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COMPARATIVE EFFECTIVENESS OF EXPOSITORY AND DISCOVERY METHODS IN THE TEACHING OF LINEAR PROGRAMMING TO HIGH SCHOOL PUPILS.

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Research proposal submitted to the Faculty of Education of the University of the Witwatersrand, Johannesburg, in part fulfilment of the requirements for the Degree of Master of Education.

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ABSTRACT

COMPARATIVE EFFECTIVENESS OF EXPOSITORY AND DISCOVERY METHODS IN THE TEACHING OF LINEAR PROGRAMMING TO HIGH SCHOOL PUPILS

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This research report investigates the comparative effectiveness of expository and discovery teaching of linear programming to high school pupils. Two groups of pupils are selected for the two teaching approaches in a non-random way: pupils of higher mathematical aptitude/ability are taught by discovery methods and pupils of lower mathematical aptitude/ability are taught by expository methods. The results would indicate that the pupils taught under expository methods benefitted far more than did pupils taught under discovery methods. Possible explanations for the results are given and the increased use of discovery methods in the teaching of mathematics is suggested in order to improve problem-solving skills.

DECLARATION

I declare that this research report is my own, unaided work. It is being submitted in partial fulfilment for the Degree of Master of Education in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

.....

20TH .....day of OCTOBER .....1986.

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CHAPTER ONE

INTRODUCTION

The teaching *and learning of mathematics* have always been central concerns in educational research. Since mathematics is a fundamental topic in school curricula, the desire to improve the effectiveness of teaching has stimulated a broad range of studies designed to understand the nature and development of mathematical abilities and those school programs or teaching strategies that produce optimal learning. Taken as a whole, recent research on mathematical abilities, learning and teaching has produced few broad or strikingly new results suggesting major improvements in effectiveness of school mathematics programs. The complexities of human learning, the mathematical subject matter and the school-societal settings for instruction seem likely to foil any search for such major breakthroughs. (Bauersfeld, 1979).

In lieu of the naive quest for simple best methods of teaching mathematics, Fey (1970) claims that we might see in the pattern of recent research the promise of solid progress in a number of specific problem areas. Research using conventional designs and data collection techniques has also improved in the last ten years. For example, the work of Good and Grouws (1977) involved traditional experimental-control group comparisons of instructional treatments. Whereas previous studies of this type made little effort to assure delivery of well-defined treatments, Good and Grouws (1977) checked very carefully (by classroom observation) to ensure that the experimental treatment features

were effected. They chose research situations with care to minimise variation due to irrelevant factors; they made special efforts to control for Hawthorne effect; they considered carefully the appropriateness of their chosen criterion measures - all problems that had methodological flaws in earlier research in mathematical education.

Recently there has been a trend among mathematics educators worldwide to try to make school mathematics more relevant and applicable to pupils' lives. In the USA, in response to "minimal competence" mandates from State and local Government, the National Council of Supervisors of Mathematics (N.C.S.M.) in 1978 drafted a position paper on basic skills that emphasized the following: problem solving; applying mathematics in everyday situations; alertness to reasonableness of results; estimation and approximation; reading, interpreting and constructing tables, charts and graphs; using mathematics to predict; computer literacy. Among the impending developments, that which offers the most certain prospect for striking influence on school mathematics is the emergence of micro-electronic technology. In twenty years the computer may be able to perform every cognitive function of human beings. Already pocket calculators can do routine computations that school mathematics programs much time teaching to young people. Here the challenge to research and development is to determine the knowledge and skill still important for various individuals to acquire and the best methods of teaching that skill and understanding in a computer-enhanced educational environment.

The mathematics educators' interest in finding improved classroom teaching, teacher preparation and curricula has been complemented by generic research in each of these questions - as a major school subject, mathematics has been involved in studies of teaching like the interaction analysis work of Flanders (1970), the studies of teacher characteristics like those of Ryans (1960) or Cooney (1980), or analyses of curriculum development like that of Walker and Schaffarzick (1974). In addition, mathematics educators need to pay close attention to research emanating from psychology. Psychologists have proposed theories of development explaining evolution of perceptual and reasoning abilities and special links between mathematics and critical affective variables. Between 1900 and 1930 the teaching of mathematics was noticeably influenced by Thorndike's "The Psychology of Arithmetic" (1922) and "The Psychology of Algebra" (1923) and Brownell's Studies of Meaning in Arithmetic (1947). The developmentalism of Piaget (1952) and the structuralism of Bruner (1960) laid important groundwork for the "New Math" thrusts and Gagne's new behaviourism (1965) provided psychological rationale for individualised instruction and school accountability initiatives. Since then psychologists have played a major role in the continuing controversy over discovery learning, while contemporary research on teaching and learning of mathematics is guided by information-processing models generated within psychology.

It would be useful to consider ideas on cognitive development in relation to human learning. One of the most difficult tasks in building a theory of cognitive development is separating the effects of *maturation* and *experiences* both in and outside of school. The early hopes that Piaget's (1952) ideas would provide a comprehensive developmental framework for curricular and instructional practice in school mathematics have given way to realisation that mathematics educators must look for more complex connections between development and the learning of specific mathematical concepts. Skemp (1979) stresses that human behaviour is goal directed and learning is geared towards survival. Carpenter (1980; p.23) sets the agenda as follows:

> "What is essential is the construction of good measures of children's thinking and the identification of specific relationships between performance on those measures and the learning of particular mathematical concepts. Whether these measures fall into an ordinal scale is not critical. It is important, however, that the measures of children's thinking predict with some accuracy children's ability to learn specific mathematical concepts and skills."

Since linear programming deals essentially with problem solving, some remarks on this topic have been included in the Introduction. Kilpatrick (1969;p.529) summarised the status of problem solving research as follows:

> "The prominence of increased problem-solving ability *as a goal* of mathematics instruction has long been admitted; but like the weather, problem-solving has been more talked about than predicted, controlled or understood.... *Problem solving* is not now being investigated systematically by mathematics educators. Few studies build on previous research; few studies have an explicit theoretical rationale".

Despite the importance of problem-solving, students find "story problems" and mathematical puzzles very difficult and teachers experience frustration in teaching problem-solving, and thus frequently give it less attention than reason suggests. Klipatrick's (1969) own thesis redirected the focus and methodology of research on problem-solving. Polya (1957) had previously outlined a general theory of key stages in problem-solving, including a variety of specific strategies useful in each stage. Klipatrick started from this framework, but instead of focusing solely on the products of problem-solving, he researched the mental processes that students used along the way to success or failure. This emphasis on process rather than product and on clinical interview research methods has had enormous influence on problem-solving research. This research has abandoned the search for a simple best method of teaching problem-solving in favour of more basic studies of component variables in the complex process. Consequently, recent research consists of small steps of progress rather than major breakthroughs. Investigation is proceeding on several classes of variables: "task variables", "subject variables" and "instructional variables". Progress has been made in identifying problem difficulty factors and the ways that problem structural features influence student processes in search of solutions (Goldin and McClintock, 1979). Soviet research has concentrated on understanding the cognitive processes of successful problem solvers (Krutetskii, 1976). A number of coordinated studies have given promise that problem-solving heuristics can be taught successfully (Lester, 1980; Schoenfeld, 1979). The fact that

the N.C.T.M. (National Council of Teachers of Mathematics) has made problem-solving the top curricular priority of the 1980's indicates the importance for continued research interest in this area.

Mathematics educators have explored the potential of discovery teaching versus expository teaching, using advance organisers and post-organisers for instruction, adjusting teaching approaches to capitalise on aptitude-treatment interactions and various instructional media. The results have seldom been unequivocal or consistent from one study to the next, but several studies stand out, indicating fundamental problems for the field. Worthen (1967) explored one facet in the discovery-expository teaching controversy in elementary mathematics, finding that trends favoured expository teaching for immediate recall and discovery teaching on retention and transfer. Olander and Robertson (1973) confirmed this pattern but found an interaction between method and student prior knowledge or ability; weaker students seemed to benefit from expository teaching and stronger students from discovery. The special value of these studies is the suggestion that short-term and long-term learning outcomes are influenced by different aspects of instruction and that adequate assessment of instruction requires multiple criteria. This is of central interest to those who are concerned about the impact of accountability-induced trends towards more frequent testing in mathematics. Therefore, it seems possible that instructional approaches that are demonstrably effective for short-term gains are far less effective when measured by

important long-term goals and that pressure for immediate measurable results distorts curriculum priorities. The recent decline in problem-solving ability has been widely attributed to such overemphasis on expository "Show-and-tell-and-drill" methods and neglect of challenging problem-solving activities.

In other directions, research based in mathematics education has tested Ausubel's (1963) theories about meaningful verbal learning. Romberg and Wilson (1973) found that presentation of post-organisers might have a negative effect on learners, perhaps conflicting with the conceptualisation of knowledge the learners work out for themselves. These results, combined with work on programmed instruction in which students successfully learned from a program presented in reverse of "logical" order, underscore the fundamental importance of student internal organisations of mathematical ideas. They suggest that knowledge structures built up by students might be very different from what most teachers imagine they are conveying.

There have been several tests of Cronbach and Snow's (1977) ideas about aptitude-treatment interaction (ATI) effects in instruction. The learning of mathematics seems to be driven strongly by innate natural abilities, the background of previously acquired knowledge and the internal structure of ideas themselves that few short-term or moderate alterations in the teaching approach have any noticeable effect on student achievement. Most ATI studies in mathematics have used relatively short treatments, often administered in the artificial

environment of programmed instructional materials, that it would not be surprising if the treatment variations are overwhelmed by background factors.

There have been some significant differences in instructional research in the form of recent process-product studies of classroom teaching in mathematics. As comparisons of novel and conventional methods repeatedly indicate "no significant difference", research attention has refocused on the specific classroom interactions that occur in teaching. The first objective was to characterise and develop procedures for describing moves that students and teachers make (Fey, 1970; Smith and Meux, 1963). Next, research sought patterns of interaction that characterise effective or ineffective classes. Several studies (Evertson, Emmer and Brophy, 1980; Good and Grouws, 1977) have been able to distinguish such patterns in elementary or junior high school mathematics classes. Good and Grouws (1977) applied those findings in experimental studies that show promising results. The pattern they found effective in elementary grades was task-focused teaching that included daily review, careful attention to development of meaning and understanding, closely monitored seatwork and regular homework assignments. They found that many of their suggestions were not regular practice in the majority of classes. The most encouraging aspect of their work is perhaps that very modest training induced the desired instructional treatment behaviour.

This research attempts to establish whether one of two alternative teaching strategies will lead to improved learning by pupils. It will establish whether or not discovery or expository teaching will lead to improved performance by pupils of different aptitude levels in mathematics.

BACKGROUND TO THE STUDY

The new mathematics syllabus for Transvaal schools will be introduced at Matric level in 1987. In keeping with world-wide trends to make mathematics more applicable, certain new topics have been included in this syllabus. "Linear Programming" has not been taught at school level before in South Africa. It is intended through this research to gain some insights into the effectiveness of expository instructional methods and discovery instructional methods with groups of high school pupils prior to the universal implementation date in the Transvaal.

It is the researcher's experience that new topics in the maths syllabus are often treated with much trepidation by the majority of high school teachers. This diffidence on the part of the teachers may be due to a number of reasons: poor qualifications, little or no exposure to the subject matter in their training, understandable concern with topics that are unfamiliar, few in-service courses that cover the new work and a lack of suitable texts which provide background information concerning the topic. As a result, teachers more often than not simply teach a new topic 'out of the textbook'. This invariably leads to rote

learning on the pupils' part and very definite expository teaching methods. Webb (1984) claims that because the educational system in South Africa is highly exam-orientated, teaching and text books opt for a stereotyped work to rule approach. The effect of a rigid syllabus and predictable examinations encourages attitudes diametrically opposed to those one wants to inculcate in a problem solver.

The subject matter in 'Linear Programming' is fairly difficult so that many teachers may find difficulty in handling the topic. It is hoped [illegible] research may assist in helping to improve sensitivity to [illegible]rnative teaching methods of new topics.

RESEARCH QUESTIONS

1. Will students taught by discovery methods or expository methods perform better in linear programming?

2. Will students of different aptitudes and/or personological variables benefit differently as a result of the alternative teaching strategies?

RESEARCH HYPOTHESES

1. There is a significant difference in performance of pupils in linear programming courses as a result of different teaching strategies.

2. Students with different aptitudes will benefit differently because of the different teaching methods.

ASSUMPTIONS

1. The teacher is competent in handling the subject matter.

2. The enthusiasm of the teacher in presenting both expository and discovery technique courses in linear programming will be the same.

3. There will be no Hawthorne effect since both experimental and control groups will receive treatment.

4. Other variables which may lead to improved performance will be controlled so as to minimize their effect. Therefore it is assumed that the results of the test will be due to treatments under investigation.

IMPORTANCE OF THE STUDY

Even though teachers would consider changing to innovative approaches, they feel under pressure to concentrate on what is in the examination syllabus. This tends to devolve to the coaching of a set of standard techniques which are then rehearsed on abstract, stereotyped problems. This tutoring for examination results is not conducive to the development of conceptual structures, strategies and an appreciation of mathematics in pupils.

In South Africa the emphasis placed on exam results is such that very little innovative teaching takes place in our schools. This study may prove to be valuable to those who wish to teach in such a way that pupils may be able to use and understand basic concepts and strategies when they are applied to realistic and novel situations. Insight gained may facilitate the implementation of more maths-applicable content in the present teaching of school mathematics.

DELIMITATIONS OF THE STUDY

The subjects in this research were students at private schools in Johannesburg. This study was limited to those schools where the support of the maths teacher and headmaster was clearly evident. Only those students who volunteered to attend sessions on "Linear Programming" were included in the sample.

LIMITATIONS OF THE STUDY

Due to time constraints the courses were taught after normal school hours. This may have had an effect on performance.

The school-societal setting with respect to private schools and the type of student frequenting such schools may limit the general applicability of the study.

Different schools may have prepared the students in varying degrees in background knowledge and problem-solving techniques.

Also, motivation in learning mathematics may be different in different schools. These factors may further limit the external validity of the proposed study.

DEFINITION OF TERMS

Expository Methods

Methods that characterise traditional procedures involving "telling" on the part of the teacher and more passive learning on the part of the pupils.

Discovery Methods

Methods that foster a condition in which pupils tend to become active participants in the acquisition of their learning.

Linear Programming

Linear programming involves the problem of determining the maximum or minimum value of a linear function subject to a system of linear equations or inequalities called constraints. The objective in linear programming is to determine the values of the variables that will satisfy all the restrictions of the constraints and will yield the optimal value.

METHODOLOGY

1. Design

The design is experimental. Homogeneous selection was made.

This may be diagrammed as follows:

$$\begin{array}{c} O_1 \times O_2 \\ \hline O_3 \times O_4 \end{array}$$

By tradition and in order to discriminate between the two groups, the conventionally taught (expository methods) group is called the control group. A diagnostic test was used in selecting subjects for the two samples. The implication in this type of design is to have a control for the non-random selection, which is a factor that limits the external validity of the research.

2. Instruments

Both a pretest and a posttest were used to guage the pupils' mathematical competence. A questionnaire was given to assess the students' motivation level.

An external evaluator had the responsibility to assess the presentation through unannounced visits. This was intended to minimise the possibility of teacher bias in presenting the different treatments.

3. Sample

A group of over 100 pupils was used from various cooperating private schools. In addition to the researcher-made pretest used to determine the level of competence of the students, use was made of the incumbent teacher's personal knowledge of the pupils: factors including recent test results, I.Q., motivation and general assessment of the pupil's aptitude, were used to select the samples for the experimental and the control groups. Comparisons made were not inter-school but rather intra-school ones.

ORGANISATION OF THE REMAINDER OF THE STUDY

Chapter I has introduced the problem statement and has focused prospects for research in mathematics education. The back-ground of the problem, research questions, hypotheses and assumptions, delimitations and limitations within the proposed methodology are indicated.

Chapter II will review the relevant literature on teaching and learning of mathematics.

Chapter III will describe in detail the methodology involved in this study as well as describe the selected samples involved in the research.

Chapter IV will contain an analysis of the findings.

Chapter V will examine the findings with a view to making recommendations from the data collected.

CHAPTER TWO

REVIEW OF THE LITERATURE

"If we view *mathematical* speculations with reference to their use, it appears that they should be divided into two classes. To the first belong those which furnish some marked advantage either to common life or to some art, and the value of such is usually determined by the magnitude of this advantage. The other class embraces those speculations, which, though offering no direct advantage, are nevertheless valuable in that they extend the boundaries of analysis and increase our resource and skill. Now since many investigations, from which great advantage may be expected, must be abandoned solely because of the imperfection of analysis, no small value should be assigned to those speculations which promise to enlarge the field of analysis."

(Leonard Euler)

Although ideal instruction has been characterised by a variety of models, from Socratic dialogue to scholar and teacher on a one-to-one basis, the predominant pattern in classroom teaching of mathematics has always been more pedestrian. While major development projects experimented with content and global organisation of school mathematics, there has been an active interest in alternative approaches in the day-to-day format of classroom instruction. Mathematics educators have explored the potential of discovery versus expository teaching, using advance organisers and postorganisers for instruction, adjusting teaching approaches to capitalise on aptitude-treatment-interactions and various instructional media. The results, however, have seldom been unequivocal or consistent from one study to the next. Several findings do, nevertheless, stand out and are elaborated on below.

DISCOVERY AND EXPOSITORY METHODS

Worthen (1967) explored one facet of the discovery-expository teaching controversy in elementary mathematics, finding that trends favoured expository teaching for immediate recall and discovery teaching on retention and transfer. This research was delimited to elementary grades. Olander and Robertson (1973) confirmed the pattern established by Worthen but also found an interaction between method and student prior knowledge or ability: weaker students seemed to benefit from expository teaching and stronger students from discovery teaching. The

research which they conducted was delimited to fourth-grade pupils. Specific conclusions from their study, based on two types of analysis of data, are:

A. Comparisons of test means of groups of pupils on variables of the study through use of multiple analysis of covariance:

   i) On both posttests and retention tests, pupils taught under the Expository Method seemed to excel on Computational Skills.

   ii) Pupils taught under the Discovery Method were seemingly superior in retaining the ability to apply mathematical knowledge after a 5 week period following the end of the experiment.

B. Interaction effects as shown through the linear-regression equation suggests that:

   i) Pupils scoring on the lower part of the range on the Computation pretest improved more when taught under the Expository procedure; pupils scoring higher improved more under the Discovery approach.

   ii) Pupils scoring lower on the Concepts pretest benefitted more from the Discovery approach; those scoring higher improved more under the Expository approach.

   iii) Pupils scoring lower on the Applications pretest improved more as a result of the Expository technique; those scoring higher profitted more when taught by the Discovery technique.

iv) On the Principles and Relationships pretest, pupils instructed under *Discovery* techniques started off better than those taught under the *Expository* approach and they continued to improve at a greater rate.

The major implication of Olander and Robertson's study is that instruction should be individualised: what is appropriate for one pupil may be inappropriate for another while that which is appropriate for one desired outcome may be inappropriate for another.

The specific value of the work of Worthen and Olander and Robertson is the idea that short-term *and long-term* learning outcomes are influenced by different aspects of instruction *and* that adequate assessment of instruction requires multiple criteria. This is of crucial interest to those who are concerned *about* the impact of accountability-induced trends towards more frequent testing in Mathematics. These critics believe that instructional approaches that *are* demonstrably effective for short-run gains are far less effective when measured by input and long-term goals. The pressure for immediate measurable results distorts curriculum priorities.

During the 1970s several major curriculum development and research projects explored the issues of content and psychological structure. At the Pittsburgh Learning Centre, the *Individually* Prescribed Instruction (IPI) program applied the

principles of behaviourist learning theory (specific objectives, hierarchies, progress towards mastery to elementary school mathematics. While evaluators found that students progressed through the objectives at very different rates, there were few overall advantages for this carefully constructed instructional system over more conventional approaches (Fey, 1980). The various tests of individualised systems for delivery of mathematics instruction have not given strong support to this alternative approach (Miller, 1976; Schoen, 1976). The main individualised factor has been challenged severely by studies suggesting that even successful students develop fundamentally flawed ideas about mathematics (Erlwanger, 1973). The collection of discouraging results and traditional resistance to change in schools has led to diminished interest in this way of individualising mathematics instruction, despite the great promise early in the 1970s.

In another direction, research based in mathematics education has tested Ausubel's (1963) theories about meaningful verbal learning. A number of studies of the advance organised hypothesis have failed to yield support for such preinstructional aids. Furthermore, Romberg and Wilson (1973) found that presentation of postorganisers might have a negative effect on learners, perhaps conflicting with the conceptualisation of knowledge the learners work out for themselves. These results, combined with some work on programmed instruction in which students successfully learned from a program presented in reverse of "logical" order, underscore again the fundamental importance

of student internal organisation of mathematical ideas. They suggest that the knowledge structures built up by the students might be very different from what most teachers believe they are conveying.

STUDENT APTITUDES

It is of interest to examine the effects of student aptitudes on "outcomes". At the age of 21, Isaac Newton formulated the basis of differential and integral calculus. At 16, C.F.Gauss contributed to the founding of non-Euclidean geometry, and at 19, he gave a complete proof for the important law of quadratic reciprocity in number theory. The works of these two mathematical giants are outstanding examples in a remarkable story of intellectual precocity. In a manner matched only in music, mathematical talent seems to emerge at a very early age (Stanley, Keating and Fox, 1974) suggesting that the mental abilities required to learn and use the subject are very close to primary, innate aptitudes. For this reason, studies of the nature and development of mathematical abilities have attracted the combined attention of mathematics educators and psychologists.

Although students and laymen commonly look on mathematical ability as a unitary phenomenon ("You can either do maths or you can't") research reveals a much more complicated picture. When students are grouped homogeneously for mathematics instruction, the most remarkable feature of the groups is how much the members

still vary in the patterns of their mathematical abilities and cognitive styles. While some pupils seem richly gifted with all the talents required for successful mathematics learning, there are component abilities that develop at different rates and in different ways for each individual. Fennema and Behr (1980) provide an organising framework for mathematical abilities that serves as a useful guide in reviewing research progress and prospects for the foundation of mathematics programs.

Starting from the definition of aptitude as a characteristic of an individual that increases (or impairs) the probability of success in learning (Cronbach and Snow, 1977), Fennema and Behr sort aptitudes into two main categories: cognitive and affective. The cognitive aptitudes are further subdivided into abilities and information-processing styles. The learning of mathematics is vitally affected by the natural or instructional development of aptitudes in each category.

COGNITIVE ABILITIES

A cognitive ability may be viewed as the intellectual power to acquire or produce new information from the facts or structure of a situation. For example, logical reasoning ability allows a learner to deduce new facts from logically related givens; spatial visualisation ability permits comparison of shapes presented in different orientations; ability to perceive abstract similarities in symbolic forms such as $s^2 - t^2 = (s-t)(s+t)$ and $9a^2 - 16b^2 = (3a-4b)(3a+4b)$ is critical in algebra;

creativity and flexibility help problem solvers generate fresh solution approaches to puzzling problems. These and other cognitive abilities involved in mathematical learning and performance have been studied extensively as part of research that tries to explain their natural maturational development, and the relation between abilities and school learning. Despite much research (Woods, Resnick & Groen, 1975; Kulm & Days, 1979; Simon and Hayes, 1976), there are many important open questions. For instance, special attention has recently been focused on the development and training of spatial abilities, concern having been sparked by some evidence that male and female students have different abilities in this area (Fennema and Behr, 1980). There is evidence that full logical reasoning develops much later than supposed, but the reasons for this late development are not clear (Sowder, 1980). After a period of limited interest in creativity, the recent concern for problem solving in mathematics seems to have sparked new concern for the nature and nurture of mathematical inventiveness.

The long-standing interest in cognitive abilities and their influence on mathematical productivity has recently extended to cognitive style, the personal preferences for various information-processing modes. Research in psychology and mathematics education has identified several dimensions of processing styles that appear to be related to performance in mathematics. Such processing preferences as visual or verbal imagery, reflective or impulsive problem solving, convergent or divergent thinking and field dependence or independence have been

studied in relation to mathematics teaching. The contributions of maturation and experience to formation of cognitive style preferences and abilities are just beginning to be understood. There have been consistent positive correlations between field independence and mathematical achievement. The availability of an established instrument (Witkin et al., 1977) for measuring the elusive cognitive style has undoubtedly also contributed to interest in field dependence-independence.

AFFECTIVE APTITUDES

Affective aptitudes related to mathematics have experienced active and sophisticated interest. Despite the plausible connection between enjoyment and success in mathematics, the first instruments to measure attitudes towards mathematics only appeared recently (Dutton, 1954; Aitken and Dreger, 1961). Swafford (1981; p.6) observed that research on affective factors in mathematics learning now regards attitude as a multidimensional variable:

> "Subscales include enjoyment, usefulness, confidence, anxiety, motivation, difficulty, perceived nature of mathematics, and attitudes towards success to name a few."

The findings emerging from this more sophisticated approach to attitude assessment (Carpenter et al., 1980; Kulm, 1980; Swafford, 1981) lead to five conclusions:

1. Attitudes towards mathematics as a school subject are most positive in early adolescence and decline through high school.

2. The effects of teachers' attitudes on students' attitudes and achievement is not as easy to confirm as one may expect.
3. There are generally no sex-related differences related to the enjoyment of mathematics. There has been some evidence that males are more self-confident, view mathematics as more useful and tend to stereotype mathematics as a male domain, but most recent data suggest that there are changing patterns in this view.
4. Positive attitudes towards mathematics and the perceived usefulness of mathematics are highly correlated with mathematics course participation.
5. Students have very different reactions to different facets of mathematics.

The work of an active group of Soviet psychologists has also focused on abilities needed in doing mathematical thought. Krutetskii (1976) suggests that students with mathematical talent commonly forget the particular facts of a situation as soon as they are unneeded, but they retain the structure of a problem and solution almost indefinitely. Furthermore, the operation of the various forms of reasoning is often so rapid and subconscious among these talented students that they arrive at correct results without recalling the thought processes or arguments used to reach these conclusions.

One of the most difficult tasks in building a theory of cognitive development is separating the effects of maturation both in and outside of school. As Carpenter (1980) noted, mathematics

educators were quick to challenge the age and sequence patterns suggested by Piaget's theory. Dozens of training studies attempted to accelerate the pace of development ahead of the time schedule which had been suggested. A large number of these studies were successful, raising questions about the extent to which cognitive development followed some inflexible biological clock. Other research (Brainerd, 1973) challenged Piaget's hypotheses in which cognitive ability and mathematical conceptual sequences emerge naturally. Hiebert (1981) has recently argued that Piagetan abilities like conservation are not required for learning most mathematical skills. It has become evident that mathematics educators must look for more complex connections between development and the learning demands of specific mathematical topics. There has been active work on this issue with basic number concepts and with measurement. However, the critical stage of formal operations, that which underlies so much of secondary school mathematics, has been explored only tentatively. The Van Hieles (Hoffer, 1981) have proposed a developmental sequence in learning geometry and recent information-processing studies of algebra learning (Carry, Lewis & Bernard, 1980) offer some promise of similar theories for that domain. At the present time, developmental theories offer useful insights into certain specific aspects of mathematics learning (for example, number and measurement) but they do not offer comprehensive guidelines for the selection, sequence or instructional presentation of most mathematical ideas.

ATI STUDIES

There have been several tests in mathematics of Cronbach and Snow's (1977) ideas about aptitude-treatment interaction (ATI) effects in instruction. The results have not been encouraging, but an optimist could discount this finding as another instance of a promising instructional idea that makes no significant impact in Mathematics. The learning of mathematics seems to be driven so strongly by innate mental abilities, the background of previously acquired knowledge and the internal structure of the ideas themselves that few short-term or moderate alterations in the teaching approach have any noticeable impact on student achievement. Most ATI studies in mathematics have used relatively short treatments, often administered in the artificial environment of programmed instructional material. Thus, it is probably not surprising that the treatment variations are overwhelmed by background factors.

The most notable exception to the pattern of no significant differences in instructional research has been recent process-product studies of classroom teaching in mathematics. As comparisons of novel and conventional methods of instruction repeatedly came up empty, research attention re-focused on the specific classroom interactions that occur in teaching. The first objective was to characterise and develop procedures for describing moves that teachers and students can and do make (Fey, 1970; Smith and Meux, 1963). Next, research sought patterns of interaction that characterise effective or ineffective classes.

Several studies (Good and Grouws, 1977; Evertson, Emmer and Brophy, 1980) have been able to distinguish such patterns in elementary and junior high school mathematics classes. Good and Grouws have applied those findings in experimental studies that show promising results (Good, 1981). The pattern they found effective in elementary grades was task-focused teaching that included daily review, careful attention to development of meaning and understanding, closely monitored seatwork and regular homework assignment. While this pattern hardly seems novel, Good and Grouws found that many of their suggestions were not regular practice in elementary school classes. The most encouraging aspect of their work is the fact that very modest training induced the desired instructional treatment behaviour. The work of Good and Grouws (1977) involved traditional experimental control group comparisons of instructional treatments. Where previous studies of this type made little effort to ensure delivery of well-defined treatments, Good and Grouws checked very carefully (by classroom observation) to see that the experimental treatment features were effected. They chose research situations with care to minimise variation due to irrelevant factors; they made special efforts to control for Hawthorne effect; and they considered carefully the appropriateness of their chosen criterion measures - all problems that have been poorly handled in earlier teaching research in mathematics education.

LINEAR PROGRAMMING AND PROBLEM SOLVING TECHNIQUES

Linear programming is a subset of mathematical programming. The central idea is the optimising of a mathematical function subject to certain given constraints. Further, when the said expression to be optimised and constraints are linear in the variables involved, then the type of mathematical programming is called linear programming. The constraints are usually inequalities. Fryer (1970) summarises the state of the art as:

1. A linear programming problem is one in which the expression to be optimised is linear, as are the constraints which usually take the form of inequalities.
2. Solutions can be obtained by graphical methods when only two (or possibly three) variables occur.
3. The feasible region (when it exists) is in the form of a convex polygon, and each vertex corresponds to a basic feasible solution.
4. An optimal solution occurs at a vertex. If optimal solutions occur at two vertices, then any point on the line joining them corresponds to an optimal solution.

Geldinhuys (1985) regards the name "linear programming" as misleading and suggests "linear optimisation" as more descriptive of what goes on mathematically. He suggests that in teaching linear programming, more exposition should be given in terms of the teacher's expanding on the method of solution in general and then application of this method to specific practical problems. The disadvantage of such a process is the possibility that

genuine practical problems may be passed over and that the value of linear programming may be completely lost. Geldinhuys believes that the most correct approach is to develop the method of solution through simple practical problems. There is another school of thought concerning the way people solve problems. The issue is to what degree the teacher intervenes in the process of student mastery of mathematical problems. The normal mathematical curriculum is almost universally based on explanation by the teacher with the use of illustrative examples followed by endless practice on exercises which are close imitations of the examples. When students have difficulty they are "coached": teachers restate explanations or break exercises down into smaller steps. Coaching is based on the idea of reinforcing a single track of learning. This is hardly conducive to the development of independent thinking. Indeed, from this perspective, giving students unfamiliar problems violates the very rules of the game.

Webb (1984: p.117) believes that the effect of a rigid syllabus and predictable examinations encourages attitudes diametrically opposed to those one wants to inculcate in pupils. The mathematics is reduced to the stereotyped applications of rigid rules and standard techniques. Webb feels that textbooks on the whole do not encourage problem solving:

> "Some South African textbooks are more imaginative than others, with offbeat and open-ended problems, but underqualified teachers see these as a threat. A teacher whose mastery of mathematics is weak will feel that his poor command of the subject is shown up, his authority will be weakened. He will

therefore follow the conventional and safe paths of drill and practice."

The introduction of mathematical contests in South Africa recently has promoted enthusiasm for problem solving. In addition various journals intended for high school mathematics pupils have been launched. The aim of these magazines is to entertain, to show that mathematics is alive, to stimulate and to challenge - in short, to get the reader "hooked" on mathematics. It is the researcher's belief that Linear Programming may be taught in such a way as to make it equally alive and entertaining without detracting from the value of independent thinking on the part of the pupil.

PROBLEM SOLVING STRATEGIES

A general consensus seems to be emerging that conceives of problem solving somewhat like this:

1. The verbal statement of the problem is an initial input.
2. Typically, not all of these input data are absorbed at once: some part of them is accepted.
3. From this partial input, an internal knowledge representation structure is created through some combination of retrieval and synthesis.
4. Now a process of "extending" takes place; from time to time more data are accepted as input; the representation structure is extended and revised; a dialogue takes place with the present problem representation; gaps and mismatches are dealt with "understanding". The problem can be

characterised, thus, as the building (through synthesis and retrieval) of a sequence of successive understandings, each one an elaboration of a earlier one.

Perhaps the central issues in problem solving are:

1. *Decisions* - which processes should the problem solver use in solving this particular problem?
2. *The construction* and the research: Nilsson (1971) believes that where a large number of alternatives must be considered or where there are several steps which must be performed in sequence, it is often possible to represent the possibilities by means of a tree diagram, in which every step is listed, below which every subsequent choice at the next step is listed (and, in diagrams, a connection is indicated by a line segment).

   It is questionable, however, whether a really complex tree can be built up in one person's mind. Furthermore, a large tree cannot usually be searched exhaustively so some method of "pruning" is required to eliminate unlikely choices. There is the further difficulty that the solver must create the tree and may possibly omit parts of it "because they didn't come to mind" (Davis, Jockush and McKnight, 1978). Still worse the feedback from a problem will often be misleading (one can seem "to be getting nowhere" or "to be going in the wrong direction"); perhaps what is then necessary is more of the same treatment. (Minsky and Papert, 1972).
3. Frame retrieval - Before one can begin to select

alternative courses of action, one must make an initial assessment of "problem type", *i.e.* the input data must be matched up with some information representation structure retrieved from memory. Students often struggle at this stage as they misrepresent the type of problem with which they are dealing and try to match up the input data with an incorrectly chosen cognitive representation structure.

4. Planning and Control - In any involved problem, one must make plans, try to carry out the plans, and exercise good judgement about when to modify the plan if difficulties are encountered.
5. Execution - Clearly after a plan has been thought out, one must carry out the plan by modifying the problem, solving algebraic equations, drawing a diagram, or doing whatever else the plan calls for.

The recent interest in process studies has made it clear that *little is known about* the differences between the behaviour of very good problem solvers versus very poor problem solvers. In the last few years some real progress has been made. Schoenfeld (1980), Larkin (1977) and Larkin et al (1980) report studies that compare the problem-solving behaviour of experts with that of novices. Some of the devices used by experts can be explicitly taught to novices with good results (Schoenfeld, 1979; 1980). These devices often take the form of "heuristics", a type of vague advice that, interpreted correctly, can often be of very great value in solving difficult problems. Examples of heuristics include: Can you identify which feature of the

problem is making it difficult? Can you modify the problem so as to eliminate this difficulty? Can you think of a somewhat similar problem that you do know how to solve? Can you break the problem up into parts, and deal separately with the various parts? Of course, heuristics are not helpful in improving problem-solving performance unless the student also possesses appropriate technical knowledge. "Breaking a problem into sub-problems" is useful only if you are able to solve the sub-problems.

This is of interest to the present research in that pupils taught under different methods may evolve different heuristics.

Matz's Extrapolation Theory (1980) involves conceptualising typical mathematical tasks as requiring the creation of a match between a known standard "rule" and a new problem input that does not perfectly match the rule. For example, one standard rule is the distributive law $a(b+c)=(ab)+(ac)$. Given the task of factorising $(ps)+(pw)+(pt)$ no direct match can be made between the task input and the memorised rule. The gap must be closed, either by extending the standard rule, or by modifying the task input. This is a very general task, occurring in a large percentage of mathematical uses. Thus Matz's theory not only explains much correct performance but also explains many of the common errors made by students.

CHAPTER THREE

METHODOLOGY

The preceding chapters have, respectively, discussed the rationale for this research and reviewed the relevant literature. This chapter will detail the source of the data, the subjects, the tests involved in the selection of experimental and control groups and the procedure adopted in presenting the subject matter.

SOURCE OF THE DATA

It was felt necessary to involve pupils at schools where the full support of both the Headmaster and Head of Mathematics Department was guaranteed. This was possible at three schools, all of which are Catholic open colleges, at which the researcher has taught previously. The researcher is known by the pupils at all three of the above-mentioned Colleges so the pupils were at ease and relaxed in the classes given.

At each school four separate days were taken to conduct the experiment. On the first day the pretest and motivation tests were given; the two groups were then taught on different days and on the fourth visit the posttests were administered. Because of time constraints in the ordinary school curriculum the

research was conducted after normal school hours and this may have had an effect on the receptiveness of students to the content matter. Also all pupils attending private Catholic open schools are involved in a school-societal setting which, being different, may limit the general applicability of this study. Special care was taken not to make inter-school but rather intra-school comparisons. This was thought necessary as different schools may have provided their students with varying degrees of background knowledge and problem-solving strategies. Also there may have been different motivation levels towards mathematics at different schools.

SUBJECTS

A total of 106 pupils were involved in the study: 36 from School A, 38 from School B and 32 from School C. Only pupils who were interested in learning the topic were included in the samples. As a result a high motivation level was expected. The absence of uninterested pupils also reduced the level of distraction.

SELECTION PROCEDURE

The pupils were divided into the conventionally taught (expository) group (henceforward called the Control Group) and the group using discovery techniques (henceforward called the Experimental Group). The cooperation of the teachers was required in selecting subjects for the two groups: use was made of I.Q's (NSAIGT), previous scholastic achievement (Standard 9

mid-year exams) as well as the teacher's personal knowledge of each individual pupil. The multiple-choice diagnostic test (See Appendix II) was used to assess pupils' background knowledge. Pupils who scored highly in the above-mentioned tests were allocated to the Experimental Group while pupils who fared worse were selected for the Control Group. The combined results of Diagnostic Test plus percentage of Standard 9 mid-year was used to obtain a mark out of 80. It was possible to have equal numbers from each school in both groups. As only pupils who were interested in learning about linear programming were included in the samples, the actual numbers involved in the research were less than had previously been anticipated. However, the levels of motivation were probably improved. A motivation test (See Appendix I) was administered before the pretest to determine the pupils' attitudes towards mathematics in both the Control and Experimental Groups. A similar test had previously been used on final year Maths teachers in training at the University of the Witwatersrand.

MOTIVATION LEVEL:

A 5 point scale for each of the questions posed in the questionnaire results in a maximum possible score of 100. The results of the motivation test were not used in selection of the groups. It was expected that some of the pupils with low academic marks might show relatively high motivational levels.

HAWTHORNE EFFECT

The fact that both Experimental and Control Groups were receiving treatment minimised Hawthorne effect.

RESEARCH DESIGN

A Quasi-Experimental Design with non equivalent control groups (Cohen and Manion, 1982) was employed. Cohen and Manion (1982; p.158) differentiate between experiments and quasi-experiments as follows:

> "The single most important difference between the quasi-experiment and the true experiment is that in the former case, the researcher undertakes his study with groups that are intact, that is to say, the groups have been constituted by means other than random selection."

Homologous selection as was made has the effect of reducing randomisation which is essential for true experimentation. Kerlinger 1970, refers to quasi-experimental situations as "compromise designs". This type of design may be represented as:

(Experimental) $O_1 \times O_2$

(Control) $O_3 \times O_4$

The dotted line separating the parallel rows in the diagram of the non-equivalent control group design indicates that the experimental and control groups have not been equated by randomisation - hence the term "non-equivalent".

The independent variable in this experiment is the different teaching strategy and the dependent variable is the performance of the pupils once treatment has been effected.

DATA

Both the pretest and posttest (see Appendices III and IV) were marked out of 20. Comparisons were then possible between both Experimental and Control groups before and after treatment.

The pretest examined knowledge of the drawing of straight lines, shading to represent inequalities and ability to translate a simple word problem into inequalities. The posttest required the candidate to determine a feasible region given certain constraints and to determine an optimum solution to a typical linear programming problem.

A comparison of the change in performance of both groups from pretest to posttest gave an indication of the relative effectiveness of the two teaching strategies.

PROCEDURE ADOPTED IN PRESENTING SUBJECT MATTER

About two hours of teaching time were needed in presenting the material to both Control and Experimental groups.

EXPOSITION (CONTROL GROUP)

Very little variation in the technique adopted towards solution of problems was permitted. All pupils were required to understand and rigorously follow a set procedure for solution of linear programming problems. A step-by-step exposure was presented in the solution of 4 typical problems. Thereafter a programmed step-by-step solution to one further problem was given, requiring the pupils to adhere to a strict order.

The steps given were:

1. Write down all variables in the problem.
2. Decide on what numerical values (real, integral, natural) these variables may have.
3. Label axes, carefully describing each variable.
4. Set up inequalities involving given constraints.
5. Set up inequalities involving implicit constraints.
6. Set up inequalities deduced from given information.
7. Obtain the feasible region by graphing all the above inequalities.
8. Obtain an equation for the optimisation requirement.
9. Plot the intercepts of the above equation on the axes, and draw a line.
10. Use a set square and ruler to obtain another line parallel to the above line to intersect the feasible region and obtain the optimal value.

DISCOVERY (EXPERIMENTAL GROUP)

A very brief introduction to systems of linear inequalities was given leading to the concept of a feasible region. It was suggested that a summary of the data be written up in table form. Pupils were deliberately not told how to determine an objective function nor were they told that optimal solutions exist at the vertices of the polygon enclosing the feasibility region. No illustrative examples were done by the teacher. Guidance towards solution was provided in terms of broad hints about the governing constraints and optimisation requirement. Pupils were allowed to work in pairs and to share their ideas in small groups. Individual ideas regarding solution were constantly encouraged. Pupils were also encouraged to reflect on their individual actions during and after the solution process. The pupils were not given any idea about the teacher's problem-solving strategy and were thus required to develop their own.

CHAPTER FOUR

RESULTS OF THE STUDY

In this chapter the results of the study are analysed. Questions derived from the research hypotheses are posed, and in the light of the data collected, are answered. The hypotheses are stated below in the null form.

Ho 1.: There is no significant difference in performance of pupils in linear programming as a result of the different teaching strategies adopted.

Ho 2.: Students with different aptitudes will not benefit differently as a result of the different teaching methods.

The Control Group and the Experimental Group were constituted by analysing the results of the Diagnostic Test and the results obtained in the standard 9 mid-year examination (previous achievement). The Motivation Level results were considered only when there was no clear-cut difference in the results of the previous two tests in determining into which group a "border-line" student should be placed. I.Q.'s were available to the researcher and were used to check on individual cases that seemed to be in the inappropriate group after analysing all the other results.

Table 1 gives the results obtained by all the pupils in the Diagnostic Test and in the standard 9 mid-year exams. A mark out of 80 was obtained by adding the raw mark obtained on the Diagnostic Test (ex 30) to the percentage obtained in the mid-year exam reduced to a mark out of 50. This arrangement was considered to be a fair reflection of the students' mathematical ability by the researcher and teachers alike.

TABLE 1: MATHEMATICAL ABILITY OF SUBJECTS IN CONTROL AND EXPERIMENTAL GROUPS.

	CONTROL GROUP			EXPERIMENTAL GROUP		
Pupil No	PA (50)	DT (30)	TOT (80)	PA (50)	DT (30)	TOT (80)
1	29	12	41	35	21	56
2	27.5	18	45.5	28	18	46
3	35.5	20	55.5*	39.5	16	55.5
4	21	13	34	32.5	12	44.5
5	25	12	37	34.5	14	48.5
6	26	10	36	30	17	47
7	25	17	42	35.5	23	58.5
8	28.5	24	52.5*	44	29	73
9	21	18	39	40	27	67
10	23	14	37	30	13	43
11	19.5	9	28.5	29.5	20	49.5
12	25	12	37	32.5	24	56.5

	CONTROL GROUP			EXPERIMENTAL GROUP		
Pupil No.	PA (50)	DT (30)	TOT (80)	PA (50)	DT (30)	TOT (80)
13	23	15	38	30	23	53
14	37.5	18	55.5*	32	21	53
15	21	20	41	32	18	50
16	24	14	38	33	19	52
17	22.5	11	33.5	30	19	49
18	25	12	37	27.5	21	48.5
19	25.5	10	35.5	29	23	52
20	22.5	13	35.5	25.5	22	47,5
21	21	17	38	32.5	19	51.5
22	14.5	8	22.5	26	20	46
23	30	20	50*	24.5	30	54.5
24	30.5	20	50.5*	29.5	16	45.5
25	26	11	37	33	13	46
26	25	12	37	35	29	64
27	29	12	41	30.5	27	57.5
28	23	18	41	25	26	51
29	22.5	13	35.5	25	18	43
30	27.5	20	47.5*	27	18	45
31	23	19	42	26.5	23	49.5
32	23	26	49*	25	26	51
33	27	15	42	24.5	25	49.5
34	20	13	33	26	24	50
35	17	13	30	29	29	58
36	30	30	60*	35	16	51
37	19	23	42	25.5	14	39.5#

	CONTROL GROUP			EXPERIMENTAL GROUP		
Pupil No.	PA (50)	DT (30)	TOT (80)	PA (50)	DT (30)	TOT (80)
38	24.5	18	42.5	32.5	27	59.5
39	26.5	14	40.5	24.5	29	53.5
40	30.5	14	44.5	45	30	75
41	16	12	28	24.5	28	52.5
42	20	11	31	27	25	52
43	25.5	19	44.5	30	24	54
44	23.5	15	38.5	29.5	16	45.5
45	28.5	18	46.5*	30.5	17	47.5
46	24	14	38	27	25	52
47	24	11	35	25	22	47
48	29	18	47*	23	27	50
49	25	16	41	35	30	65
50	12.5	9	21.5	29	19	48
51	28	14	42	27.5	23	50.5
52	24	21	45	26	20	46
53	30.5	25	55.5*	28	21	49
$\bar{x}$	24.69	15.66	40.35	29.9	20.3	50.2

KEY: PA : Previous achievement

DT : Diagnostic Test

TOT: Total

* Pupils who were regarded as Experimental Group material but whose teachers believed should be put into Control Group based on I.Q. and other knowledge.

Insisted on being placed into Experimental Group.

To test whether the difference in means of the two groups was significant, use was made of the formula

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{1/n_1 + 1/n_2}}$$

where

$$S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

and

S_1^2 = variance of control group

S_2^2 = variance of experimental group.

In the above case

$$Z = \frac{50.2 - 40.35}{S\sqrt{1/53 + 1/53}}$$

where

$$S^2 = \frac{(52)(63.13) + (52)(51.34)}{53 + 53 - 2}$$

$S = 7.57$.

The value of Z is equal to 6,70 which is significant at the 0,01 level.

Conclusion: There is a significant difference ($p < 0,01$) between the two means.

Upon close inspection of the figures in Table I it is evident that there are a few pupils (marked with asterisks) who, through their own insistence or that of their teacher, were placed in the inappropriate group in terms of their results. In the opinion of the researcher, this did not have any significant bearing on the means achieved by the two groups.

It is interesting to note that the Experimental Group mean was 10 points above that of the Control Group on the BD scale. This represents an ability difference of over 12%. It should also be noted that very weak candidates ("potential failures" in the eyes of their teachers) were not allowed (by their teachers) to take part in the experiment. It is believed by the researcher that this resulted in a mean mark for the Control Group which was substantially higher than that originally ected.

To fully appreciate the differences in ap. ude in mathematics it was thought necessary to obtain motivational levels and I.Q.'s for each pupil.

Table 2 summarises the I.Q.'s and motivational levels of the Experimental and Control groups at the three schools involved in the experiment.

TABLE 2

SUMMARY OF MOTIVATION LEVELS AND I.Q.'S OF THE PUPILS IN THE TWO GROUPS AT THE DIFFERENT SCHOOLS.

SCHOOL A:	M.L.	I.Q.
CONTROL (n = 19)	51	104
EXPERIMENTAL (n = 19)	63	116
SCHOOL B:		
CONTROL (n = 18)	55	105
EXPERIMENTAL (n = 18)	59	113
SCHOOL C:		
CONTROL (n = 16)	46	102
EXPERIMENTAL (n = 16)	61	110

From the above table, it is evident that in all three schools there was a definite difference in I.Q.'s and Motivation Levels between the Experimental Group and the Control Group. The Experimental Group was therefore regarded as having a greater mathematical aptitude than the Control Group.

The Motivation Level Test (see Appendix I) consisted of 20 questions. Marks allocated were on a [illegible]nt scale. The questions were designed to evoke positive feelings (or otherwise) towards mathematics as a subject as well as the conception of mathematics not simply as a useful tool but for more aesthetic value. It was expected that pupils with a higher aptitude and better performance (in previous maths exams) would show higher motivation levels. The raw scores of the individual pupils are given in Table 3 below.

TABLE 3

RAW SCORES OBTAINED ON THE MOTIVATION QUESTIONNAIRE BY PUPILS

IN THE CONTROL GROUP AND EXPERIMENTAL GROUP

Pupil No.	CONTROL GROUP	EXPERIMENTAL GROUP
1	46	56
2	43	53
3	55	41
4	59	78
5	38	82
6	64	91
7	56	56
8	58	39
9	33	64
10	63	71
11	56	66
12	52	59
13	50	48
14	29	65
15	37	67
16	38	63
17	59	66
18	66	62
19	70	70
20	54	60
21	63	58
22	60	63

Pupil No.	CONTROL GROUP	EXPERIMENTAL GROUP
23	48	71
24	71	49
25	53	48
26	43	55
27	48	64
28	54	67
29	56	63
30	53	61
31	54	61
32	56	72
33	55	61
34	52	55
35	63	45
36	70	53
37	42	61
38	51	53
39	48	63
40	42	64
41	47	63
42	51	63
43	56	58
44	33	61
45	38	72
46	51	48
47	53	45

	CONTROL GROUP	EXPERIMENTAL GROUP
Pupil No.		
48	45	32
49	44	49
50	50	65
51	42	80
52	41	92
53	48	69
$\bar{X}$	51	61
d^2	4734	7114

Using the previous formula

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{1/n_1 + 1/n_2}}$$

Z was calculated to be 4,87. There is a significant difference ($p < 0,01$) between the means. Although the difference is significant, there is only a 10% difference for the Experimental Group over the Control Group. The motivation level of the Control Group was certainly higher than what was expected by the researcher.

Table 4 shows the pretest and posttest marks attained by the two groups. The researcher used a colleague to moderate the tests and to check that part marks were awarded accurately and consistently.

TABLE 4

PRETEST AND POSTTEST MARKS OBTAINED BY PUPILS IN THE CONTROL GROUP AND EXPERIMENTAL GROUP

	CONTROL GROUP			EXPERIMENTAL GROUP		
PUPIL NO.	PRETEST	POSTTEST	Δ	PRETEST	POSTTEST	Δ
1	12	13	+1	13	11	-2
2	7	10	+3	17	13	-4
3	10	10	0	18	11	-7
4	7	6	-1	12	12	0
5	6	8	+2	9	12	+3
6	13	15	+2	10	13	+3
7	10	10	0	20	18	-2
8	10	9	-1	19	15	-4
9	8	12	+4	20	20	0
10	9	13	+4	10	8	-2
11	10	8	-2	11	12	+1
12	11	17	+6	14	10	-4
13	15	8	-7	16	12	-4
14	18	20	+2	13	13	0
15	11	8	-3	10	11	+1
16	9	13	+4	9	12	+3
17	8	12	+4	11	10	-1
18	14	16	+2	13	4	-9
19	10	13	+3	16	10	-6
20	9	10	+1	13	10	-3
21	8	8	0	10	8	-2

	CONTROL GROUP			EXPERIMENTAL GROUP		
PUPIL NO.	PRETEST	POSTTEST	Δ	PRETEST	POSTTEST	Δ
22	4	6	+2	12	6	-6
23	18	20	+2	19	20	+1
24	12	18	+6	9	11	+2
25	9	10	+1	6	4	-2
26	7	5	-2	20	20	0
27	10	10	0	18	16	-2
28	11	13	+2	16	12	-4
29	12	8	-4	14	10	-4
30	13	13	0	13	8	-5
31	18	15	-3	12	12	0
32	20	20	0	13	10	-3
33	8	10	+2	14	10	-4
34	6	7	+1	13	11	-3
35	5	10	+5	19	20	+1
36	14	15	+1	10	8	-2
37	11	18	+7	9	4	-5
38	13	12	-1	18	20	+2
39	9	10	+1	20	18	-2
40	12	10	-2	20	14	-6
41	8	10	+2	20	16	-4
42	7	9	+2	14	10	-4
43	14	15	+1	10	4	-6

	CONTROL GROUP			EXPERIMENTAL GROUP		
PUPIL NO.	PRETEST	POSTTEST	Δ	PRETEST	POSTTEST	Δ
44	10	11	+1	9	9	0
45	11	18	+7	11	12	+1
46	10	7	-3	15	13	-2
47	9	10	+1	13	10	-3
48	17	12	-5	18	14	-4
49	8	12	+4	16	6	-10
50	8	9	+1	13	10	-3
51	11	10	-1	18	13	-5
52	13	14	+1	12	12	0
53	11	18	+7	11	10	-1

Δ represents the change in each individual's marks attained from pretest to posttest.

The means and Standard deviations are given in Table 5.

TABLE 5.

SUMMARY OF RESULTS OBTAINED ON PRETEST AND POSTTEST

BY PUPILS IN CONTROL GROUP AND EXPERIMENTAL GROUP.

	CONTROL	EXPERIMENTAL
PRETEST	$\bar{X}$ = 10,64	$\bar{X}$ = 13,83
	S.D. = 3,1	S.D. = 3,75
POSTTEST	$\bar{X}$ = 11,77	$\bar{X}$ = 11,66
	S.D. = 3,80	S.D. = 4,1

When comparing the results of the two groups on the pretest, Z as calculated according to the previous formula:-

$$Z = \frac{\left| \bar{X}_1 - \bar{X}_2 \right|}{s\sqrt{1/n_1 + 1/n_2}}$$

has a value of 4,59. There is a significant difference ($p < 0,01$) between the means of the pretest marks.

When comparing the results of the two groups on the posttest the Z value was calculated to be 0,143. This indicates that there was no significant difference between the means of the posttest marks.

The mean difference, $\bar{D}$, representing the change in mean marks from pretest to posttest is given below:

Control Group: $\bar{D}$ = +1,13

Experimental Group: $\bar{D}$ = -2,17.

It is evident from Table 4 that pupils in the Control Group experienced a bigger improvement than those in the Experimental Group. Of the 53 pupils in the Control Group, 33 pupils improved their posttest marks over their pretest marks, 6 remained the same and 14 showed a deterioration. Of the 53 pupils in the Experimental Group, 10 improved their posttest marks over their pretest marks, 7 remained the same and 36 experienced a deterioration.

Statistical Analysis of Differences:

1. Experimental Group (n = 53)

   Pretest mean: 13,83. Posttest mean: 11,66

   Mean difference $\bar{D} = -2,17$

   Standard deviation of differences: $S_D = \sqrt{\frac{\sum d^2}{n}}$

   $= 2,92$

   Standard error of mean difference:

   $$S_{\bar{D}} = \frac{S_D}{\sqrt{n-1}} = 0,405$$

   $$t = \frac{\bar{D}}{S_{\bar{D}}} = -5,358$$

There is a significant difference ($p < 0,001$) between the mean differences of pretest and posttest marks of the Experimental Group.

2. Control Group (n = 53)

Pretest mean: 10,64 Posttest mean: 11,77

Mean difference $\bar{D} = +1,13$

Standard deviation of differences: $S_D = \sqrt{\frac{\sum d^2}{n}}$

$= 2,94$

Standard error of mean difference:

$$S_{\bar{D}} = \frac{S_D}{\sqrt{n-1}} = 0,408$$

$$t = \frac{\bar{D}}{S_{\bar{D}}} = \frac{1,13}{0,408} = 2,77$$

There is a significant difference ($p < 0,01$) between the mean differences of pretest and posttest marks of the Control Group.

It was thought necessary to analyse the results of the Experimental and Control Groups at the three different schools. This was done to see whether the trends observed were typical of all three schools and also to check (by Analysis of Variance) whether the results would be affected because of differences in individual schools. The results are summarised in Table 6:

TABLE 6.

SUMMARY OF THE RESULTS OBTAINED ON THE PRETEST AND THE POSTTEST BY THE TWO GROUPS AT THE THREE INDIVIDUAL SCHOOLS.

	CONTROL GROUP			EXPERIMENTAL GROUP	
SCHOOL A	PRETEST	POSTTEST		PRETEST	POSTTEST
$\bar{X}$	10,42	11,63	$\bar{X}$	13,74	11,95
S.D.	2,91	3,63	S.D.	4,67	3,30
D	1,21		D	-1,79	
SCHOOL B					
$\bar{X}$	10,83	12,00	$\bar{X}$	13,33	11,11
S.D.	4,39	5,61	S.D.	3,77	4,55
D	1,17		D	-2,22	
SCHOOL C					
$\bar{X}$	10,69	11,69	$\bar{X}$	14,50	11,94
S.D.	2,57	2,95	S.D.	3,82	3,96
D	1,00		D	-2,56	

From the Table it can be seen that pupils at all three schools in the Control Groups improved their posttest marks over their pretest marks while at all three schools pupils in the Experimental Groups experienced a drop in the posttest marks over their pretest marks.

An analysis of variance was done on the results according to a method in Fundamental Statistics in Psychology and Education (Guilford and Fruchter, 1973, 5th Edition).

1. Pretest Results

Total Sum of Squares was obtained using the formula $(SS)_{TOT} = \sum (X - Mt)^2$. Where X represents the raw scores obtained by the 106 pupils in the pretest and Mt is the total mean for the entire sample :

$$Mt = \frac{\sum X}{N} = 12,235$$

in this case.

The calculated value of $(SS)_{TOT}$ was 1646,8.

The data derived from the experiment may be tabled as follows:

	TREATMENT		
	CONTROL GROUP	EXPERIMENTAL GROUP	
SCHOOLS			MEAN OF SCHOOL.
	n = 19	n = 19	
A	Σ 198	Σ 261	
	$\bar{X}$ 10,42	$\bar{X}$ 13,74	12,08
B	n = 18	n = 18	
	Σ 195	Σ 240	
	$\bar{X}$ 10,83	$\bar{X}$ 13,33	12,08
C	n = 16	n = 16	
	Σ 171	Σ 232	
	$\bar{X}$ 10,69	$\bar{X}$ 14,5	12,60
Group	$\bar{X}$ 10,64	13,83	
	Σ 564	733	

The sum of squares between groups is obtained as

$$(SS)_g = n\,r(\Sigma (M_g - Mt)^2)$$

$$= 69 \times 1\,[(10,64 - 12,235)^2 + (13,83 - 12,235)^2]$$

$$= 269,66$$

The sum of squares between schools is obtained as

$(SS)_s = n (\sum (Ms - Mt)^2)$

$= 32 (12,60 - 12,235)^2 + 36 (12,08 - 12,235)^2$

$+ 38 (12,08 - 12,235)^2$

$= 6.$

The interaction variance $(SS)_I$ was obtained as follows:

$(SS)_I = n (\sum (Mss - Mt)^2) - 269,66 - 6$

$= 19 ((10,42 - 12,235)^2 + (13,74-12,235)^2))$

$+18 ((10,83 - 12,235)^2 + (13,33 - 12,235)^2)$

$+16 ((10,69 - 12,235)^2 + (14,5 - 12,235)^2))- 275,66$

$= 19 (5,56) + 18 (3,07) + 16 (7,52) - 275,66$

$= 105,6 + 55,3 + 120,3 - 275,7$

$= 5,5.$

(SS) (within sets) = 1646,8 - 269,7 - 6 - 5,5

= 1365,6.

The results are summarised below:

Source	ss	df	ms	F-ratio
Groups	269,7	1	269,7	19 76
Schools	6	2	3	0,22
Interaction	5,5	2	2,75	0,20
. thin Sets	1365,6	100	13,65	
otal	1646,8	105		

From the F-ratios obtained it can be concluded that interaction effects and differences due to the different school are not significant.

The F value obtained for the groups is well beyond the F value required for significance at the 0,01 level. It may be concluded that the differences in pretest scores were dependent upon only the groups (Experimental and Control) involved.

2. Posttest results

A similar statistical analysis on the posttest results was done. The results are summarised below:

Source	ss	df	ms	F-ratio
Groups	0,3	1	0,3	0,02
Schools	1,4	2	0,7	0,04
Interaction	8,3	2	4,15	0,26
Within Sets	1619,9	100	16,2	
Total	1629,9	105		

The results obtained confirm what was shown earlier: On the posttest scores, there are no significant differences between the groups. Also there are no significant differences between the different schools and there are no significant interactions between schools and groups.

CONCLUSIONS:

In the light of the above results both null hypotheses are rejected.

The research hypotheses generate questions which are listed below and in the light of the statistical analysis done are then answered:

Research questions derived from Ho 1.

Q. (1) Is there a difference in performance of pupils in linear programming as a result of the different teaching strategies adopted?

A. : Pupils who were taught under different strategies did perform differently.

Q. (2) Did pupils who were taught by expository methods show any significant difference in performance from pretest to posttest?

A : Pupils taught by expository methods showed a significant improvement.

Q. (3) Did pupils taught by discovery methods show any significant difference in performance from pretest to posttest?

A : Pupils taught by discovery methods showed a significant deterioration.

Research questions derived from Ho 2.

Q. (1) Did students of higher mathematical aptitude/ ability show a marked improvement in the solution of linear programming problems when taught by discovery methods?

A. : Students of higher mathematical aptitude/ ability showed no significant improvement in the solution of linear programming problems when taught by discovery methods.

Q. (2) Did students of lower mathematical aptitude/ ability show a marked improvement in the solution of linear programming problems when taught by expository methods?

A. : Students of lower mathematical aptitude/ ability showed a significant improvement in the solution of linear programming problems when taught by expository methods.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The issue of effective teaching for optimal learning is a necessary concern of every teacher. The imparting of knowledge is so important as to encourage different teaching styles that attempt to improve pupil performance. Fey (1979) believes that there is little immediate hope that instructional research will yield new ideas or convincing support for any existing strategy or technique. Fey (1979:p.496) puts it as follows :

> "There is no consistent pattern of results favouring recitation, discovery, small group or individualized approaches in mathematics teaching: there is no demonstrably superior way to identify the knowledge, experiences or personal traits of people who will be consistently effective teachers. There are certainly effective teachers of mathematics, teachers whose students enjoy and learn mathematics. But at the present time, such effectiveness appears to be the result of classroom activity that is an idiosyncratic product of a constantly changing interaction among the teacher, the student, and the mathematics being taught".

The present research has attempted to compare the effectiveness of discovery techniques and expository techniques in the teaching of Linear Programming to South African high school pupils. The controversy about discovery and expository teaching seems to centre essentially about the question of how much and what kind of guidance should be provided to students in the learning situation. Shulman (1968:p.34) sees the two positions as follows :

> "those favouring learning by discovery advocate the teaching of broad principles and problem solving through minimal teacher guidance and maximal

> opportunity for exploration and trial-and-error on the part of the student. Those preferring guided learning emphasize the importance of *carefully* sequencing instructional experiences through maximum guidance and stress the importance of basic associations or facts in the service of the eventual mastering of principles and problem solving".

Shulman (1968: p.36) examines the positions held by Gagne and Bruner on this issue and his summary of the two psychologists encapsulates the essential difference :

> "for Gagne, instruction is a smoothly guided tour up a carefully constructed hierarchy of objectives; for Bruner instruction is a roller-coaster ride of disequilibria and equilibria until the desired cognitive state is reached or discovered".

Implicit in this contrast is a difference in what is meant by the very words 'learning by discovery'. *For Gagne, learning* is the goal. How a behaviour or capability is learned is a function of the task. It may be by discovery, by guided teaching, by drill or by review. The focus is on learning and discovery is but one way to learn something. *For Bruner it is learning* by discovery. The method of learning is the significant aspect.

Instructional approaches need to be appraised for their effectiveness in terms of *short-term gains and long-term goals*. Pressure for immediate measurable results often leads to teachers "cramming" knowledge into their pupils' heads. Novak (1978:p.1) puts it bluntly :

> "Knowledge acquired by rote learning is soon lost, and even before it is forgotten, this knowledge cannot be used effectively in problem-solving. In *an effort to reduce* teaching practices that encouraged rote learning, new curriculum efforts and supporting federal agencies adopted a dogmatic adherence to 'discovery approaches'"

There was, however, no guarantee that discovery teaching approaches produced meaningful learning. Novak (1978:p.2) continues :

> "While discovery learning strategies have some *important and unique educational* values, it is obvious that our cultural heritage created by geniuses over the past three or four centuries cannot be rediscovered by our pupils in ten or fifteen years. It follows, therefore, that the central task of schools is to make expository teaching and reception learning more meaningful".

In the South African situation, we may look at mathematical education in terms of the *Beeby (1966) model of educational* systems in developing countries. Webb (1984) places South Africa in stage II: the stage of formalism, at which teachers are trained but poorly educated. The classroom is highly organised, syllabi are rigid, textbooks are fixed and there is a strong emphasis on inspection. Set in this tightly structured format, with a highly exam-oriented system encouraging a stereotyped, work-to-rule approach with the inevitable "spoonfeeding" of pupils, it is small wonder that South African students are poorly prepared *as problem solvers.*

Conclusions:

The data generated from the study and subsequent statistical analysis would indicate that pupils in the Experimental Group did not benefit from discovery teaching as much as pupils in the Control Group did from expository teaching. There seems to be an acute lack of initiative in most of the pupils in the

experimental group. Very few were able to handle the challenge of a novel situation on their own. Without any substantial guidance in the matter of solving Linear Programming problems, the majority of pupils in the Experimental Group showed no marked improvement in their ability to solve Linear Programming problems. On the other hand, pupils in the Control Group, despite being of a poorer mathematical background, were able to closely follow a set of rules in a procedure for the solution of Linear Programming problems. The majority of the pupils in the Control Group showed a significant improvement in their ability to solve Linear Programming problems.

It would appear that the South African system of mathematics education encourages expository teaching. Expository teaching would appear to allow the weaker student to rely entirely on the teacher's ability to explain set procedural rules in solving problems. The data indicates that pupils of a higher mathematical ability struggle to solve problems independently and also require guided instruction.

It is necessary to attempt to explain why the pupils taught by expository methods benefitted more than those taught by discovery methods. Perhaps in the schools involved in the experiment there has been very little discovery teaching because of the pressure to complete the syllabus. The experience of the researcher is that most pupils (and their parents) are intent on achieving the highest possible symbol in Mathematics in the Matric exam. This is normally because of pressure to achieve

the necessary symbols to qualify for entrance to restricted faculties at University and other tertiary institutions. The result is that most teachers at Matric level complete the syllabus as soon as possible and immediately set their pupils on to past examination papers. This, in turn, results in the pupils becoming expert at handling particular types of problems which regularly appear in the end-of-year examinations. In this way a diligent pupil may cover all the types of questions that appear in the examinations and, because the questions on the examination are so predictable, such a pupil stands a very good chance of passing well. Webb (1984:p.117) believes that the

> ation has a negative effect on the promotion of good
>
> s towards problem solving :
>
> "for any pupil will find the problem in the exam paper decently formulated, and can assume that the problem is soluble within a reasonable time by means of standard techniques. In South African maths exams the relevant technique is heavily cued by the position of the problem in the paper. If it's question 3 in paper 1, it must be done by graphing a quadratic".

Another possible explanation for the poor performance of the Experimental Group was the lack of time to develop an approach to problem-solving. Insufficient time may have been had to allow the pupils to develop their own strategies for problem-solving. In a similar experiment Laridon (1981) developed strategies over a longer period of time. The results of this research indicate that the Experimental Group did significantly better than the Control Group in questions where a proper understanding of basic concepts of differential calculus was required. More time was spent with the Experimental Group than with the Control Group although this was regarded as not being of appreciable effect.

It is also possible that some pupils did not give of their best as they knew that the marks obtained on the Linear Programming course would not have any effect on their end-of-year promotion marks. It seems to be the case that some pupils try their best only when there is pressure on them to achieve high marks in tests that count. However, since both the Experimental Group and the Control Group were in the same situation as regards marks not counting for end-of-year symbols, it is believed by the researcher that this explanation for poor performance by the Experimental Group does not seem likely.

It may be the case that too much was expected of the Experimental Group by the researcher. Geldinhuys (1985) believes that simply presenting problems and expecting solutions in Linear Programming is asking too much of high school pupils. He argues:

> "sekerlik sal 'n mens nie die begrip lineere programmering in die klaskamer invoer deur leerlinge met so 'n probleem te konfronteer nie. Ek dink dat 'n mens eers die oplossingsmetode moet verduidelik en dan oorgaan tot die oplos van praktiese probleme."

He acknowledges the weakness in this expository approach.

> "Die nadeel verbonde aan so 'n benadering is die moontlikheid dat werklike praktiese probleme afgeskeep sal word en dat die waarde van lineere programmering dan heeltemaal verlore sal wees".

It would be tragic to allow pupils to be "spoonfed" to the degree that they become completely inept on any form of individual problem solving strategies. Carpenter (1980) regards the situation as being very serious when:

> "students see maths classtime as a time to be passive, watching teachers demonstrate rules that they must memorize and practise".

Pupils should become more active in thinking about and developing their own strategies for problem solving. When pupils encounter difficulties, the teacher who tries to remedy the situation by solving the problem himself is not encouraging the attitudes necessary for the pupils to become good problem solvers. Brousseau (1984) believes such action on the part of the teacher is actually counter-productive as:

> "everything he does to make the pupil produce the behaviour he expects tends to deprive the latter of the conditions necessary for understanding and learning the notion concerned. If the teacher says what he wants, he can no longer obtain it".

RECOMMENDATIONS

It is believed that discovery methods are not being used in South African high schools as extensively as they could be. Research is needed on ways and means of improving pupils' and teachers' attitudes towards problem solving in mathematics. Dr A Meijer in Laridon (1981: p.15) sums up the problem :

> "The most important change that I would like to see is not strictly one in syllabus, but a change of attitude. I have gained the impression that the present teaching system is geared far too much towards getting passes in the matriculation examination. Consequently, the pupils appear to be coached in solving certain standard types of problems (namely those that appear regularly in the examination paper)."

An alternative method of assessment instead of the Matric exam needs to be researched. The overemphasis of the importance of this exam is to encourage study patterns diametrically opposed to what is expected at tertiary levels. Fletcher (1985) identifies this as one of the main reasons for the high first-year failure rate at University.

> "The ever-present spectre of final examinations places pressure on teachers to regard themselves as being personally responsible for ensuring that their pupils pass. This is clearly recognised by the pupils who can safely sit back and rely on their teachers to make sure that they do enough work to succeed."

Fletcher (1985) believes that teachers at school level are wary to accept the risk of allowing pupils to rely on their own research:

> "The pupils know perfectly well that the teacher will eventually give them all the information on a plate so that the examination results will not be jeopardised. This is particularly apparent in teachers' penchant for supplying their classes with every last detail, as opposed to the University approach of providing a broad outline in lectures, leaving it up to the student to install the padding."

With regard to the end-of-year examination it is suggested that the examination become less predictable. The exam should include more questions that require insight and involve more meaning and understanding as opposed to questions that are solved by rote techniques.

Research must be done in effective ways to train teachers in problem solving. In this regard Webb (1984:p.119) sees the

introduction of mathematical magazines as a step in the right direction. Problems appearing in such magazines would promote better problem solving strategies on the part of the teachers.:

> "I want to subvert a system which encourages pupils to view mathematics as a set of rules to be memorized. I want teachers to be threatened ('challenged' would be a more tactful word) by tricky problems."

Webb sums up the role of teachers in teaching problem-solving thus (p.119):

> "How do you teach problem-solving? You solve problems yourself and reflect on your own thought processes. That's what I want teachers to do."

It is recommended that in-service courses on Linear Programming attempt to develop the desirable attitudes towards problem solving in teachers themselves. Discovery methods will then be appreciated by the teachers who can pass on their experiences to their pupils. Beeby (1966) places the teacher at the focal point of innovation.

> "There is one thing that distinguishes teaching from all other professions except perhaps the Church - no change in practice, no change in curriculum has any meaning unless the teacher understands and accepts it. If he does not understand the new method or if he refuses to accept it other than superficially, the instructions are of no avail. At best he will go on doing in effect what he has always done, or at worst will produce some travesty of modern teaching".

The success of the weaker pupils when taught by expository methods would indicate that pupils taking Maths should be streamed. Most pupils who take Maths on the Standard Grade

should be taught by expository methods to achieve best results. For the better pupils, expository methods should be replaced to some degree by consistently used discovery methods over a longer period of time. This may help to improve the attitudes required to handle the subject at University.

Textbooks should include more discovery-type exercises. Pupils will be encouraged to become more independent in problem solving. All pupils should be encouraged to enter Mathematical competitions. Webb (1984) sees these as an important means of moving mathematical education from stage 2 to stage 3 of the Beeby model. In a study of the effect of computer assisted mathematics in schools in Soweto, Metrowich (1985) believes that the transition from stage 2 to stage 3

> "can be made by accretions of knowledge and skills without any change in educational philosophy".

Research on such knowledge and skills is urgently needed.

SUGGESTIONS FOR FURTHER RESEARCH.

Further research is needed on the effectiveness of modern learning theory in developing practices which encourage creativity, imagination and intuition in pupils' mathematical experiences.

Research is also needed on more effective ways of teaching "word problems" to pupils at different stages of their schooling (including Junior school).

It is suggested that a repetition of the practical aspects of this research be undertaken. Research on the type of "mix" of expository and discovery methods of teaching which yields optimal learning would be useful.

APPENDIX I

MOTIVATION QUESTIONNAIRE

MY IDEAS ABOUT MATHEMATICS

For each of the following 20 statements, place a tick in the column you consider to be most appropriate.

SD: Strongly Disagree

D: Disagree

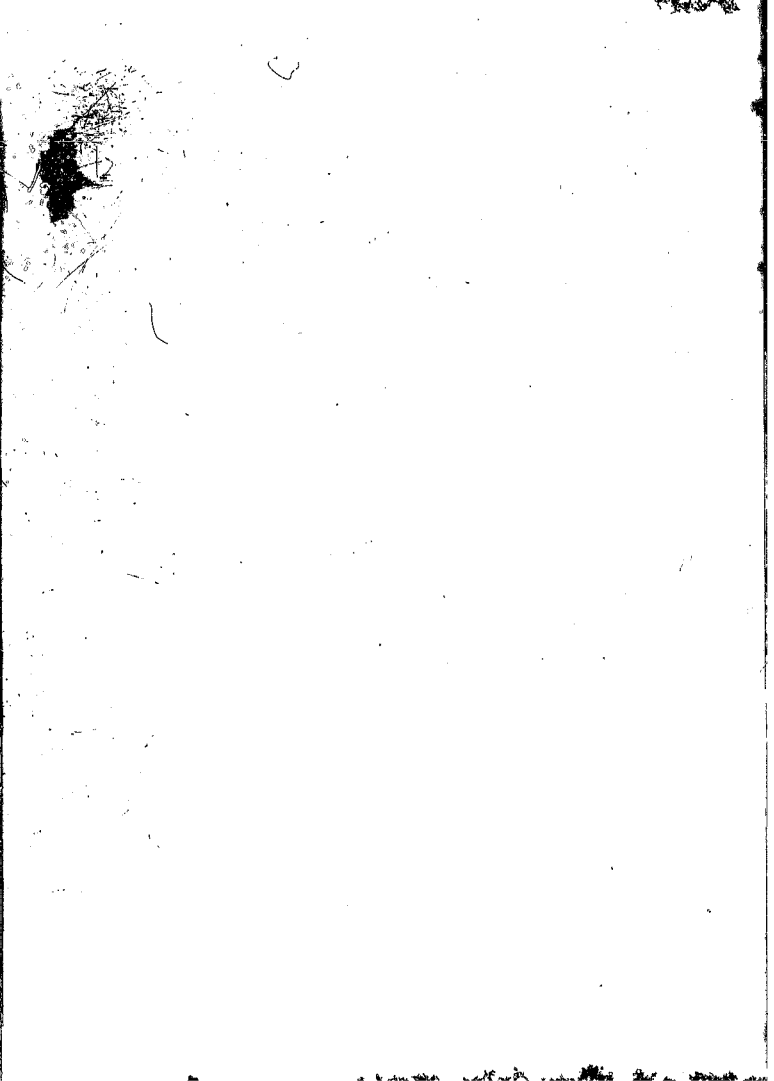
U: Undecided

A: Agree

SA: Strongly Agree

	SD	D	U	A	SA
1. Mathematics is believed to be the most repulsive of all school subjects					
2. Mathematics involves more work than most subjects					
3. Mathematics must be taught in a rigorous deductive way					
4. Pupils should be allowed to develop independent solutions to problems					
5. The study of mathematics tends to dull the imagination					
6. The important thing is to DO mathematics and get right answers and understand later what has been done					
7. All matric pupils should do Mathematics on one of the available grades					
8. Mathematical problems should be one of the most important subjects of the syllabus					
9. Mathematics at school bears little or no relation to the real world					

	SD	D	U	A	SA
10. Too many lessons each week are devoted to Mathematics					
11. "Practice makes perfect" is the most important rule in teaching Mathematics					
12. It is important that children should appreciate elegance in mathematical proofs					
13. Mathematics should be taught in such a way as to let the pupils really think for themselves					
14. Regular arithmetic practice is the most important content for pupils of below average ability					
15. Mathematics teachers should not be concerned with applications of mathematics					
16. The most important feature of mathematics is that it is useful to the individual					
17. Teaching should concentrate on the content of the examination and not on topics which may be otherwise interesting					
18. Topics not on the syllabus must be avoided completely					
19. Parents regard mathematics as being very useful					
20. Geometry problems are too difficult.					



APPENDIX II

DIAGNOSTIC TESTS

Applied towards the end of Std. 9 academic year. (Adapted from Classroom Mathematics. Laridon, et al.)
N.C.: Use of calculators not allowed.
Mark only the correct response.

1. The solution set to the system
$y - x = 3$
$x^2 - xy + 3x = 0$
is

a) ϕ

b) $\{(0; a); a \in R\}$

c) $\{(0; 3)\}$

d) $\{(a; a+3); \quad a \in R\}$

e) $\{(0; 3); (2; 5)\}$

2. A and B are the roots of $2x^2 - 6x + 15 = 0$
The value of $A^2 + B^2$ is:

a) 9

b) 6

c) -6

d) 24

e) none of the above

3. The x intercepts of the graph of $y = -3x^2 + 9x - 6$ are $(A; 0)$ and $(B; 0)$. Hence:

a) $A + B = 9$

b) $A - B = 3$

c) $AB = 2$

d) $A + B = -3$

e) $AB = -6$

4. Which one of the following statements *is false*:

   a) $1/x > 0 \Rightarrow x > 0$

   b) $1/x < 1 \Rightarrow x > 1$

   c) $x^2 < 1 \Rightarrow x > -1$

   d) $x = 1 \Rightarrow x^2 = 1$

   e) $x^2 < 1 \Rightarrow -1 < x < 1$

5. Which one of the following statements is true?

   a) $(x - 2)(x + 3) = 0 \Rightarrow x = 2$

   b) $2x > 3y \Rightarrow -6 < -9y$

   c) $x^2 - 2x = 0 \Rightarrow x = 2$

   d) $x^2 - x - 6 = 0 \Rightarrow x = 2; -3$

   e) $x^2 > 1 \Rightarrow x > 2$

6. $Ax^2 + Bx + C = 0$ has $B = 0$, $C > 0$, $A < 0$
   The roots of the above equation are:

   a) always real

   b) always non-real

   c) always rational

   d) zero or otherwise imaginary

   e) always irrational

7. If $x^2 - 16 > 0$ and $x < 4$, then:

a) $x < -4$ and $x = 4$

b) $x < -4$ and $x > 4$

c) $-4 < x < 4$

d) $x < -4$

e) $x = 4$

8. $xy = 27$; $4y - x = 3$; $x < y$

Thus $y - x =$

a) 14 1/4

b) - 6

c) 39/4

d) -12

e) -39/4

9. The graph of $y = -2(2x - 6)^2 + 1$ is symmetric with respect to the line:

a) $y = 0$

b) $x = 0$

c) $x = 3$

d) $x = +12$

e) $x = 1$

10. The point on $xy = 16$, $x > 0$ which is nearest to the origin is:

a) (2;2)

b) (4;4)

c) (-4; -4)

d) $(\sqrt{k}; \sqrt{k})$

e) (16; 1)

11. AOB is a diameter of circle ACDB.
If DAB = 30° then the magnitude of x is:

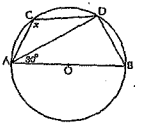
a) 30°

b) 90°

c) 120°

d) 150°

e) 60°



12. If AB = 8 units and OC = 3 units, then OB has a length of:

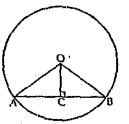
a) 6 units

b) 5 units

c) 10 units

d) 8 units

e) none of these



13. If P is the centre of the circumscribed circle of ΔABC and $P_1 = 80°$ then the size of A is:

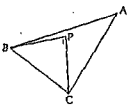
a) 40°

b) 80°

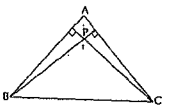
c) 120°

d) 160°

e) 100°



11. If P is the point of intersection of the altitudes of Δ ABC and $P_1 = 100^\circ$, then the size of A is:



a) 40°

b) 80°

c) 120°

d) 160°

e) 50°

15. If $\cos(-x) = 3/2$ and $x \in (0^\circ; 180^\circ)$, then the value of $\cos 2x$ is:

(NC)

a) 3

b) 1/2

c) $\sqrt{3}$

d) - 1/2

e) - 1/3

(double angle formula supplied)

16. If tan A = 3/4 and $0^{\circ} < A < 90^{\circ}$, then cos A - sin A equals: (NC)

a) $1\frac{1}{3}$

b) $1\frac{2}{5}$

c) $-1/5$

d) 1

e) $1/5$

17. If sin x = m and tan x = n, then cos x is equal to:

a) m + n

b) m - n

c) mn

d) $\frac{m}{n}$

e) $\frac{n}{m}$

18. $\cos^2 A = 1/4$ and $A \in (0°; 360°)$. Indicate which one of the following contains all the elements of the solution set:

a) $(60°; 120°; 240°; 300°)$

b) $(60°; 150°; 210°; 330°)$

c) $(60°; 120°)$

d) $(60°; 150°)$

e) $(30°; 150°; 210°; 330°)$

19. Tan A and cos A are both negative.

a) $A \in (0°; 180°)$

b) $A \notin (180°; 270°)$

c) $A \in (90°; 180°)$

d) $A \in (270°; 360°)$

e) for no value of A

20. $\cos 36° = C$, which of the following is <u>not</u> correct?

a) $\sec(216°) = -\frac{1}{C}$

b) $\tan(-36°) = \frac{-1 - C^2}{C}$

c) $\cos(-36°) = -C$

d) $\sin(54°) = C$

e) $\sin(126°) = C$

APPENDIX III

PRETEST

Assessment of pupils knowledge of linear inequalities.

1. Shade $\{(x,y) / -2x + 1 < y < x - 1\}$

2. Suppose x represents the number of girls at a coeducational school of total population 500.
   y represents the number of boys.
   If there are at least twice as many boys as there are girls show the situation graphically.

APPENDIX IV

POSTTEST

1. An oil company refines crude into Premium and Regular. Two processes are involved: distilling and cracking. Regular requires 0,3 hours cracking and 0,6 hours distilling per 100 litre and Premium requires 0,5 hours cracking and 0,4 hours distilling per 100 litre.

   The cracking plant can operate for 12 hours a day while the distillation plant operates for 18 hours a day.

   The profits to the company are 8c a litre on Regular and 12c on Premium.

   Find the mix that the company should run in order to obtain maximum profit.

2. i) Graph the feasible set. subject to the following constraints:

   $x, y > 0$

   $0,2y + 0,1x < 12$

   $0,1y + 0,2x < 10$

   ii) *Maximise the objective function*

   $P = 5x + 2y$ over the feasible set in (1) by means of *a graphical method* which you must describe fully.

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