FRACTURE OF PLAIN CONCRETE - A COMPARATIVE STUDY OF NOTCHED BEAMS OF VARYING DI

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A thesis submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Doctor of Philosophy

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DECLARATION

I declare that this thesis is my own, unaided work. It is being submitted for the degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

M. G. Alexander

24th day of September 1985

ABSTRACT

This thesis concerns the fracture of plain concrete notched beams. Its theme is the importance of understanding fracture processes in order correctly to evaluate parameters describing fracture behaviour. The major purpose of the work was to study the behaviour under load of notched beams of varying depth, including the nature and development of microcracking, the influence of macro- and microcracking on deformations, material characterisation with reference to microcracking, and the ability of various parameters to describe fracture behaviour and strength. A broad approach has therefore been adopted.

Centrally notched beams of width 100mm, depth ranging from 100mm to 800mm, and with span/depth ratio of four were tested. Notch depth ratios of 0,2, 0,25, 0,4 and 0,5 were used. Measurements were made of mid-span load and deflection, visible crack depth, strains across the fracturing section, and longitudinal ultrasonic pulse velocities. Ordinary portland cement and dense mineral aggregates were used for the concrete.

During fracture, the main crack did not extend until an extensive microcracked zone had developed ahead of it, and reached a maximum size. Thereafter, the depth of the tension zone was roughly twice that of the compression zone. Slow crack growth before maximum load amounted to 10 to 25 per cent of notch depth for larger beams, and up to three times this amount for 100mm deep beams. Relative microcracked zone depths increased with crack growth, and were initially about 40 to 47 per cent of residual beam depth, the larger beams obtaining the higher values. Relationships between various zone sizes were characteristic of material fracture behaviour.

A mean value of the Linear Elastic Fracture Mechanics parameter K_c was approximately 1,5MPa$\sqrt{m}$ for beams of depth 200mm and greater, and 0,96MPa$\sqrt{m}$ for 100mm beams which were notch-insensitive. Fundamentally, K_c could not be regarded as a valid fracture criterion, due to the presence of an extensive non-linear zone. Stresses in microcracked zones were significantly higher in larger beams, thus imparting higher fracture toughnesses. The notch-insensitivity of 100mm beams was interpreted as resulting from their inability to develop appreciable fracture zone stresses. A preliminary study of R-curves revealed aspects of the fracture process such as the stress-intensity factor required to develop the microcracked zone, and the energy-absorbing capacity of this zone.

The overall conclusion was that fracture in notched beams is governed by the presence of an extensive microcracked zone. The nature of this zone influences fracture processes, and therefore also the measured fracture parameters.

Dedicated to the memory of

my father and mother,

Daniel and Yvonne

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I acknowledge with sincere gratitude the help of the following people, without whose assistance and support this thesis could not have been completed:-

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Finally, it is with real joy that I acknowledge the grace of the One who said: "I am the resurrection and the life. He who believes in me will live... and whoever lives and believes in me will never die". To Him alone be praise.

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LIST OF SYMBOLS

The symbols below are those most commonly referred to in the text. Other less commonly used symbols, or modifications of the symbols, are defined where they occur in the text.

A	Total aggregate content (by mass)
A_c	Cross-sectional area
A_f	Area of fracture surfaces
$A_{F,\delta}$	Area under load-deflection curve
b	Beam width (breadth)
c	Cement content (by mass)
	Crack depth (or crack half-depth)
c/d	Crack depth ratio
c_{eff}	Effective crack depth
c_0	Initial notch depth
c_0/d	Initial notch depth ratio
CMD	Crack mouth displacement
COD	Crack opening displacement
d	Overall beam depth (or height)
d_{NA}	Height of neutral axis above lower face of beam
δ	Deflection
δ_{Fmax}	Deflection corresponding to maximum load
δ_m	Deflection at zero machine load
δu	Deflection at ultimate failure
E	Elastic modulus (Young's Modulus)
E_c	Static concrete elastic modulus
ε	Strain
F	Load
F_{max}	Maximum load
f_{cu}	Cube compressive strength
f_r	Modulus of rupture
f_t	Tensile strength

G	Strain energy release rate
G_c	Critical strain energy release rate (crack driving force)
γ	Fracture surface energy
γ'	Effective fracture surface energy
h	Residual ligament depth (of beam)
h_0	Initial residual ligament depth
I	Second moment of area
K	Stress-intensity factor
K_c	Critical stress-intensity factor (fracture toughness)
k_0	Fracture zone size coefficient
k_1	Neutral axis position coefficient
L	Overall beam length
λ	Compliance of beam
M	Bending moment
M_R	Moment of resistance
μ	Coefficient of friction
ν	Poisson's ratio
p_{avg}	Average stress in fracture zone
S	Major support span of beam
σ	Stress
σ_n	Net stress at root of notch
σ_t	Tensile stress
t	Ultrasonic pulse transit time
w	Water content (by mass)
W_c	Unit weight of concrete
z_{nl}	Depth of non-linear (microcracked) zone

1 INTRODUCTION

1.1 FRACTURE AND DEFORMATION IN CEMENTED MATERIALS.

Materials engineers are required to acquaint themselves thoroughly with the properties of the materials they are using. They must understand the behaviour of a material under varying conditions of load and environment, and be able to predict its response to a set of operational circumstances that may be difficult to model in a laboratory. Of the host of different material properties, two are most often of concern to the engineer : these are the strength of the material, and its deformational response under load. The important engineering properties of a material can best be obtained by subjecting it to carefully designed tests, and analysing the results in terms of a mathematical model which can itself be tested to determine the limits of its applicability. Most commonly, the mechanical testing of materials involves measuring load and deformation, and then converting these basic measurements into forms such as stress - strain curves, fracture toughness - displacement curves, and strain - time curves amongst others. Where necessary the effects of time, temperature, strain rate, and environment must be included in the analyses. The mathematical models provide the basis for predictive methods and also serve to characterise particular materials as distinct from other materials. Of late, structural analysis has taken tremendous strides with the advent of advanced numerical techniques, and it is necessary that the material models describing the constitutive relationships should keep pace.

The general subject of this thesis has to do with fracture properties of cemented materials, and of portland cement concrete in particular. This is a very broad topic and only selected aspects will be dealt with, but it is important at the outset to appreciate the inherent link between fracture, deformation, and associated load. For example, considering the growth of a crack in a cemented matrix or at a matrix-aggregate interface, the crack will progress provided that a mechanism exists for separating the atoms in the path of the crack, and that sufficient energy is supplied to satisfy the energy requirements. The first condition must involve a localised stress concentration resulting in deformation by way of the opening of crack faces, while the second condition must involve an externally applied load coupled with a global deformation in order to perform work and so produce various types of energy in the material (strain energy, surface energy, and so on). Thus fracture cannot occur without deformation, and the stress and time dependence of the deformation critically affect fracture strength.

Other examples may be quoted to show the inherent link between deformation and fracture. Considering concrete or cement paste, we now know that the material has numerous crack-like flaws and defects even before any external load has been applied. These are due to internal stresses set up by thermal and shrinkage effects, and due to the inherent microstructure of the paste phase. On applying external load the pre-existing flaws initiate microcracks which grow in the material under increasing load. This internal microcracking results in damage to the material which affects its strength and deformation properties. The degree of microcracking will affect not only the ultimate strength of the material but also its service behaviour such as fatigue resistance or the possibility of crack propagation under sustained loads leading to creep rupture. The damage occurring in concrete due to external load results in another important phenomenon - that of permanent deformation. Permanent deformation may also result from viscous movements such as shrinkage and creep, but with very different mechanisms operating in these cases. Nevertheless, the energy resulting in plastic or viscous work again links load with deformation, and creep must vitally influence fracture strength. Coupled with this is the question of strain rate effects on fracture, and once again this can be viewed in energy terms. For instance there is evidence to show that cemented materials tend to fail according to a limiting strain criterion which implies that fracture occurs when a limiting deformation has been reached. In a composite material such as concrete, this limiting deformation will be influenced by interface bond strength between aggregate and paste, which in turn also affects the stiffness of the material.

It is clear therefore that fracture and deformation of materials are closely interrelated, the application of external load playing an important part in both processes. Load and deformation are further linked through the concept of work or energy, and the energy approach to failure of materials has great value in synthesising the many different approaches to fracture, fatigue, creep, and so on that currently exist in sometimes very separate and unrelated compartments. If materials engineers can succeed in the future in bringing together these different approaches under a uniform framework such as that of energy, then a significant service will have been done to all who seek to understand material behaviour better, and apply this to improved use of materials.

1.2 DEFINITION OF MATERIALS TERMS

The material examined in the present work was concrete, but the following general definitions apply for this thesis:-

CEMENTED MATERIALS

Materials based on portland cement as a binder, and comprising mixtures of portland cement powder and water, and mineral aggregates of natural or artificial origin.

HARDENED CEMENT PASTE

The material resulting from chemical combination or hydration of portland cement powder and water, which results in an amorphous and microcrystalline gel-like substance having adhesive and cohesive properties.

MORTAR

A cemented material comprising an intimate mixture of hardened cement paste and fine aggregate particles of maximum size 4,75mm.

CONCRETE

A cemented material comprising an intimate mixture of hardened cement paste and fine and coarse aggregates, where the size of the coarse aggregate particles exceeds 4,75mm. The material may be regarded as coarse aggregate particles embedded in a mortar matrix.

1.3 OBJECTIVES OF FRACTURE STUDIES

The fracture test programme arose from the more general objectives of cemented materials fracture studies, which can be summarised as:-

1. The development of a design method based on energy considerations and fracture mechanics principles. At present direct design applications of fracture mechanics to cemented materials are lacking. However, the energy/fracture approach has the promise of allowing an integrated design method encompassing elastic, viscous, plastic, and fatigue effects within one framework.

2. The establishment of criteria and procedures for valid fracture testing. One of the most vexing problems in the concrete

fracture mechanics field has been the question of the validity of fracture mechanics parameters that have been measured. These parameters are affected by a multitude of variables, including intrinsic effects of the mix itself such as mix proportions and types of materials, extrinsic effects such as testing environment and moisture condition, specimen size effects, and influence of microcracking. These problems require resolution before the more general application of fracture approaches to design can be developed. The validity of linear elastic and non-linear fracture mechanics parameters therefore requires continued study.

3. The study of fracture processes and mechanisms in cemented materials, including interface bond effects and the influence of constituent materials. Concrete, by its nature a composite heterogeneous material, is composed of essentially brittle components and experiences cracking and fracture at every level in its structure and during its service life. Such studies will assist in the development of materials or composites with enhanced strengths and fracture toughnesses, thus leading to material economies. A recent example of this has been the development of MDF (Macro-Defect-Free) cement, in which paste voids caused by microscopic air-bubbles and capillary pores have been eliminated with a consequent dramatic improvement in strength. The reasoning which led to MDF cement was based on simple fracture concepts, and shows that the fracture approach has merit in helping to understand material behaviour.

4. The characterisation of different types of cemented materials by a study of their fracture-deformation behaviour. Fracture testing which includes related deformations can assist greatly in assessing the suitability of materials for different applications by evaluating their toughnesses, ductilities, strain capacities and energy - absorbing properties.

Objectives 2 and 4 above represent the major purpose of this thesis, which can be stated as:- "an experimental study of the fracture behaviour of notched concrete beams of varying depth , with particular attention to the influence of cracking on deformations, the nature and development of microcracking during fracture, the characterisation of the material by reference to its microcracking behaviour, and the examination of fracture parameters for their ability to describe fracture behaviour and fracture strength adequately." I shall take the viewpoint that concrete is a highly complex and heterogeneous material; therefore no single or simplistic approach will ever truly suffice for an appreciation of its behaviour or for use in analysis and design. As far as the scope of the thesis is concerned, I have intentionally kept it wide to cover aspects such as fracture mechanisms and processes, as well as evaluation of fracture parameters. If we can understand the fracture behaviour of concrete better, this will give us valuable insight into its basic nature, and therefore allow us to assess design approaches properly. Simplifications in practical situations will often be justified, but will always have limitations. We must not allow the fact that concrete fracture is complex to deter us from a diligent exploration of fracture mechanisms, and to use new knowledge to develop new design approaches.

1.4 ORGANISATION OF THE THESIS

This thesis is divided into four parts, comprising nine chapters in addition to this introductory chapter, as follows:-

PART I FUNDAMENTAL CONCEPTS

This part comprises chapters 2 and 3, and contains a resumé of the background and present status of concrete fracture studies. Literature dealing with previous investigations will be reviewed, and inadequacies and conflicts in present knowledge will be discussed. The fracture mechanics approach to the failure of materials will be outlined, and the various fracture parameters introduced. Applications of the fracture mechanics approach to cemented materials will also be reviewed.

PART II EXPERIMENTAL FRACTURE WORK

This part comprises chapters 4 and 5. Chapter 4 contains details of the fracture beams tested, mixes and materials, experimental procedures, testing equipment, and data collection. Chapter 5 covers results for materials control tests, and basic data from the fracture tests. Certain aspects salient to concrete fracture are also discussed using the basic results; these aspects cover notch-sensitivity, beam deflections at ultimate fracture, and characteristic shapes of load-deflection curves.

PART III CHARACTERISATION OF FRACTURE IN CONCRETE BEAMS:- STUDIES OF CRACK GROWTH AND FRACTURE ZONES

This part comprises chapters 6 to 8, which deal with beam compliance and slow crack growth, and with measurement of fracture zones sizes using strain techniques and ultrasonic pulse velocity techniques. The experimental results are used to develop formulations which describe or characterise the fracture behaviour of concrete when tested in the form of notched beams. The difference between the behaviour of beams of varying depth is highlighted.

PART IV FRACTURE FORMULATIONS AND CRITERIA

This part comprises chapters 9 and 10. Chapter 9 deals with the use of stress-intensity and energy parameters as criteria for concrete fracture, and also discusses fracture zone stresses and the role they play in concrete fracture. Once again a discussion of the differences between beams of varying depth form a major portion of the work. Chapter 10 is a final concluding chapter, and seeks to summarise the experimental work, to assess the implications of the results for concrete as a material and for concrete fracture testing, and to suggest future research needs.

PART I FUNDAMENTAL CONCEPTS

Part I, comprising chapters 2 and 3, will set the background for later discussion of fracture testing and interpretation of results. The historical origins of the fracture mechanics approach to the failure of materials will be sketched. The particular problems associated with the application of this approach to cemented materials will be discussed. A selection of relevant literature will be reviewed so as to highlight some of the inadequacies and conflicts in present knowledge. The various fracture parameters both linear elastic and non-linear will be introduced and critically reviewed. The background thus provided will be used to bring into clearer focus some of the more important aspects of concrete fracture requiring closer study.

2 THE FRACTURE MECHANICS APPROACH TO THE FAILURE OF CEMENTED MATERIALS

Materials science has to do with understanding why various materials behave as they do, with applying this knowledge in the more efficient use of existing materials, and in their improvement to make better materials. Sometimes, this study results in new types of materials being developed, such as the recent introduction of carbon fibres and other types of fibres to reinforce plastics and cement-based matrixes. The study of cemented materials must of necessity draw on the ideas of materials science, which include concepts of chemistry and physics often on the molecular scale, and apply these ideas to the problems of material usage in a macroscopic engineering sense. The bulk of civil engineering construction materials are non-metallic, often cement-based, and these materials provide a singular challenge to the researcher due to their heterogeneity and complex structure. One of the more promising ideas that has come out of physics in recent decades has been the fracture mechanics approach to failure; the application of this approach to the study of failure of cemented materials is still in its infancy but holds promise in helping to explain the fracture mechanisms and to assist in better material design.

2.1 HISTORICAL BACKGROUND

It has long been recognised that the theoretical and actual strengths of materials may differ by several orders of magnitude. For example, table 2.1. shows how the actual tensile strengths of some common materials fall far below their predicted strengths. There is also no definite relationship between chemical and me-

Table 2.1 Some typical values of predicted and actual strengths of materials

	Young's modulus E (MPa)	Predicted strength (MPa)	Actual strength (MPa)	Ratio of predicted to actual strength
Iron	200 000	20 000	205	98
Iron whiskers	200 000	20 000	13 110	1,5
Mild steel	210 [illegible]	21 000	410	51
High tensile steel	21[illegible] [illegible]00	21 000	1 550	13,5
Ausformed steel wire	210 000	21 000	2 100	6,8
Annealed copper	120 000	12 000	140	86
Drawn copper wire	120 000	12 000	550	22
Aluminium	70 000	7 000	70	100
Cement and concrete	15 000	1 500	4	375
Hardwood, along grain	14 000	1 400	100	14
Hardwood, across grain	14 000	1 400	3	467
Bulk glass	70 000	7 000	35 - 170	41 - 200
Parallel fibreglass	35 000	3 500	1 050	3,3
Polyester	1 400	140	3	47
Bone	30 000	3 000	140	21

(Note. The predicted strength is based on a simplified consideration of the forces binding atoms together in a solid, which gives that

$$\text{Theoretical tensile strength } f_t \simeq \frac{E}{10}$$

where E is the Young's modulus of the material.)

chanical strengths of a material. To further complicate the issue, some materials will usually fail in a ductile fashion, but may fail in a brittle way under special circumstances, while other materials are essentially brittle but can at times behave in a ductile way. Classical continuum mechanics does not provide answers to all of these problems, and this is led to the search for other ways of explaining material behaviour.

Early in this century, A.A. Griffith[1] used the concept of energy conservation to relate the strain energy stored in a loaded specimen to the surface energy of the fracture surfaces which are produced when the solid fractures. He also realised that the strengths of common materials were dependent, not primarily on their interatomic bond strengths, but on weakening mechanisms inherent in the material. These weakening mechanisms might be flaws of minute dimensions, or microscopic cracks induced during manufacture, handling, or use.

Griffith proposed that the strain energy is converted into surface energy when a brittle material fails. He showed that the stress σ_t required to just separate two layers of atoms x mm apart in the material was:-

$$\sigma_t = \sqrt{\frac{E\gamma}{x}} \qquad (2.1)$$

where E is the Young's modulus of the material,
and γ is the surface energy of the material.

Using the equation, the strength of ordinary materials is predicted to be about ten to one hundred times greater than their actual bulk strength. During his classic work on the tensile strength of thin glass fibres, Griffith found that the thinner the fibre he tested, the greater was its strength. In fact, the extrapolated strength for fibres of near zero diameter very closely approximated the theoretical strength predicted by his tensile strength equation. It was in demonstrating that the theoretical strength could be approximated experimentally in at least one case that he arrived at his postulate of the stress-raising and weakening effects of material flaws.

Later, Orowan[2] showed conclusively that the more important cracks causing weakening of the material lay on its surface rather than in the interior. Any such flaw causes severe stress concentrations around its tip, and the local bonds become overstressed and break one by one long before the bulk material is severely stressed. The crack runs through the material until total failure occurs, at stress levels far below the theoretical strength. In a brittle material a crack is really a mechanism which enables a weak external force to break even the strongest chemical bonds one by one. Such weakening mechanisms as cracks are easy to visualise in a brittle material. However, not all materials fail by the spread of a crack from some local defect. Another mechanism of fracture is that of plastic flow, where the material fails by flowing in shear. For materials that are brittle at normal temperatures, the stress needed to fracture a material by flow may be very high, and thus materials like glass, ceramics and cement pastes are more susceptible to brittle fracture at these temperatures.

2.2 NECESSARY CONDITIONS FOR FRACTURE

It is possible to identify the physical principles governing the spread of flaws in a material, and express these in mathematical terms. Physically, two conditions must be fulfilled for a crack to propagate. Firstly, it must be *energetically desirable*, and secondly there must be a *molecular mechanism* by which the energy transformation can occur.

The criterion of energetic desirability states simply that a crack will propagate only when the free energy of the system is being continually lowered by the propagation, i.e. the energy released by the propagation is sufficient to form new fracture surfaces.

This criterion leads to a relationship between crack length and critical stress required for propagation, and includes Griffiths's criterion as a special case.

The two types of energy that need to be considered, strain energy and surface energy, are both dependent on crack size, the former decreasing, the latter increasing, with increasing crack depth.* The crack propagates when the release of strain energy more than compensates for the increase in surface energy. A simple analysis using this criterion and considering the Griffith case shown in figure 2.1 gives that the stress required to just cause the crack to propagate, that is the breaking stress or bulk tensile strength of the material, f_t, is

$$f_t = \sqrt{\frac{2E\gamma}{\pi c}} \quad \text{(Griffith formula for plane stress)} \qquad (2.2)$$

where c is defined in figure 2.1 and is a characteristic crack depth.

The assumptions in deriving Griffiths's formula are:-

1. The material is perfectly elastic; in particular, it is linearly elastic right to the point of brittle failure.

2. The volume of destressed material due to the presence of the crack is equal to half the volume of an ellipse of major axis 4c.

3. The surface energy of the material is a linear function of the crack depth c.

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* The term "crack depth" is generally used throughout this thesis to indicate that cracks in notched beams are vertically orientated.

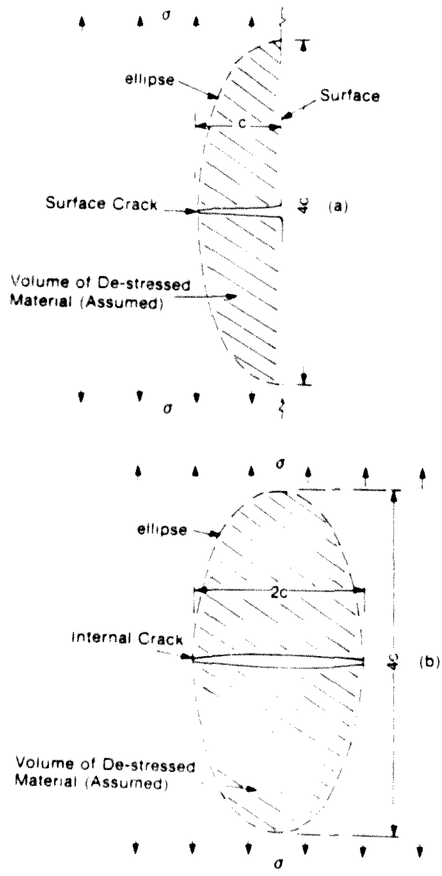


Figure 2.1 Griffith case for (a) surface crack, and (b) internal crack

Equation (2.2) is best explained by means of figure 2.2 in which the surface energy requirement and the elastic strain energy release are plotted as functions of c.

The main points are summarised below:-

1. The release of strain energy is proportional to the square of the crack depth, while the surface energy increases in direct proportion to the crack depth. Therefore, the rate of release of strain energy increases with increasing crack depth, while the rate of demand of surface energy is constant.

2. A shallow crack consumes more surface energy than it releases strain energy, until a critical crack depth c_1 is reached, called the point of instability, where the crack begins to release more energy than it consumes. Stable crack growth occurs for $c < c_1$, whereas unstable spontaneous crack propagation occurs for $c > c_1$. The point of instability represents a condition of critical release of strain energy.

3. There is a critical Griffith crack depth, often very small, for each stress level in the material.

At engineering levels of stress, all but the smallest cracks have an energetic incentive to propagate. The question is, do they have a mechanism for doing so? Is there some way of actually converting the energy from one form into another? This leads to the fact that, while the above criterion is *necessary* for fracture, it is not always *sufficient* to ensure fracture It becomes sufficient when the task of parting the atoms at the crack tip is accomplished by some form of molecular mechanism. What distinguishes a tough from a brittle material is the mechanism for implementing the energy change.

The molecular mechanism for crack propagation is the stress concentration. At the tip of a crack, the ratio between the maximum local stress σ_{max} and the bulk stress σ is given by K, the stress concentration factor, as follows:-

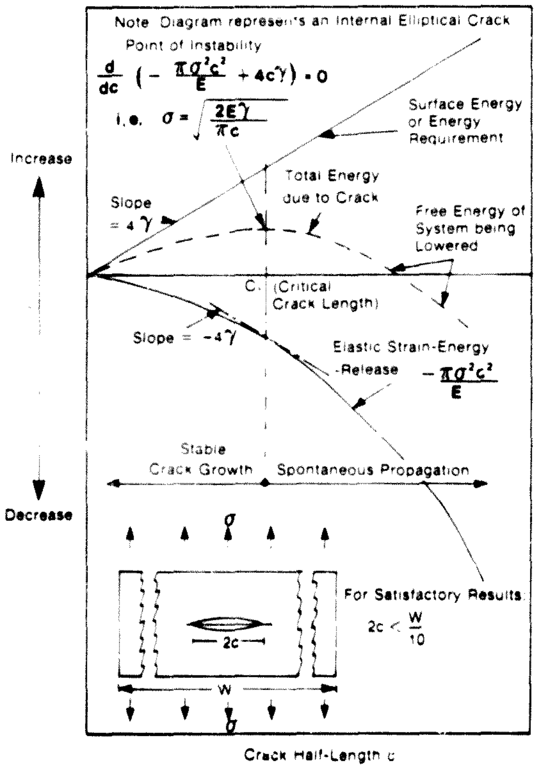


Figure 2.2 Crack propagation for the ideal Griffith case

$$K = 2\sqrt{\frac{c}{r}} \qquad (2.3)$$

where c is defined in figure 2.1, and r is the crack tip radius. In a typical brittle material, r remains constant whatever c is. As the crack grows, c increases, the stress concentration increases, and the energy balance becomes more and more favourable for rapid crack propagation. Therefore, provided the mechanism for crack propagation exists, which is usually the case in brittle materials, the crack rapidly accelerates to failure.

The two necessary conditions for fracture discussed above have given rise to two broad approaches to fracture analysis:- the energy balance approach, and the stress-intensity factor approach. The first approach represents the concepts of Griffith and their extensions, and is usually characterised by the strain energy release rate parameter G, previously introduced with reference to figure 2.2. The second approach was originally developed by Irwin[3], and utilises the parameter of stress-intensity factor, K. This factor describes the elastic stress field in the vicinity of a crack tip and its value is dependent on the applied loading. Chapter 3 will deal with the mathematical basis for determining G and K. Suffice to say that Irwin demonstrated that there was an identity between G and K, showing that if fracture may be characterised by the attainment of a critical strain energy release rate G_c, then this is equivalent to characterising the fracture event by the attainment of a critical stress environment, i.e. a critical stress-intensity factor K_c. Therefore K_c is essentially a measure of the strength of the material containing a flaw or crack, and is sometimes called "fracture toughness".

2.3 EVALUATION OF THE GRIFFITH APPROACH

Assuming random distribution of flaws and cracks of microscopic size in a material, the Griffith approach predicts that the material will fracture at a critical stress level with a corresponding critical energy level, both dependent on a characteristic flaw size for the material. The Griffith Formula (equation 2.2) has become the starting point for the engineering mechanics of fracture toughness, since it leads to a property of the material describing its fracture toughness in the presence of flaws and stress-raisers. Reasons in favour of Griffiths's concept are:-

1. It yields a correct functional relationship between stress at fracture and critical flaw size ($\sigma\sqrt{c}$ = constant).

2. It predicts a theoretical cohesive strength for materials of the right order of magnitude (0,1E), verified by experiments on whisker crystals.

Limitations of Griffith's concept are:-

1. Strictly it is valid only for ideal elastic homogeneous brittle materials.

2. A suitable value for γ is often elusive.

3. No crack-stopping processes are assumed, nor are local inelastic deformations at the crack tip allowed for.

4. It represents an oversimplification of a series of much more complicated phenomena.

The equivalence between the energy balance and stress-intensity factor approach to fracture mentioned previously is strictly true only where very limited plastic deformation occurs before fracture. Such plastic deformation may include the usual plasticity associated with metallic materials, as well as "pseudo-plasticity" associated with microcracking and heterogeneity in certain non-metals. It is the absence of the ideal conditions assumed in the linear elastic approach that has led to some of the difficulties of applying the classical formulations directly to cemented materials of construction. This point receives further attention below.

2.4 THE EFFECTIVE FRACTURE SURFACE ENERGY CONCEPT

Reference was made above to the difficulty of obtaining a suitable value for γ for use in the fracture equation (2.2). Griffith envisaged γ as being the pure free surface energy resulting from the creation of new fracture surfaces. The material is assumed to be homogeneous and linear elastic right to the point of failure, i.e. a perfectly brittle material. This is not generally true since most materials exhibit some ductility (plastic or pseudo-plastic deformation) before failure. The energy balance then actually exists between the elastic energy release and the total work done in propagating the crack as pointed out by Blight[4]. This total work includes the surface free energy of the fracture surfaces, as well as the plastic work irrecoverably consumed at the crack tip during propagation, and any other energy-absorbing mechanisms unavoidably linked with fracture[5,6]. Mathematically, we can write:

$$\gamma' = \gamma + \gamma_p \tag{2.4}$$

where γ' represents the *total work of fracture*, and is called the *effective fracture surface energy*; γ represents the free surface energy of the material; and γ_p represents plastic work plus any other work required for fracture. With γ' substituted for γ in equation (2.2), $\sigma\sqrt{c}$ = constant shows good agreement with experimental evidence for both metals and non-metals[7]. Usually, $\gamma_p \gg \gamma$, particularly for metals, and this shows up the inadequacy of using the pure free surface energy approach. Depending on the ability of the material to deform non-elastically, we can write $\gamma' \approx \gamma_p$, and changes in surface energy often have very little effect on fracture toughness. Brittle materials with their lack of plasticity cannot relieve localised high stress concentrations at crack tips, and are less tough than ductile materials. Hence, fracture toughness is best defined as the resistance of the material to crack propagation.

2.5 FRACTURE OF CEMENTED MATERIALS

2.5.1 MODELS FOR COMPOSITE HETEROGENEOUS MATERIALS

Concrete presents a good illustration of the effective fracture surface energy concept. It is a heterogeneous brittle material whose strength derives almost directly from its heterogeneity and the phenomenon of interface and matrix microcracking. Fracture surface area and energy can refer to the hardened cement paste, the aggregate, or the aggregate-paste interface. Cracking in concrete is not limited to one critical crack, but involves extensive microcracking in the highly stressed zone ahead of and surrounding the main crack*. An important consequence of this is that the area of newly formed fracture surfaces may be many times larger than the nominal fracture area*. The weak bond in concrete is at the aggregate-paste interface. Cracks form here where the energy demand is least and then spread, eventually bridging the matrix between aggregate particles. While the behaviour of the hardened paste, the aggregate, and the aggregate-paste interface is essentially linear and brittle, the effect of combining these brittle components is to create a material whose composite behaviour shows a marked degree of ductility. Fracture occurs ultimately in all the various phases, and the fracture energy term must therefore apply to the composite material rather than to discrete phases. The aggregates in concrete modify the three-dimensional stress field and impose an arresting action on the cracks, causing the fracture resistance of concrete to be greater than that of an equivalent paste[10]. In addition to microcracking of the paste, cracks meander around aggregate particles, and extensive side cracking occurs. The crack-arresting function of the aggregates and the mechanism of microcracking give to concrete a property of ductility or pseudo-plasticity, since these phenomena increase the energy-demand for crack propagation.

The effectiveness of the composite nature of concrete in imparting ductility to its stress-strain behaviour can best be illustrated by reference to Glucklich's model for the fracture behaviour of a non-homogeneous material[11]. Referring first to the Griffith case of equation (2.2), the equation can be interpreted as a term for surface energy demand rate 4γ and a term for strain energy release rate $2\pi\sigma^2 c/E$, these rates being expressed in terms of crack depth c (see figure 2.3).

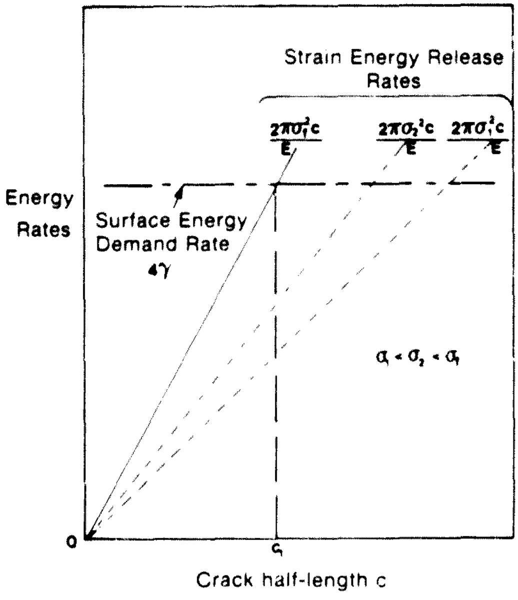


Figure 2.3 Energy rate curve for the ideal Griffith case

This ideal linear brittle material has a constant energy requirement (4γ), while the energy release rate increases linearly with crack depth c for a particular stress. This is shown in figure 2.3. A material with a crack of depth c_1 under increasing stress would remain stable until the stress level reached σ_f, at which the energy balance for spontaneous crack propagation is satisfied, and fracture would occur. The energy balance for continued crack propagation is then possible even under decreasing stress. This ideal case is probably never obtained in practice, and hence Glucklich's model becomes useful in extending the concept to real materials. This model allows us to understand the Griffith fracture concept as applied to construction materials such as concretes, mortars, asphalts, cements, soil-lime and other stabilised materials. The surface energy term γ is difficult to interpret, since it might include some plastic flow, or be modified by multiple microcracking, or refer to fracture in a bi-or multi-phase material. Figure 2.4 represents the probable energy balance for a crack propagating in such a material. As the crack grows, so too does the area of the highly stressed zone immediately ahead of the crack tip, where for non-ideal brittle materials, extensive microcracking can occur in addition to the main crack. The energy-requirement curve is therefore an increasing function of c and no longer a straight line. It is conceivable that the curve becomes linear at point A, where the highly stressed zone reaches its maximum size, determined amongst other things by the geometry of the specimen.

Consider an initial flaw or crack of depth c_0 in a material under macroscopic tensile stress σ_0. The crack begins to grow but is checked at c_1, because the energy demand has now increased. The stress must be raised to σ_1 before the crack continues to grow to c_2, where it is again checked by the increase in energy demand. This process is repeated under increasing stress until the crack reaches point A, which represents the point of instability, or the critical energy level. Both energy-requirement and energy-release curves reach their critical slopes at this point, and since there is no further increase in slope of the energy-demand curve, any increase in crack depth causes unstable crack propagation to occur. Of course, the process is not stepped as in figure 2.4, but continuous under increasing stress, shown by the dotted line in the figure. Figure 2.4 uses the concept of γ', which varies with crack depth, and is significant only at the onset of instability.

Consider also figure 2.5, which is a re-plot of figure 2.4 in terms of energy *rates*. The energy demand rate increases rapidly and reaches a uniform value at A. Slow crack growth occurs under increasing stress, and no unstable crack propagation can occur on the ascending portion of the energy demand rate curve. At point A, spontaneous crack propagation can begin and proceed even under decreasing stress. Figure 2.5 clearly explains the process of slow crack growth in a notched beam under increasing stress, as discussed in more detail in a later section.

The model discussed above could apply equally to a non-homogeneous or a homogeneous material, provided a mechanism exists to cause energy demand to grow with increasing crack depth. Referring specifically to a bi-or multi-phase material such as concrete, the above concepts can be extended to show the effectiveness of the crack-arrest mechanism provided by aggregates and aggregate-paste interfaces. Such materials consist of two or more distinct constituents or phases, each having its own energy-requirement curve represented by different slopes, such as in figure 2.6. The curve is stepped, the steep portion representing the energy demand of the "tougher" phase of the material. A crack of depth c_0 propagating in the weaker phase would progress under stress σ_0 to a depth c_1. At this point it intersects the tougher phase of the material, represented by a sharp rise in energy demand. The crack is checked and will only propagate further if the stress is raised sufficiently to overcome the energy demand of the tougher phase. This phase acts to arrest the crack, and causes the toughness of the composite material to increase. This crack-arrest mechanism does not necessarily have to be a discrete phase in the material, but could also be interfaces the crack must intercept in order to progress, such as the aggregate-paste interface in concrete. This is also the basic concept of a material like fibre-glass in which the two phases themselves are fairly weak and brittle, but when combined give a tough material whose toughness and strength stem directly from

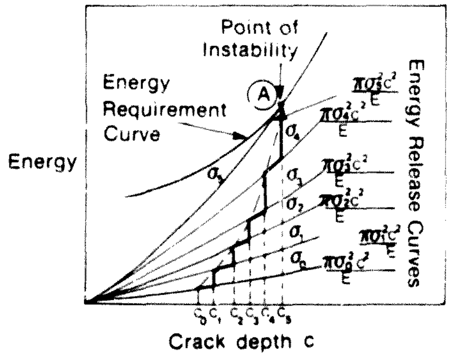


Figure 2.4 Crack propagating in a non homogeneous brittle material under monotonically increasing tension (from ref. 11)

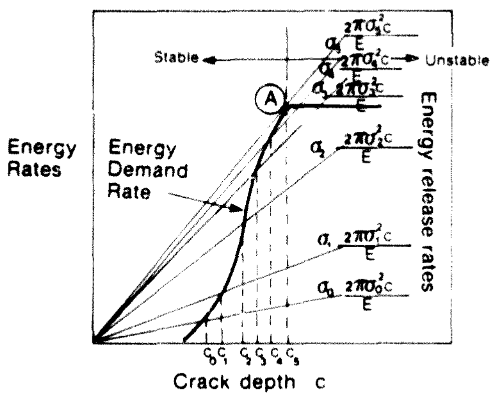


Figure 2.5 Energy rate curve corresponding to figure 2.4

interface crack-stopping. Referring again to concrete, its fracture toughness and pseudo-plasticity are due to the combined effects of increase of microcracked zone size as the main crack grows and of interface crack-arrestors.

On a macroscopic level, the effect of aggregate-paste interfaces, microcracking and crack-arrest is to give to concrete a stress-strain curve that is non-linear, the slope of which is governed at any point primarily by microcracking. This is shown in figure 2.7, in which the curve has been divided into four regions[12], according to the type and degree of microcracking present. It is obvious that a considerable amount of progressive cracking takes place as the load is increased, and this has been borne out by other investigators[8]. Cemented materials such as hardened cement paste (hcp), mortar and concrete can be thought of as essentially two-phase materials in which hard strong particles are embedded in a softer and weaker matrix: in the case of hcp, unhydrated cement grains are embedded in a gel-like matrix of calcium silicate hydrates, while for concrete, coarse aggregate is embedded in mortar. It has been possible to model the process of crack growth in these materials with the use of computer simulations, and the results clearly show the tendency of cracks to initiate at weak interfaces and eventually grow through the matrix phase[13]. Figure 2.8 shows the results of computer crack simulations; for materials such as lightweight aggregate concrete and high strength concrete, it is possible for cracks to propagate through the aggregate particles, and this leads to a more linear stress-strain curve.

2.5.2 REVIEW OF FRACTURE MECHANICS APPLICATIONS TO CEMENTED MATERIALS

This section reviews some of the work done in applying the fracture mechanics approach to civil engineering materials. Direct applications of this approach to real problems are scarce, and sometimes

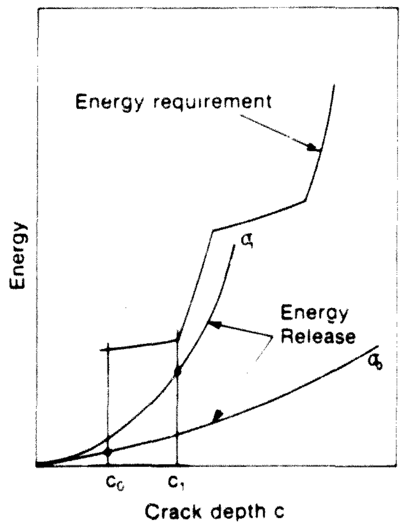


Figure 2.6 Crack progressing in a bi-phase material (from ref. 11)

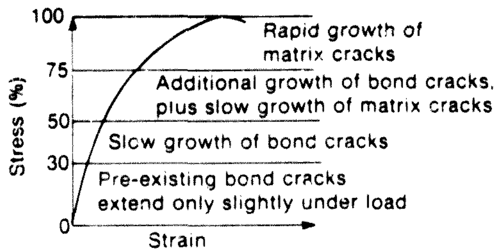
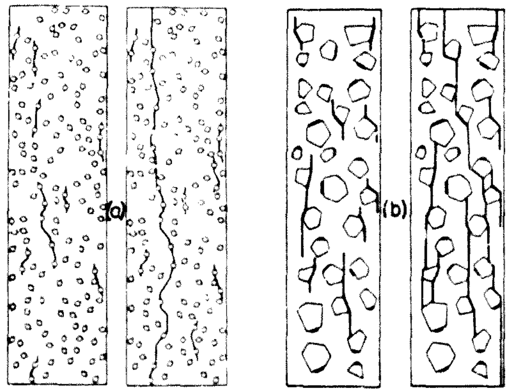
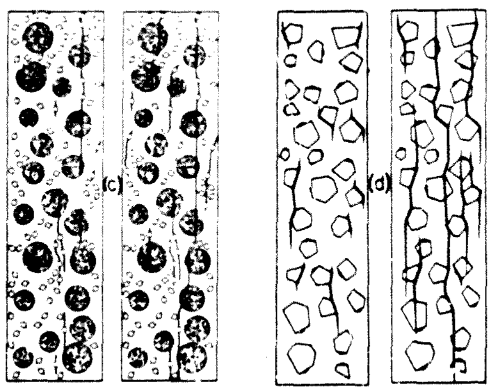


Figure 2.7 Influence of type and extent of crack growth on stress-strain curve for concrete (from ref. 12)



Hardened cement paste: cracks bridge between pores

Normal concrete: cracks go around aggregates



Lightweight concrete: cracks go through aggregate

High strength concrete: some cracks go through aggregate

Figure 2.8 Crack patterns in hardened cement paste and concrete under compressive loading produced by computer simulations (from ref. 13) the basis of the application is questionable. Materials considered will be concrete and cement mortars, soil cement, and other pavement materials.

PORTLAND CEMENT CONCRETE, MORTAR, AND PASTE

There are a number of experimental observations regarding the behaviour of concrete, mortar and hardened cement paste under load that suggest that a fracture mechanics approach may be useful with these materials:-

1. Fracture generally occurs in a brittle fashion, under tensile and flexural stresses particularly.

2. The strength of these materials is dependent on the rate of loading, indicating that slow crack growth prior to failure is occurring.

3. The ratio of tensile to compressive strength is of the order of 0,1 (depending on the method of test), which is close to the theoretical Griffith value of 0,125.

4. The presence of stress concentrations at interfaces and flaws could easily cause the local theoretical strength to be exceeded in these regions.

5. These materials are generally notch-sensitive. This is a necessary though not sufficient condition for the applicability of fracture mechanics to cemented materials.

Fracture mechanics as applied to cemented materials should really be interpreted as an endeavour to better characterise and understand the fracture process, and then model this mathematically for analysis and predictive purposes. Consequently, two main fields of study have developed:-

1. Physical studies of the cracking process. These have involved studies of mechanisms of cracking, morphologies of crack surfaces, and relation between type and extent of microcracking and the mechanical properties of the material. Hardened cement pastes, mortars, and concretes (normal, lightweight and high strength) have been studied[13,14,15].

2. Measurement of fracture parameters for concrete and cemented materials. This has generally been done in the laboratory on small specimens; the object of these tests is consistent with the basis of the fracture mechanics approach, i.e. to establish valid parameters in the laboratory and then apply these to design. However, considerable difficulty has been encountered in defining suitable parameters. Classical fracture mechanics has relied on the use of linear elastic fracture mechanics (LEFM) parameters, usually a critical stress-intensity factor K_c, and a critical strain energy release rate G_c. Mainly due to the fact that many materials and specimens, both metallic and non-metallic, exhibit considerable ductility before failure, a further group of parameters termed elasto-plastic fracture mechanics (EPFM) parameters have come into being. EPFM parameters have generally been developed for metallic materials. For cemented materials which are not truly elasto-plastic, it might be better to use the term non-linear fracture mechanics (NLFM) parameters. These parameters will be dealt with in detail in a later chapter. However the application of both of these broad groups of parameters to cemented materials will be briefly covered here.

ASSUMPTIONS AND MODIFICATIONS

Application of linear elastic fracture mechanics theory to concrete is based on the following assumptions:-

1. The Laws of Elasticity for homogeneous materials can be applied to the concrete section as a whole. (Concrete is of course

heterogeneous and discontinuous due to the presence of various phases and microcracks.)

2. The values of the elastic modulus E and Poisson's ratio ν are constant throughout the material. This criterion is satisfied by assuming an average E and ν for the material.

TENSILE FRACTURE IN CONCRETE

Under tensile stress, concrete behaves elastically until the stress $\sigma=\sqrt{2E\gamma/\pi c}$ is reached (equation (2.2)). Beyond this point, both stress and crack grow slowly, as depicted in figure 2.4, and the stress-strain curve is no longer linear. Due to decrease of the cross-section, the stress grows faster than the external load, and the strain energy release rate $\pi\sigma^2c/E$ grows faster still until it reaches a value G_c where rapid crack propagation may occur. Depending on the stiffness and mode of operation of the testing machine, it may or may not be possible to follow a descending portion of the load-deformation curve.

COMPRESSION FRACTURE IN CONCRETE (UNIFORM UNIAXIAL COMPRESSION)

Flaws and cracks in concrete under compression can lead to tensile stresses being developed near crack tips, depending on the shape and orientation of the cracks[14]. Crack propagation occurs in planes parallel to the direction of compression, and not necessarily in a plane containing the initial crack. Assuming that concrete has numerous initial flaws and cracks, the weakest of these will begin propagating under compression. However, due to the crack arrest mechanisms of microcracking and aggregate zones of high strength, these cracks will be checked by the rapid increase in energy demand. This permits a further increase in load, and another crack, next in order of weakness, starts propagating. The new crack in turn is soon arrested and another begins growing. This process is termed the "progressive cracking mechanism" by Glucklich[11].

The process of progressive cracking eventually brings about total destruction, either because the material can no longer resist shear stresses in its disintegrated state, or the value of the critical strain energy release rate G_c is attained, and the particular crack propagating at that instant runs to failure. In tension, the stress field σ_{tens} increases rapidly with the crack propagation because of the decrease in cross-section; however, in compression, this is not so. Also the progressive cracking mechanism, which effectively eliminates nuclei of fracture just as plastic flow does in metals, provides a mechanism of energy dissipation that constitutes an alternative to fracture. These two factors make compression fracture of concrete far more stable than tension fracture.

BRIEF REVIEW OF PREVIOUS WORK

Kaplan[17] was one of the first investigators in the cemented materials field. He performed tests on notched concrete beams in order to determine the critical strain energy release rate G_c. He found that smaller 75x100mm beams gave somewhat lower G_c values than 150x150mm beams, but suggested that the concept of the critical strain energy release rate being a condition for rapid crack propagation was applicable to concrete. Kaplan neglected the effects of slow crack growth prior to failure, which caused his G_c to be underestimated. He also suggested a tentative application of his fracture mechanics parameters to the failure of a simply supported plain concrete slab. However, it seems more logical in the case he used to apply simple bending theory to the failure of the slab.

Moavenzadeh and Kuguel[9] also used notched beam specimens to study the fracture of cement paste, mortar and concrete. They found that the effective fracture surface energy γ' was greater for mortars and concretes than for pure pastes, due to the introduction of solid particles and the corresponding increase in microcracking and heterogeneity. Using the technique of quantitative microscopy, they showed that γ' for cracks propagating through the aggregate-paste interface was less than γ' for cracks propagating directly through the paste. They also ignored the effect of slow crack growth prior to failure.

Naus and Lott[16] calculated the fracture toughnesses of pastes, mortars and concretes in which they varied water-cement ratio, sand-cement ratio, gravel-cement ratio, age of test, air content and gradation and type of coarse aggregate. Although they ignored the effect of slow crack growth, they employed the ideas of Lott and Kesler[18], who realised that the presence of aggregate particles near the tip of a crack modified the stress field of the crack, and increased the degree of microcracking. The latter authors argued that when concrete was analysed as a homogeneous material, a pseudo-fracture toughness resulted which was the summation of the fracture toughness of the cement paste and an arresting action of the aggregates on crack growth. They found that concretes had higher fracture toughnesses than equivalent mortars.

Glucklich[19] did a considerable amount of work studying the propagation of fatigue cracks in mortar, using both notched and unnotched beams. He introduced a compliance-crack depth relationship in order to interpret measured strains in terms of crack depth, and overcome the problem of slow crack growth prior to failure. He found that the value of G_c was approximately constant for both notched and unnotched beams, and $\sigma\sqrt{c}$ was constant for beams with equal notch depths. Generally G_c measured in fatigue tests was lower than C_c measured in static tests, which he attributed to creep. Glucklich was one of the first to point out that the object of a notch was not to simulate a crack, but to predetermine the cross-section of failure. In fact, investigators in the metallic field use the practice of growing a sharp fatigue crack from a notched specimen before fracture testing of the specimen[20].

Welch and Haisman[21,22] evaluated linear elastic parameters of a wide range of concretes, mortars and pastes, using notched and unnotched beams. They also considered the influence of different types of notches. Over a range of compressive strengths they found that fracture toughness parameters tended to increase in proportion to concrete strength properties such as compressive strength, water-cement ratio, indirect tensile (splitting) strength, and solid volume fraction of cement paste. Investigating the effect of notch sharpness (i.e. root radius), sharper notches produced flatter load-displacement curves and lower ultimate loads than blunt notches. However, if slow crack growth was taken into account, the notch type did not influence fracture toughness values by more than about 10 per cent. They pointed out the important fact that fracture toughness values are affected considerably by the assumptions used to estimate the effect of slow crack growth, the stress concentration at the root of the notch, and values of Young's Modulus. Three sets of assumptions were proposed in order to provide a better comparison of results by different research workers. These assumptions were:-

"Assumptions A" : The original notch depth c_0 is used to compute the nominal stress σ_n at the root of the notch, and for substitution in the relevant fracture equation. A value of E based on dynamic measurements on representative specimens for each concrete class is adopted.

"Assumptions B" : The original notch depth c_0 is used but the value of E is calculated directly from the load deformation curve in each case. Because of non-linear relationships in most cases, the secant modulus is adopted between zero and 70 per cent of ultimate load, and a correction is also made for shear deflection. It may be considered that the adoption of this lower value of E takes into account the slow crack growth and creep effects during loading. Consequently, it is suggested that it would be incorrect to use this value of E and to allow for slow crack growth in addition.

"Assumptions C" : The critical crack depth c_c is used to compute the nominal stress at the root of the crack, and for substitution in the relevant fracture equation. The critical crack depth is the original notch depth plus slow crack growth prior to the onset of unstable crack propagation, as determined from compliance - notch depth relationships. The dynamic value of E is used, as in "Assumptions A".

It is necessary to point out that in the present work, a slightly different set of assumptions was used to calculate linear elastic fracture parameters The assumptions used were similar to "Assumptions C", except that E was determined from the linear portion of the load-deformation curves of unnotched beams, or from static compression tests on prisms. Occasionally, the load-deflections records of notched beams were also used to determine E. The aim was to minimise the effects of creep and rate of strain, which from previous work by the author[23] had been found to have a significant effect on measured parameters. It is interesting to note that Welch and Haisman were the first to point out the need to adopt more standard practices and assumptions for the fracture testing of non-metallic materials, a subject which is continuing to receive much attention currently.

Brown[24] used two methods to measure the fracture toughness K_c of cement pastes and mortars. The first was the usual notched-beam technique, combined with compliance measurements to measure the slow crack growth prior to instability. He measured the change of toughness for separate increments of crack growth as the crack propagate , which is essentially the technique of R-curve analysis used in elastic-plastic fracture studies (see chapter 3). The second method, using a double-cantilever beam, avoided the slow crack growth problem by making a specimen of variable web width such that the length of crack front increased with and exactly compensated for the effect of crack growth. He found that the fracture toughness of cement paste was independent of crack growth, but that the toughness of mortar increased as the crack propagated, which accords with the concept of figures 2.4 and 2.5 very well. For both materials, the stress intensity required to initiate crack growth was less than that to maintain crack growth at the loading rates used.

Blight[25] used an energy approach to design in cemented materials, employing the effective fracture surface energy γ', and based on two principles:-

1. There is a simple equivalence between work done on the structure or member in applying the ultimate load and the equivalent energy of the resulting fracture surface.

2. In the absence of geometrical stress-raisers, the strength of the material will be governed by the presence of inherent starter flaws which are intrinsic to the material.

Blight argued that a direct application of the Griffith equation (2.2) to concrete and pavement materials such as cement-treated-bases gave a governing size of natural flaw of about 10mm, based on values for γ' obtained in the laboratory. This flaw size is realistic relative to bleeding channels and shrinkage cracks in concrete and to the size of flaw that could be expected to occur in most pavements due to laying or compaction procedures. Blight also applied his approach to a number of practical problems that had arisen due to severe cracking in roads with cement treated bases[6,25]. He was able to predict typical crack spacings in an orthogonal pattern, and flexural failure loads of unreinforced slabs or beams subjected to line loads or heavy moving wheel loads. The approach could be extended to flexural fatigue failures of a pavement under repeated passage of wheel loads in which the fatigue problem may be interpreted in terms of tolerable deformation rather than the number of cycles to failure. For civil engineering purposes, where magnitudes and numbers of load passes are difficult to monitor and forecast with accuracy, a record of the progress of permanent deformation with time may thus be used to assess fatigue damage. Finally, the method was successfully applied to the question of rates of crack growth in a laboratory specimen of sand/bitumen.

An interesting recent development of high-strength hydraulic cements has been brought about by an application of the simple Griffith approach[26,27]. Hydraulic cements generally have large porosities of the order of 20 per cent, and tensile strengths of between 3 to 10N/mm^2, therefore requiring reinforcement in tension. Only limited success has been achieved in correlating strength with total porosity. Recent studies by Birchall[27] and co-workers in the U.K. have shown that the maximum pore size in hardened cement paste (hcp) is about 1mm (a figure also arrived at by use of the Griffith equation). Using simple notched bend specimens of hcp these researchers were able to experimentally determine the Griffith curve, and hence a value for γ which was very similar to the value for strong brittle ceramics. Below a notch size of about 1mm, the flexural strength of the material remained constant. Therefore, the conclusions were that the strength of hcp was due not to a low intrinsic strength, but rather to the presence of macrovoids or pores. By paying attention to minimising pore sizes (using rheology control with polar admixtures and efficient mixing), they were able to achieve flexural strengths of abou 70N/mm^2, comparable with values for ceramics. Compressive strengths were also greatly increased to values of about 200N/mm^2. Figure 2.9 shows the experimental results for this material, now termed Macro-Defect-Free (MDF) cement. This development has also led to new possibilities of chemically modifying the hydration reaction to produce greater cohesion in hcp, similar to the structure of mother-of-pearl[26]. At present, the application of MDF cement in concrete or mortar seems limited by the heterogeneity introduced by the aggregates. However, MDF cement may become a serious contender as a cheap alternative for consumer uses where ceramics and metals have held sway till now.

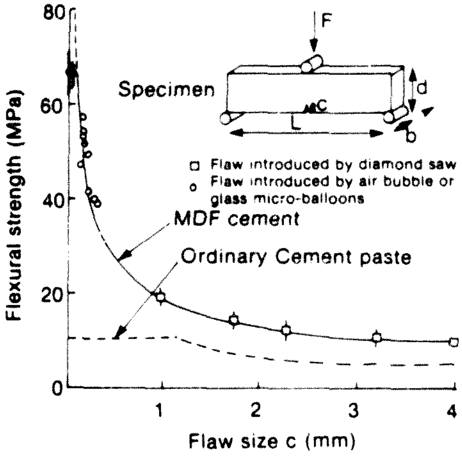


Figure 2.9 Results of flexural testing of notched MDF cement compared with OPC paste (from ref. 27)

The discussion above has generally dealt with the application to cemented materials of l[illegible] elastic fracture mechanics (LEFM) principles (except to som[illegible] the reference to Blight's work (γ')). Possibly the mos[illegible]urrent question is the limit of applicability of LEFM to [illegible]ese materials, and exactly what constitutes a valid fracture test. Mindess[28] has comprehensively reviewed this problem, and has suggested that two basic questions require to be resolved: firstly, can the strength of hcp or concrete be controlled by artificially introduced notches (i.e. the condition of notch-sensitivity) or is it always controlled by inherent flaws in the material?; and secondly is there some minimum size of specimen, or ratio of flaw size to specimen size, that is required before a fracture test can be considered to be valid? In answer to the first question, Mindess states that most recent studies indicate hcp is notch-sensitive, and mortars and concretes are also notch-sensitive though to a lesser degree. However, certain investigators[29] would not agree, and contradictions still exist. Figure 2.10, taken from Mindess, shows some of the conflicting data for notch-sensitivity, where notch-sensitivity is expressed as a ratio of the net stress at the root of the notch or crack to the flexural strength of the material.

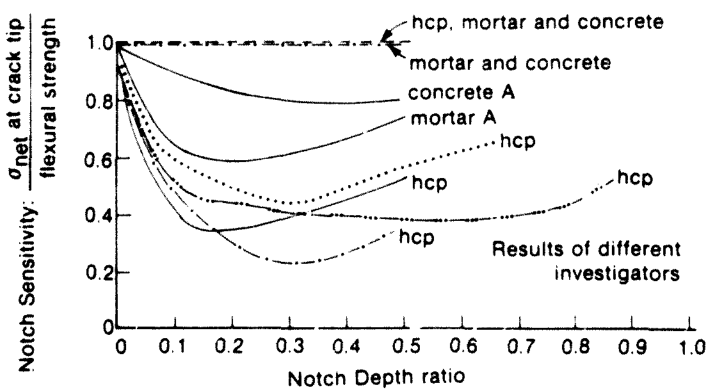


Figure 2.10 Effect of notch depth ratio on notch-sensitivity (from ref. 28)

For materials that are notch-sensitive, the above ratio appears first to decrease, and then to increase again with increasing notch depth ratio. Hence there is a critical notch depth ratio for which notch-sensitivity is most acute. For notch depth ratios greater than this critical value, notch-sensitivity becomes less marked.

The second question, that of minimum specimen size, is even more vexing to answer, and results of different investigators can be highly contradictory. For instance, while some found G_c and K_c to be substantially independent of crack depth or crack depth/specimen depth (c/d) ratio for hcp, mortar and concrete[26,30,31], others found these parameters to reduce with increasing notch depth[32], while still others found an increase in G_c and K_c with increasing crack depth[26,33]. On the other hand, the view has been expressed that the fracture parameters depend not on c/d ratio, but on absolute specimen size or specimen type[34]. Figure 2.11, taken from Mindess[28], shows some of the contradictory results that have been obtained. Some of the discrepancies can be explained as being due to different experimental techniques (for example most studies do not account for slow crack growth prior to instability) and to the influence of machine stiffness on test results. Mindess[28] has also reviewed the literature concerning absolute specimen size effects, and shown again that large differences of opinion exist. To summarise, it is appropriate to quote from him:-

> "Considering all of these contradictions, the weight of the evidence would suggest that linear elastic fracture mechanics is probably applicable to hcp. For mortar and concrete, the evidence is still ambiguous, partly because it may be that in view of the problem of specimen size discussed above, almost no valid fracture tests have ever been carried out. Thus, there is still no consensus as to whether linear elastic fracture mechanics is even a reasonable approximation when applied to concrete, or, if it is, what constitutes a valid test".

In view of the above discussion, it is not surprising that researchers are turning to the evaluation of non-linear fracture me-

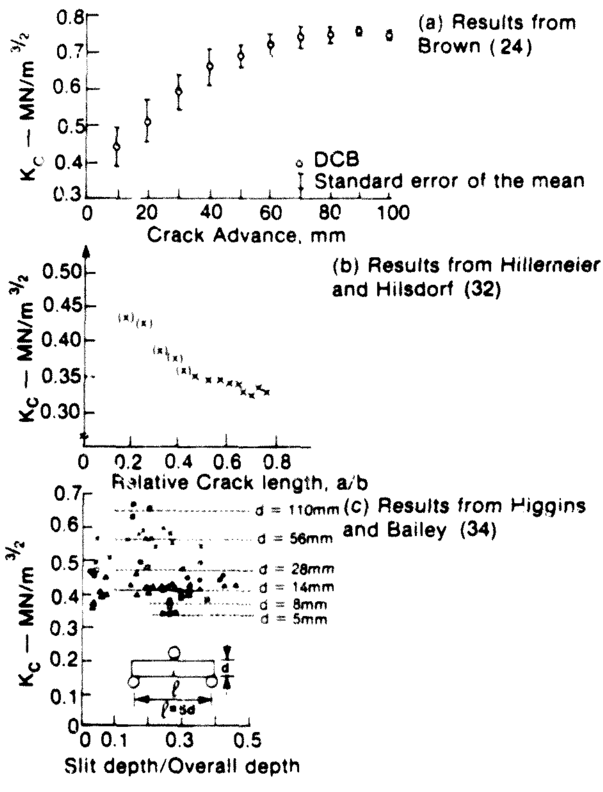


Figure 2.11 Conflicting results for fracture parameter K_c, obtained by different investigators (from ref. 28)

chanics (NLFM) parameters for cemented materials. This follows the trend in the metallic field, (i.e. the development of elasto-plastic fracture mechanics (EPFM) parameters) where ductile fracture characterised by extensive yielding before final failure has required a set of criteria other than LEFM to analyse observed behaviour. LEFM is primarily applicable to those situations where the size of the inelastic zone ahead of the crack tip is small compared with the overall specimen size. Where this no longer applies, NLFM parameters may be useful, although their application to cemented materials is still relatively new. In the concrete field, these parameters appear to have most applicability to composite or modified materials such as polymer concrete (PC) and fibre concrete (FC) in which the effect of the modification is to improve toughness, impart a considerable degree of ductility or "pseudo-plasticity" to the material, and to cause further non-linear stress-strain behaviour. The various parameters will be discussed in more detail in chapter 3 but a brief review of some of the applications to cemented materials is now given.

The application of the J-integral (J_c) to cemented materials was suggested by Mindess and others[15,16,17]. J_c is essentially the non-linear counterpart of the LEFM parameter G_c, and can therefore be interpreted as the energy available for crack extension. Tests by Carrato[17] have indicated that this parameter is independent of notch geometry. Problems exist with the J-integral however, not least of which is the large variability in experimental values. The use of J_c has been applied with some success to fibre-reinforced-cement (frc) by Mindess et al[15], who showed that while J_c was highly sensitive to fibre volume, G_c was insensitive. Halvorsen[16] found that, while there was very little effect on measured J of loading configuration (i.e. third-point or centre point beam loading), there was an appreciable effect of absolute specimen size. For instance, in third-point loading, J_c approximately doubled in value as the specimen depth increased from 75 mm to 150mm. In addition, there appeared to be no dependence of toughness on notch depth, although a very large scatter of results was obtained. Halvorsen was not entirely happy with the use of J_c as a fracture criterion for his materials, since he proposed the investigation of alternative failure criteria to see if a more consistent toughness parameter exists. Further problems with the J-integral are that certain investigators[39] have found that J is not independent of crack depth, and that according to Sih (quoted by Mindess[19]), there is a fundamental theoretical problem in applying J_c to composite cemented materials.

R-curve analysis has also been applied to composite cemented materials with some success. R-curves show the relationship between fracture toughness (expressed as K, G, or even γ') and crack growth. For non-linear materials, there is generally an increase in fracture resistance (R) with increasing crack growth, and R-curves are useful for characterising such fracture resistance in a material. The work of Brown[24] previously referred to represents essentially one of the first uses of the R-curve approach to cemented materials, although this may not have been fully appreciated at the time (1972). Mindess[19] has reviewed some of the work on fibre-reinforced concretes by Velazco et al[38] and on plain mortars by Wecharatana and Shah[40,41], which appears to show that R-curves are independent of initial notch depth and sensitive to fibre volume; however, different specimen geometries had an influence on the R-curves. The use of R-curves to characterise the fracture behaviour of asbestos cement was also found to be successful by Lenain and Bunsell[42], and by Mai et al[43]. These authors were able to distinguish various stages of crack growth, and considered the fracture toughness to be made up of a component due to the reinforced matrix and a component due to the fibres bridging the crack and thus inducing a closing stress at the crack tip. The latter authors (Mai et al[43]) also showed that R-curves were reasonably independent of specimen geometry, which is somewhat contradictory to Wecharatana and Shah's results[41].

Referring to polymer impregnated concrete (PIC), Mindess[19] has provided a valuable review of the application of fracture mechanics to these materials. In general, improvements in strength and toughness are obtained with polymer impregnation, although thermal polymerisation produced lower K_c values than did irradiation polymerisation. Cook and Crookham[44] felt that LEFM could be applied investigation of alternative failure criteria to see if a more consistent toughness parameter exists. Further problems with the J-integral are that certain investigators[38] have found that J is not independent of crack depth, and that according to Sih (quoted by Mindess[19]), there is a fundamental theoretical problem in applying J_c to composite cemented materials.

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In a somewhat different approach to the fracture of cemented materials, the basics of a Dugdale tied-crack model[45] were applied to the problem of a discrete crack in a plate-like specimen by Hillerborg and co-workers[46,47] and by Reinhardt[48]. In these models, stresses are assumed to act across a crack as long as it is only narrowly opened. In the Hillerborg model (termed the "fictitious crack model" (FCM)), the relationships are developed by considering a pure tension test, in which a fracture zone develops giving an additional component of deformation w, which in effect can be thought of as the crack width. This crack is essentially a microcracked zone with some remaining elements for load transfer, and thus a relationship exists between the tress σ across the crack and w. The area under the σ-w curve (which is a descending curve) is taken as G_c, where G_c is interpreted as the energy absorbed per unit crack area for the complete separation of the crack surfaces. In this regard, the evaluation of G_c is very similar to that used for γ' by Blight[8], except for a small discrepancy represented by the hysteresis loop of the initial loading and unloading portion of the curve. The model is suitable for finite element analysis, using stepwise linearised σ-w curves. Hence, the main difference between the FCM approach and the conventional fracture approach is the use of the stress-displacement curve.

Reinhardt[48] approached the fracture problem by treating concrete as an elastic softening material (as opposed to a strain-hardening material), i.e. the concrete has a linear stress-strain curve in tension up to the maximum stress (or tensile strength) and thereafter the stress decays with increasing deformation (similar to the FCM). With regard to the failure of a uniaxial tension specimen, a discrete crack is considered in which all the deformation is finally concentrated. Reinhardt used the Dugdale model, in which the crack consists of two sections:- one where crack faces are free (no traction), and another where a closing stress acts along the crack faces. The region in which traction operates is termed the "softening zone": at its tip the tensile strength (f_t) of the concrete applies, and at its other limit the crack tip opening displacement applies. As in Hillerborg's FCM, the stress distribution in the softening zone is a function of the deformation and the softening behaviour (i.e. the stress-deformation curve). A complex mathematical treatment using a power function for the stress distribution in the softening zone allowed the prediction of the crack opening of a tensile specimen as a function of load, and the strength as a function of specimen size and initial crack size. The mathematical predictions were tested experimentally by using narrow and wide plate specimens with edge saw cuts. The uniaxial tests were deformation-controlled, and it was possible to obtain the complete stress-deformation curve. The effect of repeated tensile and alternating tensile-compressive stresses on the envelope curve was studied and found to be negligible. Wide specimen studies showed that a linear distribution of deformation in the softening zone occurred. Reinhardt obtained good agreement between theoretical and experimental load capacity. His conclusion was that the behaviour of concrete is appropriately modelled by non-linear fracture mechanics (NLFM) based on the Dugdale model.

SOIL CEMENT

The classic work carried out on the application of fracture mechanics to the failure of soil cement was done by George[49]. He used very similar methods in determining the parameters G_c and K_c as those used for concrete or asphalt, and found that soil type, temperature and loading rate all affected the parameters. In model studies of a soil cement base, he used G_c and K_c to evaluate the crack propagation potential, and concluded that K_c rather than G_c was the parameter governing the rate of crack propagation. Crack propagation rate increased with a decrease in K_c. He also considered the spacing and configuration of cracks in a soil cement base, using the principle of minimum potential energy. His findings that the crack pattern should be one of random orthogonal polygons with a bias towards hexagons were borne out by his model studies.

LIME STABILISED SOIL

Blight[*] carried out beam bending and unconfined compression tests on soil-lime, in order to find the effective fracture surface energy γ', i.e. the actual work of fracture. Age and lime content had very little effect on γ'. He also plotted, for a number of different cemented materials, values of γ' versus strain at failure (ε_{fail}), on a log-log plot. The γ' term contains plastic work done to cause the material to fracture, and since plastic work is accompanied by strain, the plastic work increases as the strain to failure increases. The results of Blight's log-log plots are a series of straight lines of approximately constant slope, each line representing a particular material. These plots seem to indicate a relation between γ' and ε_{fail}.

2.5.3 CONCLUDING REMARKS

From the foregoing discussion, it is clear that the application of fracture mechanics to concrete and cemented materials is still very much in its infancy when compared with metallic materials. This is understandable in view of the heterogeneous and complex nature of cemented materials. Further research, both experimental and theoretical, is required into the fracture of hcp, mortars, and concretes. Firstly, the physical processes of cracking need further study, in order to give a good representation of fracture. Secondly, agreement must be reached on what constitutes a valid fracture test. Only then can we confidently use fracture mechanics as a tool in predicting crack lengths and flaw sizes, and in estimating structural performance. This approach will have to involve both stress-intensity and energy methods. Considering the energy approach to failure, fracture mechanics gives an integrated approach in which the possibility of looking at static, dynamic, and long-term failures of materials under a unified framework now exists.

2.6 SUMMARY OF THE FRACTURE MECHANICS APPROACH

Fracture mechanics pre-supposes the existence in a material of a crack-like flaw or defect that, under stress, can lead to failure. Analytical approaches have been developed that involve either an energy balance method, in which the energy required to produce new fracture surfaces must be more than compensated for by the strain energy released by the advance of the crack, or a stress-intensity factor method, in which elastic or elastic-plastic continuum mechanics theory is used to develop parameters which characterise the crack tip stress field. Provided a suitable parameter describing the fracture toughness or resistance to crack growth of the material can be defined and measured under given circumstances, then fracture mechanics allows this parameter to be regarded as a material property and to be used to calculate the behaviour of other geometrical arrangements.

Linear elastic fracture mechanics (LEFM) has had rapid and extensive development during the last few decades. Its major drawback is that it assumes a linear elastic isotropic continuum in which frac[illegible] occurs with very little or no plastic deformation ahead of and around the crack tip. The concepts of LEFM are also primarily based on a single crack propagating in the material. Despite its limitations, LEFM has had considerable application in the metallic field and has assisted greatly in understanding fracture behaviour of concrete and cemented materials. However, concrete is a highly complex, heterogeneous, composite material that contains inherent cracks and flaws at virtually every level of scale, and is complicated by the presence of interfaces between aggregate and matrix. When concrete fractures it develops a large fracture zone in which extensive microcracking occurs, and therefore the LEFM approach must have limitations in being applied to concrete fracture. Non-linear fracture mechanics (NLFM) in the form of elasto-plastic (EPFM) parameters has been developed again mainly within the metallic field, and has had some application to concrete. The truth is that at the moment we are still far from a comprehensive and uniform approach to concrete fracture. What is lacking even more are direct and useful applications of the fracture mechanics method to design and analysis problems in concrete and other cemented materials.

The work represented by this thesis covers aspects of physical fracture processes and fracture testing of concrete. Specimen type was limited to notched beams, and load-deflection relationships were carefully measured. Concrete damage was also monitored by means of strain and ultrasonic pulse velocity measurements. The aim was to study the process of fracture in bending, and apply the results to the proper evaluation of fracture tests. It was hoped that in this way some of the reported inconsistencies obtained in concrete fracture tests could be eliminated, and that positive progress could be made towards a test method or methods that would be truly representative of concrete fracture.

The considerable amount of work that has been done on the fracture of metallic and non-metallic materials has led to extensive bibliographies in both fields. Many national and international conferences have been held, and committees and research groups have been constituted to study problems in greater depth and to bring together research findings in report form. It would be virtually impossible to refer to all the fracture material that is now available, but a very brief list is given at the end of the references section. With regard to concrete in particular, two general references might be mentioned: the one has been produced by a RILEM committee (1983) and deals with cracking in concrete with respect to its structure, methods of detecting cracks, experimental determination of fracture parameters, and theoretical concepts.[50] This publication also contains a very useful fracture bibliography[51]. The second general reference has been produced by ASTM[52], representing papers presented at a symposium on fracture mechanics for ceramics, rocks, and concrete in 1980. Although only two of the papers dealt directly with concrete, the broad spectrum of other non-metallic materials reported on throws useful light on the fracture problems of these materials.

3 FRACTURE MECHANICS PARAMETERS AND TESTING

3.1 INTRODUCTION

A parameter is defined as a measurable quantity that is usually a variable, but may be constant for the particular case considered. Hence, when we talk about fracture mechanics parameters, we wish to define a quantifiable measure of fracture behaviour and fracture toughness that can be considered to be a material constant. There are two basic requirements for such a parameter:-

1. It must adequately represent the fracture process i.e. it must take into account the obvious variables that affect fracture, and allow for special features that distinguish the fracture of a particular material.

2. It must be useful for design purposes i.e. it must be reasonably independent of specimen size and geometry, related to readily observable behaviour such as load-deformation or moment-curvature relationships, and reproducible. It must also be sufficiently simple for use in design formulations.

The philosophy of fracture testing of materials follows that of general materials testing, i.e. to model the real situation in a laboratory where suitable specimens can be made and tested, and to use the results of these "model" tests to predict what is likely to happen in a real structure under various combinations of load, or to explain observed real behaviour or failures. Fracture mechanics testing of cemented materials is in many ways still in its infancy, since neither the basic parametric requirements nor the fracture testing philosophy have been treated with a satisfying degree of success to date. Many problems remain unsolved, and it is true to say that an adequate fracture parameter for cemented materials has still not been developed. However, to quote from Wells[53]:-

> "The study of crack extension is recommended to those who wish to achieve, and relish a difficult intellectual problem. There are many remaining opportunities thereby to clarify the fracture problem, which pervades the whole of technology"

With this as background, it is useful to consider those characteristics which differentiate cemented materials from other engineering materials. Swamy[54] has identified three:-

1. Cracking is irregular, tortuous and rough. The area of newly formed fracture surface is therefore many times greater than the nominal fracture area.

2. Cracking is complex, and various stages in the cracking process can be identified:- crack initiation, slow stable growth, crack arrest, critical condition of instability, and unstable crack propagation. Microcracking and branch cracking forming a fracture zone ahead of and surrounding the crack tip are essential features, and considerable stress transfer occurs across microcrack interfaces.

3. Energy dissipating mechanisms include not only surface energy, but other forms such as kinetic energy, friction, true plastic deformation, and visco-elastic effects.

Referring to concrete in particular, perhaps its most distinguishing feature is its highly complex, multi-phase, heterogeneous nature which exists at virtually every level in the material. The question may therefore be asked whether it will ever be possible or practically desirable to develop parameters that will fully explain and predict concrete's behaviour under all circumstances.

This feature of concrete as a material is already tacitly accepted in many design approaches by an undue reliance on empiricism, and it is necessary that we continually attempt to move towards more fundamental approaches. However, concrete's heterogeneity will demand very often that it be treated differently to other materials, and it may in the end prove unrealistic to impose the same limitations on obtaining "true" design parameters for concrete and cemented materials as we do for other materials.

Accepting that it is desirable to understand more about fracture processes in cemented materials, and that it would be useful to have a suitable descriptive and predictive fracture parameter for design purposes, what specimen types and test procedures are available for fracture testing? This is a vast subject in itself, and it is not proposed to deal fully with it. Other reviewers have performed this task , in both the metallic field[55,56,57] and the non-metallic field[51,54,58,59]. Referring to the plane strain fracture problem in particular, ASTM has a standard test for metals[60]. At present, no standard exists for cemented materials. The purpose of this section is to briefly review one particular specimen type for opening mode fracture that was used in the experimental programme. The formulae for various fracture parameters, related to the particular specimen type, are also presented, as well as certain of the techniques involved in the testing.

3.2 NOTCHED BEND SPECIMENS

These specimens comprise rectangular section beams with a notch in the tension face at mid-span and have been used extensively for fracture testing of cemented materials. It should be appreciated that a fabricated notch in itself does not represent an actual crack in the material. For testing of metals, it is customary to grow a sharp, natural crack from the starter notch by fatigue stressing before testing. For testing of cemented materials, slow stable crack growth prior to failure occurs, and the crack becomes naturally sharp before the point of instability is reached. Glucklich[19] states that a notch in a beam serves the following purposes:-

1. It pre-determines the cross-section of failure.

2. It serves as a sufficient stress concentration to allow a natural crack to grow from it in a stable manner before unstable crack propagation occurs. This also facilitates more readings and better accuracy.

3. It allows for deformations measured across the notch to be the deformations at the critical area of crack growth.

On the other hand, it should be recognised that introducing a notch into a beam affects the fracture behaviour when compared with an unnotched beam. A large notch does not always represent a structural flaw, and since it limits the energy dissipation to a specific region, microcracking and branch cracking is limited. Cook and Crookham[44] have argued that it is inaccurate to use large notch depths to determine fracture parameters, and have shown that an unnotched beam with slow crack growth will have a greater fracture toughness than a notched beam of the same total crack depth due to this microcracking zone size effect. These comments should be borne in mind when assessing the results of notched beam testing. The notched beam has the advantage of requiring lower loads to produce fracture than other specimens of similar dimensions. The test requires a simple arangement that can be used in a variety of test machines. The practical convenience of testing a beam also outweighs the problems of testing brittle materials in direct tension. One disadvantage of the notched beam is that the accuracy of fracture toughness measurements is inherently lower than for other types, because the sensitivity of the parameters to a small error in crack depth is greater than for other types. Beams may be loaded either in three-point or in four-point bending. Three-point bending is simpler to perform but complicates the failure by the existence of a shearing stress which changes from a positive to a negative value at the midspan loading point; this effect increases with a decreasing span to depth ratio. Four-point bending produces a uniform bending moment in the central span, with no shearing stress present - a state of pure bending. For the present tests, the simpler arrangement of three-point bending was used. Notch depths up to 50 per cent of beam depth were selected for study.

The parameters that were evaluated from the tests included linear elastic fracture mechanics (LEFM) parameters, and non-linear (NLFM) parameters. A general introduction to the common range of fracture parameters is given hereunder, although not all were evaluated in the present tests. A critical appraisal of the various parameters in the light of the test results is given in the final chapter. In addition to evaluating fracture parameters from the tests, concrete beam fracture was characterised by assessing the damaging effect of the notch using strain and ultrasonic pulse velocity measurements. These measurements could also be used to assess stresses in fracture zones. This receives attention in chapter 9.

3.3 LINEAR ELASTIC FRACTURE MECHANICS (LEFM) PARAMETERS

Fracture Mechanics in its broadest sense refers to that framework of applied mechanics which deals with the behaviour of cracked bodies under load. LEFM is the study of stress and displacement fields near a crack tip according to the normal assumptions of material isotropicity, homogeneity, and linear elasticity. The concept of a stress-intensity factor which characterises the stress field around a crack and which reaches a critical value at the onset of rapid unstable crack propagation leading to failure is central to LEFM. The use of such a factor as a criterion for failure has been and continues to be the subject of much study.

The theory of LEFM is most applicable to brittle materials in which any inelastic region near the crack tip is small compared with the crack and specimen dimensions. The assumptions inherent in the linear elastic materials approach need to be carefully appraised for cemented materials which are often far removed from the simple behaviour often assumed for them.

The mode of crack-surface displacement considered here is the opening mode, in which crack surfaces move directly apart (see figure 3.1). This mode, often denoted by the Roman numeral I, predominates for the materials that were tested, and is the most common mode even in metallic materials. The direction and plane of growth of a crack in this mode is approximately perpendicular to the maximum principal stress. In fracture testing a through-thickness crack is normally used, allowing a two-dimensional analysis to be performed.

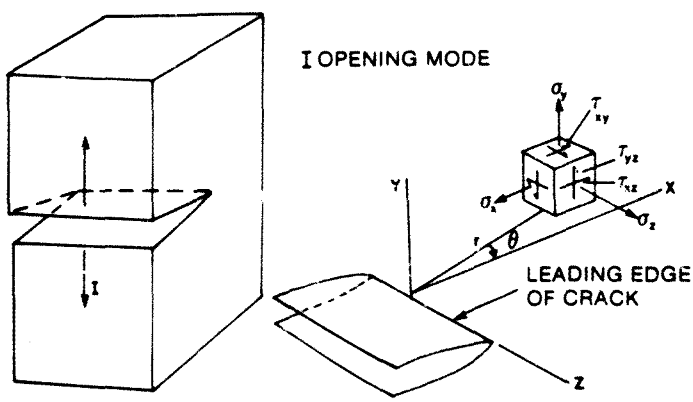

Figure 3.1 Mode of crack-surface displacement and crack tip coordinates

Two fracture parameters are considered in this section:-the Critical Stress-Intensity Factor or Fracture Toughness K_c, and the Critical Strain Energy Release Rate G_c. Both of these parameters provide design engineers with a logical basis for finding limiting stresses to avoid fracture in terms of a material property and a knowledge of stress-raising flaws in the structure.