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RESEARCH ARTICLE

Fault Estimation and Fault-Tolerant Control of Multiple Faults and Uncertain Disturbances Based on Generalized Sliding Mode Method

Junjie Zhang[1] | Fangfang Zhang[1] | Jie Li[1] | Yuanhong Liu[2] | Lei Kou[3] | Michaël Antonie Van Wyk[4]

[1]School of Information and Automation Engineering, Qilu University of Technology (Shandong Academy of Sciences), Jinan, China | [2]Northeast Petroleum University, Daqing, China | [3]Institute of Oceanographic Instrumentation, Qilu University of Technology (Shandong Academy of Sciences), Qingdao, China | [4]University of the Witwatersrand, Johannesburg, South Africa

Correspondence: Fangfang Zhang (zhff4u@qlu.edu.cn)

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ABSTRACT

Measuring element is always accompanied by uncertainty disturbances and multiple faults during the long time operation of industrial system in complex environment such as blade and pitch system of floating wind turbines. Timely detection of the fault and fault-tolerant control (FTC) perform a significant part in ensuring the stable operation of the system and saving maintenance fees. Sliding mode control is extensively applied to FTC because of its good robustness. Therefore, a sliding mode controller is constructed to guarantee the stability of the industrial plant which suffers multiple faults and uncertain disturbances. At the same time, most existing literature does not take into account several faults and uncertain disturbances. Firstly, employing generalized sliding mode method, we devise a sliding mode observer for evaluating state vector, actuator fault and sensor fault of the system. Secondly, according to the state estimation, we construct a sliding mode controller and prove its validity by Lyapunov's theorem. Our controller achieves satisfactory performance, and it is easier to be implemented in practical engineering than other controllers. Finally, we establish a SIMULINK model of blade and pitch system and make simulation experiments. Simulation outcomes validate the availability and practicability of our controller, which also provides a general scheme for fault estimation and FTC of other industrial plants.

1 | Introduction

The performance requirements of complicated industrial systems with multiple faults and uncertainty disturbances increases, fault diagnosis and fault tolerance control (FTC) is attracting more and more scholars' attention. In 1998, Zhou and Frank [1] proposed a closed-loop synchronous fault detection diagnosis and FTC strategy, where the estimated equivalent deviation was evaluated by an improved Bayes classification. In 2009, Deshpande, Patwardhan and Narasimhan [2] proposed an innovative intelligent nonlinear state estimation strategy. The strategy automatically corrects the model online to fit the diagnosed

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faults or failures. In 2018, Cho, Gao and Moan [3] introduced a sophisticated model-based approach for the detection, isolation, and mitigation of faults within the blade and pitch systems of floating wind turbines. In 2023, Zhu, Xia and Wang [4] developed a Stable Kernel Representation-inspired residual generator and designed to facilitate the online detection of actuator faults by leveraging the system's input-output (I/O) data. Subsequently, they crafted an Iterative Learning Fault Estimation algorithm, which enables the real-time reconstruction of the actuator fault. He and Jia [5] crafted a novel design strategy that decouples the estimator from the controller, relying on the measured output. Ma, Tang and Sun [6] constructed a new observer to estimate the aero-engine fault signal caused by external interference, and proposed a co-design scheme of event triggering parameters and fault tolerant controller.

In fault diagnosis, effective fault estimation is more beneficial to judge the scale and position of the fault. Gradually, FTC based on different observers has been applied in many fields. In 1996, Chen, Patton and Zhang [7] proposed a trail of fault detection by introducing a fresh perspective on designing robust filters. In 2007, Gao and Ding [8] crafted a state-of-the-art state-space observer that stands as a testament to robustness. This observer is capable of estimating the states of descriptor systems with precision, while also diagnosing actuator faults. In 2016, Gao, Liu and Chen [9] honed an effective fault estimation technology that accurately captures the system states while pinpointing the associated faults. This approach can minimize the disruptive effects of process and sensor disturbances, ensuring that the fault detection process remains robust and reliable. In 2020, Nian et al. [10] introduced a cutting-edge adaptive fault estimation observer (AFEO) in order to realize precise fault detection for quadrotors when it was subjected to actuator fault, exterior disturbances and unknown parameters. Building on the above robust fault estimation observer, they had further developed a dynamic output feedback fault-tolerant controller to stabilize the closed-loop system in the face of both faults and uncertainties. Liu et al. [11] designed a novel intermediate variable observer and a disturbance observer. In 2023, Cao and Wang [12] designed a distributed observer for the state reconstruction of continuous-time linear dynamical systems with unknown inputs.

In the complex working environment of industrial systems, uncertainty disturbances always affects the system performances. As the sliding mode control (SMC) possesses great robustness to uncertain disturbances and easy to implement, it has achieved good results, and has been widely used in practical systems. The SMC stemmed from Soviet scholar Utkin [13]. At present, SMC has made significant progress in Industrial Control Field [14], especially SMC with sliding mode observer (SMO) technology is widely used in fault estimation and FTC of real systems. In 2006, Jiang, Staroswiecki and Cocquempot [15] developed a cutting-edge fault estimation module that leverages the adaptive capabilities of estimators, which is the core of their FTC system to detect and adapt to faults. In 2007, Yan and Edwards [16] conceived an ingenious sliding mode observer, where the parameters that are calculated employing the linear matrix inequality (LMI) method. In 2008, Alwi and Edwards [17] crafted an innovative fault-tolerant SMC strategy specifically for civil aviation that includes actuator and sensor faults. In 2013, Liu and Shi [18] cooked up a SMC strategy for controlling a kind of nonlinear stochastic systems that are subject to both input and output perturbations. Based on the above SMC strategy, precise estimates of the system state, fault vector and disturbance were acquired. In 2016, Chen et al. [19] constructed a robust nonlinear controller adopting SMC technology and reverse step control technology for a four-rotor unmanned aerial vehicle. In 2023, for a nuclear power plant with actuator faults and sensor faults, Surjagade et al. [20] crafted a fault estimation and FTC strategy employing sliding mode method.

However, the research of sliding mode control is still insufficient. Firstly, the design of controllers generally only includes a single fault with uncertain disturbances [16,19] or two types of faults without uncertain disturbances [15,17,20]. In fact, there exist multiple failures and uncertain disturbances in the actual industrial plants, and so it is of great importance to research the fault estimation and FTC with multiple faults and uncertain disturbances at the same time. Secondly, a number of references only consider one type system, and the control strategy is not universally applicable to different systems [15,16]. Finally, the design process of sliding mode controller is too complicated, and it is difficult to implement in practical control engineering [20].

Inspired by the above discussions, we propose a FTC strategy on the basis of the sliding mode controller for linear and Lipschitz nonlinear systems with multiple faults and uncertain disturbances. The industrial system is expanded to a descriptor system, where the state and multiple faults produce a new state vector. For the descriptor system, the corresponding SMO is constructed, which will remove the influence of multiple faults and uncertain disturbances in the generated error system, and can obtain precise estimation of the system state, sensor fault and actuator fault, all at the same time. This way, a FTC strategy including linear matrix inequalities is designed in order to ensure the overall closed-loop system.

The main contributions of this paper are as follows.

1. By considering multiple faults and uncertain disturbances, suitable sliding mode observers are designed to achieve accurate estimation of system states and multiple faults.
2. The proposed controller is simplified by adding the exponential reaching law, which not only ensures the reachability of the sliding mode surface, but also shortens the reaching time and increases the accuracy of fault estimation.
3. The proposed sliding mode control methods applies to both linear and Lipschitz nonlinear systems and therefore provides a general scheme for FTC of linear and nonlinear systems.

The structure of the paper is as follows: we explain the layout of the proposed FTC scheme for linear systems in Section 2, and expound the process of the proposed FTC scheme for Lipschitz nonlinear systems in Section 3. In Section 4, numerical instances of blade and pitch system in floating wind turbine are supplied to prove the correctness of our controllers. Finally, we summarize this work in Section 5.

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2 | Fault Estimation and Fault Tolerance Control for Linear Systems With Multi-Fault

2.1 | Linear System

Consider the following general linear system

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + B_b(u_a(t) + f_a(t)) + B_r r_a(t) \\ y_a(t) = C_a x_a(t) + D_s f_s(t) + D_a f_a(t) \end{cases} \tag{1}$$

$x_a(t) \in \mathbb{R}^m$ is the system state vector, $u_a(t) \in \mathbb{R}^n$ is the control input, $y_a(t) \in \mathbb{R}^c$is the measurement output. $A_a \in \mathbb{R}^{m\times m}, B_b \in \mathbb{R}^{m\times n}, C_a \in \mathbb{R}^{c\times m}, B_r \in \mathbb{R}^{m\times l}, D_s \in \mathbb{R}^{c\times q}, D_a \in \mathbb{R}^{c\times n}$ are the system matrix, $f_a(t) \in \mathbb{R}^m$ $\mathbb{R}^n$is an actuator fault that exists in both input and output, $f_s(t) \in \mathbb{R}^q$ indicates that the sensor is faulty, $r_a(t) \in \mathbb{R}^l$ is an uncertain disturbance.

Throughout this paper, some assumptions for the subsequent theory are made.

Assumption 1. Sensor failure, actuator failure meets the following norm bounded constraints [18]:

$$\left\| f_s(t) \right\| \le \zeta, \left\| f_a(t) \right\| \le \vartheta, \left\| \dot{f}_a(t) \right\| \le \vartheta_1, \left\| r_a(t) \right\| \le \sigma \tag{2}$$

where $\zeta > 0, \vartheta > 0, \vartheta_1 > 0, \sigma > 0$ are known constants.

Assumption 2. (A_a, B_b) is a stabilizable pair, (A_a, C_a) is a detectable pair. There exists a constant $\delta > 0$ satisfying the condition:

$$rank \begin{bmatrix} \delta I_m + A_a & B_b \\ C_a & D_a \end{bmatrix} = m + n \tag{3}$$

We need to obtain an asymptotic estimate of $x_a(t), f_a(t), f_s(t)$ and guarantee robust stability of the system that contains uncertainty disturbances and multiple faults.

Therefore, we can construct new augmented variables and matrices:

$$\overline{x}_a(t) \triangleq \begin{bmatrix} x_a(t) \\ f_a(t) \\ D_s f_s(t) \end{bmatrix} \in \mathbb{R}^{\overline{m}}, \overline{A_a} \triangleq \begin{bmatrix} A_a & 0 & 0 \\ 0 & -\delta I_n & 0 \\ 0 & 0 & -I_c \end{bmatrix} \in \mathbb{R}^{\overline{m}\times\overline{m}},$$

$$\overline{B_b} \triangleq \begin{bmatrix} B_b \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^{\overline{m}\times n}, \overline{C_a} \triangleq \begin{bmatrix} C_a & D_a & I_c \end{bmatrix} \in \mathbb{R}^{c\times\overline{m}},$$

$$\overline{E_e} \triangleq \begin{bmatrix} I_m & \delta^{-1} B_b & 0 \\ 0 & I_n & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{\overline{m}\times\overline{m}},$$

$$\overline{B}_s \triangleq \begin{bmatrix} \delta^{-1} B_b & 0 & B_r \\ I_n & 0 & 0 \\ 0 & D_s & 0 \end{bmatrix} \in \mathbb{R}^{(m+n+c)\times(n+q+l)},$$

$$\overline{M} \triangleq \begin{bmatrix} 0 \\ 0 \\ I_c \end{bmatrix} \in \mathbb{R}^{(m+n+c)\times c}, \overline{f}(t) \triangleq \begin{bmatrix} \delta f_a(t) + \dot{f}_a(t) \\ f_s(t) \\ r_a(t) \end{bmatrix} \in \mathbb{R}^{n+q+l},$$

$$\overline{m} = n + m + c \tag{4}$$

where $\delta > 0$ is the designed parameter, making Assumption 2 hold.

Combining (1), we get an equivalent descriptor system:

$$\begin{cases} \overline{E_e}\dot{\overline{x}}_a(t) = \overline{A_a}\overline{x}_a(t) + \overline{B_b}u_a(t) + \overline{B}_s\overline{f}(t) \\ y_a(t) = \overline{C_a}\overline{x}_a(t) \end{cases} \tag{5}$$

where $r_a(t), f_a(t), D_s f_s(t)$ are the components of the descriptor state vector, augmenting the fault and state to the same vector. This implies that we can craft a state observer capable of accurately estimate both the system's state and any occurring faults with high efficiency. Then, according to these estimates, a FTC scheme is designed.

2.2 | The Design of Sliding Mode Observer for Linear System

The following lemmas are important for sliding mode observers.

Lemma 1. ([18]). *There always exists a matrix $\overline{L}_d \in \mathbb{R}^{\overline{m}\times c}$, such that the matrix $\overline{S_a} \triangleq (\overline{E_a} + \overline{L}_d\overline{C_a})$ is non-singular. Furthermore, it holds that $\overline{C_a}\,\overline{S_a}^{-1}\overline{L}_d = I_c$ and $\overline{A_a}\,\overline{S_a}^{-1}\overline{L}_d = -\overline{M}$.*

From Lemma 1, the following generalized SMO is designed for system (5):

$$\begin{cases} \overline{S_a}\dot{\overline{z}}(t) = (\overline{A_a} - \overline{L}_p\overline{C_a})\overline{z}(t) - \overline{M}y_a(t) + \overline{B_b}u_a(t) + \overline{L}_s u_s(t) \\ \hat{\overline{x}}_a(t) = \overline{z}(t) + \overline{S_a}^{-1}\overline{L}_d y_a(t) \end{cases} \tag{6}$$

where $\overline{z}(t) \triangleq \left[z_x^T(t)\ z_a^T(t)\ z_s^T(t) \right]^T \in \mathbb{R}^{\overline{m}}$ is the middle variable, $\hat{\overline{x}}_a(t) \triangleq \left[\hat{x}_a^T(t)\ \hat{f}_a^T(t)\ \hat{f}_s^T(t) \right]^T \in \mathbb{R}^{\overline{m}}$ is an estimate of $\overline{x}_a(t)$, $\overline{L}_p \in \mathbb{R}^{\overline{m}\times c}, \overline{L}_d \in \mathbb{R}^{\overline{m}\times c}$ and $\overline{L}_s \in \mathbb{R}^{\overline{m}\times(n+q)}$ are the proportional gain, derivative gain and sliding mode gain to be designed. $u_s(t)$ is a discontinuous input that eliminates the influence of input disturbance $\overline{f}(t)$. $\overline{L}_d = \left[\overline{L}_{d1}^T\ \overline{L}_{d2}^T\ \overline{L}_{d3}^T \right]^T = \left[0\ 0\ W^T \right]^T$, where $W = diag\left[g_1, g_2, \dots, g_p \right]$ and $g_i > 0$ for $i = 1, 2, 3, \dots, p$, According to Lemma 1, choose the appropriate $\overline{L}_d$.

Next we need to design the proportional gain $\overline{L}_p$. We begin by presenting a set of lemmas that will be essential in our subsequent discussion.

Lemma 2. ([18]). *Given two matrices $(\tilde{A}_a, \tilde{C}_a)$ with $\tilde{A}_a \in \mathbb{R}^{m\times m}, \tilde{C}_a \in \mathbb{R}^{c\times m}$, the following two conditions are equivalent.*

(i) The matrix $\tilde{A}_a$ is stable.

(ii) If the pair $(\tilde{A}_a, \tilde{C}_a)$ is detectable, then the Lyapunov equation $\tilde{A}_a{}^T\tilde{P}_a + \tilde{P}_a\tilde{A}_a = -\tilde{C}_a{}^T\tilde{C}_a$ has a unique solution.

Lemma 3. ([18]). *According to Assumption 2, there exists a gain matrix $\overline{L}_p$ such that the matrix $\overline{S_a}^{-1}(\overline{A_a} - \overline{L}_p\overline{C_a})$ is Hurwitz.*

We choose μ such that $\mathrm{Re}[\lambda_i(\overline{S_a}^{-1}\overline{A_a}] > -\mu, \forall i \in \{1, 2, \dots, \overline{m}\}$. It is equivalent to $\mathrm{Re}[\lambda_i(-(\mu I + \overline{S_a}^{-1}\overline{A_a}))] < 0, \forall i \in \{1, 2, \dots, \overline{m}\}$.

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It can draw the conclusion that $(-\mu I - \overline{S_{\mathrm{a}}}^{-1}\overline{A_{\mathrm{a}}}, \overline{C_{\mathrm{a}}})$ is detectable, there is a matrix $\overline{X_{\mathrm{a}}}$ such that:

$$
-(\mu I + \overline{S_{\mathrm{a}}}^{-1}\overline{A_{\mathrm{a}}})^T\overline{X_{\mathrm{a}}} - \overline{X_{\mathrm{a}}}(\mu I + \overline{S_{\mathrm{a}}}^{-1}\overline{A_{\mathrm{a}}}) = -\overline{C_{\mathrm{a}}}^T\overline{A_{\mathrm{a}}} \tag{7}
$$

Design $\overline{L}_p = \overline{S_{\mathrm{a}}X_{\mathrm{a}}}^{-1}\overline{C_{\mathrm{a}}}^T$, one can obtain:

$$
\begin{aligned}
&[\mu I + \overline{S_{\mathrm{a}}}^{-1}(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})]^T\overline{X_{\mathrm{a}}} + \overline{X_{\mathrm{a}}}[\mu I + S_{\mathrm{a}}^{-1}(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})] \\
&\quad = -\overline{C_{\mathrm{a}}}^T\overline{C_{\mathrm{a}}}
\end{aligned} \tag{8}
$$

it means that when $\mathrm{Re}[\lambda_i(\overline{S_{\mathrm{a}}}^{-1}(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}}))] < -\mu, \quad \forall i \in \{1, 2, \ldots, \overline{m}\}$, $\overline{S_{\mathrm{a}}}^{-1}(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})$ are designed as Hurwitz, $\overline{L}_p$ is obtained by solving the Lvapunov equation (7).

2.3 | Derivation of the Error Dynamics

From the descriptor system (5) and the sliding mode observer (6), combined with Lemma 1, we can get:

$$
\begin{aligned}
\overline{S_{\mathrm{a}}}\dot{\hat{\overline{x}}}_{\mathrm{a}}(t) &= \overline{S_{\mathrm{a}}}\dot{\overline{z}}(t) + \overline{L}_d\dot{y}_{\mathrm{a}}(t) \\
&= (\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})\hat{\overline{x}}_{\mathrm{a}}(t) - \overline{A_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}\overline{L}_d y_{\mathrm{a}}(t) \\
&\quad + \overline{L}_p\overline{C_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}\overline{L}_d y_{\mathrm{a}}(t) - \overline{M}y_{\mathrm{a}}(t) + \overline{B_{\mathrm{b}}}u_{\mathrm{a}}(t) \\
&\quad + \overline{L}_s u_s(t) + \overline{L}_d\dot{y}_{\mathrm{a}}(t)
\end{aligned} \tag{9}
$$

Combined with Lemma 1, the above equation is simplified to

$$
\begin{aligned}
\overline{S_{\mathrm{a}}}\dot{\hat{\overline{x}}}_{\mathrm{a}}(t) &= (\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})\hat{\overline{x}}_{\mathrm{a}}(t) + \overline{L}_p\overline{C_{\mathrm{a}}}\overline{x}_{\mathrm{a}}(t) + \overline{B_{\mathrm{b}}}u_{\mathrm{a}}(t) + \overline{L}_s u_s(t) \\
&\quad + \overline{L}_d\overline{C_{\mathrm{a}}}\dot{\overline{x}}_{\mathrm{a}}(t)
\end{aligned} \tag{10}
$$

Add $\overline{L}_d\overline{C_{\mathrm{a}}}\dot{\overline{x}}_{\mathrm{a}}(t)$ to each side of system (5)

$$
\begin{aligned}
\overline{S_{\mathrm{a}}}\dot{\overline{x}}_{\mathrm{a}}(t) &= \overline{E_{\mathrm{e}}}\dot{\overline{x}}_{\mathrm{a}}(t) + \overline{L}_d\overline{C_{\mathrm{a}}}\dot{\overline{x}}_{\mathrm{a}}(t) \\
&= (\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})\overline{x}_{\mathrm{a}}(t) + \overline{L}_p\overline{C_{\mathrm{a}}}\overline{x}_{\mathrm{a}}(t) + \overline{B_{\mathrm{b}}}u_{\mathrm{a}}(t) \\
&\quad + \overline{B}_s\overline{f}(t) + \overline{L}_d\overline{C_{\mathrm{a}}}\dot{\overline{x}}_{\mathrm{a}}(t)
\end{aligned} \tag{11}
$$

Set state variable errors

$$
\overline{e}(t) \triangleq \hat{\overline{x}}_{\mathrm{a}}(t) - \overline{x}_{\mathrm{a}}(t) = [\; e_x^T(t) \;\; e_a^T(t) \;\; e_s^T(t) \;]^T \tag{12}
$$

Subtract (11) from (10) to get the error system as below.

$$
\begin{aligned}
\dot{\overline{e}}(t) &= \dot{\hat{\overline{x}}}_{\mathrm{a}}(t) - \dot{\overline{x}}_{\mathrm{a}}(t) \\
&= \overline{S_{\mathrm{a}}}^{-1}[(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})\overline{e}(t) + \overline{L}_s u_s(t) - \overline{B}_s\overline{f}(t)]
\end{aligned} \tag{13}
$$

As $\hat{\overline{x}}_{\mathrm{a}}(t)$ in observer equation (6) is not facilitate a stability analysis of system (6), there is a necessity for us to factor out $\hat{\overline{x}}_{\mathrm{a}}(t)$.

We rewrite the formula (10) as

$$
\begin{aligned}
\overline{E_{\mathrm{e}}}\dot{\hat{\overline{x}}}_{\mathrm{a}}(t) + \overline{L}_d\overline{C_{\mathrm{a}}}\dot{\hat{\overline{x}}}_{\mathrm{a}}(t) &= (\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})\hat{\overline{x}}_{\mathrm{a}}(t) + \overline{L}_p\overline{C_{\mathrm{a}}}\overline{x}_{\mathrm{a}}(t) + \overline{B_{\mathrm{b}}}u_{\mathrm{a}}(t) \\
&\quad + \overline{L}_s u_s(t) + \overline{L}_d\dot{y}_{\mathrm{a}}(t)
\end{aligned}
$$

Rewrite the above formula as

$$
\overline{E_{\mathrm{e}}}\dot{\hat{\overline{x}}}_{\mathrm{a}}(t) = \overline{A_{\mathrm{a}}}\hat{\overline{x}}_{\mathrm{a}}(t) - \overline{L}_p\overline{C_{\mathrm{a}}}\overline{e}(t) + \overline{B_{\mathrm{b}}}u_{\mathrm{a}}(t) + \overline{L}_s u_s(t) - \overline{L}_d\overline{C_{\mathrm{a}}}\dot{\overline{e}}(t) \tag{14}
$$

Let $\overline{L}_s = \overline{B}_s$, $\overline{L}_p \triangleq [\; \overline{L}_{p1}^T \;\; \overline{L}_{p2}^T \;\; \overline{L}_{p3}^T \;]^T$, $\overline{L}_s \triangleq [\; \overline{L}_{s1}^T \;\; \overline{L}_{s2}^T \;\; \overline{L}_{s3}^T \;]^T$, $u_s(t) \triangleq [\; u_{s1}^T(t) \;\; u_{s2}^T(t) \;\; u_{s3}^T(t) \;]^T$.

It is obtained by decomposition of Equation (14)

$$
\begin{aligned}
\dot{\hat{x}}_{\mathrm{a}}(t) &= A_{\mathrm{a}}\hat{x}_{\mathrm{a}}(t) + B_{\mathrm{b}}u_{\mathrm{a}}(t) - \overline{L}_{p1}\overline{C_{\mathrm{a}}}\overline{e}(t) + \delta^{-1}B_{\mathrm{b}}u_{s1}(t) \\
&\quad + B_r u_{s3}(t) - \delta^{-1}B_{\mathrm{b}}\dot{\hat{f}}_a(t)
\end{aligned} \tag{15}
$$

2.4 | Observer Based Sliding Mode Control

This section mainly designs controller $u_{\mathrm{a}}(t)$ and discontinuous input $u_s(t)$. For discontinuous input $u_s(t)$, the sliding mode surfaces are defined as:

$$
s(t) = \overline{B}_s^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}}\overline{e}(t) \tag{16}
$$

where $s(t) \in \mathbb{R}^{n+q}$, and the Lyapunov matrix $\overline{P} > 0$ fits the following constraints:

$$
\overline{B}_s^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}} = \overline{H_{\mathrm{a}}C_{\mathrm{a}}} \tag{17}
$$

$\overline{H_{\mathrm{a}}} \in \mathbb{R}^{(n+q+l)\times c}$ is the parameter matrix.

Design $u_s(t)$ as follows

$$
u_s(t) = -(\zeta + \delta\vartheta + \vartheta_1 + \sigma + \gamma)\mathrm{sgn}(s(t)) \tag{18}
$$

where $\gamma > 0$ is the undetermined parameter, $\zeta, \vartheta, \vartheta_1, \sigma$ are shown in Assumption 1.

Next, we design the following integral sliding surface $\tilde{s}(t)$ according to state estimation $\hat{x}_{\mathrm{a}}(t)$.

$$
\tilde{s} = G_{\mathrm{a}}\hat{x}_{\mathrm{a}}(t) - \int_0^t G_{\mathrm{a}}(A_{\mathrm{a}} + B_{\mathrm{b}}K)\hat{x}_{\mathrm{a}}(\tau)d\tau \tag{19}
$$

where $G_{\mathrm{a}} \in \mathbb{R}^{n\times m}$ and $K \in \mathbb{R}^{n\times m}$ are the parameters to be designed. When $t = 0$, $\hat{s}(0) = G_{\mathrm{a}}\hat{x}_{\mathrm{a}}(0)$. We solve G_{a} to make $G_{\mathrm{a}}B_{\mathrm{b}}$ non-singular, and design K to ensure $A + B_{\mathrm{b}}K$ Hurwitz. It is obtained from Equations (15) and (19)

$$
\begin{aligned}
\dot{\tilde{s}}(t) &= G_{\mathrm{a}}[-\overline{L}_{p1}\overline{C_{\mathrm{a}}}\overline{e}(t) + \delta^{-1}B_{\mathrm{b}}(u_{s1} - \dot{\hat{f}}_a) + B_{\mathrm{b}}(u_{\mathrm{a}}(t) \\
&\quad - K\hat{x}_{\mathrm{a}}(t)) + B_r u_{s3}(t)]
\end{aligned} \tag{20}
$$

Citing exponential reaching law

$$
\dot{\tilde{s}}(t) = -\varepsilon sgn\tilde{s}(t) - k\tilde{s}(t) \quad \varepsilon > 0, k > 0
$$

we get controller

$$
\begin{aligned}
u_{\mathrm{a}}(t) &= -\varepsilon sgn(\tilde{s}(t)) - k\tilde{s}(t) - G_{\mathrm{a}}B_r u_{s3}(t) + K\hat{x}_{\mathrm{a}}(t) \\
&\quad + G_{\mathrm{a}}\overline{L}_{p1}\overline{C_{\mathrm{a}}}\overline{e}(t) - G_{\mathrm{a}}\delta^{-1}B_{\mathrm{b}}(u_{s1}(t) - \dot{\hat{f}}_a(t))
\end{aligned} \tag{21}
$$

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Substitute (21) into (15), and get

$$\dot{\hat{x}}_{\mathrm{a}}(t) = (A_{\mathrm{a}} + B_{\mathrm{b}}K)\hat{x}_{\mathrm{a}} - \varepsilon B_{\mathrm{b}} sgn(\tilde{s}(t)) - B_{\mathrm{b}} k\tilde{s}(t) + (-I_m + B_{\mathrm{b}}(G_{\mathrm{a}}B_{\mathrm{b}})^{-1}G_{\mathrm{a}})\overline{L}_{p1}\overline{C}_{\mathrm{a}}\overline{e}(t) \tag{22}$$

The combination of (22) and (13) produces an integral closed-loop system:

$$\begin{cases} \dot{\hat{x}}_{\mathrm{a}}(t) = (A_{\mathrm{a}} + B_{\mathrm{b}}K)\hat{x}_{\mathrm{a}} - \varepsilon B_{\mathrm{b}} sgn(\tilde{s}(t)) - B_{\mathrm{b}} k\tilde{s}(t) + (-I_m \\ \quad + B_{\mathrm{b}}(G_{\mathrm{a}}B_{\mathrm{b}})^{-1}G_{\mathrm{a}})\overline{L}_{p1}\overline{C}_{\mathrm{a}}\overline{e}(t) \\ \dot{\overline{e}}(t) = \overline{S}_{\mathrm{a}}^{-1}[(\overline{A}_{\mathrm{a}} - \overline{L}_p\overline{C}_{\mathrm{a}})\overline{e}(t) + \overline{L}_s u_s(t) - \overline{B}_s\overline{f}(t)] \end{cases} \tag{23}$$

2.5 | Stability Analysis of Error Dynamics

To assure the stability of a closed-loop system (23), we introduced the following theorem which delineate the precise conditions of stability.

Theorem 1. *If there exist positive definite matrices $\overline{P}_{\mathrm{a}} \in \mathbb{R}^{\overline{m}\times\overline{m}}$, $\overline{R}_{\mathrm{a}} \in \mathbb{R}^{m\times m}$, and matrix $\overline{H}_{\mathrm{a}} \in \mathbb{R}^{(n+q+l)\times c}$, such that the equality constraint (17) and the following matrix constraint*

$$\prod = \begin{bmatrix} \prod_{11} & \prod_{12} \\ \star & \prod_{22} \end{bmatrix} < 0 \tag{24}$$

hold, where

$$\prod_{11} = \overline{R_{\mathrm{a}}}(A_{\mathrm{a}} + B_{\mathrm{b}}K) + (A_{\mathrm{a}} + B_{\mathrm{b}}K)^T\overline{R_{\mathrm{a}}}$$

$$\prod_{12} = \overline{R_{\mathrm{a}}}(-I_m + B_{\mathrm{b}}(G_{\mathrm{a}}B_{\mathrm{b}})^{-1}G_{\mathrm{a}})\overline{L}_{p1}\overline{C_{\mathrm{a}}}$$

$$\prod_{22} = \overline{P_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}}) + \left(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}}\right)^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}}$$

Moreover, when the controller $u_{\mathrm{a}}(t)$ is constructed as (21), the complete closed-loop system is confirmed.

Proof. Select an appropriate Lyapunov function.

$$V(t) = V_1(t) + V_2(t)$$

where $V_1(t) = \hat{x}_{\mathrm{a}}^T\overline{R_{\mathrm{a}}}\hat{x}_{\mathrm{a}}(t)$, $V_2(t) = \overline{e}^T(t)\overline{P_{\mathrm{a}}}\overline{e}(t)$.

Taking the weak infinitesimal operator ($\mathcal{L}$) along the trajectories of system (23), we obtain

$$\begin{aligned} \mathcal{L}V_1(t) = 2\hat{x}_{\mathrm{a}}^T\overline{R_{\mathrm{a}}}[&(A_{\mathrm{a}} + B_{\mathrm{b}}K)\hat{x}_{\mathrm{a}}(t) \\ &+ (-I_m + B_{\mathrm{b}}(G_{\mathrm{a}}B_{\mathrm{b}})^{-1}G_{\mathrm{a}})\overline{L}_{p1}\overline{C_{\mathrm{a}}}\overline{e}(t) \\ &- \varepsilon B_{\mathrm{b}} sgn(\tilde{s}(t)) - B_{\mathrm{b}}k\tilde{s}(t)] \end{aligned} \tag{25}$$

$$\begin{aligned} \mathcal{L}V_2(t) &\le 2\overline{e}^T(t)\overline{P_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}[(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}})\overline{e}(t) + \overline{L}_s u_s(t) - \overline{B}_s\overline{f}(t)] \\ &\le \overline{e}^T(t)[\overline{P_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}}) + \left(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}}\right)^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}}]\overline{e}(t) \\ &\quad + 2\overline{e}^T(t)\overline{P_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}[\overline{L}_s u_s(t) - \overline{B}_s\overline{f}(t)] \end{aligned} \tag{26}$$

Let $\overline{B}_s = \overline{L}_s$, and get

$$\begin{aligned} &2\overline{e}^T(t)\overline{P_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}[\overline{L}_s u_s(t) - \overline{B}_s\overline{f}(t)] \\ &= 2\overline{e}^T(t)\overline{C_{\mathrm{a}}}^T\overline{H_{\mathrm{a}}}^T u_s(t) - 2\overline{e}^T(t)\overline{P_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}\overline{B}_s\overline{f}(t) \\ &\le 2\overline{e}^T(t)\left\|\overline{C_{\mathrm{a}}}^T\overline{H_{\mathrm{a}}}^T\right\| \cdot \|\overline{f}(t)\| \\ &\quad - 2s^T(t)(\zeta + \delta\vartheta + \vartheta_1 + \gamma)sgn(s(t)) \\ &\le 2(\zeta + \delta\vartheta + \vartheta_1 + \sigma)\|s(t)\| \\ &\quad - 2(\zeta + \delta\vartheta + \vartheta_1 + \sigma + \gamma)|s(t)| - \gamma|s(t)| \\ &\le -2\gamma|s(t)| \end{aligned} \tag{27}$$

From (25)–(27), we get

$$\begin{aligned} \mathcal{L}V(t) &= \mathcal{L}V1(t) + \mathcal{L}V_2(t) \\ &\le \hat{x}_{\mathrm{a}}^T[\overline{R_{\mathrm{a}}}(A_{\mathrm{a}} + B_{\mathrm{b}}K) + (A_{\mathrm{a}} + B_{\mathrm{b}}K)^T\overline{R_{\mathrm{a}}}]\hat{x}_{\mathrm{a}}(t) \\ &\quad + 2\hat{x}_{\mathrm{a}}^T\overline{R_{\mathrm{a}}}(-I_m + B_{\mathrm{b}}(G_{\mathrm{a}}B_{\mathrm{b}})^{-1}G_{\mathrm{a}})\overline{L}_{p1}\overline{C_{\mathrm{a}}}\overline{e}(t) \\ &\quad - \varepsilon B_{\mathrm{b}} sgn(\tilde{s}(t)) - B_{\mathrm{b}}k\tilde{s}(t)] + \overline{e}^T(t)[\overline{P_{\mathrm{a}}}\,\overline{S_{\mathrm{a}}}^{-1}(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}}) \\ &\quad + \left(\overline{A_{\mathrm{a}}} - \overline{L}_p\overline{C_{\mathrm{a}}}\right)^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}}]\overline{e}(t) - \gamma|s(t)| \\ &\le \left[\hat{x}_{\mathrm{a}}^T(t)\ \ \overline{e}^T(t)\right]\prod\left[\hat{x}_{\mathrm{a}}^T(t)\ \ \overline{e}^T(t)\right]^T \end{aligned} \tag{28}$$

If $\prod < 0$ is true, then it means that the system (23) is stable, so we have completed our proof. □

Since the LMI of Theorem 1 is difficult to solve, we rewrite the linear equality condition (17) as

$$\left(\overline{B}_s^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}} - \overline{H_{\mathrm{a}}C_{\mathrm{a}}}\right)^T\left(\overline{B}_s^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}} - \overline{H_{\mathrm{a}}C_{\mathrm{a}}}\right) < \eta I_m \tag{29}$$

where $\eta > 0$. By Schur complement, rewrite (29) as

$$\begin{bmatrix} -\eta I_{-m} & \left(\overline{B}_s^T\overline{S_{\mathrm{a}}}^{-T}\overline{P_{\mathrm{a}}} - \overline{H_{\mathrm{a}}C_{\mathrm{a}}}\right)^T \\ \star & -I_{n+q} \end{bmatrix} < 0 \tag{30}$$

Combining with (24) and (30), we will find the solutions of $\overline{P_{\mathrm{a}}}, \overline{R_{\mathrm{a}}}, \overline{H_{\mathrm{a}}}$ using the LMI toolbox of MATLAB software.

2.6 | Design Process of Fault-Tolerant Control for Linear Systems

The following is the design process of fault estimation and FTC strategy for linear systems.

i. Add the original system to the descriptor system and select the appropriate derivative gain $\overline{L}_d$. $\overline{L}_p$ is obtained by solving the Lvapunov equation (7). Design the corresponding sliding mode observer.

ii. Design $u_s(t)$ as the form of (18), we get $\overline{P_{\mathrm{a}}}, \overline{R_{\mathrm{a}}}, \overline{H_{\mathrm{a}}}$ by solving the linear matrix inequality.

iii. Choose the appropriate K, so that $A_{\mathrm{a}} + B_{\mathrm{b}}K$ is Hurwitz. Set $\overline{L}_{p1} = \left[I_{m\times m}\ \ 0_{m\times n}\ \ 0_{m\times p}\right]\overline{L}_p$, and construct $u_{\mathrm{a}}(t)$ as (21).

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3 | Fault Estimation and Fault-Tolerant Control for Lipschitz Nonlinear Systems With Multi-Fault

3.1 | Lipschitz Nonlinear System

Consider the following Lipschitz nonlinear system

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + B_b(u_a(t) + f_a(t)) + B_r r_a(t) + \Phi_a(t, x_a, u_a) \\ y_a(t) = C_a x_a(t) + D_s f_s(t) + D_a f_a(t) \end{cases} \tag{31}$$

where $\Phi_a(t, x_a, u_a) \in \mathbb{R}^m$ is a real nonlinear vector function with a Lipschitz constant matrix L, and it satisfies

$$\|\Phi_a(t, x_a, u_a) - \Phi_a(t, \hat{x}_a, u_a)\| \le \|L(x_a - \hat{x}_a)\| \tag{32}$$

$\forall (t, x_a, u_a), (t, \hat{x}_a, u_a) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^n$, the other variables has been defined in system (1). Throughout various practical applications, systems governed by Lipschitz nonlinearities, or at the very least, those exhibiting local Lipschitz nonlinearity, are commonly encountered. The analysis of locally Lipschitz systems can be adopted to the globally Lipschitz systems.

Define an augmented state vector of form (4), and

$$\bar{\Phi}_a(t, x_a, u_a) = \begin{bmatrix} \Phi_a^T(t, x_a, u_a) & 0 & 0 \end{bmatrix}^T \in \mathbb{R}^{\bar{m}} \tag{33}$$

The following equivalent augmentation system is obtained

$$\begin{cases} \bar{E}_e \dot{\bar{x}}_a(t) = \bar{A}_a \bar{x}_a(t) + \bar{B}_b u_a(t) + \bar{B}_s \bar{f}(t) + \bar{\Phi}_a(t, x_a, u_a), \\ y_a(t) = \bar{C}_a \bar{x}_a(t) \end{cases} \tag{34}$$

and other definitions are the same as (5).

3.2 | The Design of Sliding Mode Observer for Lipschitz Nonlinear System

According to Lemma 1 and (32), the following sliding mode observer is designed for system (34)

$$\begin{cases} \bar{S}_a \dot{\bar{z}}(t) = (\bar{A}_a - \bar{L}_p \bar{C}_a)\bar{z}(t) - \bar{M} y_a(t) + \bar{B}_b u_a(t) + \bar{L}_s u_s(t) \\ \qquad + \bar{\Phi}_a(t, \hat{x}_a, u_a) \\ \hat{\bar{x}}_a(t) = \bar{z}(t) + \bar{S}_a^{-1} \bar{L}_d y_a(t) \end{cases} \tag{35}$$

$\bar{\Phi}_a(t, \hat{x}_a, u_a)$ is the estimate of $\bar{\Phi}_a(t, x_a, u_a)$, and the other symbol definitions are the same as (6).

3.3 | Derivation of the Error Dynamics

From (34) and (35), combined with Lemma 1, the following is obtained

$$\begin{aligned} \bar{S}_a \dot{\hat{\bar{x}}}_a(t) &= \bar{S}_a \dot{\bar{z}}(t) + \bar{L}_d \dot{y}_a(t) \\ &= (\bar{A}_a - \bar{L}_p \bar{C}_a)\hat{\bar{x}}_a(t) - \bar{A}_a \bar{S}_a^{-1} \bar{L}_d y_a(t) \\ &\quad + \bar{L}_p \bar{C}_a \bar{S}_a^{-1} \bar{L}_d y_a(t) - \bar{M} y_a(t) + \bar{B}_b u_a(t) \\ &\quad + \bar{L}_s u_s(t) + \bar{L}_d \bar{C}_a \dot{\bar{x}}_a(t) + \bar{\Phi}_a(t, \hat{x}_a, u_a) \end{aligned} \tag{36}$$

Add $\bar{L}_d \bar{C}_a \dot{\bar{x}}_a(t)$ on each side of system (34), and get

$$\begin{aligned} \bar{S}_a \dot{\bar{x}}_a(t) &= \bar{E}_e \dot{\bar{x}}(t) + \bar{L}_d \bar{C}_a \dot{\bar{x}}_a(t) \\ &= (\bar{A}_a - \bar{L}_p \bar{C}_a)\bar{x}_a(t) + \bar{L}_p \bar{C}_a \bar{x}_a(t) + \bar{B}_b u_a(t) \\ &\quad + \bar{B}_s \bar{f}(t) + \bar{L}_d \bar{C}_a \dot{\bar{x}}_a(t) + \bar{\Phi}_a(t, x_a, u_a) \end{aligned} \tag{37}$$

Subtract (37) from (36), and get the error system as below.

$$\begin{aligned} \dot{\bar{e}}(t) &= \dot{\hat{\bar{x}}}_a(t) - \dot{\bar{x}}_a(t) \\ &= \bar{S}_a^{-1}[(\bar{A}_a - \bar{L}_p \bar{C}_a)\bar{e}(t) + \bar{L}_s u_s(t) - \bar{B}_s \bar{f}(t)] + \bar{S}_a^{-1} \tilde{\Phi}_a \end{aligned} \tag{38}$$

in which $\tilde{\Phi}_a = \bar{\Phi}_a(t, \hat{x}_a, u_a) - \bar{\Phi}_a(t, x_a, u_a)$.

Rewrite (36) as

$$\begin{aligned} \bar{E}_e \dot{\hat{\bar{x}}}_a(t) &= \bar{A}_a \hat{\bar{x}}_a(t) - \bar{L}_p \bar{C}_a \bar{e}(t) + \bar{B}_b u_a(t) + \bar{L}_s u_s(t) \\ &\quad - \bar{L}_d \bar{C}_a \dot{\bar{e}}(t) + \bar{\Phi}_a(t, \hat{x}_a, u_a) \end{aligned} \tag{39}$$

After decomposition, get

$$\begin{aligned} \dot{\hat{x}}_a(t) &= A_a \hat{x}_a(t) + B_b u_a(t) - \bar{L}_{p1} \bar{C}_a \bar{e}(t) + \delta^{-1} B_b u_{s1}(t) \\ &\quad + B_r u_{s3}(t) - \delta^{-1} B_b \dot{\hat{f}}_a(t) + \Phi_a(t, \hat{x}_a, u_a) \end{aligned} \tag{40}$$

3.4 | Observer Based Sliding Mode Control for Lipschitz Nonlinear System

If the sliding mode surface $s(t)$ is in accordance with (16), $u_s(t)$ is devised as (18), and the integral sliding mode surface $\tilde{s}(t)$ is designed the same as (19), then $\dot{\tilde{s}}(t)$ is the following expression

$$\begin{aligned} \dot{\tilde{s}}(t) &= G_a[-\bar{L}_{p1} \bar{C}_a \bar{e}(t) + \delta^{-1} B_b(u_{s1} - \dot{\hat{f}}_a) + B_b(u_a(t) \\ &\quad - K\hat{x}_a(t)) + B_r u_{s3}(t) + \Phi_a(t, \hat{x}_a, u_a)] \end{aligned} \tag{41}$$

Combined with the exponential reaching rate, controller $u_a(t)$ is obtained as follows

$$\begin{aligned} u_a(t) &= -\varepsilon sgn(\tilde{s}(t)) - k\tilde{s}(t) - G_a B_r u_{s3}(t) + K\hat{x}_a(t) \\ &\quad + G_a \bar{L}_{p1} \bar{C}_a \bar{e}(t) - G_a \delta^{-1} B_b(u_{s1}(t) - \dot{\hat{f}}_a(t)) \\ &\quad - G\Phi_a(t, \hat{x}_a, u_a) \end{aligned} \tag{42}$$

Substitute (42) into (40), and get

$$\begin{aligned} \dot{\hat{x}}_a(t) &= (A_a + B_b K)\hat{x}_a - \varepsilon B_b sgn(\tilde{s}(t)) - B_b k\tilde{s}(t) + (-I_m \\ &\quad + B_b(G_a B_b)^{-1} G_a)\bar{L}_{p1} \bar{C}_a \bar{e}(t) \end{aligned} \tag{43}$$

The combination of (38) and (43) produces an integral closed-loop system

$$\begin{cases} \dot{\hat{x}}_a(t) = (A_a + B_b K)\hat{x}_a - \varepsilon B_b sgn(\tilde{s}(t)) - B_b k\tilde{s}(t) + (-I_m \\ \qquad + B_b(G_a B_b)^{-1} G_a)\bar{L}_{p1} \bar{C}_a \bar{e}(t) \\ \dot{\bar{e}}(t) = \bar{S}_a^{-1}[(\bar{A}_a - \bar{L}_p \bar{C}_a)\bar{e}(t) + \bar{L}_s u_{s1}(t) - \bar{B}_s \bar{f}(t)] + \bar{S}_a^{-1} \tilde{\Phi}_a \end{cases} \tag{44}$$

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3.5 | Stability Analysis of Error Dynamics

The stability condition of the closed-loop system (44) is obtained

$$\prod = \begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 \\ \star & \Pi_{22} & \Pi_{23} \\ \star & \star & \Pi_{33} \end{bmatrix} < 0 \tag{45}$$

hold, where

$$\prod_{11} = \overline{R_{\text{a}}}(A_{\text{a}} + B_{\text{b}}K) + (A_{\text{a}} + B_{\text{b}}K)^T \overline{R_{\text{a}}},$$

$$\prod_{12} = \overline{R_{\text{a}}}(-I_m + B_{\text{b}}(G_{\text{a}}B_{\text{b}})^{-1}G_{\text{a}})\overline{L}_{p1}\overline{C_{\text{a}}},$$

$$\prod_{22} = \overline{P_{\text{a}}}\,\overline{S_{\text{a}}}^{-1}(\overline{A_{\text{a}}} - \overline{L}_p\overline{C_{\text{a}}}) + \left(\overline{A_{\text{a}}} - \overline{L}_p\overline{C_{\text{a}}}\right)^T\overline{S_{\text{a}}}^{-T}\overline{P_{\text{a}}} + L_{\text{a}}^T L_{\text{a}},$$

$$\prod_{23} = \overline{P_{\text{a}}}\,\overline{S_{\text{a}}}^{-1},$$

$$\prod_{33} = -I_m.$$

If there are $\overline{P_{\text{a}}}, \overline{R_{\text{a}}}, \overline{H_{\text{a}}}$ satisfying (30) and (45), and the controller $u_{\text{a}}(t)$ is designed in (42), then the entire closed-loop system is stable.

Proof. We chose the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t)$$

where

$$V_1(t) = \hat{x}_{\text{a}}^T(t)\overline{R_{\text{a}}}\hat{x}_{\text{a}}(t), V_2(t) = \bar{e}^T(t)\overline{P_{\text{a}}}\bar{e}(t).$$

Combine (32), and get

$$\begin{aligned} \mathcal{L}V_2(t) &\le 2\bar{e}^T(t)\overline{P_{\text{a}}}\,\overline{S_{\text{a}}}^{-1}[(\overline{A_{\text{a}}} - \overline{L}_p\overline{C_{\text{a}}})\bar{e}(t) + \overline{L}_s u_s(t) - \overline{B}_s\overline{f}(t) + \tilde{\Phi}_{\text{a}}] \\ &\le \bar{e}^T(t)[\overline{P_{\text{a}}}\,\overline{S_{\text{a}}}^{-1}(\overline{A_{\text{a}}} - \overline{L}_p\overline{C_{\text{a}}}) + \left(\overline{A_{\text{a}}} - \overline{L}_p\overline{C_{\text{a}}}\right)^T\overline{S_{\text{a}}}^{-T}\overline{P_{\text{a}}}]\bar{e}(t) \\ &\quad + 2\bar{e}^T(t)\overline{P_{\text{a}}}\,\overline{S_{\text{a}}}^{-1}[\overline{L}_s u_s(t) - \overline{B}_s\overline{f}(t)] \\ &\quad + 2\bar{e}^T\overline{P}_a\overline{S}_a^{-1}\tilde{\Phi} + (L_{\text{a}}\bar{e}(t))^T L_{\text{a}}\bar{e}(t) - \tilde{\Phi}_{\text{a}}^T\tilde{\Phi}_{\text{a}} \end{aligned} \tag{46}$$

The proof process of $\mathcal{L}V_1(t)$ is the same as that of (25).

We obtained from (25), (27), (46), and

$$\begin{aligned} \mathcal{L}V(t) &= \mathcal{L}V1(t) + \mathcal{L}V_2(t) \\ &\le \hat{x}_{\text{a}}^T[\overline{R_{\text{a}}}(A_{\text{a}} + B_{\text{b}}K) + (A_{\text{a}} + B_{\text{b}}K)^T\overline{R_{\text{a}}}]\hat{x}_{\text{a}}(t) \\ &\quad + 2\hat{x}_{\text{a}}^T\overline{R_{\text{a}}}(-I_m + B_{\text{b}}(G_{\text{a}}B_{\text{b}})^{-1}G_{\text{a}})\overline{L}_{p1}\overline{C_{\text{a}}}\bar{e}(t) \\ &\quad - \varepsilon B_{\text{b}}sgn(\tilde{s}(t)) - B_{\text{b}}k\tilde{s}(t)] + \bar{e}^T(t)[\overline{P_{\text{a}}}\,\overline{S_{\text{a}}}^{-1}(\overline{A_{\text{a}}} - \overline{L}_p\overline{C_{\text{a}}}) \\ &\quad + \left(\overline{A_{\text{a}}} - \overline{L}_p\overline{C_{\text{a}}}\right)^T\overline{S_{\text{a}}}^{-T}\overline{P_{\text{a}}} + L_{\text{a}}^T L_{\text{a}}]\bar{e}(t) \\ &\quad + 2\bar{e}^T\overline{P}_a\overline{S}_a^{-1}\tilde{\Phi} - \tilde{\Phi}_{\text{a}}^T\tilde{\Phi}_{\text{a}} - \gamma|s(t)| \\ &\le \begin{bmatrix} \hat{x}_{\text{a}}^T(t) & \bar{e}^T(t) & \tilde{\Phi}_{\text{a}}^T \end{bmatrix} \prod \begin{bmatrix} \hat{x}_{\text{a}}^T(t) & \bar{e}^T(t) & \tilde{\Phi}_{\text{a}}^T \end{bmatrix}^T \end{aligned} \tag{47}$$

If $\prod < 0$ is true, then it means that the system (44) is stable, thus we have completed our proof. □

3.6 | Design Process of Fault-Tolerant Control for Lipschitz Nonlinear Systems

The design process for fault estimation and FTC strategy in Lipschitz nonlinear systems is presented below.

(i) The Lipschitz nonlinear system is expanded to descriptor system and a fitted derivative gain $\overline{L}_d$ is selected. $\overline{L}_p$ is obtained by solving the Lvapunov equation (7). Design the corresponding sliding mode observer.

(ii) Design $u_s(t)$ as the form of (18), we get $\overline{P_{\text{a}}}, \overline{R_{\text{a}}}, \overline{H_{\text{a}}}$ by solving the linear matrix inequality.

(iii) Choose the appropriate K so that $A_{\text{a}} + B_{\text{b}}K$ is Hurwitz. Let $\overline{L}_{p1} = \begin{bmatrix} I_{m\times m} & 0_{m\times n} & 0_{m\times p} \end{bmatrix}\overline{L}_p$, and construct $u_{\text{a}}(t)$ as the form of (42).

4 | Simulation

4.1 | Linear System

Consider a blade and pitch system of floating wind turbine

$$\begin{cases} \dot{x}_i(t) = \begin{bmatrix} -2\varsigma\omega_n & -\omega^2{}_n \\ 1 & 0 \end{bmatrix} x_i(t) + \begin{bmatrix} \omega^2{}_n \\ 0 \end{bmatrix} (\beta_r(t) + f_a(t)) + B_r r(t) \\ y_{\text{a}}(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x_i(t) + D_s f_s(t) + D_a f_a(t) \end{cases} \tag{48}$$

We choose

$$\begin{aligned} &A_{\text{a}} = \begin{bmatrix} -13.332 & 123.4321 \\ 1 & 0 \end{bmatrix}, B_{\text{b}} = \begin{bmatrix} 123.4321 \\ 0 \end{bmatrix} \\ &C_{\text{a}} = \begin{bmatrix} 0 & 1 \end{bmatrix}, B_r = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, D_s = 0.1, D_a = 0.1, \varsigma = 0.6, \\ &\omega_n = 11.11, \beta_r = 7.11 \end{aligned} \tag{49}$$

Suppose $f_a(t), f_s(t), r_{\text{a}}(t)$ have the following form:

$$f_a(t) = \begin{cases} 0.2\sin(7.5t), & \text{for } \quad 0 \le t \le 4 \\ 4, & \text{for } \quad 4 < t \le 10 \\ -0.1976 + 0.2\sin(5(t-4)), & \text{for } \quad 10 < t \le 30 \end{cases}$$

$$f_s(t) = \begin{cases} 0.1t, & \text{for } \quad 0 \le t \le 9 \\ 0.1 + 0.3\cos(5(\text{t} - 1)), & \text{for } \quad 9 < t \le 30 \end{cases}$$

and $r_{\text{a}}(t) = 3\sin(2t)$.

It can be seen $\zeta = 0.9, \vartheta = 4, \vartheta_1 = 1.5, \sigma = 0.5$. Select $\delta = 1$, which satisfies condition (3). Select the system state vector and the initial values of the observer $x(0) = \hat{x}(0) = [1, -2]^T$, $f_s(0) = 0$, $f_a(0) = 0$ are the initial states.

To design observer (6), we choose $\overline{L}_d = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$. This means $g1 = 1$ and $W = 1$, then S_{a} can be calculated. Select $\mu = -\lambda_{\min}(\overline{S_{\text{a}}}^{-1}\overline{A_{\text{a}}}) + 0.0001 = 13.3321$, such that

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$R_e\left[\lambda_j(\overline{S_a}^{-1}\overline{A_a})\right] > -\mu$ for $j = 1, 2, \ldots, \bar{n}$. Seek the solution of Lyapunov's equation (7) and get

$$\overline{L}_P = \begin{bmatrix} -1166100 \\ 11474 \\ -9523.6 \\ 75.9928 \end{bmatrix}.$$

Given $\eta = 4.6621 \times 10^{-11}, \gamma = 2.4$, solve LMI and get $\overline{P_a}, \overline{R_a}, \overline{H_a}$. This means that the linear constraint $\overline{B}_f^T \overline{S_a}^{-T} \overline{P_a} = \overline{H_a C_a}$ is true, thus $s(t), u_s(t)$ can be calculated.

For the proposed controller $u_a(t)$, we have solved

$$K = \begin{bmatrix} 0.1004 & -1.0106 \end{bmatrix},$$
$$G = \begin{bmatrix} 0.0081 & 0 \end{bmatrix}.$$

Select $\varepsilon = 1000, k = 0.5$, we can solve $u_a(t)$.

The simulation results of state estimation and fault estimation are displayed in Figures 1–6. Figures 1–4 indicate the state vector $\{x_1, x_2\}$ and its estimated version. Figure 5 describes the actuator fault and its estimated version, and Figure 6 shows the sensor fault and its estimated result. Figure 7 shows the augmented system state vector error.

From these figures, it can be seen that the system can still operate stably despite the accompanying faults and disturbances. As shown in Figures 1 and 2, the SMO can successfully estimate

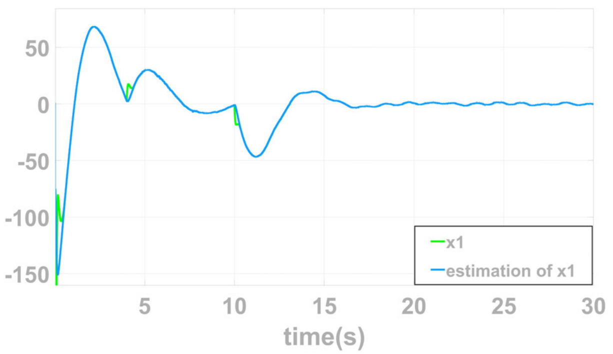

FIGURE 1 | The state x_1 and its estimation for (48).

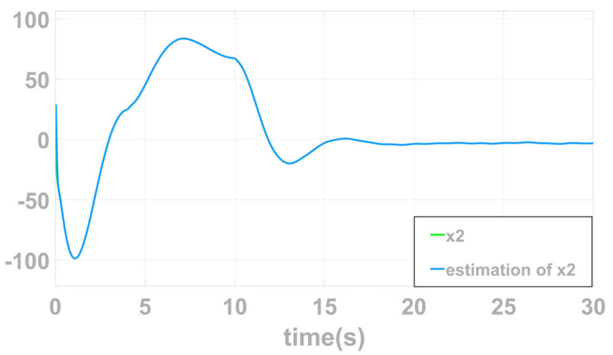

FIGURE 2 | The state x_2 and its estimation for (48).

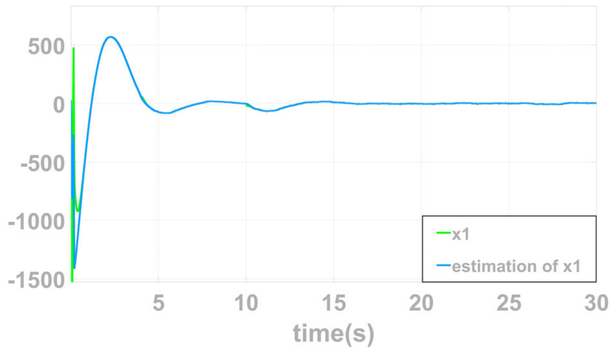

FIGURE 3 | State vector x_1 with different initial values for (48).

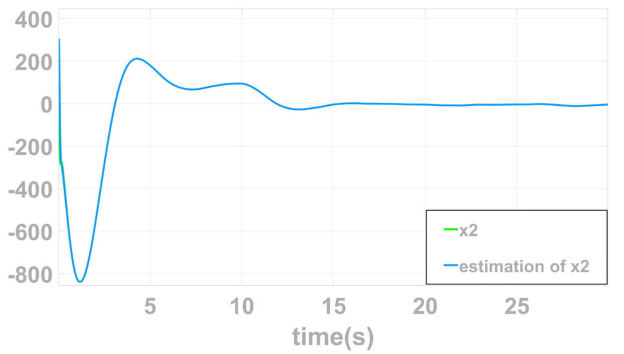

FIGURE 4 | State vector x_2 with different initial values for (48).

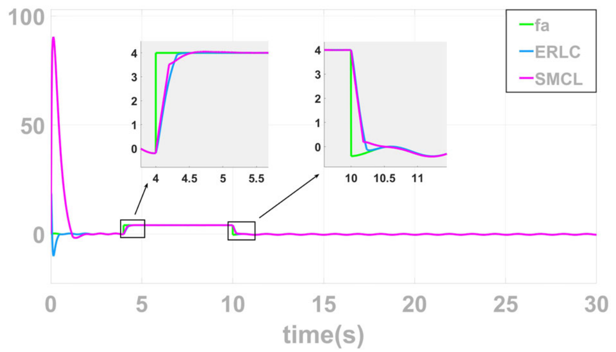

FIGURE 5 | The actuator fault $f_a(t)$ and its estimation for (48).

state vectors and multiple failures. In Figures 3 and 4, the initial values of the system state vectors x_1 and x_2 are changed to [20, −10], and the initial observer value is set to [30, −20]. By comparing Figures 1–4, when the initial value is not changed, it takes 0.469s for the state vector x_1 to achieve accurate estimation, and 18.365s for the state vector x_1 to reach a stable state. The state vector x_2 took 0.284s to achieve an accurate estimate and 18.221s to reach a stable state. After changing the initial value, it takes 0.620s for the state vector x_1 to achieve accurate estimation, and 13.053s for the state vector x_1 to reach a stable state. The state vector x_2 takes 0.367s to achieve accurate estimation and 15.217s to reach a stable state. These results show that although different initial values have an impact on the system state vector initially, they all converge under the action of the exponential reaching law controller (ERLC).

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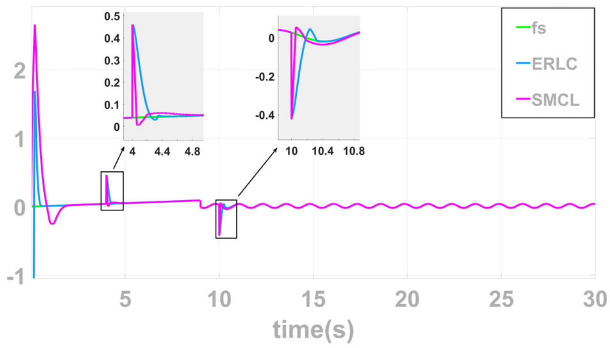

FIGURE 6 | The sensor fault $f_s(t)$ and its estimation for (48).

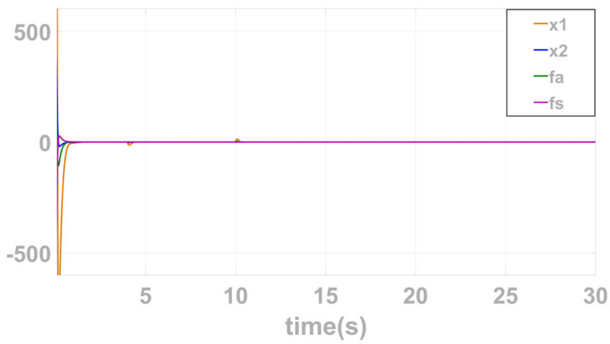

FIGURE 7 | Linear augmentation of system state vector error for (48).

Furthermore, from the tracking effects of actuator faults, sensor faults and different controllers in Figures 5 and 6, It can be seen that the accurate tracking time of the ERLC for actuator fault and sensor fault is 0.559s and 0.506s, respectively. The tracking time of the sliding mode control law (SMCL) in [18] is 2.285s and 1.852s, respectively. ERLC also tracks the fault more quickly than SMCL after being affected by abrupt faults. Compared with the SMCL, ERLC has faster approximation time and higher estimation accuracy.

4.2 | Lipschitz Nonlinear System

Blade and pitch system of floating wind turbine with Lipschitz nonlinear system is considered as

$$\begin{cases} \dot{x}_i(t) = \begin{bmatrix} -2\varsigma\omega_n & -\omega^2{}_n \\ 1 & 0 \end{bmatrix} x_i(t) + \begin{bmatrix} \omega^2{}_n \\ 0 \end{bmatrix} (\beta_r(t) + f_a(t)) \\ \qquad + B_r r(t) + \Phi_{\mathrm{a}}(t, x, u) \\ y_{\mathrm{a}}(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x_i(t) + D_s f_s(t) + D_a f_a(t) \end{cases} \tag{50}$$

We select

$$\Phi_{\mathrm{a}}(x) = \begin{bmatrix} 0 \\ -0.333\sin(x_2) \end{bmatrix}$$

and other data are the same as (49).

Suppose $f_a(t), f_s(t), r_{\mathrm{a}}(t)$ has the following form:

$$f_a(t) = \begin{cases} 0.2t, & \text{for} \quad t \le 4 \\ 0.3 + 0.3\cos(10(\mathrm{t}-3)), & \text{for} \quad 4 < t \le 30 \end{cases}$$

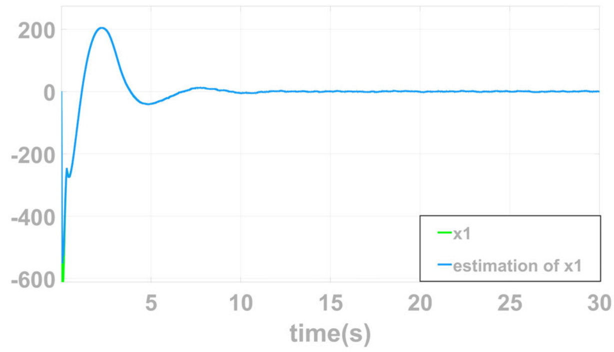

FIGURE 8 | The state x_1 and its estimation for (50).

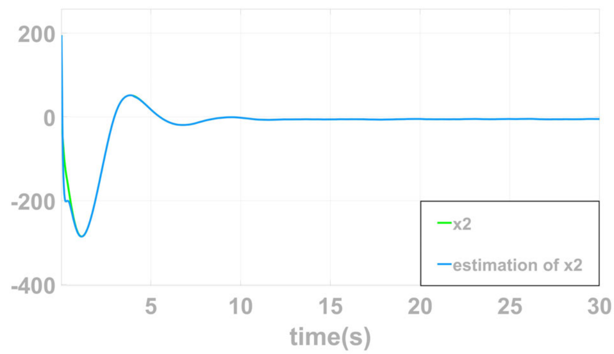

FIGURE 9 | The state x_2 and its estimation for (50).

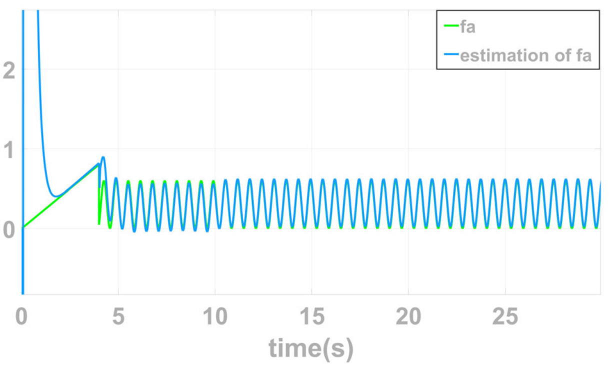

FIGURE 10 | The actuator fault $f_a(t)$ and its estimation for (50).

$$f_s(t) = \begin{cases} 0, & \text{for} \quad 0 \le t \le 2 \\ 0.5(\mathrm{t}-2), & \text{for} \quad 2 < t \le 10 \\ 1 + 0.1\sin(4t), & \text{for} \quad 10 < t \le 30 \end{cases}$$

and $r_{\mathrm{a}}(t) = 0.5\sin(2t)$, $\zeta = 4$, $\vartheta = 0.8$, $\vartheta_1 = 3$, $\sigma = 0.5$.

Select $\delta = 1$, which satisfies condition (3).

Select $\varepsilon = 3000$, $k = 0.5$, and we can solve $u_{\mathrm{a}}(t)$.

The following figures show the simulation results of state estimation and fault estimation. Figures 8 and 9 show the state vector $\{x_1, x_2\}$ and its estimated result. Figure 10 indicates the actuator fault and its estimated result. Figure 11 shows the sensor fault

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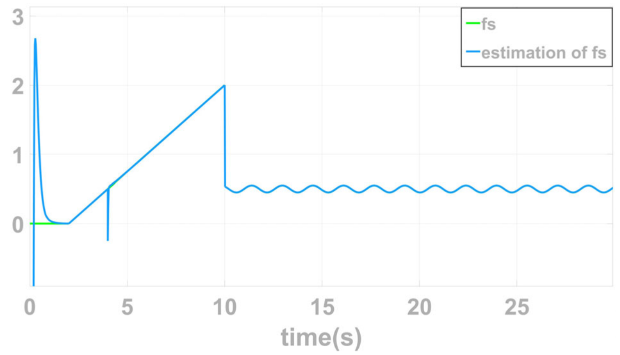

FIGURE 11 | The sensor fault $f_s(t)$ and its estimation for (50).

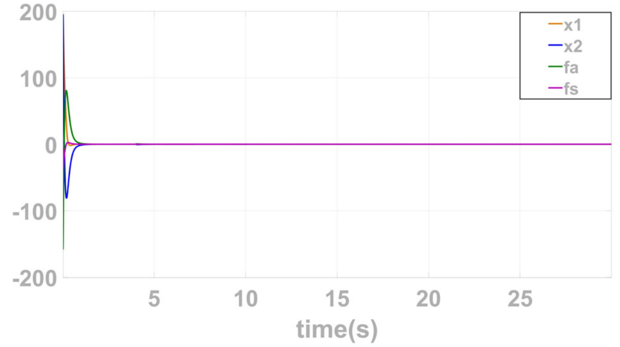

FIGURE 12 | Lipschitz nonlinear augmented system state vector error for (50).

and its estimated result and Figure 12 shows the state vector error of Lipschitz nonlinear augmented system.

From these results it is clear that both the state vector and the faults are accurately estimated from the above figures. At the same time, the nonlinear system maintains stable operation in the presence of faults and disturbances.

5 | Conclusion

Fault estimation and FTC of linear and Lipschitz nonlinear systems accompanied by actuator fault, sensor fault and uncertain disturbances were studied. An SMO based on generalized sliding mode method was constructed to accurately estimate the state vector and multiple faults. In the light of state estimation from SMO, a sliding mode controller was devised to stabilize the closed-loop system. The proposed method has been verified through a numerical study of a blade and pitch system, which holds immense importance for various industrial plants. In addition, the buffeting level can be further reduced by improving the approximation law in the near future, and the reaching law in the controller will provide a novel direction for future improvement. The proposed controller will also be applied to more practical industrial systems.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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