

Analysis of non-axisymmetric underexpanded jet flow fields

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Declaration

I declare that this thesis is my own, unaided work, except where otherwise acknowledged. It is being submitted for the degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other university.

Signed this ____ day of _____ 20__

Nandkishore Menon.

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Abstract

A study has been performed that involved the experimental testing and numerical analysis of the underexpanded free jet flows issuing from rectangular, elliptical and slot nozzles. Aspect ratios of 1, 1.5, 2, 3 and 4 were tested at pressure ratios (exit plane pressure to ambient pressure) of 2 and 3 for rectangular nozzles. In the case of the slot and the elliptical nozzles the aspect ratios of 1.5, 2, 3 and 4 were tested at the same pressure ratios. There is reasonable qualitative agreement between the experimental observations and the numerical predictions. In the case of rectangular jets, a complex system of shock waves forming the incident shock system was discovered. This shock wave system originated at the corners of the nozzle exits, and proceeded to propagate downstream. Mach reflections have been discovered on the incident shock wave surface of the underexpanded jet from rectangular nozzles. The incident shock system has also been observed to undergo a Mach reflection in the direction of the jet flow. The resulting Mach disk has had a cross-sectional shape of a square, a hexagon and octagon in some of the flow fields from the nozzles that were tested. For slot and elliptical jets the formation of the incident shock wave was not observed along the minor axis plane of the nozzle for aspect ratios > 2 . The incident shock wave was observed to originate downstream of the nozzle exit in the major axis plane. In cross-sectional slices it has been established that the incident shock wave undergoes a transition to Mach reflection as it propagates downstream of the nozzle exit. In all cases tested, the shape of the jet boundary has been significantly distorted. In rectangular jets the narrowing of the jet boundary along the diagonal axis of the nozzle exit has been observed, and in the case of the elliptical and slot jets axis switching has been observed on the cross-sectional shape of the jet boundary.

Published Work

Some aspects of this thesis have been published in the following references:

Menon N., Skews B. W., *Effect of nozzle inlet geometry on underexpanded supersonic jet characteristics*, 26th International Symposium on Shock Waves, July 15 - 20, 2007 in Göttingen, Germany

Menon N., Skews B. W., *Rectangular underexpanded gas jets: Effect of aspect ratio, pressure ratio, and Mach number*, 26th International Symposium on Shock Waves, July 15 - 20, 2007 in Göttingen, Germany

Menon N. and Skews B. W., *3-D Shock Structure of Underexpanded Gas Jets From Elliptical and Rectangular Nozzles*, Proceedings of The 24th International Symposium on Shock Waves (ISSW24), Institute of Mechanics, Chinese Academy of Sciences, Beijing, 11-16 July 2004

Menon N. and Skews B. W., *3-D Shock Structure of an Underexpanded Gas Jet*, Proceedings of the 23rd International Symposium on Shock Waves (ISSW23), Fort Worth, Texas, 22-27 July 2001

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List of Symbols and Definitions

a	Speed of sound
AR	Aspect Ratio
b_e	Distance of the Mach reflection from the nozzle exit in [46]
Barrel shock wave	The incident shock wave system within an axisymmetric jet is referred to as a <i>Barrel shock wave</i> due to the shape of the shock resembling that of a barrel
D_n	Diameter of the nozzle [53]
D_{md}	Diameter of the Mach disk [53]
γ	Ratio of specific heats
h	Distance of the Mach disk from the nozzle exit
M	Mach number
M_e	Mach number at the nozzle exit
Mach disk	The Mach reflection of the incident shock wave along the longitudinal axis of the jet
Mach corner shock	Mach reflection occurring along the incident shock wave in jet flows from rectangular nozzle exits
P	Pressure
P_e	Pressure at the exit of the nozzle in the studies carried out in [46]
P_b	Back pressure in the settling chamber in the studies carried out by [46] and [53]

P_o	Stagnation pressure in the nozzle as tested by [53]
P_{exit}	Pressure at the exit plane of the nozzles tested in this study
$P_{ambient}$	Pressure in the ambient surroundings that the jet is exiting into in the nozzles tested in this study
Triple line	The locus of triple points on the edge fo a Mach disk
V	Flow velocity

1 Introduction

Jet flows are an important part of various mixing and thrust producing propulsion systems. Problems associated with combustion and jet noise have been shown to be associated with the jet structure, particularly the existence of shock waves within the jet. The structure of jet flows within underexpanded or overexpanded jets from axisymmetric nozzles have been investigated fairly rigorously over the past half century commencing with the work carried out by Love et al. [9]. Non-axisymmetric nozzles provide improvements in mixing of the flow within the supersonic jet. The structure of the jet flows issuing from non-axisymmetric nozzles particularly rectangular and elliptical nozzles have been confined to understanding the shape of the mean jet boundary. The shock waves that may be present in such jets would be dominated by 3-dimensional effects. The shock wave shapes that have been identified within axisymmetric underexpanded or overexpanded jets cannot be used as a basis to predict the shapes of the shock waves present within similar jets issuing from non-axisymmetric nozzles.

The thesis details the work that has been carried out in order to understand the flow features that are present in underexpanded jets from rectangular, elliptical and slot nozzles. In the case of elliptical and slot nozzles the work has been extended to understand the flow fields from nozzles that have angled inlets. The characteristics of the flow fields such as the shock cell lengths, the measured Mach disk heights, etc, are presented in Chapter 7. The description of the formation and development of the flow features within the jet boundary are presented in Chapter 8, and the development of the jet boundary is discussed in Chapter 9. Conclusions are drawn at the end of each chapter to summarize the findings.

2 Objectives

1. Establish the conditions that lead to the formation of compression waves within the plumes from underexpanded sonic jets issuing from non-axisymmetric nozzle exits.
2. Investigate the phenomena as it occurs in rectangular, slot and elliptical cross-sectional exit nozzles.
3. Investigate the aforementioned phenomena at varying exit plane pressures of the jet, and aspect ratios of the nozzle exit.
4. To develop a detailed understanding of the 3-dimensional flow field that is present in these underexpanded jets.

3 Free jet flows

3.1 Overexpansion and underexpansion of supersonic gas jets

Considering a supersonic gas jet issuing from an orifice into still air, there are various different interactions which can occur depending on the ratio of the exit plane pressure of the jet to the ambient pressure.

3.2 Overexpanded jets

In the case of overexpanded supersonic jets the pressure at the exit plane is lower than the ambient pressure into which the jet is issuing. Considering this case in two dimensions, a shock front would be present at the lip of the nozzle exit, and the jet would detach at the nozzle lip. The jet boundary would be curved toward the axis of symmetry of the jet. The shock waves which form at the nozzle lip will reflect from the opposite jet boundary as an expansion fan. The expansion waves turn the flow away from the axis of symmetry and consequently the jet boundary widens. These expansion waves reflect from the jet boundary as compression waves (that steepen into shock waves) once again, and the phenomenon is repeated until viscous effects originating from the jet boundary dampen this phenomenon, and the flow is subsonic from that point onwards.

Researchers have tried to develop analytical methods to determine the flow properties within overexpanded supersonic jets. Chow and Chang [6] developed analytical methods to determine the flow properties within overexpanded jets for two dimensional and axisymmetric flow fields. Li and Ben-Dor [7] developed the Mach wave configurations and the flow fields associated with two dimensional supersonic overexpanded free jets based on the two and three shock theory along with classical

gas dynamics theory. An analytical model was developed for predicting the Mach stem height and the shock cell size of the free jet. The model developed is the only available model that is capable of dealing with Mach reflection wave configurations in supersonic jets of overexpanded nozzles.

The shock wave reflection transition in an overexpanded 2-dimensional supersonic gas jet was investigated by Kudrayavtsev et. al., [5]. Euler computations performed showed that the transition from regular to Mach reflection and the transition back occurred in agreement with von Neumann's criteria and the theoretical detachment criterion. Hence the dependence of the shock reflection type in initial conditions was established. Numerical calculations were also performed by solving the Navier-Stokes equations using the $k - \epsilon$ turbulence model for closure and to account for compressibility effects. The results obtained showed good agreement with the theoretical predictions and with the solutions from the inviscid calculations. Turbulence production was confined to the jet boundary and did not affect the core supersonic flow. The numerical study revealed that when the pressure ratio of the jet is sufficiently low, subsonic conditions appear downstream of the reflected shock for regular reflection conditions. The authors suggest that this might be a triggering mechanism for the transition from regular to Mach reflection. Further analysis was unavailable at this point.

3.3 Underexpanded jets

A new phenomenon occurs in a supersonic jet emitted from a nozzle with a diverging opening when the exit plane pressure is higher than the ambient pressure. At the rim of the opening there occurs an expansion fan tending to lower the pressure of the gas to that of the ambient surroundings, but somewhere along the outer periphery of these waves a shock front develops which cuts across them and intercepts them. If the exit plane pressure is reduced then the shock front remains at the nozzle lip as discussed earlier with overexpanded jets. Hence there is a continuous transition from intercepting shock fronts to shock fronts formed when the jet is overexpanded in nature. A schematic of the shock pattern commonly found in an underexpanded supersonic gas jet is shown in Fig. 3.1.

Courant and Friedrichs[4] suggest the possible reasons for the formation of intercepting shocks: The flow continues uninfluenced by the ambient state until it meets the first Mach line issuing from the nozzle exit, i.e. the leading edge of the expansion wave. Owing to the divergence of the jet the pressure decreases along the axis.

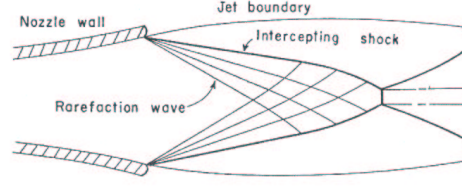


Figure 3.1: Shock pattern in a diverging jet emitted with a pressure greater than the ambient pressure (Courant and Friedrichs[4])

Across the expansion wave the pressure decreases until it drops below the ambient pressure. Hence there is a pressure gradient existing from the jet boundary where the pressure is equal to the ambient pressure. This will cause an inward curvature of the jet. All the Mach lines issuing from the jet boundary have the same angle with the jet boundary, since at the boundary the pressure, hence the sound speed and velocity remain constant. Hence these Mach lines would tend to converge with the converging jet and would have an envelope unless they were to be cut off by an intercepting shock (Fig. 3.2).

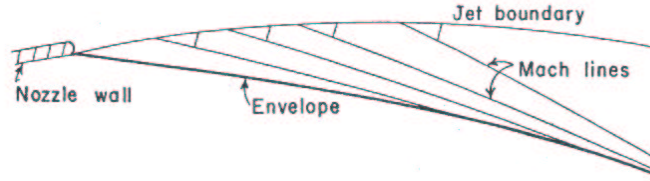


Figure 3.2: Envelope of Mach lines issuing from the jet boundary at equal angles (Courant and Friedrichs[4])

Thus an intercepting shock would have to exist to prevent the formation of the above mentioned envelope.

3.3.1 Two-Dimensional free jets

Werle et. al., [47] investigated the problem of a two dimensional jet exhausting into a quiescent medium where the jets were highly underexpanded at the entrance into the medium resulting in a classical plume configuration with the associated intercepting shock and Mach shock. The study provided a suitable means to predict the position of the Mach shock relative to the jet exit. For jet nozzle Mach numbers greater than or equal to unity, Driftmeyer [46] investigated the same problem and developed a relationship that suggested that the jet height varied linearly with Mach number for both the two dimensional and axisymmetric jet cases. The equation so developed is presented below:

$$\frac{h}{b_e} = \left[\frac{1}{j+1} \right] (\gamma_e M_e^2 \frac{P_e}{P_b})^{(\frac{1}{j+1})} (\gamma_e M_e)^{j-1} \quad (3.1)$$

where:

$j = 1$ (for axisymmetric flows)

$j = 2$ (for two dimensional flows)

The experiments for the analytical model displayed above were carried out using a slot nozzle mounted transversely between two glass side plates. The assembly was placed in a supersonic wind tunnel that served as a convenient low pressure test cell. Two supersonic nozzles were tested with Mach numbers ranging from 2.89 to 2.99. Three sonic nozzles of various slot widths were also tested. An unexpected experimental results observed was the absence of the normal shock along the centerline of the jet for the supersonic case. The observed flow field is schematically represented in Figure 3.3.

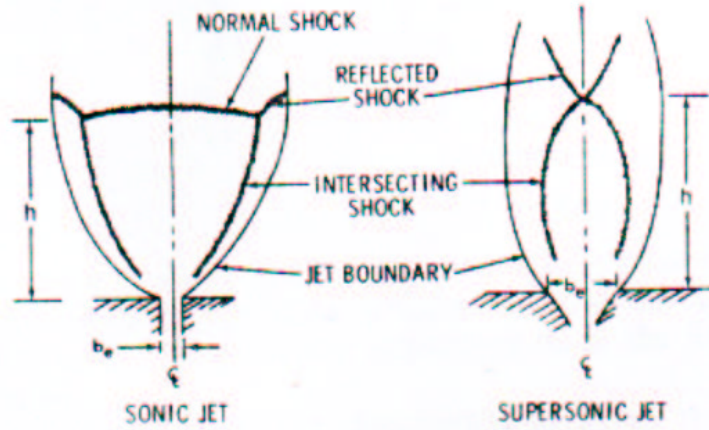


Figure 3.3: Jet plume structures [46]

The definition of shock-cell length is the distance of the normal shock from the nozzle exit. The absence of the normal shock in the supersonic case made it necessary for the authors to develop a characteristic length common to both cases. Hence the intersecting point of the incident shock in the supersonic case has been chosen to be the jet length. In figure 3.4 the experimental results are shown by the open symbols. The 45° slope of the plotted data indicates a linear variation of the dimensionless variables. The sonic free jet data matches the previous work as can be seen in the figure, though a shift was noted when the supersonic jet data was added. This shift was investigated theoretically by Driftmeyer [46] by utilising the graphical method of characteristics to establish the jet flow field. The results are plotted as solid symbols in figure 3.4.

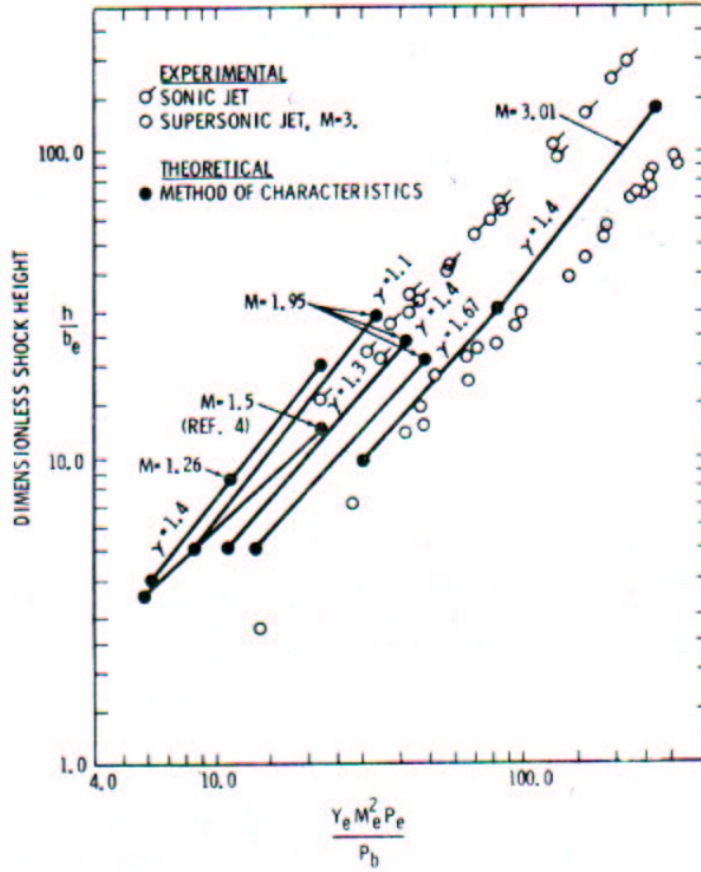


Figure 3.4: Two dimensional experimental sonic and supersonic free jet data compared with solutions obtained from the method of characteristics [46]

The data plotted in figure 3.4 collapses into a single curve when expressed in terms of the dimensionless height $\frac{h}{b_e}$ and the product of the Mach number and pressure ratios $M_e P_e / P_b$ as opposed to the dimensionless back pressure ratio $\gamma M_e^2 P_e / P_b$. This can be seen in figure 3.5.

The result so obtained implied that for the two dimensional jet the dimensionless jet height is independent of the specific heat ratio and is only linearly dependent upon the jet exit Mach number.

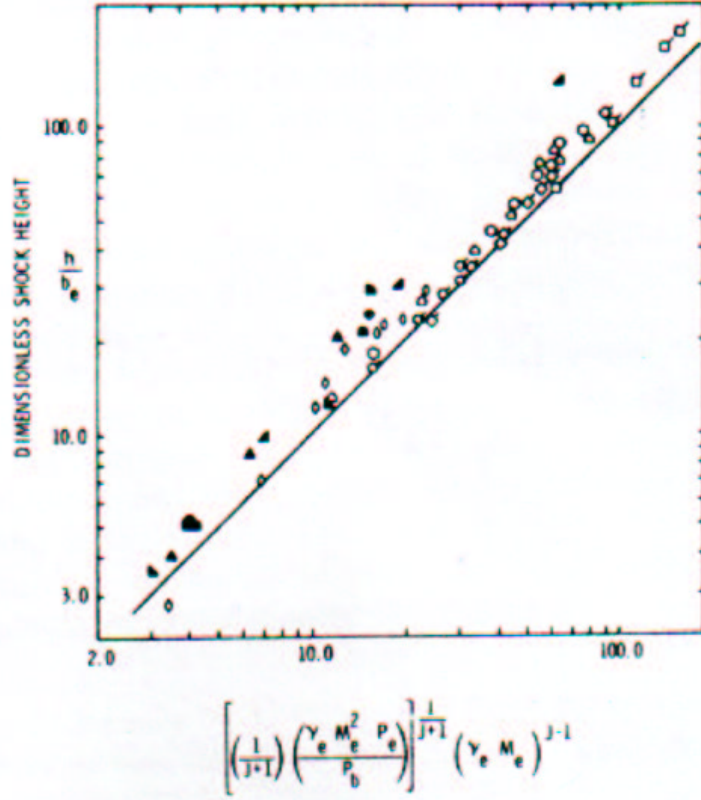


Figure 3.5: Universal correlation of free jet data [46]

3.4 Axisymmetric free jets

A considerable body of work exists in the literature on the flow field characteristics of axisymmetric underexpanded jets.

Pack [51] provided a method of characteristics approach as early as 1948 to determine the flow field characteristics of an underexpanded jet. The work was motivated by the need to infer the initial direction of the shock wave arising at or near the muzzle of a gun after it had been fired. The paper discussed the problem at hand from the point of view of the characteristics of the hyperbolic second order partial differential equations governing potential flow. Up until this point the research had been concentrated on establishing the periodic structure of the sonic jet when the pressure of the gas at the jet exit was slightly greater than the ambient pressure [51]. The periodic structure ceased to give accurate representations of the jet when the exit plane pressure of the jet was much greater than the ambient pressure. Through a method of characteristics approach Pack [51] was able to show how the point of origin of the shock wave within the jet and the shape of the shock wave formation

could be obtained through theoretical means.

A significant amount of work has also been concentrated on the production of noise as a result of the interaction of the shock wave with the turbulent shear layer of the jet (some of the work can be found in [49], [50], [48], [22], [24], [25], [26] and [27]).

In the past work has also been concentrated on understanding the initial inclination of the underexpanded supersonic jet as it exhausted from the nozzle. The inclination of the jet boundary was calculated using a method of characteristics based approach for underexpanded jets at moderate pressure ratios exhausting into a quiescent medium [10], for an underexpanded jet at large pressure ratios exhausting into a quiescent medium [11] and for the case of an underexpanded jet exhausting into a supersonic ambient stream [12], [13].

The most comprehensive work on underexpanded supersonic jets exhausting into a quiescent medium has been performed by Love et. al., [9]. Twenty-one steel nozzles of identical external geometry were constructed for the experimental studies. The throat of each nozzle was rounded and smoothly faired, and the interior surface was polished from the start of the convergence to the exit. Sixteen nozzles of the conically divergent type were designed with exit (i.e., divergence) angles of 5° , 10° , 15° , and 20° for each of four exit Mach numbers M_j of 1.50, 2.00, 2.50, and 3.00. Four nozzles were designed such that they had contoured profiles that gave essentially isentropic flow throughout and parallel flow at the jet exit. One convergent nozzle was designed for an exit Mach number of 1.00. All nozzles were designed with $\gamma = 1.4$. The experimental study was confined to schlieren observations. This is the most complete set of experimental results that can be used for validating the use of any numerical scheme for predicting the flow field within the underexpanded jet issuing from an axisymmetric nozzle. Theoretical models based on a solution of the potential flow equations using the method of characteristics was also carried out by Abbett [19], Eastman and Radke [15], and Fox [20]. These authors have utilised the body of experimental work presented by Love et. al., [9] as a basis for comparing the effectiveness of their theoretical models. Chang and Chow [16] studied the formation of the Mach disk in underexpanded flows using a method of characteristics approach and showed that the location and size of the Mach disk is governed by the appearance of a triple point shock configuration and that the central core flow would reach a choked state forming a throat like condition bounded by the slip layers from the Mach disk. The appearance of the Mach disk could hence be ascertained well from inviscid calculations.

Tam [26] developed a linear shock cell model for supersonic jets that operate at

slightly off-design conditions. The model accounted for the slow variation of the mean flow in the downstream direction and the effect of the turbulence in the mixing layer in the jet. As a result of the spatial change of the mean flow the shock cell spacing decreases gradually in the axial direction. The model so developed is only valid for the Mach number range of $|M_j^2 - M_d^2| \leq 1$. Here M_j is the fully expanded Mach number of the jet and M_d is the design Mach number of the jet. The model shows favorable agreement with experiment especially within the first three to four shock cells. This was found to be true for the gross features of the shock cells including the shock cell spacing and the pressure amplitudes associated with the shocks.

Addy [53] has provided an empirical relation to predict the location and the size of the Mach stem formed in a sonic jet issuing from a conically convergent sonic nozzle. It was observed that the pressure ratio at the onset of the Mach disk formation increased smoothly and continuously with the convergence angle of the nozzle inlet and the Mach disk structure was observed to be stable when the ratio between the stagnation pressure in the nozzle and the ambient pressure was greater than 5. The equations as developed by [53] are:

For a contoured nozzle:

$$\frac{D_{md}}{D_n} = 0.36 \left(\frac{P_o}{P_b} - 3.9 \right)^{\frac{1}{2}} \quad (3.2)$$

For a sharp edged orifice nozzle:

$$\frac{D_{md}}{D_n} = 0.31 \left(\frac{P_o}{P_b} - 5.0 \right)^{\frac{1}{2}} \quad (3.3)$$

A number of researchers have utilised the approach of solving the Euler [14] or the Navier Stokes ([21], [22], [18] and [17]) equations in cylindrical coordinates to obtain the flow field within an underexpanded axisymmetric jet.

Katanoda et. al., [14] studied the pitot pressure distributions within a supersonic underexpanded jet from axisymmetric nozzles having Mach numbers of 1.5 and 2.0. The expansion of the flow within the nozzle was achieved by means of having a contoured section and a simple conical expansion. The experimental results were compared to the experimental results of Love et. al., [9] and through means of a numerical calculation of the flow field utilising a Total Variation Diminishing (TVD) finite differencing scheme that solved the Euler equations in cylindrical coordinates.

The results obtained were as follows:

- With the correct expansion ratio, the jet from both contoured nozzles and conical nozzles showed a similar wave pattern; the compression and expansion waves were generated in the jet resulting in the cell structure in the potential core region., and the pitot pressure decreased with the increase in axial distance downstream of the potential core. The flow pattern of the contoured nozzle did not differ greatly from that of the conical nozzle. The authors have attributed this to the length of the contoured nozzles being too short to cancel out the expansion waves and compression waves generated in the nozzle and also to the axisymmetric nature of the nozzle.
- When the pitot pressure ratio across the nozzle exceeds the correct expansion ratio the pitot pressure was seen to recover significantly downstream of the potential core of the nozzle. The comparison with the numerical results indicated that although the pitot pressure in the vicinity of the jet centerline did not increase after the formation of the Mach disk within the jet, the total pressure outside the slip line is almost equal to its inviscid value. This suggested that the experimentally observed increase in the pitot pressure downstream of the Mach disk could be attributed to transfer of momentum from the region outside the slip line to the central part of the jet.

Gribben et. al., [17] successfully applied a CFD based method to solving the flow field within an underexpanded axisymmetric jet by solving the Navier-Stokes equations in cylindrical coordinates. Hysteresis was identified during the course of the analysis performed. This is incorrect as it is impossible for a Mach reflection to occur in an internal conical supersonic flow field [45]. The work has highlighted the effect of misinterpretation of results as a result of inadequate refinement of the computational mesh.

Shock oscillation

Panda [24] investigated the periodic oscillation of the shock waves in screeching underexpanded supersonic gas jets issuing from a choked axisymmetric nozzle at fully expanded Mach numbers of 1.19 and 1.42. The investigation was performed experimentally and through analytical means. The experimentation was performed using standard schlieren photography and a new shock detection technique that depends on the scattering of laser light by shock waves. The scattered light is

sensed by a photomultiplier tube (PMT). The time averaged and phase averaged statics of the PMT data revealed significant insight into the motion of the shock wave within the jet core. It was observed that the shocks moved more in the jet core and the least in the jet shear layer.

3.5 Non-axisymmetric free jets

3.5.1 Rectangular jets

There is limited literature on the detailed structure of the wave system in the near field of an underexpanded rectangular jet. Raman [34] and Alkisar [37] have investigated the shock structure toward developing an understanding of the screech tones emanating from the interaction of the compression waves within the underexpanded jet with the free jet shear layer. The investigation did not provide any clarity on the detail of the wave system or the nature of the shock system that is present in the near field of the jet.

Zaman [42] has investigated the spreading characteristics of underexpanded jets from a 3:1 aspect ratio rectangular nozzle in the Mach number range of 0.3 - 2. The research has shown that the spreading characteristics of asymmetric jets were not significantly different to that of axisymmetric jets, especially in the cases where the nozzle aspect ratio was close to unity. A noticeable increase in the spreading characteristic of the jet was only observed when the aspect ratio approached 10.

Mohamed et al., [39] performed an experimental investigation on supersonic jets from rectangular nozzles of aspect ratio 5:1, and characterised the flow regimes and the shock structure for jets in a quiescent atmosphere. The shock cell length was found to increase with the pressure ratio of the jet. The rate of increase was determined to be dependent on the nozzle design Mach number and on the shape of the jet cross section. The rate of shock cell growth was found to be greater for rectangular jets than for circular jets for the same design nozzle Mach number.

Gutmark et al., [40] investigated the spreading characteristics and the flapping modes associated with rectangular underexpanded jets issuing from a nozzle of aspect ratio 3:1 between Mach numbers of 1 and 2.4. A significant variation of the supersonic underexpanded jet was observed from near sonic velocity to higher Mach numbers. The changes in the jet structure and the flow field resulted in a corresponding variation of the near field pressure within the jet. Initially (near sonic exit

conditions) the jet appeared to be similar to axisymmetric underexpanded jets in both the major and the minor axis planes. The spreading rate of the jet was noted to be similar to that of a subsonic jet. A sudden transition to an asymmetric flapping mode resulted in increased spreading of the jet in the minor axis plane. An increase in pressure fluctuations in the minor axis plane was also noted, while the flow along the major axis plane remained symmetrical with low pressure fluctuations. The large amplitude flapping nature of the jet was suppressed with increasing Mach number and this resulted in a lower spreading rate of the jet. Gutmark et al., [40] were not able to establish the near field wave structure within the jet and have noted that the effect of the corners in rectangular jet evolution need to be investigated.

Teshima [41] performed experiments on underexpanded air jets exhausting into ambient conditions from rectangular orifices having aspect ratios of 2, 3, 4, 5 and 62 at pressure ratios of up to 500. The jet structure was visualised by using the laser induced fluorescence method. The study led to the following conclusions:

- The incident shock originating at the ends of the slit resulted in a Mach reflection or a regular reflection depending on the pressure ratio and the stagnation pressure of the jet.
- The regular reflection was noted to be stable at higher pressure ratios for a constant stagnation pressure. A range of pressure ratio for the transition from regular to Mach reflection was also observed.
- The transition was initiated by even the slightest pressure fluctuation and could be synchronised with the frequency of the rotary pump used for the experiments.
- A hysteresis for the shock reflection transition was observed for the smaller aspect ratio orifices.
- The flow conditions for the shock transition could be explained qualitatively using two dimensional theory.

Loh et al., [43] performed computational studies of an underexpanded rectangular jet utilising an unstructured space marching Euler method and was able to obtain reasonable agreement with the experimental findings by Raman [34]. The work was centered on the numerics rather than the physics of the flow field and hence did not provide any information apart from snapshots of the flow field in the major and minor axes planes. The results were however not critically discussed with regards to the marked differences in the flow field in the major and the minor axis of the

underexpanded jet. In addition the resolution of the numerical solution did not allow for a detailed examination of the results obtained.

A detailed computational study on the structure of an underexpanded jet from a rectangular nozzle was performed by Rao and Abdol-Hamid [44]. The main purpose of their work was to develop a unified method for jet flow prediction using a Navier-Stokes method with a two equation $k-\epsilon$ turbulence model. The numerical solutions were obtained for an underexpanded jet issuing from a square jet and a rectangular jet of aspect ratio 2. The jet exit Mach number was 2.0 while the ratio of stagnation pressure of the jet to the ambient pressure was set at 11.31. This corresponded to an exit static pressure ratio of 1.445. For the rectangular jet it was observed that the boundary grew quite rapidly in the minor axis plane and that the corners of the original rectangular jet get rounded due to the process of turbulent mixing. In the case of the square jet it was observed that the jet turns through angle of 45° in the direction of the flow resulting in a cross section transposed with the nozzle exit profile.

Menon and Skews [3] using the Laser Vapour Screen flow visualisation technique were able to capture 'slices' of the flow field of an underexpanded jet issuing from a square nozzle. The experimental findings produced the only images of the near field flow structure of the underexpanded jet, and the images were utilised to construct a virtual model of the shock surfaces within the jet flow field. Apart from similar observations on the jet boundary spreading they were able to establish that the jet appeared to be behaving as an overexpanded jet along the diagonal plane. The origin of the incident shock wave along the central planes of the jet was observed to occur a distance downstream of the nozzle exit. This was consistent with the nature of underexpanded jets explained in earlier sections. Along the diagonal plane of the nozzle however the incident shock wave originated from the corners of the nozzle exit. This coupled with the narrowing jet boundary along this plane resulted in the belief that the jet seemed to be behaving like an overexpanded jet in this plane. Numerical studies on underexpanded jet issuing from square and rectangular nozzles (Aspect ratio of 2) showed that the the flow field could be predicted accurately numerically [2]. A similar shock structure was observed in the case of the rectangular jet with the incident shock wave within the underexpanded jet appearing to originate at the corners of the nozzle exit. The authors were able to qualitatively explain the formation of the shock at the corners of the nozzle, but were unable to provide a substantial reason for the formation of the shock at the nozzle corner.

3.5.2 Elliptical jets

Rajakuperan and Ramaswamy [31] computed the near field structure of an under-expanded jet from elliptic nozzles utilizing a space marching shock capturing finite difference method based on McCormack's predictor - corrector numerical scheme. The method solved the 3-dimensional Euler equations with the constant pressure boundary condition for the outer free jet boundary. The computed results revealed that the method used could capture the overall flow features of the underexpanded jets including the differential jet spreading in the planes of symmetry and the axis switching. The good comparison between the computed and the numerical results validated the computational method. The axis switching phenomenon present in the underexpanded jets seemed to be governed by inviscid flow processes involving the interactions between the shock waves, the expansion waves and the jet boundary unlike the subsonic jets where the axis switching is attributed to self induction process of the vorticity developed by the viscous shear layer. The authors have suggested that for low pressure cases, the viscous effects may play a major role in the development of the jet. The computation process also captured the presence of a barrel type incident shock in the major axis plane and the absence of it in the minor axis plane. It was observed that the incident shock is confined to a narrow azimuthal region and then gradually diffuses. This was as a results of the jet boundary being more curved in the major axis plane compared to the minor axis plane, and hence the tendency for the compression waves to coalesce and form the barrel shock was greater in the major axis plane than in the minor axis plane. The cross-sectional shape of the jet was observed to change considerably along the axial direction with the formation of fine finger like structures that were formed as a result of secondary flows resembling longitudinal vortex structures. The authors suggest that the presence of these fine finger like structures in the cross-section of the jets would provide a large interface area necessary for the rapid mixing of these jets with the surrounding fluid.

Verma and Rathakrishnan [29] experimentally investigated the effect of Mach number on the acoustic field of a elliptic jet having an aspect ratio of 2. The shock cell structure was visualised in both planes of the nozzle at two pressure ratios, one at which the jet was moderately underexpanded (fully expanded Mach number = 1.5) and another at which the jet was highly underexpanded (fully expanded Mach number = 2). The visualisation technique utilised was the standard shadowgraph technique. At the moderately underexpanded condition the shadowgraph image revealed a series of oblique shocks and expansion fans impinging on and reflecting from the jet shear layers. The result was a cross over point at the end of each, giving the appearance of shock-diamonds or a braided structure within the jet core. The shock

development process was noted to be different along the two planes of the nozzle. The reason was primarily attributed to the growth of the mixing regions of the jet which was noted to be more along the minor axis plane and lesser on the major axis plane of the nozzle. The reflected compression waves coalesced to form the cross over point. The lack of the compression waves on the minor axis plane is attributed to the dissimilar growth rates of the jet along the two planes. As the pressure ratio was changed such that the jet was now highly underexpanded, the structure of the shock cell was seen to change quite considerably.

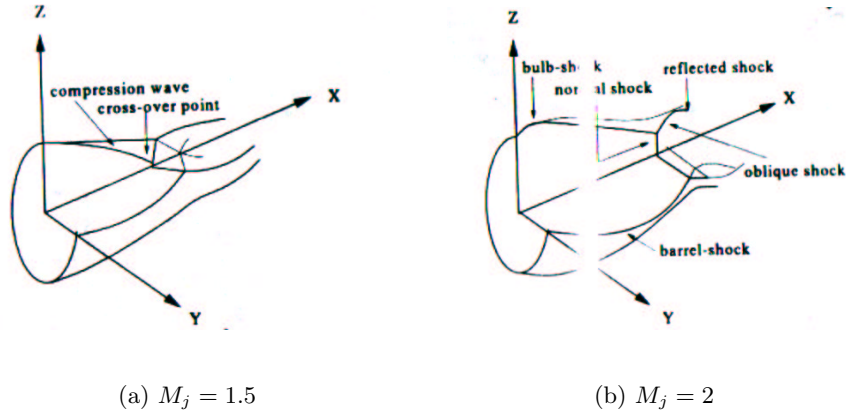


Figure 3.6: Three dimensional section view of the shock structure along the two symmetry planes of the jet flow field [29]

The pressure along the centerline of the jet decreased to a value much lower than the ambient pressure and the re-compression of the flow in the remainder of the shock cell reaches a limiting value for conical shocks and the required re-compression takes place through a normal shock. At highly underexpanded conditions the jet boundary was observed to undergo a drastic change in the shape at the beginning of the shock cell. The jet growth on the two axes being different led to the formation of a barrel type shock along the minor axis plane of the nozzle while a 'bulb' shock was seen to appear along the major axis plane. The authors suggest that this is due to the fact that along the major axis of the nozzle the jet grows slowly which makes the initial bulge in the jet boundary converge into a 'bulb' shape unlike along the minor axis plane of the nozzle where the higher growth of the jet boundary results in a barrel shock formation. The three dimensional sectional views of the shock structure can be seen in Figure 3.6. Tam [27] used a linear shock cell model to calculate the variation of shock cell spacing with the jet Mach number. Verma and Rathakrishnan [29] have obtained good agreement with the theory proposed by Tam [27].

Experiments have also been carried out on notched elliptic jets ([28] and [30]). Elliptic nozzles were constructed having triangular and square notches on the major axis of the nozzle. The purpose of the study was to determine the effect on the mixing characteristics of the jet as a result of the presence of the notches. the following conclusions were drawn by the authors:

- The introduction of the sharp edged notches on the nozzle helped enhance the mixing close to the nozzle exit (less than 10 equivalent diameters).
- The presence of the notches resulted in the axis switchover location to be moved upstream when compared to the nozzles that did not have any notches. This indicated a higher bulk mixing in the notched nozzles.
- The notch geometry was noted to be a strong parameter in controlling the mixing characteristics in the elliptic jet.
- The triangular notched nozzle was observed to have the higher jet centerline velocity decay and hence the minimum supersonic core length.

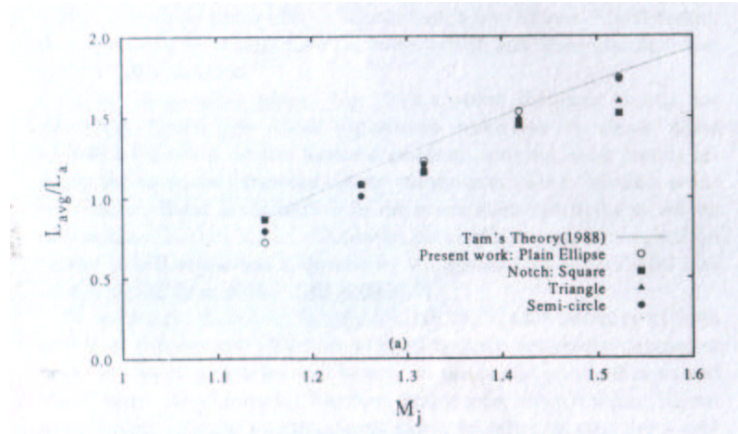


Figure 3.7: Variation of average shock cell lengths for notched elliptical jets [30]

The results obtained by Verma and Rathakrishnan [30] shows reasonable agreement with those obtained for un-notched jets when compared to the results from the theory proposed by Tam [27] (Figure 3.7).

Menon and Skews [2] investigated the near field shock structure of the jet issuing from an elliptical nozzle. The aspect ratios considered were 2 and 4, while the pressure ratios of the jets were varied between $\frac{P_{exit}}{P_{ambient}} = 2$ and $\frac{P_{exit}}{P_{ambient}} = 3$. The results were obtained by solving the Navier-Stokes equations using the k- ϵ turbulence model to provide closure to the equations. The most obvious feature of the flow was the presence of the normal shock within the jet flow under highly underexpanded

conditions ($P_{exit}/P_{ambient}$). The size of the Mach disk was seen to increase as the aspect ratio of the nozzle was increased. The orientation of the Mach disk showed a switching of axis when compared to the orientation of the nozzle exit. Due to the non-uniform azimuthal curvature of the nozzle exit the jet spreading was noted to be markedly different along the two symmetry planes of the nozzle. The presence of a shock wave originating close to the nozzle lip along the major axis plane was noted on both aspect ratios that were investigated and at both pressure ratios. At the lower pressure ratio the reflection of the shock wave along the major axis plane was noted to be regular in nature. The incident shock was seen to steepen in strength as it approached the centerline of the jet. The elliptic case being a subset of the general conical flow case, the authors have suggested that it is hence impossible for a regular reflection to occur in such a flow field citing the work performed by Mölder [45]. The inference is made from the presence of contact layers behind what appears to be a point of regular reflection along the major axis plane of the nozzle. The presence of the contact layers would suggest that there would be a normal shock wave at the reflection point. In the case of the larger aspect ratio nozzle the minor axis being much narrower than the major axis the authors suggest the formation of a quasi-planar flow condition along the minor axis plane. The Mach disk was noted to have a marked upstream curvature and that in the case of the smaller aspect ratio it was elongated in the minor axis plane. For the larger aspect ratio nozzle the jet seemed to achieve a state of quasi-axisymmetry before the end of the first shock cell. The size of the Mach disk along both axes was noted to be almost equal.

3.5.3 Slot jets

There is limited experimental knowledge on the behaviour of underexpanded jets issuing from slot nozzles. Rajakuperan and Ramaswamy [32] performed a series of detailed experiments on underexpanded jets issuing from slot nozzles. The experiments involved obtaining images of the jet flow field using schlieren photography, and measuring the pressures within the jet by means of a pitot pressure measurements. The experimental images obtained provided the authors with an image integrated through the thickness of the test section. The authors have expressed that the conjecture of the shock structure proposed by them ought to be taken with caution. However for the cases of sharp density gradients as in the case of the barrel shock present in an underexpanded axisymmetric jet the line of sight integration gives reasonable results due to the density gradients being present along the plane of symmetry of the flow field. On the basis of this Rajakuperan and Ramaswamy [32] have presented a proposed shock structure in the near field of the underexpanded

jet from a slot nozzle. It was observed that the strong barrel type of incident shocks emerged almost from the nozzle lip along the major axis plane. The incident shock thus formed underwent a Mach reflection for most of the pressure ratios and is similar to what is present in an axisymmetric jet. The shock structure in the minor axis plane was observed to be similar for the nozzle of aspect ratio of 1.4, but was different for all other aspect ratios. The incident shock was absent for the aspect ratios greater than 1.4, even though the presence of the normal shock similar to the Mach disk and oblique shocks resembling the reflected shocks were observed. The near field shock structure conjectured from the obtained results by Rajakuperan and Ramaswamy [32] is shown in figure 3.8.

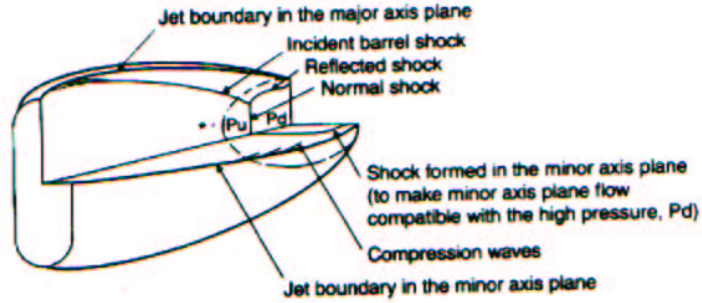


Figure 3.8: Conjectured shock structure of underexpanded jets from slot nozzles [32]

The hypothesis put forward by the authors for the shock structure observed in the major and minor axis planes is as follows: the normal shock formed in the major axis plane has to have a finite width and probably for pressure compatibility reasons the normal shock gets bent and forms an oblique shock which appears similar to a reflected shock along the minor axis plane. The absence of the incident barrel shock in the minor axis plane is due to insufficient curvature of the jet boundary in that plane for the compression waves that form at the jet boundary to coalesce and form a shock. The shock structure is quite complicated and the strength of the barrel shock formed in the major axis plane probably diffuses circumferentially in both directions and is not observed in the minor axis plane.

The location where the jet width in the minor axis plane becomes equal to the jet width in the major axis plane is known as the axis switching location. The axis switching location was observed to vary with both pressure and variation of aspect ratio of the nozzle (figure 3.9). The axis switching occurs at axial locations close to the nozzle exit for low aspect ratio nozzles ($AR = 1.4$). As the nozzle aspect ratio increases the axis switching location moves further downstream.

The conclusions reached were as follows;

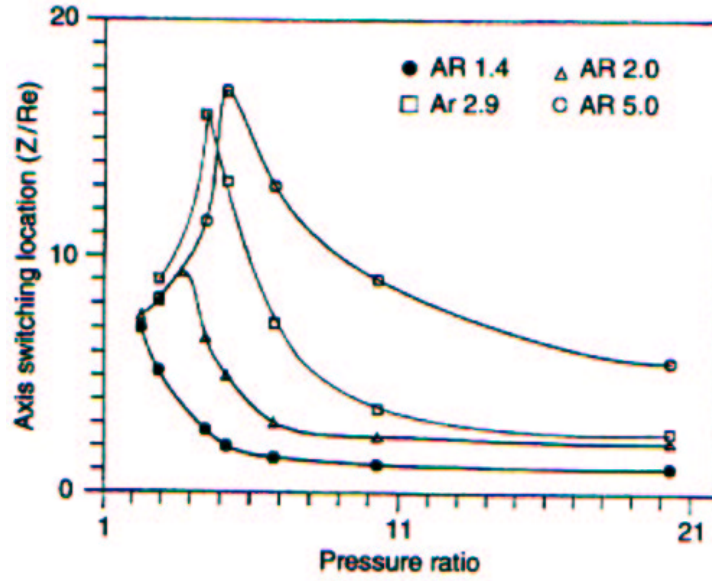


Figure 3.9: Variation of axis switching location with pressure ratio for jets from slot nozzles [32]

- The jets from oval sonic nozzles of low aspect ratio ($AR = 1.4$) exhibited a shock structure similar to that present in an axisymmetric jet, and for nozzles of higher aspect ratio the shock structure was different in the major and minor axis planes. In the major axis plane the shock structure resembled an axisymmetric underexpanded jet, but in the minor axis plane the barrel shock was not present although an oblique shock resembling a reflected shock was present.
- The jet from the slot nozzle continued to expand along the minor axis plane after the nozzle edge, whereas along the major axis plane the jet width was more or less maintained.
- The pressure ratio had a significant effect on the jet spreading characteristic along the minor axis plane, but only a marginal effect on the major axis plane.
- In the minor axis plane the spreading of the jet was noted to be at a maximum when the pressure ratio was 20.3. As the pressure ratio was reduced the spreading of the jet also reduced until a critical pressure was reached below which the spreading rate of the jet once again increased.
- The critical pressure ratio demarcated the stable and unstable operating regimes of the jets issuing from the slot nozzles. The jets exhibited a flapping mode also seen by Gutmark et. al., [40] in the experiments with rectangular nozzles at similar pressures.

- Axis switching was noted as a result of higher spreading rates experienced by the jets in the minor axis plane which was caused by the complex interaction of the expansion and compression waves within the jet. The axis switching location was also noted to correlate well with the spreading pattern in the minor axis plane for all the jets investigated except for the nozzle with $AR = 1.4$. The authors suggest that the axis switching in this nozzle is not influenced by the higher spreading but by the instability within the jet.
- The cross-sectional shapes of the jets from the slot nozzles varied significantly along the length of the jet. For the nozzles with $AR = 2$, the jet cross-section was noted to be almost circular at the axis switching location. Further downstream of this point the jet was noted to be elliptic in cross-section with the major axis of the jet now on the minor axis of the nozzle and vice-versa.
- Jets from slot nozzles were seen to have higher spreading rates when compared to axisymmetric jets and hence offer enhanced mixing potential.

4 Experimental Apparatus and Procedure

The experimental apparatus that has been used in the work described in this thesis consists of the following components

- Nozzle test assembly (including, the nozzles attached to a settling chamber)
- Compressed air supply (consisting of a reciprocating compressor, a dryer unit, and storage tank)

4.1 Compressed Air Supply

The compressed air supply for the experiments on the sonic and supersonic nozzles was provided by the same system in the laboratory that provides the compressed air supply to the supersonic wind-tunnel [1].

4.1.1 Reciprocating compressor

An electrically driven reciprocating compressor is used to pump the tunnel storage vessel up to the desired pressure.

Table A.1 in the appendix lists the technical specifications of the compressor.

4.1.2 Dryer Unit

Air is supplied to the storage vessel of the wind tunnel by the reciprocating compressor, via the dryer unit. The purpose of this device is to remove the moisture present in the air that is entering the storage vessel. This is important to prevent condensation shocks from occurring in the tunnel during its startup. The dryer consists of

a series of oil and water filters, followed by the main dryer unit - a pressure vessel containing silica crystals, which absorbs the remaining moisture in the air before it enters the storage vessel. The silica crystals can get saturated, and must be dried periodically. This is achieved by blowing hot air through the dryer over the crystals for a period of time. The technical specifications of the dryer unit are listed in Table A.2 in the appendix.

4.1.3 Storage Vessel

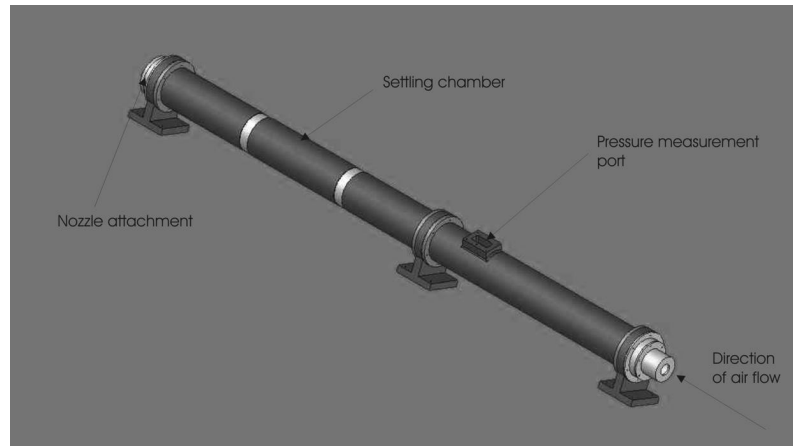
The storage vessel is a high pressure vessel designed for an operating pressure of 2.08MPa. Due to the shut off mechanism on the compressor, however, it is operated at a pressure of 1.5MPa. The technical specifications are listed in table A.3.

4.2 Nozzle Test Assembly

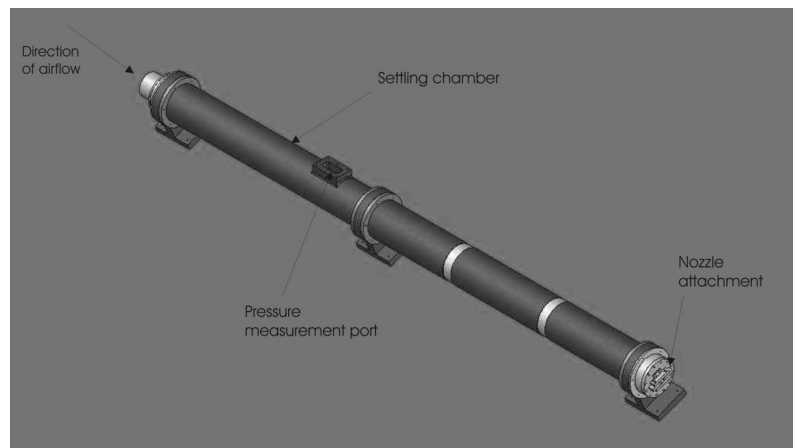
The nozzle test assembly consists of a settling chamber that is supplied with compressed air from the compressed air system described previously in this chapter, and the nozzles that are attached to the end of the settling chamber. The assembly is mounted on a rigid metal frame that is bolted to the concrete floor of the laboratory to prevent excessive vibration during the experimentation. The manufacture of all components that formed the nozzle assembly was performed in the workshops at the School of Mechanical, Industrial and Aeronautical Engineering.

4.2.1 Settling Chamber

The nozzle test assembly consisted of a settling chamber, which was made out of mild steel tubing, having an inner diameter of 102mm and a thickness of 6mm. The settling chamber had a length of 2m. It was mounted onto a stand which was fastened onto the concrete floor of the laboratory with anchor bolts. The settling chamber had much the same purpose as the settling chamber of the supersonic wind tunnel. The settling chamber was supplied with compressed air from the compressed air supply available in the lab. The nozzles are designed to be clamped onto the end of the settling chamber.



(a) Isometric View 1



(b) Isometric view 2

Figure 4.1: Settling chamber and nozzle test piece assembly

4.3 Test nozzles

The test nozzles were constructed to be able to be attached to the end of the settling chamber assembly. The square and rectangular nozzles were constructed in a different manner to that of the circular, slot and the elliptical nozzles. The detail in the design and the construction of the nozzles is outlined in the sections below. The rationale for the design of the nozzles was to ensure that the opening area of the nozzle was as close to 100 mm^2 as possible. The dimensions of the nozzles that were constructed and tested are given in Table 4.1.

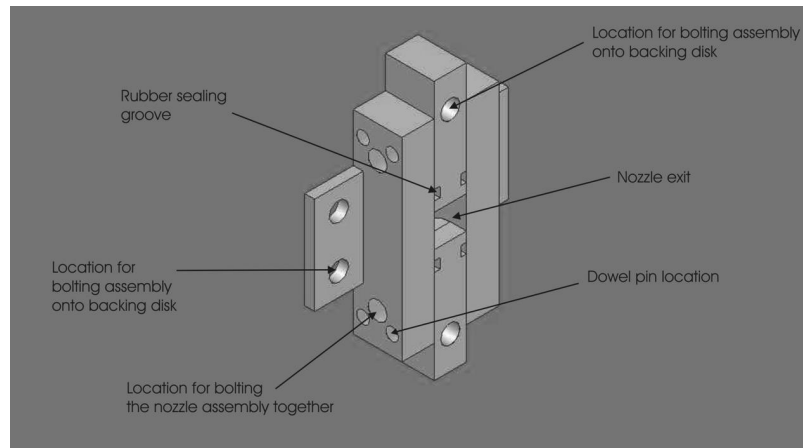
Table 4.1: Dimensions of the nozzles that have been tested in this study

Nozzle type	Aspect ratio	Major axis dimension (mm)	Minor axis dimension (mm)
Circular		10	
Rectangular	1	10	10
	1.5	12	8
	2	14	7
	3	18	6
	4	20	5
Elliptic	1.5	12	8
	2	14	7
	3	18	6
	4	20	5
Slot	1.5	12	8
	2	14	7
	3	18	6
	4	20	5

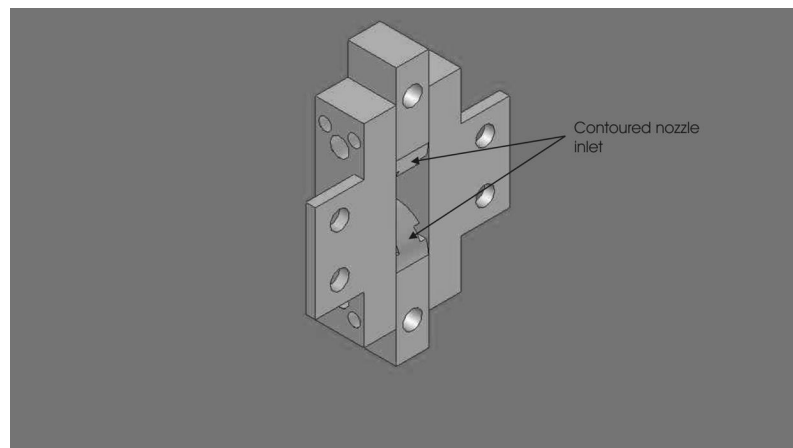
4.3.1 Square nozzles

The square nozzles that were used in this study were constructed using aluminium.

An example of the assembled nozzle is shown in figure 4.2. The figure represents the square nozzle that was used in this study. The nozzle is constructed out of four independent sections that are located together with the dowel pins that are inserted into the holes marked in figure 4.2(a). The sections forming the nozzle were bolted together, and the sealing between the section was provided by inserting cut sections of rubber "O-rings" into the sealing grooves identified in figure 4.2(a). The nozzle



(a) Isometric View

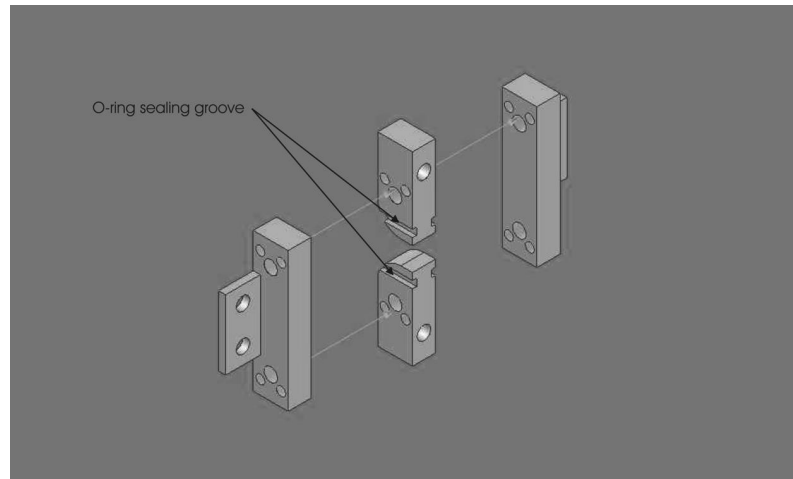


(b) Reverse view

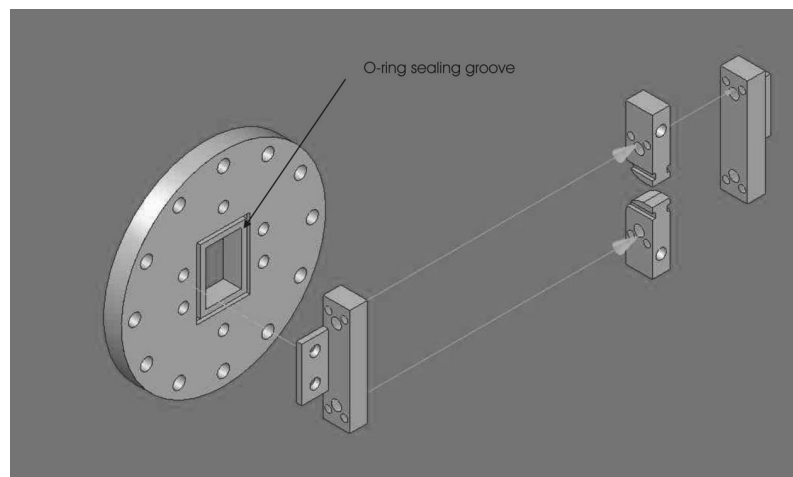
Figure 4.2: Square nozzle assembly

had a contoured inlet as can be seen in figure 4.2(b). An exploded view of the four sections that formed the nozzle assembly is shown in figure 4.3(a).

The nozzle assembly thus formed was bolted onto a backing plate. The sealing between the nozzle assembly and the backing plate was again provided by means of cut sections of rubber "O-rings" inserted into the rectangular groove in the backing plate as can be seen in the exploded view in figure 4.3(b). The nozzle assembly with the backing plate was bolted onto the settling chamber assembly to form the complete test rig. The sealing between the settling chamber assembly and the nozzle assembly with the backing plate was achieved through the use of rubber gaskets. A close-up view of the final assembly onto the settling chamber is shown in figure 4.4.



(a) Exploded view of the four sections forming the nozzle



(b) Exploded view with the backing plate

Figure 4.3: Exploded views of the square nozzle assembly

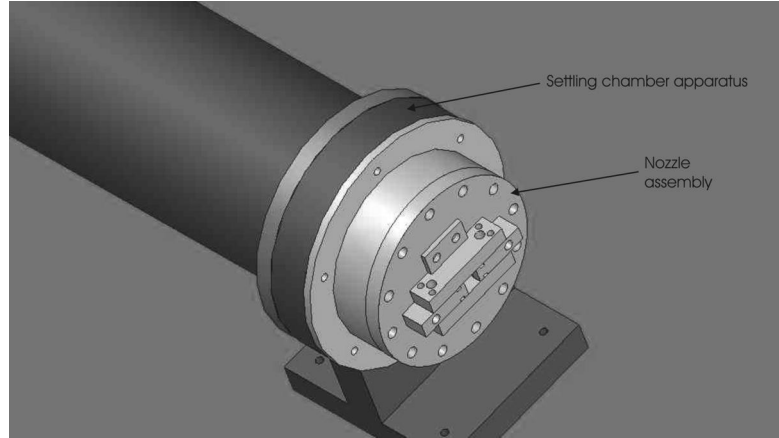


Figure 4.4: Close-up view of the final assembly of the nozzle to the settling chamber

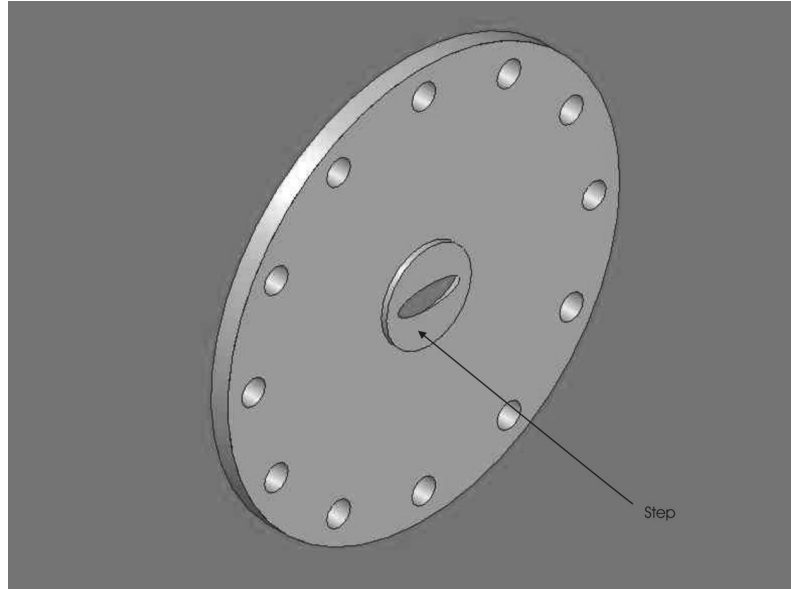
4.3.2 Elliptic and Slot nozzles

The elliptic and slot nozzles were also constructed using aluminium. The nozzles were constructed out of a single piece in a numerically controlled six axis milling machine. The slot and elliptic nozzles were also designed to be fitted onto the nozzle assembly shown in figure 4.1. Initially the nozzles were manufactured with a sharp entrance, but during the testing phase of the nozzle it was discovered that this led to spurious compression and expansion wave formation in the jet flow field. To avoid the adverse effects of a sharp entrance on the resulting flow field outside the nozzle, a 45° bevel was milled to generate a sharp edged nozzle (figures 4.5(b) and 4.6(b)).

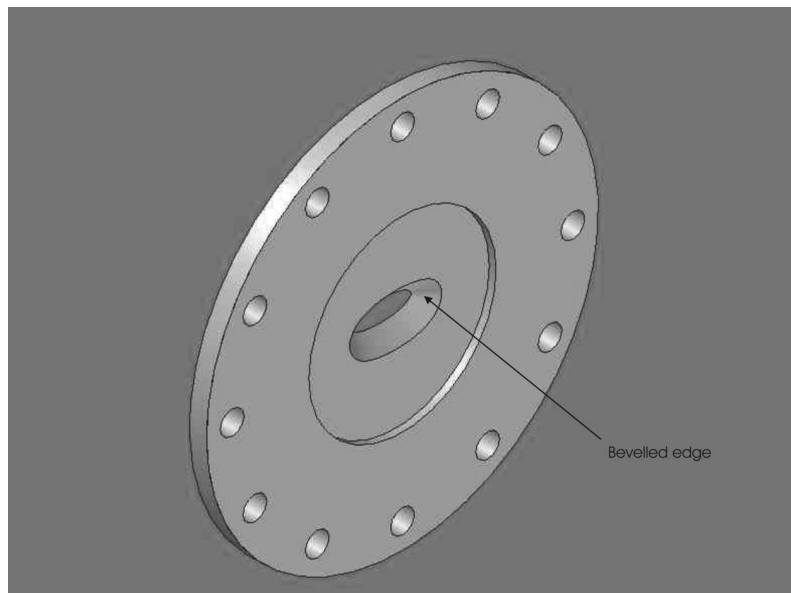
The radius of the circular sections of the slot nozzle were maintained at 5 mm (figure 4.6(b)). The flat section on each nozzle was varied to achieve the appropriate aspect ratio that was required for the test section (figure 4.6(b)). Each nozzle also had a 2mm step that was milled onto the front section of the nozzle (figures 4.5(a) and 4.5(b)). These steps allowed for the nozzle to be aligned within the shadowgraph section such that there was no reflection of the incident light from the front surface of the nozzle onto the photographic film. The step also allowed for any scaling to be performed on the experimental images.

4.3.3 Pressure measurement

Static and stagnation pressures were monitored in the settling chamber during each test. The static pressure was monitored utilising an analogue static pressure guage. The stagnation pressure in the settling chamber was monitored by attaching a digital pressure transducer at the end of a hypodermic needle. The needle had a 90° bend

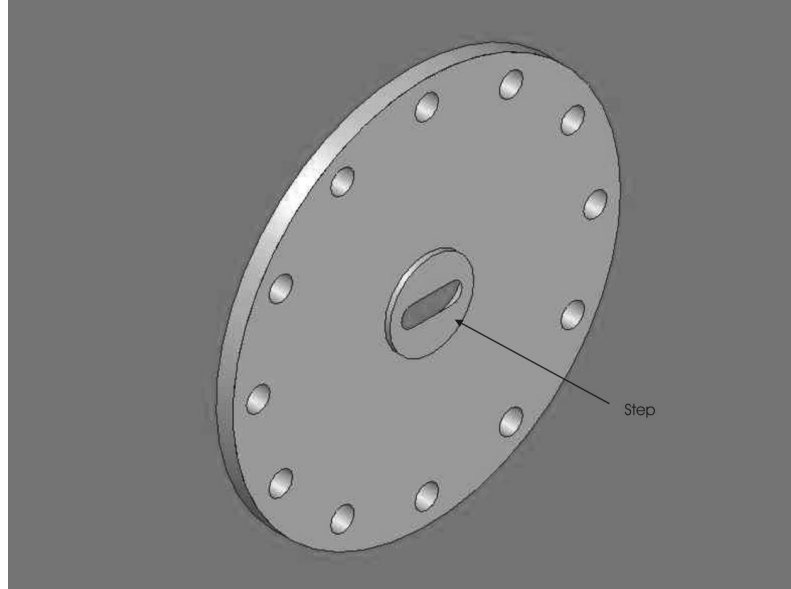


(a) isometric view of the elliptic nozzle, $AR = 3$

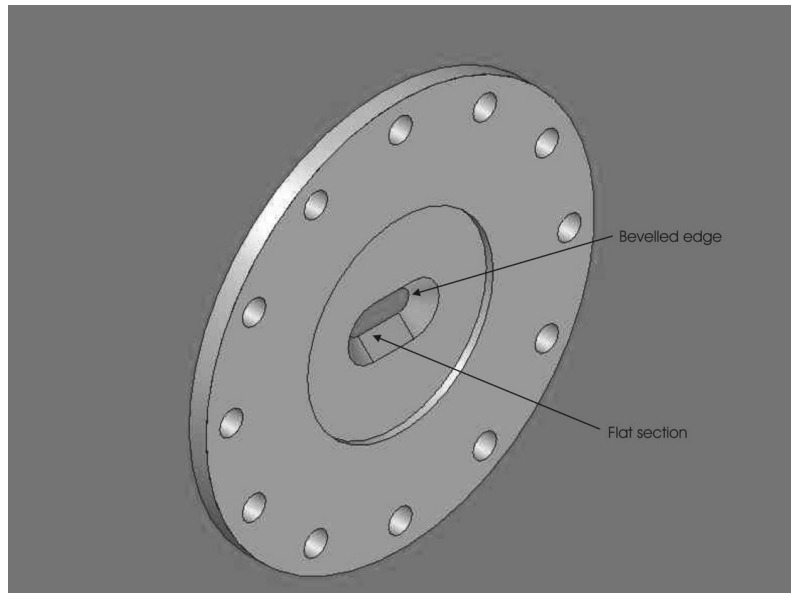


(b) Reverse isometric view of the elliptic nozzle, $AR = 3$

Figure 4.5: Views of the $AR = 3$, elliptic nozzle



(a) Isometric view of the slot nozzle, $AR = 3$



(b) Reverse isometric view of the slot nozzle, $AR = 3$

Figure 4.6: Views of the $AR = 3$, elliptic nozzle

in it, and was placed in the settling chamber such that the opening into the needle was facing opposite to the direction of the flow through the nozzle test apparatus.

4.4 Shadowgraph apparatus

A standard contact shadowgraph system has been utilised to visualise the flow fields of the jet issuing from the various nozzles. The shadowgraph system was developed using a constant light source, and the image was captured onto Black and White ISO 400 photographic film. The image was captured using a standard single lens reflex camera. The shutter speed was set at 250 ms and the shutter was opened utilizing a remote shutter opening mechanism (cable release mechanism).

Light from a constant light source consisting of a halogen lamp emitting 150 W is focused onto a double knife-edge to generate a high intensity line source. The light from the line source generated in this manner is reflected into a parallel beam utilising a parabolic mirror. The parallel beam of light is passed through the test section onto the photographic film in the camera. The details of the mirrors are listed in table A.4.

4.5 Experimental procedure

The experimental procedure for performing the tests consists of pre-test actions, performing the test, and post-test actions.

The pre-test actions are:

- Pressurising the storage vessel
- Setting up the visualisation system
- Setting up the nozzle test rig

and post-test actions are:

- Developing the photographic film

4.5.1 Pressurizing the storage vessel

Before the nozzles can be tested, the storage vessel has to be pumped up. The procedure for this is outlined below;

1. Close both valves on the dryer unit.
2. Turn on the cooling water for the reciprocating compressor (one turn of the valve).
3. At the dryer check that the air is coming out of both the oil and water filters. Note that it is a very small quantity of air which will bleed out from the filters.
4. Switch on the compressor at the electrical control board.
5. Once the compressor has pumped up to approximately 1.6MPa, it will turn off automatically. Turn the compressor switch off at the control panel and turn the cooling water supply off as well. This is to prevent the compressor from restarting as the pressure drops in the storage vessel (during a test).
6. Bleed the air from the dryer unit, by opening the top valve. Do so slowly as the pressure in the unit is of the order of 200 psi (1.37 MPa).
7. Once the air has been bled off the dryer unit, open the bottom valve and turn on the heater for approximately 1 hour.

4.5.2 Setting up the nozzle test rig

The nozzle test rig needs to be readied before each test.

1. Clamp the nozzle that is to be tested onto the end of the settling chamber. Ensure that the nozzle is firmly clamped. Take care not to disturb the visualisation apparatus during this procedure.
2. Check the pressure transducers, see that the transducer reading is 0 Pa gauge. The transducers need to be turned on at least 30 minutes prior to a series of tests to allow for them to reach thermal equilibrium.
3. Open the nozzle from the high pressure gas supply to the settling chamber

4.5.3 Setting up the visualisation system

The visualisation system needs to be tested and readied before each test sequence to ensure that the alignment of the mirrors or the knife-edges have not been disturbed.

1. Test the light source.
2. Check the mirror alignment by viewing the test section through the viewfinder of the camera. If there are reflections from the surface of the nozzle, then the light source is not aligned to be orthogonal to the nozzle and this needs to be fixed by adjusting the mirror angles.
3. Test the sensitivity of the system using a gas lighter. The lighter fluid should be visible in the test section through the camera view finder as it has a higher density than air.

4.5.4 Performing the test

The actions required to perform the test are as follows:

1. Turn the light in the room off so that the room is now dark.
2. Turn on the constant light source for the visualisation system
3. Open the valve supplying the high pressure gas to the settling chamber slowly. Allow the pressure in the settling chamber to reach the stagnation pressure required for the test.
4. Take an image of the experimental flow field using the shutter release mechanism for the camera.
5. Turn off the valve leading to the settling chamber, and record the test in the log book
6. Repeat the test at the same pressure.

4.5.5 Developing the photographic film

Precautions

- Ensure that the extraction fan is on to ensure sufficient ventilation.

- All equipment must be thoroughly rinsed in water before and after use to prevent mixing of chemicals.
- Care must be taken not to expose the film to light.

Development procedure

1. The relevant equipment was readied for use. This included a plastic spool onto which the film is wound on, a plastic shaft which is inserted into the aforementioned spool, a plastic canister into which the shaft and spool are inserted and a plastic lid which is screwed onto the canister.
2. Close the dark room door. Switch off the light and remove the film from the camera. Wind the film onto the plastic spool and follow step 1.
3. Switch on the light. Prepare the developer. The developer used was Rodinal®. The water temperature dictated the development times and these were read of a chart supplied by the manufacturer of the developer.
4. The quantity of water and developer were 500 ml and 20 ml respectively.
5. The development solution was poured into the plastic canister and agitated for one minute continuously and then for 10 seconds every minute thereafter.
6. Once the development time was reached, the mixture was poured out.
7. The canister was filled with water and agitated. This was repeated five times.
8. The fixer solution was prepared. The fixer used was Agfa®universal fixer. The quantity of water and fixer were 420 ml and 60 ml respectively.
9. The fixer fluid was poured into the canister and agitated for 15 seconds and 10 seconds thereafter for every minute for four minutes.
10. The fixer solution was poured out and step 7 was repeated.
11. The canister was opened and the film was removed from the spool. The film was then placed into a water bath and a wetting agent was added. The wetting agent used was Agepon®. The film was left for two minutes.
12. The film was then hanged in the dark room to dry. Alternatively, the film could be dried in the infrared oven.

5 Computational Approach

5.1 Pre-processing

The pre-processing tasks such as CAD model development, and the generation of the mesh was performed with the Gambit©.

The computational domain for the axisymmetric jet can be seen in 5.1(a). Due to the symmetric nature of the jet flow field, the 2-D model incorporates an axis boundary condition to reduce the computational size of the domain. The domain extends 10 nozzle diameters downstream of the nozzle exit, and 5 nozzle diameters along the radial axis. The initial mesh that is used to develop the flow field for each simulation consists of 1275 cells having a length of 1 mm along each edge of the cell. Mesh adaptation based on the density gradients in the solution was employed to improve the solution grid.

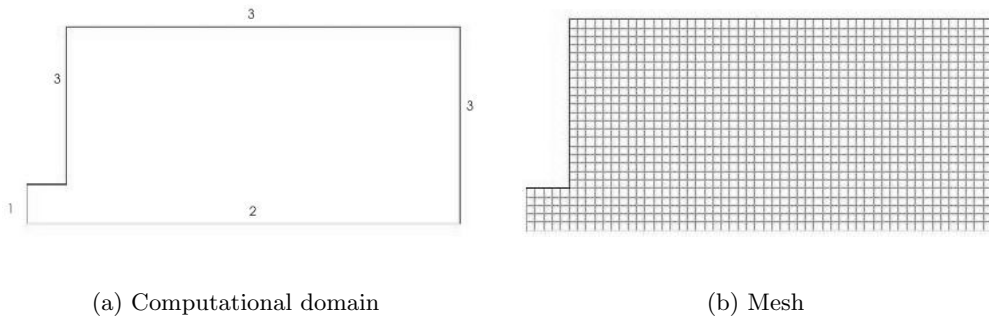


Figure 5.1: Computational region and mesh used for the simulation of the axisymmetric free jet

5.2 Mesh adaptation

The computational grid displayed in 5.1(b) is utilised to develop the initial solution for the free jet case. The adaptation of the mesh based on the gradients of density is utilised to improve the mesh so as to resolve the flow features of interest, such as the jet boundary, shock waves and expansion waves within the jet flow field. An example of the adapted mesh for the cases for the axisymmetric jet studies are shown in 5.2. The mesh adaptation was performed utilising the Fluent software, with the following parameters:

Table 5.1: Mesh adaptation parameters utilised for the modelling of the axisymmetric jet

Grid Adaptation Options	Refine/Coarsen
Adaptation Method	Gradient
Gradients of	Density
Normalization	Scale
Thresholds	Refine = 0.3 Coarsen = 0.7
Maximum level of refinement	4
Maximum number of cells	125 000

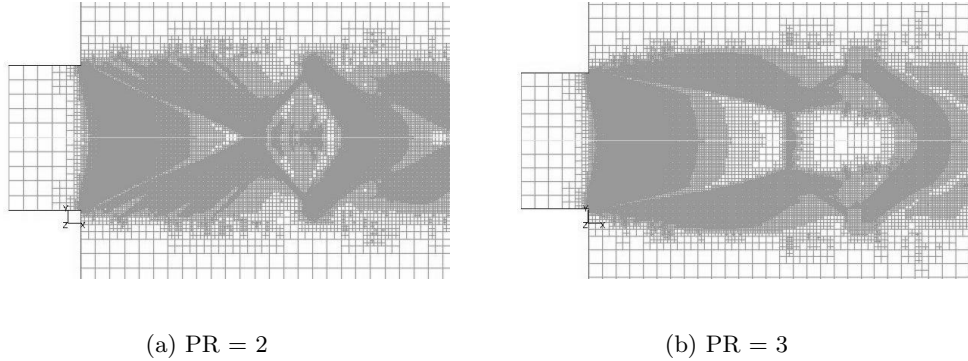


Figure 5.2: Adapted mesh for the simulation of the axisymmetric free jet

5.3 Solution methodology

The solution of the free jet cases is performed using the Computational Fluid Dynamics (CFD) solver Fluent 6.1.22©.

The solution methodology that has been pursued for the analysis for the jet flow fields

has been such that the solution for the case of a perfectly expanded sonic free jet is developed and is used as the initial conditions for the solution of increasing pressure ratio and Mach number cases. The coupled solver with a 2nd order discretisation scheme was utilised to develop the solution.

5.3.1 Boundary conditions

The boundary conditions that have been used in the development of the axisymmetric computational model is outlined in 5.1.

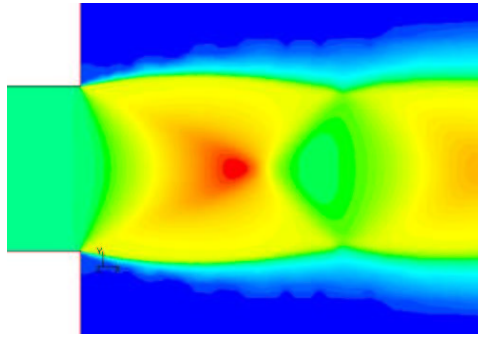
Table 5.2: Boundary conditions on the computational model for the axisymmetric jet

Boundary condition	Fundamental properties for PR = 2	Fundamental properties for PR = 3
Pressure inlet	$P_{Total} = 383604$ Pa $P_{Static} = 102325$ Pa $T_{Static} = 300$ K	$P_{Total} = 575405$ Pa $P_{Static} = 101325$ Pa $T_{Static} = 300$
Axis	-	-
Pressure outlet	$P_{exit} = 101325$ Pa	$P_{exit} = 101325$ Pa
Inviscid Wall	Wall shear stress = 0	Wall shear stress = 0

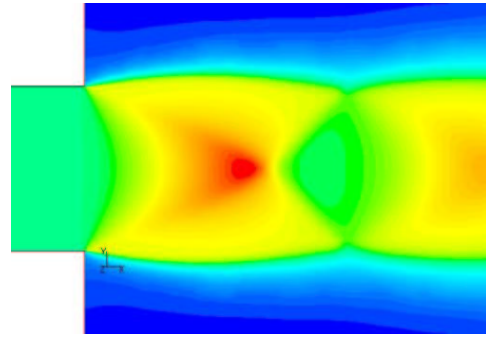
5.4 Turbulence modeling

The choice of a turbulence model was made by simulating an axisymmetric free jet operating at pressure ratios of 2 and 3. The resulting flow field that was obtained experimentally can be seen in Figures 5.3(d) and 5.4(d). The schlieren image of the jet operating at a pressure ratio of 2, shows the formation of a small but clearly defined Mach disk within the jet flow field (see Figure 5.3(d)). The shear layers emanating from the triple point of the Mach disk within the jet is more clearly visible than the Mach disk itself. Increasing the operating pressure results in the formation of a significantly stronger Mach disk (in comparison with the jet at the lower pressure ratio) within the jet flow field, with a strong barrel shock system originating at the nozzle exit. The experimental images form the basis for the comparison with the simulations to determine the most appropriate turbulence model that can be used to model the flow field.

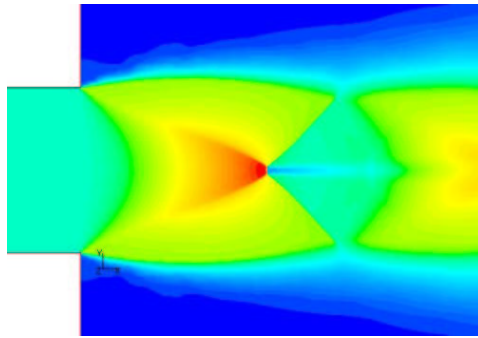
The choice of the turbulence model was based on its ability to accurately predict



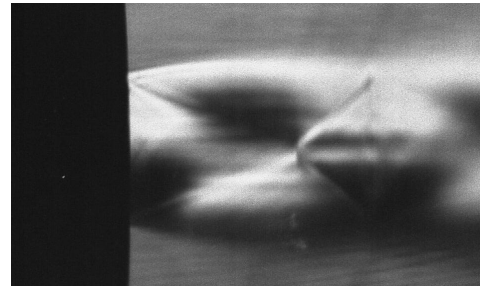
(a) $K-\epsilon$



(b) $K-\omega$

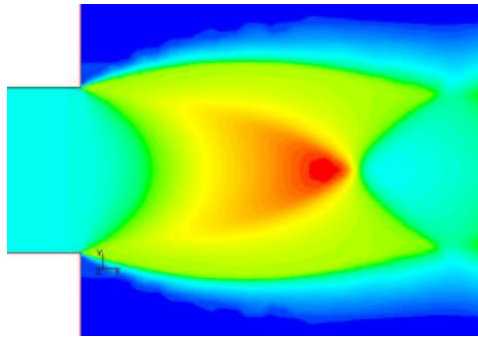


(c) Spallart - Allmaras

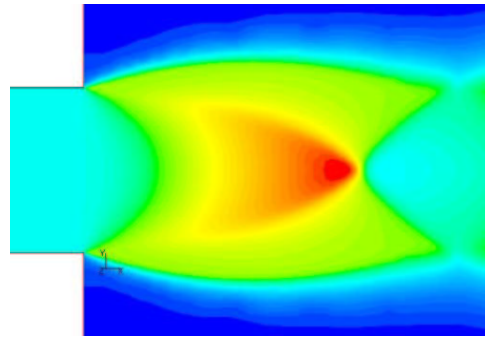


(d) Experimental Schlieren image

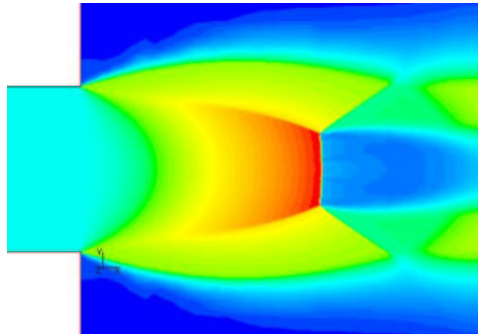
Figure 5.3: Comparison of computed and experimentally observed flow fields from axisymmetric sonic jets with $PR = 2$



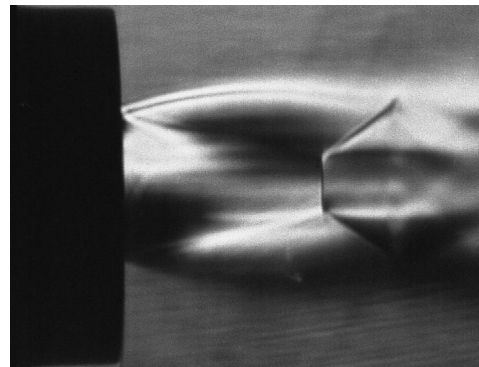
(a) K- ϵ



(b) K- ω

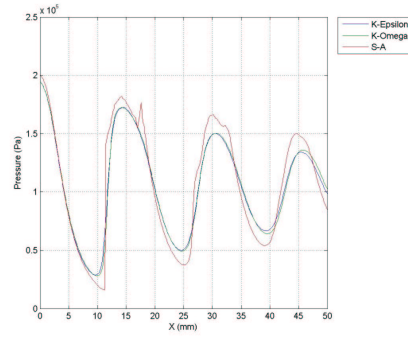


(c) Spallart - Allmaras

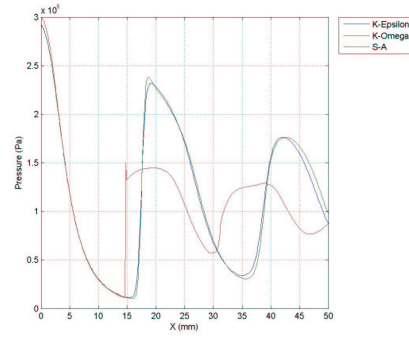


(d) Experimental Schlieren image

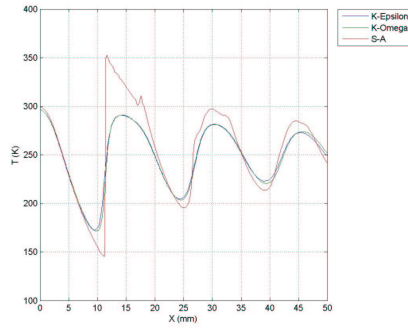
Figure 5.4: Comparison of computed and experimentally observed flow fields from axisymmetric sonic jets with $PR = 3$



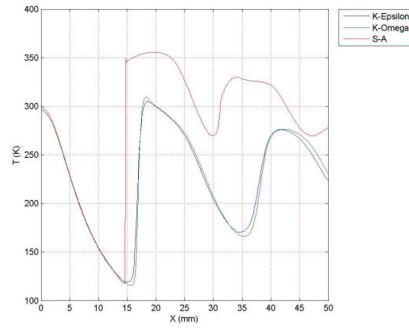
(a) Pressure: $M = 1$, $PR = 2$



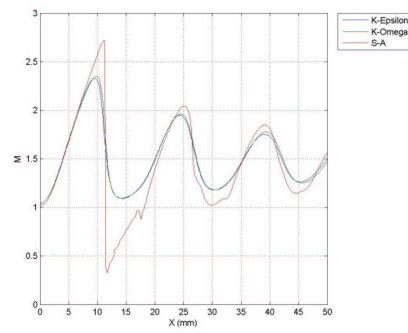
(b) Pressure: $M = 1$, $PR = 3$



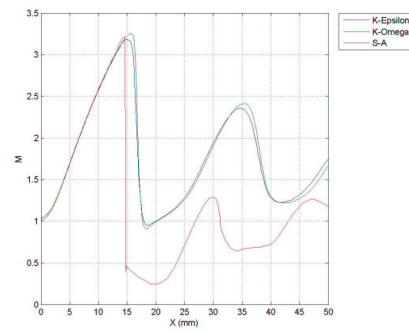
(c) Temperature: $M = 1$, $PR = 2$



(d) Temperature: $M = 1$, $PR = 3$



(e) Mach number: $M = 1$, $PR = 2$



(f) Mach number: $M = 1$, $PR = 3$

Figure 5.5: Pressure, Temperature and Mach number variation along the axis of an axisymmetric jet computed with different turbulence models

the flow field, and at the same time being computationally viable for its use. Three standard turbulence models that are fairly well understood and established in literature were evaluated for their appropriateness to model the jet flow fields. These are the K- ϵ , the K- ω and the Spallart - Allmaras turbulence models. The modeling approach that was followed to determine the appropriate turbulence model was identical to that which has been described in earlier sections.

The comparison of the flow fields that are obtained through the use of the three turbulence models are shown in Figures 5.3(a) - 5.3(c) and Figures 5.4(a) - 5.4(c). The three turbulence models predict the shape of the viscous dominated jet boundary very well, with all three turbulence models predicting almost an identical shape to it. The shock wave and expansion system within the jet flow field are predominantly inviscid interactions with the viscous effects not being the dominant feature. The K- ϵ and the K- ω turbulence models tend to distort the shock system within the jet and the experimental jet flow field is not well predicted by it. The Spallart - Allmaras model resolves the shock system very well in both cases of the experimental jet that are presented in Figures 5.3, and 5.4.

The variation of pressure, temperature and Mach number along the axis of the jet utilising the three turbulence models is shown in Figure 5.5. The presence of the Mach reflection within the jet is only reflected in the simulations using the Spallart - Allmaras turbulence model. The sudden change in the temperature, pressure and Mach number is consistent with a Mach reflection in the flow field. The isentropic expansion of the gas within the jet flow field is well modeled by the K- ϵ and the K- ω turbulence models. The variation of the pressure, temperature and Mach number within the jet until the reflection of the compression waves is fairly similar for all three turbulence models under both pressure ratios that have been evaluated (see Figure 5.5). The inability of the K- ϵ and the K- ω models to adequately resolve the shock system within the jet is evident from the incorrect pressure and temperature rise that is predicted due to the reflection of the shock waves. The Mach reflection within the jet results in subsonic flow behind it, which is only reflected in the flow field that is simulated using the Spallart - Allmaras turbulence model (see Figure 5.5(e) and 5.5(f)). The Spallart - Allmaras turbulence model is hence used for all the simulations that are presented in this thesis.

6 Axisymmetric Jets

6.1 Axisymmetric jet

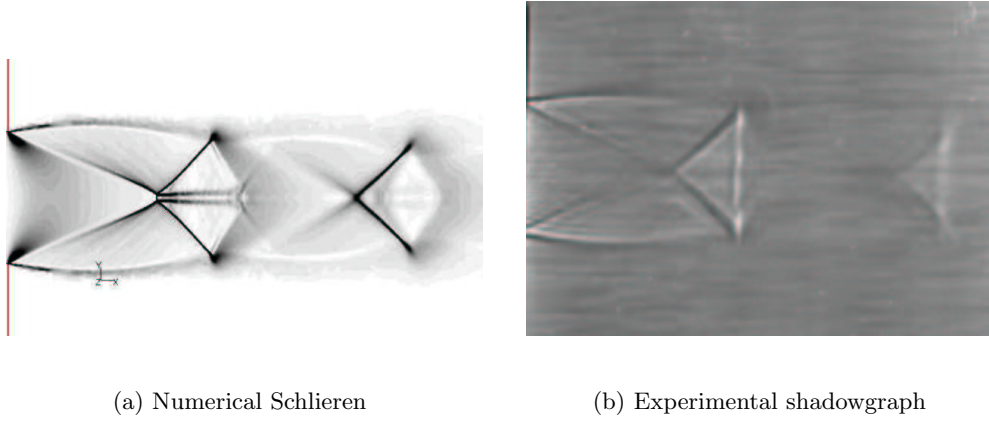


Figure 6.1: Axisymmetric nozzle, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

6.1.1 Description of the flow field

The flow field from an axisymmetric nozzle is fairly well understood and has been described in a number of references in the literature and in a previous section in this thesis (Chapter 3).

In an underexpanded jet, the gas issuing from the nozzle exit is at a higher pressure than the ambient surroundings, hence it expands as it leaves the nozzle. This phenomenon can be seen in both cases of the experimental images shown in figures 6.1(b) and 6.2(b). The extent of the spreading of the jet boundary is a function of the pressure of the jet at the nozzle exit. The larger the pressure of the jet at the nozzle exit, the larger the jet expansion into the ambient surroundings once it leaves the nozzle. The jet boundary is a shear layer and hence there is no pressure difference across it. The expansion fans that originate at the nozzle exit, reflect from

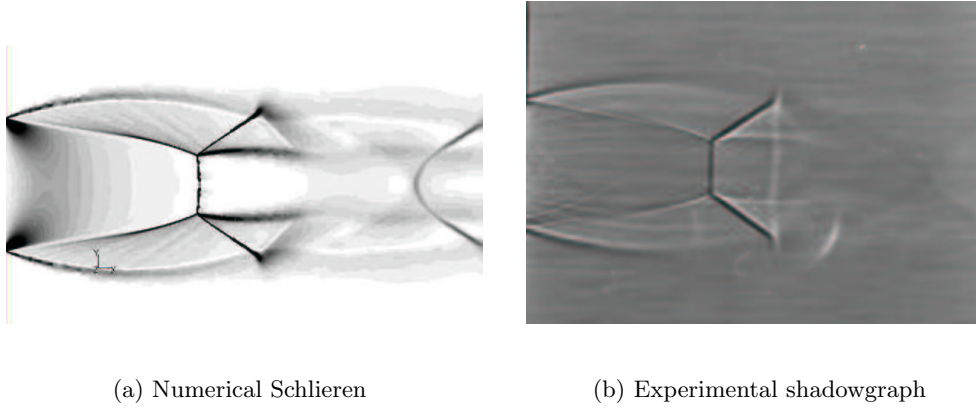


Figure 6.2: Axisymmetric nozzle, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

the jet boundary as compression waves. The compression waves coalesce to form a strong shock front that is seen to originate very close to the nozzle exit. The shock front extends downstream of the nozzle exit in both experimental cases that have been investigated, and the reflection of the shock front is irregular in nature. The jet flow field in a sonic underexpanded jet can be considered to be similar to that of an internal conical flow, and in [45] it has been proved that a regular reflection is not possible for internally converging conical flows.

Hence even though the Mach reflection is not very clear in the experimental image seen in figure 6.1(b) for the case of $PR = 2$, the Mach reflection is confirmed in the numerical simulation schlieren image (figure 6.1(a)). A closer examination of the experimental image (figure 6.3) a darkening of the image can be seen to propagate downstream from the reflection point. This is indicative of the shear layers that are forming in a tubular structure and propagating from the circular triple line¹. The vertical clear 'line' that seems to be present in the centre of the image is the image of the reflected shock wave interacting with the jet boundary. The jet is not entirely steady due to small perturbations and the inherent turbulence from the upstream flow causing a slight movement in the shock system. This is the reason why the shock system in the $PR=2$ case (figure 6.3) is not as clear and defined as in the case of the $PR = 3$ case (figure 6.4). These perturbations are also revealed as striations on the images (figures 6.3 and 6.4).

When the pressure ratio is increased such as in the case of the jet flow field shown in figure 6.2, the jet expansion into the ambient surroundings is greater as can be seen in figure 6.2(b). The flow features in the jet flow field are similar to that in the case of the lower pressure ratio jet. The increased pressure ratio results in a

¹The triple line is the locus of triple points that makes the edge of the Mach disk

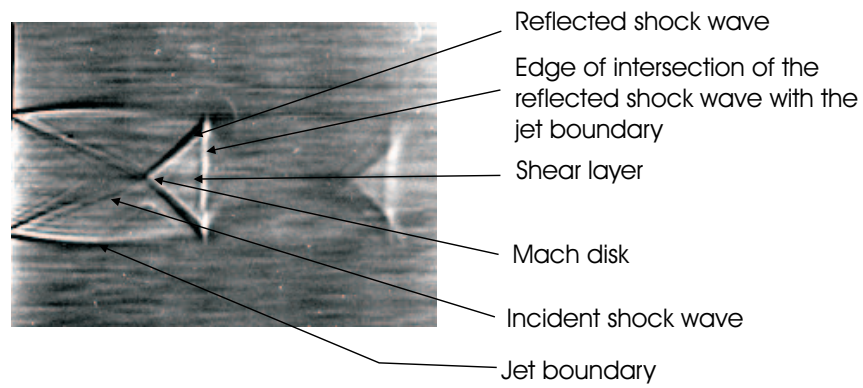


Figure 6.3: Experimental image of $PR = 2$, Axisymmetric free jet

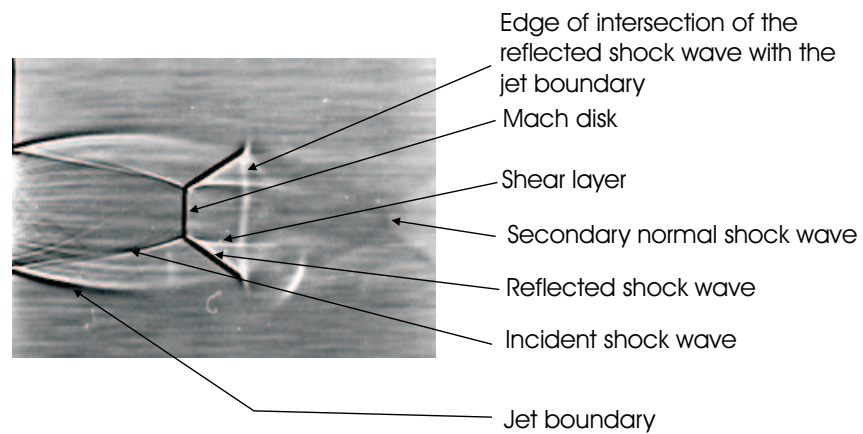
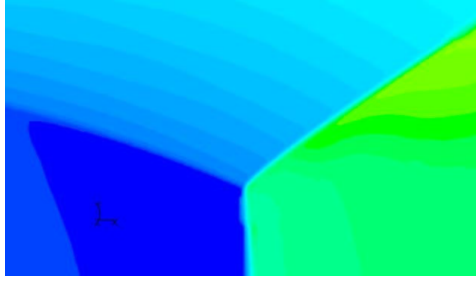


Figure 6.4: Experimental image of $PR = 3$, Axisymmetric free jet

significantly stronger incident shock wave system as can be seen in figure 6.4. The incident shock system reflects in an irregular manner and the resulting Mach disk is a lot larger than in the case of the lower pressure ratio jet. The resulting Mach disk is circular in nature, and the shear layers form a tubular structure as they propagate downstream of the triple line. The shear layers initially tend to diverge and then converge and then diverge again forming a converging - diverging nozzle like flow condition downstream of the Mach disk. This phenomenon is clearly visible in the numerical schlieren image in figure 6.2(a) The gas passing through the Mach disk is at subsonic conditions and is accelerated through the converging - diverging tube formed by shear layer to sonic conditions at which point the flow is arrested in its expansion through a second normal shock wave. This normal shock can be seen clearly in the numerical schlieren image in figure 6.2(a) as a curved compression wave in the centre of the jet flow field at the right of the image. This same pattern is visible in the experimental image figure 6.2(b), albeit significantly fainter than in the numerical image (see figure 6.4).

The shear layers that originate from the triple line, are seen to initially diverge from the triple point (see figure 6.5(d). Considering the images in figure 6.5, from the pressure contours in figure 6.5(a) it is clear that there is no pressure difference across the shear layers as would be expected. The vector plot in figure 6.5(c) shows that the flow tends to diverge as it passes through the Mach disk close to the triple point. The reason for the diverging shear layers can be inferred from the shape of the Mach disk. The Mach disk is orthogonal to the flow at the centre line of the jet, but as it approaches the triple point the Mach disk develops a curvature bulging toward the direction of the flow of the jet. This can be seen clearly in the pressure contours in figure 6.5(a) more so than in the experimental shadowgraph. The shadowgraph is an image integrated through the thickness of the flow field. The Mach disk has a shape of a concave lens. The passage of the light through the Mach disk makes it appear thicker than it actually is. In figure 6.5(d), The triple line seen on the centre of the image extending to the bottom of the image is dark on the left and light in colour on the right. The dark section in the Mach disk is the triple line, and the lighter portion of triple line is the leading edge of the Mach disk as it bulges outwards. The vector plot of the velocities of the flow in the vicinity of the Mach disk (figure 6.5(c)) shows that the angle of the flow upstream of the Mach disk is not parallel to the jet axis. The flow upstream of the Mach disk in the vicinity of the triple 'point' is diverging. The vector plot also reveals that the flow downstream of the Mach disk (in the vicinity of the triple point) also diverges. The shear layer divides the region of flow downstream of the Mach disk and the flow downstream of the reflected shock wave, and from the direction of the velocities in these two regions

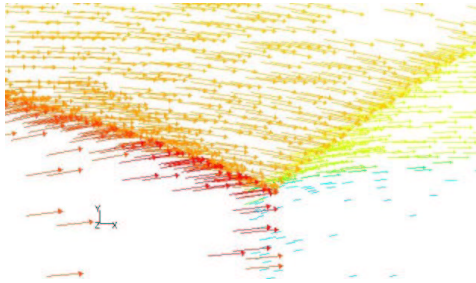
we can see that the shear layer would diverge from the triple point.



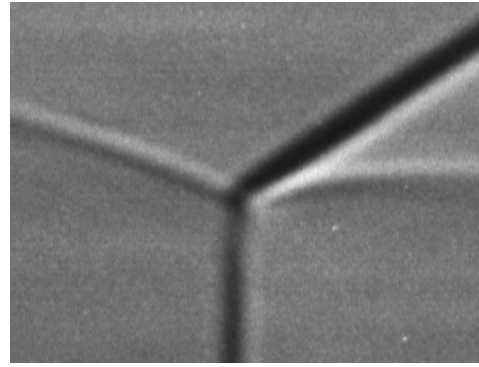
(a) Pressure contours



(b) Numerical schlieren



(c) Vector plot



(d) Experimental shadowgraph

Figure 6.5: Close up of the triple 'point', $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

6.2 Effect of the nozzle inlet angle

The following nozzle inlet geometries were analyzed to determine the effect of the inlet geometry on the underexpanded jet: contoured inlet giving axial efflux, 45° inlet and an orifice inlet (figures 6.6, 6.7 and 6.8). The nozzles were analyzed to have sonic exits, with the ratio between the exit plane and the ambient atmosphere being varied between 2 and 10 (the ratio is henceforth referred to as PR). The nozzle inlet angles were varied to asses the effect on the jet flow field as the slot and the elliptic nozzles tested (results shown later in the thesis) had a 45° beveled edge leading to the nozzle exit, such that they were sharp edged orifice nozzles.

6.2.1 General description of the flow field

For small underexpansion ratios of the jet ($PR = 2$), the contoured nozzle results in a well defined barrel shock structure appearing in the flow field as can be seen in figure 6.6(A). The barrel shock reflects along the jet centerline as a Mach reflection. For the case of the non-contoured inlets (figure 6.9 (B) and (C)), the flow field is markedly different. The expansion waves forming at the nozzle exit reflect at the sonic line in the jet boundary as compression waves. These compression waves do not coalesce to form a shock wave before the reflection point as can be seen in figure 6.6(B) and (C). As the pressure ratio is increased the strength of the barrel shock increases as can be seen in figure 6.7. The formation of a Mach disk occurs in all of the examined nozzle inlet geometries. The strength of the barrel shock is at its strongest for the case of the contoured nozzle. The formation of the barrel shock in the jet flow stream is nearest to the nozzle exit in the case of the contoured nozzle (figure 6.7 (A)). The height of the Mach disk in the nozzle flow field is lower in the case of the 45° inlet and the orifice nozzle cases (figure 6.7 (B) and (C)). Upon increasing the pressure ratio to $PR = 10$, the shape of the jet flow field remains essentially similar for the cases of the contoured nozzle and the 45° inlet nozzle (figure 6.8 (A) and (B)). In the case of the nozzle with the orifice inlet however, the expansion of the jet flow as it exits the nozzle is such that the jet boundary touches the nozzle exit face. This causes the jet to expand along the nozzle exit face and not along the jet boundary. The jet boundary in this case originates from the edge of the orifice exit nozzle as can be seen in 6.8 (C).

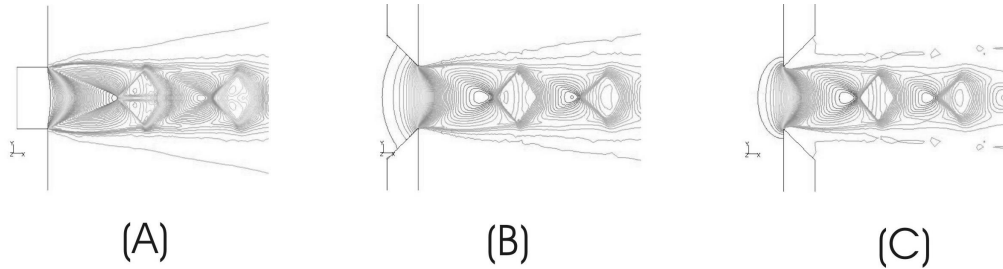


Figure 6.6: Density contours of the flow field, $M = 1$, $PR = 2$ (A) Contoured inlet (B) 45° inlet (C) Orifice inlet

6.2.2 Mach disk heights

A comparison of Mach disk heights non-dimensionalized by the nozzle exit height is shown in figure 6.9. The Mach disk height is consistently the greatest for the case of the contoured nozzle. The Mach disk height of the 45° nozzle inlet is the smallest

across the range of pressure ratios examined. The orifice inlet Mach disk height is comparable to the 45° inlet case until a pressure ratio of 5, after which the Mach disk height for the orifice inlet is greater for pressure ratios of 7 and 10 in comparison to the that of the 45° inlet case. This is as a result of the jet boundary merging with the nozzle exit at higher pressure ratios for the orifice inlet as shown in figure 6.7 (C). The analytical expressions for the orifice inlet and the contoured nozzle as developed by [53] have also been presented in figure 6.9 and it is important to note that the analytical expressions under predict the Mach disk height as compared to the simulation results. This is due to the fact that the boundary layer growth in the nozzle geometry has not been accounted for in the case of the contoured nozzle simulation resulting in a flow field that is closer to an inviscid case. The effect of viscosity is evident in the prediction with the analytical expressions as they were developed from experimental results. The effect of inlet turbulence in the development of the jet flow field needs further investigation to ascertain its exact influence.

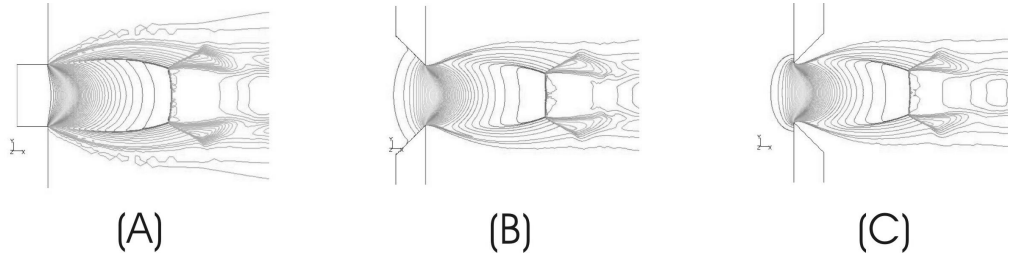


Figure 6.7: Density contours of the flow field, $M = 1$, $PR = 5$ (A) Contoured inlet (B) 45° inlet (C) Orifice inlet

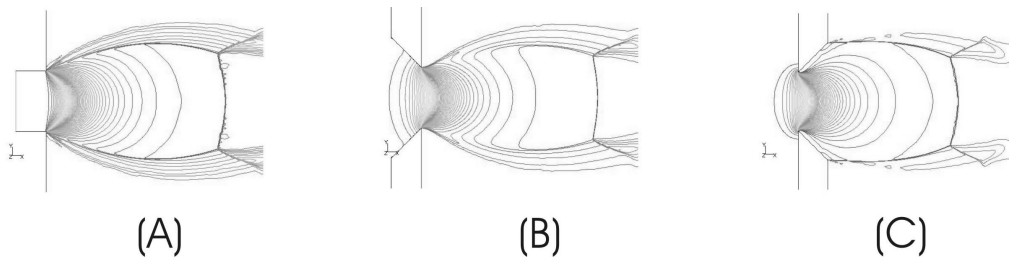


Figure 6.8: Density contours of the flow field, $M = 1$, $PR = 10$ (A) Contoured inlet (B) 45° inlet (C) Orifice inlet

6.2.3 Effect on jet boundary

The contoured nozzle results in a jet boundary that leaves the nozzle at an angle orthogonal to the nozzle exit face. In the case of the sharp edged nozzle, there is a

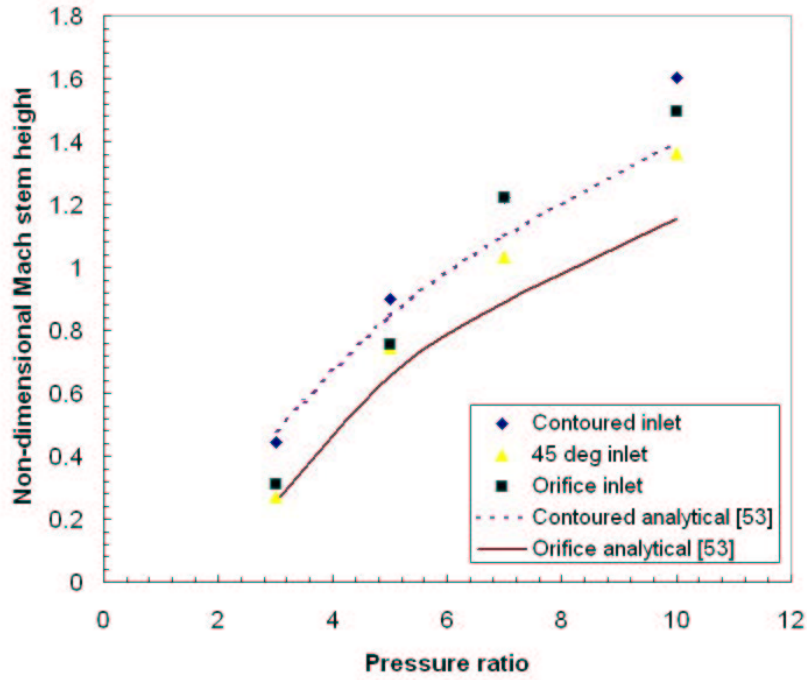


Figure 6.9: Non-dimensional Mach disk heights as a function of pressure ratio for various inlet angles

defined vena-contracta effect that can be seen at the nozzle exit. The vena-contracta effect is most pronounced for the orifice nozzle exit (figure 6.10 and 6.11). There is no discernible change in the shock structure between the jets issuing from the orifice inlet and the 45° inlet. The vena-contracta in the jet flows results in a narrowing of the jet boundary as it leaves the nozzle exit. This results in a sonic line that is not orthogonal to the nozzle exit plane, but curved in space downstream of the nozzle exit. Hence for the case of the 45° inlet nozzle and the orifice inlet nozzle the flow reaches sonic conditions a small distance downstream of the nozzle exit and not at the nozzle exit as in the case of the contoured nozzle.

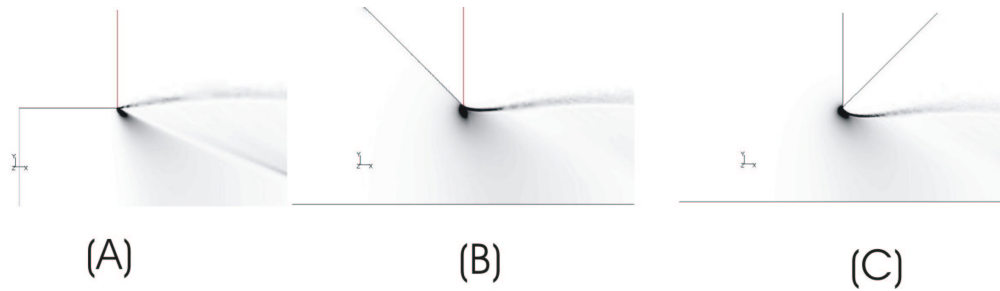


Figure 6.10: Pseudo-schlieren images of the flow field in the vicinity of the nozzle exit, $M = 1$, $PR = 2$ (A) Contoured inlet (B) 45° inlet (C) Orifice inlet

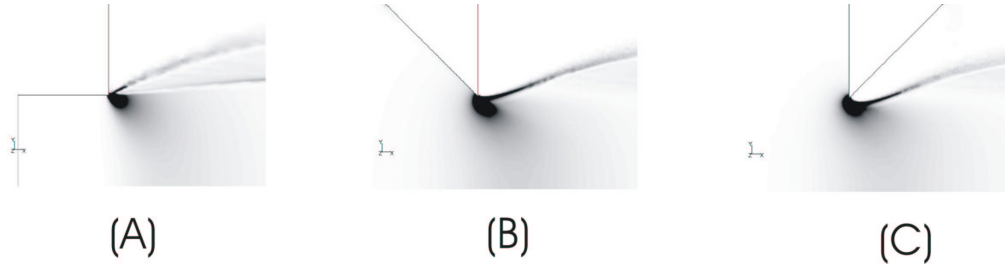


Figure 6.11: Pseudo-schlieren images of the flow field in the vicinity of the nozzle exit, $M = 1$, $PR = 5$ (A) Contoured inlet (B) 45° inlet (C) Orifice inlet

6.2.4 Conclusions

The computational study on underexpanded jet flows issuing from axisymmetric nozzles having varied inlet geometries provides the following conclusions:

- The formation of the barrel shock in the jet flow field occurs at the lower pressure ratio in the case of the contoured nozzle in comparison to the other nozzle geometries that have been investigated.
- The Mach disk height is greatest for the case of the contoured nozzle in comparison to the other nozzle geometries that have been investigated over the pressure ratio range between 2 and 10. There is a curvature of the Mach disk in the direction of the flow and the curvature is more pronounced as the pressure ratio is increased.
- The formation of a vena-contracta resulting in a narrowing in the jet boundary is observed in the case of the 45° and the orifice inlet nozzles.
- The effect of nozzle inlet turbulence needs to be investigated to account for the differences between the analytical prediction of Mach disk height in comparison to the Mach disk height predicted by simulation.

7 Characteristics of non-axisymmetric underexpanded sonic jets

7.1 Shock cell lengths

The shock cell length in non-axisymmetric jets is defined in this study to be the distance from the nozzle exit to the point where the reflected shock wave intersects the jet boundary. This length is non-dimensionalised by the nozzle minor axis length to develop a non-dimensional parameter to compare the various nozzles. The length of the shock cell is hence linked to the length scale formed by the minor axis, and for the case of nozzles with aspect ratios > 1 the non-dimensional shock cell lengths will be different along the major and the minor axes¹. The complete set of graphs describing the characteristics of the jet flows have been appended to the thesis.

7.1.1 Rectangular jets

The numerically predicted shock cell lengths are consistently larger than that observed experimentally. This is true for the shock cell lengths along the major and the minor axes and for both pressure ratios that have been investigated for all nozzles. The numerical simulation is based on the Spallart-Almaras turbulence model, and the prediction of the shock cell lengths being longer than the experimental observation is as a result of the lower turbulence in the numerically predicted flow field than that in the experimental flow field.

The variation of the shock cell lengths along the major and minor axis planes of the nozzles for increasing pressure ratios can be seen in figures 7.1 and 7.2. It can be seen from these images that while the numerical prediction of the shock cell length

¹In all the graphs that are shown in this chapter, the legend is explained as follows: Sim - Major is simulation result on the major axis, Exp - Major is the experimental result on the major axis, and so on.

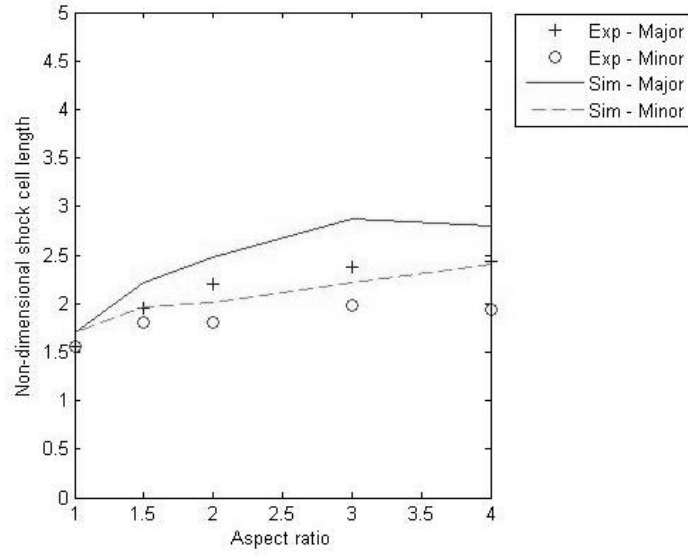


Figure 7.1: Variation of the shock cell lengths along the major and minor axis plane of the nozzle in rectangular jets when $\frac{P_{exit}}{P_{ambient}} = 2$

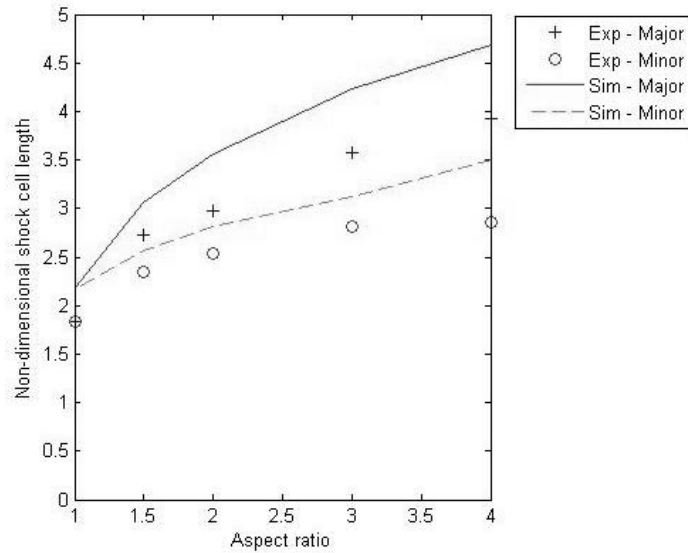


Figure 7.2: Variation of the shock cell lengths along the major and minor axis plane of the nozzle in rectangular jets when $\frac{P_{exit}}{P_{ambient}} = 3$

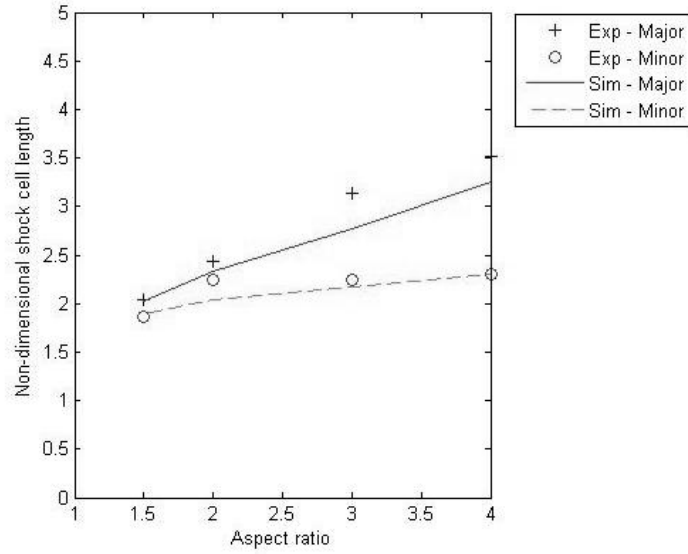


Figure 7.3: Variation of the shock cell lengths along the major and minor axis plane of the nozzle in elliptical jets when $\frac{P_{exit}}{P_{ambient}} = 2$

is greater than the experimentally observed shock cell length, there is reasonable agreement between the numerical and experimental flow field. The non-dimensional shock cell length increase along the major and the minor axis planes of the nozzle on increasing aspect ratio. The increase in the shock cell length between aspect ratios of 1.5 and 4 appears to be linear as can be seen from figures 7.1 and 7.2.

7.1.2 Elliptical jets

The variation of the non-dimensionalised shock cell lengths along the major and minor axis planes of the elliptical nozzle with a 45° inlet angle is shown in figures 7.3 and 7.4.

The experimental non-dimensional shock lengths are greater than the numerical predictions for all cases presented. This shows that the turbulence in the flow field is lesser in the experimental flow field for the elliptical nozzle in comparison to that induced by the Spallart-Allmaras turbulence model in the numerical simulation. The shock cell length increases as the aspect ratio increases when considering the major axis planes of the nozzle. There is good qualitative agreement between the experimental observation and the numerical prediction as the variation of the shock cell length with the aspect ratio of the nozzle exit in both cases is similar in nature. The discrepancies between the experimental observation and the numerical prediction is greater in the case of the shock cell length along the major axis plane of the nozzle.

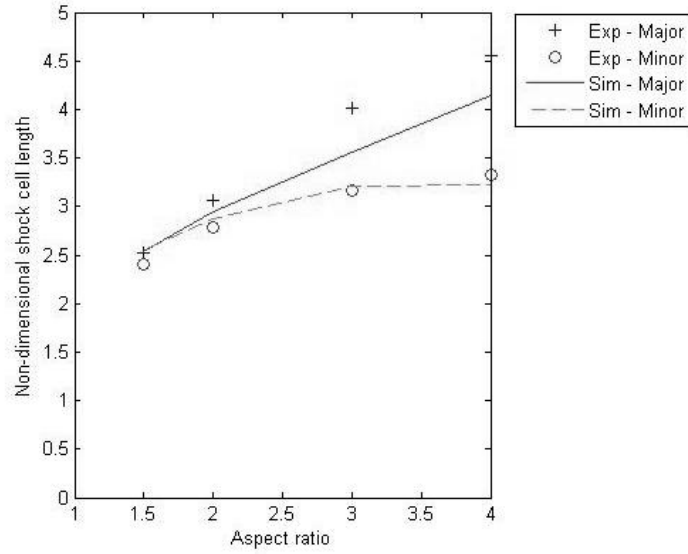


Figure 7.4: Variation of the shock cell lengths along the major and minor axis plane of the nozzle in elliptical jets when $\frac{P_{exit}}{P_{ambient}} = 3$

The shock cell length along the minor axis plane appears to be reaching a maximum length in both cases of pressure ratios that have been examined as can be seen in figures 7.3 and 7.4. The variation of the shock cell length along the major axis appears to be linear in nature.

7.1.3 Slot jets

The variation of the non-dimensional shock cell length along the major and the minor axis planes of slot nozzles with a 45° inlet angle is shown in figures 7.5 and 7.6. There is excellent agreement between the experimentally measured lengths and the numerically predicted lengths along the major axis plane of the nozzle at pressure ratios of 2 and 3. The shock cell length along the major and the minor axis increases linearly on increasing aspect ratio, and like in the case of the elliptical nozzle presented earlier, the minor axis shock cell length appears to be reaching a maximum at an aspect ratio of 3.

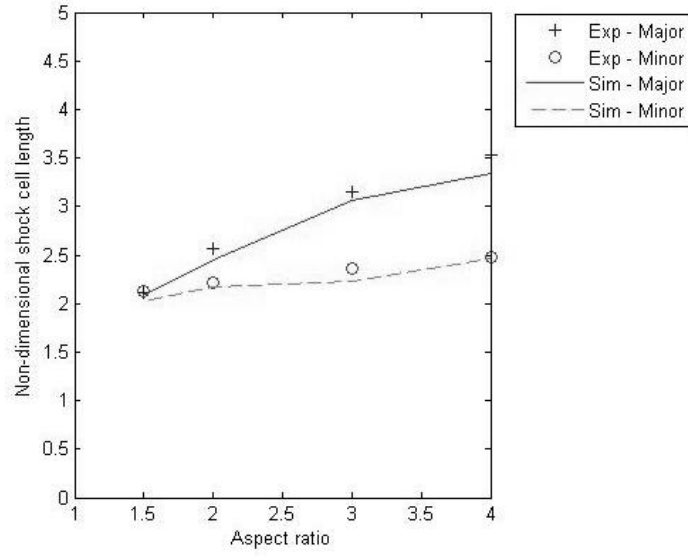


Figure 7.5: Variation of the shock cell lengths along the major and minor axis plane of the nozzle in slot jets when $\frac{P_{exit}}{P_{ambient}} = 2$

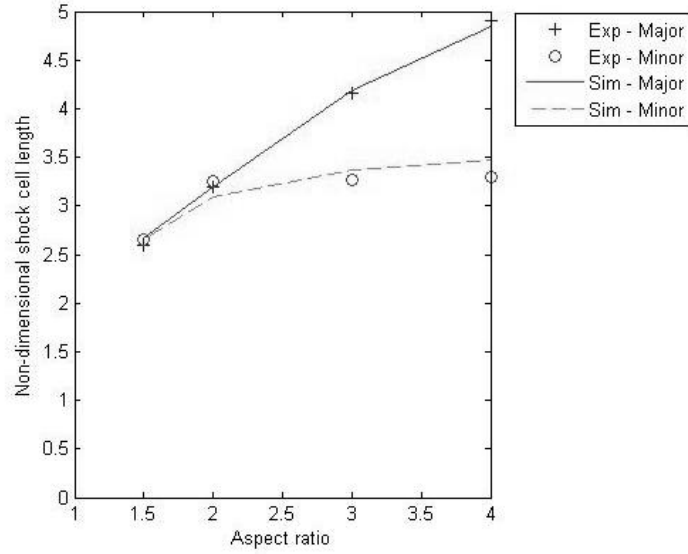


Figure 7.6: Variation of the shock cell lengths along the major and minor axis plane of the nozzle in slot jets when $\frac{P_{exit}}{P_{ambient}} = 3$

7.2 Mach reflection characteristics along the axis of the jet flow

The Mach disk heights that are presented in this chapter are measured from the experimental shadowgraph images and the numerical predictions and are non-dimensionalised

by the minor axis length of the nozzle exit.

7.2.1 Rectangular jets

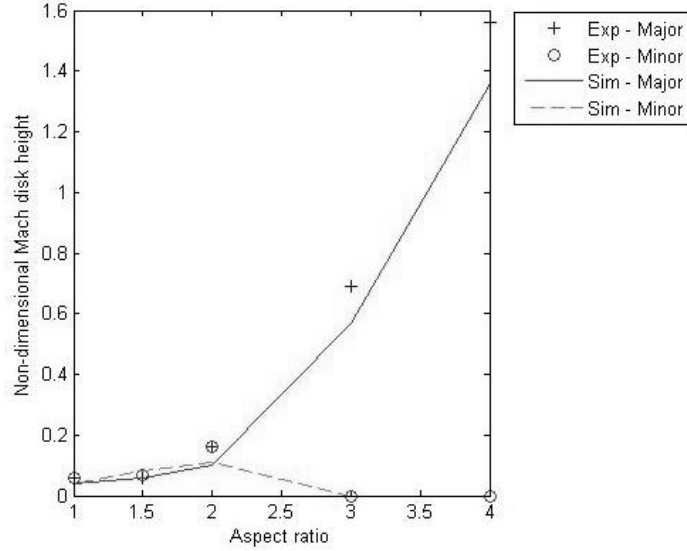


Figure 7.7: Variation of the Mach disk heights along the major and minor axis plane of the nozzle in rectangular jets when $\frac{P_{exit}}{P_{ambient}} = 2$

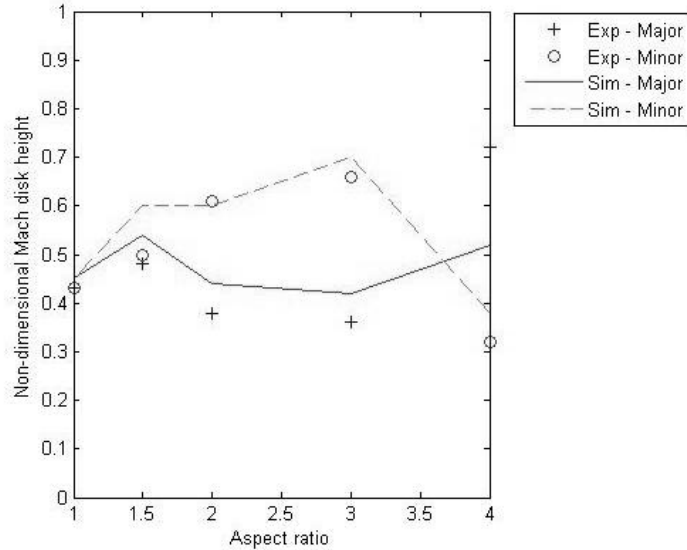


Figure 7.8: Variation of the Mach disk heights along the major and minor axis plane of the nozzle in rectangular jets when $\frac{P_{exit}}{P_{ambient}} = 3$

The variation of the non-dimensional Mach disk height in rectangular jets along the major and the minor axis for the pressure ratios that have been analysed are

presented in figures 7.7 and 7.8. Along the major axis plane at a pressure ratio of 2, the Mach disk height increases as the aspect ratio of the nozzle increases and the correlation between the experimental and simulation result is good. Along the minor axis plane of the nozzle, at the lower pressure ratio of 2, there is no Mach disk that is observed at aspect ratios of 3 and 4. At higher pressure ratios of 3 however the non-dimensional Mach disk height reduces along the major axis plane on increasing the aspect ratio as seen in figure 7.8 and appears to reach a minimum height at an aspect ratio of 3, whereas along the minor axis the non-dimensional Mach disk height appears to reach its maximum length at the same aspect ratio. This is an interesting phenomenon that is observed both numerically and experimentally. The correlation between the experimental result and the numerical prediction is not as close as that observed for the lower pressure ratio.

7.2.2 Elliptical jets

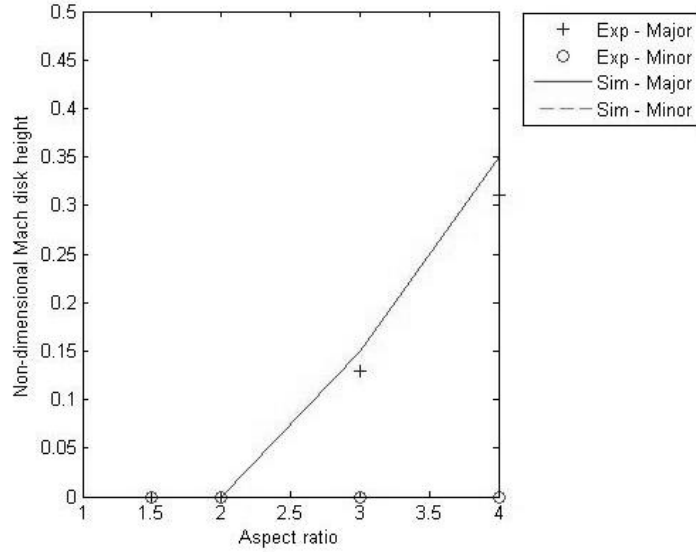


Figure 7.9: Variation of the Mach disk heights along the major and minor axis plane of the nozzle in elliptical jets when $\frac{P_{exit}}{P_{ambient}} = 2$

The variation in the non-dimensional Mach disk heights along the major and minor axes of elliptical nozzles having a 45° inlet angle is shown in figures 7.9 and 7.10. No Mach disk has been observed on the minor axis plane of the nozzle at the pressure ratio of 2 for any of the aspect ratios that have been investigated. At a pressure ratio of 2, there is no visible Mach disk along the major axis planes of the nozzle for aspect ratios of 1.5 and 2. The onset of Mach reflection in the case of the angled inlet occurs at an aspect ratio between 2 and 3 along the major axis plane of the

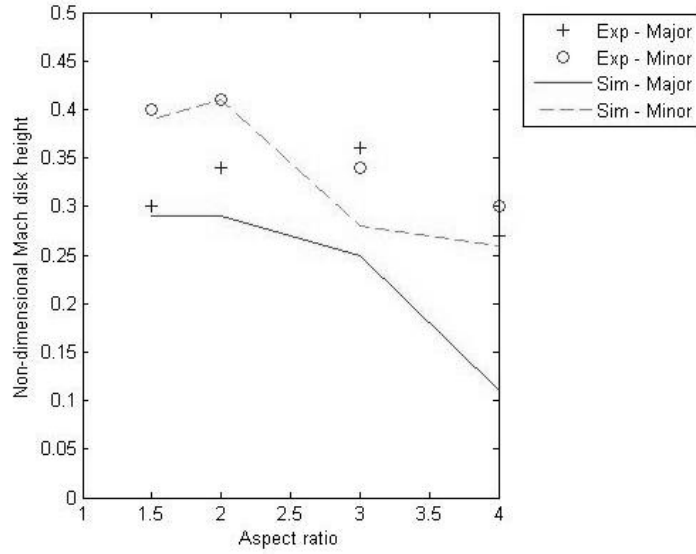


Figure 7.10: Variation of the Mach disk heights along the major and minor axis plane of the nozzle in elliptical jets when $\frac{P_{exit}}{P_{ambient}} = 3$

nozzle at the pressure ratio of 2. The Mach disk height increases on increasing the aspect ratio of the nozzle. At higher pressure ratios the Mach reflection is observed along the major and the minor axis planes of the nozzle for all the aspect ratios that have been tested in this study (see figure 7.10). There is good agreement between the numerical prediction and the experimental observation for the case of the minor axis plane, but this cannot be said for the case of the major axis. The experimental observation of the Mach disk height along the major axis plane shows an increase in the Mach disk height until a maximum is reached at an aspect ratio of 3. In the case of the minor axis the Mach disk height appears to be decreasing on increasing the aspect ratio of the nozzle.

7.2.3 Slot jets

The variation of the non-dimensionalised Mach disk height along the major and the minor axis planes of the slot nozzle with a 45° angled inlet is shown in figures 7.11 and 7.12. The formation of a Mach disk is not observed for the lower pressure ratio of 2 along the major and the minor axis planes until an aspect ratio of 2. And interestingly the formation of a Mach disk along the minor axis plane of the nozzle is not observed at aspect ratios of 3 or 4 at the pressure ratio of two as can be observed in figure 7.11. This phenomenon is observed both numerically and experimentally. Along the major axis plane of the nozzle, the Mach disk height increases significantly

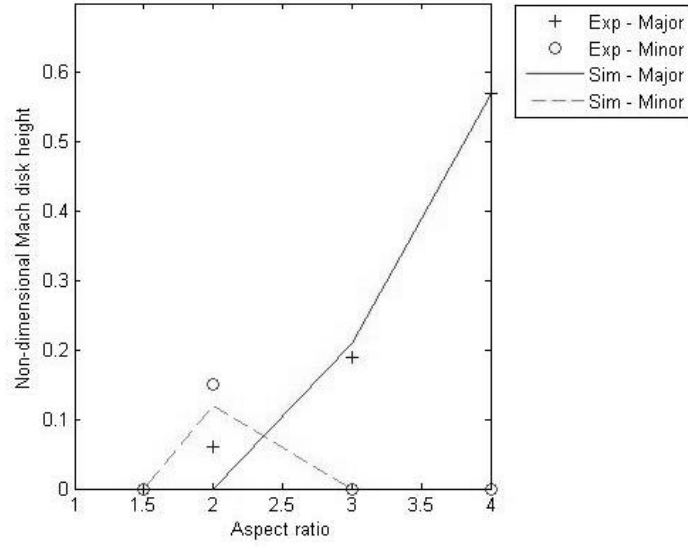


Figure 7.11: Variation of the Mach disk heights along the major and minor axis plane of the nozzle in slot jets when $\frac{P_{exit}}{P_{ambient}} = 2$

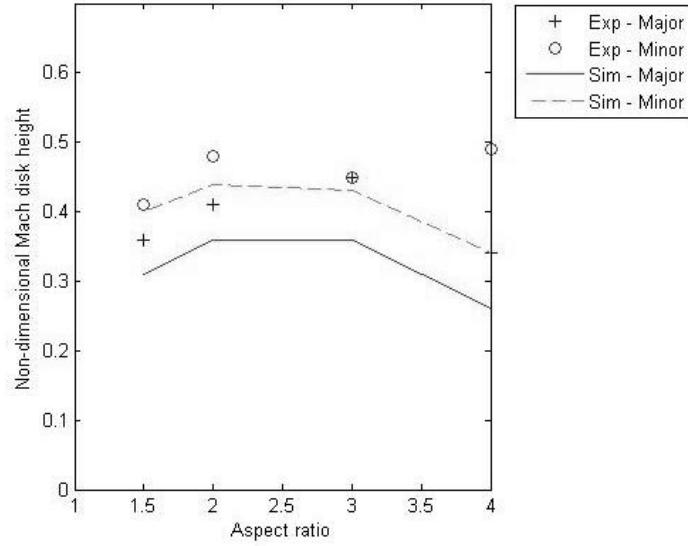


Figure 7.12: Variation of the Mach disk heights along the major and minor axis plane of the nozzle in slot jets when $\frac{P_{exit}}{P_{ambient}} = 3$

as the aspect ratio of the nozzle is increased. On increasing the pressure ratio at the nozzle exit to 3, the formation of a Mach disk is observed for all aspect ratios tested on the major and the minor axis planes of the nozzle. There is good qualitative agreement between the numerical prediction and the experimental observation as seen in figure 7.12. The Mach disk heights along the major and the minor axis planes appear to reach a maximum at the aspect ratio of 3.

7.2.4 Effect of the nozzle inlet angle

The experimental elliptic and slot nozzles that were constructed for this study had a 45° inlet angle and the numerical simulations that have been presented thus far have been representative of that. In addition numerical simulations have also been performed for the case of parallel flow inlets for these nozzle.

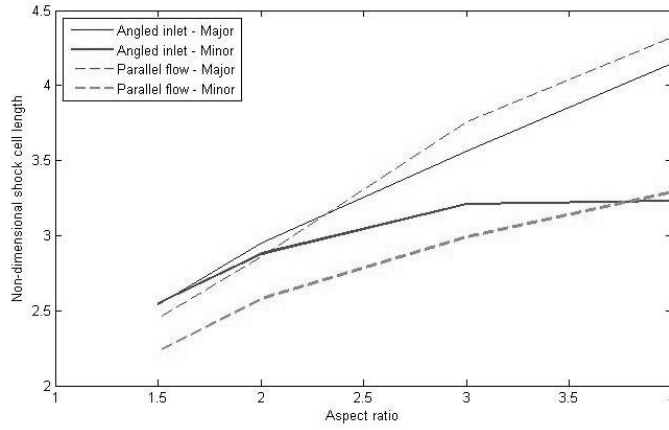


Figure 7.13: Comparison of the shock cell lengths from angled inlet and parallel flow inlet elliptical nozzles when $\frac{P_{exit}}{P_{ambient}} = 3$

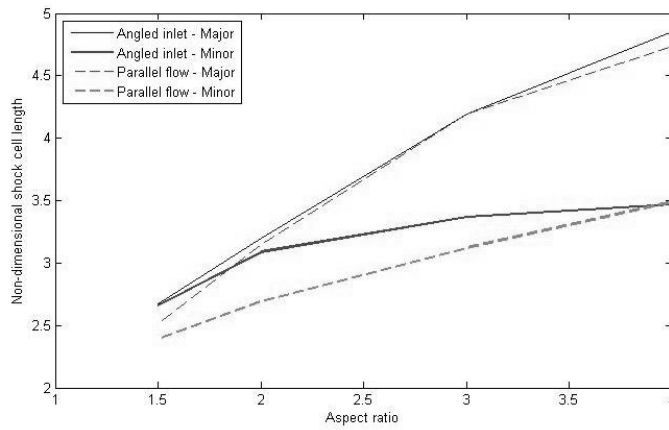


Figure 7.14: Comparison of the shock cell lengths from angled inlet and parallel flow inlet slot nozzles when $\frac{P_{exit}}{P_{ambient}} = 3$

Comparing the shock cell lengths of the angled inlet nozzle and the parallel flow inlet nozzle in figures 7.13 and 7.14 it is clear to see that the effect of the nozzle inlet angle is not significant on the shock cell length. This is the case for both pressure ratios that have been tested.

The effect of the angle of the inlet is evident though when comparing the Mach disk

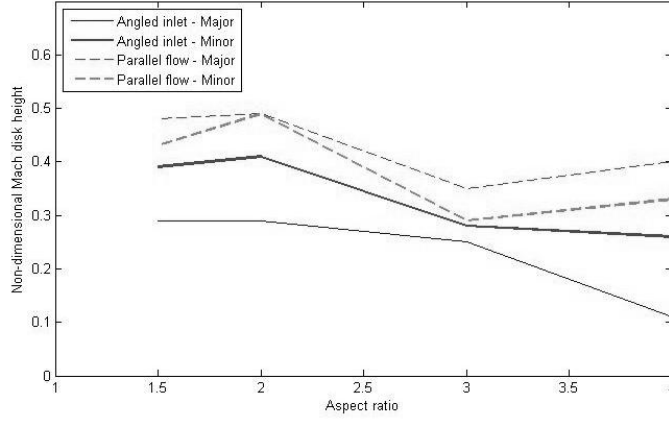


Figure 7.15: Comparison of the Mach disk heights from angled inlet and parallel flow inlet elliptical nozzles when $\frac{P_{exit}}{P_{ambient}} = 3$

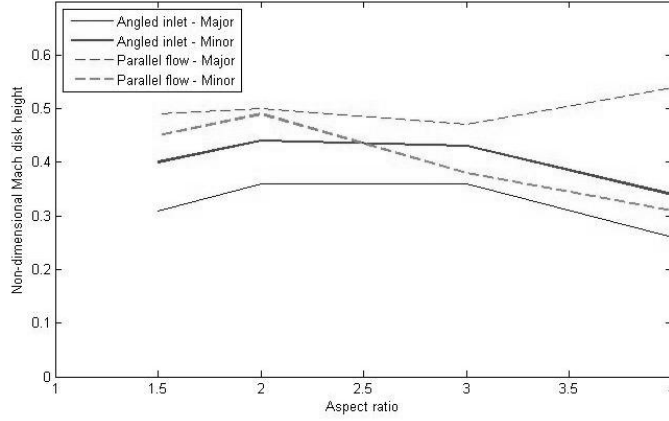


Figure 7.16: Comparison of the Mach disk heights from angled inlet and parallel flow inlet slot nozzles when $\frac{P_{exit}}{P_{ambient}} = 3$

heights of the angled inlet and the parallel flow inlet nozzles as seen in figure 7.15 and 7.16. In the case of the elliptical nozzle, the Mach disk height in the case of the parallel flow inlet nozzle does not decrease consistently as observed in the case of the angled inlet nozzle. The Mach disk height increases at an aspect ratio of 4 and at a pressure ratio of 4 when in the angled inlet nozzle it decreases. In the case of the slot nozzle a similar phenomenon is noticed though the Mach disk height is closer in magnitude to the angled inlet nozzle in comparison to the same case for the elliptical nozzle.

7.3 2-dimensionality in underexpanded non-axisymmetric jets

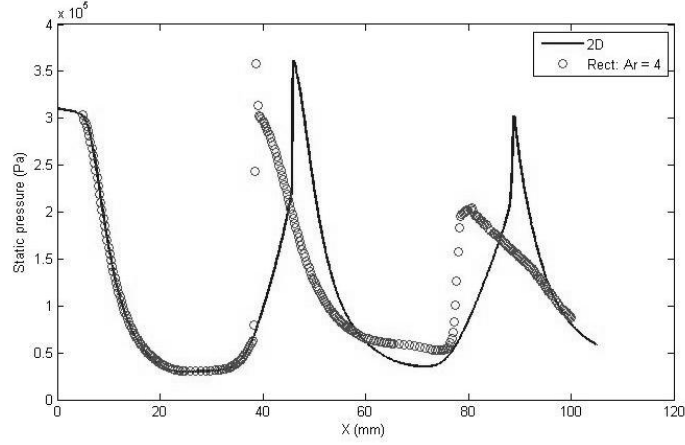


Figure 7.17: Variation of static pressure along the lengths of an aspect ratio 4 rectangular jet and a 2-d jet $\frac{P_{exit}}{P_{ambient}} = 3$

Increasing the aspect ratio of the nozzle exit does result in the flow field as observed along the minor axis plane of the nozzle to resemble a 2-dimensional jet. If one were to look at the rectangular jet as an example we can see that the variation of pressure along the jet centre-line for an aspect ratio 4 rectangular jet closely resembles that of a pure 2-d jet (see figure 7.17). The rectangular jet varies from the 2-d jet close to the formation of the shock along the jet flow direction. The 3-dimensionality of the jet causes the formation of this shock wave closer to the nozzle exit than in the case of a 2-d jet.

7.4 Conclusions

- The turbulence model that has been chosen for this study appears to have been most suitable for the slot nozzles, with the shock cell lengths of this type of nozzle being predicted the most accurately by the numerical analysis.
- The numerically predicted shock cell lengths have been consistently greater than the experimental observations for the rectangular nozzle. In the case of the elliptical nozzle the shock cell lengths have been under predicted while they have been most accurate for the case of the slot nozzle.
- The Mach disk height increases along the major axis plane for all the nozzles investigated at pressure ratios of 2.

- The effect of the inlet angle is limited to the Mach disk height, while the shock cell length appears to be relatively unaffected by the changes in the inlet angle of the nozzle.
- Higher aspect ratio jets tend toward 2-dimensionality for most of their length within the initial shock cell. This is the case for high aspect ratio flows from any nozzle exit.

8 Shock wave formation and interactions within underexpanded jets

The flow within underexpanded jets from non-axisymmetric nozzles are characterised by a complex interaction of shock waves and expansion waves with each other and with the jet boundary which encloses it.

8.1 The flow field within rectangular jets

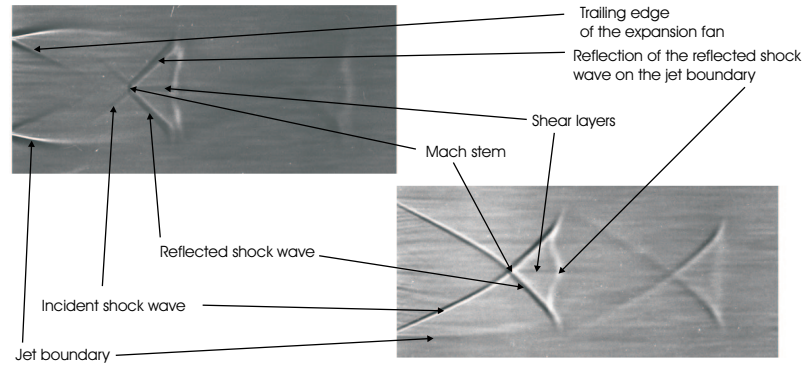


Figure 8.1: Explanation of the flow field in a free jet issuing from a square nozzle, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Major axis plane, Bottom right = diagonal plane

The flow field of an underexpanded jet issuing from a square nozzle is shown in figures 8.1 and 8.2. The flow features along the major axis plane of the square (both orthogonal axes being the same) is markedly similar to the flow field from an axisymmetric jet at the same pressure ratio as seen in figures 6.1(b) and 6.2(b) respectively. The jet flow expands into the ambient medium upon exiting the nozzle due to the positive pressure differential, and expansion fans originating at the nozzle exit also propagate outwards with the expanding jet. The expansion fans reflect off the jet boundary as compression waves which then coalesce downstream of the nozzle exit to form shock waves. These shock waves as can be seen on the major

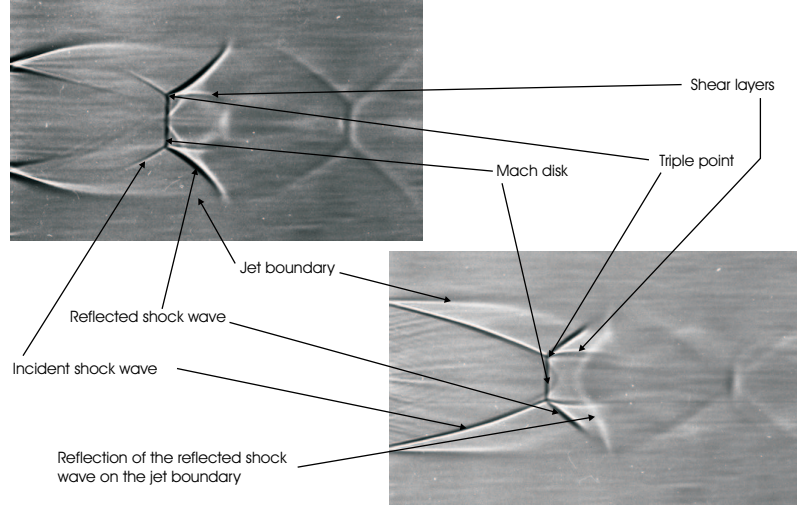


Figure 8.2: Explanation of the flow field in a free jet issuing from a square nozzle, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$. Upper left = Major axis plane, Bottom right = diagonal plane

axes in figure 8.1 and 8.2 originate some distance downstream of the nozzle exit. The incident shock waves converge onto each other from opposite sides of the flow field and reflect in an irregular manner in this case to form a small Mach reflection in the case of the lower pressure ratio flow field and a significantly larger Mach stem in the case of the higher pressure ratio flow field. The reflected shock wave that originates from the triple line also propagates downstream of the nozzle exit until it reflects at the jet boundary. This line of reflection is identified in figures 8.1 and 8.2.

The flow field along the diagonal axis of the square nozzle is where the 3-dimensionality of the flow starts to become apparent. The incident shock wave originates at the nozzle exit, unlike the case along the major axes where the incident shock wave originated some distance downstream of the nozzle exit. The incident shock wave also propagates downstream of the nozzle exit and reflects in an irregular manner forming a Mach stem as identified in figures 8.1 and 8.2.

Figures 8.3 and 8.4 show the cross-sectional schlieren image slices of the flow field developed from the numerical simulation of the flow field. It is clear from figures 8.3(b) and 8.4(b) that the incident shock wave does originate at the nozzle exit. The intersection of the incident shock wave at the major axis planes of the nozzle is what is observed experimentally to be the origin of the incident shock wave along the major axis plane. This is why it appears to originate some distance downstream of the nozzle exit in both pressure ratios that have been tested. The intersection of the incident shock wave at the major axis is in fact a reflection point as can be seen in figures 8.3(d) and 8.4(d). The reflection of the incident shock wave

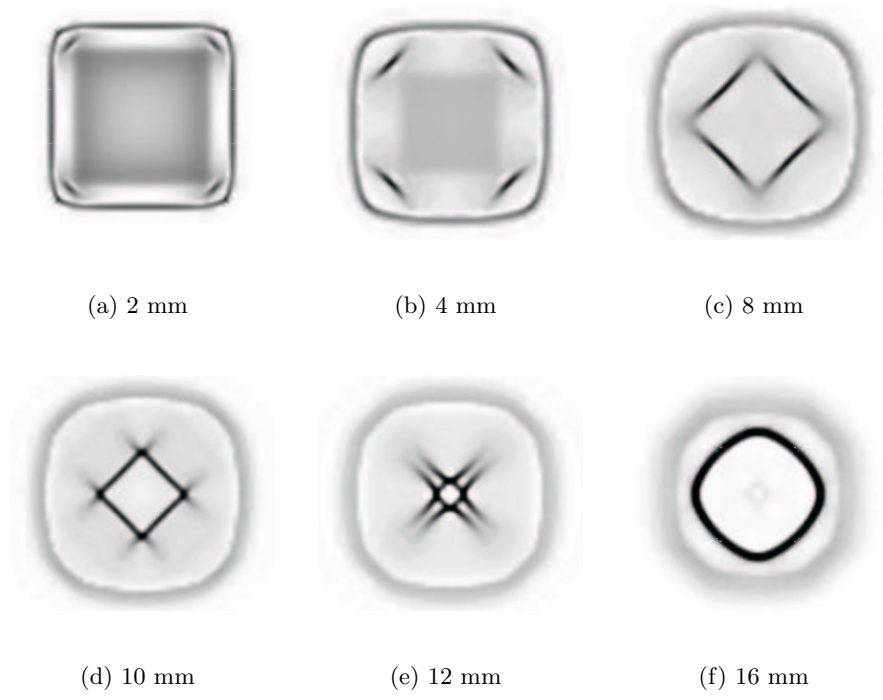


Figure 8.3: Square nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

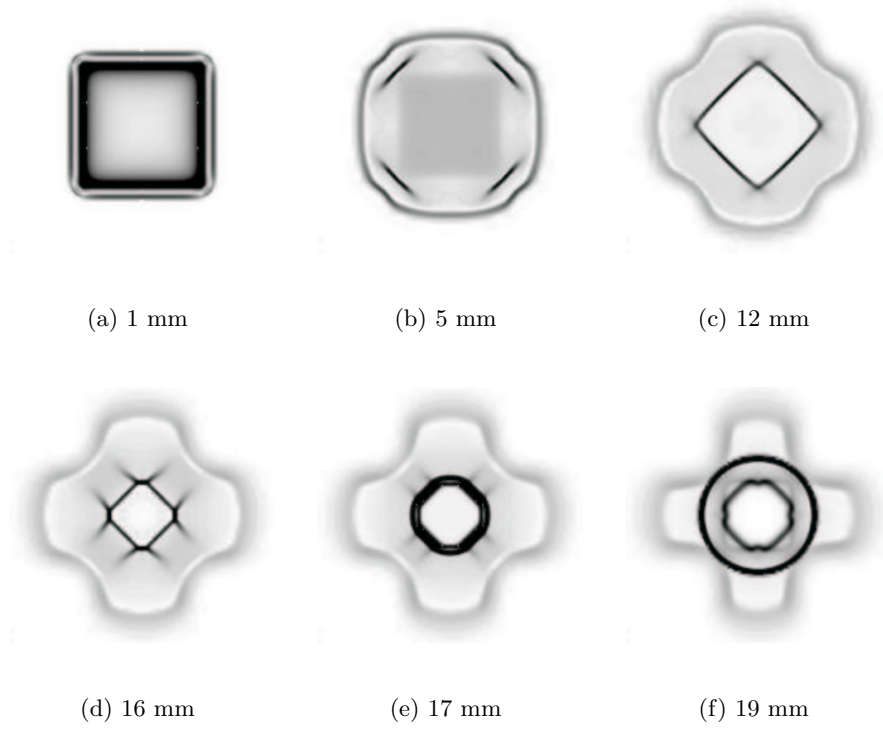


Figure 8.4: Square nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

produces a reflected portion of the incident shock wave which propagates toward the jet boundary. While in the case of the lower pressure ratio nozzle, this reflection appears to be regular in nature, the reflection in the higher pressure ratio is irregular with a Mach stem being present between the incident shock wave and the reflected portion of the incident shock wave. The incident shock surface hence consists of the incident shock wave from the corners of the nozzle exit (henceforth referred to as the incident corner shock wave), the Mach stem portion of the incident shock wave (henceforth referred to as the Mach corner shock wave) and the reflected portion of the incident shock wave (henceforth referred to as the reflected corner shock wave). Hence the incident shock wave that is observed experimentally along the major axis plane is in fact a Mach stem on the incident shock wave surface. The reflected shock wave that is formed in both cases appears to be near circular in cross-section. The jet boundary in both cases are also very different in that the jet boundary at the higher pressure ratio is more distorted downstream of the nozzle exit from its original cross-sectional shape of that of a square (this is discussed in greater detail in Chapter 9). The shear layers from the Mach stem in the case of the higher pressure jet flow field appears to be converging on themselves, and this is also observed in the cross-sectional slices of the flow field in figure 8.4. These shear layers form a converging duct through which the flow through the Mach stem (subsonic immediately downstream of the Mach stem) will be accelerated to sonic conditions. This phenomenon is confirmed by the formation of a normal shock wave in the diagonal plane downstream of the initial Mach reflection in the flow field in figure 8.2.

The cross-sectional shape of the Mach reflection in the case of the lower pressure ratio of 2, is square as the incident shocks from the corners of the nozzle have reflected in a regular manner along the major axes of the nozzle flow field. The shape of the square is however offset by 45° to the orientation of the nozzle exit (figures 8.3(c) and 8.3(d)). The Mach reflection that occurs along the direction of the jet flow has a octagonal shape (figure 8.4(d)) in the case of the jet with a pressure ratio of 3 as a result of the convergence of the incident shock wave system (which comprises of four sections of the incident shock wave that originated at the four corners of the nozzle exit, and four Mach reflection portions that propagate along the major axes planes of the nozzle).

3-dimensional representations of the flow fields from a square nozzle are shown in figures 8.5 and 8.6. The representation is generated from iso-surfaces of the gradient of density from the numerical simulation. From these representations the formation and the development of the incident shock surface can be seen clearly for both cases

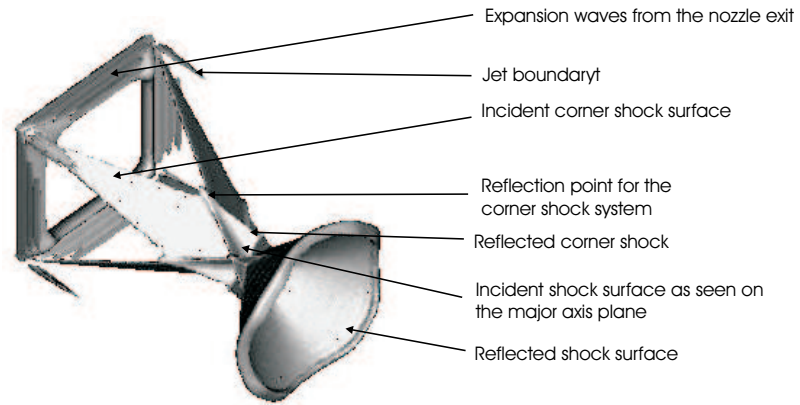


Figure 8.5: 3-dimensional representation of the flow field for the underexpanded jet from a square nozzle, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

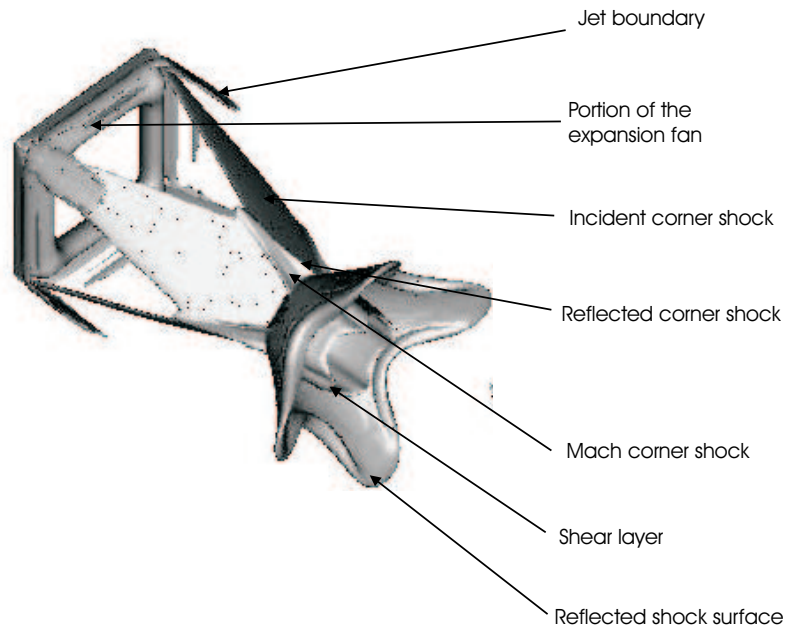


Figure 8.6: 3-dimensional representation of the flow field for the underexpanded jet from a square nozzle, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

of pressure ratios that have been investigated for the square nozzle.

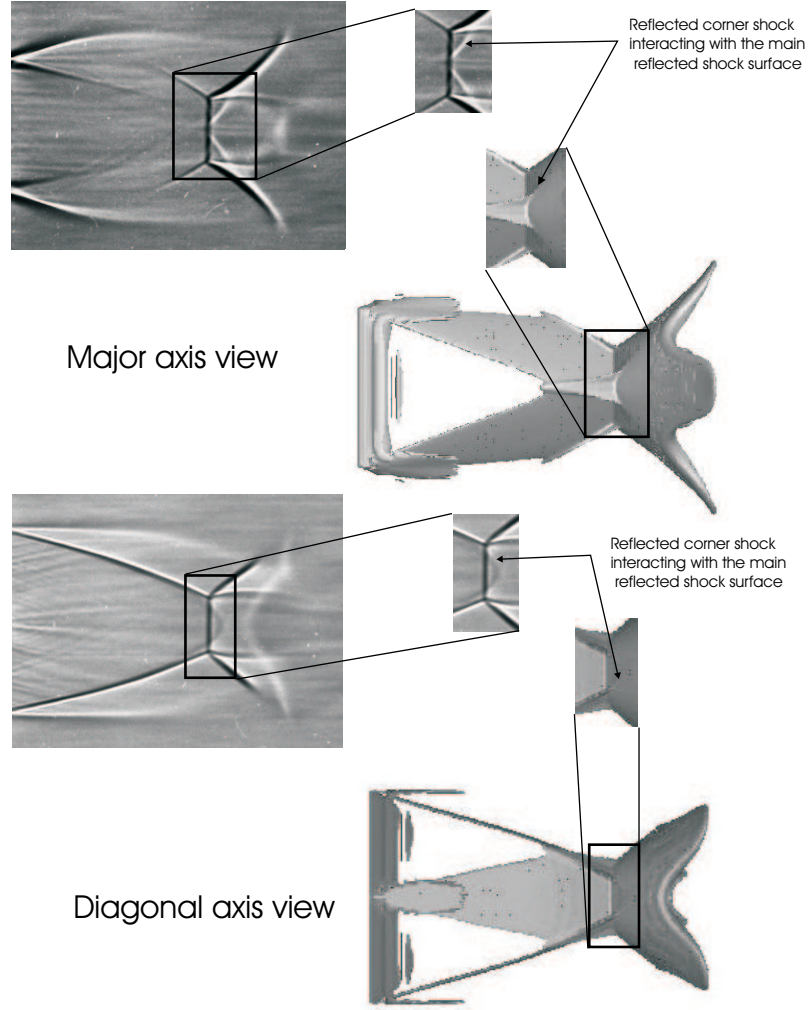


Figure 8.7: Explanation of additional flow features seen in the experimental images, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

In figure 8.2, there are two hitherto unexplained waves present immediately downstream of the Mach reflection of the incident corner shock system. These waves can now be explained when considering the 3-dimensional representation of the flow field in figure 8.7. Along the major axes and the diagonal axes the additional waves behind the Mach stem, are the intersection of the reflected corner shocks with the main reflected shock. Along the major axes these lines of intersection appear to diverge from each other, while along the diagonal axes they converge. The lines of intersection are clearly seen in the 3-dimensional representation of the flow field.

The flow field within the underexpanded jet issuing from a nozzle of aspect ratio 1.5 is similar to that of the square nozzle and is hence not discussed. These results are displayed in the appendix.

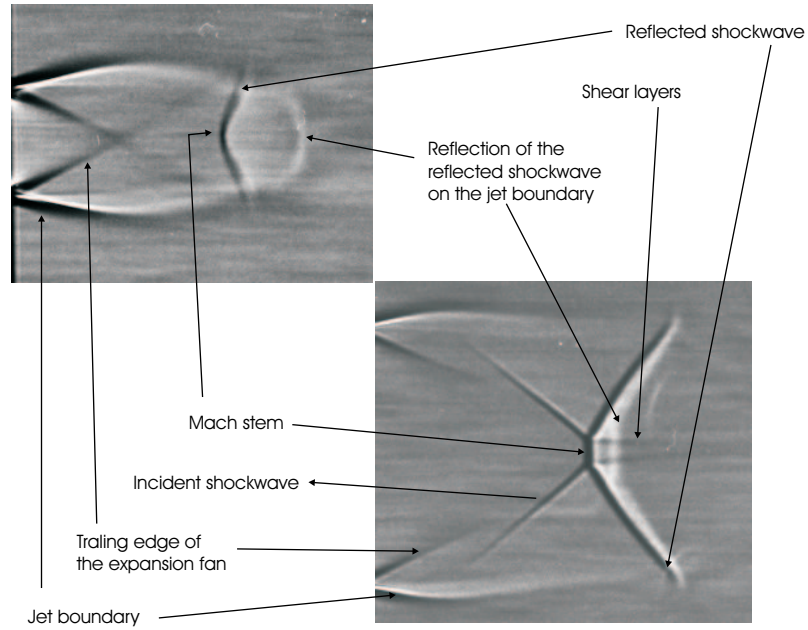


Figure 8.8: Explanation of the flow field in a free jet issuing from a rectangular nozzle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Minor axis plane, Bottom right = Major axis plane

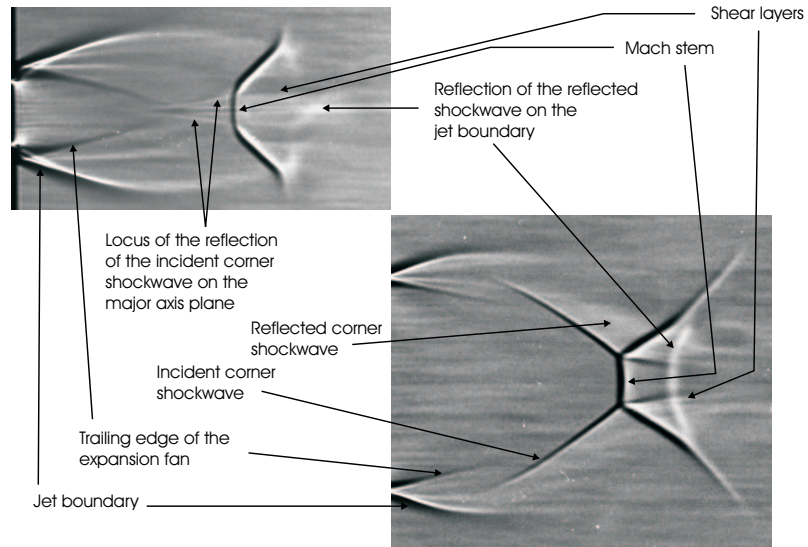


Figure 8.9: Explanation of the flow field in a free jet issuing from a rectangular nozzle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$. Upper left = Minor axis plane, Bottom right = Major axis plane

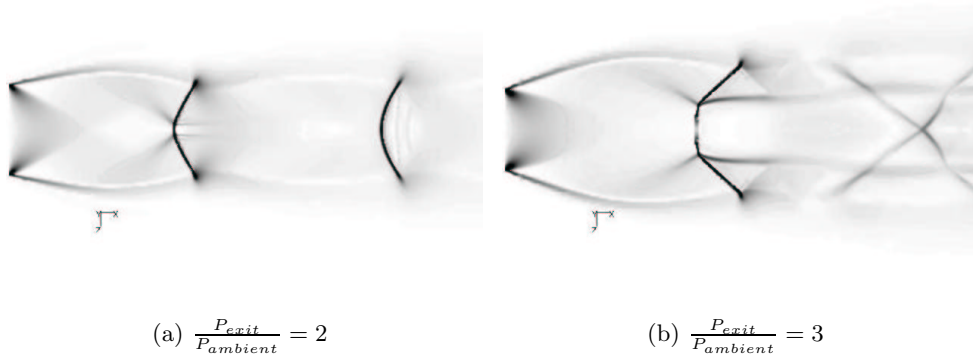


Figure 8.10: AR = 2 nozzle, images along the minor axis of the nozzle exit

The flow field within underexpanded jets issuing from rectangular nozzles having aspect ratios of 2 and 4 are displayed in figures 8.8 to 8.16. The characteristics of the flow field along the major axis planes of the two nozzles are similar to that observed in the square jet case described earlier.

As in the case of the square nozzle described earlier, in the case of the aspect ratio 2 nozzle the incident shock wave along the major axis plane originates some distance downstream of the nozzle exit. The incident shock wave undergoes a Mach reflection forming a three shock system comprising of the incident shock wave, the Mach stem and the reflected shock wave for all pressure ratios that have been investigated. The incident shock wave is straight in the lower pressure ratio case and develops a curvature toward the jet centre-line as the pressure ratio increases. This phenomenon was also noted in the case of the square nozzle.

The presence of an incident shock wave is not observed along the minor axis in figure 8.8 or figure 8.9. In the numerical simulation in figure 8.10(a), the incident shock wave is not formed completely, but seems to be a collection of compression waves. In the higher pressure ratio case in figure 8.10(b) the incident shock wave along the minor axis plane of the jet is significantly weaker than along the major axis plane. The reason the shock wave is not visible in the shadowgraph image would be as a result of them being considerably weaker than the other waves in the flow field.

The cross-sectional slices of the flow field for the two pressure ratios displayed in figures 8.11 and 8.12 reveal the formation of the incident shock wave to occur at the corners of the nozzle exit like in the case of the square nozzle. The incident shock waves formed at the corners of the nozzle exit reflect off each other along the major axis plane of the nozzle. At the lower pressure ratio (see figure 8.11), the reflection is regular in nature and incident shock wave weakens downstream of the reflection while the reflected shock wave (now curved toward the nozzle corners) gains in strength.

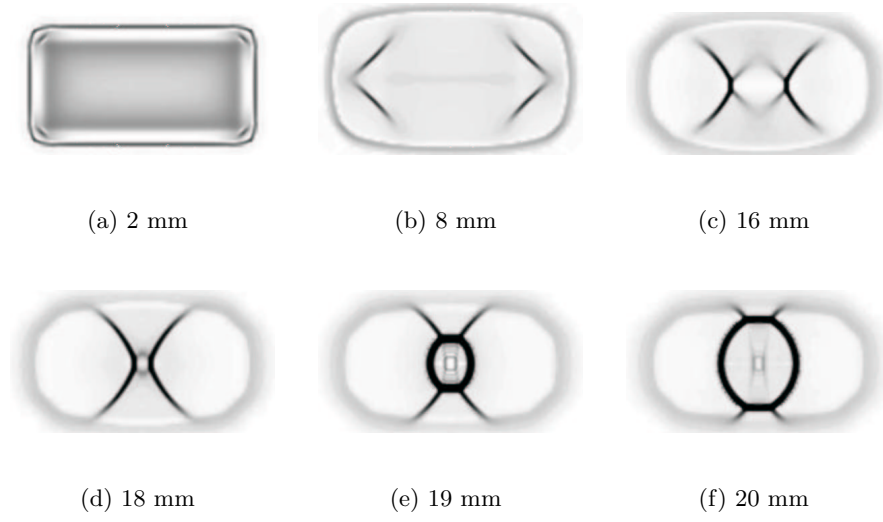


Figure 8.11: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

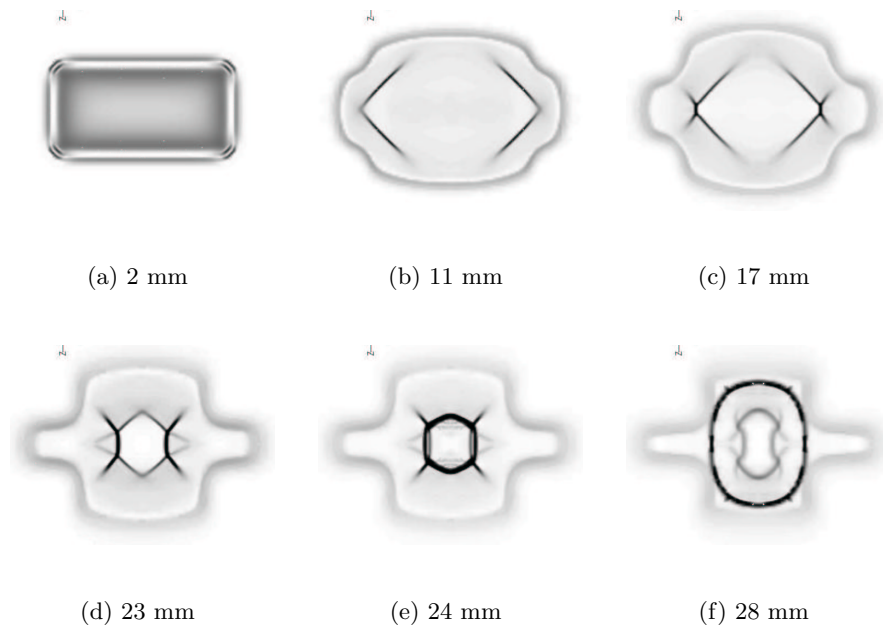


Figure 8.12: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

The lack of the incident shock wave on the minor axis plane is explained in figure 8.11(d) where just before the incident shock wave system undergoes a reflection in the direction of the jet flow, the incident shock has not reflected along the minor axis plane. The incident shock system reflects in an irregular manner forming a Mach stem that is flanked on either side by a two reflected shock waves as seen in figures 8.11(e) and 8.11(f). This is confirmed by the presence of shear layers extending between the two Mach stems and not forming a closed system as seen in the case of the square nozzle.

On increasing the pressure ratio there are two differences that can be identified compared to the case of the lower pressure ratio. These are that the reflection of the incident shock waves (also originating from the corners of the nozzle) is irregular along the major axis plane as was in the case of the square nozzle of the same pressure ratio. The formation of the three shock system (comprising of the incident shock wave, the reflected corner shock wave and the Mach corner shock wave) with associated shear layers can be clearly seen in figure 8.12(d). The incident shock system reflects along the minor axis at 23 mm downstream of the nozzle exit, forming the incident shock wave seen in the numerical simulation in figure 8.10(b). The incident shock wave is weaker after it reflects along the major axis plane as in the case of the lower pressure ratio. The Mach reflection that is observed in figure 8.9 can be seen in cross-section in figure 8.12(e). The Mach stem in this case is hexagonal in shape as compared to the octagonal shape seen in the square nozzle for the same pressure ratio. The formation of the hexagonal Mach stem is as a result of the regular reflection of the incident shock wave along the minor axis plane. The shear layers formed from this reflection are closed and appear to curl onto themselves along the minor axis plane of the nozzle. The shear layers take on a dumb-bell shape (figure 8.12(f)). The reflected shock wave that is formed takes on an elliptical cross-section with switched axes as can be seen in figure 8.12(f).

Three-dimensional representations of the flow field is shown in figures 8.13 and 8.14 based on the variation in the density gradients in the numerical simulations. The incident corner shock wave, the Mach corner shock wave and the reflected corner shock wave are all clearly observed in both cases. The locus of the triple point signifying the intersection of the Mach corner shock wave, the incident corner shock wave and the reflected corner shock wave seen in the 3-dimensional representation along the minor axis plane can also be seen in the experimental image (see figure 8.9).

Increasing the aspect ratio to 4 results in flow features that are qualitatively similar to that observed in the aspect ratio 2 case. The incident shock wave is not observed

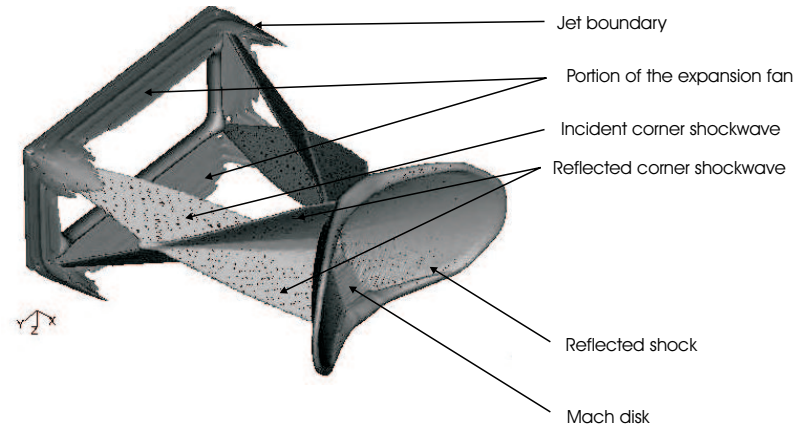


Figure 8.13: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

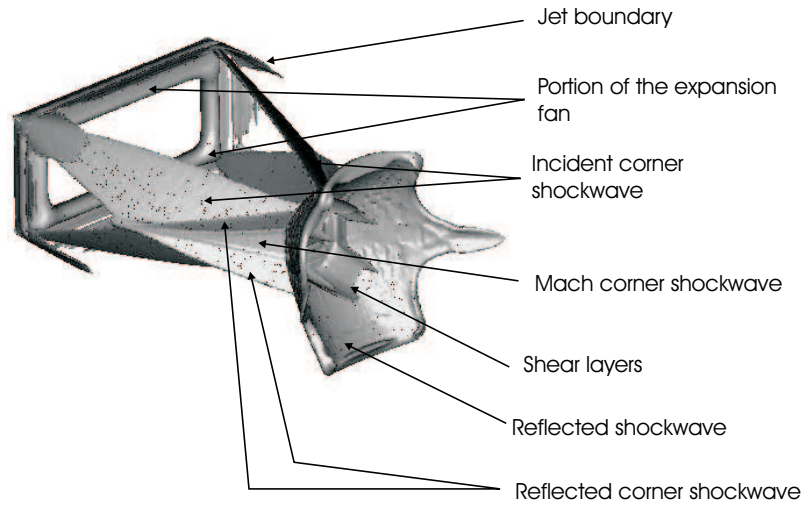


Figure 8.14: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

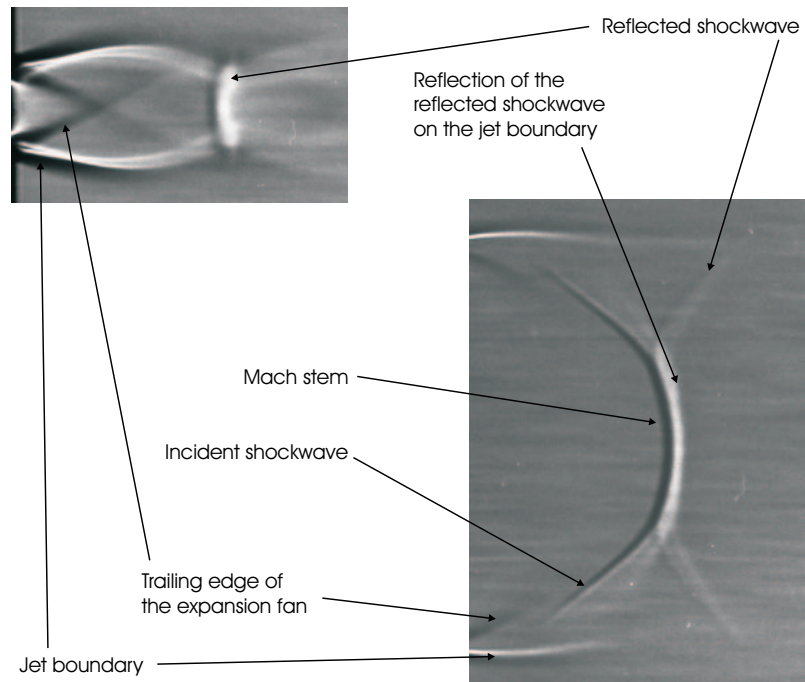


Figure 8.15: Explanation of the flow field in a free jet issuing from a rectangular nozzle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Minor axis plane, Bottom right = Major axis plane

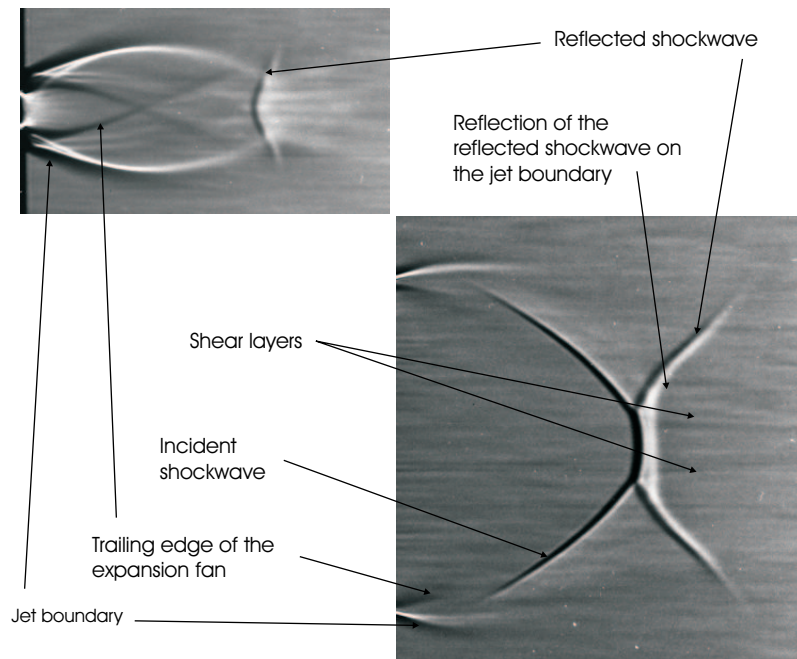


Figure 8.16: Explanation of the flow field in a free jet issuing from a rectangular nozzle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$. Upper left = Minor axis plane, Bottom right = Major axis plane

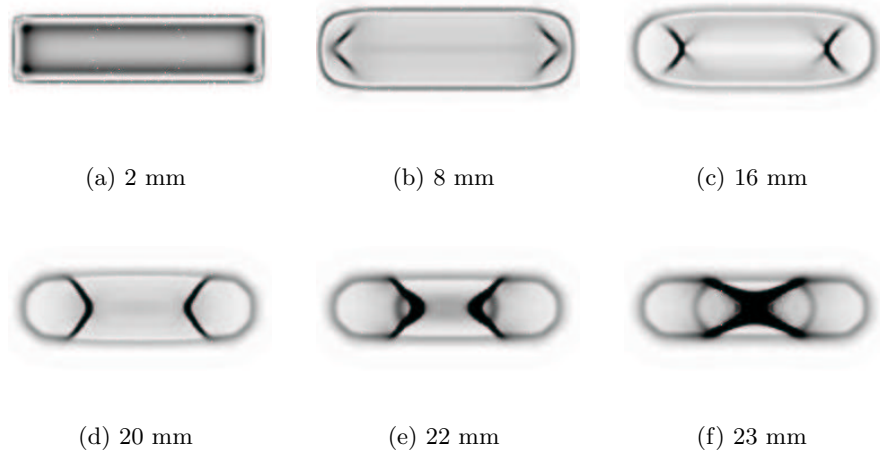


Figure 8.17: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

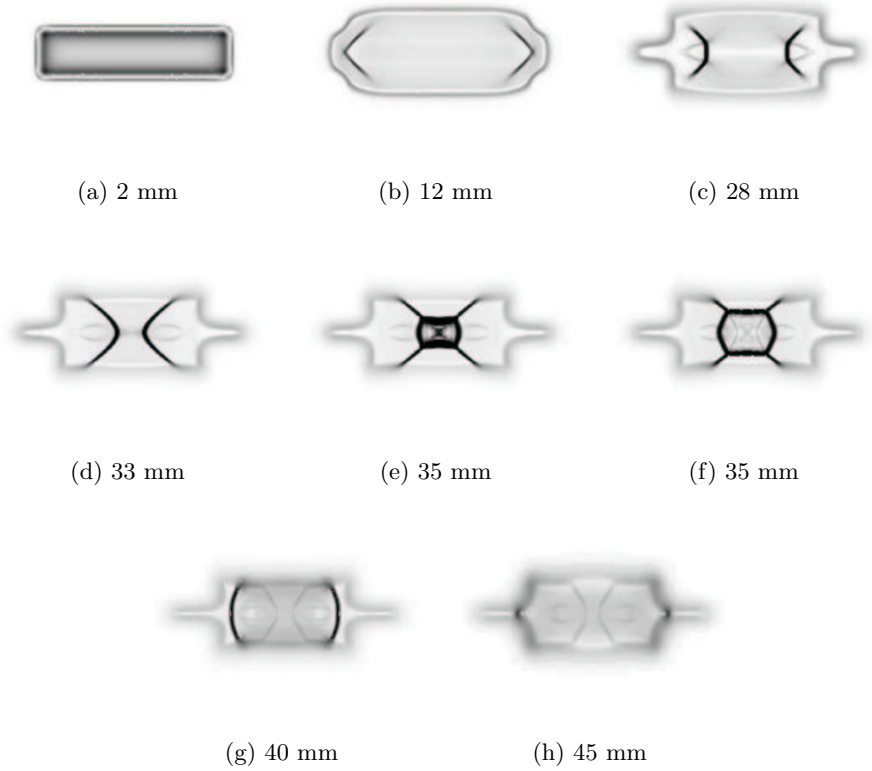


Figure 8.18: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

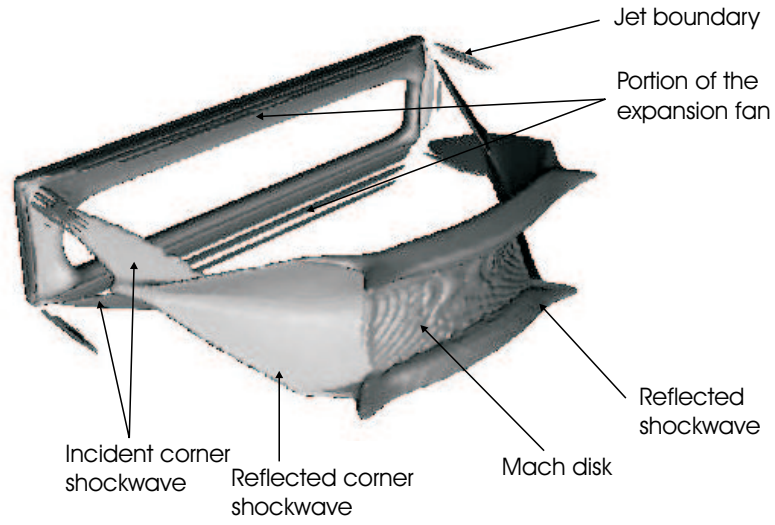


Figure 8.19: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

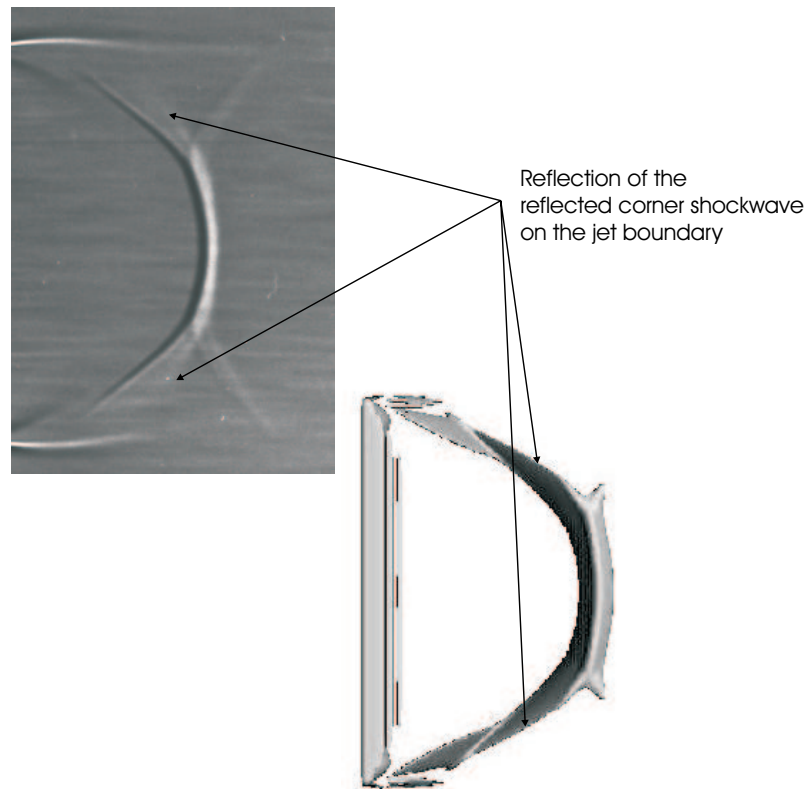


Figure 8.20: Additional flow features in the experimental images explained for the underexpanded jet from a rectangular nozzle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

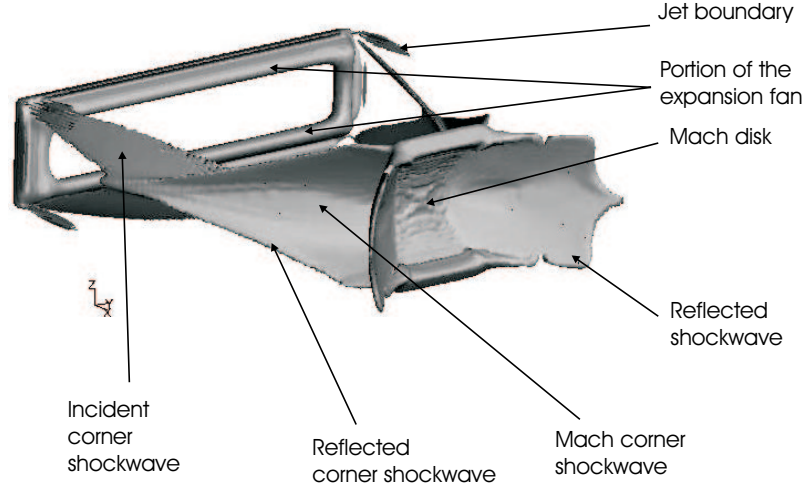


Figure 8.21: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

along the minor axis plane in both pressure ratios that were tested (see figures 8.15 and 8.16). The incident shock wave originating downstream of the nozzle exit is observed along the major axis plane. The incident shock wave undergoes an irregular reflection forming a Mach stem, and a reflected shock wave. The Mach stem formed in the lower pressure ratio case is greater than the width of the minor axis of the nozzle. The Mach stem has a pronounced outward curvature in the direction of the flow. Shear layers from the triple point of the reflection are not visible in the experimental shadowgraph in figure 8.15. The incident shock wave appears to form a single continuous wave with the Mach stem. The reflected shock wave that is formed is considerable weaker than the incident shock wave. In the higher pressure ratio case, the Mach stem is not as large in the case of the lower pressure ratio. The Mach stem in this case is also curved outward in the direction of the flow. The shear layers from the triple point propagate downstream of the Mach reflection and converge onto themselves (figure 8.16).

In the cross-sectional slices of the flow field in figures 8.17 and 8.18 the formation and the development of the shock wave system within the underexpanded jet can be observed as it occurs downstream of the nozzle exit. In the lower pressure ratio case the incident shock wave from the nozzle corner reflects along the major axis plane, and rapidly loses strength, with the reflected portion of the incident shock wave gaining in strength downstream of the nozzle exit. For the lower pressure ratio case the cross-sectional slices cut through the Mach reflection due to its pronounced curvature. This is evident in figure 8.17(e) where the Mach reflection has occurred, but is not seen completely. The occurrence of the Mach reflection can be discerned from the presence of the reflected shock wave (seen as a small curved wave between

the reflection and the jet boundary). The case of the increasing pressure ratio is similar that of the aspect ratio 2 nozzle, with the irregular reflection of the incident shock waves as observed in figure 8.18(c). The shear layers from this reflection appear to close up and form an enclosed tube-like structure as seen in the sequence of images from figure 8.18(c) to 8.18(e). The Mach reflection of the incident shock wave system is similar to that of the pressure ratio 2 case of the nozzle with aspect ratio 2. The Mach reflection consists of two Mach stems flanked by reflected shock waves. The shape of the shear layers downstream of the reflection also indicates this.

The three-dimensional representations of the flow fields are presented in figure 8.19 and 8.21. Additional waves that have been observed in the experimental image along the major axis plane (figure 8.19) have been identified; by considering the 3-dimensional representation of the flow field; as being the reflection of the reflected corner shock wave on the jet boundary.

8.1.1 Incident shock wave formation in rectangular jets

In rectangular jets expansion fans are generated at each face of the nozzle exit, and they propagate downstream of the nozzle exit. The pressure across the expansion fan is such that it equalises the high pressure at the nozzle exit with the lower pressure of the ambient surrounding. This phenomenon occurs at each face of the nozzle exit. At the corners of the nozzle though, the two expansion fans from the orthogonal faces at the corner cross over each other, and the resulting pressure is lower than that of the ambient medium. Hence a shock wave develops to provide the pressure equalisation that is needed. This shock wave propagates downstream and forms the incident shock wave identified in the preceding discussion.

8.2 The flow field within elliptical and slot jets

The flow field issuing from elliptical and slot jets having an aspect ratio of 2 at a pressure ratio of 2 is shown in figures 8.22 and 8.23. The similarity between the elliptical and the slot nozzle is in the minor axis plane where in both cases the formation of an incident shock wave along the minor axis plane of the nozzle flow field is not observed. Along the major axis planes however an incident shock wave is observed to originate downstream of the nozzle exit. The incident shock wave propagates downstream and reflects in a regular manner in the case of the elliptical

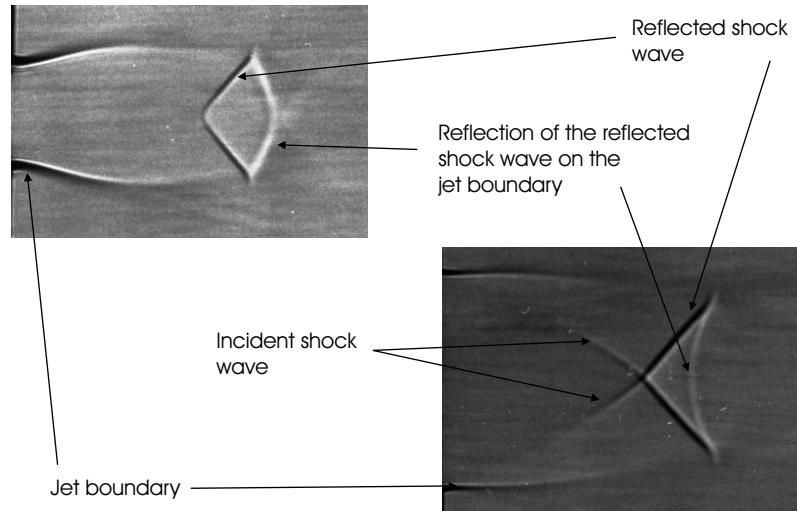


Figure 8.22: Explanation of the flow field in a free jet issuing from an elliptical nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Minor axis plane, Bottom right = Major axis plane

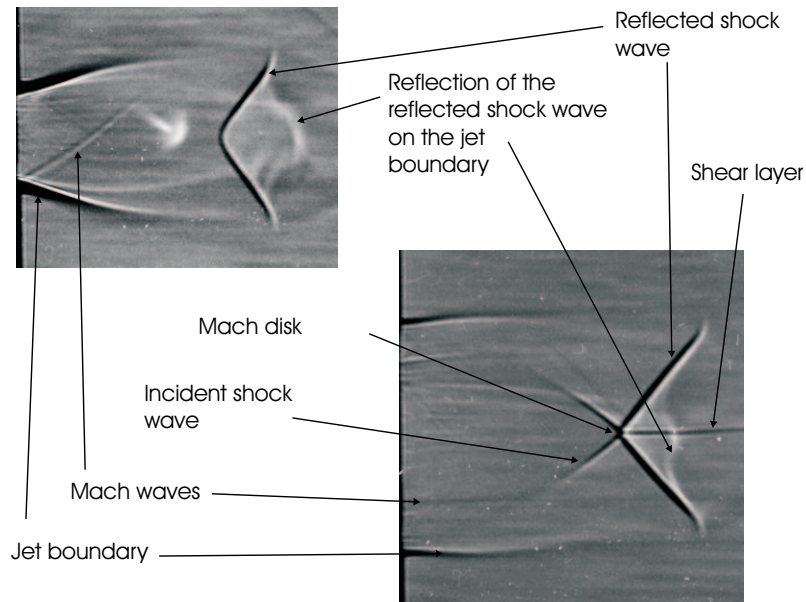


Figure 8.23: Explanation of the flow field in a free jet issuing from a slot nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Minor axis plane, Bottom right = Major axis plane

nozzle and in an irregular manner in the case of the slot nozzle forming a Mach reflection with a small Mach stem. The Mach stem in the irregular reflection in the case of the slot nozzle is not easily visible due to its size but its presence is inferred from the shear layers emanating from the reflection point as seen in figure 8.23. The formation of the incident shock wave along the major axis plane and the lack of the incident shock wave along the minor axis plane has been reported before by Rajakuperan and Ramaswamy [32] for the case of slot jets, and by Rajakuperan and Ramaswamy [31] and Verma and Rathakrishnan [29] in the case of elliptical jets.

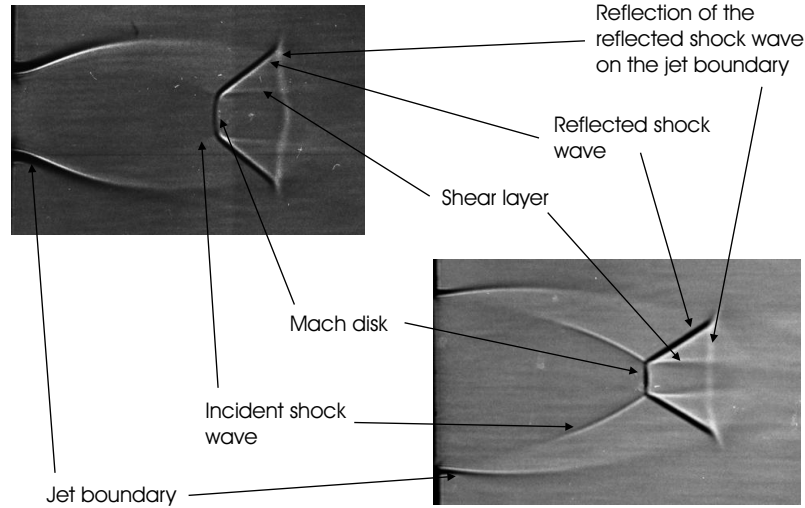


Figure 8.24: Explanation of the flow field in a free jet issuing from an elliptical nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$. Upper left = Minor axis plane, Bottom right = Major axis plane

Increasing the pressure ratio at the nozzle exit results in the flow fields shown in the images in figures 8.24 and 8.25. Along the major axis planes the flow field for the elliptical and the slot jets look similar to each other, and also similar to the case of axisymmetric jets. The incident shock wave formed along the major axis plane of the nozzle flow field is curved inwards into the jet flow, and the reflection of the incident shock wave is irregular in nature. The Mach stem height is significantly larger than that observed for the case of the slot jet at the lower pressure ratio. Along the minor axis plane of the nozzle, the incident shock wave is significantly weaker than along the major axis plane. While its presence, as in the case of the slot nozzle in figure 8.25, is not easily observed as compared to the major axis plane flow field, the incident shock wave is present from the fact that a Mach reflection occurs on the minor axis plane. The Mach stem on the minor axis plane of the nozzle is larger than in the major axis plane for both slot and elliptical jets. This shows that the cross-section of the Mach stem has switched axes as compared to the axes of the nozzle exit. The width of the reflected shock wave on the minor axis is also larger

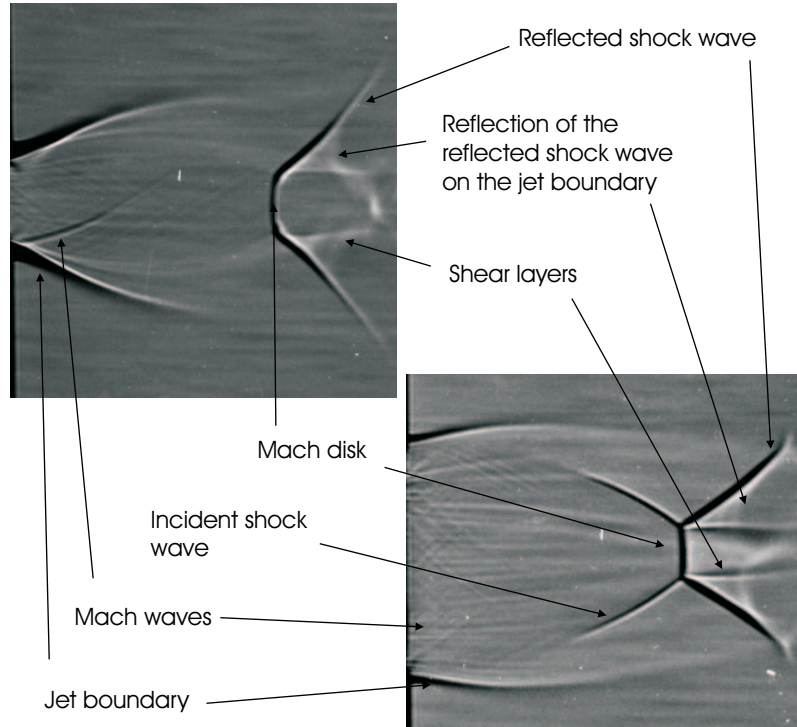


Figure 8.25: Explanation of the flow field in a free jet issuing from a slot nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$. Upper left = Minor axis plane, Bottom right = Major axis plane

than in the major axis plane for both nozzles. This suggests that the jet boundary is wider in the minor axis plane than in the major axis plane at the point where the reflected shock wave intersects with it. Comparing the reflected shock wave in the slot jet to the elliptical jet, along the minor axis the reflected shock wave is much wider at the point where it meets the jet boundary. Hence the jet boundary width for the slot nozzle is greater along the minor axis plane than for the case of the elliptical nozzle (this is confirmed and explained in greater detail in Chapter 9).

Increasing the aspect ratio of the nozzles results in flow fields displayed in figures 8.26 to 8.29. The flow field along the minor axis planes of the aspect ratio 4 nozzle for the lower and higher pressure ratio cases looks strikingly similar to that observed in the case of the rectangular nozzle. The absence of an incident shock wave is observed in both the elliptical and the slot jets for both pressure ratios. The reflected shock wave spans the width of the flow field for both the elliptical and the slot nozzles. There is no evidence of any other waves (apart from Mach lines produced by imperfections along the nozzle inlet face) between the nozzle exit and the reflected shock wave in the elliptical and slot nozzle cases, for the lower pressure ratio. Along the major axis plane of the nozzle, the flow field is familiar with the incident shock wave, which undergoes an irregular reflection forming a Mach stem, and the reflected

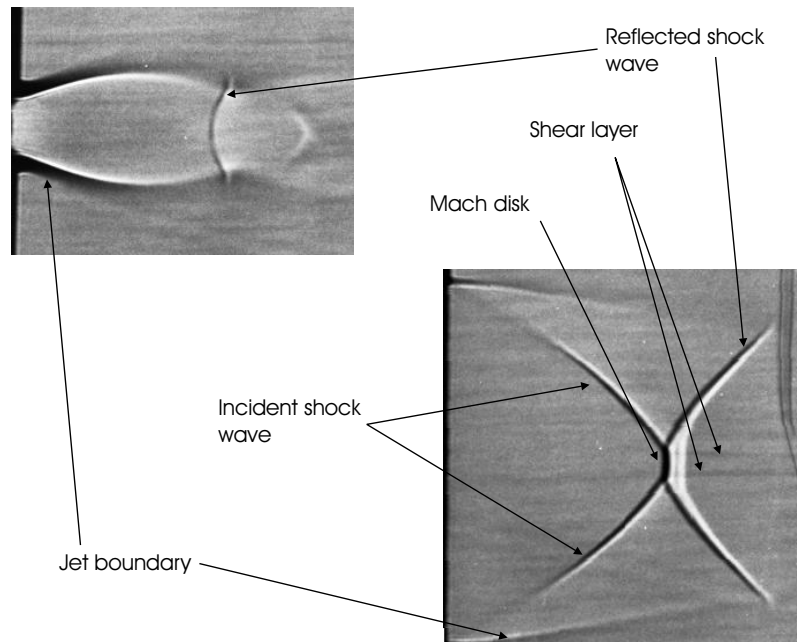


Figure 8.26: Explanation of the flow field in a free jet issuing from an elliptical nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Minor axis plane, Bottom right = Major axis plane

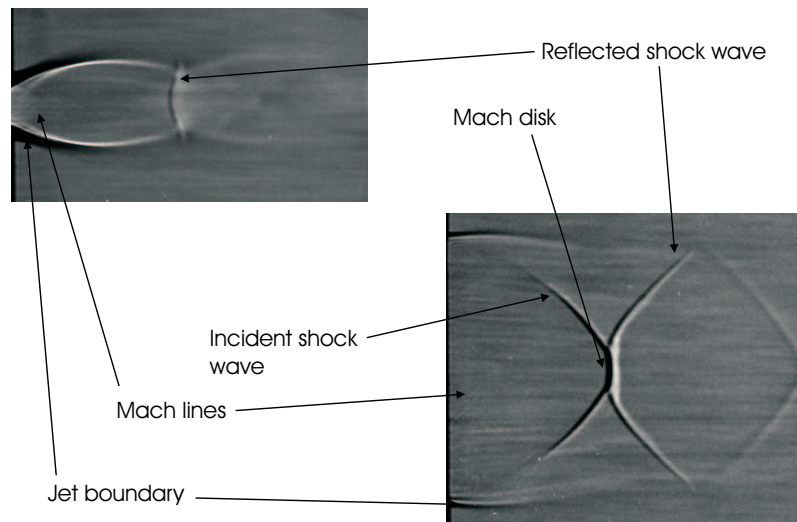


Figure 8.27: Explanation of the flow field in a free jet issuing from a slot nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Minor axis plane, Bottom right = Major axis plane

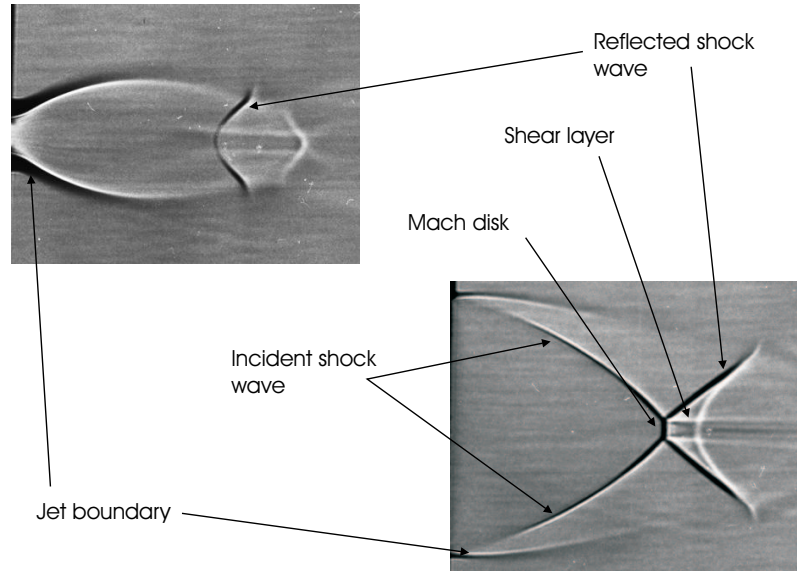


Figure 8.28: Explanation of the flow field in a free jet issuing from an elliptical nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$. Upper left = Minor axis plane, Bottom right = Major axis plane

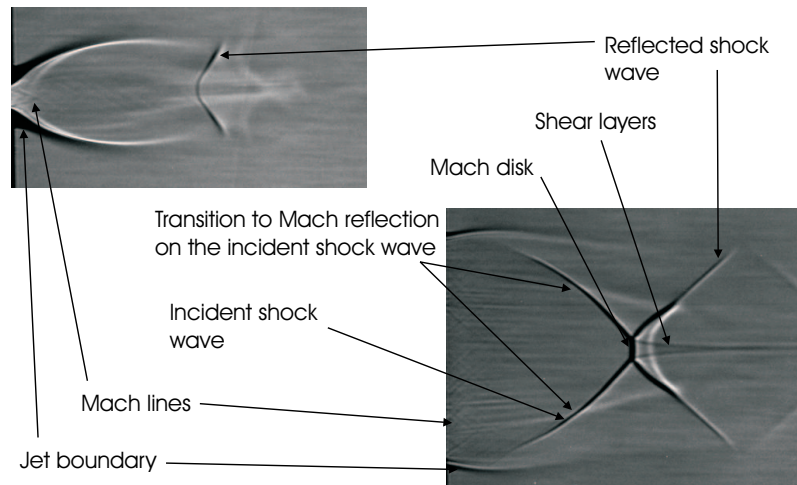


Figure 8.29: Explanation of the flow field in a free jet issuing from a slot nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$. Upper left = Minor axis plane, Bottom right = Major axis plane

shock wave. The Mach stem appears to be of similar height in the case of the slot and the elliptical nozzle. The shape of the shear layers are, however, very different. The shear layers from the elliptical nozzle stay parallel and diverge downstream of the Mach reflection, whereas the shear layers in the case of the slot nozzle converge onto themselves quickly downstream of the nozzle exit.

The cross-sectional slices of the elliptical and slot jet flow fields discussed thus far are presented in figures 8.30 to 8.37.

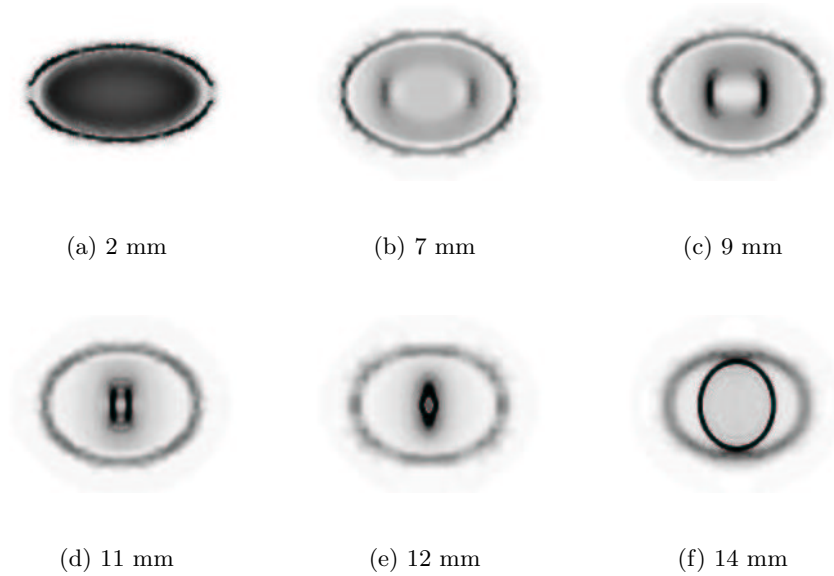


Figure 8.30: Ellipse nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

Comparing the cross-sections of the flow field for the lower pressure ratio case, there is very little difference between the two except for the shape of the jet boundary. The shape of the reflected shock wave is different as it meets the jet boundary due to the shape of the jet boundary. The cross-sectional shape of the reflected shock wave is elliptical in both cases, with the axes of the reflected shock wave cross-section being opposite to that of the nozzle exit.

For the higher pressure ratio case in figures 8.32 and 8.33, there are significant differences in the flow fields that were not easily noted in the shadowgraph experimental images presented earlier. The incident shock wave that is formed in both cases along the major axis plane of the nozzle has the curvature as that of the nozzle exit. In the case of the elliptical nozzle, the incident shock wave propagates downstream and retains its curvature as it undergoes the irregular reflection 16 mm downstream of the nozzle exit. The Mach stem that is formed is also elliptical in shape, but with its axes switched to be opposite to that of the nozzle exit (see figure 8.32(c)).

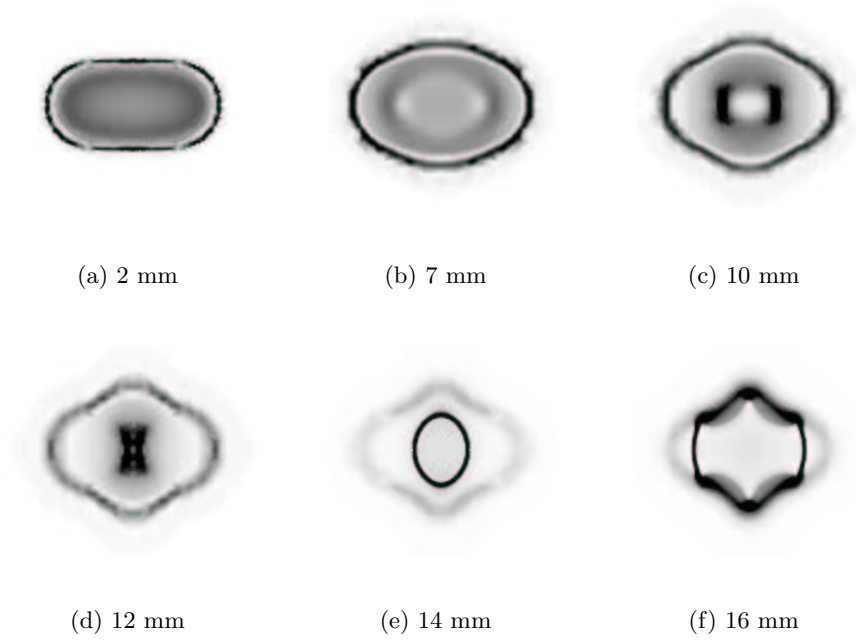


Figure 8.31: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

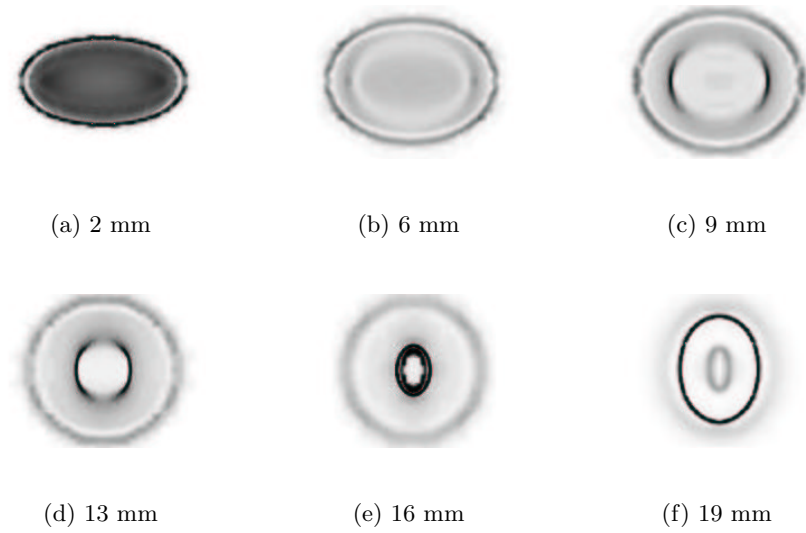


Figure 8.32: Ellipse nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

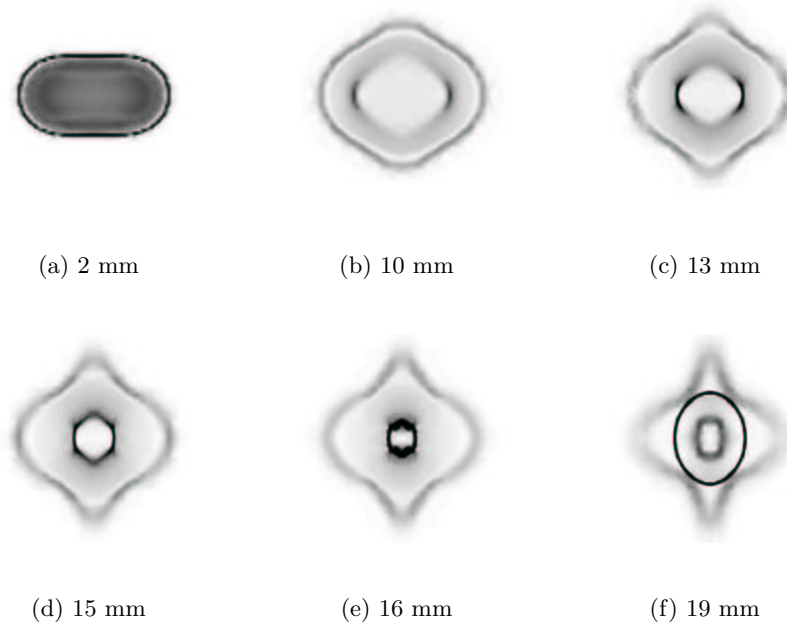


Figure 8.33: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

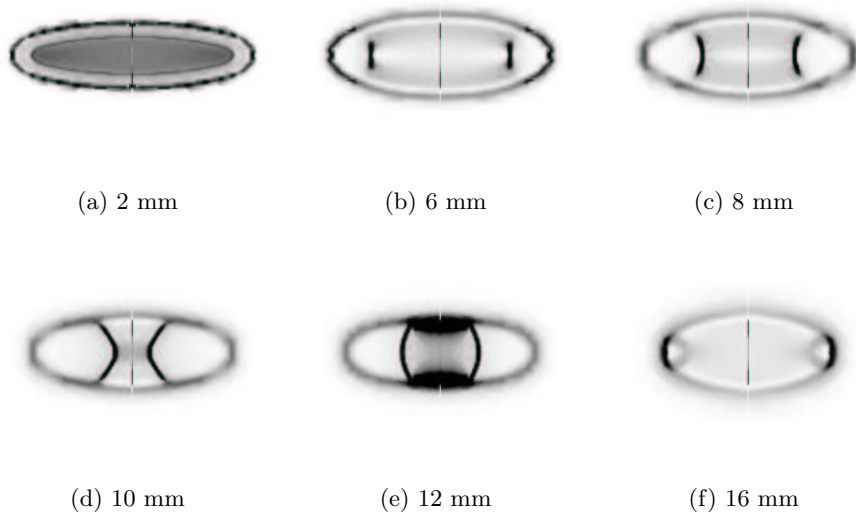


Figure 8.34: Elliptic nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

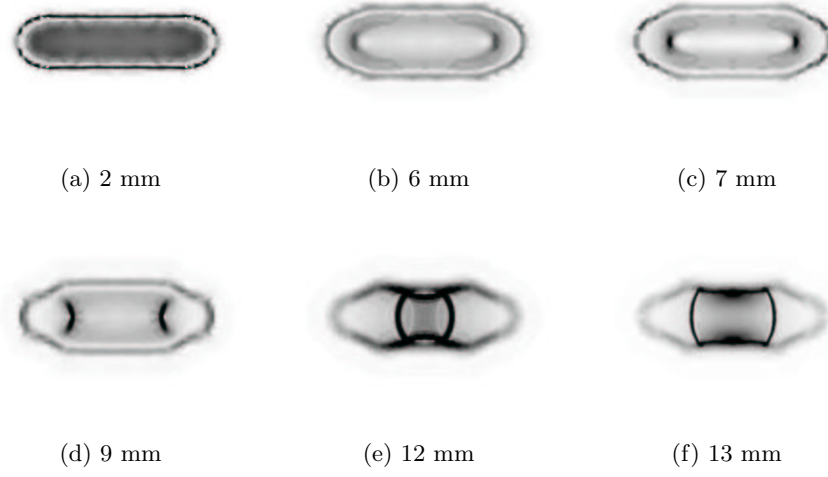


Figure 8.35: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

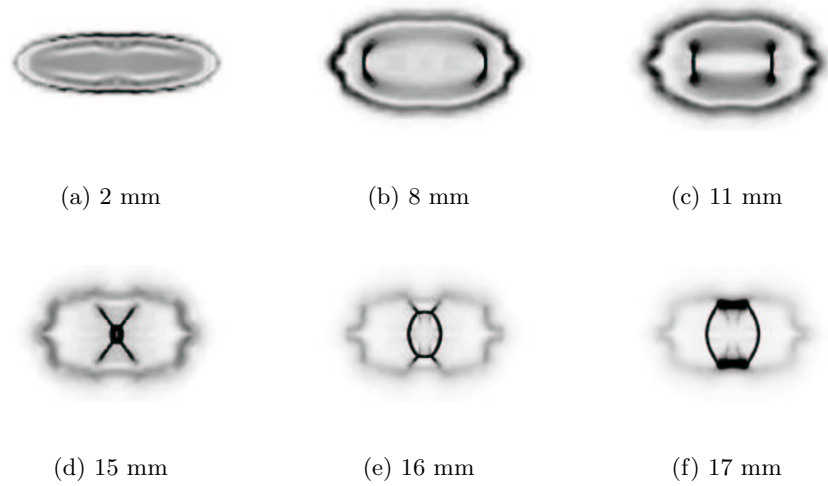


Figure 8.36: Elliptic nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

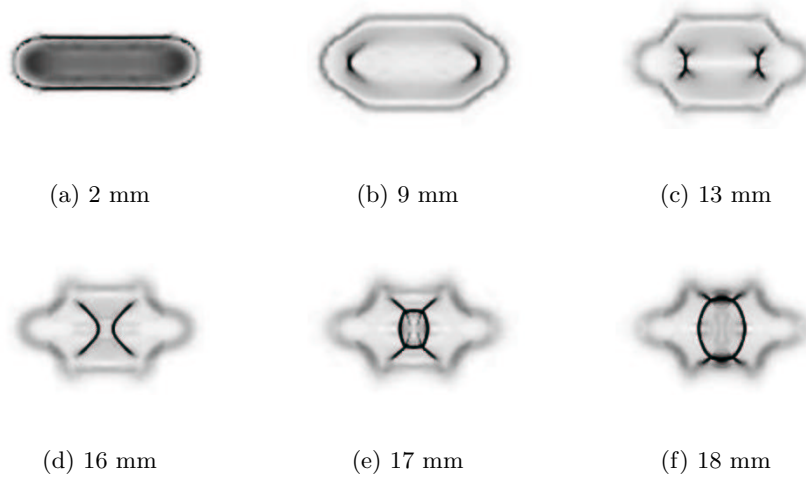


Figure 8.37: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

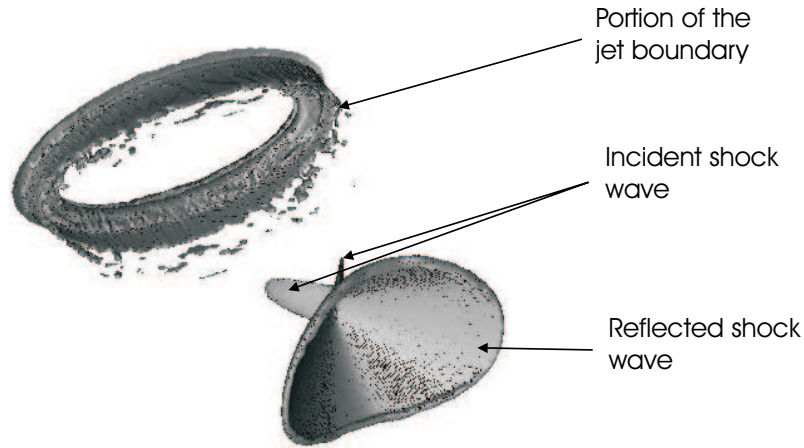


Figure 8.38: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

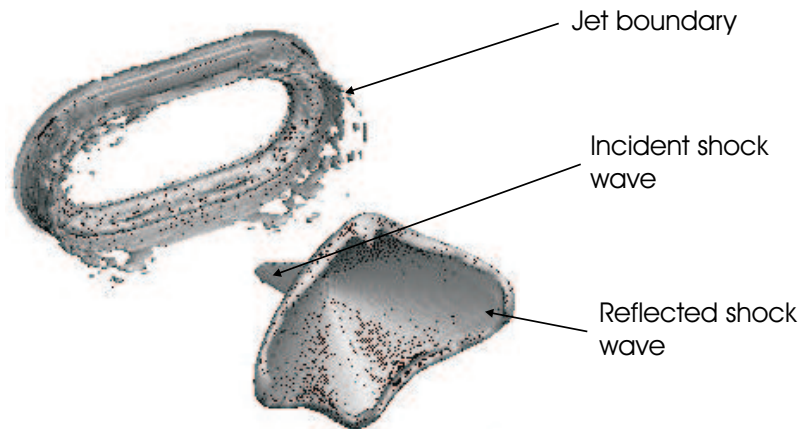


Figure 8.39: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

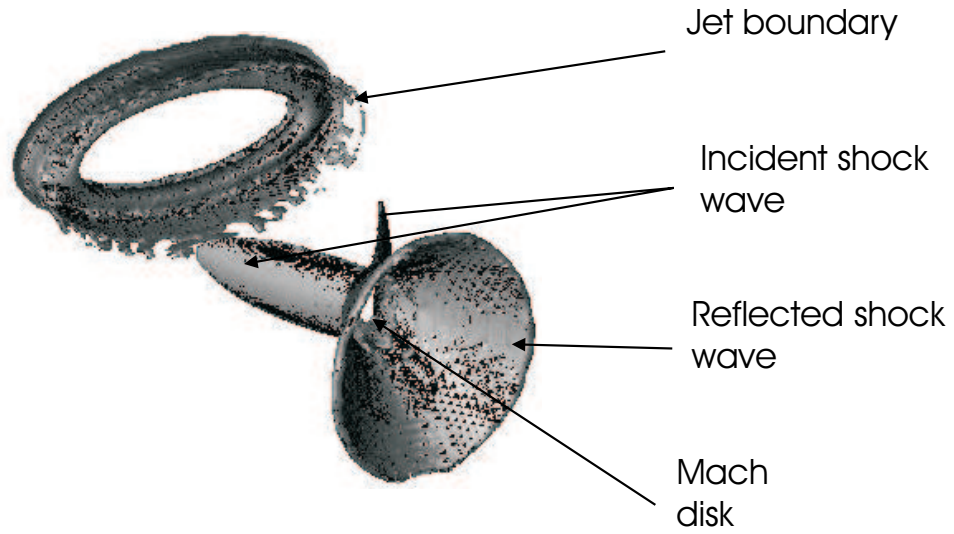


Figure 8.40: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

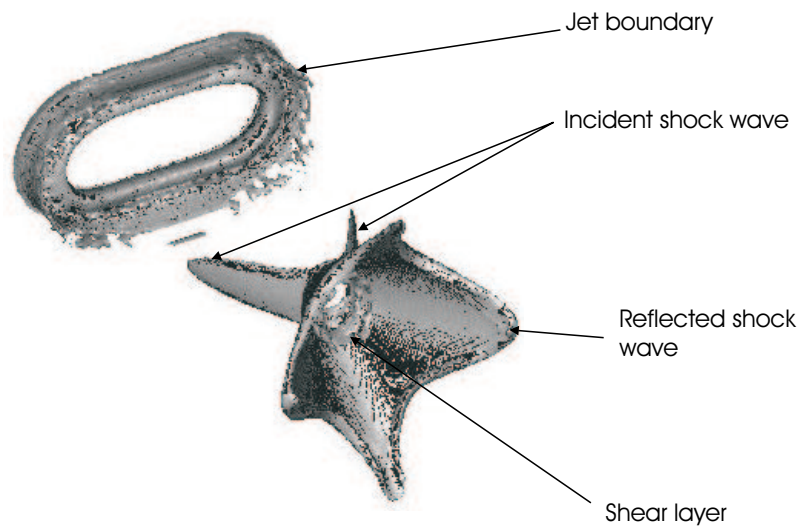


Figure 8.41: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

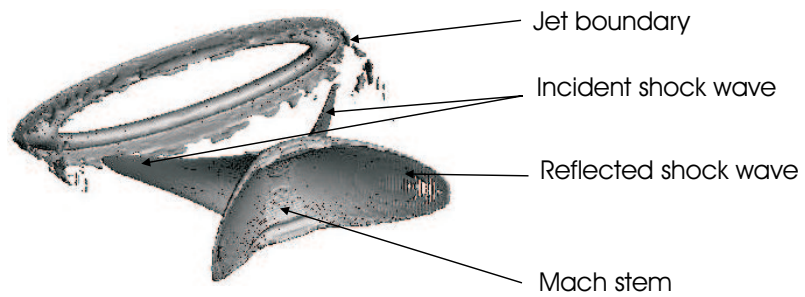


Figure 8.42: 3-dimensional representation of the flow field for the underexpanded jet from an elliptical nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

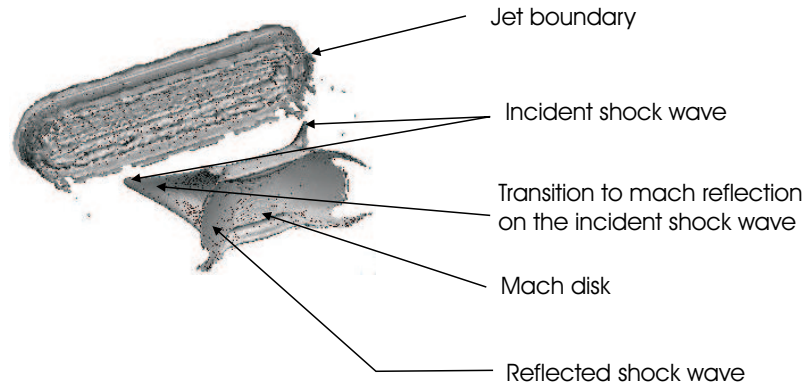


Figure 8.43: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

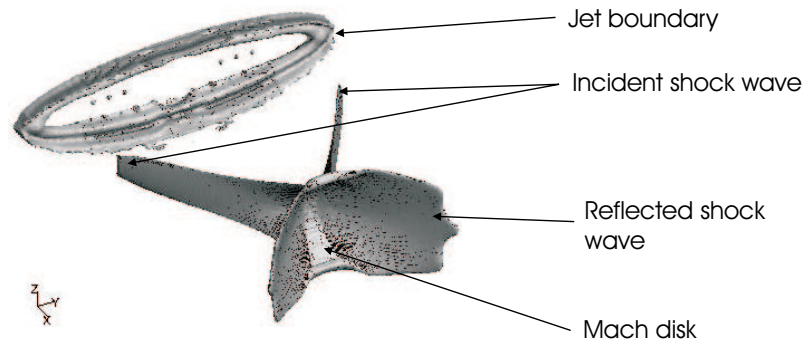


Figure 8.44: 3-dimensional representation of the flow field for the underexpanded jet from an elliptical nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

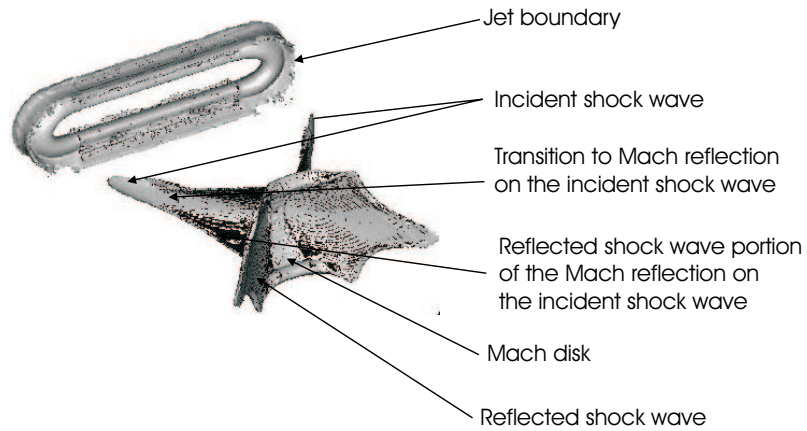


Figure 8.45: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

The reflected shock wave that is formed is also elliptical in cross-section. The shear layers; as noted in the shadowgraph images; diverges as can be seen in its increased cross-sectional size compared to the Mach stem in figure 8.32(f). The incident shock wave, in the case of the slot nozzle, retains its curvature similar to that of the nozzle exit immediately after its formation as seen in figure 8.33(b). At 13mm downstream of the nozzle exit, the incident shock wave undergoes a Mach reflection along its cross-section, breaking up into a three shock system as can be seen in figures 8.33(c) and 8.33(d). The incident shock system (now comprising of the three shock waves) undergoes a Mach reflection along the jet axis forming a Mach stem and reflected shock wave that has an elliptical cross-section. In the higher aspect ratio cases presented in figures 8.34 to 8.37, the transition to Mach reflection along the incident shock wave is observed in both pressure ratios and nozzle exit shapes. In the lower pressure ratio cases the three shock system formed after the incident shock wave has transitioned into a Mach reflection is not easily visible. The waves that are encountered in these flow field are similar to that observed earlier and are not discussed in greater detail as this has been covered in the previous paragraphs.

The three-dimensional representations of the flow fields are displayed in figures 8.38 to 8.45. In these images the flow features that have been described earlier can be seen in relation to each other as a complete representation of the shock surfaces within the flow field.

8.2.1 Incident shock wave formation in elliptical and slot jets

The origin of the incident shock wave on the major axis in the case of elliptical and slot jets is the same as that for axisymmetric jets. The characteristics that originate from the jet boundary coalesce to form the incident shock wave as was conjectured by Courant and Friedrichs [4] (and shown in figure 3.1). As the radius of curvature of the nozzle exit decreases (along the major axis plane) the characteristics merge together to form the shock wave in the same manner in which a barrel shock is formed in an axisymmetric jet. Along the major axis plane the jet boundary expands lesser than along the minor axis plane, hence the characteristics converge first along the major axis plane of the nozzle as has been observed. The incident shock wave propagates downstream and across into the jet flow field. In elliptical jets as the aspect ratio increases the radius of curvature of the minor axis increases hence delaying the onset of the formation of the incident shock wave on it. This also applies to slot jets where the curved ends of the nozzle exit are separated by straight sections.

8.3 Elliptic and slot nozzles with parallel flow inlets

Parallel flow inlets have been investigated by numerical simulation. The flow features in the case of parallel flow inlets are no different to those present in the case of the angled inlet. The spatial locations of the flow features are different due to the angled inlet, and these have been discussed in an earlier section (Chapter 7). The results of the numerical simulations have been appended.

8.4 Conclusion

1. Rectangular jets

- The incident shock wave originates at the corners of the nozzle exit and propagates downstream of the nozzle exit, and is due to the crossing over of the expansion waves from the adjacent edges of the nozzle exit.
- The incident shock wave from the corner reflects with the shock wave from another corner along the major and the minor axes forming the incident shock wave that is seen in shadowgraph images taken of the flow field along the major and the minor axes.
- The reflection of the incident shock wave from two corners can be regular or irregular.
- The cross-sectional shape of the Mach stem in rectangular jets is dependent on the nature of the reflection of the corners shock waves along the major and the minor axes of the flow field. This can result in 4 sided, 6 sided or 8 sided Mach disks as has been observed.

2. Elliptical and slot jets

The formation of the incident shock wave along the major and the minor axis planes of the nozzle are not independent processes. The formation of the shock wave along the minor axis is closely linked to the formation of the incident shock along the major axis plane and the aspect ratio of the nozzle.

The conclusions that can be drawn are:

- The incident shock wave originates downstream of the nozzle exit, and occurs initially on the major axis plane of the flow field.
- The incident shock wave is not present on the minor axis plane of the nozzle for any pressure ratio when the aspect ratio of the nozzle is > 2 . The increasing aspect ratio causes the incident shock wave to reflect along the jet axis before the reflection can occur along the minor axis plane.

- The incident shock wave undergoes a Mach reflection along its length forming a three shock system that converges onto itself downstream of the nozzle exit.

9 Jet boundary shapes in non-axisymmetric underexpanded jets

The jet boundary in underexpanded jets is defined here to be the locus of points at which the Mach number equals 1 within the jet boundary. The reflection of any waves interacting with the jet boundary occurs at the sonic line within the jet boundary. In all of the 3-dimensional representations of the jet boundary the flow direction is from the top left of the image to the bottom right.

9.1 Rectangular jets

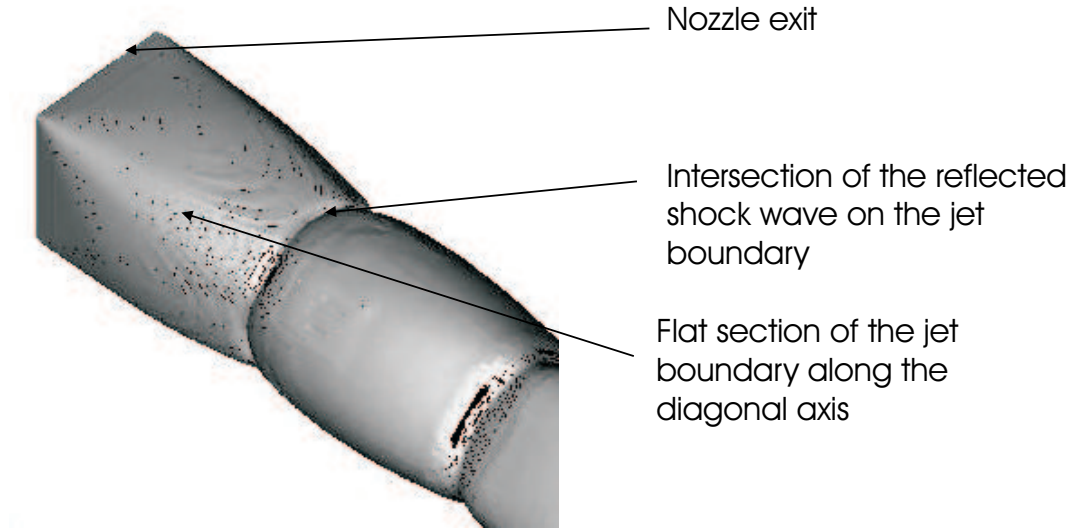


Figure 9.1: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a square nozzle. $AR = 1$, $M = 1$, $P_{exit}/P_{ambient} = 2$

The jet boundary in the square nozzle at the lower pressure ratio has a square cross-sectional shape as it exits the nozzle exit, and this is the same for that of the other nozzles with the increasing aspect ratio at the same pressure ratios (see figures 9.2, 9.4, and 9.6). The jet boundary is not markedly distorted and assumes a shape

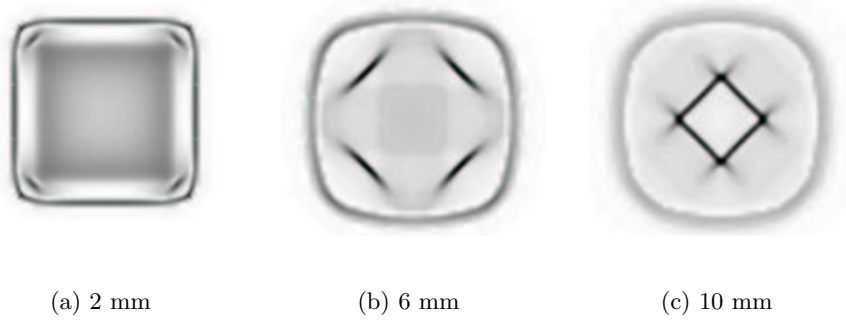


Figure 9.2: Square nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

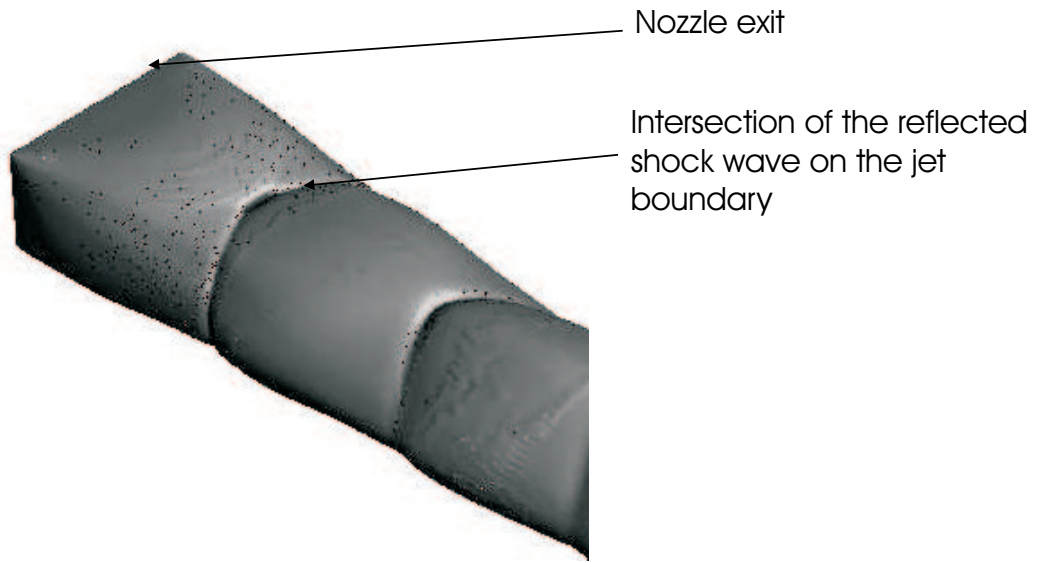


Figure 9.3: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a rectangular nozzle. $AR = 2$, $M = 1$, $P_{exit}/P_{ambient} = 2$

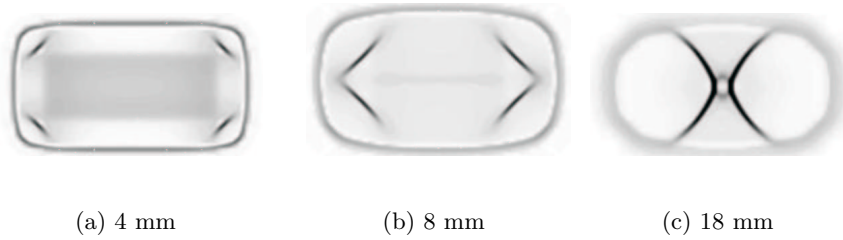


Figure 9.4: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

as one would intuitively expect it to be, based on the shape of the nozzle. In the case of the square nozzle the jet boundary appears to take up a circular cross-section in a short distance downstream of the nozzle exit. This can be seen more clearly in the

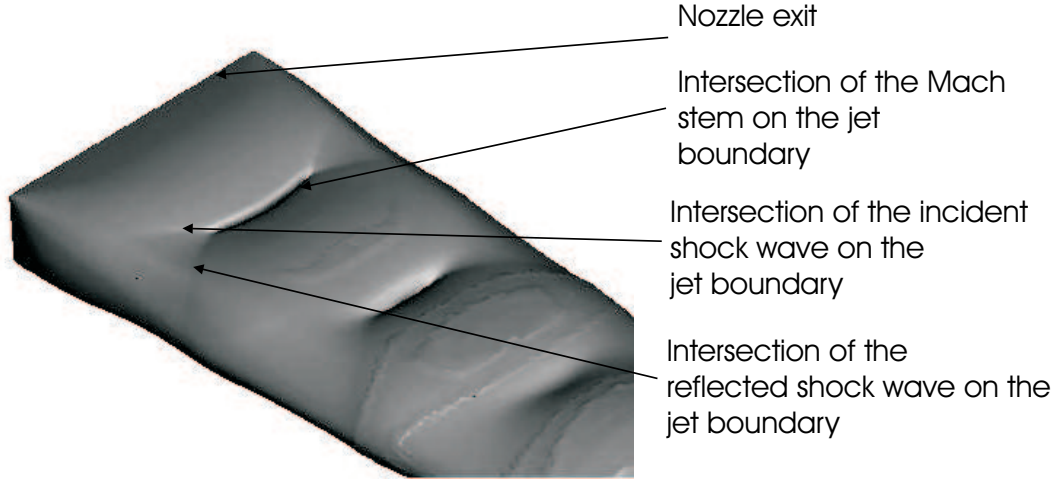


Figure 9.5: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a rectangular nozzle. $AR = 4$, $M = 1$, $P_{exit}/P_{ambient} = 2$

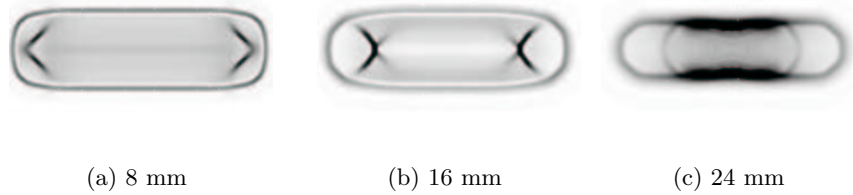


Figure 9.6: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

3-dimensional representation of the jet boundary in figure 9.1 based on the ridge that is formed on its surface. This ridge denotes the point at which the reflected shock wave intersects the jet boundary. From the cross-sectional slices of the flow field in figure 9.2 it can be observed that the only change in the jet structure is the gradual curving of the edges. The appearance of a ridge is also observed in the increasing aspect ratio cases in figure 9.3 and 9.5. Though unlike in the case of the square and aspect ratio 2 nozzles, the reflection line of the reflected shock wave on the jet boundary in the case of the aspect ratio 4 nozzle is not seen to extend around the jet boundary. The shape of the jet boundary in the case of the aspect ratio 2 nozzle also appears to be nearing a circular cross-section downstream of the reflection point of the reflected shock wave. As in the case of the square nozzle at lower pressure ratios the only effect on the jet boundary is the gradual curvature of the edges of the jet boundary toward a circular cross-sectional shape (see figure 9.4). This is expected as there has been evidence of jet flows from non-axisymmetric nozzles appearing to near quasi-axisymmetry downstream of the nozzle exit [2]. Hence as the aspect ratio increases there is a narrowing of the jet boundary downstream of the nozzle

exit along the major axis planes of the nozzle. In the case of the aspect ratio 4 nozzle, the change of the jet cross-section to a circular shape is not seen to occur. The jet boundary appears to take a slot shape as can be seen from figure 9.6(c). The narrow sides of the jet boundary are now circular in cross-section while the longer sides of the jet boundary has remained straight.

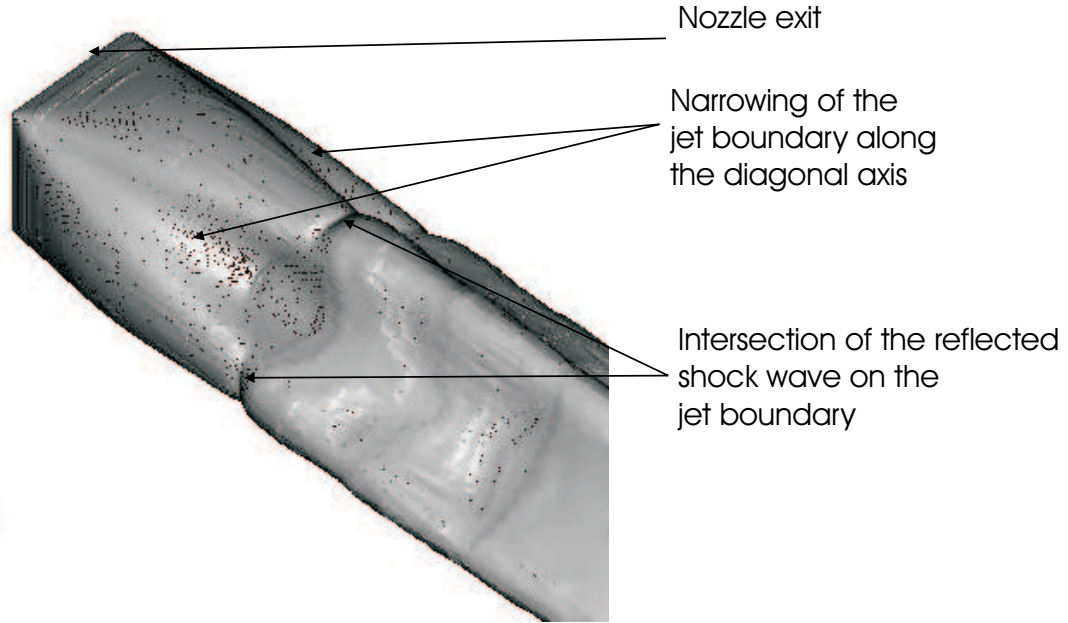


Figure 9.7: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a square nozzle. $AR = 1$, $M = 1$, $P_{exit}/P_{ambient} = 3$

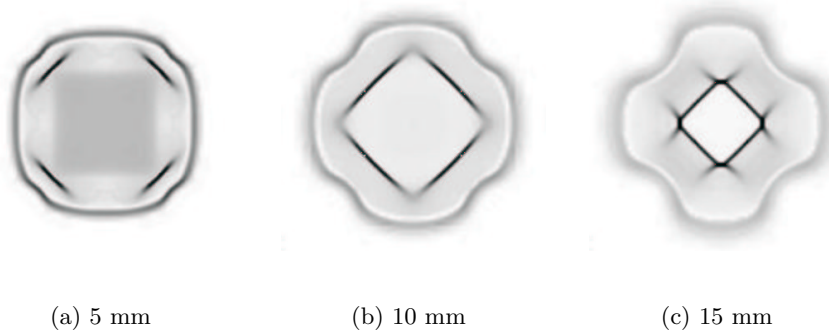


Figure 9.8: Square nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

On increasing the pressure ratio to such and extent that the exit plane pressure of the nozzle is three times that of the ambient conditions, the shapes of the jet boundary are markedly different from what is observed at lower pressure ratios. This is evident from the figures 9.7, 9.9 and 9.11. The defining feature in all three images is the narrowing of the jet boundary that can be seen along the diagonal axes of the jet flow field.

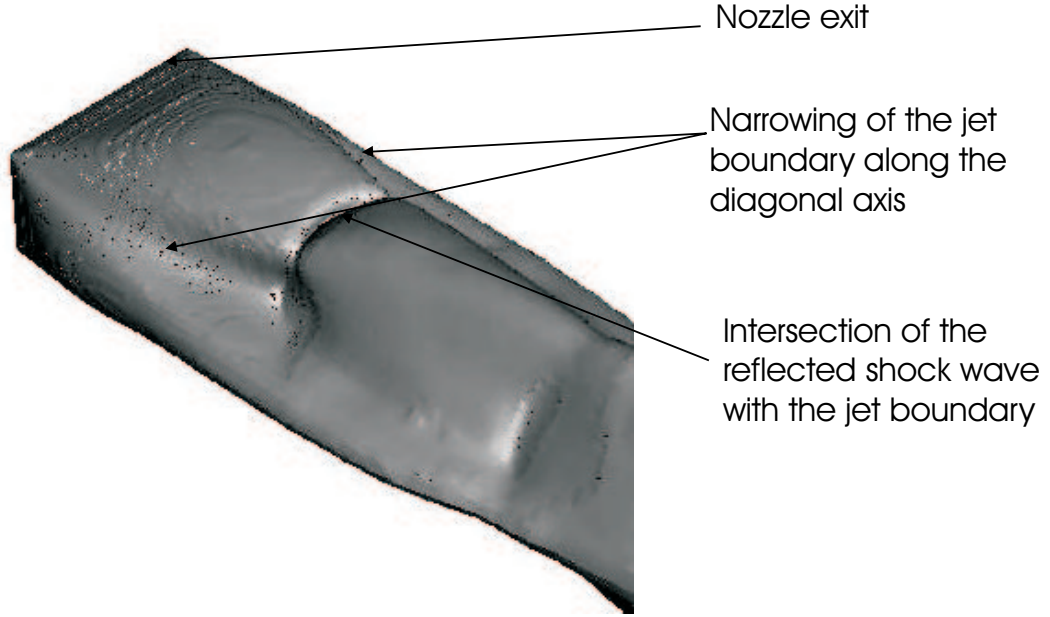


Figure 9.9: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a rectangular nozzle. $AR = 2$, $M = 1$, $P_{exit}/P_{ambient} = 3$

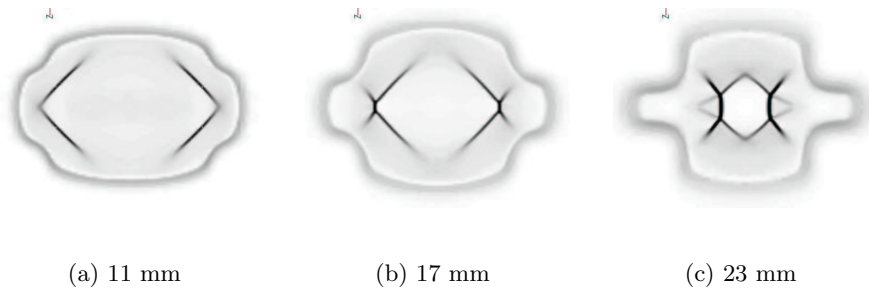


Figure 9.10: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

In the case of the square nozzle (figure 9.7), the jet boundary appears to be expanding along the orthogonal axis planes of the nozzle, whereas along the diagonal plane, the jet boundary is pulled inwards into the jet flow field (see figure 9.8). The intersection of the reflected shock wave on the jet boundary can also be seen in the 3-dimensional representation of the jet boundary in figure 9.7. The cross-sectional shape of the reflected shock wave seen in a previous section is as a result of the shape of the jet boundary. From the cross-sectional slices of the flow field in figure 9.8, the narrowing of the jet boundary along the diagonal plane of the nozzle is clearly seen. The narrowing of the jet boundary occurs from the moment the jet boundary originates at the nozzle exit.

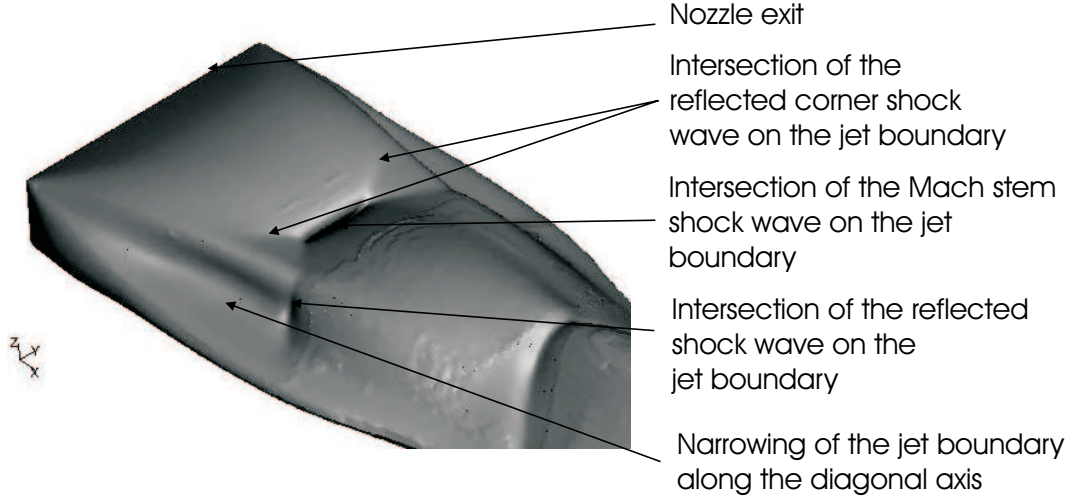


Figure 9.11: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a rectangular nozzle. $AR = 4$, $M = 1$, $P_{exit}/P_{ambient} = 3$

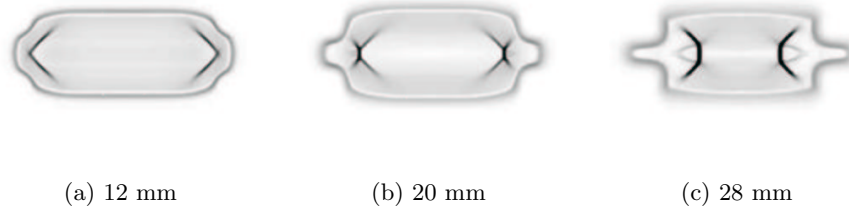


Figure 9.12: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

In the case of the increasing aspect ratio nozzle exit as seen in figure 9.9, the dominant feature is again the narrowing of the jet boundary along the diagonal axis of the nozzle. The shape of the jet boundary is a function of the aspect ratio of the nozzle. Downstream of the nozzle exit the jet boundary that started out having the same width as that of the minor axis of the nozzle is significantly narrower at the minor axis as can be seen in figure 9.10. The jet boundary that originated at the major axis of the nozzle is slightly curved in cross-section downstream of the nozzle exit.

These features are the dominant feature on increasing the aspect ratio of the nozzle exit to 4, as seen in figure 9.11 and figure 9.12.

The narrowing of the jet boundary along the diagonal axis of the jets can be explained by considering the vector plot in figure 9.13. The vectors are constructed such that they only display the in-plane velocities at a distance of 15mm downstream of the nozzle exit. The vectors have also been sized to the same length for clarity.

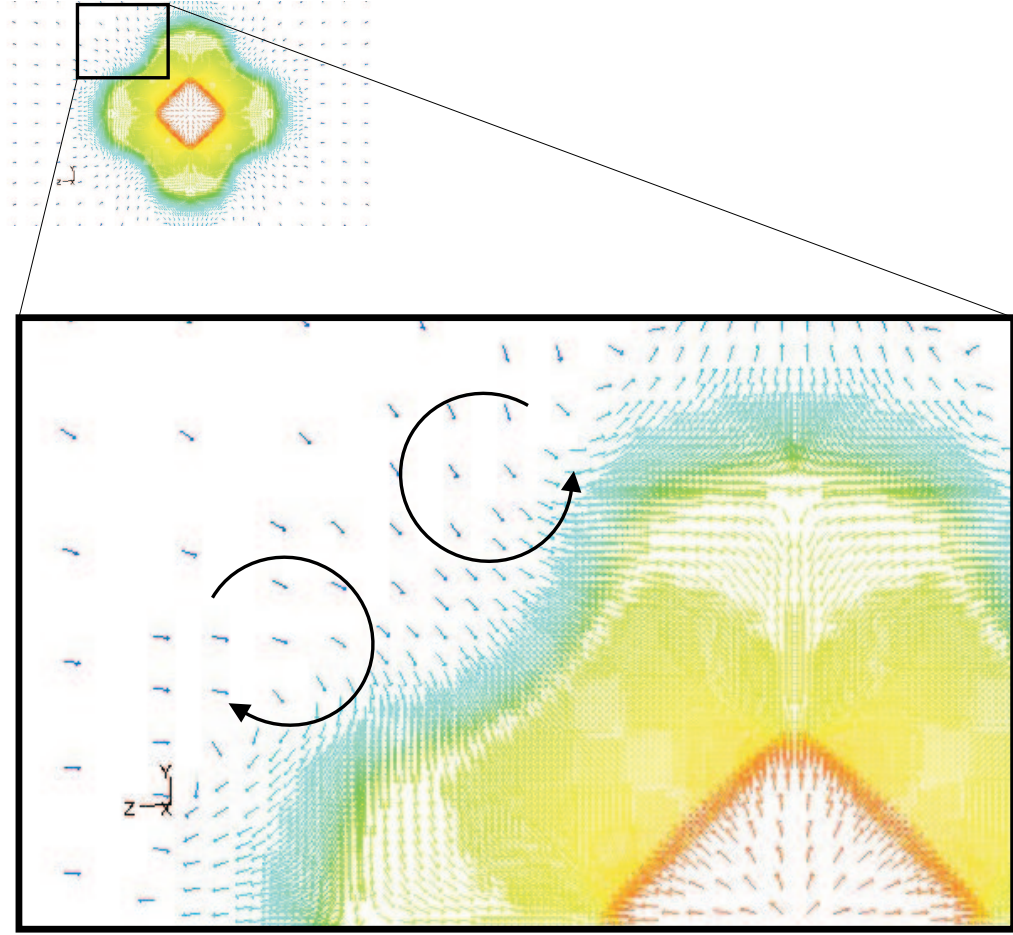


Figure 9.13: In-plane vector plot 15mm downstream of the nozzle exit. $AR = 1$, $M = 1$, $P_{exit}/P_{ambient} = 3$

The colors represent Mach numbers, with red being the highest and blue being the lowest. Within the enlarged portion of the vector plot in figure 9.13, it can be seen that the flow from within the jet (bottom right corner) tends to flow outwards from the centre of the jet. The flow in the ambient medium is flowing toward the jet. Along the flat sides of the nozzle, these flows meet, forming a stagnation point along the jet boundary and the flows are diverted to be in the direction of the jet (out of the page in the case of figure 9.13). Along the diagonal plane of the nozzle exit, the flow from the ambient surroundings and from within the jet meet, and the flow in the ambient medium is diverted to the sides, and not along the direction of the jet. This sets up a recirculating zone that travels downstream along the jet boundary in the shape of a conical spiral flow region increasing in diameter downstream of the corners of the nozzle as shown in the 3-dimensional representation in figure 9.14. These vortices result in the narrowing of the jet boundary along the diagonal plane, and the consequent diverging of the jet boundary along the major and minor axis planes of the rectangular nozzles that have been tested in this study. These vortices

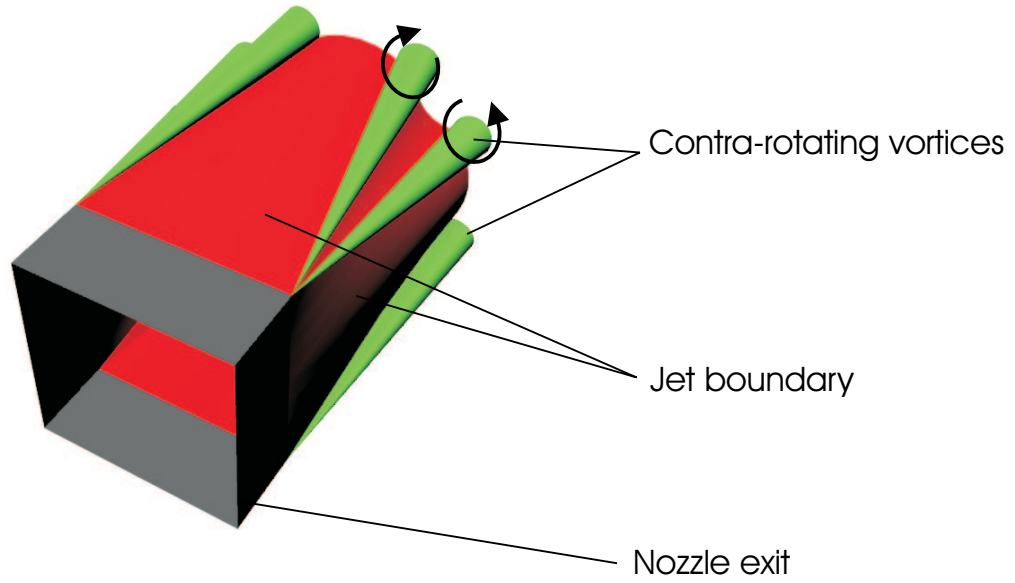


Figure 9.14: 3-dimensional representation of the vortex flows from the corner of the nozzle. $AR = 1$, $M = 1$, $P_{exit}/P_{ambient} = 3$

are generated at each corner of the nozzle exit, for all pressure ratios, but the effect of the vortex on the jet boundary is only visible as the pressure ratio increases.

9.2 Elliptical jets

The jet boundaries in elliptical jets are characterised by the switching of axes. This phenomenon is noted to occur irrespective of the inlet flow conditions of the nozzle, i.e. an angular inlet or a parallel flow inlet.

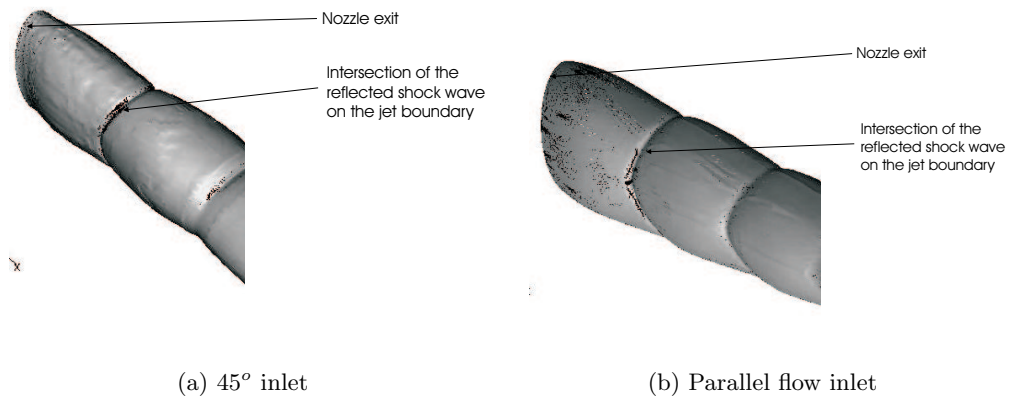


Figure 9.15: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from an elliptical nozzle. $AR = 2$, $M = 1$, $P_{exit}/P_{ambient} = 2$

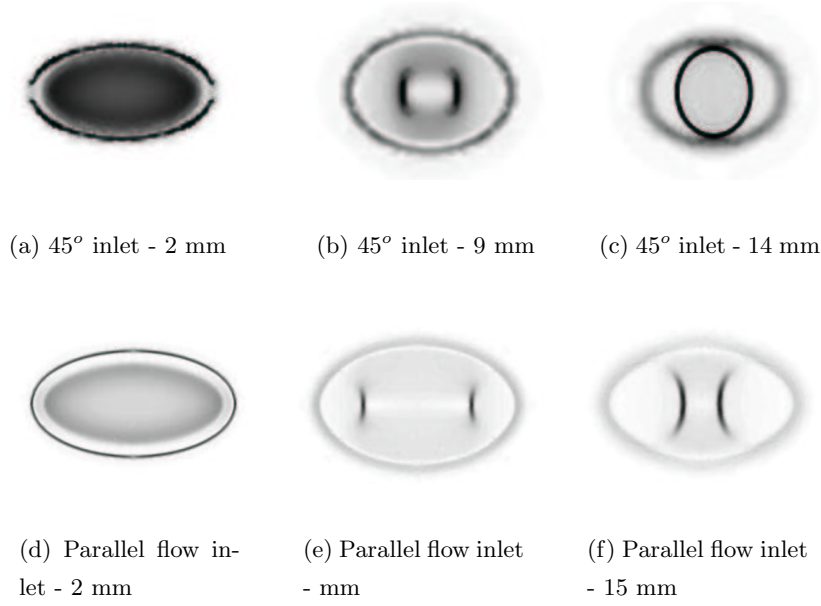


Figure 9.16: Elliptical nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

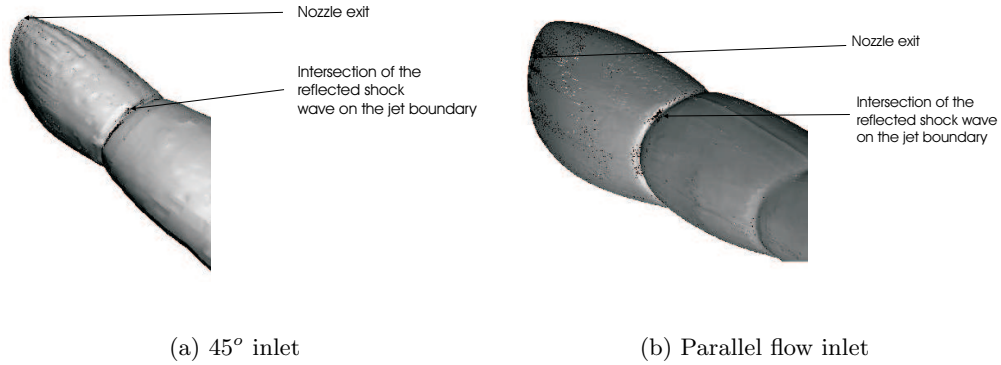


Figure 9.17: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from an elliptical nozzle. $AR = 2$, $M = 1$, $P_{exit}/P_{ambient} = 3$

In jets issuing from nozzle exits with an aspect ratio less than 2, the cross-sectional shape of the jet boundary undergoes an axis switch before the termination of the second shock cell in all cases that have been simulated as part of this study. The jet boundary has a smooth surface with the only discontinuities on it being that when the internal shock waves intersect the jet boundary (see figures 9.15 and 9.17). The jet issuing from the nozzle with the angled inlet undergoes an axis switch considerable earlier downstream of the nozzle exit than in the case of the jet issuing from the parallel flow exit (see figure 9.16). There is also a bump that appears on the major axis of the jet boundary from the parallel flow nozzle as seen in figure 9.16(f). This is not evident in the 3-dimensional representation in figure 9.15. The

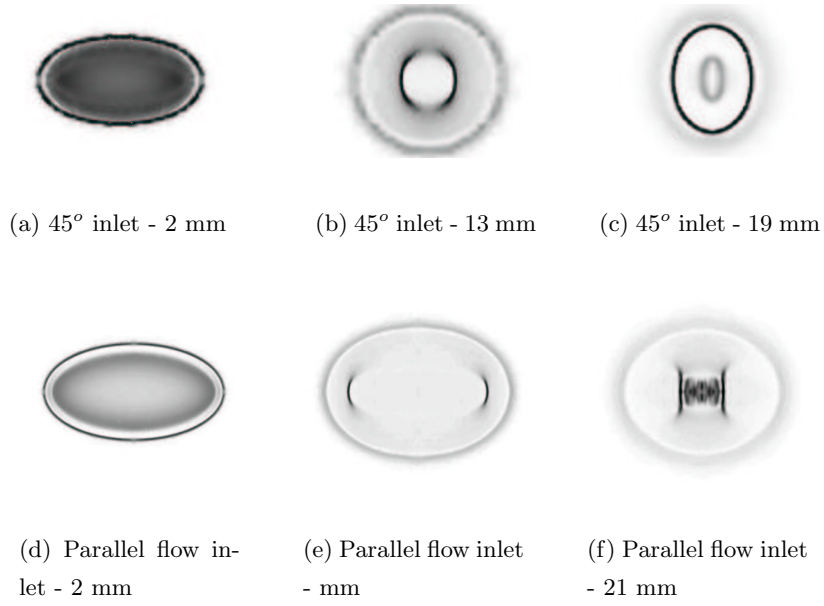


Figure 9.18: Elliptical nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

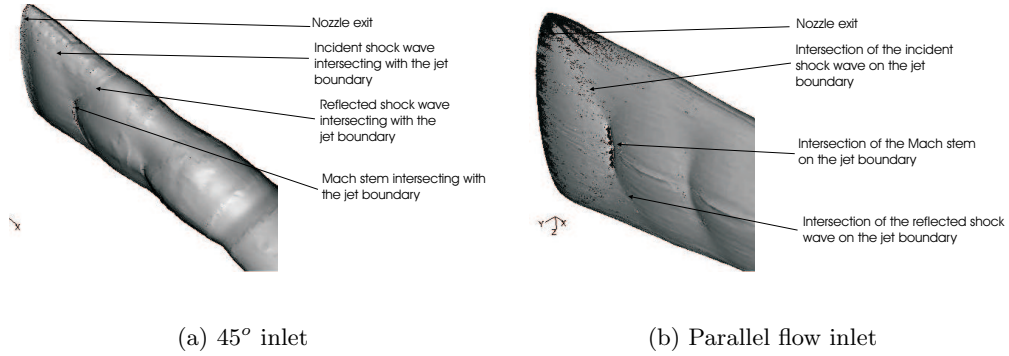


Figure 9.19: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from an elliptical nozzle. $AR = 4$, $M = 1$, $P_{exit}/P_{ambient} = 2$

bump in the jet boundary originates when the incident shock wave terminates at the jet boundary in the case of the parallel flow inlet. This phenomenon is not observed in the case of the flow from the angled inlet. The phenomenon of axis switching is even more pronounced in its rapidity of occurrence when the pressure ratio is increased as can be seen in figure 9.18. The jet boundary has reached a circular cross-section at a distance of 13mm downstream of the nozzle exit (figure 9.18(b)), and has assumed a completely switched axis cross-section by 19 mm downstream of the nozzle exit (figure 9.18(c)). In contrast the jet from the parallel flow inlet is only approaching a circular cross-sectional shape 21 mm downstream of the nozzle exit. The small perturbations on the jet boundary that can be observed close to

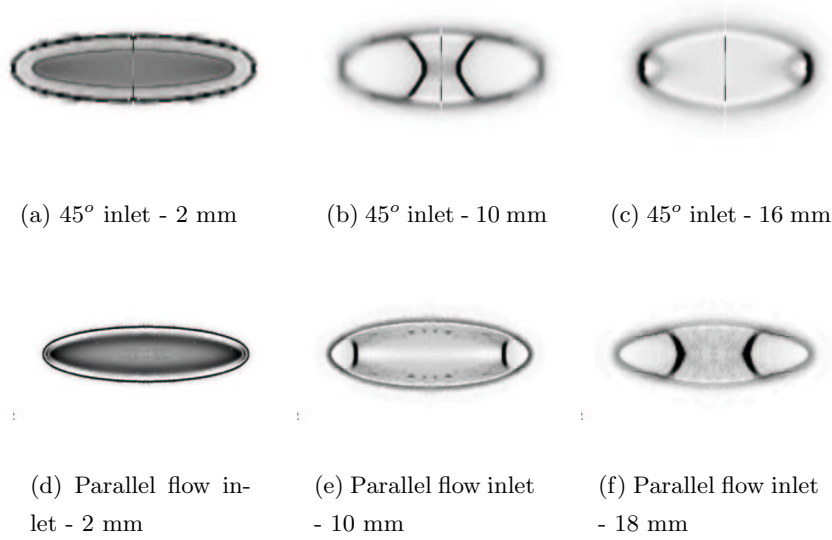


Figure 9.20: Elliptical nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

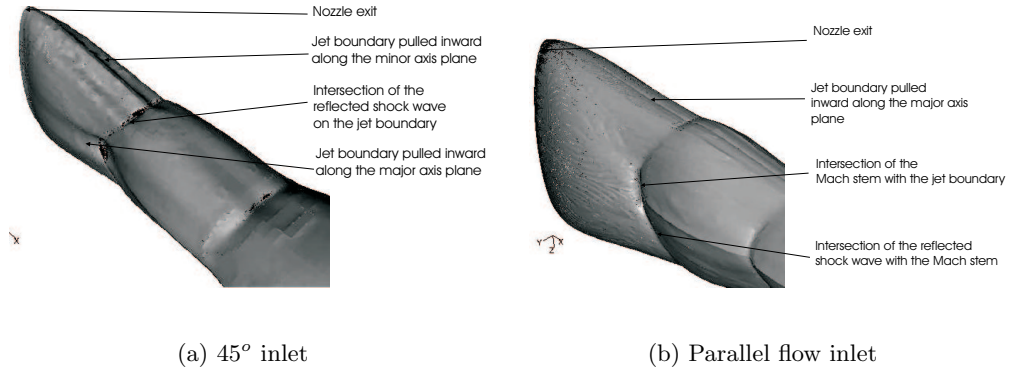


Figure 9.21: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from an elliptical nozzle. $AR = 4$, $M = 1$, $P_{exit}/P_{ambient} = 3$

the nozzle exit in figures 9.17(a), and 9.17(b) are an artifact of the resolution of the numerical simulation.

Increasing the aspect ratio does not result in a vastly different shape of the jet boundary as can be observed in figure 9.19. The switching of the axis is similar to that in the case of the aspect ratio 2 nozzle at the same pressure ratio. The axis switching however does not occur as close to the nozzle exit as in the case of the lower aspect ratio nozzle. The switching of the axis occurs only 3 - 3.5 shock cells downstream of the nozzle exit. The cross-sectional shape of the jet boundary does not seem to be greatly influenced by the pressure ratio as seen in figure 9.20. The elliptical cross-section transits smoothly to a circular cross-section. The presence of

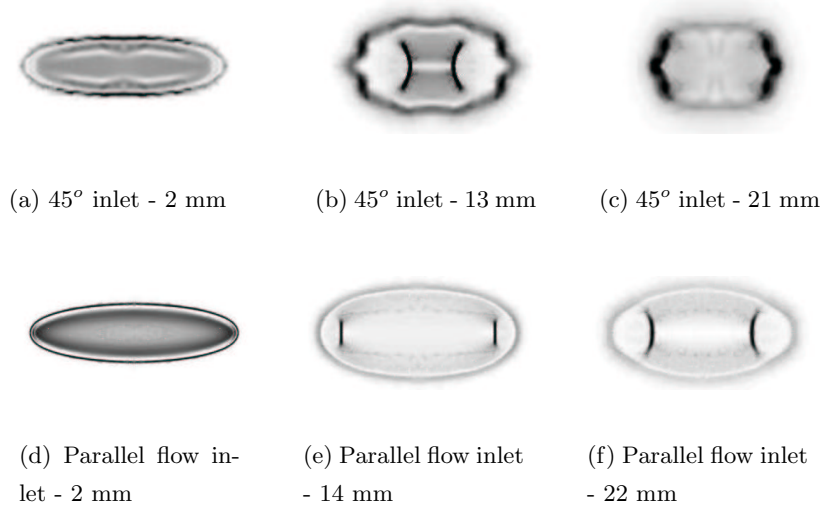


Figure 9.22: Elliptical nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

a bump on the major axis plane of the jet from the parallel flow nozzle is noted here as well (see figure 9.20(f)).

At the higher pressure ratio of 3, the larger aspect ratio nozzle does begin to exhibit differences in the jet boundary shape as observed in figure 9.21. In the case of the angled inlet nozzle 9.21(a), the jet boundary appears to be pulled inward into the jet flow field quite sharply along the major axis plane of the nozzle, and along the minor axis plane of the nozzle close to the intersection of the Mach stem with the jet boundary. This can be seen clearly in the cross-sectional slices of the flow field in figures 9.22(a) to figure 9.22(c). In the case of the parallel flow inlet nozzle, the jet boundary appears to be pulled inwards into the jet flow field much as in the case of the angled inlet nozzle, though not to the same extent. The deformation of the jet boundary is significantly more in the case of the angled inlet nozzle than in the case of the parallel flow nozzle. The switching of the axis is almost complete at 21 mm downstream of the nozzle exit in the case of the jet from the angled inlet nozzle, unlike in the case of the jet from the parallel flow nozzle.

9.3 Slot jets

The jet boundaries of slot jets are also influenced by the shape of the nozzle inlet. Unlike in the case of the elliptical nozzle where the influence was on the point at which the jet axis switched (for lower pressure ratios), in the slot jet cases there is a marked effect on the shape of the jet boundary (see figure 9.23 and figure 9.24).

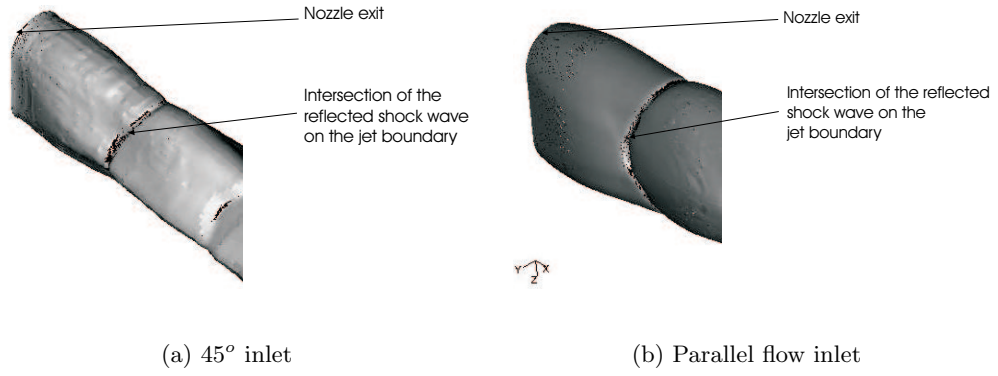


Figure 9.23: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a slot nozzle. $AR = 2$, $M = 1$, $P_{exit}/P_{ambient} = 2$

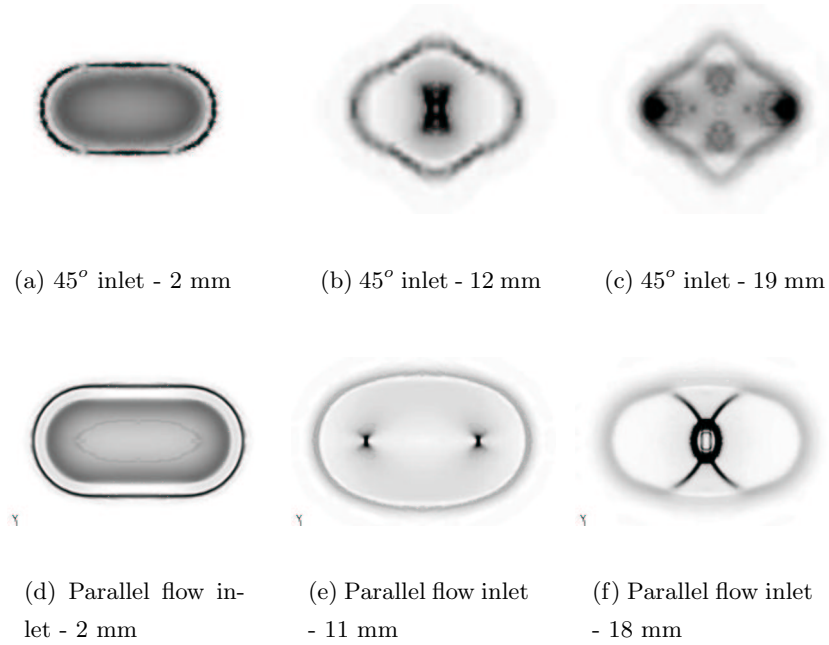


Figure 9.24: Slot nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

For jets issuing from slot nozzles from a parallel flow inlet nozzle, the formation of the bump on the major axis plane, much as in the case of the elliptical nozzle described earlier, is also identified here in figure 9.24(f). There does not seem to be a switching of axis of the jet boundary as was observed in the case of the elliptical nozzle regardless of the angle of the inlet of the nozzle. In the case of the angled inlet, the jet boundary contorts significantly as can be seen in figure 9.24(b). The jet cross-section also appears to become symmetrical along the major and the minor axes as seen in figure 9.24(c). This is also a case of axis switching where the original cross-sectional shape is also maintained. The major axis plane becomes the minor

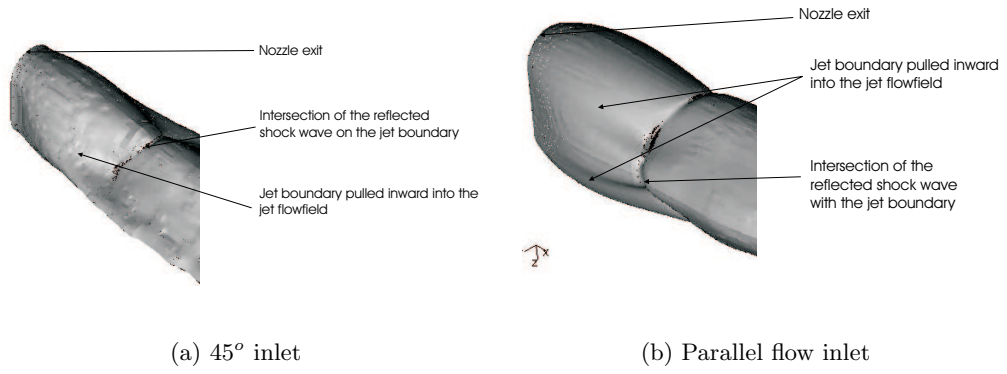


Figure 9.25: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a slot nozzle. $AR = 2$, $M = 1$, $P_{exit}/P_{ambient} = 3$

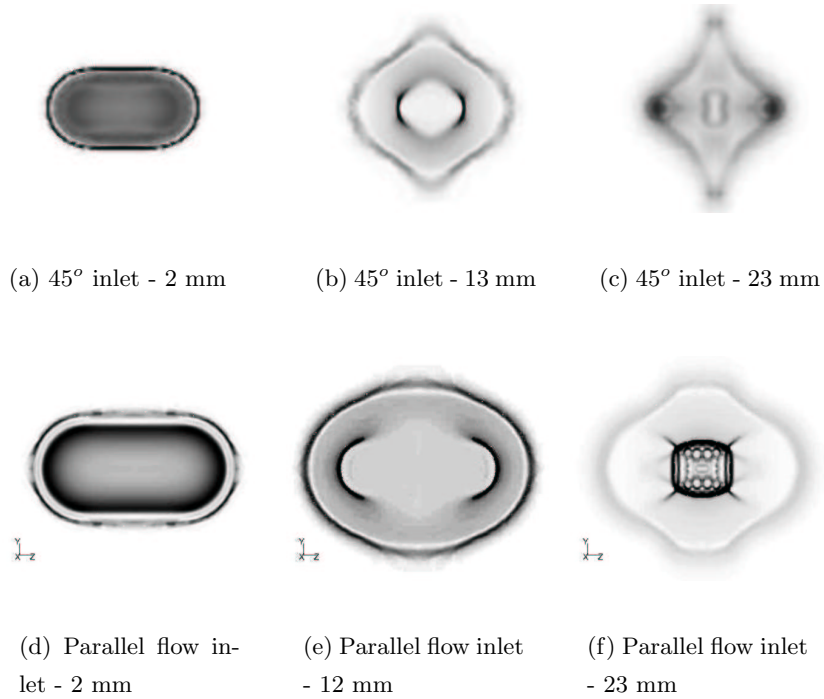


Figure 9.26: Slot nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

axis plane of the cross-section (in this case both axes appear to be the same length).

On increasing the pressure ratio of the jet such that the exit plane pressure of the nozzle is three times that of the ambient pressure, the shape of the jet boundary is more distorted in the case of the angled inlet nozzle compared to the parallel flow inlet nozzle (see figure 9.25). The jet from the angled inlet nozzle, switches its axis less than 15 mm downstream of the nozzle exit (see figure 9.26(b)). The cross-sectional shape of the jet no longer resembles that of a slot, and is even more

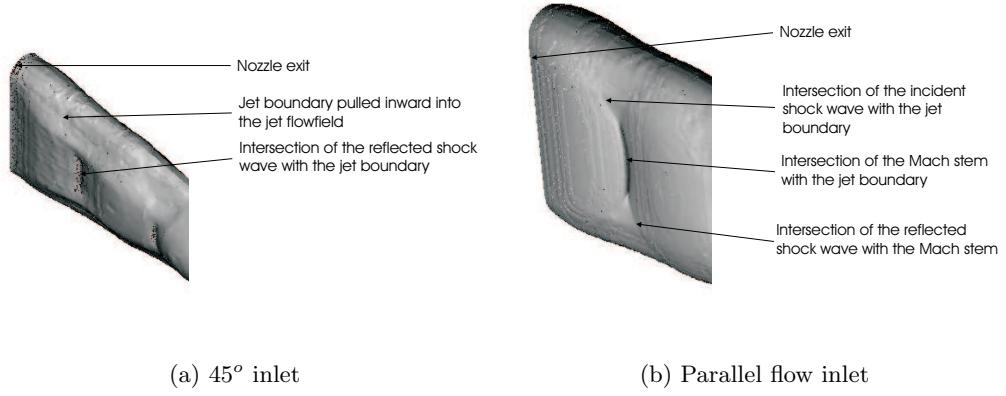


Figure 9.27: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a slot nozzle. $AR = 4$, $M = 1$, $P_{exit}/P_{ambient} = 2$

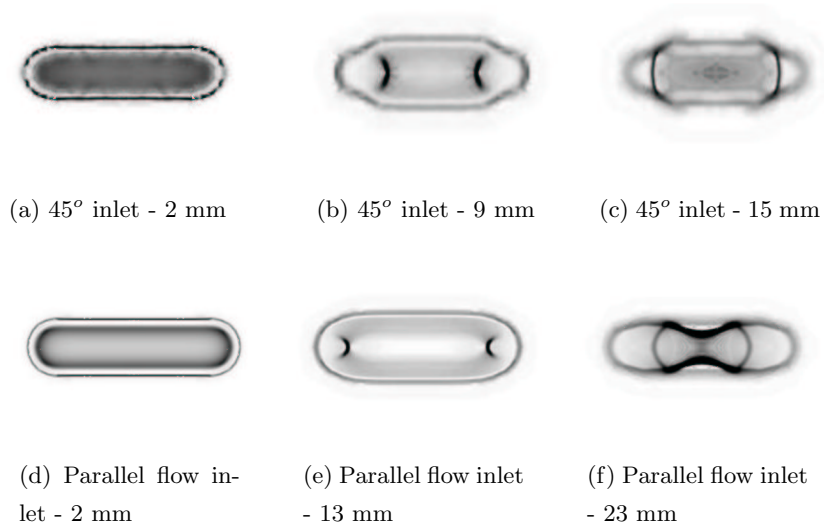


Figure 9.28: Slot nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

greatly elongated in the minor axis plane of the nozzle as can be seen in figure 9.28. The jet from the parallel flow nozzle, also undergoes as axis switch (figure 9.28(f)) though considerably further downstream than in the case of the jet from the angled inlet nozzle.

The slot jets from larger aspect ratio nozzles as seen in figures 9.27 and 9.29 are very different to those from the smaller aspect ratio nozzles. There does not seem to be any evidence of axis switching occurring. In the case of the lower pressure ratio the shape of the jet boundary from the parallel flow nozzle remains largely unaffected downstream of the nozzle exit. The jet boundary from the angled inlet begins to narrow along a diagonal axis of the jet cross-section as seen in figure 9.28(b). This

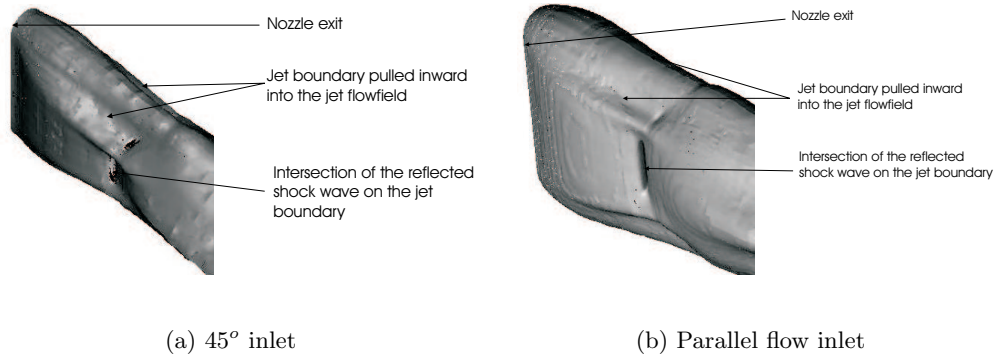


Figure 9.29: Iso-surface of the sonic line within the jet boundary of an underexpanded sonic jet from a slot nozzle. $AR = 4$, $M = 1$, $P_{exit}/P_{ambient} = 3$

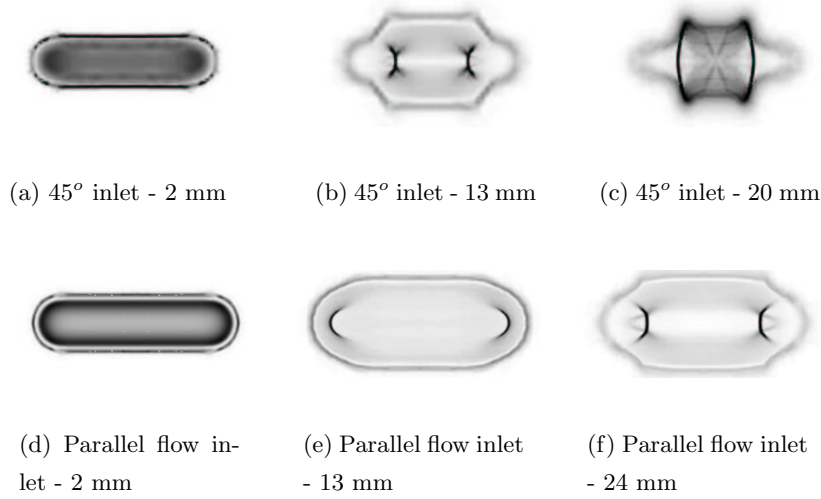


Figure 9.30: Slot nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

distortion of the jet boundary is not present in the case of the jet from the parallel flow inlet. At higher pressure ratios, the jet boundary cross-section from the angled inlet begins to resemble that of the rectangular jet from the same pressure and aspect ratio see figure 9.12. There is the same narrowing of the jet boundary along the diagonal plane that can be noted for the jet from the parallel flow exit nozzle (see figure 9.30(c)). In both cases the portion of the jet boundary issuing from the straight section along the major axis remains straight downstream of the nozzle exit. The narrowing of the jet boundary along the diagonal plane is the defining characteristic of the increase aspect ratio jet at higher pressure ratio regardless of the inlet flow conditions of the nozzle. The effect of the inlet flow is that it alters the spatial location of the phenomenon along the length of the jet, while the features observed are similar.

9.4 The *vena-contracta* effect

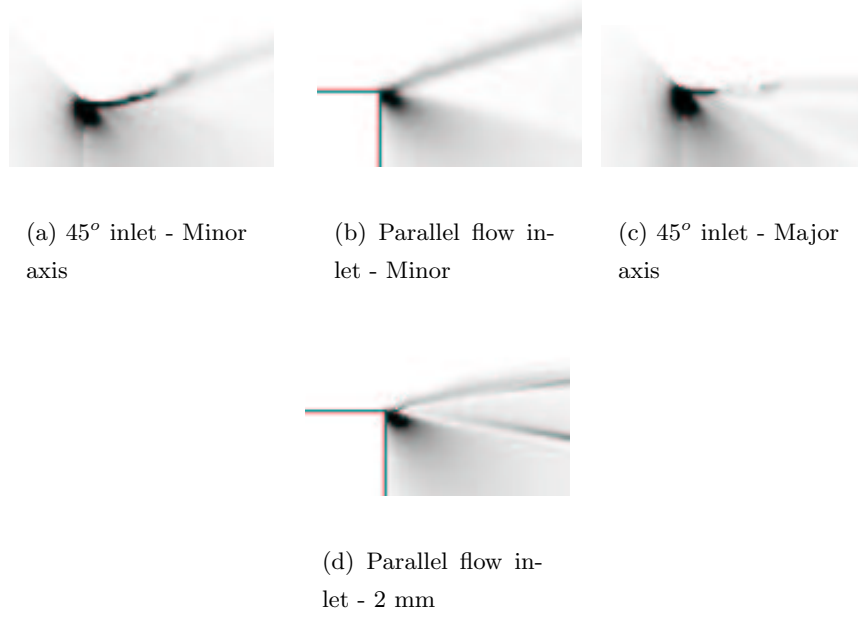


Figure 9.31: Close up of the jet boundaries at the nozzle exit of an elliptical nozzle, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

The effect of the nozzle inlet while not being significant to induce drastic changes in the flow field of elliptical and slot jets does have a significant effect on the jet boundary. In the case of parallel flow inlets the pressure differential would cause the jet to diverge the moment it exits the jet boundary. This is not the case in the flow field from a nozzle with an angled inlet. The jet boundary initially converges as the jet exits the nozzle exit. The jet boundary reaches a minimum width a short distance downstream of the nozzle exit, and then diverges. The vena-contracta effect causes the effective diameter of the nozzle to be smaller than the actual nozzle diameter, and also causes the aspect ratio to be different to the actual aspect ratio of the nozzle. The angle of the streamlines in the flow as a result of the angled inlet is responsible for the vena-contracta effect. The effect of the angled inlet is shown in the sequence of images in 9.31. It is interesting to note that the greater effect of the contraction of the jet boundary is observable along the minor axis and not along the major axis plane of the nozzle.

9.5 Conclusions

The following conclusions can be drawn with regards to the shapes of the jet boundaries in underexpanded jets from non-axisymmetric nozzles

1. Rectangular jet

- At lower pressure ratios in rectangular nozzles with aspect ratio < 3 , the shapes of the jet boundaries are as one would expect when considering a transition from a rectangular nozzle exit to a state of quasi-axisymmetry.
- Increasing aspect ratios in rectangular jets results in a jet boundary that leads to a slot like cross-sectional shape downstream of the nozzle exit.
- At higher pressure ratios in rectangular nozzles, the jet boundary narrows along the diagonal axis.
- At aspect ratios > 3 in rectangular nozzles, the cross-sectional shape of the jet boundary is more affected along the minor axis dimension than along the major axis dimension.

2. Elliptical jets

- In elliptical nozzles, the angled inlet nozzle results in the jet undergoing axis switching earlier than in the case of that of jets from parallel flow nozzles. This is true with increasing aspect ratio as well.
- This phenomenon is exacerbated on increasing the pressure ratio. Hence the mixing of the jet would be significantly greater in the case of the jet from an angled inlet.

3. Slot jets

- The effect of the angled inlet is more pronounced in slot nozzles. The jet boundary is very greatly distorted in the cases of jets from angled inlets.
- The increased distortion of the jet from angled inlets suggests enhanced mixing of the internal flow.

10 Conclusions

Underexpanded jets issuing from rectangular, elliptic and slot nozzles of varying aspect ratios under two different pressure ratios have been experimentally observed and numerically predicted. The following conclusions can be drawn from the work that has been performed:

1. The formation and development of the incident shock wave system in under-expanded jets from non-axisymmetric nozzles has been reported.
2. The development of Mach reflections on the incident shock wave in underexpanded jets from non-axisymmetric nozzles (not observed till now) have been identified and explained.
3. Complex Mach reflections along the axis of the jet flow with varying Mach stem shapes have been observed and reported on.
4. Axis switching phenomenon in slot and elliptical jets resulting in complex jet boundary shapes have been demonstrated.
5. 3-dimensional representations of the flow fields have been developed to aid in understanding the spatial location of all the flow features within the jet flow fields

11 Recommendations for further investigation

The following areas have been identified by the author to be pursued for further explorations through experimental and numerical methods:

1. Extend the work that has been presented in this thesis to supersonic nozzles. The extension of the work to cover supersonic nozzles will allow for a greater understanding of the shock interaction phenomena at higher Mach numbers while maintaining the pressure ratios to that investigated in this thesis.
2. Investigate the flow field presented in figure 9.14 through experimental methods such as PIV (Particle Imaging Velocimetry) or LVS (Laser Vapour Screen) visualisation techniques to isolate the vortices that have been conjectured through an analysis of the numerical simulation.

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APPENDIX A Technical specifications of apparatus used in the experimentation

Table A.1: Technical specifications of the reciprocating compressor

Manufacturer	Reavell and Co
Type	CSA 4
Power	18.7kW
Maximum operating delivery pressure	1.5MPa
Maximum design delivery pressure	3.1MPa
Normal operating speed	960rpm
Maximum operating speed	1000rpm

Table A.2: Technical specifications of the dryer unit

Manufacturer	Tegnon
Type	RB model
Receiver manufacturer	Servex
Receiver volume	0.06 m ³
Operating pressure	2.14MPa
Maximum test pressure	3.21MPa
Regeneration time	12Hrs

Table A.3: Technical specifications of the storage vessel

Storage tank manufacturer	Kies and Travers Ltd.
Storage tank operating pressure	1.5MPa
Storage tank design operating pressure	2.07MPa
Storage tank volume	9.04 m ³

Table A.4: Details of the parabolic mirrors

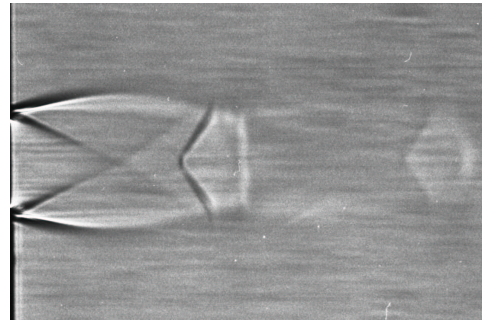
Mirror diameter	144 mm
Focal length	900 mm

APPENDIX B Flowfields in underexpanded jets issuing from rectangular nozzles

B.1 Nozzle Aspect Ratio = 1.5



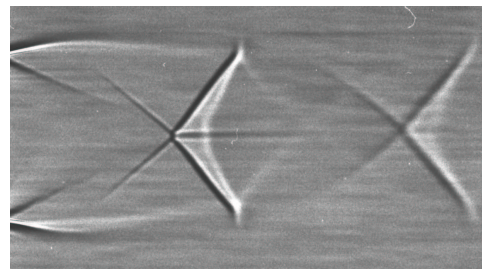
(a) Numerical Schlieren



(b) Experimental shadowgraph



(c) Numerical Schlieren



(d) Experimental shadowgraph

Figure B.1: AR = 1.5 nozzle, images along the minor and major axes of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

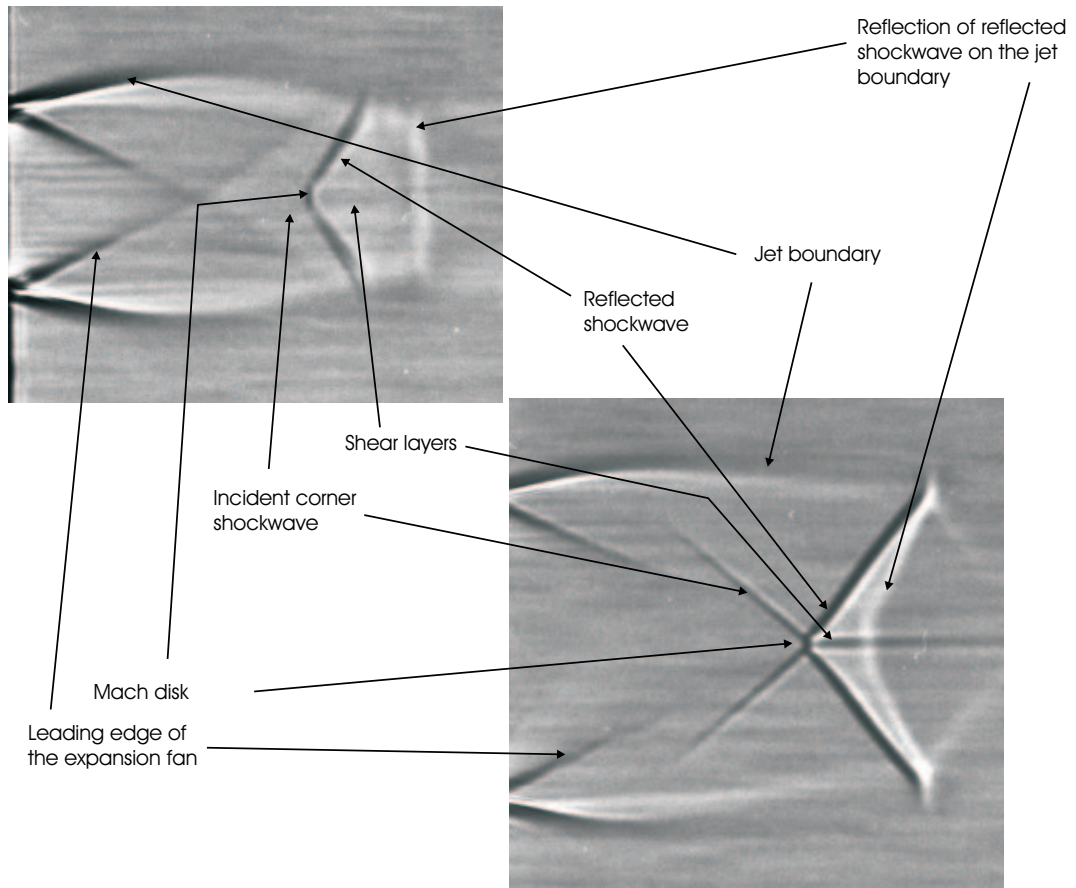


Figure B.2: Explanation of the flow field in a free jet issuing from a rectangular nozzle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$. Upper left = Minor axis plane, Bottom right = Major axis plane

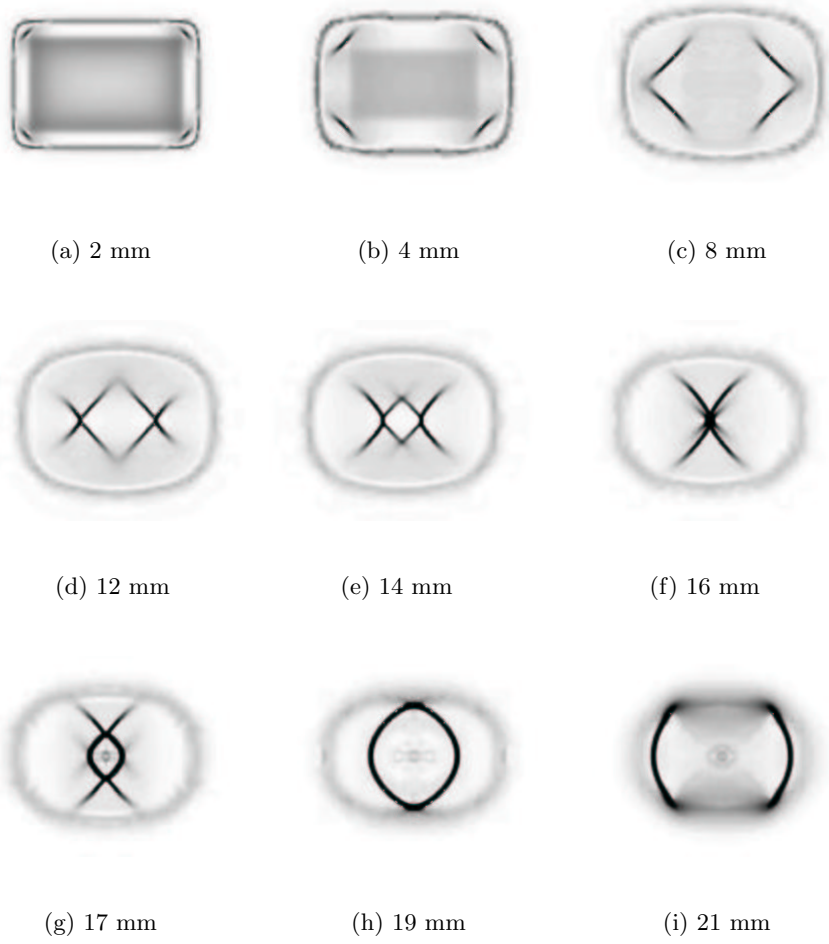


Figure B.3: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

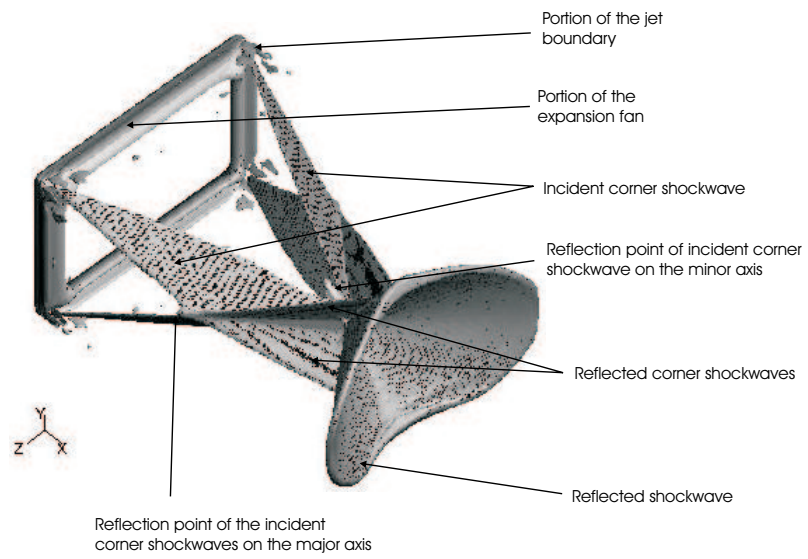
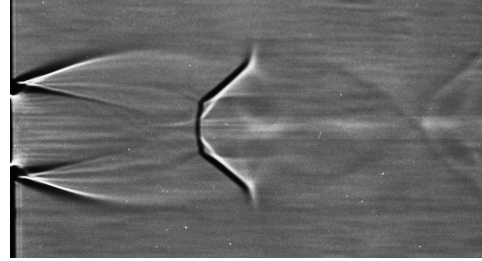


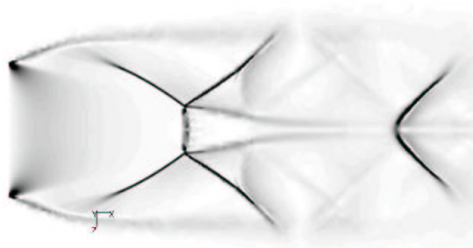
Figure B.4: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



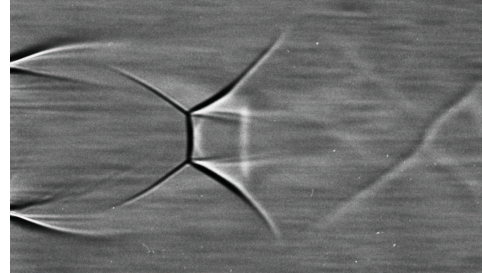
(a) Numerical Schlieren



(b) Experimental shadowgraph



(c) Numerical Schlieren



(d) Experimental shadowgraph

Figure B.5: $AR = 1.5$ nozzle, images along the minor and major axes of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

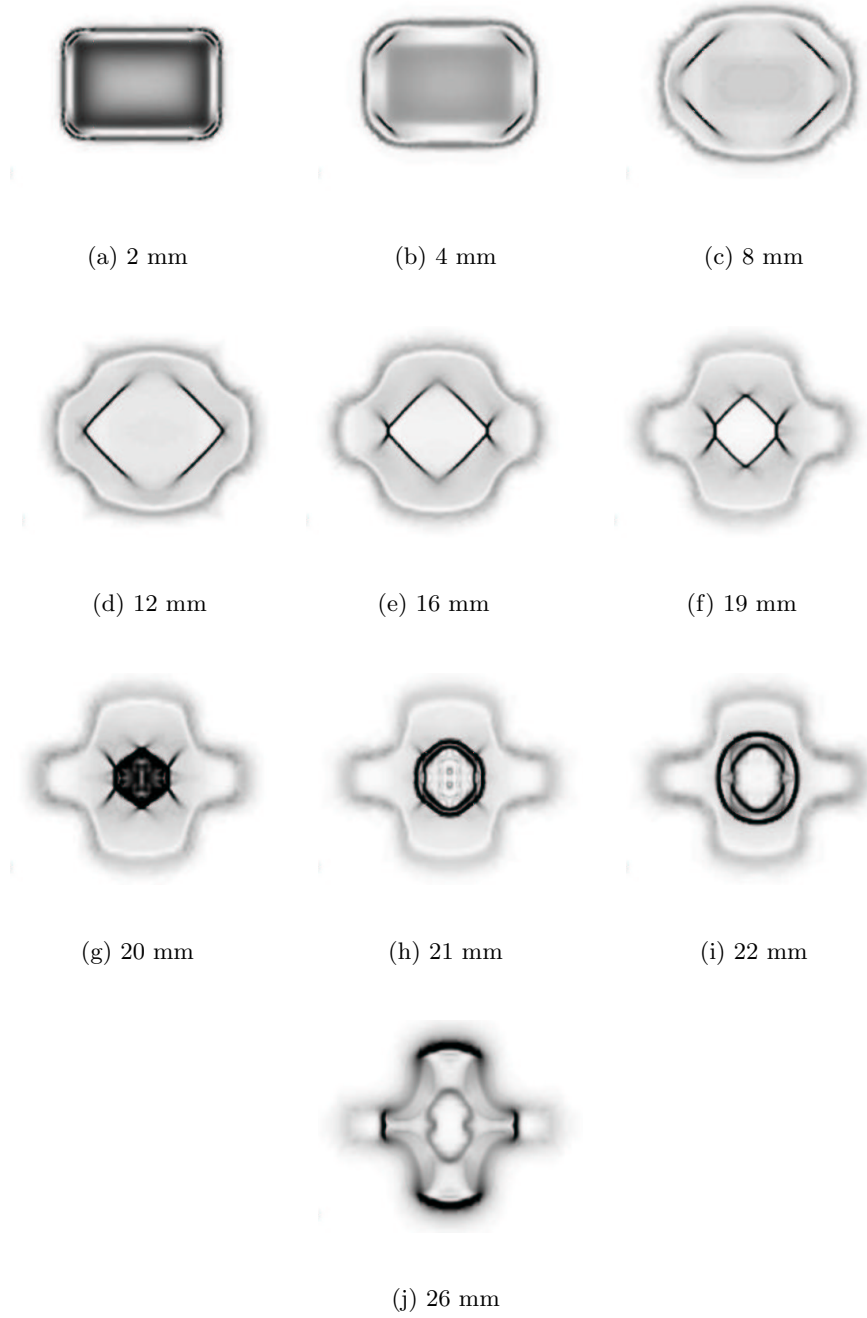


Figure B.6: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

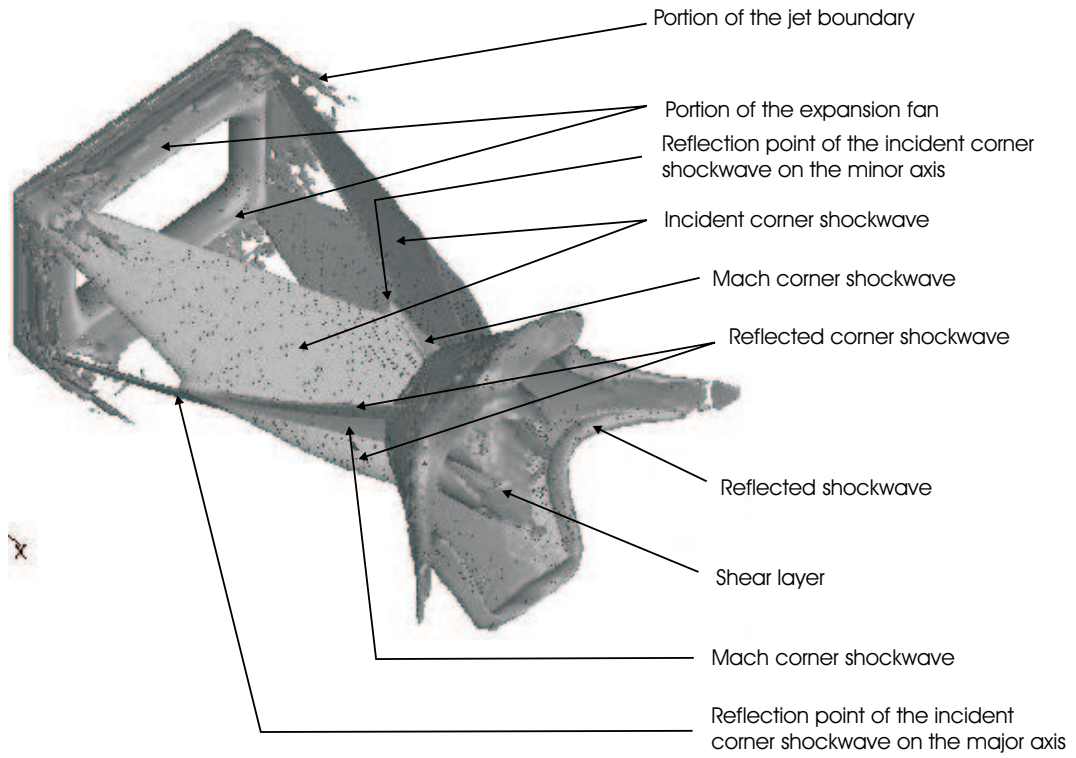


Figure B.7: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

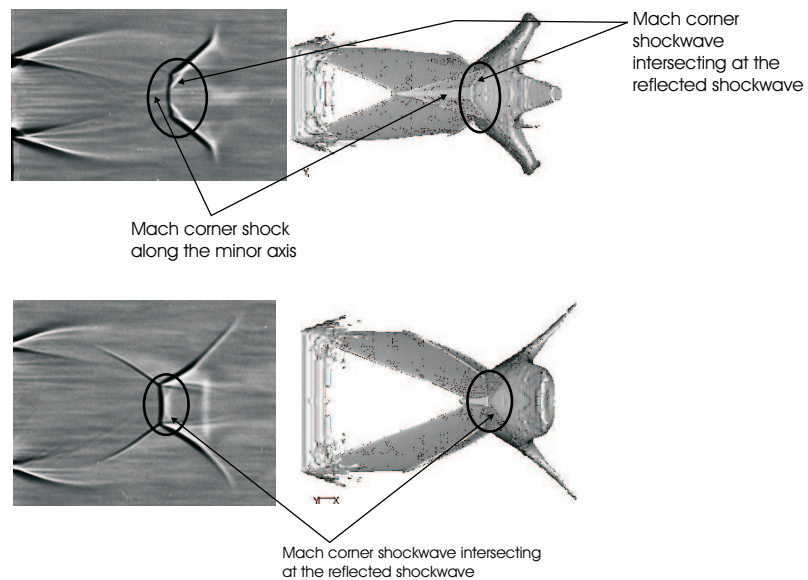
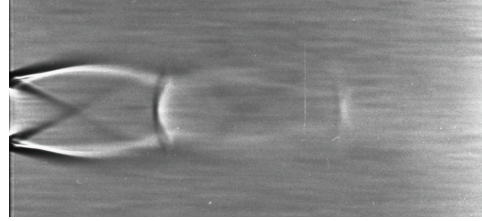


Figure B.8: Explanation of additional features present in the experimental images for the underexpanded jet from a rectangular nozzle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

B.2 Nozzle Aspect Ratio = 3



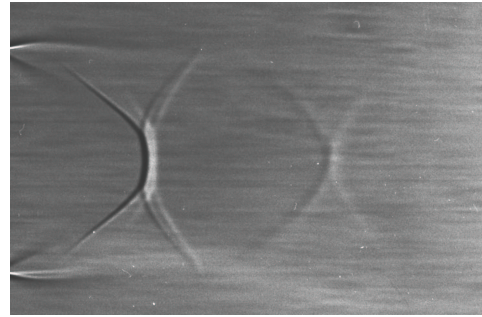
(a) Numerical Schlieren



(b) Experimental shadowgraph



(c) Numerical Schlieren



(d) Experimental shadowgraph

Figure B.9: AR = 3 nozzle, images along the minor and major axes of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

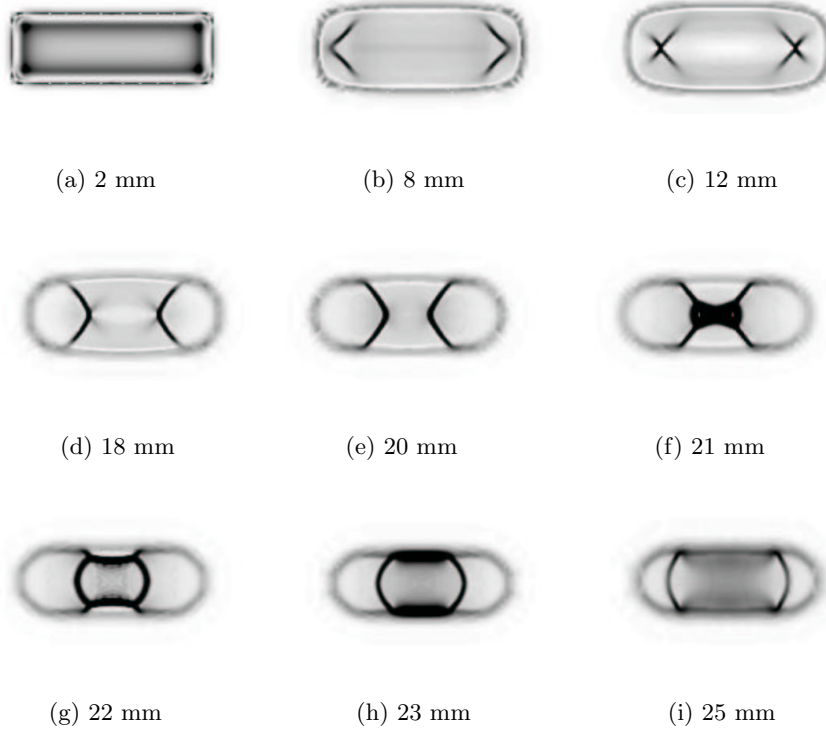


Figure B.10: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

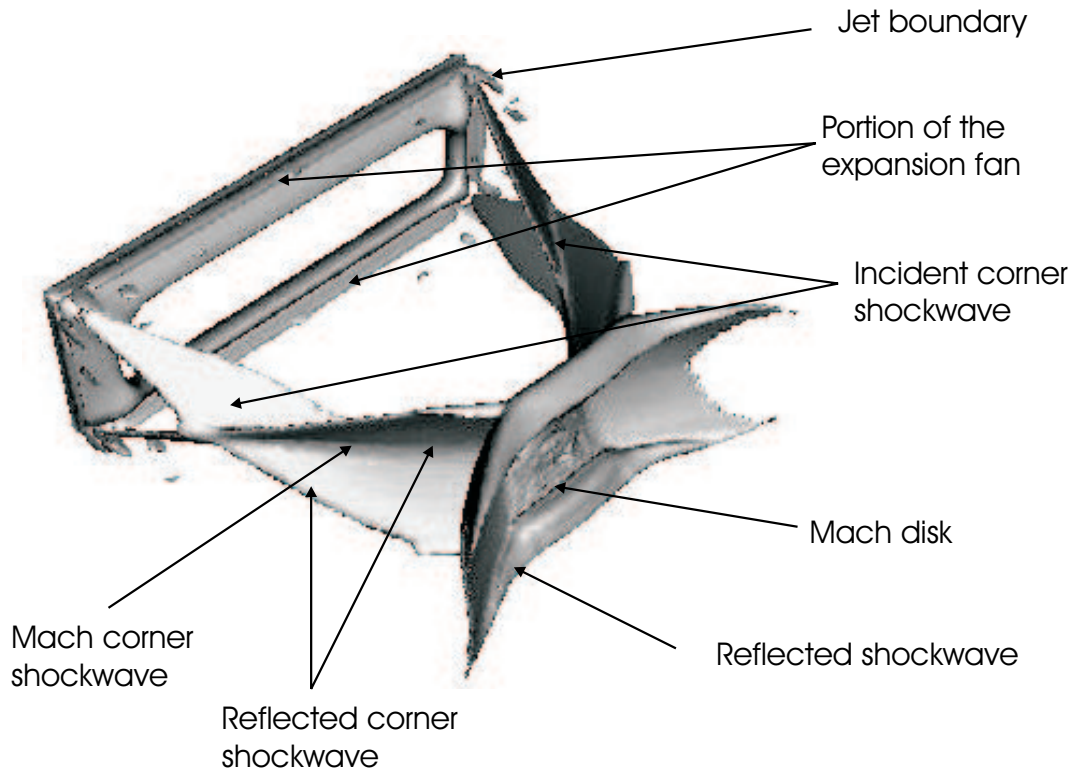


Figure B.11: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

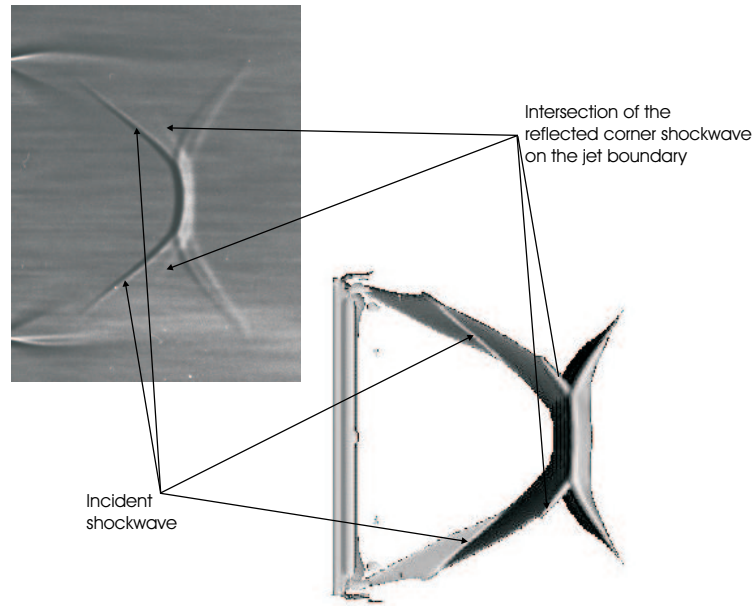
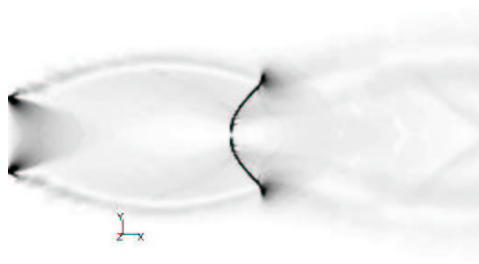
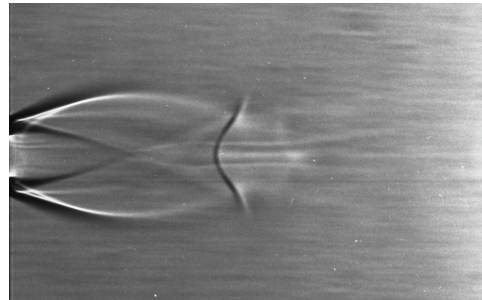


Figure B.12: Additional flow features in the experimental images explained for the underexpanded jet from a rectangular nozzle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



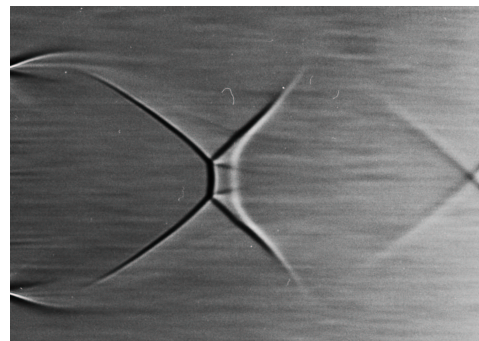
(a) Numerical Schlieren



(b) Experimental shadowgraph



(c) Numerical Schlieren



(d) Experimental shadowgraph

Figure B.13: $AR = 3$ nozzle, images along the minor and major axes of the nozzle exit, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

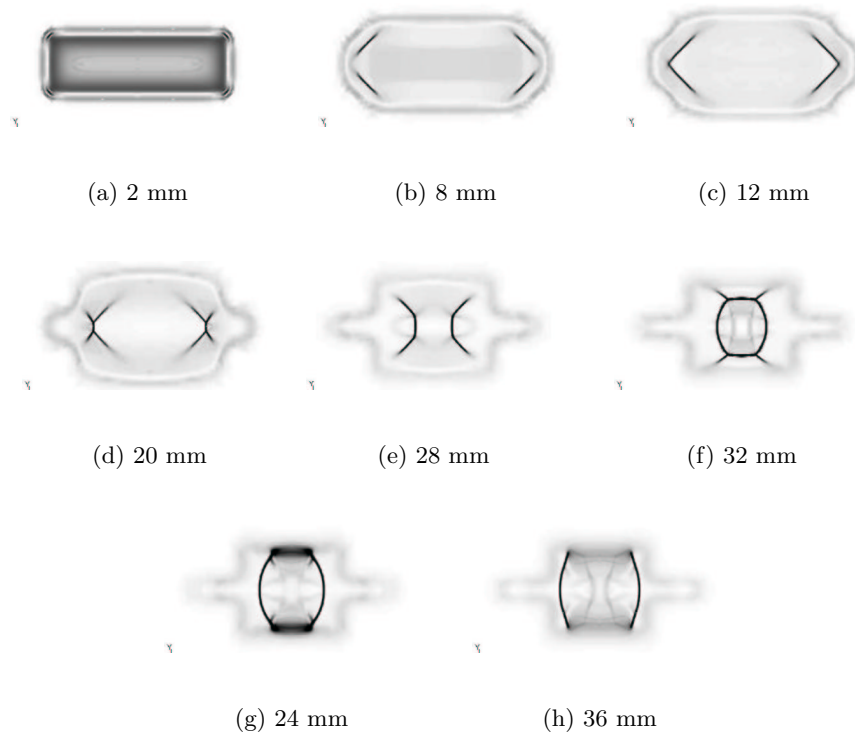


Figure B.14: Rectangular nozzle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

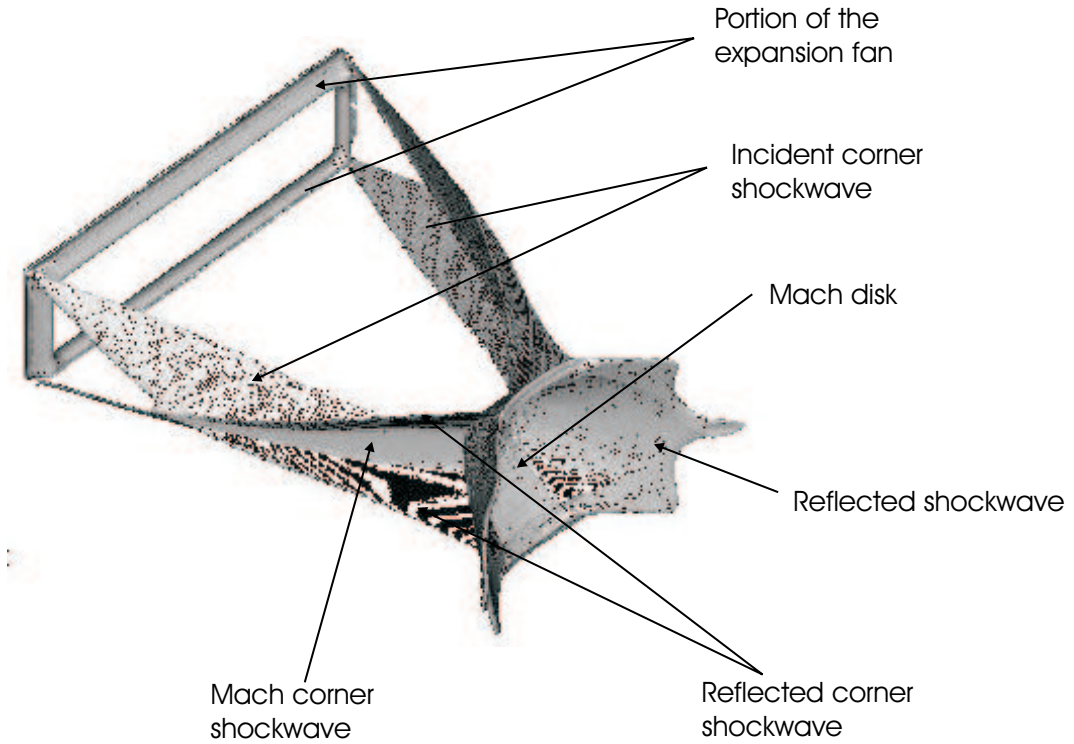


Figure B.15: 3-dimensional representation of the flow field for the underexpanded jet from a rectangular nozzle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

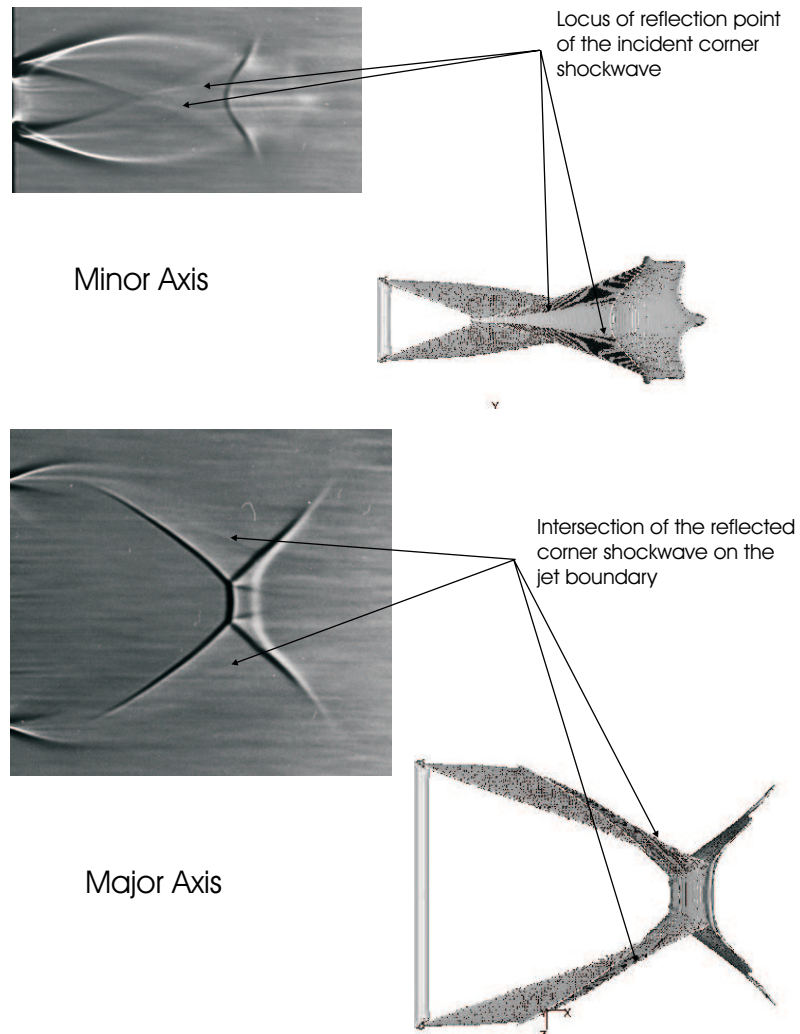


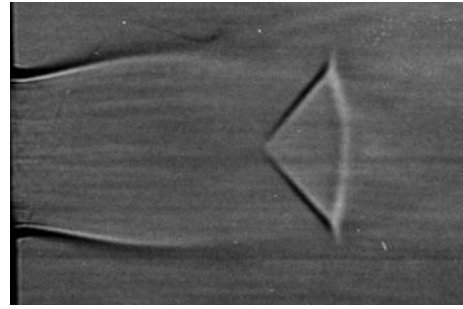
Figure B.16: Additional flow features in the experimental images explained for the underexpanded jet from a rectangular nozzle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

APPENDIX C Flowfields in underexpanded jets issuing from elliptical nozzles

C.1 Nozzle Aspect Ratio = 1.5



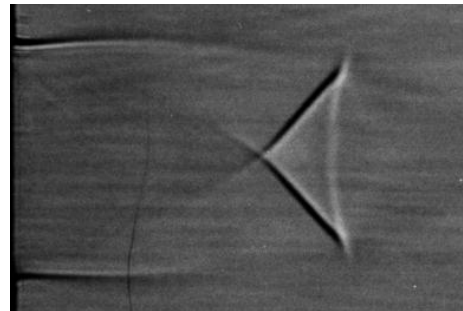
(a) Numerical Schlieren - Minor axis



(b) Experimental shadowgraph - Minor axis



(c) Numerical Schlieren - Major axis



(d) Experimental shadowgraph - Major axis

Figure C.1: Elliptical nozzle with a 45° inlet angle, images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

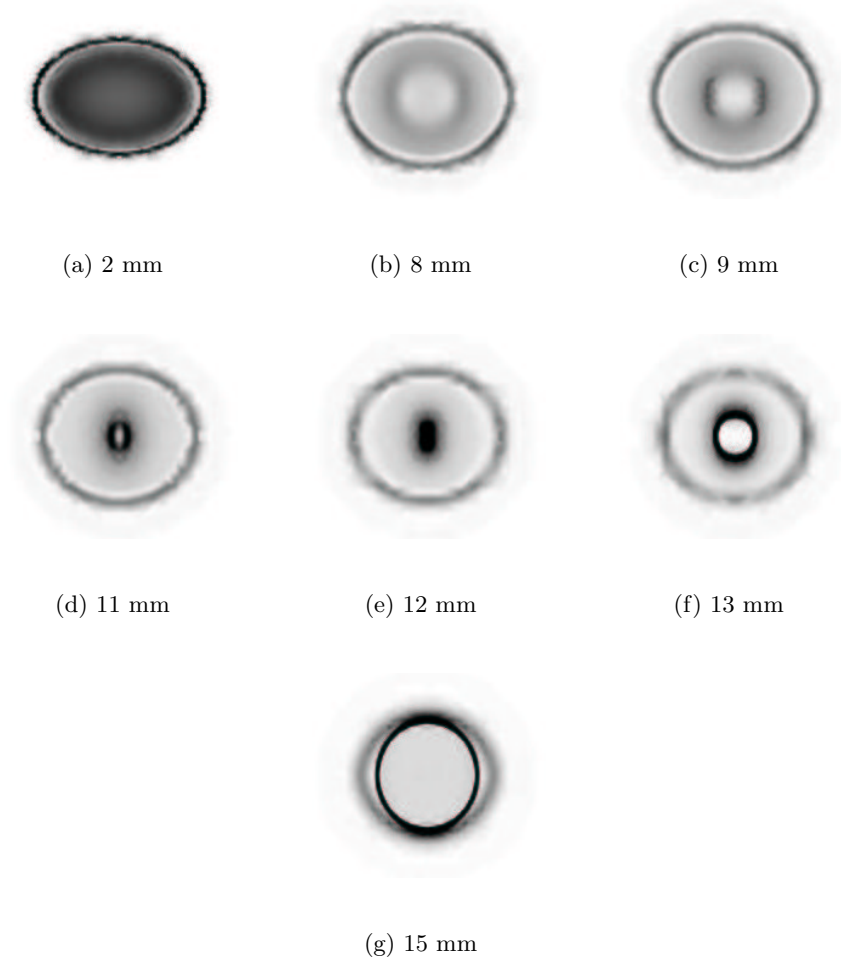


Figure C.2: Ellipse nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

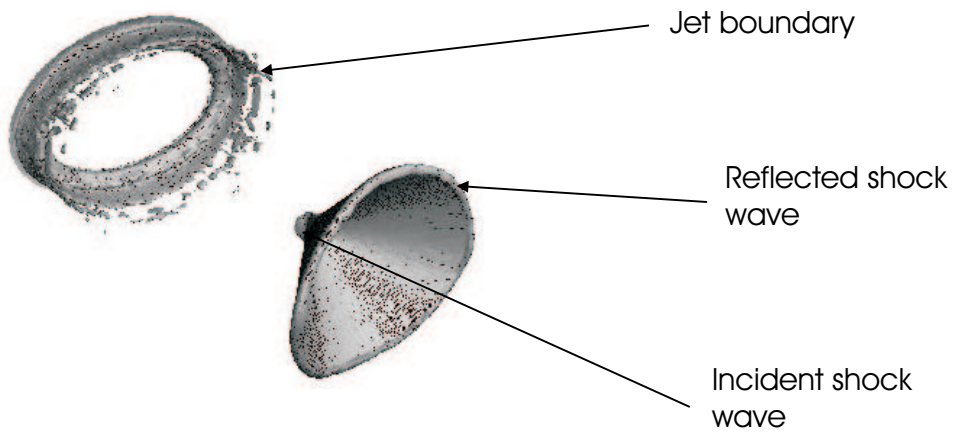
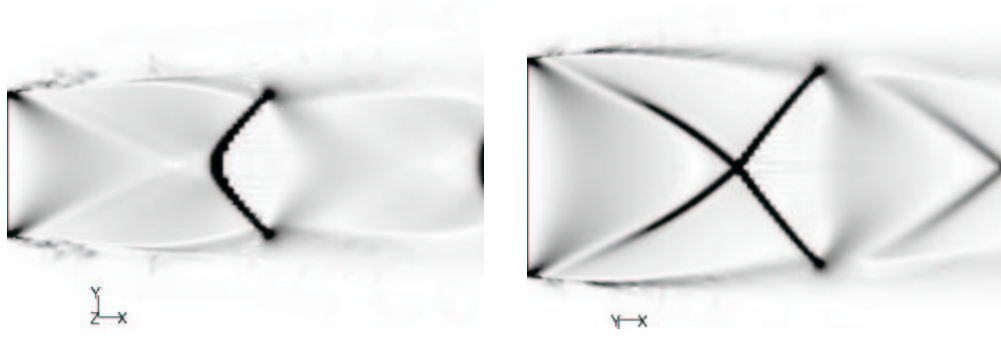


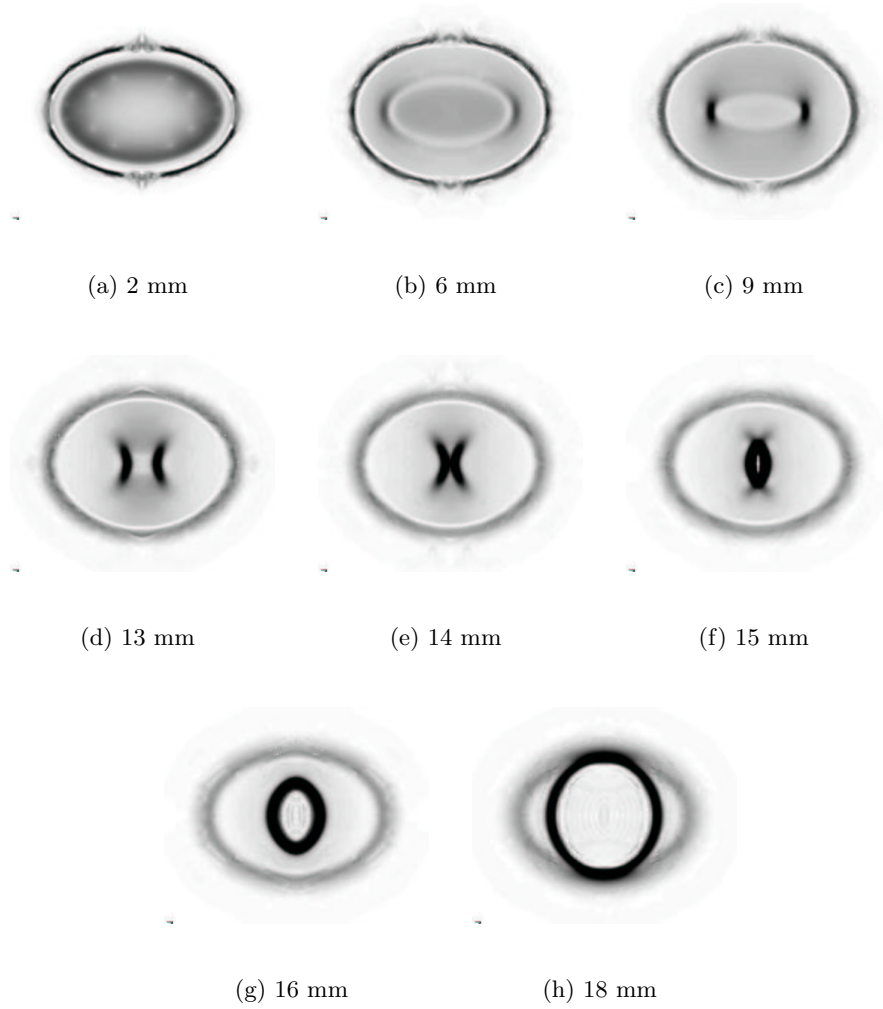
Figure C.3: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a 45° inlet angle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



(a) Numerical Schlieren - Minor axis

(b) Numerical Schlieren - Major axis

Figure C.4: Ellipse nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



(a) 2 mm

(b) 6 mm

(c) 9 mm

(d) 13 mm

(e) 14 mm

(f) 15 mm

(g) 16 mm

(h) 18 mm

Figure C.5: Ellipse nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

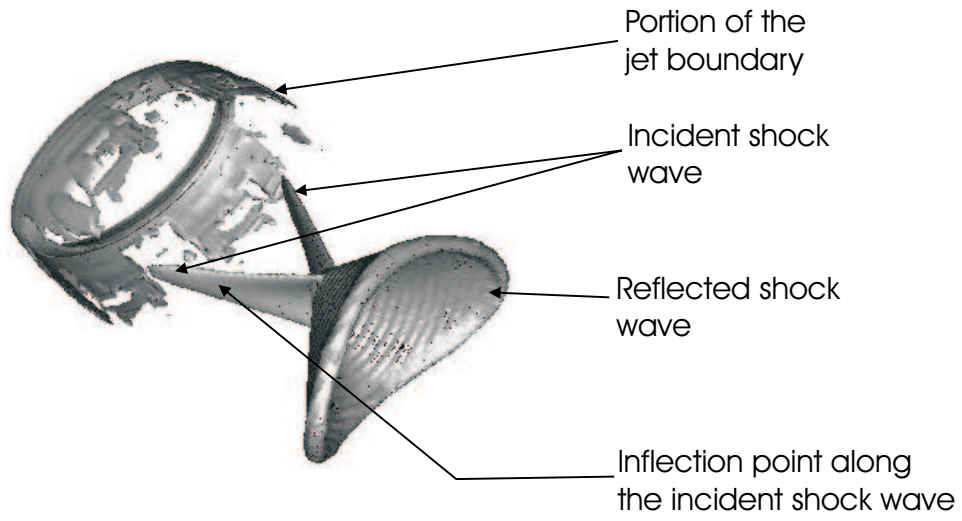
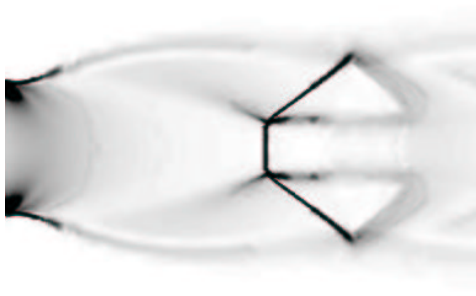
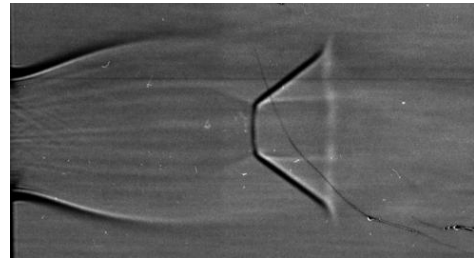


Figure C.6: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a parallel flow exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



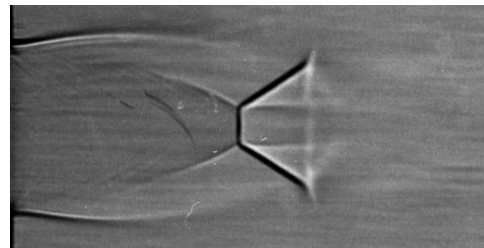
(a) Numerical Schlieren - Minor axis



(b) Experimental shadowgraph - Minor axis



(c) Numerical Schlieren - Major axis



(d) Experimental shadowgraph - Major axis

Figure C.7: Elliptical nozzle with a 45° inlet angle, images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

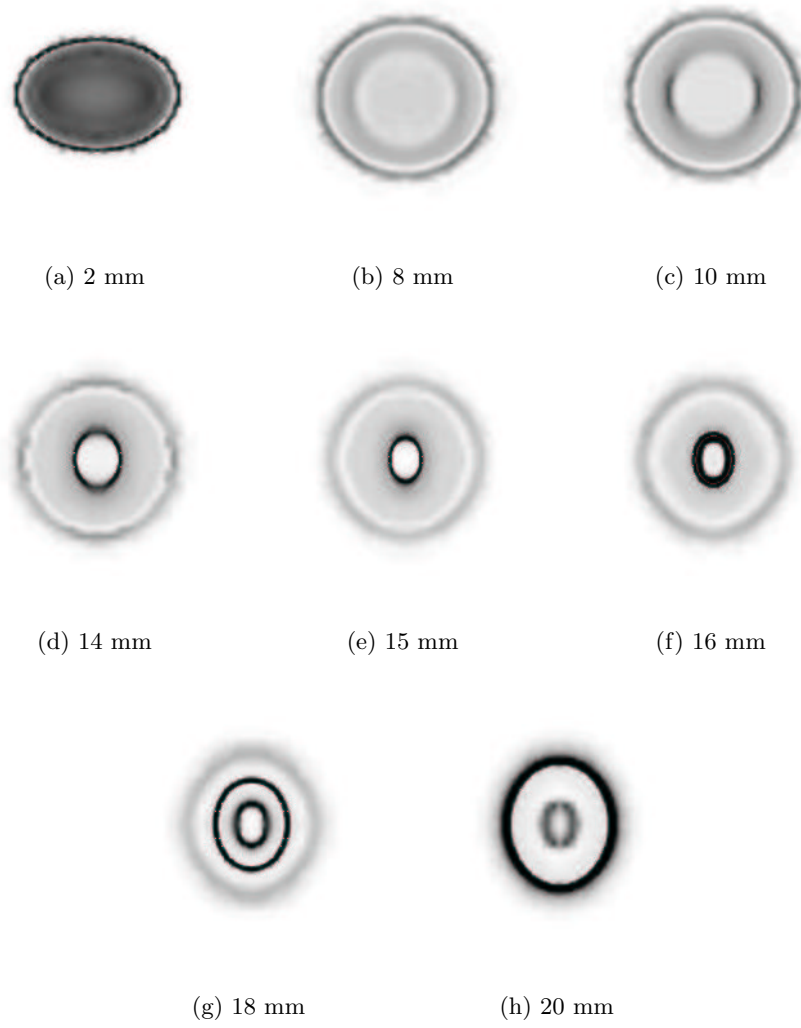


Figure C.8: Ellipse nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

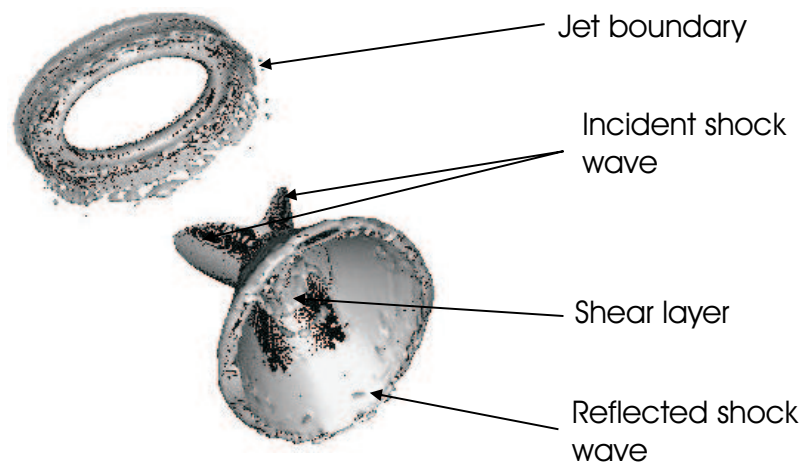
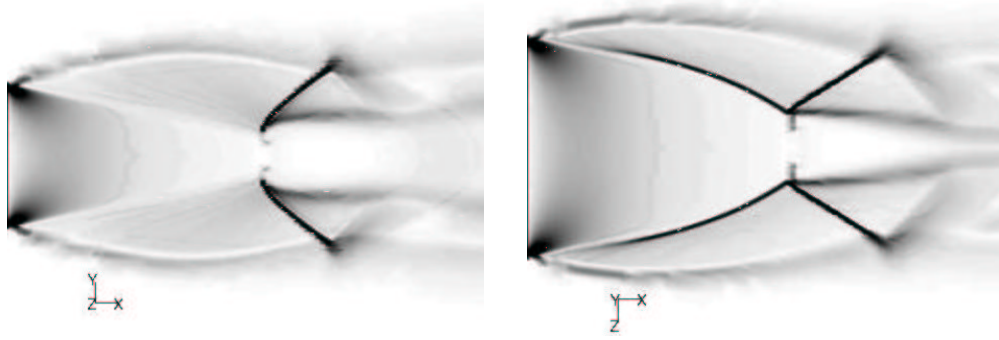


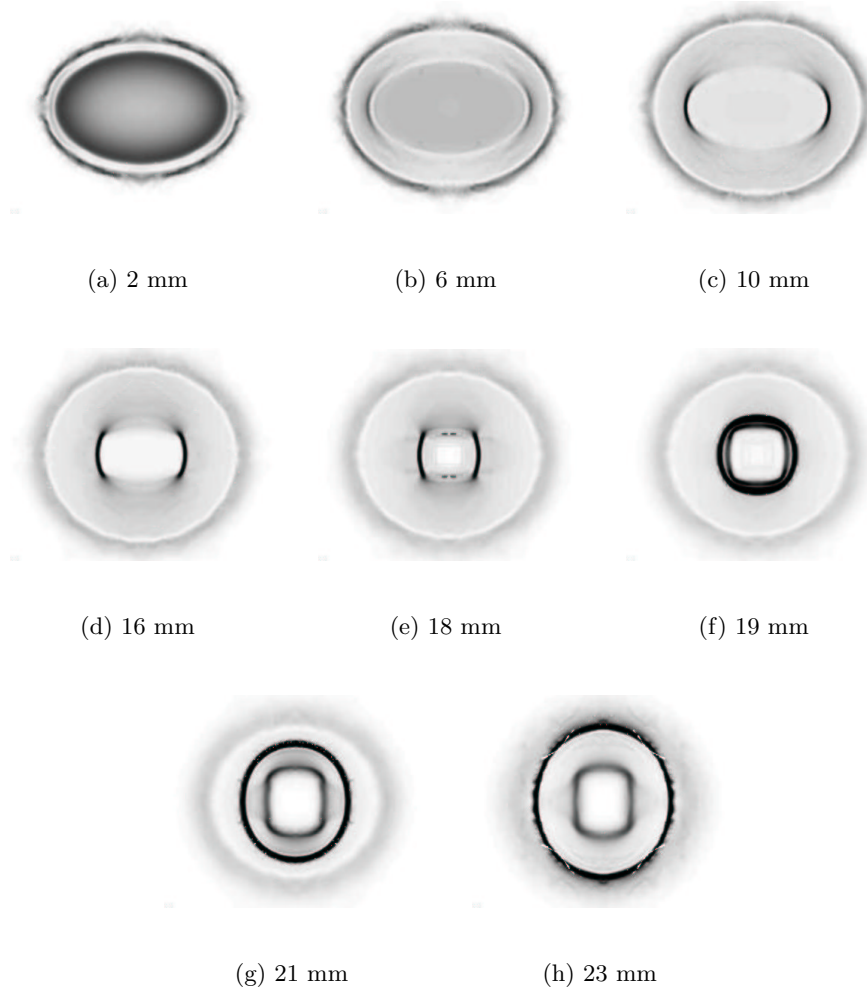
Figure C.9: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a 45° inlet angle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$



(a) Numerical Schlieren - Minor axis

(b) Numerical Schlieren - Major axis

Figure C.10: Ellipse nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$



(a) 2 mm

(b) 6 mm

(c) 10 mm

(d) 16 mm

(e) 18 mm

(f) 19 mm

(g) 21 mm

(h) 23 mm

Figure C.11: Ellipse nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

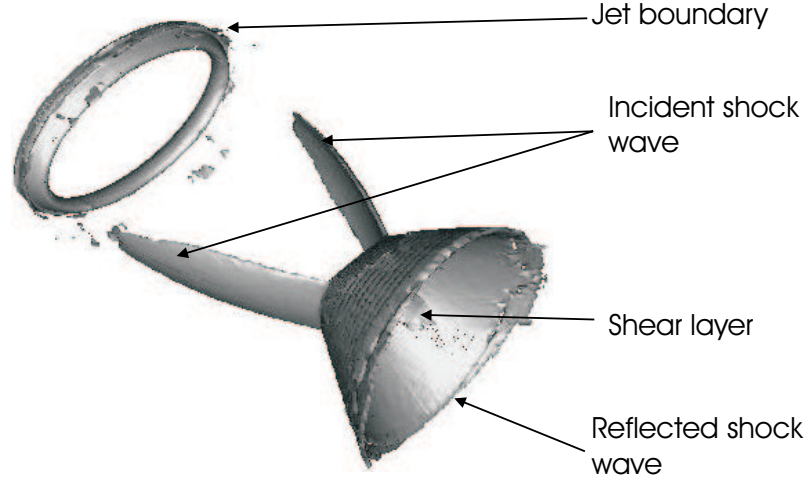


Figure C.12: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a parallel flow exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

C.2 Nozzle Aspect Ratio = 2

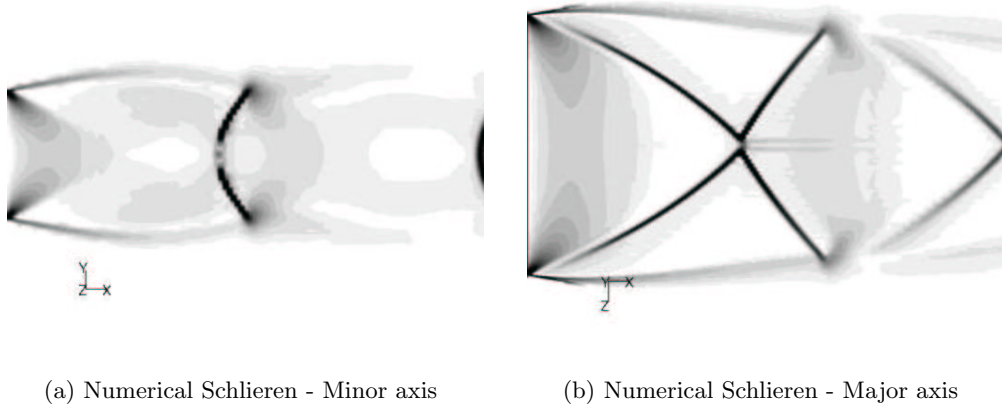


Figure C.13: Ellipse nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

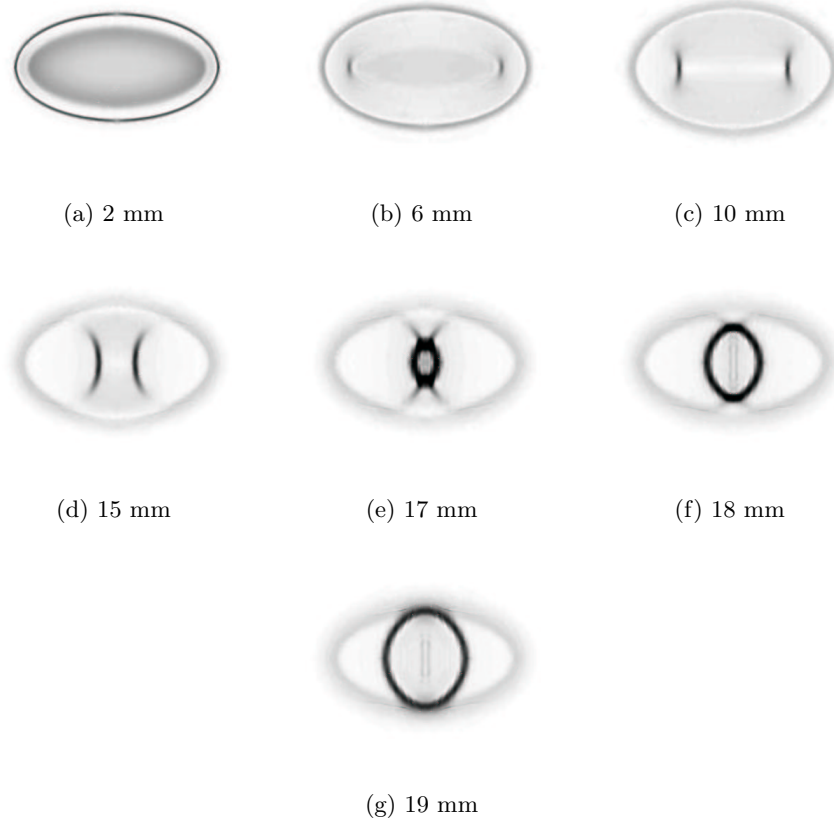


Figure C.14: Ellipse nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

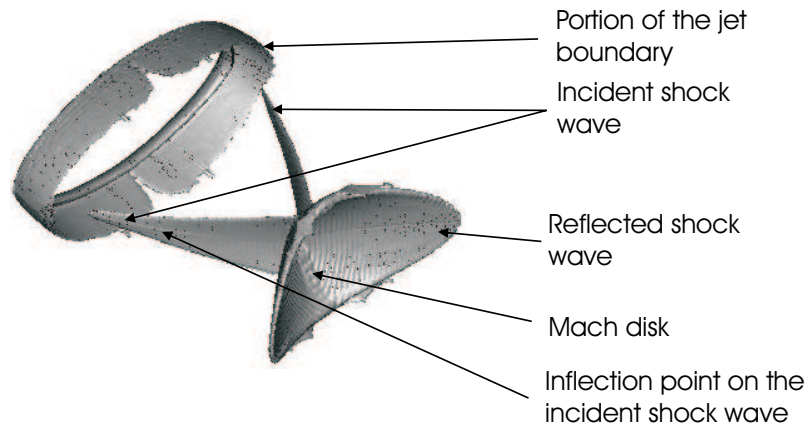
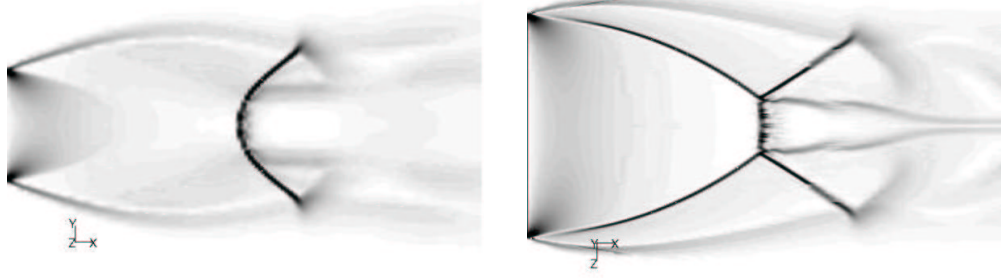


Figure C.15: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a parallel flow exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



(a) Numerical Schlieren - Minor axis

(b) Numerical Schlieren - Major axis

Figure C.16: Ellipse nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

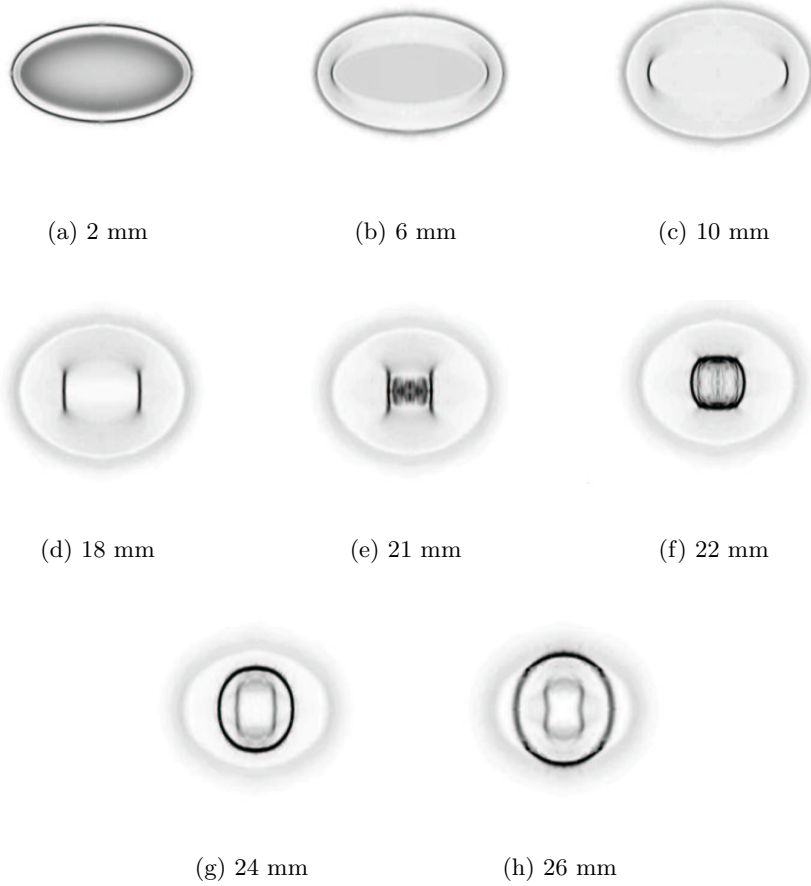


Figure C.17: Ellipse nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

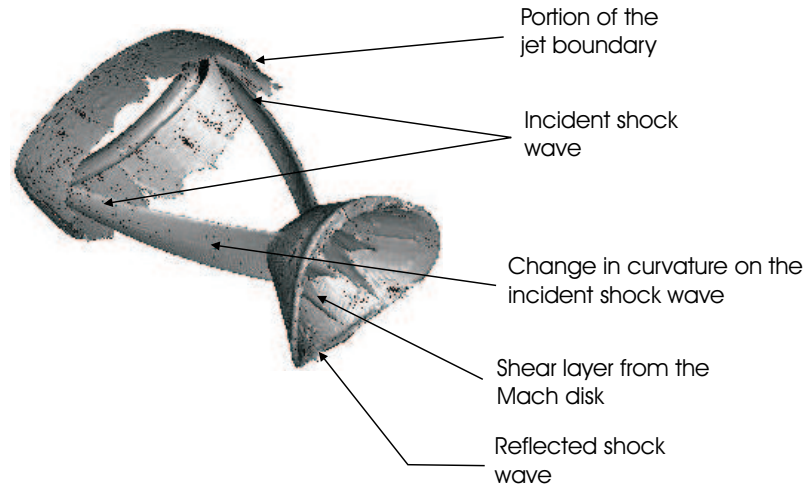
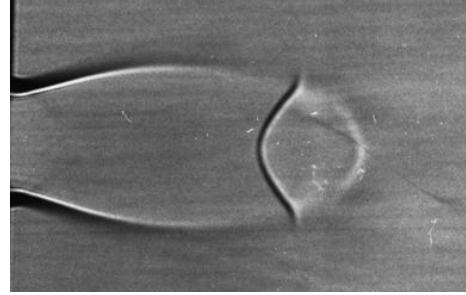


Figure C.18: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a parallel flow exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

C.3 Nozzle Aspect Ratio = 3



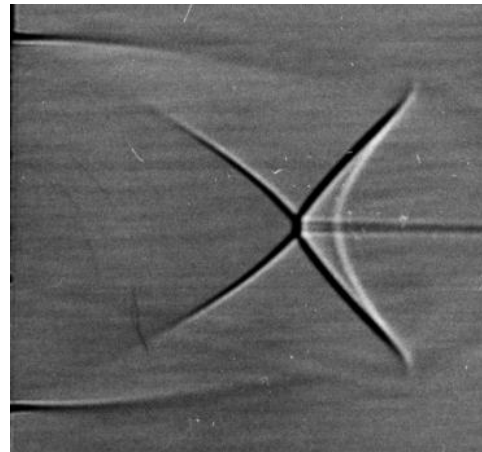
(a) Numerical Schlieren - Minor axis



(b) Experimental shadowgraph - Minor axis



(c) Numerical Schlieren - Major axis



(d) Experimental shadowgraph - Major axis

Figure C.19: Elliptical nozzle with a 45° inlet angle, images along the major axis of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

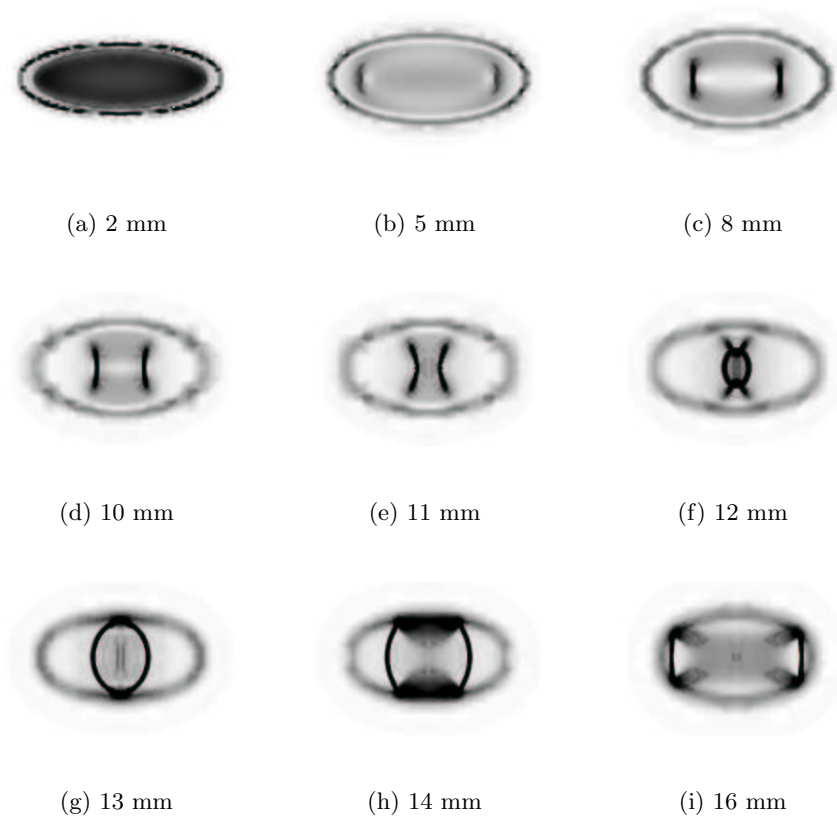


Figure C.20: Ellipse nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

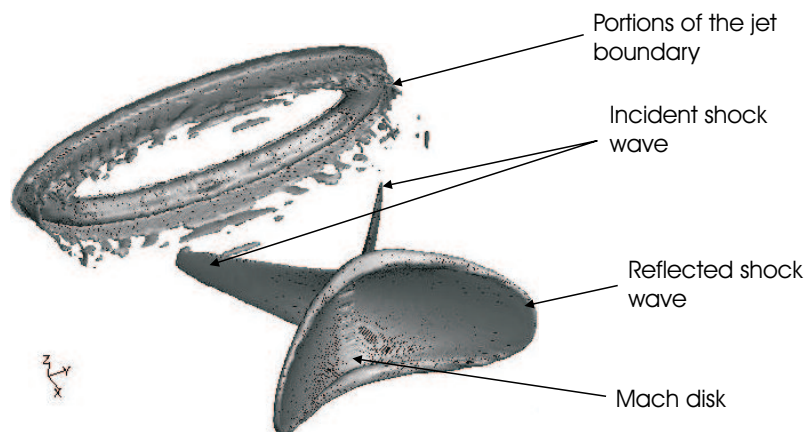
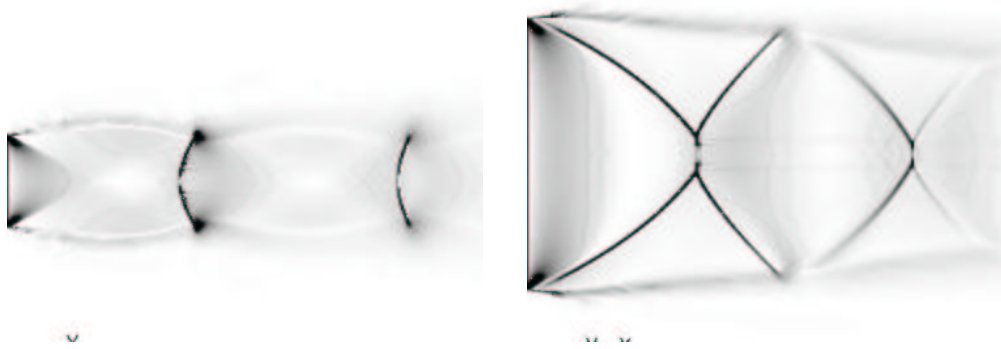


Figure C.21: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a 45° inlet angle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



(a) Numerical Schlieren - Minor axis

(b) Numerical Schlieren - Major axis

Figure C.22: Ellipse nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

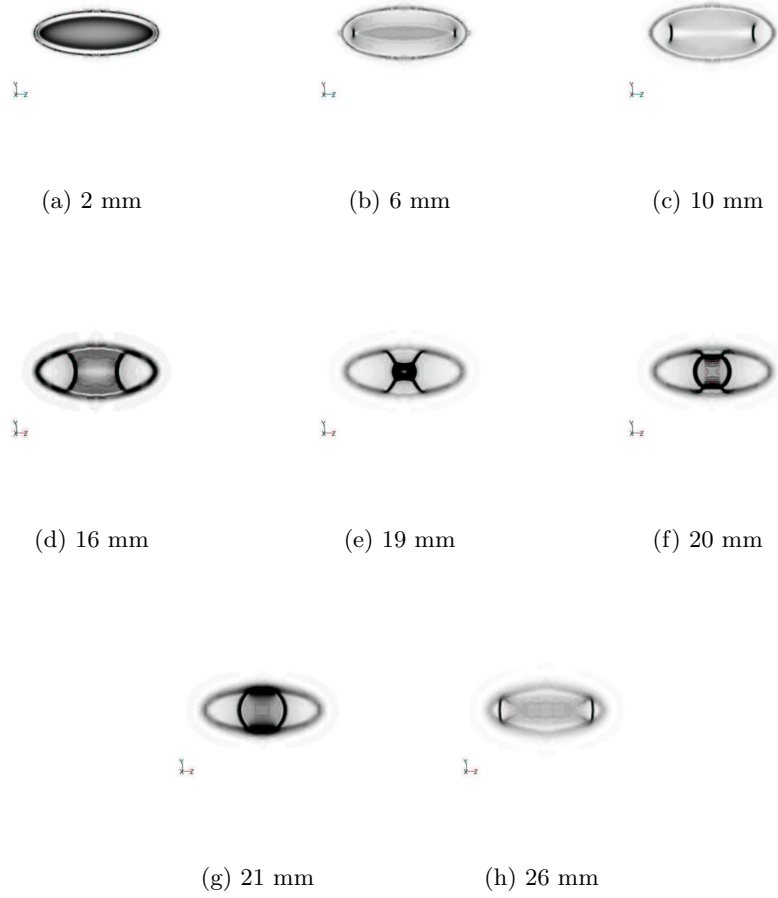


Figure C.23: Ellipse nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

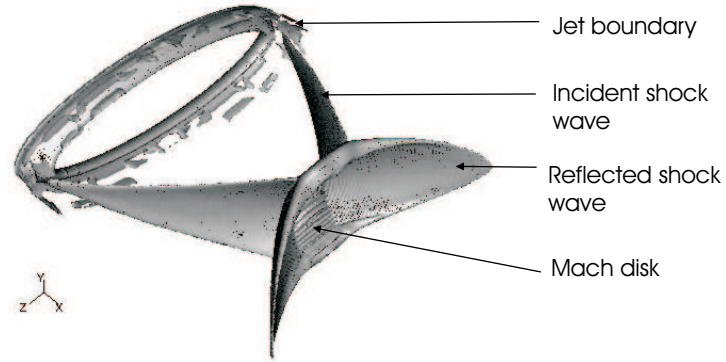


Figure C.24: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a parallel flow exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

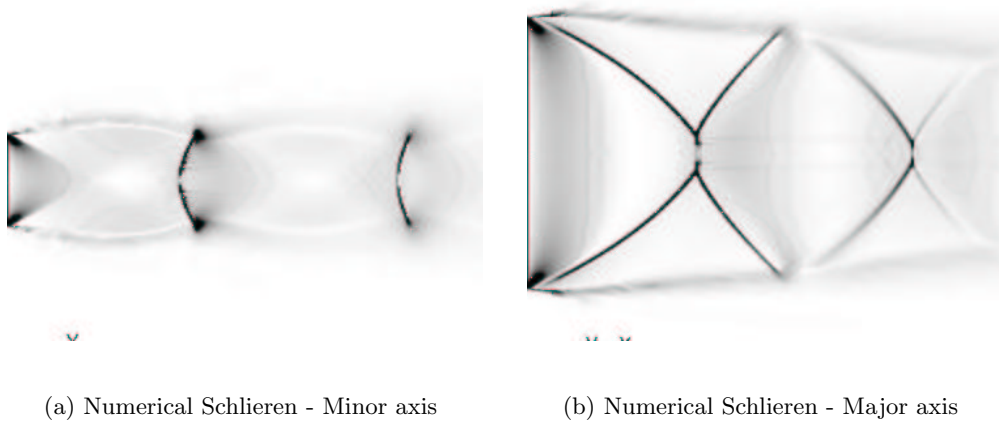


Figure C.25: Ellipse nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

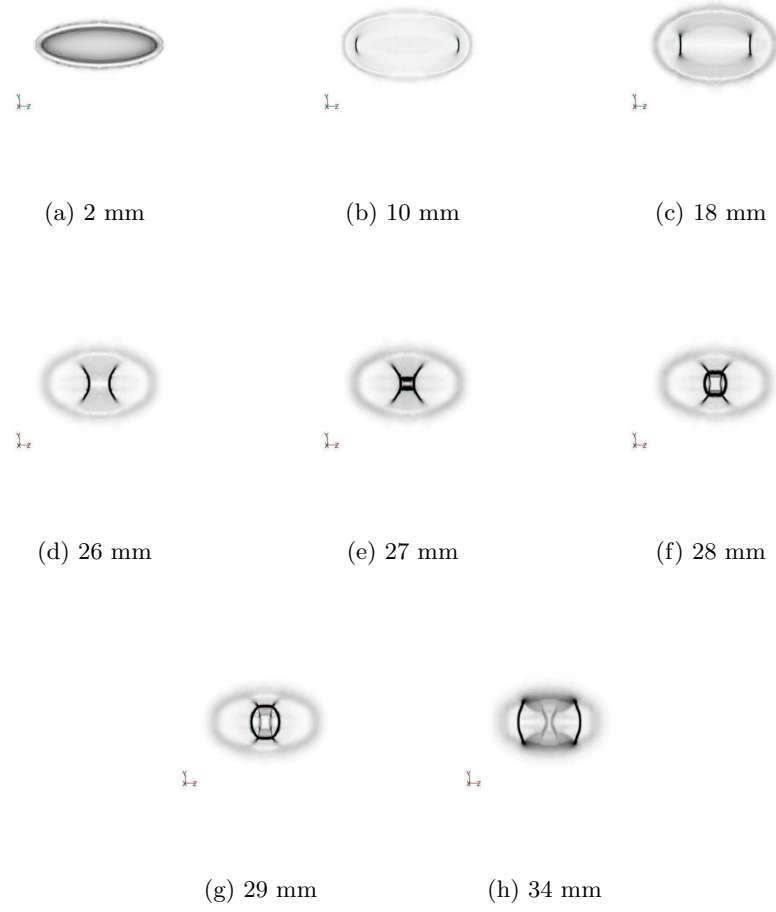


Figure C.26: Ellipse nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

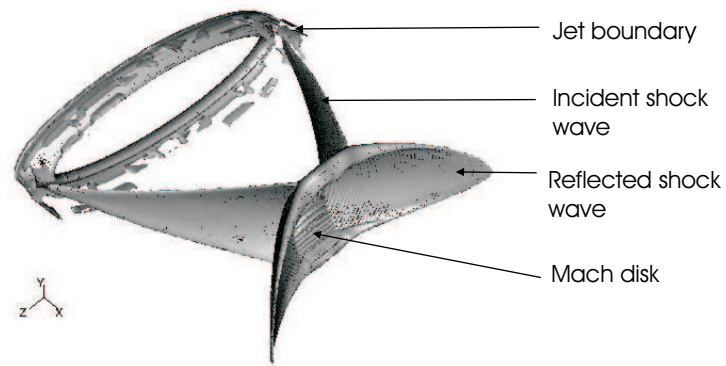


Figure C.27: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a parallel flow exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

C.4 Nozzle Aspect Ratio = 4

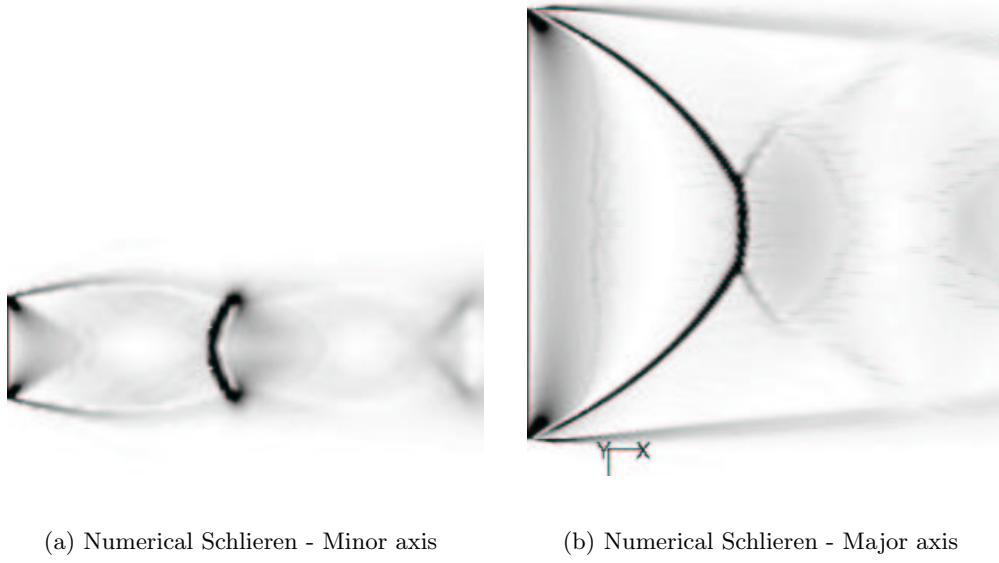


Figure C.28: Ellipse nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

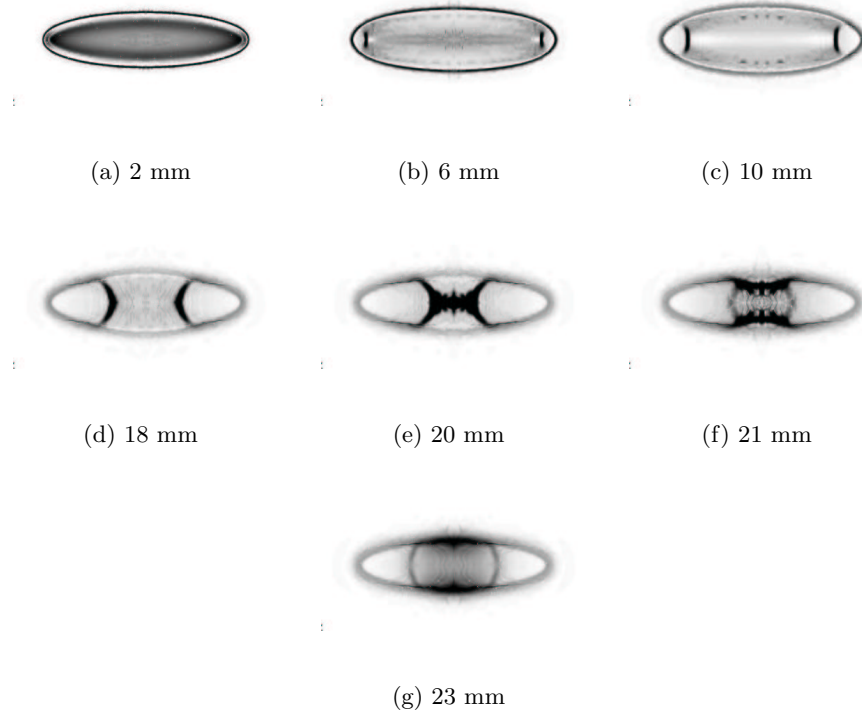


Figure C.29: Ellipse nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

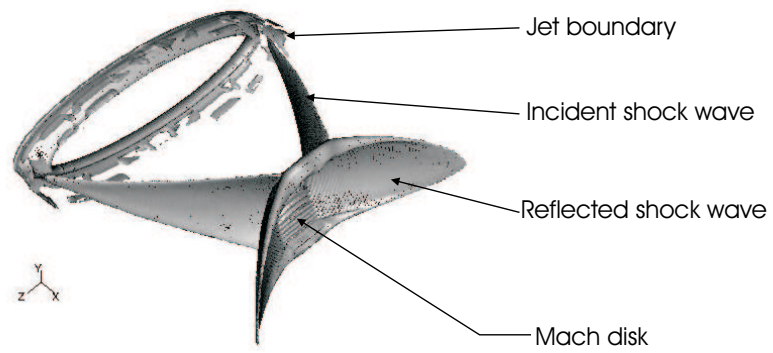
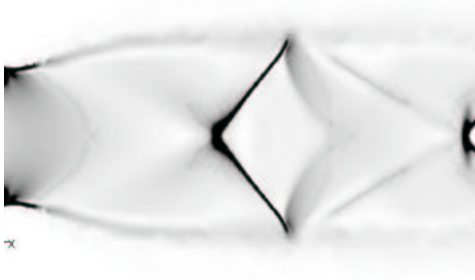


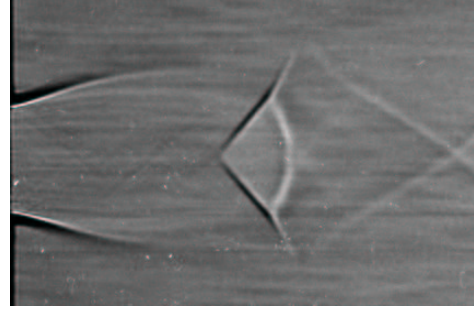
Figure C.30: 3-dimensional representation of the flow field for the underexpanded jet from a ellipse nozzle with a parallel flow exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

APPENDIX D Flowfields in underexpanded jets issuing from slot nozzles

D.1 Nozzle Aspect Ratio = 1.5



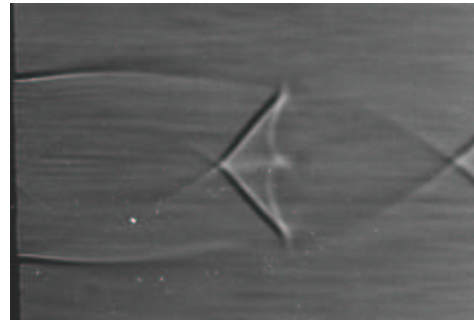
(a) Numerical Schlieren - Minor axis



(b) Experimental shadowgraph - Minor axis



(c) Numerical Schlieren - Major axis



(d) Experimental shadowgraph - Major axis

Figure D.1: Slot nozzle with a 45° inlet angle, images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

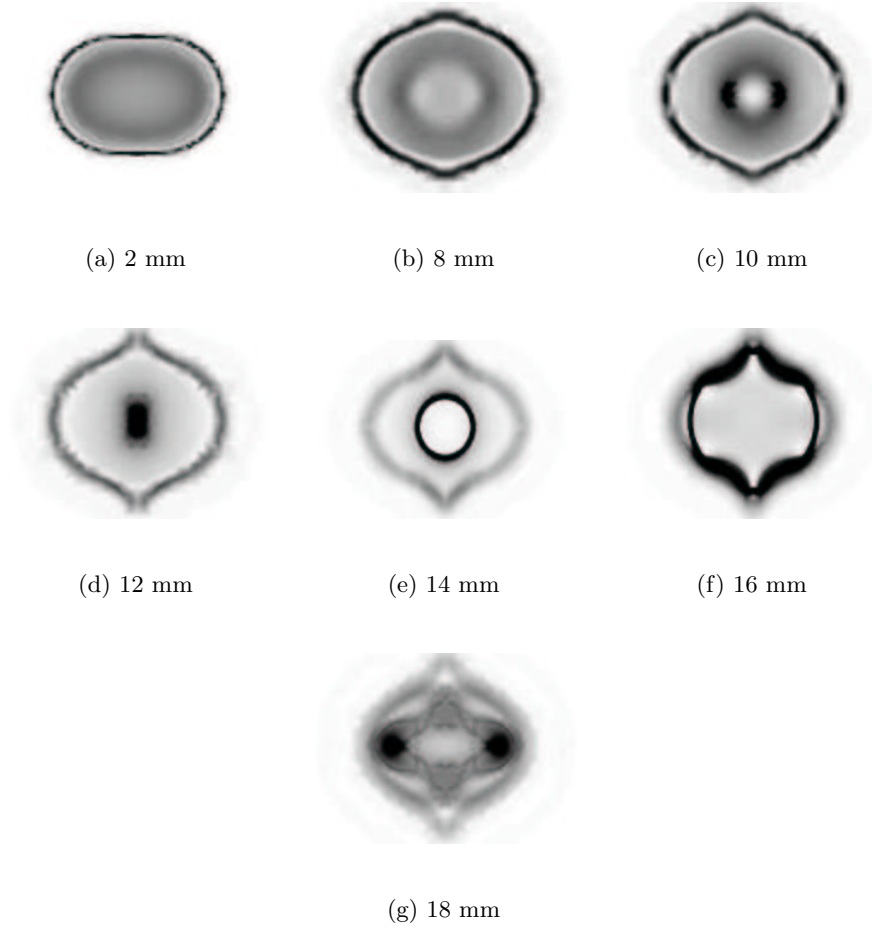


Figure D.2: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

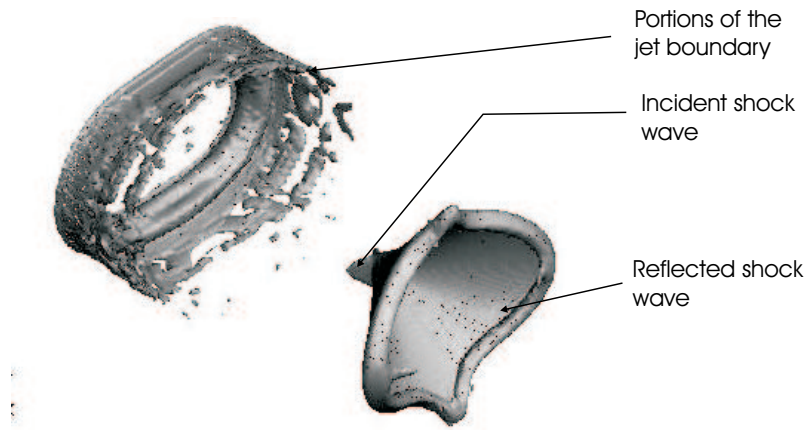


Figure D.3: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

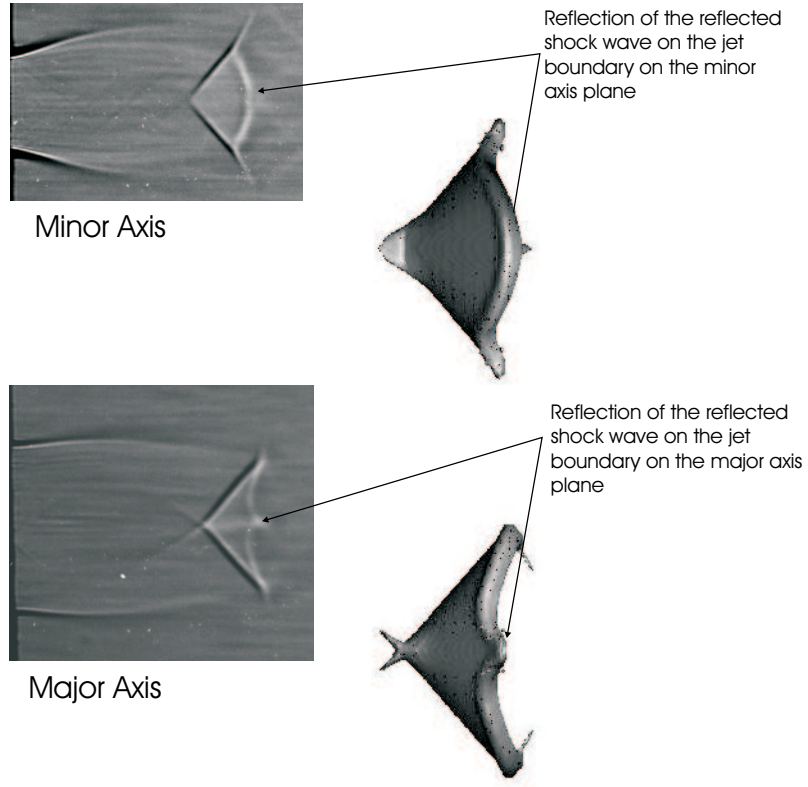


Figure D.4: Explanation of the additional waves present in the experimental images of the flow from a slot nozzle with a 45° inlet angle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

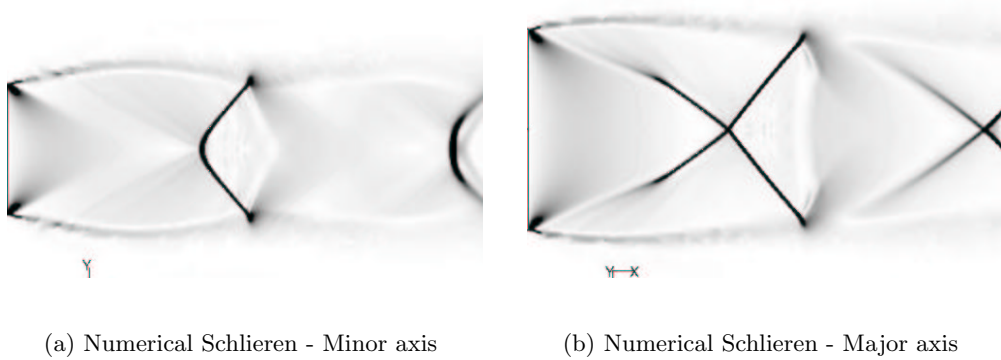


Figure D.5: Slot nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

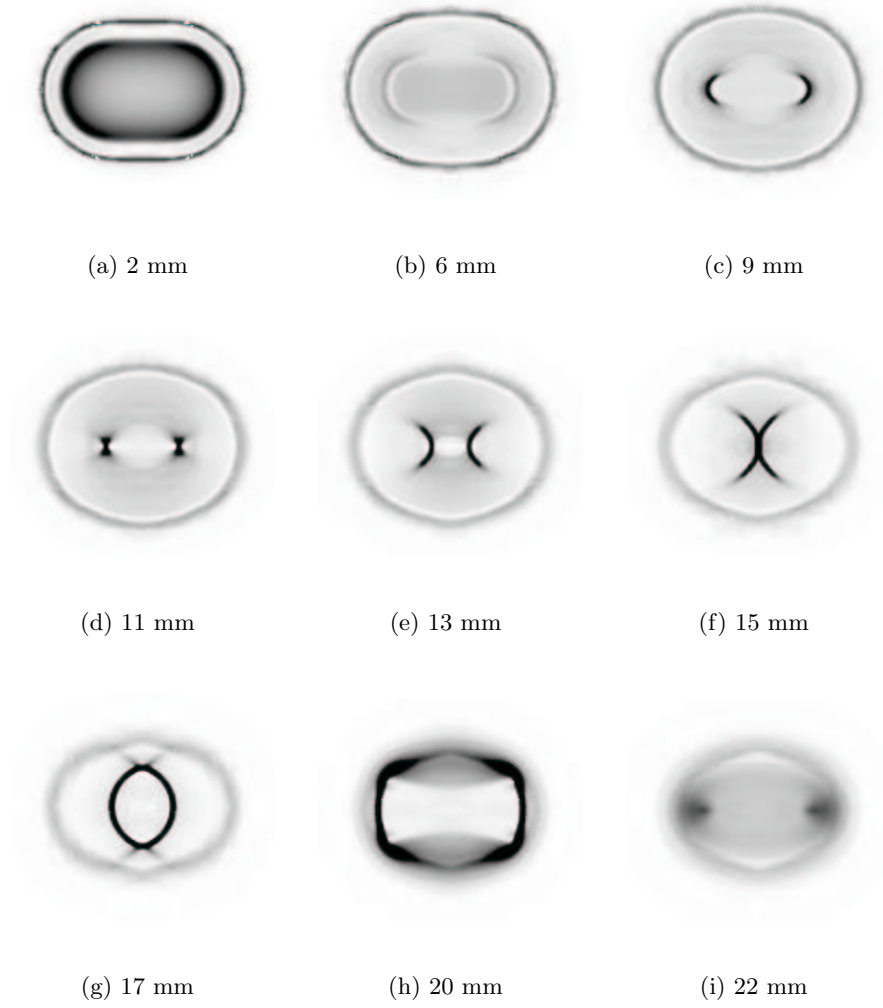


Figure D.6: Slot nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

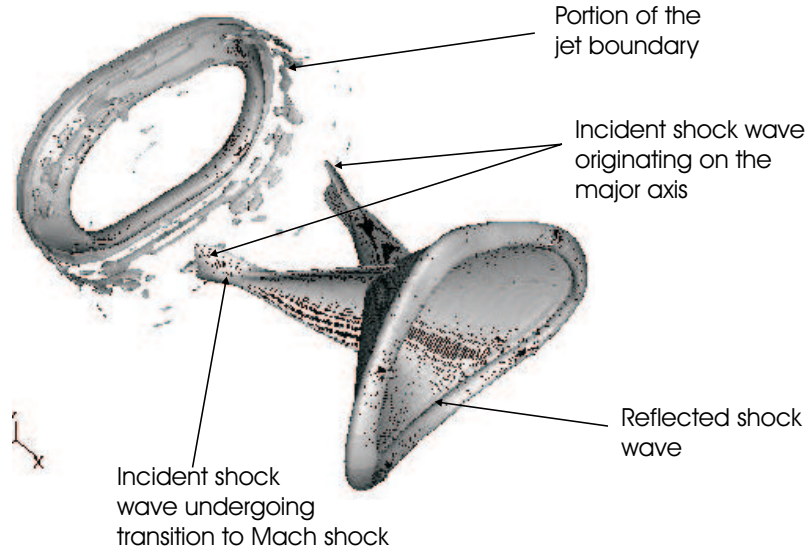
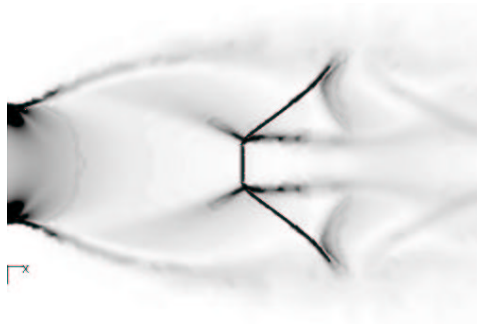
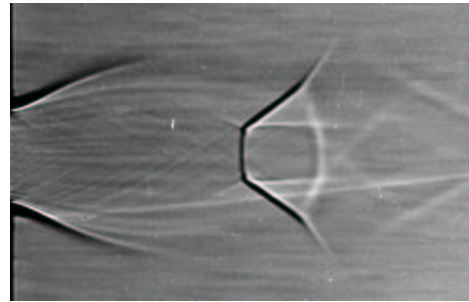


Figure D.7: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



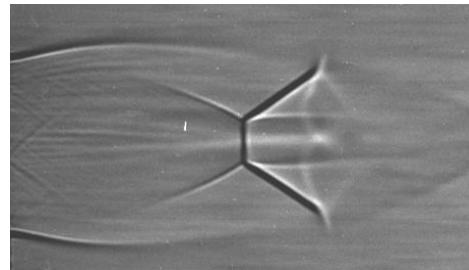
(a) Numerical Schlieren - Minor axis



(b) Experimental shadowgraph - Minor axis



(c) Numerical Schlieren - Major axis



(d) Experimental shadowgraph - Major axis

Figure D.8: Slot nozzle with a 45° inlet angle, images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

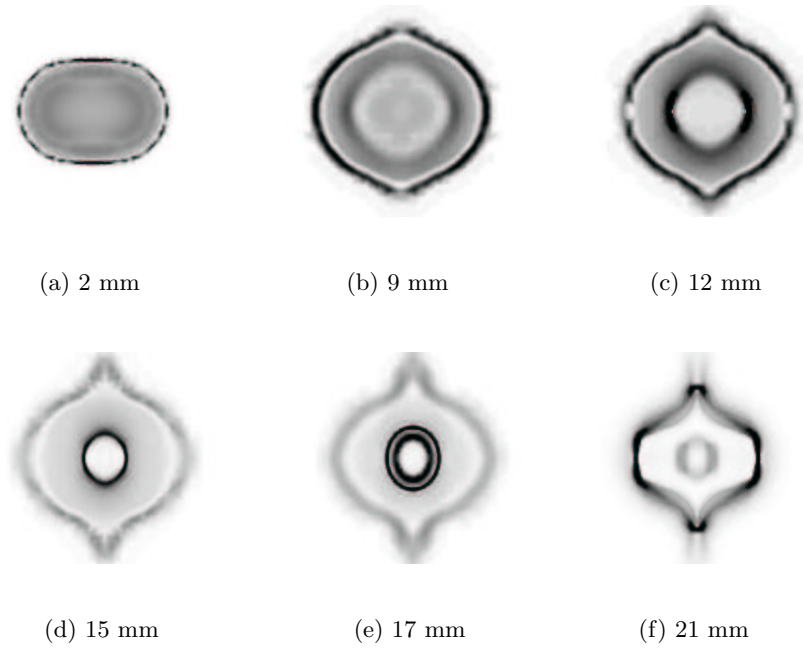


Figure D.9: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

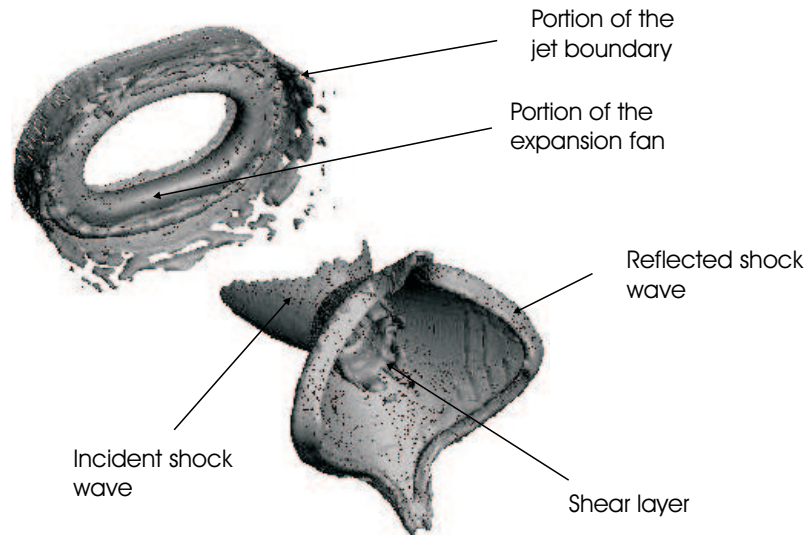


Figure D.10: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

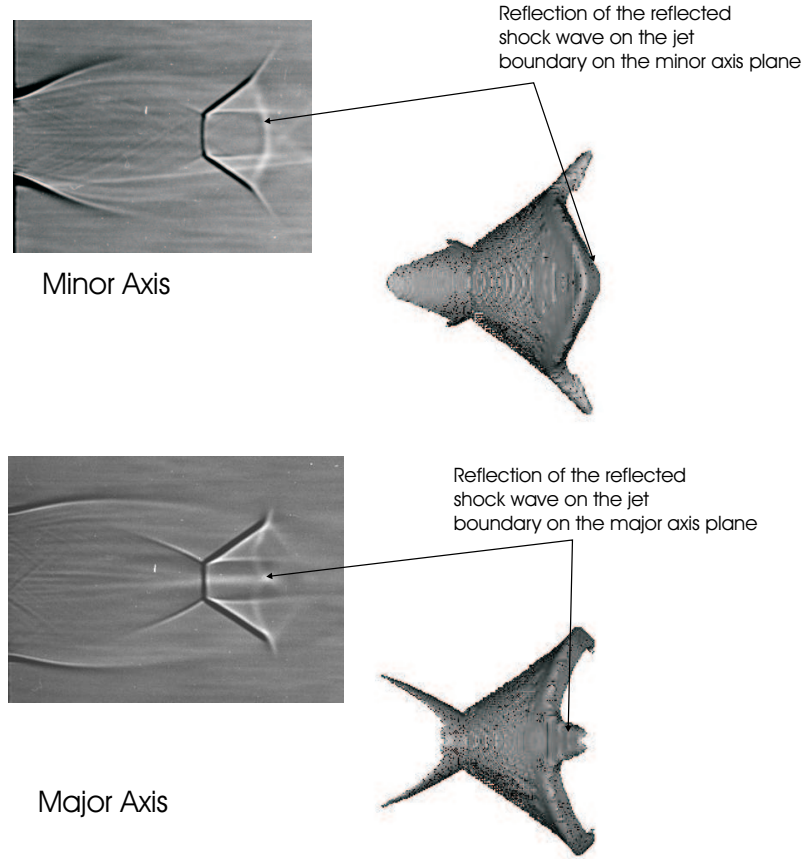


Figure D.11: Explanation of the additional waves present in the experimental images of the flow from a slot nozzle with a 45° inlet angle, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

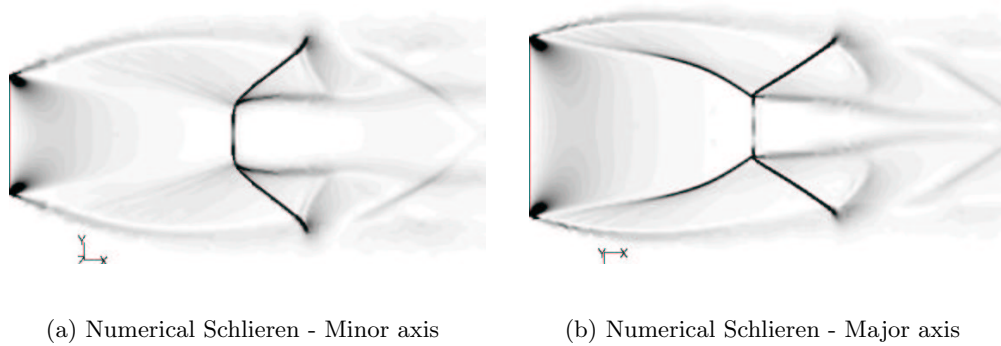


Figure D.12: Slot nozzle with a parallel exit flow, numerical schlieren images along the major axis of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



(a) 2 mm



(b) 9 mm



(c) 17 mm



(d) 18 mm



(e) 20 mm



(f) 22 mm



(g) 25 mm

Figure D.13: Slot nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

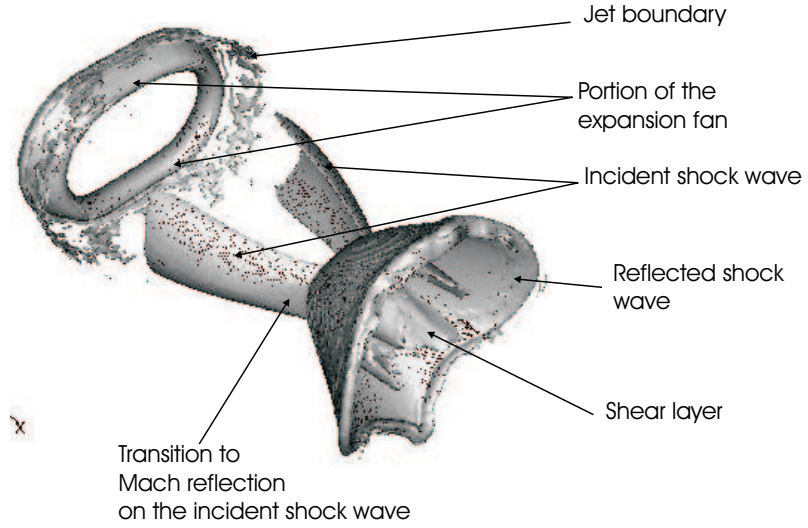


Figure D.14: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 1.5$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

D.2 Nozzle Aspect Ratio = 2

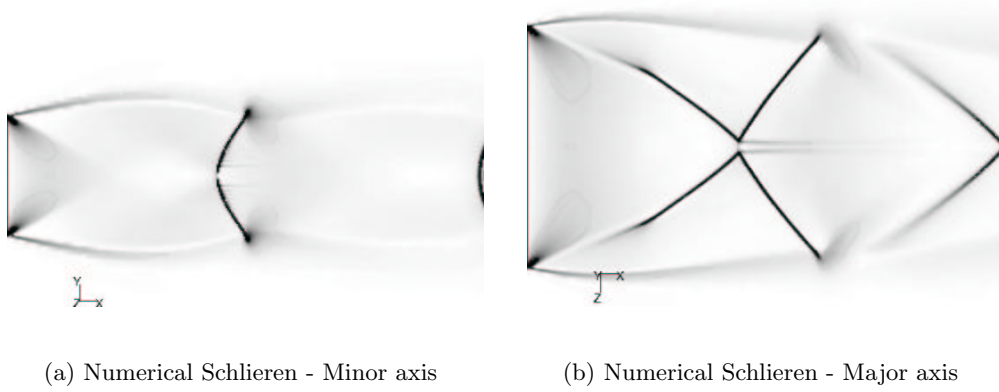


Figure D.15: Slot nozzle with a parallel flow exit; images along the major axis of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

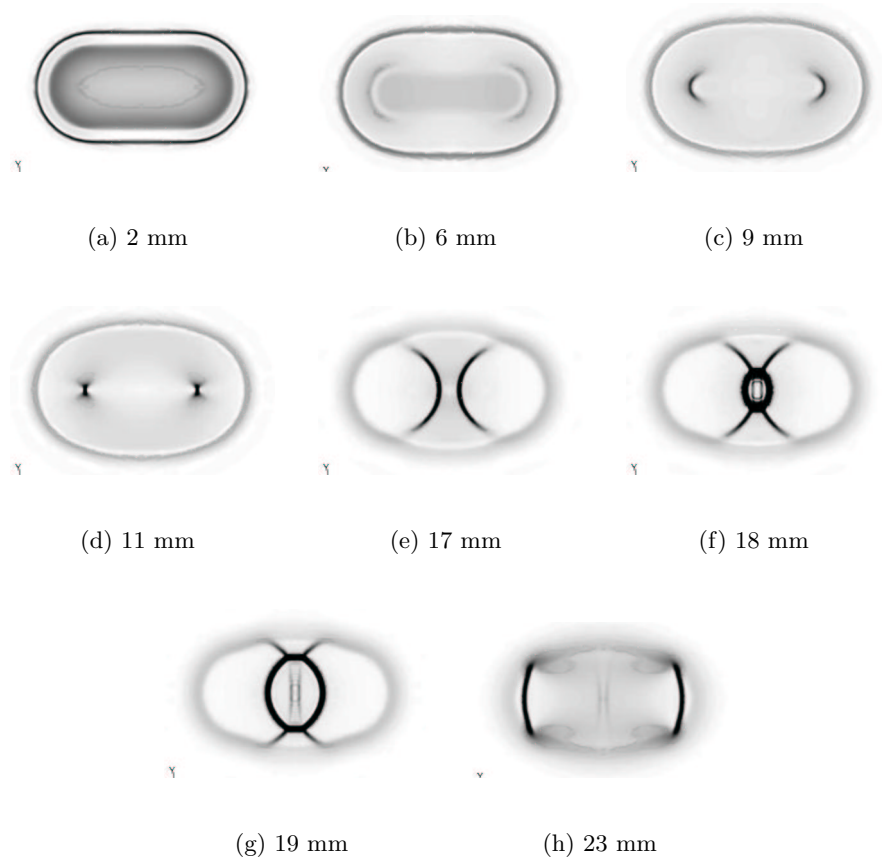


Figure D.16: Slot nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

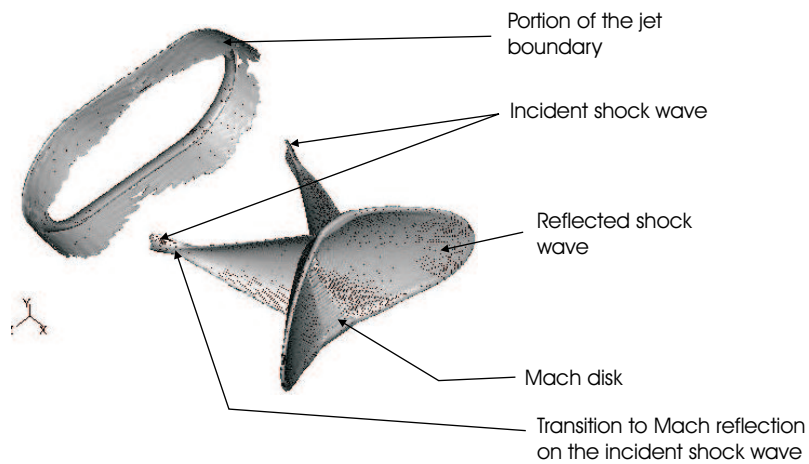


Figure D.17: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

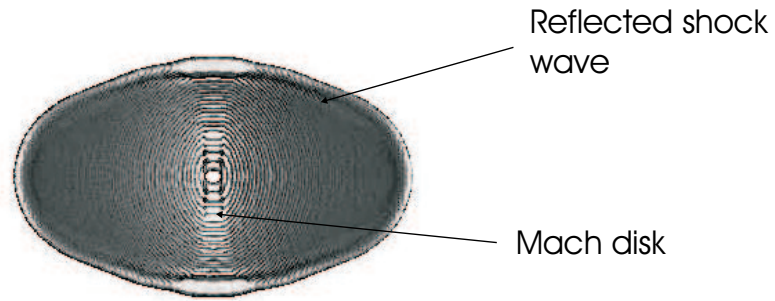
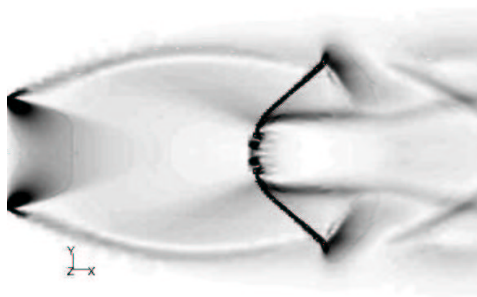
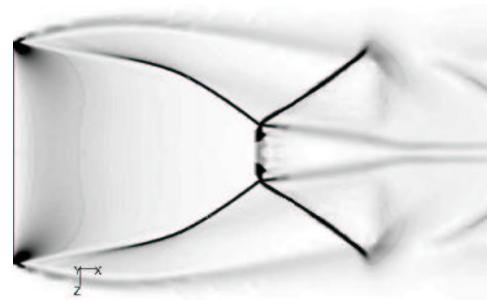


Figure D.18: Close up of the reflected shock wave in the flow field of a jet issuing from a slot nozzle with a parallel flow exit, $AR = 2$ $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



(a) Numerical Schlieren - Minor axis



(b) Numerical Schlieren - Major axis

Figure D.19: Slot nozzle with a parallel flow exit; images along the major axis of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

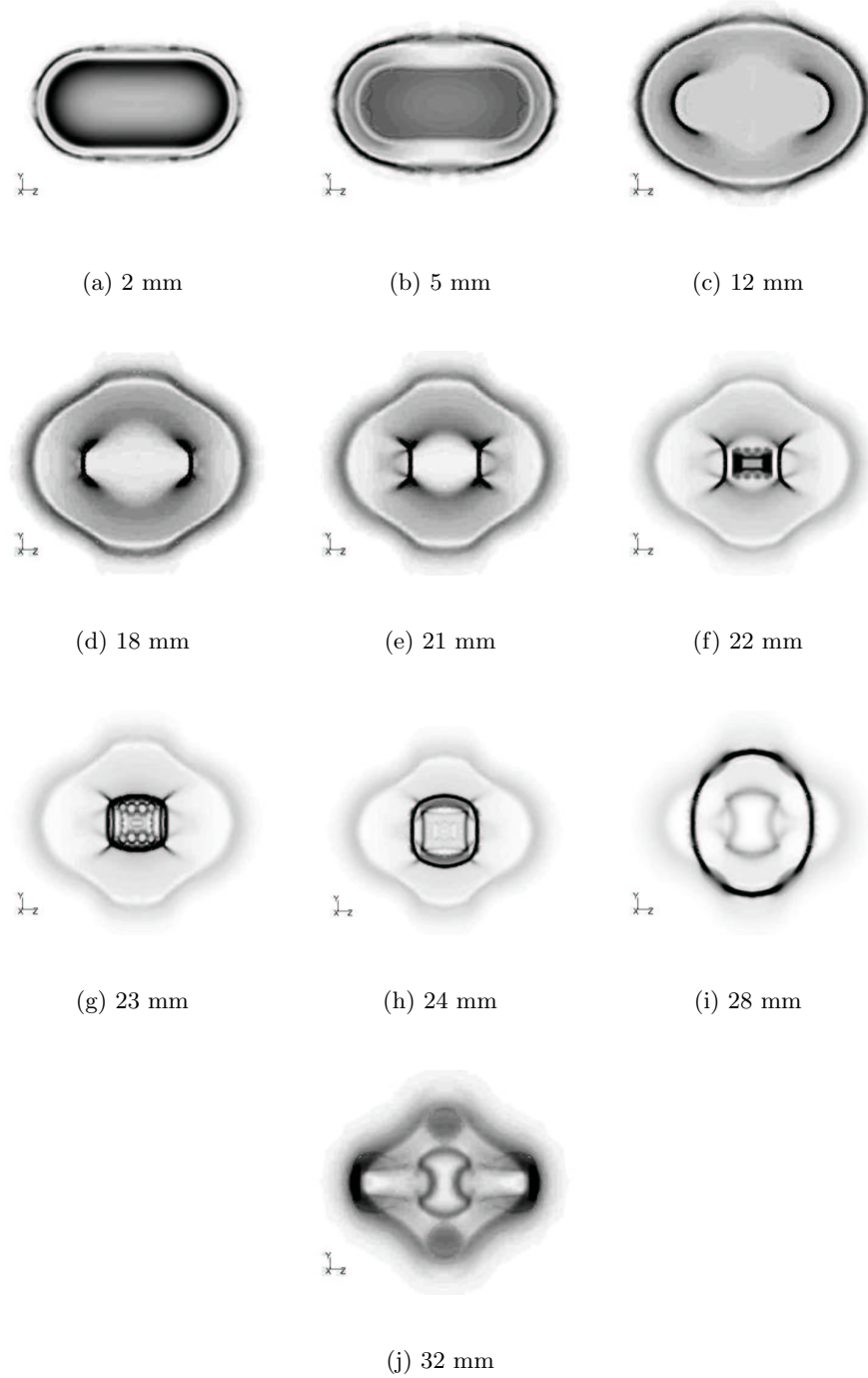


Figure D.20: Slot nozzle with parallel exit flow, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

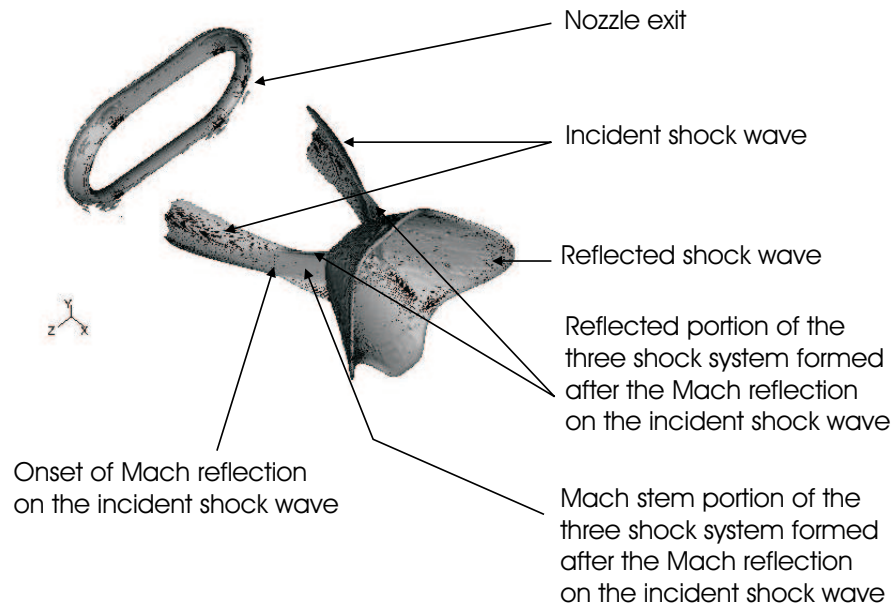


Figure D.21: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

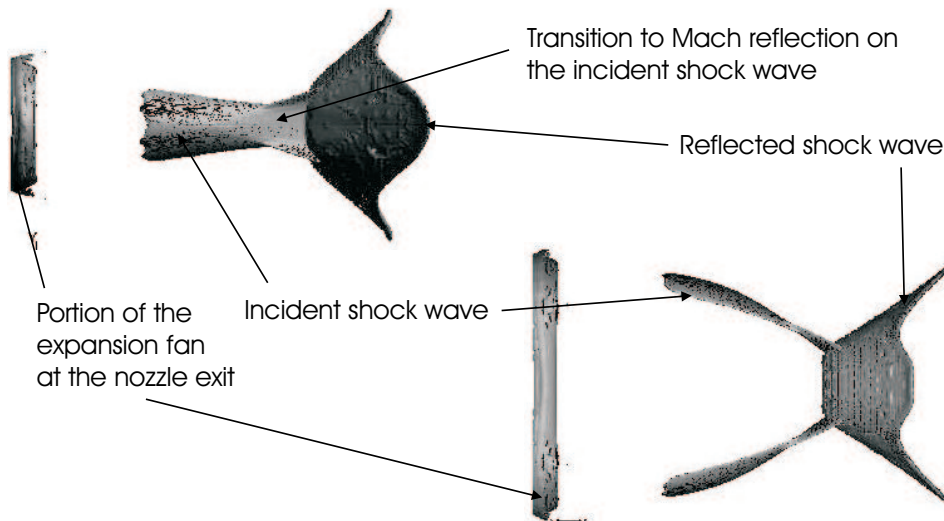
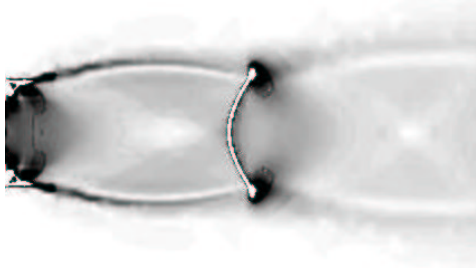
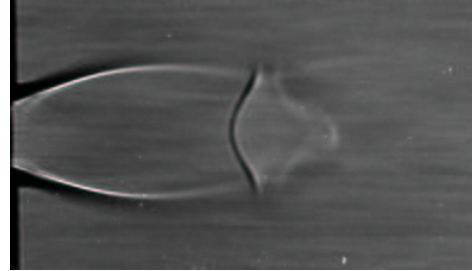


Figure D.22: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 2$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$, Minor axis view on the upper left, Major axis view on the lower right

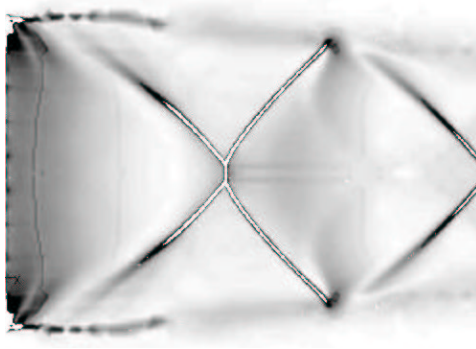
D.3 Nozzle Aspect Ratio = 3



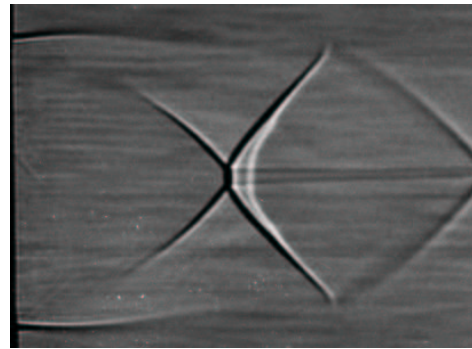
(a) Numerical Schlieren - Minor axis



(b) Experimental shadowgraph - Minor axis



(c) Numerical Schlieren - Major axis



(d) Experimental shadowgraph - Major axis

Figure D.23: Slot nozzle with a 45° inlet angle, images along the major axis of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

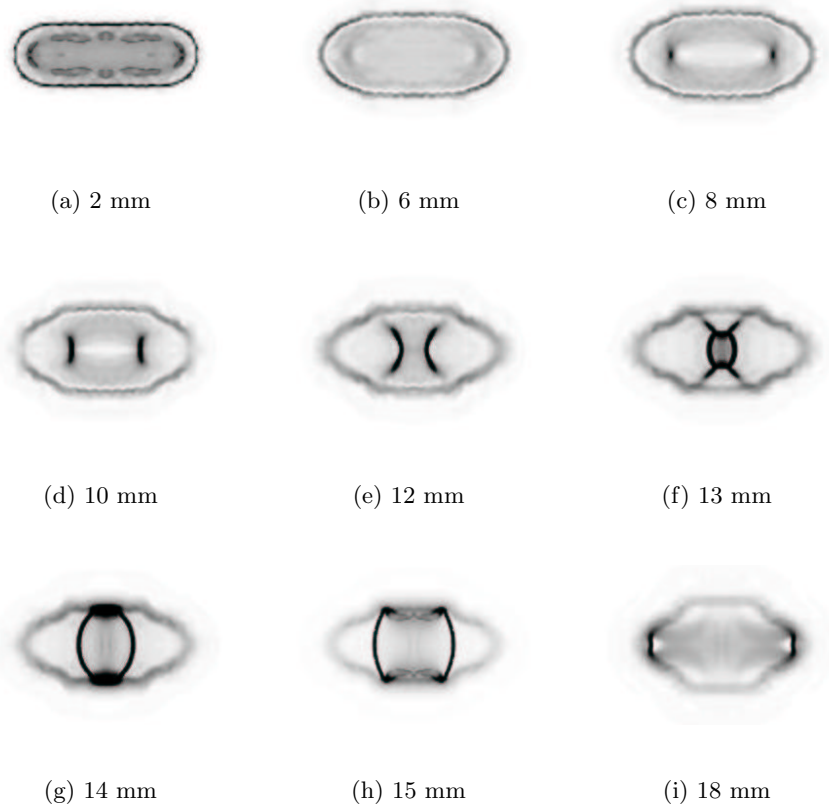


Figure D.24: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

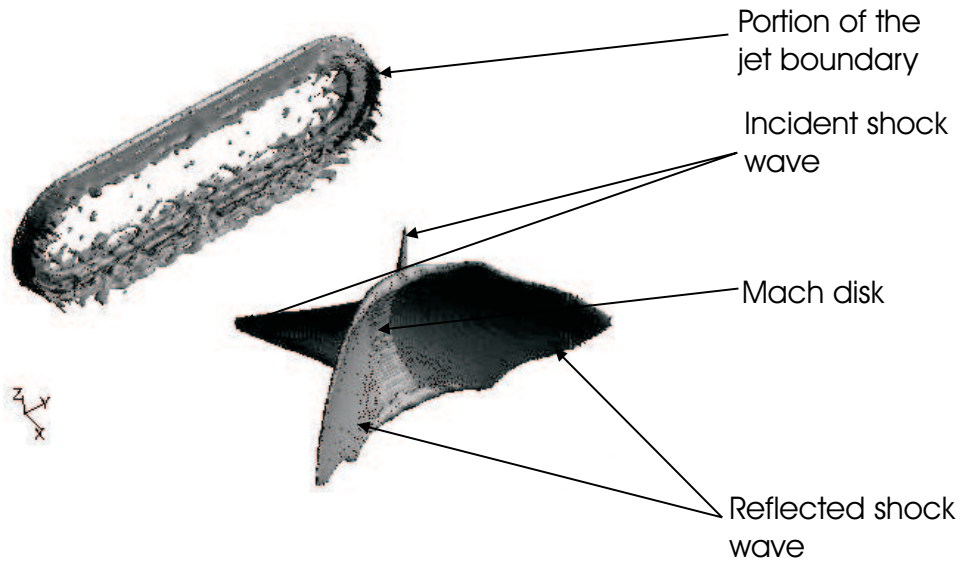


Figure D.25: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

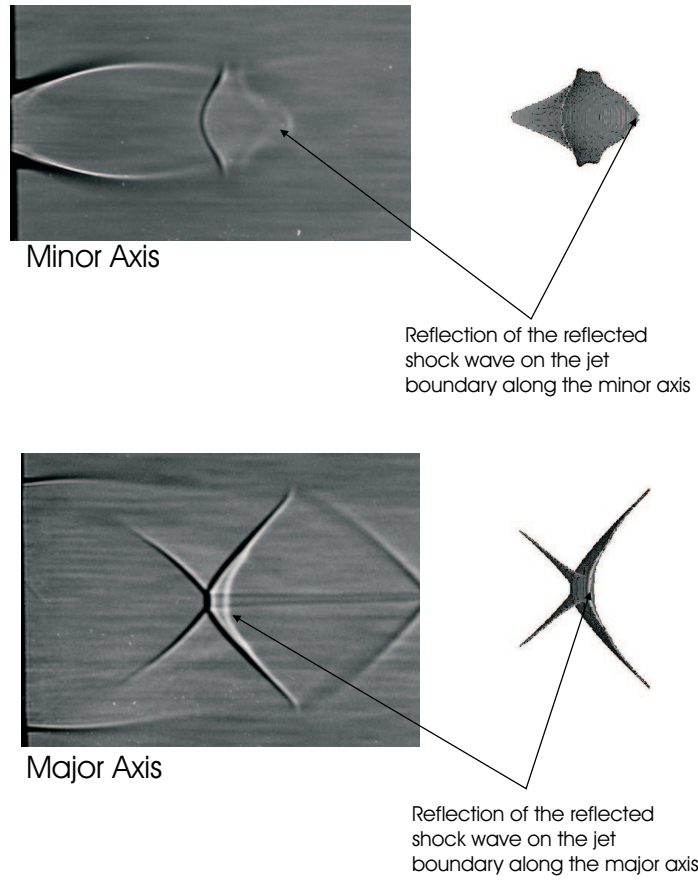


Figure D.26: Explanation of the additional waves present in the experimental images of the flow from a slot nozzle with a 45° inlet angle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

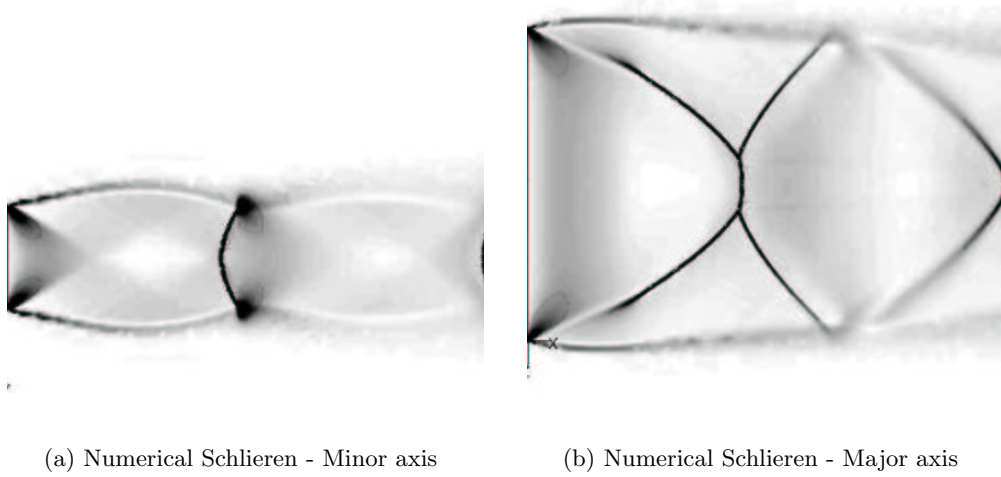


Figure D.27: Slot nozzle with a parallel flow exit, images along the major axis of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

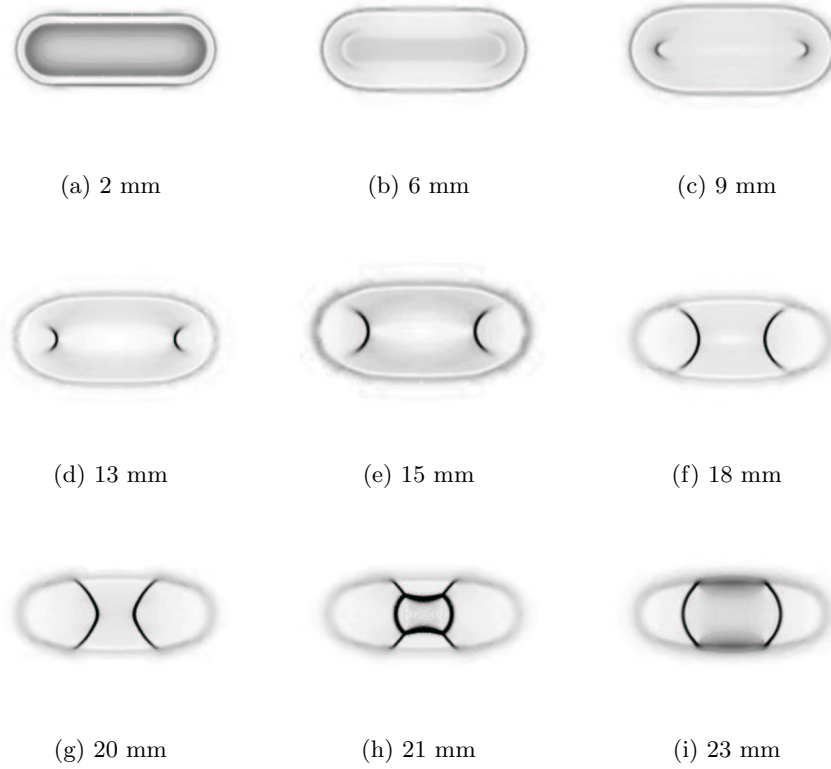


Figure D.28: Slot nozzle with a parallel flow inlet, cross sectional schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

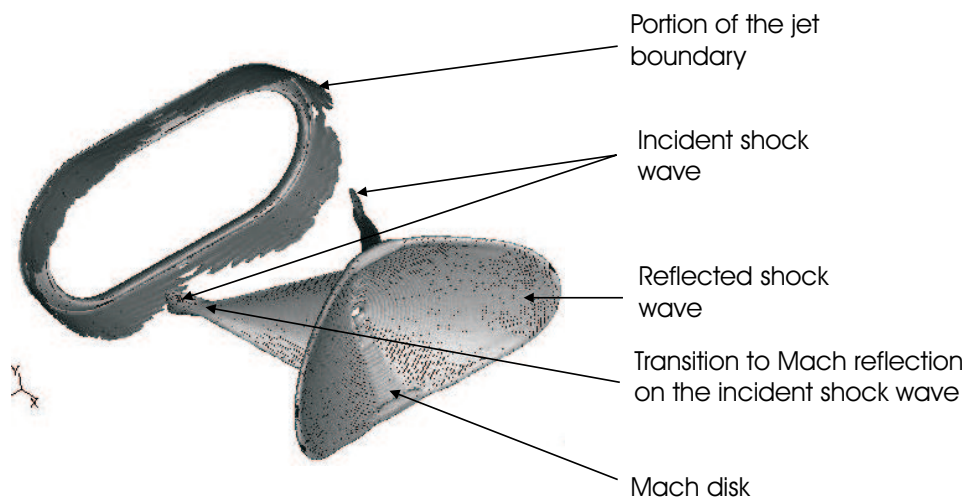
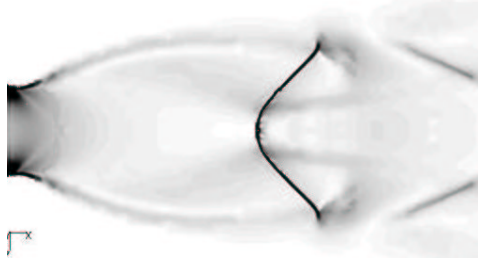
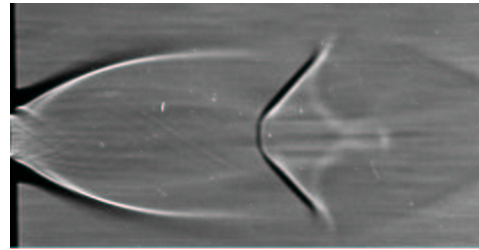


Figure D.29: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$



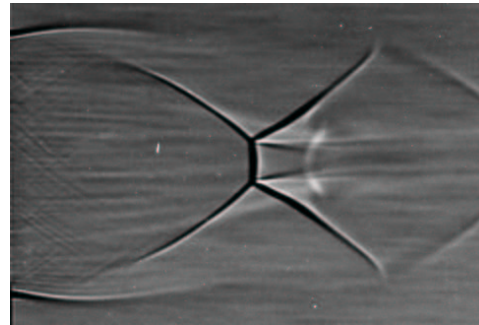
(a) Numerical Schlieren - Minor axis



(b) Experimental shadowgraph - Minor axis



(c) Numerical Schlieren - Major axis



(d) Experimental shadowgraph - Major axis

Figure D.30: Slot nozzle with a 45° inlet angle, images along the major axis of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

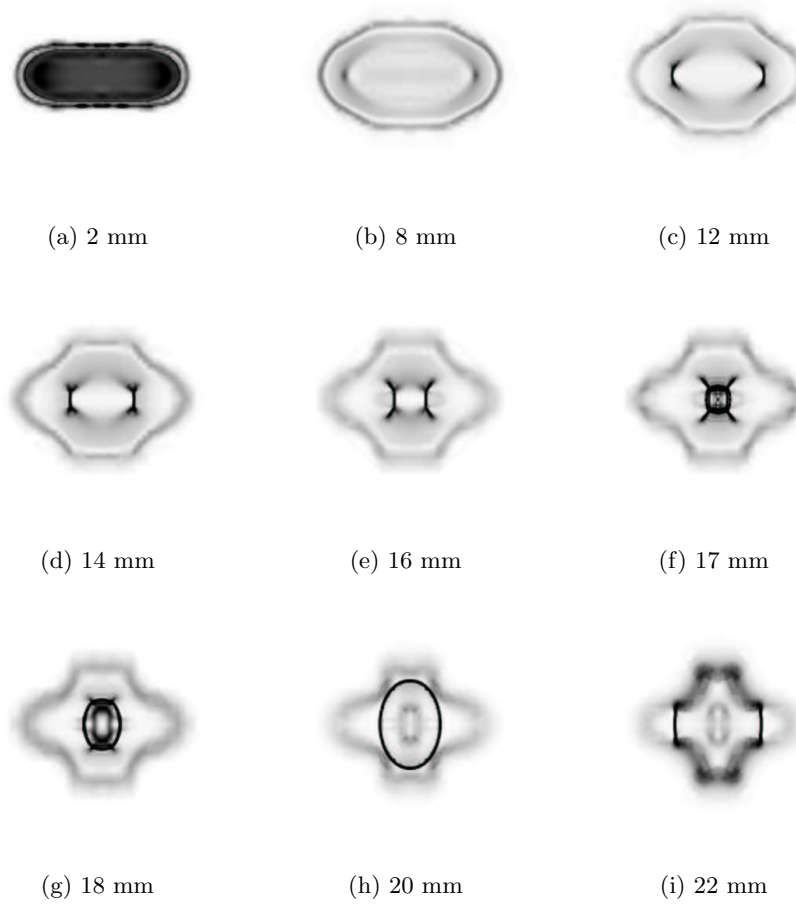


Figure D.31: Slot nozzle with a 45° inlet angle, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

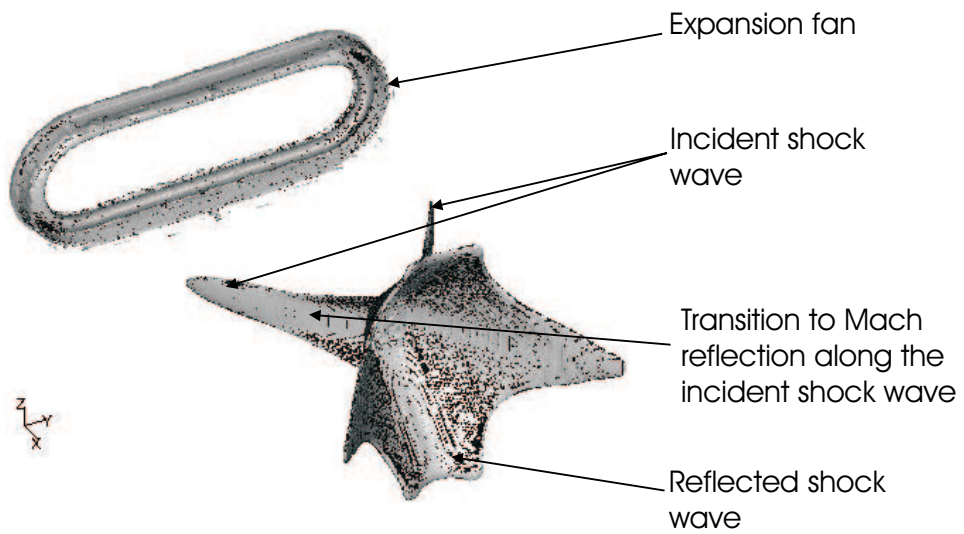


Figure D.32: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a 45° inlet angle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

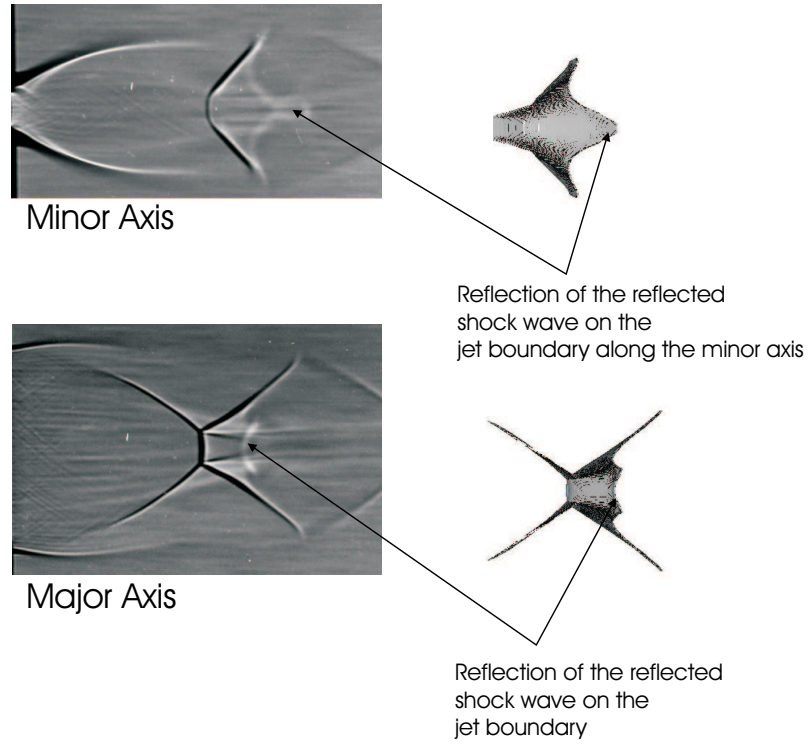


Figure D.33: Explanation of the additional waves present in the experimental images of the flow from a slot nozzle with a 45° inlet angle, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

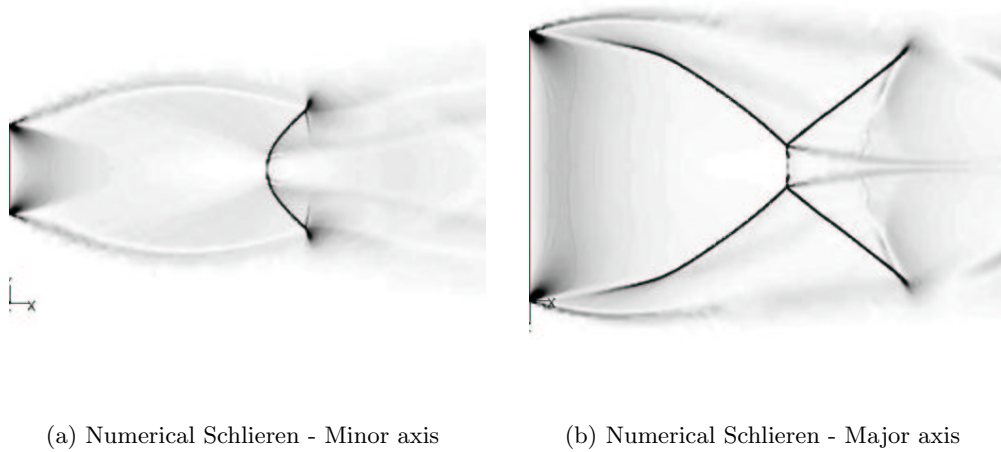


Figure D.34: Slot nozzle with a parallel flow exit, images along the major axis of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

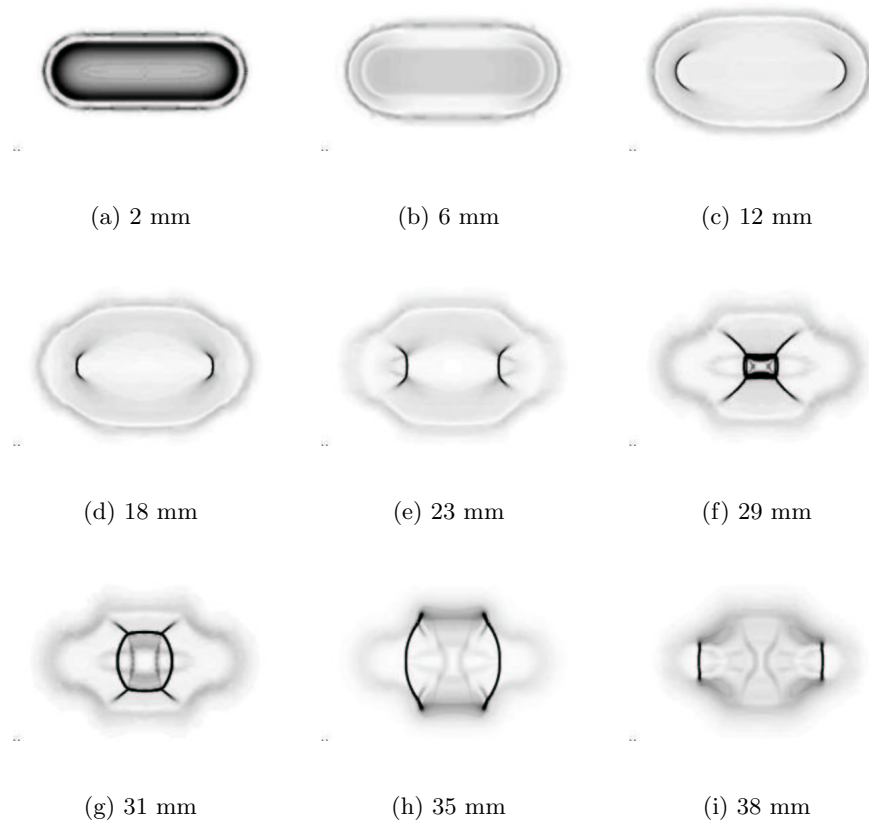


Figure D.35: Slot nozzle with a parallel flow inlet, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

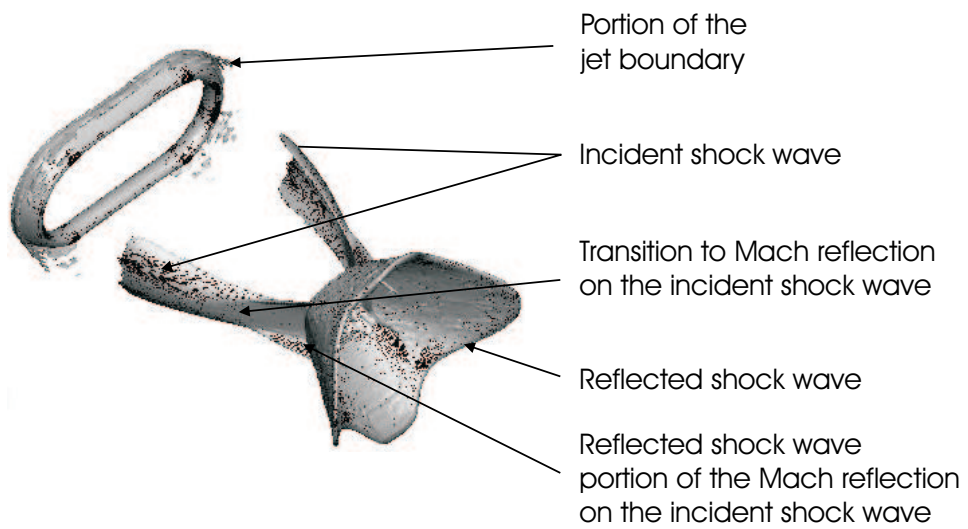


Figure D.36: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 3$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

D.4 Nozzle Aspect Ratio = 4

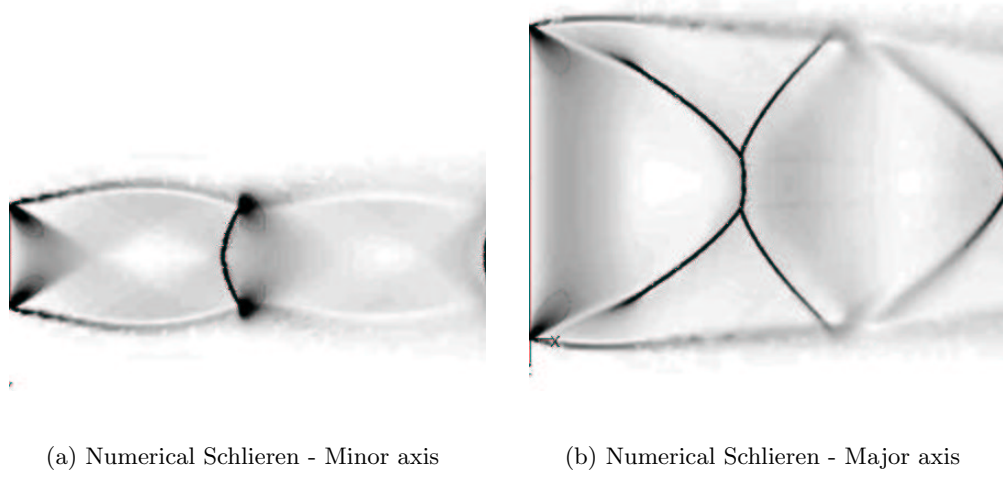


Figure D.37: Slot nozzle with a parallel flow exit, images along the major axis of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

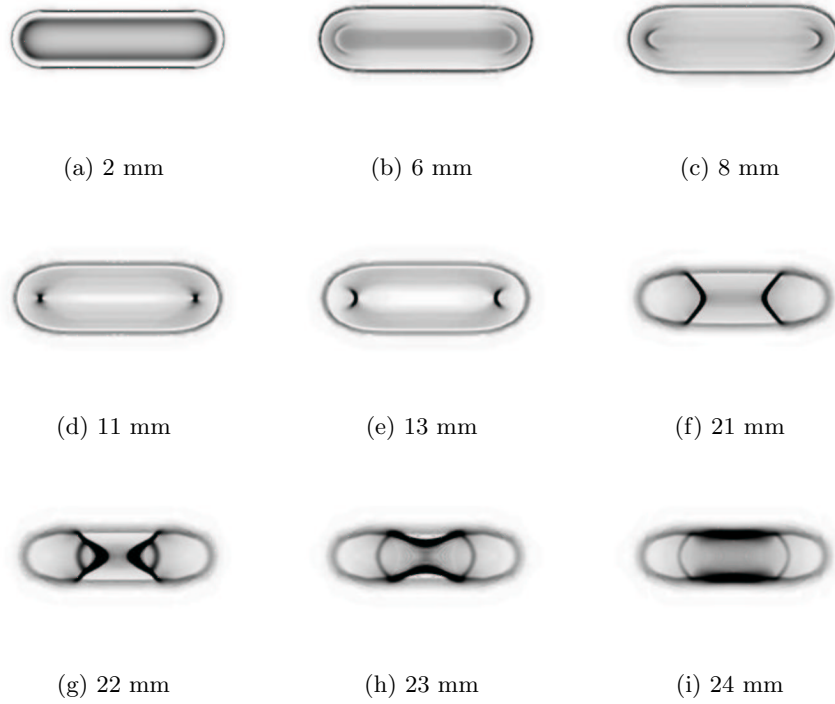


Figure D.38: Slot nozzle with a parallel flow inlet, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

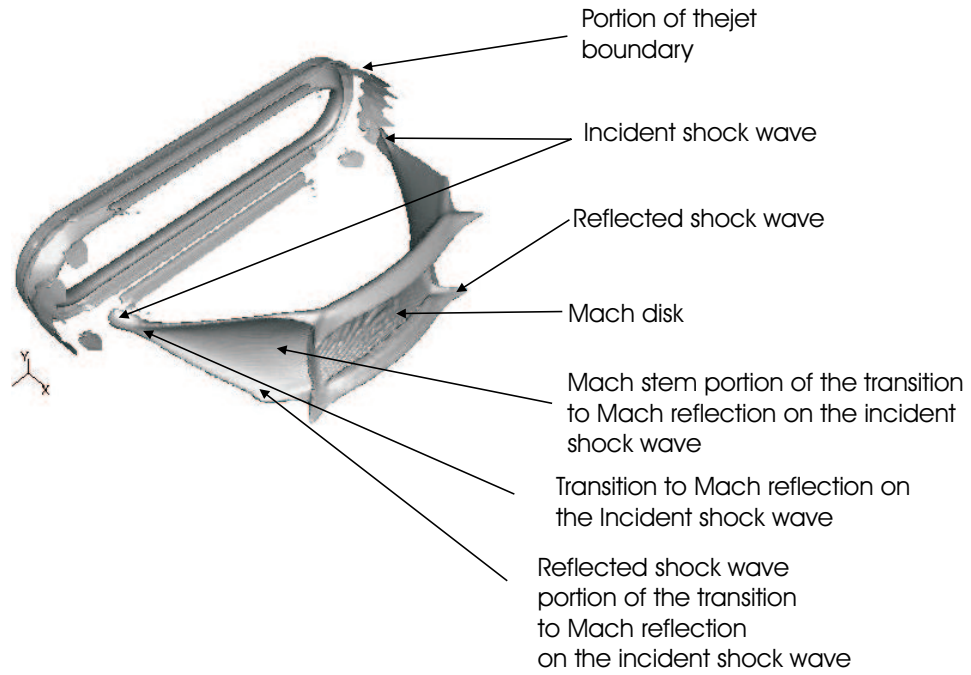


Figure D.39: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

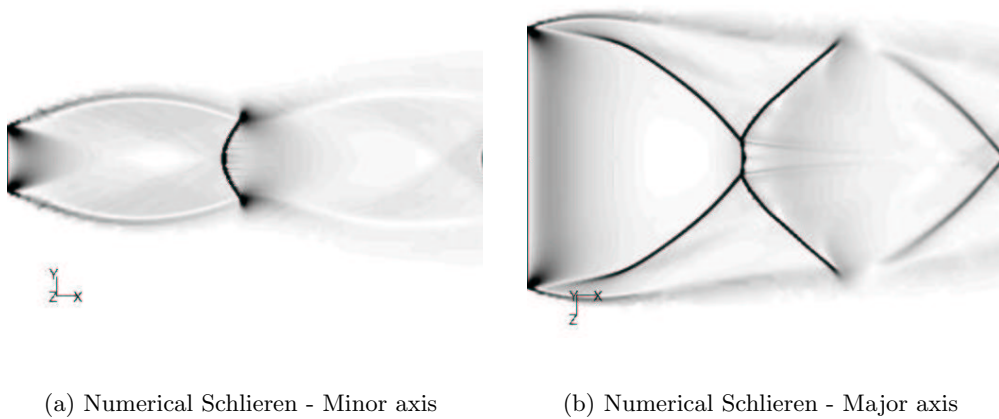


Figure D.40: Slot nozzle with a parallel flow exit, images along the major axis of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$

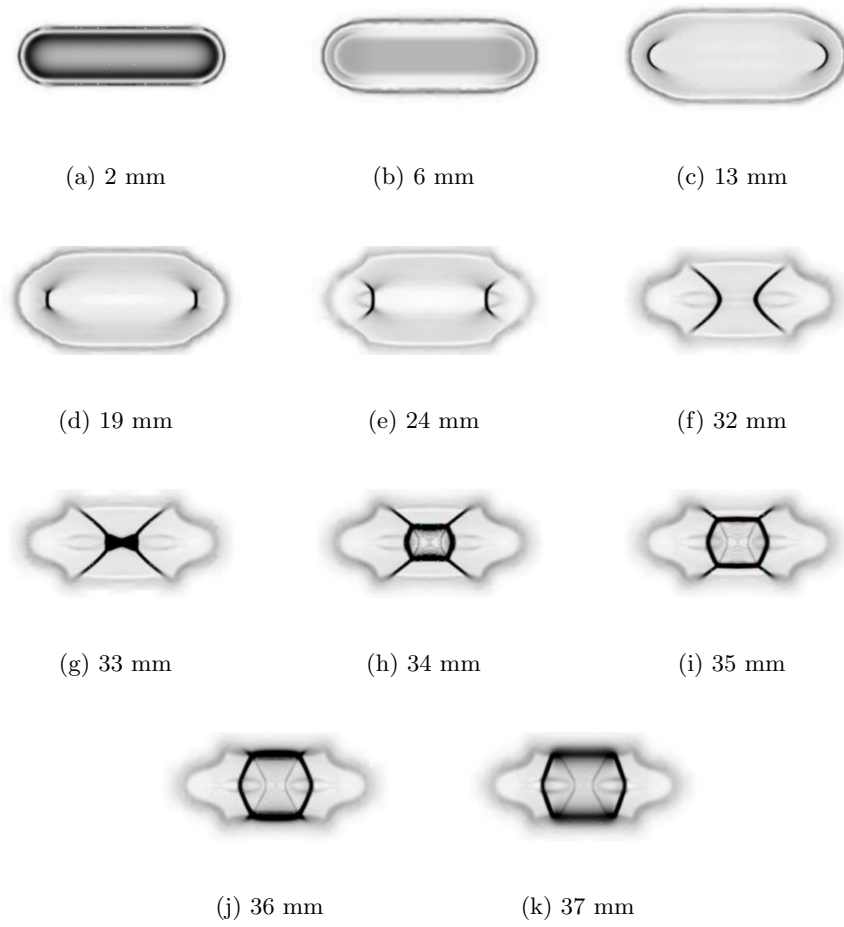


Figure D.41: Slot nozzle with a parallel flow inlet, cross sectional numerical schlieren images downstream of the nozzle exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 2$

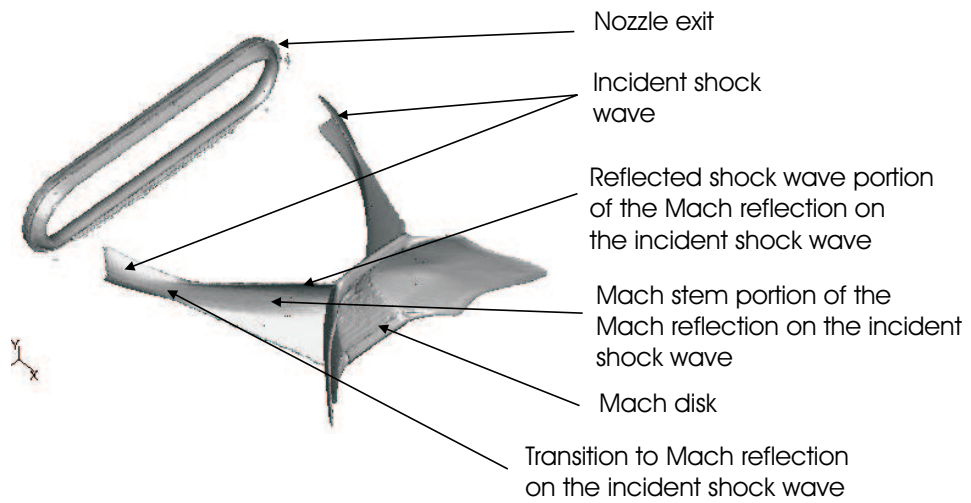


Figure D.42: 3-dimensional representation of the flow field for the underexpanded jet from a slot nozzle with a parallel flow exit, $AR = 4$, $M = 1$, $\frac{P_{exit}}{P_{ambient}} = 3$