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EXPLORING THE ENGLISH PROFICIENCY-MATHEMATICAL PROFICIENCY RELATIONSHIP IN LEARNERS: AN INVESTIGATION USING INSTRUCTIONAL ENGLISH COMPUTER SOFTWARE

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A RESEARCH REPORT SUBMITTED TO THE FACULTY OF SCIENCE, UNIVERSITY OF THE WITWATERSTRAND, JOHANESBURG, IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

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DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university

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Signature

15th day of August, 2006.

EPIGRAPH

«Mathematics learning is […] seen **not only** as developing competence in completing procedures, solving word problems, and using mathematical reasoning **but also** as developing sociomathematical norms (Cobb *et al.*, 1993), presenting mathematical arguments […], and participating in mathematical discussions […]. In general, learning to communicate mathematically is now seen as a central aspect of what it means to learn mathematics.»

Moschkovich (2002: 192; *my emphasis*).

ABSTRACT

The difficulty of teaching and learning mathematics in a language that is not the learners' home language (e.g. English) is well documented. It can be argued that underachievement by South African learners in most rural schools is due to a lack of opportunity to participate in meaningful and challenging learning experience (sometimes due to lack of proficiency in English) rather than to a lack of ability or potential. This study investigated how improvement of learners' English language proficiency enables or constrains the development of mathematical proficiency. English Computer software was used as intervention to improve the English Language proficiency of 45 learners. Statistical methods were used to analyse the pre- and post-tests in order to compare these learners with learners from another class of 48. The classroom interaction in the mathematics class before and after the intervention was analysed in order to ascertain whether or not the mathematics interaction has been enabled or constrained. The findings of this study were that proficiency in the language of instruction (English) is an important index in mathematics proficiency, but improvement of learners' language proficiency, even though important for achievement in mathematics, may not be sufficient to impact on classroom interaction. The teacher's ability to draw on learner's linguistic resources is also of critical importance.

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CHAPTER ONE

GENERAL INTRODUCTION TO STUDY

The debate surrounding the relationship that exists between language and mathematics and between language and mathematics learning is not new in research into the teaching and learning of mathematics. Neither is the debate around the value and use of technology in education (that is, how computer-based technology – for example, software - can promote learning) a recent preoccupation of mathematics education researchers. Most research into the impact of instructional computer programmes have, however, focused on the use of particular instructional mathematics programmes to ascertain the extent to which these programmes impact on children's mathematical understanding (e.g. Funkhouser, 1993; Wenglinsky, 1998). The present study makes a shift from the investigation into how mathematics computer programme shape mathematical proficiency. The study reported here investigated how the improvement of learners' English proficiency (using the English literacy computer software – ASTRALAB – designed to promote English proficiency) enables or constrains the development of mathematical proficiency in learners. The study was organised to answer the following major question:

- How does improving learners' proficiency in English enable or constrain mathematical proficiency?

The two interrelated questions below provide more depth to the key question above and serve as subsidiary questions to the study:

- How does the use of the ASTRALAB instructional English literacy software enable or constrain learners' mathematical proficiency?
- To what extent does improving English language through the use of the ASTRALAB software enable or constrain interaction in the mathematics classroom?

Purpose and Objectives of Study

The difficulty of teaching and learning mathematics in a language that is not the learners' home language (e.g. English) is well documented. It can be argued that underachievement by South African learners in most rural schools is due to a lack of opportunity to participate in meaningful and challenging learning experiences sometimes due to lack of proficiency in English rather than to a lack of ability or potential. For most South Africans, therefore, whose first language is not English, the idea of a computer software with a supposedly dual purpose of improving proficiency in English and (by so doing), of improving mathematical proficiency, is a welcome and desirable attempt to enhance mathematical understanding. The specific objectives of the study were as follows:

1. To describe the ASTRALAB program itself, to mark out the language skills it privileges and makes available to learners.
2. To conduct a comparative analysis of learner achievement between learners who used the ASTRALAB programme and those who did not.
3. To do an analysis of the teacher-learner and learner-learner classroom interaction in the mathematics class before and after the implementation of the ASTRALAB program in order to explore whether or not the mathematics communications have been enabled or constrained.

Significance of Study

Most research dealing with language issues in mathematics education have documented that proficiency in the language of learning and teaching is important for mathematical proficiency (Adetula, 1990; Gay & Cole, in Zepp, 1981; Howie, 2002; Setati & Adler, 2000; Moschkovich, 1996, 1999; Setati, Adler, Reed & Bapoo, 2002; etc.). In South Africa, where most learners learn mathematics in a language which is not their first or home language, underachievement in Matric examinations (mathematics) has been found to be more prevalent amongst learners who use English language less frequently at home (Simkins in Taylor, Muller & Vinjevold, 2003) and in areas where English is less frequently used at home. There are researchers who argue that the solution to improving second language learners' performance in mathematics is to develop their English language proficiency (Howie, 2002).

They (researchers) maintain that proficiency in English language is directly related to performance in mathematics. None of these researchers have, however, sought to directly improve English proficiency (through an experimental treatment) in the attempt to investigate how this would impact on mathematical proficiency. Thus, previous research into the relationship between English language proficiency and mathematics proficiency was not done in a classroom where there was an explicit attempt to improve learners' language proficiency using computer software. The present study seeks to investigate whether and how the development of proficiency of the one (language) through the ASTRALAB programme can enable or constrain proficiency of the other (mathematics) in a school where there was an explicit attempt to improve learners' proficiency in English.

Definitions

This section consists of the definitions of the terms I would use throughout this research report.

Instructional Learning Software (ILS): (Also called instructional learning computer programme in this study). Software developed specifically for use as resource for the instruction of learners in a particular subject or in a particular field within a subject. It usually consists one or more of the following: computerised drills, practical tutorials, standardized tests and an evaluation system which measures the learner's progress and performance (Waldron, 2004).

Mathematical Proficiency: The use of this term in this study resonates with the understanding of this term as defined by Kilpatrick, Swafford & Findell (2001). They define Mathematical proficiency by five interwoven strands necessary for learners to successfully learn mathematics:

- *Conceptual understanding* – "refers to an integrated and functional grasp of mathematical ideas" (p. 118). Conceptual understanding stands in opposition to rote learning.

- *Procedural fluency* – the "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121)
- *Strategic competence* – "the ability to formulate mathematical problems, represent them, and solve them" (p. 124)
- *Adaptive reasoning* – the capacity for logical thought, reflection, explanation and justification of the relationships among concepts and situations (p. 116, 129).
- *Productive disposition* – "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 116).

I will elaborate more on mathematical proficiency in Chapter three when I deal with the nature of mathematical knowledge.

English Proficiency: Well developed capacity in the four basic (English) language skills: listening, reading, speaking and writing, and fluency and facility thereof in the use of these skills in English.

Mathematical Understanding: This word is imbedded in our understanding of the term mathematical proficiency even though the latter is more encompassing in the definition of what it takes for learners to successfully learn mathematics. In this study, no distinction is made between the two terms (mathematical proficiency and mathematical understanding). They are used interchangeably.

Bilingualism/Multilingualism: The concept of bilingualism is an elusive one, especially when is comes to definition. Who is bilingual? Who is multilingual? As Baker (1988) notes, given the number of language skills (reading, writing, listening, speaking, etc) in all languages and the context where the language can or cannot be used, categorisation of who is or who is not bi/multilingual becomes fraught with difficulties. For example, a learner may have greater skills in reading and speaking Zulu but a poor skill in writing this language. This same learner may have a well developed skill in writing English but a poor skill in speaking English. Is this learner bilingual? Are bi/multilinguals those with well developed and equal skills (reading, writing, listening, speaking) in two/more than two languages? In this study, bilingualism is referred to as competence in more than one language (Edwards, 1994); that is, the relative ease in the knowledge and use of both the main (African) language and English language. Multilingualism refers to competence in more than two languages (Edwards, 1994); that is, knowledge of, and relative ease in the use of more than one African language and English language.

What this Study is not about

The study does not investigate into how much of each of the specific English skills (vocabulary, reading comprehension, reading speed, listening, structure and expression) are improved through the use of the ASTRALAB software and how each of these skills in English constrain or enable mathematical proficiency respectively.

CONCLUSION

This chapter has given a global picture of what the present study is about. In doing so, it has dealt with the research questions rationale for the study, the aim, objectives and significance of the present investigation into how improving learners' proficiency in English enables or constrains proficiency in mathematics.

In Chapter two, I describe the ASTRALAB English literacy software, marking out the English language skills it privileges and makes available to learners, and describing how the software was implemented in the experimental school.

Chapter three deals with theoretical orientation which informs the study and the review of related literature

The research design and the method of data collection used in the study are described in chapter four where I discuss the population, sample, control of variables, the research instruments, validity and reliability.

Chapter five provides answer to my first sub research question: How does the use of the ASTRALAB instructional English literacy software enable or constrain learners' mathematical proficiency? In dealing with this question, I provide results from the statistical analysis my data.

Chapter six provides results from the analysis of classroom interaction in the bid to respond to the question as to whether or not the mathematics interaction in the class has been enabled or constrained after the intervention with the ASTRALAB programme.

In Chapter seven, I draw conclusions from the study and make recommendations for future research.

CHAPTER TWO

THE ASTRALAB ENGLISH LITERACY SOFTWARE

INTRODUCTION

The purpose of this chapter is to describe the ASTRALAB program itself, to mark out the language skills it privileges and makes available to learners. In doing this, the chapter attempts to provide answers to the questions: what is the ASTRALAB English Literacy Software? What does it do? How is it used? In the experimental school, how was it implemented with the experimental group?

THE ASTRALAB ENGLISH LITERACY SOFTWARE

ASTRALAB provides a unique approach to the practice and reinforcement of reading comprehension by building vocabulary, spelling, and reading fluency, while testing overall comprehension.

The software is divided into lessons. The following are included in each lesson:

- Vocabulary is taught by having learners type words after they have been flashed on the screen and pronounced to the hearing of learners. This is the “Flash & Type” exercise.
- Learners are also required to use words in a short cloze exercise in which they complete a short paragraph using the listed words. This is the “Fill in the Blank” activity.
- Reading fluency is also taught in each lesson through the “Read the Story” activity. The activity provided two methods of reading: “Timed Reading” uses controlled reading to develop left-to-right fluency. Learners’ reading fluency is developed by having learners read a story which is displayed in a left-to-right fashion at various

speeds ranging from 50 to 600 words per minute; "Read At Your Own Pace" allows each learner to read the story again as quickly or as slowly as is necessary for comprehension.

- Comprehension questions then follows in each lesson to determine learners' comprehension of the story at the chosen speed. "Comprehension Check" determines how well the learner understood the story. Various comprehension skills are evaluated in each lesson. A poor score indicates the story was read too quickly and implies a slower reading speed.

- "Vocabulary Review" exercise in which learners complete a sentence using the appropriate word in a list of words, reinforces new vocabulary in used in the context of the story.

- And finally, one of the three word games – "Word Search", "Crossword Puzzle" and "Word Roll" – reinforces the spelling of the new words (Steck-Vaughn/EDL, 1998)

Why ASTRALAB?

The experimental school is running an ASTRALAB pilot project in one class and thus this was the experimental class. The ASTRALAB programme is used in this study because it is, so far, the software learners in the research school are exposed to at the moment in their attempt to improve general proficiency in different Learning Areas through the improvement of proficiency in their language of learning and teaching [LoLT]. I was approached by the ASTRALAB with their claims that mathematics is the greatest beneficiary of the ability of the ASTRALAB software to improve English proficiency. As a researcher, I was not convinced of this and thus wanted to do a scientific study.

Since it was necessary to know how the software would be implanted by the programme manager, I went to the implementation phase of the programme as an observer/researcher in the course of the first week of implementation and also in the last week. In the next section, I describe how the software was implemented by the instructor who was the programme manager himself.

Implementation Phase of the ASTRALAB Programme

Even though the ASTRALAB programme in itself was designed to be used individually as instructional learning computer software, for implementation in the research school, an adapted version which used an inbuilt projector connected to the computer was used. This enabled whole class instruction and thereby avoided the fundamental criticism of computer based programme instruction as being an individualised approach where instructional situations are cold, mechanical and dehumanizing and where interaction between the teacher and learners is highly eclipsed (Hergenhahn & Oslon, 2001). Learners were required to participate daily in the whole class "teaching" using the programme for a total of 22.5 hours consisting of 30 sessions in total.

For the three weeks in which the intervention took place, the class had two sessions each of flash and type exercise, followed by filling the blank sentences in which the words used in the first lesson were supposed to be used by learners to fill the blank spaces. A reading exercise followed and after that, questions on comprehension and finally a vocabulary review.

In the "Flash & Type" exercise, learners were expected to correctly write the words that flashed thrice on the screen. This was accompanied by an explanation of what the words meant and how the words can be used in sentences. Most of the times, the instructor asked the learners to explain the meanings of the words (or in cases of polysemous words, the various meanings and contexts where the words could be used) and then he (instructor) built on learners' explanation. This means that the flash and type exercises focused on spelling and word comprehension

In the "Fill in the Blank" activity, learners were supposed to write only the answers (that were supposed to be in the gap) in the correct order on a sheet of paper and then there was a whole class discussion on the lesson where individual learners were asked to fill the gap and then the answers checked with the computer.

Before the reading exercise, the learners were asked to look at the caption to the story (in a form of a picture) and try to figure out what the story would be about. This was done in form of a whole class discussion lasting for about 10 minutes on issues surrounding the story and most often, the instructor drew on learners' concrete life experiences and contexts to further elucidate on the picture. The reading exercise with the reading speed of the day followed.

The comprehension exercise involved writing answers to the multiple choice questions in their worksheets. This was done individually and quietly while looking at the screen. Vocabulary review followed the comprehension exercise. Learners were tested on their understanding of words in the story. Then, finally, there was a whole class discussion and correction of the comprehension and the vocabulary review tests. This whole process (flash and type, filling the blanks, comprehension and vocabulary review) was repeated for a second lesson making two lessons in a total of ninety minutes a day.

English Pre-test and Post-test

On the first day of implementation, the programme instructor gave learners in the experimental group a pre-test using the software. He also gave them a post test the week following the end of the implementation of the software. The results showed a 28.2% general improvement in English proficiency from pre-test to post test.

Nieman (2006) notes that the fact that a learner understands the educator in class and is able to, with ease, read in the language of teaching and learning does not presuppose that such a learner will understand academic texts as easily and write fluently. Earlier research by Cummins (1984, in Nieman, 2006) had led him to draw a distinction between "basic interpersonal communicative skills" (BICS) and "cognitive-academic language proficiency" (CALP). While BICS denotes language proficiency in a social situation and characterised by interpersonal interaction, CALP positions itself as second level of additional language proficiency. This second level of language proficiency is what is needed if learners are to read and understand scientific reports, tasks or academic assignments in general (Nieman, 2006). From an observer point of view, it could be argued that the intervention with the ASTRALAB software, even though very interactive in nature, was biased towards the development of learners' basic interpersonal communicative skills.

REFLECTION

From the observer's point of view, it can be perceived that the ASTRALAB English Literacy Programme is not a remedial programme for low achievers. Like a gymnasium which serves both the physically fit and those who are physically weak, the ASTRALAB programme is designed to serve both the strong and the weak in the English language; both those whose first language is English and those who learn English as an additional language.

Secondly, the implementation of the ASTRALAB ILS took place in a Grade 9. The choice of Grade 9 by the ASTRALAB instructor and the school was appropriate because, in South Africa, it is the end of the senior phase and at this level, learners sit for the Common Tasks for Assessment (CTA). The CTAs are heavy with language. There's been a lot of concern that while they (CTA) are assessing mathematics, they require learners to be fluent in English. Hence, for learners to adequately engage with the CTA, they must be proficient in both English and mathematics. Implementing the programme in Grade 9 provided, therefore, an added advantage to learners.

CONCLUSION

This chapter has described the ASTRALAB ILS and explained how it was implemented in the experimental school. In the next chapter, I provide a theoretical orientation to my study and review related literature.

CHAPTER THREE

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

INTRODUCTION

This chapter provides the theoretical stance that informed this study and locates the study in the nexus of research studies and theories dealing with language issues in the teaching and learning of mathematics. The literature review is divided into two interrelated sections. The first section would deal with findings/theories related the relationship between language and mathematics learning (including the teaching and learning of mathematics in bi/multicultural contexts). The second part of the literature review would deal with research and/or theories about the relationship between reading ability and mathematical proficiency. But before all these, theories about the nature of mathematical knowledge would be explored with the view of providing an answer to the question: Is there anything intrinsic in the nature of mathematical knowledge that necessarily links it to language?

THEORETICAL FRAMEWORK

This study is informed by the socio-cultural perspective of learning posited by Vygotsky (1978), combined with the situated perspective proposed by Lavé (1991). The socio-cultural perspective proposes that learning happens through participation in cultural practices. The emphasis in Vygotsky's theory of social interaction (socio-cultural perspective) is on the centrality of culture and of social influences in human development. For the socio-cultural perspective, cognitive development results from a dialectical process whereby a child learns through problem-solving experiences shared with someone else, usually a parent or teacher but sometimes a sibling or peer (Doolittle, 1997). Thinking, reasoning and sense-making for the socio-cultural perspective come first from the social then to the individual. That is, external social forces constitute the engine that drives intellectual development. Hence, underlying the socio-cultural perspective is the assumption that learning is essentially a social process in which learners interact with one another and share ideas to help develop understanding of concepts amongst one another.

The context of the culture in which the child is enmeshed is of crucial importance in individual development. Learning involves becoming enculturated into a community of practice in which an individual finds him/herself. By enculturation is meant the picking up of, or the adoption of behaviour, norms and belief systems of a new social group to become a member of the culture (Packer & Goicoechea, in Chernobilsky *et al*, 2004). Thus, enculturation in a way requires the active involvement by individuals and is marked by the use of conceptual tools like language. Zack & Graves (2001) capture the centrality of language in Vygotsky's theory as follows:

> According to Vygotsky (1978, 1986), learners first construct knowledge in their interactions with people and activity contexts. From this perspective, knowledge and learning are considered to be social activities which are mediated by cultural artifacts and resources both symbolic (e.g. language, numeracy systems) and material (e.g. computers). While Vygotsky (1986) writes of mediational means including both material and symbolic resources, he focused much of his empirical research on the examination of the role of language as a central mechanism of learning (p. 231).

For the socio-cultural view of learning, therefore, language is essential for participation in a community of practice. It is through shared discourse that participants in the community appropriate and negotiate socially constructed meanings for vocabulary, ideas, and methods. Language allows meanings to be constantly negotiated and renegotiated by members of a mathematics community – except for the mathematics register which has a fixed meaning across contexts (Brown *et al*; Cole and Engeström, in Chernobilsky *et al*, 2004).

A critical aspect of Vygotsky's theory (socio-cultural theory) is the notion of the Zone of Proximal Development (ZPD). Vygotsky defines the ZPD as follows:

> It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (p. 86).

The ZPD constitutes the difference between what a child can do on his own and what he can do with adult intervention. Viewed through this lens, the ZPD has for condition of possibility, full social interaction. Vygotsky criticises the assumption that "instruction must be oriented towards stages that have already been completed" (Vygotstky, in Wertsch, 1979). He argues that instruction must "proceed ahead of development" if it is to lead to intellectual development. In so doing, instruction "awakens and rouses to life those functions which are in a stage of maturing, which lie in the zone of proximal development" (Vygotsky, in Wertsch 1979: 251). Learners, with the aid of their teachers are "pulled into their ZPDs by a combination of the activity [at hand], the actors, and appropriate communication (Lerman, 2001: 103).

Vygotsky's theory is a key component of the situated learning theory. For the situated perspective, learning is more than mental processes. Rather, a full account of learning can occur only by understanding that in addition to the mental processes of learning, the activities involved in learning and the context in which learning takes place also play a vital role in determining what is learned (Putnam & Borko, in Keller, 2004). Learning is a function of the activity, context and culture in which it occurs; learning is a *social practice* (Lave, 1996). Learning requires social interaction, participation and collaboration. Knowledge for the situated perspective is contextualised. The idea that knowledge and learning are actually stretched over or distributed among people, places, artefacts, and the tools of learning is an important tenet of the situated perspective of learning (Keller, 2004). Learning involves membership in a 'community of practice' where ideas are co-produced in interaction between members. This is in radical opposition to the cognitive perspective of learning. For the situated perspective, therefore, learning is defined in terms of social co-participation by members (Lave, 1996). One learns, for example, to be a mathematician by participation in mathematical practices and becoming a better participant is the yardstick for assessment and ultimately, for measuring intellectual development. Since the production of mathematical knowledge, for example, involves participation and negotiation of meaning within a community of practice, it then means that the use of language as a communicative tool is integral to the process of mathematical enquiry (Siegel & Borasi, 1994). In consonance with the historical development of some mathematical theorem which involved the concerted efforts of mathematicians who had to carefully examine each other's conclusions and search for potential counter-examples to the theorem, it can be suggested, as does Siegel & Borasi, (1994) who towed the line of Lave, that the creation of mathematical knowledge is situated in a community of practice.

REVIEW OF LITERATURE

The Nature of Mathematical Knowledge

What does it mean to know mathematics? What is it to learn mathematics? Is it context dependent or language dependent? What counts as mathematics, is still a subject of debate amongst philosophers and to a lesser degree, amongst mathematicians (Noss, 1997). Douady (1997) contends that to know mathematics involves a double aspect. It involves firstly the acquisition, “at a functional level, certain concepts and theorems that can be used to solve problems and interpret information, and also be able to pose new questions” (p. 374). Secondly, to know mathematics is to be “able to identify concepts and theorems as elements of a scientifically and socially recognised corpus of knowledge. It is also to be able to formulate definitions, and to state theorems belonging to this corpus and to prove them” (p. 375). Mathematical thinking involves, as Stein, Grover & Henningsen (1996: 456) put it, doing what makers and users of mathematics do: framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on”.

Kilpatrick, Swafford & Findell (2001) define what counts as mathematical knowledge by using five interwoven strands necessary for learners to successfully learn mathematics:

- *Conceptual understanding* – “refers to an integrated and functional grasp of mathematical ideas” (p. 118) that involves the organisation of knowledge into a coherent whole. Conceptual understanding stands in opposition to rote learning and as such, implies that learners can monitor their own thinking, make sense of their ideas and represent situations in different ways.
- *Procedural fluency* – the flexible, quick and accurate performance of appropriate procedures and an ability, thereof, to monitor the use of procedures, pick out errors and estimate whether an answer is correct or not. Learners who are procedurally fluent can minimally adapt a procedure to new situations. Procedural fluency is needed to complement the conceptual understanding in the understanding of many mathematical concepts. Procedural fluency allows the deepening of learners’ understanding of mathematical ideas or solving mathematics problems (p. 122).

- *Strategic competence* – the ability to formulate, represent and solve mathematical problems (p. 124). A learner with strategic competence has flexibility of approach and the ability to use different methods to engage with a mathematical problem. Learners need to possess strategic competence to be able to know and interpret the demands of non-routine problems.
- *Adaptive reasoning* – Kilpatrick *et al* (2001: 129) refer to adaptive reasoning in mathematics as the "glue that holds everything together, the lodestar that guides learning". It is not enough for learners to know the algorithms involved in solving a mathematics task. They need also to be able to justify and explain the logic behind their solution process. It is the capacity to logical thought, reflection, explanation and justification of the relationships among concepts and situations (p. 116, 129). Learners display reasoning when they have sufficient knowledge base, understand the demands of the task, and are familiar with the context of the mathematics task. Learners possess adaptive reasoning when they can justify their solutions. Giving learners, therefore, the opportunity to explain their solution and solution process, to " talk about the concepts and procedures they are using and to provide good reasons for what they are doing, is therefore key in a mathematics class which aim at promoting this strand" (p. 130). It is here, more than any of the other strands, that language as a communicative tool becomes very necessary in the mathematical (explanatory) process.
- *Productive disposition* – "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 116). Learners have productive disposition when they see mathematics as sensible, useful, and worthwhile; and when they believe that mathematics is not an arbitrary set of rules but a coherent set of ideas that everyone can make sense of with effort. Coupled with this is the fact that learners see themselves as effective learners of mathematics.

What role does language play in all the above notions of what counts as mathematical knowledge? How is language linked to the nature of mathematical knowledge? From the above definitions of what counts as mathematical knowledge, it can be argued, as Rotman, (1993, in Ernest, 1994: 38) does, that mathematics is an activity which uses "written inscription and language to create, record and justify its knowledge". Language plays an important role in the genesis, acquisition, communication, formulation and justification of mathematical knowledge

– and indeed, knowledge in general (Ernest, 1994; Lerman, 2001) and it is intrinsically linked to adaptive reasoning in Kilpatrick *et al*'s strands of mathematical proficiency. Mathematics textbooks, pedagogical practices, and behaviourist view of learning have, in past and present, led learners and educators alike to portray mathematics as the acquisition of ready-made algorithms and proofs through memorisation and proofs (Siegel & Borasi, 1994) and as an activity done in isolation. More and more, new approaches in the teaching and learning of mathematics are supplanting this traditional approach to mathematics and mathematics learning. With the new approaches, the role of language is increasingly foregrounded.

Language and Cognition

The days when philosophers and psychologists attempted to draw a line of demarcation between language and cognition are obviously in the wane, and have been replaced by an era where language is recognised as intricately linked to cognitive development. But even though philosophers, psychologists and educationists are in agreement that language and cognition are related, the nature of this relationship between the two remains controversial (Hofmannová, Novotná, Moschkovich, 2004). The problematic surrounding the two concepts is best captured by Lyon's (1996: 28) questions formulated thus: "Do cognitive and communicative abilities develop independently and if not, is one a necessary precursor of the other? Can children think without language, and can they use language without some cognitive structuring of reality?"

The philosopher of language, Noam Chomsky, for example was of the opinion that language and cognition are autonomous. Following Chomsky's argument, Macnamara (1977) holds that language and thought are distinct because the former is abstract with respect to the latter. The implication of this, notes Macnamara (1977), is that a child must "develop somehow both the domain of thought and, separately, the domain of language" (p. 2).

More and more, it is generally upheld that once a child has made progress in the acquisition of language, the latter would invariably enrich the thinking process of the child (Wales, 1977). Wales (1977: 31) captures the intertwinement of language and cognition in these terms:

> It [language] certainly acts as an analyser and synthesizer, plays a role in the storage and retrieval of information, gives a flexible representational system enabling the child

> to deal with the world in its absence, and having been socially elaborated it has a notation for a whole range of intellectual tools like classification, seriation, which are used in the service of thought.

Piaget (1952, 1959, in Lyon, 1996) and Vygotsky also explored the relationship between language and cognition. Piaget was more concerned with how the development of children's language could provide insight into how children learn to think (Lyon, 1996). He concluded through his investigation into children's early verbalisation that "although language is an important factor in building logical structures, it is not the essential factor, even for children with normal hearing" (Inhelder & Piaget, 1964: 4, in Lyon, 1996:13). Thus for Piaget, as Lyon (1996) puts it, "language is a series of assimilations which accelerates the process of cognitive development", "language [is] a reflection of thought and not the shaper of thoughts" (p.13). For Piaget thus, the "child's language is determined by, rather than a determinant of, cognitive operations" (Baker, 1988: 31). Vygotsky, in contrast to Piaget, posits that language makes thoughts possible (Lyon, 1996; Baker, 1988):

> The relation of thought to word is not a thing but a process, a continual movement back and forth from thought to word and from word to thought… Thought is not merely expressed in words; it comes into existence through them (Vygotsky, 1992: 125)

In contrast also to Piaget, cognitive abilities begin through the internalisation of social exchanges such as language according to Vygotsky; hence cognition is a function of language (Lyon, 1996; Legendre, 2005). For Vygotsky therefore, development can only be understood as a social activity such as the "sign system" (speech) "which is used as a psychological tool to master higher mental processes" (Lyon, 1996: 29).

This study positions itself in the second category - that is, that language and cognition are inextricably linked together and even complementary to each other in the mathematical or intellectual development of learners.

Bi/Multilingualism and Cognition

In the area of bilingualism and cognition, as with language and cognition, differences of opinion abound as to whether or not bilingual learners are more cognitively advantaged compared to their monolingual counterparts. Research review done by Palij & Homel (1987, in

Lyon, 1996; Baker, 1988) on the relationship between bilingualism and cognition reveals that until 1962, most research reveal that bilingual learners were disadvantaged cognitively. Such research (or theory – example, Macnamara's "balance effect" theory) tended to conclude (or postulate) that this was due to the dissipation of the stock of their available intellect in knowing two languages or in learning an additional language (Cummins, 1979). Later research (Peal & Lampert, 1962; Bialystock, 1992; in Lyon, 1996; Clarkson, 1992) however proved the contrary and showed that bilingual learners consistently outperformed their monolingual counterparts. Vygotsky (1962) upholds that cognitively, bilinguals have an advantage over monolinguals.

The most significant work done in the area of bilingualism and cognition is that of Cummins (1978, in Lyon, 1996) in the development of the threshold theory. The threshold theory attempted to explain why some studies reported bilingual learners as cognitively disadvantaged compared to monolingual learners while others reported bilingual learners as more cognitively advantaged than their monolingual counterpart (Lyon, 1996). The threshold theory is therefore one theoretical position that explains negative and positive findings as far as bilingual education is concerned. It stipulates that "those aspects of bilingualism which might positively influence cognitive growth are unlikely to come into effect until the child has attained a certain minimum or threshold level of competence in his second language" (Cummins, 1978, in Lyon, 1996: 57). Distinguishing two levels of threshold, Cummins (1978) postulates that the lower threshold is sufficient to avoid the negative cognitive and academic effects of bilingualism (in the semilingualism zone) but the higher threshold is necessary to reap the positive benefits of bilingualism (Cummins, 1979). The diagrammatical representation of Cummins' threshold theory better elucidate what is said above:

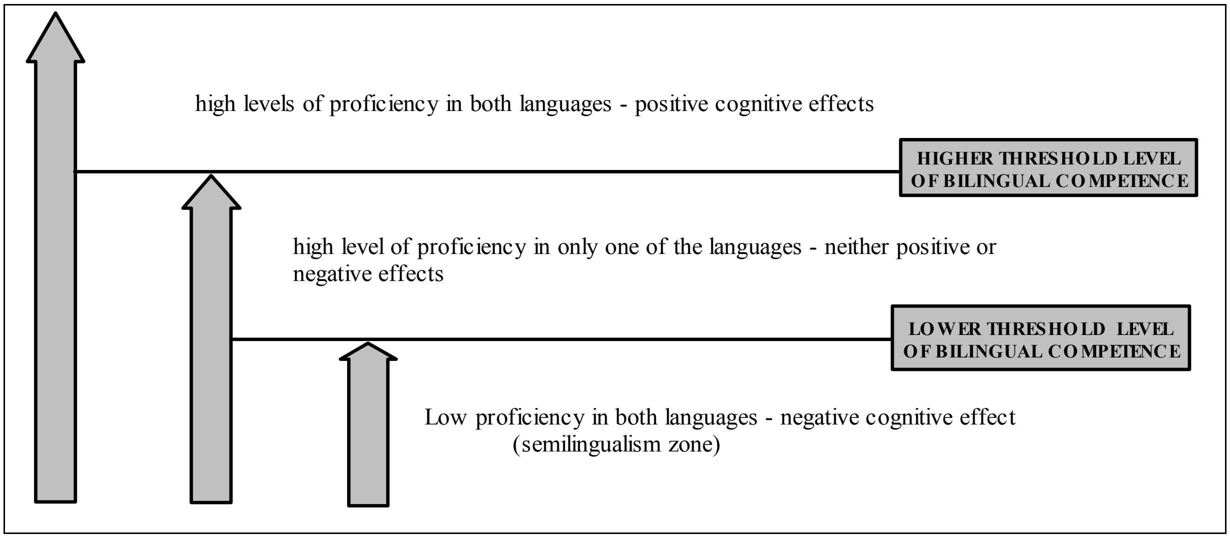

FIGURE 3.1: *Bilingualism, cognitive functioning and the thresholds theory (Adapted from Cummins, 1979)*

Cummings (1979), in explaining research studies which produced a negative relationship between bilingualism and cognitive processes, suggests that a key reason for such a relationship is inadequate level of language proficiency in the bilingual subjects (that is, bilinguals who fall below the lower threshold level – see diagram). This means that once a learner has attained a certain level of linguistic competence in his/her second or third language, positive cognitive results can occur (Baker, 1988), and the further the child progresses towards proficient bilingualism, "the greater the probability of cognitive advantages (Baker, 1988: 175). Combining the developmental interdependence hypothesis and the threshold hypothesis, Cummins (1979) also suggested that cognitively beneficial bilingualism can be achieved only when the learners' first language is adequately developed.

Clarkson's (1992) findings resonate with Cummins' model above. Clarkson (1992) investigated into the effect of bilingualism (children's own language and English language) on their capacity for learning in school. The study was prompted by the need to provide research evidence that could inform the debate according to which the use of own language by learners impeded or enhanced mathematical understanding. The subjects of the study were 232 Papua New Guinea students from five primary schools who were bilingual and 69 monolingual students, both groups in the sixth year of schooling. The research findings reveal that bilingual students with proficiency in both mother tongue and English outperformed students who were proficient in only one of either mother tongue or English, and bilingual students with low competence in both languages performed very poorly.

LANGUAGE ISSUES IN THE UNDERSTANDING OF MATHEMATICS

After decades of relative under-emphasis of the importance of language in mathematics teaching and learning, the past three decades has witnessed the mathematical education community across several countries grappling to redefine the role of language in school mathematics (Pimm & Keynes, 1994). From the notion of mathematics as dependent on proficiency in language, mathematics educators now have the role of promoting and encouraging written and oral fluency in the language of mathematics (Pimm, 1981). Any teaching or learning of mathematics involves activities of reading, writing, listening and discussing (Pimm, 1994). Language serves as a medium through which these mathematical activities are made possible. David Pimm (1994) highlights four different contexts in a mathematics classroom through which the relationship between language and mathematics are made manifest:

- The spoken language of the mathematics in classroom (including both teacher and student talk)
- The use of particular words for mathematical ends (often referred to as the *mathematics register*)
- The language of texts (conventional word problems or textbooks as a whole, including graphic material and other modes of representations).
- The language of written symbolic forms (p. 159).

As indicated earlier, various mathematics research into the interplay between language and mathematics point to the intricate link between language proficiency and mathematical aptitude. Souviney (in Clarkson, 1991) investigated with grades 2, 4 and 6 learners with various language and mathematical instruments and on eight measures of cognitive development. His study revealed that correlations between memory measures and mathematics achievements decreased as learners move to higher grades while correlations between measures in language and cognitive development, and mathematics achievement increased as learners advance in grades. From these results, Souviney (1983, in Clarkson, 1991) concluded that in higher grades, language abilities rather than memory skills are responsible for success in mathematics.

Language and Mathematics in the Curriculum

Curriculum 2005 takes an approach that is learner-centred, outcomes-based, and geared towards an integration of knowledge that traverses and transcends disciplinary boundaries (Odora, 2001). To what extent does curriculum advocate the integration of mathematics and language? The excerpt below from Curriculum 2005 document, more than any other, specifically brings to bear the recognition of the relationship between mathematics and language. It, more specifically, recognises the importance of competence in language as a necessity towards mathematical understanding and social interaction as key in the development of mathematical understanding:

> Mathematics is the construction of knowledge that deals with qualitative and quantitative relationship of space and time. It is a human activity that deals with patterns, problem-solving, logical thinking, etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction. (Department of Education, 1997, MLMMS: 2)

Embedded in the above except is the emphasis on social interaction in the mathematics class between the teacher and learners and between learners themselves where exploratory talk plays an important role as a classroom practice (Adler, 2001). Hence, in the policy document (C2005), language is seen a communicative tool in the mathematics classroom (Setati, 2005). The role of language in mathematics is also emphasised in the Revised National Curriculum Statement (RNCS). The RNCS – an upshot of the review of Curriculum 2005 – proposes that one of the skills that mathematics ought to promote in learners should be that of reasoning and communication.

Bi/Multilingualism and Mathematics Understanding

The disadvantages or advantages of bi/multilingualism for achievement especially in mathematics (and Science) education have been divisive over the century. On one end of the spectrum, a camp holds that facility in two (or more) languages leads to less room in mathematical skills. Those who uphold this thesis believe that full attainment in mathematics (and the Sciences) may be at risk in bi/multilingual children because of the demands of learning in a second language or through two languages (Baker, 1988). MacNamara's famous and much criticised research in the early 1960s on the mathematics achievement of learners from English speaking homes when taught arithmetic in the medium of Irish fall into this category. MacNamara's sample consisted of 1,084 learners from 119 schools in Ireland. Each of the learners was given tests in problem arithmetic and mechanical arithmetic. The study revealed that children from English speaking homes were behind on problem arithmetic by 11 months but not behind on mechanical arithmetic (Baker, 1988). His conclusion was that Irish bilingual education has a negative consequence (Baker, 1988).

Later researchers, notably Cummins, disputed the above findings arguing that achievement in problem arithmetic involves language as well as arithmetic skills (Cummins, in Baker, 1988).

Some research points to first language learners as having an edge over second and third language learners in that learners' first language can be a resource and help learners participate more fully in discussing mathematical concepts (Moschkovich, 1996; 1999). Other research done in the area have, however, provided overwhelming evidence to the contrary. Zepp's (in Durkin, 1991) study, to cite but one example, in which he compared the mathematics performances of learners working in different languages proved that first language learners have no inherent advantage over second or third language learners. One of his experiments consisted in giving Sesotho-speaking learners a mathematics test in their own language and in English. The results showed no evidence of superiority in the own-language version of the test (Durkin, 1991). Research done by Pirie (1998) also showed that second/third language learners do not understand mathematical concepts less than their first language counterparts do. Clarkson's (1992) findings, as indicated above, reveal that bilingual students with proficiency in both mother tongue and English outperformed students who were proficient in only one of either mother tongue or English, and bilingual students with low competence in both languages performed very poorly.

Reading and Mathematical Understanding

Research into the effects of reading on mathematical understanding has been a research topic for almost a century. A good number of studies into the relationship between reading and mathematical proficiency have clearly indicated a strong correlation between reading and mathematical understanding. Aiken (1972) argues that even though understanding the meaning of words and syntax is essential in learning to read all types of materials, training in reading is not, in his words, "invariably an important prerequisite to understanding particular aspects of mathematics"(p. 366). Henney (in Aiken, 1972) shares this opinion but brings in a nuance by making the distinction between *reading mathematics* and *reading other materials*. Henney has insightfully pointed out that with regards to the relationship between reading and mathematical understanding; learners tend to find it more difficult reading mathematics than reading other materials. Spencer & Russell (1960, in Aiken, 1972) give the following reasons to explain why reading of arithmetic is difficult:

- The names of certain numerals are confusing
- Number languages which are patterned differently from the decimal system are used
- The language of expressing fractions and ratios is complicated
- Charts and other diagrams are frequently confusing
- The reading of computational procedures requires specialized skills.

These reasons, can no doubt, be extrapolated to the reading of mathematics in general. But the question remains: Can instructions in general reading (not necessarily instructions in reading mathematics) improve or shape mathematical proficiency?

Research into the relationship between reading as a communicative tool and mathematical understanding in the early twentieth century reveal that reading ability impact positively on mathematics achievement. Aiken's (1972) review of research findings into the relationship between reading ability and mathematics achievement of children in the intermediate grades reveal a correlation ranging between 0.40 and 0.86. Notable amongst the paper reviewed was the research conducted by Linville (1970, in Aiken, 1972). Four tests were randomly administered to 408 fourth-grade learners. Linville found out that in all four tests, learners with high reading abilities scored significantly higher than learners with in low reading abilities.

Also significant is Van der Linde's (1964, in Aiken, 1972) research findings with eighteen fifth grade classes. Nine of these classes constituted the experimental group and were administered vocabulary and reading comprehension exercises for 20-24 weeks after which the achievement tests were re-administered. His analysis of the test results revealed a significant difference in scores between the experimental classes and the control classes on both arithmetic problems and word problems. The experimental group outperformed the control group. Analogous results were obtained by the research conducted by Gilmary (1967, in Aiken, 1972) on two groups of elementary school children in a six-week summer school program in remedial mathematics. The experimental group was given instructions both in reading and in arithmetic while the control group was given instructions in arithmetic only. The results from the Metropolitan Achievement Test-Arithmetic given to both experimental and control group showed that the former performed far better than the latter in the overall scores. In another experiment with high school learners, Call and Wiggin (1966, in Zepp, 1981) investigated into the effects of two different methods in the teaching of algebra. The experimental group was taught by an English trained teacher (Wiggin) with training in teaching reading and no experience in teaching mathematics and the control group was taught by an experienced mathematics teacher (Call). While the English teacher stressed understanding of the words in mathematics problems and translated the mathematical sentence into symbols (in the attempt to help learners understand the English as they read the mathematics problem), the mathematics teacher focused on the mathematics in the mathematical problem with the control group. The results of the criterion test in mathematics showed that the experimental group outperformed the control group even though both groups were initially statistically controlled for differences in reading and mathematics test scores.

A research finding whose results were contrary to the research reports above was conducted by Henney (1969, in Aiken, 1972). He divided 179 fourth-grade learners into two groups and taught both groups in an alternating sequence over a period of nine weeks. To the first group (88 learners), an instructor gave lessons in reading verbal word problems, and the second group (91 learners) were allowed to solve word problems in anyway they chose under the supervision of the same instructor. Henney found that although there was a significant improvement from the pre-test to post-test for both groups, there was no statistically significant difference between the control and experimental group in the post-test results.

More recent research into the relationship between reading and mathematical proficiency have also indicated a strong correlation between the two (Freitag,1997; Holton, Anderson, Thomas, and Fletcher, 1999, in Albert, 2001). A noteworthy research in the area of reading and mathematical understanding was carried out by Taylor (2002) on the Washington Assessment of Student Learning (WASL). Learners' scores from a wide range of mathematics topics were analysed and compared to learners' scores in a wide range of reading activities. The study provided strong evidence supporting the claim that learners' reading competence is an index in their (learners') mathematical proficiency as can be seen in the correlation coefficient in the table below:

WASL YEAR	TEST(S)	CORRELATION
2001	**WASL Math and WASL Reading (2001)**	.733
2001	**WASL Math and ITED Reading (2001)**	*.692*
	ITED Math and ITED Reading (2000)	.741

Table 3.1 *Correlation between reading and mathematics proficiency for WASL*

As Taylor (2002) notes, the results show a stronger than expected relationship between reading and mathematics scores.

Language and Interaction in the Mathematics Classroom

Gorgias' (483-375 B.C.) book entitled *On Not-being or On Nature* sparked off a debate about language and communication that would last centuries amongst philosophers, psychologists and linguists. In his book, Gorgais set out to prove that "first, nothing exists; second, that even if it does, it is incomprehensible by men; and third, that even if it is comprehensible, it is certainly *not expressible and cannot be communicated to another*" (Bormann, 1974: 17. My emphasis). Even though this position was rejected by many philosophers after him (e.g. Socrates, Plato, Aristotle, etc), it (Gorgias' work) opened up avenues for the recognition and investigation of language as a philosophical problem. A very notable work on language was done by Wittgenstein. The importance of language is a view that Wittgenstein stresses through most of his work. The crucial point for Wittgenstein philosophy is that language is a crucial part of our ability to conceptualise the world. The meaning of our thoughts and expressions do not exist independently of language. For Wittgenstein language is always practical. It is intended to do something. He says, "without language we cannot influence other people in such-and-such ways; cannot build roads and machines, etc…" (Wittgenstein, *Philosophical Investigations:* 491). Language is a tool for Wittgenstein. As such, language is a large toolbox with many instruments at our disposal and these instruments have various uses. Thus, for Wittgenstein, discourse (interaction) and language "play an essential role in the genesis, acquisition, communication, formation and justification of virtually all knowledge, including, and in particular, mathematical knowledge" (Ernest, 1994: 37).

It is now a generally accepted proposition that language is key communication tool necessary for mathematics interaction between the teacher and the learners and between learners (Setati, 2005; Ernest, 1994; Reynolds & Wheatley, 1996; Bednarz, 1996, etc.). Through this interaction, the teacher communicates mathematical knowledge to learners and learners' knowledge are ratified or certified. Through interaction in the mathematics class, learners (along with the teacher) participate in "the dialectical process of criticism and warranting of others' mathematical knowledge claims" (Ernest, 1994: 44).

Past research (e.g. Moore and Kearsley, 1996; Gokhale, 1995, Althaus, 1997, Fulford & Zhang, 1993, Garrison, 1990, Hackman & Walker, 1990 Kearsley, 1995, in Fosse, Gonzales, Houver & Ho, 2002) has shown that there is a direct relationship between learners' active engagement and learning outcomes. This active engagement of learners comes through interaction patterns facilitated by the teacher. Thus, the role of the teacher in fostering interaction (in fostering induction of learners into the mathematics community and into doing mathematics) is of crucial importance.

Mathematics teaching and learning must therefore be seen as a social activity (Moschkovich, 1996, 1999, 2002; Bednarz, 1996). As a social process, mathematics is learnt through interaction with others and the sharing of ideas to help develop understanding of concepts and by so doing, create mathematical knowledge in the mathematics community. In the process leading to the formation of mathematical concepts, teacher-learner interaction, learner-learner interaction and learner-content interaction is of crucial importance, and language plays a key role in this process. Moschkovich (2002: 192) elaborates on this point in this manner:

> Mathematics learning is […] seen not only as developing competence in completing procedures, solving word problems, and using mathematical reasoning but also as developing sociomathematical norms (Cobb *et al.*, 1993), presenting mathematical arguments […], and participating in mathematical discussions […]. In general, learning to communicate mathematically is now seen as a central aspect of what it means to learn mathematics.

To be proficient in mathematics, therefore, a minimal level of proficiency in the language of teaching and learning is necessary if learners are not to be denied a meaningful passage from the Legitimate Peripheral Participation (Lavé, 1991) to the full membership in the community of practice.

CONCLUSION

This chapter has highlighted the underlying issues and controversies in the bilingual and cognitive development research. In addition, it has explored research done in the correlation between bi/multilingualism and mathematical proficiency. While some theories and research study point to the necessity of language proficiency as a *sine qua non* for mathematical proficiency, others research/theory show/posit that mathematics proficiency is independent of language proficiency; while some research point to bi/multilingualism as a source of academic and cognitive retardation, other research show that bi/multilingualism have an edge over monolinguals when learners are proficient in these languages. It is a major aim of this study to provide research evidence that could also contribute to the above debate.

CHAPTER FOUR

METHODOLOGY

INTRODUCTION

This chapter describes the research design and method of data collection used in this study. The population, sample, control of variables, the research instruments, validity and reliability are discussed. The chapter also gives a description of the ethical issues that were taken into consideration in undertaking this study.

RESEARCH DESIGN

In order to address the critical questions presented in Chapter 1, a quasi-experimental research approach where subjects were assigned to experimental and control groups was adopted. A quasi-experimental, non-equivalent comparison group design (figure 4.1 below) was used as it was not possible to randomly assign learners to groups. In the figure below, X represents the independent variable used in the experiment (the treatment with ASTRALAB programme); O_1 and O_2 represent the pre-test and the post-test respectively.

	Pre-test measure	**Treatment with ASTRALAB**	**Post-test measure**
EXPERIMENTAL GROUP	$\mathbf{O_1}$	$\mathbf{X_1}$	$\mathbf{O_2}$
CONTROL GROUP	$\mathbf{O_1}$		$\mathbf{O_2}$

FIGURE 4.1: *Non-equivalent comparison group design*

A quasi-experimental, non-equivalent comparison group research approach was used because this approach has the best capability of establishing whether or not there was a cause-effect relationship (Fraenkel & Wallen, 1990) between improvement of English proficiency (using the

ASTRALAB programme as treatment) and mathematical proficiency. Also, given the fact that extraneous effects could really falsify outcomes in a research such as this which seeks to establish a causal relationship (Opie, 2004), quasi-experimental research approach stands out as the best option in exercising far more control of extraneous variables than other methods such as case study, survey, etc (Fraenkel & Wallen, 1990). How this study addresses the issues of validity and reliability would be discussed later under control of extraneous variables.

Population

The population that constituted my study was 1900 learners from a public school in an African School (in South Africa, an African school is generally a black township school where all learners learn mathematics in English which is their second language) in the East Rand. Most of the learners in the school come from an impoverished township in which the school is situated. The predominant home languages are Zulu, Sepedi and Sesotho.

As I explained in chapter two, the ASTRALAB ILS is not a remedial programme for low achievers. Nevertheless, given the language infra-structure in most African schools in South Africa, it can be esteemed without any reasonable doubt that the programme could be of greater benefit to learners whose home language is not English. For the purpose of the implementation of the ASTRALAB software, therefore, an African school was chosen because the study targeted learners whose main language was not the language of instruction. The research African school was chosen by the programme manager based on availability of sponsorship.

Sample

As indicated in chapter two, the research school is running an ASTRALAB pilot project in one Grade 9 class. This became the experimental group for the present study. To identify the control group, the school was asked to choose another class which was taught by the same mathematics and English teacher as the experimental group. There was no such Grade 9 class in the school. There was, however, another Grade 9 class which had the same Mathematics teacher as the experimental class. This class was therefore chosen to be the control class.

The study involved a total number of ninety-three learners in Grade Nine. The 45 learners in the Grade 9A class constituted the experimental group while 48 learners in the Grade 9G class constituted the control group (the number 45 and 48 are the number of learners in each respective class in the school). As indicated above, both classes were taught by the same mathematics teacher but different English teachers. Classes were not streamed according to academic ability but according to the African language the learners have chosen to study as a subject at first language level.

Learners in the school study an African language (Sesotho, IsiZulu, IsiXhosa, Setswana, Sepedi or Sesotho) as a subject at first language level and are fluent in one or more of these languages. The table below shows the home language distribution (of learners in the experimental and control groups) which in most cases are the language which learners were studying as first language:

	ISIZULU	**SESOTHO**	**SETSWANA**	**SEPEDI**	**XITSONGA**
EXPERIMENTAL GROUP	2	14	1	28	0
CONTROL GROUP	43	3	1	0	1

Table 4.1: *Language distribution of control and experimental groups*

Even though a majority of the learners in the control group indicated that IsiZulu was their home language, and a majority in the experimental group indicated that Sepedi and Sesotho was their home language, most of them were also fluent in at least, one other African language. Most of learners who spoke Sepedi in the experimental group were also fluent in Zulu and most who were fluent in Sesotho also indicated that they were fluent in Sepedi. It can, therefore, be argued that the learners are multilingual as their listening, speaking, reading and writing competencies are in more than two languages.

Instrumentation

The research instrument consisted of 35 questions drawn from a wide range of mathematical content and word problems which learners have covered in the class. They were made up of both multiple-choice questions and questions requiring learners to work out the answers. The test items were selected from the 2003 Third International Mathematics and Science Study (TIMSS) and were modified slightly where necessary to suit the context of learners in the study. For example, in the question below:

> *"Graham has twice as many books as Bob. Chan has six more books than Bob. If Bob has x books, which of the following represents the total number of books the three boys have?"*

The names: Graham, Bob, and Chan were replaced with Thande, Zandi and Sipho respectively to read:

> *Thande has twice as many books as Zandi. Sipho has six more books than Zandi. If Zandi has x books, which of the following represents the total number of books the three learners have?*

And in the question below,

> *A car has a fuel tank that holds 45 litres of fuel. The car consumes 8.5 litres of fuel for each 100 km driven. A trip of 350 km was started with a full tank of fuel. How much remained in the tank at the end of the trip?*

The words *fuel* and *trip* were replaced by petrol and journey because they are more familiar to the learners in South Africa.

The table below shows the distribution of questions in the instrument following the categorisation of TIMSS:

CONTENT DOMAIN	NUMBER OF QUESTIONS
NUMBER (Fraction & decimals; ratio, proportion & percent; whole numbers; integers)	15
MEASUREMENT	5
GEOMETRY	5
ALGEBRA (Equation & formulas; algebraic expression; patterns)	9
DATA	1
TOTAL	**35**

Table 4.2: *Question distribution (in pre- and post-tests) according to content domain*

While it was necessary to have both routine and non-routine questions, both algorithmic and non-algorithmic questions, it was important to choose questions which made a heavy demand on learners' understanding of the demands of the question. Such understanding comes mainly (but not solely) through the understanding of the language (both the LoLT and the mathematics language) in the question. Thus, there were more questions under the domain of number and algebra because most of the questions on these content domains were word problem questions.

METHODS OF DATA COLLECTION

Data from this study was collected over a period of four weeks. Before the commencement of the ASTRALAB programme in the first week, the pre-test was administered to both groups. The pre-test and the post-test contained the same test items. Both pre- and post-tests were written under strict examination conditions for ninety minutes. The learners were required to work individually and were not allowed to share ideas while writing the tests.

In addition to the pre-test and post-test, there were lesson observations of the mathematics class of the experimental group. The first series of the videoing of the mathematics lessons of the experimental group took place during the first two days of the implementation of the ASTRALAB ILS. At the end of the implementation phase (in the 4^{th} week), the post-test was administered and the experimental class was video-recorded. The video-recorded mathematics lessons were used to analyse the interaction and communication in the mathematics class. Two mathematics lessons in the experimental class were video-recorded at the commencement of the ASTRALAB treatment while there was only one video-recorded lesson after the treatment. It was possible to only observe one class after the implementation phase as the learners had to begin their CTA portfolio work. For analysis, the second video-recorded lesson was chosen given that in the second lesson, learners had become used to the researcher and the fact of being videoed.

The table below gives a summary of what data was collected and when they were collected.

Phase	Period	Activities
1	March 2005	Negotiating access to schools and classrooms
	May	Development of instruments
	July 20th	DATA COLLECTION - Administration of Pre-test - Commencement of ASTRALAB treatment. - Commencement of 1st phase of class observation and videoing.
	August 10th	End of ASTRALAB treatment
2	August 15th	DATA COLLECTION - Administration of post-test - Second phase of class observation and videoing.
	September to January 2006	Transcription and coding of data. Writing of the research report

Table 4.3*: Study time frame*

DATA ANALYSIS

The data from pre-test and post-test were analyzed using the Statistical Package for the Social Sciences (SPSS). For the analysis within a group, only data from learners who took both the pre-test and post-test were used. Learners who took either one or none of the two tests were systematically removed from the analysis. For inter-group analysis, only data from learners who wrote the corresponding test (pre-test or post-test) were used. The data were analyzed using measures of central tendency (means and modes only) and variability (i.e., standard deviation, and ranges). The magnitude of all relationships reported in this study used the conventions from Davis (in Waldron, 2004). These descriptors are as follows:

Coefficient	**Descriptor**
.70 or higher	Very strong correlation (relationship)
.50 to .69	Substantial correlation (relationship)
.30 to .49	Moderate correlation (relationship)
.10 to .29	Low correlation (relationship)
.01 to .09	Negligible correlation (relationship)

The correlation coefficient as well as the *p*-value were used in the comparative analysis of learner achievement between learners who used the ASTRALAB programme and those who did not.

The mathematics classroom interaction for the experimental class was transcribed and coded first, according to the learner talk and teacher talk, then according to whether or not the learner talk was justification response, procedural response, and regulatory response, asking questions or justification of answers. The teacher talk was coded according to whether or not the talk was asking justification questions, procedural questions, procedural explanation, conceptual explanation, regulation or affirmation. The coding of the whole transcript was done by the researcher and another party who was given the codes and the transcript and asked to code each utterance. The codings by from the two parties were then compared for consistency.

The figure below gives the conceptual framework of my study:

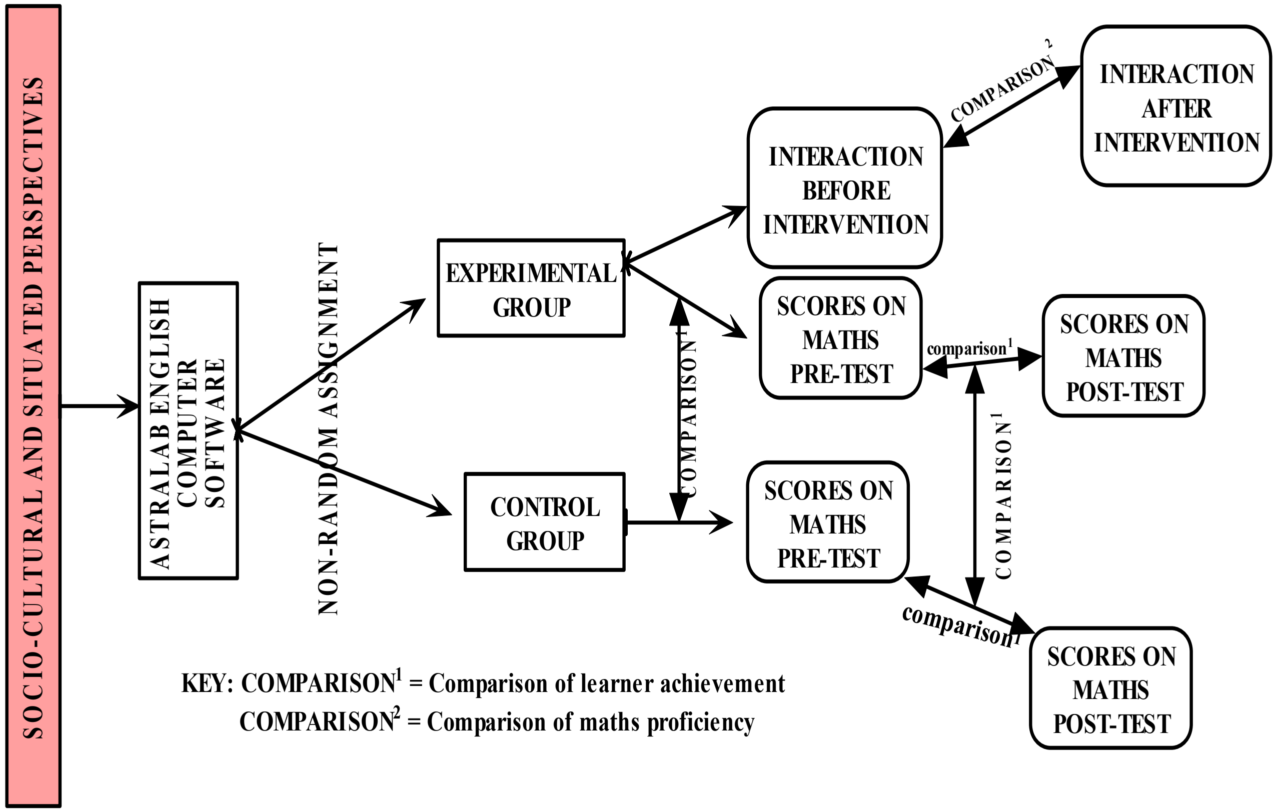

FIGURE 4.2: *Conceptual Framework of Research Study*

CONTROL OF VARIABLES

Validity

Validity is the "degree to which a method, a test or a research tool actually measures what it is supposed to measure" (Wellington, in Opie, 2004: 68). One of the key problems with doing an experimental or quasi-experimental research is the establishment of suitable control so that any change in the scores on the post-test can be attributed only to the independent variable that was manipulated by the researcher (Spector, 1981; Singleton, Straits, Straits & McAllister, 1988). The control of extraneous variables is fundamental to the validity of an experimental research (Campbell & Stanley, 1963). Campbell & Stanley (1963) identify various factors which can threaten the internal and external validity of any experimental study. Internal validity refers to the control of extraneous variables so that it can be concluded strongly that the independent variable produced the observed changes in the dependent variable. External validity on the other hand "ask the questions of generalizability (Campbell & Stanley, 1963). External validity, therefore, ask the question as to the extent in which the results from the experiment can be generalised from the sample to the population. In what follows, I discuss how factors threatening the internal validity were dealt with in my study. I deal with the generalizability of my study in the last chapter of this report.

Control of internal validity

There are eight main types of extraneous factors that can threaten the internal validity of a quasi-experimental study namely: the effects of history, maturity, statistical regression, selection, and testing, experimental mortality, instrumentation, and design contamination (Spector, 1981; Campbell & Stanley, 1963; Singleton *et al*, 1988). Campbell & Stanley (1963) however note that for the quasi-experimental non-equivalent control group design, there is inherent control of the main effects of history (events occurring within the time lag of pre-test and post-test in addition to the independent variable), maturation (biological or physiological changes that occur in the participants during implementation phase), testing (pre-test affecting the score of post-test) and instrumentation (Change in measurement method) in the design itself. This is so because the effects of these variables would be the same for both control and experimental groups (Campbell & Stanley, 1963) as was the case in the present research study. Both groups experienced the same current events, both experienced the same developmental processes, the learners did not know they were going to write the same post-test as the pre-test, and lastly, there was no change in the method of testing in both pre-test and post-test, and the class observations

Van Dalen (1973) as well as Campbell & Stanley (1963) list statistical regression as one of the key factors that could jeopardize the internal validity of a quasi-experimental design. Statistical regression refers to the tendency for extreme scorers on a test to move (regress) closer to the mean. It occurs when groups have been selected based on their extreme scores. This was not the case in the present study.

Another variable, which threatens internal validity, is experimental mortality – the drop-out rate during experimental studies. To control experimental mortality, only results from learners who took both pre-test and post-test in both groups were used in the data analysis across groups.

A seventh threat to internal validity is selection, which refers to how the subjects were assigned either to the experimental or to the control group. As indicated above, the selection was non-random. Even though the control and experimental groups in my study had the same mathematics teacher and classes in the school were not grouped according to ability, this was no guarantee that both classes were of the same mathematics ability. The pre-test results were used to match/compare the mathematical ability of both groups. The *p*-value of test (both *t*-test and nonparametric test) was used to analyse the pre-test data to determine if there was a significant difference between the mathematical ability of the control group compared to that of the experimental group.

As for control of design contamination (which is concerned with whether the control or the experimental group found out about the experiment and tried to make the experiment succeed or fail), there was no observable attempt by learners to either make the research succeed or fail in both the tests and the class observations.

Reliability

Reliability refers to the "extent to which a test, a method or a tool gives consistent results across a range of settings, and if used by a range of researchers (Wellington, in Opie, 2004: 65-66). Put differently, reliability refers to "the consistency of scores or answers provided by an instrument" (Fraenkel & Wallen, 1996). The research instruments were carefully selected from the TIMSS's. This means that the instruments, which have been tested and standardized, were used. Moreover, although learners for the study were in Grade 9, the test items from TIMSS were from Grade 8 as indicated above. The assumption was that (at the time of the year when the tests were administered) learners would be better able to deal with the Grade 8 mathematics content (which they have supposedly covered entirely in their previous class) than items from Grade 9, some of which they had not yet covered in their lesson.

There was no need for test of homogeneity of variance since the experimental and the control groups were almost equal in size. Moreover, in the two sample *t*-Test, learners who did not write *both* the pre-test and the post-test were systematically removed. This means that only learners who wrote both tests were used in the analysis of the performance for both groups before the intervention, and again, after the intervention (I will elaborate on this point in the next chapter).

The researcher alone undertook both the pre-intervention and post-intervention video recordings. This ensured that there was no increase or decrease in participation by learners because of the presence of a different person. Furthermore, the first video recording was not used for analysis as learners were still getting used to the presence of the researcher at that time.

ETHICAL CONSIDERATIONS

The key ethical question that any experimental research has to deal directly with revolves around the fact that the control group is disadvantaged in the research in the sense that the group is denied access to the treatment which could have been beneficial to them. This ethical difficulty was dealt with by making provision for the implementation of the treatment on the control group (and other classes) after the post-test for the experimental group had been accomplished. Interested educators in the school would be given a 10-hour intensive training to enable them to implement the programme without outside assistance. This second implementation, though, would not form part of the present research study.

Before attempting to explore the effect of the ASTRALAB software on the learners' mathematical proficiency, permission in writing was obtained from the Gauteng Department of Education (GDE) since the research school was situated within their jurisdiction. Clearance was also granted by University of the Witwatersrand to conduct the research (see appendix).

Access to the school was negotiated with the principal of the school and the teachers in the control and experimental classes were asked for a written consent to participate in the research.

Access to the school was negotiated with the principal of the school and the teachers in the control and experimental classes were asked for a written consent to participate in the research.

As the researcher, I had a brief contact with both the control and the experimental group to explain to them the purpose of the research, what the research was about and what was required of them. I encouraged them to participate in the research but made it clear to them that participation was not mandatory and subject to the signing of the consent form by both learners and their parents. All learners in both groups signed and returned the consent forms. There was, thus, no difficulty of systematically excluding those who declined to participate in the study.

The issue of feedback to the research community, to the teachers who participated in the study and to the research school is mainly about responsibility and trust (Setati, 2005). At the end of my study, I would first discuss the findings of my research with all the teachers who participated in the study. I would then make the report known to the rest of the school community by making a presentation at the school and giving them a copy of the research report for their library.

CONCLUSION

This chapter presented a discussion on the research design, population and sample, research instruments, and the method of data collection and methods used for data analysis. Validity and Reliability of the research, and the ethics that guided the data collection were also discussed. In the next chapter, I turn my attention to the presentation of data and data analysis.

CHAPTER FIVE

RESULTS OF THE QUANTITATIVE ANALYSIS

INTRODUCTION

The aim of this study was to investigate how improving learners English language proficiency (through the use of computer software for an accelerated English instruction) can either enable or constrain mathematical proficiency in learners. The study incorporated both qualitative and quantitative methods which guided the analysis and interpretation of data presented in this chapter. In all this, the question below would provide a focus:

- To what extent does improving learners' proficiency in English enable or constrain mathematical proficiency?

The quantitative analysis would provide answers to the first sub-question of the study below:

- ➢ How does the use of the ASTRALAB instructional English literacy software enable or constrain learners' mathematical proficiency?

The next chapter deals with the qualitative analysis and would provide answers to the second sub-question of the study.

COMPARATIVE ANALYSIS OF LEARNER ACHIEVEMENT IN TESTS

Learner Achievement in Pre-test

As stated in the previous chapter, one of the difficulties of a quasi-experimental research is matching the experimental and control groups. In this study, the pre-test results were used to compare the mathematical ability of learners in the experimental and the control groups in order to find out if there was any significant difference in ability between the two groups.

Even though learners were non-randomly assigned to the experimental group, the test for normality shows a fairly normal curve for the total scores (with a measure of the skewness of .172) as can be seen in figure 1 below:

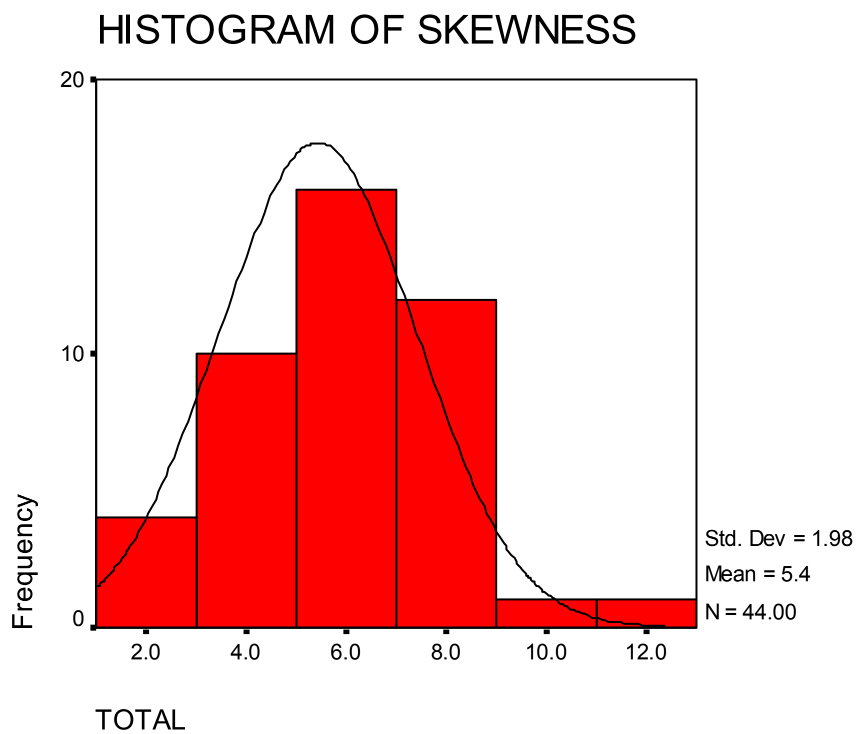

FIGURE 5.1: *Histogram showing the skewness in pre-test.*

The skewness of a distribution measures the deviation of the distribution from symmetry. If the skewness is clearly different from zero, then such a distribution is asymmetrical. The fairly normal curve above (.172) is an indication that the distribution of learners in both the control and experimental groups are fairly symmetrical.

Of the 45 learners in the experimental group, 44 of them wrote the pre-test while all 48 learners in the control group wrote the test. None of the learners in both groups obtained marks above 40% in the pre-test. The marks in the experimental group ranged from 5.7% to 31.4% while that in the control group ranged from 2.9% to 25.7%. The table below presents a summary of learners' performance in the pre-test for both groups:

GROUP	N	Mean	Std. Deviation	Std. Error Mean
EXPERIMENTAL	44	5.4318	1.9813	.2987
CONTROL	48	4.9583	2.2874	.3302

Table 5.1. *Group Statistics of control and experimental group in pre-test*

In the table above, it can be observed that the mean score for the experimental group is 5.4318 while that for the control group is 4.9583. The standard deviation from the mean is 1.9813 and 2.2874 for the experimental and control groups respectively.

The pre-test results for both control and experimental groups were also matched using the *p*-value. The probability (*p*) commonly referred to as the *p*-value of the test, is associated with an obtained statistical result that could have been produced by chance (or random error). The smaller the number (that is, the smaller this chance is), the greater the likelihood that the result expressed was not due to chance (Freedman, Pisani, Purves & Adhikari, 1991). It is conventional to draw the line at 5%. If p is < 5% (that is, $p < 0.05$), then the results (difference between the experimental and control groups) is statistically significant. If $p > 0.05$, it is tenable that both the experimental and control groups are equal. In other words there is no significant difference between the two groups, and so we fail to reject the null hypothesis (null hypothesis is Ho: $u=u_1$ where u is control group mean and u_1 is the experimental group mean and Ho is Null hypothesis)

Table 4.2 presents the summary data (pretest) of the *t*-test for the experimental and control groups.

	T-Test for equality of means						
	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference	95% Confidence interval of the difference	
						Lower	Upper
Equal variance assumed	-1.057	90	.293	-.4735	.4480	-1.3636	.4166
Equal Variance not assumed	-1.063	89.724	.290	-.4735	.4452	-1.3580	.4111

Table 5.2: *Two-sample t-Test results for experimental and control groups in pre-test*

Even though there was a difference in the mean scores and standard deviation of both groups (The mean for the experimental group was 5.4318 (Standard Deviation = 1.9813) and the mean for the control group was 4.9583 (Standard Deviation = 2.2874), a difference of .9559 points in favor of the experimental group), as can be observed from table 5.2, the p-value = .293. A nonparametric test indicates a p-value of .292. Since the p-value is > .05, there is no significant difference between the experimental and the control groups in terms of the scores of the pre-test. This decision, however, is prone to type II error (accepting *Ho* when it is false) since, in reality, it is possible that there is a significant difference between the two groups.

Analysis of Achievement in Pretest and Post-test for Control Group

Having analysed the achievement in the pre-test for both groups, I turn to the performance of learners in the control group in both the pre-test and the post-test.

All learners in the control group wrote the pre-test and 42 wrote the post-test. Learner achievement in the post-test, like in the pre-test, was low in the control group. Just like the pre-test, performance in the post-test ranged from 2.9% (1 mark out of 35) to 25.7% (9 marks out of 35). The overall test results in the post-test were, however, poorer than the test results in the pre-test. As shown in Table 5.3 below, the mean of these learners were 4.9583 and 4.9286 for the pre-test and post-test respectively, a difference of .0297 in favour of performance in the pre-test. The performance in the post-test was more homogenous than in the pre-test as indicated by the Standard Deviation in the table below.

	Mean	**N**	**Std. Deviation**	**Std. Error Mean**	**CORRELATION**
PRE-TEST	4.9583	48	2.2874	.3302	-.368
POST-TEST	4.9286	42	1.9555	.3017	

Table 5.3: *Paired Samples Statistics for control group*

The correlation between the pre-test and post-test is -0.368. This is a weak negative correlation since the performance in the post-test was poorer than performance in the pre-test. It is difficult to understand why learners in the control group performed poorer in the post-test given the fact that the same questions were used in both pre- and post-tests.

In the table below, the t-test results of both pre- and post-tests for the control group shows that the p-value is .849 and a nonparametric test gives a p-value of .809. Since $p > .05$, the results indicate that even though there is a difference in performance from pre-test to post-test, this difference is not statistically significant.

				Paired differences		
	t	df	Sig. (2-tailed)	Std. Error mean	95% Confidence interval of the difference	
					Lower	Upper
Pre-test – post-test	.192	41	.849	.3724	-.6806	.8235

Table 5.4: *Paired-sample t-test results for control group*

Analysis of Achievement in Pre-test and Post-test scores of Experimental group

A total of 44 learners in the experimental group wrote the pre-test while a total of 45 wrote the post-test.

The results from the post-test for the experimental group were also low with the scores ranging from 2.9% to 37.1%. In general, most of the scores in the pre-test ranged from 4 to 7 (32 learners in total), with the majority of these 32 learners scoring 7 out of 35 and in the post-test, the highest frequencies ranged from 5 to 9 (33 learners in total). Learners, however, improved from a mean score of 5.4318 in the pre-test to 6.3556 in the post-test, a difference of .9238 in favour of performance in the post-test. The Standard Deviation indicates that the performance in the pre-test was more homogenous than performance in the post-test.

	Mean	**N**	**Std. Deviation**	**Std. Error Mean**	**CORRELATION**
PRE-TEST	5.4318	44	1.9813	.2987	
POST-TEST	6.3556	45	2.6897	.4010	.328

Table 5.5: *Paired sample statistics for experimental group*

As shown in table 5.5, the correlation coefficient is 0.328 indicating a moderate positive relationship between scores in the pre-test and scores in the post-test. The scatter plot below shows the result (relationship) between the scores of the post-test and those of the pre-test. The post-test scores were regressed against the pre-test scores to ascertain the linearity between the two scores.

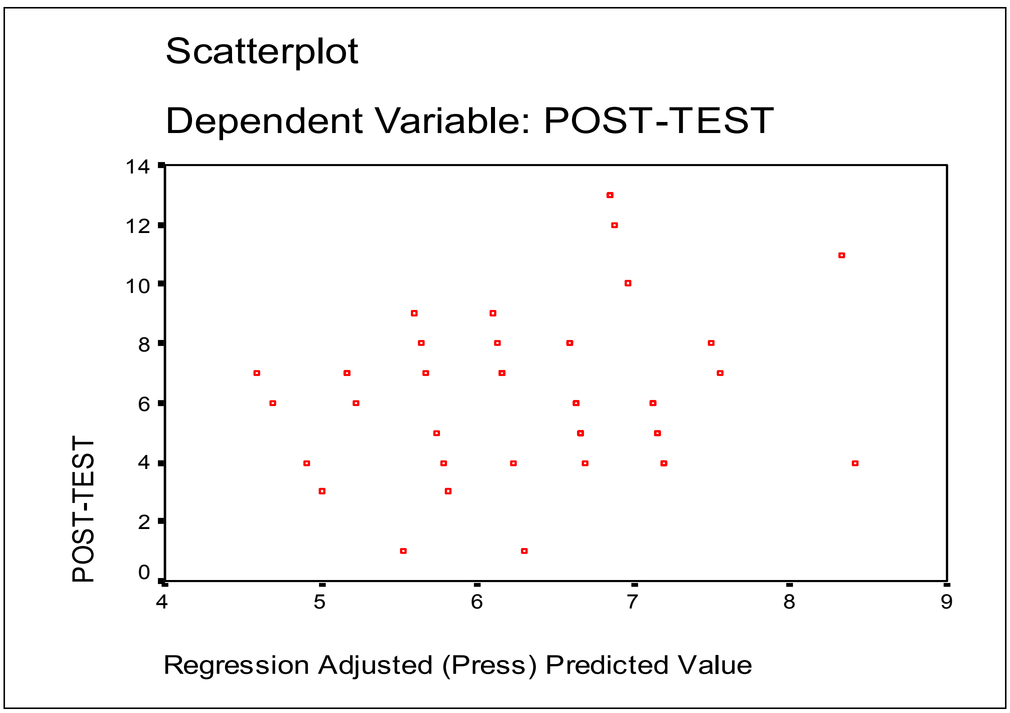

FIGURE 5.2: *Adjusted residual for post-test scores*

The scatter plot reaffirm that there is a moderate correlation between the scores of both tests. From the above, since there is a moderate correlation, it can be deduced that their previous mathematics ability has some role in the achievement in the post-test. But what is said above is not enough to explain the improvement in achievement in the post-test. The two-tailed test of difference at .05 significance shows that the intervention has some effect on the test results. The *p*-value (0.032) of the tests in the table below shows that the difference between the scores is statistically significant (since the *p*-value < .05). A nonparametric test also indicates a *p*-value (.030) that is statistically significant.

				Paired differences		
	t	df	Sig. (2-tailed)	Std. Error mean	95% Confidence interval of the difference	
					Lower	Upper
Pre-test – post-test	-2.215	43	.032	.4207	-1.7802	-.8083

Table 5.6: *Paired-sample t-test results for experimental group*

Comparative analysis of Achievement in Post-test scores for Experimental and Control groups

The 45 and 42 learners (in the experimental and control groups respectively) who wrote the post-test were compared to determine if there was any difference in learner achievement between learners who used the ASTRALAB programme (experimental group) and those who did not (control group). Tables 5.7 and 5.8 present a summary data for both groups.

GROUP	Mean	N	Std. Deviation	Std. Error Mean
EXPERIMENTAL	6.3556	45	2.6897	.4010
CONTROL	4.9286	42	1.9555	.3017

Table 5.7: *Group Statistics of control and experimental group in post-test*

The mean score for the experimental group was 6.3556 (Standard Deviation = 2.6897) and the mean for the control group was 4.9286 (Standard Deviation = 1.9555), a difference of 1.427 in favour of the experimental group.

	T-Test for equality of means						
	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference	95% Confidence interval of the difference	
						Lower	Upper
Equal variance assumed	2.813	85	.006	1.4270	1.4270	.4185	2.4355
Equal Variance not assumed	2.844	80.308	.006	1.4270	.5018	.4284	2.4256

Table 5.8: *Two-sample t-Test results for experimental and control group in post-test*

As indicated in the *t*-test results above, the *p*-value = .006. The *p*-value for the nonparametric test is .008. Since $p < .05$, the mean post-test scores are considered to be statistically unequal. That is, there is a significant difference between the scores of the experimental group's post-test and that of the control group in the population.

ANALYSIS OF PRE-TEST AND POST-TEST FOR EXPERIMENTAL GROUP BASED ON GENDER

After the first level analysis above, it was necessary to disaggregate the genders in order to understand whether or not the greater beneficiary of the ASTRALAB English software were boys or girls. It was, thus, necessary to analyse the data based on gender to see if there was any significant difference between the performance of the boys and the performance of the girls in both experimental and control classes. Out of the 44 learners who wrote the pre-test in the experimental group, 25 of them were boys while 19 of them were girls. The table below gives descriptive statistics of the performance of both groups (boys and girls) in the pre-test.

	MALE	FEMALE
MINIMUM SCORE	2	2
MAXIMUM SCORE	11	8
MEAN	5.28	5.6316
STD. DEVIATION	2.2642	1.5709

Table 5.9: *Gender statistics for pre-test*

From table 5.9, it can be noted that while the scores for boys range from 2 to 11 that for girls range from 2 to 8. Even though the performance of boys was higher in terms of marks, that of the girls was more homogeneous (SD = 1.5709) than the performance of boys (SD = 2.2642) as there were extreme scores amongst the boys. A *t*-test for both groups shows a *p*-value of .566 and a nonparametric test shows a *p*-value of .471, indicating that these differences between both groups were not statistically significant before the commencement of the treatment.

All 45 learners wrote the post-test in the experimental group. The performance in the post-test was better than that of the pre-test even though the minimum mark was lower (1) in the post test. The maximum mark was higher in the post-test (13) compared to that of the pre-test (11). Scores from 5 to 9 had the highest frequency (33 learners in total).

For descriptive analysis based on gender, even though both groups (boys and girls) had a range of 12, girls not only performed better but also the performance of girls was more homogeneous as indicated by the standard deviation in the table below.

	MALE	FEMALE
MINIMUM SCORE	1	1
MAXIMUM SCORE	13	13
MEAN	6.16	6.60
STD. DEVIATION	2.8089	2.5833

Table 5.10: *Gender statistics for post-test*

A *t*-test for both groups shows a *p*-value of .588 and a nonparametric test shows a *p*-value of .381, indicating that the differences between performance by boys and performance by girls was still not statistically significant after the treatment.

A comparative analysis of learner achievement from pre-test to post-test based on gender indicate that there was an improvement in performance in both groups. While the mean scores for the pre-test for boys and girls are 5.28 and 5.6316 respectively, the mean scores in the post-test are 6.16 and 6.60 respectively – a difference of .88 for boys and .9684 for girls. Even though the correlation coefficient is .386 for performance from pre-test to post-test for boys, the *t*-test and nonparametric test for scores in pre-test and post-test for boys show a *p*-value of .135 and .129 respectively. This indicates that there was no statistically significant difference between the scores of boys in the experimental group's pre-test scores and post-test scores. There was also no statistically significant difference between performances by girls from pre-test to post-test as the *p*-value is also .135 for the *t*-test and .127 for the nonparametric test. The correlation coefficient for girls from pre- to post-test was .206.

Analysis of pre-test and post-test for control group based on gender

A total of 22 boys and 26 girls wrote the pre-test in the control group.

	PRE-TEST		**POST-TEST**	
	MALE	**FEMALE**	**MALE**	**FEMALE**
MINIMUM SCORE	2	1	1	1
MAXIMUM SCORE	9	9	9	9
MEAN	4.9545	4.9615	5.0526	4.8261
STD DEVIATION	2.0113	2.5374	1.9571	1.9921

Table 5.11: *Statistics based on Gender for control group*

There was no difference in the performance between boys and girls in the pre-test as can be seen in the mean score and the standard deviation above (*p*-value = .992). Neither was there any statistically significant difference between the performance of boys and that of girls in the post-test.

Paired-sample analysis for gender

There was no statistically significant difference in the performance of boys in both control and experimental groups in both pre-test and post-test. The pre-test results for boys in the control and experimental groups indicate a *p*-value of .607 for the *t*-test and .637 for the nonparametric while the post-test results for boys in the control and experimental group give a *p*-value of .131 and .217 for *t*-test and nonparametric tests respectively.

There was also no statistically significant difference in the performance of girls between groups in the pre-test as the *t*-test and nonparametric test show a *p*-value of .315 and .217 respectively. There was, however, a statistically significant difference in the performance of girls between groups in the post-test. The *t*-test and nonparametric test give a *p*-value of .015 and .013 respectively.

ANALYSIS OF TEST RESULTS BASED ON QUESTION CATEGORIES

The foregoing analysis has highlighted learners' general performance in the test items. In what follows, I provide an analysis of learner performance based on the question categories that were discussed in chapter four. This was necessary to ascertain whether or not the improvement of English enabled or constrained the performances in the various question categories. The analysis is, thus, limited to the performance in the experimental group. The table below shows how learners engaged with questions on number, algebra, measurement and geometry:

NUMBER			ALGEBRA			MEASUREMENT		
QUESTION	PRE-TEST	POST-TEST	QUESTION	PRE-TEST	POST-TEST	QUESTION	PRE-TEST	POST-TEST
1	5	8	**8**	4	12	**2**	7	8
4	24	20	**9**	4	4	**5**	18	25
7	8	12	**13**	8	7	**16**	10	6
10	21	21	**15**	10	15	**23**	1	5
11	8	12	**21**	1	1	**30**	0	0
17	14	15	**24**	2	2	**TOTAL**	**36**	**44**
18	11	14	**25**	5	4			
19	10	7	**26**	3	4	**GEOMETRY**		
20	16	18	**32**	0	0	**3**	8	14
27	17	24	**TOTAL**	**37**	**49**	**6**	5	8

28	0	0				**12**	0	1
29	0	0				**22**	1	6
33	2	0				**31**	3	1
34	3	3				**TOTAL**	**17**	**30**
35	0	0						
TOTAL	**139**	**154**				**DATA**		
						14	11	16

Table 5.12: *Scores based on question categories*

Out of the 15 questions in the category of number, there was improvement (in the experimental group) in all but 3 questions and no change in 5 questions. The correlation coefficient from pre-test to post-test gives a value of .939 indicating high correlation. The *t*-test gives a *p*-value of .203 indicating that the difference is not statistically significant. This is also true for the algebra questions which have a correlation of .811 and a *p*-value of .231. In general, there was improvement (in the post-test results) in all content domains as can be seen in the total of number of correctly answered questions in each domain.

SUMMARY OF ANALYSIS OF PRE-TEST AND POST-TEST SCORES

1. A comparison of the control and experimental groups before the treatment with ASTRALAB ILS indicated that:

   - In the test for skewness, there was an even distribution of learners as far as the mathematics ability is concerned.
   - The *t*-test and the nonparametric test indicate that there was no statistically significant difference in the test results between the two groups before the treatment (even though there was a difference in the mean scores in favour of the experimental group).

2. Analysis of the performance in the pre-test and post-test for the control group indicated that there was no statistically significant difference between the performances in both pre-test and post-test (even though the performance of learners in this group was lower in the post-test).

3. Analysis of performance in pre-test and post-test scores for experimental group revealed a moderate correlation between performance in pre-test and performance in the post-test. A t-test and nonparametric test indicated a statistically significant difference between scores in pre-test and those of post-test.

4. A comparative analysis of learner performance in both control and experimental groups in post-test indicate that there was a highly significant difference between performance in the experimental group compared to performance in the control group.

5. As far as the pre-test analysis by gender was concerned

   - There was no statistically significant difference between performance of boys within and between the two groups. This was also true of the performance by girls in the pre-test.
   - In the post-test results for gender, there was no statistically significant difference between performance by boys compared to performance by girls within the two groups.
   - There was however a statistically significant difference between the performance of girls in the experimental group and the performance of girls in the control group (there was no difference in the performance of boys between the groups in the post-test).

6. As far as the content domains were concerned, there was improvement in all content domains in the experimental group but none of the domains recorded a statistically significant difference.

CONCLUSION

The study aimed at investigating how the improvement of learners' English proficiency (using the English literacy computer software – ASTRALAB – designed to promote English proficiency) enables or constrains the development of mathematical proficiency in learners. This chapter described the statistical results for the first two objectives of this study and the Mean scores, Standard Deviation, Correlation coefficient and *p*-value of tests results were presented in relation to the statistical analysis of the pre- and post-test results. In the chapter that follows, I analyse the classroom interaction (of the experimental class) before the intervention and after the intervention.

CHAPTER SIX

RESULTS OF THE QUALITATIVE ANALYSIS

INTRODUCTION

As noted in previous sections, in addition to algorithmic competence, solving word problems and using mathematical reasoning (Moschkovich, 2002), interaction in the mathematics class is also important in the teaching and learning of mathematics. If the language proficiency of learners was improved, it was also necessary to investigate whether and how such improvement of linguistic competence either enabled or constrained the interaction in the class. This chapter provides answers to the second research sub-question:

- To what extent does improving English language through the use of the ASTRALAB software enable or constrain interaction in the mathematics classroom?

Qualitative methods were used to analyse the classroom interaction and to provide answers to the question as to whether and how improvement of learners' English language proficiency enabled or constrained interaction in the mathematics classroom. Before the analysis of the classroom interaction, a description of the two chosen lessons would serve a good purpose as this would indicate in what way (or to what extent) the class before the intervention was similar/different in nature to the class after the intervention.

Pre-intervention Lesson

The class started with a revision of the previous homework (on equations) which was given to learners. The teacher began by moving from desk to desk to inspect learners' homework for about 5 minutes. Then the revision class ensued. At the end of the revision exercise, the class solved more complex questions on equations.

Post-intervention Lesson

The lesson started with the teacher moving from desk to desk checking if learners did their homework of the previous class. She reprimanded those who did not do their homework as she moved from pair to pair (learners were seated in groups of two). Then a revision of the homework problems ensued. After the revision, the teacher wrote down more problems on equations that were tackled by both the teacher and the learners in a whole class teaching.

From the description of the two lessons (see full transcript of the two lessons in the appendix), it can be seen that both the pre-intervention lesson and the post-intervention lesson were similar in a number of ways: first, they both were for a duration of 40 minutes; second, the same strategy of teaching was used by the teacher: revision of previously given homework, then new problems were tackled in a whole class discussion; third, both lessons were from the same content domain – algebra, although in the post-intervention lesson, more complex questions were solved (as one would expect).

INTERACTION IN THE MATHEMATICS CLASS

The classroom interaction before the intervention and after the intervention was analysed based on utterances by both teacher (teacher talk) and learners (learner talk). The question: was there more talk before or after the intervention was crucial to the analysis of verbalisations. The second phase of the analysis deals with language use by both learners and teacher in the mathematics class before and after the intervention. The coding system for language used by learners and the teacher would distinguish when language was used either for questioning, justification, explanation, and regulation as shown in the table below:

TEACHER TALK	
QUESTIONING	JUSTIFICATION (**QJ**): These are questions soliciting justification for learner contribution (mathematical strategy, learner answers or concepts used by learners). Example: "why is it 2p?" Questions like "How *did* you get 6" (as distinct from "how *do* you get 6) requiring learners to explain or justify their solution strategy fall into this category.
	PROCEDURAL (**QP**): These are questions soliciting procedures for solving a problem. They are questions asking learners what the next step should be in the process of solving them mathematics problems. Example: How do you get 6? what next? Then what? It could also be questions by the teacher asking learners to elaborate on the steps they used or the steps they do not understand. Example: What step don't you understand? Procedural questions are in general questions that regulate the procedural discourse.
EXPLANATION	PROCEDURAL (**EP**): Explanation by teacher of the procedures of solving a mathematics problem. They are concerned with *what* and *how* procedures are used. It is possible that learners give explanation of this kind. This was coded as Justification Response in this study.
	CONCEPTUAL (**EC**): Explanation by the teacher that elaborates on the concept under consideration (or related concepts) and aims at deepening learners' conceptual understanding. It is concerned with the *why* of procedures. It is also possible for learners to do this. This was still coded as Justification Response.
REGULATION (R)	By the teacher requesting certain comportment by learners. Example: why are you making noise? Stop guessing and think. Regulation can also be directions to move on. Example: next question
AFFIRMATION (A)	Acknowledgement of learner contribution as either correct or wrong.

Table 6.1: *Coding system for teacher talk*

LEARNER TALK	
RESPONSES	JUSTIFICATION (**RJ**): When learner contribution is a justification of either the strategy used in a solution process or justification for answers they (learners) obtained.
	PROCEDURAL (**RP**): When learners respond to procedural questions indicating the next step in the solution process. Example: T: what next? L: find the LCD. "Find the LCD" falls into the category of procedural response.
	REGULATION (**RR**): When learners respond to the teacher's request about their (learners') comportment. Example: T: will you keep quiet? L: Yes. The "yes" here is

	considered as response to regulation by the learner.
QUESTION (Q)	When learners ask questions of any sort
JUSTIFICATION (J)	When learners justify a process in solving a problem that is not part of their own solution process. Example: When learners provide justification for the question: who can tell me why it the positive sign becomes negative when taken to the other side of the equation?

Table 6.2: *Coding system for learner talk*

In most instances, and this is true for the lessons in this study, procedural questions call for procedural responses and justification questions call for justification responses. It must be noted that the above codes are not in isolation one of the other. There is, for example, a fine line between procedural questioning and procedural explanation by the teacher; conceptual explanation and procedural explanation can also be intertwined in an utterance. Depending on the context of the utterance, both codes can explain the same verbalisation by the teacher. In this study, where it was judged by the researcher that an utterance can be either one or the other, both codes were used simultaneously to code the utterance. In the teacher talk below, for example:

> *TEACHER: Look: half divided by 2. half…then you change the sign to times and this (referring to 2) becomes half and then this times this* ($\frac{1}{2}\times\frac{1}{2}$) *is …?,*

There is, in the first instance, an explanation of the procedures for dividing a fraction by a whole number. But the explanation ends with a question soliciting procedures for solving the problem. The teacher's verbalization can, therefore, be coded as both QP & EP.

In terms of Kilpatrick *et al*'s (2001) strands of mathematical proficiency, justification questions and conceptual explanation by the teacher, and justification responses from learners, in so far as they aim at giving learners a functional grasp of the concept at hand, improves conceptual understanding of learners. Questions, whether asked by learners or by the teacher can aid learners in connecting concepts, making inferences, encouraging creative and imaginative thought, aiding critical thinking processes, and generally helping learners explore deeper levels of knowing, thinking, and understanding (Wilson, 1997). Since strategic competence is about the ability to formulate, represent, solve mathematical problems, and adaptive reasoning the ability to justify and explain the logic behind their solution process, it can be argued that as far the categories above are concerned, both strands are visible in both justification questions by the teacher, and justification responses and questioning by the learner.

Both procedural questions and procedural explanations are about improving learners' procedural fluency as both aim at improving learners' ability in the use of correct procedures in solving problems.

	PRE-INTERVENTION		TOTAL	POST-INTERVENTION		TOTAL
	English	African language		English	African language	
TEACHER UTTERANCE	195	10	**205**	164	5	**169**
LEARNER UTTERANCE	179	-	**179**	157	-	**157**
TOTAL	**374**	**10**		**321**	**5**	

Table 6.3: *Talk distribution between teacher and learners*

The above table indicate that there was, first, more talk by the teacher compared to learners in the pre-intervention lesson as well as the post-intervention lesson. Second, there were more utterances (by both teacher and learners respectively) in the pre-intervention lesson when compared to the post-intervention lesson. There was also more use of the language of instruction in the pre- than in the post-intervention lesson (that is, there was less code-switching in the post-intervention lesson). The learners did not for once resort to their home language(s) in both lessons.

Note that the above table tends to depict a highly interactive class for both the pre- and the post-intervention lessons. The table tends also to portray that learners talked (almost) as much as the teacher did. What the table does not do is indicate the nature or quality of the talk by both the teacher and the learners. The table below shows a counting of the nature of the talk in both lessons:

TEACHER TALK		PRE	POST	LEARNER TALK		PRE	POST
QUESTIONING	**QJ**	4	2	RESPONSES	**RJ**	3	2
	QP	138	139		**RP**	168	148
EXPLANATION	**EP**	20	16	QUESTIONS	**Q**	-	-
	EC	8	3	JUSTIFICATION	**J**	-	-
REGULATION	**R**	28	10				
AFFIRMATION	**A**	5	-				

Table 6.4: *Talk distribution according to codes*

A careful study of the pre-intervention lesson and of the post-intervention lesson indicates that there was no difference in the interactive pattern of both lessons. This is so because what dominated the classroom interaction in both lessons was **first**, only teacher-learner interaction and learner-content interaction. In both lessons, there was no learner-learner interaction (see transcript). Even though learners were sitting in pairs, the class discussion was not structured in such a way as to encourage learners to share ideas with their partners about their solution process.

Second, in both lessons, much of the teacher talk was procedural questions requiring the learners to produce short procedural answers as can be deduced from the table above and seen in the excerpt from the pre-intervention lesson below for solving the equation: $3x - 6 = x - 4$

Teacher & Learners: 3x – 6 = x – 4 (RP)
T: Then? (QP)
Learner1: 3x – x = -4 + 6 (RP)
T: Then? (QP)
Learner 2: 3x – x equals to 2x (RP)
T: 2x (RP)
L2: equals to 2 (RP)
T: equals to 2. From there? (QP)
L2: You divide by two (RP)
T: Yes, you divide both sides by… (QP)
Learners: two. (RP)
T: 2x divided by 2 equal 2 over 2. x is equal to… (QP)
L1: 1 (RP)

Much of the pre-intervention lesson took this form of interactive pattern in solving all the homework problems and the new problems that were solved for the day. Most of the time, the teacher asked a procedural question and learners produced a corresponding procedural answer in chorus. Only a few learners were involved in any kind of direct interaction with the teacher

The post-intervention lesson was very similar to the pre-intervention lesson. Below is an example of how the interaction between the teacher and learners occurred in the post-intervention lesson.

Teacher: What is the 1st question? (QP)
Learners (in chorus): $p+\frac{3}{4}=1+\frac{p}{2}$ *(RP)*
T: What do we find first? (QP)
L: LCD (RP)
T: What's the LCD (QP)
L: 4 (in chorus) (RP)
T: Then…? (QP)
L: 4 times p plus 4 times 3 over 4 (RP)
T: Then what? (QP)
L: Equal 4 times 1 plus 4 times p over 2 (Teacher writes: $4\times p+4\times\frac{3}{4}=4\times 1+4\times\frac{p}{2}$*)*
(RP)
T: Then what? (QP)
L: 4p plus 3 equal 4 plus… (RP)
T: Plus what? (QP)
L: 2p. (RP)
T: you have to explain why it is 2p (writes 4p + 3 = 4 + 2p) (QP)
T: Then from there? (QP)
L: Group like terms (RP)
T: Group like terms…who can do that?(points at a learner) (QP)
L: (still in chorus) 4p minus 2p equals 4 minus 3 (RP)
T: Therefore? (QP)
L: 2p equals 1 (RP)
T: Then? (QP)
L1: Change the sides (RP)
L2: No, you divide (RP)
L: 2 (RP)
T: You divide by two in both sides. P equals to…? (QP)
L: half (RP)

The excerpt above from the post-intervention lesson, where the class was engaging with the problem, $p+\frac{3}{4}=1+\frac{p}{2}$ indicates that the interactive pattern in the class discussion was not different from the interactive pattern in the pre-intervention lesson.

Third, none of the learners posed any question whatsoever in both lessons and much of learners' utterances in both lessons were procedural in nature. Could this be due to the lack of confidence accrued to deficiency in the language of instruction? Could they have asked questions if they were encouraged to use their home language, and if there was more code-switching by the teacher? These questions, though important, are beyond the scope of the present study. Suffices to say that the LoLT in the school is English and while the teacher did not explicitly insist on the use of any language, the interactions were in English.

As noted earlier, language is intrinsically linked to adaptive reasoning in Kilpatrick *et al*'s (2001) strands of mathematical proficiency since it involves the reflection on, a justification and explanation of solution processes undertaken by learners. Learners need to be given the opportunity to explain their solutions and solution process and to talk about the concepts and procedures they have used. This (drawing on learners' capacity to explain and justify not only solution processes but also relationships between concepts) was not sufficiently foregrounded in both lessons as can be seen from the table above. The table also shows that learners did not have to *justify* (code J) a solution process in both lessons.

In general, it can be said that the interaction in both the pre-intervention lesson and the post-intervention lesson is reminiscent of what Young (1984, in Edwards & Westgate, 1987: 143) term a tendency for learners to be "obliged to respond within the teacher's frame of reference and at the teacher's bidding".

SUMMARY OF CLASSROOM INTERACTION

A close look at the pre-intervention lesson and the post-intervention lesson showed a number of similitudes: both were taught by the same teacher using the same strategy, the same topic was under consideration and the duration was the same for both lessons that were analyzed.

Analysis of the two lessons showed, however, that there was no difference between both lessons: Both lessons were procedural in their interactive pattern, and both had only teacher-learner interactions that were highly about procedures involved in solving the mathematics problem at hand.

CONCLUSION

The third specific objective of the study was to:

- To do an analysis of the teacher-learner and learner-learner classroom interaction in the mathematics class before and after the implementation of the ASTRALAB program in order to explore whether or not the mathematics communications have been enabled or constrained.

This chapter has provided analysis of the classroom interaction and communication for the experimental group in the bid to provide answers to this third objective of the research.

CHAPTER SEVEN

CONCLUSIONS AND RECOMMENDATIONS

INTRODUCTION

The present study, even though limited in scope, provides some useful insight into how improving learners' proficiency in English enables or constrains mathematical proficiency. This chapter draws conclusions on the research findings. The implication, limitation of the research and reflections on the research would be discussed. The study was guided by the following questions:

- How does the use of the ASTRALAB instructional English literacy software enable or constrain learners' mathematical proficiency?
- To what extent does improving English language through the use of the ASTRALAB software enable or constrain interaction in the mathematics classroom?

Suffice it to say that the purpose of the study was not to explore how ASTRALAB ILS improves mathematical proficiency, but how the improvement of English language (of which the ASTRALAB ILS was a tool used to achieve this end) enables or constrains mathematical proficiency.

SUMMARY

In South African, even though the constitution gives provision for learners to learn in any of the 11 official languages of their choice, most learners learn mathematics in English language which for most, is not their first or home language. Underachievement in Matric examinations (mathematics) has been found to be more prevalent amongst learners who use English language less frequently at home (Simkins in Taylor, Muller & Vinjevold, 2003) and in areas where English is less frequently used at home. It is against this backdrop that it was important to directly influence proficiency in English language of these learners to ascertain how this (becoming more proficient in English) would enable or constrain their mathematical proficiency

A quasi-experimental, non-equivalent comparison group design was adopted in responding to the research questions of the study. Statistical methods were used to analyse the pre-test and post-test scores for the experimental and control groups. The statistical analysis of data reveals a moderate correlation (0.328) between the learners who improved their English language proficiency (using the ASTRALAB treatment programme) and those who did not. There was also a statistically significant difference between performance in the pre-test and performance in the post-test of the experimental group and a highly significant difference between the post-test scores of the control group when compared to the post-test scores of the experimental group.

In addition to the pre-test and post-test, there was also classroom observation of the mathematics interaction in the class before and after the intervention. There was no difference between the pre-intervention interaction in the mathematics class and the post-intervention interaction.

DISCUSSIONS AND CONCLUSIONS FROM RESEARCH

When one considers the results from the qualitative and quantitative analyses of data, one is tempted to conclude that the two provide conflicting results. While analysis of test results indicated that improving learners' English proficiency level using the ASTRALAB English ILS led to improvement in mathematics performance in the tests, analysis of the classroom interaction indicated that this improvement of language proficiency did not enable mathematical communication in the classroom.

On the one hand, given that there was a highly significant difference between the post-test scores in the experimental group and those of the control group, and that the experimental group showed a statistically significant higher gains from pre-test to post-test, it can be concluded that the improvement of the performance in mathematics from pre-test to post-test was not due to chance than due to the fact of having improved the English language proficiency of learners. On the other hand, it can be deduced from the data that even though the English language proficiency level of learners was improved, such improvement had no effect on classroom interaction in the mathematics. If what is foregrounded in the development of language proficiency in learners is the basic interpersonal communicative skills (BICS) and not CALP, there is no doubt that learner interaction in the class – an enterprise which demands that learners debate, reason, criticise, analyse, evaluate and express their opinions using academic language in the class – would not be improved. Little wonder that learners did not ask questions in the post-intervention class, and the class did not become less procedural (and more conceptual and adaptive) by way of the nature of talk in the post-intervention lesson.

It can be argued, however, that this result from the study is not conflicting because the two methods (of analysis) were analysing different questions and responding to different questions. While one answered a question about achievement, the other answered a question about classroom interactions. This means that there is no causal relationship between the two (achievement and interaction). Improvement in performance does not automatically lead to improvement in mathematics communication in class.

What other conclusion can be drawn from the above seeming dichotomy between test scores and interaction in the mathematics class? The present research study is an indication of the fact that proficiency in the language of instruction (English) is closely linked to achievement in mathematics. But improving learner proficiency in English, even though necessary, is not sufficient to impact on classroom interaction. In any classroom, the teacher plays a key role in the management of the interaction in the classroom (Edwards & Westgate, 1987). The teacher's ability, therefore, to draw on learners' linguistic resources – one of which is structuring questions to allow learners to sufficiently express their thinking – , is therefore important.

RECOMMENDATIONS

Proficiency in the language of instruction (which is English in most South African high schools) has been recognised by researchers to be an important index in the teaching and learning of mathematics. Research into the interplay between proficiency in the language of instruction and Examination in Matriculation examinations in South Africa show a strong statistical correlation between marks in the language of instruction (and examination) and achievement in mathematics (CDE, 2004). The present research report also reveals that proficiency in the language of instruction is an important index in achievement in mathematics. Since English is the language of instruction and examination in most South African school, I make the following recommendations for the teaching and learning of mathematics:

The researcher takes seriously the recommendation by the Centre for Development Enterprise that all mathematics (and Science) activities be "closed linked with improved language [English] education" (CDE, 2004: 33-34).

Also, appropriate mathematics teacher training (in mathematics) must be accompanied by appropriate training of the teachers in effective English communication (Howie, 2002) and teacher development in strategies of tapping into learners' linguistic resources. Mathematics teacher must therefore be fluent in English and necessary mechanisms must be put in place by the training institutions to improve the English proficiency of second language mathematics teachers.

Finally, any attempt to improve the language proficiency of learners with the aim of improving academic proficiency should be done in such a way as to develop concurrently, both the Basic interpersonal communicative skills and the cognitive-academic language proficiency. By so doing, there would be a high possibility of learners' improvement in achievement as well as a greater classroom interaction.

LIMITATIONS OF THIS STUDY

The question as to what percentage is representative of the whole would always remain a practical problem in research. Certainly, 98 cannot be judged adequately representative of the 317 learners in Grade 9 in the experimental school. Neither can I claim that the number of learners in the study is representative of all learners who do mathematics in a language that is not their home language. I can therefore not generalise on my findings.

Akin to the above limitation was the fact that the period for the implementation of the ASTRALAB ILS was not sufficient to conclude that the English language proficiency (both BICS and CALPS) of learners was thoroughly improved. The Zone of Proximal Development certainly has implications not only for the mediation of learning, but also for the development of a second language (Nieman, 2006). Internalising a language or new vocabularies, no doubt takes time. A more prolonged intervention using the ASTRALAB software would have been worthwhile.

Even when extraneous variables are well controlled, it is still possible that some other factors be responsible for the statistical difference between the pre-test and post-test. Like any experimental study, the study assumes that all other variables remain constant for both groups in the course of the treatment with the ASTRALAB programme.

Finally, because the research instrument was limited in the number of questions in each category, it was difficult to draw conclusions about the enabling or otherwise of mathematical proficiency as far as the questions categories were concerned.

FUTURE RESEARCH

Why was there a statistically significant difference in achievement between boys and girls from the pre-test and post-test results? Was the language proficiency level of girls greatly improved compared to that of boys? What could have been responsible for the difference? Are the comprehension stories, for example, used in the ASTRALAB ILS gender biased? This could be an important area of research for future study as it could provide invaluable information for education software developers.

Secondly, future research could focus solely on two or three content domains and investigate how the improvement of the language of instruction has either enabled or constrained mathematical proficiency in those content domains. In such an enterprise, the intervention phase using the independent variable (to improve learners' language proficiency) should be such that develops both the basic interpersonal communication skills as well as the cognitive-academic language proficiency of learners.

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APPENDIX A

RESEARCH INSTRUMENT

APPENDIX B

TRANSCRIPTS OF PRE-INTERVENTION AND POST-INTERVENTION LESSONS

PRE-INTERVENTION LESSON

The class started with a revision of the previous homework on equation that was given to learners. At the end of the revision exercise, the class solved more complex questions on equations.

Teacher: Open the books (pause). (R) Teacher moves from desk to desk inspecting learners' homework for about 5 minutes). Then the revision class ensued:

Teacher: (writes x + 4 = 8). Then what? (QP)

Learner1: x + 4 – 4 = 8 – 4 (RP)

T: x = 8 – 4. Therefore? (QP)

L & T: x is equal to 4 (RP)

L: Yes, yes. (noise)

T: Ssshhh (R)

L: Who's saying ma'am? You want to say something? (R)

L: No (RR)

T: Just mark with a red pen. (R) (pause). Ok, second one. (QP)

L: 4x = 8 (RP)

T: Then what? (QP)

L1: 4x divided by 4… (RP)

T: 4x divided by 4 equal to? (QP)

L: 8 over 4 (RP)

T: Therefore (QP)

T & L: x is equal to 2. (RP)

T: x – 5 = 10 (3rd question). Then what? (QP)

L1: x – 5 + 5 = … (RP)

T: (writing) x = 10 + 5. Therefore x is equal to? (QP)

L: 15 (RP)

T & L: -5x = 10 (next question) (QP)

T & L: -5x over -5 equals to 10 over -5. (RP)

T: x is equal to? (QP)

L1: two (RP)

L: negative two (RP)
(teacher writes down next question)

T: Therefore? (QP)

L: x + 12 – 12 (RP)

T: x + 12…minus (QP)

L: Minus 12 equal to zero minus 12 (RP)

T: x is equal to? (QP)

L: 12. (RP)

T: Not 12 (inaudible). If you have written 12, it is positive.(EC)

T: 6 (question 6). botsisa nna ntho e wrong (ask me whatever you find to be incorrect)
Vernacular 38:46 (R)

L: 12x = 0 (RP)

T: 12x equals to? (QP)

L: Zero (RP)

T: Then? (QP)

L1: 12x divided by 12… (RP)

T: 12x divided by 12 equals to zero divided by? (QP)

L: 12. (RP)

T: equal to? (QP)

L1: 12. (RP)

L: zero, 12, zero (talking at the same time) (RP)

T: Next question (R)

L1: x + 2 = 1 (RP)

L: ah, ah, no, no. Yes (RP)

T: I’m not conducting a party. One person. (R)

L1: x + 2 equal to 1 (mumbling from other learners) (RP)

T: Shhh. (inaudible). (R) Then let us work it out, then you'll come (inaudible)

T: x = 1 – 2. You agree? (QP)

L: Yes (inaudible) (RP)

T: x is equal to what? (QP)

L1: x is equal to one (mumbling from other learners) (RP)

T: If you say x + 2 – 2 it is still the same thing. (EP)

T: Then number 8. (R)

T & L: 3x – 6 = x – 4 (RP)

T: Then? (QP)

L1: 3x – x = -4 + 6 (RP)

T: Then? (QP)

L2: 3x – x equals to 2x (RP)

T: 2x (RP)

L2: equals to 2 (RP)

T: equals to 2. From there? (QP)

L2: You divide by two (RP)

T: Yes, you divide both sides by… (QP)

L: two. (RP)

T: 2x divided by 2 equal 2 over 2. x is equal to… (QP)

L1: 1 (RP)

T: Number 9 (R)

L1: 3x + 8 = x + 7 (RP)

T: Hey, all of you participate (pause). Inaudible. (R)

T & L: 3x – x = 7 – 8 (RP)

T: Then? (QP)

Marku: $3x - x$ is equal to x (RP)

T: 2x equal to (QP)

Marku: 1 (RP)

T: minus 1. then from there? (QP)

Marku: divide by 2. You divide by 2 (RP)

T: From there? Divide. 2x divided by 2 is equal to -1 divided by 2. x is equal to minus over 2. (EP)

T & L: $5x - 3 = 2x + 9$ (RP)

T: Then from there? (QP)

T & L: $5x - 2x = 9 + 3$ (RP)

Silvia: $3x = 12$ (RP)

T: $3x = 12$. From there? (QP)

Djabu: You divide by 3 (RP)

T: Divide by three. Equal to 3 into 3? (QP)

L: 1 (RP)

T: time x? (QP)

L: x (RP)

T: equals to: 3 into 12? (QP)

L: 4 (RP)

T: Number 11. (R)

T & L: $2(3a + 1) = 7 - 4a$ (RP)

T: What? (QP)

L1: 6a (RP)

T: 6a. how did you get 6a? (QJ)

L1: You say 2 time 3a (RJ) & (RP)

T: 2 times 3a, which is? (QP)

L: 6a. (RP)

T: plus? (QP)

L: plus 2 (RP)

T: plus 2? (QP)

L: equal to… (RP)

T: 7 minus… (QP)

L: 4a (RP)

T: then from there? (QP)

L1: 6a + 4a (RP)

T: 6a + 4a? (QP)

L: equal to 7 minus 2 (RP)

T: then what? (QP)

L: 10a equal 5 (RP)

T: then? (QP)

L: Divide by 10 (RP)

T: 10a over 10 equal to 5 over ten. Ten into 10? (QP)

L: 1 (RP)

T: times a? (QP)

L: a (RP)

L1: 5 over 10 (RP)

T: 5 over 10. They are the multiples of 5. 5 into 5? (EP) & (QP)

L: two, one (RP)

T: 5 into 5 goes how many times? (QP)

L: one (RP)

T: 5 into 10? (QP)

L: 2 (RP)

T: If I could have arrived here (pointing to a) and say that the value of a is 5 over 10, but you have to simplify it (inaudible). This are equivalent fractions. Ne? (EC)

L: Yes (RR)

T: They have the same value. (EC) Number 12.

T & L: 6(p – 1) – p = -2(p + 1) + p – 1 (RP)

T: e right? (QP)

L: e right. (RP)

T: Then negative 3? (learners mumbling) (QP)

T: then what? (QP)

T & L: 6p – 6 – p equals to -2p. (RP)

T: minus 2p…? (QP)

L: plus… (RP)

T: Wrong. Negative times positive. Negative 2. Plus p – 1. (EC)

T: Don't forget those rules (pause) (R)

T: then what? From there what do we do? (QP)

L & T: 6p – p + 2p – p equals? (RP)

T: what? (QP)

L: -2 minus 1 plus 6 (RP)

T: 6p minus p? (QP)

L1: 5 (RP)

L2: 4 (RP)

T: 6p minus p? (QP)

L1 5, 5p (RP)

L3: 5, 4 (RP)

T: 6p minus p? (QP)

Sandile: minus 5 (RP)

T: how did you get negative 5? (QJ) 6 minus p? 6p minus p? (QP)

L : 5p (RP)

T : How do you get those things ? You have 6. You subtract 1 (EP)

L: 5 (RP)

T: (in vernacular 49:26)aoa five (not five). If you are saying five, it's wrong. Five p! (A)

T: 5p plus 2p? (QP)

L: 7p (RP)

T: 7p minus p? (QP)

L: 6p (RP)

T: If you are saying just 5, 5 what? I want to know that. (QJ)

T: Then minus 2 minus 1? (QP)

L1: negative 3. (RP)

T: Plus 6? (QP)

L1: Positive 3 (RP)

T: Then what? (QP)

L2: 6p divide by 6 (RP)

T: 6p divided by 6 and 3 divided by … (QP)

L: 6 (RP)

T: equals to 3 over 6. 6 into 6? (QP)

L: 1 (RP)

T: Then 1 times p? then what? 3 over 6. They are the multiples of … (QP)

L1: 6 (RP)

T: eh? (R)

L: 3 (RR)

T: 3 into 3 goes how many times? (QP)

L: once (RP)

T: 3 into 6? (QP)

L: 2 (RP)

T: the value of p is equal to? (QP)

L: one over two (RP)

T: Somebody clean the board. (*long pause*). Hey, clean the whole board. Leave number 10, 11, and 12. (*learners copy from the board*). Hey, you don't have to make noise, otherwise, I clean it. (R)
(*teacher copies questions on the board*)

1. $\frac{b}{4} = 3$
2. $\frac{2a}{3} = 6$
3. $\frac{x}{2} + \frac{3}{2} = 5$
4. $x + \frac{1}{3} = 2$

T: You are through with this one? (R)

L: No, Yes (RR)

T: If you are not through, we don't want to compete (R)

T: now we are going to carry on with solving problems involving fractions. Ne? You must listen. I don't want …(inaudible). (R)

T: Let us take the 1st one. (R)

L1: b over 4 equals 3 (RR)

T: How can you work out the value of $\frac{b}{4} = 3$? (pause). How can we…(inaudible). (QP)

L1: b over 4 times 4 equals 3 times 4. (RP)

T: to remove this denominator, we have to multiply both sides by? (QP)

L: 4 (RP)

T: then you are going to say b over 4 multiplied by 4. What I do on the left, I must also do to the ....?(QP) & (EP)

L: right (RP)

T: 4 divided by 4? (QP)

L: 1 (RP)

T: times b? (QP)

L: b (RP)

T: equals to? (QP)

L: 12 (RP)

T: Number two. $\frac{2a}{3} = 6$. How would you do that problem? (pause) (QP)

T: What is important to look at, you look at the coefficient of…? (QP)

L: (mumble)

T: coefficient of a. Because 2a over 3 is equal to, can be written in the form of 2 over 3 multiplied by a (writes $\frac{2a}{3} = \frac{2}{3} \times a$). Then you cancel the coefficient of that; of a which is 2 over 3. Multiply both sides with its reciprocal. (EP) If you look at the coefficient of, let's say 2a over 3, the coefficient of a is equal to what? 2 over 3. Then what would be the reciprocal of 2 over 3? (QP)

L: a, a. (RP)

T: Reciprocal! (A)

L1: 3 over 2 (RP)

T: 3 over 2. reciprocal here is going to be 3 over 2. Reciprocal of 1 over 2 is equal to what? (EP)

L: 2 over 1 (RP)

T: 2. Reciprocal of 4 over 5 is equal to what? (EP)

L: 5 over 4. (RP)

T: 5 over 4. Look at your books. Tell me, the denominator is equal to what? (QP)

L: 3 (RP)

T: there. In this case, having a problem like this, if you have a problem like this, you multiply with the reciprocal of your fraction which is? (EP)

L & T: 2 over 3 (RP)

T: Then we are going to say, 2 over 3, 2a over 3, ne? you multiply with…? (QP)

L: 3 over 2 (RP)

T: Which is equal to, what I am doing there, I must also do it there. Multiply by what? (QP)

L & T: 3 over 2 (RP)

T: 3 into 3? (QP)

L: 1 (RP)

T: 2 into 2? (QP)

L: 1 (RP)

T: 1 time 1? (QP)

L: 1 (RP)

T: times a? (QP)

L: a (RP)

T: Therefore a is equal to what? 6 times 3? (QP)

L: 18 (RP)

T: 18 over? (QP)

L: two (RP)

T: 18 over 2 is equal to what? (QP)

L: 9 (RP)

T: Then the value of a is equal to? (QP)

L: 9 (RP)

T: It makes sense? (R)

L: Yes (RR)

T: Number 3. $\frac{x}{3}+\frac{3}{2}$ equals to? (R)

L: 5 (RR)

T: What do you think would be the 1st thing to do? Just try. Yes? (QP)

L1: 2 over x (RP)

T: 2 over x (A)

L2: x over 2 times 2over 3 equal 5. (RP)

T: Inaudible

L3: x divided by 2 times 2 over 2 plus 3 over 2 times… (RP)

(other learners laugh)

L4: x…(inaudible) divided by 2 equals to 5 times 3 over 2 (RP)

T: Is that right? (QP)

T: Those steps that you need to follow when solving equations, you still apply them though you work with fraction, you still apply them. Yes. Yes, yes. Sorry, sorry. (EP)

L5: x over 2 times…em… (RP)

T: (in vernacular 01:17) O rata times (You like multiplying) unnecessarily (R)

L5: $\frac{x}{2}+\frac{3}{4}$. (RP)

T: Fraction, and a whole number. Those are…(inaudible). A natural number here or a whole number can be written in the form of… (QP) & (EP)

(silence from learners)

T: 2 and $\frac{1}{2}$. This are like terms. The reason being, we can write two in the form of a fraction. How? Can you write two in the form of a fraction? (EC)

L: Yes (RP)

T: How? (QP)

L1: 2 over 1. Then its' written as 2 over 1. Therefore, this and 1 over 2 is the form of, they are like terms. Then…you still follow those steps even when you work with fractions. Ne? (RJ)

L: Yes (RJ)

T: In the 1st example, here, we just solve. We don't have like terms that we need to put together (in vernacular 02:37)-akere (isn't it) (EC)

L: Yes. (RP)

T: Then after multiplying, you group like terms. Grouping like terms, you add or subtract those like terms. After adding or subtracting those like terms, you solve. (EP)

T: Here (referring to $\frac{b}{4}=3$), we don't have to group like terms. We don't have those like terms that we need to group, ne? But in this case (referring to $\frac{x}{2}+\frac{3}{2}=5$), we have like terms. How can you group that? As we were doing earlier, group like terms. Which ones? Identify those like terms first. (EP) & (QP)

L: (2 of them) 2 (RP)

T: What are the like terms? (QP)

L: 2 (RP)

T: eh? (R)

L: 2 and 2 (inaudible) (RP)

T: Why do you say 2 and 2? (QP)

T: I'm having $\frac{x}{2}+\frac{3}{2}=5$, group like terms. Identify the like terms for me. (QJ)

L1: x over 2 and …(inaudible) (RP)

T: (in vernacular 03:54) x ke e one ga gona e e tshwanang le yona. X over 2 e itsamaela ele one (there is only one x, there isn't any other like it. X over 2 goes on its own) (EP)

T: I don't have two. Where do you get 2? (EP) Look at that (in vernacular 04:30) nna e ka setswe mo nna (it will not appear on my face) (R)

L1: 3 over 2 and 5 (RP)

T: 3 over 2 and…? (QP)

L: 5 (RP)

T: These are like terms. Then I have to…how can I group them? I have to bring them along one side, ne? (QP)

L: yes (RP)

T: I have to bring them on one side. Then how can I do that? (pause). How can I do that? (pause). Those 2 numbers can be on one side. How can I do that? (QP)

L: I see only Grace concentrating. The rest where is your concentration? I still need that. Sandile (R)

Sandile: 3 over 2 times 5 over 5 (RP)

T: (inaudible). One side and how is it times? (QP)

L2: x over 2 equals 5 over 1 minus… (RP)

T: Good. Minus… (A)

L2: 3 over 2 (RP)

T: This is positive. When you bring it to the other side, it will change to be what? (QP)

T & L: negative (RP)

T: (in vernacular 06:01 inaudible). (learners clap) (A)

T: Then, hence, we are working with fractions, we have to change that 5, we write it in the form of… (QP)

L: Fractions (RP)

T: 5 can be written in the form, as a fraction in the form of 5 over…? (QP)

L: one (RP)

T: Then bring 3 over 2 from that side it would be? Negative 3 over …? (QP)

L: 2 (RP)

T: (in vernacular 06:41) if o e tlisa mo side e one (if you bring it to one side). If you bring it to the other side, you don't swap your fraction and you don't multiply. You multiply only if you remove it as a coefficient of x or if it would be a coefficient of b, you want to remove it, it is then that you would multiply with its reciprocal. Akere (isn't it?) (EP)

L: yes (RP)

T: Ok. But, e eikemetse gee le so (– pointing to 3 over 2)(this fraction stands on its own) (in vernacular 07:10) you just change the sign, and bring it to the next side. (EP)

T: As we were busy doing, we were saying a + 4 = 12. You bring that (+4) to the other side. Then a is equal to 12 – 4. Even working with fractions, we still follow that procedure. (EP)

T: Does it make sense? (QP)

L: yes. (RP)

T: Then, ok. You have written that … From primary (school), they say you can't add or subtract 2 fractions with different denominators. Am I wrong? (EC) & (QP)

L: NO! (RP)

T: What would you do if you have to add or subtract two fractions having two different denominators?(QP)

L1: Look for LCD (RP)

T: You find your …? (QP)

L & T: LCD (RP)

T: What is going to be your LCD? (QP)

L: 2 (RP)

T: LCD is going to be…? (QP)

L: 2 (RP)

T: Then what do you do with your LCD? Say one into 2 goes how many times? (QP)

L: 2 (RP)

T: 2 multiplied by 5? (QP)

L: 10 (RP)

T: 2 into 2 goes how many times? (QP)

L: 2 (RP)

T: multiplied by 3? (QP)

L: 3 (RP)

T: negative 3. Then x over 2 is equal to what? Eh? (QP)

L: 7 over 2 (RP)

T: No, it is time to multiply with the reciprocal…1 over 2. Immediately you don't have any number there (referring to x in $\frac{x}{2}$), you know that the number is? (EC) & (QP)

L: 1 (RP)

T: The reciprocal of 1 over 2 is equal to what? (QP)

L: 2 (RP)

T: 2 over 1. Then you are going to say x over 2 multiplied with what? (QP)

L: 2 over 1 (RP)

T: 2 over 1. Equals to? (QP)

T & L: 7 over 2 (RP)

T: Multiplied by? (QP)

L: 2 over 1 (RP)

T: 2 over 1. then, 2 divided by 2? (QP)

L: 1 (RP)

T: then multiplied by x? (QP)

L: x (RP)

T: 7 times 2? (QP)

L: 14 (RP)

T: 14 over? (QP)

L: 2 (RP)

T: Then what is the value of x? (QP)

L: 7 (RP)

T: Sipho, are you confused? (R)

Sipho: Yes (RP)

T: You are confused. As much as you are confused…(inaudible) (R)

T: Then, let us do number 4. Take number 4 as another example. $x + \frac{1}{3} = ?$ (QP)

L: 2 (RP)

T: First, group the like terms together. Which ones are the like terms? Identify like terms. What would that be? (QP)

L1: 1 over 3 and 2 (RP)

T: 1 over 3 and…? (QP)

L: 2 (RP)

T: Then, you have to bring that ($\frac{1}{3}$) to that side. Then you are going to say, equal to, write 2 in the form of…? (QP) & (EP)

L: fraction (RP)

T: Fraction. It would be…? (QP)

L: 2 over 1 (RP)

T: 2 over 1. Then minus…? (QP)

L & T: 1 over 3 (RP)

T: Then from there, x is equals to…you can't subtract different fractions with different denominators. LCD. Your LCD would be…? (EP) & (QP)

L: 3 (RP)

T: LCD is going to be 3. Then how? Then I write my LCD here. 1 into 3 goes how many times? (QP)

L: 3 (RP)

T: 3 times 2? (QP)

L: 6 (RP)

T: 3 into 3? (QP)

L: 1 (RP)

T: multiplied by 1? (QP)

L: 1 (RP)

Minus 1. then the value of x is equal to what? 5 over? (QP)

L: 3 (RP)

T: Then x is equal to…3 into 5 goes how many times? (QP)

L: 1 (RP)

T: remainder? (QP)

L: 2 (RP)

T: over? (QP)

L: 3 (RP)

T: We will continue tomorrow with the equations. (R)

POST-INTERVENTION LESSON

The lesson started with the teacher moving from desk to des checking if learners did their homework of the previous lesson. She reprimanded those who didn't go their homework as she goes from pair to pair (learners were seated in groups of two). Then a revision of the homework questions ensued.

QJ – QUESTION: JUSTIFICATION
RJ – RESPONSE: JUSTIFICATION
QP – QUESTION: PROCEDURAL
RP – RESPONSE: PROCEDURAL
EP – EXPLANATION: PROCEDURAL
Q – QUESTION
EC – EXPLANATION: CONCEPTUAL
J – JUSTIFICATION
R - REGULATION

Teacher: What is the 1st question? (QP)

Learners (in chorus): $p+\frac{3}{4}=1+\frac{p}{2}$ (RP)

T: What do we find first? (QP)

L: LCD (RP)

T: What's the LCD (QP)

L: 4 (in chorus) (RP)

T: Then…? (QP)

L: 4 times p plus 4 times 3 over 4 (RP)

T: Then what? (QP)

L: Equal 4 times 1 plus 4 times p over 2 (Teacher writes: $4\times p+4\times\frac{3}{4}=4\times1+4\times\frac{p}{2}$) (RP)

T: Then what? (QP)

L: 4p plus 3 equal 4 plus… (RP)

T: Plus what? (QP)

L: 2p. (RP)

T: you have to explain why it is 2p (writes 4p + 3 = 4 + 2p) (QP)

T: Then from there? (QP)

L: Group like terms (RP)

T: Group like terms…who can do that?(points at a learner) (QP)

L: (still in chorus) 4p minus 2p equals 4 minus 3 (RP)

T: Therefore? (QP)

L: 2p equals 1 (RP)

T: Then? (QP)

L1: Change the sides (RP)

L2: No, you divide (RP)

L: 2 (RP)

T: You divide by two in both sides. P equals to…? (QP)

L: half (RP)

T: now? (QP)

L: Check. (RP)

T: Ho do you do that? (pause) Where there is p you put what? (QP)

L: half (RP)

T & L: $\frac{1}{2}+\frac{3}{4}=1+\frac{1}{2}\div 2$ (RP)

T: And from there? (QP)

L3: 4 (RP)

T: LCD is 4. 2 into 4? (QP)

L: 2 (RP)

T: Times 1? (QP)

L: 2 (RP)

T: 2. Four into 1? (QP)

L: 4 (RP)

T: times 3? (QP)

L: 3 (RP)

T: half divide by 2, Jomo (QP)

Jomo: 1 (RP)

T: Inaudible (in vernacular 37:18). One plus…you divide half into 2 (EP)

L1: 1 (RP)

L2: 2 (RP)

L3: 4 (RP)

L4: negative 2 (RP)

L5: 2 over 4 (RP)

T: You have half, and you divide it into two, you get what? (EP)

L: one (about 3 learners); zero (about 2 learners) (RP)

T: Look: half divided by 2. half…then you change the sign to times and this (referring to 2) becomes half and then this times this ($\frac{1}{2}\times\frac{1}{2}$) is …? (QP) & (EP)

L: 1 over 4 (RP)

T: you have half and you divide that half into two, practically you get 1 over 4. Then you have the 1 over 4 here ($\frac{2+3}{4}=1+\frac{1}{4}$). (EP)

T: Then LCD? (QP)

L: 4 (RP)

T: 1 into…4 times 1? (QP)

L: 4 (RP)

T: Then, plus 1. 5 over 4 equal to 5 over 4. The left-hand-side is equal to? (QP)

L: The right hand side. (RP)

T: The questions says: Solve the following and then check if your answer is correct. If you didn't check, you didn't complete your solution. Then automatically, you get that mark for solving and another mark for checking. The second one (meaning question 2) (EP)

L: p over three (RP)

T: What? (QP)

L: $\frac{b}{3}-\frac{b}{2}=24$ (RP)

T: What? (QP)

L: LCD. (RP)

T: What is the LCD? (QP)

L: 2 (RP)

T: (very loud) LCD is going to be what? (QP)

L: (unclear) mixture of 2, 3, 4, 6, 12) (RP)

T: How do you get the LCD? (QP)

L: (say 2, 3, 4, 6, 12, 24 at the same time) (RP)

T: e e ke le boditse gore le ska bolela bothle, ha le ko musikeng (no, no, no dont all talk at the same time, this is not a music class) (vernacular 40:37) (R)

L1: 6 (RP)

T: How did you get 6? (QJ)

L1: Our LCD is 6 because…you check the multiples of… (RJ)

T: You check the multiples of 3: which is 3, 6, 9 and multiples of 2 which is 2, 4, 6, 8. 6 is the common one. (EC) Then our LCD will be…? (QP)

L: 6 (RP)

T: Then divide everything with…? (QP)

L: 6 (RP)

T: 6 multiplied by b over 3 minus 6 multiplied by b over 2 equal 6 multiplied by 4. What is it? (QP)

L & T: 2b – 3b = 24 (RP)

T: 2b minus 3b? (QP)

L1: negative b (RP)

T: negative b, which is equal to…? (QP)

L: 24 (RP)

T: divide by what? (QP)

L: Negative…(inaudible) (RP)

T: heeh? (R)

L: negative one (RP)

T: Then b is equal to? (QP)

L: negative 24 (RP)

T: Check. How do we check? (QP)

L1: Put negative 24 where there is…(RP)

T: Put negative 24 where there is b. negative 24 over 3 minus negative 24 over 2 equal ? (QP)

L: 4 (RP)

T: Ne? (R)

L: Yes (RR)

T: negative 24 divided by 3? (QP)

L: negative 8 (RP)

T: negative 24 divided by 2? (QP)

L: negative 12 (RP)

T: negative…? (QP)

L: 12 (RP)

T: negative 12 times negative? (QP)

L: positive 12 (RP)

T: positive 12 (writes -8 + 12). Which is equal to…? (QP)

L: 4 (RP)

T: negative 8 plus 12? (QP)

L: Positive 4 (RP)

T: 4 is equal to…? (QP)

L: 4 (RP)

T: Left-hand-side is equal to? (QP)

L: Right-hand-side. (RP)

T: Therefore, b is equal to…? (QP)