

equations (Pownall (1980)). A very important experimental contribution was made by Kelsall (1952) but even so this approach has not yet been fruitful.

A better understanding of cyclone performance was made possible by the adoption of the 'partition curve' which gives the fractional recovery of each particle size into each of the two products. This curve has an S-shape and does not necessarily pass through the origin. In 1953 Kelsall (1953) distinguished between recovery of particles to the underflow by the separate mechanisms of entrainment and true classification. He suggested that the recovery of particles of zero size to the underflow is equal to the recovery of the water in the feed to the cyclone to the underflow. The very fine solids in the underflow can be regarded as having 'short circuited' the classification.

Then as in sub-chapter 16.1.1, when $B = 0$

$$R_u(x) = (R_a(x) - A)/(1 - A)$$

and A is equal to the recovery of feed water in the underflow (De Kok (1977)).

Yoshioke and Hotta (1955) discovered that the corrected partition curve, under various conditions, when plotted against a dimensionless size parameter tended to coincide with a single curve which they termed the 'reduced efficiency curve'. The reference point they chose was the D_{50} size, which is the size of particle for which the corrected curve splits equally into overflow and underflow.

In over two hundred tests on cyclones varying from 100 mm to 400 mm, Lynch and Rao (1975) have confirmed Yoshioke and Hotta's (1955) observation and its general applicability. In a series of papers Lynch and his co-workers (1974, 1975, 1978) have shown that:-

- (a) The reduced efficiency curve of geometrically similar cyclones is a property of the ore and is independent of cyclone diameter.
- (b) The critical design variables are the inlet and outlet diameters, while the cyclone diameter is merely the size of the housing required to carry these diameters for normal operation.

Determining $R_u(x)$ for a cyclone requires finding three parameters A , x_{50} and C_{50} and α or β (see equations 17.11 or 17.13) all of which are dependent on the cyclone geometry and cyclone feed.

Many attempts have been made to correlate D_{50} with the cyclone geometry and feed conditions from theoretical, empirical and combined theoretical and empirical approaches. If each author can be assumed to have kept the dimensions of their test cyclones geometrically similar, so that each of the dimensions have a fixed proportionality to D_c , the cyclone diameter, then their results can be summarized in the form:-

$$D_{50} \propto D_c^a \cdot Q^b \cdot (\rho_s - \rho_L)^c \quad \text{----- (17.14)}$$

$$D_{50} \propto D_c^d \cdot P^e \cdot (\rho_s - \rho_L)^f \quad \text{----- (17.15)}$$

$$Q \propto D_c^g \cdot P^h \quad \text{----- (17.16)}$$

where Q is the volumetric feed rate

P the head or pressure drop across the cyclone

ρ_s, ρ_L the solid and liquid specific gravities and

a through h satisfy $gb+e = d, hb = e$ and $f = c$

AUTHOR	DATE	EQUATION 17.14			EQUATION 17.15			EQUATION 17.16	
		a	b	c	d	e	f	g	h
Dahlstrom	1954	1.36	-0.53	-0.5	0.408	-0.265	-0.5	1.8	0.5
Yoshioke & Hotta	1955	1.5	-0.5	-0.5					
Bradley	1955	1.5	-0.5	-0.5	0.5	-0.25	-0.5	2.0	0.5
Cheston	1958							2.0	0.5
Pavlov	1961				0.5	-0.25	-0.5		
Lilje	1982	1.5	-0.5	-0.5	0.5	-0.25	-0.5	2.0	0.5
Trzwinieki	1976	1.5	-0.5	-0.5	0.5	-0.25	-0.5	2.0	0.5
Plitt	1976	1.18	-0.45	-0.5	0.34	-0.252	-0.5	1.87	0.58

The good agreement between all authors, even though different approaches have been used, is pleasing. Lynch and Rao (1975) have also developed an empirical correlation for D_{50} , but it is not in a form suitable for comparison with the above.

The flow and water split have been carefully examined, but have so far defied theoretical treatment. The most common empirical

form has been:-

$$S = (D_U/D_O)^X$$

where $S = \frac{\text{underflow volumetric flowrate}}{\text{overflow volumetric flowrate}}$

and X varies between 3.0 and 4.4. Plitt (1976) has reviewed the literature concerning this split.

In the literature only two cyclone models exist which explicitly consider the cyclone dimensions and the feed conditions, Lynch and Rao (1975) and Plitt (1976). Both these models use the partition curve and incorporate empirical correlations of the parameters with the dimensions and feed conditions. Plitt's (1976) data incorporates that of Lynch and Rao (1975) and is thus more extensive. Plitt's (1976) model has been selected as the cyclone model for the simulator. The model can be summarized as follows:-

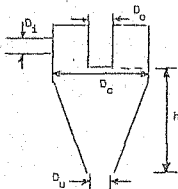
D_C = Inside diameter of cyclone measured at bottom of vortex finder

D_I = Inside diameter of cyclone inlet or $\sqrt{4A_f/\pi}$ for non-circular inlet

D_O = Inside diameter of overflow vortex finder

D_U = Inside diameter of underflow apex orifice

h = Free vortex height of cyclone, defined as the distance from the bottom of the vortex finder to the top of the apex orifice.



$$D_{50} = \frac{0.357 D_c^{0.46} D_i^{0.6} D_o^{1.21} \exp(0.063\phi)}{D_u^{0.71} h^{0.39} Q^{0.45} (\rho_s - \rho_L)^{0.5}}$$

$$S = \frac{1.9(D_u/D_o)^{3.31} h^{0.54} (D_u^2 + D_o^2)^{0.36} \exp(0.0054\phi)}{h^{0.24} D_o^{1.11}}$$

$$Q = \frac{7.0 \times 10^{-6} P^{0.56} D_c^{0.21} D_i^{0.53} h^{0.18} (D_u^2 + D_o^2)^{0.48}}{\exp(0.0031\phi)}$$

$$R_V = S/(S + 1)$$

$$R_F = (R_V - R_S\phi)/(1 - \phi)$$

$$\beta = 0.372 \exp(-1.55 R_V) \left(\frac{D_o h}{Q}\right)^{0.15}$$

and D_c, D_i, D_o, D_u, h centimetres

H	metres	pressure drop (metres of feed slurry)
Q	metres ³ /sec	volumetric flow of feed during
P	kilopascals	pressure drop across cyclone
ϕ	percent	volume fraction of solids in feed
D_{50}	micrometers	
ρ_s, ρ_L	-	specific gravity, solids & liquids
R_s, R_f, R_v	-	fractional recovery of solids, fluids, slurry to underflow

$$R_c(D) = 1 - \exp(-0.893 (D/D_{50})^\beta)$$

$$R_s(D) = (1 - R_f) R_c(D) + R_f$$

Several assumptions have been made to make the implementation of this model more efficient in terms of computer time.

- (i) The cyclone has standard dimensions

$$D_o = D_c/8$$

$$D_i = D_c/5$$

$$D_u = 0.118 D_c$$

This allows the cyclone geometry to be specified by two parameters: D_c and h/D_c .

- (ii) The feed slurry has a density of 1030 kg/m^3

In Plitt's correlation the pulp density is on average 1030 kg/m^3 and by assuming this value to be constant H can be expressed in kilopascals and is replaced by P . This helps linearize the equations while making very little difference to the results.

$$H = P/1030/9.8 \times 1000 = 0.099P$$

- (iii) $\rho_L = 1.0$

This assumes the fluid medium is water, (if brine is used then ρ_L must be changed to 1.2).

- (iv) The terms $\exp(0.0054\phi)$ and $\exp(0.0031\phi)$ equal one

This is approximately true as the maximum value of ϕ used by Plitt is $\phi = 13\%$, and this makes differences of 7% and 4% to S and P , respectively.

- (v) $R_p = R_v$

This is approximately true for dilute slurries, and if this assumption is not made internal iterations have to be done to solve for β and R_v simultaneously.

Inserting these assumptions into Plitt's model gives:-

$$D_{50} = 35.03 D_c^{0.78} h^{0.45} P^{-0.252} (\rho_g - 1)^{-0.5} \exp(0.083\phi) \quad (17.17)$$

$$S = 0.323 f_c^{-0.38} h^{0.54} P^{-0.24} \quad (17.18)$$

$$R_v = S/(S + 1) \quad (17.19)$$

$$B = 2.04 D_c^{0.044} h^{0.127} P^{-0.084} \exp(-1.58R_v) \quad (17.20)$$

$$R_c(D) = 1.0 - \exp(-0.893 (D/D_{50})^B) \quad (17.21)$$

$$R_g(D) = (1 - R_v)R_c(D) + R_v \quad (17.22)$$

where the three parameters provided by the user are:-

- (i) D_c centimetres
- (ii) h/D_c -
- (iii) P kilopascals

17.2.4 Subroutine CYCL - Cyclone Library Module

To assist the convergence of a circuit containing this highly non-linear model the derivatives of each output variable with respect to the corresponding input variable is calculated in the model subroutine. The form of the derivatives is quite involved as D_{50} is a function of ρ_g and ϕ , both of which are dependent on the feed in each category.

(i) Parameters

The parameters that follow are in the order in which they must be presented to the simulator. The values in brackets are the ranges of Plitt's data.

- (a) D_c centimeters ($3 \leq D_c \leq 50$)
- (b) h/D_c - ($0.6 \leq h/D_c \leq 2$)
- (c) P kilopascal ($50 \leq P \leq 210$)

(ii) Physical Properties

The actual size corresponding to each size interval plus the specific gravity of the material in each G-class is required. If only one G-class is used the single specific gravity provided must be that of the solids as a whole.

(iii) Units

The model calculates the fractional distribution between the two outputs, therefore the mass flows in the streams can be in any convenient set of units. However the solids and water flows must use the same unit: (e.g. both kg/hr or both tons per day) and the model parameters must be in the prescribed units.

17.3 Modelling of Gravity Separators

Gravity separators are basic to most ore dressing operations. Their roles varies from preconcentration to the production of the final concentrate. However modelling of these units has lagged behind their usage.

A dense media separator, modelled by using a partition curve, was one of the first attempts at quantifying the operation of ore-dressing equipment. The only development has been to use an empirical functional form to represent the partition curve. Loveday (1970) suggested the integral form of the Gaussian function, which was modified for easier computation. Valentik and Whitmore (1984) tried to make the model less empirical by considering the flow properties of the fluid in the dense media bath. They found that the fluids seldom have Newtonian flow properties and behave like Bingham fluids. They discuss the effects of viscosity and turbulence but do not present a model.

Much work has been done to make jigging more efficient (Zimmerman (1974,1975) and Vinogradov (1988)). Jigging can be treated as a stratification process (Meyer (1984) and Vinogradov (1988)) where the difference in potential energy before and after stratification gives a measure of the rate at which separation occurs. Meyer (1984) relates the kinetics to the jig strokes, but this does not provide a model suitable for simulation and the modeller must once again resort to the partition curve.

Improved shaking tables have been developed, like the Bartles-Mozl Concentrator and the Bartles crossbelt concentrator, but their operation has only been described in qualitative terms.

Sluices, spirals and Reichert cones are frequently used but virtually

no quantitative theory on their operation has been proposed (Naguib' (1971)).

The only quantitative theory found in the literature is by Vinogradov et al (1968). The kinetics of gravity processing in mineral separations is considered and differential equations are developed, these can be solved to give the terminal sinking rates of the particles. It may also be possible to use this theory for jigs, spirals and shaking tables once a transport model is developed to go with the separation model.

In general gravity separator models are poorly developed and it is necessary to use fixed partition curves to describe their operation at present. The development of a mechanistic model is likely to be slow as the fluid flows are complex and hence transport models difficult to find.

17.4 Modelling of Sorters and Electrostatic and Magnetic Separators

The history of separation methods using electro-magnetic radiation has been thoroughly reviewed by Hall-Carpenter (1970). It is apparent that these methods have existed for a long time and that the physics of the processes seem well understood.

Hopstock (1975) describes the physics of low-intensity dry magnetic separators. Using several simplifying assumptions he is able to balance the centrifugal and magnetic forces to obtain a relationship between the percentage of iron in the particles which stick to the drum and the speed of rotation of the drum. Reasonable agreement is found between the experimental and theoretical results.

Lewver and Hopstock (1974) review the principles of wet high intensity magnetic separation. Similarly to Hopstock (1975) they balance the magnetic and hydraulic drag forces while assuming gravitational forces to be negligible. They conclude that 25 μm is the smallest size particle that can be retained in a matrix. They also show that the apparent volume susceptibility of a mixed particle is the weighted sum (by volume fractions) of its component susceptibilities. Lewver and Hopstock (1974) give a good description of the different types of WHIMS machines available and their uses.

Certa et al (1964) conducted many experiments to determine the effect of changing electrical field, air pressure, position of electrodes, splitters, drum speed and diameter, voltage and mineral size distribution on a laboratory electrostatic separator. They found that operation at reduced air pressure is more efficient, particularly for the larger sizes, and concluded that the electrical factors are more important than the mechanical ones.

Morrison (1974) created a model of the high intensity roll separator by considering the subprocesses; selection at the earthed rotor surface, electrical force, the image force, the centrifugal force, the selection by trajectory and the effect of the corona region. However Morrison states that no solution is known for the equations he developed. On top of this field and charge density measurements are difficult to make, and Morrison suggests that a distribution of conductivities in each mineral species should be considered. Morrison eventually resorts to a simple probability approach for the conductor/non-conductor split and gets good results.

The mechanistic models discussed for both magnetic and electrostatic separators are complex and their solution requires much computational effort. Even if the equations can be solved their results only represent an ideal separator in which all particles in a particular category will report to the same output stream. So although the physics is essential for the design of the equipment, from a simulation point of view one might as well choose, on the basis of experimental work, a reasonable partition curve with the desired cut value, and then adjust the machine variables to achieve this value.

The modelling of sorters is reviewed by King (1977) and it seems that very little has been done. King proposes a model based on the sum of the product of various probabilities like: the probability of recognition and probability of deflection. This model results in a fixed partition curve for the sorter.

17.5 Modelling of Flotation

In assessing the probability of a particle being floated, Schuhmann (1942) according to Woodburn (1970) assumed that:-

$$P_{\text{overall}} = P_c \cdot P_a \cdot P_r$$

where P_c = probability of a successful bubble-particle collision
 P_a = probability of a collided particle adhering
 P_r = probability of the particle being retained by the bubble

Townlinson and Fleming (1953) redefined P_r as $P_a \cdot P_f$ where
 P_a = probability of the particle being retained while in the liquid
and P_f = probability of the particle being retained while in the froth.
Davis (1964) included another term which considers the probability of a particle being coated by collector.

The determination of each of these probabilities has been reviewed by Mika and Fuerstaneu (1969), Woodburn (1970) and Flint (1974).

Once an overall probability has been established it is possible using population balance models to compute the rates of flotation. This has been done by Kelsall (1981) and Davis (1984). The cumulative recovery of a mineral species (R) from m flotation cells is given as:-

$$R = M \{1 - (1 - P)^m\} \quad \text{----- (17.23)}$$

where M is the maximum possible recovery and P the probability of flotation for the particular species.

The complicated nature of the various subprocesses makes it impossible to obtain values for their probabilities and Davis (1984) had to be satisfied with an overall probability for each mineral in each bank of cells.

Since it is desirable to determine model parameters from laboratory tests a kinetic model has been used to account for the dynamic nature of the process. According to Mika and Fuerstaneu (1969) - 'A consensus has seems to have been reached that flotation of a sufficiently homogeneous assembly of particles occur at a rate which is proportional to the population density of particles'. This results in a 'first order law' for flotation and although it is an empirical 'law', testwork by Bull (1965), Imaizumi and Inoue (1963), Tomlinson and Fleming (1963), Jowett (1971) and Loveday and Merchant (1973) prove its validity in many cases. Then if W_i is a 'sufficiently homogeneous assembly' the 'law' can be expressed as:-

$$r_i = \frac{dW_i}{dt} = k_i W_i \quad i = 1, n \quad \text{----- (17.24)}$$

where r_i is the rate of flotation of species i

k_i is the flotation rate constant for species i

and n the number of species

Consider a single flotation cell operating at steady state

with F_i the mass flowrate of component i in the feed and $F = \sum_{i=1}^n F_i$

C_i the mass flowrate of component i in the concentrate and $C = \sum_{i=1}^n C_i$

T_i the mass flowrate of component i in the tail and $T = \sum_{i=1}^n T_i$

W_i the mass of component i in the cell and $W = \sum_{i=1}^n W_i$

A mass balance over the cell for each component gives:-

$$F_i = C_i + T_i \quad \text{-----} \quad (17.25)$$

and as the cell at steady state is equivalent to a batch cell at some instant in time (Bull (1969)); by equation 17.24

$$C_i = k_i W_i \quad \text{-----} \quad (17.26)$$

To find the steady state values of C_i , T_i and W_i for fixed k_i and F_i for each i , another relationship is required. This missing relationship is the residence time distribution of the species in the cell. Woodburn (1970) quoting Miami and Jowett finds that the assumption of perfect mixing is adequate. Harris et al (1975) found that a Denver laboratory flotation cell behaves as a perfect mixer except at flowrates much beyond the natural capacity. Applying this to a flotation cell implies that the concentration of a species in the cell must be the same as the concentration of the species in the tail.

$$\text{i.e.} \quad W_i/W = T_i/T \quad \text{-----} \quad (17.26)$$

and inserting this into equation 17.24 and 17.25 gives:-

$$\left. \begin{aligned} C_i/F_i &= \frac{k_i \tau}{1 + k_i \tau} \\ T_i/F_i &= \frac{1}{1 + k_i \tau} \\ W_i/F_i &= \frac{\tau}{1 + k_i \tau} \end{aligned} \right\} \quad \text{-----} \quad (17.27)$$

where $\tau = W/T$ and is called the mean residence time.

If k_i and τ are given then each C_i , T_i and W_i can be directly obtained using equation 17.27, however if W instead of τ is given then simultaneous solution is required. (This case will arise if the cell volume (V) and the percentage solids in the underflow ($\%S$) are specified, so that $W = V \times \%S / (\%S(\frac{1}{\rho_s} - \frac{1}{\rho_w}) + \frac{100}{\rho_w})$ and ρ_s and ρ_w are the densities of the solids and water respectively (the units of V , W and ρ must be consistent)).

Now although there may be consensus on the 'first order-law' there does not seem to be any agreement on what is a 'sufficiently homogeneous assembly' of particles. If one took this to the extreme of assigning each particle a flotation rate constant then because of the large number of particles one would expect a continuous distribution. This approach has been taken by Pogorly (1962), and Imaizumi and Inoue (1983) while Loveday (1966) has suggested that this distribution can be well represented by a gamma function.

Another method of choosing a homogeneous assembly of particles would be to assume that each mineral species has a particular rate constant. This method which discretizes particles into G-classes has been used by Davis (1984) with some success. It has however been well established by several authors including Imaizumi and Inoue (1988), Woodburn, King and Colborn (1971), Maharotra and Kapur (1975), Tomlinson and Fleming (1967) and Bartlett and Mular (1974) that particle size has a significant effect on the flotation rate. Therefore splitting the particles into categories associated with discrete size and grade (D and G) classes, as suggested by Tomlinson and Fleming (1963) and Bartlett and Mular (1974), each with its own flotation rate constant should be advantageous. Tomlinson and Fleming obtained an excellent correspondence between the experimental and predicted values for the recovery of pure apatite and hematite from a mixture of the two minerals.

Even if the particles have a fixed size and grade it is still possible for there to be a variation in surface properties and so flotation rate within the DG-class. For example the collector will not necessarily coat exactly the same fraction of the surface of each particle; or because of variation in particle shape or surface roughness the surface area of the same size particles could be different; or in mixed particles of fixed mineralogy the surface area occupied by a certain mineral will vary from particle to particle. This implies that with each D-class or G-class or DG-class a further distribution of particles into assemblies of particles with a common flotation rate constant should be done. Loveday and Marchant (1973) have done this using a gamma function to describe the distribution in each DG-class. King (1973), although analysing the problem using continuous size and grade classes, calculates with discrete DG-classes and uses the gamma function to describe the variation within the DG-classes. King (1975) later changed to a fixed discrete distribution of particles between S-classes in each DG-class. Sutherland (1977) goes the whole way and discretizes the particles into DGS-classes and associates

a flotation rate constant with every DGS-class. This is subtly different from King's (1975) method of associating each S-class with a flotation rate constant. Scrimgeour, Hamilton and Toong (1970) and Bull (1988) also associate the flotation rate constant with the S-class and let the distribution of mass in each S-class cause the difference in flotation rate between DG-classes.

The use of a fixed function or a fixed discrete distribution to describe the distribution of particles between S-classes can lead to misleading results. This happens because particles with a certain flotation rate are selectively removed and the shape of the distribution changes, so causing the fixed distribution to become more and more incorrect as the plant becomes more complex. This fault can often be obscured as the method of determining the flotation rate constants may compensate for the error. Therefore it appears that carrying discrete DGS-classes in the plant streams allows an accurate mass balance while providing sufficiently homogeneous assemblies of particles for the first order law of flotation to apply.

Whether the modeller associates the flotation rate constants with each DGS-category or only with the S-classes will depend on how he determines these rate constants. The former seems to offer more flexibility for a little extra storage, as once the rate constants for each DGS-class have been determined for one feed they will apply to any other feed, whereas for the latter a change in feed will require more testwork to determine the new mass distribution between S-classes.

It has been implicit in the preceding paragraphs that conditions of 'free flotation' exist. However it has been observed (Tomlinson and Fleming (1983), King (1973)) that there is a limit to the amount of material that can be floated in a single cell. This can be related to the carrying capacity of the bubbles and the ability of the froth to retain floated particles, which in turn can be related to the cell operating conditions; cell volume, machine type, degree of short circuiting, impeller speed, machine geometry, degree and type of aeration, froth column depth. This implies that under conditions of 'hindered flotation', k_1 of equation 17.27 is not a constant but dependent on operating conditions so that:-

$$k_1 = k_1' \cdot \psi \quad (17.28)$$

where ψ is a parameter that varies between zero and one. Tomlinson and Fleming (1983) suggest that under conditions of 'hindered flotation' the

rate of flotation will be zero order, however they do not show how or when the changeover should take place.

King (1973) sets $\Psi = \gamma A S_{AV}$ on the assumption that the flotation rate is proportional to the free bubble surface area, the concept of 'free bubble surface area' is discussed by Mika and Fuerstenau (1969). A is the bubble surface area per unit volume of pulp and S_{AV} the fraction of free bubble surface area on average, both of which are calculated from operating variables. γ is the fraction of particles which on reaching the froth phase are recovered in the concentrate, it is a function of pulp level, particle size and mineralogy. If pulp level and aeration rate are used to control the cell then γ and A can be used as control variables. S_{AV} is determined iteratively within the model.

Sutherland (1977) suggests that because of the many uncertainties inherent in plant applications that such detailed modelling is not justified. He treats hindered flotation empirically by stating that some limit must be placed on the rate of concentrate flow from a cell, and that this is linked to the saturation of bubbles with particles. This implies a lowering of the flotation rate constant. First a limit, C_{MAX} , is set and then the total concentrate flowrate, $CTOT = EC_1$, without suppression, is found.

If $CTOT \leq C_{MAX}$ $\Psi = 1$
 If $CTOT > C_{MAX}$ $\Psi < 1$

and Ψ is found iteratively so that $EC_1 = 2 \cdot C_{MAX} (1 - C_{MAX}/CTOT/2)$ is satisfied. Although EC_1 will be greater than C_{MAX} , it increases slowly with $CTOT$ and so avoids having a discontinuity.

Two library modules for flotation have been programmed for the simulator. The first is based on Sutherland's (1977) model and should be useful in design stages as it does not require much data, outside the flotation rate constants, that cannot be estimated by the modeller. The second module is based on King's (1975) flotation model in so far it uses the parameters γ , A and S_{AV} . However for the library module the variables carried in the streams are discrete DGS-class mass flows and the flotation rate constants, (although they can be obtained using King's (1975) technique of $k_{DGS} = k_S^0 \phi_D$) have the added possibility of being obtained as:

$k_{DGS} = k_S^0 \phi_D$ (The subscripts D, G and S indicate the material associated with the relevant discrete category, so that k_{DGS} is the flotation rate constant for a discrete DGS-class while k_S^0 implies that k_S^0 is related only to the discrete S-classes. ϕ_D has been defined as the fractional efficiency of flotation of particles of size D, so

that ϕ_{DG} will be the fractional efficiency of flotation of particles in each discrete DG-class relative to $k'_{G,1}$).

17.5.1 Subroutine FLTN - Flotation Library Module (Sutherland)

The subroutine has been programmed to accept a variety of data so that the user can specify his cell in whatever way is most appropriate to his problem. To completely specify the cell, for this model, any two of the following four parameters must be provided:-

- (a) Cell residence time
- (b) Cell solids holdup
- (c) Cell volume
- (d) Percent solids in the tail (underflow)

with the exception that if only residence time and solids holdup is specified this does not provide information on the water split and so another parameter is required. Zeros must be placed in the position of those parameters which are not being provided.

An advantage in using this module is that it can model a bank of cells, so that the process flow diagram will require fewer streams. Thus storage and computational effort is saved.

(i) Parameters

The full list of parameters required, in order are:-

- (a) Number of cells in bank
- (b) Cell residence time
- (c) Cell solids holdup
- (d) Cell volume
- (e) Percent solids in underflow
- (f) Maximum concentrate (float) flow in each cell
- (g) Physical property number of flotation rate constants for each OGS-class. (The order must be similar to that required in paragraph 14.4.2 (V1) when $I2 = 1$).

(ii) Physical Properties

- (a) k_{DGS} the flotation rate constant for each OGS-class (elements in the same order as stream variables).
- (b) When cell volume is an essential parameter for specifying the cell then the specific gravity of each G-class must be available as property data.

(iii) Units

Any units can be used as long as they are consistent. If the specific gravity is used care should be taken in the choice of units for mass and volume.

17.5.2 Subroutine FLTK - Flotation Library Module (King)

This module is based on the modified form of King's (1973) model as discussed in previous paragraphs.

(i) Parameters

All of the following nine parameters/operating variables must be provided in sequence:-

- (a) Cell volume V (m^3)
- (b) Aeration rate G (m^3/S)
- (c) Fractional efficiency froth recovery γ
- (d) Bubble size (m)
- (e) Bubble residence time τ (S)
- (f) Percentage solids to concentrate
- (g) Physical property number associated with k'_S
- (h) Physical property number associated with ϕ_{DG}

(ii) Physical Properties

- (a) k'_S the flotation rate for each S-class
- (b) ϕ_{DG} . This array is given as a string, in which the D-classes vary with each G-class in order
- (c) The actual size of the particles in each D-class
- (d) The specific gravity of material for each G-class as property one

(iii) Units

The units are as for the parameters and the stream mass flows should be in kg/S . The flotation rate constants must have units of S^{-1} .

17.6 Modelling of Solid/Liquid Separators

Over the past decades several techniques for finding the required area of a thickener from batch settling tests have been developed. Notable are those of Coe and Clevenger (1966), Kynch (1952), Talmage and Fitch (1955) and Yohioka et al (1957). More recently contributions related to flocculated pulps and pulp compression have been made by Scott (1968), Adorjan (1975) and Koo (1974). A very comprehensive review of

gravity thickening has been done by Pearse (1977) and detailed discussion of the pro's and cons of the above techniques is given.

In real continuous thickeners steady state does not appear to be maintained for any length of time (Fitch (1968), Hassett (1969)) due to variations in feed volume and solids concentrations. Hassett (1969) states that both solid and liquid overloading can cause a dirty overflow, therefore what has been done is to overdesign the thickener to handle surges and keep a clear overflow.

Designing a continuous thickener is often done by running a continuous pilot plant to discover what feed flowrate can be coped with by one square meter of thickener and then simply scaling this up to the full scale plant. This method may be reliable but it is expensive and time consuming. Sedimentation theory has been used by Blake and Colombara (1977) and Woude, Rietma and Ottengref (1977) to describe the particle motion in batch and continuous thickeners, while Farrar, Lim, Koppel and Gredy (1974) using Kynch's theory and a plug flow model calculate thickening capacity. Kos (1974) presents a model which can predict the solids concentration profile in a continuous thickener.

Continuous thickening involves two well studied phenomena, namely sedimentation and coagulation and although improvements in the understanding of the mechanism of thickening has evolved and many models proposed, none are able to completely predict the performance of continuous thickeners from laboratory tests. This is born out by Pearse (1977) who states that the 'state of the art' is best conveyed by the following quotation:-

'We don't as yet have any theoretically solid or empirically process way of predicting thickener performance, completely and validly from measurements made on small samples of suspension'. (B. Fitch, Chief Scientist, Dorr-Oliver Inc., 1975).

Because of the uncertainties in design procedures most thickeners are overdesigned, and operated to ensure a clear overflow. During the design stage a suitable percentage solids for the underflow is selected and the thickener is then operated to keep to the designed value, but even if there is a surge in the feed, the built in overdesign will usually allow the overflow to remain clear while the underflow retains the designed percent solids. If the thickener is overloaded the usual remedy would be to increase the underflow flow rate but this will decrease the percent solids in the underflow. However this transient overloading cannot be

handled by a steady state simulator therefore for almost any real steady state simulation specifying; the overflow must be clear and the percent solids in the underflow, will be adequate. This type of model can be handled by the MIXR subroutine, when the percentage solids in the product stream is specified. The water addition rate will be negative, in this case, indicating water removal. Filters can be similarly specified.

If the user wishes to use more mechanistic models, as described by Pearce (1977) for thickeners or by Tiller (1971), Nemeth (1967) and Wakeman (1976) using O'arcy's Law and the Kozeny-Carmen equation for filters, the user should write his own model subroutine.

By using the simple MIXR module to simulate solid/liquid separators in the overall circuit a reasonably good result can be obtained without the attendant inefficiencies of a general purpose model requiring the solution of differential equations. While the detailed user written solid/liquid separation model can be specific to the user's requirements and use the calculated mass flows from the simulation.

PART IV

TEST EXAMPLES

18 TEST EXAMPLE I

In this example many points concerning the simulator usage are demonstrated. However the main purposes are to show that: the coding is correct, the connection between plants and the operation of the converter, the variety of outputs available, and the convergence of the reduced Newton method for linear systems.

The example is divided into five sections, each of which will be used to demonstrate particular facets of the simulator. The remarks on each section should be read in conjunction with the flow diagram and computer printout.

16.1 Flow Diagram

The flow diagram is given in figure 25. From this diagram it can be seen that:-

- (i) There are three separate plants
- (ii) There are four connections between plants
- (iii) Each plant has two nodes and one of the nodes in plant 4 is simple
- (iv) The numbering of units is arbitrary
- (v) The numbering of streams is arbitrary
- (vi) Each plant has several feeds and outputs
- (vii) Some nodes have water feeds
- (viii) The nodes have various types of interlinked recycle loops

16.2 Data

The following comments should be read in conjunction with chapter 14 and computer display of the data, plus the computer echo of the data.

(a) System Data

- (i) The use of all KEYWORDS except READ and TOLM
- (ii) Plant 3 has 4 D-classes, 3 G-classes and 1 S-class; plant 2: 3,3,2 D,G and S-classes; and plant 4:3,1,1 D,G and S-classes. (Keyword NOGS)
- (iii) The converter has to drop the top size fraction in going from plant 3 to both plant 2 and plant 4. In converting from plant 3 to plant 2 the S-classes must be expanded from 1 to 2. (Keyword CNCT and NOGS)
- (iv) The mineral names are VALU and WSTE. (Keyword MINS)
- (v) The first size class is the largest. (Keyword SIZE)
- (vi) The specific gravity is given so that the grade related to volume can be calculated from the grade related to mass. (Keyword SPGR, GRDM)

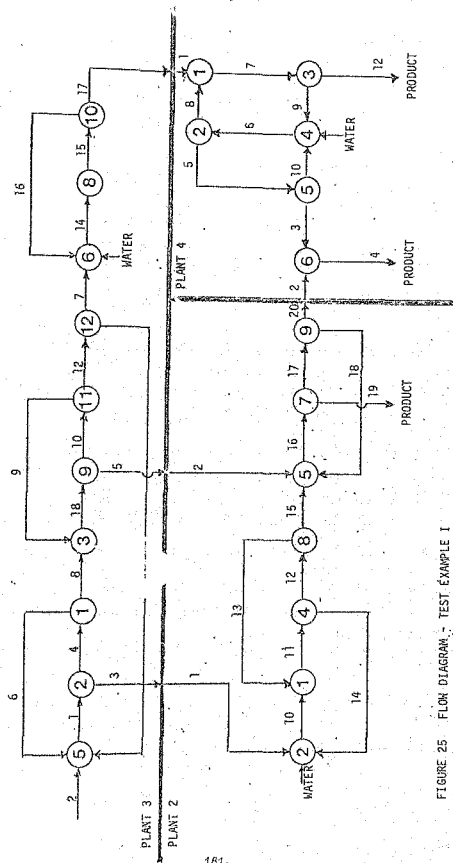


FIGURE 25 FLOW DIAGRAM TEST EXAMPLE I

- (vii) Physical property 3 is given directly. (Keyword PHYP)
- (viii) Physical property 4 is given as values for each mineral species, then related to the 3 grade classes by inverse volume fractions. (Keyword PHYP, code 1004)

(b) Plant Data

- (i) The stream connection matrix. (Keyword SCMA)
- (ii) Fixed water feed and variable water feed. (Keyword WTRF)
- (iii) Solid feed as mass fractions related to O, S and S-classes. (Keyword STRM) - plant 3 stream 2
- (iv) Solid feed used to give the converter the information of how to expand to S-classes in plant 2, streams 1 and 2. (Keyword STRM)

(c) Run Data

- (i) Plants 3 and 4 have two runs each
- (ii) Use of TYPE keyword to give unit type and parameters
- (iii) Use of COPY keyword to save repeating data
- (iv) Use of default values of CONV, ITER, TOLM and OUTC
- (v) Changing the unit parameters, convergence method, allowable iterations, and output code
- (vi) Various uses of TRMP units. Basing separation on individual and combined classes
- (vii) Variables for CRSH unit

16.3 Input Data Echo

In this section the results of many of the input data features are emphasized. The correct data reading, modification and echoing is also demonstrated. Besides the correct echoing of data which is easily verified some modifications of data by the computer are given below:-

- (i) The volume fractions in each G-class have been calculated from the mass fractions of each mineral in each G-class and the specific gravities
- (ii) The specific gravity has automatically become physical property 1, and expanded to fit the S-classes
- (iii) Physical property 4 has been expanded according to inverse volume fractions (expansion code 1004)

$$\text{e.g. } 1.0 / \left(\frac{.272}{20.0} + \frac{.728}{16.0} \right) = 16.92$$

- (iv) Stream mass flows of each category have been calculated from total mass flow and mass fractions
- (v) The correct default values CONV = 3, ITER = 8, TOLM = 10^{-4} and OUTC = 0 0 0 1 0 0 1 6 6 6 are used when the value is not explicitly specified
- (vi) The keyword COPY does copy the units requested correctly e.g. for plant 3 the parameters for unit 11 have been copied from those of unit 9, and so are identical

18.4 Results of ORDERING Subroutine

(See the computer display for these results).

Several conclusions can be drawn from these results, which have been displayed by the computer:

- (i) The program correctly groups each maximal net into a node. These are:-
 Plant 3 Node 1 Units 1,2,3,5,8,11,12
 Plant 3 Node 2 Units 6,8,10
 Plant 2 Node 1 Units 1,2,4,6
 Plant 2 Node 2 Units 5,7,9
 Plant 4 Node 1 Units 1,2,3,4,5
 Plant 4 Node 2 Unit 6
- (ii) Tears have been found for each node which are consistent with the weighting criteria. That is for plant 3 and 4 minimum number of tears, with no double tears. Plant 2, streams 13, 14 and 18 were given as preferential tears and these streams have been chosen as tears.
- (iii) The unit order calculated is feasible and consistent with the tear streams, as starting with these feeds and tears all inputs are available to each of the units when it is their turn to calculate outputs.

18.5 Results of CALCULATION Phase

(See the computer display for these results).

- (i) The important point to note from these results is the fact that the mass balance is true for the system, each plant node and each unit, and is consistent with the data. This mass

balance is true for total solids, solids in each category and water.

- (ii) Another important result is that as the plant is linear it is expected that each complex node will converge in as many iterations as there are transformation classes. This can be seen to be the case as:-

Plant 3 Node 1 Converges in 1 iteration
Plant 2 Node 1 Converges in 1 iteration
and Plant 2 Node 2 Converges in 1 iteration
while Plant 3 Node 2 Converges in 4 iterations (there are 4 size classes and the CRSH unit transforms material between the 4 classes).

Plant 4 Node 1 uses direct substitution for the first three iterations, and although it is converging it does not reach the required tolerance within the allowed three iterations. In the second run, with different unit parameters, the reduced Newton method is used and the solids converge in 1 iteration and the second iteration is required to correct the non-linear water balance. It should be noted that if the water was fed to unit 1 instead of unit 4 then even though the solids will converge in one iteration the water balance will take several further iterations to converge. This is because once the solids converge the water in stream 7 and hence stream 9 will be fixed but the loop with streams 8, 5 and 10 will not be fixed and iteration is necessary to fix the water in stream 8, (the tear stream which will then fix the water flows in all the other streams in the node). By having the water feed on unit 4, once the solids converge the water in stream 8 is fixed and so in turn are the rest, which means only one further iteration is required to complete the mass balance. From a convergence point of view it is more efficient to use fixed water feeds, however if this is not possible the variable water feeds should feed units whose outputs are tear streams or directly connected to tear streams.

- (iii) The ORDERING phase also calculated the dimensions of arrays, the paths for products of derivatives and indices for storage of arrays which are used in the calculation phase. The correct results obtained in this example show the correctness of the coding of the relevant sections of the ORDERING phase.
- (iv) The conversion of plant 3 streams 3, 5 and 17 to plant 2 streams 1 and 2 and plant 4 stream 1, respectively is done properly. Stream 1 plant 2 is identical to stream 3 plant 3 in total solids and water as there is no mass in size class 1. The other two connections do have solids in size class 1 and this mass is lost according to the simulator. When a conversion of this type is made the user should check the loss and if it is significant take steps to improve the simulation. This can be done by either changing the feeding plant to ensure less material in the dropped size classes or alternatively including the size classes in the plant that is being fed.
- (v) The operation of the converter in splitting plant 2 streams 1 and 2 from one to two S-classes is done properly.
- (vi) The variable water feed to plant 4 is negative and hence to obtain the required 20% solids in stream 5 instead adding water to unit 4 it must be removed.
- (vii) One of the main purposes of this example is to demonstrate the variety of output forms available to the user. Each run for each plant uses a different output code. In order to recognize the differences the output can be examined in conjunction with the relevant section on 'Usage of the Simulator' and the corresponding output code (chapter 14).

19 TEST EXAMPLE II

This example is based on the demonstration plant used by King (1973) in describing his flotation simulation. The flow diagram of this plant is given in figure 26.

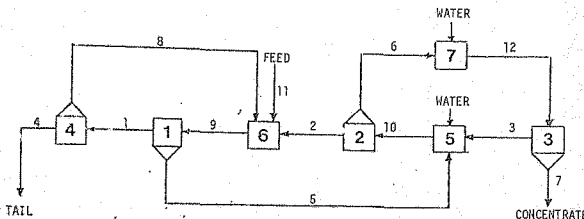


FIGURE 26. FLOW DIAGRAM - TEST EXAMPLE II

In this plant unit 1 is the rougher, unit 4 the scavenger, unit 2 the cleaner and unit 3 the recleaner. Each of these units which are modelled as single cells represent banks of cells in a real plant.

King's simulator uses a Newton Raphson procedure on the residence times of the cells to find a solution. This technique is completely different from the convergence method used in the general simulator. Using King's flotation model and the identical parameters exactly the same results were obtained. (i.e. the solids and water flows, the grades, recoveries in all streams, residence time and bubble loading in each cell). From this example it can be concluded that:-

- (i) The general simulator coding is correct, as identical mass balance results are obtained.
- (ii) The reduced Newton convergence technique works very satisfactorily on non-linear systems as well as linear ones. (This plant took 5 iterations to converge, compared to the two iterations that would have been taken if the cell model was linear - the second iteration is to calculate the variable water feeds).

- (iii) The tearing and ordering subroutines operated correctly.
- (iv) It should be noted that the King model does not provide for derivatives to the executive program and convergence is achieved using mass fractions to approximate the derivatives.

The actual computer printout is not presented as it adds nothing extra in the way of explanation.

20 TEST EXAMPLE III

In the previous two examples hypothetical plants were used to show that the coding of the simulator is correct and that the reduced Newton convergence method converges as expected on both linear and non-linear systems. This example takes a real plant to show that the simulator does work in a real situation. A convenient system is given in the literature by Sutherland (1977). He has used data measured by Cameron et al (1971) from the Broken Hill lead concentrator and added some of his own data to fit to his flotation plant simulator.

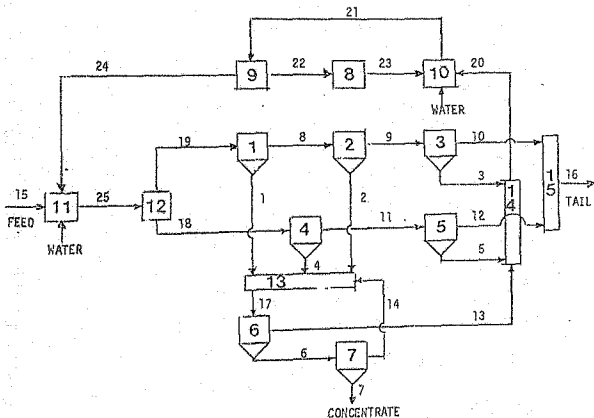


FIGURE 27 FLOW DIAGRAM - TEST EXAMPLE III

In figure 27 units 1 to 7 are banks of flotation cells. The number

of cells in each bank are 3, 3, 9, 7, 6, 4 and 3, respectively. Unit 8 is a comminution unit, unit 9 a cyclone, unit 12 a splitter and the rest mixers.

The flotation cell model is based on that of Sutherland and is described fully in the section on 'flotation modelling' (17.5). Sutherland (1977) in his paper uses a slow and fast flotation rate constant for each size/grade category. Four size and three grade classes have been used so that each stream has twenty four discrete categories ($4 \times 3 \times 2$) to describe the solids flow. The maximum concentrate flow for each cell is set, making the system non-linear.

Sutherland simply represented the regrind by defining a fixed size distribution of material being returned from the regrind, so that each mineral component feeding the regrind was redistributed according to this fixed distribution and then each size/mineral fraction was subdivided into fast and slow floating components as in the original feed material. This is clearly unsatisfactory from the point of view of a general purpose simulator. For this example the regrind is represented by a mill in closed circuit with a cyclone (units 8, 9 and 10). The mill is represented by a CRSH model with the added feature that breakage of slow floating galena results in the broken material becoming fast floating, while the breakage of fast floating zinc and gangue results in the broken material becoming slow floating. (The three G-classes being galena, marmetite and gangue).

The plant described was simulated using guessed parameters, it converged satisfactorily and the mass balance was correct. Unfortunately Sutherland did not publish the parameters used by him in his simulation so it was not possible to obtain the flows and recoveries presented by him. However the convergence and correct mass balance achieved by the simulator on a real system proves its effectiveness.

Once again the computer printouts will give no extra relevant information, so are not presented.

21 TEST EXAMPLE IV

Besides the normal design or simulation calculations the simulator can also be used to test theories about the relative merits of various processing schemes. An example of this is given by Engelbrecht (1977) who takes a conventional milling circuit (figure 28) and then claims on qualitative grounds that the circuit shown in figure 29, which has a lower total cost per ton of ore treated, will reduce the slimes produced while at the same time giving increased throughput.

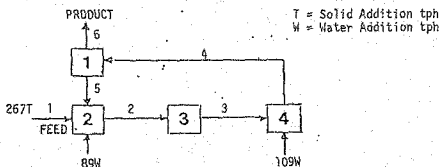


FIGURE 28 FLOW DIAGRAM - CONVENTIONAL MILLING CIRCUIT

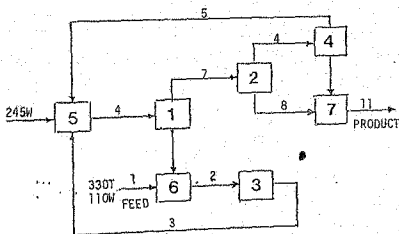


FIGURE 29 FLOW DIAGRAM - RECOMMENDED MILLING CIRCUIT

In figure 28 unit 3 is the mill and unit 1 the cyclone, unit 2 and 4 are mixers. In figure 29 unit 3 is the mill, units 1 and 2 cyclones, unit 4 a splitter and the rest mixers. For both cases the same feed size distribution was assumed. The residence time of the mill in the recommended flowsheet was set at 80% of that in the conventional circuit in accordance with the flows into each (for the same size mill). The cyclone parameters in both circuits were then adjusted until the claimed flows in both circuits were obtained. The % passing in size of the product stream was then plotted for each circuit. The results are shown in table 16 and graph 5 and it is clear that even at the increased throughput (330 t compared to 267 t) a much closer size distribution of product is obtained with far less -30 μ m material.

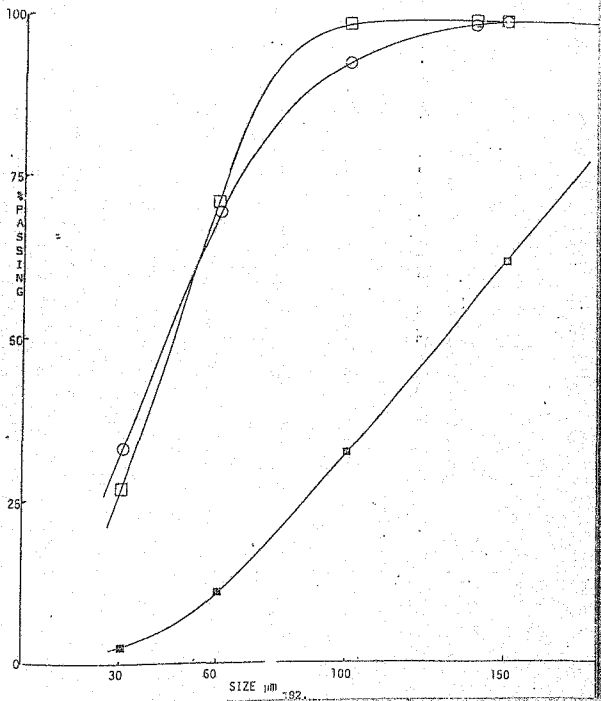
TABLE 16 RESULTS OF SIMULATION - EXAMPLE IV

SIZE μ m	FEED % PASSING	CONVENTIONAL % PASSING	RECOMMENDED % PASSING
30	2.6	33.3	28.8
60	10.1	70.0	71.4
100	32.6	93.5	99.4
150	62.5	99.7	100.0
200	100.0	100.0	100.0

Therefore by using the simulator it has been possible to quantitatively demonstrate the theory which had only qualitative backing previously.

GRAPH 5 PERCENT PASSING VERSUS SIZE (TEST EXAMPLE IV)

- CONVENTIONAL MILLING PRODUCT
- RECOMMENDED MILLING PRODUCT
- FEED



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APPENDICES

APPENDIX A CONVERGENCE OF REDUCED NEWTON METHOD

Some of the facts which are used are summarized below.

The statement $B \geq 0$ implies that each element of B is greater than or equal to zero. The statement $|B|$ is the matrix with each element the absolute value of the corresponding element in B .

(a) Neumann Lemma (Ortega and Rheinboldt (1970) p45)

Let $Bel(R^n)$ and assume $\rho(B) < 1$. Then $(I-B)^{-1}$ exists and

$$(I-B)^{-1} = \lim_{k \rightarrow \infty} \sum_{i=0}^k (B)^i \quad ((B)^i \text{ is } B \text{ to the power } i)$$

(b) Let $Bel(R^n)$ and assume $B \geq 0$. Then $(I-B)^{-1}$ exists and is non-negative if and only if $\rho(B) < 1$. (Ortega and Rheinboldt (1970) p53).

(c) Let $A, B \in L(R^n)$. Then $A=B-C$ is a 'regular splitting' of A if B is invertible, $B^{-1} \geq 0$, and $C \geq 0$; it is a 'weak regular splitting' if the condition $C \geq 0$ is replaced by $B^{-1} \cdot C \geq 0$ and $C \cdot B^{-1} \geq 0$ (Ortega and Rheinboldt (1970) p56).

(d) Let $A \in L(R^n)$ and suppose that $A=B-C$ is a weak regular splitting. Then $\rho(B^{-1} \cdot C) < 1$ if and only if A^{-1} exists and is non-negative. (Ortega and Rheinboldt (1970) p56).

(e) Let $A, B \in L(R^n)$ and $|a_{ij}| < |b_{ij}|$ for all i, j , then $\rho(A) < \rho(B)$. (Varga (1962) p 48).

(f) Let $A, B \in L(R^n)$ then $\rho(AB) \leq \rho(|A| |B|)$. (Ta. (1964) p127).

(g) As $|AB| \leq |A| \cdot |B|$, combining FACT e and FACT f implies that $\rho(|AB|) \leq \rho(|A| \cdot |B|)$.

(h) $\sum_{i=1}^m |a_{ij}| \leq \sum_{i=1}^m |a_{ij}|$ (Wyllie (1966) p642).

(i) $|x-y| \geq ||x|-|y|| \geq |x|-|y|$ (Wyllie (1966) p6+2).

(j) $|x| \cdot |y| = |xy|$ (Wyllie (1966) p642).

(k) $\sum_{j=1}^n \sum_{i=1}^m x_{ij} y_j = \sum_{i=1}^m x_i \sum_{j=1}^n y_j = \sum_{j=1}^n y_j \sum_{i=1}^m x_{ij} = \sum_{i=1}^m \sum_{j=1}^n x_{ij} y_j$

(l) Let $Bel(R^n)$. Then $\rho(B) \leq ||B||$ for any arbitrary norm. (Varga (1962) p8).

FACTS g and h are not referenced as they are self evident.

Note $\phi' = \phi'(X^*)$ and $Q = Q(X^*)$

A.1 Convergence when $\phi' \geq 0$ and $\rho(\phi') < 1$

As $\phi' \geq 0$ and Q is extracted from ϕ'

∴	$Q \geq 0$	-----	A.1
and	$(\phi' - Q) \geq 0$	-----	A.2
and	$ q_{4j} \leq \phi_{4j} $	-----	A.3
∴	$\rho(Q) \leq \rho(\phi') < 1$	by A.3, FACT e	A.4
∴	$(I - Q)^{-1} \geq 0$	by A.1, A.4 and FACT b	A.5
∴	$(\phi' - Q) \cdot (I - Q)^{-1} \geq 0$	by A.2 and A.5	A.6a
and	$(I - Q)^{-1} \cdot (\phi' - Q) \geq 0$	by A.2 and A.5	A.6b

If $A = (I - Q) - (\phi' - Q)$ then it is a weak splitting (A.5, A.6 and FACT c).

But $(I - Q) - (\phi' - Q) = I - \phi'$

∴ $A^{-1} = (I - \phi')^{-1}$
 ≥ 0 by FACT b and original conditions

∴ $\rho((I - Q)^{-1} \cdot (\phi' - Q)) < 1$ by FACT d

A.2 Convergence when $Q \geq 0$ and $\rho(\phi') < 1$

In this case choose:-

$$A = (I-Q) - |\phi' - Q|$$

But $|\phi' - Q| \geq 0$

by definition

and $(I-Q)^{-1} \geq 0$

by initial condition and FACTS b, e and f

$\therefore |\phi' - Q| \cdot (I-Q)^{-1} \geq 0$

and $(I-Q)^{-1} \cdot |\phi' - Q| \geq 0$

$\therefore A$ is a weak regular splitting by FACT C

Also

$$|\phi' - Q| = |\phi'| - Q$$

as $Q \geq 0$ and is extracted from ϕ'

$\therefore (I-Q) - |\phi' - Q| = I - |\phi'|$

$\therefore A^{-1} = (I - |\phi'|)^{-1}$

$$\geq 0$$

by initial condition and FACT b

$\therefore \rho((I-Q)^{-1} \cdot |\phi' - Q|) < 1$

by FACT d

But $\rho((I-Q)^{-1} \cdot |\phi' - Q|) = \rho(|(I-Q)^{-1}| \cdot |\phi' - Q|)$ as $(I-Q)^{-1} \geq 0$

$$\geq \rho((I-Q)^{-1} \cdot (\phi' - Q))$$

by FACTS f and g

$\therefore \rho((I-Q)^{-1} \cdot (\phi' - Q)) < 1$

A.3 Convergence when the diagonal elements of Q are the same as those of Φ' and the remainder zero and $\|\Phi'\|_\infty < 1$

Here

$$G'(X^*) = (I - Q)^{-1} \cdot (\Phi' - Q) = \begin{bmatrix} \frac{1}{1-\phi_{11}} & 0 & \dots & 0 & 0 & \phi_{12} & \dots & \phi_{1n} \\ 0 & \frac{1}{1-\phi_{22}} & & 0 & \phi_{21} & 0 & & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \frac{1}{1-\phi_{nn}} & \phi_{n1} & \dots & \dots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{\phi_{12}}{1-\phi_{11}} & \dots & \frac{\phi_{1n}}{1-\phi_{11}} \\ \frac{\phi_{21}}{1-\phi_{22}} & & & \frac{\phi_{2n}}{1-\phi_{22}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\phi_{n1}}{1-\phi_{nn}} & \dots & \dots & 0 \end{bmatrix}$$

$$\|G'(X^*)\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^n \left| \frac{\phi_{ij}}{1-\phi_{ii}} \right|$$

But when

$$\|\Phi'\|_\infty < 1$$

then

$$\sum_{j=1}^n |\phi_{ij}| < 1$$

for all i

So

$$\sum_{j=1}^n |\phi_{ij}| < 1 - |\phi_{ii}| \leq |1 - \phi_{ii}|$$

for all i by FACT 1

$$\sum_{j=1}^n |\phi_{ij}| / |1 - \phi_{ii}| < 1$$

for all i

$$\|G'(X^*)\|_\infty < 1$$

$$\rho((I - Q)^{-1} \cdot (\Phi' - Q)) < 1$$

by FACT 1

A.4. Convergence in general when $\|\phi'\|_\infty < 1$

$\phi' \in \mathbb{R}^n$ and if Q , which is block diagonal, has blocks of size $M \times M$ ($0 < M \leq n$), then $(I - Q)^{-1}$ is also block diagonal with blocks of size $M \times M$. Further if Q_i is the i th block counting from the top left corner so that when Q is partitioned:-

$$Q = \begin{bmatrix} Q_1 & 0 & \dots & 0 \\ 0 & Q_2 & & \\ \vdots & & \ddots & \\ 0 & \dots & \dots & Q_L \end{bmatrix} \quad \text{for } L = n/M$$

then

$$(I - Q)^{-1} = \begin{bmatrix} (I - Q_1)^{-1} & 0 & \dots & 0 \\ 0 & (I - Q_2)^{-1} & & \\ \vdots & & \ddots & \\ 0 & \dots & \dots & (I - Q_L)^{-1} \end{bmatrix}$$

This is readily seen from Lepidus (1962) p251-2, when the off-diagonal blocks of the matrix to be inverted are zero. When $(\phi' - Q)$ is partitioned into $M \times M$ blocks:-

$$(I - Q)^{-1} \cdot (\phi' - Q) = \begin{bmatrix} (I - Q_1)^{-1} & 0 & \dots & 0 & 0 & P_{12} & \dots & P_{1L} \\ 0 & (I - Q_2)^{-1} & & & P_{21} & 0 & & \\ \vdots & & \ddots & & & & \ddots & \\ 0 & \dots & \dots & (I - Q_L)^{-1} & P_{L1} & \dots & \dots & 0 \end{bmatrix}$$

where P_{ij} is the matrix with elements corresponding to those in rows $(i-1)M+1$ to $M \cdot i$ and columns $(j-1)M+1$ to $M \cdot j$ of ϕ' .

$$\text{therefore} \quad (I-Q)^{-1}(\Phi'-Q) = \begin{bmatrix} 0 & (I-Q_1)^{-1}P_{12} & \dots & (I-Q_1)^{-1}P_{1L} \\ (I-Q_2)^{-1}P_{21} & 0 & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ (I-Q_L)^{-1}P_{L1} & \dots & \dots & 0 \end{bmatrix} \quad \text{A.7}$$

Similarly if rows $M+1$ to $2M$ and columns $M+1$ to $2M$ of Φ' are interchanged with rows 1 to M and columns 1 to M then:-

$$(I-Q)^{-1}(\Phi'-Q) = \begin{bmatrix} 0 & (I-Q_2)^{-1}P_{21} & \dots & (I-Q_2)^{-1}P_{2L} \\ (I-Q_1)^{-1}P_{12} & 0 & \dots & (I-Q_1)^{-1}P_{1L} \\ \vdots & \vdots & \ddots & \vdots \\ (I-Q_L)^{-1}P_{L2} & \dots & \dots & 0 \end{bmatrix} \quad \text{A.8}$$

Comparing equations A.7 and A.8 it can be seen that the elements in each row are unchanged although the row position has changed. This is clearly true for any interchange of rows and columns corresponding to the $M \times M$ partitions of Φ' . Consequently in finding the l_∞ norm of $(I-Q)^{-1}(\Phi'-Q)$ it is only necessary to consider the first M rows, and if the row sums of the absolute values of elements in the rows are less than one then it can be concluded that the l_∞ norm of $(I-Q)^{-1}(\Phi'-Q)$ is less than one as any set of M rows can be moved to the first M rows without changing the elements in the rows.

Set $\phi_{ij} =$ the element in the i^{th} row and j^{th} column of Φ'

$a_{ij}^m =$ the element in the i^{th} row and j^{th} column of $(I-Q)^{-1}$ when the diagonal blocks of Q are $m \times m$

$\gamma_{ij}^m =$ the element in the i^{th} row and j^{th} column of $(I-Q)^{-1}(\Phi'-Q)$ when the diagonal blocks of Q are $m \times m$

Then considering only the first m rows of $(I-Q)^{-1} \cdot (\phi^1 - Q)$

$$y_{i1}^m = \sum_{k=1}^m \alpha_{ik}^m \phi_{k1} \quad 1 \leq m \quad 1 > m$$

$$\text{Set } G_i^m = \sum_{l=1}^n |y_{il}^m| \quad 1 \leq m \quad 1 \leq m$$

$$= \sum_{l=m+1}^n |y_{il}^m| \quad 1 \leq m$$

$$G_i^m = \sum_{l=m+1}^n \left| \sum_{k=1}^m \alpha_{ik}^m \phi_{k1} \right| \quad 1 \leq m$$

$$\text{and } G_i^{m-1} = \sum_{l=m}^{n-1} \left| \sum_{k=1}^{m-1} \alpha_{ik}^{m-1} \phi_{k1} \right| \quad 1 \leq m-1$$

Using the 'Bordering' method of matrix inversion it is possible to obtain α_{ik}^m in terms of α_{ik}^{m-1} (Faddeeva (1963) p163-8) from:-

$$\alpha_{ik}^m = \alpha_{ik}^{m-1} + \beta_{im} \cdot \delta_{mk} / \Delta_m \quad 1, k \leq m-1$$

$$\alpha_{im}^m = -\delta_{im} / \Delta_m \quad 1 \leq m-1$$

$$\alpha_{mi}^m = -\delta_{mi} / \Delta_m \quad 1 \leq m-1$$

$$\alpha_{mn}^m = 1 / \Delta_m$$

where

$$\beta_{im} = - \sum_{k=1}^{m-1} \alpha_{ik}^{m-1} \phi_{km} \quad \delta_{mi} = - \sum_{j=1}^{m-1} \phi_{mj} \alpha_{ji}^{m-1} \quad 1 \leq m-1$$

$$\Delta_m = 1 - \phi_{mm} + \sum_{i=1}^{m-1} \phi_{mi} \beta_{im} = 1 - \phi_{mm} + \sum_{i=1}^{m-1} \phi_{im} \delta_{mi}$$

Then

$$G_i^m = \sum_{l=m+1}^n \left| \sum_{k=1}^{m-1} \alpha_{ik}^{m-1} \phi_{k1} + \sum_{k=1}^{m-1} \beta_{im} \delta_{mk} \phi_{k1} / \Delta_m - \beta_{im} \phi_{m1} / \Delta_m \right| \quad 1 \leq m-1$$

$$\text{and } G_i^m = \sum_{l=m+1}^n \left| - \sum_{k=1}^{m-1} \delta_{mk} \phi_{k1} / \Delta_m + \phi_{m1} / \Delta_m \right|$$

Consider equation A.8 first

$$G_1^m = \sum_{l=m}^n |Y_{1l}^{m-1}| + \beta_{1m}/\Delta_m \left(\sum_{k=1}^{m-1} \delta_{mk} \phi_{kl} - \phi_{m1} \right) \cdot \left[-\beta_{1m} + \beta_{1m}/\Delta_m \left(\sum_{k=1}^{m-1} \delta_{mk} \phi_{km} - \phi_{mm} \right) \right]$$

$$\text{But } \sum_{k=1}^{m-1} \delta_{mk} \phi_{kl} = - \sum_{k=1}^{m-1} \phi_{kl} \sum_{j=1}^{m-1} \phi_{mj}^{m-1} = - \sum_{j=1}^{m-1} \phi_{mj} \sum_{k=1}^{m-1} \phi_{kl}^{m-1} = - \sum_{j=1}^{m-1} \phi_{mj} Y_{j1}^{m-1}$$

$$\text{and } \sum_{k=1}^{m-1} \delta_{mk} \phi_{km} - \phi_{mm} = \Delta_m^{-1}$$

$$G_1^m = \sum_{l=m}^n |Y_{1l}^{m-1}| + \beta_{1m}/\Delta_m \left(- \sum_{j=1}^{m-1} \phi_{mj} Y_{j1}^{m-1} - \phi_{m1} \right) \cdot \left[-\beta_{1m}/\Delta_m \right]$$

$$\leq \sum_{l=m}^n |Y_{1l}^{m-1}| + \left| -\beta_{1m}/\Delta_m \right| \cdot \left(\sum_{j=1}^{m-1} |\phi_{mj}| \cdot G_j^{m-1} + \sum_{l=m}^n |\phi_{ml}| \cdot 1 \right)$$

If $G_j^{m-1} \leq 1$ for all $j \leq m-1$

$$\text{then } G_1^m \leq G_1^{m-1} + \left| -\beta_{1m}/\Delta_m \right| \cdot \left(\sum_{l=1}^n |\phi_{ml}| \cdot 1 \right) \quad l \leq m-1$$

As $\|\phi'\|_\infty < 1$ then $\sum_{l=1}^n |\phi_{jl}| < 1$ for all j including $j=m$

$$\therefore G_1^m \leq G_2^{m-1} \quad \text{if } G_j^{m-1} \leq 1 \quad \text{for all } 0 < j \leq m-1 \quad \text{----- A.11}$$

Similarly for equation A.10

$$G_m^m = \sum_{l=m}^n \left| \frac{1}{\Delta_m} \right| \cdot \left(\sum_{j=1}^{m-1} \phi_{mj} Y_{j1}^{m-1} + \phi_{m1} \right) \cdot \left| -\frac{1}{\Delta_m} \right| \cdot \left(\sum_{k=1}^{m-1} \delta_{mk} \phi_{km} + \phi_{mm} \right)$$

$$= \left| \frac{1}{\Delta_m} \right| \cdot \left(\sum_{l=m}^n \sum_{j=1}^{m-1} \phi_{mj} Y_{j1}^{m-1} + \phi_{m1} \right) \cdot \left| -\frac{1}{\Delta_m} \right|$$

$$\leq \left| \frac{1}{\Delta_m} \right| \cdot \left(\sum_{j=1}^{m-1} |\phi_{mj}| \cdot G_j^{m-1} + \sum_{l=m}^n |\phi_{ml}| \cdot 1 \right) \cdot \left| \frac{1}{\Delta_m} \right|$$

If $G_j^{m-1} \leq 1$ for all $j \leq m-1$

then

$$G_m^m \leq \frac{1}{\Delta_m} \cdot \left(\sum_{j=1}^n |\phi_{mj}| - 1 + |\Delta_m| \right)$$

and hence

$$G_m^n < 1 \text{ if } G_j^{m-1} \leq 1 \text{ for all } j \leq m-1$$

----- A.12

By definition

$$\begin{aligned} G_1^1 &= \sum_{l=2}^n |\gamma_{1l}^1| = \sum_{l=2}^n |\alpha_{1l}^1 \phi_{1l}| \\ &= \sum_{l=2}^n \frac{|\phi_{1l}|}{1 - \phi_{11}} \end{aligned}$$

<1

(shown in part A.3 of this appendix)

Therefore by induction and equations A.11 and A.12 it can be concluded that:-

$$G_i^m < 1 \text{ for all } i \leq m$$

If $m=M$ then:-

$$G_i^M < 1 \text{ for all } i \leq M$$

and as before the sets of M rows and columns are interchangeable so that:-

$$G_i^M < 1 \text{ for all } i \leq n$$

----- A.13

Using the l_∞ norm:-

$$\|(I-Q)^{-1} \cdot (\phi' - Q)\|_\infty = \max_{1 \leq i \leq n} G_i^M$$

<1

by equation A.13

therefore

$$\rho((I-Q)^{-1} \cdot (\phi' - Q)) < 1 \xrightarrow{\text{by FACT 9}}$$

Corollary:-

$$|\phi_{11}| = 1 - \phi_{11} \quad \text{for all } i \leq n$$

$$\geq 1 - |\phi_{11}|$$

$$> \|\phi'\|_{\infty} - \|\phi'\|_{\infty} |1 - \phi_{11}| \quad \text{as } \|\phi'\|_{\infty} < 1$$

$$\geq \sum_{i=1}^n |\phi_{11}| - \|\phi'\|_{\infty} |1 - \phi_{11}|$$

$$\therefore \|\phi'\|_{\infty} > \sum_{i=1}^n |\phi_{11}| / |1 - \phi_{11}|$$

$$= G_1^{-1}$$

(see part A.3)

$$\text{But } G_1^{-1} \geq G_1^{-2} \geq \dots \geq G_1^{-M} \quad \text{for all } i \leq n \quad \text{by equation A.13 and A.11}$$

$$\therefore \|\phi'\|_{\infty} > G_1^{-M} \quad \text{for all } i \leq n$$

$$\therefore \|\phi'\|_{\infty} > \max_{1 \leq i \leq n} G_1^{-M} = \|G_{RN}\|_{\infty}$$

where $\|G_{RN}\|_{\infty} = \|\phi'(X^*)\|_{\infty}$ for the reduced Newton method

APPENDIX B THE SPECTRAL RADIUS OF A LOWER BLOCK TRIANGULAR MATRIX IS
THE MAXIMUM OF THE SPECTRAL RADII OF EACH OF THE DIAGONAL BLOCKS

Define

$$A = \begin{bmatrix} A_{11} & 0 & \cdots & 0 \\ A_{21} & A_{22} & & \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & \cdots & \cdots & A_{NN} \end{bmatrix}$$

where the A_{ii} are square (not necessarily equal sized) matrices and A_{ij} the lower triangular matrices (not necessarily square but determined by the sizes of A_{ii} and A_{jj}) of the partitioned matrix A.

If A is now expanded by 'Laplace's Expansion' (Wylie (1960) p406) to get the determinant of A (det(A)):-

$$\det(A) = \det(A_{11}) \cdot \det \begin{bmatrix} A_{22} & 0 & \cdots & 0 \\ A_{32} & A_{33} & & \\ \vdots & \vdots & \ddots & \vdots \\ A_{N2} & \cdots & \cdots & A_{NN} \end{bmatrix} + 0$$

$$= \det(A_{11}) \cdot \det(A_{22}) \cdot \det \begin{bmatrix} A_{33} & 0 & \cdots & 0 \\ A_{43} & A_{44} & & \\ \vdots & \vdots & \ddots & \vdots \\ A_{N3} & \cdots & \cdots & A_{NN} \end{bmatrix} + 0$$

$$= \det(A_{11}) \cdot \det(A_{22}) \cdot \cdots \cdot \det(A_{NN})$$

----- B.1

The eigenvalues of A are found by solving

$$\det(A - \lambda I) = 0 \quad \text{for } \lambda$$

which by equation B.1 is:-

$$\det(A_{11} - \lambda I) \cdot \det(A_{22} - \lambda I) \cdot \cdots \cdot \det(A_{NN} - \lambda I) = 0$$

therefore the eigenvalues are found by solving

$$\det(A_{ii} - \lambda I) = 0 \quad \text{for } i=1, N$$

and the eigenvalues of the A_{ii} are the eigenvalues of A

$$\therefore \rho(A) = \max_{1 \leq i \leq N} \rho(A_{ii})$$

APPENDIX C WEIGHTING FACTORS

To choose the tear set with the least number of double tears, each stream is simply weighted by the number of cycles it opens (m_k). Then any tear set with no double tears will have a weighted sum equal to the number of cycles, and if any cycle is opened more than once the weighted sum will be more than the minimum.

To choose between the members of the 'no double tear' family (presuming there is more than one member in the family) on the basis of the member with the least tear streams, the streams must be weighted so that streams which open fewer cycles must be weighted slightly heavier than ones which open more cycles.

The worst case that can arise when decomposing a network of NC cycles, is when the tear set containing NC streams each opening only one cycle must be chosen in preference to the tear set which contains only two streams but causes a double tear.

$$\text{Set } W_k = m_k + X \quad X > 0$$

Suppose for the two stream case that one stream opens M cycles and the other stream (NC-M+1) cycles, so causing the double tear, then the sum of weighting factors will be:

$$\begin{aligned} C' &= W_1 + W_2 = (M+X) + (NC-M+1+X) \\ &= NC+1+2X \end{aligned}$$

For the first situation $C = NC(1+X)$.

So to choose the 'no double tears' case, C must be less than C':

$$NC(1+X) < NC+1+2X$$

or

$$X < 1/(NC-2)$$

Converting to integers $W_k = A \cdot m_k + 1$

for $A > (NC-2)$

Similarly when the criterion is required to select the tear set with the least torn variables within the family of 'no double tears' then the tear set containing as many streams as there are cycles in the network, each having the maximum number of variables (NV_K) but causing no double tears, must have a weighted sum less than the tear set containing only two streams with one variable in each but causing a double tear.

Setting

$$W_K = m_K + NV_K \cdot X$$

then it is required that

$$NC(1 + NV_{\max} X) < (M + X) + ((NC - M + 1) \cdot X)$$

so that

$$X < 1 / (NC \cdot NV_{\max} - 2)$$

Converting to integers

$$W_K = B \cdot m_K + NV_K$$

$$\text{where } B > (NC \cdot NV_{\max} - 2)$$

COMPUTER DISPLAYS
FOR TEST EXAMPLE 1

(1) INPUT DATA

236

	0.4	4.C	0.0	0.2	0.0	3.2	100.0
L.0173	2000.0						
L.0174	2 FROM	3					
L.0175	CCPY	3					
L.0176	CCPY	3					
L.0177	QURC	3	0	0	3	0	
L.0178	0	3	0	0	3	0	
L.0179	STOP						

(ii) OUTPUT FROM INPUT PHASE

JP PH11

INTER-PLANT CONNECTIONS

FROM PLANT 3	STREAM 3	TO PLANT 2	STREAM 1
FROM PLANT 3	STREAM 5	TO PLANT 2	STREAM 2
FROM PLANT 3	STREAM 17	TO PLANT 4	STREAM 1
FROM PLANT 2	STREAM 30	TO PLANT 4	STREAM 2

MINERAL NAMES

WALL NSTE

SIZE DISTRIBUTION

1000.	500.0	250.0	125.0
-------	-------	-------	-------

MASS FRACTIONS IN G-CLASSES

G-CLASS 1	1.000	0.0
G-CLASS 2	0.000	1.000
G-CLASS 3	0.0	1.000

VOLUME FRACTIONS IN G-CLASSES

G-CLASS 1	1.000	0.0
G-CLASS 2	0.372	0.728
G-CLASS 3	0.0	1.000

PHYSICAL PROPERTIES

PHYS PROP 1

7.500	4.078	2.800
-------	-------	-------

PHYS PROP 3

6.000	10.00
-------	-------

16.00

NUMBER OF	CLASSES =	4
NUMBER OF	CLASSES =	3
NUMBER OF	CLASSES =	1

STREAM CONNECTION MATRIX

[illegible]

STREAM WEIGHT INCS.

USING CRITERION 2 THE FOLLOWING VALUES WERE OBTAINED:

WATER FEEDS

UNIT 6 IS FED BY 50.00 WATER AT 0.0 % SOLIDS

TOTAL MASS	SOLIDS = 100.0
PERCENTAGE	SOLIDS = 15.00

MASS IN EACH CATEGORY:

0-CLASSES

[illegible]

RUN DATA

此書係吳興金鑑序

```

CONVERGENCE METHOD= 3
MAX NUMBER ITERATIONS= 8
TOLERANCE LIMIT= 104*-4

```

242.

UNIT NUMBER 1

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

0-4000
2000

12-00 0.0 0.0

0.2000 0.5

C. 1000E CQ 400.0

UNIT NUMBER 2

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

6.4000
2300.

12.00 0.0

0.0

0.2000 0.0

C.100E CO 400.0

UNIT NUMBER 3

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 5

IS A MIXED UNIT AND NEEDS 3 PARAMETERS

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

IS A CRSM UNIT AND NEEDS 5 PARAMETERS

UNIT NUMBER 9

	0.0	2.000	0.0	1.000	0.2000	C.1000E C0	0.0000
0.4000							
5.000							

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

UNIT NUMBER 11

Category	Value	Value	Value
0.0	0.0	0.0	0.0
5.000	2.000	1.000	0.2000
0.4000			0.10000
			0.40000

IS A TRIM UNIT AND NEEDS 14 PARAMETERS

	PARAMETERS		
0.6000	2.000	0.0	1.000
300.0	2000.	0.0	0.0000
			0.8000
			8.000
			0.1000E 00

0 0 0 2 0 0 1 6 6 6

RUN DATA

CONVERGENCE METHOD= 3
MAX NUMBER ITERATIONS= 8
TOLERANCE LIMIT=10**-4

UNIT NUMBER 1
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000	12.00	0.0	0.0	0.2000	0.0	0.1000E 00	400.0
2000.							

UNIT NUMBER 2
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000	12.00	0.0	0.0	0.2000	0.0	0.1000E 00	400.0
2000.							

244.

UNIT NUMBER 3
=====

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 5
=====

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 6
=====

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 8
=====

IS A CRSH UNIT AND NEEDS 5 PARAMETERS

C.0	3.000	0.1500	0.1000E 00	0.8000E-01
-----	-------	--------	------------	------------

UNIT NUMBER 9
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000	0.0	2.000	0.0	1.000	C.2000	0.1000E 00	0.4000
S.000							

UNIT NUMBER 10
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000	2.000	0.0	0.0	0.2000	C.1000E 00	0.5000	500.0
2000.							

UNIT NUMBER 11
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000	0.0	2.000	0.0	1.000	0.2000	C.1000E 00	0.4000
S.000							

UNIT NUMBER 12
=====

IS A TRMP UNIT AND NEEDS 14 PARAMETERS

C.4000	2.000	12.00	0.0	1.000	C.8000	C.0	0.1000E 00
500.0	2000.	0.0	0.0	0.0000	0.000		

THE SELECTED OUTPUT CODES ARE:

0 0 1 C 0 0 0 0

 PLANT NUMBER 2

 NUMBER OF D-CLASSES 3
 NUMBER OF G-CLASSES 2
 NUMBER OF S-CLASSES 2

 STEAM CONNECTION MATRIX

STREAM 1	CONNECTS UNIT 10 TO UNIT 2
STREAM 2	CONNECTS UNIT 6 TO UNIT 1
STREAM 3	CONNECTS UNIT 1 TO UNIT 4
STREAM 4	CONNECTS UNIT 1 TO UNIT 8
STREAM 5	CONNECTS UNIT 4 TO UNIT 1
STREAM 6	CONNECTS UNIT 4 TO UNIT 1
STREAM 7	CONNECTS UNIT 9 TO UNIT 5
STREAM 8	CONNECTS UNIT 5 TO UNIT 7
STREAM 9	CONNECTS UNIT 7 TO UNIT 9
STREAM 10	CONNECTS UNIT 9 TO UNIT 6
STREAM 11	CONNECTS UNIT 7 TO UNIT 6
STREAM 12	CONNECTS UNIT 9 TO UNIT 6
STREAM 13	CONNECTS UNIT 9 TO UNIT 6
STREAM 14	CONNECTS UNIT 9 TO UNIT 6
STREAM 15	CONNECTS UNIT 9 TO UNIT 6
STREAM 16	CONNECTS UNIT 9 TO UNIT 6
STREAM 17	CONNECTS UNIT 9 TO UNIT 6
STREAM 18	CONNECTS UNIT 9 TO UNIT 6
STREAM 19	CONNECTS UNIT 9 TO UNIT 6
STREAM 20	CONNECTS UNIT 9 TO UNIT 6

 STREAM WEIGHTINGS

 USING CRITERION 0 THE FOLLOWING VALUES WERE OBTAINED:

1	1	0	1	1	1	1	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---

 WATER FEEDS

UNIT 2 IS FED BY 20.00	WATER AT 0.0	%SOLIDS
------------------------	--------------	---------

 STREAM NUMBER 1

TOTAL MASS SOLIDS= 1.000
 PERCENTAGE SOLIDS= 1.000

 MASS IN EACH CATEGORY:

D-CLASSES

SCL 1	GCL 1	1	0.00	2	0.95	3	1.00
SCL 2	GCL 2	1	0.00	2	0.95	3	1.00
SCL 3	GCL 3	1	0.00	2	0.95	3	1.00
SCL 4	GCL 4	1	0.00	2	0.95	3	1.00
SCL 5	GCL 5	1	0.00	2	0.95	3	1.00
SCL 6	GCL 6	1	0.00	2	0.95	3	1.00
SCL 7	GCL 7	1	0.00	2	0.95	3	1.00
SCL 8	GCL 8	1	0.00	2	0.95	3	1.00
SCL 9	GCL 9	1	0.00	2	0.95	3	1.00
SCL 10	GCL 10	1	0.00	2	0.95	3	1.00
SCL 11	GCL 11	1	0.00	2	0.95	3	1.00
SCL 12	GCL 12	1	0.00	2	0.95	3	1.00
SCL 13	GCL 13	1	0.00	2	0.95	3	1.00
SCL 14	GCL 14	1	0.00	2	0.95	3	1.00
SCL 15	GCL 15	1	0.00	2	0.95	3	1.00
SCL 16	GCL 16	1	0.00	2	0.95	3	1.00
SCL 17	GCL 17	1	0.00	2	0.95	3	1.00
SCL 18	GCL 18	1	0.00	2	0.95	3	1.00
SCL 19	GCL 19	1	0.00	2	0.95	3	1.00
SCL 20	GCL 20	1	0.00	2	0.95	3	1.00

TOTAL MASS SOLIDS = 9.000
PERCENTAGE SOLIDS = 20.00

MASS IN EACH CATEGORY:

E-CLASSES

INSTRUMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
INSTRUMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

RUN DATA

```
CONVERGENCE METHOD= 3
MAX NUMBER ITERATIONS= 3
TOLERANCE LIMIT=10**-4
```

247.

[illegible]

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

2

UNIT NUMBER 21

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 4

C.4000	0.0	0.0	4.000
H.000			

4.000

UNIT NUMBER 5

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 7
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

0.400C
17.50

0.0 13.00 0.0

4.000 C.2000 0.1000E 00 5.000

UNIT NUMBER 8
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.400C
17.50

0.0 13.00 0.0

4.000 C.2000 C.1000E 00 5.000

UNIT NUMBER 9
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.400C
2000.

1.000 0.0 0.0

0.1000E 00 C.1000E 00 3.000 250.0

THE SELECTED OUTPUT CDEES ARE:

0 0 0 1 0 0 1 6 6 6

FLANT NUMBER

NUMBER OF CLASSES =
NUMBER OF G-CLASSES =
NUMBER OF S-CLASSES =

STREAM CONNECTION MATRIX

[illegible]

STREAM WEIGHTINGS

USING CRITERION 2 THE FOLLOWING VALUES WERE OBTAINED:

—

WATER FEEDS.
牛水中に草を食す。水は牛に

UNIT	4 IS FED BY 20.00	WATER AT 20.00	%SOLIDS
1	100	100	100
2	100	100	100
3	100	100	100
4	100	100	100
5	100	100	100
6	100	100	100
7	100	100	100
8	100	100	100
9	100	100	100
10	100	100	100
11	100	100	100
12	100	100	100
13	100	100	100
14	100	100	100
15	100	100	100
16	100	100	100
17	100	100	100
18	100	100	100
19	100	100	100
20	100	100	100
21	100	100	100
22	100	100	100
23	100	100	100
24	100	100	100
25	100	100	100
26	100	100	100
27	100	100	100
28	100	100	100
29	100	100	100
30	100	100	100
31	100	100	100
32	100	100	100
33	100	100	100
34	100	100	100
35	100	100	100
36	100	100	100
37	100	100	100
38	100	100	100
39	100	100	100
40	100	100	100
41	100	100	100
42	100	100	100
43	100	100	100
44	100	100	100
45	100	100	100
46	100	100	100
47	100	100	100
48	100	100	100
49	100	100	100
50	100	100	100
51	100	100	100
52	100	100	100
53	100	100	100
54	100	100	100
55	100	100	100
56	100	100	100
57	100	100	100
58	100	100	100
59	100	100	100
60	100	100	100
61	100	100	100
62	100	100	100
63	100	100	100
64	100	100	100
65	100	100	100
66	100	100	100
67	100	100	100
68	100	100	100
69	100	100	100
70	100	100	100
71	100	100	100
72	100	100	100
73	100	100	100
74	100	100	100
75	100	100	100
76	100	100	100
77	100	100	100
78	100	100	100
79	100	100	100
80	100	100	100
81	100	100	100
82	100	100	100
83	100	100	100
84	100	100	100
85	100	100	100
86	100	100	100
87	100	100	100
88	100	100	100
89	100	100	100
90	100	100	100
91	100	100	100
92	100	100	100
93	100	100	100
94	100	100	100
95	100	100	100
96	100	100	100

RUN DATA

```

CONVERGENCE METHOD= 1
MAX NUMBER ITERATIONS= 3
TOLERANCE LIMIT= 104*-4

```

DAY NUMBER 1

IS A MIXED UNIT AND NEEDS 6 PARAMETERS

UNIT NUMBER 2
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

0.5000	3.000	0.0	0.0	0.2000	0.0	4.000	100.0
2000.							

UNIT NUMBER 3
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

0.5000	3.000	0.0	0.0	0.2000	0.0	4.000	100.0
2000.							

UNIT NUMBER 4
=====

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 5
=====

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

0.5000	3.000	0.0	0.0	0.2000	0.0	4.000	100.0
2000.							

UNIT NUMBER 6
=====

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

THE SELECTED OUTPUT CODES ARE:

3 3 0 3 3 3 0 0 3 0

RUN DATA

CONVERGENCE METHOD= 3
MAX NUMBER ITERATIONS= 8
TOLERANCE LIMIT=10**-4

UNIT NUMBER 1

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 2

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000 4.000 0.0 0.0 0.2000 0.0 3.500 100.0

UNIT NUMBER 3

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000 4.000 0.0 0.0 0.2000 0.0 3.500 100.0

UNIT NUMBER 4

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

UNIT NUMBER 5

IS A TRMP UNIT AND NEEDS 9 PARAMETERS

C.4000 4.000 0.0 0.0 0.2000 0.0 3.500 100.0

UNIT NUMBER 6

IS A MIXR UNIT AND NEEDS 0 PARAMETERS

THE SELECTED OUTPUT CODES ARE:

0 0 0 3 3 0 0 3 0

/PAGE

(iii) OUTPUT FROM ORDERING PHASE

PLANT NUMBER 3

NODE NUMBER 1

TEARS 1 9
UNIT ORDER 2 1 3 9 11 12 5

NODE NUMBER 2

TEARS 16
UNIT ORDER 6 8 10

PLANT NUMBER 2

NODE NUMBER 1

TEARS 13 14
UNIT ORDER 2 1 4 8

NODE NUMBER 2

TEARS 18
UNIT ORDER 5 7 9

PLANT NUMBER 4

NODE NUMBER 1

TEARS 6
UNIT ORDER 2 1 3 5 4

NODE NUMBER 2

TEARS 0
UNIT ORDER 6

(iv) OUTPUT FROM CALCULATION PHASE

/P PHC

PLANT NUMBER 3

RUN NUMBER 1

NODE NUMBER 1

AFTER 1 ITERATIONS THE RMS ERROR IS 0.0000

NODE NUMBER 2

AFTER 4 ITERATIONS THE RMS ERROR IS 0.0000

RESULTS FOR PLANT 3 RUN 1

THE WATER FEED TO UNIT 6 IS 50.00

>>>>>>FEED STREAM 2 CONTAINS 100.0 SOLIDS AND 566.7 WATER
MASS IN EACH CATEGORY:

		D-CLASSES			
		1	2	3	4
SCL 1	GCL 1	0.0	2.00	3.00	3.75
SCL 1	GCL 2	23.75	34.00	8.00	1.50
SCL 1	GCL 3	1.25	4.00	5.00	9.75
		VALU	WSTE		
MASSES:		42.28	57.63		
GRADE %:		42.37	57.62		

<<<<<OUTPUT STREAM 3 CONTAINS 64.00 SOLIDS AND 327.7 WATER
MASS IN EACH CATEGORY:

		D-CLASSES			
		1	2	3	4
SCL 1	GCL 1	0.0	1.57	2.93	3.59
SCL 1	GCL 2	0.0	2.72	7.76	1.44
SCL 1	GCL 3	0.0	2.54	8.59	9.38
		VALU	WSTE		
MASSES:		25.53	38.47		
GRADE %:		59.89	66.11		
% RECOVERY:		60.25	66.75		

<<<<<OUTPUT STREAM 5 CONTAINS 16.53

SCLIDS AND 15E.2

WATER

MASS IN EACH CATEGORY:

		D-CLASSES			
		1	2	3	4
SCL	1 GCL 1	0.00	0.25	0.04	0.17
SCL	1 GCL 2	12.08	2.46	0.06	0.04
SCL	1 GCL 3	0.40	0.1E	0.04	0.17

	VALU	WSTE
MASSSES:	8.12	6.41
GRADE %:	47.13	50.67
% RECOVERY:	19.17	14.55

<<<<<OUTPUT STREAM 17 CONTAINS 19.47

SCLIDE AND 133.8

WATER

MASS IN EACH CATEGORY:

		D-CLASSES			
		1	2	3	4
SCL	1 GCL 1	0.00	0.08	0.04	0.05
SCL	1 GCL 2	2.88	5.76	3.24	5.22
SCL	1 GCL 3	0.22	0.65	0.49	0.80

	VALU	WSTE
MASSSES:	8.72	10.75
GRADE %:	44.79	55.21
% RECOVERY:	20.58	18.65

*****TEAR STREAM 1 CONTAINS 129.6

SCLIDS AND 619.2

WATER

MASS IN EACH CATEGORY:

		D-CLASSES			
		1	2	3	4
SCL	1 GCL 1	0.00	2.57	3.66	4.47
SCL	1 GCL 2	28.64	46.75	5.71	1.80
SCL	1 GCL 3	1.42	5.17	10.88	11.72

	VALU	WSTE
MASSSES:	55.56	74.04
GRADE %:	42.87	47.13

*****TEAR STREAM 3 CONTAINS 11.28

SCLIDS AND 93.13

WATER

MASS IN EACH CATEGORY:

		L-CLASSES			
		1	2	3	4
SCL	1 GCL 1	0.00	0.15	0.03	0.04
SCL	1 GCL 2	0.33	1.55	0.05	0.02
SCL	1 GCL 3	0.31	0.16	0.04	0.12

	VALU	WSTE
MASSSES:	5.46	5.22
GRADE %:	48.40	51.60

*****FEED STREAM 16 CONTAINS 16.27

SOLIDS AND 89.21

WATER

MASS IN EACH CATEGORY:

		D-CLASSES			
		1	2	3	4
SCL	1 GCL 1	0.00	0.06	0.02	0.01
SCL	1 GCL 2	7.13	4.65	1.30	1.45
SCL	1 GCL 3	0.55	0.56	0.21	0.22

	VALU	WSTE
MASSSES:	7.41	2.86
GRADE 1:	42.52	54.44

RLN NUMBER 2

>>>>>>FEED STREAM 2 CONTAINS 100.0

SOLIDS AND 566.7

WATER

NODE NUMBER 1

AFTER SETTING UP, THE RMS ERROR IS 0.0000

UNIT 2 OUTPUT STREAM 3

CONTAINS: 64.00 SOLIDS AND 327.7 WATER

UNIT 2 OUTPUT STREAM 4

CONTAINS: 65.61 SOLIDS AND 491.5 WATER

UNIT 1 OUTPUT STREAM 6

CONTAINS: 21.06 SOLIDS AND 156.6 WATER

UNIT 1 OUTPUT STREAM 8

CONTAINS: 44.55 SOLIDS AND 254.9 WATER

UNIT 3 OUTPUT STREAM 12

CONTAINS: 55.52 SOLIDS AND 328.0 WATER

UNIT 5 OUTPUT STREAM 5

CONTAINS: 10.53 SOLIDS AND 155.2 WATER

UNIT 9 OUTPUT STREAM 10

CONTAINS: 29.29 SOLIDS AND 232.6 WATER

UNIT 11 OUTPUT STREAM 9

CONTAINS: 11.28 SOLIDS AND 53.13 WATER

UNIT 11 OUTPUT STREAM 12

CONTAINS: 28.01 SOLIDS AND 139.7 WATER

UNIT 12 OUTPUT STREAM 7

CONTAINS: 19.47 SOLIDS AND 63.81 WATER

UNIT 12 OUTPUT STREAM 13

CONTAINS: 6.545 SOLIDS AND 55.86 WATER

UNIT 3 OUTPUT STREAM 1

CONTAINS: 129.6 SOLIDS AND 219.2 WATER

NODE NUMBER 2
=====

AFTER SETTING UP, THE RMS ERROR IS 0.0000
=====

UNIT 6 OUTPUT STREAM 14

CONTAINS: 25.74 SOLIDS AND 223.0 WATER

UNIT 8 OUTPUT STREAM 15

CONTAINS: 25.74 SOLIDS AND 223.0 WATER

UNIT 10 OUTPUT STREAM 16

CONTAINS: 16.27 SOLIDS AND 65.21 WATER

UNIT 10 OUTPUT STREAM 17

CONTAINS: 19.47 SOLIDS AND 133.6 WATER

PLANT NUMBER 2

RUN NUMBER 1

NODE NUMBER 1
=====

AFTER SETTING UP, THE RMS ERROR IS 2.5143
=====

AFTER 1 ITERATIONS THE RMS ERROR IS 0.0000
=====

NODE NUMBER 2
=====

AFTER SETTING UP, THE RMS ERROR IS 1.8834
=====

AFTER 1 ITERATIONS THE RMS ERROR IS 0.0000
=====

RESULTS FOR PLANT 2 RUN 1

THE WATER FEED TO UNIT 2 IS 20.00

>>>>>>FEEC STREAM 1 CONTAINS 64.00 SOLIDS AND 327.7 WATER
MASS IN EACH CATEGORY:

D-CLASSES

	1	2	3
SCL 1 GCL 1	1.41	1.78	3.98
SCL 1 GCL 2	12.86	3.42	0.72
SCL 1 GCL 3	0.29	0.43	0.0
SCL 2 GCL 1	0.16	0.15	0.0
SCL 2 GCL 2	12.86	3.42	0.72
SCL 2 GCL 3	2.84	1.26	9.38

VALU WSTE

MASS: 25.53 32.47
GRADE: 35.85 40.11

>>>>>>FEEC STREAM 2 CONTAINS 3.453 SOLIDS AND 155.2 WATER
MASS IN EACH CATEGORY:

D-CLASSES

	1	2	3
SCL 1 GCL 1	0.24	0.44	0.17
SCL 1 GCL 2	1.23	0.42	0.02
SCL 1 GCL 3	0.02	0.0	0.0
SCL 2 GCL 1	0.03	0.00	0.0
SCL 2 GCL 2	1.23	0.43	0.02
SCL 2 GCL 3	0.16	0.44	0.17

VALU WSTE

MASS: 1.78 1.67 260.
GRADE: 51.00 41.40

 <<<<<<OUTPUT STREAM 15 CONTAINS 62.30
 MASS IN EACH CATEGORY:

SOLIDS AND 359.2 WATER

D-CLASSES

		1	2	3
SCL 1	GCL 1	1.66	2.72	3.25
SCL 1	GCL 2	13.67	3.43	0.57
SCL 1	GCL 3	0.31	0.40	0.00
SCL 2	GCL 1	0.18	0.14	0.00
SCL 2	GCL 2	13.67	3.43	0.57
SCL 2	GCL 3	2.72	7.61	7.10

VALU WSTE

MASSSES: 26.07 26.23
 GRADE X: 41.84 48.16
 X RECOVERY: 55.45 50.26

<<<<<<OUTPUT STREAM 20 CONTAINS 5.146

SOLIDS AND 143.7 WATER

MASS IN EACH CATEGORY:

D-CLASSES

		1	2	3
SCL 1	GCL 1	0.01	0.10	0.45
SCL 1	GCL 2	0.22	0.26	0.17
SCL 1	GCL 3	0.01	0.04	0.00
SCL 2	GCL 1	0.00	0.01	0.00
SCL 2	GCL 2	0.22	0.26	0.17
SCL 2	GCL 3	0.05	0.09	2.46

VALU WSTE

MASSSES: 1.24 2.50
 GRADE X: 24.14 72.56
 X RECOVERY: 4.55 5.72

*****TEAR STREAM 13 CONTAINS 16.18

SOLIDS AND 231.8 WATER

MASS IN EACH CATEGORY:

D-CLASSES

		1	2	3
SCL 1	GCL 1	0.11	0.43	0.64
SCL 1	GCL 2	1.99	1.24	0.27
SCL 1	GCL 3	0.08	0.16	0.00
SCL 2	GCL 1	0.01	0.02	0.00
SCL 2	GCL 2	1.99	1.24	0.27
SCL 2	GCL 3	0.47	3.10	4.18

VALU WSTE

MASSSES: 4.71 11.47
 GRADE X: 29.13 70.27

*****TEAR STREAM 14 CONTAINS 348.1

SOLIDS AND 869.2 WATER

MASS IN EACH CATEGORY:

D-CLASSES

		1	2	3
SCL 1	GCL 1	26.45	28.40	25.11
SCL 1	GCL 2	133.26	22.62	2.70
SCL 1	GCL 3	2.06	1.66	0.00
SCL 2	GCL 1	1.52	0.75	0.00
SCL 2	GCL 2	45.64	5.56	1.05
SCL 2	GCL 3	2.06	11.58	5.88

VALU WSTE

MASSSES: 200.27 147.86 281.
 GRADE X: 57.55 44.48

*****FEED STREAM 1b CONTAINS 9.764
MASS IN EACH CATEGORY:

SLIDE AND 45.79 WATER

		D-CLASSES		
		1	2	3
SCL 1	GCL 1	0.12	C.32	0.13
SCL 1	GCL 2	1.93	C.66	0.05
SCL 1	GCL 3	0.02	C.11	0.00
SCL 2	GCL 1	0.01	C.02	0.00
SCL 2	GCL 2	1.93	C.66	C.05
SCL 2	GCL 3	0.44	2.17	C.71

	VALU	WSTE
MASSSES:	3.45	6.13
GRADE X:	25.26	64.72

PLANT NUMBER 4

RUN NUMBER 1

>>>>>>>FEED STREAM 1 CONTAINS 16.37
MASS IN EACH CATEGORY:

SLIDE AND 133.8 WATER

		D-CLASSES		
		1	2	3
SCL 1	GCL 1	6.52	3.17	6.07

>>>>>>>FEED STREAM 2 CONTAINS 5.145
MASS IN EACH CATEGORY:

SLIDE AND 143.7 WATER

		D-CLASSES		
		1	2	3
SCL 1	GCL 1	0.21	1.35	3.25

NODE NUMBER 1

AFTER SETTING UP, THE RMS ERROR IS 0.3733

THE WATER FEED TO UNIT 4 IS -5.512

AFTER 1 ITERATIONS THE RMS ERROR IS 0.0783

THE WATER FEED TO UNIT 4 IS -31.01

AFTER 2 ITERATIONS THE RMS ERROR IS 0.0294

THE WATER FEED TO UNIT 4 IS -32.12

AFTER 3 ITERATIONS THE RMS ERROR IS 0.0023

THE WATER FEED TO UNIT 4 IS -32.40

***** ALLOWABLE NUMBER OF ITERATIONS EXCEEDED FOR PLANT 4 AT NODE

RUN NUMBER 2

NODE NUMBER 1

AFTER SETTING UP, THE RMS ERROR IS 0.0045

THE WATER FEE TO UNIT 4 IS -17.19

AFTER 1 ITERATIONS THE RMS ERROR IS 0.0006

THE WATER FEE TO UNIT 4 IS -17.25

AFTER 2 ITERATIONS THE RMS ERROR IS 0.0000

THE WATER FEE TO UNIT 4 IS -17.36

*****FEAR STREAM 6 CONTAINS 17.39 SOLIDS AND 69.56 WATER
MASS IN EACH CATEGORY:

D-CLASSES

	1	2	3
SCL 1 GCL 1	6.53	1.75	7.68

NODE NUMBER 2

UNIT 6 OUTPUT STREAM 4

CONTAINS: 20.15 SOLIDS AND 154.6 WATER
MASS IN EACH CATEGORY:

D-CLASSES

	1	2	3
SCL 1 GCL 1	7.03	5.15	7.97

RESULTS FOR PLANT 4 RUN 2

<<<<<OUTPUT STREAM 4 CONTAINS 20.15 SOLIDS AND 154.8 WATER
MASS IN EACH CATEGORY:

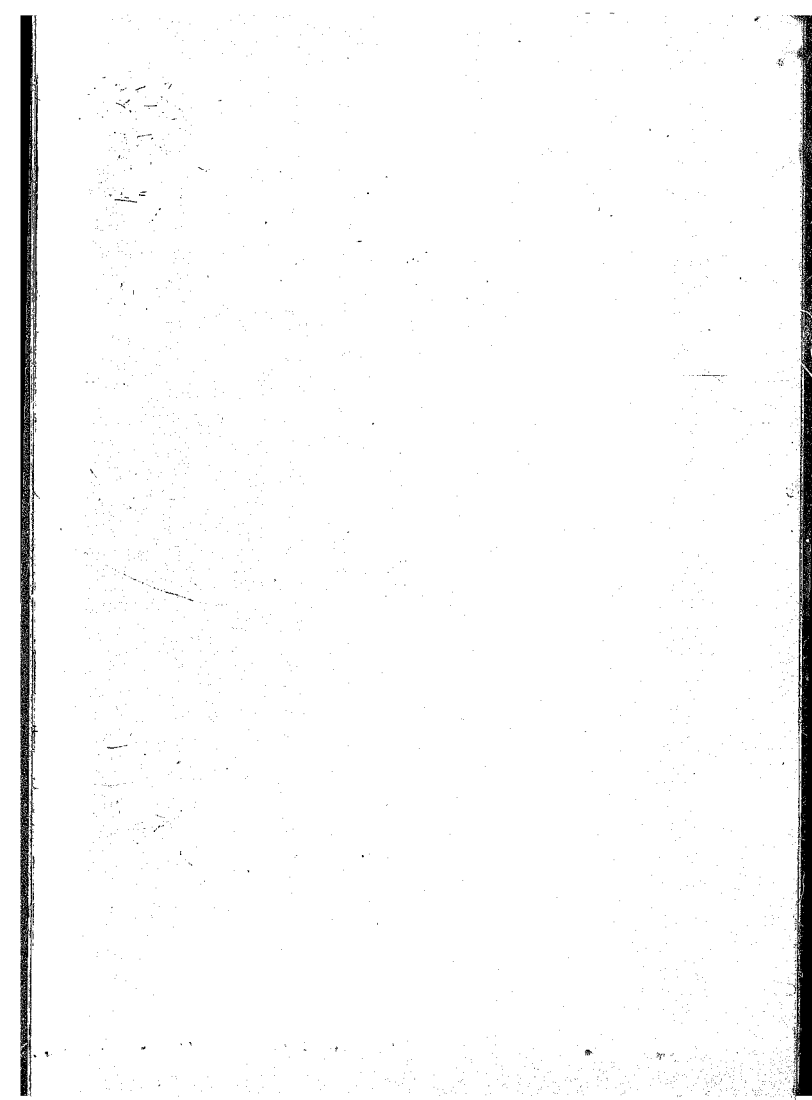
D-CLASSES

	1	2	3
SCL 1 GCL 1	7.03	5.15	7.97

<<<<<OUTPUT STREAM 12 CONTAINS 1.367 SOLIDS AND 105.3 WATER
MASS IN EACH CATEGORY:

D-CLASSES

	1	2	3
SCL 1 GCL 1	0.00	0.01	1.35



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Author Ford Merrill Anthony

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