THE SIGNIFICANCE OF POISSON'S RATIO IN THE DETERMINATION OF STRESS AND SETTLEMENT IN SOILS

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THIS IS TO CERTIFY THAT THIS THESIS IS A REPORT OF THE RESULTS OF MY OWN INVESTIGATIONS AND WORK CARRIED OUT WHILE ENGAGED INITIALLY IN FULL-TIME POST-GRADUATE STUDY ON A ONE-YEAR S.A.R. & H. RAILWAY ENGINEERING SCHOLARSHIP, AND SUBSEQUENTLY IN PART-TIME STUDY ON MY OWN.

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C O N T E N T S

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SYNOPSIS

This report describes investigations carried out in order to obtain a better understanding of the concept of Poisson's Ratio as related to the determination of stress and settlement in soils.

Since Poisson's Ratio is primarily a concept associated with the elastic property of materials and therefore generally only enters into soils work in connection with applying the elastic theory to soils, the investigations commence with a brief review of the basic principles of the elastic theory and the elastic properties of materials. It is found that Poisson's Ratio for elastic materials can only fall within the range zero - 0.50.

Consideration is then given to certain 2-dimensional and 3-dimensional problems of stress and strain in the theory of elasticity. It is found that the moduli of elasticity do not affect the stress distribution and hence also the strains in 2-dimensional problems. For the basic 3-dimensional problem viz. that of a vertical point load applied to the horizontal surface of a semi-infinite solid, the affect of various values of Poisson's Ratio on the resulting stress distribution is then considered. The results are presented mainly in the form of stress contour diagrams. These show that in the case of the Maximum Shear Stress (S_{max}) which is a major concern, the stress distribution is insignificantly affected by variation in the value of Poisson's Ratio below a certain comparatively shallow depth. Furthermore, on the axis of loading where the maximum stress values occur, the variation in S_{max} corresponding to the extreme limits of the range of Poisson's Ratio values applicable to elastic materials as mentioned above, is only a constant 14%.

Attention is then turned more directly towards the elastic theory and Poisson's Ratio in relation to soils, and current concepts of elasticity and Poisson's Ratio in soils work are then briefly reviewed. As a result of subsequent discussions, it becomes clear that for all types of saturated soils (ie. both cohesive and cohesionless), Poisson's Ratio can only have the value 0.50 for the purposes of elastic considerations.

An attempt to determine Poisson's Ratio for a soil from laboratory conducted triaxial shear tests, is then described in full. The results of 6 tests conducted show that the value of Poisson's Ratio for the silty sand test soil used (moisture content between 1-2%), lies between 0.14 and 0.24. This order of the values obtained, is found to compare favourably with the order of values obtained by overseas investigators (using methods other than that mentioned above) for various soil types at moisture contents less than saturation value.

It is finally concluded that Poisson's Ratio is only of limited significance in both the determination of stress and settlement in soils. As regards the values of Poisson's Ratio applicable to various soil types for the purposes of elastic considerations, it is concluded that Poisson's Ratio is in the first instance dependant on the moisture content of the soil; secondly, that the value of Poisson's Ratio for all types of saturated soils is 0.50; and thirdly, that in the case of partially saturated soils, the values of Poisson's Ratio for sands are less than those for clays. A suggested graphical relation between Poisson's Ratio and Degree of Saturation is then given in conclusion.

CHAPTER 1 ..... /

CHAPTER 1

INTRODUCTION

The essential requirement of any foundation is that it must be able to support any form of superimposed loading in a safe and stable manner and without detriment to the latter. A soil as the most common type of foundation material encountered in practice, must comply with this requirement. A thorough understanding of the stresses and strains in soil foundations resulting from a superimposed loading is thus essential.

There are in general two approaches to foundation design in the more specific sense of bearing capacity of a soil foundation. Firstly, there is the method known as Limit Design, wherein the safe allowable bearing capacity of a soil is obtained from a determination of its ultimate bearing capacity to which a suitable factor of safety against such catastrophic shear failure is then applied, and secondly, the method of Stress Design, wherein a suitable factor of safety against possible overstress is used to obtain the allowable stress in the soil foundation.

In the case of the latter method of foundation design, some method of determining the stress induced in a soil by an applied loading is necessary. The only means by which this is possible is by the use of some theory relating force and the deformation it produces (as the fundamental alternatives for stress and strain), since it is a mechanical property of all matter that the application of load causes deformation. In some instances, however, deformation requires a certain amount of time to be effected, so that in the general case, load, deformation and time are involved. The simplest theory relating these variables is the theory of elasticity which holds for elastic materials, but this theory has the important limitation of applying only to cases wherein stress and strain are proportional and independent of time. Furthermore, as Taylor(1.1) mentions, "solutions based on the theory of elasticity are far from easy for all but the simplest of cases, and any general theory which covers even minor variations from simple, elastic conditions becomes unbelievably complex ..... Thus the obtaining of a general stress:strain theory ..... especially for cases where time effects enter, must be acknowledged as an impossible goal."

It has therefore become practice to use the theory of elasticity to evaluate stresses and strain in soil masses, on the assumption that soils behave as perfectly elastic, homogeneous, isotropic materials. This assumption and its implications will be discussed in more detail later.

It is in the use of the elastic theory for the purposes of the design of soil foundations by the method of Stress Design that Poisson's Ratio as a property of an elastic material becomes involved, since mathematical expressions for stress and strain derived from the theory of elasticity frequently contain terms of Poisson's Ratio.

Experiments have shown(2.1) that in the case of a prismatical bar of elastic material in simple tension, the axial elongation is a constant for a bar of perfectly elastic material and is known as Poisson's Ratio, after the name of the French mathematician who was the first to determine a value for this ratio by using the molecular theory of structure of a material (see later Section 3.1). In this report, Poisson's Ratio will be denoted by the Greek letter μ.

Hence, .../

Hence, in using expressions derived from the theory of elasticity to evaluate stress and strain in soil foundations, (the latter especially in the form of settlement), the soils engineer is faced with having to decide what value of Poisson's Ratio is applicable to the particular problem he is dealing with, and as yet there is a great deal of uncertainty in making such decisions.

The work described herein is an endeavour to obtain a better understanding of the concept of Poisson's Ratio as related to the determination of stress and settlement in soils by the use of the elastic theory.

In the first Chapters to follow, some aspects of the classical theory of elasticity and Poisson's Ratio in relation to this theory will be discussed, while later Chapters are directed towards the elastic theory and Poisson's Ratio in relation to soils. In particular, a determination of Poisson's Ratio for a soil from measurements made during a triaxial compression test, will be described and discussed.

CHAPTER 2

ELASTIC THEORY CONSIDERATIONS

Whenever dealing with problems relating to the theory of elasticity, it is important to bear in mind the assumptions on which the theory is based, as well as the limits of its applicability.

Most materials possess to some extent the property of elasticity(3.1), viz. that when deformed under the action of a system of applied forces of magnitude not exceeding a certain limit known as the elastic limit, the deformation disappears on removal of the forces and the body regains its original shape. On a broader basis, elasticity implies(4) that there is a definite relation between force and deformation which is independent of any other variable such as time, so that on imposing a load a particular value of strain (or a measure of deformation) occurs, and if this load be removed and re-imposed, the same value of strain as previously will again occur. It should be noted therefore, that elasticity implies only a definite relation between force and deformation and not necessarily any particular relation, such as for instance a linear one.* +

However, Robert Hooke(22),(3.1) was the first to investigate the elastic property of materials more closely and he succeeded in establishing by experiment that in many instances the relation between the magnitude of the forces and the deformations that they produce was approximately linear up to the elastic limit. This approximately linear relation between force and the deformation it produces, is the basis of the so-called Hooke's Law which assumes the ideal material in which the relation between force and deformation is in fact perfectly linear. Hooke's Law subsequently became the basis on which the theory of elasticity was developed, so that the term 'elasticity' used in relation to the theory of elasticity actually implies linear elasticity in accordance with Hooke's Law. Throughout this report, therefore, the term 'elasticity' is intended to be understood in this sense.

For the purposes of mathematical treatment, the theory of elasticity assumes that all bodies are perfectly elastic, i.e. the theory assumes Hooke's Law as was mentioned above. Further, it assumes that the matter of such bodies is homogeneous i.e. that at every point of the body the elastic properties are identical in identical directions; and also that the bodies are isotropic i.e. that the elastic properties are the same in all directions.

The elastic theory can thus strictly be applied, in the first instance, only to materials which have properties in accordance with these assumptions on which the theory is based, and in addition, only to that range of loading during which such properties exist. However, although no material is known

which ...../

* It is well-known(4) that a material such as rubber, for example, has a non-linear force:deformation relationship, due to the fact that as it is extended, it becomes more difficult to extend as the extension proceeds.

+ See also Ref. 1, p.808.

which satisfies all these assumptions completely, the theory has been applied to many materials with reasonable accuracy and to steel with great accuracy. In general, the accuracy with which the theory can be applied to any material will depend on how closely the above-mentioned assumptions are realized.

A very important aspect of the theory of elasticity which is often overlooked, is that it only considers two variables viz. force and deformation, or alternatively, stress and strain. All other possible variables such as time and heat are considered to be constant. The magnitude of strain is thus assumed to be independent of the rate of application of stress; also, an instantaneous application of stress is considered to produce an instantaneous strain, and in all cases, during strain the body suffers no change of mass. Hence the theory is not applicable to cases where time effects or change of mass effects occur during strain. In structural materials, both instantaneous strain and constant mass during strain are invariably realized, although in the case of strain certain secondary effects(63), such as creep do also occur.

With regard to Poisson's Ratio: it was already been mentioned in Chapter 1 that it is a constant in the case of perfectly elastic materials. It should, however, be pointed out in view of the above discussions that this is actually only the case with the ideal material which behaves strictly according to Hooke's law. By implication, this also means that Poisson's Ratio for the ideal material is only constant within the elastic range of loading, i.e. while the relation between stress and strain is linear. In practice therefore, values of Poisson's Ratio are not rigidly constant(64) within the elastic range and furthermore, when the elastic limit is exceeded Poisson's Ratio becomes a variable quantity with a value which is dependent upon stress. It is known(65) that in the case of materials which contract under small loads and expand when a state of failure is approached, Poisson's Ratio shows a considerable increase in value when the failure condition is approached.

However, it is of extreme importance to realize that when applying the theory of elasticity to any material, the value of Poisson's Ratio which is applicable, is strictly that constant value pertaining to the ideal elastic state assumed by Hooke's Law. In general, Poisson's Ratio is seldom considered other than in relation to the elastic state, especially as it will be shown in the next Chapter that the concept of Poisson's Ratio was originally derived from a consideration of the elastic state of materials. In this connection also, Taylor(66) mentions that "The meaning of the elastic constants, Young's Ratio, Poisson's Ratio, Bulk Modulus, etc. seem to be restricted in connection with non-elastic materials."

A second constant which is related to the elastic state of materials, is Young's Modulus. The linear relation between stress and deformation is sufficient, or alternatively, the linear relation between stress and strain, implies that the relation between stress and strain is constant. This constant is known as the coefficient of elasticity. Most of the remarks made above in the case of Poisson's Ratio are also applicable in the case of Young's Modulus(67). For the ideal elastic material Hooke's Law applies, i.e. has a constant value within the elastic range of loading.

Tests ...../

These two constants are however not the only constants of elasticity. The Bulk Modulus (K) and the Modulus of Rigidity (G) are also such constants. It can be shown (see Section 3.2), that the constants of elasticity are inter-related. Yet, from the discussion to follow in the first Section of the next Chapter, it will be seen that for the complete definition of the elastic properties of a material, the elastic theory requires the use of only two such constants, the Modulus of Elasticity E (i.e. Young's Modulus) and Poisson's Ratio μ being the two most commonly accepted (1.1).

CHAPTER 3

POISSON'S RATIO IN RELATION TO ELASTIC MATERIALS

3.1 Historical(5)

During the initial stages of the development of the theory of elasticity at the beginning of the 19th Century, the early investigators such as Navier, Cauchy, Poisson, Lamé, St. Venant, and others, all based their work on the so-called molecular theory viz. that the elastic property of a body can be explained in terms of some kind of attractive and repulsive forces between the ultimate particles of the body.

Navier in 1821, working from the molecular theory, was the first to establish a relation between the deformations and the molecular forces for an isotropic body. These relations were in the form of a set of three differential equations of equilibrium, which notably contained only one elastic constant. This meant, that the elastic properties of an isotropic body could be completely defined by one constant, say the modulus in tension (E), was accepted by the other early investigators already mentioned.

Poisson being very interested in the theory of elasticity based on the idea of molecular structure, obtained equations of equilibrium and boundary conditions similar to those obtained by Navier. When applying his general equations to isotropic bodies, he found that for simple tension of a prismatical bar the axial elongation must be accompanied by a lateral contraction of magnitude μ, where $\mu = 0.25$. Thus, Poisson's value of $\mu = 0.25$, for the ideal isotropic material, was based on the uni-elastic-constant hypothesis of the molecular theory.

However, in 1839 George Green offered a derivation of the equations of elasticity without using any hypothesis regarding the behaviour of the molecular structure of elastic bodies. His procedure was a derivation of the stress-strain relations from a consideration of strain energy and showed that for an isotropic body two elastic constants were required, instead of one as demanded by the molecular theory. His paper started a serious controversy and gave rise to two schools of thought in the development of the theory of elasticity. As a result, several physicists became interested in the determination of elastic constants and tried to settle the argument by direct tests.

Experiments by E. Wertheim and later by A.Kupffer to determine values of Poisson's Ratio for various materials, in both cases showed values different to Poisson's theoretical value of $\mu = 0.25$, i.e. for steel a value of $\mu = 0.304$ was found. Although these and other experimental results were in conflict with the result from the uni-constant hypothesis, and could only be justified by adopting a two-elastic-constant hypothesis for isotropic bodies, the controversy could not be settled because it was always possible to argue that the materials used had not been perfectly isotropic.

George Green's result found further support when G.G.Stokes and Franz Neumann independently verified, by different theoretical approaches remote from the molecular theory, that two elastic constants were necessary to define the elastic properties of a material completely. It was only at the start of the 20th Century that W. Voigt, one of Neumann's pupils, finally settled the controversy by determining the elastic moduli of thin prisms cut out from single crystals of a material, and in

addition ...../

addition studying the compressibility of crystals under uniform hydrostatic pressure, and then showing that the results he obtained definitely disproved those relations between the elastic constants which follow from the uni-constant theory.

Thus the Navier-Poisson hypothesis regarding molecular forces was shown to be untenable and with it Poisson's value of $\mu = 0.25$ for the ideal isotropic material was shown to be inadmissible.

3.2 The possible Range of Values of Poisson's Ratio for Homogeneous, Isotropic, Elastic Materials

In discussing this problem it is useful to consider in the first instance the equations relating the various constants of elasticity (alternatively known as the moduli of elasticity). These equations are simple relationships in strength of materials and are dealt with fully in any text on the subject. Their derivations will therefore not be given again in full herein, but detailed references will be given throughout.

These relationships are obtainable by considering a cube of perfectly elastic material of unit size subjected to the action of a three-dimensional system of principal stresses as shown in Fig. 3.1 below:

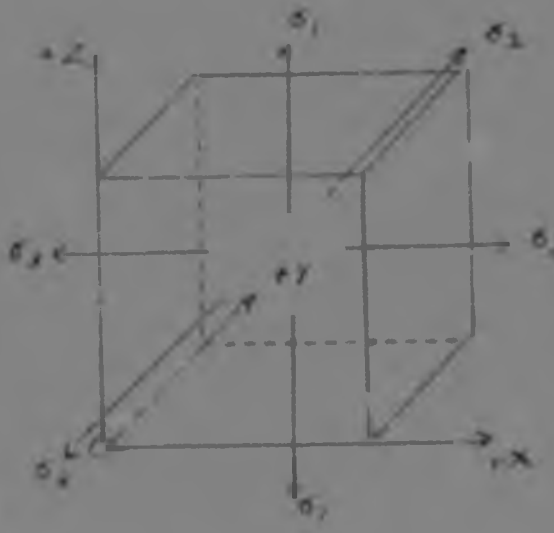


Fig.3.1

It can be shown that the sum of the strains along each of the principal axes of strain, is given by:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{1}{E}(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu) \quad \ldots\ldots\ldots 3.1$$

However, it can also shown that the unit volume expansion or Dilation (denoted by the Greek letter Δ) of the cube as a result of the action of the applied principal stresses is approximately (neglecting quantities higher than the first order) given by:

$$\Delta = \frac{\delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad \ldots\ldots\ldots 3.2$$

where V denotes the original volume of the cube and δV the subsequent change in volume.
Therefore, by combining 3.1 and 3.2:

$$\text{Dilation } (\Delta) = \frac{\delta V}{V} = \frac{1}{E}(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu) \quad \ldots\ldots\ldots 3.3$$

Bound ...../

Ref.2, p.52 and p.61-63.

Based on equation 3.3, it may be shown+ that for fluid pressure conditions when $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ say, and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$ say,

$$\frac{\sigma}{\Delta} = \frac{E}{3(1-2\mu)} = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

where K is known as the 'Modulus of Elasticity of Volume', or simply, the 'Bulk Modulus'.
Furthermore, the condition that $\sigma_1 = -\sigma_2 = \tau$ say, and $\sigma_3 = 0$; and $\varepsilon_1 = -\varepsilon_2 = \varepsilon$ say, and $\varepsilon_3 = 0$; may be shown+ to result in a pure shear state of stress, and hence a further relationship may be obtained+:

$$\frac{\tau}{\gamma} = \frac{E}{2(1+\mu)} = G \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

wherein τ is the pure shear stress, γ is the pure shear strain, and G is known as the 'Modulus of Elasticity in Shear' or simply, the 'Modulus of Rigidity'.
Finally, by combining equations 3.4 and 3.5, it can readily be shown that:

$$3K(1 - 2\mu) = 2G(1 + \mu) \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

With regard to the algebraic signs of the three moduli of elasticity E, G and K, it can be shown (see Appendix I) that whatever the algebraic signs of the principal stresses and strains and the pure shear stresses and strains (i.e. whether they be positive or negative), these moduli of elasticity always have a positive value.

Several(1) referring to equation 3.6 above, point out that "if μ were greater than 1, or if μ were less than -1, either E or G would be negative, and hence for an isotropic solid Poisson's Ratio cannot exceed $\frac{1}{2}$ and cannot be less than -1". i.e. $-1 < \mu < 0.5$. Terzaghi(2) makes no direct statement on the possible range of values of μ, but whenever discussing μ in relation to elastic behaviour, he never seems to consider any values outside the zero-0.5 range*. For instance, in dealing with Steinbrenner's approximate solution for the computation of the settlement of loads covering a finite part of the surface of elastic layers on a rigid base, he shows the extreme limits the effects of $\mu = 0$ and $\mu = 0.5$ on the settlement influence value I_ρ, and the relevant curves for I_ρ are given only for μ values in the range zero-0.5. Taylor(1) again, makes no clear statement regarding the possible range of values of Poisson's Ratio. He states(1.3) that: "Poisson's Ratio has values in elastic materials that are always between zero and 0.5."

There thus appears to be agreement as far as the upper limit of μ is concerned, viz. that μ cannot be greater than 0.5 in elastic materials. The physical significance of a $\mu = 0.5$ condition in elastic materials is apparent from equation 3.3. Substituting $\mu = 0.5$, gives the Dilation (Δ) equal to zero, i.e. $\delta V/V = 0$, i.e. $\delta V = 0$, which means that no volume change occurs

under ...../

*Ref.2, p.52 and p.61-63
+Ref.2, p.54-57.

*One instance was found(2.1) where Terzaghi mentions $\mu > 0.5$, but this was in connection with conditions existing when the material approached a state of failure, i.e. conditions outside the elastic range.

under the action of applied stresses. For the case where σ_1, σ_2 and σ_3 are negative (i.e. applied compressive stresses) and $\mu = 0.5$, the material is said to be incompressible. Practical materials of this type are not uncommon, rubber being probably the best example (see Section 3.5).

Regarding the physical implications of a condition $\mu > 0.5$: It can be seen from equation 3.3 that if $\mu > 0.5$, then $\delta V/V$ is negative, i.e. δV is negative, which implies that the material has suffered a volume decrease under the action of positive tensile stresses, or alternatively, the material has suffered a volume increase under the action of applied compressive stresses. It is inconceivable that either of these conditions can occur in practice under elastic conditions. All known materials show a volume decrease under compressive loading, and it will be shown (see discussion in Chapter 10) that soil over the linear range of the stress:strain curve is no exception. Further, as pointed out by Searle, $\mu > 0.5$ would render either the Bulk Modulus (K) or the Modulus of Rigidity (G) negative, and as already discussed, both K and G can only be positive for elastic materials.

Other examples such as a consideration of an isotropic elastic prism resting on a perfectly frictionless base as discussed by Terzaghi[6.5], also lead to impossible situations in the case of elastic materials. In the case mentioned, Terzaghi considers the 4 vertical sides of the isotropic, elastic prism to be confined between perfectly smooth vertical walls such that no lateral strain can occur due to the self weight of the material. From this condition be subsequently derives the relation

$$\frac{\sigma_h}{\sigma_z} = \frac{\mu}{1-\mu} = K \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

where Ko in the theory of earth pressure is known as the "coefficient of earth pressure at rest", and is such that no lateral strains develop. If, in the above equation 3.7, $\mu > 0.5$, then Ko > 1, which means that the lateral pressure σ_h induced by the vertical self weight of the material is greater than the vertical pressure itself due to the self weight. Such a condition is unknown in the case of elastic materials. In fact, $\mu = +1$ gives $Ko = \infty$, which is clearly an impossible condition.

It therefore seems reasonable to conclude that as an upper limit for elastic, isotropic materials, Poisson's Ratio cannot exceed 0.5.

In considering now the possible lower limit of Poisson's Ratio, the significance of a $\mu < 0$ condition will briefly be investigated. Firstly, as Searle pointed out, if μ is negative, it must have a lower limit of -1, since μ less than 1 would again render either the Bulk Modulus (K) or the Modulus of Rigidity (G) negative which is an impossible condition. Further, the physical implications of a negative Poisson's Ratio are apparent from a consideration of Poisson's Ratio itself in the first instance. From its definition, Poisson's Ratio is the ratio of the lateral contraction:longitudinal extension which is observed in the case of a bar of elastic material subjected to axial tensile loading, or the reversed conditions in the case of axial compressive loading. In both cases the lateral and longitudinal strains are considered to have the same algebraic signs, so that μ is always positive. A negative Poisson's Ratio would thus mean that either the sign of the lateral contraction is negative, which implies a lateral expansion accompanying a longitudinal extension; or, the sign of the longitudinal extension is negative, which implies a longitudinal contraction accompanying a lateral contraction. Both these concepts are inconceivable for any practical elastic material.

Also ...../

Also in the case of Terzaghi's isotropic elastic prism confined between perfectly smooth vertical walls, an impossible condition arises if μ is negative. From equation 3.7 again, if μ were negative, $\frac{\sigma_3}{\sigma_2}$ would be negative, i.e. σ_3 is negative, since the vertical pressure due to the self weight of the material always acts downwards and is therefore always positive according to the sign convention used. A negative σ_3 implies that instead of the sides of the prism being acted upon by a lateral pressure due to the confining vertical walls which prevent lateral strain from occurring, the sides of the prism are acted upon by a lateral tension which prevents lateral expansion. This again is clearly an impossible condition.

Hence, it seems reasonable to conclude that Poisson's Ratio in practice cannot be negative for elastic, isotropic, materials.

However, as previously mentioned, both Terzaghi and Taylor consider $\mu = 0$ to be the lower limit of Poisson's Ratio for elastic, isotropic materials. Poisson's Ratio equal to zero means that either a longitudinal extension is accompanied by no lateral contraction; or, a longitudinal contraction is accompanied by no lateral expansion. Such conditions are not uncommon in practice. They usually occur where lateral expansion under compressive stress is completely prevented by some other agency. The laboratory cedometer test is a good example of this condition (see section 6.3). In addition, Hendron (11) mentions that Poisson's ratio for cork can be taken as equal to zero, while the value for concrete is also close to zero, viz. in the region 0.08-0.18.

Hence it seems reasonable to accept a value of zero as the lower limit of Poisson's ratio, and it can therefore be concluded that for homogeneous, isotropic, elastic materials, Poisson's ratio falls within the range zero-0.5.

3.3 Values of Poisson's Ratio for Practical Materials

It is of interest to note the following table of values of Poisson's ratio μ, Modulus of Elasticity E, Modulus of Rigidity G, and Bulk Modulus K, given by Hyde and Ireby (10) for various materials, which are however mainly metals. Both calculated and observed values are given for μ and G; in the case of G, calculated values were obtained by using observed values of E and μ in equation 3.5 of previously. This equation can be rewritten:

$$\mu = \frac{E}{2G} - 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3.5(a)$$

and hence, using observed values of E and G, calculated values of μ were obtained. The Bulk Modulus K was obtained by calculation only, by using equation 3.6 previously. The Authors mention also that the extent of the agreement between the calculated and observed values of μ and G, gives an indication of the degree of isotropy of the material.

Table I ...../

TABLE 3

MATERIAL	POISSON'S RATIO μ		YOUNG'S MODULUS E	MODULUS OF RIGIDITY G (10^{11}dynes/cm^2)		BULK MODULUS K
	OBSERVED	CALCULATED	(10^{11}dynes/cm^2)	OBSERVED	CALCULATED	(10^{11}dynes/cm^2)
Aluminium (Worked)	0.350	0.340	7.00	2.61	2.63	7.46
Copper (Worked) pure	0.337	0.356	12.3	4.55	4.55	13.1
Gold (Worked) pure	0.412	0.425	8.0	2.77	2.80	16.8
Steel (Worked) 1%C	0.287	0.287	20.8	8.12	8.12	16.1
Lead (Cast) pure	0.446	-	1.60	-	0.562	5.00
Palladium (Cast) pure	0.395	0.401	11.3	5.11	4.04	17.6
Platignum (Cast) pure	0.387	0.360	16.8	6.16	6.04	24.7
Silver (Worked) pure	0.370	0.368	7.40	2.87	2.86	10.9
Tin (Cast) pure	0.33	-	5.43	-	2.04	5.29
Bronze (Cast) ‡	0.358	0.377	8.20	3.44	2.97	9.52
Constantin (Worked) #	0.325	0.329	16.5	6.19	6.11	15.8
Manganin (Worked)+	0.329	0.348	12.4	4.65	4.65	12.1
Zinc (Cast) pure	0.21	-	8.7*	3.8	-	-
Rubber (soft, vulcanised)	0.46-0.49	-	0.10-0.70	0.00016	-	-
Jena Glasses (crown)	0.20-0.27	-	6.5 -7.8	2.6-3.2	-	4.0-5.9
Jena Glasses (flints)	0.23-0.25	-	5.0 -6.0	2.0-2.5	-	3.6-3.8

‡ 85 Cu, 7.2 Zn, 5.4 Sn; # 60 Cu, 40 Ni;
+ 84 Cu, 12 Mn, 4 Ni; * pure Zn : E = 8.7 x 10^{11} dynes/sq.cm.

In addition to the values of Poisson's Ratio given in the above Table (which are mainly for metals), the following values are available for materials other than metals: as mentioned at the end of the previous Section, Timoshenko(2.1) states that Poisson's Ratio for cork can be taken as equal to zero, while the value for concrete is in the region 0.083-0.125. Further, Krynine and Judd(7) give the following values of Poisson's Ratio for various rock types: Granite 0.16-0.34, Limestone 0.16-0.33, schist 0.02-0.29, marble 0.25-0.30: and, from isolated tests, the value for gneiss is 0.21, monzonite 0.19, sandstone 0.19 and tuff 0.11.

CHAPTER 4

POISSON'S RATIO IN RELATION TO ELASTIC STATES OF STRESS AND STRAIN

4.1 Introduction

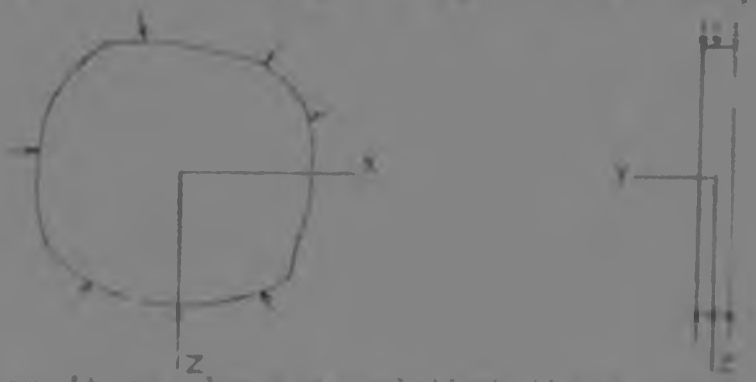
For a body in equilibrium under the action of external forces, 6 stress components are necessary(3.2) to define the state of stress at any point in the body completely. With reference to a rectangular system of coordinate axis OX, OY and OZ, these stresses are the normal stresses σ_x, σ_y, σ_z, and the shearing stresses $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$. Similarly also, 6 strain components are necessary (3.3) to define the state of strain at any point of the body completely. These are the unit strains in the three perpendicular directions ε_x, ε_y, ε_z, and the unit shear strains γ_{xy}, γ_{xz}, γ_{yz} related to the same directions.

Under two types of conditions, however, the number of stress and strain components necessary to define the states of stress and strain at a point completely reduces to only 3 in each case. These two conditions are known as a state of plane stress and a state of plane strain respectively, and in each case the stress distribution problem is reduced to a 2-dimensional problem of elasticity.

4.2 Plane Stress and Plane Strain(3.8)

If a thin plate is subjected to the action of forces applied at the boundary, parallel to the plane of the plate and distributed uniformly over its thickness (Fig.4.1), the stress components σ_y, τ_{xy}, τ_{yz} are zero on both surfaces of the plate, and without

Fig.4.1



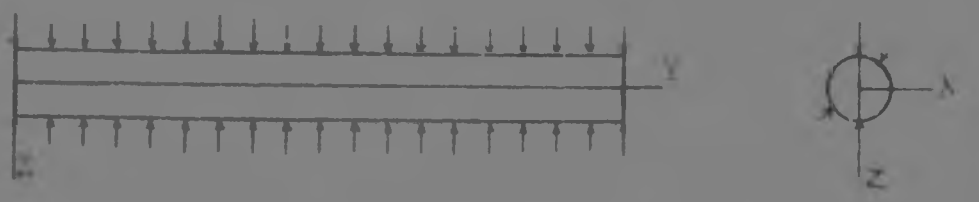
substantial error it can be assumed that these components are zero throughout the thickness of the plate. This condition constitutes a state of plane stress distribution. For its solution, a problem of plane stress therefore requires the determination of only three stress components: σ_x, σ_z and τ_{xz}. It should be noted, however, that the fact that the stresses σ_y, τ_{xy}, τ_{yz} are zero on cross-sections perpendicular to the Y-axis of the plate, does not necessarily mean that the corresponding strains ε_y, γ_{xy}, and γ_{yz} are zero, since Poisson's Ratio effects occur as a result of the stresses which are not zero.

The same simplification of the problem as in the case of thin plates is encountered when one goes to the other extreme in which the dimension of the body in the Y-direction is very large. If a long cylindrical or prismatical body be subjected to the action of forces uniformly distributed along its length and in the planes of cross-sections perpendicular to its axis (Fig.4.2), the displacements during deformation at a considerable distance from the ends, occur only in planes perpendicular to the axis of the body, and the strain components ε_y, γ_{xy}, and γ_{yz} are zero. This condition is known as a state of plane strain.

Furthermore ...../

Furthermore, the shearing stresses τ_{xy} and τ_{yz}, which are proportional to γ_{xy} and γ_{yz} are also zero, while, since $\varepsilon_y = 0$, the normal stress σ_y (which is not equal to zero) can readily be expressed in terms of σ_x and σ_z by applying basic principles of stress and strain. A problem of plane strain is therefore reduced, as in the case of problems of plane stress, to the determination of only three stress components : σ_x, σ_z and τ_{xz}, and

Fig. 4.2



hence, in both cases, the problem of stress and strain reduces to a problem in 2-dimensions. It should also be noted in these case of problems of plane strain, that conditions are not altered in any way if a uniform extension in the direction of the axis of the body is superi-posed upon the state of plane strain. Furthermore, the fact that the three strains mentioned (ε_y, γ_{xy}, γ_{yz}) are zero, constitutes an enforced condition due to the length of the body, in which the three normal stresses σ_x, σ_y, σ_z acting over cross-sections normal to the axis of the body, serve to maintain the state of plane strain.

These two conditions of plane stress and plane strain therefore constitute 2-dimensional problems in the theory of elasticity, and in all such cases the determination of stress and strain in elastic bodies is considerably simplified.

4.3 The Moduli of Elasticity in relation to States of Plane Stress and Plane Strain

In dealing with the problem of an elastic, isotropic solid in equilibrium under given body and surface forces, Michell[11] treats the following two 2-dimensional cases:

(a) That of a long cylinder with applied forces perpendicular to its length and the same at corresponding points along its length (a case of plane strain),

and (b) that of a thin plate with applied forces in its plane (a case of plane stress).

He determines the stress function which satisfies the equations of equilibrium and the boundary conditions in each case, and then shows that if the body forces are zero, the stresses in the solids are independent of the moduli of elasticity provided the resultant force on each boundary vanishes. He also points out that where there is a single boundary (such as in the case of a semi-infinite solid), this latter condition is always satisfied and the stresses are therefore always independent of the moduli of elasticity in this case, provided there is no body force. His final conclusions are that the stress distribution in elastic isotropic bodies for the cases considered (cases of plane stress and plane strain) are independent of the moduli of elasticity if, firstly, body forces are absent, and secondly, the boundary is simply connected. If the boundary is multi-connected, such as in the case of a perforated body, the stresses are independent of the moduli of elasticity only if the resultant force on each boundary separately vanishes.

It ...../

It thus appears that in 2 dimensional problems (cases of plane stress and plane strain) wherein the stress distribution in a semi-infinite, elastic, isotropic solid is required (the case of a single or simply-connected boundary), the stress distribution is independent of the moduli of elasticity provided the body forces (such as the self weight of the material) are disregarded.

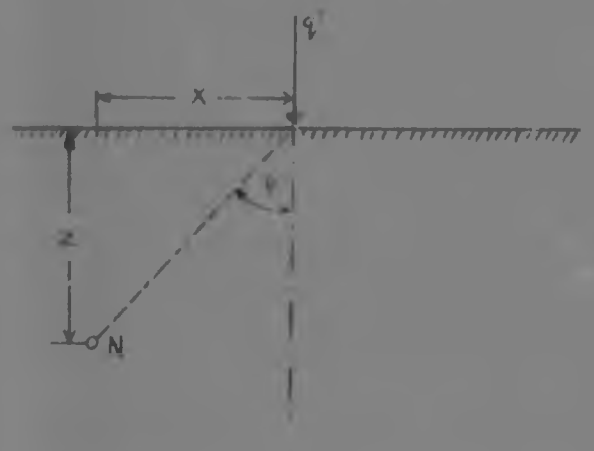
It is of interest to note[3.5] that Michell's conclusion is of great practical importance in cases where the photo-elastic method of stress analysis is used, since it enables experimental results obtained with transparent materials to be applied immediately to any other material such as steel if the external forces are the same.

4.4 Poisson's Ratio in relation to States of Stress and Strain in a Semi-infinite Elastic Solid

With a view to the later discussion of the application of the elastic theory to soils which, for such purposes, are generally considered to be semi-infinite in extent, certain 2-dimensional and 3-dimensional problems of stress and strain in a semi-infinite elastic solid will be reviewed in this Section.

Firstly, there is the case of an infinitely extended line load of constant magnitude per unit length on the surface of a semi-infinite elastic solid (Fig.4.3). From the discussion in the previous Section, it is clear that this loading condition produces a state of plane strain in the solid, and the problem is therefore essentially a 2-dimensional one. This case was originally dealt with in a paper by Flamant[12] in which he outlines the derivation of the mathematical expressions for the stresses induced at any point in the solid due to such applied loading. These expressions have also been given in polar coordinates by Terzaghi (8.4) as follows (cf.Fig.4.3):

Fig. 4.3



$$\sigma_z = \frac{2}{\pi} \cdot \frac{q'}{x} \cos^4\gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4.1$$

$$\sigma_x = \frac{2}{\pi} \cdot \frac{q'}{x} \cos^2\gamma \cdot \sin^2\gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4.2$$

$$\tau_{xz} = \frac{2}{\pi} \cdot \frac{q'}{x} \cos^3\gamma \sin\gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4.3$$

where q' is the load per unit length of the straight line. It should be noted as discussed in Section 4.2, that due to the state of plane strain, only three stress components appear in this case. Furthermore, it should be noted that the equations do not contain Poisson's Ratio for the reasons discussed in Section 4.3

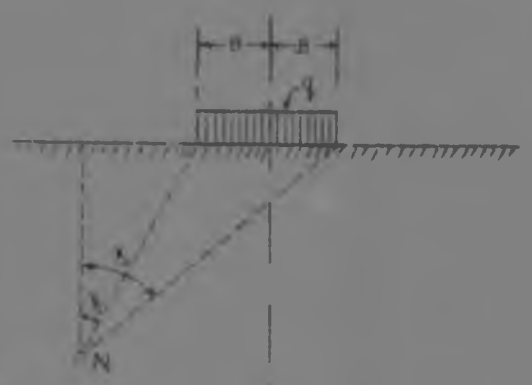
In the latter connection, it is of interest to note how Poisson's Ratio becomes eliminated from the basic expressions

of...../

of equilibrium in Flamant's original paper during his derivation of the equations 4.1, 4.2 and 4.3 given above. The actual steps in the final reduction of his mathematical expressions from which this is apparent have not been given in the paper, but are given in Appendix II of this paper.

Secondly, there is the case of a strip load of infinite extent and constant (finite) width on the surface of a semi-infinite elastic solid (Fig. 4.4). This problem was solved by Michell(13) and is actually only an extension of the problem solved by Flamant which was discussed above. It is therefore another 2-dimensional problem. The solution is obtained by integration of the basic expressions for the case of a straight line load of constant magnitude and of infinite extension on the surface of a semi-infinite solid. Terzaghi(8.4) gives these expressions for the stresses at any point in the semi-infinite solid as (cf. Fig. 4.4):

Fig.4.4



$$\sigma_z = \frac{q}{\pi}\left[\sin\psi\cos\psi + \psi\right]_{\psi_1}^{\psi_2} \qquad \text{..... 4.4}$$

$$\sigma_x = \frac{q}{\pi}\left[-\sin\psi\cos\psi + \psi\right]_{\psi_1}^{\psi_2} \qquad \text{..... 4.5}$$

$$\tau_{xz} = \frac{q}{\pi}\left[\sin^2\psi\right]_{\psi_1}^{\psi_2} \qquad \text{..... 4.6}$$

where q is the load per unit of area on the strip. It is clear that, for the same reasons as discussed in the case of the first 2-dimensional problem considered above, these expressions also only comprise 3 stress components which do not contain Poisson's Ratio.

Other 2-dimensional problems of stress and strain, such as those solved by Carothers(11) and Jurgenson(15), follow the same pattern as the two cases dealt with above in that for the complete determination of stress and strain, expressions for only the three stress components σ_z, σ_x and σ are required, all of which do not contain Poisson's Ratio.

However, the case of a vertical point load applied to the horizontal surface of a semi-infinite elastic solid (Fig.4.5), which was originally solved by Boussinesq(16), is a 3-dimensional problem of stress and strain. The expressions for the stresses at any point in the solid therefore actually comprise 6 stress components in this case, but due to radial symmetry the vertical component (τ_{tz}) and the horizontal component (τ_{tr}) of shearing stress on radial planes (see Fig.4.5) are each equal to zero and only 4 stress components are therefore involved. The relevant expressions only are given by both Taylor(1.3) and Terzaghi(8.5), but their derivation has been outlined again in full by Timoshenko(3.7). The expressions are as follows:

z...../

$$\sigma_z = \frac{Q}{2\pi}\frac{3z^3}{(r^2+z^2)^{5/2}} = \frac{Q}{2\pi z^2}(3\cos^5\theta) \quad \text{..... 4.7(a) and 4.7(b)}$$

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3r^2z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \text{.....}$$

$$= \frac{Q}{2\pi z^2}\left[3\sin^2\theta.\cos^3\theta - \frac{(1-2\mu)\cos^2\theta}{1+\cos\theta}\right] \quad \text{.....}$$

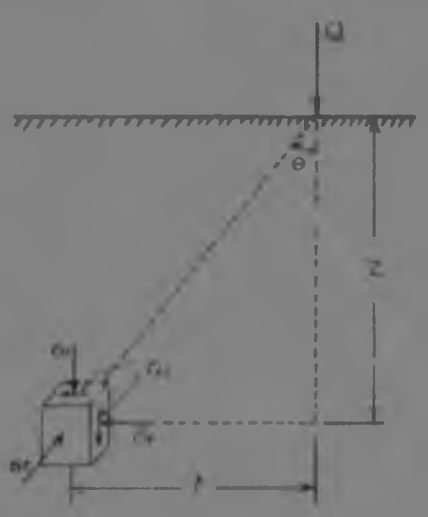
$$\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \text{.....}$$

$$= -\frac{Q}{2\pi z^2}(1-2\mu)\left(\cos^3\theta - \frac{\cos^2\theta}{1+\cos\theta}\right) \quad \text{.....}$$

$$\tau_{rz} = \frac{Q}{2\pi}\frac{3rz^2}{(r^2+z^2)^{5/2}} = \frac{Q}{2\pi z^2}(3\sin\theta\cos^4\theta) \quad \text{..... and}$$

where Q in each case is the magnitude of the applied vertical point load. Poisson's Ratio does appear in the above expressions,

Fig. 1.5



although only in the case of certain stress components viz. σ_r and σ_t. This is the case since the moduli of elasticity affect the stress distribution in 3-dimensional problems.

Extensions of the abovementioned problem in the form of a uniformly distributed loading on a finite area such as a rectangular, square or circular area, also constitute 3-dimensional problems of stress and strain, and hence, the corresponding expressions for the stress components will also contain Poisson's Ratio in certain instances. Yet, according to Terzaghi(8.6), "the computation of the stresses .....(in such cases) ..... are rather involved, and the results of the computation cannot be represented by a simple set of equations. However, the problem has been solved by Love(17) and the results have been compiled in tables which make it possible to determine the stresses at any point by means of a simple process of interpolation."

Both Terzaghi(8.6) and Taylor(1.3) point out, however,

that ...../

τ_{r3}

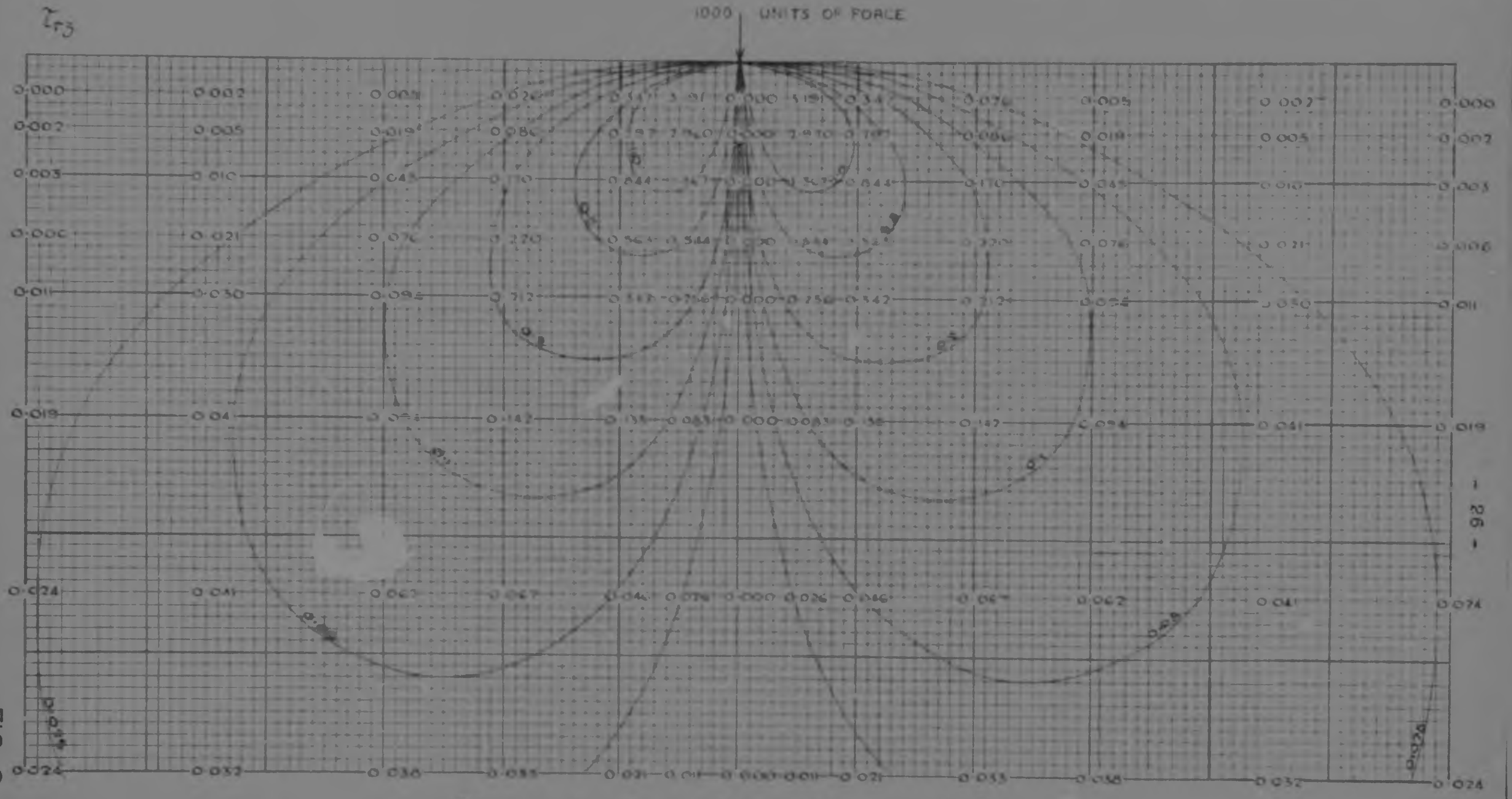


FIG. 2.

FIG 2 - CONTOURS OF VERTICAL SHEAR STRESS (τ_{rz})

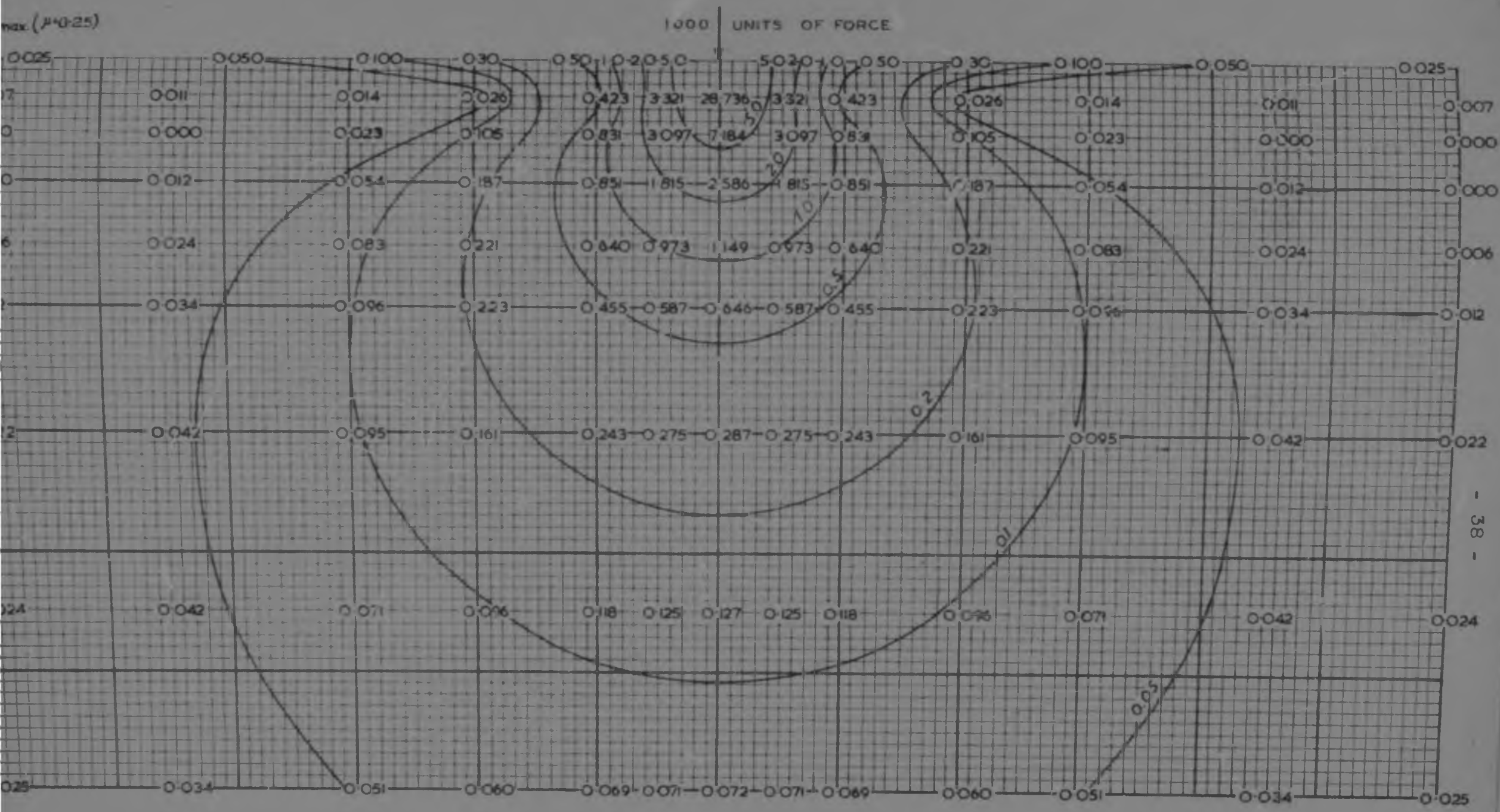
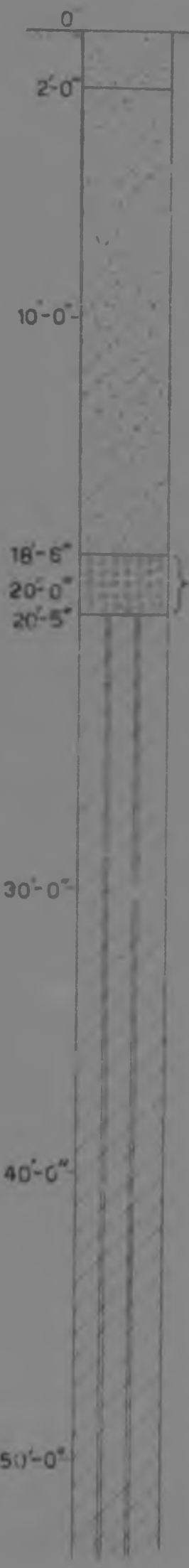


FIG 11 — CONTOURS OF MAXIMUM SHEAR STRESS (Smax) FOR CONDITION (μ=0·25)

SOIL PROFILE ON THE NORTH SIDE OF JUTA ST. NEAR BICCARD ST., BRAAMFONTEIN, J'BURG.



0 – 2'-0": Surface Rubble

2'-0" – 10'-0": Well-graded firm fine reddish-brown silty sand

Moisture Content increasing with depth

Occasional pebbles

18'-6" – 20'-0" / 20'-5": Pebble Marker containing quartz and sandstone pebble layers — maximum size of pebbles approximately 4 in.

20'-5" – 30'-0": Firm red clayey silt, — residual from shale. Requiring pick for excavation.

Joint planes inclined in all directions

30'-0" – 40'-0": Occasional pebbles

40'-0" – 50'-0":

Horizontal Radial Stress σ_r

Equation: $\sigma_r = \frac{Q}{2\pi}\left[\frac{3r^2z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$

TABLE IV

z	r	σ_r				
		$\mu = 0$	$\mu = 0.1$	$\mu = 0.25$	$\mu = 0.375$	$\mu = 0.5$
5	0	-0.[illegible]413	-3.6311	-4.1210	-4.2105	0
	5	+2.1979	+2.[illegible]	+3.[illegible]	+[illegible].5391	+[illegible].3190
	10	+0.0191	+0.3024	+0.5873	+0.8722	+1.1555
	20	-0.1703	-0.0[illegible]	0	+0.0844	+0.1687
	30	-0.10[illegible]	-0.0[illegible]	-0.0271	+0.0127	+0.05[illegible]
	45	-0.0[illegible]73	-0.0[illegible]82	-0.0207	+0.0016	+0.0159
	60	-0.[illegible]310	-0.[illegible]	-0.0111	-0.0032	+0.0064
8	0	-[illegible].2107	-1.[illegible]884	-1.1045	-0.5523	0
	5	+0.[illegible]31	+1.[illegible]624	+1.7300	+[illegible].0993	+[illegible]
	10	+0.[illegible]570	+0.[illegible]496	+0.[illegible]438	+1.1364	+1.32[illegible]
	20	+0.00[illegible]	+0.0764	+0.1464	+0.21[illegible]	+0.[illegible]01
	30	-0.0[illegible]	-0.0111	+0.0[illegible]	+0.0605	+0.[illegible]
	45	-0.0[illegible]	-0.0[illegible]	+0.00[illegible]	+0.0127	+0.[illegible]
	60	-0.[illegible]71	-0.0175	-0.0064	+0.0032	+0.0127
10	0	-0.7[illegible]	-0.[illegible]	-0.3979	-0.1[illegible]	0
	5	+0.0111	+0.17[illegible]	+0.[illegible]470	+0.5141	+0.[illegible]
	10	+0.3772	+0.[illegible]34	+0.6112	+0.7273	+0.[illegible]
	20	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	30	+0.0[illegible]	+0.[illegible]	+0.0748	+0.1050	+0.1353
	45	-0.0[illegible]	0	+0.0143	+0.0[illegible]	+0.[illegible]
	60	-0.0[illegible]	-0.[illegible]	+0.0032	+0.0111	+0.[illegible]
15	0	-0.[illegible]	-0.[illegible]	-0.3760	-0.[illegible]	0
	5	-0.1[illegible]	-0.[illegible]	+0.0191	+0.1003	+0.[illegible]
	10	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	20	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	30	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	60	-0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
20	0	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	+0.[illegible]	+0.[illegible]
	10	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	20	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	30	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	60	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
30	0	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	+0.[illegible]
	10	-0.[illegible]	-0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	20	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.0939
	30	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.0939
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	60	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
45	0	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	+0.[illegible]
	10	-0.[illegible]	-0.[illegible]	-0.[illegible]	0	+0.[illegible]
	20	-0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	30	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	60	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
60	0	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	+0.[illegible]
	10	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	+0.[illegible]
	20	-0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	30	-0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]
	60	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]

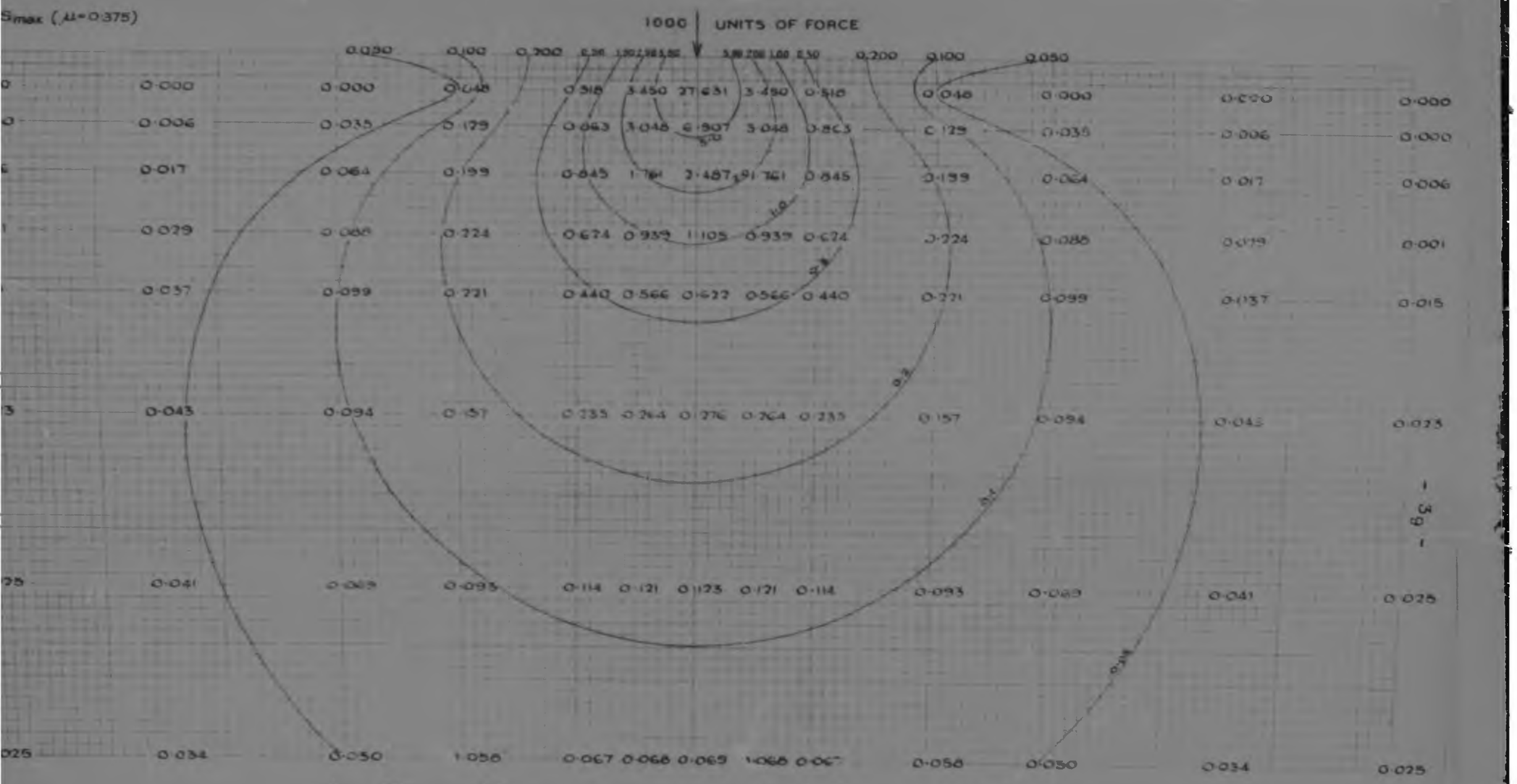


FIG 12 - CONTOURS OF MAXIMUM SHEAR STRESS (S_{max}) FOR CONDITION $\mu = 0.375$

The results of various laboratory tests on the soil are tabulated below:

TABLE VII

Atterberg Limits	: Liquid Limit	:	25.2%
	Plastic Limit	:	17.0%
	Plasticity Index	:	8.2%
	Linear Shrinkage	:	3.3%
Grading Analysis	: Gravel	:	8.5%
	Sand	:	53.0%
	Silt	:	30.0%
	Clay	:	8.5%
Specific Gravity	:	:	2.62
Void Ratio	: (Undisturbed state)	:	0.70
Compression Index C_c	: " "	:	0.26

The soil appears to fall into the class ML of the Casegrande Airfield Soil classification and into the class A4 of the Public Roads classification. A full grading curve and a pressure : void ratio curve determined from a normal 1-dimensional consolidometer test, are shown in Figs. 17 & 18 respectively. The moisture content of the soil at the time of sampling in the field is not known, but would appear to have been approximately 3% (see Appendix IV, Fig.53), thus indicating a Degree of Saturation of approximately 11%.

7.3 Design of the Experimental Work

The aim of the experimental work was to determine Poisson's Ratio for the selected test soil by conducting a series of Consolidated Undrained Triaxial Shear Tests ($\bar{c}_{cu}$) with strain control on samples of the soil. The tests were to be according to British standard practice.

In view of previous discussions, this meant that the experimental work had to be designed to enable the measurement of the change in volume of a soil sample of known initial volume, due to a known change in stress within the linear range of the soil's stress:strain curve. The information thus obtained would then be used in equation 7.5 previously derived, to enable the evaluation of Poisson's Ratio for the soil.

When a soil sample is set up inside a triaxial test cell ready for test, there is a certain volume of cell fluid inside the test cell surrounding the specimen which is equal to the difference in volume between that of the soil sample and the internal volume of the test cell. Any change in volume of the soil sample must thus cause a flow of fluid either into or out of the cell, depending upon the type of volume change occurring in the sample, which is equal in volume to the change in volume of the soil sample, if the fluid is incompressible.

However, during an actual test, change in volume of the soil sample due to compressive loading is not the only cause of cell fluid entering or leaving the test cell. Entry of the loading spindle into the test cell causes a flow of fluid out of the test cell, while leakage from the test cell, usually where the loading spindle enters the test cell, causes a flow of fluid into the cell from the fluid reservoir.

If the flow of fluid either into or out of the test cell during test be accurately measured, this flow measurement will

be ...../

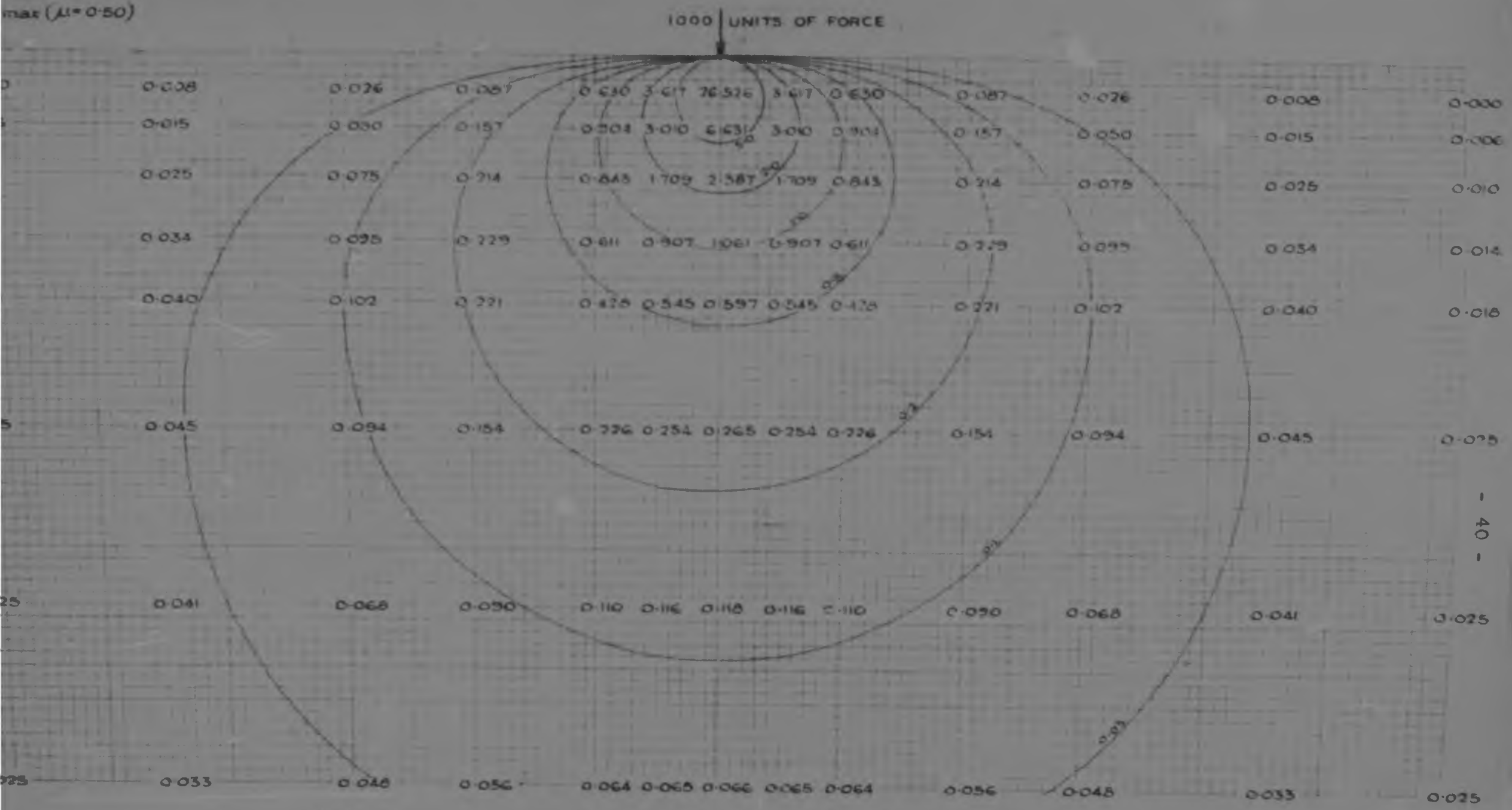


FIG 13 - CONTOURS OF MAXIMUM SHEAR STRESS (S_{max}) FOR CONDITION ($\mu = 0{\cdot}50$)

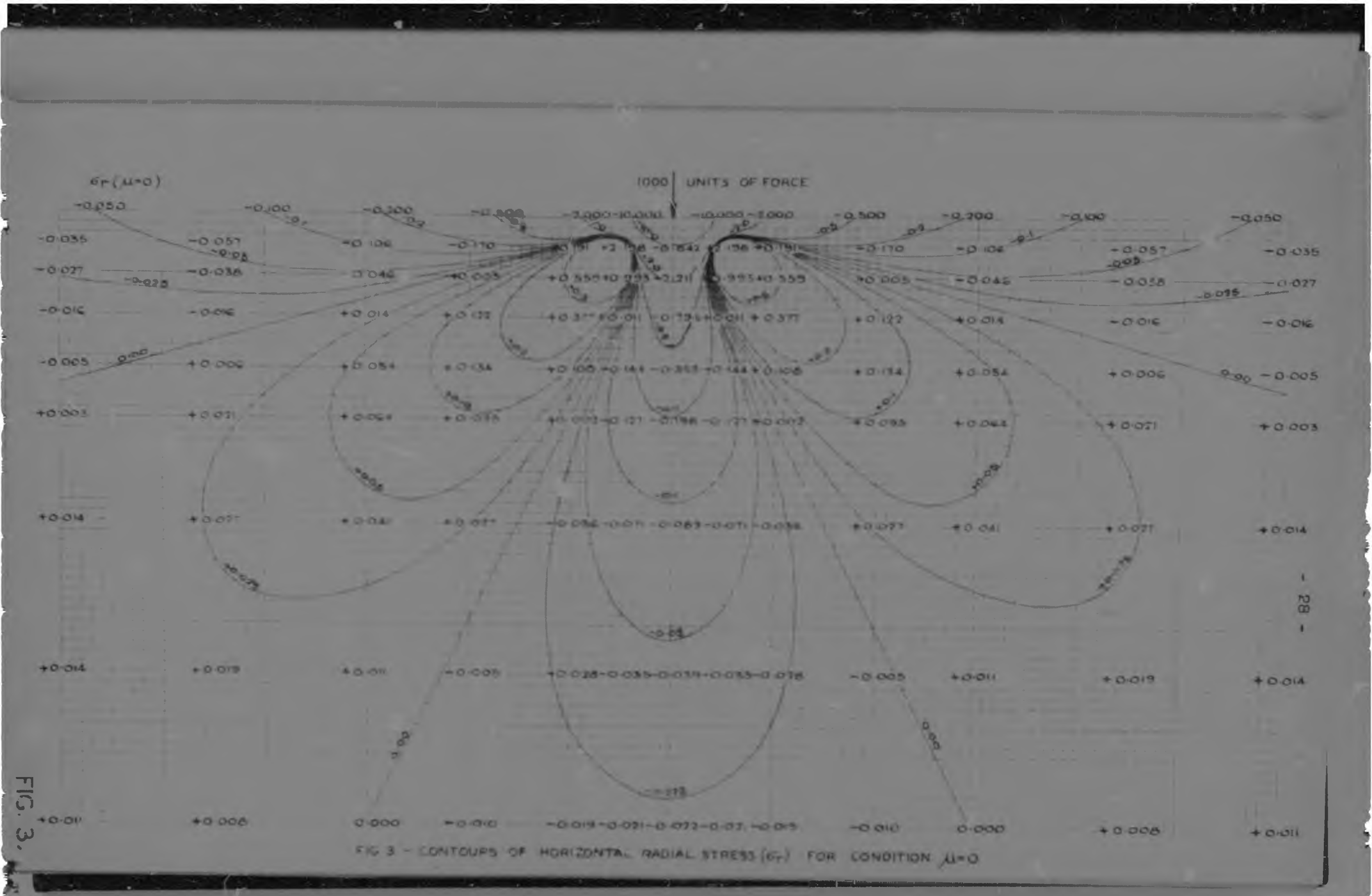
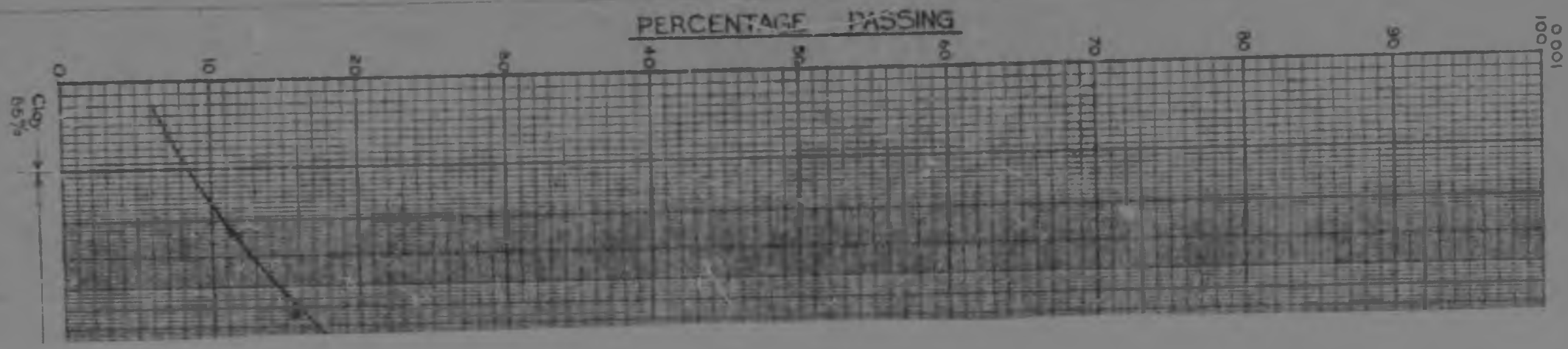


FIG. 3 - CONTOURS OF HORIZONTAL RADIAL STRESS (σ_r) FOR CONDITION μ=0


PERCENTAGE PASSING
0
10
20
30
40
50
60
70
80
90
100
Clay
65%

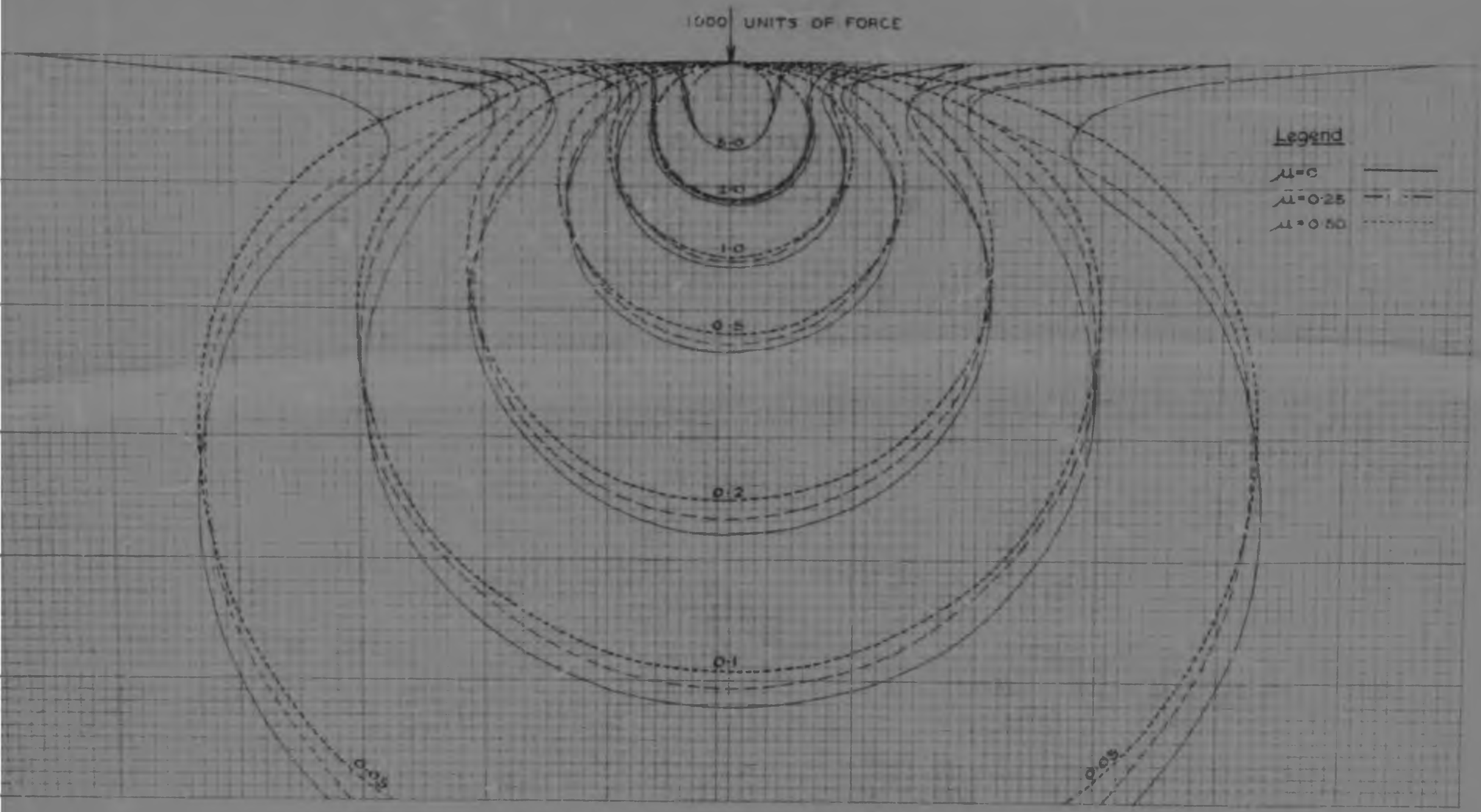


FIG 14 - COMBINED CURVES OF MAXIMUM SHEAR STRESS CONTOURS ($S_{max.}$) FOR CONDITION μ=0, 0·25, 0·50

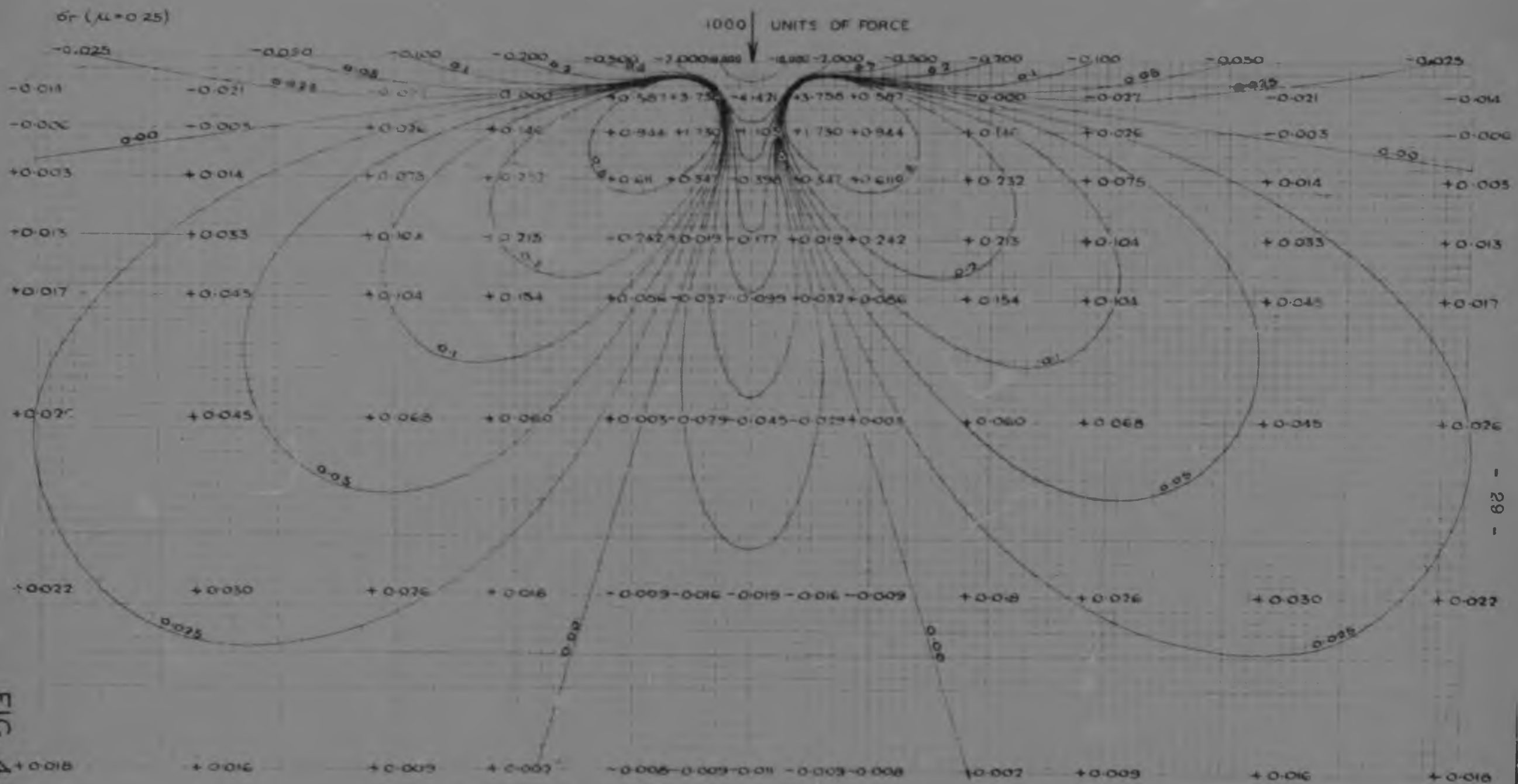


FIG. 4

FIG 4 - CONTOURS OF HORIZONTAL RADIAL STRESS (σ_r) FOR CONDITION $\mu = 0.25$

be as a result of these three combined effects, and for any one of them to be known independently, both the other two effects need to be determined.

In this case, the flow of fluid as a result of volume change of the test sample only needs to be known, so that it is necessary to determine the flow of fluid during test resulting from the effect of leakage from the test cell and entry of the loading spindle into the test cell.

The latter effect mentioned can be determined by calculation from a knowledge of the cross-sectional area of the loading spindle and the distance advanced into the test cell. If leakage effects can be totally eliminated, there would be no further difficulty in determining the nett flow of fluid as a result of volume change of the sample only.

However, if leakage cannot be eliminated completely, as is more likely to be the case, a seperate determination of the fluid flow into the test cell to replace leakage will be necessary.

Thus, the experimental work was designed to enable accurate measurements of the flow of fluid into or out of the test cell during test, due to the combined effects of leakage, spindle entry and sample volume change as well as independent measurements of the flow of fluid due to the effects of leakage and spindle entry only.

7.4 The Test Apparatus

This consisted of a standard model Triaxial Compression Test Machine designed for testing 1½"dia. x 3" length samples in a 3"dia. cell. The cell fluid was a mixture of glycerine and water in the ratio 30% : 70% by volume. Besides the usual pressure gauge on which the chamber pressure developed by pumping the foot pump could be read off directly, an adjustable head of mercury was connected to the bottom of a second 3"dia. cell, which was connected in series with the test cell and the fluid reservoir at the base of the machine, and was acting as an intermediate fluid reservoir. Hence, by measuring the height of the mercury column, a more accurate determination of the particular pressure selected for a test could be made.

The triaxial machine was operated by an electric motor through a variable speed gearbox, thus enabling the selection of one of several different rates of loading for any particular test. Values of applied load and deflection could be read off the respective dial deflection gauges provided for the purpose.

To enable the measurement of fluid flow into or out of the test cell during test, a circuit containing a small diameter capillary tube (approx. 1.6mm), fixed to a scale graduated in millimeters and containing a mercury slug of about 13mm. length, was connected in parallel with the main test cell fluid supply line between the intermediate fluid reservoir and the test cell. Suitable stopcocks were provided so that either or both the capillary tube circuit and the main fluid supply circuit could be put into use as desired. Fluid flow measurements could then be effected by cutting in only the capillary tube circuit, whence any movement of fluid into or out of the test cell, would be directly indicated by the movement of the slug, accepting that the fluid is incompressible. The arrangement of the various circuits in the test set up is shown in Fig.19.

The arrangement of the test system was such that it was

found ...../

2. Vertical Shear Stress τ_{rz}: (a) and (b) as above in (1).
3. Horizontal Radial Stress σ_r: (a) Stress data for 5 values of μ in tabular form.
   (b) Stress contour diagrams for conditions: μ=0, 0.25 and 0.50.
4. Horizontal Tangential Stress σ_t: (a) as above in (3).
   (b) Stress contour diagrams for conditions: μ=0, 0.25 and 0.375.
5. Curves of Horizontal Stresses σ_r and σ_t versus Depth on the Axis of Loading, for conditions: μ=0, 0.125, 0.25, 0.375 and 0.50.
6. Maximum Shear Stress (S_{max}): (a) as above in (3).
   (b) Stress contour diagrams for conditions: μ=0, 0.25, 0.375 and 0.50.
7. Combined Curves of Maximum Shear Stress contours for conditions: μ=0, 0.25 and 0.50.
8. Curves of Maximum Shear Stress (S_{max}) versus Depth on the Axis of Loading, for conditions: μ=0, 0.25 and 0.50.

The following points with regard to the various diagrams should, however, be noted. Firstly, in the case of item(4) above, stress contour diagrams for three values of Poisson's Ratio (μ=0, 0.25 and 0.375) are given in Appendix II — but for the condition μ=0.50, σ_t = zero (see Discussion: Section 5.3) and hence, the stress contour diagram vanishes for μ = 0.50. Secondly, regarding item(5), the values of the horizontal stresses σ_r and σ_t are equal on the axis of loading for two values of Poisson's Ratio (see Section 5.3), and hence the curves shown in Fig.9 are the same for both σ_r and σ_t versus depth on the axis of loading. Further, it needs to be mentioned that in the case of the stress contour diagrams for σ_r, σ_t and S_{max}, contour lines were frequently found to intersect the horizontal surface z = 0 of the semi-infinite solid. In these cases it was possible to calculate the exact position of the point of intersection of any stress contour line with the surface z = 0 from equations obtained by rewriting in each case the relevant stress component expression in a suitable form and at the same time applying the condition z = 0. The procedure and actual calculations are detailed in full in Appendix III Item 2.

σ_t (μ=0·375)

1000 UNITS OF FORCE

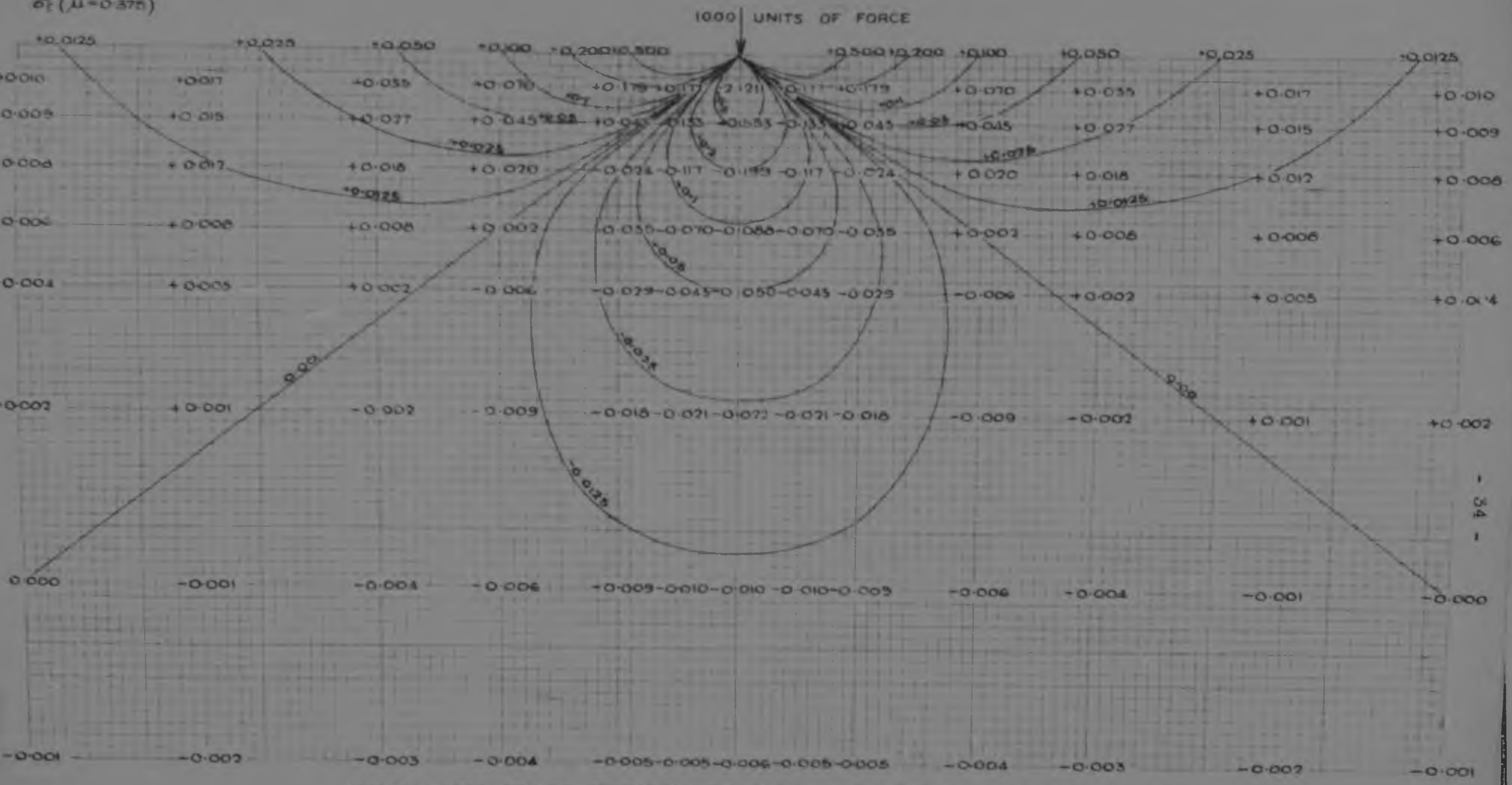


FIG. 8.

FIG 8 - CONTOURS OF HORIZONTAL TANGENTIAL STRESS (σ_t) FOR CONDITION μ=0·375

extrus'on of som of th nor wat r fro- th clay (and hence with volume duarease).

[illegible] to [illegible] it should be pointed out that it is generally not [illegible] but due to the low permeability of clay soils the consolidation process does not condition immediately after the application of loading and that during this time delay the [illegible] settlement takes place. Furthermore, the [illegible] tests the [illegible] little attention takes place without change in water content in the case of a saturated soil. On a broader scale, actually [illegible] that there is no loss of water from the soil when settlement occurs, i.e., the soil experiences no increase of shear [illegible] strength, [illegible] or otherwise [illegible] Section 6).

On this basis, namely, that [illegible] settlement in the [illegible] it should not [illegible] and is accounted by using [illegible] derived from the theory of elasticity, which is [illegible] as previously given and discussed in Section 4.21

$$A = a_{n+E} \frac{1+\mu^2}{E} \cdot \Delta p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.11$$

wherein, in this case, a_{n+E} is a [illegible] factor, μ the width of the foundation, μ the Poisson's ratio of the clay, E its Reference ratio, and Δp is fairly [illegible] settlement and [illegible] the [illegible] of the foundation [illegible] of the underlying clay.

In the case of saturated clays, the change in moisture content during the [illegible] settlement, is considered to imply no change in volume of the soil, [illegible] and only an [illegible] from the pore water and only an [illegible] Hence, for this condition $\mu = 0.5$, and the [illegible] in the above equation from [illegible] settlement of foundations.

[illegible] in the case of [illegible] soils where [illegible] of a more cohesive nature and in which [illegible] take place very slowly after the application of loading. The question arises [illegible] settlement [illegible] occur in these cases. The answer to the case of partially saturated soils is closely linked up with the [illegible] for which [illegible] is known to exist. If, in the case of saturated cohesionless soils, this [illegible] can still be considered to occur, [illegible] would also be applicable for the same [illegible] above in the case of clay soils.

4.3 The Laboratory Consolidation Test

In the normal [illegible] test specimens [illegible] the laboratory, a [illegible] of soil is confined [illegible] and is acted upon by [illegible] only vertical [illegible] being of a [illegible] consolidation test is considered to [illegible].

[illegible] wherein no horizontal strain occurs as a result of applied loading, [illegible] condition of $\mu = 0$. In this case an enforced $\mu = 0$ condition.

Vertical Normal Stress σ_z

Equation: $\sigma_z = \frac{Q}{2\pi} \frac{z^3}{(r^2+z^2)^{5/2}}$

TABLE II

z	r	σ_z
3	0	53.0516
	5	1.9152
	10	0.1040
	20	0.0037
	30	0.0005
	45	0.0003
	60	0.0000
6	0	13.26[illegible]4
	5	3.[illegible]523
	10	0.4775
	20	[illegible].025[illegible]
	30	0.003[illegible]
	45	0.0000
	60	0.0000
10	0	4.7747
	5	[illegible].7243
	10	0.8135
	20	0.0[illegible]5[illegible]
	30	0.0143
	45	0.0016
	60	0.0000
15	[illegible]	2.121[illegible]
	5	1.[illegible]313
	10	0.8467
	20	0.16[illegible]
	30	0.038[illegible]
	45	0.0064
	60	0.0016
20	0	1.1[illegible]
	5	1.0250
	10	0.682[illegible]
	20	0.2117
	30	0.0[illegible]21
	45	0.0127
	60	0.0032
30	0	0.[illegible]300
	5	[illegible].[illegible]05[illegible]
	10	0.4074
	20	0.2117
	30	[illegible].[illegible]03[illegible]
	45	0.0286
	60	0.0095
45	0	[illegible].[illegible]155
	5	[illegible].[illegible]202
	10	0.2085
	20	0.1496
	30	0.0939
	45	0.0414
	60	0.0191
60	0	0.1321
	5	0.1305
	10	0.1241
	20	0.1019
	30	0.0764
	45	0.0430
	60	0.0239

FIG 9

CURVES
of
HORIZONTAL STRESSES σ_r & σ_t
versus
DEPTH on AXIS of LOADING
for
VARIOUS VALUES OF μ.

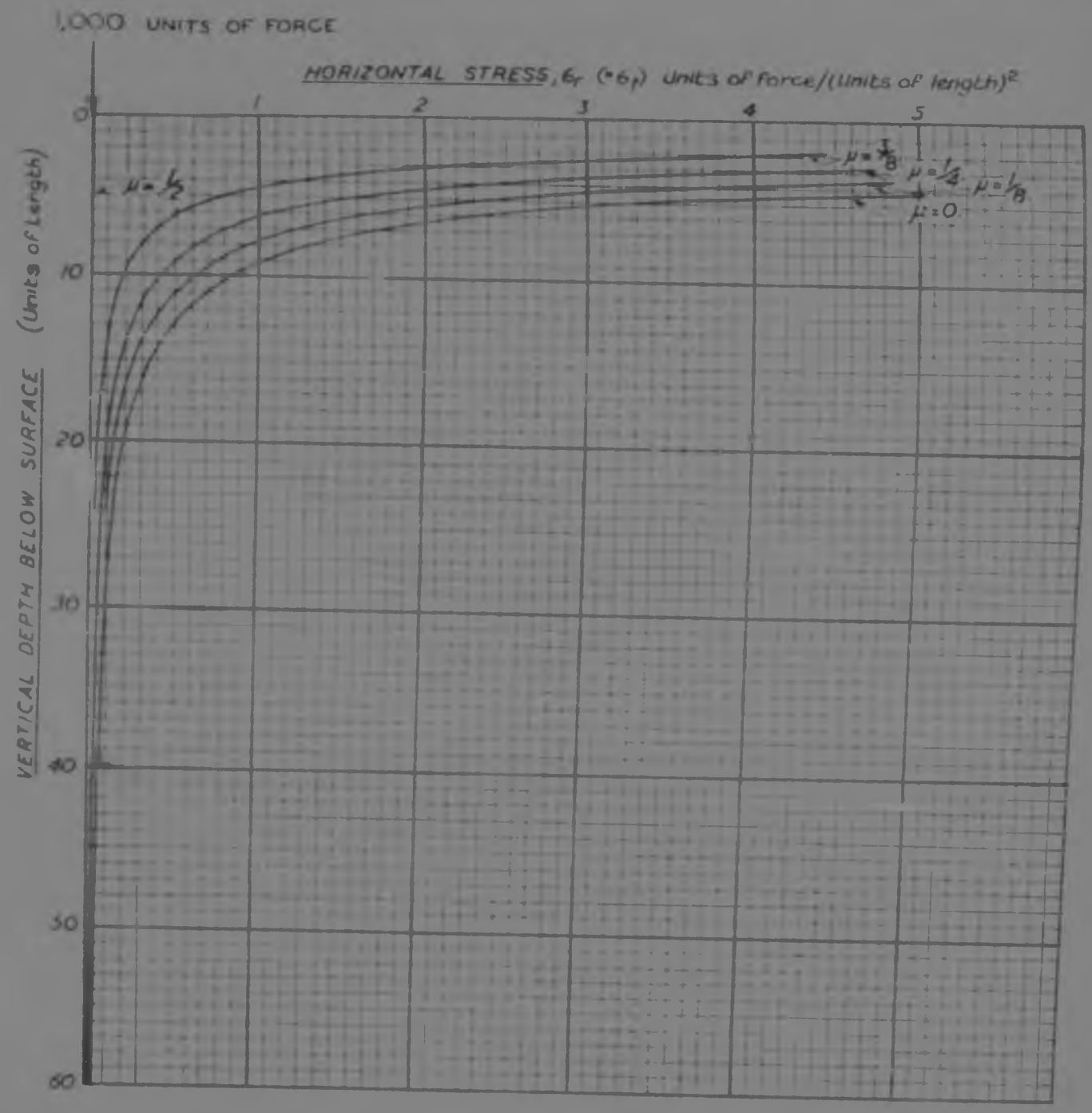


FIG. 9.

Hence, in terms of the discussion in Section 6.2, if any saturated soil be subjected to an odometer test, it can experience no elastic settlement as a result of the applied loading, since, in the case of a saturated soil, such settlement is essentially that which takes place without change in moisture content, i.e. no loss of mass, and if soil grain and pore water are assumed to be incompressible and in addition lateral strain be prevented, no such settlement can occur. Alternatively also, if soil grain and pore water are considered to be incompressible, there can be no volume change in a saturated soil without the extrusion of pore water, i.e. without consolidation taking place, which therefore implies that for calculating the elastic (or immediate) settlement of any type of saturated soil, a value of $\mu = 0.5$ only is applicable. This statement thus also verifies the statement at the end of the previous Section regarding the value of μ applicable to saturated cohesionless soils.

As a result of the above discussion, the question arises as to what the nature of the 1-initial correction (1.6) in the case of a consolidation test on a saturated soil may be. It is considered that it may possibly be due to the combined effects of very slight yield in the confining test ring (which was assumed to be zero above), and friction down of the porous plates and other parts of the test set-up. However, for a final answer, a more detailed investigation may be warranted.

6.4 Influence coefficients

The Westergaard (1.7) and Boussinesq (1.6) expressions, which is the one in use in nature, viz. that of anisotropy (1.3), in which strata of compressible material contain thin lenses of material of great lateral rigidity exists (see Fig. 1.3). This type of soil structure greatly accentuates the non-isotropic condition in these soils, and is also the cause of a greatly increased resistance to lateral strain.

Westergaard's equations constitute a solution based on the elastic theory, in which extreme non-isotropic conditions of this type are assumed. In the derivation, an elastic material is assumed to be infinitely reinforced by numerous, closely spaced, horizontal sheets of negligible thickness but of infinite rigidity, which prevent the mass as a whole from undergoing lateral strain. In his settlement expression (neglecting terms of Poisson's Ratio) for the vertical stress caused by a point load, Westergaard finds that the stresses at points directly below the load have a minimum value when Poisson's ratio has the minimum value of zero, and considers that for the case of large lateral restraint, this yields the most reliable result (see also Ref. 2.2). Hence, his final expressions for the vertical stress caused by point loading and uniform area loading, are thus obtained by substituting $\mu = 0$.

Taylor (1.6) in comparing the Boussinesq and Westergaard expressions, mentions that within certain limits (as discussed) both point loading and uniform area loading give values of vertical stresses which are approximately equal to two-thirds of the values given by the Boussinesq equations. He considers that the stratified condition upon which the Westergaard solution is based, is certainly nearer to the conditions existing in sedimentary soils than the isotropic condition assumed by Boussinesq. Furthermore, he points out that settlements of (consolidation) settlements obtained by use of the Boussinesq equations for determination of (vertical) stresses, are in the great majority of cases larger than observed settlements, which possibly indicates that the Boussinesq equations give stress values which are too large, although admittedly, such a result could also be due to other assumptions used in settlement estimates. However, the view was also expressed that the Westergaard equations tend to

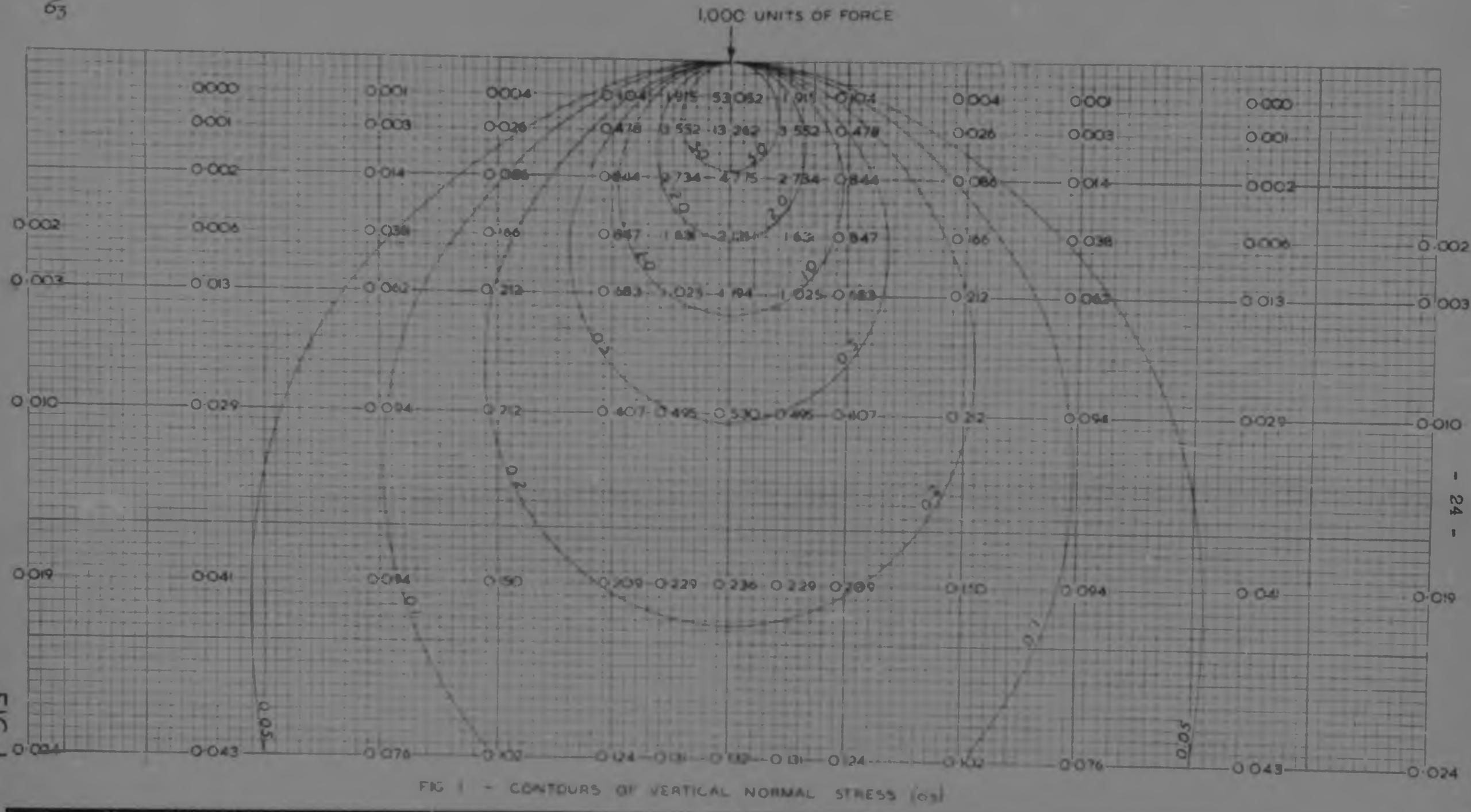


FIG. 1 - CONTOURS OF VERTICAL NORMAL STRESS (σ_z)

Maximum Shear Stress S_{max}

Equation: $S_{max} = \sqrt{(\sigma_z - \sigma_r)^2 + 4\tau_{rz}^2}$

TABLE VI

z	r	Smax				
		$\mu = 0$	$\mu = 0.125$	$\mu = 0.25$	$\mu = 0.375$	$\mu = 0.50$
3	0	30.[illegible]	29.[illegible]	[illegible].7363	27.[illegible]311	26.5258
	5	3.194[illegible]	3.2351	3.3214	3.4502	3.[illegible]166
	10	0.3[illegible]95	0.3610	0.4229	0.5176	0.6299
	20	0.0[illegible]10	0.0520	0.0264	0.0483	0.0865
	30	[illegible].0[illegible]5[illegible]	0.0336	[illegible].0138	0	0.0260
	45	0.0[illegible]	0.0[illegible]2	0.0105	0	0.0078
	60	0.0175	0.0[illegible]	0.0071	0	0
6	0	7.7365	7.[illegible]604	7.1835	6.9073	6.6312
	5	3.[illegible]0[illegible]	3.1563	3.0974	3.0[illegible]81	3.0096
	10	0.79[illegible]3	0.80[illegible]9	0.[illegible]308	0.8627	0.9039
	20	0.0[illegible]60	0.08[illegible]7	0.1051	0.1291	0.1570
	30	0.0[illegible]16	0.0[illegible]12	0.0229	0.0350	0.0502
	45	0.0103	0.0100	0	0.0063	0.0151
	60	0.0132	0.00[illegible]7	0	0	0.0063
10	0	2.7[illegible]52	2.6[illegible]57	[illegible].5863	2.4868	2.3873
	5	1.[illegible]295	1.8709	1.8149	1.7612	1.7091
	10	0.[illegible]751	0.[illegible]514	0.[illegible]514	0.8454	0.8434
	20	0.172[illegible]	[illegible].1776	0.1865	0.1990	0.2140
	30	0.0[illegible]7	0.0[illegible]71	0.0541	0.0636	0.0751
	45	0.0132	0.0100	0.0122	0.0173	0.0245
	60	0.00[illegible]0	0	0	0.0055	0.0103
15	0	1.237[illegible]	1.1929	1.1491	1.1053	1.0607
	5	1 0416	1.0072	0.9726	0.9393	0.9065
	10	0.[illegible]	0.[illegible]60	0.6395	0.6243	0.6107
	20	0.2[illegible]01	[illegible].[illegible]9[illegible]	0.2208	0.2239	0.2286
	30	0.07[illegible]7	0.0787	0.0829	0.0882	0.0948
	45	0.0[illegible]09	0.0212	0.0240	0.0287	0.0335
	60	0.0[illegible]2	0	0.0055	0.0095	0.0135
20	0	0.[illegible]6[illegible]	0.6717	0.6462	0.6215	0.5968
	5	0.[illegible]6[illegible]6	0.60[illegible]	0.5872	0.5658	0.5446
	10	0.[illegible]4[illegible]0	0.4684	0.4551	0.4402	0.4271
	20	0.[illegible]2[illegible]4	0.2[illegible]52	0.2228	0.2214	0.2209
	30	0.0[illegible]35	0.09[illegible]3	0.0960	0.09[illegible]5	0.1018
	45	0.0304	0 0316	0.0339	0.0367	0.040[illegible]
	60	0.0100	0.0112	0.0122	0.0150	0.0180
30	0	0.30[illegible]5	0.[illegible]284	0.2873	0.2761	0.2[illegible]50
	5	0.2[illegible]52	0.2845	0.2747	0.2641	0.2543
	10	0.2[illegible]00	0.2[illegible]13	0.2432	0.2346	0.2262
	20	0.1[illegible]92	0.1650	0.1607	0.1567	0.1535
	30	0.0[illegible]75	0.0960	0.0947	0.0941	0.0938
	45	0.0113	0.0[illegible]15	0.0421	0.0[illegible]33	0.0445
	60	0.0[illegible]00	0.0206	0.0218	0.0230	0.0245
45	0	0.1[illegible]77	0.132[illegible]	0.1272	0.1225	0.1177
	5	0.1347	0.1301	0.1251	0.1207	0.1161
	10	0.12[illegible]1	0.1226	0.1182	0.1139	0.1095
	20	0.1022	0.0987	0.095[illegible]	0.0925	0.0898
	30	0.07[illegible]0	0.072[illegible]	0.0712	0.0694	0.0678
	45	0.0[illegible]27	0.0421	0.0[illegible]15	0.0412	0.0412
	60	0.02[illegible]5	0.02[illegible]4	0.0244	0.0250	0.0255
60	0	0.0772	0.0740	0.0716	0.0692	0.0660
	5	0.0763	0.0731	0.0707	0.0683	0.0652
	10	0.0744	0.0712	0.0691	0.0667	0.0636
	20	0.0648	0.0628	0.0602	0.0581	0.0561
	30	0.0543	0.0526	0.0512	0.0497	0.0482
	45	0.0361	0.0353	0.0343	0.0335	0.0331
	60	0.0255	0.0250	0.0245	0.0245	0.0245

CHAPTER 7

LABORATORY TESTS

7.1 Theoretical Considerations

The basis on which a soil can be considered to approximate the elastic state of stress, was discussed in Section 6.1. As was mentioned in that Section and as was also implied by the discussions in Chapter 2, Poisson's Ratio only has a rational meaning within the linear range of loading of a soil's stress:strain curve. In this Chapter and in those to follow, a determination of Poisson's Ratio from the observed behaviour of a soil within its linear range of loading during a triaxial compression test, will be described and discussed.

Since the elastic constants are related to volumetric strains, (see Chapter 3) it is believed possible to evaluate Poisson's Ratio for a soil by measuring the change in volume of the soil resulting from a certain measured increase in loading within the linear range. This is based on the application of the three dimensional stress relationships given by equation 3.5, to the conditions of a triaxial compression test.

$$\text{Dilation } (\Delta) \quad \frac{\delta V}{V} = \frac{1}{E}\left(\sigma_1 + \sigma_2 + \sigma_3\right)(1 - 2\mu) \quad \ldots\ldots\ldots \quad 3.5$$

In the derivation of this equation, the principal stresses σ_1, σ_2 and σ_3 were considered to act in an outward direction on the cube of unit side, with a tension convention such that this direction is taken to be positive. Hence, a volume change δV, (a volume increase) caused by the action of this system of principal stresses, is considered to be positive, so that the Dilation (Δ) is positive for principal stresses acting in an outward direction.

However, in the case of a triaxial compression test, the direction of the principal stresses is opposite to that just considered and in addition, the lateral principal stresses are of equal magnitude. Hence, for convenience, the sign convention is altered so that compressive principal stresses are considered to be positive and hence, the Dilation (Δ) is positive for a reduction in volume. Furthermore, σ_3 in equation 3.5 is replaced by σ_3, the allround chamber pressure in the test cell, and hence, equation 3.5 becomes:

$$\text{Dilation } (\Delta) = \frac{\delta V}{V} = \frac{1}{E}\left(\sigma_1 + 2\sigma_3\right)(1 - 2\mu) \quad \ldots\ldots\ldots \quad 7.1$$

However, σ_1 is inclusive of the σ_3 pressure due to the allround chamber pressure, so that if σ_d, (known as the Deviator Stress), is the axial pressure to the soil-direction, we have that:

$$\sigma_1 = \sigma_3 + \sigma_d \quad \ldots\ldots\ldots \quad 7.2$$

and hence

$$\frac{\delta V}{V} = \frac{1}{E}\left(\sigma_3 + \sigma_d + 2\sigma_3\right)(1 - 2\mu)$$

i.e. $$\frac{\delta V}{V} = \frac{1}{E}\left(3\sigma_3 + \sigma_d\right)(1 - 2\mu) \quad \ldots\ldots\ldots \quad 7.3$$

If triaxial shear tests are conducted wherein the sample of initial volume V_1 is allowed to consolidate under the action of an allround chamber pressure under the pre-shear condition, then this chamber pressure causes volumetric strains in the test sample, which at the end of the consolidation process have been completely effected, thereby resulting in a new sample volume V_c. Hence, when the actual triaxial test is commenced with the application of the Deviator Stress (σ_d), the volumetric strains then produced (which constitute the δV measured during the test), are relative to the sample volume V_c attained at the end of consolidation, and furthermore, are caused by the action of σ_d only,

Vertical Shear Stress τ_{rz}

Equation: $\tau_{rz} = \frac{Q}{2\pi} \frac{3rz^2}{(r^2+z^2)^{5/2}}$

TABLE III

z	r	τ_{rz}
3	0	0.0
	5	3.1911
	10	0.3470
	20	0.0256
	30	0.0048
	45	0.0016
	60	0.0000
6	0	0.0
	5	1.9603
	10	0.7274
	20	0.0859
	30	0.0191
	45	0.0049
	60	0.0016
10	0	0.0
	5	1.3671
	10	0.6435
	20	0.1703
	30	0.0446
	45	0.0098
	60	0.0032
15	0	0.0
	5	0.7443
	10	0.3334
	20	0.1000
	30	0.0764
	45	0.0207
	60	0.0064
20	0	0.0
	5	0.4580
	10	0.2487
	20	0.0117
	30	0.0939
	45	0.0302
	60	0.0111
30	0	0.0
	5	0.2808
	10	0.1553
	20	0.1416
	30	0.0939
	45	0.0414
	60	0.0191
45	0	0.0
	5	0.1215
	10	0.0462
	20	0.0684
	30	0.0681
	45	0.0414
	60	0.0239
60	0	0.0
	5	0.0111
	10	0.0207
	20	0.0334
	30	0.0382
	45	0.0318
	60	0.0239

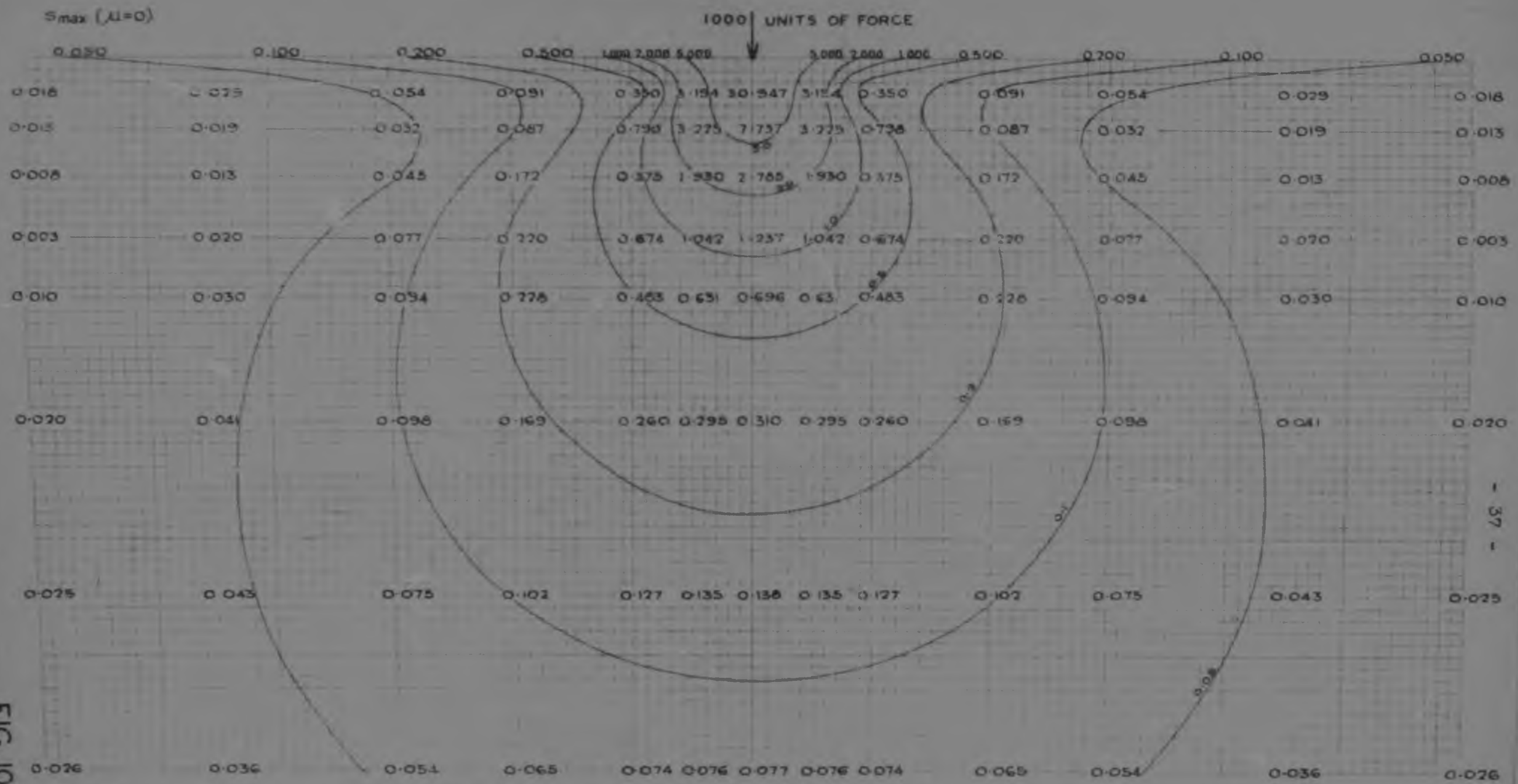


FIG 10 - CONTOURS OF MAXIMUM SHEAR STRESS (S_{max}) FOR CONDITION $\mu=0$

FIG. 10.

the chamber pressure σ_3 merely acting as a static allround pressure with no effect in so far as causing volumetric strain during test is concerned, i.e. the test conditions are similar to those of conducting a test under an allround constant air pressure, say, atmospheric pressure. In effect σ_3 thus has zero value and equation 7.3 above should be amended accordingly.

It should be pointed out, however, that the σ_3 chamber pressure controls the measured value of the volumetric strain produced by σ_d (δV as measured during the test), i.e., for different values of σ_3, the same range of σ_d results in a different measured value of δV, so that indirectly, although in the equation σ_3 has zero value, its effect is included in the calculation of Poisson's Ratio. Further, it should be borne in mind that the change in volume during the pre-shear condition resulting from the action of the σ_3 chamber pressure, is mainly as a result of consolidation effects, while the change in volume within the linear range resulting from the application of σ_d is considered to be due to elastic effects, and as such forms the valid basis of the determination of Poisson's Ratio from an equation applying to the elastic state only (cf. Chapter 2).

Hence, as it was intended to conduct Unconsolidated Undrained shear tests, σ_3, in equation 7.3, has zero value and the sample volume V, is actually the sample volume V_2 referred to in the above discussion.

Equation 7.3 thus becomes:

$$\frac{\delta V}{V} = \frac{1}{E}(\sigma_d)(1 - 2\mu) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 7.4$$

From this relationship Poisson's Ratio (μ) can be solved for by rewriting the equation as follows:-

$$(1 - 2\mu) = \frac{\delta V}{V} \cdot \frac{E}{\sigma_d}$$

$$-2\mu = \frac{\delta V}{V} \cdot \frac{E}{\sigma_d} - 1$$

$$\mu = \frac{1}{2}\left[1 - \frac{\delta V}{V} \cdot \frac{E}{\sigma_d}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 7.5$$

The values of all the unknowns on the right hand side of this equation can be obtained experimentally from a triaxial compression test, and hence, the value of μ can be calculated.

Thus, the experimental work was designed to enable the measurement during a triaxial test, of the unknowns δV, E and σ_d in the right hand side of equation 7.5, V being the volume of the test sample at the end of the consolidation process during the pre-shear condition. The method by which this volume was determined, is discussed fully in Section 7.4.

7.2 The Test Soil

The soil used for the tests was a well-graded, firm, fine, reddish-brown, silty sand, taken from a depth of 15ft. from a site on the North side of Juta Street near Viscard Street, Braamfontein, Johannesburg. The full soil profile on the site was measured down to a depth of 50ft. and is shown on the next page. (Fig. 16).

As the test soil occurs above a 2ft. thick layer of pebbles which is commonly known as the "pebble marker", it is probably of a transported origin.

The ...../

that the application to practical problems of the expressions for the stress components in 3-dimensional cases, generally requires the expression for the vertical normal stress (σ_z). Hence, the expressions for stress components containing Poisson's Ratio are seldom used. This is especially so in the case of footing loadings, since, as discussed by both the references just mentioned, an area load is very often replaced by an equivalent point load without introducing very significant errors.

An expression for the vertical normal stress distribution below a corner of a uniform rectangular area loading on the surface of a semi-infinite elastic solid, was derived by Newmark(10) by integrating Boussinesq's expression for the vertical normal stress in such a solid due to a point load (equations 4.7 (a) and (b)). This expression is given by both Terzaghi(8.7) and Taylor (1.a), and is frequently used in soils work. As is the case with Boussinesq's equation 4.7 above, Newmark's expression therefore also does not contain Poisson's Ratio.

Expressions for the displacements in a semi-infinite elastic solid in 3-dimensional problems of stress and strain, also contain terms of Poisson's Ratio, since displacements are directly related to stress distribution. Boussinesq(16) computed the vertical, horizontal and radial displacements produced by a vertical point load applied to the horizontal surface of a semi-infinite elastic solid. These expressions have also been given by Terzaghi(8.8). Using the basic equations of Boussinesq, Schleicher(19) obtained by integration an expression for the settlement of the corners of a uniformly loaded rectangular area of width B and length L. His expression is given by Terzaghi(8.9), who subsequently writes it in the form:

$$\Delta f = qB \cdot \frac{1-\mu^2}{E} \cdot I_p \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4.11$$

where q is the intensity of the uniformly distributed loading, and I_p is a pure number known as the settlement influence value. Its value depends on the ratio L:B of the sides of the loaded area. This is another expression which is frequently used in soils work as will be seen in Section 6.2.

It is thus clear that with regard to the determination of stress and strain in a semi-infinite, elastic, isotropic solid due to applied surface forces, Poisson's Ratio only enters in 3-dimensional problems of stress and strain, since as was shown in Section 1.3, the moduli of elasticity do not affect the stress distribution and hence also the strains in 2-dimensional problems. It should be noted that in the case of a semi-infinite elastic solid wherein all loadings are applied vertically on its surface (as is generally the case in soils work), a 2-dimensional problem of stress and strain is therefore also one wherein the applied loading occurs in only 2-dimensions, and similarly in the case of 3-dimensional problems.

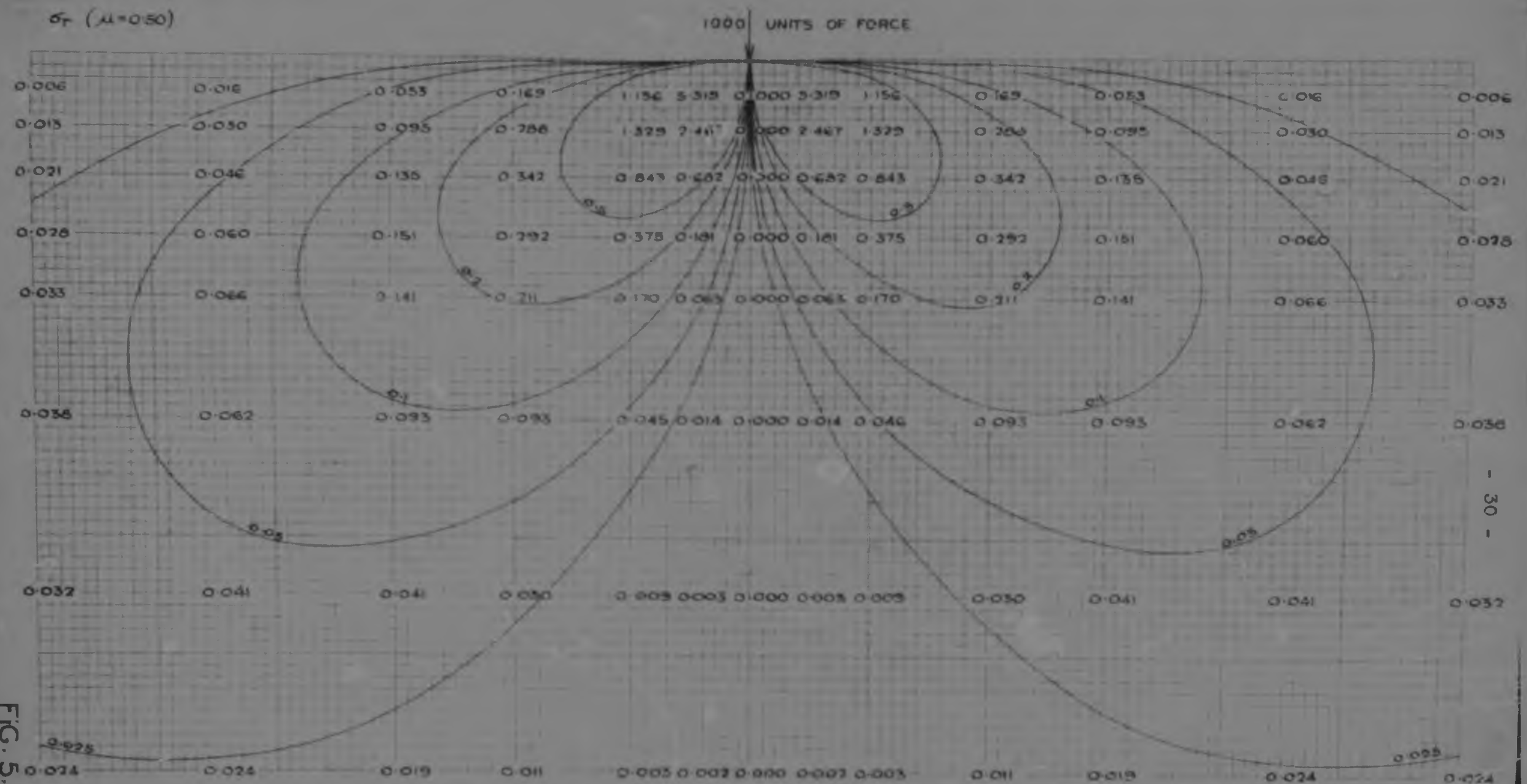
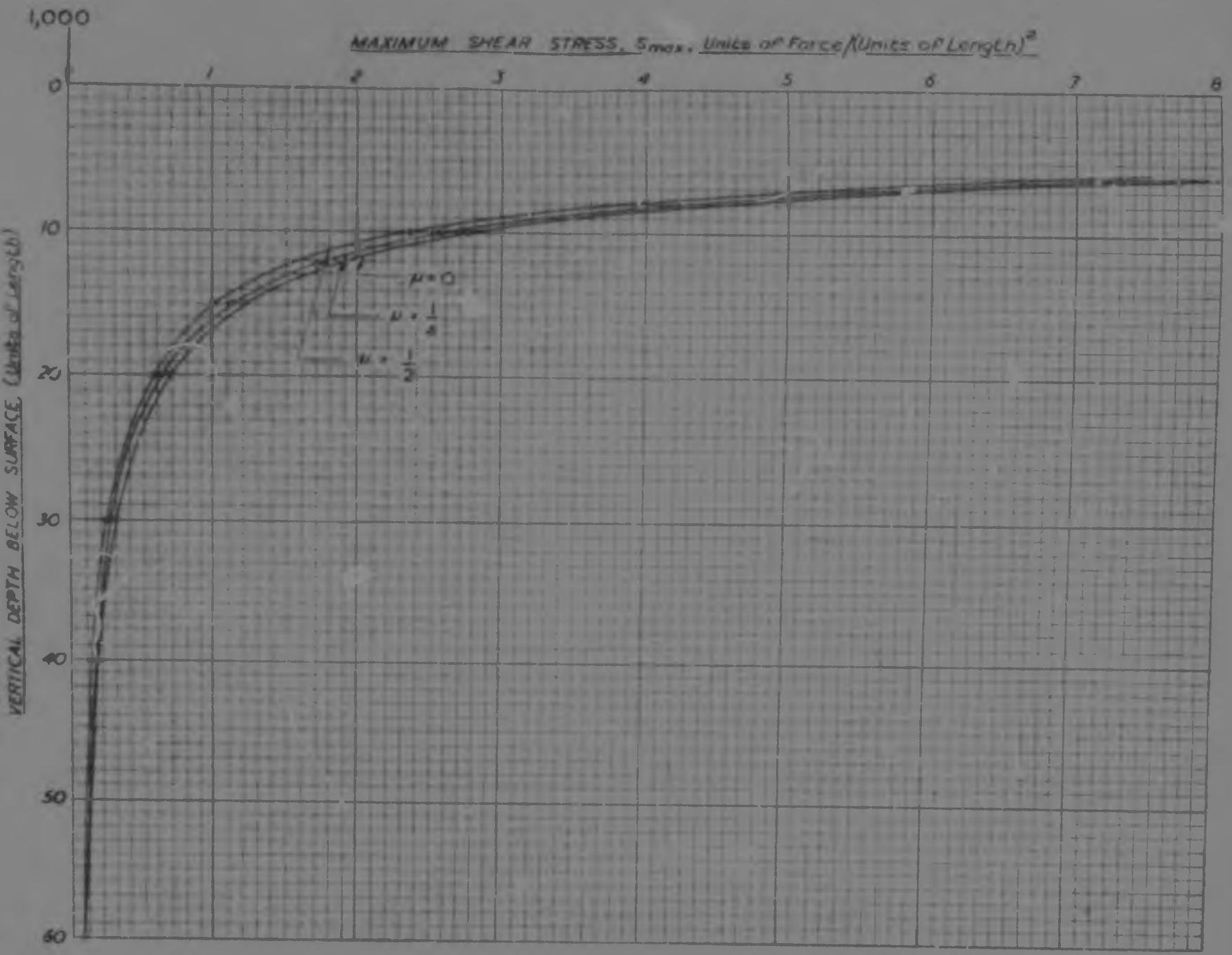


FIG 5 - CONTOURS OF HORIZONTAL RADIAL STRESS (σ_r) FOR CONDITION μ=0·50

FIG. 5.

Fig. 15

CURVES
OF
MAXIMUM SHEAR STRESS,
$S_{max.}$
VERSUS
DEPTH ON LOADING AXIS
FOR
VARIOUS VALUES OF μ



that the application to practical problems of the expressions for the stress components in 3-dimensional stress, generally only requires the expression for the vertical normal stress (σ_z). Hence, the expressions for stress components containing Poisson's Ratio are seldom used. This is especially so in the case of finite area loadings, since, as discussed by both the references just mentioned, an area load is very often replaced by an equivalent point load without introducing very significant errors.

An expression for the vertical normal stress distribution below a corner of a uniform rectangular area loading on the surface of a semi-infinite elastic solid, was derived by Newmark(18) by integrating Boussinesq's expression for the vertical normal stress in such a solid due to a point load (equations 4.7 (a) and (b)). The expression is given by both Terzaghi(9.7) and Taylor (1.8), and is frequently used in soils work. As is the case with Boussinesq's equation 4.[illegible], Newmark's expression therefore also does not contain Poisson's Ratio.

Expressions for the displacements in a semi-infinite elastic solid in 3-dimensional problems of stress and strain, also contain terms of Poisson's Ratio, since displacements are directly related to stress distribution. Boussinesq(16) computed the vertical, horizontal and radial displacements produced by a vertical point load applied to the horizontal surface of a semi-infinite elastic solid. These expressions have also been given by Terzaghi(8.6). Using the basic expressions of Boussinesq, Schleicher(19) obtained by integration an expression for the settlement of the corners of a uniformly loaded rectangular area of width B and length L. This expression is given by Terzaghi(8.9), who subsequently writes it in the form:

$$\Delta f = qB \cdot \frac{1-\mu^2}{E} \cdot I_f \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.11$$

where q is the intensity of the uniformly distributed loading, and I_f is a pure number known as the settlement influence value. Its value depends on the ratio L:B of the sides of the loaded area. This is another expression which is frequently used in soils work as will be seen in Section 6.2.

It is thus clear that with regard to the determination of stress and strain in a semi-infinite, elastic, isotropic solid due to applied surface forces, Poisson's Ratio only enters in 3-dimensional problems of stress and strain, since as was shown in Section 4.3, the moduli of elasticity do not affect the stress distribution and hence also the strains in 2-dimensional problems. It should be noted that in the case of a semi-infinite elastic solid wherein all loadings are applied vertically on its surface (as is generally the case in soils work), a 2-dimensional problem of stress and strain is therefore also one wherein the applied loading occurs in only 2-dimensions, and similarly in the case of 3-dimensional problems.

Horizontal [illegible] Stress σ_t

Equation: $\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$

TABLE V

r	z	σ_t				
		$\mu = 0$	$\mu = 0.[illegible]$	$\mu = 0.25$	$\mu = 0.375$	$\mu = 0.50$
2	0	-0.[illegible]	-0.[illegible]	-1.1910	-0.[illegible]	0
	5	+0.[illegible]	+0.[illegible]	+0.3549	+0.1774	0
	10	+0.7162	+0.[illegible]	+0.3581	+0.1790	0
	20	+0.2817	+0.2113	+0.1409	+0.0704	0
	30	+0.1416	+0.1062	+0.0708	+0.0354	0
	45	+0.[illegible]	+0.[illegible]	+0.0342	+0.0171	0
	60	+0.[illegible]	+0.[illegible]	+0.0199	+0.0100	0
4	0	-2.[illegible]	-1.[illegible]	-1.1051	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.1353	0
	10	+0.1703	+0.[illegible]	+0.0850	+0.0424	0
	20	+0.1700	+0.1[illegible]	+0.0891	-0.[illegible]	0
	30	+0.1008	+0.[illegible]	+0.0501	+0.0271	0
	45	+0.[illegible]	+0.[illegible]	+0.0295	+0.0147	0
	60	+0.[illegible]	+0.[illegible]	+0.0172	+0.0000	0
10	0	-0.7[illegible]	-0.[illegible]	-0.[illegible]	-0.1900	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.1170	0
	10	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	20	+0.[illegible]	+0.[illegible]	-0.[illegible]	+0.[illegible]	0
	30	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	-0.[illegible]	0
	60	+0.[illegible]	+0.[illegible]	+0.0181	+0.[illegible]	0
15	0	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	10	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	20	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
	30	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
	60	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.0050	0
20	0	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	10	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	20	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	30	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.0024	0
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
	60	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
30	0	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	10	-0.[illegible]	-0.[illegible]	-0.0350	-0.0175	0
	20	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	30	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	45	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
	60	+0.[illegible]	+0.[illegible]	+0.[illegible]	+0.[illegible]	0
45	0	-0.[illegible]	-0.[illegible]	-0.0191	-0.[illegible]	0
	5	-0.[illegible]	-0.[illegible]	-0.0191	-0.[illegible]	0
	10	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	20	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	30	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.[illegible]	0
	45	-0.[illegible]	-0.[illegible]	-0.[illegible]	-0.0012	0
	60	-0.0000	0	0	0	0
60	0	-0.[illegible]	-0.0167	-0.0111	-0.0056	0
	5	-0.0207	-0.[illegible]	-0.[illegible]	-0.0052	0
	10	-0.0207	-0.[illegible]	-0.[illegible]	-0.0052	0
	20	-0.[illegible]	-0.0131	-0.0088	-0.0044	0
	30	-0.0127	-0.0095	-0.0063	-0.0032	0
	45	-0.0064	-0.0048	-0.0032	-0.0016	0
	60	-0.[illegible]	-0.0024	-0.0016	-0.0008	0

5.3 Discussion

As previously indicated, the stress components σ_r and τ_{rz} in the Boussinesq equations of stress, do not contain Poisson's Ratio and hence their stress contour diagrams are unaffected by variations in μ. However, the stress components σ_r and σ_t and the Maximum Shear Stress $(\sigma_1 - \sigma_3)$ are affected by variations in μ and therefore the following discussion is directed mainly at the effect of Poisson's Ratio on these stresses.

From a closer consideration of the equations for the stress components σ_r and σ_t (which for convenience are given again immediately below), the following facts are apparent:

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{z r^2}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \qquad \ldots\ldots\ldots 5.8(i)$$

$$\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \qquad \ldots\ldots\ldots 5.8(ii)$$

Firstly, for the condition $\mu = 0.50$, σ_t disappears completely while σ_r is considerably modified in that the second term in the brackets reduces to zero. It, in addition, either $r = 0$ or $z = 0$, then σ_r is also reduced to zero. Furthermore, on the axis of loading $(r=0)$ or the vertical plane of loading, the equations for σ_r and σ_t reduce to the simple expressions, viz.

$$(\sigma_r)_{r=0} = \frac{Q}{4\pi z^2}(1+2\mu) \qquad (\sigma_t)_{r=0} = \frac{Q}{4\pi z^2}(1+2\mu) \qquad \ldots\ldots\ldots 5.9$$

Hence, the horizontal stresses σ_r and σ_t are equal at any depth z on the axis of loading for the same value of Poisson's Ratio.

These facts serve to enable a better understanding of the behaviour of the stress contour diagrams for σ_r and σ_t due to variation in the value of Poisson's Ratio. In the case of σ_r, it is observed that when increasing values of μ, the central maximum pressure bulb and the negative stress zones adjacent to the horizontal surface of the semi-infinite solid, reduce in size, while at the same time the two positive pressure bulbs increase in size. At $\mu = 0.5$, the former completely vanishes and only the two positive pressure bulb remain with zero stress throughout on both the vertical axis of loading and on the horizontal surface $z = 0$. In the case of σ_t, it is observed that at $\mu=0$ there is only one central (positive) pressure bulb with positive pressure zones adjacent to the horizontal surface of the semi-infinite solid. With increasing values of μ, both of these reduce in size until, as previously mentioned above, at $\mu=0.5$ when σ_t attains zero value, the stress contour diagram completely vanishes.

It is thus clear that Poisson's Ratio has a significant influence on the values of both of the horizontal stress components σ_r and σ_t. Fig. 9 shows that with respect to the axis of loading the effect is in both cases most marked between the horizontal surface of the semi-infinite solid and a depth of 10 units of length below the surface $z = 0$. A consideration of the numerical values of $\sigma_r (=\sigma_t)$ at the various depths on the axis of loading (Table IV or Table V), shows that the stress values corresponding to the conditions $\mu = 0$, 0.166, 0.25 and 0.375

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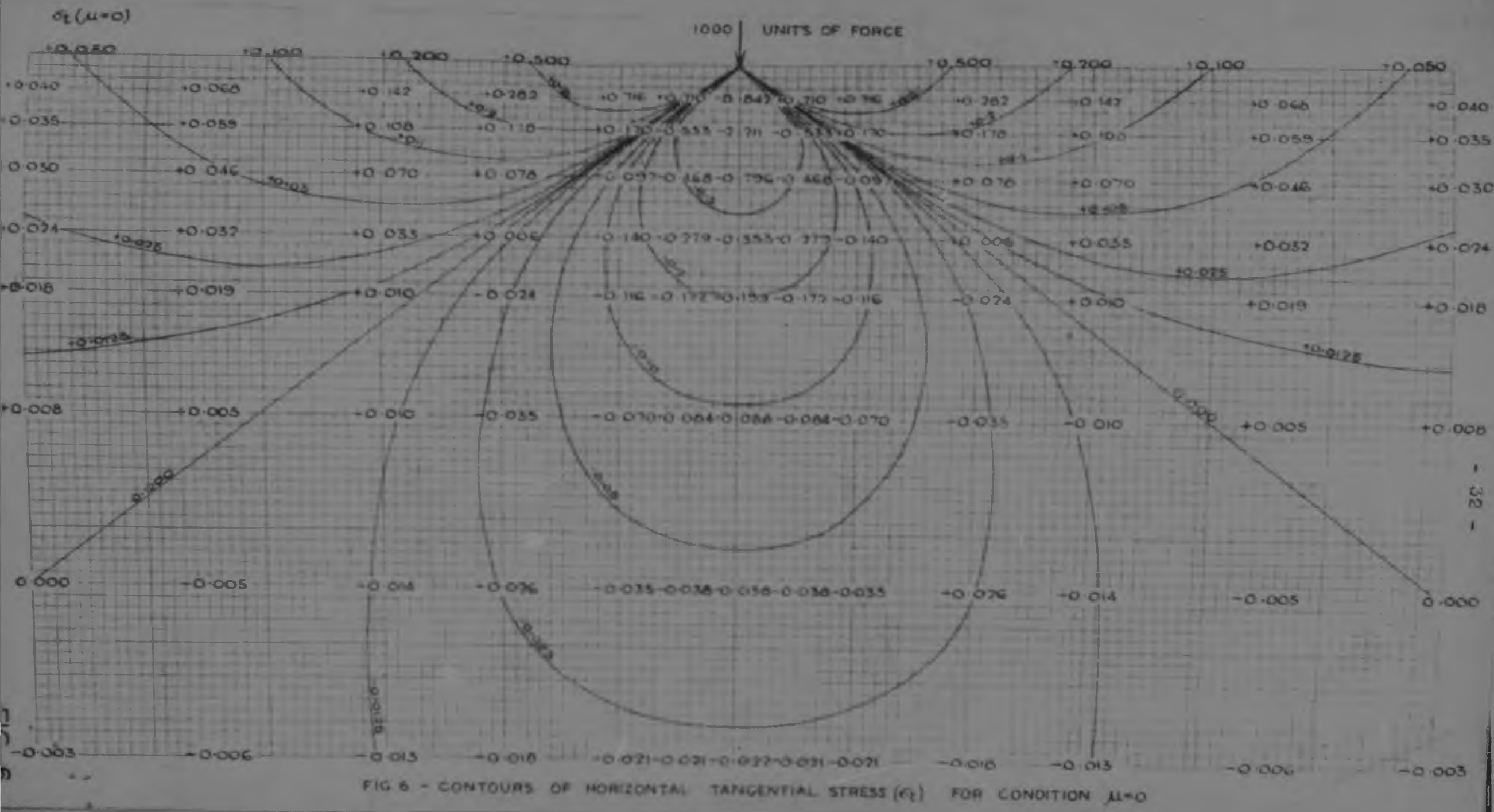


FIG 6 - CONTOURS OF HORIZONTAL TANGENTIAL STRESS (σ_t) FOR CONDITION $\mu=0$

CHAPTER 5

THE EFFECT OF VARIOUS VALUES OF POISSON'S RATIO ON THE STRESS DISTRIBUTION IN A SEMI-INFINITE ELASTIC SOLID DUE TO AN APPLIED VERTICAL POINT LOAD

5.1 Introduction

In this Chapter the effect of various values of Poisson's Ratio within the zero - 0.5 range on the stresses calculated from Boussinesq's equations as the basic 3-dimensional case, will be outlined. The equations referred to are the equations 4.7-4.10 given in the previous Chapter.

5.2 Stress Calculations and Curves

For the purposes under consideration, the value of the vertical point load Q in Boussinesq's equations was assumed to be 1000 units of force. No specific units of either force or length were adopted, as it was intended to render the results adaptable to any system of units. The units of the stresses calculated are therefore simply : units of force/(units of length)^2, the numerical value of the stress calculated being the same for any units of force and length. That this is the case, can be seen from a consideration of the form of any of Boussinesq's equations, for example, equation 4.7(a) of previously:

$$\sigma_z = \frac{Q}{2\pi} \cdot \frac{3z^3}{(r^2+z^2)^{5/2}} \qquad \text{..... 4.7(a)}$$

Whatever the units of length may be, the numerical value of the term $3z^3/(r^2+z^2)^{5/2}$ will in each case be the same, and has dimensions : (units of length)^2, while the numerical value of the term $Q/2\pi$ is also the same whatever units of force may be used. Hence σ_z, as the product of these two terms, has a numerical value which is independent of the particular units of force and length used. The only effect is that with the later presentation of the results in the form of curves, stress contour diagrams), the scale of any diagram depends on the particular units of length used, but this is of no consequence for the purposes considered herein. For this reason also, no scales are given with any of the diagrams.

For each stress component, i.e. σ_z, σ_r, σ_t and τ_{rz}, the values of the stresses have been calculated for a grid of points extending 60 units of length horizontally on either side of the axis of loading of the vertical point load, and to a depth of 60 units of length below the horizontal surface of the semi-infinite solid. Furthermore, for the grid of points just detailed, stresses were calculated for each of the stress components σ_r and σ_t (which contain terms of Poisson's Ratio) using five different values of μ, viz. μ = zero, 0.125, 0.25, 0.375 and 0.50. Stresses were in all cases evaluated by direct calculation from the relevent equations and the results have been presented, as mentioned above, mainly in the form of stress contour diagrams. The actual results of the calculations for each grid of points have been compiled in tabular form and appear in each case on the page immediately before the corresponding curves.

For the Maximum Shear Stress (S_{max}), the standard equation for 2-dimensional cases was used, although the problem being considered is essentially a 3-dimensional one. The reason for this is that the 3-dimensional case has proved to be almost insoluble (as pointed out by Kenney and Carr[20]), and the only known solution is that of Jurgenson[15] for the special case of a uniformly loaded circular area. Even in this case the solution is so complicated that Jurgenson does not give his

formulae, .../

bear a constant ratio to one another, and that the stresses for the condition $\mu = 0$ are a constant 25% of the corresponding stresses for the condition $\mu = 0.375$. Furthermore it is clear that the extreme difference in the stress values as given by the conditions $\mu = 0$ and $\mu = 0.50$ decreases rapidly with depth on the axis of loading.

In the case of the Maximum Shear Stress (S_{max}), the most serious effect of variation in μ in the stress contour diagrams appears to be only in the near vicinity of the horizontal surface $z = 0$, i.e. somewhat-less in the region which is less than 5 units of length below the surface. This can best be seen in Fig.11, in which the stress contour lines for the three conditions $\mu = 0$, 0.25 and 0.50 are shown superimposed. Fig.11 shows that the variation in values of S_{max} on the axis of loading as a result of variation in μ is comparatively insignificant throughout. A consideration again of the actual numerical values of S_{max} corresponding to the five values of Poisson's ratio considered (Table VI), shows in the first instance that at any depth on the axis of loading the five stress values show the same constant ratios to one another, and further, that in the case of the extreme conditions $\mu = 0$ and $\mu = 0.50$, the corresponding difference in the values of S_{max} is only a constant 12.5% at all points on the axis of loading.

On the whole, therefore, it appears that S_{max} is not very critically affected by variation in Poisson's Ratio throughout the limits of the range for elastic materials. This is a very important conclusion especially as it can be seen from a study of the numerical values of the various stresses at points in the axis of loading (where the greatest stress values occur), that account only to the vertical normal stresses σ_z. S_{max} follows the largest stress values and as a shear stress it may therefore be a critical factor in design. Remote from the axis of loading the values of S_{max} are quite frequently larger than the corresponding σ_z values. On the axis of loading the values of S_{max} for the condition $\mu = 0.50$ are 50% of the corresponding values of σ_z, while for the condition $\mu = 0$ this figure is 41.6%.

The horizontal stresses σ_x and σ_y and their extreme sensitivity to variations in the value of Poisson's Ratio are therefore possibly not so important in design as would appear at first sight, and it seems that S_{max} is a more important design factor with, in addition, the important property that its value is not seriously affected by Poisson's Ratio over the complete range of values applicable to elastic materials.

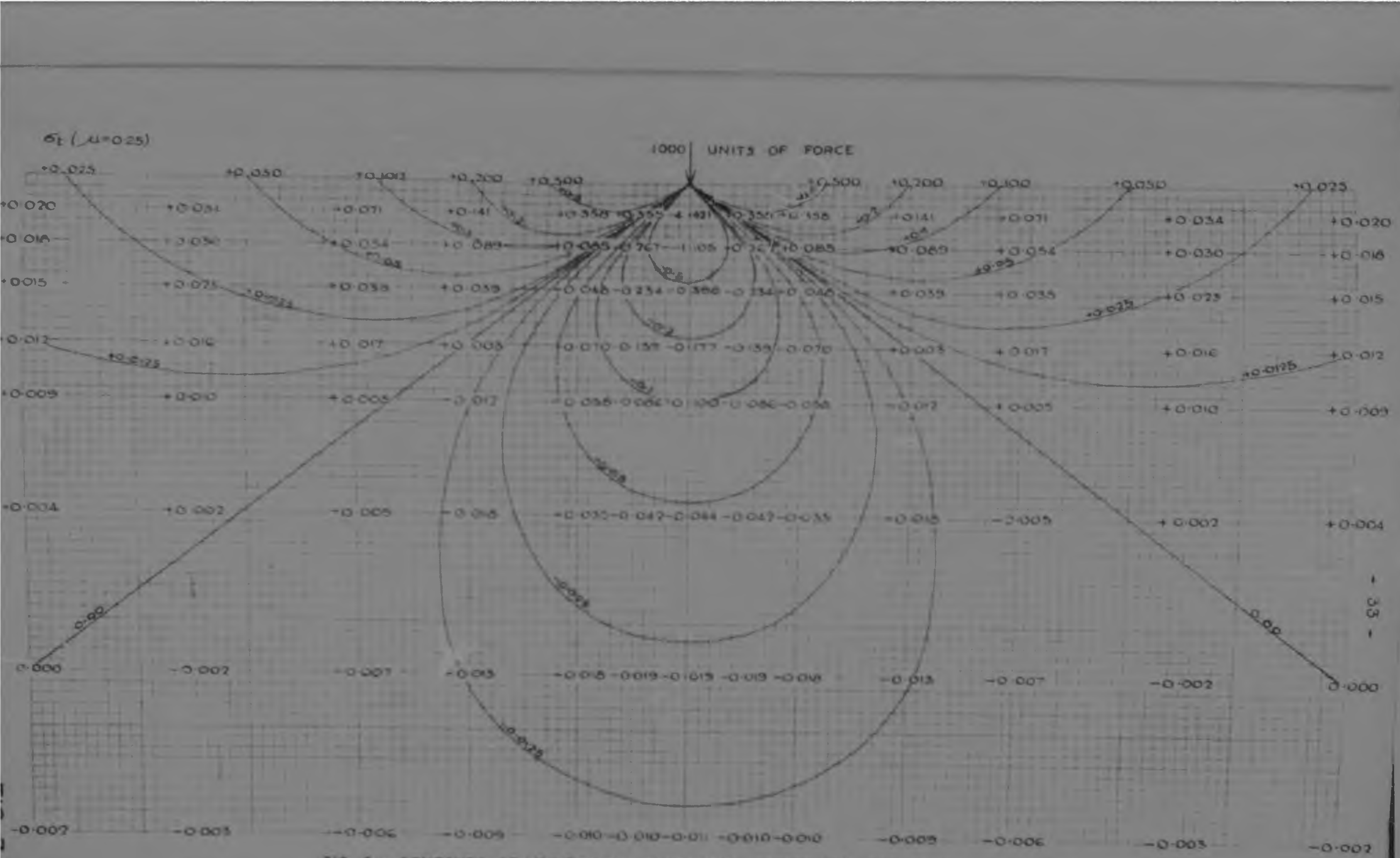


FIG 1 - CONTOURS OF HORIZONTAL TANGENTIAL STRESS (σ_t) FOR CONDITION μ=0.25

formul-, since "th y cont in functions of d rivatives of the solid ngle subtended by the loaded area at the point under consideration which are of interest to the mathematician only". The use of the formula for 2-dim nsion l c s is therefore frequently resorted t in 3-dimensional cases, especially as acc rding to K ntzy and Barr, it introduces n additional factor of safety which may b as high s 1.25.

As previously discussed (Chapter 4), 2-dim nsional cases are either problems of plane stress or plane strain. In the case of plane stress (and referring now to the same orientation of coordinate axes as in Chapter 4), σ_y, τ_{xy} and τ_{yz} are zero, while in the case of plane strain, τ_{xy} and τ_{yz} are zero, but σ_y is not zero. If the normal and shearing stresses acting on the two perpendicular planes containing the OX and OZ axes are combined to give the maximum Shearing Stress, the resulting 2-dimensional formula is:

$$S_{max} = \frac{1}{2}\sqrt{(\sigma_z - \sigma_x)^2 + 4\tau_{xz}^2} \quad \ldots\ldots\ldots\ldots 5.1$$

For this equation to hold, it therefore does not necessarily signify the assumption that $\sigma_y = zero$, i.e. the equation is valid even if the third principal stress is not zero since in the case of plane strain, which is a 2-dimensional problem, σ_y is not equal to zero. Hence also, if the normal and shearing stresses acting on the two perpendicular planes containing the OY and OZ axes be combined, the following 2-dimensional relationship is obtained:

$$S_{max} = \frac{1}{2}\sqrt{(\sigma_z - \sigma_y)^2 + 4\tau_{yz}^2} \quad \ldots\ldots\ldots\ldots 5.2$$

In the case of the 3-dimensional equations of stress, the equations 5.1 and 5.2 given above therefore become (cf. Fig. 4.5, Chapter 4):

$$S_{max} = \frac{1}{2}\sqrt{(\sigma_z - \sigma_r)^2 + 4\tau_{rz}^2} \quad \ldots\ldots\ldots\ldots 5.3$$

and $$S_{max} = \frac{1}{2}\sqrt{(\sigma_z - \sigma_t)^2 + 4\tau_{tz}^2} \quad \ldots\ldots\ldots\ldots 5.4$$

but, since $\tau_{tz} = 0$, equation 5.4 above becomes:

$$S_{max} = \frac{1}{2}(\sigma_z - \sigma_t) \quad \ldots\ldots\ldots\ldots 5.5$$

Although equation 5.3 (or basically equation 5.1) is the generally accepted expression for determining S_{max}, S_{max} was evaluated from both the equations 5.3 and 5.5 above for the purposes of comparison. Since S_{max} in both equations contains either the stress component σ_r or the stress component σ_t, both of which contain terms of Poisson's Ratio, S_{max} was also evaluated for the five conditions: μ = zero, 0.125, 0.24, 0.375 and 0.50. A tabular comparison of the results is given in Appendix III, Item 1. It was found that equation 5.3 corresponding to the generally accepted equation 5.1 for 2-dimensional cases, gave the larger values of S_{max} in all cases except a few very isolated instances.

On the following pages the results of all the abovementioned calculations and the corresponding curves are presented. These appear in the following sequence:

1. Vertical Normal Stress σ_z: (a) Stress data in tabular form.
   (b) Stress contour diagram.

2. Vertical ...../

CHAPTER 6

CURRENT CONCEPTS OF ELASTICITY AND POISSON'S RATIO IN SOILS WORK

6.1 Applicability of the Elastic Theory to Soils

The merits and weaknesses of the application of the elastic theory to soils have been discussed in full by various Authors (see for example Ref.1, p.200-201; p.250 and p.262-265, and Ref. 2, p.3-4 and p.[illegible]), and will therefore not be discussed again herein.

It need only be mentioned here that the theory of elasticity has been used for estimating stresses and settlements induced in soil masses by externally applied loads, with the important proviso that the induced stresses must be of a moderate order well clear of the state of stress tending to produce failure conditions for the results are to be of any real significance. In this connection, Terzaghi remarks (Ref.3, p.369) that: "If the factor of safety of a mass of soil with respect to failure by plastic flow exceeds a value of about 3, the state of stress in the soil is likely to be more or less similar to the state of stress computed on the assumption that the soil is perfectly elastic. Hence, the state of stress in a mass of soil under the influence of external forces can be estimated by means of the theory of elasticity. The importance of the error associated with the results of the computation depends chiefly on the extent to which the actual stress-strain relations depart from Hooke's Law."

In essence, the theory of elasticity in soils is based on the fact that the stress:strain curves derived from tests such as triaxial compression tests and field bearing tests, show a certain range, at or near the start, over which stress and strain are proportional. Soil is said to exhibit elastic behaviour over that stage of loading. In terms of the discussion in Chapter 5, the elastic theory is thus strictly only applicable to a soil mass within this restricted limit range of loading (which is usually assessed by Terzaghi's proviso of a factor of safety of about 3 against shear failure). Soil therefore also follows Hooke's postulate only for a restricted range within the range of loading.

In the next Section of this Chapter, some aspects of Poisson's Ratio in relation to currently accepted analytical and semi-empirical methods of settlement analysis will be discussed. Thereafter a brief review of the determination of Poisson's Ratio from the observed behaviour of a soil mass in the laboratory (e.g. from triaxial compression test), will be ascertained and discussed.

6.2 Settlement Analysis for Saturated Clays

In discussing a development of the existing methods of calculating settlement of structures founded on clay, Skempton and Bjerrum[61] commence by describing the principles on which such settlement analysis is based.

It is proposed that the settlement (i.e. that resulting from the increase in soil foundation pressure) at the end of construction is made up of two parts: the immediate settlement which results from deformations taking place without change in water content (and hence without volume change in a saturated clay); and the consolidation settlement arising from

extrusion ...../

SKETCH SHOWING ARRANGEMENT OF THE VARIOUS CIRCUITS IN THE TEST SET-UP

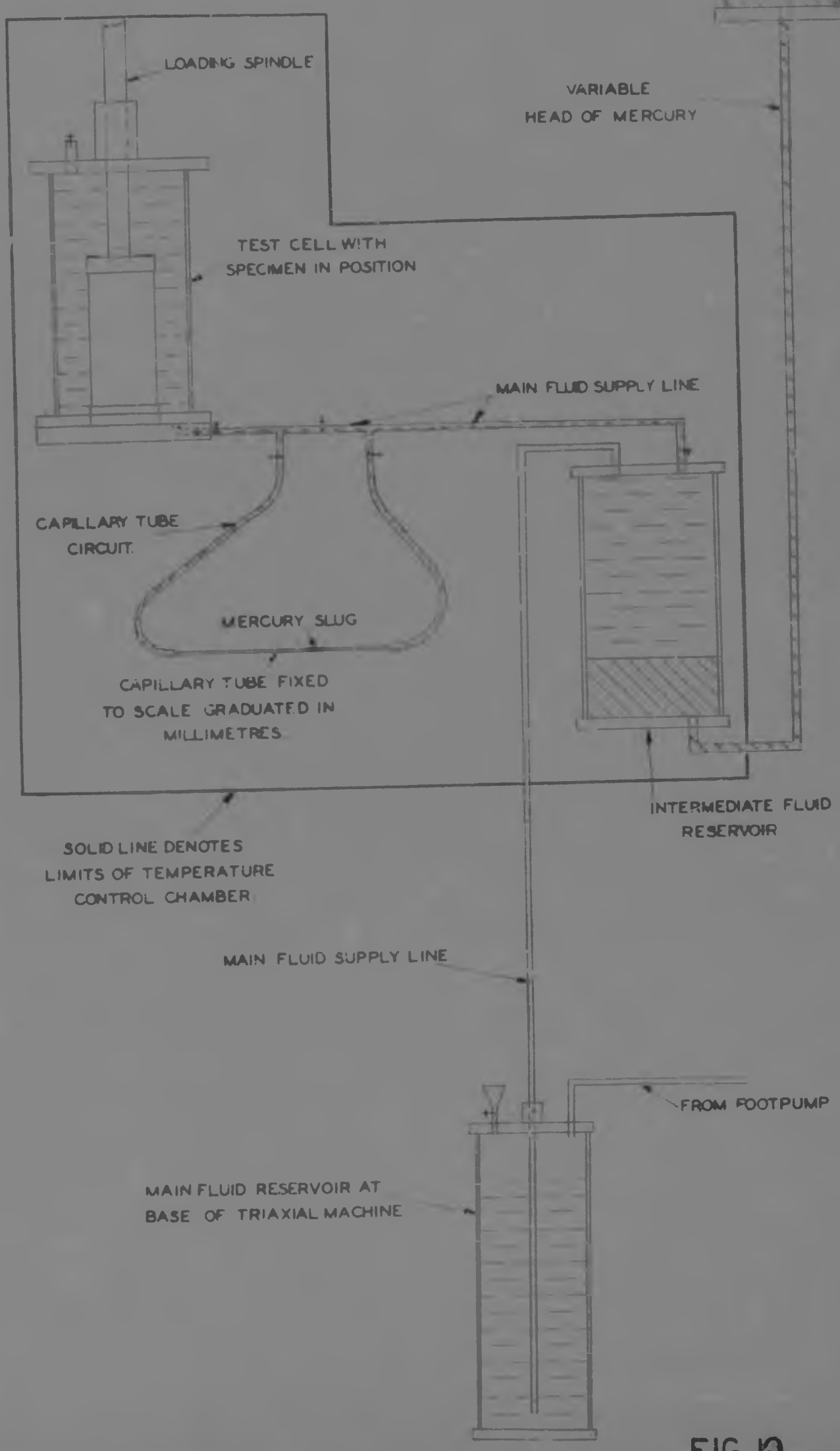


FIG. 19

found most convenient to use a tube of about 32cms. graduated length clear between end connections. In selecting a suitable tube diameter, two main factors had to be considered:

(a) The diameter would have to be small enough to enable the slug to fill the whole bore and not only part of it, otherwise it would leave a gap between its top surface and the top of the capillary tube through which fluid could flow without causing movement of the slug.

(b) The slug movement during test would have to be of a reasonable magnitude, i.e. preferably over as much of the graduated length of the tube as possible, but preferably not more than this.

A suitable balance was eventually obtained by trying out capillary tubes of different diameters in a few trial tests with the test soil.

Due consideration was given to the following matters:-

(a) Air in the test system: As the presence of air in the confining fluid system would adversely effect the flow measurements due to its compressibility it was very important to ensure that all entrapped air be expelled from the system before any test be commenced. It proved to be a very difficult matter to get the capillary tube circuit filled with fluid and its mercury slug in position with all air bubbles expelled from the circuit. However, once this was done at the commencement of the main test series, matters were so arranged by handling the stopcocks, that the circuit would remain filled with its fluid and with its slug in position over the whole series of tests.

(b) Leakage from the test cell: Leakage from the test cell was an undesirable effect for the purposes of this investigation and hence an attempt was made to eliminate it completely. However, it was found that this was not readily accomplished and hence, all that could be done was to reduce the leakage as much as possible by keeping all leakage points well greased, especially where the loading spindle enters the test cell.

(c) Temperature changes during test: Due consideration was given to the elimination of temperature changes during test as this would effect the fluid flow measurements made, and would introduce a further variable into an already complex system of test variables. For this reason the whole test bed was enclosed in a wooden box with glass windows in its access doors and a heating element and fan installed so as to enable a uniform, constant temperature to be maintained within, during test. However, it was later found that temperature conditions inside the laboratory were not very variable and that with the added protection of the wooden box against possible draughts, no significant temperature changes were liable to occur within the temperature control chamber during the comparatively short duration of a test. The heating element and fan were thus never used.

Figs. 20 to 23 inclusive are photographs showing various aspects of the test apparatus.

The test procedure would then be as follows:-

Having positioned the sample in the test cell and with the capillary tube circuit cut out, fluid is pumped from the main fluid reservoir at the base of the machine via the intermediate fluid reservoir and the main fluid supply line into the test

cell,...../

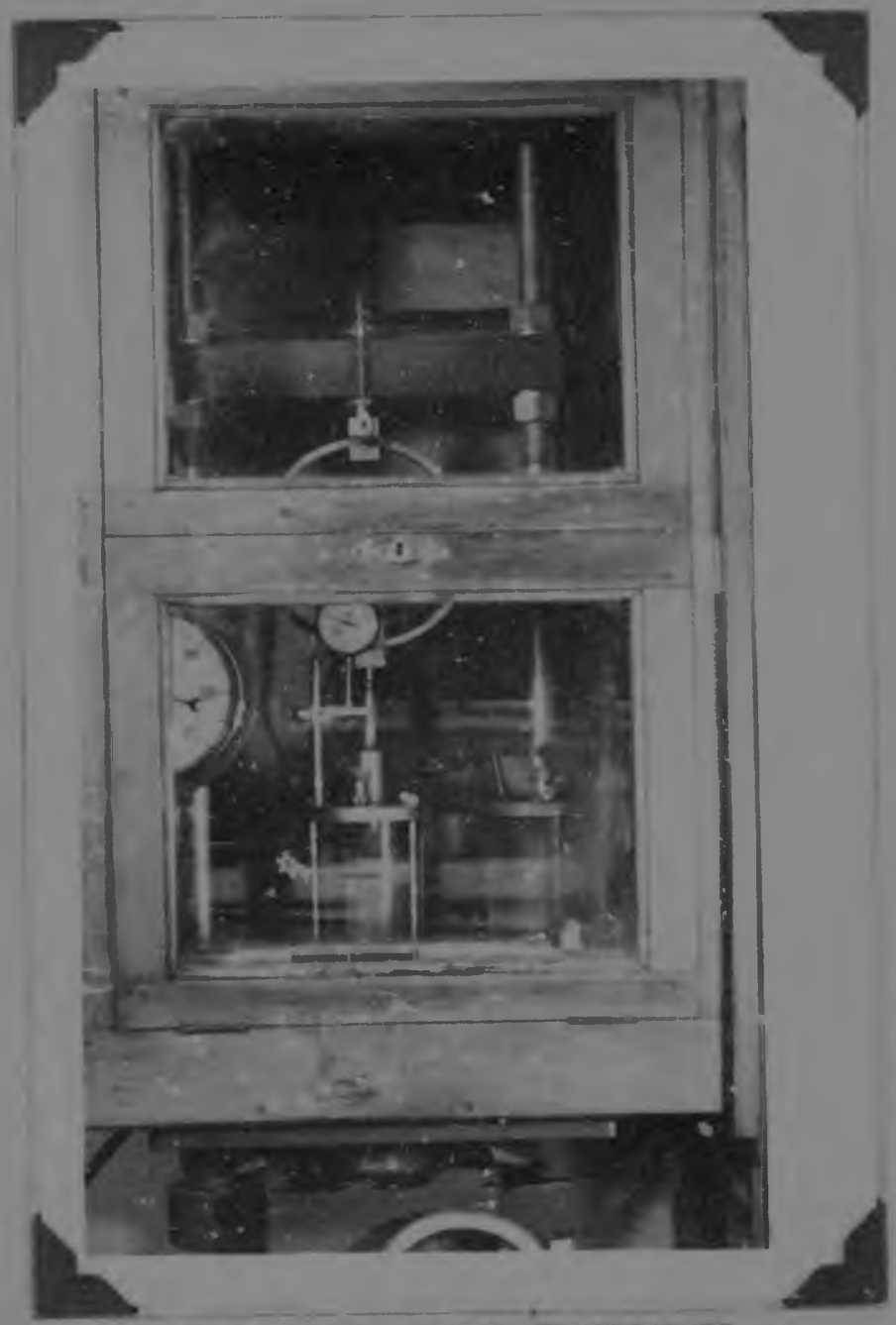


FIG 20 - OUTSIDE VIEW OF TEMPERATURE CONTROL CHAMBER ENCLOSING THE TEST BED

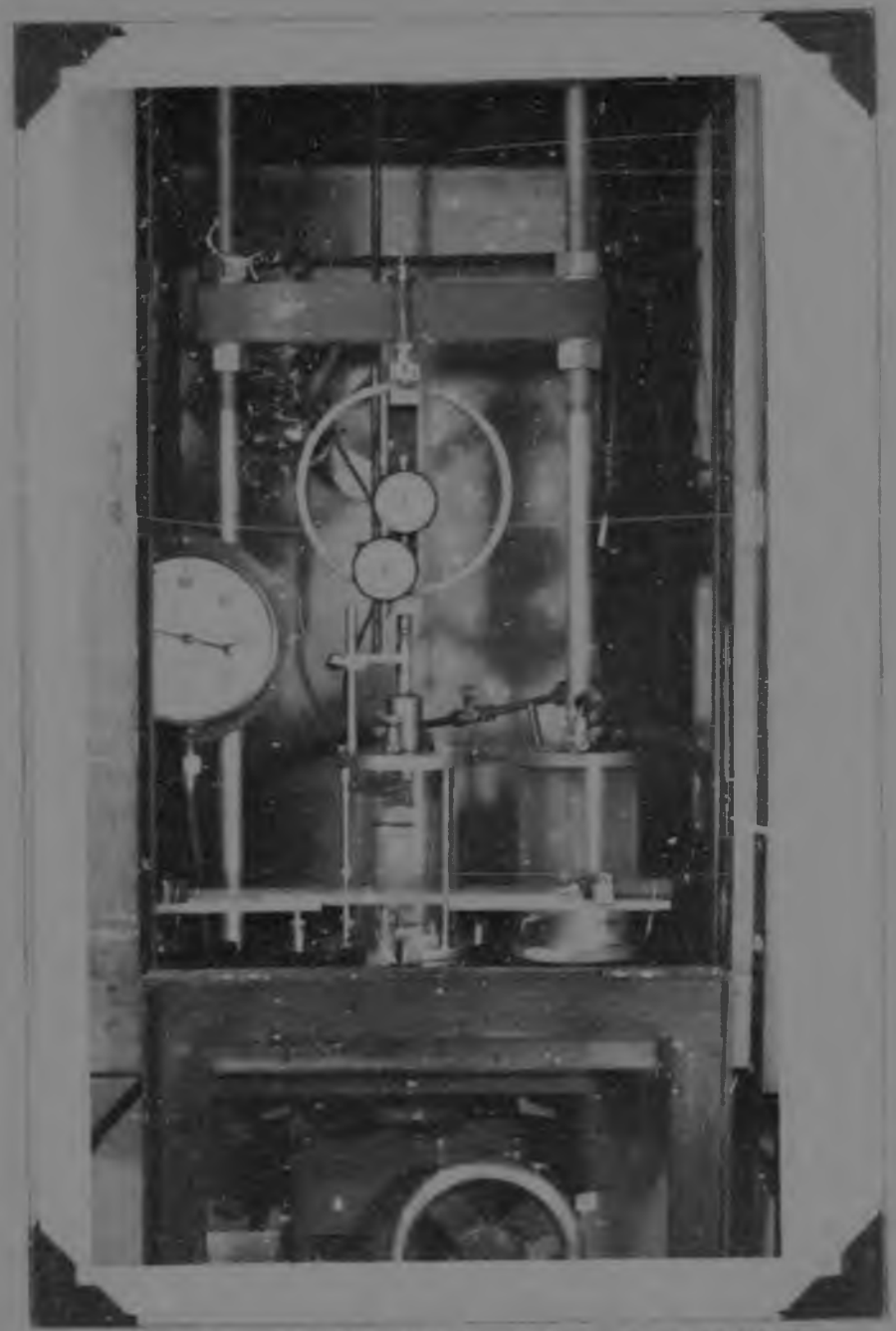


FIG 21 — ARRANGEMENT WITHIN TEMPERATURE CONTROL CHAMBER

Figs. 20 & 21

FIG. 22 - TRIAXIAL TEST CELL WITH SAMPLE IN POSITION

FIG 23 - CAPILLARY TUBE (WITH SLUG) FIXED TO GRADUATED SCALE

Figs. 22 & 23

cell, as is usual procedure with triaxial tests. Having attained the preselected chamber pressure, as indicated on the pressure gauge, the variable head of mercury is suitably adjusted, if necessary, and an accurate head measurement made.

Immediately prior to testing the sample, the capillary tube circuit is opened as well, so as to produce equilibrium conditions in all circuits throughout the whole test system. Under this condition it is possible to move the mercury slug in the capillary to any desired position along the tube, by simply tilting it up or down at one end as required. Movement of the slug is made possible by the fluid moving round in the continuous closed circuit formed by the main fluid supply line in parallel with the capillary tube circuit. This knowledge serves a very useful purpose in cases where it happens that during a test the slug tends to move off the scale, as by opening the main fluid supply circuit and tilting the tube, the slug can be rapidly reset to the desired position without stopping the test to do so. After the test, each gap in the recording can easily be bridged by plotting several points on either side of a particular gap on a large scale and then, by knowing what the change in deflection during the reset interval had been, arranging, by a little trial and error, that the two sets of points be linked with a common smooth curve which thus also straddles the gap.

At the commencement of the test the main fluid supply line is closed off so that only the capillary tube circuit is cut in, and the following effects take place:

The loading spindle advances downwards into the test cell causing the displacement of cell fluid back into the fluid reservoirs via the capillary tube circuit, thus causing a slug movement from left to right. Superimposed on this, is a movement of fluid into the test cell from the fluid reservoirs via the capillary tube circuit if a decrease in volume of the sample occurs as a result of the loading, (thereby causing a slug movement from right to left), or vice-versa in the event of a volume increase of the sample. Furthermore, superimposed on these two effects is an additional fluid movement into the test cell via the capillary tube circuit, to replace any leakage from the test cell (which usually occurs where the loading spindle enters the cell), thus also causing a slug movement from right to left. Hence, during a test, the slug movement in the capillary tube is thus the nett movement resulting from three combined effects, and in order to determine the slug movement due to volume change of the sample only, it is necessary to separate out the slug movement which has taken place due to the other two effects.

As previously mentioned, slug movement due to entry of the loading spindle into the test cell can be calculated from a knowledge of the diameter of the capillary tube and from a determination of the volume of fluid displaced by the movement of the spindle. The latter can be found from a knowledge of the diameter of the spindle and the distance the spindle has travelled into the test cell, which corresponds to the deflection of the sample as read on the relevant dial deflection gauge.

The value of slug movement as a result of leakage from the test cell during an actual test, cannot be determined directly as during the test it is not possible to eliminate the effects of slug movement due to spindle entry and sample volume change. The only circumstances under which slug movement due to leakage only can be measured is under the steady conditions existing immediately before or after test. This measurement may thus not be truly representative of the actual slug movement due to leakage occurring during a test, as the rate of leakage may be

effected ...../

affected by the disturbed conditions during test. However, as this serves as a means of obtaining some idea of the leakage during test, these measurements were taken and in each case with the full particular chamber pressure acting and only the capillary tube circuit cut in.

To obtain some idea of the leakage under disturbed conditions similar to those existing during a test, a measurement of slug movement due to the combined effects of leakage and spindle entry only was contemplated, from which, by knowing from calculation the value of slug movement due to spindle entry only, the value of slug movement due to leakage only could be determined. As such measurements would only be possible provided no sample volume change effects were present, it was considered most suitable to carry out this determination immediately after completion of a test in the following manner:

With the capillary tube circuit closed off and the main supply circuit cut in, the spindle is raised a convenient height above its previous position on the top end plate in contact with the failed sample. With the relevant chamber pressure still acting and with only the capillary tube circuit cut in, the machine is again started and a series of slug movement : deflection readings taken. This slug movement recording is now inclusive of both spindle entry and leakage effects, but exclusive of sample volume change effects. This combined measurement would be carried out directly after the leakage measurement at the completion of the test, as discussed above.

Finally, to round off these measurements of slug movement due to leakage and spindle entry effects, a further leakage measurement is made immediately after this measurement of slug movement due to the combined effects of leakage and spindle entry.

7.5 The Test Series

As previously mentioned (Section 7.3), the type of triaxial compression test selected for the test series was the Consolidated Undrained Shear Test (Scu) with strain control. In order to obtain some information about the possible effect of rate of strain and chamber pressure on the value of Poisson's Ratio determined for the soil, it was decided that the test series should comprise three tests at a comparatively fast speed, and three tests at a much slower speed. Each set of three tests would then be done at three different chamber pressures which were to be 10lb/sq.in., 20lb/sq.in., and 30lb/sq.in., thus six tests in all. The chamber pressure of 10lb/sq.in corresponded approximately to the overburden pressure of a 15ft. depth of soil, to which the test soil had been subjected in the field.

Two convenient rates of strain, one fast and one slow, were obtained by using suitable gearbox settings. An initial determination of the resulting rates of strain was made by running the machine at each of the selected gearbox settings in turn, and taking deflection:time readings. While recording, the spindle would be entering the test cell against the resistance of the cell fluid only, which was at an arbitrarily selected chamber pressure of 30lb/sq.in. It was realised that the values determined might be dependent on the particular chamber pressure used, and might also be affected by the resistance of a sample to loading during an actual test. Hence, it was decided to refer to them as the nominal rates of strain. For the fast speed this was found to be 0.0125 inches per minute and for the slow speed 0.00256 inches per minute.

As ...../

As all samples were to be consolidated in the triaxial cell under the respective chamber pressures selected, it was necessary to know within what length of time such consolidation would be completed. Hence, in conducting a consolidation test on an undisturbed sample of the test soil, settlement:time readings were taken in each case immediately after placing the loading increments corresponding to chamber pressures of 30lb/sq.in., 20lb/sq. inches, and 10lb/sq.in., respectively, i.e. approximately 2Ton/sq.ft., 1.5Ton/sq.ft. and 0.7Ton/sq.ft. respectively. These curves are shown on the next page. It was found that 90% consolidation would be reached within less than 24 hours in all cases and hence it was decided to consolidate all samples for about 24 hours before testing.

The following is a description of a typical test performed. A 3" length x 1⅜" dia., cylindrical sample was cut from an undisturbed block sample of the soil, using the cutting cylinder provided for the purpose. To standardise the volume of all test samples cut, the same cutting cylinder was used throughout. An intermittent check was kept on the initial moisture content of test samples by using cuttings from the particular test sample being trimmed.

The sample was then placed in the triaxial machine and allowed to consolidate with drainage for about 24 hours under the particular chamber pressure selected.

When consolidation was completed, the drainage cock was closed and after noting the temperature inside the closing wooden box, a leakage measurement was made by recording slug movement: time readings every 15 seconds for 5 minutes, according to the procedure already described in the previous Section.

The slug was then reset to a convenient position along the scale and the main fluid supply line closed off to leave only the capillary tube circuit cut in. After noting the time and again the test temperature, the test was commenced. Using deflection as a basis, accurate readings of load, slug movement, and time together with intermittent temperature readings were taken over the whole duration of the test, until definite shear failure of the sample had occurred. At the end of the test, the time as well as the temperature were recorded and a sketch of the failed sample made. On occasion, it may have been necessary to reset the slug during a test and this was then done on the lines previously described. In the initial tests, only deflection, load and slug movement were recorded, two observers being required in the case of free tests. Continuous time recordings were initially not considered necessary, in view of the acceptance of a constant rate of strain during test, and in addition would have required a third observer which was not readily available. However, the need for continuous time recordings throughout any test (see Section 6.3) in order to determine the actual rate of strain was realised once the fact of an uneven rate of strain during test had been noticed, and it was thus necessary to call in the assistance of a third observer. Although time readings were thus not taken in some tests, it was, however, possible to utilise the information obtained from other tests in which time readings were taken to cover such cases.

Immediately after completion of the test and with the sample and loading spindle in the same position as when the machine was stopped, the slug in the capillary was reset to a convenient position and a leakage measurement made similar to the one carried out immediately before the commencement of the test. Temperature and time were appropriately recorded.

This ...../

SETTLEMENT--TIME CURVES

(FROM NORMAL 1-DIMENSIONAL CONSOLIDATION TEST.)

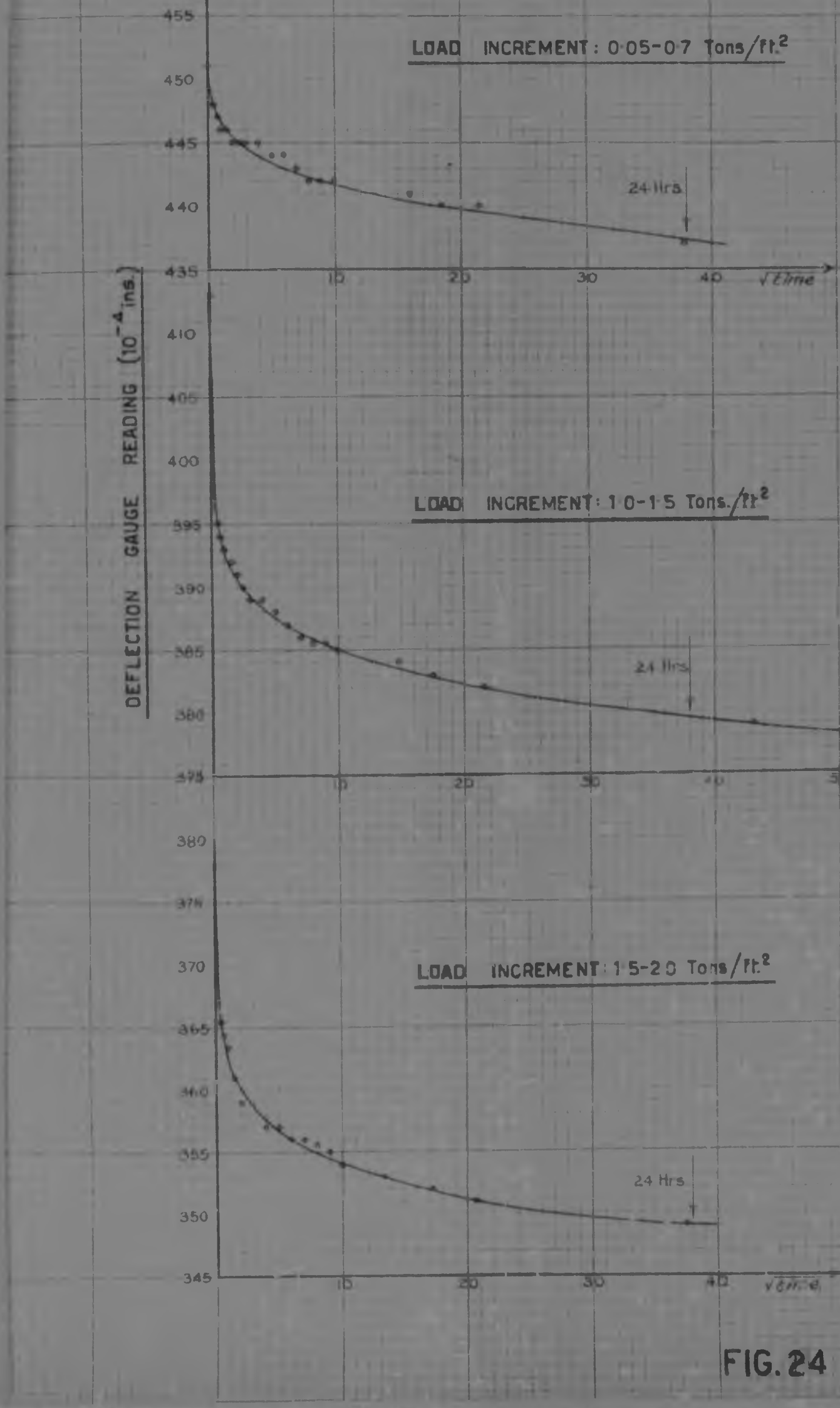


FIG. 24

SETTLEMENT–TIME CURVES

(FROM NORMAL 1-DIMENSIONAL CONSOLIDATION TEST.)

FIG 24

This having been done, a recording of slug movement due to the combined effects of leakage and spindle entry was carried out according to the procedure described under "Test Apparatus". In addition, however, time readings were also taken in order to determine the rate of strain under these circumstances and so to ascertain the possible effect of the particular chamber pressure used on the nominal rate of strain previously established. This rate of strain was determined in this manner for each test and will hereafter be referred to as the Calibrated rate of strain.

After this combined recording, a further leakage measurement was finally made, similar to the ones previously carried out.

Then with the capillary tube circuit cut out and only the main fluid supply circuit cut in, the chamber pressure was released and the cell fluid allowed to drain back into the fluid reservoirs. The failed sample was carefully removed and a moisture content determination carried out.

According to this procedure, tests falling into each of the six type categories previously mentioned were carried out. Shear strength values for tests in the same speed category were plotted in the form of Mohr circles and any test showing poor fit was discarded and a repeat test subsequently performed.

At the completion of the test series it was necessary to determine the cross-sectional area of the capillary tube used. This was done by obtaining the weight of mercury required to fill the tube nearly over its whole length and measuring the length of the tube accurately. From these measurements the volume of the tube and its average cross-sectional area could be calculated. (See Item1, Appendix V).

7.6 Test Observations and Curves

On the following pages, test observations and curves are given for the results of six tests. The results of each test appear in the following sequence:

1. Tabular presentation of the following data:
   (a) Leakage measurement immediately before test.
   Denoted: "Before test".
   (b) Leakage measurement immediately after test.
   Denoted: "After test (1)".
   (c) Combined leakage + spindle entry measurement (after test).
   (d) Further leakage measurement. Denoted: "After test(2)".

2. Presentation of the curves for the above data.

3. Tabular presentation of the main test observations under suitable headings, with reduction of these observations.

4. Graphical presentation. Rate of Strain determinations:
   (a) After test: spindle entry against resistance of solvent pressure only, i.e. Calibrated Rate of Strain.
   (b) During test: spindle entry against resistance of solvent pressure as well as resistance of sample to compression, i.e. actual Rate of Strain during test.

5. (a) Stress:Strain curve.
   (b) Young's Modulus:Strain Curve.

6. Tabular presentation: Calculation of Instantaneous values of Young's Modulus as a secant modulus over the initial range of the Stress:Strain curve.

7. Slug ...../

7. Slug movement : Deflection curve for the three combined effects during an actual test. This curve is headed: "Main slug movement : deflection curve".

Then, following on these sets of test results for each of the six tests performed, are:

8. Summary in Tabular Form of the six Triaxial Test results, i.e. the Shear Strength results required for constructing the Mohr Circles for each test.

9. Two sets of Mohr Circles, corresponding to the results of tests within the same speed categories.

The following points should, however, be especially noted:

(i) Triaxial chamber pressure values could be read on the pressure gauge provided for this purpose, but, as a check and for more accuracy, the height of a mercury column balanced by the chamber pressure was measured in centimetres. This pressure as a head of mercury was then converted to an equivalent pressure in lb/sq.in. The absolute pressure value read on the triaxial pressure gauge will hereafter be referred to as the "Nominal" chamber pressure and the value determined from the measured head of mercury will be referred to as the "Actual" chamber pressure.

(ii) With regard to the deviator stress calculation in item (3) mentioned above : The initial sample area used was taken to be the cross-sectional area of the cutting cylinder, i.e. the area of a circle 1" in diameter. This gave a figure of 1.77sq.in. The actual cross-sectional area of a sample may have been slightly less than this, thus leading to slightly greater stresses, but this is considered to be of little consequence for the purposes of these tests. However, the effect of lateral sample strain during consolidation, i.e., an increase in sample diameter and hence cross-sectional area, was taken into account by applying a factor:

$$\frac{1}{1-(\text{longitudinal strain})}, \text{ i.e. } \frac{1}{1-\varepsilon}$$

The "corrected stress" was thus obtained by using the initial sample area of 1.77 sq.in. divided by the appropriate value of this factor in each case.

(iii) Stress:Strain Curves (Item 7(i)) and Main Slug Movement:Deflection Curves (Item 7): These curves were in the first instance drawn fully expecting to use data obtained during the tests. In most cases the presence of "bedding-in" effects at the start of tests was clearly revealed by both these curve types. In the case of Stress:Strain curves, the strain due to "bedding-in" effects was determined by producing the initial approximately linear portion of the curve backwards to intersect the strain axis. The intercept on this axis then represents the strain due to "bedding-in" effects. This information was subsequently used for constructing the Young's Modulus: Strain curves. In the case of the Main Slug Movement:Deflection curves, the unusual peak-shaped "bedding-in" effect observed in some instances, is fully discussed in Section 7 of this Chapter. With the final presentation of test results (See Chapter8,Figs. 47,48,51 and 52) "bedding-in" effects have been ignored.

(iv) Young's Modulus:Strain Curves (Item 7(ii)): These curves are based on instantaneous values of Young's Modulus calculated as a secant modulus, i.e., working from point to point along the Stress:Strain curve but always from the corrected start of the curve, i.e. as obtained after eliminating "bedding-in" effects. For this reason the start of all Young's Modulus:Strain curves is displaced to the right of the actual start of the corresponding Stress:strain curves.

7.7 Discussion ...../

TEST No. L

DATE PERFORMED 18/2/57

SAMPLE CONSOLIDATED FOR 22hrs 15min TIMES TO TEST

TIME START OF TEST 4/30 P.M. TEMPERATURE 27.5°C

TIME END OF TEST 5/54 P.M. TEMPERATURE 27.25°C

NOMINAL σ_3 CHAMBER PRESSURE 30 lb/sq in

HEAD ON MERCURY 38½ inches

ACTUAL σ_3 CHAMBER PRESSURE 28.615 lb/sq in

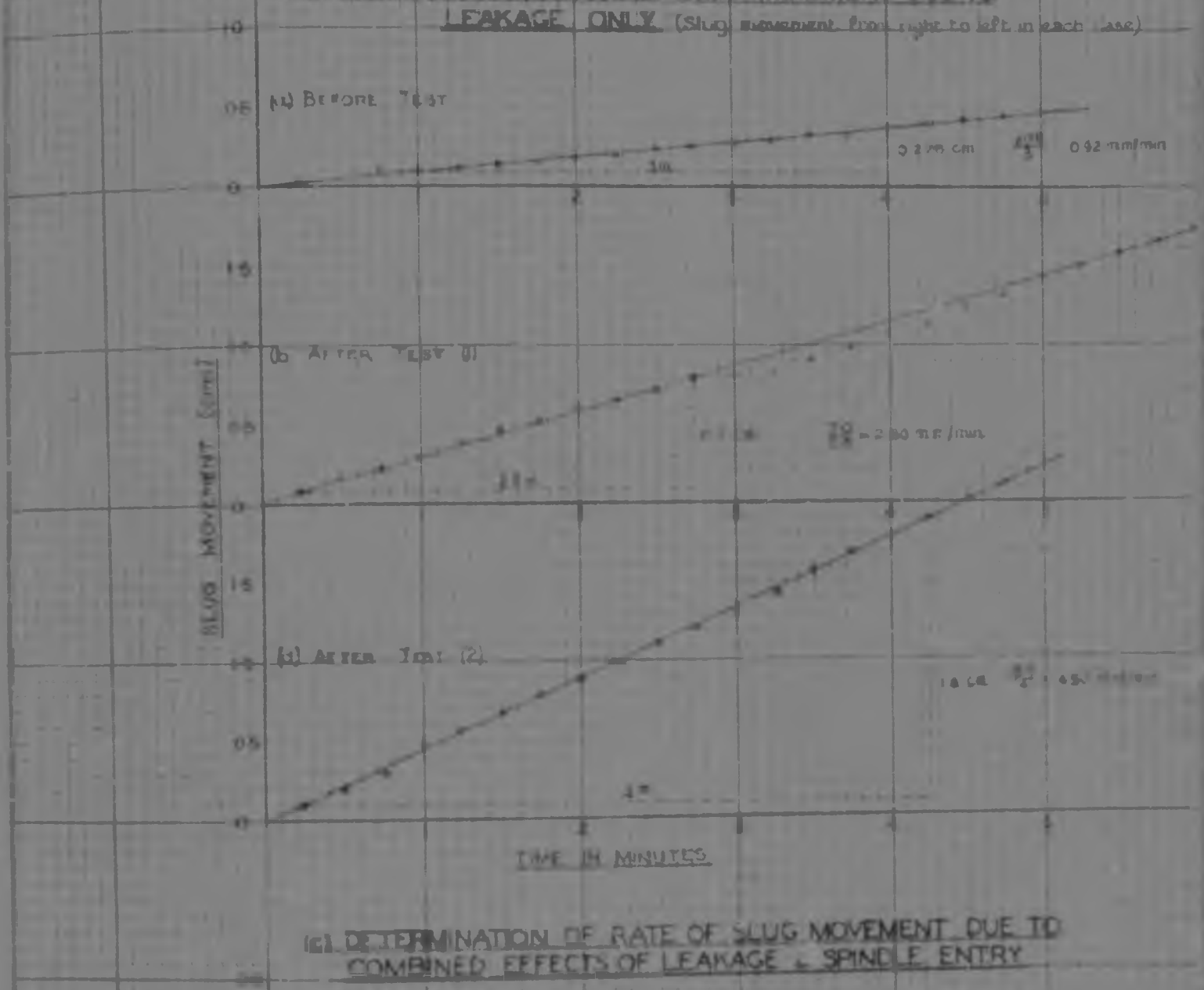
NOMINAL RATE OF STRAIN 0.00256 inches/min

TEST Nº 1 LEAKAGE MEASUREMENTS

(A) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. BEFORE TEST. TEMP 27.50C TIME 4/25 p.m.		(B) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST (1). TEMP 27.25°C TIME 5/55 p.m.		(C) MEASUREMENT OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY (AFTER TEST). TEMPERATURE 27.25°C TIME 6/05 p.m.			(D) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAG ONLY. AFTER TEST (2). TEMP 27.25°C TIME 6/[illegible] p.m.	
TIME min sec	VOLUME SCALE cm	TIME min sec	VOLUME SCALE cm	DEFLECT GAUGE [illegible]	VOLUME SCALE cm	TIME min sec	TIME [illegible]	VOLUME SCALE cm
0 00	21.60	0 00	8.90	[illegible] 40	7.40	0 00	00	8.60
15	21.575	15	8.925	41	7.35		15	8.50
30	21.525	30	8.775	42	7.30		30	8.40
45	21.50	45	8.675	43	7.50		45	8.30
1 00	21.50	1 00	8.60	44	7.30		1 00	8.125
15	21.475	15	8.50	45	7.275	2 02	15	8.025
30	21.475	30	8.425	46	7.225		30	8.025
45	21.45	45	8.375	47	7.[illegible]		45	8.-25
2 00	21.425	2 00	8.30	48	7.20		2 00	8.225
15	21.40	15	8.[illegible]	49	7.[illegible]		15	8.[illegible]
30	21.375	30	8.20	50	7.175	4 00	30	8.[illegible]
45	21.35	45	8.[illegible]	51	7.175		45	8.[illegible]
3 00	21.325	3 00	8.10	52	7.[illegible]		3 00	8.[illegible]
15	21.30	15	8.075	53	7.125		15	8.175
30	21.275	30	8.00	54	7.10		30	8.[illegible]
45	21.[illegible]	45	7.9[illegible]	55	7.10	6 06	45	4.[illegible]
4 00	21.225	4 00	7.825	56	7.075		4 00	4.8[illegible]
15	21.20	15	7.75	57	7.05		15	4.[illegible]
30	21.175	30	7.68	58	7.025		30	4.[illegible]
45	21.15	45	7.575	59	7.00		45	4.47[illegible]
5 00	21.125	5 00	7.475	60	7.00	8 04	5 00	4.[illegible]
		[illegible]	7.40	61	6.975			
		30	7.[illegible]	62	6.975			
		45	7.25	63	6.95			
		6 00	7.175	64	6.[illegible]			
				65	6.90	10.02		
				66	6.90			
				67	6.90			
				68	6.90			
				69	6.85			
				70	6.85	11 56		
				71	6.[illegible]			
				72	6.[illegible]			
				73	6.80			
				74	6.80			
				75	6.80	13 47		
				76	6.[illegible]			
				77	6.[illegible]			
				78	6.[illegible]			
				79	6.[illegible]			
				80	6.725	15 44		
				81	6.[illegible]			

TEST NO: LEAKAGE MEASUREMENTS

DETERMINATIONS OF RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY (Slug movement from right to left in each case)



(c) DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY

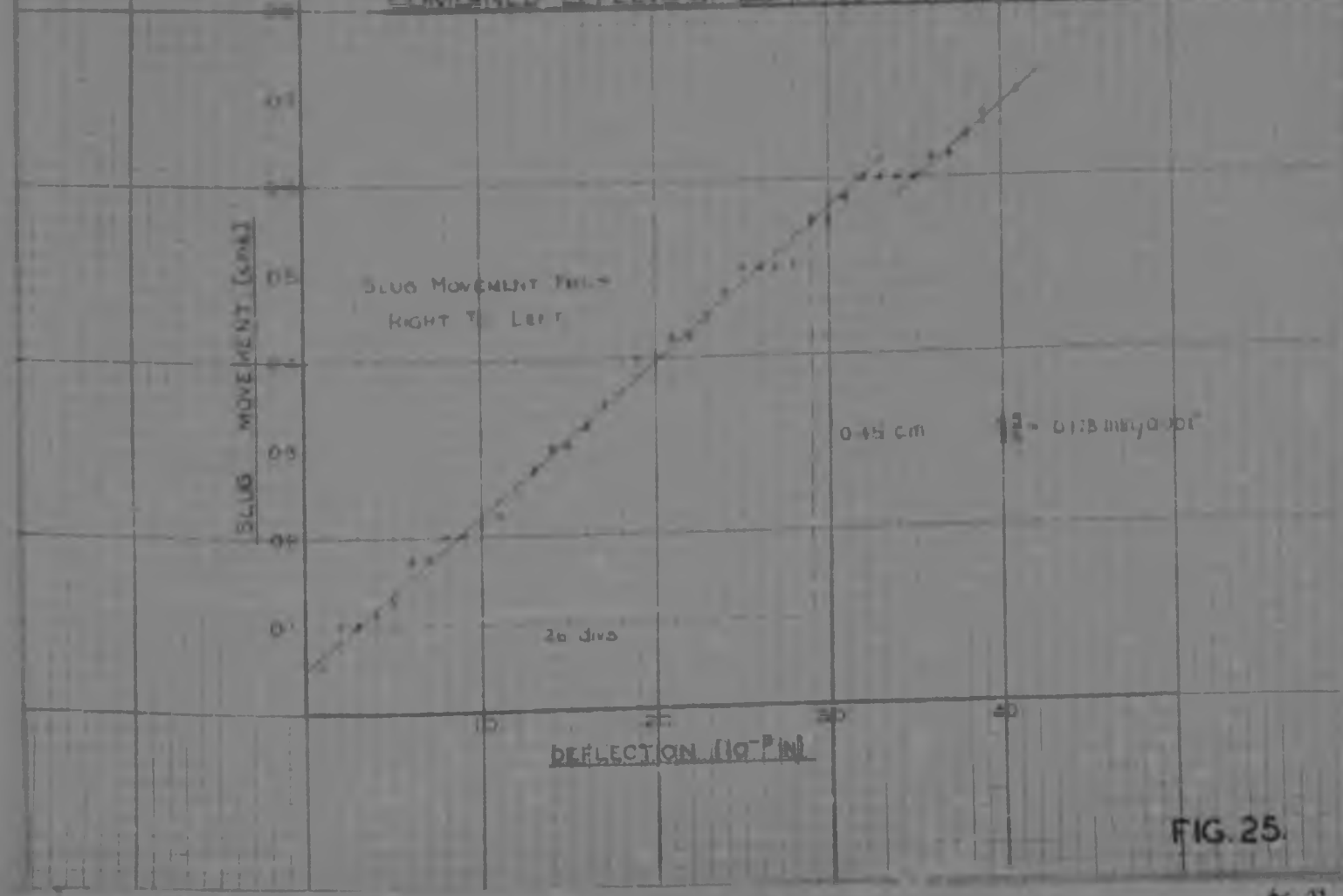


FIG. 25.

TEST No 1 MAIN TEST OBSERVATIONS

DEFLEC-TION GAUGE	LOAD GAUGE	SLUG SCALE cm	TIME min sec	TEMPERATURE	CORRECTED SLUG SCAL cm	CORRECTED LOAD lb	CORRECT DEFLECT 10^{-3} in.	STRAIN ε percent	CORRECTED STRESS $(\sigma_1 - \sigma_2)$ $lb/sq\ in$
00 30	20	[illegible]	00 00	97.5	0	0	0	0	0
31	34.5	[illegible]	00 27		[illegible]	[illegible]	1	0.035	[illegible]
32	47.5	[illegible]	00 55		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
33	32	[illegible]	01 [illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
34	37.[illegible]	[illegible]	07 [illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	14.[illegible]	[illegible]	08 40		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	49.[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
37	[illegible].1	[illegible]	10 [illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
38	[illegible]	[illegible]	10 [illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
39	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	69.0	[illegible]	18 [illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
41	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
42	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
43	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
44	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
46	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
47	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
48	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
49	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
51	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
52	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
53	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
55	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
57	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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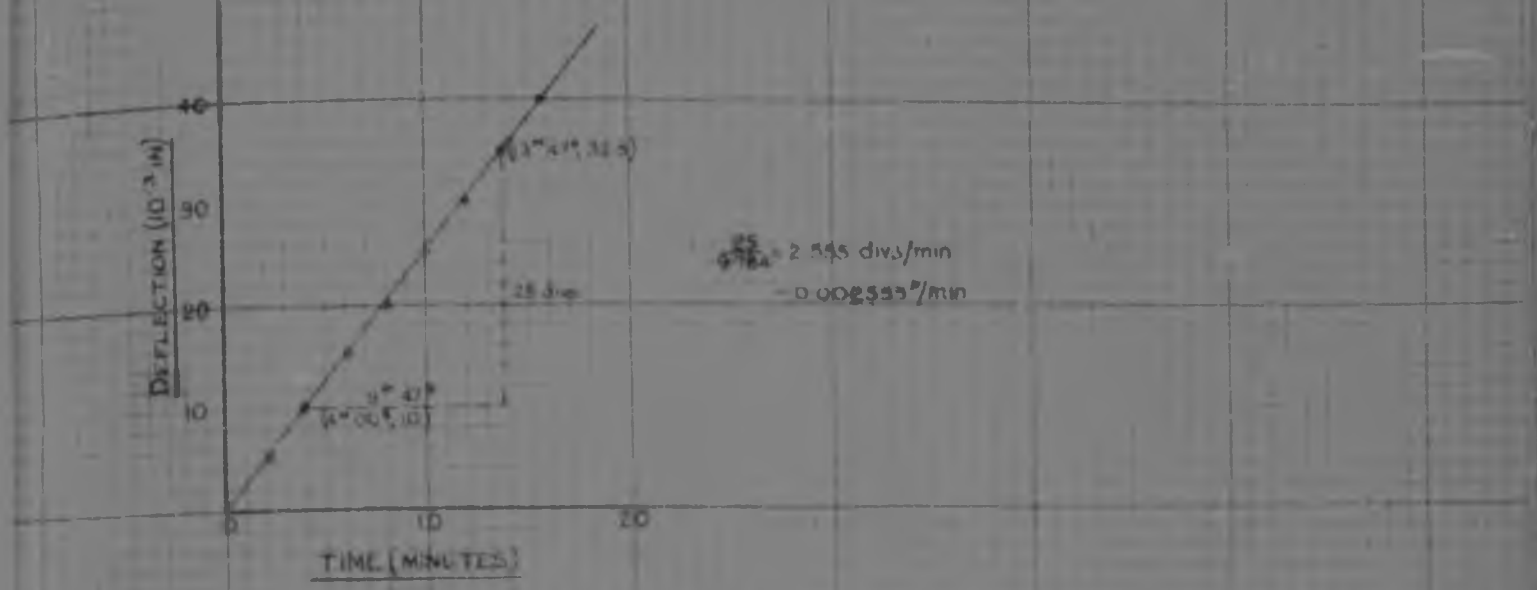
59	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
60	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
61	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
62	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
63	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
64	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
65	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
66	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
67	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
68	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
69	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
70	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
71	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
72	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
73	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
74	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
75	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
76	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
77	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
78	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
79	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
80	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
81	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
82	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
83	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
84	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
85	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
86	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
87	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
88	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
89	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
90	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
91	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
92	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
93	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
94	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
95	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
96	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
97	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
98	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
99	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
100	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
101	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
102	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
103	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
104	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
105	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
106	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
107	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
108	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
109	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
110	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
111	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
112	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
113	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
114	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TEST [illegible] MAIN TEST OBSERVATIONS (Cont.)

DEFLEC-TION GAUGE	LOAD GAUGE	SLUG SCALE cm		TEMPERA-TURE	CORRECTED SLUG SCALE cm	CORRECTED LOAD lbs	CORRECT DEFLECT 10^{-3}in	STRAIN ε μ in/in	CORRECTED STRESS (σ) lbs/sq in
117	163.0	5.425		24	41.45	163.2	87	2.900	89.63
120	163.4	5.425		31	41.45	163.4	90	3.000	92.80
123	163.5	5.426		41	41.45	163.5	93	3.100	95.91
126	163.1	5.41		50	41.425	163.1	96	3.200	98.98
129	163.5	5.425		52	41.35	163.5	99	3.300	102.02
132	163.5	5.425		12	41.35	163.5	102	3.400	105.03
135	163.4	5.8		50	41.375	163.4	105	3.500	108.02
138	163.2	5.90		55	40.975	163.1	108	3.600	111.00
141	163.0	5.400			41.0	163.0	111	3.700	114.00
144	162.9	5.20		14	40.925	162.9	114	3.800	117.00
147	162.4	5.395			40.85	162.4	117	3.900	120.17
150	162.1	5.425		18	40.45	162.1	120	4.000	123.31
153	161.9	6.4[illegible]		26	40.375	161.9	123	4.100	126.42
156	161.5	7.0		31	40.275	161.5	126	4.200	129.49
159	161.3	7.470		41	39.9	161.3	129	4.300	132.54
162	161.1	7.93		43	39.825	161.1	132	4.400	135.56
165	160.3	7.475		44	39.65	160.3	135	4.500	138.56
168	160.0	7.885		44	39.575	160.0	138	4.600	141.53
171	160.0	7.45			39.55	160.0	141	4.700	144.48
174	159.8	7.891		51	39.50	159.8	144	4.800	147.41
177	159.8	8.35		53	39.475	159.8	147	4.900	150.32
180	159.4	8.778		53	39.40	159.4	150	5.000	153.21
183	159.1	8.62		57	39.325	159.1	153	5.100	156.10
186	158.8	9.0		53	39.275	158.8	156	5.200	158.97
189	158.7	9.00		11	37.875	158.7	159	5.300	161.83
192	158.7	9.054		54	37.88	158.7	162	5.400	164.68

TEST No1 RATE OF STRAIN DETERMINATIONS

(a) CALIBRATED RATE OF STRAIN (σ_3 = 30 lb/sq in)



(b) ACTUAL RATE OF STRAIN DURING TEST

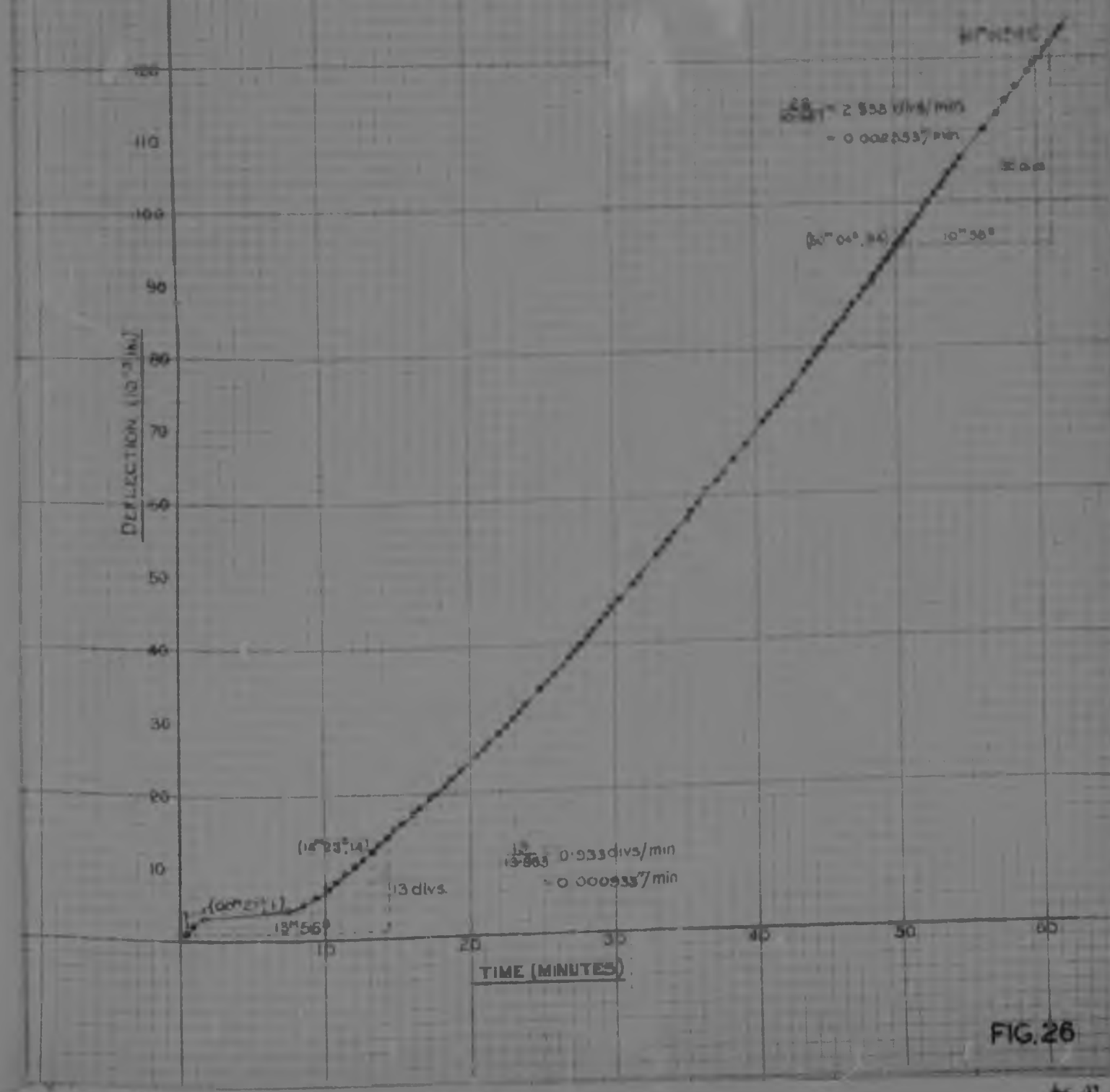


FIG. 26

TEST No 1.

(a) STRESS STRAIN CURVE

σ_3 = 30 lb/sq in
E = 0·737
ω = 0·15%
ω = 2·5%

end of linear range

STRAIN E (percent)

(b) YOUNGS' MODULUS STRAIN CURVE

YOUNGS' MODULUS (E) lb/sq in

STRAIN E (percent)

FIG. 27

TEST Nº1

CALCULATION OF INSTANTANEOUS VALUES OF YOUNGS MODULUS (E)

CORRECTED STRESS = σ lb/sq in.	STRAIN percent	STRAIN DUE TO BEDDING-IN EFFECTS percent	CORRECTED STRAIN = ε percent	YOUNGS MODULUS (E) = σ/ε lb/sq in
2.54	[illegible]	0	0.[illegible]33	7697
4.23	0.[illegible]67	0	[illegible]	6313
6.77	[illegible]	0	[illegible]	677[illegible]
10.04	0.133	0	0.133	7549
14.0[illegible]	0.187	0	0.187	8383
16.80	[illegible]	0	[illegible]	8400
19.7[illegible]	0.233	0	[illegible]	8499
22.43	0.267	0	0.267	8401
25.23	0.300	0	0.300	8401
27.[illegible]	[illegible]	0	[illegible]	8286
29.35	[illegible]	0	[illegible]	8161
32.64	0.400	0	0.400	8160
34.82	0.433	0	0.433	8042
37.[illegible]3	[illegible]	0	[illegible]	7877
39.[illegible]	[illegible]	0	[illegible]	7858
41.53	0.533	0	0.533	7792
43.20	0.567	0	0.567	7618
45.38	[illegible]	0	[illegible]	7563
47.16	[illegible]	0	[illegible]	7456
48.[illegible]	0.667	0	[illegible]	7319
50.83	0.700	0	[illegible]	7261
52.66	[illegible]	0	[illegible]	7184
54.33	[illegible]	0	[illegible]	7063
55.[illegible]	[illegible]	0	[illegible]	6990
60.02	[illegible]	0	[illegible]	6669
63.[illegible]	1.000	0	1.000	6382
67.[illegible]	1.100	0	1.100	6121
70.[illegible]	1.200	0	1.200	5861
72.[illegible]	1.300	0	1.300	5598
75.[illegible]	1.400	0	1.400	5364
76.81	1.[illegible]	0	1.[illegible]	5127
[illegible].11	1.[illegible]	0	1.600	4944
80.[illegible]	1.700	0	1.[illegible]	4713
82.61	1.800	0	1.[illegible]00	4589
83.80	1.[illegible]00	0	1.[illegible]	4410
85.04	2.000	0	[illegible]	4252

TEST No 1.

MAIN SLUG MOVEMENT DEFLECTION CURVE

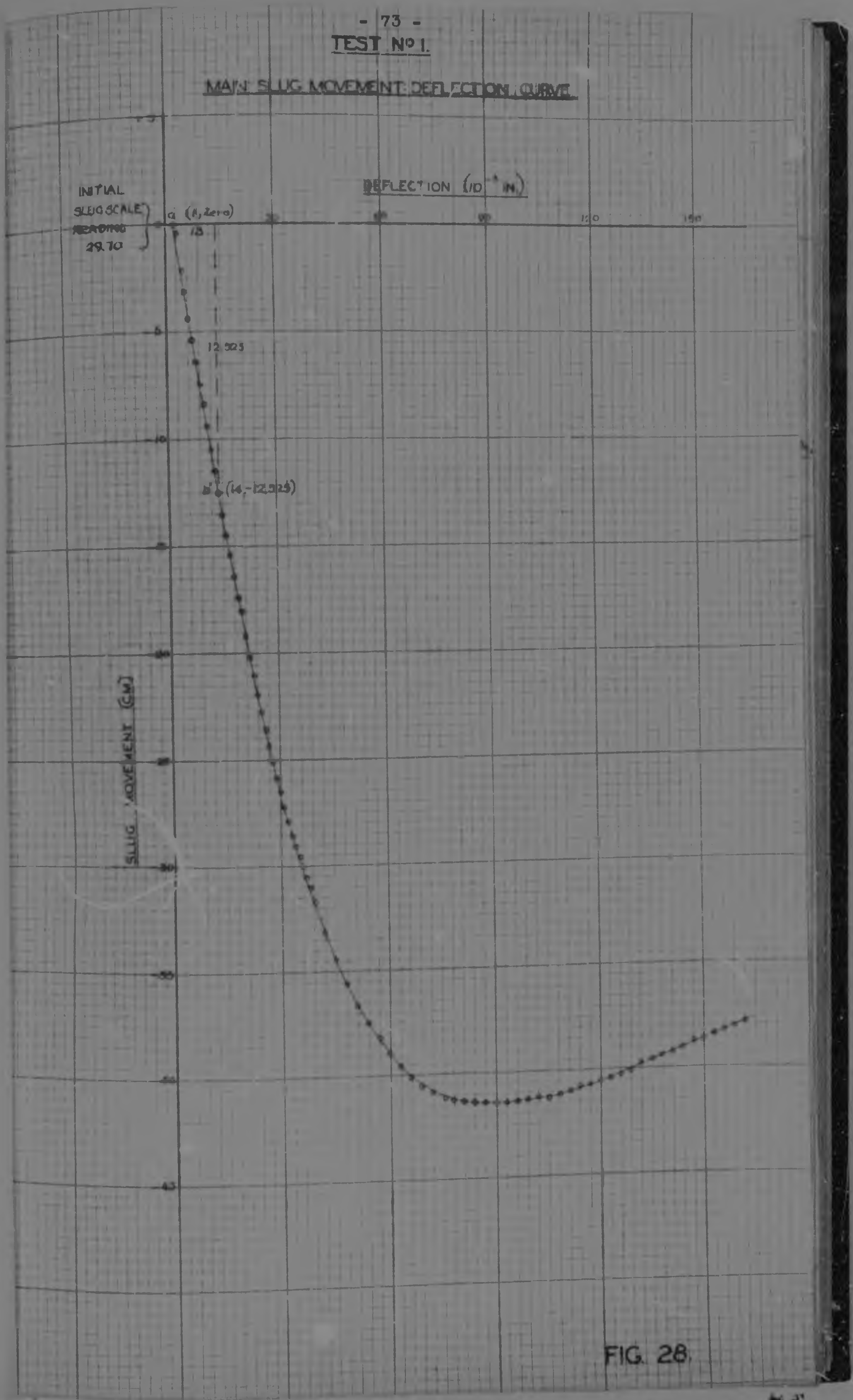


FIG. 28.

TEST No 2.

DATE PERFORMED 22/1/57

SAMPLE CONSOLIDATED FOR 24 HRS 30 MINS PRIOR TO TEST

TIME START OF TEST 2/55 P M TEMPERATURE 25 25°C

TIME END OF TEST 3/10 P M TEMPERATURE 25 25°C

NOMINAL 6_3 CHAMBER PRESSURE 30 LB/SQ IN

HEAD OF MERCURY 58⅝ INCHES, HENCE

ACTUAL 6_3 CHAMBER PRESSURE 28 736 LB/SQ IN

NOMINAL RATE OF STRAIN 0·0125 INCHES/MINUTE

TEST Nº 2 LEAKAGE MEASUREMENTS.

(A) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. BEFORE TEST TEMP 25.5°C TIME 2/25p.m.		(B) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST (1) TEMP 25.25°C TIME 3/15p.m.		(C) MEASUREMENT OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY (AFTER TEST) TEMPERATURE: 25.5°C TIME 3/25p.m.			(D) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST (2) TEMP 25.0°C TIME 3/35p.m.	
TIME min sec	VOLUME SCALE cm	TIME min sec	VOLUME SCALE cm	DEFLECT GAUGE to	VOLUME SCALE cm	TIME min sec	TIME min sec	VOLUME SCALE cm
0 00	26.00	0 00	28.00	00 50	1.60		0 00	23.7
15	25.975	15	27.95	52.5	1.80		15	23.575
30	25.95	30	27.825	55	2.15		30	23.425
45	25.925	45	27.75	57.5	2.30		45	23.3
1 00	25.90	1 00	27.65	60	2.55		1 00	23.175
15	25.85	15	27.55	62.5	2.825		15	23.075
30	25.825	30	27.45	65	3.075		30	23.00
45	25.8	45	27.325	67.5	3.40		45	22.9
2 00	25.775	2.00	27.225	70	3.65		2 00	22.80
15	25.75	15	27.125	72.5	3.90		15	22.70
30	25.70	30	27.025	75	4.20		30	22.60
45	25.675	45	26.95	77.5	4.50		45	22.475
3 00	25.675	3 00	26.85	80	4.80		3 00	22.35
15	25.625	15	26.75	82.5	5.00		15	22.225
30	25.55	30	26.65	85	5.35		30	22.10
45	25.525	45	26.55	87.5	5.65		45	22.00
4 00	25.50	4 00	26.45	90	5.95		4 00	21.90
15	25.475	15	26.325	92.5	6.20		15	21.675
30	25.45	30	26.225	95	6.50		30	21.525
45	25.40	45	26.125	97.5	6.80		45	21.40
5 00	25.375	5 00	26.00	100	7.05		5 00	21.275
				102.5	7.35			
				105	7.60			
				107.	7.85			
				110	8.20			
				112.5	8.45			
				115	8.75			
				117.5	9.05			
				120	9.325			
				122.5	9.625			
				125	9.925			
				127.5	10.225			
				130	10.525			
				132.5	10.825			
				135	11.125			
				137.5	11.40			
				140	11.70			
				142.5	12.00			
				145	12.30			
				147.5	12.55			
				150	12.85			

TEST No 2: LEAKAGE MEASUREMENTS

DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY (Slug movement from right to left in each case)

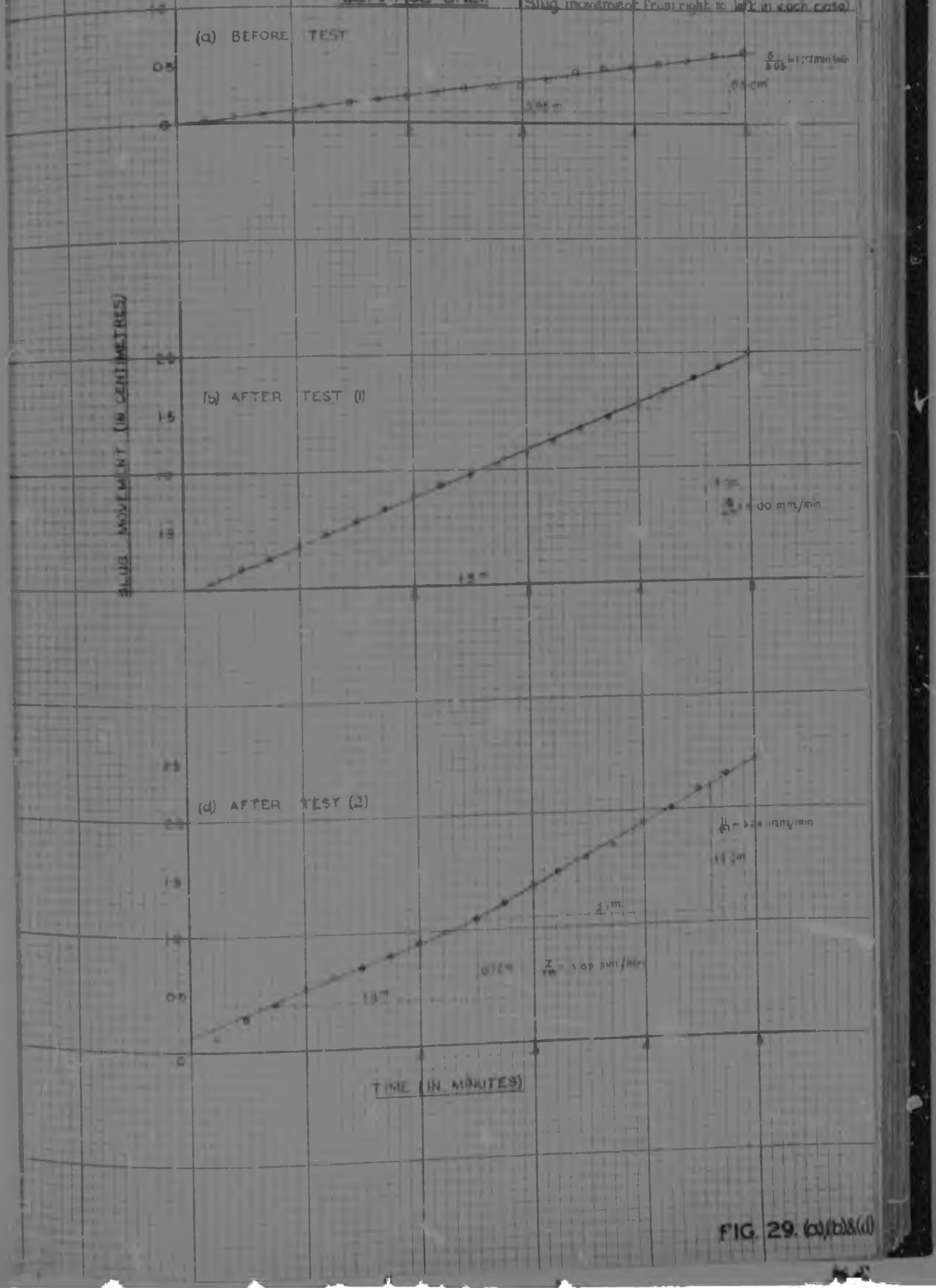


FIG. 29. (a)(b)&(d)

TEST NO.2 LEAKAGE MEASUREMENTS (CONTINUED)

(c) DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO COMBINENED EFFECTS OF LEAKAGE & SPINDLE ENTRY

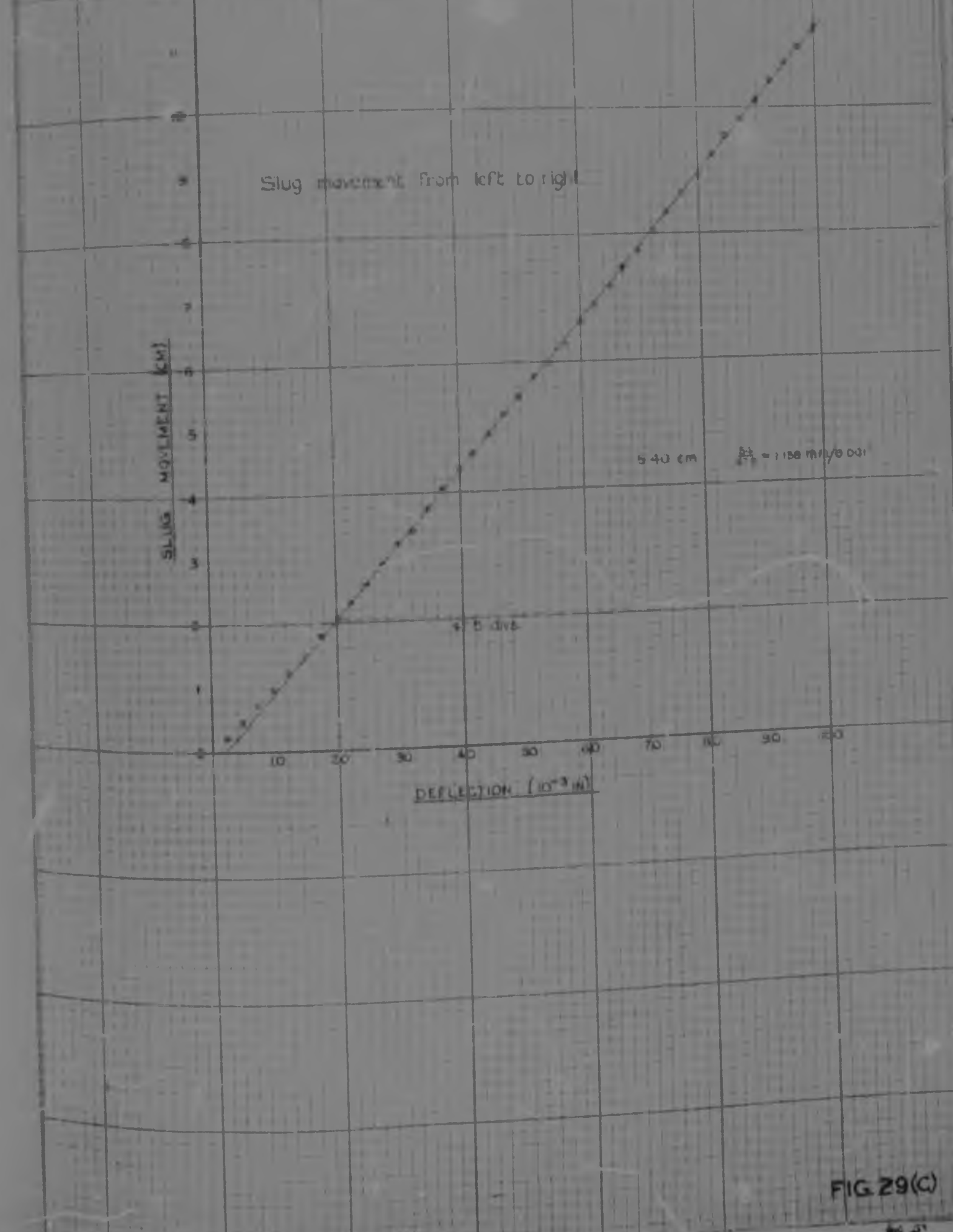


FIG. 29(C)

TEST NO3 LEAKAGE MEASUREMENTS

DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY

(a) BEFORE TEST

Slug movement from right to left in each case

(b) AFTER TEST (1)

(c) AFTER TEST (2)

SLUG MOVEMENT (IN CENTIMETRES)

TIME (IN MINUTES)

(d) DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY

Slug movement from left to right

SLUG MOVEMENT (CM)

DEFLECTION (10^{-4} IN.)

FIG 32

TEST Nº 6 : LEAKAGE MEASUREMENTS

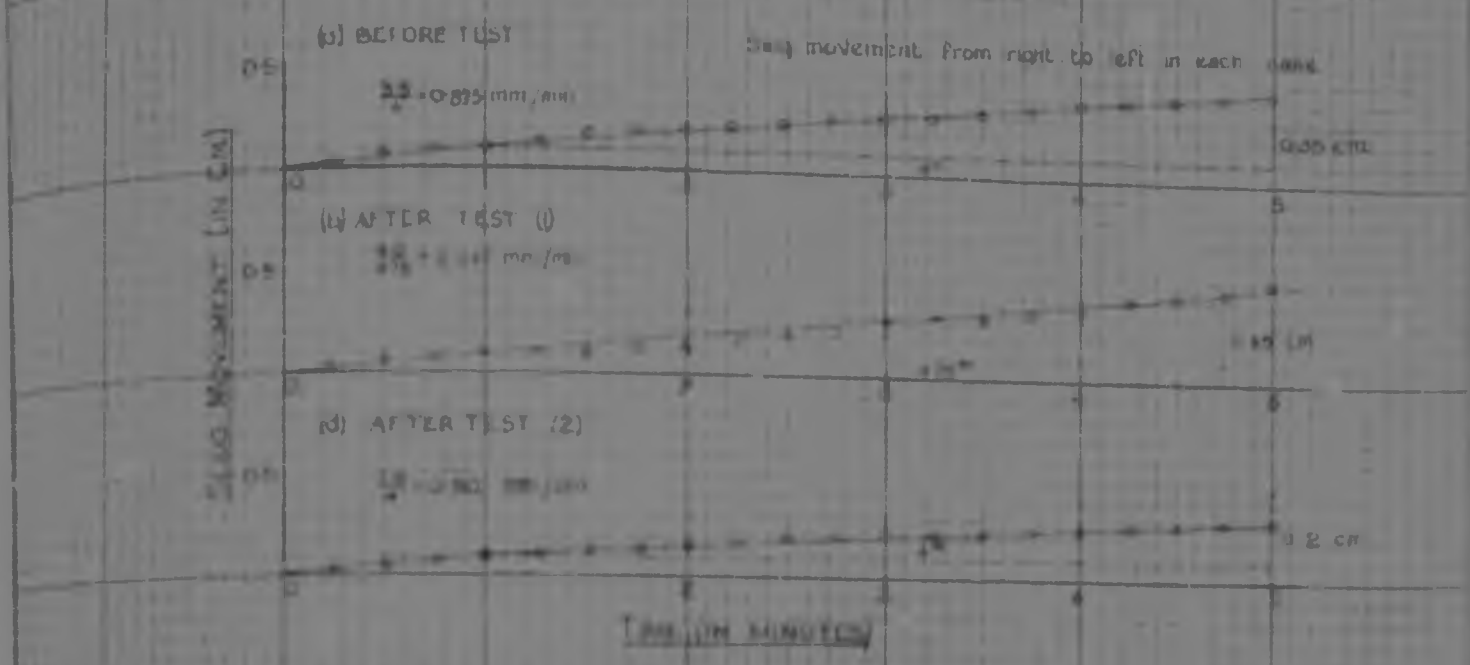
(A) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. BEFORE TEST TEMP 20.25°C TIME 11.1[illegible]		(B) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY AFTER TEST (1) TEMP 2[illegible]°C TIME 12/12[illegible]		(C) MEASUREMENT OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY (AFTER TEST) TEMPERATURE [illegible]°C TIME 12/1[illegible]			(D) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY AFTER TEST (2) TEMP[illegible] TIME[illegible]	
TIME min sec	VOLUME SCALE cm.	TIME	VOLUME SCALE cm	DEFLECT GAUGE no	VOLUME SCALE cm	TIME min sec	TIME	VOLUME SCALE cm
0 00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 0	[illegible]	[illegible]
15	[illegible]	15	[illegible]	42	[illegible]		[illegible]	[illegible]
30	[illegible]	30	[illegible]	4[illegible]	[illegible]		[illegible]	[illegible]
45	[illegible]	45	[illegible]	[illegible]	[illegible]			[illegible]
1 00	[illegible]	1 00	[illegible]	44	[illegible]		1 [illegible]	19.[illegible]
15	[illegible]	15	[illegible]	45	[illegible]	1 58	1 [illegible]	[illegible]
30	[illegible]	30	[illegible]	46	[illegible]			[illegible]
45	[illegible]	45	[illegible]	47	[illegible]		46	[illegible]
2 00	[illegible]	2 00	[illegible]	48	[illegible]		[illegible]	[illegible]
15	[illegible]	15	[illegible]	[illegible]	[illegible]		15	[illegible]
30	[illegible]	30	[illegible]	[illegible]	[illegible]	3 58	30	[illegible]
45	[illegible]	45	[illegible]	[illegible]	[illegible]		45	[illegible]
3 00	[illegible]	3 00	[illegible]	[illegible]	[illegible]		3 [illegible]	[illegible]
15	[illegible]	15	[illegible]	[illegible]	[illegible]		15	[illegible]
30	[illegible]	30	[illegible]	54	[illegible]		30	[illegible]
45	[illegible]	45	[illegible]	[illegible]	[illegible]	5 58	45	[illegible]
4 00	[illegible]	4 00	[illegible]	[illegible]	[illegible]		4 [illegible]	[illegible]
15	[illegible]	15	[illegible]	[illegible]	[illegible]			[illegible]
30	[illegible]	30	[illegible]	[illegible]	[illegible]			[illegible]
45	[illegible]	45	[illegible]	[illegible]	[illegible]		45	[illegible]
5 00	[illegible]	5 00	[illegible]	60	[illegible]	8 02	5 [illegible]	[illegible]
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			
				[illegible]	[illegible]			

TEST Nº 3 MAIN TEST OBSERVATIONS.

DEFLEC-TION GAUGE	LOAD GAUGE	SLUG SCALE	TIME	TEMPERATURE	CORRECTED SLUG SCALE cm	CORRECTED LOAD lb	CORRECT DEFLECT 10^{-3} in	STRAIN ε per unit	CORRECTED STRESS (σ1-σ3) σd lb/sq in
00 35	15	23.30	00 00	23.25	0	0	0	0	0
36	17.25	23.35			-0.05	2.25	1	0.033	
37	19.5	23.375			-0.025	4.5	2	0.067	
38	20.6	23.30			0	5.8	3	0.100	3.3
39	24.0	23.10			+0.20	9.0	4	0.133	5.1
40	26.7	23.00	02 35		0.80	11.7	5	0.167	6.6
41	29.0	22.10	03 15		1.20	13.0	6	0.200	7.34
42	33.25	20.75	08 07		2.55	19.25	7	0.233	10.29
43	37.8	20.2			3.10	22.5	8	0.267	12.68
44	42.0	19.5			4.10	27.9	9	0.300	15.66
45	48.2	17.95			5.45	33.2	10	0.333	18.70
46	52.3	16.9	11 19		6.40	37.3	11	0.367	21.00
47	56.9	15.7	11 57		7.50	41.9	12	0.400	23.58
48	60.6	14.7			8.60	45.6	13	0.433	25.65
49	64.6	13.65	13 08		9.65	49.6	14	0.467	27.92
50	68.2	12.7	13 41		10.60	53.2	15	0.500	29.91
51							16	0.533	
52	75.2	10.8	14 46		12.50	60.2	17	0.567	33.92
53	78.2	9.85			13.35	63.2	18	0.600	35.49
54	81.2	8.9			14.10	66.2	19	0.633	37.35
55	84.2	8.0			15.00	69.6	20	0.667	39.06
56	87.4	7.0	15 51		15.80	72.4	21	0.700	40.82
57	90.3	6.1			16.60	75.5	22	0.733	42.54
58	93.2	5.20			17.35	78.2	23	0.767	43.84
59	95.9	5.2			18.10	80.8	24	0.800	45.28
60	98.0	4.55	18 00		18.75	83.0	25	0.833	46.80
61	100.7	3.775			19.525	85.7	26	0.867	48.50
62	102.7	3.3			20.05	87.7	27	0.900	49.10
63	104.2	2.775			20.625	89.7	28	0.933	49.92
64	106.0	2.275			21.025	91.0	29	0.967	50.21
65	107.8	1.775	21 24		21.425	92.8	30	1.000	51.90
66	110.0	1.2			22.75	97.0	35	1.100	54.20
69	113.6	27.45			23.10	98.4	34	1.133	54.96
70	114.7	27.15	23 41		23.40	99.7	35	1.167	55.67
71	115.5	26.9			23.75	100.5	36	1.200	56.10
72	116.5	26.55			24.05	101.8	37	1.233	56.64
73	117.3	26.35			24.30	102.5	38	1.267	57.18
74	118.2	26.125			24.525	103.5	39	1.300	57.55
75	119.0	25.775	25 51		24.775	104.3	40	1.333	57.97
80	121.8	24.9	27 51		25.65	106.0	48	1.600	58.45
85	124.2	24.175	29 57		26.175	109.2	50	1.667	60.35
90	125.8	24.175	31 53		26.425	110.8	58	1.933	61.4
91	126.0	24.00	33.63		26.55	111.8	60	2.000	61.3
100	127.3	24.00	35 52		26.65	112.3	65	2.167	62.1
105	128.1	24.10	37 52		26.45	113.1	70	2.333	62.5
110	128.8	24.02			26.30	114.2	75	2.500	63.0
115	130.4	24.475	41 51		26.075	115.4	80	2.667	63.5
120	131.2	24.725	43 49		25.825	116.2	85	2.833	63.9
125	131.7	24.875	45 49		25.675	116.7	90	3.000	63.9
130	132.1	24.175	47 45		25.40	117.1	95	3.167	64.0
135	132.5	25.45	49 43		25.125	117.5	100	3.333	64.1
140	132.7	25.875	51 41		24.70	117.7	105	3.500	64.1
145	133.0	26.425	53 41		24.15	118.0	110	3.667	64.2
150	133.5	27.05	55 40		23.525	118.5	115	3.833	64.4
155	134.1	27.6	57 39		23.075	119.1	120	4.000	64.5
160	134.5	28.1	59 39		22.575	119.5	125	4.167	64.6
165	135.1	28.6	01 38		21.775	120.1	130	4.333	64.91
168	135.5	29.10			21.475	120.2	133	4.433	
170	135.7	29.50			21.275	120.7	135	4.500	65.10
171	135.8	29.70			21.175	120.9	136	4.533	

TEST No6 LEAKAGE MEASUREMENTS

DETERMINATIONS OF RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY



(c) DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY

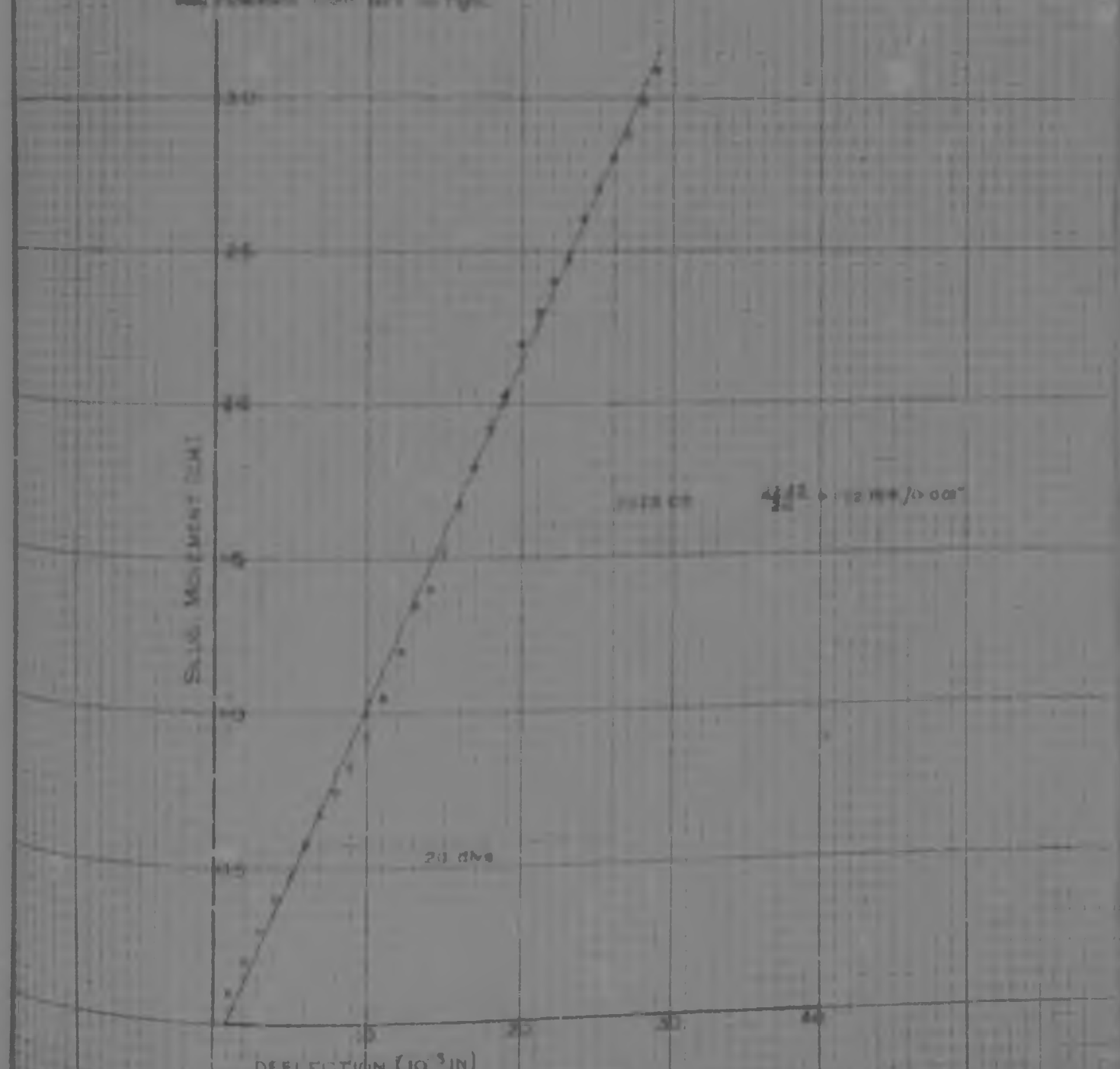


FIG 44

TEST N°3 MAIN TEST OBSERVATIONS (Cont.)

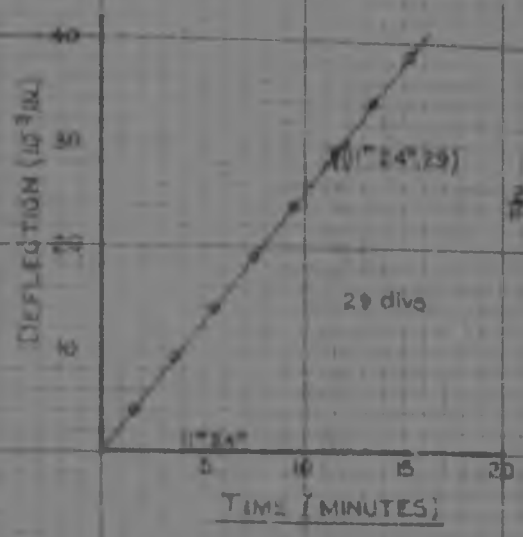
DEFLECTION GAUGE	LOAD GAUGE	SLUG SCALE	TIME	TEMPERATURE	CORRECTED SLUG SCALE	CORRECTED LOAD	CORRECT DEFLECT	STRAIN ε	CORRECTED STRESS (σ1-σ3/ σd
						lb	10^{-4} in		lb/sq in
172	135.9	3.80			21.075	120.9	137	4.567	
173	135.9	3.90			20.975	120.9	138	4.600	
174	136.0	4.00			20.875	121.0	139	4.633	
175	136.0	4.10	65 34		20.775	121.0	140	4.667	65.16
180	136.2	4.60	67 26		20.275	121.2	145	4.833	65.16
185	136.7	5.20	69 29		19.675	121.7	151	5.033	65.30
190	136.9	5.75	71 17	24.0	19.125	121.9	156	5.167	65.30
195	137.1	6.325	73 17		18.550	122.1	160	5.333	65.30
200	137.2	6.90	75 13		17.975	122.2	165	5.500	65.24
205	137.2	7.50	77 11		17.375	122.2	170	5.667	65.18
210	137.2	8.20	79 11		16.675	122.2	175	5.833	65.01
215	137.2	8.90	81 04		16.075	122.2	180	6.000	64.99
220	137.2	9.475	83 00		15.40	122.2	185	6.167	64.77
225	137.2	10.075	84 56		14.80	122.2	190	6.333	64.60
230	137.2	10.7	86 51		14.175	122.2	196	6.533	64.55
235	137.2	11.35	88 48		13.525	122.2	202	6.667	64.43
240	137.2	11.90	90 43		12.975	122.2	206	6.833	64.31
246	137.2	12.50	92 39	24.0	12.375	122.2	212	7.000	64.21

TEST No 2 MAIN TEST OBSERVATIONS

DEFLECTION GAUGE	LOAD GAUGE	SLUG SCALE	TIME	TEMPERATURE	CORRECTED SLUG SCALE	CORRECTED LOAD	CORRECT DEFLECT	STRAIN	CORRECTED STRESS
00 35	15	[illegible]	00	[illegible]	0				
36	15.0	[illegible]	01 09		0.[illegible]	[illegible]	1	[illegible]	[illegible]
37	16.4	[illegible]	01 34		[illegible]	1.4	[illegible]	[illegible]	[illegible]
38	17.3	[illegible]	01 58		[illegible]	[illegible]	3	[illegible]	[illegible]
39	18.0	[illegible]	02 22		[illegible]	[illegible]	4	[illegible]	[illegible]
40	[illegible]	[illegible]	02 49		[illegible]	[illegible]		[illegible]	[illegible]
41	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	6	[illegible]	[illegible]
42	[illegible]	[illegible]	03 08		1.80	[illegible]	7	[illegible]	[illegible]
43	[illegible]	[illegible]	03 15		[illegible]	[illegible]	8	[illegible]	[illegible]
44	[illegible]	[illegible]	10 43		[illegible]	[illegible]		[illegible]	[illegible]
45	[illegible]	[illegible]	11 [illegible]		[illegible]	[illegible]		[illegible]	[illegible]
46	46.0	[illegible]	12 26		[illegible]	[illegible]	11	[illegible]	[illegible]
47	[illegible]	[illegible]	12 [illegible]		[illegible]	[illegible]	12	[illegible]	[illegible]
48	[illegible]	18.4	12 42		[illegible]	[illegible]	13	[illegible]	[illegible]
49	[illegible]	18.[illegible]	13 17		[illegible]	[illegible]	14	[illegible]	[illegible]
50	[illegible]	[illegible]	13 [illegible]		[illegible]	[illegible]	15	[illegible]	[illegible]
51	[illegible]	[illegible]	13 54		[illegible]	[illegible]	16	[illegible]	[illegible]
52	[illegible]	[illegible]	14 12		[illegible]	[illegible]	17	[illegible]	[illegible]
53	[illegible]	[illegible]	14 [illegible]		[illegible]	[illegible]	18	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
55	[illegible]	[illegible]	15 [illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	15 54		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
57	[illegible]	[illegible]	16 [illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
58	101.[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	23	[illegible]	[illegible]
59	104.[illegible]	[illegible]	17 21		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
60	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
61	110.[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
62	111.1	[illegible]	18 56		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
63	112.[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
64	113.1	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
65	114.[illegible]	[illegible]	19 21		[illegible]	[illegible]	30	1.[illegible]	[illegible]
66	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	31	1.[illegible]	[illegible]
67	121.5	[illegible]	[illegible]		[illegible]	[illegible]	32	1.[illegible]	[illegible]
68	[illegible]	[illegible]	19 41		[illegible]	[illegible]	33	[illegible]	[illegible]
69	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	34	[illegible]	[illegible]
70	123.[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	35	[illegible]	[illegible]
71	123.[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	36	[illegible]	[illegible]
72	[illegible]	[illegible]	20 16		[illegible]	[illegible]	37	1.[illegible]	[illegible]
73	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	38	[illegible]	[illegible]
74	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	39	[illegible]	[illegible]
75	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	40	[illegible]	[illegible]
76	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	41	1.367	[illegible]
77	[illegible]	21.[illegible]	21 12		[illegible]	[illegible]	42	[illegible]	[illegible]
78	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	43	1.[illegible]	[illegible]
79	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	44	[illegible]	[illegible]
80	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	45	[illegible]	[illegible]
81	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	46	[illegible]	[illegible]
82	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	47	[illegible]	[illegible]
83	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	48	[illegible]	[illegible]
84	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	49	[illegible]	[illegible]
85	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	50	[illegible]	[illegible]
86	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	51	[illegible]	[illegible]
87	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	52	[illegible]	[illegible]

TEST No 3 RATE OF STRAIN DETERMINATIONS.

(a) CALIBRATED RATE OF STRAIN (σ_3 = 20 lb/sq in)



(b) ACTUAL RATE OF STRAIN DURING TEST (σ_3 = 20 lb/sq in)

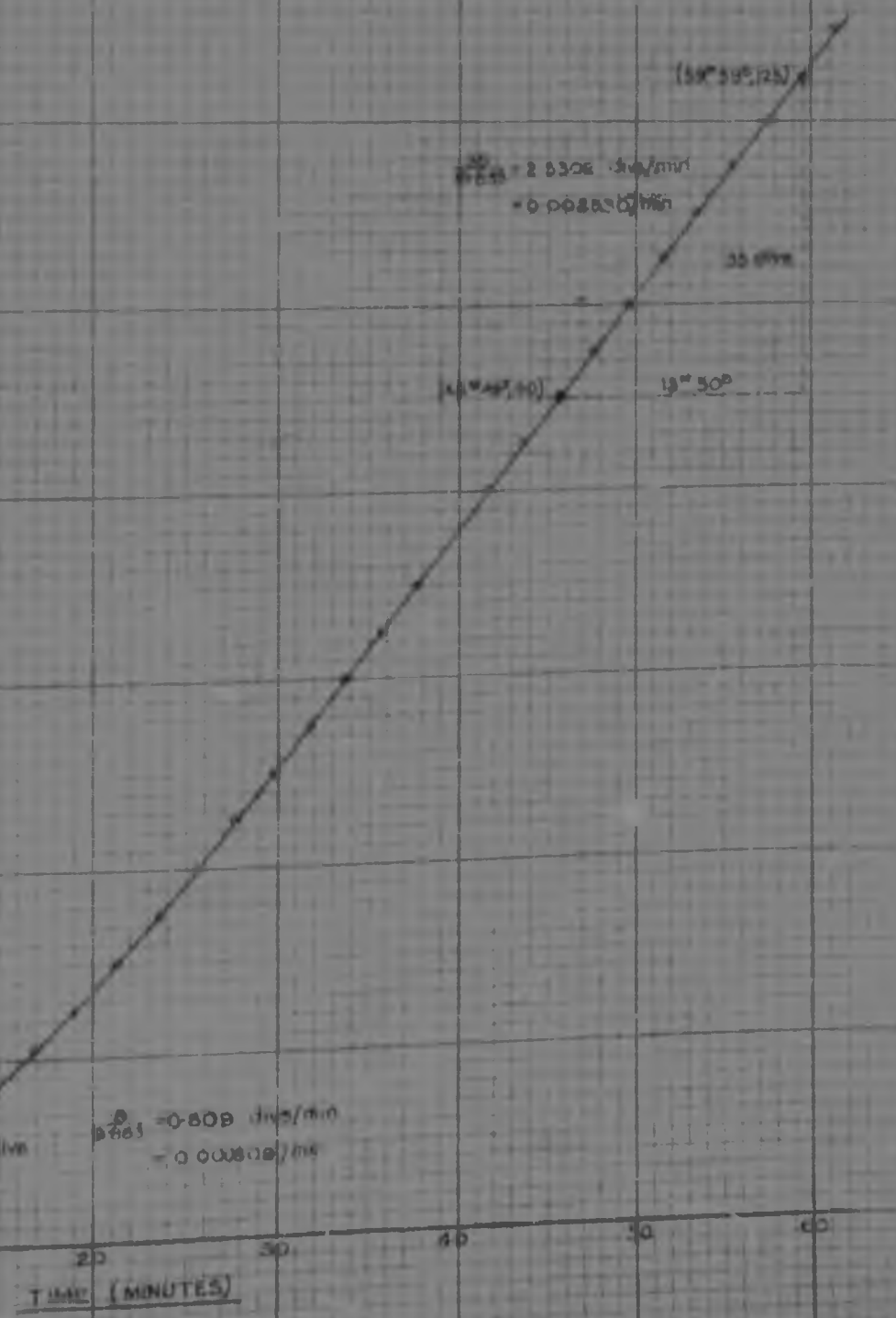
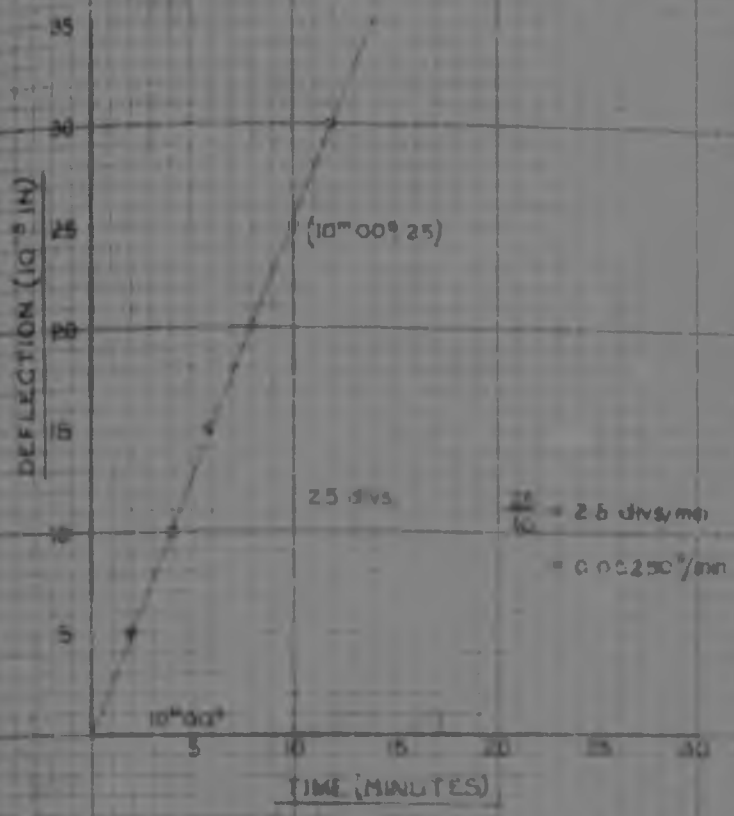


FIG. 33.

TEST No 8 RATE OF STRAIN DETERMINATIONS

(a) CALIBRATED RATE OF STRAIN (σ_3 = 10 lb/Sq in)



(b) ACTUAL RATE OF STRAIN DURING TEST (σ_3 = 10 lb/Sq in)

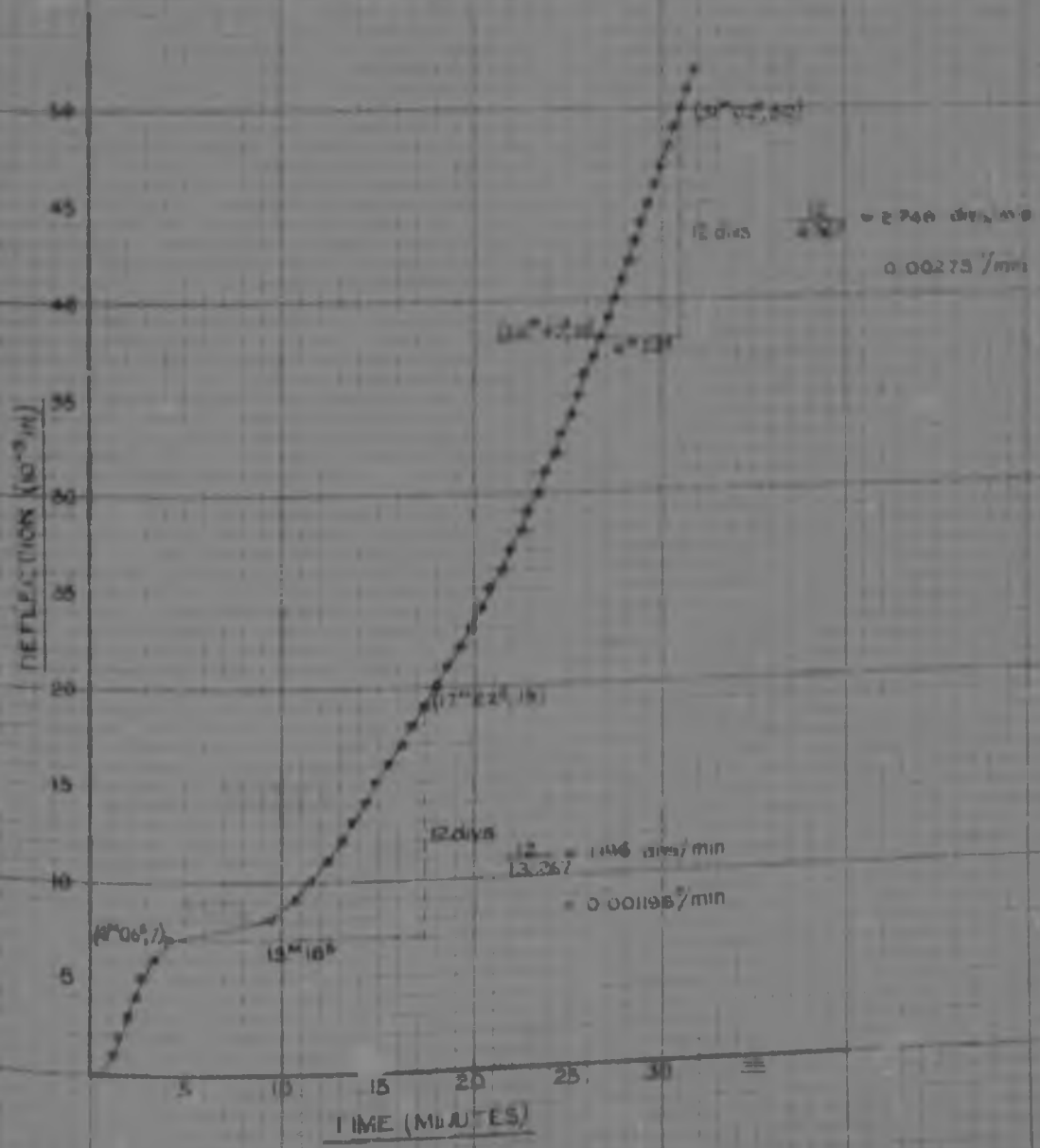


FIG 4.5

TEST No 3

(a) STRESS STRAIN CURVE

σ_3 = 2.5 lb/sq in

e = 0.764

w = 1.82%

Sr = 62.4%

$(\sigma_1 - \sigma_3)$ lb/sq in

Point of linear change

$E = \frac{20.27}{0.267 \times 10^{-2}} = 7597$ lb/sq in

STRAIN ε (percent)

(b) YOUNG'S MODULUS STRAIN CURVE

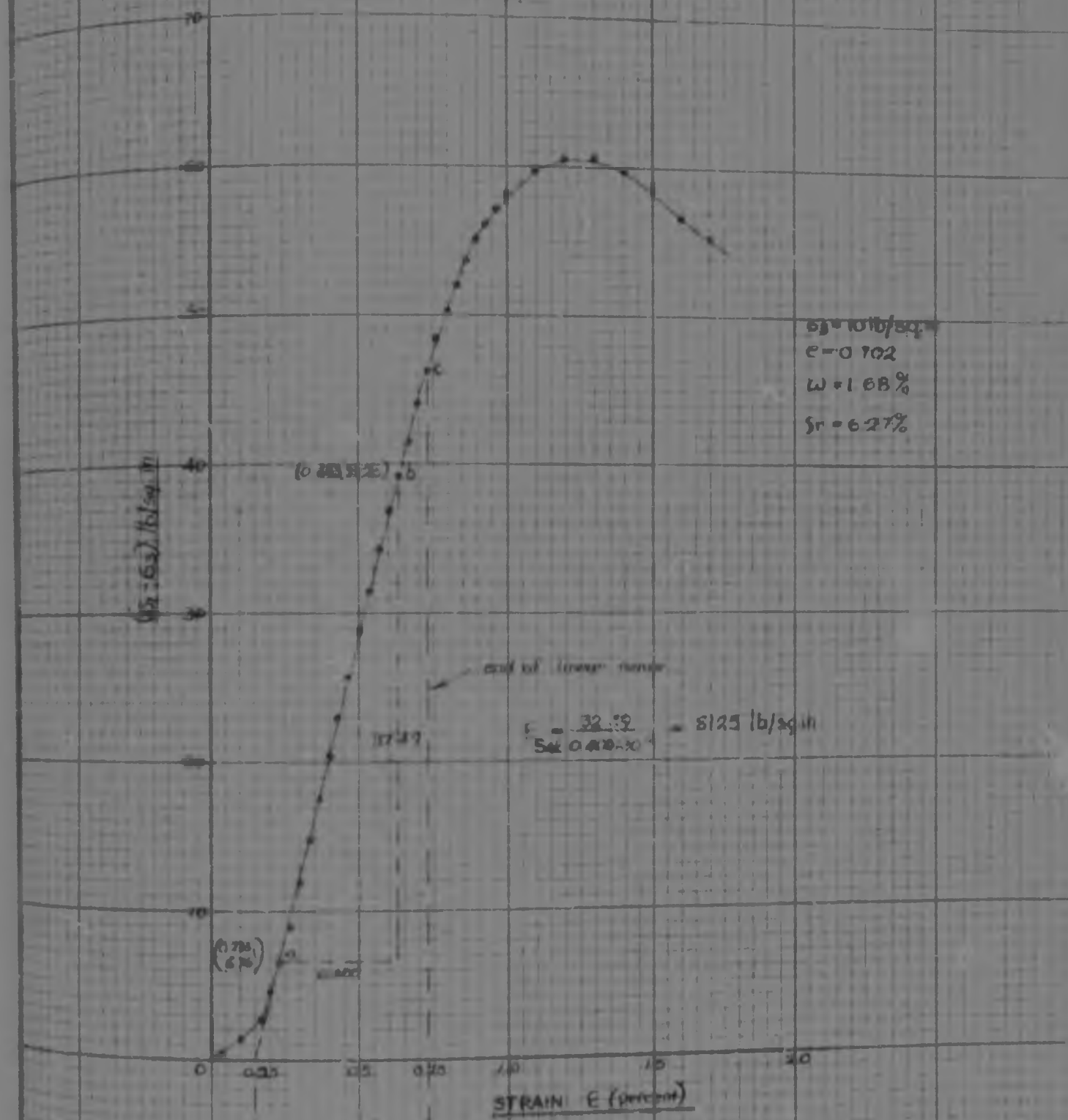
YOUNG'S MODULUS (E) lb/sq in

STRAIN ε (percent)

FIG. 34.

TEST No 6.

(a) STRESS-STRAIN CURVE



(b) YOUNGS' MODULUS STRAIN CURVE

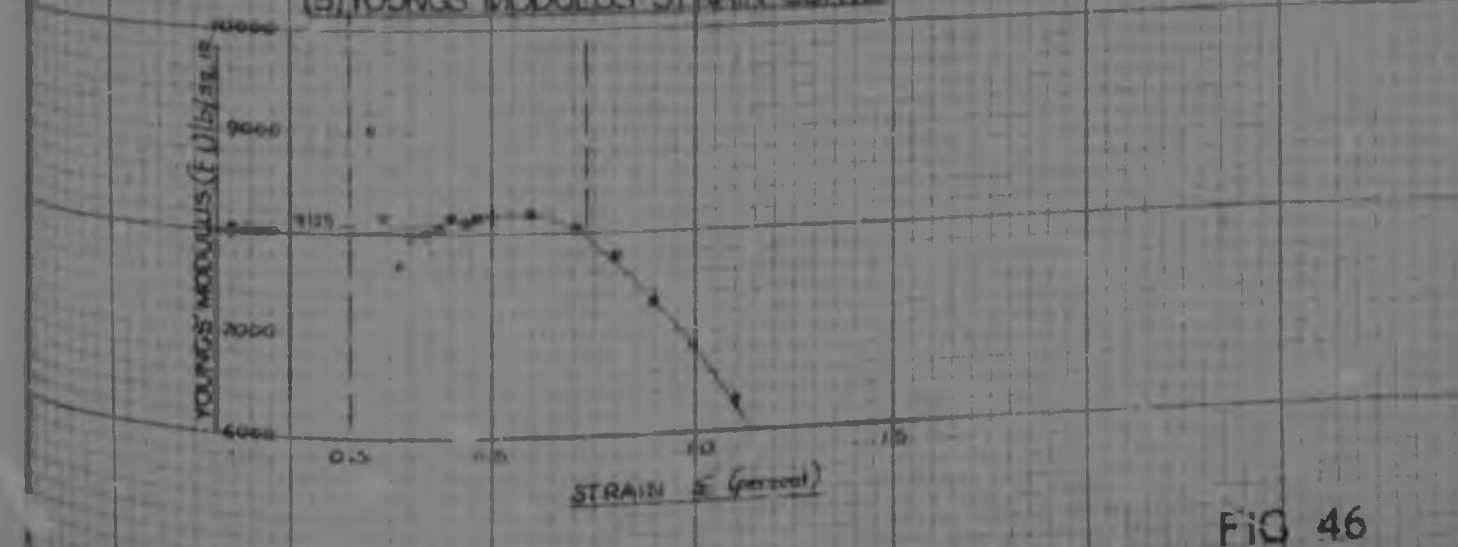


Fig 46

TEST No 3

CALCULATION OF INSTANTANEOUS VALUES OF YOUNGS MODULUS (E)

CORRECTED STRESS = a lb/sq in	STRAIN percent	STRAIN DUE TO BEDDING-IN EFFECTS percent	CORRECTED STRAIN = b percent	YOUNGS MODULUS (E) = a/b lb/sq in
7.34	0.200	0.100	0.100	7340
10.29	0.233	"	0.133	7737
12.68	0.267	"	0.167	7593
11.66	0.300	"	0.200	7830
19.20	0.333	"	0.233	8026
21.00	0.367	"	0.267	7855
23.50	0.400	"	0.400	7960
25.05	0.433	"	0.333	7703
27.84	0.467	"	0.367	7599
29.91	0.500	"	0.400	7478
33.82	0.567	"	0.467	7242
35.12	0.600	"	0.500	7098
37.53	0.633	"	0.533	7004
38.06	0.667	"	0.567	6889
40.80	0.700	"	0.600	6770
42.34	0.733	"	0.633	6689
43.96	0.767	"	0.667	6573
45.28	0.800	"	0.700	6468
46.50	0.833	"	0.733	6344
48.00	0.867	"	0.767	6258
48.10	0.900	"	0.800	6139
49.92	0.933	"	0.833	5993
50.91	0.967	"	0.867	5872
51.90	1.000	"	0.900	5767
54.50	1.100	"	1.000	5450
54.96	1.133	"	1.033	5320
56.67	1.167	"	1.067	5217
58.10	1.200	"	1.100	5100
56.64	1.233	"	1.133	4999
57.18	1.267	"	1.167	4900
57.56	1.300	"	1.200	4798
57.97	1.333	"	1.233	4702
59.43	1.367	"	1.400	4245
60.56	1.400	"	1.567	3871

TEST No 6

CALCULATION OF INSTANTANEOUS VALUES OF YOUNGS' MODULUS (E)

CORRECTED STRESS = a kg/sq in	STRAIN percent	STRAIN DUE TO BEDDING-IN EFFECTS percent	CORRECTED STRAIN = b percent	YOUNGS MODULUS (E) = a/b kg/sq in
4.51	0.200	0.150	0.050	9020
6.79	0.233	"	0.083	8144
8.96	0.267	"	0.117	7659
11.93	0.300	"	0.150	7953
14.63	0.333	"	0.183	7994
17.45	0.367	"	0.217	8041
20.30	0.400	"	0.250	8152
23.04	0.433	"	0.283	8141
25.21	0.467	"	0.317	8142
28.65	0.500	"	0.350	8186
31.48	0.533	"	0.383	8214
34.25	0.567	"	0.417	8213
36.90	0.600	"	0.450	8200
39.28	0.633	"	0.483	8126
41.53	0.667	"	0.517	8033
44.15	0.700	"	0.550	8027
46.16	0.733	"	0.583	7918
48.55	0.767	"	0.617	7869
50.33	0.800	"	0.650	7743
51.98	0.833	"	0.683	7612
53.65	0.867	"	0.717	7482
54.92	0.900	"	0.750	7323
56.08	0.933	"	0.783	7163
57.12	0.967	"	0.817	6991
58.11	1.000	"	0.850	6836
59.07	1.033	"	0.883	6661
59.42	1.067	"	0.917	6481
59.98	1.100	"	0.950	6299

TEST Nº 2 MAIN TEST OBSERVATIONS.

DEFLEC-TION GAUGE	LOAD GAUGE	SLUG SCALE cm	TIME min. sec.	TEMPERATURE °C	CORRECTED SLUG SCALE cm	CORRECTED LOAD lb	CORRECTED DEFLECT. 10^{-3} in.	STRAIN ε percent	CORRECTED STRESS (σ1 − σ3)=σ lb/sq. in.
00.50	12.7	25.5		25.25	0	0	0	0	0
52.5	17.0	25.65			- 0.15	4.3	5.0	0.083	1.83
55	17.2	25.76			- 0.25	5.2	5	0.167	4.08
57.5	23.8	25.7			- 0.20	11.1	7.5	0.250	8.86
59 pronounced time-lag sets in.									
60	32.0	25.7			- 1.8	24.3	10	0.333	14.01
62.5	53.0	25.7			4.2	40.3	20.5	0.417	23.67
65	64	25.8			6.7	53.3	22	0.500	29.05
67.5	75	16.45			9.05	65.3	17.5	0.583	34.20
70	84.5	14.55			10.95	71.5	20	0.667	41.24
72.5	94.5	13.40			13.10	81.5	21.5	0.750	46.97
75	103	10.55			14.95	90.3	25	0.833	50.64
77.5	112	8.75			16.71	98.5	27.5	0.917	54.13
80	119	7.20			18.24	105.8	30	1.000	56.45
82.5	126.5	5.55			19.91	114.2	32.5	1.083	63.22
85	133.6	4.05			21.45	120.5	35	1.167	67.45
87.5	140	2.60			22.90	127.3	37.5	1.250	72.1
90	145					132.7	40	1.333	75.2
92.5	150.3					137.4	42.5	1.417	78.7
95	155.9	25.0			27.12	143.1	45	1.500	79.4
97.5	160	25.1			27.65	147.3	47.5	1.583	81.2
100	164.2	25.7			28.35	151.2	50	1.667	84.2
102.5	168.1	25.85			29.1	155.3	52.5	1.750	86.3
105	171.7	26.10			29.8	159.0	55	1.833	88.1
107.5	175	25.45		25.5	30.6	162.3	57.5	1.917	89.8
110	178	25.60			31.05	165.2	60	2.000	91.6
112.5	180.4	24.55			31.50	167.7	62.5	2.083	92.6
115	183	24.80			31.95	170.3	65	2.167	94.1
117.5	185	23.975			32.15	172.3	67.5	2.250	95.2
120	187.8	23.85			32.4	174.6	70	2.333	96.4
122.5	189.6	23.40			32.65	176.5	72.5	2.417	97.1
125	191.0	23.325			32.85	178.1	75	2.500	98.2
127.5	192.6	23.275			33.025	179.5	77.5	2.583	98.8
130	193.5	23.275			33.125	180.6	80	2.667	99.4
132.5	194.6	23.275			33.225	181.6	82.5	2.750	99.9
135	195.2	23.30			33.30	182.3	85	2.833	100.4
137.5	196.0	23.45			33.45	183.2	87.5	2.917	100.8
140	197.3	23.60			33.60	184.6	90	3.000	101.5
142.5	198.0	23.65			33.65	185.3	92.5	3.083	101.9
145	198.7	24.05			34.05	185.9	95	3.167	102.6
147.5	199.0	24.30			34.30	186.3	97.5	3.250	101.9
150	199.1	24.35			34.35	186.4	100	3.333	101.9
152.5	199.5	25.10			35.10	186.9	102.5	3.417	102.0
155	199.7	25.70			35.70	187.0	105	3.500	102.6
157.5	199.5	26.20			36.20	186.9	107.5	3.583	101.9
160	199.5	26.60			36.60	186.9	110	3.667	101.9
162.5	199.5	27.40			37.40	186.9	112.5	3.750	101.7
165	199.5	28.00			38.00	186.9	115	3.833	101.5
167.5	199.7	28.65			38.65	187.0	117.5	3.917	101.5
170	199.3	28.30			38.30	186.6	120	4.000	101.5
172.5	199.7	30.05			39.95	187.0	122	4.167	100.7
175	199.7	31.40			41.40	187.0	125	4.250	101.1
177.5	199.5					186.9	127.5	4.333	
180	199.5					186.9	130	4.417	
182.5	199.5					186.9	132.5	4.500	
185	199.5					186.9	135	4.583	
187.5	199.5					186.9	137.5	4.667	
190	199.5					186.9	140.5	4.750	
192.5	199.5					186.9	142.5	4.833	
195	199.5					186.9	145	4.917	
197.5	199.5					186.9	147.5	5.000	
200	199.5			25.25		186.9	150	5.083	

TEST N° : MAIN TEST OBSERVATIONS

DEFLEC-TION GAUGE	LOAD GAUGE	SLUG SCALE	TIME	TEMPERA-TURE	CORRECTED SLUG SCALE	CORRECTED LOAD	CORRECT DEFLECT	STRAIN ε	CORRECTED STRESS
00 30	18			24.0					
32.5									
35									
37.5									
40									
41	drone in [illegible] time-lag sets in.								
42.5									
45									
47.5						31			
50	64.					41			
52.5		14.				48			
55		12.80							
57.5				24.0					
60		11.35							
62.5									
65	101.	11.10							
67.5									
70		11.00							
72.5									
75									
77.5									
80									
82.5									
85									
87.5									
90									
92.5									
95									
97.5	108.1								
100									
102.5									
105									
107.5									
110									
112.5									
115									
117.5									
120									
122.5									
125									
127.5									
130				24.0					

TEST N° 2.

(a) STRESS STRAIN CURVE

σ_3 = 30 lb/sq in
e = 0.695
ω = 2.29%
Sr = 8.64%

$(\sigma_1-\sigma_3)$ lb/sq in

(0.667 x 0.29) lb

end of linear range

3225

$E = \frac{32.29}{Sat\ 0.41 \times 10^{-2}} = 7750$ lb/sq in

0.417

STRAIN ε (Percent)

(b) YOUNG'S MODULUS: STRAIN CURVE

YOUNG'S MODULUS E (lb/sq in)

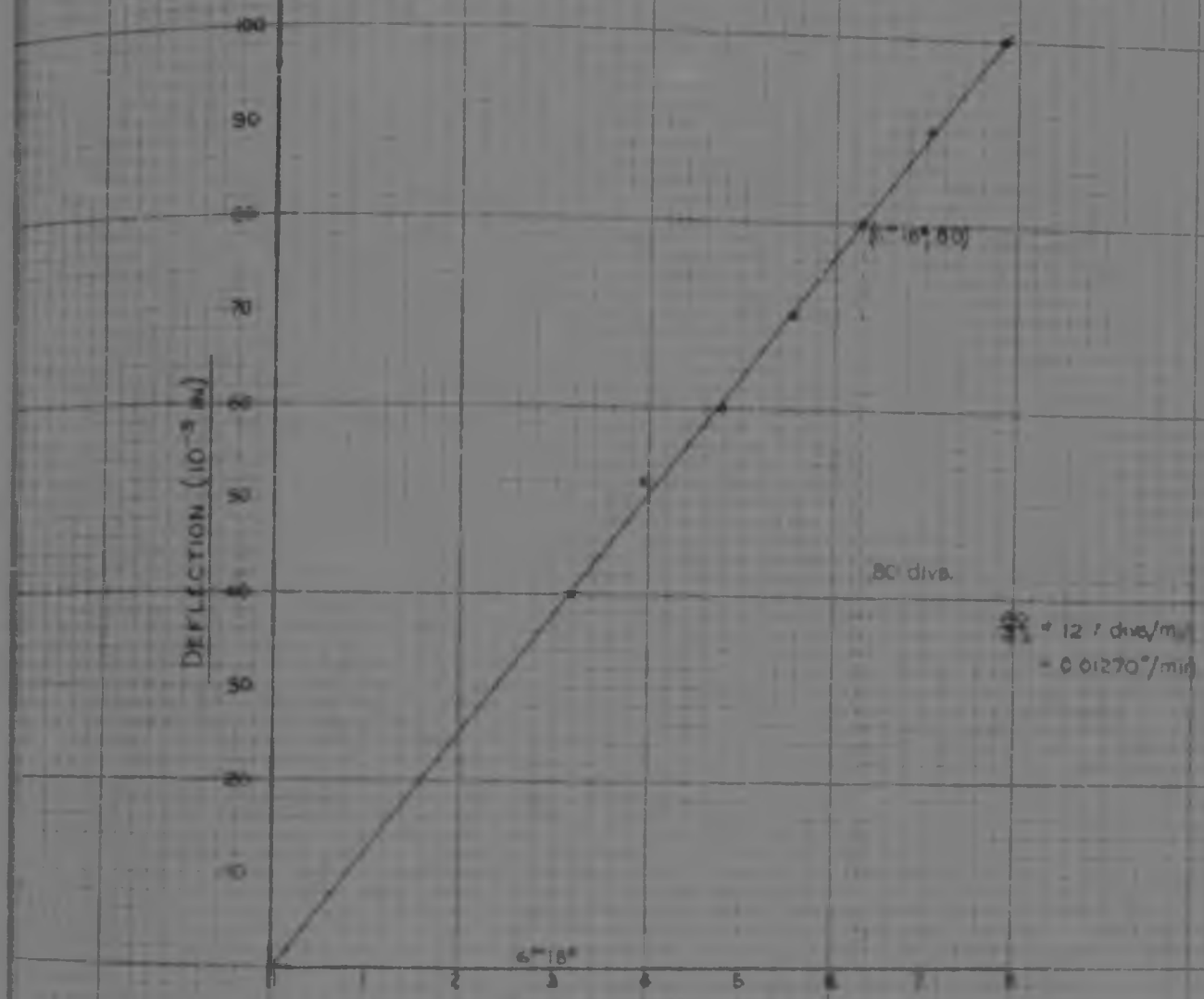
7750

STRAIN ε (Percent)

FIG. 30.

TEST No 5 RATE OF STRAIN DETERMINATION

FIG 41 CALIBRATED RATE OF STAIN (σ_a = 10 lb/sq in)



NOTE: No CURVE FOR DETERMINATION OF ACTUAL RATE OF STRAIN DURING TEST, DUE TO CONTINUOUS TIME MEASUREMENTS NOT HAVING BEEN TAKEN

F 3.41.

TEST No 9

CALCULATION OF INSTANTANEOUS VALUES OF YOUNGS' MODULUS (E)

CORRECTED STRESS = a lb/sq in	STRAIN percent	STRAIN DUE TO BEDDING - IN EFFECTS percent	CORRECTED STRAIN = b percent	YOUNGS MODULUS (E) = a/b lb/sq in
5.00	0.216	0.150	0.066	7575
14.91	0.333	"	0.183	8092
22.67	0.417	"	0.267	8492
28.84	0.500	"	0.350	8240
34.99	0.583	"	0.433	8081
40.29	0.667	"	0.517	7793
45.97	0.750	"	0.600	7645
50.59	0.833	"	0.683	7407
55.59	0.917	"	0.767	7248
59.45	1.000	"	0.850	6994
63.62	1.083	"	0.933	6819
67.45	1.167	"	1.017	6632
71.02	1.250	"	1.100	6456
73.75	1.333	"	1.183	6234
76.64	1.417	"	1.267	6049
79.63	1.500	"	1.350	5898

TEST Nº 5

A STRESS STRAIN CURVE

σ_3 = 10 lb/sq in
e = 0.714
ω = 7.75%
Sr = 6.42%

$(\sigma_1 - \sigma_3)$ lb/sq in

STRAIN ε (in percent)

B YOUNGS' MODULUS STRAIN CURVE

YOUNGS' MODULUS (E) lb/sq in

STRAIN ε (in percent)

FIG 42

TEST Nº 2

MAIN SLUG MOVEMENT : DEFLECTION CURVE

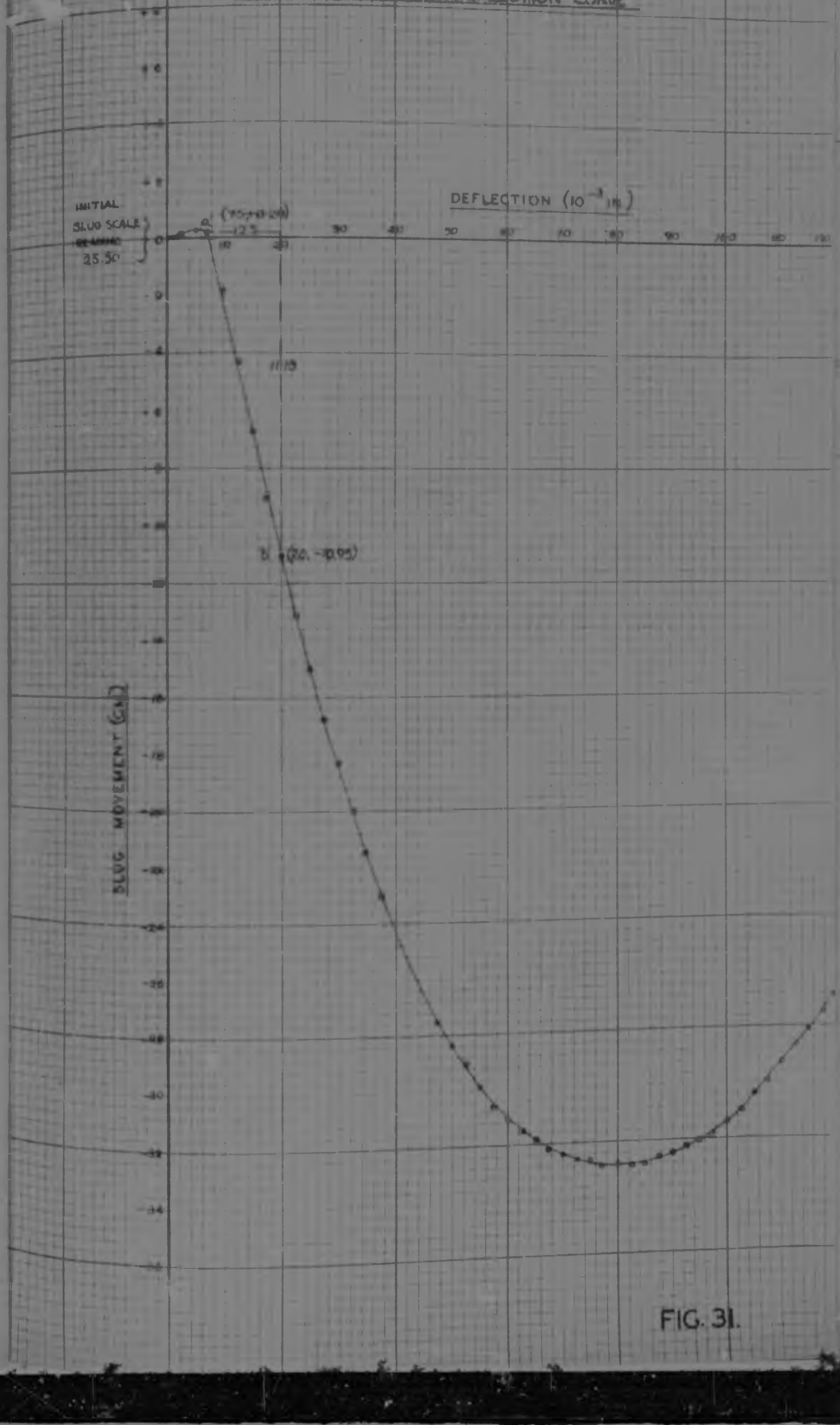


FIG. 31.

TEST No 8

CALCULATION OF INSTANTANEOUS VALUES OF YOUNGS' MODULUS (E)

CORRECTED STRESS σ_a lb/sq in	STRAIN percent	STRAIN DUE TO BEDDING IN EFFECTS percent	CORRECTED STRAIN ϵ_b percent	YOUNGS' MODULUS (E) σ_a / ϵ_b lb/sq in
5.000	0.333	0.26	0.073	6849
10.97	0.417	"	0.157	6987
17.45	0.500	"	0.240	7269
23.03	0.583	"	0.323	7130
27.50	0.667	"	0.407	6757
32.52	0.750	"	0.490	6637
36.64	0.833	"	0.573	6394
40.58	0.917	"	0.657	6177
43.63	1.000	"	0.740	5895
45.83	1.083	"	0.823	5569
48.02	1.167	"	0.907	5294
49.65	1.250	"	0.990	5015
51.01	1.333	"	1.073	4754
52.08	1.417	"	1.157	4501
52.64	1.500	"	1.240	4245

TEST No 5.

MAIN SLUG MOVEMENT DEFLECTION CURVE

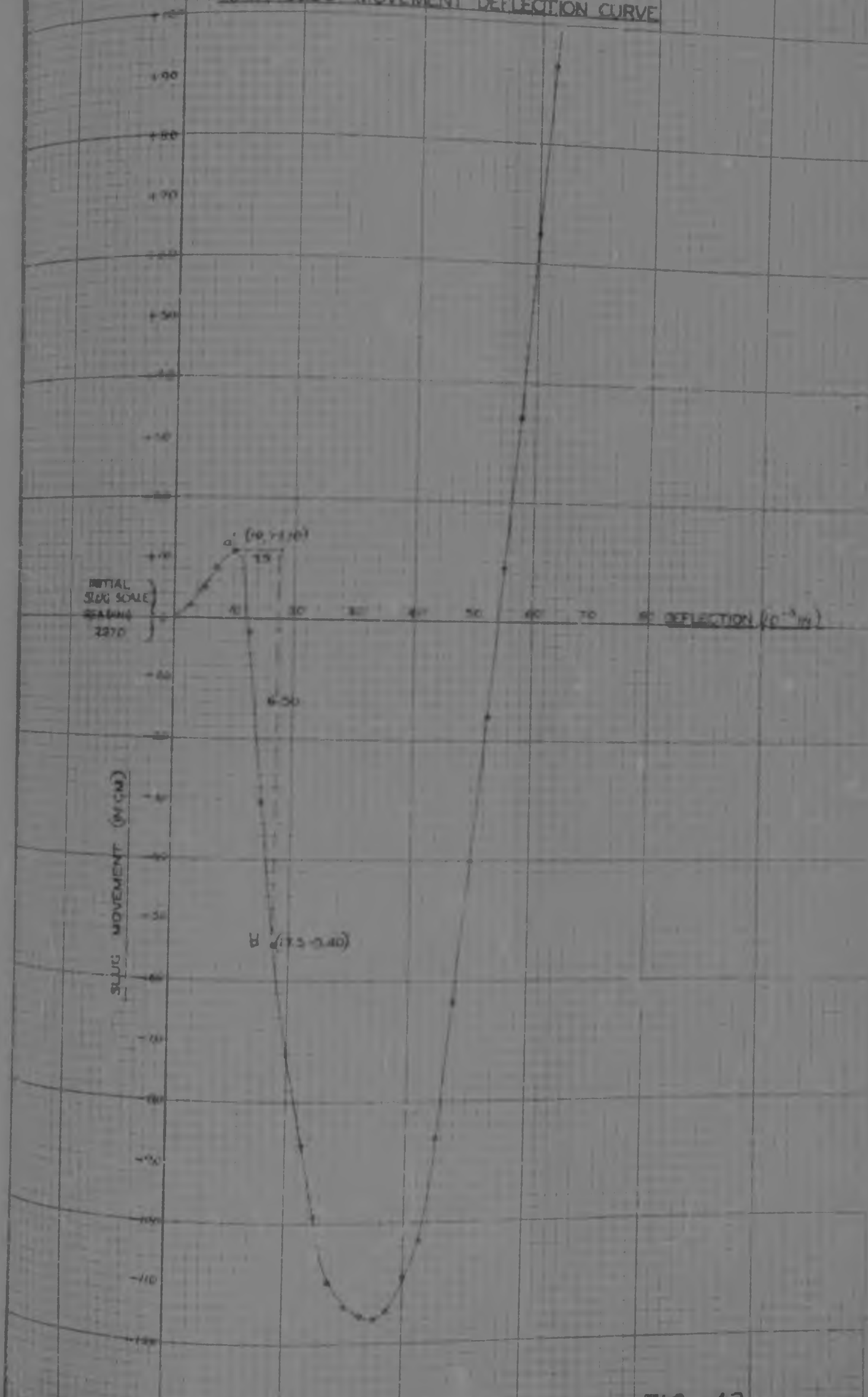


FIG 43

TEST No 3.

DATE PERFORMED 20/1/57

SAMPLE [illegible] DATED FOR 24 00 PRIOR TO TEST

TIME START OF TEST 12/32 PM TEMPERATURE 23.2°C

TIME END OF TEST 2/05 PM TEMPERATURE 24.0°C

NOMINAL I.D. DIAMETER PRESSURE 20 Lb/ sq in

HEAD OF WATER [illegible] 38½ inches [illegible]

ACTUAL [illegible] DIAMETER PRESSURE 18.569 Lb/ sq in

NOMINAL [illegible] STRAIN 0.00256 inches/inch

TEST Nº 8 LEAKAGE MEASUREMENTS.

(A) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. BEFORE TEST. TEMP [illegible] TIME [illegible]		(B) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST (1). TEMP [illegible] TIME [illegible]		(C) MEASUREMENT OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY (AFTER TEST). TEMPERATURE 24.0°C TIME [illegible]			(D) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST (2). TEMP [illegible] TIME [illegible]	
TIME min sec	VOLUME SCALE cm	TIME min sec	VOLUME SCALE cm	DEFLECT GAUGE	VOLUME SCALE cm	TIME min sec	TIME min sec	VOLUME SCALE cm
0 00	[illegible]	0 00	11.[illegible]	00 11	10.70	00 00	0 00	[illegible]
15	[illegible]	15	[illegible]	62	10.70		15	[illegible]
30	[illegible]	30	11.45	63	10.70		30	[illegible]
45	[illegible]	45	11.375	64	10.70		45	[illegible]
1 00	[illegible]	1 00	11.325	64	10.70	01 34	1 00	[illegible]
15	[illegible]	15	11.[illegible]	64	10.725		15	[illegible]
30	[illegible]	30	11.50	57	10.75		30	11.15
45	[illegible]	45	11.45	56	10.775		45	[illegible]
2 00	[illegible]	2 00	11.425	59	10.775		2 00	[illegible]
15	[illegible]	15	11.40	60	10.80	03 38	15	[illegible]
30	[illegible]	30	[illegible]	61	10.800		30	[illegible]
45	[illegible]	45	[illegible]	62	10.81		45	[illegible]
3 00	[illegible]	3 00	[illegible]	63	10.875		3 00	[illegible]
15	[illegible]	15	[illegible]	64	10.90		15	[illegible]
30	[illegible]	30	[illegible]	65	10.925	05 35	30	[illegible]
45	[illegible]	45	[illegible]	66	10.975		45	[illegible]
4 00	[illegible]	4 00	[illegible]	67	11.00		4 00	[illegible]
15	[illegible]	15	[illegible]	68	11.00		15	[illegible]
30	[illegible]	30	[illegible]	69	11.025		30	[illegible]
45	[illegible]	45	[illegible]	70	11.05	07 32	45	[illegible]
5 00	[illegible]	5 00	[illegible]	71	11.075		5 00	[illegible]
				72	11.10			
				73	11.10			
				74	11.15			
				75	11.175	09 10		
				76	11.175			
				77	11.20			
				78	11.200			
				79	11.225			
				80	11.25	11 24		
				81	11.250			
				82	11.25			
				83				
				84	11.150			
				85	11.175	13 50		
				86	11.20			
				87	11.225			
				88	11.200			
				89	11.20			
				90	11.225	15 14		

TEST No 6

DATE PERFORMED 31 / 1 / 57

SAMPLE CONSOLIDATED FOR 24 HRS 00 MINS PRIOR TO TEST

TIME START OF TEST 11 28 AM TEMPERATURE 20 75° C

TIME END OF TEST 12 00 NOON TEMPERATURE 20 75° C

NOMINAL σ_3 CHAMBER PRESSURE 10 LB / SQ IN

HEAD (M MERCURY) 18

ACTUAL σ_3 CHAMBER PRESSURE 8 825 LB / SQ IN

NOMINAL RATE OF STRAIN 0 00250 INCH / MINUTE

TEST No3.

MAIN SLUG MOVEMENT, DEFLECTION CURVE

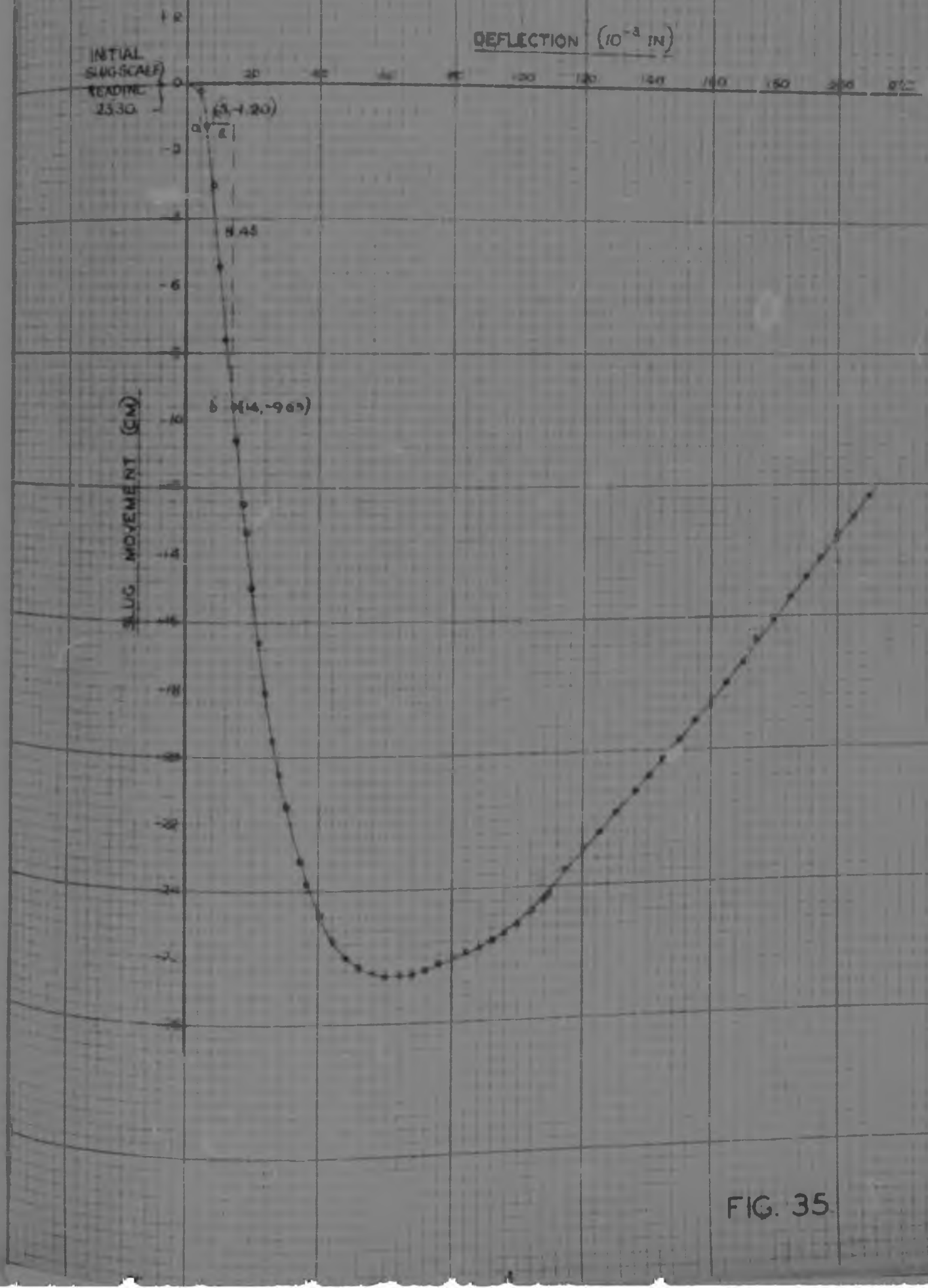


FIG. 35

TEST No 4.

DATE PERFORMED 15 / 2 / 57

SAMPLE CONSOLIDATED FOR 23^{hrs} 40^{mins} PRIOR TO TEST

TIME START OF TEST 8 46 A M TEMPERATURE 24 0°C

TIME END OF TEST 8 56 A M TEMPERATURE 24 25°C

NOMINAL 6_3 CHAMBER PRESSURE 20 LB/ SQ IN

HEAD OF MERCURY 38¼ INCHES, HENCE

ACTUAL 6_3 CHAMBER PRESSURE 18 750 LB / SQ IN

NOMINAL RATE OF STRAIN 0 0125 INCHES/ MINUTE

TEST No 4. LEAKAGE MEASUREMENTS

DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY

Slug movement from right to left in each case

(a) BEFORE TEST

0.35 cm $\frac{0.35}{4}$ = 0.875 mm/min

(b) AFTER TEST (1)

0.7 cm $\frac{0.7}{4}$ = 0.175 mm/min

0.65 cm $\frac{0.65}{0.75}$ = 1.908 mm/min

$\frac{1.75}{2}$ = 0.875 mm/min

2.000 mm/min

(d) AFTER TEST (2)

0.9 cm $\frac{0.9}{0.35}$ = 2.571 mm/min

SLUG MOVEMENT (CM)

TIME (IN MINUTES)

(c) DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE AND SPINDLE ENTRY

Slug movement from left to right

6.475 cm

$\frac{64.5 \times 1.364}{47.5}$ mm/div

1.363 mm @ 0.001"

47.5 div.

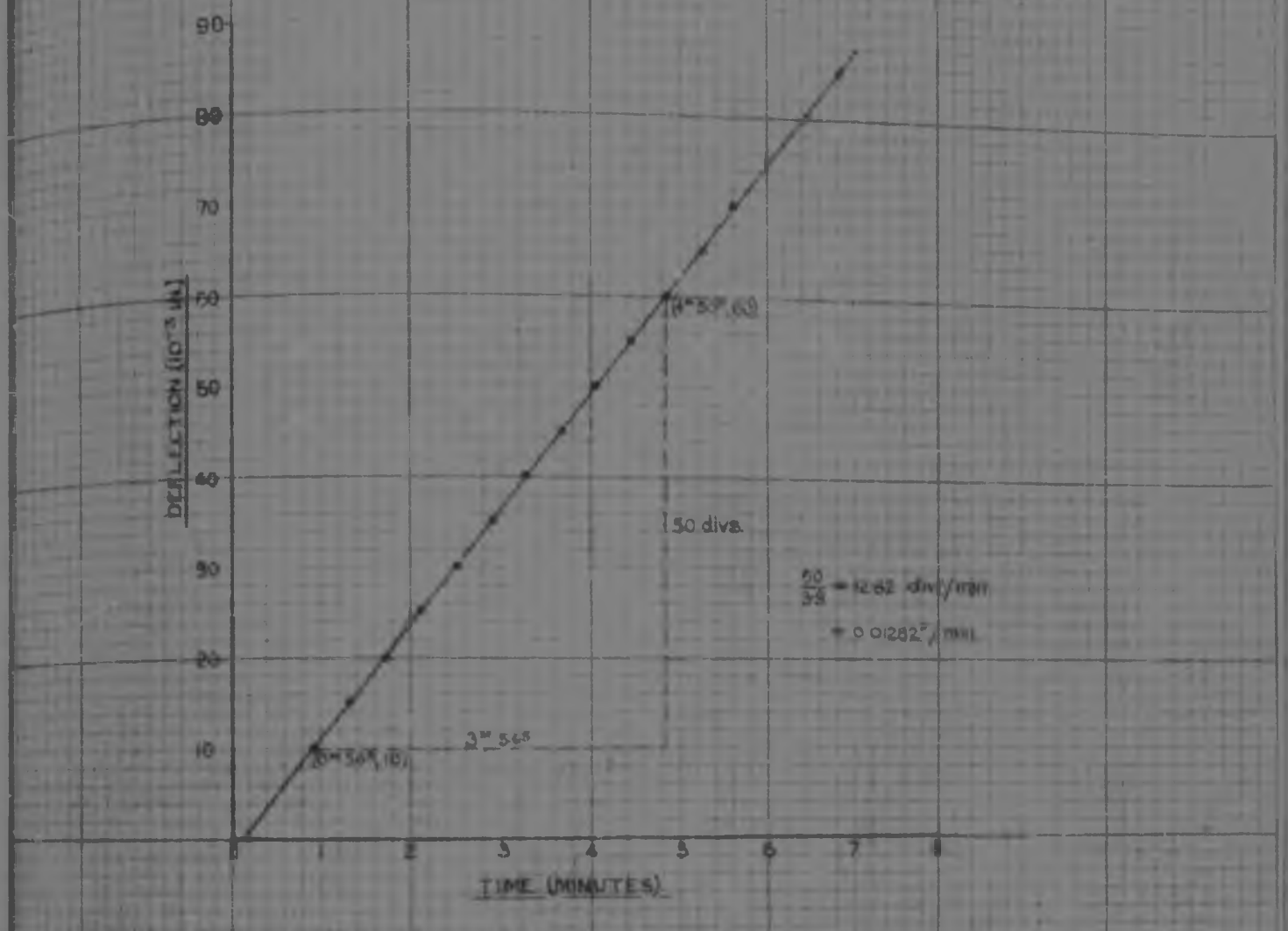
SLUG MOVEMENT (IN CENTIMETRES)

DEFLECTION (10⁻³ IN)

FIG. 36.

TEST No4: RATE OF STRAIN DETERMINATIONS

(a) CALIBRATED RATE OF STRAIN (σ_3 = 20 lb/sq in)



(b) ACTUAL RATE OF STRAIN DURING TEST (σ_3 = 20 lb/sq in)

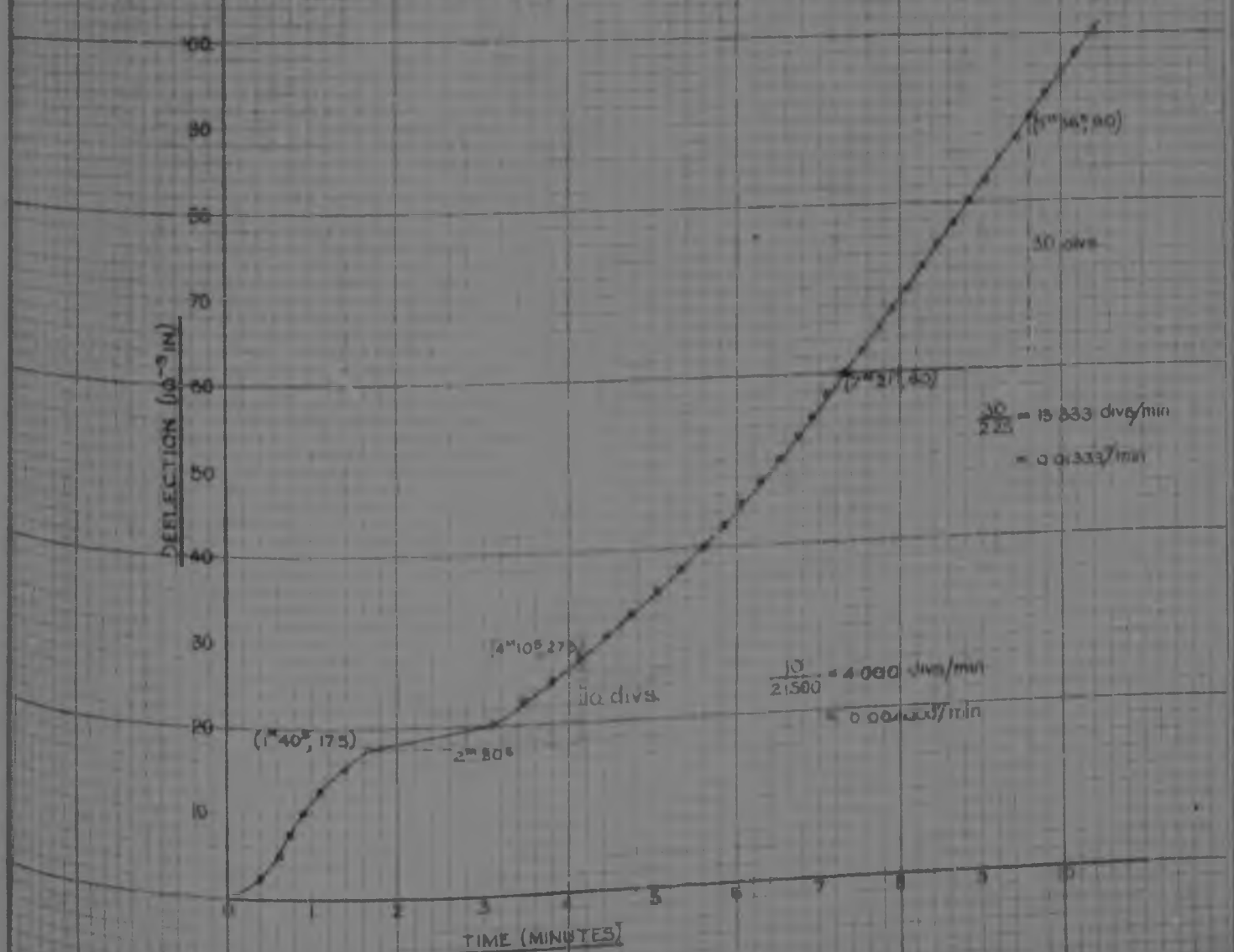


FIG. 37.

TEST N°4

(a) STRESS : STRAIN CURVE

σ_3 = 20 lb/sq in
e = 0·713
ω = 0·66%
Sr = 2·42%

(σ1 - σ3) lb/sq in

(0·97, 39·78)

end of linear range

(0·788, 7·86)

0 0·5 1·0 1·5 2·0 2·5 3·0 3·5

STRAIN ε (percent)

$E_{sec} = \frac{31.89}{0.3331 \times 10^{-2}} = 9567$ lb/sq in

(b) YOUNGS' MODULUS STRAIN CURVE

YOUNGS' MODULUS (E) lb/sq in

9000

7000

6000

9567

0·5 1·0 1·5 2·0 2·5

STRAIN ε (percent)

FIG 38

TEST No 4

CALCULATION OF INSTANTANEOUS VALUES OF YOUNGS MODULUS (E)

CORRECTED STRESS = a lb/sq in	STRAIN percent	STRAIN DUE TO BEDDING - IN EFFECTS percent	CORRECTED STRAIN = b percent	YOUNGS MODULUS E) = a/b lb/sq
7.86	0.083	0.500	0.083	9470
16.66	0.167	"	0.167	9916
24.12	0.250	"	0.25	9648
31.10	0.333	"	0.333	9330
38.75	0.417	"	0.417	9532
48.87	1.000	"	0.500	9174
53.37	1.083	"	0.583	9154
58.85	1.167	"	0.667	8823
64.72	1.250	"	0.750	8629
69.12	1.333	"	0.833	8293
73.96	1.417	"	0.917	7956
75.96	1.500	"	1.000	7596
78.96	1.583	"	1.083	7291
80.83	1.667	"	1.167	6926
82.19	1.750	"	1.250	6579
83.13	1.833	"	1.333	6241
83.67	1.917	"	1.417	5905
83.88	2.000	"	1.500	5592

TEST Nº 4

MAIN SLUG MOVEMENT DEFLECTION CURVE

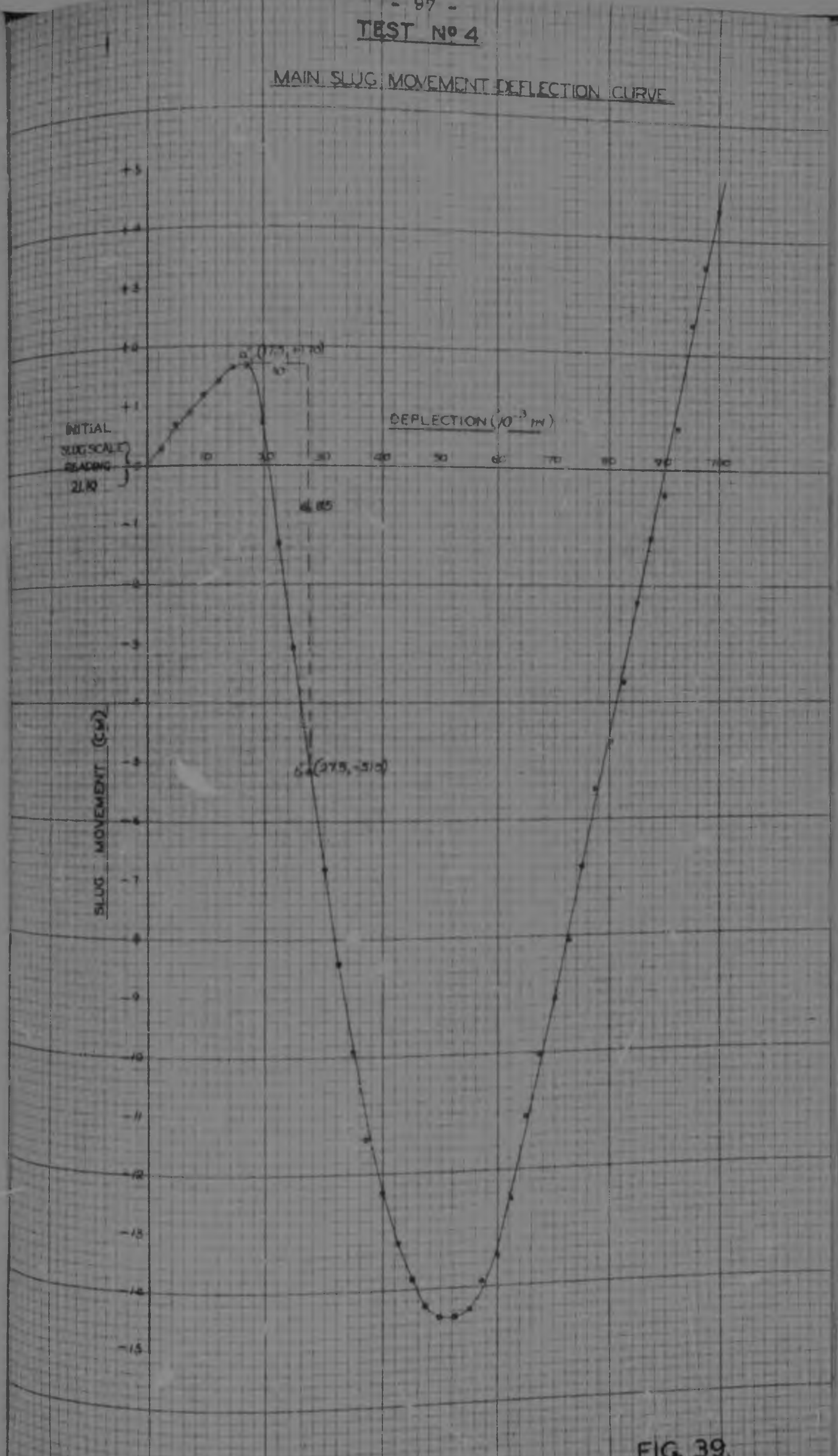


FIG. 39.

TEST No 5.

DATE PERFORMED 30/1/57

SAMPLE CONSOLIDATED FOR 23^{hrs}15^{mins} PRIOR TO TEST

TIME START OF TEST: 4/33 P.M TEMPERATURE 24.0°C

TIME END OF TEST 4/43 P.M TEMPERATURE 24.0°C

NOMINAL 6_3 CHAMBER PRESSURE 10 LB/SQ IN

HEAD OF MERCURY 18 INCHES, HENCE

ACTUAL 6_3 CHAMBER PRESSURE 8·825 LB/SQ IN

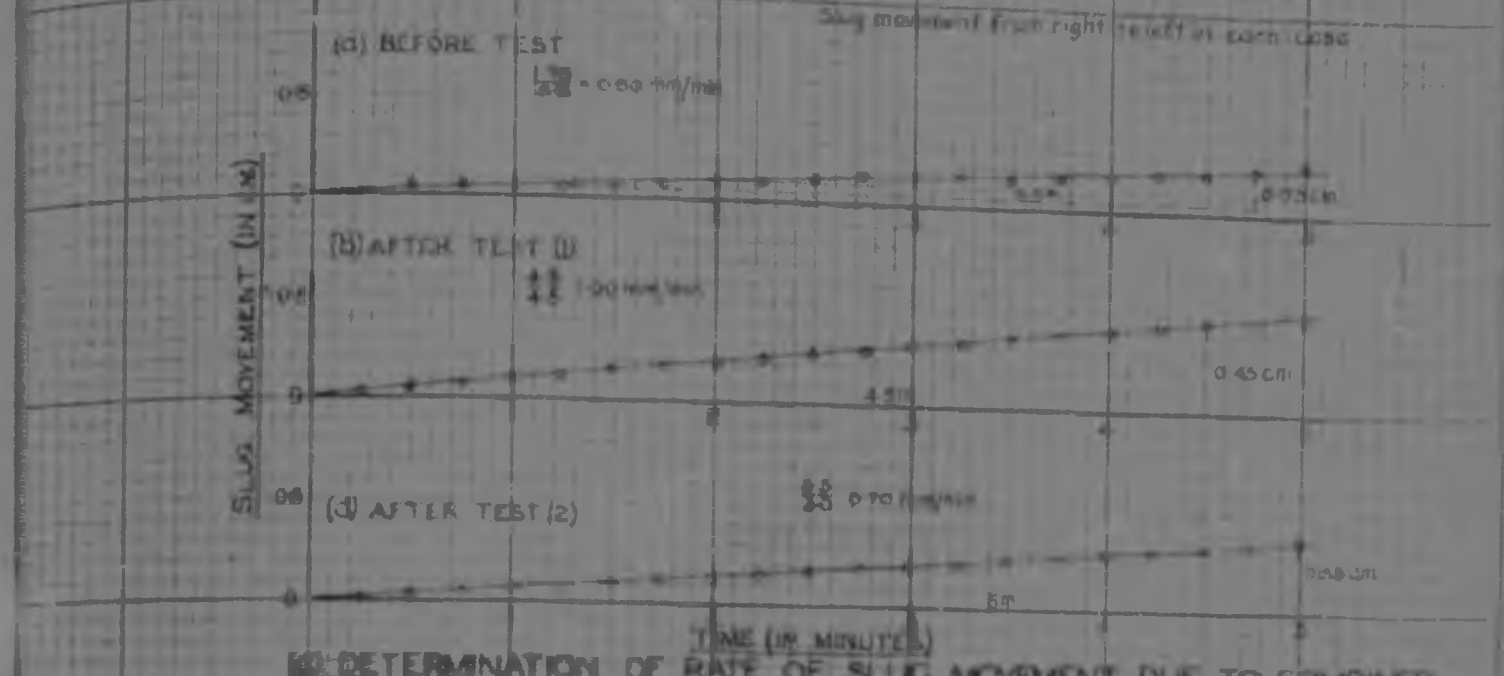
NOMINAL RATE OF STRAIN 0·0125 INCHES/MINUTE

TEST Nº 5 : LEAKAGE MEASUREMENTS.

(A) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. BEFORE TEST TEMP 24.75°C TIME 4/20 p.m.		(B) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST (1). TEMP 24.0°C TIME 4/45 p.m.		(C) MEASUREMENT OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY (AFTER TEST) TEMPERATURE 24.0°C TIME 4/55 p.m.			(D) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY AFTER TEST (2). TEMP 24.25°C TIME 5/10 p.m.	
TIME min sec	VOLUME SCALE cm³	TIME min sec	VOLUME SCALE cm³	DEFLECT GAUGE 10^{-6} m	VOLUME SCALE cm³	TIME min sec	TIME min sec	VOLUME SCALE cm³
0 00	22.75	0 00	18.00	00 10	2.30	0 00	0 00	15.70
15	22.725	15	17.975	12.5	2.65		15	15.675
30	22.70	30	17.95	15	3.00		30	15.675
45	22.70	45	17.925	17.5	3.30		45	15.65
1 00	22.70	1 00	17.90	20	3.625		1 00	15.625
15	22.70	15	17.875	22.5	3.90		15	15.625
30	22.675	30	17.85	25	4.25		30	15.60
45	22.675	45	17.825	27.5	4.60		45	15.575
2 00	22.65	2 00	17.80	30	4.925		2 00	15.575
15	22.65	15	17.775	32.5	5.30		15	15.55
30	22.625	30	17.75	35	5.60		30	15.525
45	22.60	45	17.725	37.5	5.975		45	15.50
3 00	22.60	3 00	17.70	40	6.30		3 00	15.50
15	22.60	15	17.675	42.5	6.70		15	15.475
30	22.60	30	17.65	45	7.025		30	15.45
45	22.575	45	17.625	47.5	7.50		45	15.45
4 00	22.575	4 00	17.60	50	7.90	3 10	4 00	15.425
15	22.55	15	17.575	52.5	8.25		15	15.40
30	22.56	30	17.55	55	8.70		30	15.375
45	22.525	45	17.55	57.5	9.20		45	15.375
5 00	22.50	5 00	17.525	60	9.60	3 58	5 00	15.35
		15	17.50	62.5	9.85			
				65	10.25			
				67.5	10.675			
				70	11.10	4 46		
				72.5	11.425			
				75	11.70			
				77.5	12.275			
				80	12.625	5 32		
				82.5	12.95			
				85	13.30			
				87.5	13.65			
				90	14.05	6 18		
				92.5	14.40			
				95	14.80			
				97.5	15.20			
				100	15.475	7 03		
				102.5	16.05			
				105	16.40			
				107.5	16.775			
				110	17.15	7 51		

TEST No 5 LEAKAGE MEASUREMENTS.

DETERMINATIONS OF RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY



DETERMINATION OF RATE OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY

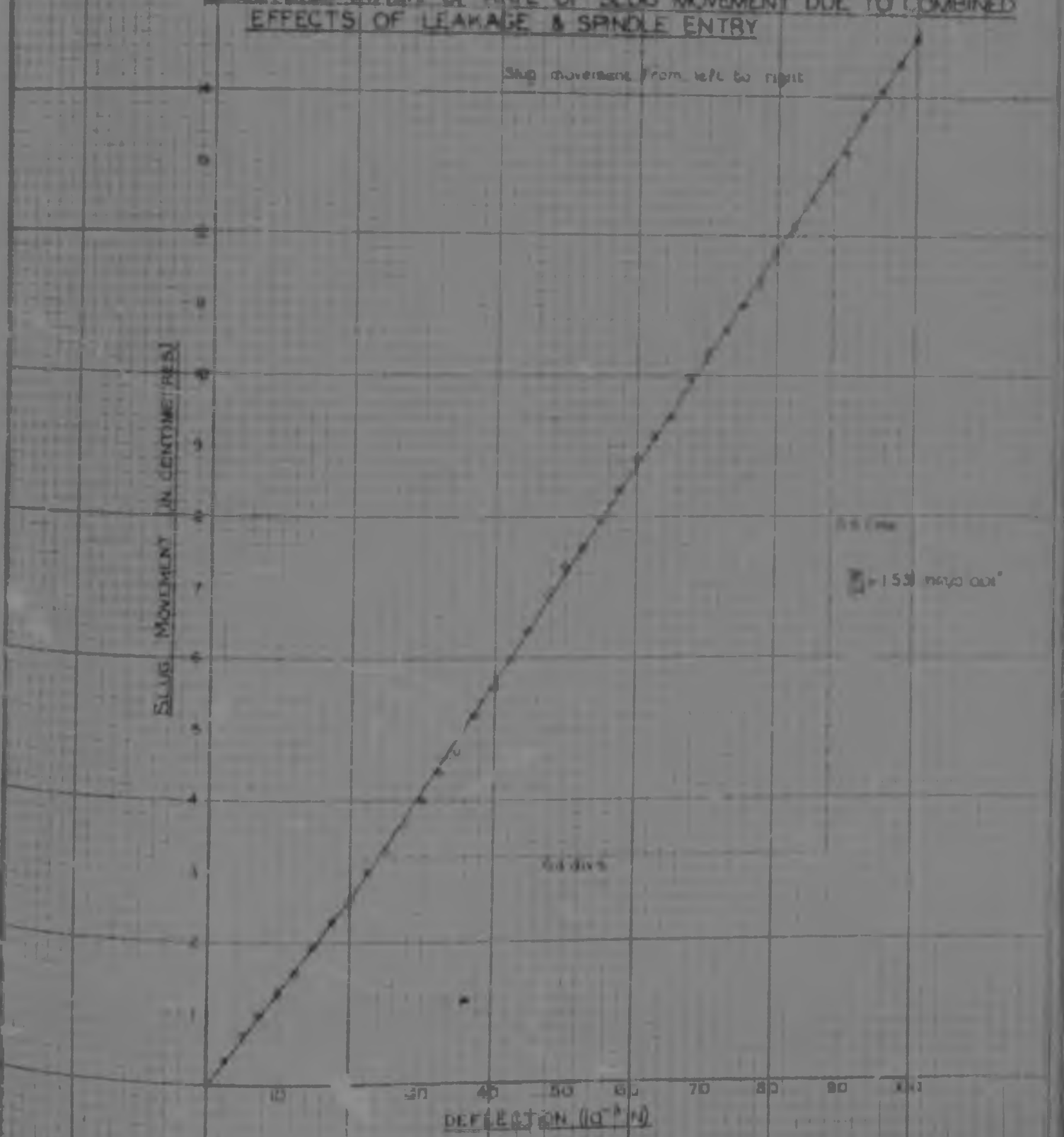


FIG. 40.

TEST Nº 1 . LEAKAGE MEASUREMENTS

(A) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. BEFORE TEST. TEMP 24.0°C TIME 8.[illegible] a.m.		(B) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST(1). TEMP [illegible]°C TIME 8.59 a.m.		(C) MEASUREMENT OF SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY (AFTER TEST). TEMPERATURE 24.25°C TIME 9.07 a.m.			(D) MEASUREMENT OF SLUG MOVEMENT DUE TO LEAKAGE ONLY. AFTER TEST(2). TEMP 24.25°C TIME [illegible].10a.m.	
TIME min sec	VOLUME SCALE cm	TIME min sec	VOLUME SCALE cm	DEFLECT GAUGE cm	VOLUME SCALE cm	TIME min sec	TIME min sec	VOLUME SCALE cm
0 00	21.70	0 00	24.7[illegible]	00 30	[illegible]	00 00	[illegible]	[illegible]
15	21.675	15	24.675	[illegible].5	[illegible]		[illegible]	[illegible]
30	21.65	30	24.60	35	[illegible]		[illegible]	15.9[illegible]
45	21.6[illegible]	45	24.[illegible]	37.[illegible]	[illegible]		45	15.[illegible]
1 00	21.60	1 00	24.5[illegible]	[illegible]	[illegible]	00 56	1 [illegible]	[illegible]
15	21.60	15	24.425	[illegible]	[illegible]		15	[illegible]
30	21.60	30	24.40	45	[illegible]	1 20	30	[illegible]
45	21.575	45	24.3[illegible]	47.[illegible]	[illegible]		45	[illegible]
2 00	21.[illegible]	2 [illegible]	24.[illegible]	50	[illegible].60	01 44	2 00	16.60
15	21.[illegible]	15	24.25	52.5	[illegible].975		15	[illegible]
30	21.[illegible]	[illegible]	[illegible]	[illegible]	[illegible].40	02 08	30	15.[illegible]
45	21.[illegible]	45	24.175	57.5	[illegible]		45	[illegible]
3 00	21.475	3 [illegible]	24.1[illegible]	60	[illegible]	02 32	3 00	16.375
15	21.45	15	24.125	62.5	[illegible]		15	15.30
30	21.40	30	24.10	65	[illegible]	[illegible]	30	[illegible]
45	21.375	45	24.075	67.5	13.15		45	[illegible]
4 00	21.35	4 00	24.05	70	[illegible]	03 19	4 00	[illegible]
15	21.325	15	24.[illegible]	72.5	13.775		15	[illegible]
30	21.30	30	24.025	75	[illegible]	03 42	30	[illegible]
45	21.[illegible]	45	24.[illegible]	77.5	14.425		45	[illegible]
5 00	21.225	5 00	[illegible]	80	[illegible]	04 04	5 00	[illegible]
		15	[illegible]	[illegible].5	[illegible]			
		30	[illegible]	85	[illegible]	04 28		
		45	[illegible]	87.5	[illegible]			
		6 00	[illegible]	90	[illegible]	04 50		
		15	[illegible]	[illegible]	[illegible]			
		30	[illegible]	95	[illegible]	05 14		
				97.5	[illegible]			
				100	[illegible]	05 37		
				102.5	17.[illegible]			
				105	[illegible]	06 01		
				[illegible].5	[illegible]			
				110	[illegible]	06 26		
				112.5	[illegible]			
				115	[illegible]	06 50		

TEST Nº 1 MAIN TEST OBSERVATIONS

DEFLECTION GAUGE	LOAD GAUGE	SLUG SCALE	TIME	TEMPERATURE	CORRECTED SLUG SCALE	CORRECTED LOAD	CORRECT DEFLECT	STRAIN ε	CORRECTED STRESS (σ₁ σ₂ σ₃)
		cm			cm	lb	10^{-3} in	percent	lb/sq in
00 30	15	21.10	00 00	24.25	0	0	0	0	0
32.5	17.5	21.375	00 24		- 0.275	2.5	2.5	0.083	1.4
35	18	21.80	00 37		- 0.70	3.0	5	0.167	1.7
37.5	18.7	22.00	00 45		- 0.90	3.7	7.5	0.250	2.1
40	19.2	22.30	00 56		- 1.20	4.2	10	0.333	2.4
42.5	21.5	22.525	01 07		- 1.425	6.5	12.5	0.417	3.7
45	24.1	22.75	01 24		- 1.65	9.1	15	0.500	5.1
47.5	29.0	22.90	01 40		- 1.80	14.0	17.5	0.583	7.86
50	44.5	21.85	03 07		- 0.75	29.6	20	0.667	16.56
52.5	58	19.80	03 30		+ 1.30	43.0	22.5	0.750	24.12
55	70.5	18.05	03 48		3.05	55.5	25	0.833	31.10
57.5	86	15.95	04 10		5.15	71.0	27.5	0.917	39.75
60	97	14.30	04 26		6.80	82.0	30	1.000	45.87
62.5	110.5	12.70	04 46		8.40	95.5	32.5	1.083	53.37
65	120.4	11.20	05 03		9.90	105.4	35	1.167	58.85
67.5	131.0	9.70	05 21		11.40	116.0	37.5	1.250	64.72
70	139.0	8.775	05 35		12.325	124.0	40	1.333	69.18
72.5	146.0	7.90	05 50		13.20	131.0	42.5	1.417	72.96
75	151.5	7.30	06 04		13.80	136.5	45	1.500	75.96
77.5	157.0	6.825	06 19		14.275	142.0	47.5	1.583	78.96
80	160.5	6.625	06 33		14.475	145.5	50	1.667	80.83
82.5	163.0	6.625	06 45		14.475	148.0	52.5	1.750	82.15
85	165.0	6.70	06 56		14.40	150.0	55	1.833	83.19
87.5	166.0	7.20	07 08		13.90	151.0	57.5	1.917	83.67
90	166.5	7.625	07 21		13.475	151.5	60	2.000	83.91
92.5	166.5	8.575	07 33		12.525	151.5	62.5	2.083	83.91
95	165.8	10.00	07 45		11.10	150.8	65	2.167	83.3
97.5	164.7	11.05	07 56		10.05	149.7	67.5	2.250	82.7
100	163.4	12.025	08 06		9.075	148.4	70	2.333	81.9
102.5	161.5	13.025	08 16		8.075	146.5	72.5	2.417	80.8
105	160.2	14.275	08 27		6.825	145.2	75	2.500	80.0
107.5	159.3	15.625	08 40		5.475	143.3	77.5	2.583	79.1
110	158.0	16.40	08 51		4.70	143.0	80	2.667	78.8
112.5	157.0	17.45	09 04		3.65	142.0	82.5	2.750	78.3
115	156.2	18.75	09 13		2.35	141.2	85	2.833	77.9
117.5	156.0	19.90	09 27		1.20	141.0	87.5	2.917	77.4
120	155.3	20.65	09 36		+ 0.45	140.3	90	3.000	76.9
122.5	154.8	21.20	09 47		- 0.70	139.8	92.5	3.083	76.5
125	154.2	22.70	09 59		- 2.20	139.2	95	3.167	76.1
127.5	153.5	23.60	10 10		- 3.10	138.5	97.5	3.250	75.6
130	153.2	24.60	10 24	24.25	- 4.10	138.2	100	3.333	75.5

TEST No 6.

MAIN SLUG MOVEMENT DEFLECTION CURVE

DEFLECTION (10^{-4} IN)

INITIAL SLUG SCALE READING: 27.10"

SLUG MOVEMENT (10^{-4} IN)

(7, -120)

(11, -240)

FIG. 47.

SUMMARY OF TRIAXIAL TEST RESULTS.

TABLE VIII

TEST NUMBER	NOMINAL RATE OF STRAIN INCHES/MINUTE	ACTUAL CHAMBER PRESSURE (σ_3) LB/SQ IN	DEVIATOR STRESS AT FAILURE $(\sigma_1-\sigma_3)_f$ LB/SQ IN	HENCE: NORMAL STRESS AT FAILURE $(\sigma_1)_f$ LB/SQ IN
1	0 00256	28 6	89·6	118·2
3	0 00256	18 7	65 3	84 0
6	0 00256	8 8	60 6	69 4
2	0 0125	28 7	102·0	130 7
4	0 0125	18 8	83·9	102 7
5	0 0125	8 8	53 0	61·8

MOHR CIRCLES FOR TRIAXIAL TESTS

Fig. 48 (a)

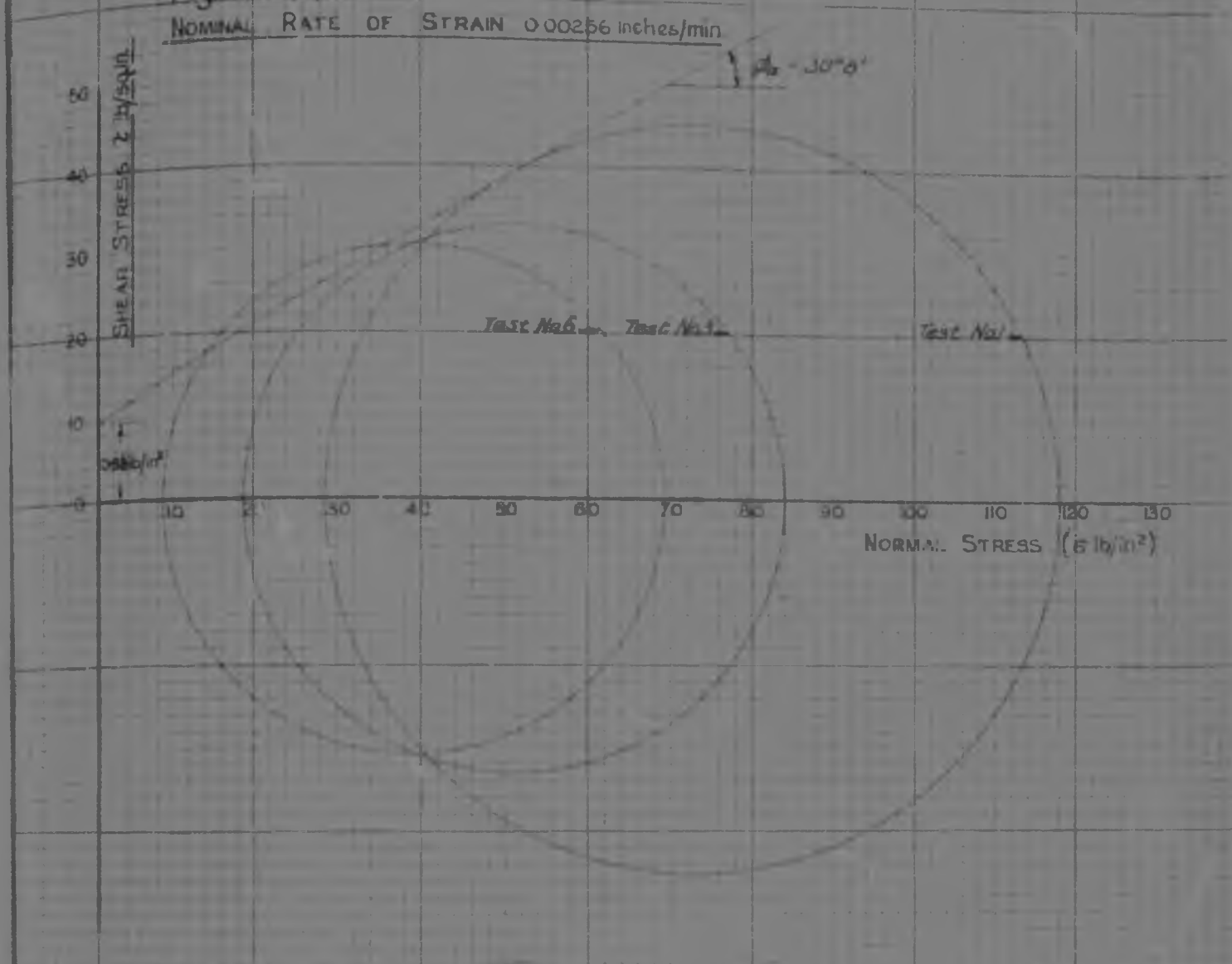


Fig. 48 (b)

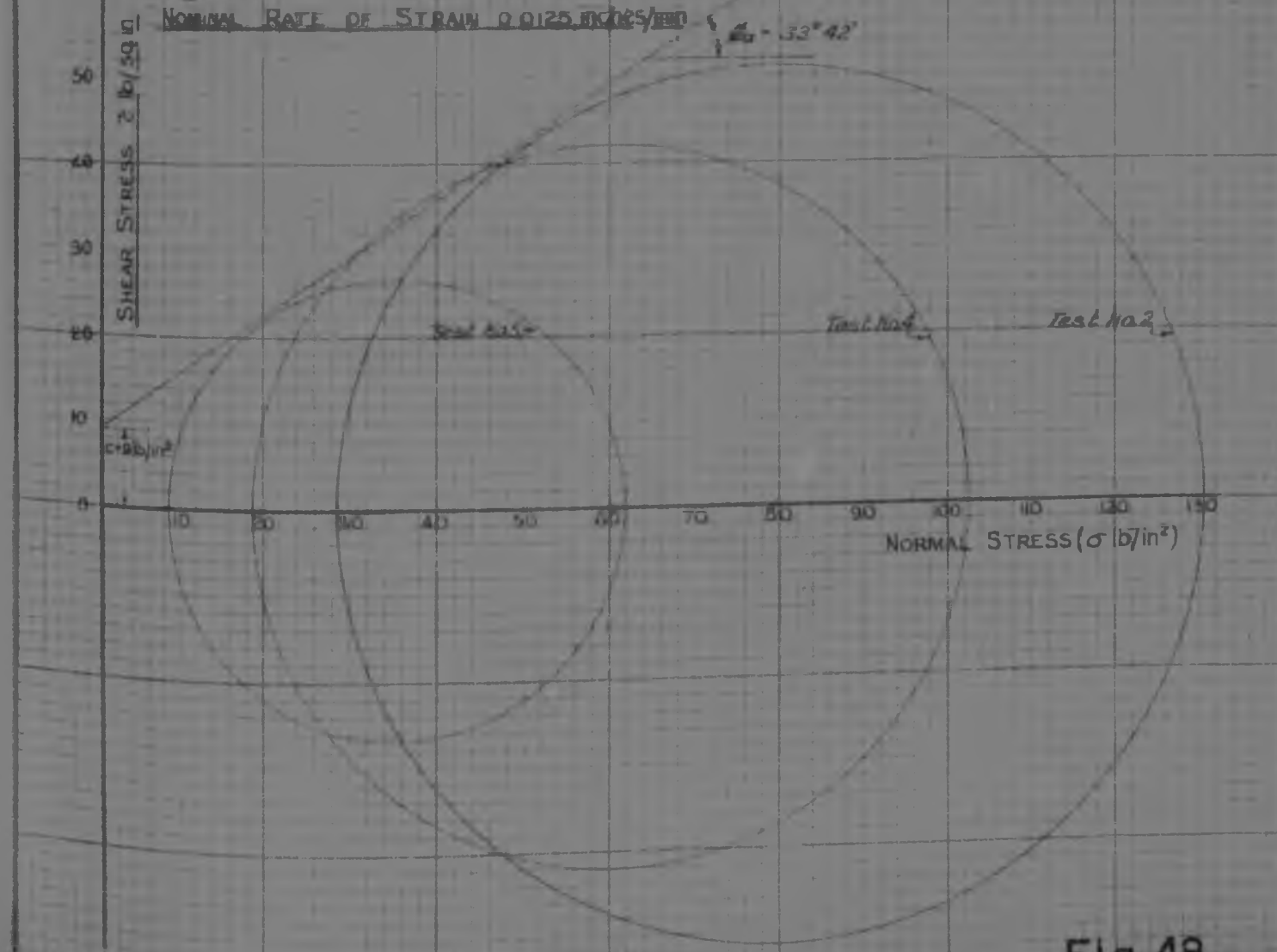


Fig. 48

7.7 Discussion

A study of the recorded data for each test shows that the variation in temperature inside the enclosing wooden box was insignificant during test in all cases, the greatest variation being 0.75°C in the case of Test No.3 which lasted for 92 min. 38 sec. It also appears that the test series as a whole was conducted within a temperature range of 20.75°C-27.5°C, ie., a maximum variation in test temperature between any two tests of 6.75°C (Tests No.1 and No.6.)

The actual time allowed for consolidation in the triaxial test cell was in each case very close to 24 hours if not exactly so, except in the case of Test No.1 in which consolidation was allowed for only 22hrs.15min. However, this probably had very little bearing (if any) on the results of the tests. The test samples were apparently so dry in each case, that no moisture was ever found in the bowl below the open drainage cock after the completion of consolidation.

From the moisture content measurements taken, it appears that:

(a) over the duration of the test series, (which lasted for about 4 weeks excluding the time taken for trial tests), the initial moisture content of test samples decreased in value from about 2.5% to 0.5%, thus indicating that drying out had been taking place in the laboratory while handling the block sample. The various values of initial moisture content:time have been plotted (Fig.53, Appendix IV) and seem to indicate that the rate of decrease in moisture content was approximately uniform during this period.

(b) the final (after test) moisture content of test samples was always slightly greater than the corresponding initial moisture content. In each case this was probably mainly due to leakage through the walls of the rubber membrane enclosing the test sample, as no special provisions were made to prevent this from happening. An additional cause may have been that some of the moisture adhering to the surface of the rubber membrane after test was imparted to the sample on removal of the membrane. It is rarely possible to prevent this from happening. Table XX, Appendix IV is a summary in tabular form of the moisture content before and after test of each test sample.

Regarding the Void Ratio and Degree of Saturation figures shown alongside the various stress:strain curves, it needs to be mentioned that they were obtained (See Table XXI, Appendix IV) by using the oven-dry weight of each test sample after test and a sample volume of 83.0cc, which is slightly less than the volume of the cutting cylinder used. A full discussion giving reasons for this procedure is given later in Section 8.4. The moisture content figure shown is in each case the initial moisture content of the particular test sample. With some tests no initial moisture content determination was made (see Table XX, Appendix IV). In such cases an estimated figure was used which was obtained by reading off the initial moisture content value corresponding to the date of the test on Fig. 53, Appendix IV.

The curves of slug movement due to leakage only as well as the curves of slug movement due to the combined effects of leakage and spindle entry only, show in some instances that leakage from the test cell could be erratic at times. In such cases more than one value of the rate of leakage was determined as shown. The results of all leakage determinations are presented

In ...../

in summarised form in Table IX, Section 8.2. In the same Section these results as well as the factors effecting the rate of leakage from the test cell are discussed in more detail.

The stress:strain curves generally show a well defined linear range which commences at or close to the start of the curve depending on the magnitude of the bedding-in effects. Some mention about "bedding-in" effects was already made in section 7.6. It was mentioned that the corrected start of the stress:strain curve was obtained by producing the approximately linear portion of the curve backwards to intersect the strain axis. In order to determine the point at which proportionality between stress and strain ceases, i.e., the end of the linear range, the curve of Young's Modulus versus strain was prepared. This curve is considered to serve as a very useful guide to the selection of the point mentioned, since, while stress and strain are proportional, Young's Modulus has a constant value (independent of strain), and therefore, under these conditions, the curve of Young's Modulus versus strain is a horizontal straight line. Hence, in the ideal case, the strain at which the Young's Modulus:strain curve starts to slope downward after the initial horizontal straight line portion, demarcates the point at which proportionality between stress and strain ceases, i.e., demarcates the end of the linear range. The actual plotted curves, however, show that this horizontal straight line portion of the ideal Young's Modulus:strain curve was seldom perfectly realised. Nevertheless, these curves were of considerable assistence in judging by careful inspection where the linear range ceased in each case. The point eventually decided on, is shown marked by the letter c .

Having determined the end of the linear range of the stress: strain curve for each test in the manner just described the "best" portion of this range in the sense of being the closest representation of truly linear conditions, was selected, and the terminal points of this selected range marked by the letters a and b. Hereafter, this range will be referred to only as the "Selected Range". The value of Young's Modulus was then determined for each selected range and denoted by the symbols Esel.

The Main Slug Movement:Deflection curves generally show a well defined linear range, nearly falling closely within the same range of strain as the linear range of the relevent stress:strain curve. The points corresponding to the limits of the Selected Range are in each case shown marked a' and b'. This information is later used in the evaluation of Poisson's Ratio (Section 8.5). These linear ranges in all cases terminate in the development of trough-shaped turning points in the curves, which demarcate the commencement of the phenomenon of sample expansion on approaching failure conditions.

The curves again reveal the presence of "bedding-in" effects, generally by an initially flat portion immediately before the main steep linear portion of the curve commences, but in some instances, (see Figs. 36 and 47) by a pronounced peak-shaped portion before the main curve commences. The phenomenon in this case can be explained as follows: During the "bedding-in" period, only portion of the applied loading is taken up by the test sample, the remainder being used up in causing the bedding-in movements in the test set-up. Hence, during the bedding-in period at the start of a test, sample compression is small and therefore the tendency for fluid flow into the test cell due to this cause is small. In contrast, the loading spindle during the same period has a partially unrestrained and hence comparatively rapid advance into the test cell with a tendency to cause fluid flow out of the test cell. It appears that during

the ...../

the bedding-in period, this latter effect outweighs the combined effects of an initial small sample compression and leakage from the test cell (which both tend to cause fluid flow into the test cell), and results in a nett fluid flow out of the test cell i.e. increasing slug scale readings and hence the initial upward trend of the curve.

Subsequently, however, when the test set-up is firmly bedded down, all the applied loading is taken by the test sample causing it to compress considerably and at the same time severely restraining the advance of the loading spindle into the test cell. As soon as this happens, the combined effects of sample compression and leakage from the test cell start to outweigh the effect of spindle entry into the test cell, and thus, result in a nett inflow of fluid into the test cell, i.e., decreasing slug scale readings and hence the commencement of a downward trend in the curve and the formation of the peak observed.

The argument presented above is strengthened by the actual calculation of Sample Dilation ($\delta V/V$) during test (Section 8.5). These calculations show that over the bedding-in range, sample compression is small compared with subsequent sample compression when bedding-in is complete. (See, for example, the calculation of Sample Dilation for Test No.4.) The pronounced peak-shaped bedding-in effect would thus occur in cases where the effect of the initial advance of the loading spindle into the test cell considerably exceeds the opposite effects caused by sample compression and leakage. This is probably only possible in the case of an initially very loosely bedded sample.

Mention was previously made about the possible effect of chamber pressure on the Calibrated Rate of Strain, i.e. the rate of strain obtained from the calibration of loading spindle entering the test cell against the resistance of the relevant chamber pressure only. (See end of Section 7.8) Table IX Section B.2. shows the values of calibrated rate of strain, obtained at the various chamber pressures for the same nominal rates of strain. The values apparently show that the calibrated rate of strain decreases as the chamber pressure increases. This is contrary to expectation and is possibly due to insufficient values being available. This suspicion is based on the fact that in the case of Test No. 8B (not included in the test series being reported on herein), a calibrated rate of strain of 0.002880 inches/min. for a chamber pressure of 10lb./sq.in. was obtained. It is considered that the calibrated rate of strain would possibly increase with decreasing chamber pressure.

Where detailed time measurements were taken, the resulting deflection:time curves show clearly that the actual rate of strain during test is very uneven, especially during the initial stages of the test. A very pronounced time lag is obtained at a point corresponding approximately to the start of the linear range of the stress:strain curve. The phenomenon is revealed by the nearly horizontal portion of the curve soon after its commencement, indicating a negligible increase in deflection during this period. In contrast, there is a rapid increase in loading during the same period. In most cases, it will also be noted (from the curves of Young's Modulus versus Strain), that Young's Modulus has a maximum value just about at the point where the time lag occurs. Subsequently the actual rate of strain steadily increases until, when sample failure is approached, it tends to take on the value of the calibrated rate of strain.

Careful investigation seems to show that the pronounced time lag occurs at the completion of the bedding-in period. It

is ...../

is possibly due to a back-lash in the loading system once the bedding down is complete and the applied loading is directed in full at overcoming the resistance of the sample to deformation, which, at this point happens to be a maximum. The subsequent steady increase in the actual rate of strain is due to a steady decrease in the value of Young's Modulus (See Young's Modulus: Strain curves). This continues until, at sample failure, the rate of strain is approximately the same as the calibrated rate of strain, which, notably, was determined for the condition of loading spindle advancing into the test cell against the resistance of the relevant chamber pressure only. The disturbing thought is that the tests were intended to have been conducted as tests subject to strain control.

Finally, it is necessary to consider reasons for the peaks in the Young's Modulus:Strain curves which occur in most cases. The basic cause of this phenomenon is the shape of the stress : strain curve. Careful inspection shows that within the accepted extent of the linear range of the stress:strain curve, the curve usually makes a slight S-shaped bend, i.e., according to actual points plotted. (See, for instance, Test No.1, Fig. 27.) The curve of Young's Modulus : Strain was prepared by calculating instantaneous values of Young's Modulus as a secant modulus, i.e. working from point to point along the curve but always from the start of the curve, which in this case, is the 'corrected' start as obtained after eliminating bedding-in effects. With an S-shaped curve and this secant modulus method of calculation, the result is clearly a peak-shaped variation of Young's Modulus.

It is considered that the slight S-shape of the curve is at its lower end probably due to the inclusion of bedding-in effects, and, at the upper end, due to a natural slight curving over to the right of the actual stress:strain curve (this being the case even within the extent of the linear range). In the absence of bedding-in effects the portion of the Young's Modulus: Strain curve to the left of its peak would probably have been much nearer horizontal and thereby would have conformed closer to the requirements of the third Young's Modulus:Strain curve As previously discussed in this Section. The fact that the respective values of E_{sel} (Young's Modulus for the Selected Range) are in all cases lower than the corresponding peak values of Young's Modulus as indicated by the Young's Modulus:Strain curves, is due to the fact that in each case the selected Range is in effect the straight line joining the ends of a slightly S-shaped curve. This clearly leads to a lower value of Young's Modulus than the maximum value obtainable by the other method.

In the following Chapter, Poisson's Ratio (μ) has for each test been calculated using both instantaneous values of Young's Modulus (yielding instantaneous values of μ, denoted μ_i) and the value of Young's Modulus obtained from the selected Range (the corresponding μ value being denoted μ_{sel}).

CHAPTER 8

REDUCTION OF TEST RESULTS AND THE DETERMINATION OF POISSON'S RATIO

8.1 Introduction

Before the evaluation of Poisson's Ratio for each test can finally be commenced, it is necessary to discuss in more detail some important factors relating to the evaluation, some of which have already been briefly discussed. These discussions follow in Sections 8.2, 8.3 and 8.4 hereafter.

8.2 Rate of Slug Movement due to Leakage only during Actual Test

For the evaluation of Poisson's Ratio, the rate of slug movement due to leakage from the test cell during actual test needs to be known. As previously discussed (Section 7.4), this measurement cannot be made directly, since it is not possible to isolate the effects of slug movement due to entry of the loading spindle into the test cell and change in volume of the test sample during test.

To enable some idea of the rate of leakage during actual test, the only circumstances under which slug movement due to leakage only can be measured directly, viz., under the steady cell conditions existing immediately before and after test, were resorted to. As the rate of leakage measured under these circumstances was likely to be different to the rate of leakage occurring under the disturbed cell conditions of an actual test, a further indirect measurement of the rate of leakage was made from a test which it was hoped simulated the disturbed cell conditions of an actual test.

Thus, having observed values of the rate of leakage under the steady cell conditions existing before and again after test, as well as having determined the rate of leakage under disturbed cell conditions similar to those of an actual test although possibly not identical, it is necessary to consider in some detail the possible value of the rate of leakage occurring under the disturbed cell conditions of an actual test.

For this purpose, it is in the first instance necessary to study carefully the results of all the various leakage measurements made during the test series. These are presented in summarised form in Table IX on the next page. Table X on the same page, shows the determination of slug movement due to leakage only under the disturbed cell conditions of the test simulating the conditions of an actual test. The basis of this determination was discussed in Section 7.4 (pages 79 and 80). Table X also shows a comparison of these results with the maximum observed slug movement due to leakage only measured under the steady cell conditions existing before and again after test. In Table IX, where more than one value appears under the heading in some instances, this happens as a result of a definite change in value having been observed during test due to a change in the rate of leakage from the test cell. The determination of the rate of slug movement due to entry of the loading spindle into the test cell, is given in Appendix V ,Item 2.

From the data shown in the Tables, it is evident that:

(1) The rate of slug movement due to leakage measured immediately after any triaxial test is generally higher than

that ...../

TABULAR SUMMARY OF ALL LEAKAGE MEASUREMENTS

NOTE AN ARROW THUS ← ABOVE A FIGURE DENOTES SLUG MOVEMENT FROM RIGHT TO LEFT
AN ARROW THUS → ABOVE A FIGURE DENOTES SLUG MOVEMENT FROM LEFT TO RIGHT

TABLE IX

TEST NUMBER	NOMINAL σ_3 CHAMBER PRESSURE LB/IN²	NOMINAL RATE OF STRAIN inches/minute	OBSERVED SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER STEADY CELL CONDITIONS. BEFORE TRIAXIAL TEST mm/minute	AFTER TRIAXIAL TEST 1.	AFTER TRIAXIAL TEST 2 mm/minute	CALCULATED SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY = N mm/div deflect	OBSERVED SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY = M mm/div deflect	CALIBRATED RATE OF STRAIN = S inches/minute
1	30	0 00256	← 0 92	← 2 80	← 4 500	→ 1 614	← 0 173	0 002555
3	20	0 00256	← 1 54	← 2 667 1 00 2 34	← 2 00 2 33	→ 1 614	→ 0 250 0 333	0 002544
6	10	0 00256	← 0 875	← 0 947	← 0 50	→ 1 614	→ 1 112	0 002500
2	30	0 0125	← 1 27	← 4 00	← 3 89 5 238	→ 1 614	→ 1 138	—
4	20	0 0125	← 0 815	← 1 909 0 875 2 000	← 2 571	→ 1 614	→ 1 363	0 01282
5	10	0 0125	← 0 500	← 1 000	← 0 700	→ 1 614	→ 1 531	0 01270

NOTE THE MAXIMUM VALUES OF SLUG MOVEMENT DUE TO LEAKAGE FOR EACH TEST, ARE SHOWN UNDERLINED IN RED

DETERMINATION OF SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER DISTURBED CELL CONDITIONS, AND COMPARISON WITH MAXIMUM OBSERVED SLUG MOVEMENT DUE TO LEAKAGE UNDER STEADY CELL CONDITIONS

TABLE X

TEST NUMBER	M = OBSERVED SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY mm/div deflect	S = CALIBRATED RATE OF STRAIN inches/minute per unit deflect mm	HENCE SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY = M X S = a mm/minute	N = CALCULATED SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY. mm/div deflect	HENCE CALCULATED SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY = N X S = b mm/minute	HENCE RESULTANT SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER DISTURBED CELL CONDITIONS = a - b mm/minute	MAXIMUM OBSERVED SLUG MOVEMENT DUE TO LEAKAGE UNDER STEADY CELL CONDITIONS = d mm/minute	DIFFERENCE c - d mm/minute
1	0 173	2 555	0 442	1 614	4 124	4 566	4 500	0 066
3	0 333	2 544	0 847	1 614	4 106	3 259	2 667	0 592
6	1 112	2 500	2 780	1 614	4 035	1 255	0 947	0 308
2	1 138	12 50 *	14 225	1 614	20 175	5 950	5 238	0 712
4	1 363	12 82	17 474	1 614	20 691	3 217	2 571	0 646
5	1 531	12 70	19 444	1 614	20 498	1 054	1 000	0 054

* NOMINAL RATE OF STRAIN USED HERE, AS CALIBRATED RATE OF STRAIN WAS NOT DETERMINED FOR THIS TEST

TABULAR SUMMARY OF ALL LEAKAGE MEASUREMENTS

NOTE AN ARROW THUS ← ABOVE A FIGURE DENOTES SLUG MOVEMENT FROM RIGHT TO LEFT.
AN ARROW THUS → ABOVE A FIGURE DENOTES SLUG MOVEMENT FROM LEFT TO RIGHT.

TABLE IX

TEST NUMBER	NOMINAL σ_3 CHAMBER PRESSURE LB/SQ IN	NOMINAL RATE OF STRAIN. inches/minute	OBSERVED SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER STEADY CELL CONDITIONS. BEFORE TRIAXIAL TEST mm/minute	OBSERVED SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER STEADY CELL CONDITIONS. AFTER TRIAXIAL TEST 1. mm/minute	OBSERVED SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER STEADY CELL CONDITIONS. AFTER TRIAXIAL TEST 2 mm/minute	CALCULATED SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY = N mm/hr deflect	OBSERVED SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY = M mm/hr deflect	CALIBRATED RATE OF STRAIN = S inches/minute
1	30	0 00258	← 0 92	← 2 80	← 4 500	→ 1 614	← 0 173	0 002555
3	20	0 00258	← 1 54	← 2 667 1 00 2 34	← (2 00 (2 33	→ 1 614	→ (0 250 (0 333	0 002544
6	10	0 00258	← 0 875	← 0 947	← 0 50	→ 1 614	→ 1 112	0 002500
2	30	0 0125	← 1 27	← 4 00	← 3 85 5 238	→ 1 614	→ 1 138	—
4	20	0 0125	← 0 875	← 1 909 0 875 2 000	← 2 571	→ 1 614	→ 1 363	0 01282
5	10	0 0125	← 0 500	← 1 000	← 0 700	→ 1 614	→ 1 531	0 01270

NOTE. THE MAXIMUM VALUES OF SLUG MOVEMENT DUE TO LEAKAGE FOR EACH TEST, ARE SHOWN UNDERLINED IN RED

DETERMINATION OF SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER DISTURBED CELL CONDITIONS, AND COMPARISON WITH MAXIMUM OBSERVED SLUG MOVEMENT DUE TO LEAKAGE UNDER STEADY CELL CONDITIONS

TABLE X

TEST NUMBER	M = OBSERVED SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY mm/hr deflect	S = CALIBRATED RATE OF STRAIN inches/minute	HENCE SLUG MOVEMENT DUE TO COMBINED EFFECTS OF LEAKAGE & SPINDLE ENTRY = M x S = a mm/minute	N = CALCULATED SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY mm/hr deflect	HENCE CALCULATED SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY = N x S = b mm/minute	HENCE RESULTANT SLUG MOVEMENT DUE TO LEAKAGE ONLY UNDER DISTURBED CELL CONDITIONS = a - b mm/minute	MAXIMUM OBSERVED SLUG MOVEMENT DUE TO LEAKAGE UNDER STEADY CELL CONDITIONS mm/minute	DIFFERENCE mm/minute
1	→ 0 173	2 555	← 0 442	→ 1 614	→ 4 124	← 4 566	← 4 500	0 066
3	→ 0 333	2 544	← 0 847	→ 1 614	→ 4 106	← 3 259	← 2 667	0 592
6	→ 1 112	2 500	← 2 780	→ 1 614	→ 4 035	← 1 255	← 0 947	0 308
2	→ 1 138	12 50 *	← 14 225	→ 1 614	→ 20 175	← 5 950	← 5 238	0 712
4	→ 1 363	12 82	← 17 474	→ 1 614	→ 20 691	← 3 217	← 2 571	0 646
5	→ 1 531	12 70	← 19 444	→ 1 614	→ 20 498	← 1 054	← 1 000	0 054

* NOMINAL RATE OF STRAIN USED HERE, AS CALIBRATED RATE OF STRAIN WAS NOT DETERMINED FOR THIS TEST

that measured immediately before the test. (Table IX)

(ii) Comparing the two leakage measurements made after the triaxial test (Table IX), it appears that they are different to one another, but of more or less the same order of magnitude.

(iii) As regards the comparison of leakage measurements made under steady conditions and those derived from the test simulating the disturbed conditions of an actual triaxial test, the values in Table X show clearly that the leakage occurring under the disturbed conditions caused by the loading spindle advancing into the test cell, is greater than even the maximum value of leakage measured at any time under steady cell conditions.

In trying to establish the causes of these experimental observations, it was necessary to examine more closely the factors affecting leakage and variations in the rate of leakage from the test cell:

Before commencement of any triaxial test, steady conditions exist in the test cell. These conditions are maintained only by the action of the particular chamber pressure selected for the test. Any leakage measurement usually takes place from the point where the loading spindle enters the test cell. This point was greased prior to positioning the test sample in order to reduce leakage during the test as much as possible. The rate of leakage under these steady cell conditions is indicated by the leakage measurement immediately before commencement of the test.

When the test is commenced, the entrance of the loading spindle into the test cell results in an increased chamber pressure which causes a tendency for fluid flow out of the test cell, as has been previously fully discussed (Section 7.4). Simultaneously however, the volume of the test sample is reduced (initially) due to the compressive loading, which results in a reduction in the chamber pressure and hence a tendency for a resulting inflow of fluid into the test cell. The nett effect may be either an increase or decrease in the chamber pressure depending on which effect is the greater, and hence, there is a consequent increase or decrease in the rate of leakage from the test cell (usually however, a decrease). As failure of the test sample is approached, the compressive loading causes an extension of the sample, resulting in an increased chamber pressure and a tendency for fluid flow out of the test cell. The nett effect then becomes a definite increase in chamber pressure giving rise to a sharp increase in the rate of leakage from the test cell. Grease conditions in the vicinity of leakage from the test cell are hereby so seriously disturbed that even when steady conditions again exist at the end of the test, the rate of leakage is usually higher than that which existed before the test was commenced. This means that the effect of the increased cell pressure at sample failure is usually greater than any initial reduction in cell pressure which could have occurred while the sample volume was decreasing. Hence, the phenomenon observed as mentioned in (i) above.

Following on the first leakage measurement immediately after the triaxial test, the test for determining the leakage under disturbed cell conditions simulating those existing during an actual test was carried out. Raising the loading spindle in preparation for this test and then subsequently the advance of the loading spindle into the test cell, again disturbs the grease conditions at the point where the loading spindle enters the test cell. Hence, the .....

the ...../

the test cell, causing the leakage measured after this test to be different to the previous two measurements, but apparently of the same order of magnitude as the leakage measured immediately after the triaxial test. This is the phenomenon observed as mentioned in (ii) above.

Further, Table X shows clearly that the value of leakage determined from this test, is in every case greater than even the maximum value of leakage measured under steady conditions before and after this test. The reason is clearly that during the test an increased rate of leakage results due to the increased chamber pressure caused by the advance of the loading spindle. The fact that the two leakage measurements after the triaxial test generally show little agreement with the measurement carried out before the triaxial test, is due to the former two measurements not being relatively affected by any sample volume change effects.

Hence, it is also clear that the test which it was hoped would simulate the disturbed cell conditions of an actual triaxial test, was not a good representation, as it did not cater for the phenomenon of an initially reduced and then increased chamber pressure due to decrease and increase in sample volume respectively.

However, although the main problem of determining the rate of leakage under the disturbed cell conditions of an actual test has thus not been solved by the information obtained from the leakage measurements that were carried out, it is nevertheless possible, as a result of the foregoing discussions, to obtain a better understanding of the problem which is likely to enable a more reliable estimation of the rate of leakage during an actual test based on the data that was obtained.

The main conclusions from the above discussions are that:

(i) as long as there is a flow of fluid into the test cell (which is indicated by a slug movement in the capillary tube from right to left, i.e., decreasing slug scale readings), there is a pressure deficiency in the test cell giving rise to this flow, and hence, the rate of leakage from the test cell is less than under the steady cell conditions existing immediately before the start of the test.

(ii) immediately the fluid flow ceases to be out of the test cell (as indicated by a reversal in the direction of the slug movement in the capillary tube, i.e., increasing slug scale readings), there is a pressure excess in the test cell giving rise to this flow and hence, there is an increase in the rate of leakage from the test cell.

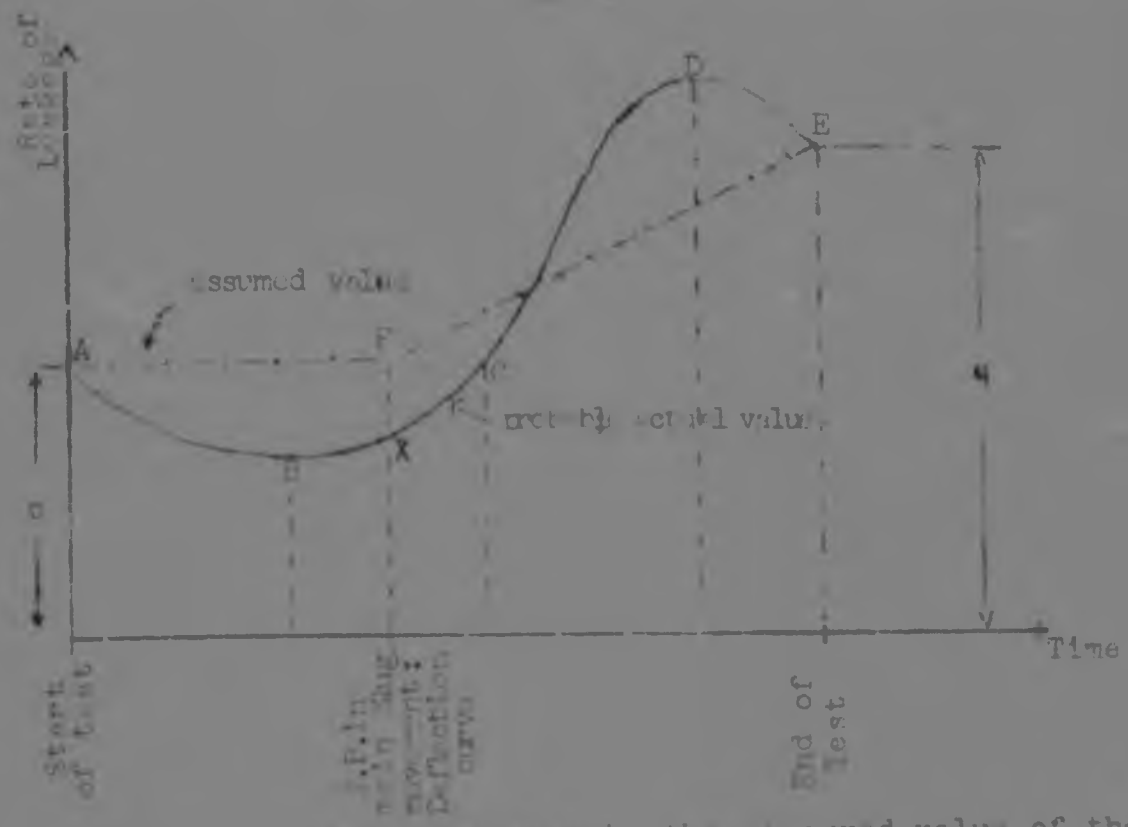
With regard to the main slug movement/direction curves, this means that in each case over the portion of the curve to the left of the trough-formed turning point, (i.e. the portion resulting from decreasing slug scale readings), there is a pressure deficiency in the test cell, while over the portion to the right of this turning point, (i.e. the portion resulting from increasing slug scale readings), there is a pressure excess in the test cell. This statement ignores the possible cell pressure increase close to the start of the curve resulting from the small peak due to bedding-in effects, which is observed in some cases immediately before the main downward portion of the curve commences — an effect which was fully discussed in Section 7.7.

Hence, it is considered that subsequent to the rate of

leakage ...../

leakage existing under the steady cell conditions immediately before the start of a test, the rate of leakage varies approximately according to the relationship shown in the diagram below:

Fig. 3.1



In this diagram, p represents the measured value of the rate of leakage under steady cell conditions immediately before test, while q represents the measured value of the rate of leakage under steady cell conditions immediately after test. As soon as the test commences there is a pressure deficiency in the test cell and therefore a resulting decrease in the rate of leakage. This pressure deficiency continues, but gradually decreases as the turning point of the Main Slug Movement:Deflection curve is approached, thereby tending to restore steady pressure conditions in the test cell. At the turning point when steady conditions exist and there is neither inflow nor outflow from the test cell, the rate of leakage would probably be a value which is greater than the minimum value attained (ordinate at B) while the pressure deficiency existed, but probably less than the value before commencement of the test (ordinates at A and C), i.e., some value between points B and C as shown (ordinate at X).

As soon as the turning point in the Main Slug Movement : Deflection curve is passed, there is a pressure excess in the test cell and hence a rapid increase in the rate of leakage to some maximum value as indicated by the ordinate at D. When the test is stopped, steady conditions again set in (pressure excess then reduced to zero), and the corresponding rate of leakage which is the value measured immediately after the test, is somewhat less than the maximum but, according to actual measurements, somewhat greater than the value for the steady conditions existing immediately before test (ordinates at B and C) i.e., possibly as represented by the ordinate at E.

Hence, as an approximation to the curve ABCDE shown, it was decided to assume that the initial measured value of the rate of leakage under the steady cell conditions before test (ordinate at A), remains constant until the turning point in the Main Slug Movement:Deflection curve is reached (which is a known point with regard to time), and then to assume that the rate of leakage increases uniformly to the value measured under the steady cell conditions immediately after test. In

cases ...../

cases where more than one value was obtained for the rate of leakage after test (See Table IX), the maximum value was used, as it is considered that this value would be closest to the actual maximum value attained at the end of the test, which is represented by the ordinate at D.

The above procedure was followed in all calculations relating to the determination of instantaneous values of Sample Dilation ($\delta V/V$) and Poisson's Ratio (μi). Although the results obtained in this way may not have been strictly accurate, it is considered that they were sufficiently accurate for the purposes required, as in actual fact, the proportion of slug movement due to leakage only in the overall measured value of slug movement due to the three combined effects is small (see calculation sheets, Section 8.5). Hence, an error in the value of slug movement due to leakage only, does not cause a serious error in the value of Sample Dilation or Poisson's Ratio.

However, in order to demonstrate this fact, the calculation of Poisson's Ratio for the Selected Range of each stress:strain curve was carried out, using both the measured value of leakage under the steady cell conditions existing before test, as well as the (maximum) measured value of leakage under the steady cell conditions existing immediately after test. The results show an insignificant difference in the value of usel obtained (see Section 9.5)

The values of usel based on the value of the rate of leakage under steady cell conditions immediately before test, are considered to be more reliable, as the actual rate of leakage during test over the Selected Range corresponds closer to this value than the one relating to conditions at the end of the test. The reason for this is that the Selected Range of the stress:strain curve always falls within that range of the Strain Slug Movement: Deflection curve which is to the left of its turning point, i.e., the range over which there is a pressure deficiency in the test cell. Therefore, the actual rate of leakage over the Selected Range is in each case a value which is lower than the value measured under the steady cell conditions immediately before the start of the test.

8.3 Time Effects during Test

The reduction in rate of strain during test, (the causes of which were discussed at the end of Section 7.7), has a very important bearing on the subsequent reduction of test results.

This statement is made, mainly as a result of the effect of a reduction in rate of strain on the calculated value of actual slug movement due to change in volume of the test sample only. The original intention with regard to the latter determination, was merely to combine:

(i) whatever value of rate of leakage during test (as a slug movement in millimetres/minute) be decided on, with

(ii) either the nominal or calibrated rate of strain (in units of divisions deflection/minute, i.e., units of 10-3 inches/minute), both of which were considered to be constant throughout the duration of a test.

Item (i) divided by item (ii) would give the rate of leakage from the test cell as a slug movement in millimetres/division deflection, which would then be in the same units as obtained for the calculated value of slug movement due to entry of the loading spindle into the test cell (see Appendix V, Item 2),

in ...../

as well as in the same units as the measured value of total slug movement due to the three combined effects during test (Section 7.3). The subsequent combination of these three rate of slug movement values in order to determine the actual slug movement in millimetres due to volume change of the test sample only over any particular range of strain, would then present no difficulty.

If the rate of strain during test remains constant, this procedure is in order. However, leakage from the test cell is in the first instance dependent on time. Hence, with the method discussed above, this determination may be seriously in error if, while the determination of rate of slug movement due to leakage only (in units of mm./div. deflection) be based on some rate of strain which is considered to be constant throughout the test, the actual rate of strain during part of the test be considerably lower than the value used. The reason for this is that, the lower the rate of strain, the greater is the proportion of actual slug movement due to leakage only in the measured value of total slug movement due to the three combined effects, the values of slug movement due to spindle entry and sample volume change only being independent of time.

Hence, it was necessary to express all measured rates of slug movement (whether they be in units of millimetres/minute or millimetres/division deflection), as actual slug movements in millimetres over the particular range of strain considered. For slug movements measured in millimetres/minute it was thus necessary to determine the actual time interval corresponding to the particular range of strain considered, in order to obtain an actual slug movement in millimetres. Hence also, the need for continuous time recordings during test mentioned in Section 7.5.

In cases where continuous time recordings during test were taken, the actual time interval corresponding to any particular range of strain could be directly obtained. In these cases it was therefore possible to evaluate Sample Dilation ($\delta V/V$) and Poisson's Ratio (μ) for each increase in strain (working from the same starting point each time - usually the corrected start of the stress:strain curve as previously discussed in Section 7.7), and so to obtain curves of Instantaneous Sample Dilation:Strain ($\delta V/V : \varepsilon$) and Instantaneous Poisson's Ratio : Strain ($\mu : \varepsilon$).

In cases where continuous time recordings during test had not been taken, (see remarks in this respect in Section 7.5), this was not possible, and it was only possible to determine the value of Sample Dilation and Poisson's Ratio corresponding to the Selected Range of the Stress:strain curve in each case. To enable even these determinations, it was necessary to estimate the value of the actual rate of strain over the Selected Range, in order to obtain the time interval corresponding to the duration of the Selected Range. The basis on which this estimate could be made, is discussed in the following paragraphs.

Careful investigation of the deflection:time curves for the tests in which detailed time recordings were taken (Nos. 1,3,4 and 6), revealed the facts shown in Table XI on the following page:

TABLE XI ...../

as well as in the same units as the measured value of total slug movement due to the three combined effects during test (Section 7.3). The subsequent combination of these three rate of slug movement values in order to determine the actual slug movement in millimetres due to volume changes of the test sample only over any particular range of strain, would then present no difficulty.

If the rate of strain during test remains constant, this procedure is in order. However, leakage from the test cell is in the first instance dependent on time. Hence, with the method discussed above, this determination may be seriously in error if, while the determination of rate of slug movement due to leakage only (in units of mm./div. deflection) be based on some rate of strain which is considered to be constant throughout the test, the actual rate of strain during part of the test be considerably lower than the value used. The reason for this is that, the lower the rate of strain, the greater is the proportion of actual slug movement due to leakage only in the measured value of total slug movement due to the three combined effects, the values of slug movement due to spindle entry and sample volume change only being independent of time.

Hence, it was necessary to express all measured rates of slug movement (whether they be in units of millimetres/minute or millimetres/division deflection), as actual slug movements in millimetres over the particular range of strain considered. For slug movements measured in millimetres/minute it was thus necessary to determine the actual time interval corresponding to the particular range of strain considered, in order to obtain an actual slug movement in millimetres. Hence also, the need for continuous time recordings during test mentioned in Section 7.5.

In cases where continuous time recordings during test were taken, the actual time interval corresponding to any particular range of strain could be directly obtained. In these cases it was therefore possible to evaluate Sample Dilation ($\delta V/V$) and Poisson's Ratio (μ) for each increase in strain (working from the same starting point each time - usually the corrected start of the stress:strain curve as previously discussed in Section 7.7), and so to obtain curves of Instantaneous Sample Dilation:Strain ($\delta V/V : \varepsilon$) and Instantaneous Poisson's Ratio : Strain ($\mu : \varepsilon$).

In cases where continuous time recordings during test had not been taken, (see remarks in this respect in Section 7.5), this was not possible, and it was only possible to determine the value of Sample Dilation and Poisson's Ratio corresponding to the Selected Range of the Strain:strain curve in each case. To enable even these determinations, it was necessary to estimate the value of the actual rate of strain over the Selected Range, in order to obtain the time interval corresponding to the duration of the Selected Range. The basis on which this estimate could be made, is discussed in the following paragraphs.

Careful investigation of the deflection:time curves for the tests in which detailed time recordings were taken (Nos. 1,3,4 and 6), revealed the facts shown in Table XI on the following page:

TABLE XI ...../

TABLE XI

Test No.	RATE OF STRAIN		AVERAGE RATE OF STRAIN DURING TEST OVER SELECTED RANGE (=a)	REDUCTION IN RATE OF STRAIN OVER SELECTED RANGE (=b/a)	REMARKS
	NOMINAL inch/min.	CALIBRATED(=a) inch/min.	inch/min.	percent	
1	0.00250	0.00255	0.000933	36.5	Selected Range for each test includes the pronounced time-lag effects soon after test commencement.
3	0.00250	0.00244	0.000800	31.9	
6	0.00256	0.002500	0.000903	36.2	
4	0.01250	0.01200	0.004000	31.2	
				Av=33.9	

It was also observed that the Selected Range for each of the four tests listed above, included the range over which the pronounced time-lag effects occurred (discussed in Section 7.7). In the case of the two tests during which continuous time recordings had not been taken (Tests No.2 and 5), the strain at which the pronounced time-lag effects commenced had been noted down during the original test observations, and hence it was known that their Selected Ranges (as chosen), also included these effects.

It therefore seemed reasonable to conclude that approximately the same reduction in rate of strain had occurred over their respective Selected Ranges, as was found to be the case with Tests No. 1,3,4 and 6. As the latter tests had shown on average reduction in rate of strain over their Selected Ranges of 33.9% (calculated with respect to the calibrated rate of strain in each case) with very little variation among individual values, it was decided to adopt a value of 33⅓% of the calibrated rate of strain, or, where this was not available, simply 33⅓% of the nominal rate of strain, as the actual rate of strain during test over the Selected Range for the two tests concerned.

8.4 Initial Volume of test samples

Under this heading, a clear definition is in the first instance considered to be essential. By initial volume of test samples is meant the volume of test samples as at the commencement of the actual triaxial test, i.e., the sample volume corresponding to zero strain. It is this volume, relative to which all subsequent sample volume changes during test are considered to occur, and hence, it is this volume which is required for substitution in equation 7.6 for the evaluation of Poisson's Ratio.

In approaching the problem of determining this volume, the two factors which have a direct bearing on the initial volume of test samples, need to be considered in detail. These are:

(i) The volume of a test sample after being extruded from the cutting cylinder. (Let this volume be denoted V_1).

(ii) the subsequent change in this volume V_1 due to consolidation of the sample in the triaxial cell during the pre-shear condition. Let the volume at the completion of the consolidation process be denoted V_2.

With regard to item (i) above: It was previously mentioned that the test soil was a rather dry sandy soil, so that in

sliding ...../

TABLE XI

Test No.	RATE OF STRAIN		AVERAGE RATE OF STRAIN DURING TEST OVER SELECTED RANGE (=b)	REDUCTION IN RATE OF STRAIN OVER SELECTED RANGE (=b/a)	REMARKS
	NOMINAL inch/min.	CALIBRATED(=a) inch/min.	inch/min.	percent	
1	0.00250	0.00258	0.000933	36.5%	Selected Range for each test includes the pronounced time-lag effects soon after test commencement.
3	0.00250	0.00244	0.000809	31.7%	
6	0.00256	0.002500	0.000903	36.2	
4	0.01250	0.01282	0.004000	31.2%	
				Av.=33.9	

It was also observed that the Selected Range for each of the four tests listed above, included the range over which the pronounced time-lag effects occurred (discussed in Section 7.7). In the case of the two tests during which continuous time recordings had not been taken (Tests No.2 and 5), the strain at which the pronounced time-lag effects commenced had been noted down during the original test observations, and hence it was known that their Selected Ranges (as chosen), also included those effects.

It therefore seemed reasonable to conclude that approximately the same reduction in rate of strain had occurred over their respective Selected Ranges, as was found to be the case with Tests No. 1,3,4 and 6. As the latter tests had shown an average reduction in rate of strain over their Selected Ranges of 33.9% (calculated with respect to the calibrated rate of strain in each case) with very little variation among individual values, it was decided to adopt a value of 33⅓% of the calibrated rate of strain, or, where this was not available, simply 33⅓% of the nominal rate of strain, as the actual rate of strain during test over the Selected Range for the two tests concerned.

8.4 Initial Volume of test samples

Under this heading, a clear definition is in the first instance considered to be essential. By initial volume of test samples is meant the volume of test samples as at the commencement of the actual triaxial test, i.e., the sample volume corresponding to zero strain. It is this volume, relative to which all subsequent sample volume changes during test are considered and hence, it is this volume which is required for substitution in equation 7.8 for the evaluation of Poisson's Ratio.

In approaching the problem of determining this volume, the two factors which have a direct bearing on the initial volume of test samples, need to be considered in detail. These are:

(i) The volume of a test sample after being extruded from the cutting cylinder. (Let this volume be denoted V_1).

(ii) the subsequent change in this volume V_1 due to consolidation of the sample in the triaxial cell during the pre-shear condition. Let the volume at the completion of the consolidation process be denoted V_2.

With regard to item (i) above: It was previously mentioned that the test soil was a rather dry sandy soil, so that in

sliding ...../

sliding the cutting cylinder downwards while trimming a test sample end, subsequently, when extruding the test sample from the cutting cylinder, a thin layer of soil viz. that which was previously in contact with the inside face of the cutting cylinder, is loosened from the test sample and is lost. Therefore, the final sample diameter is slightly less than the actual inside diameter of the cutting cylinder and hence, the final sample volume is slightly less than the internal volume of the cutting cylinder.

Regarding item (ii): The consolidation process in the triaxial test cell which test samples are subjected to during the pre-shear condition, is a 3-dimensional effect. As yet, the change in volume of a test sample during this type of consolidation process can only be evaluated with difficulty for the case of saturated samples, but cannot be evaluated in the case of partially saturated samples such as is the case with the test samples being considered.

Thus, as both the quantities concerned cannot be readily evaluated (if at all), it is necessary to consider what arbitrary reduction in the internal volume of the cutting cylinder should be allowed in order to obtain the initial volume of a test sample. Further, as the same cutting cylinder was used throughout for trimming the test samples, it will also be assumed that the initial volume of all test samples was the same throughout.

The internal dimensions of the cutting cylinder were $1\frac{1}{2}$"dia. x 3"length, thus giving a volume of 86.89 cub.cm. Assuming that on removal from the cutting cylinder a sample is 1/64inch short in diameter, i.e. only $1\frac{31}{64}$ inches in diameter, its volume would be 85.09 cub.cm., thus a reduction of 1.80 cc. As regards the additional decrease in volume due to the consolidation process during the pre-shear condition, it is considered that this reduction, as effected by the chamber pressures used viz. 10lb/sq.in., 20lb/sq.in. and 30lb/sq.in., could not have been appreciable, since, from the normal 1-dimensional consolidation test which was conducted (Section 7.5), the test soil was found to be fairly heavily preconsolidated (p_c approx. 8Ton/sq.ft. – see Fig.18), with an undisturbed compression index value of 0.26.

On the basis of this information, it was decided to adopt a value of 88.0 cub.cm., as the initial volume of all test samples used for the test series. It is considered that a slight error in this value is not likely to cause a serious error in the value of Sample Dilation ($\delta V/V$) or Poisson's Ratio subsequently determined.

8.5 Calculation of Poisson's Ratio

On the following pages in this Section, the calculation of Poisson's Ratio for each test has been set out in tabular form: in the first instance, the calculation of Poisson's Ratio as obtained from the selected Range of each stress:strain curve (giving a value denoted by μ_{sel}), and secondly, the calculation of Poisson's Ratio (denoted μ_t) for each instantaneous value of Poisson's Ratio:strain curve, but always increase in strain along the stress:strain curve, but always working from the corrected start of the curve (i.e. bedding-in effects ignored).

In both cases the calculations follow along the same lines, which are broadly as follows for each particular range of strain considered:

(i) Determination of the time elapsed over the range considered.

(ii) Determination ..../

(ii) Determination of the slug movement (in mm.) due to leakage only.

(iii) Determination of the slug movement (in mm.) due to entry of the loading spindle into the test cell — using the rate of slug movement due to spindle entry determined as shown in Appendix V , Item 2.

(iv) Determination of the slug movement (in mm.) due to change in volume of the test sample only, by combining (ii) and (iii) above with the measured total slug movement due to the three combined effects during test.

(v) Determination of the change in volume of the test sample (δV) in units of 10^{-3} cub.cm. — using the value of the cross-sectional area of the capillary tube employed during the test series which was determined as shown in Appendix V , Item 1.

(vi) Calculation of the Sample Dilation ($\delta V/V$) - using the initial volume of test samples as 85.0cc. According to the discussion in Section 8.4.

(vii) Calculation of Poisson's Ratio based on equation 7.5 and using the relevant value of Young's Modulus (i.e. either the particular instantaneous or Esel value) as well as the relevant change in deviator stress value.

More specifically, however, the first calculations appearing are those for μ_{sel}. The calculations are accommodated in 3 pages and, for each test, deal with the various steps in the following order:

First page: Determination of data relevant to the Selected Range of each test only, viz.: the value of Young's Modulus (Esel); the change in deflection (strain; the nett slug movement due to the three combined effects; and the time elapsed over the Selected Range.
Second page: Items (i) to (vi) inclusive, as detailed above.
Third page: Item (vii) above.

Then follow the calculations for μ_i. As previously discussed (Section 8.5), it was only possible to determine μ_i for tests in which continuous time recordings during test had been carried out; i.e. Tests No. 1, 3, 4 and 6. The calculations in this case are more lengthy and occupy several pages. On the first calculation pages for each test, the items (i) to (vi) inclusive appear in columns all on one page for each increase in strain being considered. On the subsequent pages, item (vii) is set out in a similar manner, using the instantaneous values of Young's Modulus shown calculated in Section 7.6. The calculation of values of μ_i was in each case not continued very far past the end of the linear range of the stress:strain curve, and particularly not past the peak, as naturally Poisson's Ratio becomes a doubtful concept once the limit of the linear range has been exceeded (See Chapter 2). However, the calculation of values of Sample Dilation ($\delta V/V$) were continued over the full range of each test.

For the results of all these calculations, it was possible to prepare curves of Instantaneous Sample Dilation : Strain and Instantaneous Poisson's Ratio : Strain as mentioned in Section 8.3. These curves are presented in each case subsequent to the μ_i calculation sheets and are shown plotted below the relevant stress:strain curve (bedding-in effects ignored) for each test.

The ...../

The following more detailed points about the calculations should, however, be noted:

An arrow above figures relating to slug movement should be interpreted in accordance with the meaning given at the head of Table IX, Section 8.2. In the case of the calculations pertaining to Appel, such arrows have been omitted, and instead, algebraic signs have been inserted in front of these figures. These algebraic signs merely serve to indicate how the figures have been combined in determining the slug movement due to change in volume of the test sample only (item (iv) on the previous page), and were arrived at as follows:

(a) Slug movement due to leakage from the test cell is from right to left in the capillary tube - denoted ⟵

(b) Slug movement due to entry of the loading spindle into the test cell is from left to right in the capillary tube - denoted ⟶

(c) Slug movement due to the combined effects of leakage, spindle entry and sample volume change, is from right to left generally (i.e. over the main portion of the Main Slug Movement : Deflection curve), and is denoted ⟵ in these cases.

Thus, numerically: (b) + (c) - (a) = M, say,

gives the slug movement due to change in volume of the test sample only.

In cases where the slug movement (c) above is from left to right (such as sometimes happens at the start of tests), the relevent figure for item (c) has been given a negative sign.

A positive value for M denotes that the slug movement due to change in volume of the test sample only is from right to left in the capillary tube, i.e., the sample has experienced a volume decrease. Hence, also, a sample volume decrease yields a positive value for the Sample Dilation $\delta V/V$, which is thus in accordance with the sign convention adopted as discussed in Section 7.1.

YOUNGS' MODULUS, SLUG MOVEMENT AND TIME CALCS. BASED ON THE SELECTED RANGE OF EACH STRESS:STRAIN CURVE.

TEST NUMBER	STRAIN (ε) percent			DEVIATOR STRESS (σ_d) lb/sq in			YOUNGS' MODULUS (E_{sec}) $= \frac{\Delta\sigma_d}{\Delta\varepsilon}$ lb/sq in	CORRECTED DEFLECTION 10^{-3} inches			CORRECTED SLUG SCALE READING millimetres			ELAPSED TIME mins secs		
	AT START OF SELECTED RANGE	AT END OF SELECTED RANGE	DIFFERENCE $=\Delta\varepsilon$ =CHANGE IN STRAIN OVER SELECTED RANGE	AT START OF SELECTED RANGE	AT END OF SELECTED RANGE	DIFFERENCE $\Delta\sigma_d$ =CHANGE IN STRESS OVER SELECTED RANGE		AT START OF SELECTED RANGE	AT END OF SELECTED RANGE	DIFFERENCE =CHANGE IN DEFLECTION OVER SELECTED RANGE $= \alpha$	AT START OF SELECTED RANGE $= P$	AT END OF SELECTED RANGE $= q$	$q-P$ = DIFF. IN SLUG MOVEMENT DUE TO THREE COMB. EFFECTS OVER SELECTED RANGE	AT START OF SELECTED RANGE	AT END OF SELECTED RANGE	DIFFERENCE =TIME ELAPSED OVER SELECTED RANGE
1.	0·033	0·467	0·434	2·54	37·25	34·71	8015	·1	14	13	0	−125·25	−125·25	00″ 27′	14″ 23′	13″ 56′
3.	0·200	0·467	0·267	7·34	27·89	20·55	7697	6	14	8	−120·00	−96·50	−84·50	03″ 15′	13″ 08′	9″ 53′
4.	0·583	0·917	0·333	7·86	39·75	31·89	9567	17·5	27·5	10	+17·00	−51·50	−68·50	01″ 40′	04″ 10′	2″ 30′
6.	0·233	0·633	0·400	6·76	39·25	32·49	8125	7	19	12	−12·00	−126·00	−114·00	04″ 06′	17″ 22′	13″ 16′

	CALIBRATED RATE OF STRAIN $=\dot{\varepsilon}$ inches/minute	$\dot{\varepsilon} \times 33\frac{1}{3}\%$ =ESTIMATED AVERAGE RATE OF STRAIN DURING TEST OVER SELECTED RANGE		$\frac{a}{m}$ =TIME ELAPSED OVER SELECTED RANGE minutes
		inches/minute	cms de Flect / minute	
5.	0·01270	0·00423	4·23	1·77″
2.	0·01250 *	0·00417	4·17	3·00″

* NOMINAL RATE OF STRAIN

CALCULATION OF SAMPLE DILATION (ΔV) BASED ON THE SELECTED RANGE OF EACH STRESS:STRAIN CURVE.

TEST NUMBER	TIME ELAPSED OVER SELECTED RANGE		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER SELECTED RANGE DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECT OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER SELECTED RANGE DUE TO SPINDLE ENTRY ONLY	% ALL MEASURED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER SELECTED RANGE	CROSSECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOLUME ΔV OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	MIN SEC	MINUTES	mm/min	millimetres	div/div		millimetres	millimetres	millimetres	sq mm	$\times 10^{-3}$ cc	cubic cm	units of 10^{-4}
1	M S 13 56	13·933	0·92 2·80	12·818 39·012	1·614	13	20·98	12·523	133·414 107·220	1·9935	265·98 213·74	85·0	3·13 2·51
2	3 00	3·000	1·27 4·00	3·810 12·000	1·614	12·5	20·175	11·150	127·865 119·675	1·9935	254·90 238·57	85·0	3·00 2·81
3	9 53	9·883	1·54 2·667	15·220 26·355	1·614	8	12·912	8·450	82·192 71·057	1·9935	163·85 141·65	85·0	1·93 1·67
4	2 30	2·500	0·875 2·000	2·187 5·000	1·614	10	16·140	6·850	82·433 79·640	1·9935	164·37 158·76	85·0	1·93 1·87
5	1 46	1·770	0·500 1·000	0·885 1·770	1·614	7·5	12·105	6·500	76·220 75·335	1·9935	151·94 150·18	85·0	1·79 1·77
6	13 16	13·267	0·875 0·947	11·60 12·564	1·614	12	19·368	11·400	121·750 120·804	1·9935	242·73 240·82	85·0	2·86 2·83

*NOTE WHERE TWO SETS OF FIGURES ARE SHOWN (ONE BELOW THE OTHER), THESE ARE INTENDED FOR COMPARISON PURPOSES:
THE UPPER FIGURES CORRESPOND TO CALCS BASED ON THE RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY MEASURED IMMEDIATELY BEFORE TEST
THE LOWER FIGURES CORRESPOND TO CALCS BASED ON THE RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY MEASURED IMMEDIATELY AFTER TEST
THE UPPER FIGURES ARE CONSIDERED TO LEAD TO MORE RELIABLE VALUES OF SAMPLE DILATION ($\frac{\Delta V}{V}$) AND POISSONS RATIO (μ)

CALCULATION OF POISSONS' RATIO (μ) BASED ON THE SELECTED RANGE OF EACH STRESS:STRAIN CURVE*

TEST NUMBER	SAMPLE DILATION $\frac{\Delta V}{V}$ units of 10^{-5}	YOUNGS' MODULUS E_{Sat} lb/sq in	$\frac{\Delta V}{V} \times E_{Sat}$ lb/sq in	CHANGE IN DEVIATOR STRESS $(\Delta\sigma_d) = \Delta(\sigma_1 - \sigma_3)$ OVER SELECTED RANGE lb/sq in	$\frac{\frac{\Delta V}{V} \times E_{Sat}}{\Delta\sigma_d} = N$	$1 - N$	POISSONS' RATIO $(\mu_{Sat}) = \frac{1}{2}[1 - N]$
1	3·13	8015	25·079	34·71	0·7225	0·2775	0·1387
	2·31		20·155		0·5807	0·4193	0·2097
2	3·00	7750	23·241	32·29	0·7198	0·2802	0·1401
	2·81		21·752		0·6736	0·3264	0·1632
3	1·93	7697	14·837	20·55	0·7220	0·2780	0·1390
	1·67		12·827		0·6242	0·3758	0·1879
4	1·93	9567	18·500	31·89	0·5801	0·4199	0·2099
	1·87		17·869		0·5605	0·439	0·2198
5	1·79	7212	12·892	18·03	0·7150	0·2850	0·1425
	1·77		12·742		0·7067	0·2933	0·1466
6	2·86	8125	23·202	32·49	0·7141	0·2859	0·1429
	2·83		23·020		0·7085	0·2915	0·1457

*SEE FOOTNOTE ON THE PREVIOUS PAGE

TEST Nº 1 CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION(W)

RANGE OF STRAIN CONSIDERED	TIME CORRESPONDING TO CENTRAL ORDINATE		RANGE OF MEASUREMENTS ENCOMPASSED BY SAMPLE LENGTH AND LEVERAGE AREA		RANGE OF MEASUREMENTS DUE TO SAMPLE LENGTH ALONE	NUMBER OF DIVISIONS OVER WHICH RANGE IS CONSIDERED	TOTAL MEASUREMENT ENCOMPASSED BY SAMPLE LENGTH ALONE	OBSERVED MEASUREMENT OVER RANGE CONSIDERED DUE TO THREE CONSECUTIVE OFFSETS	BLOCK SLIDE MOVEMENT DUE TO SAMPLE LENGTH ONLY OVER RANGE CONSIDERED	CROSS-SECTIONAL AREA OF CAPILLARY TUBE	EFFECTIVE LENGTH OF SAMPLE (LENGTH OF SAMPLE LESS MENISCUS DEPRESSION)		SAMPLE DILATION W
0.000 - 0.033	00 27	0.450	0.92	- 0.414	1.614	1	+ 1.914	0	+ 1.900	1.9935	2.59	80.0	0.05
0.000 - 0.067	00 58	0.917	0.92	- 0.844	1.614	2	+ 3.228	+ 8.0	+ 8.434	1.9935	5.28	84.0	0.08
0.000 - 0.100	01 32	1.533	0.92	- 1.410	1.614	3	+ 4.842	+ 9.0	+ 9.432	1.9935	14.99	85.0	0.17
0.000 - 0.133	07 27	7.450	0.92	- 6.854	1.614	4	+ 6.456	+ 21.0	+ 20.602	1.9935	41.07	81.0	0.48
0.000 - 0.167	08 10	8.667	0.92	- 7.974	1.614	5	+ 8.070	+ 31.0	+ 31.096	1.9935	61.99	85.0	0.73
0.000 - 0.200	06 21	9.350	0.92	- 8.608	1.614	6	+ 9.684	+ 44.0	+ 45.082	1.9935	89.87	80.0	1.08
0.000 - 0.233	10 15	10.250	0.92	- 9.430	1.614	7	+ 11.298	+ 53.75	+ 55.618	1.9935	111.87	85.0	1.30
0.000 - 0.267	10 52	10.967	0.92	- 9.998	1.614	8	+ 12.912	+ 63.75	+ 66.664	1.9935	132.89	80.0	1.56
0.000 - 0.300	11 28	11.467	0.92	- 10.550	1.614	9	+ 14.526	+ 74.0	+ 77.976	1.9935	155.48	85.0	1.83
0.000 - 0.333	12 04	12.067	0.92	- 11.102	1.614	10	+ 16.140	+ 84.0	+ 89.038	1.9935	177.80	85.0	2.09
0.000 - 0.367	12 39	12.650	0.92	- 11.638	1.614	11	+ 17.754	+ 94.0	+100.116	1.9935	199.58	85.0	2.35
0.000 - 0.400	13 15	13.250	0.92	- 12.190	1.614	12	+ 19.368	+105.0	+112.178	1.9935	223.63	85.0	2.63
0.000 - 0.433	13 40	13.800	0.92	- 12.696	1.614	13	+ 20.982	+114.75	+123.036	1.9935	245.17	85.0	2.88
0.000 - 0.467	14 23	14.383	0.92	- 13.232	1.614	14	+ 22.596	+125.25	+134.614	1.9935	268.35	85.0	3.15
0.000 - 0.500	14 50	14.917	0.92	- 13.724	1.614	15	+ 24.210	+135.25	+145.736	1.9935	290.28	85.0	3.42

TEST Nº 1 CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION(Δ) (Cont.)

RANGE OF DILATION CONSIDERED	TIME INTERVAL CONSIDERED		RATE OF MOVEMENT DUE TO LEAKAGE ONLY	MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF MOVEMENT DUE TO SAMPLE ENTRY ONLY	NUMBER OF DIVISIONS DEFLECTED OVER RANGE CONSIDERED	MOVEMENT OVER RANGE CONSIDERED DUE TO SAMPLE ENTRY ONLY	OBSERVED MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS	MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	EXPANSION-AL AREA OF CAPILLARY TUBE	EXPANSION IN VOL (CV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	RATIO SAMPLE VOLUME V	SAMPLE DIAMETER d
0.700 - 0.733	28 50	11.500	0.92	- 14.960	1.614	16	+ 82.894	+ 144.5	+152.054	1.9935	311.11	85.0	3.48
0.800 - 0.867	16 03	16.050	0.92	- 14.766	1.614	17	+ 87.438	+ 154.25	+162.902	1.9935	332.76	85.0	3.91
0.900 - 0.933	16 37	16.617	0.92	- 15.288	1.614	18	+ 29.052	+ 164.40	+178.164	1.9935	354.17	85.0	4.16
0.933 - 0.967	17 12	17.200	0.92	- 15.824	1.614	19	+ 30.666	+ 174.0	+188.342	1.9935	376.47	85.0	4.43
0.900 - 0.967	17 40	17.667	0.92	- 16.254	1.614	20	+ 32.280	+ 180.25	+196.276	1.9935	391.26	85.0	4.60
0.900 - 0.933	17 17	17.283	0.92	- 15.920	1.614	21	+ 33.894	+ 192.0	+209.074	1.9935	416.78	85.0	4.90
0.900 - 0.867	18 58	17.967	0.92	- 16.570	1.614	24	+ 38.736	+ 219.0	+239.346	1.9935	477.19	85.0	5.61
0.900 - 0.800	21 47	21.533	0.92	- 19.810	1.614	27	+ 43.578	+ 243.0	+267.2	1.9935	532.80	85.0	6.27
0.900 - 0.800	22 57	22.950	0.92	- 21.114	1.614	30	+ 48.420	+ 266.0	+298.506	1.9935	594.71	85.0	6.98
0.900 - 0.733	25 56	25.933	0.92	- 23.898	1.614	36	+ 58.104	+ 300.25	+334.696	1.9935	666.59	85.0	7.84
0.900 - 0.800	28 31	28.517	0.92	- 26.256	1.614	42	+ 67.788	+ 331.75	+371.302	1.9935	744.18	85.0	8.76
0.900 - 0.800	31 13	31.217	0.92	- 28.750	1.614	48	+ 77.472	+ 356.5	+405.252	1.9935	807.27	85.0	9.50
0.900 - 0.800	33 50	33.833	0.92	- 31.126	1.614	54	+ 87.156	+ 375.0	+431.030	1.9935	859.26	85.0	10.11
0.900 - 0.800	36 21	36.350	0.92	- 33.442	1.614	60	+ 96.840	+ 389.5	+452.992	1.9935	902.85	85.0	10.62
0.900 - 0.800	37 52	37.867	0.92	- 34.838	1.614	66	+106.524	+ 401.0	+472.686	1.9935	942.10	85.0	11.08

TEST Nº1 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$ (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECT OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS.	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS-SECTIONAL AREA OF CAPILLERY TUBE	HENCE CHANGE IN VOL (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
Division*	min sec	minutes	mm/min	millimetres	mm/division		millimetres	millimetres	millimetres	sq.cm	cubic mm (µl)	cubic cm	units of 10^{-4}
0.000 - 2.400	41 03	41.383	0.92	- 38.079	1.614	72	+116.208	+ 409.6	+ 487.134	1.4935	771.10	85.0	11.42
0.000 - 2.600	43 49	43.817	0.92	- 40.312	1.614	78	+125.892	+ 413.0	+ 498.580	1.4935	993.92	88.0	11.48
0.000 - 2.800	46 10	46.167	0.92	- 42.474	1.614	84	+136.076	+ 416.2	+ 507.602	1.4936	1011.96	85.0	11.90
0.000 - 3.000	48 31	48.517	0.92	- 44.636	1.614	90	+145.760	+ 414.6	+ 516.124	1.4935	1026.90	85.0	12.08
0.000 - 3.200	50 50	50.833	0.93	- 47.275	1.614	96	+154.944	+ 414.55	+ 521.919	1.0935	1045.44	85.0	12.24
0.000 - 3.400	53 12	53.200	1.10	- 50.500	1.614	102	+164.628	+ 412.6	+ 519.008	1.0935	1033.84	85.0	12.16
0.000 - 3.600	55 32	55.533	1.27	- 70.327	1.614	108	+174.312	+ 409.75	+ 513.538	1.0935	1025.73	85.0	12.04
0.000 - 3.800	57 54	57.900	1.44	- 43.336	1.614	114	+183.996	+ 406.75	+ 507.370	1.0935	1011.44	85.0	11.90
0.000 - 4.000	60 14	60.233	1.61	- 95.975	1.614	120	+193.680	+ 404.5	+ 501.305	1.0935	999.15	85.0	11.78
0.000 - 4.200	62 35	62.583	1.78	-111.392	1.614	126	+203.364	+ 400.75	+ 492.710	1.0935	982.23	85.0	11.56
0.000 - 4.400	64 53	64.883	1.95	-126.239	1.614	132	+213.048	+ 396.85	+ 482.775	1.0935	964.41	85.0	11.32
0.000 - 4.600	67 14	67.233	2.12	-'39.834	1.614	138	+222.732	+ 392.8	+ 473.898	1.0935	948.38	85.0	11.08
0.000 - 4.800	69 31	69.517	2.29	-153.194	1.614	144	+232.416	+ 388.0	+ 461.929	1.0935	919.45	85.0	10.80
0.000 - 5.000	71 50	71.833	2.46	-170.709	1.614	150	+242.100	+ 384.0	+ 448.391	1.0935	895.86	85.0	10.52
0.000 - 5.200	74 05	74.083	2.63	-194.838	1.614	156	+251.784	+ 380.75	+ 437.538	1.0935	872.55	85.0	10.26

TEST Nº 1 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO(μ_i)

RANGE OF STRAIN CONSIDERED	SAMPLE DILATION $\frac{\Delta V}{V}$	YOUNGS' MODULUS E	$\frac{\Delta V}{V} \times E$	CHANGE IN DEVIATOR STRESS $\Delta(\sigma_1-\sigma_3)$ OVER RANGE CONSIDERED	$\frac{\frac{\Delta V}{V} \times E}{\Delta(\sigma_1-\sigma_3)} = N$	$1 - N$	POISSONS' RATIO $\mu_i = \frac{1}{2}[1-N]$
0.000 - 0.033	0.05	7697	0.3509	7.64	0.0909	0.9191	0.4595
0.000 - 0.067	0.08	8513	0.7016	8.20	0.1936	0.8064	0.4032
0.000 - 0.100	0.17	8770	1.1109	8.77	0.1760	0.8300	0.4150
0.000 - 0.133	0.48	7549	3.6038	10.04	0.3609	0.6391	0.3195
0.000 - 0.167	0.73	8383	6.1196	14.00	0.4351	0.5649	0.2825
0.000 - 0.200	1.06	8400	8.9040	16.80	0.5300	0.4700	0.2350
0.000 - 0.233	1.30	8488	11.0357	19.78	0.5579	0.4421	0.2210
0.000 - 0.267	1.56	8401	13.1056	22.43	0.5843	0.4157	0.2078
0.000 - 0.300	1.83	8410	15.3903	25.23	0.6100	0.3900	0.1950
0.000 - 0.333	2.09	8298	17.3218	27.60	0.6276	0.3724	0.1862
0.000 - 0.367	2.36	8161	19.1784	29.95	0.6403	0.3597	0.1798
0.000 - 0.400	2.63	8160	21.4608	32.64	0.6575	0.3425	0.1712
0.000 - 0.433	2.88	8142	23.1610	34.82	0.6652	0.3348	0.1674
0.000 - 0.467	3.16	8272	25.1918	37.23	0.6766	0.3234	0.1617
0.000 - 0.500	3.42	8318	26.8744	39.29	0.6840	0.3160	0.1580

TEST Nº 1 CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO(μ) (Con't)

RANGE OF STRAIN CONSIDERED	SAMPLE DILATION $\frac{\Delta V}{V}$	YOUNGS MODULUS E	$\frac{\Delta V}{V} \times E$	CHANGE IN DEVIATOR STRESS($\Delta\sigma_d$) $=\Delta(\sigma_1-\sigma_3)$ OVER RANGE CONSIDERED	$\frac{\frac{\Delta V}{V} \times E}{\Delta\sigma_d} = N$		POISSONS RATIO (μ_s) $\frac{1}{2}[1-N]$
0.000 - 0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.000 - 0.287	3.91	7414	29.7803	43.20	0.6896	0.3104	0.1552
0.200 - 0.400	4.16	7563	31.6133	45.39	0.6964	0.3036	0.1517
0.400 - 0.600	4.43	7450	33.0035	47.16	0.6998	0.3002	0.1501
0.000 - 0.667	4.60	7319	33.6874	48.07	0.6896	0.3104	0.1552
0.000 - 0.700	4.90	7261	35.5709	50.53	0.7039	0.2961	0.1480
0.000 - 0.800	5.61	6999	39.2644	55.39	0.7087	0.2913	0.1456
0.000 - 0.900	5.97	6669	41.9146	60.01	0.6987	0.3013	0.1506
0.000 - 1.000	6.26	6989	43.7805	63.78	0.6860	0.3140	0.1570
0.000 - 1.200	7.04	6761	47.5944	70.53	0.6744	0.3256	0.1628
0.000 - 1.400	7.76	6564	50.9366	76.09	0.6695	0.3305	0.1652
0.000 - 1.600	8.40	6464	54.3456	79.11	0.6869	0.3131	0.1566
0.000 - 1.800	10.15	6189	62.8034	87.91	0.7144	0.2856	0.1428
0.000 - 2.000	10.62	6050	64.2810	89.04	0.7218	0.2782	0.1391

TEST No 1

(a)

Nom Rate of Strain = 0.0054 in/min

σ_3 = 30 lb/Sq in

e = 0.737

ω = 0.59%

S_r = 1.95%

$(\sigma_1 - \sigma_3)$ lbs/sq in

end of linear range

STRAIN E (percent)

Eqn (b)

$\frac{\Delta V}{V}$ (% per cent)

Compression

(c)

STRAIN E (percent)

POISSONS RATIO μ_d

FIG. 49.

TEST Nº 3 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		HENCE SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECT OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS-SECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	min sec	seconds	millimetres	millimetres	mm/second		millimetres	millimetres	millimetres	sq mm	cubic mm $(\times 10^{-3}$ cc)	cubic cm	units of 10^{-2}
0.000 - 0.167	00 33	2.583	1.54	- 3.978	1.614	5	+ 8.070	+ 5.00	+ 9.092	1.9935	18.12	85.0	0.21
0.000 - 0.300	03 15	3.250	1.54	- 5.005	1.614	6	+ 9.684	+ 10.00	+ 14.679	1.9935	29.25	85.0	0.34
0.000 - 0.433	08 07	8.117	1.54	- 12.500	1.614	7	+ 11.298	+ 25.50	+ 24.298	1.9935	48.44	85.0	0.57
0.000 - 0.367	11 19	11.317	1.54	- 17.428	1.614	11	+ 17.754	+ 44.00	+ 64.326	1.9935	128.23	85.0	1.51
0.000 - 0.400	11 57	11.950	1.54	- 18.403	1.614	12	+ 19.368	+ 76.00	+ 76.965	1.9935	153.43	85.0	1.80
0.000 - 0.467	13 08	13.133	1.54	- 20.225	1.614	14	+ 22.596	+ 92.50	+ 90.971	1.9935	181.30	85.0	2.12
0.000 - 0.500	13 41	13.683	1.54	- 21.072	1.614	15	+ 24.210	+ 100.00	+ 109.132	1.9935	217.57	85.0	2.56
0.000 - 0.567	14 46	14.767	1.54	- 22.741	1.614	17	+ 27.438	+ 125.00	+ 129.697	1.9935	258.55	85.0	3.04
0.000 - 0.700	16 57	16.950	1.54	- 26.103	1.614	21	+ 33.894	+ 158.00	+ 165.791	1.9935	330.50	85.0	3.89
0.000 - 0.834	19 09	19.150	1.54	- 29.260	1.614	25	+ 40.350	+ 197.50	+ 198.590	1.9935	395.89	85.0	4.66
0.000 - 1.000	21 24	21.400	1.54	- 32.956	1.614	30	+ 48.420	+ 218.25	+ 230.714	1.9935	459.93	85.0	5.41
0.000 - 1.167	23 41	23.683	1.54	- 36.472	1.614	35	+ 56.490	+ 234.00	+ 254.019	1.9935	506.39	85.0	5.96
0.000 - 1.333	25 51	25.850	1.54	- 39.809	1.614	40	+ 64.560	+ 247.75	+ 277.501	1.9935	543.03	85.0	6.39
0.000 - 1.500	27 51	27.850	1.54	- 42.899	1.614	45	+ 72.630	+ 256.50	+ 286.241	1.9935	570.62	85.0	6.71
0.000 - 1.667	29 57	29.950	1.54	- 46.133	1.614	50	+ 80.700	+ 261.75	+ 296.127	1.9935	590.73	85.0	6.95

TEST Nº 3 CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$ (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECT OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THESE COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSSECTIONAL AREA OF CAPILLERY TUBE	HENCE CHANGE IN VOL. (Δv) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
percent	(hr. min.	(hundredths	(mm./min.	millimetres	(mm./min.	(hundredths	millimetres	millimetres	millimetres	sq. mm.	cubic mm.	cubic mm.	(cubic mm./cubic mm.)
0.000 - 1.833	31 53	31.883	1.54	- 49.100	1.614	55	+ 88.770	+ 244.26	+ 303.920	1.4936	808.88	85.0	9.15
0.000 - 2.000	33 53	33.883	1.54	- 52.160	1.614	60	+ 96.840	+ 265.20	+ 310.160	1.4936	815.30	85.0	9.59
0.000 - 2.167	35 53	35.883	1.54	- 55.260	1.614	65	+ 104.910	+ 288.10	+ 315.150	1.4936	820.75	85.0	9.38
0.000 - 2.333	37.62	37.617	1.58	- 59.630	1.614	70	+ 112.980	+ 284.30	+ 317.660	1.4936	834. 4	85.0	9.45
0.000 - 2.667	41 51	41.850	1.66	- 69.471	1.614	80	+ 129.120	+ 260.75	+ 320.599	1.4936	838.78	85.0	9.51
0.000 - 2.833	43 49	43.817	1.70	- 74.489	1.614	85	+ 137.190	+ 258.25	+ 320.051	1.4936	839.20	85.0	9.53
0.000 - 3.000	45 49	45.817	1.74	- 79.722	1.614	90	+ 145.260	+ 256.75	+ 322.288	1.4936	849.48	85.0	9.56
0.000 - 3.167	47 45	47.750	1.78	- 84.995	1.614	95	+ 153.330	+ 258.00	+ 329.335	1.4936	842.87	85.0	9.55
0.000 - 3.333	49 45	49.750	1.82	- 90.545	1.614	100	+ 161.400	+ 261.04	+ 329.105	1.4936	842.12	85.0	9.55
0.000 - 3.500	51 41	51.683	1.86	- 96.130	1.614	105	+ 169.470	+ 263.50	+ 328.340	1.4936	838.60	85.0	9.51
0.000 - 3.667	53 41	53.683	1.90	-101.998	1.614	110	+ 177.540	+ 241.50	+ 317.040	1.4936	832.02	85.0	9.44
0.000 - 4.000	57 39	57.650	1.97	-113.368	1.614	120	+ 193.680	+ 220.75	+ 309.280	1.4936	817.71	85.0	9.27
0.000 - 4.167	59 39	59.650	2.01	-119.496	1.614	125	+ 201.750	+ 223.75	+ 305.804	1.4936	809.88	85.0	9.17
0.000 - 4.333	61 58	61.633	2.02	-125.240	1.614	130	+ 209.820	+ 217.75	+ 301.202	1.4936	800.49	85.0	9.06
0.000 - 4.667	65 34	65.567	2.13	-139.688	1.614	140	+ 225.960	+ 207.75	+ 294.052	1.4936	886.19	85.0	8.90

TEST Nº3 CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION ($\frac{\Delta V}{V}$) (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECT OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS.	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS SECTIONAL AREA OF CAPILLERY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
decimal	min sec	minutes	mm/minute	millimetres	mm/minute		millimetres	millimetres	millimetres	sq. mm	cubic mm (μl or cc)	cubic cm	units of 10^{-3}
0.000 - 5.000	69 22	69.367	2.21	- 153.301	1.614	150	+ 242.100	+ 190.75	+ 280.549	1.9935	559.24	85.0	6.70
0.000 - 5.167	71 17	71.283	2.25	- 160.387	1.614	155	+ 250.170	+ 191.25	+ 281.033	1.9935	560.24	85.0	6.59
0.000 - 5.333	73 17	73.283	2.29	- 167.818	1.614	160	+ 258.240	+ 185.50	+ 275.922	1.9935	550.05	85.0	6.47
0.000 - 5.500	75 13	75.217	2.32	- 174.503	1.614	165	+ 266.308	+ 179.75	+ 271.882	1.9935	541.94	85.0	6.37
0.000 - 5.667	77 11	77.183	2.36	- 182.159	1.614	170	+ 274.378	+ 173.75	+ 265.968	1.9935	530.31	85.0	6.24
0.000 - 5.833	79 11	79.183	2.40	- 190.039	1.614	175	+ 282.435	+ 166.75	+ 259.146	1.9935	516.61	85.0	6.08
0.000 - 6.000	81 04	81.067	2.44	- 197.803	1.614	180	+ 290.500	+ 160.75	+ 243.447	1.9935	505.22	85.0	5.94
0.000 - 6.167	83 00	83.000	2.48	- 205.840	1.614	185	+ 298.565	+ 154.00	+ 246.785	1.9935	491.85	85.0	5.79
0.000 - 6.333	84 55	84.933	2.52	- 214.031	1.614	190	+ 306.630	+ 148.00	+ 240.569	1.9935	479.63	85.0	5.64
0.000 - 6.500	86 51	86.850	2.56	- 222.336	1.614	195	+ 314.695	+ 141.75	+ 234.109	1.9935	466.70	85.0	5.49
0.000 - 7.000	92 38	92.633	2.67	- 247.330	1.614	210	+ 338.890	+ 125.75	+ 213.310	1.9935	429.22	85.0	5.05

TEST Nº 3 CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO(ν)

RANGE OF STRAIN CONSIDERED (STRAIN)	SAMPLE DILATION $\frac{\Delta V}{V}$	SECANT MODULUS E (kg/cm²)	$\frac{\Delta V}{V} \times E$	CHANGE IN DEVIATOR STRESS $\Delta(\sigma_1-\sigma_3)$ OVER RANGE CONSIDERED (kg/cm²)	$\frac{\frac{\Delta V}{V} \times E}{\Delta(\sigma_1-\sigma_3)} = N$	$1 - N$	POISSONS RATIO $\nu = \frac{1-N}{2}$
0.100 - [illegible]	0.39	7340	2.[illegible]	7.31	0.2812	[illegible]	.[illegible]44
0.100 - 0.333	[illegible]	7737	4.410[illegible]	3[illegible]	[illegible]	[illegible]	0.2957
0.100 - 0.[illegible]67	1.51	7865	[illegible].8722	21.[illegible]	0.1[illegible]	0.[illegible]345	[illegible].[illegible]77
0.100 - 0.400	[illegible].80	7[illegible]00	14.1480	[illegible]A.[illegible]	0.[illegible]	[illegible].[illegible]	[illegible].[illegible]000
0.100 - 0.467	2.[illegible]	7599	[illegible]7.[illegible]257	[illegible].[illegible]	0.[illegible]	[illegible].[illegible]	0.[illegible]41
0.100 - 0.[illegible]	2.[illegible]	7478	19.1437	[illegible].91	0.[illegible]	0.[illegible]	[illegible].[illegible]
0.100 - 0.[illegible]67	3.01	7[illegible]	22.0157	[illegible].[illegible]	0.[illegible]	0.[illegible]	0.1745
0.100 - 0.[illegible]	3.[illegible]	6[illegible]	26.[illegible]	[illegible].[illegible]	0.[illegible]	0.3[illegible]	[illegible].[illegible]
0.100 - 0.[illegible]	4.[illegible]	6344	29.[illegible]	[illegible].[illegible]	0.6356	0.3648	0.1[illegible]
0.100 - 1.000	[illegible].[illegible]	5787	31.1981	[illegible].[illegible]	0.6011	0.[illegible]6	[illegible].[illegible]
0.100 - 1.[illegible]	5.26	5217	[illegible].[illegible]	[illegible].67	0.[illegible]	[illegible].[illegible]	[illegible].[illegible]
0.100 - 1.333	6.[illegible]	4709	[illegible].[illegible]	[illegible].[illegible]	[illegible].6163	[illegible].[illegible]	[illegible].[illegible]
0.100 - 1.500	6.71	[illegible]248	[illegible].[illegible]	[illegible].[illegible]	[illegible].[illegible]3	[illegible].[illegible]	[illegible].[illegible]504
0.100 - 1.[illegible]	6.[illegible]	3071	[illegible].[illegible]	[illegible].[illegible]	0.4435	[illegible].[illegible]	[illegible].[illegible]

TEST No 3.

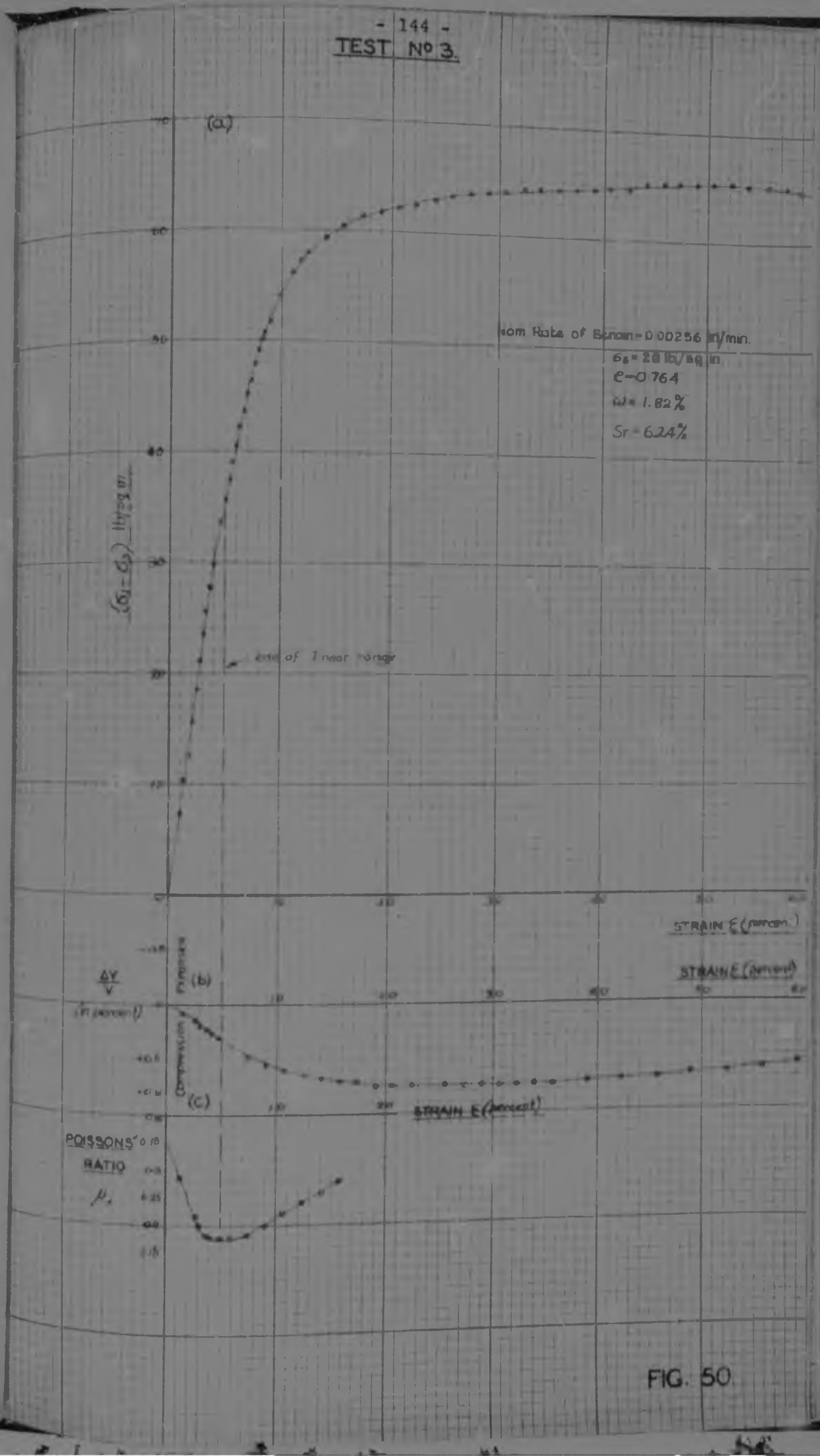


FIG. 50.

TEST No 1 CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION ($\frac{\Delta V}{V}$)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVISIONS DEFLECTED OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO COMBINED EFFECTS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSSECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
percent	min sec	minutes	mm/min	millimetres	mm/division		millimetres	millimetres	millimetres	sq mm	cu mm ($\times 10^{-3}$ cc)	cu cm	percent
0.000 - 0.063	00 24	0.400	0.875	- 0.350	1.614	2.5	+ 4.035	- 2.75	+ 0.935	1.9935	1.86	85.0	0.02
0.000 - 0.167	00 37	0.617	0.875	- 0.540	1.614	5.0	+ 8.070	- 7.00	+ 0.530	1.9935	1.06	85.0	0.01
0.000 - 0.250	00 45	0.750	0.875	- 0.656	1.614	7.5	+ 12.105	- 9.00	+ 3.449	1.9935	4.86	85.0	0.06
0.000 - 0.333	00 56	0.933	0.875	- 0.816	1.614	10.0	+ 16.140	- 12.00	+ 3.304	1.9935	6.63	85.0	0.08
0.000 - 0.417	01 07	1.117	0.875	- 0.977	1.614	12.5	+ 20.175	- 14.25	+ 4.948	1.9935	9.86	85.0	0.12
0.000 - 0.500	01 24	1.400	0.875	- 1.225	1.614	15.0	+ 24.210	- 16.50	+ 6.485	1.9935	12.93	85.0	0.15
0.000 - 0.583	01 40	1.667	0.875	- 1.459	1.614	17.5	+ 28.245	- 17.00	+ 9.796	1.9935	19.61	85.0	0.23
0.000 - 0.667	03 47	3.117	0.875	- 1.727	1.614	20.0	+ 32.280	- 7.80	+ 22.053	1.9935	41.96	85.0	0.52
0.000 - 0.750	03 30	3.500	0.875	- 3.062	1.614	22.5	+ 36.315	+ 15.00	+ 46.253	1.9935	92.00	85.0	1.08
0.000 - 0.833	03 48	3.800	0.875	- 3.325	1.614	25.0	+ 40.350	+ 30.50	+ 67.525	1.9935	134.61	85.0	1.58
0.000 - 0.917	04 10	4.167	0.875	- 3.646	1.614	27.5	+ 44.385	+ 51.50	+ 92.269	1.9935	183.88	85.0	2.16
0.000 - 1.000	04 26	4.433	0.875	- 3.879	1.614	30.0	+ 48.420	+ 68.00	+112.541	1.9935	224.38	85.0	2.64
0.000 - 1.083	04 46	4.767	0.875	- 4.171	1.614	32.5	+ 52.455	+ 84.00	+132.284	1.9935	263.71	85.0	3.10
0.000 - 1.167	05 03	5.050	0.875	- 4.419	1.614	35.0	+ 56.490	+ 98.00	+151.071	1.9935	301.16	85.0	3.54
0.000 - 1.250	05 21	5.350	0.875	- 4.681	1.614	37.5	+ 60.525	+114.00	+169.944	1.9935	338.58	85.0	3.98

TEST Nº 6 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO(ν)

RANGE OF STRAIN CONSIDERED percent	SAMPLE DILATION $\frac{\Delta V}{V}$ unit of 10^{-4}	TOUNGS MODULUS E lb/sq in	$\frac{\Delta V}{V} \times E$ lb/sq in	CHANGE IN DEVIATOR STRESS $\Delta(\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED lb/sq in	$\frac{\frac{\Delta V}{V} \times E}{\Delta(\sigma_1 - \sigma_3)} = N$	$1 - N$	POISSONS RATIO (μ_s) $\frac{1}{2}(1-N)$
0.150 - 0.200	0.26	9020	2.3450	4.51	0.5000	0.5000	0.2500
0.150 - 0.233	0.46	9144	4.2482	6.76	0.6542	0.4458	0.2229
0.150 - 0.267	0.72	7658	5.5138	8.96	0.6154	0.3846	0.1923
0.150 - 0.300	0.94	7853	7.4758	11.23	0.6656	0.3334	0.1667
0.150 - 0.333	1.18	7934	9.4329	14.63	0.6448	0.3552	0.1776
0.150 - 0.367	1.45	8041	11.6585	17.65	0.6682	0.3318	0.1659
0.150 - 0.400	1.76	8156	14.3425	20.38	0.7040	0.2960	0.1480
0.150 - 0.433	2.09	8141	16.4448	23.04	0.7138	0.2862	0.1431
0.150 - 0.467	2.29	8142	18.6452	25.81	0.7224	0.2776	0.1388
0.150 - 0.500	2.44	8186	19.9738	28.65	0.6972	0.3028	0.1514
0.150 - 0.533	2.71	8214	22.2599	31.46	0.7076	0.2924	0.1462
0.150 - 0.567	2.96	8213	24.3105	34.26	0.7098	0.2902	0.1451
0.150 - 0.600	3.13	8200	25.6660	36.90	0.6956	0.3044	0.1522
0.150 - 0.633	3.34	8196	26.9783	38.25	0.6873	0.3127	0.1564
0.150 - 0.667	3.66	8033	27.7942	41.53	0.6692	0.3308	0.1654

CHAPTER 10

CONCLUSIONS

In conclusion, it is necessary to return to the question of the significance of Poisson's Ratio in soils work. For this purpose it is appropriate to consider in the first instance again the applicability of the elastic theory to soils, in the light now also of the information which has come to hand as a result of the practical work conducted, as was described and discussed in Chapters 7, 8 and 9.

A number of additional practical tests, besides those indicated at the beginning of Section 8.1, can now be mentioned which tend to reassure the contention that the elastic theory can be applied to soils.

Firstly, the values of Poisson's Ratio obtained from the tests conducted, were found by using a formula based on elastic theory. Without exception, all the values obtained (i.e. both μ_t and μ_{sel}) fall within the range of values applying to homogeneous, isotropic, elastic materials, viz. zero-0.5 as was discussed in Chapter 3. The values obtained also fall into line with those applying to materials other than metals, such as concrete (μ between 0.083 and 0.125 — see Section 3.3), to which the elastic theory has been applied with reasonable success. That values of Poisson's Ratio in excess of 0.5 were not obtained is due to the fact that in all cases, test samples showed a decrease in volume within the extent of the linear range of the stress:strain curve as a result of the application of compressive loading, which is in accordance with the requirements of the elastic state (as discussed in Section 3.2). Had values of Poisson's Ratio outside the range zero-0.5 been obtained, it would have served as a strong indication that a soil behaves more in a non-elastic manner and the application of the elastic theory to soils should be treated with reserve.

With regard to the fact that the curves of instantaneous Poisson's Ratio:Strain and Instantaneous Young's Modulus:Strain showed (respectively) that both Poisson's Ratio and Young's Modulus are not constant within the linear range of the stress:strain curve as in accordance with the requirements of the elastic theory (see Chapter 2) : The causes of these variations have already been fully discussed; in the case of Young's Modulus at the end of Section 7.7 and in the case of Poisson's Ratio in Chapter 9. With regard to Young's Modulus it should be remembered that in each case E was determined as a secant modulus based on actual points plotted, and not on any smooth curve drawn through the plotted points so as to obtain the optimum conditions for enabling a constant value to be determined. It is therefore considered that the variation of both Poisson's Ratio and Young's Modulus within the extent of the linear range as obtained in the case of the tests conducted, does not necessarily tend to invalidate the application of the elastic theory to soils.

However, besides considerations of laboratory and field tests, recent results of actual practical measurements and their comparison with the results of calculations by the theory of elasticity, lend increasing evidence to the contention that the elastic theory can be applied to soils. Firstly a comparison[25] of measured and calculated vertical stresses (σ_z) developed in a uniform layer of sandy clay as a result of applied surface pressures, showed very good agreement. The applied pressures were of a transient type similar to those produced by road

traffic ...../

TEST Nº 6 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO(μ_2). (Cont.)

RANGE OF STRAIN CONSIDERED	SAMPLE DILATION $\frac{\Delta V}{V}$	YOUNGS MODULUS E	$\frac{\Delta V}{V} \times E$	CHANGE IN DEVIATOR STRESS($\Delta\sigma_d$) = $\Delta(\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED	$\frac{\frac{\Delta V}{V} \times E}{\Delta\sigma_d} = N$	$1 - N$	POISSONS RATIO (μ_2) $\frac{1}{2}[1 - N]$
0.100 - 0.[illegible]	3.58	8037	28.8169	44.16	0.6527	0.3473	0.1736
0.100 - 0.750	3.73	7714	28.8341	46.16	0.6248	0.3602	0.1801
0.100 - 0.787	3.86	7769	30.3743	48.50	0.6256	0.3744	0.1872
0.150 - 0.300	3.96	7583	30.0623	50.33	0.5972	0.4028	0.2014
0.150 - 0.833	4.04	7512	30.7525	51.99	0.5915	0.4085	0.2042
0.150 - 0.867	4.10	7482	30.6762	53.65	0.5716	0.4284	0.2141
0.150 - 0.900	4.14	7323	30.3172	54.92	0.5520	0.4480	0.2240
0.150 - [illegible]33	4.17	7162	29.8655	53.[illegible]	[illegible]326	0.4674	0.2337
0.150 - 0.967	4.20	6991	29.3622	52.12	0.5640	0.4360	0.2180
0.150 - 1.000	4.22	6836	28.8479	50.11	0.4964	0.5036	0.2518

traffic. Jones[29], in this connection, expresses the view that the basic assumption of elasticity is not correct for soil when it is subjected to slowly applied loads. This is possibly because it is felt that too much consolidation effects can result under these loading conditions. It is not possible to express a view on this opinion on the basis of the tests conducted. However, the results of practical measurements under static loading conditions made by the U.S. Army Corps of Engineers, Waterways Experiment Station[26],[27],[28], which were specifically directed towards the purpose of comparing calculations by the theory of elasticity with the equivalent soil measurements, showed that much better agreement was obtained than seemed generally to have been expected.

It can therefore be concluded that although soils do not comply rigidly in all respects with the requirements of the elastic theory, there is ever-increasing evidence to strengthen the belief that the theory can be applied to soils with reasonable confidence and possibly in all cases of loading, provided (according to the discussion in Section 6.1) the induced stresses are low compared to those required to produce failure i.e. provided the induced stresses are sensibly within the elastic range of loading.

With regard to Poisson's Ratio therefore, it can be concluded that since it is primarily an elastic concept and its meaning is seldom considered other than in relation to the elastic state, it rarely enters in soils work other than in connection with applying the elastic theory to soils. Furthermore, when this is the case, Poisson's Ratio can only enter in 3-dimensional problems of stress and strain, since it was shown in Chapter 4 that the elastic constants do not effect the stress distribution in 2-dimensional cases. The 3-dimensional problem can result from any type of concentrated point loading or finite area loading, such as a rolling wheel load due to rail or road traffic or a structural foundation in the form of a rectangular, square or circular footing. However, as was pointed out in Section 4.4, in the case of stress distribution in 3-dimensional problems, Poisson's Ratio only appears in the expressions for certain of the strain components, the vertical normal stress σ_z which is most frequently used in soils work, being unaffected by it. The Maximum Shear Stress (S_{max}) in 3-dimensional cases is also affected by Poisson's Ratio as was shown in Chapter 6. But, for the basic case of a vertical point load, Poisson's Ratio was in general found to be of insignificant effect below a certain comparatively shallow depth in the soil, as the stress distribution (S_{max}) below such depth was not seriously affected by a complete range of values between zero and 0.5 (Fig.14). Furthermore, on the axis of loading, which is actually of prime importance since the greatest S_{max} values occur on the axis of loading, μ has no very great effect on the values of S_{max} obtained at all depths, so the limits of the complete range of μ values applicable to elastic materials only caused a 11% variation in the corresponding values of S_{max}.

As far as stress distribution in soils is therefore concerned, it appears that Poisson's Ratio is generally not of very great significance. The immediate (elastic) settlement of foundations on soils is, however, more affected by variation in the value of Poisson's Ratio, since the factor $(1 - \mu^2)$ in the equation 4.11 for the settlement of a uniformly loaded rectangular area, (a factor which also appears in the case of the expression for the settlement of the surface of a semi-infinite solid due to a vertical point load - see Ref.8, p.376), varies between 0.75 and unity for μ varying between zero and 0.5, and is therefore liable to cause a maximum variation of 25% in the

immediate ...../

TEST No 6.

(a)

Nom. Rate of Strain = 0.00256 in/min.

σ_3 = 40 lb/sq in

ϵ = 0.702

σ_d = 1.68%

S_r = 6.27%

$(\sigma_1 - \sigma_3)$ lb/sq in.

end of linear range

STRAIN ϵ_1 (percent)

(b)

$\frac{\Delta V}{V}$ (in percent)

Compression ⊕ Expansion ⊖

STRAIN ϵ_1 (percent)

(c)

STRAIN ϵ_1 (percent)

POISSON'S RATIO μ

FIG. 52

immediate settlement value obtained. Yet, this effect is ultimately also not very great, since the immediate settlement only forms a part of the total settlement of any foundation, the other part being that due to consolidation effects. Therefore as far as the significance of Poisson's Ratio in the determination of both stress and settlement in soils is concerned, Poisson's Ratio is only of limited significance.

As a result of the discussions in Chapters 6 and 9, the following conclusions can be drawn with regard to the value of Poisson's Ratio applicable in various cases. In the first instance, it was shown in Sections 6.2 and 6.3 that for all types of saturated soils (i.e. both cohesive and non-cohesive), Poisson's Ratio = 0.50 should be used for all calculations by the theory of elasticity. Secondly, in the case of partially saturated soils, the following information is available from Chapter 9:

(a) D.S.I.R. findings:

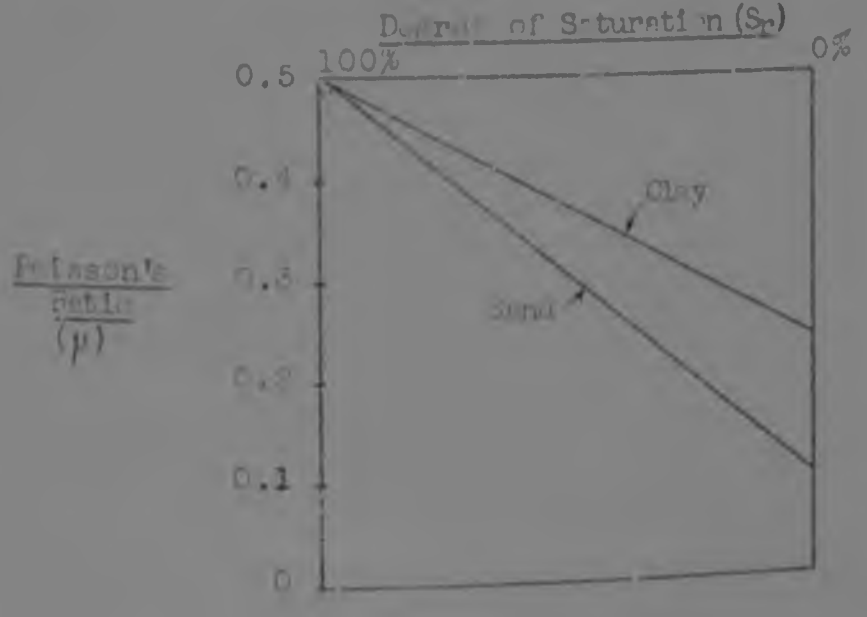
(i) For a silty clay at a moisture content of approximately 20% (plastic limit), $\mu = 0.45$
(ii) For the same soil at a moisture content of 5%, $\mu = 0.25$.
(iii) No values for sands were given, but the view was expressed that the values of μ for sands were below those for clays.

(b) Findings by the U.S. Army Corps of Engineers:

(i) For a lean clay at a moisture content near the optimum $\mu = 0.50$.
(ii) For a dry, fairly fine, uniform sand, $\mu = 0.30$.
(iii) Results (i) and (ii) in this case also indicate that μ values for sands are below those for clays.

(c) Findings from the laboratory work discussed in this report: For a silty sand at a moisture content of 1-2%, μ falls within the range 0.14-0.24.

From the above data it is thus clear that Poisson's Ratio is dependent on moisture content in all types of soils, ie., according to indications so far, μ decreases with moisture content. Furthermore, it is clear that in the case of partially saturated soils, μ values for sands are below those for clays. Hence, it is suggested that the Poisson's Ratio:Degree of Saturation relationship may be of the nature shown in the diagram below:



In ...../

CHAPTER 9

FINAL DISCUSSION OF TEST RESULTS

The curves of Instantaneous Poisson's Ratio (μ_i): Strain (Figs.49,50, 51 and 52) for each of the four tests in which it was possible to prepare these curves, show the same basic pattern in each case, viz. that of a trough-shaped curve, but with a short, approximately horizontal portion forming the turning point.

This approximately horizontal portion of the curve indicates that μ_i attains a value which is more or less constant and can be considered as such with little detrimental effect. For convenience, this constant value will be denoted μ_i(const.) Its actual numerical value was for each test obtained from the relevant calculation sheets as the average of the values for the range over which μ_i can be considered to be constant. This range of values has in each case been marked on the calculation sheets by a vertical line alongside the relevant figures, which extends between arrow heads demarcating the limits of the range.

The curves show clearly that the range of strain over which μ_i is constant, does not always fall fully within that range of strain corresponding to the linear range of the stress:strain curve. In three out of four instances (Figs.49,50 and 51), the bulk of the range of strain over which μ_i is constant falls outside the limit of the linear range of the stress:strain curve. Furthermore, before commencement of this range of strain, μ_i decreases in each case from a value generally between 0.3 and 0.5 to the respective μ_i(const.) value in the region of 0.14 - 0.24, this decrease taking place within the linear range of the stress: strain curve. Subsequent to the μ_i(const.) range, μ_i again steadily increases in value.

Regarding the latter points mentioned, their causes can be explained by considering again equation 7.5 from which Poisson's Ratio was evaluated:

$$\mu = \frac{1}{2}\left[1 - \frac{\delta V}{V}.\frac{E}{\delta\sigma_d}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots 7.5$$

or, more accurately,

$$\mu = \frac{1}{2}\left[1 - \frac{\delta V}{V}.\frac{E}{\delta\sigma_d}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots 9.1$$

but Young's Modulus $(E) = \frac{\delta\sigma_d}{\delta\varepsilon}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9.2$

$$\therefore \quad \mu = \frac{1}{2}\left[1 - \frac{\delta V}{V}.\frac{\delta\sigma_d}{\delta\varepsilon}.\frac{1}{\delta\sigma_d}\right]$$

$$= \frac{1}{2}\left[1 - \frac{\delta V}{V}\,\frac{1}{\delta\varepsilon}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots 9.3$$

which means that, as V is a constant and $\delta\varepsilon$ is the independent variable, the value of Poisson's Ratio obtained is dependent only on the measured value of δV (i.e. the change in volume of the test sample during test) and it cannot be affected by any peak-shaped variation in Young's Modulus (E) as would seem to be possible from equation 7.5.

Further, with regard to the initial decreasing portion of the μ_i:E curve, the same equation (9.3) shows that values of μ_i which are too large can be obtained if the value of δV used is too small, i.e., if the value of slug movement due to leakage only which was used for the determination of δV was larger than the actual value during test, since the slug movement due to spindle entry only is a function only of the independent variable deflection (i.e. strain), and hence, cannot affect the value

of ...../

In the case of partially saturated soils therefore, it may be necessary, depending on the nature of the purpose for which a value of Poisson's Ratio is required, to conduct a special test in order to determine the value applicable to the particular soil concerned, or it may suffice to use an approximate value based on the information given above. However, on occasion there may also be overriding considerations, such as when the condition of a soil mass as a whole is considered and it becomes necessary to adopt a particular value of Poisson's Ratio (such as in the case of the Westergaard equations), in order to obtain a result in close agreement with actual conditions.

Finally, it should again be stressed that in all cases the concept of Poisson's Ratio in soils only has a rational meaning when viewed in relation to soil conditions which can be considered to approximate the elastic state to which the theory of elasticity is applicable. Such soil conditions therefore constitute the only justifiable basis on which the significance of Poisson's Ratio in the determination of stress and settlement in soils can on any occasion be considered.

---oOo---

of δV. In effect, this means that if the assumed value of rate of leakage during test is too large, the resultant value of μ_i is too large and vice versa.

This was actually the case with the reduction of the test results, since as can be seen in Fig.(a), Section 8.1, the rate of leakage during the initial portion of a test was assumed to be a constant value equal to the rate of leakage measured immediately before the start of test, and, as was discussed in Section 8.2, this is a value which is most probably in excess of the actual value of rate of leakage occurring immediately after test commencement. Hence, the fact that values of μ_i which are too large were initially obtained. It also seems likely that while, soon after test commencement, the slug movement due to sample volume change only is still comparatively small in relation to the slug movement due to leakage only (see for example Test No.1), an error in the assumed value of rate of leakage could cause a significant error in the value of μ_i obtained. As the test progresses, and the slug movement due to sample volume change increases in relation to the slug movement due to leakage, an error in the rate of leakage has continually less significant affect on the values of δV and μ_i obtained. On the curve of μ_i:c this is manifested by steadily decreasing μ_i values until the μ_i(const.) range is obtained. It is therefore considered that the values of μ_i(const.) are the more reliable and will therefore be used.

With regard to the fact that μ_i again increases in value subsequent to the μ_i(const.) range, it will be noted that this subsequent increase in value occurs simultaneously with the turning point in the Main Slug Movement:Deflection curve being approached, i.e. it occurs simultaneously with the commencement of the phenomenon of sample expansion on approaching failure conditions (see Section 9.7). This phenomenon results in decreasing values of δV (relative to the original sample volume V) and, as can be seen from equation 9.3, is the cause of the subsequent increasing values of μ_i.

However, the fact that the μ_i(const.) range which precedes the range of increasing μ_i values, extends outside the limits of the linear range of the stress:strain curve, is considered to be due to inclusion of sample volume change due to consolidation effects in the measured value of δV. These consolidation effects occur as a result of the application of the deviator stress σd and possibly only become of significant effect near and beyond the end of the linear range and prior to the commencement of the phenomenon of sample expansion on approaching failure conditions. The curving over to the right of the stress:strain curve which commences from the end of the linear range, is considered(6) to be indicative of the inclusion of consolidation effects resulting from the application of the deviator stress. This can possibly be appreciated if it be considered that in some tests the linear range of the stress:strain curve lasted for about 15 minutes. As the test soil was partially saturated, in fact, nearly completely dry (w generally less than 2%), and was also of a cohesionless nature, it is difficult to conceive especially as a result of the discussions in Sections 8.2 and 8.3, how some consolidation effects (possibly not in the form of expulsion of pore water from the test sample, but air), could have been avoided within the linear range of the stress:strain curve.

As these consolidation effects represent loss of mass during strain (see Chapter 2), they cannot be included in sample volume changes due to elasticity effects for the purposes of calculating Poisson's Ratio. The effect of their inclusion is to cause a larger value of δV than would otherwise have been the case, and which, according to equation 9.3, is therefore

the ...../

APPENDIX I

The possible Algebraic Signs of the Moduli of Elasticity E, G and K

(a) Young's Modulus (E):
By Hooke's Law (Chapter 2), $E = \frac{\sigma}{\varepsilon}$ ..... (a)

Since, if σ is +ve, ε is essentially +ve,
and if σ is -ve, ε is essentially -ve,

Therefore, E is always positive.

(b) Modulus of Rigidity (G):
From basic strength of Materials (Ref.2,p.55),

$$G = \frac{\tau}{\delta} \qquad \text{..... (b)}$$

Since, if τ is +ve, δ is essentially +ve,
and if τ is -ve, δ is essentially -ve,

Therefore G is always positive.

(c) Bulk Modulus (K):
From equation 3.2 : For fluid pressure conditions
when $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$ say,

$$\Delta = 3\varepsilon \qquad \text{..... (c)}$$

and as discussed in (a) above:
if σ is +ve, ε is +ve, therefore in (c): Δ is +ve when σ is +ve,
and if σ is -ve, ε is -ve, therefore in (c): Δ is -ve when σ is -ve.

Hence, in equation 3.4: $K = \sigma/\Delta$

if Δ is +ve when σ is +ve, then σ/Δ is +ve,
and if Δ is -ve when σ is -ve, then σ/Δ is +ve.

Therefore K is always positive.

the cause of a smaller value of μ_i being obtained, and a delay in the start of the range of increasing μ_i values. Hence, the phenomenon that the μ_i(const.) range extends outside the limits of the linear range of the stress:strain curve.

Table XII on the next page shows the determination of the proportion of the constant μ_i range falling within the limit of the linear range of the stress:strain curve for each test. The table also compares in adjacent columns the values of μ_{sel} and μ_i(const.) obtained in each case. It appears that the values of both μ_{sel} and μ_i(const.) fall within the range 0.138-0.238, the values of μ_i(const.) being slightly larger than the corresponding values of μ_{sel} in each case.

These values show reasonable agreement with those obtained by Research workers of the D.S.I.R.'s Road Research Laboratory, Harmondsworth, Middlesex, England,(23),(24),(25) as a result of in-situ measurements made of the mechanical properties of soil by vibration methods. This research work is as yet unpublished. It was found that Poisson's Ratio for a silty clay with a liquid limit of 50%, was approximately 0.45 at a moisture content quite near to its plastic limit of 30%. For the same silty clay, Poisson's Ratio was 0.35 at 5% moisture content, thus indicating a decrease in value with decreasing moisture content. For a heavy clay a value a little above 0.49 seemed to be indicated. No data are as yet available for sands, but the opinion was expressed(23) that the values will be below those for clays.

Information obtained from the Director, Waterways Experiment Station, U.S. Army Corps of Engineers, Vicksburg, Mississippi,(26) (27),(28), substantiates these findings of the D.S.I.R., England. Their work has been directed at comparing calculations by the theory of elasticity with the equivalent soil measurements. For a lean clay at a moisture content near the optimum, a Poisson's Ratio of 0.5 seemed to agree very well, while for a dry, fairly fine, uniform sand, a value of 0.5 was indicated, which thus verifies the opinion of the D.S.I.R., workers that the Poisson's Ratio values for sands are below those for clays.

Hence, if, according to the D.S.I.R. findings previously mentioned, the value of Poisson's Ratio for a silty clay drops from 0.45 to 0.35 for a drop in moisture content from near to 20% to 5%, and if, in addition, the values of Poisson's Ratio for sands are below those for clays, then values between 0.138 and 0.238 for a silty sand at 15% - 2% moisture content would seem to be in reasonable agreement. It should especially be noted that all the values of Poisson's Ratio given above apply to partially saturated soil conditions.

In order to ascertain the possible effect of moisture content on the values of Poisson's Ratio determined from the test series, Table XIII was arranged in order of decreasing moisture content values, with the relevant values of Degree of Saturation

(S_r) ...../

APPENDIX II

Steps in the Final Reduction of the Mathematical Expressions in Flamant's paper (Ref.12)

It is useful to commence by considering in the first instance some basic relationships of the theory of elasticity which are involved in the work to follow in this Appendix. These relationships will only be quoted without their actual derivations being given. However, reference will be made to pages in Timoshenko's "Theory of Elasticity" (Ref.3) wherein the derivations are outlined in full. It is important to point out that Timoshenko uses the same notation as used by Love and other masters of the elastic theory, in that Poisson's Ratio is denoted by the Greek letter ν (Nu), whereas in this report Poisson's Ratio is in all cases denoted by μ. However, since Lame's elastic constants which appear in Flamant's paper are denoted by λ and μ, Poisson's Ratio will in this Appendix only be denoted by ν, to avoid confusion in the work to follow. Furthermore, the unit volumetric strain or Dilation which throughout this report is denoted by Δ, is denoted by θ in Timoshenko's book as well as in Flamant's paper. Therefore the Dilation also will in this Appendix only be denoted by the symbol θ.

The first set of basic relationships which are involved in Flamant's work are those given by Timoshenko(3) in his equations (2),p.7, viz., that if with respect to a set of three mutually perpendicular axes OX,OY and OZ the small displacements of particles of a deformed body be resolved into components u, v, and w parallel to these respective coordinate axes, then:

(i) the linear strain components ε are expressible by:

$$\varepsilon x = \frac{\partial u}{\partial x} \; ; \; \varepsilon y = \frac{\partial v}{\partial y} \; ; \; \varepsilon z = \frac{\partial w}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

and (ii) the shear strain components γ are expressible by:

$$\gamma xy = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \; ; \gamma xz = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \; ; \gamma yz = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad \ldots\ldots\ldots \quad (b)$$

The second set of basic relationships which are involved are those given by Timoshenko(3) in his equations (11),p.11, relating the stresses and strains in a 3-dimensional system:

$$\begin{aligned} \sigma x &= \lambda\theta + 2\mu\varepsilon x \\ \sigma y &= \lambda\theta + 2\mu\varepsilon y \\ \sigma z &= \lambda\theta + 2\mu\varepsilon z \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (c)$$

wherein:

(i) λ and μ are Lame's elastic constants as given by:

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (d)$$

$$\text{and } \mu = G = \frac{E}{2(1+\nu)} = \frac{\tau}{\gamma} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (e)$$

where E = Young's Modulus
ν = Poisson's Ratio
G = Modulus of Rigidity
τ = shear stress
γ = shear strain

(ii) θ is the unit volumetric strain as given by:

$$\theta = \varepsilon x + \varepsilon y + \varepsilon z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (f)$$

Equations (e) and (f) above are identical to equations 3.5 and 3.2 which appeared earlier in Chapter 3 of this report.

From equation (e) above, it should be noted that:

$$\tau = \mu . \gamma$$

whence .....

TEST Nº 4 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION ($\frac{\Delta V}{V}$) (Cont.)

...or ...ERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECTED OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS.	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS-SECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	hrs.	mins.	mm./hour	millimetres	cm./hour		millimetres	millimetres	millimetres	Sq. cm.	cubic cm.	cubic cm.	cubic cm.
1.333	05 35	5.583	0.875	- 4.885	1.614	40.0	+ 64.560	+123.25	+182.925	1.9935	364.86	86.0	4.24
1.417	05 50	5.833	0.875	- 5.104	1.614	42.5	+ 68.595	+130.00	+193.691	1.9935	386.71	88.0	4.38
1.500	06.04	6.067	0.875	- 5.309	1.614	45.0	+ 72.630	+136.00	+205.321	1.9935	409.31	89.0	4.59
1.583	06 19	6.317	0.875	- 5.527	1.614	47.5	+ 76.665	+142.75	+213.088	1.9935	426.39	88.0	4.84
1.667	06 33	6.550	0.875	- 5.731	1.614	50.0	+ 80.700	+144.75	+219.719	1.9935	438.91	85.0	5.16
1.833	06 56	6.933	0.988	- 6.850	1.614	55.0	+ 88.770	+144.00	+225.220	1.9935	449.87	82.0	5.49
2.000	07 21	7.350	1.000	- 8.085	1.614	60.0	+ 96.840	+134.75	+223.505	1.9935	445.56	85.0	5.24
2.167	07 45	7.750	1.218	- 9.393	1.614	65.0	+104.910	+111.00	+206.517	1.9935	411.68	89.0	4.64
2.333	08 08	8.100	1.325	-10.732	1.614	70.0	+112.980	+ 90.75	+192.998	1.9935	384.74	85.0	4.53
2.500	08 27	8.450	1.438	-12.151	1.614	75.0	+121.050	+ 68.25	+177.144	1.9935	353.15	81.0	4.36
2.667	08 51	8.850	1.550	-13.718	1.614	80.0	+129.120	+ 47.00	+162.403	1.9935	323.76	80.0	4.05
2.833	09 13	9.217	1.662	-15.319	1.614	85.0	+137.190	+ 23.50	+145.371	1.9935	289.80	85.0	3.41
3.000	09 36	9.600	1.775	-17.040	1.614	90.0	+145.260	+ 4.50	+132.720	1.9935	264.58	85.0	3.11
3.167	09 57	9.983	1.888	-18.848	1.614	95.0	+153.330	- 25.00	+109.482	1.9935	218.22	82.0	2.67
3.333	10 24	10.400	2.000	-20.800	1.614	100.0	+161.400	- 45.00	+ 96.600	1.9935	192.60	85.0	2.24

TABLE XII: COMPARISON OF VALUES OF POISSONS' RATIO μ_{sec} AND μ_{mt}

TEST NUMBER	STRAIN AT END OF LINEAR RANGE OF STRESS STRAIN CURVE = a percent	STRAIN DUE TO BEDDING-IN EFFECTS = b percent	DIFFERENCE a - b = CORRECTED STRAIN AT END OF LINEAR RANGE OF STRESS STRAIN CURVE = c percent	CURVE OF μ_L VS E CORRECTED STRAIN* VALUES (percent)		PROPORTION OF CONSTANT μ_L RANGE FALLING WITHIN THE LIMIT OF THE LINEAR RANGE OF THE STRESS STRAIN CURVE		POISSONS RATIO μ_{sec}	POISSONS' RATIO $\mu_{t(mean)}$	DIFFERENCE $\mu_{t(mean)} - \mu_{sec}$
				AT START OF CONSTANT μ_L RANGE = d	AT END OF CONSTANT μ_L RANGE = e	$\frac{c-d}{e-d}$	PERCENTAGE			
1	0·600	ZERO	0·600	0·500	1·000	$\frac{0·100}{0·500}$	20%	0·1387	0·1535	0·0148
2	0·833	0·150	0·683	--	—	—	—	0·1401	—	—
3	0·600	0·100	0·500	0·367	0·733	$\frac{0·133}{0·381}$	36%	0·1390	0·1793	0·0403
4	1·083	0·500	0·583	0·417	0·833	$\frac{0·167}{0·417}$	40%	0·2099	0·2372	0·0273
5	0·750	0·260	0·490	—	—	—	—	0·1425	—	—
6	0·750	0·150	0·600	0·230	0·483	$\frac{0·350}{0·235}$	100%	0·1429	0·1484	0·0035

* STRAIN DUE TO BEDDING-IN EFFECTS SUBTRACTED

TEST Nº 4 : CALCULATION OF INSTANTANEOUS VALUES OF POISSONS RATIO(μ).

RANGE OF STRAIN CONSIDERED (in cm)	SAMPLE DILATION $\frac{\Delta V}{V}$ (units = 10^{-3})	YOUNGS MODULUS E (lb/sq in)	$\frac{\Delta V}{V} \times E$ (lb/sq in)	CHANGE IN DEVIATOR STRESS($\Delta\sigma_d$) = $\Delta(\sigma_1 - \sigma_3)$ OVER RANGE CONSIDERED (lb/sq in)	$\frac{\Delta V}{V} \times \frac{E}{\Delta\sigma_d} = N$	$\frac{1}{2}(1 - N)$	POISSONS RATIO (μ_d) $\frac{1}{2}(1 - N)$
0.500 - 0.583	0.23	3470	2.1791	7.86	0.2771	0.7229	0.1385
0.500 - 0.667	0.62	9916	5.1563	14.86	0.3114	0.6886	0.1443
0.500 - 0.750	1.06	9648	10.4198	24.12	0.4320	0.5680	0.2840
0.500 - 0.833	1.88	9330	14.7556	31.10	0.4744	0.5256	0.2628
0.500 - 0.917	2.16	9531	20.5891	39.75	0.5180	0.4820	0.2410
0.500 - 1.000	2.64	9174	24.2194	45.87	0.5280	0.4720	0.2360
0.500 - 1.083	3.10	9154	28.3774	53.67	0.5317	0.4683	0.2342
0.500 - 1.167	3.54	8823	31.2334	58.85	0.5307	0.4693	0.2346
0.500 - 1.250	3.98	8629	34.3434	64.72	0.5306	0.4694	0.2347
0.500 - 1.333	4.29	8298	35.5884	69.12	0.5150	0.4850	0.2425
0.500 - 1.417	4.58	7956	36.4385	72.96	0.4994	0.5006	0.2503
0.500 - 1.500	4.82	7580	36.5156	75.96	0.4820	0.5180	0.2590
0.500 - 1.583	5.02	7291	36.6008	78.96	0.4635	0.5365	0.2682
0.500 - 1.667	5.18	6986	35.6549	80.93	0.4413	0.5587	0.2794
0.500 - 1.833	5.30	6241	33.0773	83.19	0.3976	0.6024	0.3012

(S_r) alongside. From the data presented it is clear, however, that no conclusion can be drawn as to the effect of moisture content or Degree of Saturation on the value of Poisson's Ratio. Insufficient values are available and the range of moisture content is too small to show up any significant trend such as that found by the Road Research Laboratory of the D.S.I.R. For the same reasons no conclusion can be drawn about the effect of void ratio on the value of Poisson's Ratio (Table XIV).

However, Table XIV clearly indicates the important relationship between chamber pressure (σ_3), void ratio (e) and sample dilation ($\delta V/V$) which has also been discussed by Taylor (17), viz., that for sandy soils, the nature and magnitude of the volumetric strains ($\delta V/V$) during test, depend on both the chamber pressure used and the void ratio of the sample. In the first instance, the volumetric strain ($\delta V/V$) for a reduction in volume, increases as the chamber pressure increases, and secondly, the lower the void ratio the greater the tendency for a volume increase after the initial volume decrease at the commencement of the test. Hence, for the two effects combined, the lower the void ratio and chamber pressure the less the initial volume decrease of the sample and, subsequently also, the more rapid the tendency to a volume increase. In the case of Tests No. 1 & 3 ; the effect of the higher chamber pressure in Test No. 1 apparently outweighed the effect of the lower void ratio and resulted in a larger volume decrease and a lesser tendency for a volume increase than in the case of Test No.3. The effect of a lower void ratio at the same chamber pressure is clearly indicated in the case of Tests No. 3 & 4. In the case of Test No. 6 with both low void ratio and chamber pressure, the phenomenon of sample expansion subsequent to an initial compression to the extent of yielding a sample volume in excess of its original volume at the commencement of the test, was clearly realised. i.e., a change in sign of $\delta V/V$ from that corresponding to a decrease in volume to that corresponding to an increase in volume, was clearly effected.

The effect of variation in chamber pressure at a constant rate of strain, on the shear strength obtained, is firstly presented in Table XV. The Mohr circles for each of the three chamber pressures for the two respective rates of strain used, are shown in Fig. 16, and give the values of cohesion (c) and apparent angle of friction (ϕ_a) shown in Table (XV) in the column adjacent to the Shear Strength values. The values of c and ϕ_a were slightly different for the two rates of strain, the higher rate of strain yielding the higher value of apparent angle of friction and the lower value of cohesion. Individual Mohr circles for the case of the faster rate of strain, fit in better with the relevant Mohr rupture line, than in the case of the lower rate of strain. From the following discussion of the data presented in Table XVI, it will be seen that chamber pressure and rate of strain probably have no influence in causing this phenomenon.

The effect of both variation in chamber pressure at a constant rate of strain and variation in rate of strain at a constant chamber pressure (Tables XV and XVI), on the values of

Young's ...../

TEST No 4

(a)

Nom. Rate of Strain = 0.0125 in/min
σ_3 = 20 lb/sq in
e = 0.713
w = 0.66%
Sr = 2.42%

$(\sigma_1 - \sigma_3)$ lbs/sq in.

end of linear range

STRAIN ε (percent)

(b)

$\frac{\Delta V}{V}$ (in percent)

(Compression)

STRAIN ε (percent)

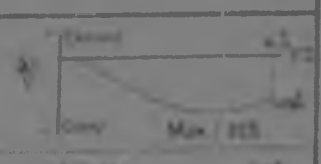
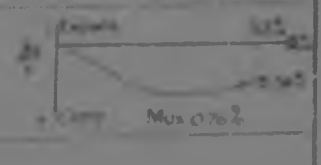
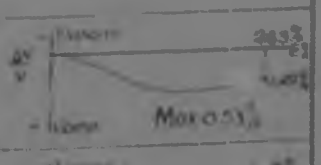
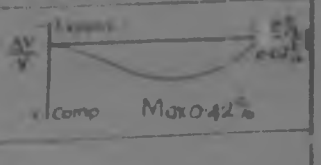
POISSONS RATIO

STRAIN ε (percent)

TABLE XIII

TEST NUMBER	INITIAL MOISTURE CONTENT ω percent	INITIAL DEGREE OF SATURATION S_r percent	POISSONS RATIO μ_{set}	POISSONS RATIO $\mu_{(unloading)}$
2	2 290	8 64	0 1401	—
3	1 820	6 24	0 1390	0 1793
5	1 750	6 42	0 1425	—
6	1 680	6 27	0 1429	0 1464
4	0 658	2 42	0 2099	0 2372
1	0 548	1 95	0 1387	0 535

TABLE XIV

TEST NUMBER	NOMINAL CHAMBER PRESSURE σ_3 lb/sq in	INITIAL VOID RATIO e	POISSONS RATIO μ_{set}	POISSONS RATIO $\mu_{(unloading)}$	DETAILS OF $\frac{\Delta V}{V}$ ε CURVE
1	30	0 737	0 1387	0 1535	
3	20	0 764	0 1390	0 1793	
5	10	0 714	0 1425	—	—
4	20	0 713	0 2099	0 2372	
6	10	0 702	0 1429	0 1461	
2	30	0 695	0 1401	—	—

TEST Nº 6 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$

E OF AIN ERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECT OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED EFFECTS.	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSSECTIONAL AREA OF CAPILLERY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
	HRS MINS			millimetres			millimetres	millimetres	millimetres				
- 0.033	01 09	1.150	0.875	- 1.006	1.614	1	+ 1.614	+ 0.75	+ 1.368	1.9935	2.71	85.0	0.03
- 0.067	01 34	1.567	0.875	- 1.371	1.614	2	+ 3.228	+ 0.75	+ 2.607	1.9935	5.20	85.0	0.06
- 0.100	01 58	1.967	0.875	- 1.721	1.614	3	+ 4.842	+ 0.25	+ 3.371	1.9935	6.72	85.0	0.08
- 0.133	02 22	2.317	0.875	- 2.027	1.614	4	+ 6.456	+ 0.25	+ 4.679	1.9935	9.33	85.0	0.11
- 0.167	02 49	2.817	0.875	- 2.465	1.614	5	+ 8.070	+ 0.50	+ 6.150	1.9935	12.17	85.0	0.14
- 0.200	03 20	3.333	0.875	- 2.916	1.614	6	+ 9.684	+ 3.75	+ 10.618	1.9935	20.97	85.0	0.25
- 0.233	04 06	4.100	0.875	- 3.589	1.614	7	+ 11.298	+ 12.00	+ 19.710	1.9935	39.29	85.0	0.46
- 0.267	05 18	5.300	0.875	- 4.638	1.614	8	+ 12.912	+ 26.00	+ 30.774	1.9935	61.35	85.0	0.72
- 0.300	10 40	10.667	0.875	- 9.334	1.614	9	+ 14.526	+ 35.00	+ 40.212	1.9935	80.19	85.0	0.94
- 0.333	11 29	11.483	0.875	-10.048	1.614	10	+ 16.140	+ 44.25	+ 50.342	1.9935	100.36	85.0	1.18
- 0.367	12 26	12.467	0.875	-10.909	1.614	11	+ 17.754	+ 55.00	+ 61.445	1.9935	123.20	85.0	1.45
- 0.400	13 05	13.083	0.875	-11.448	1.614	12	+ 19.368	+ 67.00	+ 74.820	1.9935	149.35	85.0	1.76
- 0.433	13 49	13.700	0.875	-11.988	1.614	13	+ 20.982	+ 77.00	+ 85.994	1.9935	171.43	85.0	2.02
- 0.467	14 13	14.317	0.875	-12.527	1.614	14	+ 22.596	+ 87.50	+ 97.569	1.9935	194.60	85.0	2.29
- 0.500	14 55	14.917	0.875	- 13.052	1.614	15	+ 24.210	+ 93.00	+104.158	1.9935	207.74	85.0	2.44

TABLE XV

TEST NUMBER	NOMINAL RATE OF STRAIN inches/min	NOMINAL CHAMBER PRESSURE σ_3 lb/sq in	SHEAR STRENGTH S lb/sq in	COHESION(C) AND FRICTION(ϕ) COMPONENTS OF SHEAR STRENGTH	YOUNGS' MODULUS E_{sat} lb/sq in	POISSONS' RATIO μ_{sat}	POISSONS' RATIO μ_s (constant)
1	0 00256	30	89 6	C = 9 5 lb/sq in	8163	0 1387	01535
3	0·00256	20	65 3	ϕ = 3 0°08	7697	0 1390	0 1793
5	0 00256	10	60 6		8125	0 1429	0 1464
2	00125	30	102 0	C = 9 lb/sq in	7750	0 1401	—
4	0 0125	20	83 9	ϕ = 33° 42	9567	0 2099	0 2372
5	0 0125	10	53 0		7212	0 1425	—

TABLE XVI

TEST NUMBER	NOMINAL CHAMBER PRESSURE σ_3 lb/sq in	NOMINAL RATE OF STRAIN inches/min	SHEAR STRENGTH S lb/sq in	YOUNGS' MODULUS E_{sat} lb/sq in	INITIAL MOISTURE CONTENT w percent	INITIAL VOID RATIO e
1	30	0·00256	89 6	8163	0·548	0·737
2	30	0 0125	102 0	7750	2·290	0 695
3	20	0·00256	65 3	7697	1·820	0·764
4	20	0 0125	83 9	9567	0 65E	0 713
6	10	0·00256	60 6	8125	1·680	0 702
5	10	0·0125	53·0	7212	1 750	0·714

TEST Nº 6 : CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION $(\frac{\Delta V}{V})$ (Cont.)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ONLY	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SPINDLE ENTRY ONLY	NUMBER OF DIVS DEFLECT OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SPINDLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE CC. /BINED EFFECTS.	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED.	CROSSSECTIONAL AREA OF CAPILLERY TUBE	HENCE CHANGE IN VOL. (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL SAMPLE VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
percent	min sec.	minutes	mm per sec.	millimetres	Quarters		millimetres	millimetres	millimetres	sq. mm	cubic cm $(\times 10^{-3})$	cubic cm	units of 10^{-4}
0.400 - 0.433	16 34	16.567	0.875	- 13.681	1.614	16	+ 25.824	+ 103.50	+ 115.705	1.9935	230.68	85.0	2.71
0.000 - 0.567	16 12	16.200	0.875	- 14.175	1.614	17	+ 27.438	+ 112.75	+ 126.013	1.9935	251.21	85.0	2.96
0.000 - 0.600	16 48	16.800	0.875	- 14.700	1.614	18	+ 29.052	+ 119.00	+ 133.352	1.9935	265.84	85.0	3.13
0.000 - 0.633	17 22	17.367	0.875	- 15.196	1.614	19	+ 30.666	+ 126.00	+ 141.470	1.9935	282.03	85.0	3.32
0.000 - 0.667	17 58	17.967	0.875	- 15.721	1.614	20	+ 32.280	+ 131.00	+ 147.559	1.9935	294.16	86.0	3.46
0.000 - 0.700	18 34	18.567	0.875	- 16.246	1.614	21	+ 33.894	+ 135.50	+ 153.149	1.9935	305.30	85.0	3.59
0.000 - 0.733	19 09	19.150	0.875	- 16.756	1.614	22	+ 35.508	+ 140.50	+ 159.252	1.9935	317.47	85.0	3.73
0.000 - 0.767	19 47	19.783	0.875	- 17.310	1.614	23	+ 37.122	+ 144.75	+ 164.562	1.9935	328.05	85.0	3.86
0.000 - 0.800	20 21	20.350	0.875	- 17.806	1.614	24	+ 38.736	+ 148.00	+ 168.990	1.9935	336.76	85.0	3.96
0.000 - 0.833	20 51	20.850	0.875	- 18.244	1.614	25	+ 40.350	+ 150.00	+ 172.106	1.9935	343.00	85.0	4.04
0.000 - 0.867	21 25	21.417	0.875	- 18.740	1.614	26	+ 41.964	+ 151.50	+ 174.724	1.9935	348.31	85.0	4.10
0.000 - 0.900	21 56	21.933	0.875	- 19.191	1.614	27	+ 43.578	+ 152.00	+ 176.397	1.9935	351.65	85.0	4.14
0.000 - 0.933	22 23	22.383	0.875	- 19.585	1.614	28	+ 45.192	+ 152.12	+ 177.727	1.9935	354.30	85.0	4.17
0.000 - 0.967	22 52	22.867	0.875	- 20.009	1.614	29	+ 46.806	+ 152.12	+ 178.917	1.9935	356.67	85.0	4.20
0.000 - 1.000	23 21	23.350	0.884	- 20.641	1.614	30	+ 48.420	+ 152.00	+ 179.779	1.9935	358.39	85.0	4.22

Young's Modulus and Poisson's Ratio found, cannot readily be assessed from the data shown. In the case of Young's Modulus (E_{sec}), it seems probable, (also as a result of the discussion in the next paragraph on the data presented in Table XVI), that both the effects mentioned have little influence on the variation in value of Young's Modulus. As regards Poisson's Ratio, insufficient data is available to enable any definite conclusion to be drawn.

Table XVI shows values of Shear Strength (S) and Young's Modulus (E_{sec}) for tests conducted at the same chamber pressure (although not at the same rate of strain), with the corresponding values of initial moisture content (ω) and initial void ratio (e) for each case. From the data shown, it can clearly be seen that the differences in Shear Strength and Young's Modulus values, can be accounted for very satisfactorily by the respective values of initial void ratio and initial moisture content. In the case of each of the test pairs: Nos. 1 & 2 (σ_3 = 30lb/sq.in.); Nos. 3 & 4 (σ_3 = 20lb/sq.in); and Nos. 5 & 6 (σ_3 = 10lb./sq.in.), the higher Young's Modulus value occurs together with the lower moisture content value, while in the case of the Shear Strength values for the same test pairs, the higher Shear Strength value occurs together with the lower void ratio value. Where the higher value of moisture content and void ratio occur simultaneously in any one of a pair of tests, the reduction in both Shear Strength and Young's Modulus values is very marked (Tests No. 3 & 5), and conversely (Tests No. 4 & 6). It is thus clear that both the values of Shear Strength and Young's Modulus are affected by the initial moisture content and initial void ratio of the sample being tested, with void ratio having a particular effect on Shear Strength and moisture content a particular effect on Young's Modulus.
Hence, also, in the case of the test pair Nos. 1 & 2, wherein the higher moisture content is accompanied by the lower void ratio in the case of Test No.2 and conversely in the case of Test No.1, the test with the lower void ratio has the higher Shear Strength, and the test with the lower moisture content has the higher value of Young's Modulus. It is thus not necessarily the higher rate of strain which leads to the higher Shear Strength, as would appear to be the case by comparing the Shear Strength values for the test pairs Nos. 1 & 2; and Nos. 3 & 4, especially as the test pair Nos. 5 & 6 shows the higher value of Shear Strength at the lower rate of strain.

As a result of this discussion it is also possible to explain the slight misfit on the Mohr envelope (Fig.18) in the case of the test group Nos. 1, 3 & 5 which were conducted at a non-constant rate of strain. The Mohr envelope is tangential to the Mohr circle for Test No.1, with Tests Nos. 3 & 5 showing the misfit. Test No.3 has a higher void ratio (0.744), and Test No.5 a lower void ratio (0.708) than Test No.1 (0.737), and hence the relatively lower and higher strengths respectively obtained for these tests, leading to the Mohr circles passing below and above the Mohr envelope respectively. In the case of the test group Nos. 2, 4 & 6, the void ratios were in much closer agreement and hence gave a better fit on the Mohr envelope.

The average void ratio for the test group Nos. 1, 3 & 5 was 0.731 compared with an average of 0.722 for the test group Nos.2, 4 & 5. Hence, the higher average strengths in the case of the latter test group and thus the steeper angle of the Mohr envelope in this case, leading to the difference in apparent angle of friction (ϕ_a) between the two test groups.

An ...../

TEST NP 6 CALCULATION OF INSTANTANEOUS VALUES OF SAMPLE DILATION(δV) (Cont'd)

RANGE OF STRAIN CONSIDERED	TIME ELAPSED OVER RANGE CONSIDERED		RATE OF SLUG MOVEMENT DUE TO LEAKAGE ACCOMPANYING	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO LEAKAGE ONLY	RATE OF SLUG MOVEMENT DUE TO SINGLE ENTRY ONLY	NUMBER OF CYCLES OVER RANGE CONSIDERED	HENCE SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO SINGLE ENTRY ONLY	OBSERVED SLUG MOVEMENT OVER RANGE CONSIDERED DUE TO THREE COMBINED ATTACKS	HENCE SLUG MOVEMENT DUE TO SAMPLE COMPRESSION ONLY OVER RANGE CONSIDERED	CROSS-SECTIONAL AREA OF CAPILLARY TUBE	HENCE CHANGE IN VOL (ΔV) OF SAMPLE OVER RANGE OF STRAIN CONSIDERED	INITIAL TRIAXIAL VOLUME V	SAMPLE DILATION $\frac{\Delta V}{V}$
percent	hr. min.	minutes	mm/hr.	millimetres	% per cycle		millimetres	millimetres	millimetres	sq. mm.	cubic mm. (ΔV)	cubic mm.	percent
0.000 - 1.100	24 41	24.683	0.893	-22.042	1.614	33	+ 53.289	+ 143.00	+ 174.293	1.9935	347.31	85.0	4.08
0.000 - 1.300	27 04	27.067	0.911	-24.658	1.614	39	+ 69.948	+ 98.00	+ 132.288	1.9935	263.72	85.0	3.10
0.000 - 1.400	28 12	28.120	0.920	-25.944	1.614	42	+ 67.788	+ 52.00	+ 99.844	1.9935	199.04	85.0	2.34
0.000 - 1.500	29 14	29.233	0.929	-27.157	1.614	45	+ 72.830	+ 22.00	+ 67.473	1.9935	134.51	85.0	1.58
0.000 - 1.600	30 19	30.317	0.938	-28.437	1.614	48	+ 77.872	- 14.00	+ 36.055	1.9935	69.84	85.0	0.82
0.000 - 1.700	31 23	31.383	0.947	-29.720	1.614	51	+ 82.914	- 45.00	+ 9.594	1.9935	19.12	85.0	0.22

An effect which has not been discussed so far, is the restraint at the ends of a sample during test, as a result of the end platens used. These end platens tend to prevent the ends of the sample from stretching laterally whilst the compressive loading is being applied, so that the sample deforms as a cylinder bulging over its middle portion only. This causes a different change in volume to be measured than if the ends had been unrestrained. The phenomenon has also been discussed by Fisher and Kunkel (22). We feel that at small strains, end restraint is of no importance. As the limit of the linear range in the case of every test in the reported test programme occurred at less than one percent strain, it is considered that the effect of end restraint on the values of Poisson's Ratio determined would not have been serious.

Finally, with regard to the possible effect of test temperature (see Section 7.7) on the values of Poisson's Ratio determined, Table XVII below shows the tests arranged in order of decreasing average test temperature.

TABLE XVII

TEST NUMBER	AVERAGE TEST TEMPERATURE °C	POISSONS' RATIO μ_{sel}	POISSONS' RATIO μ_{v}(const.)
1	[illegible]	0.1387	0.1535
2	[illegible]	0.14[illegible]1	-
4	[illegible]	0.20[illegible]9	0.[illegible]372
5	24.0	0.1425	-
3	[illegible]	0.1[illegible]90	0.1793
6	20.7[illegible]	0.1429	0.1464

It is clear that no conclusion can be drawn as to the possible effect of the particular test temperature on the values of Poisson's Ratio determined.

whence equations (b) above can also be written:

$$\tau_{xy} = \mu \gamma_{xy}; \quad \tau_{xz} = \mu \gamma_{xz}; \quad \tau_{yz} = \mu \gamma_{yz} \ldots\ldots\ldots (g)$$

Turning now to Flamant's paper: The following is a closer investigation of the steps omitted in the paper in obtaining the expressions for the stresses N_x, N_z and T_y which in the notation used in Chapter 4 of this report, are the stresses σ_x, σ_y and τ_{xz}. On page 1467 Ref.12, Flamant firstly obtains the expressions for the displacements u, v, w of any point (x,y,z) below the horizontal surface of the semi-infinite solid subjected to an infinitely extended line loading of P per unit length. These are given as:

$$u = \frac{P}{2\pi\mu}\left(\frac{xz}{x^2+z^2} - \frac{\mu}{\lambda+\mu} \text{ arc tan } \frac{x}{z}\right)$$

$$v = 0$$

$$w = \frac{P}{2\pi\mu}\left(\frac{z^2}{x^2+z^2} + \frac{\lambda+2\mu}{\lambda+\mu} \lg \frac{2\ell}{\sqrt{x^2+z^2}}\right)$$

Hence, based on equations (e):

(i) $\varepsilon_x = \frac{\partial u}{\partial x} = \frac{P}{2\pi\mu}\left[(x^2+z^2)^{-1}.z + xz\left\{-1(x^2+z^2)^{-2}\right\}2x - \frac{\mu}{\lambda+\mu}\left(\frac{1}{z}.\frac{1}{1+\frac{x^2}{z^2}}\right)\right]$

$$= \frac{P}{2\pi\mu}\left[\frac{z}{x^2+z^2} - \frac{2x^2z}{(x^2+z^2)^2} - \frac{\mu}{\lambda+\mu}\left(\frac{1}{z}\,\frac{z^2}{x^2+z^2}\right)\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{(x^2+z^2)z - 2x^2z - \frac{\mu}{\lambda+\mu}.z(x^2+z^2)}{(x^2+z^2)^2}\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{z(x^2+z^2)(1-\frac{\mu}{\lambda+\mu}) - 2x^2z}{(x^2+z^2)^2}\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{\lambda}{\lambda+\mu}.\frac{z}{x^2+z^2} - \frac{2x^2z}{(x^2+z^2)^2}\right]$$

(ii) $\varepsilon_y = \frac{\partial v}{\partial y} = 0$

(iii) $\varepsilon_z = \frac{\partial w}{\partial z} = \frac{P}{2\pi\mu}\left[(x^2+z^2)^{-1}.2z + z^2\left\{-1.(x^2+z^2)^{-2}\right\}2z + \frac{\lambda+2\mu}{\lambda+\mu}\left\{\frac{(x^2+z^2)^{\frac{1}{2}}}{2\ell}.2\ell(-\frac{1}{2})(x^2+z^2)^{-\frac{3}{2}}.2z\right\}\right]$

$$= \frac{P}{2\pi\mu}\left[\frac{2z}{x^2+z^2} - \frac{2z^3}{(x^2+z^2)^2} - \frac{\lambda+2\mu}{\lambda+\mu}.\frac{z}{x^2+z^2}\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{(x^2+z^2).2z - 2z^3 - \frac{\lambda+2\mu}{\lambda+\mu}.z(x^2+z^2)}{(x^2+z^2)^2}\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{z(x^2+z^2)(2-\frac{\lambda+2\mu}{\lambda+\mu}) - 2z^3}{(x^2+z^2)^2}\right]$$

i.e. $\varepsilon_z = \frac{\partial w}{\partial z} = \frac{P}{2\pi\mu}\left[\frac{\lambda}{\lambda+\mu}.\frac{z}{x^2+z^2} - \frac{2z^3}{(x^2+z^2)^2}\right]$

Now by equation (f): $e = \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\therefore e = \frac{P}{2\pi\mu}\left[\frac{\lambda}{\lambda+\mu}.\frac{2z}{x^2+z^2} - \frac{2x^2z}{(x^2+z^2)^2} - \frac{2z^3}{(x^2+z^2)^2}\right]$$

$$= \frac{P}{\pi\mu} \ldots\ldots/$$

$$= \frac{P}{\pi\mu}\left[\frac{\lambda}{\lambda+\mu}\cdot\frac{z}{x^2+z^2} - \frac{z(x^2+z^2)}{(x^2+z^2)^2}\right]$$

$$= \frac{P}{\pi\mu}\left(\frac{z}{x^2+z^2}\right)\left(\frac{\lambda}{\lambda+\mu} - 1\right)$$

$$= \frac{P}{\pi\mu}\left(\frac{z}{x^2+z^2}\right)\left(-\frac{\mu}{\lambda+\mu}\right)$$

$$= -\frac{P}{\pi(\lambda+\mu)}\left(\frac{z}{x^2+z^2}\right)$$

Hence, by equations (c):

(i) $\sigma_x = \lambda\epsilon + 2\mu\varepsilon_x$

$$= \lambda\left[-\frac{P}{\pi(\lambda+\mu)}\cdot\frac{z}{x^2+z^2}\right] + 2\mu\left[\frac{P}{2\pi\mu}\left\{\frac{\lambda}{\lambda+\mu}\cdot\frac{z}{x^2+z^2} - \frac{2x^2z}{(x^2+z^2)^2}\right\}\right]$$

$$= -\frac{P}{\pi}\cdot\frac{\lambda}{\lambda+\mu}\cdot\frac{z}{x^2+z^2} + \frac{P}{\pi}\cdot\frac{\lambda}{\lambda+\mu}\cdot\frac{z}{x^2+z^2} - \frac{P}{\pi}\cdot\frac{2x^2z}{(x^2+z^2)^2}$$

i.e. $$\sigma_x = -\frac{2P}{\pi}\cdot\frac{x^2z}{(x^2+z^2)^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

(ii) $\sigma_z = \lambda\epsilon + 2\mu\varepsilon_z$

$$= \lambda\left[-\frac{P}{\pi(\lambda+\mu)}\cdot\frac{z}{x^2+z^2}\right] + 2\mu\left[\frac{P}{2\pi\mu}\left\{\frac{\lambda}{\lambda+\mu}\cdot\frac{z}{x^2+z^2} - \frac{2z^3}{(x^2+z^2)^2}\right\}\right]$$

$$= -\frac{P}{\pi}\cdot\frac{\lambda}{\lambda+\mu}\cdot\frac{z}{x^2+z^2} + \frac{P}{\pi}\cdot\frac{\lambda}{\lambda+\mu}\cdot\frac{z}{x^2+z^2} - \frac{P}{\pi}\cdot\frac{2z^3}{(x^2+z^2)^2}$$

i.e. $$\sigma_z = -\frac{2P}{\pi}\cdot\frac{z^3}{(x^2+z^2)^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For the calculation of the Shear Stress Ty (ie. τ_{xz}), the equation $\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$ is firstly used (equation (h))

$$u = \frac{P}{2\pi\mu}\left\{xz(x^2+z^2)^{-1} - \frac{\mu}{\lambda+\mu}\tan^{-1}\frac{x}{z}\right\}$$

$$\frac{\partial u}{\partial z} = \frac{P}{2\pi\mu}\left[(x^2+z^2)^{-1}.x + xy(-1)(x^2+z^2)^{-2}.2z - \frac{\mu}{\lambda+\mu}\cdot\left(-\frac{x}{z^2}\right)\left(\frac{1}{1+\frac{x^2}{z^2}}\right)\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{x}{x^2+z^2} - \frac{2xz^2}{(x^2+z^2)^2} + \frac{\mu}{\lambda+\mu}\cdot\frac{x}{x^2+z^2}\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{x}{x^2+z^2}\left\{1+\frac{\mu}{\lambda+\mu}\right\} - \frac{2xz^2}{(x^2+z^2)^2}\right]$$

$$= \frac{P}{2\pi\mu}\left[\frac{\lambda+2\mu}{\lambda+\mu}\cdot\frac{x}{x^2+z^2} - \frac{2xz^2}{(x^2+z^2)^2}\right]$$

$w = \frac{P}{2\pi\mu}$ ...../

$$\omega = \frac{F}{2\pi\mu}\left[z^2.(x^2+z^2)^{-1} + \frac{\lambda+2\mu}{\lambda+\mu}\log\left\{2\ell(x^2+z^2)^{-\frac{1}{2}}\right\}\right]$$

$$\text{So } \frac{\partial\omega}{\partial x} = \frac{F}{2\pi\mu}\left[z^2(-1)(x^2+z^2)^{-2}.2x + \frac{\lambda-2\mu}{\lambda+\mu}\left\{\frac{(x^2+z^2)^{\frac{1}{2}}}{2\ell}.2\ell.(-\tfrac{1}{2})(x^2+z^2)^{-\frac{3}{2}}.2x\right\}\right]$$

$$= \frac{F}{2\pi\mu}\left[-\frac{2xz^2}{(x^2+z^2)^2} - \frac{\lambda+2\mu}{\lambda+\mu}\left(\frac{x}{x^2+z^2}\right)\right]$$

Further, by equations (g):

$$\tau_{xz} = \mu.\gamma_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\text{So } \tau_{xz} = \mu.\frac{F}{2\pi\mu}\left[\frac{\lambda+2\mu}{\lambda+\mu}.\frac{x}{x^2+z^2} - \frac{2xz^2}{(x^2+z^2)^2} - \frac{\lambda+2\mu}{\lambda+\mu}.\frac{x}{x^2+z^2} - \frac{2xz^2}{(x^2+z^2)^2}\right]$$

$$\text{ie. } \tau_{xz} = -\frac{2F}{\pi}\frac{xz^2}{(x^2+z^2)^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

It should be noted that in each case the stresses σ_x, σ_z and τ_{xz} finally do not contain terms of Lamé's constants, which in turn contain the moduli of elasticity including Poisson's Ratio.

The equations (1), (2) and (3) can be transformed into Polar coordinates, to bring them into the form given by Terzaghi Ref.8 p.376, (also equations 4.1, 4.2 and 4.3 of this report) as follows:

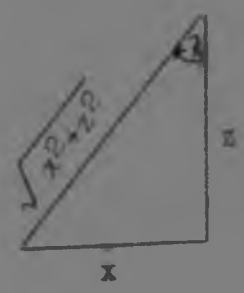
$$\cos\psi = \frac{z}{\sqrt{x^2+z^2}} \; ; \quad \sin\psi = \frac{x}{\sqrt{x^2+z^2}}$$

$$\therefore \cos^4\psi = \frac{z^4}{(x^2+z^2)^2} \; ; \; \cos^2\psi\sin^2\psi = \frac{x^2z^2}{(x^2+z^2)^2} \; ; \; \cos^3\psi\sin\psi = \frac{z^3x}{(x^2+z^2)^2}$$

Hence in equation (1) : $\sigma_x = -\frac{2F}{\pi}\frac{x^2z}{(x^2+z^2)^2}$

$$= -\frac{2F}{\pi z}\frac{x^2z^2}{(x^2+z^2)^2}$$

$$\text{i.e. } \sigma_x = -\frac{2I}{\pi z}\cos^2\psi.\sin^2\psi$$



in equation (b) : $\sigma_z = -\frac{2I}{\pi}\frac{z^3}{(x^2+z^2)^2}$

$$= -\frac{2F}{\pi z}\frac{z^4}{(x^2+z^2)^2}$$

$$\text{ie. } z = -\frac{2F}{\pi z}\cos^4\psi$$

in equation (c) : $\tau_{xz} = -\frac{2F}{\pi}\frac{xz^2}{(x^2+z^2)^2} = -\frac{2F}{\pi z}\frac{xz^3}{(x^2+z^2)^2}$

$$\text{ie. } \tau_{xz} = -\frac{2F}{\pi z}\cos^3\psi\sin\psi$$

It should be noted, however, that Terzaghi's sign convention is such that the expressions for the stresses are in each case taken to be positive.

APPENDIX III

(1) Comparison of values of Smax as determined from:

(1) $S_{max} = \frac{1}{2}\sqrt{(\sigma_z - \sigma_r)^2 + 4\tau_{rz}^2}$ ..... Eqn 5.3

(2) $S_{max} = \frac{1}{2}\sqrt{(\sigma_z - \sigma_t)^2 + 4\tau_{tz}^2} = \frac{1}{2}(\sigma_z - \sigma_t)$ ..... Eqn 5.5 ($\tau_{tz} = 0$)

TABLE XVII

z	r	μ = 0		μ = 0.125	
		Eqn 5.3	Eqn 5.5	Eqn 5.3	Eqn 5.5
3	0	30.9468	30.9468	29.8415	29.8415
	5	3.1942	0.6027	3.2351	1.6915
	10	0.3495	0.3061	0.3610	0.2165
	20	0.0910	0.1390	0.0520	0.1038
	30	0.0536	0.0706	0.0336	0.0529
	45	0.0288	0.0340	0.0192	0.0255
	60	0.0175	0.0199	0.0128	0.0150
5	0	7.7365	7.7365	7.4604	7.4602
	5	3.2250	2.0427	3.1563	1.9761
	10	0.7983	0.1536	0.8089	0.1749
	20	0.0882	0.0763	0.0897	0.0540
	30	0.0316	0.0525	0.0212	0.0390
	45	0.0193	0.0294	0.0100	0.0221
	60	0.0132	0.0175	0.0087	0.0131
10	0	2.7852	2.7852	2.6857	2.6817
	5	1.6298	1.6011	1.8709	1.5426
	10	0.8751	0.4703	0.4614	0.4581
	20	0.1724	0.0040	0.1776	0.0137
	30	0.0447	0.0278	0.0471	0.0191
	45	0.0132	0.0222	0.0100	0.0165
	60	0.0080	0.0151	0	0.0113
15	0	1.2374	1.2374	1.1929	1.1932
	5	1.0414	0.9549	1.0072	0.9201
	10	0.6736	0.4933	0.6560	0.4758
	20	0.2201	0.0795	0.2196	0.0803
	30	0.0767	0.0024	0.0787	0.0066
	45	0.0200	0.0127	0.0212	0.0087
	60	0.0032	0.0111	0	0.0081
20	0	0.6983	0.6983	0.6717	0.6715
	5	0.6305	0.5984	0.6088	0.5770
	10	0.4820	0.3995	0.4684	0.3850
	20	0.2204	0.1183	0.2262	0.1148
	30	0.0938	0.0263	0.0943	0.0275
	45	0.0304	0.0032	0.0316	0.0008
	60	0.0100	0.0072	0.0112	0.0050
30	0	0.3098	0.3088	0.2984	0.2978
	5	0.2952	0.2897	0.2845	0.2791
	10	0.2600	0.2387	0.2513	0.2300
	20	0.1692	0.1234	0.1650	0.1190
	30	0.0975	0.0517	0.0960	0.0505
	45	0.0413	0.0119	0.0415	0.0125
	60	0.0200	0.0008	0.0206	0.0017
45	0	0.1377	0.1369	0.1328	0.1320
	5	0.1347	0.1337	0.1301	0.1289
	10	0.1271	0.1218	0.1226	0.1173
	20	0.1023	0.0876	0.0987	0.0843
	30	0.0750	0.0541	0.0728	0.0523
	45	0.0427	0.0231	0.0421	0.0225
	60	0.0245	0.0096	0.0244	0.0095
60	0	0.0772	0.0772	0.0740	0.0744
	5	0.0763	0.0756	0.0731	0.0730
	10	0.0744	0.0724	0.0712	0.0698
	20	0.0648	0.0597	0.0628	0.0575
	30	0.0543	0.0446	0.0526	0.0430
	45	0.0361	0.0247	0.0353	0.0239
	60	0.0255	0.0136	0.0250	0.0131

		μ = 0.25		μ = 0.375	
		Eqn 5.3	Eqn 5.5	Eqn 5.3	Eqn 5.5
5	0	[illegible].7363	[illegible].7363	27.6311	27.6311
	[illegible]	3.3214	0.7801	[illegible]	0.8689
	[illegible]	0.4229	0.1270	0.5176	0.0375
	[illegible]	0.0264	0.0685	0.0483	0.0333
	30	0.0138	0.0351	0	0.0175
	45	0.0105	0.0170	0	0.0084
	60	0.0071	0.0100	0	0.0050
6	0	[illegible]	7.1939	6.[illegible]073	6.[illegible]075
	5	[illegible].0971	1.9094	[illegible]	1.[illegible]428
	10	0.[illegible]30	0.1[illegible]61	0.8027	0.2174
	20	0.10[illegible]1	0.031[illegible]	0.1[illegible]1	0.[illegible]098
	30	0.0229	0.025[illegible]	0.0350	0.0120
	45	0	0.014[illegible]	0.[illegible]023	0.0023
	60	0	0.00[illegible]7	0	0.[illegible]044
10	0	[illegible]	[illegible]	2.4868	2.4868
	5	1.[illegible]14[illegible]	1.4[illegible]1	1.7612	1.[illegible]256
	10	0.[illegible]514	0.4460	0.8454	0.4339
	20	[illegible].1[illegible]	0.0234	0.1390	0.0332
	30	0.0341	0.010[illegible]	0.0636	0.0016
	45	0.01[illegible]	0.0107	0.0173	0.0050
	60	0	0.0075	0.0055	0.0038
15	0	1.1491	1.1[illegible]91	1.1053	1.1049
	5	0.9726	0.8852	0.939[illegible]	0.8504
	10	[illegible].6536	[illegible].4585	0.6243	0.4408
	20	[illegible].2208	0.0811	0.223[illegible]	0.0820
	30	0.0699	0.0107	0.0882	[illegible].0160
	45	0.0240	0.0048	0.0287	[illegible].0009
	60	[illegible].0045	[illegible].0012	0.0095	0.0022
20	0	0.[illegible]27	0.64[illegible]	0.6215	0.6217
	5	[illegible].287	0.5555	0.[illegible]758	0.5340
	10	0.4551	[illegible].4704	0.[illegible]402	0.3559
	20	[illegible].2028	0.1118	[illegible].2014	0.1088
	30	0.0960	[illegible].0[illegible]9[illegible]	[illegible].0[illegible]94	0.0298
	45	[illegible].0330	0.0015	0.0367	0.0010
	60	[illegible].0122	0.0028	0.0150	0.0006
30	0	0.2873	0.2[illegible]69	0.2761	0.2760
	5	0.[illegible]747	0.2685	0.2641	0.[illegible]580
	10	0.2[illegible]32	0.[illegible]212	[illegible].2[illegible]46	[illegible].2[illegible]95
	20	0.1607	0.111[illegible]	[illegible].1[illegible]67	0.1102
	30	[illegible].0[illegible]47	0.0493	0.0941	0.0481
	45	0.0421	0.0131	0.043[illegible]	0.0137
	60	0.0218	0.002[illegible]	0.0230	0.0037
45	0	0.1272	0.1273	[illegible].1[illegible]25	0.1225
	5	0.125[illegible]	0.1241	0.1207	0.1194
	10	0.1182	0.1130	0.1139	0.1086
	20	0.0958	0.0812	0.0925	0.0780
	30	0.0712	0.0505	0.0694	0.0487
	45	0.0[illegible]15	0.0219	0.041[illegible]	0.0213
	60	0.021[illegible]	0.0095	[illegible].0[illegible]50	0.0095
60	0	0.0716	0.0716	0.0692	0.0688
	5	0.0707	0.0701	0.0683	0.0678
	10	0.0691	0.0672	0.0667	0.0646
	20	0.0602	0.0553	0.0581	0.0531
	30	0.0512	0.0413	0.0497	0.0398
	45	0.0343	0.0231	0.0335	0.0223
	60	0.0245	0.0127	0.0245	0.0124

ε	r	μ = 0.50	
		Eqn 5.3	Eqn 5.5
3	0	26.5251	26.5259
	5	3.6165	0.9576
	10	0.629[illegible]	0.0120
	20	0.0265	0.0018
	30	0.0260	0.0002
	45	0.0078	0.0001
	60	0	0
6	0	6.6312	6.6312
	5	3.0096	1.7761
	10	0.3037	0.2387
	20	0.1570	0.0127
	30	0.0502	0.0016
	45	0.0151	0
	60	0.0063	0
10	0	2.3873	2.3873
	5	1.7091	1.3671
	10	0.8434	0.4217
	20	0.2140	0.0430
	30	0.0751	0.0071
	45	0.0215	0.0008
	60	0.0104	0
15	0	1.0607	1.0607
	5	0.[illegible]	0.[illegible]
	10	0.6107	0.4233
	20	0.2[illegible]	0.0827
	30	0.[illegible]	0.0191
	45	0.0335	0.[illegible]
	60	0.0135	0.0008
20	0	0.[illegible]	0.[illegible]
	5	0.5446	0.5125
	10	0.4271	0.34[illegible]4
	20	0.2[illegible]	0.10[illegible]
	30	0.10[illegible]	0.0310
	45	0.04[illegible]	0.0063
	60	0.0180	0.0016
30	0	0.2650	0.2650
	5	0.2643	0.2475
	10	0.2762	0.2037
	20	0.1535	0.1058
	30	0.0[illegible]	0.0470
	45	0.0[illegible]5	0.01[illegible]
	60	0.0245	0.0047
45	0	0.1177	0.1177
	5	0.1161	0.1146
	10	0.1095	0.1042
	20	0.0898	0.07[illegible]8
	30	0.0678	0.0[illegible]70
	45	0.0412	0.0207
	60	0.0255	0.0095
60	0	0.0660	0.0660
	5	0.0652	0.0620
	10	0.0636	0.0510
	20	0.0561	0.0382
	30	0.0482	0.0215
	45	0.0331	0.0120
	60	0.0245	

2) Determination of the Points of Intersection of Stress Contour lines with the Surface z = 0 of a semi-infinite Solid in the case of Stress Contour Diagrams for σ_r, σ_t and S_{max}

(i) The Horizontal Radial Stress σ_r is given by the expression:

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3r^2 z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \ldots\ldots\ldots 5.8(a)$$

On the plane z = 0, this becomes:

$$\sigma_r = \frac{Q}{2\pi}\left[-\frac{(1-2\mu)}{r^2}\right]$$

$$= -\frac{Q}{2\pi r^2}(1-2\mu)$$

$$\text{i.e. } r^2 = -\frac{Q}{2\pi\sigma_r}(1-2\mu)$$

on the plane z = 0, σ_r is found by calculation to have negative values for all values of μ.

Hence:

$$r = \pm\sqrt{-\frac{Q}{2\pi\sigma_r}(1-2\mu)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

where σ_r = $-Q_1$, $-Q_2$, $-Q_3$ etc. which therefore renders the term inside the square root a positive quantity.

(ii) The Horizontal Tangential Stress σ_t is given by the expression:

$$\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right] \quad \ldots\ldots\ldots 5.9(a)$$

On the plane z = 0, this becomes:

$$\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left(-\frac{1}{r^2}\right)$$

$$= +\frac{Q}{2\pi r^2}(1-2\mu)$$

$$\text{i.e. } r^2 = +\frac{Q}{2\pi\sigma_t}(1-2\mu)$$

On the plane z = 0, σ_t is found by calculation to have positive values for all values of μ.

Hence:

$$r = \pm\sqrt{\frac{Q}{2\pi\sigma_t}(1-2\mu)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

(iii) The Maximum Shear Stress S_{max} is given by the expression:

$$S_{max} = \frac{1}{2}\sqrt{(\sigma_z-\sigma_r)^2 + 4\tau_{rz}^2} \quad \ldots\ldots\ldots\ldots\ldots 5.3$$

On ...../

On substituting the values of σ_z, σ_r and τ_{rz} as given by equations 4.7(a), 4.8(a) and 4.10(a) in the above equation 5.3, and in addition substituting z = 0, this gives the Maximum Shear Stress S_{max} on the plane z = 0 as:

$$S_{max} = \left[\left\{-\frac{Q}{2\pi}\left(-\frac{1-2\mu}{r^2}\right)\right\}^2\right]^{\frac{1}{2}}$$

$$= -\frac{Q}{2\pi}\left(-\frac{1-2\mu}{r^2}\right)$$

$$= +\frac{Q}{2\pi r^2}(1-2\mu)$$

$$\text{i.e.} \quad r^2 = +\frac{Q}{2\pi S_{max}}(1-2\mu)$$

On the plane z = 0, S_{max} is found by calculation to be positive for all values of μ

Hence:

$$r = \pm\sqrt{\frac{Q}{2\pi S_{max}}(1-2\mu)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(c)}$$

It will be seen that the equations (a), (b) and (c) reduce to the same equation if:

firstly: $-\sigma_r = \sigma_t = S_{max} = K$ is substituted;

secondly: the same value of μ is used in each case.

Hence, for a particular numerical value of any contour, say the contour 1.0 units of force/(units of length)^2, and the same value of μ, only one calculation is needed to find the value of r for the contour line $-\sigma_r = \sigma_t = S_{max} = 1.0$ units of force/(units of length)^2.

The various calculations are shown in tabular form on the next page.

Equation ...../

Equation: $r = \sqrt{\frac{Q}{2\pi K}(1-2\mu)}$ where $-\sigma_r = \sigma_t = S_{max} = K$

TABLE XVIII

K	1-2μ	$\frac{Q}{2\pi}$	$\frac{Q}{2\pi K}$	$\frac{Q}{2\pi K}(1-2\mu)$	$r=\sqrt{\frac{Q}{2\pi K}(1-2\mu)}$
Condition: μ = 0					
0.05	1	[illegible]	[illegible]	3143.0[illegible]	56.4
0.1	1	"	[illegible]	[illegible]	[illegible]
0.2	1	"	[illegible]	785.7747	28.2
0.5	1	"	[illegible]	[illegible]	17.8
1.0	1	"	[illegible]	[illegible]	12.6
[illegible].0	1	"	[illegible]	79.5773	8.9
[illegible].0	1	"	[illegible]	[illegible]	[illegible].6
[illegible].0	1	"	[illegible]	[illegible]	4.0
Condition: μ = [illegible]					
[illegible]	0.[illegible]	[illegible]	[illegible]	[illegible]	56.4
[illegible]	"	"	[illegible]	[illegible]	[illegible]
0.1	"	"	[illegible]	[illegible]	[illegible].2
0.2	"	"	[illegible]	[illegible]	[illegible].[illegible]
0.5	"	"	[illegible]	[illegible]	12.6
1.0	"	"	[illegible]	79.5773	8.9
[illegible].0	"	"	[illegible]	[illegible]	6.3
[illegible].0	"	"	[illegible]	[illegible]	4.0
10.0	"	"	[illegible]	[illegible]	2.8
Condition: μ = [illegible]					
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	"	"	[illegible]	[illegible]	[illegible]
[illegible]	"	"	[illegible]	[illegible]	[illegible]
0.1	"	"	[illegible]	[illegible]	[illegible]
0.2	"	"	[illegible]	[illegible]	14.1
0.5	"	"	[illegible]	79.5773	8.9
1.0	"	"	[illegible]	[illegible]	[illegible]
[illegible].0	"	"	[illegible]	[illegible]	[illegible]
[illegible].0	"	"	[illegible]	[illegible]	2.8
[illegible].0	"	"	[illegible]	[illegible]	[illegible]

TABLE XX

DATE	TEST NO.	INITIAL MOISTURE CONTENT			MOISTURE CONTENT AFTER TEST		
		WT OF OVEN DRIED SOIL = W_{ds} (gm)	WT OF WATER = W_w (gm)	MOISTURE CONTENT w%	WT OF OVEN DRIED SOIL = W_{ds} (gm)	WT OF WATER = W_w (gm)	MOISTURE CONTENT w%
22 1 57	2	—	—	—	131 4	3 8	2 740
28 1 57	CONSOLOO-METER	178·3	3·5	·581	—	—	—
29 1 57	3	—	—	—	126 2	3 0	2 377
30 1 57	5	—	—	—	128 5	2 7	2 079
31 1 57	6	—	—	—	130 9	2 5	1 910
31 1 57	*6B	33·77	0 50	1 481	126 1	2 1	1 665
15 2 57	4	48 01	0 32	0 1050	130 0	4 8	3 538
18 2 57	1	23 74	0 13	0 548	28 2	4 2	3 276

* This test was not included in the test series reported on

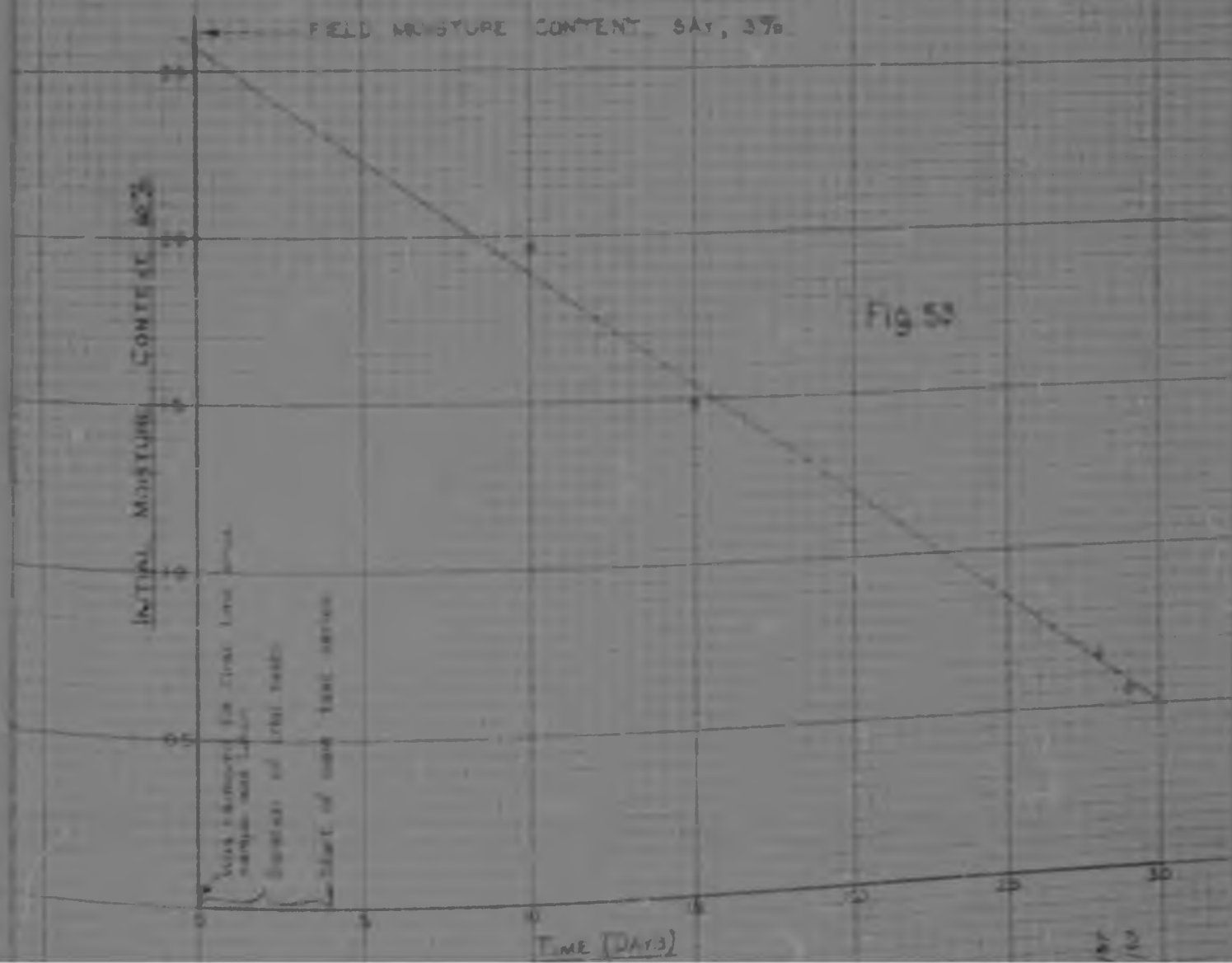


Fig 53

2) DETERMINATION OF INITIAL VOID RATIO (e) AND DEGREE OF SATURATION (S_r) OF EACH TEST SAMPLE

TABLE XXI

TEST NUMBER	WT OF OVEN DRIED SOIL = W_s gm.	SPECIFIC GRAVITY OF SOIL GRAINS = G_s	HENCE VOLUME OF SOIL SOLIDS = $V_s = \frac{W_s}{G_s}$ cc	OVERALL VOL OF SOIL SAMPLE = V cc	HENCE VOLUME OF VOIDS = $V_v = V - V_s$ cc	HENCE INITIAL VOID RATIO = $e = \frac{V_v}{V_s}$	INITIAL MOISTURE CONTENT = ω [From Fig 53] percent	HENCE INITIAL DEGREE OF SATURATION = $S_r = \frac{\omega G_s}{e}$ percent
1	128 2	2·62	48 74	85·00	36·08	0·737	0·548	1·948
2	131 4	2 62	50 16	85·00	34 84	0 695	2 230	8 636
3	126 2	2 62	48 17	85·00	36 83	0 764	1·820	6 237
4	130 0	2·62	49 62	85·00	35·38	0·713	C·858	2·417
5	129 9	2 62	49·60	85 00	35·40	0 714	1 750	6 422
6	130·9	2 62	49 95	85·00	35·05	0 702	1·880	6 272
CONSOLIDOMETER	179 5	2 62	68·13	113·85	47 72	0 700	1·951	7·331

APPENDIX V

1) Calibration of Capillary Tube used.

Measured length of capillary tube = 15 9/32 inches.
Weight of porcelain dish used = 20.17 gms.

Expt. No.	Wt. of Mercury to fill Capillary tube + Wt. of basin.	Temperature
1	1[illegible] gms.	
2	4[illegible].64 gms.	28.5°C
3	4[illegible].67 gms.	
4	4[illegible].67 gms.	36.7°
5	[illegible] gms.	
Average:	[illegible] gms.	[illegible]

Wt. of Mercury to fill capillary tube = (45.58 - [illegible]) gms.
= 10.4[illegible] gms.

From Ref. 31, page [illegible], by interpolation:
Density of Mercury at 28.5°C = 13.[illegible] gms/cc.

Hence, Volume of Mercury to fill capillary = $\frac{10.4^{-}}{13.[illegible]}$ = [illegible] cc.

From above, length of capillary = $15\frac{9}{32}$" = [illegible] x 2.54 cms.

Cross-section area of capillary = A = $\frac{10.4^{-}}{[illegible]}$ $\frac{[illegible]}{[illegible]}$ [illegible] sq.cms.

= $\underline{1.[illegible] \times 10^{-2}}$ sq.cms.

Hence, Diameter of Capillary = D = $\sqrt{4A/\pi}$

= $2\sqrt{\frac{1.[illegible]\times10^{-2}}{\pi}}$

= 0.[illegible] cms.

say, = $\underline{1.[illegible]}$ cms.

2) Calculation of Slug Movement due to Spindle Entry only.

Diameter of Loading Spindle = [illegible] inches.

Hence, Area of Spindle = $\frac{\pi \times (0.[illegible])^2}{4}$ sq.ins.

Fluid displaced per 1 division deflection (0.001")

= $\frac{\pi \times (0.1)^2}{4} \times 10^{-3}$ cub.ins.

= $\frac{\pi \times (0.1)^2}{4} \times 10^{-3} \times 16.[illegible]$ cub.cms.

= $\pi \times \frac{16.[illegible]}{16} \times 10^{-3}$ cub.cms.

= $\underline{0.21[illegible] \times 10^{-3}}$ cc.

From Item (1) of this appendix
Cross-sectional area of capillary tube = 1.[illegible] x 10^{-2} sq.cms.
i.e. 1.[illegible]x10^{-2} cub.cms. of fluid displaced from the [illegible] causes a slug movement of 1cm. In the apparatus under

[illegible] ...../-

Hence, 3.21[illegible]x10^{-3} cub.cm. of fluid displaced from the test cell causes a slug movement of:

$$\frac{3.2102\times10^{-3}}{1.0055\times10^{-2}} \times 1.0 \text{ cm.}$$

$$= 1.2143 \times 10^{-1} \text{ cm.}$$

$$= \underline{1.214 \text{ cm.}}$$

inches, per unit reflex ion (10^{-3}in.), the leading spindle advances a distance of 1.214×10^{-3}inches into the test cell, thereby causing a flow of fluid out of the test cell resulting in a slug movement of 1.214cm. in the capillary tube, from left to right.

i.e. slug movement due to spindle entry only

$$= \underline{1.214 \text{ cm. per division deflection.}}$$

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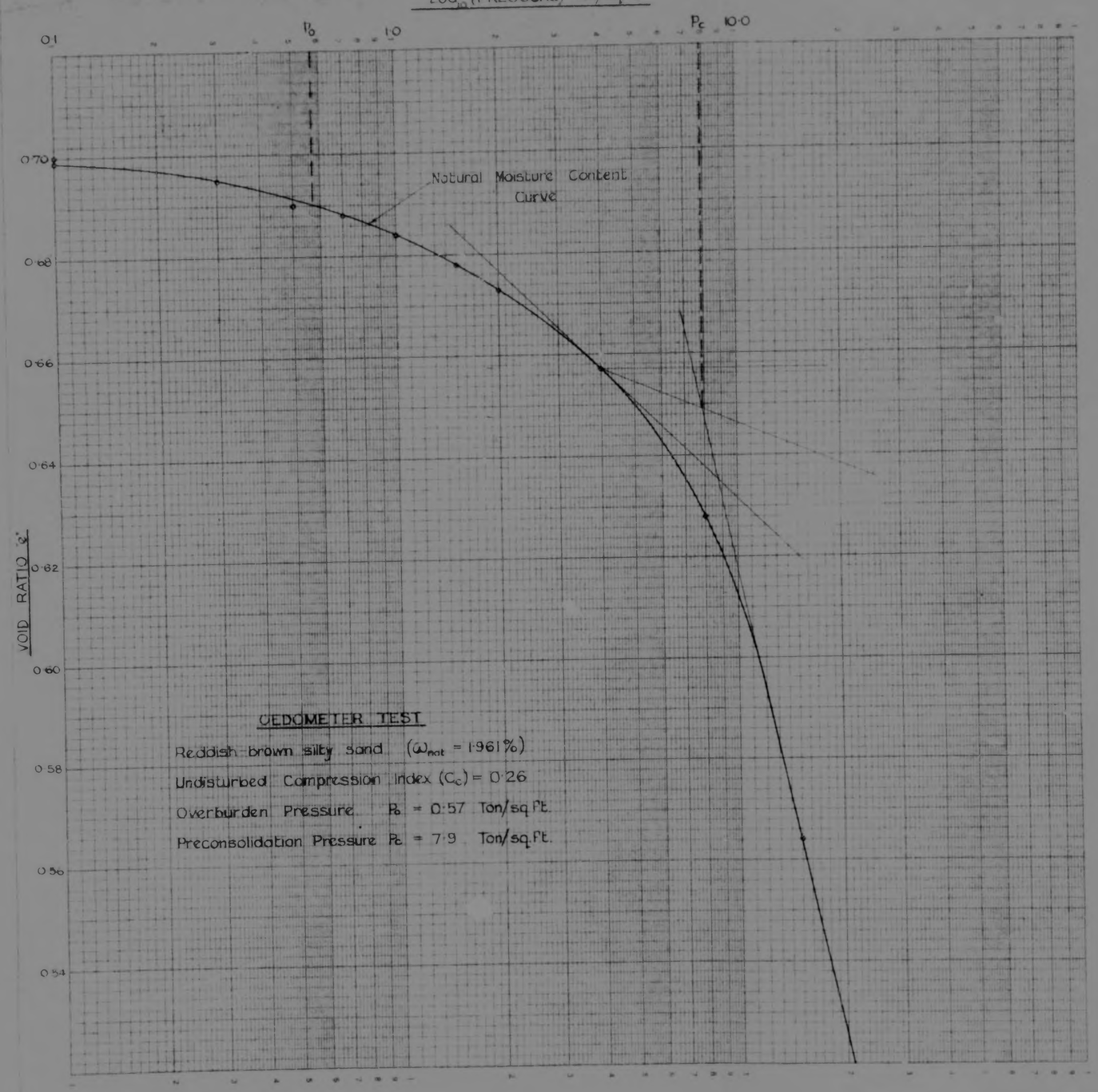

Fig. 18 — Pressure : Void Ratio Curve for the Test Soil
(Normal 1-Dimensional Consolidation Test)

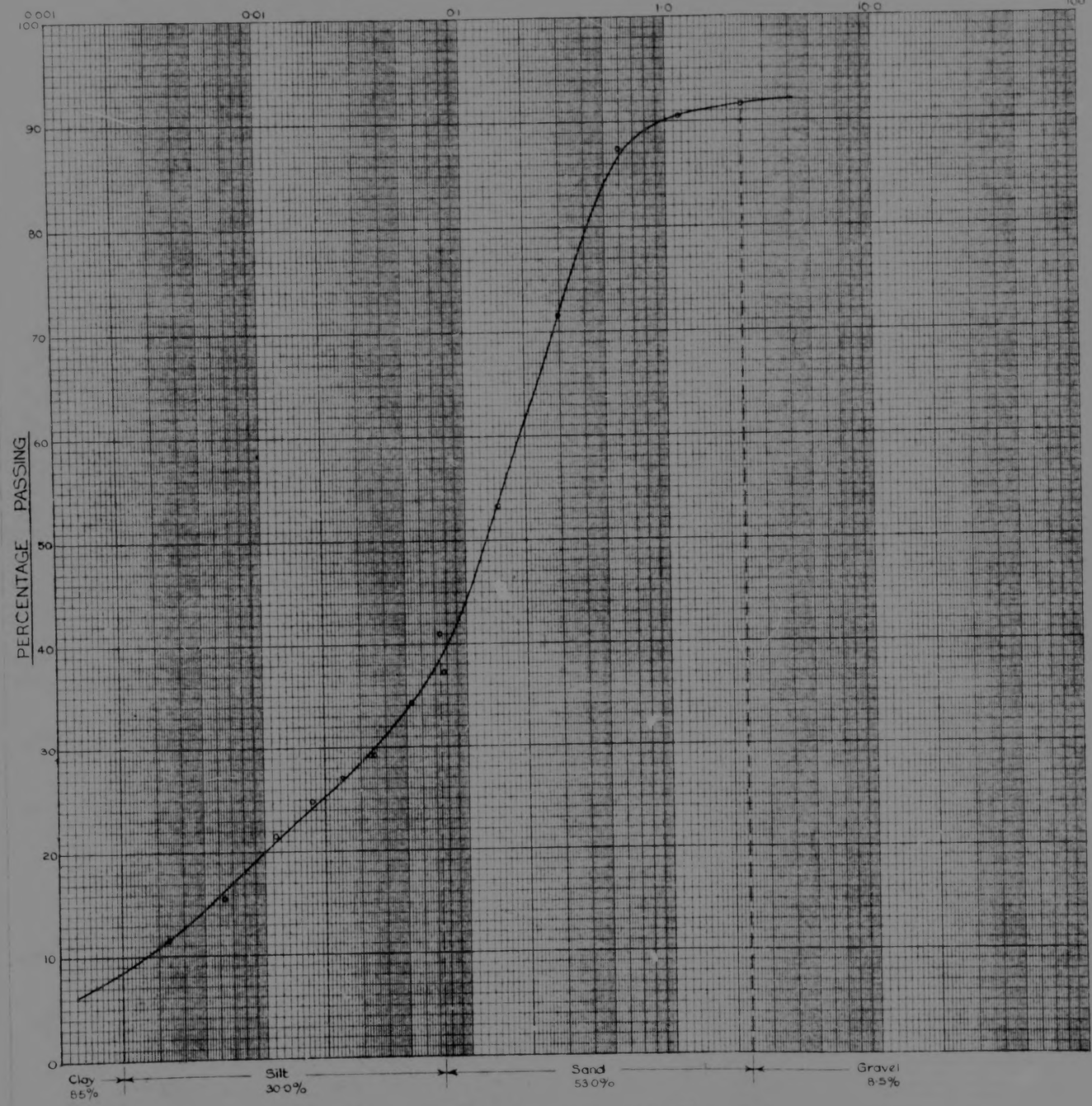

Fig. 17 Grading Curve for the Test Soil

THE SIGNIFICANCE

OF

POISSON'S RATIO IN THE DETERMINATION

OF

STRESS AND SETTLEMENT IN SOILS

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A thesis presented to the University of the Witwatersrand, for the degree Master of Science in Civil Engineering.