

A COMPUTER AND EXPERIMENTAL SIMULATION OF STIRLING CYCLE MACHINES

DAVID MONTAGUE BERCHOWITZ

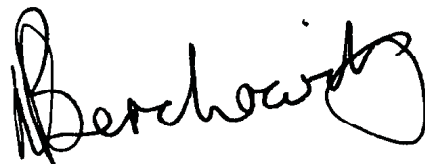
11K
1980/164.

A Dissertation Submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, for the degree of Master of Science.

Johannesburg, March 1978.

DECLARATION

I, DAVID MONTAGUE BERCHOWITZ, hereby declare that this thesis is my own work and that the material herein has not been submitted by me for degree purposes at any other University.

A handwritten signature in black ink, appearing to read "Berchowitz", with a large, stylized flourish at the end.

ABSTRACT

A critical evaluation on recent advances in the field of Stirling machine simulation is conducted. It is shown that the approach developed by Urieli [Ur77] is the most rigorous.

The transport equation set (which describes the behaviour of the working gas) is derived in both the integral and differential forms. Only the integral equation set is used in the simulation. The differential equation set is used to investigate the structure of individual terms in the integral equation set. This procedure allows these terms to be more accurately understood and hence modelled. The analytical derivation of the transport equation sets is different to that used by Urieli, and it is shown that certain assumptions made by Urieli are not necessary.

The simulation model presented includes viscous dissipation in the working gas. Also, the quasi-steady assumption for the viscous friction is modified to include non-steady effects.

An experimental rig has been constructed to investigate the validity of the simulation model. This rig is based on an alpha arrangement with separate heater and cooler. No attempt has been made to transfer heat at the working spaces. Experimental data have been obtained for the pressure profiles, wall temperature distributions and overall energy balances. These data are presented for three speeds: low, high and moderate. Air has been used exclusively as the working gas.

Comparison of the experimental results with the simulation results indicates that the simulation tends to underpredict the heat transferred at the cooler and heater by approximately 13%. The pressure profiles, wall temperature distributions and the indicated work rate profiles are fairly accurately predicted.

Two ideal analyses are also presented for comparison with the simulation. These are the Schmidt analysis and the second order ideal pseudo-Stirling analysis.

A unique method for calculating the phase displacement between the working space pressure profiles is presented. This calculation is based on acoustic theory and agreement with the simulation predicted results is good.

Qualitative three-dimensional graphs indicating the detailed behaviour of the working gas pressure, temperature and mass flux density are also included.

To my brother

Mark

ACKNOWLEDGEMENTS

I would like to express my gratitude to the following people and organisations:

My supervisor, Professor Costa J Rallis, for whose help, encouragement, criticism and concern I will always be indebted.

Dr Israel Urieli whose imaginative work formed the basis of this research. Also with whom I spent many hours of fruitful discussion during the period October 1975 to February 1977. Much of the work presented here has the vantage point of hindsight.

My family and friends, in particular David Rosenthal, who were always sympathetic and helpful.

My colleagues and members of staff at the School for their interest and advice.

Frans Bezuidenhout, Mick Carrol and all the technicians in the workshop who built the experimental rig.

The Bursary and Scholarships Committee and the CSIR who made the necessary funds available.

The university Computer Center.

Fernanda Andrade, who spent many weekends and overtime hours typing this thesis.

And Gilly, whose sense of humour and moral support was my greatest encouragement.

TABLE OF CONTENTS

	PAGE
DECLARATION	(i)
ABSTRACT	(ii)
ACKNOWLEDGEMENTS	(iv)
TABLE OF CONTENTS	(v)
LIST OF FIGURES	(xi)
LIST OF TABLES	(xvii)
NOTATION	(xviii)
1 INTRODUCTION	
1.1 General	1
1.2 The Stirling Cycle	2
1.3 Problem Areas	5
1.4 Modern Analytical Techniques	7
1.5 Purpose of this Study	9
1.6 Conventions Used	9
1.6.1 Operators	9
1.6.2 References	11
2 REVIEW OF RECENT STIRLING MACHINE ANALYSIS	
2.1 Introduction	12
2.2 The Schmidt Cycle Analysis	14
2.3 The Ideal Pseudo-Stirling Cycle Analysis	17
2.4 Computer Simulation Analyses	18
2.5 Finkelstein's Model	19
2.6 Urieli's Model	25
2.7 Statement of the Problem	32
3 THE SYSTEM MODEL	
3.1 Introduction	33

3.2	Numerical Simulation Techniques	36
3.3	A Critical Investigation of Urieli's Analysis	40
3.4	The Fundamental Transport Equations	46
3.5	The Variable Volume Working Spaces	51
3.6	Empirical Factors	53
3.7	System Algorithms	55
3.8	Method of Solution	56
3.9	Summary and Conclusions	57

4 THE EXPERIMENTAL SIMULATION RIG

4.1	Introduction	58
4.2	Description of the Experimental Rig	58
4.3	Geometric Specifications of the Experimental Rig	61
4.4	Operating Conditions	62
4.5	Instrumentation	63
4.6	Sensitivity and Highest Expected Errors of the Experiments	72

5 EXPERIMENTAL PROCEDURE

5.1	Introduction	73
5.2	Preliminary	73
5.3	Pressure Profiles	74
5.4	Temperature Measurement	78
5.5	Energy Balance	79

6 EXPERIMENTAL AND SIMULATION RESULTS

	Introduction	81
--	--------------	----

7 IDEAL ANALYSES AND SIMULATION RESULTS

	Introduction	119
--	--------------	-----

8 DISCUSSION OF RESULTS

8.1	Introduction	130
8.2	Experimental and Simulation Results	130
8.3	Ideal Analyses and Simulation Results	136

9 CONCLUSIONS, DEVELOPMENTS AND RECOMMENDATIONS

9.1	Conclusions	139
9.2	Developments	141
9.3	Recommendations for Future Work	141

APPENDICES

A: THE IDEAL SECOND ORDER STIRLING CYCLE (SCHMIDT)

A.1	Introduction	143
A.2	The Analysis	145
A.3	The Exact Solution for the Heat Transferred and the Work done in the Schmidt Analysis	159

B: THE IDEAL SECOND ORDER PSEUDO-STIRLING CYCLE

B.1	Introduction	163
B.2	Analysis	164
B.3	Computer Programme	171

C: THE PHASE DISPLACEMENT OF THE PRESSURE PROFILES

177

D: THE FUNDAMENTAL TRANSPORT EQUATIONS-CONTINUITY, MOMENTUM, ENERGY AND THE ENTROPY INEQUALITY

D.1	Introduction	180
D.2	Continuity	182

D.3	Momentum	184
D.4	The Mechanical Energy Equation	186
D.5	The Energy Equation	189
D.6	The Entropy Inequality	197
D.7	The Final Transport Integral Equation Set	197
D.8	Turbulence	198
E: EMPIRICAL FACTORS		
E.1	Introduction	201
E.2	The Frictional Resistance	201
E.3	The Dissipation Work	209
E.4	The Heat Transfer Coefficient	215
F: TRANSPORT PROPERTIES - VISCOSITY, THERMAL CONDUCTIVITY		
F.1	Introduction	218
F.2	Dynamic Viscosity	218
F.3	Thermal Conductivity	219
G: NORMALISING FACTORS, COMPUTER AND NUMERICAL NOTATION		
G.1	Introduction	221
G.2	Prefix Set	226
G.3	Suffix Set	227
G.4	Subscript Set	230
G.5	Dimensionless Groups	232
H: SYSTEM ALGORITHMS		
H.1	Introduction	233
H.2	Continuity	234
H.3	Momentum	237
H.4	Energy Equation of the Working Gas	249

H.5	Eddy Dissipation	257
H.6	Energy Balance of the Cooler (Heater) Wall	260
H.7	Energy Balance of the Regenerator	263
H.8	Viscosity	265
H.9	Reynolds Friction Factor and Flow Tag	266
H.10	The Heat Transfer Coefficient	269
H.11	Volume Variations	270
I: METHOD OF SOLUTION		
I.1	Introduction	273
I.2	Method of Forcing System to Converge	274
I.3	The Solution Flow-Chart	274
J: THE SIMULATION COMPUTER PROGRAMME		
J.1	Introduction	277
J.2	Main Programme Listing	277
J.3	Subroutine Listing	295
K:	ENGINEERING DRAWINGS	301
L:	MEASURING AND RECORDING EQUIPMENT	305
M: ENERGY BALANCE EQUATIONS		
M.1	Introduction	309
M.2	The Cooler	309
M.3	The Heater	313
N: CALIBRATION OF INSTRUMENTS		
N.1	Pressure Measuring and Recording Equipment	317
N.2	Temperature Measuring and Recording Equipment	317
N.3	Industrial Gas Meter	324
N.4	Digital Frequency Meter, Wattmeter and Scale Balance	324

O: ERROR AND SENSITIVITY ANALYSIS OF EXPERIMENTS

O.1	Introduction	325
O.2	Error and Sensitivity in the Experimental Heat Transferred at the Cold End	325
O.3	Error and Sensitivity in the Experimental Heat Transferred at the Hot End	328
O.4	Error in the Experimental Pressure Profiles	331
O.5	Error in the Experimental Indicated Work Rate and Cyclic Work	335
O.6	Summary	337

P: SAMPLE COMPUTER RESULTS

P.1	Full Printout	338
P.2	Partial Printout	352

Q: EXPERIMENTAL DATA

Q.1	Pressure Profiles	355
Q.2	Temperature	356
Q.3	Energy Balances	358

R: SAMPLE CALCULATIONS FOR THE ENERGY BALANCES

R.1	Introduction	360
R.2	Cooler	360
R.3	Heater	361

S:	REFERENCES	364
----	------------	-----

LIST OF FIGURES

FIGURE	TITLE	PAGE
1.1	pV and TS Diagrams	3
1.2	Piston and Displacer Motions	4
1.3	The Pseudo-Stirling Cycle Engine	5
2.1	The Various Order Analyses in Stirling Machine Development	13
2.2	g Versus $pr(pr - pc)$ (Normalised)	21
2.3	g^2 Versus $pr(pr - pc)$ (Normalised)	21
2.4	Representation of the Nodal Network for Digital Computer Simulation of a Small Stirling Cycle Cryogenic Cooling Engine (after Finkelstein [Fi75])	23
2.5	The Gas Node (after Finkelstein [Fi75])	24
2.6	The Cellular Model (after Urieli [Ur77])	26
2.7	The Generalised i 'th Elemental Cell (after Urieli [Ur77])	28
2.8	The Test Engine (after Urieli [Ur77])	30
3.1	Alpha Arrangement	34
3.2	Beta Arrangement	34
3.3	Gamma Arrangement	34
3.4	The Pseudo-Stirling Machine (Alpha Configura- tion)	36
3.5	The Control Volume	40
3.6	The Generalised i 'th Cell	49
3.7	The Generalised Compression Space	52
4.1	Overall Pictorial Representation of the Rig	60
4.2	Overall View of the Rig, Indicating Temperature and Pressure Measurement Points	64
4.3	Thermocouple Installation	66
4.4	Thermocouple Circuit	66

LIST OF FIGURES

FIGURE	TITLE	PAGE
1.1	pV and TS Diagrams	3
1.2	Piston and Displacer Motions	4
1.3	The Pseudo-Stirling Cycle Engine	5
2.1	The Various Order Analyses in Stirling Machine Development	13
2.2	g Versus $pr(pr - pc)$ (Normalised)	21
2.3	g^2 Versus $pr(pr - pc)$ (Normalised)	21
2.4	Representation of the Nodal Network for Digital Computer Simulation of a Small Stirling Cycle Cryogenic Cooling Engine (after Finkelstein [Fi75])	23
2.5	The Gas Node (after Finkelstein [Fi75])	24
2.6	The Cellular Model (after Urieli [Ur77])	26
2.7	The Generalised i 'th Elemental Cell (after Urieli [Ur77])	28
2.8	The Test Engine (after Urieli [Ur77])	30
3.1	Alpha Arrangement	34
3.2	Beta Arrangement	34
3.3	Gamma Arrangement	34
3.4	The Pseudo-Stirling Machine (Alpha Configura- tion)	36
3.5	The Control Volume	40
3.6	The Generalised i 'th Cell	49
3.7	The Generalised Compression Space	52
4.1	Overall Pictorial Representation of the Rig	60
4.2	Overall View of the Rig, Indicating Temperature and Pressure Measurement Points	64
4.3	Thermocouple Installation	66
4.4	Thermocouple Circuit	66

4.5	Pressure Recording Apparatus	67
4.6	Pressure Profiles Triggering Switch	68
4.7	Transistor Switch Circuit	68
4.8	Speed Measuring Transducer	69
4.9	The Gas Meter Used for Determining the Volume of Carbon Dioxide Gas	70
4.10	Condenser Installation	71
5.1	Example of a Recorded Pressure Trace (Compression Space)	76
5.2	Example of a Recorded Pressure Trace (Expansion Space)	77
5.3	Example of a Recorded Decaying Pressure Trace	78
5.4	Temperature Printout	79
6.1	Simulation Pressure Profiles at 2,50 Hz	84
6.2	Simulation Pressure Profiles at 3,35 Hz	85
6.3	Simulation Pressure Profiles at 4,35 Hz	86
6.4	Simulation and Experimental Compression Space Pressure Profiles at 2,50 Hz	87
6.5	Simulation and Experimental Compression Space Pressure Profiles at 3,35 Hz	88
6.6	Simulation and Experimental Compression Space Pressure Profiles at 4,35 Hz	89
6.7	Simulation and Experimental Expansion Space Pressure Profiles at 2,50 Hz	90
6.8	Simulation and Experimental Expansion Space Pressure Profiles at 3,35 Hz	91
6.9	Simulation and Experimental Expansion Space Pressure Profiles at 4,35 Hz	92
6.10	Wall Temperature Distribution at 2,50 Hz	93
6.11	Wall Temperature Distribution at 3,35 Hz	94
6.12	Wall Temperature Distribution at 4,35 Hz	95
6.13	Work Rate and Cyclic Work at 2,50 Hz	96
6.14	Work Rate and Cyclic Work at 3,35 Hz	97
6.15	Work Rate and Cyclic Work at 4,35 Hz	98
6.16	Heat Rates and Cyclic Heat Transferred at 2,50 Hz	99

6.17	Heat Rates and Cyclic Heat Transferred at 3,35 Hz	100
6.18	Heat Rates and Cyclic Heat Transferred at 4,35 Hz	101
6.19	Regenerator Heat Rate at 2,50 Hz	102
6.20	Regenerator Heat Rate at 3,35 Hz	103
6.21	Regenerator Heat Rate at 4,35 Hz	104
6.22	Working Spaces Temperature Profiles and Average Temperatures at 2,50 Hz	105
6.23	Working Spaces Temperature Profiles and Average Temperatures at 3,35 Hz	106
6.24	Working spaces Temperature Profiles and Average Temperatures at 4,35 Hz	107
6.25	Simulation Pressure Profiles at 4,35 Hz (Central Difference)	108
6.26	Simulation and Experimental Compression Space Pressure Profile at 4,35 Hz (Central Difference)	109
6.27	Simulation and Experimental Expansion Space Pressure Profiles at 4,35 Hz (Central Difference)	110
6.28	Wall Temperature Distribution at 4,35 Hz (Central Difference)	111
6.29	Work Rate and Cyclic Work at 4,35 Hz (Central Difference)	112
6.30	Heat Rates and Cyclic Heat Transferred at 4,35 Hz (Central Difference)	113
6.31	Regenerator Heat Rate at 4,35 Hz (Central Difference)	114
6.32	Working Spaces Temperature Profiles and Average Temperatures at 4,35 Hz (Central Difference)	115
6.33	Pressure versus Theta and Position in Machine at 4,35 Hz	116

6.34	Gas Temperature versus Theta and Position in Machine at 4,35 Hz	117
6.35	Mass Flux Density versus Theta and Position in Machine at 4,35 Hz	118
7.1	Schmidt and Simulation Pressure Profiles at 4,35 Hz	120
7.2	Ideal Pseudo-Stirling and Simulation Pressure Profiles at 4,35 Hz	121
7.3	Schmidt and Simulation Work Rates	122
7.4	Ideal Pseudo-Stirling and Simulation Work Rates at 4,35 Hz	123
7.5	Schmidt and Simulation Cyclic Heats and Heat Rates at 4,35 Hz	124
7.6	Schmidt and Simulation Regenerator Heat Rates at 4,35 Hz	125
7.7	Ideal Pseudo-Stirling and Simulation Cyclic Heats and Heat Rates at 4,35 Hz	126
7.8	Ideal Pseudo-Stirling and Simulation Regenerator Heat Rates at 4,35 Hz	127
7.9	Ideal Pseudo-Stirling and Simulation Working Spaces Temperature Profiles at 4,35 Hz	128
7.10	Schmidt and Simulation Stagnation Planes at 4,35 Hz	129
8.1	Phase Relationship of the Experimental Pressure Profiles	132
A.1	The General Alpha Arrangement	144
A.2	The Temperature Distribution in the Regenerator	146
A.3	The Ideal Isothermal Working Space	150
A.4	The Ideal Regenerator	153
A.5	The Mass Flow Rate in the Cooler	155
A.6	The Mass Flow Rate in the Regenerator	157

B.1	The Adiabatic Working Space	165
B.2	The Cooler	169
D.1	The Generalised Control Volume	181
E.1	Non-isothermal Flow	203
E.2	A Changing Cross Section	206
E.3	Approximate Friction Factor Diagram [Ur77]	209
E.4(a)	Flow in an Abrupt Enlargement	210
E.4(b)	Flow in Abrupt Contraction	210
E.5	Entrance and Exit Pressure Loss Coefficients [KL64]	212
H.1	The Generalised i 'th Continuity and Energy Elemental Cell	234
H.2	The Generalised Compression Space Continuity and Energy Cell	236
H.3	The Generalised i 'th Momentum Cell Defined by Urieli [Ur77, UR77]	238
H.4	The Generalised i 'th Momentum Cell Without the Use of Upstream Differences	241
H.5	The Generalised 1st Momentum Cell as Defined by Urieli [Ur77, UR77]	244
H.6	The Generalised 1st Momentum Cell Without the Use of Upstream Differences	247
H.7	Displacement of Crank and Connecting Rod	270
I.1	The Algorithm of Solution (after Urieli [Ur77])	275
M.1	Cooler Control Boundary	310
M.2	Heater Control Boundary	314
N.1	Calibration of Pressure Transducer (Compression Space)	318
N.2	Calibration of Pressure Transducer (Expansion Space)	319
N.3	Calibration of Temperature Inputs (Stations 0 to 9)	320
N.4	Calibration of Temperature Inputs (Stations 10 to 19)	321
N.5	Calibration of Temperature Inputs (Stations 20 to 29)	322

N.6	Calibration of Industrial Gas Meter	323
O.1	Instantaneous Worst Fractional Error in the Pressure Profiles (at 4,350 Hz)	334
O.2	Instantaneous Highest Expected Error in the Work Rate Profiles (at 4,350 Hz)	336
Q.1	Temperature Measurement Stations	358

LIST OF TABLES

TABLE	TITLE	PAGE
8.1	Experimental and Simulation Cyclic Heats	133
8.2	Overall Energy Balances	136
F.1	Values of the Viscosities of Gases [Br71]	219
G.1	Normalised and Actual Parameters	224
G.2	Dimensionless Groups	232
0.1	Experimental Errors at the Cold End	326
0.2	Evaluation of Overall Error in Heat Extracted	327
0.3	Experimental Errors at the Hot End	329
0.4	Evaluation of Overall Error in Heat Addition	330
Q.1	Experimental Pressure Profile Data	355
Q.2	Experimental Temperature Data	356
Q.3	Energy Balance Data	359

NOTATION

Two basic notation sets are used in this work; one for the analyses and the other for the computer programmes.

The notation used in the analyses is made up of a base parameter set which is suffixed. The suffixes are used to indicate location and whether certain parameters are time dependent or not.

The notation used in the simulation computer programme involves both dimensional and normalised parameters. This notation follows a rigid format which is explained in Appendix G. In this section only the more important simulation computer parameters are listed. The computer programme for the ideal pseudo-Stirling analysis uses only dimensional parameters, these are listed in this section. Dummy variables used in both computer programmes are not listed.

All the parameters and variables used in the analyses are italicised, those used in the computer programmes are upper case Roman.

QMAN

combination of terms (Schmidt analysis); constant in heat transfer formula.

(A) area: free flow area; control surface area.

$A(i)$ free flow area at node i .

$\bar{c}$ average value of Fourier series.

c constant in Fourier series.

NOTATION

Two basic notation sets are used in this work: one for the analyses and the other for the computer programmes.

The notation used in the analyses is made up of a base parameter set which is suffixed. The suffixes are used to indicate location and whether certain parameters are time dependent or not.

The notation used in the simulation computer programme involves both dimensional and normalised parameters. This notation follows a rigid format which is explained in Appendix G. In this section only the more important simulation computer parameters are listed. The computer programme for the ideal pseudo-Stirling analysis uses only dimensional parameters, these are listed in this section. Dummy variables used in both computer programmes are not listed.

All the parameters and variables used in the analyses are italicised, those used in the computer programmes are upper case Roman.

Roman

a combination of terms (Schmidt analysis); constant in heat transfer formula.

A (A) area: free flow area; control surface area.

A_i ($A(I)$) free flow area at node i .

A_0 average value of Fourier series.

A_1 constant in Fourier series.

A_n constant in Fourier series.

A_H heater free flow area.

A_K cooler free flow area.

A_p (PA,AP) piston face area.

A_r regenerator free flow area.

A_s smallest area of flow.

A_{n_i} free flow area associated with the momentum equation.

A_{w_i} (AW(I)) cross sectional area of heat exchanger wall in cell i .

A_{wr} cross sectional area of regenerator wall

A_{wcyL} cross sectional area of cylinder wall

A_{wg_i} (AWG(I)) heat exchanger wall surface area in cell i wetted by the working gas.

b constant in heat transfer formula.

B prefix used in computer programme to denote a combination of terms.

B_n constant in Fourier series.

c suffix indicating time dependent compression space parameter; specific heat capacity.

C suffix indicating constant compression space parameter; heat capacity; local speed of sound.

C_p (CPS) specific heat capacity at constant pressure.

C_v (CVS) specific heat capacity at constant volume.

c_w (CSW) specific heat capacity of the heat exchanger wall material.

C_w (CW) heat capacity of the heat exchanger wall material contained in a single cell.

d (D) hydraulic diameter.

DIA heat exchanger pipe diameter ratio, computer programme only.

D (D) prefix used to denote total time derivative operator d/dt .

D cylinder bore.

DO normalised external diameter of heat exchanger pipes.

DTIME time increment for numerical integration.

DYDT dummy variable increment.

e specific total energy; total specific internal energy; suffix indicating time dependent expansion space parameter.

E prefix to denote dimensioned parameter.

E suffix indicating constant expansion space parameter; dissipation terms.

$EA[m^2]$ area normalising factor.

$ED[m]$ actual internal diameter of heat exchanger pipes.

EK[J/msK] thermal conductivity normalising factor.

EM[kg] mass normalising factor.

ER[J/kg K] specific heat capacity normalising factor.

EU[m/s] velocity normalising factor.

EX[m] length normalising factor.

EDO[m] actual external diameter of heat exchanger pipes.

EKW[J/ms K] actual thermal conductivity of wall material.

EMU[kg/ms] dynamic viscosity normalising factor.

ERO[kg/m³] density normalising factor.

ETH[K] actual hot source temperature.

ETK[K] temperature normalising factor; actual cold sink temperature.

EVS[m³] volume normalising factor.

EXP[m] actual length of heat exchanger pipe bundle.

ECSW[J/kg K] actual specific heat capacity of the heat exchanger wall material.

EROW[kg/m³] actual density of wall material.

ENRGY[J] energy normalising factor.

$ETIME[s]$ time normalising factor.

$f(EFREQ)[Hz]$ operating frequency.

F forces on solid surfaces; general function.

F_i forces on solid surfaces at node i .

Ff fanning friction factor.

Fp pressure forces.

Fr (FR) Reynolds friction factor.

Ff_1 fanning friction factor from Blasius.

Ff_2 fanning friction factor from von Kármán

Fr_1 Reynolds friction factor from Blasius.

Fr_2 Reynolds friction factor from von Kármán.

g mass flux density or momentum per unit volume.

g_i (G(I)) mass flux density at node i .

G_i mass flux at node i .

gr gravitational constant.

h heat transfer coefficient; specific enthalpy; suffix indicating time dependent heater parameter.

H enthalpy; suffix indicating constant heater parameter.

h_i (H(I)) heat transfer coefficient at node i .
 Hk enthalpy transport from cooler.
 Hh enthalpy transport to heater.
 hfg specific latent heat of vaporisation for water.
 Ic the cyclic work and heat at the compression space.
 Ie the cyclic work and heat at the expansion space.
 $Iflo_i$ (IFLO(I)) signed integer indicating the direction and regime of flow at node i .
 j specific latent heat of vaporisation of CO_2 .
 $JCYCL$ integer indicating how many cycles have been performed.
 $JPRNT$ two valued integer {0,1} specifying a partial or full printout respectively.
 k working gas thermal conductivity; suffix indicating time dependent cooler parameter.
 K suffix indicating constant cooler parameter.
 Kc contraction pressure loss coefficient.
 Ke expansion pressure loss coefficient.
 kr thermal conductivity of regenerator material.
 kw (RKW) thermal conductivity of wall material.

k_{cyl} thermal conductivity of working space cylinder material.

l (EXCR) connecting rod length.

L half period in Fourier analysis; length.

LH length of heater.

LK length of cooler.

Lr length of regenerator.

m working gas mass; integer.

M (EM) [kg] actual total mass of gas in the system.

m_c mass of gas in the compression space.

m_e mass of gas in the expansion space.

m_h mass of gas in heater.

m_k mass of gas in cooler.

m_r mass of gas in regenerator.

m_t (RMT) total mass of gas in the system.

m_i (RM(I)) mass of gas in cell i .

n integer; n 'th harmonic; ratio of connecting rod length to crank throw.

N unit normal vector; number of sample points.

n_c (NC) total number of cells in the simulation model.

n_h (NH) number of cells in the heater.

n_k (NK) number of cells in the cooler.

n_r (NR) number of cells in the regenerator.

NP number of heat exchanger pipes.

Nu Nusselt number.

NKR sum of cooler and regenerator cells.

NINC number of integration time increments per cycle.

NPRNT number of increments per printout.

o suffix indicating reference parameter.

p pressure.

P power.

p_i (P(I)) pressure in cell i .

p_c (PC) pressure in compression space.

p_e (PE) pressure in expansion space.

p_k (EPK) [Pa] average pressure in the system at cyclically steady state operation (charge pressure).

p_{kk} (EPKK) [Pa] pressure factor.

Pr (PR) Prandtl number.

p_{chg} charge pressure.

q heat transfer per unit area.

Q heat.

Q_i ($Q(I)$) convective heat transfer to gas in cell i .

Q_c heat transferred to gas in compression space.

Q_e heat transferred to gas in expansion space.

Q_h (Q_H) heat transferred by heater.

Q_k (Q_K) heat transferred by cooler.

Q_r (Q_R) heat transferred by regenerator.

Q_{r_i} ($Q_R(I)$) heat flow from the regenerator solid surfaces to the gas in the i 'th regenerator cell.

Q_{hg} heat transferred by heater to working gas only.

Q_{kg} heat transferred by cooler to working gas only.

Q_{wr_i} ($Q_{WR}(I)$) axial conductive heat flow across the wall of the i 'th regenerator node.

Q_{CO_2} heat removed by CO_2 gas.

Q_{cyl} heat conducted to/from working space cylinder.

Q_{ele} heat supplied by electrical heater.

Q_{ins} heat conducted through insulation.

Q_{reg} heat conducted to/from regenerator solid surfaces.

Q_{stm} heat removed by steam.

r suffix indicating regenerator parameter (constant or time dependent); crank throw (EXCT).

R suffix used to convert parameters which have integer mnemonics to real mode.

R gas constant.

$Re(RE)$ Reynolds' number.

$Ro(RO)$ relative roughness factor.

Re_v oscillatory flow Reynolds' number.

reg suffix indicating regenerator parameter.

RKW normalised thermal conductivity of wall material.

RMU normalised dynamic viscosity.

RMSQR normalised root mean square net regenerator heat.

s specific entropy.

S combination of terms (Schmidt analysis)

$S1, S2$ combination of terms (ideal pseudo-Stirling).

St Stanton number.

t (TIME) time.

T absolute temperature.