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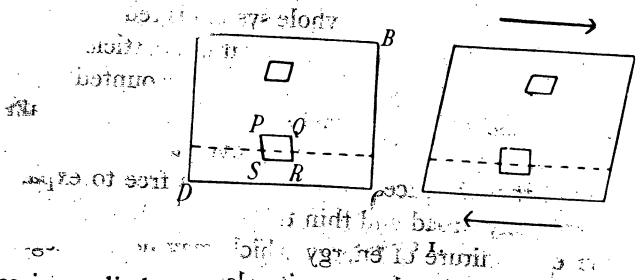
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AN ATTEMPT TO ESTIMATE THE DEGREE OF PRECIPITATION HARDENING, WITH A SIMPLE MODEL

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ABSTRACT. The process of diffusion being assumed to result simply in an interchange of atoms, an estimate is made of the internal strains produced by precipitation. According to the dislocation theory these strains are responsible for the hardness of the material, and so the degree of hardening can be predicted.

According to the dislocation theory of G. I. Taylor and Orowan[1], slip in a solid takes place through the motion of dislocations, which are generated in some way which remains obscure, but once generated will move under the influence of a small applied stress. According to Peierls[2] this stress must exceed a certain small value, which may however be neglected here. In cold-worked or age-hardened alloys the cause of hardness in crystals is the existence of internal strain. In figure 1, *ABCD* represents a solid possessing internal strains, so that a small block of the material *PQRS*, which would take the form of a cube if cut out of the block, has the distorted form of a parallelogram shown in the figure. Clearly,



if a stress is applied as shown by the arrows, no dislocation can travel along the dotted line until the strain is sufficient to bring *PQRS* back to the form of a cube. If the solid contains random internal strains, slip can only take place when the external strain is so great that the internal strain along the yield plane is everywhere in the same direction.

Let then S be the critical shear stress for the material, and let s (d[illegible] s) be the strain for this stress. Then the dislocation theory leads one to expect that s will be approximately [illegible] to the mean internal strain in the material.

Our aim in this note is to calculate the hardness of an alloy in which a number of spherical precipitates have formed. Assuming that metallic diffusion consists

of a simple interchange of atoms, each nucleus will contain as many atoms of the precipitate as it originally contained of the original alloy. If the atomic volumes are different in the two cases, a state of strain will be set up.

Figure 2 shows the deformation of the crystal planes of an isotropic solid round a spherical occlusion. It will be seen that any piece of material in the matrix will suffer a shear strain; a piece which was originally a rectangle will now be a parallelepiped.

Consider then a spherical particle of radius r_0 embedded in the middle of an infinite matrix. As has been explained above, the number of atoms taken out from the hole is equal to the number put in; we shall assume that under zero pressure

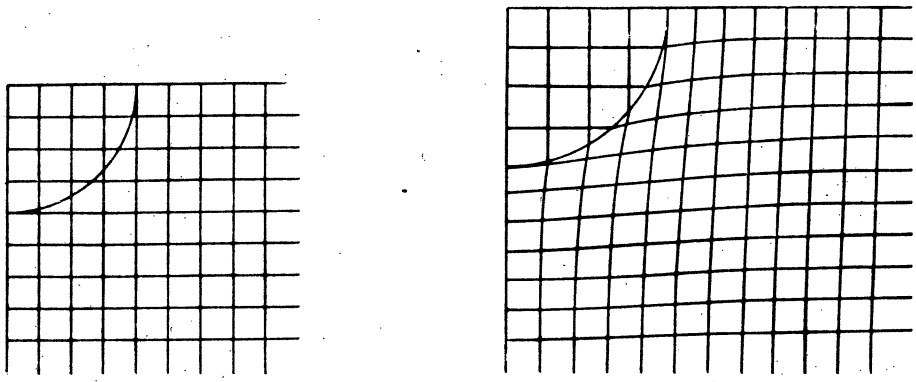


Figure 2. State of strain in a material due to an occlusion with a larger volume than the available hole (*a*) with the occlusion compressed to the atomic volume of the matrix, and (*b*) in equilibrium. The distortion illustrated here is that caused by an infinite cylindrical occlusion: this clearly illustrates the fact that the volume of a unit cell in the matrix remains constant during the expansion.

the atomic volume Ω_N of the material of the occlusion bears a ratio to that of the matrix Ω_M given by

$$\frac{\Omega_N}{\Omega_M} = \frac{(1+\delta)^3}{1}.$$

If δ is positive, the material of the occlusion will be under hydrostatic pressure; let its lattice parameter be reduced by a fraction $\delta - \epsilon$, where $\epsilon < \delta$, so that under pressure

$$\frac{\Omega_N}{\Omega_M} = \frac{(1+\epsilon)^3}{1}.$$

Let $x(r)$ represent the displacement of the medium at a distance r from the centre of occlusion. Elasticity theory gives as the most general form for $x(r)$ with spherical symmetry

$$x(r) = Ar + \frac{B}{r^2}.$$

In our case we may write

$$x(r) = \epsilon r \text{ when } r < r_0,$$

$$x(r) = \epsilon r_0^3/r^2 \text{ when } r > r_0.$$

Let the bulk modulus of the nucleus be K, and let the matrix have Young's modulus E and Poisson's ratio σ. The strain in the occlusion is a uniform hydrostatic compression $3(\delta-\epsilon)$, and hence the pressure p is given by

$$p=3K(\delta-\epsilon). \qquad \text{.....(1)}$$

In the matrix the radial and tangential strains are

$$-2\epsilon r_0^3/r^3 \quad \text{and} \quad +\epsilon r_0^3/r^3$$

respectively. Let the stresses be F_r, F_t, F_t. Then

$$\frac{-2\epsilon E r_0^3}{r^3}=F_r-2\sigma F_t$$

$$\frac{\epsilon E r_0^3}{r^3}=-\sigma F_r+(1-\sigma)F_t.$$

Equating p to F_r at the boundary

$$\epsilon=\frac{3K\delta}{3K+2E/(1+\sigma)}.$$

We may note that the strain in the matrix is a shear without dilatation. The shear at a distance R from the occlusion is

$$\epsilon r_0^3/R^3,$$

where ϵ depends on the materials only.

Consider now a single crystal containing N occlusions per unit volume, each of radius r_0. The average distance R of a point in the matrix from the nearest of them will be given by

$$R\simeq N^{-\frac{1}{3}}.$$

Thus the average shear strain s in the matrix is given by

$$s\simeq\epsilon r_0^3 N.$$

Thus if f is the ratio of the volume precipitated to the volume of the matrix, the average shear strain is

$$s=\epsilon f.$$

This gives, as we have seen, the shear strain at the yield point. The critical shear stress S is given by

$$S\simeq E\epsilon f.$$

We note that the critical shear stress is independent of the size of the occlusions and depends only on the quantity of metal deposited. This may not, however, be true for very small nuclei. We may expect that our strained regions must have a certain minimum size before they are effective in hardening the metal; a Taylor dislocation might jump a very small strained region.

The total strain energy is also independent of the size of the particles, for the energy associated with each sphere is proportional to

$$\int_{r_0}^{\infty}(\epsilon r_0^3/R^3)^2\,4\pi R^2\,dR=\tfrac{4}{3}\pi r_0^3\epsilon^2.$$

and this is directly proportional to the amount of matter in the particle of precipitate. We are attempting at the present time to apply these ideas to the non-spherical occlusions obtained in real cases, and a preliminary discussion is given in the next paper.

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