A COMPUTER AND EXPERIMENTAL SIMULATION OF STIRLING CYCLE MACHINES

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A Dissertation Submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, for the degree of Master of Science.

Johannesburg, March 1978.

DECLARATION

I, DAVID MONTAGUE BERCHOWITZ, hereby declare that this thesis is my own work and that the material herein has not been submitted by me for degree purposes at any other University.

ABSTRACT

A critical evaluation on recent advances in the field of Stirling machine simulation is conducted. It is shown that the approach developed by Urieli [Ur77] is the most rigorous.

The transport equation set (which describes the behaviour of the working gas) is derived in both the integral and differential forms. Only the integral equation set is used in the simulation. The differential equation set is used to investigate the structure of individual terms in the integral equation set. This procedure allows these terms to be more accurately understood and hence modelled. The analytical derivation of the transport equation sets is different to that used by Urieli, and it is shown that certain assumptions made by Urieli are not necessary.

The simulation model presented includes viscous dissipation in the working gas. Also, the quasi-steady assumption for the viscous friction is modified to include non-steady effects.

An experimental rig has been constructed to investigate the validity of the simulation model. This rig is based on an alpha arrangement with separate heater and cooler. No attempt has been made to transfer heat at the working spaces. Experimental data have been obtained for the pressure profiles, wall temperature distributions and overall energy balances. These data are presented for three speeds: low, high and moderate. Air has been used exclusively as the working gas.

Comparison of the experimental results with the simulation results indicates that the simulation tends to underpredict the heat transferred at the cooler and heater by approximately 13%. The pressure profiles, wall temperature distributions and the indicated work rate profiles are fairly accurately predicted.

Two ideal analyses are also presented for comparison with the simulation. These are the Schmidt analysis and the second order ideal pseudo-Stirling analysis.

A unique method for calculating the phase displacement between the working space pressure profiles is presented. This calculation is based on acoustic theory and agreement with the simulation predicted results is good.

Qualitative three-dimensional graphs indicating the detailed behaviour of the working gas pressure, temperature and mass flux density are also included.

To my brother

Mark

ACKNOWLEDGEMENTS

I would like to express my gratitude to the following people and organisations:

My supervisor, Professor Costa J Rallis, for whose help, encouragement, criticism and concern I will always be indebted.

Dr Israel Urieli whose imaginative work formed the basis of this research. Also with whom I spent many hours of fruitful discussion during the period October 1975 to February 1977. Much of the work presented here has the vantage point of hindsight.

My family and friends, in particular David Rosenthal, who were always sympathetic and helpful.

My colleagues and members of staff at the School for their interest and advice.

Frans Bezuidenhout, Mick Carrol and all the technicians in the workshop who built the experimental rig.

The Bursary and Scholarships Committee and the CSIR who made the necessary funds available.

The university Computer Center.

Fernanda Andrade, who spent many weekends and overtime hours typing this thesis.

And Gilly, whose sense of humour and moral support was my greatest encouragement.

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NOTATION

Two basic notation sets are used in this work: one for the analyses and the other for the computer programmes.

The notation used in the analyses is made up of a base parameter set which is suffixed. The suffixes are used to indicate location and whether certain parameters are time dependent or not.

The notation used in the simulation computer programme involves both dimensional and normalised parameters. This notation follows a rigid format which is explained in Appendix G. In this section only the more important simulation computer parameters are listed. The computer programme for the ideal pseudo-Stirling analysis uses only dimensional parameters, these are listed in this section. Dummy variables used in both computer programmes are not listed.

All the parameters and variables used in the analyses are italicised, those used in the computer programmes are upper case Roman.

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combination of terms (Schmidt analysis); constant in heat transfer formula.

(A) area: free flow area; control surface area.

$_i$ (A(I)) free flow area at node i.

$_c$ average value of Fourier series.

$_j$ constant in Fourier series.

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a combination of terms (Schmidt analysis); constant in heat transfer formula.

A (A) area: free flow area; control surface area.

A_i (A(I)) free flow area at node i.

A_0 average value of Fourier series.

A_1 constant in Fourier series.

A_n constant in Fourier series.

AH heater free flow area.

AK cooler free flow area.

Ap (PA,AP) piston face area.

Ar regenerator free flow area.

As smallest area of flow.

An_i free flow area associated with the momentum equation.

Aw_i (AW(I)) cross sectional area of heat exchanger wall in cell i.

Awr cross sectional area of regenerator wall

$Awcyl$ cross sectional area of cylinder wall

Awg_i (AWG(I)) heat exchanger wall surface area in cell i wetted by the working gas.

b constant in heat transfer formula.

B prefix used in computer programme to denote a combination of terms.

B_n constant in Fourier series.

c suffix indicating time dependent compression space parameter; specific heat capacity.

C suffix indicating constant compression space parameter; heat capacity; local speed of sound.

Cp (CPS) specific heat capacity at constant pressure.

Cv (CVS) specific heat capacity at constant volume.

cw (CSW) specific heat capacity of the heat exchanger wall material.

Cw (CW) heat capacity of the heat exchanger wall material contained in a single cell.

d (D) hydraulic diameter.

DIA heat exchanger pipe diameter ratio, computer programme only.

D (D) prefix used to denote total time derivative operator d/dt.

D cylinder bore.

DO normalised external diameter of heat exchanger pipes.

DTIME time increment for numerical integration.

DYDT dummy variable increment.

e specific total energy; total specific internal energy; suffix indicating time dependent expansion space parameter.

E prefix to denote dimensioned parameter.

E suffix indicating constant expansion space parameter; dissipation terms.

EA[m^2] area normalising factor.

ED[m] actual internal diameter of heat exchanger pipes.

EK[J/msK] thermal conductivity normalising factor.

EM[kg] mass normalising factor.

ER[J/kg K] specific heat capacity normalising factor.

EU[m/s] velocity normalising factor.

EX[m] length normalising factor.

EDO[m] actual external diameter of heat exchanger pipes.

EKW[J/ms K] actual thermal conductivity of wall material.

EMU[kg/ms] dynamic viscosity normalising factor.

ERO[kg/m^3] density normalising factor.

ETH[K] actual hot source temperature.

ETK[K] temperature normalising factor; actual cold sink temperature.

EVS[m^3] volume normalising factor.

EXP[m] actual length of heat exchanger pipe bundle.

ECSW[J/kg K] actual specific heat capacity of the heat exchanger wall material.

EROW [kg/m^3] actual density of wall material.

ENRGY[J] energy normalising factor.

ETIME [s] time normalising factor.

f(EFREQ) [Hz] operating frequency.

F forces on solid surfaces; general function.

F_i forces on solid surfaces at node i.

Ff fanning friction factor.

Fp pressure forces.

Fr (FR) Reynolds friction factor.

Ff_1 fanning friction factor from Blasius.

Ff_2 fanning friction factor from von Kármán

Fr_1 Reynolds friction factor from Blasius.

Fr_2 Reynolds friction factor from von Kármán.

g mass flux density or momentum per unit volume.

g_i (G(I)) mass flux density at node i.

G_i mass flux at node i.

gr gravitational constant.

h heat transfer coefficient; specific enthalpy; suffix indicating time dependent heater parameter.

H enthalpy; suffix indicating constant heater parameter.

h_i (H(I)) heat transfer coefficient at node i.

Hk enthalpy transport from cooler.

Hh enthalpy transport to heater.

hfg specific latent heat of vaporisation for water.

Ic the cyclic work and heat at the compression space.

Ie the cyclic work and heat at the expansion space.

$Iflo_i$ (IFLO(I)) signed integer indicating the direction and regime of flow at node i.

j specific latent heat of vaporisation of CO_2.

JCYCL integer indicating how many cycles have been performed.

JPRNT two valued integer {0,1} specifying a partial or full printout respectively.

k working gas thermal conductivity; suffix indicating time dependent cooler parameter.

K suffix indicating constant cooler parameter.

Kc contraction pressure loss coefficient.

Ke expansion pressure loss coefficient.

kr thermal conductivity of regenerator material.

kw (RKW) thermal conductivity of wall material.

kcyl thermal conductivity of working space cylinder material.

l (EXCR) connecting rod length.

L half period in Fourier analysis; length.

LH length of heater.

LK length of cooler.

Lr length of regenerator.

m working gas mass; integer.

M (EM) [kg] actual total mass of gas in the system.

mc mass of gas in the compression space.

me mass of gas in the expansion space.

mh mass of gas in heater.

mk mass of gas in cooler.

mr mass of gas in regenerator.

mt (RMT) total mass of gas in the system.

m_i (RM(I)) mass of gas in cell i.

n integer; *n*'th harmonic; ratio of connecting rod length to crank throw.

N unit normal vector; number of sample points.

nc (NC) total number of cells in the simulation model.

nh (NH) number of cells in the heater.

nk (NK) number of cells in the cooler.

nr (NR) number of cells in the regenerator.

NP number of heat exchanger pipes.

Nu Nusselt number.

NKR sum of cooler and regenerator cells.

NINC number of integration time increments per cycle.

NPRNT number of increments per printout.

o suffix indicating reference parameter.

p pressure.

P power.

p_i (P(I)) pressure in cell i.

pc (PC) pressure in compression space.

pe (PE) pressure in expansion space.

pk (EPK)[Pa] average pressure in the system at cyclically steady state operation (charge pressure).

pkk (EPKK)[Pa] pressure factor.

Pr (PR) Prandtl number.

$pchg$ charge pressure.

q heat transfer per unit area.

Q heat.

Q_i (Q(I)) convective heat transfer to gas in cell i.

Qc heat transferred to gas in compression space.

Qe heat transferred to gas in expansion space.

Qh(QH) heat transferred by heater.

Qk(QK) heat transferred by cooler.

Qr(QR) heat transferred by regenerator.

Qr_i (QR(I)) heat flow from the regenerator solid surfaces to the gas in the i'th regenerator cell.

Qhg heat transferred by heater to working gas only.

Qkg heat transferred by cooler to working gas only.

Qwr_i (QWR(I)) axial conductive heat flow across the wall of the i'th regenerator node.

Qco_2 heat removed by CO_2 gas.

$Qcyl$ heat conducted to/from working space cylinder.

$Qele$ heat supplied by electrical heater.

$Qins$ heat conducted through insulation.

Qreg heat conducted to/from regenerator solid surfaces.

Qstm heat removed by steam.

r suffix indicating regenerator parameter (constant or time dependent); crank throw (EXCT).

R suffix used to convert parameters which have integer mnemonics to real mode.

R gas constant.

Re(RE) Reynolds' number.

Ro(RO) relative roughness factor.

Re_v oscillatory flow Reynolds' number.

reg suffix indicating regenerator parameter.

RKW normalised thermal conductivity of wall material.

RMU normalised dynamic viscosity.

RMSQR normalised root mean square net regenerator heat.

s specific entropy.

S combination of terms (Schmidt analysis)

S1,S2 combination of terms (ideal pseudo-Stirling).

St Stanton number.

t (TIME) time.

T absolute temperature.