.1
- an estimate of the friction head per metre of beach (friction gradient) of the slurry emerging from the plunge pool determined using the computer program developed by Verkerk (1990) as described in Chapter 3, Section 3.5.2

TABLE 4.1 : PRINTOUT OF SPREADSHEET MODEL

BEACH PROFILE AND PARTICLE SIZE DISTRIBUTION ANALYSIS SPREADSHEET

INPUT PARAMETERS

PIPE DIAMETER (m)	0.2	
VELOCITY WITHIN PLUNGE POC	0.35887	m/s
ENERGY SLOPE AT DISCHARGE	-0.01890	
S G SOLIDS	2.89	
Cw TOTAL SOLIDS	38.7%	
t/h(solids)	1.63399	
L	350	m
DISCHARGE ELEV'N	1428.75	m

▨ FROM PLUNGE SPREADSHEET

PTCLE SIZE DISTR-EX MILL

BAND NO.	SIZE (mm)	CUM % PASSING
MIN CALC	0	0
1	0.002	9
2	0.006	25
3	0.02	49
4	0.063	75
5	0.075	80
6	0.15	96
7	0.3	99
8	0.425	100
9	1.18	100

DATA FOR GRAPH OF GRADINGS AT ANY X (INCL NON-CUMUL)

0.001	0.08974161	0.08974	0.09	0.09
0.004	0.15954064	0.24928	0.16	0.25
0.013	0.23931103	0.48859	0.24	0.49
0.0415	0.25926109	0.74785	0.26	0.75
0.069	0.04986754	0.79772	0.05	0.8
0.1125	0.15979147	0.95751	0.16	0.96
0.225	0.03067529	0.98819	0.03	0.99
0.3625	0.01181133	1	0.01	1
0.8025	0	1	0	1

EX

TOTAL FLOW RATE 3973.5296

CALCULATED FLOW PARAMETERS	
RHO SUSP	1.3388493
l/s/t/h(solids)	0.5404
FLOW RATE (m3/s)	0.000883
VELOCITY HEAD	0.0065641

SOLVING FOR MU		
P0	0.076127	1
MU	82.85285	
EQUATION TO SOLVE FOR MU	[illegible]	

35
1
0

FROM:
TO:
AVERAGE SIZE (mm)
PARTICLE WT (kg)
SUBMERGED WT (N)

X/L		POWER (watts)	POWER SLOPE	BEACH ELEVATION (mAMSL)	CONC BY WT (AVE)	CONC BY VOL	RHO SUSP (t/m3)	KINEMATIC VISCOSITY (kg/m/s)	VELOCITY HEAD (m)	SIGMA DENOMS* POWER/NO.	DIFF SIGMA & SIGMA	TOT NO. PTCLES PER SEC	ALPHA	SIGMA DENOMS/ NUMERS
0	0	0.0761272	-0.01888	1428.75	0.387	0.17929	1.33885	0.00021939	0.0065641	5.2875E-06	9.8E-11	14397.5	-30950.4	5.2876E-06
0.05	17.5	0.0357463	-0.00067	1428.41954	0.18172	0.07136	1.13487	2.231E-05	0.00308224	6.1237E-06	5.3E-06	5837.36	-19382.2	1.1408E-05
0.1	35	0.0275949	-0.00034	1428.4079	0.140281	0.05344	1.10101	1.173E-05	0.00237938	4.2767E-06	3.3E-06	6452.43	-13193.7	7.6146E-06
0.15	52.5	0.0227734	-0.00023	1428.40197	0.115771	0.04334	1.08191	7.679E-06	0.00196365	3.3837E-06	2.2E-06	6730.27	-9314.54	5.5614E-06
0.2	70	0.0193376	-0.00017	1428.39799	0.098304	0.03635	1.06871	5.568E-06	0.00166739	2.9063E-06	1.5E-06	6653.63	-6654.11	4.4196E-06
0.25	87.5	0.0166662	-0.00014	1428.395	0.084724	0.03104	1.05866	4.296E-06	0.00143705	2.621E-06	1.1E-06	6358.59	-4713.1	3.7259E-06
0.3	105	0.0144802	-0.00011	1428.39261	0.073612	0.02676	1.05058	3.46E-06	0.00124856	2.4352E-06	8.4E-07	5946.3	-3232.49	3.2711E-06
0.35	122.5	0.0126301	-9.8E-05	1428.39061	0.064207	0.02319	1.04383	2.879E-06	0.00108904	2.3058E-06	6.5E-07	5477.65	-2083.61	2.9544E-06
0.4	140	0.0110263	-8.6E-05	1428.38889	0.056053	0.02013	1.03805	2.458E-06	0.00095075	2.2111E-06	5.1E-07	4986.85	-1116.58	2.7235E-06
0.45	157.5	0.0096108	-7.6E-05	1428.38739	0.048857	0.01746	1.03301	2.142E-06	0.0008287	2.139E-06	4.1E-07	4493.19	-332.345	2.5485E-06
0.5	175	0.008344	-6.9E-05	1428.38606	0.042418	0.0151	1.02853	1.901E-06	0.00071947	2.0821E-06	3.3E-07	4007.41	380.668	2.4115E-06
0.55	192.5	0.0071977	-6.2E-05	1428.38485	0.03659	0.01297	1.02452	1.712E-06	0.00062062	2.0363E-06	2.7E-07	3534.65	898.03	2.3018E-06
0.6	210	0.0061508	-5.7E-05	1428.38376	0.031268	0.01105	1.02088	1.561E-06	0.00053036	1.9986E-06	2.1E-07	3077.61	1389.74	2.2121E-06
0.65	227.5	0.0051375	-5.3E-05	1428.38276	0.026371	0.00929	1.01755	1.44E-06	0.0004473	1.9668E-06	1.7E-07	2637.49	1821.49	2.1374E-06
0.7	245	0.0042955	-4.9E-05	1428.38183	0.021837	0.00767	1.01449	1.34E-06	0.00037038	1.9398E-06	1.3E-07	2214.41	2204.01	2.0742E-06
0.75	262.5	0.0034649	-4.6E-05	1428.38097	0.017614	0.00617	1.01165	1.258E-06	0.00029876	1.9164E-06	1E-07	1807.99	2545.82	2.0201E-06
0.8	280	0.0026878	-4.3E-05	1428.38017	0.013663	0.00477	1.00902	1.189E-06	0.00023175	1.896E-06	7.7E-08	1417.58	2853.57	1.9732E-06
0.85	297.5	0.0019577	-4E-05	1428.37942	0.009952	0.00347	1.00655	1.13E-06	0.0001688	1.878E-06	5.4E-08	1042.42	3132.52	1.9321E-06
0.9	315	0.0012692	-3.8E-05	1428.37871	0.006452	0.00224	1.00424	1.081E-06	0.00010944	1.862E-06	3.4E-08	681.657	3386.91	1.8959E-06
0.95	332.5	0.0006179	-3.6E-05	1428.37804	0.003141	0.00109	1.00206	1.037E-06	5.328E-05	1.8476E-06	1.6E-08	334.457	3620.17	1.8636E-06
1	350	-2.68E-18	-3.4E-05	1428.3774	-1.4E-17	-4.7E-18	1	1E-06	-2.307E-19	1.8346E-06	1E-10	-1.5E-12	3835.1	1.8347E-06

DATA FOR MULTIPLE CUMULATIVE GRADINGS - STREAM FLOW (AS OPPOSED TO BEACH)

11.7			DISTANCE DOWN BEACH																	
23.4	diameter	EX MILL	0	17.5	35	52.5	70	87.5	105	122.5	140	157.5	175	192.5	210	227.5	245	262.5	280	297.5
46.8		CUMULATIVE GRADINGS																		
70.2	0.001	0.09	0.08315	0.08148	0.0852	0.087074	0.088114	0.08876	0.0891944	0.08951	0.08974	0.08993	0.09007	0.090195	0.090297	0.09038	0.0904602	0.09053	0.09059	0.09064
93.6	0.004	0.25	0.23097	0.22633	0.23668	0.241873	0.244762	0.24655	0.2477623	0.24863	0.24928	0.24979	0.2502	0.250541	0.250825	0.25107	0.2512783	0.25146	0.25163	0.25177
-MILL	0.013	0.49	0.4527	0.4436	0.46388	0.474071	0.479735	0.48325	0.4856144	0.48731	0.48859	0.48959	0.4904	0.49106	0.491617	0.49209	0.4925053	0.49287	0.49319	0.49347
	0.0415	0.75	0.69299	0.67907	0.7101	0.725673	0.734329	0.73969	0.7433082	0.7459	0.74785	0.74938	0.75061	0.751617	0.752465	0.75319	0.7538189	0.75437	0.75486	0.75529
	0.069	0.8	0.73931	0.72448	0.75754	0.774128	0.783341	0.78905	0.7928916	0.79565	0.79772	0.79934	0.80064	0.801716	0.802616	0.80339	0.8040511	0.80463	0.80515	0.80561
	0.1125	0.96	0.88997	0.87244	0.91142	0.930752	0.941352	0.94784	0.9521598	0.95523	0.95751	0.95928	0.9607	0.961853	0.962817	0.96364	0.9643412	0.96496	0.9655	0.96598
	0.225	0.99	0.9276	0.91059	0.948	0.965765	0.97509	0.98056	0.9840632	0.98646	0.98819	0.98949	0.99049	0.991295	0.991949	0.99249	0.992949	0.99334	0.99368	0.99398
	0.3625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	0.8025	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

SIZE	% FINER	
0	0	
0.002	9	
0.001	9	AVE RETAINED
2.3E-09		
1.5E-08		

D'	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENO	M-B NUM
			0.09	0.09			
0.00041	2.65E-07	3.9E-15	0.08315	0.08315	1197.13	0.09	3.47E-16
0.00139	4.7E-07	6.8E-15	0.08148	0.08148	475.615	0.09	6.16E-16
0.00193	5.14E-07	7.5E-15	0.0852	0.0852	549.766	0.09	6.73E-16
0.00239	5.4E-07	7.9E-15	0.08707	0.08707	586.033	0.09	7.07E-16
0.00279	5.58E-07	8.1E-15	0.08811	0.08811	586.281	0.09	7.31E-16
0.00315	5.73E-07	8.3E-15	0.08876	0.08876	564.386	0.09	7.5E-16
0.00346	5.84E-07	8.5E-15	0.08919	0.08919	530.377	0.09	7.66E-16
0.00373	5.94E-07	8.7E-15	0.08951	0.08951	490.286	0.09	7.79E-16
0.00395	6.03E-07	8.8E-15	0.08974	0.08974	447.528	0.09	7.91E-16
0.00413	6.11E-07	8.9E-15	0.08993	0.08993	404.05	0.09	8.01E-16
0.00426	6.18E-07	9E-15	0.09007	0.09007	360.959	0.09	8.1E-16
0.00435	6.24E-07	9.1E-15	0.09019	0.09019	318.807	0.09	8.18E-16
0.00438	6.3E-07	9.2E-15	0.0903	0.0903	277.899	0.09	8.25E-16
0.00436	6.35E-07	9.2E-15	0.09038	0.09038	238.388	0.09	8.32E-16
0.00429	6.4E-07	9.3E-15	0.09046	0.09046	200.316	0.09	8.38E-16
0.00417	6.44E-07	9.4E-15	0.09053	0.09053	163.671	0.09	8.44E-16
0.00397	6.48E-07	9.4E-15	0.09059	0.09059	128.412	0.09	8.5E-16
0.00369	6.52E-07	9.5E-15	0.09064	0.09064	94.4827	0.09	8.55E-16
0.00329	6.56E-07	9.6E-15	0.09069	0.09069	61.8163	0.09	8.6E-16
0.00266	6.6E-07	9.6E-15	0.09073	0.09073	30.3447	0.09	8.65E-16
0	6.63E-07	9.7E-15	0.09077	0.09077	-1.3E-13	0.09	8.69E-16

SIZE	% FINER	
0.002	9	
0.006	25	
0.004	16	AVE RETAINED
3.6E-08		
2.3E-07		

D'	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENC	M-B NUM
			0.16	0.25			
0.00164	4.233E-06	9.9E-13	0.14782	0.23097	2128.22	0.16	1.58E-13
0.00554	7.518E-06	1.8E-12	0.14485	0.22633	845.537	0.16	2.8E-13
0.00772	8.217E-06	1.9E-12	0.15147	0.23668	977.362	0.16	3.06E-13
0.00955	8.633E-06	2E-12	0.1548	0.24187	1041.84	0.16	3.22E-13
0.01116	8.93E-06	2.1E-12	0.15665	0.24476	1042.28	0.16	3.33E-13
0.01259	9.161E-06	2.1E-12	0.15779	0.24655	1003.35	0.16	3.42E-13
0.01384	9.351E-06	2.2E-12	0.15857	0.24776	942.893	0.16	3.49E-13
0.01492	9.512E-06	2.2E-12	0.15912	0.24863	871.62	0.16	3.55E-13
0.01581	9.651E-06	2.2E-12	0.15954	0.24928	795.606	0.16	3.6E-13
0.01653	9.774E-06	2.3E-12	0.15987	0.24979	718.312	0.16	3.64E-13
0.01705	9.884E-06	2.3E-12	0.16013	0.2502	641.705	0.16	3.68E-13
0.01738	9.983E-06	2.3E-12	0.16035	0.25054	566.768	0.16	3.72E-13
0.01752	1.007E-05	2.3E-12	0.16053	0.25083	494.042	0.16	3.76E-13
0.01745	1.016E-05	2.4E-12	0.16068	0.25107	423.802	0.16	3.79E-13
0.01717	1.024E-05	2.4E-12	0.16082	0.25128	356.117	0.16	3.82E-13
0.01666	1.031E-05	2.4E-12	0.16094	0.25146	290.97	0.16	3.84E-13
0.01588	1.037E-05	2.4E-12	0.16104	0.25163	228.288	0.16	3.87E-13
0.01476	1.044E-05	2.4E-12	0.16113	0.25177	167.969	0.16	3.89E-13
0.01316	1.05E-05	2.4E-12	0.16122	0.2519	109.896	0.16	3.91E-13
0.01063	1.055E-05	2.5E-12	0.16129	0.25202	53.9462	0.16	3.93E-13
0	1.061E-05	2.5E-12	0.16136	0.25213	-2.4E-13	0.16	3.95E-13

SIZE	% FINER	
0.006	25	
0.02	49	
0.013	24	AVE RETAINED
3.8E-07		
2.5E-06		

D'	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL
			0.24	0.49
0.00533	4.471E-05	1.1E-10	0.22173	0.4527
0.01801	7.941E-05	2E-10	0.21727	0.4436
0.02511	8.679E-05	2.1E-10	0.22721	0.46388
0.03105	9.118E-05	2.2E-10	0.2322	0.47407
0.03628	9.432E-05	2.3E-10	0.23497	0.47973
0.04092	9.677E-05	2.4E-10	0.23669	0.48325
0.04499	9.877E-05	2.4E-10	0.23785	0.48561
0.04848	0.0001005	2.5E-10	0.23868	0.48731
0.0514	0.0001019	2.5E-10	0.23931	0.48859
0.05371	0.0001032	2.5E-10	0.2398	0.48959
0.05542	0.0001044	2.6E-10	0.24019	0.4904
0.0565	0.0001054	2.6E-10	0.24052	0.49106
0.05694	0.0001064	2.6E-10	0.24079	0.49162
0.05672	0.0001073	2.6E-10	0.24103	0.49209
0.05581	0.0001081	2.7E-10	0.24123	0.49251
0.05415	0.0001089	2.7E-10	0.2414	0.49287
0.05161	0.0001096	2.7E-10	0.24156	0.49319
0.04799	0.0001103	2.7E-10	0.2417	0.49347
0.04277	0.0001109	2.7E-10	0.24183	0.49373
0.03454	0.0001115	2.7E-10	0.24194	0.49397
0	0.000112	2.8E-10	0.24205	0.49418

315	332.5	350
0.09069	0.090728	0.090768
0.2519	0.252023	0.252132
0.49373	0.493965	0.494179
0.75569	0.756042	0.756367
0.80603	0.806407	0.806752
0.96641	0.966806	0.967162
0.99424	0.99447	0.994679
1	1	1
1	1	1

SIZE	% FINER	
0.02	49	
0.063	75	
0.0415	26	AVE RETAINED
3.9E-06		
2.5E-05		

SIZE	% FINR	
0.063	75	
0.075	80	
0.069	5	AVE RETAINED
1.1E-05		
6.9E-05		

SIZE	% FINER
0.075	80
0.15	96
0.1125	16
2.9E-05	
0.00018	

NO. PTCLES	M-B DENC	M-B NUM	D*	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENC	M-B NUM	D*	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENO	M-B NUM	D*	SETTLING VELOCITY (m/s)
						0.26	0.75							0.05	0.8					
3192.35	0.240001	2.64E-11	0.01703	0.0004556	1.1E-08	0.24029	0.69299	3459.59	0.260092	2.97E-09	0.02831	0.0012595	8.7E-08	0.04632	0.73931	666.87	0.0501353	4.38E-09	0.04616	0.0033482
1268.31	0.240001	4.69E-11	0.05749	0.0008093	2E-08	0.23547	0.67907	1374.54	0.260102	5.28E-09	0.09559	0.0022372	1.6E-07	0.0454	0.72448	265.026	0.0501505	7.78E-09	0.15586	0.0059472
1466.05	0.240001	5.13E-11	0.08015	0.0008845	2.2E-08	0.24621	0.7101	1588.68	0.260076	5.77E-09	0.13325	0.0024451	1.7E-07	0.04744	0.75754	306.109	0.0501119	8.49E-09	0.21726	0.0064998
1562.76	0.240001	5.39E-11	0.09912	0.0009292	2.3E-08	0.2516	0.72567	1693.35	0.260056	6.06E-09	0.16481	0.0025688	1.8E-07	0.04845	0.77413	326.115	0.050083	8.92E-09	0.26871	0.0068287
1563.42	0.24	5.57E-11	0.11582	0.0009612	2.4E-08	0.25459	0.73433	1693.97	0.260042	6.27E-09	0.19257	0.0026572	1.8E-07	0.04901	0.78334	326.111	0.0500613	9.22E-09	0.31397	0.0070638
1505.03	0.24	5.72E-11	0.13062	0.0009861	2.5E-08	0.25645	0.73969	1630.64	0.26003	6.43E-09	0.21717	0.0027261	1.9E-07	0.04935	0.78905	313.827	0.0500446	9.46E-09	0.35409	0.0072468
1414.34	0.24	5.83E-11	0.14361	0.0010065	2.5E-08	0.25769	0.74331	1532.33	0.260021	6.56E-09	0.23877	0.0027825	1.9E-07	0.04958	0.79289	294.838	0.0500312	9.65E-09	0.3893	0.0073968
1307.43	0.24	5.93E-11	0.15477	0.0010238	2.6E-08	0.25859	0.7459	1416.46	0.260014	6.68E-09	0.25734	0.0028303	2E-07	0.04975	0.79565	272.492	0.0500203	9.82E-09	0.41957	0.0075238
1193.41	0.24	6.02E-11	0.16408	0.0010388	2.6E-08	0.25926	0.74785	1292.9	0.260008	6.77E-09	0.27281	0.0028717	2E-07	0.04987	0.79772	248.682	0.0500111	9.96E-09	0.44479	0.0076339
1077.47	0.24	6.1E-11	0.17147	0.001052	2.6E-08	0.25979	0.74938	1167.27	0.260002	6.86E-09	0.2851	0.0029083	2E-07	0.04996	0.79934	224.487	0.0500034	1.01E-08	0.46483	0.0077311
962.558	0.24	6.17E-11	0.17691	0.0010639	2.7E-08	0.26021	0.75061	1042.76	0.259998	6.94E-09	0.29414	0.002941	2E-07	0.05004	0.80064	200.519	0.0499966	1.02E-08	0.47958	0.0078181
850.152	0.24	6.23E-11	0.18036	0.0010746	2.7E-08	0.26056	0.75162	920.976	0.259994	7.01E-09	0.29987	0.0029706	2.1E-07	0.0501	0.80172	177.082	0.0499908	1.03E-08	0.48892	0.0078968
741.063	0.24	6.28E-11	0.18176	0.0010844	2.7E-08	0.26085	0.75247	802.789	0.25999	7.07E-09	0.30221	0.0029977	2.1E-07	0.05015	0.80262	154.344	0.0499856	1.04E-08	0.49274	0.0079687
635.702	0.24	6.34E-11	0.18107	0.0010934	2.7E-08	0.2611	0.75319	688.643	0.259987	7.13E-09	0.30106	0.0030225	2.1E-07	0.05019	0.80339	132.387	0.0499809	1.05E-08	0.49086	0.0080349
534.175	0.24	6.39E-11	0.17817	0.0011017	2.8E-08	0.26131	0.75382	578.655	0.259984	7.18E-09	0.29623	0.0030456	2.1E-07	0.05023	0.80405	111.235	0.0499767	1.06E-08	0.48299	0.0080962
436.455	0.24	6.43E-11	0.17286	0.0011095	2.8E-08	0.2615	0.75437	472.793	0.259982	7.23E-09	0.2874	0.0030671	2.1E-07	0.05027	0.80463	90.879	0.0499729	1.06E-08	0.46859	0.0081532
342.432	0.24	6.47E-11	0.16477	0.0011167	2.8E-08	0.26167	0.75486	370.939	0.259979	7.28E-09	0.27395	0.0030871	2.1E-07	0.05029	0.80515	71.2966	0.0499695	1.07E-08	0.44667	0.0082066
251.954	0.24	6.51E-11	0.15319	0.0011236	2.8E-08	0.26182	0.75529	272.926	0.259977	7.33E-09	0.25469	0.003106	2.2E-07	0.05032	0.80561	52.455	0.0499663	1.08E-08	0.41526	0.0082567
164.843	0.24	6.55E-11	0.13653	0.00113	2.8E-08	0.26195	0.75569	178.563	0.259975	7.37E-09	0.22699	0.0031238	2.2E-07	0.05034	0.80603	34.3172	0.0499633	1.08E-08	0.3701	0.008304
80.9192	0.24	6.58E-11	0.11027	0.0011361	2.8E-08	0.26208	0.75604	87.6535	0.259973	7.41E-09	0.18335	0.0031406	2.2E-07	0.05036	0.80641	16.8449	0.0499606	1.09E-08	0.29893	0.0083487
-3.5E-13	0.24	6.62E-11	0	0.0011419	2.9E-08	0.26219	0.75637	-3.8E-13	0.259971	7.44E-09	0	0.0031565	2.2E-07	0.05038	0.80675	-7.3E-14	0.0499581	1.09E-08	0	0.0083911

AVE RETAINED

POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENC	M-B NUM
	0.16	0.96			
6.2E-07	0.15067	0.88997	2169.26	0.163085	1.01E-07
1.1E-06	0.14796	0.87244	863.692	0.163435	1.79E-07
1.2E-06	0.15388	0.91142	992.93	0.162548	1.95E-07
1.3E-06	0.15662	0.93075	1054.12	0.161887	2.04E-07
1.3E-06	0.15801	0.94135	1051.35	0.161392	2.1E-07
1.3E-06	0.15879	0.94784	1009.69	0.16101	2.15E-07
1.4E-06	0.15927	0.95216	947.057	0.160707	2.19E-07
1.4E-06	0.15958	0.95523	874.118	0.160458	2.22E-07
1.4E-06	0.15979	0.95751	796.857	0.160252	2.25E-07
1.4E-06	0.15994	0.95928	718.652	0.160076	2.28E-07
1.4E-06	0.16005	0.9607	641.399	0.159924	2.3E-07
1.5E-06	0.16014	0.96185	566.028	0.159791	2.33E-07
1.5E-06	0.1602	0.96282	493.035	0.159674	2.35E-07
1.5E-06	0.16025	0.96364	422.66	0.159569	2.36E-07
1.5E-06	0.16029	0.96434	354.948	0.159475	2.38E-07
1.5E-06	0.16032	0.96496	289.859	0.159389	2.4E-07
1.5E-06	0.16035	0.9655	227.305	0.159311	2.41E-07
1.5E-06	0.16037	0.96598	167.17	0.159239	2.42E-07
1.5E-06	0.16038	0.96641	109.327	0.159173	2.44E-07
1.5E-06	0.1604	0.96681	53.6465	0.159111	2.45E-07
1.5E-06	0.16041	0.96716	-2.3E-13	0.159054	2.46E-07

SIZE	% FINER
0.15	96
0.3	99
0.225	3 AVE RETAINED
0.00011	
0.00074	

D*	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENC	M-B NUM
			0.03	0.99			
0.09231	0.01339283	9.9E-06	0.03762	0.9276	541.664	0.040722	4.02E-07
0.31171	0.02378868	1.8E-05	0.03815	0.91059	222.717	0.042145	7.39E-07
0.43452	0.02599925	1.9E-05	0.03657	0.948	235.974	0.03863	7.4E-07
0.53741	0.02731494	2E-05	0.03501	0.96576	235.645	0.036189	7.29E-07
0.62795	0.02825519	2.1E-05	0.03374	0.97509	224.481	0.03446	7.18E-07
0.70818	0.02898739	2.1E-05	0.03272	0.98056	208.063	0.033179	7.09E-07
0.77859	0.02958711	2.2E-05	0.0319	0.98406	189.707	0.032192	7.02E-07
0.83914	0.030095	2.2E-05	0.03123	0.98646	171.085	0.031405	6.97E-07
0.88958	0.03053545	2.3E-05	0.03068	0.98819	152.973	0.030764	6.93E-07
0.92967	0.03092428	2.3E-05	0.0302	0.98949	135.708	0.030228	6.89E-07
0.95916	0.0312723	2.3E-05	0.0298	0.99049	119.406	0.029772	6.86E-07
0.97784	0.03158727	2.3E-05	0.02944	0.9913	104.07	0.029379	6.84E-07
0.98547	0.0318749	2.3E-05	0.02913	0.99195	89.6568	0.029036	6.82E-07
0.98171	0.03213956	2.4E-05	0.02886	0.99249	76.1064	0.028733	6.81E-07
0.96597	0.03238463	2.4E-05	0.02861	0.99295	63.3493	0.028462	6.8E-07
0.93718	0.03261281	2.4E-05	0.02838	0.99334	51.3178	0.028219	6.78E-07
0.89333	0.03282628	2.4E-05	0.02818	0.99368	39.948	0.027998	6.78E-07
0.83052	0.03302681	2.4E-05	0.02799	0.99398	29.1815	0.027797	6.77E-07
0.7402	0.03321588	2.4E-05	0.02782	0.99424	18.9655	0.027612	6.76E-07
0.59787	0.03339472	2.5E-05	0.02766	0.99447	9.25244	0.027442	6.76E-07
0	0.03356438	2.5E-05	0.02752	0.99468	-4E-14	0.027284	6.75E-07

SIZE	% FINER
0.3	99
0.425	100
0.3625	1 AVE RETAINED
0.0003	
0.00191	

D*	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENO
			0.01	1		
0.14873	0.03476348	6.7E-05	0.0724	1	1042.43	0.0783702
0.5022	0.06174779	0.00012	0.08941	1	521.925	0.0987633
0.70006	0.0674857	0.00013	0.052	1	335.557	0.0549327
0.86583	0.07090082	0.00014	0.03424	1	230.412	0.0353854
1.01169	0.07334141	0.00014	0.02491	1	165.74	0.0254428
1.14095	0.07524197	0.00014	0.01944	1	123.606	0.0197109
1.2544	0.07679865	0.00015	0.01594	1	94.765	0.0160807
1.35194	0.07811696	0.00015	0.01354	1	74.1606	0.0136134
1.43322	0.07926024	0.00015	0.01181	1	58.9014	0.0118453
1.4978	0.0802695	0.00015	0.01051	1	47.2458	0.0105237
1.54531	0.08117286	0.00016	0.00951	1	38.0992	0.0094995
1.5754	0.0819904	0.00016	0.0087	1	30.7678	0.0086858
1.5877	0.082737	0.00016	0.00805	1	24.7793	0.008025
1.58165	0.08342397	0.00016	0.00751	1	19.8044	0.0074768
1.55629	0.0840601	0.00016	0.00705	1	15.6137	0.0070151
1.5099	0.0846524	0.00016	0.00666	1	12.0401	0.0066207
1.43925	0.08520649	0.00016	0.00632	1	8.95991	0.0062797
1.33806	0.085727	0.00016	0.00602	1	6.27974	0.0059818
1.19254	0.08621776	0.00016	0.00576	1	3.92814	0.0057191
0.96323	0.08668197	0.00017	0.00553	1	1.84952	0.0054855
0	0.08712234	0.00017	0.00532	1	-7.8E-15	0.0052763

SIZE	% FINER	
0.425	100	
1.18	100	
0.8025	0	AVE RETAINED
0.00146		
0.00938		

M-B NUM	D'	SETTLING VELOCITY (m/s)	POWER/ PTCLE (watts)	Pi	PI CUMUL	NO. PTCLES	M-B DENO	M-B NUM
				0	1			
5.21E-06	0.32925	0.17037162	0.0016	0	1	0	0	0
1.17E-05	1.11177	0.3026185	0.00284	0	1	0	0	0
7.09E-06	1.5498	0.3307393	0.0031	0	1	0	0	0
4.8E-06	1.91676	0.34747642	0.00326	0	1	0	0	0
3.57E-06	2.23967	0.35943745	0.00337	0	1	0	0	0
2.84E-06	2.52582	0.36875185	0.00346	0	1	0	0	0
2.36E-06	2.77697	0.37638096	0.00353	0	1	0	0	0
2.03E-06	2.99292	0.38284184	0.00359	0	1	0	0	0
1.8E-06	3.17284	0.3884449	0.00364	0	1	0	0	0
1.62E-06	3.31581	0.3933912	0.00369	0	1	0	0	0
1.48E-06	3.42099	0.39781844	0.00373	0	1	0	0	0
1.36E-06	3.48761	0.40182513	0.00377	0	1	0	0	0
1.27E-06	3.51484	0.40548412	0.0038	0	1	0	0	0
1.19E-06	3.50144	0.40885085	0.00383	0	1	0	0	0
1.13E-06	3.44531	0.41196847	0.00386	0	1	0	0	0
1.07E-06	3.3426	0.41487124	0.00389	0	1	0	0	0
1.02E-06	3.18621	0.41758679	0.00392	0	1	0	0	0
9.81E-07	2.96218	0.42013774	0.00394	0	1	0	0	0
9.44E-07	2.64004	0.42254287	0.00396	0	1	0	0	0
9.1E-07	2.13239	0.4248179	0.00398	0	1	0	0	0
8.8E-07	0	0.42697614	0.004	0	1	0	0	0

4.2.2 Calculate the Beach Profile

The beach profile is calculated using the equations derived in Chapter 3 by means of the following algorithm:

- Calculate the stream power P_0 at the plunge pool from the equation:

$$P_0 = \rho Q g(\frac{v^2}{2g}) = \rho q \frac{v^2}{2} \quad (1)$$

where v is the velocity in the plunge pool.

- Solve for μ by putting the friction gradient at the plunge pool, S_0, equal to the slope of the stream power equation given by:

$$S_0 = - \frac{(1 - \exp^{-\mu P_0})}{L \mu \exp^{-\mu P_0}} \quad (2)$$

where L is the length of the beach. S_0 is estimated from a calibrating beach profile or using Verkerk's program (Chapter 3).

- Calculate the stream power P(x) at any point distant x down the beach from the equation:

$$P(x) = -\frac{1}{\mu} \ln \left[(1 - \exp^{-\mu P_0})\frac{x}{L} + \exp^{-\mu P_0}\right] \quad (3)$$

- Calculate the slope of the stream power curve, $S_B(x)$, (which is equal to the slope of the beach) at any point x along the beach from the equation:

$$S_B(x) = -\frac{(1 - \exp^{-\mu P_0})}{L \mu \exp^{-\mu P(x)}} \quad (4)$$

- Calculate the beach profile by calculating the elevation of the beach surface from the equation:

$$y = y(x + \Delta x) + S_B(x)\Delta x \qquad (5)$$

4.2.3 Calculate Slurry Properties along the Beach

- Calculate the concentration by weight, $C_w(x)$ at any point x along the beach from the equation:

$$C_w(x) = \frac{P(x)}{P_0} C_0 \qquad (6)$$

where C_0 is the concentration at the discharge.

- Calculate the volumetric concentration $C_v(x)$ at any point x along the beach from the equation:

$$C_v(x) = \frac{\frac{C_w(x)}{SG}}{(1 - C_w(x)) + \frac{C_w(x)}{SG}} \qquad (7)$$

where SG is the specific gravity of the solids.

- Calculate the slurry density $\rho_{susp}(x)$ at any point x along the beach using the equation:

$$\rho_{susp}(x) = \frac{1}{(1 - C_w(x)) + \frac{Cw(x)}{SG}} \qquad (8)$$

- Estimate the kinematic viscosity $\nu(x)$ of the slurry at any point x along the beach from the equation:

where k is a curve-fitting constant from viscosity testing. If no viscosity tests are available k is taken as 2,5

$$\nu(x) = \frac{\nu_{water}\ \rho_{water}}{\rho_{susp}(x)} (1 + kC_v(x) + k^2C_v^2(x) + k^3C_v^3(x)) \quad (9)$$

- Calculate the velocity head at any point x along the beach from the equation:

$$\frac{v^2(x)}{2g} = \frac{P(x)}{P_0} \frac{v_0^2}{2g} \quad (10)$$

4.2.4 Determine Average Characteristics for Particle size Bands

If there are i bands of particle sizes, then for each band, at any distance x along the beach, determine the following:

- Mean diameter, d_i, of the band
- Average particle weight, W_i, for each band
- Average submerged particle weight, W_{subi}, for each band
- The average settling velocity w_i for each band corrected for viscosity effects using the equation (Raudkivi (1990)):

$$w_{vi} = w_i (1 - C_v)^{\beta} \quad (11)$$

where w_i is determined using the methods outlined in Section 2.2 of Chapter 2, and the factor β is 4.65 for all values of the non-dimensional grain size D_i^* and D_i^* is given by:

$$D_i^* = \left[\frac{\left(\frac{\rho_{susp}(x) - \rho}{\rho} \right) g d_i^3}{\nu_{susp}(x)^2} \right]^{1/3} \quad (12)$$

When D^* is greater than 800, β is taken as 2.35 and for the transition between the two limits, β is given by:

$$\beta = 7.478\ D^{*-0.129} \qquad (13)$$

4.2.5 Determine the Proportion of Each Particle Size Band

At any point x along the beach the proportion, p_i, (where i = 1, 2, 3,...n) of each particle size band making up the distribution of particles in the flow stream and therefore most likely to be deposited on the beach. The proportions are moderated according to the distribution of particles in the mill tailings (as discussed in Chapter 3) and q_i is the equivalent proportion of each particle band in the mill tailings.

At each distance, x, the parameter α is determined through iteration of the following two equations:

$$p_i(x) = \frac{q_i \exp^{-\alpha e_i}}{\left(\sum_{i=1}^{n} q_i \exp^{-\alpha e_i}\right)} \qquad (14)$$

(where e_i is the stream power to maintain the particle in suspension), and:

$$\frac{\sum_{i=1}^{n} q_i e_i \exp^{-\alpha e_i}}{\sum_{i=1}^{n} q_i \exp^{-\alpha e_i}} - \bar{e} = 0 \qquad (15)$$

where $\bar{e}$ is given by :

$$\frac{P(x)}{N_t(x)} = \frac{\left(\sum_{i=1}^{n} p_i(x) W_i\right) \frac{v(x)^2}{2g}}{C_w(x)} \qquad (16)$$

and $N_t(x)$ is the average number of particles in suspension at x. W_i is the average weight of each band of particles.

4.3 INITIAL TEST RUNS TO CHECK ON THE OPERATION OF THE BEACHING MODEL

This section sets out a series of test runs conducted using available data from the literature on the beaching of tailings. The objectives of the runs were as follows:

- to check that the beaching equation gives reasonable estimates of the beach profile
- to check that the model is able to predict the segregation potential of a tailings product
- to obtain an indication of the degree of reliability of the segregation predictions

Unfortunately the available data does not have rheological measurements with which to estimate the initial slope of the beach. It is not possible, therefore, to consider predictions of the velocity of the flow exiting the plunge pool other than to compute the critical velocity. This problem has been overcome by scaling off the beach profiles the initial slope of the beach. As a check against the predicted critical velocity in the plunge pool the model has been run repeatedly to determine a value of the velocity in the plunge pool that yields a reasonable approximation to the measured beach profile.

4.3.1 TEST RUN ON THE DATA OF KÜPPER (Syncrude Tailings)

In her PhD dissertation Küpper describes a series of 7 field tests in which zones of the Syncrude tailings dam were demarcated and surveyed. Beach profiles were measured before and after depositions. Samples were recovered for particle size determination. Deposition took place through spigot pipes teed off a main delivery. By varying the orientation of the

spigot and varying the location of the spigot on the circumference of the main delivery it was possible to obtain a range of source particle distributions and slurry relative densities due to the fact that the solids settle out in the pipe and move by a combination of suspended flow and saltation. A number of the tests were troubled by thickener problems, unstable flow conditions, and burying and freezing of the spigots. Only two tests are suitable for inclusion in this analysis:

- Test 1 - tailings drawn off the bottom of the main delivery resulting in a slightly coarser higher relative density product. Section 3 has been selected as being representative

- Test 4 - tailings drawn off the top of the main delivery pipe resulting in a finer lower relative density product. Section 1 was selected as being representative.

The parameters for the above tests are as follows:

	test 1		**test 2**
• Flow rate	60 l/s		60 l/s
• spigot pipe dia	150 mm		150 mm
• Representative C_w	68 %		34 %
• beach profile		as per Figs 4.1 and 4.2	
• beach slope at plunge pool	-0.165		-0.054
• grading ex main pipe		unknown[1]	
• grading on beach[2]		as per Figs 4.3 and 4.4	
• distance to pool	260 m		227 m
• final beach elevation at discharge	341 m		334 m

Notes 1 : A broad grading is given and noted as being a composite of several samples recovered from the beach.

2 : The tailings is exceedingly uniform and undergoes little segregation

The measured beach profiles and particle size distributions are indicated in Figures 4.1 to 4.4 below:

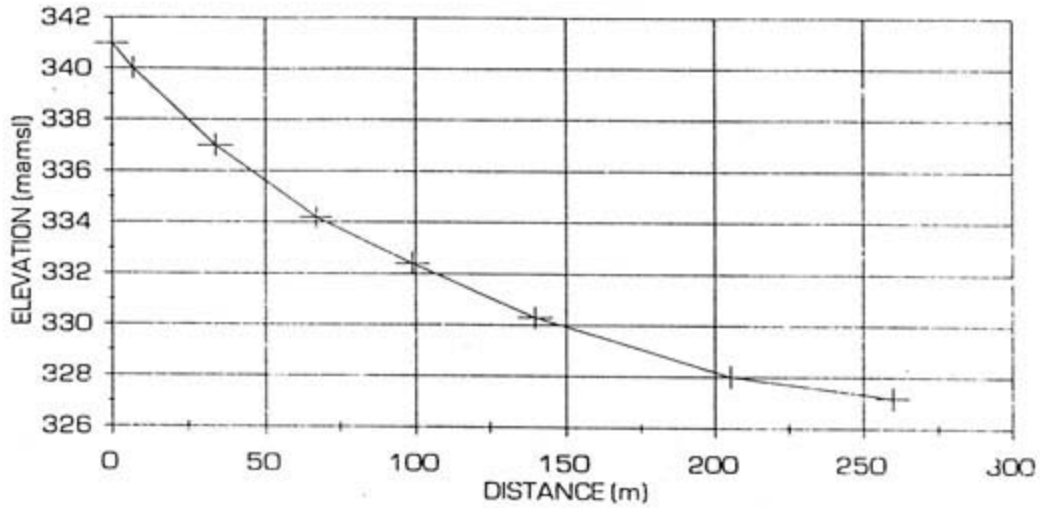


FIGURE 4.1 : KÜPPER:TEST 1 SECTION 3 - MEASURED BEACH PROFILE

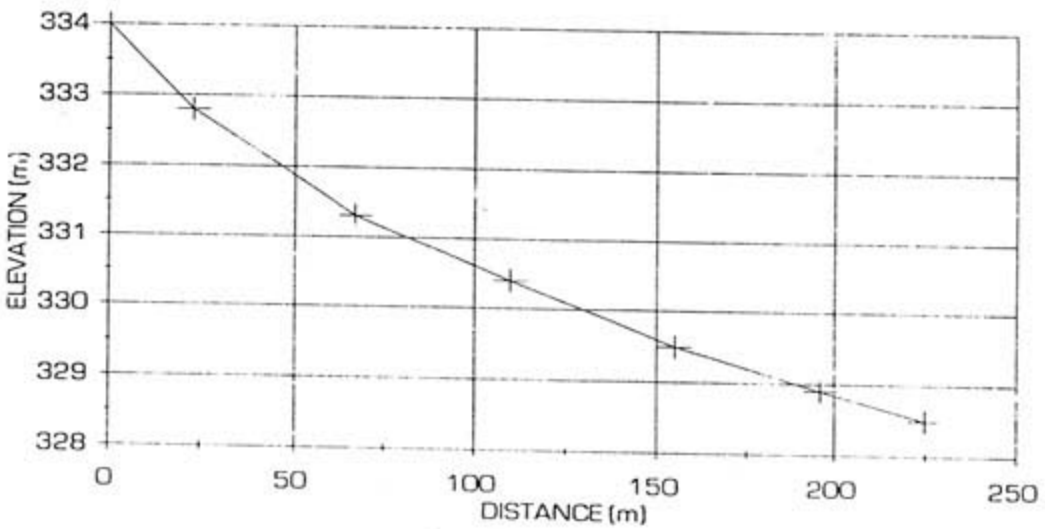


FIGURE 4.2 : KÜPPER:TEST 4 SECTION 1 - MEASURED BEACH PROFILE

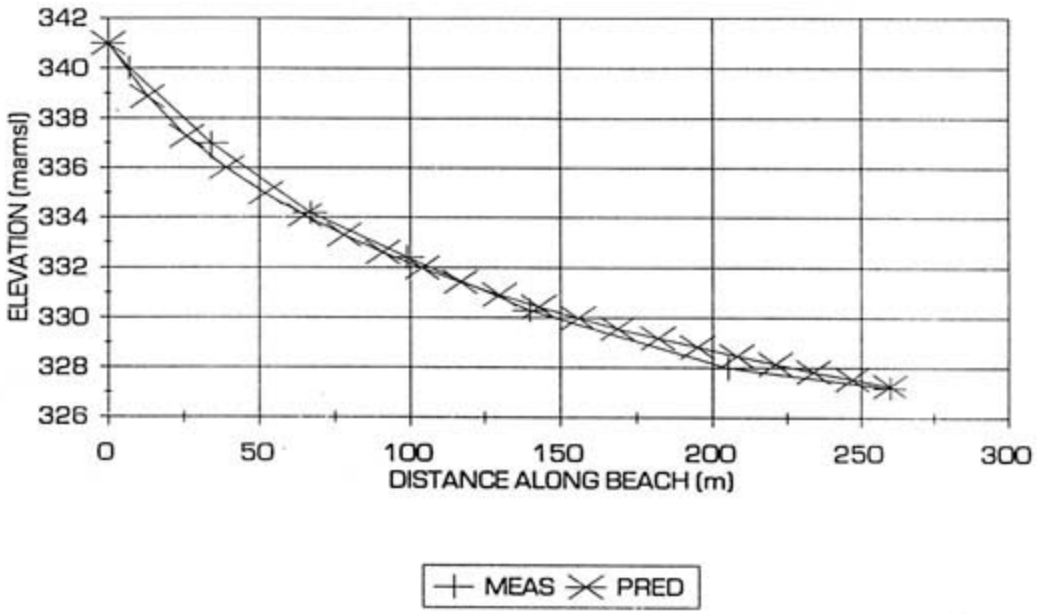


FIGURE 4.3 : KÜPPER:TEST 1 SECTION 3 - PREDICTED BEACH PROFILE

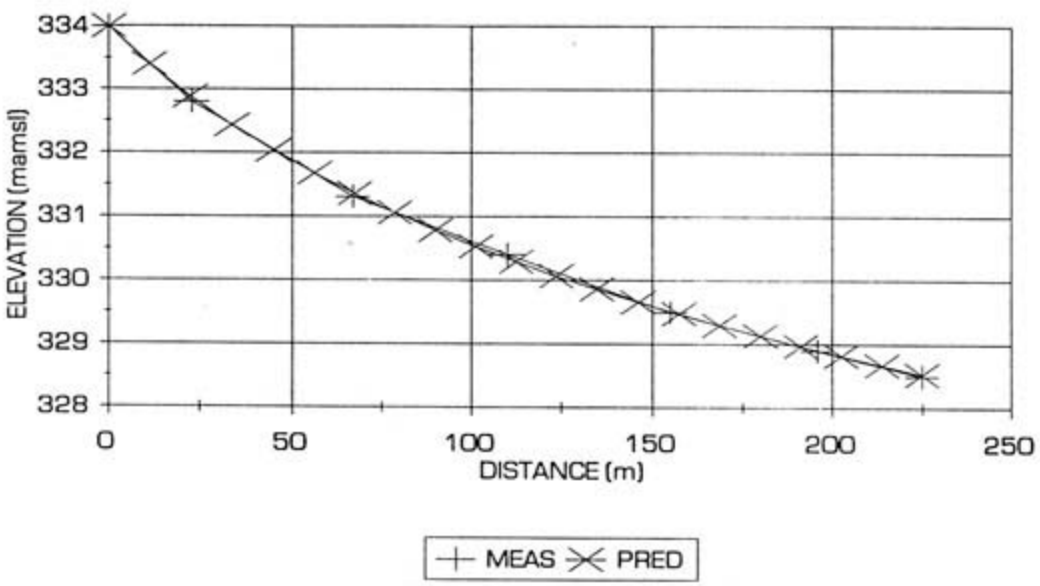


FIGURE 4.4 : KÜPPER:TEST 4 SECTION 1 - PREDICTED BEACH PROFILE

Figures 4.5 and 4.6 show the beach profiles for the above tests as predicted by the model on the following basis: The measured profiles are superimposed on Figures 4.5 and 4.6 for comparison.

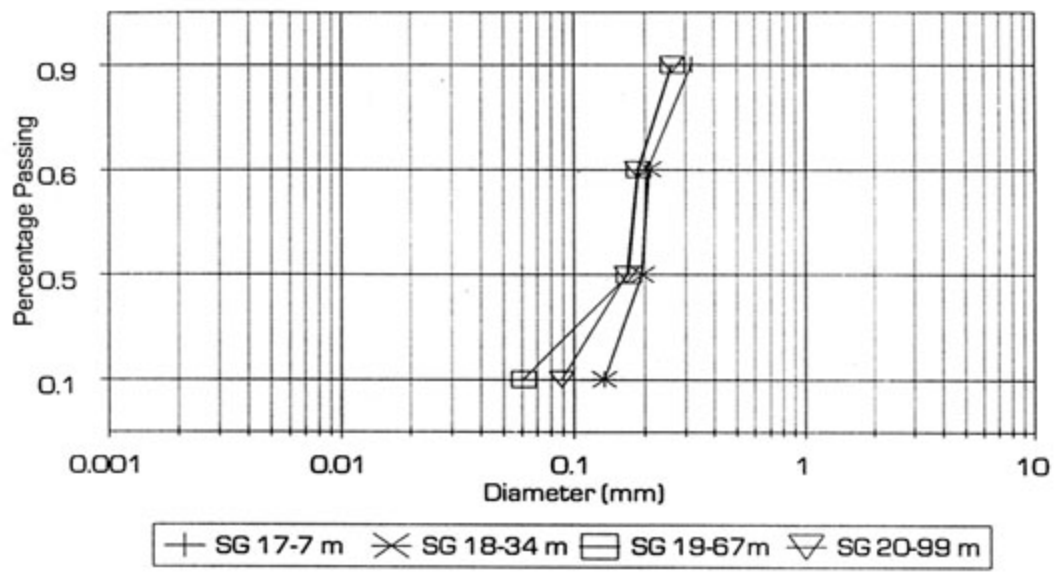


FIGURE 4.5 : KÜPPER:TEST 1 SECTION 3 - MEASURED PARTICLE SIZE DISTRIBUTIONS

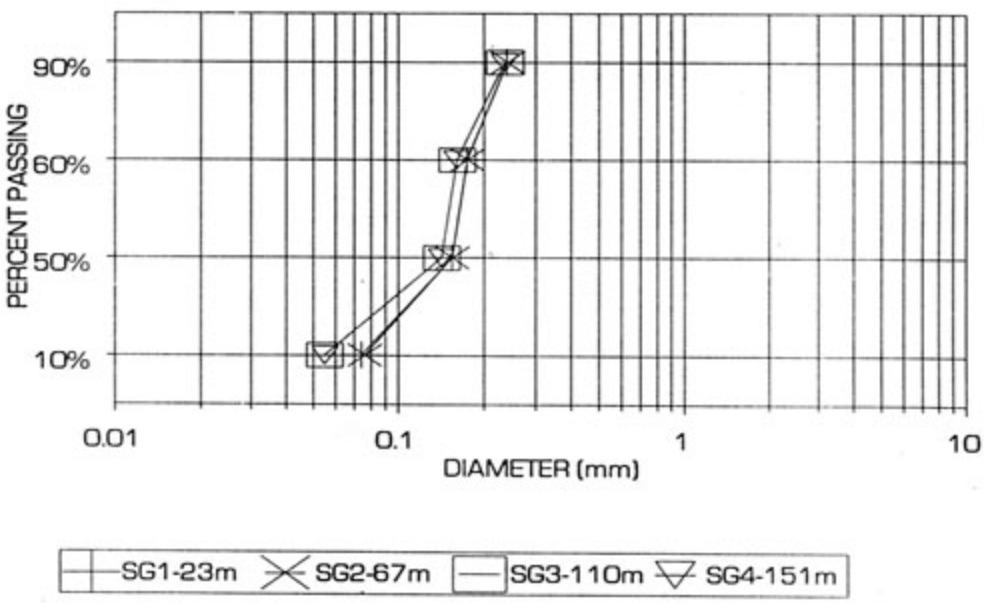


FIGURE 4.6 : KÜPPER:TEST 4 SECTION 1 - MEASURED PARTICLE SIZE DISTRIBUTIONS

The velocities at the discharge and as fitted from the plunge pool are as follows:

	test 1	test 4
• from discharge pipe (calculated for 60 l/s)	3,4 m/s	3,4 m/s
• from plunge pool (fitted	)0,49 m/s	0,37 m/s
• calculated critical velocity	1,17 m/s	0,77 m/s

It is evident from the above that the losses in the plunge pool need to be very high in order for the shape of the beach profile to match the measured profile. The model has correctly predicted relatively little change in the particle size distribution which would be characteristic of the uniform particle size distribution of the tailings. This is not, however sufficient to demonstrate the ability of the model to predict gravitational segregation.

4.3.2 TEST RUN ON THE DATA OF THOMSON (1984) (Gold Tailings)

Thomson (1984) recorded measurements of the beach profile of a gold mine tailings dam east of Johannesburg, South Africa, for the purposes of comparing these with the results of beaching measurements made in a laboratory flume. The following information is available from Thomson (1984):

- The beach profile as plotted in Figure 4.9.
- Three particle size distributions described as "total", "coarse" and "fine" as plotted in Figure 4.10. The "coarse" sample was recovered adjacent to the delivery point and the "fine" sample was taken close to the pool (as far from the delivery point as was possible)
- The particle specific gravity was measured as 2.78.

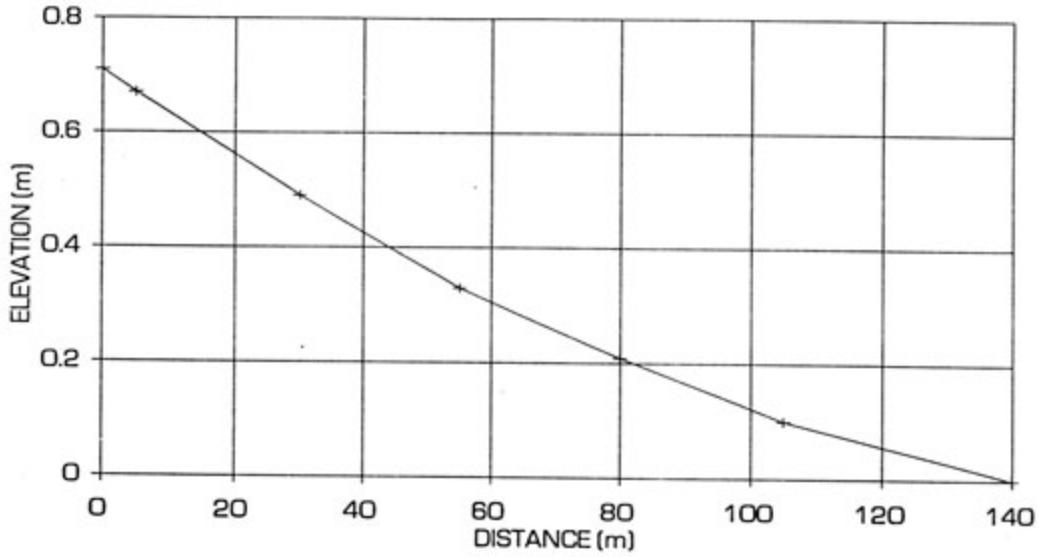


FIGURE 4.9 : THOMSON GOLD DATA - MEASURED BEACH PROFILE

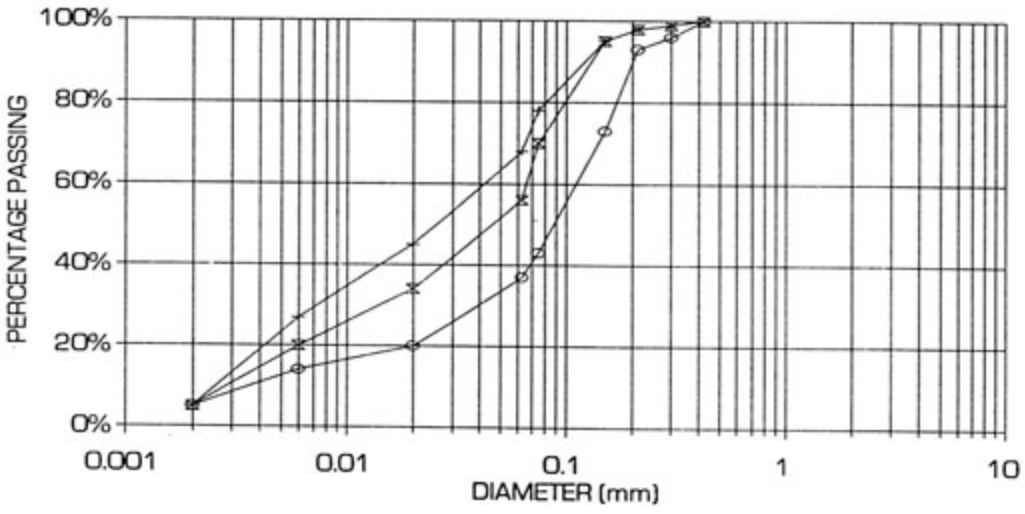


FIGURE 4.10 : THOMSON GOLD DATA - MEASURED PARTICLE SIZE DISTRIBUTIONS

Although not recorded by Thomson, this author was busy with consulting work at the mine during the period immediately preceding Thomson's work and the following approximations are reasonable:

- Tailings was being delivered to the dams at approximately 160 000 tonnes per month at a slurry relative density of, on average, 1.35.

- The delivery pipe diameter was a nominal 200 mm (unlined mild steel pipe)

Figure 4.11 shows a plot of the beach profile as predicted by the beaching model and Figure 4.12 plots of the predicted particle size distributions. These plots are based on:

- an initial beach slope of -0.008 as scaled off the measured beach profile

- a calibrated plunge pool discharge velocity of 0.095 m/s. This is considerably lower than the calculated critical velocity of 0,96 m/s.

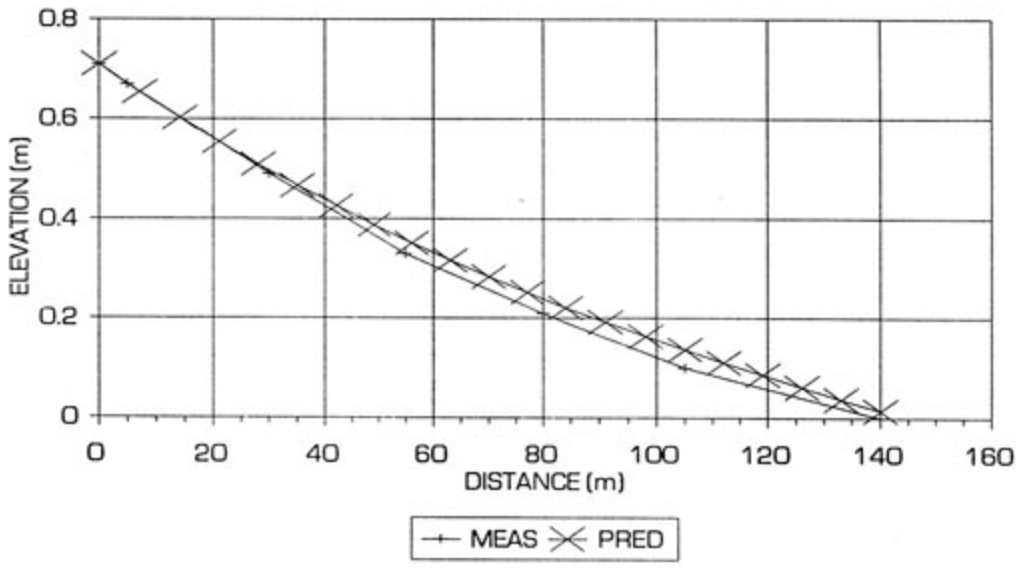


FIGURE 4.11 : THOMSON GOLD DATA - PREDICTED BEACH PROFILE

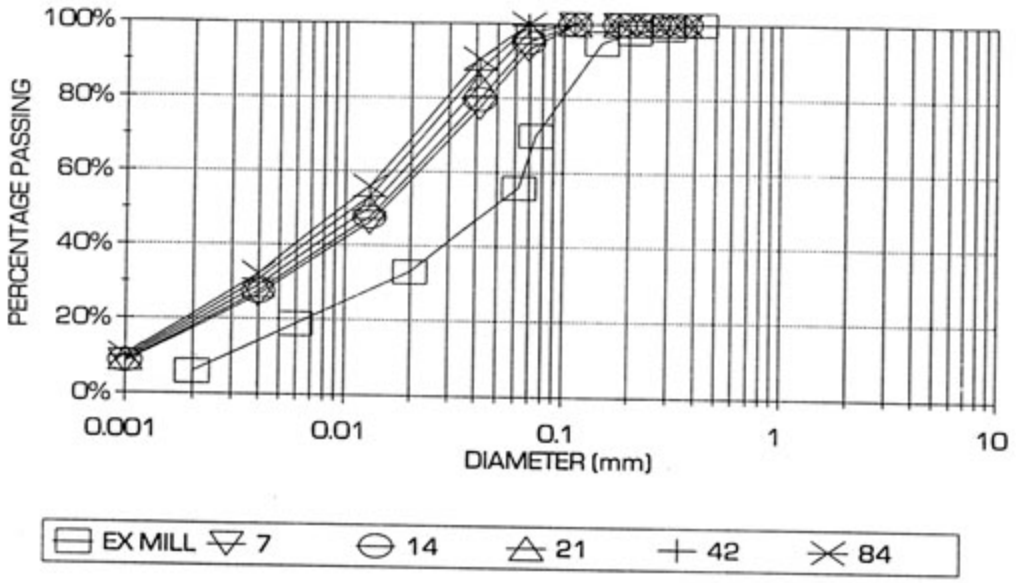


FIGURE 4.12 : THOMSON GOLD DATA - PREDICTED PARTICLE SIZE DISTRIBUTIONS

It is evident from the above plots that as a result of the velocity calibration the beach profile spans the correct elevation. The profile also reasonably closely approximates the measured profile. The particle size distributions are closer to the fine material measured by Thomson. In view of the low plunge pool velocities it is suspected that the fine distribution actually predominates down the beach with the coarse material settling out in the immediate proximity of the plunge pool.

4.3.3 TEST RUN ON THE DATA OF BENTEL (1981) (Platinum Tailings)

Bentel (1981) recorded beaching measurements on platinum tailings for which:

- The particle size distribution at the discharge and at various locations along the beach were measured as indicated in Figure 4.13.

- The beach profile was measured as indicated in Figure 4.14.

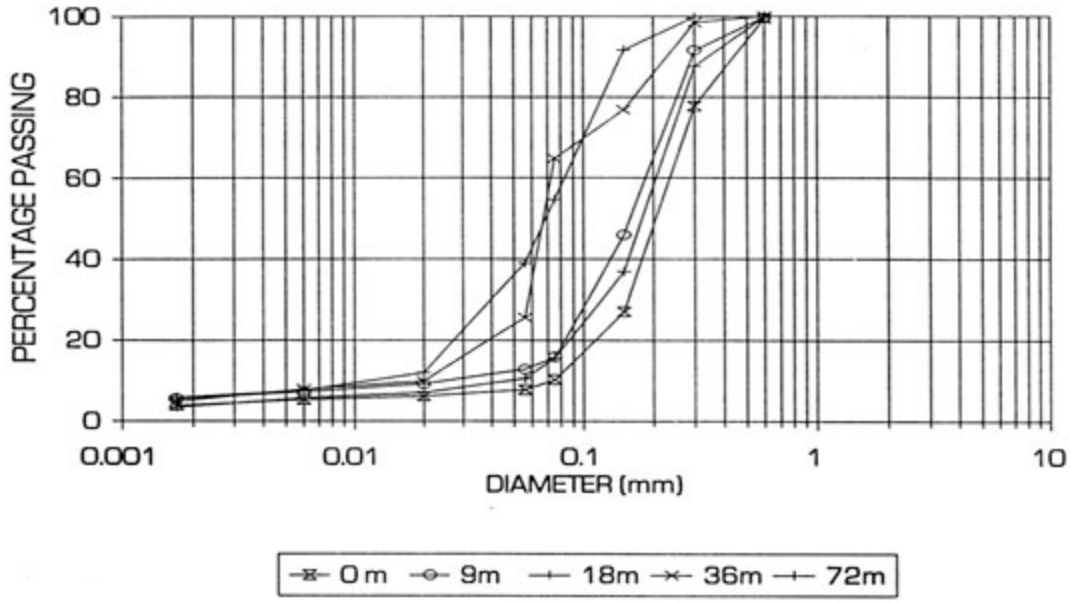


FIGURE 4.13 : BENTEL PLATINUM DATA - MEASURED PARTICLE SIZE DISTRIBUTIONS

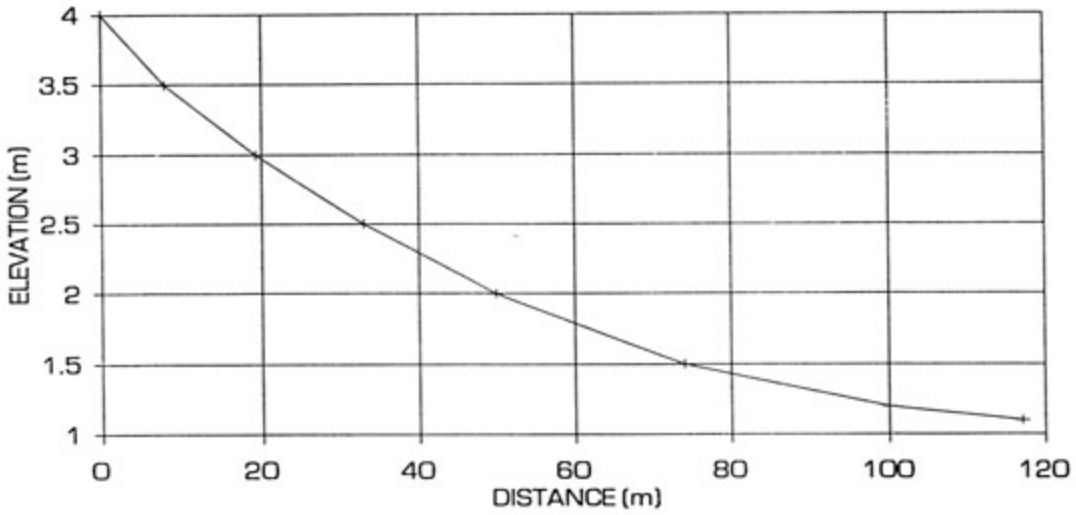


FIGURE 4.14 : BENTEL PLATINUM DATA - MEASURED BEACH PROFILE

This author was consulting on the Platinum Mine at the time of Bentel's research and was able to obtain measurements of deposition tonnage and water:solids ratios from the Mine. Deposition was by means of a spray-bar system with spigot openings at 2 m centres. Approximately 20 spigots would be used at a time for a tonnage of 550 000 tonnes per month and an average water:solids ratio of 45:55 by mass. In the field, while the discharge would take place through multiple spigots, after 10 to 50 m several streams would re-combine. It is considered reasonable to assume that the 20 spigots re-group into approximately five streams.

The above parameters have been included in a model which was calibrated by:

- utilising the slope of the initial portion of the measured beach. This is approximately -0.08

- adjusting the initial velocity along the beach until the measured change in elevation was approximated. This velocity emerged as 0,27 m/s compared with the calculated critical velocity of 1,02 m/s

The resulting beach profile as predicted by the model is indicated in Figure 4.15 and the predicted particle size distributions along the beach in Figure 4.16.

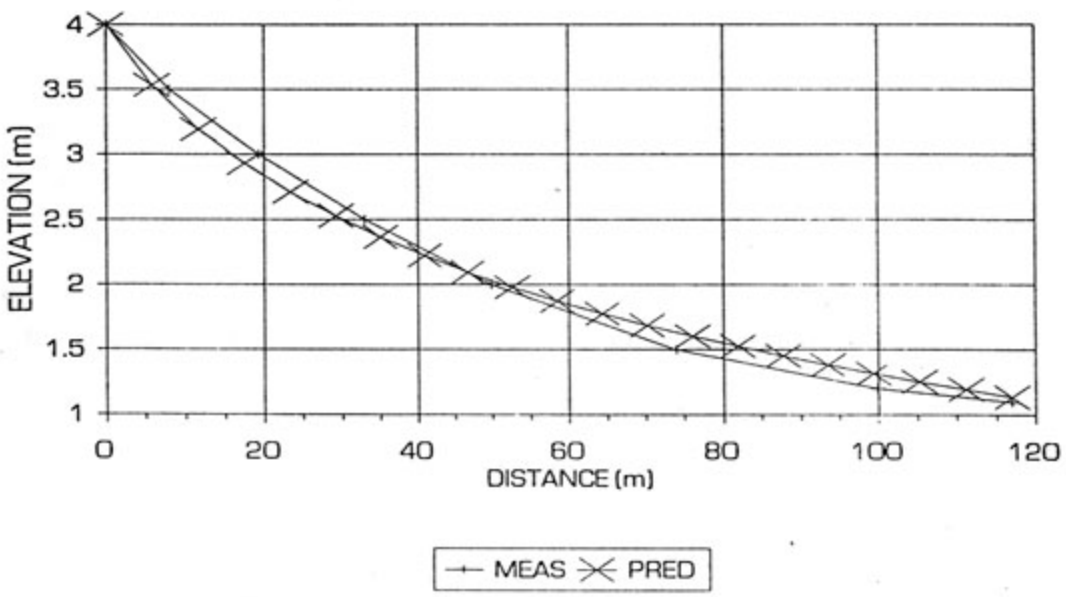


FIGURE 4.15 : BENTEL PLATINUM DATA - PREDICTED BEACH PROFILE

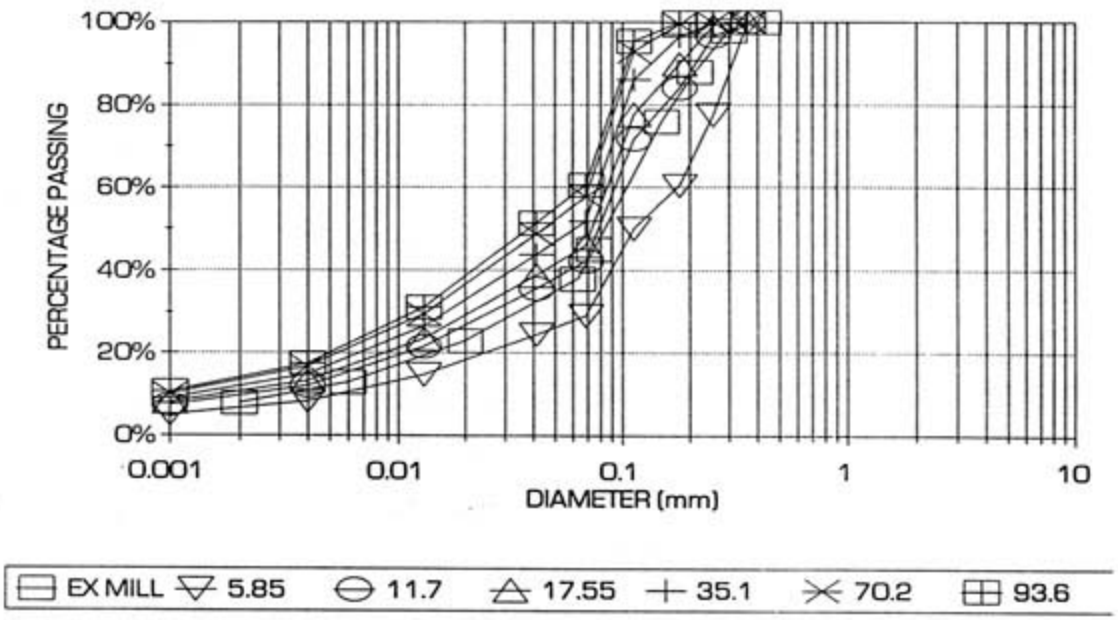


FIGURE 4.16 : BENTEL PLATINUM DATA - PREDICTED PARTICLE SIZE DISTRIBUTION

The following points are evident from comparison of the measured and predicted results:

- calibration of the beach profile has yielded a predicted profile which is very similar to that measured

- the model has predicted that significant gravitational segregation will take place and the progression from coarse to fine material with distance down the beach is in line with both field observations and the data recorded by Bentel.

- as in the case of the Thomson gold predictions the model has predicted considerably finer particle size distributions than those actually measured.

4.4 DISCUSSION

The reasonable success with which the beaching model has predicted the trends in particle size distribution with distance along the beach is encouraging. The test runs have, however, been

based on a number of assumptions relating to the discharge flow rate, tonnage per hour, slurry density, discharge conditions. Frictional resistance characteristics have been effectively incorporated by measuring the slope of the beach immediately after the plunge pool. The runs are calibrated to yield an end-of-beach elevation which is similar to that measured. The predicted beach profiles are remarkably similar to those measured. Indeed the predictions are considerably closer than those made using the "master beach" equations developed by Bentel (1981) and Blight (1987) which would indicate that the form of the beaching equation is approximately correct.

Predictions of particle size distribution have met with varied but generally less success. For the more singular sized products of Küpper the predicted gradings demonstrated a low tendency to segregation down the beach. For the gold and platinum products there was a tendency to predict a finer particle size distribution in the flow stream. In all cases the predicted distributions were finer than the discharged tailings (ie prior to this contacting the beach).

In all of the above test cases the calibrated velocities were considerably lower than the critical velocity at the outlet to the plunge pool. In both Thomson's measurements and Bentel's measurements this author can confirm from observations made over the period during which their measurements were taken that:

- the flow from the gold tailings plunge pool was indeed subcritical

- the flow from the spigoted platinum tailings was subcritical

Observations on many other operations indicates that velocities from the plunge pool vary from sub to supercritical depending on the discharge conditions, flow rate and slurry flow characteristics.

Chapter 5 documents a sensitivity analysis in the course of which the tailings, slurry and discharge characteristics are varied in order to check that the model reacts in a manner in accordance with observations.

CHAPTER 5

CONTENTS

Sec	Description	Page
5	SENSITIVITY ANALYSIS	5-1
5.1	INTRODUCTION	5-1
5.2	PART 1 : VARIATION OF PRIMARY AND SECONDARY VARIABLES IN ISOLATION	5-2
5.2.1	METHODOLOGY	5-2
5.2.2	INFLUENCE OF PARTICLE SIZE DISTRIBUTION	5-4
5.2.3	INFLUENCE OF SOLIDS SPECIFIC GRAVITY	5-8
5.2.4	INFLUENCE OF FLOW RATE	5-11
5.2.5	INFLUENCE OF SLURRY CONCENTRATION	5-15
5.2.6	INFLUENCE OF BEACH LENGTH	5-18
5.2.7	INFLUENCE OF VELOCITY EX-PLUNGE POOL	5-23
5.2.8	INFLUENCE OF SEDIMENT FRICTIONAL RESISTANCE	5-26
5.3	PART 2 : CORRELATED VARIATION OF PRIMARY AND SECONDARY VARIABLES	5-29
5.4	DISCUSSION	5-34

CHAPTER 5 SENSITIVITY ANALYSIS

5.1 INTRODUCTION

Chapter 4 has indicated that the entropy-based beaching model has yields results that are in line with measurements of beaches as recorded in the literature. This chapter documents a series of test runs carried out to check that the model yields logical results as variables are varied individually.

The primary variables in the model are:

- Particle size distribution of the tailings ex-mill
- Particle specific gravity
- Slurry flow rate on discharge
- Slurry water to solids ratio
- Beach length

Secondary variables, although no less influential than the primary variables, are determined by the primary variables. These are:

- The velocity of the slurry as this leaves the plunge pool
- The frictional resistance of the slurry at the velocity it leaves the plunge pool

It is important to note at the outset that the primary function in the model is the stream power which is the multiplicand of several primary variables. These variables are themselves correlated in that a change in one will result in a change in another. For example, for a constant tonnage rate, coarsening of the tailings product will require a higher velocity in the delivery pipe to avoid settling out of the coarse particles en route to the tailings dam. Pipelines are manufactured in size bands such as 150 mm, 200 mm etc. For a constant diameter pipeline an increase in velocity will require an increase in the flow rate which is

usually accomplished by bulking the flow with water (not a good bulking agent in view of its negative effects on the rheology of the slurry) since this is the most readily available and convenient commodity. This means that the slurry density will reduce. Since the stream power is the product of density, flow rate and velocity head, and all three variables will have changed, the resulting change in stream power will be considerably amplified.

This sensitivity analysis has therefore been split into two parts. The first part evaluates the effects of the variation of each of the primary and secondary variables in isolation ie without taking into cognisance the concomitant variations that will occur in correlated variables. The purpose of this exercise is to observe that the model demonstrates the anticipated sensitivity and reacts in the manner expected. The second part attempts to simulate a number of situations where changes in grind could occur, such as during periods in a mining situation when plant through-put is temporarily increased with a reduction in milling, or particle specific gravity could occur, such as when second reef is mined, or slurry density could occur, such as when a thickener comes out of operation. The results of these simulations will be compared qualitatively with observations and discussed.

5.2 PART 1 : VARIATION OF PRIMARY AND SECONDARY VARIABLES IN ISOLATION

5.2.1 METHODOLOGY

Table 5.1 below sets out the primary and secondary variables and the values assumed in the course of 21 simulations in which the variables were varied in isolation. Each horizontal line describes the values of the variables assumed. For example, in evaluating particle size distribution, three runs were carried out for three products, coarse, medium and fine, and for each of the three runs the remaining six variables are as listed horizontally across the table. These variables were assigned the middle value of each of their individual ranges, for example, in the simulations to evaluate the effects of variables other than particle size distribution, the medium particle size distribution was assumed consistently.

The middle values were selected somewhat arbitrarily as being representative as a whole of mining operations in South Africa, and the upper and lower bound values in the sensitivity assessment were selected as being typical of the ranges experienced in the South African mining industry.

TABLE 5.1 : MATRIX OF VARIABLES ASSUMED IN SENSITIVITY ANALYSIS

	PARTICLE SIZE DISTRIBUTION	PARTICLE S G	FLOW RATE (l/s)	WATER TO SOLIDS	BEACH LENGTH (m)	VELOCITY EX-PLUNGE POOL (m/s)	FRICTIONAL RESISTANCE (initial slope)
PARTICLE SIZE DISTRIBUTION	coarse (1) medium (2) fine (3)	3	77,8	50:50	250	0,25	-0,1
PARTICLE S G	medium	5 3 2	77,8	50:50	250	0,25	-0,1
FLOW RATE (l/s)	medium	3	155,6 (4) 77,8 (5) 25,9 (6)	50:50	250	0,25	-0,1
WATER TO SOLIDS	medium	3	varies	70:30 50:50 25:65	250	0,25	-0,1
BEACH LENGTH (m)	medium	3	77,8	50:50	1000 250 100	0,25	-0,1
VELOCITY EX-PLUNGE POOL (m/s)	medium	3	77,8	50:50	250	0,5 (7) 0,25 (7) 0,1 (7)	-0,1
FRICTIONAL RESISTANCE (initial slope)	medium	3	77,8	50:50	250	0,25	-0,5 -0,1 -0,01

NOTES : 1 30 % minus 0,075 mm, maximum size 5 mm

2 50 % minus 0,075 mm, maximum size 2 mm

3 80 % minus 0,075 mm, maximum size 0,425 mm

4 Calculated from tonnage rate of 300 000 tonnes per month at 50:50 water:solids ratio

5 Calculated from tonnage rate of 150 000 tonnes per month at 50:50 water:solids ratio

6 Calculated from tonnage rate of 50 000 tonnes per month at 50:50 water;solids ratio

7 Implies varying discharge conditions/spigot size/spigot orientation

The results of the simulations are documented in the sections below.

5.2.2 INFLUENCE OF PARTICLE SIZE DISTRIBUTION

Figure 5.1 indicates the particle size distributions selected for inclusion in the sensitivity analysis . These have been selected on the following basis:

- The fine product has 80 % minus 0,075 mm and would be representative of a mildly segregating grind similar to that from South African gold tailings which is known to be mildly segregating with a top size of 0,425 mm - a medium sand size

- The medium product has 50 % minus 0,075 mm and would be representative of a segregating grind similar to that from South African platinum, copper and lead-zinc mines with a top size of 2 mm, a coarse sand/fine gravel size

- The coarse product was selected as being representative of a heavily segregating grind with a top size of 5 mm, a fine gravel.

In line with most hard rock milling products the 0,002 mm size which is the rock flour component of the tailings was kept constant at 3 %. Representative "S" shaped curves were then "fitted" to achieve the distribution curves indicated in Figure 5.1

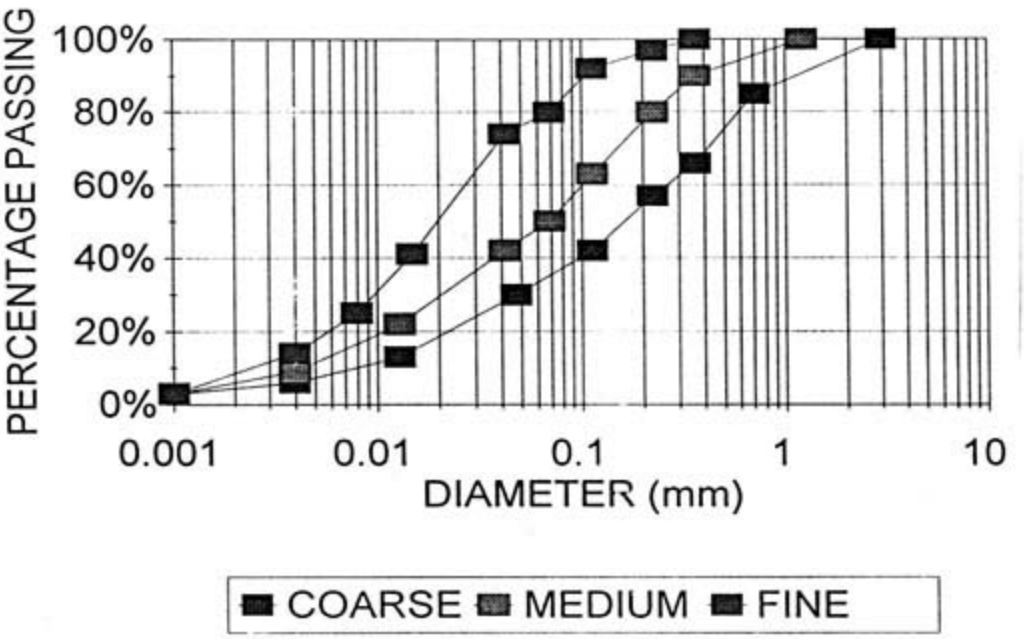


FIGURE 5.1 : COARSE , MEDIUM AND FINE PRODUCTS USED IN SENSITIVITY ANALYSIS

The results of the simulations are indicated in the following figures:

- Figure 5.2 - predicted beach profiles
- Figure 5.3 - predicted particle size distributions for the coarse product at various distances along the beach
- Figure 5.4 - predicted particle size distribution for the medium product at various distances along the beach
- Figure 5.4 - predicted particle size distribution for the fine product at various distances along the beach

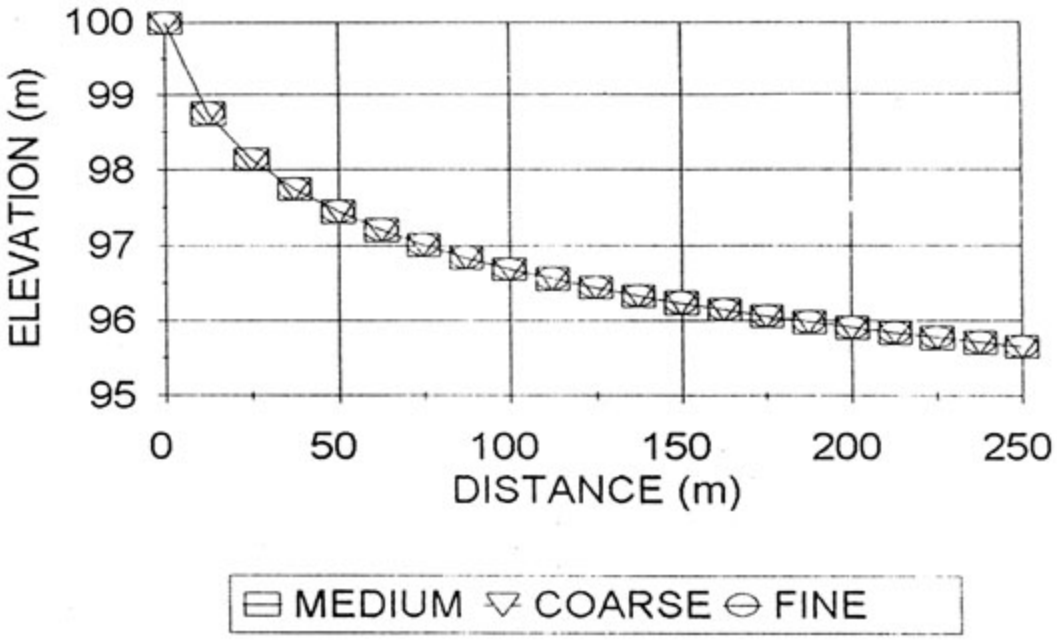


FIGURE 5.2 : PREDICTED BEACH PROFILES - COARSE, MEDIUM AND FINE PRODUCTS

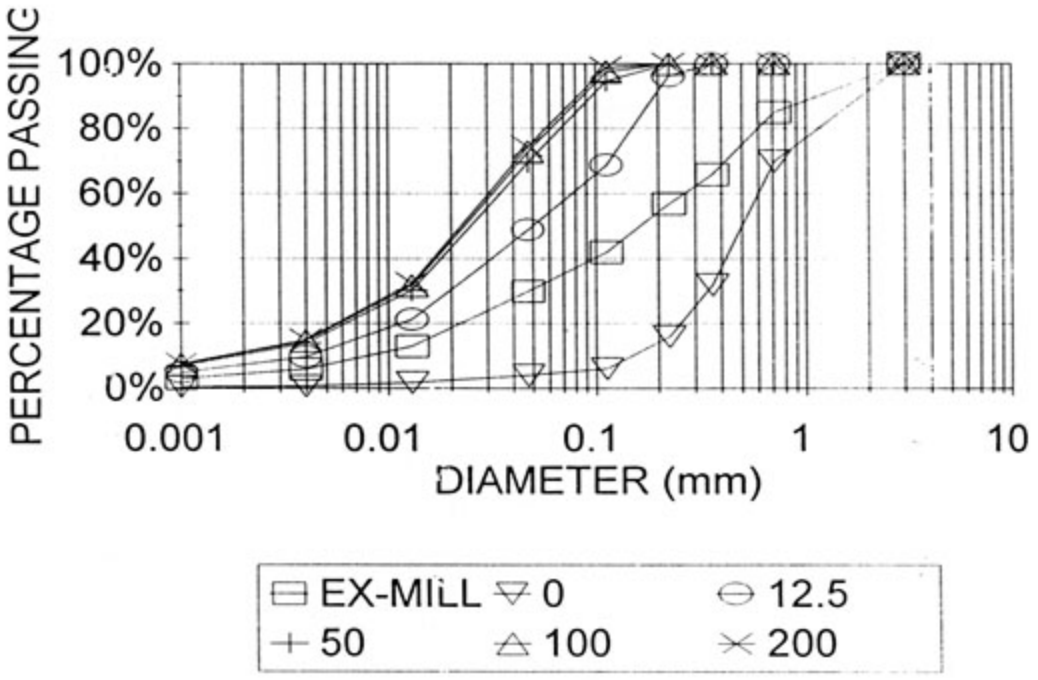


FIGURE 5.3 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - COARSE PRODUCT

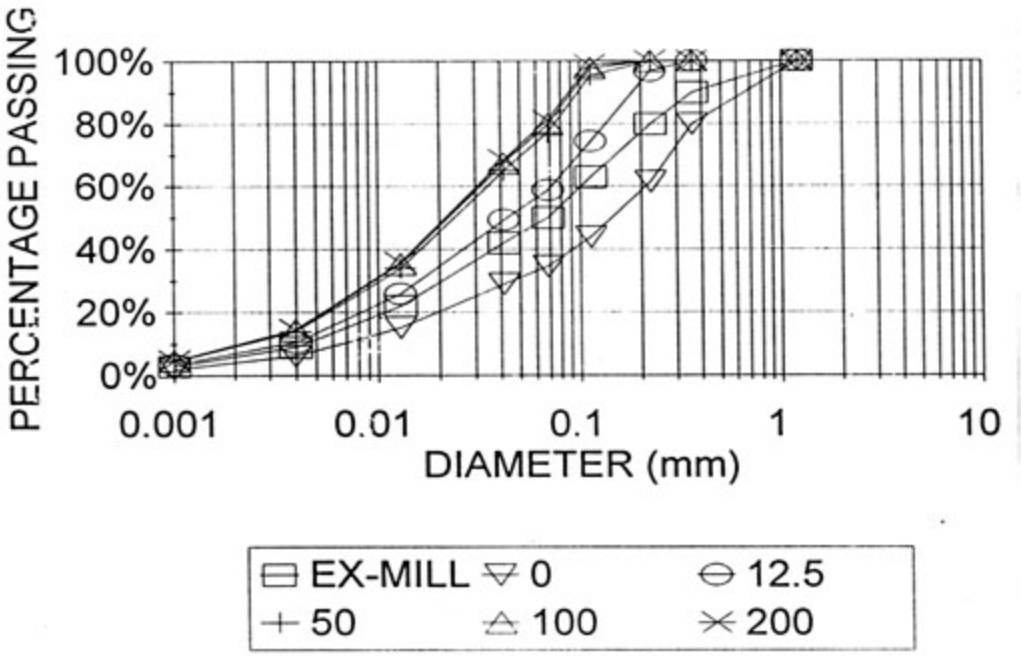


FIGURE 5.4 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - MEDIUM PRODUCT

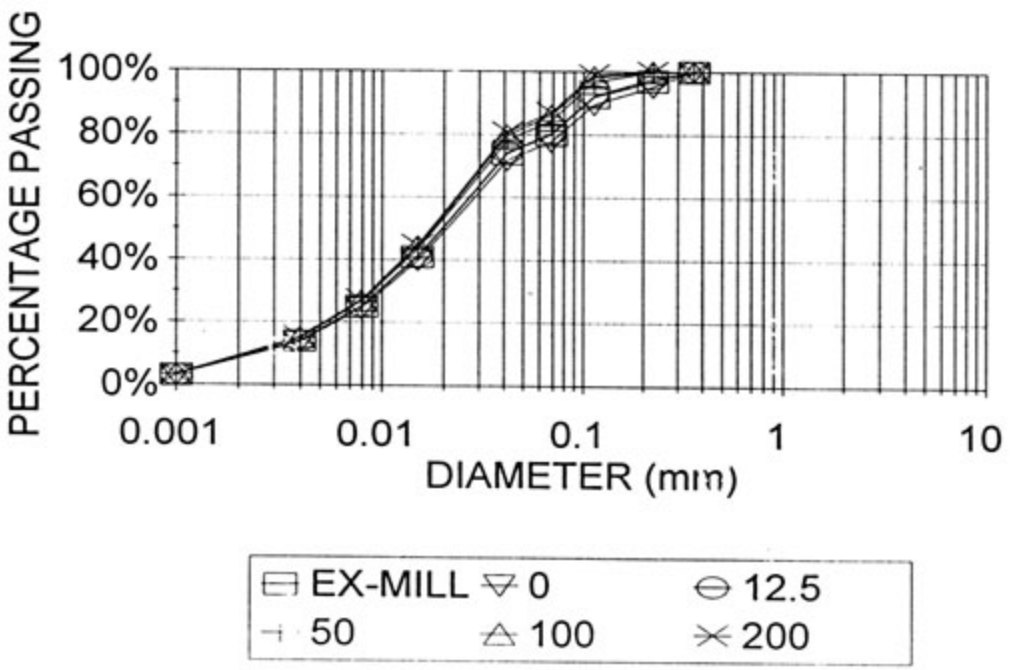


FIGURE 5.5 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - FINE PRODUCT

The equations for the prediction of the beach profile do not incorporate details of the grind and consequently a constant beach profile for the three simulations. is reflected in Figure 5.2. At first this may not appear reasonable, but it should be borne in mind that the simulations have varied the grind in isolation. In practice the various grinds would require differing flow velocities and it is contended that it is these effects which determine the variations in beach profile as opposed to the grind. This conclusion is in line with that of Küpper namely that grain sorting is not the main factor leading to beach concavity.

Figures 5.3 to 5.5 indicate that the products range form heavily segregating for the coarse product to mildly segregating for the fine product. It is of interest to note in the two segregating products that the predicted particle size distribution changes only slightly from 100 m to 200 m out of a 250 m long beach. This is in line with expectation since segregation occurs until a residual particle size distribution in the flow stream is developed which is similar to the finer product which is known to be mildly segregating.

5.2.3 INFLUENCE OF SOLIDS SPECIFIC GRAVITY

A range of particle specific gravities from 2, representative of some ash products from coal fired power stations through 3 which would be representative of platinum mine tailings to 5 which would be representative of the upper limit of magnetite and lead tailings.

The results of the simulations are indicated in the following figures:

- Figure 5.6 - predicted beach profiles
- Figure 5.7 - predicted particle size distributions at the discharge
- Figure 5.8 - predicted particle size distribution at 125 m (midway along the beach)
- Figure 5.9 - predicted panicle size distribution at 225 m (90 % of the beach length)

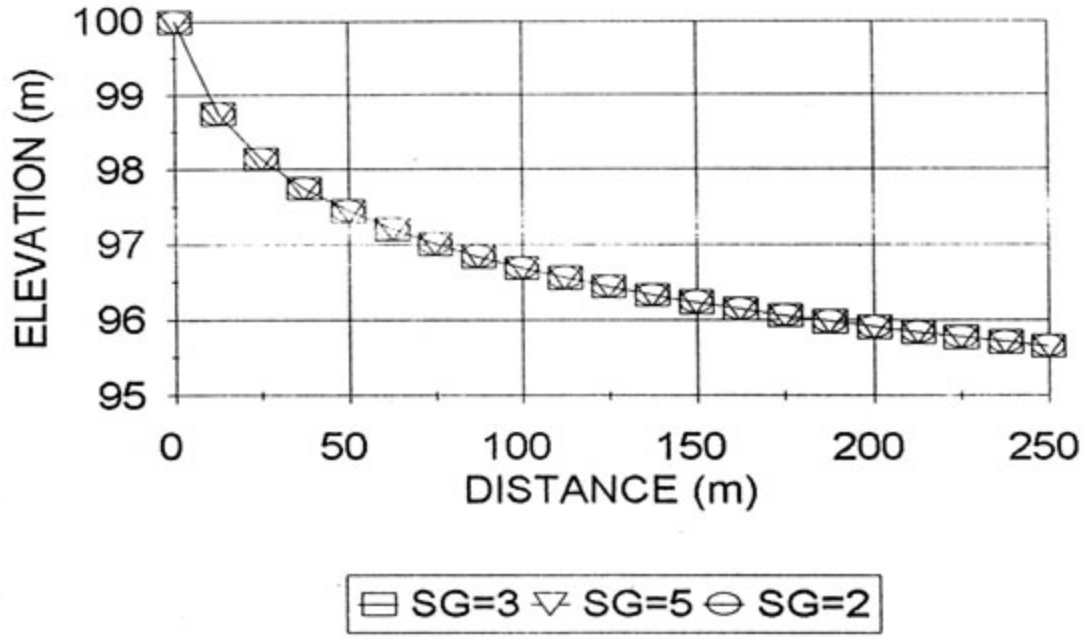


FIGURE 5.6 : PREDICTED BEACH PROFILES FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY

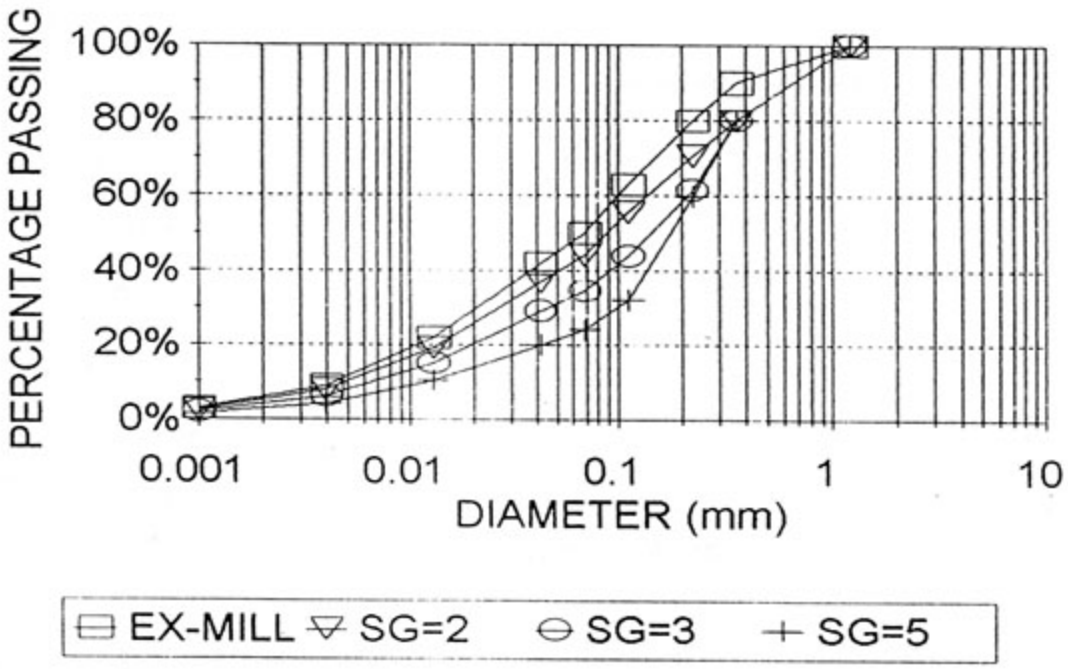


FIGURE 5.7 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY - AT DISCHARGE

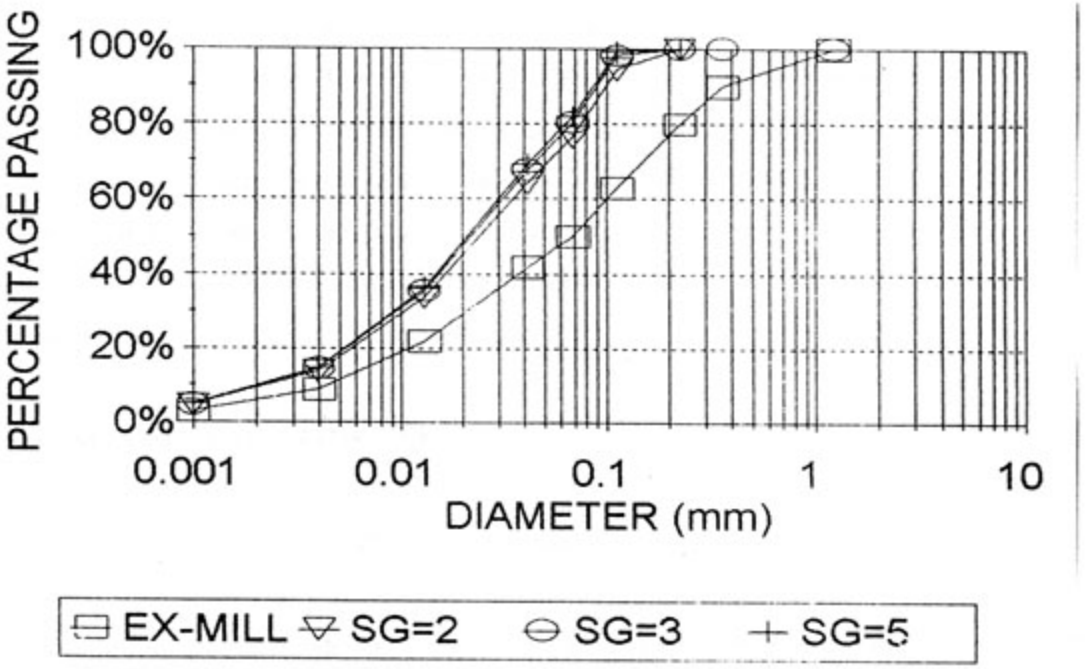


FIGURE 5.8 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY - AT 125 m

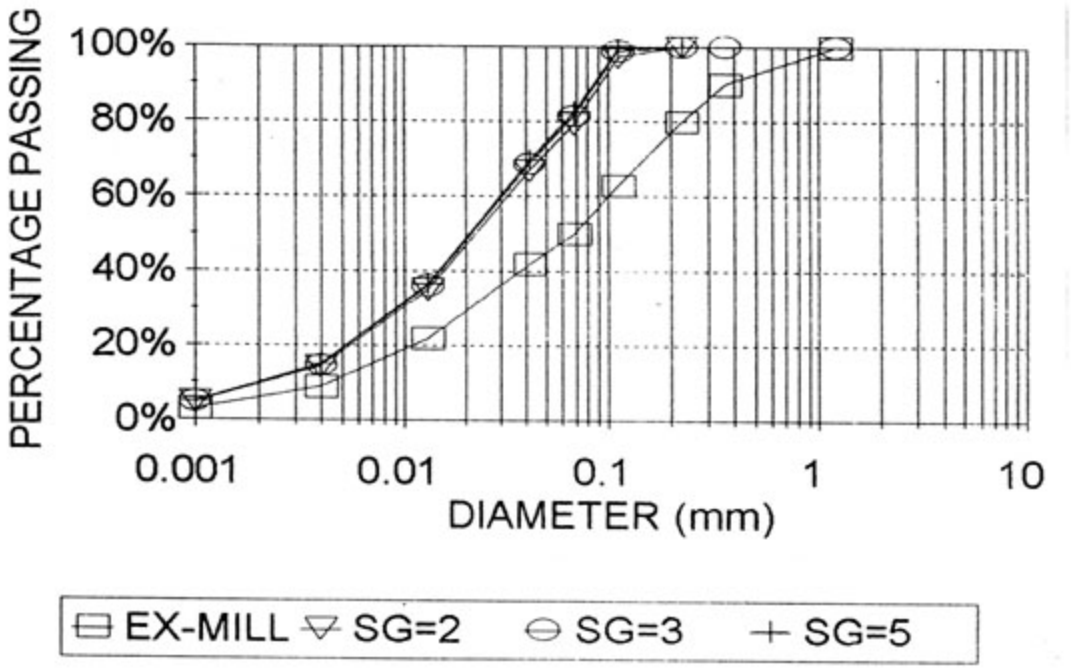


FIGURE 5.9 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN PARTICLE SPECIFIC GRAVITY - AT 225 m

In a similar argument to that presented for the simulations on variations in particle size distribution, variations in particle specific gravity do not influence the beach profile if taken in isolation. This is reflected in Figure 5.6. Figure 5.7 indicates that, in the region where the slurry is of a grind which is segregating, the products undergo increased particle segregation with increasing particle specific gravity which is in line with expectations. Figures 5.8 and 5.9 indicate that as the slurry approaches a particle size distribution which is mildly segregating with distance along the beach, the settled material on the beach adopts a distribution which is less dependent on the particle specific gravity.

5.2.4 INFLUENCE OF FLOW RATE

A range of flow rates was selected for the sensitivity analysis on the basis of typical tonnages experienced on gold and platinum mining operations in South Africa and ranged from an upper bound of 300 000 tonnes per month to a lower bound of 50 000 tonnes per month with a middle value of 150 000 tonnes per month. At a 50:50 water to solids ratio these tonnages translate into flow rates of 155,6 l/s, 77,8 l/s and 25,9 l/s respectively.

The results of the simulations are indicated in the following figures:

- Figure 5.10 - predicted beach profiles
- Figure 5.11 - predicted particle size distributions at the discharge
- Figure 5.12 - predicted particle size distributions at 12,5 m
- Figure 5.13 - predicted particle size distribution at 25 m
- Figure 5.14 - predicted particle size distribution at 125 m (midway along the beach)
- Figure 5.15 - predicted panicle size distribution at 225 m (90 % of the beach length)

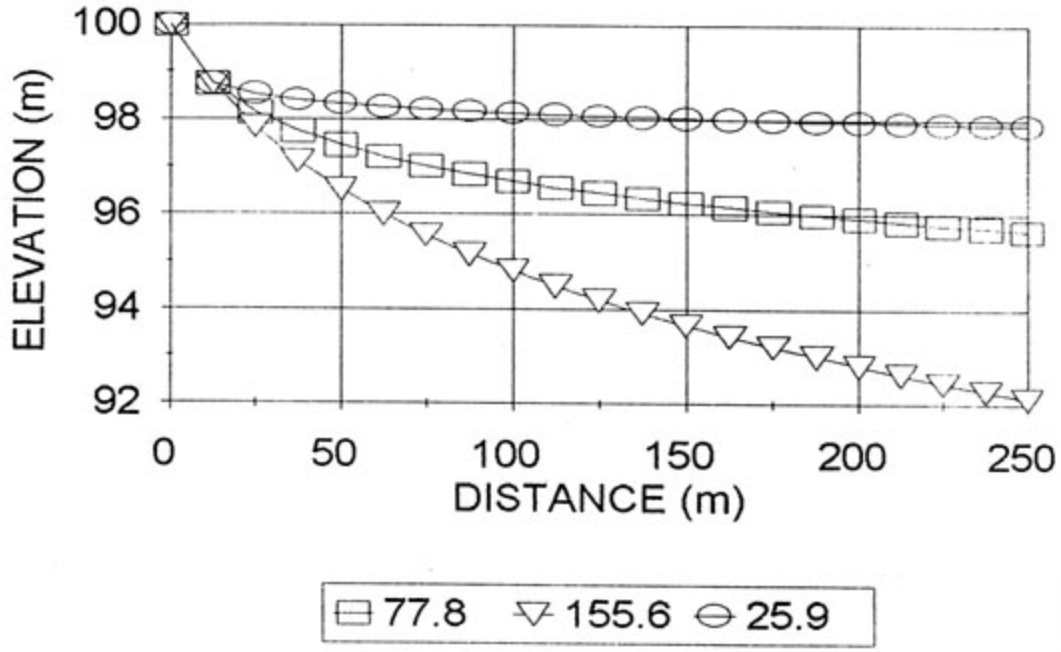


FIGURE 5.10 : PREDICTED BEACH PROFILES FOR VARIATIONS IN FLOW RATE

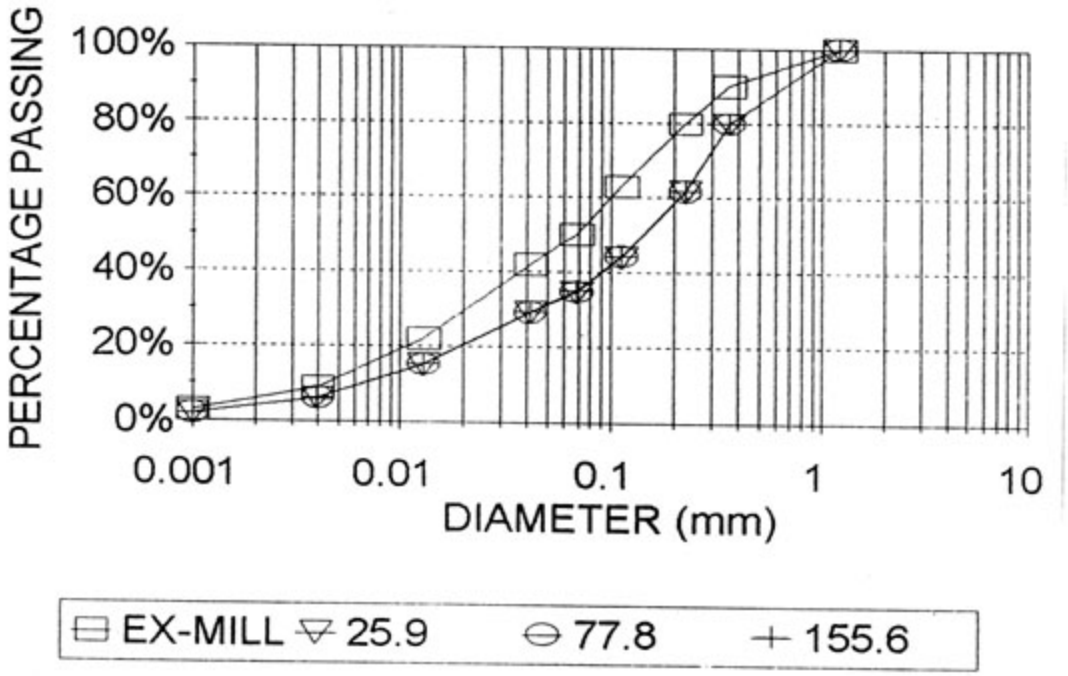


FIGURE 5.11 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATIONS IN FLOW RATE - AT DISCHARGE

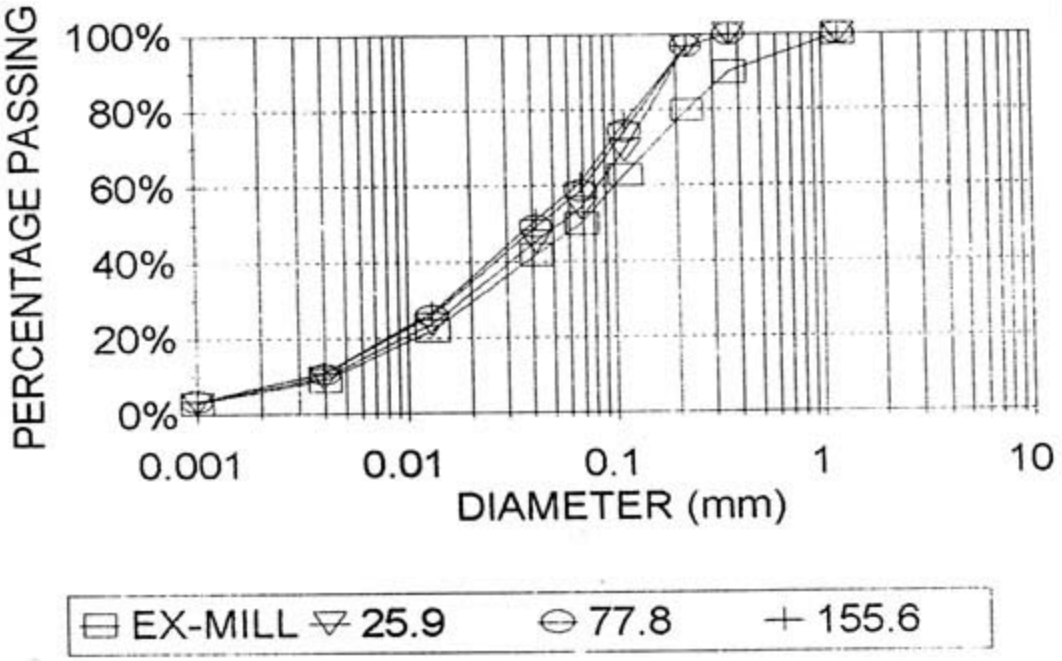


FIGURE 5.12 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 12,5 m

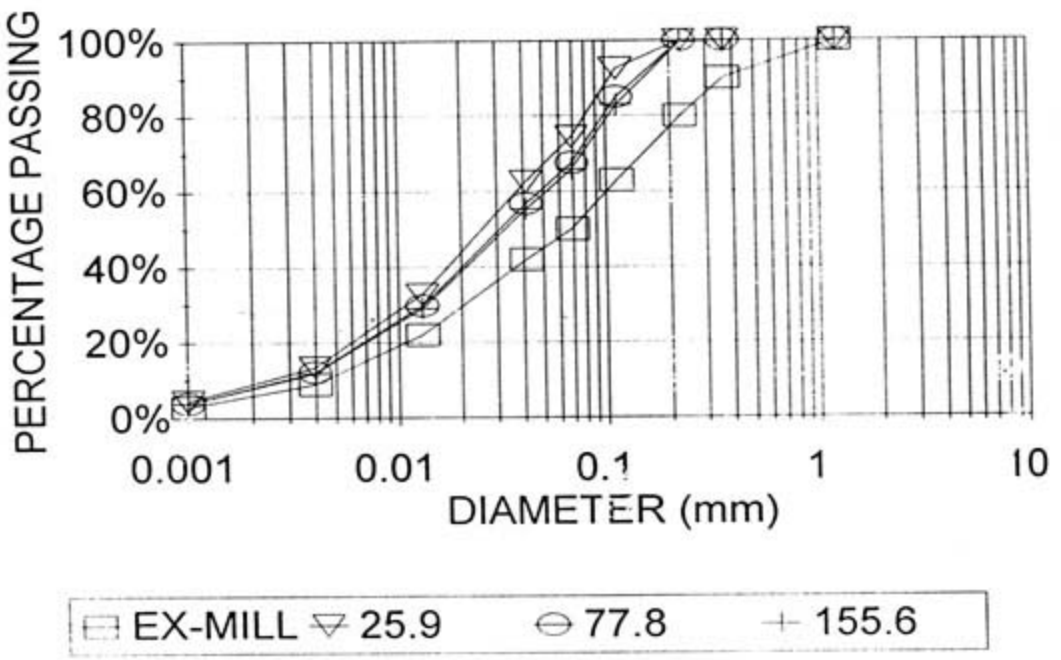


FIGURE 5.13 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 25 m

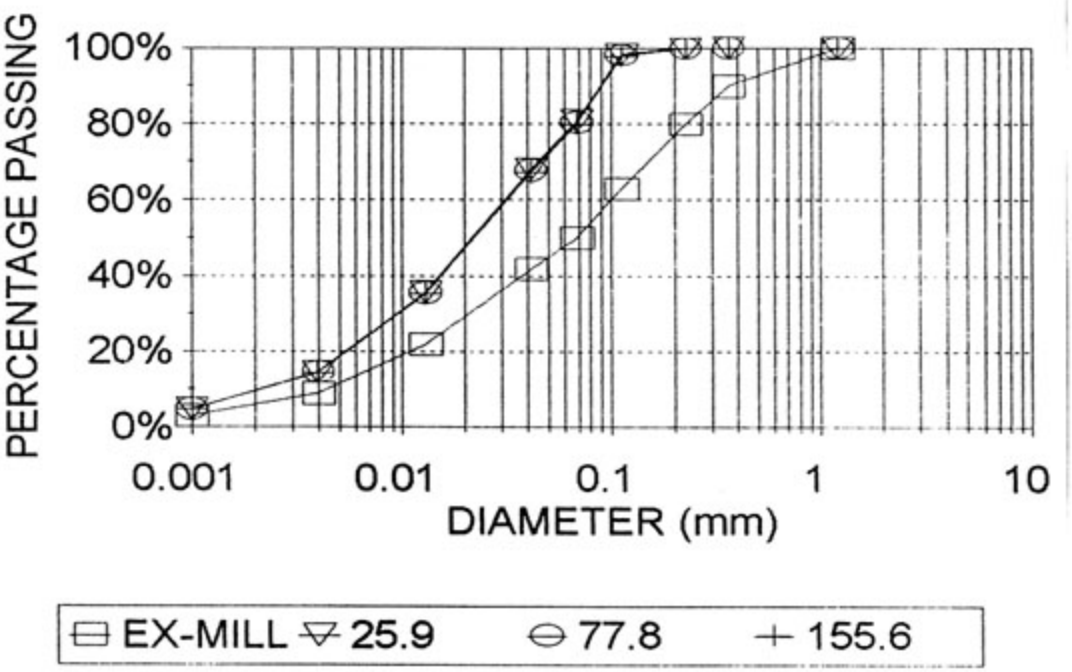


FIGURE 5.14 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 125 m

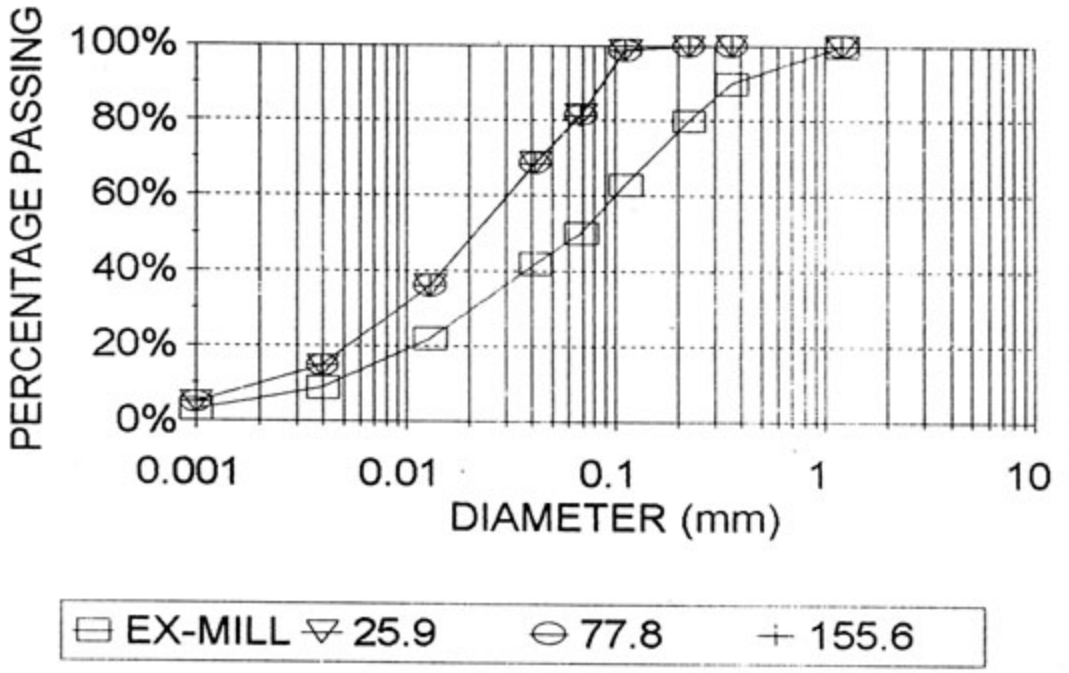


FIGURE 5.15 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN FLOW RATE - AT 225 m

The beach profiles in Figure 5.10 indicate a significant variation in beach profile for the variations in flow rate simulated and are in line with expectations. At lower flow rates, and therefore lower stream power, there is less energy available to maintain the particles in suspension and consequently an increased rate of deposition, hence a flatter overall beach. Conversely, at higher flow rates, with higher stream power, there is a lower tendency for settling out and the beach profile is steeper.

The results indicated in Figure 5.11 show there is negligible difference in the particle size distribution with variation in flow rate. This indication continues into Figures 5.12 and 5.13 where differences are slight. It would logically be anticipated that the lower stream power characteristic of the low flow rate should result in more rapid settling out of the coarser particles but the results from the simulations indicate only limited sensitivity to variations in flow rate. It is suspected that the full effect of the change in flow rate is being dissipated by the fact that the flow rate has been varied in isolation. In reality the velocity of the slurry exiting the plunge pool would vary in sympathy with the flow rate as would the frictional resistance characteristics of the slurry. If these sympathetic vacations were incorporated, as they will be in the second part of this sensitivity analysis, it is suspected that the variations in particle size distribution and beach profile would be amplified.

5.2.5 INFLUENCE OF SLURRY CONCENTRATION

The influence of variations in water to solids ratio (taken in isolation) was evaluated by means of simulations where the slurry concentratataon was taken from a lower bound of C_w = 30 % (water:solids ratio of 70:30).through C_w = 50 % (w:s = 50:50) to an upper bound of C_w = 65 % (w:s = 35:65). This range would be typical of tailings slurries in South Africa.

The results are indicated on the following figures:

- Figure 5.16 - predicted beach profiles
- Figure 5.17 - predicted particle size distributions at the discharge
- Figure 5.18 - predicted particle size distribution at 125 m (midway along the beach)

- Figure 5.19 - predicted panicle size distribution at 225 m (90 % of the beach length)

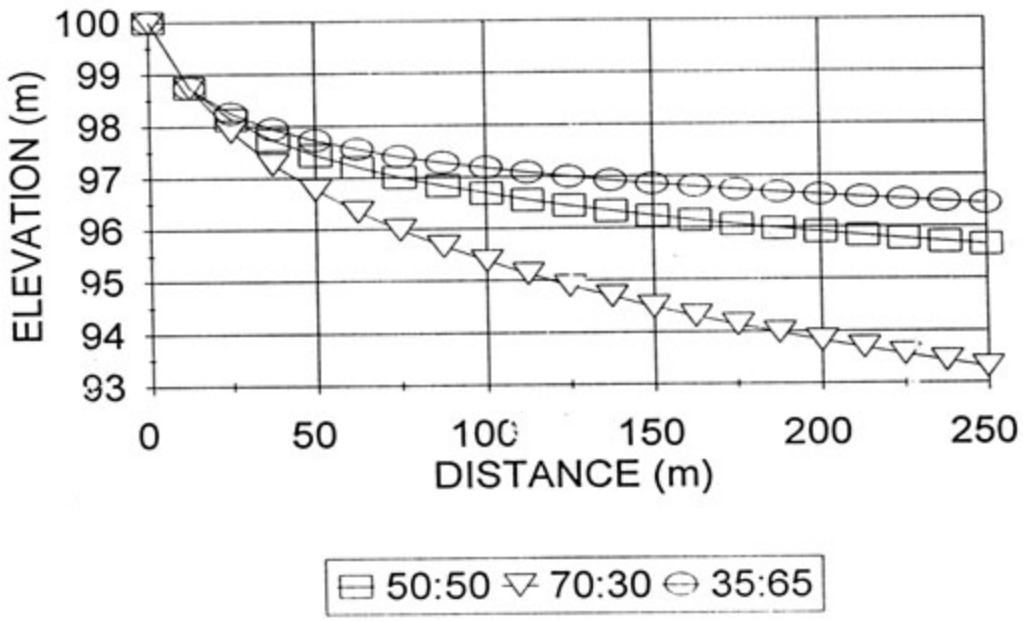


FIGURE 5.16 : PREDICTED BEACH PROFILE FOR VARIATIONS IN WATER:SOLIDS RATIO

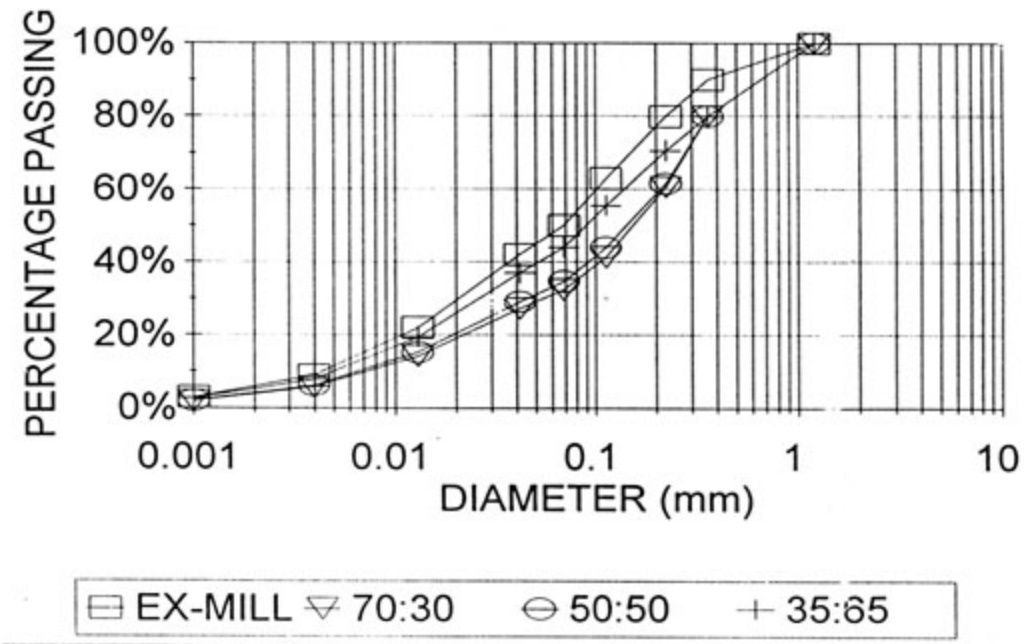


FIGURE 5.17 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN WATER:SOLIDS RATIO - AT DISCHARGE

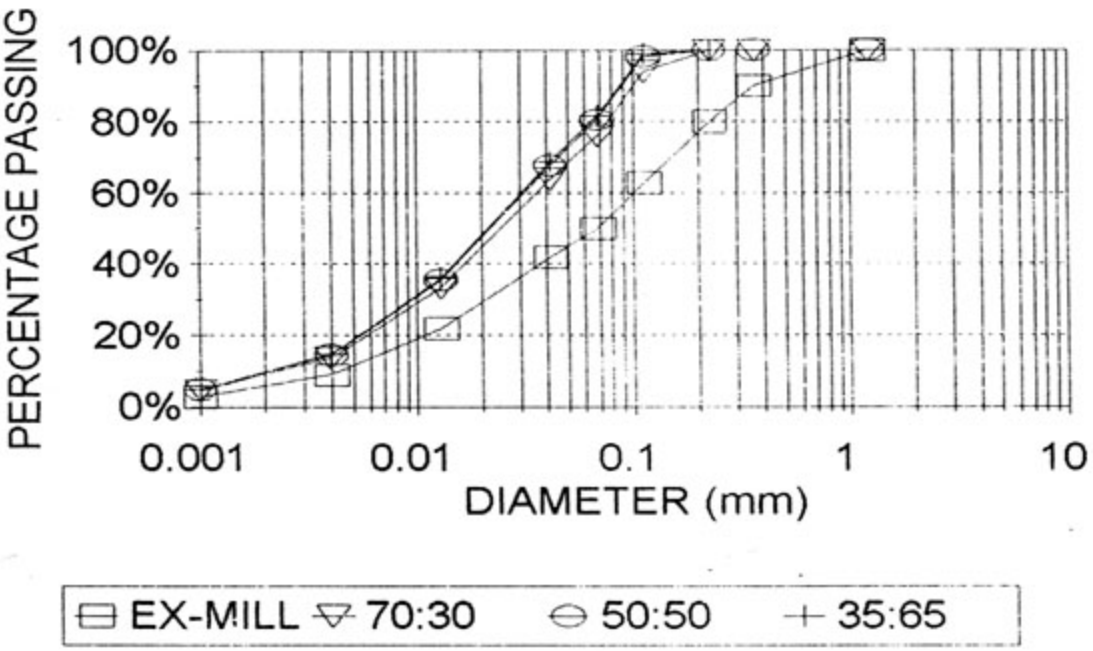


FIGURE 5.18 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN WATER:SOLIDS RATIO - AT 125 m

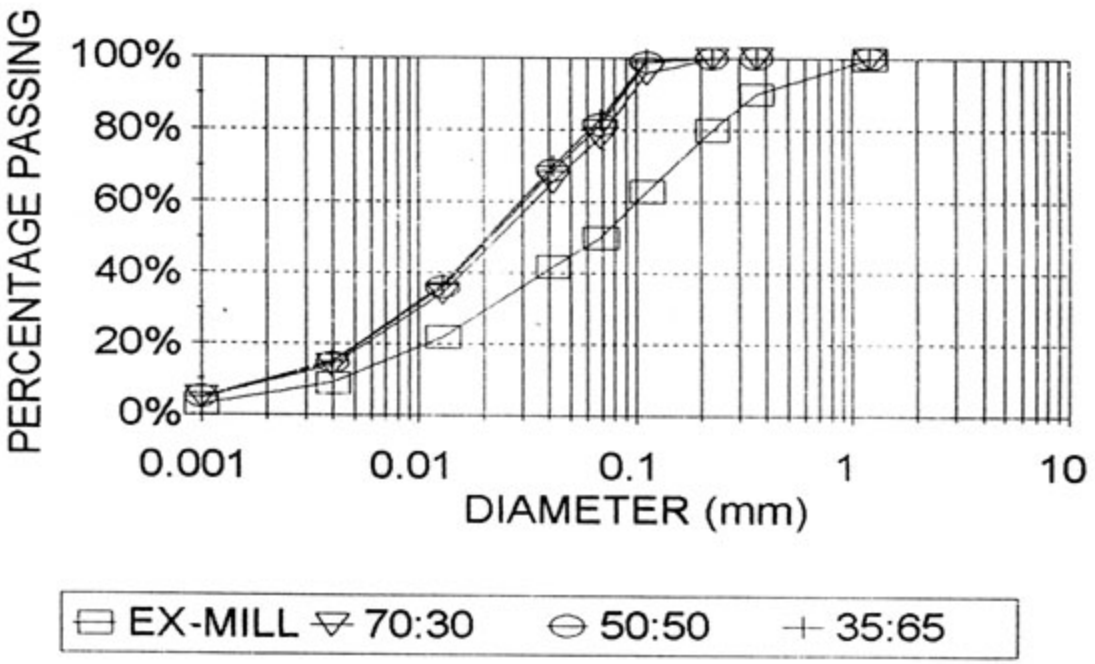


FIGURE 5.19 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN WATER:SOLIDS RATIO - AT 225 m

The variations in beach profiles predicted in Figure 5.16 are in line with expectations bearing in mind that the changes in water to solids ratio as simulated imply concomitant variations in tonnage rates so as to keep flow rates constant - an unlikely real life situation. Figure 5.17 show that the higher water:solids slurry has a greater tendency to segregate while that 35:65 slurry undergoes negligible segregation. This would be due to the differences in stream power between the lower density, high water content, tailings and the higher density, low water content slurry.

5.2.6 INFLUENCE OF BEACH LENGTH

One set of simulations that is not influenced by sympathetic variations of variables is that to assess the ability of the model to simulate the influence of beach length. A range of beach lengths representative of the tailings dams in South Africa was selected. These were an upper bound of 1 000 m, a lower bound of 100 m and a middle length of 250 m which was applied to all other simulations.

The results of the simulations are indicated on the following figures:

- Figure 5.20 - predicted beach profiles (standard scales)
- Figure 5.21 - predicted beach profiles (normalised scales)
- Figure 5.22 - predicted particle size distribution at the discharge
- Figure 5.23 - predicted particle size distribution at 125 m (midway along the beach)
- Figure 5.24 - predicted particle size distribution at 225 m (90 % of beach length of 250 m)

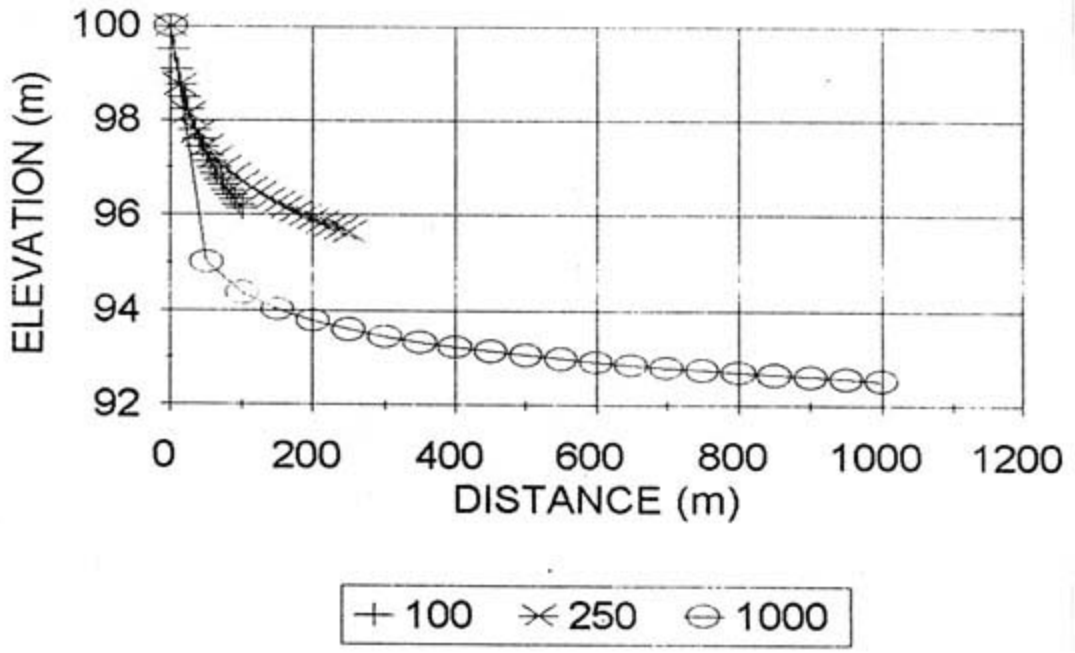


FIGURE 5.20 : PREDICTED BEACH PROFILES FOR VARIATION IN BEACH LENGTH

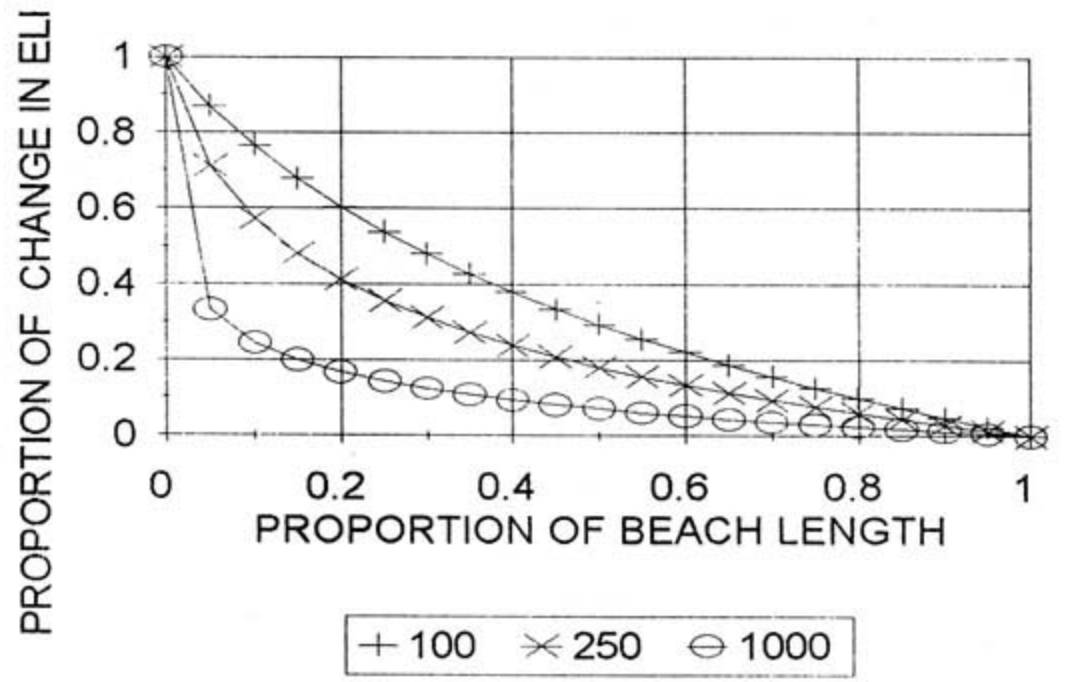


FIGURE 5.21 : PREDICTED NORMALISED BEACH PROFILES FOR VARIATION IN BEACH LENGTH

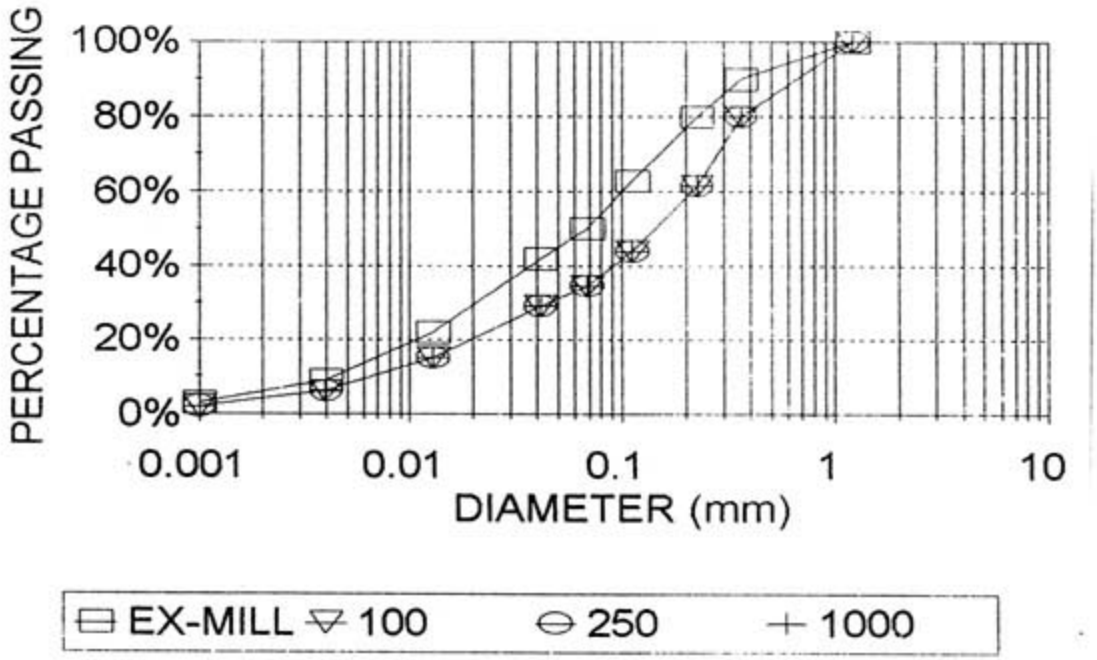


FIGURE 5.22 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATIONS IN BEACH LENGTH - AT DISCHARGE

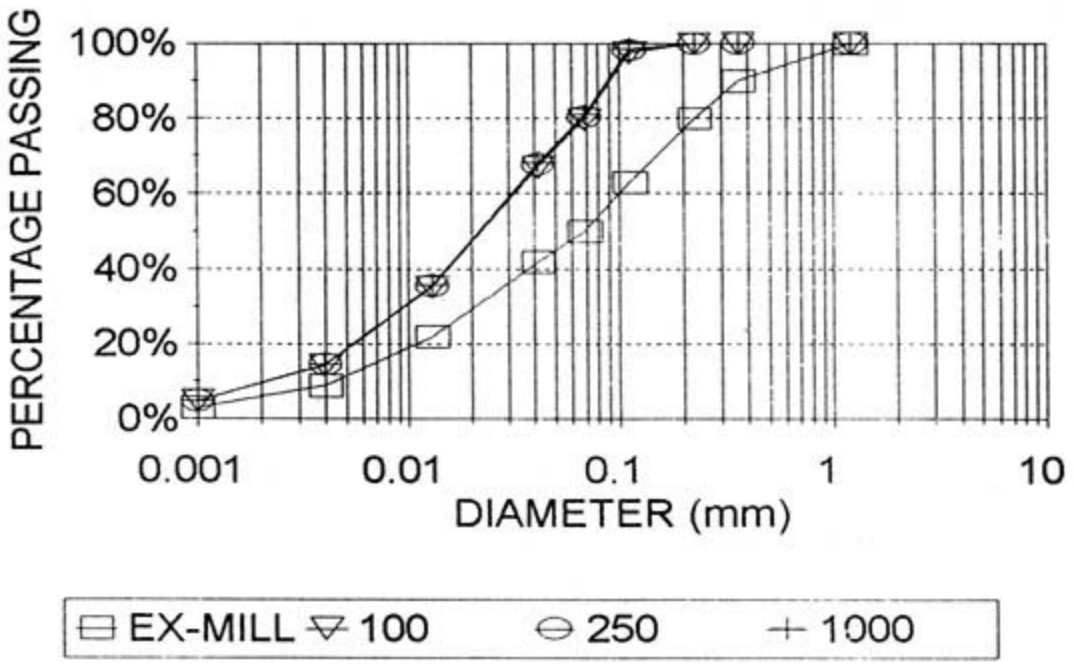


FIGURE 5.23 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATION IN BEACH LENGTH - AT 125 m

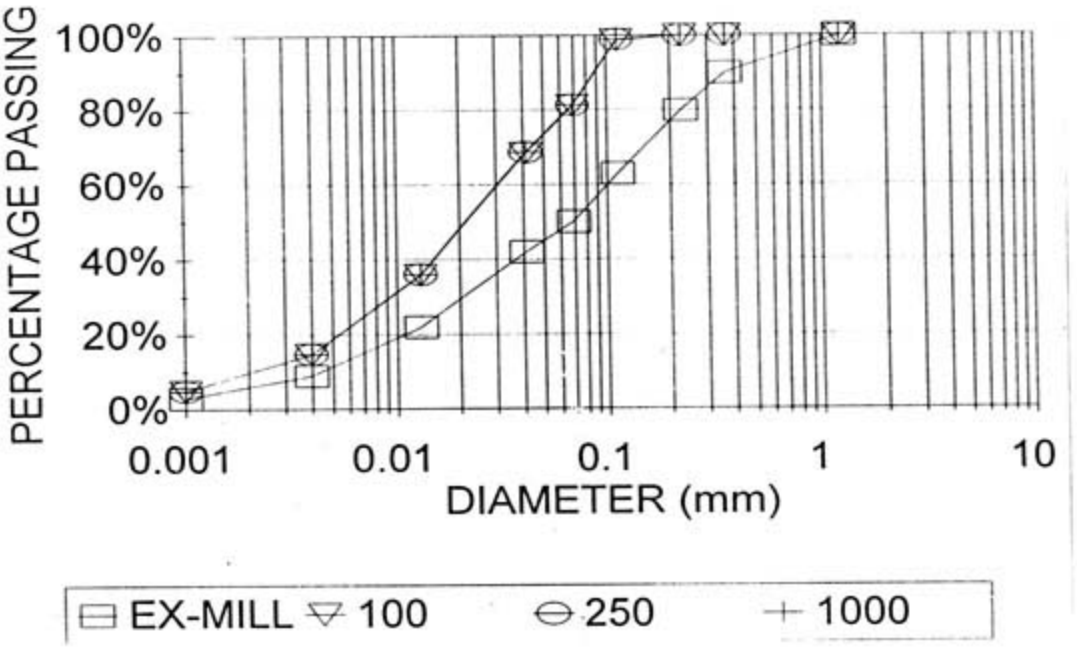


FIGURE 5.24 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN BEACH LENGTH - AT 225 m

A fundamental assumption in the derivation of the entropy-based equation for the variation of stream power with distance along the beach is that the stream power is monotonically decreasing function of the beach length. It therefore follows that the final beach profile will be a function of the beach length. Observations of beach build-up on tailings dams indicate that at the start of a deposition the slurry flows directly to the beach without significant build-up. Only after the pool has been reached does beach build-up proceed in earnest which indicates that beach length is fundamental in determining the deposited equilibrium profile. The variation of beach profile with beach length as indicated in Figure 5.20 is in line with these observations and appears reasonable.

A normalised plot of the beach profiles does not, however, indicate that the beach profile can be modelled with a master profile equation as has been suggested by several researchers (Melent'ev et al (1973), Bentel (1981), Blight (1987)). Given the demonstrated sensitivity of the stream power-entropy model to density, flow rate and velocity it would be logical to expect a wide variation in beach profiles for products which differ only slightly in particle size distribution, slurry concentraton and method of discharge. It should be noted that the range of beach profiles simulated is from 100 m to 1 000 m whereas the range covered by Bentel

(1981) and Blight (1987) covers beach lengths from 40 m to 140 m on a range of tailings dams and mining operations which did not have identical slurries, particle size distributions, discharge methods etc.. Close examination of the data of Bentel and Blight indicates that there is indeed differentiation between the profiles for the various beach lengths documented as indicated in Figure 5.25.. The stream power-entropy approach therefore shows that the master beach profile represents a practical approximation but no more. The stream power-entropy approach also represents an approximation but it is suggested that the approximation has a rational basis in its derivation and as such is able to differentiate more clearly between identical slurries discharged in identical ways, but over varying beach lengths.

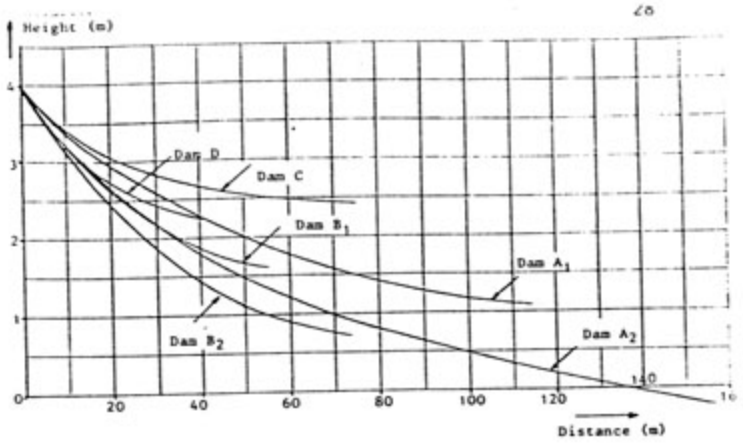


Figure 2.6: Measured Beach Profiles (Platinum)

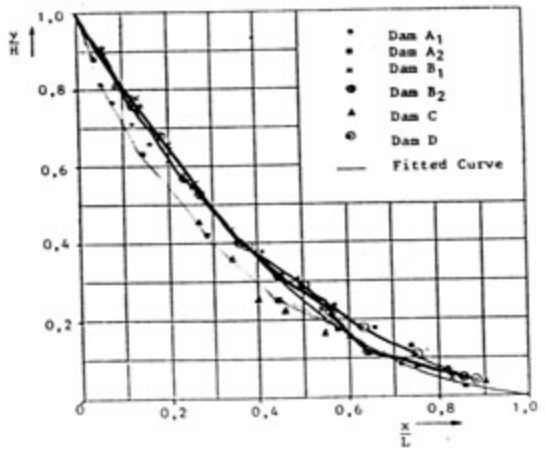


FIGURE 5.25 : NORMALISED BEACH PROFILE PLOTS AFTER BENTEL (1981) AND BLIGHT (1987)

The particle size distribution at any point along the beach is determined by the stream power at that point which, in terms of the derivation of the stream power relationship, is a function of the ratio of distance along the beach, x, and the total beach length L. Therefore at similar ratio's of x/L similar particle size distributions will be predicted by the model. This is verified in the curves in Figures 5.22 to 5.24.

On a tailings dam where beach lengths vary as a result of the particular dam's geometry it is necessary to compensate for the differing height drops over the beach lengths if a level crest is to be maintained (refer to Figure 5.20). This is can be accomplished if the discharge method is varied around the dam so as to regulate the velocity ex-plunge pool. The sensitivity of this parameter is examined in the next section.

5.2.7 INFLUENCE OF VELOCITY EX-PLUNGE POOL

The range of velocities ex-plunge pool assumed in the analyses covers that observed on several platinum and gold tailings operations in South Africa from an upper velocity of 0,5 m/s to a lower velocity of 0,1 m/s. Figures 5.26 to 5.29:

- Figure 5.26 - predicted beach profiles
- Figure 5.27 - predicted particle size distributions at the discharge
- Figure 5.28 - predicted particle size distribution at 125 m (midway along the beach)
- Figure 5.29 - predicted panicle size distribution at 225 m (90 % of the beach length)

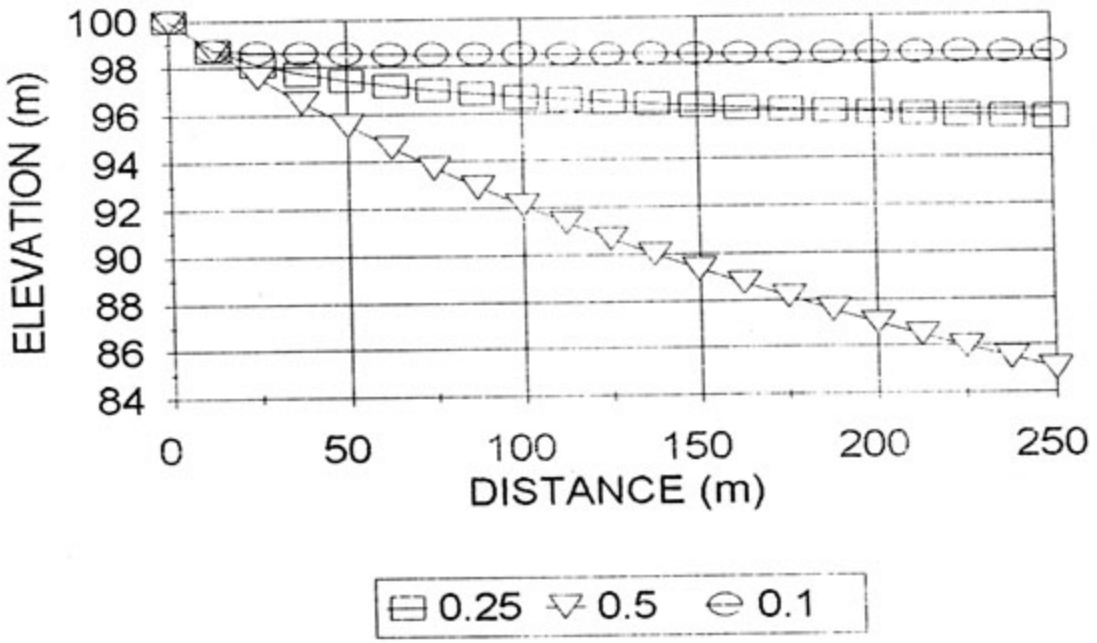


FIGURE 5.26 : PREDICTED BEACH PROFILES FOR VARIATION IN VELOCITY EX-PLUNGE POOL

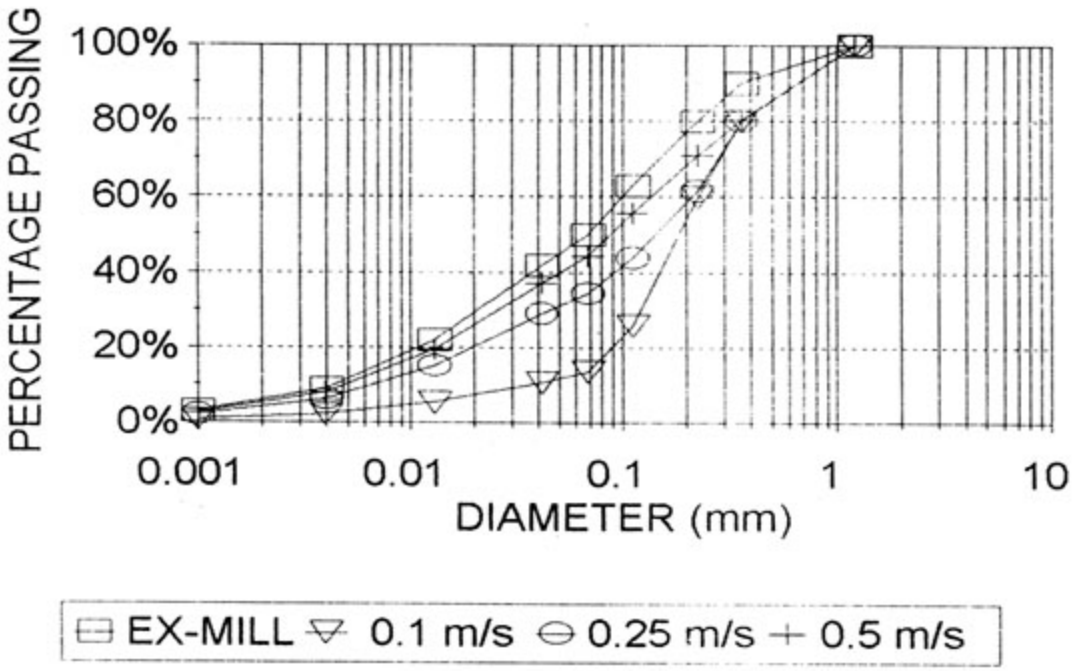


FIGURE 5.27 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN VELOCITY EX-PLUNGE POOL - AT DISCHARGE

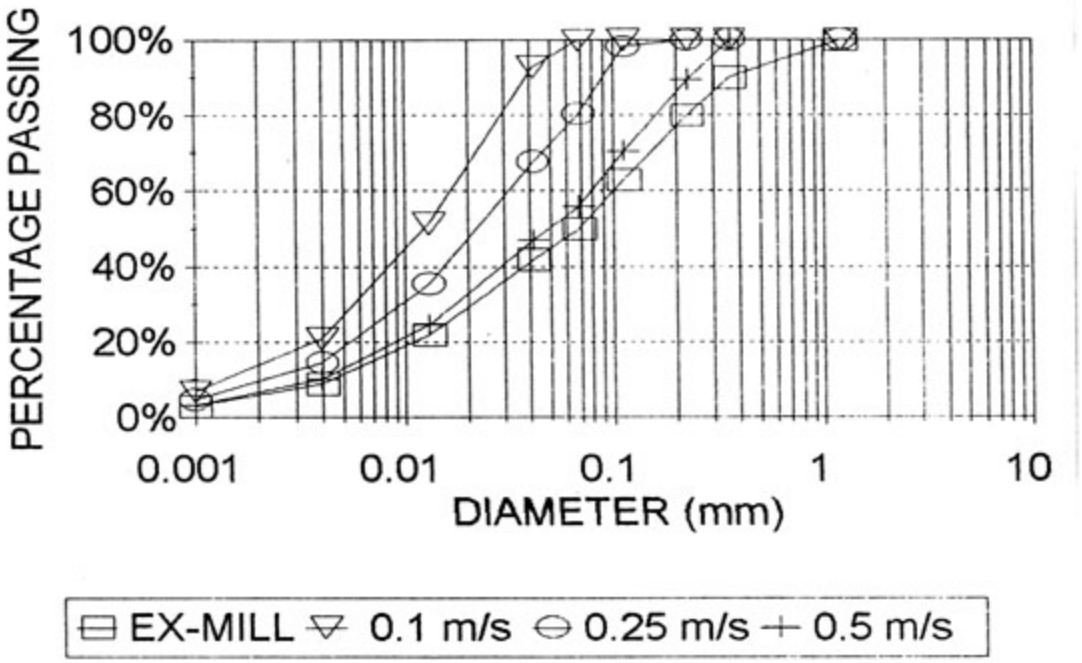


FIGURE 5.28 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATION IN VELOCITY EX-PLUNGE POOL - AT 125 m

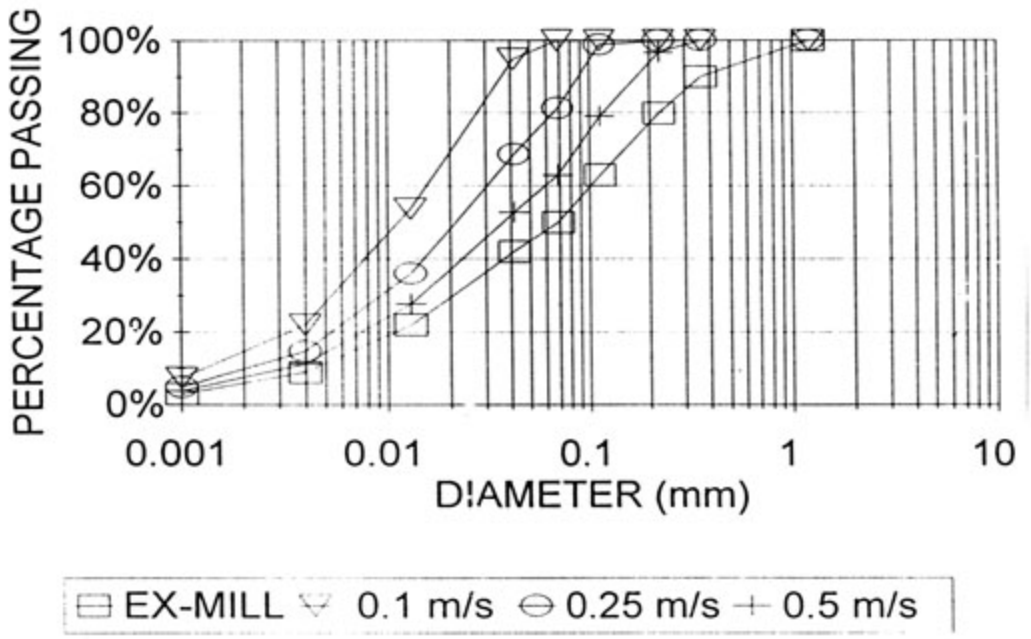


FIGURE 5.29 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN VELOCITY EX-PLUNGE POOL - AT 225 m

Variations in beach profile appear to be particularly sensitive to variations in the velocity ex-plunge pool as illustrated in Figure 5.26. This is understandable since the stream power is a function of the square of the velocity. This is unfortunate since this velocity, along with the frictional resistance characteristics of the slurry are the most problematic to estimate.

Sensitivity to the velocity is carried through to the particle size distributions given in Figures 5.27 to 5.29. The trends predicted by the model are in line with expectations.

5.2.8 INFLUENCE OF SEDIMENT FRICTIONAL RESISTANCE

The frictional resistance of a slurry will depend on the host rock, particle size distribution, water to solids ratio and, since most slurries are non Newtonian, on the velocity of the slurry. The range of slurry frictional characteristics assumed in the sensitivity analysis is in line with the range of measurements for pipeline rheology as documented by Verkerk (1990) and covers head losses per metre from an upper limit of 0,5 m/m to 0,01 m/m. This head loss has been included as the slope of the velocity head variable at the discharge, and also represents the slope of the beach profile at the plunge pool. The following figures illustrate the results of the simulations:

- Figure 5.30 - predicted beach profiles
- Figure 5.31 - predicted particle size distributions at the discharge
- Figure 5.32 - predicted particle size distribution at 125 m (midway along the beach)
- Figure 5.33 - predicted panicle size distribution at 225 m (90 % of the beach length)

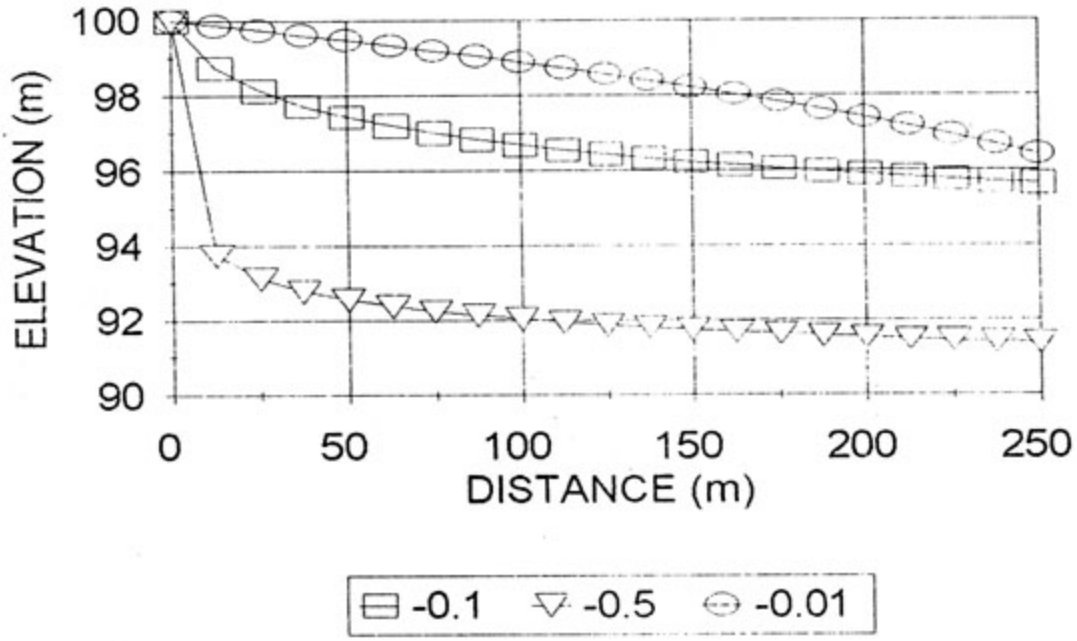


FIGURE 5.30 : PREDICTED BEACH PROFILES FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope)

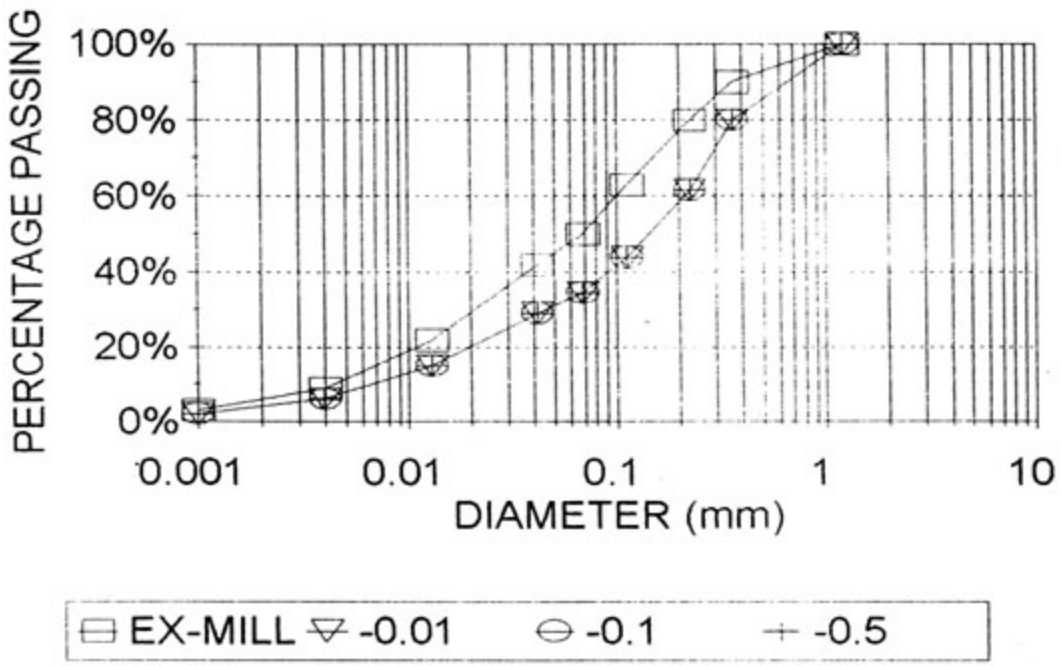


FIGURE 5.31 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope) - AT DISCHARGE

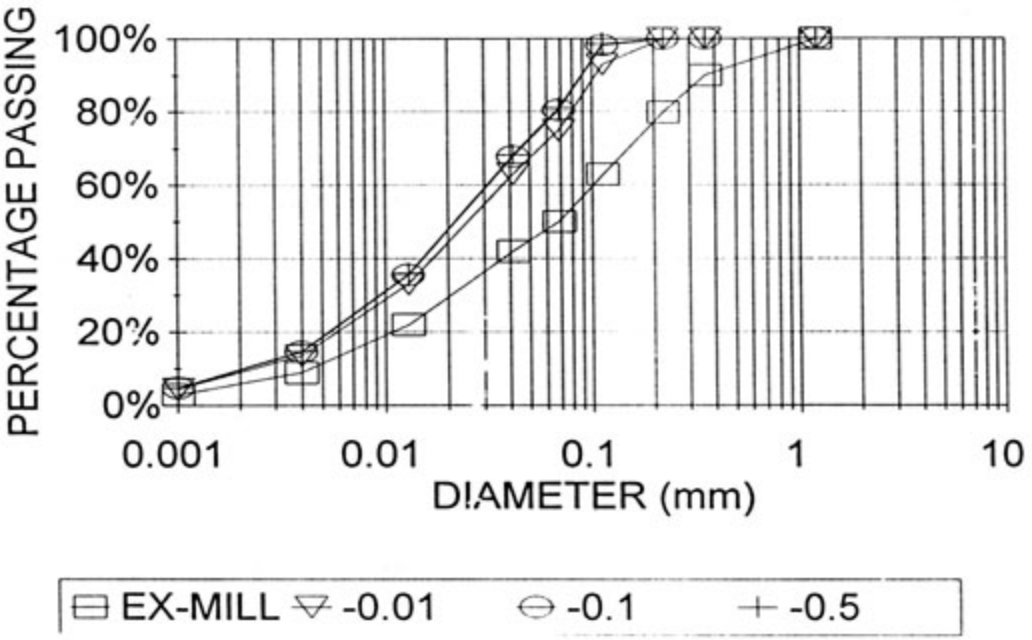


FIGURE 5.32 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope) - AT 125 m

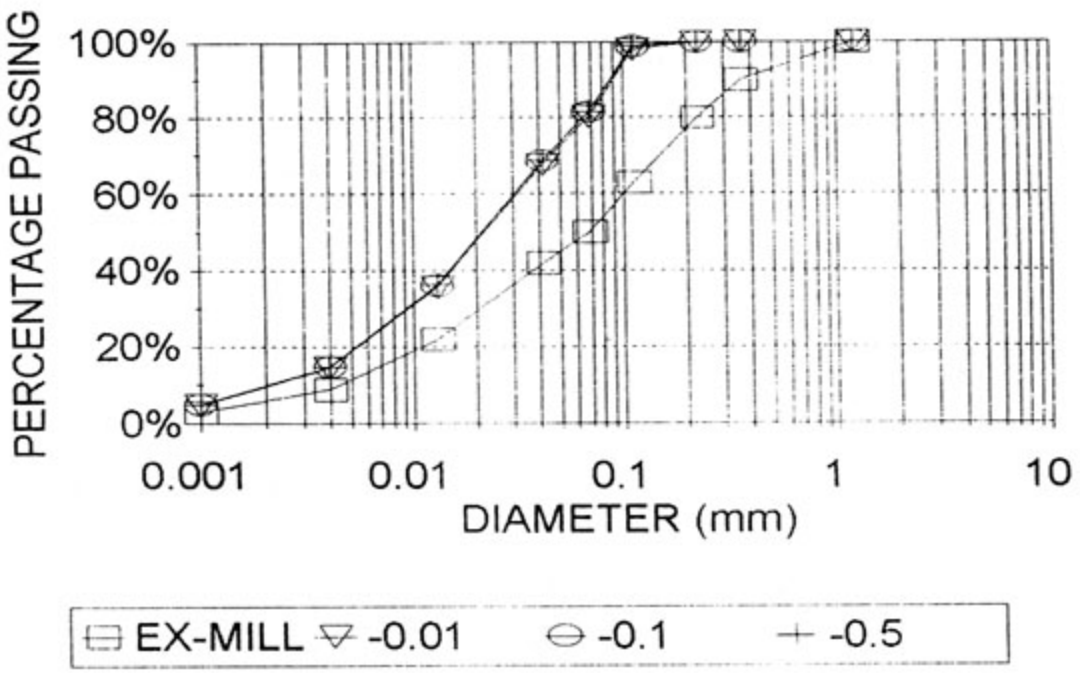


FIGURE 5.33 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN FRICTIONAL RESISTANCE (initial beach slope) - AT 225 m

Figure 5.30 indicates a large degree of variation in beach profile. A convex beach profile is predicted for the lower bound frictional resistance. This is in line with observations on a tailings slurry comprising phyllite described in Chapter 6.

Since the frictional resistance variable is applied to the initial slope of the beach only the model does not predict a variation in particle size distribution as indicated in Figures 5.31 to 5.33.

5.3 PART 2 : CORRELATED VARIATION OF PRIMARY AND SECONDARY VARIABLES

The reaction of the beaching model to correlated variations in primary and secondary variables has been assessed by carrying out simulations for an assumed slurry and comparing the output with that for the following simulations:

- a typical operational condition where a thickener is taken out of operation. Under these conditions it is unlikely that the Mine would reduce the milling rate and the tonnage of tailings delivered to the tailings dam would remain unchanged. As a result of the thickener being out of commission:

  - the water to solids ratio would increase, say to 70:30
  - the slurry flow rate would increase
  - the velocity ex-plunge pool would increase
  - the frictional resistance characteristics of the slurry would decrease slightly

  Table 5.2 summarises the assumed values for the above and the remaining variables.

- a situation where a new reef is to be mined, or a new mine developed where the grind is likely to be similar to a previous tailings product but the particle specific gravity is higher and an initial impression of the change in beaching characteristics is required. Under this change the following related changes are likely to occur:

- the velocity in the delivery pipeline will need to be increased to prevent settling out in the pipeline. This would most likely be achieved though an increase in the water:solids ratio of the slurry which would bulk the flow

- the flow rate would increase

- the velocity ex-plunge pool would increase.

The values assumed/calculated for the purposes of this analysis are listed in Table 5.2.

TABLE 5.2: SUMMARY OF PRIMARY AND SECONDARY VARIABLES ASSUMED IN CORRELATED SENSITIVITY ANALYSIS

	PARTICLE SIZE DISTRIBUTION	PARTICLE S G	FLOW RATE (l/s)	WATER TO SOLIDS	BEACH LENGTH (m)	VELOCITY EX-PLUNGE POOL (m/s)	FRICTIONAL RESISTANCE (initial slope)
INTIAL SLURRY	medium (1)	3	77,9 (2)	50:50	250	0,25	-0,1
THICKENER GOES DOWN	medium	3	155,6 (3)	70:30	250	0,35	-0,08
HIGHER SG TAILINGS	medium	4	95,14 (4)	58:42	250	0,30	-0,1

NOTES :
1 50 % minus 0,075 mm, maximum size 2 mm
2 Calculated from tonnage rate of 150 000 tonnes per month at 50:50 water:solids ratio
3 Calculated from tonnage rate of 150 000 tonnes per month at 70:30 water:solids ratio
4 Calculated from tonnage rate of 150 000 tonnes per month at 58:42 water:solids ratio

5.3.1 RESULTS OF SIMULATIONS FOR CORRELATED VARIATION OF PRIMARY AND SECONDARY VARIABLES

Figures 5.34 to 5.37 illustrate the results of the simulations:

- Figure 5.34 - predicted beach profiles
- Figure 5.35 - predicted particle size distributions at the discharge
- Figure 5.36 - predicted particle size distribution at 125 m (midway along the beach)
- Figure 5.37 - predicted particle size distribution at 225 m (90 % of the beach length)

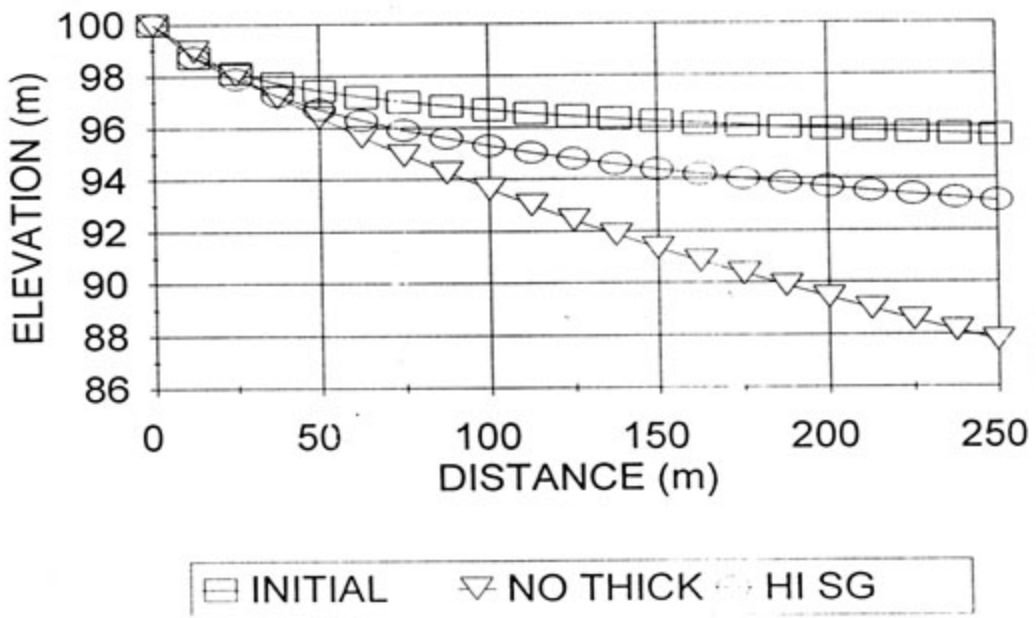


FIGURE 5.34 : PREDICTED BEACH PROFILES FOR VARIATIONS THICKENER OPERATION AND ORE TYPE

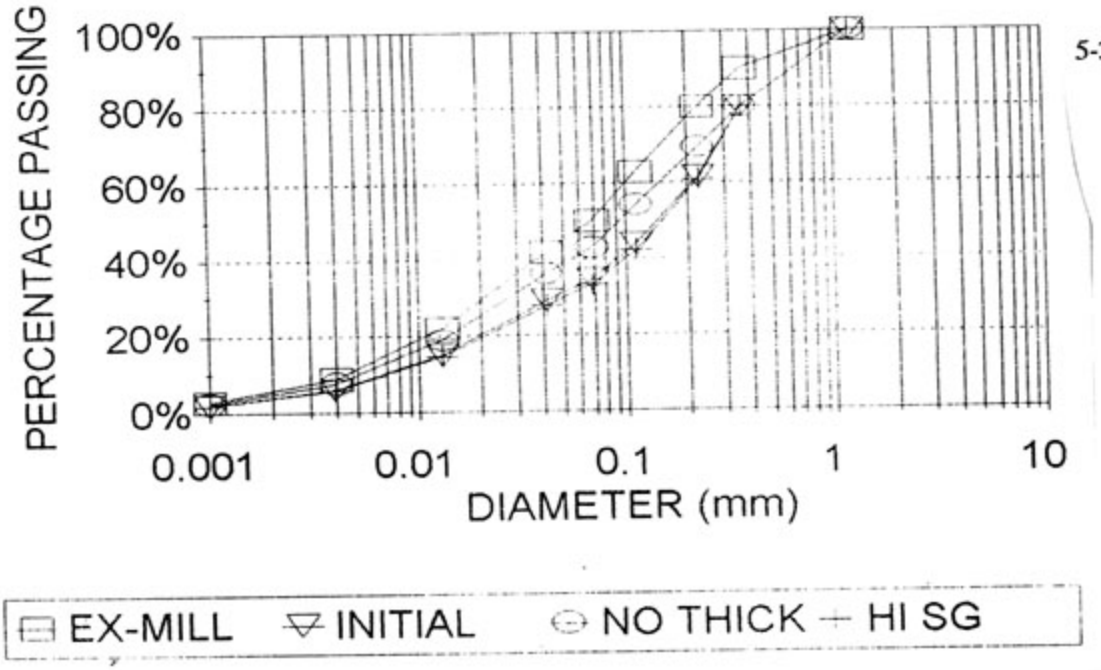


FIGURE 5.35 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR VARIATION OF THICKENER OPERATION AND ORE TYPE - AT DISCHARGE

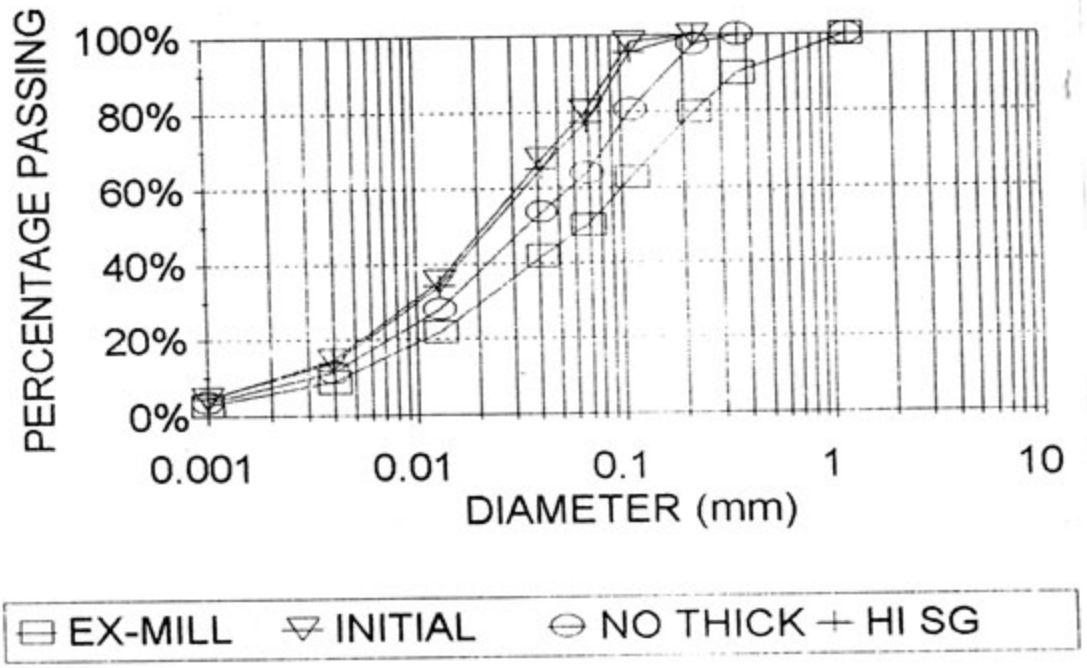


FIGURE 5.36 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN THICKENER OPERATION AND ORE TYPE - AT 125 m

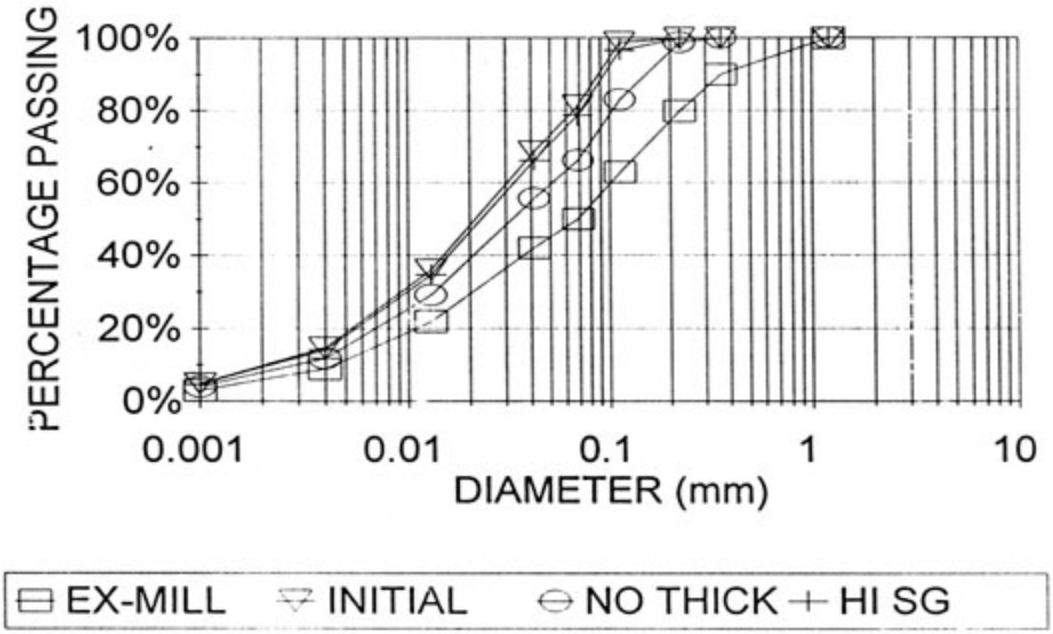


FIGURE 5.37 : PREDICTED PARTICLE SIZE DISTRIBUTION FOR VARIATION IN THICKENER OPERATION AND ORE TYPE - AT 225 m

Figure 5.34 indicates the following:

- the beach profile will be steepened considerably as a result of the thickener coming off line and severe cutting of the beach will occur. This is in line with observations on site

- the beach profile of the higher SG product will be significantly steeper and result in significant improvements in freeboard for storm containment. This trend is in line with observations.

Figures 5.35 to 5.37 indicate the following:

- as a result of the increased flow rate in the unthickened slurry there is markedly less segregation down the beach which means that a greater proportion of coarse material will be carried to the pool area and pool capacity and storage characteristics will be adversely affected

- the increased particle specific gravity as only a slight effect on the gravitational sorting of the slurry.

It should be borne in mind that the above analyses are based on assumptions with regard to the changes in the primary and secondary variables. The exact nature of these changes would depend on the actual tailings product and mining operation.

5.4 DISCUSSION

The simulations to evaluate sensitivities of the variables in isolation indicate that over practical ranges of primary and secondary variables, significant variations in beach profile and particle size distribution along the beach can result. For a given segregating tailings product it is evident that a change in water to solids ratio at a constant tonnage through-put which would result in an increase in the flow rate and the velocity from the plunge pool would effect the most significant variation. This is because the flow rate and the square of the velocity from the plunge pool are multiplied to obtain the stream power. Such fluctuations are common in tailings disposal systems where variations in thickener performance, pump sump levels and surges in delivery as a result of partial settling out at dips in the pipe route result in periods of high flow as well as momentary periods during which no flow occurs at all !

Simulations to assess the ability of the beaching model to predict realistic results for correlated changes in the primary and secondary variables indicate that the model predicts the anticipated and observed trends correctly. The extent to which it will be able to predict the changes in absolute terms will depend on the accuracy with which the slurry properties in particular those which determine the secondary variables of velocity ex-plunge pool and the frictional resistance of the slurry on the initial part of the beach (or initial beach slope) can be estimated. As discussed in Chapter 3, the estimation of these properties presents a particular problem. One potentially viable method is that of model testing in which it would be possible to vary discharge rates, slurry properties etc and observe and measure the resulting beach profiles. Once the model has been calibrated for the model it could be used for extrapolating the profile into the field situation.

Chapter 6 assesses the application of the spreadsheet model to the results of laboratory-scale model testing in the literature and pilot scale testing as conducted by this author. The relative success with which the flume-based parameters can be extrapolated to the field situation will be evaluated.

CHAPTER 6

CONTENTS

Sec	Description	Page
6	APPLICATION OF ENTROPY TO FLUME TESTING	6-1
	6.1 INTRODUCTION	6-1
	6.2 FLUME SCALING CRITERIA	6-1
	6.3 KÜPPER FLUME TESTS	6-7
	6.4 THOMSON FLUME TESTS	6-14
	6.5 FLUME TESTS ON BRAZILIAN GOLD TAILINGS	6-14
	6.6 FOURIE FLUME TESTS	6-22
	6.7 DISCUSSION	6-22

CHAPTER 6 APPLICATION OF ENTROPY TO FLUME TESTING

6.1 INTRODUCTION

Preceding chapters have demonstrated that an entropy-based stream power approach responds to variation of the major variables generally in the manner observed in the field. To be able to apply the approach on a fully predictive basis in cases where a similar full scale beach does not exist from which to obtain the initial slope or frictional characteristics, it is necessary to obtain reasonable estimates of this parameter. One method of estimating the slurry frictional resistance characteristics is by means of laboratory-scale or pilot-scale flume tests. Several researchers referenced in the literature survey in Chapter 2 have published the results of flume tests. These include Küpper (1991), Thomson (1984), and Fourie (1988). Some of these published results provide adequate data on which to base predictive runs using the entropy-stream power model and compare flume test results with full scale operations. In addition to the published flume test results, this author has conducted a series of pilot scale tests on a consultancy basis and these too are amenable to comparison with the field scale situation.

6.2 FLUME SCALING CRITERIA

The principles of flow similitude and models has been comprehensively set out by Albertson et al (1960) who describe the types of similarity in terms of *geometric*, *kinematic* and *dynamic* criteria.

Geometric similarity requires that all length ratios from the flume (model) to the full scale operation (prototype) and all area ratios be the same. Since area is the product of two lengths, area will scale according to the square of the length scale, ie, if:

$$L_R = \frac{L_p}{L_m} = \frac{H_p}{H_m} \qquad (1)$$

then:

$$A_R = \frac{A_p}{A_m} = \left(\frac{L_p}{L_m}\right)^2 = L_R^2 \qquad (2)$$

where m and p refer to the model and the prototype respectively.

Kinematic similarity requires that the velocity ratio and the acceleration ratio be the same from point to point in the model as in the prototype so that:

$$V_R = \frac{V_{0p}}{v_{0m}} = \frac{V_{1p}}{V_{1m}} = \frac{V_{2P}}{V_{2m}} = ... \qquad (3)$$

and

$$a_R = \frac{a_{0p}}{a_{0m}} = \frac{a_{1p}}{a_{1m}} = \frac{a_{2p}}{a_{2m}} = ... \qquad (4)$$

By combining equations 3 and 4 the discharge ratio and the time ratio can be determined as:

$$Q_R = A_R\, V_R = L_R^2\, V_R \qquad (5)$$

and

$$T_R = \frac{L_R}{V_R} \qquad (6)$$

Dynamic similarity requires that the shear ratio, the pressure ratio and the force ratio be the same from point to point in the flume and in the field. In the case of the force ratio where the force relates to pressure over an area this yields the following relation:

$$F_R = \frac{F_p}{F_m} = p_R A_R = p_R L_R^2 \qquad (7)$$

Basic fluid properties such as density ρ, viscosity μ, specific weight γ, surface energy σ, and elasticity E have a considerable influence on the dependent variables such as pressure and shear.

The inertia force, based on Newton's equation can be expressed dimensionally as:

$$F = \rho L^3 \frac{L}{T^2} \tag{8}$$

Similar expressions can be developed for the viscous, gravity, surface energy and elastic forces acting on a system and if complete dynamic similarity is to be established in the flume, each force ratio from the flume to the field should be the same at all points. This is not possible.

In most problems of flow the force of inertia is the primary force and this has led to the development of a number of standard ratios of forces. These are referred to as the shear number, S_n, the Reynolds number, Re, the Froude number, Fr, the Mach number, Ma, and the Weber number, We, which represent the ratios of the inertia forces per unit area to the shear, viscous, gravity, elastic and surface energy forces per unit area respectively. The numbers are the ratios of force per unit area and are therefore dimensionless. In modelling, the forces which are observed to be the most important are identified and the model is set up in such a way that the ratios of the dimensionless numbers appropriate to the type of force are kept as close to unity as possible.

Two dimensionless numbers are of overriding importance in flume testing:

- the Reynolds number, Re, expressed as :

$$\frac{\rho V L}{\mu} \tag{9}$$

- the Froude number, Fr, expressed as :

$$\frac{V}{\sqrt{L\frac{\gamma}{\rho}}} \tag{10}$$

The Reynolds number is particularly important in flow situations where viscous forces predominate such as under laminar flow conditions. Quiescent settling of a particle in a stagnant fluid is controlled by viscous forces associated with the flow past the boundary between the fluid and the particle and is estimated using the Reynolds number. As flow conditions become more turbulent and viscous forces become less dominant the Reynolds

number becomes less important and inertial forces such as those controlled by gravity become more important. The Froude number relates the forces of gravity to the inertial forces and hence takes over where the Reynolds number leaves off. The Froude number dominates where the flow involves materials of different specific weight which would be the case in slurry flow. For most tailings, in laminar flow situations the Reynolds number would dominate and in turbulent flow situations the Froude number would dominate. What is important at the outset is to make sure that the flume has the same flow characteristics as the field situation. A flume which demonstrates laminar flow will be dominated by viscous forces and will not model a field situation where flow conditions may be highly turbulent.

Modelling on the basis of the Reynolds and Froude numbers involves setting up geometric, kinematic and dynamic scaling ratios in such a way that the ratio of either the Reynolds number or the Froude number is kept at unity. (It may be necessary to satisfy both numbers simultaneously. This is theoretically possible by selecting and appropriate fluid of the required density and viscosity. Practically, however, such fluids either do not exist or they are too difficult or expensive to use.)

Table 6.1 lists the scaling ratios that apply to each of the parameters listed down the first column given a length scale, L, a density scale, ρ, a specific weight scale, γ, and a viscosity scale, μ, for situations where either the Reynolds number or the Froude number is dominant. The significance of the ratio information in Table 6.1 is as follows:

Assume that flow is highly turbulent and therefore that the Fr number is of most significance. Assume also that the same tailings at the same slurry density is used in the flume as in the field. This implies that the viscosity will also be unchanged between flume and field. Under these constraints the scale ratio's in the 6th column apply. column 6 applies. If a length scale, L, is selected as, for example, 1/100, then in order to satisfy the similitude requirement that Fr be the same at all times and points in the flume as in the field:

- the time scale will be $L^{1/2}$ or 1/3,16
- the velocity scale will need to be $L^{1/2}$ or 1/3,16
- the flow scale will need to be $L^{5/2}$ or 1/100 000

- the stream power scale will therefore be $L^{7/2}$ or 1/10 000 000

Conversely, if viscous forces dominate:

- the time scale will be L^2 or 1/10 000
- the velocity scale will need to be L^{-1} or 100/1
- the flow scale will need to be L or 1/100
- the stream power scale will therefore be L^{-1} or 100/1

TABLE 6.1 : SIMILITUDE SCALING RATIOS

Characteristic	Dimension	Ratio Scale			
		L, ρ, γ, μ variable		ρ, γ, μ constant	flume to field
		$(Re)_R$	$(Fr)_R$	$(Re)_R$	$(Fr)_R$
Length	L	L	L	L	L
Area	L^2	L^2	L^2	L^2	L^2
Volume	L^3	L^3	L^3	L^3	L^3
Time	T	$\rho L^2/\mu$	$(L\rho/\gamma)^{1/2}$	L^2	$L^{1/2}$
Velocity	L/T	$\mu/L\rho$	$(L\gamma/\rho)^{1/2}$	L^{-1}	$L^{1/2}$
Discharge	L^3/T	$L\mu/\rho$	$L^{5/2}(\gamma/\rho)^{1/2}$	L	$L^{5/2}$
Density	M/L^3	ρ	ρ	1	1
Specific Weight	M/L^2T^2	γ	γ	1	1
Viscosity	M/LT	μ	μ	1	1
Momentum	ML/T	$L^2\mu$	$L^{7/2}(\rho\gamma)^{1/2}$	L^2	$L^{7/2}$
Energy	ML^2/T^2	$L\mu^2/\rho$	$L^4\gamma$	L	L^4
Power	ML^2/T^3	$\mu^3/L\rho^2$	$(L^{7/2}\gamma^{3/2})/\rho^{1/2}$	L^{-1}	$L^{7/2}$

As flow moves from viscous control to gravitational control ie from fully laminar to highly turbulent flow states there is a transition. With increasing turbulence the importance of the Reynolds number diminishes while that of the Froude number increases. In the majority of the flume tests on tailings evaluated in the course of this research few have achieved fully turbulent conditions event though these conditions may be occurring in the field. It is suspected therefore that it will be necessary to interpolate between Reynolds similitude requirements and Froude similitude requirements.

Yang (1980) reports that he established on the basis of a series of flume tests that it was possible to satisfy both the Fr and the unit stream power ratio's from tests conducted using the field sediment at field slurry concentrations in a flume. In this case the slope ratio can be established from the unit stream power ratio as follows:

$$\frac{V_R S_R}{w_R} = 1 \qquad (11)$$

but for the Fr criterion and with $w_R = 1$ (since the same sediment is used in the flume as in the field):

$$V_R = L^{1/2} \qquad (12)$$

and

$$S_R = L^{-1/2} \qquad (13)$$

This means that, in the example quoted above, the slope of the beach in the field would be equal to $(1/100)^{1/2}$ times the slope measured in the model. This is in line with Küpper's conclusion that the model beach slope is always steeper than the field beach slope.

To satisfy both the Reynolds and the unit stream power criteria, following the same process as in the above derivation, the slope would scale according to:

$$S_R = L \qquad (14)$$

The above describes how to arrive at the scaling parameters in the case where a set of flume tests is to be set up. The scaling requirements need to be fitted into practical limitations and are unlikely to be possible in each instance. The above scaling requirements have not been observed in the important testing by Küpper nor was it practical to observe them in the pilot-scale flume testing conducted by this author and it is therefore necessary to derive a way of interpreting the flume test results for extrapolation to the field in the best way practicable. This is set out under the sections below.

6.3 KÜPPER FLUME TESTS

Kùpper (1991) comprehensively documented an extensive suite of laboratory-scale tests conducted in a 5 m long by 300 mm wide (per run) flume. The tests were conducted on three products one of which was a sample of the beach tailings at Syncrude for which Chapter 4 of this dissertation set out field-scale predictive runs for comparison against field measurements. Küpper's flume tests covered a range of solids concentrations and discharge rates and were structured to evaluate the influence of these two variables on the beaching (and beached) characteristics of the tailings. Figures 6.1 to 6.4 are reproduced from Kùpper's dissertation and illustrate the nature of the results obtained from the flume tests for the tailings sample from Syncrude.

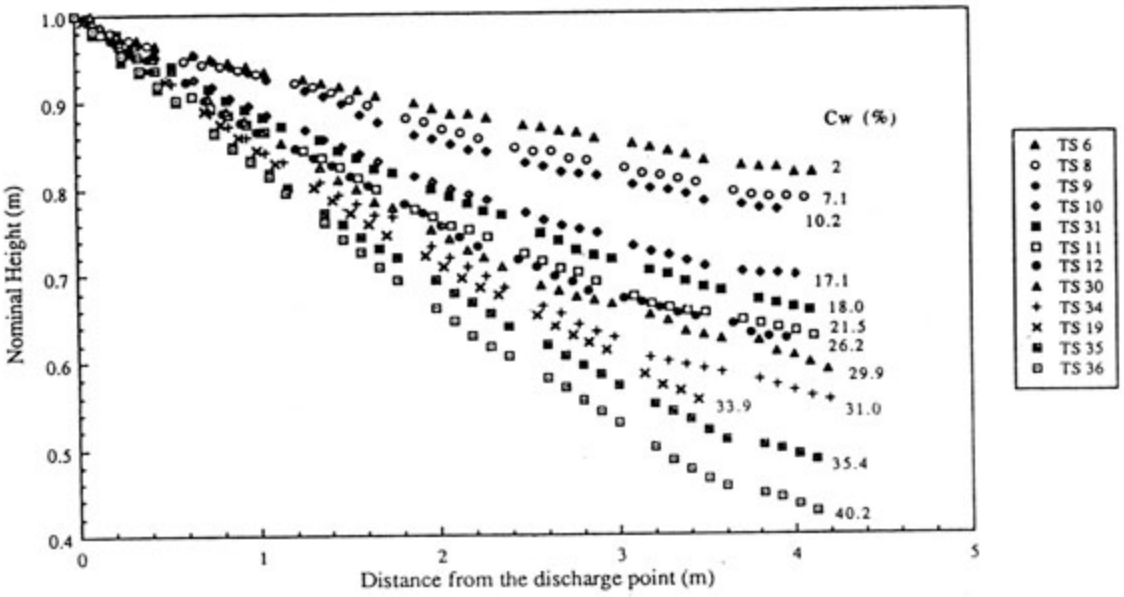


FIGURE 6.1 : PROFILES OF TS TESTS WITH Q_t = 5 l/min AND VARIOUS Cw (After Kùpper (1991)

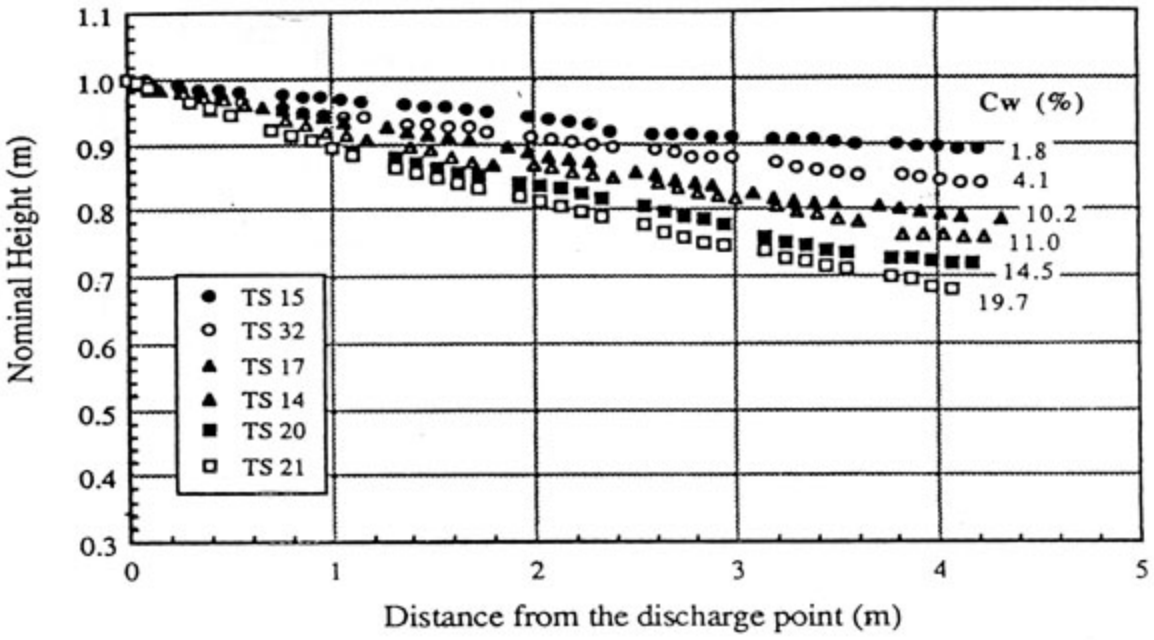


FIGURE 6.2 : PROFILES OF TS TESTS WITH Q_t = 10 l/min AND VARIOUS Cw
(After Kùpper (1991)

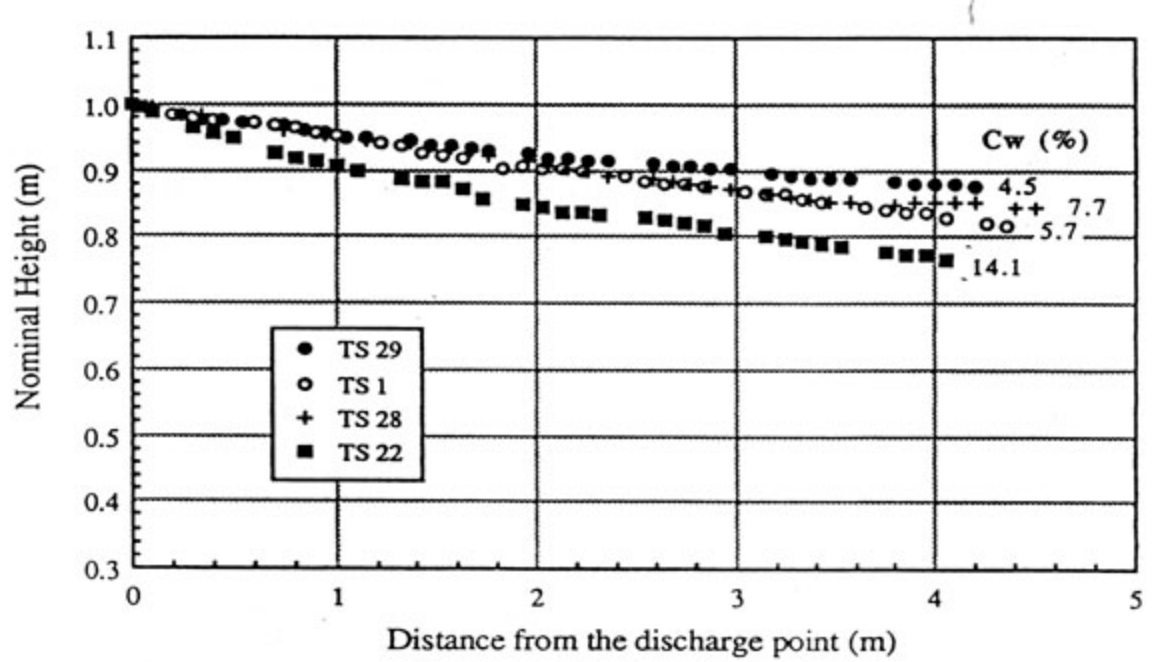


FIGURE 6.3 : PROFILES OF TS TESTS WITH Q_t = 15 l/min AND VARIOUS Cw
(After Kùpper (1991)

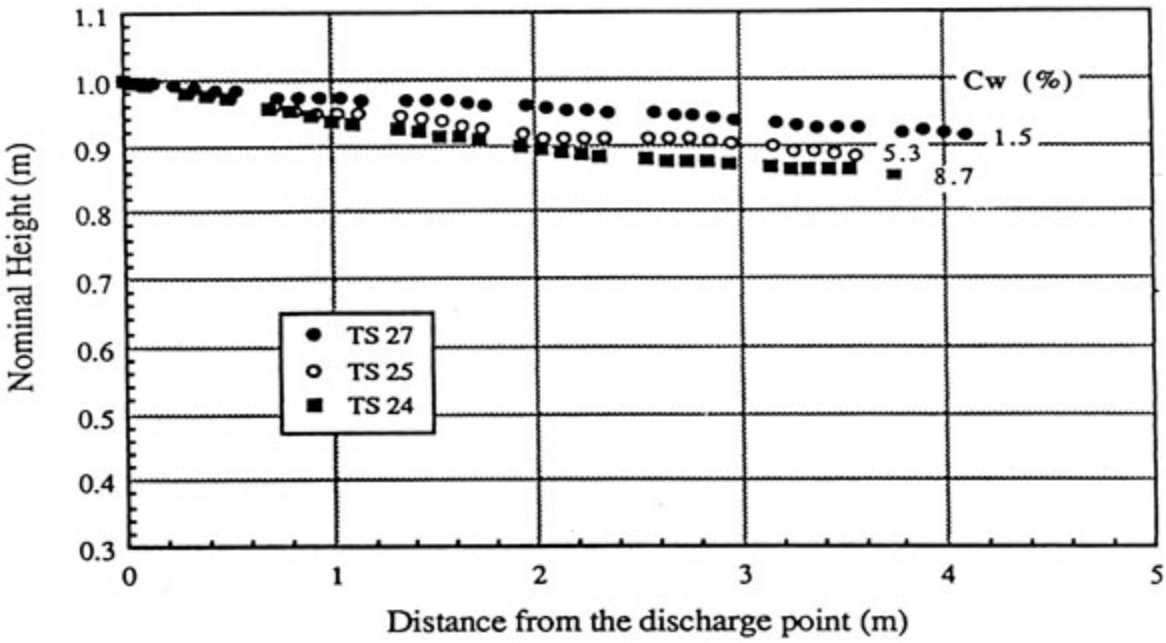


FIGURE 6.4 : PROFILES OF TS TESTS WITH Q_t = 20 l/min AND VARIOUS Cw (After Kùpper (1991)

The plots in Figures 6.1 to 6.4 indicate an increasingly steep beach profile for an increasing concentration over the range from 2 % solids to 40 % solids. Of the tests, only test TS 19 is most representative of a field test - Test 4, Spigot 1. This field test was identified in Chapter 4 as being relatively free of mishaps such as thickener problems, freezing of spigots etc. It is also representative of the range of concentrations achieved in the flume tests which extend to a maximum of 40.2 % whereas this concentration is more representative of the lower bound of the field tests. Field test 4 was conducted with the spigots at the top of the pipe and therefore is representative of the finer, higher water content spigot product tested.

Figure 6.5 is a plot showing the measured and calibrated beach profiles in flume test TS 19 for which the following calibrating parameters were established:

- velocity close to discharge — 3,1 m/s
- flow rate — 5 l/min (0,0833 l/s)
- stream power — 0,512 watts
- initial slope — -0,1657

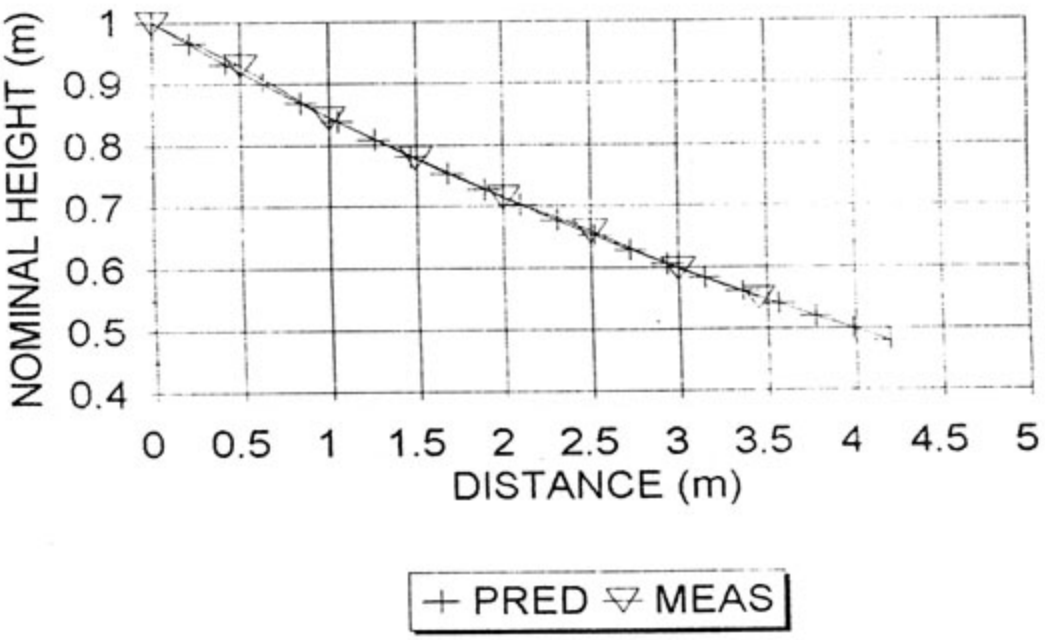


FIGURE 6.5 : MEASURED AND CALIBRATED BEACH PROFILES FOR FLUME TEST TS 19

Table 6.2 sets out the relevant scaling figures for Froude similitude in the flume, what these represent in the field and the actual field figures as calibrated in Chapter 4:

TABLE 6.2 : SCALING FACTORS FOR FROUDE SIMILITUDE

Characteristic	Scale relative to geometric scale	Scaling factor	Laboratory flume	Scaled-up field	Actual field
Geometric scale (beach length)	L	53,57	4,2 m	225 m	225 m
Time scale	$L^{1/2}$	7,32	1 s	7,32 s	1 s
Velocity scale	$L^{1/2}$	7,32	3.1 m/s close to discharge	22,69 m/s	0,35 m/s
Flow scale	$L^{5/2}$	21 004	0,0833 l/s	1 750 l/s	60 l/s
Stream power	$L^{7/2}$	1 125 190	0,512 watts	576 063 watts	5,23 watts

Characteristic	Scale relative to geometric scale	Scaling factor	Laboratory flume	Scaled-up field	Actual field
Slope	$L^{-1/2}$	0,1366	-0,1657 (initial slope)	-0,0226 (initial slope)	-0,0539 (initial slope)

Table 6.3 sets out the scaling factors for Reynolds similitude:

TABLE 6.3 : SCALING FACTORS FOR REYNOLDS SIMILITUDE

Characteristic	Scale relative to geometric scale	Scaling factor	Laboratory flume	Scaled-up field	Actual field
Geometric scale (beach length)	L	53,57	4,2 m	225 m	225 m
Time scale	L^2	10 000	1 s	10 000 s	1 s
Velocity scale	L^{-1}	0,019	3.1 m/s close to discharge	0,058 m/s	0,35 m/s
Flow scale	L	53,57	0,0833 l/s	4,46 l/s	60 l/s
Stream power	L^{-1}	0,019	0,512 watts	0,01 watts	5,23 watts
Slope	L	53,57	-0,1657 (initial slope)	-8,87 (initial slope)	-0,0539 (initial slope)

It is evident from the above tables that direct scaling in strict accordance with similitude requirements will not be representative of field conditions and that neither Reynolds nor Froude similitude criteria are being modelled by the flume tests.

It has been the contention of this study that stream power is the overall controlling parameter in determining beaching characteristics of tailings. If this were the case it would be possible to obtain a reasonable estimate of the beaching characteristics by adopting stream power as a scaling criterion over and above the geometric scaling. A methodology

is set out below.

The following equation can be used to scale up the stream power at any point along the flume beach to the field condition:

$$\left(\frac{P_p}{P_m}\right)_{for all \frac{x}{L}} = \frac{\left[-\frac{1}{\mu}\ln\left((1 - \exp^{-\mu P_0})\frac{x}{L} + \exp^{-\mu P_0}\right)\right]_p}{\left[-\frac{1}{\mu}\ln\left((1 - \exp^{-\mu P_0})\frac{x}{L} + \exp^{-\mu P_0}\right)\right]_m} = \frac{(P_0)_p}{(P_0)_m} \qquad (15)$$

The variables in equation 14 can be determined as follows:

- The parameter μ in the flume is estimated by fitting the initial slope of the profile to the equation:

$$\left(\frac{dP}{dx}\right)_m = \left[\frac{-(1 - \exp^{-\mu P_0})}{L\ \mu\ \exp^{-\mu P_0}}\right]_m \qquad (16) \qquad (16)$$

- Using this value of μ the stream power at a number of points along the beach of the flume is calculated using the equation derived in Chapter 3:

$$P_m = \left[-\frac{1}{\mu}\left(\ln\ (1 - \exp^{-\mu P_0})\frac{x}{L} + \exp^{-\mu P_0}\right)\right]_m \qquad (17)$$

- P_0 for the field condition is determined from plunge pool calculations, and the ratio $(P_0)_p/(P_0)_m$ is calculated. This ratio effectively forms the stream power scaling factor.

- The parameter μ for the field condition is determined by calibrating equation 15 against the model stream power.

- The initial slope of the field condition determined using equation 16 but applied to the field data

- These values of μ and slope are applied to the field velocity and flow rates and the applicable beach profile determined

The above approach has been applied to the results of TS 19 and the field measurements. The beach profiles in Figure 6.6 illustrate the measured beach profile for Test 4, Section 1 and the beach profile as predicted according to the above methodology.

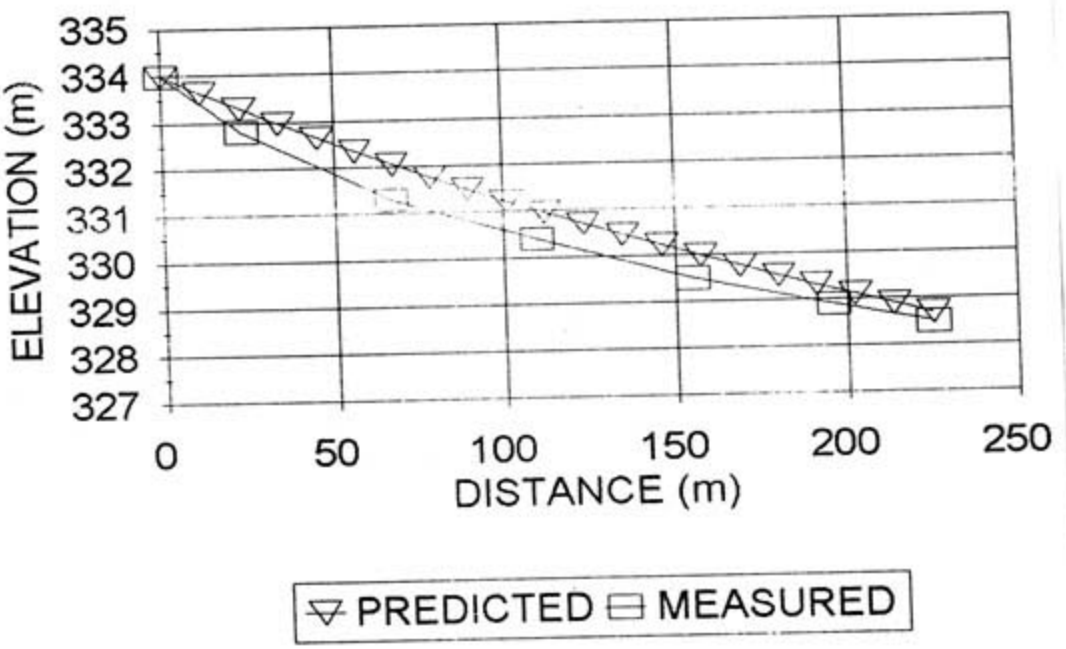


FIGURE 6.6 : MEASURED AND PREDICTED BEACH PROFILES FOR TEST 4 , SECTION 1 BASED ON FLUME DATA

It is evident from Figure 6.6 that the initial slope has been under-estimated in the above process although the overall slope of the beach is predicted reasonably well. In assessing the results of the prediction a number of points need to be borne in mind:

- discharge conditions in the flume were set up specifically to avoid a plunge pool. This is unrepresentative of the field condition and could effect a difference in initial slope

- the tailings sample was not fully representative of the field tailings since the sample was recovered on the beach and comprised segregated tailings. The differences were evidenced in particle size distribution determinations and were noted by Küpper as affecting the d_{10} size which was smaller in the field This difference is accentuated for Test 4 since the spigots were located at the top of the delivery pipe. The differences in d_{10} size would affect the frictional characteristics of the slurry that are of primary importance in determining the initial slope of the beach.

6.4 THOMSON FLUME TESTS

There is, unfortunately, insufficient data provided in the documentation of the flume tests by Thomson (1984) to facilitate runs on the entropy-stream power model.

6.5 FLUME TESTS ON BRAZILIAN GOLD TAILINGS

In March 1992 this author conducted pilot scale flume tests in Brazil in the flume indicted in Figure 6.7. The flume was 18 m long and approximately 1 m wide. Flow was diverted from the tailings delivery channel to the flume and regulated using a valve. Since the feed to the tests comprised mill feed in a production situation it was possible to vary only the flow rate in a controlled manner, and not the concentration. The tests were conducted in such a way that a pond was formed at the decant end to simulate the pond in the field. The tests were run until equilibrium of deposition was achieved. Beached profiles were surveyed on the day following a deposition. The flow from the distribution box was measured and concentrations were determined for each outlet from the box. Velocities down the beach were estimated at regular intervals along the beach by timing the movement of matches floating on the surface of the flow.

A number of trial tests were carried out to establish the control mechanisms and iron out problems with the equipment whereupon two tests were carried out and carefully documented. The results of the measurements for the two tests are included in Appendix A.

The tailings comprise milled phyllite which is of soft rock consistency and is mined without blasting or crushing. The tailings slurry has a "soapy" feel which is in line with the shale-

like nature of the rock. It was therefore deduced that the frictional characteristics of the slurry would be representative of the lower bound of hard rock milled products. The particle size distribution of the tailings is indicated in Figure 6.8 and the field beach profile in Figure 6.9.



FIGURE 6.7 : PHOTOGRAPHS OF THE PILOT-SCALE FLUME

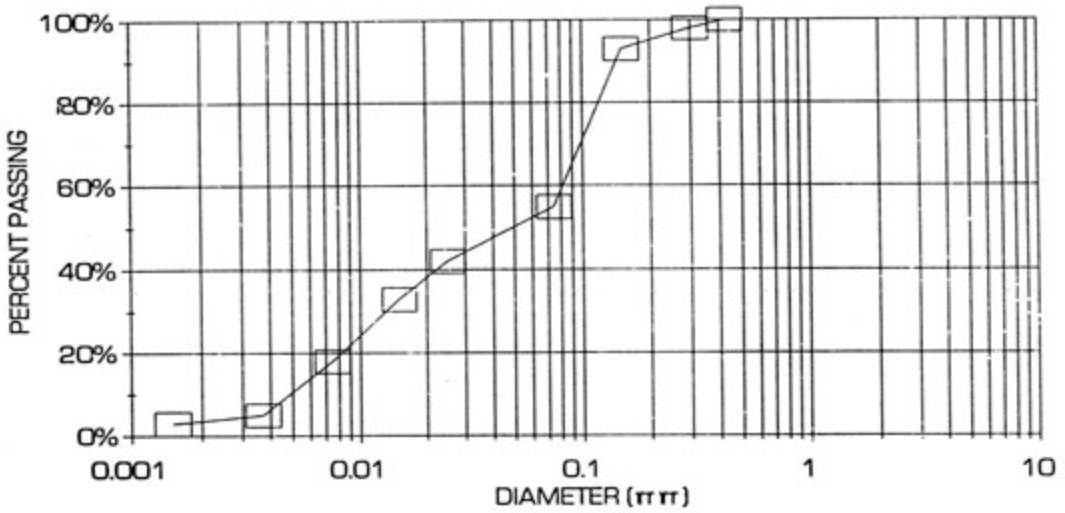


FIGURE 6.8 : PARTICLE SIZE DISTRIBUTION OF BRAZILIAN GOLD TAILINGS PRODUCT

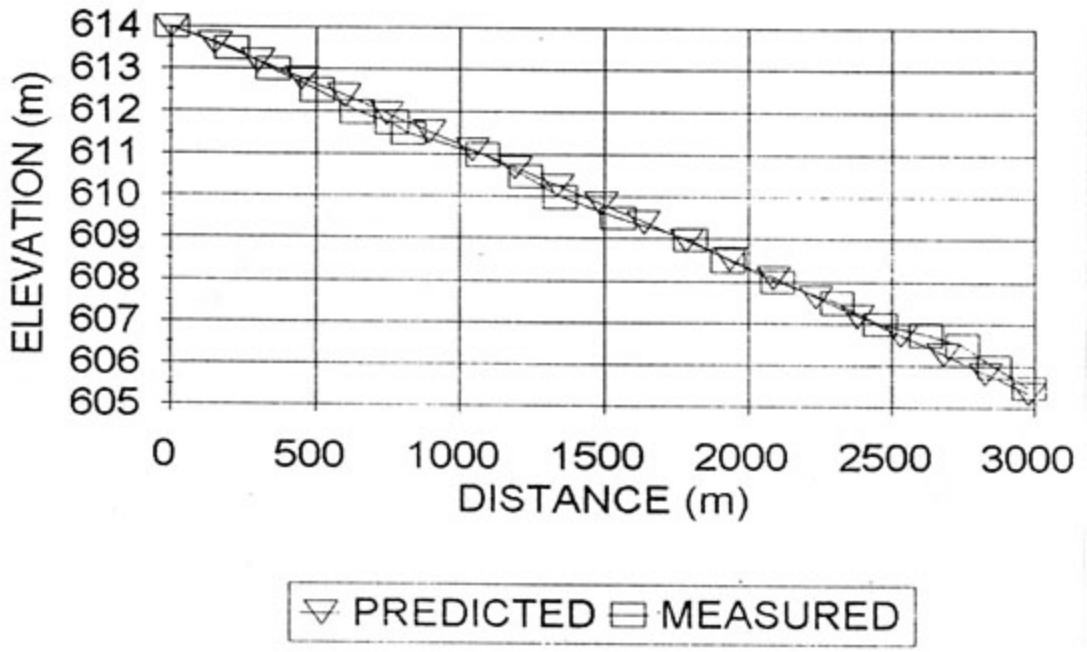


FIGURE 6.9 : MEASURED BEACH PROFILE ON THE TAILINGS DAM

The measured beach profiles for flow rates of 2,6 l/s and 0,44 l/s are indicated in Figures 6.10 and 6.11.

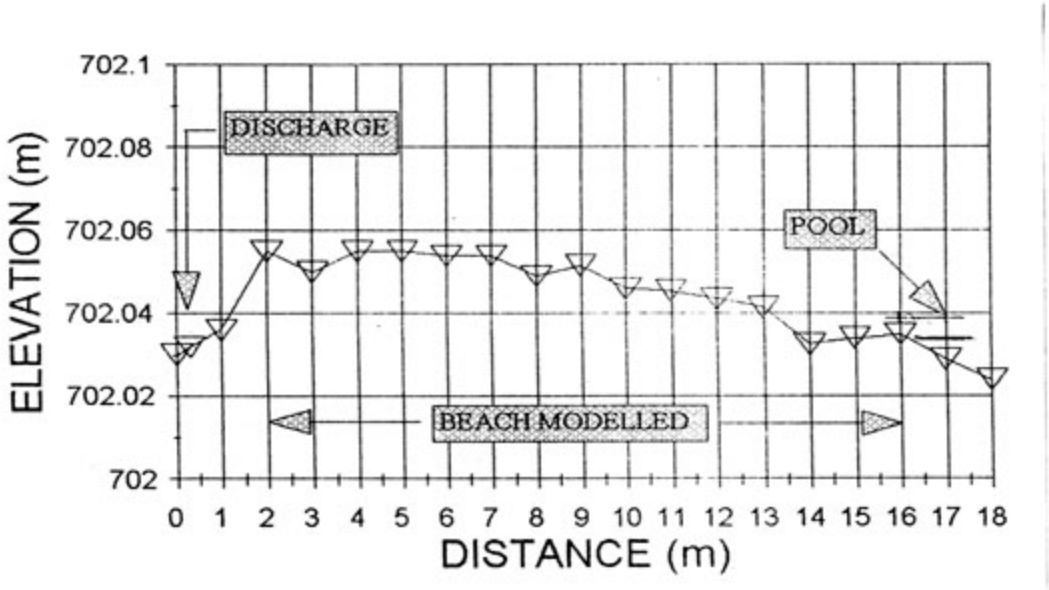


FIGURE 6.10 : MEASURED BEACH PROFILE FOR FLOW RATE OF 2,6 l/s

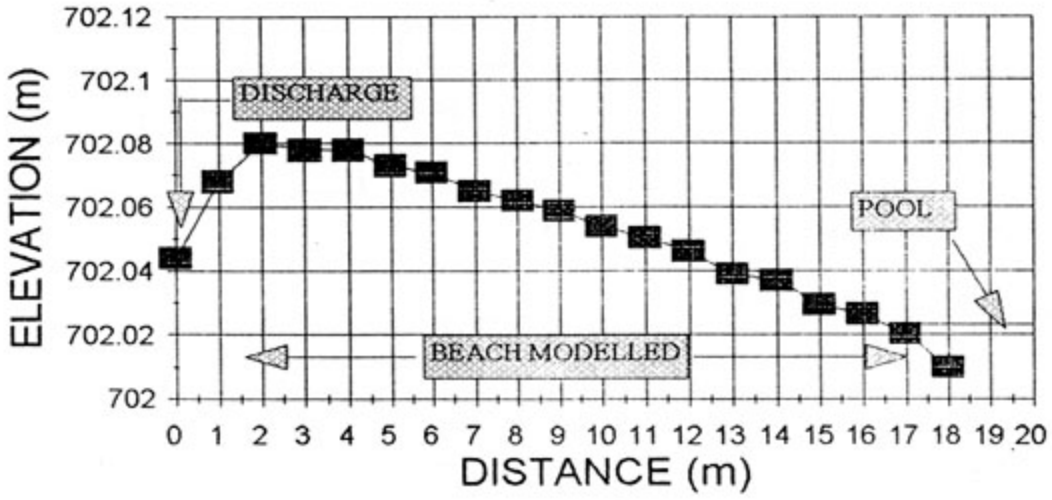


FIGURE 6.11 : MEASURED BEACH PROFILE FOR FLOW OF 0,44 l/s

It was observed that the flow on the tailings dam is predominantly laminar. Flow conditions in the flume were therefore selected to ensure that the laminar flow characteristic was modelled. In this case, therefore, Reynolds similitude scales would be applicable. The relevant scales for the two tests emerged as set out in Tables 6.4 and 6.5:

TABLE 6.4 : SCALING FACTORS FOR 2,6 l/s FLOW RATE

Characteristic	Scale relative to geometric scale	Scaling factor	Laboratory flume	Scaled-up field	Actual field
Geometric scale (beach length)	L	200 m	15 m	3 000 m	3 000 m
Time scale	L^2	40 000	1 s	40 000 s	1 s
Velocity scale	L^{-1}	0,005	0,12 m/s	6.10^{-4} m/s	0,13 m/s
Flow scale	L	200	2,6 l/s	520 l/s	725 l/s
Stream power	L^{-1}	0,005	0,0266 watts	$1,3.10^{-4}$ watts	8.67 watts
Slope	L	200	-0,0009 (initial slope)	-14,2 (initial slope)	-0,0028 (initial slope)

Table 6.5 sets out the scaling factors for the second flume test in which the flow rate is 0,44 l/s:

TABLE 6.5 : SCALING FACTORS FOR 0,44 l/s FLOW RATE

Characteristic	Scale relative to geometric scale	Scaling factor	Laboratory flume	Scaled-up field	Actual field
Geometric scale (beach length)	L	200	15 m	3000 m	3000 m
Time scale	L^2	40 000	1 s	40 000 s	1 s
Velocity scale	L^{-1}	0,005	0,014 m/s	7.10^{-5} m/s	0,13 m/s
Flow scale	L	200	0,44 l/s	88 l/s	725 l/s
Stream power	L^{-1}	0,005	0,00006 watts	3.10^{-7} watts	8,667 watts

Slope	L	200	-0,001 (initial slope)	-0,2 (initial slope)	-0,0028 (initial slope)

The calibrated flume beach profiles are indicated in Figures 6.12 and 6.13:

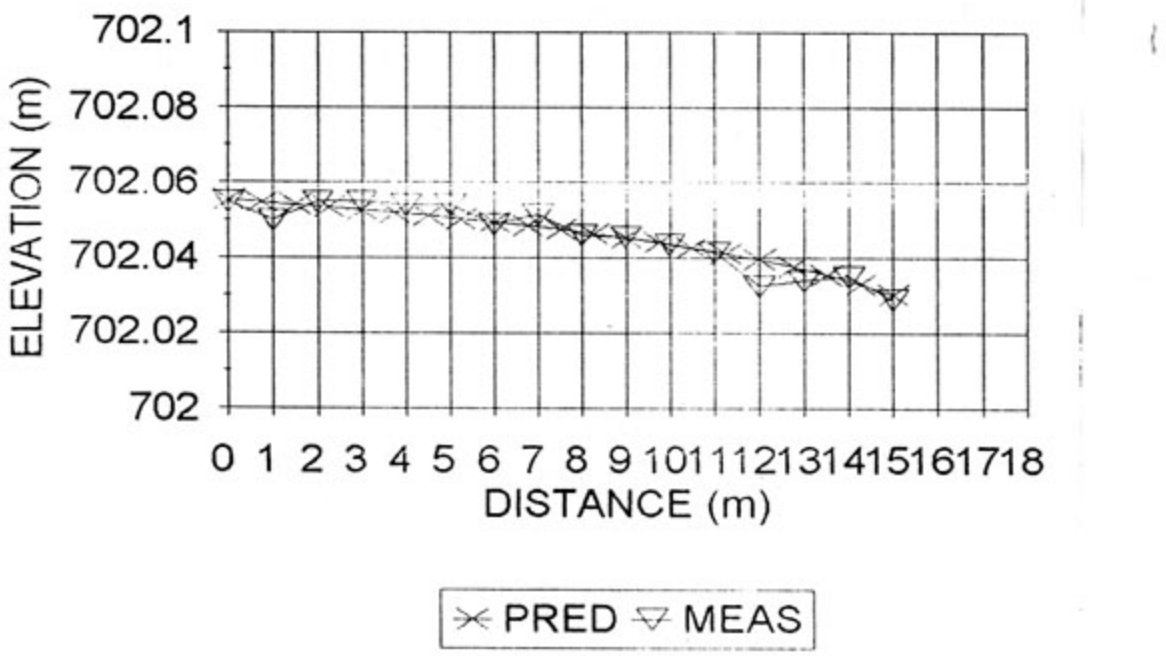


FIGURE 6.12 : CALIBRATED BEACH PROFILE FOR FLUME TEST AT FLOW OF 2,6 l/s

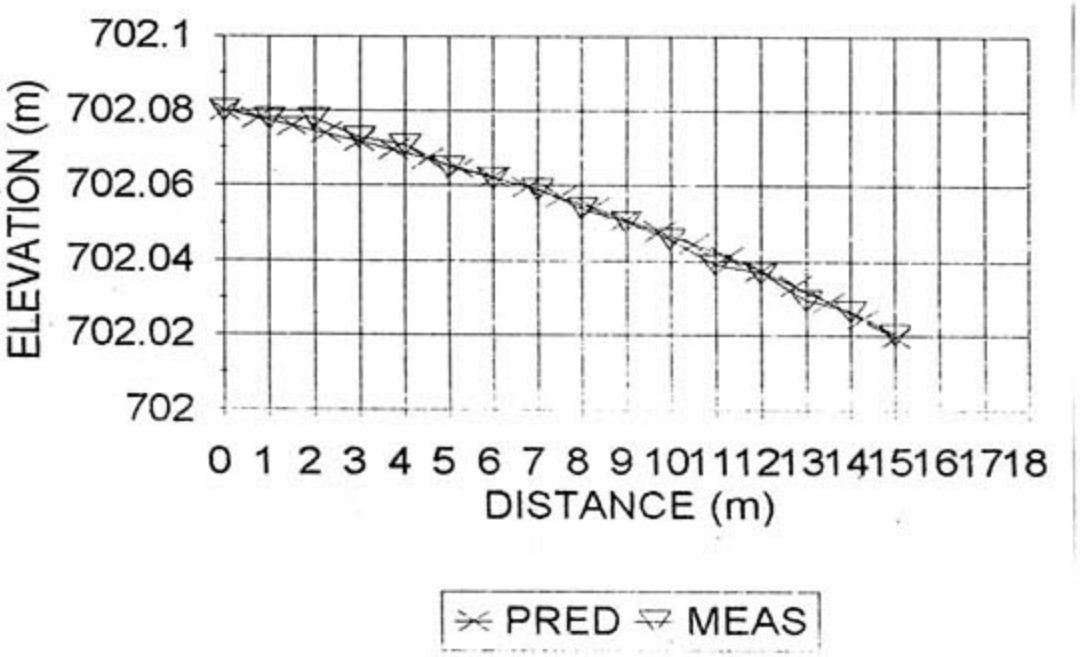


FIGURE 6.13 : CALIBRATED BEACH PROFILE FOR FLUME TEST AT FLOW OF 0,44 l/s

While the above calibrations are relatively accurate it is important to note that they were only achieved on the assumption of higher velocities at the discharge point than measured. In the case of the 2,6 l/s test the velocity calibrated was 0,13 m/s while that measured was 0,065 m/s. In the case of the 0,44 l/s test the discrepancy was not only greater but the calibrated velocity is implausibly high at 0,43 m/s as opposed to a measured 0,014 m/s. These differences would seem to indicate that stream power is not the controlling factor in the flume tests and that viscous forces, as opposed to the inertial forces intrinsic in the stream power approach, are dominant.

Notwithstanding the above the methodology outlined in Section 6.3 was applied to the results for the two flume tests and the profiles in Figures 6.14 and 6.15 were determined:

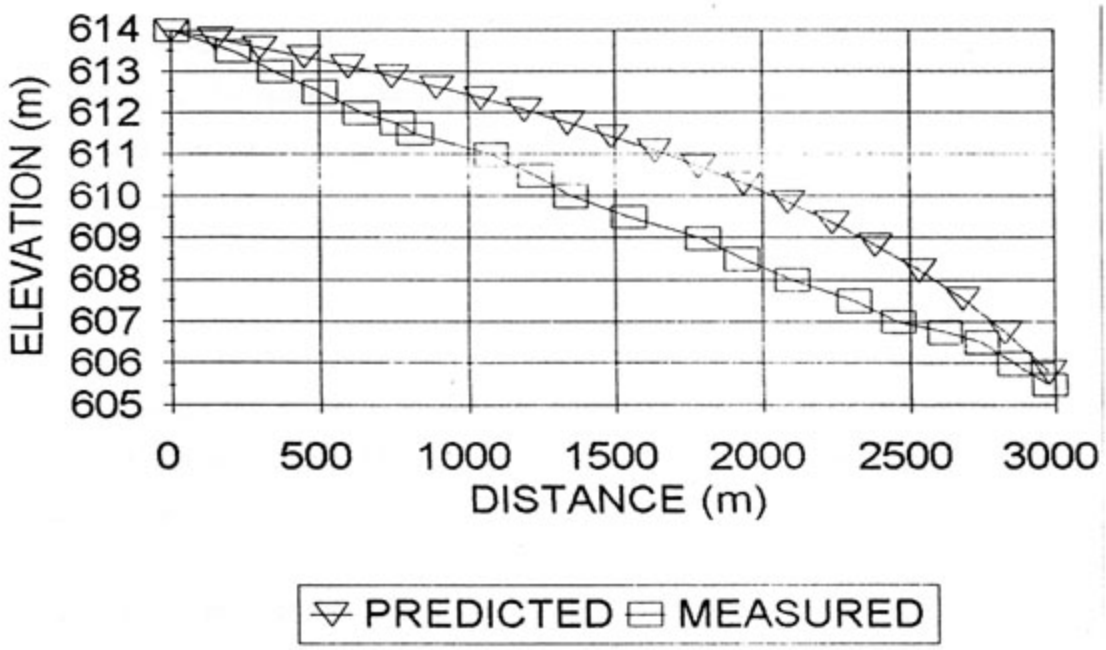


FIGURE 6.14 : PREDICTED BEACH PROFILE USING DATA FROM THE FLUME TEST FOR A FLOW RATE OF 2,6 l/s

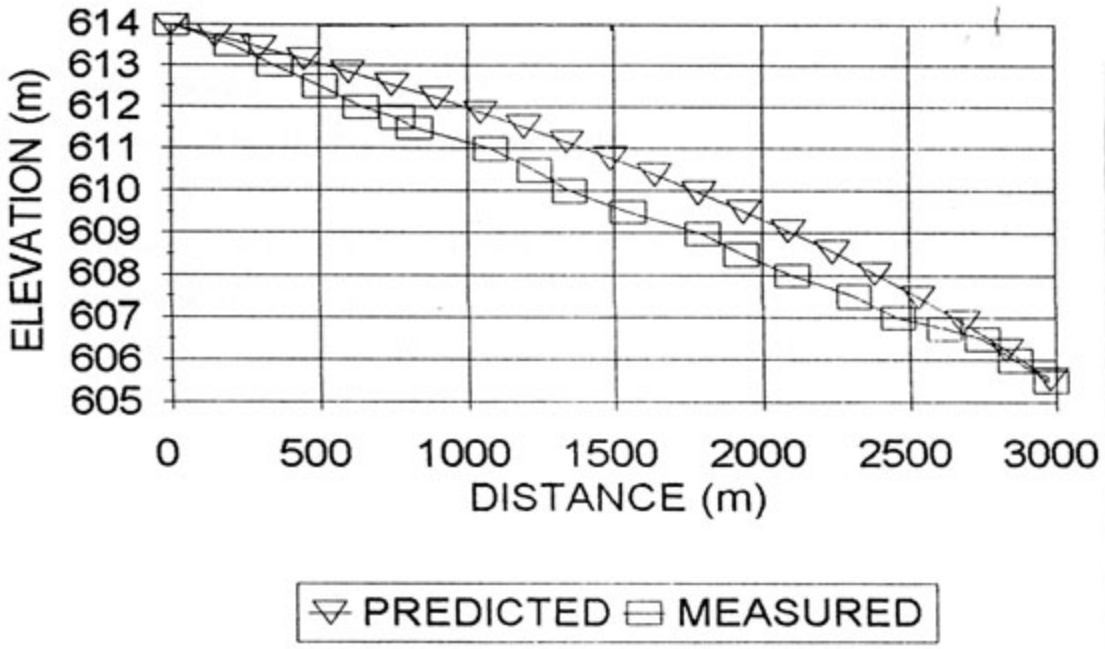


FIGURE 6.15 : PREDICTED BEACH PROFILE USING DATA FROM THE FLUME TEST FOR A FLOW RATE OF 0,44 l/s

The following points should be noted in regard to the above tests:

- It was observed in the 2,6 l/s test that the maximum velocity in the flow profile did not occur at the surface where the velocity measurements were being measured. This was evidenced by graphite particles characteristic of the ore which were suspended 3 to 5 mm below surface and were observed as moving faster than the surface.

- In the course of the 0,44 l/s test a problem occurred at the pond end soon after beginning the test The slats forming the end leaked and some water and fines were lost. The leak was quickly stemmed and the test results were taken as being acceptable.

6.6 FOURIE FLUME TESTS

There was no reference to any field beach measurements for the test work by Fourie (1988) and therefore this data would not be adequate for the flume-field comparisons required.

6.7 DISCUSSION

The flume tests evaluated in this Chapter have been set up according to practical constrains of space and cost and fall far short of meeting formal scaling criteria. Tables 6.6 and 6.7 below set out the flow rates, velocities and stream power requirements for meeting Froude criteria in Küpper's test TS 19 and in meeting the Reynolds criteria in the Brazilian gold tailings testing respectively for a range of scales some of which meet practical constraints of velocity and flow but are not of practical geometric scale and vice versa.

TABLE 6.6 : IDEALISED SCALING FACTORS FOR KÜPPER'S TEST TS 19

Characteristic	Scale relative to geometric scale	Actual field	Alternative Scale 1:10	Alternative Scale 1:25	Alternative Scale 1:33,33
Geometric scale (beach length)	L	225 m	22,5 m	9 m	6,75 m
Time scale	$L^{1/2}$	1 s	0,316 s	0,2 s	0,17 s
Velocity scale	$L^{1/2}$	0,35	0,111 m/s	0,07 m/s	0,061 m/s
Flow scale	$L^{5/2}$	60 l/s	0,19 l/s	0,019 /s	0,0094 l/s
Stream power	$L^{7/2}$	5.23 watts	0,0017 watts	$6{,}69.10^{-5}$ watts	$2{,}44.10^{-7}$ watts

TABLE 6.7 : IDEALISED SCALING FACTORS FOR THE BRAZILIAN FLUME TESTS

Characteristic	Scale relative to geometric scale	Actual Field	Alternative Scale 1:10	Alternative Scale 1:30	Alterative Scale 1:100
Geometric scale (beach length)	L	3 000 m	300 m	100 m	30 m
Time scale	L^2	1 s	0,01 s	0,0011 s	0,0001 s
Velocity scale	L^{-1}	0,13 m/s	1,3 m/s	3,9 m/s	13 m/s
Flow scale	L	725 l/s	72,5 l/s	24,17 l/s	7,25 l/s
Stream power	L^{-1}	8,667 watts	86,67 watts	260 watts	866,7 watts

The slope relation derived from the unit stream power criterion as proposed by Yang (1980) has been eliminated from the above tables as it proved ineffective in Tables 6.2 to 6.5. It is apparent from Tables 6.6 and 6.7 that for Froude modelling the largest possible scale should be chosen. This implies pilot scale testing. Conversely, for Reynolds

modelling a small scale should be chosen in order to keep velocities and flow rates within manageable bounds.

This author therefore concurs with the Küpper (1991) who concluded that the difficulties associated with formal scale modelling of the formation of fill by slurry deposition in which all similitude requirements are met make it necessary to regard flume tests as small scale systems and not scale models of prototypes and to evaluate the results of flume tests on an informal criterion of "similarity of process". To achieve this the basic requirements of gross scaling relationships and reproduction of morphological characteristics need only be met. This has been the case both with the flume testing documented by Küpper and those conducted by this author. In view of the fact that formal modelling criteria are not met direct extrapolation of the results from the flume test to the laboratory is precluded although it is possible to draw qualitative conclusions.

Two aspects in this regard emerged from the evaluation of the test results:

- application of a stream power scaling up procedure can facilitate reasonable estimates of the overall (average) slope of the beach. This procedure was set out in Section 6.3.

- an indication of the dependency of the initial slope of the beach and therefore of the frictional resistance of the slurry can be obtained from a flume testing programme. Figure 6.16 below is a plot of the data presented by Küpper for tests on the tailings product and indicates that beyond a concentration of 20 % by mass the frictional characteristics are determined primarily by concentration; flow rate and velocity diminish in influence beyond this point for this particular tailings. Moreover, extrapolation of the points in Figure 6.16 suggests that over the range of field concentrations (35 % to 65 %) the initial slope may be relatively insensitive to concentration.

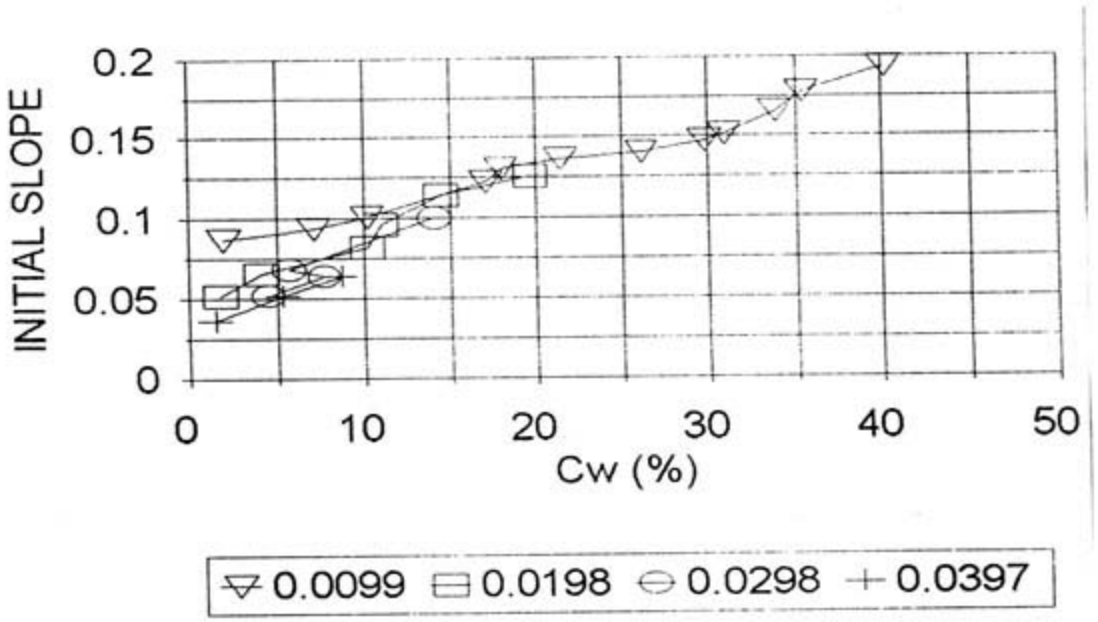


FIGURE 6.16 : CONCENTRATION VS INITIAL SLOPE FOR THE RANGE OF TS TESTS DOCUMENTED BY KÜPPER (1991)

Reliable estimates of the frictional resistance of the slurry, an important calibrating variable for the entropy-based stream power method cannot practically be obtained from flume testing. At best the testing will serve to confirm frictional resistance predictions from viscosity testing and the methods of Wasp et al (1977) and Verkerk(1990). The application of these methods will be illustrated in the next chapter which documents application of the beaching model to field and laboratory measurements made by this author on actual operations.

CHAPTER 7

CONTENTS

Sec	Description	Page
7	APPLICATION OF ENTROPY TO FIELD MEASUREMENTS	7-1
	7.1 INTRODUCTION	7-1
	7.2 URANIUM TAILINGS	7-1
	7.2.1 Rössing ST Paddock	7-4
	7.2.2 Rössing Old Beach	7-11
	7.3 GOLD TAILINGS	7-15
	7.4 COPPER TAILINGS	7-19
	7.5 MAGNETITE TAILINGS	7-27
	7.6 DISCUSSION	7-32

CHAPTER 7 APPLICATION OF ENTROPY TO FIELD MEASUREMENTS

7.1 INTRODUCTION

In Chapter 4, following development of the entropy-based stream power equations and the spreadsheet model, a number of runs were conducted using available data in the literature. This data contained no details of:

- velocity measurements in and around the plunge pool
- rheology

It was therefore possible only to check that the approach was able to make predictions of the beach profile and trends in gravitational segregation.

This Chapter documents field and laboratory testing conducted by the author and application of the model to actual operations to check on the predictive capabilities of the approach. Where appropriate data is available for other operations in project files this data has been incorporated The operations cover a range of grinds from coarse and a range of particle specific gravities.

7.2 URANIUM TAILINGS

Measurements were conducted on Rössing tailings dam in Namibia where uranium is extracted from granitic rock which is milled to a coarse grind. The tailings product is a re-combination of sands and slimes which have been separated in the reduction process. The sands make up some 80 % of the tailings and the slimes the remaining 20 % . Figure 7.1 indicates the particle size distributions of the two tailings. Superimposed onto Figure 7.1 is the particle size distribution of the combination. The top size of the sands is 10 mm which would represent an upper-bound of the range of tailings products usually found in Southern Africa.

The tailings is pumped through an elevation of 55 m and a distance of some 7 km to the tailings dam which has, in recent years, been modified to operate as a system of paddocks each approximately 40 ha in area. Within these paddocks beach lengths vary form 50 m to 400 m. Prior to this modification the dam was operated as a single entity with beach lengths of up to

2,5 km. Figure 7.2 is a photograph of the dam as at July 1993.

Measurements have been taken as follows:

- within paddock ST where the beach is some 300 m long. The beach profile has been measured immediately following a deposition, samples have been recovered and deposition has been recommenced for a limited period deposition to facilitate measurement of velocities and observation of depositional behaviour. The samples have been tested to determine the particle size distribution and particle specific gravity.

- along an old beach which was deposited some years previously but for which estimates of tonnage, slurry concentration and flow characteristics are available. This beach is some 1200 m long and represents an excellent opportunity to sample and survey levels over the entire length of the beach from discharge to within the pool area - something which is seldom possible on an operating dam of this size.

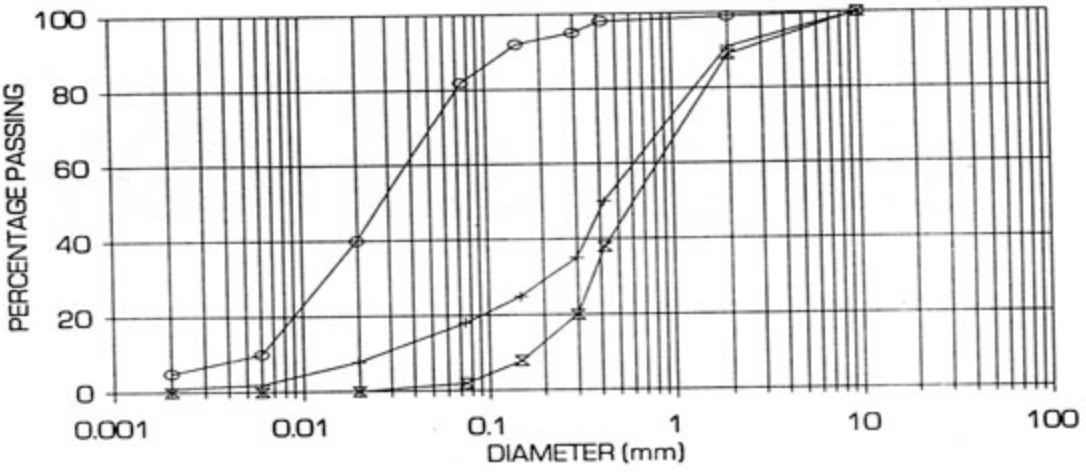


FIGURE 7.1 : PARTICLE SIZE DISTRIBUTION OF SANDS, SLIMES AND TOTAL PRODUCT AT RÖSSING URANIUM



FIGURE 7.2 : AERIAL VIEW OF THE RÖSSING TAILINGS DAM

Rheological data was available from studies carried out by the University of Cape Town in 1985 through to 1987. The data has been reviewed and updated recently by Dr N Verkerk in terms of a consulting brief to Rössing (Rössing Uranium Mine (1993)). This means that comprehensive data on which to base the beaching predictions is available.

7.2.1 Rössing ST Paddock

At the time of the testing on the ST paddock two out of four mills were operating. Since the pumping system is designed to handle four mill operation some instability of the delivery stream was experienced. This instability was compounded by the fact that the ST paddock is the further-most paddock from the plant - a distance of some 7 km. Over this distance the tailings takes approximately 25 to 30 minutes to reach the discharge point from the primary pumping station.

The instability of the slurry stream manifested itself as surges in flow rate and in segregation of the sands in the pipeline and resulted in alternating slugs of very coarse material comprising almost entirely sands followed by periods during which only slimes were delivered. During periods of coarse slug deposition velocities would reduce drastically and the plunge pool would build vigorously. Immediately the lower density slimes slug discharged the plunge pool would be cut through and the beach would be eroded. Velocity measurements were taken by observing the time taken for the beach stream to flow between stakes placed on the beach in the immediate vicinity of the plunge pool. During periods of coarse slug deposition the velocity ex-plunge pool would reduce to 0,5 to 0,7 m/s and during periods of slimes deposition the velocity would increase to up to 2.2 m/s. The average velocity of the stream leaving the plunge pool over the duration of the test was 1,5 m/s.

Figure 7.3 summarises the rheological data for the tailings for a range of pipe diameters form 192 mm to 432 mm. It is evident from Figure 7.3 that the friction gradients are relatively insensitive to pipe diameter. The data covers a range of concentrations and blends of sands and slimes.

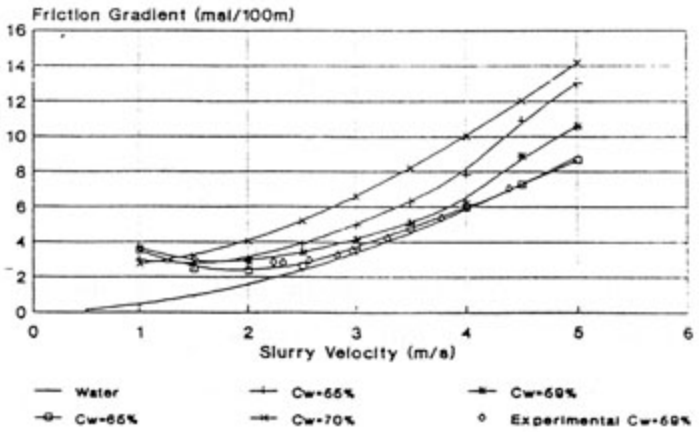


Figure 10 *194mm Friction Gradients*

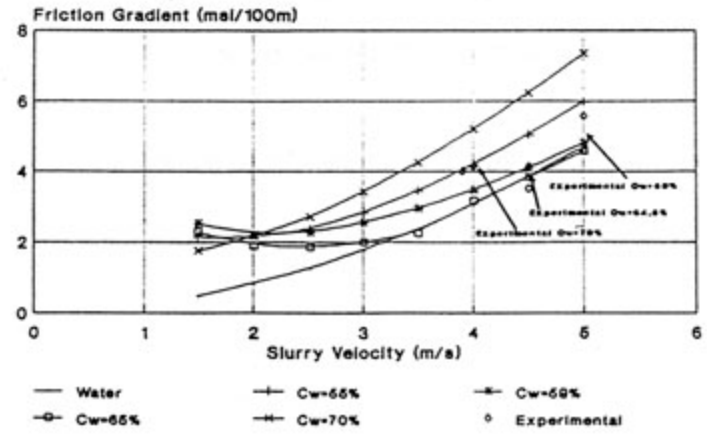


Figure 11 *332mm Friction Gradients*

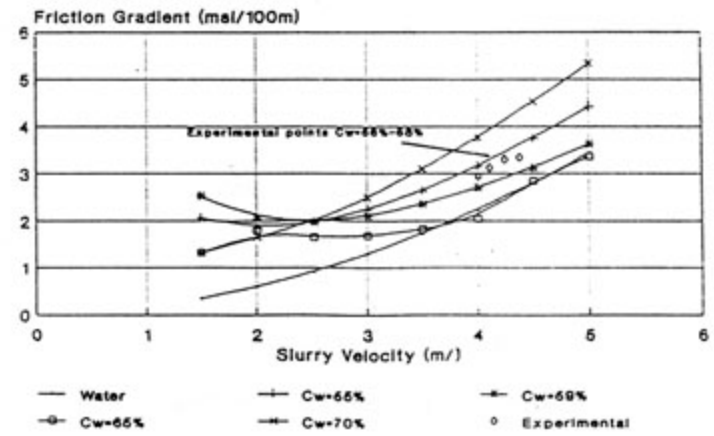


Figure 12 *432mm Friction Gradients*

FIGURE 7.3 : HEAD LOSS DETERMINATIONS FROM RHEOLOGY (after Rössing Uranium Limited (1993))

The critical velocity at the plunge pool calculates as 1,51 m/s and the critical flow depth as 0,33 m. This is in line with observations at the plunge pool and lends credence to the equations proposed in Chapter 4.

The hydraulic radius for a semi-circular flow channel of depth 0,33 m is 0,165 m. If this is taken as the equivalent pipe diameter the friction gradients may be estimated by extrapolating the curves in Figure 7.3. For the concentration recorded by Rössing's Central Process Control on a continuous basis over the duration of the test (50 % solids on average and a total tonnage rate of 1035 tph) the minimum head loss is approximately 0,02 m of slurry per metre of flow length at between 1,8 and 2,2 m/s. This head loss in metres of slurry per metre of flow length increases to 0.04 at 1 m/s. Extrapolation of the curves suggests that the head loss could increase to 0,07 metres of slurry per metre of flow length at 0,2 m/s.

The following parameters emerged from calibration against the measured results:

Tonnes/hr	1035	
Flow rate	0,26	m^3/s
Pipe velocity	1,6	m/s
Concentration (mass)	65	%
Slurry Density	1,69	t/m^3
Length of beach	300	m
Height of pipe	1	m
Angle of pipe	10	degrees upwards
Plunge pool velocity	0,14	m/s
Initial slope (friction gradient)	-0.03	m_{water}/m
Stream power	4,51	watts

The velocity that achieves a reasonable approximation of the beach profile yields a velocity of 0,14 m/s. This corresponds with a delivery pipe height of 1 m which is approximately that observed on site as shown in Figure 7.4. The measured and predicted beach profiles are shown in Figure 7.5. Measured particle size distributions are indicated in Figure 7.6 and the predicted distributions in Figure 7.7.

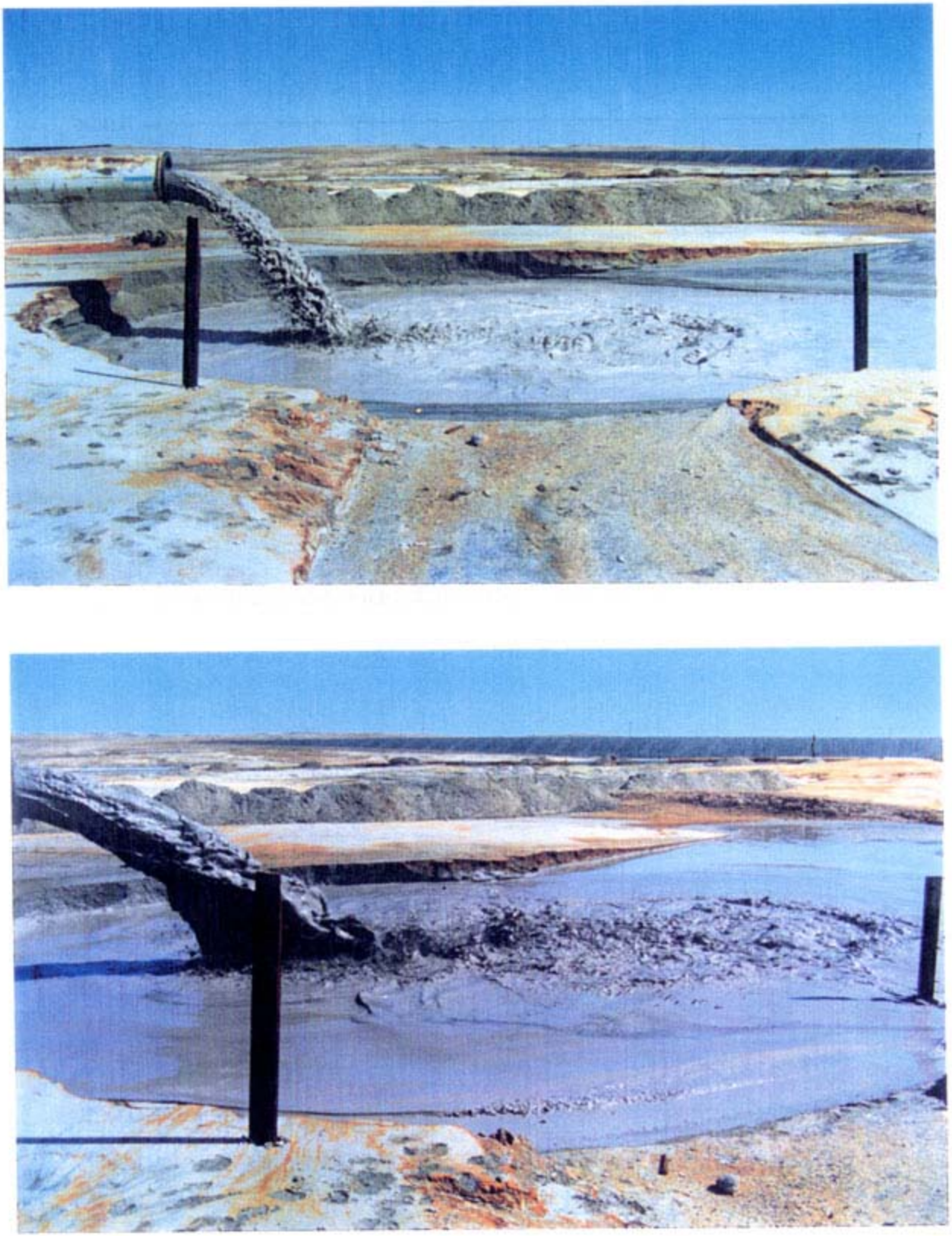


FIGURE 7.4 : PHOTOGRAPHS SHOWING THE DISCHARGE OF TAILINGS DURING THE TEST ON PADDOCK ST

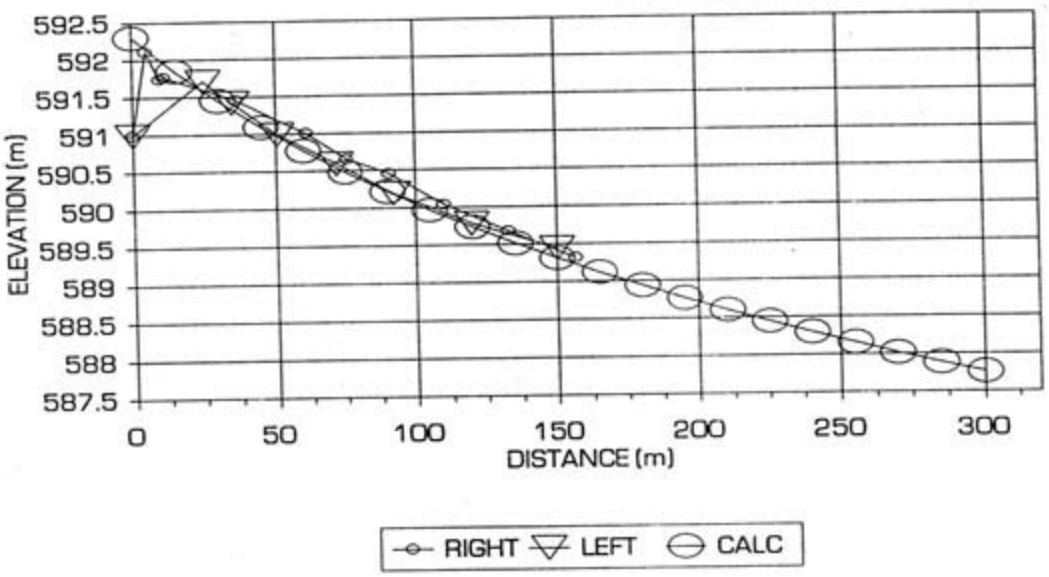


FIGURE 7.5 : MEASURED AND PREDICTED BEACH PROFILES

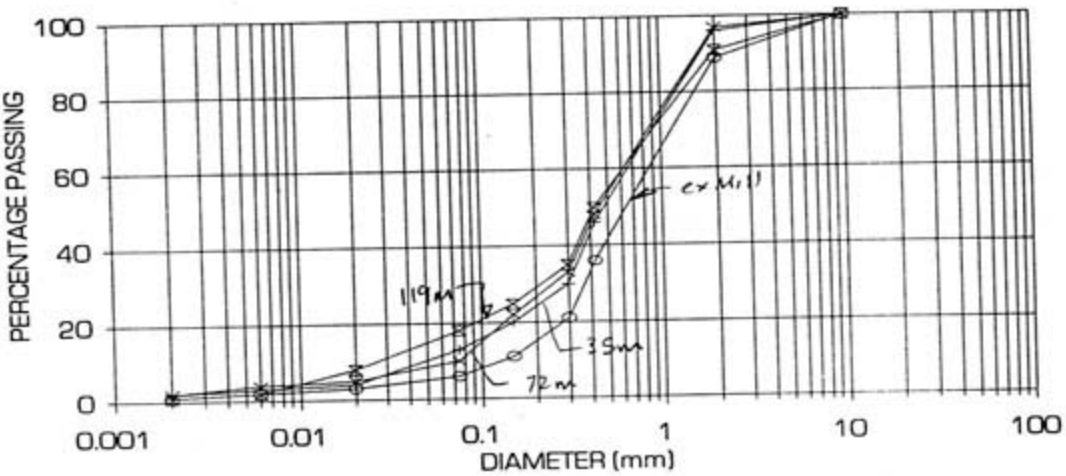


FIGURE 7.6: MEASURED PARTICLE SIZE DISTRIBUTIONS

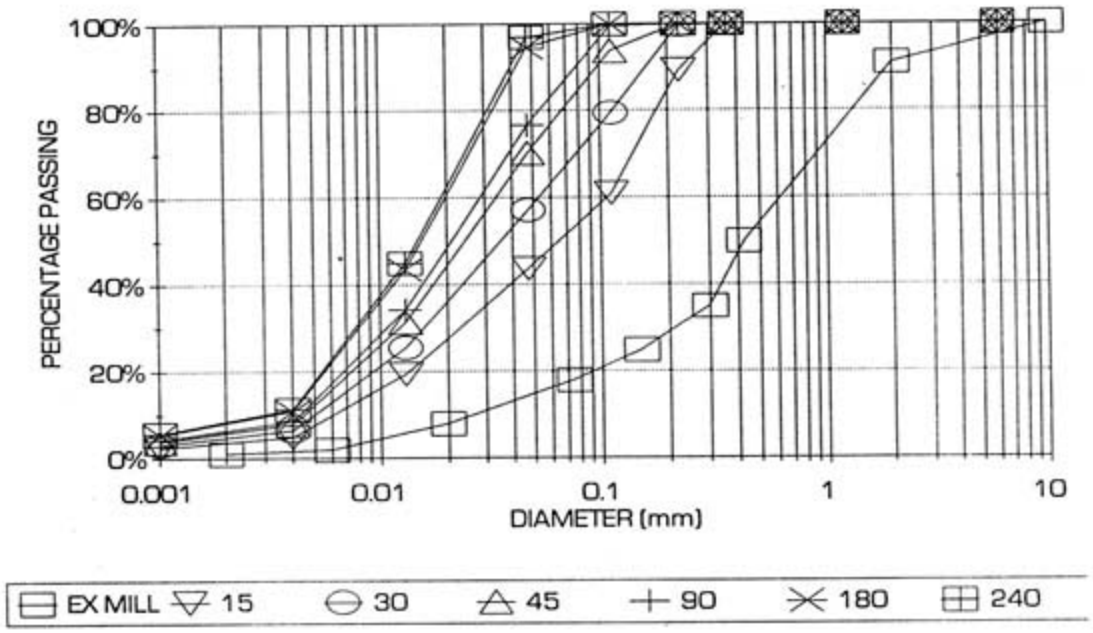


FIGURE 7.7 : PREDICTED PARTICLE SIZE DISTRIBUTIONS

The following points are noteworthy:

- the predicted beach profile for the ST beach has been calculated with an initial slope of 0,02 m of slurry per m (approximately 0,03 m water /m). The initial beach slope in Figure 7.3. agrees reasonably with this estimate. This indicates that the method of determination of the initial slope using the approach of Verkerk (1990) modified for beaching slurry is by substitution of the hydraulic radius for diameter is reasonable.

- the measured particle size distributions indicate negligible segregation down the beach over a distance of some 140 m. This is unexpected and may be attributable to the surging and slugs of coarse and fine delivery

- the predicted particle size distributions indicate marked segregation should be expected. However, segregation appears too severe to be regarded as entirely plausible.

The following points were noted during the test:

- accelerated deposition on the beach occurs when the flow stream from the plunge pool meanders. This occurs when deposition at the pool-end of the stream causes the flow stream to change flow direction and meander so as to maintain the profile dictated by the stream power. As the stream meanders there is rapid build-up on the inside edge of the meander where velocities are the lowest. This is in line with river morphology.

- deposition in the immediate vicinity of the plunge pool only occurs once the outflow stream has been forced to meander by changing conditions downstream.

On the basis of the above it is postulated that there are two deposition mechanisms at work:

- that of the main stream at the pool end of the beach. The main stream carries the majority of the solids to the pool-end by virtue of its higher velocity and the consequent lower tendency to deposit

- that of the main stream as it meanders. The inside edge of the meander moves at a considerably lower velocity than the main stream and deposits heavily as a consequence

The flow section of the stream is constantly changing. Indeed with distance down the beach the stream begins to split, then to braid and ultimately becomes sheet flow close to the pool.

Figure 7.8 shows the sensitivity of the plunge pool velocity to the height of the delivery pipe above the plunge pool. As this height decreases there is less opportunity for the vertical component of the discharge velocity to develop. It is this component that is responsible for the development of the plunge pool. It is of interest to note that at pipe heights of 1,5 m there is little change in value of plunge pool velocity, and by 2 m the velocity is essentially constant. It should be noted, however, that the stream power is exceedingly sensitive to the value of this velocity and even the small differentials noted are can significantly impact on the reliability of the predictions.

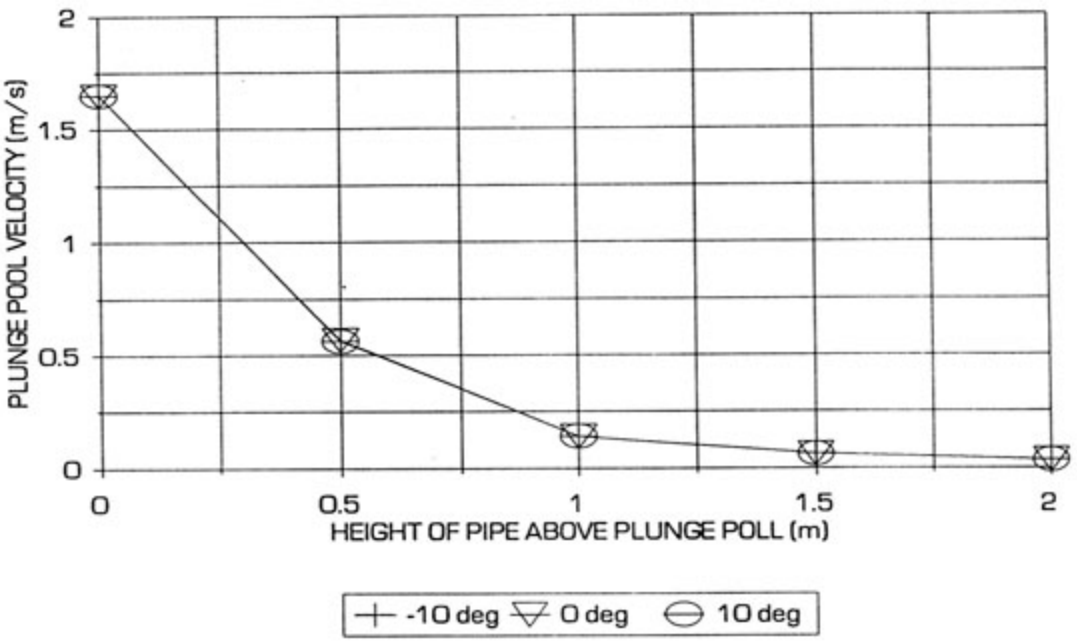


FIGURE 7.8 : VARIATION OF PLUNGE POOL VELOCITY WITH HEIGHT OF DISCHARGE PIPE

7.2.2 Rössing Old Beach

At the time that deposition on the old beach at Rössing took place the tailings production rates were considerably higher than for the ST beach since four mill production was in progress. In addition, to reduce problems with of silting up of the delivery pipes, very high velocities were maintained through the addition of water. Slurry concentration ratios were lower than for the ST test. The calibrated beaching data is as follows:

Tonnes/hr	1 700	
Flow rate	0,57	m^3/s
Pipe velocity	3,56	m/s
Concentration (mass)	55	%
Slurry Density	1,53	t/m^3
Length of beach	1 000	m
Height of pipe	1,25	m
Angle of pipe	10	degrees upwards

Plunge pool velocity	0,23 m/s
Initial slope (friction gradient)	-0,08 m_{water} per m of beach
Stream power	23,49 watts

The calibrated beach profiles are indicated in Figure 7.9, the measured particle size distributions in Figure 7.10 and the predicted distributions in Figure 7.11.

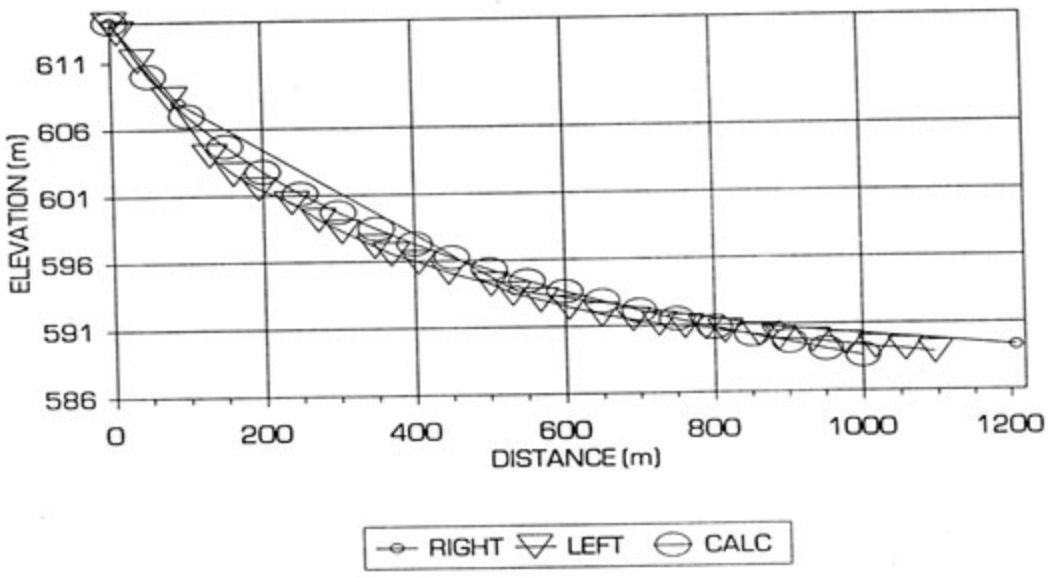


FIGURE 7.9 : PREDICTED BEACH PROFILES FOR RÖSSING OLD BEACH

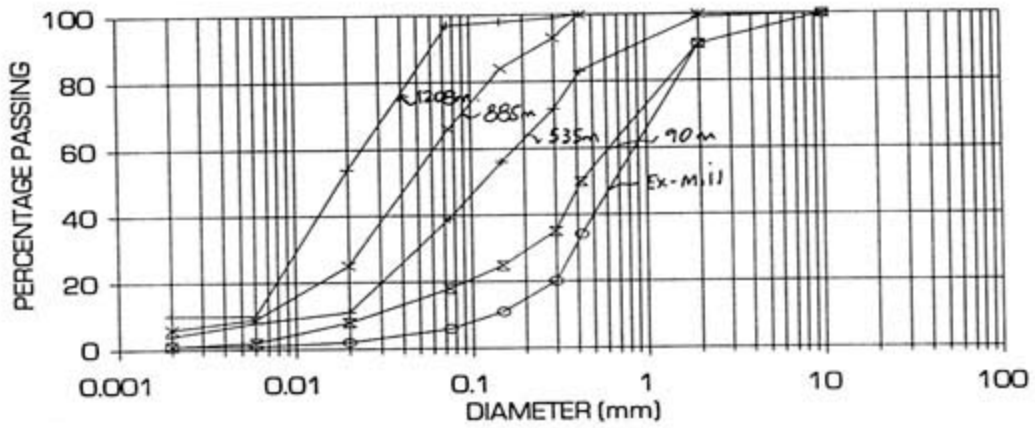


FIGURE 7.10 : MEASURED PARTICLE SIZE DISTRIBUTIONS FOR RÖSSING OLD BEACH

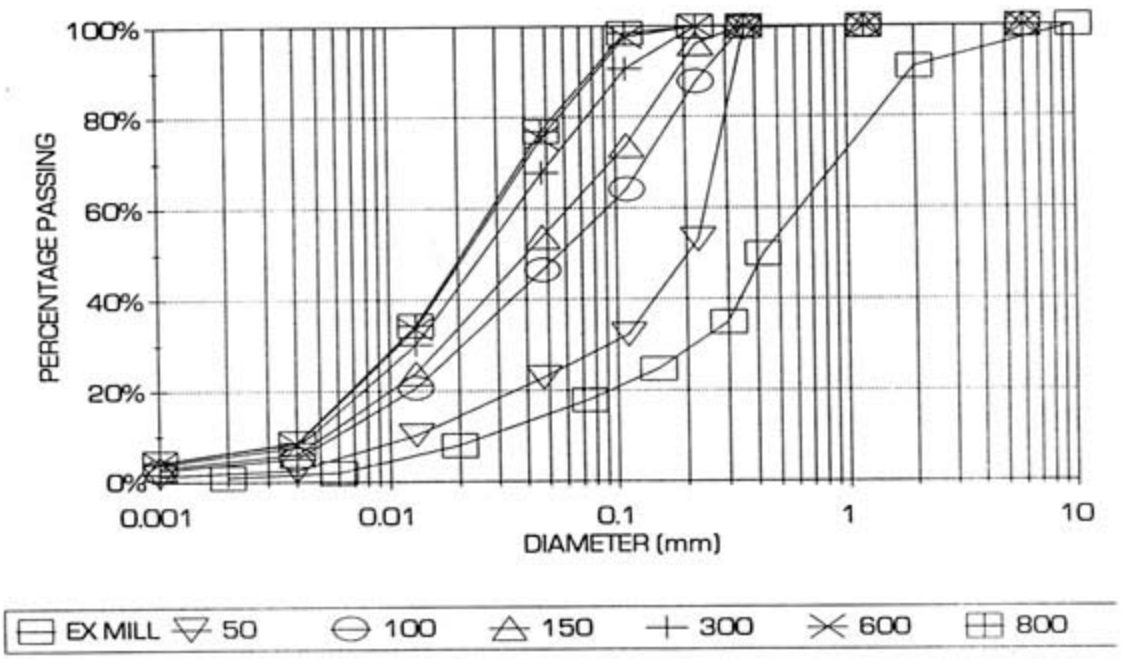


FIGURE 7.11 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR RÖSSING OLD BEACH

The graph of plunge pool velocity versus height of pipe above the plunge pool is indicated in Figure 7.12

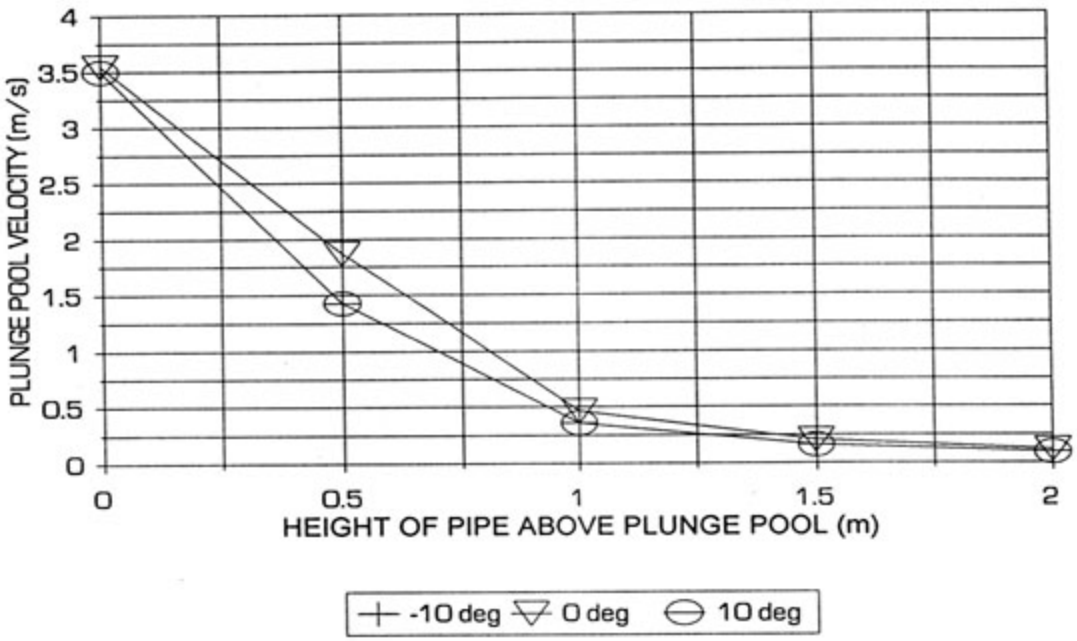


FIGURE 7.12 : PLUNGE POOL VELOCITY VERSUS PIPE HEIGHT ABOVE PLUNGE POOL FOR RÖSSING OLD BEACH

As in the case of the ST beach the calibrated beach agrees very well with the measured beach which was surveyed on two separate occasions, the first to obtain the profile and the second to locate the samples for particle size distribution tests. The measured particle size distributions show a marked gradation down the beach and this is relatively well predicted by the model. To achieve the above it was necessary to adopt a friction gradient of 0,08 m_{water} / m beach. This represents a gradient of 0,052 metres of slurry per metre of beach which is within the range of the rheological results in figure 7.3 but is certainly at the upper end of the range. The slurry exhibits the same degree of sensitivity to pipe height although velocities are higher due to the higher flow rate.

7.3 GOLD TAILINGS

The model has been applied to the Brazilian gold tailings described in Chapter 6 for which limited viscosity testing has been carried out. Figure 7.13 shows the results of the viscosity tests calculated using Bingham plastic and power law models. Figure 7.14 shows a plot of the shear stress versus the rate of shear. Plotted on Figure 7.14 are two best-fit lines representing Bingham plastic and a Newtonian models of the rheology. The Bingham fluid is represented by the following equation:

$$\tau = \tau_0 + \eta\left(-\frac{du}{dr}\right) \qquad (1)$$

where τ is the shear stress, τ_0 the yield shear stress, η the viscosity, u the angular velocity in a rotating cylinder viscometer and r the radius. From Figure 7.14 τ_0 is 2,65 Pa and η as 0.0095 Pa-s.

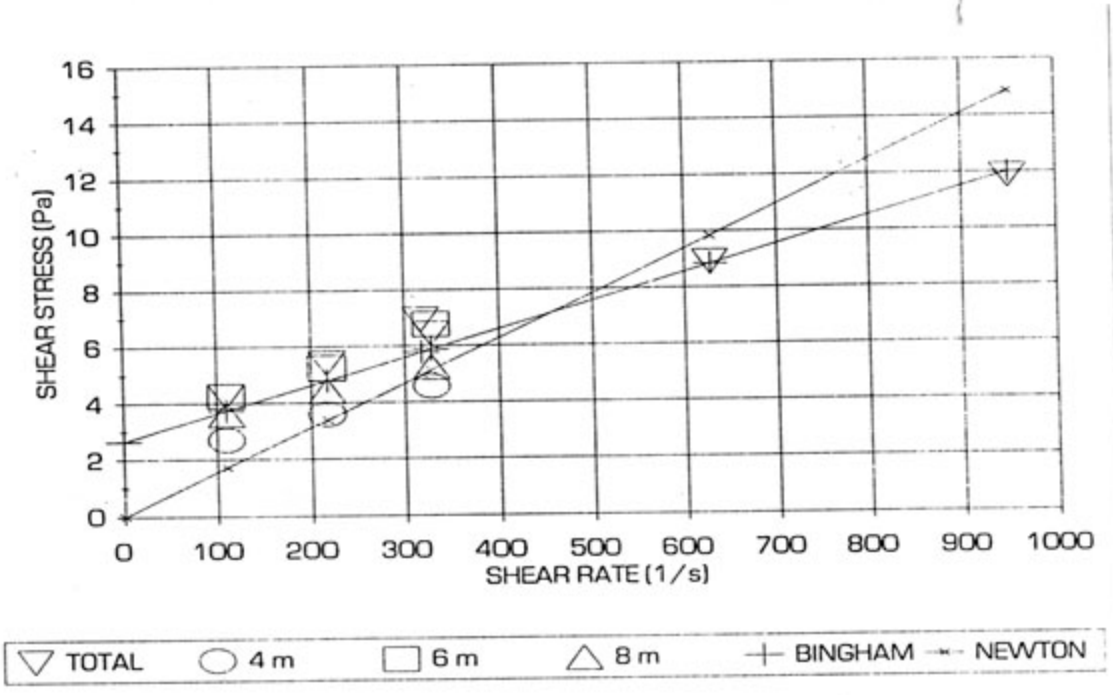


FIGURE 7.13 : RHEOGRAMS FOR THE BRAZILIAN GOLD TAILINGS

Figure 7.14 shows a plot of the Fanning friction factors resulting from rheological modelling using Verkerk's (1990) program for a range of hydraulic radii from 0,5 m to 1,5 m.

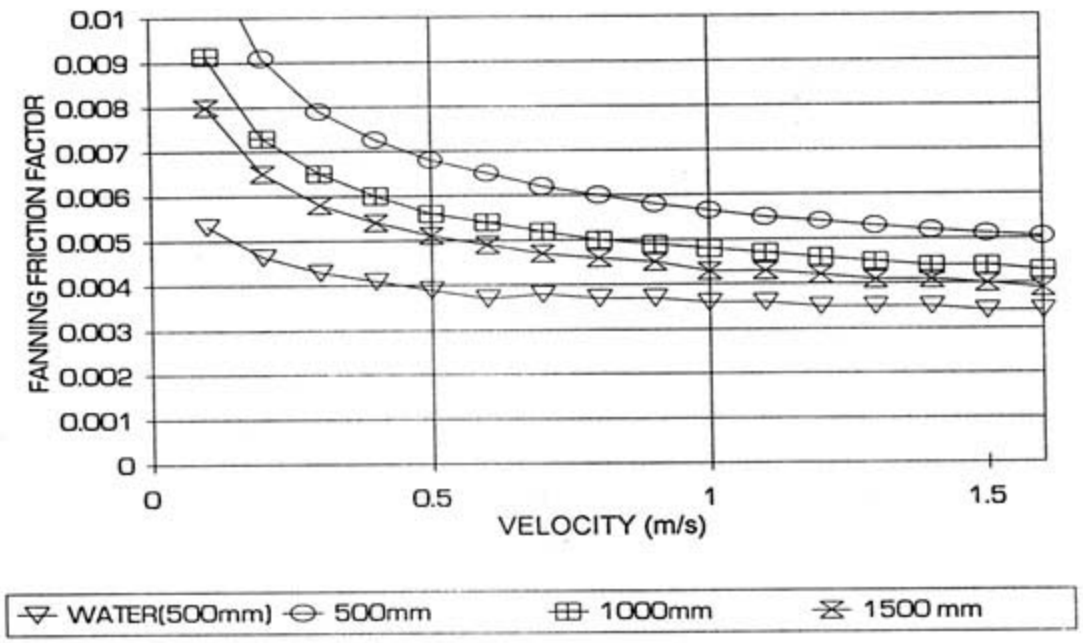


FIGURE 7.14 : FANNING FRICTION FACTORS

Tailings is generated at a rate of 40 000 tonnes per day and discharged at a concentration of approximately 45 % by weight down a natural gulley into a tailings dam of some 800 ha. The tailings beaches 3 km to a pond which is retained by a compacted earth embankment. Since the tailings is delivered in a natural gulley in which the flow section is highly variable. It is not practical to carry out plunge pool calculations. Therefore the beach has been calibrated using the beaching model and the plunge pool velocity and initial slope determined by back analysis. The predicted and measured beaching profiles are shown in Figure 7.15.

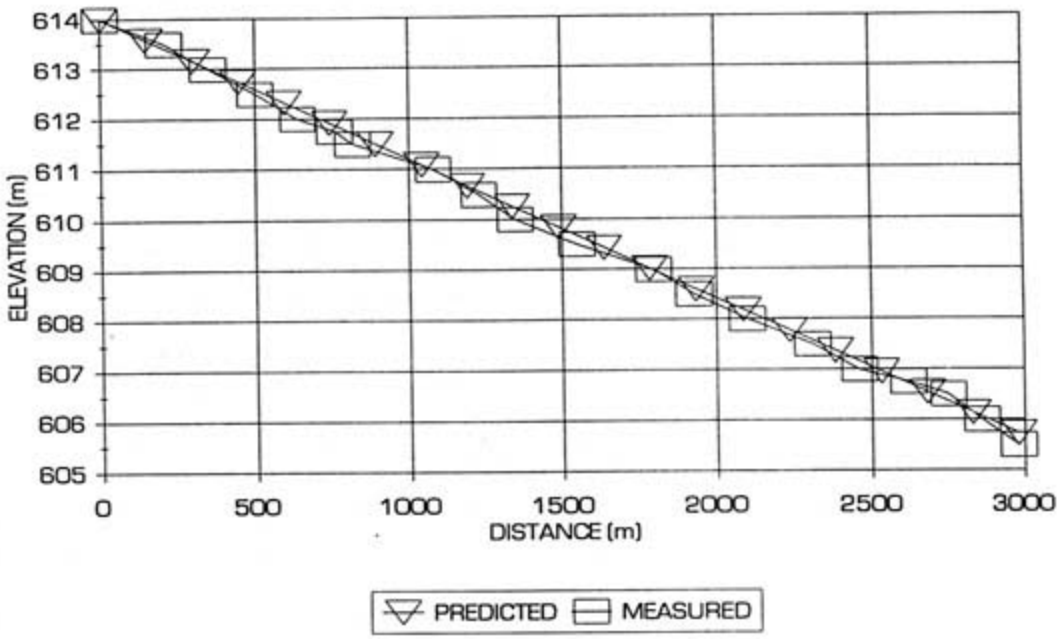


FIGURE 7.15 : CALIBRATED AND MEASURED BEACH PROFILES

The calibrations yielded the following parameters:

Tonnes/hr	1 667
Flow rate	0,.723 m^3/s
Pipe velocity	Not applicable
Concentration (mass)	45 %
Slurry Density	1,41 t/m^3
Length of beach	3 000 m
Height of pipe	Not applicable
Angle of pipe	Not applicable
Plunge pool velocity	0,127 m/s (Calibrated)
Initial slope (friction gradient)	-0,0028 m_{water} per m of beach (Calibrated)
Stream power	8,27 watts

The velocity of the flow stream close to the point where the natural channel intersects the tailings dam was estimated by the author as approximately 1 m per second. Assuming a flow depth of 1 m and using the Darcy Weisbach equation:

$$S_f = \frac{4 f v^2}{2 g R}$$

where f is the Fanning friction factor, v is the velocity and R is the hydraulic radius, for values of 0,0045, v at 1 m/s , and a flow depth of 1 m, S_f calculates as 0,0036 which is of the same order as the calibrated friction slope

The friction data in the above figures indicates that the calibrated friction slope and velocity agree with the calculated friction gradient at the corresponding velocity (within the accuracy of the calculations). It is interesting to note the low frictional resistance of the tailings. This is in line with the "soapy" feel of the tailings which, as noted in Chapter 6, are sourced from phyllite rock. This low resistance is a major factor in determining the beaching characteristics of the tailings in particular the almost linear beach profile as indicated in Figure 7.15.

Figure 7.16 shows the particle size distributions predicted on the basis of the calibrated slope and plunge pool velocity. The sudden change in particle size distribution predicted between chainages 298 and 447 is a consequence of the large steps between the beach calculation distances. The changes would be more gradual if more points on the length beach were calculated. This would, however, necessitate considerable modification of the computer model.

The beach is inaccessible as a result of the wet conditions on the beach and field samples could not, unfortunately, be recovered for comparison.

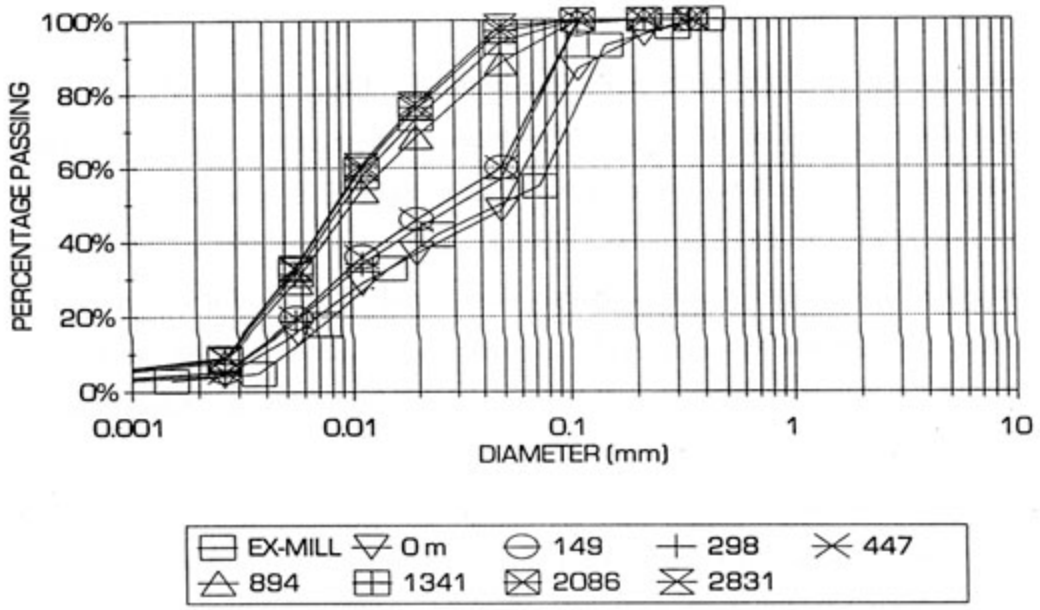


FIGURE 7.16 : PREDICTED PARTICLE SIZE DISTRIBUTIONS FOR THE BRAZILIAN TAILINGS

7.4 COPPER TAILINGS

The copper tailings dam at Palabora Mine in South Africa is formed using two by-products of the copper tailings stream. These by-products are generated through a cyclone plant located at the copper tailings dam site which yields:

- a coarser drier underflow product which is used to form the outer crest wall and the initial beach by discharging through 3 to 4 spigot pipes at any location around the dam

- a finer, wetter underflow product which is discharged at the upper part of the tailings dam and allowed to beach to a pond located on one end of the dam. Until recently the underflow product has been discharged through two separate pipelines, referred to as "berm" lines, of 500 mm diameter on a point-discharge basis ie the stream is not split into several streams.

This method of discharge is under review in order to modify the dam geometry and to

accommodate a change in the particle size distribution and tonnage of the copper tailings product and forms the subject of an extensive and protracted consulting brief.

In the course of this brief several measurements have been made of the beaching characteristics of the copper tailings discharged through the "berm" lines prior to changes to the particle size distribution of the tailings. This information was gathered in order to obtain a calibration of the beaching model for use in predicting the potential changes in beaching characteristics which will be brought about by a change in the tailings and a consequent modification of the method of discharge. These calibration runs form the basis of the application described hereunder.

Figure 7.17 shows a rheogram of the tailings developed from Ferrante rotating cylinder viscometer tests on the minus 150 micron fraction of the tailings. The tests have been carried out at a range of water contents and shear rates as indicated. It is evident from the best-fit linear regression lines that the slurry can be modelled using a Bingham Plastic model.

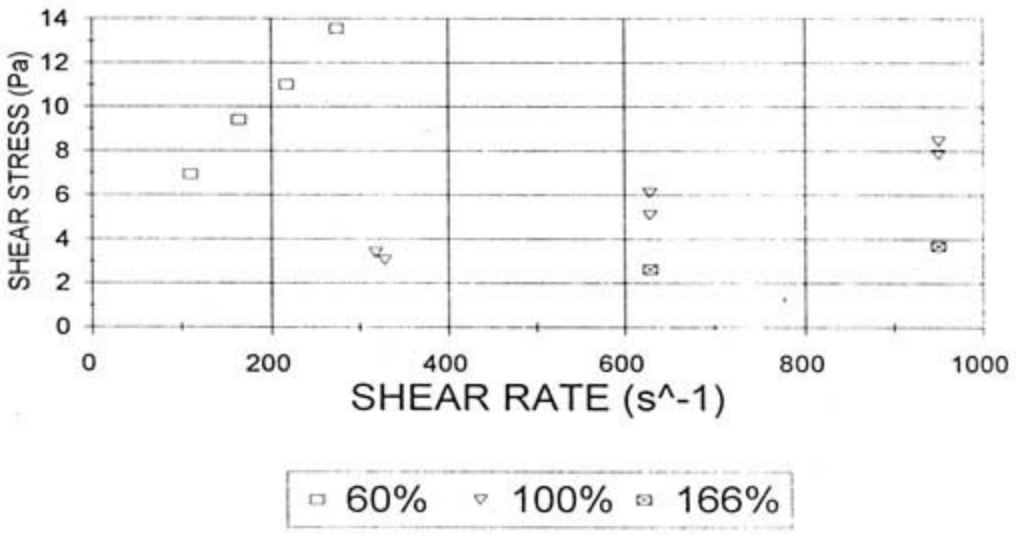


FIGURE 7.17 : COPPER TAILINGS RHEOGRAMS

The Bingham model parameters that emerge from the linear regression are:

moisture content	τ_0 (Pa)	η (Pa-s,Ns/m^2)
60 %	0,272	0,0039
100 %	0,092	0,0008
166 %	0,051	0,0003

Figure 7.18 shows a plot of Bingham viscosity against moisture content and Figure 7.19 a plot of the yield stress against moisture content.

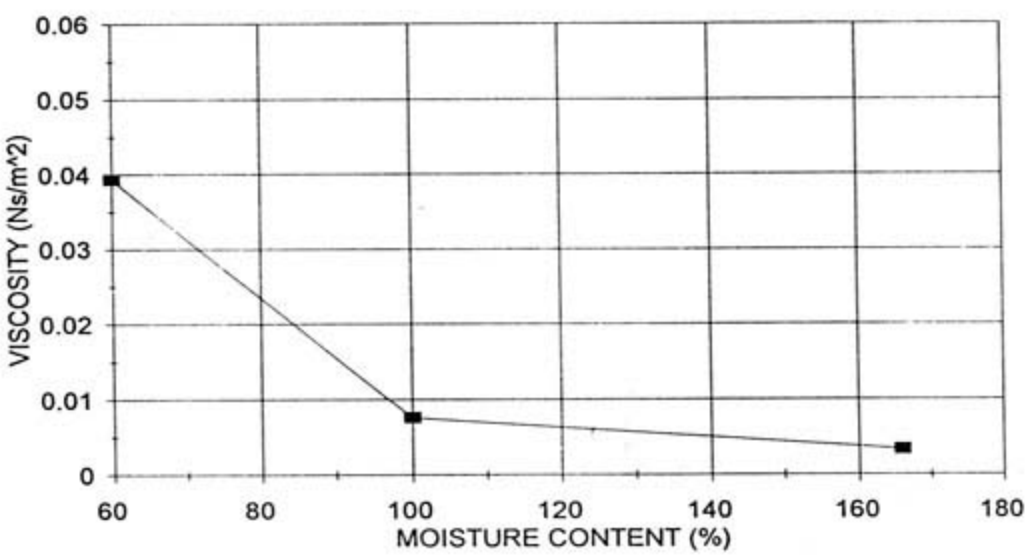


FIGURE 7.18 : BINGHAM VISCOSITY vs MOISTURE CONTENT

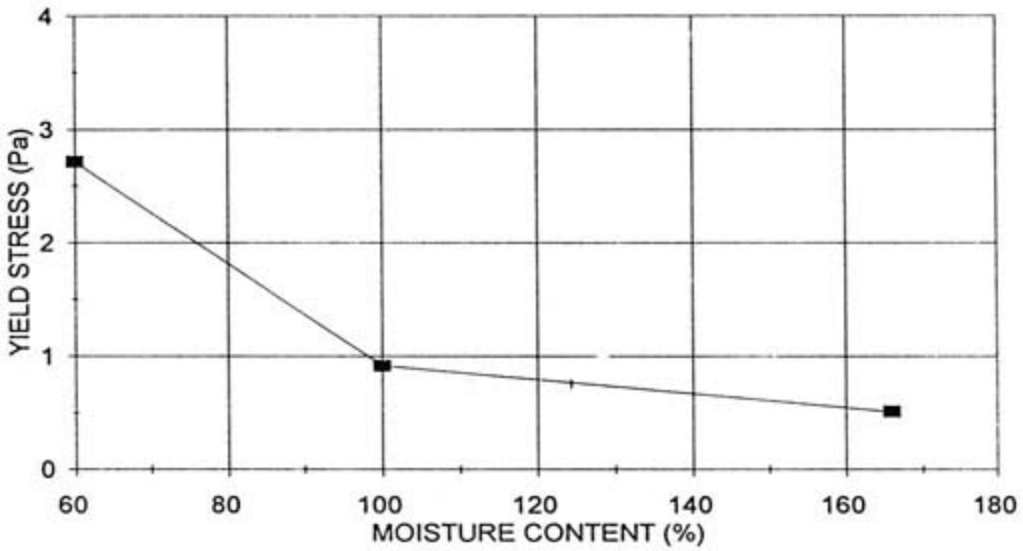


FIGURE 7.19 : YIELD STRESS vs MOISTURE CONTENT

Figure 7.20 shows the Fanning friction factors determined using Verkerk's programme and the appropriate Bingham parameters. Figure 7.21 shows the measured and calibrated beach profiles for berm line tailings. The calibrated parameters were based on the following:

Tonnes/hr	350	(overflow of 60% of 28 000 tpd split between 2 berm lines)
Flow rate	0,16 m^3/s	
Concentration (mass)	44 %	
Slurry Density	1,39 t/m^3	
Length of beach	2 300 m	
Height of pipe	0,6 m (Calibrated)	
Angle of pipe	10 degrees upwards	
Plunge pool velocity	0,25 m/s	
Initial slope (friction gradient)	-0,0187 m_{water} per m of beach	(Calibrated)
Stream power	6,86 watts	

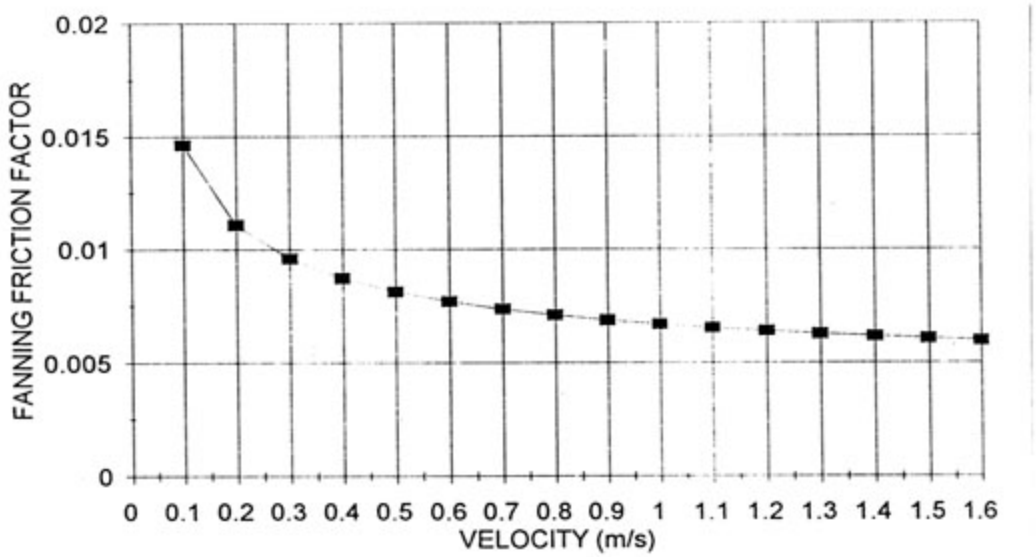


FIGURE 7.20 : FANNING FRICTION PARAMETERS

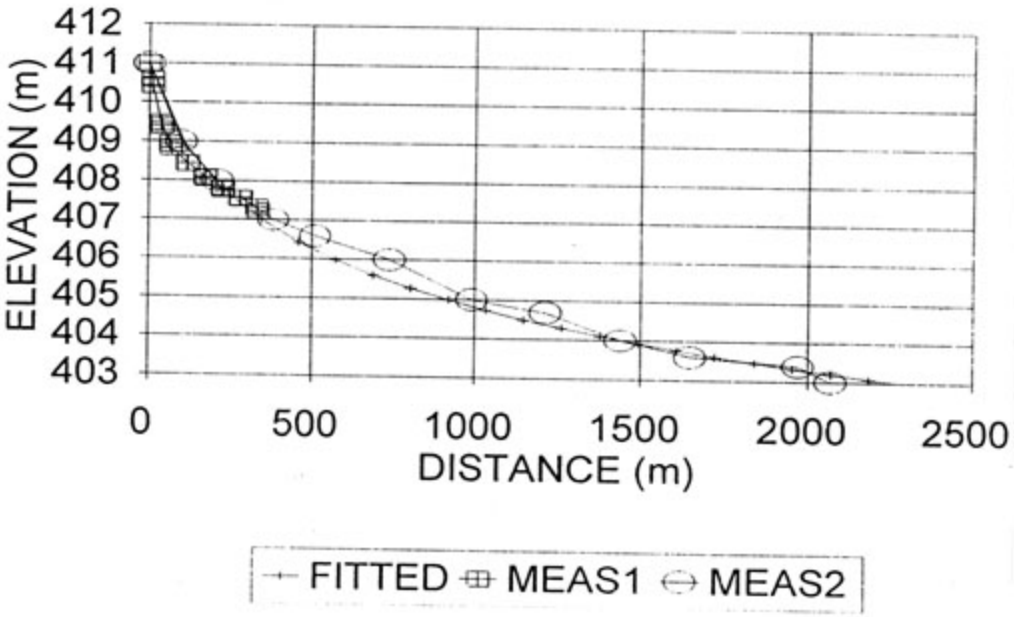


FIGURE 7.21 : MEASURED AND CALIBRATED PROFILES

The measured beach MEAS1 is from topographical survey which extended over only a limited distance due to access problems, and MEAS2 is from an aerial survey taken some 6 months prior to MEAS1 and adjusted to yield the same starting elevation at the head of the beach.

The calibrated profile in Figure 7.21 necessitated adjustment of the height of the pipe and the Fanning friction factor which determines the initial beach slope. The height of the pipe needed to be calibrated as this was not measured at the time that the beach was formed although it has been observed to vary between almost zero to 2,5 m. The Fanning friction factor needed to be set to 0,012 as opposed to the 0,0065 which would apply for the 1 m/s calculated critical velocity from the plunge pool. If a Fanning friction factor of 0,0065 were to be used with a height of pipe of 0,6 m, the profile would be as indicated in Figure 7.22.

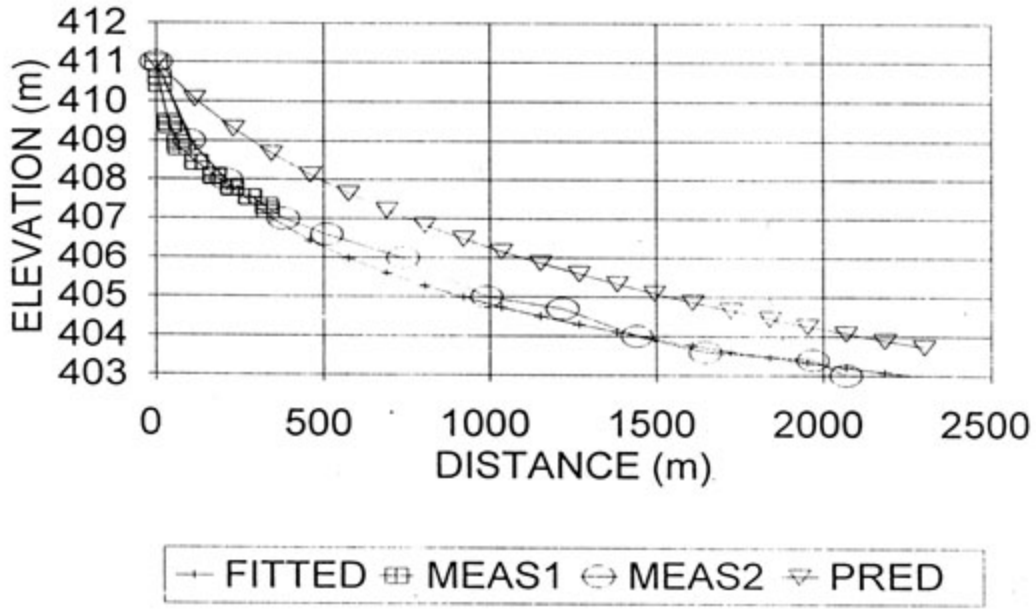


FIGURE 7.22 : MEASURED, CALIBRATED AND PREDICTED PROFILES

Figure 7.23 shows the predicted particle size distributions based on the fitted parameters while Figure 7.24 shows the measured particle size distributions over the length over which access could be gained.

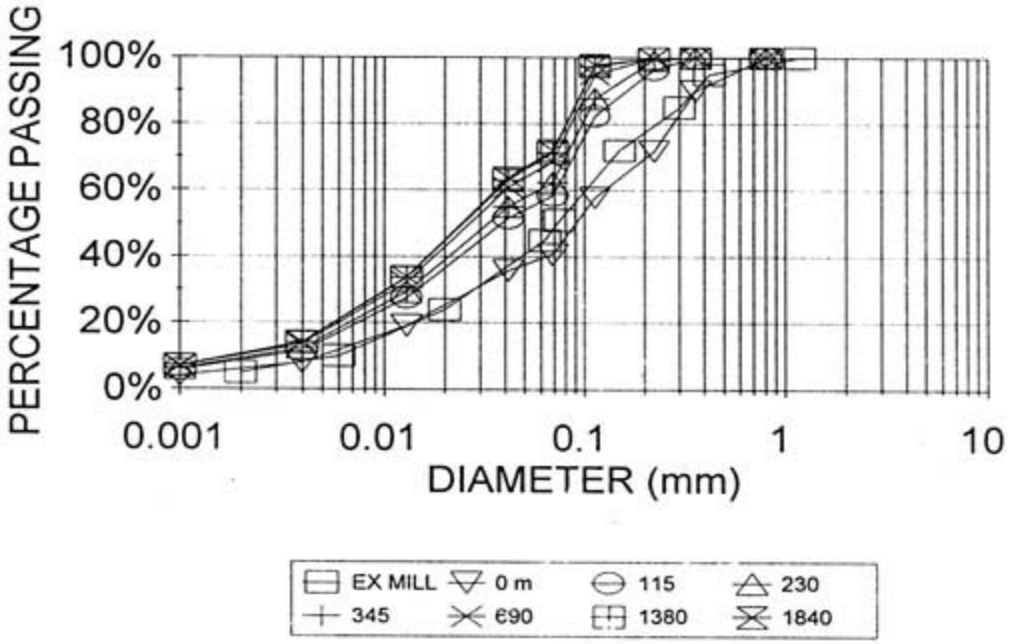


FIGURE 7.23 : PREDICTED PARTICLE SIZE DISTRIBUTIONS

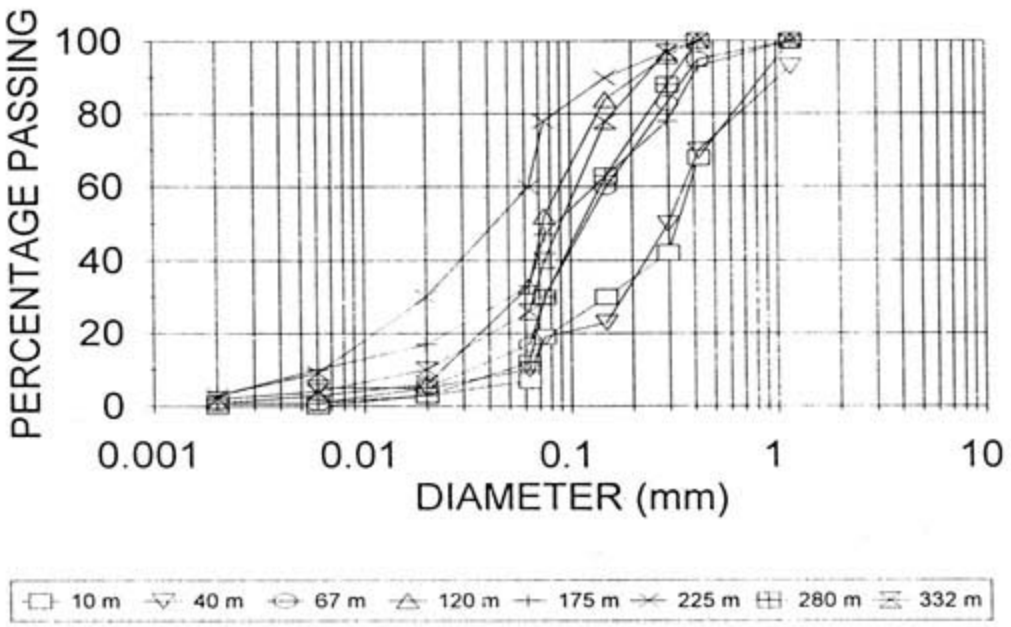


FIGURE 7.24 : MEASURED PARTICLE SIZE DISTRIBUTIONS

Figures 7.25, 7.26 and 7.27 show the variation of the d_{90}, d_{50} and d_{20} sizes respectively with distance from the discharge.

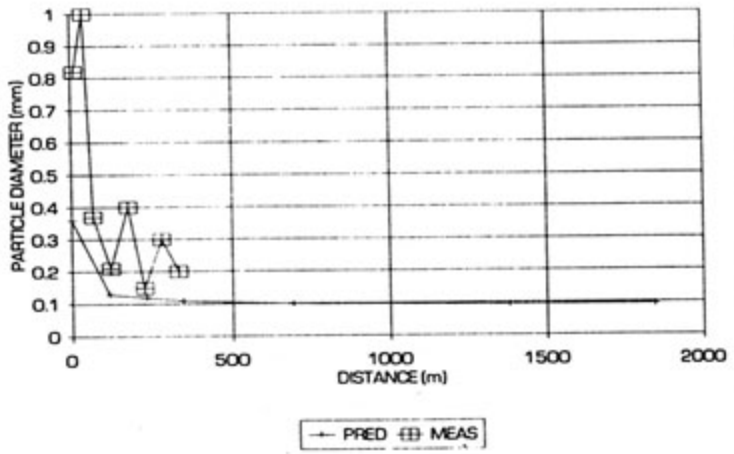


FIGURE 7.25 : COMPARISON OF d_{90} SIZE WITH DISTANCE

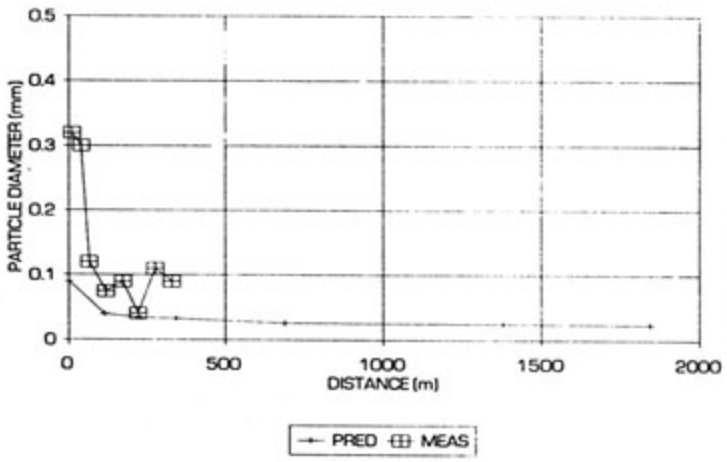


FIGURE 7.26 : COMPARISON OF d_{50} SIZE WITH DISTANCE

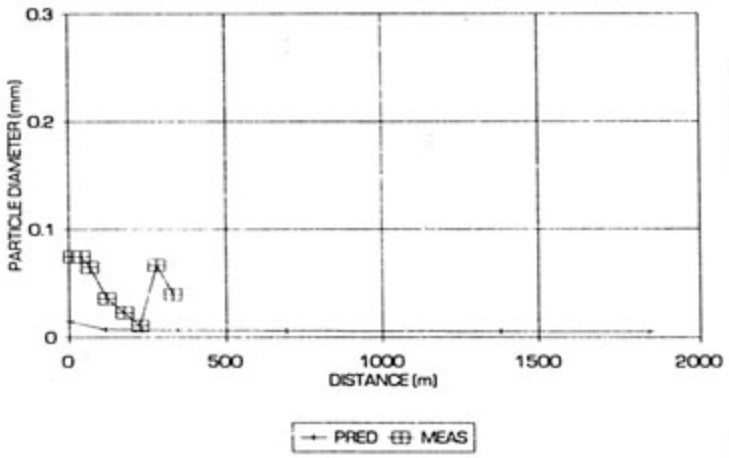


FIGURE 7.27 : COMPARISON OF d_{20} SIZE WITH DISTANCE

The following points are noteworthy in respect of Figures 7.23 to 7.27:

- the measured particle size distributions in Figure 7.24 do not show a continuous decrease in coarseness with distance down the beach, specifically the distributions at 280 and at 332 m are coarser than that at 225 m. This is ascribed to cross-flow from the cyclone underflow (sands line) depositions from the crest of the dam which may have covered the berm line tailings. This would also imply some interference in the beach profile but it is not possible to determine the full extent of this. Also, there are significant periods of operation during which the cyclone plant is by-passed and the total product us routed through the berm lines. Under these conditions the particle size distribution of the tailings would be coarser and the slurry density higher. Once again it is not possible to determine the extent to which these by-pass periods will influence the beach profile and the deposited particle size distributions.

- While the predicted beach profiles and particle size distributions do not match the measured parameters there is nonetheless a definite correlation in the results and sufficient plausible explanation for the differences in the measured and predicted parameters.

7.5 MAGNETITE TAILINGS

The Palabora Mine in South Africa also produces a magnetite tailings product which is stockpiled in a hydraulically formed dam within the copper tailings dam. The magnetite tailings is characterised by a relatively coarse particle size distribution and a high particle specific gravity of 4,69. The tailings is pumped simultaneously to three locations around the dam in a 250 mm diameter HDPE pipeline and discharged over the crest onto the beach via 250 mm diameter spigots as indicated in Figure 7.25.



FIGURE 7.28 : PHOTOGRAPH OF A TYPICAL DISCHARGE ON THE MAGNETITE DAM

Ferrante rotating viscometer viscosity tests have been carried out on the minus 150 micron fraction of the magnetite tailings however the results of the tests are erratic due to difficulties experienced in keeping the fines in suspension during the test. The high specific gravity of the magnetite results in rapid settling out of the solids during a test to the extent that at higher concentrations the particle-wall friction interferes with the measurements while at lower concentrations an unrepresentatively low viscosity is measured. Approximating the viscosity measurements with a Newtonian model results in a variation of viscosity with volumetric concentration as indicated in Figure 7.26.

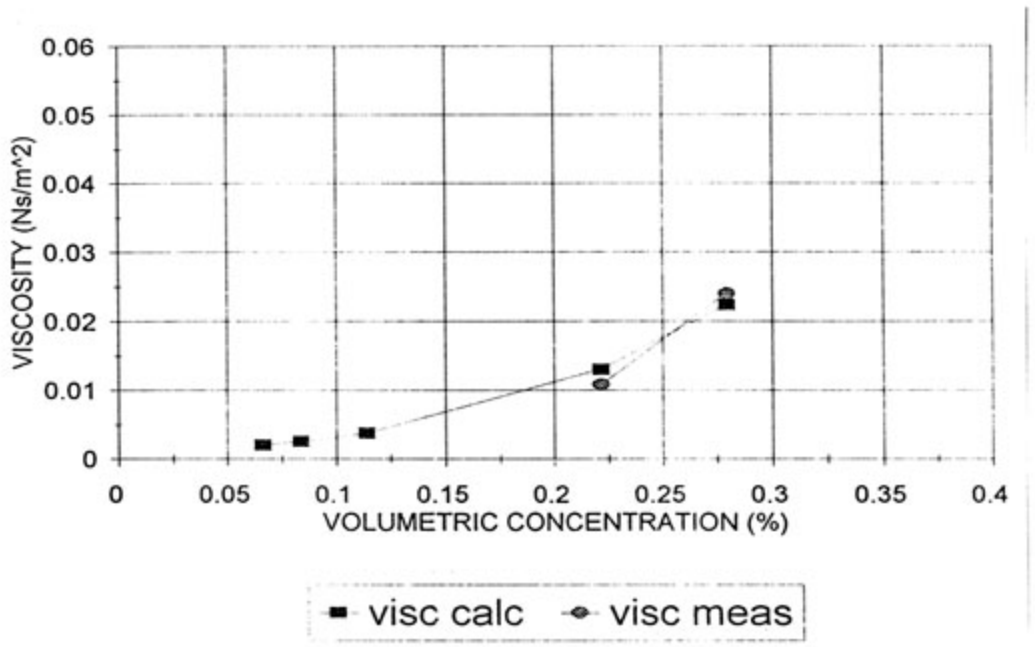


FIGURE 7.29 : VARIATION OF NEWTONIAN VISCOSITY WITH VOLUMETRIC MOISTURE CONTENT

The beaching simulation model was calibrated against beach profile measurements by varying the plunge pool velocity, the initial slope and the discharge height. The reasons for calibrating against all three of these parameters are as follows:

- it is evident from Figure 7.25 that the stream breaks up as it leaves the pipe and the assumption of an approximately continuous stream on which the model is based as set out in Chapter 3 is distinctly invalid. Consequently the plunge pool depths were considerably over-estimated with the result that the plunge pool velocity was considerably under-estimated.

- the initial slope of the beach was considerably under-estimated possibly as a result of problems experienced with the viscosity measurements. The inability of the model to cater for the hydraulics of the discharge stream also resulted in under estimation of the critical velocity at the crest of the plunge pool which would cause under-estimation of the friction.

- the height of the discharge needed to be set to compensate for shortcomings in the

modelling of the hydraulics of the discharge stream

The calibrated parameters are set out below and the calibrated profiles shown in Figure 7.30.

Tonnes/hr	319,4	(23 000 tpd split between 3 discharges)
Flow rate	0,07 m^3/s	
Concentration (mass)	63,6 %	
Slurry Density	2,00 t/m^3	
Length of beach	300 m	
Height of pipe	0,06 m (Calibrated)	
Angle of pipe	35 degrees upwards	
Plunge pool velocity	0,46 m/s (Calibrated)	
Initial slope (friction gradient)	-0,082 m_{water} per m of beach (Calibrated)	
Stream power	14,75 watts	

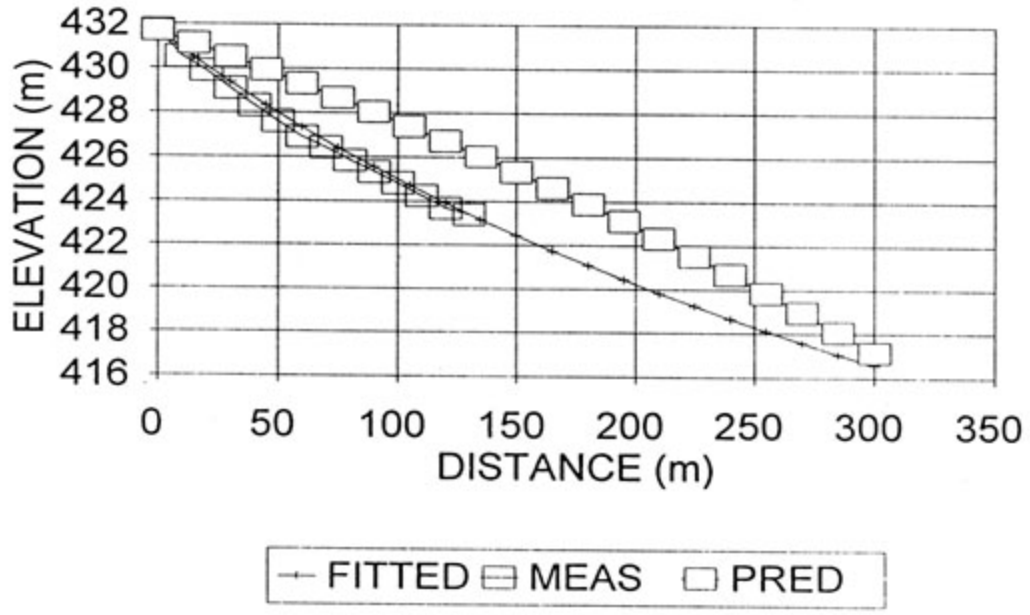


FIGURE 7.30 : CALIBRATED, MEASURED AND PREDICTED BEACH PROFILES

Figure 7.30 also shows a predicted beach profile determined on the basis of friction factors shown in Figure 7.31 and derived from Verkerk program runs using the viscosities in Figure 7.29. The model predicts a critical velocity of 1.34 m/s for a discharge at 0,06 m which, from Figure 7.31 implies a Fanning friction factor of 0,012. The calibrated factor is 0,02.

Figure 7.32 shows the predicted particle size distributions down the beach based on the calibrated parameters. The predicted distributions do not indicate significant segregation into a coarse fraction close to the discharge which is contrary to observations on site.

It is evident that the beaching simulation system has not successfully modelled the magnetite tailings. To improve the reliability of the model it will be necessary to :

- use a method of viscosity measurement which prevents particles settling out
- improve hydraulic modelling of the discharge stream where this breaks up

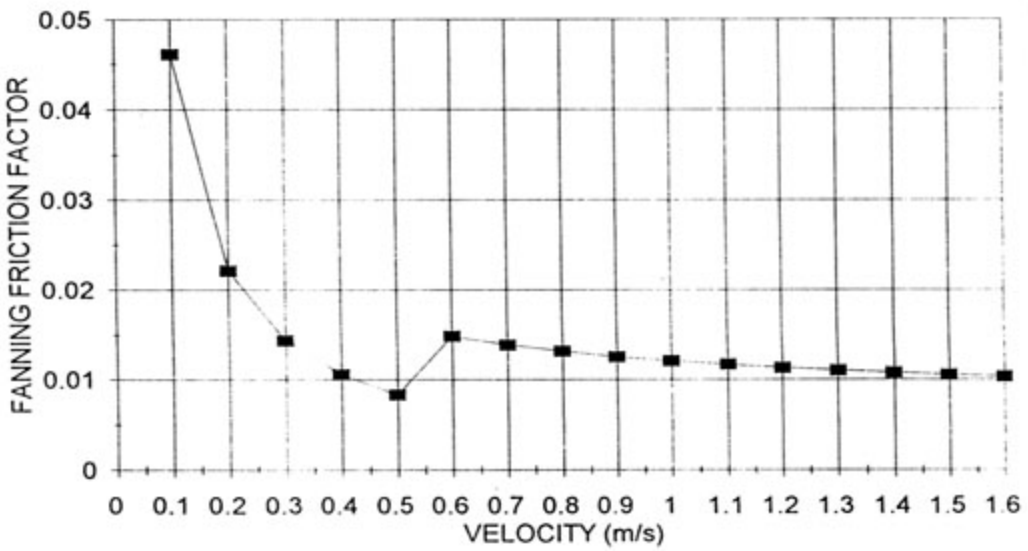


FIGURE 7.31 : FANNING FRICTION FACTORS

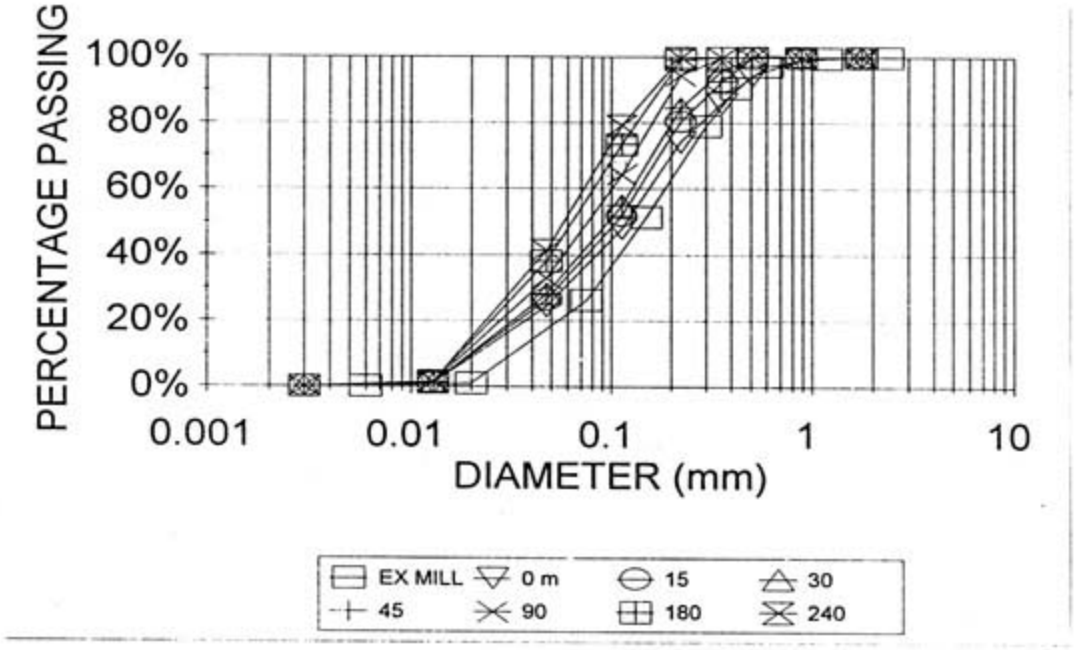


FIGURE 7.32 : PREDICTED PARTICLE SIZE DISTRIBUTIONS DOWN THE BEACH BASED ON CALIBRATED PARAMETERS

7.6 DISCUSSION

The analyses documented above indicate the following:

- the entropy-based stream power model provides reasonable estimates of the beach profile provided the input parameters of plunge pool velocity and frictional resistance can be determined reliably

- the methods of predicting the plunge pool velocity as set out in chapter 3 provide realistic results provided the discharge stream remains intact enroute from the pipe to the plunge pool. However, as can be noted from the photographs on in Figures 7.33 and 7.34 the discharge stream breaks up. The extent to which the stream breaks up appears to be related to the discharge pressure, discharge velocity and the height of the pipe above the plunge pool. Wind also plays a significant role. Some refinement through specific research into the hydraulics of the discharge stream under all conditions of flow is warranted in view of the importance of predicting the plunge pool velocity as

accurately as possible; this parameter determines the stream power and therefore all of the beaching characteristics

- the use of hydraulic transport analysis methods based on the rheological characteristics of the slurry and the methods of Wasp et al (1977) as verified by Verkerk (1990) is a reliable method of determining the initial slope of the beach. The substitution of hydraulic radius of the flow channel for pipe diameter and the assumption of a semi-circular flow channel at critical velocity reasonably approximates the unsteady nature of the stream flowing from the plunge pool. This conclusion is conditional on control of the settling out of particles in the viscometer when measuring viscosity.

- the model provides a more quantitative predictive estimate of the particle size distribution than has been possible up to the present.

It is therefore evident that the entropy-based stream power model provides a useful predictive tool with which to assess the implications of changes in slurry, grind and deposition method, and for the design of new tailings operations. The method of tailings placement can now be optimised in the design office. This, coupled with verification measurements in the field, will facilitate optimisation of tailings dam performance through controlled deposition practice. This aspect is elaborated on in the next chapter.



FIGURE 7.33 : PHOTOGRAPHS SHOWING TYPICAL DISCHARGE STREAMS AND THEIR EFFECT ON THE PLUNGE POOL VELOCITY





FIGURE 7.34 : PHOTOGRAPHS SHOWING TYPICAL DISCHARGE STREAMS AND THEIR EFFECT ON THE PLUNGE POOL VELOCITY

CHAPTER 8

CONTENTS

Sec	Description	Page
8	OPTIMISATION OF TAILINGS DAM PERFORMANCE THROUGH CONTROLLED DEPOSITION PRACTICE	8-1
8.1	INTRODUCTION	8-1
8.2	PERFORMANCE OF A TAILINGS DAM	8-3
8.3	CONTROLLED DEPOSITION OF COARSE MATERIAL	8-7
8.4	ENHANCEMENT OF UNDERDRAINAGE OPERATION	8-9
8.5	CONTROLLED PLACEMENT OF FINE MATERIAL	8-13
8.6	CONTROL OF POOL LOCATION	8-16
8.7	MAXIMISED TAILINGS STORAGE CAPACITY	8-19
8.8	CASE STUDY : PLATREEF TAILINGS DAM	8-22
8.8.1	Introduction	8-22
8.8.2	Platreef Tailings Product	8-23
8.8.3	Beaching Characteristics	8-25
8.8.4	Description of a Hybrid Paddock-Spray bar Deposition system	8-36
8.8.5	Review of the Hybrid System following Initial Deposition Trials	8-44
8.8.6	Review of Method of Main Drain Formation and Covering	8-45
8.8.7	Review of Method of Formation of Outer Toe	8-46
8.8.8	Long Term Deposition Method	8-47
8.8.9	Summary of Conclusions	8-47
8.9	CASE STUDY : LEAD ZINC TAILINGS	8-48
8.9.1	Introduction	8-48
8.9.2	Pering Tailings Product	8-50
8.9.3	Beaching Simulations	8-52
8.9.4	Conclusions	8-64

CHAPTER 8 OPTIMISATION OF TAILINGS DAM PERFORMANCE THROUGH CONTROLLED DEPOSITION PRACTICE

8.1 INTRODUCTION

The primary aim of this research has been to derive a tool for practical application in providing answers to a number of problems that presently cannot be answered to any degree of reliability without first constructing the tailings dam. These problems occur through changes of slurry and tailings characteristics typically brought about through:

- Changes in tailings particle size distribution as a result of :
  - mine backfill operations in which the coarse fraction is scalped for placement underground
  - Finer milling as a result of improvements in metal recovery
  - "high grading " or "low grading" in the mining operations. This would depend on market conditions and would arise from variations in mining rates. This, in turn, would influence the mill residence time and therefore the grind
  - variation in the mineralogy and mineral hardnesses in the ore
- Changes in the particle density as a result of geological intrusions or differing ore seams
- Changes in process plant operation as new process units are brought into or taken out of operation
- Down time of process units.

There is similarly no reliable method for determining the optimum operating method for marginally segregating slurries on new mining operations for which no track record of experience exists.

The above problems will impact on the previous or anticipated behaviour of the tailings after

this has discharged from the delivery pipe and can affect:

- the geotechnical properties of the placed tailings specifically drainage and consolidation rates which, in turn, determine slope stability

- access conditions on the tailings dam which may impact on routine operating cycles and may ultimately lead to problems of depositonal control, loss of freeboard and blinding of drainage structures

- changes in freeboard as a result of changes in beach profile

Changes in slurry and grind characteristics therefore impact on the safety and security of the tailings dam. They can also lead to increased costs of disposal since failure to recognise timeously the impacts of the changes can make it necessary to carry out unplanned and unbudgeted remediation. The remediation will, in turn impact on dam operations since this will frequently require removal of fine material deposited over drainage structures, excavation to break up layering in the embankment area of the dam, mechanical raising of embankments that are usually hydraulically placed etc

The problem to date has been an inability to assess the impact, both short term and long term, of grind or slurry changes the tailings dam. The only guide has been past experience and the "gut" feel of operators. The model described in this dissertation will go a long way towards enabling industry to assess the implications of the changes, make adjustments to operations accordingly, and adequately budget for any remediations.

To carry out an assessment the following information would be required concerning the new slurry:

- the new particle size distribution, solids concentration and delivery rates

- the rheology of the tailings slurry

- the geometry of the tailings impoundment, specifically topographical data and beach lengths

The assessment would consider the following:

- the segregation characteristics of the tailings and the extent to which it will influence the method of tailings dam operation

- the preferred discharge method so as to maximise freeboard, minimise operator effort and optimise the performance of drainage control structures and decants

The above aspects are of particular importance in South Africa where upstream construction methods predominate and therefore where the geotechnical characteristics of the placed tailings are paramount in determining the success of the tailings operation.

There are other considerations relating to the control of beaching characteristics than can lead to an improvement of the performance of tailings disposal operations even if there are no changes in slurry or grind. Prior to describing these considerations it is first necessary to define what is meant by tailings dam performance and identify those factors which would be influenced by beaching characteristics and could therefore be optimised through controlled deposition practice. This definition is set out in the next section. In the sections following the next section a number of methods of deposition control which have been employed on South African mining operations to optimise performance will be described.

8.2 PERFORMANCE OF A TAILINGS DAM

From a mine-owner perspective a tailing dam is part of the cost of mining and a necessary evil. Against this background the owner's primary requirements for optimal performance of a tailings dam are that it:

- have the lowest possible operating cost

- achieves this operating cost at acceptable risk of failure and damage to health and the environment

- is seldom responsible for down-time of the plant

- seldom needs replacement or preferably can last the life of the mine

- is cost effective to establish and can be developed on a phased basis to minimise cash flow impact

The upshot of the above requirements is that the tailings dam needs to:

- be as close to the plant as possible to reduce the capital costs of establishing the dam and the operating costs in pumping the slurry to the dam. This is a function of land availability and topography.

- utilise the smallest possible area and achieve the maximum possible height within the bounds of stability and reliability taking environmental constraints of dust and seepage control into consideration.

It is this last point that requires specialist input.

A small area implies a higher rate of rise. To accommodate a high rate of rise the tailings needs to drain and consolidate more rapidly in the embankment area of the dam to avoid the build-up of pore pressures. A build-up of pore pressures will result in lower shear strengths and lead to slope instability.

To reduce the build-up of pore pressures it is necessary to provide filter drains within the dam. These drains should be located in such a way that the drains are surrounded by free draining material and effectively keep the embankment tailings below saturation. This is illustrated Figure 8.1. Segregating tailings present both a problem and an opportunity in this regard. If the drain is placed at ground level and tailings is discharged without consideration of the fact that the tailings will segregate the drain could be blinded through the deposition of fine materials or layers over the drain. On the other hand judicious placement of the tailings to ensure that coarse material is placed over and around the drain would enhance drain performance and provide greater assurance that the embankment area remains unsaturated. More importantly for the

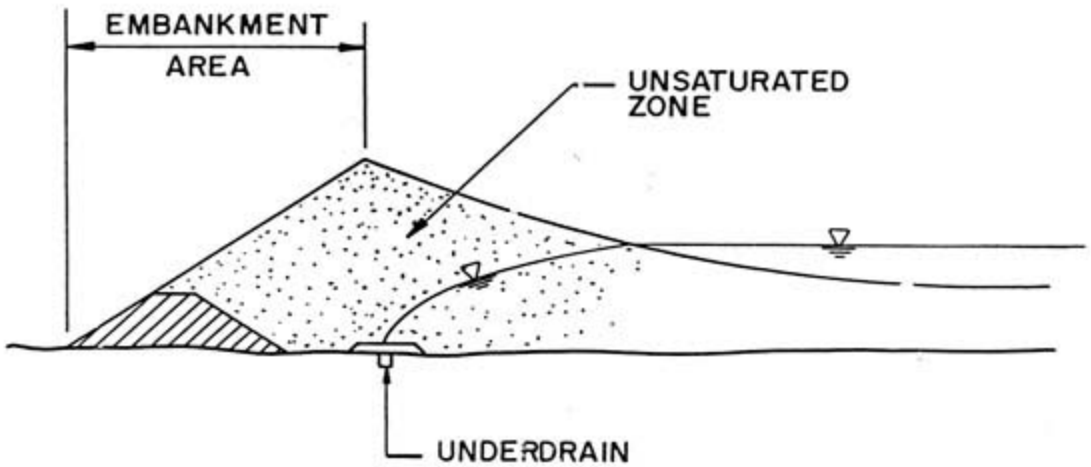


FIGURE 8.1 : INFLUENCE OF UNDERDRAINAGE

Owner the enhanced drain performance would translate into smaller drainage systems and therefore lower capital cost. This is because the size of a drain is determined not by its outlet capacity but by its inlet capacity and the inlet capacity is determined by the drainage characteristics of the tailings in contact with and immediately surrounding the drain.

A further important factor in determining the height and rate of rise of a dam is the location of the pond relative to the toe. The pond is a continuous source of seepage and is responsible for the development of a phreatic surface in the tailings dam. The geometry of the phreatic surface is determined by the location of the underdrainage, if such features are present, and by the distance between this underdrainage and the edge of the pond. If, as is the case on most of the older tailings dams in South Africa, there is no filter drainage, or if there is filter drainage but this is located close to the toe, the resulting geometry of the phreatic surface will affect the slope stability of the tailings dam. This is illustrated in Figure 8.2. The location of the edge of the pool relative to the toe or drainage is a function of the volume of water stored on the dam, the beaching characteristics which determine the basin area-storage characteristics, and the location of the decant facility.

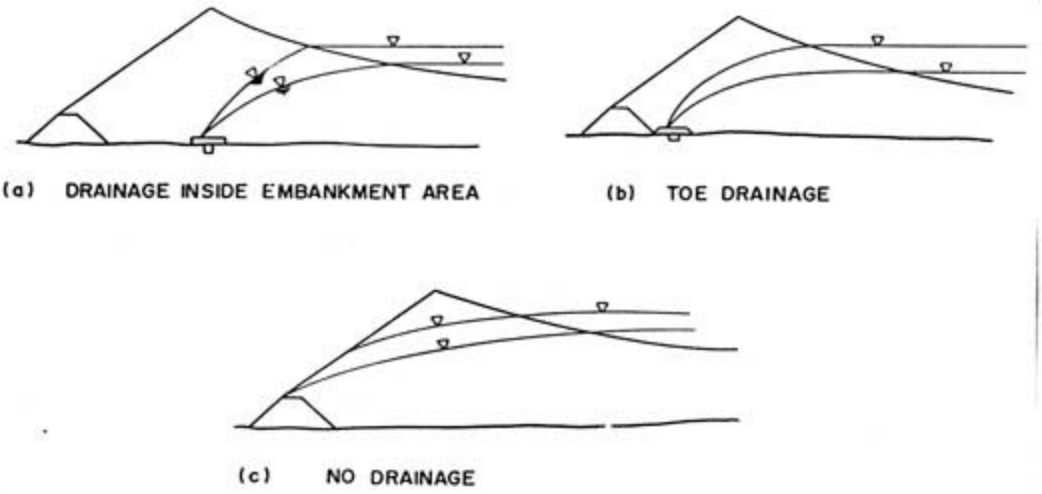


FIGURE 8.2 : INFLUENCE OF THE POND ON THE PHREATIC SURFACE IN THE EMBANKMENT AREA

The seepage losses from the tailings dam are dependent not only on underlying geology but also on the drainage characteristics of the tailing material immediately in contact with the foundation (Nelson and McWhorter (1977)). If initial beaching of the tailings is controlled it is possible to place finer material in contact with the foundation. The tailings deposited in underwater in the pond area at the end of the beach is always the finest and slowest draining and if deposition is carried out in such a way that the pond covers as much of the natural ground surface in the developmental stages of the dam as possible, then it will be possible to form an hydraulically placed "liner". During the developmental stages there may be an increase in foundation seepage as a result of this deposition practice but this will reduce as the dam develops and reaches maturity.

To achieve the lowest practical operating cost it is necessary to limit the number and size of mechanical equipment on the dam that are required for the formation of the crest confining wall. For non-segregating slurries it has been established that the paddock system as developed in South Africa is the most cost effective method of disposal requiring only a tractor and plough implement to effect wall building. This is because the volume of hydraulically

placed tailings in the crest confining embankment is maximised and the volume of mechanically worked material is minimised. (This method of dam construction was described in detail in Chapter 1). For segregating slurries, the most cost effective method of placement is through the use of spigot deliveries, or, for finer segregating slurries, through the use of a spray bar system located on the crest. These methods of delivery encourage controlled hydraulic segregation so that coarse material is deposited in the embankment area and beach flow takes place directly from the delivery pipe to the pond in the central part of the dam. The use of hydrocyclones is not as cost effective since a higher, and more importantly, consistent delivery pressure is required at the hydrocyclones. This usually implies a booster/control pump station close to the tailings dam particularly in cases where delivery lines follow undulating topography en route to the discharge point. The hydrocyclones require continuous and careful management to ensure that they can be moved at-will, overflow pipes are not buried and, during periods of low slurry density, that the underflow embankment is not eroded away.

In South Africa the majority of tailings generated on non gold mining operations is segregating. This applies specifically to platinum and copper mining. On the majority of these operations upstream operation using point, spigot or spraybar delivery takes place. Aspects of beach control as set out above are therefore particularly relevant and are singled out for detailed discussion in this dissertation as set out below.

8.3 CONTROLLED DEPOSITION OF COARSE MATERIAL

The need to control gravitational segregation so as to encourage the deposition of the coarser fraction in the embankment area of the tailings dam has been long recognised. What have not been fully identified are the mechanisms controlling the extent of segregation and the correlated effects of variations in discharge rate and orientation, slurry concentration and tailings particle size distribution. The sensitivity analysis described in Chapter 4 utilised the model to check whether variation of the primary and secondary variables produced the effects observed in the field. The information obtained is useful also in assessing the impact of the variables and determining which discharge methods will enhance the deposition of coarse materials. The sensitivity analysis shows that to deposit coarser fraction sooner down the beach the stream power should be lower. There will then be insufficient energy in the stream to sustain suspension of these particles. For segregating slurries that are delivered at velocities above 1,5 m/s (to avoid settling out of the coarse fraction in the pipe) this implies splitting the

stream into a number of smaller deliveries through spigots or multiple discharges in a spraybar system. To further enhance settling out the spigot/spraybar discharge should be directed downwards. The resulting plunge pool at each discharge will be deeper, and the losses greater, with the result that the stream power of the beaching stream will be lower.

The above measures should be balanced with beach profile requirements since a very low beach velocity will also result in a beach profile which is steep only in the immediate vicinity of the delivery and flattens very quickly as the beach slurry loses coarse material.

The height of the discharge above the beach plays an interesting and important role in determining the stream power and beach profile. Commonly the discharge pipe at the start of a deposition is 1 to 2 m above the beach. This results in a deep plunge pool, low stream power in the plunge pool, rapid deposition of coarse in the vicinity of the discharge and the rapid build up of the plunge pool sides. The beach profile is flat. The slurry flowing down the beach is finer as a consequence of losing coarse fraction close to the discharge with the result that over the initial part of the deposition the beached tailings is finer beyond the confines of the plunge pool. As the deposition progresses and the tailings beach builds up, the height of the discharge above the beach decreases. With this decrease will come a reduction in the vertical component of the velocity of the discharge jet, a shallower plunge pool, and higher stream power. The beach profile will become more rounded and coarser particles will be carried further along the beach. In areas where, at the start of the deposition, the deposited tailings was finer, the deposited tailings now becomes somewhat coarser. The degree of difference between the fine and coarse will depend on the discharge rates and the height of the discharge pipe above the beach at the start of the deposition. It is therefore important to manage the height of the discharge above the beach in the coarse of a deposition to ensure that a more uniform material is deposited over the period of deposition.

A higher discharge rate will encourage the placement of coarse material further from the discharge and could be important in ensuring the placement of coarse material over the underdrainage as set out in the next section.

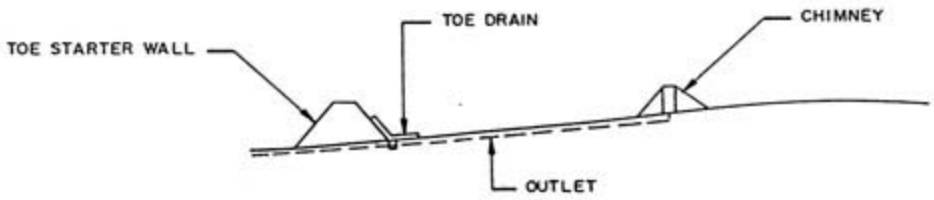
8.4 ENHANCEMENT OF UNDERDRAINAGE OPERATION

Underdrainage in a tailings dam has the most beneficial effect if it is located inside of the final crest and at ground level. This poses a problem for dams constructed of segregating slurry since during the commissioning of the dam deposition begins at the topographically lowest region of the dam. With each deposition a pond is formed at the contact between the tailings and natural ground. Since the stream power in the pond is effectively zero, within a very short distance into the pond the beaching stream loses all remaining coarse fraction leaving only the clay and fine silt size fraction in suspension. This fraction requires the stillness of the pond to settle out. The result is that a very fine layer is deposited over the base of the pond. If during the commissioning and development of the dam the pond is allowed to pause over the underdrain the drain will be blinded and will be rendered ineffective.

To satisfactorily cover the underdrainage with coarse material, several noteworthy methods of dam development have been developed and this author has been fortunate to be involved with these developments. These are described below

Figure 8.3 illustrates the method of establishing a blanket drain in an upstream dam of segregating slurry. The development procedure is as follows:

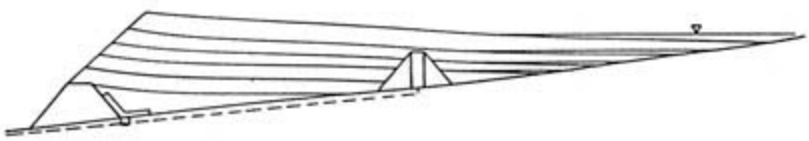
- during initial construction a toe starter wall is formed with a toe drain (Figure 8.3 (a)). This drain will encourage drainage of the tailings in the vicinity of the toe starter wall during the first few depositions and facilitate rapid access for construction of the crest embankment. At the same time a chimney drain is constructed at the preferred location of the main drainage blanket. The outlet to service the drain is constructed.



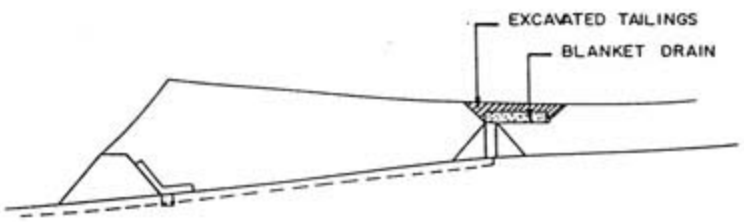
(a) INITIAL CONSTRUCTION



(b) DEPOSITION COMMENCES



(c) DEPOSITION PROGRESSES



(d) ELEVATED BLANKET INSTALLED

FIGURE 8.3 : CONSTRUCTION OF ELEVATED DRAINS WITH CHIMNEYS

- deposition commences with care to avoid washing away and blinding of the toe drain (Figure 8.3 (b) and (c))

- once the pond has been pushed permanently beyond the chimney, and the beached material in the vicinity of the chimney is adequately coarse, deposition activity is temporarily stopped and the beach is allowed to dry to permit access. The tailings is then excavated to, and between, the chimneys and an elevated blanket drain installed and mechanically covered with the excavated tailings to reinstate the beach. Excess tailings is heaped to avoid creating an obstruction behind which new ponding may occur at the recommencement of deposition.

- construction of the elevated blanket is phased to reduce the disruptions to deposition activity and to accommodate natural ground topography.

Using the above methodology it has been possible to limit the elevation of the drain to 4 m above natural ground level in the topographically lowest parts of the dam and to generally keep this to 1 to 2 m.

Where topography is flatter or is sloped away from the toe starter wall it is possible to avoid the use of chimneys and to excavate to the pre-installed trench as illustrated in Figure 8.4. This method facilitates installation of the blanket closer to natural ground but is more sensitive to access for construction of the blanket.

In the case of high tailings dams where the blanket drain will be located at large a distance into the dam where there is concern that the beached tailings may be too fine the method illustrated in Figure 8.5 has been employed where the topography slopes away from the toe starter wall. The method of development is as follows:

- initial construction involves placement of all of the underdrainage components (Figure 8.4 (a)). An inner starter embankment is formed immediately outside of the main blanket and a sacrificial penstock decant is provided at the outer edge of the inner starter embankment. The outlet pipe for this decant is taken beyond the main blanket.

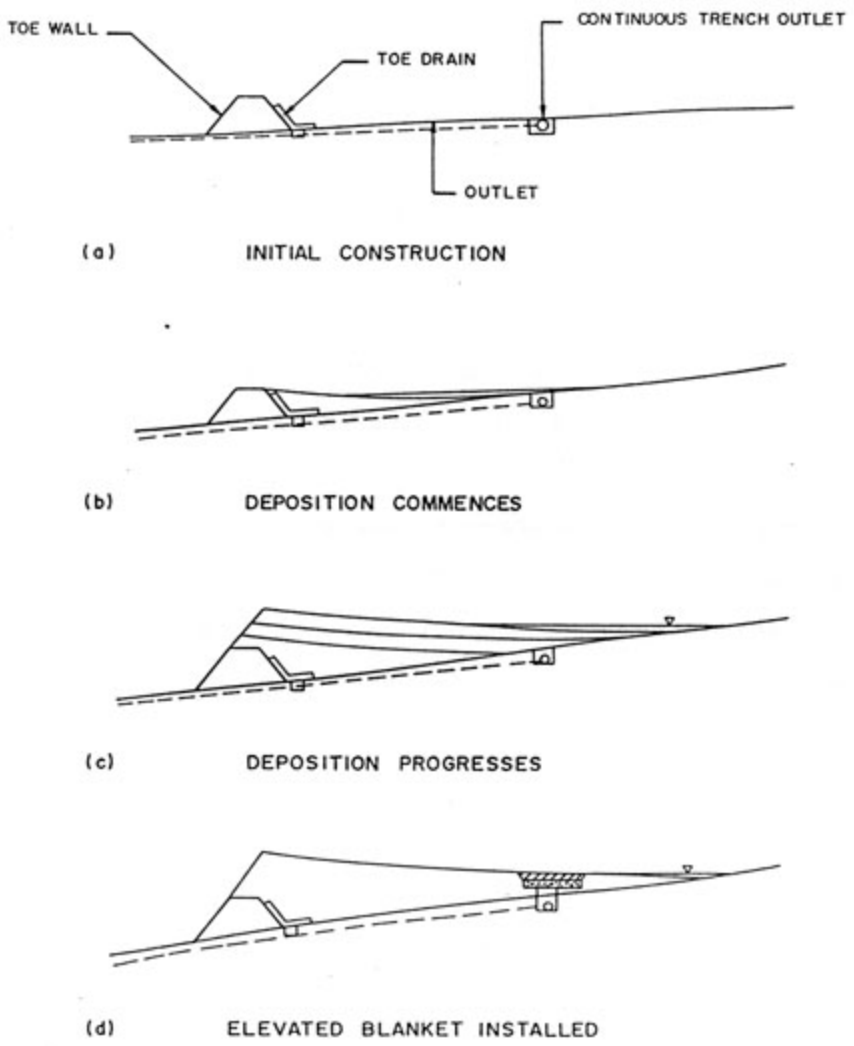


FIGURE 8.4 : CONSTRUCTION OF ELEVATED DRAINS WITHOUT CHIMNEYS

- deposition commences from the outer starter embankment (Figure 8.5 (b)). Pond water is decanted through the inner embankment beyond the main drain

- deposition then commences from the inner starter embankment and the relative heights of the inner and outer crests maintained until 3 to 5 m of coarse tailings has been deposited over the main drain (Figure 8.5 (c)).

- at the appropriate height deposition at the inner crest is stopped and the outer beach allowed to envelop the inner crest by channelling through initially (Figure 8.5(d)).

- deposition continues only from the outer crest.

To remove the effects of finer layers that tend to form even with careful discharge the material over the main drain is rotovated or tilled yielding a uniform coarse cover to the drain which effectively forms an extension of the drainage structure.

An approach to the design of a high tailings dam where deposition is against the contours is illustrated in Figure 8.6. In this case a smaller "buttress" dam was formed at the toe of the dam using the methods described above. The main drain was then constructed behind the buttress dam and covered mechanically by pushing the coarse material from the buttress dam over the main drain (Figure 8.6 (a)). Sufficient material was pushed to allow the pre-formation of a beach. Deposition from behind the buttress dam into the main dam was then commenced beginning at the foot of the pre-formed beach (Figure 8.6 (b)).

8.5 CONTROLLED PLACEMENT OF FINE MATERIAL

An important consideration in deciding on the spacing of discharge points is the problem of mounding at the discharge points and the consequent formation of valleys between them. Apart from generating an uneven crest elevation there is a tendency for ponding in the valley as the beached material cuts off drainage from the valley. Fine layers which can cover an extensive area are left in the embankment area. These layers will hinder

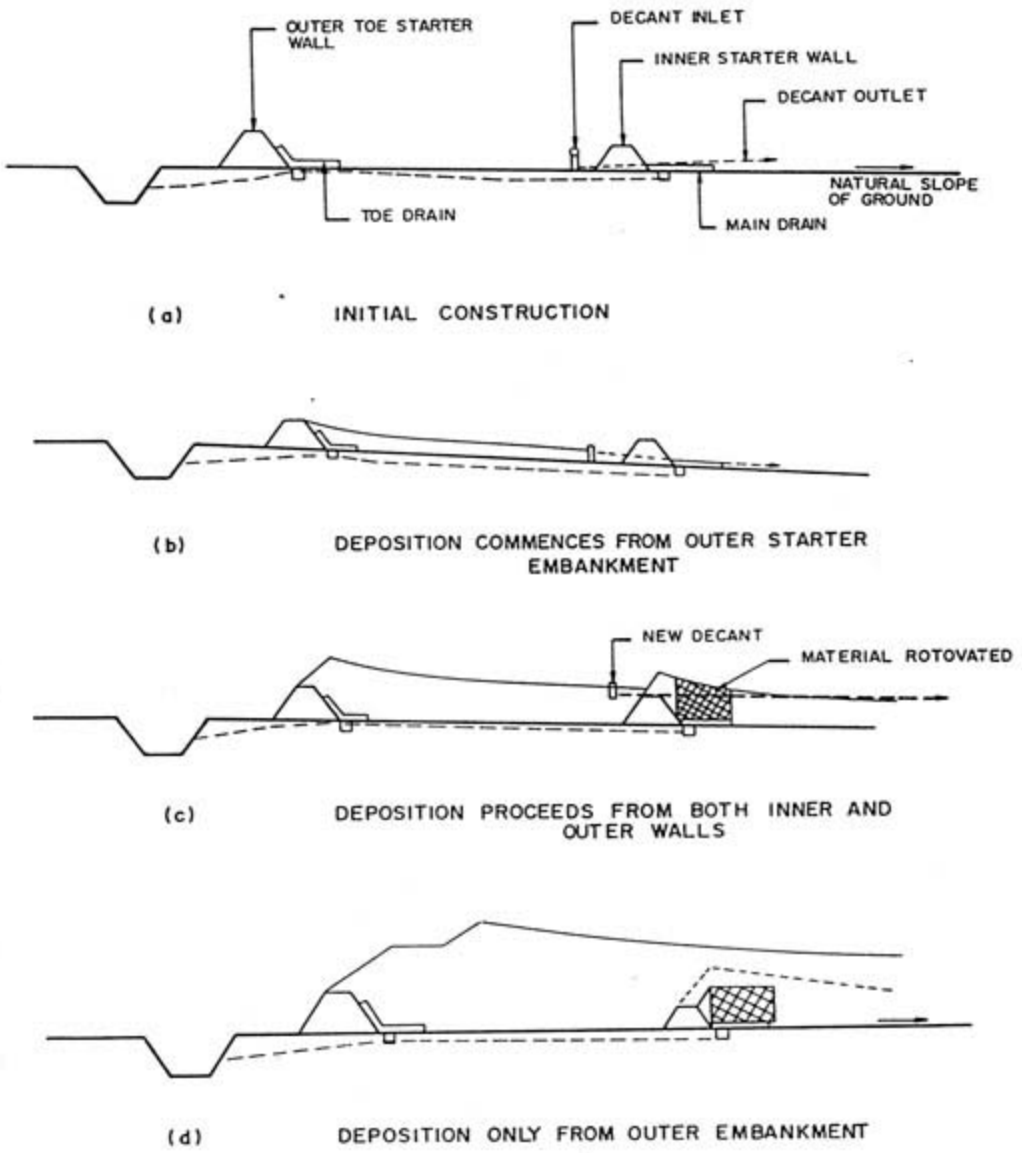


FIGURE 8.5 : DRAIN CONSTRUCTION USING THE INNER-OUTER WALL METHOD

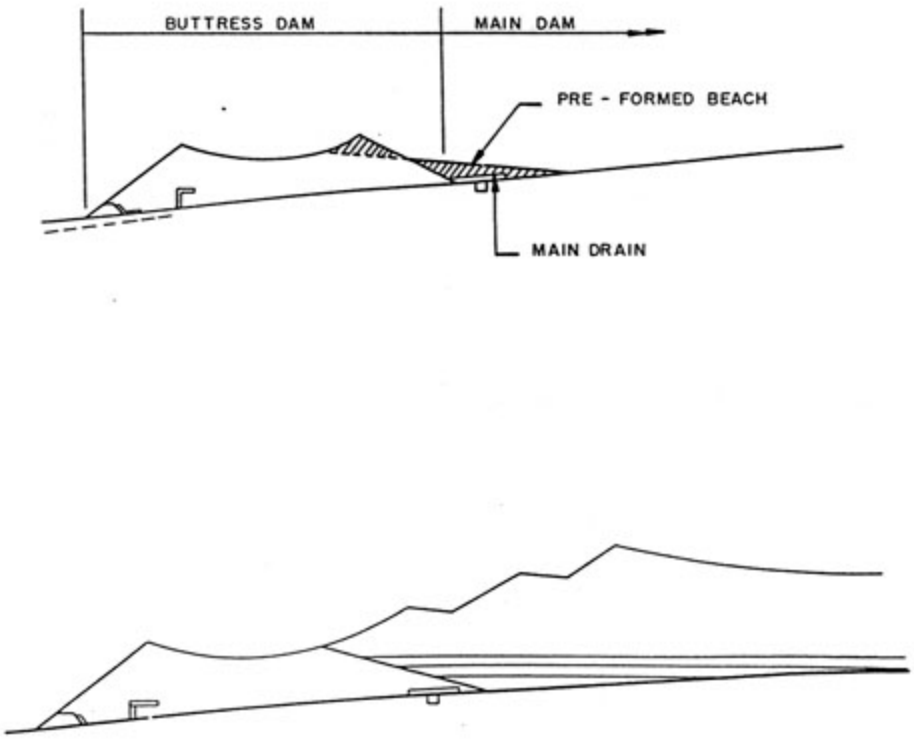


FIGURE 8.6 : DRAIN CONSTRUCTION BY MEANS OF THE BUTTRESS DAM CONCEPT

the passage of wetting fronts that are generated with each deposition, and which must pass to the base of the dam, and will reduce the safe rate of rise of the dam. These layers were determined as the principal cause of the Bafokeng failure of 1974 (Jennings (1978)). It was in an attempt to prevent further layering that the mining company concerned has subsequently converted to a spraybar system with baffled discharges so that the discharge jet is directed at the beach thus encouraging the deposition of coarse material close to the delivery.

It is possible to hydraulically place a liner over up to 60 % of the base of the dam through careful location of temporary cross walls parallel to the natural ground contours at regular centres and control of the formation of temporary ponds against the walls. Since the underdrainage system maintains an unsaturated embankment area which can comprise some 20 % of the tailings dam it is possible to significantly reduce the hydraulic gradients at natural ground level and therefore potential seepage losses.. Figure 8.7 illustrates a potential layout for the tailings dam and temporary cross walls (Figure 8.7 (a)) and a typical cross-section (Figure 8.7 (b)). The walls are breached in multiple locations prior to overtopping to encourage the flow of pond fines into the next (up contour) section.

The selection of discharge method to maximise the above needs to be carried out in conjunction with the selection of the discharge method to maximise the placement of coarse material in the embankment areas.

8.6 CONTROL OF POOL LOCATION

Suitable design and regulation of the discharge method can facilitate improved pond control. This point is elaborated on by referring to the tailings dam layout illustrated in Figure 8.8. Figure 8.8 shows a plan of a tailings dam with an off-centre decant. The figure indicates a "pool wall" formed using the same methods as the employed at the crest of the dam with the objective of lengthening and thereby equalising the beach lengths. By placing the decant in the nick of the pool wall the maximum depth of pond is ensured at this point since the length of beach to this point from any point on the crest is maximised.

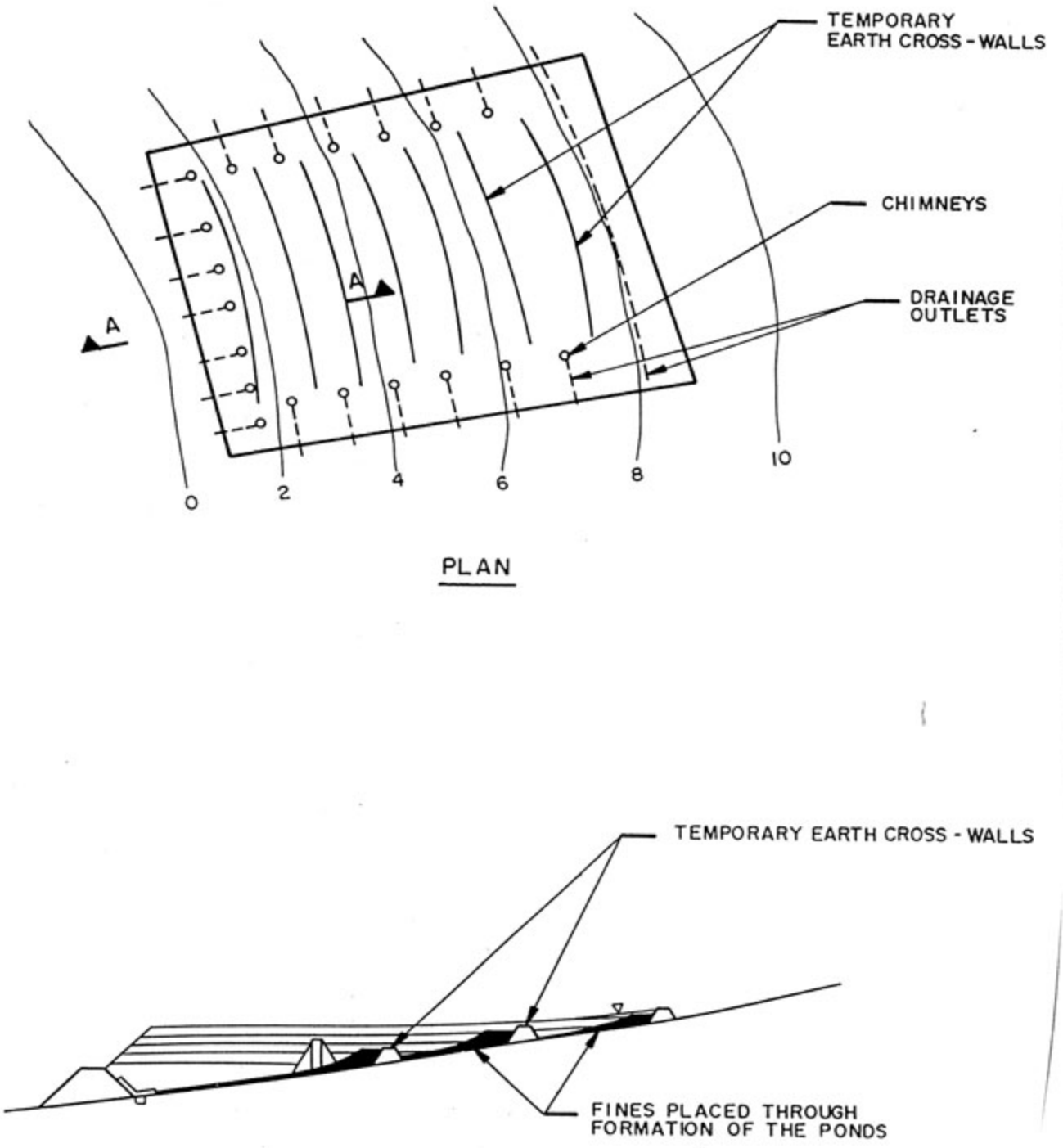
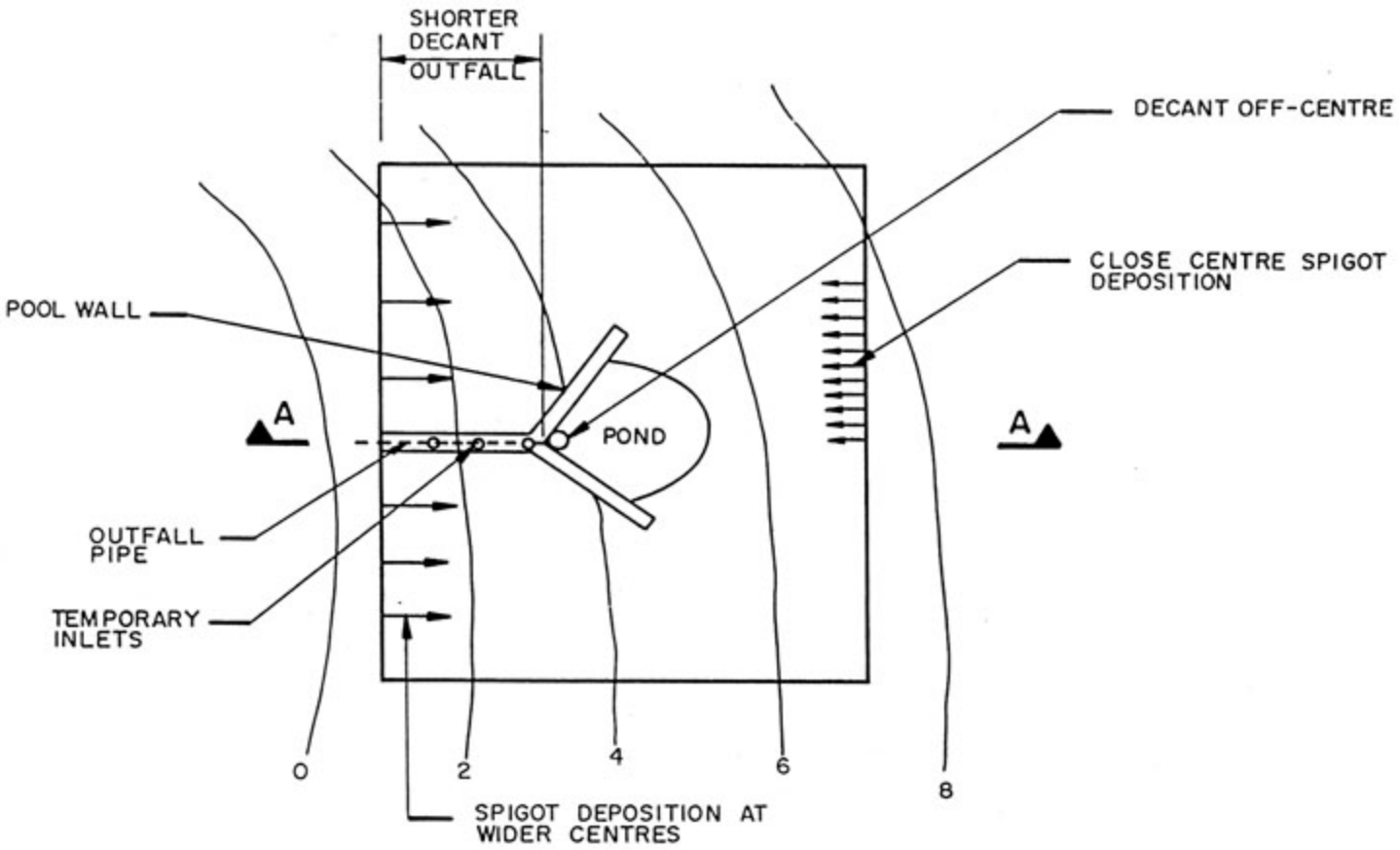


FIGURE 8.7 : PLACEMENT OF FINES OVER THE FLOOR OF THE TAILINGS DAM



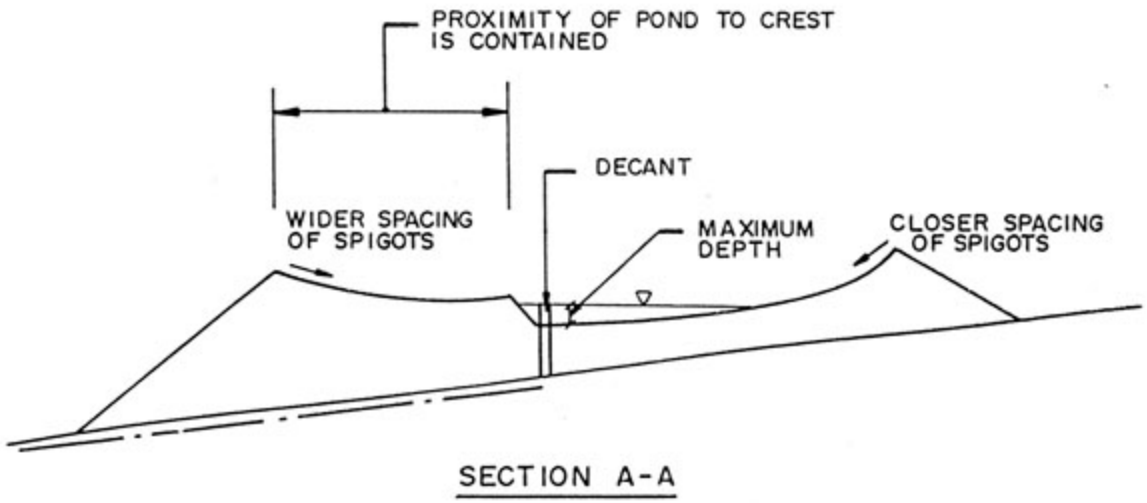


FIGURE 8.8 : ILLUSTRATION OF POOL WALL OPERATION

The reasons for establishing an off-centre decant are relate to safety and economy. By carefully locating the pool wall it is possible to control the proximity of the edge of the pool relative to the outer crest and at the same time minimise the length of the penstock outfall. The centroid of the pond is therefore centrally located in the basin of the dam without having to locate the penstock at this point.

The pool wall concept evolved from operations on South African gold tailings dams where beaching angles are shallow and the area storage characteristics of the basin are poor. This hampers the decanting of clear water. By placing a pool wall around the inlet the depth of the pond at the inlet is maximised and the clarity of the decant water is improved. A further advantage offered by the pool wall is that it controls the extent of expansion of the pool during storm events. It was recognised that the advantages of the pool wall could also be obtained on dams constructed of segregating tailings. Unlike paddock dam construction where freeboard is provided not only by the beach but also by the daywall in extreme events, all freeboard provision on dams constructed of segregating tailings has to be provided by beach. There is therefore a greater need to maintain a level crest. The pool wall provides an opportunity to control beach lengths and therefore beach freeboard and offers considerable improvement in dam performance if operated in conjunction with regulated discharge from spigots or spraybar deliveries.

Regulation of the number of, and therefore flow rate from, spigot deliveries makes it possible to control the beaching profile such that a flatter profile is obtained on long beaches and a steeper profile is obtained on shorter beaches as illustrated in Figure 8.8.

8.7 MAXIMISED TAILINGS STORAGE CAPACITY

On larger tailings dams where beach lengths exceed 500 m there can be a considerable loss of tailings dam capacity as a result of the basin formed by the beach profile. Controlled deposition practice presents opportunities for utilising this volume as discussed below. An additional important related consideration is the fact that the finer beached tailings has a lower in-situ density than the coarser tailings as a result of the poorer packing characteristics of the finer tailings. On tailings dams where the beach lengths exceed 1 km there can be significant loss of capacity as a result of this lower density. Moreover, as a result of the lower density there can be a significant lock up of water in the voids of the low density tailings. The density

can be improved and the volume of water lost can be reduced if coarser, heavier tailings can be placed over the fines.

On tailings dams with beach lengths up to 1 km part of the basin volume can be used up by :

- stepping the deliveries in from the crest wall as illustrated on Figure 8.9 (a).

- regulating the rates of discharge to achieve a flatter beach profile. This would require alternating between closely spaced spigot discharge of low flow rates, to attain a steep outer beach while filling and levelling the basin, and open ended delivery with high discharge rates. This will cause the stream to cut into the beach but the depth of cut will be limited by the elevation of the pond area. The coarse material will be transported to the inner parts of the basin as a result of the high stream power of the stream.

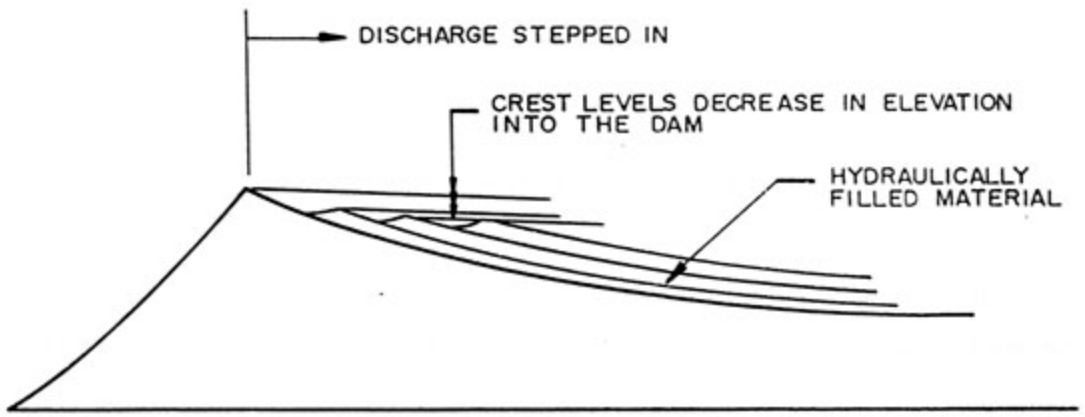


FIGURE 8.9 : FILLING THE BASIN BY STEPPING IN THE DELIVERY LINES

The degree of filling of the basin is determined by post closure storm water control requirements.

On tailings dams of longer beach lengths it is preferable to establish smaller paddocks within the basin and regulating the height differences between the paddocks too reduce potential slope stability problems associated with placement of coarse tailings over the fine material in the basin. Figure 8.10 illustrates this principle. It is evident from Figure 8.10 that the formation of the paddocks requires regulation of the deposition using spigot or spray-bar discharge so as to achieve uneven beach lengths.

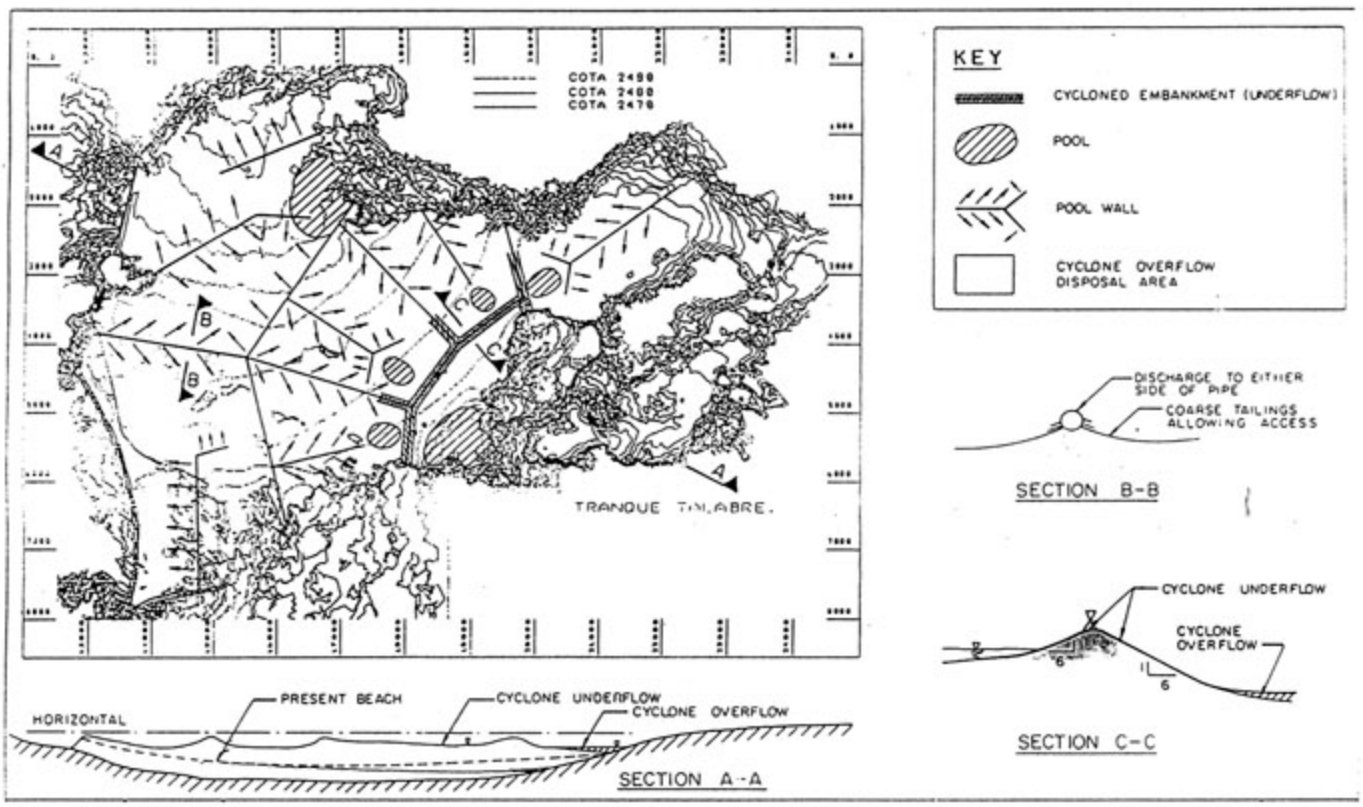


FIGURE 8.10 : LAYOUT OF PADDOCKS ON A LARGE TAILINGS DAM

The following two sections of this chapter describe two case studies where the deposition model has been used to determine a future deposition strategy so as to enhance the performance of the tailings dam. In both cases changes in deposition practice were necessitated through a change in tailings product.

8.8 CASE STUDY : PLATREEF TAILINGS DAM

8.8.1 Introduction

The ore body at the newly commissioned Potgietersrust Platinums Mine located in the Bushveld Igneous Complex in the northern Transvaal province of South Africa is characterised by serpentinised zones within the pyroxenite host rock. The serpentinised zones are variably high in talcs and calc silicates. These minerals differ significantly in hardness both in comparison with each other and in comparison with the pyroxenite ore body. Moreover, the serpentinites contain saponite which may be described as "soft, like butter or cheese but brittle on drying" (Dane and Ford (1966)). The mineral is described as having a soapy feel as indicated by its name which is derived from *sapo*, soap and piotine, greek for fat. The specific gravity of this mineral at 2,24 to 2,3 is low especially in comparison to the host rock which has a specific gravity in excess of 3.11. As a consequence of this low specific gravity the mineral tends to "float" on the tailings stream surface and forms fine layers wherever it settles out.

Milling rates in the reduction plant have been set so as to ensure adequate break down of the pyroxenites which in unoxidised form are of relatively high hardness. During the early stages of mine development variably oxidised rocks are mined and the milling rates require tempering to ensure that excessive grinding does not occur.

Notwithstanding fine tuning of the milling rates these are set to suit metallurgical requirements which will always result in a fine fraction due to the softness of the talc-type minerals in the serpentinised zones.

A further complicating factor is the fact that the serpentinised zones occur as lenses and are scattered through the ore body. Even with extensive blending it has not been possible todate to minimise the tendency for periods of predominantly high talc content (soft) rocks to be processed, and equally, the tendency for predominantly low talc (hard) rocks to be processed. The result is that for periods of some 1 to 2 hours a tailings product dominated by either hardness characteristic occurs. This results in layers of fine and coarse material being formed. Even during "average" conditions the soft, low density minerals in the serpentinites result in layering with distance down the beach as the deposition streams meander.

The layering presents two problems which, if not negated, will reduce the capacity of the tailings dam potentially to half that for which it has been designed. The costs for constructing a replacement dam (not to mention the environmental constraints that dictated the location and geometry of the present dam and that would play a still more significant role in the new dam), would be of the order of R10 m. The two problems are:

- the potential for precipitating a "piping" type failure as occurred in the Bafokeng disaster of 1974 a failure that had its root in the layering of that dam.

- the potential for blinding the main elevated blanket drain which, in the case of the platinum tailings dams is constructed after commissioning of the dam to avoid blinding during initial development of the dam.

The problem of layering will not easily be overcome. However it is possible to reduce the impact of this layering through adjustments to depositional procedures. Trial depositions using a hybrid deposition method described in Section 8.8.5 have been conducted. The trials are supported by beaching simulations carried out on the basis of geotechnical tests undertaken on tailings products sampled during the trials. These trials will be on-going over the next two to three years.

8.8.2 Platreef Tailings Product

The make up of the tailings product and the influence of serpentinitic lenses has been set out in the preceding section. In this section the results of particle size distribution testing and viscosity testing are described.

- **Particle size distribution testing**

As commissioning of the mine and metallurgical reduction plant has progressed milling rates have been adjusted to improve recoveries. This has resulted in a general coarsening of the tailings product with the minus 75 micron fraction reducing from 73 percent to 58 percent. These particle size distributions are illustrated in Figure 8.11.

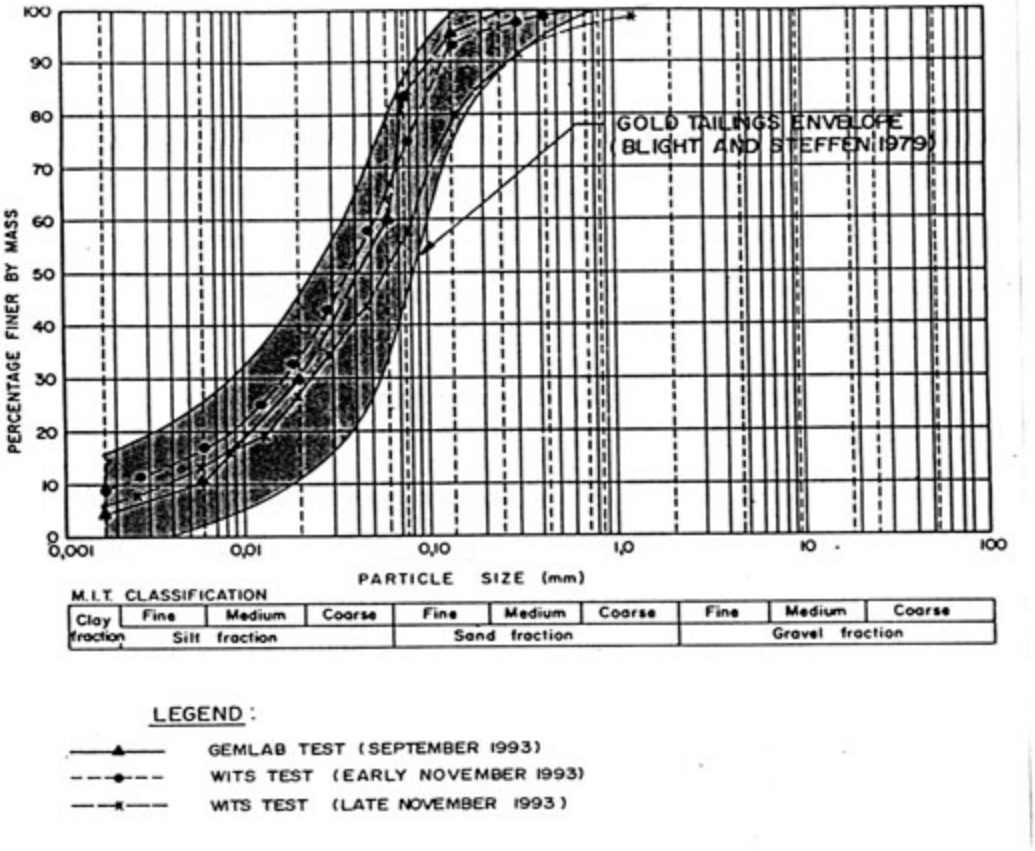


FIGURE 8.11 PARTICLE SIZE DISTRIBUTIONS OF THE COARSE AND FINE PRODUCTS

It is significant that the reduction in milling rate has not produced a concomitant reduction in the minus 2 micron fraction, the fraction that has the greatest impact on layering and the permeability of the tailings.

At 73 percent minus 75 micron the tailings falls within the gold tailings envelope of Blight and Steffen (1979) and it can be expected that the overall beach would be relatively flat and yield poor freeboard. At 58 percent minus 75 micron the standard paddock system of operation can be expected to be less suitable due to more significant gravitational segregation.

- **Slurry and Viscosity Characteristics**

Viscosity testing was carried out at the University of the Witwatersrand on a range of slurry concentrations and shear rates. The results were analysed to determine the most representative rheological model. The analyses indicate that the slurry tends to be very slightly Bingham plastic but can be adequately modelled using Newtonian equations. Figure 8.12 shows the variation of the Newtonian viscosities with concentration. Also indicated on Figure

8.12 are the viscosities calculated using a fitted equation adopted in the beaching simulations in line with the equations in Chapter 2. Friction factors were calculated using Verkerk's programme as described in Chapter 4 and the resulting factors entered into each appropriate beaching simulation. The friction calculations have been carried out at two slurry densities:

- 1,45 t/m^3 (45,7% solids) representing the present slurry density
- 1,75 t/m^3 (63.2% solids) representing a maximum achievable allowing for coarsening of the tailings product and a consequent improvement in thickener performance.

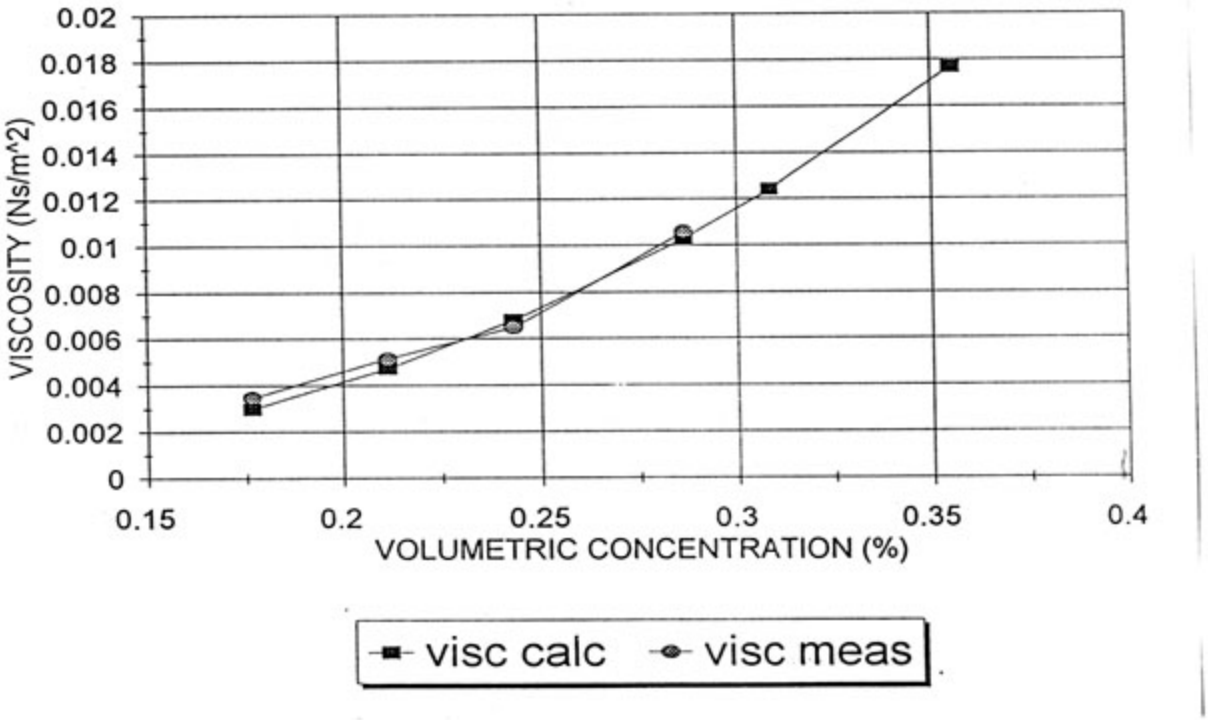


FIGURE 8.12 : CONCENTRATION vs VISCOSITY

8.8.3 Beaching Characteristics

Beaching simulations have been based on a horizontal discharge from the spigots of a spray-bar pipe. The height of the discharge above the beach level has been varied to provide an indication of the beach that is likely to be characteristic of each height. Figure 8.13 shows the simulated beaches for slurry densities of 1,45 t/m^3 for the 58 percent minus 75 micron product. Figure 8.14 shows the measured beach profile which would be representative of a discharge

height of 0,4 m as indicated in the photo in Figure 8.15. Comparison of the measured beach with those in Figure 8.13 shows that the simulations yield realistic results.

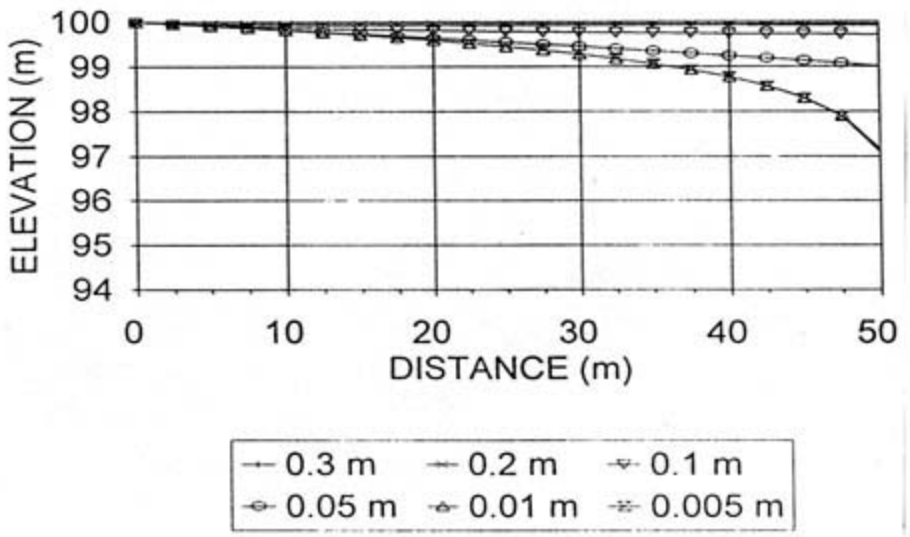


FIGURE 8.13 : 50 m BEACH, COARSE PRODUCT, DENSITY 1,45 t/m^3

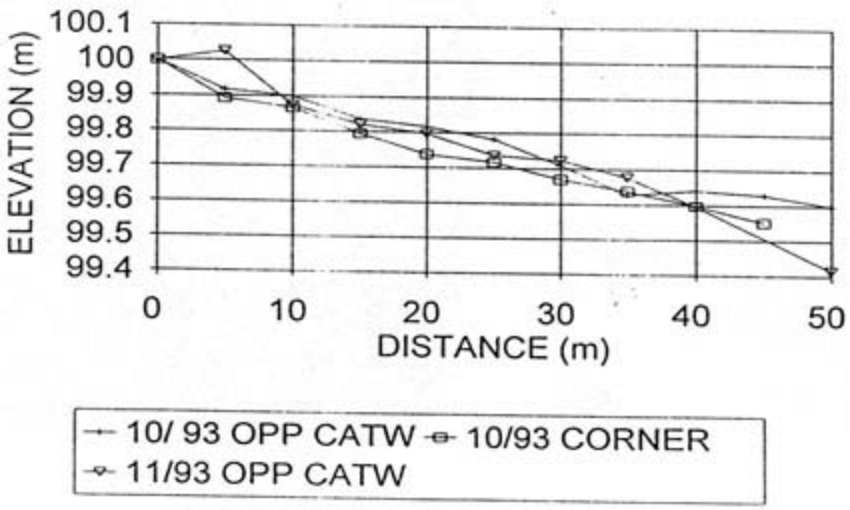


FIGURE 8.14 : MEASURED BEACH

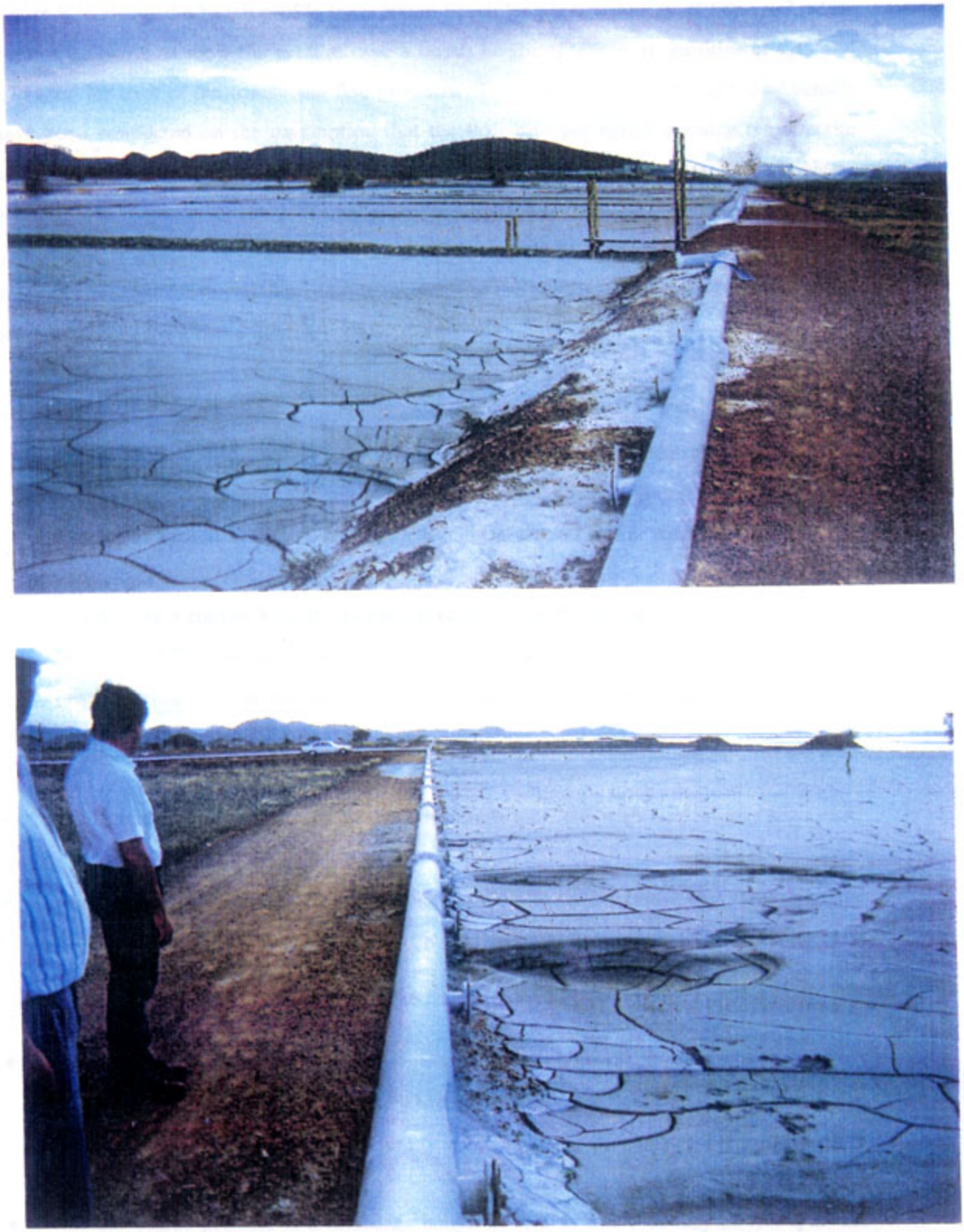


FIGURE 8.15 : PHOTOGRAPHS SHOWING TYPICAL DISCHARGE CONDITIONS DURING DEPOSITION TRIALS

Figure 8.16 shows the beach profile for a 200 m beach length at a density of 1,45 t/m^3, while Figures 8.17 to 8.20 show the profiles over a beach length of 700 m at densities of 1,45 and 1,75 t/m^3 for each of the coarse and fine products. The simulations for the 1,75 t/m^3 density have been conducted on the assumption that the flow rate per spigot opening remains the same as for the 1,45 t/m^3 simulation. For a constant milled tonnage this means that 7 spigots openings are required for the 1,75 t/m^3 slurry as opposed to 12 spigot openings for the 1,45 t/m^3 slurry. Figures 8.21 to 8.28 show the predicted particle size distributions with distance down the beaches for the coarse and fine products at the two densities. The following points are noteworthy from these figures:

- the beach profile is exceedingly sensitive to the discharge height. This is due to the fact that at shallow discharge heights less stream power is lost in the plunge pool. The stream therefore begins flowing at a higher stream power. Since this stream power reduces to zero at the pond the higher the initial stream power the steeper the beach.

- by adopting a routine operating practice of ensuring that depositions proceed until the newly deposited tailings reaches the spigot pipe it will be possible to generate adequate beach freeboard. This conclusion would need to be confirmed through trial depositions on site.

- the beach profiles for the denser slurries are steeper, although only slightly so.

- more significant segregation occurs at higher discharge heights. This is because the stream power of the flow is lower and there is less energy available to maintain particles in suspension.

- at a discharge height of 0,01 m and a density of 1,45 t/m^3 the fine and coarse products undergo less gravitational segregation than for a discharge height of 0,2 m. This is due to the lower stream power.

- for the coarse product at a discharge height of 0,01 m and a density of 1,75 t/m^3 the coarse material is deposited over a greater distance down the beach. This is due to a combination of a higher stream power and a greater quantity of coarse product.

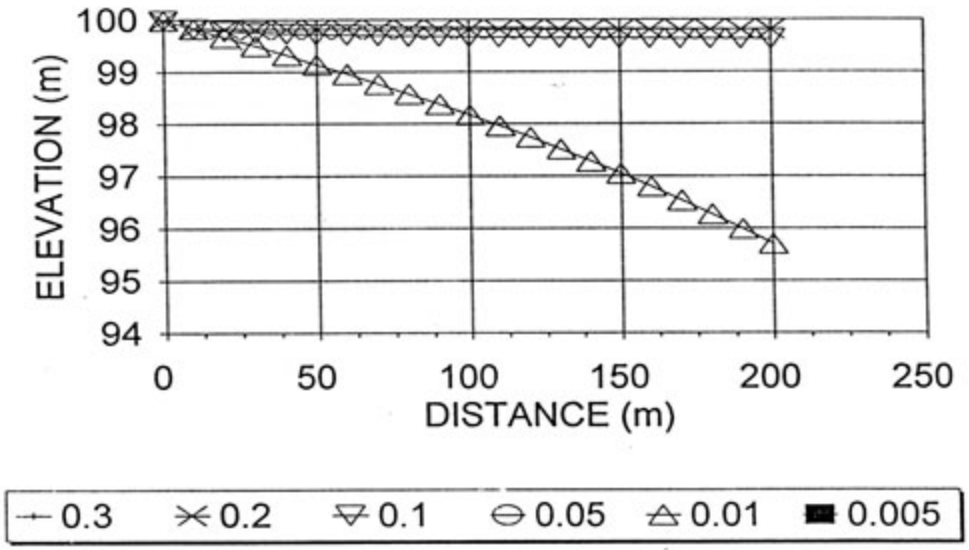


FIGURE 8.16 : 200 m BEACH, COARSE PRODUCT, DENSITY 1,45 t/m³

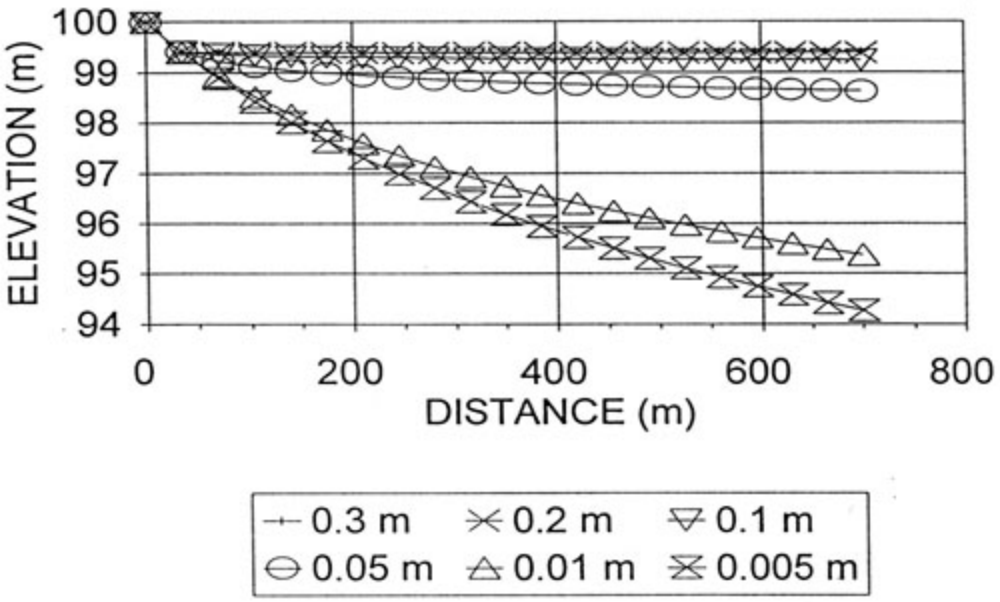


FIGURE 8.17 : 700 m BEACH, COARSE PRODUCT, DENSITY 1,45 t/m³

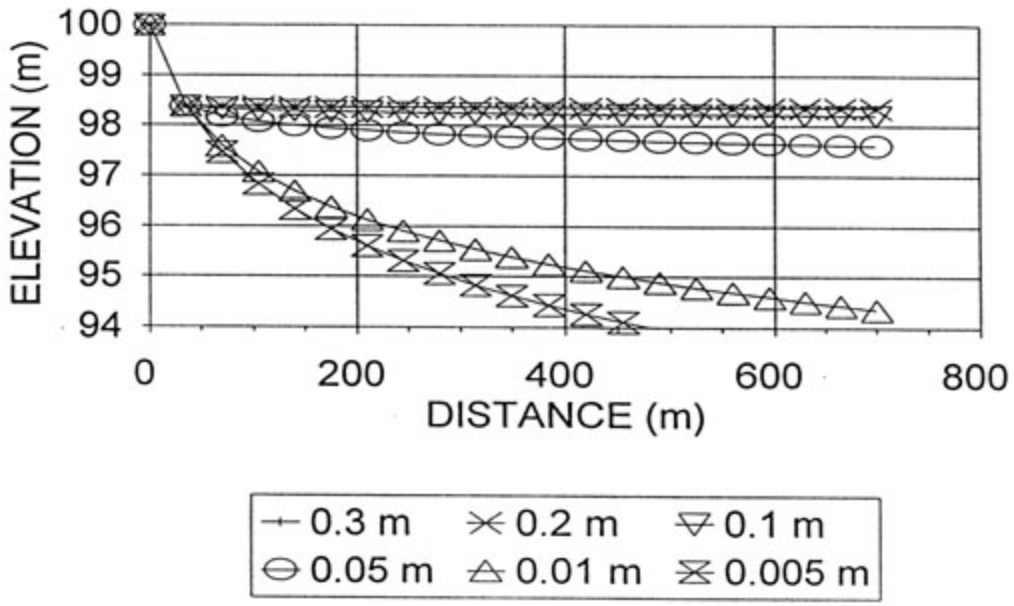


FIGURE 8.18 : 700 m BEACH, COARSE PRODUCT, DENSITY 1,75 t/m³

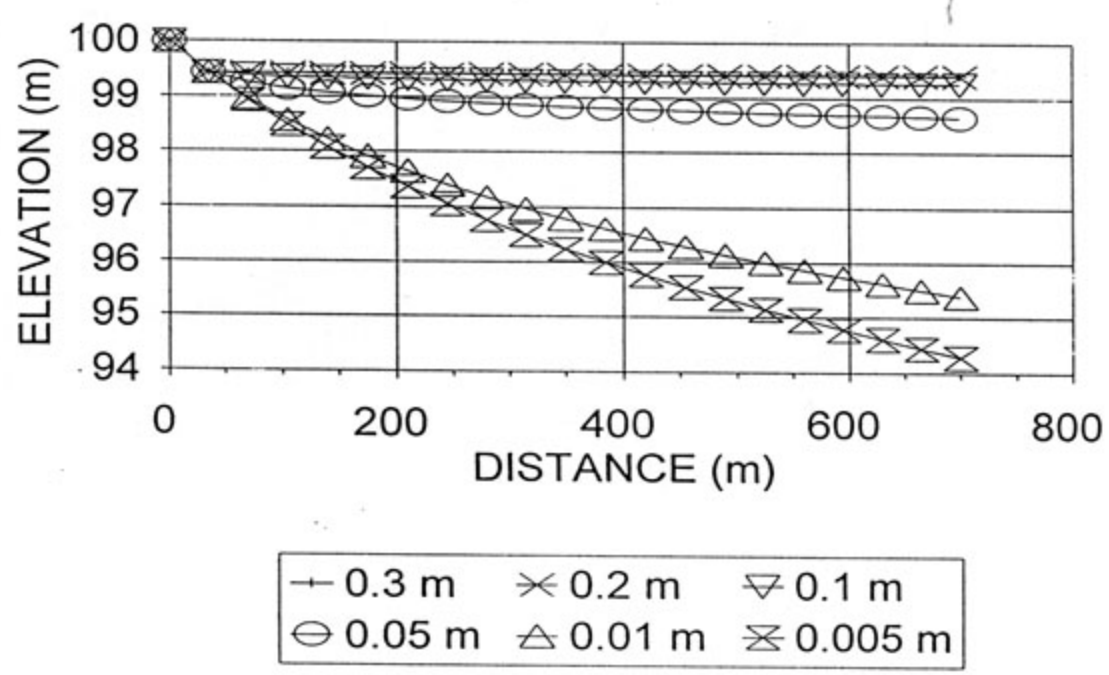


FIGURE 8.19 : 700 m BEACH, FINE PRODUCT, DENSITY 1,45 t/m³

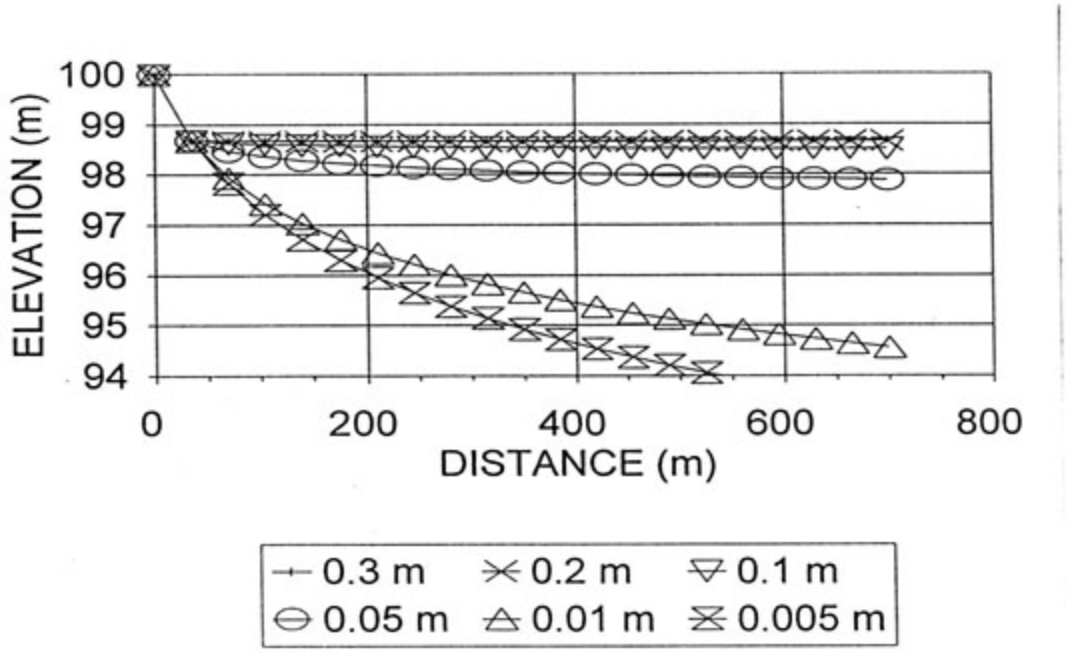


FIGURE 8.20 : 700 m BEACH, FINE PRODUCT, DENSITY 1,75 t/m³

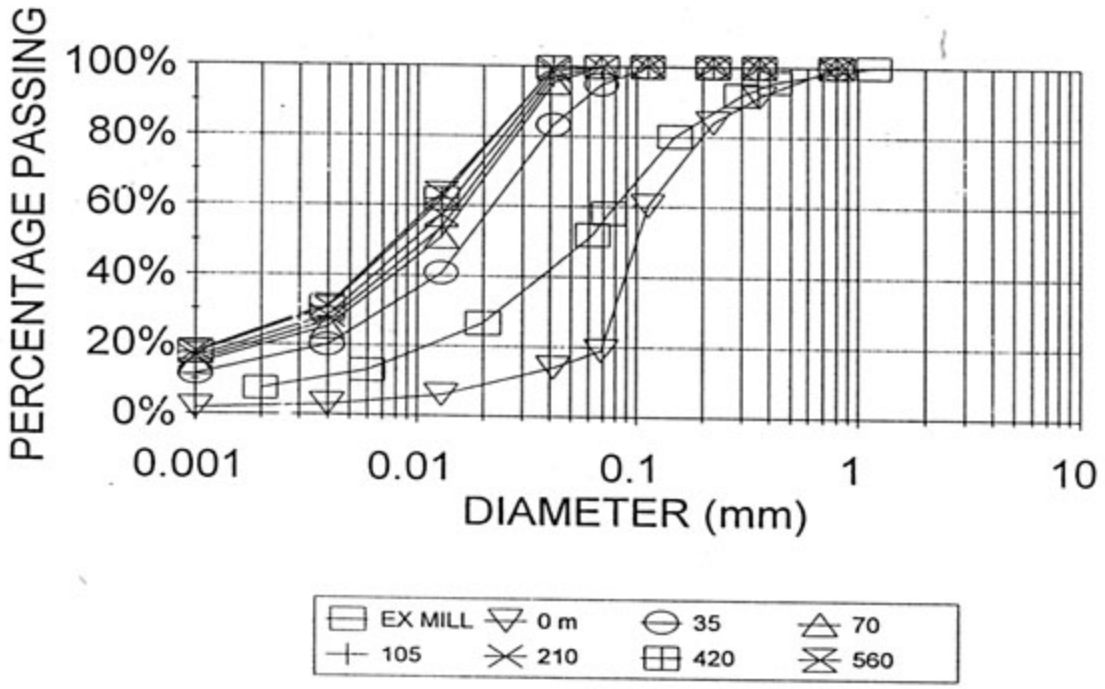


FIGURE 8.21 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,2 m, DENSITY 1,45 t/m³

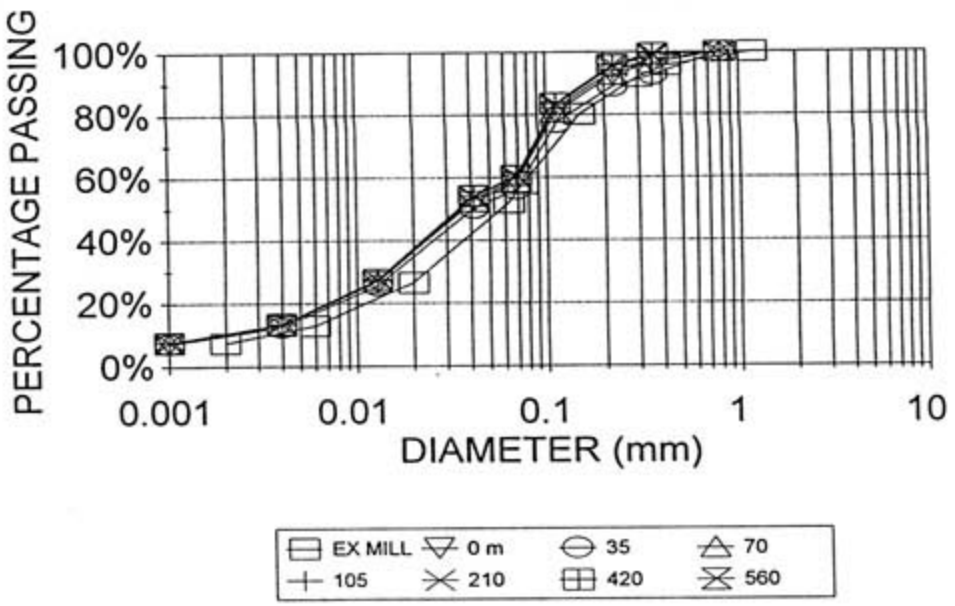


FIGURE 8.22 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,01 m, DENSITY 1,45 t/m³

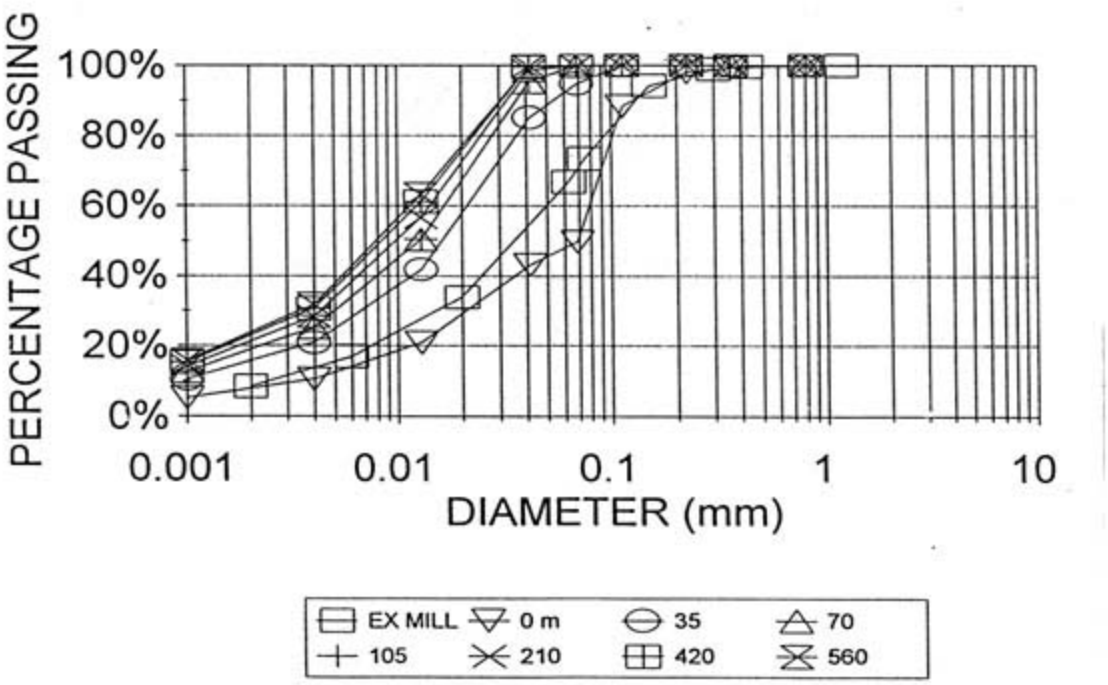


FIGURE 8.23 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,2 m, DENSITY 1,45 t/m³

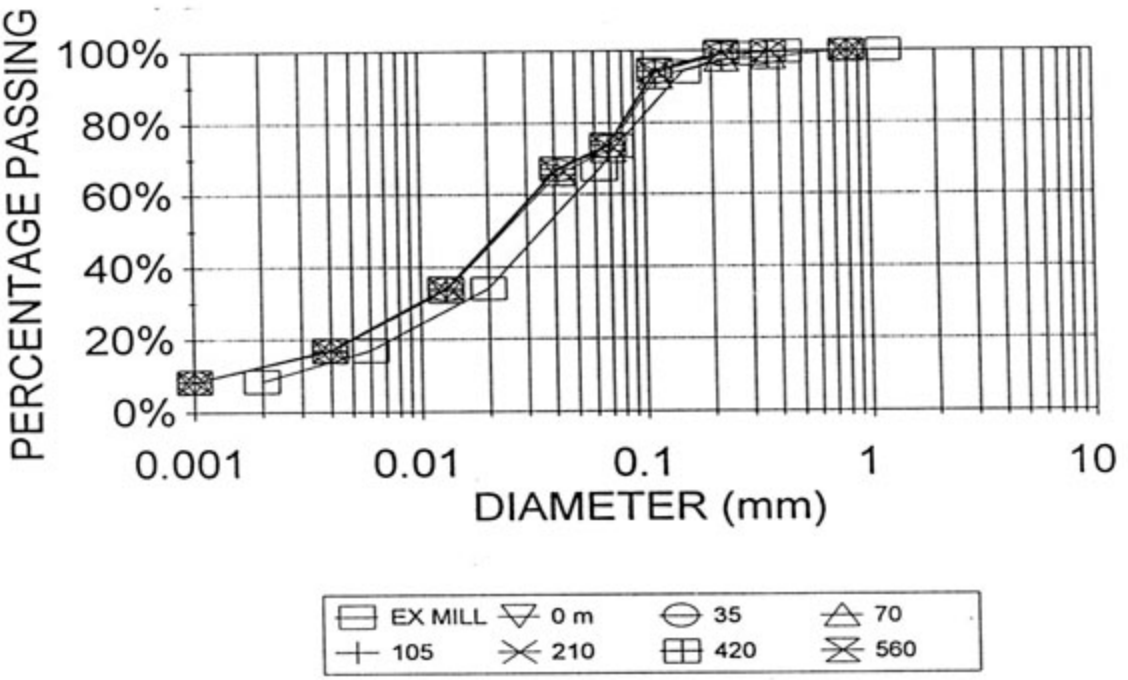


FIGURE 8.24 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,01 m, DENSITY 1,45 t/m^3

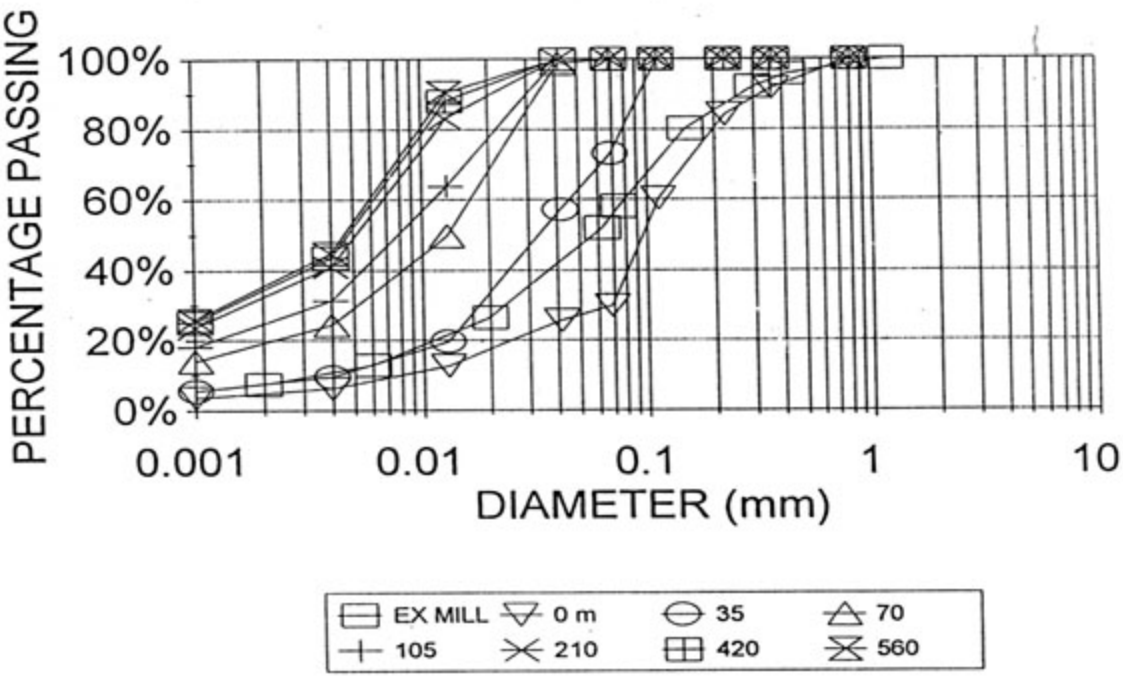


FIGURE 8.25 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,2 m 1,75 t/m^3

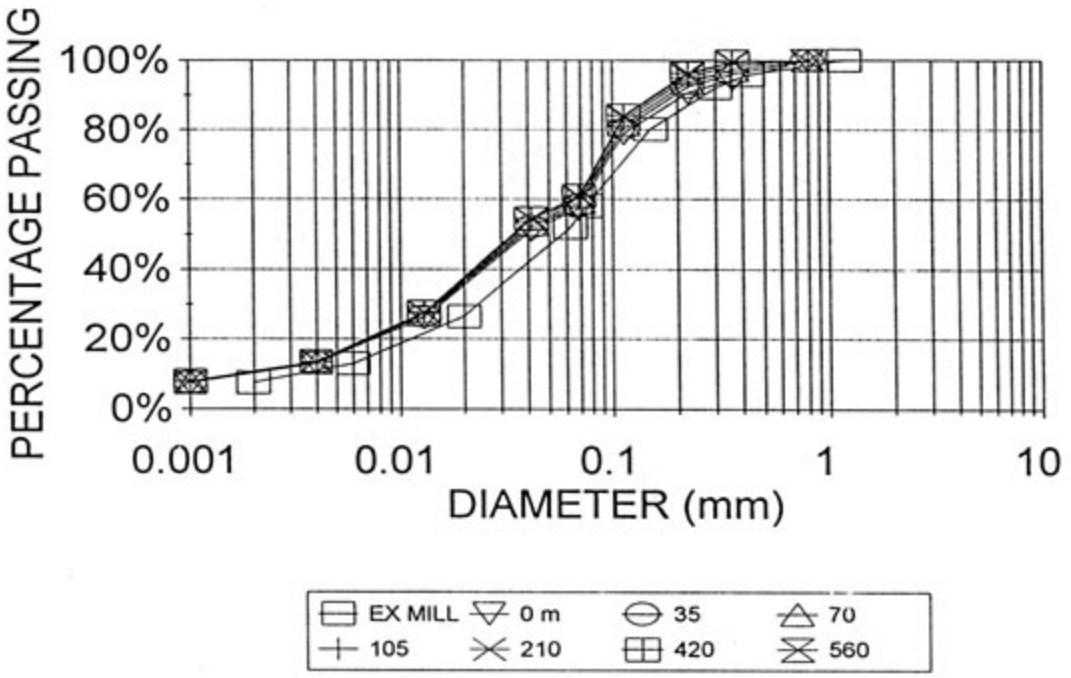


FIGURE 8.26 : 700 m BEACH, COARSE PRODUCT, DISCHARGE HEIGHT 0,01 m, DENSITY 1,75 t/m^3

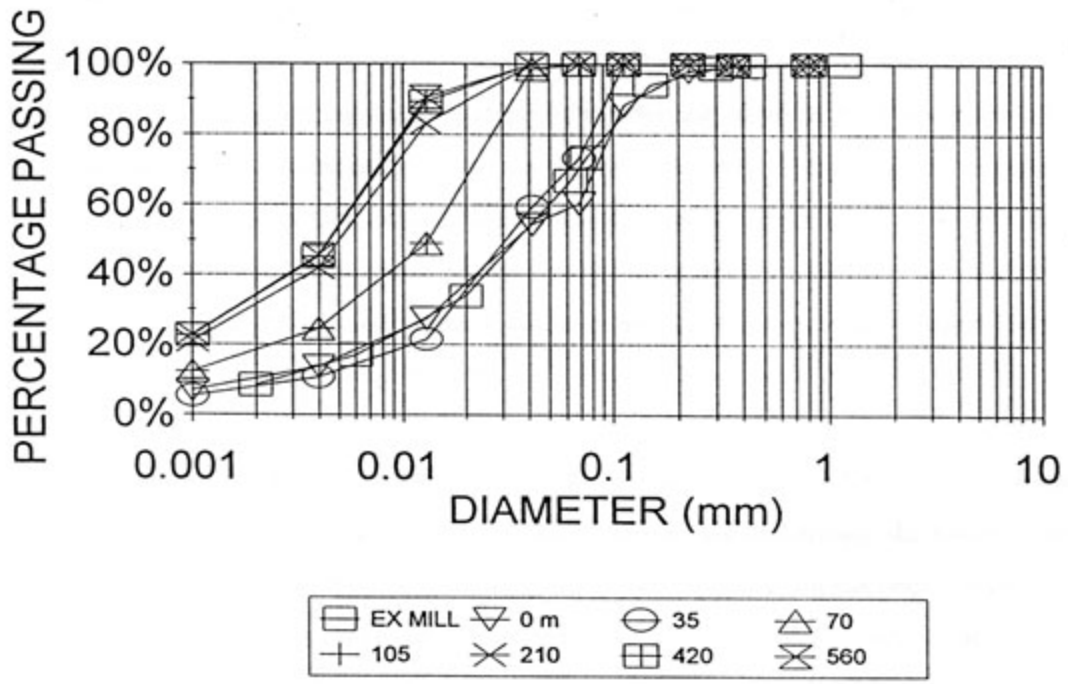


FIGURE 8.27 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,2 m, DENSITY 1,75 t/m^3

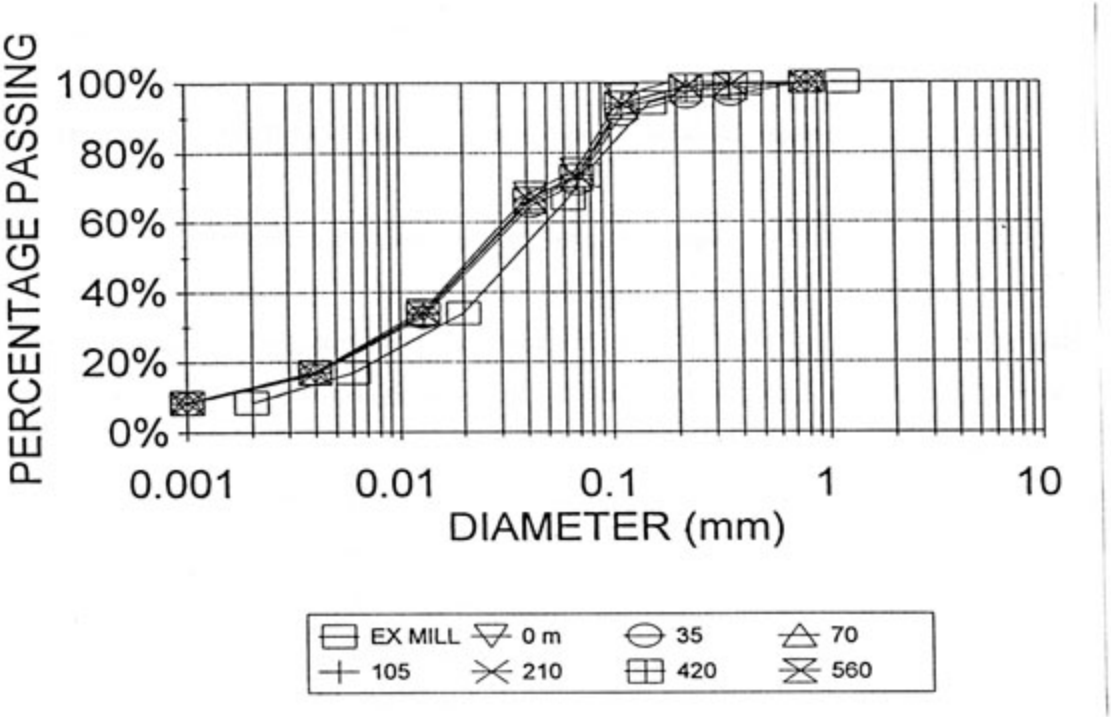


FIGURE 8.28 : 700 m BEACH, FINE PRODUCT, DISCHARGE HEIGHT 0,01 m, 1,75 t/m³

The results of the simulations indicate that it will be inadvisable to adopt a conventional paddock system as suggested by the particle size distributions in view of the gravitational segregation that will occur during the early stages of the deposition when the discharge heights are at a maximum and of the order of 0,2 m. It will, however, be possible to achieve a reasonable difference in elevation between the crest and the pond using the conventional spigot deposition system provided depositions proceed until the deposited tailings reaches the base of the spigot pipe. Under these conditions it will be unnecessary to adopt the paddock system as a means for generating greater freeboard.

The simulations do not take account of variations in the tailings product that will cause layering. To negate the effects of layering on drain performance and reduce the potential for a piping-type failure it will be necessary, initially and until the main drain has been constructed and covered with a significant amount of unlayered coarse material, to implement specific deposition methods such as those outlined in Section 8.8.5. These methods may be described as a hybrid of the spigot and paddock systems.

These aspects are discussed in more detail in the next section.

8.8.4 Description of a Hybrid Paddock-Spray bar Deposition system

- **Method of Deposition Proposed at the Time of Design**

The tailings dam at Potgietersrust Platinums Ltd was designed on the basis of available information to be deposited using a spigot method of deposition commonly used on platinum mines notwithstanding prior knowledge that the product was likely to be of finer grind than found on most other mines. This was done because past experience with tailings operations on base metal mines has shown that it is easier to convert a spigot dam to a paddock dam if the product is found to segregate less than anticipated. Moreover, it was anticipated that the relatively higher particle specific gravity (3.14 compared to 2.76 on most gold paddock operations) would induce a greater tendency for gravitational segregation than is experienced on gold mining operations.

What was not apparent at the time of design was the existence of the serpentinised zones and the consequent influence of the talc-related minerals. These minerals and their variable occurrence in the processed ore necessitated a modification of the spigot system due to the following effects:

- there is unlikely to be adequate beach freeboard generated during initial operations to provide the required stormwater holding capacity on the dam. This would imply a need for a method of operation which generates freeboard by formation of an outer wall. The paddock system permits the formation of this outer wall hydraulically.

- the fine slimes material is placed at a high moisture content as a result of its high specific surface and tends to drain and dry slowly. This will influence cycle times if routine paddock methods of operation were to be employed notwithstanding the layering problems noted above.

- layers of fine and coarse tailings are likely to form in the tailings deposited in the vicinity of the discharge.

- unless deposition of fine and coarse material can be controlled there is a significant danger that underdrainage systems will be blinded and rendered ineffective which would have very severe effects on the slope stability of the dam and limit the maximum height of the dam

In order to address each of the above concerns a method of operation which may be described as a hybrid of the paddock method and the spigot method was designed. This is described below.

- **Description of hybrid method of operation**

Figures 8.29 and 8.30 show a typical plan and cross-section through the dam once the proposed hybrid system is established. Noteworthy features of the section in Figure 8.30 are:

- a relatively wide (50 m) daywall formed by beaching from spigot discharges directly across the daywall. At the inner wall of the daywall a number of pipe decants are used to draw supernatant water into the basin as well as some fines so as to encourage the deposition of coarser material at the inner wall and speed up drying times and density.

- the use of overflow pipes to ensure that daywall formation is controlled and systematic Only a portion of the flow is taken through the spigots ie the spigots are throttled down to restrict the flow per spigot. Only a limited number of paddocks are operated at a time again to improve control. The overflow pipe is therefore used to transfer excess slurry over the day wall requirements into the floor area of the dam. This pipe therefore assists in ensuring that an adequate proportion of tailings is placed in the main body of the tailings.

- the use of paddock cross walls to confine the flow to manageable areas and facilitate control of the tendency of the tailings to flow laterally along (parallel to) the daywall as opposed to across the daywall.

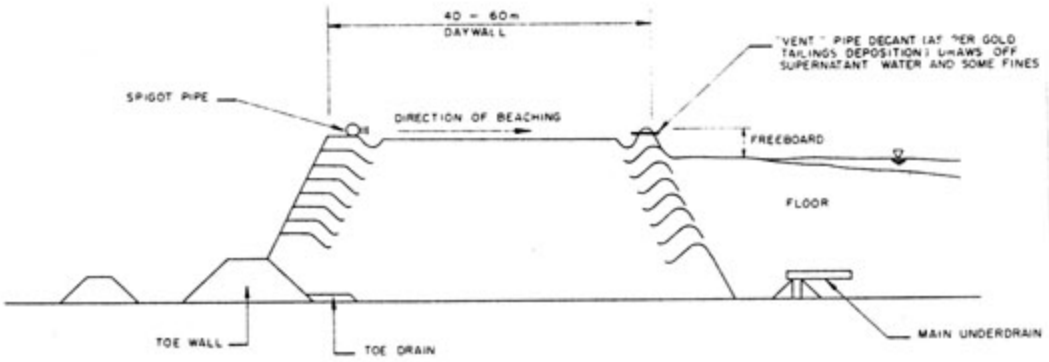


FIGURE 8.29 : TYPICAL SECTION THROUGH THE DAYWALL

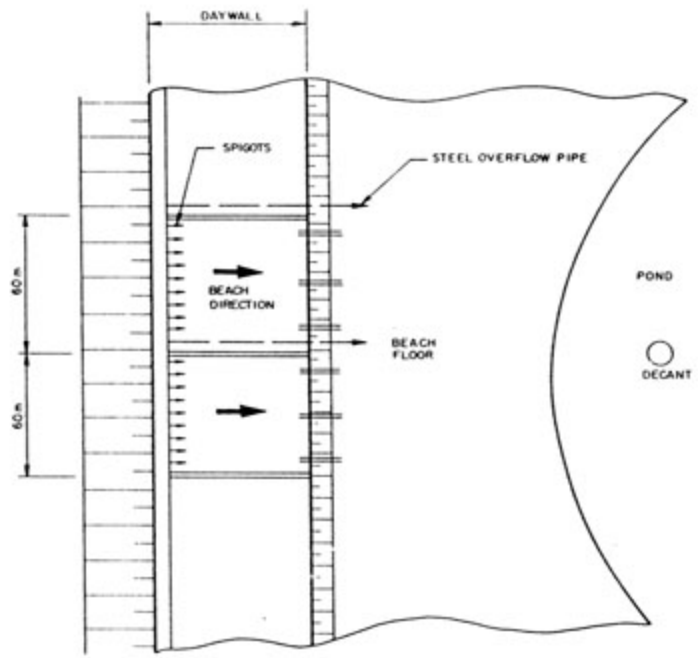


FIGURE 8.30 : TYPICAL PLAN ON THE DAYWALL

- **Placement of Coarse Material over the Main Drain**

To further enhance the quality of the tailings deposited in the daywall the method of operation of the spigot flow control gate or "spectacle" gate is modified. Figure 8.31(a) illustrates the typical method of operation of the spectacle gate. The deposition of coarse product is enhanced by utilising a fan spray method of discharge whereby the fan is directed vertically downward as illustrated in 8.31(b). At the point of contact of the spray with the beached tailings a deep plunge pool is formed, most energy is dissipated allowing coarse particles to settle out more readily.

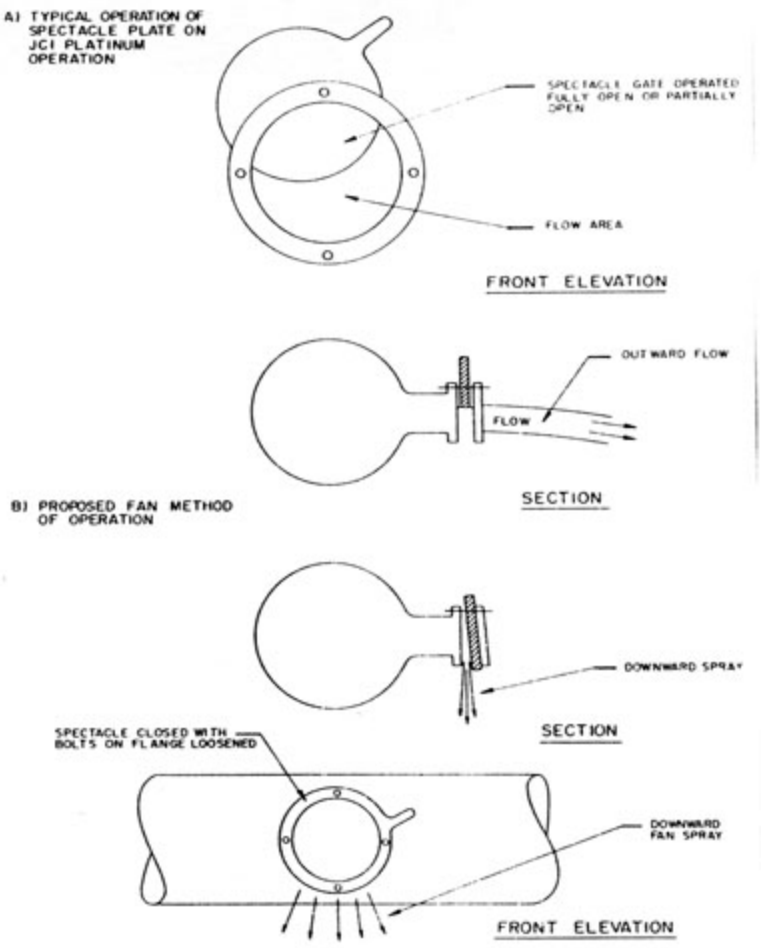


FIGURE 8.31 : SPECTACLE PLATE ARRANGEMENT



FIGURE 8.32 : PHOTOGRAPH ILLUSTRATING TYPICAL SPECTACLE PLATE FAN SPRAY OPERATION

The methods illustrated above indicate the method of operation once the hybrid system is established. To ensure that the operation of underdrainage systems is not hindered by layering over the drain a modification on the above method of operation is proposed. This method would apply in the period immediately preceding the formation of the main elevated blanket drain and would continue until a significant (5 m thick) layer of coarse product has been formed over the drain. It is proposed to employ a spray-bar pipe to be located at the inner wall of the daywall. This pipe will be equipped with 25 mm holes at 1 m centres and would discharge the tailings in the form of a jet. The jets would be angled downwards and would meet the beach just outside of the main drain. The method of discharge would encourage the deposition of coarse material over the main drain. This is illustrated in Figure 8.32. Figure 8.33 shows the pipe set-up in plan; the spray bar pipe is connected to an overflow pipe which is connected to one of the spigots in the spigot pipe. The spectacle plate is used to regulate flow.

Deposition over the drain is alternated with deposition in the daywall. Daywall decant pipes are extended beyond the drain to prevent blinding of the drain.

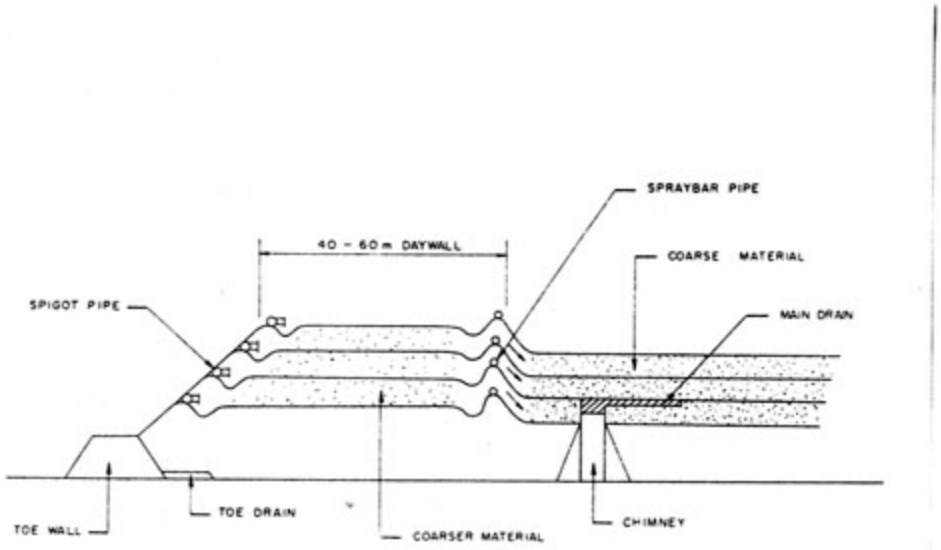


FIGURE 8.33 : SECTION THROUGH HYBRID DEPOSITION SYSTEM ILLUSTRATING COVERING OF THE MAIN DRAIN

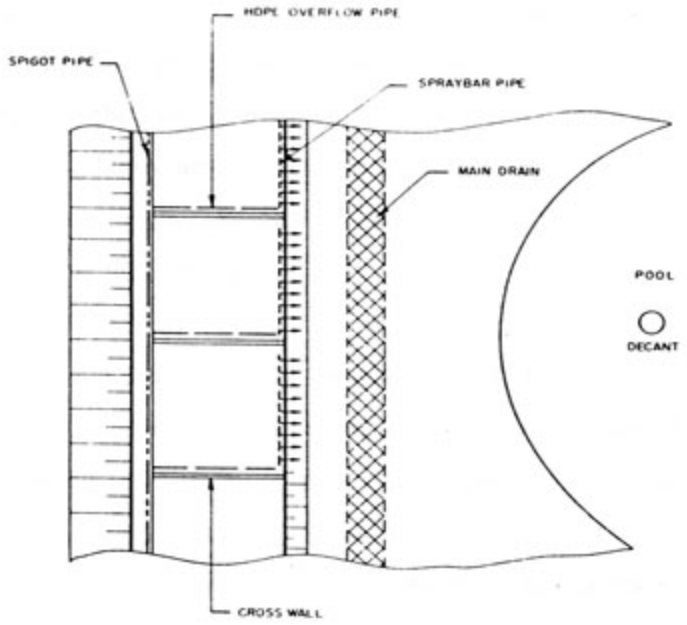


FIGURE 8.34 : PLAN ON HYBRID DEPOSITION SYSTEM ILLUSTRATING COVERING OF THE MAIN DRAIN

- **Placement and formation of the toe drain**

It is common in South Africa to equip platinum tailings dams with double toe walls, an inner toewall to facilitate commissioning, and an outer toe wall to form the outer slope. (refer to Figure 8.35). Usually the zone between the inner and outer toe walls is filled using cyclone underflow with the overflow being discharged down the beach. The beach profile is maintained using spigot deposition with cyclone deposition taking place on a part-time, low-key basis. This method of development ensures the placement of a good drainage medium relatively free of layering in the embankment area of the tailings dam. The underflow material intercepts any layers that may have been formed in depositing from the inner toe wall.

The inner toe wall is equipped with a temporary filter drain at the inside edge of the toe wall to facilitate commissioning. This filter drain runs the risk of becoming partially blinded during commissioning since it is difficult to control deposition during the initial stages. To compensate for the potential loss of this drain a final toe drain is placed at the inside edge of the outer toe wall immediately before covering with the cyclone underflow.

In the case of Potgietersrust Platinums it may not be practical to form the outer zone of cyclone underflow material due to variations in grind. A slug of finer higher water content slurry would cause the underflow product to change and scour the newly deposited underflow away. Moreover, the risk of the inner toe drain becoming blinded is increased as a result of the product and plant commissioning problems. There is therefore a need for an outer toe drain. The following method of formation of this drain and the material over the drain is proposed (refer to Figure 8.35):

- a main toe drain is formed on the crest of the toe wall and 2 m up the face of the tailings dam. The purpose of this arrangement is to intersect seepage along horizontal layers and control the phreatic surface above the toe wall. The drain is connected into the existing outlets. Once in place the drain is covered by mechanically pushing coarse tailings from the daywall area over the drain as illustrated.

- a small (3 m) wide drain is placed at the outer toe wall and deposition using a 100 mm diameter spray-bar pipe equipped with 25 mm holes at 1 m centres is carried out on a gradual basis by beaching from the outside inwards. The blanket and its coarse cover

(from the push-down) will act as a drainage/decant facility thereby negating the need for a decant. It is emphasised that this operation is carried out on a gradual basis over several years which will allow the fine material in the buttress adequate time to drain, dry and desiccate.

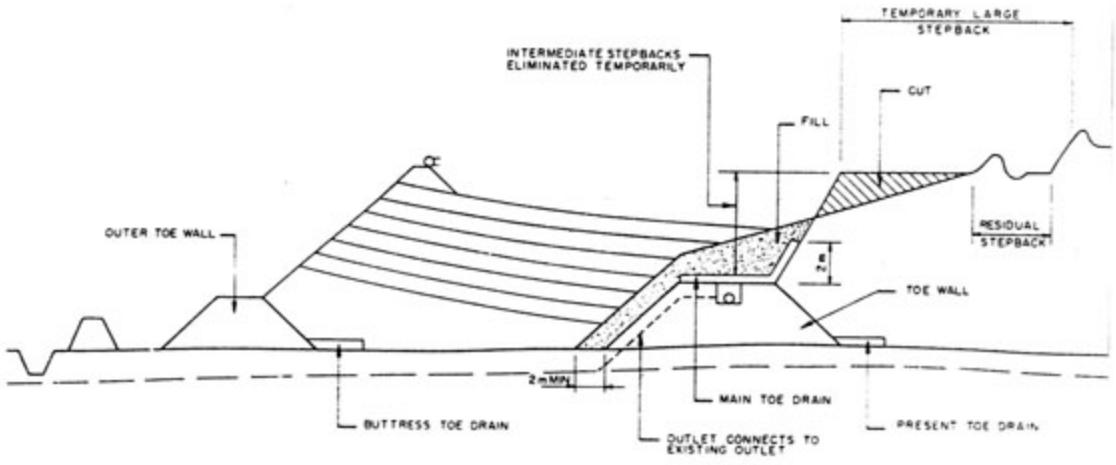


FIGURE 8.35 : TYPICAL SECTION THROUGH TOE EMBANKMENT

- **Formation of the Toe Embankment**

Once the main dam has been established, the main drain constructed and the requisite cover of coarse material placed over the drain it is proposed to utilise the 100 mm diameter spray-bar pipes to deliver tailings from the spigot pipe to the toe paddock area to facilitate the formation of the toe embankment.

8.8.5 Review of the Hybrid System following Initial Deposition Trials

Trial construction of the hybrid system using only the existing spigot pipe and daywall decant pipes has indicated that this is indeed a valid method of forming the dam. Observations show that the material at the inner paddock wall can be packed out satisfactorily and that it is possible to generate good freeboard using this method.

Observations on site indicate that the conclusions from the simulations that a spigot deposition procedure could be made to work are correct. It is intended to continue the deposition trials to further confirm these conclusions. If the trials prove successful it will be possible to revert from the hybrid system to the spigot system once the main drain has been installed and reliably covered with coarse, mixed material.

To facilitate routine operation whereby the tailings is filled to the spigot pipe without a risk of overtopping the outer crest wall the configuration illustrated in Figure 8.36 is proposed and will be tested on site.

It is concluded that the hybrid system is viable and will achieve the desired effect.

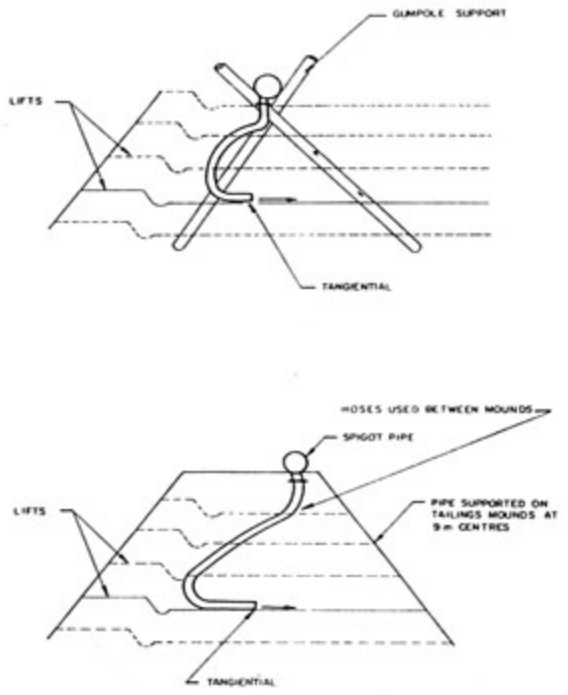


FIGURE 8.36 : SPIGOT PIPE SUPPORT AND OUTER PADDOCK WALL GEOMETRY

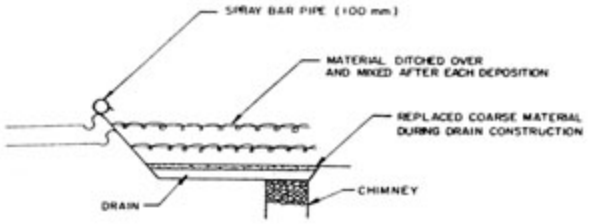
8.8.6 **Review of Method of Main Drain Formation and Covering**

The trials have confirmed that the method of drain covering described in Section 8.8.5 is necessary. However, the trials have also indicate that it will be necessary to ditch and dig over the tailings placed over the drains so as to mix the coarse material hydraulically deposited over the drains. This is because while there will be a reduced tendency for layers to be formed as a result of flow meanders, those which will form as a result of variations in product will continue to occur.

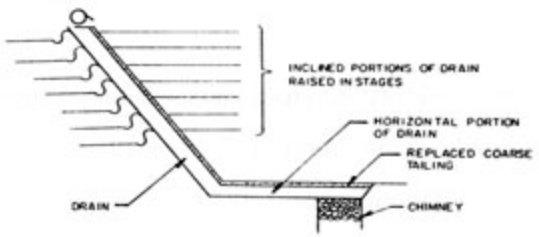
An alternative to ditching of the material over a horizontally orientated drain as indicated in Figure 8.37(a) will be to use a modified drain layout as illustrated in Figure 8.37(b). This shows an inclined section of drain which is constructed in stages. The drain will intercept

layers and will allow drainage of the coarse tailings between the layers.

Selection between the alternatives will depend on costs and accessibility.



(a) HORIZONTALLY ORIENTATED DRAIN



(b) INCLINED DRAIN CONSTRUCTED IN STAGES

FIGURE 8.37 : ALTERNATIVE METHODS OF DRAIN FORMATION

8.8.7 **Review of Method of Formation of Outer Toe**

As the tailings product becomes coarser the possibility of implementing the cyclone method of formation of the outer toe improves. It is intended that this process would only begin once the main drain has been installed and there is therefore opportunity to carry out cyclone trials once the grind has stabilised to a greater extent. It will be important to ensure that the overflow product is deposited well beyond the main drain.

Should it not be possible to implement the cyclone option the methodology described in the method statement in Section 8.8.5 will form a viable alternative.

8.8.8 Long Term Deposition Method

The spigot method of deposition remains the most cost effective method of disposal over the long term in the case of platinum tailings. At this stage it appears that it may be possible to revert to this method of disposal over the majority of the life of the tailings dam beginning once the main drain has been installed and adequately covered with mixed coarse material.

8.8.9 Summary of Conclusions

The following conclusions have been drawn at this stage of the deposition trials:

- Serpentinised lenses in the ore body will continue to influence the grind and characteristics of the tailings product notwithstanding extensive blending, and metallurgically-motivated adjustments to milling rates and screen sizes

- As the tailings product becomes coarser the focus of depositional problems becomes layering rather than slurry SG, grind or resultant beaching characteristics. This conclusion is, however, dependent on further deposition trials in which the influence of discharge height on the beaching characteristics of the tailings are to be confirmed

- Failure to take adequate cognisance of the negative effect of layering on the dam will result in a considerable reduction in the safe operating capacity of the dam. It will be more economical to effect adjustments to the method of operation as opposed to replacing the present tailings dam at an earlier date.

- From the trials carried out to date the hybrid method of disposal is practical and will achieve the desired effect of rapidly producing freeboard and facilitating the formation of the main drain. This method appears to be the only practical means of placing coarse material over the main drain hydraulically. The trials to date have proceeded without the full compliment of piping originally envisaged and have confirmed that these pipes will be required.

- it will be necessary to either ditch and mechanically mix the coarse layered tailings which will be deposited over the main drain or adopt a more expensive drain configuration

which will extend across the near horizontal layers.

- It appears from the beaching simulations that it will be possible to revert to the spray-bar method of operation once the main drain has been covered provided deposition to the bottom of the spigot openings becomes the operating norm.

8.9 CASE STUDY : LEAD ZINC TAILINGS

8.9.1 Introduction

Pering Mine is located on the Kaap Plateau in the northern Cape province of South Africa. The plateau comprises a dolomite plain with sand and tufa deposits between sparse grass. There are sporadic beds of chert and outcrops of dolomite. Pering mine was commissioned in 1987. At that time this author was the design engineer for the tailings dam and has been fortunate to continue an involvement with the dam throughout its development and operation.

The design of the tailings dam was based on a grind from pilot metallurgical testing. The particle size distribution indicated on Figure 8.38 fell within the envelope of gold tailings as reported by Blight and Steffen (1979). The particle specific gravity at 2,86 was of the order of that for the quartzitic gold bearing rocks on which Blight and Steffen based their testing and it was concluded that a paddock system of operation should be designed.

On commissioning of the mine the grind was similar to that anticipated at the time of design and the development of the dam proceeded as intended. After some months, following fine tuning of the grind to suit metallurgical recoveries the tailings product became significantly coarser. The coarsening of the grind introduced problems of gravitational segregation which the paddock system was unable to accommodate.

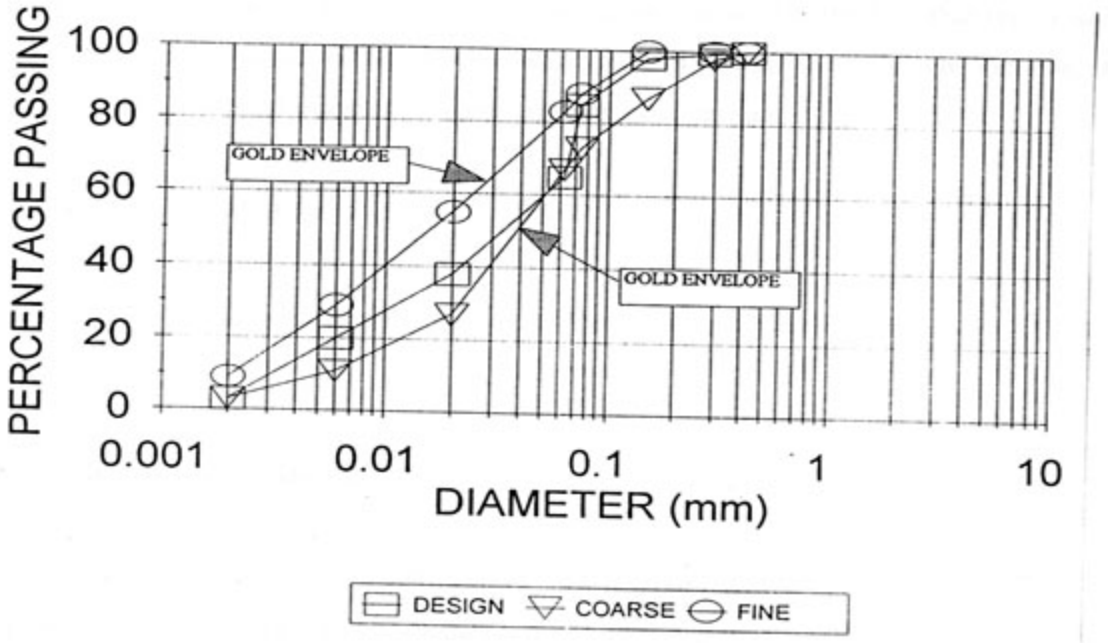


FIGURE 8.38 : DESIGN PARTICLE SIZE DISTRIBUTION OF PERING TAILINGS RELATIVE TO THE BLIGHT AND STEFFEN GOLD TAILINGS ENVELOPE (Blight and Steffen (1979))

After some 6 months of operation the particle size distributions of tailings, samples taken at the discharges and over the main blanket drain indicated that continued operation as a paddock system would result in blinding of the main underdrainage systems and it was therefore decided to convert to a spigot system and monitor the beaching characteristics. It was necessary to break up the layers that had formed over the main drain by excavating to the drain, stockpiling the excavated tailings on the side of the excavation and then backfilling the tailings.

At the time of reaching the decision convert to the spigot (spray-bar) system the milled product included particles of up to 1,18 mm and as a result the beach profile proved adequately steep to accommodate the spigot method. With time, however, further metallurgically motivated adjustments were made to the milled grind making this finer. The beach profile flattened somewhat introducing a concern of overtopping. Storm simulations were carried out to check on the storm retention characteristics of the dam and it was established that beach freeboard was adequate to facilitate continued operation. The milled

grind has fluctuated between close to the design grading at 80 percent minus 75 micron and 60 percent minus 75 micron. Yet, despite the fluctuations, as well as the fact that the product is not as coarse as it was when the decision was taken to change to the spigot method, the beach profile has remained adequately steep against expectations.

8.9.2 Pering Tailings Product

Particle size distributions and viscosity testing on the tailings product has been carried out as described below:

- **Particle Size Distributions**

The fluctuation of the particle size distribution over the period of available records from the Mine is indicated on Figure 8.39 which shows a plot of the percentage passing the 75 micron sieve size. Figure 8.40 shows the range of particle size distributions typical of the variations in Figure 8.39.

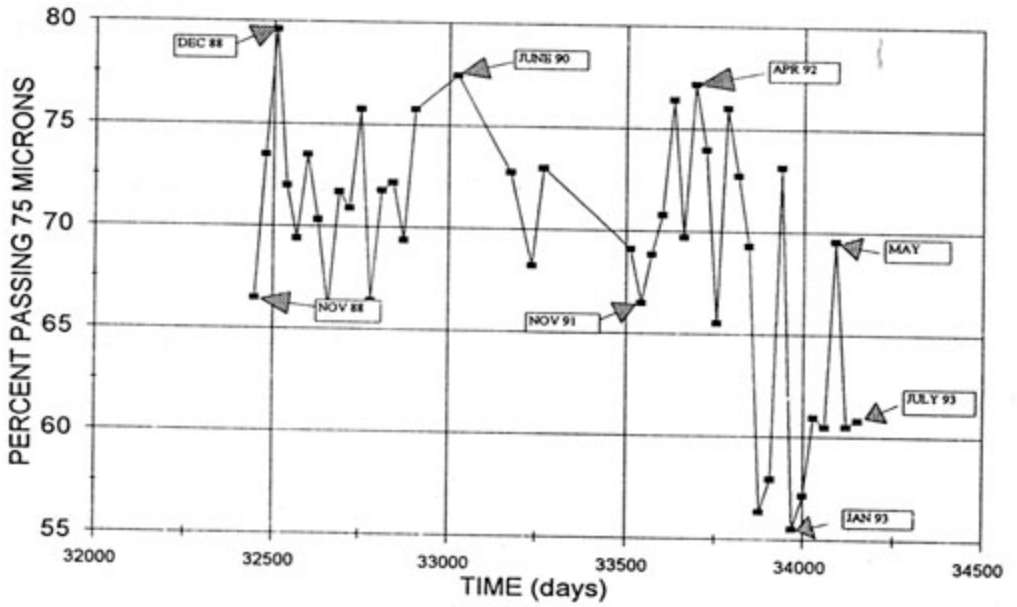


FIGURE 8.39 : PARTICLE SIZE DISTRIBUTIONS OVER THE PERIOD SINCE COMMISSIONING

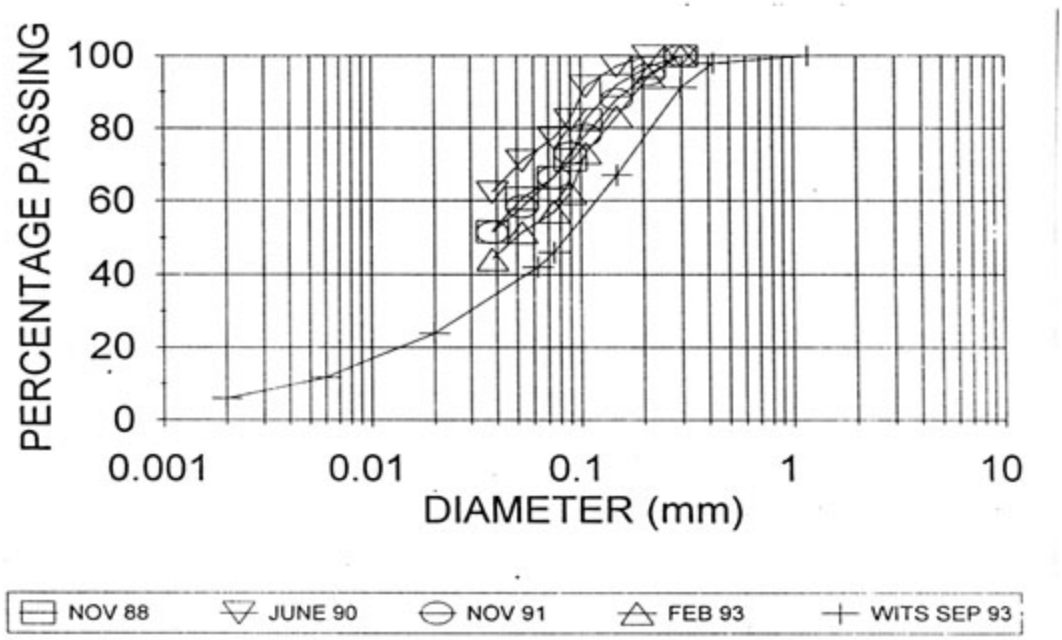


FIGURE 8.40 : PARTICLE SIZE DISTRIBUTIONS UTILISED IN BEACHING ANALYSES

- **Slurry and Viscosity Characteristics**

Tonnage rates and water contents are unlikely to change significantly as the tailings grind varies. The following slurry characteristics have therefore been kept constant in all simulations:

- particle specific gravity 2.86
- concentration of solids (mass) 38.7 %
- slurry density 1,35 t/m^3

Viscosity testing has been conducted at the University of the Witwatersrand using a Ferrante viscometer over a range of concentrations and shear rates. A curve has been fitted to the viscosities in accordance with the equations in Chapter 2 and the resulting measured and fitted viscosities are indicated in Figure 8.41. The slurry is slightly Bingham plastic at high concentrations of solids but in the range of interest may be modelled using a Newtonian model.

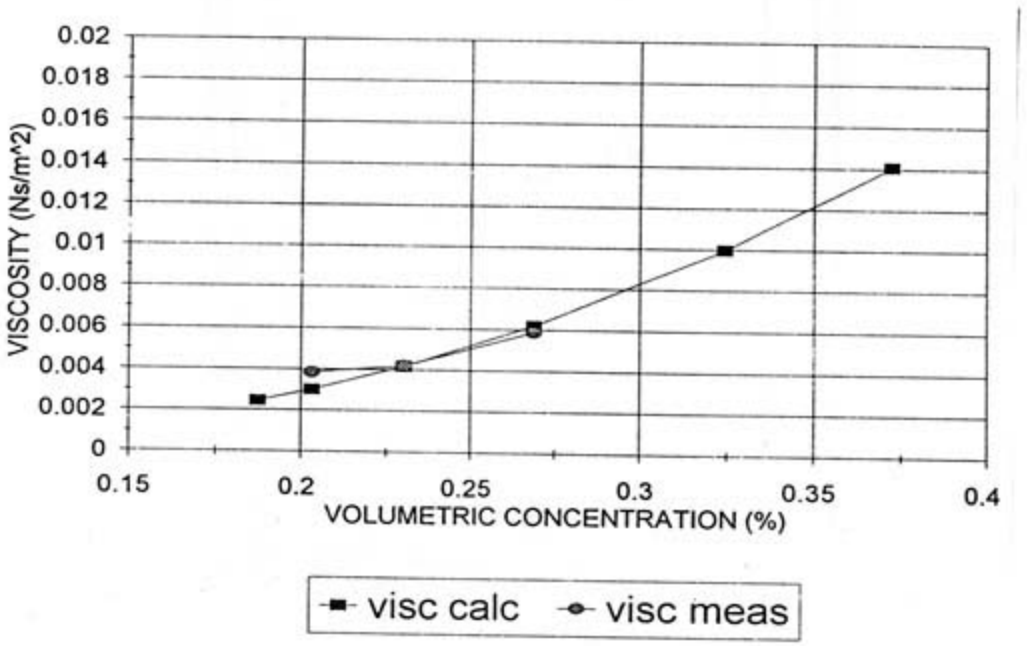


FIGURE 8.41 : CONCENTRATION vs VISCOSITY

8.9.3 Beaching Simulations

The tailings dam covers an area of some 44 hectares. The dam is almost square with side lengths of 700 to 800 m. The pool located centrally by means of a pool wall as indicated on Figure 8.42 which shows the layout at the time of commissioning when a paddock system was in operation using point discharges at the locations indicated. When the conversion to the spigot (spray-bar) system took place the spray-bar pipe was connected to the daywall deliveries.

Beaching simulations have been conducted on the basis of horizontally aligned openings on a spray-bar pipe as illustrated in the photograph in Figure 8.43. The pipe has a diameter of 200 mm and 37,5 mm openings at 1 m centres. The openings are closed using wooden bungs when not in use. A minimum of 75 openings are used at a time with this number often approaching 85. Only the profile at the end of each deposition has been considered since it is established practice to carry out deposition at a location until the tailings reaches almost the level of the pipe openings as indicated in Figure 8.43.

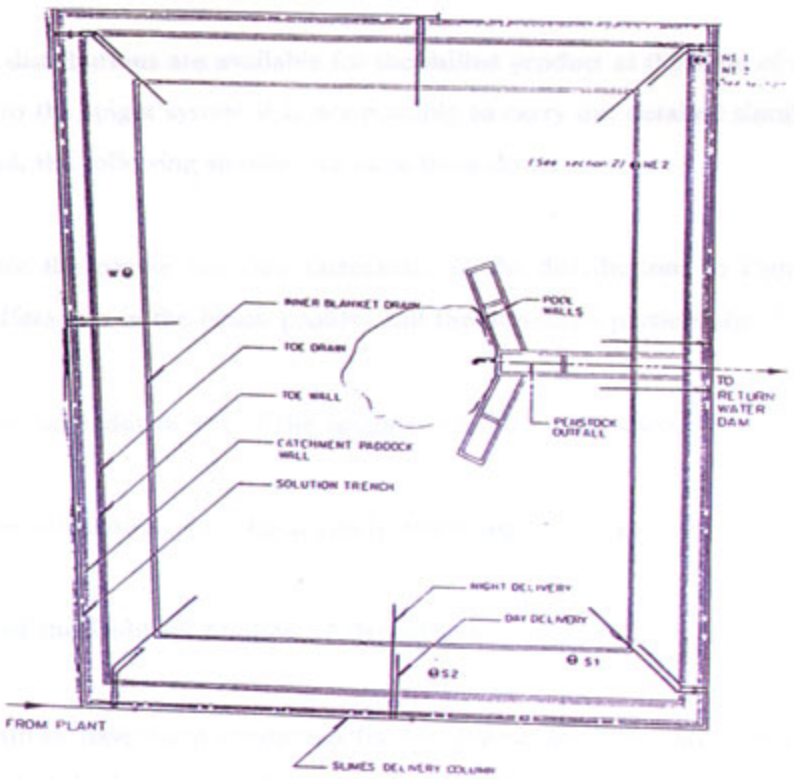


FIGURE 8.42: PLAN OF TAILINGS DAM SHOWING UNDERDRAINAGE STRUCTURES AND DELIVERY LOCATIONS



FIGURE 8.43 : PHOTOGRAPH OF THE SPRAY-BAR DISCHARGE PIPE AT PERING MINE

As no particle size distributions are available for the milled product at the time of converting from the paddock to the spigot system it is not possible to carry out detailed simulations for this period. Instead, the following simulations have been done:

- simulations for the coarse and fine extremities of the distributions in Figure 8.40 to determine differences in the beach profiles and the deposited particle size distributions

- simulations of the paddock and of the spigot methods of operation.

Frictional resistance calculations have been conducted using Verkerk's (1991) program.

- **Simulations of the Paddock Method of Deposition**

  These simulations have been conducted for the coarse and fine extremities using the design method of discharge, namely, an open-ended discharge from a 200 mm diameter pipe angled at upwards at 35 degrees to the horizontal to represent an in-wall delivery in which the delivery pipeline is buried in the daywall or night wall and rises at the natural angle of repose of the tailings. This method of operation is common on the gold mines in South Africa. The open end is taken 0,5 m above the beach/daywall level. For this method of discharge the Verkerk frictional resistance calculations show a variation of the Fanning friction factor as shown in Figure 8.44:

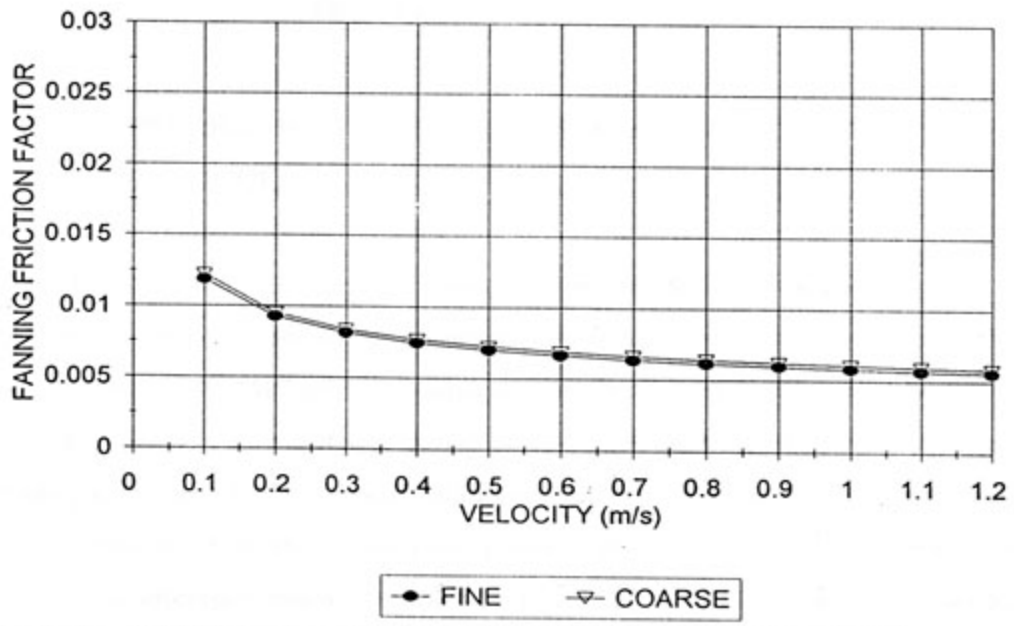


FIGURE 8.44 : FANNING FRICTION FACTORS FROM VERKERK RUNS

Figure 8.44 shows that the difference in frictional resistance of the coarse and fine products as a result of particle size differences is marginal. Figure 8.45 shows the predicted beach profiles for the coarser and finer products. The coarse and finer products differ only to the extent that the frictional characteristics differ

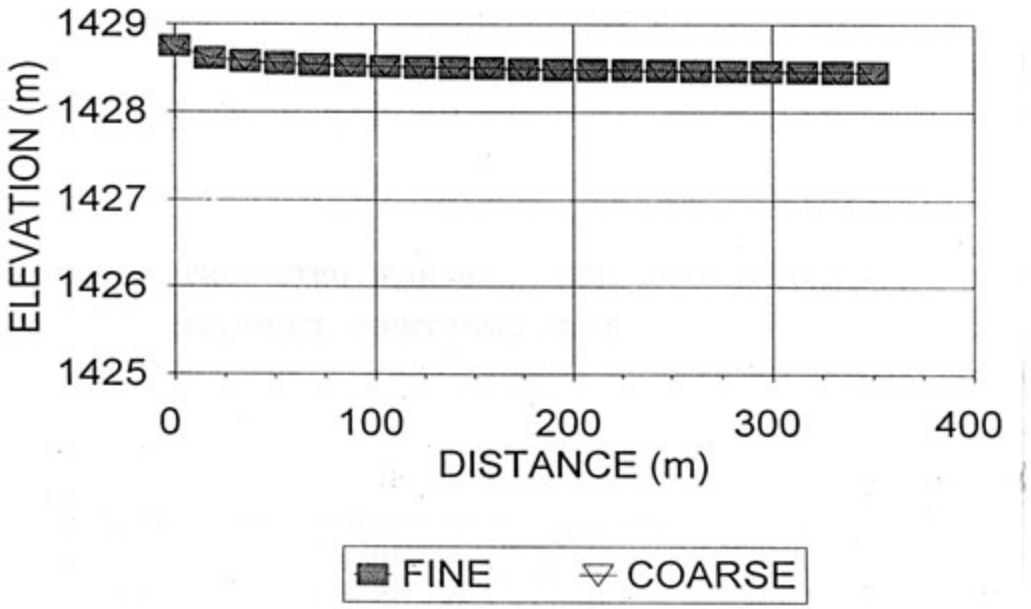


FIGURE 8.45 : PREDICTED BEACH PROFILES FOR THE COARSE AND FINE PRODUCTS

Figure 8.46 shows the predicted deposited particle size distribution along the beach for the coarse product. Figure 8.47 shows the results of particle size testing in 1987 at the time of conversion. Note that the 1987 product contained 1,18 mm size fraction. Comparison of the figures indicates that the extent of segregation is reasonably well modelled. The difference between the particle size distributions of tailings deposited close to the discharge and that at distances in excess of 100 m shows that these products will have very different permeability characteristics, a difference which could not be tolerated along the daywall of a paddock system.

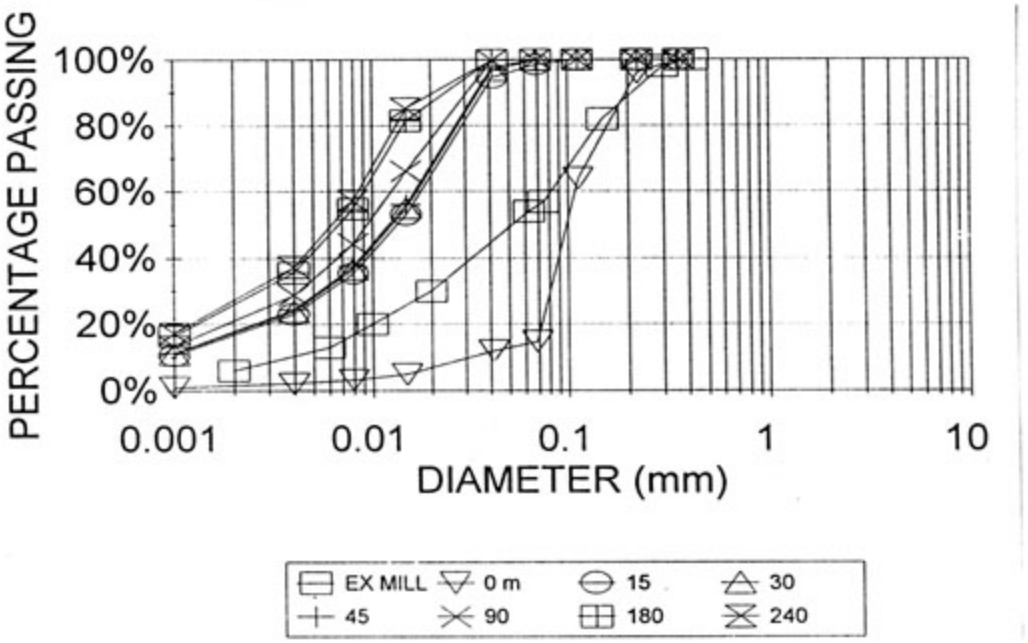


FIGURE 8.46: PREDICTED PARTICLE SIZE DISTRIBUTIONS - COARSER PRODUCT : POINT DISCHARGE

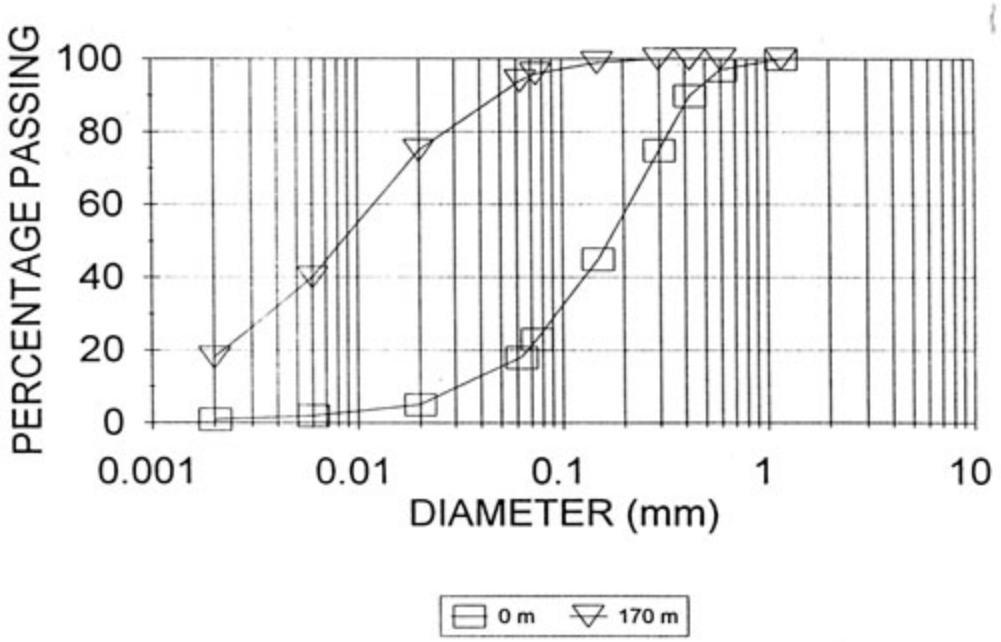


FIGURE 8.47: MEASURED PARTICLE SIZE DISTRIBUTIONS IN 1987 : COARSE GRIND

Figures 8.48 and 8.49 show the equivalent particle size distributions to those in Figures 8.45 and 8.46 but for the finer product.

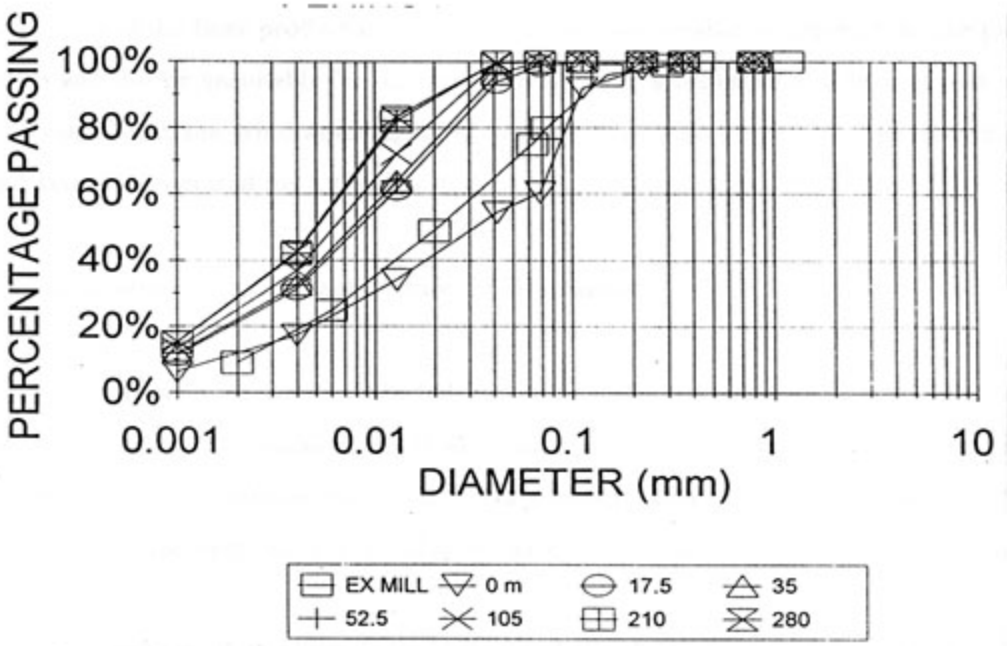


FIGURE 8.48: PREDICTED PARTICLE SIZE DISTRIBUTIONS - FINE PRODUCT : POINT DISCHARGE

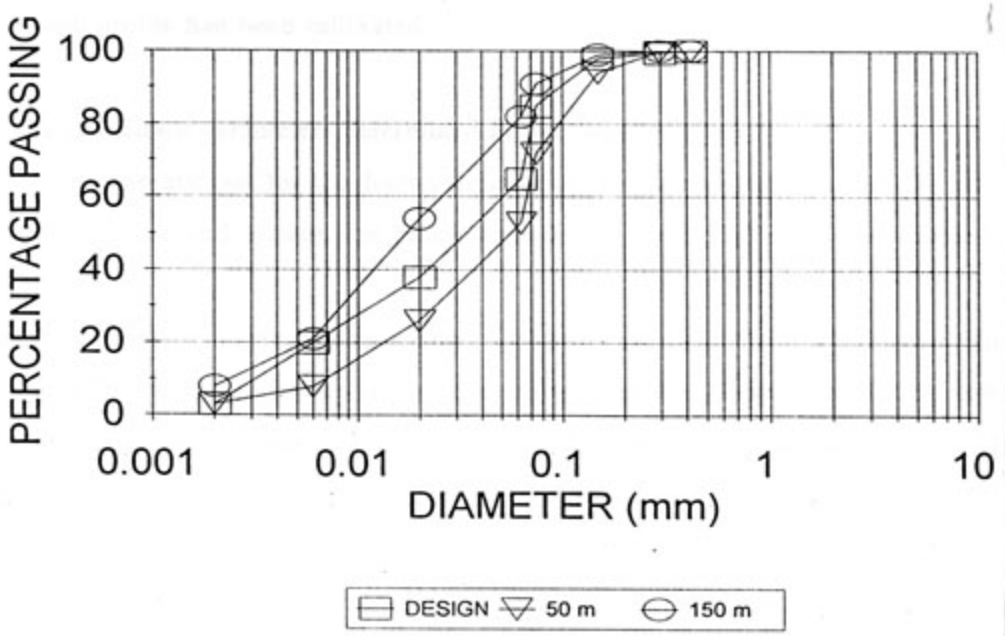


FIGURE 8.49: MEASURED PARTICLE SIZE DISTRIBUTIONS IN 1987 - FINE PRODUCT : POINT DISCHARGE

It is evident from Figure 8.49 that the grind in 1987 during the period over which a fine grind predominated is finer than that represented in Figure 8.46 which accounts for the greater degree of segregation in Figure 8.46. Despite the reduced tendency for gravitational segregation of the finer product in relation to the coarser product in Figure 8.46 the paddock system will still be unsuitable for the finer product. For a resumption of the paddock system to be considered, the grind would again need to approach that predicted at the time of design (and actually generated for a time) in 1987.

- **Simulations of the Spigot Method of Deposition**

These simulations have been conducted for the fine and coarse extremities of the record of mill grinds as in the paddock method simulations. The spigots have been orientated horizontally and depositions simulated for a situation where the tailings is filled to within millimetres of the spigots. The simulations have been conducted on the following basis:

- The number of openings in operation and the discharge height have been calibrated against measured beach profiles for material deposited in April 1993. The grinds for this period have been used. This grind is in fact representative of the coarse limit of the recorded grinds. The predicted particle size distributions were determined.once the beach profile had been calibrated

- the deposition parameters determined in the calibration runs have been used to carry out the simulations for the finer extremity of the recorded grinds and the predicted beach profile and particle size distributions for the deposited tailings determined.

Figure 8.50 shows the Fanning friction factors calculated using Verkerk's program for the coarse and fine products. Note that the flow rates cover the laminar, transitional and fully turbulent phases of flow, and Figure 8.50 bears the required resemblance to the Moody diagram.

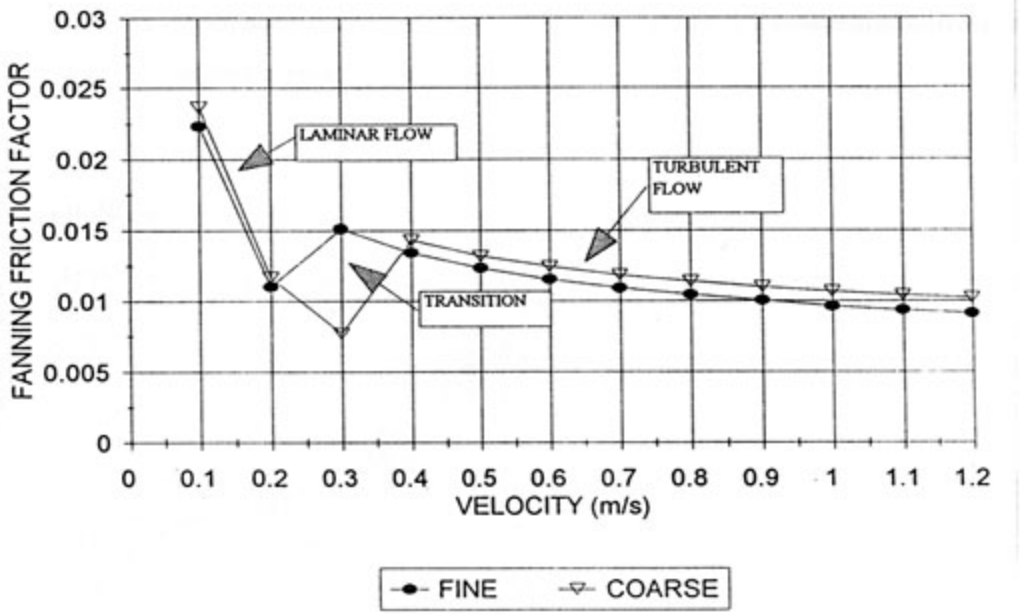


FIGURE 8.50 : FANNING FRICTION FACTORS FROM VERKERK RUNS

Figure 8.51 shows the calibrated profile for the coarser product against the measured beach profiles. To achieve this level of calibration 85 openings discharging at a height of 7 mm above the beach level (plunge pool level) have been used.

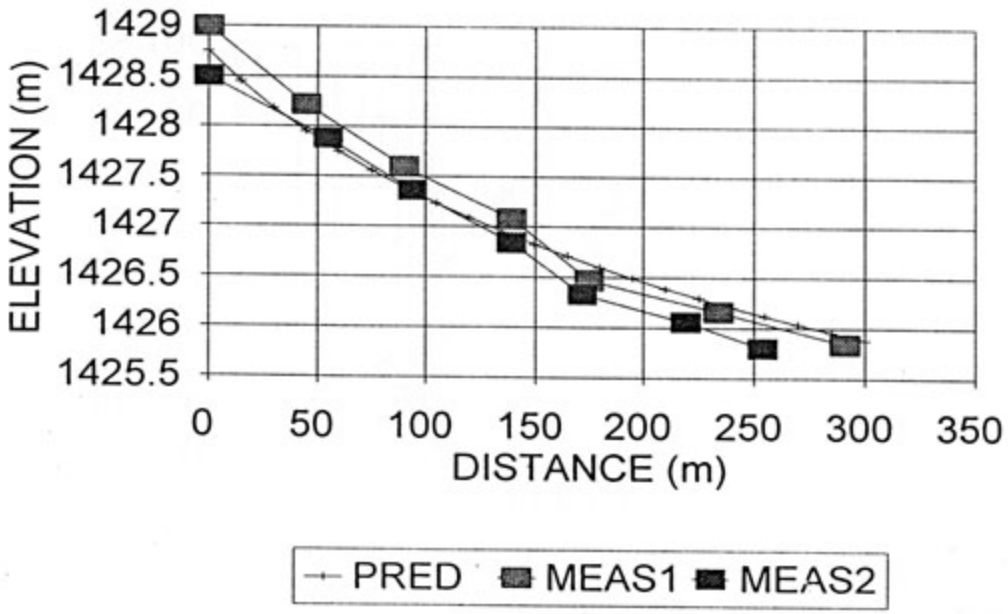


FIGURE 8.51 : CALIBRATED BEACH PROFILE - COARSER PRODUCT

Figure 8.52 shows the predicted particle size distributions with distance down the beach and Figure 8.53 shows the measured particle size distributions. There is a reasonable correlation between the predicted and measured distributions.

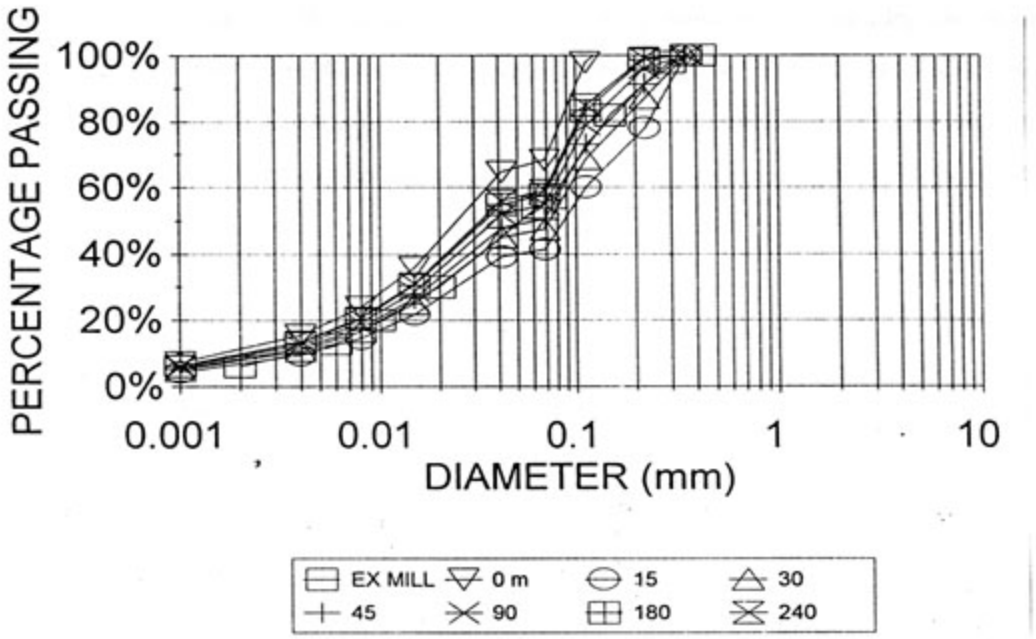


FIGURE 8.52 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - COARSER PRODUCT WITH INDIVIDUAL DISCHARGES COMBINING DOWN THE BEACH

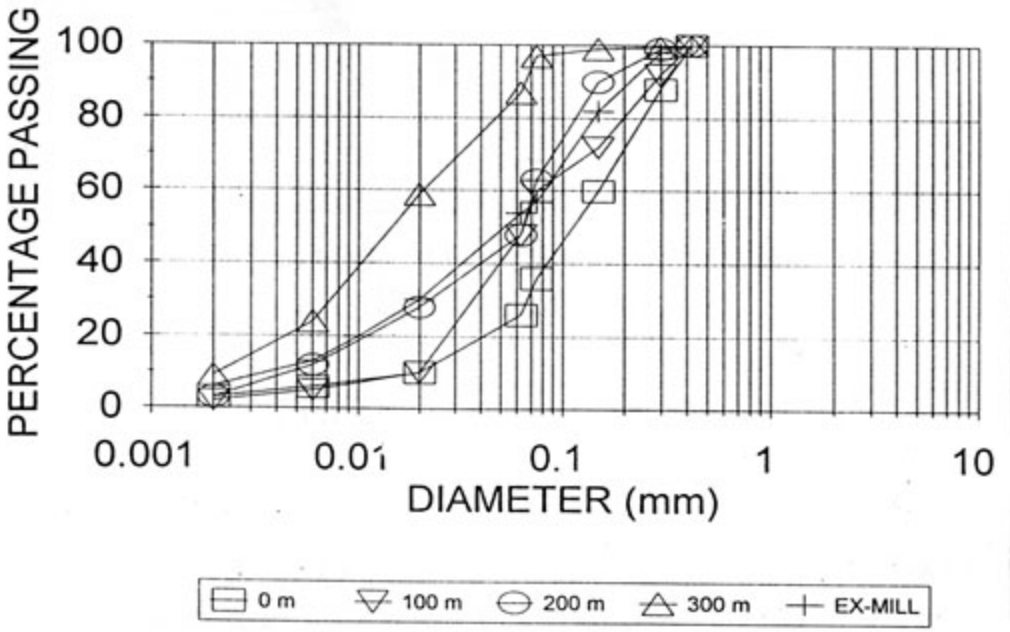


FIGURE 8.53 : MEASURED PARTICLE SIZE DISTRIBUTIONS

It should be noted that as in the case of the Platinum tailings in Chapter 7, to match the measured beach profile it was necessary to simulate the situation observed in the field whereby groups of individual streams from the spigot openings combine with distance down the beach. In the case of Pering it was necessary to assume that the flows from 35 spigots combine with distance down the beach. This has been catered for by numerically summing the stream powers of the individual spigot flows. The particle size distribution in Figure 8.52 is therefore not representative of the situation close to the spigot pipe. For this reason additional runs assuming 85 openings but considering an individual spigot were conducted. The resulting beach profile is indicated in Figure 8.54 and the particle size distribution is in Figure 8.55.

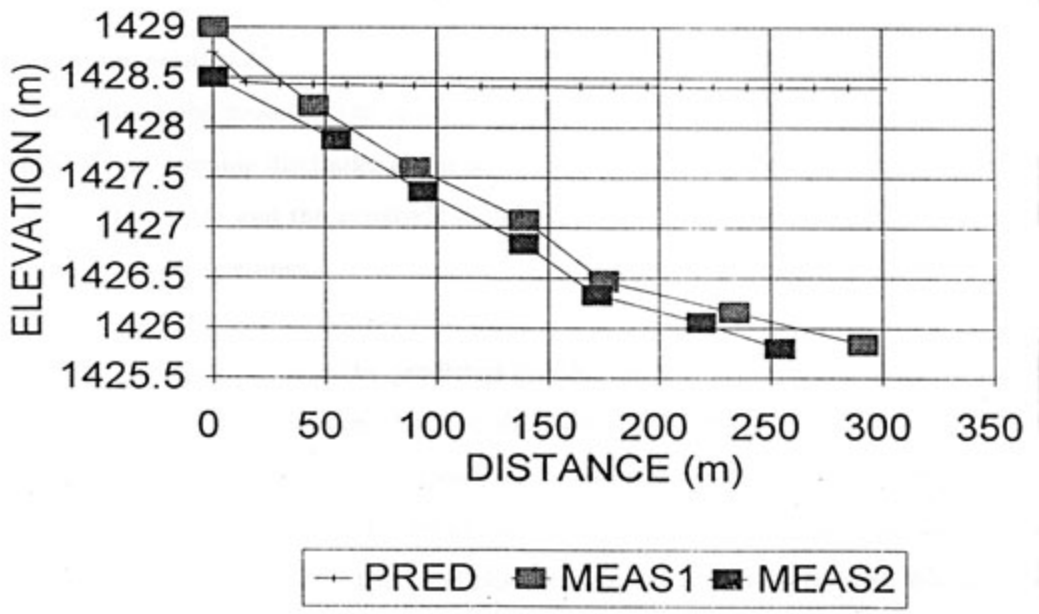


FIGURE 8.54 : SINGLE OPENING BEACH PROFILE - COARSER PRODUCT : SPRAY-BAR DISCHARGE

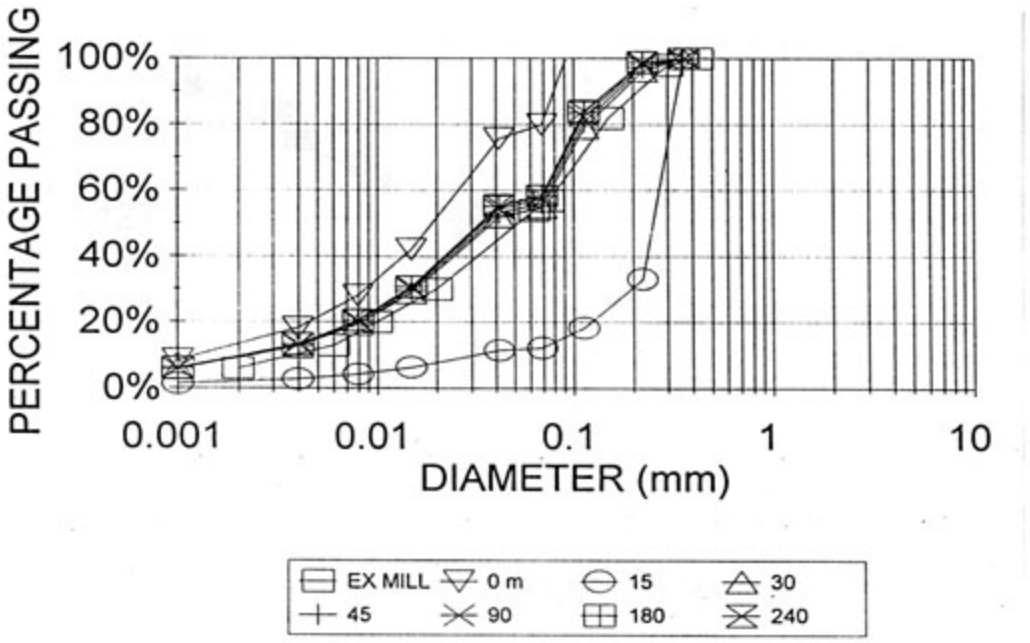


FIGURE 8.55: PREDICTED PARTICLE SIZE DISTRIBUTION FROM A SINGLE OPENING

A spurious prediction of the particle size distribution at locations close to the discharge has been made by the model. The spurious prediction occurs both for the combined and for the single spigot opening discharge and is ascribed to the inability of the model to accommodate the low flow rates and the relatively high velocity of the tailings discharging from the small diameter spigot openings.

Figures 8.55 and 8.56 show the predicted profiles and particle size distributions for the finer product simulated on the same basis as the calibrated beaches for the coarser product ie 85 openings, 7 mm above the plunge pool, and combined flow down the beach of 35 openings. The predictions show that there will be marginal difference in beaching profile, a foreseeable result in view of the similar frictional characteristics of the two products, but that there will still be a significant degree of gravitational segregation albeit less than for the coarser product.

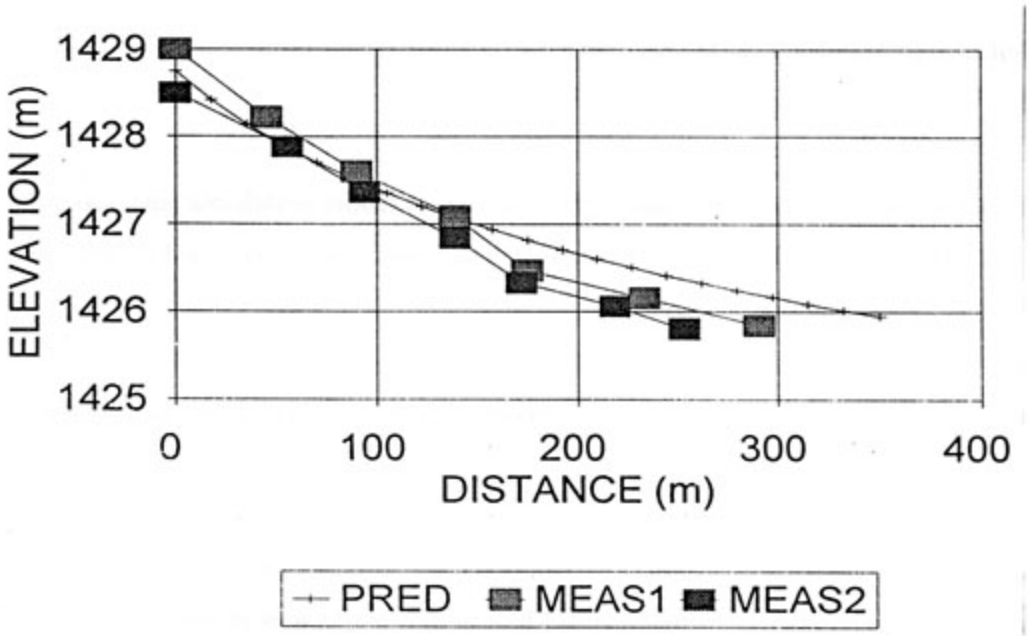


FIGURE 8.56 : PREDICTED BEACH PROFILE - FINER PRODUCT COMBINED SPIGOT FLOWS

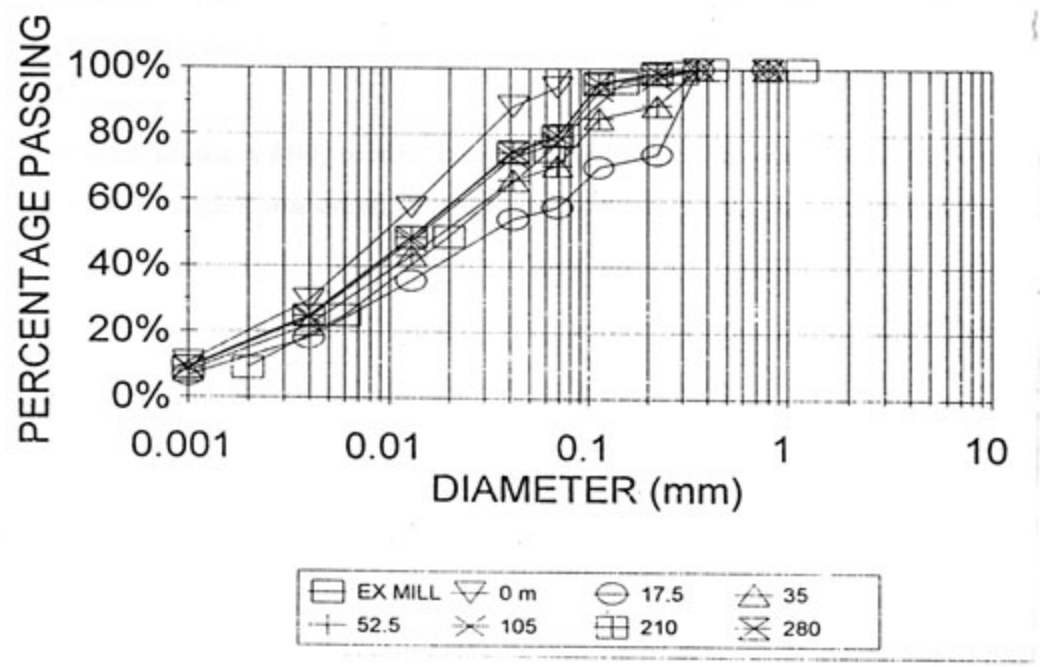


FIGURE 8.57 : PREDICTED PARTICLE SIZE DISTRIBUTIONS - FINER PRODUCT COMBINED SPIGOT FLOWS

8.9.4 Conclusions

As a result of the simulations documented in the above case study the following conclusions are drawn:

- the beaching simulation model representatively simulates the depositional behaviour of the Pering lead zinc tailings where this is deposited from a single point discharge

- the need for the change to the spigot system could have been predicted using the beaching model if representative particle size distributions (and the model) were available at the time of the design

- given the present range of mill grind it would be inadvisable to consider reinstating the paddock system in view of the anticipated extent of gravitational segregation.

- the present spigot (spray-bar) depositional system can be adequately simulated using the model but only if the depositional characteristics are first calibrated. Predictions on the beaching and segregation characteristics can then be reasonably modelled using the calibrated system provided the number of spigot openings and the extent to which individual streams from the discharges combine are kept constant.

- the beach profile is likely to remain adequately steep to provide sufficient freeboard to withstand design storm events over the range of mill grinds presently experienced.

CHAPTER 9

CONTENTS

Sec	Description	Page
9	CONCLUSIONS	9-1

CHAPTER 9 CONCLUSIONS

The objective of the research documented in this dissertation was to consider the influence of depositional practice on the performance of tailings dams in South Africa where the method of formation is primarily upstream deposition in ring-dyke impoundments. More specifically the research was to consider the mechanisms of natural gravitational segregation of the tailings as this moves down the beach and in so doing to develop a predictive method of estimating the resulting beach profile and the particle size distribution at any point along the beach. The benefits of achieving this objective would be an ability to optimise depositional practice so as to enhance tailings dam performance measured in terms of cost, safety and impact on the environment.

It was recognised at the outset that variations in the flow regimes of the stream down the beach would preclude analysis through classical hydrodynamic approaches. A new approach was therefore developed which would be insensitive to the detailed variations but which would accommodate the global effect of these variations. The approach is based on entropy which is defined as probabilistic uncertainty. This is a relatively new development in science and mathematics and has, to date, found favour with only a few researchers. Application of entropy as a measure of probabilistic uncertainty occurs in conjunction with a complimentary principle that all natural systems will automatically maximise entropy or probabilistic uncertainty. The combination of entropy and the principle that entropy is maximised at all points along the beach provides additional tools with which to analyse the mechanisms of deposition.

To apply entropy and the principle of maximum entropy it was necessary to define a function which incorporated all of the significant variables known to influence the beaching characteristics of tailings slurries. The function selected, that of stream power, brought together, in a scalar function, the key variables of flow rate, slurry density and flow velocity. Through consideration of the factors which influence the change of stream power down the beach it has been possible to incorporate the frictional resistance characteristics of the slurry. The stream power is an important controlling factor in determining the ability of the beaching stream to transport and maintain in suspension particles of varying size. By deriving an expression describing the variation of the stream power down the beach using entropy it has been possible to make estimations of the particle size distribution of the solids remaining in suspension. From changes in these particle size distributions with distance it has been possible to deduce the particle size distribution of the tailings that settles out on the beach.

The functions have been programmed into a spreadsheet model which has been checked against observed data and found to :

- predict the beach profile excellently
- predict the particle size distributions with moderate success

Sensitivity analyses conducted using the model have confirmed that variation of the most important parameters of flow rate, concentration, particle size distribution, solids density characteristics and frictional characteristics is as observed on tailings operations

It has been found that the beaching characteristics are extremely sensitive to variations in the slurry concentration, flow rate and the velocity at the head of the beach immediately below the discharge point. This is because the stream power is the multiplicand of these variables. Moreover the stream power is particularly sensitive to velocity since it is effectively proportional to the cube of the velocity. This sensitivity, and the fact that variations in any one of these variables automatically effects the others, explains why researchers to date have been largely unsuccessful in pinning down the effect of such variations on the beaching characteristics.

The entropy-based stream power approach provides a vital new tool with which to assess the impacts of variations in the parameters on mining operations and, equally importantly, with which to optimise the development and performance of tailings dams. To apply this tool it will be necessary to conduct testwork to determine the rheology of the tailings from which the frictional characteristics of the tailings may be determined. Thereafter a range of deposition methods, flow rates, and concentrations can be evaluated in detail and varied in isolation or in conjunction with one another to determine optimum method of discharge.

It is concluded that small scale flume testing is of very limited use as a result of significant divergence from similitude requirements. It is generally impractical to meet the similitude requirements due to limitations imposed by the solids. Flume testing will continue, therefore, to be useful only in obtaining a qualitative assessment of depositional behaviour.

An aspect which merits further research relates to the modelling of the plunge pool. While the calculation system developed in the course of this research is adequate and useful it is confined to

situations where the discharge stream remains relatively intact after it leaves the pipe and experiences negligible wind resistance so that the stream's gain in energy as a result of the free-fall is relatively unaffected. Where this stream breaks up due to excessive discharge energy or wind factors the calculated velocities at the level of the plunge pool are over-estimated. It is suggested that the method of calculation could be improved through detailed model studies. This will improve the ability to predict the plunge pool velocity which is paramount in determining the beaching characteristics.

LIST OF REFERENCES

Abadjiev, C B (1985), *Estimation of the Physical Characteristics of Deposited Tailings in the Tailings Dam of Nonferrous Metallurgy*, Proc XI Int Conf on Soil Mech and Found Engrg Vol 1, pp 1231-1234.

Abadjiev, C B (1990), *Improvement of a Tailings Dam Stability after Nine Years of Operation*, Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Ackers, P and White, R (1973), *Sediment Transport : New Approach and Analyses*, Journal of Hydraulic Engineering, ASCE, Vol 99, No. 11, pp 2041-2060

Ahuja, L R (1974), *Division S-1 - Soil Physics. Unsaturated Hydraulic Conductivity from Cumulative Infow Data.* Proceedings of Soil Science Society of America, Vol. 38, No. 5, September-October 1974, pp 695 - 699.

Aitchison. G D. ed, (1965), *Moisture Equilibria and Moisture Changes in Soils beneath covered areas*, General Review, P1.1 : Gen : Aitchison, 1965.

Albertson, M L, Barton, J R and Simons, D B, (1960), *Fluid Mechanics for Engineers*, Publ. Prentice-Hall, 1960

Annandale G (1987), *Reservoir Sedimentation*, Developments in Water Science Series, Publ. Elsevier Science Publishers B V, Amsterdam, 1987

Bennett, J S and Nordin, C F (1977), *Simulation of Sediment Transport and Armouring*, Hydrological Sciences Bulletin, No. XXII, 4 Dec. 1977, pp 555-569

Bentel, G M (1981), *Some Aspects of the Behaviour of Hydraulically Deposited Tailings*, MSc Thesis, University of the Witwatersrand, Johannesburg, 1981, pp 1-145.

Blight, G E (1987), *The Concept of the Master Profile for Tailings Dam Beaches*, Proc. Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987, pp 95-100

Blight, G E and Bentel, G M (1983) *The Behaviour of Mine Tailings During Depostion*, Jnl South African Instiution of Mining and Metallurgy, April, 1983, pp 73-86

Blight, G E, Thomson R R, and Vorster, K (1985), *Profiles of Hydraulic-Fill Tailings Beaches, and Seepage through Hydraulically Sorted Tailings*, Jnl South African Institute of Mining and Metallurgy, Vol 85, (1985), No. 5, pp 157-161

Blight, G E and Steffen, O K H (1979), *Geotechnics of Gold Mine Waste Disposal*, paper in Current Geotechnical Practice in Mine Waste Disposal, ASCE, pp 1-52

Blight, G E, McPhail, G I and Fourie, J (1986), *Possible Difficulties with Building Tailings Dams of Cyclone Overflow Product*, Association of Mine Managers of South Africa, Symposium on Backfill, Johannesburg, 1986, 14pp

Borah, D K and Bordoloi, P K (1989), *Stream Bank Erosion and Bed Evolution Model*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 612-617.

Bradley, J B (1989), *Field Data Collection and Analysis of Hyperconcentrations and Mud Flows*,

Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 687-692.

Bridge, J S and Dominic D D (1984), *Bed Load Grain Velocities and Sediment Transport Rates*, Water Resourc. Res. Vol 20, No. 4, pp 476-490

Bumb, A C, Murphy, C L and Everett, L G (1992), *A Comparison of Three Functional Forms for Representing Soil Moisture Characteristics*, Ground Water, Vol. 30, No. 2, March-April 1992, pp 117-185.

Byers, E and Stephens, D B (1983), *Statistical and Stochastic Analysis of Hydraulic Conductivity and Particle-Size in a Fluvial Sand*, Proceedings of Soil Science Society of America, Vol. 47, 1983, pp 1072-1080.

Capuzzo, D (1990), *Discussion: Static Stability including Seepage Pressures and Properties of Deposited Tailings*, Vol. 2, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 47-63.

Cassidy, J J and Hui, S L (1990), *Flood Criteria and the Safety of Tailings Dams*., Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Cassidy, J J and Vick, S (1990), *General Discussion Contribution and Reply*. Static Stability including Seepage Pressures and Properties of Deposited Tailings, Vol. 2, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 78-79.

Celik, I and Rodi, W (1988), *Modeling Suspended Sediment Transport in Non-equilibrium Situations*, Journal of Hydraulic Engineering, Vol 114, No. 10, pp 1157-1190

Celik, I and Rodi, W (1991), Suspended Sediment-Transport Capacity for Open Channel Flow. *Journal of Hydraulic Engineering*, Vol. 117, No. 2, February 1991, pp 191-204.

Chang, H H (1984), *Regular Meander Path Model*, Journal of Hydraulic Engineering, Vol. 110 No. 10, October 1984, pp 1398-1411.

Chang, H H (1985), *River Morphology and Thresholds*, Journal of Hydraulic Engineering, Vol. 111102 No. 3, March, 1985, ppp 503- 519.

Chang, H H and Hill, J C (1976), *Computer Modeling of Erodible Flood Channels and Deltas*, Journal of Hydraulic Division, Proceedings of the American Society of Civil Engineerings, Vol. 102 No. HY10, October 1976, pp 1461-1477.

Chang, H H (1982), *Modeling of River Channel Changes*, Journal of Hydraulic Engineering, Vol. 110 No. 2, November 1982, pp 157-172.

Chang, H H and Hill, J C (1977), *Minimum Stream Power for Rivers and Deltas*, Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings, Vol. 103, No. HY12, December 1977, pp 1375-1389.

Chang, H H (1985), *Water and Sediment Routing through Curved Channels*, Journal of Hydraulic Engineering, Vol. 111, No. 4, April 1985, pp 644-658.

Chang, H C. *Stable Alluvial Canal Design*, Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings, Vol. 106, No. HY5, May 1980, pp 873-891.

Chang, H H (1980), *Geometry of Gravel Streams*, Journal of Hydraulic Division, Proceedings of the American Society of Civil Engineerings, Vol. 106 No. HY9, September 1980, pp 1433-1456.

Chang, H H (1982), *Mathematical Model for Erodible Channels*, Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings, Vol. 108, No. HY5, May 1982, pp 678-689.

Chang, H H (1989), *Background and Applications of Fluvial-12*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 648-652.

Chang, H H. *Energy Expenditure in Curved Open Channels.* pp 1012-1022

Chang, H H. *Variation of Flow Resistance through Curved Channels.* pp 1772-1782

Chang, H H. *Design of Stable Alluvial Canals in a System.* pp 36-43.

Chang, H H (1984), *Analysis of River Meanders*, Journal of Hydraulic Engineerings, Vol. 110, No. 1, January 1984, pp 37-50.

Chiu, C-L (1987), *Entropy and Probability Concepts in Hydraulics*, Journal of Hydraulic Engineering, Vol. 113, No. 5, May 1987, pp 583-600.

Chiu, C-L (1988), *Entropy and 2-D Velocity Distribution in Open Channels*, Journal of Hydraulic Engineering, ASCE, 114, No. 7, July 1988, pp738-756

Chiu, C-L (1989), *Velocity Distribution in Open Channel Flow*, Journal of Hydraulic Engineering, ASCE, 115, No. 5, May 1989, pp 576-594

Chiu, C-L (1991), *Application of Entropy Concept in Open-Channel Flow Study*, Journal of Hydraulic Engineering, ASCE, Vol 117, No 5, May 1991, pp 615-628

Cooke, R and Lazarus, J H (1988), *Isokinetic Sampling Probe for Slurry Flows* , Hydrotransport 11, 11th International Conference on the Hydraulic Transport of Solids in Pipes, Stratford-Upon-Avon, UK, 19-21 October 1988, pp 117-130.

Cooper, D, (1988), *Tailings Management in Australia*, Hydraulic Fill Structures, ASCE Geotech. Specialty Publication No. 21 Ed. by D J A van Zyl and S G Vick pp 130-141.

Corey, A T (1957), *Measurement of Water and Air Permeability in Unsaturated Soil*, Reprinted from Soil Science Society of America Proceedings, Vol 21, No. 1, January-February 1957, pp 7-10.

Corey, A T and Brooks, R H (1975), *Drainage Characteristics of Soils*, Soil Sc. Soc. Amer, Vol. 39, 1975, pp 251-255.

Crilly, A J, Earnshaw, R A and Jones, H (1991), *Fractals and Chaos*, New York, Springer-Verlag, 1991, pp 1-277.

Dane, J H and Hruska, S (1983), *Division S-1 - Soil Physics. In-Situ Determination of Soil Hydraulic Properties during Drainage*. Proceedings of Soil Science Society of America, Vol. 47, No. 4, July-August 1983, pp 619-624.

Dane, J H (1980), *Division S-1 - Soil Physics. Comparison of Field and Laboratory Determined Hydraulic Conductivity Values*, Proceedings of Soil Science Society of America, Vol. 44, 1980, pp 228-231.

de Groot, M B, Heezen, F T, Mastbergen, D R, and Stefess, H, (1988), *Slopes and Densities of Hydraulically Placed Sands*, Hydraulic Fill Structures, ASCE Geotech. Specialty Publication No. 21 Ed. by D J A van Zyl and S G Vick pp 32-51,

Donaldson, G W.(1960), *Practical Observations and the Results of Research on the Stability of Slimes Dams for the Gold Mining Industry* Jnl. South African Inst. Mining and Metallurgy, Vol. 61, No. 3, 1960, pp 183-199.

Donalsdon, G W, (1965), *The effectsof Capillary Action on the Consolidation and Shear Strength of Silt in a Hydraulic Fill Dam*. In : Proceedings, Sixth International Conference on Soil Mechanics and Foundation Engineering, Montreal, pp 459-465.

Donaldson, G W, (1973), *Theoretical and Engineering Aspects of Slimes Dam Construction*. J. S. Afr. Inst, Min. Metall, Vol. 74, No. 5, 1973, pp 175-177.

Donaldson, G W, Admason, R J and Clausen, H T, (1976), *Slimes Dams for Gold Mine Tailings and Other Residues in South Africa*. Commission Internationale Des Grands Barrages, Couzieme Congres des Grands Barrages, Mexico. 1976, pp 709-730.

Durand, R (1953), *Basic Relationships of the Transporation of Solids in Pipes - Experimental Research*, Proc 5 th Congress IAHR, Minneapolis, 1953.

Durand, R and Condolis, E (1957), *The Hydraulic Transport of Coal and Solid Materials in Pipes*, Proc, Colloquium on Hydraulic Transportation, London.

Eagleman, JR and Jamison, V C. *Soil Layering and Compaction Effects on Unsaturated Moisture Movement*, Soil Sc. Soc. Amer, Vol. 26, No. 6, pp 519-522.

Einstein, H A, (1950), *The Bed Load Function for Sediment Transport in Open Channel Flows*, Technical Bulletin No. 1028, Soil Conservation Service, U S Department of Agriculture, Washington DC, pp 1-78

Elzeftawy, A and Mansell, R S (1975), *Division S-1 - Soil Physics Hydraulic Conductivity Calculations for Unsaturated Steady-State and Transient-State Flow in Sand*. Proceedings of Soil Science Society of America, Vol. 39, No. 4, July-August 1975, pp 599 - 603.

Fan, X and Masliyah, J (1990), *Laboratory Investigation of Beach Profiles in Tailings Disposal*, Journal of Hydraulic Engineering, Vol. 116 No. 11, November 1990, pp 1357-1373.

Feddes, R A, Neuman, S P and Bresler, E (1975), *Finite Element Analysis of Two-Dimensional Flow in Soils Considering Water Uptake by Roots : II. Field Applicatons* Proceedings of Soil Science Society of America, Vol. 39, 1975, pp 231-237.

Feddes, R A, Kabat, P, Van Bake. P J T, Bronswijk, J J B and Halbertsma, J (1988), *Modelling Soil Water Dynamics in the Unsaturated Zone - State of the Art*, Institute for Land and Water Management Research (ICW), Wageningen (The Netherlands), March 16, 1988.

Fredlund, D G (1989), *Soil Suction Monitoring for Roads and Airfields*, Presented to the SYMPOSIUM State-of-the-Art of Pavement Response Monitoring Systems for Roads and Airfields, West Lebanon, NH, March 6-9, 1989.

Fredlund, D G and Wong, D K H (1989), *Calibration of Thermal Conductivity Sensors for Measuring Soil Suction*, Geotechnical Testing Journal, September 1989, pp 188-194.

Fredlund, D G and Rahardio, H (1988), *State-of-Development in the Measurement of Soil Suction*,

Engineering Problems in Regional Soils. Proceedings of the International Conference, Beijing, China, August 11-15, 1988. pp 582-588.

Garcia, M and Parker, G (1991), *Entrainment of Bed Sediment into Suspension*, Journal of Hydraulic Engineering, Vol. 1117 No. 4, April 1991, pp 414-435.

Glysson, G D (1989), *Criteria for a Sediment Data Set*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 675-680.

Gowan, M J. *Civil Engineering Aspects of Tailings Disposal in the Orange Free State*, Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Gowan, M J and Williamson, J R G (1987), *A Review of Tailings Deposition Techniques in South Africa and Appropriate Selection of Application*, Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Graham, J and Williams, D J (1992), *Cracking in Drying Soils.* Can. Geotech. Journal, No. 29, 1992, pp 263-277.

Graf, W H, (1971), *Hydraulics of Sediment Transport*, McGraw-Hill, 1971

Green, R E and Corey, J C. *Division S-1 - Soil Physics. Calculation of Hydraulic Conductivity A Further Evaluation of Some Predictive Methods*. Proceedings of Soil Science Society of America, pp 3 - 8.

Griffin, P M (1990), *Control of Seepage in Tailings Dams*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Hanks, R W and Dadia, B H (1971), *Theoretical Analysis of the Turbulent Flow of Non-Newtonian Slurries in Pipes*, Jnl of the AIChE, No. 17, 1971.

Hanks, R W (1978), *Low Reynolds Number Turbulent Pipeline Flow of Pseudohomogeneous Slurries*, Proc 5th Int. Conf. on Hydraulic Transport of Solids in Pipes, Hannover, Germany, 1978, pp C2 - C25

Harr, M E, (1987), *Reliability-Based Design in Civil Engineering*, McGraw-Hill Book Company, pp 1-290

Harr, M E. *Two-Point Probabilistic Estimates for Multivariate Analyses*. School of Civil Engineering, Purdue University, West Lafayette, IN 47907, USA, pp 1-22.

Head, K H, (1980), *Manual of Soil Laboratory Testing - Volume 1*, Pentech Press Ltd, Plymouth, Devon, UK, (1980)

Highter, W H and Page, G L, (1976), *Innovations in Tailings Disposal: Discussion*, Can. Geot. Jnl. Vol. 13, 1976, pp 334-336

Hillel, D. ed (1980), *Applications of Soil Physics*, Publication Acadamic Press, 1980, pp 5 - 355.

Holly, F M and Rahuel, J L (1989), *Proceedings of Int. Symposium on Sediment Transport Modelling*, 14-18 August, 1989, ASCE Hydr. Div, pp 58-63.

Hodge, W E, (1988), *Construction Method for Improving Underwater Sand Fills*, Hydraulic Fill Structures, ASCE Geotech. Specialty Publication No. 21 Ed. by D J A van Zyl and S G Vick pp

105-129.

Ingram, J J, Abt, S R and Richardson, E V, (1989), *Total Sediment Discharge Measurement*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 693-699.

Ingram, J J, Abt, S R and Richardson, E V, (1991), *Sediment Discharge Computation using Point-Sampled Suspended-Sediment Data*. Journal of Hydraulic Engineering, Vol. 117, No. 6, June 1991, pp 758-773.

International Atomic Energy Agency, (1981), *Current Practices and Options for Confinement of Uranium Mill Tailings*, Technical Reports Series No. 209, Vienna, 1981, pp 1-102

International Commission on Large Dams, (1982), *Manual on Tailings Dams and Dumps*, Bulletin No. 45, pp 1-237

International Commission on Large Dams, (1989a), *Tailings Dam Safety - Guidelines*, Bulletin No. 74, pp 1-107

International Commission on Large Dams, (1989b), *Bibiliography Mine and Industrial Tailings Dams and Dumps*, Bulletin No. 44A, pp 1-321

Ivanov, P L (1990), *Stability Increase and Volume Expansion of Tailings Dams due to Draining and Compaction of Dam Soils by Charge Explosions*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

James C S (a) (1992), *A Review of the Bottom Boundary Conditions for Suspended Sediment Distribution Models*, Draft Paper, Personal communication

James C S (b) (1992), *Modelling the Interaction between Bed Material and the Different Modes of Transport*, Draft Paper, Personal communication.

Jennings, J E, (1979), *The Failure of a Slimes Dam at Bafokeng*, The Civil Engineer in South Africa, Vol 21, No. 6, 1979, pp 135-141.

Jian, Y and Gourein, D (1989), *A New Tubulence Model : the* K-ϵ *Model*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 184-189.

Jones, G A, van Zyl, D J and Rust, E (1981) *Mine Tailings Characterisation by Piezometer Cone*, Symposium on Cone Penetration Testing and Experience, ASCE, St Louis, 1981, pp303-324

Julien, P (1989), *Laboratory Analysis of Hyperconcentrations*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 681-686.

Julien, P Y and Lan, Y (1991), *Rheology of Hyperconcentrations,* Journal of Hydraulic Engineering, Vol. 117, No. 3, March 1991, pp 346-353.

Kapur, J N and Kesavan, H K (1992), *Entropy Optimisation Principles with Applications*, Academic Press, Inc, publ. Harcourt Brace Jovanonvich, 1992.

Karim, M F and Holly, F M Jr (1986), *Armoring and Sorting Simulation in Alluvial Rivers*, Journal of Hydraulic Engineering, Vol. 112, No. 8, August 1986, pp 705-715.

Kilpachkova, A B and Kulchitskaja, T G (1990), *Effects of Technological Parameters on Tailings Dam Stabilities*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings

Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Knight, R B, and Haile J P, (1983), *Sub-aerial Tailings Deposition with Underdrainage*, Proc. 7th Pan American Conf. on Soil Mech. and Found. Eng., Vancouver, B C, 1983 P

Kunze, R J and Kirkham, D (1962), *Simplified Accounting for Membrane Impedance in Capillary Conductivity Determinations*, Proceedings of Soil Science Society of America, Vol. 26, 1962, pp 421 - 426.

Küpper, A M A G, (1991), *Design of Hydraulic Fill*, PHD Thesis, University of Alberta, Department of Civil Engineering, Spring 1991 pp 1-524

Kynch, G J, (1952), *A Theory of Sedimentation*, Trans. Faraday Soc. 48, pp 166-176

Lee, R K C and Fredlund, D G (1984), *Measurement of Soil Suction using the MCS 6000 Sensor,* Proceedings of the 5th International Conference on Expansive Soils, Adelaide, South Australia, May 21-23, 1984, pp 50-54.

Lee, H and Odgaard, A J (1986), *Simulation of Bed Armoring in Alluvial Channels*, Journal of Hydraulic Engineering, Vol. 112, No. 9, September 1986, pp 794-801.

Leopold, L B and Langbein, W B (1962), *The Concept of Entropy in Landscape Evolution*, U S Geological Survey, Professional paper 500 A (1962)

Libardi, P L, Reichardt, K, Nielsen, D R and Biggar J W (1980), *Division S-1 - Soil Physics. Symplified Field Methods for Estimating Soil Hydraulic Conductivity*, Proceedings of Soil Science Society of America, Vol. 44, 1980, pp 3 - 7.

Low, H S (1989), *Effect of Sediment Density on Bed-Load Transport*, Journal of Hydraulic Engineering, Vol. 115, No. 1, January 1989, pp 124-138.

Lu, J-Y and Shen, H W (1986), *Analysis and Comparisons of Degradation Models*, Journal of Hydraulic Engineerins, Vol. 112 No. 4, April 1986, pp 281-299.

Lyn, D A (1992), *Turbulance Characteristics of Sediment-Laden Flows in Open Channels*, Journal of Hydraulic Engineering, Vol. 118, No. 7, July 1992, pp 971-1001.

Lyn, D A (1991), *Resistance in Flat-Bed Sediment-Laden Flows*, Journal of Hydraulic Engineering, Vol. 117, No. 1, January 1991, pp 94-114.

Lyn, D A (1987), *Unsteady Sediment-Transport Modelling*, Journal of Hydraulic Engineering, Vol. 113, No. 1, January 1987, pp 1-15.

Management of Wastes from the Mining and Milling of Uranium and Thorium Ores, A Code of Practice and Guide to the Code, International Atomic Energy Agency, Vienna, 1976, pp 1-49.

Melent'ev, V A,Kolpashnikov, N P, and Volmin, B A, (1979), *Hydraulic Fill Structures*, Energy, Moscow, (1973), English Translation, Ed D van Zyl, pp 1-240

McPhail, G I and Dorman, S A (1987), *Increased Tailings Dam Capacity through Controlled Deposition and Pore Pressure Observation : A Case History*, Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987, pp 161-174.

McRoberts, E C and Nixon J F (1976) *A Theory of Soil Sedimentation*, Can. Geot. Jnl. Vol 13, Number 3, 1976, pp 294-310

McWhorter, D B and Nelson, J D (1980), *Seepage in the Partially Saturated Zone Beneath Tailings Impoundments*, Mining Engineering. April 1980, pp 432-439.

McWhorter, D B and Nelson, J D (1979), *Unsaturated Flow Beneath Tailings Impoundments*, Journal of Geogechnical Engineering Division, Vol. 105, No. GT11, November 1979, pp 1317-1334.

Meyer-Peter, E and Muller, R (1948), *Formulas for Bed Load Transport*, Proc. 2nd Congress of the Int. Assoc. for Hydraulic Research, Stockholm, Sweden, pp 39-64

Meynink, W J C, *Development of a Tailings Transport Model*, publication uncertain pp1-3.45.

Miller, E E and Elrick, D E (1958), *Dynamic Determination of Capillary Conductivity Extended for Non-Negligible Membrane Impedance*, Proceedings of Soil Science Society of America, Vol. 22, 1958, pp 483 - 486.

Minns, A, (1988), *A Review of Tailings Disposal Practice in North America and Australia*, Hydraulic Fill Structures, ASCE Geotech. Specialty Publication No. 21 Ed. by D J A van Zyl and S G Vick pp 52-68

Misri, R L, Garde, R J, and Ranga Raju, K G (1984), *Bed Load Transport of Coarse Non-uniform Sediments*, Journal of Hydraulic Engineering, ASCE, Vol 110, No. 3, pp 312-328

Mittal, H K and Morgenstern, N R (1975) *Parameters for the Design of Tailings Dams*, Can. Geot. Jnl. Vol. 12, Number 2, pp 235-261

Mittal, H K and Morgenstern, N R (1976), *Seepage Control in Tailings Dams*, Can. Geot. Jnl. Vol 13, Number 3, 1976, pp 277-293

Morgenstern, N R (1985) *Geotechnical Aspects of Environmental Control*, Proc. XI Int. Conf. Soil Mechanics and Foundation Engineering, Vol 1, San Francisco, 1985, pp 155-185

Morris, P H, Graham, J and Williams, D J, (1992), *Cracking in Drying Soils*, Can. Geot. Jnl. Vol. 29, No. 2, April, 1992, pp 263-277.

Mrost, M, (1974), *Slimes Disposal at South African Gold Mines*, South African Institute of Mining and Metallurgy, 1974

Mualen, Y (1976), *A New Model for Predicting the Hydraulic Conductivity of Unsaturated Porous Media*, Water Resources Research, Vol. 12, No. 3, June 1976, pp 513 - 522.

Murphy, S and Williams, P (1990), *The Application of Finite Strain Consolidation Theory to the Design of Tailings Disposal Systems*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

National Building Research Institute, (1959), *An Investigation into the Stability of Slimes Dams with Particular Reference to the Nature of the Material of their Construction and the Nature of their Foundation*, Council for Scientific and Industrial Research, Pretoria, South Africa, pp 1-127.

Nelson, J D and Davis L A (1987), *State of the Art - Drainage of Tailings Impoundments*, Proceedings of the International Conference on Mining and Industrial Waste Management, Johannesburg, 1987, pp 195 - 204.

Nelson, J D and Davis, L A (1987), *Drainage of Tailings Impoundments - State of the Art*, Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Nelson, J D and McPhail, G I. *Design and Construction of Gold Mill Tailings Impoundments*, Gold Mining 87, pp 363 - 381.

Neuman, S P (1973), *Saturated-Unsaturated Seepage by Finit Elements*, Journal of Hydraulic Division, Vol. 99 No. HY12, December 1973, pp 12233-2250.

Panteleev, V G (1990), *Mechanisms of Sluicing Dump Stop Prisms*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Panteleev, V G (1990), *On Some Aspects of Tailings Dikes Stability Analysis*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Park, I and Jain, S C (1987), *Numerical Simulation of Degradation of Alluvial Channel Beds*, Journal of Hydraulic Engineering, Vol. 113, No. 7, July 1987, pp 845-859.

Phene, C J, Hoffman, G J and Rawlings, S L (1970), *Measuring Soil Matric Potential in situ by Sensing Heat Dissipation within a Porous Body : I. Theory and Sensor Construction,* Univ. of California, Riverside, 12 June, 1970, pp 27-33.

Phene, C J, Hoffman, G J and Rawlings, S L (1970), Measuring Soil Matric Potential in situ by Sensing Heat Dissipation within a Porous Body : II. Theory and Sensor Construction, Univ. of California, Riverside, 12 June, 1970, pp 225-229.

Powell, R D, and Morgenstern, N R, (1985), *The Use and Performance of Seepage Reduction Measures*, Seepage and Leakage from Dams and Impoundments, Ed R L Volpe and W E Kelly, ASCE Geot Eng Div, 1985.

Rahardjo, H, Loi, J and Fredlund, D G (1989), *Typical Metric Suction Measurements in the Laboratory and the Field using Thermal Conductivity Sensors*, Proceedings of the Indian Geotechnical Conference, Rome, December 14-16, 1989, pp 1-5.

Raudkivi, A J. *Loose Boundary Hydraulics*, 3rd Edition, Pergamon Press, pp 1-538.

Register of Mine and Industrial Tailings Dams, 1st Edition, Stockholm, May 1981, pp 1-159.

Richards, L A and Richards P L (1962), Division S-1 - Soil Physics. Radial-Flow Cell for Soil-Water Measurements. *Proceedings of Soil Science Society of America*, Vol. 26, No. 6, November-December 1962, pp 515 - 518.

Richards. B G. ed, *Moisture Equilibria and Moisture Changes in Soils beneath covered areas*, Pschyrometic Paper, pp 39 - 46.

Richie, E B and Schnabel, R R (1992), *Parameter Estimation of the Coefficients in Soil Hydraulic Equations from Steady-State Unsaturated Flow*, Ground Water, Vol. 30, No. 4, July-August 1992, pp 515-522.

Rijtema, P E (1959), *Calculation of Capillary Conductivity from Pressure Plate Overflow Data with Non-Negligible Membrane Impedance*, Neth. Jnl. of Aric. Science, Vol. 7, 1959.

Risso, D and Bresler, E. *Field Determination of Soil Hydraulic Properties for Statistical Analysis*, Proceedings of Soil Science Society of America, Vol. 44, 198075, pp 697-702.

Robinsky, E I, (1975), *Tailings Disposal By the Thickened Discharge Method for Improved Economy*

and Environmental Control, Proc. 2nd Int. Tailings Symposium, Miller Freeman, San Francisco, pp 75-95.

Rooseboom, A (1975), *Sediment Transport in Rivers and Rsevoir Basins*, (in Afrikaans), DSc(Eng) Thesis presented to the University of Pretoria, South Africa.

Round, G F (1986), *Design and Operation of Hydraulic Conveying Systems*, Course notes, University of the Witwatersrand Technology Centre, 1986.

Royer, J M and Vachaud, G (1975), *Division S-1 - Soil Physics. Field Determination of Hysteresis in Soil-Water Characteristics,* Proceedings of Soil Science Society of America, Vol. 39, No. 2, March-April 1975, pp 221-223.

Samaga, B R, Raju, K G R and Garde, R J (1986), *Suspended Load Transport of Sediment Mixtures*, Journal of Hydraulic Engineering, Vol. 112, No. 11, November 1986, pp 1019-1035.

Sattler, P and Fredlund, D G (1989), *Use of Thermal Conductivity Sensors to Measure Matric Suction in the Laboratory*, Canadian Geotechnical Journal, Vol. 26, No. 3, 1989, pp 491-498.

Schiffman, R.L and Carrier, W D (1990), *Large Strain Consolidation used in the Design of Tailings Impoundments*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Schuh, W M and Bauder, J W (1986), *Effect of Soil Properties on Hydraulic Conductivity-Moisture Relationships*, Proceedings of Soil Science Society of America, Vol. 50, 1986, pp 848-855.

Scott, J D and Cymerman G J (1984), *Prediction of Viable Tailings Disposal Methods*, Proc. Symposium on the Prediction and Validation of Consolidation, ASCE, San Francisco, pp 522-544

Shields, D H, (1975), *Innovation in Tailings Disposal*, Can. Geot. Jnl., Vol. 12, pp 320-325

Shiqiang, W and Ren, Z (1989), *Sediment Transport Rate for Non-uniform Sand*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div,588-593.

Smith, G M, (1984), *Predicting Beach Slopes on Hydraulically Deposited Mill Tailings*, MSc Thesis, Colorado State University, Fort Collins, Colorado, pp 1-128

Smith, M E, Robbertze, J B and Bentel, D L (1987), *The Repair and Recommissioning of Gold Tailings Dams*, Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Smith, M E, Bentel, D I and Robbertze, J R (1987), *Management Guidelines for the Construction of Gold Tailings Dams*, Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Soderberg, R L and Busch R A (1977), *Design Guide for Metal and Nonmetal Tailings Disposal*, U S Bureau of Mines Information Circular 8755

Song, C C S and Yang, C T (1976), *Velocity Profiles and Minimum Stream Power,* Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings, Vol. 105, No. HY8, August 1976, pp 981-998.

Song, C C S and Yang C T (1980), *Minimum Stream Power : Theory*, Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings, Vol. 106, No. HY9, September 1980, pp 1477-1487.

Song, C C S and Yang C T (1980), *Discussions of Minimum Stream Power : Theory*, Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings, Vol. 106, No. HY9, September 1980, pp 877-878.

Soni J P, Garde R J and Ranga Raju K G (1980), *Aggradation in Streams due to Overloading*, Journal of Hydraulic Engineering, ASCE, Vol 106, No. 1, pp 117-182

Steffen, Robertson and Kirsten (Mining) Inc (1981 to 1985), *Various Reports and minutes of meetings concerning a Gold Mining Operation to the east of Johannesburg*, Confidential Reports.

Swamee, P K and Ojha, C S P (1991), *Drag Coeffieient and Fall Velocity of Nonspherical Particles*, Journal of Hydraulic Engineering, Vol. 117, No. 5, May 1991, pp 660-667.

Swamee, P K Ojha, C S P (1991), *Bed-Load and Suspended-Load Transport of Nonuniform Sediments*, Journal of Hydraulic Engineering, Vol. 117, No. 6, June 1991, pp 774-787.

Swarbrick, g e, (1992), *Transient Unsaturated Consolidation in Desiccating Mine Tailings*, PhD Thesis, University of New South Wales, Australia, 1992 pp 1-1 - 6-3.

Thomson, R R,(1984), *Depositional Characteristics of Tailings as Observed from Laboratory Studies*, MSc Degree, University of the Witwatersrand, Johannesburg, 1984, pp 1-75.

Thomson, R R (1984), *Depositional Characteristics of Tailings as Observed from Laboratory Studies*, MSc Thesis, University of the Witwatersrand, Johannesburg, 1984, pp 1-75.

Van Raadt, P, Fredlund, D G, Clifton, A W, Klassen, M J and Jubien, W E (1987), *Soil Suction Measurements at Several Sites in Western Canada*, Presented to Session on Geotechnical Problems in Arid Regions, Transportation Research Board, Washing, D.C. 1987, pp 1-31.

Van Genuchten, M Th. *A closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils*, Soil Sci. Soc. Am. J 44, pp 892-898.

Van Rijn, L C (1984), *Sediment Ransport Part I: Bed Load Transport*, Journal of Hydraulic Engineering, ASCE, Vol 110, pp 1431-1456, *Part II: Suspended Load Transport*, Journal of Hydraulic Engineering, ASCE, Vol 110, pp 1613-1641, *Part III: Bed Forms and Alluvial Roughness*, Journal of Hydraulic Engineering, ASCE, Vol 110, pp 1733-1754

Van Rijn, L C (1989), *The State of the Art in Sediment Transport Modelling*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 13-32.

Van Zyl, D (1987),. *Seepage and drainage analysis of tailings impoundments - Is it really that simple?*, Proceedings of the International Conference on Mining and Industrial Waste Management, Johannesburg, 1987, pp 211 - 222.

Vauclin, M, Vieira, S R, Vachaud, G and Nielsen, D R (1983), *The Use of Cokriging with Limited Field Soil Observations*, Proceedings of Soil Science Society of America, Vol. 47, March-April 1983, pp 175-184.

Velich, K (1990), *Safety of Tailings Dams In Czechoslovakia*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Verkerk, C G (1987), *The Hydraulic Transportation of Gold Mine Waste Materials* Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Verkerk, C G (1990), *The Role of the Particle Size and Distribution and its Allied Physical Characteristics on the Transportation of Medium and Dense Phase Slurries in South Africa*, Phd Thesis submitted to the University of the Witwatersrand, Johannesburg, South Africa, 1990, 648 pp

Verkerk C G (1990), *Pipeline Design and Friction Loss Estimations*, Short Course on Advanced Pumping and Pipeline Design, S A Inst of Mechanical Engineers, September 1990.

Vick S G, (1990), *Planning Design and Analysis of Tailings Dams*, BiTech Publishers Ltd, Vancouver, B C, Canada, pp 1-369

Vollmers, H-J (1989), *The State of Art of Physical Modelling of Sediment Transport,* Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 7-12.

Voogt, L, van Rijn, L E and van den Berg, J H (1991), *Sediment Transport of Fine Sands at High Velocities*, Journal of Hydraulic Engineering, Vol. 117, No. 7, July 1991, pp 869-890.

Wasp E J, kenny, J P and Gandhi, R L (1977), *Solid-Liquid Flow Slurry Pipeline Transportation*, Trans. Techn. Publications, Clausthal, W Germany, 1977.

Ward, A Dm, Wells, L G and Phillips, R E. *Infiltration through Reconstructed Surface Mined Spoils and Soils,* American Society of Civil Engineerings, 0001-2351/83/2603-0821502.00, pp 821-832.

Wates, J A, Sevenson, C, and Purchase, A R, (1987), *The Effect of Relative Densities on Beaching Angles and Segregation on Gold and Uranium Tailings Dams*, Proc. Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987, pp 89-93

Watson, K K, Reginato, R J and Jackson, R D (1975), *Division S-1 - Soil Physics. Soil Water Hysteresis in a Field Soi*l, Proceedings of Soil Science Society of America, Vol. 39, March-April 1975, pp 242-246.

Webber, N B (1971), *Fluid Mechanics for Civil Engineers*, Publ. Chapman and Hall Ltd, London, 1971.

White, W R, Milli, H and Crabbe, A D, (1975), *Sediment Transport Theories : A Review*, Proceedings of Institution of Civil Engineers, Part 2, 59, June 1975, pp 265-292, *Additional Discussion: Sediment Transport Theories : A Review*, Proceedings of Institution of Civil Engineers, Part 2, 61, Dec 1976, pp 803-810

Wiberg, P L and Smith, J D (1989), *Model for Calculating Bed Load Transport of Sediment*, Journal of Hydraulic Engineering, Vol. 115, No. 1, January 1989, pp 101-123.

Williams, D T (1989), *The Relationship of Milligrams per Liter to Parts per Million*, Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 428-433.

Williams, J G R (1987), *The Hydraulic Transportation of Solids in Pipelines - State of the Art,* Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Williams, D J and Morris, P H (1987), *Effect of Placement Technique on the Properties of Slurried Fine Grained Coal Mine Tailings*, Int. Conf. on Mining and Industrial Waste Management, Johannesburg, 1987.

Williams, D J and Morris, P H, (1988), *Properties of Slurried Coal Tailings*, Hydraulic Fill

Structures, ASCE Geotech. Specialty Publication No. 21 Ed. by D J A van Zyl and S G Vick pp 410-429.

Williamson, J R G (1990), *South African Tailings Dam Safety*. Vol. 1, ICOLD International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, 23 May 1990, pp 146-155.

Winterwerp, J C, de Groot, M B, Mastbergen, D R and Verwoert, H. (1990) *Hyperconcentrated Sand-Water Mixture Flows Over a Flat Bed*, Journal of Hydraulic Engineering, ASCE, Volume 116, January 1990

Wipplinger, O (1990), *Soil Moisture Flow Theory*. Technical Paper, The Civil Engineer in South Africa, March 1990.

Wong, D K H, Fredlund, D G, Imre, E and Putz, G (1988), *Evaluation of AGWA-II Thermal Conductivity Sensors for Soil Suction Measurement,* Submitted to Transportation Research Board, Washington, D.C., June 1988, pp 1-

Wösten, J H M and Van Genuchten, M Th (1976), *Division S-6 - Soil and Water management and Conservation. Using Texture and Other Soil Properties to Predict the Unsaturated Soil Hydraulic Functions,* Proceedings of Soil Science Society of America, 1976, pp 1762-1770.

Yalin, M S (1977), *Mechanics of Sediment Transport*, 2nd Edition, Pergamon Press, pp 1-298

Yang, C T (1971), *Potential Energy and Stream Morphology*, Journal of Water Resources Research, Vol. 7, No. 2, April 1971, pp 311-322.

Yang, C T (1976), *Minimum Unit Stream Power and Fluvial Hydraulics,* Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings,. Vol. 102, No. HY7, July 1976, pp 919-933

Yang, C T (1989), *Sediment Transport Modelling,* Proceedings of Int. Symposium on Sediment Transport Modelling, 14-18 August, 1989, ASCE Hydr. Div, pp 653-658.

Yang, C T (1972), *Unit Stream Power and Sediment Transport*, Journal of Hydraulic Engineering, Vol. 35, No. HY10, October 1972, pp 1806-1826.

Yang, CT. *Sediment Transport and River Engineering,* International Symposium on River Sedimentation, March 24-29, 1980, Beijing, China, pp 349-389.

Yang, C T (1984), *Unit Stream Power Equation for Gravel*, Journal of Hydraulic Engineering, Vol. 110, No. 12, December 1984, pp 1783-1797.

Yang, C T and Mollinas, A (1982), *Sediment Transport and Unit Stream Power Function*, Journal of Hydraulics Division Proceedings of the American Society of Civil Engineerings, Vol. 1086, No. HY69, June 1982, pp 774-793.

Yang, C T (1983), *Rate of Energy Dissipation and River Sedimentation,* Paper to be published in the Proceedings of Second International Symposium on River Sedimentation, Nanjing, China, October 11-16, 1983.

Yang, C T and Song, C C S (1986), Chapter II : Theory of Minimum Energy and Energy Dissipation Rate. Reprinted from the Encyclopedia of Fluid Mechanics, 1986.

Yong, R N, (1984), *Particle Interaction and Stability of Suspended Solids (Status Position No. 2),*

Proc. Symposium on the Prediction and Validation of Consolidation, ASCE, San Francisco, pp 30-59.

Yong, R N and Elmonayeri D S, (1984), *Convection-Diffusion Analysis of Sedimentation in Initially Dilute Solids-Suspensions*, Proc. Symposium on the Prediction and Validation of Consolidation, ASCE, San Francisco, pp 260-274

Zhang, H and Kahawita, R (1987), *Nonlinear Model for Aggradation in Alluvial*, Channels, Journal of Hydraulic Engineering, Vol. 113, No. 3, March 1987, pp 353-369.

3010069298

THESIS
TN
525
MACP

McPHAIL, G. I.

Prediction of the beaching characteristics of hydraulically placed tailings

University of the Witwatersrand, Johannesburg

The Library

This book may *not* be retained after the last date stamped below. A renewal for a further period of seven days may be granted unless the book is in demand. *Fines, as scheduled in the Rules and Regulations, are incurred on overdue books.*

17 JAN 1996
1 JUL 1996
IHL 42926
25 NOV 1997
IHL 65238
SAB721668
27.09.97
30.10.97
11 SEP 1998
26 MAR 2001
16 MAY 2001
03 DEC 2001

This book is *not transferable.* It will remain your charge until returned by you in person.

REFERENCE LIBRARY



CIVIL ENGINEERING

