

C Christie

MISCONCEPTIONS ABOUT THE SYMBOLS OF MATHEMATICS

To most people mathematics is a foreign language.

There is no doubt that an important prerequisite for being able to learn the language adequately is an ability to perform adequately in the skills of reading, writing and understanding verbal instructions in one's home language. So, before one can even begin to study the language of mathematics, one must have a sound background of one's mother-tongue. The onus for this rests heavily on one's parents, one's peers and one's teachers. Fortunately, much opportunity exists for a person to develop his skills in his home language. Even so, some people do not develop these skills to any acceptable degree for reasons into which I shall not go. What is important is that such people can rarely, if ever, perform adequately in the field of mathematics. They are hamstrung before they even begin.

Because mathematics is usually conceived of as a foreign language, most people have little opportunity to practise the skills of the language; and even when such opportunities do arise, these people tend to abuse the opportunity

either by preferring to practise their mother-tongue or by doing something completely different. They rationalise their actions by saying, for example, that mathematics is 'not as important' as their home language, or whatever else it is that they choose to do instead.

Such notions are, I submit, fallacious in the technological world in which we live today. More and more, our lives are being ruled by things mathematical, and unless we have at least a basic grasp of the language, we lay ourselves open to unscrupulous exploitation by machines and men whose language of control is beyond our comprehension.

But I digress. Mathematics teachers, if they regard themselves as teachers of a foreign language, ought perhaps to follow the example of their colleagues who are German, French, Latin or African language teachers, and devote much more time to practising the verbalisation of the language. The beauty of mathematics as a written language lies in the conciseness of its symbolism. But this elegance is also a major source of students' difficulties in coming to grasp the verbal language. People tend to abbreviate the verbalisation, in accordance with the brevity of the symbolism. For example:

Written: $AB = 5 \text{ mm}$

Incorrect verbalisation: AB equals five millimetres

Correct verbalisation: The length of the line segment AB is equal to five millimetres

But you are being pedantic, I hear most people, even mathematics teachers, crying! I disagree. One has only to pick up any mathematics script to see how, in particular, the '=' symbol is horribly abused to appreciate that what I am suggesting is vitally necessary in the teaching of the language of mathematics.

Another example of a symbol which is abused is the '+', or addition symbol. Consider

$$3 + 4 = 7$$

read by many people as three and four equals seven. Such people take the process one step further, and write

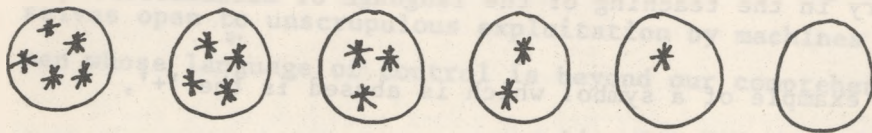
Sugar + Water, boiled = Toffee

They mix their languages with gay abandon, probably because their lack of ability in either language renders them insensitive to the damage they are inflicting on both.

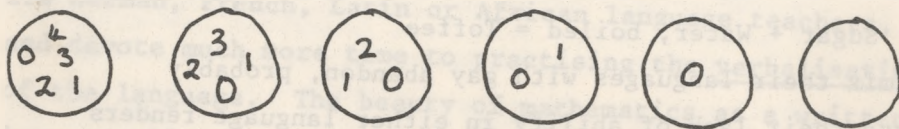
In both examples above, the colloquial meaning of the symbols has been preserved to some extent, even if the literal meaning has been lost through a lack of appreciation, or through some other misconception. In my

final example, I wish to pinpoint a misconception regarding the meaning of an important symbol in mathematics, which is totally inconsistent with its correct meaning. I refer to the numeral 0.

The misconception that '0 means nothing' is widely held. It arises, I believe, from the way in which the concept is developed in the grades, namely by asking the children to write down the number of 'things' in sets such as the following:



The misconception could easily be avoided by slipping in an example such as



in an attempt to make it clear to the pupils that zero is not a 'no-thing', but a number which counts how many things there are in an empty set. The notion of an empty set is very abstract, so to introduce this idea to young children one should rather use an example such as 'Let us count the number of tigers in this room...We have counted the number

of tigers in the room. The number is 0 (zero).

The misconception that '0 means nothing' is reinforced by examples such as

$$\begin{array}{r} 14 \\ 3 \overline{) 42} \\ \underline{3} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

where the pupils are taught that 'there is no remainder'!

Of course, there is a remainder, and if the algorithm were completed

$$\begin{array}{r} 14 \\ 3 \overline{) 42} \\ \underline{3} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

where the remainder is clearly shown to be zero, the misconception could be avoided or, if it already exists, possibly corrected.

Misconceptions in respect of mathematical symbols are not as unimportant as many teachers tend to believe. Their effects can be quite bizarre, and frighteningly

misleading, laying their holders open to all sorts of exploitation. My next example is taken from the examination script of a student here at College. She was asked to describe how to use Dienes' Blocks in order to explain to children how to work out $32 \div 100$:

Now we say that we have 100 people and 32 blocks. We can't share the 32 blocks between 100 people so we add on a '0'. A zero is nothing, so we can do this. (My underlining.)

100 | 32 So: 100 | 320
can't do.

On reading this response, a colleague remarked that he wished the Director of Education held the same misconception. Then he would add a couple of zeroes to each teacher's salary, and it would cost the government nothing, and the teachers would be happy.

Unfortunately, not all educationalists are that ignorant.