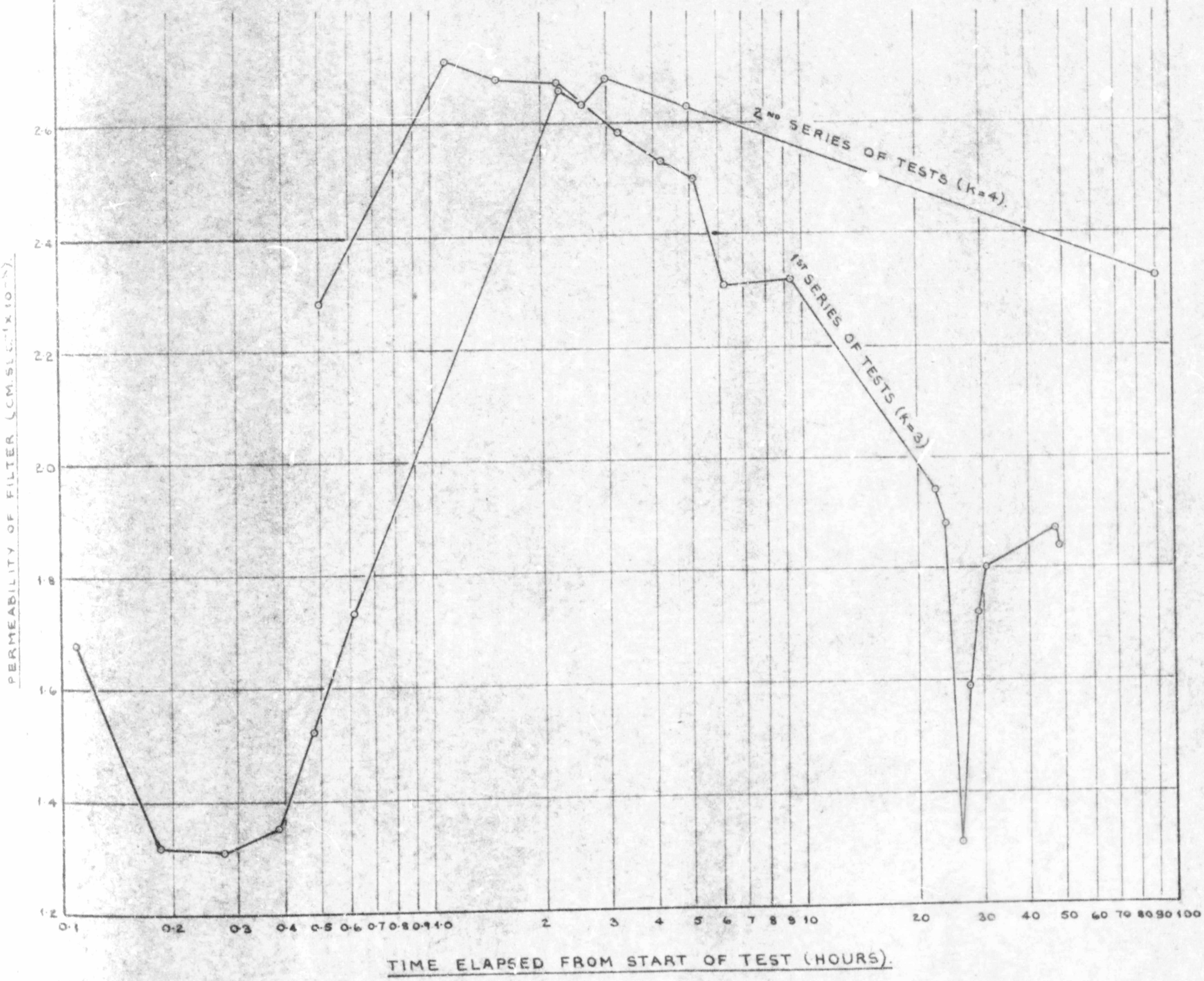
PERMEABILITY-TIME CURVES FOR TESTS ON MODEL FILTERS.

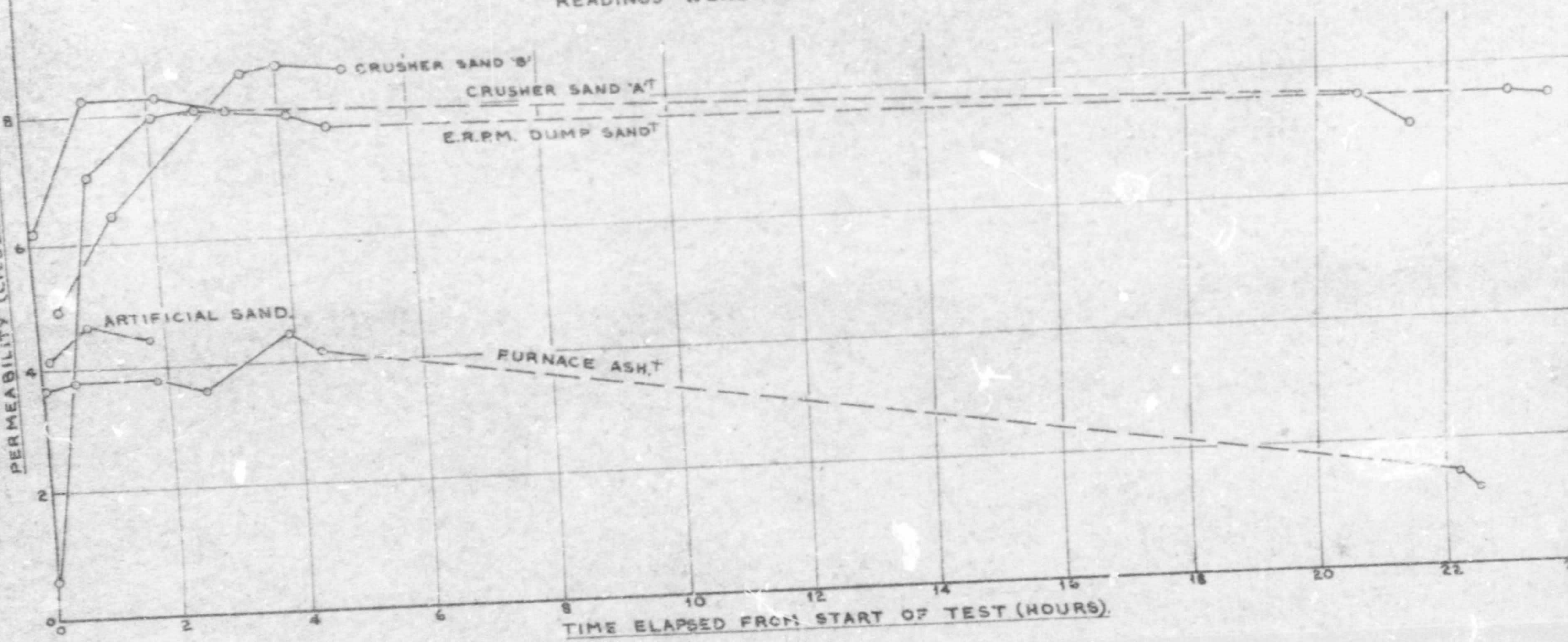
THESE CURVES SHOW HOW THE PERMEABILITY OF A LAYERED FILTER OPERATING UNDER A HIGH GRADIENT VARIES WITH TIME AS ALTERNATE PIPING AND RE-STABILISATION OF THE FILTER GRAIN STRUCTURE PROCEED.

PEAKS REPRESENT MAXIMUM PIPING CONDITIONS.
TROUGHS REPRESENT CONDITIONS OF STABLE GRAIN STRUCTURE.



PERMEABILITY-TIME CURVES FOR SANDS WITH UNSTABLE GRADINGS.

THESE CURVES SHOW TYPICAL VARIATIONS IN PERMEABILITY FOR THE FIRST 24 HOURS OF FLOW THROUGH SEVERAL FRESHLY PLACED SANDS SANDS MARKED† UNDERWENT EXTENSIVE CHANGES OF GRADING. THE DOTTED PORTIONS OF THE CURVES REPRESENT THE NIGHT HOURS DURING WHICH NO READINGS WERE TAKEN.



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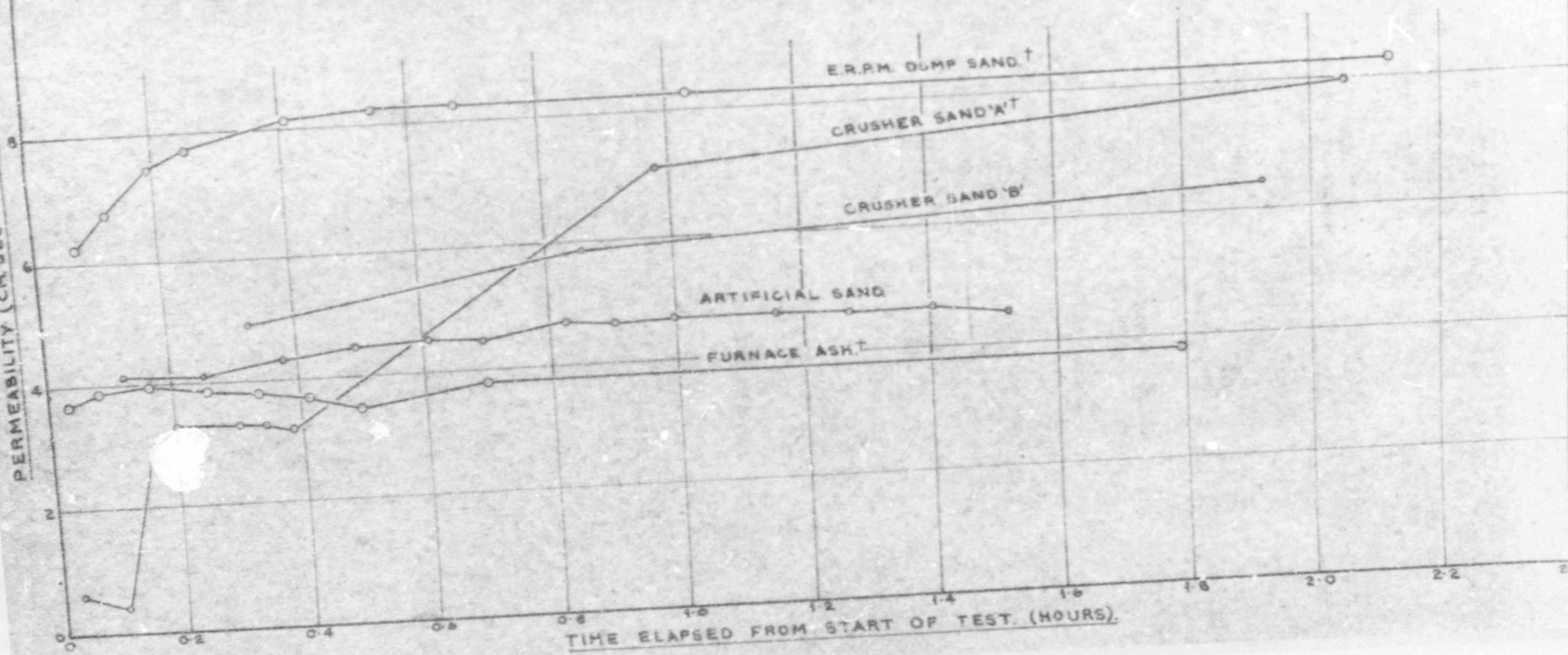
PERMEABILITY-TIME CURVES FOR SANDS WITH UNSTABLE GRADINGS.

THESE CURVES SHOW TYPICAL VARIATIONS IN PERMEABILITY FOR THE FIRST 2 HOURS OF FLOW THROUGH SEVERAL FRESHLY PLACED SANDS. SANDS MARKED † UNDERWENT EXTENSIVE CHANGES OF GRADING.



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extensive changes of grading, but also of those in which the grading changes were insignificant. As in nearly every case, the changes in void ratio due to compression were very small, this strengthens the conviction that particle migration takes place to some extent in all freshly placed soils when first subjected to the action of percolating water.

The Permeability-Time graphs for five typical filter sands are shown in the accompanying two illustrations:

(i) to an enlarged time scale for the initial two hour period;
(ii) for the entire 24 hour period of the tests.

The breaks in the curves represent the night hours during which no readings were taken.

Of the sands considered, those marked * underwent extensive changes of grading.

It is typical of the results obtained for all sands tested that those sands which show the greatest and most sudden variation in permeability with time also show greatest ...nges in grading. It was generally observed that the ...eability of all sands tended initially to increase with ... as the percolating water sought out weak spots in the mass of sand and widened its flow channels through those weak spots.

GENERAL DISCUSSION AND CONCLUSIONS:

None of the empirical relationships developed in this chapter have any direct practical significance. They are interesting in that they describe for the first time, the hydraulic behaviour of a filter sand while in an unstable condition with changes in void ratio, and hydraulic gradient, and Particle Migration taking place.

The calculations necessary to produce the curves contained in this chapter are extremely laborious and as the object was mainly to show that the various relationships did exist and to illustrate their form, the number of sands investigated was kept to a minimum.

It has been shown that:

(i) D'Arcy's Coefficient of Permeability for the flow of water through filter sands which are undergoing changes of grading can be expressed as:

$$k = C\left\{\frac{\mu D}{\gamma}\right\}^{(p-1/23)}$$

(ii) The general equation to the flow of water through a sand which is undergoing changes of grading is:

$$\frac{\gamma dk}{\mu} = a\left\{\frac{ds}{\gamma k^2}\right\}^{-(0.53 + p)}$$

(iii) The relation between the Area under a Grading Curve and the corresponding Specific Surface Area is:

$$A = 98.\ S^{-0.47}$$

The relation for the change in Specific Surface Area of a filter sand due to Particle Migrati which directly relates the grading of the sand to its hydraulic properties is:

$$\frac{(\sigma_i - \sigma_f)^{1.26}}{(S_i^{-0.47} - S_f^{-0.47})} = 4.5$$

Where $\sigma = \frac{i^{1/3}}{q^{2/3}} \left\{\frac{1+e}{G-1}\right\}^{4/3}$

All the constants quoted have been calculated from experimental data measured in c.g.s. units.

CHAPTER VIII

SUMMARY AND CONCLUSION TO THE THESIS

(i) Summary - The Thesis Program.

The object of this thesis was to carry out as complete an investigation of filter sands as the available time and facilities allowed. A special emphasis was placed on the property which some sands have of changing their grading. The enquiry has probed into both the hydraulic and grain structure characteristics of filter sands.

The investigation started with a consideration of the effects of percolating water on the grading of a filter sand when operating as a single layer and went on to see how the behaviour varied when the sand was used as one layer of a zoned filter. After this, changes of grading due to physical and chemical weathering of the sand particles were investigated.

This work covered the aspect of changes of grading and structure fairly fully and was followed by a study of the hydraulic characteristics of sands undergoing changes of grading, the object being to relate corresponding changes in structural and hydraulic properties.

(ii) Conclusions - Changes of Grading in Sands.

There are a number of factors which decide whether a sand will behave as a filter within itself or not:-

The geological history of the sand has an important effect and sands which in their recent past have been acted on by percolating water are in general most resistant to changes of grading.

The original grading of a sand is also important and the smaller the scatter of the grading curve and the more regular the shape of the curve the more stable does the sand tend to be. For satisfactory behaviour of the normal type of graded natural sand the ratio D_{85}/D_{15} should not exceed 4 to 5, and for permeable sands and gravels the percentage of particles passing the No. 200 sieve should not exceed 5%.

For sands of comparable gradings and void ratios a sand consisting of angular fragments tends to be more stable against changes of grading than does a sand composed of rounded particles.

The initial density of a sand does not affect the ultimate extent to which changes of grading will proceed, but may affect the rapidity with which the changes take place. Also, a densely compacted layer of sand is safer against failure by local piping. Piping is a very destructive agent and can initiate a general failure of an entire filter layer.

Carelessly placed layers of sand in which segregation of fines has been allowed to take place are more prone to changes of grading than are layers of completely homogeneous sand.

The final extent to which changes of grading will proceed is not influenced by the magnitude of the hydraulic gradient. Provided the gradient is high enough to transport the soil grains bodily, the hydraulic gradient affects only the speed with which the grading alters.

The tests on model filters showed that provided the particle size ratios between successive layers of a filter are correctly designed, it does not matter if the sands forming the filter layers do not behave as filters within themselves. A stable internal grain structure is usually established rapidly and, once formed, a "non-filter" gives as good a protective filtering action as does a sand which behaves as a filter within itself. The most important thing to be guarded against is failure by internal piping. Piping can occur regardless of the quality of the filter sands if the filter layers are not strong and properly compacted.

The Terzaghi-Bertram criteria for the design of filters appear to give satisfactory results when applied to layers of graded sands.

Apart from the effects of percolating water, chemical and physical weathering may also have a destructive effect on the grading of a filter sand and on the filter incorporating the sand. In designing a filter consideration must be given to the chemical properties of the filtrate and its possible deleterious effects on the minerals composing the filter sand. If the filter is liable to be subjected to violent temperature fluctuations, care must be taken to select a sand which is not likely to break down due to differential thermal expansion of its constituent minerals, etc.

<u>Hydraulic Properties of Unstable Sands</u>.

D'Arcys coefficient of permeability for a sand with unstable grading has been shown to be a rather complex function of the viscosity of the percolating fluid and the grading and density of the sand.

A general relation was derived which describes the flow of water through a soil which is undergoing changes of grading and shows that the flow was of the same type for each soil tested despite very different values of permeability.

Correlations have been found between the area under the grading curve of a sand and its specific surface area, and the change in the area beneath the grading curve due to changes in the grading of the sand has been found to be related to the hydraulic properties of the sand. From this it has been possible to formulate a relation which links directly changes of specific surface area and hydraulic properties.

Sands with unstable gradings have been found to show very sudden and marked changes in permeability which indicate that the process of changes of grading is usually one of sudden rapid breakdown rather than a gradual, uniform change.

A P P E N D I X N O. 1

APPENDIX TO: A STATISTICAL INVESTIGATION OF GRADING CURVES

THE EFFECT OF SIEVING TECHNIQUE ON THE ACCURACY OF SIEVING ANALYSIS

Sand Used: Jukskei River Sand

Location: Johannesburg

Sieving Method: 'Ro-Tap' Mechanical Sieve Shaker

Sieving Time: 5 Min.

Sieve No.	Sieve Opening MM.	Weight of Sieve GM.	Weight of Sieve + Soil GM.		Weight of Soil Retained GM.		Weight of Soil Finer Than GM.		Percent. of Wt. Finer Than GM.		Difference in percent. due to sieving Technique, etc.
			Sieving 1	Sieving 2	Sieving 1	Sieving 2	Sieving 1	Sieving 2	Sieving 1	Sieving 2	
0.371"	9.423	634.8	666.1	662.1	26.3	27.3	1576.5	1575.8	98.36	98.30	+0.06
4	4.760	600.6	658.6	658.0	58.0	57.4	1518.5	1518.4	94.74	94.72	+0.02
9	2.000	498.7	696.7	693.4	198.0	194.7	1320.5	1323.7	82.39	82.57	-0.18
14	1.168	479.4	842.8	852.9	363.4	373.5	957.1	950.2	59.71	59.27	+0.44
28	0.590	439.5	1118.6	1120.0	679.1	680.5	278.0	269.7	17.34	16.82	+0.52
48	0.295	352.7	477.7	481.6	125.0	128.9	153.0	140.8	9.55	8.78	+0.77
100	0.147	424.0	572.8	559.9	148.8	135.9	4.2	4.9	0.26	0.31	-0.044
200	0.074	392.4	394.1	394.8	1.7	2.4	2.5	2.5	0.16	0.16	0.0
Pan		386.1	388.6	388.6	2.5	2.5	0.0	0.0	0.0	0.0	0.0
					1602.8	1603.1					

THE EFFECT OF NATURAL VARIATION OF A SAND
FROM SAMPLE TO SAMPLE

Calculation of 95% Confidence Limits for Natural Variation of Sand

Sand Used: Jukskei River Sand   Location: Johannesburg

Sieve No. Tyler Sieves	Sieve Opening MM.	Percentage Finer By Weight Sample 1 x_1	Sample 2 x_2	Sample 3 x_3	Sample 4 x_4	Sum % Finer by Weight	Estimated Mean % Finer $\bar{x}$	Sum of Square Deviations S.S.D.	Estimated Variance S^2	Range of True Mean % Finer $t = 3.18$	95% Limits For Grading Curve
0.371"	9.423	97.6	99.4	98.9	94.4	390.3	97.58	15.17	5.06	± 3.57	94.00 - 100.00
4	4.760	94.8	96.0	96.4	91.8	379.0	94.75	12.99	4.33	± 3.31	91.4 - 98.1
9	2.000	85.4	85.2	86.9	81.4	338.9	84.73	16.47	5.49	± 3.72	81.0 - 88.5
20	1.168	44.1	41.8	44.7	39.1	169.7	42.45	19.43	6.44	± 4.04	38.7 - 46.8
35	0.590	9.4	8.5	9.4	7.5	34.8	8.70	2.46	0.82	± 1.44	7.3 - 10.1
60	0.295	1.0	1.3	1.4	1.0	4.7	1.18	0.128	0.033	± 0.29	0.9 - 1.5
100	0.147	0.3	0.4	0.4	0.2	1.3	0.325	0.0275	0.0092	± 0.15	0.18- 0.48
200	0.074	0.0	0.2	0.2	0.1	0.5	0.125	0.0275	0.0092	± 0.15	0.00- 0.30

SPECIMEN CALCULATIONS FOR CRUSHER SAND 'A'

Calculations of Means and Standard Deviations.

(i) Original Sand

Particle size x MM.	$Log_{10}x$ z	Percent of size x f%	f.z	$(z-\hat{\xi})$	$(z-\hat{\xi})^2$	$(z-\hat{\xi})^2 f$
2.362	0.373	0.0	-	0.817	-	-
1.168	0.066	20.0	1.32	0.510	0.260	5.20
0.589	-0.231	30.4	-7.02	0.213	0.045	1.37
0.295	-0.531	17.6	-9.35	-0.087	0.0076	0.13
0.147	-0.833	20.8	-17.31	-0.389	0.151	3.14
0.104	-0.984	6.7	-6.59	-0.540	0.291	1.95
0.074	-1.132	2.4	-2.72	-0.688	0.472	1.13
0.055	-1.280	1.4	-1.76	-0.716	0.511	0.72
0.040	-1.389	0.5	-0.70	-0.954	0.909	0.45
0.028	-1.553	0.2	-0.31	-1.109	1.22	0.24
0.020	-1.699	0.0	-	-1.255	-	-
		100.0	-44.44			14.33

$$\hat{\xi} = \frac{\sum fz}{\sum f} = \frac{-44.44}{100.0} = \underline{\underline{-0.444}}$$

$$\underline{\underline{\bar{x} = 0.36 \text{ MM.}}}$$

$$\hat{\sigma}_z^{\,2} = \frac{\sum(z-\hat{\xi})^2 f}{\sum f} = \frac{14.33}{100.0} = \underline{\underline{0.143}}$$

$$\underline{\underline{\hat{\sigma}_z = 0.378}}$$

(ii) Lower Third of Test Cylinder

Particle size x MM	$Log_{10}x$ z	Percent of size x f%	f.z	$(z-\hat{\xi})$	$(z-\hat{\xi})^2$	$(z-\hat{\xi})^2 f$
2.380	0.376	0.0	-	-	-	-
1.168	0.066	14.3	0.95	0.610	0.371	5.30
0.590	-0.230	27.2	-6.26	0.314	0.0985	2.68
0.295	-0.531	10.2	-5.41	0.013	0.00017	0.00
0.147	-0.833	36.6	-30.45	-0.289	0.083	3.04
0.074	-1.132	9.3	-10.53	-0.588	0.345	3.21
<0.074	-1.132	2.4	-2.72	-0.588	0.345	0.83
			-54.42			15.06

$$\hat{\xi} = \frac{\sum fz}{\sum f} = \frac{-54.42}{100.0} = \underline{\underline{-0.544}}$$

$$\underline{\underline{\bar{x} = 0.29 \text{ MM.}}}$$

$$\hat{\sigma}_x^{\,2} = \frac{\sum(z-\hat{\xi})^2 f}{\sum f} = \frac{15.06}{100.0} = \underline{\underline{0.151}} \qquad \underline{\underline{\hat{\sigma}_z = 0.389}}$$

(iii) Upper Third of Test Cylinder

Particle size x MM.	$Log_{10}x$ z	Percent of size x f%	f.z	$(z-\hat{\xi})$	$(z-\hat{\xi})^2$	$(z-\hat{\xi})^2 f$
1.651	0.218	4.4	0.960	0.635	0.403	1.77
0.840	-0.076	29.3	-2.22	0.341	0.116	3.40
0.417	-0.380	27.4	-10.42	0.037	0.0014	0.04
0.246	-0.609	19.4	-11.81	-0.192	0.037	0.72
0.149	-0.833	12.8	-10.66	-0.416	0.173	2.22
0.074	-1.132	5.2	-5.89	-0.715	0.510	2.66
<0.074	-1.132	1.5	-1.70	-0.715	0.510	0.76
		100.0	-41.74			11.57

$$\hat{\xi} = \frac{\sum fz}{\sum f} = \frac{-41.74}{100.0} = \underline{\underline{-0.417}} \qquad \underline{\underline{\bar{x} = 0.38}}$$

$$\hat{\sigma}_z^2 = \frac{\sum(z-\hat{\xi})^2 f}{\sum f} = \frac{11.57}{100.0} = \underline{0.116} \qquad \underline{\underline{\hat{\sigma}_z = 0.341}}$$

TESTS OF SIGNIFICANCE - Between Upper and Lower Thirds of Test Cylinder

(i) Significant Differences Between the Means

$\hat{\xi}_1 = -0.417 \qquad \hat{\xi}_2 = -0.544 \qquad (\hat{\xi}_1 - \hat{\xi}_2) = 0.127$

$n_1 = 7 \qquad n_2 = 7$

$$\frac{\hat{\sigma}_{z_1}^2}{n_1 - 1} = \frac{0.116}{6} = 0.0193 \qquad \frac{\hat{\sigma}_{z_2}^2}{n_2 - 1} = \frac{0.151}{6} = 0.0252$$

For a Significant Difference

$(\xi_1 - \xi_2)$ lies between $0.127 \pm k\sqrt{0.0193 + 0.0252}$

$(\xi_1 - \xi_2)$ lies between $0.127 \pm k \times 0.211$

Value of k for Significance : $k < \frac{0.127}{0.211} = 0.61$

Level of Significance (Student's Distribution, 12 D.F.) 46%

Degrees of Freedom: $n_1 + n_2 - 2 = 14 - 2 = 12$

(ii) Significant Differences Between the Variances

$$\hat{\sigma}_{z_1}^2 = 0.151 \qquad \hat{\sigma}_{z_2}^2 = 0.116 \qquad v^2 = \frac{\hat{\sigma}_{z_1}^2}{\hat{\sigma}_{z_2}^2} = 1.30$$

Degrees of Freedom: 7 and 7

Result : Not significant at 5% level.

χ^2 Test for Normality of Original Grading of Crusher Sand 'A'

z	$\frac{z-\hat{z}}{\hat{\sigma}_z} = u$	$\Phi(u)$	$\Delta\Phi(u) = p$	$n\Delta\Phi(u) = np$	f	$f-np$	$(f-np)^2$	$\frac{(f-np)^2}{np}$
0.373	2.16	0.983	0.017	1.7	0.0	-1.7	2.9	1.7
0.066	1.35	0.913	0.070	7.0	20.0	13.0	169.0	24.2
-0.231	0.564	0.715	0.198	19.8	30.4	10.6	106.0	5.4
-0.531	-0.230	0.410	0.305	30.5	17.6	-12.9	166.0	5.4
-0.833	-1.03	0.150	0.260	26.0	20.8	-5.2	27.0	1.0
-0.984	-1.43	0.078	0.072	7.2	6.7	-0.5	0.25	0.0
-1.132	-1.82	0.039	0.041	4.1	2.4	-1.7	2.9	0.5
-1.260	-1.90	0.031	0.008	0.8	1.4	0.6	0.36	2.4
-1.389	-2.52	0.005	0.026	2.6	0.5	-2.1	4.4	1.7
-1.553	-2.93	0.001	0.004	0.4	0.2	-0.2	0.04	0.1
-1.699	-3.32	0.0	0.001	0.1	0.0	-0.1	0.01	0.1
								42.5

Prob ($\chi^2 > 42.5$, with 11 d.f.) < 0.01

Hence conclude that distribution is not normal.

MEASURES OF SCATTER FOR SANDS TESTED

(i) Sands which failed:

Sand	D_{85}	D_{15}	D_{85}/D_{15}	D_{90}	D_{10}	D_{90}/D_{10}
E.R.P.M. Dump Sand	0.42	0.094	4.5	0.46	0.076	6.1
Cullinan Sand	1.20	0.18	6.7	1.50	0.13	11.5
Bartlett's Sand	0.59	0.033	17.9	0.71	0.023	30.8
E.R.P.M. Ash	9.5	0.06	158.2	12.0	0.027	444.0
Wit Deep Sand	4.1	0.10	41.0	4.5	0.047	95.9
Witbank Sand	0.64	0.089	7.2	0.75	0.060	12.5
Decomposed Sandstone	7.5	0.083	90.4	14.0	0.041	341.0
Van Ryn Sand	2.3	0.31	7.4	2.6	0.23	11.3
Crusher Sand 'A'	1.4	0.16	8.8	1.9	0.14	13.6
	$\bar{r}_{85}$	Mean	38.0	$\bar{r}_{90}$	Mean	107.5

(ii) Sands which did not fail:

Sand	D_{85}	D_{15}	D_{85}/D_{15}	D_{90}	D_{10}	D_{90}/D_{10}
Yellow River Sand	1.1	0.36	3.1	1.4	0.32	4.4
Decomposed Granite	0.80	0.24	3.3	0.99	0.20	4.9
Artificial Sand No.1	2.3	0.099	23.2	2.9	0.079	36.7
Crusher Sand 'B'	1.98	0.040	49.5	2.5	0.022	113.6
White River Sand	1.90	0.14	13.6	2.1	0.098	21.4
Graaf Reinet Sand	1.97	0.31	6.4	2.8	0.24	11.7
	$\bar{r}_{85}$	Mean	16.5	$\bar{r}_{90}$	Mean	32.1

(i) Sands which failed:

$r_{85}-\bar{r}_{85}$	$(r_{85}-\bar{r}_{85})^2$	$r_{90}-\bar{r}_{90}$	$(r_{90}-\bar{r}_{90})^2$
-33.5	1,122	-101.4	10,282
-31.3	979	-96.0	9,216
-20.1	404	-76.7	5,883
120.2	14,400	336.5	113,232
3.0	9	-11.6	135
-30.8	949	-95.0	9,025
52.4	2,748	233.5	54,522
-30.6	936	-96.2	9,254
-29.2	853	-93.9	8,817
	$\sum$: 22,400		$\sum$: 220,366

(ii) Sands which did not fail:

$r_{85}-\bar{r}_{85}$	$(r_{85}-\bar{r}_{85})^2$	$r_{90}-\bar{r}_{90}$	$(r_{90}-\bar{r}_{90})^2$
-13.4	179	-27.7	769
-13.2	174	-27.2	739
+6.7	45	+4.6	21
+33.0	1,089	+81.5	6,642
-2.9	8	-10.7	114
-10.1	102	-20.4	415
	$\sum$: 1,597		$\sum$: 8,700

(iii) Summary:

(a) Sands which failed:

$\hat{\xi}_1$ = Mean D_{85}/D_{15} = 38.0

$\bar{x}_1$ = Mean D_{90}/D_{10} = 107.5

$\hat{\sigma}_1^2$ = Variance for D_{85}/D_{15} = $\frac{22,400}{9}$ = 2,490 $\hat{\sigma}_1$ = 49.9

S_1^2 = Variance for D_{90}/D_{10} = $\frac{220,366}{9}$ = 24,460 S_1 = 156.5

(b) Sands which did not fail:

$\hat{\xi}_2$ = Mean D_{85}/D_{15} = 16.5

$\bar{x}_2$ = Mean D_{90}/D_{10} = 32.1

$\hat{\sigma}_2^2$ = Variance for D_{85}/D_{15} = $\frac{1,597}{6}$ = 266 $\hat{\sigma}_2$ = 16.3

S_2^2 = Variance for D_{90}/D_{10} = $\frac{8,700}{6}$ = 1450 S_2 = 38.1

(iv) Tests for Significant Diffs. Between Means:

D_{85}/D_{15}

$\hat{\xi}_1 - \hat{\xi}_2 = 21.5$

$\frac{\hat{\sigma}_1^2}{n_1-1} = 312 \qquad \frac{\hat{\sigma}_2^2}{n_2-1} = 53.1$

$\sqrt{\frac{\hat{\sigma}_1^2}{n_1-1} + \frac{\hat{\sigma}_2^2}{n_2-1}} = 19.1$

$\xi_1 - \xi_2$ lies between $21.5 \pm t \times 19.1$

No. of D.F: 6+9-2 = 13

For Significance $t < \frac{21.5}{19.1} = 1.13$

Level of Significance: 72%

Tests for Significant diffs. between means:

D_{90}/D_{10}

$\bar{x}_1 - \bar{x}_2 = 75.4$

$\frac{S_1^2}{n_1-1} = 3060 \qquad \frac{S_2^2}{n_2-1} = 290$

$$\sqrt{\frac{S_1^2}{n_1-1} + \frac{S_2^2}{n_2-1}} = 57.9$$

$\xi_1 - \xi_2$ lies between $75.4 \pm t \times 57.9$

No. of D.F. = 13

For significance $t < \frac{75.4}{57.9} = 1.30$

Level of significance: 78%

(v) Estimation of Means:

D_{85}/D_{15}

ξ_1 lies between $38.0 \pm t_{(.95,\ 8)} \times 17.7$

ξ_1 lies between $38.0 \pm 17.7 \times 2.31$

ξ_1 lies between 38.0 ± 40.9

ξ_2 lies between $16.5 \pm t_{(.95,\ 5)} \times 7.4$

ξ_2 lies between $16.5 \pm 7.4 \times 2.57$

ξ_2 lies between 16.5 ± 19.0

Estimation of Means:

D_{90}/D_{10}

ξ_1 lies between $107.5 \pm t_{(.95,\ 8)} \times 55.4$

ξ_1 lies between $107.5 \pm 2.31 \times 55.4$

ξ_1 lies between 107.5 ± 128

ξ_2 lies between $32.1 \pm 2.57 \times 17.0$

ξ_2 lies between 32.1 ± 43.7

With 60% Confidence

ξ_1 lies between 38.0 ± 15.7 $\quad$ ξ_1 lies between 107.5 ± 49.1

ξ_2 lies between 16.5 ± 6.8 $\quad$ ξ_2 lies between 32.1 ± 15.6

APPENDIX NO. 2

APPENDIX TO: THE EFFECTS OF PERCOLATING WATER ON THE GRADINGS OF FILTER SANDS

TEST OF THE HYPOTHESIS: SANDS WHICH FAIL HAVE LARGER % OF FINES

(i) Sands which fail:

Sand	% < #200 Sieve x	$(x-\bar{x})$	$(x-\bar{x})^2$
E.R.P.M. Dump Sand	9.8	-1.4	2.0
Cullinan Sand	4.6	-6.6	43.6
Bartlett's Sand	28.2	+17.0	289.0
E.R.P.M. Ash	13.4	+2.2	4.8
Wit Deep Sand	14.8	+3.6	12.9
Witbank Sand	11.4	+0.2	0.0
Decomposed Sandstone	14.3	+3.1	9.6
Van Ryn Sand	3.1	-8.1	65.4
Crusher Sand 'A'	1.5	-9.7	94.0
	$\sum x$: 101.1	$\bar{x} = 11.23$	$\sum(x-\bar{x})^2$: 521.3

$\bar{x}_1 = 11.23$ $\quad S_{x_1}^2 = \dfrac{521.3}{9} = 58.0$ $\quad S_{x_1} = 7.63$

(ii) Sands which do not fail:

Sand	% < #200 Sieve x	$(x-\bar{x})$	$(x-\bar{x})^2$
Yellow River Sand	0.3	-5.8	33.6
Decomposed Granite	0.1	-6.0	36.0
Artificial Sand No.1	8.9	+2.8	7.8
Crusher Sand 'B'	21.7	+15.6	243.4
White River Sand	4.7	-1.4	2.0
Graaf Reinet Sand	0.9	-5.2	27.0
	$\sum x$: 36.6	$\bar{x} = 6.10$	$\sum(x-\bar{x})^2$: 349.8

$\bar{x}_2 = 6.10$ $\quad S_{x_2}^2 = \dfrac{349.8}{6} = 58.4$ $\quad S_{x_2} = 7.65$

(iii) Test for Significance:

$\bar{x}_1 - \bar{x}_2 = 5.13$ $\quad \dfrac{S_{x_1}^2}{n_1-1} = \dfrac{58.0}{8} = 7.25$ $\quad \dfrac{S_{x_2}^2}{n_2-1} = \dfrac{58.4}{5} = 11.69$

$$\sqrt{\frac{S_{x_1}^2}{n_1-1} + \frac{S_{x_2}^2}{n_2-1}} = \sqrt{7.25 + 11.69} = 4.36 = S$$

$$U = \frac{(\bar{x}_1 - \bar{x}_2) - (\xi_1 - \xi_2)}{S} = \frac{5.13 - (\xi_1 - \xi_2)}{4.36}$$

$(\xi_1 - \xi_2)$ lies between $5.13 \pm t \times 4.36$

For significance $t < \frac{5.13}{4.36} = 1.18$

No. of D.F.: $n_1 + n_2 - 2 = 14$

Level of Significance (Student's Distribution, 14 D.F.): 74%

(iv) Estimates of the Means (with 60% confidence)

ξ_1 lies between $11.23 \pm (t_{60,\ 8})\ 2.70$

ξ_1 lies between 11.23 ± 2.40

ξ_2 lies between $6.10 \pm t_{(60,\ 5)}\ 3.42$

ξ_2 lies between 6.10 ± 3.04

APPENDIX NO. 3

APPENDIX TO:

TESTS ON MODEL FILTERS - FIRST SERIES.

TIME-FLOW MEASUREMENTS

Elapsed time from start of Test Hours	Pressure drop across filter Lb/□"	Hydraulic Gradient	Rate of Flow CC/SEC	Mean Permeability of Filter CM/SEC
0.5	22	28.5	74.1	2.28×10^{-3}
1.1	21	27	83.5	2.71×10^{-3}
1.5	21	27	82.9	2.68×10^{-3}
2.2	21	27	82.4	2.67×10^{-3}
3.2	21	27	82.5	2.58×10^{-3}
4.2	21	27	78.0	2.53×10^{-3}
5.1	21	27	77.4	2.50×10^{-3}
6.1	21.5	28	74.0	2.31×10^{-3}
9.3	21.5	28	74.1	2.32×10^{-3}
22.2	21.5	28	61.9	1.94×10^{-3}
23.8	21.5	28	60.3	1.88×10^{-3}
26.0	22	28.5	42.6	1.31×10^{-3}
27.5	22	28.5	51.7	1.59×10^{-3}
29.0	22	28.5	55.8	1.72×10^{-3}
30.1	21.5	28	57.6	1.80×10^{-3}
47.2	21.5	28	59.9	1.87×10^{-3}
48.2	21.5	28	58.9	1.84×10^{-3}

TESTS ON MODEL FILTERS - SECOND SERIES

TIME-FLOW MEASUREMENTS

Elapsed time from start of Test Hours	Pressure drop across filter Lb/□"	Hydraulic Gradient	Rate of Flow CC/SEC	Mean Permeability of Filter CM/SEC
0.11	23	30	5.77	1.68×10^{-4}
1.83	23	30	4.52	1.32×10^{-4}
2.72	23	30	4.48	1.31×10^{-4}
3.80	22.5	29	4.62	1.35×10^{-4}
4.75	22.5	29	5.04	1.52×10^{-4}
6.18	22.5	29	5.75	1.73×10^{-4}
22.3	23.5	30.5	9.29	2.66×10^{-4}
25.8	23.5	30.5	9.16	2.63×10^{-4}
29.7	23.5	30.5	9.32	2.68×10^{-4}
49.5	24	31	9.32	2.63×10^{-4}
192	26.5	34.5	8.84	2.24×10^{-4}

APPENDIX NO. 4

APPENDIX TO:

A STUDY OF SOME EMPIRICAL RELATIONS FOR THE FLOW OF WATER THROUGH FILTER SANDS

(i) The Estimation of the Best Value of an Observed Constant

The theory which will be briefly summarised here has been used in this chapter to estimate the values of the constants m (in the modification of D'Arcy's Law) and n (in the General Relationship for Flow Through a Filter Sand).

Assume that a statistical variable x (in this case the observed constant) has an observed mean $\bar{x}$ obtained from the results of n observations. Then it can be shown that an unbiassed estimate of the variance of the n observations is:

$$\hat{\sigma}^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

Suppose that the true mean of the observed constant is ξ, then $t = \frac{\bar{x}-\xi}{\hat{\sigma}/\sqrt{n}}$ will have a frequency function f(t) where

$$f(t) = \frac{C}{(1 + t^2/n-1)^{n/2}}$$

f(t) is known as Students 't' Distribution and tabulations of t for various values of n are available.

If t_p is the value of t such that the probability:

$$P\,(|t| \leq t_p) = p, \text{ then:}$$

$\bar{x}$ will lie between $\xi \pm t_p\sqrt{\frac{\sum(x-\bar{x})^2}{n(n-1)}}$ with a confidence of 100p%.

Whenever this is true:
ξ will lie between $\bar{x} \pm t_p\sqrt{\frac{\sum(x-\bar{x})^2}{n(n-1)}}$ with a confidence of 100p%.

Using this relationship and a tabulation of values of t for various values of n and p, the best value of the observed constant and its limits can be estimated with any desired confidence.

(ii) Estimation of a Line of 'Best Fit' by the Method of Least Squares

This method of fitting a straight line to a group of correlated data has been used to establish the lines of best fit in the Specific Surface Area - Area Under Grading Curve Correlation and the relation between Particle Migration and changes in Specific Surface Area.

If the several points of a set of correlated data (x and y), when plotted against each other show up a strong trend towards linearity so that average y is roughly linear in x and the scatter of the y values about the line is about the same for all x, then it is worthwhile to find that line which fits the data most closely.

Suppose that this line of best fit has the equation:

$$y = a(x-\bar{x}) + b$$

Then for any observation y_i:

$$y_i = a(x_i-\bar{x}) + b + d_i$$

Where d_i represents the deviation of the observed line from the line of best fit. The line of best fit is now chosen so that the series of d_i values have zero mean and minimum variance.

$$d_i = y_i - a(x_i-\bar{x}) - b$$

$$\sum d_i = \sum y_i - \sum a\ (x_i-\bar{x}) - nb$$

but since $\sum(x_i-\bar{x}) = 0$ and if $\sum d_i = 0$

$$0 = \sum y_i - nb \quad \text{i.e. } b = \frac{\sum y_i}{n}$$

$$b = \bar{y}$$

$$\sum d_i^2 = \sum\left\{(y_i-\bar{y}) - a(x_i-\bar{x})\right\}^2$$

$$\frac{d(\sum d_i^2)}{da} = -2\sum\left\{(y_i-\bar{y}) - a(x_i-\bar{x})\right\}(x_i-\bar{x}) = 0$$

for a minimum $\sum d_i^2$ i.e. a minimum variance.

i.e. $\sum(y_i-\bar{y})(x_i-\bar{x}) - a\sum(x_i-\bar{x})^2 = 0$

$$a = \frac{\sum(y_i-\bar{y})(x_i-\bar{x})}{\sum(x_i-\bar{x})^2}$$

Hence the line of best fit by the method of least squares is $y = a(x_i-\bar{x}) + b$

Where: $$a = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}$$

$$b = \bar{y}$$

(iii) <u>The Correlation Coefficient</u>

The correlation coefficient r is defined as $r = \frac{S_{xy}}{S_x S_y}$

Where $S_{xy} = \frac{1}{n-1}\sum(x_i-\bar{x})(y_i-\bar{y})$ is known as the co-variance of x and y

$$S_x^2 = \frac{1}{n-1}\sum(x_i-\bar{x})^2 \text{ is the variance of x}$$

$$S_y^2 = \frac{1}{n-1}\sum(y_i-\bar{y})^2 \text{ is the variance of y}$$

r is a measure of the degree to which x and y are lineally interrelated.

If an exactly linear relationship exists such that $y = ax + b$:

Then $r = +1$ for a +ve
$r = -1$ for a -ve

If $r = 0$, then no relationship exists between x and y.

For practical observed data, the closer r approaches unity, the better is the correlation between the two sets of data.

The equation of the line of best fit by the method of least squares can be transformed to give:

$$\frac{y-\bar{y}}{S_y} = r \cdot \frac{x-\bar{x}}{S_x}$$

MODIFICATION OF D'ARCY'S LAW - CALCULATIONS

Sand	$\frac{G-1}{1+e} = b$	d	μ	$\frac{\mu . d}{b . \gamma_w}$	k $CMSEC^{-1} \times 10^{-3}$
Crusher Sand 'B'	1.051	0.0214	12.1	0.246	0.414
			12.1	0.246	0.488
			12.0	0.244	0.541
			11.8	0.240	0.629
			11.4	0.232	0.764
			11.4	0.232	0.775
			11.2	0.228	0.776
E.R.P.M. Furnace Ash	0.396	0.0663	11.6	1.94	11.9
			11.3	1.89	12.4
			11.3	1.89	12.1
	0.398		11.5	1.91	12.1
			11.4	1.90	12.0
	0.398		11.5	1.91	12.7
	0.399		11.7	1.95	9.89
			11.6	1.93	9.76
E.R.P.M. Dump Sand	0.894	0.0141	11.7	1.85	5.35
			11.5	1.82	5.97
	0.898		11.5	1.81	7.29
	0.898		11.4	1.79	7.36
			11.2	1.76	7.26
			11.1	1.74	7.24
			11.1	1.74	7.04
			11.5	1.81	6.58
			11.5	1.82	7.21
	0.894				
Crusher Sand 'A'	0.944	0.036	11.5	0.439	11.4
			11.5	0.439	15.9
			11.6	0.443	11.6
			11.3	0.431	15.3
			11.7	0.446	15.2
			11.6	0.443	14.9
Artificial Sand No. 1	1.101	0.0307	12.1	0.337	3.49
	1.107		11.6	0.322	3.64
			11.8	0.328	3.74
			11.8	0.328	4.04
			11.7	0.324	3.93
			11.7	0.324	4.05
			11.6	0.322	4.05

VALUES OF THE CONSTANT 'm' FOR THE MODIFIED D'ARCY LAW FOR FLOW THROUGH SOILS SUBJECT TO GRADING CHANGES,

$$\underline{\text{viz}}: \ q = C.i.A \left\{ \frac{\mu.d}{\gamma_w} \cdot \frac{1+e}{C-1} \right\}^{-1/m}$$

Sand	Value of m'	m' - $\bar{m}$'	(m' - $\bar{m}$')^2
Crusher Sand 'A'	50.1	27.4	749
Artificial Sand No.1	13.9	-8.8	77
Crusher Sand 'B'	7.9	-14.8	219
E.R.P.M. Dump Sand	19.1	-3.6	13
	91.0		1058

$\bar{m}' = \frac{91.0}{4} = 22.7$ $\qquad \hat{\sigma}^2_{m'} = \frac{1058}{3} = 353 \qquad \hat{\sigma}_{m'} = 18.8$

m' lies between $\bar{m}' \pm t_{(80,\ 3)} \times \hat{\sigma}_{m'}/\sqrt{4}$ with 80% confidence

m' lies between $22.7 \pm 1.64 \times 18.8/_2$

m' lies between 22.7 ± 15.4 with 80% confidence.

DIMENSIONLESS RELATIONS FOR THE FLOW OF WATER THROUGH SANDS

Calculations of: $\pi_1 = \frac{G-1}{1+e} \cdot \gamma_w \cdot \frac{dk}{\mu}$; $\pi_2 = \frac{1+e}{G-1} \cdot \frac{d.i}{k^2}$

Sand	$\frac{G-1}{1+e}=b$	d CM.	k $CM \times 10^{-3}$ SEC^{-1}	μ Poise $\times 10^{-3}$	$\pi_1 = \frac{b.\gamma_w dk}{\mu}$	$k^2 \times 10^{-6}$	i	$\pi_2 = \frac{d.i}{b.k^2} \times 10^{-6}$
Crusher Sand 'B'	1.051	0.0214	0.414	12.1	.000771	0.172	14.1	1.670
			0.488	12.1	.000909	0.238		1.206
			0.541	12.0	.001015	0.292		0.984
			0.629	11.8	.001200	0.394		0.729
			0.764	11.4	.00151	0.581		0.494
			0.775	11.4	.00153	0.600		0.478
			0.776	11.2	.00156	0.601		0.477
E.R.P.M. Furnace Ash	0.396	0.0663	11.9	11.6	.0270	142	14.3	.00169
			12.4	11.3	.0288	154		.00155
			12.5	11.3	.0290	156		.00153
			12.4	11.3	.0288	154		.00155
			12.3	11.3	.0286	151		.00158
			12.2	11.3	.0284	148		.00162
			12.1	11.3	.0281	146		.00164
	0.398		12.3	11.3	.0287	151		.00158
			12.1	11.5	.0278	146		.00163
			12.0	11.4	.0278	144		.00165
			12.7	11.5	.0292	161		.00148
			12.5	11.5	.0287	156		.00153
	0.399		9.89	11.7	.0224	97.4	14.4	.00246
	0.399		9.76	11.6	.0223	95.0	14.4	.00252
E.R.P.M. Dump Sand	0.894	0.0141	5.35	11.7	.00576	28.6	14.2	.00784
			5.97	11.5	.00655	35.6		.00630
			6.53	11.5	.00715	42.5		.00527
			6.80	11.5	.00746	46.1		.00486
			7.14	11.5	.00781	50.9		.00441
			7.21	11.5	.00790	51.9		.00432
			7.21	11.5	.00794	51.9	14.3	.00432
	0.898		7.29	11.5	.00802	53.0		.00423
			7.36	11.4	.00819	54.0		.00416
			7.26	11.2	.00821	52.7		.00426
			7.24	11.1	.00826	52.1		.00431
			7.04	11.1	.00804	49.4		.00454
			6.65	11.5	.00731	44.1		.00509
			6.58	11.5	.00724	43.1		.00521
Crusher Sand 'A'	0.944	0.036	9.22	11.6	.0270	84.8	14.1	.00635
			9.07	11.5	.0268	82.0		.00656
			11.4	11.5	.0337	130		.00414
			11.6	11.5	.0343	134		.00401
			12.4	11.5	.0367	153		.00352
			11.6	11.5	.0343	134		.00401
			11.6	11.6	.0340	134		.00401
			11.5	11.5	.0340	132		.00408
			15.3	11.3	.0460	234		.00230
			15.8	11.5	.0467	249		.00216
			15.9	11.5	.0471	252		.00214
			15.2	11.7	.0441	231		.00233
			14.9	11.6	.0437	221		.00244