# THE ROLE OF TECHNOLOGY IN THE ZONE OF PROXIMAL DEVELOPMENT AND THE USE OF VAN HIELE LEVELS AS A TOOL OF ANALYSIS IN A GRADE 9 MODULE USING GEOMETER'S SKETCHPAD

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# ABSTRACT

In 2010 a course called MathsLab was designed and implemented in a Johannesburg secondary school, aimed at Grade 9 learners, with the objective of using technology to explore and develop mathematical concepts. One module of the course used Geometer's Sketchpad to explore concepts in Euclidean geometry. This research report investigates whether technology can result in progression in the zone of proximal development as described by Vygotsky. Progression was measured through the use of a pre- and post-test designed to allocate Van Hiele Levels of geometric thought to individual learners. Changes in the Van Hiele Levels could then verify movement through the zone of proximal development.

The results of the pre- and post-tests showed a definite change in learners' Van Hiele Levels, specifically from Van Hiele Level 1 (visualisation) to Van Hiele Level 2 (analysis). This observation is in line with research that places learners of this age predominantly at these levels. Some learners showed progression to Van Hiele Level 3 (ordering) but this was not the norm. The value of using technology in an appropriate and effective manner in mathematics education is clear and is worthy of further research.

**DECLARATION**

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

________________________________________________

Karen Hulme

_________ day of _______________ in the year __________

## DEDICATION

To my father

Arbutus Johannes Janse van Rensburg

from whom I inherited a passion for mathematics

and in loving memory of my mother

Pamela-Ann Janse van Rensburg

from whom I inherited a deep love of learning.

## ACKNOWLEDGEMENTS

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# 1. INTRODUCTION

## 1.1. Context

This study encompasses research in the field of technology in mathematics education. In particular, the study considers the zone of proximal development (ZPD), as originally described by Lev Vygotsky (Vygotsky, 1978) in the first part of the twentieth century, as well as possible effects that technology might have on the model of geometric thought known as the Van Hiele Levels (VHL) - a theory which distinguishes "five discrete thought levels in respect to the development of pupils' understanding of geometry" (de Villiers, 2010, pg. 1). The study investigates whether a particular use of technology (Geometer's Sketchpad) can be effectively used as a mediating tool in the ZPD and attempts to track possible movement of individual learners through the ZPD. An adaptation of the Van Hiele Levels is used as a tool of analysis of progress through the ZPD.

## 1.2. Research objectives and questions

To this end, a technology-based course that has been developed and implemented in the high school in which I am currently employed is considered. The course forms part of a year-long computer-based instruction programme for Grade 9 learners, aged between 14 and 16 years old. This course, called MathsLab, aims to use computers to explore mathematical concepts. The specific learning outcomes and assessment standards covered in the geometry module of MathsLab are given in Appendix 1.

The school is a well-resourced, independent school, situated north of Johannesburg in an affluent area. The school is coeducational and the majority of learners (from all race groups and with a number of non-South Africans) can be classified as privileged. Classes are small and teachers are well-qualified, experienced and enthusiastic. The work ethic among the learners at the school is strong and mathematical results in terms of international benchmark tests and the final Grade 12 examinations are excellent. The use of technology in the school is actively encouraged and all learners have access to state-of-the-art computer centres.

The aim of MathsLab is two-fold:

- to expose learners to computer technology, and
- to encourage the mathematical processes of exploration, conjecturing, reasoning and proof.

The course comprises two modules. The first uses Microsoft Excel spreadsheets to explore number patterns and for application to simple and compound interest problems. The second module, used in this study, uses Geometer's Sketchpad (GSP) to introduce learners to the concepts of drawing shapes, identifying and defining shapes, sorting shapes (hence classification of shapes), establishing the properties of quadrilaterals and using formal, deductive reasoning. The tasks in the module have been specifically chosen to reflect the descriptions of the Van Hiele Levels as presented by Burger and Shaughnessy (1986, pg. 34) and are explained in more detail in the chapter on Methodology. The advantage of using dynamic geometry software such as GSP in a study which is examining progress through the ZPD is that instructional activities and tasks can be "designed to emphasise students' learning through explorations instead of teaching a specific mathematical content" (Olkun, Sinoplu & Deryakulu, 2005, pg. 88) – this was the philosophy that was followed in the design of the module. The course was implemented for the first time in 2010.

Learners were given pre- and post-tests to write in an attempt to ascertain their Van Hiele Level before and after the implementation of the module. The primary aim of this research study is to observe whether technology in the form of dynamic geometry software could mediate within the ZPD. The instrument of analysis as an attempt to determine this is an adaptation of the Van Hiele Levels which has drawn significantly on the work of Burger and Shaughnessy (1986). A rubric was designed and used to assign Van Hiele Levels to each learner in both the pre- and post-tests. Specifically the critical question of the research is:

- Do the Van Hiele Levels of learners change when measured before and after the implementation of the MathsLab module?

Research of this sort is exciting and relevant, and opens the door to further research and investigation, e.g.

- How do GSP and MathsLab mediate in the ZPD?
- What are the roles of GSP and the MathsLab module in the ZPD of different learners?

It must be noted, however, that both of these latter research questions are beyond the scope of this research project.

## 1.3. Outline of the research report

Chapter 2 looks briefly at the reasons for undertaking a study looking at an alternative methodology using technology in the teaching of Euclidean geometry. The advantages of including this type of geometry in curricula are outlined, as well as government policies with regard to the inclusion of geometry in the South African school context over the last 10 years. The development of dynamic geometry software (specifically Geometer's Sketchpad) is considered as well as the merits of using such software. The importance of geometric reasoning as a skill to be developed in learners is particularly examined.

In Chapter 3 the theoretical framework for this study is explained. There are three areas that are studied, viz. the use of technology in education and semiotic mediation, the theory propounded by Lev Vygotsky and particularly the concept of the zone of proximal development, and finally the model of geometric thinking developed by the van Hieles in the 1950s and expanded upon subsequently.

Because the instrument being used to test whether there has been any improvement in learners' geometry understanding is based on the van Hiele model, Chapter 4 is a literature review looking at other studies that used the Van Hiele Levels to determine geometric ability. The initial studies of the early 1980s are considered in some detail, as these studies developed the first techniques and instruments to test Van Hiele Levels, and the chapter concludes with a look at more recent literature.

Chapter 5 outlines the methodology and method of this study. The study made use of a per- and post-test to determine the Van Hiele Levels of the learners before they were exposed to the GSP module, and once the module had been concluded. The test is explained, and the methods used to collate the results are given. Issues of reliability, validity and ethics are also considered in this chapter.

In Chapter 6 I look at some of the limitations of the study – both anticipated limitations as well as those that emerged when the results were being analysed. Analysis of the data is given in Chapter 7, with the pre-test and the post-test results being compared. The final chapter of the study, Chapter 8, summarises the findings and discusses the results.

## 2. RATIONALE

The need for and advantages of teaching geometry, including Euclidean geometry, are important, particularly in terms of the development of reasoning and proof skills in learners. Further, teaching geometry "trains the mind in clear and rigorous thinking" (Bursill-Hall, 2002, pg. 3). According to Marrades and Gutiérrez (2000), there is general agreement on the educational value of developing mathematical proof skills, and the specific objectives of mathematical proof are

> *to verify or justify the correctness of a statement, to illuminate or explain why a statement is true, to systematise results obtained in a deductive system (a system of axioms, definitions, accepted theorems, etc.), to discover new theorems, to communicate or transmit mathematical knowledge, and to provide intellectual challenge to the author of a proof.*
>
> Marrades & Gutiérrez; 2000, pp. 87 - 88

In many countries, "geometry is the first course in which students are introduced to and expected to practise deductive reasoning, which sometimes includes proof" (Hollebrands, Laborde and Strasser; 2008, pg. 157). In South Africa (as with many other countries, e.g. Namibia), the value of Euclidean geometry and its emphasis on formal proof and reasoning processes were downgraded in the current curriculum, resulting in it being placed in the optional Paper 3 of the Further Education and Training phase (Grades 10, 11 and 12). This placement as an optional course de-emphasised the value of the topic. However, the National Curriculum Statement for Grades 10 – 12 is under review, and will be incrementally replaced in South Africa, starting in 2012. The new document is called the National Curriculum Statement Grades R – 12 and the current Subject Assessment Guidelines (SAGs) will be replaced by a Curriculum and Assessment Policy Statement (CAPS). An interesting feature of the new curriculum is the re-introduction of Euclidean geometry as one of 10 main content areas and the proposal that it will form one-third of Paper 2 in the final examinations. Importantly, one of the specific aims listed in the document for mathematics is "to provide the opportunity to develop in learners the ability to be methodical, to generalise, make conjectures and try to justify or prove them" (DBE, 2011, pg. 11).

This has come at a time when very exciting work has been developed using dynamic geometry software such as Geometer's Sketchpad (GSP). GSP is a software package that enables learners to discover geometric concepts through exploration and experimentation. Explorations of the properties of geometric shapes and configurations are facilitated by the dynamic features of the software. Figures can be dynamically transformed while preserving important geometric relationships. GSP thus "encourages a process of discovery where pupils or students first visualise and analyse a problem, and make conjectures before attempting a logical explanation (proof) of why their observations are true" (De Villiers, 1998, pg. 11).

This clearly has implications for the teaching of Euclidean geometry, as "mathematics should be a human activity in which the process of guided invention takes the learner through the various stages and steps of the discovery of mathematical ideas and concepts" *(*Mudaly, 2007, pg. 64). Research suggests that there are many reasons for the poor performance of learners in geometry, particularly in formal proofs, one of which is that "learners may be working at the incorrect Van Hiele Level when attempting proof" (ibid, pg. 65) and that many school curricula "focus on having students learn lists of definitions and properties of shapes" (Olkun, Sinoplu & Deryakulu, 2005, pg. 1). It is thus important for further research to be done on ways to improve the understanding of geometry by learners and on ways to facilitate the development of the Van Hiele Levels. The use of technology provides one such instrument, which is both visual and relevant, to achieve these aims.

Research suggests that reasoning needs to be developed in learners in order to enable them to begin the processes involved in mathematical proof (Brodie, 2000; Kilpatrick, Swafford, and Findell, 2001; Portnoy, Grundmeier and Graham, 2006; Evans, 2007). Simply put, the acquisition of reasoning skills is necessary for the development of proof skills, as opposed to just learning proofs. De Villiers (2003) has done research into the role and functions of proof in a learning context, and believes that "we need to see proof as part of mathematical activity, as formulating, justifying, and proving conjectures" (De Villiers, 2003, pg.76). The reason proof is important is because it "helps to convince, communicate and explain mathematical statements and conjectures" (ibid, pg. 76). The development of formal geometric reasoning as a mathematical process in learners is thus important and needs to start with playful exploration of concepts before proceeding to formal proof processes. This is emphasised by Brodie (2000, pp. 43 - 87), who states: "the need for a proof is a process, which probably first needs a lot of informal reasoning**.** We should probably do more work on informal reasoning with learners, at all levels, before we introduce them to formal proofs". Informal reasoning incorporates the mathematical processes of exploration, conjecturing, reasoning and proof.

All these processes can be facilitated using appropriate technology. Evans (2007, pg. 38) has noticed that "pupils tend to base their confidence on a finite number of cases, being able to show by example but not to create a generalised argument". With GSP, learners are theoretically able to explore very large numbers of examples. They will also notice that other learners work with different examples. This should enable them to see the possibility of the generalisability of concepts more readily: however, although both generalisability and the ability to develop longer sequences of statements in a proof are important mathematical skills, they are usually only evident in learners who have reached the higher levels of reasoning (de Villiers, 1996, pg. 9).

The use of dynamic geometry software has revolutionised the teaching of Euclidean geometry in recent decades (ibid, pg. 25), but there is little evidence of a corresponding improvement in learners' geometric abilities. It is possible that learners' poor performance in geometry can be analysed in terms of the Van Hiele Levels. Many learners have under-developed visualisation skills and, certainly in the past, were introduced to formal proof-writing too early. Paper-and-pencil methods of developing geometric concepts have been avoided by many teachers, due both to the time-consuming nature of such tasks, and to the fact that figures drawn in this manner remain static. In packages such as GSP, geometric figures can be constructed and manipulated with ease and speed. Important explorations and concepts are facilitated by this, e.g. "the software allows one to easily repeat experiments in many different orientations and thereby checking which geometric properties stay invariant" (ibid, pg. 26). GSP is a valuable and probably grossly under-utilised tool and one of the aims of this research is to see how it can be effectively integrated into a teaching environment.

The use of GSP is also appealing to learners, as it represents a break from a conventional classroom situation. McClintock, Jiang and July (2002, pp. 739 – 754) reported on a series of studies undertaken in Florida, U.S.A., where they investigated the development of Van Hiele Levels in middle and high school learners using GSP. They noted that the learners had overwhelmingly positive attitudes:

> *They found it interesting and dynamic. They found it a friendly environment in which they could play and experiment. Using GSP to learn geometry in an experimental fashion has contributed to the subjects' enthusiasm for geometric exploration. In some sense, GSP was not considered a geometric environment so much as an artistic and creative environment by these subjects.*
>
> McClintock et al, 2002, pg. 744

# 3. THEORETICAL FRAMEWORK

## 3.1. Introduction

There are a number of different aspects of research that need to be explored in a study of this nature. The use of technology needs to be examined in a number of ways – technology in general, and GSP specifically. The role of *instrumental genesis* is an important aspect that needs to be considered and links directly to the advent of technology in education. The link between technologies and *semiotics* must also be considered, specifically in terms of semiotic mediation on the part of a knowledgeable other. This links directly to the Vygotskian notion of the ZPD. The impact of technology on actual learning and its role in sociocultural contexts can be considered, with particular reference to the notion of the ZPD and the development of the Van Hiele Levels in learners. It is also important to look at both the computer and GSP as semiotic mediators. An extensive literature search was undertaken to investigate these aspects of the research. As a result of this, a large part of the literature review is covered in the theoretical framework.

This research study specifically wanted to investigate the development of geometric abilities in learners using technology, and used the Van Hiele Levels as a tool to analyse the learners' grasp of concepts before and after the implementation of the module. Research has shown that "most students who finish secondary school achieve only the first or second Van Hiele Level, and … progress from the second level to the fourth level is very slow" (Marrades & Gutiérrez, 2000, pg. 87). It is thus crucial to identify the Van Hiele Levels of each learner before and after the implementation of the module. For this purpose, drawing substantially on the work of Burger and Shaughnessy (1986, pp. 34 – 45), an instrument was developed to be used as a pre- and post-test (see Appendix 2), and this instrument is the focus of this research study. In particular, the instrument has been developed as an analytic tool to tease out learners' knowledge of geometry. Pre- and post-testing has the advantage of assessing the progress of each individual learner with a high degree of reliability (Scaife, 2004, pg. 67). The module itself was developed during 2010, and implementation began at the beginning of July 2010.

### 3.2. Technology in education

The implementation of technology in education has been researched extensively but Goos, Soury-Lavergne, Assude, Brown, Kong, Glover, Grugeon, Laborde, Lavicza, Miller & Sinclair (2010, pp. 311 – 328) point out that "the predicted integration of digital technologies into mathematics teaching and learning has proceeded much more slowly" (ibid, pg. 312) than the fast pace of technological change since the 1980s. In fact, researchers "have difficulty in providing improved learning with technological means, as well as in understanding the influence of technology on learning" (Kieran & Drijvers, 2008, pg. 206). This is in spite of government support in many nations around the world, including France: "these difficulties are indeed persistent in France in spite of the continuous governmental support given to integration for more than 20 years now" (Artigue, 2002, pg. 253).

An important feature of technology in mathematics education is that, "as technology involved in mathematics education embodies mathematics, the technical and conceptual parts are intrinsically intertwined" (ibid pg. 245). This implies that the very use of technology in mathematics education "shapes the knowledge constructed by students" (Goos et al, 2010, pg. 313). Some may argue (as in the case of every day ordinary discourse) that the use of technology distracts learners from focussing on the core object of instruction. The point being stressed here is that the use of technology does not occur in a vacuum – it occurs in the context of exploring specific mathematical concepts – and therefore should aid learning.

Artigue (2002) focussed on a number of important issues around the educational use of computer technology, drawing on a large amount of research done by institutions in France. The development of mathematics throughout human history has always depended on the use of both “material and symbolic tools available for mathematical computations” (Artigue, 2002, pg. 245) – from the invention of the abacus, through the development of logarithmic tables, to the widespread use of pocket calculators and computer technology. What is important to realise is that the introduction of new tools throughout history does not mean that the tools immediately become “efficient mathematical instruments for the user” (ibid, pg. 245). Even so, the introduction of computer-based technology has changed many mathematical practices, and this has led to a growing body of research linked to the development of software packages.

There is a tension that exists when considering the introduction of technology as a teaching tool in mathematics. This is because current curricula are primarily concerned with the transmission of concepts of mathematics and mathematical culture. The aim is not to develop mathematical practice assisted by computational tools. The tension largely exists because current software and computational tools have been designed to be “pedagogical instruments for the learning of mathematical knowledge and values which were defined in the past, mostly before these tools existed” (ibid, pg. 246). The tension between secondary school mathematical knowledge and the use of technology in developing that knowledge is reflected in the South African context. There is no explicit suggestion of using technology, other than where the soon-to-be-defunct National Curriculum Statement requires learners to be able to “use science and technology effectively” (NCS, 2003, pg. 12) in general, and “use available technology … in calculations and in the development of models” (ibid, pg. 20) for learners of mathematics in particular. The new proposed CAPS document does not refer to the use of technology explicitly, though both the specific aims and the specific skills allow for the use of technology to develop both aims and skills (CAPS, 2011, pp. 11 - 12), and the generation of graphs using point-by-point plotting with available technology is encouraged.

Artigue approaches research in the field of technology in mathematical education from a socio-cultural perspective (Artigue, 2002, pg. 247), with a focus on an anthropological approach. This is emphasised in the acknowledgement that mathematics is a product of human activity and is thus dependent on social and cultural contexts. This means that mathematical objects are "entities which arise from the practices [praxeologies] of given institutions" (ibid, pg. 247). Practices in this sense refer to tasks, techniques, technologies and theories undertaken in mathematics education. Techniques in particular refer to manners in which tasks are solved, both pragmatically and epistemically.

It is further important to acknowledge that the advance of knowledge, specifically mathematical knowledge, requires the routinisation of techniques, although knowledge based on routine can be considered as low-level knowledge. The ages of the participants in this study, however, suggest that routinisation of techniques is important for internalisation (in a Vygotskian sense) of basic concepts and skills in geometry to be effected.

For Artigue, an instrument is part artefact or object and part cognitive scheme (Artigue, 2002, pg. 250). The object initially has no instrumental value, until a process known as *instrumental genesis* takes place, transforming the object or artefact into an instrument with a purpose. Instrumental genesis involves both the construction of personal schemes of use and the appropriation of existing schemes of use in the development of artefact to instrument. Goos et al (2010) further describe this process of constructing schemes of use for a particular technological tool, showing that instrumental genesis is the bidirectional relationship between a tool and its user. This will be discussed more fully later.

Hoyles et al (2004) attempt to interpret and synthesise the work down by French researchers such as Artigue (2001), Lagrange (1999), Guin and Trouche (1996) and Cuoca (2002) in "the developing theory of computationally mediated mathematical knowledge" (Hoyles, Noss & Kent, 2004, pg. 309). They too considered the process of instrumental genesis, describing it succinctly as "the mutual transformation of learner and artefact in the course of constructing knowledge with technology" (ibid, pg. 309), as well as the issues of orchestration, i.e. problems around the integration of technology in classrooms. Importantly they note that computer technology has a poor educational legitimacy but high social and scientific legitimacies. This has led to "an inevitable vicious circle of disillusion" (ibid, pg. 311).

Drijvers and Trouche (2008, pp. 363 – 391) summarised and examined three essential issues that have developed in the field of technology in education research. These were linked to three important points. The first issue is the need for analysis of how learning is happening in a technology environment, and this is linked to the concept of instrumental genesis. The next two points are of a more practical bent: "the question of how the teacher can organise the teaching in such an environment, and the need to describe the development of resources for professional development of new teaching practices" (Drijvers & Trouche, 2008, pg. 386). These are linked directly to the systems of instruments used by the learners and the metaphor of orchestration. Development of software (e.g. GSP) and learning activities (e.g. MathsLab) can only be effective "if it is *related to its use* in the classroom" (ibid, pg. 387; authors' emphasis).

The instrumental approach itself has been "formed by the ideas of Vygotsky" (Drijvers & Trouche, 2008, pg. 366); the instrument mediates the activity. The tools or artefacts have an active role (they are not just "waiting to be used") and this influences mental processes. The tool or artefact is the "bare tool", which, through the process of instrumental genesis, becomes part of a "valuable and useful instrument which mediates the activity" (ibid, pg. 367). It is important to remember that the tool does not become an instrument until a meaningful relationship develops between the artefact and the user for a specific task. Through the process of instrumental genesis and as the tool becomes an instrument, mental schemes are developed, which organise problem-solving strategies.

Drijvers and Trouche (2008) use the metaphor of an orchestra when discussing the implementation of technology in a mathematics classroom. The process of instrumental genesis is largely an individual process, likened to a solo performer in an orchestra, and can result in different learners developing different mental schemes for the same task. However, these mental schemes are developed in the context of a classroom, and hence instrumental genesis has a social dimension as well. Learners are not “isolated individual(s) facing the world” but are “deeply embedded” in highly structured environments (Hollebrands, Laborde & Sträβer, 2008, pg. 157).

An important factor here is the guidance of the teacher – analogous to that of the conductor of an orchestra. The onus falls on the teacher to ensure that the technological environment has been well-designed, that mathematical situations are created that manage both mathematical and instrumental knowledge, and that a *didactic exploitation system* is established to link the technological environment and the mathematical situations (Drijvers & Trouche, 2008, pg. 381). A didactic exploitation system is:

> *an essential element concerned with making relevant use of the potential resources of a given environment and with achieving both the coordination and integration of the environment components and the mathematical situations* ibid, pg. 381

In the field of mathematical education, there has been a specific worry that computer use can lead to the “rejection of mathematical meanings and discourse” (Hoyles et al, 2004,, pg. 311). Hoyles et al note that the opposition between the technical and conceptual dimensions of any mathematical activity relates directly to instrumental genesis and its complexity. Further, because the process of instrumental genesis is highly complex, there is a need to find ways to connect to current curricula. Hoyle et al emphasise that there is a need to create effective use of technology in classrooms; specifically, it is important to help “learners engage with, develop and articulate understandings of mathematical procedures, structures and relationships *through* [authors’ emphasis] the technology” (ibid, pg. 311). They also note the importance of allocating both time and status to this process – limiting factors in many current mathematics classrooms.

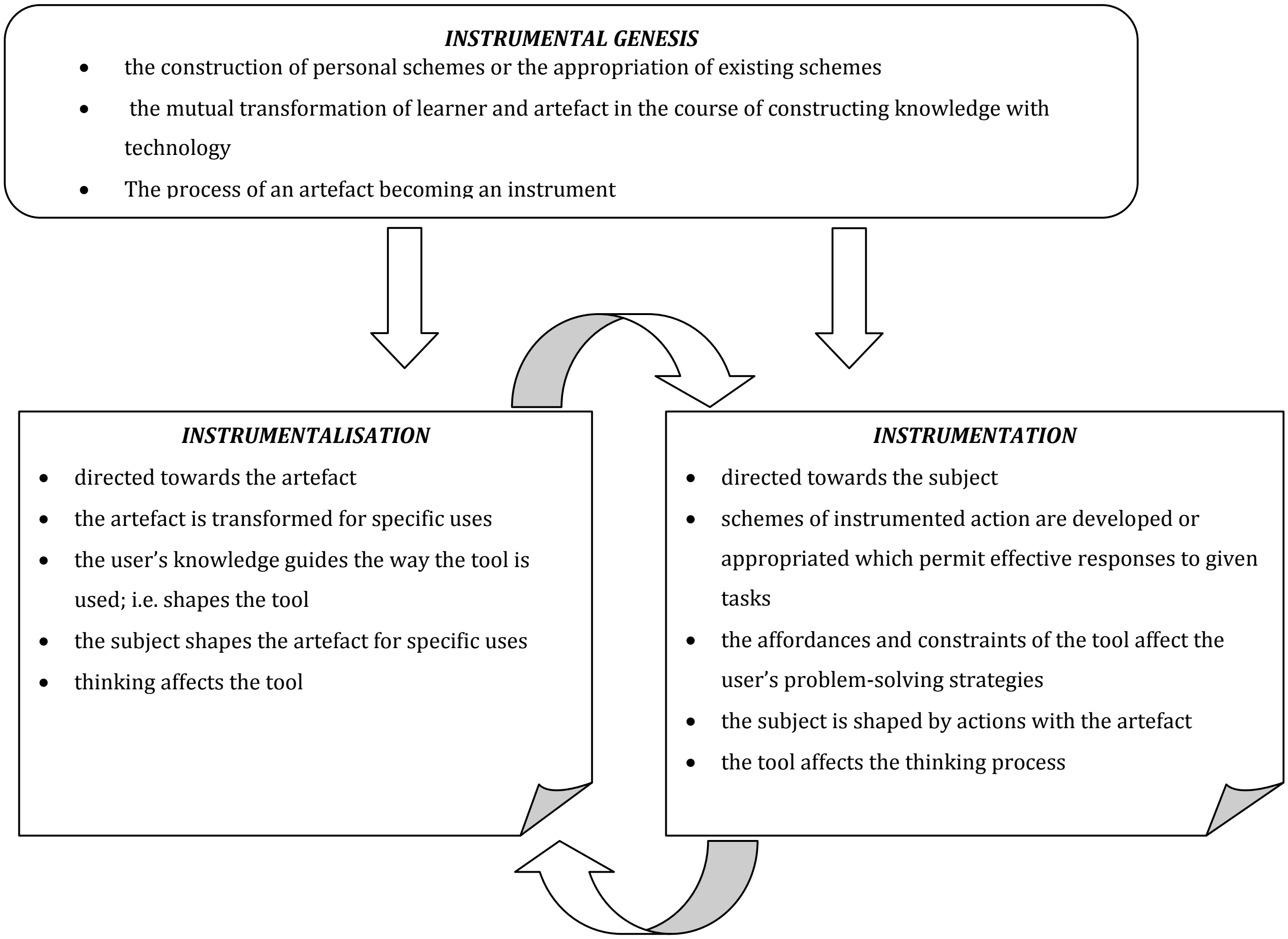
Hoyle and other researchers are particularly interested in how mathematical knowledge is transferred by the presence and use of computers in classrooms. An algebraic example is how learners experience variables by using them as inputs in software such as Logo. In terms of the development of geometric understanding, "conjectures can be tested by making constructions in dynamic geometry systems, thus transforming early encounters with proof from procedural exercises in validation to exploratory exercises in explanation" (ibid, pg. 312). Unfortunately, as Hoyle et al point out, there has been considerable marginalisation of technology by a number of educational institutions, often because the potential that lies in the use of technology has been overstated. The implementation of technology or orchestration is also questioned - "all too often, students are left to their own devices, exploring mathematical tasks for themselves in their computer interactions as teachers struggle to interpret what they do in mathematical terms" (ibid, pg. 313). The important question that needs to be asked when researching the growth of mathematical knowledge in technology-based lessons is whether what the students express is recognised as being mathematical "within the discourse of the [particular] institutional learning system for mathematics" (ibid, pg. 313).

Clearly the process of instrumental activity or instrumental genesis needs to be understood. Hoyle et al describe this as "activity that employs and is shaped by the use of instruments" (ibid, pg. 313). When an attempt is being made to integrate technology into classroom situations, the process of instrumental genesis needs to be fostered. Mathematical knowledge needs to be specifically developed.

Hoyle et al discuss at some length the socio-cultural idea of the formation of collaborative communities in classrooms. Their research indicates that "computer-based collaboration … has been found to encourage a shift in relationships between teacher and student and … enhanced task-based interactions between students and students and teachers" (ibid, pg. 318). This is one of the explicit aims of the MathsLab course at my school. This means that technological tools become not only cognitive tools, but also "genuine mediator(s) of social interaction through which shared expressions can be constructed" (ibid, pg. 318).

At this stage, it becomes important to analysis more closely the concept of instrumental genesis. There are two sides to the process of instrumental genesis – how the artefact is shaped by the user, called *instrumentalisation,* and how the user is shaped by the artefact, called *instrumentation*. I have developed the following illustration as a means to understand the difference between and the interdependence of the two processes, drawing on the work of Artigue (2002), Hoyle et al (2004), Kieran and Drijvers (2006) and Goos et al (2010).

**Figure 1: The processes of instrumental genesis**




Simply: with instrumentalisation, thinking affects the use of tool, and the result is that the technological artefact becomes a tool for learning. On the other hand, with instrumentation, the tool affects the thinking processes that are developed as a result of the use of the tool. Instrumental genesis is thus the process by which a new technological artefact (or an existing artefact used in a new way) becomes a functional instrument with which learners can do mathematics. This can particularly be illustrated using dynamic geometry software:

> *The drag mode can be seen as an instrument to identify geometrical properties of a figure. The pupil must learn how to drag points (instrumentalisation), which is rather easy, but*

*also why to drag points (instrumentation), which is strongly related to his/her conceptualisation of geometrical properties. Classroom observations have revealed that several weeks are needed until pupils decide to drag points on their own with a mathematical intention and not only to see objects moving. This gives evidence of the instrumental genesis and the need for the teacher to support it.*

Goos et al, 2010, pg. 313

The tool needs to become an instrument for learning, but at the same time, "the possibilities and constraints of the artefact shape the techniques and conceptual understanding of the user" (Drijvers and Trouche, 2008, pp. 368 - 369).

Another useful concept discussed by Hoyle et al is that of "situated abstraction", which can be used in the analysis of developing mathematical knowledge in learners. Learners interacting with a microworld (e.g. the MathsLab module) "reconstruct the knowledge underlying the microworld by modelling the phenomena observed on the screen of the computer" (Hollebrands et al, 2008, pg. 175). A "situated abstraction" is thus the invariant that has been constructed by the learner and shaped by the specific situation. Different situated abstractions in a community of mathematical learners can be aligned through negotiation around a "boundary object" (a shared object which acts as a means for discussion within communities of practice without one necessarily adopting the perspectives of others [Hoyle et al, 2004, pg. 320]). In the MathsLab course, the Excel module and the GSP module become the boundary objects. Situated abstractions become "the concrete (visible and audible) expression of the different communities' views of the boundary object" (ibid, pg. 321) and mutual negotiation and meaning-construction are facilitated through the use of the boundary object. In the context of this study, the boundary object is the GSP module itself, and the design of the module and anticipated interactions should lead to the construction of mathematical meaning for the learners.

Goos et al (2010, pp. 313 – 314) further describe four different modes of technological integration into mathematical teaching, which are summarised in Table 1. These comprise the process of "instrumental integration", which provides a means to describe how conditions for instrumental genesis are presented to learners and how mathematics learning is fostered through the process of instrumental genesis.

**<u>Table 1: Modes of technology integration into mathematical teaching</u>**

*from Goos et al, 2010, pp. 313 - 314*

| Level of instrumental integration | Mode of technology integration | Description |
|---|---|---|
| lowest | Instrumental initiation | • Learning new technology<br>• Focus on how to use the technology<br>• Minimal relation between know-how and mathematical knowledge |
| | Instrumental exploration | • Aim is to improve both know-how and mathematical knowledge<br>• Technology explored through mathematical tasks |
| | Instrumental reinforcement | • Pupils face instrumental difficulties while solving mathematical tasks<br>• Aim is primarily improving mathematical knowledge |
| highest | Instrumental symbiosis | • Mathematical tasks that are designed to improve both know-how and mathematical knowledge<br>• Relation between know-how and mathematical knowledge is maximal |

These "modes of integration" indicate that the integration of technology depends on both the teacher and the learners' knowledge. The first two modes (the stage of instrumental initiation and instrumental exploration) occur when learners are beginners at the technology, and is the stage at which the learners in this research study were at when introduced to GSP. The third and fourth modes (the stage of instrumental reinforcement and instrumental symbiosis) become apparent once learners have been introduced to the instrument. The allocation of lessons set aside specifically for using technology in teaching enables both stages of instrumental integration to occur. The teacher must foster the evolution of instrumental integration, resulting in "an increasing interdependency between the know-how and the mathematical knowledge into the management of teaching" (ibid, pg. 316). These are important considerations that need to be taken into account in the design of any technology-based learning sequences. Modes of integration can also be used to describe actual practices, as the gap between the planned mode of integration and the actualisation of the technology in the classroom can reveal incomplete instrumental genesis.

From the above, it is clear that it is valuable to use technology specifically in mathematics education, both from an instrumentalisation aspect and an instrumentation aspect. Both aspects can be effectively developed in the mathematics environment using tools such as GSP and MathsLab. The growth of technology and the ease with which 21st century learners utilise technology "ha[ve] opened up diverse routes for learners to construct and comprehend mathematical knowledge and to solve problems" (Sacristán et al, 2010, pg. 179). This has important implications for teaching and learning, as it "implies a revision of the pedagogical landscape in terms of the ways in which students engage in learning, and how understandings emerge" (ibid, pg. 179). Technology may have a mediating role in the learning of mathematics; learning resulting from the cognitive activity that, in this instance, depends on the continual interaction between user (the learner) and tool (the module). A computer-based learning environment has been described by Noss and Hoyles (Sacristán et al, 2010, pg. 183) as a *domain of abstraction* that provides an environment where generalisations of abstract concepts are made possible: "a domain of abstraction supplies the tools so that exploration may be linked to formalisation" (ibid, pg. 183).

The specific advantages of GSP for the exploration of geometrical concepts is a field in which there is potential for ongoing research, particularly because the use of “dragging provides learners with an interactive way of validating their own constructions [and encourages] learner conjecturing and [development of] sequences of tasks that move pupils from conjectures to proofs” (Jones, Mackrell & Stevenson, 2010, pg. 55). Dynamic geometry software also provides researchers with windows onto learner reasoning, particularly for “investigating the nature of the representations and cognitive objects that students use in geometric thought” (Battista, 2008, pg. 341), where *cognitive objects* are mental entities that are operated on during reasoning and *representations* are “something that stand(s) for something else” (Battista, 2008, pg. 342). Research cited by Clements, Sarama, Yelland & Glass (2008, pg. 131) shows that “students can progress from the first van Hiele level (visual) to the second and third van Hiele levels (descriptive / analytic and abstract / relational) while using the [GSP] environments to construct figures that retain certain properties”.

An advantage in using geometry in a technological environment is that *didactical transposition* comes into play with relative ease. Didactical transposition refers to the change in knowledge from the way it was created (from the context of developing, exploring and proving) to a form in which it can be taught and learnt (Hollebrands, Laborde & Sträβer, 2008, pg. 156). This is particularly facilitated using dynamic geometry software such as GSP. In GSP, diagrams result from sequences of constructions with specific geometric properties. This means that “when an element of such a diagram is dragged with the mouse, the diagram is modified while all the geometric relations used in its construction are preserved” (ibid, pg. 167). The diagrams are hence “quasi-independent” of the user – modification of the diagrams (as a result of dragging) is according to the geometry of the original constructions and not the wishes of the user. An obstacle in the effective use of GSP is that learners initially have difficulty constructing diagrams that use the drag function. They attempt to use static figures, or resist dragging over large regions. This is evident in a number of cases cited by researchers (ibid, pp. 168 – 170) – however, our experience with the MathsLab module suggested that, possibly due to the time allocated to it (about 4 months) and its routine nature (regular sessions on a fortnightly basis), learners adapted quickly and with confidence in using the drag function. Learners of the 21st Century are also familiar and comfortable with using technology – in fact, they can be described as "digital natives", unlike older generations, who are "digital immigrants".

In terms of learning, research indicates that technological environments can enhance metacognitive activity:

> *Technological environments have two features that may enhance metacognitive activity: When used as tools they have the capacity to offload some of the routine work associated with mathematical activity leaving more time for reflection, and with their strict communication requirements they may help to bring to consciousness mathematical ideas and procedures.*
>
> Heid & Blume, 2008, pg. 427

### 3.3. A Vygotskian perspective

As the potential benefits of the use of technology in mathematics education were researched in the latter part of the 20th century, application of the theoretical writing of Vygotsky became more evident (Drijvers et al, 2010, pg. 99). Vygotsky (1978) proposed that an essential feature of learning is that it creates the Zone of Proximal Development (ZPD); that is:

> *Learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers.* Vygotsky, 1978, p.90

Vygotsky differentiates these developmental processes in reference to actual and potential developmental levels. He explains that when a learner successfully completes a task while working independently, the actual development of the learner is evident, whereas when a learner solves a problem under adult guidance or in collaboration with his peers, potential development can be ascertained. The learner gains more in a cognitive sense when working with others in the zone of potential or proximal development. Vygotsky emphasised that the same skill that a learner develops with assistance will be mastered independently at a later stage; thus, a new actual development will be achieved and a new stage of potential development will be reached (Vygotsky, 1978).

An important aspect of the Vygotskian notion of the ZPD is that good learning must take place slightly in advance of development. Good teaching must “awaken and rouse to life those functions which are in a state of maturing, which lie in the zone of proximal development” (Wertsch & Stone, 1985, pg. 165z). This has clear implications for teaching, as the ZPD needs to be defined in terms of the students’ actual development level and their potential development level. Further, the onus lies with teachers to ensure that work that is given is “slightly in advance of development”. Also, teachers need to provide assistance at appropriate points within the students’ ZPD (Hufferd-Ackles, Fuson and Sherin, 2004, pg. 83). Students need to move from “other-regulation” to “self-regulation”. Cole (1985, pg. 155) describes the ZPD as “the structure of joint activity in any context where there are participants who exercise differential responsibility by virtue of differential expertise” – the expertise in the context of this research is supplied by both teacher and the tasks (developed by the teacher) for use with computer technology. In a computer-mediated environment, this takes place through the process of instrumental genesis.

One of the most important concepts in Vygotskian theory is that of *internalisation*, where external cognitive processes, which are initially interpersonal, are transformed into internal processes, which are intrapersonal. The external processes are cultural skills, and as these are internalised, gradual control over them emerges. As the use of technology becomes a significant “cultural skill” of the 21$^{st}$ Century, the tension between instrumentalisation and instrumentation, and the internalisation of these processes, needs careful analysis. While this will not be considered directly in this study, it does form an area of possible future research.

An implication for teaching is that individuals have different abilities to internalise processes; hence, a need arises for situations of "mediated learning" (Meadows, 2004, pg. 170). Mediation, which is also core to Vygotskian theory, requires the use of psychological tools or signs – of which language is particularly important. Language becomes increasingly useful as a tool for abstract reflection – language is used "to communicate, to make reference, to represent ideas, to regulate one's own actions, initially within a context of social interaction and shared knowledge but increasingly independently of social partner and of supportive context" (ibid, pg. 173). Language mediates and creates learning as, according to Vygotsky, human forms of labour activity result from the semiotic mediation of tool use (Lee, 1985, pp. 68 – 69). Vygotsky is quoted by Lee (ibid, pg. 75) as stating:

> *The most significant moment in the course of intellectual development, which gives birth to the purely human forms of practical and abstract intelligence, occurs when speech and practical activity, two previously completely independent lines of development, converge.*

This incorporation of speech into human consciousness is fundamental to cognitive development (ibid, pg. 75). In sociocultural theory, scientific learning (as opposed to everyday learning) does not just happen – it is a conscious activity. Vygotsky's point is that participation must be mediated through more knowledgeable others, tools, etc. Learning drives development, and specific social interaction (or mediated participation) is the source of learning. Thus, learning cannot happen unless it is in a social context. For the 21st Century learner, computer laboratories in schools become areas of social interaction. This is due in no small measure to the fact that they are fundamentally different to conventional classrooms in terms of layout, teacher-learner interaction, learner-learner interaction, etc.

Zeuli (1986) explains the difference between everyday and scientific learning in terms of Vygotsky's ZPD. He describes the ZPD as "a phase of development where a person is unable to perform a task alone but can eventually accomplish and internalise it with the help and someone more experienced" (Zeuli, 1986, pg. 2). Everyday concepts are unsystematised and there is clearly a lack of conscious awareness. This is because attention is focussed on objects rather than on the act of thought. Control over everyday concepts is achieved through contact with scientific concepts. Scientific concepts begin in a very different way to everyday concepts – they are used non-spontaneously, i.e. the concept itself is worked on. The development of consciousness and deliberate control occurs "through learning concepts separate from the [learner's] immediate, concrete experiences" (ibid, pg. 8). For this to be achieved, the learner needs to have reached a certain level already in terms of everyday concepts. Also, collaboration between teacher and learner is essential for cognitive growth. Zeuli emphasises the role of the teacher, specifically by support through hints, props and scaffolding.

Zeuli emphasises that instruction must proceed ahead of maturing abilities. The more experienced person or knowledgeable other must

- structure the interaction,
- lead the learner through the steps of the task,
- provide any support necessary until the learner is able to complete the task independently.

The MathsLab course was designed with these three processes in mind, with support in the third step provided by the teacher supervising the lessons.

Analysis within a Vygotskian framework "with its emphasis on mediated activity via signs and tools (semiotic mediation) is ideally suited to address the relationship between the mathematical learner and the different sign systems" (Berger, 1997, pg. 160) that can be found in technology such as the graphics calculator and, by extension, the use of appropriate software using computers. The two agents of mediation are signs and tools. Berger (ibid, pg. 160) distinguishes them as follows:

> *Signs are internally oriented and aimed at mastering oneself whereas tools are externally oriented and aimed at mastering the environment. All higher human mental functions are products of mediated activity in which the role of the mediator is played by a psychological tool or sign, such as language, graphs, algebra, or a technical tool, such as a computer or a graphic calculator.*

Participation occurs in different forms in mathematics classrooms, of which a technology-based classroom is one example. For socioculturalists, knowledge is located in society and is internalised by the individual, where internalisation is "the transformation of external activity into internal activity" (Wertsch & Stone, 1985, pg. 162). Internalisation is primarily concerned with social processes and is mediated by semiotic mechanisms, e.g. language, dynamic geometry, computer algebra systems. Semiotic mediation hence provides a bridge that "connects the external with the internal and the social with the individual" (ibid, pg. 164). In Wertsch and Stone's words, semiotic mechanism is "the emergence of control over external sign forms" (ibid, pg. 167).

In Vygotskian theory, learning is social; hence, social influences and factors affect learning. As such, language in particular is an important sociocultural psychological tool and mediates and creates learning from a socioculturalist viewpoint. Hanks (1991, pp. 13 – 24) explains that, in terms of constructivist theory, "the individual mind … acquires mastery over processes of reasoning and description, by internalising and manipulating structures … Two people may well learn the same thing … yet this is a matter of coincidence, not collaborative production" (Hanks, 1991, pg. 15). Put simply – learning and development are separate; learning is purely external and is a social process. Socioculturalists believe that learning results from a need to achieve social goals. It results from increasing organisation of both inter- and intra-mental functions, and leads to increased intersubjectivity. Other perspectives on the nature of learning also exist that do not foreground the social aspect of learning.

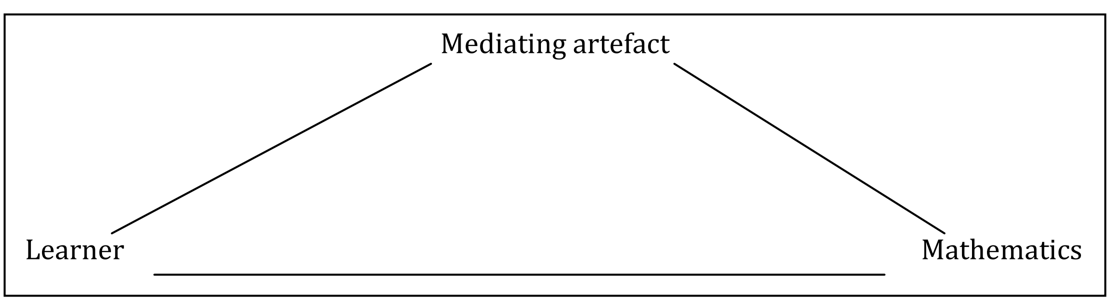
These include the work of Piaget and Ausubel, who take the view that learning is first and mostly individual and idiosyncratic. A discussion of these learning theories, however, is beyond the scope of this research report.

Importantly, teaching is not separate from learning. The teacher acts as a mediator, without whom it would be difficult to progress through the zone of proximal development. The teacher or knowledgeable other provides the conceptual tools needed for learning and hence development to take place. The organisation and mediation of tasks are key for learning to take place. Learning and development, although separate, are mutually dependent and interactive.

Technology can be considered one mediating artefact or even a knowledgeable other. Mediation is vital in mathematics education – "because of its epistemological nature any immediate relationship with mathematics is impossible; any relation passes through a mediation process" (Drijvers et al, 2010, pg.114). Importantly, interaction between learners and technology is based on symbolic interpretations and hence a semiotic approach is appropriate. Figure 2 illustrates Drijvers et al's model of the sociocultural approach to learning (ibid, pg. 116), which focuses on the use of an artefact and is expressed in terms of mediation.

**<u>Figure 2: A model of the process of mediation by an artefact</u>**

*From Drijvers et al, 2010, pg. 116*




Artefacts in this sense are not just a means to accomplish an action, e.g. GSP used to construct a parallelogram, but also a means for learning, e.g. GSP used to establish the invariant properties of parallelograms. Radford (cited in Drijvers et al, 2010, pg. 117) states that artefacts and semiotic systems contribute both to accomplishing tasks and constructing knowledge. This involves the linking of "several tools, signs and linguistic devices" (ibid, pg. 117). Once again, the role of the teacher is vital. The teacher is no longer the primary semiotic mediator. The teacher has to play the role of *cultural mediator*, where the artefact is acting as *semiotic mediator* and not simply as *mediator*. The use of the artefact as a semiotic mediator must be intentional on the part of the teacher.

Dynamic geometry software has obvious semiotic potential. This "resides in the correspondence between the logic of the stability of dragging … and the logic of the geometrical validity of the corresponding construction procedure" (Drijvers et al, 2010, pg. 119). Similarly, dynamic software can be used as a cultural artefact within the ZPD. Learners generally enjoy using computer software for learning purposes and this can aid learners as they learn within the ZPD: "… the ZPD is the difference between what a person can achieve when acting alone and what the same person can accomplish when acting with support from someone else and/or cultural artefacts …" (Lantolf, 2000, pg. 17). GSP is an example of a technological tool which embodies mathematical knowledge. Hollebrands, Laborde and Sträβer (2008, pg. 176) state:

> *For the learner, the underlying mathematics is not visible but by using technology, especially as a tool for solving relevant problems, a process of internalisation may occur and the technology may become a sign in the Vygotskian sense.*

In particular, some of the features of GSP, such as the drag mode, may be considered as signs and hence contribute to mathematics learning.

## 3.4. <u>The Van Hiele model and the Van Hiele Levels (VHL)</u>

The Van Hiele model is used to identify the stages of geometric thinking that learners attain. In the literature, levels are described variously as from Level 0 to Level 4, or Level 1 to Level 5. Originally, the van Hieles used the levels 0 to 4, but American researchers began to use the numbers 1 to 5 instead. This was to allow for a pre-recognition level to be called Level 0 (Mason, 1998, pg. 5). In this research study, the latter classification (levels 1 to 5) is used. The importance of understanding the Van Hiele Levels is apparent world-wide, e.g. the geometry curriculum in Taiwan has been developed and designed according to the model (Wu & Ma, 2006, pg. 409). Different researchers describe the attributes of each level differently, but fundamentally (as will be seen below), the level descriptions are analogous.

Importantly, progression between the Van Hiele Levels depends on instruction, not biological maturation. This links directly with the Vygotskian notions of everyday and scientific concepts discussed above, as well as with progress through the ZPD. What was scientific knowledge in one Van Hiele Level becomes everyday knowledge in the next Van Hiele Level. This is described by Usiskin (1982, pg. 5) as the property of *adjacency*, viz. that "at each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level". In understanding geometry, the levels need to be completed in order. This is known as the *fixed sequence* property.

As well as these two properties, other properties of the levels include *distinction* (each level has its own linguistic symbols and network of relationships connecting the symbols) and *separation* (two persons who reason at different levels cannot understand each other). This last property of the levels explains why students frequently do not understand their teachers during lessons on geometry. A property introduced by Usiskin is that of *attainment*, who believes that "the learning process leading to complete understanding at the next higher level has five phases, approximately but not strictly sequential" (ibid, pg. 6). These phases are entitled *inquiry*, *directed orientation*, *explanation*, *free orientation* and *integration*. The van Hieles themselves found that the process of moving from one level to another takes a lot of time, e.g. Dina van Hiele reported that it took 20 lessons to get from VHL1 to VHL2, and 50 lessons to get from VHL2 to VHL3, in a group of 12-year-olds (ibid, pg. 6).

The Van Hiele Levels were chosen as an instrument to test whether there was any progression in learners' geometrical understanding both before the implementation of the GSP module in the MathsLab course, and once the module had been completed. The pre- and post-tests administered to the learners before and after the implementation of the GSP module of MathsLab comprise six tasks, which have been adapted from the work of Burger and Shaughnessy (1986, pp. 31 – 48). Each task involves a different mathematical skill related to geometry. The aim of the test is to categorise the learners in terms of the first four Van Hiele Levels. These levels are (ibid, pp. 31 – 48):

- Level 1 – visualisation / recognition
- Level 2 – analysis
- Level 3 – abstraction / ordering
- Level 4 – deduction.

VHL5 (rigour) is not included, as this is "a level that requires the ability to compare different geometries" (ibid, pg. 34); clearly a level beyond the capabilities of Grade 9 learners. As Burger and Shaughnessy state: "The drawing, identifying, and sorting tasks … were expected to tease out characterisations of VHL [1] – [3] from the protocols. … The formal reasoning tasks were intended to obtain data about Levels [3] and [4]" (ibid, pg. 34). Of interest is the fact that P. M. van Hiele himself began to doubt the existence of VHL5 in his later writings (Usiskin, 1982, pg. 79; Senk, 1982, pg. 12).

De Villiers (2010, pg. 2) points out that progression from Level 1 to Level 2 involves more than merely verbalising intuitive knowledge – "the verbalisation goes together with a restructuring of knowledge" (ibid, pg. 3). Similarly, where Level 2 involves a focus on the properties of figures, at Level 3 the focus is on the "logical relationships between the properties of figures" (ibid, pg. 3). This involves the realisation that properties follow from each other as well as the identification of definitions as equivalent.

Two-dimensional geometry has some limitations in developing geometric concepts using GSP and a number of researchers have used three-dimensional geometry as a basis for their studies. McClintock et al (2002) used three-dimensional shapes in researching the question "what role can the GSP dynamic instructional environment play in the development of students' geometric thinking as defined by the van Hiele theory?" (McClintock et al, 2002, pg. 740). A very important observation made by these researchers was that the nature of the instruction given to the students in their study determined their progress from one level to the next. The MathsLab module uses GSP in combination with a specially designed module, in the expectation that this will enable learners to move up the levels.

McClintock et al described the levels as follows:

- Level 1 – visual (figures recognised by appearance alone)
- Level 2 – analysis of properties (properties perceived but isolated and unrelated)
- Level 3 – ordering / hierarchy (relationships; implications and class inclusions)
- Level 4 – deduction / proof (deduction is meaningful; can construct proofs)
- Level 5 – rigour (formal aspects of deductions; symbols manipulated)

Progression through the Van Hiele Levels depends on educational experiences rather than age or maturation, and instruction needs to be organised into five phases of learning (Mason, 1998, pg. 5; Hoffer, 1983, pg. 206):

- *Information/inquiry*: Through discussion, the teacher identifies what students already know about a topic and the students become oriented to the new topic.
- *Guided orientation*: Students explore the objects of instruction in carefully structured tasks … The teacher ensures that students explore specific topics.
- *Explicitation*: Students describe what they have learned about the topic in their own words. The teacher introduces relevant terms.
- *Free orientation*: Students apply the relationships they are learning to solve problems and investigate more open-ended tasks
- *Integration*: Students summarise and integrate what they have learned, developing a new network of objects and relations

The above sequence of phases was specified by the van Hieles in order to facilitate movement from "very direct instruction to the student's independence of the teacher" (Hoffer, 1983, pg. 207). At the close of the fifth phase (integration), a new Van Hiele Level will have been attained.

Mason believes that if a teacher is teaching at a level of thought that is above the learner's level, the learner will not understand the content that is being taught. However, in Vygotskian terms, for learning to happen, instruction must be ahead of development. This is a seemingly contradictory view of the Van Hiele Levels; however, instruction should not be at a different Van Hiele Level to that of the learner, but ahead of the learner's understanding within a level. Once internalisation of the type of thinking characterised at a specific level has occurred, the learner will progress to the next level.

Senk (1985) developed a means of testing the Van Hiele Levels of students in the USA by using pre- and post-tests. She describes the Van Hiele Levels as follows (Senk, 1985, pg. 2):

- Level 1 – knowledge is obtained by observation
- Level 2 – knowledge is obtained from reasoning about the properties of figures

- Level 3 – knowledge is derived via short chains of deduction about the relations between the properties of figures
- Level 4 – knowledge is derived by formal proofs in a deductive system, i.e. by relating the relations derived at level 3
- Level 5 – knowledge is derived by reasoning about logical thinking itself, or by relating various deductive systems

Senk corroborates that two persons reasoning at different Van Hiele Levels will probably not understand each other. The discrepancy between a teacher's Van Hiele Level and a learner's Van Hiele Level could explain why many learners do not achieve in geometry. If the teacher is teaching at too high a level, this will be beyond the ZPD of the learner. Senk's studies were based specifically on students' abilities in proof-writing, and she categorises Level 3 as a transitional level between informal and formal geometric reasoning. Importantly, "a student who has attained only level $n$ will not understand the thinking of level $n + 1$ or higher" (Senk, 1989, pg. 310). Identification of learners' Van Hiele Levels thus is important when designing an instructional programme.

Blair (pp. 1 - 7) researched a way of describing the Van Hiele Levels in terms of objects of thought and noted that "the different levels of thinking … shift towards increasingly more complex objects of thought" (ibid, pg. 2). While this initially appears to be a break-away from the more conventional way of describing the Van Hiele Levels via descriptions of each level, Blair believes that specific objects of thought are embedded in each level:

> *At the third level, the properties of figures are used to think about relationships between properties, which become objects of thought. At the fourth level, these relationships are seen as definitions and theorems, and they in turn are related to each other. Hence, at the fourth level, "networks of theorems" become an object of thought. Note that the objects of thought at both of these levels are relational*

*in type, but clearly different in complexity.*

Blair, pg. 2

As the objects of thought become more complex, the learner moves up through the Van Hiele Levels. Blair related the Van Hiele Levels to types of reasoning, stating that "the third Van Hiele Level has often been associated with 'informal' reasoning in order to contrast it from the 'formal' deduction seen at the fourth level" (ibid, pg. 3). This corresponds to Senk's observations. The following table represents Blair's characterisation of the Van Hiele Levels:

**Table 2 - An object of thought characterisation of the five Van Hiele Levels**

*from Blair, pg. 2*

| Van Hiele Level | Object of Thought |
|---|---|
| 1 Visual | Figure(s) as a whole |
| 2 Descriptive | Individual Properties of Figures |
| 3 Abstraction | Relationships between Properties |
| 4 Deduction | Networks of Relationships within a System |
| 5 Rigour | Relationships between Systems |

Blair also noted that descriptive defining, i.e. "choosing a minimal, sufficient set of conditions to characterise a known object" (ibid, pg. 6), indicates third level reasoning, while constructive defining, which "involves constructing new objects by examining changes to a known definition" (ibid, pg. 6), implies the systematisation of existing knowledge, which is characteristic of fourth level reasoning. In Blair's study, some participants even worked within axiomatic systems when formulating definitions, which indicates fifth level reasoning. Another important observation was that students' thinking is a dynamic progression through the levels, which changes during the course of a task, and is often related to changing objects of thought, usually from objects of thought changing from individual properties (second level) to relationships between these properties (third level).

Fuys and Geddes (1984, pg. 2) also discuss the differences in the objects of thought in each level:

> *At level [1], the objects of thought are geometric figures. At level [2]the student operates on certain objects, namely, classes of figures, which were products of level [1] activities and discovers properties for these classes. At level [3], these properties become the objects that the students act upon, yielding logical orderings of these properties. At level [4], the ordering relations become the objects on which the student operates and at level [5], the objects of thought are the foundation of these ordering relations.*

Another important factor pointed out by Fuys and Geddes (1984, pg. 2) is that each level has its own linguistic symbols. In the words of Pierre van Hiele himself: "A relation which is "correct" at one level can reveal itself to be incorrect at another" (ibid, pg. 2).

Burger and Shaughnessy (1986, pp. 31 – 48) researched the possibility of characterising the Van Hiele Levels operationally based on observations of student behaviour when dealing with shapes. They developed a number of level indicators, which they used to assign the first four Van Hiele Levels to the participants of their study, in each case assigning levels that represented the predominant level of thinking displayed by the participant. They achieved a degree of success with this process, and their studies were based on tasks involving triangles and quadrilaterals, and hence their level indicators refer directly to the shapes involved. In other words, the objects of thought in their characterisation of levels are shapes. Table 3 summarises their level indicators.

**Table 3 – Level indicators for the Van Hiele Model**

*Adapted from Burger and Shaughnessy, 1986, pp. 43 – 45*

| LEVEL NUMBER | LEVEL NAME | INDICATORS |
|---|---|---|
| 1 | Visualisation | • use of imprecise properties (qualities) to compare drawings and to identify, characterise and sort shapes<br>• references to visual prototypes to characterise shapes<br>• inclusion of irrelevant attributes when identifying and describing shapes, such as orientation of the figure on the page<br>• inability to conceive of an infinite variety of shapes<br>• inconsistent sortings, i.e. sortings by properties not shared by the sorted shapes<br>• inability to use properties as necessary conditions to determine a shape; e.g. guessing a shape from too few clues, as if the clues triggered a visual image |
| 2 | Analysis | • comparing shapes explicitly by means of properties of their components<br>• prohibiting class inclusions among general types of shapes, such as quadrilaterals<br>• application of a litany of necessary properties instead of determining sufficient properties when identifying shapes, explaining identifications, etc<br>• descriptions of types of shapes by explicit use of their properties, rather than by type names<br>• explicit rejection of textbook definitions of shapes in favour of personal characterisations<br>• treating geometry as physics when testing the validity of a proposition<br>• explicit lack of understanding of mathematical proof |
| 3 | Abstraction | • formation of complete definitions of types of shapes<br>• ability to modify definitions and immediately accepts and use definitions |

| | | |
|---|---|---|
| | | of new concepts<br>• explicit references to definitions<br>• ability to accept equivalent forms of definitions<br>• acceptance of logical partial ordering among types of shapes, including class inclusions<br>• ability to sort shapes according to a variety of mathematically precise attributes<br>• explicit use of “if, then” statements<br>• ability to form correct informal deductive arguments<br>• confusion between the roles of axiom and theorem |
| 4 | Deduction | • clarification of ambiguous questions and rephrasing of problem tasks into precise language<br>• frequent conjecturing and attempts to verify conjectures deductively<br>• reliance on proof as the final authority in deciding the truth of a mathematical proposition<br>• understanding of the roles of the components in a mathematical discourse, such as axioms, definitions, theorems, proof<br>• implicit acceptance of the postulates of Euclidean geometry |

These are the level indicators that were adapted and used in the rubric designed to assess learners’ Van Hiele Levels before and after the implementation of the MathsLab module.

The use of formal and economical definitions is another aspect of geometry that lends itself to classification in terms of the Van Hiele Levels, although it is not an aspect that is examined in this research study. Michael de Villiers (1998, pg. 3) noted that understanding of formal definitions is only possible at VHL3, as it is at this level that learners start to notice inter-relationships between the properties of a figure.  De

Villiers described the Van Hiele Levels in terms of the ability to define geometric figures:

- VHL1 – visual definitions
- VHL2 – uneconomical definitions
- VHL3 – correct, economical definitions

De Villiers endorses the use of dynamic geometry software such as GSP to enable hierarchical classification of geometric figures, as it is possible to, for example, drag a parallelogram into the shape of a square or a rectangle, and then accept that squares and rectangles are special cases of parallelograms.

## 4. LITERATURE REVIEW

An attempt was made to find other research studies where the Van Hiele Levels of students or student teachers were investigated, using both traditional teaching methods and using dynamic geometry software. Before the 1980s, there is little evidence of such research in English-speaking countries, although the Van Hiele model was being studied and implemented in the Netherlands and the Soviet Union. It was only in the 1970s and early 1980s that the Van Hiele model began to be a focus of research for Western educationists (Usiskin, 1982, pg. 3; Hoffer, 1983, pp. 209 - 214).

An early and important report was authored by Zalman Usiskin in 1982 as a result of the CDASSG (Cognitive Development and Achievement in Secondary School Geometry) project undertaken by the University of Chicago. 2 700 Grade 10 students in 13 high schools throughout the USA were selected, and were assessed at the beginning and the end of a school year. The assessments were designed to determine what changes in Van Hiele Levels, if any, took place after a year's study of geometry by the subjects. Of interest is the fact that, at the time of this project, geometry was studied in a single year in the USA, normally in the tenth grade (Usiskin, 1982, pg. 1). Researchers at the time were aware of two important aspects of the Van Hiele theory – firstly that students who were having trouble with geometry were being taught at a level higher than they were ready for, and secondly, that the remedy was to work through the sequence of Van Hiele Levels in specific ways.

The specific questions that the CDASSG project addressed are listed below (Usiskin, 1982, pp. 1 – 2):

- How are entering geometry students distributed with respect to the levels in the Van Hiele scheme?

- What changes in the Van Hiele Levels take place after a year's study of geometry?
- To what extent are Van Hiele Levels related to concurrent geometry achievement?
- To what extent do Van Hiele Levels predict geometry achievement after a year's study?
- What generalisations can be made concerning the entering Van Hiele Levels and geometry knowledge of students who are later found to be unsuccessful in their study of geometry?
- To what extent is the geometry being taught to students appropriate to their Van Hiele Levels?
- To what extent do geometry classes in different schools and socio-economic settings differ in appropriateness of the content to the Van Hiele Levels of the student?

The CDASSG project recognised that the Van Hiele theory needed to be rigorously tested. For this to happen, the levels needed to be accurately identified. The researchers involved examined a total of nine works, both in English and translated into English from the original Dutch, German and French, in order to accurately define each level. They concluded that the first three Van Hiele Levels are easily testable, though VHL4and VHL5 were of questionable testability. MathsLab is aimed at Grade 9 learners, who are generally only operating at VHL1 and VHL2, and occasionally VHL3.

The CDASSG project included two pre- and three post-tests: Entering Geometry Test and Van Hiele Level Test were given in the first week of school and Van Hiele Test, Comprehensive Assessment Programme Geometry Test and Proof Test were given three to five weeks before the end of school. Tests were piloted and items rejected or modified. Specific Van Hiele Tests were designed, as not all questions in standardised tests could be assigned to a level. Burger and Shaughnessy (1986) also designed their own tasks, and it is these tasks that were adapted for the pre- and post-test for the MathsLab module.

Reliability for the tests as described by Usiskin was generally good (Usiskin, 1982, pg. 29), but it was noted that the low reliabilities at VHL4 and VHL5 were probably “a by-product of the lack of specification of the van Hiele theory at these levels” (ibid, pg. 29). This was not an issue in terms of this research, as it is doubtful that Grade 9 learners would be reasoning at these Van Hiele Levels. Crowley (1990) expressed some doubts about the validity and reliability of the CDASSG tests used, noting that “the dependence of the final Van Hiele Level assignment on an individual’s performance on each subtest requires that all subtests meet the desired criteria” (Crowley, 1990, pg. 240). Usiskin and Senk (1990, pp. 242 – 245) replied to these criticisms, noting that the Van Hiele Geometry Test developed by the CDASSG had been used far more extensively that originally foreseen. Acknowledging the low reliability coefficients, Usiskin and Senk believed that the results of the CDASSG study were robust and noted that the results that they had received from instances where the test had been duplicated and used did not differ significantly from their own study.

The CDASSG was a three-year project, and the results are discussed in great detail in the research report authored by Usiskin. Interesting conclusions were reached, e.g. the non-existence or “non-testability” of VHL5. This conclusion “substantiates P. M. van Hiele’s disavowal of the existence of this level in his more recent writings” (ibid, pg. 79). The study found that over two-thirds (and in some cases as many as nine-tenths) of the students responded to test items in such a way that it was easy to assign Van Hiele Levels to them. However, arbitrary decisions made by the researchers in assigning levels affected the level assigned to many students. This possible weakness was also evident in the MathsLab study.

Over the course of the year, researchers found that there was great variability in changes in Van Hiele Levels in students – about one-third of students stayed at the same Van Hiele Level or even went down, about a third went up a level and about a third went up two or more levels. Researchers also found that the Van Hiele Levels were very good predictors of concurrent performance in multiple choice tests, specifically in content tests and less so in proof tests. In fact, they found that VHL2 was a good "guidepost" regarding success in proof – "Above these levels, success in proof is likely. Below these levels, failure in proof is just as likely" (Usiskin, 1982, pg. 82).

The CDASSG project was also described by Sharon Senk (1982, pp. 1 – 28). The focus of her discussion was the attainment of proof-writing skills, a skill not comprehensively covered in the MathsLab module. Senk found that about 70% of the Grade 10 students who were part of the CDASSG study were able to complete simple proofs requiring a single deduction. Achievement on proofs requiring longer chains of reasoning was found to be substantially lower. Even "after a full year of a geometry course with proof, only about half the students can do any more than simple proofs" (ibid, pg. 7). Particularly those students who entered the year at a low Van Hiele Levels did not succeed in learning to write proofs. Data from the research further corroborated the view of the Van Hieles themselves that "only students who have reached levels 4 or 5 are able to write their own proofs" (ibid, pg. 11).

In 1984, David Fuys and Dorothy Geddes (1984, pp. 1 – 30) reported on research that used sixth and ninth graders from inner city schools to investigate the Van Hiele Levels. Their aim was threefold: to develop a working model based on the Van Hiele Model, to characterise geometry thinking in terms of the Van Hiele Levels in the subjects of their study, and to analyse the geometry strand in three textbooks in use at the time.

The first objective of these researchers was achieved following intensive research of original van Hiele source material, and van Hiele himself validated the working model that was developed (Fuys & Geddes, 1984, pg. 4). The second objective was carried out in several phases – modules dealing with quadrilaterals were developed, comprising instructional tasks as well as key assessments "correlated with specific level descriptors" (ibid, pg. 4). The research tool took the form of clinical interviews which were all video-taped. Finally, the three textbooks were reviewed by the use of data forms, which determined the Van Hiele Levels in introductory test, exercises and test questions, using the level descriptors developed by the researchers.

Fuys and Geddes found that there was a wide range of thinking in the subjects, though VHL1 was dominant. This was attributed to the fact that, at the time of the research, the subjects had a significant lack of experience in doing geometry in school. They further found that, where students had been exposed to some geometry, "the text material did little to encourage higher levels of thinking" (Fuys & Geddes, 1984, pp. 9 – 10). There was also little progress through the levels. This was attributed to a lack of general ability, specifically in terms of language (ibid, pg. 10). The instructional materials and the interviews were designed to help students progress through the levels. Interviewers typically guided student responses in order to improve the quality of the responses, specifically to "observe relationships between parts of a figure and to make generalisations (level [2]) or to give deductive explanations (level [3])" (ibid, pg. 11). They observed that if students were operating at a specific level, they realised what the procedure was, but if they were in transition between levels, they needed guidance in terms of the expectations of the interviewers. The interviewer is thus described as an "interviewer-teacher" who used a specific meta-language to communicate their expectations to the students.

Of interest to this research project is the following comment made by Fuys and Geddes (1984, pg. 11): "... these results indicate that some sixth graders and *almost all ninth graders* [my emphasis] are quite capable of engaging in geometry activities that call for level [2] or even level [3] thinking which is probably a higher level than they experienced in school".

Reference has already been made in this study about the work of Burger and Shaughnessy (1986, pp. 31 – 48) as this work was used predominantly in the design of the pre- and post-test included in the GSP module of the MathsLab course. Burger and Shaughnessy undertook a three-year project investigating the Van Hiele Levels in school geometry. The subjects of the research were 45 students from three states in the USA and ranged from early primary pupils to college mathematics majors. The procedure was in the form of clinical interviews administered by one of four researchers. Eight tasks were given to the students, each dealing with geometric shapes, and were coded by the researchers. Van Hiele Levels were assigned by the researcher for each student on each task and had to represent the predominant level of thinking exhibited by the student. This was in order to identify a preferred level of reasoning, and the same procedure was followed in the assessments of the pre- and post-tests in this research study.

Burger and Shaughnessy reported comprehensively on the results of a sample of six students, from a Grade 3 learner to a university mathematics major (Burger & Shaughnessy, 1986, pp. 37 – 41). Some interesting features of the Van Hiele Levels emerged during the course of their study. Levels were seen to involve the development of both concepts and reasoning processes. This development was noted to be "highly dependent on instruction and much less dependent, if at all, on age" (ibid, pg. 45). It was also observed that, although the van Hieles theorised that the levels were discrete structures, the study questioned the discrete nature of the levels. Of importance was that students exhibited different levels on different tasks, and even oscillated from one Van Hiele Level to another within a task. This was also observed in this research study. Burger and Shaughnessy thus concluded that the Van Hiele Levels are both dynamic and of a more continuous nature than previously believed.

The van Hiele theory has been revisited a number of times in the years between the 1980s and the present. Anne Teppo (1991, pp. 210 – 221) compared the van Hiele theory to the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) and found a good correlation between sample tasks and the Van Hiele Levels. In 1995, Bethe McBride and James Carifio found that an analytic scoring procedure for assessing students' performances in doing geometry proofs was not well-aligned with the van Hiele theory and recommended that their analytic rating scale was a better model than the Van Hiele Levels. This does seem to have been a personal belief of these researchers (albeit backed by convincing data) and was not considered relevant for this study.

A study of interest is that of Regina Mistretta (2001, pp. 1 – 13), who described a field trial of a geometry unit that was developed to raise the Van Hiele Levels in a group of eighth-grade students. In this study, a pre-test was administered in order to assign Van Hiele Levels to the subjects. The pre-test comprised Van Hiele Levels1, 2 and 3 questions, in accordance to the belief that it is at these levels that learners of this age operate. Mistretta points out that "a student is classified as being at a particular van Hiele thinking level if he or she successfully answers 60% or more of the questions at and below that level" (ibid, pg. 3). This was the principle that guided the assessment of the tasks in both the pre- and post-test in the MathsLab GSP module.

Mistretta also conducted a post-test at the completion of the geometry unit, which comprised questions similar to those on the pre-test. In general, the results of the post-test indicated that there had been a significant improvement in the reasoning abilities of the students involved in the study. Mistretta's concluding comments note the importance of a strong foundation being laid in primary school, specifically in VHL1 and VHL2. She states that "if students are encouraged to develop their cognitive skills, they will be more likely to accept the challenges of analytical thinking" (ibid, pg. 6).

The use of dynamic geometry software, specifically GSP, in activities based on the van Hiele model was reported on by Olkun, Sinoplu and Deryakulu (2005, pp. 1 – 12). The instructional activities studied were designed to "emphasise students' learning through explorations instead of teaching a specific mathematical content" (ibid, pg. 1). They pointed out that if school geometry is presented in an axiomatic fashion (as in the context of the old South African curriculum), it is based on the assumption that learners already are able to think on a formal deductive level. Hence their endorsement of the use of the van Hiele model in the design of geometric tasks.

One of the advantages of using GSP in these types of instructional activities is that learners explore geometric concepts according to their individual geometric thinking levels. The aim of these activities is that learners "continue their explorations with little help from their teachers" (ibid, pg. 3). In this case, the technology becomes the "knowledgeable other" in Vygotskian terms.

Olkun et al observed that learners enjoyed these activities and learned a lot (pg. 11). An important aspect of the tasks was that learners had to reflect on the work, and this is an important part of each lesson in the GSP module of MathsLab. Learners were certainly observed to have moved from one Van Hiele Level to another, higher Van Hiele Level.

Adaptations of the work by Usiskin (1982) and Burger and Shaughnessy (1986) were done in Taiwan in 2005(?) by Der-bang Wu (….). Wu noted that both Usiskin and Burger and Shaughnessy consistently used triangle and quadrilateral definitions and classifications in their work. Further, these researchers looked to assign VHL1 to VHL4 to students in their studies. Wu developed a test aimed at testing the first three Van Hiele Levels – the last two levels were not considered. Wu also included the circle in his geometry tests and administered the tests as a pre-test and a post-test.
As with other researchers (Mistretta, 2001), Wu's tests were multiple-choice tests. The tests also were guided by the curriculum requirements of the Ministry of Education in Taiwan. At the time of the printing of the article in question, the tests had not yet been administered.

Researchers have also used the van Hiele model in studies of possible gender differences in student teachers (Halat, 2008, pp. 1 – 11). While these studies have no specific reference to the MathsLab study, the study carried out by Halat is included in this literature review to show the various possible uses of the Van Hiele Levels in ascertaining the effects of variables on performance, such as gender, age, etc.

The reason many studies include student teachers is that "teachers' mathematical and pedagogical content knowledge plays vital roles in student learning" (ibid, pg. 1). In a literature review of his own in Halat's paper, he noted that many factors seem to affect the performance and motivation of pre-service mathematics teachers, of which gender is but one. Research quoted by Halat shows that "there [are] statistically significant sex-related differences between male and female students' attitudes towards mathematics" (ibid, pg. 3) and Halat researched possible gender-related differences in Van Hiele Levels in a study of 281 pre-service teachers in central Turkey. The result of the study was (ibid, pg. 8):

> *…although there was no statistically significant difference in terms of reasoning stages between male and female pre-service elementary teachers, a statistically significant sex-related difference was found among the pre-service secondary mathematics teachers favouring males*.

Michael de Villiers (2010, pg. 1) presented a plenary in Croatia in 2010, where he reflected on the van Hiele theory, specifically research on the Van Hiele Levels over the previous 30 years, the design of learning activities using dynamic geometry, and issues of further research. This is an appropriate conclusion to this literature review as it brings together a number of strands with regards to the van Hiele theory.

De Villiers discusses the acquisition of technical language as an indicator of the attainment of VHL2. However, the attainment of VHL2 also involves both rearrangement of relations and refinement of concepts – it is not "merely a verbalisation of intuitive knowledge" (de Villiers, 2010, pp. 2 – 3). VHL3 represents a different network of relations: where VHL2 involves the association of properties with specific types of figures, VHL3 involves logical relationships between the actual properties (ibid, pg. 3).

De Villiers looked closely at the South African context in terms of geometry education. His conclusion is profound, with potentially far-reaching consequences: “It seems clear that no amount of effort and fancy teaching methods at the secondary school will be successful, unless we embark on a major revision of the primary school geometry curriculum along Van Hiele lines” (de Villiers, 2010, pg. 7). Comparison with the geometry curricula in both Japan and Taiwan shows that learners in these countries attain VHL3 as early as Grade 6.

The de Villiers article looks in detail at some activities using GSP for exploring concepts. He extends the Van Hiele Levels to include definitions and even other branches of mathematics, such as Boolean algebra, trigonometry and functions. De Villiers concludes by noting that more research can be done on the use of dynamic geometry software to enhance the development of geometric thinking as well as the concern about the poor levels of geometric thinking amongst teachers themselves.

The levels of thought in geometry in pre-service educators were the focus of research done by Sonja van Putten of the University of Pretoria (2008). The lack of confidence in dealing with Euclidean geometry (specifically in terms of the solutions of geometry riders) extends beyond the learner in the classroom to the actual teacher. Van Putten points out that “the logic, ability to reason and insight exercise demanded by this discipline render its pursuit worthwhile since these skills are not only essential in all the mathematical disciplines, but also in life itself” (ibid, pg. 16) and carried out a case study of 32 third-year students at the University of Pretoria who were studying to become teachers of mathematics in the FET phase. A van Hiele test (based on the work of Usiskin, ibid, pg. 61) )was used as a source of evidence to address how the students apply their knowledge of Euclidean geometry to solve riders, with the expectation that students should be able to solve problems at van Hiele Level 4 (i.e. using principles of formal deduction). This was then followed by an interview schedule with selected students.

The results of the pre-test in van Putten's research indicated that performance on van Hiele levels 2, 3 and 4 was low, and particularly low on items that tested van Hiele Level 4. This was of concern, as teachers need to be operating at this level in order to teach Euclidean geometry effectively. An improvement was noted after the post-test was administered, which was at the end of an intensive geometry module. The conclusion of the research was that it is essential to provide in-service teachers with modules that deal specifically with the rigour and content necessary to teach Euclidean geometry.

Atebe and Schafer (2008) investigated the levels of understanding of 36 learners from grades 10 to 12 and from both South Africa and Nigeria. Similarly to this research study, concepts of triangles and quadrilaterals were investigated and tasks involved identifying and naming shapes, sorting shapes, stating the properties of shapes, defining shapes and establishing understanding of the concept of class inclusion. The results of this study showed that the majority of the participants were operating at van Hiele Level 1, which is lower than that expected of learners in these grades.

The conclusion from these last studies is that, in the South African context, there is profound lack of conceptual geometric understanding. This is a concern, and studies such as these mentioned and this research report should be undertaken wherever possible.

# 5. METHODOLOGY AND METHODS

## 5.1 Data collection

In order to assess whether the use of technology could be successful in moving learners up through the Van Hiele Levels, it was necessary to establish their Van Hiele Levels both before and after the implementation of the GSP module. This was done through a pre- and post-test.

The GSP module itself comprises six lessons. Each Grade 9 group was timetabled for MathsLab once a fortnight and learners were expected to complete the given tasks and reflections before the next session. Learners worked in pairs and were encouraged to discuss their findings with each other. One concern that arose from this manner of implementation was that results would reflect group rather than individual progress; however, as the learners were assessed individually, and acknowledging the social nature of learning, it was decided to proceed with the group working in pairs rather than individually.

The type of teaching that occurred in this context was not traditional, front-of-class teaching. The teacher moved about the classroom, helping with both the technical aspects of GSP and facilitating discussion between and with the learners as necessary. Having teacher intervention when necessary does not imply that learners were passive participants in these lessons; on the contrary, learners were actively encouraged to explore and analyse the tasks in their pairs. Communication between the learners was specifically significant when the tasks required them to start conjecturing. Again, teacher input at this stage of each task was anticipated to be important initially, and it was hoped that, as learners progressed through the ZPD, teacher input would become less.

Each lesson concluded with a section which allowed for individual learner reflection on the task and skills learnt. The first lesson introduced the learners to GSP and they learnt the basic uses of the tools in the software. In the design of the module, the principle was to use investigations of circles and/or polygons to develop reasoning. The module ended with the post-test, the results of which were compared with the pre-test and used to answer some of the research issues. Neither the pre-test nor the post-test used GSP. Constructions were made manually and both tests were written under strict test conditions. The test can be found in Appendix 2.

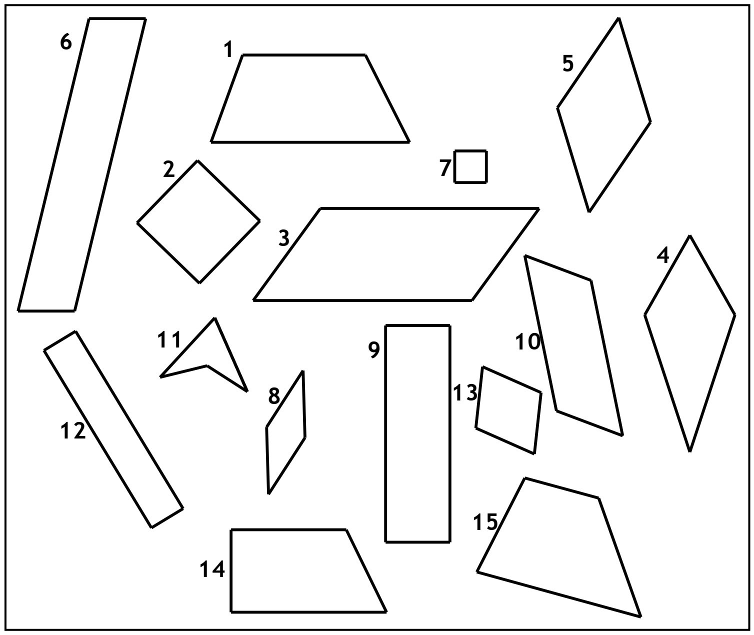
The test was administered to every Grade 9 learner both before and after the implementation of the module. Lessons were 45 minutes long, and it was anticipated that this might present a problem, as the purpose of the test is not whether tasks can be completed in a certain time frame, as is the case with conventional classroom tests. The post-test was administered during the November examination session at the school, and the time constraint was effectively nullified.

The tasks in the pre- and post-test were identical and were based on the work of Burger and Shaughnessy (1986, pp. 31 – 48). The tasks involved "drawing shapes, identifying and defining shapes, sorting shapes, and engaging in both informal and formal reasoning about geometric shapes" (ibid, pg. 34) because it was believed that these would reflect accurate descriptions of the Van Hiele Levels.

Task 1, the drawing task, required learners to draw a succession of triangles, changing only one aspect of the triangle each time and explaining what changes occurred as a result. The aim was to see whether learners appreciated that changing one attribute at a time would result in changes in other attributes. The final question of the task asked how many possible triangles could be drawn, to see whether learners appreciated that there an infinite number of triangles that could be drawn.

Task 2 involved identifying types of quadrilaterals. Learners were given a sheet of quadrilaterals as shown in Figure 3 below:

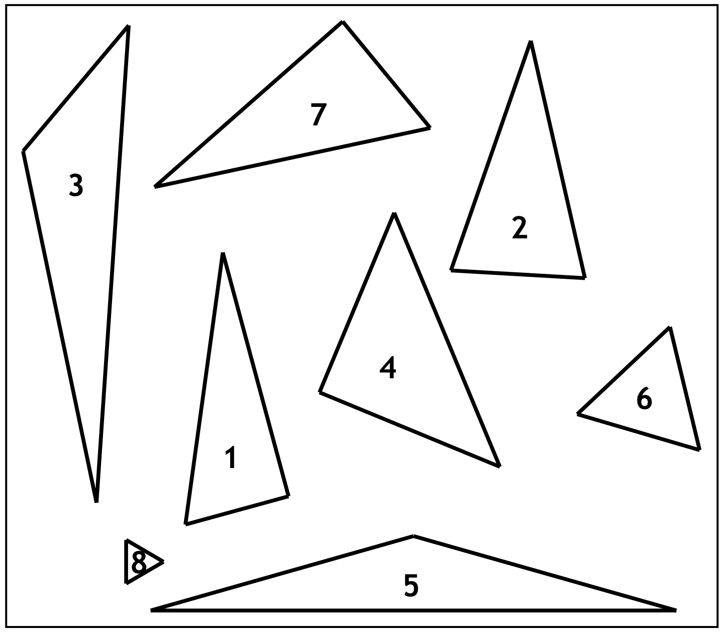
**Figure 3 – Quadrilaterals in identifying task**




Learners then had to classify each shape using the letters S (square), R (rectangle), P (parallelogram) and B (rhombus). They were also required to explain the choices that they had made. This activity explored learners' understanding of definitions and class inclusions.

Task 3 involved defining shapes. There were four questions, all variations of: "What would you tell someone to look for in order to pick out all the RECTANGLES on a sheet of quadrilaterals?" The other three questions related to squares, parallelograms and rhombuses. Task 4, the sorting task, involved learners sorting the triangles in Figure 4 in as many different ways as possible.

**Figure 4 – Triangles in sorting task**




Burger and Shaughnessy expected these first tasks (drawing, identifying and sorting) to "tease out characterisations of van Hiele levels [1] to [3]" (Burger & Shaughnessy, 1986, pg. 34). These first tasks comprised Section A of the test.

At the beginning of both the pre- and the post-test, each learner was given Section A to complete and submit before they were given Section B. The reason for this is that completing the task in Section B (which involves listing the properties of various quadrilaterals) could influence the answers in Section A. By collecting Section A before administering Section B, learners were not able to go back and modify their responses in Section A. This is an important factor as it relates directly to the validity of their responses in Section A.

The tasks in Section B differed substantially from the work of Burger and Shaughnessy, which was conducted in the form of clinical interviews. In Task 5, properties of quadrilaterals were tested by giving shapes to the learners and asking them to list as many properties as possible. The final task, Task 6, attempted to test the level of formal reasoning in each learner and was written as follows: "Suppose you have a quadrilateral with both pairs of opposite sides equal. Will the opposite sides also be parallel? In your answer, you need to explain your reasoning. Use diagrams to assist your explanation". These formal reasoning tasks were designed to give information about Van Hiele Levels 3 and 4. The full pre- and post-test is included in Appendix 2.

The assessment of the tasks and the allocation of the appropriate Van Hiele Levels were crucial to answering the research questions. The challenge was to link the learners' progress within the ZPD to the potential effects of GSP and the MathsLab module. The possibility of other influences on the learners' progress needed to be considered. However, as a result of the implementation of MathsLab, there was minimal class time allocated to the teaching of the geometry involved, so that it is fair to assume that the majority of learning resulted from the module itself. It was debated whether a control group should be created to compare the progress of the learners' understanding of concepts, but it was agreed that it would not be ethical in the context of the school to discriminate between groups of learners. The module was designed to be an integral part of the teaching process; not merely as a research tool. A rubric (based substantially on the work of Burger and Shaughnessy) was developed in order to assess the pre- and post-tests in terms of the Van Hiele Levels and this is shown below:

## TABLE 4 - RUBRIC FOR ASSESSING TASKS

| **TASK** | **Van Hiele Levels 1 VISUALISATION** *The student reasons about basic geometric concepts primarily by means of visual, imprecise considerations of the concept as a whole without explicit regard to properties of its components* | **Van Hiele Levels 2 ANALYSIS** *The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concepts are established* | **Van Hiele Levels 3 ABSTRACTION** *The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of a set of properties in determining a concept* | **Van Hiele Levels 4 DEDUCTION** *The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions, and theorems* |
|---|---|---|---|---|
| 1 – Drawing shapes | • Visual qualities only varied, e.g. orientation on page, "skinniness" of object<br>• Belief that only a finite number of triangles exist | • Focus on components of objects (angles, sides)<br>• Realisation that components can be varied in an infinite number of ways<br>• Use of technical language | • Triangles drawn as representatives of general types<br>• Ability to interrelate shapes | NOT RELEVANT FOR DRAWING TASKS |
| 2 – Identifying shapes | • Imprecise visual qualities used in describing shapes<br>• Irrelevant attributes included in descriptions of shapes<br>• Some relevant attributes omitted<br>• References to visual prototypes | • Shapes identified explicitly by means of their properties<br>• Too many properties given<br>• Avoidance of class inclusions (e.g. squares and rectangles seen as disjoint) | • Shapes characterised by referring to other shapes<br>• Minimal characterisations of shapes | • Evidence of conjecturing and formal reasoning |
| 3 – Defining shapes | • Imprecise visual qualities used in describing shapes<br>• Irrelevant attributes included in descriptions of | • Shapes identified explicitly by means of their properties<br>• Too many properties given<br>• Personal | • Shapes characterised by referring to other shapes<br>• Minimal characterisations of shapes<br>• Definitions complete | • Evidence of conjecturing and formal reasoning |

| | | | | |
|---|---|---|---|---|
| | shapes<br>• Some relevant attributes omitted<br>• References to visual prototypes<br>• Easily misled by orientation of figures | characterisatio ns | • Definitions result from modification of previous definitions<br>• Definitions economic and correct | |
| 4 – Sorting shapes | • Use of imprecise, visual qualities only<br>• Incomplete or inconsistent sortings | • Sortings using properties of shapes, even if properties imprecise<br>• Sorting by single attributes or properties | • Sorting refers explicitly to a variety of types<br>• Hierarchical classification of figures | NOT RELEVANT FOR SORTING TASKS |
| 5 – Properties of quadrilaterals | • Visual attributes only | • Properties listed without reference to other shapes<br>• Some irrelevant or incorrect properties listed | • Properties listed with reference to other shapes | NOT RELEVANT |
| 6 – Formal reasoning | • Attempted answer by drawings only<br>• Question rephrased as an assertion | • Use of a variety of diagrams<br>• Validity of statement tested inductively or empirically<br>• Lack of understanding of mathematical proof | • Attempt at informal, deductive arguments<br>• Explicit use of "if, then" reasoning | • Deductive argument, with proof as final authority<br>• Sufficient conditions for a quadrilateral to be a parallelogra m |

Each learner was assessed and the conclusions were completed in a table for each individual (see Appendix 3). For the sake of anonymity, each learner analysed was allocated a pseudonym, which was unrelated to gender or ethnicity. An example is shown in Table 5:

**TABLE 5 – EXAMPLE OF INDIVIDUAL SUMMARY**

**NAME: PAUL** **PRE-TEST**

| TASK | PREDOMINANT Van Hiele Levels | RESEARCHER'S COMMENTS (if appropriate) |
|---|---|---|
| 1 | 1 | Generally only lengths of sides changed. In no. 5 realisation that changing lengths also changed angles. Realisation of infinity of possibilities. No. 6 – VHL1: only visual quality varied (size) |
| 2 | 1 | Little understanding of quadrilaterals ("This is a kite [not asked for] but also a rhombus because it has four sides that are unequal"). Only visual attributes ("this is a square because it has four equal sides"). |
| 3 | 1 | Insufficient conditions given for rectangle ("two pairs of equal sides") and square ("all of its sides are equal") |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only 3 out of 4 completed correctly. Confusion between terms "isosceles" and "scalene" but sorted correctly in terms of properties. |
| 5 | 2 | No reference to other shapes. Incorrect lines of symmetry listed. |
| 6 | 1 | No evidence of formal reasoning at all |

Results for all learners were then summarised and presented as shown in Table 6:

**TABLE 6 – VAN HIELE LEVELS ALLOCATIONS FOR PRE-TEST**

| **TEST RESULTS:** | | **PRE-TEST** | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Tasks:** | | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 | ***average*** |
| 1 | Lynn | 1 | 2 | 2 | 2 | incomplete | incomplete | 1.8 |
| 2 | Paul | 1 | 1 | 1 | 2 | 2 | 1 | 1.3 |
| 3 | Tim | 2 | 2 | 2 | 1 | incomplete | incomplete | 1.8 |
| 4 | Darren | 2 | 1 | 1 | 1 | incomplete | incomplete | 1.3 |
| 5 | Ronald | 2 | 2 | 2 | 2 | incomplete | incomplete | 2.0 |
| 6 | Sarah | 1 | 2 | 3 | 3 | incomplete | incomplete | 2.3 |
| 7 | Garth | 1 | 2 | incomplete | incomplete | incomplete | incomplete | 1.5 |
| 8 | David | 1 | 2 | 1 | 2 | 2 | incomplete | 1.6 |
| 9 | Jeff | 1 | 1 | 1 | 3 | 2 | 1 | 1.5 |
| 10 | Rosie | 1 | 2 | 2 | 1 | incomplete | incomplete | 1.5 |
| 11 | Matt | 1 | 1 | 1 | 1 | 1 | incomplete | 1.0 |
| 12 | Anne | 1 | 1 | 1 | 1 | 2 | 1 | 1.2 |
| 13 | Mike | 1 | 2 | 2 | 2 | 1 | 1 | 1.5 |
| 14 | Dean | 1 | 2 | 2 | 2 | incomplete | incomplete | 1.8 |
| 15 | Talya | 1 | 1 | 1 | 1 | incomplete | incomplete | 1.0 |
| 16 | George | 2 | 2 | incomplete | incomplete | incomplete | incomplete | 2.0 |
| 17 | Tarryn | 1 | 2 | 2 | 1 | 2 | incomplete | 1.6 |
| 18 | Gloria | 1 | 1 | 1 | 1 | 1 | 1 | 1.0 |
| 19 | Ted | 1 | 1 | 1 | 1 | 2 | 1 | 1.2 |
| 20 | Andrea | 1 | 1 | 1 | 2 | incomplete | incomplete | 1.3 |
| 21 | Kate | 1 | 1 | 1 | 2 | 2 | 1 | 1.3 |
| 22 | Claud | 2 | 2 | 1 | 2 | 2 | incomplete | 1.8 |
| 23 | Cathy | 1 | 1 | 1 | 2 | 2 | incomplete | 1.4 |
| | Mean | 1.22 | 1.52 | 1.43 | 1.67 | 1.75 | 1.00 | 1.50 |
| | SD | 0.42 | 0.51 | 0.60 | 0.66 | 0.45 | 0.00 | 0.34 |

A similar table (see Table 7 in Chapter 7) was completed for the results of the post-test and comparisons between the results for each test were done using bar graphs. These analyses are discussed in detail in Chapter 7.

## 5.2 Reliability, validity and credibility

The research procedures were carefully selected in order to provide necessary rigour and to provide a check against possible bias. Scaife argues that the pre-/post-test process of data-collecting is reliable (Scaife, 2004, pg. 67). The quantitative data is in the form of the pre- and post-testing, or "test-retest". This is an example of the use and re-use of an instrument with the same subjects, where the results of each are compared. Timing between the two tests of this nature is important: "If the interval is too short, subjects' memories of the test may influence the retest. If it is too long, the subjects' beliefs, behaviours and so on may have changed" (ibid, pg. 67). In this study, the time interval was in the region of about 5 months; this was believed to be appropriate – neither too long nor too short.

The fact that the MathsLab module is designed for Grade 9s is also significant, as these learners are likely to be operating at VHLs 1, 2 and 3 only. The CDASSG study of the early 1980s indicated that these three levels are particularly "testable" (Usiskin, 1982, pg. 13) and that results can then be considered both reliable and valid.

Validity is "the degree to which a method, test or a research tool actually measures what it is supposed to measure" (Wellington, 2000, quoted by Scaife, 2004, pg. 68). The rubric to establish the Van Hiele Levels of the learners before and after the implementation of the module was designed with a great deal of care. As it is based on actual research on Van Hiele Levels completed by Burger and Shaughnessy (1986), it was anticipated that it would prove effective in this study. The validity of the study was also explored by the literature review. The use of technology and specifically GSP is well-researched and it was hoped that it would be possible to cross-check the results of the data collection. Many researchers have also used the Van Hiele Levels as a tool for analysis, as described in the literature review.

## 5.3 Ethical Issues

The University's code of ethics for researchers on human subjects was strictly adhered to. Parents or legal guardians were requested to sign consent forms, as were the individual learners (see Appendices 4 and 5). The form was accompanied by a short description of the study. An information sheet outlining the study was given to the principal (see Appendix 6), teachers and learners, and the participants were assured that they could withdraw at any point in the research.

The MathsLab module was a compulsory part of the teaching programme of the school in 2010, but participation in the research was optional. As it turned out, all learners participated willingly in the module. A control group was not implemented, as it was decided that no learners could be disadvantaged by not being exposed to the module. All 75 learners in the Grade 9 cohort completed the MathsLab modules, though a sample of 23 was chosen for in-depth analysis (see Chapter 7).

## 6. LIMITATIONS OF THE STUDY

The role of the teacher is an important consideration, both in terms of technology in mathematics education and in this research study in particular. The role of the teacher becomes increasingly one of mediator, with the function of bridging individual and social perspectives. However, this role has not been studied for this research report. It is important, though, to recognise that the tension between teaching the concepts and simply mediating the process could be a limiting factor in this particular study. Research has shown that appropriate facilitation by teachers coupled with the use of technology "can enable the learner to not only explore problems but to make links between different content areas that may otherwise have developed discretely" (Sacristán, 2010, pg. 192). Linked to this is the development of the tasks involved in the module. The tasks in the geometry module of MathsLab are in a process of ongoing development, and need to take into account important aspects as pointed out by Sacristán et al (2010), viz. the pedagogical setting, the context of inquiry, the level of openness of the activity, the sequencing of tasks within the activity and the mathematical content (ibid, pp. 188 – 191).

New demands on teachers and teacher educators have resulted from the incorporation of technology into mathematics education. Research in the field suggests a number of considerations that need to be taken into account. Research that "included observation of students' interactions while at the computer [revealed] that those interactions were more social and less predictable than was expected" (Blume & Heid, 2008, pg. 450). Using technology allows teachers to engage both themselves and their learners in exciting exploratory tasks, but other research has shown that this can lead to classroom management issues (Zbiek & Hollebrands, 2008, quoted by Heid & Blume, 2008, pg. 421). This did not happen in this study, as there is an established culture of effective working in the school where the research was implemented. The social nature of the work during the module was encouraged, in line with the importance of learning as a social activity.

These researchers also point out that the use of technology does not in itself guarantee learning (Heid & Blume, 2008, pg. 424) and internalisation of key concepts acquired through the use of the technology is of utmost importance. During the course of the module, learners gained control over the external signs and tools implicit in the use of technology, which refers to the instrumentalisation process within instrumental genesis (Goos et al, 2010, pp. 311 – 328), particularly in terms of instrumental initiation and instrumental exploration (see Table 1, pg. 15) . Technology in itself can afford learners the time for deeper reflection, allowing learners time to develop "inner speech" in Vygotskian terms, whereby speech becomes self-regulating and planning, as opposed to merely social and communicative (Wertsch & Stone, 1985, pg. 172). The success of the module thus depended on learners completing both the tasks and, importantly, the reflections at the end of each lesson.

Learners completed the module in late October 2010 and the post-test was administered to them during the school's November 2010 examination session. The results from the post-test were subjected to and compared to the same analysis as the results from the pre-test. It was anticipated that this would yield interesting, relevant data.

A limitation of the study which became apparent as the data was being analysed was that all conclusions were based solely on written evidence (the pre- and post-tests). The Burger and Shaughnessy study was able to assess levels more accurately due to the interviews conducted in the study. The written responses from the learners were reviewed a number of times to try to ensure consistency in the allocation of levels, and the rubric was referred to frequently, but it was difficult to ensure absolute objectivity in the allocation of levels. This could have been improved if more than one researcher (who was familiar with the van Hiele theory) had analysed the responses.

## 7. DATA ANALYSIS

The quantitative data (the results from the pre- and post-tests) collected in this study was subjected to analysis for comparison purposes. The rubric used for assessing the Van Hiele Levels of the learners for each task is given in Chapter 5, and the relevant sections of the rubric introduce the discussions of each task below. The sample size analysed was appropriate, comprising 23 learners randomly chosen from the 75 learners who participated in the study, all of similar age and comparable socioeconomic backgrounds. Each learner in the entire cohort had a statistically equal chance of being chosen for analysis and the 23 learners also represented varying mathematical abilities. All 23 learners wrote the pre-test, though a number did not complete it due to time constraints, and two learners were absent for the post-test. Time constraints limited the number of learners chosen for analysis to 23. Each task was assessed and commented on, and this is presented in Appendix 3. The test itself forms Appendix 2. Summaries and analyses of each task follow.

### 7.1. Task 1 – Drawing shapes

The section of the rubric used for analysis is indicated below:

| | VHL 1<br>VISUALISATION | VHL 2<br>ANALYSIS | VHL 3<br>ABSTRACTION | VHL 4<br>DEDUCTION |
|---|---|---|---|---|
| 1 – Drawing shapes | • Visual qualities only varied, e.g. orientation on page, "skinniness" of object<br>• Belief that only a finite number of triangles exist | • Focus on components of objects<br>• Realisation that components can be varied in an infinite number of ways<br>• Use of technical language | • Triangles drawn as representative s of general types<br>• Ability to interrelate shapes | NOT RELEVANT FOR DRAWING TASKS |

### 7.1.1. <u>Pre-test</u>

The first task involved sketching a triangle and then analysing the effects that result when changing one aspect at a time of the triangle. Six changes were required and it was interesting to note that many learners did not complete all six changes. Some changes even went back to original constructions. In most cases this reflected a poor grasp of the concept that changing one component results in the change of other components, and hence the interrelated nature of the properties of triangles. This analysis put the majority of learners at VHL1.

What also became immediately evident was an observation that emerged in the Burger and Shaughnessy study (Burger et al, 1986, pg. 45) that "… although the van Hieles have theorised that the levels are discrete structures, this study did not detect that feature". Another observation made in the Burger and Shaughnessy study was also reinforced in this project, namely that learners seemed to "oscillate" between levels within the task. Many learners showed different levels of thinking within tasks. This made it difficult to assign levels to each learner, and the results were reviewed a number of times. The predominant level exhibited in each task was then recorded. One learner, Anne, was assigned VHL1 from her constructions, but showed understanding of class inclusion in the question about the number of possible triangles that could be drawn ("I think as many different triangles that you want but they will always fall under the categories of right angled triangle, isosceles, equilateral etc."), which is a characteristic of VHL3 reasoning (ordering).

In most cases, the level assignment was VHL1, because there was a focus on the visual attributes of the changes. In a number of cases, there was a realisation that there were infinitely many triangles that could be drawn as well as some realisation of the properties of triangles (VHL2). There was also some acknowledgement that changing one component of a triangle resulted in changes in other components, e.g. "I decreased the length of side c by 1 cm this made all the angles change" (Ronald), an indication of VHL2 thinking.

Language used was frequently technical in nature (indicative of Level 2), but it was also clear that many learners were not using the language with any significant degree of understanding (e.g. "exterior triangle" – Rosie). 18 out of 23 learners were allocated VHL1, while 5 learners were categorised as reasoning at VHL2.

### 7.1.2. Post-test

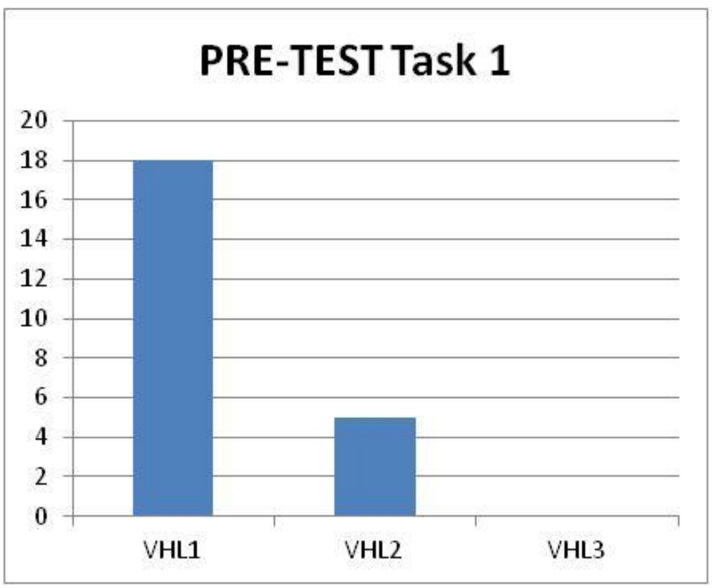
The post-test was written at the conclusion of the MathsLab module, and was written during the November examination period in 2010. The learners were told that they would be writing "a MathsLab examination" but were not aware that it was the same test as they had written about 5 months previously. They were also told that there was no point in preparing or studying for the examination. The pre- and post-tests were identical, and the same rubric was used for both to assign levels. Anecdotal evidence suggests that some learners attempted to prepare for the "examination" by reviewing the GSP module of the MathsLab course, while many did not prepare at all. They had been told that the results of the MathsLab examination would not be included in their final promotion mark.

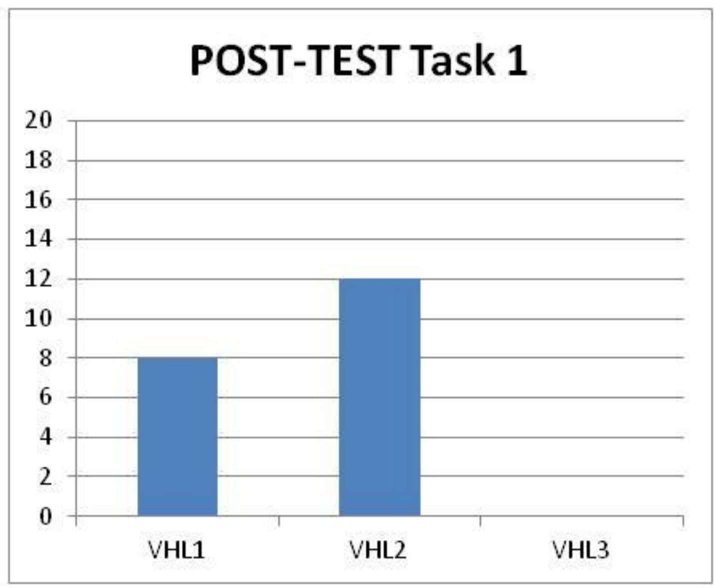
The spread of learners who achieved VHL1 and VHL2 in Task 1 changed considerably. In the pre-test, 18 learners were allocated VHL1 and 5 learners VHL2, while in the post-test, 8 learners were allocated VHL1 and 12 learners VHL2. One learner did not complete Task 1 while 2 learners did not write the post-test. That means that 10 learners moved up a Van Hiele Level after the completion of the GSP module. No learners moved down a level and no learners moved from VHL2 to VHL3.

An examination of individual results reinforces this change in Van Hiele Levels. One learner (Paul) showed a realisation that change in one component results in changes in others and the subsequent creation of a new category of triangles. This thinking was not evident in his pre-test. In particular, he realised that components can be varied in an infinite number of ways "… an infinite amount can be drawn because you can always change the length of sides or degrees of an angle by the slightest amount". This particularly indicates the power of the use of dynamic software, where it is possible to vary components infinitely and "by the slightest amount". This concept was articulated by other learners as well, e.g. George who stated that: "I think you can draw triangles for an infinite amount of time because you keep increasing or decreasing angles or lengths of lines"; and Ted who wrote ""You can draw an infinite of triangles (sic) because you can enlarge or change it a little bit each time". Claud indicated an understanding of class inclusion (characteristic of VHL3) in stating: "you can create an infinite number of triangles. But there will always be 6 categories: acute, obtuse, equilateral, scalene, isosceles and right-angled".

Figure 5 shows the results for Task 1 for both the pre-test and the post-test.

**Figure 5: Comparative results for task 1**




The mean for the pre-test was 1,22 with a standard deviation of 0,42 (giving a range for the mean of 0,80 to 1,64) while the mean for the post-test was 1,63 with a standard deviation of 0,50 (giving a range for the mean of 1,13 to 2,13). There is thus a clear progression between Van Hiele Levels in this task. This can also be interpreted as movement through the individual ZPDs for a number of the learners – specifically as described by Usiskin (1982, pg. 5): "at each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level". It needs to be remembered that, as De Villiers (2010, pg. 2) points out, progression from VHL1 to VHL2 involves more than merely verbalising intuitive knowledge – "the verbalisation goes together with a restructuring of knowledge" (ibid, pg. 3). In the assessment of this task, it was not possible to determine with any sense of certainty whether any restructuring of knowledge had taken place.

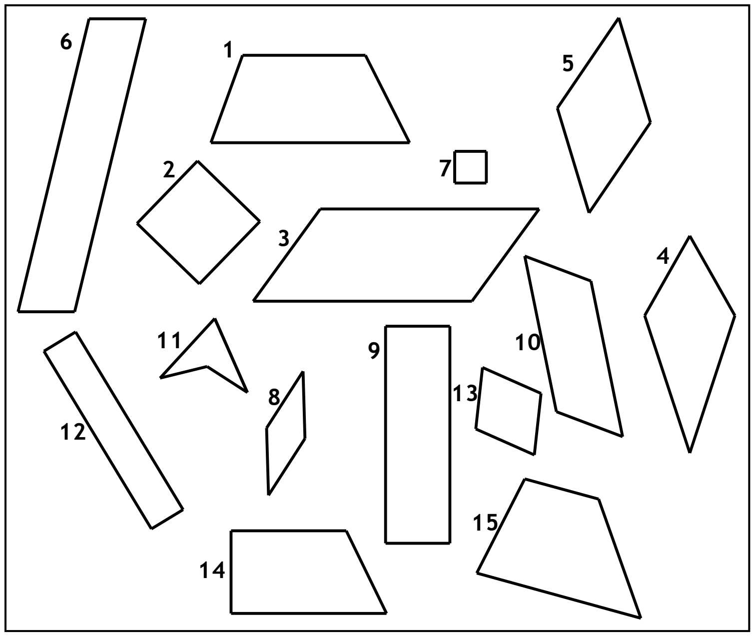
### 7.2 Task 2 – Identifying shapes

The section of the rubric used for analysis is indicated below:

| TASK | VHL 1 VISUALISATION | VHL 2 ANALYSIS | VHL 3 ABSTRACTION | VHL 4 DEDUCTION |
|---|---|---|---|---|
| 2 – Identifying shapes | • Imprecise visual qualities used in describing shapes<br>• Irrelevant attributes included in descriptions of shapes<br>• Some relevant attributes omitted<br>• References to visual prototypes | • Shapes identified explicitly by means of their properties<br>• Too many properties given<br>• Avoidance of class inclusions (e.g. squares and rectangles seen as disjoint) | • Shapes characterised by referring to other shapes<br>• Minimal characterisatio ns of shapes | • Evidence of conjecturin g and formal reasoning |

The task involved identifying a number of quadrilaterals, as shown in Figure 6 below:

**Figure 6: Task 2 – Identifying shapes**




Learners were told to identify whether shapes were squares, rectangles, parallelograms and rhombuses, and to justify their choices. The aim of the task was to explore learners' understandings of definitions of quadrilaterals and to examine their grasp of class inclusion (Burger et al, 1986, pg. 34).

### 7.2.1. Pre-test

Overwhelmingly, the learners assigned only one quadrilateral type to each shape. Many also named shapes other than those that were required, implying that they were not comfortable leaving shapes unnamed. In this task, there were many more VHL2 assigned than in the previous task, as many of the learners used the properties of quadrilaterals to define the type of quadrilateral, e.g. Sarah identified a rectangle as a figure in which "opp[osite] sides are equal and angles are equal". However, almost all of the other learners used more than sufficient properties and only in a few instances were sufficient properties used. Learner Ronald recognised class inclusion, identifying shapes 2 and 7 as rectangles and squares. 11 out of 23 learners were allocated VHL1 while 12 learners were believed to be reasoning at VHL2.

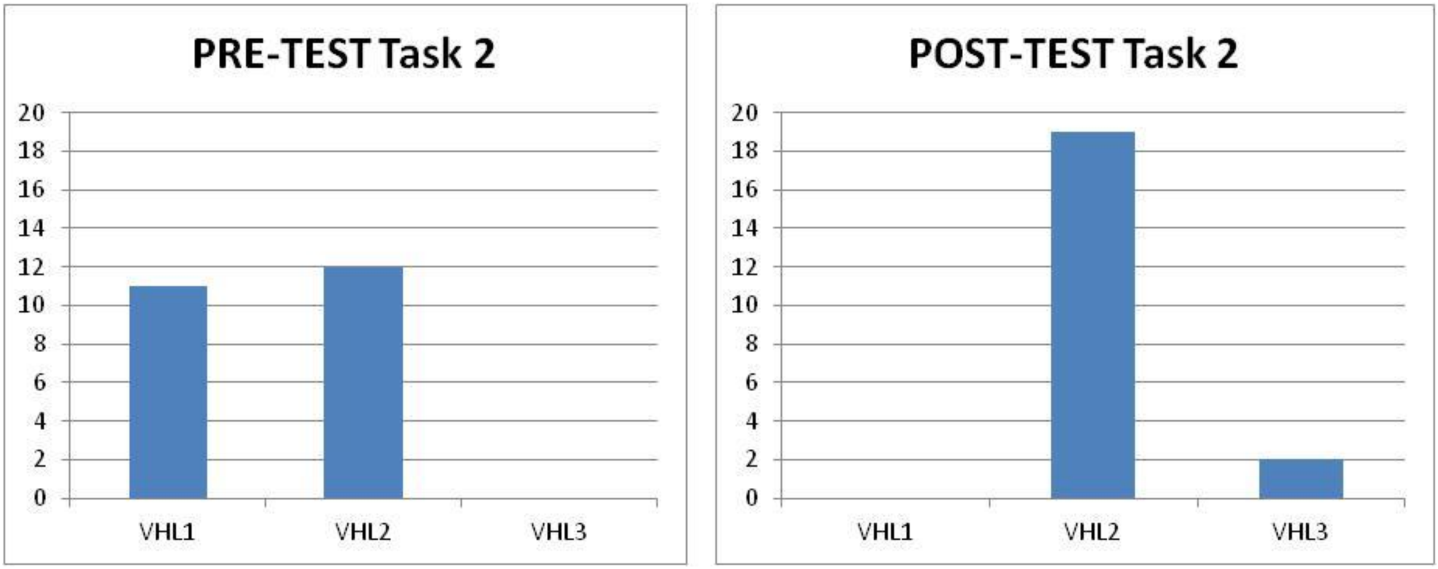
A weakness of this particular task is that it is prejudiced towards eliciting "partition" or non-inclusive thinking and can be misinterpreted by the participants. It is thus not possible to state that learners are unable to classify hierarchically simply from their results on this task.

### 7.2.2. **Post-test**

There was an overall movement up within the Van Hiele Levels. Many learners who were allocated VHL2 in the pre-test submitted almost identical answers in the post-test. The most significant change was that all learners who were operating at VHL1 in the pre-test (focussing predominantly on visual attributes of the shapes) used properties to identify and define shape in the post-test, and hence indicated movement to VHL2. In almost all cases, too many properties were used and listed. Furthermore, there was little evidence of class inclusion – the majority of learners only allocated one quadrilateral type to each example.

Individual results proved interesting. In the pre-test, Lynn used too many properties to identify shapes: "This is a rectangle because it has 2 sets of parallel lines and opposite sides are equal and it has 4 right angles", but was becoming more economical in the post-test: "square, because it has 4 right angles and all sides are equal". Darren moved from VHL1 in this task to VHL2, as he used properties to identify shapes as opposed to visual attributes only. In the pre-test, he made the following observations: "This is a rectangle because it looks like one"; "This is a parallelogram because it is like a rectangle pushed on its side"; "This is a rhombus because it resembles a square but pushed over". However, in the post-test, Darren identified the quadrilaterals correctly using properties of individual quadrilaterals: "it is a parallelogram because all 4 sides are equal and two sides are longer than the other two but it has no 90° angles". This was evident in almost all the learners who moved from VHL1 to VHL2 in the post-test.

**Figure 7: Comparative results for task 2**




The mean for the pre-test was 1,52 with a standard deviation of 0,51 (giving a range for the mean of 1,01 to 2,03) while the mean for the post-test was 2,10 with a standard deviation of 0,30 (giving a range for the mean of 1,80 to 2,40). The small standard deviation in the post-test verifies the small spread of results about the mean. As in Task 1, there is a very clear movement up between VHL1 and VHL2. This is significant in the context of the task, as learners moved almost completely away from using visual prototypes to identify quadrilaterals and focussed extensively on the properties of quadrilaterals. In Senk's terms (Senk, 1985, pg. 2), "knowledge is obtained from reasoning about the properties of figures". It is also clear that in this task, restructuring of knowledge has occurred. Two learners were allocated VHL3 on the post-test. Lynn used definitions that were largely economical and Ronald showed an understanding of class inclusion.

## 7.3 Task 3 – Defining shapes

The section of the rubric used for analysis is indicated below:

| TASK | VHL 1 VISUALISATION | VHL 2 ANALYSIS | VHL 3 ABSTRACTION | VHL 4 DEDUCTION |
|---|---|---|---|---|
| 3 – Defining shapes | • Imprecise visual qualities used in describing shapes<br>• Irrelevant attributes included in descriptions of shapes<br>• Some relevant attributes omitted<br>• References to visual prototypes<br>• Easily misled by orientation of figures | • Shapes identified explicitly by means of their properties<br>• Too many properties given<br>• Personal characterisations | • Shapes characterised by referring to other shapes<br>• Minimal characterisations of shapes<br>• Definitions complete<br>• Definitions result from modification of previous definitions<br>• Definitions economic and correct | • Evidence of conjecturing and formal reasoning |

In the Burger and Shaughnessy study, tasks 2 and 3 were combined into one task and participants were interviewed about identifying and defining shapes. In this study it was decided to split the task, as all responses were written and not in the form of an interview. This was appropriate due to the "test" nature of the pre- and post-tests. A serious limitation of this modus operandum, as discussed in Chapter 6, was that learners could not be prompted or asked to expand on their responses, unlike in an interview process. The process relied completely on written responses to questions regarding defining quadrilaterals. Clarification on responses could not be sought as in an interview.

The task comprised four questions:

- What would you tell someone to look for in order to pick out all the RECTANGLES on a sheet of quadrilaterals?

- What would you tell someone to look for in order to pick out all the SQUARES on a sheet of quadrilaterals?
- What would you tell someone to look for in order to pick out all the PARALLELOGRAMS on a sheet of quadrilaterals?
- What would you tell someone to look for in order to pick out all the RHOMBUSES on a sheet of quadrilaterals?

### 7.3.1. <u>Pre-test</u>

Many learners used “visual, imprecise considerations” to define the quadrilaterals. It was common for learners to describe parallelograms and rhombuses as “slanted” (or similar terminology) rectangles and squares, e.g. Rosie described a parallelogram as a “slanted rectangle”. Conventionally, good definitions of rectangles and squares come from modifications of parallelograms and/or rhombuses (e.g. a rectangle is a parallelogram with one right angle and a square is a rhombus with one right angle), and this would be indicative of VHL3 reasoning. In this study, learners were modifying the “special cases” of rectangles and squares to derive definitions for parallelograms and rhombuses, e.g. parallelogram – “rectangle with 2 sides converging inwards” (Ted). This was not accepted as being consistent with the criterion in the rubric for VHL3 thinking: “Shapes characterised by referring to other shapes”. Most learners also included a number of redundancies, e.g. rhombus – “all sides must be the same and opposite angles must be equal” (Dean). An economical definition for a rhombus would simply be that all sides are equal.

13 out of 21 learners were allocated VHL1, while 7 learners were believed to be reasoning at VHL2. Two learners did not complete the task. One learner (Sarah) was assigned a Van Hiele Level of 3 - minimal characteristics of the four types of quadrilaterals were listed and definitions were complete although not always economical (rectangle – “opposite sides equal in length and all angles are the same”; square – “all sides and angles are equal”; parallelogram – “the opposite sides are parallel”; rhombus – “the opposite sides are equal in length”). In no cases did learners

define shapes by referring to other shapes (e.g. a rectangle is a parallelogram with one right angle). Once again the observation by Burger and Shaughnessy that learners oscillate between levels in a given task was substantiated.

### 7.3.2. <u>Post-test</u>

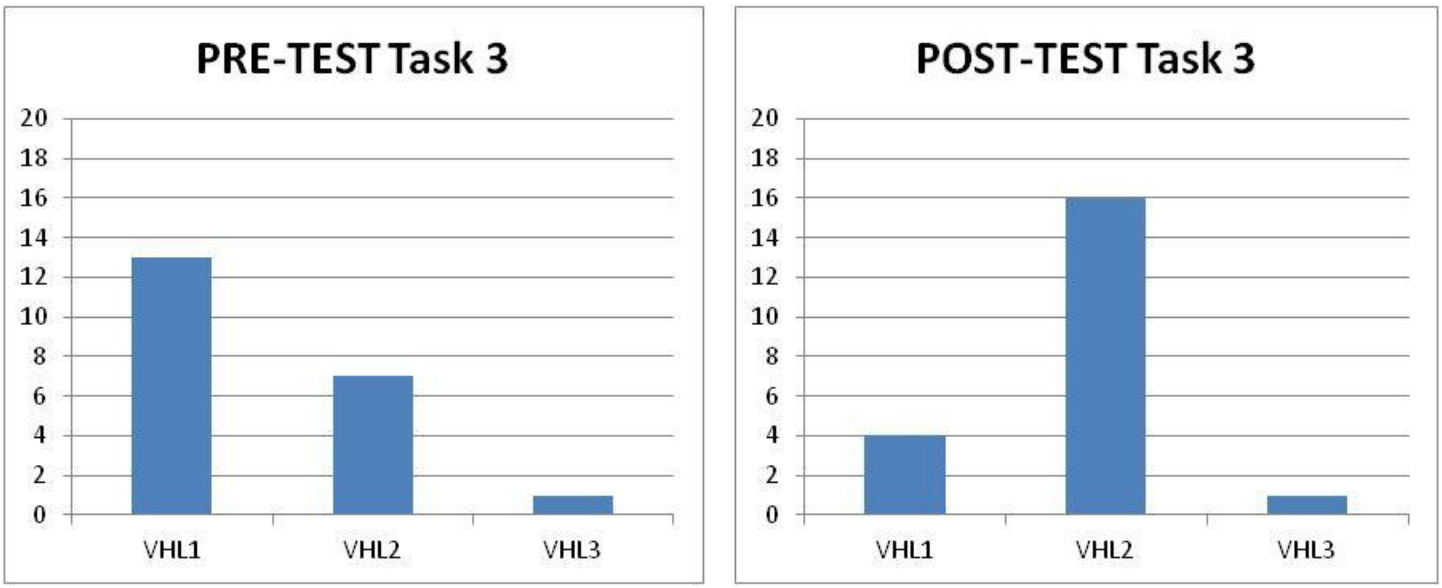
Little movement up the Van Hiele Levels was evident. Movement that did occur was from VHL1 where learners focussed on visual prototypes to VHL2 where the focus shifted to properties of quadrilaterals, e.g. in the pre-test Darren's response to the question on parallelograms was: "Look for a rectangle without 90° angles", which was improved on in the post-test: "Look for all four sides being parallel but not being the same length and it must have no 90° angles". In this instance (and many others) there was a mere listing of the properties of the relevant quadrilateral. There also seemed to be a lack of understanding of the concept of class inclusion, e.g. in a number of instances rectangles were specifically excluded from the category of parallelograms. Four learners were allocated VHL1, 16 VHL2 and one VHL3.

In two cases, learners Ronald and Tarryn moved from VHL2 to VHL1. In the pre-test, in response to the questions on parallelograms and rhombuses, Ronald defined the quadrilaterals by listing properties. In the post-test, however, his responses to these two questions were "look for a rectangle that is falling to one side" and "look for a square that is falling to one side". Tarryn defined a rectangle as "[t]he shapes with 2 short sides and 2 long sides". A change from fairly economical definitions in the pre-test to merely listing properties in the post-test resulted in Sarah moving from VHL3 in the pre-test to VHL2 in the post-test.

It was difficult to assign a level to Rosie in the post-test. In the pre-test she was clearly operating at VHL1. However, in the post-test she focussed on properties, and even hinted at an understanding of class inclusion: "Most properties of parallelograms are also applied to rectangles except no right angles". Kate went from VHL1 in the pre-test to VHL3 in the post-test. VHL3 was allocated to her for economical definitions (for a square: "… the shape that has all 4 sides that equal each other and … *has at least one*

*90° angle*" [my emphasis]) as well as for characterising rhombuses by referring to parallelograms ("a parallelogram, but all sides are equal").

**Figure 8: Comparative results for task 3**




The mean for the pre-test was 1,48 with a standard deviation of 0,60 (giving a range for the mean of 0,88 to 2,08) while the mean for the post-test was 1,86 with a standard deviation of 0,48 (giving a range for the mean of 1,38 to 2,34). There was very clearly a move from relying on what shapes looked like (VHL1) to the properties of shapes (VHL2). However, too many properties were listed in the majority of cases, resulting in uneconomical definitions. Class "exclusion" was strongly evident in that definitions for parallelograms and rhombuses frequently explicitly excluded rectangles and squares. In only one case (Kate) did a learner refine an existing shape in order to create a new definition.

## 7.4. <u>Task 4 – Sorting shapes</u>

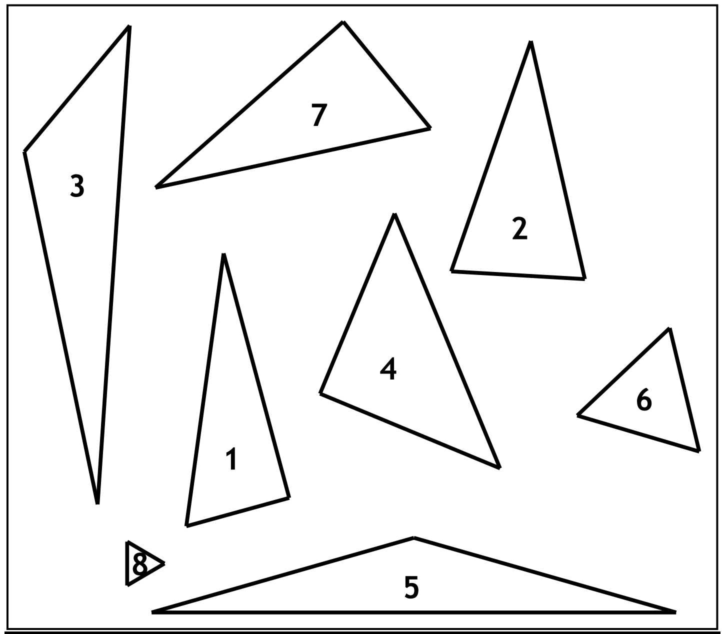
The section of the rubric used for analysis is indicated below:

| TASK | VHL1 VISUALISATION | VHL 2 ANALYSIS | VHL 3 ABSTRACTION | VHL 4 DEDUCTION |
|---|---|---|---|---|
| 4 – Sorting shapes | • Use of imprecise, visual qualities only<br>• Incomplete or inconsistent sortings | • Sortings using properties of shapes, even if properties imprecise<br>• Sorting by single attributes or properties | • Sorting refers explicitly to a variety of types<br>• Hierarchical classification of figures | NOT RELEVANT FOR SORTING TASKS |

The aim of this task was to see how many different sorting properties learners could come up with. Incomplete sortings or sortings which used imprecise or visual qualities only were assigned VHL1. If sorting was done using properties of the shapes, these learners were assigned VHL2. If any sortings referred to a variety of types and were explicit in this regard, they were assigned VHL3. More than one attribute or property needed to be used in the sorting for VHL3 to be allocated and most learners only considered the lengths of the sides of the triangles when sorting. Based on the work of Burger and Shaughnessy, it was decided that "sorting tasks did not elicit reasoning beyond Level 3" (ibid, pg. 43).

The triangles that needed sorting are shown in Figure 9:

**Figure 9: Task 4 – Sorting shapes**



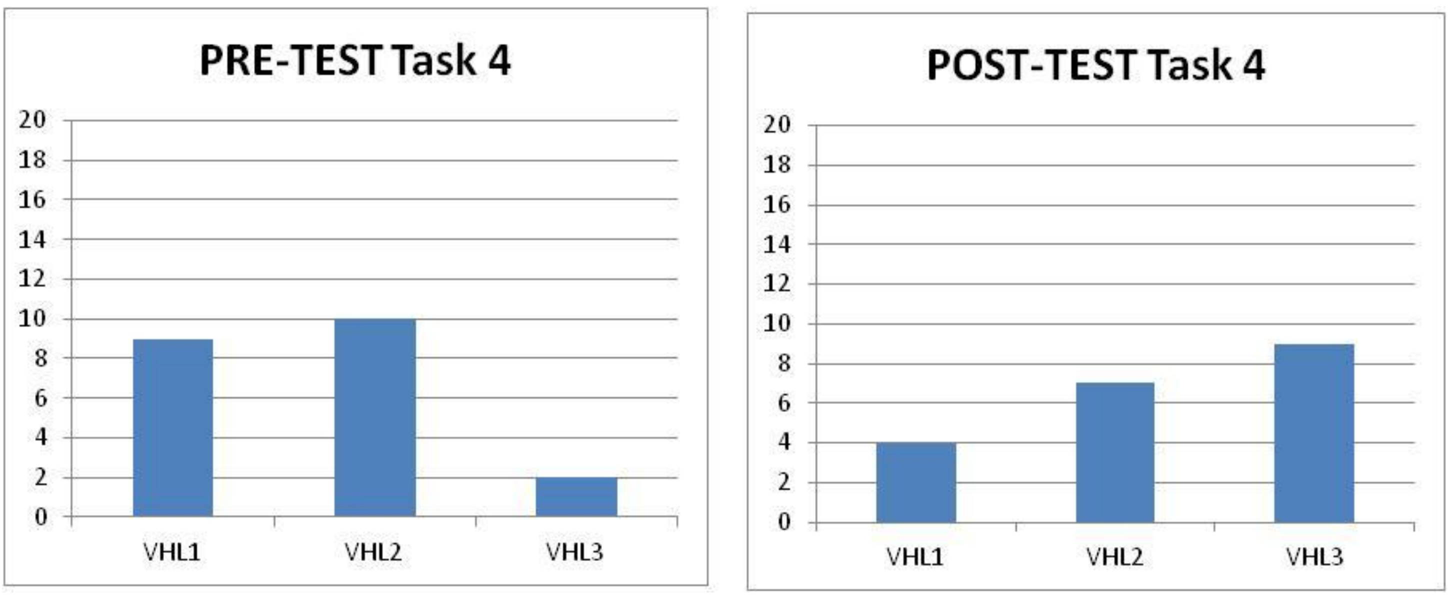

### 7.4.1. Pre-test

It was expected that the sorting task would show higher Van Hiele Levels than the previous three tasks but there were only two learners who were assigned VHL3 in the pre-test. In these cases, the learners referred explicitly to triangle types (equilateral, isosceles, scalene and right-angled) and looked at both the lengths of the sides of the triangles and the sizes of the angles (though only right-angled triangles were sorted - not acute-angled or obtuse-angled triangles). Most of the other learners sorted the triangles based on a single attribute, viz. the lengths of the sides. This resulted in them being allocated VHL2 instead of VHL3. A surprise from the data was that 9 out of the 21 learners who completed this task used imprecise visual qualities only, and were awarded VHL1, e.g. Rosie had no sorting other than "they are all triangles". She then listed the properties of triangles. Listing of properties was common amongst those learners awarded VHL1. Another learner, Anne, showed confusion in her response: "They can be alike in terms of different things being the same". Gloria used very basic visual attributes: "you would have to rotate or make them bigger"; "you would have to change their angles"; and "you would have to modify them to fit each other". 9 learners were allocated VHL1, 10 learners VHL2 and 2 learners VHL3.

### 7.4.2. Post-test

There was a significant shift in the number of learners who were allocated VHL3. 9 learners sorted the triangles based on more than one attribute – in all cases the sortings were in the categories equilateral, isosceles, scalene and right-angled triangles. There were only four sortings that were requested and it would have been interesting to see if, had there been more sortings required, learners would have sorted the triangles into acute-angled and obtuse-angled triangles as well. This would constitute an improvement in the instrument.

**Figure 10: Comparative results for task 4**




The mean for the pre-test was 1,67 with a standard deviation of 0,66 (giving a range for the mean of 1,01 to 2,33) while the mean for the post-test was 2,25 with a standard deviation of 0,79 (giving a range for the mean of 1,46 to 3,04).

### 7.5 Task 5 – Properties of quadrilaterals

The section of the rubric used for analysis is indicated below:

| TASK | VHL 1 VISUALISATION | VHL 2 ANALYSIS | VHL 3 ABSTRACTION | VHL 4 DEDUCTION |
|---|---|---|---|---|
| 5 – Properties of quadrilaterals | • Visual attributes only | • Properties listed without reference to other shapes<br>• Some irrelevant or incorrect properties listed | • Properties listed with reference to other shapes | NOT RELEVANT |

In Task 5, properties of quadrilaterals were tested by giving shapes to the learners and asking them to list as many properties as possible. This task was only given to the learners after they had completed and submitted tasks 1 to 4 and was not based on any task in the Burger and Shaughnessy study. The aim of the task was to see how deeply learners understood the properties of quadrilaterals. This is linked to the concept of defining quadrilaterals in that any referrals to other quadrilaterals would suggest reasoning was at VHL3.

#### 7.5.1. Pre-test

The majority of learners were assigned VHL2 and included cases where properties were listed, there was no reference to other quadrilaterals, and some irrelevant or incorrect properties were included. Many learners listed insufficient properties, but the rubric specified that VHL1 only be allocated when only visual attributes were listed. As assessment of the learners' responses progressed, it was felt that the rubric was too general for this task. Some consideration of incorrect responses needed to be included.

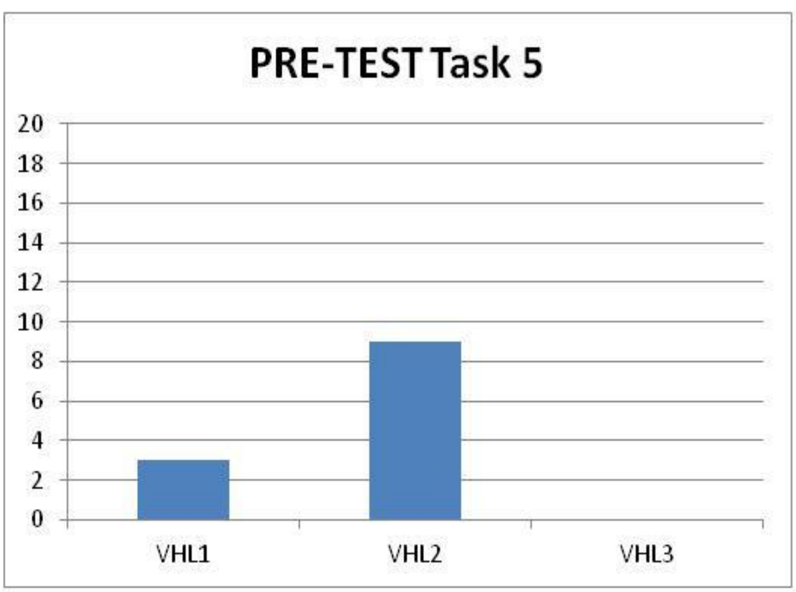
The majority of learners showed no concept of class inclusion. Properties of quadrilaterals excluded some possibilities, e.g. the definitions of parallelograms frequently included comments excluding shapes with 90° angles. The majority of learners also included the comment that the non-parallel lines of trapeziums were always equal. This led to them stating that trapeziums had a single line of symmetry as well. The diagram given, in fact, did look like an isosceles trapezium, which meant that learners were considering visual attributes when listing properties. This was seen as another weakness in the test and it is recommended that in future a diagram of a trapezium that is clearly not isosceles is included.

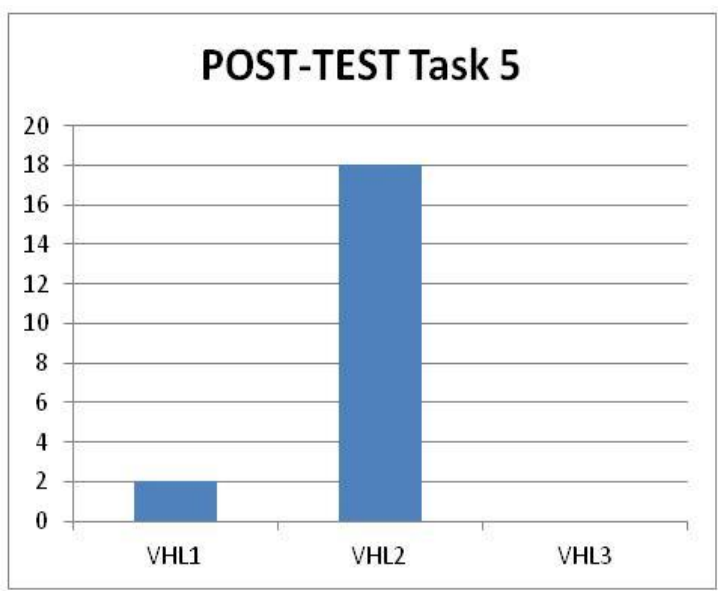
### 7.5.2. Post-test

In most instances, there was very little difference between the pre- and post-test results. In some cases, where learners had listed insufficient properties in the pre-test, they listed more properties in the post-test. Unfortunately, the rubric did not allow for differentiation between levels in these instances.

One learner (Mike) relied totally on measurements of the figures in the post-test. He measured each angle and the length of each side. This implies a reliance on the visual attributes of each quadrilateral and not an understanding of each type in general terms. For this, he was allocated VHL1. Another learner (Tarryn) described a rhombus as "… being in the shape of a diamond".

**Figure 11: Comparative results for task 5**



The post-test indicates that no learners were allocated VHL3 and that the majority of learners were believed to be operating at VHL2. A statistical comparison between the results is not relevant here, as, due to time constraints, so few learners actually completed the task in the pre-test, but the statistical parameters are given for the sake of completeness. The mean for the pre-test was 1,75 with a standard deviation of 0,47 (giving a range for the mean of 1,28 to 2,22) while the mean for the post-test was 1,90 with a standard deviation of 0,31 (giving a range for the mean of 1,59 to 2,21). The low standard deviation for the post-test indicates the consistency of the learners being allocated VHL2.

## 7.6 Task 6 – Formal reasoning

The section of the rubric used for analysis is indicated below:

| TASK | VHL 1 VISUALISATION | VHL 2 ANALYSIS | VHL 3 ABSTRACTION | VHL 4 DEDUCTION |
|---|---|---|---|---|
| 6 – Formal reasoning | • Attempted answer by drawings only<br>• Question rephrased as an assertion | • Use of a variety of diagrams<br>• Validity of statement tested inductively or empirically<br>• Lack of understanding of mathematical proof | • Attempt at informal, deductive arguments<br>• Explicit use of "if, then" reasoning | • Deductive argument, with proof as final authority<br>• Sufficient conditions for a quadrilateral to be a parallelogram |

This task consisted of the following question: "Suppose you have a quadrilateral with both pairs of opposite sides equal. Will the opposite sides also be parallel? In your answer, you need to explain your reasoning. Use diagrams to assist your explanation". It was not expected that any learners would be allocated high VHLs, as, at this stage of their school career, they had not had any introduction to formal reasoning at all. The task was included to see whether there was any form of informal reasoning, or even attempts at verification and justification.

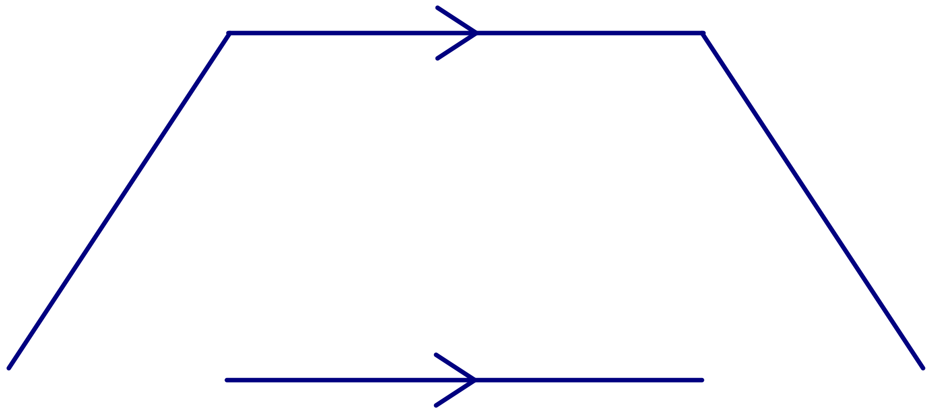
Kilpatrick et al (2001, pg. 130) allude to the development of proof skills as a consequence of the development of reasoning skills, particularly in terms of justification as a function of proof: “One manifestation of adaptive reasoning is the ability to justify one’s work”. However, they also point out that justification is not necessarily proof: “Proof is a form of justification, but not all justifications are proofs. Proofs (both formal and informal) must be logically complete, but a justification may be more telegraphic, merely suggesting the source of the reasoning” (ibid, pg. 130). It was anticipated that, in this study, “proof” would not go beyond justification. Justification can, however, be considered a starting point for the development of more sophisticated proof skills.

### 7.6.1. Pre-test

None of the learners showed any evidence of formal reasoning. This was not unexpected for this age group as the learners had never been exposed to any form of formal proof. Any analysis of the given shape was informal and focussed on component parts and attributes, e.g. Anne drew a square in an attempt to justify the statement. A number of learners also did not complete Section B.

### 7.6.2. Post-test

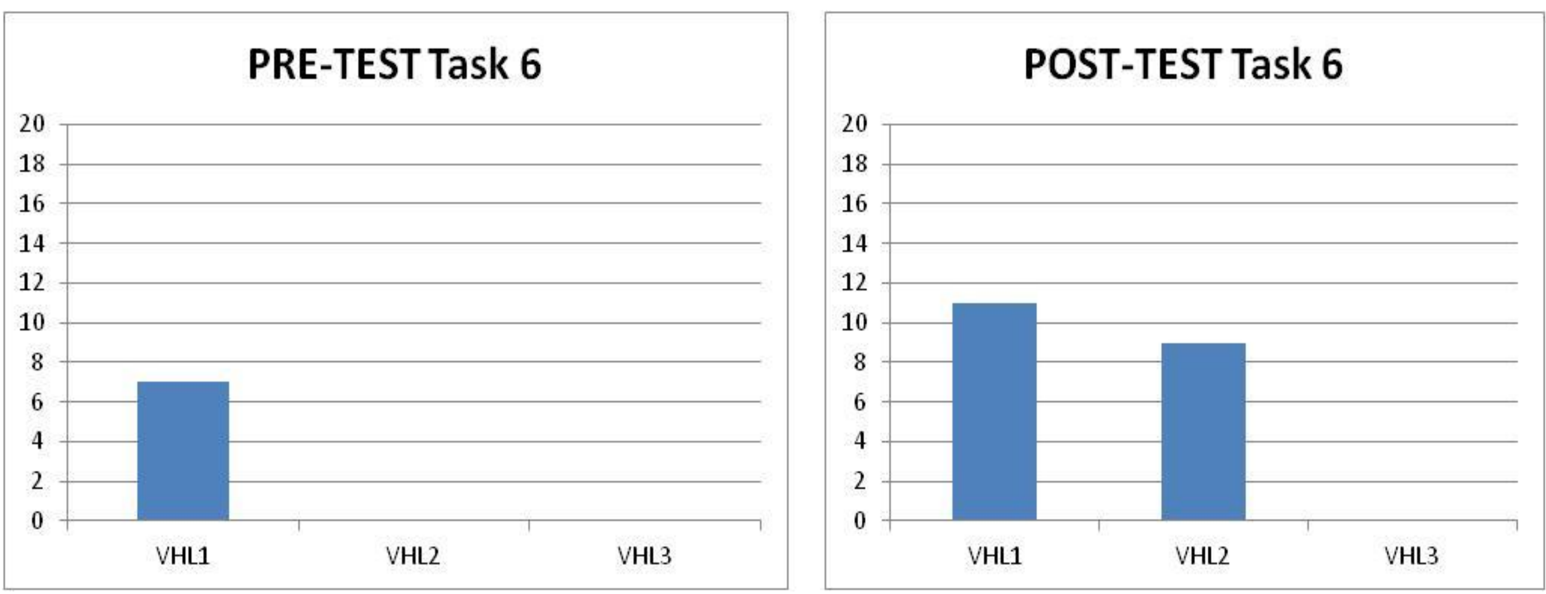
Some learners attempted to verify the statement using diagrams. One learner (Darren) drew two diagrams with opposite sides equal and then drew a “counter-example” with opposite sides unequal. Jeff drew a diagram similar to the one below and stated that the statement was true because otherwise “… the lines couldn’t connect so they could have to parallel”:



This shows a degree of inductive reasoning as well as experimentation – again important stages before the development of formal reasoning.

A number of learners investigated "counter-examples", drawing diagrams with equal opposite sides verifying the statement and including diagrams where sides were not equal to show that opposite sides could be not equal. One learner (George) linked his diagrams to the properties of quadrilaterals: "If a quadrilateral has [two] sets of opposite sides that are equal it would form a square or a rectangle. We know that the properties of the different shapes are that they are parallel lines".

**Figure 12: Comparative results for task 6**



Again, a statistical comparison in task 6 is not particularly relevant due to the low number of learners (7) that completed the pre-test. The post-test does indicate an almost even spread between learners allocated to VHL1 and those allocated to VHL2 with a mean of 1,45 and a standard deviation of 0,51. Attempts at reasoning were similar throughout the cohort and the major difference in allocation of levels depended on the number of diagrams drawn and the amount of text used by the learners to explain their reasoning. The results for the post-test were not unexpected, due to the fact that formal reasoning had not been introduced to learners in this grade.

## 7.7. General analysis

The following table was generated in order to analyse individual results as well as trends within tasks; in particular, any evidence of positive movement through the Van Hiele Levels.

## Table 7 – Analysis of individual learner results

| TEST RESULTS: | | PRE-TEST | | | | | | | POST-TEST | | | | | | | %age incr in average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tasks: | | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 | average | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 | average | |
| 1 | LB | 1 | 2 | 2 | 2 | incomplete | incomplete | 1.8 | 1 | 3 | 2 | 3 | 2 | 2 | 2.2 | 23.8 |
| 2 | PB | 1 | 1 | 1 | 2 | 2 | 1 | 1.3 | 2 | 2 | 2 | 2 | 2 | 1 | 1.8 | 37.5 |
| 3 | TC | 2 | 2 | 2 | 1 | incomplete | incomplete | 1.8 | 2 | 2 | 2 | 3 | 2 | 1 | 2.0 | 14.3 |
| 4 | DC | 2 | 1 | 1 | 1 | incomplete | incomplete | 1.3 | 2 | 2 | 2 | 3 | 2 | 2 | 2.2 | 73.3 |
| 5 | RE | 2 | 2 | 2 | 2 | incomplete | incomplete | 2.0 | 2 | 3 | 1 | 2 | 2 | 1 | 1.8 | -8.3 |
| 6 | SG | 1 | 2 | 3 | 3 | incomplete | incomplete | 2.3 | 2 | 2 | 2 | 3 | 2 | 1 | 2.0 | -11.1 |
| 7 | GH | 1 | 2 | incomplete | incomplete | incomplete | incomplete | 1.5 | incomplete | 2 | 2 | 1 | 2 | 1 | 1.6 | 6.7 |
| 8 | DK | 1 | 2 | 1 | 2 | 2 | incomplete | 1.6 | 1 | 2 | 2 | 3 | 2 | 1 | 1.8 | 14.6 |
| 9 | JK | 1 | 1 | 1 | 3 | 2 | 1 | 1.5 | 1 | 2 | 1 | 2 | 1 | 2 | 1.6 | 6.7 |
| 10 | RL | 1 | 2 | 2 | 1 | incomplete | incomplete | 1.5 | 2 | 2 | 2 | 1 | 2 | 1 | 1.7 | 11.1 |
| 11 | ML | 1 | 1 | 1 | 1 | 1 | incomplete | 1.0 | absent | absent | absent | absent | absent | absent | absent | |
| 12 | AM | 1 | 1 | 1 | 1 | 2 | 1 | 1.2 | 2 | 2 | 2 | 1 | 2 | 2 | 1.8 | 57.1 |
| 13 | MM | 1 | 2 | 2 | 2 | 1 | 1 | 1.5 | 2 | 2 | 2 | 2 | 1 | 1 | 1.7 | 11.1 |
| 14 | DM | 1 | 2 | 2 | 2 | incomplete | incomplete | 1.8 | 2 | 2 | 2 | 3 | 2 | 2 | 2.2 | 23.8 |
| 15 | TO | 1 | 1 | 1 | 1 | incomplete | incomplete | 1.0 | 1 | 2 | 1 | 2 | 2 | 1 | 1.5 | 50.0 |
| 16 | GP | 2 | 2 | incomplete | incomplete | incomplete | incomplete | 2.0 | 2 | 2 | 2 | 1 | 2 | 2 | 1.8 | -8.3 |
| 17 | TP | 1 | 2 | 2 | 1 | 2 | incomplete | 1.6 | 1 | 2 | 1 | 3 | 2 | 2 | 1.8 | 14.6 |
| 18 | GQ | 1 | 1 | 1 | 1 | 1 | 1 | 1.0 | absent | absent | absent | absent | absent | absent | absent | |
| 19 | TS | 1 | 1 | 1 | 1 | 2 | 1 | 1.2 | 1 | 2 | 2 | 2 | 2 | 2 | 1.8 | 57.1 |
| 20 | AS | 1 | 1 | 1 | 2 | incomplete | incomplete | 1.3 | 1 | 2 | 2 | 3 | 2 | 1 | 1.8 | 46.7 |
| 21 | KV | 1 | 1 | 1 | 2 | 2 | 1 | 1.3 | 2 | 2 | 3 | incomplete | 2 | incomplete | 2.3 | 68.8 |
| 22 | CM | 2 | 2 | 1 | 2 | 2 | incomplete | 1.8 | 2 | 2 | 2 | 3 | 2 | 2 | 2.2 | 20.4 |
| 23 | CV | 1 | 1 | 1 | 2 | 2 | incomplete | 1.4 | 1 | 2 | 2 | 2 | 2 | 1 | 1.7 | 19.0 |
| | Mean | 1.22 | 1.52 | 1.43 | 1.67 | 1.75 | 1.00 | 1.50 | 1.63 | 2.10 | 1.86 | 2.25 | 1.90 | 1.45 | 1.87 | |
| | SD | 0.42 | 0.51 | 0.60 | 0.66 | 0.45 | 0.00 | 0.34 | 0.50 | 0.30 | 0.48 | 0.79 | 0.30 | 0.51 | 0.22 | |
| | | | | | | | | %age increase: | 34 | 38 | 30 | 35 | 9 | 45 | | |

An interesting factor in the analysis of the data was the change in individual learners' "average" Van Hiele Level, e.g. Darren went from an average of 1,3 to an average of 2,2 – an increase of 73%. 18 learners overall showed an increased Van Hiele Level while 3 actually showed a decrease. The overall averages for all learners over all tasks differed marginally – from 1,50 for the pre-test to 1,87 for the post-test. This represents a shift within VHL1 but not from VHL1 to VHL2. However, it is arguable whether this analysis is statistically sound, and I believe the earlier analyses of individual tasks and individual learners are more telling.

In general, the overall trend indicated a substantial rise in Van Hiele Levels in tasks 1 to 4, a small rise in Task 5, and, because very few of the learners completed Task 6 in the pre-test, the result for Task 6 was not considered to be statistically viable. Table 8 summarises the changes in means for the tasks.

**Table 8 - Comparison of means per task**

| | **PRE-TEST** | | | | | | **POST-TEST** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 |
| **Mean** | 1,22 | 1,52 | 1,43 | 1,67 | 1,75 | 1,00 | 1,63 | 2,10 | 1,86 | 2,25 | 1,90 | 1,45 |
| **Std dev** | 0,42 | 0,51 | 0,60 | 0,66 | 0,45 | 0,00 | 0,50 | 0,30 | 0,48 | 0,79 | 0,30 | 0,51 |
| ***%age increase in mean*** | | | | | | | 34 | 38 | 30 | 35 | 9 | 45 |

The significant rise in the means of the first 4 tasks was not unexpected, as the tasks involved drawing, identifying and sorting polygons. The means for tasks 2 and 4 both indicated a general move from VHL1 to VHL2 in the learners, and there was clear evidence that restructuring of knowledge had occurred. These two tasks will be discussed further.

Task 2 involved identifying quadrilaterals and the predominant change from pre-test to post-test was the result of learners shifting away from considering visual attributes to using properties to identify the shapes. A typical example is the case of Darren, who wrote statements in the pre-test such as: “This is a rectangle because it looks like one”; “This is a parallelogram because it is like a rectangle pushed on its side”; and

“This is a rhombus because it resembles a square but pushed over”. In the post-test, Darren identified quadrilaterals based on properties, e.g. “it is a parallelogram because all 4 sides are equal and two sides are longer than the other two but it has no 90° angles”. This is not a correct definition of a parallelogram and it excludes rectangles and squares. It is also confused (all four sides are equal but two sides are longer than the other two), but it does represent a movement away from a definition based on what the shape looks like towards a (clumsy) definition based on the properties of the shape. In Senk’s terms (Senk, 1985, pg. 2), Darren has moved from acquiring knowledge by observation to acquiring knowledge by “reasoning about the properties of figures”.

Task 4 required learners to sort triangles into different categories. The change in the mean from 1,67 to 2,25 was a result of the increased number of learners that were allocated VHL3. In the pre-test, many sortings were based on a single attribute, which was lengths of sides. In the post-test, a significant number of learners sorted the triangles based on more than one attribute, viz. lengths of sides and sizes of angles. An improvement in the instrument would be to make provision for more sortings, in order to establish if learners would consider other attributes as well.

# 8. CONCLUSIONS

The study was exciting in a number of areas. The mathematical content was interesting and pertinent to the new South African curriculum (to be implemented in 2012), and the implications for the development of mathematical reasoning significant. The tool for the implementation of the module, viz. GSP, is fun to use, and has enormous potential in mathematics education including and beyond Euclidean geometry. The effective use of technology as a mediator in the zone of proximal development is a subject worthy of research and has interesting pedagogical implications.

This type of research study has come at a significant time in the South African context, with Euclidean geometry being reintroduced into the core Mathematics curriculum in Grade 10 in 2012. The value of dynamic geometry software in the development of generalising, conjecturing and justifying skills is clear from the literature review. The emphasis on exploration as a means for cognitive development is the subject of much research; however, further research on the effective implementation of technology in mathematics education is necessary, as effective implementation has lagged behind technological development (Goos et al, 2010, pg. 312). As this research study proceeded, it became clear that refinements to both the GSP module in the MathsLab course and the tool of analysis (the pre- and post-test) are necessary. This is in line with Artigue who noted that as tools have been introduced throughout mathematical history, they have not immediately become “efficient mathematical instruments for the user” (2002, pg. 245).

Three important issues raised by Drijvers and Trouche (2008, pp. 363 – 391) have been illustrated in this research. The first issue involves the need for accurate analyses of how learning happens in technological environments, but this research investigated specifically whether (and not how) learning happens in the MathsLab environment. In this specific context, learning should lead to a change in the nature of geometric reasoning and this was tested by means of the pre- and post-test. The second issue raised by Drijvers and Trouche related to the organisation by the teacher in such an environment. In this study it was clear that the MathsLab course's structure allowed for efficient organisation. The modules in MathsLab were designed to be user-friendly and allowed learners to proceed at individual paces. Once learners had grasped the skills required to use GSP, there were few requests for assistance during the implementation of the module. MathsLab is potentially a particularly effective teaching tool. The ongoing development of teaching resources such as MathsLab relates directly to the third issue raised by Drijvers and Trouche, viz. "the development of resources for professional development of new teaching practices" (ibid, pg. 386).

The role of the teacher during the MathsLab course was essentially that of a facilitator. Drijvers and Trouche (2008, pg. 381) emphasise the responsibilities of teachers in a technological environment; particularly in terms of the design of the environment so that mathematical situations exist that manage both mathematical knowledge and instrumental knowledge. This was evident in the MathsLab classes through the design of the actual modules and the function of the teacher. A *didactic exploitation system* (see Chapter 3) was established – potential resources were fully used, and the "components of the environment" (in this case, GSP and the general technology environment) were coordinated and integrated with the specific "mathematical situation" (in this case, Euclidean geometry).

In terms of the role of technology in mathematics education, this research study wanted to see to what extent the instrument (GSP) mediated the activity, and whether GSP could be classed as a "knowledgeable other" in a classroom environment, supporting progress through an individual's zone of proximal development (in Vygotskian terms). The fact is that GSP has to be actively engaged with for the process of instrumental genesis to happen. Engagement with the MathsLab module seemingly led to the development of mental schemes, as learners moved increasingly away from a reliance on visualisation and recognition towards an approach that was increasingly analytical in nature in terms of the Van Hiele Levels. This trend was borne out in the comparison between the pre- and post-tests. Whether this can be explicitly stated as being as a result of the MathsLab is difficult to ascertain, but some measure of instrumental genesis was positively experienced by the majority of the learners. Also, although the affective dimension of working with technology was not formally researched, I suggest that the learners enjoyed the dynamic nature of GSP and were confident about exploring as they worked. This is an area which requires further research.

The results of the post-tests show that learners developed both mathematical meaning and discourse. This was due to, amongst other factors, the connection to current mathematics curricula. It seems safe to conclude that a process of instrumental genesis did take place. It is also clear that a *domain of abstraction* (Sacristán et al, 2010, pg. 179) evolved, and that the exploration implicit in the module led to increasing formalisation of meaning and discourse. A domain of abstraction is an environment in which it is possible to develop generalisations of abstract concepts and the MathsLab environment was designed with this in mind.

The use of GSP confirmed other aspects explored in the theoretical framework (Chapter 3), e.g. the ease with which didactical transposition comes into play (Hollebrands, Laborde & Sträβer, 2008, pg. 156) through use of the drag function. The drag function allows a change in geometric knowledge from the way in which it was created to a form which allows for both effective teaching and effective learning. The results from the post-test also confirm Heid & Blume's observation that the use of technology helps to "bring to consciousness mathematical ideas and procedures" (Heid & Blume, 2008, pg. 427).

An important aspect of the research was to ascertain whether there was any movement within the zones of proximal development of individual learners. It was thus important to establish the actual development levels of the learners both before and after the implementation of the GSP module. Good learning must take place slightly in advance of development and the MathsLab module was designed with this purpose in mind. The Van Hiele model proved to be successful in measuring the learning and a clear positive movement within and between Van Hiele Levels is evident from the analyses of the results of the pre- and post-tests.

The concept of *internalisation* is key in Vygotskian theory, and it appears that internalisation of concepts did occur. This could have been established with more certainty if a form of interview process had been possible. Another fundamental aspect in this theory is that of mediated participation and the importance of social interaction. The role of the teacher is again important in this respect, specifically by supporting the learners through hints, props and scaffolding. The experience in the MathsLab classes was that teacher intervention became less as learners grew in confidence and conscious awareness. GSP acted as a mechanism through which semiotic mediation developed – this being the bridge that "connects the external with the internal and the social with the individual" (Wertsch & Stone, 1985, pg. 162). The role of the teacher was that of cultural mediator – implying intentional use of the artefact (GSP) as semiotic mediator. A further social interaction in the implementation of MathsLab was that of the pairing of the learners. Learners worked together and were encouraged to discuss their progress together. This resulted in reflection of the tasks and reflection is one of the key processes leading to internalisation of concepts.

One of the most interesting implications to arise from this research project was the change in geometric reasoning or thinking of the learners as measured by the Van Hiele Levels. The MathsLab module was implemented in a four-month period, so it can be claimed that progression between the levels was due to instruction and not biological maturation. Usiskin (1982, pg. 6) described five phases of the learning process, viz. *inquiry*, *directed orientation*, *explanation*, *free orientation* and *integration*. The first three phases were specifically targeted in the module as part of the design of the module. The latter two processes were observed in a non-scientific way during the actual lessons. The nature of these five phases could be explored more thoroughly, but this was not part of this research project.

The rubric developed from the work of Burger and Shaughnessy proved to be a useful tool of assessment. In some tasks, learners fluctuated between levels in terms of this rubric, and in all cases, the predominant Van Hiele Level was allocated to the learner. The greatest movement between the Van Hiele Levels happened in Task 2 (from a mean of 1,52 to a mean of 2,10) and Task 4 (from a mean of 1,67 to a mean of 2,25). In Task 2, many learners moved from predominantly identifying quadrilaterals using visual prototypes to recognition of properties of quadrilaterals. Task 4 was related to this skill in that triangles needed to be sorted into different categories. Initially triangles were sorted based on the single attribute of lengths of sides, while the post-test showed sorting based on this attribute as well as a second attribute, viz. sizes of angles. Both tasks illustrated a movement from VHL1 (visualisation) to VHL2 (analysis).

The critical question of the research was: "*Do the Van Hiele Levels of learners change when measured before and after the implementation of the MathsLab module?*" Emphatically the answer to this question is "yes". The analyses of the pre- and post-tests showed progression within and between the Van Hiele Levels. What is less clear is whether this progression can be attributed solely to the implementation of the MathsLab module. An attempt was made to ensure this in that this section of the curriculum was deliberately not taught during formal classroom lessons. This implies that the only exposure learners had to the concepts was through MathsLab. No other worksheets or examples on the concepts were given to the learners. Certainly MathsLab constituted the major tool for teaching and learning for this section of work.

There are other aspects to the implementation of MathsLab that are not related to the use of technology. Learners completed the module by working in pairs and this may have been a contributing factor to progress in the work. Learners may also have had a more positive attitude towards MathsLab as it represents a change from traditional teaching methods – this possible affective factor was not researched.

Allied to the critical question is whether technology can be used as a "knowledgeable other" and a semiotic mediator in the learning of Euclidean geometry. This research study was not able to arrive at a conclusion in this regard. What the study does show is that the potential for ongoing research in the field of technology in mathematics education is great. This includes research on working in pairs or groups as opposed to working individually, possible gender differences in the approach to technology, the nature of the learning process (the "how") in a MathsLab type setup and certainly the affective factors involved. The MathsLab module, the use of the pre- and post-tests and the allocation of Van Hiele Levels as a measurement of progress through the zone of proximal development are but a small part of this research.

# 9. <u>APPENDICES</u>

## <u>Appendix 1 – Relevant learning outcomes and assessment standards</u>

**Learning Outcome 3: Space and Shape (Geometry)**

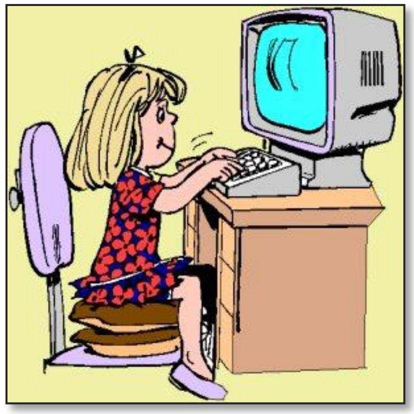
*The learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions.*

**Assessment standards**

We know this when the learner:

- Recognises, visualises and names geometric figures … including: regular and irregular polygons …
- Uses geometry of straight lines and triangles to solve problems and to justify relationships in geometric figures.
- Uses transformations, congruence and similarity to investigate, describe and justify … properties of geometric figures and solids, including tests for similarity and congruence of triangles.

**Appendix 2 – Pre- and post-test** (modified for typographical purposes)



# Grade 9 MathsLab

**NAME:**

……………………………………………………………………………………………………………

**MATHS TEACHER:**

………………………………………………………………………………………………

**INSTRUCTIONS:**

- There are two sections to this assessment. You will only be issued with Section B once you have completed Section A
- All questions must be answered on the question papers
- Work neatly and answer all questions as fully as possible
- Have fun!

**SECTION A**

**TASK 1 – Drawing shapes**

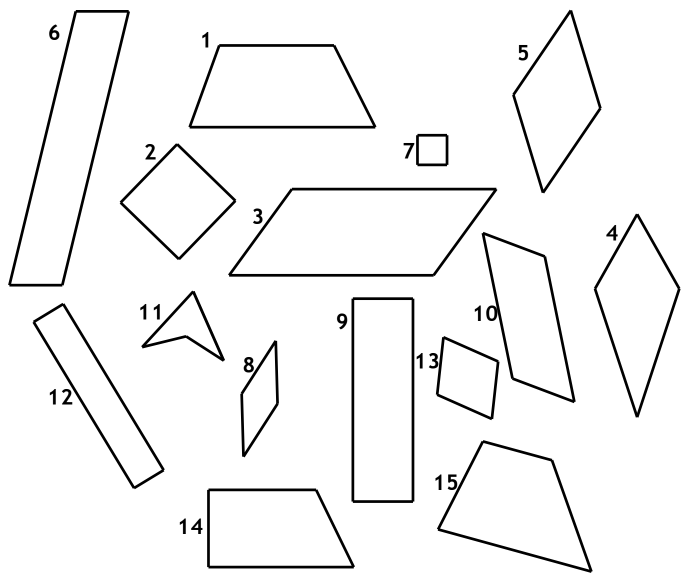
| | | |
|---|---|---|
| 1. Draw any triangle: | | Describe this triangle: |
| 2. Draw another triangle, by changing ONE THING in your first triangle: | | Describe the change that you made: |
| 3. Draw a third triangle, by changing ONE THING in your second triangle (it must be different to what you changed in the first triangle): | | Describe the change that you made: |

| | | |
|---|---|---|
| 4. Draw another triangle, by changing ONE THING in your previous triangle (it must be different to what you changed in all previous triangles): | | Describe the change that you made: |
| 5. Draw another triangle, by changing ONE THING in your previous triangle (it must be different to what you changed in all previous triangles): | | Describe the change that you made: |
| 6. Draw another triangle, by changing ONE THING in your previous triangle (it must be different to what you changed in all previous triangles): | | Describe the change that you made: |

| How many different triangles do you think can be drawn? |
| --- |
| |

**TASK 2 – Identifying shapes**

Consider the following quadrilaterals:




*Adapted from Burger, W. & Shaughnessy, J. (1986)*
*Characterising the Van Hiele Levels of development in geometry.*
*Journal for Research in Mathematics Education. Vol. 17, no. 1, pp. 31 – 48*

a) Put an S on each square, an R on each rectangle, a P on each parallelogram and a B on each rhombus.

b) Explain each of your choices by completing the following. If you did not choose any of S, R, P or B, explain why not.

| QUADRILATERAL NUMBER | EXPLANATION OF YOUR CHOICE(S) OF QUADRILATERAL TYPES |
| --- | --- |
| 1 | |

| | |
|---|---|
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |

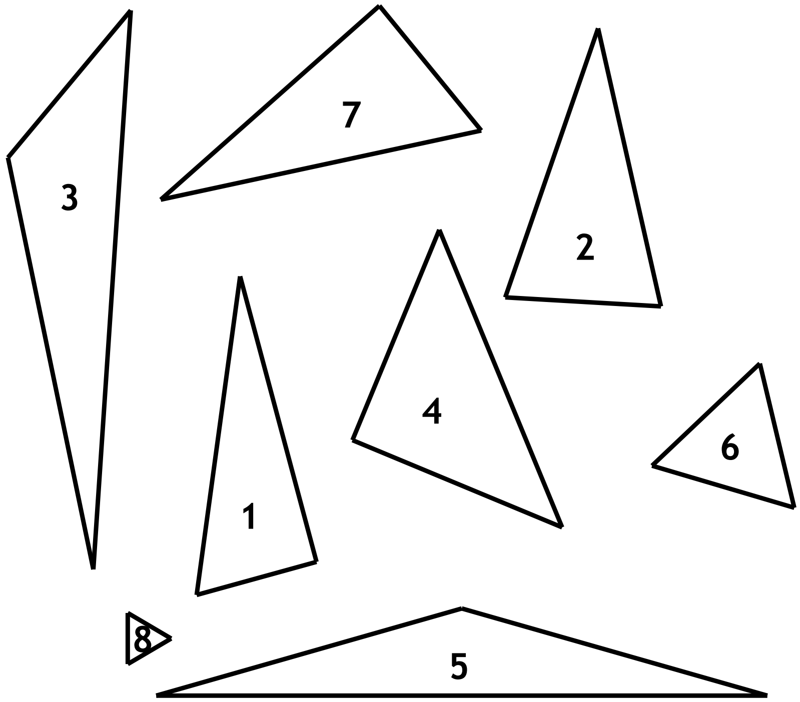
| | |
|---|---|
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |

**TASK 3 – Defining shapes**

| |
|---|
| What would you tell someone to look for in order to pick out all the RECTANGLES on a sheet of quadrilaterals? |
| What would you tell someone to look for in order to pick out all the SQUARES on a sheet of quadrilaterals? |
| What would you tell someone to look for in order to pick out all the PARALLELOGRAMS on a sheet of quadrilaterals? |
| What would you tell someone to look for in order to pick out all the RHOMBUSES on a sheet of quadrilaterals? |

## TASK 4 – Sorting shapes

Consider the following triangles:




*Adapted from Burger, W. & Shaughnessy, J. (1986)*
*Characterising the Van Hiele Levels of development in geometry.*
*Journal for Research in Mathematics Education.*
*Vol. 17, no. 1, pp. 31 – 48*

| |
|---|
| Can you put some of these triangles together that are alike in some way? Explain how they are alike. |
| Can you put some of these triangles together that are alike in a different way? Explain how they are alike. |
| Can you put some of these triangles together that are alike in another way? Explain how they are alike. |
| Can you put some of these triangles together that are alike in another way? Explain how they are alike. |

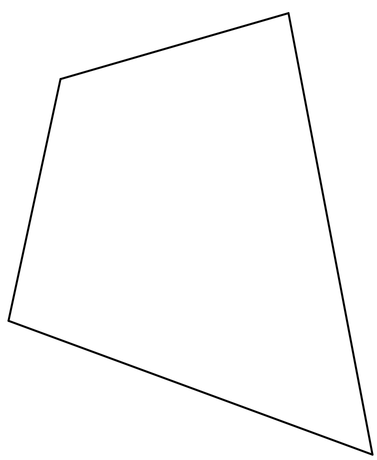
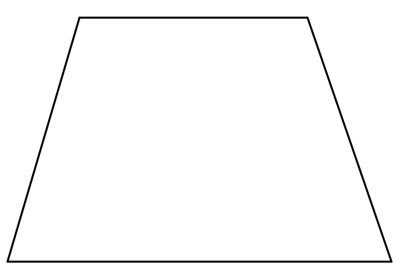
Once you have completed this section of the test, hand it to your teacher, who will give you Section B to complete.

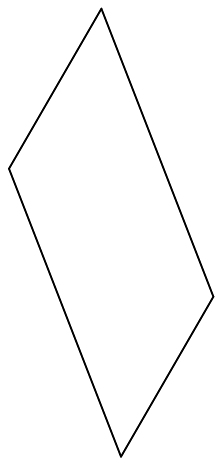
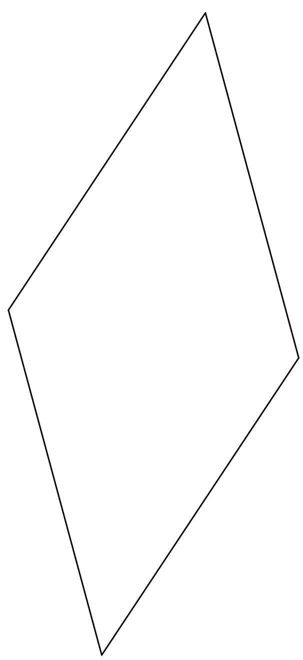
**<u>NAME</u>: …………………………………………………………………………………………………………**

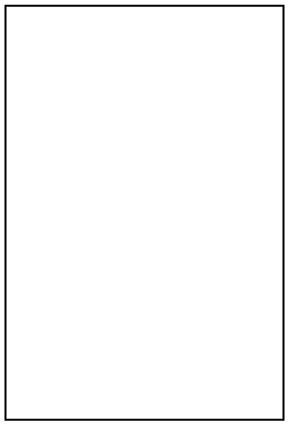
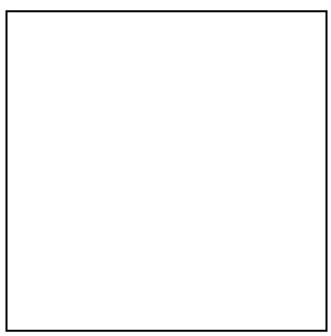
**<u>SECTION B</u>**

**<u>TASK 5 – Properties of quadrilaterals</u>**

For each of the following quadrilaterals, list as many properties that you can think of. Consider sizes of angles, lengths of sides, diagonals, relationships between sides, lines of symmetry, and anything else you think is important.

| **QUADRILATERAL** | **PROPERTIES** |
| --- | --- |
|  | |
| Trapezium:  | |

| | |
|---|---|
| Parallelogram:  | |
| Rhombus:  | |

| | |
|---|---|
| Rectangle:  | |
| Square:  | |

**TASK 6 – FORMAL REASONING**

Suppose you have a quadrilateral with both pairs of opposite sides equal. Will the opposite sides also be parallel? In your answer, you need to explain your reasoning. Use diagrams to assist your explanation.

## Appendix 3 – Individual summaries

NAME: LYNN PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Limited changes made. Some changes went back to earlier constructions. Angles and sides varied – hence focus on components. Realisation of infinite possibilities |
| 2 | 2 | Shapes defined by properties. No class inclusion (only one type of quadrilateral per shape). Too many properties ("This is a rectangle because it has 2 sets of parallel lines and opposite sides are equal and it has 4 right angles"). |
| 3 | 2 | Predominantly defined using component parts, but too many properties listed and one clear example of visual considerations (rhombus – "square on its side") |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only single attribute considered (lengths of sides) |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: LYNN POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on properties and realisation of infinite number of triangles but much more focus on visual aspects, e.g. triangle rotated, reflected and enlarged |
| 2 | 3 | Shapes defined by properties again, but in most instances economical definitions (square, because it has 4 right angles and all sides are equal"). No evidence of class inclusion. |
| 3 | 2 | Responses better than pre-test but still too many properties given. Good defn for rhombus: "a quadrilateral with two pairs of parallel lines and all sides are equal" |
| 4 | 3 | Specific referral to explicit types. More than one attribute considered. |
| 5 | 2 | Properties listed without reference to other shapes. |
| 6 | 2 | Diagrams used to inductively show validity of statement. Attempt at analysis. |

NAME: PAUL PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Generally only lengths of sides changed. In no. 5 realisation that changing lengths also changed angles. Realisation of infinity of possibilities. No. 6: only visual quality varied (size) |
| 2 | 1 | Little understanding of quadrilaterals ("This is a kite [not asked for] but also a rhombus because it has four sides that are unequal"). Only visual attributes ("this is a square because it has four equal sides"). |
| 3 | 1 | Insufficient conditions given for rectangle ("two pairs of equal sides") and square ("all of its sides are equal") |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only 3 out of 4 completed correctly. Confusion between terms "isosceles" and "scalene" but sorted correctly in terms of properties. Only lengths of sides considered |
| 5 | 2 | No reference to other shapes. Incorrect lines of symmetry listed. |
| 6 | 1 | No evidence of formal reasoning at all |

NAME: PAUL POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Realisation that change in one component results in changes in others and creation of new categories of triangles. Specific realisation that components can be varied in an infinite number of ways "… an infinite amount can be drawn because you can always change the length of sides or degrees of an angle by the slightest amount" |
| 2 | 2 | Good recognition of quadrilaterals but no class inclusion and too many properties listed. Parallelogram: "its opposite sides are parallel and equal and it does not contain any 90° angles". Does not recognise that a rectangle is a parallelogram |
| 3 | 2 | Consideration of properties of quadrilaterals but too many properties listed. |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only single attribute (sides) considered |
| 5 | 2 | Good listing of properties. No class inclusion e.g. parallelogram: "… the shape does not contain any 90° angles". Economic defn for rhombus: "this shape has 4 equal sides". Trapezium ltd to isosceles trapezium |
| 6 | 1 | Diagrams used to attempt to reason. No evidence of inductive reasoning. |

NAME: TIM PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Focus on components ("this is scalene as it has 3 different sides and angles"). Incomplete realisation that as one component varies, other components change ("I made the bottom side longer and so two angles and a side also changed"). Occasional focus on visual qualities only. |
| 2 | 2 | Shapes defined by properties. No class inclusion (only one type of quadrilateral per shape). Too many properties (e.g. for a rhombus: "all sides equal. Opp sides parallel. Opp angles equal"). |
| 3 | 2 | Sufficient conditions for rectangle ("… all equal angles of 90°") but too many properties for other quadrilaterals |
| 4 | 1 | No sorting. Visual qualities only considered. |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: TIM POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Good focus on changes to component parts but no acknowledgement of resulting changes. Realisation of infinite number of triangles (no response this question in pre-test). |
| 2 | 2 | Too many properties listed. No economic definitions and no class inclusion |
| 3 | 2 | Mere listing of properties |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, right-angled and scalene) – more than one attribute considered |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case: "Only one pair of opposite sides must be parallel. The other two sides must also be equal in length" |
| 6 | 1 | Use of single diagram. Incorrect response. |

NAME: DARREN PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Focus on components but varies one and all change (changed from right-angled triangle to equilateral triangle). Finite number of triangles ("more or less a 1000 triangles could be drawn") |
| 2 | 1 | Only visual attributes considered: "This is a rectangle because it looks like one"; "This is a parallelogram because it is like a rectangle pushed on its side"; "This is a rhombus because it resembles a square but pushed over". |
| 3 | 1 | Too many constraints (rectangle – "… a shape that has 4 90° angles with 2 sides longer than each other") and reference to visual prototypes (parallelogram – "look for a rectangle without 90° angles"). |
| 4 | 1 | No sorting. Visual qualities only considered. |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: DARREN POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Good focus on changes to component parts but no acknowledgement of resulting changes. Realisation of infinite number of triangles (contrast to pre-test: "more or less a 1000 triangles could be drawn"). |
| 2 | 2 | Correct identification of quadrilaterals based on properties. No class inclusion: e.g. "it is a parallelogram because all 4 sides are equal and two sides are longer than the other two but it has no 90° angles" |
| 3 | 2 | Focus on and listing of properties |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, right-angled and scalene) – more than one attribute considered |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case |
| 6 | 2 | Use of diagrams to verify statement, including the use of a counter-example where lengths of sides are not equal. |

NAME: RONALD PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Variation of one component and realisation that more than one other component changes as a result ("I decreased the length of side c by 1 cm this made all the angles change"). Realisation of infinite number of possibilities |
| 2 | 2 | Shapes identified by properties or "criteria". Some class inclusion evident: shapes 2 and 7 identified as squares and rectangles. |
| 3 | 2 | Too many properties for quadrilaterals; specifying 2 sets of equal and parallel lines |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only single attribute (sides) considered |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: RONALD POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Good focus on changes to component parts but no acknowledgement of resulting changes. Realisation of infinite number of triangles as in pre-test. |
| 2 | 3 | VHL3 allocated because of evidence of class inclusion and minimum (though not economic) properties listed for definition |
| 3 | 1 | Movement towards visual attributes: "look for a rectangle that is falling to one side" |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only 3 out of 4 completed correctly. |
| 5 | 2 | In one case insufficient properties listed. Isosceles trapezium |
| 6 | 1 | Simply states "yes" and draws 2 diagrams |

NAME: SARAH PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on and listing of the properties of specific triangles – visualisation. More than one component changed but no realisation of this. Finite number of triangles. |
| 2 | 2 | Oscillation between visual prototypes ("It is a leaning over square") and good economic definitions (rectangle: "Opp sides are equal and angles are equal") |
| 3 | 3 | Minimal characteristics listed and definitions complete (rectangle – "opposite sides equal in length and all angles are the same"; square – "all sides and angles are equal"; parallelogram – "the opposite sides are parallel"; rhombus – "the opposite sides are equal in length"). |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, scalene and right-angled). 4 out of 4 completed correctly. |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: SARAH POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Explicit focus on properties. No response to question re possible number of triangles |
| 2 | 2 | Not relying on visual attributes as in pre-test. Identification by listing of properties. Non-economic. |
| 3 | 2 | Listing of properties |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, right-angled and scalene) – more than one attribute considered |
| 5 | 2 | Properties listed with no reference to other shapes. Isosceles trapezium |
| 6 | 1 | Use of two diagrams. Simply states "yes". |

NAME: GARTH PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Components varied but no recognition of resultant changes in other components. Equilateral triangle with 3 sides equal to 5cm was changed by making one length 6cm, but angles still labelled as 60°. Task was not completed |
| 2 | 2 | Focus on properties of quadrilaterals ("I chose square because it had all the properties of a square"). Incorrect answer for shape 1: "I chose rhombus, I did a process of elimination, it wasn't a square because it didn't have 4 equal sides, it wasn't a rectangle because the top line was greater in length than the bottom line". |
| 3 | - | Task not completed |
| 4 | - | Task not completed |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: GARTH POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | - | The task was not completed. Only 2 changes made. |
| 2 | 2 | Listing of properties of quadrilaterals. Correction from pre-test in shape 1. No concept of class inclusion evident: "It is a parallelogram because it has 2 sets of opposite and parallel sides and the sides aren't all equal or 90° angles like a square or rectangle" |
| 3 | 2 | Oscillation between levels e.g. parallelogram: "2 sets of opposite parallel sides [that] looks a bit like a rectangle" |
| 4 | 1 | No reference to explicit types: "they all have three sides"; "all have three vertices"; "2 sides will always join at a point" |
| 5 | 2 | Properties listed with no reference to other shapes. Correct trapezium defn |
| 6 | 1 | Uses single example of square to justify statement |

NAME: DAVID PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Only visual qualities varied. No recognition that change in one cpt causes change in others. Task not completed |
| 2 | 2 | Recognition of properties but no class inclusion. Some confusion: "Square – all four sides equal (90°); all four angles equal" |
| 3 | 1 | Too many conditions given. No response to question re parallelograms |
| 4 | 2 | Sorting by both single attributes only (VHL2) and by explicit types (VHL3). Predominantly VHL2 |
| 5 | 2 | No reference to other shapes. Incorrect lines of symmetry listed. |
| 6 | - | incomplete |

NAME: DAVID POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | More focus on properties. Task not completed. Belief that only 6 different triangles could be drawn. |
| 2 | 2 | No improvement on pre-test. Defn of rhombus not good enough: "opposite lines are equal, opposite angles are equal, opposite lines are parallel" – could be a parm. Same as in pre-test |
| 3 | 2 | Listing of properties |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only 3 out of 4 completed correctly. Angles and sides analysed |
| 5 | 2 | Excludes some properties but predominantly VHL2 |
| 6 | 1 | Poor attempt at justification |

NAME: JEFF PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on visual aspects of triangles. Some listing of properties. Answer to number of triangles that can be drawn: "I think that 5 angles can be drawn" |
| 2 | 1 | Some acknowledgement of properties but insufficient, e.g. "square because all the sides are equal to each other". Visual: "rhombus because all the sides are equal but the lines turn outwards". |
| 3 | 1 | Little understanding of quadrilaterals evident. No differentiating characteristics given. |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, scalene and right-angled). 4 out of 4 completed correctly. |
| 5 | 2 | Incorrect, too few or irrelevant properties listed. |
| 6 | 1 | Attempt to reason using visual attributes only "Yes both the opposite sides will be parallel because the length will force it to level out" |

NAME: JEFF POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Only visual qualities varied. Belief that only four triangles could be drawn (down from pre-test) |
| 2 | 2 | Slight improvement on pre-test. Tendency towards more economical definitions: "Parallelogram because opposite sides are parallel and lengths the same" |
| 3 | 1 | Focus on visual attributes e.g. rhombus: "Look for a square that has been squashed a little and doesn't have 90° angles anymore" |
| 4 | 2 | Only lengths of sides considered |
| 5 | 1 | Many incorrect and insufficient properties listed. No understanding of lines of symmetry |
| 6 | 2 | Use of counter-example to illustrate statement |

NAME: ROSIE PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Clear confusion with technical terminology e.g. "exterior triangle"; isosceles and scalene confused. Listing of properties. Final question: "There are 13 triangles" |
| 2 | 2 | Focus on both visual ("Parallelogram – slanted rectangle") and properties, but too many properties ("Square – all sides equal, 4 right angles, all angles equal, all sides parallel"). |
| 3 | 1 | Reliance on visual attributes (parallelogram – "it is a slanted rectangle"). Too many constraints on definition for rectangle. |
| 4 | 1 | No sorting other than "they are all triangles". Properties then listed |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: ROSIE POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | More focus on components and some realisation that change in one component results in changes in other components. Belief that 9 triangles could be drawn (down from 13 in pre-test). |
| 2 | 2 | Almost exactly the same as in pre-test. Listing of properties; focus on visual attributes ("shifted rectangle") |
| 3 | 2 | Difficult to assign level. Properties listed. Reference to their shapes and their properties: "most properties of parallelogram are also applied to rectangles except no right angles" – hint of class inclusion |
| 4 | 1 | No change from pre-test |
| 5 | 2 | Visual attributes considered e.g. parallelogram a "shifted rectangle" and a rhombus "a shifted square". Properties listed with no reference to other shapes. |
| 6 | 1 | Attempt at reasoning but incorrect |

NAME: MATT PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Only sides varied throughout. No observation of other components changing. Realisation of infinite number of possibilities. |
| 2 | 1 | Poor grasp of quadrilaterals. Visual attributes and some properties considered. Incorrect idea of rhombus. Definition of square: "All sides are equal and quadrilateral". |
| 3 | 1 | Confusion of terms and properties (rectangle – "all sides are 90° and parallel to each other"). Reliance on visual attributes (square – "all same size") |
| 4 | 1 | Complete lack of understanding of the task's requirements |
| 5 | 1 | In some cases, too few properties listed |
| 6 | - | incomplete |

NAME: MATT POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | | absent |
| 2 | | absent |
| 3 | | absent |
| 4 | | absent |
| 5 | | absent |
| 6 | | absent |

NAME: ANNE PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Changes all based on visual attributes and no description given of changes. Interesting understanding of class inclusion in question re number of possible triangles: "I think as many different triangles that you want but they will always fall under the categories of right angled triangle, isosceles, equilateral eTc." – VHL3 |
| 2 | 1 | Some focus on properties but largely visual and imprecise ("Rhombus – equal in length, slanted square" and "Diamond [not given as an option] – all four sides same but pushed further than a rhombus"). Also referred to "rectangular prism". |
| 3 | 1 | Some VHL2 definitions with too many attributes; some reliance on visual prototypes (rhombus – "slanted square") |
| 4 | 1 | No understanding of sorting. "They can be alike in terms of different things being the same" |
| 5 | 2 | In some cases all properties listed, in other cases too few. |
| 6 | 1 | A single special case (square) drawn in attempt to answer question. |

NAME: ANNE POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Good realisation that change in one component results in changes in other components. Explicit reference to properties. Ambiguous answer to question re number of possible triangles: "3 different types but 6 different triangles" |
| 2 | 2 | Almost total focus on properties and no suggestion of visual attributes. Too many properties listed. |
| 3 | 2 | Listing of properties |
| 4 | 1 | No change from pre-test |
| 5 | 2 | Much better than pre-test in that more properties listed |
| 6 | 2 | Attempt at justification using diagrams and "counter-examples" |

NAME: MIKE PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on visual aspects of changes. Observed that change to isosceles triangle by changing angles meant change in length of side but stated that this was not possible. No further analysis. Realisation that infinite number of triangles possible |
| 2 | 2 | Shapes defined by properties. No class inclusion (only one type of quadrilateral per shape). Too many properties (e.g. for a parallelogram: "Opp sides equal. Opp sides parallel."). |
| 3 | 2 | Too many properties listed (rhombus – "all sides are equal and opposite sides are parallel"). |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only 3 out of 4 completed correctly. |
| 5 | 1 | In all cases too few properties listed – based on what diagram looks like. No mention of lines of symmetry. |
| 6 | 1 | Specific quadrilateral drawn (rectangle) and irrelevant assertion made: "Yes, because if opposite sides are equal in length [then] opposite angles are also equal" |

NAME: MIKE POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Simple listing of measurements. Realisation that infinite number of triangles possible |
| 2 | 2 | Exactly as for pre-test |
| 3 | 2 | Exactly as for pre-test |
| 4 | 2 | Only sides considered |
| 5 | 1 | Total reliance on measurements completed on each diagram. |
| 6 | 1 | A number of diagrams drawn, but no attempt to explain reasoning |

NAME: DEAN PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Visual aspects of change. Realisation of infinite number of possible triangles. |
| 2 | 2 | Shapes defined by properties. No class inclusion (only one type of quadrilateral per shape). Too many properties (e.g. for a rhombus: "all sides equal. Opp angles equal"). |
| 3 | 2 | Too many properties listed (rhombus – "all sides must be the same and opposite angles must be equal"). |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only based on lengths of sides |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: DEAN POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Accurate measurement (visual) of changes. No reference to types of triangles. Realisation that infinite number of triangles possible. |
| 2 | 2 | Very little change from pre-test |
| 3 | 2 | Little change from pre-test |
| 4 | 3 | Correctly sorted but without explicit names for types of triangles. |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case. |
| 6 | 2 | Good attempt at justification using diagrams and couner-examples |

NAME: TALYA PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on visual attributes only. Confusion between sides and angles "I made the side of the triangle = 90°". Task incomplete. Final question not answered |
| 2 | 1 | Largely incorrect. Attempt at class inclusion for shape no. 3 (a parallelogram) but completely incorrect – stated it was a rhombus and a rectangle. |
| 3 | 1 | Confusion between sides and angles (parallelogram – "they have to equal 180° and have parallel sides"). Reliance on visual prototypes (rhombus – "… it looks like a square except it is slanted one way"). |
| 4 | 1 | Imprecise visual attributes only. Error: "2, 6, 8 alike because they are all squares" |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: TALYA POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Limited changes and changes back to earlier figures. Visual aspects considered. Task incomplete. Belief that only 6 triangles are possible. |
| 2 | 2 | Much improved but listing of too many properties. Sometimes too few properties for defn, e.g. rhombus: "opposite sides are parallel and equal" |
| 3 | 1 | Reliance on visual attributes. |
| 4 | 2 | Correctly sorted but without explicit names for types of triangles. Only lengths of sides considered |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case. |
| 6 | 1 | Very little evidence of reasoning |

NAME: GEORGE PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Realisation in each construction that a change in one component forces a change in others ("I have move[d] a side inwards creating a smaller angle. The other line must link up. 2 lines will change by moving one to create a triangle"). Realisation of infinite number of triangles |
| 2 | 2 | Shapes defined by properties. Some idea of class inclusion (shape 3 – "the two sides that are shorter or not at 90° so not … a square or rectangle"), but only one type of quadrilateral per shape. |
| 3 | - | incomplete |
| 4 | - | incomplete |
| 5 | - | Incomplete |
| 6 | - | incomplete |

NAME: GEORGE POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Realisation that change in component results in changes in other components. Infinite number of triangles possible. |
| 2 | 2 | As in pre-test – shapes defined by properties but too many properties listed |
| 3 | 2 | Listing of properties |
| 4 | 1 | Idea that specific types exist but no sorting |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case. |
| 6 | 2 | Use of diagrams and attempt to link to properties of quadrilaterals |

NAME: TARRYN PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on visual changes but with some realisation of properties. Answer to question re number of triangles: “About 15. Depending on what variations you use”. |
| 2 | 2 | Shapes defined by properties. No class inclusion (only one type of quadrilateral per shape). Too many properties (e.g. for a parallelogram: “4 sides equal and parallel”) contrasts with earlier economic definition of parallelogram (“the sides are parallel”). |
| 3 | 2 | Too many properties given; restrictive in case of rectangles (“two equal sides that are shorter than the other two sides”) |
| 4 | 1 | No sorting. Listing of properties |
| 5 | 2 | Properties for parallelogram and rhombus include “slanted”. Many errors, e.g. rhombus: “angles – 2 equal and 2 unequal”. Properties listed |
| 6 | - | Incomplete, but a general quadrilateral drawn |

NAME: TARRYN POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Some realisation that change in component results in change in other components. Task not completed. |
| 2 | 2 | Very little change from pre-test. Similar listing of properties |
| 3 | 1 | Visual attributes, e.g. rectangle: “the shapes with two short sides and 2 long sides” |
| 4 | 3 | Correctly sorted but without explicit names for types of triangles. |
| 5 | 2 | Many instances of visual attributes considered e.g. the rhombus is described as “… in the shape of a diamond”. |
| 6 | 2 | Good use of diagrams to justify statement. Analysis evident. |

NAME: GLORIA PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on visual aspects. Realisation of infinite number of triangles. |
| 2 | 1 | Visual aspects only considered. Definitions incomplete, e.g. "square – it has 4 right angles" |
| 3 | 1 | No understanding of quadrilaterals, e.g. rectangle – "two sides that face each other with different lengths". Visual aspects only considered, e.g. square – "a shape that all sides look the same". |
| 4 | 1 | Very basic visual attributes: "you would have to rotate or make them bigger"; "you would have to change their angles"; "you would have to modify them to fit each other" |
| 5 | 1 | Too few properties listed |
| 6 | 1 | Specific quadrilateral used: "Yes because the sides will all be equal and the angles will be right angles so it will be parallel and it's a square" |

NAME: GLORIA POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | | absent |
| 2 | | absent |
| 3 | | absent |
| 4 | | absent |
| 5 | | absent |
| 6 | | absent |

NAME: TED PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Started with equilateral triangle and changed to isosceles by making one side shorter. In next step changed to right-angled triangles by making angle 90° but lengths of sides not changed. Question re number of triangles: "9 because a triangle has 3 sides" |
| 2 | 1 | Many shapes identified incorrectly. Solely visual aspects considered: "slanted rectangle" |
| 3 | 1 | Some reference to properties (square – "4 sides, all equal, 90° angles") but mostly visual prototypes (parallelogram – "rectangle with 2 sides converging inwards"; "slanted rectangle"). |
| 4 | 1 | No sorting. Properties listed. |
| 5 | 2 | Most properties were listed correctly. Lines of symmetry incorrect. |
| 6 | 1 | Single diagram drawn and question simply rephrased as an assertion: "Yes, opposite sides are parallel" |

NAME: TED POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Initial focus on visual attributes only (enlargement and rotation) but then change of component resulting in further changes. Significant difference in last question: "You can draw an infinite of triangles (sic) because you can enlarge or change it a little bit each time". |
| 2 | 2 | Significant improvement from pre-test. No more "slanted rectangles". Use of properties to define shapes. |
| 3 | 2 | Improvement from pre-test but oscillations between levels evident. |
| 4 | 2 | Only sides considered |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case. |
| 6 | 2 | Number of diagrams used |

NAME: ANDREA PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus on visual attributes. Too many properties listed. Belief that 100 triangles can be drawn |
| 2 | 1 | Mixture of solely visual attributes (“rhombus as it is like a slanted square”) and economic definitions based on properties (“parallelogram, opposite sides are equal”). Predominantly visual |
| 3 | 1 | Some properties – mostly visual (parallelogram – “it is like a rectangle but the opposite left and right sides are slanted”) |
| 4 | 2 | Sorting by single attributes. Properties imprecise |
| 5 | - | incomplete |
| 6 | - | incomplete |

NAME: ANDREA POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Focus solely only visual aspects, including rotations and enlargements. Belief that only 6 triangles could be drawn. |
| 2 | 2 | Definitions based on properties |
| 3 | 2 | Properties listed. |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only 3 out of 4 completed correctly. Sides and angles considered |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case. |
| 6 | 1 | One case only considered. |

NAME: KATE PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Visual properties varied but realisation that change in one component forces changes in other components. Answer to final question; "My imagination is not that big" – clearly understands that an infinite number of triangles is possible |
| 2 | 1 | Visual aspects dominate ("… a parallelogram because it looks like a squashed square"). Where properties are focussed on, too many are given ("… a square because it has four equal sides and four interior angles of 90° each"). |
| 3 | 1 | Too many properties listed resulting in restriction (rectangle – "2 [sides] opposite one another ill be smaller than the other two"). Visual attributes – squashed rectangles and squares. |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only 3 out of 4 completed correctly. |
| 5 | 2 | Insufficient properties listed |
| 6 | 1 | No formal reasoning – "logical" assertion made: "It is impossible to make equal opposite sides unparallel in this shape. Suffice to say that is why it is called a 'parallelogram'" |

NAME: KATE POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Good link between changes in components and properties of triangles: "I changed [one angle] and by the law of '180° interior angles', the other angles changed accordingly". Interesting response to final question: "An infinite number, provided that length of sides is relevant, Other wise, due to interior angles of a triangle having to = 180°, there is a limited number of ways to split 180° into 3" |
| 2 | 2 | Properties focussed on. Some realisation of class inclusion: "I cannot say for certain that Quadrilateral 2 is a square but it is definitely a rectangle because opposite sides are equal, parallel and at least one angle = 90°" |
| 3 | 3 | Reference to other shapes: rhombus – "a parallelogram but all sides are equal". Economic definition: square – " all 4 sides equal and at least one 90° angle" |
| 4 | - | incomplete |
| 5 | 2 | Properties listed with no reference to other shapes. |
| 6 | - | Not attempted |

NAME: CLAUD PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Good realisation that change on one component forces changes in others. Interesting response to question re number of triangles: “There is no limit to the no. of triangles if you continuously change angles and sides. But if you can only change 1 angle or side then only 7 can be drawn”. |
| 2 | 2 | Shapes defined by properties. No class inclusion (only one type of quadrilateral per shape). Too many properties (e.g. for a parallelogram: “opposite sides equal and parallel”). |
| 3 | 1 | Incorrect definitions for parallelograms and rhombuses – in both cases it was stated that angles must be equal. |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only lengths of sides considered |
| 5 | 2 | Incomplete listing of properties |
| 6 | - | incomplete |

NAME: CLAUD POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 2 | Final question: “you can create an infinite number of triangles. But there will always be 6 categories: acute, obtuse, equilateral, scalene, isosceles and right-angled” – class inclusion. |
| 2 | 2 | Very little change from pre-test |
| 3 | 2 | Listing of properties |
| 4 | 3 | Specific referral to explicit types (isosceles, equilateral, right-angled and scalene) |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case. |
| 6 | 2 | Good use of diagrams |

NAME: CATHY PRE-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Task not completed – only 3 changes made. Focus on visual changes. |
| 2 | 1 | Only visual aspects considered and frequently incorrectly. |
| 3 | 1 | Visual aspects considered. Language confused ("opposites are equal to each other"; "each side is the same length as its opposite and corresponding side"). Incorrect definitions (rhombus – "only two sides are of the same length"). |
| 4 | 2 | Specific referral to explicit properties. 3 out of 4 completed correctly. |
| 5 | 2 | Insufficient properties listed |
| 6 | - | incomplete |

NAME: CATHY POST-TEST

| TASK | PREDOMINANT VHL | COMMENT (if appropriate) |
|---|---|---|
| 1 | 1 | Some focus on visual only (rotation) but realisation that change in one component results in changes in others. Last question: "a lot with the correct measurements and method" |
| 2 | 2 | Significant improvement from pre-test. Properties focussed on, although too many properties listed. |
| 3 | 2 | Listing of properties |
| 4 | 2 | Specific referral to explicit types (isosceles, equilateral, and scalene) but only lengths of sides considered |
| 5 | 2 | Properties listed with no reference to other shapes. Trapezium ltd to isosceles case. |
| 6 | 1 | Poor attempt at reasoning |

## Appendix 4 – Letter of permission: parents

DAINFERN College

*A World of Education…an Education for the World!*

CELEBRATE

25 July 2010

Dear Parent or Guardian and Learner

Research Project

A study of the role of Technology in the zone of proximal development and in defining van Hiele levels in a grade 9 module using Geometer's Sketchpad

University of the Witwatersrand School of Education

In addition to teaching at Dainfern College, I am also a student at the University of the Witwatersrand working towards an M.Sc. in Mathematics Education. I am currently engaged in research that focuses on learning and teaching Euclidean geometry. This is an optional topic in the FET mathematics curriculum but one that all the mathematics learners at Dainfern College study. We believe that our research, which involves the use of a dynamic software package called Geometer's Sketchpad, will make a meaningful contribution to enabling teachers to teach this mathematics content efficiently and enhancing both learners' understanding of the concepts involved and their reasoning abilities.

To this end, I would like to involve your child in research about geometric reasoning. The research uses a topic known as MathsLab in which all the Grade 9 learners already participate. Lessons take place within the existing school timetable and no further work is required by each individual learner. I need your consent to use your child's written work for analysis purposes in the research project. Some learners will also be audio recorded and their work on the computer screen will be screen-recorded.

The data collected will be used for analysis in the research study and destroyed afterwards. The study will be written up for degree purposes and presented to an academic audience at the University of the Witwatersrand, and possibly at academic conferences.

Your child's name and school will not be made public - pseudonyms will be used. I must stress that participation in the research is voluntary, but participation in MathsLab is compulsory and forms an integral part of the mathematics work covered this year. Your child will not be discriminated against in any way should he/she choose not to take part in the research. I would be very grateful for this opportunity, however, and if you agree to this process, please read and complete the accompanying consent form and return it to me.

If you have any questions or concerns, or would like to discuss the aims of the research in more detail, please do not hesitate to contact me or the supervisor of the research, Professor Margot Berger.

With thanks

Karen Hulme
HOD Mathematics
Dainfern College
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011 469 0635

Margot Berger
Supervisor
University of the Witwatersrand
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011 717 3411

**Appendix 5 – Consent forms**

# CONSENT FORM (parent)

I, ________________________________ parent or legal guardian of ______________________________, hereby give / do not give permission for my child to participate in the research project as explained above (please circle where applicable).

I give consent / do not give consent (please circle where applicable) to the researcher to

1. use my child's written work and / or audio recordings for analysis purposes, and
2. use the results in a research report and disseminate it in academic forums.

I understand that participation is voluntary and that my child's identity will be protected.

Signed: __________________________ (Parent) Date: ________________________

# CONSENT FORM (learner)

I, ________________________________ am willing to participate in the research project as explained above.

I give consent to the researcher to

1. use my written work and / or audio recordings for analysis purposes, and
2. use the results in a research report and disseminate it in academic forums.

I understand that participation is voluntary and that my identity will be protected.

Signed: __________________________ (Learner) Date: ________________________


Karen Hulme

HOD Mathematics

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## Appendix 6 – Letter of information: headmaster

DAINFERN College

*A World of Education…an Education for the World!*

CELEBRATE

10 May 2010

Dear Mr West

Research Project - University of the Witwatersrand School of Education

As you are aware, I am a part-time student at the University of the Witwatersrand working towards an M.Sc. in Mathematics Education. I am currently engaged in research that focuses on learning and teaching Euclidean geometry. This is an optional topic in the FET mathematics curriculum but one that all the mathematics learners at Dainfern College study. I believe that my research, which involves the use of a dynamic software package called Geometer's Sketchpad, will make a meaningful contribution to enabling teachers to teach this mathematics content efficiently and enhancing both learners' understanding of the concepts involved and their reasoning abilities.

To this end, I would like to involve the Grade 9 learners in research about geometric reasoning. The research uses a topic known as MathsLab in which all the Grade 9 learners already participate. Lessons take place within the existing school timetable and no further work is required by each individual learner.

The data collected will be used for analysis in the research study and destroyed afterwards. The study will be written up for degree purposes and presented to an academic audience at the University of the Witwatersrand, and possibly at academic conferences. Each learner's name and the name of the school will not be made public - pseudonyms will be used. Participation in the research is voluntary, but participation in MathsLab is compulsory and forms an integral part of the mathematics work covered this year. The learners will not be discriminated against in any way should he/she choose not to take part in the research.

If you have any questions or concerns, or would like to discuss the aims of the research in more detail, please do not hesitate to contact me or the supervisor of the research, Professor Margot Berger.

With thanks

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# 10. REFERENCES


Atebe, H. U. & Schäfer, M. (2008). "As soon as the four sides are all equal, then the angles must be 90°". Children's misconceptions in geometry. African Journal of Research in Science, Mathematics & Technology Education, 12(2), 47 - 66

Battista, M. T. (2008). Representations and cognitive objects in modern school geometry., In Blume, G. W. & Heid, M. K. (eds) Research on Technology and the Teaching and Learning of Mathematics. Vol 2, pp. 341 – 362

Berger, M. (1997). A mediating role for the graphic calculator. SAJHE / SATHO Vol. 11 no. 1 pp. 159 - 166

Berger, M. (2010). A semiotic view of mathematical activity with a computer. Relime, Vol 13 no. 2 pp. 1 – 27

Blume, G. W. & Heid, M. K. (2008). The role of research and theory in the integration of technology in mathematics teaching and learning. In Blume, G. W. & Heid, M. K. (eds) Research on Technology and the Teaching and Learning of Mathematics. Vol 2, pp. 449 - 464

Brodie, K. (2000). Mathematical reasoning Chap. 2 pp. 43 – 87

Brodie. K. & Pournara, C (2005). Towards a framework for developing and researching groupwork in mathematics classrooms

Burger, W. & Shaughnessy, J. (1986). Characterising the Van Hiele Levels of development in geometry. Journal for Research in Mathematics Education. Vol. 17, no. 1, pp. 31 – 48

Cole, M. (1985). The zone of proximal development: where culture and cognition create each other. In Wertsch, J. V (ed) Culture, Communication and Cognition: Vygotskian perspectives. Cambridge University Press, pp. 146 - 161

De Villiers, M. (1996). The future of secondary school geometry. SOSI Geometry Imperfect Conference, UNISA, Pretoria

De Villers, M. (2010). Some reflections on the Van Hiele theory. Invited plenary presented at the *4th Congress of teachers of mathematics* of the Croatian Mathematical Society, Zagreb, 20 June – 2 July 2010

Drijvers, P,; Kieran, C.; Mariotti, M-A.; Ainley, J.; Andreson, M.; Chan, Y. C.; Dana-Picard, T.; Guedet, G.; Kidron, I.; Leung, A. & Meagher, M. (2010). Integrating technology into mathematics education: theoretical perspectives. In Hoyles, C. & Lagrange, J-B. (eds) Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study. Springer Science+Business Media, New York

Evans, R. (2007). Proof and geometric reasoning. Mathematics teaching incorporating micromath 201, pp. 38 – 41

Goos, M.; Soury-Lavergne, S.; Assude, T.; Brown, J.; Kong, C.M.; Glover, D.; Grugeon, B.; Laborde, C.; Lavicza, Z.; Miller, D. & Sinclair, M. (2010). Teachers and teaching: theoretical perspectives and issues concerning classroom implementation. In Hoyles, C. & Lagrange, J-B. (eds) Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study. Springer Science+Business Media, New York

Hanks, F. (1991). Foreword to Lave, J. & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. (pp. 13 – 24). Cambridge, Cambridge University Press

Heid, M. K. & Blume, G. W. (2008). Technology and the teaching and learning of mathematics: cross-content implications. In Heid, M. K. & Blume, G. W. (eds) Research on Technology and the Teaching and Learning of Mathematics. Vol 1, pp. 419 – 434

Hollebrands, K,; Laborde, C. & Strasser, R. (2008). Technology and the learning of geometry at the secondary level. In Heid, M. K. & Blume, G. W. (eds) Research on Technology and the Teaching and Learning of Mathematics. Vol 1, pp. 155 – 205

Hufferd-Ackles, K.; Fuson, K. & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. Journal for research in mathematics education, vol. 35, no. 2 pp. 81 - 116

Jones, K.; Mackrell, K. & Stevenson, I. (2010). Designing digital technologies and learning activities for different geometries. In Hoyles, C. & Lagrange, J-B. (eds) Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study. Springer Science+Business Media, New York

Lantolf, J. P. (2000). Introduction to sociocultural theory. In Lantolf, J. P. Sociocultural theory and second language learning. Oxford: Oxford University Press

Lee, B. (1985). Origins of Vygotsky's semiotic analysis. In Wertsch, J. V (ed) Culture, Communication and Cognition: Vygotskian perspectives. Cambridge University Press, pp. 66 - 93

Marrades, R. & Gutiérrez, Á. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. Educational studies in mathematics, 44, pp. 87 - 125

Meadows, S. (2004). Models of cognition in childhood: metaphors, achievements and problems. In H. Daniels and A. Edwards. The Routledge Falmer Reader in Psychology of Education. London: Routledge/Falmer

Mudaly, V. (2007). Proof and proving in secondary school. Pythagoras 66 pp. 64 – 75

Olkun, S.; Sinoplu, N. B. & Deryakulu, D. (2005). Geometric explorations with Dynamic Geometry Applications based on van Hiele Levels. International Journal for Mathematics Teaching and Learning, Vol. , pp. 88 - 100

Sacristán, A. I.; Calder, N.; Rojano, T.; Santos-Trigo, M.; Friedlander, A.; Meissner, H; Tabach, M.; Moreno, L. & Perrusquía. (2010). The influence and shaping of digital technologies on the learning – and learning trajectories – of mathematical concepts. In Hoyles, C. & Lagrange, J-B. (eds) Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study. Springer Science+Business Media, New York

Scaife, J. (2004). Reliability, validity and credibility. In Opie, C. (2004) Doing Educational Research- A Guide to First Time Researchers. London. Sage Publications. Pp. 58 – 72

Van Putten, S. (2008). Levels of thought in Geometry of Pre-service Mathematics educators according to the van Hiele Model. Unpublished Master's thesis, University of Pretoria. http://upetd.up.ac.za/thesis/available/etd-05202008-130804/unrestricted/dissertation.pdf

Vygotsky, L. (1978). Internalization of higher psychological functions. Mind in Society, Cambridge, MA, Harvard University Press

Wertsch, J. V. & Stone, C. A. (1985). The concept of internalisation in Vygotsky's account of the genesis of higher mental functions. In Wertsch, J. V (ed) Culture, Communication and Cognition: Vygotskian perspectives. Cambridge University Press, pp. 162 - 179