

School of Mining Engineering



UNIVERSITY OF THE
WITWATERSRAND,
JOHANNESBURG

**A PROPOSED METHOD FOR
OPTIMIZING COAL PILLAR DESIGN
USING COALFIELD SPECIFIC
UNIAXIAL COMPRESSIVE STRENGTH**

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A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, 2021

DECLARATION

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ABSTRACT

The research described considers whether the variability in coal material strength as expressed through a series of UCS tests, could be used to indicate the variability in coal pillar strength. The aim of the research is to be able to use a distribution of UCS tests as an input into the coal pillar strength calculation. This will allow the pillar design to be expressed in terms of a probability of failure rather than as the commonly used safety factor. Using a regional coal material strength curve as a baseline, coalfields which are comparatively stronger than the regional mean allows for optimisation. This is based on the stronger coalfield achieving lower probabilities of failure at similar safety factors. The research has considered actual UCS data from multiple mines in the Mpumalanga coalfields, and has proved that the variability in material strength between coalfields could allow for some optimisation using the proposed approach. Further research will be required to validate the results of the study in an underground environment.

DEDICATION

Thank you to my loving, patient and incredibly supportive wife.

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This research was made possible by the contributions of Mr. Trevor Rangasamy from Middindi Consulting who supplied UCS test results for use in the analysis; Professor Nielen van der Merwe who provided the South African coal pillar database as well as guidance and insight on the matter of coal pillar design and Professor Thomas Stacey who supervised the research and provided feedback and support.

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LIST OF SYMBOLS

1	ARMPS	Analysis of Retreat Mining Pillar Stability
2	UCS	Uniaxial Compressive Strength
3	TAT	Tributary Area Theory
4	MPa	Megapascals
5	σ_s / σ_p	Pillar Strength
6	k	Pillar bulk strength
7	w	Pillar width
8	h	Pillar height

1 INTRODUCTION

1.1 Purpose of the Study

This research described in this report investigated whether the variability in Uniaxial Compressive Strength (UCS) of a specific coalfield could be used to define the variability in the pillar strength for the coalfield. Defining the variability in coal pillar strength would allow a pillar design to be expressed in terms of a probability of failure instead of the commonly used safety factor. This approach would not only quantify the uncertainty associated with the pillar design, but, by accounting for the actual material strength of the coal, allow for potential optimisation of planned pillar dimensions.

1.2 Research Background/Context

The pillar strength formulae currently used in the coal mining industry, make use of a statistically determined bulk strength factor to describe the strength of the pillar. The strength factors were obtained from the analysis of pillar failure data and provide a single strength value which is applied to all coalfields that formed part of the analysis. Although this approach has provided significant progress in designing stable pillars, these methods do not allow for the optimisation of pillar dimensions between stronger or weaker coalfields. Currently, a coal pillar failure database exists, consisting of 337 “unfailed” pillar cases and 87 “failed” cases (Van der Merwe, 2021). This database includes pillar failures in multiple coalfields

and various seams within South Africa, but does not provide sufficient data to allow for a confident analysis of the individual coalfields.

In line with the work conducted by Van der Merwe (2003; 2016; 2019), the terms “failed” and “unfailed” are used to distinguish between the “failed” and “stable” datasets. These terms were introduced based on the argument that the “stable” pillars could deteriorate over time to the extent that failure may occur and therefore cannot be assumed to be indefinitely “stable”.

1.3 Research Motivation

Currently no empirical methods exist that allow for a pillar design to be based on the material strength of a specific coalfield. The development of such a method would allow for pillars in stronger coalfields to be optimised relative to weaker coalfields. There have been multiple attempts to extrapolate coal pillar strength from UCS test results, however none of these studies was able to accurately correlate pillar strength with the strength of the intact rock mass.

Unlike these studies, the research discussed in this report did not attempt to extrapolate pillar strength from UCS results, but rather investigated whether the variability in coal material strength, as observed in UCS data, could be used to estimate the variability in the strength of pillars. In so doing, the strength of a coal pillar can be expressed as a probability

density function rather than a single strength value. This allows for the design to be expressed in terms of its probability of failure rather than the more commonly used safety factor. As a result, coalfields with notably stronger material strengths could be optimised in term of the pillar size required to ensure stability.

1.4 Problem Statement

In order to determine whether pillar optimisation between coalfields would be possible, the differences in the material strengths of the coalfields have to be quantified. With coal being an inhomogeneous, anisotropic material, such a comparison must account for the variability in material strength instead of using an abstract strength value for all coalfields. This can be achieved by introducing a distribution of strength data into the pillar strength calculation, something which cannot be achieved with the current pillar strength formulae. To address this problem, the study proposes a method in which a distribution of UCS values can be used to determine the probability of failure for a specific pillar design. This is achieved by developing a correlation between the mean UCS of the coal material and the *bulk coal strength* values of the existing pillar strength formulae.

From Hoek's (2007) notes on *Practical Rock Engineering*, a probability of failure can be determined by expressing multiple safety factors in the form of a probability density function. Safety factor provides the ratio between the capacity of a design and the demand or load induced on that design. A

safety factor above '1' would therefore indicate that the capacity is greater than the demand and the design will be stable. A safety factor below '1' would indicate instability. The concept is mathematically expressed in *Equation 1-1*.

$$\text{Safety Factor} = \frac{\text{Capacity}}{\text{Demand}} \quad \textbf{Equation 1-1}$$

By introducing a data distribution into either the capacity or the demand (or both), the resulting safety factor can be expressed as a probability density function. This function indicates the distribution of results based on the variability introduced by the dataset and can be mathematically expressed as seen in *Equation 1-2* (specific to a normal distribution).

$$f_x(x) = \frac{\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]}{\sigma\sqrt{2\pi}} \quad \textbf{Equation 1-2}$$

Where $f_x(x)$ is the probability density function of a normal distribution curve in terms of the variable x (see Figure 1-1), μ is the mean of the dataset and σ is the standard deviation of dataset. Using the probability density function, the probability of failure is obtained by calculating the area below the distribution curve for a safety factor less than '1' (therefore instability). This is illustrated in Figure 1-1 below.

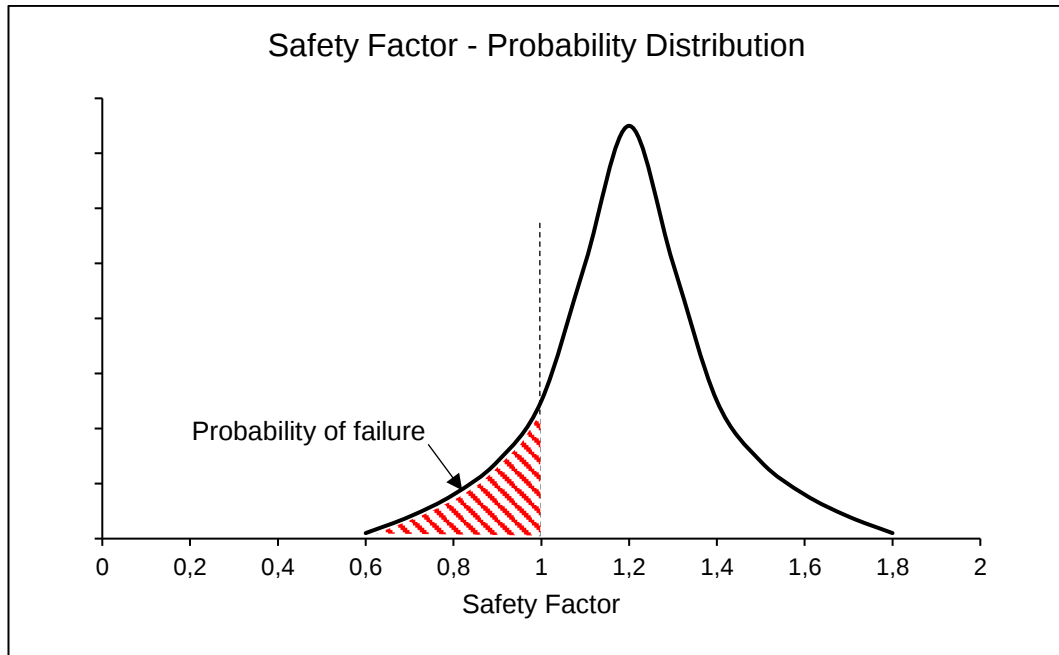


Figure 1-1. Illustration of the probability of failure as obtained from a normal probability density function.

The data distribution used in this study will be based on numerous UCS tests describing the strength of the coal material within a number of coalfields.

Multiple studies have found that the strength of coal material, as obtained from UCS tests, is much higher than the strength of a coal pillar. As such, the strength obtained from the UCS tests would have to be reduced to an acceptable level to accurately represent the strength of a coal pillar. Empirical pillar strength equations make use of a bulk strength value, k , to express the strength of a unit cube of coal. In the current pillar strength formulae, this bulk strength is multiplied with an adjusted width to height ratio to estimate the strength of a pillar. The common power law formula

on which many of these pillar strength equations are based, is given in *Equation 1-3*.

$$S = kh^\alpha w^\beta \quad \textbf{Equation 1-3}$$

Where S is the pillar strength, k is the strength of a unit volume of coal, h is the pillar height, w is the pillar width and α and β are dimensionless constants. As part of the study, a factor was required with which the UCS strength could be reduced to provide a strength value comparable to the k -values of each of the pillar strength equations. In addition to this, an acceptable probability of failure threshold was required to ensure that a design based on this approach will meet the current industry accepted design criteria.

1.5 Assumptions

To allow for a quantitative assessment of the proposed method, the study made use of the following assumptions:

- Pillar loading is accurately expressed by the application of the Tributary Area Theory (TAT), where every similar sized pillar carries an equal load of the overburden; and
- The set of UCS results used in the study, obtained from various South African collieries within the Mpumalanga province, provides a realistic indication of the variability in, and strength of, the coal material in Mpumalanga coalfields.

2 LITERATURE REVIEW

In order to ensure the approach proposed by this study expands on the research previously conducted in the field, a review was conducted of the historic pillar strength estimations and pillar design methodologies.

2.1 History of coal pillar strength estimations

The search for a reliable method with which coal pillar strength could be determined, is largely attributable to the Coalbrook disaster that occurred in January 1960. In 1962, Denkhaus (1962) conducted a review of the science of coal pillar strength and concluded that no method existed that could estimate pillar strength to a satisfactory level.

The initial attempts to use the rock strength of small-scale samples to describe the strength of a much larger pillar did not bear much fruit and soon the focus shifted to the testing of larger coal samples. One of these attempts was conducted by Bieniawski (1968) and the Council for Scientific and Industrial Research (CSIR) in South Africa who initiated a testing program within the Witbank coalfields. The idea was to determine whether the strength obtained from the “larger” coal samples could be used to effectively estimate the strength of a pillar through extrapolation in size. The testing program was conducted underground to ensure temperature and humidity differences, usually associated with surface testing, would not affect the results. Unfortunately, due to the amount of uncertainty in the extrapolation process, no clear link could be found

between sample size and strength. The results did however confirm that the strength of a sample decreases with increasing size (Figure 2-1).

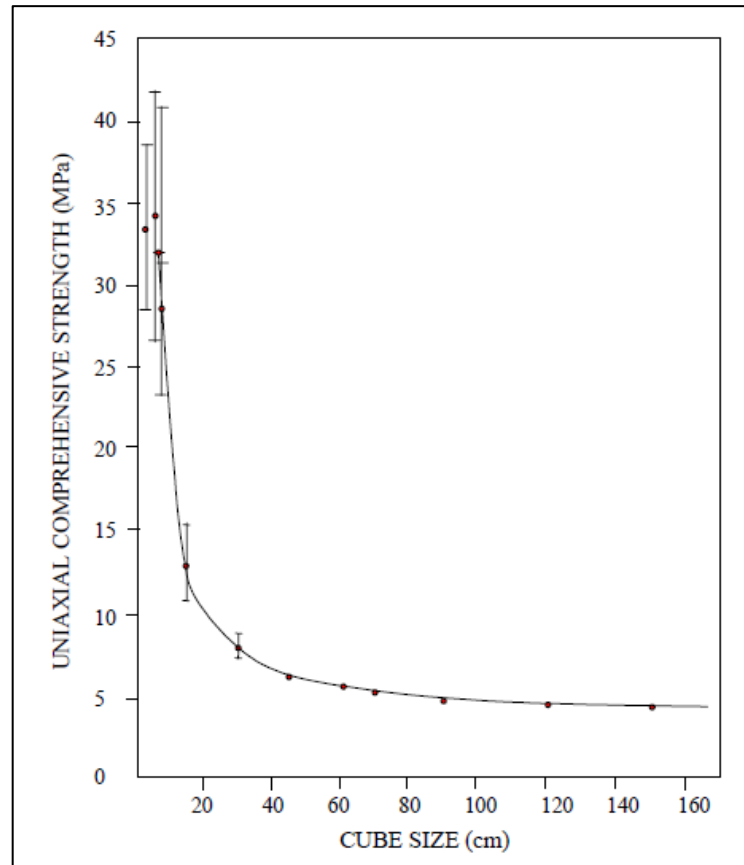


Figure 2-1. Illustration of the reduction in scatter obtained with increasing sample size (Bieniawski, 1968).

Following his study, Bieniawski (1968) published a correlation between the estimated strength of a pillar and its width-to-height ratio, with the correlation expressed in *Equation 2-1* below:

$$\sigma_p = 2.76 + 1.52 \frac{w}{h} \quad \text{Equation 2-1}$$

Wagner (1974), along with the Chamber of Mines, also conducted tests on “larger” coal samples at Usutu Colliery in 1974 (Wagner, 1974). Fitting a curve to the results, he was able to produce the following strength formula (*Equation 2-2*)

$$\sigma_p = 11 * \sqrt{\frac{w}{h}} \quad \textbf{Equation 2-2}$$

Applying a similar approach, as used by Wagner (1974), to Bieniawski’s (1968) results, produces a comparative strength formula as seen in *Equation 2-3*.

$$\sigma_p = 4.5 * \sqrt{\frac{w}{h}} \quad \textbf{Equation 2-3}$$

Similar testing was conducted by Van Heerden (1975) at New Largo Colliery, which also investigated the post failure characteristic of “larger” coal samples. Van Heerden’s (1975) approach made use of hydraulic jacks installed either on top of, or within an in-situ block of coal. Measuring instruments were installed on the coal sample, and the samples loaded to failure. Figure 2-2 shows a photograph of one of Van Heerden’s (1975) test-setups after the hydraulic jacks and instrumentation were installed. The strength formula obtained by Van Heerden (1975) is seen in *Equation 2-4*.

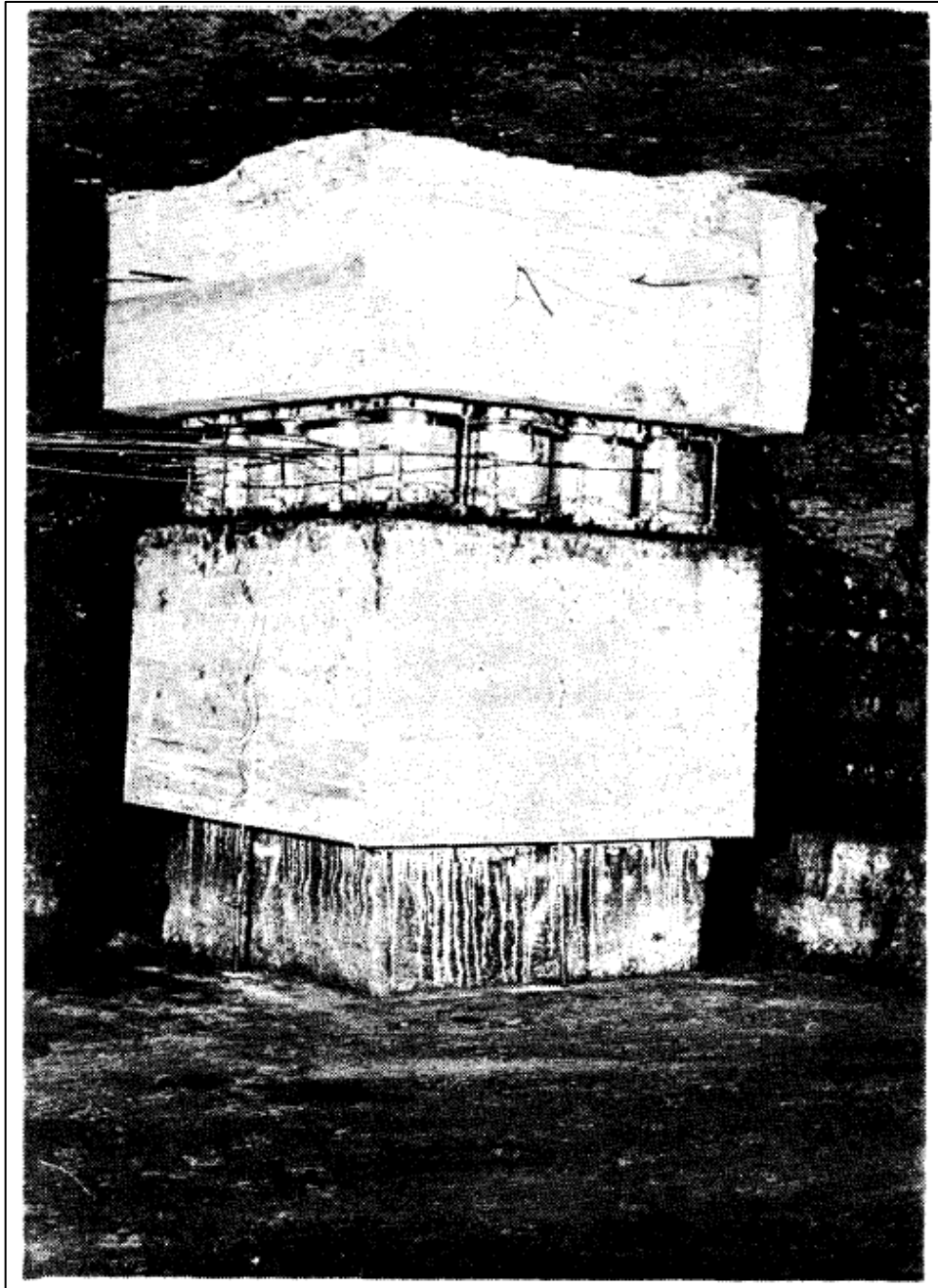


Figure 2-2. Photo taken of the test-setup as used by Van Heerden in his study. The photo shows the hydraulic jacks installed in between two cement blocks, with the coal sample located on the floor (Van Heerden, 1975).

$$\sigma_p = 13.3 * \sqrt{\frac{w}{h}}$$

Equation 2-4

Up to this stage, none of the “larger” sample tests provided definitive results in estimating coal pillar strength. As such, Salamon and Munro (1967), opted for an empirical solution. In order to accurately quantify the strength of a coal pillar, Salamon and Munro (1967) compiled a database consisting of 125 pillar “cases”. The word “cases” was used, as each case referred to a panel or segment of a mine in which similar sized pillars were excavated, providing a more definitive indication of pillar behaviour compared to individual pillars. From the 125 data entries, 98 represented stable cases (therefore “unfailed”) with the remaining 27 representing areas where pillars failed. Using the two datasets, Salamon and Munro (1967) considered a probabilistic approach to describe the data. Instead of focussing purely on pillar strength, the concept of a safety factor was selected for their analysis. This approach relied on actual pillar behaviour and provided not only an indication of pillar strength, but pillar loading as well.

Their analysis described both the failed and unfailed datasets using a set of probability density functions. In essence a probability curve was produced from the “failed” cases, with a second curve for the “stable” cases. Each curve described the range of safety factors within each dataset. The pillar load was determined as a function of the extraction percentage as seen in *Equation 2-5* below:

$$p = \frac{1.1H}{1-e} \quad \text{Equation 2-5}$$

Where p is the load induced on the pillar, H is the depth below surface and e is the extraction percentage. The extraction percentage (e) is calculated using *Equation 2-6*.

$$e = 1 - \left(\frac{w}{w+B} \right)^2 \quad \text{Equation 2-6}$$

Where w is the width of a square pillar and B is the bord width. The final pillar strength is obtained from *Equation 2-7*:

$$\text{Strength} = kw^\alpha h^\beta \quad \text{Equation 2-7}$$

The variables (α) and (β) are constants selected to best describe the probability density distribution relative to the critical safety factor (S_c) at which failure occurs. The statistical method used to obtain (α), (β) and (k) is described as the “method of least squares”, later referred to as the *maximum likelihood* method. The approach made use of the assumption that the critical safety factor (S_c), is located between the range of safety factors where the pillar remains stable (S), and a safety factor of ‘1’, therefore; $1 < S_c < S$. This statement thus assumes that pillar failure will occur once the safety factor reaches ‘1’ and pillar load starts exceeding the pillar strength.

Based on the assumption that failure will occur at a safety factor of '1', the values of (α) , (β) and (K) should be such that the curve describing the 27 failed cases, centres around a safety factor of 1. The final pillar strength formula proposed by Salamon and Munro (1967) is seen in *Equation 2-8*. Using this equation, the safety factors for each of the 125 cases in the database were calculated, with the results illustrated graphically in Figure 2-3. As seen from Figure 2-3, the failed pillar data clusters around the line along which pillar strength is equal to pillar load (safety factor = 1), thus meeting the criterion stipulated by Salamon and Munro (1967). The stable cases on the other hand, all plot below the threshold line, falling within the "stable" sector of the graph.

$$Strength = 7.17 \left(\frac{w^{0.46}}{h^{0.66}} \right) \quad \textbf{Equation 2-8}$$

A few years after the Salamon and Munro (1967) formula was published, Salamon and Oravecz (1976) made the comment that the Salamon and Munro (1967) strength formula underestimates the strength of a pillar when the width to height ratio exceeds a range of 5 to 6, as it does not account for the additional confinement of the pillar core at large width to heights. Salamon and Wagner (1985) subsequently proposed a formula which allowed for a rapid increase in pillar strength above a critical width to height ratio (*Equation 2-9*).

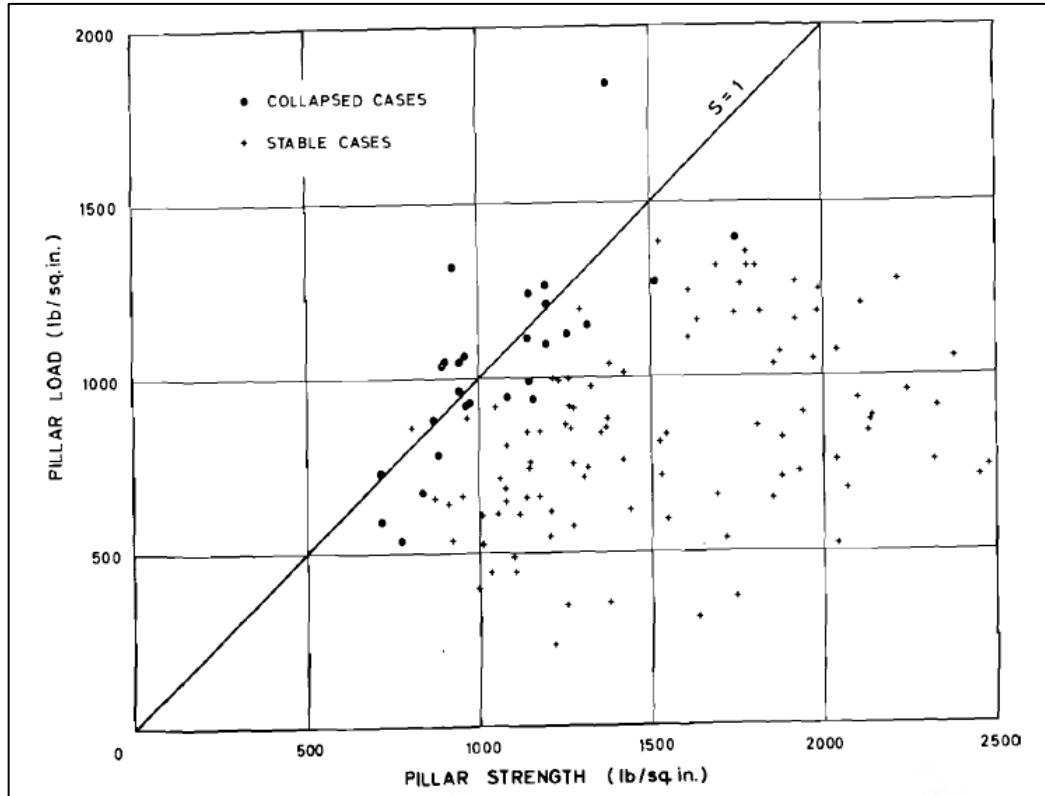


Figure 2-3. Salamon and Munro's stable and failed datasets (Salamon & Munro, 1967).

$$Strength = 7.2 \frac{R_0^{0.5933}}{V^{0.0667}} \left\{ \frac{0.5933}{\epsilon} \left[\left(\frac{R}{R_0} \right)^\epsilon - 1 \right] + 1 \right\} \quad \text{Equation 2-9}$$

From *Equation 2-9*; R_0 is the critical width to height ratio at which the “squat effect” becomes relevant, R is the actual width to height ratio of the pillar, V is the pillar volume and ϵ is the parameter that controls the rate of strength increase. The critical width to height ratio was suggested as $R_0 = 5$ and the rate of strength increase parameter as $\epsilon = 2.5$. This formula later became known as the *squat pillar formula*.

Madden (1991) conducted further research to substantiate the validity of the squat pillar formula based on additional data available at the time. He also found that the strength of a coal pillar increased almost linearly up to a width to height range of 5 to 6, after which the strength starts increasing almost exponentially. Contrary to this, Mathey and Van der Merwe (2016) discourage the use of the squat pillar formula on the basis that the formula provides a misleading interpretation of the mechanical behaviour of large width to height pillars. Mathey and Van der Merwe (2016) considered data from analytical and numerical models, field observations as well as laboratory tests and concluded that the high load bearing capacity of coal pillars with a width to height ratio in excess of 10 to 12 is more plausibly as a result of the transition from brittle to ductile behaviour in the pillar core.

Madden *et al.* (1995) conducted a re-analysis of an updated pillar failure database which contained 17 additional failures compared to the database used by Salamon and Munro (1967). The results correlated well with those obtained by Salamon and Munro (1967), and although a different strength formula was obtained, the difference was not sufficient to warrant the use of the Madden *et al.* (1995) formula instead of the original Salamon and Munro (1967) formula.

The failure database was again updated by Van der Merwe (2003) with an additional 28 failures, while the “unfailed” dataset consisted of the same 98 cases described in Salamon and Munro’s (1967) study. In addition to conducting a re-analysis of the pillar strength formula, Van der Merwe also

proposed a method with which to estimate the timespan for which a pillar will remain stable, based on the scaling rate of the pillar. This concept is also referred to as the *Pillar Life Index*, and expresses the time required for the pillar to scale from its initial mined dimensions to the point where failure occurs. The study found that the scaling rate is directly proportional to the mining height, however the rate of scaling decreases over time.

What is perhaps of more interest from the pillar life study, was that Van der Merwe had to split the various coalfields into groups in order to obtain meaningful correlations. The Vaal Basin, Utrecht, Klip River and South Rand fields were separated as these indicated a distinctly shorter pillar life compared to the remaining coalfields. In subsequent studies, this group would be referred to as the Vaal Basin Coalfields. Based on these findings Van der Merwe (2003) opted to separate the Vaal Basin and Witbank groups during the re-analysis of the strength formula, with his 2003 study focussing on the Witbank coalfields.

Instead of making use of the *maximum likelihood* method, Van der Merwe (2003) approached the data analysis with what has become known as the *overlap reduction* method. This method presented both the failed and unfailed datasets as probability distributions, arguing that the ideal formula would result in a complete separation between the two curves. The concept is illustrated in Figure 2-4 and makes use of an iterative process to find the minimum overlap between the failed and unfailed probability density functions.

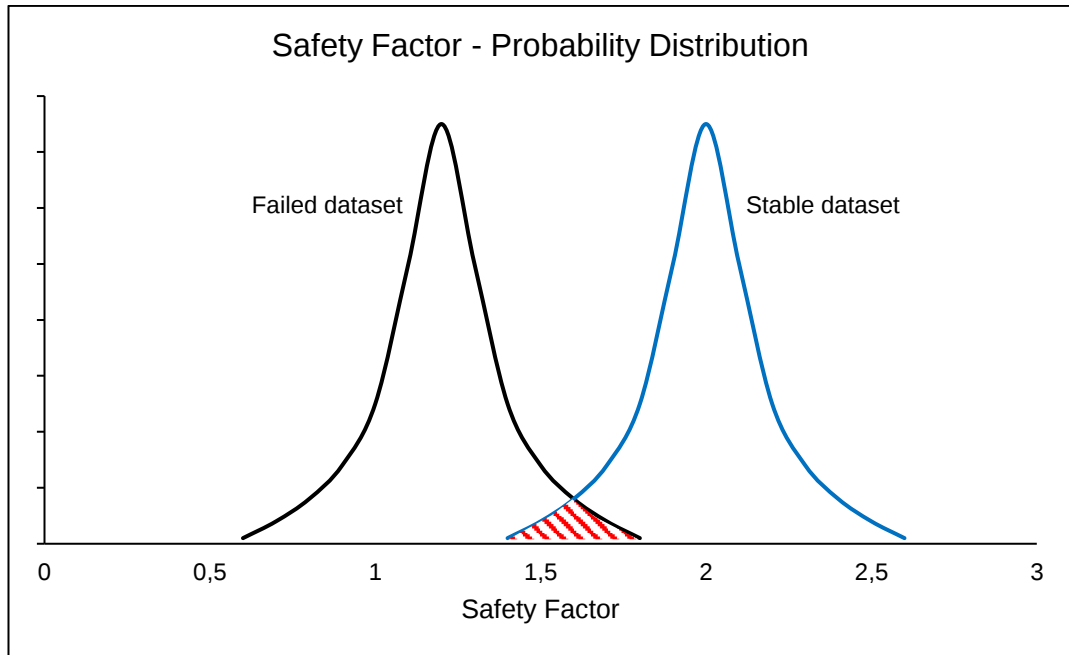


Figure 2-4. Illustration of the “overlap” between the stable and failed distributions as investigated by the overlap reduction method.

The analysis resulted in the pillar strength formula seen in *Equation 2-10*.

$$Strength = 3.5 \left(\frac{w}{h} \right) \quad \text{Equation 2-10}$$

In 2013 another update was made to the pillar failure database, this time by Van der Merwe and Mathey (2013^a). The updated database consisted of a total of 86 failed cases and 337 unfailed cases and the analysis made use of both the overlap reduction and maximum likelihood methods. As a result, two formulae were produced. The *maximum likelihood* method provided *Equation 2-11* while the *overlap reduction* approach resulted in *Equation 2-12*.

$$Strength = 6.61 \left(\frac{w^{0.5}}{h^{0.7}} \right) \quad \text{Equation 2-11}$$

$$Strength = 5.47 \left(\frac{w^{0.8}}{h} \right) \quad \text{Equation 2-12}$$

In his latest study Van der Merwe (2019) accounted for the effect of pillar scaling and the associated reduction in pillar size, considering the influence this would have on pillar design. In order to describe the rate of scaling, Van der Merwe made use of the equation developed as part of his 2016 study that focussed on the *pillar life index*. According to the study, the rate of scaling can be described by *Equation 2-13*.

$$dT = mh^x T^{1-x} \quad \text{Equation 2-13}$$

Where h is the mining height, m is the constant 0.1799, x is the constant 0.7549 and T is the age of the pillars in years. The reduced pillar width is subsequently obtained using *Equation 2-14*.

$$w_T = w_0 - dT \quad \text{Equation 2-14}$$

From these equations, the pillar sizes within both the “failed” and “unfailed” datasets were reduced according to their time since mining. The updated datasets were then used to obtain an updated pillar strength formula, with the formula based on the overlap reduction method described by Van der Merwe and Mathey (2013^a) in their pillar strength study. It should be noted

that in determining the updated pillar strength formula, the loads on the pillars were also recalculated according to the expected reduction in pillar size resulting from scaling over time. The pillar strength formula obtained from this analysis is seen in *Equation 2-15*.

$$\sigma_{S,T} = 10.2 \frac{w_T^{0.74}}{h^{0.85}} \quad \textbf{Equation 2-15}$$

Where $\sigma_{S,T}$ is the pillar strength accounting for the effect of time and w_T is the pillar width at time of failure.

From the formulae discussed in this section of the report, the four most commonly used formulae in industry include Salamon and Munro's (1967) formula, Van der Merwe's (2003) linear formula and Van der Merwe and Mathey's (2013^a) *maximum likelihood* and *overlap reduction* formulae. Figure 2-5 illustrates a comparison between the different formulae indicating the difference in strength increase associated with increasing width to height ratio for each.

2.2 Pillar design methods for rock types other than coal

Pillar design has been a contentious aspect in rock mechanics for many years including outside the coal mining fraternity. Following the Coalbrook disaster, coal pillar design received markedly more attention compared for example with hard rock pillar design.

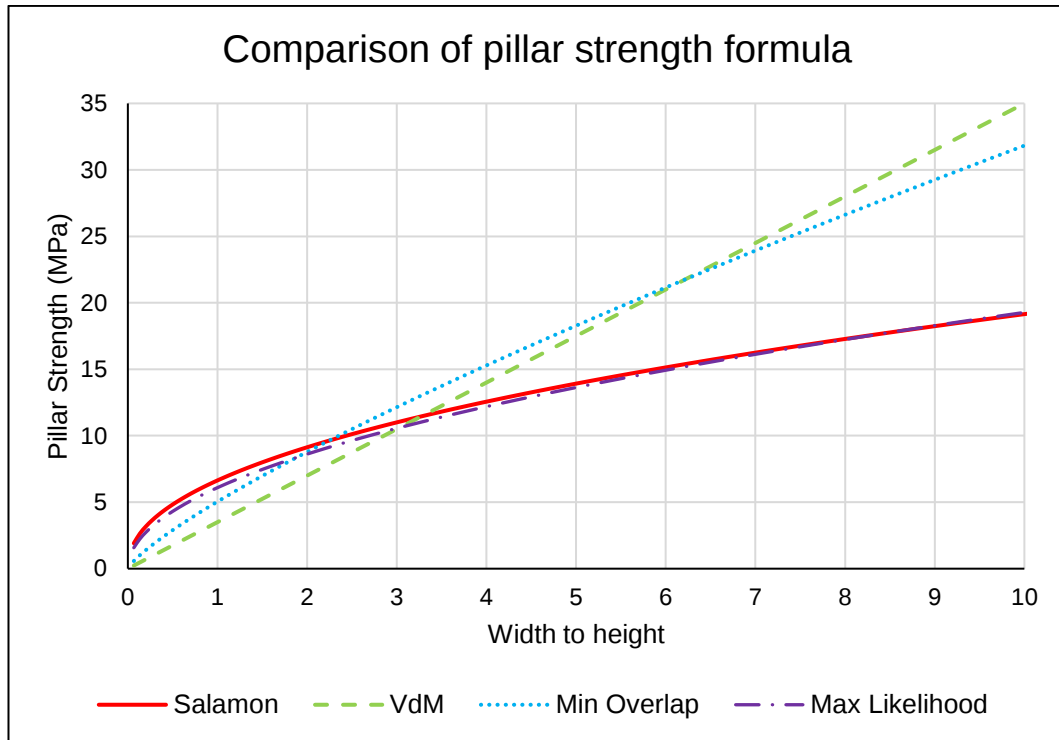


Figure 2-5: Comparison of the four commonly used pillar strength formulae in the South African mining industry.

As a result, there has been little drive to ensure pillars are not “under-designed”. One of the first attempts to define hard rock pillar strength, was the research conducted by Hedley and Grant (1972), who analysed 28 rib-pillars developed in quartzites and conglomerates at the Elliot Lake uranium mine. The approach adopted by Hedley and Grant (1972) was similar to that of Salamon and Munro (1967), using back analysis from a combination of stable and failed pillar cases and fitting a power curve to best represent the data. The resulting formula is seen in *Equation 2-16*.

$$\sigma_p = k \frac{w^{0.5}}{h^{0.75}} \quad \text{Equation 2-16}$$

The bulk strength adopted for the Hedley and Grant (1972) formula was initially selected as 179 MPa and later reduced to 133 MPa.

Since the work conducted by Hedley and Grant (1972), several other studies investigated the strength of hard rock pillars and provide formulae with which to calculate pillar strength. These studies include the work by Von Kimmelman *et al.* (1984), Krauland and Soder (1987), Potvin *et al.* (1989), Sjöberg (1992) and Lunder and Pakalnis (1997). The pillar strength formulae proposed by these authors are summarised in Table 2.1. What becomes apparent from the various formulae, is that the strength of a pillar is in all cases a function of the pillar width and height and makes use of a combination of single or multiple constants to describe the bulk strength. The only exception to this, is the formula published by Lunder and Pakalnis (1997), who expressed the pillar strength as a function of the field stress ratio between σ_1 and σ_3 . This being the case, the ratio between σ_1 and σ_3 , representing the average confinement in a pillar, remains a function of the pillar width and height as seen in *Equation 2-17* (Lunder & Pakalnis, 1997):

$$\frac{\sigma_3}{\sigma_1} = 0.46 \left[\log \left(\frac{w}{h} + 0.75 \right) \right]^{\frac{1.4}{(w/h)}} \quad \textbf{Equation 2-17}$$

Table 2.1: Hard rock pillar strength formulae proposed by different authors.

Pillar strength formula	Reference
$PS = 133 \frac{w^{0.5}}{h^{0.75}}$	Hedley & Grant (1972)
$PS = 65 \frac{w^{0.46}}{h^{0.66}}$	Von Kimmelman <i>et al.</i> (1984)
$PS = 35.4 \left(0.778 + 0.222 \frac{w}{h} \right)$	Krauland & Soder (1987)
$PS = 0.42 \sigma_c \frac{w}{h}$	Potvin <i>et al.</i> (1989)
$PS = 74 \left(0.778 + 0.222 \frac{w}{h} \right)$	Sjöberg (1992)
$PS = 0.44 \sigma_c (0.68 + 0.52K)$	Lunder & Pakalnis (1997)

In Table 2.1, PS is the pillar strength and σ_c the uniaxial compressive strength of the rock. The symbol K , used in Lunder and Pakalnis' (1997) equation, indicates the ration between the far field horizontal and vertical stress within the rock mass. A comparison of the different formulae from Table 2.1 was conducted by Martin and Maybee (2000), which produced the graph in Figure 2-6. In producing Figure 2-6, the UCS was normalised to allow for a direct comparison between formulae.

What is interesting to note from Table 2.1, is that both Potvin *et al.* (1989) as well as Lunder and Pakalnis (1997) opted for a similar approach as proposed by this study, in that the UCS is multiplied by a factor to describe pillar bulk strength.

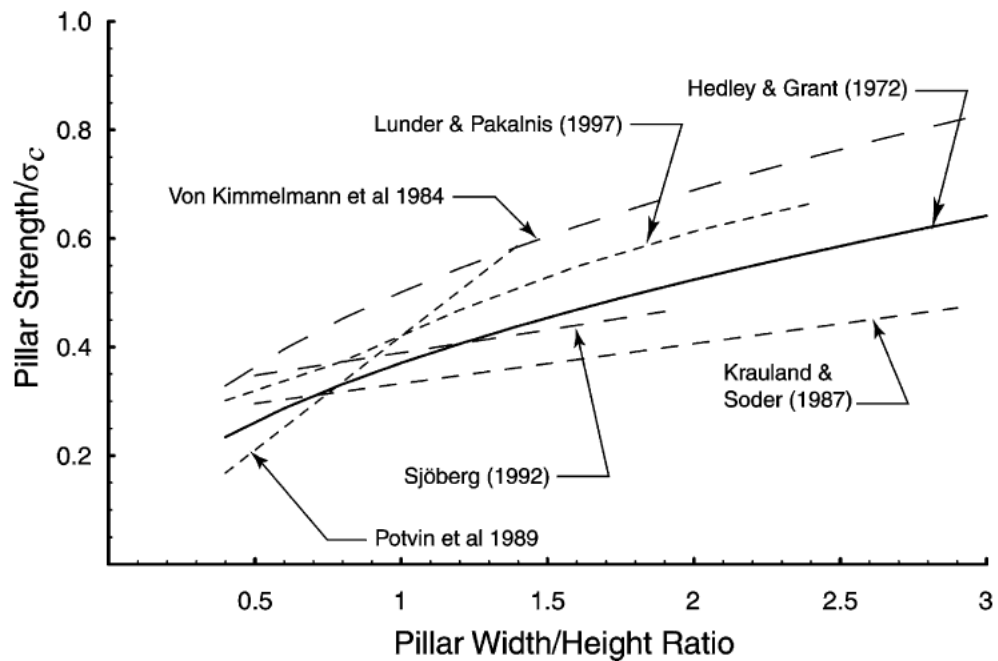


Figure 2-6: Comparison of the empirical pillar strength formulae published for hard rock pillars (Martin and Maybee, 2000).

Another example of using the UCS to describe pillar strength is the estimation of the (k) parameter for use in the Hedley and Grant (1972) formula, as being a third of the UCS (Ozbay *et al*, 1995). It is therefore not uncommon for the bulk strength of a pillar to be dependent on the UCS of the rock mass.

2.3 Current methods to calculate pillar probability of failure

As part of their initial work, Salamon and Munro (1967) inferred a “probability of having a stable layout”. This was based on the frequency distribution of the failed pillar dataset as used by Salamon and Munro (1967). Although not strictly a probability of failure, it did provide a

quantified estimation of pillar stability in terms of probability. Van der Merwe and Mathey (2013^b) derived a similar probability, however in their approach the probability of failure was based on both the failed and unfailed datasets. From their study, the probability of failure is expressed in *Equation 2-18*.

$$\text{Probability of Failure} = \left(\frac{N_{\text{failed}}}{N_{\text{failed}} + j * N_{\text{stable}}} \right) \quad \textbf{Equation 2-18}$$

The parameter j from *Equation 2-18* represents an extrapolation factor used to estimate the total number of unfailed cases. The analysis was conducted on the assumption that the failed dataset is a true reflection of “all” failed cases in South Africa, however the unfailed dataset is only a representation of the total number of stable pillars ever mined in South Africa. Van der Merwe and Mathey (2013^b) estimated the total number of unfailed cases using historically mined coal volumes for Southern Africa, as well as baseline assumptions for pillar sizes and bord widths. The total numbers of cases were subsequently compared to the unfailed dataset, and the extrapolation from the dataset to the total number produced the extrapolation factor j . Van der Merwe and Mathey (2013^b) argued that in calculating a probability of failure, considering the failed database on its own will not provide a realistic indication of probability as the stable cases obtained at similar safety factors cannot be disregarded.

Van der Merwe and Mathey (2016) further expressed their probability of failure in terms of a *probability of survival* (POS). The relation between the

probability of survival and the probability of failure is seen in *Equation 2-19*.

$$POF = \frac{100-POS}{100} \% \quad \textbf{Equation 2-19}$$

Expressing the probability of survival in terms of safety factor, Van der Merwe and Mathey (2013^b) used both their *maximum likelihood* and *overlap reduction* pillar strength equations. The *maximum likelihood* formula produced the relation seen in *Equation 2-20*, while the overlap reduction formula resulted in *Equation 2-21*.

$$POS_{ML} = 100 - 100\exp(-2.7SF_{ML}^{1.9}) \% \quad \textbf{Equation 2-20}$$

$$.POS_{OR} = 100 - 100\exp(-2.5SF_{OR}^{1.8}) \% \quad \textbf{Equation 2-21}$$

In *Equation 2-20* and *Equation 2-21*, POS_{ML} is the probability of survival based on the maximum likelihood approach, POS_{OR} is the probability of survival based on the overlap reduction approach, SF_{ML} is the safety factor obtained from the maximum likelihood strength formula and SF_{OR} is the safety factor obtained from the overlap reduction strength formula. Based on these relations, Van der Merwe (2016) proposed a design method which allows the design of a coal pillar to be based on the probability of survival rather than the commonly used safety factor. The approach considers the intended excavation life, designing to a minimum required

probability of survival as outlined by Van der Merwe (2016) and indicated in Table 2.2.

Table 2.2. Panel categories with the minimum probability of survival of each (Van der Merwe, 2016)

Category	Minimum POS %	Remarks
Production Panel	99.0	Short term, limited access, low traffic
Secondary Development	99.5	Medium term, general access, medium traffic
Main Development	99.9	Long term, general access, high traffic
Surface Structure (1)	99.99	Low sensitivity to subsidence
Surface Structure (2)	99.995	High sensitivity to subsidence, public access

The Van der Merwe (2016) probability of survival method provided the first real attempt to produce a pillar design based on probability rather than safety factor. In addition to the updated pillar strength formula produced by Van der Merwe (2019), an updated probability of failure estimation was derived. Using the same approach as the Van der Merwe and Mathey 2013^b study, the probability is expressed as the percentage of “failed” cases within the entire dataset for a specific safety factor. From the updated analysis, the probability of failure of a pillar can be calculated using *Equation 2-22*.

$$PoF = \exp(-0.93SF_T^{4.28})$$

Equation 2-22

In *Equation 2-22*, SF_T is the safety factor accounting for the reduction in pillar size over time. It should be noted that for this latest study, Van der Merwe (2019) again focussed on the Witbank and Highveld no. 2 and 4 seams. As such the formula is not intended for application to coalfields outside of this data range.

2.4 Difference in strength between coalfields

To determine whether the strength differences between coalfields and seams provide any meaningful opportunity for pillar optimisation, a literature review on the subject was conducted.

Initial indications that coalfields have different strengths were obtained from the testing of 'larger' coal samples by Bieniawski (1968), Van Heerden (1975) and Wagner (1974), each of which obtained a different strength curve from their respective test results. The tests were conducted on different coalfields and on different mines with the results graphically illustrated in Figure 2-7 below. York and Canbulat (1998) provided several possible reasons for this, chief amongst which was the different approaches used during testing, suggesting that the difference in material strength only partly contributes.

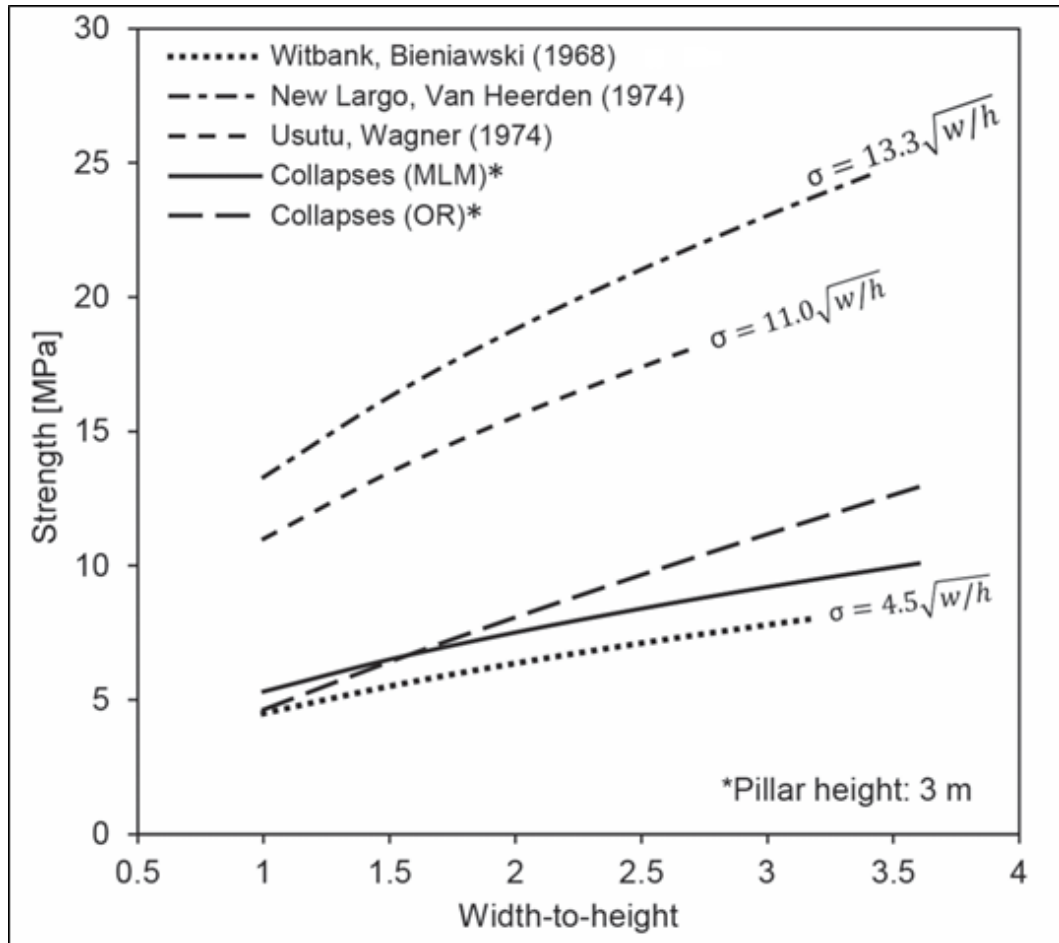


Figure 2-7. Illustration of the difference in strength curves obtained between different coalfields based on the testing of large-scale coal samples (Mathey and Van der Merwe 2016).

Madden and Hardman (1992) conducted a comparison between their pillar strength formula and that of Salamon and Munro (1967) using data from different seams. Interestingly, it was found that the differences between individual seams were statistically so little, they concluded that an average strength value could be used to effectively represent the different seams. Bertuzzi *et al.* (2016) found in a comparison of coal strengths between Australia, South Africa, USA, India and the UK, that the coal strengths

between the different reserves, although not exactly equal, are very similar.

Contrary to this, Van der Merwe (2016) states that there is sufficient evidence indicating the strength of coal to be highly variable. A study by Salamon, Canbulat and Ryder (2006), aimed at determining seam-specific strengths by means of back analysis, also concluded that significant variations exist between coalfields. Van der Merwe and Mathey (2013^a, 2013^b) have also highlighted the fact that the strength of coal varies between different coalfields and coal seams.

2.5 Use of uniaxial compressive strength results in defining pillar strength

As mentioned earlier in the report, there have been many attempts to use the uniaxial compressive strength of coal to estimate pillar strength, none of which returned usable results. York and Canbulat (1998) provide a detailed summary of these attempts, all of which lead to the conclusion that the extrapolation of pillar strength from a laboratory sample is highly affected by the ‘scaling’ effect. The ‘scaling’ effect refers to the estimated reduction in sample strength obtained when increasing the size of the sample. One such method which received wide acceptance in the United States, is Gaddy’s (1956) formula. Gaddy (1956) proposed a relationship between the decrease in strength and increase in sample size based on multiple tests conducted on a range of different sized coal samples. The relationship is given in *Equation 2-23*.

$$\sigma_{cl} = KD^{-0.5}$$

Equation 2-23

In *Equation 2-23*, K is known as the *Gaddy constant* which is estimated as the strength of a 2.5 cm cube of coal, and D is the specimen dimensions in inches. York and Canbulat (1998) concluded that the estimation of pillar strength by means of scaling only provides an estimation of pillar strength at best.

A study of particular relevance to this research report, is that of Mark and Barton (1996) which investigated whether pillar strength can effectively be extrapolated from laboratory coal samples. In addition to this, they investigated the influence of coal seam strength and whether a stronger seam makes for a stronger pillar when extrapolating strength according to volume. This was achieved by using UCS results from various seams to estimate the specific pillar strength associated with each seam. The pillar strengths were estimated using *Equation 2-23*. Factors of safety were calculated for the case studies from which the Analysis of Retreat Mining Pillar Stability (ARMPS) formula was developed by Mark *et al.* (1995). In calculating the safety factors, the pillar strengths obtained for the various seams were used with the assumption being that the stronger coal material would make for stronger pillars and therefore fail at a higher safety factor. The opposite was however observed, with the stronger coal seams failing at lower safety factors. Another analysis conducted by Mark and Barton (1996) showed that the use of a uniform bulk pillar strength,

provides a significantly better fit to the back analysis of case histories, compared to the results based on the individual coal seam strengths. From this, Mark and Barton (1996) concluded that the use of seam specific strength becomes meaningless when designing coal pillars, and that the use of a uniform bulk strength value of 6.2 MPa is preferred. Mark and Barton (1996) argued that one of the key reasons laboratory results are not suited to the estimation of pillar strength, is because a pillar is significantly influenced by geological features not represented in the sample. As a result, the laboratory sample will not only be significantly stronger, but the failure mechanism between sample and pillar will differ.

This being the case, full scale pillar testing is currently not feasible, and even if it were, multiple pillar tests would be required to provide sufficient understanding of the variability in pillar strength within a specific coalfield. Additionally, to conduct a fair comparison between the strength of various coalfields, a common testing approach is required. Currently one of the most common and relatively inexpensive indications of rock strength remains the UCS test. Considering all its shortfalls in describing pillar strength, it still provides an indication of the strength of the material. The use of a single bulk strength value in the design of pillars, also limits the ability for any optimisation between reserves.

3 RESEARCH METHODOLOGY

In order to determine whether the variability in coal pillar strength can effectively be described by the variability in UCS data, the study required a method with which UCS strength data could be introduced into the pillar strength calculations. From the literature study it is clear that historic attempts to extrapolate pillar strength from UCS results by means of scaling did not provide reliable results. The approach adopted for this study, was therefore to keep to the bulk strength values determined for each of the formulae by the original authors, introducing UCS results only as a means of expressing variability. To achieve this, the equations required some adjustment. This section of the report deals with the methodology on how the adjusted formulae were determined as well as the data used.

3.1 Data used in study

The study made use of both coal UCS and pillar data. The detail of the various datasets is discussed in the following sub-sections.

3.1.1 Coal strength data

Coal, being naturally inhomogeneous and anisotropic, differs in strength from one position in a coalfield to the next. It is this variability which requires design engineers to make use of a safety factor when defining a pillar design. To effectively understand the variability in a coal seam, sufficient data points (UCS test results) are therefore required.

As discussed under the literature review, coal deposits in South Africa differ in strength. Most notably the strength of the Vaal basin coalfields has been found to be markedly less than that of the Mpumalanga coalfields. With this in mind, Van der Merwe (2003) and Van der Merwe and Mathey (2013) opted to exclude the Vaal basin pillar data during the development of the *linear*, *maximum likelihood* and *overlap reduction* strength formulae. Taking this into account, applying these formulae to data sourced from the Vaal Basin is outside of the intended scope of the formulae and could provide unrealistic results. Based on this, the data used in this study was only sourced from mines located within the Mpumalanga coalfields.

The total dataset consisted of 83 coal UCS results and covered multiple mining operations. As part of the agreement to use the data, the mining operations are not mentioned by name in the study. Most operations had limited UCS data available, however three mines, referred to as Mine A, B and C, provided sufficient data to allow for the variability in coal strength at each operation to be defined to a higher confidence level. Table 3.1 below provides a summary of the UCS data used in the study.

In order to calculate a probability of failure from the UCS data, the data was expressed as probability density functions. To assist with analysis, the function curves were produced using the software program *@Risk*. The program allows the user to select the statistical function which best fits the distribution of data in a specific dataset.

Table 3.1. Summary of the UCS dataset used in the study (All data provided in MPa).

Source	Number of datapoints	Mean	Min	Max	Std. Dev
Mine A	9	22.4	12.3	40.5	8.9
Mine B	21	21.9	9.3	40.5	7.9
Mine C	20	29.4	22.0	38.5	5.3
Multiple sources	33	24.1	7.6	48.8	9.7
Total Dataset	83	24.6	7.6	48.8	8.7

The functions selected for the study were all Lognormal distributions as this provided the best representation of the actual distribution of data in each of the datasets. A comparison between the original data and the Lognormal functions is seen in Table 3.2. The resulting distributions are illustrated in Figure 3-1 to Figure 3-4.

Table 3.2. Comparison of original and best-fit datasets.

Dataset	Mean		Standard Deviation	
	Original	Lognormal curve	Original	Lognormal curve
Mine A	22.4	22.8	8.9	12.3
Mine B	21.9	21.9	7.9	8.0
Mine C	29.4	29.5	5.3	5.8
Total Dataset	24.6	24.7	8.7	8.8

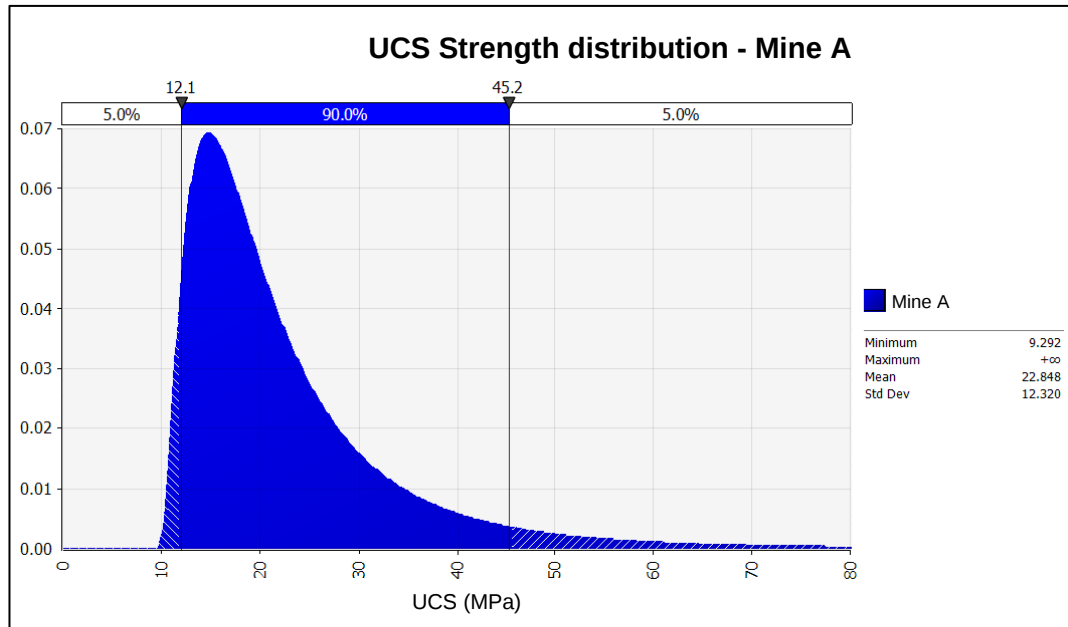


Figure 3-1. UCS Strength distribution for data obtained from Mine A.

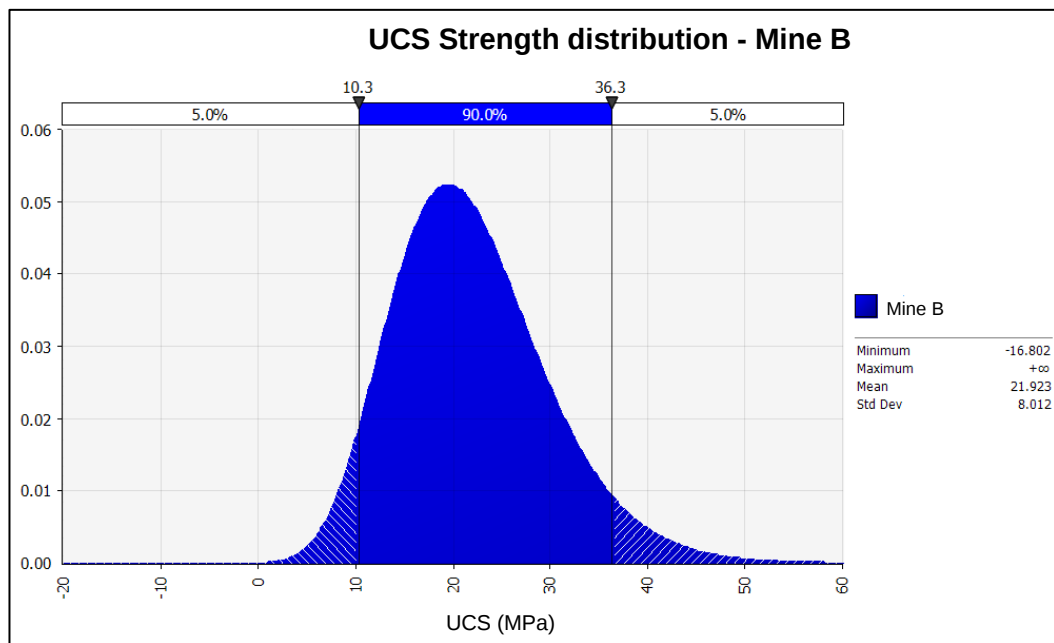


Figure 3-2. UCS Strength distribution for data obtained from Mine B.

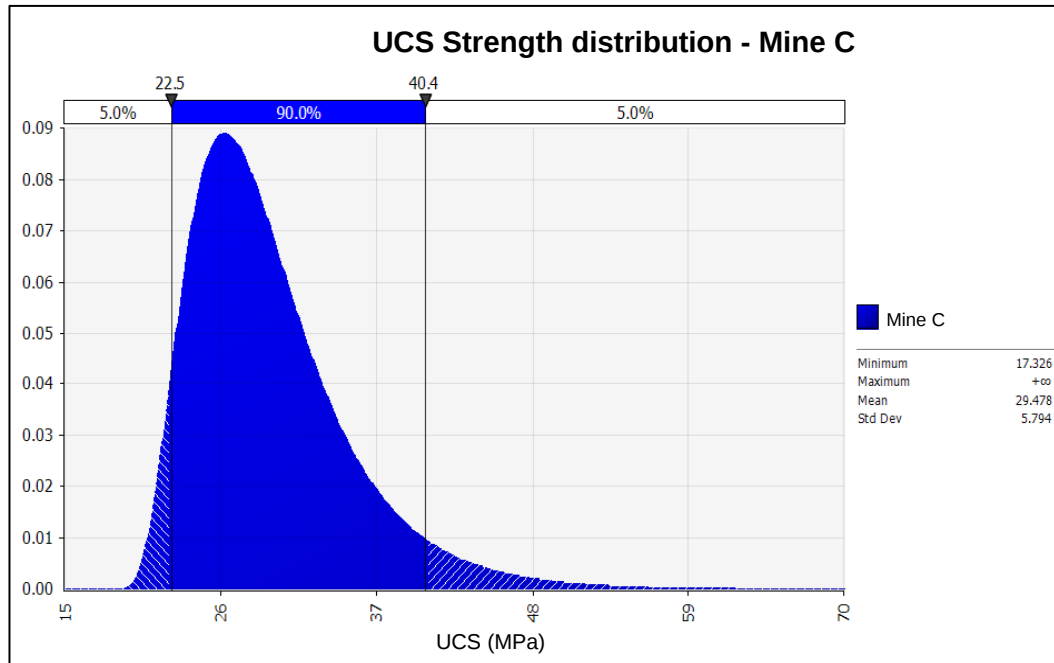


Figure 3-3. UCS Strength distribution for data obtained from Mine C.

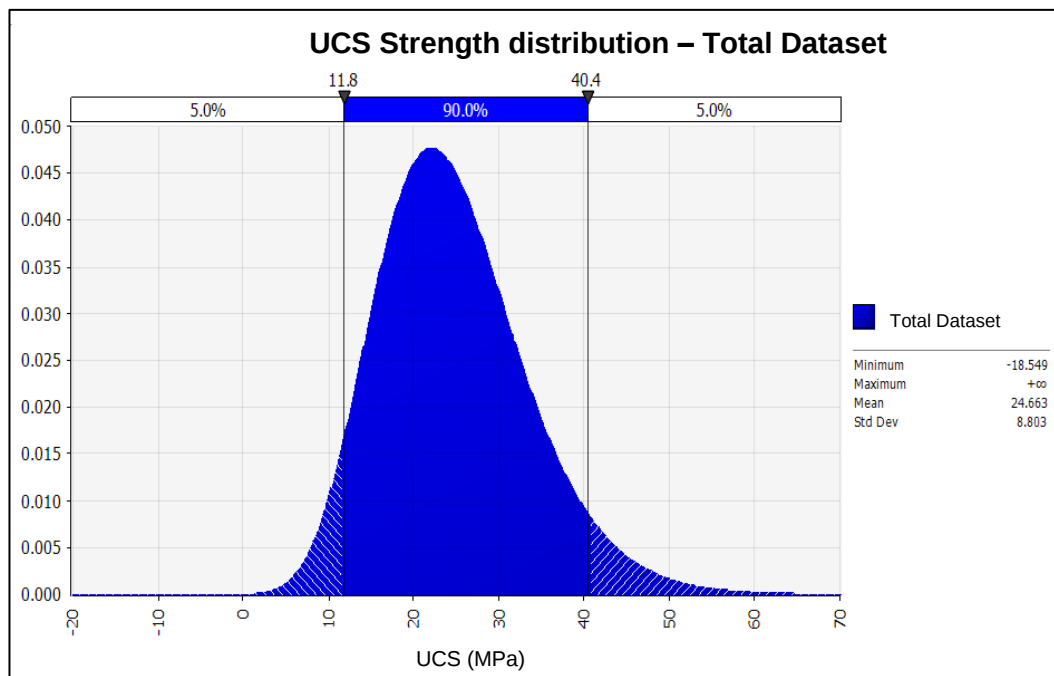


Figure 3-4. UCS Strength distribution of the total dataset.

3.1.2 Pillar database

In order to allow for a comparison between the results from the study and the results from the original “unadjusted” pillar strength formulae, a set of pillar examples was required which represented both stable and failed pillars. The original pillar strength formulae were developed through the statistical analysis of the South African pillar failure database originally compiled by Salamon in 1967. Throughout the years the database was expanded by a number of authors as discussed under the literature review in section 2.1. This database was therefore ideally suited to the study as it would allow for a direct comparison between the calculation method proposed in the study, and the design methods currently used in industry.

The latest iteration of the database contains a total of 424 pillar cases and covers a range of pillars at multiple depths. Table 3.3 summarises the detail of the database.

Table 3.3. Summary of the data contained in the South African Pillar failure database at the time of the study.

Dataset	Number of pillar cases	Depth range (m)		Mining height range (m)	
		Min	Max	Min	Max
Failed	87	19.00	205.00	1.35	6.50
Unfailed	337	13.22	254.45	1.00	6.45

3.2 Adjustments to pillar strength formulae

The current pillar strength formulae make use of a predetermined bulk strength factor (k) to describe pillar strength. This factor however is constant regardless of the coalfield being designed for. In order to introduce the variability of the UCS data into the pillar calculation, adjustments were required to the existing pillar strength equations. It should be reiterated that the study is not aimed at extrapolating pillar strength from UCS results, but rather to describe the variability in pillar strength using the variability in UCS results. The intent was therefore to develop an approach which introduced the variability but returns a similar pillar strength as would be obtained when using the original formulae. This meant that the UCS strength had to be divided by a factor to ensure that the resulting bulk strength (k) used in the calculation is unchanged from the original formulae. The resulting strength parameter (k) used in the study, was therefore calculated using *Equation 3-1*.

$$\text{Bulk Strength } (k) = \frac{UCS}{\text{Factor } (f)} \quad \textbf{Equation 3-1}$$

In selecting an appropriate adjustment factor for each formula, the factors were calculated using the entire dataset of UCS results. The reasoning behind this was to ensure that the final strength factor would be appropriate for use with any of the Mpumalanga coalfields without being biased to any specific mine or seam. This was achieved by using the mean strength from the total Mpumalanga dataset (24.66 MPa). The final

adjustment factor calculated for each of the formulae, was therefore based on the most likely UCS strength from the Mpumalanga dataset, which when divided by the factor, matches the original bulk strength value (k) from the unadjusted equations. The adjusted formulae are seen in *Equation 3-2* to *Equation 3-5*.

Adjusted Salamon and Munro (1967) formula:

$$Strength = \left(\frac{UCS}{3.43} \right) \times \left(\frac{w^{0.46}}{h^{0.66}} \right) \quad \textbf{Equation 3-2}$$

Adjusted Van der Merwe (2003) formula:

$$Strength = \left(\frac{UCS}{7.05} \right) \times \left(\frac{w}{h} \right) \quad \textbf{Equation 3-3}$$

Adjusted Maximum Likelihood formula:

$$Strength = \left(\frac{UCS}{3.73} \right) \times \left(\frac{w^{0.5}}{h^{0.7}} \right) \quad \textbf{Equation 3-4}$$

Adjusted Overlap Reduction formula:

$$Strength = \left(\frac{UCS}{4.51} \right) \times \left(\frac{w^{0.8}}{h} \right) \quad \textbf{Equation 3-5}$$

Using the adjusted formulae, it is possible to calculate pillar strength using the strength of the coal material associated with a specific coalfield. Additionally, expressing the material strength as a distribution of UCS

values, the adjusted pillar strength formulae allowed for the probability of failure to be calculated for different pillar scenarios.

3.3 Calculation of pillar failure probabilities and validation of adjusted pillar strength formulae

The four pillar strength formulae discussed in the previous section are all acceptable methods of calculating pillar strength and have been implemented by various mining houses in South Africa. In order to determine whether the adjustments to the formulae would alter the outcome of the results, a comparison was conducted between the result from the adjusted formulae and the result of the formulae in their original form. For a direct comparison to be made, the results were stated in terms of safety factor, as probability of failure is beyond the application of the original formulae.

Keeping in mind that the adjusted formulae will produce a distribution of safety factors in order to determine a probability of failure, expressing the results from the adjusted formulae in terms of a single safety factor would require the distribution of safety factors to be reduced to a single representative value. This can easily be achieved by using the mean of the distribution as it represents the safety factor that most frequently occurs in the resulting data distribution (the peak of the distribution curve). Figure 3-5 illustrates a distribution of safety factors, graphically indicating the difference between the probability of failure obtained from a distribution and the mean of the distribution. In Figure 3-5, the probability

of failure is indicated as the area below the curve for a safety factor less than one, and is expressed as a percentage of the total area below the curve as shown in *Equation 3-6*.

$$\text{Probability of Failure} = \left(\frac{\text{Shaded Area}}{\text{Total Area}} \right) \quad \text{Equation 3-6}$$

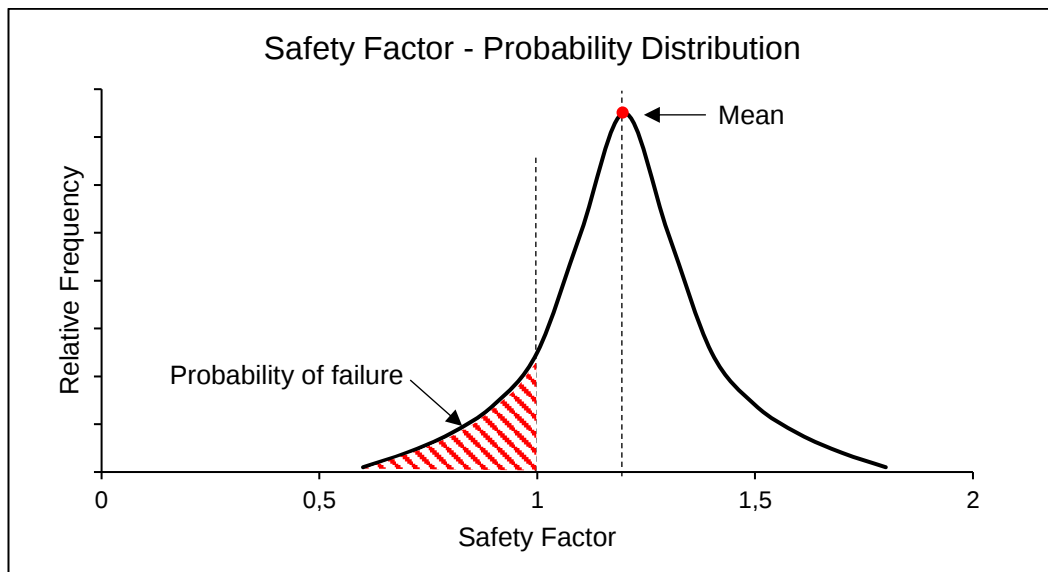


Figure 3-5. Illustration of a distribution of safety factors indicating the probability of failure and the mean safety factor for the distribution.

In calculating the distribution of safety factors using the adjusted formulae, the total dataset of 83 UCS values was used. With the adjustments based on the total dataset, the resulting distribution of safety factors should return similar results compared to the unadjusted formulae.

In order to conduct the comparison, a set of baseline pillar scenarios was selected from the South African pillar failure database. The pillar parameters obtained from the database included the pillar width, height,

bord width and depth below surface. In keeping with the industry norm, an overburden density of 2500 kg/m³ was used in all calculations. Subsequently, the only parameter that changed in the comparison was the bulk strength factor (*k*).

The calculations of pillar safety factor using the original formulae are discussed at length in the earlier sections of the report and will not be repeated. The calculations of the safety factors using the adjusted formulae were conducted as follows:

The software program *@Risk*, allows the user to calculate a probability density function using a distribution of values as an input to an equation. The program then simulates results for a pre-set number of iterations. The UCS distributions discussed in section 3.1.1 were used as the input to the UCS parameter of the adjusted formulae. All calculations made use of 1000 iterations to produce a probability density function for each pillar scenario. The resulting distributions indicate the range of pillar strengths expected for each scenario. Using the distribution of pillar strengths as the input into the “capacity” parameter in *Equation 3-7*, a distribution of safety factors is subsequently obtained in *@Risk*.

$$Safety\ Factor = \frac{Capacity}{Demand} \qquad \qquad \qquad \textbf{Equation 3-7}$$

These distributions of safety factors are finally used to determine the mean safety factor for each scenario as well as the probability of failure. An

example of one of the safety factor distributions obtained from @Risk is seen in Figure 3-6. The probability of failure is automatically calculated in @Risk with the area below the curve in the example, being 0.7 % of the total area (based on a dividing line equal to a safety factor of one).

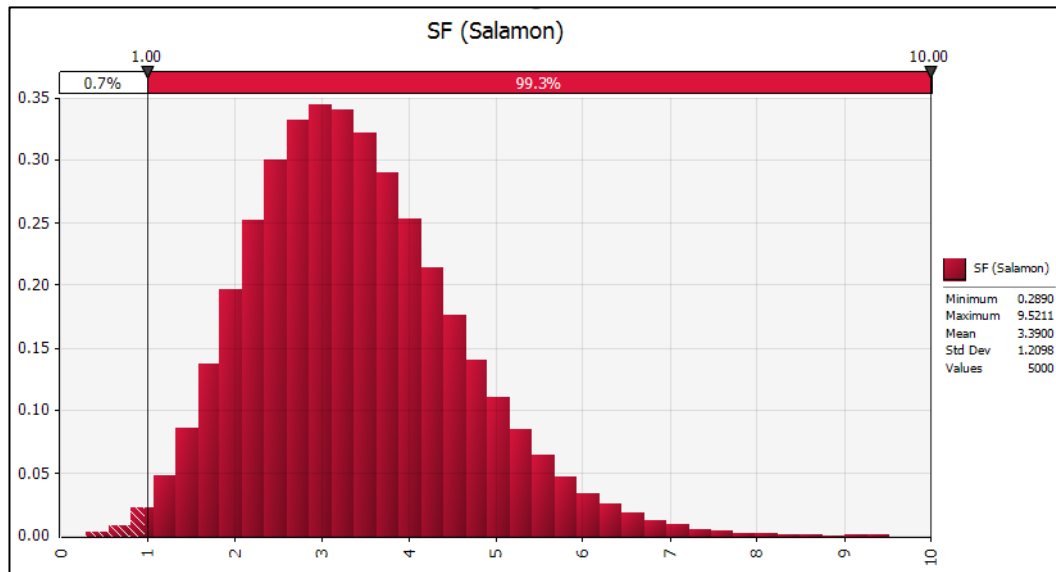


Figure 3-6. Example of a distribution of safety factors calculated using @Risk. The probability of failure for the distribution is 0.7% with a mean safety factor of 3.39.

Using this process, probability density functions were calculated for all pillars in the South African pillar failure database (both failed and unfailed datasets). Table 3.4 summarises the results of five randomly selected pillars from the failed dataset and five from the unfailed dataset. From these results, it can be seen that, although isolated scenarios did not provide the exact same safety factor, the difference is never more than 0.01. From the results, the adjusted pillar strength formulae were subsequently validated for use in the study.

Table 3.4. Comparison of the safety factors obtained from the adjusted and original pillar strength formulae as calculated using the total UCS dataset.

Database	Pillar case number	SF - Salamon		SF - Van der Merwe		SF - Maximum Likelihood		SF - Overlap Reduction	
		Original formula	Adjusted formula	Original formula	Adjusted formula	Original formula	Adjusted formula	Original formula	Adjusted formula
Failed	5	4.85	4.84	4.45	4.45	4.58	4.59	4.75	4.75
	29	1.26	1.25	0.95	0.94	1.16	1.16	1.03	1.03
	51	0.92	0.92	0.63	0.63	0.85	0.85	0.76	0.76
	66	1.01	1.01	0.71	0.71	0.94	0.94	0.88	0.88
	83	0.77	0.77	0.63	0.63	0.72	0.72	0.67	0.67
Unfailed	71	2.48	2.47	3.82	3.82	2.47	2.47	3.85	3.84
	140	2.29	2.29	3.21	3.21	2.25	2.25	3.05	3.05
	228	2.35	2.35	3.82	3.82	2.34	2.34	3.50	3.50
	284	4.02	4.02	6.69	6.68	3.98	3.98	5.77	5.77
	328	4.01	4.01	5.18	5.17	3.92	3.92	5.10	5.10

With the adjusted formulae validated for use, the calculated probabilities of failure were considered. Figure 3-7 and Figure 3-8 provide a graphical illustration of the calculated probabilities relative to the safety factor for both the failed and unfailed pillar datasets. The graphs include the results of all four formulae, and interestingly, the relation between safety factor and probability of failure remains constant regardless which formula is used. This could however be expected as the same distributions of variables were applied to all formulae.

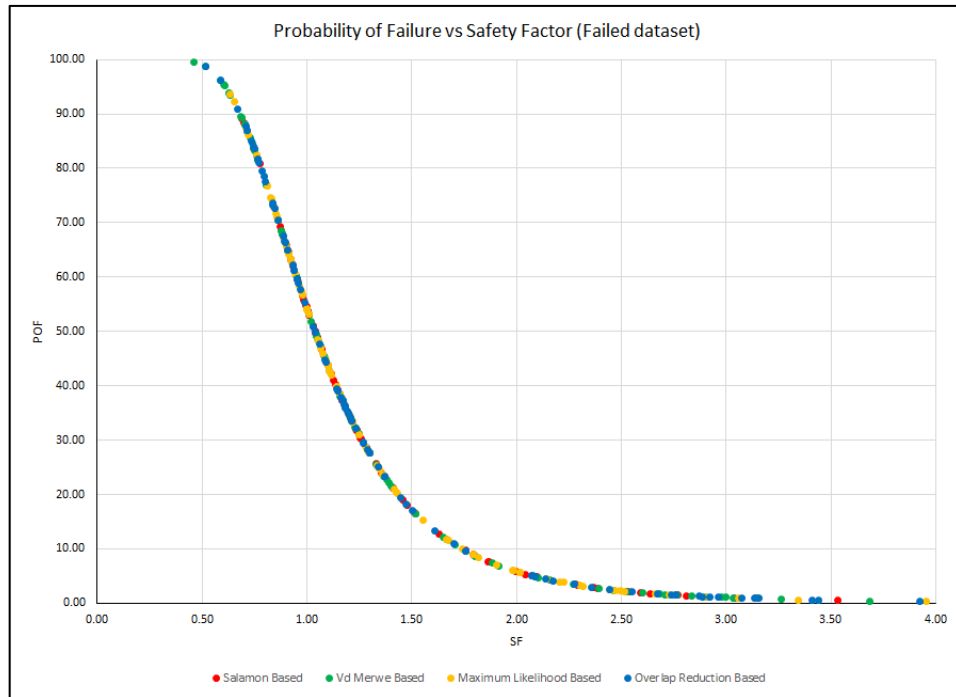


Figure 3-7. Probability of failure plotted against safety factor for the failed pillar dataset.

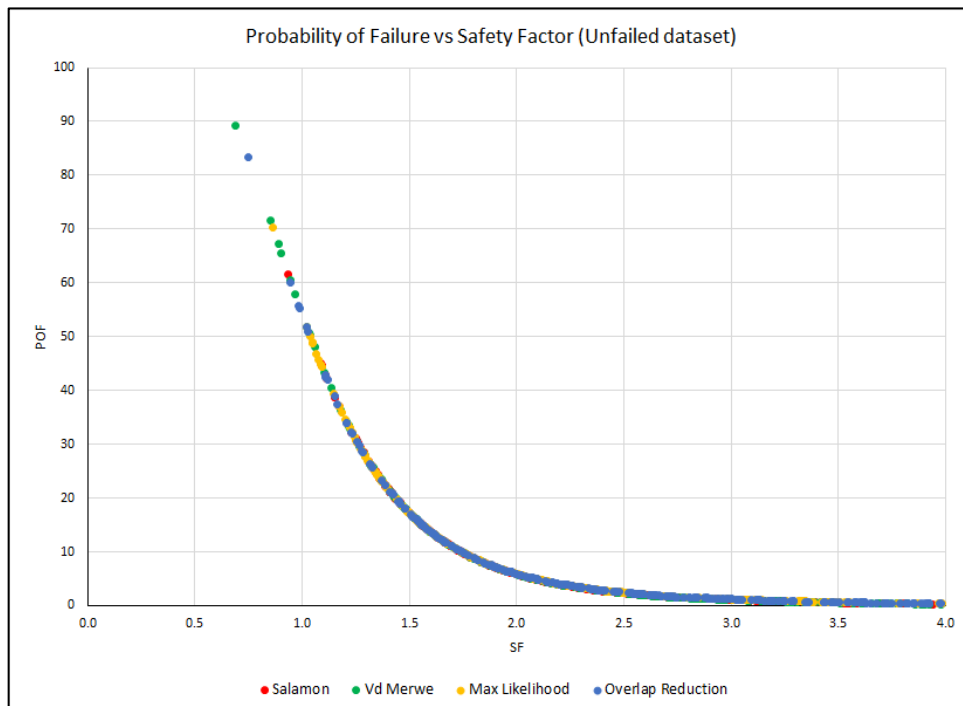


Figure 3-8: Probability of failure plotted against safety factor for the unfailed pillar dataset.

From Figure 3-7 and Figure 3-8, a probability of failure of 50 % equates to a safety factor of 1.04. This aligns well with the assumption that a safety factor of 1, taken as the divide between stable and unstable, should equate to a probability of failure of 50 %. For both graphs, a safety factor of 1.6 is achieved at a probability of failure of 13.8 %. This was therefore selected as the probability threshold for the interpretation of the results discussed in the following section.

4 RESULTS AND DISCUSSION

4.1 Results obtained from the proposed method

In the previous section, the study proposed a method with which pillar probability of failure can be calculated based on the laboratory strength of a specific coal seam or mine. The calculation of the probability of failure is key to the study as this forms the basis with which pillar optimisation can be justified. This is achieved by designing a pillar to the same probability of failure as the baseline pillar obtained when using the total UCS dataset (therefore the same pillar strength as would be obtained from the original unadjusted formulae). As a result, the final pillar would have a similar probability of failure as the baseline but be designed to a lower safety factor provided the coal material is of a higher strength.

To determine whether the variability in coal strength would allow for sufficient optimisation in pillar size, the proposed method was applied to the data from mines A, B and C discussed in section 3.1.1. From section 3.3, it was confirmed that the “failed” and “unfailed” datasets from the pillar failure database provide similar probabilities of failure across a range of safety factors. The decision was therefore made to use only the “failed” dataset for this section of the study, therefore reducing the number of calculations required while still having sufficient data from which to draw conclusions. The failed dataset consists of 87 pillar scenarios and would therefore produce 87 datapoint with which distributions could be plotted.

In calculating the safety factors and probabilities of failures for each of the mines, the UCS distributions discussed in section 3.1.1 were applied to the adjusted pillar strength formulae. Figure 4-1 provides a graphical comparison of the safety factor distributions obtained from each of the four datasets, being the total Mpumalanga dataset as well as the data from the three individual mines A, B and C. The distributions in Figure 4-1, were not graphed using *@Risk*, as the distributions would not be used as input into further calculations. The distributions are therefore compiled by dividing the results into bins of safety factors and plotting the number of data points per bin across the range of safety factors applicable.

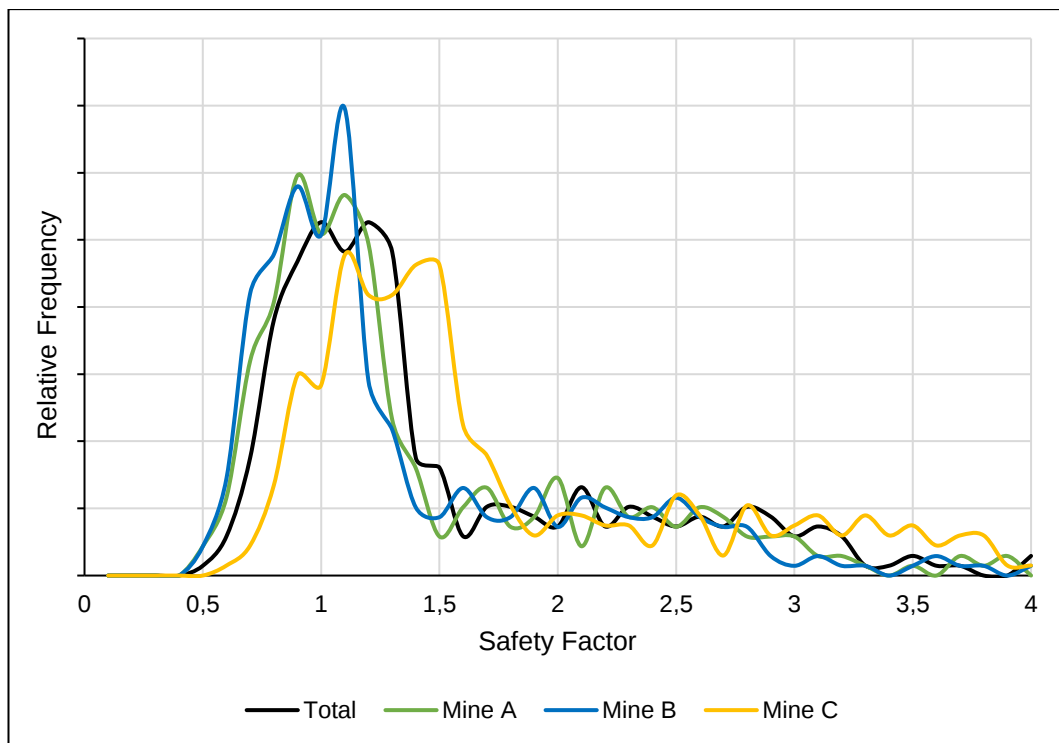


Figure 4-1: Comparison of safety factors and probabilities of failure obtained from the different datasets.

To determine whether the different distributions would allow for any pillar optimisation, the distribution for each mine was compared to the original distribution from the total dataset (also referred to as the baseline distribution). Should the distribution move towards a higher range of safety factors compared to the baseline distribution, the adjusted formulae predicted higher pillar strengths, and pillar optimisation would be possible. This would occur when the average UCS from the mine database is higher than the Mpumalanga mean, therefore inferring a stronger pillar material. Should the distribution move towards a lower range of safety factors, the adjusted pillar formulae predicted lower pillar strengths and a larger pillar would be required to maintain the probability of failure indicated by the baseline distribution.

Figure 4-2 to Figure 4-4 provide graphical comparisons of the individual distributions to the baseline. Due to the distributions returning a bi-modal curve, the harmonic mean from each distribution was used to provide a quantifiable measure for optimisation.

From Figure 4-2 to Figure 4-4, it can be seen that the datasets from Mine A and B, return lower safety factors compared to the baseline. Mine C on the other hand returns higher safety factors. This is in line with expectation when considering the mean UCS from the various datasets (Table 3.1).

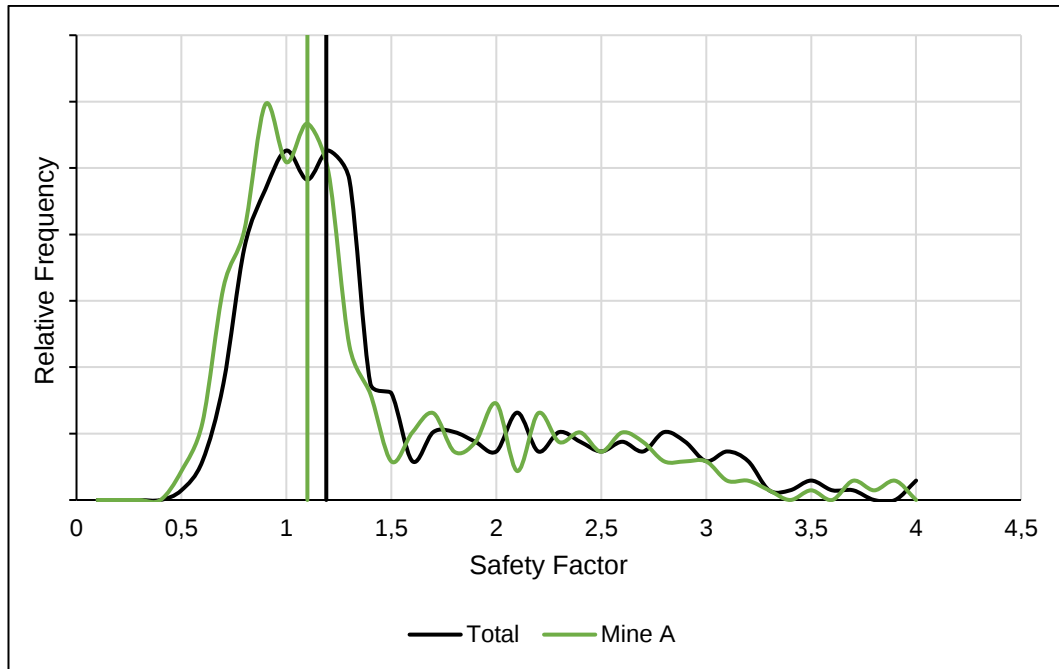


Figure 4-2: Comparison of the Mine A distribution to the baseline obtained from the total UCS dataset.

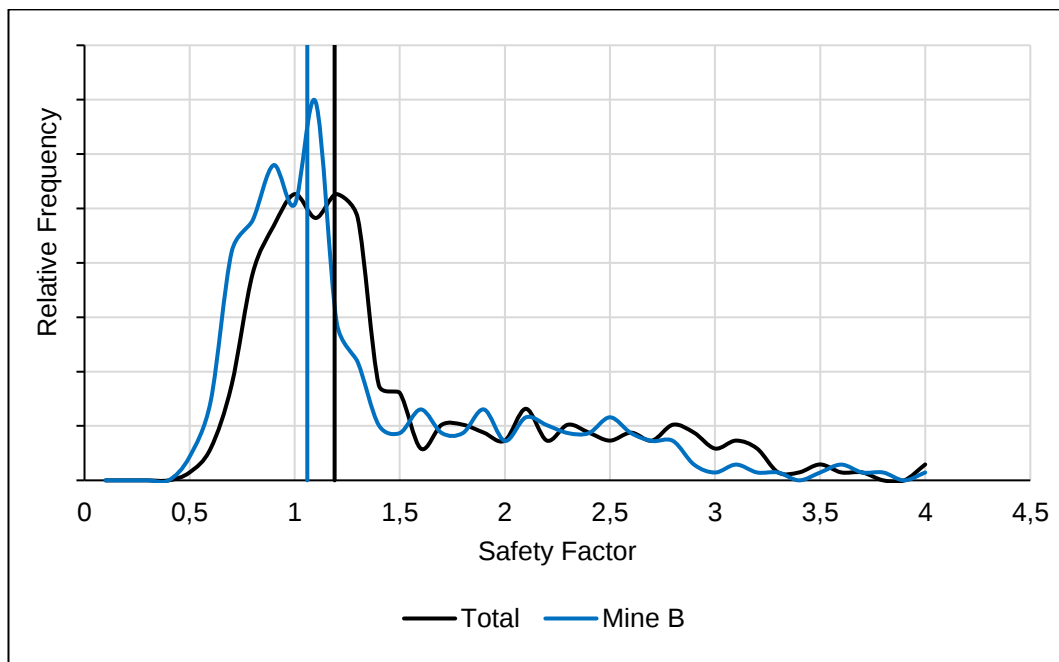


Figure 4-3: Comparison of the Mine B distribution to the baseline obtained from the total UCS dataset.

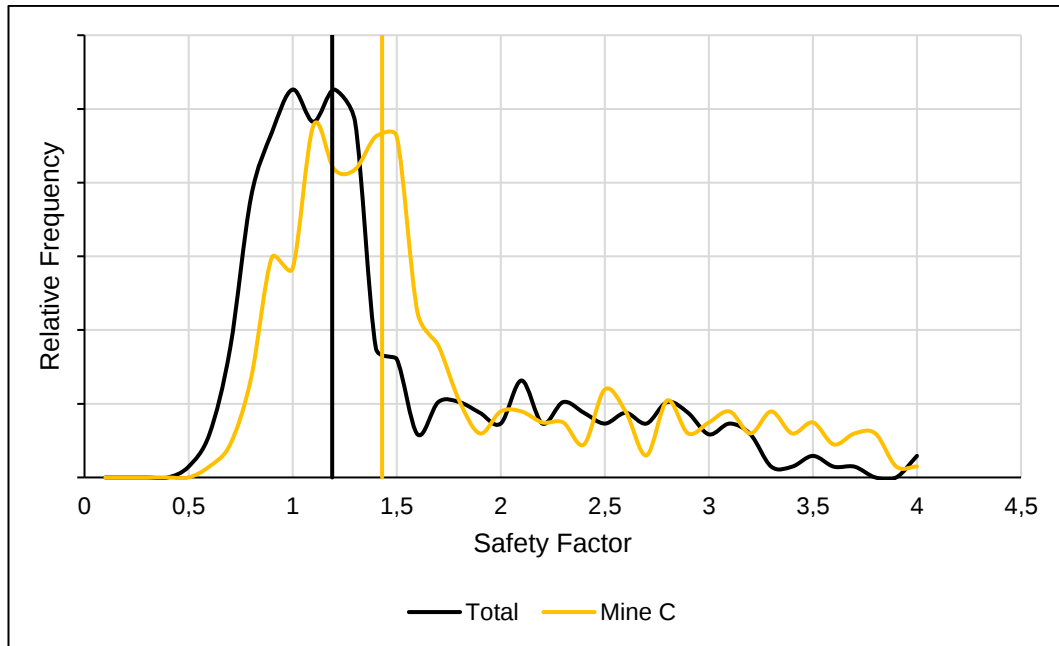


Figure 4-4: Comparison of the Mine C distribution to the baseline obtained from the total UCS dataset.

Mine A and B have mean UCS values of 22.4 MPa and 21.9 MPa compared to the total dataset mean of 24.6 MPa. Mine C, however, has a mean UCS of 29.4 MPa, indicating a stronger coal material compared to the Mpumalanga average. Based on these results, the pillars from Mine C, could potentially be optimised in size without increasing the probability of failure. Figure 4-5 to Figure 4-7 graphically illustrate this effect. In plotting these graphs, the adjusted formulae are compared with the results from the original formulae relative to the probabilities of failure obtained for each dataset. To determine the actual shift in safety factor, a probability of failure of 13.8 % was used, as determined from the baseline distribution to represent a safety factor of 1.6 prior to any adjustment of the formulae.

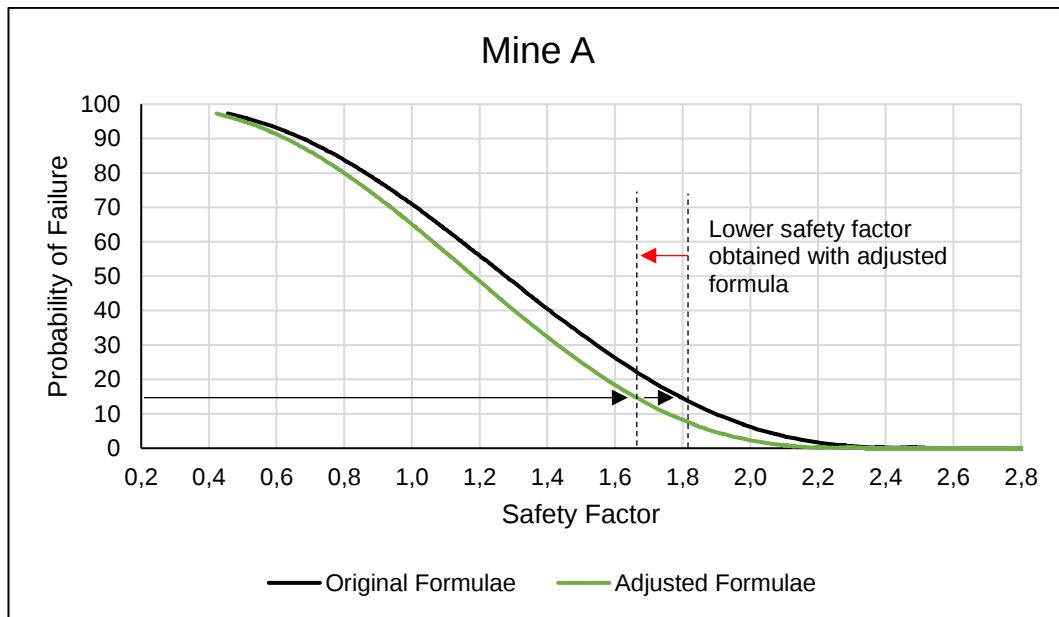


Figure 4-5: Comparison of safety factors from the original and adjusted formulae using the Mine A dataset.

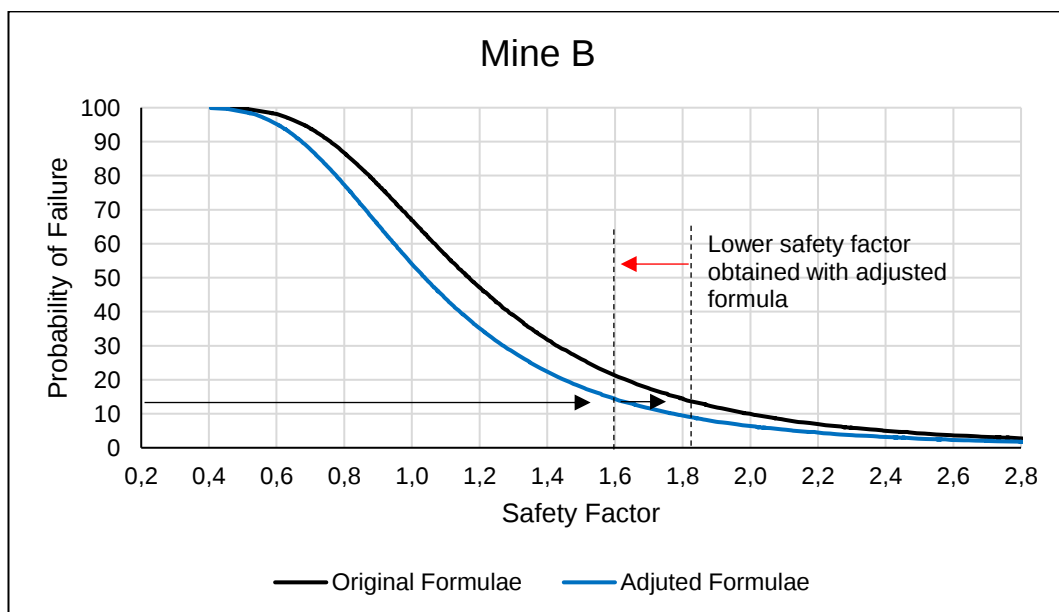


Figure 4-6: Comparison of safety factors from the original and adjusted formulae using the Mine B dataset.

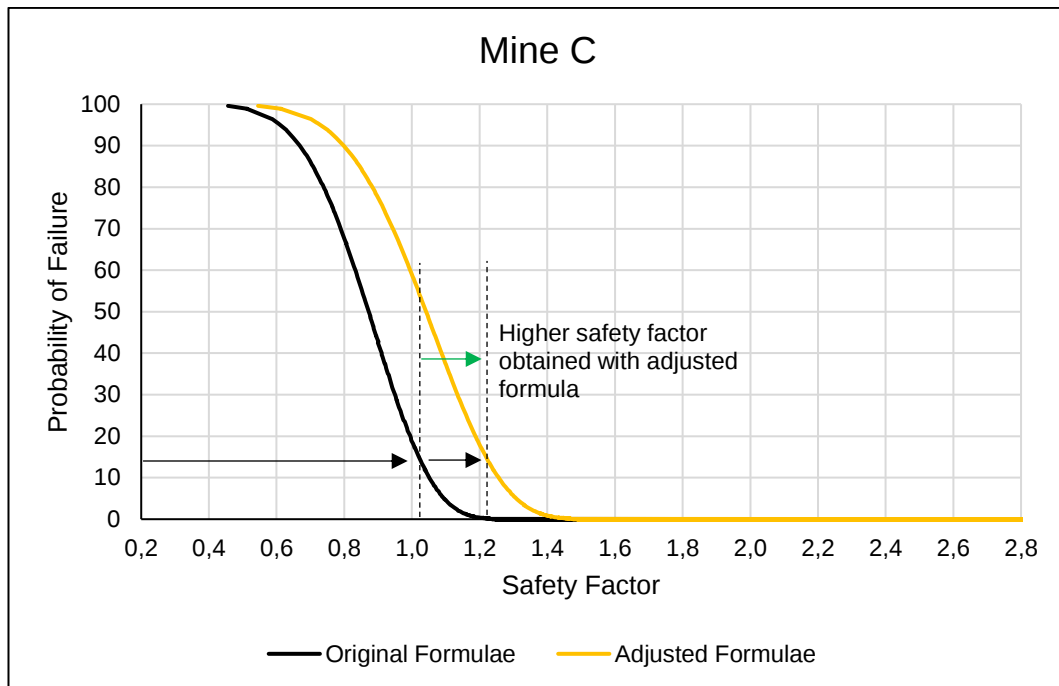


Figure 4-7: Comparison of safety factors from the original and adjusted formulae using the Mine C dataset.

It should be noted that the graphs in Figure 4-5 to Figure 4-7 are only used to determine the difference in safety factor at the threshold probability of 13.8 %. The intent is therefore not to match the curve to a safety factor threshold of 1.6. The adjustment to the final pillar dimensions is conducted using the unadjusted formulae and is discussed later in the report. To quantify the potential optimisation using the data from the various mines, a single mining scenario was required. This would allow for the actual change in pillar dimensions to be calculated and compared. This comparison was conducted using the mining parameters from Table 4.1.

Table 4.1: Mining scenario used for comparison.

Depth (m)	Pillar width (m)	Bord width (m)	Mining height (m)	Load (MPa)
100	11	6.5	3	6.33

From the parameters in Table 4.1, the pillar load as calculated based on TAT is 6.33 MPa. The safety factors obtained from the original pillar strength formulae for this scenario are summarised in Table 4.2.

Table 4.2: Safety factors obtained for the mining scenario using the original pillar strength formulae.

Formula	Salamon	Van der Merwe	Maximum Likelihood	Overlap Reduction
Safety Factor	1.65	2.03	1.61	1.96

Using the difference in harmonic mean between the datasets as illustrated in Figure 4-2 to Figure 4-4, the allowable difference in safety factor was determined. Table 4.3 summarises the differences in harmonic means between the total and individual mine datasets.

The 'optimised' pillar design would therefore be designed to the original pillar safety factor, plus or minus the difference, depending on whether the safety factor should increase or decrease.

Table 4.3: Difference in safety factors determined from the harmonic mean of the distributions.

Dataset	Harmonic mean	Difference in SF
Mine A	1.10	0.09
Mine B	1.06	0.13
Mine C	1.43	-0.24
Harmonic mean of total dataset – 1.19		

Table 4.4 provides a summary of the updated pillar widths determined from the analysis. The reason for using the original pillar formulae for optimisation is that the safety factors obtained from the adjusted formula include the effect of the difference in material strength. Using the adjusted formulae to find the updated pillar width will therefore overestimate the potential gain or loss in pillar width. The difference in using the original and adjusted formulae is illustrated in Table 4.5. In calculating the updated pillar widths, the calculations were based on perfectly square pillars. Parameters such as depth below surface, mining height and bord width were unchanged.

As stated above, Mines A and B have lower material strengths compared to the Mpumalanga mean, and would therefore require an increase in pillar width to meet the probability of failure threshold of 13.8 %. Mine C on the other hand has a stronger material strength, which would allow for some optimisation in the pillar design.

Table 4.4: Updated pillar widths obtained from the analysis.

Dataset	Formula	Original width (m)	Updated width (m)	Difference (m)
Mine A	Salamon	11	11.5	+ 0.5
	VdM	11	11.3	+ 0.3
	ML	11	11.5	+ 0.5
	OR	11	11.3	+ 0.3
Mine B	Salamon	11	11.7	+ 0.7
	VdM	11	11.4	+ 0.4
	ML	11	11.8	+ 0.8
	OR	11	11.5	+ 0.5
Mine C	Salamon	11	9.7	- 1.3
	VdM	11	10.2	- 0.8
	ML	11	9.7	- 1.3
	OR	11	10.1	- 0.9

Based on the results from Table 4.4, the potential gain involves a reduction in pillar width that ranges between 0.8 m and 1.3 m. Due to practical mining considerations, a pillar width will not be adjusted by a value of 0.8 m or 1.3 m. The change would rather be made in factors of 0.5 m. This would mean that the pillar design values using the *Salamon* and *Munro* and *Maximum Likelihood* formulae would reduce by 1 m, and the *Van der Merwe* and *Overlap Reduction* formulae by 0.5 m.

Table 4.5: Overestimation of change in pillar width obtained from the adjusted formulae compared to the original formulae.

Dataset	Formula	Change in pillar width using original formulae (m)	Change in pillar width using adjusted formulae (m)
Mine A	Salamon	+ 0.5	+ 1.5
	VdM	+ 0.3	+ 1.0
	ML	+ 0.5	+ 1.5
	OR	+ 0.3	+ 1.5
Mine B	Salamon	+ 0.7	+ 2.0
	VdM	+ 0.4	+ 1.5
	ML	+ 0.8	+ 2.0
	OR	+ 0.5	+ 1.5
Mine C	Salamon	- 1.3	- 2.5
	VdM	- 0.8	- 1.5
	ML	- 1.3	- 2.5
	OR	- 0.9	- 1.5

To link this to the expected change in a production profile, Table 4.6 provides a summary of the additional tonnes that could be mined per pillar based on these adjustments.

Table 4.6: Gain in production tonnes per pillar based on the optimised pillar sizes for Mine C.

Formula	Salamon	VdM	ML	OR
Pillar width original (m)	11.0	11.0	11.0	11.0
Pillar width new (m)	10.0	10.5	10.0	10.5
Difference in width (m)	- 1.0	- 0.5	- 1.0	- 0.5
Gain in tonnes per pillar	157.50	80.63	157.50	80.63

The reduction in pillar sizes stipulated in Table 4.6 increases the extraction percentage as indicated in Table 4.7, providing an additional 2.78 % for the *Salamon* and *Maximum Likelihood* formulae, and 1.36 % for the *Van der Merwe* and *Overlap Reduction* formulae. Although the increase appears marginal, it would allow for better resource utilisation and could aid in extending the life of a mining operation.

Table 4.7: Increase in extraction percentage for each of the strength formulae compared to the base case used in the study.

Formula	Salamon	VdM	ML	OR
Original extraction %	60.49 %	60.49 %	60.49 %	60.49 %
Updated extraction %	63.27 %	61.85 %	63.27 %	61.85 %
Gain in extraction %	+ 2.78 %	+ 1.36 %	+ 2.78 %	+ 1.36 %

4.2 Discussion

From the results, it seems plausible that the method proposed by the study would allow for optimisation in pillar design where the coal material is comparatively stronger than the regional average. There have been multiple studies which concluded that pillar strength cannot be extrapolated from UCS results, the most notable being the study by Mark and Barton (1996). This being the case, the method proposed by this study does not attempt to extrapolate pillar strength from smaller samples by means of scaling, but rather matches the bulk pillar strength of the original formulae using the regional average UCS. The strength of a pillar is subsequently adjusted based on the UCS of the coal material specific to a coal seam or mine. A similar approach has been adopted in the hard-rock pillar design fraternity, with both Potvin *et al.* (1989) and Lunder and Pakalnis (1997) defining the pillar bulk strength by means of the UCS divided by a factor. Another approach commonly used in South African hard rock mines, which supports the link between UCS and pillar strength, is to estimate the pillar bulk strength as a third of the UCS (Ozbay *et al.*, 1995). It is therefore not uncommon to use the UCS of a rock type to describe the expected pillar strength. What is different in the approach proposed by the current research, is the introduction of a distribution of UCS values in describing not only pillar strength, but variability in pillar strength. This allows for a pillar design to be expressed in terms of a probability of failure, which forms the basis on which optimisation can be conducted.

A question that remains unanswered however, is whether the behaviour of a pillar can effectively be estimated using only the UCS of a material without considering the effect that jointing has on the stability of the pillar? The study by Mark and Barton (1996) indicates this as unlikely based on the inability of a laboratory UCS test to account for natural discontinuities and imperfections in the larger mass. Esterhuizen (2000) studied the effect of jointing on pillar stability using field data from South African coal mines. He concluded that the weakening effect of jointing in a pillar reduces with increasing width to height and is largely dependent on the joint orientation. This presents a challenge as the orientation of jointing in a reserve is generally not well defined prior to start of mining and based on the above mentioned studies, should be accounted for in the pillar design prior to start of mining.

Another consideration that should be taken into account however, is that the pillar strength formulae discussed in this report have been implemented in industry for many years without pillars failing abnormally as a result of jointing. This could be attributed to the fact that the pillars recorded in the South African pillar failure database provide an indication of actual pillar behaviour, including the effects of material imperfections, cleating and discontinuities. Taking this into consideration, it begs the question whether further adjustment to the pillar strength formulae to include the effect of discontinuities are in fact required?

Regardless of whether the effect of discontinuities is sufficiently accounted for in the current pillar strength formulae, the use of UCS test data in expressing pillar strength seems prudent. Additionally, it provides a logical and viable means with which to express pillar probability of failure, based not on the statistical back analysis of pillar failure data, but on the variability of the coal material as determined by existing laboratory testing methods. These failure probabilities subsequently provide a means with which to potentially optimise the pillar design on a mine.

The method proposed by the research does however require further validation, ideally by means of underground trials in a controlled environment, to ensure underground stability is maintained over the life of a mine.

5 CONCLUSION AND RECOMMENDATION

The research was conducted on UCS data from actual mines, and proved the capacity for optimisation between reserves by considering the difference in material strength as obtained from UCS tests. Based on the datasets considered, a potential increase in extraction of up to 2.78 % could be obtained without any change to the calculated probability of failure. The results however assume that the variability in pillar strength can be effectively described by the variability in UCS strengths of a specific reserve. The probability of failure is determined from a distribution of safety factors, which in turn is calculated from a distribution of UCS results used in pillar strength formulae adopted from existing methods.

The results of the research confirm on a theoretical basis the applicability of the proposed optimisation method. The additional extraction resulting from the optimisation would aid in resource utilisation as well as extending the life of mine. The results however, do not confirm the proposed method as one that can be implemented without due consideration. To ensure this approach does not compromise long-term pillar stability, further research is required. It is recommended that the optimised pillar sizes obtained from this approach, be trialled in a controlled environment where abnormal pillar behaviour will not affect the rest of the mine. Once trial mining confirms stable behaviour is maintained, the optimised design can systematically be introduced across the remainder of the mine.

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