Chapter 1
Introduction

1.1 A motivation for undertaking this study

The change in the world, the growth and development of knowledge and technology required improvement to the existing South African curricula. At the same time South Africa as a country has changed and the values and principles enshrined in the country's constitution should be reflected in the curricula. According to the reform vision of teaching and learning, mathematics is no longer seen as a fixed collection of facts and procedures; it is a dynamic body of knowledge that is continually enriched through conjecture, exploration, analysis, and proof (Smith, 1996: 393).Confrey (1990: 110) states:

> I am teaching them how to develop their cognition, how to see the world through a set of quantitative lenses which I believe provide a powerful way of making sense of the world, how to reflect on those lenses to create more and more powerful lenses and how to appreciate the role these lenses play in the development of their culture. I am trying to teach them to use one tool of the intellect, mathematics.

The National Curriculum statement or NCS for short, the first version of the new curriculum in South Africa, similarly summarizes mathematics as "a powerful conceptual tool" that helps the learners to analyse situations, make and justify critical decisions, reconstruct and develop new ideas and engage with socio-relations (Department of Education, 2008:7).

These dramatic changes in the conceptions of the subject are paralleled by significant shifts in the ways that teachers are to carry out their work. Teaching by demonstrating and practice is no longer acceptable and the teacher is no longer a "knowledge provider". Teachers should pay attention not only to the content, what learners learn, but how the learners learn, the process of learning. The NCS suggests that teachers can act only indirectly, by creating settings in which learners learn mathematics through their own activity. The Department of Education (1997:11) called for teachers "to think and prepare interesting and appropriate learning activit[ies]…" in which learners investigate and learn new mathematical concepts. Furthermore the NCS for mathematics is based on the following view of the nature of the discipline:

> Mathematics enables creative and logical reasoning about problems in the physical and social world and the context of Mathematics itself. It is a distinctly human activity practised by all cultures. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. (Department of Education, 2003: 9)

Thus, mathematics is described as the outcome of social processes with the main characteristic of generating critical thinking and understanding.

Focusing on the goal of Mathematics teaching and learning practice and developing learners' thinking, the mathematics education community looks for practical applications of the various learning theories: Cognitive, Socio-cultural and Situated. From different perspectives the important topic, the new role of the teacher in a mathematical classroom, has been discussed. Using various tenets many researchers (e.g. Cobb, Wood & Yackel, 1993; Shifter, 2001; Smith, 1996) identified multiple new teaching actions and renamed the teacher as "facilitator". This notion is, however, too broad and is not yet complete. In connection with this there is an interesting investigation in Ben-Zvi & Sfard's article "Ariadne's Thread, Daedalus' Wings, and the learner's autonomy" (2007). In their paper they use two metaphors from Greek mythology to illuminate two contradictory instructional approaches:

> Just think about the striking contrast between the learning processes induced by the mythological heroes Daedalus and Ariadne when they were trying to help their loved-ones to escape King Minos' prisons. Ariadne, to guide her beloved Theseus through Minotaur's labyrinth, provided the young man with a thread which he was told to follow faithfully and without questions. Daedalus, on the other hand, armed his son Icarus with wings and let him choose his own trajectory (Ben-Zvi & Sfard, 2007: 1).

Facing two opposite ways of the teaching-learning process, Ben-Zvi & Sfard argue that the dynamics of discursive lead-taking and lead-following is a topic for further research. They state that with the new phenomenon, the participatory classroom, theoreticians and practitioners are still in the dark about various aspects of school learning.

Furthermore in the article "When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint", Sfard (2007) attempts to answer two questions: what are the features of new mathematical discourse and what are learners' and teachers' efforts toward the necessary discursive transformation? She identifies specific cases in collaborative learning where the learners cannot manage a situation by themselves. In the end she arrives at one possible solution, that collaborative learning requires the active lead of an interlocutor and needs to be fuelled by a learning-teaching agreement between the interlocutor and the learners.

My interest is to explore discursive lead-taking and lead-following in collaborative learning in the current South African context. A further reason to undertake this study is my curiosity as a foreign teacher. As a teacher coming from Europe (specifically from Bulgaria), I received mathematical knowledge in a strongly structural way and practised teaching in the accepted traditional method of that period. It is not immediately obvious to me what it means for a teacher to organise her/his class for collaborative learning in mathematics, and also how a teacher learns this practice – becoming a *facilitator*. I thus ask the question: how are mathematics teachers implementing the new reform in the South African classroom? In relation with this, the focus of my study is **the new role of the teacher in the mathematics classroom.**

1.2 The research topic and research questions

This study adopts Sfard's theory in order to set up a theoretical framework to establish a better understanding of the role of the teacher in the mathematics classroom and particularly where collaborative learning is valued. In her theory of commognitive development Sfard distinguishes two types of learning – object-level and meta-level learning.

Together with Ben-Zvi (2007) in the article "Ariadne's Thread, Daedalus' Wings, and the learner's autonomy" object-level learning is described as a straightforward type of learning. They state that it is a product of logical necessity. Object-level learning increases the set of "known facts" (Ben-Zvi & Sfard, 2007: 6) about the investigating objects and the goal of this type of learning is to get better acquainted with the object, with the properties of the object and the mathematical narratives. Object-level learning leads simply to the extension of a discourse. For example, the commutative law of multiplication and addition are object-level rules (A + B = B + A and A x B = B x A). The principles that regulate this kind of routine are usually explicit. In this type of learning the discussant is able to discover the property of the mathematical operation – multiplication and addition; in object-level learning the participant is able to resolve the mathematical problems and make independent decisions.

In contrast, meta-level learning is distinctive from object-level learning; it is not a straightforward type of learning. Sfard (2008:161) considers mathematics as a "multi-layered recursive structure of discourses". This means that mathematics as a discourse consists of sub-discourses, which relate to each other in various ways: some are isomorphic, and some subsume others, while some are incommensurable. For example, in the discourse of positive numbers the addition rule that makes numbers bigger is mutually exclusive in the discourse of negative numbers. (2 + 5 = 7; (-2) + (-5) = (-7)). Brodie and Berger (2010:173) illustrate meta-level learning beautifully with an example:

> [t]he rational numbers discourse subsumes the whole-number discourse but some aspects of these two discourses are incommensurable: in the whole number discourse, multiplication makes numbers bigger while in the rational numbers discourse, this generalization does not always hold true.

The relationship that we are familiar with is no longer valid in the new discourse, the discourse of rational numbers. This type of learning that results in an incommensurable discourse is meta-level learning. Meta-level learning involves changes in rules and endorsed rules and mathematical laws of the old discourse may sound contradictory compared with the rules of the new discourse and may also be mutually exclusive. At the same time meta-level rules are difficult to discover; they are the result of historically sanctioned custom and are thus contingent rather than inevitable.

All these important differences in object-level and meta-level learning challenge learners and teachers in different ways. Exploring the learners' autonomy, Ben-Zvi & Sfard (2007: 25) make the conjecture that learner autonomy is possible only in object-level learning. They state that "[i]n the case of meta-level learning there is no room for the learners' autonomy". Looking at the problem from the learners' perspective the authors distinguish between the learners' roles in these two types of learning. Focusing on teaching the intention of my study is **the role of the teacher in object-level and meta-level learning**.

The following research questions will be considered:

1. How does a teacher mediate instruction during object-level and meta-level learning?
2. What enables and constrains her/his facilitative mediation in the case of Congruency in Grade 9?
3. What can we learn about the practical efficacy of Sfard's discourse theory?

It can be observed that technical language in mathematics education needs to be used before it (the technical language/term) is clearly elaborated on. Sfard's ideas of different types of learning are described in detail in Chapter 3; they are used here as they are important in motivating and framing the study.

1.3 Congruency discursive shift

In the topic Congruency in Grade 9 there is a discursive shift between object-level and meta-level learning. In previous grades the learners are able to spontaneously recognise that two triangles are 'the same'. They can even state that "[i]f two triangles are 'the same', then the three sides and three angles of the one triangle are equal to the three sides and three angles of the other triangle." The learners easily transfer the notion of equality from algebra to geometry. Therefore, at this stage they have to compare six pairs of equal measurements.

However, the type of learning changes when they need to answer the question: "[i]f we want to determine whether two triangles are the same or not, do we need to know all six measurements?" To distinguish between necessary and sufficient conditions for two triangles to be congruent is the next level in the development of geometrical thoughts. According to van Hiele's theory of level of thought in geometry (2004) the learners in grades lower than Grade 9 are at a descriptive level or Level 2 (more details are provided in Chapter 2). At this level, Analysis, the learners are reasoning about a geometric shape in terms of its properties but they do not understand the relationships between these properties and between different figures. For van Hiele this is Level 3. From Grade 9 the learners need to understand the four conditions for Congruency. Furthermore the learners need to construct their own proofs, which is Level 4. Sfard (2007: 599) accepts this description about thought levels in geometry in van Hiele's theory and transfers the idea to the commognitive perspective. She states: "van Hiele's levels may be interpreted as a hierarchy of mutually incommensurable geometric discourses".

Applying this statement, Congruency is a topic that involves mutually incommensurable geometric discourses differing in their use of words and mediators, in their process to endorse narratives and construct routines; these are key concepts in Sfard's theory of mathematical discourse and each is elaborated on later.

The table below compares two types of learning in the topic Congruency:

Table 1 Comparison of two types of learning

Variables	Object-level learning	Meta-level learning
Goal	Compare two triangles	Compare two triangles
Procedure	Spontaneous recognition	Four conditions for congruency
Words	'the same'; different	'corresponding', 'congruent', included angles
Visual Mediators	The length of the sides is indicated with numbers and the sides of angles with degrees	• For equal sides  • For equal angles 
Narratives	Six pairs of measurements	Three pairs of measurements
Routines	Visual comparison. Cut two shapes and put on top of each other to see whether they are the same or not.	Geometrical proof. Use standard form to record the solution.

Thus Congruency is one of the possible topics wherein teachers and learners practice two types of learning (like many others such as BODMAS, negative numbers, fractions and ratios).

In summary, the first chapter of this study explores the reasons for undertaking this study. My curiosity as a foreign teacher was inspired by, on the one hand the changes in values and principles in the South African Constitution parallel with conceptual changes of the subject (mathematics identified a new role of the teacher in mathematical classroom) and by how the new reform is applied in the mathematical classroom on the other hand. In the Introduction the three research questions that will be focused on in this study will be formulated and it will be shown that Congruency is a suitable topic for this research.

The first question of the research study is how the teachers mediate instruction; this important issue in classroom practice has a long history and with the reconceptualization of the role of the teachers there is no singular answer to the above question. In Chapter 2, the Literature review, the field of research and claims relevant to the issue of the mediation of the teacher in the reform curriculum will be elucidated; claims that reflect strong debate in this regard. In Chapter 3, the Theoretical framework, three positions of the nature of learning will briefly be discussed and the fourth learning theory – commognitive theory (the perspective that will be used in this study) will be focussed on. Chapter 4 discusses the methodology and research design of the study, the results are presented in Chapter 5 and Chapter 6 concludes the study by returning to Sfard's work and considering how this study informs and has been informed by her key insights and theoretical elaborations.

Chapter 2
Literature review

How teachers manage collaborative learning is a much debated topic in mathematics education. The dilemma of telling/non-telling becomes a key problem in the teaching process.

2.1 Research view against the transmission model of teaching

On one side several researchers criticize the transmission model of teaching in which the teacher stands in front of the class and imparts facts and procedures to learners. According to Lobato, Clarke and Ellis (2005: 103) the "teaching as telling" practice is undesirable because it

> (a) minimizes the opportunity to learn about students' ideas, interpretations, images, and mathematical strategies; (b)focuses only on the procedural aspects of mathematics; (c) emphasizes the teacher's authority as the ultimate arbiter of mathematical truth rather than developing the students' responsibility for judgments of mathematical correctness and coherence; (d) minimizes the possibility of cognitive engagement on the part of students; (e) communicates to students that there is only one solution path; and (f) represents premature closure of mathematical exploration.

Undermining the role of the teacher as a *knowledge provider* the National Research Council (in Smith, 1996: 394) states that "[i]n reality no one can teach mathematics. Effective teachers are those who can stimulate students to learn mathematics." Doyle, Sanford and Emmer (1983) examined learners' view on the 'academic work' in the traditional classroom and found that, the teachers may inadvertently mediate against the development of higher cognitive skills. In relation with this Confrey (1990) in her article "What constructivism implies for teaching" criticizes direct instruction. "[t]here has recently appeared an increasing amount of evidence that direct instruction may not provide an adequate base for student's development and for student use of higher cognitive skills"(Confrey,1990: 107). Other challenges to direct instruction come from research on misconceptions (Confrey, 1987, in Confrey 1990). According to Confrey teaching concepts as a form of communication is not a simple process of passing on the information. Considering the fact that teachers' and learners' view of mathematical ideas are quite different, the teacher needs to assist the learners in restructuring those views in order to become more adequate to acceptable mathematical cannons (Confrey, 1990: 109).

Furthermore Confrey (1990) developed an alternative model of instruction with six components: the promotion of learner autonomy, the development of reflective processes, the construction of case histories, the identification and negotiation of tentative solution paths, the retracing and group discussion of the paths, and the adherence to the intent of the materials. To increase the level of learners' autonomy the teacher insists that the learners be engaged with the problem questions, their answers and explanations and she/he emphasizes the importance of their contributions. The development of the learners' reflective processes is linked with three questioning strategies: asking learners to discuss their interpretation of the problems, to describe their methods of finding a solution and to define their answers. A construction of a case history is based on learners' performance over time: "[b]y interacting with the students primarily in one-on-one settings, the teacher was able to form a powerful model of the student's characteristic approaches to solving problems."(Confrey, 1990: 119) Reviewing the problem also provided the opportunity for reflection and developed learners' sense of accomplishment.

It can be concluded that, in this new form of teaching Confrey (1990) shares a commitment to the importance of an active view of the learners. Many researchers share the same opinion, which will be elaborated on in the next section.

2.2 New view of the roles of the teacher and the learner in the learning-teaching process.

Firstly, Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier & Hiebert (1997: 36) notes: "[p]eriodically, educational reformers have advocated presenting less information, shifting more responsibility to the students to search for or invent the information they need." Furthermore, Sfard (1994, in Cobb, 2000) states that the central metaphor of learners as processors of information has been displaced by learners' who are acting purposefully in an evolving mathematical reality of their own making. Brodie (2007) also agrees that learners need to participate in mathematical lessons to express their mathematical ideas. Further, she elaborates:

> Learners' talk is seen to be important because it (i) shows that learners are attending to the lesson; (ii) allows learners to express and clarify their own ideas; (iii) enables learners to share ideas with each other; and (iv) provides teachers with information about what learners know and don't know, and how learners are thinking and trying to make sense of ideas. Teachers are encouraged to make their lessons more learner-centred by encouraging learners to contribute to the lesson. (Brodie 2007:1)

Shifter (2001) and McNair (1998) are interested in reforming teaching actions and characterize a teacher as a 'facilitator' of mathematical discourse. They state that a teacher has to guide a well-chosen-problem-solving activity, rather than present the information. Wood (1995 in Lobato, Clarke & Ellis, 2005: 104) adds that "[t]he teachers should generate situations that stimulate children's mathematical activity, and should realize that substantive learning occurs through interaction, conflict, and surprise".

In relation to this, Wendel (1973) explains the positive side of open, inquiry-oriented teaching. In the article "The teacher's dilemma with the open classroom" Wendel introduces this teaching strategy as a fundamental model for facilitating individual growth. The teacher in this process delivers an open-ended learning activity and invites learners to think creatively; instead of explaining the common-known fact and procedure the teacher engages learners in participation in solving mathematical problems and designed learning activities take into account the learners' level and interest in a topic or concept. Wendel (1973: 185) states: “[t]he teacher’s role in this process is critical; he must facilitate questioning, be trusting and respectful, encourage cooperative problem-solving processes that humanize learning and help students to understand and accept the tentativeness of knowledge.”

In the end Wendel (1973:187) concludes:

> The teacher’s role is not one of explaining “content”, but one of guiding the group inquiry process toward reflective self-awareness using perplexing subject matter as a means for developing clearer insights into the nature of whatever subject is under consideration.

Wood and Turner-Vorbeck (2001) express the same opinion when they state that a teacher’s ability to tell can be reduced to what the learners have discovered, or the teachers should explain to the class the learners’ solutions.

Furthermore many projects and studies have been conducted with the common feature to reconceptualise the role of the teacher in ‘reformed’ classrooms. For instance, Wood, Cobb &Yackel (1990: 502) described a case study of one teacher who was “no longer the authority and sole source of knowledge whose role was to transmit information, but instead was actively involved with students’ learning by negotiating meaning with them.” Mathematics was seen as a “community project”. The learners and the teachers were fellow players, co-workers in the learning process. They respected each other’s thinking and worked collaboratively on the instructional activity. The teacher in this project acted as facilitator and encouraged learners to take responsibility for their own improvement.

The work of Lampert (1990) aimed to create “a community of discourse.” In her classroom, learners’ ideas were brought into the public forum and she attempted to sanction any intuitive use of mathematical principles. The Cognitively Guided Instruction (CGI) program is another example where teachers make decisions based on their knowledge of individual children’s thinking. The learners share their strategies for problems and the teacher relies on it to build on mathematical knowledge. Fennema, Carpenter, and Franke (1992: 1) claim that “the climate in a CGI classroom is one in which each person’s thinking is important and respected by peers and teachers.”

In the Middle Grade Mathematics Project (MGMP, 1988) researchers reported that more emphasis on open-ended questions focused on and valued what learners think.

In the teaching experiment at Whitnall High School in the Netherlands one teacher described the classroom environment as follows: “We had to listen to students, examine their work, and try to learn what they were thinking as they solved a problem… Communication becomes an integral part of classroom dynamics…” (De Lange, van Reeuwijk, Burrill & Bomberg, 1993)

The Reality in Mathematics Education (RIME) project for secondary school teachers in Australia was intended to assist teachers in building problem solving into their teaching practice. Exemplary lessons were developed by teachers and shared and improved in local networks of teachers. In the study 18 teachers were interviewed concerning their perceptions of the change in the teaching role ‘demanded’ by the use of these materials. The teachers encouraged learners to see themselves as 'fellow players' with the teachers. They also convinced the class that learners' answers were valued (Lowe & Lovitt, 1984).

All these different studies claim a variety of components of the new role of the teacher in a reformed classroom. Clarke (1997: 280) summarizes the components as the use of non-routine problems as the starting point, the adaptation of materials and instructions according to local context, the use of a variety of classroom organizational styles, the development of a "mathematical discourse community" with the teacher as "fellow player" who values and builds on learners solutions and methods, and the identification and focus on the big ideas of mathematics. What the teacher needs to do in the classroom adds important characteristics to the image of 'reform' teaching.

Increasing the possibility of cognitive engagement of the learners and providing the opportunity to learn about learners' ideas enables the teachers to put more emphases on lead-following as construct models of teaching. At the same time the attention of the teacher is moved to another action that was deemed to be marginal in the traditional way of teaching, but very visible in collaborative learning – listening.

2.3 Listening as a new action in collaborative learning

With regard to these new pedagogical actions Davis (1997) adds another role of the educator to the new model of teaching, as qualified listener. In the research project he recognizes three different types of listening: evaluative listening, interpretive listening and hermeneutic listening. In his description of each of these Davis explains the different ways of listening; for instance: **Evaluative Listening** is listening that evaluates the correctness of the contribution by judging it against a preconceived standard. The role of the teacher in this sort of listening is to assess the learners' answers. In this interaction, however, the teacher does not use enough of the learners' contribution and learners have feelings that their responses are not valued. In contrast, in **Interpretive Listening** the model of communication is replaced with awareness that an active interpretation is involved. Learners' articulations are increased dramatically. The type of questions posed in this type of discussion is information-seeking, not response-seeking. They require more elaborate answers and very often, some sort of demonstration or explanation. The teacher understands what the learners are saying and decides which answer is adequate, which is wrong and which requires clarification. Furthermore the interpretive listener summarizes the learners' work and elicits the new idea into the discussion. The listening becomes an equally vital component of the teacher's action. In this case the teacher "is not listening to assess the knowledge students have acquired, but to access the subjective sense being made" (Davis, 1997:365)

In the third type of listening, in **Hermeneutic Listening** the teacher becomes a participant in the exploration of a certain piece of mathematics. The manner of listening is completely different.

“[i]n this case, learning[is] a social process, and the teacher’s role [is] one of participating, of interpreting, of transforming, of interrogating – in sort, of listening” (Davis, 1997:371).

In conclusion, it can be summarized that in different situations during the lesson, the teacher can use different types of listening and this idea reflects the dynamics of discursive lead-taking and lead-following in collaborative learning. Furthermore, listening in a ‘reform’ way of teaching plays a beneficial role, in contrast to the marginal role it plays in traditional teaching. This new action is a key component in this analysis. It should be noted that time constraints can be reason for incorrect application.

2.4 Never *telling* is a misconception: Reformulating *telling* as initiating and eliciting.

In contrast Clement (1997), Cobb (1994) and Ernest (1995) in Lobato et al. (2005) note that, avoiding proactive behaviour such as *telling* is a misconception. To illustrate this Jaworski (1994:137) gives a solution to the dilemma of *telling/non-telling* when she explains that “learners will construct for themselves, whatever the teacher does”. To confirm the necessity of *telling* it is stated that“[learners] cannot be expected to reinvent [an] entire body of mathematics regardless of how well each concept is problematized by a well-chosen tasks.” (Clarke, 1994 and Romagnano, 1994 in Lobato et al., 2005:106)

Furthermore, looking at the practical side of reform teaching, Adler (1997) argues that, in a multilingual classroom, a participatory-inquiry approach to teaching and learning mathematics creates dilemmas of mediation for the teacher. She elaborates: “working to meet the dual goals of validating diverse pupils’ perspectives (which entails working with informal expressive language and learners’ conception) together with developing mathematical communicative competence (which in turn entails access to formal mathematical concepts) is extraordinarily complex with the time-space relation in the classroom” (Ibid.:236). Instead of leading the discussion in formal language of mathematics the teacher is shaping it in informal, sometimes incomplete and confusing, language using learners’ perspectives. Illuminating the case using an analytic narrative vignette Adler (1997) concludes that a participatory-inquiry approach can inadvertently constrain mediation of mathematical activity and access to mathematical concepts.

To confirm Adler’s’ finding Smith (1996) says that by focusing on what not to do, teachers are left with an inadequate model for how to move learners forward during times when learner-learner interaction fails to generate the ideas necessary for mathematical growth. Romagnano (1994, in Lobato et al., 2005: 105) states that *never telling* can engage learners at a superficial level.

In response to this Lobato et al. (2005: 102) conclude that *telling* is instructionally important and reformulate *telling* in the three ways: “…in terms of the *function* rather than the *form* of teachers’ communicative act… in terms of the conceptual rather than the procedural content of the new information… in terms of its relationship to other actions rather than as an isolated action.”(Ibid: 102)

Furthermore they reformulate *telling* as *initiating.* Lobato et al. (2005: 110) define *initiating* as the set of teaching actions that serve the function of stimulating learners' mathematical constructions via the introduction of new mathematical ideas into a classroom conversation. The goal of initiating is to provoke disequilibrium in learners' thinking. Instead of making declarative statements the teacher can describe a new concept, ask the learners for ideas to solve a mathematical task, summarize a learner's' work, generate counterexamples and "insert a new voice" (Ibid: 110) via questions and comments in order to change the direction of the discussion. All these actions focus on the conceptual rather than the procedural context. The function of initiation is going to be incomplete without learners' engagement with the new information. "[learners] should be conceptually able and motivated to make sense of the teacher's utterance."(Ibid: 111) By considering initiating as part of a system of actions, we focus our attention on the development of the learners' mathematics rather than on the communication of the teacher's mathematics.

Lobato et al. (2005) consider *initiating* not in isolation, but in conjunction with *eliciting.* Thus *eliciting* occurs when the teacher's actions serve the function of drawing out learner's images, ideas, strategies, conceptions and ways of viewing the mathematical situation. "Eliciting actions occur when the teacher arranges for situations in which students articulate, share, discuss, justify, reflect upon, and refine their understanding of the mathematics." (Lobato et al, 2005:112). The teacher elicits the new information from the learners' response.

Initiating and eliciting interact together, if a mathematical idea originates with the teacher initiating occurs, if an idea to solve a mathematical task originates with the learner, then the teacher is likely operating as an elicitor. In order to promote lead-taking the teacher has to use these two categories of action that are not in conflict but complement each other. Moreover, Lobato et al. (2005) believe that initiating is most profitably used in conjunction with eliciting.

2.5 Sfard's view of teaching.

Sfard (2007) also supports the idea of teachers' leadership in collaborative learning. In the article she builds a commognitive framework, which is elaborated on theoretically in Chapter 3. Briefly, for our purpose here, Sfard proceeds to answer the questions: what are the features of new mathematical discourse and what are learners' and teachers' efforts toward the necessary discursive transformation? After analysing two empirical studies she arrives at the conclusion that special cases in collaborative learning require an active lead of an interlocutor and needs to be fuelled by a learning-teaching agreement between the interlocutor and the learners. Sfard (2007: 607) argues "that proactive participation of the expert interlocutors is critical to the success of learning" (Ibid: 607). She also continues that for the communicational process the three aspects of the learning-teaching agreement are essential. The first aspect of this set of unwritten understandings is the issue of leadership in discourse. The teacher needs to be trusted and the discourse that he or she presents must be valued. Secondly, the learners need to show the acceptance of the roles of the discourse and show interest in the new discussion. Thirdly, Sfard (2007: 609) says: “If learning is to succeed, all participants have to have a realistic vision of what can be expected to happen in the classroom. In particular, all the parties to the learning process need to agree to live with the fact that the new discourse will initially be seen by the participating students as somehow foreign, and that it will be practiced only because of its being a discourse that others use and appreciate.”

Ben-Zvi and Sfard (2007: 7) take this idea further. They explain that “meta-level learning, can only happen in the process of scaffolded individualization: the student joins experienced discussant in implementing discursive task, acting first only as a spectator and then as peripheral participant.” In the process of learning the learners become more and more independent. Over a period of time, the type of discourse is transferred from meta-level learning to object-level learning. In this article Ben-Zvi and Sfard not only explain the process of learning in the meta-level. Using Sfard’s (2007) findings for teaching-learning agreement they also explore the relation between the quality of a learning-teaching agreement and the effectiveness of meta-level learning. In four classroom episodes the authors show that if one or more of the three components of a learning-teaching agreement are missing meta-level learning will not happen. For example, if the leading discourse is different from the one that is practiced by experienced authoritative interlocutors; or when there is no leading discourse; or when the leading discourse is in place but no expert support is available to the learners. Looking at the problem from the learners’ perspective they suggest that learners’ autonomy is possible only in object-level learning.

In summary, we can conclude that the dynamics of discursive lead-taking and lead-following in collaborative learning is a controversial topic in the mathematical community. Thus it will be interesting to investigate how South African teachers implement new reforms. To answer the research questions, namely how a teacher mediates instruction during object-level and meta-level learning and what enables and constrains her/his facilitative mediation in the case of Congruency in Grade 9, the van Hiele model of learning about shapes will be described.

2.6 The van Hiele Levels of Geometric Understanding

The best-known theoretical model for learning about shapes continues to be the van Hiele model (1986). Pierre van Hiele together with his wife Dina van Hiele-Geldof developed a theory involving levels of thinking in geometry that learners pass through as they progress from merely recognizing a figure to being able to write a formal geometric proof.

There are five hierarchical levels in the model:

Level 1, also known as the *Level of Visualisation*, in which learners recognize and name geometric shapes by comparing it with their prototype. At this level the learners are aware of the shapes as a whole and they do not perceive the property of the shapes.

In Level 2, the *Level of Analysis,* the learners recognize and name the properties of geometric figures, but they do not see the relationship between these properties. At this level the learners describe an object, but they do not discern which properties are necessary and which are sufficient to describe the object.

At Level 3, the *Abstraction level,* the learners perceive relationships between the properties of the object. They define the shapes based on these properties. At this level the learners start reasoning and understand some proof.

At level 4, Deduction, the learners understand the definitions and axioms and are able to use them and construct proofs.

At Level 5, Rigor, the learners can understand the use of indirect proof and proof by contra positive. They also understand non-Euclidean systems.

According to the van Hiele model, progress from one level to the next level is more dependent on educational experiences than on the age or maturation of the learner. In contrast to Piagetian ideas of the child's "natural" development, van Hiele is of the opinion that each level of thought is the result of instruction that is organised in five phases of learning: Information, Guided orientation, Explicitation, Free Orientation, and Integration. Van Hiele's position is commensurable with the communicational approach and the transition of his model into communicational terms will be explained in the next section.

2.7. A model of geometric discourse development

A new model is constructed on the basis of the foundational communicational tenets combined with van Hiele's ideas about the level of geometric thinking. Within the communicational framework "[t]hinking becomes a form of communication, whereas [the] level of thinking becomes levels of discourse" (Sinclair & Moss, 2012: 30).In the next paragraph these levels of geometric discourse will be discussed. The key to defining these three levels of geometrical discourse is the nature of *saming* (to be willing to give one name to a number of things that have not been considered as being 'the same' [Ibid: 29], for instance the images of two triangles, a long skinny triangle and an equilateral triangle) and the routine of identifying geometrical shapes and naming them.

The first level of geometric discourse is the discourse of elementary discursive objects (corresponds to van Hiele's Level 1): "[a]t this point, the routine of identification is purely visual." (Sinclair & Moss, 2012: 31). The learners need to recognize the permanence of the objects, the process similar to recognising different images of a human face as presenting the same person. The second level of geometric discourse is the discourse of concrete discursive objects (a counterpart to van Hiele's Level 2): "[a]t this level, transformability is still the only criterion for calling two things by the same name" (Ibid: 31). At this level the wide range of geometrical shapes hold one name called a family name. The third level of geometric discourse is the discourse of abstract objects. The transformability is no longer the 'official' reason for saying that two shapes can be called by the same name. Instead, such a decision is grounded in the recognition of a commonality of verbal descriptions of the shapes: "two shapes are considered as deserving one label because they fit the same verbal descriptions" (Ibid: 31).

The transition from Level 2 to Level 3, from visual recognition to discourse-mediated identification, is a slow process and Sinclair's and Moss's conjecture of the study is that software products such as the Sketchpad environment will speed up the process. Hence, the issue of mediation also becomes critical in the case of this study when the learners from Grade 9 need to move from Level 2 to Level 3 and then to Level 4.Therefore it is important to explore the role of the teacher in this process.

In the next chapter Learning (according to Sfard's perspective) will be theorized.

Chapter 3
Theoretical framework

There are various learning theories, namely the Cognitive, Socio-cultural, Situated and Commognitive theories. Coming from different angles they introduce four theoretical positions of the nature of learning and development. The Commognitive theory, the most recent theoretical view of learning was chosen for this study because it is engaged with mathematical context. At the same time knowing the historical development of learning theories helps us to see that Sfard's theory is not a new theory but a discursive perspective of Vygotsky's ideas.

3.1 Review of the Cognitive, Socio-cultural, Situated and Commognitive theories.

3.1.1 The Constructivist/ Cognitive Theory

The constructivist theory explains cognition as a form of adaptation between an organism and the environment. Piaget, the father of the constructivist learning theory, shows similarities in the process in which the person understands the world around him/her and the process of adaptation of the individual to the surrounding environment. For Piaget (1964) *adaptation* in cognition is preceded by means of *assimilation*; it is the integration of any sort of reality into a structure. He argues that knowledge in not just a copy of the living word transmitted from the teacher to the learner; the essence of knowledge is action and in order to obtain knowledge the learner must act on it. This means that new facts taught by the teacher have to be modified and transformed by the learner, in order for the learner to form structures which can enable her/him to understand.

Other constructivists Smith, Disessa and Rochelle (1993) interpret knowledge not in terms of the presence or absence of a single element but they discuss knowledge as systems composed of many interrelated elements. To compose such a system Piaget (1964) introduces three levels of assimilations: "[t]he first is assimilating object to schemes,…the second is assimilation between different schemes… and the third and highest level is assimilation between subschemas and the totality which integrates them into a coherent whole" (Meadows, 2004: 138). Hatano (1996) elaborates on construction and reconstruction and argues that to reconstruct includes interpreting, enriching and connecting to prior knowledge. Once the knowledge is assimilated it now has to be accommodated into these old existing structures (which can bring about that the old structure must be reorganized into a new structure). Meadows (2004) claims that the development into the new structure happens under pressure and that it is the result of new information or problems with such information or from pressure of internal contradictions by incompatible structures.

This process is driven by the need to reach equilibrium (Piaget, 1964). Piaget explains equilibration as a force for stability, via a self–regulation that balances external and internal changes. He perceived it as the most fundamental factor for evaluation of biological structures. Defined as an active compensation, the first constructivist belief is that the equilibrium must be relevant for cognitive development. The learner tries to make sense of the unknown knowledge; when he/she confronts it, it leads to the conflict that needs to be resolved. The solution of the conflict reaches equilibrium. Meadows (2004:140) confirms this with the following explanation: "[t]he changes and demands of the outside world produce in the thinker small 'perturbations' or 'conflicts' which leads the cognitive systems to small automatic adjustments (of assimilation and accommodation) to cope with the conflicts and return either to the original cognitive equilibrium or to a new and better one."

Piaget (1964) concludes that learning of structures obey the same laws as the natural development of these structures; the first constructivist claims that assimilation is a relationship in two processes, development and learning. Equilibration, the most important factor that explains development, is also a factor in the learning process. Lastly Piaget (1964) states that learning is subordinated to development.

3.1.2. The Socio-Cultural Theory

In contrast with Piaget, who based cognitive development on the logical necessity of operational thought, Vygotskian theory rests on the social factors in learning and development, on the interaction between the social environment and individual. All new concepts – internalization, mediation and the Zone of Proximal Development (ZPD) – describe cognitive development based on social factors.

Instead of seeing cognitive development by building individual construction Vygotsky, the Russian psychologist places emphasis on social interaction. According to Vygotsky (1978) the adult who is competent in solving specific problems can help the learners to reach the solution successfully by providing guidance and support. This is done step-by-step in order for learners to eventually become independent. At the start the adult provides almost all the cognition necessary for solving the task but in the process of interaction the learners' understanding develops. The increasing competence of the learner reduces the assistance of the adult. Eventually adults' support is removed and the learner is able to solve the activity alone. This practice based on the social interaction is called "scaffolding".

Furthermore Vygotsky (1978) introduces the new concept of internalization. He formulates internalization as an internal reconstruction of an external operation and describes the stages of the transformation process; external activity becomes internal activity; an interpersonal process becomes an intrapersonal process. According to Vygotsky "[a]ny function in [a] child's cultural development appears twice or on two planes. First it appears on [the] social plane and then on the psychological plane. First it appears between people as an inter-psychological category and then within the child as an intra-psychological category" (Vygotsky, 1981, in Meadows 2004:166). This notion is taken by Sfard and it is developed in her theory as will be explained in section 3.2.2.

The most powerful concept in Vygotskian theory is the Zone of Proximal Development (ZPD): "It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers"(Vygotsky:1978: 86).The valuable thought of this notion is that Vygotsky recognizes the possible potential of the learner and the level that they have to be taught in: "Children can imitate a variety of actions that go well beyond the limits of their own capabilities." (Vygotsky: 1978: 88).

With emphasis on the importance of learning with and from other people, Vygotsky's theory has its core in the notion of 'mediation'. Meadows (2004: 171) confirms Vygotsky's view that "[m]ediation, or the use of communicable systems for representing reality as well as acting on it, is at the foundation of cognitive processes". With regard to this notion Vygotsky describes two terms, 'tool' and 'sign'. Signs are products of the history of the culture; it can be whole collections of symbols or ways of problem-solving which shape our thinking. A good example for a sign is a knot tied in a piece of string.

Vygotsky, the Russian psychologist, focuses on language and argues that it is the most important 'psychological' tool: "[l]anguage, for example, changes the relation of human beings to each other and to the non-human world from what those relations are in those who cannot use language" (Vygotsky, 1981 in Meadows: 2004: 171). Vygotsky perceives speech as the beginning of social contact in childhood. Expressing the emotions is the first step to show social communication and it is followed by the use of language to represent ideas, to explain actions and to share knowledge. Integration with other psychological tools such as counting systems, techniques, writing diagrams and maps take cognitive processes to a high level.

Sfard adopts all these notions in different ways, some of them are used as is (e.g. internalization), some are rearranged (e.g. tools and sign) and some create a strong link between mediation and commognition. A detailed description will be provided in section 3.2.

3.1.3 The Situated Theory

Lave (1993) considers learning not as a process of socially shared cognition that results in the end in the internalization of knowledge by individuals, but as a process of becoming a member of a sustained community of practice. Hanks (1991) states that when working with learning, the focus must be on the relationship between learning and the social situation in which it occurs. Her view is compared with the cognitive perspective; the question is not what kind of cognitive processes and conceptual structure are involved, but what kind of social engagements provide the proper context for learning to take place. In conclusion, while the previous two learning theories are interested in the individual mind (what goes into the mind), the socially situated theory shifts the focus between the individual and the others, and explains learning as "legitimate peripheral participation …of communities of practice" (Lave, 1993: 64). The mechanism is participation in community and thus it is a distributed view of cognition.

The new terms (oldtimer, newcomer, full participants, and legitimate peripheral participant) show the social relation of each person in these communities. A newcomer develops knowledgeable skill and a change in understanding of practice, when participating in on-going activity of the community. When the cognition is complete and identity as a practitioner formed the newcomer becomes a full participant, an oldtimer. Newcomers and oldtimers depend on each other and the relationship is complicated. Hanks (1991) states, that legitimate peripheral participation (LPP) is not a simple participation structure, in which an apprentice occupies a particular role in the process; each apprentice has several roles, implements different types of responsibilities, has different interactive involvement. Lave (1993) describes two planes of LPP: on the one hand it develops a person (local plane) and on the other hand (in the global plane) it develops a community of practice.

Brown, Collins and Duguid (1989) argue that knowledge is situated and progressively developed through activity. They state that activity, concept and culture are dependent on each other and learning must involve all three. In connection with this they propose an alternative model of teaching, namely cognitive apprenticeship, which honours the situated nature of knowledge. Apprenticeship tries to enculturate learners into authentic practice through activity and social interaction in a way similar to that evident and evidently successful in craft apprenticeship. Brown et al. (1989) explain that people adopt the behaviour and beliefs of a social group. When in a social group with practitioners it can enable a person to become a practitioner, being an apprentice provides the opportunity to adopt the behaviour of the practitioner until the apprentice can become a practitioner and an apprentice develops concepts out of and through authentic activity. According to Brown et al. (1989) the cognitive apprenticeship model stands in contradiction to current school practice; schools prepare learners to participate in the culture of schooling, how to write exams, behave in class, but they are not prepared for what practitioners, for example mathematicians or scientist, do.

The situated perspective focuses on the activity in the communities of practice. This activity helps an apprentice on LPP to move from peripheral participant towards full participant and increasing participation will assist the developing process.

In conclusion, while the constructivist learning theory, named the acquisitionist theory, is interested in what goes into the human mind, the situated theory or the participationist theory, focuses on social engagement in the learning process. Brodie and Berger (2010:173) clarify:

> While the differences between acquisitionist and participationist theories may seem small, in fact participationist theories radically shift understandings of learning. Participationist theories do not merely claim that learning happens through participation, as do acquisitionist theories …, but that learning is participation.

3.2 Commognitive Theory

In this section Sfard's theory of commognition will be elaborated on. Why it is the appropriate orientation to frame this study and enable engagement with these research questions will be illuminated.

3.2.1 Early publication of Sfard

New technologies afford opportunities to the researcher to record social phenomena as teaching. With audio and video-recorders the study can produce high resolution evidence of the complexity of classroom discussion. However, quality documented conversations are not sufficient to gain insight into human phenomena, such as teaching and learning. Thus Sfard (2007) introduces a powerful theoretical apparatus that is applicable for all school subjects. She states that "we need an analytic lens that extends our field of vision so as to include both the 'how' and the 'what' of teaching and learning"(Ibid: 568). Sfard started developing a new learning approach in 1998 in the paper "On two metaphors for learning and the dangers of choosing just one". She conceptualizes the notion of learning using two metaphors, the acquisition metaphor (AM) and the participation metaphor (PM). Her goal is to answer the question "[w]hat is this thing called learning?"(Sfard, 1998: 7). According to Sfard (Ibid: 5) "[t]he language of 'knowledge acquisition' and 'concept development' makes us think about the human mind as a container to be filled with certain materials and about the learner as becoming an owner of these materials." Two main points here are knowledge as a commodity and learning as an act of gaining knowledge and becoming an owner of cognition. The key words generated by the acquisition metaphor are concepts, notions, misconceptions, context and meaning and the researchers have offered different mechanisms through which mathematical concepts can turn into the learner's private property: firstly by actively constructed knowledge (Piaget's theory); secondly by an internalized process where the knowledge, initially interpersonal, becomes intrapersonal and a never-ending process of emergence in a continuing interaction with the teacher, competent peer or textbook. The second metaphor is the participation metaphor. It explains learning as participation, 'taking part and being a part'. The author states that learning is a process of becoming a member of a certain community, the ability to communicate in the language of this community and act according to its particular norms (the work of Lave and Wenger [1991] is an example of participationist theories).While the AM is interested in the individual mind and what goes into the mind, the PM shifts the focus between individuals and others. Sfard (1998) explains the nature of learning interaction as a part-whole relation, and emphasizes that the *whole* and the *part* affect and inform each other and the *whole* is fully dependent on the *parts.* Furthermore, she makes associations with different organs combined to form a living body. In the same way the learners contribute to the existence and functioning of a community of practitioners.

In closing Sfard recommends that both metaphors be used and that the rejection of any one particular metaphor can lead to distortions and undesirable practice. Sfard (1998:11) states that "[a]n adequate combination of the acquisition and participation metaphor would bring to the fore the advantages of each of them, while keeping their respective draw-backs at bay…It seems that the most powerful research is the one that stands on more than one metaphor leg."

However, in later research Sfard adopts a more radical approach and presents a participationist approach against the traditional acquisitionist approach. The origins of participationism can be traced to acquisitionists' unsuccessful attempt to deal with the long-standing dilemma about human thinking. Participationism appears to provide a more reasonable explanation to the learning paradox "[l]earning a new thing is inherently impossible." "How can we want to acquire a knowledge of something that is not yet known to us?" (Sfard, 1998: 7) While acquisitionists struggle to solve this contradiction, the participationists reconceptualise the view of human development by extending the boundaries of individual life. They shift understanding of human development from the development of an individual to the development of the collective. Sfard (2007:571) states that the developmental transformation is "[t]he result of two complementary processes, that of individualizations of the collective and that of communalization of the individual." These two processes are interrelated and as a result is a bi-directional transition. On one hand learning to speak and to solve mathematical problems is a transition from being able to take part in a collective implementation of a task to becoming capable of implementing such a task individually. On the other hand the collective activities are the primary model for the individual form of acting, where individuals contribute into the collective forms of doing the activities. This reconceptualization of human development resolves the problem of historical change in human forms of doing, at the same it time brings a new view of basic terms such as thinking, mathematics and learning, which will be discussed later.

Another advantage of participationism is that it does not consider knowledge as a commodity, in which case people can be drawn apart. Sfard (1998:8) says that "[k]nowledge and material possessions are likely to play similar roles in establishing people's identities and in defining their social positions." Giving the power and privileged position to the people in society separate them instead of bringing them together. Participationism shifts the talk about private possessions to discourse about shared activities and the vocabulary of participation brings the democratic message of togetherness and collaboration.

The participationist version of human development is a new lens that undermined the biological makeup of the individual and emphasises effective human communication. These perspectives also link to the new South African curriculum and its values for collective activities. This approach views all the uniquely human capacities as resulting from the fundamental fact that humans are social beings, engaged in collective activities from the day they are born and throughout their lives.

Following this direction the role of the teacher becomes more important and in the teaching process the question ‘how the teacher mediates learning’ is critical.

3.2.2 Basic Commognitive Tenets. What is thinking?

One of the basic commognitive tenets is that thinking is individualization of interpersonal communication and that human skills are products of individualization of historically established collective activities. According to Sfard (2007: 571) young children develop the ability to speak, read or cook by gradually turning from peripheral participants who can only implement small parts of the job in collaboration with others into independent performers, who can do the task on their own. Thinking as an inherently human activity has developed as another form of human’s doing from collective activity. In fact, human thinking can be defined as an individualized form of the activity of communication. Treating thinking as a special type of communication, self-communication, is a big shift with the general acceptable view that thinking grows from inside the person and it is biologically determined. Therefore, thinking is no longer a self-sustained process; it is an act of communication. The main point in this definition is that thinking has a dialogical nature.

Sfard has been inspired by Austrian-British philosopher Wittgenstein, who believes that “thought is not an incorporeal process which lends life and sense to speaking, and which it would be possible to detach from speaking”(1953, in Sfard, 2006: 159). Sfard takes this claim and replaces the limiting word *speaking* with the more general term *communicating*. In this way she proposes to combine the terms *cognitive* and *communicational* into the new adjective *commognitive*. The new word *commognitive* includes the term cognition, but at the same time is associated with interpersonal exchanges. Here the link between Vygotsky’ term of mediation and Sfard’s term of cognition can be established because both notions are associated with the communication system during the cognitive process.

Furthermore Sfard explains that communication is a collectively-performed, rules-driven activity that mediates and coordinates other activities. Individuals perform actions followed by a certain type of reactions and the reactions can be either practical actions or communicational moves. For example, the instruction in a mathematical classroom ‘simplify the expressions’ involves the learners in practical actions. They start to apply the rules that they are familiar with in order to implement the operations while the question ‘[w]hat is the relationship between these two variables, a and b?’ engages the learners in a meaningful discussion. In this case, in a long chain of communicational interaction, the participants play both roles of actor and reactor. In classroom practice there is no order of practical action and communicational moves.

The definition of communication speaks about the rules regulating interaction. Communicative actions, both action and reaction, are a matter of construction and are performed according to rules that constrain but do not dictate. For instance, when one discussant talks the rest of the participants listen. At the same time the participants need to know the commognition rules.

The dynamic of these rules is complicated, sometimes naturally but mostly historically established. The issue of commognitive rules will be clarified in the discussion on the two types of learning.

Considering that the goal of Mathematics teaching and learning practice is a developing learners' cognitive thinking, the definition described above plays an important role in this study. The development depends on the effectiveness of human communications: more specifically the teacher's mediation in classroom is an important issue for learners' mathematical development.

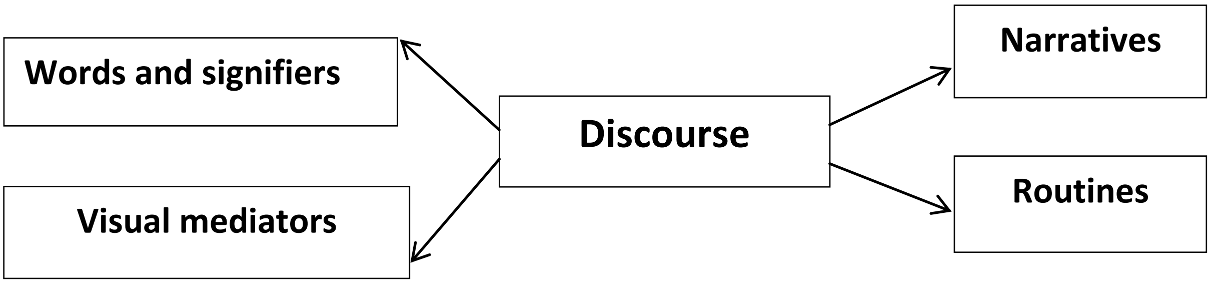
3.2.3 What is Mathematics?

Sfard (2007: 573) provides a definition of discourse: "[t]he different types of communication that bring some people together while excluding some others." From the commognitive perspective, thinking is a special form of communication and mathematics is a "multi-layered recursive structure of discourse-about-discourse, and its objects are therefore in themselves discursive constructs" (Sfard, 2008: 161). Berger (2010) explains this definition and says that mathematics is a discourse consisting of sub-discourses, which relate to each other in various ways; some are isomorphic; some subsume others, while some are incommensurable. An example for isomorphic discourses is discourses of positive and negative numbers. In positive number discourse addition makes the number bigger, while in the negative number discourse this generalization is not valid even if the opposite makes the number smaller (3 +5 = 8; (-3) + (-5) = (-8)).

This view of discursive learning is significant in Sfard's theory. In this specific frame she makes us see the explicitly essential necessity of an experienced discussant. Firstly, a crucial feature of mathematical discourses is that each discourse creates its own objects, which most often are only used in that particular discourse. For instance, in Trigonometry *sine* and *cosine* ratios are used mostly in this discourse and the only way to come to know such objects is through talking and thinking about them as two parts of the same process, commognition. Learning to communicate by communicating involves the imitation of more experienced interlocutors and in so-doing we develop ways of talking and thinking in this specific discourse. Secondly, in these collaborative activities, Sfard distinguishes that learners are involved in two types of learning – object-level and meta-level. Very often the narratives of a new discourse are mutually excluded from old-well known discourses. Based on this important feature of the process of learning the intention of this study is to determine how the teacher mediates instructions during these two types of learning.

According to Sfard (2008) a mathematical discourse is characterized by four key concepts that are represented in the diagram below.

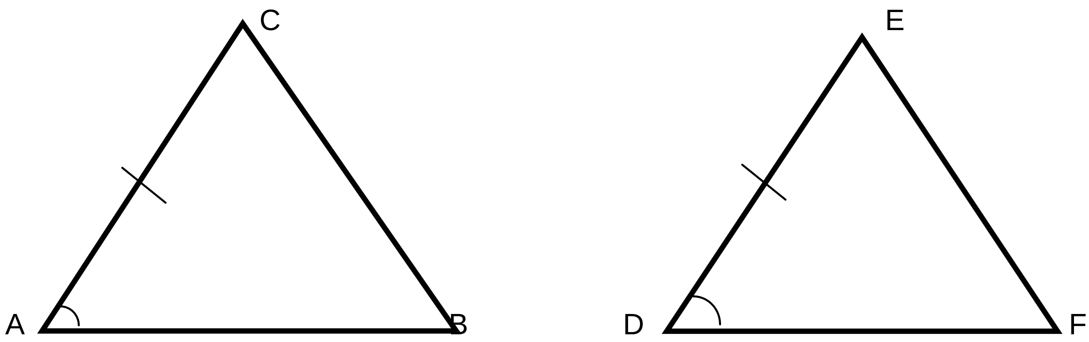
Diagram 1 Four key concepts of mathematical discourse



Firstly, mathematical discourses use mathematical words, related with quantities and shapes, for instance negative numbers, squares and pentagons. The participants can use words that they know from every-day situations but also need to add terms that they have never used before such as *logarithm* and *sine* and *cosine*. Simultaneously, well-known words from real life can appear with different meanings in mathematical discourse for example, ‘supplementary’ and ‘complementary’. It is essential for participants to understand and properly use discursive terminology.

The second feature of mathematical discourse is visual mediation. The participant of mathematical discourses is involved with symbolic artefacts such as mathematical formulae, graphs, drawings and diagrams. These symbolical presentations are always part of mathematical conversation. For instance, in the topic Congruency the participants need to read geometric schemas (e.g. Diagram 2); they need to understand the visual representations “one corresponding side of each triangle is equal” (e.g. AC=DE) and “one corresponding angle of each triangle is equal” (e.g. $\sphericalangle C = \sphericalangle F$) when they are engaged to prove congruency.

Diagram 2 Geometric schema



Sfard (2007: 574) provides a definition for the third characteristic of discourse: “*Narrative* is any text, spoken or written, that is framed as a description of objects, or of relations between objects or activities with or by objects, and that is subject to endorsement or rejection, that is, to being labelled as true or false.” She further specifies that “[i]n the case of scholarly mathematical discourse, the consensually endorsed narratives are known as mathematical theories, and this includes such discursive constructs as definitions, proofs, and theorems.”According to Sfard (2008) mathematical narratives can be divided into object-level and meta-level. The examples are again from the topic Congruency. The statements, *congruent figures differ only in position or orientation in space* or *we can test whether figures are congruent by sliding, rotating or flipping them until they fit exactly on top of each other,* are object-level narratives. This property of the congruent figures is straightforward; it is a product of logical necessity. It is a narrative without contradiction with any fact related with congruency. The opposite narrative, *if in two triangles the lengths of the three sides in one triangle are equal to the lengths of the three sides in the other triangle, the triangles will be congruent,* is a meta-level narrative. This condition of congruency is mutually excluded from the well-known narrative that *figures are congruent when all sizes and all angles are the same*.

Routines, is the fourth characteristic of mathematical discourses; they are well-defined repetitive patterns in an interlocutor's actions. Routines include mathematical procedures and mathematical practices such as looking for similarities and differences, methods of proving and generalization. Two key aspects of routines are important to consider: how and when routines are generated. The routines are categories that partially overlap with the other three characteristics (words, visual mediators and narratives) and these repetitive patterns are found in almost every mathematical discourse. In other words to practice the routine – proving congruent triangles – you need to know the mathematical meaning of the words 'congruent' and 'corresponding', visual mediators such as in Diagram No 2 and the condition for congruency, which is narrative.

In this particular study how the teacher introduces words and visual mediators in the new mathematical discourse – Congruency – will be observed. Furthermore this study is interested in how the teacher as facilitator will mediate instructions in the teaching process of solving geometric problems, is he/she taking the lead in order to show the standard narratives and routines or giving the chance to the learners to work on activities and so develop narratives and the routines themselves.

3.2.4 What is learning?

From the commognitive perspective "[l]earning mathematics is individualizing mathematical discourse, that is, as the process of becoming able to have mathematical communication not only with others, but also with oneself" (Sfard, 2007:575). The participants are familiar with everyday discourse and learning mathematics seems to be transforming these spontaneous discourses into scientific ones. This notion of learning defines learning as becoming a better participant in practice or discourses.

In the context of learning however, Sfard is not the creator of this idea as the roots thereof lie in theories like Lave and Wenger's Social Practice Theory (1991), Mercer's Theory of Practice (1995) and Vygotsky's Sociocultural Theory (1978, 1986).

For Lave and Wenger, becoming knowledgeable means becoming a full participant in the practice, and this involves, in part, learning *to talk* in the manner of the practice. They divide learning *to talk* into *talking within* and *talking about* practice (1991: 109).

Mercer (1995:82) also classifies discourses, produced in the context of schooling. He distinguishes between *educational discourse* (the discourse of teaching and learning in the classroom) and *educated discourse* (new ways of using language, 'ways with words' which will enable pupils to become active members of wider communities of educated discourse).

Mercer explains the role of the teacher as follows:

> Teachers are expected to help their students develop ways of talking, writing and thinking which will enable them to travel on wider intellectual journeys, understanding and being understood by other members of wider communities of educational discourse; but they have to start from where learners are, to use what they already know, and help them go back and forth across the bridge from 'everyday discourse' into 'educated discourse' (Mercer, 1995: 83).

While Lave, Wenger and Mercer distinguish discourses on a superficial level Vygotsky recognises different kinds of concepts that learners can understand during the learning process. The father of Sociocultural Theory talks about 'scientific' concepts and 'spontaneous' concepts; for Vygotsky, scientific concepts are part of a system of concepts, and they are deliberate and self-conscious. Spontaneous concepts are inundated with experience and are not systematic. These two concepts, scientific and spontaneous, interact with and influence each other:

> One might say that the development of the child's spontaneous concepts proceeds upwards and the development of his scientific concepts downwards, to a more elementary and concrete level… The inception of a spontaneous concept can usually be traced to a face-to-face meeting with a concrete situation, while a scientific concept involves from the first, a 'mediated' attitude towards its object (Vygotsky, 1986: 172).

In order to operate with scientific and spontaneous concepts, Vygotsky introduces the well-known notion of the Zone of Proximal Development (ZPD). In conclusion Vygotsky highlights the necessity of scaffolding educated discourse in the ZPD.

How the teacher mediates instruction in a South African mathematics classroom will be explored by transferring the same idea of different discourses in the learning process. Changing the discourse, moving from old discourse to new discourse (and vice versa) is a challenging task for the teachers. Using the commognitive perspective this study investigates the role of the teacher in two types of discourses.

The picture of basic commognitive tenets will be incomplete if the commognitive conflict is not described. In the next section this term in Sfard's theory will be clarified.

3.2.5 What is commognitive conflict?

Commognitive conflict is "a situation in which communication is hindered by the fact that different discussants are acting according to different meta-rules" (Sfard, 2007:576). The source of the conflict is the fact that different participants endorse contradicting narratives and this conflict is defined as the phenomenon that occurs when conflicting narratives come from different discourses (such discourses are called incommensurable). Sfard (2007) states that commognitive conflict is the main opportunity for meta-level learning and continues that this type of conflict is significant for learning while the cognitive conflict is optional and useful only when learners display misconceptions. To emphasise the importance of commognitive conflict she found two more differences between cognitive and commognitive conflict – in the “locus of the conflict” and in resolving the conflicts. While in cognitive conflict the learners hold on to contradicting beliefs about the word, the commognitive conflict is defined in incommensurable discourses. The cognitive vision of conflict resolution is grounded in non-contradicting principles, incompatible discourse. “Commognitive conflict, in contrast, is defined as the phenomenon that occurs when seemingly conflicting narratives come from different discourses”. There are many examples of commognitive conflict in mathematical practice such as in algebra (two rules of operation from left to right and BODMAS) and in geometry (two meanings for the terms ‘congruency’ and ‘corresponding’).

This type of conflict will be the object of observation and the manner in which the teacher will resolve it will be analysed.

In summary, after a brief explanation of the three theoretical perspectives of learning − the Cognitive, Socio-cultural, Situated Theories – the latest perspective of learning theory, the commognitive theory, was introduced. Simultaneously the thought development of Sfard, the creator of this theory, was followed. Furthermore the basic tenets such as thinking, learning, mathematics, the four key concepts of mathematical discourse and commognitive conflict have been described in detail. This mathematical orientated framework is the most suitable framework for collaborative learning.

Chapter 4
Methodology and design

4.1 Methodology

The object of this study is a social phenomenon; how the teacher needs to teach the learners in order to promote learning. The interpretation of the social phenomenon links with qualitative research. Silverman (2001:41) states that, a researcher does not need to “follow a purely statistical logic in order to replace commonsense understandings by scientific explanations.” And then continues: “[q]uantitative research would simply rule out the study of many interesting phenomena,relating to what people actually do in their day-to-day lives.” (Ibid.: 43)

A case study, according to Cohen, Manion and Morrison (2000:79) enables the researcher to “[c]atch the complexity of behaviour” by an in-depth study of specific instances. Merriam and Simpson (1984: 95) have the following to offer: “[a] case study tends to be concerned with investigating many, if not all, variables in a single unit. By concentrating upon a single phenomenon or entity, this approach seeks to uncover the interplay of significant factors that are characteristic of the phenomena.”

The focus of this study is to investigate the actual teaching process. A case study constituted from two teaching practices on one topic, Congruency, at a College in Johannesburg (which is equivalent to high school in South Africa) will be presented and will be considered. I consider the work of two teachers, not for controversial reasons or to compare their ability. The purpose of observing and interviewing two teachers on the same lesson is to obtain a greater variety of conversation on object-level and meta-level learning. At the same time analysing their teaching processes in-depth creates an opportunity to have different possibilities of mediating collaborative learning. Thus, choosing only one teacher leads to an insufficient amount of data and it is somewhat risky. However, choosing more than two teachers creates a situation with a large amount of complex data, which is impractical and beyond the scope of this research report.

This study aims to address the three research questions through two related activities – non-participant observation and semi-structural interviews with teachers (in order to provide an opportunity for teachers to express their opinion).

To conduct the observation a teaching experiment in collaboration with teachers was used. This particular type of teaching experience – a **classroom teaching experiment** – was used because it allows the exploration of the teachers’ mediation in a natural setting. While a prime purpose for using this teaching experiment methodology is for researchers to experience learners’ mathematical learning and reasoning, one of the possible goals of a classroom teaching experiment is to investigate the teachers’ beliefs and instructional practice. Compared to the constructivist teaching experiment where the researcher acts as a teacher and usually interacts with learners either one-on-one or in small group, the classroom teaching experiment is designed at classroom level in collaboration with the practicing teachers who are members of the research and development team. In other words, in this type of experiment the observed teachers and researchers are all responsible for the quality of the learners' mathematical development.

The second reason for choosing this methodology is the theoretical framework of this research study.

> The strength of design studies lie in testing the theories in the crucible of practice; in working collegially with practitioners, co-constructing knowledge; in confronting everyday classroom, school, and community problems that influence teaching and learning and adapting instruction to these conditions; in recognizing the limits of theory; and in capturing the specifics of practice and the potential advantages from iteratively adopting and sharpening theory in its context. (Shavelson, Phillips, Towne and Feuer, 2003: 25)

Sfard shifts understanding of human development from the development of an individual to the development of the collective. As noted Sfard (2007:571) states that the developmental transformation is "[t]he result of two complementary processes, that of individualizations of the collective and that of collectivization of the individual"; these two processes are interrelated and as a result is a bi-directional transition. Furthermore Eisenhart (1988) elaborates on this idea by considering the development of cognitive skill as central to human development. Thus, this new participatory view in mathematical learning, reconceptualised in Sfard's theory, is the one reason for conducting the classroom teaching experiment.

Another reason why the classroom teaching experiment is appropriate for my research study is that it is generally acknowledged that the classroom is the primary learning environment for teachers to show their beliefs and instructional practice. Cobb (2000: 312) concludes that teachers' mediation, as it occurs in a social context, can become a direct focus of investigation in a teaching experiment. Additionally, the teachers who participate in the teaching experiment can illustrate effective reform teaching. This can make an important contribution to reform teaching.

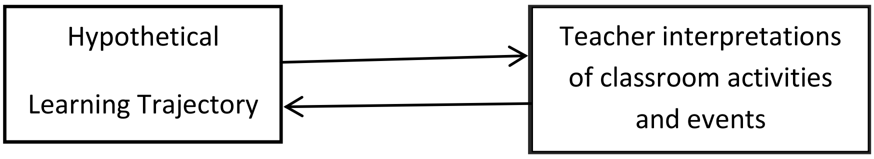
Cobb (2000: 333) confirms the right methodological choice as follows: "the classroom teaching experiment allows researchers to address some issues better than others"; especially investigating the implications of reform as they play out in interactions between teachers and learners in classrooms.

The process of conducting a classroom teaching experiment can be characterized by the developmental research cycle as described by Gravemeijer (1995). This cycle consists of two aspects:

4.1.1 First aspect of the cycle

The first focus is concerned with instructional development and planning and is guided by an evolving instructional theory. This cycle is similar to the mathematical teaching cycle developed by Simon (1995).

Diagram 3 Mathematical teaching cycle



A classroom teaching experiment begins with defining mathematical learning goals and with a thought experiment in which the research team envisions how the teaching-learning process might be realized in the classroom (Gravemeijer, 1995). The development of a hypothetical learning process is also included in planned learning and instructional activities. It is important for teachers to see the possible process of learning in an assumed classroom micro-culture. The instructional theory outlines three tenets when planning a teaching experiment and designing instructional activities. The first tenet is that the starting point of instructional sequences should immediately engage learners in meaningful mathematical activity (Streefland, 1991). The second tenet is the potential ending point of the learning sequence. Ball (1993, in Cobb, 2000) states that in educational reform building bridges between the experiences of the child and the knowledge of the expert is important. The third tenet in instructional sequences should contain activities in which learners create and elaborate symbolic models of their informal mathematical activity.

4.1.2 Second aspect of the cycle

The second aspect involves the on-going analysis of classroom activities and events, and is guided by an interpretive framework. According to Simon (1995: 133) “the only thing that is predictable in teaching is that classroom activities will not go as predicted.” Thus the interpretive framework for analysing individual and collective activity at the classroom level provides a way of coping with the messiness or disorder and complicity of classroom life. The framework contains classroom social norms as well as socio-mathematical norms.

The classroom social norms are not specific to mathematics; they are social constructions of participation. Examples of such norms for whole-class discussion are explaining and justifying solutions, indicating agreement and disagreement and questioning alternatives in solutions. The teachers and learners know their authority in action.

In contrast socio-mathematical norms constitute norms in mathematical activities. These norms help to judge, reject or accept mathematical contributions.

4.2 Generalizability and Trustworthiness

The theoretical analysis is a result of a complex, purposeful, problem solving process. Therefore, one would not expect that different researchers would develop similar theoretical constructs when analysing a data set created in the course of a teaching experiment. This implies that the notion of replicability is not relevant in this context. The relevant criteria are generalizability and trustworthiness of the analysis.

The importance is the view that presents classroom events as a broader phenomenon. The theoretical analysis starts with the understanding of one case and moves to the interpreting of another case. Thus, what is generalized is a way of interpreting. The notion of trustworthiness is concerned with the reasonableness and justifiability. This requires a systematic analytical approach. It is important to document all phases of the analysis process. Then, final claims can be justified by theory being used as a framework for the study.

4.3 Collaborating with teachers

The type of classroom teaching experiment that was conducted entails that the researcher collaborates with two teachers. Cobb (2000:330) states: “The researchers who collaborate with teachers probably have less flexibility in pursuing particular vision of reform than do researchers who act as teachers.” The role of the researcher is that of the leader in a local development community. He continues: “His/her primary responsibility is to guide the development of this community as it seeks to arrive at taken-as-shared decisions and judgments.” (Cobb, 2000: 330)

With regards to this an effective basis for communication needs to be established in order for the teachers and research to constitute a community united by a common purpose. The role of the collaborating teachers is to translate research decisions and judgments into practice. Separating the implementers from the researcher makes the teaching experiment more objective.

The analysis of this study will place emphasis on the data collected from the observations and the data collected from interviews with teachers will be used as additional information. The most important instrument of this study – data for my observation – is direct evidence on how the teacher mediates object-level and meta-level learning. Instead of asking what the teacher thinks about this issue, or how the teacher acts in a specific situation, I as a researcher will observe what actually happens when the learners are familiar with topics and can solve the problems themselves or when they cannot participate in classroom discussion without an expert interlocutor. Another advantage of observation is that the researcher collects the data from fieldwork. Gathering the information from a real life situation determines the natural setting. It is better to observe the classroom situation as it normally happens, rather than creating a scenario and looking at the reaction of the actors.

For going in depth to understand the concept of mediation, more specifically to answer the second question [w]*hat enables and constrains her/his facilitative mediation in the case of Congruency in Grade 9*, video-stimulated interviews with the two teachers will be conducted. Semi-structured interviews in which a large amount of discussion will be generated by watching a replay of the videos will be used. In this way opportunities for participants to raise their opinion about "what did not happen in the real situation, but can be done better in the teaching process?" will be provided.

4.4 Description of the designed lessons

Design research in education necessarily involves creating some form of deliverable product. The goals of the product design are two-fold: to advance a new theory (Simon, 1989) (in this case Sfard's theory of learning) and to solve some practical problems in education (Middleton and Gorard: 2008). At the same time according to DiSessa and Cobb (2004) the theory provides a framework for analysing the collected data.

As already discussed, Sfard and Ben-Zvi (2007) distinguish between two types of learning – object-level and meta-level learning. Following these two authors I, as a researcher, together with two teachers, wanted to explore the role of the teacher when mediating learning across these two levels of learning. Three of us as a team deliberately designed four lessons on the topic Congruency in ways we considered to be suitable for this exploration. Using the idea of collaborative learning in the first three lessons the learners had to implement the activity by working in groups. Simultaneously, lesson one and lesson four were designed to contain two activities; one on object-level and one on meta-level learning.

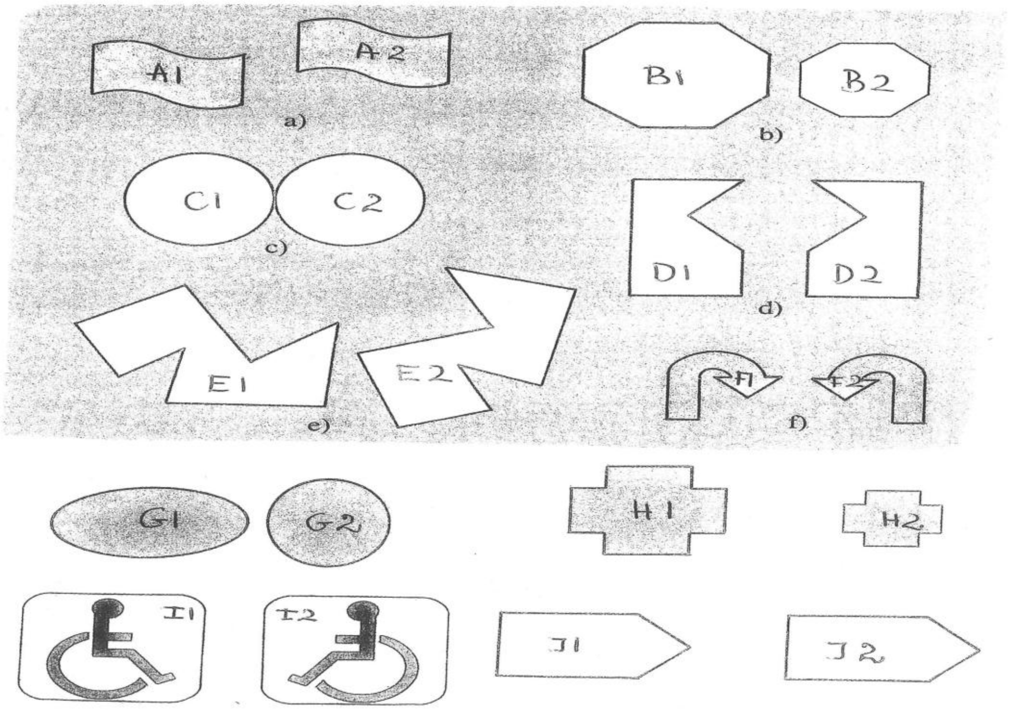
4.4.1 First lesson: First Activity

As a first lesson of the topic the teachers decided to do a revision activity with the learners. In this activity they were given the worksheet with ten pairs of shapes.

LESSON 1

ACTIVITY 1 **WHEN ARE FIGURES CONGRUENT?**

1. Which of the following pairs of figures is congruent? Why?
2. What can we say about
   - the shape,
   - size and
   - position in space of congruent figures.



Conclusion:

1. Figures that are exactly the same in shape and size are congruent.
2. Congruent figures differ only in position or orientation in space.
3. We can test whether figures are congruent by sliding, rotating of flipping them until they fit exactly on top of each other.

Getting them involved in the geometric topic the teachers chose visual mediation to engage learners in this ‘first-step’ problem. The nature of shapes from the list was non-geometrical such as A1 and A2, I1 and I2, F1 and F2, and geometrical for instance circles C1 and C2, octagons B1 and B2 and ellipses G1 and G2. In this way the teachers planned to show that identical figures are taken from the real world and it is not used only in one specific subject – mathematics. The purpose of the task was to recognize whether the pairs of figures were congruent or not, based on the statement *figures that are exactly the same in shape and size are congruent.*

The first activity of the lesson was designed to involve learners in object-level learning. According to Sfard object-level rules regard the properties of the object (2008:201). She describes object-level learning as a straightforward type of learning, a product of logical necessity (2007: 569). Furthermore Sfard elaborates that object-level learning is an “expansion of the existing discourse, attained through extending a vocabulary, constructing new routines and producing new endorsed narratives" (2008:575).The design of the first activity of the first lesson followed these descriptions. Firstly the teachers had to recall the previous knowledge that for the phenomenon "two shapes are the same" mathematicians use the special term *congruency*. Secondly, the teachers intended to increase the spector of words related with the mathematical term congruency – "fit on top of each other", "the same", "identical geometric figures". Thirdly both teachers involved the learners in routine − classifying pairs of shapes. Lastly and most importantly they have to increase the set of known facts about congruency; the learners needed to understand that the shape of two congruent figures is the same, the size is the same but the position in space is not necessarily the same. The goal of this investigating activity was that the learners needed to construct this new narrative.

4.4.2 First lesson: Second activity

According to the syllabus for Grade 9 the learners needed to learn the necessary conditions for congruency. Therefore the two participating teachers, and myself as researcher (also a practicing mathematics teacher) constructed a series of investigating activities to involve learners in this process. The teachers considered the learners' previous level of knowledge and created the patterns of activities that involved learners in meta-level learning.

The learners in grades lower than Grade 9 following the van Hiele levels of Geometric Understanding are at a descriptive level. At this level – Level 2: Analysis – "they can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object" (Mason, 1998).So at this stage the term congruency still holds the meaning that all sides of the shape have to be the same and all angles have to be the same. Therefore the learners had to compare six pairs of measurements. The learners actually transferred the notion of equality from algebra to geometry.

The curriculum for Grade 9 states "3: SPACE AND SHAPE (Geometry)

[1] Recognises, visualises and names geometric figures and solids in natural and cultural forms and geometric settings.

[2] In contexts that include those that may be used to build awareness of social, cultural and environmental issues, describes the interrelationships of the properties of geometric figures and solids with justification, including:

- congruence and straight line geometry;
- transformations.

[3] Uses geometry of straight lines and triangles to solve problems and justify relationships in geometric figures.

[4] Draws and/or constructs geometric figures and makes models of solids in order to investigate and compare their properties and model situations in the environment.

[5] Uses transformations, congruence and similarity to investigate, describe and justify (alone and/or as a member of a group or team) properties of geometric figures and solids, including tests for similarity and congruence of triangles." (Department of Education, 2012)

The development team of three teachers focused on [4] and engaged learners in a series of constructing activities to investigate the minimum conditions for congruency:

<u>ACTIVITY 2</u> ARE TWO MEASUREMENTS ENOUGH?

In this activity we will use only two measurements to create a triangle. We can explore the following possibilities:

- **Given only two sides**
- **Given only two angles**
- **Given only one angle plus one side.**

1.1 Create your own triangle using the line segments 4 cm and 6 cm. Compare your triangle with others in your group.

1.2 How many different triangles can we create if we know only two sides of a triangle?

2.1 Create your own triangle using two angles equal to 60^o and 80^o. Compare your triangle with others in your group.

2.2 How many different triangles can we create if we know only two angles of a triangle?

3.1 Create your own triangle using one angle of 65^o plus one side of 5 cm. Compare your triangle with others in your group.

3.2 How many different triangles can we create if we know only one angle and one side of a triangle?

In this series of investigations we plan to involve learners in meta-level learning. The endorsed rules and mathematical laws of the old discourse for congruency, that six pairs of measurements define identical geometric figures, may sound contradictory compared with the rules of the new discourse that only three measurements are enough to draw a unique triangle. The three teachers planned to engage learners in new discourse when the learners needed to answer the question: "[i]f we want to determine whether two triangles are the same or not, do we need to know all six measurements?" To distinguish between necessary and sufficient conditions for two triangles to be congruent is the next level in the development of geometrical thought.

In the second activity of the first lesson the teachers decided to engage learners to construct triangles using two measurements. The teachers planned to explore all three possibilities – given only two sides, given two angles and given one side and one angle. In this group activity each member of the team had to compare their own triangle that they constructed with classmates' triangles. The purpose of this activity was to answer the question "[a]re two measurements enough to create a unique triangle?" The conclusion that the activity was designed to achieve was that "[i]f we know that only two corresponding parts of two triangles are equal, it is not enough information to conclude that all the other parts are equal."

The team of three teachers designed the second activity following the direction from the Department of Education (1997: 11) that teachers need to "prepare interesting and appropriate learning activit[ies]…" in which learners investigate and discover new mathematical concepts. In the meeting the teachers rejected the possibility to open the discussion in front of the class by drawing one side a of triangle then drawing second side of a triangle and asking a series of questions to convince the learners that without knowing the angle between the two sides a unique triangle cannot be drawn. Undermining the role of the teacher as a knowledge provider the team planned to organize the proper learning environment for developing geometric thinking – Level 3 – *the Abstraction level.* The new role of the teacher is to elicit the finding from the learners and elaborating on it.

In the next lessons, lesson two and three, the teachers together with learners planned to continue looking for the conditions of congruency. They planned to co-construct that three pairs of measurements are enough to create a unique triangle. As the investigation activity progressed they planned to find the three conditions for congruency – three sides, two sides and an included angle, two angles and a corresponding side. These two lessons were not the object of my observation because it was similar to the second activity of the first lesson. The focus of my observation was lesson four, where the learners needed to construct their own proof. Lesson four was also guiding according to Sfard's theory of learning; it contained two activities; one involved object-level and the other meta-level learning.

4.4 Lesson four

The teachers organized this lesson with the assumption that meta-level rules for congruency are transformed to object-level rules for discourse. According to Sfard (2008: 202) metarules (opposite to object-level rules of mathematics, which once formulated, remain immutable) may evolve over time. She clarifies that "mathematics is an autopoietic system that grows by annexing its own meta-discourses, and this means, among others, that what counts as a meta-rule in one mathematical discourse will give rise to an object-level rule as soon as the present meta-discourse turns into a full-fledged part of the mathematics itself."(Ibid:202) The design of the next, follow-up activities was based on this important point.

4.4.3 Fourth Lesson: First activity

As introduction for the first activity the teachers planned to do a summary of the three conditions for congruency as follows: If in two triangles:

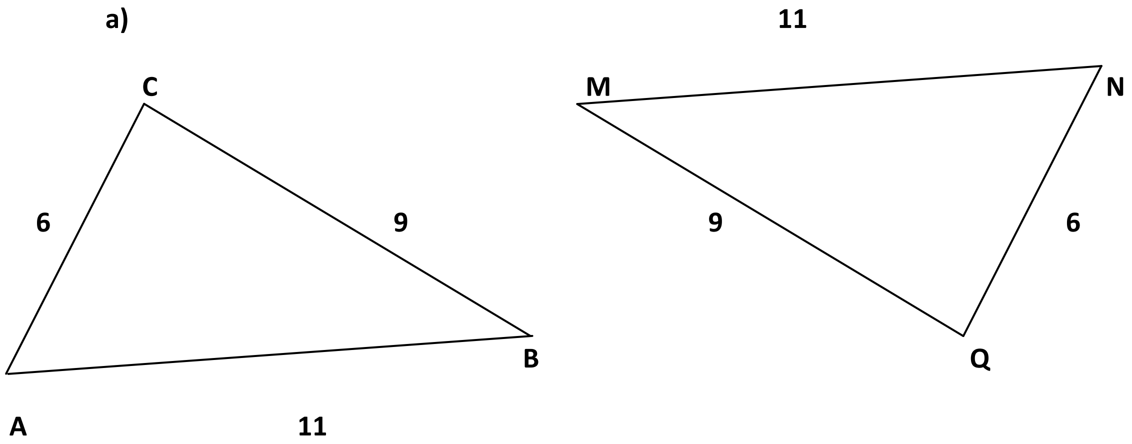
- the lengths of the three sides in one triangle are equal to the lengths of the three sides in the other triangle, the triangles will be congruent (S, S, S).
- the lengths of two sides and the size of the angle between them in one triangle equals the length of two sides and the included angles in the other triangle, the triangles will be congruent (S, A, S).
- the sizes of two angles and the length of one side, equals the sizes of two angles and the length of the corresponding side in the other triangle, the triangles will be congruent (A, A, S).

Then the learners were given the task to explain how they would convince someone whether or not the two triangles in each case were congruent.

LESSON 4

ACTIVITY 1

a)



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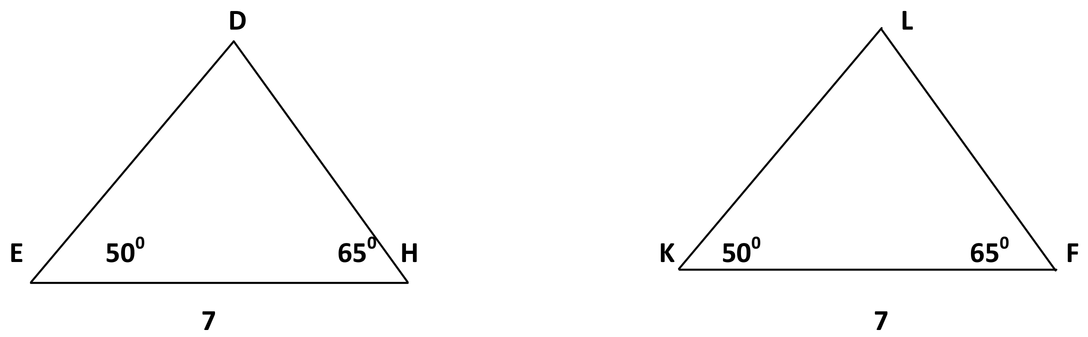
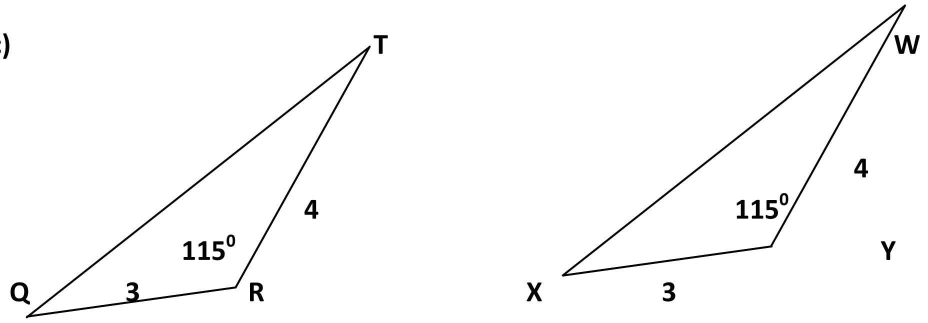


c)



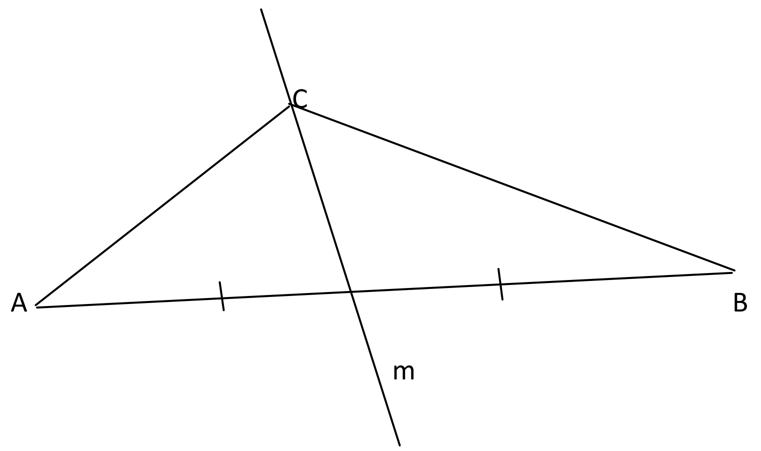
In this task the teachers chose one example from each condition of congruency. The measurements that will guarantee congruency of two triangles (three pairs of measurements) were explicitly noted on the drawing in each case. The actual process of solving the first activity of the fourth lesson was straightforward and involved object-level learning. Constructing proofs by using the necessary conditions for congruency was actually applying conditions for congruency in different examples; in other words the teachers showed the learners a new routine. Another argument that in this activity the teachers organized the learners in object-level learning is that there is no change in meta-level rules and no commognitive conflict. How to record the solution, however, is meta-level learning. For the first time in learning geometry they learned about geometrical proof. Until now the learners did not know mathematical artefacts for congruency ($\equiv$), visual representations "two sides are equal" and "two angles are equal" and in general how to formally write proof (for instance each statement is followed by an explanation; the proof of congruency has specific schema).

4.4.4 Fourth Lesson: Second activity

Activity two contained the following question:

Does the line m passing through the vertex C and middle of opposite side AB in triangle ABC divide the given triangle in to two congruent triangles? If the answer is yes, prove it.

If the answer is no, what are the conditions that would make these two triangles congruent?



This task required higher order thinking. Firstly, it is needed to analyse the conditions of the problem by breaking the information up into parts. Then the participants in the discourse needed to compare what was given with the conditions for congruency. After that they needed to notice what was missing and construct the solution of the problem. Even though the activity was not straightforward it can be classified as object-level learning. This high level task expected of the learners to know the conditions for congruency and how to formally record the geometrical explanation. Thus the actual process of solving the second activity is not meta-level learning because it does not involve changing the previous rules, but applying the conditions for congruency. Even though the task expected of learners to do probing, teacher intervention was not compulsory.

4.5 Data collection

The main data collection instrument for this research is video recordings. Firstly, the same planned lessons of two teachers on the topic of Congruency were intentionally observed. Lesson one and lesson four in the series of four geometrically designed lessons was videotaped. The reason of selecting these two lessons is that they contain activities involving two types of learning, namely object-level and meta-level learning.

The research study was conducted in one private secondary school in Johannesburg, which is located in an urban setting in the biggest city in South Africa. It was a multi-racial group of learners – white, black, coloured, Indian and Chinese – with middle socio-economic status. The two observed classes differed; the first teacher's class was a class containing 27 learners with good linguistic background and middle level mathematical knowledge; the class of the second teacher contained only 15 learners identified by the school as having problems and the process of understanding of mathematical concepts were slow in this class.

The teaching experience of the two teachers differs. The first teacher has 5 years' experience in High School teaching (and an Honours degree in Mathematics Education) while the second teacher has 20 years teaching experience in Primary and High school teaching (and a Foundation phase degree with special courses in Mathematics such as Statistics, Functions, and Transformations). Simultaneously they demonstrate different levels of mathematical knowledge according to the degree that they obtained. As a result of the diversity of experience and qualification I as a researcher observed a variety of instructional actions developed by both teachers during these two lessons.

During the process of the observation the video camera followed the teachers and captured what they said and did during the lessons, including writing on the board. The learners' responses were also recorded.

After all the classroom observations were completed a post-lesson interview was conducted. The nature of the interview was semi-structured; the main questions that directed the interview are recorded in Appendix C. Follow-up questions and contingency questions that improve the quality of the data collection are included. The focus of the discussion between researcher and individual teacher was to answer the second research question "[w]hat enables and constrains [the teacher's] facilitative mediation in the case of Congruency in Grade 9?" The consideration of particular episodes from the observations and raising the general opinion about the 'reform' in teaching and learning process provide complete answers to this question.

This data analysis is based on an analytical framework which will be described in the next section.

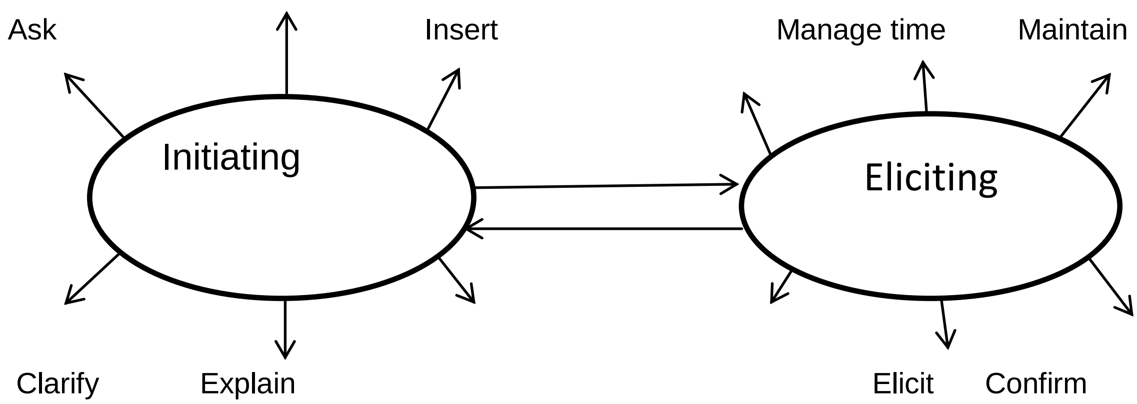
4.6 Analytical Framework

Focusing on the goal of Mathematics teaching and learning practice, developing learners' thinking, the teachers from South Africa try to implement the curriculum expectations. In these new teaching practices, the teachers are expected to be facilitators and use a variety of teachers' actions to manage classroom discussion. Lobato et al. (2005) group all of these actions into two sets – *initiating and eliciting*. They define *initiating* as the set of teaching actions that serve the function of stimulating learners' mathematical constructions via the introduction of new mathematical ideas into a classroom conversation. Instead of making declarative statements the teacher can describe a new concept, ask the learners for ideas to solve a mathematical task, summarize a learner's work, generate counterexamples and "insert a new voice" (Ibid: 110) via questions and comments in order to change the direction of the discussion. All these actions focus on the conceptual rather than the procedural context. The function of initiation will be incomplete without learners' engagement with the new information. "Learners should be conceptually able and motivated to make sense of the teacher's utterance."(Ibid: 111) Lobato et al. (2005: 112) consider *initiating* in conjunction with *eliciting.* Thus eliciting occurs when the teacher's actions serve the function of drawing out learners' images, ideas, strategies, conceptions and ways of viewing mathematical situations. "Eliciting actions occur when the teacher arranges for situations in which learners articulate, share, discuss, justify, reflect upon, and refine their understanding of the mathematics" (Ibid: 112). The teacher elicits the new information from the learners' response.

Initiating and *eliciting* interact together; these two categories of action are not in conflict but complete each other. Moreover, Lobato et al. (2005) believe that initiating is most profitable used in conjunction with eliciting.

These two major ideas are expressed in the diagram below.

Diagram 4 Initiating and eliciting



The metaphoric expression of *Initiating* and *Eliciting* are two suns, two main sources to mediate classroom discussion. Each of these suns has subcategories, which are rays of the sun (some of the subcategories are not yet discovered and thus not yet named and left blank on the diagram; they need further investigation). The large set of rays is called tools, codes or moves, respectively. All these tools function differently. The most relevant was selected in order to answer the research question **"how does the teacher mediate instructions during object-level and meta-level learning?"**

A short description for each of these is provided.

Ask– the teacher asks the main question in the discussion or the lesson, thereby initiating thinking.

Elicit – the teacher tries to get something from the learner. The eliciting move is very often an authentic question (information-seeking not respond-seeking).

Clarify – to make the learner's response more clear.

Very often it is difficult to understand what the learners say. According to Chapin, O' Connor and Anderson (2003) deep thinking and powerful reasoning do not always correlate with clear verbal expression. To achieve the goal of effective mathematical discussion all learners need to understand learners' contributions that are unclear. Therefore the teacher has to deal with the lack of clarity of learners' contributions by repeating the learners' comments. The reformulation of learner ideas has been called *re-voicing.* Another solution to the problem is asking learners to restate someone else's reasoning. Rephrasing what another learner says and applying her/his own reasoning to someone else's reasoning is a third way to clarify a learner's idea.

Command – the teacher's instruction that shows what learners need to do in the current activity.

Confirm– the teacher confirms that the learner's contribution is correct.

Give facts – the teacher provides the definitions, meaning of the words necessary for certain discourse. She/he also explains the axioms that give the background of a new discourse.

Describe a new concept – the teacher explains main ideas, proves theorems. Without this information the discourse cannot function. In a short time the learners cannot manage to discover themselves.

Summarize – the teacher or learner make the necessary conclusion during the discussion or at the end of the lesson.

To see the full picture of teacher' mediation **Social regulation** is included in the analysis; the learners need to obey the rules of the classroom environment. Thus two more moves that help to control the classroom – time management and disciplinary management – are added to the set of teacher's moves; a short description of each follows:

Time management – when the teacher gives a certain amount of time to the learners to organize her or his thoughts. Providing waiting time after asking the questions is an important code of mediating classroom discussion. Another way to manage the discussion is to push the learner to explain her/his own ideas.

Disciplinary management – when the teacher reminds the learners to focus their attention on the task at hand and to behave in class.

Another characteristic of teacher's mediation is **teacher's listening**. In this section the three types of listening – evaluative, interpretive, hermeneutic – that can be possibly used in lesson's mediation will not be described; these kinds of listening can be found in the Literature review.

Going beyond traditional teaching the teachers engage the learners in different kinds of interaction. **The interaction patterns** will be the third characteristic that I, as researcher, will use to analyse the teacher's mediation in the classroom.

The first kind of interaction pattern is Initiation-Response-Feedback/Evaluation (IRF/E). Identified by Sinclair and Coulthard (1975) and Mehan (1979) in this conventional key structure, the teacher makes an *initiation* move, a learner *responds*, the teacher provides *feedback* or *evaluates* the learner response and then moves on to the new *initiation.* Sometimes *feedback* and *evaluation* are combined into one term or it is not explicit. This conversion extends sequences into *initiation-response* pairs. Because it is similar to traditional teaching, teachers usually use this particular kind of interaction more often and feel comfortable with it. According to Brodie (2007: 3)

> Although this structure requires a learner contribution at every other turn (the response move), and therefore apparently gives learners time to talk, much research has shown that because teachers tend to ask questions to which they already know the answers (Edwards & Mercer, 1987) and to 'funnel' learners' responses toward the answers that they want (Bauersfeld, 1980), space for genuine learner contributions is limited.

Therefore in order to achieve the goal of learner engagement and inquiry Brodie suggests three new possibilities of interaction.

One of these is **reversing the IRE** in which learners ask questions and the teachers respond to them. The teacher does not only tell the answer. She/he can choose different teachers moves to respond to the question such as (a) re-voicing so that others can contribute to the answer; (b) asking for justification; (c) clarifying the question that was unclear; (d) using the opportunity to ask the question and reverse the dialogue to traditional IRE mode. "Exchanges like this are both learner- and teacher-directed." (Brodie, 2007: 7) These exchanges are learner-directed because learners' questions drive the dialogue and teacher-directed because the teacher is still in control and can choose how to respond. In this type of interaction the teacher shows interest in the learners' questions, takes their contribution serious and uses different responding strategies to make valuable points in the lesson.

Another interaction pattern is **learner-learner dialogue.** The two learners take the roles of asking and answering questions to clarify each other's thinking seriously, and were taking up each other's ideas in ways that teachers rarely do (Nystrand et al, 1997). The role of the teacher is to facilitate these interaction patterns, to get learners to talk and listen to each other, to ensure that learners are not miscommunicating.

The third kind of interaction is **whole-class dialogue**. More than two learners contribute to the conversation. They express their views, hear each other and build on each other's ideas. Now the teacher's roles are to recognise the debate question, engage learners in the discussion and support the conversation around the initiated contradictions. She/he participates in the discussion in a 'neutral' tone by repeating learners' contributions, repeating learners' questions, or by challenging them as to why they are claiming that a certain statement is correct.

In summary, to analyse the data from the observation for each dialogue, the following questions will be considered:

Table 2 Summary table from analytical framework

<table>
<tr><th>Forms</th><th>Context</th></tr>
<tr><td>1. In collaborative learning, what kind of interaction patterns do the teachers use and are the learners involved in?
• Traditional IRF/E
• Reversing IRE
• Learner-learner dialogue
• Whole class dialogue</td><td rowspan="4">Looking from the commognitive perspective, what components of the discourse have been changed?
• Words
• Visual mediators
• Narratives
• Routines</td></tr>
<tr><td>2. Which teachers' move has been used most frequently in the dialogue?
• Eliciting move –elicit, confirm.
• Initiating move – ask, clarify, explain, command, give facts, describe a concept, summarize.</td></tr>
<tr><td>3. Are there any social regulations and what kind? Yes
• Time management
• Disciplinary management</td></tr>
<tr><td>4. What kind of listening does the teacher practice?
• Evaluative listening
• Interpretive listening
• Hermeneutic listening</td></tr>
</table>

This analytical framework is developed particularly for data obtained from this study's observation.

Chapter 5
Data Analysis

5.1 Unit of analysis

In the context of learning, Ben-Zvi and Sfard (2007) distinguish between object-level and meta-level learning. Thus, the role of the teacher (particularly with respect to mediation across these two levels of learning) was explored. The activities of the two lessons were purposefully designed according to Sfard's theory: each lesson intended and included one object-level activity and one meta-level activity. Therefore, the unit of analysis will be the activity of the lesson. Because of the activity's time span being lengthy, each unit is divided into dialogues for a more detailed description. The segments correspond to the specific idea exchanged during the event; they are numbered according to the sequence in which they occur.

Two summary tables of each activity in each lesson are provided below. Appendix A and B provide the full details of the lesson with a table consisting of the full transcript of the lessons and short descriptions of the teachers' mediation.

5.2 .1 First teacher (T1). First Lesson: First Activity.

The Table 3 below gives short description of each dialogue in first activity of the first lesson:

Table 3 T1: Short description of first lesson, first activity

Dialogue 1	T1 provides a worksheet and instructions on what the learners need to do in the beginning of the lesson. He defines the term congruency and sets up the main question of the activity.
Dialogue 2	T1 gives additional information on what learners can do with shapes, shows how to deal with the first two pairs of shapes. After that the learners work on their own.
Dialogue 3	
Dialogue 4	
Dialogue 5	T1 does correction of the first activity.
Dialogue 6	T1 summarizes the results and does conclusion.

The Table 4 below, which is based on the final table from the analytical framework, is a summary of interaction patterns, teachers' moves and type of listening that T1 used in the first activity.

Table 4 T1: Summary table for first lesson, first activity

Lesson 1 Activity 1	**Interaction patterns**	**Teacher's moves**	**Type of listening**	**Social regulation**
Dialogue 1	No interaction patterns. The teacher only talks providing instruction to the learners.	Prior teacher's move − ask command.	None	None
Dialogue 2				Disciplinary management
Dialogue 3				Time management
Dialogue 4				Time management
Dialogue 5	Traditional IRE/F structure	Elicit, Confirm	Evaluative listening	None
Dialogue 6	Traditional IRE/F structure	Summarize, clarify, explain	Evaluative listening	None

Participation

In the first four dialogues of the first activity the participation of T1 was dominant. He started with procedural instructions on how to implement the task. Some of them were:

> **[00:01:26.02]I asked you to write two columns there. Figures that fit on each other, congruent and figures that do not fit on each other, not congruent.**
> **[00:01:46.11]You are going to cut out each of the shapes ... each of the pairs. You got A1 and A2; you got B1 and B2, C1 and C2 etc. Right?**
> **[00:01:57.19] When you cut them out, then you are going to see are they fit one on top of other one. If you can it will be considered to be congruent and you going to put it ... you going to paste it in your book in left-hand column. Correct?**
> **[00:02:15.29] If they cannot fit on each other then we gonna say that they are not congruent, they are not fit to each other and you going to paste them in the right-hand column**.(Appendix A)

If we have to classify the instructional approach of T1 in the first activity we can associate it with "Daedalus' behaviour which armed his son Icarus with wings and let him choose his own trajectory" (Ben-Zvi & Sfard: 2007:1).T1 did not do scaffolding to show how to solve the problem step-by-step. As the teacher adopted Daedalus' approach he provided only the commands on how to implement the task. The learners were working in groups independently and there is even no evidence for verbal participation; the learners were busy cutting, recognising and pasting the pairs of shapes in the correct column and their actions show learners' autonomy in this activity.

While in the first four dialogues hardly observed any interaction patterns, in Dialogue 5 and 6 the teacher was involved in a traditional IRE structure with the whole class. In Dialogue 5 he did corrections together with the learners. The teachers' moves that he used most were Elicit and

Confirm. At the same time T1 listened for the correctness of learners' contributions. According to Davis (1997) he did Evaluative listening. For example:

[00:14:05.19]T: Ok. A1 and A2. Are they congruent or not?
[00:14:11.02]L: Yes. They are.
[00:14:14.07] T: What about B1 and B2? Are they congruent?
[00:14:16.27] L: No.
[00:14:20.05]T: The one is the most...So that will be in the second column.
[00:14:22.05]T: What about C1 and C2?
[00:14:23.05]L: Yes.
[00:14:24.05]T: They are congruent, right? (Appendix A)

Looking at Sfard's characteristics of mathematical discourse in activity one the development team used a visual mediator when designing the lesson; the worksheet with different pairs of figures consolidated the mathematical concept of congruency. In the lesson presentation T1 also used a visual mediator; he not only did corrections verbally but he also recorded the solution in the table in front of the class using the overhead projector.

Sfard's second characteristic of mathematical discourse is words. T1 and the learners in the first activity were communicating using informal and formal language. As a proof T1 defined the term congruency as "figures that fit each other" in the first dialogue. In Dialogue 3 he was more specific when he asked "[c]an you fit them on top of each other exactly?" T1 elaborated further:

[00:07:09.26]T: Guys, if you look at the shapes, you can do anything you like to the shapes except fold it. You can flip over, you can turn it, and you can rotate it.
[00:11:16.07]T: You can turn it, you can slide it you can flip it. (Appendix A)

By giving the instructions on how to use this narrative − the definition for congruency in Dialogue 4− the teacher used informal language. He extended the vocabulary, one of the features of object-level learning, of the endorsed narrative − congruency. In contrast, T1 used formal mathematical language such as size, shape and orientation, rotation and reflection in Dialogue 6. It can be summarized that T1 lead the discussion firstly by using informal mathematical words such as **fit, turn, flip, fold** and at a later stage using formal mathematical words like **congruency, rotate, reflect.**

Learners are familiar with mathematical discourse as is evident in the learner's participation in Dialogue 5with the following: **[00:15:42.24]L: If you reflected, Sir.** (Appendix A)

The fourth feature of the mathematical discourse according to Sfard's is routine. The learners recognise congruent figures by practicing the same actions – cutting the pairs of shapes, fitting them on top of each other pasting them in the right column.

<u>Social regulation</u>

It is a fundamental fact that humans are social beings, engaged in collective activities from the day they are born and throughout their life. In connection with this the reforms of education affect the power relations in school society. While in a traditional classroom the power structure was supposed to be fully determined by the institutional context − the teacher was the leader by default − in a reformed classroom the issue of leadership is open to negotiation. Minimizing the authority of the teacher creates disciplinary problems (Sfard, 2007: 608); to balance the situation T1 said the following in Dialogue 2:

> **[00:04:22.17] T: I am going to take these, I think I'm gonna take your books and I think I am gonna use this activity for the week as part of your assessment for the term. So make more effort.** (Appendix A)

He encouraged the learners to take the work seriously and make more of an effort.

The time management of the lesson was well defined. T1 planned how long it would take to finish the first activity; he informed the learners on a regular basis how many minutes they had to complete the task. The following examples of teacher utterances indicate this:

> **[00:05:34.12]T: We have to work out quickly because we have another activity. That is more interesting one. [00:05:39.28] T: So, I am only gonna to give you about ten more minutes for this one. [00:12:22.21] T: Guys, we going to use five more minutes for this activity. Do as many as you can, put them in a column so we can go to the next activity.** (Appendix A)

Even though the learners were involved in an investigating activity the time-frame was satisfied.

At the end it can be concluded that the first activity, designed as an example for object-level learning, is an expansion of existing discourse and that it produces a new endorsed narrative. The learners should understand the new narrative that two figures are congruent when they have the same size, the same shape but different positions. T1explained clear conditions of the problem and then checked the correctness of implementation. In this activity, he did not use a variety of moves; in his mediation the priority teaching moves were **Command, Ask and Explain**.

In addition the teacher asked three key questions in Dialogue 6, the last dialogue in activity one:

> **[00: 16: 39.26] T: What can you say about the shape of the one that are congruent?**
> **[00: 17: 02. 21] T: What can we say about the size?**
> **[00: 17: 12. 02] T: Ok. What can we say about the position?** (Appendix A)

One learner responds with the wrong answer to the last question. (**[00:17:14.15] L: The position ....is the same],** Appendix A) T1 gave the correct answer straight away. He summarized: "[s]o, the shape is the same, the size is the same and the position is different." He did not open the floor for discussion. The questions remain whether the learners implemented the routines that were asked for, whether they understood why they were doing it. Did the learners put any meaning in the activity? What is Sfard's explanation for these results?

5.2.2 Second Activity

The Table 5 below gives short description of each dialogue in the second activity of the first lesson:

Table 5 T1: Short description of first lesson, second activity

Dialogue 1	Before the second activity T1 asks the question "[h]ow many measurements do I need to give you in order to draw a unique square?" In the open discussion he compares the necessary conditions to draw a unique square and to draw a unique

	triangle.
Dialogue 2	T1 set up the main question of the activity.
Dialogue 3	T1 provides instruction to the whole class on how to implement the second activity.
Dialogue 4	T1 shows on the board how to draw a triangle when two measurements are given.
Dialogue 5	T1 repeats the procedure of drawing.
Dialogue 6	T1 shows a particular learner how to draw a triangle.
Dialogue 7	T1 shows on the board for the third time how to draw a triangle when two measurements are given.
Dialogue 8	T1 concludes at the end of the lesson.

The Table 6 is a summary of interaction patterns, teachers' moves and type of listening that T1 have been used in second activity.

Table 6 T1: Summary table for first lesson, second activity

Lesson 1 Activity 2	**Interaction patterns**	**Teachers' moves**	**Type of listening**	**Social regulation**
Dialogue 1	Traditional IRE/F structure	Elicit and Clarify	Evaluative listening	None
Dialogue 2	Traditional IRE/F structure	Ask	No listening	None
Dialogue 3	Teacher talks	Command	No listening	None
Dialogue 4	Teacher talks	Describe	No listening	Time management
Dialogue 5	Teacher talks	Describe	No listening	None
Dialogue 6	Teacher talks	Describe	No listening	None
Dialogue 7	Teacher talks	Describe	No listening	Disciplinary issues
Dialogue 8	Traditional IRE/F structure	Elicit Summaries	No listening	None

In the second activity the learners needed to construct triangles using two measurements. The goal of this activity was to answer the question "[a]re two measurements enough to create a unique triangle?" Finding the conclusion that "[i]f we know that only two corresponding parts of two triangles are equal, it is not enough information to conclude that all the other parts are equal" (lesson conclusion) involved meta-level learning. Distinguishing between necessary and sufficient conditions for two triangles to be congruent is the next level in the development of geometrical thought.

T1 started the second activity by initiating the discussion. He posed the question “[h]ow many measurements do I need to give you in order to draw a unique square?” and involved the learners’ previous knowledge about the square. In Dialogue 1 in activity two the learners’ participation confirmed that they are familiar with the properties of the square and most importantly, that they know the answer to the main question “[i]f I give you one measurement of the square you can get the same one?” Coming from known narrativeT1focused the second activity on the question “[a]re two measurements enough to draw unique triangle?” In this way T1 did contextual scaffolding; he expected from learners to make a shift from the well-known necessary conditions of drawing a unique square to unknown necessary conditions of drawing a unique triangle. He provoked meta-level thinking and then the teacher involved the learners in the investigating activity.

In Dialogue 1 the T1again used the traditional structure of classroom discourse, Initiation-Response-Feedback/Evaluation (IRF/E). His listening can be characterized as evaluative listening.

He asked specific questions such as “[w]hat are the properties of the square?” … “[a]nd what about the angles?”, “what angles are equal of the square?” and he expected straightforward answers. The priority of the teacher’s moves in this exchange was to elicit and confirm. In Sfard’s terminology the property of the square is a narrative; this is what the class discussion was about.

In Dialogue 2 the teacher asked the question “[h]ow many different triangles can we create if we know only two measurements of a triangle?” Through this he provoked meta-level thinking and left the discussion open. (**[00: 20: 44. 03] T: All right. Just think about it?** Appendix A)

Dialogue 3 was dominated by the teacher; he provided instruction to the whole class on how to implement the investigation. He divided the class into three groups and each of these groups had to draw a triangle; the first group using two sides, the second group using two angles and third group using one angle and one side. Each member of the group had to draw a triangle with the given specifications for that particular group. After that the learners from one group had to compare the triangles that they drew to check whether they were the same or not. The dominant teacher’s move in this part of the lesson was Command.

However, although the main question of the activity two was well-defined and the whole class was well organised, problems emerged. The focus of the task was how to find the narrative, but was changed to drawing a triangle. In general, T1 identified that learners’ prior knowledge was very weak. In this particular classroom the constructing skills of the most of the learners were not yet developed; the learners did not know how to draw two lines from the same point − one 4 cm and one 6 cm long −and they did not know how to draw an angle. Although the topic Angles is in the syllabus from Grade7, the learners in the Grade 9 classes still struggle with this topic.

Furthermore they do not have routines to draw a triangle by using given measurements, the skills that they have to acquire in Grade 7 and 8. That is why predominant findings in the second activity of the first lesson are teacher's scaffolding in the drawing. In order to conduct the lesson the teacher explained to the learners thrice on board how to create their own triangle using the line segments of4 cm and 6 cm, and the angles of 60° and 80° degrees. He showed them how to perform the routine step-by-step verbally and using a visual mediator. He also paid individual attention to one group and showed them how to construct triangles with given specifications. Therefore from Dialogue 4 to Dialogue 7 there are no interaction patterns; no listening; the prior teacher's move that had been used is Explain. T1 focused in activity to support all learners to draw one triangle each

In Dialogue 8 there is the following exchange:

[00:05:28.15]T: Now the question is "Do you all have the same triangle or not?"[00:05:30.25]Learners: The answer is "No".[00:05:32.23]Learners: What is the reason?[00:05:33.11]T: In other words you can create much different types of triangles with only given two sides.[00:05:39.20]T: So in other words if I give you one side of the square it is enough for everybody to have the same square.[00:05:46.08]T: But now if I give you two things of the triangle it is not enough to all of you to have the same triangle? You agree? [00:05:50.25]Learners: Yes, Sir. [00:05:53.26]T: Because you all got different triangles. You all got triangles with 6 and 4 for the sides, but they are not the same. [00:05:54.13]Learners: But why?
[00:05:59.13]T: What about over here? You have two angles, one 60 and one is 80.[00:06:04.01]T: Are all got the same triangle?[00:06:07.00]Learner1: Yes.[00:06:07.02] Learner2: No.[00:06:08.16]T: You got different triangles, but they all got 60 and 80.[00:06:14.02]T: So two angles by themselves it is not enough so all got the same triangle.[00:06:14.04]T: Over here you got 65 and 5. Do all have the same one? [00:06:21.18]Learners: No. [00:06:23.21]T: You all have got different ones. So there is a different possibility with 65 and 5. It is not enough to give you all the same triangle, all the congruent triangle. [00:06:31.11]Learners: I do not understand why? (Appendix A)

The participationist version of human development is a new lens that emphasises the effective human communication. Sfard (2007) explains the nature of learning interaction as a part-whole relation and emphasises that the whole and the part affect and inform each other and the whole is fully dependent on the parts. In contrast the above exchange shows communication on superficial level. In this dialogue the learners do not understand the answer to the question: "[w]hy are two measurements not enough to create a unique triangle?" On one hand they ask the teacher:

[00:05:32.23] Learners: What is the reason?
[00:05:54.13] Learners: But why?
[00:06:31.11] Learners: I do not understand why? (Appendix A)

And on the other hand the teacher ignores these questions by providing the statement without explanation: **[00:05:33.11] T: In other words you can create [many] different types of triangles with only given two sides.** (Appendix A) At the end in the second activity the goal of the investigation – to find the new narrative, namely that two measurements are not enough to draw unique triangle −was not identified by the learners. Furthermore the focus of the task was changed to developing drawings skills. Taking this direction the teacher taught the class how to draw a triangle by using the given two measurements in the second half of the lesson.

5.3.1 Second teacher (T2). First Lesson: First Activity.

The Table 7 below gives short description of each dialogue in first activity of the first lesson:

Table 7 T2: Short description of first lesson, first activity

Dialogue 1	T2 revises Similarity.
Dialogue 2	T2 explains the new narrative for congruency and compares it with similarity. She also gives instructions how to implement the first activity.
Dialogue 3	T2 pays attention to each group and helps with class work.
Dialogue 4	T2 does corrections of activity one.

The Table 8 corresponds to Table 7 for T2.

Table 8 T2: Summary table for first lesson, first activity

Lesson 1 Activity 2	**Interaction patterns**	**Teachers' moves**	**Type of listening**	**Social regulation**
Dialogue 1	Traditional IRE/F structure	Elicit, Confirm	Evaluative listening	None
Dialogue 2	Teacher talks	Command	No listening	Time management
Dialogue 3	Reversing IRE Traditional IRE/F structure	Elicit, Clarify and Ask	Evaluative and interpretive listening	None
Dialogue 4	Traditional IRE/F structure	Elicit, Confirm and Clarify	Evaluative listening	Time management

The implementation of the first activity by T2 is different compared to T1. Firstly, she used the following thought: in order to achieve structural understanding you need to build new knowledge based on the previous knowledge. Therefore, she started the new lesson by revising the previous material – Similarity. In Dialogue 1 the teacher involved learners in a discussion related to the main point in this concept:

[00:02:01.08]T: Ok, we'll discuss similarity. What can you tell me about similarity?
[00:02:10.24]T: What was very important about the similarity? (Appendix B)

The teacher used mathematical words such as shape and size of mathematical discourse to clarify the meaning of similarity:

[00:02:17.28]T: . . . The shape was the same, but? [00:02:29.22]T: They were different sizes, remember… (Appendix B)

The learners' participation in the initiated dialogue shows their good knowledge of mathematical narrative – the definition of similarity. The teacher used Initiation-Response-

Feedback/Evaluation structure (IRF/E); she asked questions and the learners provided correct answers. Furthermore the teacher's moves observed in this dialogue were Elicit and Confirm.

In Dialogue 2 she introduced the new narrative of the lesson Congruency. She gave the new definition using mathematical words of the previous mathematical discourse – shape and size as follows:

> **[00:02:55.06] T: Important with congruency, the shape is the same and the size should be the same.** (Appendix B)

After that T2, similar to T1, provided instructions to the learners on how to perform the task. That is why the teacher's dominant move in this dialogue is Command.

In the next dialogue – Dialogue 3 – the learners classified the pairs of figures in the correct column. Considering that the class contains learners with special needs the teacher provided more individual attention to each member of the groups. In general, the facilitation of the second teacher can be characterized by the diversity of teachers' moves and the variety of the exchange structure. For instance, in the first group the learner asked a question and the teacher did not provide a straight answer; she responded with a question for justification.

> **[00:06:19.20]L: So mam, that is similar and that congruent? [00:06:29.28]T: Why do you say that is similar?** (Appendix B)

After that the reversed IRE mode changed to traditional IRE mode in which the learner discovered the answer by himself. In the next dialogue an inverse IRE structure can also be recognised - the learner initiated the question. This time the teacher made a statement that helped to answer it.

> **[00:08:04.06]L: Congruent? No, similar?**
> **[00:08:05.06]T: Similar is size doesn't matter, shape must be…**
> **[00:08:09.15]L: The same. [00:08:13.11]T: The same.** (Appendix B)

In the conversation below the teacher used different teachers' moves such as Elicit, Ask, Command and Confirm:

> **[00:06:19.20]L: So mam, that is similar and that congruent?**
> **[00:06:29.28]T: Why do you say that is similar? Elicit**
> **[00:06:36.05]L: 'Cause the shape is the same, just a different position.**
> **[00:06:38.05]T: Ok, same shape, different position. Do I say anything about position? Are they allowed to be in different positions to be congruent? (()) Ask**
> **[00:06:48.00]T: Now you put them on top of each other (()) and now which one (()) Elicit**
> **[00:06:53.23]L: Cut out the shape like this? [00:06:59.22]T: Hmm, doesn't have to be perfect, perfect ... put this one on top of... Command**
> **[00:07:07.10]L: Oh, so they don't have to be perfect... [00:07:08.00]L: They're congruent.**
> **[00:07:09.28]T: Ja, paste it under congruent. Confirm**
> (Appendix B)

Instead of providing straight answers, she engaged learners in mathematical reasoning. Interactions with another group also confirmed this:

> **[00:11:38.20]T: (()) Yes, but I want to know why she put them under congruent. They don't look the same to me. (())**
> **[00:11:42.28] L: Oh, no no ... different pictures. (()).**
> **[00:11:52.28]T: Ah, ah, ah.**
> **[00:11:57.21]L: They are they're facing...**
> **[00:12:00.18]L: No it's not [00:12:01.16] L: (())**

[00:12:03.16]T: She must tell me why she put it there. She's got s reason why she did. She must give me the reason why she put it under congruency.
[00:12:06.19]L: They fit, they're same, and they're just different positions.
[00:12:10.19]T: The positions change. (Appendix B)

The learners needed to give the answer for their actions.T2 finished the first activity in this classroom in the same way as T1– with correction. For that she also used the conventional structure of classroom discourse (IRF/E), in which she provided additional explanation on some answers. For instance:

[00:14:27.08]T: Congruent, because if we flipped it they fit onto each other. E?
[00:15:18.27]L: Uhm, it's congruent, 'cause they're both the same.
[00:15:22.15]T: What did I do to...[00:15:25.19]L: ... when you flip them ... facing opposite
[00:15:27.25]T: When I, you said when I flipped them then they'll be the same. Ok. Good, and J? (Appendix B)

In this interaction the teacher increased the vocabulary such as "congruency is … fit onto each other", "congruent, 'cause they're both the same", "when you flip them ...facing opposite…and then they'll be the same". Following the flow of words (the first characteristic of discourse according to Sfard) and different registers (level of formality of vocabulary used) used in the activity, no logical development is evident; it is messy, starts from the use of formal register and goes to informal register. As emphasised in the case of T1, visual mediators and routines were also employed.

With the utterance the second teacher summarized:

[00:15:59.14] T: Ok, so what if we... similar… Size is not the same but the shape and the position is the same, most of the time. Congruency doesn't matter on the position, I can flip it, I can slide it, I can rotate it and then I can have the same shape, but it has the same shape, size, but the position is different. Ok. (Appendix B)

She did not involve learners in the discussion to show that they understood the purpose of the activity. Again, in the first activity of the second teacher's first lesson there is no evidence that learners discovered the new narrative –**In Congruency the position does not matter**.

As conclusion for the first activity the role of T2 was similar to T2 – providing the instructions on how to perform the task and checking the correctness of the implementation. Even though the participants were learners with special needs and the teacher provided additional explanation, scaffolding was not observed. The learners, with little help from teacher, were working in groups independently; they were busy cutting, recognising and pasting the pairs of shapes in the correct column and their actions show learners' autonomy in this activity.

5.3.2 Second activity

The Table 9 below gives short description of each dialogue in the second activity of the first lesson:

Table 9 T2: Short description of first lesson, second activity

Dialogue 1	T2 explains the implementation of the second activity.
Dialogue 2	T2 helps a particular learner to draw a triangle with 6 cm and 4 cm.
Dialogue 3	T2 initiates a situation where the learners compare their triangles and ask why they are not the same.
Dialogue 4	T2 compares two triangles.
Dialogue 5	T2 concludes the first task.
Dialogue 6	T2 explains the next task. She shows how to draw triangle with angles 60° and 80°
Dialogue 7	T2 helps individual learners with drawing.
Dialogue 8	The class discuss drawing triangles. T2 gives homework.

Table 10 corresponds to Table 9:

Table 10 T2: Summary table for first lesson, second activity

Lesson 1 Activity 2	**Interaction patterns**	**Teachers' moves**	**Type of listening**	**Social regulation**
Dialogue 1	No interaction	Command	None	None
Dialogue 2	No interaction	Describe and Command	None	None
Dialogue 3	Reversing IRE	Describe, Ask and Command	Interpretive listening	None
Dialogue 4	Reversing IRE	Ask and Confirm	Interpretive listening	None
Dialogue 5	Traditional IRE/F structure	Elicit, Describe, Summarize	Evaluative listening	None
Dialogue 6	Teacher talks	Describe and Command	No listening	None
Dialogue 7	Teacher talks	Describe and Command	No listening	None
Dialogue 8	Traditional IRE/F structure	Elicit Summarizes	No listening	None

In the first dialogue of the second activityT2provided the instruction on how to draw a triangle with two given sides, 4 cm and 6 cm. No interaction was observed and the teachers' move that was used was Command. The lack of drawing routines from the learners' side led the teacher to do scaffolding not only on the board but in each group in the class. That is why the dominant teachers' move in Dialogue 2was Describe. Because the class contains less than 10 learners she was able to pay individual attention to each of them and every learner implemented the task successfully.

In Dialogue 3 the teacher created the situation where the learners compared the triangles that were drawn. In the following exchange conflict in learners' minds is evident:

[00:25:00.14] T: But I gave you a four centimetre and a six centimetre.
[00:25:00.28] L: Why are they not the same?
[00:25:03.18] T: I'm asking you.
[00:25:09.28] L: Who has theirs cut out?
[00:25:14.07] T: How come these are not the same? Why isn't it the same? (Appendix B)

The learners' previous knowledge is that two figures are congruent when they are 'the same', the length of the sides is the same and the sizes of the angle are the same. Now in activity two, they drew triangles with two equal sides, all triangles are different and they need to know why.

There was no clarification of the question. The answer "[a]s the lesson progresses you'll see why, ok?" did not resolve the main problem of the second activity. Furthermore when the learners needed to draw a triangle with the two given angles, 60 and 80, in the second task and one learner recognised that the class drew similar triangles because the baseline is not given the teacher did not clarify the issue. This is evident in the following conversation in Dialogue 8:

[00:04:17.19] Right boys, have you cut out your triangle? We got one here one here and one there. Cut them out let's see if we've got congruent triangles or have you got lots of different ones? You think they're similarities?[00:04:29.10] L: Yes ma'am.[00:04:31.16] T: Why do you think they're similar?[00:04:31.20] L: Because the base (()).[00:04:34.17] T: You think so?[00:04:36.05] L: Yes ma'am.[00:04:37.05] T: What do you think changes it?[00:04:38.08] L: Your baseline.[00:04:39.08] T: Your baseline?[00:04:39.58] L: Yes ma'am.[00:04:41.22] T: Do you think that's what's making a difference? [00:04:43.23] L: Ja, I think so. (Appendix B)

The learner had an idea on how to answer the question: "[are] two angles enough to draw a congruent triangle?" He found that two angles are not enough and the baseline is the third component to draw a unique triangle. However the teacher did not emphasise the learners' findings which could have helped the whole class understand the problem; thus the potential of the discussion was not recognised by the teacher.

Although two teachers have different approaches in second activity, it can be concluded as follows:

- the goal of the investigation – to find the new narrative, namely that two measurements are not enough to draw unique triangle − was not clarified. There is no conclusion discussion on why drawn triangles are different.
- There is no evidence that learners understand the meaning of the activity.
- The lack of learners' previous knowledge changes the focus of the task to developing drawings routines.

<u>Social regulation</u>

In the following utterance social regulation can be observed:

[00:04:41.12]T: What can you say about the shape, the size and the position? We're going to discuss it as a class. I want you to cut each shape out and paste it under the correct heading. You don't have a lot of time to do this. Maybe get your friend to do one half of the questions and then you so the top half, ok? And they don't need to be cut out perfectly, because...[00:08:19.27]T: Quick, quick...[00:13:41.01]T: Right, we've got three more minutes

than we've gonna start with the next bit. Right, let's quickly stop what we're doing, ok. (Appendix B)

Similar to T1, T2 implements time management.

5.4.1 First teacher (T1). Fourth Lesson: First Activity.

The Table 11 below gives a short description of each dialogue in first activity of the fourth lesson:

Table 11 T1: Short description of fourth lesson, first activity

Dialogue 1	T1 explains the purpose of the lesson.
Dialogue 2	T1 stands in front of the class and tells the learners the conclusions of previous lessons – the three conditions for congruency.
Dialogue 3	Discussion about the meaning of the word 'corresponding side'.
Dialogue 4	T1 hands out the worksheet and solves the first task together with learners.
Dialogue 5	T1 concludes the first task.
Dialogue 6	Solving the second task.
Dialogue 7	Solving the third task.
Dialogue 8	T1 shows an example for corresponding side.

The Table 12 below, based on the final table from the analytical framework, is a summary of interaction patterns, teachers' moves and type of listening that T1 have been used in the first activity.

Table 12 T1: Summary table for fourth lesson, first activity

Lesson 2 Activity 1	**Interaction patterns**	**Teachers' moves**	**Type of listening**	**Social regulation**
Dialogue 1	Traditional IRE/F structure	Ask and Describe	Evaluative listening	None
Dialogue 2	Teacher talks	Describe	None	None
Dialogue 3	Traditional IRE/F structure	Ask and Describe	Evaluative listening	None
Dialogue 4	Traditional IRE/F structure	Ask, Confirm and Describe	Evaluative listening	None
Dialogue 5	Teacher talks	Summarize	No listening	None
Dialogue 6	Traditional IRE/F structure	Ask, Confirm and Describe	Evaluative listening	None
Dialogue 7	Traditional IRE/F structure	Ask, Confirm and Describe	Evaluative listening	None
Dialogue 8	Teacher talks	Summarize	No listening	None

In Dialogue 1 T1was getting into the topic of the lesson. He explained the purpose of the lesson – the application of three conditions for congruency.

> **[00:01:14.11]T: So we are going to look at the three measurements that will guarantee the congruency of triangles. We are going to look at those three. We are going to see how [we] formally go [about] proving triangles [are congruent].** (Appendix A)

In Dialogue 2 the teacher continued to dominate. He mediated learning by providing worksheets with a summary of the previous two lessons – the three conditions for congruency – and explained these conditions to the class. The first discussion with learners took place in Dialogue 3 and it was about the meaning of the term 'corresponding side'. The teacher asked questions in order to elicit information from the learners:

> **[00:03:54.15]T: What is corresponding side? [00:03:59.11]T: Correspond? How do you know that two sides are corresponding?**
> (Appendix A)

The teacher provided answers because the learners did not give any reasonable answers.

> **[00:04:22.05]T: Corresponding means, corresponding side means if you look at the angle opposite. It must be opposite the same size angle.** (Appendix A)

The learners should be familiar with the mathematical term 'corresponding side' from previous lessons of the topic. After recalling previous knowledge the current lesson started with three example-questions, for example: "[e]xplain how you will convince someone whether or not the two triangles in each case below are congruent". During the whole-class discussion in Dialogue 4 the learners did not understand what they had to do and did not contribute much. Most of the time they asked questions from which a lack of orientation can be conducted:

> **[00:07:43.41]L: Sir, must be in order? [00:07:57.14]L: What about if you rotate it?**
> **[00:07:43.06]L: Should we write that down?** (Appendix A)

The teacher explained the solution verbally and mediated learning by recording the solution in front of the class. During the exchange he used the move Describe to emphasise the important point in the dialogue.

> **[00:07:17.14] T: So triangle will be NMQ. It is important to get order correct when you are doing congruency. [00:09:31.05] T: I will say triangle ABC is congruent...Do you know the symbol for congruency?** (Appendix A)

In Dialogue 5 the teacher summarized the formal proof for Congruency. In the next two dialogues the learners' participation of the lesson dramatically increased. Although the teacher used the IRF/E structure the learners mostly provided all the answers. Using scaffolding the teacher helped the learners to become full participants in the discourse specifically. However T1 took the lead in Dialogue 7 using the Eliciting move by asking the question; the learners showed understanding of what they were doing. For instance the following exchange is evidence for that:

> **[00:16:14.09]T: Another C. Let got the order first. In triangle QRT. [00:16:30.13]L: They are congruent [00:16:36.12] L: They are so congruent. It is true. [00:16:40.26]T: In triangle QRT. What will be the order of next one? [00:16:42.28]L: YXW. [00:16:49.12]T: YXW. Right! [00:16:56.26]T: Ok. Number one. [00:16:58.03]T: What can we say? [00:17:06.27]L: XY equals to RQ. [00:17:07.18]T: RQ first. RQ equals to XY equals to 3 cm. [00:17:19.26] T: Given. Right!**

[00:17:19.26]L: TQ equals to WY. [00:17:23.22]L: So angle QRT is equals to ... [00:17:29.09] T: QRT... [00:17:36.01]T: So angle QRT.QRT. [00:17:45.13]L: Is equal to YXW. (())[00:17:46.03]T: YXW equals to what?[00:17:50.05]T: 30 degrees.[00:17:51.07]L: Where did you get that from?[00:17:55.21]T: Where did I get what from?[00:18:06.08]T: What is the last one?[00:18:09.15]L: TQR.[00:18:11.02]T: TQR.[00:18:11.06]L: TQR is equal to WYX.[00:18:15.21]T: TQR is equal to WYX is equal to 125 degrees.[00:18:28.12]T: What is the reason?[00:18:28.52]T: Given.[00:18:33.09]T: Therefore. What can we say? [00:18:36.09]L: They are congruent. (Appendix A)

In conclusion, in this activity the learners did not learn a new narrative, but they needed to apply a new narrative – the three conditions for congruency, more specifically how to formally prove that two triangles are congruent –as well as a routine that they had never practiced before. From this perspective it can be concluded that the activity was successful. The Imitation that the first teacher practiced was the appropriate method to mediate this type of learning. Sfard (2008) also confirms:" Imitation, which evidently is a natural human property, is the obvious, indeed, the only imaginable way to enter new discourse" (p.250)

5.4.2 Second activity

The Table 13 below gives short description of each dialogue in second activity of the fourth lesson:

Table 13 T1: Short description for fourth lesson, second activity

Dialogue 1	T1 introduces the second activity to the class.
Dialogue 2	T1 asks a question and gives one learner the chance to show the solution.
Dialogue 3	Another suggestion from learners' side.
Dialogue 4	T1 records the solution on the board.
Dialogue 5	Second method to solve the questions

The Table 14 below, which is based on the final table from the analytical framework, is a summary of interaction patterns, teachers' moves and type of listening that T1 have been use in second activity.

Table 14 T1: Summary table for fourth lesson, second activity

Lesson 2 Activity 2	Interaction patterns	Teachers' moves	Type of listening	Social regulation
Dialogue 1	Traditional IRE/F structure	Ask and Describe	Evaluative listening	None
Dialogue 2	Traditional IRE/F structure	Elicit	Evaluative listening	None
Dialogue 3	Whole – class dialogue	Ask, Confirm and Describe	Evaluative listening	None

Dialogue 4	Traditional IRE/F structure	Ask, Confirm, Describe	Evaluative listening	None
Dialogue 5	Whole – class dialogue	Ask and Describe	Evaluative listening	None

In second activity the learners were engaged in the non-standard question, namely the application of three conditions for congruency. Using the worksheet with sketches of triangles showed that this activity also included visual mediators.

In Dialogue 1 T1 introduces the task; explains in detail the given conditions and asks the main question of the problem. The teachers' moves used in this dialogue were Ask and Describe. In the next dialogue one learner shows the solution on the board. The interaction pattern that was observed in Dialogue 2 was the traditional IRF/E structure; the teacher asks questions and the learner responds. After the idea failed the teacher opened a class-discussion and different learners contributed different ideas. The teacher took one of these suggestions and clarified it by recording the solution in front of the class. In Dialogue 4of the activity the learners' participation was fairly good; they answered the teachers' questions.

5.5.1 Second teacher. Fourth Lesson: First Activity

The Table 15 below gives short description of each dialogue in the first activity of the fourth lesson:

Table 15 T2: Short description of fourth lesson, first activity

Dialogue 1	Not relevant to the lesson
Dialogue 2	T2 revises the meaning of congruency. She involves the learners in conversation about three conditions for congruency.
Dialogue 3	T2 shows rules related with geometric proofs using the overhead projector.
Dialogue 4	T2 together with learners prove the first example.
Dialogue 5	T2 checks for homework. T2 pays individual attention to the learner who was absent in the previous lessons.
Dialogue 6	After explanation of the first example T2 gives the next three questions to solve.
Dialogue 7	T2 communicates separately with different learners. She helps them to prove congruency.
Dialogue 8	T2 provides individual help to another learner. T2 gives homework.

Table 16 is a summary of interaction patterns, teachers' moves and type of listening that T2 have been used in first activity.

Table 16 T2: Summary table of fourth lesson, first activity

Lesson 2 Activity 1	**Interaction patterns**	**Teachers' moves**	**Type of listening**	**Social regulation**
Dialogue 1	Not relevant to the lesson			
Dialogue 2	Conventional IRE/F structure Whole-class discussion	Elicit, Ask, Confirm, Describe	Interpretive listening	Disciplinary issues
Dialogue 3	Teacher talks	Describe	No listening	None
Dialogue 4	Traditional IRE/F structure	Elicit, Confirm, Describe	Evaluative listening	Time management
Dialogue 5	Not relevant to the lesson			
Dialogue 6	Traditional IRE/F structure	Elicit, Confirm, Describe	Interpretive Listening	None
Dialogue 7	Traditional IRE/F structure	Ask, Confirm, Describe	Evaluative listening	None
Dialogue 8	Traditional IRE/F structure	Ask, Confirm, Describe	Evaluative listening	None

The actual lesson started in Dialogue 2 where the second teacher involved learners in a whole-class discussion. The teacher revised the meaning of congruency; she used the conventional IRE/F structure in the discussion. Very good learner's participation indicated their understanding of the terms and the most used moves were Elicit and Confirm. For instance:

[00:02:24.12]T: What have we discovered about congruency?
[00:03:42.12]T: What else did we discover about congruency?
[00:02:01.04]T: Is the orientation the same Rodney? (Appendix B)

Then T2 involved the learners in a conversation about three conditions for congruency. She used informal language in the traditional IRE/F structure to encourage their participation in the whole-class discussion. Showing good facilitation, she asked the questions, elicited information from different learners and concluded. Simultaneously, the teacher applied two tools of mediation, writing on the board and discussion. In conclusion two different ways to start this lesson was observed. While T1 repeated the summary of the previous lessons, the second teacher elicited the same information from the learners.

In Dialogue 3 T2 read the summary from the overhead projector. The teachers' move that she used was Describe. Her utterances are evidence for that:

[**00:06:48.17]T: I just want to remind you how we write it down. Then I will do an example. Firstly we got to get the reason for each (()) action. Remember you have your two triangles and you're going to label your two triangles. So you can't just say that A equals E (()) you got give me a reason why they ..., hey? [00:07:12.10]T: You've got to mark each new piece of information on the copy of the sketch. So if you've used the information, you mark off that**

you've used it, but you can't use it twice, ok? And always start with the information that is given on the sketch. Lastly, you need at least three pieces of information to prove congruency. So I'm going to have three sides, I'm going to have two angles and a corresponding side or I'm going to have two sides and the included angle. (Appendix B)

In Dialogue 4 T2 together with the learners, solved the first task; the teacher was scaffolding and in the process emphasised the important part of formal proof; starting with the given information, always providing reasons for the given statement and indicating how to write the new sign for congruency. She used the conventional IRE/F structure in the discussion as well Elicit, Confirm and Describe.

In dialogues 6, 7 and 8 T2 had conversations with individual learners using the traditional IRE/F structure and teachers moves such as Elicit, Confirm and Describe. It can be noted that in these exchanges the learners still struggle with solution of the task. For this particular class, with learners that slow understand mathematical concepts, one example was not enough to become participant of the discourse.

5.5.2 Second activity

The Table 17 below gives short description of each dialogue in the second activity of the fourth lesson:

Table 17 T2: Short description of fourth lesson, second activity

Dialogue 1	T2 introduces the second activity. She explains the conditions of the problem.
Dialogue 2	Whole–class discussion about the problem.

The Table 18 below corresponds to Table 17 for T2:

Table 18 T2: Summary table of fourth lesson, second activity

Lesson 2 Activity 2	**Interaction patterns**	**Teachers' moves**	**Type of listening**	**Social regulation**
Dialogue 1	Teacher talks	Describe	No listening	None
Dialogue 2	Traditional IRE/F structure	Elicit, Confirm, Describe	Evaluative listening	Time management

In Dialogue 1 the teacher introduces the second activity. She describes the condition of the problem.

In Dialogue 2 the teacher opens whole–class discussion. In the traditional IRE/F structures she asks the learners questions and elicits information from them. The participation shows that they do not use the conditions for congruency yet they still use the definition for congruency.

It should be noted that the study's main focus is not social regulation; this should be further explored in future studies.

Findings

This research study explores mediation of two teachers across two levels of learning – object-level and meta-level learning. Two main findings can be summarized: Firstly, the ways the teacher manages instruction originates from their teaching style. The main criteria of the analytical framework, that characterize a way of teaching, such as interaction patterns, teacher's moves and type of listening, social regulation are different for both teachers. The second tables of each activity from the data analysis clearly confirm that mediation of the two teachers on the topic Congruency does not differ according object-level and meta-level learning, but according to the teachers.

For the first teacher (T1) in summary-tables 4, 6, 12, 14, under the second column "Interaction patterns" there are only two statements: "Teacher talks" or "Traditional IRE-structure". Consequently because the priority of the dialogues T1 explains, describes or summarizes that there is no listening or that there is one type of listening – Evaluative Listening. Therefore his way of teaching is close to traditional teaching. The responses of the questions from his interview also confirm the above information, which is contained in the following chapter.

The results from summary-tables 8, 10, 16, 18 for the second teacher (T2) draw a different picture. Under the second column "Interaction patterns" there is variety of statements "Traditional IRE/F structure", "Reversing IRE", "Whole-class dialogue", "Teacher talks", which shows that T2 uses different structures to communicate with learners. Simultaneously there is a diversity of teachers' moves that she uses in her own teaching practice, such as Ask, Elicit, Describe, Confirm and Summarize. She also uses two types of listening – Evaluative and Interpretive.

The second finding is related with Sfard's theory, more specifically with two key concepts of discourse – narratives and routines. The design of Lesson one and Lesson four was to make learners learn a new narrative and develop new routines. In Lesson one in both activities, the first activity involves object-level learning and the second activity involves meta-level learning. The teaching mediation for both teachers did not meet expectations. There is no evidence in the first activity that learners in both classes understand the meaning of the narrative "**In congruency the position does not matter.**" The same conclusion can be made for the second activity. The new narrative, namely that two measurements are not enough to draw unique triangle, was not identified by the learners. Furthermore the focus of the second activity was changed to developing drawings skills in both classes. Chapter 7, Interpretation of the lessons according to Sfard's theory provided good explanation. Sfard developed new terms such as deed that explain empirical phenomenon.

Chapter 6

Answer to second research question

Analysis of data from the interview

To answer the second main question of the research project "[w]hat enables and constrains her/his [the teacher's] facilitative mediation in the case of Congruency in Grade 9?" the data analyses of the observations will be considered. At the same time the data from the interviews of the both teachers will provide additional information.

Going through the lessons, the first factor that constrained good facilitative mediation can be noted. The implementation of the first lesson's second activity became impossible because of **learners' lack previous knowledge.** Both teachers found themselves in the situation where the learners did not know how to construct a triangle with two given elements; the knowledge that they should have gained in grade 6 and7. This constrained the classwork and the focus of the lesson changed from finding the minimum conditions for congruency to drawing a triangle with two given measurements. The interview with the first teacher confirms this:

> **and sometimes you are in a situation where, where the kids, their knowledge is so poor that you have to make up so much knowledge that they don't know from previous grades or for whatever reason that they don't know, you end up having very little time to actually do the kind of investigation that you would like to do… (Appedix D: 2)**

The **teacher's own personality and teaching experience** can also be factors that affect facilitative mediation. The second teacher confirms this fact: "I think it has a lot to do with your personality as well… [a]nd experience…" (Appendix E: 3) For instance, the good mathematical knowledge of the first teacher enabled him to link what the learners know in geometry with new knowledge that he had to present. In the first lesson's second activity he opened the discussion by posing the question "[h]ow many measurements do I need to give you in order to draw a unique square?" and involves the learners' previous knowledge about the square. Coming from known facts the teacher focused the second activity on the question "[a]re two measurements enough to draw a unique triangle?" In this way the teacher did contextual scaffolding; he expected of learners to make a shift from well-known necessary conditions of drawing a unique square to unknown necessary conditions of drawing a unique triangle. He provoked meta-level thinking and then involves the learners in the investigating activity.

The teaching experience of the second teacher enabled her to lead the discussion successfully. In most dialogues she used a variety of interacting patterns and different teachers' moves. In the interview she shares the secret:

> **I've learnt through the years that kids don't just understand and you need to ask them when they don't understand and we need to take time to listen to what they're saying to you. And as teachers we don't always have the time to sit and listen, you know, and sometimes we, we assume that they understand what we want them to do, but they don't. So you need to take time to make sure that they really do understand what you want them to do. (Appendix E: 3)**

In addition the second teacher acknowledges that the relationship with learners can stimulate learning mathematics: "I think if you've built a relationship with your learners and they understand that you're here to help them, you're not going to criticise them when they make a mistake in your class, then learning takes place easily" (Appendix E: 10). Her foundation-phase training also had a positive effect on the work in class: "I had foundation phase training, OK, and I've learnt that small children learn through pictures and I've noticed through my years of teaching that if you… if a child doesn't understand and you draw it – not physically draw it as a picture, but paint a picture for them, they seem to understand it better." (Appendix E: 3)

Furthermore both teachers recognize that their effectiveness will vary across different situations. They point out two constraints that limit their facilitative mediation – **range of the classes (large number of learners) and subject variation (Maths, Mathematical Literacy, and Extra Maths).** The first teacher shares the following:

> **How could I actually provoke more discussion, you know? I struggle to… If it's an extra class it's different. I'm much more Socratic in extra classes. ..Yes small groups. Ja, much better. I mean if it's one-on-one I'm very… Or even like I've got some very small classes down here where some of the classes are like 10. I'm extremely Socratic. I never give answers. I always ask and I ask and I ask and I ask and I ask, you know. (Appendix D: 13)**

He continues: "But as I say, if it was a smaller group in a different situation I think I would have because I do often do that ... especially in Maths Literacy where I've got more time to do that kind of thing" (Appendix D: 14). Considering that the second teacher has a Grade 9 class of only 15 learners she states: "Here you've actually got enough time to go and see that each one has grasped what you've done. There aren't so many in the class that you can't get around to everybody during the teaching time. In the big class … I did teach in big classes, I didn't get around to everyone" (Appendix E: 5)

Another difficulty to implement collaborative learning is **the diversity of learners' knowledge:**

> **Sometimes when you decentralise, um, when you start making things more learner-centred, um, you know you have a problem. I sometimes will try and put weaker people with stronger people so that I know at least there's one person that knows what's going on and he can help the others. But sometimes it's even not possible to do that. So sometimes if you leave things to be learner-centred, nothing happens." (Appendix D: 6)**

The **contradiction between teachers' sense of efficacy and the reform conception of teaching and learning mathematics** can be another constraint in mediation. On the one hand Ashtor (1985, in Smith, 1996: 338) defines teachers' sense of efficacy as "their belief in their ability to have a positive effect on student learning". Teaching by telling determines this sense of efficacy clearly because the teacher knows the mathematical context and her/his own role as a knowledge provider. On the other hand "[t]eaching by demonstration and practice is no longer acceptable, because learners cannot learn mathematics as passive listeners. More deeply, the reform challenges the fundamental assumption that teachers can be direct causal agents in student learning" (Smith, 1996: 388). The reform way of teaching expects of the teacher to create settings in which learners learn mathematics through their own activity. However, the first teacher is concerned with the practical implementation of this process.

> **How do you know that the kids are not ... messing around because you're all over the place now, you can't control everything from the front because now, you know … But sometimes it's even not possible to do that? So sometimes if you leave things to be learner-centred – nothing happens – OK because… So for me I think having too much decentralised or learner-focussed stuff is… there's too much of a risk of things not happening. (Appendix D: 6)**

At the end he concludes: "If you leave… it too much nothing might happen, you know. And nothing often does happen, that's the problem." (Appendix D: 7)

Furthermore, **the limitation of past mathematical experiences of the teachers in reform teaching** can be added to the list of constraints of teachers' mediation**.** Smith (1996: 394) poses the relevant question, how teachers who have learnt by listening to traditional mathematics and have taught by telling can then achieve a sense of efficacy that is consistent with the reform. The first teacher confirms the lack of own previous experience "When I was... at school... we did very, very little investigative type of work... and the way I do now is there's very, very little group work in my, in my classes like the one that we did in the first lesson."(Appendix D: 2) He continues: "Well, the new collaborative type of learning is something that I'm not familiar with…but I'm not trained in it, OK, I haven't experienced it myself when I was at school and... and so I don't practise it really" (Ibid.: 3) In other words he does not know how to exactly implement the new reform: "I don't have the resources for it … and disciplinary policy." (Ibid.: 3)

The first teacher makes the following suggestions for implementation of collaborative learning. In order to answer the question: "How are we going to implement? How are we going to make it practical? OK, so how is it going to be practical for the teachers and for the pupils?" he said: "So you need to find people that actually know how to do it and it needs to then be disseminated properly. Now I mean most of the… most ways that people, those things get disseminated to teachers is through textbooks" (Ibid.: 9). His personal opinion is that teachers need to participate in writing the textbooks or designing the media. The information on the Internet is another resource for teachers and can be used by all teachers for collaboration. He states: "The other thing is, is that you've got millions of teachers now all doing the same thing and not collaborating. You see you want pupils to collaborate in their learning environment but teachers aren't collaborating. I mean why don't teachers start collaborating?" (Ibid.: 10)

Going beyond the frame of the four lessons the teachers find more constraints for facilitative mediation: **the contradiction between time management and the wide range of topics in the syllabus.** The first teacher says:

> **the syllabus... is extremely full and you are constantly aware that you are under pressure to complete the syllabus. At the same time Maths is about understanding. You cannot be successful in Maths if you don't understand… obviously understanding comes with thinking something through, finding it out, understanding, investigating it. That's all part of the understanding process. (Appendix D:2).**

Then he clarifies: “So you need to put in investigation where you can... but it needs to be balanced because of your time constraints…” (Ibid.: 2). He is of the opinion that providing an explanation to the learners is a quicker way of working through the lesson.

Another mitigating factor is **culture** and can be a major constraint or tool, depending on the level of priority that education fills in a particular culture. The excerpt from the first teacher’s interview shows that

> **I think... you know I’ve heard of schools in China where you have like 100 or 200 students and you’ve got one lecturer in the front and he says, ‘Pens up’ and everybody picks up their pen and they do the work and he says, ‘Pens down’, everybody puts down their pen and they’ve got maybe some of the best Maths marks in the world, I don’t know. But they do… they do teaching in a very disciplined and structured way and I mean they’re apparently producing lots of engineers and whatever. OK, that’s one way of doing it. It seems like there’s quite a lot of discipline in the culture and, um, and they, they learn in that kind of way to, to learn the knowledge that they must learn and to process it. (Appendix D: 15)**

In conclusion, in this chapter described many factors that constrain facilitative mediation: a lack of previous learners’ knowledge, a range of classes, the diversity of learners’ knowledge, the contradiction between teachers’ sense of efficacy and the reform conception of teaching and learning mathematics, the limitation of past mathematical experiences of the teachers in reform teaching, the contradiction between time management and the wide range of topics in the syllabus and cultural issues. Simultaneously, teaching experience and teachers’ competence can be factors that have positive effects on facilitative mediation.

These highlighted factors, however, are localized; they emerged from observed data of two lessons and from conducted interviews with two teachers.

The selection of these factors is, however, subjective as it consists of two teachers’ utterances and the researcher’s selection of those utterances.

There are, in fact, many more problems related to reform teaching, not only regarding object-level and meta-level learning and these should be investigated further for expert solutions.

Chapter 7
Conclusion

7.1. Interpretation of the lessons according to Sfard's theory

In this chapter the results of the lesson observation will be explained using Sfard's theory about rules and routines of discourse. This part of her overall theory was not used when the development team (three teachers) designed the lessons. Its relevance only became fully visible at a later stage (during the analysis of the lessons).

Firstly, the main point in Sfard's theory of learning will be elucidated. In Chapter 7 the author states that "[h]uman communication has been defined as a rule-regulated activity, and the preceding observations about the repetitive, patterned nature of discourses convey the same message" (Sfard, 2008: 200). She goes on to say that "it is important to distinguish between metadiscursive and object-level rules" (Ibid.: 201).

Sfard (2008: 208) defines routine as "a set of metarules that describe a repetitive discursive action". Further she elaborates:

> This set of pattern-defining rules may be divided into two subsets:
> - The *how* of a routine, which is a set of metarules that determine, or just constrain, the course of the patterned discursive performance(*the course of action or procedure*, from now on) and
> - The when of a routine, which is a collection of metarules that determine, or just constrain, those situations in which the discussant would deem this performance as appropriate.(Sfard, 2008: 208)

While the routine of *how* is a straightforward task and is performed well in the classroom environment, the routine of *when* is neglected in school teaching.

Sfard (Ibid.) divides discursive routines into three types – explorations, deeds and rituals – and explores how each of these participates in the teaching-learning process. She distinguishes between them according to the final tasks that they will accomplish.

For instance, exploration is a mathematical routine that produces *endorsable* narratives. The term *endorsable* shows that the narrative can be endorsed or rejected according to well-defined rules of the given mathematical discourse. Sfard emphasises that *endorsable* narratives are narratives "labelled as true and become known as 'mathematical facts'" (Ibid: 223). They are generally accepted rules by the whole mathematizing community, such as definitions, axioms and theorems. Closed sets of different types of endorsed narratives combine into well-organized systems called mathematical theories. On the other hand, Sfard continues:

> [a]ll the exploratory routines can be divided into three types: *construction,* which is a discursive process resulting in new endorsable narrative; *substantiation,* the action that helps mathematics decide whether to endorse previously constructed narratives; and *recall,* the process one performs to be able to summon a narrative that was endorsed in the past. (Ibid: 225).

In some cases, the process of construction is an act of substantiation. With other types of narratives conjecture and proof are two separate processes. Sfard also recognizes different ways to construct new narratives – deduction, induction and abduction. She explains: "[d]eduction takes place when a new narrative is obtained from previously endorsed narratives with the help of well-defined inferring operations." (Ibid: 229) While the deduction is a manipulation that moves from common to individual, the induction, in contrast, is a process in which a new narrative is common and originates from a specific instance. For the third manipulation – abduction – endorsability appears as the necessary consequence. Using abduction can be an endorsed narrative without additional substantiation.

The ultimate goal of the second type of routine – deed – is to produce or change a mathematical object (a deed is an operational result of practical action [Sfard, 2008: 236]). A good example of deeds is a solution of equation. The goal of the process is to transform the object (equation), not produce the narrative. Sfard mentions in some cases that what for one person is an implementation of exploration is an implementation of a deed for another (Ibid.: 239). Lesson one, activity one confirms this statement and will be analysed later.

While both of these mathematical performances – exploration and deed – are related with mathematical objects, transforming or getting to know it better, the third type of routine – ritual – is socially orientated. Ritual "is a way of getting attention and approval of others and becoming a part of a social group." (Ibid.: 241). To distinguish rituals from deeds and explorations Sfard creates a table

Table 19 Deeds, explorations and rituals – comparison (Sfard, 2008: 243)

	Deed	Ritual	Exploration
Closing condition/Goal	A change in environment	Relationships with others (improving one's positioning with respect to others)	Description of the world (production of endorsed narrative about the world)
By whom the routine is performed For whom the routine is performed	No special requirements No special requirements	With (scaffolded by) others *Others* (authoritative discourse)	No need for scaffolding – can be performed individually *Others and oneself* (internally persuasive discourse)

Applicability (changing the *when*, keeping the *how* constant)		Restricted – the procedure is highly *situated*	Broad – the procedure is applicable in a wide range of situations
Flexibility (changing the *how*, keeping the *when* constant) Correctibility		Almost no degrees of freedom in the course of action. Cannot be locally corrected – has to be reiterated in its entirety	The procedure is a whole class of equivalence of different courses of action. Parts can be locally replaced with an equivalent subroutine.
Acceptability condition	The *result* – the change in environment – must count as adequate; no need for human mediation of the acceptance –it depends on the environment	The activity has to be shown to adhere strictly to the rules defining the routine procedure – the acceptance depends on other people	The narrative produced through the performance must be *substantiable* in such a way that the acceptance is independent of other people
Words' and mediators' use	Possibly no active use of keywords	Phrase-driven use of keywords – as descriptors of extra discursive mediators	Objectified use of keywords – as signifying objects in their own right

The table confirms that the primary goal of rituals is creating and sustaining a bond with other people, acting with others in harmony, doing exactly what these other people do; in other words building relationships with others. In the section "By whom the routine is performed" and "From whom the routine is performed" under ritual it is written: "With (scaffolded by) others" and "Others (authoritative discourse)." The ritual action is defined as that group of different people who perform identical operations, possibly together. Therefore rituals are highly reproductive. The ritual actions are also associated with constancy and homogeneity and it is opposite to variation and diversity, as written in the table "[a]lmost no degrees of freedom in the course of action". Another characteristic that is indicated in the table is applicability conditions. Because a ritual is a routine that is not performed by one person, this community-building activity is highly situated. Rituals are restricted; the implementation depends on specific situational attributes. In addition Sfard says: “In the case of ritual, which is about performing, not about knowing, there is no room for substantiating.” (Ibid: 244)

After introducing these three types of discourses routine Sfard explains the mechanism of discursive change. She states that “[t]he starting point for any discursive development is deed.” (2008: 245) Continuing to look for discussing possible trajectories of routine development she states that: “[t]he idea of ‘growing’ explorations directly from deeds may not always be feasible.” (Ibid: 246) She elaborates:

> The direct jump from deed to exploration is particularly unlikely in those cases in which new metarules are involved. This follows directly from the participationist vision of learning. The argument goes as follows. Metarules of mathematical discourses, rather than being “laws of nature”, are historically established customs which survived because of their usefulness. This is the case, for example, with the rules for mediated identification of geometric figures, as well as with the metarule for conjuring new mathematical objects from sets of axioms, as opposed to deriving them from concrete models. <u>One cannot expect a child to learn the corresponding routines by independent reinvention.</u> (Ibid: 246. Own emphasis added)

Rejecting the investigating activity she suggests another way to individualize meta-level rules:

> Individualization of other people’s discourse, however, are more likely to result in rituals than in explorations, and this is true even if the learner is already familiar with deeds that the new discursive routine is supposed to enhance. (Ibid: 246)

The learner could not possibly appreciate the value of the new discourse until she/he was aware of its advantages. She/he could only understand the value of new routine when using it. Sfard states:

> The answer, it seems, lies in the child’s propensity for imitation. Imitation, which evidently is a natural human property, is the obvious, indeed, the only imaginable way to enter new discourse. The tendency to imitate others occurs hand in hand with the need to communicate, a need so strong that it would often lead to what may appear as the reversal of the ‘proper’ order of learning.(Ibid: 250)

Furthermore Sfard describes the process of transforming rituals to exploration as follows: the thoughtful imitator constantly performs what others do and asks the reason for doing it. Through following in other people’s footsteps the process of individualization emerges; in other words, the growth of proficiency corresponds with gradual deritualization and eventually a change to exploration. She continues:

> This latter transformation can happen quite abruptly, so that the stage of ritualization is hardly noticeable, or it can last for a long time, perhaps even forever. The transitory phase of ritualization corresponds to the period of individualizing – the period during which the learner can participate in the collective implementation of the routine but is not yet capable of independent performance. (Ibid: 253)

To summarize the following conclusions in Sfard’s theory can be made:

- All three types of routines play a role in the development of discourses.
- Natural continuity of discourse is implemented in this direction:

Diagram 5 Deed, ritual and exploration

Contingent Discursive development starts with deeds and then, likely, moves to rituals. In some cases rituals eventually evolve into explorations (to ensure that the process happens is highly consolidated discourse).

- "In the case of *metalevel* learning, when the routine to be learned involves new metarules or new mathematical objects, its reinvention by the learner is highly unlikely. In this case, the learning would typically occur through scaffolded individualization, that is, through interaction with mathematics who are already insiders in the target discourse" (Ibid: 259).

- Sfard clarifies: "[t]he participationist vision of human development implies that any substantial change in individual discourse, one that involves a modification in meta-rules or introduction of whole new mathematical object, **must be mediated by experienced interlocutor.**" (Ibid: 254. Own emphasis added).

- Another condition for effective mediation for meta-level learning is *commognitive conflict* – a situation in which different discussants are acting according to different metarules.

- School teaching focuses on *how* routines should be performed but the question *when* this performance would be most appropriate is not developed.

The observation of two lessons of both teachers confirms Sfard's conclusions.

In the first activity of the first lesson the task can be divided into two sections: the act of recognition (involving a recall routine of certain past experience associated with the present one) and the act of naming (of attaching a word to the recognized shape). This identification procedure involves previous endorsed narrative: "[t]wo figures are congruent when they are the same." From the observation both teachers intend to discover the new narrative: "[t]wo congruent figures have the same shape, the same side, but not the same orientation." While the teachers plan to practice mathematical exploration, the learners implement another type of routine – deeds. The evidence of this conclusion is when the teacher asks the question: "[w]hat can we say about the position?" The learner gives the wrong answer: "[t]he position… is the same." The routines that they are doing are cutting the pairs of shapes, fitting them on top of each other and pasting them in the right column, they are dealing with object congruency, not discovering the new narrative. In the second teacher's first activity there is, again, no evidence that learners discovered the new narrative – **In Congruency the position does not matter.** The teacher summarized, concluding as follows:

[00:15:59.14] Ok, so what if we... similar… Size is not the same but the shape and the position is the same, most of the time. Congruency doesn't matter on the position, I can flip it, I can slide it, I can rotate it and then I can have the same shape, but it has the same shape, size, but the position is different. Ok. (Appendix B)

He did not involve learners in the discussion to show that learners understood the purpose of the activity. The first teacher also concluded:

[00:17:50.14] T: So, the shape is the same the size is the same and the position is different. Right? (Appendix A)

The manipulation of producing this new narrative is abduction. Using this way the new narrative can be endorsed without an additional substantiation.

In the second activity of the first lesson both teachers intended to involve learners in the same routines as the first activity – exploration. They asked learners to answer the question whether "two measurements [are] enough to create unique triangle" The learners, through investigating activity two, need to find the new narrative, that two measurements, whatever they are, two sides, two angles or one side and one angle, are not enough to draw a unique triangle.

When the activity started both teachers found that the learners did not have basic drawing skills. So discovering the new narrative became impossible. The learning process had been transformed to drawing a triangle. Both teachers used routine – ritual – to engage learners in this meta-level learning. The teachers showed the drawing procedure on the board and thereafter they paid individual attention to different groups in order make sure that learners learned how to draw a triangle. In this collective activity, on one hand the teachers did scaffolding, on the other hand learners imitated the teacher in the drawing procedure. Other evidence that shows that learners are doing rituals, is that their actions can be characterised with "[a]lmost no degrees of freedom " (Table 7.1); the learners repeated exactly what the teachers do. In addition Sfard (2008: 244) says about this type of routine: "[i]n the case of ritual, which is about *performing*, not about *knowing*, there is no room for substantiating." No substantiation was observed in the second activity of the first lesson.

In the first activity of the second lesson both teachers did scaffolding to show formal proof of Congruency. At the same time the learners were involved in thoughtful imitation. This process can be characterised as a ritualization, a community building activity in which the entire class and the teacher participate. In the beginning (first pair of triangles) the teacher performed most of the steps of the problem's solution. In the next examples gradual deritualization was observed, in which the learners' participation increased. Although the teachers lead all discussions the transformation of ritualization and phase of individualization became obvious where the learners' implementation of the task moved to independent performance.

This activity follows Sfard's theory and was successful.

7.2 Final Thoughts

Practice and theory influence each other. Their relationship becomes more crucial, in the recent times of changes in values and principles in the Constitution in South Africa. The new reforms reflect on the conceptual view of subjects such as mathematics and identify the new role of teachers and learners in the mathematical classroom. The Department of Education called for teachers to prepare investigative activities in which learners learn mathematical concepts and the teacher facilitates the process. The theoretical background of this directive and practical implementation is a problematic discussion topic.

Because of spacial limits, this study only touches on the above problem in the bigger topic of learning − the role of the teacher in collaborative learning. Looking from Sfard's theoretical perspectives, that distinguish between object-level and meta-level learning and states:

> In the case of meta-level learning, when the routine to be learned involves new metarules or new mathematical objects, its reinvention by the learner is highly unlikely. In this case, the learning would typically occur through scaffolded individualization that is through interaction with mathematics who are already insider in the target discourse" (2008: 259)
>
> and "[t]he participationist vision of human development implies that any substantial change in individual discourse, one that involves a modification in meta-rules or introduction of whole new mathematical object, **must be mediated by experienced interlocutor.**"(2008:254. Own emphasis added)

it is concluded the Sfard's perspective is contradictory to the Department of Education's directive. On the one hand the Department recommends investigative activities, whilst, on the other hand, Sfards' theory states that reinvention by the learner is highly unlikely. Therefore the practical efficacy of Sfard's theory is that in meta-level learning investigative activities are not appropriate and the role of the teacher should be dominant, not necessarily as facilitator; thus answering the third research question.

Furthermore this research study is not only an empirical proof of the Department of Education's misleading requirements but a confirmation of the validity of Sfard's theory. The second activity of the first lesson was designed as an investigative activity and it was expected of learners to find that two measurements are not enough to draw a unique triangle. This meta-level learning activity did not meet expectations. At the same time activity one in lesson four (also involving meta-level learning) both teachers successfully used imitation as mediation to complete the goal.

Sfard's theory is a fruitful learning theory. Forman continues the exploration of discursive development by presenting the ACT2 documents: seven rich applications of the commognitive framework to mathematical learning in a range of settings (in special issue of the International Journal of Education). This special issue "travels" to different countries (the United States, Korea, Canada and Israel) to understand the use of this new framework. This series of investigations have been conducted in a variety of educational contexts (preschool, elementary school, middle school and high school) and addresses a range of topics in mathematics (learner learning, curricular implementation and teacher's instruction). In addition it highlights the commognitive framework across different mathematical objects (rational numbers, functions, geometrics figures, algebra and infinity). (Forman, 2012: 152).

Articles by Nachlieli and Tabach (2012) and by Caspi and Sfard (2012) provide important contributions by investigating trajectories of the development of algebraic objects such as functions and algebraic equations. “Smith and Stein (2011) recommend that teachers follow five practices for their discussions: anticipating, monitoring, selecting, sequencing and connecting. All of those practices depend upon understanding the developmental trajectory of key mathematical object” (Forman, 2012: 152). The Commognitive perspective has been applied in another research project that explores the impact of the dynamic geometry environment (DGEs) on the development of learners’ geometric thinking. The conjecture of Sinclair and Moss (2012) is that software products such as the Sketchpad environment will speed up the transition from the visual recognition of geometric shapes to discourse-mediated identification.

In the Newton’s (2011) study the notions of the commognitive framework is applied again in order to answer the question “[h]ow faithful is the implementation of a given mathematical curriculum to the intention of curriculum developers?”

These seven studies, combined, raise a new view of teaching and learning mathematics that goes against widespread beliefs. There are many topics in this regard for further investigation.

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