Novel Energy-Centric Analysis for Random Access Networks

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***Abstract*—In this letter, we revisit the notion of energy efficiency widely adopted in age-centric network-based systems. To this end, we consider two traditional ALOHA random-access protocols (unslotted ALOHA and slotted ALOHA) wherein their total energy consumption is analyzed. The importance of considering the total energy needed to deliver timely information stems from the increasing data consumption through resource-constrained devices, such as sensors in an application of the Internet of Things (IoT). We propose a novel perspective on the energy consumption, taking into account synchronization costs for a fair comparison between the access schemes.**

***Index Terms*—Age of information, energy dissipation, packet generation, random access protocols.**

I. Introduction

INFORMATION freshness is becoming crucial in large status-update networked systems, such as Internet of Things (IoT) and massive machine-type communications (mMTC), given the increasing adoption of data-based decision-making and acting. In this regard, access of a shared communication channel by numerous users is a long-standing problem that requires considerable coordination efforts for successful transmissions in the context of timeliness of information updates. To characterize the timely delivery of information updates, age-of-information (AoI) has been introduced as a key metric to measure the freshness of information updates [1], [2]. With the spiralling growth of massive connectivity of devices that require a timely delivery of information in the same channel, random access protocols have gained renewed attention as a viable age-centric approach to address the frequent packet transmission failures that may lead to the degradation of the information updates owing to the fierce channel contention in large-scale networks.

Capitalizing on the minimum coordination and distributed control, traditional ALOHA policies, such as unslotted ALOHA (UA) and slotted ALOHA (SA), have widely been adopted in medium access control protocols for communication networks. While the SA protocol requires a global synchronization to increase the transmission success rate, it is more suitable for delay-sensitive applications than its UA counterpart, particularly in large-scale networks. Despite these advantages, SA still exhibits a relatively low-access efficiency due to multiple collisions. To address this issue, recent studies have led to the development of a new class of advanced random access schemes built upon the classic SA, including, e.g., threshold ALOHA and frameless ALOHA (see [3], [4], [5]).

Common to the aforementioned SA protocols is synchronization, which plays a crucial role in reducing the packet collision rate, and consequently, in the decrease of the AoI. While optimizing AoI, it is crucial to keep in mind the energy efficiency, which is a pressing concern in the design of future large-scale wireless networks [6]. In this letter, we present a novel perspective on energy efficiency and its effect on the timely delivery of status updates using random access protocols. Specifically, we formulate the total energy consumed by the underlying system to include the energy cost due to synchronization to perform the actual work, i.e., deliver timely information. To the best of the authors' knowledge, no prior work has considered the energy efficiency of network-based systems from that perspective, and this is the knowledge gap that the present work aims to partially fill.

II. Background and Motivation

The proliferation of small energy-constrained devices motivated the definition of the bits-per-Joule capacity as a performance metric for the transmission of a limited amount of bits through a network with limited energy available [7]. This letter assumes a fully connected network and shows that the bits-per-Joule capacity increases with the size of the network, in contrast with the scaling laws of throughput capacity. The work in [7] shows that large-scale deployments of energy-limited devices may be suitable for delay-tolerant applications. We use this finding as a motivation to study large-scale networks when the application requires timely information. There is no shortage of applications where timeliness of

Received 27 September 2024; accepted 30 October 2024. Date of publication 4 November 2024; date of current version 10 January 2025. This work was supported in part by the Research Council of Finland through EnergyNet Fellowship under Grant 321265, Grant 328869, and Grant 352654, through the X-SDEN Project under Grant 349965, and through ECO-NEWS under Project 358928; in part by the EU MSCA Project "COALESCE" under Grant 101130739; in part by the Business Finland 6G REEVA Project under Grant 10278/31/2022; and in part by RNP, with resources from MCTIC under Grant 01 245.010604/2020-14. The associate editor coordinating the review of this article and approving it for publication was L. Zhang. *(Corresponding author: Jules M. Moualeu.)*

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Digital Object Identifier 10.1109/LWC.2024.3490695

information is crucial. When the received information is used to make a decision, or perform a specific task, timeliness may determine the quality of the outcomes, for example, when the communication process is part of control automation, vehicular networks, or industrial automation. The evaluation of average AoI in an uncoordinated multiple access collision channel was presented in [8], with the focus on limiting cases as the number of sources and the update transmission rate grow. The limiting case is shown to provide an accurate individual age approximation, even for a small number of sources. The use of the SA protocol was considered in [4] and was shown to be outperformed by a threshold-based version of this policy with respect to average AoI. A detailed steady-state analysis of the threshold-based SA policy (Lazy Policy) was discussed in [9]. An alternative policy based on the SA protocol was proposed in [10], using the notion of age gain of a packet to compute transmission probability according to the estimated number of active users, which requires constant monitoring of collisions. With the Lazy Policy, users may enter the sleep mode when inactive, which is a desirable feature in energy-limited networks. The importance of proper network configuration to address the energy-AoI trade-off has previously been addressed under different assumptions. The work in [11] focused on the maximum allowable transmission times in a truncated automatic repeat request scheme. The optimal power allocation for a weighted utility of AoI and energy is obtained as the solution to a Markov Decision Process in [12]. The study in [13] considered energy efficiency under an AoI constraint in an SA-based Poisson bipolar network with the energy efficiency based on lifetime throughput, showing its decline as the node density increases. In contrast to previous works, our energy-centric investigation also takes into account the energy spent on synchronization where applicable.[1] We study the proposed energy model by leveraging an energy characterization adopted from thermodynamics where the communication system requires a given amount of energy, which encompasses the useful and dissipated energy associated with the transmission of data packets.

III. System Model

We consider a large homogeneous wireless network with n sources. The sources update their status through the *generate-at-will* model, i.e., a fresh sample is generated just before the transmission.[2] Thus, there is no retransmission of packets and each packet takes one fixed unit of time[3] to be transmitted.

Let $u > 0$ be the average use of the channel by the system, which can be calculated as the average number of transmissions during one packet duration (one unit of time), and $\mathcal{E}$ be the average energy used by the system during one unit of time normalized by the energy necessary to transmit one packet through the channel, i.e., $\mathcal{E}$ is dimensionless. Considering the functionality of the system to transmit packets from the sources to the receivers successfully, it is possible to classify the energy as either used to directly perform actual work or dissipated. The actual work consists of the energy directly used to transmit successful packets, and the dissipated energy consists of the energy to maintain the system added to the energy directly used to transmit unsuccessful packets. In what follows, we present the models investigated in this letter: a general model and UA/SA models.

A. General Model

The average use of the channel is denoted by $u \in U$, where $U \subset \mathbb{R}_+^*$ is an open interval of admissible values. Let $p : U \to (0, 1)$ be a differentiable function that represents the packet transmission success probability in stationary state.[4] This probability is assumed to be the same for all nodes given a homogeneous network. Let $\mathcal{C} : U \to \mathbb{R}_+^*$ be the normalized energy cost to maintain the communication system. For example, $\mathcal{C}$ can represent synchronization costs.

In this context, the total energy used by the system per unit of time is equal to the energy cost to maintain the system added to the use of the channel u multiplied by the energy cost to transmit a packet. After dividing by the energy cost to transmit a packet, we obtain the normalized quantity $\mathcal{E}$. The (normalized) dissipated energy $\mathcal{D}$ consists of the energy cost to maintain the system added to the unsuccessful packet transmission energy, then divided by the energy cost to transmit a packet. We define the throughput $\mathcal{T}$ as the number of packets transmitted successfully per unit of time. Then, the throughput $\mathcal{T}$, the energy dissipation $\mathcal{D}$, and the system energy $\mathcal{E}$ can be mathematically defined by

$$\mathcal{T}(u) = u\,p(u), \tag{1}$$

$$\mathcal{D}(u) = u\,(1 - p(u)) + \mathcal{C}(u), \tag{2}$$

$$\mathcal{E}(u) = u + \mathcal{C}(u), \tag{3}$$

respectively, which after some algebraic manipulations, yields

$$\mathcal{E}(u) = \mathcal{T}(u) + \mathcal{D}(u). \tag{4}$$

It follows from (4) that the total energy consumed by the system is equal to the sum of the useful work and the dissipated energy. Subsequently, we study the energy efficiency from the novel energy model obtained in (4) by focusing on two cases, i.e., the traditional UA and SA schemes.[5]

B. Unslotted ALOHA

In the UA protocol, each source waits a backoff time that is independent and exponentially distributed with mean $1/\beta > 0$ and then attempts one single transmission that takes 1 unit of time. This process repeats indefinitely [14]. The use of the channel is given by $u = n/(1+1/\beta)$ since there are n sources

[1] Namely, the energy consumed is given by the energy used to perform useful work plus the energy cost due to synchronization. Consequently, the addition of the latter provides a robust representation of the energy consumption in wireless systems, and therefore, it cannot be neglected. In this regard, it will be worthwhile revisiting the energy consumption from the proposed perspective in scenarios where the node density increases.

[2] This is applicable to wireless sensor networks where sensors generate new samples whenever they decide to transmit.

[3] The reader may think, e.g., of 2 ms as the "unit of time" (and packet duration) throughout the whole manuscript.

[4] Due to the *generate-at-will* assumption and no queueing in the model, the stationary state is reached instantly.

[5] Despite the superiority of the advanced SA policies over the traditional ALOHA schemes, our work is limited to the latter policies to better understand this novel approach on the characterization of the energy efficiency. Moreover, the ensuing results can be used as benchmarks for the advanced SA schemes, which can be investigated in future studies.

that transmit for 1 unit of time each $(1+1/\beta)$ unit of time on average. Consequently, $\beta = u/(n-u)$ for $u \in U = (0, n)$. In this scenario, for $n \to \infty$, the packet success probability function is given by $p_{\mathrm{UA}}(u) = \mathrm{e}^{-2u}$. Assuming that the energy cost to maintain the system is negligible compared with the packet transmission costs, we have $\mathcal{C}_{\mathrm{UA}} \equiv 0$. Substituting the packet success probability $p_{\mathrm{UA}}(u) = \mathrm{e}^{-2u}$ in (1), (2), and (3) for every $u \in U$, the resulting expressions of the throughput, energy dissipated, and energy consumed pertaining to the UA protocol are respectively given by

$$\begin{aligned}\mathcal{T}_{\mathrm{UA}}(u) &= u\,\mathrm{e}^{-2u},\\ \mathcal{D}_{\mathrm{UA}}(u) &= u\left(1-\mathrm{e}^{-2u}\right),\\ \mathcal{E}_{\mathrm{UA}}(u) &= u.\end{aligned}$$

C. Slotted ALOHA

In the case of the SA protocol, the time is divided into slots of duration equal to 1 unit of time, and each source transmits independently with a probability equal to the mean backoff rate $\beta \in (0,1)$ [15]. The use of the channel is given by $u = n\beta$. Thus, $\beta = u/n$ for $u \in U = (0,n)$. For this protocol, the packet success probability function is given by $p_{\mathrm{SA}}(u) = \mathrm{e}^{-u}$ as $n \to \infty$. Assuming that the energy cost to maintain the system is primarily composed of the energy used to synchronize the system, we have $\mathcal{C}_{\mathrm{SA}} \equiv \mathcal{E}_s$, where $\mathcal{E}_s$ is a positive constant representing the energy cost to keep all sources synchronized. Thus, after substituting the expressions of $p_{\mathrm{SA}}(u) = \mathrm{e}^{-u}$ and $\mathcal{C}_{\mathrm{SA}} \equiv \mathcal{E}_s$ in (1), (2), and (3) for every $u \in U$, we obtain

$$\begin{aligned}\mathcal{T}_{\mathrm{SA}}(u) &= u\,\mathrm{e}^{-u},\\ \mathcal{D}_{\mathrm{SA}}(u) &= u\left(1-\mathrm{e}^{-u}\right)+\mathcal{E}_s,\\ \mathcal{E}_{\mathrm{SA}}(u) &= u+\mathcal{E}_s.\end{aligned}$$

IV. Results for the General Model

Under regularity conditions, we derive properties and provide an expression of the mean AoI for the general model of Section III-A. From now on, we assume that $\mathcal{C}$ is a differentiable nondecreasing function because the energy cost to maintain the system cannot decrease with traffic. Moreover, it is also assumed that p satisfies the conditions of the theorem presented below, which guarantees that the throughput $\mathcal{T}$ has a unique local (global) maximum.

Theorem 1: Let U be an open interval of $\mathbb{R}_+^*$ and the function $p : U \to (0,1)$ be continuously differentiable. In addition,

$$\psi : U \longrightarrow \mathbb{R},\ u \longmapsto -u\frac{p'(u)}{p(u)}.$$

If ψ is a strictly increasing function and its image contains a neighborhood of 1, then the function $\mathcal{T} : u \mapsto u\,p(u)$, $u \in U$, has a unique local maximum, which is the global maximum.

Proof: It is easy to show that $\mathcal{T}'(u) = (1-\psi(u))p(u)$ is a continuous function for every $u \in U$. Note that the neighborhood condition guarantees that there exists $u_0 \in U$ such that $\psi(u_0) = 1$ and, thus, $\mathcal{T}'(u_0) = 0$. Since p is a positive function and ψ is a monotonously increasing function, then for every $u < u_0$, $\mathcal{T}'(u) > 0$. On the other hand, for every $u > u_0$, $\mathcal{T}'(u) < 0$. This concludes the proof. ∎

In ALOHA-based systems, for example, we have $p(u) = e^{-\alpha u}$, $u \in U = \mathbb{R}_+^*$, for some constant $\alpha > 0$. In this case, $\psi(u) = \alpha u$. Then, it is easy to see that ψ is a strictly increasing function, and any neighborhood of $1/\alpha$ guarantees that the image $\psi(U)$ contains a neighborhood of 1.

Lemma 1: The functions $\mathcal{D}$ and $\mathcal{E}$ are strictly increasing.

Proof: Let $u \in U \subset \mathbb{R}_+^*$.

$$\begin{aligned}\mathcal{D}'(u) &= (1-p(u)) - up'(u) + \mathcal{C}'(u),\\ \mathcal{E}'(u) &= 1 + \mathcal{C}'(u).\end{aligned}$$

By hypothesis, we have $1-p(u) > 0$, $-up'(u) > 0$, and $\mathcal{C}'(u) \geq 0$, which concludes the proof. ∎

For every $u \in U$, Lemma 1 guarantees that $\frac{\partial \mathcal{T}}{\partial \mathcal{D}} \triangleq \frac{\mathcal{T}'(u)}{\mathcal{D}'(u)} \geq 0$ if and only if $\mathcal{T}'(u) \geq 0$. Thus, the throughput increases with dissipation if and only if the throughput increases with the use of the channel. We define the efficiency of a system as

$$\eta \triangleq 1 - \frac{\mathcal{D}}{\mathcal{E}}, \tag{5}$$

which represents the proportion of the total energy that was directly used to transmit successful packets.

Using the fact that $\mathcal{E}(u) \geq \mathcal{T}(u) + \mathcal{C}(u)$ for every $u \in U$, we obtain a simple but fundamental upper bound on efficiency as a function of throughput as

$$\eta \leq \frac{\mathcal{T}}{\mathcal{T}+\mathcal{C}}. \tag{6}$$

Theorem 2: Suppose that a node transmits packets of 1 unit time duration after independent and identically distributed (i.i.d.) backoff times $(I_k)_{k\in\mathbb{N}}$ with a success probability given by p. Then, its mean AoI is given by

$$\bar{\Delta} = 1 + \frac{\mathbb{E}[I_1+1]}{2}\left(\frac{2-p}{p} + \frac{\mathbb{V}[I_1]}{\mathbb{E}[I_1+1]^2}\right), \tag{7}$$

where $\mathbb{E}[\,\cdot\,]$ and $\mathbb{V}[\,\cdot\,]$ denote the expectation and variance operators, respectively.

Proof: From [2], the mean AoI is given by $\bar{\Delta} = 1 + \frac{\mathbb{E}[J^2]}{2\,\mathbb{E}[J]}$, where J is the waiting time between successful updates and

$$J = K + \sum_{k=1}^{K} I_k = \sum_{k=1}^{K}(I_k+1),$$

where K follows a geometric distribution of the parameter p and represents the number of transmissions between successful updates, and I_k is the kth backoff time between transmissions.

Since $\{I_k\}$ are i.i.d., it is not hard to show that

$$\begin{aligned}\mathbb{E}[J] &= \mathbb{E}[K]\mathbb{E}[I_1+1],\\ \mathbb{E}\left[J^2\right] &= \mathbb{E}\left[K^2\right]\mathbb{E}[I_1+1]^2 + \mathbb{E}[K]\mathbb{V}[I_1].\end{aligned}$$

Hence, $\bar{\Delta} = 1 + \frac{\mathbb{E}[K^2]}{2\mathbb{E}[K]}\mathbb{E}[I_1+1] + \frac{\mathbb{V}[I_1]}{2\mathbb{E}[I_1+1]}$.

Substituting $\mathbb{E}[K] = 1/p$ and $\mathbb{E}[K^2] = (2-p)/p^2$ in the previous equation concludes the proof. ∎

V. Results for the ALOHA Models

This section compares the traditional ALOHA policies assuming a large population of nodes ($n > 40$, [14]). This simplifying assumption captures the fundamental behavior of the system, deliberately ignoring subtleties that would complicate the analysis, hindering an analytical comparison.

Corollary 1: Let $u > 0$. The mean AoI of UA and SA satisfy

$$\bar{\Delta}_{\mathrm{UA}} = \frac{\mathrm{e}^{2u}}{u} n + \mathcal{O}(1),$$
$$\bar{\Delta}_{\mathrm{SA}} = \frac{\mathrm{e}^{u}}{u} n + \mathcal{O}(1),$$

as $n \to \infty$, where $\mathcal{O}$ stands for the big O notation.[6]

Proof: A direct application of Theorem 2 along with the fact that $\mathbb{V}[I_1] = \mathbb{E}[I_1]^2$ for both the unslotted (exponential distribution) and slotted (geometric distribution) ALOHA scenarios, and the fact that $\mathbb{E}[I_1] = \frac{n}{u} + \mathcal{O}(1)$. ■

A consequence of Corollary 1 is that, under the specified assumptions,

$$\bar{\Delta} = \frac{n}{\mathcal{T}(u)} + \mathcal{O}(1), \tag{8}$$

as $n \to \infty$. Thus, minimizing the mean AoI is equivalent to maximizing the throughput in the large population regime. We highlight that the two metrics are not equal, even though their optimizations can be interchanged under the current assumptions. A more detailed model (e.g., with queuing or updating costs) would certainly capture the difference between the two metrics.

Now, let us consider optimizing the AoI (or throughput) for SA and UA under energy constraints. Let the set of admissible traffic values be $U_0 = \{u \in U \mid \mathcal{E}(u) \leq \mathcal{E}_0\}$, where $\mathcal{E}_0$ is a positive constant that represents the maximum total energy that can be used. We denote the minimum mean AoI achievable for UA and SA by $\bar{\Delta}^*_{\mathrm{UA}} = \min_{u \in U_0} \bar{\Delta}_{\mathrm{UA}}$ and $\bar{\Delta}^*_{\mathrm{SA}} = \min_{u \in U_0} \bar{\Delta}_{\mathrm{SA}}$, respectively.

Theorem 3: In the large population regime ($n \to \infty$), we have $\bar{\Delta}^*_{\mathrm{UA}} < \bar{\Delta}^*_{\mathrm{SA}}$ if and only if one of the following conditions hold:

1) $\mathcal{E}_0 < \frac{1}{2}$ and $\mathcal{E}_0 - \mathcal{E}_s < -W(-\mathcal{E}_0 \mathrm{e}^{-2\mathcal{E}_0})$,

2) $\mathcal{E}_0 \geq \frac{1}{2}$ and $\mathcal{E}_0 - \mathcal{E}_s < -W(\frac{-1}{2\mathrm{e}}) \approx 0.232$,

where $W(\cdot)$ is the Lambert W function defined in [16].

Proof: For ease of exposition, we omit the order term $\mathcal{O}(1)$. Let us assume that $\mathcal{E}_s < \mathcal{E}_0 < \mathcal{E}_s + 1$, otherwise the proof is trivial. Using Corollary 1, if $\mathcal{E}_0 \geq 1/2$, then $\bar{\Delta}^*_{\mathrm{UA}} = 2\mathrm{e}n$ and $\bar{\Delta}^*_{\mathrm{SA}} = \frac{\mathrm{e}^{\mathcal{E}_0 - \mathcal{E}_s}}{\mathcal{E}_0 - \mathcal{E}_s} n$. In this case, $\bar{\Delta}^*_{\mathrm{UA}} < \bar{\Delta}^*_{\mathrm{SA}}$ if and only if

$$2\mathrm{e} < \frac{\mathrm{e}^{\mathcal{E}_0 - \mathcal{E}_s}}{\mathcal{E}_0 - \mathcal{E}_s},$$

which is equivalent to condition 2 above.

On the other hand, if $\mathcal{E}_0 \leq 1/2$, then $\bar{\Delta}^*_{\mathrm{UA}} = \frac{\mathrm{e}^{2\mathcal{E}_0}}{\mathcal{E}_0} n$ and $\bar{\Delta}^*_{\mathrm{SA}} = \frac{\mathrm{e}^{\mathcal{E}_0}}{\mathcal{E}_0} n$. In this case, $\bar{\Delta}^*_{\mathrm{UA}} < \bar{\Delta}^*_{\mathrm{SA}}$ if and only if

$$\frac{\mathrm{e}^{2\mathcal{E}_0}}{\mathcal{E}_0} < \frac{\mathrm{e}^{\mathcal{E}_0 - \mathcal{E}_s}}{\mathcal{E}_0 - \mathcal{E}_s},$$

[6] Let f and g be positive functions. We write $f(n) = \mathcal{O}(g(n))$ as $n \to \infty$ if there exist $M > 0$ and $n_0 > 0$ such that $f(n) \leq M g(n)$ for all $n \geq n_0$.

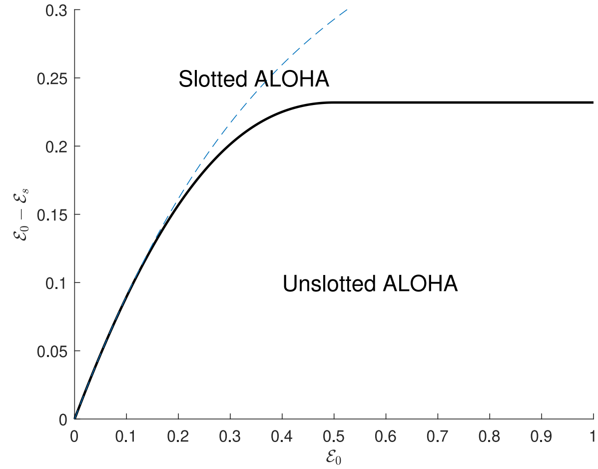

Fig. 1. Optimum operating regions between SA and UA the dashed curve is the upper bound (9) of the exact boundary given by Theorem 3.

which is equivalent to condition 1). ■

Furthermore, if

$$\mathcal{E}_0 > \sqrt{\mathcal{E}_s}\sqrt{\mathcal{E}_s + 1}, \tag{9}$$

then $\bar{\Delta}^*_{\mathrm{SA}} < \bar{\Delta}^*_{\mathrm{UA}}$. This is an upper bound on the necessary and sufficient conditions, and it is very accurate for small $\mathcal{E}_0$.

Fig. 1 presents the best protocol in the sense of minimizing the mean AoI between UA and SA given the available energy $\mathcal{E}_0$ and the available energy for transmission of packets $\mathcal{E}_0 - \mathcal{E}_s$ in slotted ALOHA according to Theorem 3.

Considering the same traffic $u \in \mathbb{R}^*_+$, the following proposition presents the condition for which the dissipated energy $\mathcal{D}$ of UA is always smaller than SA.

Proposition 1: If $\mathcal{E}_s > \mathcal{E}^*_s \approx 0.26$, then $\mathcal{D}_{\mathrm{UA}}(u) < \mathcal{D}_{\mathrm{SA}}(u)$ for every $u > 0$.

Proof: The first step is to analyze the function $f = \mathcal{D}_{\mathrm{UA}} - \mathcal{D}_{\mathrm{SA}}$,

$$f(u) = u\,\mathrm{e}^{-u}\big(1 - \mathrm{e}^{-u}\big) - \mathcal{E}_s, \quad u \in \mathbb{R}^*_+. \tag{10}$$

This function is concave in the region of interest, and therefore, its maximum point can be obtained by equating its derivative $f'(u)$ to zero and solving for the resulting equation

$$f'(u) = \mathrm{e}^{-2u}(u + (1-u)(\mathrm{e}^{u} - 1)). \tag{11}$$

The solution of $f'(u) = 0$ yields $u^* \approx 1.4456$. From (10), it is possible to see that the function may have no root, one root, or two roots, depending on the value of $\mathcal{E}_s$. By computing the value of $\mathcal{E}_s$ that leads to $f(u^*) = 0$, we have $\mathcal{E}^*_s \approx 0.2604$. Thus, $\mathcal{D}_{\mathrm{UA}} < \mathcal{D}_{\mathrm{SA}}$ when $\mathcal{E}_s > \mathcal{E}^*_s$ because (10) has no root. ■

Considering the synchronization cost for the SA protocol, the following proposition provides the achievable upper bound for the efficiency $\eta_{\mathrm{SA}} = 1 - \mathcal{D}_{\mathrm{SA}}/\mathcal{E}_{\mathrm{SA}}$.

Proposition 2: Let $\mathcal{E}_s > 0$. The efficiency of the SA protocol is upper-bounded by

$$\eta_{\mathrm{SA}} \leq (1 - \phi(\mathcal{E}_s))\mathrm{e}^{-\phi(\mathcal{E}_s)},$$

where $\phi(\mathcal{E}_s) \triangleq \frac{1}{2}\sqrt{\mathcal{E}_s}(\sqrt{\mathcal{E}_s + 4} - \sqrt{\mathcal{E}_s})$. The equality is achieved when the use of the channel $u = \phi(\mathcal{E}_s)$.

Proof: Let $u > 0$. Recall that the efficiency is given by

$$\eta_{\mathrm{SA}}(u) = 1 - \frac{\mathcal{D}_{\mathrm{SA}}(u)}{\mathcal{E}_{\mathrm{SA}}(u)} = \frac{u\,\mathrm{e}^{-u}}{u + \mathcal{E}_s}.$$

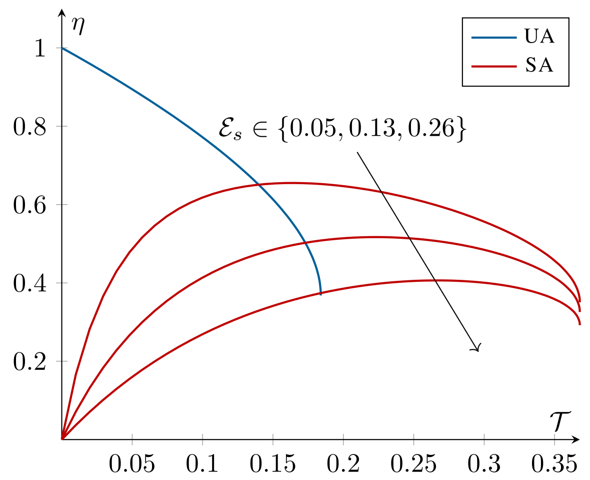

Fig. 2. η versus $\mathcal{T}$ for UA and SA for various values of $\mathcal{E}_s$.

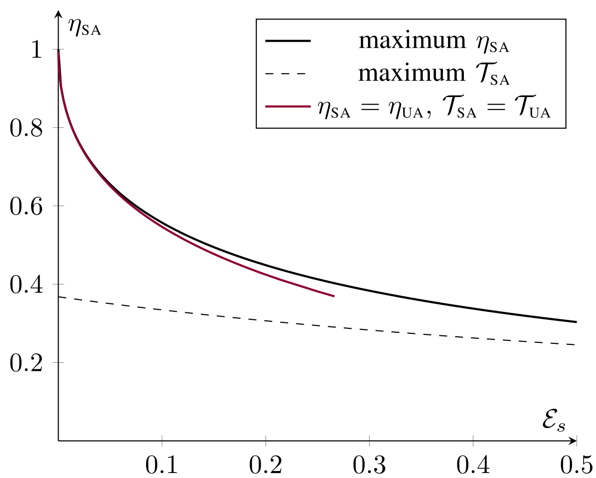

Fig. 3. Efficiency η of SA versus $\mathcal{E}_s$ for some scenarios.

Then, $\frac{\partial \eta_{\mathrm{SA}}}{\partial u} = \frac{\mathcal{E}_s(1-u)-u^2}{(\mathcal{E}_s+u)^2} \mathrm{e}^{-u}$. One can possibly check that if $u < \phi(\mathcal{E}_s)$, then $\frac{\partial \eta_{\mathrm{SA}}}{\partial u} > 0$ and if $u > \phi(\mathcal{E}_s)$, then $\frac{\partial \eta_{\mathrm{SA}}}{\partial u} < 0$. Thus, $u = \phi(\mathcal{E}_s)$ achieves the absolute maximum. ■

In Fig. 2, we show the efficiency of both the SA and UA schemes as a function of throughput for some values of the synchronization cost $\mathcal{E}_s$ of SA. It can be seen that the system becomes more efficient as the dissipated energy due to synchronization is reduced or as the actual work increases. Note that the efficiency of UA tends to 1 as $\mathcal{T} \to 0^+$ because we assumed that the energy cost $\mathcal{C}_{\mathrm{UA}}$ to maintain UA is negligible compared with the energy to transmit packets and, in this case, the assumption is no longer valid. Attributing any small constant value to $\mathcal{C}_{\mathrm{UA}}$ would make $\eta_{\mathrm{UA}} \to 0$ as $\mathcal{T} \to 0^+$.

Fig. 3 shows the efficiency η as a function of energy due to synchronization $\mathcal{E}_s$ for the SA policy considering the cases of maximizing the efficiency η, maximizing the throughput $\mathcal{T}$, and achieving the same efficiency and throughput as UA. For small $\mathcal{E}_s$, the operating regime that maximizes throughput is different from the one that maximizes efficiency. However, as $\mathcal{E}_s$ increases, they converge to the same operating regimes. Indeed, it can be noted from Proposition 2 that $u = \phi(\mathcal{E}_s) < 1$ achieves the optimum efficiency, and $u = 1$ achieves the maximum throughput. Nevertheless, $\phi(\mathcal{E}_s) \to 1^-$ as $\mathcal{E}_s \to \infty$. Furthermore, maximizing the efficiency in SA is almost equal to the operating point that has the same efficiency and throughput as UA. The plot stops at $\mathcal{E}_s \approx 0.266$ because, past this point, SA cannot simultaneously reach the same efficiency and throughput as UA.

VI. Conclusion

This letter presents a novel energy-centric approach for wireless networked systems, taking into account the energy dissipated due to synchronization that ensures a timely delivery of status updates in random access protocols. We present results for a general model, and further refine our results for unslotted and slotted ALOHA as benchmark models. Both timeliness and energy consumption are crucial to many applications, including sensor networks and the Internet of Things. Therefore, we provide an objective comparison of the random access schemes. Due to the complexity of large-scale networks, the trade-off between the novel energy-centric approach and timeliness of advanced SA protocols would be an interesting avenue for future work.

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