An exploration into Grade 3 learners' mathematical proficiencies as seen using the Mental Starters Assessment Project (MSAP) and the Learning Framework in Number (LFIN).

A dissertation submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfilment of the requirements for the degree of Masters of Education

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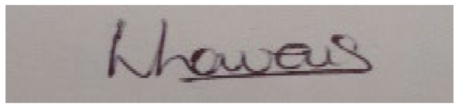
ABSTRACT

South African learners are consistently underperforming in mathematics, and it appears as if this downward trend is not changing. Much research has been undertaken in recent years in search of possible solutions to remedy this dire situation. Part of this foci into research has led to the formulation and development of the Mental Starters Assessment Project (MSAP) through a collaborative process from various stakeholders from Wits Math's Connect-Primary Team, Rhodes University and Gauteng Depart of Education (GDE). This current study forms part of that broader research focusing on possible solutions to the challenges faced by Foundation Phase learners. And what started simply as an effort to assess grade three learners' Additive Reasoning strategies through a two-week mental mathematics intervention study morphed into a study with a much broader focus. While starting and conducting the study with that single focus, I became aware of more significant implications that ultimately led me to expand the area of focus. As such when I completed the MSAP intervention, I then proceeded to assess six of the learners from the same sample through conducting interviews with them based on the Learning Framework in Number. The LFIN is an assessment tool developed by Wright et al. (2006) as an intervention model to remediate underperforming learners. When I completed that aspect of the study, I was further intrigued with the findings that it yielded in that the learners with the highest gains in the MSAP correlated with the learners demonstrating high levels of mathematical proficiencies in the LFIN as they were located on the higher stages of the SEAL. And the learners with the lowest to no gains in the MSAP correlated with the learners in the LFIN that demonstrated an over-reliance on concrete forms of counting as being located on the lower levels of the SEAL. As such, this study's findings seem to indicate a strong correlation between the MSAP and the LFIN. The implications for the South African Foundation Phase mathematics classroom are that the MSAP is easier to administer than the LFIN and requiring no specialized skill and can be administered nationally. Although the findings of this study were insightful and exciting more research is needed to verify these findings. It has however created an awareness that there is an alternative other than the LFIN that is more suitable for the South African context in the form of the MSAP.

Keywords: Additive Relations, Empty Number Lines, Jump Strategy, Assessments.

DECLARATION

I declare that this dissertation is my own unaided work. It is being submitted for the degree of Masters of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.



Lincoln Lavans

10th March in the year 2023

DEDICATION

To Jesus Christ, who sees us for what we can be in and through Him and not for what we are.

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LIST OF ABBREVIATIONS

BTT – *Bridging Through Ten*

BNWS – *Backward Number Word Sequence*

ENL – *Empty Number Line*

FNWS – *Forward Number Word Sequence*

FNW – *Forward Number Word*

GDE – *Gauteng Department of Education*

JS – *Jump Strategies*

LFIN – *Learning Framework in Number*

MSAP – *Mental Starters Assessment Project*

NWB – *Number Word Before*

NID – *Numeral Identification*

PPW – *Part-Part-Whole*

SEAL – *Stages of Early Arithmetical Learning*

WMC-P – *Wits Math's Connect - Primary*

CHAPTER 1 - CONTEXT AND BACKGROUND OF STUDY

Learner underperformance in mathematics in South Africa has been extensively documented in the past 25 years, and the causes that gave rise to it (Moloi & Strauss 2005; Spaull 2013; Spaull & 2015). However, when learners do not master the elementary concepts of number work in the Foundation Phase of their formal schooling, their deficiencies in number work will only be exacerbated in the higher grades as the number range increases and more complex mathematical concepts are introduced.

One of the primary causes for this learner's underperformance from a purely educational perspective is the learner's weak or poorly developed Number Sense (Graven & Venkat, 2019). Stemming from this situation, there is an overreliance on concrete forms of unit counting (Schollar, 2008 & Hoadley, 2012), usually characterized by tally counting, as noted by Weitz and Venkat (2013). These forms of counting are slow, error-prone and inefficient significantly when the number range increases into the higher grades (Askew, Bibby & Brown, 2001). Moreover, as young learners grapple with conceptualizing numbers as an abstract entity, the necessary and required shifts from concrete forms of counting into more efficient mental calculating strategies cannot be made. The result is that young learners struggle with mental math and become over-reliant on concrete forms of counting.

In this dire situation of learner underperformance, the Wits Math's Connect-Primary (WMC-P) Project were initially launched in 2011 in one school district in Gauteng to collaborate with ten schools to remedy this downward trend of learner underperformance. The project is in its third cycle of five years and has grown to include fifteen school districts with five schools per district. This current study forms part of the scope of research that has already been successfully undertaken in the broader Wits

Math's Connect-Primary Project. The approach to mental starters has been taken up nationally in grade three classrooms in the Mental Starters Assessment Project (MSAP).

1.1 Rationale

As a Foundation Phase mathematics researcher, this study aims to enable me to explore the shifts in learner performance in solving additive tasks once they have completed a two-week mental math developmental intervention program. The intervention is based on using Jump Strategies on the Empty Number Line. After intervention completion, I would like to understand learners' performance about types of calculations. I would also like to observe whether those results could be correlated through another assessment tool (LFIN) of identical learners.

1.2 Research Questions

A case study of exploration into additive reasoning with grade three learners using Jump Strategies on an Empty Number Line and LFIN assessment. I will attend to the research question by responding to the following set of questions:

1. *What were, if possible, the gains from pre to post-test when making use of the Jump Strategy on the Empty Number Line interventions?*
2. *What was the profile of the 6 selected learners after the LFIN interview was administered on them?*
3. *What is the correlation between the Jumps Strategies and the LFIN, if any?*

1.3 Literature Review

As the focus of this study is to investigate grade 3 learner's additive reasoning on the *Empty Number Line* with the use of *Jump Strategies*, I have identified and selected to review literature about the following specific aspects, which include but are not limited to, additive reasoning, additive representations the use of the *Empty Number Line* and *Jump*

Strategies. Furthermore, I have selected to review literature by the advocators of a sequential approach to computation as it fits well within *Jump Strategies* on the *Empty Number Line*. Although I will foreground a sequential approach towards computation in this study, I will also acknowledge the importance of a structured approach.

1.3.1 Additive reasoning

Additive reasoning in young learners indicates progression in terms of their ability to work with greater efficiency and flexibility with number. As formal schooling commences, young learners' initial strategies to solve number problems are characterized by counting strategies, as noted by Thompson (2000) and Carpenter, Fennema, Franke, Levi & Empson (1999). As such, they will initially solve addition and subtraction problems by counting in ones. However, as learners develop stronger number sense, they soon start to display additive reasoning proficiencies. Additive reasoning, therefore, is the learner's increasing capacity to work with greater efficiency and flexibility with number, and it incorporates, amongst other elements, the use of prior knowledge or known number facts to solve number problems. Therefore, from a sequential approach to computation, counting strategies eventually form the foundation on which additive reasoning is constructed.

1.3.2 Additive representations

The two additive representations that are of particular interest in this study are the Part-Part Whole diagram and Number Lines. Although Number Lines will be foregrounded in this present study, the importance of the Part-Part Whole diagram will be acknowledged. Both representations have significant advantages in enhancing learner development of mental computational skills. However, each additive representation foregrounds different computational and teaching approaches, which will be discussed in the following section.

1.3.3 Part-whole diagram

The Part-Part-Whole diagram is a rectangular bar that is used to represent the whole and its constituent parts in picture form. The Part-Part Whole diagram is grounded in a structural approach, and as such, counting is avoided by starting with measurement and comparing relations (Ekdahl, 2019). The focus of the part-whole relation is also on the principles of de-composing and re-composing numbers. In the literature, the approach to de-compose and re-compose number fits well within the 1010 approach (or split strategy) that splits both addends into tens and units and ends off by compressing them again to arrive at an answer. This computational approach contrasts the N10 approach (or jump strategy) in which the first addend is kept whole, and only the second addend is split into its smaller parts. Moreover, in this way, the parts are seen as making up the whole. Advocates of this approach towards computation acknowledge the importance of counting but suggest that counting strategies are not necessarily the natural pathway into structural thinking of number and that there are alternative methods that can be used to achieve the same objective (Ekdahl, 2019).

1.3.4 The Number Line

The literature identifies three types of number lines: structured, semi-structured and empty number lines. The number line foregrounds counting as the natural pathway that leads to structural ways of thinking about number (Ekdahl, 2019), as opposed to the 1010 approach, which avoids counting and focuses instead on number relations. It is furthermore considered a sequential approach to computation and fits well within the N10 approach to number work. A characteristic of the N10 approach is that the first addend is kept whole, and the second addend is split into tens and units to derive the answer usually enacted in jumps on the number line. For the purposes of this study, the empty number line will be foregrounded, which is discussed next.

1.3.5 The Empty Number Line

According to Blote, Klein & Beishuizen (2000), the empty number line can be employed as a powerful tool for supporting the development of children's mental strategies up to 100. Gravemeijer (1994) noted three benefits of using the empty number line: its linear representation of the number. Secondly, it reflects more closely intuitive mental strategies used by children, and lastly, it's the potential to foster the development of more sophisticated strategies. Bobis and Bobis (2005) added a fourth benefit in that it can serve as a powerful tool to enhance classroom communication, as learners can explain their processes of solving number problems. Moreover, it also allows for observing learners' thinking processes (Bobis, 2007). Further advantages of the empty number line, as noted by Gray and Tall (1994), another advantage of the empty number line is that it is graphic, gives the value of the numbers and their position relative to other numbers and as such, numeral recognition and identification are essential skills that are needed and developed.

1.3.6 Jump Strategies

Bobis & Bobis (2005) assert that the Jump Strategy is a straightforward strategy, and the main characteristic of the strategy is that the first addend or minuend is kept whole, and the second addend or minuend is established in parts. Wright, Ellemor-Collins and Tabor (2012) described the strategy as a sequence of jumps on a straight line, and for this reason, the jump strategy is described as a sequential or linear approach to computation referred to in the literature as the N10 approach as discussed above. As opposed to the base ten or 1010 computational procedure in which the numbers are decomposed into tens and ones, then processed separately and then put back together again on completion of the process (Torbeyns & Verschaffel, 2013). The N10 procedure is most suitable for the Jump Strategy on the empty number instead of the 1010 procedure (Blote et al., 2000).

Using the Jump Strategies, the following set of number skills is a prerequisite for working on the empty number line. This set of skills is also evident in the Learning Framework in Number (LFIN), which will serve as the theoretical framework for this study that will be discussed in more detail in the following chapter.

- Numeral recognition and identification
- Forward number word sequence (FNWS) and number word after
- Backward number word sequence (BNWS) and number word before
- Have a linear understanding of number and their relation relative to other numbers
- How to plot on an empty number line
- Making a 10 – use of a decuple
- 1010
- Bridging through ten (BTT)
    - Need number bonds
    - How to make a ten
    - How to select the most suitable combination of number sets.

Young learners need to be able to enact all these underlying number skills with the Jump Strategy on the empty number line. Any limitation in these related skills will adversely affect a learner's ability to work successfully on the number line using the Jump Strategy.

1.3.6 Thompson and Carpenter et al. and the advocators of the sequential approach to computation.

Thompson (2000) and Carpenter et al. (1999) are advocators of the sequential approach to computation. Moreover, a key characteristic of their work is the models they propose that contain different stages young learners' progress through as they shift from less sophisticated counting to more sophisticated calculating strategies (Ekdahl, 2019). As such, I will review the work of Thompson (2000), which formulated a model containing a set of five different counting strategies and *seven different calculating strategies*. According to Thompson (2000), the learner's selection of a particular counting or calculating strategy indicates the learner's fluency level. On the other hand, Carpenter et al. (1999) proposes a model with three different stages; these stages are also intended to be indicators of a learner's fluency with number work that is established based on the learner's selection of a particular counting or calculating strategy.

In the Thompson (2000) model, the five different counting strategies learners can employ are: *counting on from the first number*, *counting on from a larger number*, *counting back from a given number*, *counting back to the subtrahend, and counting from the missing number*. As noted by Thompson (2000), the seven different calculation strategies are *doubles, facts for subtraction*, *near doubles for subtraction, near doubles for addition*, *subtraction as the inverse of addition, bridging through ten for addition, bridging through ten for subtraction and compensation*. According to Thompson (2000), the list of possible calculating strategies learners can select speaks to the learner's ability to use already known number facts to derive new facts in each problem.

The three-stage model Carpenter et al. (1999) proposed is the *Direct Modelling* stage, *Counting Strategies* stage and *Number Facts* stage. As I am conducting my study in a grade three class, my assumption is that learners are well past the *Direct Modeling* stage and that most of them are vacillating between stages two and three, which are the *Counting Strategies* stage and *Number Fact* stage. As noted from the above reflection on the two different models, the Thompson (2000) model does not incorporate perceptual counting as reflected in the Carpenter et al. (1999) model but moves from the premise that learners have shifted past that elementary form of counting. The second observation is that in the Carpenter et al. (1999) model, stage two (Counting Strategies) relates to the five different counting strategies in the Thompson (2000) model. Thirdly, Thompson's (2000) model allows for a much deeper analysis of a learner's level of proficiency. Finally, that stage three of the Carpenter et al. (1999) model, which is the Number Facts stage, relates to the calculating strategies of the Thompson (2000) model. As Thompson noted, calculating strategies speak to the learner's ability to use known number facts, which is what the Carpenter et al. (1999) model accentuates.

1.4 Theoretical Framework

Due to the nature of my research, I employed a blended approach by merging two theoretical lenses. Firstly, to locate the present study in literature, I employed Askew (2012) as the overarching theoretical lens as it speaks to how learners approach mathematics learning. Furthermore, the test instrument employed for my study was developed by Askew (2012) and incorporated some mathematical proficiencies. These mathematical proficiencies were translated into Fluency *(answer calculations instantly)*, Strategic Calculating *(use efficient strategies to answer problems),* and Strategic Thinking *(use number relations to answer problems),* which informed the design of the test items (Graven, Venkat, Askew, Bowie, Morrison and Vale, 2020). Secondly, within the overarching theoretical lens, I have incorporated certain aspects of the Learning Framework in Number (LFIN) developed by Wright, Martland and Stafford (2006), as it assisted me in focusing in greater depth and detail to ascertain the sample group's mathematical proficiencies and strategies when solving mathematical tasks.

1.4.1 Askew four types of proficiencies

Askew (2012) proposes four types of proficiencies concerning mathematics. Fluency, Reasoning, Problem-Solving and Understanding are the proficiencies described by Askew (2012). However, in the test item design criteria, only Fluencies, Reasoning and Problem-Solving were used. The four proficiencies formed the basis of the pre-test and post-test items and the eight starter lessons to develop Fluencies, Strategic Calculations, and Strategic Thinking within the intervention. According to Askew (2012), the four proficiencies are intertwined and develop in tandem and not necessarily in succession. According to the traditional approach regarding mathematics, *Fluency* should be developed first, followed by other successive proficiencies at such a time when the learner is deemed to be ready, which means that mathematical development is considered a progression in or through different stages. However, according to Askew (2012), becoming mathematically proficient requires learners to work with all four proficiencies from the beginning and engage in specific actions before even displaying complete competence in those actions. Such mathematical proficiency is not the end goal but part of the entire learning trajectory.

According to Askew (2012), *Fluency* refers to knowledge of procedures, implying the capacity to know when and how to employ suitable and appropriate procedures with flexibility and accuracy. In the design of the test items, "Fluency" was identified as answering calculations instantly. Problem-solving entails the ability to formulate mathematical problems, then represent them with suitable and appropriate strategies and solve them accurately. According to the test design, *problem-solving is* referred to as "Strategic Calculations", which use efficient strategies to answer problems. *Reasoning* refers to the capacity to think logically, reflect on your thoughts, and then explain and justify your choice mathematically. Again, in the test design, the *reasoning was embedded in* "Strategic Thinking". *Understanding* involves the construction of complete and whole mathematics knowledge forms by which the learner can adapt and transfer mathematics principles and thereby make connections between related mathematical concepts. Whilst understanding is an essential proficiency, the test's designers did not incorporate it explicitly.

1.4.2 Learning Framework in Number (LFIN) framework

Wright et al. (2006) developed the Learning Framework in Number (LFIN) as a mathematics recovery program for learners lagging in the mathematics classroom. I will not employ the complete prescribed interview schedules of the LFIN for my study to assess learner strategies in the different aspects relating to number work. I will, however incorporate the specific aspect of the LFIN, which includes the following elements:

- Stages of Early Arithmetical Learning (SEAL)
- Base ten arithmetical strategies
- Forward number word sequence and number word after
- Backward number word sequence and number word before
- Numeral identification (Wright et al., 2006)

These elements of the LFIN will assist me in developing insights into the way the learners make sense of the additive mathematical tasks and what types of calculation strategies they employ.

1.5 Proposed Methodology

For this current study, I have adopted a non-traditional mixed methods approach, which combines and integrates both qualitative and quantitative methods in the same study. According to Creswell (2012), a mixed methods approach provides a better understanding of the research problem than either approach on its own. A perceived disadvantage of the mixed methods approach is that it requires more work and, thus, more time and, in some instances, more significant financial input. Another disadvantage is that the researcher must acquire the necessary skill and experience to employ both approaches. However, the advantage is that the researcher can use the results of one method to help develop the use of the second method (Greene, Caracelli & Graham, 1989). In a non – traditional mixed methods approach, I focus mainly on the qualitative aspect of the research while using the numbers to tell my qualitative story. In the initial phase of the study, I conducted a pre-test (see appendix A) and post-test (see appendix B) on 42 grade 3 learners followed by a schedule of 8 intervention lessons (see appendices C to J) over a 2-week period. Based on the results of the first phase, I then selected 6 learners with whom I proceeded to administer the LFIN interviews. As such, the quantitative approach fed into the qualitative approach, which afforded me the opportunity to gain better insights concerning the phenomena I was seeking to investigate, which is grade 3 learner's additive reasoning representations and strategies both before and after an intervention.

1.5.1 Research design

My study utilized experimental and exploratory research styles, involving 2 tests and a schedule of 8 intervention lessons over a 2-week course. This chosen design assisted me in determining if the intervention assisted in the conceptual development of the mathematical concept of additive reasoning and evaluating if learners developed fluency in their mental mathematics ability. After the intervention, I conducted the LFIN interview schedule on 6 selected learners.

1.5.2 Target population

The selected participants for this study were from 1 class of grade 3 learners comprised of 20 girls and 22 boys between the ages of 8-10 years old in a formerly disadvantaged school in the South of Johannesburg. I have selected grade 3 as it is the final year of the Foundation Phase, in which young learners are expected and required to make the necessary shifts from concrete forms of counting into abstraction. I have also selected grade 3 as it fits with my interest in Foundation Phase school mathematics. In addition, I selected 6 learners, 2 strong-attaining learners, 2 average-attaining learners and 2 low-attaining learners.

1.5.3 Research instrument

The selected research instrument for this study was a pre and post-test, a schedule of 8 intervention lessons, 4 lessons on addition and 4 lessons on subtraction and 6 interviews that were video recorded. Furthermore, the research instrument consisted of a combination of 15 addition and 15 subtraction items in the mid to high number range, see appendices A and B and was designed by Askew through a collaborative process with members of the Wits Math's Connect-Primary Project. The test items were designed to start with more accessible items and shift into more complex items. In total, there were 16 *Result unknown* items, 10 *Change unknown*, and 2 *Start unknown* items. Each intervention lesson started with a 2-minute mental warm-up exercise, that stimulated cognitive function and conceptual understanding. Thereafter the focus of the first 4 lessons were on addition, and the following set of 4 lessons focused on subtraction. After the completion of each lesson, learners had to complete, a short exercise comprised of 2 to 3 items. These exercises were used as ongoing assessments of learner take up of the concepts. After completion of the post-test, I select 6 learners from across the attainment band, 2 strong, 2 average and 2 low-attaining learners and interviewed each of them, in turn, to ascertain how they solved test items. The 6 interviews conducted were based on an adapted version of the LFIN assessments schedule that served to shed light on the learner's strategies in solving selected test items. These interviews did not exceed 15 minutes and served to aid me in answering the third research question of the study which was: *What is the correlation between the Jump Strategies and the LFIN?*

1.5.4 Pre and post-test

Utilizing tests (pre and post-tests) as my primary research instrument aided me in assessing whether or not the 42 grade 3 learners, on completion of the intervention, made any gains in relation to additive reasoning. Pre and post-tests were comprised of the same problem items and were administered to the same sample group under the same conditions. According to the design of the research instrument part one of the test were comprised of 20 Fluency items, and 2 minutes have been assigned to learners to complete it. Part two of the test contained 5 Strategic Calculating items and 5 Strategic Thinking items; learners were assigned 3 minutes to complete them. Bertram and Christiansen (2014) assert that researchers often test learners before an intervention to ascertain their achievement and then re-test them again after the intervention. This method shifts the focus away from comparing learners against a norm or standard towards understanding how they can reason in various additive situations. Cohen, Manion, and Morrison (2011) maintain that using a pre-test before an intervention program helps identify starting abilities and achievements in students and adds value to teaching and learning.

1.5.5 Intervention lessons

A schedule of 8 intervention lessons focusing on additive reasoning were utilized. Each of the 8 lessons were approximately 20 minutes long spread over a 2-week period with one lesson per day. A typical lesson was comprised of a 2-minute mental warm-up that involves the full participation of each learner. The mental warm-up required that learners shout out the next multiple of ten on or off the decade; it also required learners to shout out the closest multiple of ten to any given number. It is a quick thinking and quick responsive strategy to stimulate the learners' thinking and reinforces conceptual development.

Thereafter the lesson of the day was presented to the learners with the use of the board, and learners were taken slowly through the steps of solving a given number problem. An example of this type of problem is 34 + 27 =☐. Learners were taught the steps of working on the empty number line as prescribed in the teacher's manual. Upon completion of the lesson, learners were given a sheet containing 2 to 3 number problems to solve independently. During this exercise, I walked amongst the learners to observe and assist them where needed. These learner sheets were collected, and learner uptake were assessed and monitored based on their work. The first 4 lessons dealt exclusively with addition, and the last 4 lessons dealt exclusively with subtraction problems; both operations have increased complexity. The increase in complexity of the lessons was in relation to the fact that lesson three of both the additive and subtraction sections included a bridging through ten step. In addition, lesson four of addition and subtraction includes a change unknown step.

1.5.6 Interviews

Interviews were conducted with 6 selected learners based on their overall performance from the pre to post-test. These 6 learners were selected on the following set of criteria: 2 weak, 2 average and 2 strong performers. Cohen et al. (2011) state that interviews are employed to follow up on unexpected test results, to validate other methods or, at times, to delve deeper into the motivations of respondents for their reasons for responding in the manner they have. All 6 interviews were video recorded and later employed in an in-depth analysis to confirm or clarify misconceptions in the analysis process.

1.5.7 Data collection and analysis

The data collection tools used for this study were the pre and post-test, daily classroom exercises based on the day's lesson, and interviews of 6 learners, as noted earlier in the discussion and my field notes. As the interviews were video recorded, I positioned the camera as close as possible to the learners to see what they were writing but not too close so I can see their facial expressions and body language, as these provided insights into their strategy selection processes. The advantage of video recording is that it captures everything that is taking place during the interview, and as such, it provides the researcher with the facility to revisit aspects that are of relevance in the pursuit of the answers sought. Another advantage is that it allows the researcher the time to make considerable judgement calls which can be revisited at will. A weakness of video recording is that because you deal with the greater sensory perception, you can spend hours watching learner actions and gestures that are irrelevant to your study (Pirie, 1996). As such, much experience in the use of video recording of interviews is essential.

The analysis process involves developing initial insights from the pre and post-tests and classroom exercises. Thereafter interpreting learners' work and drawing implications from it. Parts of the video-recorded interviews were transcribed, they were read and re-read to extract and describe initial insights to reflect on their overall meanings. Thereafter the sample learners' problem-solving strategies and models were analyzed and compared using an adapted version of the Learning Framework in Number (LFIN) developed by Wright et al. (2006). The LFIN encapsulates possible stages and levels of number learning that young learners progress through as they develop number knowledge.

1.5.8 Rigor

Reliability in one's research is vital in that it ensures confidence and credibility. Reliability is synonymous with consistency and replicability over time, instruments, and groups of respondents (Cohen et al., 2011). However, using the test/re-test method, care should be taken regarding the timing between the tests since situational factors may occur, and participants may tend to remember the questions in the test (Cohen et al., 2011). This, in addition to questions being ambiguous, unstandardized tests and participants feeling uneasy or misinterpreting questions, could potentially affect the reliability of the test (Creswell, 2012). To resolve these issues, tests will be conducted two weeks apart so that learners are not too familiar with the questions, clear instructions will be provided, and the same questions for both tests will be used.

In terms of ***validity*** – the data itself does not ensure validity. Rather validity relies on the relationship between the claim and the data-gathering process (Opie, 2004). To ensure validity, I have chosen a sample group of 42 learners, which I believe is sufficient. Additionally, the level of the tests also needs to be appropriate, non-discriminatory, non-biased and neither too long nor short (Cohen et al., 2011). Items in the tests will also be clearly legible and vary in cognitive demand level, including low and high number ranges.

1.5.9 Ethical considerations

Ethical considerations for this study were that informed consent forms were given to the participants and received back from them. A brief containing all relevant information were given and discussed with the teacher and the participants involved in the study. Data collected were treated with confidentiality as it will not be discussed or used outside of the purposes of this study, and the anonymity of participants and the school will be maintained, as different names were given to participants and the school. The teacher and all participants received feedback on the study. Moreover, data collected will only be utilised for the study; it will be stored in a safe location and kept in a secure cloud for further research.

1.5.10 Limitations

Time: This study was conducted over a 2-week period, which may not be enough time to see significant gains, making it difficult to generalize. A small-scale study cannot provide a comprehensive understanding of the nature of additive relations. Thus, more research and evidence will be needed before strong claims can be made. Although this study is on a small scale, it provided valuable insight that can provide motivation for a longitudinal study later with the entire Foundation Phase in one school.

Small sample size: The limitation of a qualitative method is that it focuses on meaning and experiences rather than on contextual sensitivities. Smaller sample sizes raise the issues of generalizability to the whole research population, and the results cannot be generalized to the larger population (Flick, 2011).

1.6 Chapter Summaries

In this section, I will show the structure of the study, and I will also provide a brief summary of each chapter.

Chapter One:

In the introduction to the study. I lay out the following: The background to the study, I identify the possible cause of the perceived problem of learner underperformance. The purpose of the study, locating the study within the broader Wits Math's Connect Primary project research and development work and the research questions. Also, I will look at the reasons for my selection of how to conduct the research together with the validity and generalizability and then ethical considerations and limitation of the study.

Chapter Two:

In this chapter, I present a review of the literature in which the study is embedded. I have selected to review literature dealing with relevant and specific aspects of additive reasoning, jump strategies and the number line. I have furthermore selected to review materials of theorists that have made a considerable contribution in the area of a sequential approach to computation, although the structured approach to computation is also acknowledged.

Chapter Three:

This chapter discusses the two theoretical frameworks that underpin this study, which will be a combination of Askew's (2012) three learner proficiency and the Learning Framework in Number (LFIN) by Wright et al. (2006). For the purposes of this current study, I will only incorporate the first three aspects of the LFIN as these aspects relates to Additive Reasoning (AR) and exclude the fourth aspect as it relates to Multiplicative Reasoning (MR).

Chapter Four:

This chapter presents the methodology I used to answer the research questions of the study. I discuss the research sample, methods of data collection, validity and generalizability, and ethical considerations.

Chapter Five:

In this chapter, I present a discussion of the finding of the analysis of the intervention on one class and the LFIN assessment of six learners use of calculation strategies. I furthermore reflect on the possible implications it holds for the teaching and learning of Foundation Phase mathematics.

Chapter Six:

Contains some concluding remarks regarding the study with some recommendations for further research and a few recommendations for the teaching and learning of Foundation Phase mathematics.

CHAPTER 2 - LITERATURE REVIEW

2.1 Introduction

My study focused on investigating grade three learners' additive reasoning on the empty number line with the use of jump strategies. I have identified and selected to review literature about the following specific aspects; firstly, I will describe Number Sense and how to develop it, then reflect on two different approaches to computation, followed by a reflection on additive reasoning, additive representations, and the empty number line. I will include a reflection on mental strategies and then jump strategies. Furthermore, I have selected to review literature produced by the advocators of a sequential approach to computation as it fits well within the use of jump strategies on the empty number line. Although I will foreground a sequential approach toward computation in this study, I will also acknowledge the advocators' contribution of the structured approach to computation.

2.2 Number Sense

Many learners do not perform well in mathematics in the Foundation Phase and in successive grades due to a poorly developed number sense (Dyson, Jordan & Glutting, 2013). McIntosh, Reys & Reys (1992, p8) states that number sense is "… nebulous and difficult to describe, although it is recognizable." Therefore, although difficult to describe, the following can be said concerning it. Number sense is the ability first to recognize and identify numbers. It incorporates aspects of recognizing measurement and distance, implying the ability to locate numbers on a number line in relation to other numbers. However, it also incorporates an understanding of the size of the number that 36+30=☐ cannot equal 80. Learners where number sense is poorly developed and who do not have the inclination to understand the size aspect of numbers will produce nonsensical answers to number sentence questions, as indicated in figure 1 below. Furthermore, it also includes the ability to make a ten from any combination of two single digits and the related ability to transfer that knowledge and number principles into the teens and higher number range. It also carries an awareness of counting forward and backwards on and off the decuple in tens.

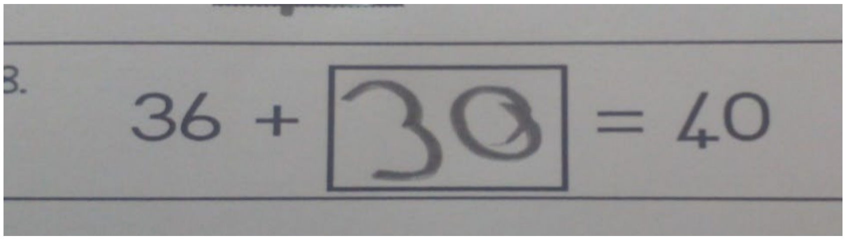


Figure 1: Nonsensical answers when learners do not understand the size of numbers.

Number sense encompasses proficiency in recognizing different aspects of numbers and incorporating that knowledge and principles into problem-solving skills. In my study, I made some observations while generating data. Those learners who have been identified as having a poorly developed number sense struggle to cope with the tasks of performing the jump strategy on the empty number line. As they have not yet developed the desired and required level of number sense that will enable them to apply themselves skillfully when working with the jump strategy on the empty number line. As a result, due to their inability to cope with the higher cognitive demand that is being placed on them, their only recourse is to select a default strategy that is, unfortunately, error-prone, ineffective, and time-consuming—resulting in learners employing elementary forms of unit counting strategies such as the triple count strategies as reflected in figure 2 below. The triple-count strategy is seen in the fact that the learner has a sum of 3 + 3 = □, and the learner first counts the first three (first count) and then counts the second three (second count) and then counts 3 and 3 to produce the answer (third count).

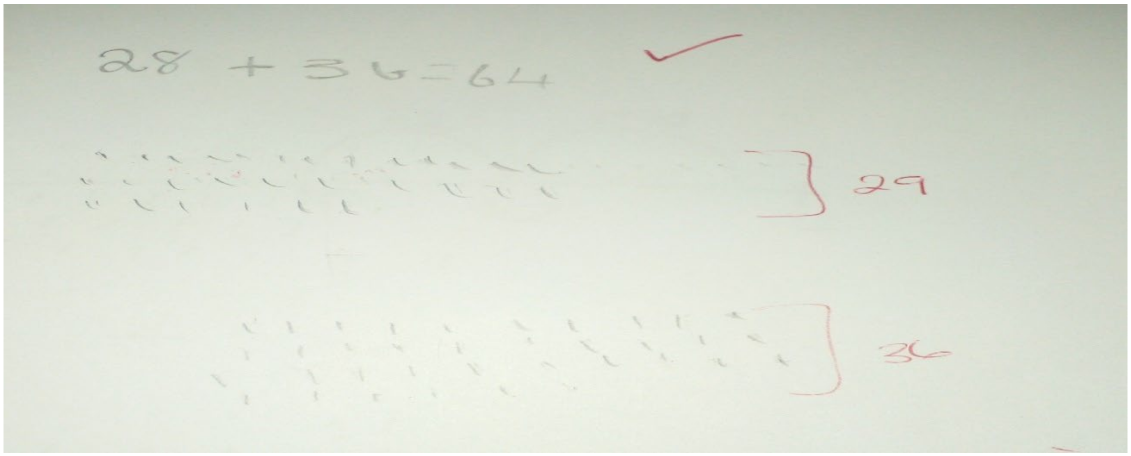


Figure 2: Low performing learner 6 depiction of a triple count strategy

As I have presented a reflection on number sense and the importance of developing a strong number sense. I will now proceed to present a discussion on the two different approaches to computation: the sequential and the structural.

2.3 Two Different Approaches

Additive reasoning includes both mathematical operations of addition and subtraction. Furthermore, two broad approaches to computational work have been identified in the literature. The first approach has been labelled as a sequential approach that is based primarily on counting strategies. The second approach has been labelled as a structured approach that is based on de-composing and re-composing strategies that emphasize the relational aspect of number (Ekdahl, 2019). Each approach to computation accentuates different philosophies of mathematics with different pedagogic practices. The purpose of this study is to research and develop effective means of enhancing the development of mental mathematics in young learners. As such, the purpose of this study is not to draw a distinction between these two approaches to advocate one over the other. Moreover, due diligence requires that all relevant aspects and facets of the issue under consideration needs to be explored to find the strengths of both approaches and to employ these for the academic advancement of learners. I will, therefore, first present a discussion on the sequential approach followed by a discussion on the structured approach towards computation.

2.3.1 Sequential Approach

Advocators of the sequential approach (Carpenter, Hiebert & Moser, 1981; Carpenter et al., 1999; Thompson, 1999) suggest that counting strategies are the most natural developmental pathway along which young learners develop number knowledge. Furthermore, they assert that young learners' counting strategies eventually develop into calculating strategies. Ultimately, according to this perspective, the starting point for young learners is in counting, which leads to calculating and other successive mathematical concepts. From this perspective, counting becomes the foundation for subsequent mathematical concepts. As such, greater care should be taken to ensure that learners firmly grasp foundational concepts. That will ideally lead to stronger and greater mathematical achievement in successive levels of number understanding.

2.3.2 Structured Approach

Before commencing with the reflection on a structured approach, I wanted to clarify my use of the term structure, as much has been produced concerning this idea. As such, for the purposes of this present study, the notion of structure fundamentally involves the arrangement or re-arrangement of numbers to be worked with into some mathematically appropriate relation to each other, as expressed by Venkat, Askew, Watson, & Mason (2019). Meaning that structure can be seen in the following set of numbers 5, 5 and 25 and that arranging them into an appropriate mathematical arrangement involves the following $5 \times 5 = 25$; however, 5×25 is not equal to 5. As such, a structured approach to computation emphasizes an appropriate mathematical relationship between elements. The underlying number skills required to facilitate such an undertaking include a fluent understanding of the base ten number system (Venkat, Askew & Morrison, 2021) that solving 16 + 8 = involves moving to the next multiple of 10, which is 16 + 4 = 20 + 4 = 24.

The advocators of the structured approach as mentioned by Schmittau and Morris (2004) acknowledge and appreciate the role and place of counting in young learners' number knowledge. However, the advocators of a structured approach emphasize the development of a structured way of thinking about number from a relationship between quantities. They argue against a sequential counting developmental process but posit that there are alternative methods to achieve the same goal of learner proficiencies in mathematics. Furthermore, they assert that many young learners in the sequential perspective develop an unhealthy over-reliance on unit counting practices and fail to make the shifts into more efficient strategies successfully. The reality of this dilemma is evident within the South African primary school landscape, where an unhealthy over-reliance on counting in ones is evident, as well as an over-emphasis on unit counting by educators that is re-enforced by pedagogic practices that place a higher premium on counting in ones. Concerning this situation, Mason, Stephen & Watson (2009) argue that unless young learners are encouraged to attend to structure and to engage in structural ways of thinking about a number, they will be blocked from thinking productively and deeply about mathematics. As such, the advocators of the structured approach place a much higher premium on developing and cultivating a culture that emphasizes the relational aspect of numbers instead of the sequential aspects. The part-whole relations of number speak to a structured approach in which the constituent parts and the whole can be composed and decomposed. Or alternatively arranged into a mathematically appropriate relationship. Furthermore, the advocators of the structured approach posit that a structured approach has far more significant long-term benefits for young learners than the sequential approach.

2.4 Additive Reasoning

Additive reasoning in young learners is the gradually increasing capacity for greater flexibility with numbers and quantities. As formal schooling commences, young learners' initial capacity reasoning competencies are limited to elementary forms of perceptual unit counting strategies. Perceptual unit counting strategies are when learners enact their counting operations through one-to-one correspondence, and it is characterized by their dependence on concrete forms of counting. However, as young learners progress in Number Sense which, according to Green (2005), refers to a learner's general understanding of number and operations, their additive reasoning competencies will also incorporate both counting and calculating strategies. This progression in competencies, according to the models developed by Thompson (2000) and Carpenter et al. (1999) will be reflected in the choice of counting and or calculating strategies young learners choose when attempting to solve given number problems.

According to Ching & Nunes (2017) additive reasoning is a young learner's increased competencies to reason logically about quantities. The increased capacity to reason logically about numbers will start to incorporate an understanding of the part-whole relationships of quantities, which includes the two central properties of part-whole relations that involve commutativity and inverse relations between addition and subtraction. As such, additive reasoning at different levels of a learner's developmental path incorporates different elements of number knowledge.

2.5 Additive Representations

Representation is a commonly used idea in the teaching and learning of mathematics. We have different types of representations, namely, objects, diagrams, graphs, and symbolic representations. Due to mathematical concepts' abstract and complex nature, researchers and educators employ representations to communicate dense mathematical concepts to young learners. A representation is a pedagogic device or tool used in the place of something that is not yet understood by young learners. As such, a representation resembles the object represented (Kami, Kirkland & Lewis, 2001). According to the NCTM, "representations are placeholders, which allow the children to carry out and record the intermediate steps of a task without being overwhelmed by the rest of the problem" (NCTM, 2000). While representations might serve the means for which they were intended, careful and systematic instruction regarding the use of these tools is nonetheless essential. The following is not an exhaustive list of representations used in the mathematics classroom. Pictorial diagrams (Part-Part-Whole diagrams, horizontal number sentences, vertical (column) addition, tallies and number lines) and manipulatives (dienes blocks and bead strings). Part-part-whole diagrams and number lines are the two representations supporting additive reasoning that is particularly interesting in this study. Although number lines will be foregrounded in this study, the importance of the part-part-whole diagram is acknowledged. Both representations have significant advantages in enhancing learner development of mental math proficiencies. However, each of the two additive models foregrounds different computational approaches and teaching approaches.

2.5.1 Part-Whole Diagrams

While part-whole schemas fall within the purview of this current discussion, due diligence dictates a short reflection on the clear distinction drawn in the literature between part-whole and part-part schemas about computational work. While part-whole schemas represent the relationships of a part to a whole, part-part schemas represent the relationship of two parts to each other (Singer & Resnick, 1992). In the following example, the distinction is apparent. In a collection of 10 marbles, 6 are blue, and 4 are red. A part-to-whole relationship is expressed by stating 6/10 blue to the whole or 4/10 red to the whole. The part-to-part relationship is expressed as 6/4 blue to black. So, in this study, we will focus on the part-to-whole relationship for both addition and subtraction. Part-whole relationships of numbers speak to the understanding that quantities can be interpreted as being composed of other numbers where a quantity can be broken into two or more parts (Sinnakaundan, Kulda, Hasim & Ghazali, 2016). Furthermore, part-whole thinking is considered a significant transition from concrete to abstract thinking in computational work Steinke (2001). Cheng (2012) noted that helping children build and understand part-part whole relationships of number work improves their ability to solve addition problems. According to Hunting (2003) and Sinnakaundan et al. (2016), understanding part-part-whole relationships can enhance the concept of place value. As such, part-whole relational knowledge and understanding holds a great advantage for young learners. The part-part whole diagram is a rectangular bar that is used to represent the whole and its constituent parts in picture form, as depicted in figure 3 below.

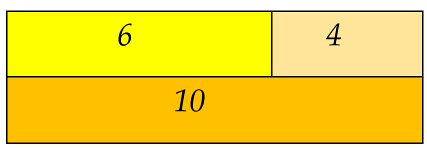


Figure 3: A part-part-whole diagram

Furthermore, the part-part whole diagram is grounded in a structural approach toward computation and as such, counting in units is avoided by starting by comparing the number relations (Ekdahl, 2019). Furthermore, the part-whole relation focuses on the principles of decomposing and recomposing numbers (Plunkett, 1979). According to Cheng (2012), the underlying skill that enables decomposing and composing numbers is fluency in the number range 1-10. These fluencies should be known and recalled without any counting involved. Moreover, this approach to computational work of decomposing and composing numbers fits well within the 1010 approach that splits both addends into tens and units and ends off by compressing it again at the end to arrive at an answer. The 1010 computational approach contrasts the N10 approach in which the first addend is kept whole, and only the second addend is split into smaller parts. An example of the N10 approach can be seen when solving the number problem 24+35=. The first addend is kept whole (24). Thereafter the second addend is split into tens and ones (30 and 5). In this case, it will be 24+30=54+5, producing the answer of 59. On the other hand, an example of the 1010 approach can be demonstrated when trying to solve 24 + 35 = □. Both the first and second addends are split into tens and ones (20+4) and (30+5). Thereafter the tens are compressed (20+30), and the ones are compressed (4+5). Resulting in (50+9), leading to the answer of 59. In this way, both addends are split into tens and ones and are compressed again at the end to arrive at the correct answer. Moreover, in this way, the parts are seen as making up the whole. Advocates of this approach acknowledge the importance of counting but suggest that counting strategies are not necessarily the natural pathway to structural thinking of numbers and that there are alternative methods that can be used to achieve the same objective (Ekdahl, 2019). In using the jump strategy, we apply the N10 approach to find the answer.

2.5.2 The Number Lines

The literature identifies three different types of number lines; these are structured, semi-structured and empty number lines. From an educational perspective, the number line has proven to be a powerful tool in teaching various mathematical concepts. These include estimation, fractions, multiplication, measuring length, and measuring time for giving access to possible solution strategies; it also allows the representation of numbers. Furthermore, it is the most basic way to picture a mathematical concept as it does not require any words (Skoumpourdi, 2010). The use of the number line foregrounds counting, and it is furthermore considered a sequential approach to computation and fits well within the N10 approach to number work (Ekdahl, 2019). A characteristic of the N10 approach is that the first addend is kept whole, and the second addend is split into tens and units to derive the answer usually enacted in jumps on the number line. An example of the N10 approach can be seen when solving the number problem 24 + 35 = □. The first addend is kept whole (24) and plotted on the number line; the second addend is split into tens and ones (30 and 5). In this case, it will be enacted on the number line, plot 24 on the number line, then jump in 30, landing on 54, and then jump in 5, producing the answer 59. For the purposes of this study, the empty number line will be foregrounded as the preferred representation to make sense of the additive tasks.

2.5.3 The Empty Number Line

The empty number line was first recorded in the Netherlands in the 1970s (Gravemeijer, 1994). Moreover, it is considered a powerful tool to support the development of children's mental strategies for addition and subtraction up to 100 Beishuizen (2001), unlike the structured and semi-structured number lines with number markings on them. On the empty number line, students only mark the numbers they will use for their calculations. Gravemeijer (1994) noted three benefits of using the empty number line: its linear representation of numbers. Secondly, its reflection is more closely intuitive mental strategies children use, and it's the potential to foster the development of more sophisticated strategies. Bobis and Bobis (2005) added a fourth benefit in that it can serve as a powerful tool to enhance classroom communication, as learners can explain their processes of solving number problems. Moreover, it also allows for observing learners' thinking processes (Bobis, 2007). Further advantages of the empty number line, as noted by Gray and Tall (1994), the advantages of the empty number line are that it is graphic, gives the value of the numbers and their position relative to other numbers and, as such, numeral recognition and identification are essential skills.

2.6 What are mental strategies and why is it important?

Firstly, mental mathematics refers to any mathematical calculation that is performed mentally without using a calculator, pen and paper or any other form of assistance to complete. Mental strategies, on the other hand, refer to the ability to mentally select the most suitable and appropriate strategy or method and thereby solve mathematical problems mentally. Thompson (1999a) proposes the following four reasons why the ability to perform mental mathematics is essential to the early development of young learners. Firstly, most calculations in adult life are conducted by mental means. Secondly, mental work develops insight into the number sense. Thirdly, mental work develops problem-solving skills. Fourthly, mental work promotes success in later written calculations. Above all, mental strategies should not only be effective, but they need to be efficient. In other words, solving $5 + 6 = \square$ can be solved by keeping 5 in one's head and counting in ones until you get to the answer 11. Another way could be doubling five and adding 1 to make 11. While both strategies are effective, the doubling strategy is both effective and efficient.

2.7 Jump Strategies

Bobis & Bobis (2005) asserts that the jump strategy is a straightforward strategy, and the main characteristic of the strategy is that the first addend or minuend is kept whole and the second addend or minuend is established in parts. Wright et al., (2012) described the strategy as a sequence of jumps on a straight line, and for this reason, the jump strategy is described as a sequential or linear approach to computation referred to in the literature as the N10 approach. As opposed to the base ten or 1010 computational procedure in which the numbers are decomposed into tens and ones then processed separately and then put back together again on completion of the process (Torbeyns & Verschaffel, 2013).

The N10 procedure is most suitable for the jump strategy on the empty number instead of the 1010 procedure (Blote et al., 2000).

2.8 Prerequisite knowledge for working with Jumps Strategies on the Empty Number Line

The purpose of using jump strategies on the empty number line is to enhance the development of mental mathematics in young learners. As such to be able to make the shifts into mental math the following set of number skills are considered prerequisite skills for working with jump strategies on the Empty Number Line. If learners are not well on their way to having a firm grasp on these number skills, then their ability to work in any meaningful way on the number line will be negatively affected, this also implies that if young learners do not possess these number skills, then they are still challenged in understanding number as an abstract entity.

- Numeral identification
- Forward Number Word Sequence (FNWS) and number word after
- Backward Number Word Sequence (BNWS) and number word before
- Have a linear understanding of number
- How to plot on an empty number line
- Place value of 10's and 1's
- Making a 10 – use of a decuple
- N10 and 1010
- Bridging through ten (BTT)

Young learners need to be able to enact all these underlying number skills with the jump strategies on the empty number line. Any limitation in these related skills will adversely affect a learner's ability to work successfully on the number line using the jump strategy.

2.9 Thompson (2000) and Carpenter et al., (1999) and the advocators of the sequential approach to computation.

My study focuses on the use of jump strategies on the empty number line, so it aligns itself with the sequential approach toward computation. Therefore, I furthermore selected to review the writings of two authors that are considered advocators of the sequential approach as it will further aid me in gaining insights into the different counting and calculating developmental stages young learners progress through. It will also assist me in assessing where my sample size is located in terms of their levels of proficiencies.

The two authors I selected to review are considered advocators of the sequential approach because the following two aspects characterize their work. Firstly, according to Ekdahl (2019), a key feature and characteristic of their work are the proposed developmental stages young learners progress through, from least efficient to most efficient. The emphasis is on the stages of development as being measurable. Secondly, both authors consider counting as learners' starting point and most natural developmental pathway. Thompson (2000) and Carpenter et al. (1999) are the two authors. Both authors assert that learners start with counting or transition into more efficient calculating strategies. But the shift should not be considered binary but always in a state of flux in that although learners can use more efficient strategies, they can still employ counting strategies when needed. The emphasis is not on an over-reliance on counting but on the awareness of having a greater repertoire of strategies at their disposal.

2.9.1 Thompson

Thompson (2000) formulated a set of five different counting stages: counting *on from the first number*, *counting on from a larger number*, *counting back from a given number*, *counting* *back to the subtrahend, and counting up from the missing number*. As well as a set of eight different calculating stages; *double, facts for subtraction, near doubles for subtraction, near doubles for addition, subtraction as the inverse of addition, bridging through ten for addition, bridging through ten for subtraction and compensation.* According to Thompson (2000), the learner's selection of either a particular counting or a particular calculating strategy indicates the learner's proficiency level in number work.

2.9.2 Carpenter et al.

On the other hand, Carpenter et al. (1999) proposed a model with three different stages; these stages are also intended to be indicators of a learner's fluency with number work, which is established based on the learner's selection of a particular counting or calculating strategy. Carpenter et al. (1999) proposed the three-stage model: the Direct Modelling stage, the Counting Strategies stage and the Number Facts stage. As I am working in a grade three class, I assume that learners are well past the Direct Modeling stage and that most of them are vacillating between stages two and three, the Counting and Number Facts stages.

Thompson (2000) Model	***Carpenter et al., (1999) Model***
	1. Direct Modeling Stage
1. Counting on from the first number *2. Counting on from a larger number* *3. Counting back from the given number* *4. Counting back from the subtrahend* *5. Counting up from the missing number*	*1. Counting Strategies Stage*
1. Doubles *2. Facts for subtraction* *3. Near double facts for subtraction* *4. Near double for addition*	*2. Number Facts Stage*

5. Subtraction as the inverse of addition *6. Bridging through ten for addition* *7. Bridging through ten for subtraction* *8. Compensation*	

Table 1: Comparison between Carpenter et al. and Thompson models

For the purposes of my study, I will keep in mind the Carpenter et al. (1999) model as that highlights the two major strategies young learners can resort to when solving problems. At the same time, I will remember Thompson's (2000) model as it allows me access to the finer nuances of the counting and calculating strategies learners can employ when solving mathematical problems (see Table 1 above).

As can be noted from the above reflection on the two different models is that the Thompson model does not incorporate the Direct Modeling stage as it is a perceptual counting stage. The second observation is that in the Carpenter et al. (1999) model, stage two relates to the five different counting strategies in the Thompson (2000) model. Finally, that stage three of the Carpenter et al. (1999) model, which is the number facts stage, relates to the calculating strategies of the Thompson (2000) model. As Thompson (2000) noted, calculating strategies speak to the learner's ability to use known number facts, which is what the Carpenter et al. (1999) model accentuates. And lastly, although Thompson (2000) and Carpenter et al. (1999) may be considered advocators of a sequential approach to computation, it is noteworthy to mention that both models incorporate the relationship aspect of number but seek to start off with counting strategies. There is also a correlation between the two models, although different names have been assigned to designate different stages

2.10 Conclusion

Literature is replete in stating the absolute necessity for young learners to develop strong number sense, as a diminished capacity causes poor mathematical performance. A strongly developed number sense is also connected to the development of mental mathematics in young learners. I selected to employ the sequential approach to arithmetic, as the type of representation most suitable for the selected intervention is the jump strategy on the empty number line. The advocators of the sequential approach assert that counting is the most natural developmental pathway along which young learners develop number sense. However, the advocators of the structured approach to arithmetic in which the relational aspect of number is foregrounded in mathematics learning in the Foundation Phase. The following chapter will discuss the selected theoretical framework in which my study is located.

CHAPTER 3 - THEORETICAL FRAMEWORK

3.1 Introduction

For the purposes of this study, I selected to employ a blended approach by utilizing two theoretical lenses. Firstly, to locate this study in the broader field of literature, I selected to employ Askew (2012) as the first theoretical lens as Askew (2012) proposes a mathematical proficiency model containing four proficiencies: however, for the purposes of this study. I will only focus on three of the proficiencies, namely, *fluency, problem-solving*, and *reasoning*. The Askew (2012) lens is a suitable lens through which these proficiencies can be assessed and enhanced. The second theoretical lens I selected is the Learning Framework in Number (LFIN), developed by Wright et al. (2006) as an intervention tool for students lagging behind in mathematics. The purpose of selecting the LFIN as a second theoretical lens is due to its applicability in assessing the finer nuances of the sample group's mathematical proficiencies. Although the LFIN is comprised of four different parts, namely, *(1) Stages of Early Arithmetical Learning (SEAL) and Base Ten Thinking (2) Forward Number Word Sequence (FNWS) and Backward Number Word Sequence (BNWS), (3) Structuring Number 1-20 and (4) Early Multiplication and Division.* For the purposes of this study, I will only incorporate the first three parts of the LFIN as it directly bears relevance to the current study and the fourth aspects which relates to Multiplicative Reasoning (MR) does not fall within the perimeters of the focus of this current study.

3.2 What are mathematical proficiencies?

In mathematical terms, proficiency does not simply imply knowing the bare minimum but speaks to an ability to move fluently and flexibly between mathematical concepts and make connections between concepts. It is through this notion of proficiency that Cowan, Donlan, Cole-Fletcher, Saxton and Hurry (2011) intimate that mathematical proficiency means the accurate solution by any efficient strategy. Implying that learners considered proficient have access to a rich variety of mathematical strategies combined with conceptual and procedural knowledge whereby they can select the most appropriate and efficient strategy and execute a problem accurately. Khairanis, Sahari & Nordin (2011) propose that mathematical proficiencies incorporate expertise, competence, knowledge, and facility aspects. Phaniew, Junpeng & Tang (2021) intimate that mathematical proficiency is defined as a "student's capability to search, speculate, and think logically in the cognitive process to comprehend how to solve a mathematical problem by using appropriate strategies to solve problems and replicate the procedure used to solve the problems". From the above reflection, mathematical proficiency is a notion that is absolutely nuanced and multi-layered, requiring a wide range of words to describe it.

3.3 How does mathematical proficiency develop?

From a practical point of view and of primary importance on how mathematical proficiencies are developed in young learners, considering teacher content knowledge is essential. In research conducted by Venkat and Spaull (2015) and (Venkat, Rollnick, Loughran, & Askew, 2014). There is a strong correlation between teacher content knowledge and the level of proficiency achieved by mathematics learners. Implying that if teacher content knowledge of mathematical concepts is weak, it will be negatively reflected in learners' mathematical achievements.

Furthermore, from a theoretical perspective on how mathematical proficiency develops, I have decided to discuss three different views. The first is that of the *traditional view*, *the conventional wisdom view* and the *number sense view* (Fithratu, Prihatmanto, Gitarana & Muis 2020). Each of these views proposes slightly different developmental pathways for mathematical proficiency development. Moreover, although each of the three views has a different developmental pathway, they nonetheless have similar or connected elements constituting each of them. In the *traditional view,* proficiency develops through rote memorization and practice, as solutions are stored in long-term memory. According to Conlan et al (2011), the first use of source, long-term memory, can be challenging as learners' mental capacities differ, and many learners have cognitive function challenges. Furthermore, according to the traditional view, the formation of mathematical proficiency dictates that learners' mathematical proficiencies develop in stages. This means that only when a learner has been deemed proficient enough in one stage will they then proceed to the next stage of development. Therefore, according to this view, one stage of proficiency is considered necessary as a gateway into the next or higher stage of proficiency. Meaning that mathematical proficiency was considered as a progression in or through stages.

The *conventional wisdom view* posits that mathematical proficiency develops along a three-step developmental process described as *counting, decomposition and retrieval*. According to Ashcroft (1995), "retrieval refers to the direct retrieval of an answer from a store of facts held in long-term memory". According to this view, learners start their formal education by counting with their fingers. They solve problems through de-composition strategies and, lastly, through long-term retrieval of known facts. According to Hopkins & Bayliss (2014), mathematical proficiency develops through the accurate use of retrieval and de-composition strategies. What is common to both of these views is that both emphasize rote memorization and practice as key to its development.

However, the *Number Sens*e view advocated by Baroody (2006) and concurred by Cowan (2011) proposes that mathematical proficiency results from an understanding of number operations, patterns, and principles. According to this view, rote memorization is not emphasized, but a conceptual understanding of number and number operations is crucial. This approach is strongly advocated by Venkat et al. (2019) as the most efficient manner of achieving learner proficiency, in a similar vein to advocating the development of strong Number Sense Askew (2012) proposes that becoming mathematically proficient requires learners to work with all four proficiencies from the beginning and by engaging in specific actions before even displaying complete competence in those actions. As such, a substantial number sense is essential. According to Askew (2012), mathematical proficiency is not the end goal but part of the entire learning trajectory. Askew (2012) furthermore proposes that learner mathematical proficiencies develop in tandem and not necessarily in succession as traditional views propose. The goal of the educational enterprise and mathematics in particular, and by implication, my study, is to develop means to enhance learners' mathematical proficiency.

By synthesizing the strengths of each of the three different views, an amalgam and infusion of the strengths of the three approaches can potentially develop mathematical proficiencies in young learners. Initially, all young learners commence their formal education by counting in ones on their fingers, and educators incorporate pedagogic practices of rote memorization of practices, as it is one of the ways that young learners grasp early number concepts. In time, they develop a conceptual understanding of the structure of numbers that enables them to compose and decompose numbers. In time, they can generalize principles by transferring their knowledge to other domain areas. As such, incorporating the strengths of all three views can develop mathematical proficiency in young learners.

3.4 Askew's Proficiencies

Pedagogic practices regarding mathematics teaching and learning have undergone considerable transformations over the past 50 years. Traditional pedagogic practices placed a high premium on rote memorization of mathematical concepts that were reinforced by practicing a series of procedural steps. This approach toward mathematics teaching and learning resulted in learners possessing procedural knowledge of concepts but lacking conceptual understanding of why they must follow a set of pre-determined steps. Furthermore, this approach also resulted in learners not acquiring the required mathematical knowledge that would enable them to generalize and apply mathematical principles that can be transferred from one learning domain to the next.

However, in the current model of mathematical pedagogic practices, there is a new focus on developing learners' mathematical proficiencies in a balanced and holistic manner. As such, while traditional approaches focus on procedural knowledge as an essential proficiency, current models focus on enhancing all possible proficiencies. As such, researchers developed various models to describe the inner mechanisms of young learners' developmental processes in developing mathematical proficiencies. There has also been a wide range of ideas regarding assessing the notion of mathematical proficiency, especially in the face of the current downward trend of learner performance as intervention programs and remedial work must be undertaken.

Within this rich maze of literature, various models have been developed and proposed regarding mathematical proficiencies. The most well-known and used model was proposed by Kilpatrick, Swafford & Findel (2001), who proposed a model of five strands of interwoven and interdependent mathematical proficiencies: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Furthermore, in a similar vein, in a study conducted in Thailand researchers develop a model that can identify and assess the development of learner mathematical proficiencies in geometry research conducted in Thailand (Phaniew, Junpeng & Tang 2021). Researchers employed a model developed by Junpeng, Krotha, Chanayota, Tang, and Wilson (2019) that contains five proficiency levels, namely non-response/irrelevance, unrecalled memory, basic memory and basic memory and reproduction, simple skills and concept, and strategic/extended thinking. While the above two models indicate the complexity of trying to propose models whereby learner mathematical proficiencies can be assessed, I have selected to use the Askew (2012) model for this current study.

I have decided to draw on the Askew (2012) model for the purposes of this current study, for the following reasons. Firstly, much of the research undertaken by Askew (2012) has been formulated and developed with the unique challenges of the South African education landscape in mind. The implications are that the Askew (2012) model has been contextualized to fit the specific needs of the challenges within the South African context. Secondly, the two-week Mental Starters Assessment Project (MSAP) intervention program used in this study, as well as the assessment tools for the intervention was developed by Askew in collaboration with researchers from the University of Witwatersrand, Rhodes University, Department of Basic Education (DBE) and Provincial Education Departments (Venkat, Graven, Morrison, Vale, Askew, Bowie, Westaway, Chikiwa, Hlokonya, Chetty and Mathebe, 2021). The implications are that the MSAP intervention program, along with the assessment tools with its particular focus on the three types of calculation strategies of *Fluency, Strategic Calculations and Strategic Thinking,* was formulated and developed by the same team members that worked on the formulation of the intervention program. Lastly, the MSAP intervention program has been tried and re-evaluated through numerous iterations. I will use the approach set up in the MSAP intervention to underpin my study.

The Askew (2012) model for mathematic proficiency contains four proficiencies: fluency, *reasoning, problem-solving and understanding*. According to Askew (2012), *Fluency* refers to a learner's knowledge of procedures, implying the capacity to know when and how to employ suitable and appropriate procedures with flexibility and accuracy. According to Askew (2012), this descriptor of fluency incorporates elements of using retrieval strategies acquired through rote memorization and practice. Fluency also speaks to the ability to instantly provide answers to problems. *Reasoning* refers to learners' capacity to think logically, reflect on their thoughts, and then explain and justify their reasoning about the perceived relationships between mathematical concepts (Askew, 2012). *Problem-solving* refers to strategic competence, which entails the ability to formulate mathematical problems and then represent them with suitable and appropriate strategies and by solving them accurately (Askew, 2012). *Understanding* involves the construction of complete and whole mathematics knowledge forms by which the learner can adapt and transfer mathematics principles and thereby make connections between related mathematical concepts; furthermore, it refers to applying previous knowledge to develop new ideas. In addition, Askew (2012) proposes that the fourth proficiency of *Understanding* should be seen as the outcome of doing the other three proficiencies.

The abovementioned reflection discusses each of the four-proficiency proposed by Askew (2012). In the following section, I will reflect on the correlation between the Askew (2012) model and my study in which I utilized the (MSAP) tool to collect data. Moreover, in this way, I will be able to present the applicability of the Askew (2012) model in conjunction with the assessment tool used in my study and how aspects of the theoretical lens inform the assessment tool to provide in-depth insights into learners' proficiencies.

3.5 Correlating Askew's model with the MSAP intervention tool

For the purposes of this study, the above-mentioned reflection on the four proficiencies as proposed by Askew (2012) has been translated into three different types of calculation strategies in the pre-intervention assessment and post-intervention assessment that have been employed in this study (see Table 2 below). The three different types of calculation strategies are *Fluency/Rapid Recall, Strategic Calculating* and S*trategic Thinking* (Venkat et al., 2019). The first proficiency of *Fluency/Rapid Recall* according to the Askew (2012) model, the first proficiency of Fluency/Rapid Recall correlates to the first calculation strategy in the pre and post-test items, *Fluency/Rapid Recall* (Venkat et al., 2021). The second proficiency of the Askew (2012) model, which is Problem-Solving, correlates to the second calculation strategy of the pre and post-test items, which is *Strategic Calculations* (Venkat et al., 2021). The third proficiencies in the Askew (2012) model of *Reasoning* correlate to the third calculation strategy of the pre and post-test items, which is *Strategic Thinking* (Venkat et al., 2021).

Askew (2012) Proficiency Model	***Types of calculation strategies in the (MSAP) pre and post-test items***
Fluency	*Fluency*
Problem Solving	*Strategic calculations*
Reasoning	*Strategic thinking*

Table 2: Correlation between the Askew (2012) Model and the MSAP Assessment Tool (Venkat et al, 2012)

An inference based on this proposal is that learners should be able to display consistent strength throughout different types of problems, ranging from relatively easy to complex problem types. However, although these four proficiencies are deemed to develop in tandem, observation during the data collection process indicated that learners that are weak with *Fluency* items are, in turn, weak in *Strategic Calculations* items and even much weaker in *Strategic Thinking* items. So, for the purposes of this research, I will focus only on Fluency, Strategic Calculations and Strategic thinking. This observation will be discussed at length in the following chapter.

In the above section, I have presented a reflection on the four proficiencies proposed by Askew (2012), with a particular emphasis on three of the proficiencies. I have also included a reflection on some of the relevant aspects regarding the notion of proficiency. In the following section, I will discuss the second theoretical lens of the LFIN.

3.6 Learning Framework in Number (LFIN) framework

The Learning Framework in Number (LFIN) was developed by Wright et al. (2006) as a mathematics recovery program for learners that were lagging behind in the mathematics classroom. However, the LFIN is organized into four main parts: Part A – the Stages of Early Arithmetical Learning (SEAL) and Base-ten Arithmetical Strategies. Part B – Forward Number Word (FNW) and Backward Number Word (BNW) sequencing and Numeral Identification. Part C – which is Structuring Number 1-20. Part D – early multiplication and division. However, for the purposes of the current study, I will only employ Parts A, B and C to establish the finer nuances of learner levels of calculation strategies (see Table 3 below). I will furthermore not incorporate Part D as it pertains to multiplicative reasoning and bears no relevance to the current study. The LFIN and the following elements of the framework are of particular relevance to my study:

Part A	***Part B***	***Part C***
Stages of Early Arithmetical Learning (SEAL)	*Forward number word sequence and number word after*	*Structuring number 1 - 20*
Base ten arithmetical strategies	*Backward number word sequence and number word before*	
	Numeral identification	

Table 3: Part A, B and C of the LFIN.

3.6.1 Part A: Stages of Early Arithmetical Learning (SEAL) and Base-Ten Arithmetical Strategies

Part A of the LFIN contains two primary essential aspects for my study—the Stages of Early Arithmetical Learning (see table 4 below) and the Base-Ten Arithmetical Strategies (see table 4 below). The SEAL sets forth the six progressive stages young learners progress through as their proficiency in number-related work progress.

Part A
Early Arithmetical Strategies
Base-ten Arithmetical Strategies
SEAL Stages:
0 - Emergent Counting
1 - Perceptual Counting
2 - Figurative Counting
3 - Initial Number Sequence
4 - Intermediate Number Sequence
5 - Facile Number Sequence
Levels:
Base-Ten Arithmetical Strategies
1 - Initial Concepts of Ten
2 - Intermediate Concept of Ten
3 - Facile Concept of Ten

Table 4: SEAL stages and Base-ten levels

The model within this aspect of the LFIN is divided into six strategies, and selecting a particular strategy indicates the learner's level of proficiency. The six stages within the model are *Emergent Counting, Perceptual Counting, Figurative Counting, Initial Number Sequence, Intermediate Number Sequence and Facile Number Sequence* (Wright et al., 2006). Emergent Counting is when learners cannot count visible items or they cannot connect numbers words with items. Perceptual Counting is when learners can only count items they can touch as they cannot count screened items. Figurative Counting is when a learners will count from one every time. Initial Number Sequence is when learners can use count on strategies instead of count from one every time. Intermediate Number Sequence this learner can use countdown to strategies to solve problem items. And Facile Number Sequence is when learners use none-count-by-one strategies to solve problem and they can select the most efficient calculating strategies. At the same time, young learners develop fluency in their counting strategies and develop into Stages 4 and 5 of the SEAL; they also begin to understand the Base-Ten Arithmetical system, in that they begin to see ten as constituted of ten ones. Moreover, as they begin to develop base–ten understanding, their counting strategies improves drastically. In the Base-ten, arithmetical aspect of the LFIN, there is a three-stage model of learner understanding. These three stages are the *Initial Concept of Ten, the Intermediate Concept of Ten and the Facile Concept of Ten* (Wright et al., 2006). The *Initial Concept of Ten* is when a learner cannot see ten as a unit of any kind and focuses on the items that comprise a ten. The *Intermediate Concept of Ten* is when a learner sees ten as a unit comprised of ten ones, but it is still dependent on materials to solve problem items. The facile *Concept of Ten* is when a learner can solve Additive Reasoning problems without using materials or representations.

3.6.2 Part B: Forward Number Word Sequence and Number Word After/Backward Number Word Sequence and Number Word Before/Numeral Identification and Recognition

Within Part – B of the LFIN, three aspects are of primary importance, and each of these aspects will be reflected upon in turn. The first two aspects of the LFIN are the Forward Number Word Sequence (FNWS) and Number Word After and the Backward Number Word Sequence (BNWS) and Number Word Before. Each of these two aspects of Part – B of the LFIN has been divided into models containing six learner proficiency levels. These are the Emergent FNWS or BNWS, Initial FNWS or BNWS up to ten, Intermediate FNWS or BNWS up to ten, Facile with FNWS or BNWS up to ten, Facile with FNWS or BNWS up to thirty and lastly, Facile with FNWS or BNWS up to one hundred (Wright et al., 2006). The third aspect of Part – B is Numeral Identification. I think it is best to clarify and draw a distinction between numeral identification and numeral recognition, as these two terms have to be used with precision.

According to Young-Loveridge (2011), numerals are the labels we use to identify and distinguish different quantities. Furthermore, according to Wright et al. (2006), numeral identification speaks to a learner's ability to state the name of a displayed numeral. Numeral recognition, on the other hand, according to Wright et al. (2006), is a learner's ability to select a named numeral from a randomly arranged group of displayed numerals. As such, numeral recognition and identification are essential abilities young learners have to acquire early on in their learning trajectories, as without them, they will not be able to function mathematically. These numeral recognition and identification skills are developed as learners are exposed to ever-increasing number ranges. Only through practice and social interaction can these complex concepts be mastered.

3.6.3 Part – C Structuring Number 1-20

As young learners commence their formal education, they initially employ count-by-one strategies to solve problems. Later on, they develop more effective and more efficient calculating strategies. This includes recognizing and understanding that $5 + 6 = 11$. They can know that $5 + 5 = 10 + 1 = 11$. Meaning that they are beginning to see the structure of numbers. According to Wright et al. (2012), structuring number relates to learners seeing that numbers are constructed of smaller parts, and by organizing these numbers in accessible ways, they can solve problems easier. When learners have the ability to see that numbers are constituted of smaller parts, they will no longer need to count in ones, as they will see combinations of numbers and relate that knowledge to other knowledge forms to solve problems. With this ability, learners can see the part-whole construction of numbers, which is a major developmental step for them. Learners who do not have this ability are greatly handicapped, as they will continue to use counting in one's strategies to solve mathematical problems.

3.6.4 Correlation between demarcation for the MSAP and LFIN

In the above sections, I have discussed the correlation between Askew's (2012) four proficiencies and the MSAP calculating strategies employed for this study. In the following section, I will reflect on the correlation between the demarcations between the MSAP and the LFIN. For the purposes of this study and mainly due to the knowledge gaps between learners in this study, I decided to use the following demarcations for the MSAP as I established the criteria for the three attainment bands. For learners that demonstrated -30% to 0% shifts as low-performing learners, for learners that demonstrated 3% to 13% shifts as average-performing learners (see Appendix U). And learners that demonstrated 20% to 33% shifts as strong-performing learners. For the LFIN, I established the following criteria: learners located on SEAL stages 1 and 2 were classified as low performing. Learners in SEAL stage 3 are average-performing learners, and learners in SEAL stages 4 and 5 are strong-performing learners, as depicted in table 5 below.

MSAP	**LFIN**
20% to 33% - strong shifts	SEAL Stages 4 and 5
3% to 13% - average shifts	SEAL Stage 3
-30% to 0% - low shifts	SEAL Stages 1 and 2

Table 5: Correlation of demarcations between the MSAP and the LFIN

3.7 Conclusion

In this chapter, I have first presented a reflection on the Askew (2012) model that contains four proficiencies, namely *Fluency, Reasoning, Problem Solving and Understanding;* however, for the purposes of this study, I will not include the proficiency of *Understanding*. Thereafter I have included a reflection on the pre and post-tests in which the three proficiencies have been translated into three types of calculations, namely *Fluency, Strategic Calculations and Strategic Thinking,* through which learner mathematical proficiencies are assessed. Along with that reflection, I have also included a reflection on mathematical proficiencies and how they develop in young learners. Thereafter I presented a reflection on the LFIN as the second theoretical lens that will enable me to focus more on the finer nuances of leaner proficiencies. Lastly, I showed how I matched the outcomes from the MSAP tests and the assessment of individual learners' performance on the LFIN. In the following section, I will discuss the proposed methodology for this study.

CHAPTER 4 – METHODOLOGY

4.1 Introduction

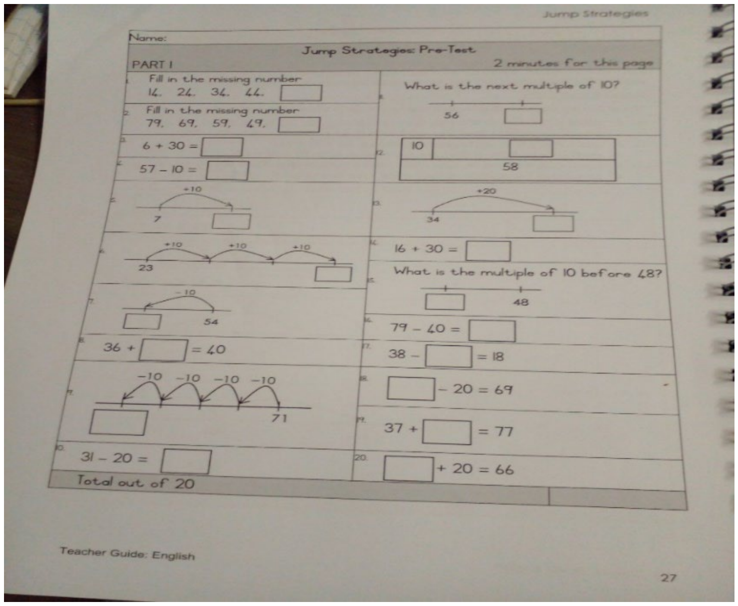
In this chapter, I describe the ways and methods of data collection and analysis. I will first examine my choice for employing a non-traditional mixed methods approach and what it means for this study. Thereafter I will discuss aspects of the target population, the research instrument of a pre and post-test as well as a reflection on the ethical considerations that include rigor validity and the limitations of the study.

4.2 Non-Traditional Mixed methods approach

The first part of the collection and analysis is quantitative, and the second part of the collection and analysis is qualitative. I employed the two methods in the same study as mixed methods approach, combining and integrating quantitative and qualitative methods. Whilst I use these two methods, I use them in a non – traditional sense, as I am using the numbers to shed light on the qualitative aspect of the study George (2023). The integration of both methodologies in the same study is not simply the collection of quantitative and qualitative data, but more so, it is an indication that somewhere along the line, research data will be integrated and mixed to bring the researcher closer to the research problem which is under consideration (Creswell, Fetter & Ivankova, 2004). According to Creswell (2012) and Creswell et al. (2004), the purpose of a mixed methods approach is to provide the researcher with a better understanding of the research problem than either approach on its own. Furthermore, for the purposes of this study, integrating both methodologies call for the collection and analysis of quantitative data and then the collection and analysis of qualitative data sequentially (Stange, Miller, Crabtree, O'Conner & Zyzansky, 1994). Therefore, in this present study, as a non-traditional approach to mixed methods, I will use the numbers from the pre-test and post-test to shed light on the learners' performance of the different types of calculations. I will first apply quantitative methods in the collection and analysis of data. I also make use of number within my LFIN assessment schedule to determine the level of strategic calculations of learners. Based on the numbers' findings, I will apply a qualitative lens to look at the data set, which will be analyzed to bring me close to understanding and answering my research questions.

Although I am using a non- traditional mixed methods approach the disadvantages of a mixed method paradigm is still applicable. A perceived disadvantage of the mixed methods approach is that it requires more work and, thus, more time and, in some instances, more significant financial input (Creswell et al., 2007). Another disadvantage is that the researcher has to acquire the necessary skill and experience to employ both approaches. However, the advantage is that the researcher can use the results of one method to help develop the use of the second method (Greene, Caracelli & Graham, 1989).

In the first phase of my study, which is the quantitative phase, I administered a pre-test (see Appendices A and B) on 42 grade 3 learners, followed by a schedule of 8 intervention lessons (see Appendices C to J) over a 2-week period, which was followed by the administration of a post-test. Pre- and post-test (see appendices A and B) will comprise the same test items. Based on the findings of the quantitative approach, after the completion of the two interventions, I proceeded with the second phase of data collecting, in which I proceeded to select six learners, two learners from each attainment band, two strong, two average and two low performing learners on whom I conducted 15- minute interviews based on selected items extracted from the pre and post-test. As such, the quantitative methods fed into the qualitative methods, which afford me the opportunity to gain better insights into the finer nuances of the phenomena I was seeking to investigate, which was grade three learner’s additive reasoning strategies after an intervention that sought to enhance the development of mental math calculations and identify them on the LFIN test.


Jump Strategies

Name:

Jump Strategies: Pre-Test

PART I — 2 minutes for this page

1. Fill in the missing number 14, 24, 34, 44, ☐	11. What is the next multiple of 10? 56 ☐
2. Fill in the missing number 79, 69, 59, 49, ☐	
3. 6 + 30 = ☐	12. 10 ☐ 58
4. 57 – 10 = ☐	
5. +10 7 ☐	13. +20 34 ☐
6. +10 +10 +10 23 ☐	14. 16 + 30 = ☐
	15. What is the multiple of 10 before 48? ☐ 48
7. –10 ☐ 54	16. 79 – 40 = ☐
8. 36 + ☐ = 40	17. 38 – ☐ = 18
9. –10 –10 –10 –10 ☐ 71	18. ☐ – 20 = 69
	19. 37 + ☐ = 77
10. 31 – 20 = ☐	20. ☐ + 20 = 66
Total out of 20	

Teacher Guide: English

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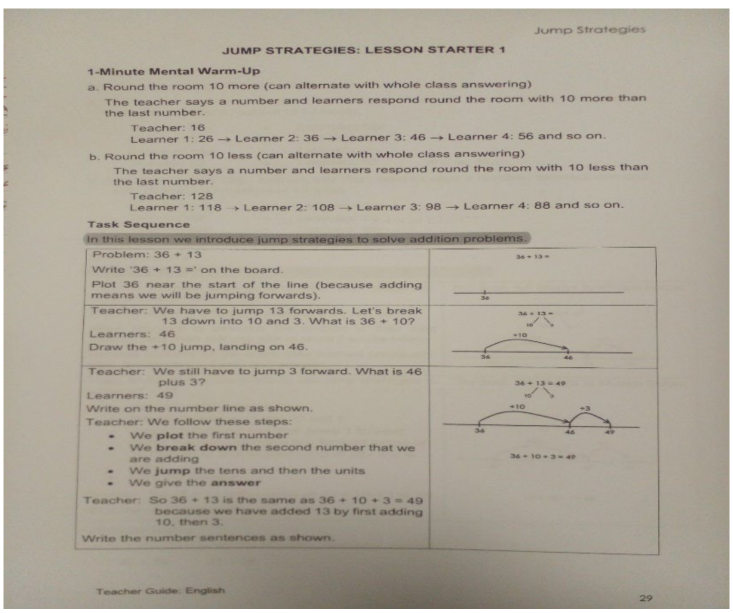
Figure 4: MSAP Pre and Post-Test


Jump Strategies

JUMP STRATEGIES: LESSON STARTER 1

1-Minute Mental Warm-Up

a. Round the room 10 more (can alternate with whole class answering)

The teacher says a number and learners respond round the room with 10 more than the last number.

Teacher: 16
Learner 1: 26 → Learner 2: 36 → Learner 3: 46 → Learner 4: 56 and so on.

b. Round the room 10 less (can alternate with whole class answering)

The teacher says a number and learners respond round the room with 10 less than the last number.

Teacher: 128
Learner 1: 118 → Learner 2: 108 → Learner 3: 98 → Learner 4: 88 and so on.

Task Sequence

In this lesson we introduce jump strategies to solve addition problems.

Problem: 36 + 13 Write '36 + 13 =' on the board. Plot 36 near the start of the line (because adding means we will be jumping forwards).	36 + 13 = 36
Teacher: We have to jump 13 forwards. Let's break 13 down into 10 and 3. What is 36 + 10? Learners: 46 Draw the +10 jump, landing on 46.	36 + 13 = 10 3 +10 36 46
Teacher: We still have to jump 3 forward. What is 46 plus 3? Learners: 49 Write on the number line as shown. Teacher: We follow these steps: • We **plot** the first number • We **break down** the second number that we are adding • We **jump** the tens and then the units • We give the **answer** Teacher: So 36 + 13 is the same as 36 + 10 + 3 = 49 because we have added 13 by first adding 10, then 3. Write the number sentences as shown.	36 + 13 = 49 10 3 +10 +3 36 46 49 36 + 10 + 3 = 49

Teacher Guide: English

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Figure 5: MSAP Lesson 1

4.3 Research Design

My study utilized both experimental and exploratory styles of research. It is exploratory in that it sought to gain new insights into the phenomenon under consideration (Akhtar, 2016). And it was experimental in that it sought to test a causal relationship under controlled situations. It should be noted that an experiment is an observation that takes place under controlled conditions. It can also be further stated that an experimental design necessitates that some of the variables that are being studied are manipulated. Seeing that the experiment is taking place under controlled conditions it meant that for reliability and neutrality the conditions should not be allowed to change while the experiment is taking place (Akhtar, 2016). This chosen design aided me in determining if the intervention on the enhancement of grade three learners' mental math proficiencies enhanced the conceptual development of the mathematical concept of additive reasoning and also to evaluate if learners developed fluency in their mental mathematics ability.

4.4 Target Population

Sampling in field research involves the selection of a research site, people, time and events (Burgess, 1982). The selected participants for my study are from one class of grade 3 learners comprised of 20 girls and 22 boys between the ages of 8-10 years old in a formerly disadvantaged Quintile 1 school located in the South of Johannesburg. The Minister of Education has identified quintile 1 schools as the poorest in a region and Quintile 5 as the least poor. The school is situated in a low socio-economic context with several social challenges. Because I employed a blended approach in a sequential manner, starting with quantitative and then qualitative methodology, it speaks to the fact that I used non-probability sampling methods. Non-probability sampling methods, according to Honigmann (1982), are the logical methods for qualitative research as long as the investigator expects mainly to use his data to answer qualitative problems, such as discovering what is occurring and the implications of what is occurring and not to answer *how much* or *how often* questions. I will not use my findings to generalize the phenomena over a broader population, as with probability sampling. The language of instruction at the school is English, and the test was also conducted in English. A large percentage of the learners are from bilingual homes in which English and Afrikaans are the preferred languages. While a smaller percentage of learners' home language is other than English or Afrikaans, they were comfortable conducting the test in English. I have selected grade 3 as it is the final year of the Foundation Phase in which South African learners, according to the CAPS document, are expected and required to make the necessary shifts from concrete forms of counting into more efficient and effective calculating strategies*"…learners need to exit the Foundation Phase with a secure* ***number sense*** *and operational fluency…Foundation Phase Mathematics forges the link between the child's pre-school life…on the one hand, and the abstract mathematics of the later grades on the other hand."* (DBE, 2011:12). I have furthermore also selected grade 3 as it fits well with my interest in Foundation Phase school mathematics.

4.5 Research Instrument

The selected research instrument for this study was a pre, and post-test, a schedule of 8 intervention lessons and 6 interviews that was conducted on completion of the intervention, and these were video recorded for further analysis. Furthermore, the research instrument comprises a combination of 15 addition and 15 subtraction items in the mid to high number range. The representations employed in the research instrument are number sentences, part-part-whole diagrams, and unstructured number lines. Appendices A and B were developed and designed through a collaborative process by Wits Math's Connect-Primary Project members. Each intervention lesson starts with a five-minute mental warm-up exercise, stimulating cognitive function and conceptual understanding. The initial 4 lessons focused on addition, and the subsequent set of 4 lessons focused on subtraction. Each lesson was developed to progress from easy to more complex. After the completion of each lesson, learners had to complete a short exercise comprised of two to three items. These exercises were used as ongoing assessments of learner take up of the concepts. After completion of the post-test, I selected 6 learners across the attainment band, 2 strong, 2 average and 2 low-attaining learners and interviewed each of them, in turn, to ascertain their reasoning when solving test items. The 6 interviews were based on an adapted version of the LFIN interview schedule (see Appendices M to P), which shed light on the learner's reasoning in solving the selected problem items. These interviews did not exceed 15 minutes and served to aid me in answering the third research question of my study.

4.6 Pre and Post-Test

Utilizing tests (pre and post-tests) as my primary research instrument aided me in assessing whether or not the 42 grade 3 learners, on completion of the intervention, made any gains in relation to additive reasoning. Both pre and post-tests was comprised of the same problem items and were administered to the same sample group. Bertram and Christiansen (2014) assert that researchers often test learners before an intervention to ascertain their achievement and then re-test them again after the intervention. This method shifts the focus away from comparing learners against a norm or standard towards understanding how they can reason in various additive situations. Cohen, Manion, and Morrison (2011) maintain that using a pre-test before an intervention program helps identify starting abilities and achievements in students and adds value to teaching and learning.

4.7 Intervention Lessons

A schedule of 8 intervention lessons focusing on additive reasoning were utilized (see Appendices C to J). Each of the 8 lessons were approximately 20 minutes long, spread over a 2-week period with 1 lesson per day. A typical lesson comprises a 2 -minute mental warm-up that involved the full participation of each learner. The mental warm-up requires that learners shout out the next multiple of 10 on or off the decade; it also required learners to shout out the closest multiple of 10 to any given number. It is a quick-thinking and quickly responsive strategy that stimulates learners' thinking and reinforces conceptual development. Thereafter the lesson of the day was presented to the learners with the use of the board, and learners were taken slowly through the steps of solving a given number problem. An example of this type of problem is 34 + 27 = □. Learners were taught the steps of working on the empty number line as prescribed in the teacher's manual. Upon completion of the lesson, learners were given a sheet containing 2 to 3 number problems to solve independently. During this exercise, I walked amongst the learners to observe and assist them where needed. These sheets were collected, and learner uptake of concepts were assessed based on their work. The first four lessons dealt exclusively with addition, and the last 4 lessons dealt exclusively with subtraction problems, both with increased complexity.

4.8 Data Collection and Analysis

Before the commencement of data collection, I met with the school principal, the grade 3 teacher and the 42 learners explaining my research and the process to be undertaken. And having gained informed formal consent from the Gauteng Department of Education (GDE), the school principal, the teacher, the learners, and the parents, I commenced the data collection process. This study's primary and central data collection tool was a structured instrument (pre-test and post-test see figure 6 below). administered face-to-face. The pre-test was administered on the first day of the 2-week intervention to establish a baseline of the learner's levels of fluency. On completing the 2-week intervention that focused on enhancing mental math, I administered the post-test, which comprised the same test items as in the pre-test. This part of the data collection describes the quantitative aspects of my study, as its purpose was to establish if a class of grade 3 learners made any gains after a 2-week intervention on the development of mental math. This data was entered into an excel spreadsheet for data analysis to generate learner shifts from pre to post-test (see figure Q). The second part of the data collection process is the qualitative approach that was based on the findings of the analysis gained from the quantitative aspect of the study. In the second part of the data collection process, to fit the rationale for my study, I selected 6 learners from the sample of 42 to conduct open-ended semi-structured interviews employing an adapted version of the LFIN interview schedule (see figure 7 below) to ascertain grade 3 learners reasoning when attempting to solve additive reasoning problems as well as gain insight into the strategies they selected when attempting to solve these test items. The 6 learners were selected based on the following criteria: 2 strong, 2 average and 2 low-performing learners. These 6 interviews were videotaped for subsequent analysis. Part of the data collected in the study that supported my initial findings was the daily individual classroom exercises learners had to do after each lesson each day.

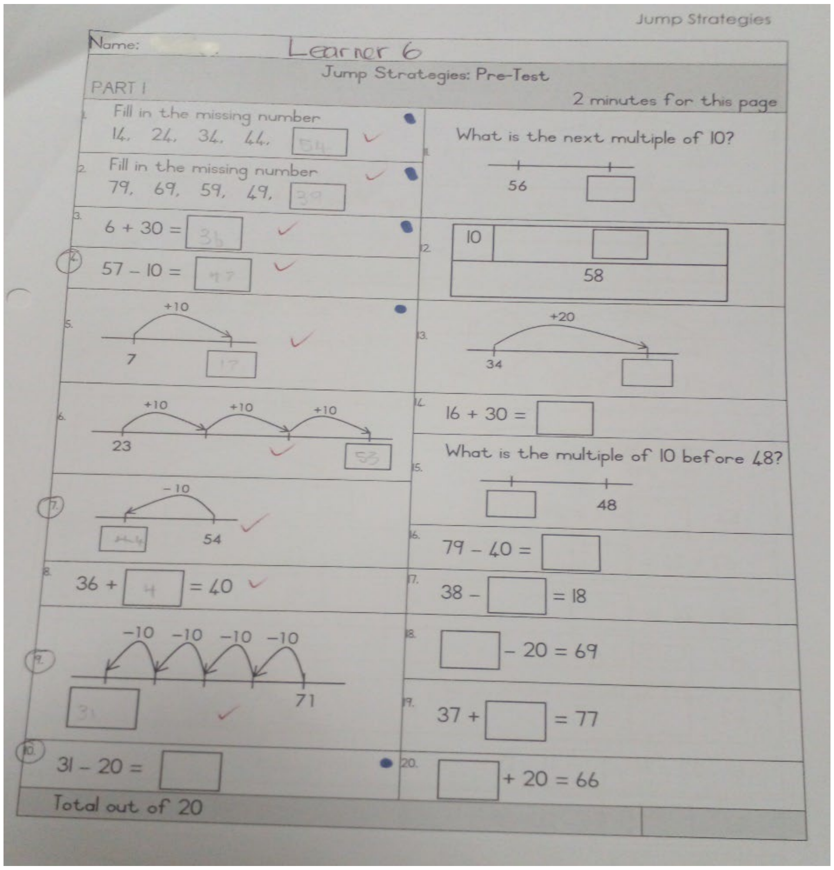
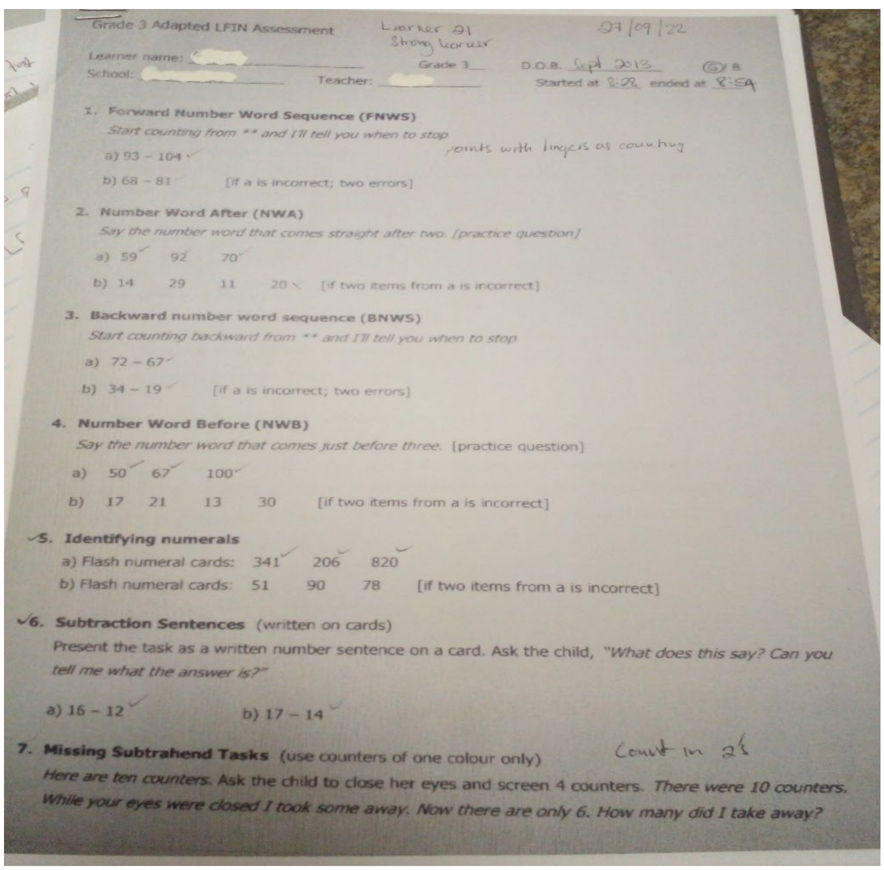


Figure 6: Learner 6 MSAP Pre-test data collection tool

As the interviews were video recorded, I positioned the camera as close as possible to the learners to see what they were writing but not too close so I could see their facial expressions and body language, as these body gestures could potentially provide valuable insights into their reasoning and strategy selection processes. The advantage of video recording was that it captured every detail that is taking place during the interview, and as such, it provided me with a facility to revisit aspects that were of relevance in the pursuit of the answers sought. Another advantage is that it allows the researcher the time to make considerable judgement calls which can be revisited at will. A weakness of video recording is that because you deal with the greater sensory perception, you can spend hours watching learner actions and gestures that have little relevance to your study (Pirie, 1996). As such, much experience in the use of video recording of interviews is essential.

The data analysis process for the quantitative part of the study involved marking and establishing who the strong, average, and low-performing students were.

Analyzing the qualitative aspects of the data involves developing initial insights from the pre and post-tests and classroom exercises. Thereafter interpreting learners' work and drawing implications from it. Parts of the video-recorded interviews was transcribed, in order to extract and describe initial insights on their overall meanings. Thereafter the sample learners' problem-solving strategies and models were analyzed and compared using the Learning Framework in Number (LFIN) by Wright et al. (2006). The LFIN encapsulates possible stages and levels of number learning that young learners progress through as they develop number knowledge, as discussed in Chapter 3.


Grade 3 Adapted LFIN Assessment Learner 21 Strong learner 27/09/22

Learner name: ____ Grade 3 D.O.B. Sept 2013 G/B

School: ____ Teacher: ____ Started at 8:28 ended at 8:59

1. **Forward Number Word Sequence (FNWS)**
   *Start counting from ** and I'll tell you when to stop* points with fingers as counting
   a) 93 – 104
   b) 68 – 81 [if a is incorrect; two errors]
2. **Number Word After (NWA)**
   Say the number word that comes straight after two. [practice question]
   a) 59 92 70
   b) 14 29 11 20 [if two items from a is incorrect]
3. **Backward number word sequence (BNWS)**
   *Start counting backward from ** and I'll tell you when to stop*
   a) 72 – 67
   b) 34 – 19 [if a is incorrect; two errors]
4. **Number Word Before (NWB)**
   Say the number word that comes just before three. [practice question]
   a) 50 67 100
   b) 17 21 13 30 [if two items from a is incorrect]
5. **Identifying numerals**
   a) Flash numeral cards: 341 206 820
   b) Flash numeral cards: 51 90 78 [if two items from a is incorrect]
6. **Subtraction Sentences** (written on cards)
   Present the task as a written number sentence on a card. Ask the child, *"What does this say? Can you tell me what the answer is?"*
   a) 16 – 12 b) 17 – 14
7. **Missing Subtrahend Tasks** (use counters of one colour only) Count in 2's
   Here are ten counters. Ask the child to close her eyes and screen 4 counters. *There were 10 counters. While your eyes were closed I took some away. Now there are only 6. How many did I take away?*

Figure 7: LFIN interview schedule of learner 21

																				PRE-TEST																						
	PART ONE																				PART TWO										PART ON			PART TW			TOTAL			ch	OVERALL	%
Learner	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	✓	✗	bla	✓	✗	bla	✓	✗	blank			
1	1	1	1	1	1	1															1		0	0	0						6	0	14	1	3	6	7	3	20	30	23.3333	
2	1	1	1	1	1	1															1		0								6	0	14	1	1	8	7	1	22	30	23.3333	
3	1	1	0	1	0	0															1	0	0								3	3	14	1	2	7	4	5	21	30	13.3333	Average
5	1	1	1	1	1	0															1	0	0								5	1	14	1	2	7	6	3	21	30	20	
6	1	1	1	1	1	1	1	1	1												1	1	1								9	0	11	3	0	7	12	0	18	30	40	
8	1	1	1	1	0	0															0	0	0	0	1						4	2	14	1	4	5	5	6	19	30	16.6667	
9	0	1	1	0	0																1	0	0		0						2	3	15	1	3	6	3	6	21	30	10	
10	1	1	1	0	1		0														0	1	0								4	2	14	1	2	7	5	4	21	30	16.6667	
11	1	1	1	0	0	0	1	0													0	0	0	0	0	0	0	0			4	4	12	0	8	2	4	12	14	30	13.3333	
12	1	1	0	1	1	0		0													0	0	0	0							4	3	13	0	4	6	4	7	19	30	13.3333	
13	0	0	0	0	0																0	0	0	0	0	0	0	0	0	0	0	5	15	0	10	0	0	15	15	30	0	Low
15	1	1	1	1	1	1	1	1													1	1	0	0	1	0					8	0	12	3	3	4	11	3	16	30	36.6667	Low
16	0	1	1	0	0																0	0	0	0	0						2	3	15	0	5	5	2	8	20	30	6.66667	
17	1	1	1	1	1	1	0	0													0	1	0								6	2	12	1	2	7	7	4	19	30	23.3333	Strong
18	1	1	0	1	1	0															0	0	0								4	2	14	0	3	7	4	5	21	30	13.3333	
19	0	0	0	0	0	0	0	0	0												0	0	0	0	0	0	0	0	0	0	0	9	11	0	10	0	0	19	11	30	0	
21	1	1	1	1	1	1	0	1	0	1	0		1	1							1	0	1								10	3	7	2	1	7	12	4	14	30	40	Strong
22	1	1	1																		0	0	0								3	0	17	0	3	7	3	3	24	30	10	
23	1	1	0	1	0	0															0	0	0	0	1						3	3	14	1	4	5	4	7	19	30	13.3333	
24	1	0	0	1	0	0	0	0	0	0											0	0	0	0	0	0	0	0			2	8	10	0	8	2	2	16	12	30	6.66667	
25	1	1	0	0	0	0	0	0	0	0	0	0	0	0							0	0	0	0	0	0	0	0	0	0	2	12	6	0	10	0	2	22	6	30	6.66667	
26	1	1	1	1	1																1	0	0								5	0	15	1	2	7	6	2	22	30	20	
27	1	1	1	0	1	1	1	1													1	1			0	0					7	1	12	2	2	6	9	3	18	30	30	
28	1	1	1	1	1	0															1	0	0	0	1						5	1	14	2	3	5	7	4	19	30	23.3333	
29	1	1	1	0	1	1	0	0	0	0											1	1	0	0	0	0	0	0			5	5	10	2	6	2	7	11	12	30	23.3333	
31	0	0	0	0	0																0	0	0	0	0	0	0	0			0	5	15	0	8	2	0	13	17	30	0	
32	1	0	1	0																	0	0									2	2	16	0	2	8	2	4	24	30	6.66667	Average
33	1	1	1	1	1	1	1	1	1	1	0	0	1	1							1	1	0	0	0						12	2	6	2	3	5	14	5	11	30	46.6667	
34	1	1	1	1	1																1	0									5	0	15	1	1	8	6	1	23	30	20	
37	1	1	1	1	1	1	0	1	0	1	0	1	1	1							1	0			1		0	0	0	0	11	3	6	2	5	3	13	8	9	30	43.3333	
38	1	1	1	1	1	0	1	1	1	1	0	1	1	1	0						1	1	0	0	1	0	0	0	0	0	12	3	5	3	7	0	15	10	5	30	50	
39	1	1	1	1	1	1	0	1													1	1	0	0							7	1	12	2	2	6	9	3	18	30	30	
40	0	0	1	0																	0	0	0	0	0	0	0	0	0	0	1	3	16	0	10	0	1	13	16	30	3.33333	
41	1	1	1	1	0	0	0	1	0	0	0										0	0	0	0	1	0	0	0			5	6	9	1	7	2	6	13	11	30	20	
42	0	1	1																		0	0	0	0	0						2	1	17	0	5	5	2	6	22	30	6.66667	
	28	29	26	21	19	11	6	9	3	4	0	2	4	4	0	0	0	0	0	0	17	9	2	0	7	0	0	0	0	0												

Figure 8: MSAP Pre-test data analysis spreadsheet

																			POST-TEST																							
	PART ONE																				PART TWO										PART ON			PART TW			TOTAL			ch	OVERALL	L %
Learner	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	✓	✗	bla	✓	✗	bla	✓	✗	blank			
1	1	1	1	1	1	0	1	0	1	1	0	0									1	1	0	0	0	0	0	0	0	0	8	4	8	2	8	0	10	12	8	30	33.333	
2	1	1	1	1	1	0	1	1	1	1	0										0	1		0	1	0	0	0	0	0	9	2	9	2	7	1	11	9	10	30	36.667	
3	1	1	1	0	1	1	0		0												0	0	0	0	0						5	3	12	0	5	5	5	8	17	30	16.667	Average
5	1	1	1	1	1	0	1				0		1								0	0			0	0					7	2	11	0	4	6	7	6	17	30	23.333	
6	1	1	1	1	1	0	1	1	0	0	0	0	0								0	1	0	1	0	0	0	0	0	0	7	6	7	2	8	0	9	14	7	30	30	
8	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	14	6	0	1	9	0	15	15	0	30	50	
9	1	1	1	1	1	1	1	1	1	1	0										1	1			1	0	0				10	1	9	3	2	5	13	3	14	30	43.333	
10	1	1	1	0	1	1	1	1	1	1		0									0	1	0	0	0	1	0				9	2	9	2	5	3	11	7	12	30	36.667	
11	1	1	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	15	0	0	10	0	5	25	0	30	16.667	
12	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0					0	1	0	0	0	0	0	0	0	0	12	4	4	1	9	0	13	13	4	30	43.333	
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	10	0	0	30	0	30	0	Low
15	0	1	1																		0										2	1	17	0	1	9	2	2	26	30	6.6667	Low
16	0	1	1	1	1	1	1	1	1	0		0	1	1							1	1	0	0	0	0					10	3	7	2	4	4	12	7	11	30	40	
17	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0				0	1	0	0	0	0	0	0	0		13	4	3	1	8	1	14	12	4	30	46.667	Strong
18	0	0	0	0	0	0															0	0	0	0							0	6	14	0	4	6	0	10	20	30	0	
19	0	0	1	0	0	0	0	0	0	0	0										0	0	0	0	0	0	0	0	0	0	1	10	9	0	10	0	1	20	9	30	3.3333	
21	1	1	1	1	1	1	1	1	1	1	0		1	1	0	1	1	0	1	1	1	1		0	1	1		1	0	0	16	3	1	5	3	2	21	6	3	30	70	Strong
22	1	1	1	0	1	1	1	0													1	1	0	0	1						6	2	12	3	2	5	9	4	17	30	30	
23	0	1	1	0	1	0	0	0	0												0	0	0	0	0	0	0	0	0	0	3	6	11	0	10	0	3	16	11	30	10	
24	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	17	0	0	10	0	3	27	0	30	10	
25	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0					0	0	0	0	0	0	0	0	0	0	4	12	4	0	10	0	4	22	4	30	13.333	
26	1	1	1	0	1	0	1	0													1	0	1	1							5	3	12	3	1	6	8	4	18	30	26.667	
27	1	1	1	1	1	1	0	1													0	0	0	0	1	0					7	1	12	1	5	4	8	6	16	30	26.667	
28	0	1	1																		0										2	1	17	0	1	9	2	2	26	30	6.6667	
29	1	1	0	0	0	0					0	0	0	0							0	0	0	0	0	0	0	0	0	0	2	8	10	0	10	0	2	18	10	30	6.6667	
31	0	0	0	0	0	0	0	0	0	0											0	0	0	0	0	0	0	0	0	0	0	10	10	0	10	0	0	20	10	30	0	
32	1	1	1	1																	0	0	0	0	0						4	0	16	0	5	5	4	5	21	30	13.333	Average
33	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	17	3	0	1	9	0	18	12	0	30	60	
34	1	1	1	0	1	1	0														0	0	0	0	0	0	0	0	0	0	5	2	13	0	10	0	5	12	13	30	16.667	
37	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	1	1	0	0	0		1	0	0	0	17	3	0	3	6	1	20	9	1	30	66.667	
38	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	0	0	1	0	1	0	0	1	1	1	0			15	5	0	4	4	2	19	9	2	30	63.333	
39	1	1	1	1	1	1	1														0	1	0	0	0						7	0	13	1	4	5	8	4	18	30	26.667	
40	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	19	0	0	10	0	1	29	0	30	3.3333	
41	1	1	1	1	1	1	0	1	1	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	8	0	0	10	0	12	18	0	30	40	
42	0	0	0	0																	0	0	0	0	0	0	0	0	0	0	0	4	16	0	10	0	0	14	16	30	0	
	25	28	29	17	22	17	19	14	14	12	1	3	10	9	0	6	4	1	4	3	8	14	1	2	6	3	2	1	0	0												

Figure 9: MSAP Post-Test data analysis spreadsheet

4.9 Rigor

Reliability in one's research is vital in that it ensures confidence and credibility. Reliability is synonymous with consistency and replicability over time (Lowhorn, 2007 & Cohen et al., 2011). However, using the test/re-test method, care should be taken regarding the timing between the tests since situational factors may occur, and participants may tend to remember the questions in the test (Cohen et al., 2011). This, in addition to questions being ambiguous, unstandardized tests and participants feeling uneasy or misinterpreting questions, could potentially affect the reliability of the test (Creswell, 2012). To resolve these issues, tests will be conducted two weeks apart so that learners are not too familiar with the questions; clear instructions will be provided, and the same questions for both tests will be used.

In terms of ***validity*** – the data itself does not ensure validity. Rather validity relies on the relationship between the claim and the data-gathering process (Opie, 2004). To ensure validity, I have chosen a sample group of forty-two grade three learners, which I believe is sufficient. Additionally, the level of the tests also needs to be appropriate, non-discriminatory, non-biased and neither too long nor short (Cohen et al., 2011). Items in the tests will also be clearly legible and vary in cognitive demand level, including low and high number ranges.

4.10 Ethical Considerations

The ethical processes were followed, and ethics clearance was granted for my study before I entered the school (see appendices R, S, T and U). Ethical considerations for this study are that informed written consent forms were given to the participants (principal, parents and learners) and received back from them. A brief containing all relevant information were given and discussed with the teacher and the participants involved in the study. Data collected were treated with confidentiality and will not be discussed or used outside of the purposes of this study, and the anonymity of participants and the school were maintained, as different names will be given to participants and the school. The teacher and all participants received feedback on the study. Moreover, data collected was only utilised for the study; it was stored in a safe location and kept in a secure cloud for further research.

4.11 Limitations

Time: Part of the study was conducted over a 2-week period, which may not be considered sufficient time to see significant gains, making it difficult to generalize. A comprehensive understanding of the nature of additive relations cannot be made from a small-scale study; thus, more research and evidence will be needed before strong claims can be made. Although this study is on a small scale, it can provide valuable insight that can provide motivation for a longitudinal study later on with the entire Foundation Phase in one school. *Small sample size:* The limitation of a qualitative method is that it focuses on meaning and experiences rather than on contextual sensitivities. Smaller sample sizes raise the issues of generalizability to the whole research population, and the results cannot be generalized to the larger population (Flick, 2011).

4.12 Conclusion

In this chapter, I have provided a description of the proposed research design for this study, along with the reasons for this choice. I have also described the target population and the reasons for selecting this sample group. Furthermore, I have included a description of the research instrument, the data collection process, and the data analysis. The chapter concluded with a discussion of some of the ethical considerations and the limitations thereof. In the following chapter, I will present the findings of the analysis of the data. The process of presenting the findings will be conducted in the following manner. I will first present the findings of the MSAP intervention, thereafter, the findings of the LFIN and the placement of the six learners on the SEAL stages. I will conclude with a reflection on the correlation between the two assessments.

CHAPTER 5 – FINDINGS AND ANALYSIS

5.1 Introduction

The process of presenting the findings and reflecting on the analysis of the study will be conducted in the following manner. I will first present the findings of the analysis of the quantitative aspects of the study, which will focus on the MSAP intervention using the Jump Strategies. Thereafter I will present the findings of the analysis of the qualitative aspects of the study, which will focus on the LFIN and the assignment of the 6 selected learners into the 5 different SEAL stages. An analysis of the relationship between the MSAP intervention and the LFIN will follow. The presentation of the findings and analysis thereof will respond to the three research questions that are under consideration in this study. The three research questions I will endeavor to answer are:

1. *What were, if possible, the shifts from pre to post-test when using the MSAP (Jump Strategy) interventions?*
2. *What was the profile of the 6 selected learners after the LFIN interview was administered on them?*
3. *What is the correlation between the MSAP (Jump Strategies) and the LFIN, if any?*

In presenting the findings of the quantitative analysis, the following should be noted. Firstly, of the original sample size of 42 learners, I could only match the pre- to post-data of 35 learners, as 7 learners were absent either on the day of administering the pre or the post-test. Secondly, of the 30 items in the test, there is an even balance of 15 addition and 15 subtraction tasks.

5.2 Research questions:

5.2.1 Research Question 1: What were, if possible, the shifts from pre to post-test when using the MSAP (Jump Strategy) interventions?

I will present the findings of the analysis of the MSAP intervention in the following manner firstly, of learner gains from pre to post-test; secondly, of gains made across the three proficiencies; thirdly, the top eight performing learners; fourthly, the thirteen lowest-performing learners and lastly of overall gains made from pre to post-test. I will conclude by presenting a summary of the findings.

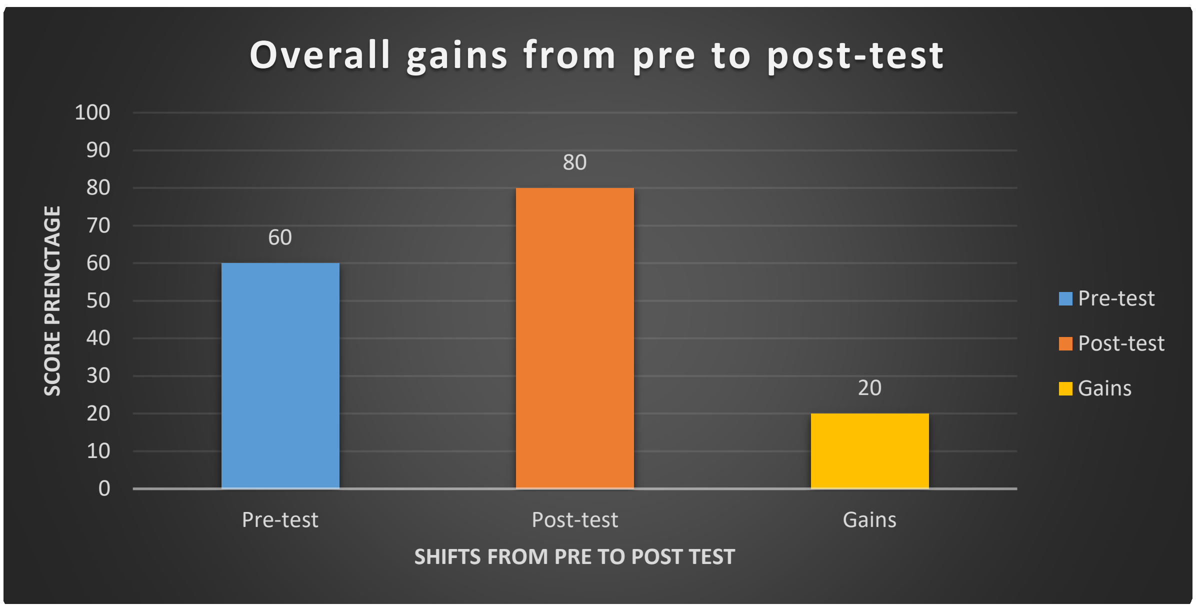


Figure 10: Overall gains from pre to post-test

Figure 10 above examines the overall gains made from the pre to post-test. We have the pre-and post-test results on the horizontal axis and the average percentage gains on the vertical axis. The pre-test showed that the learners had achieved a 60% average. After eight intervention lessons, learners were given a post-test. The post-test results were 80 %. Firstly, the learner gains analysis findings indicate an overall average improvement of 20 % in learner performance from pre to post-test. Furthermore, these overall average gains of 20% in learner performance can further be broken down into the three types of calculations the MSAP intends to assess: Fluencies, *Strategic Calculations and Strategic Thinking* (Askew, 2012).

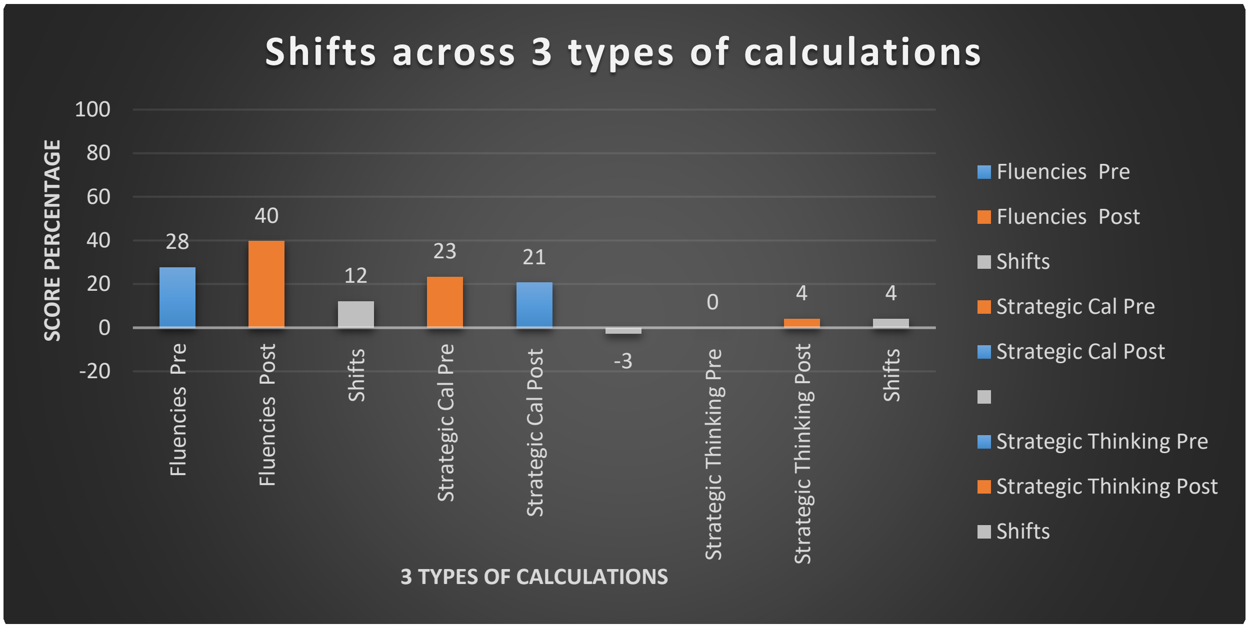


Figure 11: Learner average gains made across the 3 types of calculations.

In figure 11 above, we can see learner gains across two types of calculation categories. On the horizontal axis, we see the three calculation types with their performance depicted and the average percentage gains on the vertical axis. In the *Fluencies (Questions 1 to 20)* category, there was an overall learner increase of 12% from 28% to 40%% from pre to post-test, which is the category with the most gains made by learners. In the Strategic Calculations category, there was a decrease of 3% from 23% to 21% from pre to post-test. This finding confirms the work of Graven and Venkat (2018). In most of their samples, they noticed that learners increased with Fluencies while decreasing in performance in Strategic Calculating and Strategic Thinking (*Questions 21 -30*). However, in this study, learner performance increased in the Strategic Thinking category by 4% from 0% to 4% from pre to post, and this finding is different to the work of Graven and Venkat (2018).

The decrease in learner performance in Strategic Calculating is understandable as it is considered more difficult for learners.

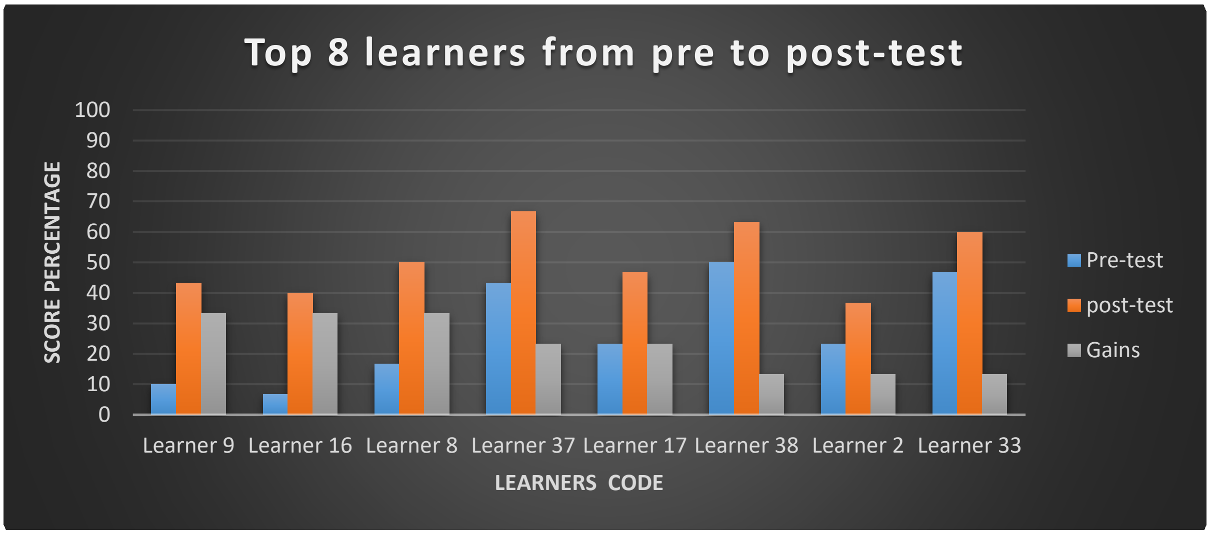


Figure 12: Top 8 learners' gains from pre to post-test.

Furthermore, in figure 12 above, the top 8 performing learners demonstrated small to significant improvement between pre to post-test. The average percentages are on the vertical axis, and the 8 top performing learners are on the horizontal axis. Learners 9,16, and 8 improved each by 33% from pre to post-test. While learners 37 and 17 each demonstrated improvements of 23%. Learners 38, 2 and 33 demonstrated the lowest improvement of 13% from the learners in the top-performing band. For this project, learners 9, 16 and 8 made the most significant gains from the pre to post-test.

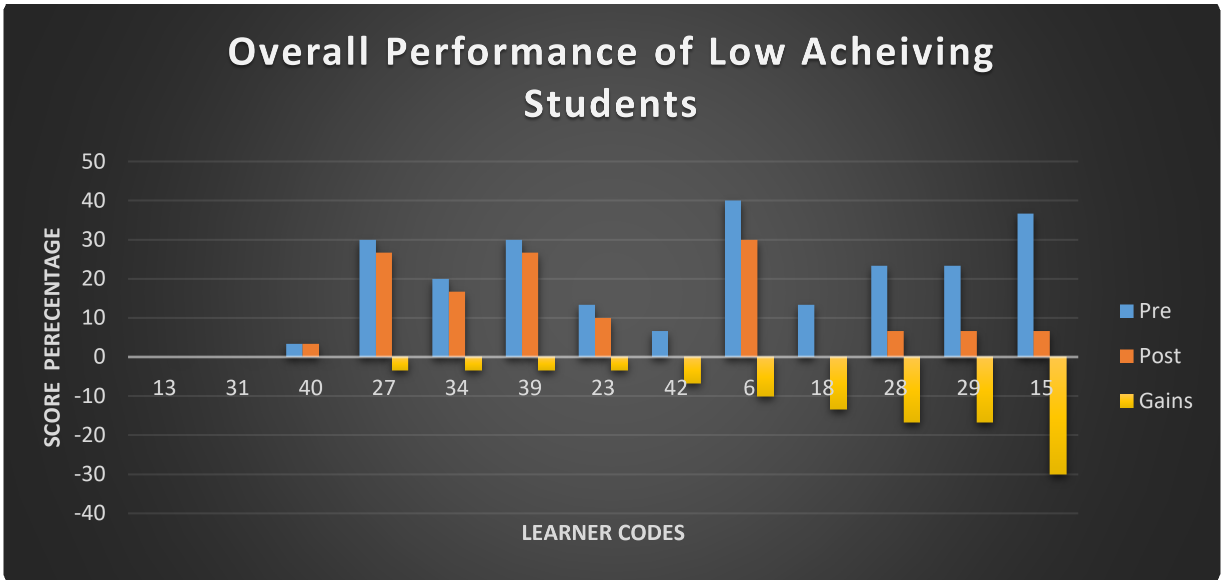


Figure 13: Thirteen lowest performing learners from pre to post-test

In figure 13 above, we have the thirteen lowest-performing learners on the horizontal axis; we have the average percentages on the vertical axis. The thirteen lowest-performing learners demonstrated no shifts from the pre-test to the post-test (Learners 13,31,40) or decreased from pre-test to post-test (Learners 27,34,39,23,42,6,18,28,29,15). What came into focus for me was that the learners with the lowest gains were learners 28, 29 and 15.

Although findings of the analysis indicate an overall improvement of 20% in learner performance from pre to post-test, the following anomalies were also noted. Firstly, 3 test items (15, 29 and 30) showed no improvement or decline in learner performance from pre to post-test. Two of the test items are equivalence problems *(**item 29**. 61-32=61-☐-2* and ***item 30.** 7, -☐=74-20-5)*, and an analysis of the data indicates that the majority of the learners did not attempt to solve these two items, and those that did attempt got it incorrect. According to Carpenter, Franke & Levi, (2003) and McNeil (2014), mathematical equivalence is needed for success in algebra, and many children have trouble understanding it (Baroody & Ginsburg, 1983; Falkner, Levi & Carpenter, 1999). Furthermore, it also indicates that learners demonstrated an average ability in their pre-test results in solving items 1, 2, 4 and 21, with slight declines in their post-test results. These four items are all *addition result unknown* (12 + 24 = □) problem types in the low to a high number range, and the findings of this study indicate that of the three different types *(result unknown, change unknown and start unknown)* of addition problems *result unknown* is the least challenging for young learners to solve. As such, these study findings correlate with the findings in the international literature (Olkun, & Toluk, 2002).

Whereas findings furthermore indicate that in items 23 and 25, learners demonstrated very low ability in their pre-test results with slight declines in their post-results. Item 25 also adds *unknown* problem types in the mid to high number range. Learners demonstrated a better ability in solving it when compared to item 23, which is of a different sub-type of an addition problem. Item 23 is an addition *change unknown* problem with three numbers to be added instead of two as with the other problem types. And as a result, learners demonstrated a very low ability in their pre and post-test results. These findings also agree with international literature that *change unknown* problem types are more challenging for a learner to solve when compared to *result unknown* problem types (Stigler, Fuson, Ham, and Sook Kim, 1986).

	Addition	***Subtraction***
Result unknown	*4,6%*	*7,7%*
Change unknown	*3%*	*2,3%*
Start unknown	*1,5%*	*1%*

Table 6: Addition and subtraction problem types with significant to smallest gains made.

Furthermore, the findings indicate that with addition problems, the greatest gains were made with results *unknown* with an improvement of 4,6% from pre to post (see table 6 above). And the lowest improvement was made with a *start unknown* with an increase of 1,5% from pre to post. With subtraction problems, the greatest gains were made with results *unknown* with a 7,7% increase and the lowest improvement was made with *start unknown* problems with an increase of 1% in post-test results. These findings of the study align with international literature that makes unknown problem types easier to understand when compared to starting *unknown* and changing *unknown* problem types (Stigler et al., 1986; Peterson, Fennema and Carpenter, 1989). As such, the gains are higher in this sub-type of question.

Item no.	**Pre-test score**	**Post-test score**
Item 1 - (Fill in the missing number) 14, 24, 34, 44, ☐	28	25
Item 2 - (fill in the missing number) 79, 69, 59, 49, ☐	29	28
Item 11 - (what is the next multiple of 10) 56 ☐	0	1
Item 15 - (what is the multiple of 10 before 48) ☐ 48	0	0

Table 7: *Test scores of items with written word instructions*

Four of the 30 test items (see Table 7 above) were designed to include written word instruction and a combination of one of the selected representational forms used for the test (number sentence, number line and part-part-whole). In 2 of these items (items 1 and

2), the pre-test recorded high scores of 28 and 29, respectively, with minor drops in the post-test to 25 and 28, respectively. However, in items 11 and 15, the pre-test scores were zero, with only one gain reflected in the post-test for item 11, as reflected in figure 6 above.

Item	Representation	Pre-test	Post-test	Difference
Item 4	Number sentence	21	17	-4
Item 5	Number line	19	22	3
Item 6	Number line	11	17	6
Item 7	Number line	6	19	13
Item 9	Number line	3	14	11
Item 12	Part-part whole	2	3	1

Table 8: Six items that add or subtract ten off the decuple

These 4 test items share the following characteristic, all 4 items either add or subtract a ten of the decuple (Wright et al. 2012). When considering the rest of the items in the test that share that one characteristic, there are six items: test items 4, 5, 6, 7, 9 and 12. When considering the findings of the analysis of these 6 test items (4,5,6,7,9 and 12), the following can be noted, these test items reflect small to significant pre-test scores with small to significant gains made in the post-test, as reflected in Table 8 above. These findings indicate that although ten test items share one common characteristic of adding or subtracting a ten of the decuple, the six items (4, 5, 6, 7, 9 and 12) in figure 13 above reflect higher pre-test scores and higher post-test scores when compared to items 11 and 15 that recorded zero pre-test scores with one gain in the post-test for item 11. These findings indicate that although ten test items share one common characteristic of adding or subtracting a ten of the decuple, the six items (4, 5, 6, 7, 9 and 12) in table 13 reflect higher pre-test scores and higher post-test scores when compared to items 11 and 15 that recorded zero pre-test scores with one gain in the post-test for item 11.

Furthermore, the 6 test items (4, 5, 6, 7, 9 and 12) in figure 13 above all use the exact representation of the number line except item 12, which uses the part-part-whole representation. Singer & Resnick (1992) note that the part–part whole diagram shows the whole part of a part to a whole, and part-part schemas represent the relationship of two parts to each other. In this study, the part-part whole representation was challenging for learners to solve in both pre and post-test as learners obtained low test results in both tests. However, the pre-test and post-test scores for item 12 (the part-part whole representation) were marginally higher when compared to items 11 and 15 (written word instructions). This finding further indicates that although learners were challenged in solving the part-part whole problem, their proficiency was slightly better compared to items 11 and 15. And the only difference between items 11 and 15 and items 4, 5, 6, 7, 9 and 12 is that written word instructions accompanied them. The finding indicates that learners were challenged in understanding what was required of them to do in items 11 (see figure 14 below) and items 15 (see figure 15 below), as their responses to both questions verify their uncertainty, as can be seen in the two inserts below. Learners assumed that they were expected to do in items 11 and 15 as was required of them to do in the 6 other items (4, 5, 6, 7, 9, and 12) with the similar characteristic of adding or subtracting a ten off the decuple (Wright et al., 2012).

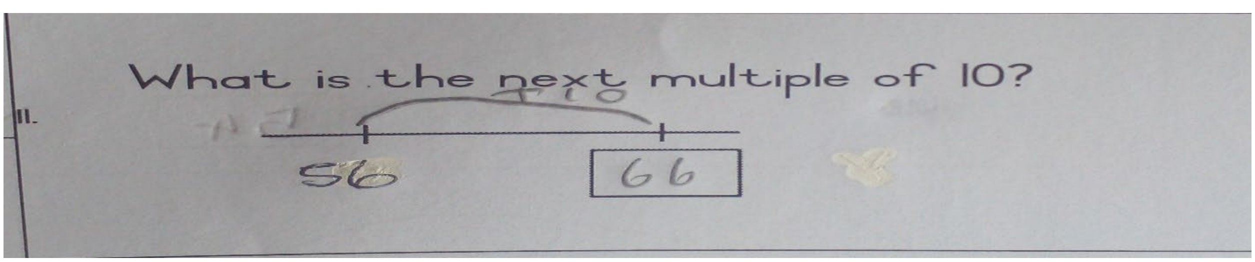


Figure 14: Test item 11

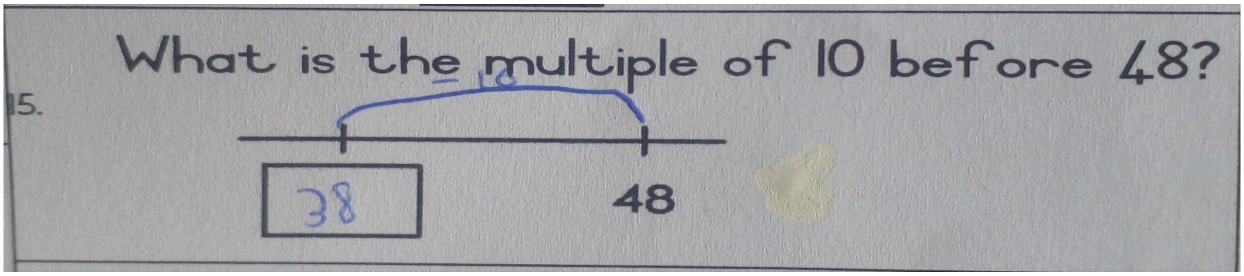


Figure 15: Test item 15

5.2.2 Concluding remarks for research question 1

The analysis of the MSAP findings indicates that there was an overall improvement of 20% in learner performance from pre to post-test with small to significant gains (3% - 33%) made. That 29% of learners reflected a small to significant decline (10% - 30%) in performance from pre to post-test, and 8% of learners remained the same as there reflected no improvement or decline. The breakdown of learner improvements across the three calculating categories assessed in the MSAP indicates that learner *Fluencies* increased by 3,6% while S*trategic Calculations* decreased by 0,8% a*nd Strategic Thinking* proficiencies increased by 1,2%. These findings, as far as Fluencies and Strategic Calculations are concerned, confirm the work of Graven and Venkat (2018); however, the findings of this study also challenge the conventional understanding that learner performance with complex concepts such as Strategic Thinking does not always decrease but can demonstrate an increase. Findings also indicate an improvement made across 70% of test items from pre to post-test, a decline in 20% of test items from pre to post-test, and no increase or decline across 10% of test items. Furthermore, the highest improvement was recorded with subtraction test items of the *result unknown* problem types, and the least improvement was made with subtraction *start unknown* problem types. The findings also indicate that when test items incorporate written word instruction, it adds an extra layer of complexity for young learners to solve, and finally, young learners do not understand equivalence problems. In conclusion, we noticed that there were shifts from pre to post-test; we could ascertain learners who obtained high gains, average gains and lower gains. We also noticed that our top learners all had 33 gains from pre to post, while our lowest learners either remained the same or decreased from pre to post.

5.2.3 Research Question 2: What was the profile of the 6 learners after the LFIN interview was administered on them?

Once I completed the analysis of the pre to post-test results, I could identify and select the 6 learners I was going to interview for the study. As such, from the smaller sample size of 35 learners, I could select two strong, two average and two low-performing learners. As a result of the significant knowledge gaps in learner understanding, I used the following criteria to establish the three attainment bands. 37% of learners demonstrated no improvement to negative gains and were classified as low-performing learners. 23% of learners demonstrated an increase of 3% to 10% that were classified as average attainment learners. The 40% of learners who demonstrated a significant increase from 11% to 33% were classified as strong attainment learners. For anonymity's sake, I employed two different labelling referencing systems for the study's MSAP and LFIN aspects. For the MSAP aspect of the study, I employed numbers 1 to 6; however, for the LFIN aspect of the study, I employed the chronological sequencing of numbers as they reflect on the class list. I commenced the analysis process by reviewing the video-recorded interviews and transcribing all six of the interviews. In the analysis process, I employed an adapted version of the LFIN interview schedule to establish the different SEAL stages each of the learners was located on. The three attainment bands were established as SEAL stages 4-5 as strong attainment learners. SEAL stage 3 as average attainment learners and SEAL stages 1-2 as low attainment learners. The following section is a presentation of the findings of the qualitative analysis. I will commence with learner 1 from the MSAP; however, in the LFIN, learner 1 has been labelled as learner 21, and I will conclude with learner 6 in the MSAP and in the LFIN, it is learner 15.

LFIN - learner 21 (MSAP learner 1)

FNWS	5
BNWS	5
Numeral Identification	4
Base ten	3
SEAL	5

Table 9: LFIN Learner 21

On completing the analysis of the data of Learner 1 (LFIN learner 21), I made the following presentation. With Forward Number Word Sequence (FNWS) and Number Word After (NWA), I found that Learner 1 was on level 5, being the highest level facile with FNWS up to one hundred as the learner was able to produce the FNWS and NWA any number without dropping back. The learner also demonstrated an ability to provide numbers beyond 100. With the Backward Number Word Sequence (BNWS) and Number Word Before (NWB), Learner 1 was on level 5 as the Learner could produce without dropping back any number word up to a hundred. With Numeral Identification (NID), the Learner was on level 4, the highest level, as the Learner could quickly identify two- and three-digit numbers. Regarding Base-ten arithmetical strategies, the Learner demonstrated abilities compliant with level 3 as the Learner had a facile concept of ten (Wright et al., 2012). Based on these findings and the Learner's ability to solve Additive Reasoning (AR) problems in the LFIN assessment with a range of non-count-by-one strategies, I located learner 1 on SEAL stage 5. In other words, this learner has demonstrated confidence in all the criteria as set out in the LFIN. Table 9 above reflects the findings of the data analysis of Learner 1.

LFIN - learner 17 (MSAP learner 2)

FNWS	4
BNWS	3
Numeral Identification	3
Base ten	3
SEAL	4

Table 10: LFIN Learner 17

The findings of the analysis of the data of Learner 2 (LFIN learner 17) indicate that with the FNWS and NWA, the Learner was on level 4 as the Learner was able to produce the FNW and BNA beyond thirty but could not do it beyond sixty and thus failed to meet the criteria of a level 5. With the BNWS and NWB, the Learner was on level 3 as the learner could produce the NWB beyond ten but not beyond twenty and, as such, failed to meet the criteria of level 4. With NID, the Learner was on level 3 as the Learner could not identify with fluency three-digit numbers and failed to meet the criteria of level 4. Regarding Base-ten arithmetical strategies, the learner was on level 3 as the Learner could solve written number sentences involving ones and ten (Wright et al., 2012). These findings, along with the fact that Learner 2 made use of count-down-to strategies to solve problems in the interview, the Learner were found to be on SEAL stage 4, as reflected in table 2 below. In other words, learner 2 was located on SEAL stage 4 as the learner demonstrated a high level of proficiency regarding Base-ten thinking but did not meet all the set criteria of the LFIN regarding FNWS, BNWS and NID as reflected in Table 10 above.

LFIN - learner 32 (MSAP learner 3)

FNWS	5
BNWS	5
Numeral Identification	4
Base ten	2
SEAL	3

Table 11: LFIN Learner 32

The Findings of the analysis of Learner 3 (LFIN learner 32) indicate that with FNWS and NWA, the learner was on level 5 as being facile with FNWS up to one hundred. Regarding BNWS and NWB, the learner was on level 5 as being facile in easily producing the backward numbers from one hundred to one. Regarding NID, the Learner was on level 4, comfortably identifying three-digit numbers. Regarding Base-ten arithmetical strategies, the Learner was found to be on level 2 as having an intermediate concept of ten (Wright et al., 2012). All of the above, coupled with the fact that the learner employed counting on strategies (Thompson, 2000) to solve problems in the interview, indicates that the Learner is on SEAL stage 3, as reflected in figure 3 above. In other words, although this student demonstrated a high level of proficiency with FNWS, BNWs and NID the learner was not able to demonstrate a high level of proficiency with regards to Base ten thinking and was located on SEAL stage 3 as reflected in Table 11 above.

LFIN - learner 3 (MSAP learner 4)

FNWS	5
BNWS	4
Numeral Identification	4
Base ten	3
SEAL	3

Table 12: LFIN Learner 3

The data analysis findings of Learner 4 (LFIN learner 3) regarding FNWS and NWA indicated that the learner is located on level 5 as the learner demonstrated to be facile with FNWS and NWA up to one hundred and even beyond. Regarding the BNWS and NWB, the learner was located on level 4 as the learner demonstrated to be facile with the number range beyond thirty but not beyond sixty and thus does not meet the requirement for a level 4. Regarding NID, the Learner demonstrated level 4 capabilities as the learner could identify with fluency one-, two- and three-digit numbers. The findings of the Base-ten arithmetical strategies indicate that the Learner is on level 3, possessing a facile concept of ten as the learner could solve AR problems involving 'tens' and 'ones' without using materials or representations (Wright et al., 2012). These findings and the fact that the learner employs count-on strategies instead of counting from one strategy when solving AR problems indicate that the learner is on SEAL stage 3, as reflected in Table 12 above. In other words, learner 4 demonstrated a high level of fluency with FNWS, NID and base ten thinking by employing count-on strategies and is located on SEAL stage 3.

LFIN - Learner 13 (MSAP learner 5)

FNWS	4
BNWS	4
Numeral Identification	4
Base ten	2
SEAL	2

Table 13: LFIN Learner 13

The findings of the analysis of Learner 5 (LFIN learner 13) indicated that the learner is on level 4 for both the FNWS, NWA and, BNWS, NWB as the learner demonstrated facile abilities with numbers beyond thirty but not beyond seventy and thus does not meet the requirement for a level 5 allocation. Regarding NID, the findings indicate that the learner is located on level 4 as the learner demonstrated facile abilities in identifying three-digit numbers up to one thousand. The findings of the Base-ten arithmetical strategies indicate that the learner is located on level 2 as the learner can see ten as composed of ten ones but fails to demonstrate flexibility when solving test items (Wright et al., 2012). These findings, coupled with the fact that the learner makes use of counting from one each time to solve problems, indicate that the Learner is located on SEAL stage 2 as a figurative counter. The findings of learner 5 are reflected in figure 5 above. In other words, Learner 5 demonstrated an average ability with FNWS and BNWS and their knowledge of base ten and employed count from one strategy to solve test items and was located on SEAL stage 2 with a reliance on figurative counters (see Table 13).

LFIN - learner 15 (MSAP learner 6)

FNWS	3
BNWS	3
Numeral Identification	2
Base ten	1
SEAL	2

Table 14: LFIN Learner 6

According to the findings of the data analysis, Learner 6 (LFIN learner 15) is located on level 3 with regards to FNWS and NWA, furthermore, with regards to BNWS and NWB, the learner is also located on level 3 as the learner could not demonstrate facile ability in the number beyond twenty. Regarding NID, the analysis findings indicate that the learner is located on level 2, as the learner demonstrated a facile ability in identifying numbers beyond twenty but not beyond sixty. Regarding Base-ten arithmetical strategies, the learner is located on level 1 as the learner does not see ten as a unit composed of ten ones. The learner cannot solve the AR problem item when it is presented in a number sentence (Wright et al., 2012). In other words, these findings indicate that learner 6 demonstrated weak ability with FNWS, BNWS and base ten thinking. Furthermore, Learner 6 employs count from one strategy to solve test items, indicating that the learner is located on SEAL stage 2, as reflected in Table 14 above.

5.2.4 Concluding remarks for research question 2

On completing the qualitative aspects of the data, I could allocate each of the six learners to their respective SEAL stages based on each of the learners' proficiencies (see Table 13 below). SEAL stage 5 is the highest level of proficiency, and SEAL stage 1 is the lowest level of proficiency. SEAL stage 5 learners demonstrated high competence by solving problem items with various highly efficient calculating strategies. However, in SEAL stage 1, learners demonstrated low levels of calculating strategies as being highly dependent on the concrete form of counting. In other words, Learners 21 and 17 were located on SEAL stages 5 and 4, respectively, as they demonstrated a high level of proficiency in solving problems. While learners 13 and 15 were both located on SEAL stages 2 as being learners that demonstrated an over-reliance on inefficient and error-prone concrete forms of counting.

Learner in LFIN	**FNWS, NWA**	**BNWS, NWB**	**NID**	**Base ten**	**SEAL**
Learner 21	5	5	4	3	5
Learner 17	4	3	3	3	4
Learner 32	5	5	4	2	3
Learner 3	5	4	4	3	3
Learner 13	4	4	4	2	2
Learner 15	3	3	2	1	2

Table 15: Placement of learners on the SEAL stages.

5.2.5 Research Question 3: What is the correlation between the Jump Strategies and the LFIN, if any?

The following reflects the findings of the correlation between the MSAP and the LFIN. I will present the findings in the following manner firstly, of the 2 strong learners, followed by a discussion on the 2 average performing learners and lastly, of the 2 low performing learners.

MSAP				**LFIN**		
Learner	**Attainment band**	**Shifts**		**Learner**	**Attainment band**	**SEAL**
Learner 1	Strong	30% gains		Learner 21	Strong	SEAL 5
Learner 2	Strong	23% gains		Learner 17	Strong	SEAL 4

Table 16: Strong performing learners' correlation between the MSAP and the LFIN

In the MSAP intervention, learners 1 and 2 demonstrated the highest gains from pre to post-test, with gains as high as 30% and 23%, respectively. These two learners demonstrated a high level of efficiency and speed in completing their individual daily classroom exercises as well as in the pre and post-test. They also demonstrated a high level of classroom participation. When conducting the LFIN interview, these 2 learners employed none-count-by-one strategies to solve problems, and they also demonstrated a high level of fluency in their base ten thinking. They were located on SEAL stages 5 and 4, respectively, as reflected in Table 16 above.

MSAP				**LFIN**		
Learner	**Attainment band**	**Shifts**		**Learner**	**Attainment band**	**SEAL**
Learner 3	Average	6% gains		Learner 32	Average	SEAL 3
Learner 4	Average	3 % gains		Learner 3	Average	SEAL 3

Table 17: Average performing learners' correlation between MSAP and LFIN

In the MSAP intervention, learners 3 and 4 demonstrated average abilities with small gains made from pre to post-test. These 2 learners were not confident in their classroom interactions and daily classroom exercises but were nonetheless responsive in class; they also had an average ability in their pre and post-test. Regarding the LFIN interview conducted on these 2 learners, they demonstrated average problem-solving abilities by employing less sophisticated count-on strategies instead of none-count-by-one strategies. Therefore, they were located on SEAL stages 3, respectively, as reflected in Table 17 above.

MSAP				**LFIN**		
Learner	**Attainment band**	**Shifts**		**Learner**	**Attainment band**	**SEAL**
Learner 5	Low	0 % gain		Learner 13	Low	SEAL 2
Learner 6	Low	-30% gains		Learner 15	Low	SEAL 2

Table 18: Low-performing learners' correlation between MSAP and LFIN

In the MSAP intervention, learners 5 and 6 demonstrated no shifts to negative decrease from pre to post-test. Both learners furthermore demonstrated weak ability in their base ten thinking and their ability to solve test items in the MSAP. These two learners were non-responsive and evasive in class during lessons. Furthermore, in the LFIN, both learners were located on SEAL stage 2 (see Table 18 above), respectively, as both employed count from one strategy when solving problem items and demonstrated low ability when trying to solve problems during the interviews.

5.2.6 Concluding remarks for research question 3

These findings indicate that according to the MSAP intervention, the 2 learners that demonstrated the highest gains achieved across pre to post-test also demonstrated a high level of proficiency in the set criteria of the LFIN. And the 2 learners that demonstrated average gains achieved in the MSAP also demonstrated an average level of proficiency in the LFIN. And the learners that demonstrated a decrease in performance to negative 30% in the MSAP also demonstrated low levels of proficiencies in the LFIN.

5.3 Conclusion

The above discussion of the presentation of the findings and the analysis thereof answered each of the three research questions and indicates that the MSAP intervention can be utilized as a significant catalyst to improve young learners' mental mathematics as there was a 20% improvement in learner performance from pre- to post-test—the highest gains reflecting in subtraction problems with improvements in 70% of test items. The Findings furthermore indicate a strong correlation between the MSAP and the LFIN, as there is a match with the selected six learners. Two of the strong attainment learners (learners 1 and 2) from the MSAP match with the 2 strong attainment learners in the LFIN. In the average attainment band of the MSAP, the two learners correlate with the findings of the LFIN. The low attainment band, as selected from the MSAP learner 5, matches with the findings of the LFIN as a low-attaining learner as being located on a SEAL stage 2. As these findings indicate a 67% correlation between the MSAP and the LFIN, further studies will have to be conducted to establish with greater certainty the validity of the claims put forward in this study.

CHAPTER 6 – CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

South African learners underperform in mathematics, and researchers continue to explore avenues to remedy this dire situation. Part of this exploration effort was the development of the Mental Starters Assessment Project (MSAP), a model that seeks to enhance the development of mental mathematics in young learners. In addition, using an alternative model, the LFIN also aims to identify learners' levels of use of calculation strategies. However, due to contextual challenges, the MSAP model would be more advantageous within the South African context. As such, the purpose of this study was to establish if there is a correlation between the MSAP and the LFIN. To achieve this objective, this study had to pursue a twofold path. Firstly, to explore the shifts in learner performance after completing a two-week mental math developmental intervention program using Jump Strategy on the Empty Number Line based on the MSAP model. Secondly, select six learners from the MSAP data set on the criteria of being two strong, two average and two low-performing learners and assess their mathematical proficiencies on the LFIN by locating each learner on a SEAL stage based on their level of proficiency. In the chapter, I will address the focus of the area of study, the summary of the findings of the survey, contributions of the study, implications of the study for mathematics teaching, limitations of the research and self-reflection.

6.2 Main focus of the area of study

The focus area of my study was to assess grade three learners' mental mathematics development after a two-week intervention program using Jump Strategy on the Empty Number Line. The purpose was to see whether the types of calculations could be improved from a pretest to a post-test. The three types of calculation strategies looked at were fluencies, strategic calculations, and strategic thinking. Whilst the Jump Strategy was the focus of the study, the LFIN test was also introduced to see the level of calculation strategies six learners possessed. Based on the intervention and the assessment tool, the study tried to ascertain if there could be any correlations between the MSAP and the LFIN.

6.3 Summary of the findings of the study

Upon completion of the MSAP intervention and the analyses of the data, I discovered the following findings, ten learners demonstrated substantial gains (between 20% to 33%), twelve learners showed average gains (between 3% to 6%), and thirteen learners demonstrated no gains to negative decreases (0% to -30%). However, the most significant shifts observed were in the Fluency items. With a marginal negative decrease of 0,8% in Strategic Calculating items from pre to post-test. And a minor increase of 1,2% in Strategic thinking items. These findings are consistent with previous research conducted in the South African terrain in which there was an increase in the Fluency category with no shifts in the Strategic Calculating. However, in this study, there was an increase of 1,2% in the Strategic Thinking category. From the different categories in attainment, I selected six learners, two from each attainment band, to assess their level of proficiency on the LFIN by locating each of them on an appropriate SEAL stage based on their level of calculation strategies. The findings indicated that the two learners that achieved the highest gains in the MSAP were the identical two on the highest SEAL stages in the LFIN. These learners could solve problem items through various efficient calculating strategies. And the two learners that demonstrated no increase to negative decrease in the MSAP were the identical two learners located on the lower stages of the SEAL in the LFIN as learners grounded and dependent on concrete forms of counting. These findings indicate a strong correlation between the MSAP and the LFIN. And that the LFIN can be replaced with the MSAP as an intervention model.

6.4 Contributions of the study

The MSAP intervention is essential because it is scalable and is currently being implemented nationally. The MSAP provides ordinary teachers with a tool to assess their learner's levels of calculation strategies so that they can focus on how to remedy those learners who are still prone to counting in ones to find answers to additive tasks. The LFIN as an assessment tool is very precise in identifying levels of learners' calculation strategies. However, administering the LFIN test requires a specialist interviewer and is time-consuming. The study's contribution to the Foundation Phase numeracy field can be very beneficial as the study has shown a significant link between the MSAP and the LFIN. The link is that both the intervention and the LFIN test show the same level of calculation strategies of the same learner. In other words, the MSAP intervention could be a reliable replacement for the LFIN test at a national level. Another contribution towards the terrain is that, in some instances, learners' fluencies may drop in this case slightly. Yet, an increase in the more complex types of calculations, such as strategic calculations and strategic thinking, could improve. This finding adds to the nuance of how interventions express themselves in different contexts and classrooms.

6.5 Implications of the study for mathematics teaching

The implications can significantly benefit the South African primary school mathematics classroom in that the MSAP learners that lag in mathematics can be assessed and assisted. Secondly, the MSAP can be administered by the classroom teacher as it requires no extra training from the educator nor requires a specialized skill. Thirdly, due to its relatively easy administrative use, it can be rolled out nationally to remedy learners' underperformance in the mathematics classroom.

6.6 Limitations of the study

I worked with a very small sample size, and one of the implications is that the findings of this study need to be generalized across large numbers of the population. However, this small-scale study's findings established a correlation between the MSAP and the LFIN and that further research could be conducted.

6.7 Self-reflections

When I started the study, I built on some insights I gained from my Honors project. That process taught me essential skills in identifying learners' calculation strategies. While working on the broader Wits Math's Connect-Primary Project, I never had the opportunity to teach a class independently. One of the essential experiences was my exposure to being able to teach intervention lessons. It was in the teaching of the intervention lessons that I began to see how important different types of teaching and learning theories came into play. At the same time, I could relate to many Foundation Phase teachers' experiences of teaching mathematics. I discovered the complexity and the nuance of working with mathematics in a face-to-face experience. What also excited me was when I found that the results of the MSAP and the LFIN were relatively similar. The strength of that discovery allowed me to embrace the power of research in a very different way than before. Before my research confirmed the findings of other studies, I discovered a new contribution to the broader math community in this study. The process of conducting this study was highly informative as well as insightful.

. REFERENCE LIST

Akhtar, D. M. I. (2016). Research design. *Research Design (February 1, 2016)*.

Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and Summary of new directions. Mathematical Cognition, 1(1), 3-34

Askew, M. (2012). Professor's page. Is understanding a proficiency? Australia Primary Mathematics Classroom, 17(1), 19-20.

Askew, M., Bibby, T, & Brown, M. (2001). *Raising Attainment in Primary Number Sense: From Counting to Strategy. BEAM Research Papers (RES01).* London: BEAM.

Baroody, A. J., & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the "equals" sign. Elementary School Journal, 84,199–212.

Baroody, A. J. (2006). Why children have difficulty mastering the basic number combinations and how to help them. Teaching Children Mathematics, 13, 22–31

Beishuizen, M. (2001). Different approaches to mastering mental calculation processes. In J. Anghileri (Ed.), *Principles and Practices in Arithmetic Teaching* (pp. 119-130). London: Open University Press.

Bertram, C., & Christiansen, I. (2014). *Understanding research. An introduction To reading research*. Pretoria: Van Schaik Publishers.

Blöte, A. W., Klein, A. S., & Beishuizen, M. (2000). Mental computation and conceptual understanding. *Learning and instruction*, *10*(3), 221-247.

Bobis, J., & Bobis, E. (2005). The empty number line: Making children's thinking visible. *Making mathematics vital*, 66-72.

Bobis, J. (2007). The empty number line: A useful tool or just another procedure? Teaching Children Mathematics, 13, 410–413. Retrieved from http://www.nctm.org/publications/article.aspx?id = 22065

Carpenter, T. P., Moser, J. M., & Hiebert, J. (1981). Problem structure and first-grade children's initial solution processes for simple addition and subtraction problems. *Journal for research in mathematics education*, *12*(1), 27-39.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). Children's mathematics. *Cognitively Guided Instruction. Postmourth, NH: Heinemann. Trad. de C. De Castro y M. Linares: Las Matemáticas que hacen los niños*.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003) Thinking mathematically: Integrating Arithmetic and algebra in elementary school. Portsmouth, NH: Heinemann.

Cheng, Z. J. (2012). Teaching young children decomposition strategies to solve addition problems: An experimental study. *The Journal of Mathematical Behavior*, *31*(1), 29-47.

Ching, B. H. H., & Nunes, T. (2017). The importance of additive reasoning in children's mathematical achievement: A longitudinal study. *Journal of Educational Psychology*,*109*(4), 477.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research Methods in Education* (7th Ed.). Milton Park. Abingdon, Oxon, England: Routledge.

Cowan, R. J. (2011). *The development and importance of proficiency in basic calculation*. Institute of Education, University of London.

Cowan, R., Donlan, C., Shepherd, D. L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology*, *103*(4), 786.

Creswell, J. W., Fetters, M. D., & Ivankova, N. V. (2004). Designing a mixed method Study in primary care. *The Annals of Family Medicine*, *2*(1), 7-12.

Creswell, J. and Plano Clark, V. (2007) Designing and Conducting Mixed Methods Research, Sage, Thousand Oaks.

Creswell, J. W. (2012). *Educational Research. Planning, Conducting and Evaluating Quantitative and Qualitative research* (4, Illustrated). University of Nebraska-Lincoln: Pearson.

Dyson, N. I., Jordan, N. C., & Glutting, J. (2013). A number sense intervention for low-income kindergartners at risk for mathematics difficulties. *Journal of learning disabilities*, *46*(2), 166-181.

Ekdahl, A. L. (2019). *Teaching for the learning of additive part-whole relations: The power of variation and connection* (Doctoral dissertation, Jönköping University, School of Education and Communication).

Falkner, K. P., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. Teaching Children Mathematics, 6, 232– 236.

Fithratu, C., Prihatmanto, A. S., Gitarana, G. R. E., & Muis, A. N. (2020, December). Arithmatopia: Using MOBA Game to Increase Basic Calculation Proficiency. In *2020 6th International Conference on Interactive Digital Media (ICIDM)* (pp. 1-4). IEEE.

Flick, U. (2011). *Introducing research methodology: A beginner's guide to doing a research project*. London: SAGE Publications Ltd.

George, T. (2023, June 22). *Mixed Methods Research | Definition, Guide & Examples.* Scribbr. Retrieved October 30, 2023, from https://www.scribbr.com/methodology/ mixed-methods-research/

Gravemeijer, K. (1994). Educational development and educational research in mathematics education. *Journal for Research in Mathematics Education, 25* (5), 443–71.

Graven, M., & Venkatakrishnan, H. (2018). Promoting teaching and learning of strategic Calculation and thinking through diagnostics assessments. Integrated Education For The Real-World 5th international STEM in Education Conference (pp. 123-129). Brisbane: Queensland University of Technology.

Graven, M., & Venkat, H. (2019). Piloting national diagnostic assessment for strategic calculation. *Mathematics Education Research Journal*, 1-20.

Graven, M., & Venkat, H., & Askew, M., & Bowie, L., & Morrison, S., & Vale, P. (2020) Grade 3 Mathematics Mental Starters Assessment Project (MSAP).

Gray, E. & Tall, D. (1994). Duality, ambiguity and flexibility: a proceptual view of simple arithmetic. *Journal for Research in Mathematics Education,* 25, 116-140.

Green, J. (2005). Improving mental computation skills. *Review process*, 139.

Greene, J. C., Caracelli, V. J., & Graham, W. F. (1989). Toward a conceptual Framework for mixed-method evaluation designs. Educational Evaluation and Policy Analysis, 11 (3), 255-274.

Hoadley, U. (2012). What do we know about teaching and learning in South African primary schools? Education As Change, 16, 187-202

Hopkins, S., & Bayliss, D. (2014). The Importance of Achieving Proficiency with Simple Addition. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Hunting, R. P. (2003). Part-whole number knowledge in preschool children. *The Journal of Mathematical Behavior*, *22*(3), 217-235.

Junpeng, P., Krotha, J., Chanayota, K., Tang, K. N., & Wilson, M. (2019). Constructing progress maps of digital technology for diagnosing mathematical proficiency. Journal of Education and Learning, 8(6), 90 − 102. https://doi.org/10.5539/jel.v8n6p90

Kamii, C., Kirkland, L., & Lewis, B. A. (2001). Representation and abstraction in young children's numerical reasoning. *The roles of representation in school mathematics*, *2001*, 24-34.

Khairani, A. Z., & Sahari Nordin, M. (2011). The development and construct validation Of the mathematics proficiency test for 14-year-old students. *Journal of Educators & Education/Jurnal Pendidik dan Pendidikan*, *26*.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding It Up: Helping Children Learn Mathematics. Washington DC: National Academy Press.

Lowhorn, G. L. (2007, May). Qualitative and quantitative research: How to choose the best Design. In *Academic Business World International Conference. Nashville, Tennessee*.

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. Mathematics Education Research Journal, 21(2), 10–32.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A Proposed Framework for Examining Basic Number Sense. For Learning of Mathematics, 12(3). 2-8, 44.

McNeil, N. M. (2014). A change–resistance account of children's difficulties Understanding Mathematical equivalence. Child Development Perspectives, 8(1), 42-47.

Moloi, M., Strauss J. (2005). A study of the conditions of schooling and the quality of education. South Africa working report. Harare and Pretoria: SACMEQ and the Ministry of Education.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: NCTM

Olkun, S., & Toluk, Z. (2002). Textbooks, word problems, and student success on addition And subtraction. International Journal for Mathematics Teaching and Learning, 11(18), 162-170.

Opie, C. (2004). *Doing Educational Research*. London: SAGE.

Peterson, P. L.; Fennema, E. & Carpenter, T. (1989). Using knowledge of how students Think about mathematics. Educational Leadership, 46(4), 42-46.

Phaniew, S., Junpeng, P., & Tang, K. N. (2021). Designing Standards-Setting for Levels Of Mathematical Proficiency in Measurement and Geometry: Multidimensional Item Response Model. *Journal of Education and Learning*, *10*(6), 103-111.

Pirie, S. E. (1996). Classroom Video-Recording: When, Why and How Does It Offer a Valuable Data Source for Qualitative Research?

Plunkett, S., (1979). Decomposition and all that rots. *Mathematics in Schools,* 8, 2-5.

Schmittau, J., & Morris, A. (2004). The development of algebra in the elementary Mathematics curriculum of VV Davydov. The Mathematics Educator, 87(1), 60-87

Schollar E. (2008). Final report of the Primary Mathematics Research Project. Presentation to the Conference What's Working in School Development, JET Education Services, February 2008, available at http://www.jet.org.za/item.php?i_id=263

Singer, J. A., & Resnick, L. B. (1992). Representations of proportional relationships: Are children part-part or part-whole reasoners? *Educational Studies in Mathematics 23*(3), 231-246.

Sinnakaundan, S., Kuldas, S., Hashim, S., & Ghazali, M. (2016). Enabling pupils to conceive part-whole relations of numbers and develop number sense: Year one of primary schools in Malaysia. *International Journal for Mathematics Teaching and Learning*, *17*(3).

Skoumpourdi, C. (2010). The number line: An auxiliary means or an obstacle. *International Journal for Mathematics Teaching and Learning,270*.

Spaull, N. (2013). Poverty & privilege: Primary school inequality in South Africa. *International Journal of Educational Development*, *33*(5), 436-447.

Spaull, N., & Kotze, J. (2015). Starting behind and staying behind in South Africa: *The case of insurmountable learning deficits in mathematics. International Journal of Educational Development*, 41, 13-24

Spaull, N. (2015). Schooling in South Africa: How low-quality education becomes a poverty traps. *South African child gauge*, *12*(1), 34-41.

Stange, K. C., Miller, W. L., Crabtree, B. F., O'Connor, P. J, Zyzanski, S. J. (1994). Multimethod research: Approaches for integrating qualitative and quantitative methods. J Gen Intern Med. 9:278-282.

Steinke, D. A. (2001, June). Does "Part-Whole Concept" understand correlate with success in basic math classes? Paper presented at the 8th Annual Adults Learning Mathematics Conference. Roskilde, Denmark.

Stigler, J. W., Fuson, K. C., Ham, M., & Sook Kim, M. (1986). An analysis of addition and Subtraction word problems in American and Soviet elementary mathematics Textbooks. Cognition and instruction, 3(3), 153-171.

Thompson, I. (1999a). Mental calculation strategies for addition and subtraction. Part 1. *Mathematics in school*, *28*(5), 2-4.

Thompson, I. (2000). Mental calculation strategies for addition and subtraction: *Part 2. Mathematics in school,* 29(1), 24-26.

Torbeyns, J., & Verschaffel, L. (2013). Efficient and flexible strategy use on multi-digit sums: A choice/no-choice study. *Research in Mathematics Education*, *15*(2), 129-140.

Venkat, H., Rollnick, M., Loughran, J., & Askew, M. (Eds.). (2014). *Exploring Mathematics And Science Teachers' Knowledge: Windows into Teacher Thinking*. Routledge.

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' Mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development*, *41*, 121-130.

Venkat, H., Askew, M., Watson, A., & Mason, J. (2019). Architecture of mathematical structure. *For the Learning of Mathematics*, *39*(1), 13-17.

Venkat, H., Askew, M., & Morrison, S. (2021). Shape-shifting Davydov's ideas for early number learning in South Africa. *Educational Studies in Mathematics*, *106*(3), 397-412.

Venkat, H., Graven, M., Morrison, S., Vale, P., Askew, M., Bowie, L., Westaway, S., Chikiwa, W., Hlokonya, M., Chetty, D., Mathebe. Going national with the mental starter's Assessment project 2021: model, outcomes and provincial Foundation Phase math's coordinators' experiences. *Electronic Record of*, 13.

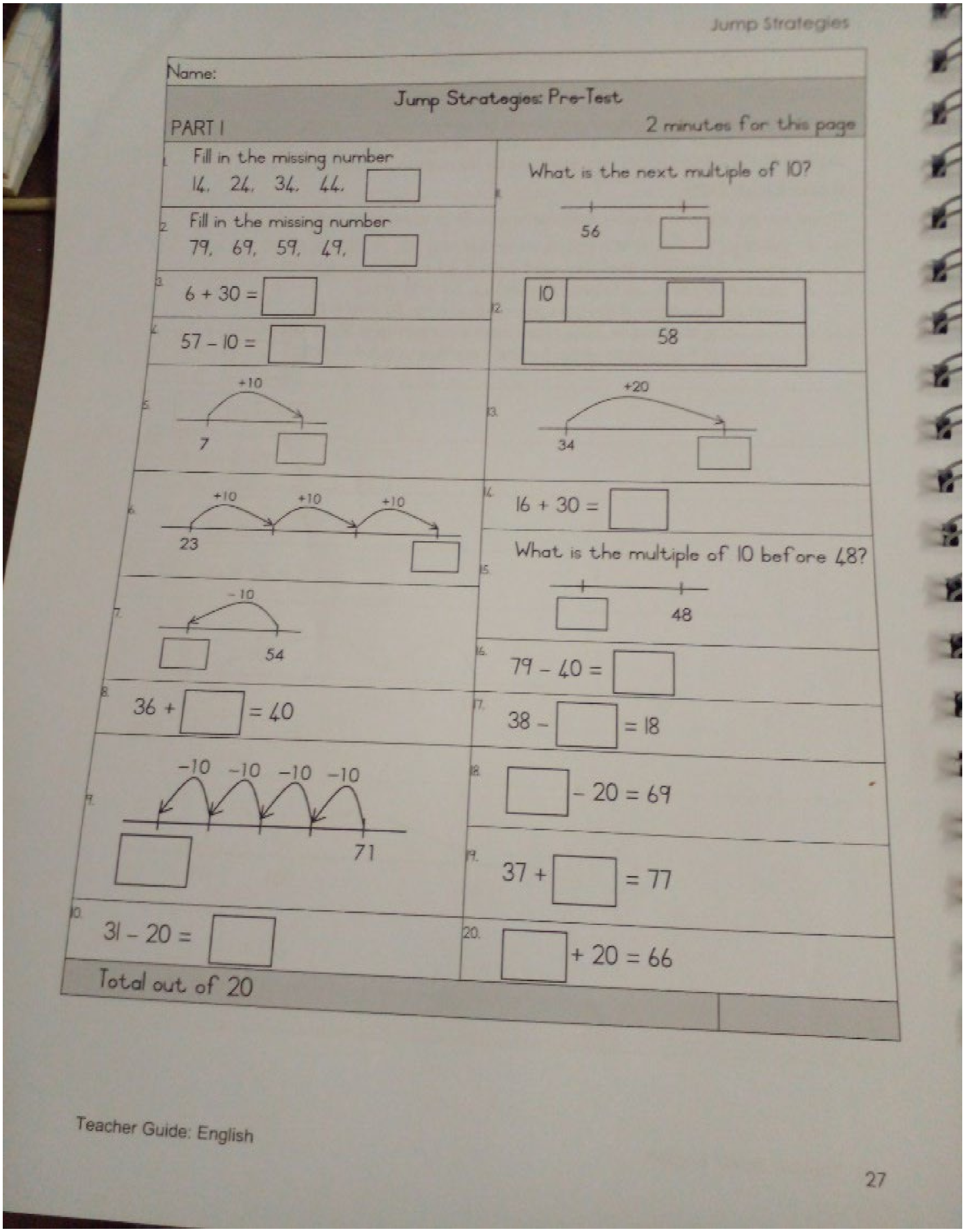
Wright, R. J., Martland, J., & Stafford, A. K. (2006). *Early numeracy: Assessment for teaching and intervention*. Sage.

Wright, R. J., Ellemor-Collins, D., Tabor, P. D. (2012) Developing Number Knowledge. Sage: London

Weitz, M. & Venkat, H. (2013). Assessing early number learning: How useful is the Annual National Assessment in Numeracy? Perspectives in Education, 31(3), p49-65.

Young-Loveridge, J. (2011). Assessing the mathematical thinking of young children in New Zealand: the initial school years. *Early Child Development and Care*, *181*(2), 267-276

Appendix A



Jump Strategies

Name:

Jump Strategies: Pre-Test

PART I

2 minutes for this page

1. Fill in the missing number
14, 24, 34, 44, $\square$

2. Fill in the missing number
79, 69, 59, 49, $\square$

3. 6 + 30 = $\square$

4. 57 – 10 = $\square$

5. +10
7 $\square$

6. +10 +10 +10
23 $\square$

7. – 10
$\square$ 54

8. 36 + $\square$ = 40

9. –10 –10 –10 –10
$\square$ 71

10. 31 – 20 = $\square$

11. What is the next multiple of 10?
56 $\square$

12. 10 $\square$
58

13. +20
34 $\square$

14. 16 + 30 = $\square$

15. What is the multiple of 10 before 48?
$\square$ 48

16. 79 – 40 = $\square$

17. 38 – $\square$ = 18

18. $\square$ – 20 = 69

19. 37 + $\square$ = 77

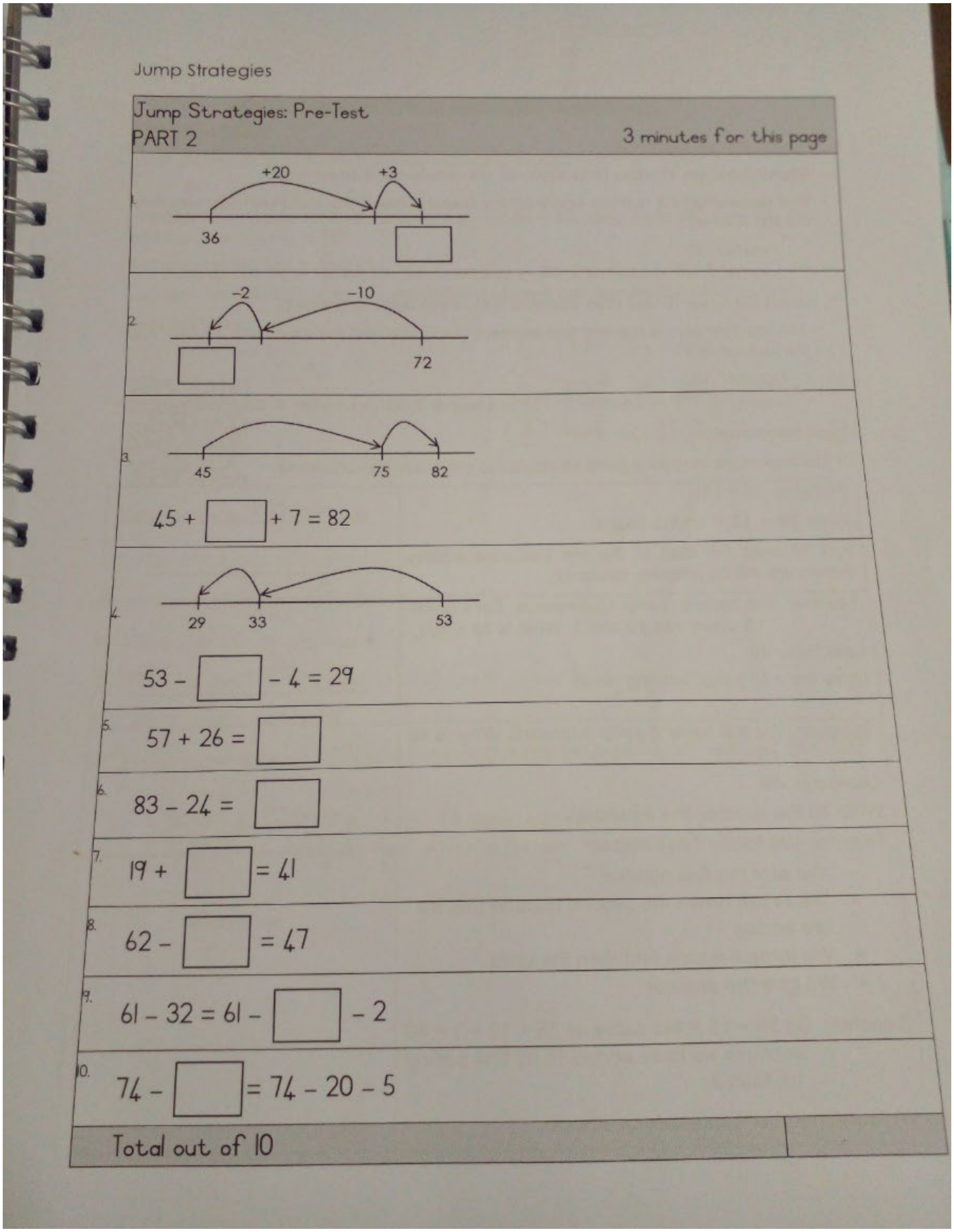
20. $\square$ + 20 = 66

Total out of 20

Teacher Guide: English

27

Appendix B


Jump Strategies

Jump Strategies: Pre-Test
PART 2 — 3 minutes for this page

1. +20 +3
36 $\boxed{}$

2. −2 −10
$\boxed{}$ 72

3. 45 75 82
45 + $\boxed{}$ + 7 = 82

4. 29 33 53
53 − $\boxed{}$ − 4 = 29

5. 57 + 26 = $\boxed{}$

6. 83 − 24 = $\boxed{}$

7. 19 + $\boxed{}$ = 41

8. 62 − $\boxed{}$ = 47

9. 61 − 32 = 61 − $\boxed{}$ − 2

10. 74 − $\boxed{}$ = 74 − 20 − 5

Total out of 10

Appendix C

JUMP STRATEGIES: LESSON STARTER 1

1-Minute Mental Warm-Up

a. Round the room 10 more (can alternate with whole class answering)

The teacher says a number and learners respond round the room with 10 more than the last number.

Teacher: 16
Learner 1: 26 → Learner 2: 36 → Learner 3: 46 → Learner 4: 56 and so on.

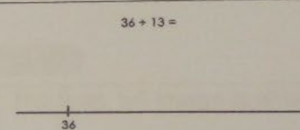
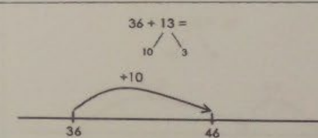
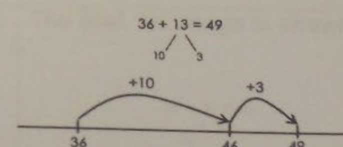
b. Round the room 10 less (can alternate with whole class answering)

The teacher says a number and learners respond round the room with 10 less than the last number.

Teacher: 128
Learner 1: 118 → Learner 2: 108 → Learner 3: 98 → Learner 4: 88 and so on.

Task Sequence

In this lesson we introduce jump strategies to solve addition problems.

Problem: 36 + 13 Write '36 + 13 =' on the board. Plot 36 near the start of the line (because adding means we will be jumping forwards).	
Teacher: We have to jump 13 forwards. Let's break 13 down into 10 and 3. What is 36 + 10? Learners: 46 Draw the +10 jump, landing on 46.	
Teacher: We still have to jump 3 forward. What is 46 plus 3? Learners: 49 Write on the number line as shown. Teacher: We follow these steps: • We **plot** the first number • We **break down** the second number that we are adding • We **jump** the tens and then the units • We give the **answer** Teacher: So 36 + 13 is the same as 36 + 10 + 3 = 49 because we have added 13 by first adding 10, then 3. Write the number sentences as shown.	 36 + 10 + 3 = 49

Appendix D

JUMP STRATEGIES: LESSON STARTER 2

1-Minute Mental Warm-Up

Pop Fizz: 10 more or 10 less

a. The teacher says 'pop', learners say 'fizz'; teacher says a number, learners respond with **10 more** (or a multiple of 10 more):

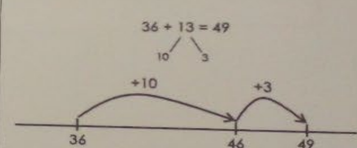
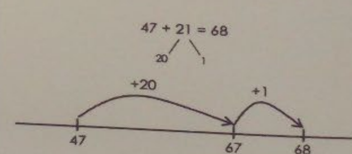
Teacher: pop	→	Learners: fizz	
Teacher: 3	→	Learners: 13	
Teacher: pop	→	Learners: fizz	
Teacher: 53	→	Learners: 63	and so on...

b. The teacher says 'pop', learners say 'fizz'; teacher says a number, learners respond with **10 less** (or a multiple of 10 less):

Teacher: pop	→	Learners: fizz	
Teacher: 49	→	Learners: 39	
Teacher: pop	→	Learners: fizz	
Teacher: 78	→	Learners: 68	and so on...

Task Sequence

In this lesson we use jump strategies to solve addition problems.

Ask the learners to try to remember from yesterday: How did we solve 36 + 13? Start by drawing a line and marking the point '36'. Allow learners to describe the method to their seat partner, and then ask one pair to show it on the board. Remind learners of the steps they learned previously: **plot, break down, jump** and **answer**.	The final, full image is shown below: 
Then, show on the board how to use jump strategies to solve: 47 + 21 • **Plot** 47 on the number line. • **Break down** the 21 into 20 and 1 • **Jump** 20 forward and then **jump** 1 forward. Some learners will make two forward jumps of 10 instead of one forward jump of 20 - this is okay. • Give the **answer**. Teacher: So 47 + 21 is the same as 47 + 20 + 1 = 68 Write the number sentences as shown.	The final, full image is shown below:  47 + 20 + 1 = 68

Individual Tasks

Learners should now try the following examples *mentally*:

43 + 24 31 + 25

Appendix E

JUMP STRATEGIES: LESSON STARTER 3

1-Minute Mental Warm-Up

Pop-Fizz: 10 more and 10 less; 20 more and 20 less

Task Sequence

In this lesson we extend jump strategies to include a bridging through ten step.

Show on the board how to use jump strategies to solve: 35 + 16

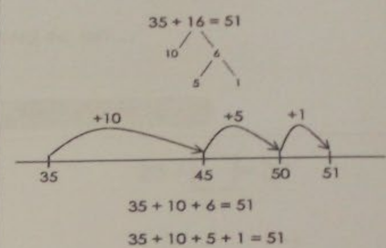
Write the number sentence on the board and draw an empty number line.

- **Plot** 35 on the number line.
- **Break down** the 16 into 10 and 6
- **Jump** 10 forward to reach 45. **Jump** the remaining 6 by bridging through the next multiple of 10 (this is 50). So the 6 needs to be broken down into 5 and 1. **Jump** forward 5 and 1.
- Give the **answer**.

Teacher: So 35 + 16 is the same as 35 + 10 + 6 = 51 or 35 + 10 + 5 + 1 = 51.

Write the number sentences as shown.

The final, full image is shown below:



Individual Tasks

Learners should now try the following examples *mentally*:

39 + 23 68 + 35

Learners should explain their thinking, e.g. "for 39 + 23, I add 20 to 39 to get 59, then add 1 to get to 60, and then 2, so the answer is 62."

Tell learners NOT to count in 1s.

Give early finishers more to practice:

36 + 28 47 + 34

Appendix F

JUMP STRATEGIES: LESSON STARTER 4

1-Minute Mental Warm-Up

Jumping to the **next** multiple of ten

This is not rounding to the nearest ten but jumping to the **next** multiple of ten on the number line.

"What is the **next** multiple of ten **after**...?"

Teacher: 47	→	Learners: 50	
Teacher: 55	→	Learners: 60	
Teacher: 32	→	Learners: 40	and so on...

Task Sequence

In this lesson we use jump strategies to solve missing number problems.

Show on the board how to use jump strategies to solve 23 + □ = 37 as follows:

Write the number sentence on the board and draw an empty number line.

Plot '23' on the number line.

Teacher: We need to jump forward to 37.

Mark 37 on the number line.

Teacher: What tens jump, and what units jump, should we make?

Learners: Jump 10 to get to 33 and jump 4 to get to 37.

Draw these jumps on the number line.

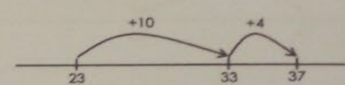
Teacher: How much have we jumped in total?

Learners: 14

Write 14 in the block.

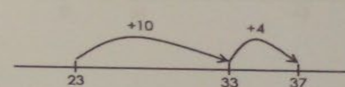
Teacher: So 23 + 10 + 4 is the same as 23 + 14 = 37.

23 + □ = 37



23 + $\boxed{14}$ = 37

10 4



Individual Tasks

Learners should now try the following examples *mentally*:

45 + □ = 67 67 + □ = 81

Learners should explain their thinking, e.g. "for 45 + □ = 67, I add 20 to 45 to get 65, then add 2 to get 67, so the total jump is 22."

Tell learners NOT to count in 1s.

Give early finishers more to practice:

45 + □ = 68 67 + □ = 83

Appendix G

Jump Strategies

JUMP STRATEGIES: LESSON STARTER 5

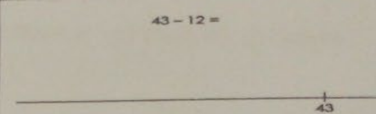
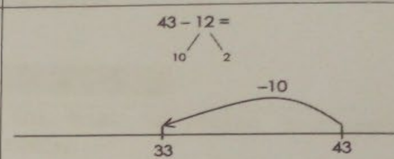
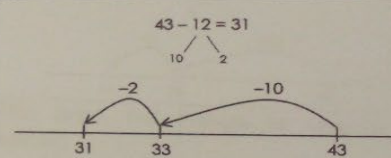
1-Minute Mental Warm-Up

a. Round the room 10 more

b. Round the room 10 less

Task Sequence

In this lesson we use jump strategies to solve subtraction calculations.

Problem: 43 – 12 Write '43 - 12 =' on the board. Plot '43' near the end of the line (because subtracting means we will be jumping backwards).	
Teacher: We have to jump 12 backwards. Let's break 12 down into 10 and 2. What is 43 - 10? Learners: 33 Make the –10 jump, landing on 33.	
Teacher: We still have to jump 2 backwards. What is 33 minus 2? Learners: 31 Write on the number line as shown. Teacher: We have subtracted 12 by subtracting 10 and subtracting 2. • We **plot** the first number • We **break down** the second number • We **jump** the tens and then the ones (backward jumps because we are subtracting) • We give the **answer** Teacher: So 43 – 12 is the same as 43 – 10 – 2 = 31. Write number sentences as shown.	

Individual Tasks

Learners should now try the following examples *mentally*:

62 – 12   53 – 11

Learners should explain their thinking, e.g., "for 62 – 12, I subtract 10 from 62 to get 52, then subtract 2, so the answer is 50."

Tell learners NOT to count in 1s.

Appendix H

JUMP STRATEGIES: LESSON STARTER 6

1-Minute Mental Warm-Up

a. Add 10 (or add a multiple of 10)

Same method as '10 more', but this time the teacher offers an 'add 10' problem.

Teacher: 16 + 10 → Learners: 26
Teacher: 84 + 10 → Learners: 94
Teacher: 96 + 10 → Learners: 106 and so on...

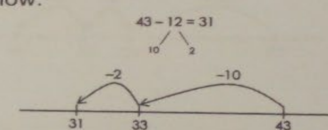
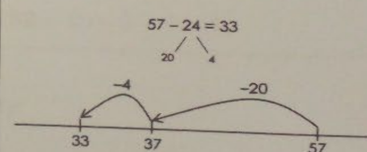
b. Subtract 10 (or subtract a multiple of 10)

Same method as '10 less', but this time the teacher offers a 'subtract 10' problem.

Teacher: 56 – 10 → Learners: 46
Teacher: 84 – 10 → Learners: 74
Teacher: 95 – 10 → Learners: 85 and so on...

Task Sequence

In this lesson we use jump strategies to solve subtraction calculations.

Ask the learners to try to remember from yesterday: How did we solve 43 – 12? Allow learners to describe the method to their seat partner, and then ask one pair to show it on the board. Remind learners of steps they learnt previously: **plot, break down, jump** (backwards) and **answer**.	The final, full image is shown below: 
Show on the board how to solve: 57 – 24 Plot '57' on the number line. Break down the 24 into 20 and 4. Jump backward 20 and then jump backward 4. Some learners will make two backward jumps of 10 instead of one backward jump of 20 - this is okay. Write the answer. Teacher: So 57 – 24 is the same as 57 – 20 – 4 = 33. Write the number sentence as shown.	The final, full image is shown below: 

Individual Tasks

Learners should now try the following examples *mentally*:

95 – 23 43 – 22

Learners should explain their thinking, e.g., "for 95 – 23, I subtract 20 from 95 to get 75, then subtract 3, so the answer is 72."

Tell learners NOT to count in 1s.

Learners who struggle to do this mentally can draw rough number lines to help them.

Appendix I

JUMP STRATEGIES: LESSON STARTER 7

1-Minute Mental Warm-Up

Jumping to the multiple of ten **before**

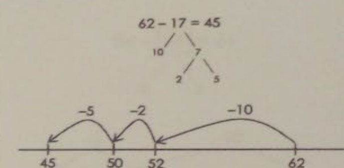
This is not rounding to the nearest ten but jumping to the multiple of ten **before** on the number line.

"Give me the multiple of ten that comes **before**…"

Teacher: 26	→	Learners: 20	
Teacher: 53	→	Learners: 50	and so on…

Task Sequence

In this lesson we extend jump strategies to include a bridging through ten step.

Show on the board how to bridge through ten to solve: 62 – 17 Plot '62' on the empty number line. Break down the 17 into 10 and 7. Jump backward 10 to reach 52. Jump back the remaining 7 by bridging through the multiple of 10 before (this is 50). So the 7 needs to be broken down into 2 and 5. Jump backward 2 and 5. Write the answer.	The final, full image is shown below: 
Teacher: So 62 – 17 is the same as: 62 – 10 – 7 = 45 or 62 – 10 – 2 – 5 = 45	62 – 10 – 7 = 45 62 – 10 – 2 – 5 = 45

Individual Tasks

Learners should now try the following examples *mentally*:

75 – 18   93 – 14

Learners should explain their thinking, e.g. "for 75 – 18, I subtract 10 from 75 to get 65, then subtract 5 to get 60, then subtract 3, so the answer is 57".

Tell learners NOT to count in 1s.

If any learners finish these tasks quickly, give them more to practice:

73 – 28   62 – 35

Take-Home Task – Worksheet 2

At the end of today's session give learners Worksheet 2.

You do not need to time learners as they do this worksheet. The aim is to give learners some written practice of the work they have done mentally.

Appendix J

JUMP STRATEGIES: LESSON STARTER 8

1-Minute Mental Warm-Up

a. Jumping to the multiple of ten **before**

b. Subtract multiples of 10

Task Sequence

In this lesson we use jump strategies to solve missing number problems.

Problem: 84 – □ = 61

Plot '84' on the number line.

Teacher: We need to jump back to 61.

Mark '61' on the number line.

Teacher: What tens jump and what units jump should we make?

Learners: Minus 20 to get to 64 and minus 3 to get to 61

Draw these jumps on the number line.

Ask: How much did we jump back in total?

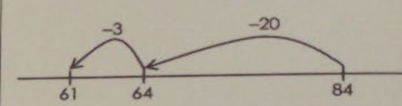
Learner: 23

Write the answer in the block.

Teacher: So 84 – 20 – 3 is the same as 84 – 23 = 61

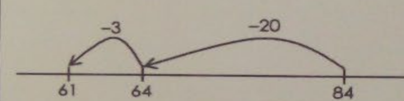
Write number sentence as shown.

84 – □ = 61



84 – $\boxed{23}$ = 61

20 3



Individual Tasks

Learners should now try the following examples *mentally*:

75 – □ = 62 93 – □ = 69

Learners should explain their thinking, e.g. "for 75 – □ = 62, I subtract 10 to get 65, then subtract 3 to get 62, so the total jump is 13."

Tell learners NOT to count in 1s.

If any learners finish these tasks quickly, give them more to practice:

75 – □ = 63 94 – □ = 69

Appendix K

Fluency Shifts Pre to Post-Test																						
Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Gains	%
Pre-test fluency items	28	29	26	21	19	11	6	9	3	4	0	2	4	4	0	0	0	0	0	0	166	8,3
Post-test fluency items	25	28	29	17	22	17	19	14	14	12	1	3	10	9	0	6	4	1	4	3	238	11,9
Shifts across items																					72	3,6

Strategic Calculating Shifts Pre to Post-Test							
Item	21	22	23	24	25	Gains	%
Pre-test Strategic calulations	17	9	2	0	7	35	7
Post-test strategic calculations	8	14	1	2	6	31	6,2
Shifts across items							-0,8

Strategic Thinking Shifts Pre to Post-Test							
Item	26	27	28	29	30	Gains	%
Pre-test strategic thinking	0	0	0	0	0	0	0
Post-test strategic thinking	3	2	1	0	0	6	1,2
Shifts across items							1,2

Appendix L

10 Strong Performing Learners

Learner Number	Pre	Post	Gains	
9	10	43,3333333	33,3333333	
16	6,66666667	40	33,3333333	
8	16,6666667	50	33,3333333	
12	13,3333333	43,3333333	30	
21	40	70	30	Selected learner
17	23,3333333	46,6666667	23,3333333	Selected learner
37	43,3333333	66,6666667	23,3333333	
10	16,6666667	36,6666667	20	
22	10	30	20	
41	20	40	20	

13 Low Performing Learners

Learner no.	Pre	Post	Gains	
13	0	0	0	Selected learner
31	0	0	0	
40	3,33333333	3,33333333	0	
27	30	26,6666667	-3,3333333	
34	20	16,6666667	-3,3333333	
39	30	26,6666667	-3,3333333	
23	13,3333333	10	-3,3333333	
42	6,66666667	0	-6,6666667	
6	40	30	-10	
18	13,3333333	0	-13,333333	
28	23,3333333	6,66666667	-16,666667	
29	23,3333333	6,66666667	-16,666667	
15	36,6666667	6,66666667	-30	Selected learner

Appendix M

Grade 3 Adapted LFIN Assessment

Learner name: ______________________ Grade 3__ D.O.B. ____________ G / B

School: __________________ Teacher: _____________ Started at ____ ended at ____

1. **Forward Number Word Sequence (FNWS)**

   *Start counting from ** and I'll tell you when to stop*

   a) 93 – 104

   b) 68 – 81 [if a is incorrect; two errors]

2. **Number Word After (NWA)**

   Say the number word that comes straight after two. [practice question]

   a) 59 92 70

   b) 14 29 11 20 [if two items from a is incorrect]

3. **Backward number word sequence (BNWS)**

   *Start counting backward from ** and I'll tell you when to stop*

   a) 72 – 67

   b) 34 – 19 [if a is incorrect; two errors]

4. **Number Word Before (NWB)**

   Say the number word that comes just before three. [practice question]

   a) 50 67 100

   b) 17 21 13 30 [if two items from a is incorrect]

5. **Identifying numerals**

   a) Flash numeral cards: 341 206 820

   b) Flash numeral cards: 51 90 78 [if two items from a is incorrect]

6. **Subtraction Sentences** (written on cards)

   Present the task as a written number sentence on a card. Ask the child, *"What does this say? Can you tell me what the answer is?"*

   a) 16 – 12 b) 17 – 14

7. **Missing Subtrahend Tasks** (use counters of one colour only)

   Here are ten counters. Ask the child to close her eyes and screen 4 counters. *There were 10 counters. While your eyes were closed I took some away. Now there are only 6. How many did I take away?*

Appendix N

Grade 3 Adapted LFIN Assessment

10 – [] = 6 [practice question]

a) 15 – [] = 11

b) 27 – [] = 23

8. Doubles (with double ten frame cards)

Explain the structure of the double ten frame. Show double ten frame cards for two seconds.

i) How many dots on the top/bottom row?

ii) How many altogether?

a) 7 + 7 b) 8 + 7

c) 9 + 9 d) 8 + 9

9. Combining to make ten

a) *Tell me two numbers that add up to 10.*

b) *Tell me two other numbers that add up to 10.*

c) *Can you tell me another two?*

10. Incrementing by ten off the decuple (using strips)

Place the four dot strip on the desk. *Can you count how many dots there are?*
Place a ten strip to the right of the four strip. *How many dots are there altogether?*
If the child starts counting remind her that a strip has ten dots – we don't need to count it.

Continue placing ten strips to the right of the four strip. *How many dots are there altogether?*

24 34 44 54 64 74 84

11. Counting by ten on the decuple (using strips) ***** **Only if Q10 is incorrect** ****

Explain to the child that each strip like this (show ten strip) has ten dots. She does not have to count it.

a) Put down a ten strip. *How many dots do we have?* If the child counts in one remind her that there are ten dots on each strip.

b) Put down 1 ten strip at a time to 9 strips, asking *How many altogether?*
10 20 30 40 50 60 70 80 90

c) Pick up at the strips. How many dots do we have?

d) How many strips are here?

2

Appendix O

Grade 3 Adapted LFIN Assessment

12. Base-ten thinking (using strips)

a) Place a ten strip on the desk. *How many dots are there?*

b) Place a three strip to the right of the ten strip. *How many dots now?*

c) Repeat with 10 and 8

d) *I have [total] dots on this paper. Here are [] dots* (uncover one side), *how many are under here?*

10+[]=13   6 +[]=16

e) Place out a ten strip. *How many dots are there?*

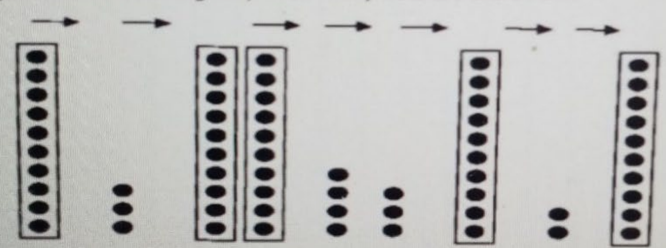
Place a five strip to the right of it. *How many dots do we have now?*

If the child answered 15, then ask how many more dots are needed to make 25.

10+5 = []   15+[] = 25

13. Uncovering Task (Board 1)

Upon each uncovering ask, *"How many dots are there now?"*



___ ___ ___ ___ ___ ___ ___ ___

14. Written number sentences

Can you read this card and tell me the answer please?

a) 16+10   [if correct as known fact]   So what is 16+9?

If 16+10 was answered correctly as a known fact without unit counting. (a structured number line or paper and pencil are allowed)

b) 42 + 23   [if correct without unit counting]   38 + 24

c) 56 – 23   [if correct without unit counting]   43 – 15

****** DO Q15 AND Q16 ONLY IF LEARNER CANNOT DO Q14 ******

Appendix P

Grade 3 Adapted LFIN Assessment

15. Additive Tasks (use counters of two colours and two screens)

There are three red counters under here (flash and screen) and two yellow counters under here (flash and screen). How many are there altogether?

3 + 2 [both collections screened – practice question]

a) 5 + 4 9 + 6 [both collections screened – entry tasks]

b) 5 + 2 9 + 4 [first collection screened – less advanced tasks]

16. Missing Addend tasks (use counters of two colours and one screen)

Here are four red counters. Now close your eyes. While your eyes were closed I put some more yellow counters under here. Now there are 6 counters altogether. How many yellow counters did I put under here?

4 + [] = 6 [practice question]

a) 7 + [] = 10

b) 12 + [] = 15

17. Missing Addend/Subtrahend (a structured number line or paper and pencil are allowed)

Can you read this card and work out the answer please?

a) 65 + __ = 82 [if correct without unit counting] 57 + __ = 100

b) 94 – __ = 78 [if correct without unit counting] 82 – __ = 67

18. Start Unknown (a structured number line or paper and pencil are allowed)

Can you read this card and work out the answer please?

a) __ + 12 = 37 [if correct without unit counting] __ + 35 = 79

4

Appendix Q

		Strong Learner	Strong Learner	Average Learner	Average Learner	Low Learner	Low Learner
LFIN	Item	Learner 21	Learner 17	Learner 32	Learner 3	Learner 13	Learner 15
FNWS	93-107	rapid recall	incorrect response	rapid recall	incorrect response	incorrect response	rapid recall
	68-89	rapid recall	rapid recall	rapid recall	rapid recall	incorrect respone	rapid recall
NWA	59 - 60	rapid recall	rapid recall	rapid recall	rapid recall	incorrect respone	rapid recall
	92-93	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall
	70-71	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall
BNWS	72-65	rapid recall	incorrect respone	rapid recall	rapid recall	incorrect respone	rapid recall
	34-27	rapid recall	incorrect respone	rapid recall	rapid recall	incorrect respone	rapid recall
NWB	3 to 2	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall
	50-49	rapid recall	rapid recall	rapid recall	rapid recall	incorrect respone	rapid recall
	67-65	incorrect response	rapid recall	rapid recall	rapid recall	incorrect response	rapid recall
	100-99	rapid recall	rapid recall	rapid recall	rapid recall	incorrect respone	rapid recall
N. ID	341	rapid recall	rapid recall	rapid recall	rapid recall	incorrect respone	rapid recall
	206	rapid recall	rapid recall	rapid recall	rapid recall	incorrect respone	rapid recall
	820	rapid recall	rapid recall	rapid recall	rapid recall	incorrect respone	rapid recall
	16 minus 12	Strategic thinking	Strat calcul (unit counting)	Strat calcul (unit counting)	Strat calcul (unit counting)	incorrect respone	strategic thinking
	10-_=6	rapid recall	Strat calcul (unit counting)	Strat calcul (unit counting)	Strat calcul (unit counting)	rapid recall	rapid recall
	15-_=11	rapid recall	rapid recall	incorrect response	rapid recall	Strat calcul (unit counting)	Strat calcul (unit counting)
	27-_=23	rapid recall	Strat calcul (unit counting)	incorrect response	Strat calcul (unit counting)	incorrect respone	Strat calcul (unit counting)
		rapid recall	rapid recall	Strat calcul (unit counting)	rapid recall	incorrect response	rapid recall
		rapid recall	no response	Strat calcul (unit counting)	rapid recall	incorrect response	rapid recall
		rapid recall	no response	incorrect response	rapid recall	incorrect response	rapid recall
	10,13,23,33,37	rapid recall	Strat calcul (unit counting)	incorrect response	Strat calcul (unit counting)	incorrect response	Strat calcul (unit counting)
	10+3=	rapid recall	rapid recall	rapid recall	rapid recall	Strat calcul (unit counting)	rapid recall
	10+8=	rapid recall	Strat calcul (unit counting)	rapid recall	rapid recall	incorrect response	rapid recall
	16+10=26	Strategic thinking	rapid recall	rapid recall	Strat calcul (unit counting)	Strat calcul (unit counting)	strategic thinking
		see relational aspect	see relational aspect	do not see relational aspect	do not see relational aspect	do not see relational aspect	do not see relational aspect
	16+9=25	rapid recall	rapid recall	no response	Strat calcul (unit counting)	incorrect response	Strat calcul (unit counting)
	14, 24, 34, 44, 54	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall	rapid recall
Number line	16+30=	Strategic thinking	strategic thinking	rapid recall	Strat calcul (unit counting)	incorrect response	rapid recall
	6+30=36	rapid recall	rapid recall	rapid recall	rapid recall	incorrect response	rapid recall
	7+10=17	rapid recall	rapid recall	rapid recall	rapid recall	incorrect response	rapid recall
	Fluency	27 Rapid Recall	19 Rapid Recall	21 Rapid Recall	22 Rapid Recall	5 Rapid Recall	24 Rapid Recall
	Strategic Calculatin	SEAL 5 (none-count-by-ones)	SEAL 4 (count-down-to strat)	SEAL 3 (count-on & count-down-from)	SEAL 3 (count-on)	SEAL 2 (count from one)	SEAL 2 (count from one & count on)
	Strategic Thinking	3	1	0	0	0	2

Appendix R

LETTER TO THE PRINCIPAL, SGB CHAIR

Cavendish Primary
School
6520 Cavendish Street
Extension 6
Eldorado Park
1812
Date: 29
March 2022

Dear: Mr. Louw

My name is ***Lincoln Lavans*** and I am a ***Master's of Education*** student in the School of Education at the University of the Witwatersrand. I am doing research on ***exploring additive reasoning with grade three learners through the use of jump strategies on an empty number line in a government school in Gauteng.***

My research will focus on the strategies that Grade 3 learners use when working with additive reasoning problems. My aim is to investigate this issue through the design and implementation of a small-scale intervention program. This study will comprise of tests (a pre-test and post-test) as well as 8 intervention lessons and virtual interviews with 6 learners. Both the pre and post-test will be carefully administered and analyzed. These tests will analyze learner's strategies of working with additive reasoning problems. In between the pre-test and post-test, I will be conducting 8 intervention lessons. These lessons will be conducted and form part of the first 20 minutes of each day for a total of eight days. The intervention lessons will equip learners with different reasoning skills of working with additive reasoning problems.

After the pretest and intervention lessons, I will conduct a post-test which will be further analyzed. After every intervention lesson, I will collect samples of learners work in the learner exercises they do after the lessons. Part of my research will comprise of me using learner's tests, written work, video recorded interviews. The focus of my analysis will be on the reasoning that learners use when working with additive reasoning problems. Interviews will be conducted virtually during school time and will be conducted in the staff room for no longer than 15 minutes. These interviews will be done with learners from the sample group (6 learners from the sample group) and will be done to gain further insight into the models and strategies that learners use.

The reason why I have chosen your school is because I was a learner at your school and I am greatly interest in improving mathematics education. I believe that this study will help benefit me as a researcher which will in turn benefit children in general. Therefore, I am inviting the school to participate in this research. The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Pseudonyms will be used in the place of the learners' names and of the school. Privacy will be maintained in all published and written data resulting from the study. The findings from this study may be used in academic journals, by other post graduate students and at conferences.

All research data will be kept safely under lock and key at my home and on my personal computer in a separate folder which requires a password. All research data will be stored on a virtual cloud after completion of the project. Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Please contact us if you require any further details. If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Yours sincerely,
Lincoln Lavans
Lincoln.lavans@wits.ac.za
0849958217

Permission Form

Jump Start Strategy Intervention

Declaration by participant

I am happy for the *Jump Start Strategy Intervention* to be run in my school, and for the learner data and interview to be collated for research purposes alongside the development purposes detailed above.

Principal name: ________________________

School: __________________

Province: ______________________________

Date: ________________

Signature: ______________________

Please add the school stamp in the box below confirming your permission for the school to be in the Master's project.

Appendix S

INFORMATION SHEET - PARENT

Cavendish Primary School
6520 Cavendish Street
Extension 6
Eldorado Park
1812
DATE: 29 March 2022

Dear Parent

My name is ***Lincoln Lavans*** and I am a ***Master's of Education*** student in the School of Education at the University of the Witwatersrand. I am doing research on ***exploring additive reasoning with grade three learners through the use of jump strategies on an empty number line in a government school in Gauteng.***

My research will focus on the additive reasoning that Grade 3 learners use when working with additive reasoning problems. My aim is to investigate this issue through the design and implementation of a small-scale intervention program. This study will comprise of a pre-test and post-test. Both tests will be carefully administered and analyzed. These tests will analyze learner's strategies of working with additive reasoning problems. In between the pre-test and post-test, I will be conducting an intervention program which will be made up of eight lessons. The intervention lessons will equip learners with different models and strategies of working with additive reasoning problems.

After every intervention lesson, I will collect samples of learners' work. The focus of my analysis will be on additive reasoning that learners use when dealing with number problems. Thereafter, I will conduct a post-test which will be further analyzed. This study will take place during the second and third term and will form part of the first 20 minutes of each day for a total of eight days. The reason why I have chosen your child's class is because I feel passionate about improving Mathematics Education and would like to support your child together with other children so that they can improve their understanding and performance in additive reasoning problems. Further I would like to develop myself through this process for the benefit of the learners.

I was wondering whether you would mind if your child participates in this study. Your child will need to complete a pre-test and post-test on additive reasoning number problems as well as possibly answer a few short questions in the form of a virtual interview which will be video recorded. Not all learners will be video recorded. Learner's approaches to word problems will be videotaped so that I could gain an in-depth understanding of their additive reasoning practices. Interviews will be done during school time in the staff room one learner at a time and will not last longer than 15 minutes. Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

The findings from this study may be used in research reports, academic journals and at conferences. However, your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study. Pseudonyms will be used in the place of his/her name. All research data will be kept safely virtual cloud with relevant passwords after the completion of the project. Please let me know if you require any further information. Thank you very much for your help.

Please contact us if you require any further details. If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Yours sincerely,
Lincoln Lavans
Lincoln.lavans@wits.ac.za
0849958217

PARENT CONSENT FORM

Kindly fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called:
Exploring additive reasoning with grade three learners through the use of jump strategies on an empty number line in a government school in Gauteng.

I, ________________________ the parent of ______________________

Permission to review/collect documents/artifacts
Circle one
I agree that my child's (Pre-Test, Post-Test and Written work) can be used for this study only. YES/NO

Permission for test
I agree to my child writing test for this study.
YES/NO

Permission to be virtually interviewed
I agree that my child may be interviewed for this study.
YES/NO
I know that he/she can stop the interview at any time and doesn't have to answer all the questions asked. YES/NO

Permission to be videotaped
I agree that my child can be videotaped during the interview.
YES/NO
I know that the videotapes will be used for this project only.
YES/NO

Informed Consent
I understand that:

- My child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- He/she does not have to answer every question and can withdraw from the study at any time.
- All the data collected during this study will be stored on a virtual cloud after completion of my project.

Sign_______________________________ Date____________________________

Appendix T

INFORMATION SHEET – LEARNERS

Cavendish Primary School
6520 Cavendish Street
Extension 6
Eldorado Park
1812
DATE: 29 March 2022

Dear Learner
My name is ***Lincoln Lavans*** and I am a ***Master's of Education*** student in the School of Education at the University of the Witwatersrand. I am doing research on ***exploring additive reasoning with grade three learners through the use of jump strategies on an empty number line in a government school in Gauteng.***

I am interested in the methods that Grade 3 learners use when working with addition and subtraction problems. I would like to investigate this problem through providing you with a program of eight lessons where different ways of working with additive problems will be shown to you. After every lesson, I will collect samples of your work. I am interested in looking at the different methods that you use when doing additive problems. Before and after the lessons, you will need to write a test on additive problems. These tests will not be for marks but will only be used for my research. This study will take place during the second and third term during the first 20 minutes the school day.

I was wondering whether you would mind if I invited you to participate in this research. I need your help with participating in completing the tests that I give to you as well as possibly answering questions in the form of a virtual interview which will be video recorded. These interviews will help me gain a better understanding of some of the work. Interviews will be done during school time and will not last longer than 15 minutes.

Remember, though there is a test, it is not for marks and it is completely voluntary, which means that you don't have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and you will not be advantaged or disadvantaged in any way. Also, no money will be paid to you for participating. I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept private in all my writing about the study. I will only be using your work for my university research project.

The findings from my research may be used in journals, post graduate students and at conferences. Also, all collected information will be stored safely and privately. All data used for my project will be kept on a virtual cloud after I have completed my project. Your parents have also been given an information sheet and consent form, but at the end of the day it is entirely your decision to join me in the study. I look forward to working with you. Please feel free to contact me if you have any questions.

Please contact us if you require any further details. If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Yours Sincerely,
Lincoln Lavans
Lincoln.lavans@wits.ac.za
0849958217

LEARNER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called:
Exploring additive reasoning with grade three learners through the use of jump strategies on an empty number line in a government school in Gauteng.

My name is: ______________________________________

Permission to review/collect documents/artifacts
Circle one
I agree that (Pre-Test, Post-Test, Your written work) can be used for this study only.
YES/NO

Permission for test
I agree to write a test for this study.
YES/NO

Permission to be virtually interviewed
I agree to be interviewed for this study.
YES/NO
I know that I can stop the interview at any time and don't have to answer all the questions asked. YES/NO

Permission to be videotaped
I agree to be videotaped during the interview.
YES/NO
I know that the videotapes will be used for this project only.
YES/NO

Informed Consent

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be video recorded.
- My test and videos can be used by other post graduate students and researchers in the field.
- All the data collected during this study will be stored on a virtual cloud after completion of the project.

Sign_____________________________ Date___________________________

Appendix U

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	13 July 2022
Validity of Research Approval:	08 February 2022– 30 September 2022 2022/294
Name of Researcher:	Lavans L
Address of Researcher:	20 Ferreira Street
	Discovery
	Roodepoort
Telephone Number:	085 995 8217
Email address:	Lincoln.lavans@wits.ac.za
Research Topic:	Exploring additive reasoning with grade three learners through the use of jump strategies on an empty number line in a government school in Gauteng.
Type of qualification	Masters of Education
Number and type of schools:	1 Primary School
District/s/HO	Johannesburg Central

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

14/07/2022

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below are met. Approval may be withdrawn should any of the conditions listed below be flouted:

Making education a societal priority

1

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za

Appendix V

1. The letter would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. **Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools, or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.**
4. **The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.**
5. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s has been granted permission from the Gauteng Department of Education to conduct the research study.
6. A letter/document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs, and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
7. The Researcher will make every effort to obtain the goodwill and cooperation of all the GDE officials, principals, and chairpersons of the SGBs, teachers, and learners involved. Persons who offer their cooperation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
8. Research may only be conducted after school hours so that the normal school program is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
9. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
10. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
11. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
12. The researcher is responsible for supplying and utilising his/her research resources, such as stationery, photocopies, transport, faxes, and telephones, and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
13. The names of the GDE officials, schools, principals, parents, teachers, and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
14. On completion of the study, the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
15. The researcher may be expected to provide short presentations on the purpose, findings, and recommendations of his/her research to both GDE officials and the schools concerned.
16. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a summary of the purpose, findings, and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr. Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: ....14/07/2022..............................

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Making education a societal priority

Office of the Director: Education Research and Knowledge Management
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Appendix W

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER:** 2022ECE021M
PROJECT TITLE	Exploring additive reasoning with grade three learners using jump strategies on an empty number line in a Government school in Gauteng.
INVESTIGATOR	Lincoln Lavans
SCHOOL/DEPARTMENT OF INVESTIGATOR	**WSOE**
DATE CONSIDERED	24 June 2022
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	No Risk
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	**CHAIRPERSON** Mbokane Dr. Batseba Mofolo-Mbokane

cc: Dr. Corin Mathews

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

Signature ____________________ Date ____/____/____

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Appendix X

Criteria for 3 categories				
	Learner	Pre	Post	Shifts
	9	10	43.333	33.333
	16	6.6667	40	33.333
	8	16.667	50	33.333
Strong	21	40	70	30
	12	13.333	43.333	30
	37	43.333	66.667	23.333
Strong	17	23.333	46.667	23.333
	10	16.667	36.667	20
	41	20	40	20
	22	10	30	20
	38	50	63.333	13.333
	2	23.333	36.667	13.333
	33	46.667	60	13.333
	1	23.333	33.333	10
	26	20	26.667	6.6667
	25	6.6667	13.333	6.6667
Average	32	6.6667	13.333	6.6667
	5	20	23.333	3.3333
Average	3	13.333	16.667	3.3333
	11	13.333	16.667	3.3333
	19	0	3.3333	3.3333
	24	6.6667	10	3.3333
	40	3.3333	3.3333	0
Low	13	0	0	0
	31	0	0	0
	27	30	26.667	-3.333
	39	30	26.667	-3.333
	34	20	16.667	-3.333
	23	13.333	10	-3.333
	42	6.6667	0	-6.667
	6	40	30	-10
	18	13.333	0	-13.33
	28	23.333	6.6667	-16.67
	29	23.333	6.6667	-16.67
Low	15	36.667	6.6667	-30