UNIVERSITY OF THE WITWATERSRAND

Title: An investigation into Grade 6 teachers' understanding of geometry according to the Van Hiele level of geometric thought

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DECLARATION

I, Elspeth Khembo, declare that this research report is my own, unaided work. It is being submitted for the degree of Master of Education in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination at any other university.

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Thank you, one and all.

DEDICATION

To my mum, Sarah Lucas for giving me life…and to my son Mpho and my brother William for their encouragement and unwavering support.

ABSTRACT

A qualitative three phase research study was undertaken to explore the geometric thinking of Grade 6 teachers in the context of low performance of the Gauteng learners in the International Competitions and Assessments for Schools (ICAS) tests in 2006. The van Hiele theory of geometric thinking was used as a theoretical framework. Data was collected by means of document analysis, teachers responding in writing to ICAS items and task based interviews. This research shows that the ICAS items map very well with the National Curriculum Statement (NCS) Assessment Standards and the van Hiele levels. The mapping of the 82 geometry ICAS items to the (NCS) Assessments Standards and the van Hiele levels revealed that 59% of the ICAS items were at van Hiele level 2 across all grades and therefore accessible to the majority of the learners. The written responses conducted with 40 ACE teachers and the task based interviews conducted with six teachers indicate that the majority of teachers are not at the required level of geometric thinking as expected by the NCS Assessment Standards and the van Hiele levels. This finding has serious implications for classroom practice in the learning and teaching of geometry in primary schools. The challenge for teacher education is to take into consideration the relationship between language and the van Hiele levels; the use of van Hiele levels in the development of teachers' geometric thinking and the possible use of the van Hiele levels in identifying, explaining and rectifying geometric misconceptions.

TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION

1.1 INTRODUCTION

I can safely attribute my passing matric mathematics to a good knowledge of algebra and trigonometry and I remember very little about high school geometry. Geometry was only taught late in the year shortly before the final examinations. Richard Bach (1977) says that "we teach best what we most need to learn" hence my fifteen years of teaching primary school mathematics, in particular geometry, was also a learning opportunity for both me and my learners, to actively engage in developing "our" spatial skills. I always scheduled to teach geometry during the first term so that I could have enough time to deal with and revisit important concepts throughout the year.

In 2008, I joined a teacher development and research project focused on data gathered from international mathematics test results, written by Gauteng Grade 3-11 learners. As part of my research focus in the project it was only natural that I would be interested in the performance of the learners and teachers' knowledge of geometry.

1.2 CONTEXT OF THE STUDY

The International Competitions and Assessments for Schools (ICAS) tests were written in Gauteng schools from 2006 to 2008. For purposes of this study, I only used the 2006 results. Data gathered from the results is being used by the Gauteng Department of Education (GDE) in partnership with the Wits School of Education in a project called the Data Informed Practice Improvement Project (DIPIP), to inform teaching and learning practice in Gauteng schools (Brodie et al, 2010). The mathematics test for each grade is made up of 35-40 multiple-choice items, including two free-response items. The content of the items covers core curriculum areas such as number sense, measurement, chance and data, geometry and algebra.

The results showed that the overall performance across the grades was low and that the percentages of learners who chose correct answers decreased from grades 3 to grade 11. The learners' performance was also low across all categories. For example in Grade 6, the percentage of correct responses is: Number 40%, Data Handling 25%, Measurement 22%, and Geometry 30%. In geometry, the percentage of correct responses ranged from 34% in Grade 3 to 26% in Grade 11.

It seems that the geometric skills of learners across grades have not been adequately developed. This suggests that there is a challenge in the teaching and learning of space and geometry in our classrooms. Research over the years has indicated that many learners encounter difficulties and perform poorly in geometry (e.g. Fuys, Geddes, & Tischler, 1988, Gutierrez, Jaime, & Fortuny, 1991). Many learners fail to grasp the fundamental concepts in geometry and leave geometry classes without learning basic terminology (Usiskin, 1982). The reason for this underachievement of learners, according to the Human Sciences Research Council (HSRC, 2008), who conducted research on Grade 6 teachers in Gauteng is that the majority of teachers appear to have a limited understanding of the mathematical content that they teach. They found that the errors made by teachers were in the area of geometry. They conclude therefore that, geometry has been neglected and poorly taught in the past (p. 15).

This study is an investigation of Grade 6 teachers' understanding of geometry according to the Van Hiele level of geometric thought. This theory has been widely used by researchers to categorise learners' geometric thinking to "examine the content knowledge of teachers and prospective teachers. It indicates a way in which a learner's geometric thinking can be measured" (van Hiele, 1959:1-31)

This study is therefore aimed at finding out how teachers understand their learners' errors focusing on their own reasoning about their learners' geometric thinking.

1.3 RESEARCH QUESTIONS

My study is guided by the following research questions:

1. What level of thinking do the geometry items in the ICAS tests expect?

2. What is the level of geometric thinking of selected Grade 6 teachers?

3. In what ways do Grade 6 teachers understand their learners' geometric errors?

A lot has been said about the low levels of achievement of learners generally in mathematics as indicated by the Systemic Evaluation in 2003 and 2005 by the Department of Education reports and the much publicized TIMSS 2003 study. However, not much research has been done on the utilization of the data generated from these studies as a tool for teacher professional development.

The premise of the DIPIP project is that only when teachers respond to the items themselves, engage in discussion about how they solved the problem, and most importantly, analyse their learners' errors, can they begin to develop professionally in the most important areas of recognizing misconceptions and in the concept development of their own geometric thinking skills.

1.4 THE REPORT

This report is divided into five chapters. In this chapter (chapter 1), I have provided the background to the study and the research questions that have shaped my analysis. Chapter 2 develops a conceptual framework for my study by reviewing literature on the van Hiele levels. Chapter 3 provides a motivation for the methodological approach adopted in this study, and a discussion of my methods of data collection. Chapter 4 presents the analysis and interpretation of the research results. In chapter 5, I discuss the findings that emerged that have implications for the important issues that affect teachers' geometric thinking. This chapter also makes suggestions that are supported by ongoing research.

CHAPTER 2 THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, I will discuss the theoretical framework and related literature that has guided my research. The framework is based on the van Hiele model of geometric thought. The framework will assist me in trying to understand the level of thinking expected by the ICAS tests, the level of teachers' geometric thinking as well as how they understand their learners' geometric errors. I will also review related literature on learners' geometric thinking, teachers' geometric thinking, geometric misconceptions, and spatial reasoning in the curriculum.

2.2 VAN HIELE MODEL OF GEOMETRIC THOUGHT

The van Hiele model of geometric thought originated from Piaget's cognitive development theory. Jaime and Gutierrez (1995) argue that "the van Hiele model of mathematical reasoning has become a proved descriptor of the progress of students' reasoning in geometry and is a valid framework for the design of teaching sequences in school geometry" (p. 592). The van Hieles were Dutch educators experiencing challenges in their own classrooms that inspired their focus for their doctoral dissertations. Their goals were to try and categorise student thinking in geometry by levels, hereafter referred to as the van Hiele levels of geometric thought.

2.2.1 The van Hiele Levels of Geometric Thought

According to van Hiele (1986), there are five levels of geometric thought that are sequential and hierarchical. For learners to function at any given level, they must have mastered the reasoning needed at the preceding level. Progression from one level to another is based more on instruction and experiences than on age or physical development. Each level of geometric thought has its own vocabulary (Fuys & Liebov, 1997).

At the first van Hiele level, known as the **recognition/visual level**, the learner identifies names and compares geometric figures on the basis of their appearance as a whole, but does not recognise properties of the figures, such as the fact that there are right angles in a square.

At the second van Hiele level, **analysis level**, the learner analyses figures in terms of their properties, establishes the properties of figures empirically and uses the properties to solve problems, but cannot explain the interrelationships of properties between figures. For example the learner at this van Hiele theory level, does not realise the fact that a square is a rectangle.

The third van Hiele level is called the **informal deduction level**, where the learner understands the relations within and between figures, gives informal deductive arguments and formulates and uses definitions - for example, a square is a rectangle because it has all the properties of a rectangle.

The fourth van Hiele level is the **formal deduction level** where the learner understands the interrelationship and role of undefined terms, axioms, definitions, theorems and formal proof. The possibility of developing a proof in more than one way is seen.

At the fifth van Hiele level, **rigour**, a learner can compare different axiom systems and even non-Euclidean geometry can be studied. Geometry is seen in the abstract with a high degree of rigour, even without concrete examples.

According to the van Hiele theory, the five levels are not associated with any age range. “An important characteristic of mathematical reasoning is that growth in age does not necessarily imply growth in students’ level of reasoning. Instruction plays a central role in students’ progression throughout the levels”, Jaime & Gutierrez (1995, p. 592).

Van Hiele further claimed that the levels are discrete, that each learner can only be classified at a specific level, but researchers have reported evidence questioning the discrete nature of the levels.
Researchers found that learners in transition are difficult to classify (Fuys et al, 1988; Usiskin, 1982). Difficulties in deciding between levels were considered as evidence questioning the discrete nature of the levels. For instance, Burger and Shaughnessy (1986) found that the levels were more continuous because they found it difficult to choose between levels and often identified learners oscillating between levels, even on the same question. They suggested that this kind of behaviour made sense coming from learners transitioning between levels, particularly between level 1 and 2.

2.2.2 Comparison of the van Hiele model and theories of learning and teaching

(a) Piaget's developmental theory

Before the van Hiele model, Jean Piaget also described the nature of children's thinking and learning, including in the area of shape and space. Piaget's theory is structured using stages of cognitive development associated with certain ages. The stages are **sensori-motor** (infancy), **preoperational** (early childhood through preschool), **concrete operational** (childhood to adolescence) and **formal operational** (early adulthood). Piaget assumed that these stages of cognitive growth were inevitable based on a person's mental structures developing and were not linked with or necessarily influenced by instruction (Lehrer et al, 1998). He did however think that instruction was important, but not primary.

The most helpful part of Piaget's theory is his central tenet that learners construct their own knowledge, i.e., a learner's representation of space is developed through her own activity and interaction within the environment and other people. Although Piaget's theory has contributed immensely to the field of education by giving description to learners' thinking, in terms of geometry, there has not been evidence to suggest that Piaget's stage theory has been effective in influencing teaching (Clements & Battista, 1992). However van Hiele and Piaget's ideas about children constructing their own knowledge and making sense of their environment through active manipulation continues to be stressed as essential in mathematics learning.

Clements and Battista (1992) claim that van Hiele and Piaget's models promote students' ownership in building understanding; and that a critical dilemma in both theories is teaching about objects that are not yet objects of reflection for students (p. 437). This goes back to the idea of level mismatch; students will struggle to make sense of ideas being taught at higher levels than they have reached. Clements and Battista suggest that in both cases, van Hiele and Piaget believe that the conflicts that arise for students in trying to think at opposing levels are an essential and healthy part of the learning process. Clements and Battista also point out that van Hiele and Piaget do not support the notion that clear explanations are what define good teaching. Lastly, Clements and Battista say that neither van Hiele nor Piaget view their theories as an avenue to speed up development. This means that they do not support the attempt by teachers to force students quickly through succeeding levels once the current level has been established.

(b) Language skills

Looking at the differences between van Hiele and Piaget, Pandiscio and Orton (1998) claim that one difference is the two theorists' stance on students' movement among levels or stages. They say that Piaget would suggest that this is dependent on activity, whereas van Hiele would suggest that it is dependent on classroom instruction and language. Van Hiele believes that language skills are particularly critical for creating and linking new ideas to past experiences and prior knowledge. Vygotsky (1962) also maintained that language plays a central role in cognitive development. He argued that language was the main tool for determining the way a child learns how to think, because complex concepts are conveyed to the child through words. So in this aspect van Hiele is closer to Vygotsky than Piaget.

Pandiscio and Orton say that the purpose of Piaget's stage theory and van Hiele model are quite different. They suggest that van Hiele was trying to help teachers to improve instruction by describing levels of thinking of learners. Alternatively, Piaget was interested in describing the progression of thinking and when it could be expected to occur. This means that the van Hiele theory can inform instruction whereas Piaget's is less useful for this purpose. Pandiscio and Orton (1998) say that Piaget's developmental theory is more general than van Hiele's and can be applied more extensively. They also suggest that Piaget's ideas can give us guidance in understanding why a learner may have difficulty understanding a geometric concept. The corresponding drawback is that Piaget's theory does not offer help in alleviating the problem whereas van Hiele's model would.

Clements and Battista (1992) also make a distinction in the way Piaget and van Hiele believe learners develop in their thinking about reasoning and proof. They indicate that van Hiele would say this growth is dependent on increasing understanding of geometric relationships. According to Piaget, certain logical operations develop in learners "independent of the content to which they are applied" (p. 441). This suggests that van Hiele would say that a learner was ready for proof if her understanding of the content is at an appropriate level (formal deduction). Thus, instruction is an important factor that can prepare them for this kind of reasoning. For that reason the teacher is positioned as the central mediator of learning in the classroom. The teacher is the one who is crucial to supporting learners' development.

(c) Vygotsky vs. Piaget

The above descriptions of van Hiele' theory fits more with Vygotsky's theory than Piaget's. Vygotsky (1978) proposes that an essential feature of learning is that it creates the Zone of Proximal Development

(ZPD). He defines ZPD as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (1978, p. 86). This explains that learners will oscillate between levels depending on the nature of the tasks and instruction, and that the teachers’ role is central in helping learners move levels.

The above discussion shows that the van Hiele model is consistent with both Piagetian and Vygostykian theories and places the teacher at the centre of my study as the one individual who has to have the necessary level of geometric thinking to be able to understand the learners’ geometric errors. Research has shown that there are gaps in the level of thinking of both teachers and learners in primary school and yet the future of the secondary geometry curriculum depends on primary school geometry. Research has also shown that poor performance in geometry can be explained using the van Hiele model of geometric thought. In my study I focused on a group of teachers, their geometric thinking and their understanding of learners’ errors.

2.2.3 South African perspectives on van Hiele model of geometric thought

In my communication with Prof. Michael De Villiers (August 2008) on the van Hiele model, he said that his long standing problem with the van Hiele model was the assumption that children naturally first learn to partition quadrilaterals into disjoint sets at the visual level, i.e. viewing squares not as special rectangles, but disjoint from them, in other words, viewing a rectangle as a figure with right angles but having two long and two short sides (not allowing them to be equal). He argues that this occurs as a result of the way in which children in primary schools are first introduced to quadrilaterals with static visual prototypes. He suggests that learners never seem to realise when they first encounter these quadrilaterals how a parallelogram, for example, can be transformed into a rectangle, using geometric software such as a sketchpad, or a wire model.

De Villiers argues that the assumption that children naturally first learn to partition quadrilaterals into disjoint sets at the visual level results from a limited experience of quadrilaterals that children have had in the primary school. This partition view of quadrilaterals creates problems later on when learners need to formally organise geometry and deal with formal definitions of quadrilaterals, which are hierarchical (inclusive) and hence is problematic if learners have partitional views. De Villiers suggests that firstly,

teachers need to have a hierarchical classification of quadrilaterals, and understand the mathematical and pedagogical need for learners to develop this early on, even at the visualisation level.
De Villiers and Njisane (1987) found that about 45% of black pupils in Grade 12 in KwaZulu Natal had only mastered van Hiele level 2 or lower, whereas the examination assumed mastery at level 3 and beyond. Malan (1986) and Govender (1995) also found similar low van Hiele levels among secondary school learners. Feza and Webb (2005) in their study in the Eastern Cape found that none of the Grade 7 learners in their study attained the requirements of the Revised National Curriculum Statement (RNCS) assessment standards or could be categorised at van Hiele level 2. They suggest that van Hiele levels and Assessment Standards should be provided to teachers at pre and in-service training, by engaging teachers in activities that require classifying of answers by van Hiele levels and by also challenging them to match the responses to the NCS Assessment Standards.

Considering the low attainment of the learners in geometry in the ICAS tests, I did expect the learners to score low on the van Hiele levels as well. But of most importance is the fact that the van Hiele levels are a good indication of where the problems in the geometry curriculum lie for relevant interventions to focus on. Although in this study I did not engage teachers in activities that require them to classify learners' answers by van Hiele levels, as Feza and Webb suggest, I did get to understand the teachers' geometric levels and their understanding of their learners' errors.

Research was also sought to test the effectiveness of the van Hiele model by using interventions with both learners and teachers. There is support to indicate that the van Hiele levels are an appropriate way to characterise geometric reasoning, although research has raised doubts about the discrete nature of these levels. Different forms of assessment, which might indicate van Hiele levels, include a multiple-choice test and clinical interviews. The assessment of students has indicated that many are not prepared for success in high school geometry, possibly because of their lack of background with lower levels prior to high school. The assessment of teachers at the primary level has also shown evidence that the gap in learners' understanding could be partially due to the lack of geometric knowledge and low van Hiele levels of teachers in the early grades.

It is for this reason that I decided to work with Grade 6 teachers in order to gather information regarding their geometric thinking level and their learners' geometric errors. I hoped for the interviews to reveal the teachers' van Hiele level of geometric thought through discussion about the written responses, and how they think about their learners' errors. My focus in the latter was on distractors in the multiple choice ICAS items that were deliberately selected to represent particular misconceptions or errors that learners usually make. The distractors did help us see the teachers' geometric thinking as well as help them to see their learners' geometric errors.

Research on the van Hiele model can be divided into three main categories. Researchers aimed towards determining if the van Hiele levels can be used to categorise learners' geometric ability, whether they are reasonable in describing geometric reasoning and through what means these levels could be assessed. The success of these efforts made it possible to assess van Hiele levels of learners in geometry as well as teachers in training and those currently teaching.

2.2.4 Learners' geometric thinking

Generally, empirical research has confirmed that the van Hiele levels are useful in describing learners' geometric concept development (Burger & Shaughnessy, 1986; Fuys et al, 1988; Han, 1986; Hoffer, 1983; Wirzup, 1976). A lot of research has been done which both validates and challenges the van Hiele model. One of the first major studies with the van Hiele model was by Usiskin (1982). Usiskin developed a multiple-choice test to measure learners' van Hiele levels and to find out if the tests could predict achievement in geometry in the United States. He consistently found that most grade 10 students were at level 1 and 2 and not ready for high school geometry.

Burger and Shaughnessy (1986) wanted to find out if the levels were reasonable for classifying students' thinking in geometry. They found that their results supported the notion that the levels were reasonable in assigning students' level of thinking and were able to assign certain behaviours to each level. In identifying quadrilaterals, learners who made reference to visual prototypes, e.g., "a rectangle looks like a door" were assigned van Hiele level 1.

Learners who said that rectangles have two sides equal and parallel to each other, two longer sides are equal and parallel to each other, and they connect at 90 degree" , (p. 29) were assigned van Hiele level 2. Learners who characterised shapes by referencing other shapes were assigned van Hiele level 3 for example; a square is a parallelogram that has all the properties of a rhombus and a rectangle. One learner frequently made conjectures and attempted to verify these conjectures by means of formal proof, indicating level 4 thinking. The levels appear to exist and describe learners' geometric development, validated through both interviews and written assessments.

Another strong area of research using the van Hiele model that informed my study is research done with in-service and pre-service teachers.

2.2.5 Teachers' geometric thinking

Mayberry (1983) was one of the first to study United States teachers' van Hiele levels of geometric reasoning. On interviewing primary pre-service teachers he found that the teachers' responses indicated a lack of readiness for a formal deductive geometry course. Mayberry's other objective was to evaluate whether the questions posed to the participants formed a hierarchy that would correspond to the van Hiele levels. The purpose of assessing teachers' levels of reasoning is to identify where deficiencies exist, so that instruction can be catered to the needs of the teachers, to strengthen those particular problem areas.

Mason and Schell (1988) also studied the levels of reasoning and misconceptions of pre-service primary and secondary teachers and in-service secondary teachers. It was found that the secondary in-service teachers tended to score highest of the three groups, in terms of van Hiele levels. The results of the secondary in-service teachers showed that over 75% were at level 4 or higher. At the same time, some misconceptions were noted with this group; for example, some did not indicate that parallel lines had to lie in the same plane. Others applied theorems of isosceles triangles to equilateral triangles; however their previous definition of isosceles triangles stated that exactly two sides had to be congruent. For the pre-service primary teachers, 38% were found to be operating below van Hiele level 2. Some indicated that rectangles were squares, whereas some had trouble noting congruence of corresponding angles with similar figures.

Fuys, Geddes and Tischler (1988) interviewed grade six through to grade nine, eight pre-service and five in-service teachers to see if the teachers could learn how to assign van Hiele levels to characterise their students' thinking. The teachers got a brief introduction to the van Hiele model and then completed some of the same instructional tasks as the sixth grade students. Then, the teachers watched video clips of students completing the same tasks and were guided in matching student responses to the van Hiele level descriptors. They also examined some pages from a textbook and assigned van Hiele levels to the explanation used as well as thinking required by students to successfully complete the tasks.

Fuys et al. claimed that it was possible for teachers to learn how to assess van Hiele levels with student responses. Sharp (2001) found that the act of teachers analysing van Hiele levels improved their content knowledge of geometry. Jacobson and Lehrer (2000) further suggested that teachers be exposed to current research on student reasoning, but this was after an intervention programme with teachers. The researchers wanted to investigate how content and theory would ultimately impact student instruction. In this study, I have shown how the van Hiele model gives us a framework for analysing teachers' geometric level and their understanding of their learners' geometric errors. In the next section, I will present a review of literature that will help us to understand the thinking levels expected by the ICAS geometry test items when mapped with the van Hiele levels and the National Curriculum Statement.

2.3 CURRICULUM

Researchers seem to agree that the primary cause of poor performance in geometry might be the United States mathematics curriculum, both in what topics are included and how they are treated. The major focus of primary school geometry is on recognizing and naming geometric shapes (van Hiele level 1); writing the proper symbolism for simple geometric concepts; developing skill with measurement and construction tools such as a compass and protractor, and using formulae in geometric measurements (Porter, 1978; Thomas, 1982), but little is done in terms of geometric concept development.

Usiskin (1987), after looking at the United States mathematics curriculum documents concluded that there was no geometry curriculum at the elementary school, hence students enter high school not knowing enough geometry to succeed and that only about half of the students encounter geometry in high school, and only about a third of these students understand it (p. 29). I think he means that the primary school curriculum lacks enough content to sufficiently prepare learners for the more formal high school content.

2.3.1 National Curriculum Statement (NCS)

In South Africa in the Foundation Phase (Grade R-3), learners begin by recognizing and describing objects and shapes in their environment that resemble mathematical objects and shapes. Clearly the focus in the Foundation Phase is on the naming of shapes and objects. The focus of the study of shapes and objects in the Intermediate Phase (Grade 4-6) moves from recognition and simple description to classification and more detailed description of features and properties of 2-D shapes and 3-D objects. There are clear similarities between the Foundation Phase focus and the Intermediate Phase focus and the van Hiele levels 1 and 2 respectively as illustrated in the figure below:

Table 2.1 National Curriculum Statement and Van Hiele levels

National Curriculum Statement Focus	Van Hiele Levels
Foundation Phase Recognizing and **naming shapes** and objects	**Level 1: Recognition/Visual level** **Naming** and comparing geometric figures on the basis of their appearance as a whole
Intermediate Phase Classification and description of features and **properties** of shapes and objects	**Level 2: Analysis/Descriptive Level** Analysing figures in terms of their **properties** and using properties to solve problems The image begins to fall into the background and properties are established experimentally
Senior Phase Classification and description of geometric figures and solids extended to include similarity and congruence. Learners develop the ability to reason and justify appropriately Drawing and constructing geometric figures and solids using appropriate instruments.	**Level 3: Informal deduction Level** Understanding the relations within and between figures and also give informal deductive arguments; formulates and uses definitions. Reason about properties of classes of figures, "properties are ordered, e.g., the figure is a rhombus if it satisfies the definition of quadrangle with four equal sides"(van Hiele, 1986, p. 109)
Further Education and Training Phase Explore relationships, make and test conjectures, solve problems involving geometric figures and geometric solids Investigate geometric properties of 2-D and 3-D figures in order to establish, justify and prove conjectures	**Level 4: Formal deduction Level** Learners can reason formally by logically interpreting geometry statements such as axioms, definitions, and theorems. They are capable of constructing original proof, i.e., they can produce a sequence of statements that logically justifies a conclusion as a consequence of the "givens"

In the Senior Phase (Grade 7-9), the shift is from using measurement as a basis for determining properties to the interrelationship between figures and solids and their properties (The Mathematics Learning Programme, 2003, pp. 40-41). This is in contrast to the claim made by Usiskin above, because Usiskin is talking about another country and not South Africa. The NCS in the Senior Phase requires of the learners to analyse properties of shapes and solids and to understand relations within and between figures, using definitions and also reasoning (van Hiele level 3**-** Informal deduction Level).

Learning Outcome 3 in the Further Education and Training band (Grade 10-12) states; "The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification." (National Curriculum Statement Grades 10-12, (General) Mathematics, 2003, p13). The focus is on location, visualization and transformation. The learners engage with new tools that can be used in a range of applications, and they should become more proficient in the process leading to proof. The Content Focus in the Intermediate, Senior Phase as well as the Further Education and Training band (FET) is summarised below:

Table 2.2 Intermediate Phase, Senior Phase and FET Curriculum Focus

Intermediate Phase	Senior Phase	FET
Shapes and Objects Transformations Position	Shapes and Objects Transformations, Congruence and Similarity Position Straight line Geometry (Grade 8 and 9 only)	Surface Area and Volume Euclidean Geometry Analytical Geometry Transformation Geometry Trigonometry

The content summarised above shows progression of content across grades and phases. Ideally, the learners leaving the intermediate phase should be at van Hiele level 2, and enter high school with some ability of analysing shapes and objects. Problems outlined by Usiskin above, that "students enter high school not knowing enough geometry to succeed and that only about half of the students encounter geometry in high school, and only about a third of these students understand it", allude to the fact that challenges lie with the quality of teaching and learning in the development of spatial reasoning.

2.4 SPATIAL REASONING

Positive correlations have been found between spatial ability and mathematics achievement at all grade levels (Fennema & Sherman, 1977; Fennema & Sherman, 1978; Guay & McDaniel, 1977). Also in agreement with the van Hiele model, researchers have found that visual representations of mathematical ideas are especially important at the elementary school level (Stigler et al., 1990) because learners rely more heavily on imagery than do adults Kosslyn (1983).

Johnson (1987) describes imagery as what enables us to use our bodily experiences to structure all thought, not just mathematics. Johnson further says that we make use of patterns that help us to develop our more abstract thinking, through metaphor. For example, learners high in spatial ability and low in verbal ability tended to translate problems into pictures or diagrams more easily than learners who are low on spatial ability and high in verbal ability. This point is supported by Lean and Clements (1981) who suggest that learners who process mathematical information by verbal-logical means outperform learners who process this information visually. Hershkowitz (1989) warned that the use of imagery in mathematical thinking can sometimes cause difficulties because if a concept is tied too closely to a single image, its critical attributes might not be recognised because of over reliance on this image.

Despite the different views on the development of geometric thinking, in van Hiele level 1, learners rely on visual processing. At van Hiele level 2 and higher, learners' use of visual images is constrained by their verbal knowledge. Images and transformations of images incorporate this knowledge and, as a result, might behave differently at the different levels. Employing such a synthesis of analytic and verbal processes to construct robust concepts is possible, especially for students in grade 5 and beyond (Hershkowitz et al, 1990).

Vinner and Herschkowitz (1980) claim that in thinking; learners do not use definitions of concepts, but rather use concept images, combinations of all the mental pictures and properties that have been associated with the concept. They warn that such images could be adversely affected by inappropriate instruction. For example, for many learners, the concept image of an obtuse angle having a horizontal side might result from the limited set of examples they see in textbooks and a gravitational factor (i.e., a figure is stable only if it has one horizontal side, with the other side ascending). Other researchers also agree that learners limit concepts to studied exemplars and consider inessential but common features as essential to the concepts, (Burger & Shaughnessy, 1986; Fisher, 1978; Fuys et al., 1988; Kananova Meller, 1979; Zykova, 1969).

Research also suggests that learners have to master the skill of defining concepts to avoid errors in using terms that signify them because at each van Hiele level, new vocabulary is experienced and Van Hiele believes that language skills are particularly critical for creating and linking new ideas to past experiences and prior knowledge. Acquisition of mathematics vocabulary will help learners to synthesize definitions of concepts more easily because a word acquires its sense from the context in which it appears in different contexts, words change their meanings.

Researchers highlight pitfalls with the use of diagrams or illustrations when learning. Learners often incorporated information contained in a specific diagram as part of a theorem (for instance, thinking that the exterior angle of a triangle must be obtuse because the diagram given with the theorem pictured an obtuse exterior angle). Vladimirrskii (1971) concluded that the diagram accompanying the discussion of a geometric statement is not always helpful in reasoning. Learners might mistake features of the geometric relationship being considered, thus introducing irrelevant ideas into the concept.

For the learners, the figure drawn on paper represented pictures of geometric objects; they were not considered as geometric objects themselves. In mathematics, pictures are meant to be symbols for concepts. Research has established a wide discrepancy that can exist between a learner and teacher or textbook interpretations of a simple diagram. Although there are numerous accounts of learners arriving at incorrect solutions to problems due to improper problem representations (Davis, 1984), teachers should not overlook misconceptions that could arise out of learners' interpretations of the diagrams themselves.

2.5 CONCLUSION

In this chapter, I have developed a theoretical and analytic framework for understanding teachers' geometric thinking and their learners' errors. Geometry is important as it offers us a way to interpret and reflect on our physical environment. Spatial thinking is essential to creative thought in all high level mathematics. Research has shown that there is potential for many errors in geometry due to many factors, including how we visualise objects, curricula, teaching methods and teacher knowledge. In the next chapter, I discuss the research process used to answer my research questions.

CHAPTER 3: DATA COLLECTION AND METHODOLOGY

3.1 INTRODUCTION

In the previous chapter, I discussed the theoretical and analytical framework that informed my research. In this chapter I will map out the route I took in planning and collecting information concerning teachers' geometric thinking about learners' errors. I discovered that this kind of research is not easy; therefore I needed a three-phase data collection. A careful selection of methodology and methods became a very important issue for the validity and reliability of the findings.

3.2 METHODOLOGY

This study sought to understand teachers' geometric thinking as well as their understanding of their learners' geometric errors, thus a qualitative research methodology was suitable as it involves "a real situation, with real people in an environment often familiar to the researcher. Its aim then was to provide a picture of a certain feature of social behaviour or activity in a particular setting and the factors affecting that situation" (Opie, 2004, p. 74).

The qualitative research method allowed me to analyse the ICAS test items, study curriculum policy documents, analyse teachers' written responses, and also get the perspective of the teachers in the task-based interviews. The experiences were rooted within a context (Merriam, 1998) and this method allowed me also to focus on a specific situation and to explore the various interactive processes at work within that situation (Verma & Mallick, 1999).

3.3 DATA COLLECTION

The aim of this study was to reveal teachers' understanding of their learners' geometric errors focusing on their own reasoning about learners' geometric thinking, in order to answer the following research questions:

1. What level of thinking do the geometry items in the ICAS tests expect?
2. What is the level of geometric thinking of selected Grade 6 teachers?
3. In what ways do Grade 6 teachers understand their learners' geometric errors?

In order to answer these questions, I used three methods of data collection. First I mapped the Grade 3-11 geometry ICAS items to the van Hiele levels and the NCS assessment standards. Having mapped the items, I chose items in the range of levels 1, 2, and 3 van Hiele levels to give to a group of teachers to solve. The chosen test items included two van Hiele level 1 items, eight van Hiele level 2 items, four van Hiele level 3 items, two van Hiele level 4 items and two ICAS items that could not be classified according to van Hiele levels.

I gave the eighteen chosen geometry ICAS items to a class of in-service General Education and Training (GET) teachers to solve. After analysing their responses, I used the results to categorise the teachers into the various van Hiele levels of geometric thought, (this will only be a rough categorisation, as you will see in Chapter 4, that there are many factors that influence classification of teachers into van Hiele levels). I then chose 4 items representing van Hiele levels 1, 2, and 3, as the expected levels in the intermediate phase. These four items were used with the six interviewed teachers.

The use of different sources and different methods helped to produce results that are more comprehensible than would be the case with fewer methods and sources.

3.3.1 Document analysis

In order to find out exactly what level of thinking the geometry items in the ICAS tests expected, I mapped all the 82 geometry ICAS items found in the Grade 3-11 test papers to the National Curriculum Statement (NCS) assessment standards and then categorised them according to the van Hiele levels.

I used the NCS for Grades R-9 developed by Scheiber (2005) to map the ICAS items with the assessment standards. The document made it easy for me to find the relevant assessment standards as they have been numbered and also arranged on one page from Grade R-9 (see attached **Appendix A**). The progression of the content of mathematics within the assessment standards can also easily be detected. For Grade 10-11, I used the NCS for Grade 10-12, Department of Education (2003) which also has numbered assessment standards and shows progression of the content across grades from Grade 10-12.

Mapping the test items allowed me also to find the appropriate ICAS items to use in the written responses with the teachers. I came across a number of problems associated with the document analysis. Analysing 82 test items was a lot of work, especially since some of the questions assumed knowledge, so I had to map back to lower grades and sometimes the question asked at a particular grade was not suitable for the grade, so I had to look forward in the higher grades in order find the relevant assessment standard.

With regard to the van Hiele levels, I used the level indicators for the first four van Hiele levels developed by Burger and Shaughnessy (1986) (see attached **Appendix B**), after reading a range of literature on van Hiele levels. This particular model was chosen because it had been constructed in consultation with van Hiele and other mathematics educators and because the descriptors were elaborated clearly and in detail. In order to make it easy for me to map, I added numbering to the van Hiele level descriptors as shown in Appendix B.

It was a difficult process trying to map the different ICAS items to the van Hiele levels, so I tried to link the ICAS items with the assessment standards first then later link the assessment standards with the van Hiele level descriptor using the key words in the questions and the assessment standards and the van Hiele descriptors as guidelines.

3.3.2 Written responses

For purposes of this study, I worked with an original sample of about 120 in-service teachers participating in the Advanced Certificate in Education (ACE) programme at Wits University. My interest in ACE teachers is because of their practical teaching and learning experience gathered over the years. Lecturers in the ACE programme selected the class and of the 120 teachers who received the written items, 89 responded and only 40 gave consent to participate in the study, and these were the responses I analysed.

In order to see the link between the geometry content focus in the GET and FET geometry, I consulted the National Curriculum Statement (NCS) documents and found that "the teaching and learning of space and shape... in the FET band must build on experiences from the GET band to make more formal and extended levels of knowledge accessible. Aspects that are important for the attainment of this Learning Outcome include location, visualisation and transformation (NCS Grade 10-12, Mathematics, p. 13). The teachers in the intermediate and senior phase need to teach and lay a proper foundation for the learners to deal with aspects of location, visualisation and transformations later on in the FET phase, so I bore these points in mind when I selected the eighteen ICAS items to be used as written responses by teachers. (see attached **Appendix C**). It is important to note here that of the 82 geometry ICAS items analysed, I found that 27 items focused on transformation geometry, whilst 47 items focused on Euclidean geometry ICAS items and 8 ICAS items focused on Location. The location questions could not be mapped to the van Hiele levels as they do not have polygons but are an important part of the curriculum, so, I included two of them in the selection of eighteen items.

I chose two level 1 van Hiele questions, eight level 2 van Hiele questions, four level 3 van Hiele questions, two level 4 van Hiele questions and two questions that could not be categorised according to van Hiele levels because, as mentioned, they do not have polygons, but deal with location and position, an important link to geometry of coordinates in the FET phase.

I chose items from the ICAS tests across the grades to allow the teachers to experience a variety of content and help us understand their geometric thinking and included items that the learners did exceptionally well in and those that the learners did not do well in to allow the teachers to expose their skills in explaining their learners' geometric errors. I included two ICAS items that were not multiple choice so as to give the teachers an opportunity to reveal their thinking as they were at liberty to use any suitable method to solve the problems. The teachers were given 30 minutes to respond in writing to the items.

I analysed the teachers' written responses and compared them to that of the learners in the ICAS tests (2006), for the purposes of understanding the teachers' geometric thinking and to help me in selecting the teachers to be interviewed, and the items I would use in the task based interviews. In the interviews I used five items, which will be discussed later in the next section.

3.3.3 Interviews

Interviewing can be a valuable way of gaining a description of actions and events (Maxwell, 1996) as well as participants' perspectives. Task-based interviews with these six teachers were used to further understand the teachers' geometric reasoning and to also find out how they understand their learners' geometric errors. The task-based interviews focused on how the teachers solved the problems and imagined how the learners might have got the correct and incorrect answers using the various distractors in the multiple-choice questions. The distractors in the multiple choice questions are specifically selected to represent misconceptions learners might have. Therefore interrogating the distractors with the teachers helped me to get them talking about their own teaching and learners in a more meaningful way. The wrong options learners selected revealed some misconceptions and errors of learners or that learners might have misread the question, have problems with language or did not have the necessary geometry knowledge and skills to solve the problem.

I conducted the teacher interviews after having analysed the written responses. I was hoping to get more information from the interview on teachers' geometric thinking and learners geometric errors that could not be revealed by the written responses. Four of the six teachers interviewed worked in pairs in task-based interviews to understand their own geometric thinking and the GDE learners' geometric errors. The other two teachers worked individually on the tasks whilst being interviewed.

Task-based interviews focus on carrying out a mathematical task or tasks. The whole idea was to have the teachers working in pairs, whilst "talking aloud" (Newell and Simon, 1972) to the solutions and the distractors. Talking aloud involves a description of the problem-solving process Teachers are encouraged to work in pairs so that they can discuss their own thinking and the learners' responses with each other. This revealed some of their thinking that might not easily come out if they could have been interviewed alone. But the teachers who were interviewed alone were encouraged to speak to the researcher, explaining their thinking patterns how they solved the problem and why the learners could not solve some of the items.

Before the interview I explained to the teachers that I was not trying to assess their geometric knowledge, that all I needed was their honest opinion about the ICAS items as they were obviously different from the normal exercises we teach within our classrooms every day. I asked them to refer back to their scripts and explain to each other how they solved the problems, if they would use the items to teach with and if they would, at what grade level. I did not correct them or reveal to them whether their answers correct or incorrect. Rather, I encouraged them to explain how they arrived at their answer, focusing on the methods used. Once I gathered that they were not sure of a problem, how to solve it, I would move on to the next item. I did this to make the teachers feel at ease because I was aware that the teachers' responses were likely to be influenced by the views of the researcher and in so doing they might fabricate their answers (Bassey, 1995).

The interviews were semi-structured as they "allow for a depth of feeling to be ascertained by providing opportunities to probe and expand the interviewee's responses. It also allows for deviation from a prearranged text and to change the wording of questions or the order in which they are asked", (Opie, 2004. p. 118). The interview schedule consisted of twelve questions; six questions focused on the teachers' geometric thinking and the other six questions focused on the learners' geometric errors **(See**

Appendix F). The teachers' interview schedule focused on the teachers' reasoning associated with van Hiele levels. The teachers referred back to their written papers and talked through how they solved the problem, how they would use the item to teach with, at which Grade and what the purpose of the lesson would be.

For the purpose of getting the teachers to talk about the learners' geometric errors, they were given the items with the achievement of the learners in the GDE sample. I also made available the achievement of learners in percentages focusing on the responses of the learners on each of the four distractors **(See Appendix G).** The teachers had to talk about the conceptual knowledge and skills necessary for the learners to get the correct answer. They also had to think about the problems learners experienced that made them choose the wrong answers and also analyse the wrong answers to check for misconceptions or if they were just plain errors or miscalculations. Using the questions as a stepping stone, I probed more deeply into the responses given focusing more on learners' errors, which allowed the teachers to engage more openly.

The interviews were tape recorded and later transcribed. Bassey (1995) explains that "the advantage of recording for the researcher is that she can attend to the direction rather than the detail of the interview and then listen intently afterwards". It was interesting to see that when the teachers talked about how they solved a problem, they were very quick to give an answer as to which level or grade they would teach using the item, but as soon as they saw the low achievement of the learners, their responses became more realistic and also enabled them to realise the unique type of questioning of the items as opposed to what they were used to in their classrooms.

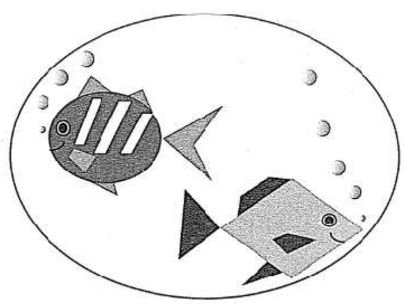
Opie (2004) states that the purpose of an interview is to encourage respondents to say what they think and to do so with richness and spontaneity. I anticipated that there might be problems with paired interviews if one of the pair dominates and the other feels "silenced" in the interview. Some might tried to defend their answers and justify them using non-mathematical reasons. I did make sure that each teacher gets an equal opportunity to engage with each other's ideas from their own experience of teaching over the years by asking probing questions to those who were more silent. I looked out for when the teachers compared answers and explored suggestions in order to gather information about their geometric thinking and the learners' geometric errors.

According to Sikes (2004, p. 21 in Opie), the reason for doing research is to develop knowledge and to communicate that knowledge, with the ultimate view of informing practice and/ or policy and, improving these in some way. Sikes (2004) further argues that if the assumption is that knowledge is experiential, personal and subjective, then questions will have to be asked of the involved participants. I therefore interviewed each pair separately on different days, sitting with them and probing whilst looking for each teacher's knowledge with regard to geometry and learners' geometric errors. I focused on the reasons given by the teachers on how they approached the questions and on the mathematical thinking processes applied by the GDE learners in solving the questions. For the purposes of this study, I used the interview transcripts of all six interviews which I will elaborate more fully in Chapter 4.

(a) Interview items

I selected five ICAS items from the original eighteen written responses to use for the interviews. Seeing that my interest is in intermediate phase teachers, I chose questions that were at van Hiele level 1, 2 and 3 because as already established by Feza & Webb (2005) "the exit level outcomes for learners in the intermediate phase of the NCS can be related to the expectations of van Hiele level 3". The first item for interview I used came originally from a Grade 3 question paper (question number 18). I mapped it to van Hiele level 1 and assessment standard 3.3.1 [1]which requires the learners to *recognise, identify 2D shapes and 3D objects in pictures/environment including triangles, squares and rectangles (four-sided) shapes.*

18.



How many four-sided shapes are in this picture?

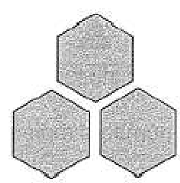
(A) 5
(B) 6
(C) 7
(D) 8

Figure: 3.1 Interview Item no. 1

[1] 3.3.1 Refers to Grade 3, Learning Outcome 3, Assessment Standard 1; which explains that we will know this when learners: Recognises, identifies and names 2-D shapes and 3-D objects in the environment and in pictures including boxes (prisms), balls (spheres) and cylinders, triangles, squares and rectangles, circles, cones and pyramids as outlined in Scheiber (2005).

The item is a multiple choice question with various 2D shapes in a context of a fish tank. This question originally from a Grade 3 paper required the recognition and selection of four sided shapes amongst triangles and spheres. I was curious when only 29% of the GDE learners answered correctly because it seemed like such an easy question. But when only 30% of the 40 ACE teachers who responded to the ICAS items in writing managed to score the correct answer, I was really surprised **(See Appendix E).** I had categorised this question at van Hiele level 1 and wanted to find out why the achievement for both the learners and the teachers was so low, I therefore chose the item as the first interview question. The second item selected for interviews, originally from a Grade 5 question paper, was mapped to van Hiele level 2 and assessment standard 6.3.2 which requires the learners to *describe and classify 2D shapes in terms of properties including faces, vertices and edges.*

8. Jade had these three tiles.



She joined the tiles together with no gaps or overlaps to make a new shape.

What is the smallest number of edges Jade's new shape could have?

(A) 6
(B) 12
(C) 16
(D) 18

Figure 3.2 Interview Item no. 2

The item referred to tessellations – no gaps or overlaps, but I suspected issues of language would arise because the question was asking for the number of edges in a 2-D shape. I wanted to find out if the teachers do use precise mathematics language and also how they prepare their learners to use proper mathematical language. The performance of Grade 5 GDE learners in this item was low compared to that of the 40 ACE teachers, and that made me to want to find out what made the learners achieve such low scores. I thought this was an ideal opportunity for the teachers to describe learner errors. The third item chosen, originally from a Grade 11 paper, question number 2, was mapped to van Hiele level 3 and assessment standard 9.3.3 which required the learners to *use geometry of straight lines and triangles to solve and justify relationships in geometric figures.*

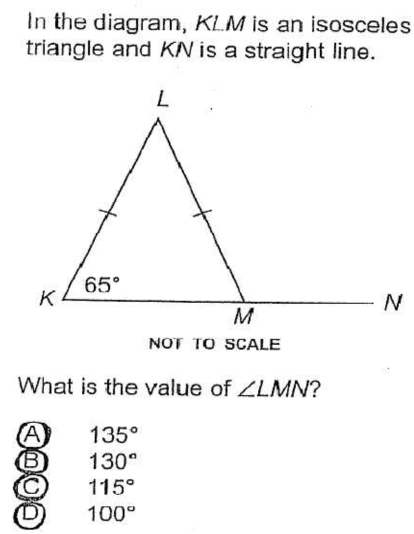


Figure: 3.3 Interview item no. 3

I was inspired by the fact that 80% of the 40 ACE teachers scored the answer correct whilst only 49% of the GDE learners managed to get the correct answer. The item required knowledge of properties of a triangle integrated with properties of angles on a straight line. I wanted to find out how the teachers teach the concepts of base angles of an isosceles triangle; angles on a straight line and the exterior angle of a triangle. I classified this as a van Hiele level 3 item looking at all the properties that needed to be deduced from the figure provided. I was also interested in finding out how the teachers would analyse the GDE learners' errors. The next chosen item, originally from a Grade 6 paper question number 39, required the number of diagonals of a regular 15 sided shape, given the number of diagonals of four other polygons, i.e., triangle, square, etc. I mapped it to van Hiele level 3 and assessment standard 8.3.2 for it required the learners to *describe and classify geometric figures in terms of their properties, including sides, angles and diagonals.*

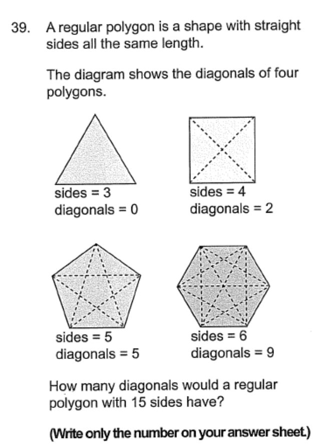


Figure 3.4 Interview Item no. 4

It was not a multiple choice question so I thought it would make for good conversation plus the fact that none of the GDE learners answered the item correctly and only 2 (5%) of the 40 ACE teachers managed to calculate the answer correctly. The item required one to study the numeric and geometric pattern that develops in order to find a relationship that would help generate a general rule. I had classified this particular question at van Hiele level 3 for it requires the learners to deduce from the information given to find the general rule. The item combines aspects of Learning Outcome 2 and Learning Outcome 3 that deal respectively with patterns, functions and algebra and space and shape. The last item used for the interviews, originally from the Grade 7 paper, question number 39, interested me because it was not well answered by both teachers and learners. I mapped it to van Hiele level 3 and Assessment Standard 7.3.2 as it required the learners to *describe and classify geometric figures and solids in terms of properties including sides and angles of polygons.*

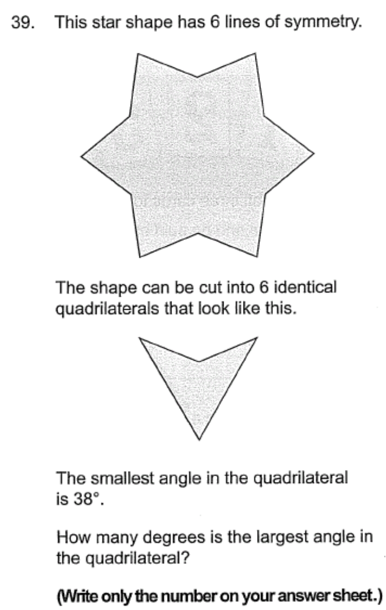


Figure 3.5 Interview item number 5

In fact none of the GDE learners nor the 40 ACE teachers could find the correct answer in the written responses. This item above also dealt with the elusive concave quadrilateral, lines of symmetry and the calculation of the largest angle, so I classified it at van Hiele level 3. This problem integrated lots of concepts that I thought it would make interesting material for interview with the teachers, where their thought patterns would have to come through very clearly to get to the correct answer. Below is the summary of the items selected for interviews:

Table 3.1 Summary of interview items

ICAS Items	Grades	Percentage of Learners that got the items correct	Percentage of Teachers that got the items correct	Van Hiele Levels	Assessment Standards
1	3	29%	30%	1	AS 3.3.1a
7	5	24%	80%	2	AS 6.3.2
13	11	49%	80%	3	AS 9.3.3
15	6	0	5%	3	AS 8.3.2
17	7	0	0	3	AS 7.3.2

3.4 THE TEACHERS

3.4.1 Written Responses participants

The ACE class consisted of 120 ACE students. Of the 120 students given the written responses, only 89 of them responded in writing to the ICAS Items, 46 of them did not give consent, 3 did not return their consent forms and 40 gave consent to participate in the study. Of the 40 teachers who gave consent, 22 were female and 18 male. I have given further details about the teachers in the study in the tables 3.2 and 3.3 below. These teachers were in their first year of an in-service training course, from the schools in the east part of Johannesburg, studying towards an Advanced Certificate in Education specialising in GET mathematics. They responded to the ICAS items at the Wits School of Education during their normal ACE class session.

Table 3.2: Frequency Table for Years of Teaching Experience

Teaching Experience in Years	N
1 – 4 years	14
5 – 10 years	13
11 – 20- years	9
20 – 30 years	4
TOTAL	**40**

According to table 3.2 above, the majority of teachers had up to ten years teaching experience, with very few at the top end with more than 20 years teaching experience. The participants in the study had enough teaching experience to give useful data for in-depth analysis of their own geometric thinking and their thinking about their learners' possible errors.

Table 3.3: Teacher Qualifications

Qualification	N
National Primary Diploma in Education (NPDE)	2
Senior Primary Teachers Diploma (SPTD)	28
Secondary Teachers Diploma (STD)	3
Further Diploma in Education (FDE) Higher Diploma in Education (HDE)	2
B.Tech/B.A/B.Ed	5
TOTAL	**40**

From table 3.3 above, most teachers have an SPTD (Senior Primary Teachers Diploma) qualification obtained at various Colleges of Education in South Africa. With these qualifications and teaching experience, I thought these teachers would be able to clearly articulate their own geometric thinking as well as their thought processes while looking at the learners' responses, specifically focusing on the required conceptual skills needed to answer the questions correctly. After analysing the written responses and realising that the performance of the teachers was not as good as I had expected, I decided to select teachers who achieved well above the others with the hope that they would readily be able to articulate their thoughts. But I soon realised that it was also not going to be easy to get hold of teachers due to their work and study commitments.

3.4.2 Interview participants

I had initially planned to work with five pairs of teachers in task-based interviews from the 40 teachers who had responded in writing to the eighteen ICAS items. The challenge of getting teachers to commit to attend interview sessions became a problem because of their work and study commitments. At the end I only managed to interview only one teacher (Teacher A) from the original 40 and paired her with teacher B from the second Maths GET ACE class. Teacher A had achieved the highest of all the 40 teachers in the written ICAS items. Teachers B, C, D, E and F volunteered to participate in the study even though they did not write the original eighteen ICAS items. All the six teachers gave consent in writing to participate in the study and to respond in writing to the five interview ICAS items chosen and to be interviewed. It was by sheer coincidence when Teacher A and B came together for the interview that they realised that they knew each other as they were together in the HOPE project and they had worked together before. Teacher C and D were also interviewed as a pair using task-based interviews. Teacher E and F were interviewed individually.

The interviews took 60 – 90 minutes and it was amazing how the teachers were committed till the end. As much as I wanted to do a study of Grade 6 teachers, I realised that it is not easy to find a teacher who teaches only Grade 6. The majority of teachers teach multiple grades and so most teachers teach across Grade 4 to 6 and even up to Grade 7. Below, in table 3.4 is the profile of the teachers that were interviewed:

Table 3.4: Interview participants

Teachers	Qualifications	Class Teaching	Teaching Experience	Profile
A	Higher Diploma in Education (HDE)	Grade 6/7	14 years	She is involved in the Hope project –group sessions that help teachers develop their geometric skills. Qualified as a teacher and taught under the former TED. Her average[2] in the ACE mathematics specialisation is 80%. The last AMESA conference attended was in 2003 (Cape Town)
B	Senior Primary Teachers Diploma (SPTD)	Grade 6	20 years	He is also involved in the Hope project and qualified as a teacher at a former "Coloured College of Education". He averages 80% in the ACE mathematics specialisation
C	B.Ed in Educational Management	Grade 6	15 years	She qualified as a teacher in the former 'Indian College of Education" and upgraded her qualifications to obtain a B.Ed. Her average is 80% in the mathematics specialisation
D	Senior Primary Teachers Diploma	Grade 6	14 years	She qualified as a teacher at a former 'DET" College of Education and averages 60% in the ACE mathematics specialisation
E	B.Ed in Educational Management	Grade 6/7	14 years	He qualified as a teacher at a former "TED" College of Education and averages 75 % in the ACE mathematics specialisation. He studied further to obtain his highest qualification (B.Ed)
F	Senior Primary Teachers Diploma	Grade 6	14 years	He qualified as a teacher at a college of education in Zimbabwe and averages 75% the ACE mathematics specialisation

Teacher A had a passion for geometry "I like geometry, and …when you like something, you teach it well…" were actual words during the interview. She was one of the very few teachers who managed to find the answer to the item requiring calculation of diagonals of a 15 sided regular polygon. This teacher initially trained at a "white" college of education under apartheid and went on to acquire a lot of experience whilst teaching for many years. She has also participated in a community of practice forum provided by the Association for Mathematics Education (AMESA) conferences and is involved in experiential learning offered by the Helping our People to Excel (Hope) project, which focuses on improving the geometric teaching skills of primary school teachers.

Teacher B has a wealth of experience, having taught for 20 years in Grades 6 and 7 in one school and also involved in the HOPE Project where as a "Junior Facilitator –they do new things and it's hands-on, that's what I like about …I mean 3D objects. We make the children build with straws, etc." He is a group

[2] Mathematics ACE averages obtained from lecturer of the teachers.

leader[3] in the ACE class and a top performer. He worked well together with Teacher A. The interview with Teacher A and B was conducted in the afternoon at a primary school in Kempton Park where teacher A is the deputy principal. The interviews for Teachers C, D, E and F took place during school holidays at the Wits School of Education.

Teacher C has taught for the past 15 years. She did not feel well on the day of the interview but managed to share some interesting thoughts the ICAS items. She qualified as a teacher at an Indian college of education prior to 1994. She has studied further and has a B.Ed in educational management systems and an incomplete M.Ed (Educational Management) coursework and mini dissertation. She tended to dominate the interview and sometimes did not give teacher D an opportunity, unless I specifically requested teacher D to explain her thoughts.

Teacher D, comes from a background of a former Department of Education and Training College of Education (Matric + 3 years), where teachers were taught all the subjects offered in school because of the belief that a primary school teacher must be able to teach every subject. Therefore had no specialized training in the pedagogic content knowledge of mathematics and low content knowledge of mathematics. She enjoys the ACE programme opportunities to immerse herself in mathematics content and methodology. She displayed a submissive attitude during the interview and agreed with basically everything Teacher C said, unless I specifically asked her first to elaborate a concept or thinking pattern. Although she has an overall teaching experience of 14 years, she had only two years experience teaching mathematics in Grade 6. She teaches at a township primary school with few educational resources.

Teacher E qualified as a teacher at a “white” college of education (Matric + 4 years). He gave very long explanations outlining the particular mathematical strategies he employed in solving the problems. He teaches at a former model C primary school which is better resourced and well managed. He is highly computer literate and often searches the internet for free teacher resources and information. He is also one of the top performing teachers in the ACE programme.

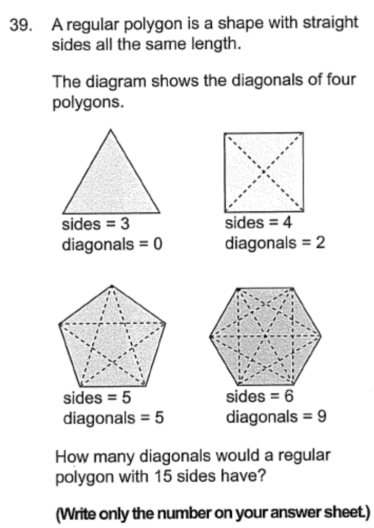
Teacher F qualified as primary school teacher at a college of education in Zimbabwe and has been teaching in South Africa for the past six years. He has 14 years mathematics teaching experience and also worked well with the National Curriculum Statement and knew his way around the assessment standards.

[3]ACE lecturer provided information that he was a group leader in the ACE class

I do realise that this is a skewed sample because the six interviewed teachers are relatively top achievers in the ACE programme as revealed by their profiles. However they did provide interesting data revealing their geometric thinking which when taken together with the 40 ACE teachers provides interesting insights.

3.5 DATA ANALYSIS

Qualitative research is not concerned with verification of existing themes and hypotheses but rather with discovery of new knowledge (Hitchcock and Hughes, 1989). Qualitative data analysis as explained by Hitchcock and Hughes (1989) "is not closed; it is creative, emergent and developmental". Research reveals that important information lies in the data and by systematically asking the right questions of the data, that information can be revealed. I began my analysis by doing a document analysis of mapping the geometry ICAS Grade 3-11 items to the van Hiele levels of geometric thought and the RNCS Assessment Standards in order to determine the geometric levels expected by the ICAS test items. I used the level indicators for first four van Hiele levels developed by Burger and Shaughnessy (1986) (see attached **Appendix B**), because it gave me a clear structure to classify the various ICAS items in line with the literature on the van Hiele levels and their distinct classification.



ICAS item 39 (Grade 6)

In order to analyse the items I looked at what the question required in terms of assessment standards, then referred to table 3.5 below to classify according to the van Hiele levels. For example, ICAS Item 39 above, in the Grade 6 question paper required the learners to find the number of diagonals in a given polygon.

Table 3.5 ICAS Item vs. Assessment Standards vs. van Hiele levels

Grade 6, no. 39	Assessment Standard 8.3.2	Van Hiele level
Find the number of diagonals in a regular polygon with 15 sides	Describes and classifies geometric figures in terms of their properties, including sides, angles and diagonals	Frequent conjecturing and attempts to verify conjectures deductively

I started with the item, focusing on the key words, in this case, diagonals and searched through the policy document to find the relevant assessment standard. I then searched through the van Hiele descriptors to find the relevant level. I soon realised that it was not a very easy process. In order to verify my classification, after mapping the assessment standards, I checked my mapping against the DIPIP[4] analyses. When I encountered a different DIPIP mapping, it helped me to go back and double check the question what concepts were required and if indeed the correct assessment standard was assigned to the question. I found that in most cases my individual mapping and the DIPIP analyses matched well.

In coding the scores obtained by the ACE teachers in the written responses, I had to make decisions about how to show and match their correct scores according to the van Hiele levels. But before I analysed the written responses, I started by first marking the scripts of the 40 ACE teachers, after which I recorded the scores of the teachers in Appendix E and arranged their correct scores according to the van Hiele levels. The first two questions at van Hiele level 1, the next eight questions at van Hiele level 2, the next six questions at van Hiele level 3 and the last two were those items that could not be mapped to the van Hiele levels.

I substituted the names of the teachers with letters A, B, C up to Z and continued with AA, BB, CC up to NN in order to protect their identity as per the consent forms signed. I then recorded every correct score as van Hiele level 1, 2 or 3 so that I could see which van Hiele levels the teachers had scored correctly on. The teacher with the highest correct score was recorded as A and the teacher who scored the least correct score was recorded as NN. Lastly, I calculated how many teachers had scored a particular ICAS item correct and finished off by calculating the average score of the correct answers obtained by the 40 ACE teachers.

After recording all the correct scores, I then attempted to categorise the teachers into van Hiele levels by looking at how many ICAS items each teacher had scored correctly and at what van Hiele level. After

[4] Teachers involved in the Data Informed Practice Improvement Project (DIPIP) mapped all the ICAS test items with the relevant NCS assessment standards (Brodie et al, 2010).

coding the correct scores into van Hiele levels, I had to make a decision about what qualified as a van Hiele level 1, 2 or 3 teacher. This turned into a great challenge because the scores achieved by the individual teachers did not allow for easy categorisation into van Hiele levels because the items achieved correct were not consistent at each van Hiele level. I then decided to analyse the errors made by the teachers in the ICAS tests. This would allow me to look into specific conceptual errors made by the teachers that would possibly reveal their geometric conceptual understanding.

I also had to make a decision as to which ICAS items I was going to analyse. After I had closely looked at the individual scores of the teachers for each ICAS item to find the suitable items for error analysis, I decided to choose the same five ICAS items to be used for interviews for the analysis of teacher errors in the written responses. Analysing these five items would give me an opportunity to see what geometric topic the question was dealing with and what geometric concepts it required the teachers to apply in order to solve the problems. The error analysis would also help me to better understand how the teachers could have got the answers wrong and possibly get a glimpse of what to expect during the interviews.

The analysis of the interview transcripts in order to try and understand the geometric thinking of the teachers and how they understand the learners' geometric errors also proved to be very challenging. I transcribed the interviews of the six teachers, and as a result had a lot of data to work through. Inductive analysis was used to categorise the data collected from the teachers' discussions about the learners' errors. Inductive data analysis is a search for patterns of meaning in data so that general statements about phenomena under investigation can be made. Inductive analysis begins with an examination of the particulars within data, moves to looking for patterns across individual observations, then arguing for those patterns as having the status of general explanatory statements (Hatch, 2002, p. 161). I will explain more in detail in chapter 4.

3.6 RIGOUR

One of the most difficult aspects of qualitative research lies in establishing and maintaining rigour. Opie (2004) identifies credibility, reliability and validity in establishing the quality of statements and relationships. Credibility can be achieved by the researcher checking with the participants that what is analysed and interpreted represents their actual views.

Reliability is the extent to which a method gives consistent results over a range of settings. Reliability is difficult to achieve as the study is qualitative and the researcher herself is central to interpretation. The researcher reflects on her experiences in the description of what she perceives. The triangulation from my data sources and the careful recording and encoding of each step of the research process and all decision points provides for readers to assess the reliability of my study. I also checked my initial interpretations with the members of my research group made up of my supervisor, who is an expert in the field and two other Masters students. Many of my interpretations were supported and I was prompted to reconsider some, thus adding to the reliability of the study.

Validity ensures that the method measures what it is supposed to measure. Once again a triangulation as well as member check is important for ensuring validity. I have tried to point out my own assumptions as influenced by my experiences and also indicate any shifts in thinking in my research report. I have also given evidence with transcripts of the teachers interviewed where appropriate. During the interview I tried to probe the teachers' viewpoints and experience by referring to the written responses that helped reveal their perspectives influenced by their experience and teaching background. Teachers A, B C and E checked the initial interview transcripts to ensure that their views were correctly captured.

3.7 ETHICAL ISSUES

I have abided by the code of ethics of Wits University that governs research on human subjects. The universities ethics committee approved the study: Protocol 2008ECE78. I applied to the Head of the Wits School of Education for permission to conduct research with the mathematics GET ACE students before collecting data and this permission was granted. An information sheet outlining the study was given to the teachers and they were asked to sign informed consent forms. Of the 120 teachers in the GET ACE Mathematics class, only 40 provided consent for their written responses to be analysed and used for the study. All the teachers I interviewed also signed consent forms and agreed to the terms of data collection. These forms informed the participants of the study and assured them that they could withdraw at any point in the course of the research.

In addition to the required ethics procedures, it is important for the researcher to act ethically at all times. I tried as hard as possible to act ethically by coding the names of the teachers who responded to the ICAS items as well as the interviewed teachers into letters of the alphabet in order to keep the identity of the teachers confidential. The teachers' permission was sought before I recorded them with a tape recorder and I made them aware what the data was going to be used for. During the interviews, whenever I realised that the teacher did not know how to solve a particular problem, I never pressed on to embarrass them, but moved along and talked about the concepts they felt comfortable talking about.

3.8 LIMITATIONS

The study cannot be generalised because the study was restricted to geometry in the ICAS test items with special focus on Grade 6, and the findings only reflect the experiences and views of the teachers interviewed. The number of teachers who responded to the items and the teachers interviewed are too few to give a generalised view about the geometric thinking of intermediate phase teachers and their understanding of learners' errors. As it is a qualitative study, the results are not generalisable in the statistical sense. However it is hoped that the findings will illuminate curriculum issues in geometry and factors contributing to learner geometric errors and possible ways of dealing with them.

3.9 CONCLUSION

In this chapter, I described the process that enabled the research: the methods chosen and the instruments used to conduct the study. I have given a description of the context in which the study took place. Issues of validity, reliability and ethics were also discussed. In the next chapter I will describe the findings and analyses of the ICAS items, van Hiele levels and NCS Assessment Standards, the analyses of the teachers' written responses to the items and my understanding of teachers' geometric thinking and their understanding of their learners' geometric errors.

CHAPTER 4: DATA ANALYSIS

4.1 INTRODUCTION

In this chapter I present the analysis of the three phase qualitative study. First, I present the mapping of the geometry ICAS Grade 3-11 items to the van Hiele levels of geometric thought and the NCS Assessment Standards for the purposes of determining the geometric levels expected by the ICAS test items. Second, I present the analysis of the teachers' written responses for the purposes of understanding teachers' geometric thinking according to the van Hiele levels and to select suitable items for the interviews. Third, I present the analysis of the interview transcripts in order to explore further the geometric level of thinking of the teachers and how they understand learners' geometric errors.

4.2 MAPPING THE GRADE 3-11 GEOMETRY ICAS ITEMS TO THE VAN HIELE LEVELS

Using the Level Indicators for the first four van Hiele levels developed by Burger & Shaughnessy (1986), (see **Appendix B**), I categorised the ICAS items according to the four van Hiele levels as indicated in table 4.1 below:

Table 4.1: Summary of the mapping of the ICAS items and the van Hiele levels

Grade	No. of Items	VHL1	VHL2	VHL3	VHL4	*N/A
3	10	3	5			2
4	10	1	8			1
5	10	1	9			1
6	10		6	1		2
7	10		8	1		1
8	7		5	1		1
9	8		2	4	2	
10	8		2	4	2	
11	9		2	4	3	
TOTAL	**82**	**5**	**47**	**15**	**7**	**8**

***N/A indicates items that could not be mapped to the van Hiele levels.**

According to table 4.1 above, there are only five out of 82 ICAS items at van Hiele level 1(visualization), found only in the lower Grades 3-6 tests. At this **recognition/visual level**, the learner identifies names and compares geometric figures on the basis of their physical appearance as a whole, but does not recognise their properties (van Hiele, 1986). Looking at figure 4.1 below, the learners are required to use the physical features of the shape to find the one that looks exactly like it from the choice given. The learners do not have to know the properties of this shape, but only to identify it using only its outside appearance.

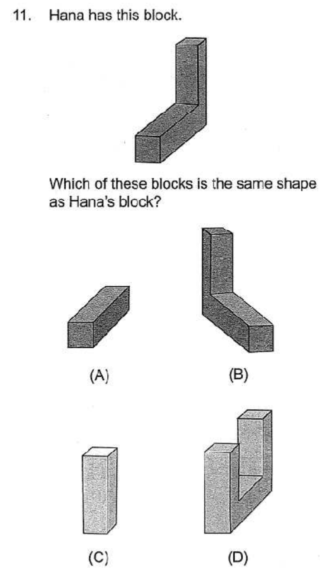


Figure 4.1 Recognition/Visual Level

Table 4.1 shows that there are forty seven ICAS items at van Hiele level 2, **analysis level**, where the learner analyses figures in terms of their properties, and uses the properties to solve problems. Van Hiele level 2 (analysis), is evident across all the Grades, from Grade 3-11. The example below asks of the learners to apply their knowledge of tessellations and edges in order to answer the question correctly.

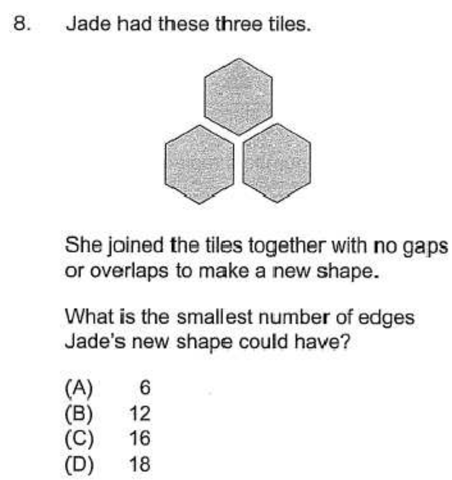
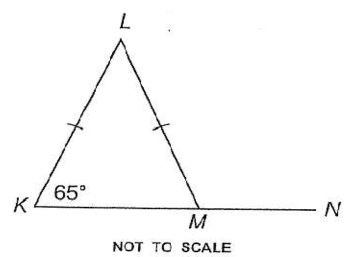


Figure 4.2 Analysis Level

I classified this question at van Hiele level 2 because it requires the learners to solve a problem in which properties of shapes are important components. So, the largest number of questions was asked at van Hiele level 2 across all the grades. Fifteen van Hiele level 3 (informal deduction) ICAS items start appearing from Grade 6 to 11 according to table 4.1. The example below, was classified at van Hiele level 3 because it requires the learners to calculate the value of angle LMN using the properties of a triangle in relation to the properties of angles on a straight line because at this level, learners are expected to establish the interrelationships of properties both within figures and among figures (van Hiele, 1986).

In the diagram, *KLM* is an isosceles triangle and *KN* is a straight line.



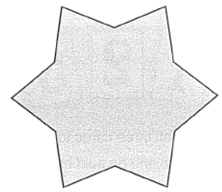
What is the value of $\angle LMN$?

(A) 135°
(B) 130°
(C) 115°
(D) 100°

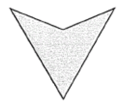
Figure 4.3 Informal deduction level

At this **informal deduction level**, the learners understand the relations within and between figures and also give informal deductive arguments, formulate and use definitions. An isosceles triangle is defined as a triangle which has two equal sides and equal base angles. When the learners know that, then they will know and deduce that angle LMK must also be 65˚. They will then have to formulate informal deductive arguments by applying their knowledge of angles on a straight line to be able to calculate that angle LMN is 180˚subtract 65˚which equals 115˚. Seven van Hiele level 4 (deduction) ICAS items appear from Grades 9 to 11. Looking at the example below, one can see that the question requires the learners to use the given information to make a conjecture and verify it.

39. This star shape has 6 lines of symmetry.



The shape can be cut into 6 identical quadrilaterals that look like this.



The smallest angle in the quadrilateral is 38°.

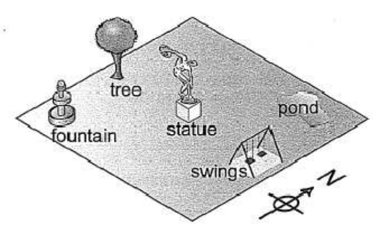
How many degrees is the largest angle in the quadrilateral?

(Write only the number on your answer sheet.)

Figure 4.4 Formal Deduction Level

I classified this item at the fourth van Hiele level (**formal deduction level)** where the learners have to recognise and flexibly use the components of an axiomatic system (undefined terms, definitions, postulates, theorems); create, compare, contrast different proofs and do not compare axiomatic systems (van Hiele, 1986). In Figure 4.4 above, the learners have to find the largest angle in the concave quadrilateral, making use of given information. Table 4.1 also reveals that not all the items could be classified according to the van Hiele levels. Eight ICAS items that could not be classified to the van Hiele levels are found in Grade 3 to 8. For instance, in Grade 6, item number 3 as shown below, learners are required to recognise direction from a plan on a map, cannot be accommodated by the van Hiele levels.

3. This map shows a statue in the middle of a park.



What is west of the statue?

(A) fountain
(B) swings
(C) pond
(D) tree

Figure 4.5 ICAS item that cannot be mapped to the van Hiele levels

This implies a limitation of the van Hiele theory as described because it does not accommodate issues of location and orientation, but rather focuses on shapes. The van Hiele levels were developed at a time before the introduction of direction and location as an important part of the geometry curricula worldwide and in South Africa. I will now compare the ICAS items with the assessment standards in greater detail looking at the level of thinking expected by the ICAS items.

4.3 VAN HIELE LEVELS AND THE ASSESSMENT STANDARDS

Learning Outcome 3 in the General Education and Training band (Grade R-9) states that "the learner is able to describe and represent the characteristics and relationships between 2-D shapes and 3-D objects in a variety of orientations and positions." (Department of Education, 2002:6). The focus of the study of shapes and objects in the Intermediate Phase (Grade 4-6) moves from the recognition and naming of shapes and objects in the Foundation Phase (Grade R-3); to the properties of these shapes as well as their classification. Foundation phase as well as the van Hiele level 1 (visualisation), are both characterised by the naming and visualising of shapes and objects in natural and cultural forms. Both van Hiele and the NCS assessment standards characterise this level by recognition of the shape as a whole. For example the Foundation Phase assessment standard 3.3.1 states: Recognises, identifies and names 2-D shapes and 3-D objects in the environment and in pictures including: boxes (prisms), balls (spheres) and cylinders; cones and pyramids; squares and rectangles; triangles and circles.

The Intermediate phase assessment standards require the analysis of figures in terms of their properties and their relationships. I found that this relates very well with van Hiele level 2 which requires the definition and identification of shapes and objects using their properties. Drawing of 2-D shapes and making models of 3-D objects is encouraged. For example, assessment standard 5.3.8 states: Makes 2-D shapes and 3-D objects and patterns from geometric shapes and describe these in terms of tessellations, line and rotational symmetry and movement, including: rotations, reflections and translations

At van Hiele level 3, informal deduction level, properties are logically used to give informal arguments such as drawing, interpreting, reducing and locating positions, similar to what the assessment standards in the senior phase (Grade 7-9), indicate. For example assessment standard 9.3.7 states that the learners should; use transformations, congruence, and similarity to investigate, describe and justify (alone and/or as a member of a group or team) properties of geometric shapes and solids, including tests for similarity and congruence of triangles. In the Senior Phase the shift is from using measurement as a basis for determining properties to the interrelationship between figures and solids and their properties (The Mathematics Learning Programme (2003: p. 40-41).

For example, assessment standard 9.3.2 requires the learners to describe , in contexts that include those that may be used to build awareness of social, cultural and environmental issues, the inter-relationships of the properties of geometric figures and solids with justification including; transformation, congruence and straight line geometry. Van Hiele further explains that at level 3 “the student can define a figure using minimum (sufficient) sets of properties; gives informal arguments, and discovers new properties by deduction; follows and can supply parts of a deductive argument; does not grasp the meaning of an axiomatic system, or see the interrelationships between networks of theorems.

With regard to the Further Education and Training band (Grade 10-12) the NCS states: “the learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.” (National Curriculum Statement Grades 10-12, (General) Mathematics (2003: p. 13). The focus is on location, visualisation and transformation. The learners engage with new tools that can be used in a range of applications, and they should become more proficient in the process leading to proof. Van Hiele (1986) contends that at deduction level, “the student recognises and flexibly uses the components of an axiomatic system (undefined terms, definitions, postulates, theorems); creates, compares, contrasts different proofs and does not compare axiomatic systems. The two link well with van Hiele level 4 where construction of proof, understanding the role of axioms and definitions, and the meaning of necessary and sufficient conditions are realised.

Focusing on geometric topics in the NCS, I found that the study of properties of shapes was a main focus in the ICAS tests. 47 out of 82 items focused on the study of shapes and objects and their properties. The importance of the teaching of properties of shapes and objects in primary school builds a solid and necessary foundation for the study of Euclidean Geometry in the FET Phase. It is also important to note that the level of engagement with the properties of shapes in the intermediate phase is a crucial foundation for congruence and similarity of triangles in the senior phase.

The teaching of transformation geometry is an important concept in the NCS across all the grades and is also evident in the ICAS items. I found that 27 of the 82 geometry ICAS items focused on rotations, reflections, translations and symmetry. The teaching and learning of transformation geometry in the GET Phase forms an important basis for the future study of transformation geometry in the FET Phase.

I also found that the teaching of location and position is prescribed in the GET phase is in line with the ICAS items which also had questions from Grade 3 to 8 (refer to Table 4.1) on location and position but as mentioned earlier, these could not be mapped to the van Hiele levels. Location and position in the GET phase forms the basis for the study and development of coordinate geometry.

The process of mapping all 82 ICAS items to the assessments standards and the van Hiele levels revealed the thinking levels expected by the ICAS tests. The majority of the test items were at van Hiele level 2, therefore accessible to the majority of the learners and the main content focus was on naming and applying properties of 2-D shapes and 3-D objects across all grades. To give the reader a glimpse of the process I undertook in mapping the ICAS items to the van Hiele levels and the assessment standards, take a look at Table 4.3 below: The first column represents the ICAS item numbers from the Grade 6 test. Followed by the description of the item, indicating the concept that the learners are being asked to respond to. The item is then mapped to the appropriate assessment standard and a reason is given as to why the particular assessment standard was allocated to the question. Lastly, the ICAS item is mapped to the relevant van Hiele level as described by Burger & Shaughnessy (1986).

Table 4.2 Mapping of ICAS Items, Assessment Standards and van Hiele levels

ICAS Item	Description	Assessment Standard	Reasoning	Van Hiele Level	Reasoning
2	Identify the missing shape to complete a pattern based on rotation	5.3.8 Makes 2D shapes and patterns from geometric shapes and describe in terms of movement; including rotations	Mapped back to Grade 5. Learners are required to identify a shape to complete a pattern	VHL2 (d)[5]	Application of a litany of necessary properties instead of determining sufficient properties
3	Recognise west from a plan in an unfamiliar orientation	6.3.11 Locates positions on a map	Learners are required to recognise direction from a plan on a map	N/A	Locates position
9	Select the correct image to complete a given picture	5.3.2 Describes, sorts, and compares 2D shapes from the environment and pictures according to properties including shape of faces	Learners are required to complete a given picture image	VHL2 (a)	Comparing shapes by means of properties
11	Identify the correct picture given the coordinates of it position	6.3.11 Locates positions on coordinate systems using horizontal and vertical change	Learners required to identify a picture given the coordinates of its position	N/A[6]	Locates position
15	Use symmetry to select the reflection which completes a	5.3.8 Makes 2D shapes and patterns from geometric shapes and describe these	Mapped back to Grade 5. Learners are required to select the reflection	VHL2 (d)	Application of a litany of necessary properties instead of determining sufficient

[5] Refers to the van Hiele classification by Burger & Shaughnessy (1986)
[6] Refers to ICAS items that cannot be classified to van Hiele levels

	picture	in terms of line symmetry and reflections	which completes a picture		properties...
27	Compare areas of a shaded pattern	6.3.4 (a) Investigates and compares 2D shapes by drawing shapes on a grid paper	Learners required to compare areas of shaded pattern on a grid	VHL2 (d)	Application of a litany of necessary properties instead of determining sufficient properties...
29	Select the shape which is exactly one-fifth of the given 5–pointed star	5.3.8 Makes 2D shapes from geometric shapes and describes these in terms of line symmetry	Learners are required to identify a shape from the pointed star	VHL2 (b)	Sorting by attributes of line symmetry
33	Identify the net of a given square pyramid	6.3.4 Investigates and compares 2D shapes by making use of nets provided	Learners are required to identify the net of a given square pyramid	VHL2 (a)	Comparing shapes by means of properties of their components
35	Use visual clues and measurement to select half of a given cylinder	6.3.1 Recognises, visualises and names 3D objects in geometric settings	Learners are required to use visual clues to select half of a given cylinder	VHL1	References to visual prototypes to characterise shapes
39	Find the number of diagonals in a given polygon	8.3.2 Describes and classifies geometric figures in terms of their properties, including sides, angles and diagonals	Learners are required to find the number of diagonals in a given polygon	VHL3 (e)	Ability to sort shapes according to a variety of mathematically precise attributes

In summary, I found that the ICAS items map very well with the NCS assessment standards and the van Hiele levels. I found that 47 of the 82 ICAS items focused on Euclidean Geometry, the study of geometrical objects and shapes in 2-dimensional space. Transformation Geometry, defined as the study of geometric objects that undergo translation, reflection, rotation and enlargement changes was covered by 27 of the 82 ICAS items. The remaining 8 ICAS items that could not be mapped to the van Hiele levels focused on location and position on labeled or coded grids, which form the basis for the study and development of geometrical objects and their relationships with each other and with other objects on a Cartesian coordinate system (Coordinate Geometry).

I also found that most of the ICAS items were at van Hiele level 2, which suggests that the ICAS items were accessible to the majority of the learners and also expected the learners to analyse properties of 2-D shapes and 3-D objects. The low achievement of the GDE learners in the ICAS tests despite the fact that most of the questions were at van Hiele level 2, I think is well explained by one of the teachers interviewed commenting on the inability of our learners to solve the ICAS items:

Teacher C: *"…what we do is use too much of a structured format to teach…er…shapes*
Researcher: *Structured format, what does that mean?*
Teacher C: *In terms of , …like in 3 sides (triangle) – 3 means tri, and then four sides(quadrilaterals), etc and then we draw a shape, …so, they don't have experience of placing those shapes in real life situations, so I think that is what is lacking in our teaching…*

In this section, I analysed and mapped the ICAS items to the assessment standards and van Hiele levels and found that there were similarities between the assessment standards and van Hiele levels. The NCS expects the learners to leave the intermediate phase at van Hiele level 2. I also found that the ICAS items mapped very well with the NCS assessment standards and the van Hiele levels, indicating that the NCS has content that satisfies international tests. What was expected of the learners by the ICAS tests was within their reach as provided by the NCS. I also found that the study of Euclidean geometry was the main focus in the ICAS items, followed by transformation geometry. This shows the importance of the primary school learners acquiring and developing the necessary geometric concepts in the study of properties of shapes and objects in preparation for high school Euclidean geometry. Lastly, I found that most of the questions were at van Hiele level 2, which suggests that the ICAS items were accessible to the majority of learners because at this level, learners are required to use properties of shapes to solve problems. This exercise also enabled me to select eighteen items I used to give to the teachers as written tasks.

4.4 WRITTEN TASK ANALYSIS

The written responses consisted of eighteen questions. I was restricted in my selection because research has discussed the van Hiele theory in terms of shapes. For the purpose of my study, I used 2 of those ICAS items that could not be classified to the van Hiele levels as they also form an important part of primary school geometry necessary for the development of coordinate geometry in high school. As mentioned earlier, the study of locations and position is found in the intermediate and senior phase because it is a prerequisite for understanding coordinate and transformation geometry in the FET phase.

I also felt that the teachers would manage to easily reveal their geometric thinking in line with the van Hiele theory whilst working mostly with shapes and their properties. I used two van Hiele level 1items, eight level 2 items, four level 3 items and two level 4 items and two items that could not be classified according to the van Hiele levels. Table 4.3 below outlines the ICAS items used as written responses with the ACE teachers. The second column shows which grade the question came from, e.g., Grade 3 ICAS number 15. The third column tells us the van Hiele level of the ICAS item and column four, the relevant assessment standard. I have also included the correct answer and the percentage of learners who got the answer correct as compared to the percentage of the 40 ACE teachers who got the answer correct.

Table 4.3: The selected ICAS items used as written responses and the teachers' results

ICAS Item	Grade and ICAS number	Van Hiele Levels	Assessment Standards	Percentage of Learners that got the items correct	Percentage of Teachers that got the items correct
1	Gr3 no.15	VHL1	3.3.1c	29%	30%
2	Gr3 no.11	VHL1	3.3.1a	77%	95%
3	Gr7 no.29	VHL2	5.3.8	15%	7.5%
5	Gr4 no.39 & Gr5 no.38	VHL2	5.3.8	Gr4: 20% & Gr5: 18%	20%
6	Gr7 no.28	VHL2	5.3.8	19%	15%
7	Gr5 no.8	VHL2	6.3.2	24%	80%
8	Gr7 no.19	VHL2	6.3.2	24%	25%
16	Gr7 no.8	VHL2	7.3.2	30%	32.5%
18	Gr7no.10	VHL2	7.3.7	33%	57.5%
9	Gr11 no.19	VHL3	9.3.7	10%	25%
11	Gr6 no.33	VHL3	6.3.4	22%	35%
12	Gr9 no.33 & Gr10 no.33	VHL3	8.3.6	Gr9: 21% & Gr10: 22%	17.5%
13	Gr11 no.2	VHL3	9.3.3	49%	80%
14	Gr9 no.13 & Gr10 no.13	VHL3	10.3.2	Gr9: 31% & Gr10: 33%	45%
15	Gr6 no.39	VHL3	8.3.2	0%	5%
17	Gr7 no.39	VHL3	6.3.4	0%	0%
4	Gr5 no.14	N/A	5.3.11	39%	75%
10	Gr6 no.3	N/A	6.3.11	35%	45%

I chose this selection of items because I wanted to find out how the 40 teachers would perform at different van Hiele levels. Most of the ICAS items I analysed were at van Hiele level 2, the minimum geometric thinking level expected at the intermediate phase. I also used a high number of van Hiele 3 items because I wanted to see whether the geometric thinking level of the teachers was at a higher level than that of the learners. Sixteen of the items were multiple choice type questions and two were free response to give teachers an opportunity to do the calculations that would possibly reveal their geometric thinking skills. The teachers were at liberty to use any method to solve the problems.

I started by analysing the scores of the GDE learners on the items chosen as written responses for the ACE teachers. I found that the learners achieved well only in ICAS item 2, a van Hiele level 1 question. The learners' scores were below 50% across all other items and higher van Hiele levels. The overall scores of the learners convinced me that they were operating between van Hiele level 1 and 2. I then analysed the written items of the 40 ACE teachers who participated in the study. These scores were recorded in **Appendix E** and Figure 4.1 below:

Figure 4.1 The achievement of ACE Teachers per ICAS item

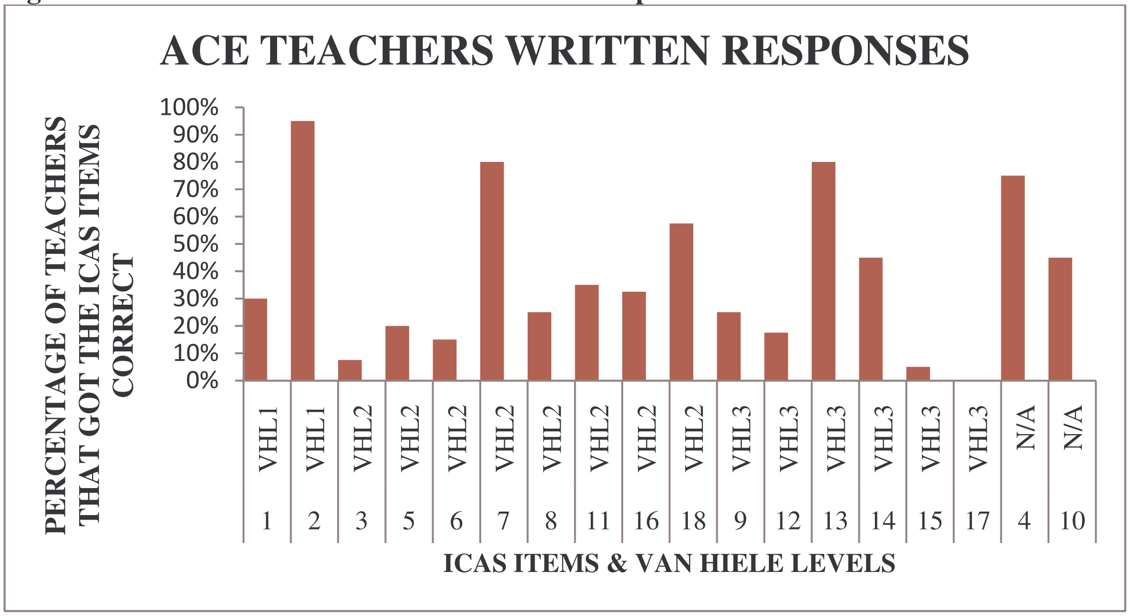


Figure 4.1 illustrates how many teachers got a particular ICAS item right. The graph shows the same achievement of the 40 ACE teachers as shown in table 4.3 above. For instance, the teachers also scored very highly on item 2, similarly to the GDE learners. The reason being that ICAS item 2 was a Van Hiele level 1 question that focused on visualisation of a 3-D object. I found that the two free response ICAS items 15 and 17 were the lowest achieved by both the learners and the teachers. Although the learners scored higher than the teachers in items 3, 6 and 12, Table 4.3 above also reveals that the achievement of the teachers was higher than that of the GDE learners in most of the written responses. The ACE teachers performed well in questions 2, 4, 7 and 13 according to graph 4.1 above. It was at this stage not easy to categorise the teachers into van Hiele levels, even though I could see that most of them could mostly answer questions at van Hiele levels 1and 2. I therefore decided to look into the specific errors the teachers made in the scores of the written responses in order to get a glimpse of their geometric thinking.

4.4.1 Analysis of teachers' scores in written responses

I first of all drew up a table that showed the 40 ACE teachers' choice of distractors for each of the multiple choice questions and then also analysed the free response answers given by the teachers as outlined in Table 4.4 below. The table below shows the ICAS items arranged according to the van Hiele levels 1, 2, 3 and the last two that could not be mapped to the van Hiele levels plus the percentage of teachers who made a choice between the four distractors, A, B, C, D or left the score sheet blank. The correct answer is highlighted and also indicated in a special column showing the percentage of the 40 ACE teachers who chose the correct answer.

Table 4.4 Itemised scores of ACE Teachers in the Written Responses

ICAS NO.	VH Levels	Teachers' Correct Scores	Correct Answers	A	B	C	D	Blank
1	VHL1	30%	D	12,5%	12,5%	40%	**30%**	5%
2	VHL1	95%	B	0%	**95%**	0%	2,5%	2,5%
3	VHL2	7,5%	A	**7,5%**	10%	42,5%	20%	20%
5	VHL2	20%	C	22,5%	35%	**20%**	15%	7,5 %
6	VHL2	15%	D	20%	10%	40%	**15%**	15%
7	VHL2	80%	B	15%	**80%**	2,5%	0%	2,5%
8	VHL2	25%	B	10%	**25%**	37,5%	17,5%	10%
11	VHL2	35%	D	12,5%	32,5%	20%	**35%**	-
16	VHL2	32,5%	B	15%	**32,5%**	22,5%	27,5%	2,5%
18	VHL2	57,5%	B	12,5%	**57,5%**	12,5%	7,5%	10%
9	VHL3	25%	C	25%	30%	**25%**	20%	-
12	VHL3	17,5%	D	22,5%	40%	20%	**17,5%**	-
13	VHL3	80%	C	2,5%	15%	**80%**	2,5%	-
14	VHL3	45%	C	27,5%	12,5%	**45%**	10%	5%
15	VHL3	5%	**90**	-	-	-	-	-
17	VHL3	0%	**224°**	-	-	-	-	-
4	N/A	75%	B	2,5%	**75%**	7,5%	5%	10%
10	N/A	45%	D	27,5	15%	10%	**45%**	2,5%

Looking at the general observations on the scores as revealed by Table 4.4 above, I found that van Hiele level 1 items content focus was on visualisation of 2-D shapes and 3-D shapes. Item 2 required the identification of a shape in a different orientation and was correctly answered by 95% of the teachers. We can confidently say the teachers are operating at van Hiele level 1 in ICAS item 2. I think the situation is a bit different for ICAS item 1 which required the teachers to identify 4-sided shapes within a context of a fish tank and was not well achieved at 30%. Of greatest concern is the 40% who chose the wrong distractor because they missed the concave quadrilateral (see Figure 4.6 page 51) and possibly mistook it for a triangle.

Does it mean that 70% of the teachers on item 2 are not operating at van Hiele level 1? This particular question might seem like an easy van Hiele level 1 question but the response of the teachers suggests that in order for them to get the correct answer, they had to go deeper and consider the properties of the shapes in order for them to realise that the shape was also a 4-sided shape and not a triangle as they possibly thought. The evidence suggests that in item 2 the ACE teachers are operating at van Hiele level 1 since research has shown that item 2 is in fact a van Hiele level 2 question as it requires proper engagement with the properties of a concave quadrilateral. The achievement of the teachers in ICAS item 1 and 2 ranged from 30% to 95% which makes it difficult to assign van Hiele level 1 to all the teachers because some got item 1 wrong and some got item 2 right while others got both items right.

I found that six of the eight van Hiele level 2 ICAS items focused on transformation geometry – tessellations, rotations, translations, reflections, lines of symmetry and this includes the most achieved item 7 at 80% from a Grade 5 paper and the least achieved item 3 at 7.5% from a Grade 7 paper. The two remaining van Hiele level 2 items, 8 and 16 focused on Euclidean geometry, specifically on 3-D objects and their nets, were achieved very low at 32.5% and 25% respectively by the teachers. The teachers achieved poorly at items 3 and 6 in van Hiele level 2. Only 7.5% of the teachers chose the correct answer in item 3, whereas the other 42.5% of the teachers chose the wrong distractor C, a parallelogram instead of an equilateral trapezium. Does this mean that 42.5% of the teachers have not engaged enough with tessellating with equilateral trapeziums?

The teachers' scores in van Hiele level 2 items range from 7.5% to 80%, this suggests that these teachers know the properties of shapes but cannot use the properties to solve problems, nor explain the interrelationships of properties between figures. I can therefore conclude that they are oscillating between van Hiele level 1 and 2, depending on the problem at hand.

All of the six van Hiele level 3 questions focused on various content knowledge in Euclidean geometry. Item 13, scored at 80% focused on the exterior angle of a triangle, and angles on a straight line from a Grade 11 paper. Item 14 scored at 45% focused on the interior angle of a hexagon in the context of trapeziums from the Grade 10 paper. Item 9 scored at 25% focused on congruency of triangles form a Grade 11 paper. Item 12 scored at 17.5% focused on 3-D object and nets from a Grade 10 paper. Item 15 scored at 5% focused on regular polygons, diagonals and geometric pattern from a Grade 6 paper. Item 17 scored at 0% focused on a star shape, lines of symmetry, and angles of a concave quadrilateral from a Grade 7 paper.

The achievement of the teachers at van Hiele level 3 ranged from 0% to 80% for items 17 and 13 respectively. In ICAS item 9, whilst 25% of the teachers chose the correct answer, the other 30% chose the wrong distractor B, showing a lack of content knowledge in congruency of triangles. It is quite evident that the teachers did not achieve well at van Hiele level 3 in five out of the six ICAS items. We therefore cannot confidently say that the teachers are able to solve problems by recognizing relationships between and among properties of shapes and also follow logical arguments using such properties. In this instance, the teachers are not operating at van Hiele level 3 because they scored very low marks (80%, 45%, 25%, 7.5%, 5%, and 0%), in five of the six van Hiele level 3 items.

The items that could not be mapped to the van Hiele levels were scored at 75% and 45% from the Grade 5 and 6 papers respectively. ICAS item 4 focused on finding position using a columns and rows whilst ICAS item 10 focused on finding position using a map. ICAS item 10 was not achieved correct by 55% of the teachers showing a possible lack of content knowledge in location and position.

I also found that a number of questions were not answered by the teachers who left the answer sheet blank. For example, in ICAS item 3 the teachers who answered correctly achieved very low at 7.5%, whilst 42.5% chose the wrong distractor, the other 20% did not supply an answer. This may show a lack of content knowledge of tessellations or a challenge of another kind, maybe language. After looking at the general observations as revealed by Table 4.4, I realised that I still could not confidently classify teachers according to the van Hiele levels that I decided to also look at the individual scores of the teachers on different items as captured in **Appendix E**.

According to **Appendix E**, only one teacher scored 4 of the 6 van Hiele level 3 ICAS items correct and I can safely classify this teacher at van Hiele level 3 because she seems to understand the relations within and between figures and give informal deductive arguments. The four teachers who scored 3 items correct I believe are oscillating between van Hiele level 2 and 3 because they only achieved half of the level 3 items correct and therefore cannot explain the interrelationships of properties between figures. I think the nineteen teachers who scored 2 items correct and the fourteen teachers who scored 1 item correct are operating at van Hiele level 2 and only get to operate at van Hiele level 3 depending on the question to be solved. The two teachers who did not achieve any item correct at van Hiele level 3, are definitely not operating at van Hiele level 3.

Teachers A and K in **Appendix E** are the only two teachers who achieved ICAS item 15 correct. This item required the teachers to recognise the relationship between the regular polygons and their number of diagonals given, in order to follow a logical pattern revealed in order to find the 90 diagonals of a 15-sided regular polygon without drawing it physically. Allow me to focus on the achievement of these two teachers in brief.

Teacher A, as mentioned earlier, was the only one who got ICAS items 12, 13, 14 and 15 at van Hiele level 3, ICAS items 3, 7, 8, and 18 at van Hiele 2 and ICAS items 1 and 2 at van Hiele level 1 correct.

Coupled with the two ICAS items that could not be mapped to the van Hiele levels, she achieved the highest mark at 12 out of 18 ICAS items. However, she did not manage to solve ICAS item 17 which required the use of lines of symmetry of a star shape and properties of a concave quadrilateral to find the biggest angle. It seems that Teacher A is the only teacher who has thus far displayed the ability to solve difficult problems that we can confidently classify her at van Hiele level 3. She would be a good candidate for interview to further probe her geometric thinking skills.

Teacher K managed to solve two van Hiele level 1 ICAS items, one van Hiele level 2 and two van Hiele level 3 problems. He achieved a total score of 7 out 18 ICAS items correct if we also include the two items that could not be mapped to the van Hiele levels. I find it interesting that Teacher K did not get ICAS item 7 at van Hiele level 2 correct as this item was scored correct by 80% of the teachers but he managed to score ICAS items 13 and 15 at van Hiele level 3 correct. ICAS item 15 was not a multiple choice question and required him to seriously apply his mind to the properties given of the regular polygons for him to calculate the 90 diagonals of a 15-sided regular polygon. It seems Teacher K makes it very difficult to classify him into a specific van Hiele level because his total score of 7 out of 18 means that he only managed to get 39% of the ICAS items correct. I think it would be safe to classify him at van Hiele level 1. What does this say about his competence at the various van Hiele levels, particularly level 3 where he managed to solve a difficult problem that other teachers who achieved three and two ICAS items at van Hiele level 3 correct could not solve and also what does it say about his geometric thinking skills.

The two teachers who did not manage to score even one item correct at van Hiele level 1, Teachers EE and GG according to **Appendix E**, surprisingly managed to score items correct at van Hiele levels 2 and 3. Teacher EE achieved ICAS items 7, 11, 18 at van Hiele level 2 and ICAS items 13 and 14 at van Hiele level 3 and one item that could not be mapped. Whereas Teacher GG scored correctly in ICAS items 7, 11 and 18 at van Hiele level 2 and ICAS items 12 and 14 at van Hiele level 3. Teachers EE and GG achieved total scores of 33% and 27% respectively, which suggests that they are both operating at van Hiele level 1. But if they both could not solve the two van Hiele level 1 items but each solved three van Hiele level 2 and two van Hiele level 3 problems respectively, what then does it say about the two teachers' geometric thinking skills and are they really competent at van Hiele level 1 only?

The findings revealed above indicate that it was not easy to classify the 40 ACE teachers who responded in writing to the ICAS items into van Hiele levels because even their low overall performance does not necessarily suggest that all of them are operating at van Hiele level 1. But in order to confirm my earlier decisions of classifying teachers at various van Hiele levels, I decided to analyse the errors of the teachers on specific ICAS items in the written responses.

4.4.2 Analysis of teachers' errors in selected ICAS items

I selected polygons that were promising to reveal some interesting thinking patterns of teachers as indicated by their choice of distractors. But before I analyse the items in detail, I will first outline how I chose the items to be used in the error analysis. It would also be interesting to think about how the teachers conceptualised the visual and verbal aspects of the ICAS items and also to think about the possible thinking patterns with regard to the items that were not successfully solved. Based on my analysis so far, I was particularly interested in ICAS items 1, 7, 13, 15 and 17 to analyse what was required to solve the problems, what the correct answers chosen and what the wrong distractors chosen reveal about the geometric thinking levels of the teachers.

Problem solving at any van Hiele level may be understood as an ability to formulate a mathematical problem, represent the problem and then solve it (Kilpatrick et al, 2001). In order to successfully solve the problem, one needs to understand what the question requires. This calls for one to conceptualise both the visual and verbal mathematical elements of the problem and discard unnecessary information and use appropriate procedures to solve the problem. I will now analyse specific items in detail and try to infer what the teachers might have been thinking when they chose particular distractors.

Table 4.4 reveals that in ICAS item 1 **(see Figure 4.6, page 51),** the teachers chose the following distractors: A 5 - (12,5%); B 6 - (12,5%), C 7 - (40%); D 8 - (30%); Blank (5%). The correct answer is 8. Naturally, this item was my first choice, because, as seen earlier, both the GDE learners and the 40 ACE teachers scored 29% and 30% respectively even though it looked like a very simple question asking for recognition of four sided shapes, a van Hiele level 1 question. It would also be interesting to find out why only 30% of the teachers chose the correct answer D when 70% of them chose the wrong options.

I was surprised by the fact that the teachers did not do well in this item below which required the teachers to say how many four sided shapes were found in the picture, in the context of a fish tank. All they needed to do find the correct answer was to simply count the number of 4 sided shapes in the fish tank. Only 30% of the teachers gave the correct answer, 8. It seems 40% of the teachers who chose C (answer 7) completely missed the concave quadrilateral because they are unfamiliar with it. Researchers also agree that learners limit concepts to studied exemplars and consider inessential but common features as essential to the concepts, (Burger & Shaughnessy, 1986; Fisher, 1978; Fuys et al., 1988; Kananova Meller, 1979; Zykova, 1969). These teachers are not used to engaging with the concave quadrilateral in their own teaching practice.

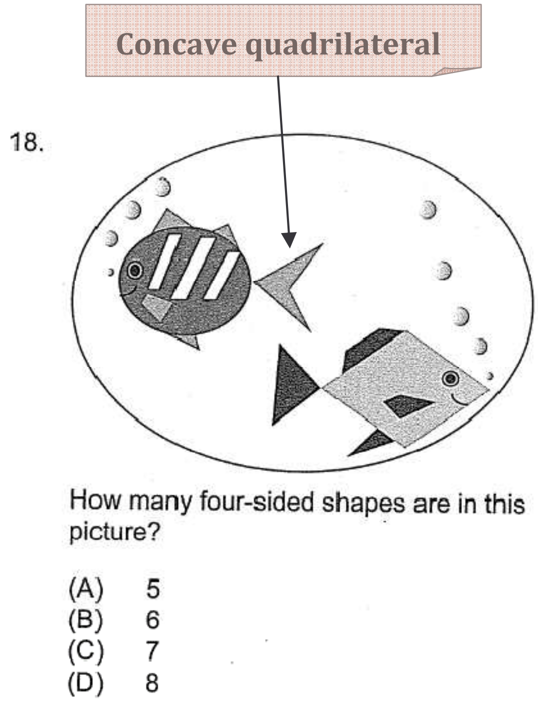
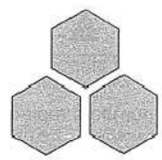


Figure 4.6 The elusive concave quadrilateral

They may have thought that it was a triangle or they may have only been attending to the visual aspects of the shapes, a level 1 response. They did not let the overall visual aspect of the figures fade into the background, so that they could attend to the shape' properties, a level 2 response (Clements & Battista 1992, p. 432) .This means that the question, although requiring mere recognition of quadrilaterals, also required application of the properties of a quadrilateral to be able to recognise it, especially as it is not a regular quadrilateral, but a concave one. The teachers who chose A, answer 5, probably did not see the concave quadrilateral, the trapezium inside the rhombus and the trapezium below the three rectangular shapes in the round headed fish. Those who chose B, the answer 6 might have missed the concave quadrilateral and one of the smaller trapeziums inside the rhombus and the round headed fish. 5% of the teachers left a blank space on this seemingly easy question originally from a Grade 3 paper. Researchers had claimed in their study that the concave quadrilateral required a van Hiele level 2 response; it therefore required me to classify the 30% of the teachers who answered correctly at van Hiele level 2. I decided to classify the 70% of the teachers who got the answer wrong at van Hiele level 1 because they did not let the visual aspect of the concave quadrilateral to fade into the background and attend to the its properties, a level 2 response (Clements & Battista, 1999).

Table 4.4 above again reveals that in ICAS item 7 (see Figure 4.7, page 52), the teachers chose the following options; A 6 - (15%); B 12 - (80%); C 16 - (2,5%); D 18 - (0%); Blank (2,5%). The correct answer is 12. It would be interesting to find out how teachers who chose the wrong options could have missed the integration of tessellations and counting of edges, a van Hiele level 2 question. Seeing that 80% of the teachers answered correctly I also wanted to find out how the other 20% of the teachers could manage to give their answers as A and C whilst the remaining teachers left the answer sheet blank.

8. Jade had these three tiles.



She joined the tiles together with no gaps or overlaps to make a new shape.

What is the smallest number of edges Jade's new shape could have?

(A) 6
(B) 12
(C) 16
(D) 18

Figure 4.7

80% of the teachers answered this question that comes from the Grade 5 paper correctly because they managed to put together the three tiles together in their heads leaving no gaps or overlaps (tessellation) and thus covering the inside edges for them to be able to count the 12 (B) correct number of edges. 15% of the teachers gave the answer as 6 (A). They could have been confused by the "smallest number of edges" and decided to just focus on the edges of one hexagon. This indicates a problem of language as all the ACE teachers use English as a language of instruction and first additional language. Issues of language are a serious impediment to most learners who first have to deal with the specific language of mathematics – "edges" and also comprehend what the question is asking for – the "smallest" number of edges. As mentioned earlier in chapter 2, van Hiele also agrees that language skills are particularly critical for creating and linking new ideas to past experiences. Language plays a central role in cognitive development and Vygotsky (1962) further argued that language was the main tool for determining the way a child learns how to think.

The teachers who gave the wrong answer 16, seemingly did not mentally alter the picture to apply the properties of tessellations, imagine the hexagons with no gaps or overlaps and count the outside edges, but also went ahead to miscalculate. In this case, the teachers could have been looking for the answer 18, because the answer 16 does not make sense. The teachers who gave the wrong answer 18 also did not mentally alter the picture and imagine the hexagons with no gaps and overlaps. That would have allowed them to count the edges on the outside only and not inside instead of going ahead counting all the sides of the three hexagons to come up with the wrong answer 18.

I claim that in ICAS item 7, 80% of the teachers who answered the item correctly are operating at van Hiele level 2. The other 20% who answered wrongly cannot use the properties of shapes to solve problems, thus are at van Hiele level 1.

After analysing the two ICAS items at van Hiele level 1 and 2, I needed more examples at van Hiele level 3 to see if the teachers could recognise relationships between and among properties of shapes and were able to follow logical arguments using such properties observed or given. I thought this would allow me to better classify teachers into van Hiele levels better. As shown earlier, the NCS requires teachers teaching in the intermediate phase to at least have van Hiele level 3 geometric thinking. I therefore analysed three more items at van Hiele level 3.

My third choice for error analysis in the written responses was ICAS item 13 (see Figure 4.8, page 54) scored by the teachers as follows: A (2,5%); B (15%); C (80%); D (2,4%); Blank (0%). This item promised to reveal the geometric thinking levels needed because it was answered correctly by 80% of the teachers as against 49% of the GDE learners. The wrong options A, B and D chosen by the teachers were promising to reveal misconceptions or lack of content knowledge of angles on a straight line, the sum of the interior angles of a triangle and the exterior angle of a triangle. The question was classified at van Hiele level 3 because it required problem solving in which properties of a triangle and the angles on a straight line are important components.

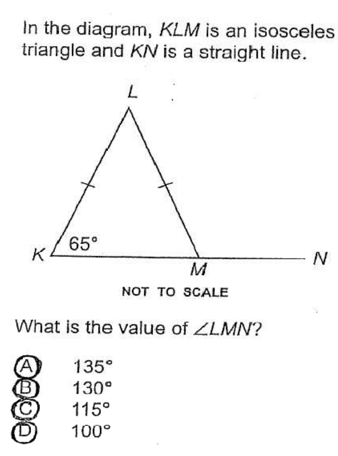


Figure 4.8

This particular question came from a Grade 11 paper. To be able to calculate the value of angle LMN, the teachers had to analyse the isosceles triangle and realise that the size of angle KML should also be 65˚. They could then calculate the angle on a straight line (180˚ - 65˚ = 115˚), or they could have added the two base angles and considered the sum of interior angles of a triangle (180˚ - (65˚ + 65˚) = 50˚. Now that all the angles of the triangle are known, then the two opposite angles are added together to give the exterior angle (65˚ + 50˚ = 115˚).

2.5% of the teachers chose option A (135˚). To get this answer, they could have assumed that the base angle of the isosceles triangle was 45˚ and on subtracting from 180˚, one would end up with 135˚. 15% chose option B (130˚). To get this answer, they could have incorrectly assumed that the angle at KML in the triangle was 50˚, so that when they subtract 50˚ from 180˚, the answer would be 130˚. This indicates that 15% of the teachers did not know the properties of an isosceles triangle, that if the sides of a triangle are equal, then the two base angles are also equal, therefore angle KML should also be 65˚. Only 2.5% chose the incorrect answer D (100˚). They could have incorrectly calculated that the exterior angle as 100˚ or used a protractor to measure the angle even though it was specified that the diagram was not drawn to scale. 80% of the teachers are at this ICAS item operating at van Hiele level 3 because they managed to solve this item correctly. The other 20% that failed to give the correct answer, are maybe operating in this question, at van Hiele level 1 because they do not seem to know the properties of a triangle, angles on a straight line and the outside angle of a triangle.

My fourth choice, ICAS item 15 (see Figure 4.9, page 55), classified at van Hiele level 3, was selected because the teachers and the GDE learners scored 5% and 0% respectively. Only two teachers answered correctly that a 15 sided shape had 90 diagonals. The 33 wrong answers given, ranged from 2 to 135 (2, 5,5, 7, 7, 8, 13, 15, 15, 15, 15, 15, 15, 15, 16, 16, 17, 18, 18, 21, 23, 30, 32, 36, 36, 44, 44, 45, 75, 85, 90, 90, 135) and 7 of the 40 teachers did not give an answer. As this question integrated regular polygons with algebraic patterns I was interested in finding out why the teachers could not find the number of diagonals of a regular 15 sided polygon using the information given. The question was not multiple choice and in error analysis I worked with the wrong answers given to find out how the teachers failed to use the properties given to investigate certain relationships among polygons to look for a pattern and find a general rule.

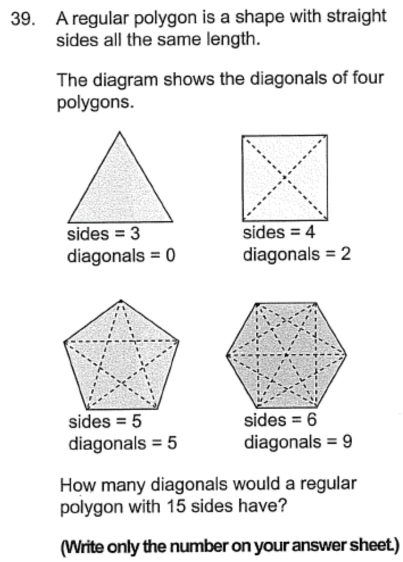


Figure 4.9

The correct answer for this free response question is 90 diagonals. The question instructed the teachers to only write down the answer, so the 33 teachers who gave their answers did not at all show how they found their answers. Of the 33 answers analysed, 31 of them were incorrect.
Only 2 out of the 40 ACE teachers got the correct answer by finding a pattern from the regular shapes given as shown below:

No. of sides	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**
No. of diagonals	0	2	5	9	14	20	27	35	90

I noted with interest that seven teachers gave the wrong answer, 15, assuming that if the question is asking for diagonals of a regular 15 sided figure, then the diagonals should also be 15. Vladimirskii (1971) has shown that sometimes the diagram accompanying a question is not always very helpful in mathematical reasoning because learners might mistake features of the geometric relationship being considered, thus introducing irrelevant ideas into the concept. I would say this was a misconception because the teachers simply assumed that since it was a 15 sided regular polygon, then it should have 15 diagonals without considering all the information given to enable them to solve the problem.

It is difficult to understand how one of the teachers could have come up with the answer 2. The teacher simply did not engage with all the information given in the question. Research has warned about mastering the skill of defining concepts to avoid errors in using terms that signify them, because at each van Hiele level, new vocabulary is experienced. Maybe the definition of a diagonal is not understood by these teachers because they are not at the required van Hiele level where engaging with diagonals would make sense.

As mentioned earlier, the two teachers who managed to solve this question also got ICAS item 13 correct, but it does not necessarily mean that they can both be classified at van Hiele level 3. Although Teacher K managed to solve ICAS item 15, a difficult van Hiele level 3 question that was not a multiple choice, his total score was only 7 out of 18 ICAS items. I think Teacher K might possibly be oscillating between van Hiele levels 2 and 3 depending on the difficulty of the problem at hand. The other 38 teachers, who failed to get the correct answer, are in this question also not operating at van Hiele level 3. I believe that the error analysis in this item reveals that the teachers have knowledge gaps in their geometric thinking.

My fifth choice, ICAS item 17 (see Figure 4.10, page 57), also classified at van Hiele level 3, was selected for error analysis because both the teachers and the GDE learners could not solve the problem and I had to find out why this was so. Looking at the scripts, I noticed that the teachers gave the following wrong answers; 322˚, 284˚, 246˚, 228˚, 228˚, 208˚, 208˚, 180˚, 180˚, 154˚, 152˚, 148˚, 142˚, 142˚, 122˚, 122˚,120˚, 110˚, 110˚, 104˚, 104˚, 100˚, 90˚, 90˚, 90˚, 76˚, 76˚, 74˚, 72˚, 45˚, 45˚, 38˚, 38˚, and 7 out of the 40 teachers did not give an answer. In the error analysis I used the wrong answers given by the teachers to see how they failed to establish a relationship between the quadrilateral, the star shape with its 6 lines of symmetry and the smallest angle given to determine the largest angle in the quadrilateral.

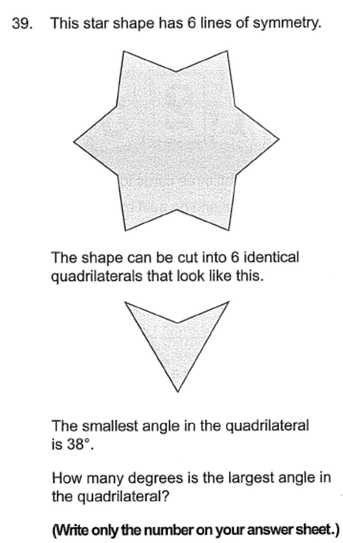


Figure 4.10

None of the teachers managed to get the correct answer, 224˚. In order to get the correct answer, they would have to consider all information given. A star shape has 6 lines of symmetry. The shape is then cut into six identical, quadrilaterals in a form of a concave quadrilateral. They are also told that the smallest angle is 38˚ and asked to calculate the biggest angle in the quadrilateral. The six lines of symmetry clearly signify that the six pointed shapes each measure 60˚ in order to make up 360˚, given by (60˚ x 6 = 360˚). Then the biggest angle will be given by; 360˚ - (38˚ + 38˚ + 60˚) = 224˚. In order to find the biggest angle in the quadrilateral, they would have to consider the 6 lines of symmetry, and deduce that the pointed angle in the star is 60˚ and also use the smallest angle given as 38˚ for them to find the correct answer.

The teachers, who got the wrong answers, had different reasons and clearly did not consider all the given information to be able to come up with the correct answer. Looking at a few examples, it seems that the teachers who got 208˚ as the answer thought that the biggest angle was given by:
360˚ - (38˚ +38˚+ 76˚) = 208˚. They did not consider the lines of symmetry that rendered the angle at the outer end of the star to be 60˚ and just assumed that since the smallest angle given was 38˚, the other angle would naturally be twice the smallest angle (38˚ +38˚ = 76˚), a misconception one might expect from learners but not teachers. Therefore the lines of symmetry bisected the angle into two 38˚ angles and thus dividing the quadrilateral into two similar angles. The two triangles would add up to 180˚ each and enable them to reach the answer of 208˚.

The assumption that the since the smallest angle is 38˚, then the other angle would also be 38˚ also indicates a misconception that further assumes that the quadrilateral is divided into two similar triangles, revealing knowledge gaps in what properties qualifies similar triangles. The teachers, who got 246˚ as the answer, just assumed that all the three angles of the quadrilateral were given as 38˚, also indicating a misconception in assuming that all the smaller angles of the quadrilateral should necessarily be 38˚. A same kind of response one would expect from the learners who never consider the properties but would assume that since the smallest angle was given, then all the angles that "look" small, must be 38˚; 360˚ - (38˚ +38˚+ 38˚) = 246˚.

The teachers who gave the answer as 284˚ did not consider the third angle in the quadrilateral, revealing a lack of knowledge and application of properties of a quadrilateral; 360˚ - (38˚ +38˚) = 284˚. Usiskin (1982) found, that "students enter high school not knowing enough geometry to succeed and that and only about a third of these students understand it. The teachers who got the answer 322˚ did not even realise that they needed all three angles to find the fourth unknown angle in the quadrilateral, also revealing a lack of knowledge and application of properties of a quadrilateral; 360˚ - 38˚ = 322˚. The explanation is simple; challenges lie with the quality of teaching and learning in the development of spatial reasoning by the teachers.

In summary, the findings of the error analysis revealed that there are gaps in the level of thinking of teachers in primary school and yet the future of success of the secondary geometry curriculum depends on Primary school geometry. In ICAS item 1, 30% of the teachers, classified at van Hiele level 2, could deal with irregular quadrilaterals and the other 70%, classified at van Hiele level 1, who could not identify the concave quadrilateral showed a lack of knowledge of dealing with irregular quadrilaterals.

In ICAS item 7, 80% of the teachers who answered correctly were classified at van Hiele level 2 whereas the other 20%, who cannot use the properties of tessellation of hexagons to solve a geometric problem, were classified at van Hiele level 1. This has implications for the learners they teach. In ICAS item 13, 80% of the teachers were classified at van Hiele level 3 and the other 20% that failed to give the correct answer, were classified at van Hiele level 1 because they recognised the triangle but did not seem to know the properties of a triangle, angles on a straight line and the application of the outside angle of a triangle.

In ICAS item 15, only 5% of the teachers could be classified at van Hiele level 3, whereas the other 95%, who failed to get the correct answer, were classified at van Hiele level 1. The 95% of the teachers revealed a lack of content knowledge in using a geometric pattern given to find an algebraic number pattern.

In ICAS item 17, 0% of the teachers required to solve a geometric problem to find an unknown angle in a quadrilateral could be classified at van Hiele level 3. The teachers failed to use the properties of lines of symmetry to find the outer angle as 60˚ and further assumed that since it was given that the smallest angle is 38˚, then all the other angles that looked small would also be 38˚, showing a misconception of angles. The teachers in this question were all classified at van Hiele level 1 because they failed to solve the item using all the necessary information given.

After analysing the ACE teachers' errors of selected ICAS items, I think I was justified earlier in classifying Teacher A at van Hiele level 3 and the rest of the ACE teachers at van Hiele level 1 and 2 because the error analysis has revealed that the teachers display a lack of content knowledge of basic geometric concepts. Sixteen of the ICAS items used in the written responses were multiple choice and I cannot rule out guess work especially in the case of Teachers EE and GG whose total score was 6 and 5 out of 18 items respectively and could not solve van Hiele level 1 items but managed to solve items at van Hiele level 2 and 3 correct as shown above.

The only teacher I had a problem categorising into van Hiele levels was Teacher K, who managed to solve ICAS item 15, a difficult van Hiele level 3 question that was not a multiple choice but his total score was only 7 out of 18 ICAS items. I think that Teacher K could be the kind of person who does not do well in multiple choice type questions and could possibly be oscillating between van Hiele levels 2 and 3 depending difficulty of the problem at hand.

Even after the error analysis, I still realised that the written responses were clearly not enough to categorise the teachers into van Hiele levels, because learners will oscillate between van Hiele levels depending on the nature of the tasks and instruction, and the teachers' role is very central in helping learners move levels. So, the next phase of the study was to interview some of the teachers in order to find their geometric thinking as they explained how they solved the problems and also by observing them analyse the GDE learners' errors. The findings of the error analysis encouraged me to target the same five ICAS items to use in the interviews in order to see if the teachers being interviewed would also display the same misconceptions and a lack of content knowledge of basic geometric concepts.

4.5. INTERVIEW ANALYSIS

Of the six teachers interviewed, only one of them (Teacher A) actually responded to the eighteen ICAS items and as mentioned, scored 12 items correct out of 18 items which gave her an average of 67% in the written responses. Teacher A has already been classified at van Hiele level 3. The other five teachers from the ACE teachers, who volunteered to be interviewed, focused only on the items used specifically for interviews. I gave them the opportunity to respond to the five items before the interview. I did not mark their scripts and also did not focus on what answer they had come up with. I allowed them to explain their strategies used to solve the problems.

As outlined in chapter 3, the items chosen for interviews were at different van Hiele levels. The teachers' geometric thinking interview schedule consisted of six questions whilst the interview schedule for understanding learners' geometric errors' also had six questions. The two sets of interview questions were used in the same interview. The teachers' interview schedule focused on the teachers' reasoning associated with van Hiele levels. The teachers referred back to their written papers and talked through how they solved the problem, how they would use the item to teach with, at which Grade and the purpose of the lesson.

The learners' interview schedule focused on whether and how the teachers thought that their own learners' responses would be different from that of the learners, what strategies the learners would use to solve the ICAS items and also what they thought the underlying problems would be if the learners came up with incorrect responses and how they would go about assisting their own learners' incorrect geometric thinking. The two categories revealed by the interviews are teachers' geometric thinking and teachers' understanding of learners' errors.

4.5.1 Teachers' geometric thinking

(a) ICAS item 1

All the six interviewed teachers said at first that they found ICAS item 1 (see Figure 4.6 on page 56) to be quite easy as it only required the recognition of four sided shapes, but only Teacher A, B, C and E correctly identified 8 quadrilaterals whilst Teacher D and E identified 7 quadrilaterals and totally missed the concave quadrilateral thinking it was a triangle.

Teacher D: *"For me it was much easy because I just looked at the shapes and counted all those that are four sided. The answer is 7…"*

Teacher F: *"Recognise, identify and basically, …, to recognise 4-sided shapes."*
…this triangle given and this other bottom fish, made me think it's not 4-sides, but actually, it's 4 sided, you see…

It was interesting that the teachers thought that the problem could be taught to Grade 4 and 5 learners. But we have seen earlier that ICAS item 1 was categorised at van Hiele level 1 and in Grade 3 as per the NCS assessment standards. Teacher A thought that teaching this kind of question using the fish tank context would be fun for learners. All the teachers agreed to the fact that learners should be taught and encouraged to practically and physically count the number of sides without just observing mentally. They also suggested that teachers should use a more integrated way of teaching shapes so that learners can relate what they learn in mathematics with real life.

Teacher C: *Ja, I should think it's more challenging, what we do is we use too much of a structured format to teach,…er,…shapes. In terms of , …like in 3 sides – triangle, 3 means tri, and then four sides – quad, five, …and then we draw a shape, …so, they don't have experience of placing those shapes in real life situations so I think that that is what is lacking in our teaching , in real life…*

The teachers warned against teaching shapes (quadrilaterals) in isolation but suggested that from a young age learners should be taught to combine shapes.

Teacher F: "Then now maybe you need to combine 2 shapes together so that children can know that you have already dealt with them and now … they can identify them amongst others…"

It was only teacher A and C who described interesting lessons of which ICAS item 1 might be part of possibly because of the passion that they have in working with shapes. It was also clear that they do try out innovative ways of teaching shapes to their learners.

Teacher A: *"At Grade 3, I would first let them build their own picture and talk to one another about their pictures and then bring it here-it could be about animals-it could be a day at amphibian or reptile farm "*
At Grade 6, not exactly the same, I would change it-I would go for say name the different quadrilateral shapes that you see- use the same picture, but change the question-or I would say how many different rectangles do you see-how many different parallelograms do you see- erm -for Grade 7 I would go for your convex and concave angles.."

Teacher C*: "so by the 3rd lesson, I gave them a task – a group assignment where they had to use different shapes to create a theme, ..erm - going sailing and ..erm - ..so.. the requirement of the assessment was to use all the types of shapes and some of them made a boat using triangles for the sail,…they created a sea that is 4-sided,…that kind of thing, …so they created a picture, …not like this…using shapes of their own in different colour paper and come out and place it together and some of them I said built a school and so they created a school structure and then you could …the rectangles, the squares, the circles, triangles were used…It helps them (learners) to contextualize…the …understanding of shapes and to see the applicability of shapes in real life*

ICAS item 1, as mentioned asked for the number of four sided shapes in the context of a fish tank completely misled many teachers and learners into thinking that it's an easy question but the results tell us a different story. The concave quadrilateral again proved to be a serious problem. Of the six teachers interviewed, only four managed to solve the problem correctly at first and the other two missed the concave quadrilateral. For example, Teacher F remarked: "this other bottom fish, made me think it's not 4-sides, but actually, it's 4 sided, you see…". He was not focusing on the whole shape and so missed the fact that it is a quadrilateral and not a triangle. Research has shown that, he needed to "let the overall visual aspect of the figures fade into the background, attending instead to the shape' properties, a level 2 response (Clements & Battista, 1992).

(b) ICAS item 7

The teachers found the question very easy as they all readily provided the correct answer as 12.

Teacher D*: "I just closed the gaps that appears here and count the number of …er…edges…that are on the outside of the new shape*

The teachers identified the problem of mathematics vocabulary as a key factor in developing concepts in geometry. In the NCS, in the study of 2D shapes we use the term sides as opposed to the international arena, where the outer part of 2D shapes is referred to as the edge.

Teacher D: *"…that…in 2D shapes we refer to them as sides and when we're talking 3D objects, we refer to them as edges…so that when they come across this question they encounter the problem of edge…*

All the teachers knew the concept of tessellation well and it was clear that they also do teach it to their learners. They mentioned that tessellations have to be taught using regular shapes and those issues of leaving no gaps and no overlap were very key.

Teacher E: *"er- tessellations, I suppose is another concept that you could use there – I mean tessellations – you teach them – there's no gaps there – you know there are spaces in between in tessellations, so these shapes are moving directly into each other – so you know if you went out there would be no gaps and you could tessellate it*

It was also evident that the teachers do not only advocate for learners doing mathematics using practical methods but teacher E strongly suggested that learners in high school should also be taught using manipulatives and hands on activities before we can expect learners to internalize and make sense of the abstract the concepts.

Teacher E*: …manipulatives and also uhm …but – they've got to start playing around with it …Yeah, hands on activities, I think for children up to Grade 12 still have to use manipulate or else they're gonna battle especially Geometry they're gonna battle with it*

(c) ICAS item 13

The teachers could easily find the correct answer 115˚ and could identify the concepts needed to solve the question as angles on a straight line, isosceles triangle and teachers A, B and C even referred to the theorem of the exterior angle of a triangle.

Teacher B: *"Yes to Grade 7- this goes immediately over to Grade 8 as well where you do the outside angle of a triangle*

Teacher C*: "But I also used the double check on the answer by saying that 180˚subtract the addition of the two known that will say 130˚ leaves us with 50˚ to give 180˚ because I wanted to apply that rule of the exterior angle…*

Only teacher F could not solve the problem.

Teacher F: *"angazi ukuthi ngi zo iyenza kanjani lena… (I don't know how to do this one…in Zulu)*
Okay, right in fact to me the simplest way will be to start with a straight line and then you are done with this side is 65, this side is 65, these are equal…The sides are equal and this is 65˚ - 65˚ subtract from 180, which is 105, the answer is 105, yeah, sure

It was interesting to find that having been exposed to the van Hiele levels in the Hope project, teacher A and B could classify the questions into van Hiele levels.

Teacher A: *"Van Hiele level -2 application of knowledge*

The teachers placed the question at Grade 6 or 7 when it originally came out of a Grade 11 paper, but I mapped it back to Grade 9 in the NCS assessment standards. Teacher E mentioned that ICAS item 13 integrated learning outcome 3(shape and space) and learning outcome 4 (measurement) and also pointed to the fact that the two learning outcomes come together in the FET phase.

Teacher E: *"Uhm these are…angles – which disappear in LO3 still when it comes to measuring of angles, LO4, at high school LO4 actually disappears – we don't ask them to measure – We just have four LO's – but it's definitely a mixture of 3 and 4 in this case for up to Grade 6*

It came out very strongly again that the teachers believe that the best way to teach the learners is by allowing them to practically measure, draw, using appropriate instruments as a way of investigating the key concepts in the study of geometry.

Teacher B: "*I would give this to a child and let them cut these two angles up and join them together…fit them…fold. I've done this with my learners where they actually had to measure and from there go onto the properties-so-… you see that it is investigation once again-…because once this side and that side are equal those two angles will be the same now …they measure it physically-I had an exercise where they measured physically measure with a ruler, measure with a protractor to…"*

(d) ICAS item 15

The teachers agreed that the question was challenging and only teachers A, B and C managed to get 90 diagonals as the correct answer because they realised that there was a pattern being formed by the given regular shapes. Teacher D was very surprised that the answer could be 90 and teacher F remarked that it needed high level of thinking.

Teacher A: "*I saw there was a pattern. There was no way I was going to draw 15 sides and diagonals, so I worked with a pattern 1st one for 3 sides, zero diagonals 4 sides +2 diagonals gave you the 2. Then it was 2 + 3, 3 + 4 and the next one 0+2+3+4+5+6+7+8+9 its nine up to 10,11,12,13 and I just added up*

As much as Teacher A placed the question at Grade 7, Teacher C said she would only use the question as an extended activity for her learners. But Teacher E would only use it if it was modified to include a table and clearly indicate to the learners that there is a pattern to be worked out.

***Teacher E**: "Uhm…yeah, I don't think I'd use the shapes as such em, unless I can put it in a form of a table like this…I think I would teach it together – rather than try to work out what is gonna come next – rather teach them patterns that would be forming – when the pattern starts and also the diagonals…Look, it's a difficult one to teach I think – when it comes to this – many teachers try to stay clear of this because in their own minds they're thinking…*

The development of problem solving skills was raised by teacher C who indicated that the biggest challenge was the way that teachers teach their learners in a procedural manner focusing on rules and regulations and little is done in developing investigative and problem solving skills of learners.

***Teacher C:** "The pedagogy is not placed on problem solving – it's based on content and rules and regulations. Application on…when I reflect – we don't teach problem solving…*

(e) ICAS item 17

The teachers agreed that this problem was challenging and none of them got the right answer, 224˚. Teachers A, B, and E after calculating thought the correct answer was 208˚, whilst teacher D and F did not know where to begin. Teacher C after calculating ended up with 254˚.

***Teacher B:** "Interior angles are 180 degrees… But you can also see that it is an isosceles triangle which makes the other angle 38 degrees and together 38+38=76 if you go now to the other angle of the triangle that I have cut 38 and 38 is 76, subtract that from 180 which is 104 and now I've cut that angle in half…so if that is 104+104 then the answer is 208…I like working with triangles*
To look at it another way-you can also see that it makes sense to say if that is 104 and104 reflex

***Teacher C:** "I think both of these angles are 38. The properties of a quad, there are 4 sides, 4 angles and the angles add up to 360˚. Yeah, I tried to figure the centre of the centre point of the star, because we said the diagonals and then I looked at what the angles around the centre point, how many they are and then I discovered that there is 12. I said 360˚ divided by 12 is 30˚, so then this angle here had to be 30˚, and these I assumed – and I think I'm correct, they have to be equal because this is a line of symmetry. So then, these are equal – so if that is to be 38˚, this is 38˚…that is what, 30˚ 38˚ + 38˚ is 76˚ + 30˚ is 106˚ - that's what the 3 angles add up to and the final one has to be 360˚ - 106˚, which gives me 254˚*

Teacher E thought this was an exciting question and not necessarily difficult for the learners

***Teacher E**: "For me I did not find these questions difficult – but erm, we are thinking of a child erm, they see it with very different eyes…Uhm, the quadrilateral again is 360˚and the way I teach my children is that if you have a quadrilateral, like this – shaped like this – If I draw a line from the base there to there…I'm gonna have two triangles – I know that a triangle has 180˚ each If I extend this line here, going down here, and I have 2 triangles – I know that a triangle has 180˚ in it –and so if I've got 2 – then it means its 360˚*

You see that's how you can teach them about a quadrilateral – it's 360˚. So just by extending that line down there – you have 180˚ and 180. You have 38˚ there ok, and this is a regular shape so this is 38, which is 76, so this angle here is also 76 'cause it's 38 there and 38 there – so as I said there are various ways of going about doing this – but if it's 38 - 38 and this is gonna be 76 it's gonna be 104 – 104 plus 104 – because it's that angle plus that angle is

208.But the thing is that they see one shape and they don't learn to break it down – it's very nice shape. I love concave quadrilateral

Teacher E raised the issue of important geometric skills that learners need to be taught – breaking down complicated shapes into smaller manageable pieces in order to can easily solve them; practically working with shapes using tangrams to explore and analyse their properties and to expose them to irregular quadrilaterals like the concave quadrilateral. The teachers interviewed also confirmed its difficulty when two of the teachers did not recognise it as a quadrilateral and also suggested that it had to be taught to Grade 4 or 5 learners, even though it came from a Grade 3 paper. I have classified Teachers A, B, C, E at van Hiele level 2 in this question and Teachers D and F at van Hiele level 1 since they could only identify the regular quadrilaterals.

In summary, the overall geometric thinking level of the interviewed teachers appeared much better than that of the teachers in the written responses. This is due to the fact the interviewed teachers were all high achievers in the ACE programme as revealed by their profiles in Chapter 3. For instance, earlier I could not confidently classify the teachers into van Hiele levels using the written responses because the teachers' performance was not consistent as seen from the example of teachers EE and GG who failed to solve van Hiele level 1 items but went on to solve van Hiele 2 and 3 items.

It does not seem like the interviews also make it easier to classify the teachers into van Hiele levels because the teachers operate at different van Hiele levels even on one question. But I think I can safely say that Teachers A, B and E are operating at van Hiele level 3 as they displayed geometric thinking skills above that of Teachers D and F who I believe are operating at van Hiele level 2 because they struggled to articulate their geometric content knowledge.

The interviews confirmed my earlier classification of Teacher A at van Hiele level 3 due to the fact that she had been the only teacher in the written responses who achieved the highest marks. In the interviews, she confidently talked about the lessons that she would design, and also expressed her passion for geometry. She showed extensive knowledge in the manner that she solved the items. She was very certain that she had found the correct solution for ICAS item 17 that she neglected to use the important information given by the six lines of symmetry. She showed a misconception in assuming that if the smallest angle was 38˚, then the other angle that bigger would be twice 38˚, and therefore 76˚. This is the same kind of misconception readily found in learners despite the fact that was involved in the HOPE project and could even classify the items into van Hiele levels.

Teacher B managed to solve all the items except the last one. He expressed himself well and easily related to the geometric concepts because of his involvement in the HOPE project. He could even classify the ICAS items into van Hiele levels. His understanding of geometric concepts was also good in that Teacher A did not silence him with all her passion and love for geometry. He is also operating at van Hiele level 3.

Teacher C was also very good in articulating geometric concepts that I was impressed when she revealed high content knowledge by solving ICAS item 13 using the theorem of the outside angle;"the outside angle of a triangle is equal to the sum of the angles". She completely took over the task-based interview. Teacher D would not say anything unless I directly asked her a question. I can easily classify Teacher C at van Hiele level 3 because even though she also missed the significance of information given such as the lines of symmetry and could not solve ICAS item 17, she did however manage to articulate her thoughts around the problem using the angle at the centre.

Teacher D was mostly quiet during the interview with Teacher C. She failed to solve item 15 and when she revealed that she would never have found the 90 diagonals, I was surprised to notice that she was not at all thinking about finding a pattern. But she easily solved item 13 using the sum of the interior angles of a triangle and the angles on a straight line. She did not attempt to solve item 17. She can be classified at van Hiele level 2 because she knows the shapes and their properties but could not use the properties to find the unknown angle, showing a lack of skills to apply learnt knowledge to solve problems.

Teacher E also showed high content knowledge and managed to solve all items except ICAS item 17. He described useful teaching strategies that should be taught to learners to deal with complicated shapes such as breaking down the shape into manageable smaller pieces. He failed to consider all the information given thus revealing the same kind of errors found in the learners. He is also a van Hiele level 3 teachers as he revealed a lot of knowledge higher than that needed in the intermediate phase.

Teacher F could not solve items 1, 13, 15 and 17 at the first instance and only after seriously thinking about the problems and considering all the information given did he manage to solve items 1 and 13. He also described good strategies that learners could be taught to solve geometric problems such as good reading skills and using a pen to underline difficult words. He is operating between van Hiele level 1and 2 depending on the difficulty of the problem at hand.

The van Hiele levels do not necessarily tell us more about the teaching skills of the teachers but the interviews revealed some of their views on teaching and the impact it has on their learners. Research has shown that when teachers are exposed to van Hiele levels, they become more aware of the geometric content knowledge they teach and also manage to pitch the content at the correct van Hiele level for the learners as Feza and Webb (2005) suggested that teachers needed to engage in activities that require them to classify learners' answers by van Hiele levels. It was very encouraging to see that Teachers A and B, having been exposed to the van Hiele levels in their HOPE project could easily classify the ICAS items into appropriate van Hiele levels. It was interesting though that they both did not reveal that they had ever used the van Hiele levels to categorise their learners or in assessment tasks used in their classrooms to deal with learners of diverse ability.

These teachers' views give useful hints that we need to keep in mind as we now get into the teachers' understanding of GDE learners' errors. The teachers are shown the same ICAS items they were interviewed with but this time with the actual scores of the GDE learners so that they could tell if they thought that their own learners could achieve the same or better results. They should also think about how their own learners could have solved the problems and also explain what skills the learners would have gained by solving the problems. The teachers were also required to explain what they would say were the underlying difficulties of the learners who gave incorrect responses and further explain how they would go about assisting the learners' incorrect geometric thinking.

4.5.2 Teachers' understanding of learners' errors

In order to answer my third research question, the same five ICAS items used to analyse the teachers' errors in the written responses were used to interview the teachers with to further explore their understanding of the learners' errors in the ICAS tests. The teachers were requested to look at the actual performance of the learners as displayed in the table below and to explain the learners' errors or misconceptions so as to reveal their own geometric thinking levels.

Table 4.5.2.1 below shows the achievement of the learners at specific grade levels and the different distractors chosen. I have also included the actual answers of the learners so that we can easily see their errors. The correct answer is shaded. ICAS items 15 and 17 were not multiple choice questions and as indicated, none of the learners scored them correct. ICAS item 1 was scored correct by 29% of the Grade 3 learners. ICAS item 7 was scored by 24% of the Grade 5 learners and ICAS item 13 was scored correct by 49% of Grade 11 learners.

Table 4.5.2.1 GDE learners' achievement in the ICAS test

ICAS item	Original Paper	Correct Answer	A	B	C	D	BLANK
1	Grade 3	(D) 29%	26%	17%	21%	29%	4%
7	Grade 5	(B) 24%	43%	24%	10%	16%	3%
13	Grade 11	(C) 49%	15%	28%	49%	6%	1%
15	Grade 6	90 diagonals	0% of the learners scored item correct				14%
17	Grade 7	224°	0% of the learners scored item correct				12%

I started off by showing the teachers being interviewed the scores of the learners who wrote the ICAS tests in the year 2006 as indicated in the table above. It was surprising how when they saw the actual scores of the learners they suddenly changed their perception of their own learners to realistically speak about the problems hampering problem solving skills in their learners.

(a) ICAS item 1

The table above shows that ICAS item 1 was scored very low by the Grade 3 learners. Only 29% found the correct number of quadrilaterals. 26% chose option A, 17% option B and 21% chose option C and 4% had completely refrained from answering this question by leaving the answer sheet blank:
The teachers seemed to agree that their own learners would not easily solve the problem and also mentioned that their Grade 4 learners would not cope with the item and only their Grade 6 learners would even try to solve it. This was a complete turnaround now that they had seen the actual scores of the learners. Teacher C who had earlier said that this was an easy question that her Grade 4 learners could easily solve, suddenly changed and said that her Grade 4 learners would never manage to solve it.

***Teacher C**: "Yeah, from the classes that I teach, I can tell that they won't …I think at least 30% …at Grade 6 level…not Grade 4. Nooo…The Grade 4 learner is never going to do this …there's the odd chance that the Grade 4 learner can be able to do this*

It emerged very clearly that the biggest problem was again the concave quadrilateral. Learners have not been exposed to this irregular quadrilateral.

***Teacher E:** "the one that that got 7 – possibly did not recognise that (concave) as a quadrilateral – that's why this % is very high, they didn't see the concave*

The teachers indicated that learners needed analytical skills to be able to identify and recognise the four sided shapes. It seems that teachers are in agreement with Clements and Battista (1992) who say that this concave quadrilateral question is not a mere recognition van Hiele level 1 question but an analysis van Hiele level 2 question.

Teacher F: *"skills of recognition, maybe identifying and also analysing…very analytic and critical."*

The underlying problems of the learners' errors were identified as a lack of exposure to problem solving strategies, lack of observation skills, lack of language proficiency and reading skills (English). Learners also develop misconceptions in geometry due to the manner in which concepts are first taught to them because teachers tend to teach in the similar way that they were taught by their own teachers in school – in a rigid very structured way where shapes are dealt with in a separate isolated manner.

Teacher A*: "…maybe they were not taught strategies to answer these type of questions and then a big problem-a reading problem- if they answered 7-there is a misconception… that 21% because they don't see this as a four sided shape. Yeah they think it's a triangle or a triangle that is not closed at the bottom because we don't teach this normally to Grade 3s - A lot of the teachers is also stagnant-we teach the ones that we have been taught*
Ja - that's what is happened-but if you move to your triangles – er… children say an equilateral is a triangle but a right angled triangle is half triangle…that has been cut in the center –if you put the "other half then it's a triangle…Because only an equilateral is a triangle if you look at your Tupperware Toys and things all the triangles used are equilaterals. So, … that is what a child perceives as a triangle and nothing else.

So, basically, the problem is that we teach them right sided rectangles quadrilaterals. They don't see strange quadrilaterals and think that it as a quadrilateral and it's also strange that teachers also don't see this as a quadrilateral .But the teachers were not taught that this is a quadrilateral. We very often tend to teach them a square is a quadrilateral because in the NCS document it says: quadrilaterals such as… because some teachers of the phase where they don't specialize in mathematics

Teacher C*: "It does – because it kind of streamlines their thinking – it makes them think that, okay,…I'll have to look on the page and …er…and then I'll see an isolated 4-sided shape*
This is not misconception – just a lack of experience for that kind of thing – it's not a misconception

But it was interesting to see that teacher E disagreed that learners who did not recognise the concave quadrilateral because they have never been exposed to it are not suffering from a misconception, but just lack of experience that leads to learners guessing the wrong answer.

In assisting the learners' incorrect geometric thinking, the teachers suggested that practical work was necessary for learners to actually do the mathematics using concrete material and exploring different kinds of irregular shapes and also to allow learners to create their own shapes.

Teacher A: *"I would actually ask them to draw over, you know, I would cut these little ask them to take crayons, pencil crayons, anything…Koki pens and to draw on the lines and tell me how many lines they see or to touch the finger on the outside of the edge one edge because you have to use the correct terminology even if they are small…angle, for them I would say from corner to corner go with your finger count 1,2,3,4, to quadrilateral ones…umm…not just the shape of a tail of fish but I would let them try and create their own quadrilaterals…strange ones and also again…on um… chalkboards, paper, cut it out pass it on using different materials to built the* *quadrilaterals and then I would do this in groups and perhaps let one group swop with another group to see how many sides I would not even stop with quadrilaterals, I would then let them build strange shapes and let them count how many sides*

In summary, the teachers agreed that the learners missed the concave quadrilateral because they had a misconception and could only think of the shape as a triangle and not a four sided shape". They also pointed to the fact that the problem might be the way that teachers teach shapes to their learners, in a very rigid, structured way with no room for applying shapes in context and also the manner in which the learners are socialized into shapes. De Villiers (1987) agrees with that and argues that we need to use geometry software like geometer sketchpad so that learners can experience different orientations of shapes and also to realise that shapes are not static.

I also found that the teachers seemed to know the curriculum well in terms of learning outcomes and assessment standards, but Teacher A identified the lack of content specification of the NCS assessment standards as a stumbling block to teaching geometry in innovative ways because teachers "…*very often tend to teach learners that only a square or rectangle is a quadrilateral because in the NCS document it says: quadrilaterals such as…square, rectangle, etc*". Teachers end up teaching their learners only a square and a rectangle and the learners never being exposed to irregular quadrilaterals; simply because they are not mentioned in the assessment standards. A fact supported by De Villiers (1987) that the van Hiele levels are very dependent on the curriculum.

The teachers agreed that learners need to be taught geometry using manipulatives since it has been proven that learners learn best through discovery and experiencing the geometry first hand. For instance, the concave quadrilateral can be cut out from cardboard and used by learners to play with so that they can learn its properties. They also realised the importance of teaching learners observation skills by exposing them to real life question type using the application of properties of shapes learnt in geometry lessons.

(b) ICAS item 7

Only 24% of the GDE learners scored the correct answer, 12. The majority of the learners in this question (43%) chose option A (6 edges), 10% of the learners thought that the edges were 16 by choosing option C (16 edges), whilst 16% thought that the answer was option D (18 edges) and 3% of the learners left the answer sheet blank. The teachers seemed to agree that learners needed to understand concepts of tessellations (no gaps or overlap) and really understand the question (reading skills) in order to get the correct answer.

Teacher E: *"Okay, if it is given in this form – if they correctly answered 12 then I would say –er- okay, in this child' mind – he or she could put together 3 shapes that are the same and they could make it into a new shape and you have to understand that you have to count all the edges that are outside of the new shape, so if they get that right then I know that they can piece together in their own mind…the 3 pieces – they're integrating basically there's 3 shapes into one and they're saying now I recognise that there are now 12 sides on this new shape going around so I would know that they can successfully do that…*

With regard to the problems learners encountered, the teachers thought that the learners committed stupid mistakes by not focusing on what the question was asking for.

Teacher C: *"You see that I'm analysing this… the one that chose D as the answer, didn't read the joining of the gaps, that's the only way you can come up with 18. If you ignore the initial instruction…*
If you look at those that answered 6, they just took 1, 2, 3, 4, 5, 6, of an individual – they didn't consider the others, can you see that , because they possibly thought that it's the same thing – you can't add them over and over again – the same thing and 43% of them did that…
So, they possibly thought – well, if the one is like that and then they are all the same – when you join them… it's still that…I'm just trying to analyse …and 16 I'm not sure how they came to that answer 16 … (calculating)…Yes, they just guessed

The teachers also agreed that the learners had problems with language that lead to the misconception that made them to choose options A and D, but option C was just a miscalculation or guessing on the part of the learners.

Teacher E: *"smallest number of a new shape – they must understand that …of a new shape – the 3 being joined together, but if you look at the shape, it's er 6 if you count the edges – so I can understand why 43% of them –er- cause the moment they see smallest number of edges, then they're looking 1, 2, 3, 4, 5, 6…*
yeah, definitely that is a misconception and also language thing, I think you know, what would have been helpful – if they had this here and the new shape next to it with a new joined shape…
I don't know how you get 16….(pause)… laugh…I don't know how you get 16…it's just 10% - I'm sure they guessed

The teachers suggested the following strategies to help the learners; reading to them focusing on difficult words and encouraging them to underline key words and also allowing learners to explore the concept of tessellation using concrete materials to avoid misconceptions.

In summary, I found that the teachers do teach the concept of tessellation to their learners, although issues of language proved to be a stumbling block in allowing the learners to achieve the correct answer. I found that they do teach tessellations but I gathered that they teach it alone or separate from other concepts in mathematics not in the manner that this question integrates tessellation and geometric patterns. The teachers also agreed that learners might have been misled by "edge" and "smallest" that led to 43% of GDE learners giving the answer 6 instead of 12. Thus, teachers saw the need to focus on teaching learners geometric vocabulary from an early age to avoid misconceptions.

With regard to the "edge" it was revealed that in other parts of the world, they refer to length of a side as "edge length" but in our South African context, in 2D we refer to "sides". The teachers suggested that the relationship between 2D and 3D should always be emphasised so that the learners can differentiate between sides and edges. They also agreed that the learners had a misconception of what tessellation is hence the majority of the learners could not get the correct answer. I also found that the teachers do not teach tessellations using problem solving.

(c) ICAS item 13

49% of the GDE learners managed to get the correct answer, 115˚. Option B (130˚). was chosen by 28% Option A (135˚) was chosen by 15% . Option D (100˚) was chosen by only 6% and a mere 1% did not bother answering the question. The teachers agreed that their learners would easily manage to solve ICAS item 13. But teacher C contradicted herself when she said that only a few of her Grade 6 learners would be able to solve this question.

Teacher C: *"Not in Grade 6 level – I would not get a higher percentage, maybe I'll get 3 or 6 learners out of my class. You must remember even in Grade 6, you need a lot of repetition around isosceles for them to understand fully and remember that 2 angles are equal and 2 sides are equal – it's difficult… for some learners*

They all agreed that the learners need knowledge of properties of isosceles triangle in order to can successfully solve the problem.

Teacher C*: "And also manipulation – you've gotta look in geometry you look at erm, different things that will provide an answer to some questions … and the possibilities of using that, to determine that, also knowledge of properties so that you can exploit the information available*

In outlining the learner strategies used by the learners to get to the answers, the teachers agreed that guesswork, miscalculation and definite misconception played a role.

Teacher A: *"… Guess work …65-can't be 135-also a straight line …100-guess work, 135-miscalculation, 130-misconception*

Teacher F: *okay, what they did – they were able to identify the two bottom angles of an isosceles triangles and just added this, I think 65 + 65 make 130…they recognised that this bottom angles are equal – 65 and this one is also 65 and they just added. It's not a misconception as such – they didn't critically look at the question such – they just thought – okay, it's 65 yes! I know the other side must be 65 – we are done – they didn't go back and read carefully which angle is exactly required by the question*

Teacher F: *"but basically they must understand that a straight line has got 180 – of course, using a protractor to measure a straight line, you'll see how you get 180 – now coming to the question as it is – it has specified that not drawn to scale – meaning that they must use the properties of angles on a straight line, properties of an Isosceles triangle*

In summary, it was very clear from the discussions with the teachers that they do teach their learners concepts of the sum of the interior angles of a triangle, base angles of an isosceles triangle, angles on a straight line and teachers C and E displayed knowledge of the exterior angle of a triangle. In the written responses the teachers achieved 80% in this question and in the interviews only teacher F could not solve the problem.

The teachers could identify the learners' errors and misconceptions with regard to the isosceles triangle and angles on a straight line. Again issues of using hands on strategies and allowing learners to explore and investigate the concept of angles on a straight line came out very strongly from the teachers. They suggested that the level of geometry teaching at the intermediate phase might be the problem in learners having misconceptions and the lack of teaching and learning geometry using manipulatives**.** Thus, they suggested that learners needed to develop geometric skills through drawing, measuring using appropriate tools, estimating and actually being given opportunities of solving problems to be able to grasp important concepts.

(d) ICAS item 15

None of the GDE learners could solve this problem and 14% of the GDE learners did not even attempt to answer it. The teachers all agreed that this question was too challenging for the learners and were amazed that it came out of a Grade 6 paper. Teacher C actually thought it came out of a Grade 11 paper.

Teacher C*: "This is a Grade 10-12 question. When you came up with 90 – it was like I thought…this is too much… and then you started doubting your answer so it's basically even more time consuming to check so where is it Grade 11 level?...They could have provided a formula or data sheet – so you can apply a formula – this is a sequences and series formula*

Teacher E: *"I don't think any of my children would…*

The teachers agreed that their learners would not manage to solve the problem but could be used as an extended activity for a few bright learners.

Teacher C: *"It will be a bit much – confusing – so you can't in actual fact, you're wrong in thinking that I'd never do this with my learners – you can't underestimate them. There may be one child in the class that can… so as an extended opportunity… it will take another 3weeks to explain to the rest of the children – time wasted….*

The important concepts learned from this problem were summed up by Teacher E well when he said that the concept of regular polygons, coupled with patterns is one of the most important aspects and ultimate skills in the teaching and learning of mathematics.

Teacher E*: "Yeah, to work with patterns as well…the fact that it's a regular polygon - it's not all 4 or 3 sided shapes that will have this – only the regular polygon and I think they have to understand the concept of regular polygons all the edges are the same*

To me, they can recognise patterns which is so important in Geometry – and not only do they have – do they recognise the pattern – but they can also follow a logical conclusion as to what 15 would then be – in terms of this particular thing – so, they can correctly computate the answer for me

On the matter of learner errors, Teacher A remarked that the learners were inclined to just guess the answer because the question requested an answer only without encouraging them to show their calculations, thus promoting guess work. The time issue was also brought up by the teachers that this problem would require a lot of time for the learners to successfully attempt to solve it.

Teacher A: "*maybe because the question asked them to write only the answer, they wrote whatever number. came to their mind…if they found the pattern, then they would make calculation errors… Gr6-not surprised they did not do well. The reading was not that much my learners would basically just thumb suck they found it too difficult…your brighter child would have thought , …I'll do this later because its time consuming…*

Teachers A, B and E suggested that because learners of today are visual, the 15 regular polygon could have been drawn for them because they simply cannot visualise it and maybe a table could have been provided to assist the learners work out the pattern from there and not just assume that they would work out a pattern all by themselves…maybe the question was asking for too much from Grade 6 learners.

Teacher E: *"Tables, I find are the easiest ways in which to do this erm…we would have – depends on what we're gonna be looking at erm…Drawing a table I would say these too should be in this question – I would have asked this question – you would say record these in the table below…and then you would have the 15 sided shape and say what would the…*

In summary, none of the GDE learners got the answer right and only 2 of the 40 teachers (5%) managed to solve the problem in the written responses. This question required one to work with the given information to come up with a pattern that would help solve the problem of diagonals of a 15 sided polygon. I got different reactions from the teachers but they all agreed that it was too challenging. It integrated concepts of regular shapes with patterns. The teachers thought that strategies of actually giving the Grade 6 learners the 15 sided regular polygon and a table indicating to them how working out a pattern would have helped since learners of today are visual and cannot successfully use their imagination. The teachers all agreed that learners needed to learn geometry using hands on activities.

On the learners' geometric errors, the teachers could not comment much as they did not have access to the learners' work, the question required an answer only. Teacher E mentioned the importance of learners solving such a problem to the learning and teaching of geometry and patterns and summed it thus:

Teacher E*: "To me, they can recognise patterns which is so important in Geometry – and not only do they have – to recognise the pattern – but they can also follow a logical conclusion…"*

Teacher A commented that because the learners were required to write the answer only, it gave them an opportunity to guess and come up with any answer. It was also interesting that she said that even if they could have come up with the pattern, they would have made wrong calculations, which shows a lack of confidence in their learners being able to solve this problem.

(e) ICAS item 17

None of the GDE learners gave the correct answer, 224˚ and 12% of the learners did not even attempt the question. This ICAS item required the integration of lines of symmetry, with properties of a quadrilateral and the sum of angles of the interior angle of a quadrilateral. The teachers found the question challenging and difficult to solve. The concave quadrilateral as in the earlier question posed a problem for both the teachers and the learners that they did not know what to do with it or go about solving the problem.

They suggested that the easiest way of helping learners would be to teach them to break the diagram given into two triangles and work out the biggest angle from there. It was interesting to note that teachers A, B and E arrived at the same answer (208˚) but also interesting to see that all three of them totally missed the significance of the lines of symmetry mentioned in the question because that is where they would have found the other angle in the quadrilateral, 60˚.

Teacher A decided to cut the concave quadrilateral into triangles and so did teachers B and E in order to find the solution whereas teacher C tried a different method of using the angle at the centre and came with a different solution (254˚) which was also wrong. I did not go into whether the answer was correct or not because I was interested in their thinking patterns. Teachers A, B and E used the same method of the sum of the angles of a quadrilateral and the given smaller angle 38˚. The other teachers D and F did not even attempt to solve the problem it was too difficult for them. The teachers all agreed that doing hands-on practical work, using play with the learners was very key at all stages of developing geometric skills in the learners.

In summary, the teachers agreed that this was a difficult question that combined concepts of lines of symmetry with the calculation of the biggest angle in a quadrilateral. The teachers did not have any learners' data for them to analyse but the teachers agreed that the teaching and learning of properties of shapes including lines of symmetry should be taught using the correct vocabulary, hands-on activities or manipulatives and also through play. Issues of reading skills and utilising all the information given were highlighted as key problem areas. Teaching strategies of allowing learners to explore shapes using hands-on activities and using investigations were mentioned by the teachers.

But, I found that the teachers do not as readily as they give answers, understand their learners' geometric errors. They could not differentiate between a simple error or a misconception, which may be why they cannot diagnose their own learners' errors easily. It was only Teachers A, B and C who managed to solve ICAS item 15, a van Hiele level 3 question. The other three teachers could not figure out exactly what needed to be done to get the correct answer. The teachers also agreed that none of their own learners would manage to solve it as was the case with the learners in the ICAS test written by Gauteng learners. It was rather encouraging to see that teacher A immediately saw that there was a pattern in ICAS item 15.

In conclusion, I analysed the data I gathered from document analysis, written responses and interviews with the teachers. I found that the ICAS items map very well with the NCS assessment standards and the van Hiele levels. I also found that most of the questions expected the learners to analyse properties of 2D and 3D objects (Euclidean and Transformation geometry).

In trying to answer the question; what is the level of geometric thinking of selected teachers, I found that the teachers' geometric thinking was lower than it was required at the level of the primary school as per the requirements of the curriculum which is in line with the van Hiele levels. This suggests that these teachers do not have the necessary geometry knowledge to teach at the primary school level. This finding was supported by Mayberry (1983) that pre-service teachers had inadequate reasoning levels in seven geometric concepts that were squares, right triangles, isosceles triangles, circles, parallel lines, similarity and congruence.

The achievement of the teachers who responded in writing to the eighteen ICAS items ranged from 5 to 12 out of 18 ICAS items. This enabled me to classify them into van Hiele levels with teacher A achieving the highest mark. After interviewing her, I realised that she was mostly operating at van Hiele level 3. Teachers B, C and E, after interviewing them I found that they displayed van Hiele level 3 in some items and van Hiele level 2 in some items, a fact that Burger & Shaughnessy (1986), agree with that they also found students oscillating between levels, sometimes even on the same question.

I also found that teaching experience and the love of geometry as a mathematical subject, was an important factor in the teachers' geometric thinking levels where teachers with more experience teaching coupled with the quality of their initial training as in the case of Teachers A, B, C and E who expressed more knowledge of the concepts in the higher van Hiele levels.

I found that the teachers do not as readily as they give answers, understand their learners' geometric errors. They could not differentiate between a simple error or a misconception, which may be why they cannot diagnose their own learners' errors easily. The teachers who taught more of geometry like teacher A, B, C and E could easily articulate the concepts needed and what the learners were lacking but the main factor that came up was the style of teaching, and the level of engagement of the teachers in their own professional development in mathematics teaching and learning.

I found teacher A and B complementing one another in the interview because of their work in the Hope project where they work as facilitators, helping other teachers, I believe, has given them experience of teaching of geometry in a very practical and meaningful way. It was also interesting to note that teacher A and B knew the van Hiele levels well and even classified the last two questions used in the interviews at van Hiele level 3 and 4

***Teacher A**:* *definitely -analysis-synthesis, between 3 and 4*
***Teacher B**:* *agree*
Researcher: *Van Hiele level on ICAS item 5 of the interviews)*
Teacher B: *level 3 and 4*

The second item used for interviews was classified at van Hiele level 2 or 3 because it required application of properties of tessellations and 2D shapes.

Teacher A: *Van Hiele level -2 application knowledge, maybe 3-apply knowledge*

It was also revealed that teachers do not teach the topics that they themselves struggle with, for example, diagonals, number patterns and solving problems using the application of properties of geometric concepts. A lot of the problems pointed to the management of teachers as a resource in the schools, where school principals did not realise the special kind of knowledge required by a mathematics teacher and that not just anybody can teach the primary school geometry. Mention was also made of the teachers' initial training which has a lot of impact on their teaching of learners as the teachers tend to teach the learners the same way that they were initially taught.

***Teacher A**:* *"I find that teachers are not teaching patterns very well…patterns are taught the way that they (teachers) were taught 2 4 6 8 10…clap clap…that's when they really go overboard…maybe apple pear … If you look at how we teachers were taught - chalk and talk*

I agree with teacher A who remarked that the most important thing is to have the love for what you do so that you can get the best out of it.

Teacher A: *I like geometry…when you like something you teach it well*

4.6 CONCLUSION

In this chapter, I have analysed the data gathered from document analysis that revealed that the ICAS items mapped well with the NCS assessment standards and the van Hiele levels. The majority of the ICAS items were at van Hiele level 2, therefore accessible to the majority of the learners. The written responses revealed that Teacher A could be classified at van Hiele level 3 and the rest of the teachers at van Hiele level 2. The written responses also revealed the low content level of the teachers which was confirmed by the error analysis of teacher errors in selected ICAS items. The error analysis also revealed teachers' misconceptions in geometric concepts that were same as that of the learners.

The interviews helped me to confidently classify Teacher A at van Hiele level 3. Teacher B, C and E were mostly operating at van Hiele level 3 but also at van Hiele level 2 at some instances depending on the difficulty of the problem at hand. Teachers D and F were operating at van Hiele level 2, below the required level of the NCS and the intermediate phase assessment standards. The written responses and the interviews revealed that both the learners and the teachers displayed a lack of key geometric content knowledge and problem solving skills.

In the next chapter, I discuss the findings of the study as well as implications and suggested recommendations for teacher development in geometry.

CHAPTER 5: CONCLUSION

5.1 DISCUSSION OF FINDINGS

This study has explored the geometric understanding of Grade 6 teachers according to the van Hiele levels looking at the following research questions:

1. What level of thinking do the geometry items in the ICAS tests expect?
2. What is the level of geometric thinking of selected Grade 6 teachers?
3. In what ways do Grade 6 teachers understand their learners' geometric errors?

The geometric understanding of Grade 6 teachers according to the van Hiele levels was explored. Although van Hiele focuses on classifying figures as topological or Euclidean, research has shown that the van Hiele levels are also useful in describing students' geometric concept development (Burger & Shaughnessy, 1986; Fuys et al,., 1988, Han, 1986; Hoffer, 1983; Wirszup, 1976).

The mapping of the 82 geometry ICAS items to the (NCS) Assessments Standards and the van Hiele levels revealed that 59% of the 82 geometry ICAS items were at van Hiele level 2 across all grades and therefore accessible to the majority of the learners and yet the achievement of the learners who wrote the test in geometry ranged from 34% in Grade 3 to 26% in Grade 11. Other research has also found that most secondary students who studied geometry formally, were on levels 0 to 2, not level 3 or 4, and almost 40% finish high school geometry below level 2 (Burger and Shaughnessy, 1986; Suydam, 1985; Usiskin, 1982).

I found that the ICAS items map very well with the NCS assessment standards and the van Hiele levels. The Intermediate phase assessment standards require the analysis of figures in terms of their properties and their relationships. This relates very well with the van Hiele level 2 which requires the definition and identification of shapes and objects using their properties. Drawing of 2-D shapes and making models of 3-D objects is encouraged. The study shows that there are clear similarities between the foundation phase and the intermediate phase content focus and the van Hiele levels 1 and 2.

I found that the study of properties of shapes was a main focus in the ICAS items, 45 out of 82 items focused on the study of shapes and objects and their properties. The study has eluded to the importance of empirical teaching of properties of shapes and objects in the primary school that build a solid and necessary foundation for the study of Euclidean geometry in the FET phase.

The teaching of transformation geometry is an important concept in the NCS and I also found that transformation geometry was evident in the ICAS tests because I found that 28 of the 82 geometry ICAS items I mapped to the assessment standards focused on transformations – rotations, reflections, translations and symmetry. It is important to note that the level of engagement with the properties of shapes in the intermediate phase is a crucial foundation for congruence and similarity of triangles in the senior phase. The teaching and learning of transformation geometry in the intermediate phase forms an important basis for the future study of transformation geometry in the FET phase.

The focus of geometric topics in the NCS reveals that the teaching of location and position is prescribed at the GET phase. This is in line with the ICAS items which also had questions from Grade 3 to 8 which focused on orientation location and position but as mentioned earlier, but these could not be mapped to the van Hiele levels. This implies a limitation of the van Hiele theory of geometric thought as described which does not accommodate issues of location and orientation, but rather focuses on shapes. The van Hiele levels were developed at a time before the introduction of direction and location as an important part of the geometry curricula worldwide and in South Africa.

The teachers who I interviewed alluded to the fact that their learners struggled with language, an important tool to conceptual understanding. Van Hiele (1986) believes that language skills are particularly critical for creating and linking new ideas to past experiences and prior knowledge. A fact Vygotsky (1962) agrees with that language plays a central role in cognitive development of the learners. He argued that language was the main tool for determining the way a child learns how to think, because complex concepts are conveyed to the child through words. The teachers agreed that their learners needed to be taught from an early age using the correct geometric vocabulary and terminology so as to enhance concept development in geometry.

I also found that the van Hiele levels do not necessarily tell us anything about the teaching skills of the teachers whereas the interviews revealed some of the strategies teachers use in teaching geometry in the primary school. The study revealed that the teachers were failing to systematically develop the necessary geometric concepts in their learners mainly due to the way that they teach their learners; in a procedural manner focusing on rules and regulations. The rigid way of teaching also fails in developing the investigative and problem solving skills of learners. Teacher C explained it well when she said that; "*pedagogy is not placed on problem solving – it's based on content and rules and regulations … when I reflect – we don't teach problem solving…"*

In order to address the low level of achievement for teachers as revealed by the analysis of the written responses, teaching strategies need to change. Teachers in the interviews agreed with researchers that manipulatives are an essential aid in learning geometry (Fuys et al., 1988). The use of manipulatives allows learners to try out their ideas, examine and reflect on them, and modify them. The physical approach helps the learners in creating definitions and new conjectures, and to help them in gaining insight into new relationships within or among shapes.

The mapping of the 82 geometry ICAS items to the (NCS) Assessments Standards and the van Hiele levels had earlier revealed that 59% of the 82 geometry ICAS items were at van Hiele level 2 across all grades and therefore accessible to the majority of the learners and yet the achievement of the learners in geometry ranged from 34% in Grade 3 to 26% in Grade 11. This finding makes sense seeing that the teachers scored well in only 2 of the 8 van Hiele level 2 ICAS items. The study therefore reveals that the low content knowledge of the teachers has some impact on the achievement of the learners.

The written responses revealed important insights with regard to the geometric thinking skills of selected teachers and my second research question. 40 teachers solved eighteen geometry ICAS items in the study. Their achievement of correct scores, although higher than that of the learners in most of the written responses had a total average of only 38%. Only one teacher achieved the highest total score (67%) and the lowest score (28%) was achieved by 9 of the 40 teachers.

This study has also revealed the teacher' role as central in helping learners move up the van Hiele levels, but if 70% of the teachers could not identify four sided shapes, then the achievement of the learners cannot be expected to be anything more than the 30 % achieved by the teachers in ICAS item 1. Both the learners and teachers could not identify the concave quadrilateral due to the lack of exposure to working with irregular quadrilaterals. A fact that Burger & Shaughnessy (1986) agrees with that learners limit concepts to studied exemplars and consider inessential but common features as essential to the concepts.

The analysis of the ACE teachers' errors of selected ICAS items, justified my earlier classification of Teacher A at van Hiele level 3 and the rest of the ACE teachers at van Hiele level 1 and 2 because the error analysis has revealed that the teachers display a lack of content knowledge of basic geometric concepts. Sixteen of the ICAS items used in the written responses were multiple choice questions and I cannot rule out guess work especially in the case of some teachers who could not solve van Hiele level 1 items but managed to solve items at van Hiele level 2 and 3 correct as revealed by the teacher error analysis.

I had a bit of a problem with categorising Teacher K in the general written responses analysis. He managed to solve ICAS item 15, a difficult van Hiele level 3 question that was not a multiple choice but his total score was only 7 out of 18 ICAS items. I think that Teacher K could be the kind of person who does not do well in multiple choice type questions and could possibly be oscillating between van Hiele levels 2 and 3 depending on the difficulty of the problem at hand.

I found that the teachers in the written responses were not operating at van Hiele level 3 as required by the NCS, except Teacher A, because the teachers' performance was very low in five of the six van Hiele level 3 items. The teachers cannot solve problems by recognising relationships between and among properties of shapes and also follow logical arguments using such properties. For example in ICAS item 9, 30% of the teachers chose the wrong distractor B, showing a lack of content knowledge in the congruency of triangles. It is quite evident that the low performance of teachers at van Hiele level 3 proves that their geometric content knowledge is very low and not at the expected and required level of the NCS.

I found that the ICAS items that could not be mapped to the van Hiele levels were scored correct by 75% and 45% of the teachers respectively. ICAS item 4 focused on finding position using columns and rows whilst ICAS item 10 focused on finding position using a map. ICAS item 10 was not achieved correctly by 55% of the teachers showing a possible lack of content knowledge in location and position. This has serious implications for the learners who will not get a proper grounding in preparation for the study of coordinate geometry later in high school.

I found that the interviews also did not make it easier to classify the teachers into van Hiele levels because the study revealed that the teachers operate at different van Hiele levels even on one question. This finding also casts a doubt on the discrete nature of the van Hiele levels as other research has indicated earlier in Chapter 2. But I think I can safely say that in the interviews, Teachers A, B, C and E are operating at van Hiele level 3 as they displayed geometric thinking skills above the requirement of Grade 6 teachers. Teachers D and F are operating at van Hiele level 2 as they failed to solve the van Hiele level 3 questions.

Other research has shown that when teachers are exposed to van Hiele levels, they become more aware of the geometric content knowledge they teach and also manage to pitch the content at the correct van Hiele level for the learners as Feza and Webb (2005) suggested that teachers needed to engage in activities that require them to classify learners' answers by van Hiele levels. It was very encouraging to see that Teachers A and B, having been exposed to the van Hiele levels in their HOPE project could easily classify the ICAS items into appropriate van Hiele levels. I got a sense though that this was just head knowledge for them which they did not find necessary to utilise in identifying their own learners' van Hiele levels. They also did not find it necessary to use van Hiele levels to categorise the different type of questions in order to cater for learners with barriers to learning mathematics or learners with content gaps in geometry as well as in designing extended tasks for their high achieving learners in order to further develop their problem solving skills. But I think that more research needs to be done to find out whether learners taught by teachers with high geometric thinking skills and content knowledge will necessarily perform better in geometry assessment tasks.

The findings of the error analysis revealed and confirmed the geometric misconceptions and content knowledge gaps among the teachers. 70% of the teachers could not identify the concave quadrilateral and showed a lack of knowledge in dealing with irregular quadrilaterals. De Villiers agrees with this fact that the assumption that children naturally first learn to partition quadrilaterals into disjoint sets at the visual level results from a limited experience of quadrilaterals that children have had in primary school. In this case, the teachers do not support learners' geometric reasoning and concept development.

But, I found that the interviewed teachers do not as readily as they give answers, understand their learners' geometric errors. They could not differentiate between a simple error or a misconception, which may be why they cannot diagnose their own learners' errors easily. Teachers have to understand that sometimes learners make mistakes through errors, because of lack of concentration, quick reasoning, or by failing to notice the important features of a problem. Once learners realise their mistake, they can easily correct it by themselves and it doesn't necessarily hinder their progress in learning mathematics.

But sometimes mistakes are due to a misconception a learner has about a topic. In this instance, the mistake is the result of a consistent, misinterpretation of a mathematical concept, an indication that there is something that has not been clearly understood about the mathematics being learnt. For example, a student may focus on irrelevant parts of a diagram when trying to solve a problem. Or when learners are used to experiencing 2-D shapes only in certain orientations can lead to students being unable to recognise shapes in other orientations. For example the learners failed to identify the concave quadrilateral as a 4-sided shape because they were probably used to engaging with squares and rectangles oriented in the "normal" way and not on its side. This makes the learners as we have seen in the study to fail to apply their knowledge to a shape presented in an unfamiliar orientation or context.

Although the study was limited to the data in the DIPIP study of Gauteng learners and a few ACE teachers, there are important implications that we need to take into consideration when continuous professional development of teachers is designed for primary school teachers, namely, the relationship between language and the van Hiele levels; the use of van Hiele levels in the development of teachers' geometric thinking; the possible use of the van Hiele levels in identifying, explaining and rectifying geometric misconceptions.

5.2 Implications

My research adds to other research that indicates that the majority of teachers are not at the required level of geometric thinking as expected by the NCS assessment standards and the van Hiele levels. The findings are in agreement with Ball et al. (2001, p. 444) that elementary and secondary, pre-service or experienced teachers all revealed universal weakness in the understanding of basic fundamental mathematical ideas and relationships.

The study reveals that as much as it seems that the teachers have embraced the principles of the NCS that promote learner centred teaching and learning mathematics such as group work and alternative methods of assessment like investigations in mathematics, they struggle to understand their learners' geometric errors and fail to develop their problem solving skills. Teachers still teach geometric concepts in isolation, the way that they were taught by their own teachers (Teacher A) hence they fail to promote problem solving in the learning of geometry in the primary school. The analysis in Chapter 4 revealed that the teachers do teach particular geometric content (tessellations), but often pitch the content at a very low level, the main reason for underperformance of South African learners (Taylor and Vinjevold, 1999).

My research is set in a context of the low levels of achievement of learners generally in mathematics as indicated by the Systemic Evaluation in 2003 and 2005 by the Department of Education reports and the much publicised TIMSS 2003 study. However, not much research has been done on the use of the data generated from these studies as a tool for teacher professional development. This study shows how the ICAS items changed the roles of the teachers by allowing them to engage with the geometric thinking expected by the ICAS items. Being confronted by the actual scores of the learners revealed to teachers in a powerful way how they have been teaching geometry to the learners in a disabling manner that does not build their confidence in problem solving.

These teachers are still in the process of studying towards an ACE qualification. The GET ACE Space and Shape course aims to develop teachers' learning and teaching of geometry by investigating shapes using a hands-on approach that should encourage all who engage with it to appreciate and enjoy the study of geometry. The focus is on the applications of the study of shapes in real contexts that will involve experimenting with shapes (Sapire et al, 2004).

My analysis in chapter 4 showed that the teachers displayed an understanding of some of the purposes and goals of the new curriculum. They seemed to understand that learners should be taught using investigative and hands-on approach as proclaimed by the ACE programme and the van Hiele theory. However, the pedagogical aspects (problem solving skills and integration of mathematical concepts) were more difficult for the teachers to implement. The teachers were aware of the errors the learners made but could not explain the learners' thinking or even differentiate between an error and a misconception. The study reveals that teachers need to be socialised in learning how to take learner errors as a normal part of the learning process (Smith et al., 1993). Learner errors analysis helps the teachers to develop an understanding of errors as indicators of deeper misconceptions (Brodie, et al., 2010).

My involvement in the DIPIP project exposed me to an innovative way of professional development, especially for the teachers I worked with. They were taken on a journey of self discovery, where they mapped the ICAS items to the NCS assessment standards, analysed learner errors, read literature about learner errors and misconceptions and how to develop problem solving skills in their learners. They planned micro- lessons and presented in front of their colleagues and were required to reflect on moments where they handled a situation well or badly with learners during the lesson. This I believe is a unique yet powerful way of teacher professional development that is setting the scene for new trends in teacher professional development.

This study suggests that in-service teacher training needs to think seriously about how to help teachers develop new skills in the teaching and learning of geometry in order to avoid misconceptions. The training should encourage the teachers to analyse learner errors and to allow them to discuss different geometric solutions in their ACE tutorials in order to promote key problem solving skills. The discussions will also promote a community of practice among the teachers in their quest to find the best solution to a geometric problem. The training could also include allowing teachers to respond to international geometry test items so as to develop the necessary problem solving skills because these are not skills that teachers and learners necessarily come to class with. This study directs us to the fact that new ways of teaching and developing new skills in geometry need to be understood through further research.

In the analysis of teacher errors in selected ICAS items in chapter 4, it was evident that the teachers' lack of content knowledge in geometry had a somewhat direct impact on the low achievement of the learners in the ICAS test items. The same items the learners did not do well in were the same that the teachers struggled to solve. The study on Grade 6 mathematics teachers also found that most of the errors made by teachers were in the space and shape content area, …an area that has been poorly taught in the past (HSRC, 2008).

Professional development of teachers should also help teachers to develop geometric content knowledge in order to help build their confidence in exploring new ways of teaching the content that they avoid teaching for instance, using the application of properties of shapes to find unknown angles. Then can we begin to experience an improvement in the performance of learners in international and systemic tests. Mayberry (1983) suggests in support of this study that the purpose of assessing teachers' levels of reasoning where deficiencies exist is to make sure that the needs of the teachers are catered for. Thus, professional development has to specifically target the areas of need identified in order to better position the teacher as the central mediator of learning in the classroom.

5.3 CONCLUSION

This study indicates that in light of the low geometric thinking levels of the teachers, the development of new practices in the teaching and learning of geometry will take time before any significant increase in performance of learners in international tests can be experienced. The study suggests that we need to break away from traditional ways of teaching geometry in our classrooms shown by the manner in which teachers struggled to integrate geometric concepts within the mathematics curriculum. This research also suggests that the geometric content knowledge and pedagogic content knowledge of the teachers needs to be developed through tapping into their learners' geometric errors to address learner misconceptions in the learning and teaching of geometry.

This suggests that teachers can only make the necessary shifts when they can begin to use their learners' errors as opportunities for learning and also engage them in solving geometric problems with integrated concepts, and as such challenge them to develop the much needed problem solving skills that will hopefully translate into improved learner performance in international tests and systemic evaluation. Further research is also needed to indicate how exposing teachers to van Hiele levels and allowing them to categorise their learners' responses according to van Hiele levels will benefit the teachers as a new way of developing important geometric skills.

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APPENDIX A

LEARNING OUTCOME 3 – Space and Shape

The learners will be able to describe and represent characteristics and relationships between 2-D shapes and 3-D objects in a variety of orientations and positions

LEARNING OUTCOME FOCUS

The study of space and shape improves understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions and transformations of 2-D shapes and 3-D objects.

The study of space and shape enables the learner to:

- Develop the ability to visualise, interpret, calculate relevant values, reason and justify
- Interpret, understand, classify, appreciate and describe the world through 2-D shapes and 3-D objects, their location, movement and relationships

Learners should gain these skills from experiences with concrete objects, through drawing and construction, and in the abstract justification of spatial relationships. It is important that the study of 2-D shapes and 3-D objects be contextualised to include the study of natural and cultural forms and artifacts.

Select contexts for learners to study space and shape in a way that builds awareness of other Learning Areas, human rights, social, economic, cultural, political and environmental issues.

For example:

- Use of national flags to demonstrate transformations and symmetry in designs.
- Investigate and recognise the geometrical properties and patterns in traditional and modern architecture (e.g. construction and paining of Ndebele homes)
- Use of maps in geography as specific forms of grids
- Investigate geometric patterns in art (e.g. African and Islamic art)

FOUNDATION PHASE FOCUS

The study of space and shape in the Foundation Phase is very practical and hands-on. Learners begin by recognising and describing objects and shapes in their environment that resemble mathematical objects and shapes. Learners should handle these objects and shapes, cut out and draw sketches, and describe them with appropriate and expanding vocabulary.

Learners should be given opportunities to follow and give directions as well as describe their own positions and the positions of objects in space using appropriate vocabulary.

INTERMEDIATE PHASE FOCUS

Learners' experience of space and shape in this phase moves from recognition and simple description to classification and more detailed description of features and properties of 2-D shapes and 3-D objects.

Give learners opportunities to:

- Draw 2-D shapes and make models of 3-D objects
- Describe location, transformations and symmetry

SENIOR PHASE FOCUS

In the Senior Phase, we refer to 2-D shapes and 3-D objects as geometric figures and solids. In this phase learners draw and construct a wide range of geometric figures and solids using appropriate geometrical instruments. Learners should develop an appreciation for the use of constructions to investigate the properties of geometric figures and solids. Classification and description of geometric figures and solids should be extended to include similarity and congruence. Learners should develop the important ability to reason effectively and justify appropriately, when required.

LEARNING OUTCOME 3

The learner is able to describe and represent characteristics and relationships between 2-D shapes and 3-D objects in a variety of orientations and positions.

Grade R We know this when the learner:	**Grade 1** We know this when the learner:	**Grade 2** We know this when the learner:	**Grade 3** We know this when the learner:	**Grade 4** We know this when the learner:	**Grade 5** We know this when the learner:	**Grade 6** We know this when the learner:	**Grade 7** We know this when the learner:	**Grade 8** We know this when the learner:	**Grade 9** We know this when the learner:
R.3.1 Recognises, identifies and names 3-D objects in the classroom and in pictures including: • boxes (prisms) • balls (spheres)	**1.3.1** Recognises, identifies and names 2-D shapes and 3-D objects in the classroom and in pictures including: • boxes (prisms) and balls (spheres) • triangles and rectangles • circles	**2.3.1** Recognises, identifies and names 2-D shapes and 3-D objects in the school environment and in pictures including: • boxes (prisms), balls (spheres) and cylinders • triangles, squares and rectangles • circles	**3.3.1** Recognises, identifies and names 2-D shapes and 3-D objects in the environment and in pictures including: • boxes (prisms), balls (spheres) and cylinders • cones and pyramids • triangles, squares and rectangles • circles	**4.3.1** Recognises, visualises and names 2-D shapes and 3-D objects in the environment including: • rectangular prisms, spheres, cylinders, and other objects • prisms and pyramids • circles and rectangles • polygons in terms of the number of sides up to 8-sided figures	**5.3.1** Recognises, visualises and names 2-D shapes and 3-D objects in natural and cultural forms and geometric settings including those previously dealt with and focusing on: • similarities and differences between cubes and rectangular prisms • similarities and differences between squares and rectangles	**6.3.1** Recognises, visualises and names 2-D shapes and 3-D objects in natural and cultural forms and geometric settings, including those previously dealt with and focusing on: • similarities and differences between tetrahedrons and other pyramids • similarities and differences between rectangles and parallelograms	**7.3.1** Recognises, visualises and names geometric figures and solids in natural and cultural forms and geometric settings, including those previously dealt with as well as focusing on: • similarities and differences between different polyhedra • similarities and differences between all quadrilaterals including kites and trapeziums	**8.3.1** Recognises, visualises and names geometric figures and solids in natural and cultural forms and geometric settings, including • those previously dealt with the platonic solids (tetrahedron, cube, octahedron, dodecahedron icosahedron)	**9.3.1** Recognises, visualises and names geometric figures and solids in natural and cultural forms and geometric settings, including: • regular and irregular polygons and polyhedra • spheres • cylinders
R.3.2 Describes, sorts and compares physical 3-D objects according to • size • objects that roll • objects that slide	**1.3.2** Describes, sorts and compares physical 2-D shapes and 3-D objects according to • size • objects that roll or slide • shapes that have straight or round edges	**2.3.2** Describes, sorts and compares 2-D shapes and 3-D objects in pictures and the environment according to • size • objects that roll or slide • shapes that have straight or round edges	**3.3.2** Describes, sorts and compares 2-D shapes and 3-D objects in pictures and the environment including • 2-D shapes in or on the faces of 3-D objects • flat/straight and curved/round surfaces and edges	**4.3.2** Describes, sorts and compares 2-D shapes and 3-D objects from the environment according to geometric properties including • shapes of faces • number of sides • flat and curved surfaces / straight and curved sides	**5.3.2** Describes, sorts and compares 2-D shapes and 3-D objects from the environment and from drawings or pictures according to properties including: • number and/or shape of faces • number and/or lengths of sides	**6.3.2** Describes and classifies 2-D shapes and 3-D objects in terms of properties including: • faces, vertices and edges • length of sides • angle size of corners	**7.3.2** In contexts that include those that may be used to build awareness of social, cultural and environmental issues, describes and classifies geometric figures and solids in terms of properties including: • faces, vertices and edges • sides and angles of polygons (with focus on, but not limited to, triangles and quadrilaterals) • parallel and	**8.3.2** In contexts that include those that may be used to build awareness of social, cultural and environmental issues, describes and classifies geometric figures and solids in terms of their properties including: • sides, angles and diagonals and their inter-relationships with focus on triangles and quadrilaterals (e.g. types of triangles and quadrilaterals)	**9.3.2** In contexts that include those that may be used to build awareness of social, cultural and environmental issues, describes the inter-relationships of the properties of geometric figures and solids with justification including: • congruence and straight line geometry • trans-formations

Grade R We know this when the learner:	**Grade 1** We know this when the learner:	**Grade 2** We know this when the learner:	**Grade 3** We know this when the learner:	**Grade 4** We know this when the learner:	**Grade 5** We know this when the learner:	**Grade 6** We know this when the learner:	**Grade 7** We know this when the learner:	**Grade 8** We know this when the learner:	**Grade 9** We know this when the learner:
							perpendicular sides		
R.3.3	**1.3.3**	**2.3.3**	**3.3.3**	**4.3.3**	**5.3.3**	**6.3.3**	**7.3.3**	**8.3.3** Uses vocabulary to describes: parallel lines cut by a transversal, perpendicular lines, intersecting lines and triangles in terms of angle relationships (e.g. vertically opposite, corresponding, **etc)**	**9.3.3** Uses geometry of straight lines and triangles to solve problems and to justify relationships in geometric **figures**
R.3.4 Builds 3-D objects using concrete materials (e.g. building blocks)	**1.3.4** Observes and builds given 3-D objects using concrete materials (e.g. building blocks and construction sets)	**2.3.4** Observes and creates given 2-D shapes and 3-D objects using concrete material (e.g. building blocks, construction sets and cut-out 2-D shapes)	**3.3.4** Observes and creates given and described 2-D shapes and 3-D objects using concrete materials (e.g. building blocks, construction sets, cut-out 2-D shapes, clay and drinking straws)	**4.3.4** Investigates and compares (alone and/or as a member of a group or team) 2-D shapes and 3-D objects studied in this grade according to the properties listed above by: • making 3-D models using cut-out polygons (supplied) • drawing shapes on grid paper	**5.3.4** Investigates and compares (alone and/or as a member of a group or team) 2-D shapes and 3-D objects studied in this grade according to the properties listed above by: • making models of geometric objects using polygons they have constructed and cut out • cutting open models of geometric objects (e.g. boxes) to trace their nets • drawing shapes on grid paper	**6.3.4** Investigates and compares (alone and/or as a member of a group or team) 2-D shapes and 3-D objects studied in this grade according to the properties listed above by: • making 3-D models of geometric objects using ◦ drinking straws to make a skeleton ◦ nets provided by the teacher • drawing shapes on grid paper • using a pair of compasses to draw circles, patterns in circles & patterns with • circles •	**7.3.4**	**8.3.4**	**9.3.4**
R.3.5	**1.3.5**	**2.3.5**	**3.3.5**	**4.3.5**	**5.3.5**	**6.3.5**	**7.3.5**	**8.3.5** Uses a pair of	**9.3.5** Draws and/or

Grade R We know this when the learner:	Grade 1 We know this when the learner:	Grade 2 We know this when the learner:	Grade 3 We know this when the learner:	Grade 4 We know this when the learner:	Grade 5 We know this when the learner:	Grade 6 We know this when the learner:	Grade 7 We know this when the learner:	Grade 8 We know this when the learner:	Grade 9 We know this when the learner:
								compasses, ruler and protractor to accurately construct geometric figures for investigation of their properties and design of nets	constructs geometric figures and makes models of solids in order to investigate and compare their properties and model situations in the environment
R.3.6	**1.3.6**	**2.3.6**	**3.3.6**	**4.3.6**	**5.3.6**	**6.3.6**	**7.3.6** Designs and uses nets to make models of geometric solids studied up to and including this grade	**8.3.6** Designs and uses nets to make models of geometric solids studied up to and including this grade	**9.3.6**
R.3.7 Recognises symmetry in themselves and their environment (with focus on front and back)	**1.3.7** Recognises symmetry in themselves and their environment (with focus on left, right, front and back)	**2.3.7** Recognises symmetry in 2-D shapes and 3-D objects	**3.3.7** Determines lines of symmetry in 2-D shapes using paper folding and reflection	**4.3.7** Recognises and describes lines of symmetry in 2-D shapes, including those in nature and in cultural art forms	**5.3.7** Recognises, describes and performs rotations (turns), reflections (flips) and translations (slides) using geometric figures and solids	**6.2.7** Uses the vocabulary & properties of rotations, reflections & translations to describe the relationships between distinct 2-D shapes & 3-D objects within patterns (including transformations and symmetry)	**7.3.7** Uses transformations (rotations, reflections & translations) and symmetry to investigate (alone and/or as a member of a group or team) properties of geometric figures	**8.3.7** Uses transformations (rotations, reflections & translations) and symmetry to investigate (alone and/or as a member of a group or team) properties of geometric figures	**9.3.7** Uses transformations, congruence, and similarity to investigate, describe and justify (alone and/or as a member of a group or team) properties of geometric shapes and solids, including tests for similarity and congruence of triangles
R.3.8 Describes one 3-D object in relation to another (e.g. in front of or behind)	**1.3.8** Describes one 3-D object in relation to another (e.g. in front of or behind)	**2.3.8** Recognises 3-D objects from different positions	**3.3.8** Recognises and describes 3-D objects from different positions	**4.3.8** Makes 2-D shapes and 3-D objects and patterns from geometric objects (e.g. Tangrams) and shapes with a focus on tiling (tessellation) and line symmetry	**5.3.8** Makes 2-D shapes and 3-D objects and patterns from geometric shapes and describe these in terms of • tessellations • line and rotational symmetry • movement, including: **rotations,** reflections and translations	**6.3.8** Draws enlargements and reductions of 2-D shapes (at least quadrilaterals and triangles) using grid paper to compare their size and shape	**7.3.8** Recognises and describes the properties of similar and congruent figures and the differences between them	**8.3.8** Use proportion to describe the effect of enlargement and reduction on properties of geometric figures	**9.3.8**
R.3.9 Follow directions (alone and/or as a	**1.3.9** Follow directions (alone and/or as a	**2.3.9** Positions him/herself within	**3.3.9** Reads, interprets and draws informal	**4.3.9** Recognises and describes natural	**5.3.9** Recognises and describes natural	**6.3.9** Recognises and describes natural	**7.3.9**	**8.3.9**	**9.3.9**

Grade R We know this when the learner:	Grade 1 We know this when the learner:	Grade 2 We know this when the learner:	Grade 3 We know this when the learner:	Grade 4 We know this when the learner:	Grade 5 We know this when the learner:	Grade 6 We know this when the learner:	Grade 7 We know this when the learner:	Grade 8 We know this when the learner:	Grade 9 We know this when the learner:
member of a group or team) to move or place him/herself within the classroom (e.g. at the front or at the back)	member of a group or team) to move or place him/herself within the classroom or 3-D objects in relation to each other	the classroom or 3-D objects in relation to each other	maps of the school environment or of an arrangement of 3-D objects and locates objects on the map	and cultural 2-D shapes, 3-D objects and patterns in terms of geometric properties	and cultural 2-D shapes, 3-D objects and patterns in terms of geometric properties	and cultural 2-D shapes, 3-D objects and patterns in terms of geometric properties			
R.3.10	**1.3.10**	**2.3.10** Describes positional relationships (alone and/or as a member of a group or team) between 3-D objects or him/herself and a peer	3.3.10 Describes positional relationships (alone and/or as a member of a group or team) between 3-D objects or him/herself and a peer	**4.3.10** Describes changes in the view of an object held in different positions	**5.3.10** Describes and sketches views of a simple 3-D object in different positions	**6.3.10** Draws and interprets sketches of simple 3-D objects from different positions (perspectives)	**7.3.10** Draws and interprets sketches of solids from different perspectives	**8.3.10** Draws and interprets sketches of geometric solids from different perspectives with attention to the preservation of properties	**9.3.10** Recognises and describes geometric solids in terms of perspective, including perspective drawings
R.3.11	**1.3.11**	**2.3.11**	**3.3.11**	**4.3.11** Locates position on a coded (labeled) grid, including • maps from given instructions • column and rows	5.3.11 Locates position on a coded (labeled) grid including maps and traces a path between positions following verbal and written instructions	**6.3.11** Locates positions on a coded grid, describes how to move between positions on the grid, and recognises maps as grids	**7.3.11** Locates positions on co-ordinate systems (ordered grids), and maps using: • horizontal and vertical change • compass directions	**8.3.11** Locates positions on co-ordinate systems (ordered grids), Cartesian plane (1st quadrant) and maps, and describes how to move between positions using: • horizontal and vertical change • ordered pairs • compass directions	**9.3.11** Uses various representational systems to describe position and movement between positions, including: • ordered grids • Cartesian plane (4 quadrants) • compass directions in degrees • angles of elevation and depression

Scheiber (2005) NCS Grades R-9, page 19-24

APPENDIX B

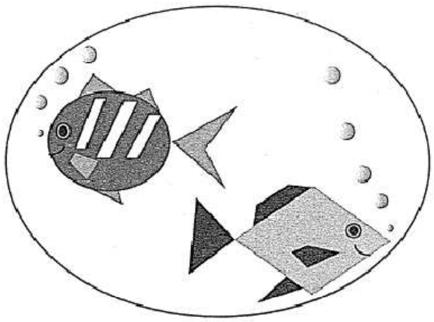
Level Indicators for first four van Hiele levels (Burger & Shaughnessy, 1986, pp. 43-45)

Level 1	a) Use of imprecise properties (qualities) to compare drawings and to identify, characterise, and sort shapes b) References to visual prototypes to characterise shapes c) Inclusion of irrelevant attributes when identifying and describing shapes, such as orientation of the figure on the page d) Inability to conceive of an infinite variety of types of shapes e) Inconsistent sortings; that is, sortings by properties not shared by the sorted shapes f) Inability to use properties as necessary conditions to determine a shape; for example, guessing the shape in the mystery task after far too few clues, as if the clues triggered a visual image
Level 2	a) Comparing shapes explicitly by means of properties of their components b) Prohibiting class inclusions among general types of shapes, such as quadrilaterals c) Sorting by single attributes, such as properties of sides, while neglecting angles, symmetry, and so forth d) Application of a litany of necessary properties instead of determining sufficient properties when identifying shapes, explaining identifications, and deciding on a mystery shape e) Descriptions of types of shapes by explicit use of their properties, rather than by type names, even if known. For example, instead of rectangle, the shape would be referred to as a four-sided figure with all right angles. f) Explicit rejection of textbook definitions in favour of personal characterisations
Level 3	a) Formation of complete definitions of types of shapes b) Ability to modify definitions and immediately accept and use definitions of new concepts c) Ability to accept equivalent forms of definitions d) Acceptance of logical partial ordering among types of shapes, including class inclusions e) Ability to sort shapes according to a variety of mathematically precise attributes f) Explicit use of 2 if, then" statements g) Ability to form correct informal deductive arguments, implicitly using such logical forms as the chain rule (if p implies q and q implies r, then p implies r) and the law of detachment (modus ponens) h) Confusion between the roles of axiom and theorem
Level 4	a) Clarification of ambiguous questions and rephrasing of problem tasks into precise language b) Frequent conjecturing and attempts to verify conjectures deductively c) Reliance on proof as the final authority in deciding the truth of a mathematical proposition d) Understanding of the roles of the components in a mathematical discourse, such as axioms, definitions, theorems, proof e) Implicit acceptance of the postulates of Euclidean geometry

APPENDIX C

ICAS ITEMS USED AS WRITTEN RESPONSES

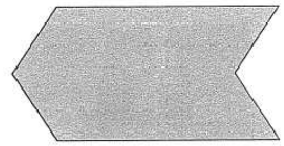
18.



How many four-sided shapes are in this picture?

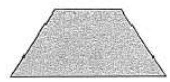
(A) 5
(B) 6
(C) 7
(D) 8

29. This shape has been made out of **four** identical tiles exactly.

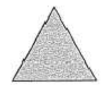


The tiles are blue on one side and red on the other.

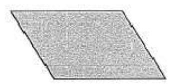
Which of these tiles could have been used?



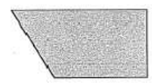
(A)



(B)

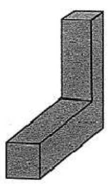


(C)



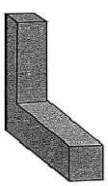
(D)

11. Hana has this block.



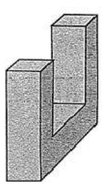
Which of these blocks is the same shape as Hana's block?

(A)



(B)

(C)



(D)

14. Gemma created a spreadsheet to list information about her friends.

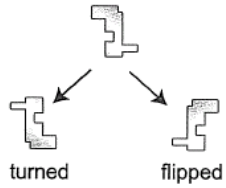
	A	B	C	D
1	**Surname**	**First Name**	**Address**	**Email**
2	King	Joe	123 Humour Rd	joek@maill.com
3	Leigh	Sarah	45 Dessert St	yum@otpus.com
4	Donald	Ronald	678 Pickle Ave	burger@otpus.com
5	Ver	Claire	9 Bright St	smarty@maill.com

Each section in the spreadsheet is called a cell.

Which one of these cells contains the first name of one of Gemma's friends?

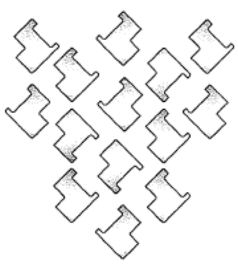
(A) A1
(B) B5
(C) C2
(D) D4

38. The picture shows how a shape can be 'turned' or 'flipped'.



Brian made copies of this shape.

To make this design he turned some shapes once, he flipped some shapes once and some shapes he left the same.

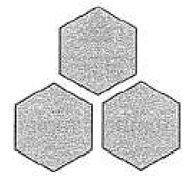


How many shapes were flipped?

(A) 3 (B) 5

(C) 6 (D) 7

8. Jade had these three tiles.

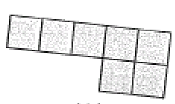


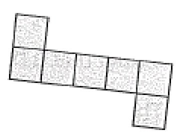
She joined the tiles together with no gaps or overlaps to make a new shape.

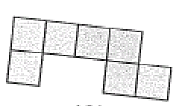
What is the smallest number of edges Jade's new shape could have?

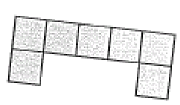
(A) 6
(B) 12
(C) 16
(D) 18

28. Which of these shapes does **NOT** tessellate?


(A)

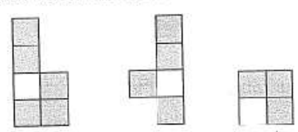

(B)


(C)

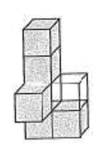

(D)

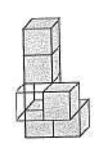
24. A solid is made out of seven blocks, two of which are clear plastic.

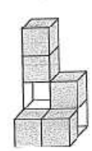
Here are three views of the solid.

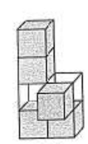


Which of these is the solid?

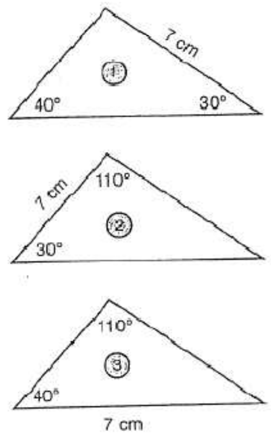

(A)


(B)


(C)


(D)

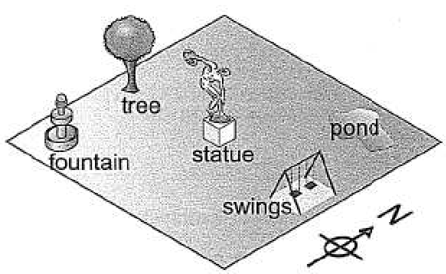
These three triangles are **NOT** drawn to scale.



Which triangles are congruent (exactly the same shape and size)?

(A) 1 and 2
(B) 1 and 3
(C) 2 and 3
(D) 1, 2 and 3

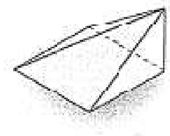
3. This map shows a statue in the middle of a park.



What is west of the statue?

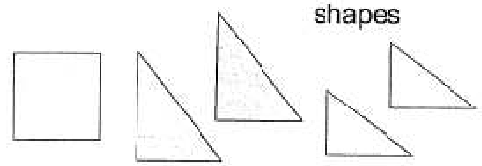
(A) fountain
(B) swings
(C) pond
(D) tree

33. This diagram shows the shapes that were used to make a square pyramid.

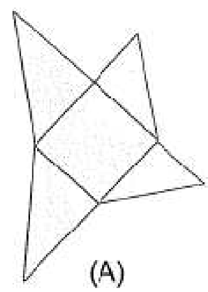


pyramid

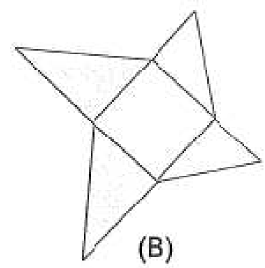
shapes



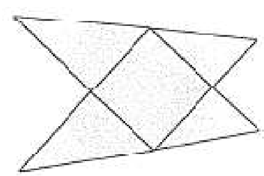
Which of these could be the net of the pyramid?



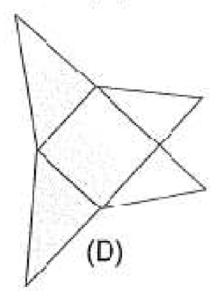
(A)



(B)

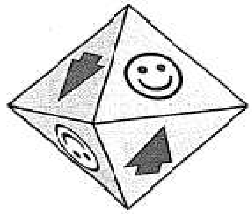


(C)

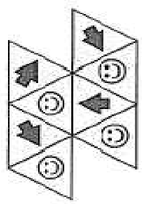


(D)

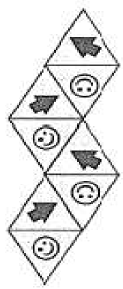
33. Jane made this solid using equilateral triangles.



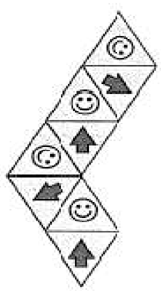
Which of these nets did Jane use to make the solid?



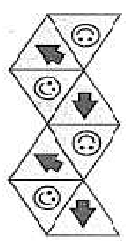
(A)



(B)

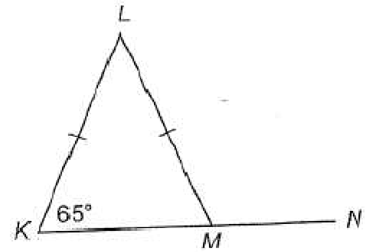


(C)



(D)

In the diagram, KLM is an isosceles triangle and KN is a straight line.

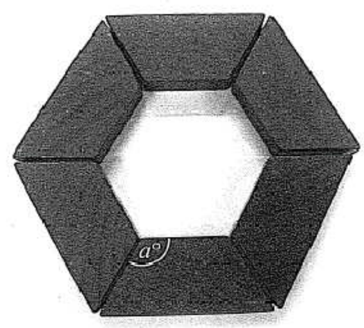


NOT TO SCALE

What is the value of $\angle LMN$?

(A) 135°
(B) 130°
(C) 115°
(D) 100°

25. Jack arranged six identical blocks, as shown.

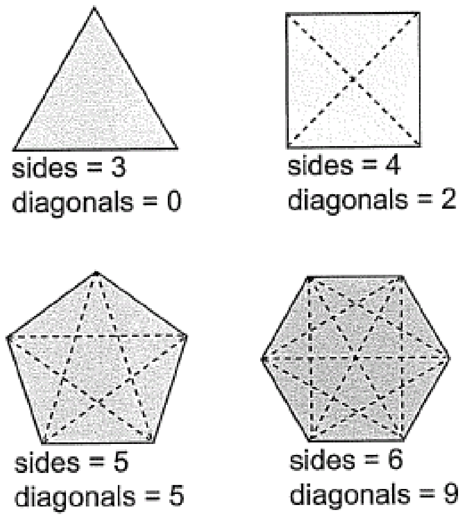


What is the value of a?

(A) 60 (B) 100
(C) 120 (D) 240

39. A regular polygon is a shape with straight sides all the same length.

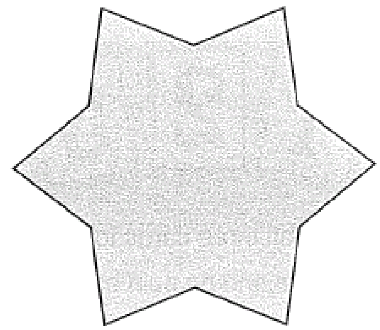
The diagram shows the diagonals of four polygons.



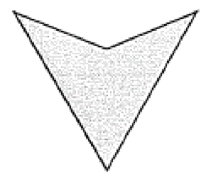
How many diagonals would a regular polygon with 15 sides have?

(Write only the number on your answer sheet.)

39. This star shape has 6 lines of symmetry.



The shape can be cut into 6 identical quadrilaterals that look like this.



The smallest angle in the quadrilateral is 38°.

How many degrees is the largest angle in the quadrilateral?

(Write only the number on your answer sheet.)

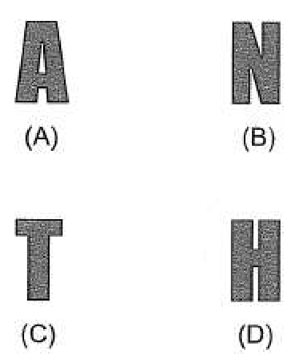
8. Nicole had 8 identical straws.

She made an object using all 8 straws without cutting or bending any of them.

Which object did she make?

(A) cube
(B) square pyramid
(C) triangular pyramid
(D) rectangular prism

10. Which of these letters **CANNOT** have any lines of symmetry?



APPENDIX D

Grade 6 Assessment Standards, ICAS Tests and Van Hiele Mapping Table

ICAS Item	Description	Assessment Standard	Reasoning	Van Hiele Level	Reasoning	Learner Achievement				
						A	B	C	D	Blank
2	Identify the missing shape to complete a pattern based on rotation	5.3.8 Makes 2D shapes and patterns from geometric shapes and describe in terms of movement; including rotations	Mapped back to Grade 5. Learners are required to identify a shape to complete a pattern	VHL2 (d)	Application of a litany of necessary properties instead of determining sufficient properties...	**59**	10	22	6	1
3	Recognise west from a plan in an unfamiliar orientation	6.3.11 Locates positions on a map	Learners are required to recognise direction from a plan on a map	N/A	Locates position	22	20	21	**35**	1
9	Select the correct image to complete a given picture	5.3.2 Describes, sorts, and compares 2D shapes from the environment and pictures according to properties including shape of faces	Learners are required to complete a given picture image	VHL2 (a)	Comparing shapes by means of properties	**67**	18	5	8	2
11	Identify the correct picture given the coordinates of it position	6.3.11 Locates positions on coordinate systems using horizontal and vertical change	Learners required to identify a picture given the coordinates of its position	N/A	Locates position	28	**50**	13	7	1
15	Use symmetry to select the reflection which completes a picture	5.3.8 Makes 2D shapes and patterns from geometric shapes and describe these in terms of	Mapped back to Grade 5. Learners are required to select the reflection which	VHL2 (d)	Application of a litany of necessary properties instead of determining sufficient	**44**	24	18	10	3

		line symmetry and reflections	completes a picture		properties...					
27	Compare areas of a shaded pattern	6.3.4 (a) Investigates and compares 2D shapes by drawing shapes on a grid paper	Learners required to compare areas of shaded pattern on a grid	VHL2 (d)	Application of a litany of necessary properties instead of determining sufficient properties...	**20**	21	9	46	2
29	Select the shape which is exactly one-fifth of the given 5–pointed star	5.3.8 Makes 2D shapes from geometric shapes and describes these in terms of line symmetry	Learners are required to identify a shape from the pointed star	VHL2 (b)	Sorting by attributes of line symmetry	37	33	10	17	3
33	Identify the net of a given square pyramid	6.3.4 Investigates and compares 2D shapes by making use of nets provided	Learners are required to identify the net of a given square pyramid	VHL2 (a)	Comparing shapes by means of properties of their components	20	26	27	**22**	3
35	Use visual clues and measurement to select half of a given cylinder	6.3.1 Recognises, visualises and names 3D objects in geometric settings	Learners are required to use visual clues to select half of a given cylinder	VHL1	References to visual prototypes to characterise shapes	29	**7**	53	6	5
39	Find the number of diagonals in a given polygon	8.3.2 Describes and classifies geometric figures in terms of their properties, including sides, angles and diagonals	Learners are required to find the number of diagonals in a given polygon	VHL3 (e)	Ability to sort shapes according to a variety of mathematically precise attributes	No learner achievement was recorded due to lack of correct answers displayed				

APPENDIX E: ICAS WRITTEN RESPONSES ANALYSIS

TEACHERS	Teacher VHLevel	1 VH1	2 VH1	3 VH2	5 VH2	6 VH2	7 VH2	8 VH2	11 VH2	16 VH2	18 VH2	9 VH3	12 VH3	13 VH3	14 VH3	15 VH3	17 VH3	4 N/A	10 N/A	Total
A	VHL3	1	1	2			2	2			2		3	3	3	3		N/A	N/A	12 (67%)
B	VHL2	1	1	2		2	2	2	2					3	3			N/A	N/A	11 (61%)
C	VHL2		1		2		2	2	2		2		3	3	3			N/A	N/A	11 (61%)
D	VHL2	1	1		2		2	2		2	2			3				N/A	N/A	10 (56%)
E	VHL2	1	1				2			2	2			3	3			N/A	N/A	9 (50%)
F	VHL2	1	1		2		2		2	2				3	3			N/A		9 (50%)
G	VHL2		1			2	2		2	2	2		3	3				N/A		9 (50%)
H	VHL2	1	1				2	2		2	2			3				N/A	N/A	9 (50%)
I	VHL2		1				2	2	2			3		3				N/A	N/A	8 (44%)
J	VHL2		1	2			2		2	2				3	3			N/A		8 (44%)
K	VHL2	1	1				2				2			3		3		N/A	N/A	8 (44%)
L	VHL2		1			2	2			2	2			3					N/A	7 (39%)
M	VHL2		1			2	2		2			3		3					N/A	7 (39%)
N	VHL2		1			2	2					3		3				N/A	N/A	7 (39%)
O	VHL2	1	1				2				2			3				N/A	N/A	7 (39%)
P	VHL2	1	1				2		2		2			3	3					7 (39%)
Q	VHL2		1		2		2				2		3		3			N/A		7 (39%)
R	VHL2		1				2			2	2			3				N/A	N/A	7 (39%)
S	VHL2		1				2	2			2	3			3			N/A		7 (39%)
T	VHL2		1					2			2	3		3	3					7 (39%)
U	VHL2		1		2		2				2	3						N/A	N/A	7 (39%)
V	VHL2	1	1								2		3	3	3			N/A	N/A	7 (39%)
W	VHL2	1	1				2					3		3				N/A	N/A	7 (39%)
X	VHL2	1	1		2		2		2									N/A		6 (33%)
Y	VHL2		1					2	2			3		3	3					6 (33%)
Z	VHL2		1				2			2				3				N/A	N/A	6 (33%)
AA	VHL2		1		2	2	2	2						3						6 (33%)
BB	VHL2		1		2				2					3						6 (33%)
CC	VHL2		1							2	2			3	3			N/A		6 (33%)
DD	VHL2		1				2					3		3				N/A		6 (33%)
EE	VHL2						2		2		2			3	3			N/A		6 (33%)
FF	VHL2		1				2					3			3			N/A		5 (28%)
GG	VHL2						2		2		2		3		3					5 (28%)
HH	VHL2		1								2			3				N/A	N/A	5 (28%)
II	VHL2		1				2		2	2				3						5 (28%)
JJ	VHL2		1				2				2				3			N/A		5 (28%)
KK	VHL1/2		1								2			3				N/A		4 (23%)
LL	VHL1/2		1							2			3	3						4 (23%)
MM	VHL1/2		1				2			2				3						4 (23%)
NN	VHL1/2		1				2				2									3 (16%)
40 Teachers		12	38	3	8	6	32	10	14	13	23	10	7	32	18	2	0	28	18	274/720
		30%	95%	7.5%	20%	15%	80%	25%	35%	33%	57.5%	25%	17.5%	80%	45%	5%	0%	75%	45%	Class Average 38%

APPENDIX F

Interview Schedule

Teachers' geometric thinking questions:

1. What made this type of question easy/challenging to answer for you? Explain.

2. Reflect on your own geometric thinking. What mathematical strategies did you use to solve the problem?

3. Would you use this problem to teach with? If not, why not? If yes, at what level?

4. How would you plan to teach this problem? As part of which topic?

5. What would your goals be in using this problem?

6. Describe a lesson of which this problem might be part of.

Learners' geometric thinking questions:

1. Would your learners' responses be similar or different? Explain.

2. Think about how your learners might solve this problem. What strategies might they use and what might they find difficult?

3. How would you know what the learners had gained from solving the problem?

4. If some learners come up with incorrect responses, what would you say are the real underlying difficulties of your learners' errors?

5. What do you think the learners might be thinking to get the wrong answers?

6. How would you go about in assisting your learners' incorrect geometric thinking?

APPENDIX G

Interview Transcripts

1. Interview transcript with Teachers A and B

ICAS ITEM No. 1

Researcher: What made this type of question easy for you?

Teacher A: question 1- visualization
Well, it's extremely straight forward just looking at the little shapes and counting them, so you can see hear I ticked off as I was counting shapes with 4 sides. Also interesting here was that it wasn't only eh …your normal regular four-sided shapes
Even this little tail has got four sides, very easy…just recognizing the shapes-counting them

Researcher: What mathematical strategies did you use to solve the problem?

Teacher A: Actually not much-I suppose for a grown up because basically it was just four-sided shapes, not really…erm… also making sure that you count the big ones and the small ones, so it was basically just finding them…I started on one side working through to the other side

Researcher: Would you use this problem to teach with?

Teacher A: Yes, I would-definitely - for lower grades-this would be very cute to get them-it's a statement…to show that Maths can be fun-brings Maths alive and it also teaches the child to distinguish between your normal standard-you know just the way that we always do it and this could be fun-colourful or could be for your child that finds artistic things easier to remember this could be a very cute question

Researcher: Very contextualized…

Teacher A: Yes, that's a big word…

Researcher: So, you said at the lower level, which level

Teacher A: I would say…Gr4, maybe Gr5

Researcher: Interesting that this comes from the Gr3 question paper

Teacher A: Yes, but I would even do this for the Gr5 child as entry level not assessment for - who of you can still remember, and to get them in the mood for Geometry

Researcher: How would you plan to teach this problem?

Teacher A: To a Grade 3 class now, all right-to a Gr3 class it would be a completely different ball game because there they still have to find out what are three-four-and five sided shapes so I would definitely teach them a strategy where you put your finger on the 1st one to show the starting point and to then count 1,2,3,4, erm - and to keep the finger there

Researcher: Make it hands-on…

Teacher A: Absolutely, I don't think that Geometry can be taught to children at that age any other way my thinking erm I would also tell them make a mark with a pencil and draw little lines through each of the sides to help them

Researcher: This can be a part of which topic?

Teacher A: Erm… this could be interlinked with their Arts/Culture which is part of their Life Skills or could ask them to use geometric shapes to build a picture…I would take it further than this-I would first let them build their own picture and talk to one another about their pictures and then bring it here-it could be about animal-it could be a day at amphibian/reptile farms or fish or part of my pets-definitely something or pets a big part of that

Researcher: Interesting-do you normally think of quadrilaterals when you see fish?

Teacher A: No, you don't-but here you can definitely you could bring it alive-makes me want to teach Maths again

Researcher: What would your goals be in using this problem?

Teacher A: What would my goals be…eh to make sure to a Grade 3 class that they recognise the shapes and to ensure that they don't only look at a square and rectangle…because that's normally what we show them at Grade 3, a square or a rectangle as eh-quadrilaterals-but if you look at it here, it comes to life and you also testing that the child can observe in a group…where you have to read and scan certain things

Researcher: So, for Grade 6, using the same problem…

Teacher A: Not exactly the same, I would change it-I would go for say name the different quadrilateral shapes that you see-can use the same picture, but change the question-eh I would say how many different rectangles do you see-how many different parallelograms do you see-what is the name of no. a-I would number them-eh-for Gr7 I would go for your convex and concave angles

Researcher: So, this can actually grow

Teacher A: Yes, it can-and the more I look at it the more I can do a lot with it-so you can use this-but I can change it a little bit to get the level higher

Researcher: Whilst we are still on the same question-lets go to the learners' achievement on this question

Learners' questions

Researcher: Achievement of the learners-look at achievement obviously the correct answer is D

Teacher A: Only 29% of learners got it right at Gr3 level

Researcher: So, what do you think happened?

Teacher A: Oh-maybe eh, maybe the shapes in the first little fish all look the same-maybe they misread it as how many different four-sided different shapes and so they counted that as 1, 2, 3, 4-eh which is not even an option answer 5, but there is only 26%there so, if you look between 26% with the answer 5 and 29% you sitting there with 55% of the learners if they misread it. How they got to 7, erm…negligence-maybe too excited maybe they
were not taught strategies to answer these types of questions and then a big problem-do the children read the question themselves or was it read to them.

Researcher: English

Teacher A: Not just English-reading skills-I find my class-50% actually 70%-if I read and explain the problem, they can do the Maths

Researcher: So, would you think your own learners would get the same kind of result

Teacher A: Oh-difficult to say-erm-I would hope to believe they would get better results, erm…

Researcher: Because you've taught them different…

Teacher A: I taught them and I taught them more tactile, I taught them shapes and I enjoy Geometry therefore I teach Geometry more
Eh and as I said-we've cut it out we've pasted, we've coloured, so I try to cover all the different learning styles when I teach Mathematics as well-therefore I would really hope that the results would be better than this-it would be interesting to actually write this with my children this question to see how they would fare

Researcher: What strategies they might use…

Teacher A: mmmm I find that children struggle to verbalize their strategies-you actually have to stand and watch because if you ask them-what did you do?
Aah-I just counted it

Researcher: The teachers also did not do well on this question. Because….
That I can't understand

Researcher: Probably what you said earlier that they get excited thus that's an easy question

Teacher A: so…bang bang lets breeze through it

Researcher: Would you say they are when children come up with5 here as the answers instead of 8-would you say there are underlying misconception or just negligence

Teacher A: Oh, I think there is a reading problem-how many I think read to themselves different 4-sided shapes in this picture

Researcher: there are 5 different kinds of quadrilaterals in this picture

Teacher A: Which ones - 1, 2,3,4,5 - that to me that's the problem or misconception if they answered 7-there is a misconception that 21% because they don't see this as a four sided shape

Researcher A: They think it's a triangle…

Teacher A: Yea they think it's a triangle or a triangle that is not closed at the bottom because we don't teach this normally to Grade 3s - A lot of the teachers is also stagnant-we teach the ones that we have been taught

Researcher: Interesting

Teacher A: Ja - that's what is happened-but if you move to your triangles – erm… children say an equilateral is a triangle but a right angled triangle is half triangle…that has been cut in the center –if you put the "other half then it's a triangle…
Because only an equilateral is a triangle if you look at your Tupperware Toys and things all the triangles used are equilaterals
So, that is what a child perceives as a triangle and nothing else.

Teacher A: I think it's extremely important that teachers get to have more knowledge because if you have more knowledge, then you won't teach the way that you were taught… I think that's why older teachers struggle with OBE…and I enjoy change…for me change is a challenge. I don't see change as an obstacle…I like change…I get bored very quickly with doing the same thing over and over.

Researcher: Would you help the learners like those at 21% who got the sum wrong?

Teacher A: I would actually ask them to draw over, you know, I would cut these little ask them to take crayons, pencil crayons, anything…Koki pens and to draw on the lines and tell me how many lines they see or to touch the finger on the outside of the edge one edge because you have to use the correct terminology even if they are small…angle, for them I would say from corner to corner go with your finger count 1,2,3,4, to quadrilateral ones…umm…not just the shape of a tail of fish but I would let them try and create their own quadrilaterals…strange ones and also again…on um… chalkboards, paper, cut it out pass it on using different materials to built the quadrilaterals and then I would do this in groups and perhaps let one group swop with another group to see how many sides I would not even stop with quadrilaterals, I would then let them build strange shapes and let them count how many sides

So, basically, the problem is that we teach them right sided rectangles quadrilaterals. They don't see strange quadrilaterals and think that it as a quadrilateral and it's also strange that teachers also don't see this as a quadrilateral .But the teachers were not taught that this is a quadrilateral. We very often tend to teach them a square is a quadrilateral because in the NCS document it says: quadrilaterals such as… they name your rectangle, etc but there is a name for this…quadrilaterals such as…and teachers stick with such as because some teachers of the 7phase where they don't specialize in mathematics or in one learning area don't really like mathematics…but they have to teach something which they are not 100% comfortable with and unfortunately that impacts later on Grade 4, Grade 5 when they(learners) get to us…you know it impacts

ICAS ITEM No. 15

Researcher: What does it ask for a regular polygon…diagonals…

Teacher A:: I really enjoyed this

Researcher: The funny thing about this it didn't have any multiple choices…they expected you to work it out yourself write only the answer on the answer sheet

Teacher A: I saw there was a pattern. There was nowhere I was going to draw 15 sides and diagonals, so I worked with a pattern 1st one for 3 sides, zero diagonals 4 sides +2 diagonals gave you the 2. Then it was 2 + 3, 3 + 4 and the next one 0+2+3+4+5+6+7+8+9 its nine up to 10,11,12,13 and I just added up

You can write a little equation x+1 or something plus x+1 that Johnny is equals its n+2 N+ the previous answer plus 1 - I remember the equation…so I was not going to "sekel" to do the equation ha…ha…ha (can you give me the equation) - I just used the equation in my head I literally went and wrote 7, 8, 9,10,11,12

Teacher B: And you just added according to that one

Researcher: Guess how many people got it right…2 out of 89

Teacher A: 45 of them have been on hope…but you know what I know, - scary…

Researcher: We calculating…no of diagonals if 15 sides 3-0 how many diagonals; 4-2…15 sides, quite a lot. First of all make sure that you know again what a diagonal is.
Know it's from one angle to the opposite angle. I think the child…

Researcher: How did u do this?

Teacher A: It was a bit challenging…quite nice it was difficult to a point where it was challenging and let me look at the terminology. Use my ZPD - Slightly more difficult…

Teacher B: Study one that they give a & find the pattern Triangle- no diagonals Hexagon –
Square- 2 diagonals…from 0-2 Pentagon- 5 diagonals
3-0
4-2
5-5
Sides get larger by 2, the diagonals get larger by
Looking at a pattern here 7 sides you have your 9 10 sides that you
Came to 15 which gave me 90 diagonals

Researcher: Have u ever drawn a regular 15 sided polygon

Teacher B: Start with your circle…divide 360 by 15 and you what each angle will be…start drawing your angles and your lines on the circle and you will get your 15 sides by the way when we said well give it names your you take your pentagon (5sides) and decagon(10 sides) and create your own name "Pentecagon"

Researcher: Would you use this problem to teach with?

Teacher A: I think so…

Researcher: At what level?

Teacher A: I wouldn't try this before Grade 7…I agree (Johnny)

Teacher A: Maybe Gr6 class as an extension activity but I think this is just going to keep them for too long in class and the syllabus is exceptionally long and I wouldn't like to give them… Maybe with an Octagon but not 15sides

Researcher: The assessment standard, which topic?

Teacher A: Polygons, 2D shapes (both)

Researcher: How would you use it in class under new/other things tessellations, colour

Teacher B: Unless you use the formula

Researcher: What is the formula?

Teacher A: Not today…nie vandag nie I need a bit more time to come up with a formula

Researcher: Is there a lesson you can do using this question?

Teacher A: Yes absolutely, this links onto symmetry… Find other polygons just drawing

Teacher B: There already you have a rhombus trapezium, with colour…

Teacher A: you could make a star there …then there's a small….pentagon inside

Researcher: help with visualization

Teacher A: There is a…what do we call it…where a get your 6 sided star basically on the outside and inside smaller hexagons…that is when they take a hexagon and they…em…I cant remember the word now, I know there is terminology for it…remember we did it
Even if you take this hexagon and you see little triangles inside and large with equilateral triangles and the blocks that me and Johnny have in our classes with beautiful-pattern blocks

Teacher B: which we use for fractions-works like a bomb
Learners Grade 7-none got it right

Teacher A: I'm not surprised …

Teacher B: Maybe if they didn't go up to 15-you might have got a better result.

Teacher A: maybe because the question asked them to write only the answer, they wrote whatever number… came to their mind

Teacher A: I find that with learners as soon as they get a slightly challenging question, they give up…I don't know…I rather just write any number. that comes into my mind…if it doesn't count for the report…it doesn't matter I don't' really need to stretch my brain I just write something
I think the challenge was too big for them a lot of them would actually try and draw it and draw their lines in

Teacher B: If it was a hexagon-octagon they might have found the pattern

Teacher A: if they found the pattern, then they would make calculation errors
Gr6-not surprised they did not do well
The reading was not that much

Researcher: the beginning of the question leads up to the pictures…the child can just come up with a number

Teacher A: I think…too… my learners would basically just thumb suck they found it too difficult…your brighter child would have thought , …I'll do this later because its time consuming…was there a time limit? Just an hour…no 45 min.

Teacher A: I teach my children if your writing something and there is a time limit rather leave it for later carry on with something you know you can get your marks.

Teacher B: By the way-how many marks-they were all I mark obviously it is its one mark its about 1 minute

Teacher A: Because this was for research that is why it was for they did not differentiate between questions so basically they took 2 and ½ minute per question

Researcher: why do you think the teachers did not do well also?

Teacher A: I don't think that teachers teach diagonals to children I think they find it difficult to teach and I don't think teachers realize the importance of teaching diagonals it's something that I don't have to do…Addition and Subtraction is far more important…

Researcher: Foundations for learning…do you think it emphasises such type of things or it gives more attention to the four operations

Teacher A: The four operations are extremely important, but I find what is even more important than the four operations in my view is place value and number sense and for me that is most of our children fail…they don't have number sense-the teachers work so hard to drill this into the children from a white background I can only speak from where I come from…

Teachers did not like geometry and its always been perceived as difficult…and …I was not taught geometry at primary school-with that shape never…I never heard of diagonals in primary school and I really don't think the teachers are teaching it because it can't be that important when do you work with diagonals…and teachers don't tell you

Researcher: Look at this type of question it moves from diagonals and moves on to patterns it links up very nicely with L02-which is your patterns and the development of algebra

Teacher A:	but again there in prim school-I don't know with you…
Teacher B:	I find that teachers are not teaching patterns very well…patterns are taught the way that they (teachers) were taught 2 4 6 8 10…clap clap…that's when they really go overboard…maybe apple pear
Researcher:	this requires of you to come up with a formula
Teacher A:	I don't think …
Teacher B:	Why did teachers get it wrong?
Teacher A:	Many times teachers are not well trained for Math…somebody is taken…let's say u have a shortage-u go into that class-ag man its prim school…its 1plus 1, use the textbook (Elmarie) Maybe the teacher himself doesn't have the good Maths background and that is also very important…for teacher to understand to be able to teach it because if a teacher doesn't know…how is she going to teach it to a class Many times you find that teachers are just placed there because there is a shortfall…15yr teacher has resigned - …training of teachers…
Teacher A:	I had matric Maths…but I was never taught this the textbook doesn't really tell me how to teach it
Teacher B:	Today's children are visual…are we all agreed…see computer…cell phone-abstract kind of problem maybe if the 15 sided polygon was drawn-he cannot visualise-(Elmarie) THEY DON'T HAVE IMAGINATION ANYMORE like I said…they sit in front of a computer they see everything…they CANT VISUALISE INSIDE THE SHAPE WHAT THE DIAGONALS WOULD BE LIKE
Teacher A:	A lot of teachers wouldn't also…
Teacher B:	If you look at how we teachers were taught - chalk and talk
Teacher A:	Smacked into us …but the type of children…
Researcher:	You as a good teacher, can you come up with a formula…
Teacher A:	3-0; …4-2; … 3 4 5 6 7 8 9 10 11 12 13 14 15; …0+2 2+3 5+ 9 14 20 27; …N=n-3 …….N=n-2 ……N=2n … erm …another day, not today…
Researcher:	Van Hiele level-
Teacher A:	definitely -analysis-synthesis, between 3 and 4
Teacher B:	agree

ICAS ITEM No. 17

Researcher: Was it easy

Teacher A: again not that easy… Something nice to do…I was really interested how you cut it out to get that quadrilateral-I couldn't figure how- where you got it from…that pickled my brain… I cut it… Then I said I want to work with triangles so I cut it

Researcher: the smallest angle is 38 degrees

Researcher: How many angles is the largest angle

Teacher A: I started out… If you then cut it through, along there you end up with 2 triangles…yaa

Teacher B: Interior angles are 180 degrees… But you can also see that it is an isosceles triangle which makes the other angle 38 degrees and together 38+38=76 if you go now to the other angle of the triangle that I have cut
38 and 38 is 76, subtract that from 180 which is 104 and now I've cut that angle in half…so if that is 104+104 then the answer is 208…I like working with triangles

Teacher B: To look at it another way-you can also see that it makes sense to say if that is 104 and104 is a reflex angle

Teacher A: You can also work with the whole quadrilateral and 360 …interior angles of a quad is 360

Teacher B: I keep telling the kids to explore more

Teacher A: 76

Researcher: At which grade

Teacher A: I wouldn't do this before Grade 7

Researcher: Before your learners can be able to solve this problem what should they have learnt first?

Teacher A: How to use a Protractor…
Teach them all the properties of quadrilaterals and triangles
Teach them sum of interior inside angles of triangle is 180 and even then it's difficult for the child even the question at no.13 they'll find it difficult
Where you just extend the line for them to see that the 2 adjacent angles are
For a child-very challenging-Grade 6- …no
Grade 7-with a push and a shove
Not to stretch that with more extended activity for one or 2 brighter child in the class
Look at this as a hexagon
I would have had to teach them extensively about Hexagons. If they know what the angles of a Hexagon are then you would have to do a lot of previous work before

Teacher B: -especially investigating 2-sided shapes - senseless also telling them about interior angles they have to discover for themselves –
Like Elmarie said; - lots of work with Protractors they've got to measure

Gr7s are … you give them an acute angle triangle and they say it's 175 - they read it wrong

Researcher: This would work well with investigation

Teacher A: Yes, a group investigation even where they can

Teacher B: help each other

Researcher: only your brighter kids might be able to do this

Teacher A: I find putting them in a group with weaker children-where they can bounce ideas off where they can lift those up-works well with a difficult question
Not that the other children are holding them back that much

Teacher B: That why I always tell them the purpose of group work is not to sit back and receive but to be active…even if you are wrong that's the only way that the brighter ones can pick up your way of thinking and a way of doing things
But I think children find multiple choice questions even more difficult because they don't calculate the answer

Researcher: Van Hiele level

Teacher B: level 3 and 4

ICAS ITEM No. 13

Researcher:	KLM isosceles; KM is straight line … What value of angle LMN , outside angle multiple
Teacher A:	First thing; you have to know your rules about Isosceles triangle - Easy-for teachers it shouldn't be challenging
Researcher:	Geometric strategies
Teacher A:	Two angles at the bottom of 2 equal sides are equal because isosceles sides And your two adjacent angles is equal to 180 two angles on a straight line is 180 This was quite easy 180 subtract 65 gives you 115 degrees
Researcher:	At what level
Teacher A:	Van Hiele level -2 application knowledge
Teacher B:	You've got to know your knowledge…first, Isosceles
Teacher A:	Maybe 3-apply knowledge
Researcher:	Teach to learners
Teacher B:	Yes to Grade 7- this goes immediately over to Grade 8 as well where you do the outside angle of a triangle
Researcher:	Sometimes we compartmentalize knowledge When you do the sides of a triangle you don't mention that the angles are also equal
Teacher A:	We scared to mention it. Before OBE - We were very rigid in the way we teach I haven't been a conventional teacher and I've been in trouble for using colour-like with Mrs Rottenmeier, the Head of Department …We would teach things informally…its incidental learning but now… With OBE-we have been given carte blanche you can teach without overstepping maybe somebody else is going to teach this next year I like geometry…when you like something you teach it well
Teacher B:	I would give this to a child and let them cut these two angles up and join them together…fit them…fold
Teacher A:	This question I would do later on because you have said that the angles of a triangle is 180- when you take that they see it as a straight line so then they remember that a straight line is equal to 180 because a straight line equals 180 means squat to those children until they have physically seen it-they cut the angle and put them together and you get a straight line and they go WOW you didn't lie-or wow you clever how did u know Or that is magic-it only works with your triangle I had a child who for two solid days he was doing nothing but drawing different triangles, he wanted to prove me wrong… that child will never ever forget that the inside angles of a triangle is 180

	Tried to prove me wrong
Researcher:	49% of learners got it right At which level-Gr11 question 2
Teacher A:	My children would have done better
Teacher B:	It comes back to foundations not laid in primary school
Teacher A:	There is a perception out there that Geometry is too difficult-don't do Maths-geometry is difficult. My own child was told-if you have done well-rather remark paper 1 because you might in any case not get anything from geometry. He likes his geometry
Researcher:	Miscalculated…
Teacher B:	If they take 135 and then 65+65=130
Researcher:	Misconception…
Teacher B:	Misunderstood what an isosceles triangle is - calculated 130 …
Researcher:	28% in Grade 11
Teacher B:	At least they knew that the bottom angles of an isosceles are equal…somebody taught them something in primary school
Teacher A:	Iny minie…miny…more … Guess work …65-can't be 135-also a straight line …wow-that the correct answer 100-guess work 135-miscalculaton 130-misconception
Researcher:	Correcting it?
Teacher A:	To work with properties of an Isosceles triangle
Teacher B:	I've done this with my learners where they actually had to measure and from there go onto the properties-so-… you see that it is investigation once again-…because once this side and that side are equal those two angles will be the same now …they measure it physically-I had an exercise where they measured Physically measure with a ruler, measure with a protractor to find
Teacher A	I would again work with colour. Draw the line KL in red also draw LM in red then I would ask them that they've got those two angles at the bottom that are the same and to fold and put the red lines on top of each other to see what happens to those angles, so they can see-measure it as well… Problem is that my kids buy these extremely cheap Protractors

ICAS ITEM No. 6

Teacher B: What is the smallest number … they look here- then they stop if you look here 6 is the smallest no. it's a reading problem not a misconception

Teacher A: Grade 5 …time …45 min-they were under pressure-maybe be a reading problem

Researcher: 16…How did they come up with 16 …what if they did not understand the word tessellate

Teacher A: but they don't use tessellate

Teacher B: join with no gaps… vertex to vertex… not edge to edge

Researcher: But that would give those 18

Teacher B: …tiled This is misconception on what tessellations are…with these kids, I would…cover a floor using hexagon … tessellate also in a straight line

Teacher A: They all play soccer where they have pentagons & hexagons- are used on the soccer ball with us we are used to working from Gr1 they play with pattern blocks

Teacher B: I'm trying to figure out how they got 16
Have no idea how to tessellate without overlaps or gaps
Now they counting one on one vertex to vertex instead of edge to edge

Teacher A: Maybe question was slightly misleading; maybe the word tessellation should have been here

Teacher B: Although it explains tessellation-with no gaps we teach names of shapes

Researcher: cover for tessellation

Teacher B: Remember what we do we take a whole lot of triangles and build and start counting the sides without counting the ones on the inside and you come up 3 triangles might give you eight sides and become, irregular octagon-erm-… and that's what we do, … play games-who of you can build something with more sides than me … but with the same amount of triangles or whatever shapes

Researcher: Are you going to try it (Both, were doing it already)

Teacher A: That is right at the beginning where you start teaching them the names of shapes

Teacher B: Now the need to know that if it's got 4 sides-it's a quadrilateral or a special name

Teacher A: We teach them up to dodecagon then the kids give the shapes their own names or build an animal or just a picture

Teacher B: Yes-and –erm- then you count –and get beautiful…

Researcher: When do you teach Geometry?

Teacher A: 1st Term. …but both our districts work on a standardised work schedule
Term 1 –Number, Term 2 is fractions, Term 3 is Geometry, Term 4 data handling

Teacher B: Sometimes …Especially if there is a correlation between the Maths and the Geometry

Teacher A: Something that should be taught more

Teacher B: With the course that we got with being Junior Facilitators, like I told you …does new things and it's hands-on…
That's what I like about-I mean-3D shapes
We make the children build with straws, etc so that
We can actually count the faces, it's not just how many because it's difficult for some children to see a 3D object on a 2D picture but if they build them-shape them …they can physically see and we make them built them

Teacher A: I use wets to build them-I have strings of them in my class …all the hexagons, all the pentagons –erm-…star-rated so that

Teacher B: Then the children see the relationship between the 2D-shape and 3D because in a 3D … if it's got 10 faces, it's also dodeca-but not gon … dodecahedron-they can then see the relationship between 2D and 3D and its only when you build it when the child is physically sitting with that shape in his hand, then he can see the relationship between them –because there is a relationship like I said-the no. of edges, for your 2D but the no of faces for your 3D

Researcher: Get one on Transformation Geometry …

Teacher B: Paper folding to teach them how to build a cube
Turning … Flipping … it's that way
So those vocabularies are coming in-but once again get a piece of paper and fold it…

Teacher A: There they actually tell them this is a flip and a turn- the picture shows-
Brian uses this shape –looks like a chair to me
To make this design he flips some shapes one and some shapes he leaves the same...

Question: How many shapes were flipped?

Researcher: How would you tackle this problem?

Teacher B: Once again it's the terminology, the child needs to understand what flip means…
Rotate is to turn
We actually make them stand up –rotate 90… then the child has got to turn –erm- it's difficult to flip a child though (Laugh)… but flip a book-a pancake…
Yes-you see

Teacher A: Coming back to the question…I have to first … figure out what's what … I think this is too many –so you take this one and …
You say all those that are left the same throughout- so … That's one, another one-so let's get rid of all of them

Teacher B: There's another one and another one … No-no-no

Researcher: What are you looking for …?

Teacher A: I'm still looking for the ones that were left the same … 1, 2,3,4,5 that he left like that

Teacher B: So now we look at the ones that he flipped … So it would be that one-maybe give it a dot on … The inside-one, two-here's another one

Teacher B: He flipped those ones-turned them once …Last year this lady told us the best thing to do is … To trace this thing and place it there and you turn … It-that's what I did … I took a piece of trace paper in the exam room-I … Traced it to do my flipping and turning-yes for… The reflections-even now I'm trying to count this thing in my head … visualising it…it's a bit difficult so … so … so … what you can do is give the children some tracing paper or if you get a clever child he is…going to cut this thing out from there … and he is going to put it somewhere there-then it …once and see what shape does it take on …because … It's difficult to see being turned once … These are flipped…

Teacher A: Yes I agree … But you can also flip from bottom up…

Teacher B: From this position …I'm trying to establish that … If you flip it over-it's going to sit up there

Teacher A: Ja-those are flipped once

Teacher B: What if you flipped it up it would have still been the same if you would flipped it over here…That is why I taught-if you turn it once …This is actually a turn 'cause if you look at this-what they consider a turn-that's a 90 degrees to the left … Then the back is gonna show up here

Teacher A: 90 degrees to the left-so … it's basically going to look like that. Those are turned, not flipped … Answer, so the rest are flipped

Researcher: How many are they?

Teacher A: 1,2,3,4,5,6 … Ja it can be confusing-I would say 6

Researcher: How would you teach the children?

Teacher A: Wow-very difficult-I wouldn't start with something as…as this drawing because the difference between flipped and turned is difficult … do you always flip from left to right or from right to left or do you flip up and down or what and erm if you look at it-the correct answer erm-18% of learners got that-which is scary because that is actually very difficult, because I think the shape made it difficult for the learners

Teacher B: Yes too many curves

Teacher A: If they would have used something slightly easier eh-maybe like a kite where you can still see the longer and the shorter one-it would be easier for them to see a flip and a turn

Researcher: There according to the results, which ones got the most 22,23,28,18 is all very level-which tells me that the children…

Teacher B: difficult to work it out…confusing.

Researcher: But using the process of elimination helps-yes

Teacher B: Yes that's what she started with, she eliminated those that stayed the same 1,2,3 were turned so if you look at no(9) there is a misconception they don't know the difference between flipping and turning

Researcher: But this is also very confusing (shape) and this is at Grade 5

Teacher A: No, I wouldn't do this in Grade 5, maybe Grade 7s

Researcher: When I looked at all the questions from Gr3 to Gr11, 21 of them are in Learning Outcome 3, Assessment Standard 2.8 (Transformation Geometry)

Teacher A: It's a difficult concept for learners … But this more confusing

Teacher A: The learners (people) that said B, took the same picture …because that was the same

Teacher B: Ja … 3 were turned, not flipped … 7 I don't know how they got 7 … I think it was just the next number

Teacher A: I think its confusion more than misconception

Teacher B: Lots of confusion

Teacher A: If the one little foot was coloured in black, then you can see which one is which but to be honest with you-check for time and you look at this…it looks like a whole lot of…

Researcher: These questions come from Australia, the type of questioning … how this is different from our South African way of questioning

Teacher A: I think this is more interesting-I think we can get there…

Researcher: When we talking integration, normally when we plan our lesson, we talk Life Orientation, Science but we never talk integration of mathematics within mathematics-like integration of Learning Outcome 2 and 3Diagonals-patterns

Teacher A: What I think we must remember is that children in Australia start school much earlier than our children and what they have as Grade 5-are children much younger than our Grade 5 learners, already … New Zealand-year 4…When a child turns a year-they link year 3 and 4 together … South Africa Gr5 & Australia Gr5… But Australia start their children formal education at age(year) 4 … We start formal education at age 6
Its not wishy, washy plastic play they actually teach these children

Researcher: At what VHL is this

Teacher A: For me now 12…laugh… I would say this is level 3

Researcher: What we normally do is ask this differently… ask the question on rotation separate … not all of them together and sort out …you think this is bold…

Teacher A: But I think this is good. I think its stretches the child, and I think if we teach like this…they will be able to do it.
I think our kids are more held backwards I think our teachers –we as teachers are scared that we are going to make a mistake which we don't allow our learners to try it because what if this child asks me a question I can't answer
but this gives me an idea

Researcher: This is at Grade 5-normally you would say that my Grade 5s won't be able to do this

Teacher A: This shape I think was confusing, but this type of question with another shape can definitely work …Maybe because it was too small

2. **Interview Transcript**

Interview transcript with teacher C and D

Researcher: What made this type of question easy or challenging to answer?

Teacher D: For me it was much easy because I just looked at the shapes and counted all those that are four sided

Researcher: So, what's the answer?

Teacher D: 7

Researcher: Would you say this is an easy question?

Teacher C: I think easy because the examiner could have said how many quadrilaterals are there and that may have stopped the learners who are not familiar with the terminology so in this sense here they made it easy for the learner, I think by saying four sided, but the trick in here was that uhm…concave shape, Ja, quadrilateral

Researcher: Reflecting on your own geometric thinking,…what mathematical strategies did you use?

Teacher C: I just looked for four-sided shapes and I eliminated the triangular shapes and then I concentrated on the closed shapes so,…that's how I proceeded.

Researcher: and how did you do it Nomkhosi

Teacher D: yeah, ugh…, more or less the same as Rukhshana, I just focused on the closed shapes, then counted the number of sides

Researcher: okay, …, and the concave thing did not mess you up?

Teacher D: no, not really…

Researcher: would you use this problem to teach with?

Teacher C: Ja, I should think it's more challenging, what we do is we use too much of a structured format to teach,…erm,…shapes

Researcher: structured format, what does that mean?

Teacher C: In terms of , …like in 3 sides – triangle, 3 means tri, and then four sides – quad, five, …and then we draw a shape, …so, they don't have experience of placing those shapes in real life situations so I think that that is what is lacking in our teaching , in real life…

Researcher: context… Nomkhosi, what do you think... Would you use this problem to teach with?

Teacher D: Ja, I think I can…

Researcher: But at the beginning or when doing, erm, ..as consolidation

Teacher D: consolidation…

Researcher: and at what level?

Teacher C: I could enter them right at Grade 4 level because at Grade 4 level they do have, erm,…erm,…even in Foundation Phase they're learning names of polygons,…so I could introduce this even right at Grade 4 level…

Teacher D: I think Grade 4, uhm,…yeah…

Researcher: so, your Grade 6's will just fly and be able to recognise four sided shapes

Teacher C: the problem,…we'll have a problem with that, concave kite, but everything can't e that easy they've got to provide challenges that is a challenge

Researcher: Then how would you plan to teach with this problem, and as part of which topic?

Teacher C: Ja, it is Learning outcome 3, space and shape and then assessment standard will be properties of shapes,…2-D shapes, …yeah…
Yeah, 2-D shapes, and also that we gonna have to go into the , er, experiential knowledge of shapes because they may come in with a knowledge of circles, uhm,…that kind of thing so you would have to go into that, and then I think teach them why a 3 sided figure is called a triangle, …erm, …well,…what is this new terminology based on what, …hexagon,…And then to show you would have to demonstrate that kite shape,…can be reflected differently and so you gonna have to teach them the concept of convex and concave

Researcher: okay, so is it something that we readily do … at what level?
At Grade 4, 5, or 6 level?
At Grade 4 we don't teach concave, but at Grade 6 – I do teach concave
Researcher: okay… Nomkhosi
Because what I see is we normally teach, erm …like you said …in isolation triangles, in isolation 4 sided shapes, quadrilaterals in isolation, we never put them together like this…we …never…

Teacher D: we never do this, what we do …we do the properties of the 4-sided alone

Researcher: separately …yeah …separately
Do you think that affects the learners in any way?

Teacher C: It does – because it kind of streamlines their thinking – it makes them think that, okay,…I'll have to look on the page and …erm…and then I'll see an isolated 4-sided shape

Researcher: that's interesting…

Teacher C: Yeah – but what I did to overcome that is…at the end of the …roughly..erm..it was just a week – 3 lessons. In 3 lessons I had to … I don't have enough time to dwelve in this for weeks,…so by the 3rd lesson, I gave them a task – a group assignment where they had to use different shapes to create a theme, ..erm..Going sailing and..erm..so.. the requirement of the assessment was to use all the types of shapes and some of them made a boat using triangles for the sail,…they created a sea that is 4-sided,…that kind of thing, …so they created a picture, …not like this…using shapes of their own in different colour paper and come out and place it together and some of them I said built a school and so they created a school structure and then you could …the rectangles, the squares, the circles, triangles were used…

Researcher: by doing that, how does that help the learners?

Teacher C: It helps them to contextualize…the …understanding of shapes and to see the applicability of shapes in real life
Uhm…how important is all that around here, Nomkhosi, what you think,

Teacher D: Yeah, I agree

Teacher C: I'm sorry for talking too much,…laugh,…

Researcher: It's wonderful what you are saying…, okay, next question, number 7
Jade had 3 tiles…
Let's just check what is the answer?

Teacher C: 12

Teacher D: 12

Researcher: So, what is this topic all about…

Teacher C: It's tessellations

Teacher D: It's just,…erm…tessellations

Researcher: Was it easy for you to do?

Teacher D: Yeah, it was easy…

Researcher: because you've done tessellations before?

Teacher C: Oh! Learners do tessellations from Grade,…erm, …foundation phase,…one of the assessment standards includes tessellations, that is taking shapes and putting them together without spaces between them and in a repeated way…

Researcher: what about the question,…what is the smallest number of edges,…think, that could pose a problem?

Teacher C: Yeah, the word edges could pose a problem if a child doesn't know what you're referring to by an edge …erm…he may not know what you're looking for…
When do we talk about edges, when we're dealing with which topic?

Teacher D: I think when we're dealing with 3D objects and relationships

Researcher: But when we're dealing with 2D shapes, we never talk of edges, we call them sides…

Teacher C: uhm…

Researcher: Interesting, erm…
So, reflecting on your own geometric thinking, what mathematical strategies did you use to solve the problem…Nomkhosi? How did you come up with the answer 12?

Teacher D: I just closed the gaps that appears here and count the number of …erm…edges…that are on the outside of the new shape

Teacher C: but you have to recognise that an edge is from one point to the next
Is that the definition of an edge?
We talk about the vertex and we talk about the edge and the face
Ja, but just like …erm…questioning differently…you come up…with all these things…yeah…interesting…

Researcher: so would you use this problem to teach with?

Teacher D: I think we should use it so that they will be able to know how to differentiate…that…in 2D shapes we refer to them as sides and when we're talking 3Dobjects, we refer to them as edges…so that when they come across this question they encounter the problem of edge…

Teacher C: That's the thing that…erm…this kind of problem where the main focus is on the fact that the spaces lend the shapes to have more edges…if you count…the total number individually gives a different total figuratively if we had to look at the total of all the 3 figures…they would be more than the combined so they need to understand that we have collapsed some of the edges by joining them together

Researcher: even when you ask your learners…
would you put the shape together or leave it as it is 'cause leaving it as it is like you say…erm…it asks for them to think…it's more challenging for them that they must eliminate those edges that are going to be joined together and just focus on the ones that are going to be on the outside

Teacher C: uhm…a hasty learner will just go ahead and count all at once so they have to be aware that they must read the question and instructions. It is clear in the question…

Researcher: how do you normally teach tessellations, Nomkhosi?

Teacher D: erm…I firstly define the term what is tessellation and then use different shapes to demonstrate the examples of tessellations so that they will be able to see that in tessellations, I mean the shapes must not overlap or leave gaps

Teacher C: I alert them to practical things that works better…erm…a lot of them have…I cannot say all of them…but even in the school situation…some floors are tiled in squares …so you alert them to the practical situation that all those squares are placed together to create a pattern and …erm…they are placed together to create a pattern and …erm…they actually enjoy that what I actually do as well is cut out accurately exact shapes, and then I give them a piece of paper and say create your own pattern…now there's interesting things that can happen with triangles

Researcher: Hmm…such as …

Teacher C: Like when they pattern it, most of them won't just put the pattern straight across they will go diagonally across and all of that and …erm…some of them wouldn't have triangles just the border of the triangle creates a different visual impact and then …erm…what I do as well is …erm…have them create their own collage with the shapes…I mean focusing on the number of gaps.
I must say that when I tried to tessellate with circles, I wasn't as successful…I had only about 50% of my learners that were able to create a circular pattern in terms of the 16:52 tessellation then I just support those who didn't manage…I guided them, giving them the accurate measurement of the radius and saying now place your compass on that point and…then we created a little flower with the inner circle and that kind of thing … but… I must say that I can't regard that lesson to have been successful…yeah

Researcher: But like you say…its better when you do it practically…

Teacher C:: Yeah…with shapes
'Cause I find that there's tessellations all around us
Yes, instead of using only pictures there is paving outside, CTM adverts…
So you can actually ask them to bring in…and it's actually accessible for them to bring in examples of tessellations
…and create something?
What about the animals,…when they're bringing shapes do they stick to like regular
17:56 shapes, that they do in the classroom or they move on and create animals like…or something

Teacher C: No, no, no…they do that erm…recognise or something because you ask them with the butterfly and then they say…there is a bit of a tessellation there because it's the same
18:15 shape – repeated…by the spaces around them – so they do recognise the repetition of the shape, and actually you could integrate this with your life sciences Natural Sciences being observant of the properties of certain insects and what they look like…
Let's do the next one before we come to the learners…the next question – diagram – Isosceles

Researcher what is the value of angle LMN which is that outside angle…
Is this an easy or challenging question?
What grade are we talking about…'cause if we look at a Grade 4 learner I'm not sure that a learner will even be…

Teacher C: At Grade 6…

Researcher: Ja, at Grade 6 level I think it would be …erm…not entirely challenging because you do teach them that the interior angles of a triangle…add up to 180˚. If two are equal that you add them and you subtract from 180˚ to get that angle. What about the properties of an isosceles triangle. Yeah…you have to teach them…
So, this question requires of them to know which concepts?

Teacher D: To know that erm…for instance a triangle is equal to 180˚

Researcher: the sum of the angles inside here are equal to 180˚,…what about isosceles…

Teacher C: Two sides are equal and 2 angles are equal and then you could also teach them the properties of a line…it …erm, …all angles on a line and it adds up to 180˚

Researcher: they need to know three things here…

Teacher C: Although that this could be extended to a higher level of thinking when we look at…if we look at M1 within the isosceles triangle and M2…M2 actually is the exterior angle that adds up to the sum of the opposite interior angles but at grade 6 level they …wouldn't be much too difficult
So all they need there are interior angles…
Interior angles and an angle on a straight line

Researcher: wow…so, looking at your own geometric thinking, what strategies did you use to solve this – what the answer anyway…

Teacher C: it's 115˚

Researcher: Nomkhosi, how did you get 115˚, tell us

Teacher D: 21:37 I used the properties of an isosceles that the 2 sides are equal and the 2 opposite angles are equal…SK here is equal to M1

Researcher: so that angle M is also equal to 65˚

Teacher D: uhm…

Researcher: and using the angles on a straight line that's how you got 115˚

Teacher C: But I also used the double check on the answer by saying that 180˚subtract the addition of the two known that will say 130˚ leaves us with 50˚ to give 180˚ because I wanted to apply that rule of the exterior angle…

Researcher: because you want to brag…ha…ha…ha
Where did you learn it …in high school?

Rukhshana: Yeah…ha…ha…ha

Researcher: So would you use this kind of question to teach with because I think normally when I was teaching I would use …erm,… angles on a straight line. I would never integrate it with a triangle. I would just use the angles on a straight line and just do it separately

Teacher C: You see when I teach I make use of the protractor to measure angles – I start off with a straight line and I show them how if you can place your protractor on the point that you've made on your straight line, all those angles if you add them up 5, 10 up to 180, so let's draw all those angles…so we draw the protractor in the book

Researcher: okay

Teacher C: and then make up those lines copy …I say can you see that all those lines – all those minute angles that connect at the point, make up an angle, and they will always add up to 180˚. So, then I set them as an exercise another line but maybe three…draw it let me see…
And I give them the measurements of the two and I say work out the angles and I

Challenge them and I draw exactly 2 same angles …so then I say if you add those you just subtract from 180˚
So that kind of …erm gets them into a pattern of thinking …

Researcher: Nomkhosi, angles on a straight line – how do you teach them?

Teacher D: 25:04 More or less as Rukhshana has said, but I teach them that angles on a straight line all add up to 180˚
Do you ever cut them up and put them back together…

Teacher C: I use the cutting up of angles in a triangle. see when you teach them that angles on a straight line and I say cut the edges – now place them on a straight line and they find that they all fit perfectly on top of a straight line - that's how I teach them that the angles inside a triangle must be equal to 180˚. We give them more than 1 triangle – different triangles and then repeat that activity on the straight line …I actually enjoy it
Learners probably don't believe it…
At first they don't, yeah

Researcher: Interesting…and just for interest' sake this one…so at which level would you teach that…

Teacher C: erm…, Grade 6
Let's look at this one talking about diagonals. All questions were multiple choice – but this one is not

Researcher: How many diagonals would a regular polygon with 15 sides …
Nomkhosi, explain to us, Rukhshana, has got a very big number…how did you do it and figure this one out.
Was it challenging or easy?

Teacher D: challenging because I just add this one to this one…

Researcher: You add what?

Teacher D: Erm…the 3 sided to the six sided to the six sided…
So how did you come up with…will give you 12 sides and then plus 3 I come up to 18

Researcher: You think it's right?

Teacher D: I don't know…

Researcher: 18…diagonals of a 15 sided regular shape. Normally in Grade 6 up to which shape – number of sides do you go

Teacher C: Right up to 8 sides

Researcher: So, was this also easy for you?

Teacher C: No, it was a bit challenging…(the pictures are not clear) If you're focusing on the diagonals your pictures must show the diagonals, number 1 then secondly, I felt tempted to start drawing …

Researcher: Why didn't you

Rukhshana: I thought no, the time constraints… and they can't ask me about 15 sides if they were
expecting me to come up to the answer by drawing each one of them…so I stopped 30:11
there, then I said perhaps there's a pattern…
So then I missed it – by putting the number of sides on one row and below that the
corresponding diagonals and then by the time I came to the 6th one – I saw a pattern,
30:29 that its plus 2, plus 3, plus 4, plus 5, so I went on but couldn't remember the formula for
30:44 the number patterns – so because its only 15 sides, I carried on adding – it could have
been tedious if it was 30 or 40 because then I would have had to write the whole forty
31:00 then …I worked it up to 90 – I'm not sure if I'm 100% correct

Researcher: I also don't know…ha…ha…ha…

Teacher C: I must actually find the time to go and draw the 15 sided figure what do you think it will look like…

Researcher: go do it with your learners and bring it along, …it would be interesting… actually the correct answer is 90
Nomkhosi, the answer is 90…uhm…is it too much?

Teacher D: How do you reach 90?

Researcher: Listen to what Rukhshana said, Rukhshana you were saying…

Teacher C: I worked a pattern
For 3 sides, how many diagonals,

Teacher D: no diagonals

Researcher: 4 sided

Teacher D: two

Researcher: 5 sided

Teacher D: add 3 to your previous answer is 5
Plus 4 to your previous answer is 9

Researcher: so, already you can see there's a pattern
Its 0 + 1 = 2
2 + 3 = 5
5 + 4 = 9
9 + 5 = 14
14 + 6 = 20 and so forth…
20 + 7 = 27
27 + 8 = 35
35 + 9 = 44
44 + 10 = 54
54 + 11 = 65
65 + 12 = 77

77 + 13 = 90
Interesting…
So would you use this kind of question to teach with…

Teacher C: Actually, it's a very challenging question for a bright learner who can …, but we don't teach the equation for the formula for patterns at Grade 6 level yet – we just ask them,…why?
The assessment standard does not ask you to do that so you can introduce them
I don't know how many learners can in Grade 6 understand a formula and how to apply it
Even if they don't get the formula – just to get to see the pattern
Ja, we do teach them patterns and we do teach them to observe the relationships between the previous one and the one you're working with to determine the pattern

Researcher: So, would you give this to your learners as a form of or an investigation or a task

Teacher C: yeah, as an extended activity

Researcher: Whilst you are busy working – doing admin work…

Rukhshana: I don't manage to do any marking – I've got 50 in my class and busy with the learners throughout the lesson
And there's no time to mark?
It's creating a problem in terms of assessing whether my learners are able to achieve the outcomes

Researcher: How many learners do you have Nomkhosi?

Teacher D: I've got 46 in my class

Researcher: Terrible

Teacher C: Huge numbers are problematic because the marking in the class gives you a bit of understanding of where your learners are at quickly
so, if you just started marking a sample of five or six and say hey, there is a misconception, then you can get to the board and quickly deal with it…
the reliability of the answer that it was justified – 90 seems like a big number – but it makes sense…

Researcher: Yeah, it makes sense

Researcher:	Item no. 17 Star shape with 6 lines of symmetry – the shape can be cut into 6 identical quads. The smallest angle is 38˚. How many degrees is the Is this easy or challenging?
Teacher C:	It's challenging
Researcher:	Where would you put this, Grade 6, 7 and 8
Teacher C:	seven
Researcher:	What does it require the learners to do – or know before they can answer this question?
Teacher C:	All angles in a quadrilateral add up to 360˚
Researcher:	Okay, let's look at how you solved it… If the smallest angle in the quad is 38˚, so which angle is 38˚
Teacher C::	I think both of these angles are 38˚ Now looking at your properties of a quadrilateral, how did you go about doing this? The properties of a quad, there are 4 sides, 4 angles and the angles add up to 360˚
Researcher:	How did you go about it, solving this…I see you've tried some nice things here
Teacher C:	Yeah, I tried to figure the centre of the centre point of the star, because we said the diagonals and then I looked at what the angles around the centre point, how many they are and then I discovered that there is 12 I said 360˚ divided by 12 is 30˚, so then this angle here had to be 30˚, and these I assumed – and I think I'm correct, they have to be equal because this is a line of symmetry. So then, these are equal – so if that is to be 38˚, this is 38˚…that is what, 30˚ 38˚ + 38˚ is 76˚ + 30˚ is 106˚ - that's what the 3 angles add up to and the final one has to be 360˚ - 106˚, which gives me 254˚
Researcher:	If this is a line of symmetry here what does that do to the diagram?
Teacher C::	It bisects it…
Researcher:	Yes, and what does it do to the angles of the two angles? The triangles then …equal, isosceles like…
Teacher C::	They're not isosceles because that one is 38˚ and this becomes 15˚. They're not isosceles triangles. Isosceles triangles you need two angles that are equal.
Researcher:	Are you sure that it's 15˚?
Teacher C:	If you're bisecting that angle - the angle is 30˚.
Researcher:	Are you sure that angle is 30˚

Teacher C: Now, I'm not so 100%

Researcher: Maybe it's not 30˚

Teacher C: So, how do you determine that I used the centre point of a circle?
They said there are 6 lines of symmetry
So' let's assume it's 1, 2, 3… you see that's why I was frustrated with that diagram…
Yeah, it is 12…around the centre (doing calculations)
So, looking at this, if this angle is 38˚and that is 38˚
I hope so that I can determine that this angle is at the centre (doing calculations)
I have a problem about 38˚ - I said if these at the edge ,…how many have we got, …1, 2, 3, 4, 5, 6 and 6 six angles and if you have and you draw a circle, then each angle ought to be 36˚, not 38˚

Researcher: so, what's the answer?

Teacher C: I then went back to the examiner and worked with 38˚ but my logic tells me that's it's 36˚ each 'cause if you put your compass there and measure your radius and put a circle around it, there are six angles touching the circumference and 360˚ ÷ 6 is 36˚
No, you know I'm used to a calculator, it's not 36˚, it's …uhm…60˚
I may be wrong, but initially had a problem with that angle there - 38˚

Researcher: So, what's the answer?

Teacher C: I came up to 254˚

Researcher: Are you sure?

Teacher C: 254 + 38 + 38 (calculating) yeah, I'm sure if 38 is right

Researcher: Okay, let's go on to the learners
You never told me if you would teach with this problem…

Teacher C: It's too complicated
I think it will confuse my learners…

Researcher: Okay

Learners' Geometric Thinking Questions

ICAS ITEM NO. 1

Researcher: Looking at the achievement of the learners…
26% of them got 5
17% said 6
21% said 7
Only 29% got 8, the correct answer
So, would your own learners' responses be similar?
Think about your own 50 learners

Teacher C: Yeah, from the classes that I teach, I can tell that they won't
Yeah, 20% would get it right

Researcher: Would your learners get the same results
29% of them got the right answer, 8
26% of them got the wrong answer, they said 5
17% of them got 6
21% of them got 7
So, do you think there are problems there?

Teacher C: With the question

Researcher: with the learners or with the question…laugh…

Teacher C: I think the question is fine

Researcher: the question is fine, so why didn't they say …agh… 4 sided shapes and just count them?

Teacher C: I think that if the teacher did not teach them this … uhm… shape, which is not regular

Researcher: Irregular shape

Teacher C: Yeah

Researcher: so, we dwelt mostly on regular shapes – square, rectangle, parallelogram, rhombus, kite…kite, even the kite we never use that kind of kite

Teacher C: Yeah

Researcher: mmmm…interesting…so how would your own learners solve this kind of question?

Teacher C: I think at least 30%

Researcher: At which Grade?

Teacher C: At Grade 6 level

Researcher: laugh…uhm…not at Grade 4

Teacher C: Not Grade 4. Nooo…The Grade 4 learner is never going to do this …there's the odd chance that the Grade 4 learner can be able to do this

Learners' geometric thinking

ICAS Item No. 1

26% of them said 5
17% said 6
21% said 7
Only 29% got 8 , the correct answer…okay

Researcher: so would your own learners' responses be similar? Think about your own 50 learners…

Teacher C: Yeah, from the classes that I teach, I can tell that they won't…yeah, 20% would get it right

Researcher: would your learners get the same results…do you think there are problems there…

Shana: Okay, with the learners, I think the question is fine

Researcher: The question is fine? So why didn't they say agh, four sided shapes and just count them…

Teacher C: I think that if a teacher didn't teach them this –erm – shape, which isn't regular…

Researcher: irregular shape…

Teacher C: yeah, we dealt mostly on regular shapes – square, rectangle, parallelogram, rhombus, kite…

Researcher: even the kite, we never use that kind of kite…(Concave quadrilateral)

Teacher C: Yeah

Researcher: interesting, so how would your own earners solve this kind of question

Teacher C: at what range, I should think at least –erm- about 30%

Researcher: Which grade

Teacher C: at Grade 6 level

Researcher: (laugh)… not at Grade 4…

Teacher C: not Grade 4
The Grade 4 learners are never going to be able to do this – there's the odd chance that the grade 4 learners can be able to do this…

Researcher: By solving this kind of question, what would they gain…what skills would they acquire…

Teacher C: They will be able to look at four sided figures outside of the box

Researcher: In context

Teacher C: yeah, in context

Researcher: and then those who come up with wrong responses, how would you go about –erm- helping them

Teacher C: I would actually draw a concave kite and count the sides

Researcher: Like these one's who got 5 – how did they come up with 5

Teacher C: I think they looked at the very explicit one's – like those 3 are quite explicit and the fourth one and that one on the outside is four prominent and on that side

Researcher: and they didn't see that… then the ones who came with 6…

Teacher C: I think that one is likely to be ignored because it's so small. … oh we felt, Grade 7…

Researcher: But it's interesting how lot of learners did not get this, but this was at Grade 3 level

Teacher C: I'm not sure that Grade 4 could identify these, so how would a Grade 3 learner do this, because they do not teach them… I know… I've seen Grade 3 textbooks and worksheets

Researcher: Looking at the wrong answers, 5, 6, 7, would you say there are misconceptions or they just didn't count well

Teacher C: This is not misconception – just a lack of experience for that kind of thing – it's not a misconception

Researcher: Okay, and you have to attribute negligence - you know, you get hasty learners …yeah, I know and they didn't look for anything further in the picture. So, it's not even a lack of knowledge of quadrilaterals. In fact what is this question intended to test whether they can identify a 4 – sided figures, so you can't say a child doesn't know 4-sided figures because he can't count four sides

Researcher: Okay

ICAS ITEM No. 7

Researcher: Look at the percentages
A – 6 – 43%
B – 12 – 24% (Correct Answer)
C – 16 – 10%
D – 18 – 16%

Teacher C: The answer is 12, I can see that those that answered 18, didn't join…

Researcher: So, would your own learners be able to do this

Teacher C: Which Grade, Grade 6 ones…

Researcher: What is it that they found difficult – looking at the question – language…

Teacher C:: You see that I'm analysing this… the one that chose D as the answer, didn't read the joining of the gaps, that's the only way you can come up with 18. If you ignore the initial instruction …

Researcher: and then…

Teacher C:: If you look at those that answered 6, they just took 1, 2, 3, 4, 5, 6, of an individual – they didn't consider the others, can you see that , because they possibly thought that it's the same thing – you can't add them over and over again – the same thing and 43% of them did that…
So, they possibly thought – well, if the one is like that and then they are all the same – when you join them… it's still that…
I'm just trying to analyse …and 16 I'm not sure how they came to that answer 16 … (calculating)
Yes, they just guessed

Researcher: So, there are no misconceptions here…

Teacher C: There can't be a misconception here – the only thing is when the child doesn't know what to do about edges

Researcher: edges, that is a conceptual issue

ICAS ITEM No. 13

This comes from the Grade 11 paper:

A – 135 – 15%
B – 130 – 28%
C – 115 – 49%
D – 100 – 6%

Researcher: so, will your learners be able to do this

Teacher C: Not in Grade 6 level – I would not get a higher percentage, maybe I'll get 3 or 6 learners out of my class
You must remember even in Grade 6, you need a lot of repetition around Isosceles for them to understand fully and remember that 2 angles are equal and 2 sides are equal – it's difficult… for some learners

Researcher: 'because it requires a lot of things a lot of things – you must know properties of the isosceles, angles on a straight line…

Teacher C: And also manipulation – you've gotta look in geometry you look at erm, different things that will provide an answer to some questions … and the possibilities of using that, to determine that, also knowledge of properties so that you can exploit the information available

Researcher: mmmm…interesting

ICAS ITEM No. 15

Teacher C: The 90, the answer is right… (Laugh)

Researcher: The answer is correct, it's 90, but …, is there a simpler way of arriving at it

Teacher C: This is a Grade 10-12 question,

Researcher: Are you sure, it was in a Grade 7 paper

Teacher D: It was in the Grade 7 paper, so most of them have failed

Researcher: None of them got it right

Teacher C: This is telling you why because , say for instance you had no. 30 here, I mean I can't go on for the next… it's just not adding to a pattern…it's acknowledging that your number that you adding is increasing by what number…
The question comes from a Grade 6 paper, I don't think it's appropriate level, because at Grade 6 level, I noticed that answering questions about patterns – if you were to use the form that determining of a pattern to find the answer. I noticed that they respond quickly to counting in natural numbers, better even numbers and into multiples of 3 or dividing - you can say it's halved, or quarter or whatever, or dividing by 10. But this one here – to identify that easily is not so easy for Grade 6 learners… and let's say for instance this is 30, I was hoping in the exam, I'm allocating 3 min to this and this is taking me more than 3 min
When you came up with 90 – it was like I thought…this is too much… and then you started doubting your answer so it's basically even more time consuming to check so where is it Grade 11 level, they could have provided a formula or data sheet – so you can apply a formula – this is sequences and series formula

Researcher: So, would you do this with your learners

Teacher C: It will be a bit much – confusing – so you can't in actual fact, you're wrong in thinking that I'd never do this with my learners – you can't underestimate them. There maybe one child in the class that can… so as an extended opportunity… it will take another 3 weeks to explain to the rest of the children – time wasted….

Researcher: At Grade 7, your learners won't be able to do this

Teacher C: At Grade 7, we do teach them to place regular polygons in a circle, how to draw them and how to determine the angles so they may cope if they identify the radius of the circle to determine

Researcher: But none of the learners… what do you think of these types of questions

Teacher C: The pedagogy is not placed on problem solving – it's based on content and rules and regulations. Application on…
When I reflect – we don't teach problem solving…

3. Interview Transcript

ICAS ITEM No. 1

Speaker	Dialogue
Researcher:	Looking at this one particular question – how did you perceive it …was it difficult for you or was it easy
Teacher E:	For me was not difficult well for me it is just about recognizing four sided shapes of there's one in particular that will catch most of the students out and that is the tail of the one fish on the left there um it's a concave quadrilateral its tail of the fish and many of them when they see it they think triangle rather than thinking counting four sides but if they don't literally count four sides… so my answer was 7 and I'm sure most of them - they got 6.
Researcher:	How did you get 7
Teacher E:	Uhm, erm that's, 1 big one 2,3,4,5,6,7, is there something here, okay, I couldn't see - it's a bit light there it would be 8 then.
Researcher:	No, no, no, it's just that this is a copy of a copy, the tests were originally in full colour But again, that would immediately show me that a child would be looking at –er-no, no, were talking about you, now
Teacher E:	No, no for me I immediately saw that I looked for all the four sided shapes for some I had to look more carefully than others, trapeziums and so on
Researcher:	The concave one…
Teacher E:	So this question is not so difficult – they must just pay attention to the answer, recognise the 4-sided shapes
Researcher:	Would you use this kind of question to teach with
Teacher E:	Uhm, -erm- it might have to be a bit clearer – I don't want to confuse children –er- when you give them shapes –erm- I don't know if I'll use fish exactly, yeah…I would use shapes – but not fish. I would try and use them in context, in context that what I'm trying to say..
Researcher:	At what Grade would you…
Teacher E:	For me this is more Grade 5, we need to understand at Grade 5 level, where they recognise shapes; this is all about just recognizing shapes. They're not asked to give any properties, just recognise 4-sided shapes
Researcher:	So, Grade 5
Teacher E:	Grade 5
Researcher:	You say you wouldn't use this to teach with

Teacher E: Uhm, -erm- not necessarily. I would not specifically use that – but for me if a child can individually identify the –erm- shape – what I would do then is give them –erm- something like this to have a combination of different shapes and ask them to identify how many triangles there are, how many squares, how many – whatever, but –erm- uhm, not for Grade 7, not something like this, no…

Researcher: So, this falls under which topic

Teacher E: space and shape, -erm- more importantly, the LO's – I don't know -4 which is space and shape

Researcher: 3

Teacher E: Yeah, 3, LO4 is measurement, yeah, so it would be LO3, yeah…

Researcher: Because what I see is the question – the shapes are in context of a fish pond and the different shapes have been included – they must take out 4-sided shapes

Teacher E: Yeah, no, it's just that a bit different and fun, I suppose if you are in Grade 7 – it's a difficult question to start with – we should start a test with easy questions – and you move from easy to more challenging because they have to sort out …yeah in the context of fish, then I could look at this, yeah…

Researcher: Let's just finish this off with the learners. Would your learners be able to do this and achieve the same results?

Teacher E: No, I don't think I've ever taught a class where all the learners' achievement are all the same – there's always somebody gets it immediately and there's always somebody who doesn't

Researcher: Let's look at the achievement of these learners – we agreed that the right answer is 8…

Teacher E: Yeah

Researcher: Uhm, so look at how many learners got the percentage that got the correct answer, the question papers are in colour

Teacher E: Possibly the one that that got 7 – possibly did not recognise that (concave) as a quadrilateral – that's why this % is very high, they didn't see the concave

Researcher: and these that said 5…, and it's higher than the 7

Teacher E: I'm just trying to see where they got the 5 from – that would definitely have been 1, -er-2, 3, 4, and the 5th one I assume they would have identified this…or miss the big one and this trapezium here as well, it's not always easy to identify trapezium and especially if it's lying with a point like that ---now this time it would be because it is straight – it's a trapezium that's you know…put down a bit – it would be a triangle – they could possibly have missed that one as well…

Researcher: and then those who got 6…

Teacher E: Yeah, okay, 1, 2, 3, 4, 5, 6 – they missed those two – so some of the shapes are not all that obvious to get and some –erm-

Researcher: Is it a good thing or a bad thing…in terms of challenging the learners' thinking

Teacher E: -erm-yeah, look,-erm- in Maths, you've got to learn to teach them to observe carefully – and if you don't get across to them they'll just glance at it and say okay,… there are 5 of those once there, --- when I missed that out I didn't see that one there…so

Researcher: this one,…can you see easily…

Teacher E: No, the concave quadrilateral is definitely –erm- erm-one that you could miss here

Researcher: so, there wouldn't be any like misconceptions here, it's just recognition

Teacher E: It's just recognition and observation – they must observe very carefully

Researcher: and –erm-, so the one's that get incorrect answers did not simply pay attention

Teacher E: And also when children don't know, they guess, especially with

Researcher: so those who got wrong answers here it's not crucial, like they do not have misconceptions or anything like that

Teacher E: Yeah, they just look once and some children I know, they do this test , they rush through in twenty minutes

ICAS ITEM No. 7

Researcher:	Now, let's look at the next one, number 7, which is asking for , -er-… Jade had 3 tiles…
Teacher E:	12 is the correct answer
Researcher:	and then have 43% of the learners saying 6
Teacher E:	I think they just took the one shape – I think they just counted the sides of the one hexagon instead of counting –erm- drawing, outlines where the hexagons join together, so I think they just counted 1, 2, 3, 4, 5, 6
Researcher:	so, would you say maybe they were misled by the "smallest" number…
Teacher E:	Uhm, yeah, yes, I think, I think that is there –erm- it's a wording thing – I think they were caught out by the wording
Researcher:	what is the smallest…
Teacher E:	smallest number of a new shape – they must understand that …of a new shape – the 3 being joined together, but if you look at the shape, it's erm 6 if you count the edges – so I can understand why 43% of them –erm- cause the moment they see smallest number of edges, then they're looking 1, 2, 3, 4, 5, 6…
Researcher:	the smallest number there is 6
Teacher E:	is 6 - yeah
Researcher:	so, would you say that is a …
Teacher E:	yeah, definitely that is a misconception and also language thing, I think You know, what would have been helpful – if they had this here and the new shape next to it with a new joined shape…
Researcher:	that's an idea…
Teacher E: where sides	you understand, then that's –erm- in B, S is where the shapes are apart and B id now the shapes are together with no gaps so now how many …what are the minimum being , see how the answer would have been differed from this…
Researcher:	would you have done it with your learners
Teacher E:	Well, we could try…I could use my own class as a control class and give it to them and give it to the other two classes and then we can see how they would answer that question
Researcher:	but I think it was left deliberately like that,

Teacher E: you see, I think what they're trying to do is, can a child in his or her mind put the shape together and then count

Researcher: In their mind

Teacher E: In their minds, what the new shape is – but it depends on what you're testing here, what do you…

Researcher: Do they even know an edge is…

Teacher E: Well, that's another thing – yeah, but I mean if they got 43% of them got 6, they could have chosen 6 because it's the smallest number and those that got 18 counted all of them together, 6, 6, 6, counted all the –erm- smallest sides together and then added them

Researcher: So, they didn't put anything together and make a new shape

Teacher E: yeah, they didn't make a new shape

Researcher" 10% of them got 16, how do you get 16

Teacher E: I don't know how you get 16…. (Pause)… laugh…I don't know how you get 16…it's just 10% - I'm sure they guessed

Researcher: How do you know what the learners had gained from solving this kind of problem?

Teacher E: Okay, if it is given in this form – if they correctly answered 12 then I would say –er- okay, in this child' mind – he or she could put together 3 shapes that are the same and they could make it into a new shape and you have to understand that you have to count all the edges that are outside…

Researcher: Uhm

Teacher E: of the new shape, so if they get that right then I know that they can piece together in their own mind…the 3 pieces – they're integrating basically there's 3 shapes into one and they're saying now I recognise that there are now 12 sides on this new shape going around so I would know that they can successfully do that…

Researcher: This topic is what…

Teacher E: erm- uhm…

Researcher: putting shapes together without any gaps or overlaps

Teacher E: erm- tessellations, I suppose is another concept that you could use there – I mean tessellations – you teach them – there's no gaps there – you know there are spaces in between in tessellations, so these shapes are moving directly into each other – so you know if you went out there would there be no gaps and you could tessellate it

Researcher: so, would you say those learners who got especially like these ones because they are many – 43% - who said 6 – would you say they've got underlying problems in their minds…

Teacher E: Erm- for me I think it's –erm- it's like they didn't get the concept of these 3 shapes are to forming one because it's not there – it's not obvious, you have to conclude in your mind that these 3 shapes are spilled into each other

Researcher: when they're together

Teacher E: when they're together – it's a new shape – it's no longer 3 hexagons – it's a new shape now – it's a 12-sided shape – it's a dodecagon

Researcher: How would you correct their errors

Teacher E: In this case, so, yeah, okay, here you gotta be –erm- again there's terminology that comes in and these 3 shapes are gonna be together to form a dodecagon – so now if they don't know what a dodecagon is, being a 12-sided shape, you say there is 6 sides, 6 sides, 6 sides, but when we join them together now, becomes a 12-sided shape, 'cause these edges in the middle are now joined together and no longer exist as such, they no longer exist as such, they no longer exist – yeah – I say most of them know only up to decagon- so for them it will be difficult to see a concept but until they see that it it's a new ting being created then they can understand if there's going to be new properties of that particular shape

Researcher: it's like they didn't read the whole question – they just picked, smallest

Teacher E: Okay, if we say what is the smallest no. of edges Jade's new shape could have –erm- I think also what would be useful tool is to say uhm- introduce the concept, introduce the word – Jade's new shape would form a dodecagon – how many edges would the q dodecagon have - you know what I'm saying. So, if they count 12 – okay, means, 12 sides means dodecagon – so they're learning something new in that thing – I mean that's how I would go about doing it…

Researcher: I like that you would have a complete new shape here…

Teacher E: Yes, A and B. A would be the 3 shapes, B the dodecagon – you see then they could also explore what a dodecagon has

Researcher: How do you normally teach tessellations?

Teacher E: Right, tessellation –erm- we first of all say it is not possible to have tessellations – if you don't have a regular shape – to create a pattern – the shape has got to be regular shape and as I explained – there has to be no spaces – nothing left over – no overlapping, so, first of all they've got to understand that concept and uhm what I do is – we – I've got different shapes – I've got rectangles, triangles and I've got regular triangles – do

Researcher: Do you use manipulatives

Teacher E: yes, manipulatives and also uhm what I do is – what I've done before – I've given them a sheet and they have to cut out different shapes and they have to cut out different shapes and say can you tessellate these shapes
Is it possible to say tessellate the shapes into one line – if you can, if you turn it upside down, you can – like some triangles – you can do that – and so they have to understand

the concept that tessellations means it's got to be a regular – so you ask the question, can you tessellate a square, yes – a triangle… now be careful because some triangles – if it's a regular triangle – yes you can – uhm – but, but – they've got to start playing around with it …

Researcher: The most important thing is play..

Teacher E: Yeah, they must play around with the shapes and –erm- even I use a lot of paper uhm and folding an A4 paper into half, you know, what shapes do you now have and then so even with those –erm- I did that last week – I was doing folding of paper with them to get different shapes and also we were looking specifically but they were also shapes there were made in that -erm-

Researcher: so, hands on activities are key…

Teacher E: Yeah, hands on activities, I think for children up to Grade 12 still who are very Piaget' level of concrete…they're still on the concrete level of thinking so they don't have stuff to manipulate they're gonna battle especially Geometry they're gonna battle with it Even in Grade 7 at 13, they're starting to go to semi-concrete and then to abstract, but they're still pretty much rooted in there so we ask the children to do this which is in a lot of senses…abstract, you gotta picture – you gotta imagine something like the one with the shapes with the blocks – children have to see that – they're not old enough, some of them to see that…

Researcher: Interesting…

Teacher E: Let's go to the next one number…

ICAS ITEM No. 13

Researcher: This is a diagram – KLM is an Isosceles triangle and KN is a straight line and it's not drawn to scale – you can't just measure and come with an answer – what is the value of angle KMN

Teacher E: It will be 115˚, cause you see the thing is that again children have to understand that in an Isosceles triangle,

Researcher: before you talk about that – how did you find the question?

Teacher E: The question, to me, this is pretty – for Grade 7's, pretty straight forward

Researcher: Okay

Teacher E: In an Isosceles triangle

Researcher: You say Grade 7? What about Grade 6, they won't be able to…

Teacher E: No, this is a Grade 7 question. For Grade 5, not so easy – Grade 6 – they must know this here look in Grade 5, we're introducing the Isosceles triangle and we say 2 sides and 2 angles are equal – but there's no application as such. They're just learning the property that an Isosceles triangle means 2 sides and 2 triangles are equal at Grade 6 – we expect them to apply that – Grade 7 – they certainly must know it --- easy for Grade 7…

Researcher: When you looked at it did you say it's a difficult or an easy question

Teacher E: -erm- look – for me it was pretty straight forward question

Researcher: because it's asking what the properties of Isosceles and Straight line

Teacher E: and a straight line yeah, and this also …this little sign here indicates angle – some children don't understand that sign – so they might be looking for something that we don't know – what they're looking for – so I would prefer to say what is the value of an angle in words

Researcher: so, what mathematics strategies would you use to find 115˚, how did you find 115˚

Teacher E: Erm- for me - mean in the triangle, this should be 65˚, therefore this angle LMK would also be 65˚ and then in the straight line – a straight line is 180˚ - so if you subtract 65˚ from 180˚, you land up with 115˚

Researcher: so would you use this type of question to teach with

Teacher E: Yeah, this would be excellent to use because all you giving children is basically – one angle – but you're giving information here which is additional – those two lines indicate that this is an Isosceles triangle – we would be expecting them to apply their knowledge of what they have done – the first thing we would teach them is that a straight line is 180˚and therefore we would then go on to say if I gave that angle at 65˚, you would be

able to work that angle is what – and if they added that angle and that angle, because these are co-exterior, those 2 add up to this one here angle… angle K and angle M are equal to angle N. As I said I did Maths at High School so for me these are basic concepts. But to teach it to children – and we were just taught that –er-theorems – we were taught that to learn it and know it – we didn't actually work out why they are equal – but I just know that they were…(laugh)…but now I'm trying to get my children to work out hat these angles are equal. So, if I give them this angle here, how would you go about working out what these 2 angles are… and then I said to them explain to me what you are going to do – and I like them to explain – say this is 50˚ - the angles inside the triangle is 180˚ - I subtract 50˚ from 180˚, which leaves me with 130˚ - these two angles if they are the same, then I must divide 130 by 2
This is how we teach, erm, around these particular concepts –erm- I like this – I like working with this particular shape

Researcher: Yeah, interesting – and this is – under which topic is this?

Teacher E: Uhm these are…angles – which disappear in LO3 still when it comes to measuring of angles, LO4, at high school LO4 actually disappears – we don't ask them to measure – We just have four LO's – but it's definitely a mixture of 3 and 4 in this case for up to Grade 6

Researcher: Interesting – so let's look at what the learners got. Can you guess in which paper this comes from…

Teacher E: Grade 7

Researcher: Eleven

Teacher E: Eleven, this for me is too easy for Grade 11. My children in Grade 7 can do this…

Researcher: Really and they can do much better than this – look at the scores
A - 135˚ - 15%
B - 130˚ - 28%
C - 115˚ - 49%
D - 100˚ - 6%
You say your own learners can do this…

Teacher E: No, Grade 7 – my Grade 7's will be able to do this because…

Researcher: But Grade 6, can you find 65˚

Teacher E: And when you use the straight line to now that you have a straight line – because that's how you –erm- tell them right at the beginning what these things are, straight line is 180˚
- properties of – when it comes also to – I'll just tell you how I go about teaching the properties of a triangle – angles add up to 180˚ - you draw 3triangles of different shapes, they mustn't look the same and I get to measure the angles of those things and I want you to right down in pencil on the shape one and what it adds up to, and then they'll have 179˚ - but it's all similar to 180˚ - I say can you see a number that's there on all 3 – it should be more or less the same. Okay – look at this one…it's more or less the same – so why still the same – so I say find somebody who's got triangles very different from yours and see what their answers are – so now we're collaborating it's all more or less the

same…so why still the same – so why still the same - so I say find somebody who's got triangles very different from yours and see what their answers are – so now we're collaborating it's all more or less the same… then we talk of accuracy of measurement and all of that but interesting stuff this… I love this particular aspect of theories, if they know how to apply…an Isosceles triangle is two equal sides and angles but…I have to find out what this angle is – I can't…that they can say angle K is equal to 65˚ because it's an Isosceles triangle and these two sides are the same and these two angles at the base – have to be equal…

Researcher: So, which of these incorrect answers reveal misconceptions on the part of the learners, the once who said 135˚, 100˚

Teacher E: for me, the 100˚doesn't make sense, I mean and the 130˚ there's no – I mean I can see 65 + 65 is 130 – I can understand where they get the 130 from, but this 100

Researcher: But would you say that's a misconception – why would they add …

Teacher E: Look…I don't know why they would choose a 100 out of the 4…-erm- because let's just say if you think of it logically – how do you arrive at a 100, but erm at all, cause even if you look at the interior angles of a triangle being 180 – then that's 65˚, that's 65˚ is 130˚ there is 50˚ over there. So, 50˚ and 65˚ they don't make 100˚ - they don't even understand the theory of the Isosceles triangle – because they should be able to work out what these here – interior angles are equal to co-exterior angle there – they can't even apply theories, they just maybe know it – or they don't even know it – but they do know it – they still not applying this very successfully

Researcher: Get them through practical's…

Teacher E: I think also – I think these guys made mistakes on the very concrete stage of development, which is taking the shape and playing around with the shape and playing around with the shape –erm- the Isosceles triangles getting them to measure, investigating - measure the angles

Researcher: Properties on their own…

Teacher E: It takes a lot longer to do especially if children have to cut …understand it more…when they've done it when I did that other thing last week on shapes and they were folding … One of the children actually recognised similar to that char of fractions… and I said – that's exactly what we're doing …so they're making a connection and I think a lot of these children – doing a 100 – missed out on the that particular stage of their development

Researcher: Uhm, interesting

Teacher E: Yeah, you must give me these examples – so that I can know exactly which to do. But I'm also working on baseline tests for Grade 4, 5, 6, 7, so beginning of the year…

Researcher: When do you teach Geometry?

Teacher E: Normally, third term. You know first and second term – they're just dealing with numbers and tallies maybe graphs with them but we're doing percentages, number concept the first two terms…

ICAS ITEM No. 15

Researcher: The diagram shows the diagonals of polygons, triangle, square, pentagon, hexagon, they're all regular – that's what it says. How many diagonals would a regular polygon with 15 sides have

Teacher E: that would be 15

Researcher: you say it would be 15

Teacher E: Yeah

Researcher: First of all, this question – is it difficult or easy

Teacher E: It's erm, you see, if the next…or edges would be 7 and even number of sides can only split down in the number of sides that it has. So this one is 5 – you can't get 10 out of this, you can't do it with a pentagon

Researcher: what about that one with 6

Teacher E: It's different – even number of sides – so there are different ways in which you can do this – you can do this – you can halve it – you can go …diagonals this…so there are different ways of doing it – so, like you see and hear you come down straight on the middle there – you can go straight across the middle there – so those are ones that you wouldn't get – with a pentagon – you can

Researcher: working with a pattern – it says here a 3-sided shape, no diagonals, a four sided in this case has got a 2 and from 0 to 2, 3 to 4 and then here there's 5 sides
0 – 3 – 5, then to 6 – 9 and is there a pattern there
There's 2 there – there's 3 there, then 5, 6, 7 as you would go on it would increase

Teacher E: Would you say there's a pattern

Researcher: There is a pattern there - yeah

Teacher E: Look here – 0 – 2 and then it goes to 0 + 2 is 2; 2 + 3 is 5; 5 + 4 is 9 , so …

Researcher: So for 7 it will be 7 + 5 is 12

Teacher E: I don't think …

Researcher: but according to this pattern

Teacher E: Oh yeah, according to this pattern – yeah – they would be…

Researcher: for 8 it would be plus 6 and so on…so how do you handle questions like these where you find this pattern…don't you draw a table for instance…

Teacher E: Tables, I find are the easiest ways in which to do this erm…we would have – depends o what we're gonna be looking at erm…

Researcher: looking for the number of diagonals…

Teacher E: okay, here would be our shapes here…so it's triangle, it will be square, it will be pentagon, erm- hexagon, so you would have –erm…their sides there, diagonals – so the triangle, square, pentagon, hexagon, heptagon which is 7, then according to this pattern it would be 7 + 5 be 12. The octagon we gonna have …it would be 12 +6 is 18, it would be going according to this pattern that we're getting here
And there can only be a minimum of 2 diagonals erm you can't have 1 diagonal in a shape – so there is a minimum of only 2 diagonals – yeah, but this is not an easy question, erm – conceptually the child has to ---I mean if they can move those 3 shapes together to form one new shape

Researcher: So, would you use this to teach with

Teacher E: Uhm…yeah, I don't think I'd use the shapes as such erm, unless I can put it in a form of a table like this…I think I would teach it together – rather than try to work out what is gonna come next – rather teach them patterns that would be forming – when the pattern starts and also the diagonals…
Look, it's a difficult one to teach I think – when it comes to this – many teachers try to stay clear of this because in their own minds they're thinking…

Researcher: When do you do diagonals … Grade 5, 6

Teacher E: Yeah, in Grade 6, they definitely should know diagonals because on our wall, we've got properties of your different shapes – and – er – definitely the diagonals are in it, yeah --- so Grade 6 they're learning – they're going to the next level – saying what the properties of a rectangle are, triangle. In Grade 7, they've started now to apply these shapes – they have to identify…so this I will only use during Grade 7

Researcher: This question was in the Grade 6 paper

Teacher E: Sure not, not easy

Researcher: Let's say you would use this – your goal would to help them to work out diagonal and also to work patterns

Teacher E: Yeah, to work with patterns as well…the fact that it's a regular polygon - it's not all 4 or 3 sided shapes that will have this – only the regular polygon and I think they have to understand the concept of regular polygons all the edges are the same

Researcher: What about stretching it to 15 sides – we normally go up to ten sides

Teacher E: Yeah, look in Grade 6 we're not – we haven't introduced them to 15 sides – but if they follow the pattern which is important – they should be able to extrapolate what is going to happen if the same thing we do with 10 sided shape, with 15 sided shapes – they could even as long as you figure out what the pattern is

Researcher: Do you think this is a good question with regard to problem-solving

Teacher E: Yeah, higher-order thinking skills - definitely

Researcher: the learning outcomes…

Teacher E: and also theories of learning. I mean if you're looking at shapes – how are these shapes the same – how are they different –erm- because children also – we have to teach them geometry how this is the same – how it is different –erm and when they recognise similarities – they can also start making patterns easier and to form patterns

Researcher: definitely drawing a table

Teacher E: drawing a table I would say these too should be in this question – I would have asked this question – you would say record these in the table below…and then you would have the 15 sided shape and say what would the…

Researcher: …easily find it with the table

Teacher E: with a table yes…

Researcher: The question paper says just write down the answer

Teacher E: Yeah, no exactly erm, yeah, and it's wrong to just expect children to write down the answer. In Maths you've got to systematically work through something they can't just write down the answer otherwise they are gonna guess and just write down the answer…

Researcher: okay, so how many do you think got it right

Teacher E: I wouldn't say…I don't think any…

Researcher: none, with the teachers, only two got it right – they got the pattern

Teacher E: Yeah

Researcher: The answer is 90

Teacher E: what...90, yeah if you follow the pattern then

Researcher: to draw a table like this

No of Sides	3	4	5	6	7	8	9	10	15
No of Diagonals	0	2	5	9	12	18	26	34	90

Teacher E: I don't think any of my children would …

Researcher: we haven't finished about the children – would your learners be able to do this

Teacher E: No, no you see the thing is that erm I'll ask a question – how many of you can see a pattern and the temptation is to give them the pattern but even when they discover…

Researcher: But if you can start the pattern like that for them – they can be able to finish it off…

Teacher E: If you go up to 15 sides…and so…erm… and again only the very few in each class that I would imagine that would logically conclude where you would end with the …

Researcher: so you would use this as expanded

Teacher E: definitely expanded…and I would not do this towards the end of the year let's say in Grade 6 – I would never give it to them immediately when I start teaching – because to me they're still ---gaining the properties of –er-then, when you revisit again in the 4th term you're hoping that they're going to refresh their memory about what they did last time
Yeah, we learnt this last term – now we're going to go further and we're gonna try to look for the pattern and apply the pattern to what we're gonna do here

Researcher: by solving this problem – those learners what would they have gained

Teacher E: To me, they can recognise patterns which is so important in Geometry – and not only do they have – do they recognise the pattern – but they can also follow a logical conclusion as to what 15 would then be – in terms of this particular thing – so, they can correctly computate the answer for me

Researcher: So, the visual – what they see – the diagonals and work out

Teacher E: And then work out what

Researcher: higher order

Teacher E: definitely higher order thinking – but this is lovely, lovely question

ICAS ITEM No. 17

Researcher: 6 lines of a star – 6 lines of symmetry the shape can be cut into identical quadrilaterals

Teacher E: …concave quadrilaterals again…(laugh)

Researcher: the answer would be…

Teacher E: 208˚

Researcher: First when you saw, the question – was it difficult or easy

Teacher E: For me I did not find these questions difficult – but erm, we are thinking of a child erm, they see it with very different eyes

Researcher: I want your geometric strategies …how did you arrive at 208˚?

Teacher E: Uhm, the quadrilateral again is 360˚and the way I teach my children is that if you have a quadrilateral, like this – shaped like this – If I draw a line from the base there to there…I'm gonna have two triangles – I know that a triangle has 180˚ each
If I extend this line here, going down here, and I have 2 triangles – I know that a triangle has 180˚ in it –and so if I've got 2 – then it means its 360˚

Researcher: Uhm

Teacher E: You see that's how you can teach them about a quadrilateral – it's 360˚. So just by extending that line down there – you have 180˚ and 180. You have 38˚ there ok, and this is a regular shape so this is 38, which is 76, so this angle here is also 76 'cause it's 38 there and 38 there – so as I said there are various ways of going about doing this – but if it's 38 - 38 and this is gonna be 76 it's gonna be 104 – 104 plus 104 – because it's that angle plus that angle is 208
But the thing is that they see one shape and they don't learn to break it down – it's very nice shape. I love concave quadrilateral

Researcher: So would you use this to teach with

Teacher E: Yeah, I love this

Researcher: what is this, the topic

Teacher E: Look; you're still working with shapes – properties of 2-D shapes and I don't think most children are familiar with concave quadrilateral – when they see this shape – they think – star – diamond, they're not questioning what shape they're looking at…

Researcher: and they throw in lines of symmetry

Teacher E: Yeah, lines of symmetry

Researcher: Is this confusing or what – the lines of symmetry

Teacher E: It's 6 lines of symmetry…for me as I said it's quite easy to come to 208 if I take into account 2 triangles – 38 – 38, this is a regular shape because it has - 6 lines of symmetry and they are divided equally

Researcher: So, how did you go about assisting learners to…

Teacher E: Looking at…okay… I…

Researcher: …you're using…

Teacher E: where would the 6 lines of symmetry go when we talk about lines of symmetry? So, I said just show me in pencil – they'll go down there, down there, so they must do it practically – so that they understand what the lines of symmetry actually mean. This is assuming that the children know what lines of symmetry are

Researcher: and this question was in the Grade 7 paper – none of the children got it right

Teacher E: Yeah, I can understand that

Researcher: What about your learners

Teacher E: Well, I just said hopefully what I do is I try and teach them to break down shapes that they don't recognise into shapes that they do recognise – there are also other questions that have 2 shapes together – they say for instance – what is the area of this shape – that doesn't have that line there…

So, I said you don't know this shape – there is no shape…but you can draw that and you'll be left with a triangle and a rectangle and you can work with something you know…understand…so – so – this would be a shape that they didn't know but if you draw one – it means you'll end up with two triangles

Researcher: So, what would your learners find difficult

Teacher E: In doing this…I don't know – first of all erm- most of them would say how do you fit this into this – I always teach them to make a connection how exactly would these fit into this here – and – so for me – look here this shape fits in there, but how many children can visualise that, it sounds strange – you've got like these shapes here and you give them like 20 of these shapes – see if you can form this particular shape – you know what I'm saying – they're playing around with shapes – and again what's that Japanese game…with the different shapes – where they gotta build…Tangrams

Something similar to Tangrams, but you – now you're giving them this,… see if yu can form this shape with…using these. So if they're clever – they'll see that – that corresponds to that and all they have to do is put 6 of them around…1, 2, 3, 4, 5, 6, you see, erm- but a difficult concept to teach if you leave it as one big piece like that – and you don't teach them the children to break it down and have a concept of putting these into there – but if I give them 6 of these things I say – try to form this shapes you can work it out – you know what I'm saying – mine is – I suppose – analysis –I'm strong on analysis – they must be able to analyse when something is apart …here is the big shape – what is it made up of…

This big shape is made up of smaller parts and what are these parts and I'm saying…So how does this fit into shape, so but if they recognise it, it's fantastic – but as I said, it's again a small percentage of children – 2 maybe 3 in a class would get it if they were given

Researcher: expanded work

Teacher E: expanded work for them

Researcher: for those few…

Jeffrey: and again they can logically deduce this from there like smallest angle 38…erm…and they are going to take that as the smallest angle – not angles – but there are two that are the same size. See that can also be confusing to children…
When you say smallest angle – then you think – there must be only one – they'll think that one is 38 and that one is not 38…

Researcher: then you can use your visual sense – because that angle can't be 38 because they say the smallest angle

Teacher E: It's a language thing – angle!

Researcher: the smallest angle can't be that because…

Teacher E: No, you and I will see it, but with children when you say angles – then they'll look at more than one – given – that must be 38 – if it's just angle – singular – then they're thinking one angle – these 2 are the same – so this cannot be the smallest angle

Researcher: so, the smallest angle

Teacher E: then they come to a different question – so language here – I would say the smallest angles in the quadrilateral are 38

Researcher: basically, they need to know what…the properties of a quadrilateral…

Teacher E: quadrilateral

Researcher: and also the properties a tri…

Teacher E: triangle, yeah – if they know to –er- because in a regular quadrilateral, you've got diagonals which will form your triangle, understand, because it bisects it, so evenly, erm, when you go either side, so they have to logically follow the steps – they miss out and they say I can't see how you got those…from that thing there, you understand – so –erm- but if you play around with it…you say…okay – now you have 6…but playing in Geometry is crucial – they must play…

ICAS ITEM No. 14

Researcher: What do you think about this question?

Teacher E: This is not an easy question – again you've got a new shape - they've given you some indication that it's made up of 6 –erm- trapeziums and otherwise it would have been that – otherwise they could have completely joined – okay this is made up of 6 identical blocks – and then you would but they have given you some indication of – in a sense – they've made it easier and not many children know the name of a trapezium, this is the Isosceles trapezium…

Researcher: This means that 2 sides are equal…

Teacher E: are equal, so coming again to this, I got to…

Researcher: How did you solve it

Teacher E: uhm – again I know the angles of a trapezium which is 360˚
uhm, I always say to children, if you draw a line going straight down – that from there to there is 90˚ - so it's obviously more than 90˚. You can figure out, how much more than 90 is that particular angle going there so which ones here are more than 100 or is it 120˚... I said to him look erm – that looks more to me like 20˚ than 10˚…10˚ is actually smaller – so erm-this 120, 120, 60 – it will be 60 there – so that value over there, but what's confusing is because it's the short side – so you think the angle will be smaller

Researcher: what's your answer then?

Teacher E: it's 120˚

Researcher: isn't that –erm- that hexagon helping…

Teacher E: Yeah, again, look – the – I don't know if it's particularly helpful
You see –erm- 720˚, there you've got to take into account – I don't know particularly that it helps

Researcher: the learners know that angles of a…the inside angles of a regular hexagon…is 120, so…

Teacher E: if a child thinks differently – that might help them but,

Researcher: If I make triangles

Teacher E: Like I said – for children, maybe one – sometimes more than one – you must find out which way you understand this which way – you most comfortable with this – I said no, that's a- let's say you must use this particular method, otherwise you can as you understand it. Call to me as you understand it and I will not mark you wrong for something that's correct, but …your brother/sister a another school may have leant something from a teacher there they taught you – but I didn't teach you, but no, I can't mark it wrong, I said no, I can't because the answer is correct – then you've explained how you got it, yeah

Researcher: What level – Grade, would you teach this
Teacher E: I would say Grade 6

Researcher: it was in the Grades 8, 9, 10 and 11 papers

Teacher E: but Grade 6, when they're leaving – at the end of Grade 6 I would really like to test them on things like this, because for me, it's not good teaching them only about theory or giving opportunities to apply 'cause what's the point then of just teaching them the theory – in your 4^{th} term you're doing work that you want to move on to next or the following year – so once they've…I find that the concepts in Grade 6 are much more difficult than in Grade 7 – to go over it again when they've…once they've caught it in Grade 6

Researcher: What is this topic here

Teacher E: It's still space and shape and angles, measuring of angles which is LO4

Researcher: look what the learners got – this is from the Grade 8 paper

Teacher E: It's that angle, the side is shorter

Researcher: what's the angle – you can see the angle is bigger than 90˚

Teacher E: Yeah, correct

Researcher: …46% of learners…

Teacher E: that's almost half the class – as I explained to you I said – when they see the short side , they assume the smaller number –goes with the shorter side – which is not the case
As I said, what I teach my children is to draw a line to the base – then you know it's bigger than 90˚- and all you have to check out is how much bigger in degrees is this angle really – look at your options, if it's 100 – certainly more than 10

Researcher: so, if you look at it – 46% of the learners got this wrong

Jeffrey: Yeah, you see – at Grade 6 – this you can trace back to Grade 4 and 6 where they didn't understand these concepts and teachers have moved on

Researcher: They've got gaps

Teacher E:: Yeah, they've got gaps in their learning, that's why I said - I refuse to listen – when children tell me other children are doing this and that at other schools, I say – no – no – no- I don't move on to that until I know that you understand what the properties of this triangle is – what erm, and they have to get – 'cause you can't answer these questions unless you understand

Researcher: You can't even visualise

Teacher E: Yeah, you can't visualise that angle is bigger than 60 and if you can't see that, that angle is bigger than 90, you're going nowhere because you – yeah, this angle you can say is bigger than 90 – straight from there and I ask you – what's this angle here – it's less, you can see it's less

Researcher:	240˚ - 7% of them got it wrong
Teacher E:	…it's an obtuse angle – yeah, it goes up to 270, but it's no 270 They're guessing – they know an obtuse angle is more than 90 but they don't know up to where it goes, yeah – so this is 180 up to 179 -180 is your straight angle , but that's why I'm saying – they've learnt concepts and facts in isolation. We need to apply them - for me, it's a big thing – get them to apply as quickly as possible what they have learnt, otherwise they can't see what the application is – 'cause again, research says – children learn facts in isolation - we have to integrate and show them how to apply these things - yeah -
Teacher E:	I'm interested that 46% of the learners got 60. Clearly you can see that angle is more than 90
Researcher:	Yeah, just visualizing – at Grade 6 level, already they should be able to visualise
Teacher E:	but certainly this is the stuff that we deal with
Researcher:	How do you begin to help them to deal with these concepts
Teacher E:	let's say the whole idea of teaching children to measure angles Mmatladi, tell me; what is this angle here – I think it's 40 okay, now take your protractor and you measure it for me, and also by them doing hands on
Researcher:	uhm
Teacher E:	Guess first, then you measure – they don't like to guess, you say estimate
Researcher:	they don't like it
Teacher E:	Yeah, a friend of mine … and they poke fun at them, when they answer something that's incorrect – they're "gwara", they say – they "gwaring" us, - but I'm saying that even the best of mathematicians, I mean Einstein – took a guess and said I think – this theory was – I say even he was wrong on occasion and he had to rework and redo – it just doesn't – the answers – don't just come like that
Researcher:	why they don't like estimating, I wonder…
Teacher E:	giving them the chance to estimate is very important – when you estimate – you're not asking to be correct, it's more or less… Yeah you're not saying if it's 78 and you say 80 – when it's 2 out – that's nothing – but if you're telling me a 90 angle is 50 , then I'd say look at these angles – why do you think that's 50
Researcher:	Basically, you must start with measuring…
Teacher E:	measuring, yeah
Researcher:	then they can be able to estimate

Teacher E: estimate yeah. But as I say – when they draw their own triangles – not triangles – they gonna draw their own triangles so in other words

Researcher: This is 30 – 60 – 9-. That's 120. When you work with the protractor, and you draw and you draw, you can actually like at an angle and say that's 130

Teacher E: Exactly, when – even in Technology – they say to me, Sir, this is 75. I say that's not 75 – I say – I don't have to measure it to know that it's not 75. They say Sir, how do you know…I say give me a protractor and you measure and you find it's not what it's supposed to be and they can't figure out how you know that but also in the angles of a triangle I can say this is that…and that angle can't be – what do you say 34 or that – it's gotta be bigger or smaller – and then they ask; - how does he know, and I say I know the theorem that says that the interior angles of a triangle add up to 180°and the properties of a triangle – but once you play around with these things, once you use it –

I say if you go into a butcher and you say to the guy, I want R40 of Mince – I mean he can grab more or less something that's close to R40 and maybe just take a little bit away – doesn't it ever make you wonder – how does this guy know – that's because he works with it every day. If you say no I want R20 – he knows he must half the R40 mince – that's why it's good for us to estimate and to learn how to do these things – even a tailor will look at you and say –erm – you're gonna need about 1,5m for your academic gown, …they will measure eventually – but I say yeah, it gives them a rough idea – to check if they have enough material needed.

Yea, it's very important for me that they learn to estimate without feeling that I'm a complete idiot because I gave you the wrong answer now – then I say to…there are times when I ask you to guess – In Maths, not everything that you can just guess – always work it out – unless we ask you to estimate before you start measuring – you just don't go wild – if I ask you the length of a rugby field – I mean you're not gonna tell me 400m – no field is that big. Then if you don't know the concept of metre, then you we can begin to address all sorts of problems – so one of the things I did in a project with them – they had to weigh 60 teachers, but estimate before – when you look at somebody – you must estimate how much you think they weigh. Estimate me at 68kg, so when you go to Mr Mluleki – he is not gonna weigh 70kg – he's much bigger than I am – he's not even gonna weigh 80kg – more than that – so that's how you estimate – you have to give them benchmarks…

Researcher: What do you think of such questions – in our own curriculum – we work with questions like these – we ask our learners to do these kind of thing?

Teacher E: Yeah, I don't think we set up questions like these in a multiple choice erm,…

Researcher: Even if it's not like that but the kind of questions

Teacher E: Look, the concepts are definitely – we teach the concepts – I definitely teach this stuff – but not in this way – you know, when they ask questions – that's another thing that we need to address – children get used to particular wording and description

Researcher: what do you think about integrating concepts – because here they integrate concepts – look at this one question – there's integration of concepts – lines of symmetry, quadrilaterals, diagonals and patterns?

Teacher E: No, no… that's where we hopefully want to move with our children –erm-…I mean you can't learn about angles in isolation – you can't learn about symmetry in isolation – at some stage, you must learn how to integrate all of this – what does it mean that I know angles, etc…erm…if I'm a tiler – what are the things I need to know …how to …area – I must be able to work out area if I want to tile a room I need to know how big is this room and how many tiles I'm gonna need…
That's what makes Maths for me – interesting

Researcher: …, thanks very much.

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Researcher:

Teacher E: No, they come to me and give me memorandums of tests that I have to correct. Then I say, what they are teaching the children…

Researcher: If they themselves can't understand this, so what do you think is the problem?

Teacher E: I think the teachers in the Primary schools – if a Maths teacher leaves a primary school – the chances of them getting another teacher to replace that teacher are often not very good. So, whoever is coming in has to learn to teach Maths if they haven't already taught Maths – or they take one of the existing teachers – I mean by the time I came to that school where I teach, they already 4 teachers – I was the 4th teacher to teach them Maths and some of them were teaching the wrong concepts – to unlearn , like fractions – Oh – and they say Sir, Mam before you said… see you can't add denominators of different kinds by simply adding them together – you can't do that – it is wrong – get it out of the mind – we can't do it like that – but again…

Researcher: Normally, how do you deal with misconceptions that are built over the years – all these wrong things? Do you start from the beginning…

Teacher E: Again, it's going back to –erm- like with fractions when you take –erm- this piece of paper – we're gonna fold it in half and fold it in half again and we're gonna open it up and fold it – I say this is called landscape – and if you're talking Microsoft word – this is landscape and this is portrait. We 're folding it like that 4 times and I mean the –okay- what do we have now – count how many squares there are… others can say there's 4 lines – there are 8 across, 32 Sir. I say correct, that's when I make them realise multiplication table – important 4 x 8 is 32. It should take you 30 seconds and not 3 minutes because you can't count fast enough…

When we go out to the properties – sort them out into groups and they do it hands on – no longer working individually – for me it's important that they do it individually – so you can see who is working and who is not working – because when I put them into groups eventually – I expect them to give a contribution so I put somebody who understands in the group by looking and observing, they are actually learning a lot – when they see somebody else doing something – they may not be able to do it themselves, they battle, but when they see somebody else do it it's like easy for them to say – you see how folded and cut it there and so on…

The thing is to get everybody working. In my class I never sit and the children that come to me for substitution – you can't come to my class and sit here and do nothing – we are doing Maths – do something. Yes – if you don't want to do Maths – you are doing something else – English whatever – you're doing something…

4. Interview Transcript
ICAS Item no. 1

Interview transcript with teacher F

Speaker	**Dialogue**
Researcher:	So, what did you record your answer as?
Teacher F:	The answer is 7.
Researcher:	The question says
Teacher F:	How many 4 sided shapes are in this picture
Researcher:	Show me the 7
Teacher F:	1, 2, 3, 4, 5, 6, 7
Researcher:	And that's it?
Teacher F:	Yeah, that's it
Researcher:	So, would you say this type of question is difficult or easy
Teacher F:	Depending on the level of the children - to me this is easy
Researcher:	What does it require?
Teacher F:	It just requires for me to recognise 4 sided shapes
Researcher:	only recognition
Teacher F:	recognise, identify and basically that's it, to recognise
Researcher:	and be able to sort out other sided shapes from 4-sided shapes. Are you sure the answer is 7
Teacher F:	Yeah, at this moment I think the answer is 7
Researcher:	Check again…
Teacher F:	1, 2, 3,4,5,6…
Researcher:	The big one and what about this
Teacher F:	Yeah, okay, that one it's sort of confused me but not really confusing as such. If I were to get a piece of paper to sketch this one but…
Researcher:	What made you miss it out?

Teacher F:	Ah, in fact this triangle given and this other bottom fish - but that made me think it's not 4-sided but is actually 4 sided, you see…
Researcher:	Looking at this type of question, would you use it as a problem that you can teach with
Teacher F:	Depending on the level of learners, maybe, Grade 4, 5, 6, 7
Researcher:	Interesting, this comes from a Grade 3 paper
Teacher F:	Yeah, depending on the groundwork done in preparing your kids
Researcher:	So, do you think it's a good kind of problem
Teacher F:	Yeah, is good, like I said depending on the level – it's good
Researcher:	Then how would you plan to teach with it
Teacher F:	Ah, in fact, this one would be…
Researcher:	under which topic
Teacher F:	Space and Shape
Researcher:	Specifically under which section
Teacher F:	shapes, yeah, 2-Dimensional shapes
Researcher:	Using this type of question, what would be your goal, 'cause you missed that one (concave quadrilateral)
Teacher F:	Yeah, in fact, this question must be taught in a gradual form the educator must start with the simplest shape say Grade 4's – you don't just do everything at the same time. Start with a square, identify that and then, maybe er…say parallelograms, then now – maybe you need to combine 2-D shapes together dealt with them and now they can identify them amongst others
Researcher:	Interesting, research has shown that most teachers teach shapes separately – you teach triangles, you teach quadrilaterals and you focus on these type of quads (4-sided shapes), these ones – that what you not realise that this one (concave) is also a 4-sided shape.
Teacher F:	unfortunately…
Researcher:	but normally we teach them separately, different types of quadrilaterals
Teacher F:	but from my experience, I'm well aware of such shapes if we were to be given this one in a text book as such – you would find that learners… Learners can you count – my mistake was I did not count. I should have counted and say 1, 2,…it's not clever to just look at the shape and …
Researcher:	So, let's look at the learners - and see what levels of achievement did the learners get. Would you say your own learners would do well in this question?

Teacher F: Ah, definitely – ah, definitely – in fact, unfortunately I was not prepared when I came – I would have brought you –erm, resource, a book, a different book – check and you probably find such things

Researcher: Does it mean that you also do different things with your learners

Teacher F: Yeah, I do so…

Researcher: But look at the achievement of the learners – the way that the learners achieved in this question

Teacher F: So, you are saying that, erm…

Researcher: 26% of them got 5, 17%...what is the correct answer by the way…

Teacher F: 8

Researcher: 29% of them got it right…

Teacher F: Ah, it's too low – it's too low

Researcher: What do you think is the problem, looking at this one answer – the ones who chose 8 – others 5 – what is it that they missed

Teacher F: I think they did not see this one tail of the first fish and that one which is very faint – probably the big one…

Researcher: You just looked at the question and said the answer is 7, so what strategies do you think the learners used

Teacher F: on their own – like I said, learners used counting of lines at the same time they would separate these shapes into individual shapes – suppose they draw this one – they draw that part and then they tick, they'll discover that there are so many 4-sided shapes here – 'cause looking at this combined like this – definitely on your left – some will get the answer without scrutinizing what it is…

Researcher: Would you think the learners find this difficult when they do not do all those things mentioned, they might get the wrong answer

Teacher F: Yeah, if learners did not do that – you'll find they experience problems

Researcher: they really need to pay attention…

Teacher F: and maybe we can just …to them – I think from a mathematical point of view, my understanding as such, learners need to be sort of groomed in a way – to say when you think in a mathematical way – don't just …think simply but go further – think critically – missed some points which are important to take

Researcher: and if learners do this type of question –erm- what skills are they going to learn by solving – doing this

Teacher F: uhm…skills of recognition, maybe identifying and also analysing – they'll analyse

Researcher: because they have to sort out all 4-sided shapes out of all these shapes

Teacher F: right

Researcher: so, it makes them to be very analytic and critical

Teacher F: very analytic and critical

Researcher: only 29% got the correct answer – how would you deal with all other learners, 17%...

Teacher F: to me this is very simple – what you do – according to what I understand – you'll have to dwell on that –er- sort of concept that approaching this from different angles get the children to do practicals – they cut shapes – they put them together they sort them – they draw them, they also describe them – to say this is a 4-sided shape because of this and this and this and er even to the extent of saying, naming them if possible – to say – this is a square, this is a trapezium, it's a rectangle, a rhombus and then they'll tend to understand this one and develop for their future help as such

Researcher: so you're talking about hands on activities…

Teacher F: definitely – sort, write,

Researcher: how about using different colours

Teacher F: very correct, sorting, maybe also they can be assisted by colours – you know the visual ability of learners is different – others, they don't just see what is happening – but if colours are added – it will let them see – erm…

Researcher: and what do you think learners might be thinking when they get the wrong answer – maybe they might not be thinking at all…

Teacher F: -erm- they'll be just thinking so –erm- I will say just simple, they did not go to the extent of critically looking at the problem, definitely it would not be their first time to see this thing so as long as they just say I know this then; they can give correct answer, sometimes

Researcher: not even thinking..

Teacher F: yes - it will say agh- I missed this one – come on – but if they critically think – they'll say but I know this, but what about it, what if I do this which one is a bit complicated amongst this one – they'll get the answer

Researcher: think critically and look at the whole picture

Teacher F: whole picture, really, and look even breaking it into pieces so that you also look at it and get it right – so that's what they can do

Researcher: Last question, how do you go about assisting your learners' incorrect geometric thinking?

Teacher F: right –erm-, one I think it will be like –erm- sort of starting –erm- to look at the basics erm- shapes as such 2-D shapes and later on –erm-help them to recognise the different kinds of shapes that fall under that category right and also ask them to describe at the same time – even allow them to draw to manipulate yeah, yeah, and then –erm- I think they will have the basic….

Researcher: so, the most important thing is to use manipulatives

Teacher F: exactly

Researcher: Hmm…yeah, interesting

ICAS ITEM No. 7

Researcher: Looking from this question…

Teacher F: Jade had 3 tiles…what is the smallest no. of edges…

Researcher: so, which concepts are they looking for here

Teacher F: space and shape, this one

Researcher: but which concepts are they looking for

Teacher F: tessellations

Researcher: how did you do this question?

Teacher F: Hmm, I think I got 12

Researcher: you got 12, what concepts are they looking for

Teacher F: Tessellations

Researcher: So the learners must be able to be able to do this – do you think this type of question is easy or difficult

Teacher F: In the first place this question you see is the answer correct

Researcher: before we get to whether the answer is correct or not

Teacher F: I thought it was easy

Researcher: It's easy

Teacher F: Yeah, easy

Researcher: Why

Teacher F: It's just a question of just joining those 3 shapes

Researcher: so how did you do it, what mathematical strategies did you use to solve this problem?

Teacher F: ah, just, just, -erm- probably this could be separate shapes at that can be manipulated and you can just put them together

Researcher: but how did you do it – how did you come up with…

Teacher F: so, if these shapes are sort of joined together, and just check the outer part of the 3 shapes combined

Researcher: and how would you plan to teach with something like this – do you just use plain shapes or do you use other strategies as well

Teacher F: Hmm, just plain shapes

Researcher: basically, at the beginning plain shapes

Teacher F: Hmm, they can use now – let me just say if you've got shapes like –erm maybe 3-D shapes, I'm thinking about –erm-

Researcher: in tessellations, we use 2-D shapes

Teacher F: Okay, okay, I've got to find –erm- what else –yeah, even erm you know what comes to my mind, we can look at the tile now tile, if you look at this tile, it is thin, flat, it has got some height, and learners must recognise that if that is the case, as long as they are doing this kind of concept, so those tiles will fit in, then what do we say about that if the tiles have got height, what do we say

Researcher: In real life there are 3-D objects around us, 2-D shapes is only what the eye sees

Teacher F: I think that's right

Researcher: so, what about using animal shapes

Teacher F: animal shapes, that of a tortoise

Researcher: Like you take, erm- Tangrams – in the Assessment Standards – we talk about Tangrams – using Tangrams to make different shapes

Teacher F: Oh yes, I remember that one – that works as well – do you know the tortoise shell comes to mind – the tortoise shell is translation

Researcher: Hmm, and look at the learners, what they have achieved. Only 24% of the learners got 12 45% said 6, why would they say 6.

Teacher F: Can I ask, is that answer correct, if the answer is correct – what learners could have done here…
They say 6, I think they didn't understand that the shapes or…

Researcher: would you say if you can give this to your learners – will they make the same mistakes

Teacher F: like this one –erm- ah, no

Researcher: ok, why

Teacher F: let's say just a few, in fact you know what happens –erm- what I normally understand is that if the learners have been given the basics – and they have mastered that, it's not easy for them to find it difficult to do this thing – the problem is, they've been given correct --- they must do they really understand - instead

Researcher: so, how would your own learners solve this problem?

Teacher F: Ah, in fact it's simple – I need to give them something like –erm- let's say these shapes, would be drawn on a large scale, right then cut out – then when they do this problem, they just – do practicals

Researcher: solving this problem, do you think …

Teacher F: ah no, drawing now comes only in Grade 7, but the little ones in Grade 4 and 5, they need to put them together and then…

Researcher: okay, and the learners who would be able to do this type of question – what skills would they have…

Teacher F: they'll have that –erm- one, this is a regular shape, this one can be joined, two, they understand that those strategies used to join the shapes – now the edge

Researcher: and if some learners come with incorrect responses – what would you say, - are they really experiencing underlying difficulties, like these ones – what is the problem here – they come with 6

Teacher F: Ask them how did you come up with this

Researcher: but looking at it – how can you come up with a 6 here because each individual shape has got 6 sides?

Teacher F: Oh, yes, yes, yes, ok, what would have happened here is that they just looked at one shape and counted 6

Researcher: look at the question

Teacher F: what is the smallest no. of edges…

Researcher: look at that…

Teacher F: okay, I think the term smallest to them meant that each shape, there as individual shapes as combined shape that would be a stumbling block on their part

Researcher: because the shape hasn't been put together – asking them to visualise them

Teacher F: In fact, it's a question of not understanding the question – but if they understood, and then they would say a number…

Researcher: would you say language is a problem

Teacher F: correct, language, not only language, also the skill of analysing the language, -erm- the time when I was teaching at …(inaudible), my learners came up…
Learners need to underline key words

Researcher: so that they understand what

Teacher F: don't think that it's only for those –erm-senior learners, the intermediate phase learners they would underline key words…if they learn to do that many questions will be answered correctly

Researcher: do you mean to say that's what you teach your learners to underline

Teacher F: they underline – see if they say how many edges are in this shape – then they must understand how the edges and many link together…you're looking at the question as it is and simply have a wrong impression of what it means, then if you break it into pieces and look at keywords, if I know the answer, I know it, if I don't know - I don't know it

Researcher: so what about this one - 16% said 18 – how would you come up with 18

Teacher F: 18, see, see, and see

Researcher: they counted

Teacher F: they must have counted wrong

Researcher: maybe they counted everything

Teacher F: let me just check, you know 3, 3, 3 – 9 ok, maybe they counted everything

Researcher: would you say these kinds of learners – you know the other ones would say they didn't understand the question. These ones who counted everything – do they know what tessellations are…

Teacher F: is either they don't understand what tessellation is right, at the same time, they might not be able to understand the question is asking them to do

Researcher: so there wouldn't be any misconceptions or anything like that…

Teacher F: very correct, yes misconceptions will be there – say that when they look at this one they understand it in a different way as compared to what the question is requiring - misconception

Researcher: it's a misconception when they don't realise that when you do tessellate – these edges are going to come together and they can't form the edge – it's the outside edge and how would you help such learners…

Teacher F: Like I said earlier on you need to make sure that they first recognise these shapes as individuals, and maybe describe them, these are regular shapes, let them cut the shapes, maybe, let them handle the shapes – let them put the shapes together…all edges of the shape – you see, how this edge and that one, --will it make the same kind of formation and all that…until they are sure that this means this then they'll be right

Researcher: it means that if you put shapes together, the edges…

Teacher F: and to recognise that this is a regular shape 'cause at times they can use any regular shape and find that it's not working – they must check…

ICAS ITEM NO. 13

Researcher: What made this type of question challenging…

Teacher F: It was easy – angles on a straight line

Researcher: then reflecting on your own geometric thinking what mathematical strategies did you use to solve this

Teacher F: erm…

Researcher: question says – what is the value of angle LMN, and in the diagram KLM is an Isosceles triangle, KN is a straight line

Teacher F: erm- in fact in this case, one needs to know that –erm- erm- that is the …I don't know…in the Isosceles triangle…

Researcher: You made some calculations…

Teacher F: angazi ukuthi ngi zo iyenza kanjani lena…
Okay, right in fact to me the simplest way will be to start with a straight line and then you are done with this side is 65, this side is 65, these are equal…

Researcher: The sides are equal

Teacher F: The sides are equal and this is 65 _65 _ subtract from 180, which is 105

Researcher: you are sure the answer is 105

Teacher F: yeah, sure

Researcher: The answer is 115

Teacher F: Er..er…let me check it out

Researcher: At what Grade will you teach the learners?

Teacher F: Aah…

Researcher: Isosceles triangle, equal sides, equal angles, at which level…

Teacher F: Aah, Grade 5, 6, 7

Researcher: In Zimbabwe…

Teacher F: Grade 5 – I may not be sure, but I remember…

Researcher: So…would you use this type of question to teach your learners with…

Teacher F: yeah,

Researcher: this one is nice, it's integrating triangles and a straight line…Do we normally ask learners things like this or do we teach them triangles separately

Teacher F: I would also ask them to integrate

Researcher: A lesson on this will be under which topic

Teacher F: erm…geometry, angles, straight angles, something like that

Researcher: Let's look at the learners…would your learners be able to do this

Teacher F: Hai, they might have problems – but it will depend on the practice that I would have given them … but if they haven't done this in a long time, they'll get problems here

Researcher: So, let's look at these problems

Look at this – 15% of the learners say the answer is 135, 28% said 130; 49% said 150%, only 6% said 100. The biggest number is 28 – where they got the answer wrong – why would 28% of learners say the answer is 130 – how do you get 130

Teacher F: erm…

Researcher: would you say it's a misconception or just an error made….

Teacher F: okay, what they did – they were able to identify the two bottom angles of an isosceles triangles and just added this, I think 65 + 65 make 130…they recognised that this bottom angles are equal – 65 and this one is also 65 and they just added
It's not a misconception as such – they didn't critically look at the question such – they just thought – okay, it's 65 yes! I know the other side must be 65 – we are done – they didn't go back and read carefully which angle is exactly required by the question

Researcher: So, there's 15 %...said 135 – where did they get it

Teacher F: (calculating)… it doesn't make sense

Researcher: but what do you think they were thinking… how do you deal with it…

Teacher F: we go back to properties of and constructing shapes and identifying angles that make a particular triangle – especially given such triangles before until understand right, erm… in fact, maybe at this point you can use a protractor – they must know how to place a protractor on a straight line

Researcher: this question says this is not drawn according to scale, meaning you can't measure to find the correct answer

Teacher F: but basically they must understand that a straight line has got 180 – of course, using a protractor to measure a straight line, you'll see how you get 180 – now coming to the question as it is – it has specified that not drawn to scale – meaning that they must use the properties of angles on a straight line, properties of an Isosceles triangle

Researcher: so, these are integrated, would you say this is a clever way of teaching learners – or is there a different way

Teacher F: depending on the level – I think that this should be appropriately really

Researcher: at which level

Teacher F: I think Grade 6, Grade 7

Researcher: And this question was in the Grade 11 paper

Teacher F: very correct, what is interesting is that -erm, as an educator, -erm-, I don't rule out that the question was maybe in Grade 4, 5 because Maths is the same. You need to find out whether the learners understands, at whatever level

ICAS ITEM No. 17

Researcher: What makes the question difficult/easy?

Teacher F: uhm…

Researcher: this question is… socialization in diagonals, patterns – when do your learners do diagonals

Teacher F: Grade 5

Researcher: So, this question is rightly placed at Grade 7 – what made it difficult for you to answer

Teacher F: needs high level of thinking this one…

Researcher: so, when you're teaching you would do it with which learners…

Teacher F: suppose Grade 4, but – go up to 8 sides… agh no, and such patterns also …have a formula – the formula is there to say

Researcher: what do you mean there is a formula

Teacher F: the formula is there – then that's where you can also …

Researcher: with learners, when do you teach them patterns?

Teacher F: agh, Grade 7, upwards

Researcher: so, what Maths strategies did you try to use solving this problem?

Teacher F: erm, probably, I was trying to link the incidence of sides and diagonals
Such problems, if you are an educator – you need to practice them yourself before you go to the learners… let them do practicals
What I have discovered when you can research on this one – at the same time give learners such kind of a problem – see how they fare and then you can develop and give them a challenge

Researcher: What about you bright learners

Teacher F: yeah, they'll do it
It must be routine work for all learners as such, but anyway, if you go to this kind of the question, yes, you can go further now and test all, …..

Researcher: or you can give them as an investigation

Teacher F: if they understand this question…