EXPLORING HOW GRADE 7 LEARNERS SOLVE ADDITIVE WORD PROBLEMS

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DECLARATION

l, Karabo Mochosa, student number: 1436430, declare that this Research Report is my own original work. At the University of Witwatersrand in Johannesburg, it is being presented for a master's degree in education. It hasn't been turned in to any other universities for a degree or test.

Signature:

Date : 15 March 2023

ABSTRACT

This report focuses on the role that the dynamic model of the bar model and number line can play in redirecting learners from a non-formal understanding associated with the real or imagined reality to an understanding of formal systems (Van den Heuvel Panhuizen, 2003). My investigation is located within two grade 7 classes in a school at Johannesburg, one class was the control group and the other was the intervention group, the control group was not exposed to any tools that can help improve and support the learner's informal solution strategies, whereas the intervention had an exposure. In addition to researching the models that learners use before the intervention courses, I was the teacher of the six intervention lessons on additive relation problems. This report presents evidence that learners struggle with working with abstract calculations. However, with proper intervention, learners can make important changes toward more abstract calculations. In addition, the most important finding is that most of the problems were approached using the column model in the pre- and post-test of the control group, and the pre-test of the intervention group. This confirms that this model has been encouraged by most schools for learners to use from the beginning of the intermediate phase. Also, the results show that most questions in both the control group's pre-test and posttest as well as the intervention group's pre-test had a significant percentage of wrong answers due to the usage of the column model. On the other hand, the post-test of the intervention group had very fewer problems that were tackled using the column model. The use of the bar model and number line in the post-test of the intervention group was the one mostly used and it was associated with a proportion of correct answers, this suggests that improvements can be achieved in a short period of time.

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ABBREVIATIONS

ANA	Annual National Assessments
BM	Bar model
BD	Breaking down
CM	Column model

HNS	Horizontal number sentence
MM	Mathematical Modelling
NL	Number line
NM	No Model
RME	Realistic Mathematics Education
WS	Word Sentence

CHAPTER ONE: INTRODUCTION

1.1 Introduction

One of the most crucial components of problem-solving is the capacity to model situations and solve additive word problems. Today's researchers concur that the cognitive processes involved in problem-solving and mathematical reasoning are exceedingly complicated, and that further study is necessary to better understand these processes at the phenomenological level (Polotskaia, 2015). Teachers are unable to successfully fulfill the requirements of their learners due to a lack of knowledge about the types of reasoning learners use to analyse and model word problems. This study explored how Grade 7 learners solve additive problems to answer the following questions: What kind of models do learners use to solve additive word problems, what models did they initially use to analyse additive word problem tasks and what was their performance on these tasks? After the intervention, what models were used and what were the shifts in their performance?

1.2 Problem statement

One of the most crucial components of teaching and studying mathematics in schools is mathematical problem solving. Learning how to analyse a problem statistically equips learners to think critically and engage with a range of social and political topics (Polotskaia, 2015). Teachers are unable to effectively address the requirements of learners because they lack a comprehensive knowledge of the types of reasoning that learners may use to analyse and model a situation. Chirove and Mogari (2017) point out that word problems are a crucial, yet challenging component of early-grade mathematics. For instance, word problems have been noted as a persistent shortcoming in the South African Annual National Assessments (ANAs) in the country (Department of Basic Education, 2012, 2014, 2015).

Similarly, Roberts (2016) asserts that learner difficulty with word problems has been identified as a recurrent weakness in South Africa. In the senior grades, however, teaching is still generally examination driven, where learners are subjected to drill and rote learning with an emphasis on mastery of procedures rather than developing conceptual reasoning strategies and high order thinking skills (Chirove & Mogari, 2017). Suffice it to state that South African learners' problem-solving ability is compromised due to such teaching. For example, grade 12 learners could not successfully solve problems that involved high order thinking and reasoning (Chirove & Mogari, 2017).

Mathematical problem-solving is crucial for the growth of reasoning and thinking abilities, encourages higher-order thinking, and aids in conceptual comprehension and meaningful learning. Poor teaching in lower grades contributes to learners' inadequate comprehension of the fundamentals and weak foundational abilities, which prevent them from solving cognitively demanding problems like non-routine difficulties (Chirove & Mogari, 2017). For instance, when operating whole numbers, schools encourage learners to use the column addition model. This model is used to deal with additive operations, mostly used on whole numbers. As a result, more models are needed to deal with other topics that have additive operations involved in. When learners are using the column model, they operate the numbers individually and this becomes a problem in that they will not understand place value and will lack number sense (Kamii & Dominick, 1989). What this model does is that it places little value on learners having to understand the positions of the numbers. The column addition/subtraction model thus, as sated by Thompson (1997) underplays the need to understand the quantity values of the numbers involved, and instead, Wright et al. (2006) points out that it presents addition and subtraction as a series of rules for manipulating individual digits.

1.3 Possible Solution

Using new models should improve and support learners' problem-solving strategies and remediate the incorrect strategies they used to work with. My conjecture as I begun this study was that learners that master the use of the bar model and the number line will be at a better position to improve their performance on problem solving. This conjecture was based on the hypothesis made by Polotskaia and colleagues that learners are likely to have fewer difficulties with analysing and making sense of word problems when taught problem solving following a relational approach (Polotskaia, 2015).

Learners must be taught mathematics in a way that will enable them to identify situations in which it might be used to interpret data or solve practical problems, apply their knowledge appropriately in situations where they will need to use mathematical reasoning processes, select mathematics that makes sense in the given situation, make assumptions, resolve ambiguity, and determine what is reasonable.

Between relational understanding and instrumental understanding, Skemp (2006) draws a contrast. Relational understanding is the theoretical component of issue solving and is also content based; it is the knowledge of what and why. Instrumental understanding is defined as knowing the set of rules of how to solve a mathematical problem; "the knowing how to" (Skemp, 2006).Relational teaching and learning permits and encourages children to apply thinking to their learning, in contrast to instrumental teaching and learning, which entails following rules and procedures without explanation. According to Skemp (2006), relational teaching and learning fosters the deep knowledge of mathematical concepts in both learners and teachers as well as the development of both parties' meta-language and capacity for explanation.

Although some teachers are hesitant to teach relational understanding because it would take too long to acquire, these learners are probably only going to need to be able to use a certain strategy. Although learners still need this relational understanding for exam purposes, it is too difficult to achieve (Skemp, 2006). Relational understanding has the advantage of being simpler to remember. It is definitely tougher to learn, which seems to be a paradox. For learners to learn a formula to solve a specific problem is undoubtedly simpler than understanding the reasoning behind it. However, learners must subsequently learn specific rules associated with that formula; in contrast, relational knowledge involves seeing all of these in relation to a specific topic (Skemp, 2006). As a result, it helps learners link the components of a problem rather than teaching them the rules for each individual formula independently, which makes it easier for them to remember (Skemp, 2006).

The type of instruction that prepares learners for instrumental mathematics is teaching them how to solve problems in a variety of methods as they arise and instructing them on how to proceed. They are unaware of the general connection between the various phases and the solution (Skemp, 2006).In contrast, learning relational mathematics entails creating a conceptual framework, or "schema," from which the learner can (theoretically) generate an infinite number of solutions to various issues contained inside the schema (Skemp, 2006).

The number line and bar models both involve creating a conceptual framework. As a result, learners will have the chance to learn mathematics in contexts where they will need to use mathematical reasoning processes to interpret information or solve practical problems, apply their knowledge appropriately, choose mathematics that makes sense given the circumstances, make assumptions, clear up ambiguity, and determine what is reasonable. rather than blindly adhering to laws and procedures. Additionally, learners will have the chance to connect the dots while tackling specific problems rather than blindly following instructions without understanding how the various procedures relate to one another.

1.4 Motivation for the study

Learners should be flexible in the way they interact with word problems and make relationships. They should be able to analyse the word problem and build a holistic and flexible understanding from the beginning. These are necessary for successful word problem-solving. They should see word problems as a process to finding the solution and be able to represent the text or the word problem into an equation and find the answer

(Polotskaia, 2015). Knowledge of operations is critical and serves as the foundation for successful problem-solving. Most problems encountered by learners in problem-solving are typically described as an incorrect choice of arithmetic operation or an inability to solve them (Giroux & Ste-Marie, 2001). Operations knowledge should be transformed into conceptual understandings. The structure of the question also contributes to learners' difficulty in solving word problems. If what the learners are required to do differs from what the operation is, the learners are likely to struggle to solve the problems (Polotskaia, 2015). Learners lack a numerical sequential understanding of operations as well as a relational understanding of problems. As a result, there are inconsistencies in word problem-solving (Polotskaia, 2015). In the problem-solving context, this means that one should first grasp the additive relationship described in the problem and then derive from this relationship the arithmetic operation needed to find the unknown element (Polotskaia, 2015). Furthermore, I want to see the possible outcomes of shifting learners to use the bar model and number line as an alternative to column addition, breaking down and expanded notation. I also want to see if learners that use these two models will be better positioned to solve word problems.

Addition-and-subtraction tasks and additive word problems are two examples of additive tasks that learners encounter in the classroom, according to the literature (Polotskaia, Savard, & Freiman, 2015). For the later type, the learner must first interpret the additive connection in the problem situation before attempting to compute the addition or subtraction that relationship demands. The former type merely requires calculation.

To assist in the solution of word problems involving additive relations, Polotskaia et al. (2015) created a conceptual framework that adheres to the Relational Paradigm's guiding principles. This conceptual framework describes the process of solving additive word problems as a cycle that moves the word problem from the context of daily life to the context of mathematics and back again. The use of my relational models, which are presented as an alternative to solving additive word problems, was informed by this conceptual framework.

The interpretation of additive word problems for Savard and Polotskaia (2017) focuses on comprehending the problem situation by building a visual representation of the relationships between the quantities involved. The arithmetic operation to be employed in the subsequent computation sequence is then informed by this graphical representation. Then, depending on the relationship shown in the visual model, the calculation sequence is carried out with an eye towards efficient calculation. The interpretation of the additive relationship involved, and the calculation required by the additive relationship should be improved as part of any intervention targeted at enhancing learners' proficiency with solving additive word problems. These two crucial components of solving additive word problems, interpreting the word problem, and deciding, are integrated into the relational paradigm.

According to Freiman, Polotskaia, and Savard (2017), the term "relational paradigm" refers to learning and teaching strategies that give priority to quantitative relationships and relational thinking in mathematics. The primary role of the Relational Paradigm is to view problem-solving situations as systems of relationships and build models of those interactions, which sets it apart from traditional approaches. According to the Relational Paradigm, learners' success in solving additive word problems depends on their capacity to understand the additive relationship by modelling it, deriving the operation involved, and then using the number sentence incorporating the operation as the foundation for calculating the answer to the word problem. Relational model can support the sense making and efficient calculation to solve additive word problems. The bar model and number line were selected in this study to support for sense making of additive word problems as well as efficient calculation.

1.5 Context of the study

Word issues are difficult for students in South Africa's Senior Phase (Grades 7-9). The inability to solve a problem or erroneous selections of arithmetical procedures are typical descriptions of these issues. A vital ability needed to improve problem solving is a thorough and adaptable comprehension of the additive structures in word problems. There are a variety of circumstances that could make it difficult for students to master this ability. The capacity to mathematically examine and solve a word problem is one of the most difficult to achieve. There is a pressing need for improvement in this field of educational study (Checkley, 2006). More particular, research is required to more fully define the phenomenological level cognitive processes involved in the act of comprehending a word problem and mathematising (Hestenes, 2010).

1.6 Research aims

The aim of this study is to explore how senior phase learners solve additive word problem tasks.

1.7 Research objectives

The objective of this study is to explore how grade 7 learners solve additive word problems.

The research questions that guided this exploration were the following:

1.8 Research question

1.8.1 What models did Grade 7 learners initially use to analyse additive word problem tasks and what was the learners' performance on these tasks?

1.8.2 What models did these Grade 7 learners use to analyse additive word problem tasks following the intervention and what were the shifts in their performance?

1.9 Methodology

To answer my research questions, I went to the school where I teach and requested two grade 7 classes to avail themselves for the purpose of my study. I approached the school management and grade 7 learners for consent to participate in the study. The language of teaching and learning in the school is English, this made it easier because I did not have to translate the contents of the intervention and the tests to their language of teaching and learning. Two grade 7 classes with approximately 40 learners in the class were used for the purpose of this study. Learners were doing word problems at school by the time this study was being conducted, and most of them where struggling. An alternative way was needed to help learners on how to deal with word problems. The data that I worked with came from the pretest and post-test learners wrote, the work learners produced during the intervention, the field notes gathered during the intervention.

1.10 Conclusion

This chapter outlined the motivation of the study, the objectives of the research along with the problem statement and the research question and the possible solution of the study. The main problem to be explored is how grade 7 learners solve additive word problems. In the next chapter I will focus on the literature review and my theoretical framework.

CHAPTER 2 LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

The ability to solve additive word problems and model a situation is one of the most important aspects of problem-solving. How learners understand word problem situations and how they calculate the solution to the problem are two essential factors that comprise the solutions of additive relations word problems that must be supported if learners are to solve additive word problems (Giroux & Ste-Marie, 2001). The following chapter reviews literature regarding word problems and possible solutions to help learners be better prepared to solve different types of word problems. It also examines the literature on the many sorts of additive word problems. This literature base served as the foundation for task selections for pre-and post-tests, as well as the intervention sessions.

2.2 The two paradigms

Additive problem-solving has been studied for over a hundred years. Researchers from different countries and different schools of knowledge have looked at learner's solving strategies, teaching approaches and many other aspects related to additive problems (Giroux & Ste-Marie, 2001). To better navigate through the sea of research in this domain, I started by distinguishing two paradigms in which problem-solving can be seen (Giroux & SteMarie, 2001).

Problem-solving can be seen within two paradigms: the Operational and the Relational paradigm (Polotskaia, 2017). The operational see the easiest problem-solving tasks as those that have the placeholder after the equal sign. Learners find problem-solving difficulties when the placeholder is before the equal sign or before the operation, as well as when the action that needs to be done is not consistent with the language being used. However, from the viewpoint of the relational paradigm, the analysis and identification of the additive relationship mentioned in the word problem are both steps in the issue-solving procedure. Planning or building the operation to compute the unknown element is also included in this (Polotskaia, 2017).

The relational approach allows for the allocation of time and space to create an explicit representation and modelling of the mathematical relationship between known and unknown elements, and it offers greater support for flexible holistic reasoning growth. The kind of reasoning that the relational method encourages is another significant potential advantage. It takes a form of reasoning akin to algebraic thinking to comprehend a situation in a relational fashion and operate on known and unknown parts of the connection. As a result, learners can gradually acquire conceptual knowledge and abilities that will enable the ongoing growth of their algebraic thinking (Polotskaia & Savard, 2018).

2.3 Mathematical structures of problem-solving

Mathematical structures have an impact on learners' ability to solve word problems. In other words, learners must be exposed to a range of additive problem-solving types. According to Askew (2004) additive word problems can be classified into the following categories: Simple Combine, Simple Compare, and composition of two transformations. The simple combine problems include three quantities that are connected by an additive relationship, they can also be seen as one structure that is made up of three elements. The simple compare category includes more than three mental elements that represent quantities and more than two relations between them (Polotskaia, 2015). As a result, simple compare category issues frequently need the employment of greater working memory or more potent control mechanisms for their mental representation and solution, making them more complicated and challenging than simple combine problems. The composition of two transformations category includes a combination of both the simple compare and simple combine categories. Yet, representing, analysing, and solving connected issues need for a powerful working memory or a complex method. Several mathematical constructs may be present in additive problems and may heighten the mental burden that learners must overcome in order to solve them (Polotskaia, 2015).

2.4 Thinking processes involved in problem solving

There are different types of thinking that learners are involved in when solving word problems. When learners can see the word problem as a process of events and are able to represent the word problem into a text, this is called the sequential thinking process (Polotskaia, 2015). Another type of thinking process that seems to have worked is one in which learners are able to consider the quantities in the word problem, make sense of the relationships and choose the correct arithmetic operation. This type of thinking can be seen as holistic and flexible. According to Polotskaia (2015), these two types of thinking are a manifestation of the duality of human understanding of mathematical concepts. The two ways of thinking identified above correspond well to this duality, with sequential thinking being associated with understanding a situation as a process/event, and holistic thinking is associated with understanding the situation as a system of relationships or a structure (Polotskaia, 2015). In line with this thinking duality, Polotskaia (2017) proposes two types of understanding: relational versus operational understanding. Word problem solving falls under this dichotomy as well, though from a different angle. Some learners attempt to interpret a situation as a system of relationships when given a word problem to solve in a relational manner. They eventually arrive at a relational knowledge of the issue that is backed by a comprehensive view of the circumstances. Some students attempt to jump right to the action. Their sequential mastery of mathematical processes including addition, subtraction, and addition and multiplication helped them to understand the dilemma and provide an effective solution. So, the learners' understanding of the problem varies depending on their initial intentions (Polotskaia, 2017).

2.5 Tasks fostering understanding of mathematical relationships in word problems

According to Giroux and Ste-Marie (2001), tasks should have the following principles so to enhance learners' engagement with them: (1) there should be no explicit or immediate questions that may be resolved by finding a single number, and the task should, (2) employ a sociocultural setting where learners can identify themselves as active agents. This criterion is added to prevent learners from immediately calculating the answer; however, it is further pointed out that the task should include an intriguing element, which would support learners' natural interest and commitment (Giroux & Ste-Marie, 2001). The objective of the task, which is for learners to comprehend how to analyse the scenario, should be made clear to them. The task's text has to be brief and should only use common, uncomplicated phrases and concepts that the learners are already familiar with. The mathematical discussion of the situation ought to ensure that appropriate graphical representations are integrated as a method of analysis (Giroux & SteMarie, 2001).

To solve complex problems, learners should be able to see the mathematical structure of the problem in a flexible holistic way (Polotskaia, 2015). The use of different representations can help learners develop efficient problem-solving strategies. Using diagrams to represent word problem, helps learners to have a holistic view of the problem. Feedback on the solved problem may provoke coordination between representations and procedures and may lead learners to reorganise their thoughts. The role of the teacher in task management, especially in attracting learners' attention to various elements of the situation, is a crucial part of the success

(Giroux & Ste-Marie, 2001).

2.6 Davydov's approach and the South African curriculum

According to Venkat, Askew, and Morrison (2020), counting-based strategies—counting in ones or counting in multiples to determine the solutions to additive and multiplicative problems—are common and persistent in the South African educational system. According to Van den Heuvel-Panhuizen (2008), the persistence of counting-based strategies demonstrates a lack of transition from calculation-by-counting to calculation-by-structuring. Calculating-bystructuring entails leveraging number relationships to perform more efficient calculations. Davydov's approach to early number emphasises quantities in a relational rather than a counting-based sense (Venkat, Askew & Morrison, 2020). Carpenter et al. (1999) do not provide multiple ways of representing single additive word problems in their work on additive situations. Alternative structural representations of additive links are given less weight, while upfront models of the relationships suggested in problematic circumstances are given greater weight. Davydov's method of comparing physical items and then algebraically encoding the comparison in a range of structural representations of the comparative connection is different from other research's typical approaches to additive problem-solving (Venkat et al., 2020).

According to Venkat et al. (2020), in South Africa, concrete counting methods predominate well into the higher elementary schools, claim (grade-appropriate students are 13 years old by grade 7, the last year of primary schooling). Wright, Martland and Stafford (2006) found concrete counting approaches that were prevalent in the schools that they were working with in the mathematics recovery programme. They discovered that, when dealing with early additive circumstances, over three-quarters of the sample used count-all methods. For instance, to figure out 5 + 4, a learner might count out 5 counters, separate out 4, and then add up all of the counters from 1 to 9 (Wright et al., 2006).

Nonetheless, there is more proof that instruction focused on resolving the current issue reinforces unit-counting techniques. Here, neither the processes nor the outcomes are viewed as having been established over time; rather, teachers and learners repeatedly solve each case using count-all procedures (Venkat & Naidoo, 2012). Progression from counting into structuring (making use of number relationships and properties for more efficient calculation) is thus limited. With additive relationships, the curriculum suggests three avenues of progression: (1) increasing the number range, (2) expanding the types of number being worked with, and (3) moving towards more efficient calculation strategies (DBE, 2011). It provides little attention to progression to number structure. The South African curriculum does not include generalised algebraic forms in any of the basic grades, and when they are discussed (such as number lines), they are described as helpful tools for solving problems (Venkat et al., 2020).

According to Mostert (2019), word problems are a significant component of South Africa's early-grade mathematics curriculum. Word problems are challenging for learners in South Africa's Senior Phase (Grades 7-9), regardless of whether they are presented in English or their local language (Sepeng, 2014). In support of the statement, it is stated that there is evidence of learner issues primarily connected to word problems at the Intermediate Phase

(Grades 4-6), where the connection between word problems and English language competency has been made clear (Roberts, 2016). Sepeng (2014) argues that word problem difficulties among learners start to appear in elementary school and persist through secondary school. These difficulties are related to both language and mathematics. Moreover, the errors that learners construct when attempting to solve word problems come into play. Instead of choosing the erroneous arithmetic operation, as was previously generally assumed, learners are more likely to make mistakes while answering word problems with additive relationships when they inaccurately depict the problem situation (Mostert, 2019).

In Table 2.1 below, Askew (2004) summarises a classification of word problems that divides addition and subtraction problems into the following categories: change, combining and separating, and comparison:

Root situation	***Change***		***Combining*** *and* ***Separating***		***Comparison***
Specification	*Increase*	*Decrease*	*Combining*	*Separating*	*Comparing*
A description of the situation.	*Situations where there is an initial quantity and this is increased or decreased in some way.*		*Situation where two sets are put together to create a new set that did not previously exist*	*The reverse of combining, where a single set is split into two.*	*Situations where nothing actually changes or is combined but where two sets are compared.*
Example	*I have 10c in my purse and put another 5c. How much is in my purse now?*	*I have 10 jelly beans and eat 6. How many jelly beans do I have left?*	*Gran gave me 10c and granddad gave me 5c. How much do I have altogether?*	*I have 10 books in a pile and put 6 on the shelf. How many books are left in the pile?*	*I have 10c, Penny has 5c. How much more do I have?* *I have 5c, Mark has 10c. How much more do I need to have the same as Mark?*
Calculation implication	*Change 10 by* ***adding*** *5.*	*Change 10 by* ***taking away*** *6.*	***Combine*** *10 and 5.*	***Separate*** *the 10 into 6 and . . .*	*What is the* ***difference*** *between 10 and 5?*

Table 2.1: Classification of word problems

Source: Askew (2004)

Mostert (2019) claims that some word problem kinds are easier for learners to solve than others, with compare type problems being the most challenging. It is crucial to comprehend the relative difficulty of various compare-type problems to assist learners in comprehending and solving them (Mostert, 2019). The fact that many compare-type problems feature quantifiers like "greater than" and "less than" is at least one factor in why learners have trouble solving them (Mostert, 2019). Another reason why compare-type problems are difficult for learners is that they tend to confuse the traditional comparison question "How many more?" with other questions such as "How many?" or "Who has more?" (Mostert, 2019). An additional perspective on how learners can advance their understanding of solving arithmetic problems is provided by Davydov (1990). He suggests that additive relationships consist of two elements producing a third element. All additive word problems have three elements, which can be explained by this part-whole relationship (Schmittau & Morris, 2004). Below is Davydov's diagram that explains the relationships between elements involved in a word problem.

<table>
<tr><td>PART A</td><td>PART B</td></tr>
<tr><td colspan="2">WHOLE</td></tr>
</table>

Table 2.2: Relationships between elements involved in a word problem

Source: Davydov (1990)

Whole = Part A + Part B	Whole −Part A = Part B
Part A + Part B = Whole	Part B = Whole −Part A
Whole = Part B + Part A	Whole −Part B = Part A
Part B + Part A = Whole	Part A = Whole −Part B

According to Roberts (2016), any one of the eight equivalent number sentences that all express the same additive relationship can be used to describe any one of the many additive relations situations (specified in respect to the problem kinds defined above). This is a crucial finding since the relationship between situations and symbols is not a straightforward one-to-one mapping but rather a many-to-many mapping (Roberts, 2016). This is likely to help learners' abstract relationships between quantities. Learners' ability to distinguish and analyse quantitative relationships, and to apprehend the differences between various types of relationships between quantities is likely to develop. The use of the diagram helps learners to identify the action that they need to perform to solve the unknown (Davydov, 1990).

2.7 Modelling

There are two types of modelling, according to research (Blum et al., 2007; Gravemeijer, 2007; Lesh & Harel, 2003), namely mathematical modelling, and emergent modelling. For example, Lesh and Harel (2003) defined Mathematical Modelling (MM) as the process of identifying quantifiable patterns in a phenomenon and generalizing those patterns, while Blum et al. (2007) defined mathematical modelling as the process of learning mathematics to gain proficiency in using mathematics and creating mathematical models for domains and purposes that are essentially extra mathematical. The former is often used to translate the problem scenario into mathematical terms that can serve as a model, whereas in the latter, the model arises as part of an organising activity that involves structuring the problem situation (Gravemeijer, 2002). The model and the scenario are separate in the case of mathematical modelling, whereas, in the case of emergent modelling (EM), the model and the situation modelled cross and are mutually constituted during the modelling activity**.**

2.8 Dynamic models

Models are regarded widely in the context of the Realistic Mathematics Education (RME) approach to learning and teaching as tools to elicit and promote the transition from a nonformal understanding associated with the real or imagined reality to an understanding of formal systems (Van den Heuvel Panhuizen, 2003). A model can take on several forms in this manner, including objects, sketches, paradigmatic scenarios (such as repetitive subtraction), systems, schematics, and even symbols. Therefore, it is necessary that a model support such a process for something to be considered emergent. Models must therefore meet the following prerequisites to facilitate learning processes as intended: (1) they need to be grounded in plausible, realistic situations on the one hand, and (2) flexible enough to be used on a more basic, or general level, on the other. In this sense, the emergent models employed can be linked to mathematical expansion, where the development of learners' cognition is seen as the main objective of learning (Gravemeijer, 1997).

2.9 Part - part - whole

In a part part-whole diagram, a rectangular bar is used to graphically depict the whole and the parts, with the length of the bar representing the whole being equal to the sum of the lengths of the parts (Davydov,1990). Although it can be useful in helping learners understand the part-part-whole link, it does not build on the counting procedures that have helped learners advance into the senior phase, thus its adoption is not natural.

2.10 Column model

The column model is one of the widely and most used model in the South African educational setting. It is introduced in grade 4 and used throughout the intermediate and senior phases. The caps document encourages the usage of this model. For the effective use of the column model, learners need to have prerequisite skills, such as them having a good understanding of the place value. In the classroom, it is a different story. Learners in schools do not get the opportunity to learn place values, to the extent of showing adequate proficiency, this is mainly because, they are presented as algorithms (Dominick & Kamii, 1989; Fuson, 1992). Learners are most likely not to think independently when working with algorithms, and this is argued to be the danger that these algorithms bring (Kamii & Dominick, 1989). They are depriving learners the opportunity to develop number sense because they unteach place value.

There are learners that arrive to correct answers when using them. However, these learners show little understanding of the procedure they are using. Working with column models is reduced to dealing with single digit numbers when the necessary understandings of the decimal system are not in place, offering fertile ground for errors caused by a tendency to treat tens and units as separate from one another. Typical examples include learners subtracting the smaller number from the bigger number, even though the smaller number being subtracted is part of the big number that is subtracting the number that the big number is part of. Learners are most likely to make such errors. Another error that is most likely made by learners is when they get a two-digit number after adding the units, they tend to write the two-digit number, or carry the one to the tenths, when adding the tens they then forget to add the one they carried (Kamii & Dominick, 1989).

Fosnot and Dolk (2001) agree with Kamii and Dominick (1989) that applying an algorithm to all problems is counterintuitive to calculating with number sense. They argue that calculating using number sense entails first looking at the numbers and then deciding on an appropriate technique. Given what has been said, it is evident that the column model advances a technique that ignores number and operational linkages. The part-part whole and the number line can assist lessen the lack of support that learners require to acquire number sense.

2.11 Simple Combine Problems

It is crucial to visualise three quantities coupled in an additive manner - i.e., as three components of one structure to appropriately grasp an issue falling under this category. Learners can solve simple combine issues using a variety of approaches and tactics, depending on how the questions are phrased, according to Nesher et al. (1982). However, to resolve the most challenging of these word problems, a comprehensive analysis is necessary. For complicated problems, a comprehensive representation is required. The Relational paradigm states that this comprehensive analysis guarantees the resolution of any problem falling under this heading Nesher et al.., (1982). The three indicated values and the part-part whole relationship that connects them in working memory should therefore be treated as three mental elements by the problem solver in order to examine the situation effectively.

2.12 Simple Compare problems

Since they call on the employment of more working memory or more potent control mechanisms for their mental representation and solution, simple compare category problems are frequently more complicated and challenging than simple combine problems (Nesher et al, 1982). Simple compare problems are more challenging than other problems from the relational paradigm perspective because it takes more mental effort to understand the additive relationship at play. Problems involving simple change and simple combine are less difficult compared to the compare problems (Nesher et al, 1982).

2.13 Conclusion.

This chapter summarised what other authors had to say about additive word problems. It described the present strategies used by learners in the classroom and it detailed the difficulties that learners have when dealing with additive word problems. However, more importantly, the chapter proposed models that can help improve and support learners' problem-solving skills, as well as remediate the faulty strategies they employed. The following chapter describes the study's methodology as well as the intervention that aided and supported learners' problem-solving.

CHAPTER 3: RESEARCH METHODOLOGY, THE INTERVENTION, ITS RESOURCES, AND RATIONALES

3.1 Introduction

This chapter discusses the methodology, study design, intervention and data collection performed by the researcher. Proper research methodology is essential for the success of the research, and it is for that reason that the research questions of the study are answered appropriately.

3.2 Objectives of the research study

The main objective of my study was to explore how senior phase learners make sense of and solve additive word problems. The bar diagram and number line, two relational models, were introduced in the study. I presented it to a group of learners who had never used it before. They might be useful for more than only the intermediate level of the South African education system. The models are meant to offer a more durable alternative rather than to completely replace or surpass current ones. My main hypothesis for this study was that learners who have comprehended and demonstrated mastery of the bar model and number line for addition and subtraction will be better prepared for answering and comprehending word problems, given the long history of use of these tools. It is less likely for learners to have difficulty interpreting and responding to word problems. This is because the models will help them break down the word problem in a way they will easily interpret and answer efficiently.

3.3 The test

The test was created in accordance with the classification listed in the literature review. There were ten-word problems, with three problem types distributed across. All the questions were word problems (see Appendix A: The Test).

3.4 Qualitative analysis

The study applies qualitative methods, the epistemological aspect is interpretive. Interpretivism theory is concerned with the individual's worldview of how a person approaches the reality of the world he is studying (Myers, 1997). In other words, an interpretive researcher tries to provide deep meaning to social phenomena related to the person or environment he or she is studying.

The research follows a qualitative approach because it provides possibilities to understand how learners understand a concept.

According to Creswell J. (1998), qualitative research is an inquiry process of understanding based on distinct methodological traditions of investigation that explores a social or a human problem, from which the researcher builds a complex, holistic picture, analyses words, reports detailed views of informants, and conducts the study in a natural setting. To put it another way, the researcher should be out in the world or in a social situation, obtaining information that will be investigated to help answer the questions raised by the study.

The information under investigation is obtained in a social context (Denzin & Lincoln, 2011). The aim of the study is to explore how senior phase learners solve additive word problem tasks. Thus, I adopted a qualitative approach as to explore the research problem

The focus of my research was not on whether the learners provided the correct answer to addition and subtraction word problems, but more importantly, on the quality of the answer according to the types of strategies they use to arrive at the answer. In addition, what models did learners use to analyse additive word problem tasks and what was the learner's performance on these tasks. Lastly, what models did learners use after the intervention and what were the shifts in their performance. I tracked this through the solutions of tasks in learners' books and their pre- and post-test scripts. At the end of the intervention, I purposefully selected one learner from each of the three attainment levels in my sample space to ask them why they solved the problem the way they did and if they have another way of solving it.

As part of my analysis on how learners performed on word problems. Learners from both the control and intervention group were selected. These learners exemplify the typical shifts in performance from both the pre and post-test. They had little improvements on the change problems, medium improvements on the combine problems, and significant improvements on the compare problems. These learners were given the following pseudonyms : Learner JM, RM , RP and KK, these were learners from the intervention group. The following learners reflect the typical shifts in performance of the pre- and post-test in the control group. The following pseudonyms names were used : Learner SM, MM and MDM.

My second layer of analysis looked at the learners performance from both the control and intervention group across the test two test sittings. Learner's work was marked, this helped me to see the significant shifts that happened in the performance across the pre and post-test, this was for both the control and intervention group. The mark was allocated for the model used as well as the correct answer. This was for both the control and intervention group. It was one mark for the correct answer as well as the model used to arrive to the answer, this was done in the pre and post-test. For the correct answer the code 1 was used, and for the incorrect answer the code 2 was used. The marked scripts gave the opportunity to compare the control and intervention group performances. I compared the average performance of both the control and intervention group across both tests. The average was calculated by finding the percentage of correct responses for each question, added all the correct percentages for each question and divided by the number of questions in the task. This was done for both groups across the pre and post-test. I then looked at the average of the intervention group, looked at the shifts in correct answers across both tests. In addition, analysis was also made on the shifts across the three categories (change, combine and compare) for the intervention group as well as the control group, across the two tests.

3.5 Data generation

There was a total of 72 learners divided into two groups, one the control and the other the intervention group. The control group had a maximum of 34 learners and the intervention group had 38 learners in total. The study was undertaken in a primary school in Johannesburg, Mahlangu primary school.

The language of learning and teaching is English. The learners in the school are doing English as their first additional language and are not as proficient in it. Most of the learners in the school are from a low socio-economic status. The school has approximately 2000 from grade 1 – 7. The school averages about 40 learners a class. There are limited resources that learners can use or teachers to help enhance learning and teaching. There is one computer lab at the school, and it has 10 computers, most out of that 10 are not working; there is a library at the school. There are no sports fields at the school. However, there is a big playground that learners normally use for sporting activities. The home languages offered in the school are Isizulu, Sesotho and Xitsonga. Isizulu is the most spoken language in the school, and teachers normally code-switch to it when teaching in class.

The study made use of a pre- and post-test for data collection. The test was created in accordance with the classification listed in the appropriate section. There were ten-word problems, with the three problem types distributed across. All the questions were word problems (see Appendix A: The Test). Both the control and intervention group took the same set of tests. The pre-test helped in exploring the different models and strategies used by learners to solve additive word problems. Before the intervention, I designed the pre-test to discover the models and strategies used by learners. The post-test was the same as the pre-test. Learners in the control group used the same strategies and models in the post-test that they had used initially. Learners in the intervention group used different models and strategies in the post-test compared to the ones they used in the pre-test.

3.6 Data analysis procedures

Beginning with the tests, I concentrated on areas where learners struggled to answer questions in the pre-test to see how they handled the same tasks in the post-test, this gave an indication of the impact that the intervention had on the learners. What followed was looking at the solutions in learners' exercise books to understand whether there was a change in their use of strategies and models during the intervention. I then selected episodes in which I found significant shifts, and these became the basis of my report. One of the things that showed progress was the efficient use of the bar model and the number line as a model to solve challenging problems.

The shifts in the different models that learners used were tracked through the answers they gave in their pre- and post-tests as well as in their exercise books during the intervention lessons. I compared the pre- and post-test of the control group to see how the learners performed in both tests. It made more sense for me to compare the pre- and the post-test of the intervention group for shifts because the post-test was written after the intervention when the bar model and number line were already introduced. Learners being able to solve the problems that they initially could not, served as an indicator of success.

The process began by seeking to understand and get a sense of the models and strategies that the learners in the selected Grade 7 classes initially used to solve various kinds of addition and subtraction word problems. The next objective was to gauge their fluency about these models and strategies, as was already mentioned in the first chapter of the report. Being able to examine the learners' scripts from the pre-test of both the control and intervention group, made it easier to answer the research question. For the purposes of this study, there were six 45-minute sessions that made up the intervention, and my conclusions regarding one of my research questions will be addressed throughout the six lessons that made up the intervention. To determine the impact that such an intervention can have on learners' use of models and strategies, a post-test was given at the conclusion of the intervention to understand learners' usage of strategies and models.

Learner's solutions in the pre-test from both the control and intervention group, provided answers to the types of models and strategies that were in learners range or that leaners are mostly using for solving additive word problems, these were prior to the intervention. The intervention was then designed and implemented once these were identified. After the intervention lessons, the learners from the intervention group as well as the control group sat for the post-test, where they were encouraged to solve the same additive word problems that were in the pre-test with the use of the models and strategies with which they were most comfortable.

My first layer of analyses discussed the different models that learners in both groups employed, across the pre and the post-test to solve the word problems. It further discussed the use of each model across the three categories of word problems and delved on the success rate associated with the use of each model for both groups in the pre- and post-test. I conducted analyses that helped answer one of my research questions that has been outlined in the previous sections. To help me with the analyses I used codes (words) that described the ways that learners used to find solutions to the word problems. The learners in the intervention group used five different models in the pre-test. These are: column model (CM), horizontal number sentence (HNS), no model (NM), word sentence (WS), as well as a combination of the column model (CM) with the word sentence (WS) used together. Across the responses provided by the learners in the control group, six different models were visible in the pre-test.-Part of the six models, three were the same as the ones that learners in the pre-test of the intervention had used. The other three models were the breaking down model and two types of combinations, where the learner used more than one model to solve the problem. The two combinations where the learner used more than one model to solve the question are as follows: in the first combination, the learner used a combination of the number line (NL) and the horizontal number sentence (HNS), the second combination is where the learner used two models to answer the question, and this was a combination of the column model (CM) and breaking down model

3.7 Intervention

The intervention was done by the researcher. It was composed of six sessions that were done over a period of six days; each session was forty-five minutes. The intervention was conducted at the school where I am teaching. Before the intervention learners wrote a pre-test and then the intervention started, thereafter learners wrote the post-test.

3.7.1 Session 1

The first session was to introduce the bar model to the learners.

C	B

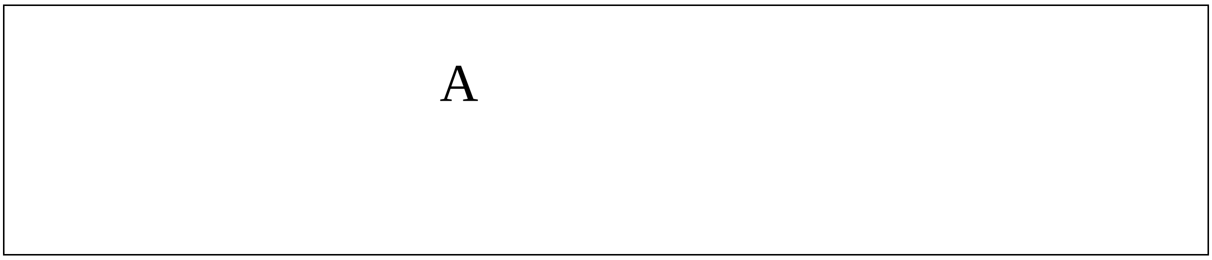


Table 3.1: Example of Bar Model Introduced to Learners

Source: Author [adopted from Davydov (1990)]

I explained to the learners that the bar model will be used to solve additive word problems, but for the purpose of this lesson, we used to link addition and subtraction. An explanation was given on the bar model. It was made clear that the biggest bar is named BIG which is A, the smaller block is named B and the smallest block is named C. It was made clear that A is the whole and is made up of two parts, which is B and C. Meaning to find the value of A you need to add C and B; to find the value of B you must subtract C from A; to find the value of C you must subtract B from A - this was emphasised during the lesson. To help enhance learners' understanding of the bar model, I explained to them that the three letters can be thought of as a number family. In a number family bar diagram, the two small numbers/boxes together are the same size as the big number/box. After explaining the bar diagram to learners, it was more logical for us to have mathematical equations of the diagram.

Learners were asked to give a number sentence based on the bar diagram. They became silent, and it seemed like they did not understand the question. I then wrote the first equation on the board which is A = B + C then explained again that for us to get A we need to add B and C. Then l asked learners again to give number sentences based on the bar model. The following responses were then given by learners: (1) B = A – C; (2) C = A – B; (3) A = C + B. Thereafter, leaners were asked to refute the following statements: A + C = B, C – A= B and give reasons if they are true or false.

A + C = B learners answered false. To explain the reasoning behind it being false, learners' responses were as follows: You cannot add two numbers if you are looking for the middle number (B). You only add when looking for the value of A. If you are looking for B you need to subtract C from A.

A = C + B learners said this statement is true, learners responded by saying the two parts which are (B) and (C) when added together make the whole (A).

For C – A = B, most of the learners said this was false, though some said its true, but after hearing the explanation of other learners responding with false, they were convinced that it was false. Learners explained that we cannot subtract a bigger number from a smaller number and get the middle number. I asked learners if we can subtract a bigger number from a smaller number, learners answered yes. I then asked why it is not true in this case. One learner answered: "*we are looking for the middle number (B) so if we subtracted the bigger number from the smallest number we would get the smallest number instead of the middle number, hence the statement cannot stand*". Most of the learners reasoning showed that they understood the relationship of the numbers in the bar model. After these interactions, the following activity was given to learners:

(i) Activity

1. **Make all the possible number sentences using the following numbers: 1140; 351; 789 (Draw the bar model)**
2. **Given the diagram below, answer the questions that follow:**

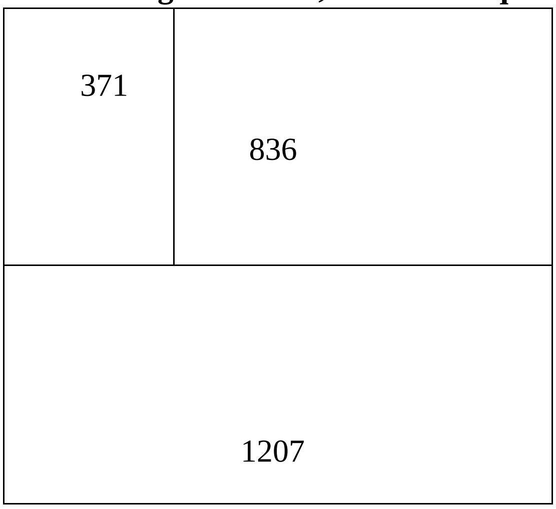


Table 3.2: Activity (i) Diagram

Source: Author's own compilation sample

(a) State if the following is true or false by using the bar to help you

371 + 836 = 1207	836 – 371 = 1207
1207 = 836 + 371	1207 – 371= 836
836 + 1207 = 371	371 = 836 - 1207
836 + 371 = 1207	1207 – 836= 371

These are the answers that learners gave for the first question:

1) 351 + 789 = 1140	1140 = 351 + 789
789 + 351 = 1140	1140 = 789 + 351
1140 – 789= 351	789 = 1140 - 351

1140 – 351 = 789 351 = 1140 – 789

These are the answers learners gave for the second question:

2) 371 + 836 = 1207 **T** 836 – 371 = 1207 **F**

1207 = 836 + 371 **T** 1207 – 371= 836 **T**

836 + 1207 = 371 **F** 371 = 836 – 1207 **F**

836 + 371 = 1207 **T** 1207 – 836= 371 **T**

3.7.2 Session 2

In this lesson, we used the number line to link addition and subtraction in which to show the relationship between the parts within a whole. I started the lesson by writing a sum on the board that had to be solved: 237 + 89 = ___. I further wrote the sum with the empty number line as shown below:

237 + 89 = ___

Figure 3.1: Session 2 number line

Source: Author’s own compilation sample

I explained to the learners that we add using the number line, the first number (big number) or the number we are adding onto should be to the furthest left because we are moving to the right when adding. An explanation was also given on how we subtract on the number line, we write the number we are subtracting from on the furthest right and move to the left. Thus, I wrote 237 at the furthest left on the number line as shown below:

237 + 89 = ___

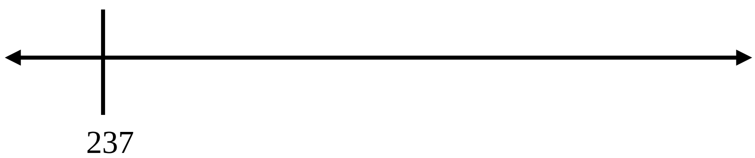


Figure 3.2: Session 2 number line

Source: Author’s own compilation sample

I emphasised to the learners that we are adding 89 onto 237, but to make it easier we will break up the smaller number into place values. I asked learners to break 89 to its place values and learners wrote 89 in the form 80 + 9. We started from 237 to indicate the addition of 80, which will lead us to 317 as shown below:

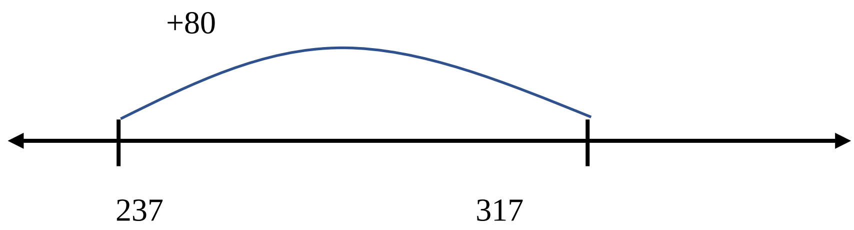


Figure 3.3: Session 2 number line

Source: Author's own compilation sample

After adding 80, we still had 9 to add, so we started from 317 and added 9, e.g., 237 + 80 + 9 = 326. The addition of 9 is shown below:

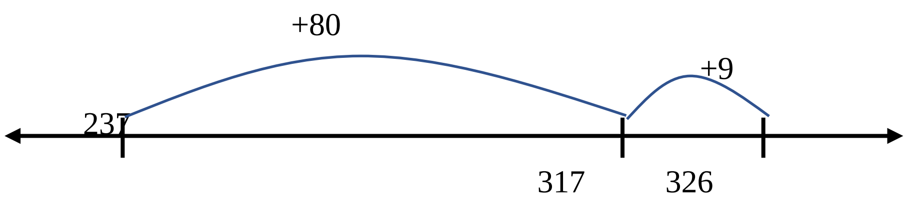


Figure 3.4: Session 2 number line

Source: Author's own compilation sample

I emphasised that leaners need to first identify the operation, then identify if they are moving to the right or left, which will inform them where to put the bigger digit on the number line. Thereafter, emphasis was that they need to break the smaller number to its place value and then add it in the order of the place values. We only did one example to show how they need to use the number line for addition and subtraction. Learners were given an activity to use the number line for addition and subtraction.

(ii) Activity

Use the number line to calculate the following

1. **5878 + 3328**
2. **1227 – 1209**

3.7.3 Session 3

The purpose of this session was to practice linking addition and subtraction using the bar model. The session was also an integration of the first and second session. A recap what was done in the first and second sessions was done before proceeding with the session. I wrote the three numbers on the board and asked learners to make links between the three numbers using the bar diagram and represent the calculations on a number line. l suggested that we do this one together before I can give them an activity so they can do themselves. I asked learners which of the three numbers is going to the whole, and which two numbers constitute the two parts. Learners answered the big number is the whole and the other two number will form the two parts. Some learners had difficulties understanding which of the two numbers will form the smaller part and the middle part. I explained to the leaners that as long as we know the whole then it does not matter which of the number is a smaller part or middle part. These were the three numbers: 287, 507 and 872.

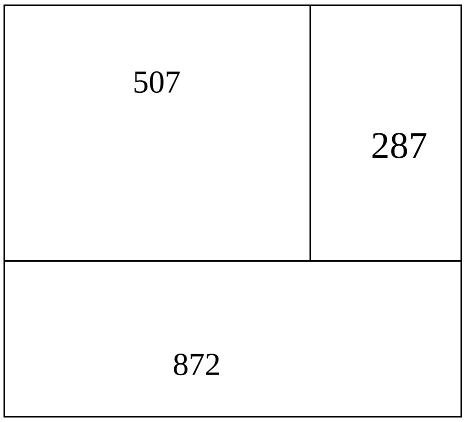


Table 3.3: Session 3 Activity Diagram

Source: Author's own compilation sample

+ calculations	- calculations
507 + 287 = 872	872 – 287 = 507
287 + 507 = 872	507 = 872 - 287
872= 507 + 287	872 – 507 = 287
872= 287 + 507	287 = 872– 507

Table 3.4: Calculations Table

Source: Author's own compilation sample

Learners will be required to represent all these calculations on a number line

These are the calculations that we came up with from the numbers in the bar model. The first column these are the addition calculations and the second are subtraction calculations. Most of the calculations came from learners and I scaffolded learners to come up with other calculations. The first two calculations were straight forward for learners to understand. The third calculation was somewhat confusing to the learners. The confusion was why are we starting with the answer and having the operation on the right side of the equation. This confusion can be argued to stem from learners having an operational understanding of the equal sign. I explained to the learners that having the answer before or after equal sign does not make any difference and emphasized that the equal sign serves as symbol of equivalence or showing two sides are equal. I emphasized and encouraged learners to refer to the answer of the calculation as the whole and the other numbers as parts of a whole.

Learners were given 3 numbers: 498, 1075 and 1573. I then drew an empty bar diagram as shown below:

Table 3.5: Session 3 Activity Diagram

Source: Author's own compilation sample

Learners were required to suggest links between the numbers 498, 1075 and 1573 using the bar model. This was the activity given to learners, they were further required to represent their answers on a number line.

3.7.4 Session 4

We first had a recap of what was done in the previous session before starting with the session. In this lesson we found easier related calculations for addition calculations where one of the numbers added is missing. I wrote the following number sentence on the board 863+ __ = 2780. This was the problem that had to be solved. I asked learners to solve the problem using their own method of calculation or the ones they previously used. One of the answers that came up was 3643, almost half of the learners in class were able to obtain this answer. When asked how they obtained this answer, the learners' responses were: "*we added the two numbers and wrote the answer*". This suggests that these learners have an operational view of the equal sign. Some of the leaners got 2123 as their answer. When asked how they obtained the answer, the learners responded by saying: "*we used the column addition method and subtracted the smaller number from the bigger number*". The learner's incorrect response suggests that they subtracted the smaller number from the bigger number without considering where the smaller number was. They were subtracting 863 from 2780, but they subtracted 0 from 3 instead of borrowing one from 8. The other learners gave the correct answer, which is 1917. When asked how they were able to obtain the answer, they responded by saying: "*we looked for a number that when added to 863 can give us 2780*". A probing question was asked to understand how long it took for them to find the answer; their response was as follows: "*Yoh Sir!!! It took us very long*". The learners were then required to use the bar model to solve the equation. We drew a bar diagram.

They were asked which number was the whole, and the learners responded by saying 2780.

They were also asked which number was the part, and the learners responded by saying 863. The learners were then asked to state what it is they are looking for, and thy responded by saying: "*the part*". From this discussion or probing it was clear what leaners had to do. The learners were then asked to explain how they are going to find the other part. Learners' response was: "*we will subtract the given part from the whole to find the other part*". Below is how the bar diagram was labelled.

Table 3.6: Session 4 Activity Diagram

Source: Author's own compilation sample

After filling in the diagram with the numbers, it was easy for learners to find the missing number that has been added. Learners subtracted 863 from 2780 and obtained 1917. I asked learners which method was easier to use to find the answer, learners responded emphatically by saying the model is easier to use.

Learners were given following questions to solve using the bar model:

1760+ __ = 4537

__ + 5321 = 7190

6521 = 4378 + __

3.7.5 Session 5

We had a quick recap of what was done in the previous sessions. The aim of this lesson was to enhance learners understanding of the words more and less when they are used in word problems. We also aimed to help learners analyse word problem questions using the bar model.

After the recap, we had a warmup session. The warmup session was as follows:

Leaners were asked to give 36 more of the following numbers:

130 = 166

148 = 184

196 = 232

108 = 144

Learners were then asked to give 120 less of the following numbers:

248 = 128

1287 = 1167

2365 = 2245

1789 = 1689

After the warm-up session, I wrote a word problem on the board. Before learners could attempt to solve the word problem. I explained and outlined possible words that they should look for when solving word problems. For instance, words like more, less, and altogether. I emphasised the importance of why learners should understand these words. We also explored the different contexts in that these words are likely to be used. Thereafter, we went back to the word problem I had written on the board. The learners had to use the bar model to solve the word problem and represent their answers on the number line. The problem is depicted below:

Jabu scored 289 points in the first half of a game, he scored another 378 goals in the second half. How many goals did he score altogether?

Table 3.7: Session 5 Activity Diagram

Source: Author's own compilation sample

Firstly, learners were asked what the key words in this word problem are. They responded by saying altogether, the number of points scored in the first half and second half. Learners were further asked to share what the key words inform them to do. There was silence in the class, one learner said the word altogether means addition. I asked learners to fill the bar model with what the values information they are given. It was easy for learners to know that we are looking for the whole, because the word altogether implied addition. I told learners to put the variable x where a number is not known. I then asked the learners where we are going to put the variable. Learners responded by saying, since we are looking for the sum of two numbers, this suggests we are looking for the whole, because addition of two parts gives us a whole. So, the variable should go into the big box, then I told learners to fill the bar model. This is how the diagram was labelled:

<table>
<tr><td>289</td><td>378</td></tr>
<tr><td colspan="2">x</td></tr>
</table>

Table 3.8: Activity Diagram using variable

Source: Author's own compilation sample

I then took learners back to the first session where we had different equations that were made up of big = small + small, small= big – small and so forth. Then I asked learners which number sentence is acceptable for this problem. Then learners said, big = small + small. Learners modelled their equations. I gave learners more word problems for them to model using the bar diagram. The main objective of the lesson was for learners to identify which number goes where in the bar model and how it relates to the other numbers. Below is the activity that learners had to complete:

Julia had 1278 sweets; she gave away 589 sweets. How many sweets does she have now?

Learners were expected to use the bar model to model their equations.

3.7.6 Session 6

This session was focused on learners modelling their own equations based on a completed bar model. We first had a recap of what was done in the previous sessions before we could start with the lesson. In this session learners were given a bar model with numbers and a variable. Learners were expected to describe the relationships in the bar model and write the relationships in their own words and model possible equations.

x	480

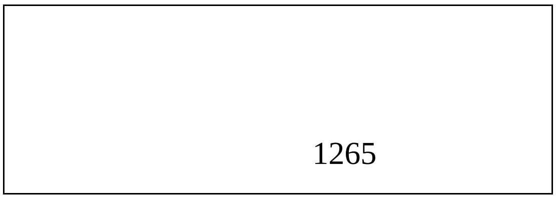


Table 3.9: Session 6 Activity Diagram using variable

Source: Author's own compilation sample

Learners were expected to describe the relationships in the above bar model and model the number sentence represented on it. The main thing was for them to understand and see that we have a whole and we need to take out a part so we can find the other part, the words they used to describe it were not important. Learners were also expected to model their equations. These are the responses that learners gave: (1) 1265 = x + 480; (2) $x = 1265 - 480$.$1265 - x = 480$; (3) 480 + x = 1265. They are all correct. I took learners back to the first session, where we had different equations that were made up of big = small + small, small= big – small and so forth. Then learners were asked which number sentence would be the most effective to solve this problem:

1265 = x + 480. for this number sentence, before they could start with the calculation, they still needed to find the additive inverse of 480 and then start working out the answer.
$x = 1265 - 480$, for this number sentence, you can immediately start with the calculation and without finding the additive inverse of any number.

$1265 - x = 480$, for this number sentence you still need to find the additive inverse of 1265 and multiply the variable with negative one, then start with the calculation.

For 480 + x = 1265, same applies with this number sentence, you need to find the additive inverse of 480 and add it on both sides of the equation then start with the calculation, by the time you start with the calculation chances are you would have probably made a mistake elsewhere with adding the additive inverses.

These are the explanations I gave to the leaners from the number sentences they gave. I then asked learners, which number sentence can be the most effective to use. Learners emphatically responded by saying $x = 1265 - 480$. I then asked why because it leads us straight to the calculation. The emphasis was you always use the number sentence that leads to finding the answer straight away or that enables you to start with the calculation first. More examples were done on the board. Learners were given the following activity:

608	x
2185	

Table 3.10: Activity Diagram using variable

Source: Author's own compilation sample

Describe the relationships represented in the bar model using your own words, then model the equation for the relationship represented.

3.8 Sampling techniques

The study employed purposeful and convenient sampling. Grade 7 learners were doing word problems in term 3 and this was the time I was collecting my data, so this was a good time for learners to provide the information needed for the study. The data was collected at the school I am teaching in, this made it convenient because learners will easily be accessible.

3.9 Ethical considerations

Protecting human participants in research is critical and there needs to be appropriate implementation of ethical principles when conducting a study (Arifin, 2018). The researcher is responsible for ensuring that human subjects have the power of freedom of choice, that their identity is respected and protected throughout the study process, and that there is promotion of clear and honest reporting of the research (Arifin, 2018). Thus, is doing so, an ethics clearance document was submitted and approved to ensure the study would be accomplished in an ethical manner. Thereafter, consent was sought and obtained from the school's principal, the teachers and the parents on behalf of the learners that were involved in taking the tests. The participants protected from harm and the necessary precautions were taken to guard against treating participants as objects. Their right to privacy and confidentiality was respected and thoroughly explained to them to ensure that they only took part in the study if they wanted to. Participants were also made aware of their right to withdraw from taking part in the study when they wanted to and were not coerced into doing anything that they did not part – meaning that their participation was fully voluntary.

3.10 Trustworthiness

According to Lincoln and Guba (1985) trustworthiness concerns the truth value of qualitative data, analysis, and interpretation. It encompasses four criteria: Credibility refers to the truth in the collected data, dependability refers to the stability of data over time, confirmability refers to neutrality, the potential for congruence between two or more people about data accuracy, transferability refers to the extent to which the findings can be transferred to other settings or groups and dependability refers to the extent to which the researcher fairly and faithfully show a range of different realities that the participants conveyed during the interviews and in the learners scripts (Lincoln & Guba, 1985). I work at the school where data was collected, this meant that I am mostly involved with the learners, and they trust me. This suggests the findings have a higher possibility to be valid. To ensure trustworthiness and credibility, two or more people checked the accuracy of the data, to make sure there is no bias (Lincoln & Guba, 1985) Furthermore the data was looked at again and again to ensure accuracy. Lastly, this was a study of 72 people and the school where the data was collected is a quintile 1 school. The school is under-resourced, and the conditions of learning and teaching are not the best. While the learners in this school can find the test challenging, it might be different for schools in the upper quintiles.

3.11 Conclusion

The study adopted qualitative method and interpretive as its paradigm. The data generation and procedures of analysing the data were discussed, the intervention sessions were discussed in detail, the sampling techniques, ethical considerations, trustworthy and credibility of the study were also discussed. The next chapter will present the findings of the study.

CHAPTER 4: MODEL USE AND ASSOCIATED SUCCESSES

4.1 Introduction

The following chapter discusses the different models that learners employed in both groups, across the pre- and post-test. It further discusses the average use of each model across the three categories of word problems and delves on the average success rate associated with the use of each model for both groups in the pre- and post-test.

4.2 Models used by learners in the intervention and control groups across the pre- and post-tests

The learners in the intervention group used five different models in the pre-test. These are: column model (CM), horizontal number sentence (HNS), no model (NM), word sentence (WS), as well as a combination of the column model (CM) with the word sentence (WS) used together. Across the responses provided by the learners in the control group, six different models were visible in the pre-test.-Part of the six models, three were the same as the ones that learners in the pre-test of the intervention had used. The other three models were the breaking down model and two types of combinations, where the learner used more than one model to solve the problem. The two combinations where the learner used more than one model to solve the question are as follows: in the first combination, the learner used a combination of the number line (NL) and the horizontal number sentence (HNS), the second combination is where the learner used two models to answer the question, and this was a combination of the column model (CM) and breaking down model.

Single models

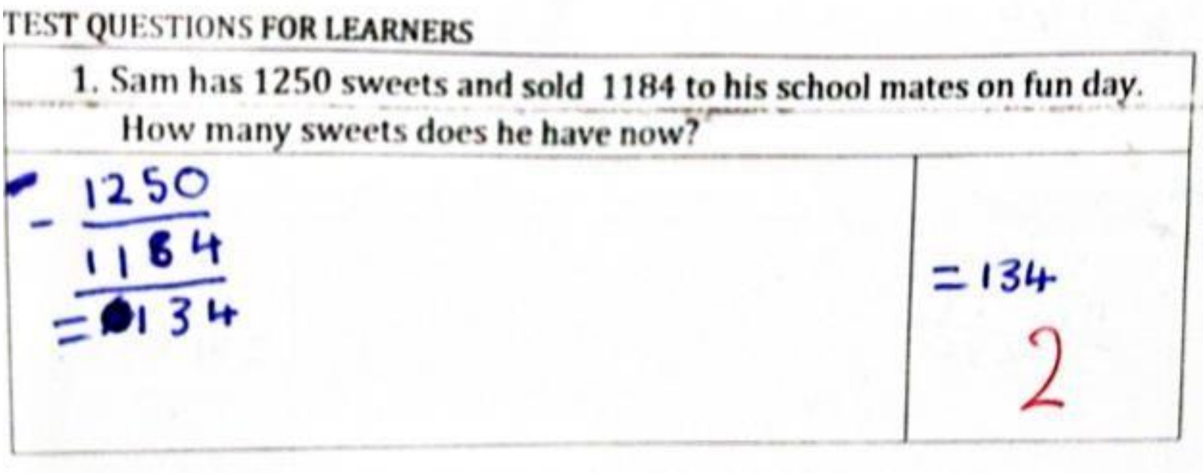
Most learners in the control and intervention groups resorted to the use of a single model to solve the problems, it was only in the post-test of the intervention group that the use of a single model saw a decline. The most used single model was the column model, as it was discussed above, and the success rate of the model became erroneous when the word problems became more difficult. The problem with the use of the single model was that the model had to help make sure learners mathematise correctly, and they can use the same model to calculate correctly. The single models did manage to do both on some categories of word problems but could not do on other categories.

Combination of models

There were not a lot of learners that used more than one model across the intervention and control groups, it was only in the post-test of the intervention group that most learners used combined models, and the success rate associated with the combination was high. The combination of models across the tests of the control and pre-test of the intervention group always had a good success rate. However, the average use of the combination was not as good as its success rate, because on other categories the combination was not used. Looking at learners that used a combination of models in the control and pre-test of the intervention group, the two models were complementing each other, one model helped learners to write the equation and the other model helped with the calculation process. This was the same with learners in the post-test of the intervention group that used two models. For instance, the bar model would help learners write the correct equation by choosing the correct additive inverse, and the number line assisted learners to complete the calculation. The following are examples of the single and combine models that learners in the intervention and control group used in their pre- and post-tests.

Column Model (CM)

This was one of the most used models in the responses of the learners. Learners are exposed to this model at the start of their intermediate phase; it is the most utilised model in school to help learners add and subtract numbers with more than three digits.


TEST QUESTIONS FOR LEARNERS

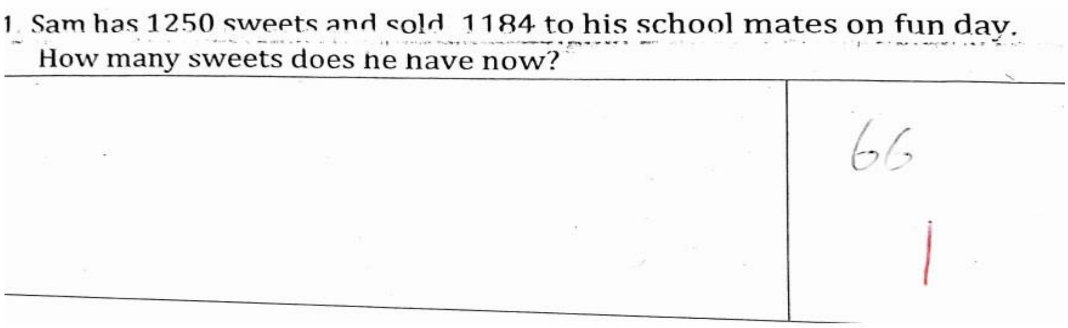
1. Sam has 1250 sweets and sold 1184 to his school mates on fun day. How many sweets does he have now?

$-\ 1250$ $\ \ \ 1184$ $=0134$	$=134$ 2

Table 4.1: Learner responses CM

No Model (NM)

Learners that used this model did not show any workings on the section that was provided for them to work out the answer. They only wrote the answer for the given question.

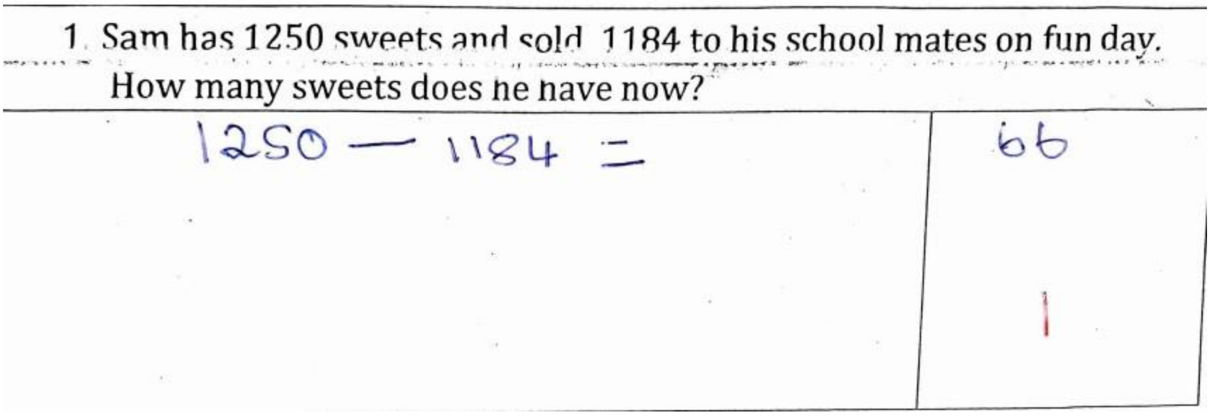

1. Sam has 1250 sweets and sold 1184 to his school mates on fun day. How many sweets does he have now?

	66 1

Table 4.2: Learner responses NM

Horizontal Number Sentence (HNS)

Learners who utilised this model just wrote the numbers in the word problem and chose the addition operation before writing the answer; they did not demonstrate any workings:



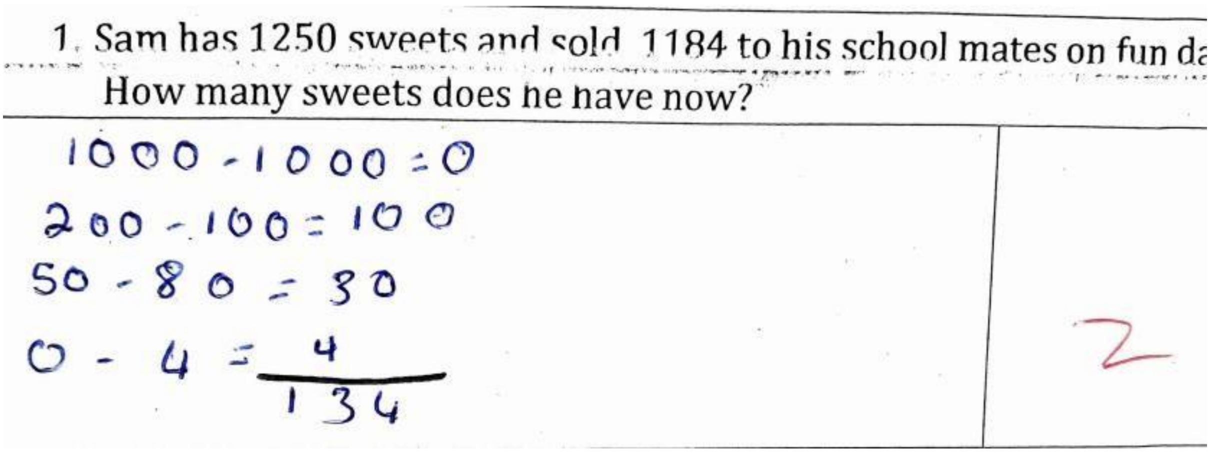
1. Sam has 1250 sweets and sold 1184 to his school mates on fun day. How many sweets does he have now?

1250 — 1184 =	66 1

Table 4.3: Learner responses HNS

4.2.6 Breaking down (BKD)

Breaking down is a frequent strategy taught in schools, often known as expanded notation. Learners who utilised this model converted the numbers in the word problems into their place values. They then calculated each place value, added or subtracted units, and progressed to tenths until they obtained the result. This way of processing is also taught in school with the column model.



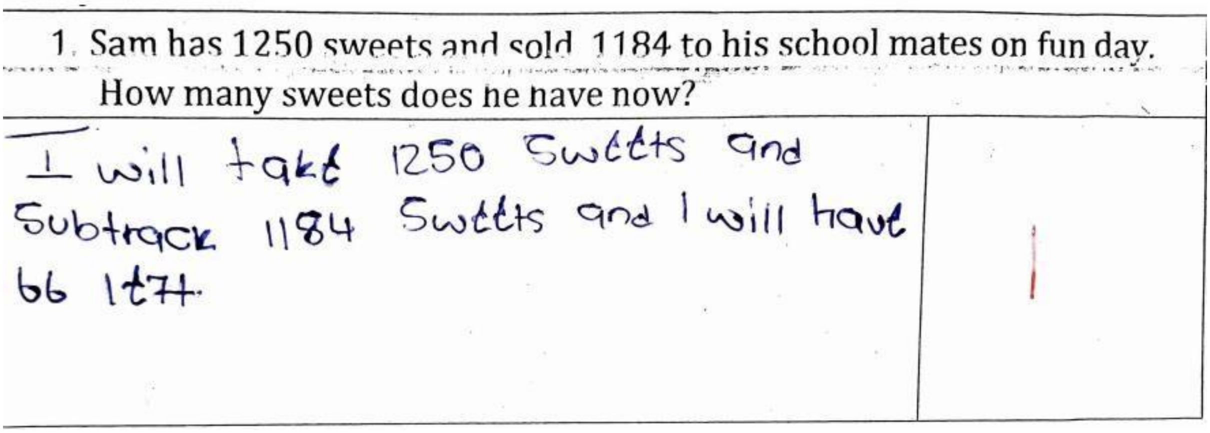
1. Sam has 1250 sweets and sold 1184 to his school mates on fun da
How many sweets does he have now?

1000 - 1000 = 0 200 - 100 = 100 50 - 80 = 30 0 - 4 = 4 134	2

Table 4.4: Learner responses BKD

4.2.6Word sentence (WS)

Learners who employed this model did not demonstrate any workings. They responded to the question using words, detailing how they will obtain the solution and then providing the answer.



1. Sam has 1250 sweets and sold 1184 to his school mates on fun day.
How many sweets does he have now?

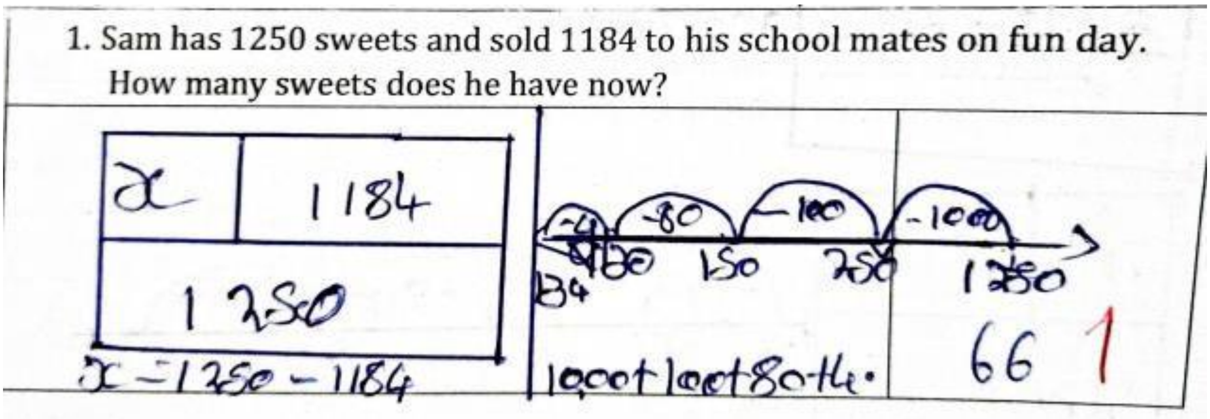
I will take 1250 Sweets and Subtrack 1184 Sweets and I will have 66 left.

1

Table 4.5: Learner responses WS

4.2.7Bar model and Number line (BM/ NL)

The intervention group was introduced to a combination of these models in order to facilitate the employment of more sophisticated tactics than the learners had previously been exposed to.

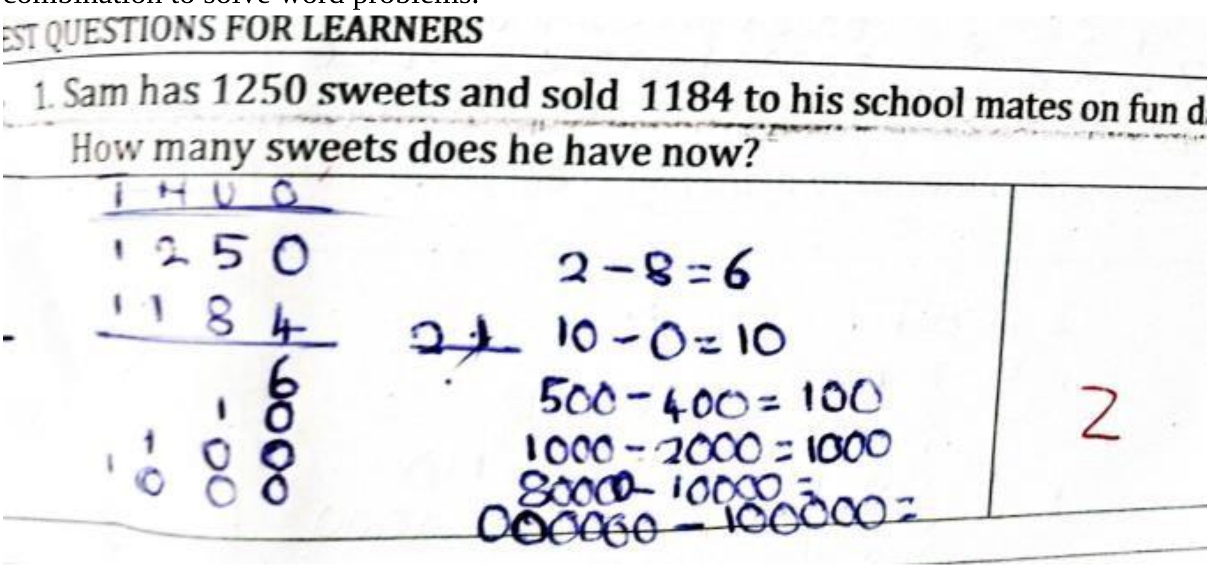


1. Sam has 1250 sweets and sold 1184 to his school mates on fun day.
How many sweets does he have now?

Table 4.6: Learner responses BM/NL

4.2.8Column model and breaking down (CM/BD)

This is a hybrid of two models: column and breaking down. Some learners utilised this combination to solve word problems.



EST QUESTIONS FOR LEARNERS

1. Sam has 1250 sweets and sold 1184 to his school mates on fun d
How many sweets does he have now?

Table 4.7: Learner responses CM/BD

Combination of three models

One of the learners employed a combination of models, including the bar model, number line, and breaking down model.

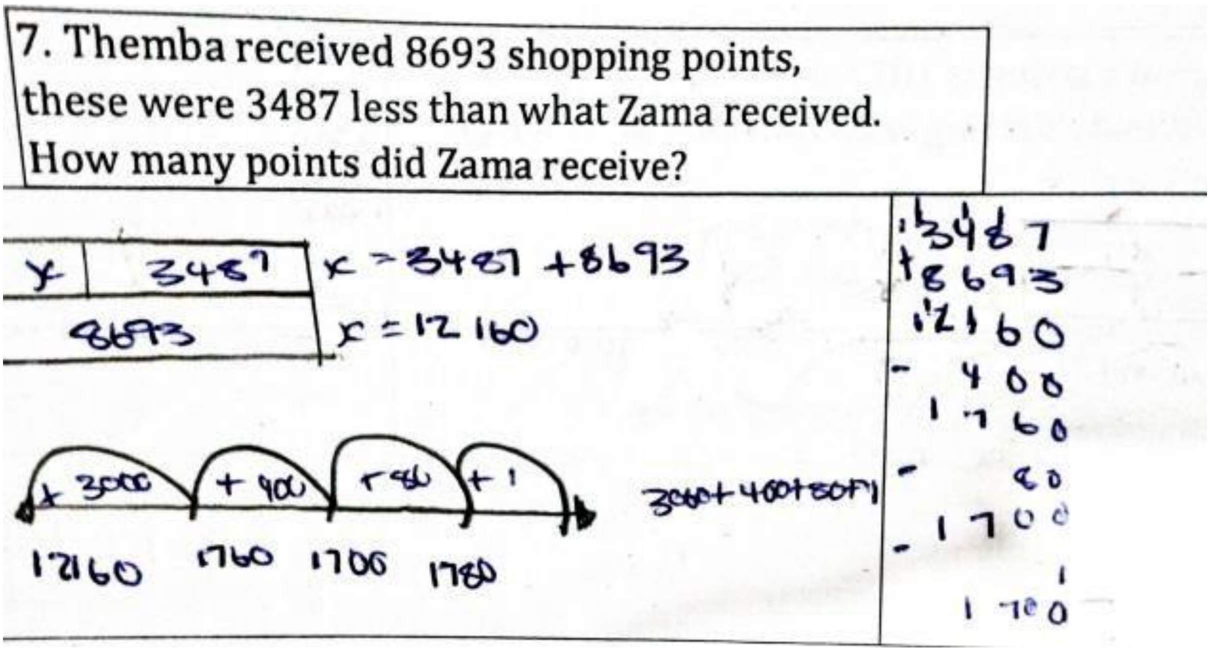


Table 4.8: Learner responses combination of three model

4.3 Models that learners in the intervention and control groups used during tests

This section will go over the models that learners in the intervention and control groups used during the pre- and post-tests. It will initially examine the models used by learners in the intervention group before moving on to the models used by learners in the control group.

4.3.1 Comparison of the models that were used by learners in the intervention group across the pre, and post-test

The following is a comparison of the models employed by learners in the intervention group in the pre- and post-test. First, the models employed in the pre-test by learners in the intervention group will be examined. Following that, these models will be compared to those utilised by learners in the intervention's post-test.

There were five different models that were employed by learners in the pre-test of the intervention group, and one model had a combination of two models that were used to solve word problems. These are the models: column model (CM), horizontal number sentence (HNS), no model (NM), word sentence (WS), and a combination of the column model with the word sentence used together to solve a particular problem. The column model was again the most used with a good success rate on the change problems, and the success rate associated with its use showed a significant decline on the combine and compare problems. The following table will provide a detailed and overall distribution of the models that were used by the intervention group in the pre-test. It will also provide the success rate the model had on each question:

PRE-TEST INTERVENTION																				
WORD PROBLEM (SUCCESS RATE)																				
MODEL	**Q1**	**%**	**Q2**	**%**	**Q3**	**%**	**Q4**	**%**	**Q5**	**%**	**Q6**	**%**	**Q7**	**%**	**Q8**	**%**	**Q9**	**%**	**Q10**	**%**
CM	24/31	77.4	27/31	87.1	29/32	90.1	25/32	78.1	16/33	48.5	12/31	38.7	2/31	6.5	1/32	3.1	1/32	3.1	9/33	27.3
HNS	4/4	100	3/5	60	4/4	100	3/3	100	1/3	33.3	0/2	0	0/2	0	0/2	0	0/1	0	0/1	0
NM	2/2	100	1/1	100	0/1	0	2/2	100	0/1	0	2/4	50	0/5	0	0/3	0	0/4	0	0/3	0
WS	1/1	100	1/1		1/1	100	0		1/1	100	0/1	0	0		0/1	0	0/1	0	1/1	100
CM/WS							1/1	100												
Totals	31/38		32/38		34/38		31/38		18/38		14/38		2/38		1/38		1/38		10/38	

Table 4.8: Models used and success rate in the pre-test of the intervention group

Source: Author's own compilation sample

Table 4.8 above shows the models learners in the intervention group used in the pre-test when answering the word problems and the success rate of the models' learners used. In some of the questions, learners used more than one method to find the solution to the question. Two methods were used to solve one problem, the models were the number sentence and column model. Only one learner used the combination of these models, and the success rate was 100%. One of the interesting issues about Table 4.8 above, is the use of the word sentence model (WS), this model was never used in the control group in either test. This model seems to be the least favored, with only one learner using it across 8 questions. It is again surprising that learners did not use the breaking down model (BD) which is a model that has been taught simultaneously with the column model from grades 4 to 6.

Another interesting aspect about Table 5.8 is the use of the column model; it has an average of 30 learners using it across all questions. This suggests that this is the most preferred model of usage. The average success rate of using the column model on the first four questions is 83.2%, for the last 6 questions it averages a success rate of 21.2%. This is a decline of 62%. This decline suggests that when learners are using the column model while working with complicated word problems, they are more likely to make errors. The decline also suggests that there is a high proportion of incorrect answers when using the column model on more complicated word problems. The focus will now be turned to the models used by learners in the intervention's post-test.

The post-test of the intervention group had four models, three of the models were not used in the pre-test of the intervention group. A combination of the bar model and number line was one of the models, the bar model, the column model, and a combination of the number line. These were the predominant models in the post-test of the intervention group. The use of these models was associated with great success rate on the change, combine, and compare problems. The following table (Table 4.9) will give an overall distribution and success rate of the models used by the intervention group in the post-test:

POST-TEST INTERVENTION GROUP																				
WORD PROBLEM (SUCCESS RATE)																				
MODEL	**Q1**	**%**	**Q2**	**%**	**Q3**	**%**	**Q4**	**%**	**Q5**	**%**	**Q6**	**%**	**Q7**	**%**	**Q8**	**%**	**Q9**	**%**	**Q10**	**%**
BM/NL	31/37	83.8	33/38	86.8	28/36	77.7	32/38	84.2	30/38	78.9	30/37	81.1	27/38	71.1	26/37	70.3	19/37	51.4	16/36	44.4
BM	0		0		1/1	100	0		0		0		0		0		0		0	
CM	1/1	100	0	0	0/1	0	0	0	0	0	0/1	0	0	0	0/1	0	1/1	100	1/1	100
BM/NL/CM																			1/1	100
Totals	32/38		33/38		29/38		32/38		30/38		30/38		27/38		26/38		20/38		18/38	

Table 4.9: Models and success rate of the intervention group in the post-test

Source: Author's own compilation sample

Table 4.9 above shows the models learners in the intervention group used in the post-test when answering the word problems. In some of the questions, learners used more than one model to find the solution to the question. A noteworthy issue about Table 5.9 is that at most, three models were used to find one solution and this combination was used on question 10. The combination of models was the column model, number line, and the bar model, this combination of models had a success rate of 100% and it was mostly used by one learner. It is important to note that the intervention group was introduced to alternative ways of calculating rather than using the column model and their previously taught models, this was done during the intervention sessions.

Another issue that stands out is the use of the column model being the least used, with only one learner using the model on 5 questions. What is worth highlighting from Table 5.9 above is the predominant use of the combination of the bar model and number line. This seems to be the preferred model of usage, with an average of 37 learners using this combination across all questions. The average success rate of the bar model and number line for the first four questions is 83.1%, for the last 6 questions this combination had average success rate of 66.2%.

4.3.2 Comparison of the models that were used by learners in the control group across the pre- and post-test

The following is a comparison of the models employed by learners in the control group in the pre- and post-test. First, it will examine the models employed by learners in the control group's pre-test. Following that, it will compare these models to those employed by learners in the control group's post-test.

There were six models that learners in the control group used in the pre-test. Four out of the six models were used in the post-test of the control group. The column model was again the most used with a good success rate on the change problems. A great decline in the use of the column model on the combine and compare problems were evident. The following table will give an overview of the models and their success rate in the pre-test of the control group.

PRE-TEST CONTROL GROUP																				
WORD PROBLEMS																				
MODEL	**Q1**	**%**	**Q2**	**%**	**Q3**	**%**	**Q4**	**%**	**Q5**	**%**	**Q6**	**%**	**Q7**	**%**	**Q8**	**%**	**Q9**	**%**	**Q10**	**%**
CM	17/23	74	20/22	95.5	12/23	52.17	15/20	75	8/23	35.8	7/24	29.1	6/24	25	10/21	47.6	6/20	30	6/21	28.6
HNS	4/6	57.1	6/9	66.7	4/7	57.1	7/11	63.6	3/7	37.5	2/6	28.6	5/7	28.6	1/9	11.1	3/9	33.3	2/9	22.2
BD	0/1	0	0/1	0	0/1	0	0/1	0	0/1	0	0/1	0	0/1	0	0/1	0	0/1	0	0/1	0
NL/HNS	1/1	100	0		0		0		0		0		0		0		0		0	
NM	0/2	0	0/1	0	0/3	0	0/2	0	0/3	0	0/3	0	0/2	0	0/3	0	0/4	0	0/3	0
C/N/BD	1/1	100	1/1	100	0		0		0		0		0		0		0		0	
Totals	23/34		27/34		16/34		22/34		11/34		9/34		11/34		11/34		9/34		8/34	

Table 4.10: Models used and their success rate of the control group in the pre-test

Source: Author's own compilation sample

The above table shows the models learners in the control group used in the pre-test when responding to the word problems. In some of the questions, learners used more than one method to find the solution to the particular question. At most, three methods were used to solve one particular problem. Table 4.10 illustrates the number of learners that used a particular model to a specific question and the success rate of the model used on a particular question. Looking at Table 4.10 above, seeing that there were three models used on the first two questions is noteworthy, and this combination was used by only one learner and the success rate for those questions was 100%. The combination of these three models (column model, number line and breaking down) only appears twice in the pre-test of the control group. There is another combination of models that appears on Table 4.10, the use of a number line and horizontal number sentence. This combination of models appears once and has a success rate of 100%.

The second issue that stands out is the use of the column method. It seems to be the most preferred method of usage. It has an average usage of 64.5% from question 1 to the last question. The least used model is the BD, also known as the expanded notation. It is surprising why this is the least used method, in that from grade 4 to 6, learners are taught this method in school simultaneously with column addition method. The third issue that stands out is that the success rate of the column method declines from the fifth question till the last question. The success rate of the column method from question 1 to 4 was at an average of 65.7 %. The success rate of the column method from question 5 to 10 was at an average of 28.6 %. This is a decline of 37.1%. This decline suggests that learners are most likely to make mistakes when using the column method with more complicated word problems. The discussion will now move to the models that were used by leaners in the post-test of the control group.

The post-test of the control group had four different models that learners used. Three of the models are the same as the ones that appeared in the pre-test of the control group. The column method and the horizontal number sentence where the most used models in the post-test. The column method was again the most used with a good success rate on the change problems, and it showed a significant decline on the combine and change problems. Table 4.11 below will show the models used and the success rate of each of the models across all questions:

POST-TEST CONTROL GROUP																				
WORD PROBLEM																				
MODEL	**Q1**	**%**	**Q2**	**%**	**Q3**	**%**	**Q4**	**%**	**Q5**	**%**	**Q6**	**%**	**Q7**	**%**	**Q8**	**%**	**Q9**	**%**	**Q10**	**%**
CM	14/22	63.6	16/20	80	13/24	54.2	14/25	56	11/23	47.8	10/21	47.6	9/23	39.1	11/22	50	8//22	36.3	5/21	23.8
HNS	3/6	50	5/8	62.5	3/5	60	4/5	80	3/7	42.9	2/7	28.6	2/6	33.3	3/7	42.9	2/7	28.6	3/7	42.9
NM	0/3	0	0/2	0	½	50	½	50	1/3	33.3	¼	25	0/2	0	0/2	0	1/1	100	0/2	0
BD	0		0		0		0		0		0		0		0		0		0	

CM/BD	2/3	66.7	4/4	100	3/3	100	2/2	100	1/1	100	½	50	1/3	33.3	2/3	66.7	0/4	0	0/4	0
Totals	19/34		25/34		20/34		21/34		16/34		14/34		12/34		16/34		11/34		8/34	

Table 4.11: Models used and their success rate of the control group in the post-test

Source: Author's own compilation sample

Table 4.11 above shows the models learners in the control group used in the post-test when answering the word problems. In some of the questions, learners used more than one method to find the solution to the question. At most, two models were used to solve one problem. Table 4.11 illustrates the number of learners that used a particular model to a specific question and the success rate of the model used on a particular question. In the post-test of the control group, there are no learners that used the BD method, whereas in the pre-test there were. This, further leaves one with questions because this method has been taught from grade 4 to 6 simultaneously with the column method. Looking at the Table 5.11, it is riveting to observe that there is a combination of two models used in all questions, the models used are the CM and the BM methods. This combination was at most used by four learners.

Moreover, this combination had the highest success rate of 100% on questions 2, 3 and 4. Table 4.11 also illustrates that the least used model was the NM. Although this model was used on all the questions, it had a maximum of only 4 learners using it, which is similar to the number of learners that used it in the pre-test. Another distinct issue is the use of the column method. It seems to be the most preferred method of usage, with an average of 25 learners using it across all questions. The average success rate for the first four questions using the column method is 63.45%. For the last 6 questions, the column method averages a success rate of 38.13 %. This decline suggests that as word problems get more complicated, the use of the column method becomes more error prone. This is like the pre-test where the most preferred model was the column method and the success rate of using the model declined as the word problems became more complicated.

Overall, the most used model in the pre-test of the intervention group was the column model, it was the most preferred model of usage in the pre-test of the intervention group, as well as in the pre-test of the control group. The model also showed a good success rate for the change problems which are questions: 1,2,3 and 4 of the pre-tests of the control group, as well as the pre-test of the intervention group. However, on the combine problems which are questions 5, 6 and10; and compare problems which are questions 7, 8 and 9, there was a significant decline in the success rate from using this model. This was evident in both the pre-tests of the intervention and control groups. This suggests that the model became error-prone when it was used to solve complicated word problems.

There were four different models that learners used in the post-test of the intervention group, with two of the four models having a combination of two models used to solve the word problems. These are the models that learners used in the post-test of the intervention group: a combination of the bar model and number line (BM/NL), bar model (BM), column model, and a combination of the bar model, number line and column method (BM/NL/CM). The control group had five different models, four of the five were the same models they had used in the pre-test. The only model that was new from the ones they had used in the pre-test was the combination of the column model and the breaking down model. The least used model in the post-test of the control group was the NM, with a maximum of two learners using the model across all questions.

However, there seemed to be a difference when comparing with learners from the intervention group in the post-test. The column model was the least used model across all questions, with only one learner employing this model. The most used model from the post-test of the control group was the column model, with an average of 24 learners using this model across all questions. The success rate for this model was good for the change problems (questions 1, 2, 3 and 4), but had a significant decline in performance on the combine questions (questions 5, 6 and 10) and compare problems (questions: 7, 8 and 9). On the other hand, the most used model for the intervention group in the post-test was the combination of the bar model and the number line. This had an average of 37 learners using these models across all questions. This combination also had a good success rate for the first four questions but did not have a significant decline on the last six questions.

The most used model in the pre-test of the intervention and control group is the column model, which suggests that this was the most preferred model of usage. This model had a good success rate on questions 1, 2, 3 and 4, which are change problems. The success rate of the model showed a great decline on questions 5, 6 and 10, which are combine problems, and on questions 7, 8 and 9 which are change problems. The decline in the success associated with using the column model on these questions was in the pre-tests of both the intervention and control group. This was similar with the post-test of the control group, where the most used model was the column model. It also had a good success rate from question 1, 2, 3 and 4 which are change problems, but had a great decline on questions 5, 6 and 10 which are combine problems and questions 7, 8 and 9 which are change problems. However, the post-test of the intervention had the column model as the least used model, the most used model was a combination of the bar model and the number line. This combination of models had a great success rate associated with its use on the combine problems (questions: 1, 2, 3 and 4), and did not have a significant decline compared to the column model on the combine problems (questions: 5, 6 and 10) and compare problems (questions 7, 8 and 9).

The column model was the one that was most frequently utilised in the pre-tests of the intervention and control groups, indicating that this was the model that learners chose to employ. The success rate associated with the use of the model showed a significant fall on questions 5, 6 and 10 which are combine problems as well as on questions 7, 8 and 9 that are compare problems. This model performed well on questions 1, 2 and 3 which are change problems. Both the intervention group and the control group's pre-tests showed a reduction in the questions under the compare and combine category. The column model was the most popular model during the post-test for the control group. It also had a high success rate for the change-related questions 1, 2 and 3, but a very low success rate for the combine-related questions 5, 6 and 10, as well as the change-related questions, such as questions 7, 8 and 10. The column model was, however, the least frequently used model in the post-test of the intervention; instead, a model that combined the number line and the bar model was most frequently utilised. The combine problems (questions 1, 2 and 3) were solved with a high degree of success using this set of models, while the combine problems (questions 5, 6 and 10) and compare problems were solved with a similarly high degree of success using this set of models (questions, 7, 8 and 9).

4.4 The average use of models and the average success rate associated with the use of the models on different categories of word problems for the pre- and post-test in the intervention group.

The following will discuss the average use of the models used across the three-word problem categories, and it will discuss the average success rate associated with the use of the model in the three categories. The averages were calculated in the following way: the average use of each model was calculated by adding the number of learners that used that model in that category and then divided by the number of questions in the category of word problems. For instance, the average use of the column model under the change category was calculated by adding the number of learners that used the model on questions 1, 2, 3 and 4 and the sum was divided by 4, because there are four items under the change category. This was the same with the averages of the success rate associated with the use of the models. For instance, the average of the success rate associated with the use of the column model was calculated by adding the success rate on question 1, 2, 3 and 4, and then divided by 4. The table below indicates the different models used in each category, the average use of the model as well as the average success rate associated with using the model in the pre-test of the intervention group.

PRE-TEST INTERVENTION GROUP			
Category	**Models**	**Average use**	**Average success rate**
Change (Qs1, 2, 3	Column model	32 (84%)	83.2 %
	Horizontal number sentence	4 (11%)	90 %
	Word Sentence	0.75 (8%)	75%
	Combination of CM and WS	0.25 (3%)	25 %
Combine Qs 4, 5, 10	Column model	32 (84%)	38%
	Horizontal number sentence	2 (6%)	11%
	Word sentence	1 (3%)	67%
	Combination of CM and WS	0 (0)	0
Compare Qs 7, 8, 9	Column Model	32 (84%)	4%
	Horizontal number sentence	2 (6%)	0
	Word sentence	1 (3%)	0
	Combination of CM and WS	0 (0)	0

Table 4.13: The average use of models and the average success rate associated with the use of the models for the pre-test in the intervention group

Source: Author's own compilation sample

What is interesting about Table 4.12 above is the average use of the column model. Across the three categories it has the highest average use, accounting for 32 learners using this model across all questions in the three categories. The average success rate associated with using this model is decreasing across the three categories, with the highest average success rate being for the items in the change category, followed by the combine category and the compare category having the poorest average success rate from using the column model. There is a big gap in the average success rate from using the column model between the change category and the compare category. Another interesting aspect of the above table is the average success rate associated with using the horizontal number sentence, it is the highest in the change category and only has an average of four learners using this model across the change category. However, there was a significant drop in the average success rate associated with using this model on the combine and compare problems. The combination of models had the least average use in the change category and the least success rate associated from using this combination. Both the average use and the average success rate from using this combination of models declined on the combine and compare problems. The following table is a distribution of different models used in each category, the average use of the model as well as the average success rate associated with using the model in the post-test of the intervention group.

POST-TEST INTERVENTION GROUP			
Category	**Models**	**Average use**	**Average success rate**
Change	Combination of the Bar model and number line	37 (97.4%)	83.1%
	Bar model	0.5 (1.3%)	25%
	Column model	0.5 (1.3%)	25%
	Combination of the bar model, number line and column model	0 (0)	0
Combine	Combination of the Bar model and number line	37 (97.4%)	68.1%
	Bar model	0 (0)	0
	Column model	0.3 (0.8%)	33%
	Combination of the bar model, number line and column model	0.3 (0.8%)	33%
Compare	Combination of the Bar model and number line	37 (97.4%)	64%
	Bar model	0 (0)	0
	Column model	0.7 (1.8%)	33.3%
	Combination of the bar model, number line and column model	0 (0)	0

Table 4.13: The average use of models and the average success rate associated with the use of the models for the post-test in the intervention group

Source: Author's own compilation sample

It is important to note that the intervention group was introduced to alternative ways of solving word problems. It is interesting to see the column model still being used across the three categories. The combination of the number line and bar model have the highest average use across the three categories. Accounting for 37 learners using this combination across all question on each category. The average success rate associated with using this combination of models is the highest in the change category, followed by the combine category and the compare category having the least success rate. However, the gap between the categories in the average success rate associated with using this combination is not significant. The average success rate associated with using the bar model was the second highest in the change category. However, this model was not used in the combine and compare categories. Another interesting aspect of the above table is the average use of the combination of three models. This combination was used only on the combine category and had an average success rate of 33%.

4.6 The average use of models and the average success rate associated with the use of the models on different categories of word problems for the pre- and post-test in the control group

The next sections will go through the average use of the models across the three-word problem categories, as well as the average success rate linked with the employment of the model in the three categories. Table 4.14 below shows the various models used in each category, the average use of the model, and the average success rate associated with employing the model in the control group's pre-test:

PRE-TEST CONTROL GROUP			
Category	**Models**	**Average use**	**Average success rate**
Change	Column model	22 (64.7%)	74.2%
	Horizontal number sentence	8.3 (24.4%)	61.1%
	Breaking down	1 (2.9%)	0%
	Combination of a number line and a horizontal number sentence	0.25 (0.7%)	0%
	Combination of the column model, number line and breaking down	0.5 (1.5%)	50%
Combine	Column model	22.7 (66.8%)	31.2%

	Horizontal number sentence	7.3 (21.5%)	29.4%
	Breaking down	1 (2.9%)	0%
	Combination of a number line and a horizontal number sentence	0 (0)	0
	Combination of the column model, number line and breaking down	0 (0)	0
Compare	Column model	21.7 (63.8%)	34.2%
	Horizontal number sentence	8.3	24.3%
	Breaking down	1 (2.9%)	0%
	Combination of a number line and a horizontal number sentence	0 (0)	0
	Combination of the column model, number line and breaking down	0 (0)	0

Table 4.14: The average use of models and the average success rate associated with the use of the models for the post-test in the intervention group

Source: Author's own compilation sample

Looking at Table 4.14 above, the column model had the highest average use across the three categories. The highest average success rate across the three categories is associated with the use of the column model. The change category has the highest success rate from using the column model, then followed by the combine and the compare category. This suggests that the success rate of using this model declines across the categories, whereas the average use of the model is consistent across the categories. There is a significant decrease in the success rate associated with using the column model between the change and combine categories. The horizontal number sentence has the second highest average use across the three categories and the second highest success rate across all three categories is associated with its use. There were two combinations of models used in the change category, the first combination had an average use of 0.7% and a success rate of 0%, which means the learners that used this combination did not find any success. This combination was only used in the change category. The second combination of models had an average use of 0.5% and a success rate of 50%. This combination of the model was not used in the combine and compare category. Table 4.15 is a distribution of different models used in each category, the average use of the model as well as the average success rate associated with using the model in the post-test of the control group.

POST-TEST CONTROL GROUP			
Category	**Models**	**Average use**	**Average success rate**
Change	Column model	22.8 (67.1%)	63.5%
	Horizontal number sentence	6 (17.6%)	63.1%
	Breaking down	0 (0)	0
	Combination of column model and breaking down	3 (8.8%)	91.8%
Combine	Column model	21.6 (63.5%)	39.7%
	Horizontal number sentence	7 (20.6%)	38.1%
	Breaking down	0 (0)	0
	Combination of column model and breaking down	2.3 (6.8%)	50%
Compare	Column model	22.3 (65.6%)	34.9%
	Horizontal number sentence	6.7 (19.7%)	34.9%
	Breaking down	0 (0)	0
	Combination of column model and breaking down	3.3 (9.7%)	33.3%

Table 4.15: The average use of models and the average success rate associated with the use of the models for the post-test in the control group

Source: Author's own compilation sample

The change category has the column model with the highest average use, and the success rate associated with its use is the second highest. The combination of models in the change category has the highest success rate associated with its use and the lowest average use. The combine category still has the column model being the one with the highest average use. The success rate associated with this model in this category is still the second highest, and it has seen a significant decrease compared to the success rate it had in the change problems whereas the average use has remained slightly the same. The combination of models still has the highest success rate and the lowest average use in the combine category. The horizontal number sentence has the second highest average use in the change and combine category, the success rate associated with the use of this model decrease significantly from the change category to the combine category. The average use of the column model in the change category was slightly higher than in the combine category. However, the success rate associated to the use of the column model in the compare category was lower than in the combine and change categories. The following table will show the differences between the two groups about the average use of models and the success rate associated with the models. Table 4.16 will give a comparison of the average use of models and the success rate associated with the use of the models across the control and intervention group.

		CONTROL GROUP				**INTERVENTION GROUP**			
		Pre-test		**Post-test**		**Pre-test**		**Post-test**	
Category	**Models**	**Average use**	**Average success rate**	**Average use**	**Average success rate**	**Average use**	**Average success rate**	**Average use**	**Average success rate**
Change Qs 1, 2, 3, 4	Column model	22 (64.7%)	74.2%	-	-	32 (84%)	83.2%	0.5 (1.3%)	25%
	Horizontal number sentence	8.3 (24.4%)	61.1%	-	-	4 (11%)	90 %		
	Word Sentence	-	-	-	-	0.75 (8%)	75%		
	Breaking down	1 (2.9%)	0	-	-				
	Combinatio n of number line and horizontal number sentence	0.25 (0.7%)	0%	-	-	-	-	-	-
	Combinatio n of CM, number line and breaking down	0.5 (1.5%)	50%	-	-	0.25 (3%)	25%	-	-
	Bar model and number line	-	-	-	-	-	-	37 (97.4%)	83.1%
	Bar model	-	-	-	-	-	-	0.5 (1.3%)	25%
	Bar model, number line and column model	-	-	-	-	-	-	0 (0)	0
Combine Qs 5, 6, 10	Column model	22.7 (66.8%)	31.2%	-	-	32 (84%)	38%	0.3 (0.8%)	33.3%
	Horizontal number sentence	7.3 (21.5%)	29.4%	-	-	2 (6%)	11%	-	-
	Word Sentence	-	-	-	-	1 (3%)	67%	-	-
	Breaking down	1 (2.9%)	0%	-	-	-	-	-	-

	Combinatio n of CM, number line and breaking down	0	0	-	-	0 (0)	0	-	-
	Bar model and number line	-	-	-	-	-	-	37 (97.4%)	68.1%
	Bar model	-	-	-	-	-	-	0 (0)	0
	Bar model, number line and column model	-	-	-	-	-	-	0.3 (0.8%)	33.3%
Compare Qs 7, 8, 9	Column model	21.7 (63.8%)	34.2%	-	-	32 (84%)	4%	0.7 (1.8%)	33.3%
	Horizontal number sentence	8.3	24.3%	-	-	2 (6%)	0	-	-
	Breaking down	1 (2.9%)	0%	-	-	1 (3%)	0	-	-
	Combinatio n of CM and HNS	0	0	-	-	0 (0)	0	-	-
	Bar model and number line	-	-	-	-	-	-	37 (97.4%)	64%
	Bar model	-	-	-	-	-	-	0 (0)	0
	Bar model, number line and column model	-	-	-	-	-	-	0 (0)	0

Table 4.16: The average use of models and the success rate associated with the models for the control and intervention group

Source: author's own compilation sample

In conclusion, the above table shows the different models that were employed by both groups in the two tests, it indicates the average use of the model and the average success rate associated with the use of the model. A noteworthy aspect of Table 5.16 above is that the column model had the highest average use in the pre-test of the intervention, the control group, and the posttest of the control group. However, it had the least average use in the post-test of the intervention group. In both tests of the control group and the pre-test of the intervention group, the average use of the column model was always the highest across the three-word problem categories and it always had an average use of over 60% across all categories.

However, the success rate associated with the use of the column model was not as consistent as the average use of the model in terms of having the highest success rate across all three categories. In the pre- and post-test of the control group, the average use of the column model was the highest in the compare category, as well as the success rate associated with its use. In the combine category, the success rate associated with the use of the column model saw a great decline, whereas the average use of the model was more or less at the same level as in the change category. In the compare category of the pre- and post-test of the control group, the average use of the column model was slightly higher than in the combine and change category, whereas the success rate associated with its use saw a further decline in the compare category. This trend was the same in the pre-test of the intervention group, where the average use of the column model was relatively constant across the three categories, but the success rate associated with its use saw a great decline across the three categories. It is only in the post-test of the intervention group, that the average use of the column model was the lowest across all three categories. The model with the highest average use across all three categories was the combination of the bar model and the number line. The average success rate associated with the use of the combination of the bar model and number line slightly decrease across the three categories.

CHAPTER 5: CONTROL AND INTERVENTION GROUP PERFORMANCES

5.1 Introduction

In the first layer of analysis, the different models that learners employed in both groups was discussed across the pre and post- test. It discussed the use of each model across the three-word problem categories and delved at the success rate associated with the use of each model for both groups across the two tests. In this chapter, the pre- and post-test average performance for the control and intervention groups have been compared in this chapter. The objective of this chapter is to examine changes in performance for both groups throughout the three-word problem categories (change, combine, and compare). The shifts in the correct responses for both groups will also be examined.

5.2 Averages of the intervention group versus averages of the control group

	Control group		**Intervention group**	
	Pre-test	Post-test	Pre-test	Post-test
Average	43%	47%	46%	73%

Table 5.1: Comparison of learner performance for intervention and control group across the pre, and post-tests

Source: Author's own compilation sample

The intervention group had an average of 46% in the pre-test, which is slightly higher than the average of the control group in the pre-test. An average of 73% was achieved by the intervention group in the post-test. Which is a big improvement compared to the average of the control group in the post-test. The intervention group had an improvement of 27 percentage points whereas the control group had an improvement of only 5 percentage points. This difference of 22 percentage points that an improvement comes even though the averages of the two groups in the pre-test are almost the same, with a difference of only 3%. So, whilst the two groups' initial performances scored similarly, there was a big difference in their shifts in performance following the post-tests. Learners in the control group did very well on the first 3 – 5 questions but struggled with the last 5 -6 questions. Similarly, learners in the intervention group did well on the first 3-5 questions and struggled with the last 5 – 6 questions. In the post-test, learners in the control group still struggled with the last 5 – 6 questions and did well on the first 3 – 5 questions. However, learners in the intervention group showed improvement on the last 5 – 6 questions in the post-test and still did well on the first 3 – 5 questions.

The intervention and control group did relatively well on questions 1 – 4 in the pre-test. These questions are change increase/decrease problems. On the last 6 questions which are combine/separate and compare problems, both groups did not do as well. In the post-test, the control group still did well on the first four questions though they had a decrease in performance compared to the pre-test performance, the decrease was on Questions 1, 2 and

4. They showed no significant improvement on the last 6 questions. However, the intervention group showed significant changes in performance on the last 6 questions which are combine/separate and compare problems.

5.2.1Averages of the intervention group

The intervention group had 38 learners that wrote the pre- and post-test. The intervention group had an average of 46% in the pre-test and an average of 73% in the post-test. This is an increase of 27 percentage points, a big improvement in performance. The 27 percentage points increase in performance of the intervention group also suggests that the number of correct answers provided by the learners increased on average by 2 to 4 correct answers in the post-test as compared to the pre-test. The average improvement also suggests learners' shifts in performance across the test items. The findings are as follows:

1. There is little improvement on the change problems - items initially associated with high performance in the pre-test (Qs 1, 2, 3 and 4),
2. There are medium improvements on the combine problems - items that had medium performance in the pre-test (Qs 5,6 and 10).
3. There are big improvements on the compare problems - items that were associated with low performance in the pre-test (Qs 7, 8 and 9).

The following table illustrates learners' shifts on the pre- and post-tests of the intervention group:

Intervention group					
Q#	**Pre-test correct**	**Pre-test correct %**	**Post-test correct**	**Post-test correct %**	**Shifts in correct answers**
Q1	31	81.58	32	84.21	1
Q2	32	84.21	33	86.84	1
Q3	34	89.47	29	76.32	-5
Q4	31	81.58	32	84.21	1
Q5	18	47.37	30	78.95	12
Q6	14	36.84	30	78.95	16
Q7	2	5.26	27	71.05	25
Q8	1	2.63	26	68.42	25
Q9	1	2.63	20	52.63	19
Q10	10	26.32	18	47.37	8

Table 5.2: Comparing learners' performance between the pre- and post-test for the intervention group

Source: Author's own compilation sample

The first four questions were change increase/decrease problems. The shift in correct answers from the pre- to post-test is a maximum of one - which is an improvement of 2.63%. Questions 5, 6 and 10, are combine/separate problems. This category of problems saw a maximum shift of 16 correct responses from the pre- to the post-test which was on question 6. In the pre-test, this category of problems achieved an average of 36.84 % of correct responses. Whereas, in the post–test they achieved an average of 68.41% of correct responses. This suggests that the category of problems had an average improvement of 31.57% shift in correct answers from the pre- to the post-test.

Questions 7, 8 and 9 compare problems. This category of problems had a maximum shift of 25 correct responses, and these were on questions 7 and 8. In the pre-test, this category achieved an average of 10.52% of correct answers and followed by an average of 64.03% of correct responses in the post-test. This suggests this category of problems had an average improvement of 53.51 % shift in correct answers from the pre- to the post-test. The first category of problems had an average improvement of 2.63%, this had the smallest shift in correct answers compared to the other two categories. The second category of problems had an average shift of 31.57%, which is bigger than the average shift of the first category. The last category of problems had an average shift of 53.51% of correct answers, this is the largest shift in the three categories.

Table 4.3 below summarises the shifts across the three categories for the intervention group:

Low pre-test performance, **High** post-test performance items				**Low** pre-test performance, **Mid** post-test performance items				**Mid** pre-test performance, **Mid** final performance items			
Item	Type	Pre-	Post	**Item**	Type	Pre-	Post	**Item**	Type	Pre-	Post
Q7	compare	2	27	**Q5**	combine	18	30	**Q1**	change	31	32
Q8	compare	1	26	**Q6**	combine	14	30	**Q2**	change	32	33
Q9	compare	1	20	**Q10**	combine	10	18	**Q3**	change	34	29
								Q4	change	31	32

Table 5.3: Summary of shifts across the three categories, for the intervention group

Source: Author's own compilation sample

Table 5.3 above shows a summary of learners' shifts across the three categories in the intervention group. The first column shows that there was a significant shift on the compare problems category. Learners had a low performance in the pre-test of this category and high performance in the post test of the same category of problems. The second column shows the category of problems where learners performed poorly in the pre-test and had an average performance in the post-test. This is the combine category, there were no significant shift from the pre- to the post-test. The last column shows a category of problems where there were slight shifts in performance between the pre- and post-test. This is the change problem category, the performance in the pre- and post-test is slightly different with learners slightly improving in the post-test.

This section reflects the performance of learners in the intervention group. These learners exemplify the typical shifts in performance. They had little improvements on the change problems, medium improvements on the combine problems, and significant improvements on the compare problems. Learner JM obtained a mark of 4 out of 10 in the pretest and a mark of 9 out of 10 in the post-test. Based on the provided information, it can be argued that this suggests an improvement in the learner's performance. The incorrect responses that learner JM provided in the pre-test were on questions 4, 6, 7, 8, 9 and 10. These incorrect responses were on the combine/separate and compare problems, except for question 4. The learner's answers on questions 1, 2, 3 and 5 were correct. These correct responses are on the change increase/decrease problems except for question 5. In the post-test, the incorrect response was only on question 10 - which is the only question that the learner got incorrectly in both tests. The questions that were initially wrong, were correct in the post-test. The learner's correct answers in the pre-test were still correct in the post-test. This can thus be argued to suggest a shift on how the learner answered the questions. This also suggests an improvement in answering the combine/separate and compare problems.

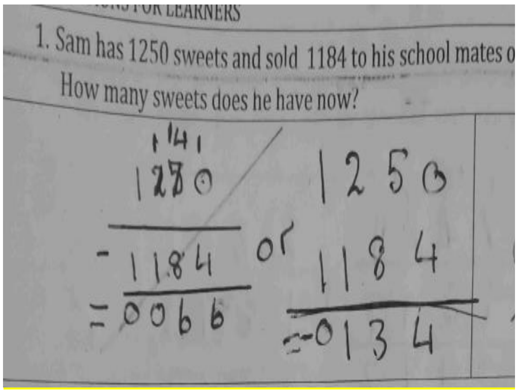
Learner RM obtained 6 out of 10 in the pre-test, and the questions where the learner's responses were incorrect are questions 7 - 10. From the test, questions 7 – 9 are compare problems and question 10 is a combine/separate problem. The learner was able to provide correct answers for questions 1 – 6, while most of these were change problems except for question 5 and 6. In the post-test, the learner obtained a mark of 9 out of 10. The incorrect response was on question 10. The questions that were initially wrong from the learner's responses in the pre-test were compare problems, and the learner was able to provide correct answers in the post-test. The questions that the learner initially got right in the pre-test, were still correct in the post-test except for question 10. This suggests an improvement in the learner's performance.

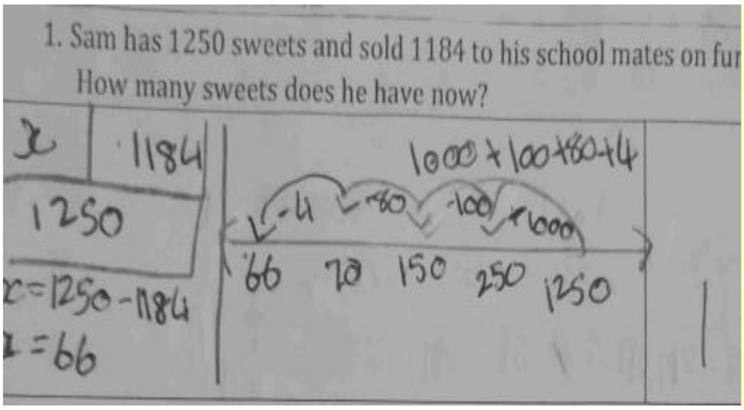
Another learner (learner KK) obtained a mark of 7 out of 10 in the pre-test. The incorrect responses were on question 7, 8 and 9, and these were compare problems. In the post-test, the learner obtained 100% (10 out of 10). For both the pre-test and the post-test, the learner's correct answers remained the same. However, the learner was able to make an improvement in the post-test on questions that were initially incorrect in the pre-test. This suggests a clear improvement in the learner's performance. Learner RP obtained a mark of 3 out of 10 in the pre-test. The incorrect answers were on questions 1, 5, 6, 7, 8, 9 and 10; and these questions were combine/separate and compare problems except for the first question. The correct responses were on questions 2 – 4, and these are change increase problems. In the post-test, the learner obtained a mark of 9 out of 10 and the correct responses were on questions 1, 2, 3, 4, 5, 6, 7, 8 and 10. This showed an improvement from the mark that the learner obtained in the pre-test. It was also observed that the learner's performance on the combine/separate and compare problems improved in the post-test. The change increase/decrease problems that the learner was correct in on the pre-test, were still correct in the post-test. This also suggests that there was a significant improvement of the learner's performance on the compare problems. The following table shows learner RP's work for the pre and post-tests. The table shows the shift in performance in learner RP's work and the different models that the learner employed in the post test of the intervention.

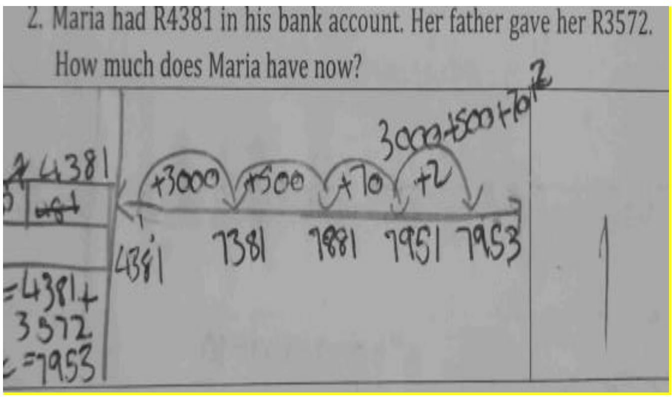
The table shows learner RP's work for the pre and post test

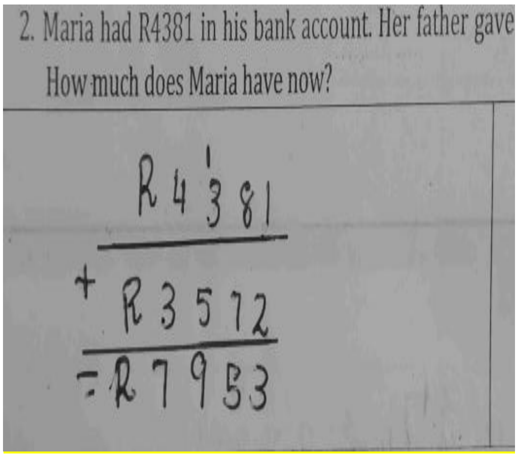
Pre-test

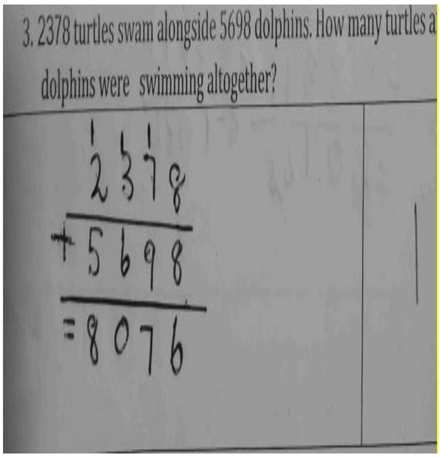
Post-test

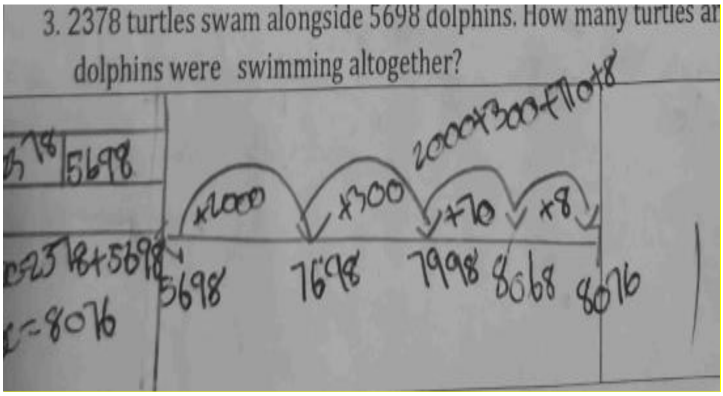


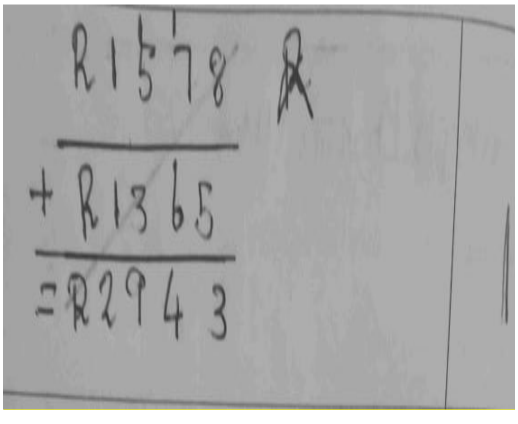


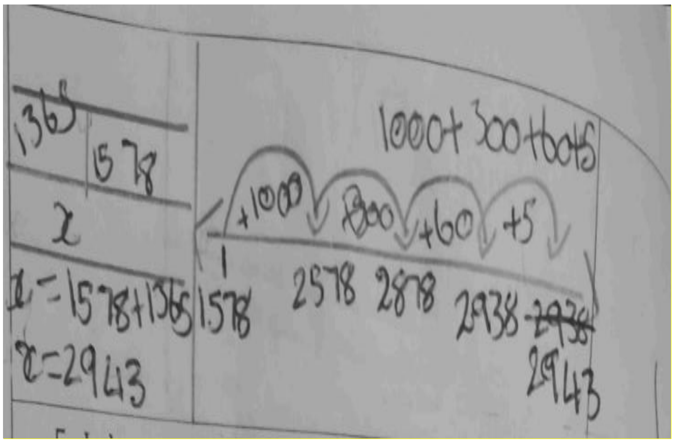


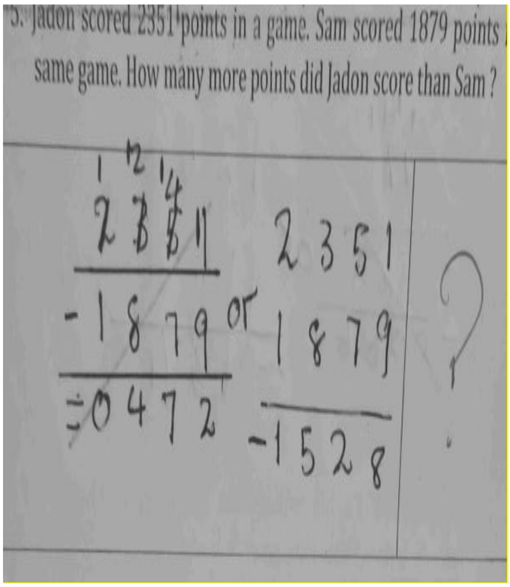

2. Maria had R4381 in his bank account. Her father gave
How much does Maria have now?
R4381
+ R3572
= R7953

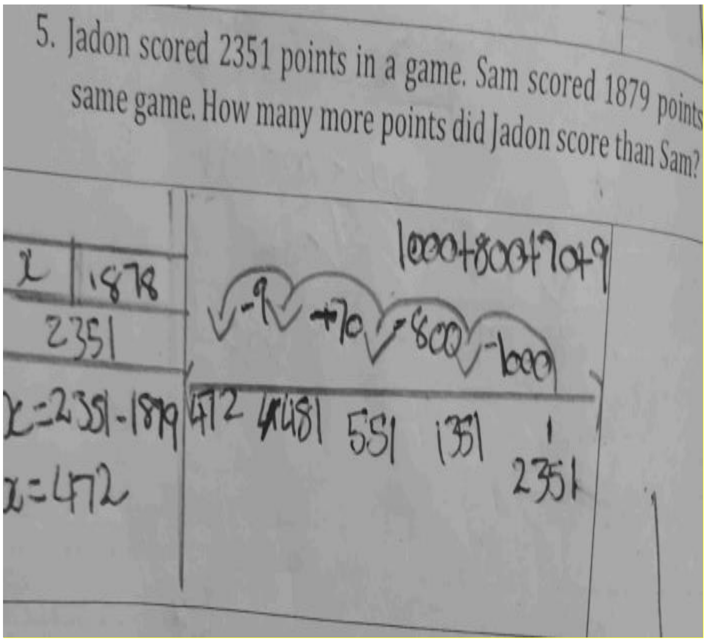

3. 2378 turtles swam alongside 5698 dolphins. How many turtles a
dolphins were swimming altogether?
2378
+ 5698
= 8076

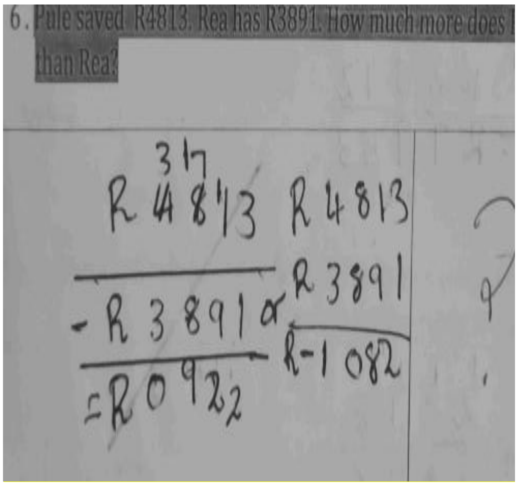

3. 2378 turtles swam alongside 5698 dolphins. How many turtles ar
dolphins were swimming altogether?
5698
7698
7998
8068
8076

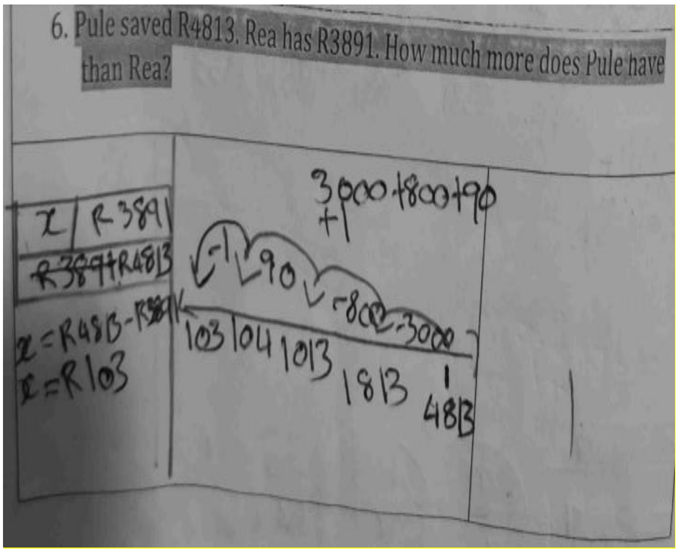

R1578
+ R1365
= R2943

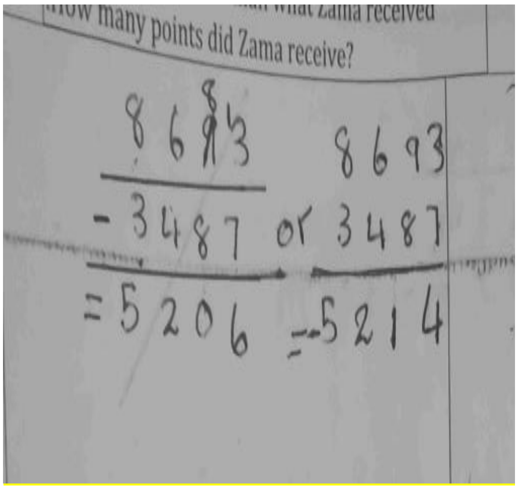

1578
2578
2878
2938
2943


5. Jadon scored 2351 points in a game. Sam scored 1879 points
same game. How many more points did Jadon score than Sam?

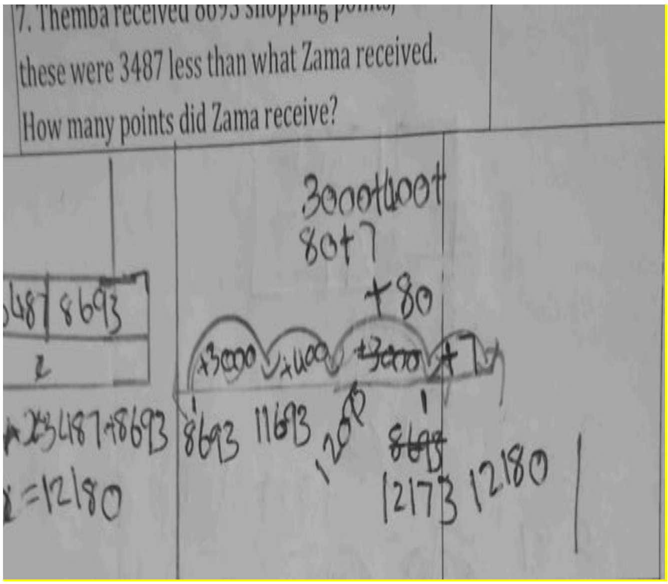

5. Jadon scored 2351 points in a game. Sam scored 1879 points
same game. How many more points did Jadon score than Sam?


6. Pule saved R4813. Rea has R3891. How much more does
than Rea?


6. Pule saved R4813. Rea has R3891. How much more does Pule have
than Rea?


How many points did Zama receive?

8693 − 3487 = 5206 or 8693 − 3487 = 5214


7. Themba received 8693 shopping points, these were 3487 less than what Zama received. How many points did Zama receive?

3000+400+80+7

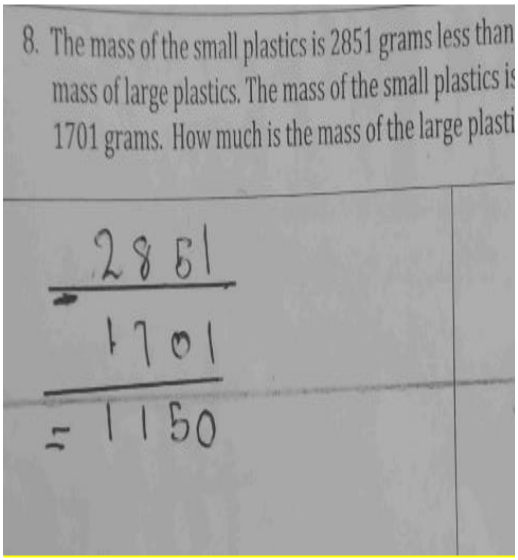
+80

+3000 +400 +80 +7

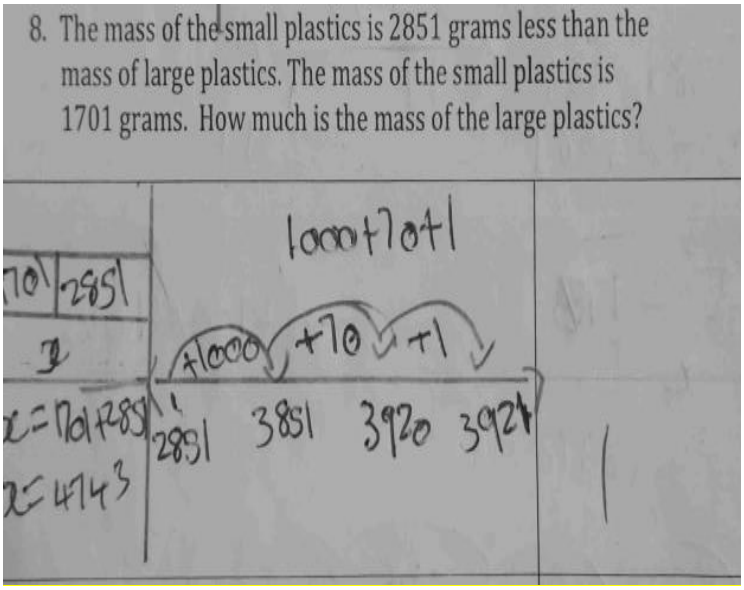
8693 11693 12093 12173 12180

x = 3487 + 8693

x = 12180


8. The mass of the small plastics is 2851 grams less than the mass of large plastics. The mass of the small plastics is 1701 grams. How much is the mass of the large plasti

2851 − 1701 = 1150


8. The mass of the small plastics is 2851 grams less than the mass of large plastics. The mass of the small plastics is 1701 grams. How much is the mass of the large plastics?

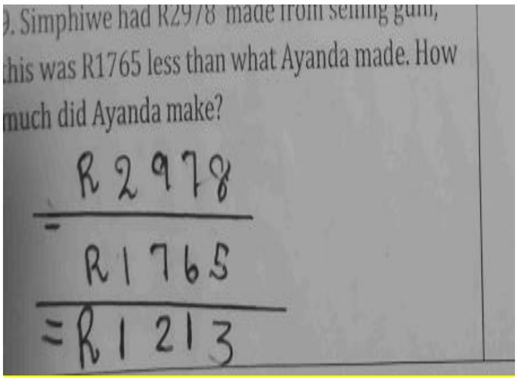
1000+70+1

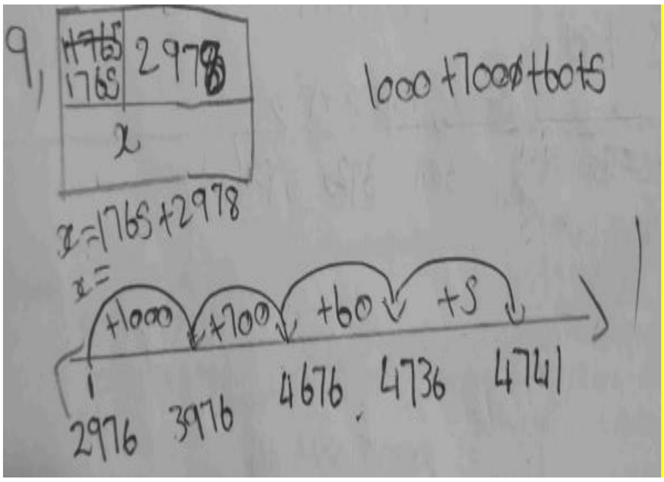
+1000 +70 +1

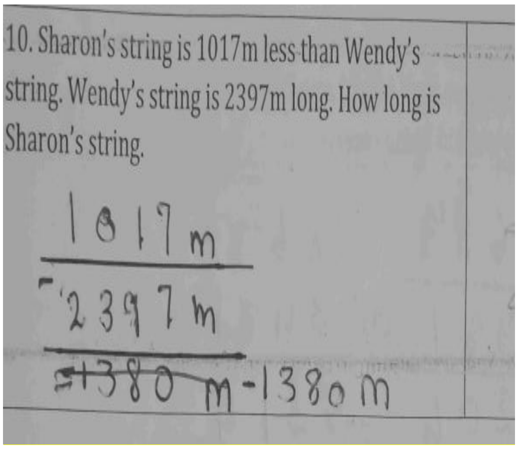
2851 3851 3920 3921

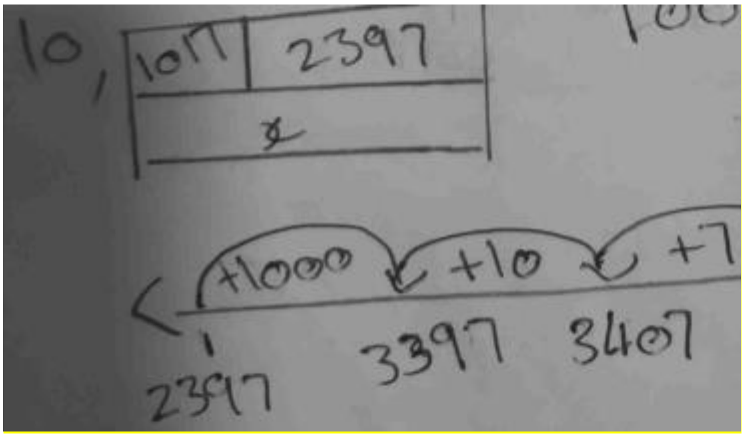
x = 1701 + 2851

x = 4743









5.2.2 Averages for the control group

This section is a discussion of the performance by the control group in the pre-test and posttest. A total of 34 learners wrote the pre-test from the control group and achieved an average of 43% with the highest learner getting a mark of 10 out of 10 and the lowest mark being 0 out of 10. The control group had an average of 47% in the post-test, which suggests a 4% increase

from the pre-test average. The 4% increase suggests that the correct answers produced by learners did not improve by large margins. The highest mark in the post-test for the control group was 9 out of 10 and the lowest mark was 0 out of 10. A total of 34 learners wrote the post-test in the control group. Table 5.4 below illustrates learners' shifts on the pre- and posttests for the control group:

Q#	Pre-test correct	Pre-test correct %	Post-test correct	Post-test correct %	Shifts in correct answers
Q1	23	67,65	19	55,88	-4
Q2	27	79,41	25	73,53	-2
Q3	16	47,06	20	58,82	4
Q4	22	64,71	21	61,76	-1
Q5	11	32,35	16	47,06	5
Q6	9	26,47	14	41,18	5
Q7	11	32,35	12	35,29	1
Q8	11	32,35	16	47,06	5
Q9	9	26,47	11	32,35	2
Q10	8	23,53	8	23,53	0

Table 5.4: Comparison of learners' performance between the pre- and post-test for the control group

Source: Author's own compilation sample

In Table 5.4 above, the first four questions are change increase/decrease problems. This category of problems was inconsistent with regards to performance. They achieved an average of 64.71% in the pre-test and an average of 62.50% in the post-test. This suggests a decline of 2.21% in this category of problems for learners in the control group. Question 5, 6 and 10 are combine problems. This category had an average of 27.45 % in the pre-test and an average of

37.26% in the post-test. This suggests an average shift of 9.81% in learner's correct answers. The last category of problems is question 7 to 9 and these are compare problems. This category had an average of 30.39% in the pre-test and an increased average of 38.23% in the post-test. This suggests an average shift of 7.84% in learners' correct answers. The first category of problems had a decline of 2.21%, while the second category of problems had an average shift of 9.81% - which is bigger than the average shift of the first category. The last category of problems had an average shift of 7.84% of correct answers. For the control group, the second category which is the combine problems had the largest shift in correct answers compared to the other two categories. Table 5.5 below summarises the shifts across the three categories:

Low pre-test performance, **High** post-test performance items				**Mid** pre-test performance, **Mid** post-test performance items				**High** pre-test performance, **Low** final performance items			
Item	Type	Pre-	Post	**Item**	Type	Pre-	Post	**Item**	Type	Pre -	Post
Q5	combine	11	16	**Q7**	compare	11	12	**Q1**	change	23	19

Q6	combine	9	14	**Q8**	compare	11	16	**Q2**	change	27	25
Q1O	combine	8	8	**Q9**	compare	9	11	**Q4**	change	22	21

Table 5.5: Summary of shifts across the three categories for the control group

Source: Author's own compilation sample

The first column shows learners that had a low performance in the pre-test and a high performance in the post-test. This shift in performance was on the combine problems category. It is only question 10 that saw learners have the same performance in the pre- and post-test items. The compare problems category did not have any significant shifts in performance. It is only question 5 that saw an improvement of 5 correct responses. The last column shows learners that had a high performance in the pre-test and performed poorly in the post-test, this is the change problems category.

The following learners reflect the shift in the averages of the pre- and post-test in the control group. These learners exemplify how learners in the control group did not improve significantly but had slight improvements on the combine and compare problems. Learner SM obtained a mark of 4 out of 10 in the pre-test and 5 out 10 in the post-test. In the pre-test, learner SM obtained incorrect answers on questions 5, 6, 7, 8, 9 and10, and these were combine/separate and compare problems. The correct responses were on questions 1 – 4, which are change increase problems. In the post-test, learner SM had correct answers on questions 5, 6, 8, 9 and 10, and these are also combine/separate and compare problems. It was observed that the learner still struggled with the combine/separate and compare problems except for question 7. This suggests that the learner had a slight improvement on the combine problems.

Learner MM obtained a mark of 4 out of 10 in the pre-test and 5 out of 10 on the post-test. This suggests a slight improvement in the learner's marks. The incorrect responses that the learner got in the pre-test are from questions 5 – 10, and these questions include combine/separate and compare problems. The first four questions were change increase/decrease problems and they were correct. In the post-test, the learner was able to get the first three questions and the last question correct. The incorrect responses were on questions 4, 5, 7, 8 and 9. The learner got the same questions incorrectly both in the pre- and post-test, except for question four and ten in the post-test.

This shows no change, and it can be argued that the learner struggled with the combine/separate and compare problems in both the pre- and post-tests. The learner also had a decline in performance in the change category problems. Learner MDM obtained a mark of 4 out of 10 in the pre-test and 4 out of 10 in the post-test. The incorrect responses that the learner got in the pre-test were on questions 5, 6, 7, 8, 9 and 10. In the post-test, the learner obtained a mark of 5 out of 10. The learner's incorrect answer in the post-test were on questions 5, 6, 8, 9 and 10. For both the pre- and post-test, the learner got the same incorrect answers. It is only question seven in the post-test that the learner was able to provide correct answers, whereas in the pre-test the learner's answer was incorrect.

5.3 Conclusion

In conclusion, the intervention group's performance significantly improved from the pre-test to the post-test, whereas the control group's performance did not. When it came to the combine and compare problems, learners in the intervention group performed much better than those in the control group. The control group's combine and compare categories performed poorly in both the pre-test and the post-test, remaining unchanged. The many models that learners used, and their corresponding levels of success are examined in the section that follows.

CHAPTER 6: CONCLUSION AND RECOMMENDATIONS

This study had two groups that wrote the same test twice, the control group and the intervention group. The intervention group was introduced to alternative ways of calculating, whereas the control group was not. The control group had an average performance of 43% in the pre-test and an average of 47% in the post-test. On the other hand, the intervention group had an average of 46% in the pre-test and a 73% average in the post-test. This was an average improvement of 27% for the intervention group, which was a significant improvement in performance compared to the 3% average improvement by the control group. The significant shift in performance from the intervention group can be attributed to the intervention sessions that the group had.

This study has also demonstrated that even in a brief intervention like this one based on 6 lessons, learners can retain some knowledge. It was discovered that learners use different models to solve additive word problems, some used single models, and some used a combination of models. In both groups, the column model was the most used model, except in the post-test of the intervention group. Where the most used model was a combination of the bar model and number line. Most crucially, it was discovered in this study that when learners use the column model to solve challenging word problems, it is incorrect and that utilising this model to tackle more challenging word problems has a very low success rate. Moreover, using relational models like the bar model and number line helped learners to better understand additive relationships. The combination of the bar model and number line assisted learners to efficiently solve word problems, with the bar mode helping learners to correctly model the equation and choose the correct additive inverse, the number line helped learners to complete the calculation modelled from the bar model. The two models complemented each other in this regard.

Moreover, encouraging the use of the column model in schools seems to be detrimental in that the average success rate associated with using this model in the combine problems was 31.2% and 34.2% for the compare problems, this was in the pre-test of the control group. In the posttest of the control group, the average success rate associated with the use of the column model was 39.7% for the combine problems and 34.9% for the compare problems. In the pre-test of the intervention group, the average success rate associated with the use of the column model was 38% for the combine problems and 4% on the compare problems. However, the success rate associated with the use of the number line and bar model was 68.1 % on the combine problems and 64% on the compare problems, this was in the post-test of the intervention group. There is a 30.1% difference in the success rate of the combine problems and a 60% difference in the success rate of the compare problems. When comparing the success rate associated with the use of the bar model and number line, to the success rate associated with the use of the column model on the combine and compare problems, the results are noteworthy. The success rate of the bar model and number line had an increased success on the combine and compare problems, which was not the same as the column model. However, the use of the column model did not have a significant drop in the combine and compare problems, this was for both groups except the post-test of the intervention group. This could be because the column model is introduced from grade 4 and it is the prescribed model of use.

Finally, with literature identifying the compare and combine class of problems as being the most difficult to work with where learners are concerned, this study reveals that a bigger proportion of learners mathematised to arrive at the correct answer when using the bar model and the number line as compared to other models. In the post-test of the intervention group, an average of 25 learners that used the bar model and number line to answer the combine problems did so successfully. Moreover, an average of 24 learners that used the bar model and number line to answer the compare problems did so successfully. This is substantiative considering that there was a 38% success rate on the combine problems and a 4% success rate on the compare problems in the pre-test of the intervention group.

The column model ,horizontal number sentence, word sentence and a combination of the word sentence and column model were the models used by grade seven learners to analyse word problems before the intervention. The column model was the dominant model across both groups and for both test sittings, except in the post-test of the intervention group. Learners in the intervention group performed better in the post-test compared to the learners in the control group across the two test sittings. The shift in and improvement in performance was attributed to the relational models that learners in the intervention group used for the post-test.

This study reveals that a bigger proportion of learners mathematised to arrive at the correct answer when using the relational models (bar model and the number line)as compared to other models, this aligns with my theoretical assumption that relational models can support the sense-making and efficient calculation necessary to arrive at the correct answers. The relational models provided learners with the ability to make sense of the additive relationships, learners were able to interpret the additive word problem and choose the correct operation. Learners were able to create graphical representations of the relationships between the two quantities involved. This shows that they were working relationally as the relational paradigm, integrates two important elements that are involved in solving additive word problems, interpreting the word problem and choosing the correct additive symbol. This is evident in the high performance that learners in the intervention group achieved in the post-test compared to the other test sittings.

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APPENDIX A: THE TEST

Test

Use the bar model and number line to solve the following problems

Pre-Test

Name :______________________________

Use the bar model and number line to solve the following problems

TEST QUESTIONS FOR LEARNERS

1. Sam has 1250 sweets and sold 1184 to his school mates on fun day. How many sweets does he have now?	
2. Maria had R4381 in his bank account. Her father gave her R3572. How much does Maria have now?	
3. 2378 turtles swam alongside 5698 dolphins. How many turtles and dolphins were swimming altogether?	
4. On Wednesday Thabo made a profit of R1578 from selling , on Thursday he made R1365. How much profit did Thabo make in the two days?	

5. Jadon scored 2351 points in a game. Sam scored 1879 points in the same game. How many more points did Jadon score than Sam ?

6 . Pule saved R4813. Rea has R3891. How much more does Pule have than Rea?

7. Themba received 8693 shopping points, these were 3487 less than what Zama received .How many points did Zama receive?

8. The mass of the small plastics is 2851 grams less than the mass of large plastics. The mass of the small plastics is 1701 grams. How much is the mass of the large plastics ?	
9. Simphiwe had R2978 made from selling gum, this was R1765 less than what Ayanda made. How much did Ayanda make?	
10. Sharon's string is 1017m less than Wendy's string. Wendy's string is 2397m long. How long is Sharon's string.	

APPENDIX B: CONSENT FORMS

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

June 2022

Dear Principal

Research project

My name is Karabo Mochosa and I am a master's student in the School of Education at the University of the Witwatersrand. I am doing my research project to investigate how learners in grade 7 analyze additive word problems. I would like to administer a mathematics test to your 7 learners (two classes). The purpose of the test is to investigate how learners in grade 7 analyse additive word problems. The test results will assist teachers at the school with regard to teaching strategies on word problems . The test is scheduled for 20 - 40 minutes. I realize the potential complications of collecting data in the context of COVID-19. However, l expects that the standard COVID protocols which your school has in place will be adequate for administering the test. I do not expect learners to prepare for the test. The learners will write the pre- and post-test. The pre-test will happen early in the current year and the post-test towards the end of the year. The learner's math teachers will have access to learners' scripts. The learners will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission for me to use their test responses at any time, during or after the test/s, and without any penalty. There are no foreseeable risks in participating in this study. Learners will not be paid for their participation, and the names of the learners and identity of the school will be always kept confidential and in all academic writing about the study. If you have any question about this research, feel free to contact me on the details listed below. The data collected from this research project will be stored in a secure location for five years. With your permission, the data collected from this research project may be used by other researchers in future research to improve the teaching of mathematics in South Africa. If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (NonMedical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Should you agree to participate in this study, I request that you complete the attached permission form and stamp with the school stamp. Please feel free to contact me if you require any further information.
Yours sincerely,

Karabo Mochosa 1436430@students.wits.ac.za 073 377 3882
Supervisor: Herman Tshesane Herman.Tshesane@wits.ac.za/ 082 411 9164

APPENDIX C: PARENTS/GUARDIANS CONSENT FORM

Research Project : Exploring how Grade 7 learners solve additive word problems

Please fill in and return this section to your child's Maths teacher as soon as possible.

School: ______________________________

Your first name and surname: ______________________________

Your child's name: ______________________________

Your child's surname: ______________________________

I give consent for my child to participate in this research project. The research has been explained to me and I understand what my child's participation will involve.

I agree to the following:

(Please circle YES or NO below).

I consent for my child's test responses to be used in the research project

YES NO

I consent that my child's responses may be used anonymously after this project has ended. I understand that it will be used for academic purposes by other researchers and that they must obtain their own ethics clearance to use it.

YES

NO

Informed Consent

I understand that:

- My child's name and information will not be made public in research and presentations related to this study
- I can withdraw my consent at any time.

All the data collected during this study will be destroyed 5 years after the completion of the

APPENDIX D: SCHOOL PERMISSION FORM

wits maths connect supporting secondary maths

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

SCHOOL PERMISSION FORM

Research project: Exploring how Grade 7 learners solve additive word problems

Researcher: Karabo Mochosa

I hereby give permission for data to be collected at our school during June/ July 2021 for the purpose of the above research study.

I understand that:

- The school's name and the names of participating learners will not be revealed in any research and writing that is presented or published.
- All the data collected during this research study will be destroyed 5 years after completion of the project.

School name:	
School/principal's email	
Principal's name:	
Signed:	
Date:	

School stamp

Exploring how Grade 7 learners solve additive word problems June 2022

Dear Parent/Guardian

My name is Karabo Mochosa and I am a master's student in the School of Education at the University of the Witwatersrand. I am doing my research project to investigate how learners in grade 7 analyse additive word problems.

I have requested and been granted permission from your school to conduct this study. Since the school is participating, your child's teacher will require him/her to write a math's test in June/ July. I would like to request that you allow us to use your child's test responses as part of the research project.

The test will take approximately 20 – 40 minutes and will be written at a time agreed with the school. Your child is not expected to study for the test. The school's standard COVID protocols will apply when the test is written to ensure that there is no compromise of your child's safety.

There will be no personal costs to your child if she/he participates in this project (that is, if we are allowed to use his/her test responses in our research). Your child will not receive any direct benefits from participation but there are no disadvantages or penalties if you do not allow us to use your child's test responses. You may withdraw your permission at any time.

Your child's teacher will know the test responses, and this is important for supporting your child in his/her learning of mathematics during the year. I will not share the details of your child's test responses with anyone outside my supervisor. I will use a pseudonym (false name) if I refer to your child's test responses in any writings about this research.

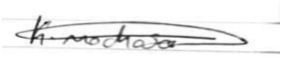
If you have any question about this research, feel free to contact me on the details listed below.

The data collected from this research project will be stored in a secure location for five years.

If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnonmedical@wits.ac.za

Your child has also been given an information sheet and consent form. At the end of the day it is your child's decision to participate in the research.

Yours sincerely,



Supervisor: Herman Tshesane Herman.Tshesane@wits.ac.za/ 082 411 916