

https://dspace.org/

A time-dependent green element formulation for solution of potential flow problems in 3 dimensional domains

Nyirenda, Edwin

2011-05-11
http://hdl.handle.net/10539/9724

Downloaded from a DSpace repository. https://dspace.org/

ABSTRACT

In this work we develop a generalised methodology for the solution of time dependent second order parabolic differential equation of potential flow in heterogeneous media using the Green element method. Parabolic differential equations are one class of differential equations, the others being elliptic partial differential equations and hyperbolic differential equations. Since elliptic differential equations generally arise from a diffusion process that has reached equilibrium, they can also be solved using the methodology developed, and represent a simplification because of the steady state situation.

Potential flow problems are of great interest in many engineering applications such as flow in aquifers, heat transfer processes, electromagnetic field problems, etc. Traditionally, the finite difference method and the finite element method have proved to be powerful techniques to solve such potential flow problems, but each has limitations and challenges which have led to continued research in numerical methods. The finite difference method is more applicable to domains with regular boundary, and the finite element method, though extremely versatile, exhibits unacceptable inaccuracies with coarse meshes, thus requiring fine meshes with the associated high computation costs.

In view of some of the limitations with these earlier methods, several numerical schemes are now being developed as viable alternatives to these conventional methods. Among such methods are the boundary element method, finite volume method, and the analytic element method. The boundary element method has been particularly promising because of its domain-reduction feature and the second order accuracy that can generally be achieved. The domain-reduction feature of the boundary element method, though achieved for restricted class of problems, lends it to efficient grid generation algorithm, while its second order accuracy ensures reliability and consistency of the numerical solutions.

The boundary element method in its original formulation is unable to deal with heterogeneities in the domain. For physical problems, especially in groundwater flow, heterogeneities and anisotropy are a natural and frequent occurrence, and this has fuelled research into boundary element techniques that are capable of accommodating these features.

The Green element method is one technique which is based on the boundary element theory and which has been proven to be very effective in handling heterogeneities and anisotropy in 1D and 2D domains. However, development of techniques to implement the Green element method in 3D domains has remained largely unexplored. This work represents an effort in this direction.

We have investigated the adoption of the general tetrahedral and hexahedra elements for use with the Green element method, and found that the large number of degrees of freedom generated precludes retention of the internal normal direction as in 1D and 2D formulations. Furthermore, some of the complicated surface and domain integrations with these elements can only be addressed with quadrature methods. The compatibility issues that arise between element faces, which present considerable challenges to multi-domain boundary element techniques, are innovatively addressed in the computer code that has been developed in this work.

The Green element method is implemented for steady and time-dependent problems using regular hexahedra elements, and the results show that the performance is slightly better than the results obtained using FEMWATER. FEMWATER is an established finite element method software. No attempt is made to compare the computation efficiencies of the 3D GEM code and FEMWATER because the two codes were not developed on a common platform.